

Connecting Math to Our World: **The Relevance of Math**

Student Reader



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The Relevance of Math

Student Reader



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The Relevance of Math

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We can observe the relevance of math in many aspects of our lives.

Math isn't just about numbers in a book—it's used in finances, sports, science, and planning.

Math helps us see the relationship between things.

Math lets us express extremely large or tiny numbers in a way that is easy to read.

We use math to make smart decisions—like which smartphone plan is cheapest, which camera angles are best for a scene, or when creating art.

Using our mathematical skills helps us make sense of the world.

How else can we use math?

Why Do We Need Numbers?

1

Elijah squints at the strange object in the museum glass case.

“What is that?” he asks.

His older cousin Maya, a college anthropology major, leans in and says, “That’s a replica of the Ishango bone. It’s approximately 20,000 years old, and it’s thought to be one of the first examples of humans using math.”

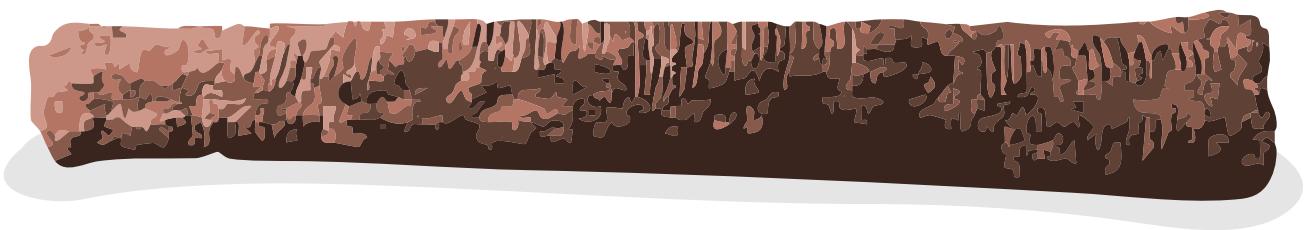
Elijah’s eyes widen. “Math? I didn’t know math was this old!” he exclaims.

Maya nods. “It shows that people were solving real problems with numbers way before calculators, even before writing,” she says.

Elijah glances around the museum, suddenly realizing that the story of math began long before textbooks and whiteboards.

“The Ishango bone,” continues Maya, “was found in Africa and might be the oldest known math tool.”

Maya explains that the Ishango bone has notches carved into it. Some of the notches are in groups of 3, 6, 4, and 8. The notches show that people could add, subtract, double, and halve numbers. They used these skills to solve math problems.



“It is like a 20,000-year-old math puzzle,” Maya says. “Early humans may have used the Ishango bone to track the moon, count items, or even experiment with patterns.”

Elijah imagines holding it in his hand, scratching lines into it to count something important, such as days, animals, or food. Early humans used what they had—bones, stones, and sticks—to think about numbers.



It amazes him that such a small object shows such a big idea: People have always used numbers to try to make sense of the world.

Maya explains, “Before paper or digital devices, people needed ways to keep track of things. As a way of recording, early humans created tally systems. For example, a trader might use a tally stick to count how many goats were traded. Each mark represented a specific quantity.”

"But tally marks only work for small numbers," Elijah says.

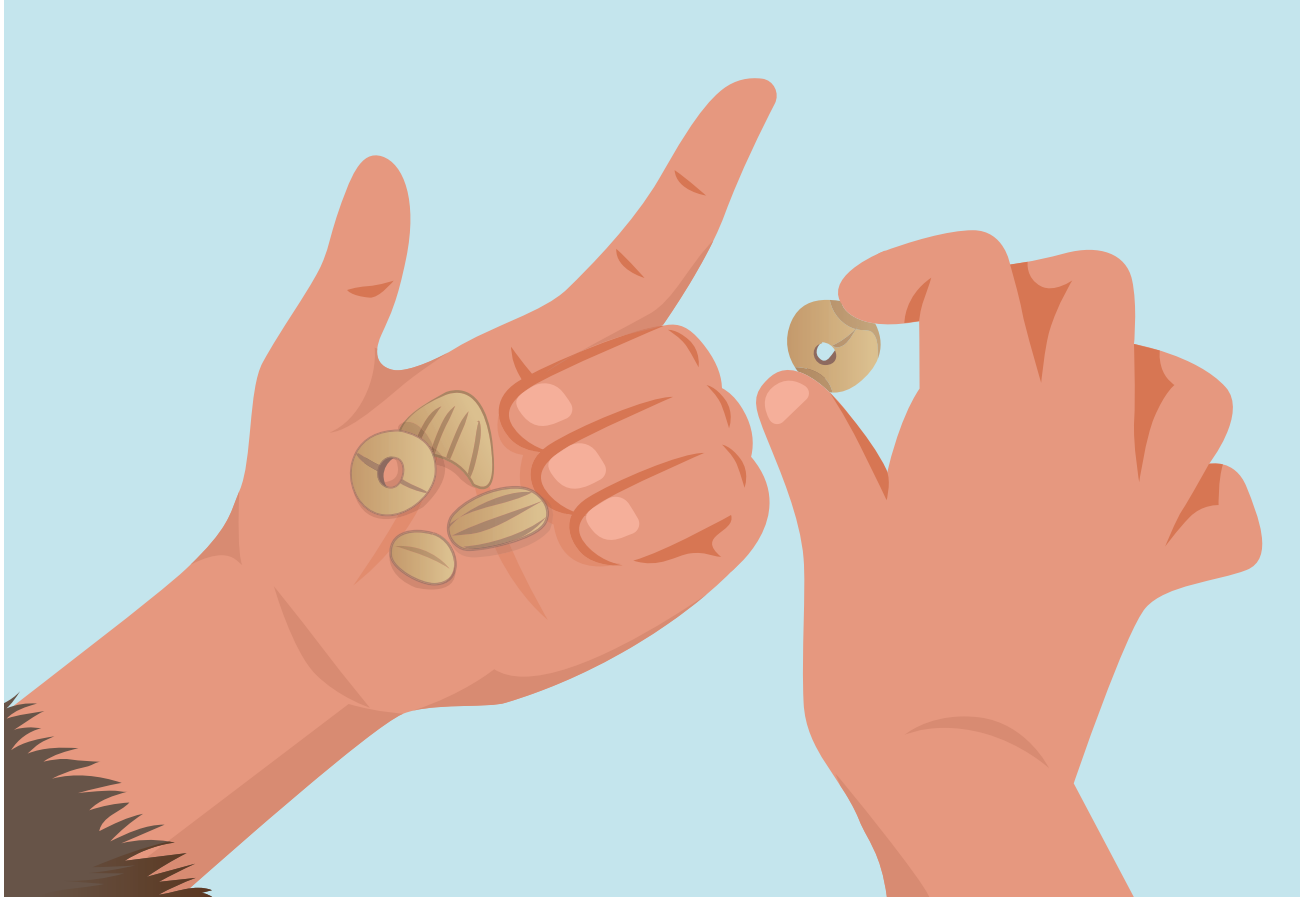
"That's right," Maya replies. "As trade and cities grew, people needed better systems."

Elijah pictures early markets filled with livestock and grain, people calling out prices. He wonders if early humans ever lost count when they were trying to tally things. What would they do if the stick broke or they miscounted? They needed to come up with a better system.

"Eventually, symbols for tracking items were created," Maya says. "It wasn't just counting; it was also recording."



Maya explains that, around 8000 BCE, the Sumerians began using clay tokens to count goods. Around 3500 BCE, the tokens began to have markings or images on them. A farmer might have used a cone-shaped token to represent 1 bag of grain and a ball-shaped token for 1 goat. Over time, they began drawing these shapes onto clay tablets.



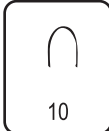
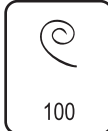
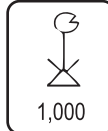
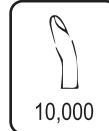
“Sumerians used tokens to represent numbers long before written language was invented,” explains Maya. “This was the beginning of arithmetic—adding, subtracting, and organizing quantities.”

Elijah tries to picture what it would be like to remember everything without writing it down. Numbers gave people a way to count, measure, compare, and keep track of things in the world.

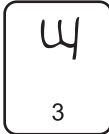
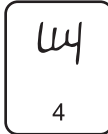
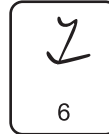
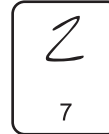
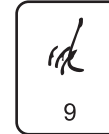
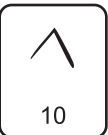
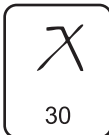
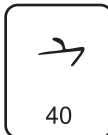
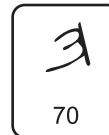
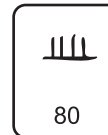
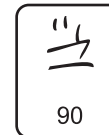
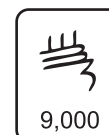
Maya leads Elijah to a display showing Egyptian hieroglyphics and tells him that by 3100 BCE, Egyptians had developed a full numeral system. They needed to measure land, build pyramids, and manage food supplies. Their symbols represented numbers using lines and shapes.

“Egyptians used symbols to measure and build,” Maya says. “They didn’t use decimals like we do, but they understood how to separate things into parts.”

Egyptian Hieroglyphics Numeral System

						
1	10	100	1,000	10,000	100,000	1,000,000

Egyptian Hieratic Numeral System

								
1	2	3	4	5	6	7	8	9
								
10	20	30	40	50	60	70	80	90
								
100	200	300	400	500	600	700	800	900
								
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000

“What do you mean?” asks Elijah.

“Think about the Nile River,” answers Maya. “When the river flooded, they’d think of the farmland as being separated into 2 parts—flooded and not flooded. Then they would measure how much farmland was covered by the floodwater.”

As Elijah thinks about how the ancient Egyptians used their number system to solve practical problems, he realizes how math has been connected to real life.

“So numbers weren’t just abstract,” he says.
“They were essential for survival and building.”

Next, Elijah and Maya move on to an exhibit that explains how real numbers have come to be classified.

“The Egyptian number system, with its distinct symbols and use of parts of a whole, is connected to how we represent and organize numerical information today,” Maya tells Elijah.

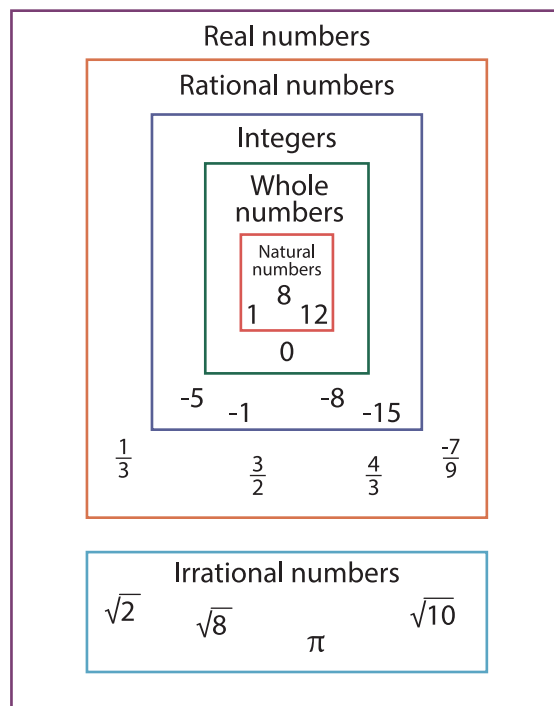
The exhibit explores real numbers and shows how they are divided into rational and irrational numbers. It highlights the differences between the 2 groups and provides some examples.

“We just learned about this!” exclaims Elijah.

Maya continues, “Natural numbers, also called counting numbers, were created because people needed to count things. Later, zero was created to represent having nothing. When grouped together, counting numbers and zero are called whole numbers. The next group, called integers, includes all whole numbers and their opposites, which were needed to represent situations like debt or imbalance. For example, they can show when somebody owes something. As the Egyptian economy grew, numbers were needed that represented parts of a whole. This led to rational numbers, which include fractions in addition to integers.”

“And irrational numbers are numbers that go on forever. Did ancient Egyptians need to use irrational numbers?” asks Elijah.

“No,” Maya answers, “but they are a group of real numbers that were eventually discovered and are used today.”



Next to the illustration of the number classification system is an interactive digital exhibit. Elijah sees a prompt asking him to tap on a number. He taps on 0.5. On the screen, the orange square lights up, indicating that this number falls into the classification of a rational number.

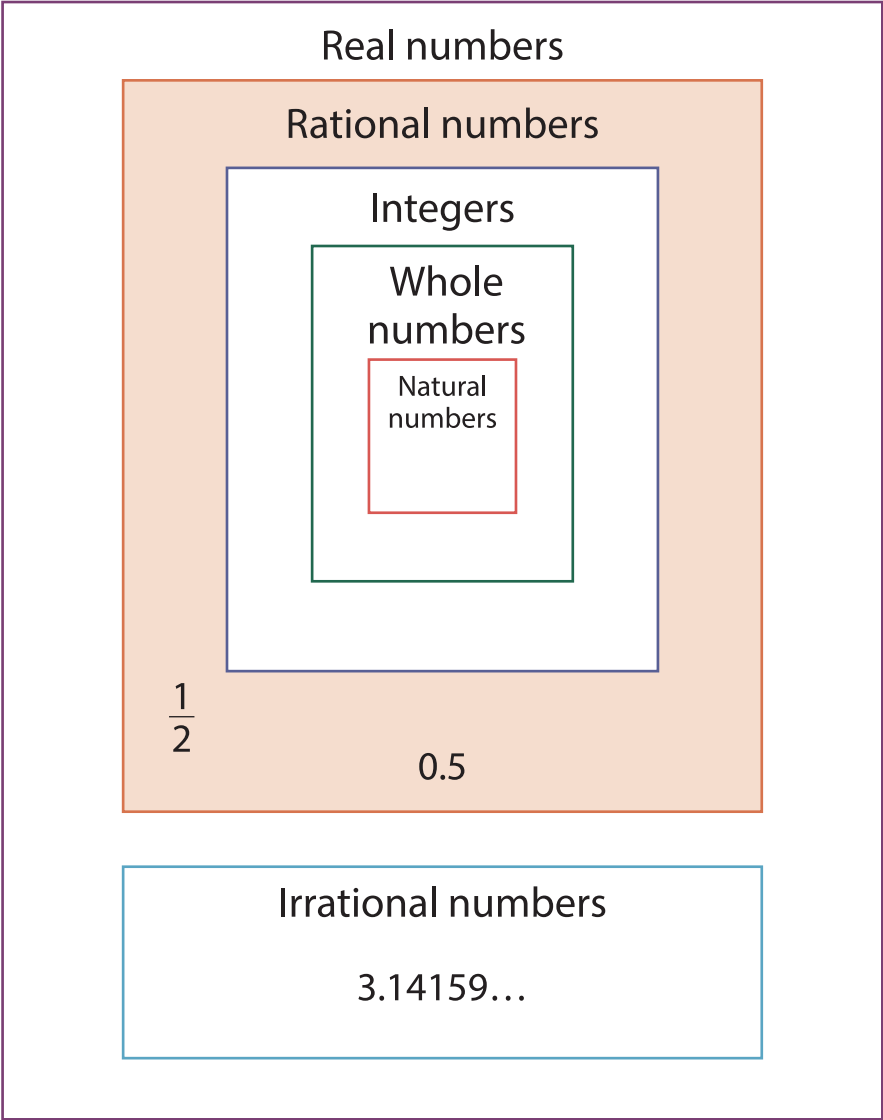
Maya nods. “That makes sense. It can also be written as the fraction $\frac{1}{2}$. All rational numbers can be written as a fraction.”

Maya takes a turn. She taps on 3.14159..., the number that represents pi. The light blue rectangle lights up.

“This number is irrational, and it can’t be turned into a fraction,” explains Maya.

For Elijah, it is exciting to realize that math didn’t stop evolving thousands of years ago. Instead, it continues to grow and connect across cultures and time.

“Even today,” Maya continues, “understanding number classification helps us solve real problems, like converting repeating decimals into fractions or figuring out whether a number makes sense in a situation.”





Elijah never expected he would see a classification system for numbers when he went to the museum or learn that ancient Egyptians were using math. He is in awe when he considers that people have been able to develop number systems throughout history.

As they leave the museum, Elijah looks back at the Ishango bone display. “That bone doesn’t look like much,” he says, “but it started a whole journey.”

Maya smiles and replies, “That’s why we need numbers. They help us count, measure, build, trade, and understand our world. Math is more than just the basics of learning math skills. It is the ultimate form of connection in our world, from past to present and powering development into the future.”

Elijah takes one last glance at the exhibit, and he is inspired to look at math not just as required schoolwork but as a part of human history.

Main Idea

Humans have developed a number system that uses rational and irrational numbers to count, measure, compare, and understand quantities in the world.

Human Calculators

2

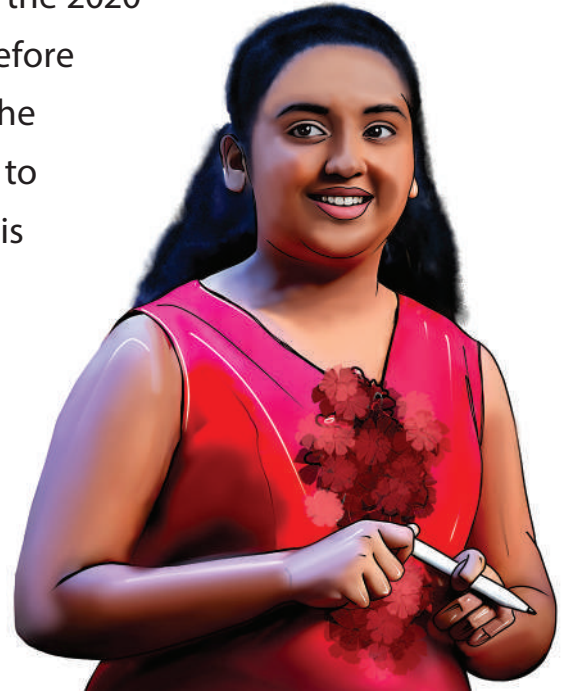
Imagine being one of only 100 people selected for a global award and invited to attend an award ceremony in New Delhi, India. Now, imagine this happening when you are only 10 years old! That's what happened to Miguel Secillano, a resident of Ocala, Florida. He was 10 when he received the 2020 Global Child Prodigy Award. What is even more amazing is that he was nominated for the award when he was only 7 years old!



A prodigy is a young person who is extraordinarily talented in a certain field. They can often do important work in their area of talent just as well as an adult expert. Started in 2020, the Global Child Prodigy Award is an annual award to recognize child prodigies from around the globe in a wide array of fields. The award is meant to promote and support the children so that they can have a positive impact on society.

In Miguel's case, he has a special gift for advanced math. By the time he was 4 years old, he could already solve complicated math problems. Miguel is a magic square expert and lover of puzzles. He was featured on *Little Big Shots* with Steve Harvey when he was 7 years old. By the time he was 10, he could find solutions in his head and explain how he reached his answers. Just think: At 10, he was solving the same level of math problems as college professors!

Fourteen-year-old Nimna Hiranya also received the 2020 Global Child Prodigy Award for Mathematics. Before that, at age 11, she had amazed the judges on the show *Sri Lanka's Got Talent*. She used her ability to solve any math problem using mental math. This meant she could solve complex problems all in her head. Nimna was 7 years old when she learned an ancient Indian system of numerical calculations, called Vedic mathematics. Vedic math uses smart shortcuts and patterns to solve any math question easily. In Nimna's case, she can use these shortcuts while visualizing the equations in her mind.



Rodrigo Medinilla, a resident of Miami, Florida, was 13 years old when he received the 2020 Global Child Prodigy Award. He is famous for being both a math and spelling prodigy. He was reading both English and Spanish by age 3. He also speaks around 11 languages. Rodrigo excels in science and music as well. Coding and programming come easily to him, and he puts these skills to good use on a competitive robotics team. When his IQ was tested at age 6, the result was high enough for him to be accepted into Mensa, an international high-IQ society.

Miguel, Nimna, and Rodrigo are just a few of the math prodigies recognized by the Global Child Prodigy Award over the last few years. However, young people being recognized as math prodigies is not a recent development. Math prodigies have existed throughout history.



Zerah Colburn was born in Cabot, Vermont, in 1804. He began showing signs of being a math genius at an early age. When Zerah was just 6 years old, his father overheard him reciting the multiplication tables to himself. He had only been in school for 6 weeks. Zerah's father asked him to multiply 13 by 97 in his head. Zerah did not hesitate with the answer: 1,261. Zerah and his father traveled the United States and parts of Europe, amazing and astounding audiences with his abilities as a "mental calculator."



Another math prodigy, Shakuntala Devi, was born in 1929 in Bangalore, India. Like Zerah Colburn, she demonstrated a knack for math at a very young age. She was just 3 years old when people noticed her math abilities. Shakuntala did not have a formal mathematics education. Despite this, she was able to solve random math problems posed to her.

Shakuntala traveled to the United States in the 1970s. While she was in the United States, she visited the State Bank of India in New York City. The bank used computers that were very advanced for the time. When they asked Shakuntala to solve math problems, they were able to check her answers using the computers. Most of the time, she knew the answer as soon as she heard the question.

Shakuntala was especially good at solving square roots and cube roots in her head. It was an incredible feat because many of the answers were irrational numbers. An irrational number is a number that cannot be written as a fraction and cannot be made into a terminating or repeating decimal. Shakuntala's skill earned her the nickname "Human Computer," a nickname she did not like.



The Human Brain

How and why are human calculators able to do what they do? According to scientists, the answer is not clear. One idea is that math prodigies process numbers in the same way most people process language.

Think about how you communicate with those around you. Do you need to actively think about what word to use? No! That information comes to you naturally, without thinking about it. It is in your brain. It just comes to you when you need it. Some scientists think prodigies' brains work the same way with math. That could partially explain why so-called human calculators show their incredible abilities at such an early age. It's the same way we learn our native language when we are very young.

Another reason may be that the brains of math prodigies use memory to store information and perform calculations like computers do. According to one study from 2014, math prodigies have larger working memories. Working memory is the part of the brain that holds onto or stores the equations. At the same time, other parts of the brain process other pieces of information.

Math prodigies may not have to work hard at solving complex math problems. On the other hand, many people do need to give it more effort. Some people require more time or need to write out a problem to solve it. But they can still get the correct answer. Just because math comes more easily to some people doesn't mean you can't learn to solve complex problems too! There are many strategies people can use to improve their math skills.



One of the easiest strategies used to solve problems is to look at patterns. Looking for patterns helps us find a way to separate a math problem into smaller steps. Place value is a pattern based on tens. Breaking numbers down by place value can make them easier to add and subtract.

Look at the equation $37 + 19$. The number 19 can be broken down into 10 (tens) and 9 (ones). In the first step, we add 10 (from 19) to 37, or $37 + 10 = 47$. This leaves 9 (from 19.) The second step is $47 + 9$. Using what we know about place value, 9 can be written as $10 - 1$.

	$x = 37 + 19$
<i>Break 19 into 10 + 9.</i>	$x = 37 + 10 + 9$
	$x = 47 + 9$
<i>9 is the same as 10 - 1.</i>	$x = 47 + 10 - 1$
	$x = 57 - 1$
	$x = 56$

So $47 + 9$ is the same as $47 + 10 - 1$ or $57 - 1$, which equals 56.

Mathematicians use patterns like place value to solve problems. Although recognizing patterns adds steps to the math problem, it helps the brain find the solution faster. Adding 37 and 19 showed how to use a pattern based on 10s.

Another strategy used to solve problems faster is replacing numbers with numbers that are easier to use. Look at the problem 39×5 . Instead of multiplying by 39, the number 40 can be used. This produces an easier equation to solve. The answer must be adjusted to account for the change. Because 39 is 1 less than 40, an extra group of 5 was added. Subtract 5, and the answer is 195.

	$x = 39 \times 5$
<i>Change 39 to 40.</i>	$x = 40 \times 5$
<i>Product</i>	$x = 200$
<i>Subtract 5.</i>	$x = 200 - 5$
<i>Actual product</i>	$x = 195$

The human brain is a fascinating organ. It is capable of many ways of thinking. It can process new information and move it into storage to use later. In fact, that is what learning is all about. You know that your brain can store memories of personal experiences. Now, you know that it can solve math equations without pencil and paper, too.

Beyond the Calculator: Thinking Like a Mathematician

Not every famous mathematician was a math prodigy. In school, they may have made the same errors that you might make on your math homework. Mathematicians rely on something much deeper than just memorizing their multiplication tables or being able to quickly do mental math. They use a deep understanding of what they know and apply that to solving difficult math problems. Deep understanding is understanding the *why* behind math concepts. They think about ideas and patterns instead of specific numbers. Instead of solving problems the usual way, they find new paths to get answers.

Amit Garg

Amit Garg is another remarkable mathematician. Born in 1978 in India, he always had an interest in numbers. He taught himself math algorithms. Algorithms are procedures used to solve complex problems quickly and correctly. When Garg began working as an engineer, he developed his own algorithms that made it easier and more efficient to solve equations related to work. One of Garg's most impressive abilities is calculating the square root of 100-digit numbers. He received a Guinness World Record for correctly dividing a 10-digit number by a 5-digit number in under 6 minutes.



Benoit Mandelbrot: Seeing the Unseen Patterns

Benoit Mandelbrot (1924–2010) was a French American mathematician who struggled with simple math but was a genius at other kinds of math, specifically geometry. He was extremely curious about where math exploration could lead.

In his work *The Fractalist: Memoir of a Scientific Maverick*, Mandelbrot wrote:

I found myself in the position of that child in a story who noticed a bit of string and—out of curiosity—pulled on it to discover that it was just the tip of a very long and increasingly thick string . . . and kept bringing out wonders beyond reckoning.



Mandelbrot described a type of geometry that did not rely on smooth lines and perfect circles like traditional geometry. He called it “fractal geometry.” This came from the Latin word *fractus*, meaning “broken or uneven.” In fractals, the same pattern is repeated over and over inside itself.

Have you ever looked through a kaleidoscope? As you turn the kaleidoscope, the mirrors in the toy create a repeated pattern. Fractals are similar. Instead of using mirrors, fractals are created with repeated geometric patterns. In a fractal, you are not creating the pattern; it’s already there. Instead, you can zoom in on the pattern and see the same intricate shapes repeat themselves, no matter how far you zoom in. Fractals can be found in nature. Think of tree branches, leaf veins, snowflakes, and ice crystals!

Ernst Kummer: The Power of Abstract Ideas

Ernst Kummer was a German mathematician (1810–1893). He was well known for his work in number theory and algebra. He spent years trying to prove Fermat’s last theorem. That was a difficult problem that had baffled mathematicians for centuries. While Kummer never solved the problem, his attempt helped future mathematicians who did eventually solve it. In the process of working on Fermat’s last theorem, he developed a revolutionary new concept called “ideal numbers.” Ideal numbers are like invisible helpers that make sure every number, even complex ones, can be broken down into unique factors, or parts. Kummer used math in a way that had never been used before.



Stephen Smale: Challenging the Status Quo

Stephen Smale was born in the United States in 1930. He became an important mathematician and worked in a type of math called topology. He won a big award called the Fields Medal—it's like the Nobel Prize but for math!

Smale wasn't known for being extremely fast at solving problems or doing quick math like some other math geniuses. He even made mistakes and had difficulty memorizing things. But Smale had a special way of thinking. He is good at looking at hard problems in new and creative ways. His big ideas show that you don't have to think like everyone else to do something amazing. Sometimes, the best discoveries come from people who think differently!

Bringing It All Together

To sum it up (pun intended), being good at math goes far beyond quick calculations. Being good at math relies on understanding concepts and reasoning. Both famous mathematicians and math prodigies use math skills and strategies that most can learn. You can make sense of problems, and you can persist in working on a problem, even after making a mistake. You might look instead at the big picture for patterns and relationships. You do not have to rely on memorization or rapid calculation. Mathematicians rely on understanding and reasoning. True mathematical strength comes from learning skills and strategies that prepare you to tackle complex challenges and understand the world around you. You don't need to be a math genius to be good at math!



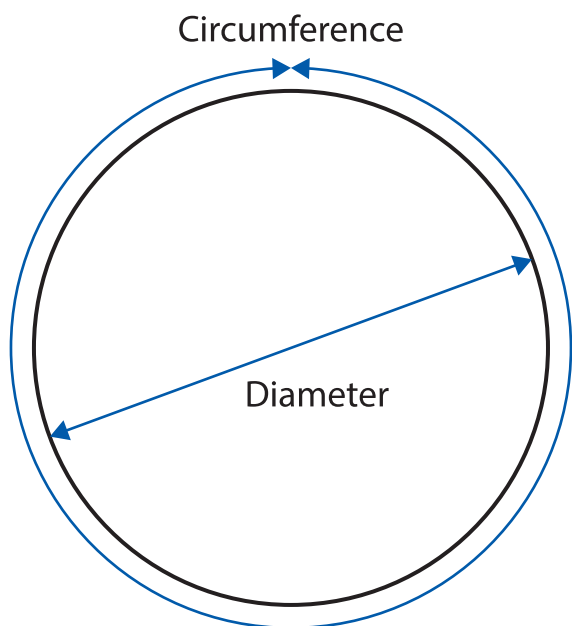
Main Idea

Some people are good at math because they practice math skills, know number patterns, and use smart strategies to solve problems.

A Slice of Pi

3

Picture yourself as a NASA engineer calculating the path for a spacecraft to reach Mars. Or imagine designing the next popular video game, where characters jump through spinning circles. Maybe you're an architect designing a stadium dome with a perfectly curved roof.



What do all these jobs have in common? They all use a special number that people have been exploring for thousands of years. That number is pi (pronounced like *pie*).

Pi, or π , is a simple and powerful number that is also a ratio. It shows how the distance around a circle (the circumference) compares with the distance across the circle (the diameter). No matter how large or small the circle, this ratio stays the same.

Pi is also an irrational number, meaning it cannot be written as a simple fraction.

We estimate pi as $\frac{22}{7}$. This helps us solve equations that use fractions. As a decimal, pi is commonly written as 3.14. But because it is an irrational number, pi never ends and never repeats as a pattern. It goes on and on. As of 2025, computers have been used to calculate pi to 300 trillion decimal places. We know it keeps going! The first 21 digits of pi are shown below.

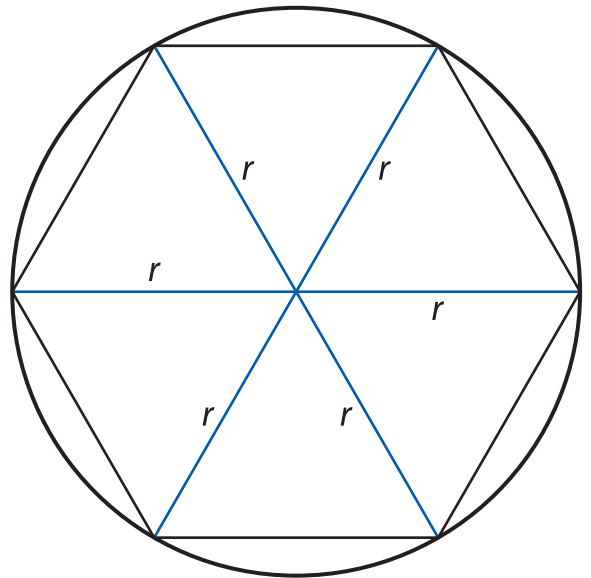
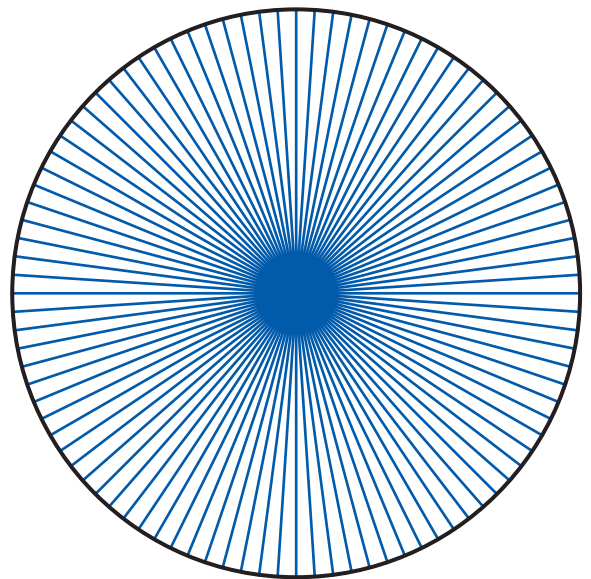
3 . 1 4 1 5 9 2 6 5 3 5 8 9 7 9 3 2 3 8 4 6

Over 4,000 years ago, architects in the Babylonian Empire (1890 BCE–539 BCE) used pi to construct impressive buildings. The Babylonians first calculated pi as 3, which made their math easier. But clay tablets show they also used 3.125 when building round towers and storage containers.

The Babylonians found these values for pi by comparing hexagonal shapes to circles. By drawing a hexagon inside a circle and measuring both shapes, they discovered this important relationship. The Babylonian innovation in calculating pi was to relate the area of the circle to the circumference.

Around 250 BCE, the Greek Archimedes refined the calculation of pi by switching the hexagon inside to polygons with more than six sides. Archimedes ended up with a 96-sided polygon. He calculated the area of each of the 96 triangles and then multiplied by 96. He calculated that pi was between 3.14084 and 3.14285.

Advancement in mathematics has allowed us to be more exact in the way we represent pi. To keep it simple, today we typically use 3.14 as the value for pi. Despite that, the Babylonian and Greek calculations of pi are impressive, especially since they did them without calculators!



Many other groups of people have also been calculating the value of pi for thousands of years. While all these different cultures developed and used the value of pi, no one had a name for the term. In 1706, William Jones, a mathematician in Wales, used the Greek letter π to represent the value of pi. This symbol represents the Greek word for *periphery*. The word *periphery* represents the outer boundary of an object.

Mathematicians have had to use reasoning

and modeling to estimate pi. This challenge led to some of the most important mathematical discoveries. These discoveries are evidence of how solving problems advances math.

The 1600s were a remarkable time of mathematical discoveries. Mathematicians like Isaac Newton used new ideas to find better estimates for pi. Newton was even able to use one of his formulas to calculate pi to 15 decimal places.



William Jones (mathematician)

Why do mathematicians keep calculating more digits of pi? After all, most practical problems only need a few decimal places to get the job done. But for scientists and mathematicians, calculating digits of pi isn't just about getting a more precise number—it's about exploration. It pushes the limits of what we can do and, along the way, helps researchers develop faster algorithms and uncover patterns that could lead to unexpected discoveries.

In the early 20th century, Indian mathematician Srinivasa Ramanujan developed formulas that made it easier for computers to calculate pi to millions of digits. Later work would make it easier for computers to calculate pi to trillions of digits.

Ramanujan had a gift for spotting patterns. This is helpful when working with math. He discovered complex formulas for pi that seemed impossible to find. His formulas were so advanced that they're still used by modern computers today!

The value of pi is used for more than math problems. Today, we use pi for many things. Engineers use pi to calculate how far skateboard wheels travel with each rotation. The speakers you listen to your music through use pi. This is because sound waves follow circular patterns.



Srinivasa Ramanujan (mathematician)

For amusement parks, designers use pi to calculate the size of loops and angles on roller coasters. Engineers solve hard problems about circular motion, using pi to find the right speed and forces for each part of the ride.

For example, when designing a loop-the-loop, engineers must calculate the exact radius of the turn using pi formulas to make sure riders feel excited but stay safe. If a loop is too small, the roller coaster will need to slow down before the loop. This will make the ride less exciting. If the loop is too large, the roller coaster will need a lot of speed to complete the loop. Engineers would have to design longer drops before the loop.



The Ferris wheel is another perfect example of using pi to engineer a ride. Its shape is already a perfect circle. Each spoke connecting the axle to the perimeter is a radius of the circle. Every two spokes in a row form a diameter.

Pi is used to determine the dimensions, motion, and structural integrity of the Ferris wheel. Using the

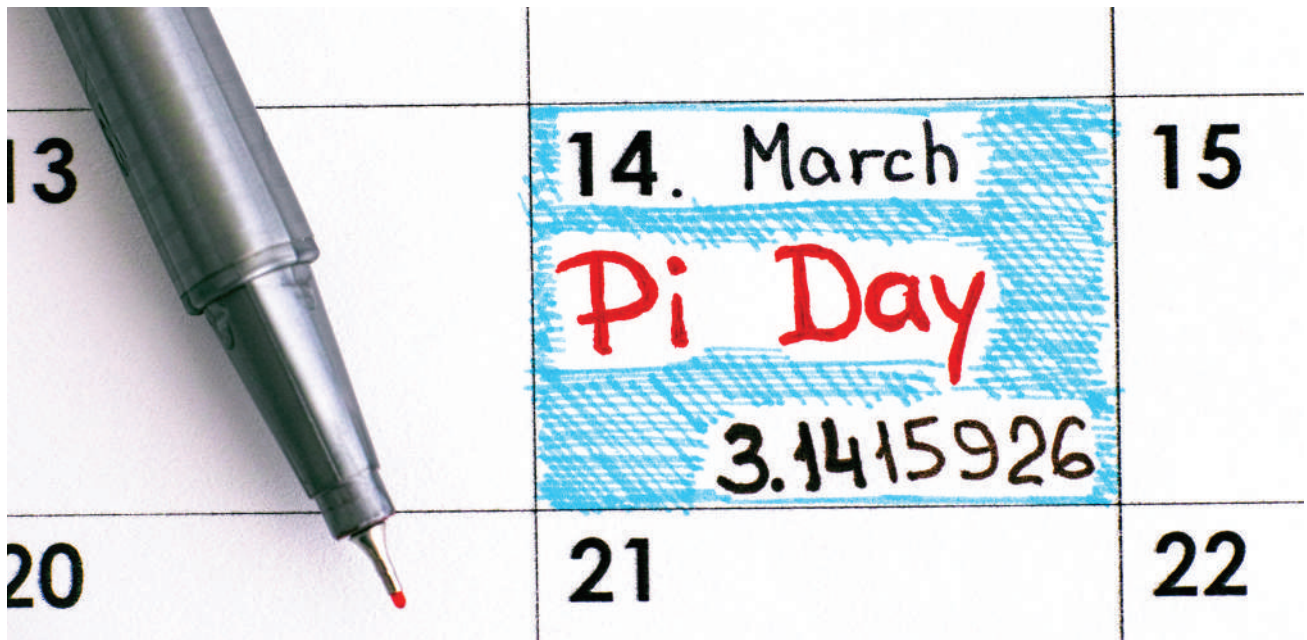


circumference of the wheel, engineers can calculate how far a seat travels in one full turn. Once they know the time it takes for each turn, they can calculate the speed at which the wheel will turn. This is important because the point of a Ferris wheel is to get high up in the air to see the surrounding area. Engineers can use their calculations to determine the speed of the wheel to make sure people have enough time to look around and see the sights. They also want to make sure the speed is fast enough to keep people from getting bored.

Engineers can also use pi to space the seats evenly so the load on the Ferris wheel is the same throughout the wheel. An unbalanced wheel would wobble and be dangerous. Lastly, engineers can calculate the forces acting on the different parts of the Ferris wheel, like the axle, frame, and pivots for each seat. This ensures that the Ferris wheel is safely constructed.

Pi Day

Pi is so special that it has its own day of celebration worldwide! While ancient mathematicians certainly appreciated pi, it took thousands of years for someone to give pi its own celebration. That someone was Larry Shaw, a physicist at San Francisco's Exploratorium, a science museum. In 1988, Shaw had an idea to create a celebration in honor of pi on March 14 (3/14). You might have guessed that it's because the date matches the first three digits of the number pi. Shaw and his coworkers served fruit pies and tea to visitors on this day at the museum. The very first Pi Day was a simple and fun way for people to connect to math in a new way.



A few years later, they realized that March 14 was also Albert Einstein's birthday. After this discovery, they included Einstein's birthday in their celebration. Museum workers sing the song "Happy Birthday to You" after parading in a circle around the museum. Many Pi Day celebrations now include learning about Einstein and how he used pi in his groundbreaking work on space and time.

Today, Pi Day is celebrated in schools, museums, and communities around the globe. It has become a day that connects math to our everyday lives through learning and fun. Schools often celebrate with games, contests, and challenges. Students solve circle problems or build projects that use pi in creative ways. Others try to memorize as many digits of pi as they can.

Pi Day started in the United States and is now celebrated worldwide. In 2009, the U.S. House of Representatives made March 14 “National Pi Day.”

As you continue to learn math in school, you will see pi again and again. In geometry, you’ll use pi to find the area and circumference of circles, plus the surface areas and volumes of cylinders, spheres, and cones.

The next time March 14 comes around, celebrate Pi Day and take a moment to appreciate this awesome number that helps us understand and shape our world.



Main Idea

Pi is used to solve problems that involve circles and curves.

Famous Unsolved Problems

4

Would it surprise you to learn that not every math problem has been solved yet? You might think all math was figured out hundreds or thousands of years ago, but there are still problems we have yet to solve!

Many of these math problems involve conjectures. A conjecture is an educated guess that always seems to be true, but we don't have proof yet to show it is true in every case. Instead, it's based on what we've seen so far but not on complete evidence. If someone wants to say a conjecture is *always* true, they must prove it.

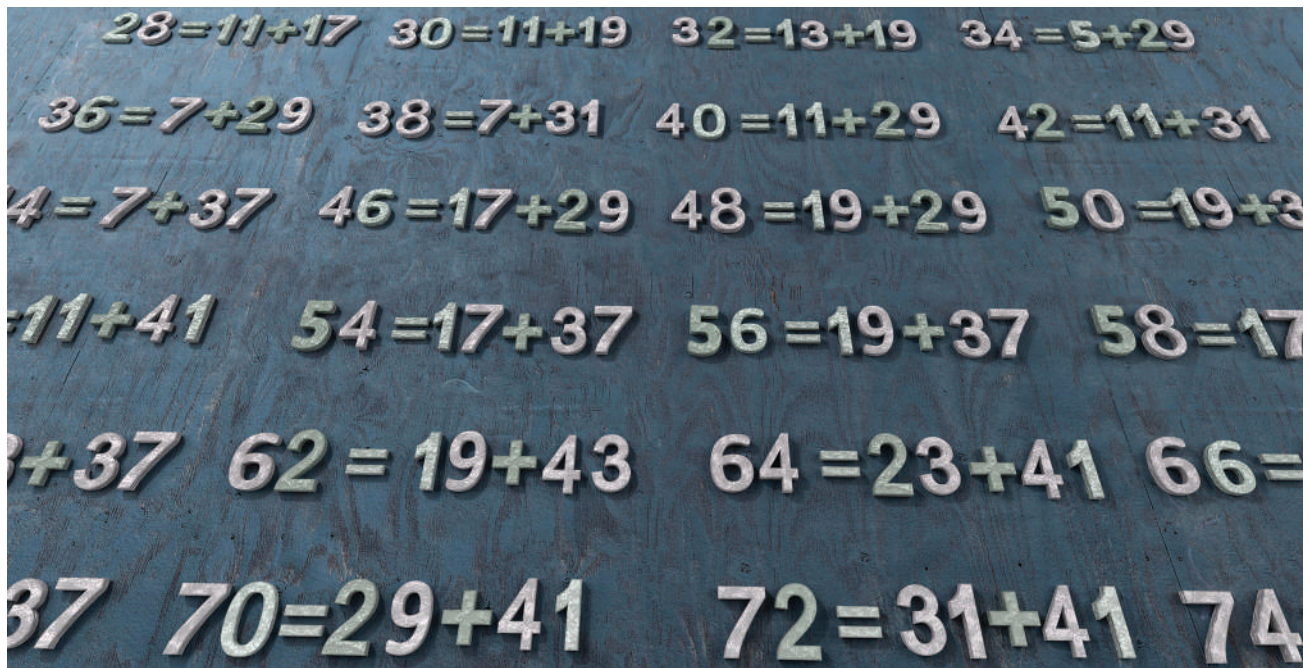
Goldbach's Conjecture

The year was 1742. A German mathematician, Christian Goldbach, wrote a letter to a Swiss scholar, Leonhard Euler. "At least it seems," Goldbach wrote, "that every even, natural number greater than 2 is the sum of two prime numbers." This statement would later become known as Goldbach's conjecture, one of the oldest unsolved problems in mathematics.

haben, nicht bezeugen, ob nicht aber schon mal gesehen haben,
wenn nicht series tantum numeros uno modo in duo quadrata
divisibiles habet auf solche Weise will ich auf eine conjecture
hazardsieren: daß jede Zahl welche aus zweyen primis
zusammengesetzt ist ein aggregatum severalium numerorum
primorum sey als man will: die unitatem mit dazu genommen
bey sich die conjectur omnium unitatum? ganzes Beispiel
 $4 = \begin{cases} 1+1+1+1 \\ 1+1+2 \\ 1+3 \end{cases} \quad 5 = \begin{cases} 2+3 \\ 1+1+3 \\ 1+1+1+2 \\ 1+1+1+1+1 \end{cases} \quad 6 = \begin{cases} 1+5 \\ 1+2+3 \\ 1+1+1+3 \\ 1+1+1+1+2 \\ 1+1+1+1+1+1 \end{cases} \quad \text{etc}$
Künftig folgen mir ganz observationes so demonstrant unter
den Poisson:
Si v. sit functio ipsius x. cuiusmodi ut facta $v = c$. numero cui-
cunque, determinari possit x per c. et reliquis constantes in functio-
ne expressas, poterit etiam determinari valor ipsius x. in æ-
quatione $v^{n+1} = (2v+1)(v+1)^{n-1} \cdot \frac{v^{n-1}-v^{n-2}}{v^{n-1}-v^{n-2}} \dots$ d. h. v. v. v. v.
Si accipiat curvæ cuius abscissa sit x. applicata erit sit
summa seriei $\frac{x^n}{1.2} + \frac{x^n}{2.3} + \frac{x^n}{3.4} + \frac{x^n}{4.5} + \dots$ dico, si fuerit
applicata $= 1$, applicatum fore $= \frac{1}{2} = \frac{1}{2}$: sed hoc applicatum = $\frac{1}{2}$
2 - - - - - $\frac{1}{2}$.
3 - - - - - $\frac{1}{2}$.
4 vel major - - - - - infinitum.
Ich verhoffe mit aller ansehnlichkeit die Versicherung
zu geben daß ich die Ehre habe
Moscov 7. Jun. st. 12. 1742. J.
Goldbach

Let's test this conjecture.

Goldbach's letter to Euler



Additional even numbers that are calculated by adding 2 prime numbers

Goldbach’s conjecture is like a math puzzle. His problem asks: Can every even number greater than 2 be calculated by adding two prime numbers? So far in the examples below, it is true.

<i>Sum of 2 prime numbers?</i>	
$4 = 2 + 2$	✓
$6 = 3 + 3$	✓
$8 = 3 + 5$	✓
$10 = 3 + 7$	✓
$12 = 5 + 7$	✓
$50 = 3 + 47$	✓
$100 = 3 + 97$	✓

Amazingly, even though this idea seems true, in over 300 years, no one has been able to prove it is *always* true. There are too many possible calculations. We can only prove it is true in the calculations we have done.

More recently, mathematicians have used modern technology they can use to help prove this conjecture. They are using supercomputers to check calculations for even numbers up to 4×10^{18} (that’s 4 followed by 18 zeros!). Goldbach’s conjecture holds every single time. Yet nobody can prove it will work for all even numbers, which is why it is still a conjecture. It is one of the most well-known problems in math that remains unsolved.

Collatz Conjecture

Instead of focusing on prime numbers like Goldbach’s conjecture, the Collatz conjecture breaks apart counting numbers, or whole numbers larger than zero, into evens and odds. The conjecture is simple: By applying 2 rules to a counting number, you will always reach the number 1.



The Collatz conjecture’s 2 rules are:

Rule 1: For any even counting number, divide by 2.

Rule 2: For any odd counting number, multiply by 3 and add 1.

These rules are applied as many times as necessary until eventually the result is the number 1. Here are sample calculations starting with the counting numbers 1 through 3. How many times did the rules need to be applied for starting numbers 1, 2, and 3? What patterns can you find?

Starting Number	Rule Applied	Calculations	Result
1	None		1
2 (even number)	Rule 1	$2 \div 2 = 1$	1
3 (odd number)	Rule 2	$3 \times 3 + 1 = 10$	10 (even number)
	Rule 1	$10 \div 2 = 5$	5 (odd number)
	Rule 2	$5 \times 3 + 1 = 16$	16 (even number)
	Rule 1	$16 \div 2 = 8$	8 (even number)
	Rule 1	$8 \div 2 = 4$	4 (even number)
	Rule 1	$4 \div 2 = 2$	2 (even number)
	Rule 1	$2 \div 2 = 1$	1

The Collatz Conjecture was introduced by the German mathematician Lothar Collatz in 1937, using these two rules seems to result in the number 1. But no evidence has been found to show it *always* works. There are two parts of these calculations that make finding a proof hard.

First, the number of calculations is hard to predict. If you start with the number 2, only 1 calculation is used. When starting with the number 3, 7 calculations are used. However, if you start with the number 4, only 2 calculations are needed.

Second, finding a proof is hard because the results from the initial starting numbers can vary greatly. It may seem that starting with an even number will mean a small number of calculations, but that is not always true. Consider using 20 as the starting number.

Starting Number	Rule Applied	Calculations	Result
20 (even number)	Rule 1	$20 \div 2 = 10$	10 (even number)
	Rule 1	$10 \div 2 = 5$	5 (odd number)
	Rule 2	$5 \times 3 + 1 = 16$	16 (even number)
	Rule 1	$16 \div 2 = 8$	8 (even number)
	Rule 1	$8 \div 2 = 4$	4 (even number)
	Rule 1	$4 \div 2 = 2$	2 (even number)
	Rule 1	$2 \div 2 = 1$	1

After using Rule 1, where numbers are divided by 2, the number gets smaller and closer to 1. But when you start with an odd number, like 5, this means you use Rule 2, which is tripling and adding. The resulting number is 16, which is farther away from 1. That increases the number of calculations that are needed.

These two different paths numbers can take make a proof difficult to find because it's hard to know how this pattern will work with every number. So far, a number eventually gets to 1. Nobody has found an example that does not work.

The 196 Algorithm

Have you heard of palindromes? Maybe you have seen some words that are palindromes, such as *kayak* or *race car*. Look closely, and you will see that these words are spelled the same forward and backward!

Dates can be palindromes, too. An example is December 02, 2021, which is in the twelfth month of the year, as shown at right.



Some whole numbers that are palindromes include:

1,331
12,321
809,908

The 196 algorithm (also known as the reverse-then-add process) involves reversing the digits of a number and adding the result to the original number, then repeating this process until a palindrome is formed.

Here is how the algorithm works:

Step	Example
1. Start with any number.	87
2. Reverse the digits of the number.	78
3. Add the numbers together.	$87 + 78 = 165$
4. Is the number a palindrome yet? If not, repeat the process until you get one.	165 is not a palindrome. $165 + 561 = 726$ $726 + 627 = 1,353$ $1,353 + 3,531 = \mathbf{4,884}$

The 196 algorithm is a fun way to play with numbers, but why is it named for that number? The number 196 is famous because even after thousands of steps, no one has found a palindrome when starting with that number.

It's a mathematical mystery!

In 1995, a computer programmer ran nearly 2.5 million reverse/add steps starting with the number 196, but he did not encounter a palindrome. Twenty-five years later, another programmer got up to 1 billion steps. The resulting number had over 4 million digits and would be 1,000 kilometers long if printed on paper—all without a single palindrome.

Even with 1 billion steps, however, mathematicians cannot prove that a palindrome of 196 will not be found, so it remains a mystery to this day.

Odd Perfect Numbers

Mathematicians have been searching for odd perfect numbers for hundreds of years, but so far, no one has found one. It's another one of the big mysteries in math that while we know that perfect numbers exist, we don't know if there are any odd ones.

What's a perfect number? A perfect number is a positive whole number that is equal to the sum of all its divisors, excluding the number itself.

For example, 6 is a perfect number. The divisors of 6 (the numbers that divide evenly into 6) are 1, 2, 3, and 6. So 6 is a perfect number because the sum of its divisors (excluding itself) equals 6.

$$1 + 2 + 3 = 6$$

Another example of a perfect number is 28. The divisors of 28 are 1, 2, 4, 7, 14, and 28.

$$1 + 2 + 4 + 7 + 14 = 28$$

This means that 28 is also a perfect number.

So what about odd perfect numbers? An odd number is a number that is not divisible by 2. So far, no one has found any odd perfect numbers! All the perfect numbers we know are even, like 6, 28, 496, and 8,128.

No one knows for sure whether an odd perfect number exists. Some mathematicians believe odd perfect numbers don't exist. Others think that they might be out there somewhere but that they are just really hard to find.

Prime Number Theorem

Not all mathematical mysteries go unsolved. For many years, mathematicians wondered whether prime numbers followed a pattern or just appeared randomly in the sequence of counting numbers.



During the late 1800s, mathematicians Jacques-Salomon Hadamard and Charles de la Vallée-Poussin worked independently to answer this question. Both proved that prime numbers do follow a pattern! Essentially, the pattern is that the average distance gap between two consecutive prime numbers increases as the values of the prime numbers grow.

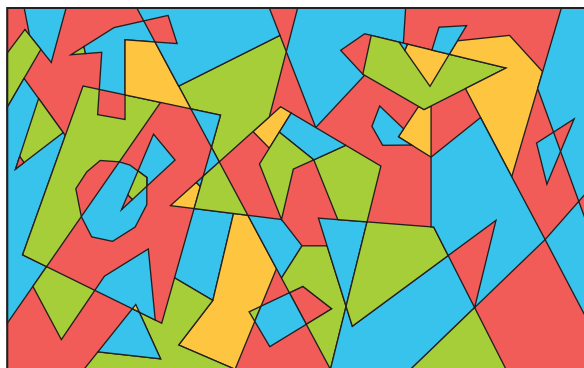
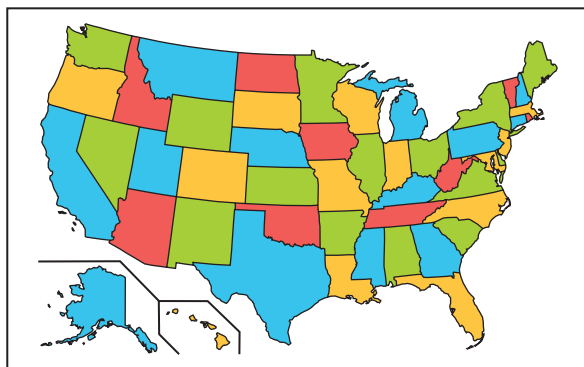
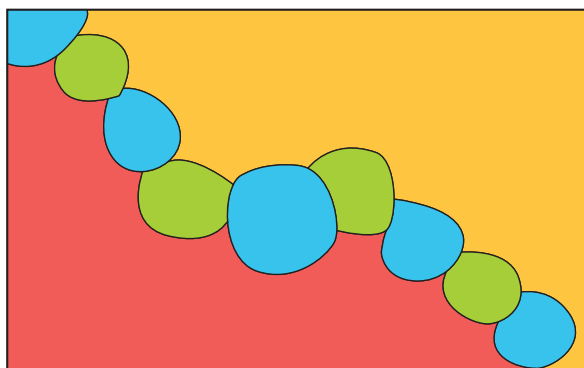
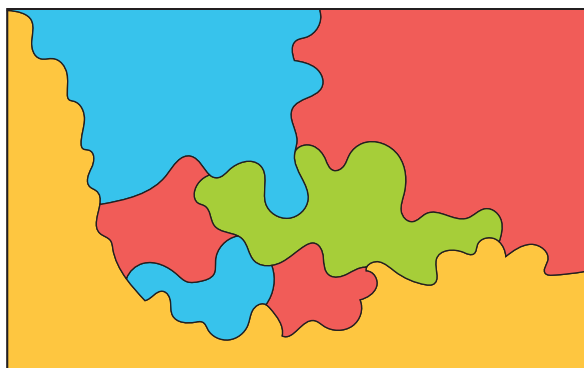
The 4-Color Theorem

Have you ever noticed that the countries next to each other on a map are shaded differently so you can more easily see their borders?

In 1852, mathematician Francis Guthrie conjectured that you only need 4 colors to color any map so that no two countries that share a border are the same color.

Even ignoring shared corners, where countries were already clearly distinguishable from one another, it took over 100 years for mathematicians to prove this conjecture is true. The proof was extraordinary because it was the first big math problem solved with the help of a computer!

Mathematics is more than what you find in a textbook. It is full of mystery and discovery. It is a way of thinking. We don't have all the answers. There is always something to solve. We continue to question, explore, wonder, and learn using math.



Main Idea

A math conjecture is an idea or statement that seems to be true but hasn't been proven yet.

Writing Large and Small Numbers

5

You use numbers every day to describe things. Your friend is 62 inches tall. The soccer field is 110 meters long. A 5K race is 5,000 meters long. But there is another way to write these numbers called *scientific notation*. It's like a shortcut or abbreviation for writing really big or really small numbers. Instead of writing out many zeros, we use a shorthand system.

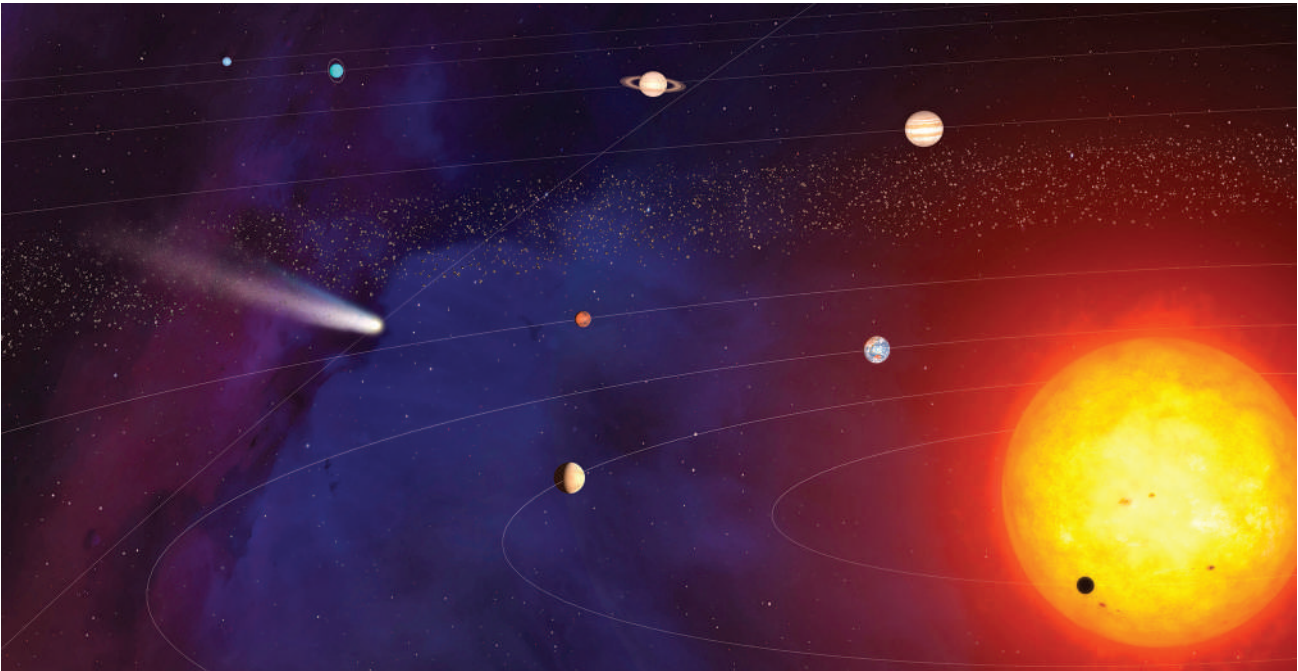
Here's how it works: Take any number, and write it as a number between 1 and 9, then multiply it by 10 raised to some power. That power tells you how many places to move the decimal point. Let's convert the samples above into scientific notation.

	Measurement	Unit of Measure	Scientific Notation	Unit of Measure
Friend	62	inches	6.2×10^1	inches
Soccer field	110	meters	1.1×10^2	meters
5K race	5,000	meters	5.0×10^3	meters

For the measurement of your friend's height, the decimal is moved to the left 1 place, from 62 to 6.2. So the power is 10^1 . For the soccer field, the decimal is moved 2 places to the left. So the power is 10^2 . For the 5K race, the decimal is moved 3 places to the left, from 5,000 to 5.0. So the power is 10^3 . Notice that using scientific notation represents a number and just a number. The unit of measure does not change, so your friend's height is in inches, whether it's 62 inches or 6.2×10^1 inches.

Scientific notation is useful when dealing with very large or very small numbers. For example, it's easy to make a mistake when writing the correct number of zeros. When using scientific notation, you don't have to write out every digit in a large number.

Think about this: The distance from Earth to the sun is about 149,000,000 kilometers. That's 149 million! What if you had to write that number over and over in your homework? You might incorrectly write those zeros at least once. Let's look at some large numbers that an astronomer might work with and convert them into scientific notation.



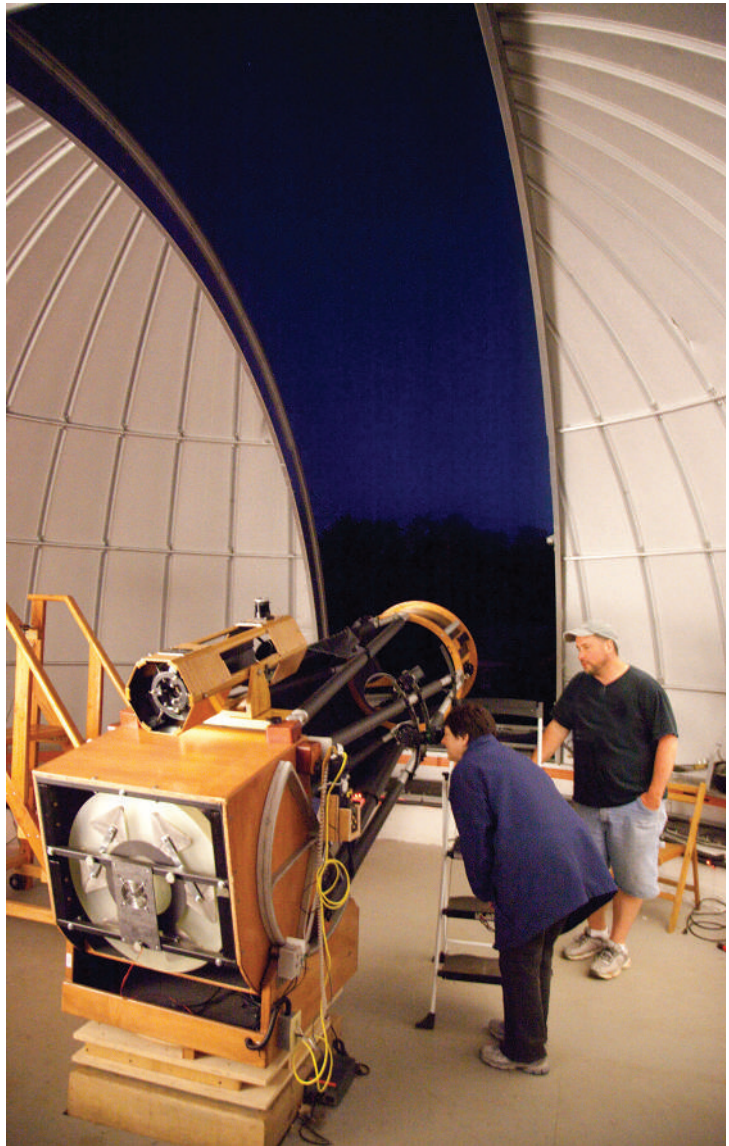
Distance	Measurement	Scientific Notation
Earth to moon	384,400 km	3.844×10^5 km
Earth to sun	149,000,000 km	1.49×10^8 km
Earth to Neptune	4,500,000,000 km	4.5×10^9 km

For Earth to the moon, the decimal was moved 5 places to the left, so the power is 10^5 . For Earth to the sun, the decimal was moved 8 places, so the power is 10^8 . For Earth to Neptune, the decimal was moved 9 places, so the power is 10^9 .

Do you ever wonder where astronomers get their data? It's collected from powerful telescopes in observatories on mountaintops located in places like Hawaii, Chile, or China. Other data comes from space-based telescopes orbiting Earth, the sun, or other objects. Astronomers examine images of the cosmos, or universe. They are looking for patterns, unusual objects, or changes over time.

The data astronomers collect about the universe and the objects in it, such as planets, solar systems, and galaxies, is all described by very large numbers. Astronomers use scientific notation when they are talking about the sizes of galaxies, stars, comets, planets, and asteroids. Distances between objects in space are vast, and objects in space are traveling at incredible speeds. All these measurements involve enormous numbers that are more easily understood when written in scientific notation.

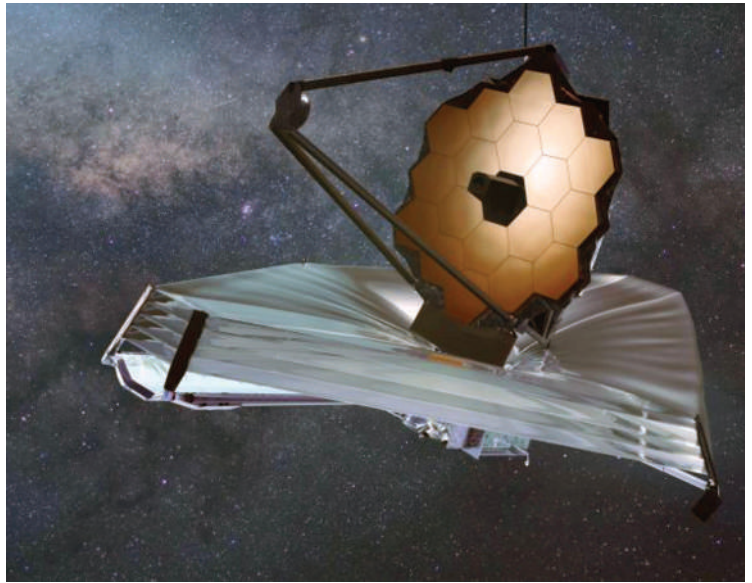
Astronomy is genuinely a global science. Astronomers share information with other astronomers all over the world. Clear communication is key. When astronomers write numbers in reports, they use scientific notation. This ensures that astronomers in other countries have data in the same format!



Many nights, astronomers go to observatories to make observations of the sky. They might bring a group of students in order to show them objects in the universe and teach the students how planets move. They are surrounded by high-tech equipment and computers, spending hours watching the night sky. They might track the movement of a comet, search for new planets, or study the light from distant galaxies.

One fun fact about seeing stars at night is that the light you see coming from the star is many years old. Light travels at 299,792,458 meters per second, or, in scientific notation, 3.0×10^8 m/s. But the stars are very, very far away. The nearest star system to ours is Alpha Centauri. Alpha Centauri is a star system with 3 suns. In scientific notation, Alpha Centauri is 4.1×10^{16} m from Earth. Those are numbers so large it's hard to imagine the speed and distance each represents.

When you know speed and distance, you can calculate how long it takes the light from Alpha Centauri to reach observatories on Earth. Instead of writing both numbers out with all the zeros, you can use the number in scientific notation to calculate how long it takes the light from Alpha Centauri to reach Earth.



James Webb Space Telescope



Hubble Space Telescope

Let's do the math. We need to divide the distance to Alpha Centauri by the speed of light to find out how long it takes for Alpha Centauri's light to reach Earth.

$$(4.1 \times 10^{16} \text{ meters}) \div (3.0 \times 10^8 \text{ meters/second})$$

Step 1. Divide the decimals of each number.

$$4.1 \text{ m} \div 3.0 \text{ m/s} = 1.4 \text{ s}$$

Step 2. Subtract the exponents.

$$10^{16} \div 10^8 = 10^{(16-8)} = 10^8$$

Step 3. Multiply the answers from Step 1 and Step 2.

$$1.4 \times 10^8$$

Step 4. Write the result in scientific notation.

$$(1.4 \times 10^8 \text{ s}) \text{ or } 140,000,000 \text{ seconds} = 4.4 \text{ years}$$

The light we see coming from Alpha Centauri's 3 stars was generated 4.4 years ago. In a way, when we see the light from Alpha Centauri, we are looking back in time. Eventually, we might have telescopes that are strong enough to clearly look at the planets around star systems like Alpha Centauri. If that happens, we will be looking back in time at the events on those planets.

Now you've seen how very large numbers are written and calculated using scientific notation. But how are very small numbers written and calculated using scientific notation?



Alpha Centauri A and B are a binary pair that orbit each other. Proxima Centauri, a small, dim red dwarf star, orbits the pair at a great distance. This picture was taken with the Hubble Space Telescope.

While astronomers look up at the huge universe, physicists look down at the incredibly small world of atoms and particles. They use special microscopes that can show individual atoms! The measurements they make are so tiny that scientific notation is essential. Like astronomers, physicists use scientific notation to record their numbers. Unlike astronomers, the numbers used by physicists are very small.

For example, the average teenager has a mass of about 40 kilograms. When written in scientific notation, the decimal moves 1 place to the left, from 40 to 4.0. Because the decimal moves 1 place to the left, the power is 10^1 . The measurement of the mass of a sheet of paper is 0.0045 kilograms. Using scientific notation, the decimal is moved 3 places. But this time the decimal moves to the right. So the power is 10^{-3} . For the human hair, the decimal moves to the right 7 places, so the power is 10^{-7} .



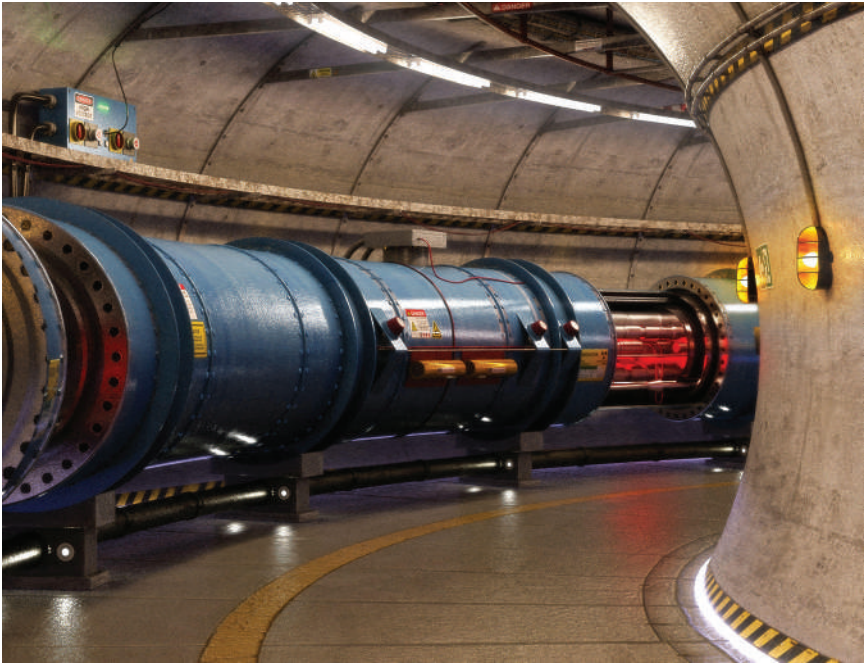
	Measurement	Unit of Measure	Scientific Notation	Unit of Measure
Teenager	40	kilograms	4.0×10^1	kilograms
Sheet of paper	0.0045	kilograms	4.5×10^{-3}	kilograms
Human hair	0.00000015	kilograms	1.5×10^{-7}	kilograms

Here’s something mind-blowing: Scientists used to think atoms were the smallest particles that make up matter. Then they discovered that atoms are made of even smaller particles called protons, neutrons, and electrons. But wait—it gets smaller! Those protons and neutrons are made of even tinier particles called quarks.

Without scientific notation, these numbers would be written with 27 or 30 decimal places. Using scientific notation makes the numbers much more manageable.

Particle	Mass (kg)
“Up” quark	3.6×10^{-30}
“Down” quark	8.9×10^{-30}
Proton	1.7×10^{-27}
Neutron	1.7×10^{-27}

To calculate these masses, physicists use a particle accelerator. This is a machine used to make tiny particles, like protons or electrons, move extremely fast. Electric fields push the particles faster and faster, and magnetic fields steer the particles in the right direction. When the particles are moving fast enough, scientists smash particles into each other to study what happens. By identifying the pieces left over from the collision, physicists can calculate the masses of the particles before and after the collision.



Underground particle accelerator machine used to conduct research on colliding atoms.

Suppose you're a physicist and someone asks you to calculate how many uranium atoms would make up the mass of a human hair. The mass of a uranium atom is 3.9×10^{-25} kilograms and the mass of a human hair is 1.5×10^{-7} . In this case, we'll divide the mass of the human hair by the mass of the uranium atom to get our answer.

$$(1.5 \times 10^{-7} \text{ kilograms}) \div (3.9 \times 10^{-25} \text{ kilograms})$$

Step 1. Divide the decimals of each number:

$$1.5 \text{ kg} \div 3.9 \text{ kg} = 0.38$$

Step 2. Subtract the exponents. Note that the -25 , because it is subtracted, becomes a positive 25:

$$10^{-7} \div 10^{-25} = 10^{(-7 - -25)} = 10^{18}$$

Step 3. Multiply the answers from Step 1 and Step 2:

$$0.38 \times 10^{18}$$

Step 4. Write the result in scientific notation:

$$(3.8 \times 10^{17})$$

Note that because we moved the decimal one place to the right, from 0.38 to 3.8, the power is reduced by 1, from 10^{18} to 10^{17} .

So if your strand of hair had the same mass but was made up of uranium atoms, you would have 380,000,000,000,000,000 uranium atoms. That's 380 quadrillion atoms!

And that number of uranium atoms is why scientific notation is so useful. Scientific notation writes very big or very small numbers in a simple way, making them easier to read and work with. This makes it less likely for you to make errors as you calculate numbers.

Main Idea

Scientific notation allows us to write and use very large or very small numbers.

Pick a Plan!

6



Felipe could hardly contain his excitement; he was finally getting a smartphone. It had not been easy—both of his parents required lots of convincing. He went so far as to create a presentation filled with facts about the benefits of smartphones. Felipe made sure to mention the convenience of not having to borrow one of their phones. He

reminded them of times when his soccer team had an away game and he needed to let them know when the bus was headed back to the school. He concluded his presentation by saying, “Really, letting me have my own phone is better for you than it is for me.”

During his presentation, Felipe had also made the point that, since he now has an after-school job, he can pay for the smartphone and the plan without their help. All he needs is a ride to the smartphone store.

Mr. Perez agrees to drive Felipe and his best friend, Jessie, to the shopping center on Saturday. He’ll shop for new pickleball shoes while the teens shop for a phone.

"Felipe, you know that a smartphone comes with a lot of responsibility," his dad reminds him as they drive to the mall.

"Dad, I know. Trust me. I won't let having a phone interfere with school or sports. I won't stay up all night texting and playing games! I've even picked out a smartphone that's virtually indestructible, so it won't break if I drop it!" Felipe explains.

Jessie's face lights up, and she begins to laugh. "If you drop it? Oh, buddy, you are going to drop your new phone! You're one of the best goalies in the state, but if it's not a soccer ball, you can't seem to hold onto it!"

Mr. Perez chuckles, too. "Alright, alright, but have you done any research about different phone plans or given any thought to how much you're comfortable spending each month?" he asks.

"A little, but I'm not too worried about it—I'll figure it out once we get to the smartphone store," Felipe replies.

Mr. Perez rolls into the parking lot and pulls the car into a spot. "Because you are under 18, I will need to sign the contract. I will come back to the store when I am done and we can talk about your decision."

"Thanks, Dad," Felipe says as he hops out of the car, "but I've got this; *trust* me!"

"See you in a bit, Mr. Perez!"
Jessie calls over her shoulder as she follows Felipe to the store entrance.



There was only one smartphone store to choose from: WeConnectU. But that wasn't an issue; Jessie's grandma had given her a smartphone from WeConnectU as a birthday gift the year before, and she'd had a very positive experience so far.

Jessie giggles as they walk into the store. "It's weird how happy my memories are of this place. You should have seen Gran talking to the salesperson. He couldn't believe how tech-savvy she was!"

"Your gran really is awesome," says Filipe. "My mom can barely text, but your 92-year-old grandma has her own streaming channel!"

One of the WeConnectU salespeople approaches the friends. "Hi, I'm Alycia. How can I help you today?"

"I'm here to buy my first smartphone," Felipe replies excitedly.

"I'd be happy to help you with that; do you know what phone you want?" asks Alycia.

"Definitely the SMRT1 with a shatterproof screen," says Felipe.

"... And the matching waterproof case if you have it!" adds Jessie.

"Let's see here," Alycia says as she looks up the store's inventory on her tablet.

"You're in luck—not only do we have both in stock, but we're also running a special on plans for all SMRT phones!"



“Great!” Felipe responds eagerly. “What are my options?”

“We have 2 plans to choose from: Plan A requires you to pay \$350 up front, and then your monthly cost would be \$100, and Plan B is \$50 up front and \$150 a month.”

Alycia notes the surprised look on Felipe’s face. “I have to head into the back to grab your phone and the case,” Alycia smiles and says. “I’ll give you a few minutes to talk through the options.”

“Before you go,” Jessie asks, “any chance you have paper and a pencil we could use?”

Alycia walks over to the tech desk and pulls a spiral notebook from a drawer, then passes it and a pen to Jessie.

“Thanks!” says Jessie as she quickly begins scrawling numbers on the pad before Alycia has even stepped away. “OK, there are a couple of things to think about. Let’s start with which one of these plans is the better value. This is like the system of equations math problems we have done in class. I think we can represent both plans as linear equations.”

Jessie shows Felipe what she has written.



	Plan A: $y = 100x + 350$
	Plan B: $y = 150x + 50$

Felipe glances at the figures Jessie scrawled in the notebook. “I see what you’re getting at—the value of x is the number of months, and y is the total cost over a certain period of time,” Felipe says.

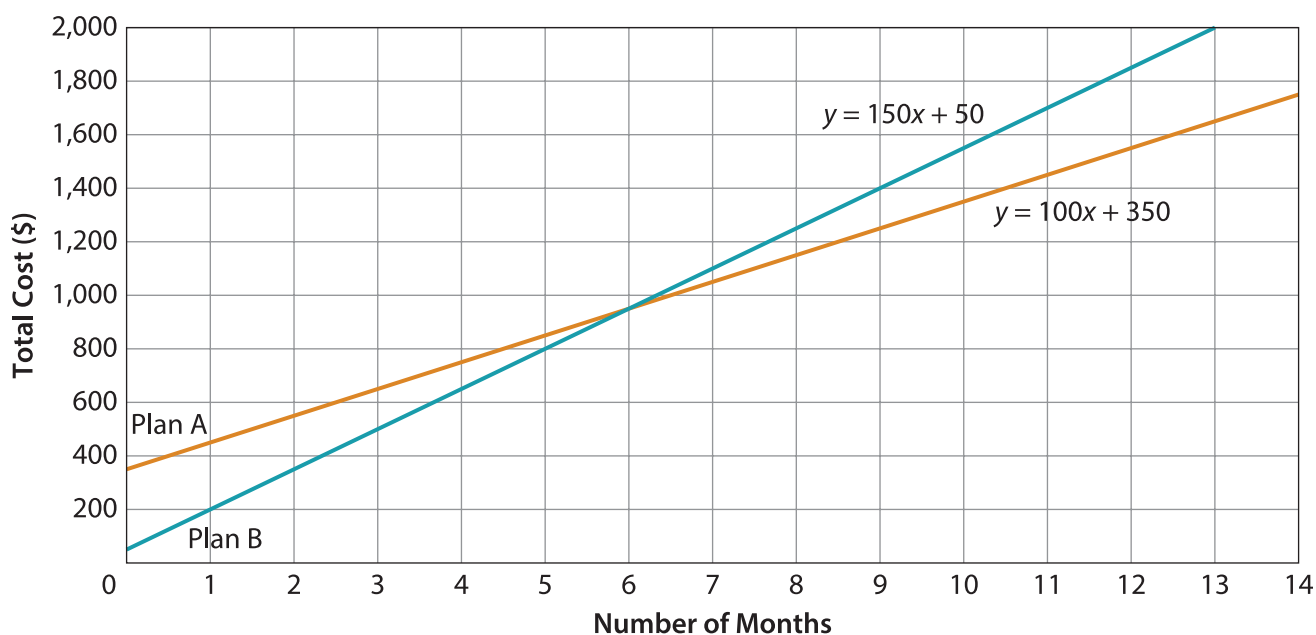
“That’s exactly it!” Jessie replies. “The first month is easy—your total cost for Plan A is \$350 up front plus the \$100 monthly fee, so it’s \$450. This is more than Plan B, which is only \$200—\$50 up front and the \$150 monthly fee.”

“But,” says Felipe, “with the \$150 monthly fee, it seems like Plan B might get more expensive than Plan A at some point.”

Jessie’s eyes light up. “Let’s use an online graphing calculator! It will do all the plotting for us. Then, we can look at the lines!”

They pull up an online graphing calculator on Jessie’s smartphone. Felipe types the equations into the calculator. Then he labels the x -axis “Number of Months” and the y -axis “Total Cost (\$).”

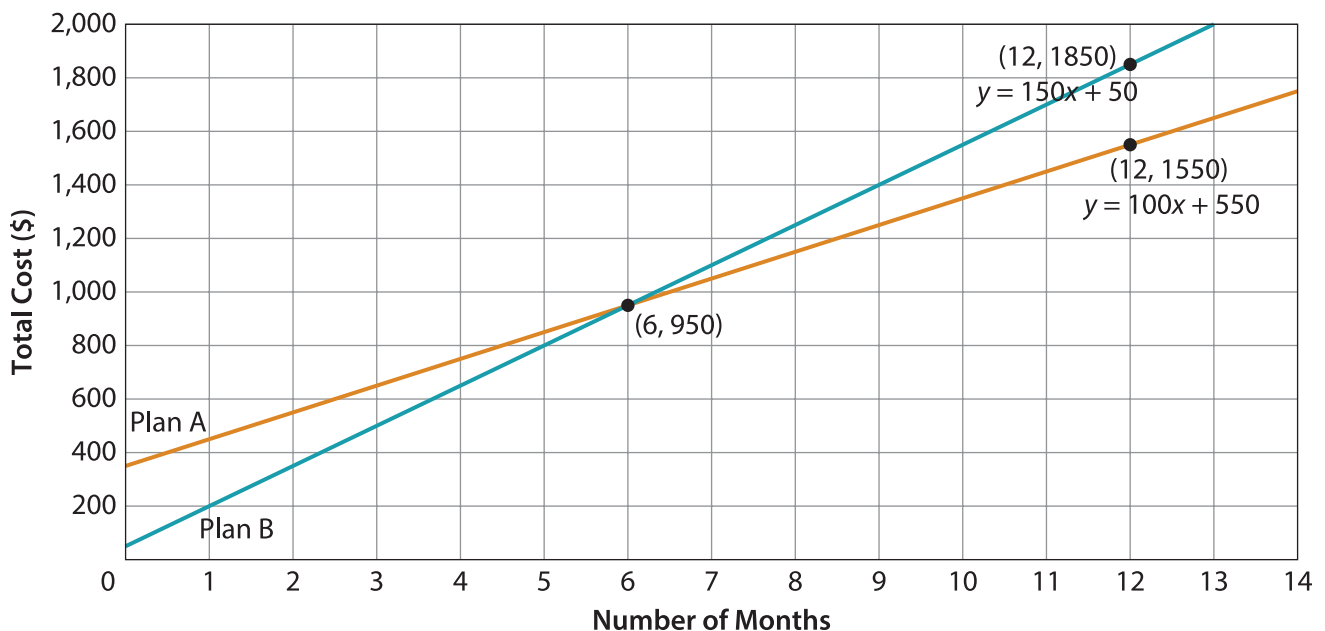
They lean over the screen with curiosity. “Look!” Jessie points to the graph. “The calculator even shows us where the 2 lines cross. That’s the point of intersection!”



The online calculator clearly illustrates the point where the lines meet. The lines intersect at 6 months and \$950 Felipe exclaims. “That means that after I have the phone for 6 months, both plans will have cost me \$950!”

“But look what happens after 6 months!” Jessie points to the line for Plan A. “Plan A’s line is below Plan B’s—and the difference between them keeps increasing. That means that after 6 months, Plan A is the cheaper option.”

Felipe points to each line at 12 months. “One year from now, I will have spent \$1,850 if I go with Plan B and only \$1,550 if I go with Plan A,” he says. “That’s a \$300 difference!”



“This is amazing, Felipe,” Jessie says. A wide smile spreads across her face. “By putting these 2 plans into those equations and graphing them, we can actually see the better choice.”

Felipe nods. “It is cool that we can see which plan is cheaper at different times. Before the lines cross, Plan B is better. After they cross, Plan A is better.”

“It only takes 6 months for the plans to equal the same amount. That’s not a lot of time, and I’d definitely be saving money in the long run if I choose Plan A,” Felipe thinks aloud.

Jessie bobs her head. “You are right. Plan A is the better option of the 2, but you also need to think about what you’ve saved so far and how much money you make each month.”

“Good point! I make \$12 an hour and work about 5 hours a week, so after taxes, that’s about \$214 each month, and I’ve saved \$400 so far. Plan A would take most of my savings.”

“I think you need to weigh the pros and cons of the two plans,” Jessie replies as she quickly works through the options in the notebook.

<i>Savings: \$400</i>
<i>Monthly income: \$214</i>
<i>Plan A: \$400 – \$350 up front = \$50 left in savings</i>
<i>\$214 – \$100/month = \$114 left/month</i>
<i>Plan B: \$400 – \$50 up front = \$350 left in savings</i>
<i>\$214 – \$150/month = \$64 left/month</i>

“With Plan A, my monthly cost would be lower, which means that I could still put money aside each month, but it will take some time to rebuild my savings! Plus, there are some other things I need to buy in the next few months, like new soccer cleats. And with Plan B, I’d still have a lot of money left over in my savings.”

“Yes, but think about how little you’d be able to save each month with those higher payments!” adds Jessie.

"Very true," Felipe says as he glances at his watch. His dad should be here any minute.

Alycia emerges from the storeroom with the new smartphone and case and asks, "Have you made a decision?"

"I believe we have," says Felipe.

Mr. Perez is surprised to see Felipe and Jessie standing in the store without a phone. "Oh

no, were they out of the smartphone you wanted?" he asks. "I'm so sorry. I know how excited you were to make such a big purchase!"

"They had the phone I wanted," Felipe explains, "but I decided to wait a few months to save a little more money before I commit to a smartphone plan.

"And, fortunately, WeConnectU has special pricing on plans through the end of the year!"

Mr. Perez smiles at the teens. Felipe was right; he really was responsible enough for a smartphone after all.



Main Idea

Using a graph of multiple equations can help you make decisions.

Math on the Big Screen

7

Hannah stares at the massive green screen in front of her, her notebook clutched tightly in her hands. Today is the first day of her internship at Stellar Studios, and she can barely contain her excitement.

"Welcome to the magic of filmmaking!" announces Jake, the visual effects supervisor, to Hannah and the other new interns. "Before we dive into the fun stuff, I want to share the most important tool in filmmaking."

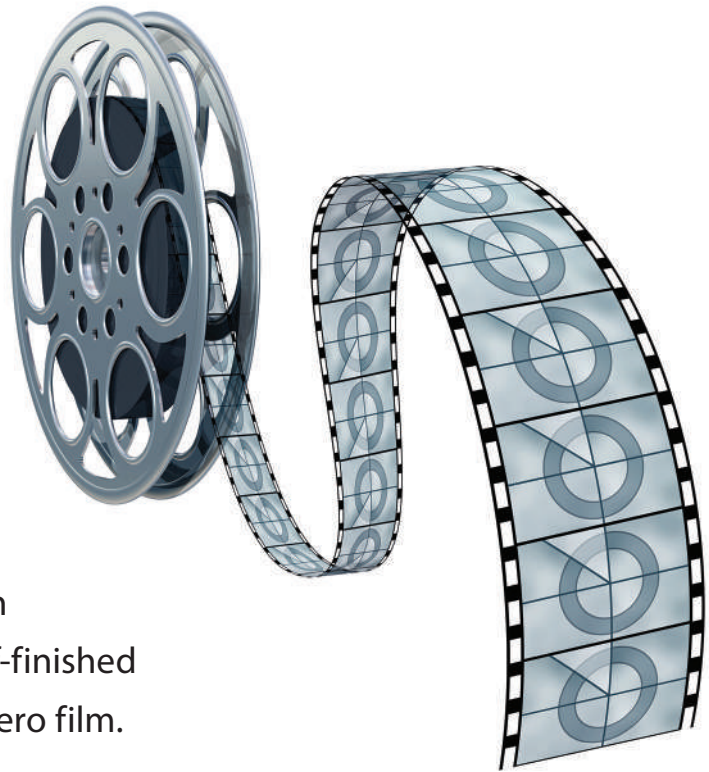
Jake grins and exclaims, "Math! Every single thing you see on screen involves math in some way."

Jake leads the group to a workstation where several monitors display a half-finished scene from the studio's latest superhero film.

"See that character flying through the air?" Jake points to a figure soaring between skyscrapers. "We use math to figure out how fast he should move and where he should be in each frame."

Hannah knows that any movie is really a series of still images, referred to as frames. In old cartoons, the artists had to draw each frame! Now, computers help with transitioning images from one frame to another.

"Your first assignment is to work with different departments and see how they use math. Hannah, you're starting with the camera crew," Jake announces.



Hannah follows cinematographer Elena to the main sound stage, where an elaborate medieval castle set towers above them. The director shows actors how they should move and turn as they say their lines during the scene.



"People think we just point the cameras and shoot," Elena explains, adjusting a camera mounted on a mechanical arm. "But every shot requires a good understanding of math."

"Cinematography is all about understanding how camera angles, light, and the geometry of the frame affect the scene. I need to choose the correct angles for all the cameras to create the dramatic effects the director wants. I also need to have a complete understanding of camera lenses and light to choose the correct ones for each shot for all the different cameras."

Hannah nods as Elena continues.

"Let's consider some geometry and lighting for this shot," Elena gestures to the actors walking up and down the stairs and points to the camera. "Look through the camera." Hannah looks through the camera to see that the actors are visible from the back, directly in the middle of the stairs. "Now look through the camera at this angle," Elena says as she swings the camera so that it shows the actors from the side. This catches the expressions on their faces when they turn their heads. She continues, Do you see how the angle of the camera changes the viewpoint?"

Hannah's next stop is to shadow lighting designer Lin Chow. Since her conversation with Elena, Hannah has been thinking about how making a change to the lights in a scene can affect the mood, but she's not sure how math is involved.

Lin takes Hannah to a set on a smaller soundstage where the cast and crew are filming a nighttime scene. Lin points out the positions of some of the different light setups.

"When the director wants to achieve a certain effect, it's up to us to make sure we know exactly where to put the lights to make that happen. One of the most

important calculations lighting designers make is something called the throw of the light. The throw of a light has to do with how far the light can travel from the bulb and how bright it is when it gets there. We calculate how intense the lights will be similar to the equation $I = \frac{P}{A}$, in which I is intensity of the light, P is power of the light source, and A is area that the light illuminates. The effects of the lights need to look real in the context of the situation. So calculating the throw of each light and putting it in exactly the right place is the only way to do that."

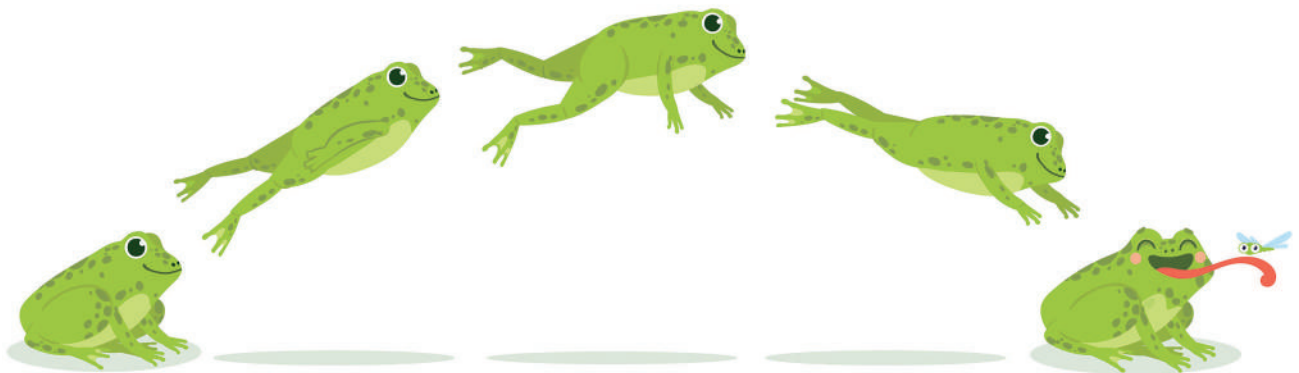
Hannah can see how using more or fewer lights and changing the position of the lights would alter the scene.



Hannah moves to the animation department, where she meets Carlos, the lead animator. His workspace has whiteboards covered with both detailed drawings and smaller sketches.

"Animation is math," Carlos explains. He shows Hannah a scene in which a cartoon dragon breathes fire. "Every second of animation involves many different kinds of math. We use geometry to understand the shape of the dragon and the environment in the scene. We also use complex calculations to determine how the fire should come out of its mouth and move around the screen to look real.

"Now we use computers to do animation," Carlos continues, "but before computers, each frame, or each picture, had to be drawn by hand!" He shows Hannah how movies work by having her flip through an old stack of drawings. As she flips the drawings, she sees that they show the sequence of a frog jumping and catching a fly.



"Movies project 24 frames per second, or fps. We can use a simple equation to calculate the fps: frame rate equals total frames divided by time in seconds. Let's say this cartoon was going to be drawn by hand. I want the dragon to breathe fire for 3 seconds. At 24 frames per second, I would need to draw 72 different pictures! That's a lot of pictures. Cartoon animators will often hold a drawing longer than 1 frame to reduce the number of unique drawings they have to draw. So even though the cartoon is set to 24 fps, animators only need to draw 8 pictures and hold each drawing for 3 frames or draw 12 pictures and hold each drawing for 2 frames."

Hannah has never thought about how math could be used in animation! She thinks about how realistic some cartoons look.

“Carlos,” Hannah asks, “how do the animators make everything look so real?”



“Well,” Carlos says as he points back to the dragon animation on the screen.

“There are lots of details that go into each image that make up an animation. We are lucky enough that computers do most of the math for us, but it’s important to understand what they are doing and why. We use coordinate planes to track the movement of characters in a space. We use math, science, and computer code to create animations that look real and believable. Sometimes this means understanding how gravity works when a character runs, jumps, flies, or throws something.”

Carlos continues, “For example, we use equations and computer simulations to understand how high and far a character can jump to make sure it’s believable. If it isn’t believable, then the audience won’t like it. Complex equations also help us understand how different things move in the real world, such as how the dragon’s fire might be affected by wind or how a fish would move underwater.”

Hannah’s next stop is the set building department, where she meets Maria Santos, the carpenter on the project.

“To build all the sets for each movie, we need to use math,” Maria says.

She leads Hannah to where a team is constructing a set of stairs that will be used for an inside scene in the movie with the castle that Hannah saw being filmed earlier.

Maria explains that the set had stairs but that the director was concerned about safety. “Each step was 7.5 inches high, and it was causing some accidents during filming. So we’re rebuilding the stairs to make each step a bit lower. This means we’ll need to have more stairs, and they’ll have to come out a bit farther into the room.”

“How do you know how many stairs to make?”

“Great question!” Maria says. She writes out the calculations so Hannah can see.

“Well, this staircase must reach a height of 15 feet. We had 24 steps before. We’re going to lower the step height to 6 inches. First, we convert 15 feet to inches, since

$$15 \times 12 = 180$$

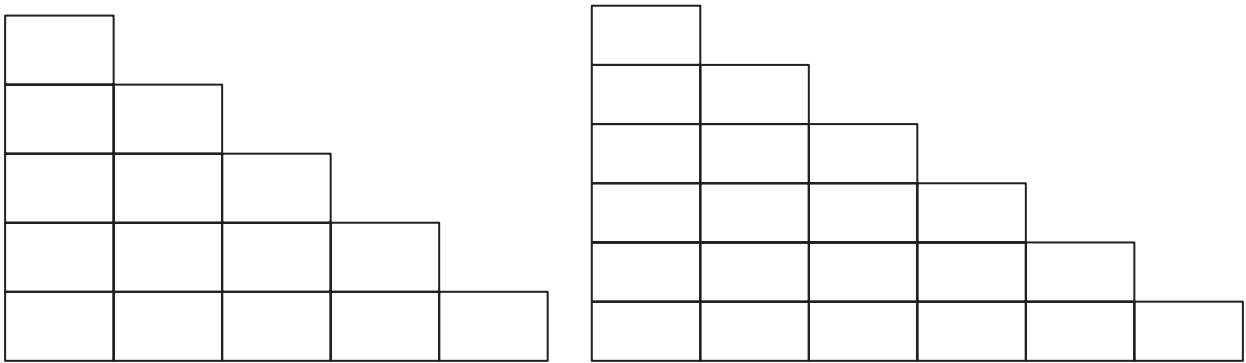
The total rise of the stairs is 180 inches.

$$180 \div 6 = 30$$

Divide the rise of the stairs by the height of each step. So we need 30 steps.

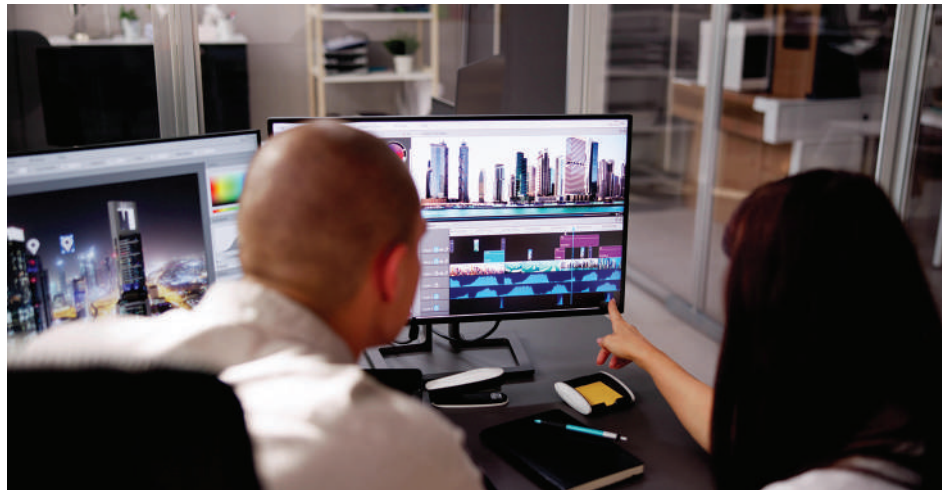
the height of our stairs is in inches. It’s important to use the same unit. Then we can divide the total rise of the stairs by the height we want each step to be. So we’ll need 30 steps this time.

“You can see in this model how making the height of each step just a little bit lower makes a difference!”



Hannah’s final department visit takes her to the editing room, where editor Lisa Thompson works with multiple monitors showing strips of video footage and sound clips.

“Most finished films are about 2 hours long,” Lisa explains. “Sometimes we see some movies that are longer or shorter, but mostly they are about that long. During filming, a lot more video footage



is created than will ever fit in a film. Depending on lots of things, the ratio of how much extra footage there is to the length of the final movie might be different. Sometimes it’s 3:1, meaning for every 1 hour of the final movie, there are 3 hours of extra footage. Sometimes it’s 10:1, and sometimes it’s even more!”

Hannah is shocked. “So that means there could be 20 hours of footage that has to be cut for a 2-hour movie! How do you decide what gets to stay in the movie and what gets cut?”

"Well," Lisa says, "there are lots of decisions and lots of calculations involved. We know we have a certain amount of time to tell a story, and we have to think carefully about how much time we spend on each scene, each moment, and each part of the story. We must choose which shots to include carefully."

"How many shots make up a movie?" Hannah asks.

"That's changed over the years," says Lisa. "In the early days of filmmaking, each shot was about 12 seconds. Now each shot is about 2.5 seconds."

Hannah thinks for a minute. "Wow! A 2-hour movie has 120 minutes, or 7,200 seconds. If each shot is 2.5 seconds, then that's about 2,880 different shots! That's a lot of careful editing!"

As Hannah's first day of her internship comes to an end, Jake gathers all the interns in a conference room.

"So what did you discover about math and movies?" Jake asks the group.

Hannah raises her hand enthusiastically.

"Math is everywhere! Even in places I didn't expect!"

"Exactly!" Jake beams. "Filmmaking is art. But to achieve any vision—whether it's cinematography, lighting, animation, set design, editing, or any other department—everyone uses math to bring their vision to reality. You will learn many technical skills this summer, but think about math as being the most important item in your toolbox."



Main Idea

Solving linear equations helps us figure out answers when things depend on each other.

Ready for Liftoff

8

I have been flying jets for a long time, but standing at the entrance to the space center feels completely different. The facility stretches out before me. I watch the engineers and scientists buzzing around doing their jobs. After years of dreaming about this moment, it is finally the day before launch. The mission objectives are clear: dock with the base camp, conduct several spacewalks to make some repairs to the base camp, and return safely to Earth after two weeks.

My name is Abigail Park. I was born in Seoul, South Korea, and immigrated to the United States when I was 7 years old. My dream was to become a pilot. Now, in 2041, I am about to take my first flight as an astronaut.

During my childhood, looking up at the stars always sparked a sense of wonder in me. I have always had the urge to explore the unknown. At a young age, I learned all the constellations. I often dreamed about what it would be like to experience seeing Earth from space. As a member of the next class of astronauts, I am so excited that I will get that chance.



I have worked hard to prepare for this. The physical preparation involved grueling workouts that build muscle and stamina. I have pushed my body to new limits. Every morning began with endurance tests, followed by time in the centrifuge that spun me around until the g-forces made my stomach lurch. Then I spent hours in massive water tanks. The water tanks simulate the weightlessness of space. In them, I practiced the movements that I would make in space. My muscles have learned to adapt to an entirely new way of moving. This all prepares me for the extreme forces of space launch and the environment of zero gravity.

The technical training is just as challenging as the physical training. I have spent hours learning the spacecraft's intricate systems, thousands of switches, buttons, and displays that control everything from life support to navigation. These systems are all automated, but we astronauts still need to be able to solve problems in case systems fail.

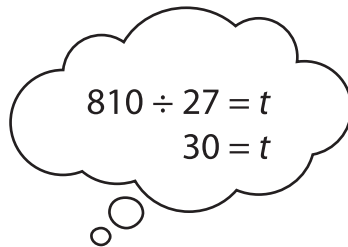
"Remember: In space, there's no up or down," my instructor reminded me once as I struggled to orient myself during a simulated equipment repair. "Your brain will try to find the ground. Trust your training."

I have memorized emergency protocols for every kind of emergency. Virtual reality simulations provide realistic scenarios. I use these simulations to make sure that I am prepared for all the things I will need to do in the spacecraft. For example, these simulations allow me to practice maneuvers like docking with the base camp.

During one particularly intense simulation, mission control radioed: "Abigail, you need to begin slowing down before you dock. You're traveling at 27 meters per second, and you're currently 810 meters away."

I quickly wondered how much time that meant I had. My mind immediately went to work. I knew I could use a formula that relates distance, rate, and time: $d = r \times t$. To calculate distance, I multiply my rate, or speed, by time.

Since I needed to know the time, I knew I could plug in the values for the rate and the distance and divide to get the time because distance divided by rate equals time. I did some quick calculations in my head.


$$\begin{aligned} 810 \div 27 &= t \\ 30 &= t \end{aligned}$$

Because the rate had units of meters per second and the distance was in meters, I knew that the time was in seconds. So at my current rate, I would only have 30 seconds before reaching the dock! Time to slow down!

Mastering equations like this isn't just academic—it is essential for mission safety. As a pilot, it's my job to make sure the spacecraft stays safe during docking or if there are emergencies. The ability to solve linear equations could mean the difference between a successful docking and a dangerous collision.

In addition to the physical and technical training sessions, I had to learn to work in a team with my crewmates. Astronauts all come from different backgrounds, and everyone has specific jobs on the mission. Our training includes many kinds of team-building exercises and communication drills. This training helps all of us work through various situations and take quick action. We must feel comfortable with each other, trust each other, and rely on each other.

Finally, the day is here. The initial anxiety has given way to confidence this morning. I woke up last night in a cold sweat, wondering if I am ready for this. But I know I am. I have prepared my whole life for this moment.

“The weather looks good,” announced the flight director. “All systems are ready to go for launch.”

My crewmates and I give the flight director a thumbs-up.

My family flew in from California. They are excited but are trying to hide their worry. My brother Jake asks technical questions about the rockets. My parents look proud but are very quiet. They know how hard I have prepared for this journey. I feel honored that they recognize my work and support my dream. When I say goodbye to my family, the goodbyes are brief but emotional.

It is time for the ritual of suiting up. Each layer of the space suit feels like a second skin to me. It is a complex system of life-support devices. The suit is designed to regulate temperature, as well as to protect me from cabin decompression and fire. I know that these space suits are much easier to wear than the space suits the first astronauts wore. Changes in technology have made it possible for these suits to be light yet efficient. This amazing suit will keep me alive in space.

I use this quiet time to contemplate. All the physical and technical work I have done prepares me for this moment. I think of those astronauts who went before me. How they must have felt to be first! I stand on their brave shoulders. I hope they somehow can see me now and know how much I appreciate them and their accomplishments.

Finally, we ride to the launchpad. I can't believe that in just a few moments, I will be in space! It is so surreal. I look up at the massive spacecraft. The view is a reminder of the massive forces that are soon to be unleashed! I feel so small, yet I know that my crewmates and I are ready to take control of this massive ship.

Inside the spacecraft, I am strapped to my seat next to my fellow astronauts. I am surrounded by an array of controls, lights, and switches. The air is thick with anticipation. I focus on deep, steady breaths.

"T-minus 10 minutes . . . "

Each passing minute seems like forever and like a blink of the eye at the same time. The final system checks are completed. Green lights flash across the control panels.

"T-minus 5 minutes . . . "

I exchange a few last words with ground control. I confirm my readiness one last time. Then a profound silence descends over the crew.

"T-minus 1 minute . . . "

The anticipation is palpable. Adrenaline rushes through my veins.

"T-minus 10 seconds . . . "

My hands grip the armrests, and my heart races. Mission control counts out the final seconds—nine, eight, seven, six, five, four, three, two, one.

Liftoff!

A deep, guttural rumble shakes the entire spacecraft. Every bone in my body vibrates. The engines ignite with a blinding flash. Suddenly, I feel the thrust from the engines pushing against the launchpad with incredible force. The sound gets louder until it grows into a deafening roar.

The force of acceleration presses me deep into the seat. Through the small window, I can tell that the spacecraft is charging through the atmosphere. The sky slowly changes from blue to purple. Soon it will change to black, and we will be in space!

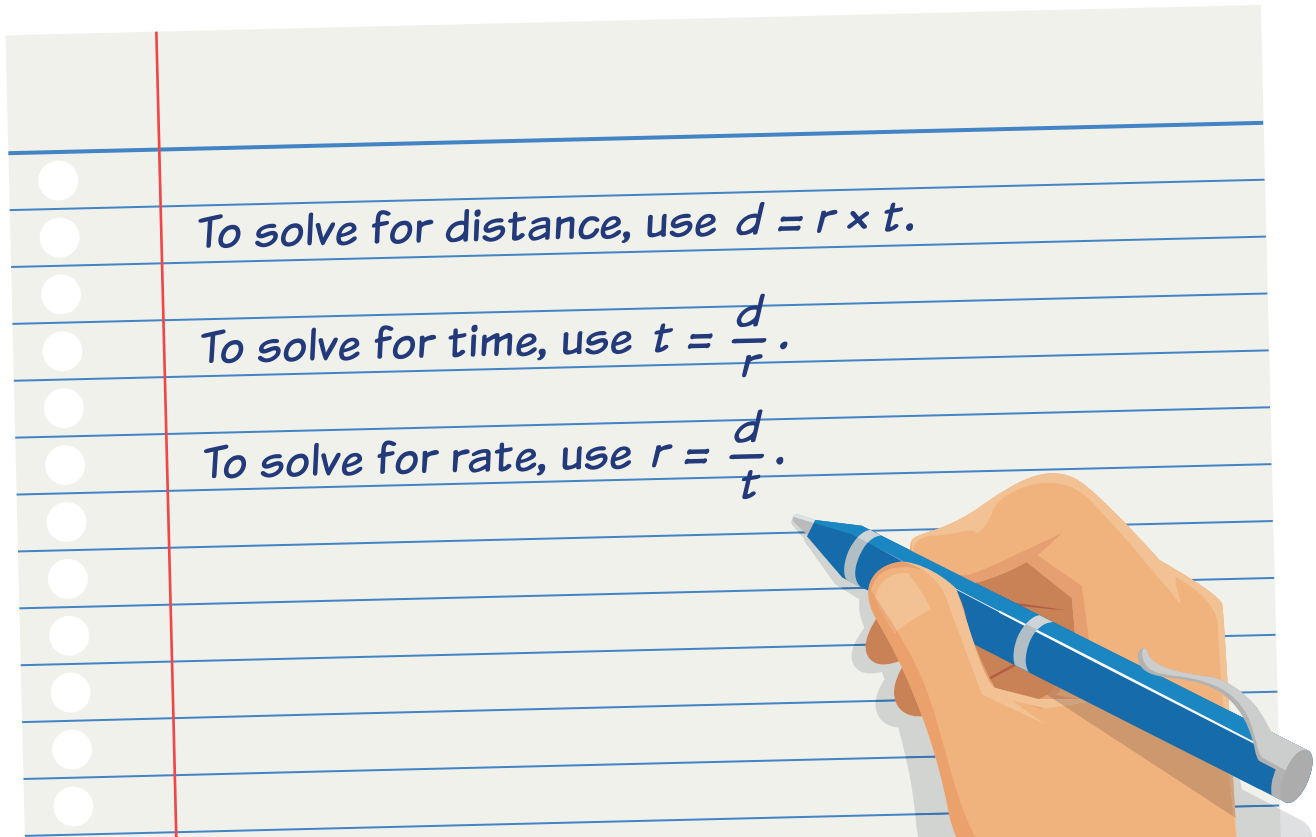
Earth shrinks away. Finally, it is black outside the window, and I can look back at our planet. It looks like a vibrant blue marble receding into the vast darkness. I feel an overwhelming wave of awe and pride. The dream is real. When the spacecraft reaches a certain altitude, it begins its orbit around Earth. I watch as the view transforms into a breathtaking expanse of space.



The next critical phase in our journey will be in two days when we dock with the base camp. This isn't a simple parking job. The maneuvers involved require incredible precision and timing.

In space, everything is always moving, even the base camp! That makes calculating the path of the spacecraft from Earth to base camp very important and very difficult! How do mission control and the astronauts ensure two objects moving at thousands of miles per hour meet perfectly in the vacuum of space? To calculate the exact time and place the spacecraft will reach base camp, engineers use complex equations.

On Earth, the measures of speed or rate (r), distance (d), and time (t) are all related. The relationship can be shown in three ways, depending on which variable is unknown.



Engineers use equations that are more complicated than these to determine the exact intersection point of the spacecraft's path and the base camp orbit. Multiple equations work together to pinpoint exactly where and when the two paths will meet. This is simple to visualize when things move in straight lines, but in space, anything in orbit, like the base camp, moves in a circular path! That makes the math very difficult! This is an incredible example of how mathematics can be used to model situations and ensure precision in complex real-world scenarios.

Two days later, I find myself at the controls during the final approach to the base camp. The massive structure grows larger in the windscreen, looking like something from a science fiction movie.

“Station Control, this is Starflight,” I say into my headset. “Beginning final approach sequence.”

“Copy, Starflight. You’re looking good from here.”

We determine the exact point where the spacecraft’s trajectory will intersect with the base camp’s orbital path. As the two spacecraft slowly come together, I hold my breath. Docking is one of the most dangerous parts of the mission. I trust my training and put all my focus to the task at hand. There is a solid but gentle *thunk*, and the spacecraft is docked at base camp.

Cheers erupt from both the spacecraft and the base camp. I feel tears floating away from my eyes in zero gravity—tears of relief, joy, and accomplishment.

Inside base camp, we float in zero gravity. It feels just like the water tanks I trained in on Earth. We are ready to eat, drink, sleep, and work with no gravity!

There are so many people who work to make a mission to space possible. There are those at mission control and so many scientists, mathematicians, and engineers who have devoted many years to planning and calculating. They have tested their equations using complex computer modeling, all to make this moment possible. The astronauts may be the face of a mission to space, but it’s mathematics that makes it all possible.

Main Idea

Linear equations let us turn real-world problems into math we can solve step-by-step.



When Amara moved to the United States five years ago, she was only one of many people moving to a new place.

“My family moved here from Nigeria,” she explains to her classmates during

their migration lesson. “People move to new countries every year. My family was part of the migration that year.”

“It must be difficult to move to another place,” says Dylan. “Did you need to learn a new language? It must be so different here in Atlanta than in Nigeria.”

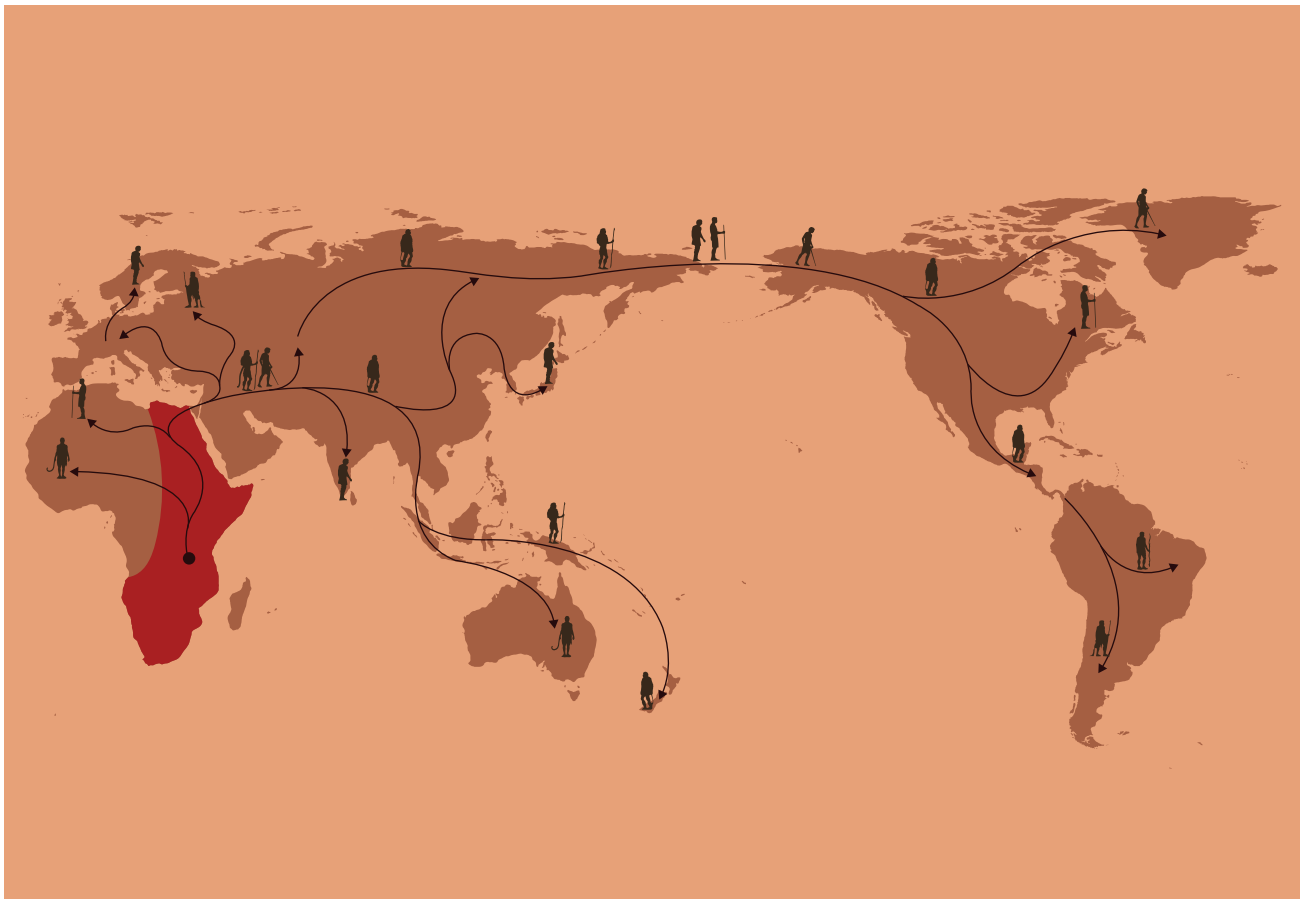
“It is very different,” answers Amara. “I had to adjust to how people speak, what they eat, and what they wear. The weather is different, too! And there are some cultural differences that I have had to get used to. Where I lived before, people were like me.”

Tara was quietly listening to the conversation. Then, she shyly says, “I didn’t move from Nigeria, but I did move from Maine to Georgia. It was hard, too. Even though I moved within the United States, I had to leave behind all my friends. Fitting into a new place can be challenging.”

"Marcus just moved across town, but he had to change schools," states Dylan. "He can't even play on the same baseball team now!" Dylan shakes his head. "I don't think I could do it!"

"I have been so lucky," Amara says. Many people here helped my family adjust. Making new friends can be difficult. But I got involved in many clubs and activities, and that helped a lot!"

Mr. Jones suddenly speaks up behind them, continuing the lesson on migration. "Humans have been on the move since our earliest days. Sometimes they moved close to where they originally lived, and sometimes they moved far away. The same thing happens today. In fact, with airplanes, ships, and trains, people can move farther faster. Up until a few hundred years ago, people had to walk with all their belongings when they moved from one area to another."



Migration Through History

Scientists think the first humans lived in Africa. Small groups began migrating, or moving, out of Africa around 70,000 to 100,000 years ago. They may have been looking for food or better places to live.

These early people migrated into Europe and Asia. Later, about 50,000 years ago, they traveled by boat to Australia. Much later, during the Ice Age, people crossed a land bridge from Asia to North America (where Alaska is today) and gradually populated the Americas. In the late 1400s, Europeans arrived in the Americas and met those people who had come during the Ice Age. These were now the Native peoples who had lived here for many thousands of years.



Throughout history, migration has meant change for both the regions that people move to and the regions they leave. People moving to a new area are called immigrants. But these same people are called emigrants in the places they left. Immigrants move to a place, and emigrants move from a place. But they are all migrants.

The Great Atlantic Migration in the 1800s and early 1900s brought millions of Europeans to the Americas. They came in search of better lives. People from Asia came to the United States for new opportunities.

Tracking Population Changes

People are still moving around today. They can move to a different town, city, state, or country.



It is important to know how a population is changing in a certain location. Knowing how the population changes each year helps communities watch trends in populations and manage resources and infrastructure. If more people are moving to an area, there must be an increase in things like housing, roads, power lines, school buildings, public transportation, and hospitals. When people leave an area, there needs to be fewer of these things. When a lot of people leave an area, it can hurt the local economy. Businesses may struggle to survive with fewer people.

Mathematicians have tools to track the movement of people. They're called migration functions. Functions show patterns and the relationship between variables. The net migration rate is a function that shows how migration changes the number of people in a certain place. Some places may lose people, and other places might gain people.

$$\text{Net Migration Rate} = (\text{Immigrants} - \text{Emigrants}) \div \text{Total Population} \times 1,000$$

For this function:

Immigrants: These are people who move INTO your area.

Emigrants: These are people who move OUT OF your area.

Total Population: This is how many people live in the area you're studying. This is usually measured midyear, around July.

Step 1. Count how many people moved in (immigrants).

Step 2. Count how many people moved out (emigrants).

Step 3. Subtract the number who left from the number that came in.

Step 4. Divide that number by the total number of people in the designated area.

Step 5. Multiply by 1,000 to set the number per 1,000 people.

To find the net migration rate, find the number of immigrants, and subtract the number of emigrants. For example, think of a room with 10 people in it. If 6 people enter the room and 3 people leave, the number of people in the room increases by 3.

The net migration rate identifies how many immigrants there are per 1,000 people. Some cities have a population of a million people, and some only have a population of 100. By looking at the number of immigrants per 1,000 people, it is easy to compare the populations of two different-sized places.

When the net migration rate is positive, more people are moving in than out of an area.

For example, if your town has a population of 10,000 people and, this year, 100 people move into your town while 40 people move out, what is the net migration rate for your town?

Let's use the function. The input is the city's population of 10,000. There are 100 immigrants and 40 emigrants.

$$\text{Net Migration Rate} = (\text{Immigrants} - \text{Emigrants}) \div \text{Total Population} \times 1,000$$

$$\text{Net Migration Rate} = (100 - 40) \div 10,000 \times 1,000$$

$$= (60) \div 10,000 \times 1,000$$

$$= 6$$

This means, for every 1,000 people, 6 people are joining the town.

When the net migration rate is negative, more people are leaving the area than moving into it.

For example, if a city has 100,000 people and 1,000 new people move in while 2,000 move away this year, what is the net migration rate?

$$\text{Net Migration Rate} = (\text{Immigrants} - \text{Emigrants}) \div \text{Total Population} \times 1,000$$

$$\text{Net Migration Rate} = (1,000 - 2,000) \div 100,000 \times 1,000$$

$$= (-1,000) \div 100,000 \times 1,000$$

$$= -10$$

This means there are 10 people leaving the city for every 1,000 people. A negative number means that the population is declining.

"Can we use functions to predict who might migrate to our city in the future?" Lily eagerly asks as Mr. Jones finishes explaining migration.

"Great question," Mr. Jones replies. "Demographers are scientists who study population changes. They create functions that look at many reasons people move. They look at economic opportunities, housing costs, climate, political stability, and even cultural connections."

"My parents chose this city because there was already a Nigerian community here," Amara notes. "That made it easier to adjust to a new country and culture."

"Social connections definitely play a big role in where people move," Mr. Jones agrees. "Once some people establish themselves in a new place, friends and family often follow. Mathematicians call this 'chain migration' and include it in their models."

Devon is intrigued. "So functions help us understand not just how many people move but why they choose certain destinations?" he asks.

"Absolutely," Mr. Jones replies. "The beauty of math is that it helps us look at human behaviors and find patterns in them."



As the class wraps up the migration discussion, Mr. Jones emphasizes how math helps people understand human movement patterns. He stresses that functions provide a powerful way to analyze migration data, track population changes, and even predict future trends.

“When we understand these patterns, we can better prepare for changes in our communities and world,” he says confidently.

“I never realized math could help us understand why people move from place to place,” Amara reflects.



“That’s the beauty of mathematics,” Mr. Jones replies. “It gives us tools to make sense of our world.”

“And it could even help us solve problems,” adds Devon. “If we understand migration patterns better, maybe we can create more welcoming communities.”

Mr. Jones nods proudly at his students. “That’s exactly right. Math isn’t just about numbers—it’s about using those numbers to improve lives.”

Main Idea

Functions help us understand cause and effect, make predictions, and solve real-world problems.

How Old Am I?

10

Imagine digging in your backyard and finding a bone buried deep in the dirt. How would you find out how old it is? There's no expiration date stamped on it like a carton of milk!

Scientists use a variety of methods to estimate the age of artifacts. Relative dating is a method of comparing the age of objects based on how deep they are found in the rock layers below Earth's surface. However, relative dating just tells you whether one artifact is older than another. It does not tell you the actual age of an artifact.



Fossil embedded in rock

In the 1940s, scientists developed a way to use carbon to determine the age of organic materials. Organic materials contain carbon. Carbon is found in anything that was once living. Organisms use carbon as a building block to make molecules that are needed to live. Carbon is found in DNA and proteins. Carbon is needed to build and repair tissues. Organisms use carbon to generate energy. But how does carbon get into living organisms?



All organisms have both carbon-12 and carbon-14 in their bodies. Carbon-12 and carbon-14 are found in carbon dioxide. Plants absorb carbon dioxide during photosynthesis to make sugar. These carbon atoms become part of the plant's tissue. When animals eat plants or eat animals that eat plants, the carbon enters their bodies, too. Thus, all living things have carbon inside them.

As long as an organism is alive, it keeps taking in carbon. But when it dies, it stops absorbing carbon because it is no longer eating or performing photosynthesis. Carbon-12 is stable, so it stays at the same quantity after the organism dies. But over time, carbon-14 breaks down, or decays. Scientists discovered that this decay happens at a steady rate.

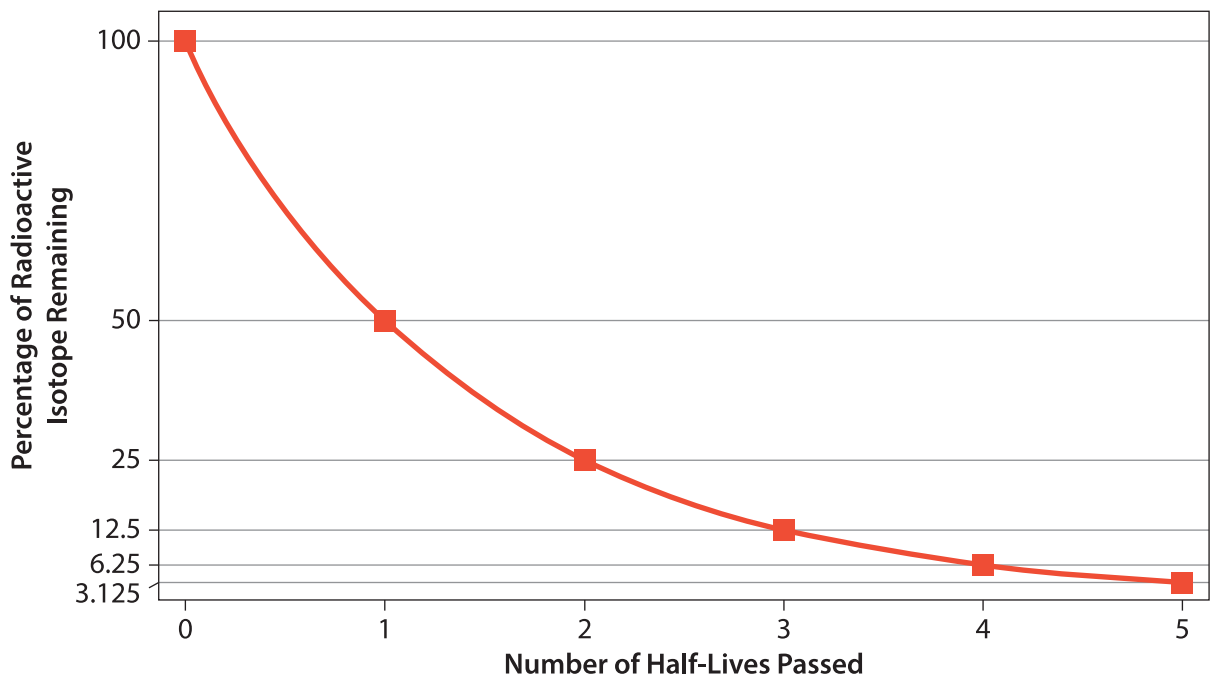
By looking at how much carbon-12 is in a sample, scientists know how much carbon-14 was in an organism when it died. They compare how much carbon-14 was in the organism when it died to how much is still present today. This allows them to estimate how long ago an organism died. The longer the organism has been dead, the less carbon-14 it contains. This process is called radiocarbon dating or carbon-14 dating.

Carbon-14 has a half-life of about 5,730 years. A half-life is the amount of time it takes for half of the carbon-14 in a sample to decay. This means that every 5,730 years, the amount of carbon-14 in a sample decreases by $\frac{1}{2}$. This steady reduction creates a predictable pattern.

- 1 half-life (5,730 years): $\left(\frac{1}{2}\right)^1 \cdot 100\% = 50\%$
- 2 half-lives (11,460 years): $\left(\frac{1}{2}\right)^2 \cdot 100\% = 25\%$
- 3 half-lives (17,190 years): $\left(\frac{1}{2}\right)^3 \cdot 100\% = 12.5\%$
- 4 half-lives (22,920 years): $\left(\frac{1}{2}\right)^4 \cdot 100\% = 6.25\%$

Scientists can use this pattern like a clock, counting backward from the amount of carbon-14 still in the object. The graph shows the pattern of decay for carbon-14 over time.

On the graph, the x-axis shows time in half-lives. The y-axis shows the amount of carbon-14 remaining in a sample. The graph shows that as time increases, the amount of carbon-14 remaining decreases.



Scientists use a function to perform radiocarbon dating. A function is a math rule that assigns each input to exactly one output.

	<i>Input: Time (years since the organism died)</i>
	<i>Output: Amount of carbon-14</i>

Scientists write this function as $f(t)$, where t is time in years, and $f(t)$ is the number of units of carbon-14 remaining. For example, if a sample begins with 16 units of carbon-14, the function would be:

- $f(0) = 16$ (units of carbon-14 present when the organism dies)
- $f(5,730) = 8$ (units of carbon-14 remaining after 1 half-life)
- $f(11,460) = 4$ (units of carbon-14 remaining after 2 half-lives)
- $f(17,190) = 2$ (units of carbon-14 remaining after 3 half-lives)

After each half-life of 5,730 years, the units of carbon remaining are divided in half. This mathematical relationship is incredibly powerful. Let's say scientists measure a sample and find it has only 25%, or $\frac{1}{4}$, of its original amount of carbon-14. They can confidently say it died about 11,460 years ago. No guessing needed!

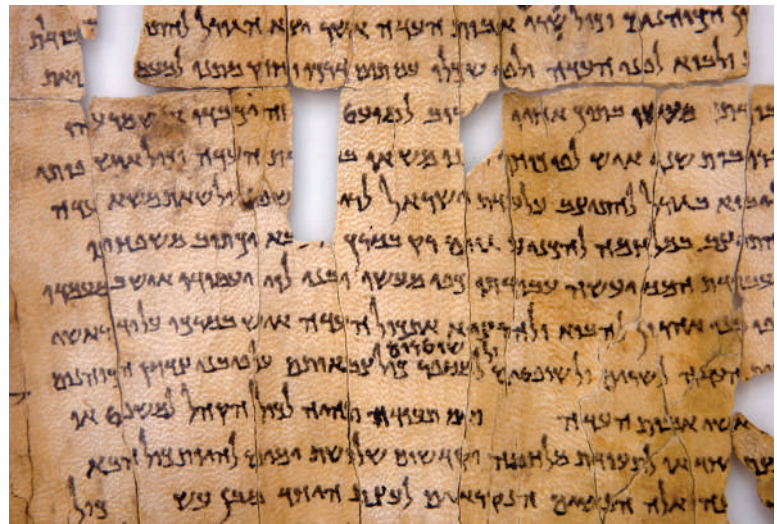


Preparing samples for carbon dating

Functions are more than just math tools. Functions are rules that help explain how the world works. If someone knows the input, they can always predict the output.

By using carbon-14 dating, scientists can learn about history. They can date objects to reconstruct timelines of ancient civilizations.

The Dead Sea Scrolls are ancient manuscripts. They were found in caves along the shore of the Dead Sea. They were found in



The Dead Sea Scrolls

thousands of fragments from an estimated 800 to 900 original individual scrolls. Some manuscripts were written on parchments made from animal skins. Other manuscripts were written on papyrus paper made from papyrus plants. Scientists could use radiocarbon dating because the scrolls contained parts of plants and animals that were once living. They determined that most of the materials used to make the scrolls died over 2,000 years ago, placing their origin in the time when ancient Jewish communities lived near the Dead Sea.

Many years ago, a volcano named Mount Vesuvius erupted and buried the ancient city of Pompeii. The ash and volcanic rock preserved household objects and food for centuries. Scientists tested a preserved loaf of bread. They determined that wheat used to make the bread was harvested almost 2,000 years ago. This confirms the historical written accounts that Mount Vesuvius erupted in 79 CE.



Bread from Pompeii

Ötzi the Iceman is the name given to a mummy found in the Alps by hikers in 1991. Ötzi was preserved and mummified by the ice on the mountain range. When archaeologists found the mummy, they wanted to know how long ago he had died. By using carbon-14 dating and calibrating the findings, they determined that Ötzi lived about 5,300 years ago, between 3370 and 3100 BCE.

The village of Jarmo is an important archaeological site in the foothills of the Zagros Mountains in northern Iraq. Archaeologists have found evidence that this is one of the earliest known farming settlements.

Artifacts such as bone fragments, charcoal, and pottery have been tested over the years. These tests, along with other archaeological dating methods, revealed that this site was occupied over 9,000 years ago, around 7000 BCE.



Ötzi the Iceman



Zagros Mountains

Carbon-14 dating is useful, but it has some limits. For example, it only works on material that came from living organisms. This includes artifacts made from wood, bones from animals, or fabric from cotton, wool, or silk. Also, objects older than 50,000–60,000 years have too little carbon-14 to measure. Fortunately, scientists can use other radioactive substances, such as potassium-40 or uranium-235, to date items that are billions of years old.

Another challenge is that the amount of carbon-14 in the air has changed slightly over time. To fix this, scientists calibrate, or adjust their findings, to account for this shift. One of the simplest ways to calibrate radiocarbon dating is to use tree rings.

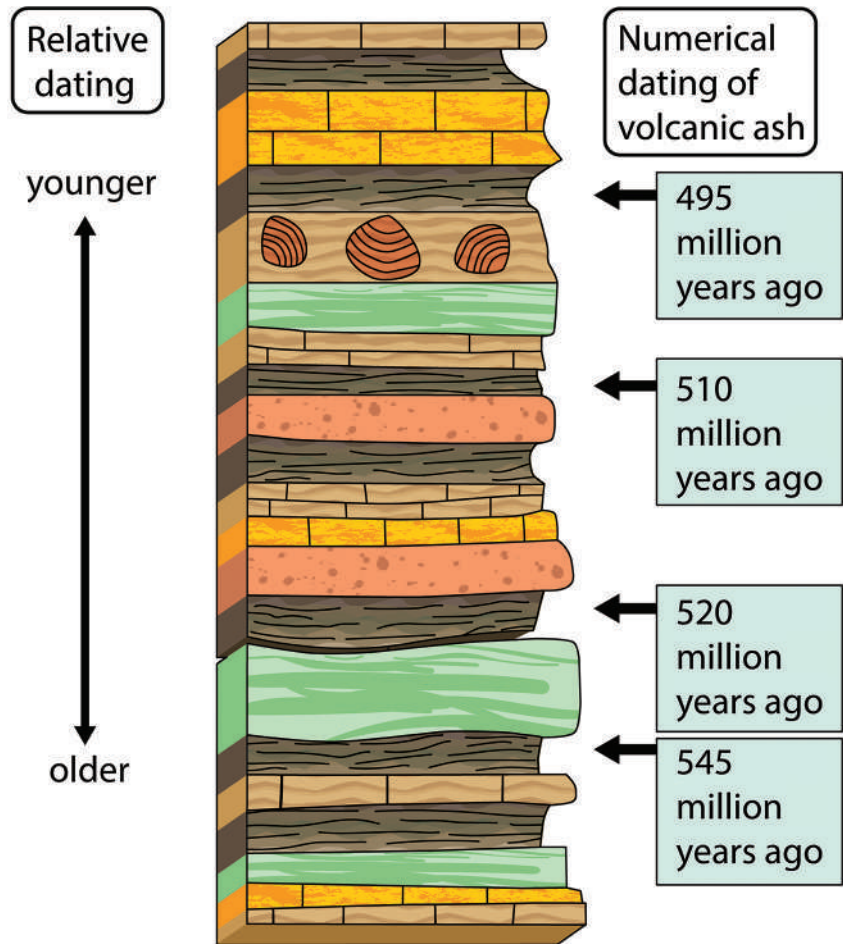
Inside a tree is a pattern of rings. For every year a tree grows, it forms a new ring. By counting these rings and measuring the carbon-14 in each ring, scientists build a calibration chart. This lets them adjust radiocarbon dates to match real calendar years. Tree-ring calibration makes it possible to be more exact when estimating dates of important artifacts.



Scientists can also use a combination of dating methods. For example, volcanic eruptions release hot ash into the atmosphere. The ash settles in layers on the ground, much like the rock layers used for relative dating. The volcanic ash contains potassium-40, a radioactive isotope that breaks down into a gas called argon. Scientists can

estimate the age of fossils found in ash layers using their relative location to each other and the amount of potassium and argon present in each layer.

Radiocarbon dating is just one way to show that math isn't just something you do in school—it's a powerful tool that helps us unlock the secrets of the past. The simple concept of a mathematical function, combined with the natural radioactive decay of carbon-14, has become an important dating method in archaeology and science.



Main Idea

A function gives one output for each input and can be graphed.

My First Paycheck

11

Hey, Celia! Great job this week. Don't forget your paycheck before you leave!

Of course, Marcus! How could I forget something as exciting as my first paycheck!



I worked 20 hours, so I should earn \$300! I'm planning to put the money toward a new paddleboard.

A paddleboard sounds like a great investment!

It's my favorite pastime, but I need \$900 for the one I want. I should have enough saved in a few weeks!

That sounds like a lot of fun!! Here's your paycheck.



Hey, are you OK? It looks like something is wrong

Our manager must have made an error. This paycheck is for \$203.84, not \$300!



Let me take a look. I don't think that's an error.

What do you mean?

Remember: You asked for a Super Scoops sweater in addition to your regular uniform.

But that's only \$25. What about the rest of the missing money?



The money isn't actually missing.

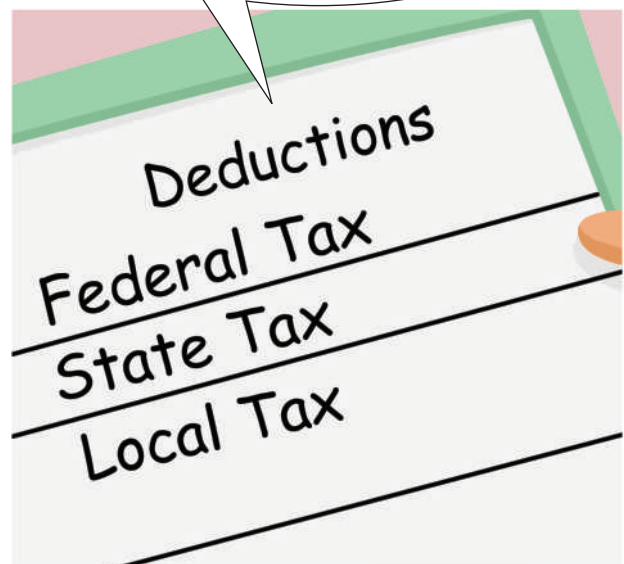
It certainly looks like it's missing to me!



See this list on your pay stub? This includes all the deductions that are taken from your paycheck.

Deductions?

Yeah, deductions. They're the different taxes you must pay on the money you earn. Plus any special fees, like your sweater, that are *deducted* from your paycheck.

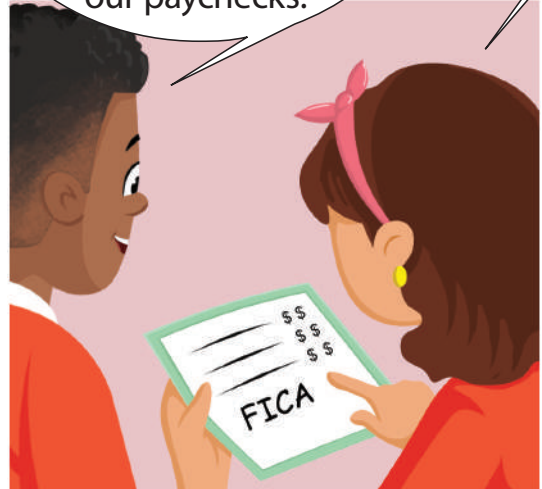


Super Scoops deducts money for federal, state, and local income taxes from each paycheck. That way, you don't have to pay it later.

Oh, I completely forgot about having to pay taxes! Getting all \$300 would be nice, but I guess it's easier this way than me figuring it out when my yearly taxes are due.

FICA is short for the Federal Insurance Contribution Act, which is a 15.3% tax on workers' earnings. But it's not all on you! Workers and employers actually split this tax—so only 7.65% is deducted from our paychecks.

But wait, what's this deduction here—FICA?



So \$300, the total amount you earned, is called your *gross* pay. The amount left after any deductions—\$203.84—is called your *net* pay or take-home pay. It would look like $\$300 - x = \203.84 , where x is the combination of those deductions we are talking about. But remember: This could change depending on how much you work each week.



The English word **gross** has its roots in the French word *gros*, meaning “large.” That can help you remember that your gross income is larger than your net pay.

I guess with all of these extra deductions, it's going to take me a little longer to save up for my new paddleboard!

Maybe, but don't forget about holiday pay; after all, we'll both be working over the Fourth of July!

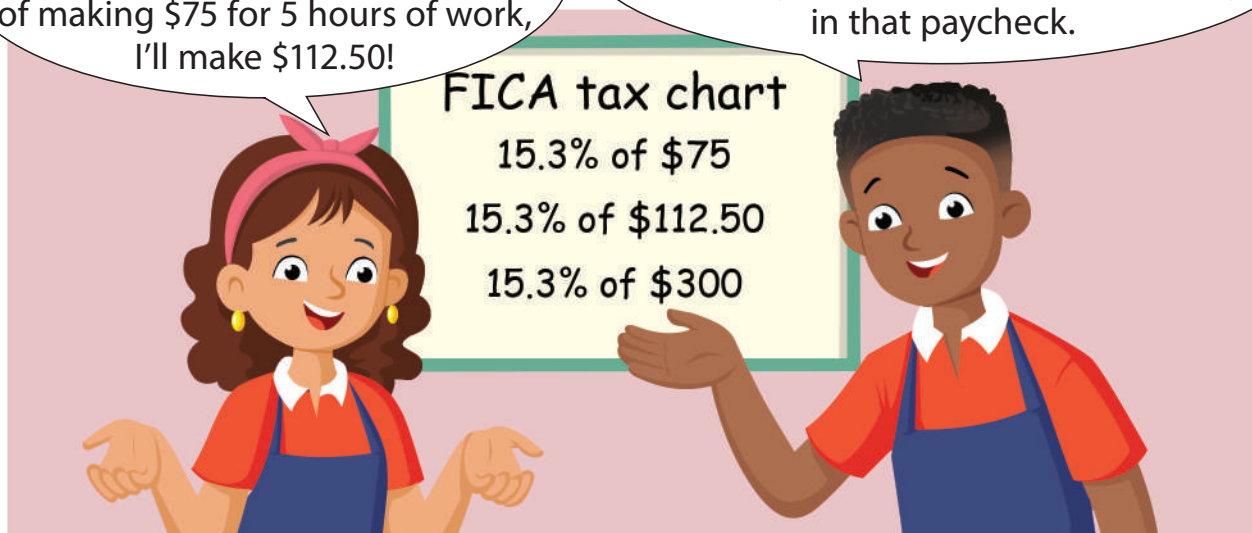
That's right. I am working this weekend!

Which means we'll be earning time and a half. That's 1.5 times your hourly pay rate for holiday pay.



That means I'll earn 1.5 times \$15, or $\$15 \times 1.5$. That equals \$22.50 per hour. So instead of making \$75 for 5 hours of work, I'll make \$112.50!

Right! But keep in mind that your deductions for taxes will be higher because you'll be making more money in that paycheck.



Gross income is one example of a function. The input is the number of hours worked at a given rate, and the output is the gross pay. This can be represented as an equation in which x represents the number of hours and y represents the gross pay amount: $y = 15x$

July and August are also our busy season. Would you like to earn some more money each week to get you to your goal faster?

Absolutely. How can I do that?



Do you think you could work another hour each day?

Sure! That would definitely help me get my paddleboard sooner.

If I work another hour each day, I will make an extra \$75 a week!

Don't forget about the deductions!



I know saving money is your goal this summer, and luckily, the company wants to encourage employees to do that, too.

How is it going to do that?



It has a savings plan where it will match you dollar for dollar for each dollar you put into a savings account up to 3% of your weekly pay.

That sounds like a great deal.

Do you think you'll join the plan?

3% of \$375 is \$11.25. I'd be saving \$22.50 each week. That adds up over time.



Did you know that Super Scoops also offers bonuses to its best scoopers. A bonus is a payment in addition to your hourly wages.

Yes! Sandi earned a bonus last month for scooping over 1,000 cones in one week. It's a new record for Super Scoops!

Super Scooper

In 2018, a man named Dimitri Panciera set the world record for the number of ice cream scoops balanced on a cone with 125 scoops!



Plus, if the company likes your work, you can get a raise! Last year I got 2 raises, so my wages went up 7%, from \$15 per hour to \$16.05 per hour.

That would bring my gross paycheck up to \$321. I wonder what my net would be.

I think we should probably have some ice cream before we work out that math!



Main Idea

A function makes it easier to determine outcomes in a clear and consistent way.

Transformations in Art

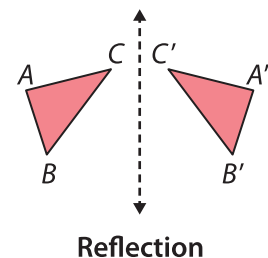
12

Artists have been using transformations as an art technique since prehistoric people made paintings and drawings in caves. Street art is a visual art form that is usually found outdoors. It includes graffiti, stencils, stickers, posters, and murals.

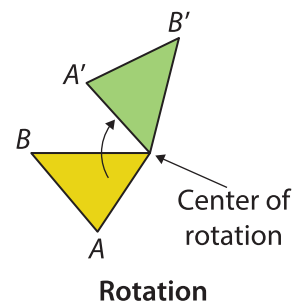
Some street art, such as graffiti, may not be wanted or even be legal in some locations. However, some cities will set up areas where graffiti is allowed. Many cities also fund community murals. These works of art often reflect the culture, stories, and voices of their communities. This art is meant for everyone to see.

Today, many street artists also use transformations in their work. In math, *transformation* means changing the appearance of a shape in different ways, including dilation, reflection, rotation, or translation.

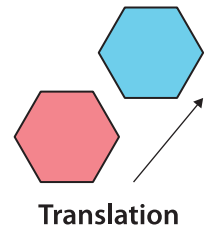
A **reflection** is a flip over a line of reflection. The image is the same shape as the original but a mirror image.



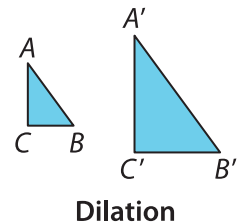
A **rotation** is a turn around a fixed point. The image is the same size and shape as the original.



A **translation** is a slide in a certain direction with no turns or flips. The image is the same size and shape as the original.



A **dilation** enlarges or reduces the size of a figure without changing its shape. The image is the same shape as the original but not the same size.



Reflections

Ardif is a well-known French street artist. He is based in Paris, but you can see his art in other parts of the world. Ardif uses pasteups which are paper-based drawings that he glues to the wall. His art often combines machinery and nature, such as his series of “mechanimals.”

This mechanical wolf is in Düsseldorf, Germany. Half of the animal’s face is on one side of the line. A reflection and symmetry are used to create the other half using mechanical images. This work symbolizes the balance of technology and nature as well as the sometimes wild and primitive nature of urban life.



Murals are common forms of street art. A mural is painted directly on a wall. It often takes up the space of the entire wall. One famous American street artist, Keith Haring, was known for his bright, cartoonlike figures and repeated patterns. Occasionally, Haring reflected characters



to create mirrored versions on either side of a central line. An example of this is his graffiti art of a red heart over two yellow humans in Tuscany, Italy. The symmetry and balance of the picture might symbolize harmony among people.

Rotations

Another of Haring's murals is in Pisa, Italy. Haring used rotation to add energy and create movement in the mural. In this mural, the blue figures appear to be rotated to face different directions. The two figures form a pair of scissors. They work together to cut the snake. The mural is meant to symbolize peace and solidarity.



American Gothic is a famous painting by Grant Wood. The original painting symbolizes the values of rural America and the spirit of the Midwest. Artists Steve Galgas and Mike Altman created a fun twist on the painting. They rotated the woman around the farmer to the other side, making her upside down. This playfulness is meant to represent the spirit of the Short North Arts District in Columbus, Ohio.



American Gothic



Translations

Aberdeen, Scotland, is famous for its street art. Every year, the city invites artists from all over the world to the Nuart Aberdeen festival to create street art throughout the city. These international artists bring a piece of their culture to the festival.

In this mural, Lebanese Egyptian artist Bahia Shehab uses an Arabic inscription inside a watermelon. She uses translations to create watermelon seeds. Then she translates the shape to create dots in the Arabic letters.



Portuguese artist Diogo Machado, who works under the name Add Fuel, uses tiling in the tradition of Portuguese azulejo ceramic tiles. This work combined patterns found on local tile floors. Several different images are repeated and translated to create the tile patterns.



Dilations

Street art often uses dilations to create objects that are larger or smaller than real life. Artists may even use projectors to scale photos up or down from their original size. Some artists take dilating objects to the extreme.

Pablo Delgado is a Mexican-born artist who is famous for his miniature



artwork on the walls of London's streets. He uses dilations to create very small figures. He is known for adding black shadows on the pavement. The shadows help to make his miniature work stand out so it is not missed as people walk by. The leaf on the sidewalk shows the scale of his drawings.

It is more common to find art done on a large scale, even covering entire walls. Scottish artist Kerry Wilson painted this gigantic 30-foot mural of a boy playing with a truck. The mural can stop traffic as passersby admire the details of the painting. The car to the left of the wall shows the scale of the mural.



Combining Transformations

Artists often use a combination of transformations for more effect. John Grider and Mike Fitzsimmons, known as Broken Crow, have achieved worldwide recognition for their use of large-scale stencils. Stencils are paper or cardboard cutouts used to create images quickly on walls. They used a combination of translations and dilations to create these cheetahs at the Cans Festival in London. Each cheetah is a smaller version of the cheetah to its left.



This public art display was completed by Okuda San Miguel on the Faro de Ajo lighthouse in Cantabria, Spain. Originally painted all white, this lighthouse was the first in Spain to be transformed into a work of art. The lighthouse stands 16 meters, or just over 50 feet, tall. San Miguel used a variety of transformations and colors to represent cultural diversity.

Toward the top, horizontal and vertical reflections create a kaleidoscope look. A series of horizontal translations of a triangle decorate the top rim of the bottom section. A pattern of black and white rectangles appears a little lower.

The middle of the tower displays a striped bird and other wild animals that are part of the animal life in Cantabria. The base of the lighthouse features numerous horizontal and vertical reflections of triangles.



This last piece of art utilizes all 4 geometric transformations. Sarah Morris, a London-born, New York-based artist, has been creating geometric works since the 1990s. Many of her works are in urban areas in major cities around the world, including Abu Dhabi, United Arab Emirates; Osaka, Japan; and Paris, France. In 2009, Morris completed *Hornet*, a mosaic in Paul Klee Square in Düsseldorf, Germany. The mosaic uses vibrant colors and square tiles to create an abstract image modeled on Japanese origami.



The next time you see art, look for translations, reflections, rotations, and dilations. Think about how the artist used transformations to achieve certain effects.

Main Idea

Changing a shape's position or size helps show its properties and relationships to other shapes.

Robin Hood and the Golden Arrow

13

Ellis leans forward in his seat as the class starts reading about the famous archery contest in *Robin Hood*. He's always loved action stories, but today he notices something new. "They're not just aiming," he whispers to himself. "They're calculating."

Ellis continues to silently read about Robin Hood and the archery contest.

Not far from the crowd, a cloaked figure watches the archery contest with interest. The air is tense with rivalry. Contestants are lined up, each standing behind a white line in the grass. Eighty yards away, brightly painted targets stand in the sun. The rules are clear: There is one arrow per archer, and the best archers in each round advance until the top shooter wins the prize.

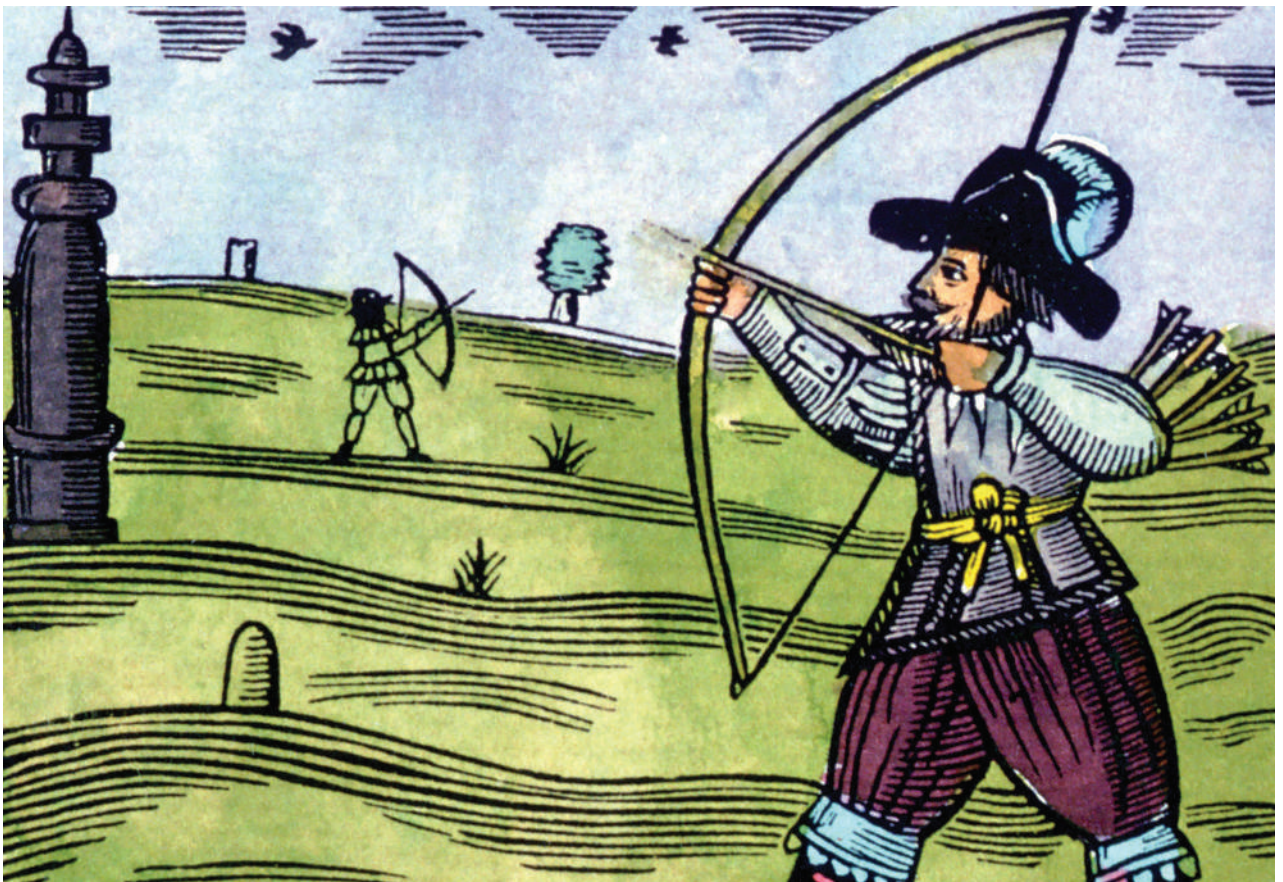
Robin Hood adjusts his ragged cloak. As he goes unrecognized, he blends into the crowd. Around him, villagers whisper excitedly about who might win the town sheriff's golden arrow. Children sit on their parents' shoulders for a better view, and merchants pause their sales to watch the contest unfold.



Each archer steps forward and lifts his bow. Hushed silence fills the air. They raise their arms, estimate the distance, and aim carefully. Some aim too low, and their arrows dig into the grass. Others overshoot. Hitting the target's center takes more than just strength; it requires understanding angles.

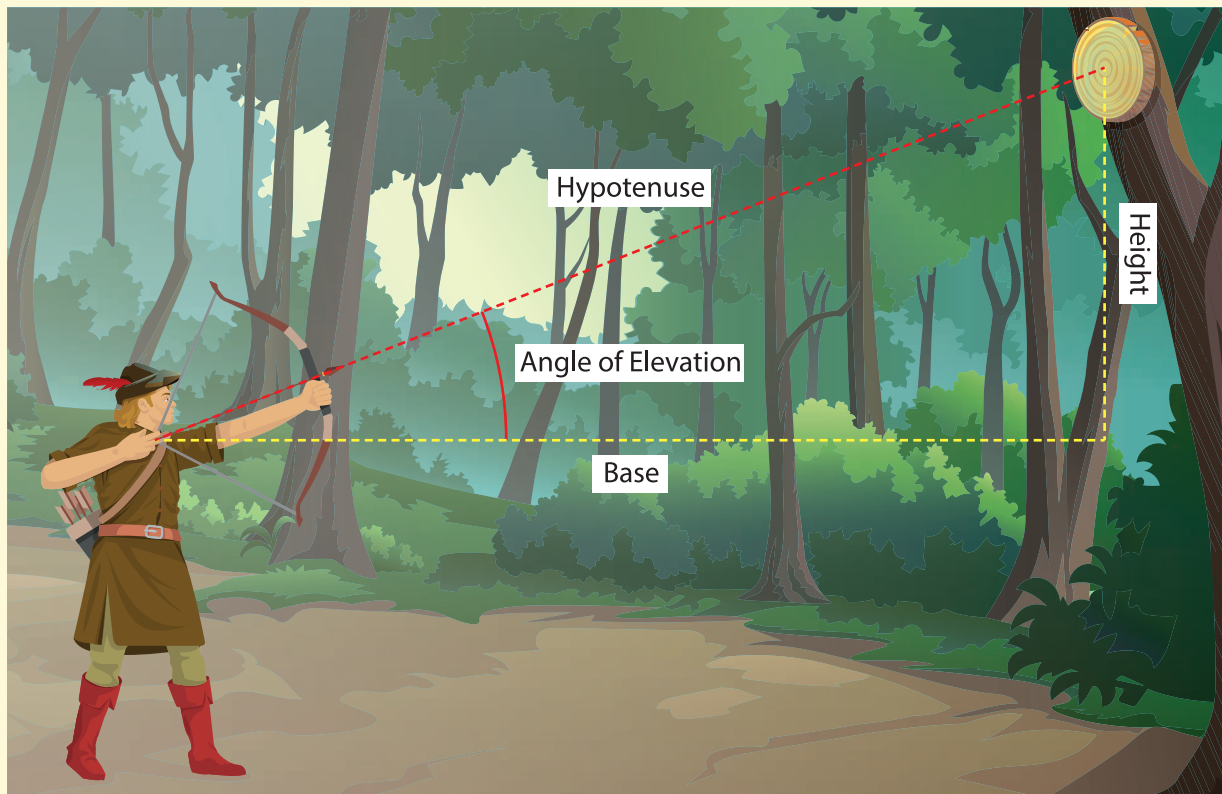
To hit the target's center, Robin Hood must consider the distance to the target and whether the target is higher or lower than or level with where he stands. He also must consider the pull of gravity on the arrow as it moves toward the target. The aiming angle, or how high or low the archer tilts the bow, changes the arrow's path. If the angle is too high or too low, the arrow will miss the target. If he gets any of this wrong, even a strong shot can fail.

Ellis thinks, "Math is used for every calculation of range and height! The angle you use controls how far and how high an arrow flies."



Ellis continues to read about Robin Hood at the tournament.

Robin's turn comes. He steps forward silently. The sheriff, who is offering the prize, doesn't recognize him. Robin's eyes scan the target, 80 yards away. He feels the wind, judges the slope of the ground, and adjusts his stance.



He lifts the bow at the perfect angle—high enough to carry the arrow far but not too high that it misses the target. This is not a guess; it's based on math. Robin uses an aiming angle called the angle of elevation. This angle is formed where the horizontal line between Robin and a spot below the target meets the line along which the arrow travels when it leaves the bow. He makes a calculated decision based on his experience with angles.

Robin knows his aim creates a triangle between himself and the target. He imagines that the base of the triangle creates a right angle with the height of the triangle. The base is the horizontal line from where he is to the point below the target. The height is the vertical line that goes from the base up to the target. The hypotenuse is the line that connects the base to the height, or the path his arrow must take to reach the target. Robin lets the arrow fly. It travels and hits the target directly in the center. The crowd gasps. He steps back as if nothing has happened.

Behind him, two archers whisper. "How did he do that? That takes a perfect calculation," one says.

"That angle was no accident. It was perfect precision!" the other archer quietly exclaims.



The archers are right. Such a perfect shot requires careful mathematical planning. An arrow follows a path that depends on the angle of elevation and the force of the shot. Even small changes in the angle can change the entire outcome.

As the contest dwindles to 10 finalists, each remaining archer fires 2 arrows. Robin's arrows both strike close to the center. When only 3 archers remain, they each shoot 3 arrows.

Robin uses mental geometry with his understanding of angles within triangles to refine his aim. He slightly adjusts the angle of his bow. His final shot splits a competitor's arrow, already stuck in the target.



To win, Robin had pictured the lines in his mind. He started with the imaginary line that followed the ground from where he stood to the target. By thinking about the angle of elevation, Robin got a clear idea of how to aim and hit the target just right. Then, he raised his bow and arrow to create enough of an angle to get his arrow to the target.



Robin Hood wins the golden arrow! But he says nothing. He nods politely, accepts the prize, and disappears into the crowd.

One young archer, amazed by the shot, whispers, "How did he do it?"

Another replies, "He knew how to aim. He understood the math."

The contest may be over, but the lesson remains: Precision comes from understanding. Geometry, particularly angle relationships, can turn a good archer into a legendary one. The golden arrow glints in Robin's hand, a quiet reward for silent calculation.



Back in class, Ellis closes his book with a smile. "Robin Hood had to understand angles to win," he says to himself.

Ellis flips to his geometry notes, now seeing more than numbers on a page. Every triangle, every angle, tells a story—just like Robin’s arrow.

Whether in ancient forests or modern classrooms, triangles, angles, and lines still guide decisions from archery to architecture.



Main Idea

You can use angles and triangular relationships to predict the path of an object.

The Mathematics of Billiards

14

Alex holds his billiards stick as his eyes trace the familiar felt of the billiards table. A huge junior tournament is coming up in Dixon City, Pennsylvania, Alex's hometown. He dreams of winning the tournament, but he needs to improve his skills.



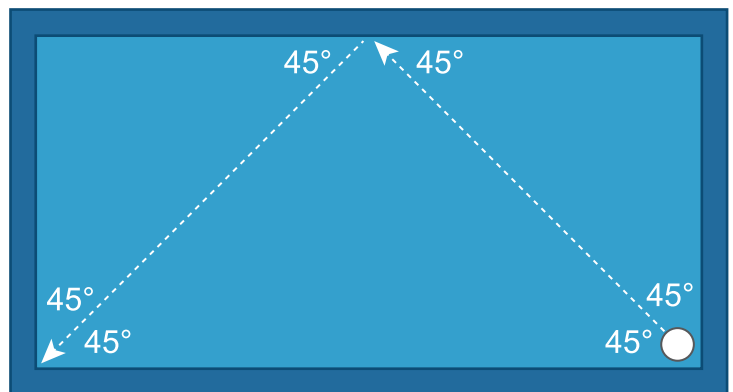
When he was younger, Alex's dad taught him about billiards. Billiards is a group of games that uses a table, balls, and a stick. Each version uses a different number of balls. In this tournament, Alex is playing a game called 3-cushion billiards. The sides of the billiards table are called rails, or cushions. The game uses 3 balls. The white ball is the cue ball. It's the only ball Alex can strike with the stick. The other 2 balls are called target balls. One target ball is yellow, and the other is red. The goal in each turn is to strike the cue ball so that it hits both target balls, while making contact with at least 3 rails before hitting the second target ball.

Alex is practicing how to strike his cue ball. He is good, but winning a major tournament takes more, something extra. Alex knows that the placement of the balls on the table will determine how to strike the cue ball.

To get good at billiards, Alex needed to understand how balls move. Alex learned about the reflections of angles in math class. The law of reflection in geometry states that if a ball hits a flat surface, it will bounce away from the surface at exactly the same angle.

Alex thinks, "The billiards table is a rectangle, and the sides are a flat surface. The ball's path forms angles. It's a giant math puzzle! I can use angles and their reflections to predict the ball's path."

He decides to test the law of reflection. Alex puts his cue ball in the corner. He aims to shoot the cue ball at a 45° angle toward the rail. Because the table is a rectangle, the corner is a 90° angle. So cutting the corner in half perfectly is a 45° angle. If Alex aims precisely, the ball will



hit the rail in the middle of that side and then bounce off at a 45° angle. The ball bounces away from the rail following a mirror-image line to the path it traveled to the rail.

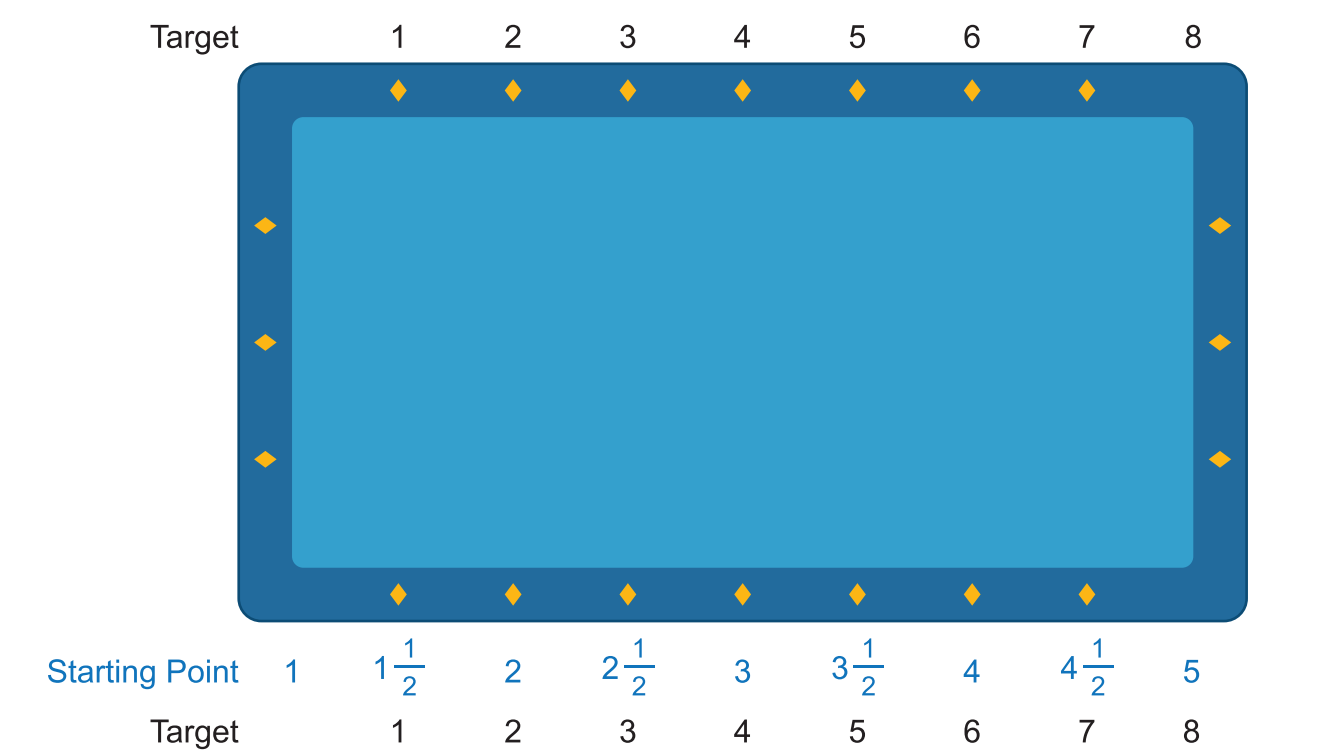
Alex places his cue ball at a different spot and aims it at the rail. Before he takes the shot, he tries to predict where the ball will go. He knows that if he aims straight at a rail, the ball bounces straight back. If he aims at a slight angle, the cue ball bounces off at the same angle. The 2 lines the ball follows are a mirror image. No wonder they're called reflections!

Alex uses a piece of chalk to mark where his cue ball hits the rail and then where it ends up. He continues practicing shooting over and over to see how the cue ball will bounce. He reinforces that the law of reflection is true. It feels like magic, but it is really just math!

Alex knows there are diamonds on the billiards table's rails. Alex studies his practice table. He examines the 7 diamonds on each of the long rails and 3 diamonds on the short rails of the billiards table.

Alex knows that the diamonds help players identify positions on the billiards table. By assigning numbers to the diamonds, a number system is created. This system can help players figure out where the ball will go after hitting the rails.

Alex knows that if he aims at a diamond at a certain angle, the ball will bounce off the rail at the same angle. He labels the billiards table to help him use the diamond numbering system.



The table has two sets of numbers. The blue numbers are used to identify the location of the cue ball before he strikes it. Alex thinks of these numbers as starting point numbers. The black numbers are used while the ball is in play. Alex uses the bottom-right corner as a starting point for his cue ball. He will start with the cue ball on the blue 5.

Alex knows that the core of the diamond numbering system is a basic equation:

Starting Point (S) – Aiming Target Point (A) = Finish Target Point (F)

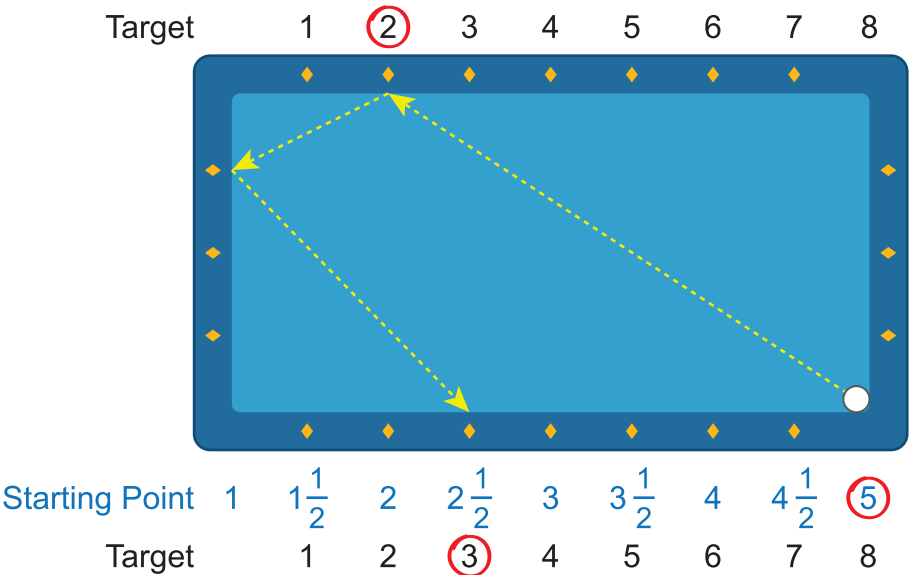
- The starting point (S) is where the cue ball is before the shot.
- The aiming target point (A) is the diamond on the first rail you're trying to hit with the cue ball.
- The finish target point (F) is the last rail you want the cue ball to hit.

This equation helps players figure out where to aim their cue ball to hit the target point. Alex decides to try a shot using the equation. He writes some notes on a piece of paper to help him plan his shot.

$S - A = F$
$5 - 2 = 3$

If he uses the equation correctly, Alex can predict where the cue ball will end up. Because his cue ball is starting at the blue 5, he uses 5 in the equation. He wants his last target to finish at black 3. If he subtracts 3 from 5, he sees that he must aim at black 2.

He shoots. It works! Alex is excited. His cue ball ends up right where he wants it to—at black 3!

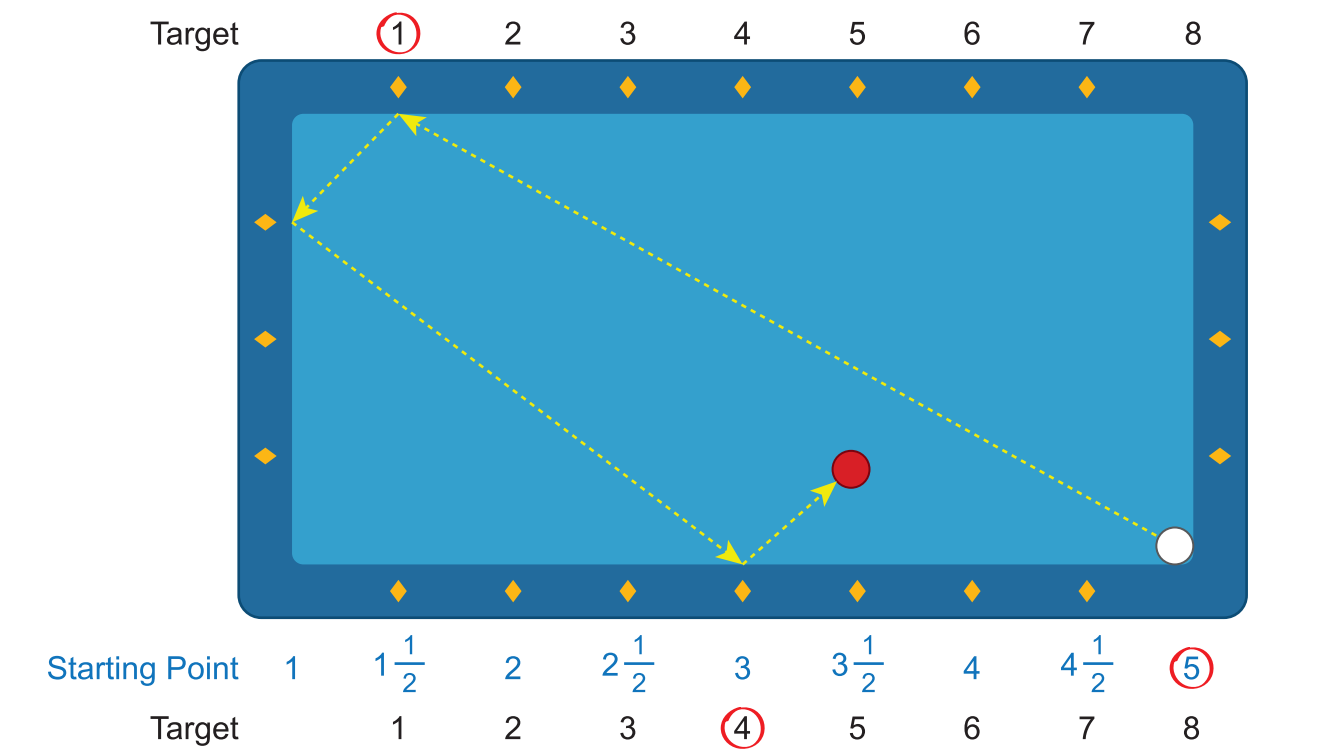


If Alex knows where his cue ball will start and where he wants it to end up, he can solve the equation for where he should aim the cue ball. Alex decides to practice hitting 3 rails and just 1 target ball.

He places his cue ball at the blue 5 position.
 He thinks that if he can target his cue ball to finish on the rail at black 4, he can hit the red ball. He writes some more equations.

$S - A = F$
$5 - A = 4$
$A = 1$
$5 - 1 = 4$

Now Alex sees that he must aim for black 1.
 If he aims precisely, the ball should bounce from black 1 on the opposite side to the left short rail and be reflected to black 4 on the bottom rail.
 Will his cue ball hit the red ball? He gives it a try. His aim is about right but not perfect, but he hits the ball, and he is thrilled!

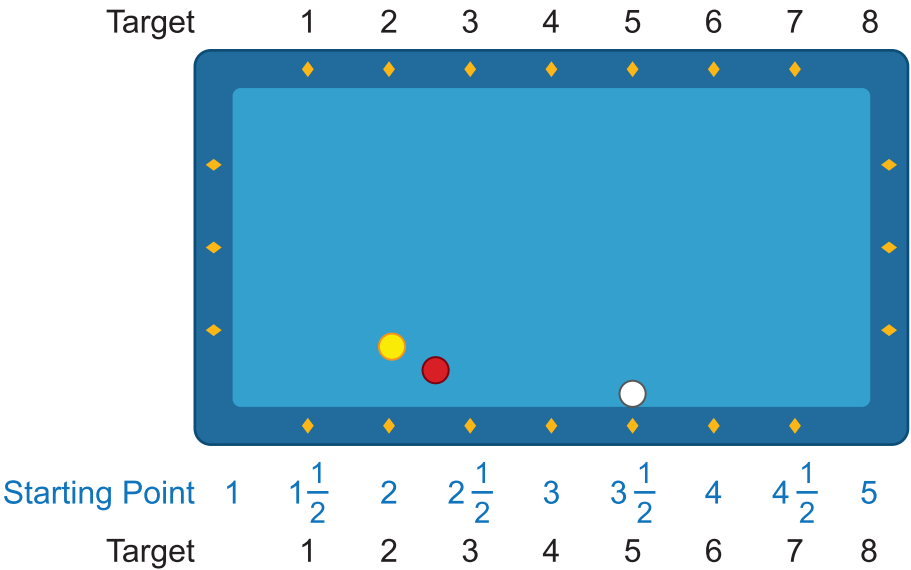


It's one thing to make his cue ball go where he wants and hit at least 3 rails, but Alex knows the real goal is to also strike the other 2 balls. This means that the balls may not always be in a position that makes it easy to hit the 3 rails and 2 balls, one of which needs to be hit after the cue ball bounces off at least 3 rails!



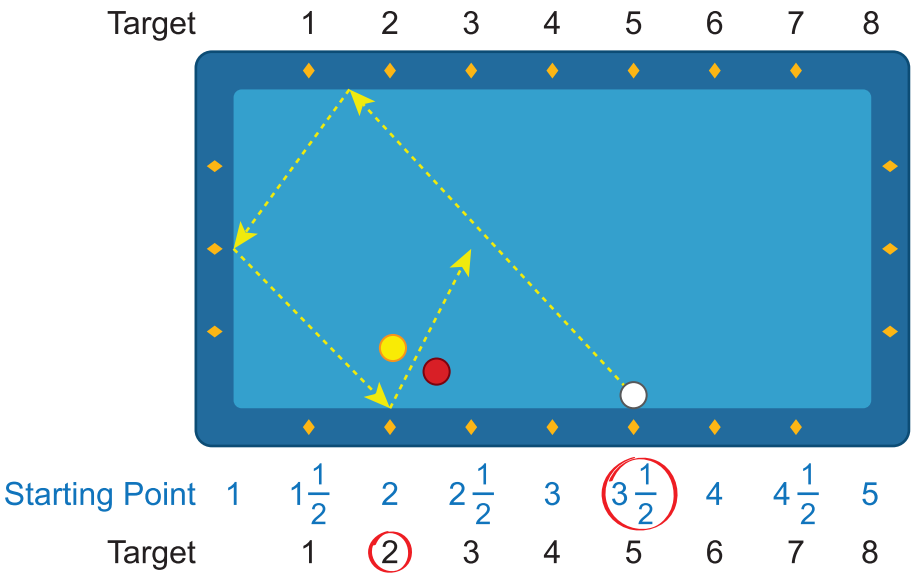
Alex practices on making a few shots in a row. He realizes that using his understanding of the law of reflection and the equation takes so much guesswork out of billiards! He sees that he must adjust his shots so that the cue ball does not always hit exactly at a diamond. Sometimes the best place to aim is in between the diamonds.

Alex sees the target balls are in a tricky arrangement. He thinks he can hit both balls after bouncing off 3 rails if the final bounce is at the black 2. How can he do it?



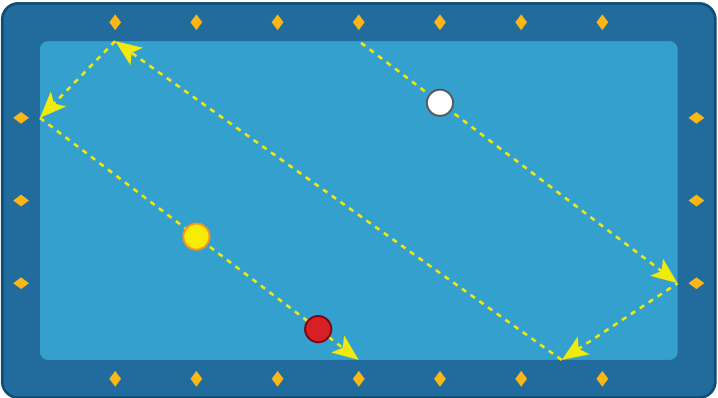
Alex’s cue position starts at blue $3\frac{1}{2}$, and he wants the cue ball to finish at black 2. He subtracts to find his aiming target point. His target point is black $1\frac{1}{2}$. He realizes that the space between diamond positions 1 and 2 can also represent diamond position $1\frac{1}{2}$, so he decides to give it a try.

His cue ball bounces off the rail between black 1 and black 2 and then off the left side rail. It’s all going according to plan. Alex holds his breath as the cue ball bounces off the black 2 and taps both target balls. He did it! Alex jumps up in excitement, his hands in the air.



Alex tries one more shot. He analyzes the positions of the balls after his previous shot. He realizes that he can’t hit the 2 balls at the same time. He can't find a shot where the cue ball touches only 3 rails before hitting the balls. He thinks about angle reflections and sees a way to hit 4 rails. He realizes that the equation he knows won’t work with 4 rail

bounces, so Alex decides to rely on his visualization of angle reflections. As his cue ball bounces off the last rail and gently taps the balls, he feels ready for the tournament. He knows he is connecting math to strategy in his favorite pastime.



Finally, the tournament is here! The game is down to the final round. Alex is tied with his opponent, and it's his turn. The cue ball is in an awkward position. Alex walks around the table, examining the situation. He thinks a shot using only 3 rails is impossible. This is where all his practice and all his geometry come together.

He lines up the shot, focusing intently. He uses his eyes to trace the angles he expects the cue ball to take, using the diamonds. He takes a deep breath, brings the cue stick back, then pushes forward smoothly. *CRACK!* His cue ball flies across the table, hits the diamond on the opposite rail and bounces off at the exact angle Alex predicts. It streaks across the felt and hits another rail and then 2 more rails, striking the red ball and then the yellow ball with a satisfying thump.



The crowd erupts in cheers. Alex won! He looks down at the table, a smile spreading across his face. He knows that his practice and instincts are important, but his new understanding of geometry gives him a true edge. He realizes that math isn't just for tests and textbooks; it is a powerful secret weapon that can help him achieve his goals, even on the billiards table.

Main Idea

Players use the mathematical law of reflection to predict and plan when playing games.

What do you notice when you look at this drawing? We know, of course, that it's a 2-dimensional illustration, but it looks 3-dimensional! This means that we know it's a flat image but that it feels like it pops out of the page. How did the artist do it?

Believe it or not, the artist used math! My name is Jaz, and in this tutorial, I'm going to teach you how to make your own art that looks like it pops off the page!

Before we get started, think about what you know about buildings on a street.

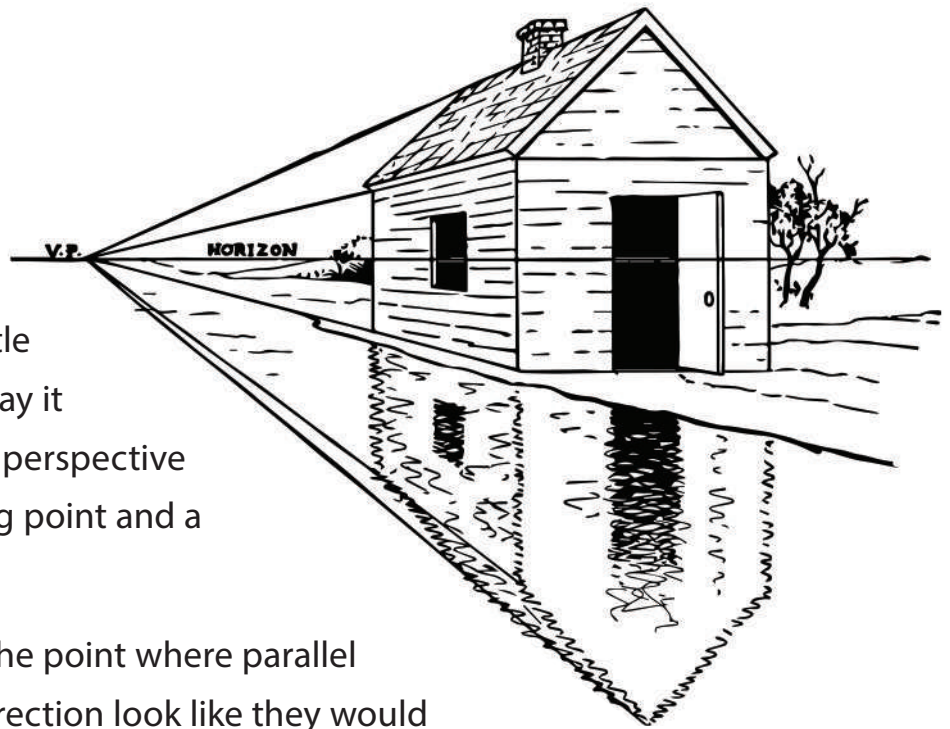


We know that most buildings are generally like rectangular prisms. What do you notice about how the rectangular prisms are drawn here? What about the angles of the windows? How did the artist know what angle would make the buildings look so real? Let's find out!

Let's start with a little background. To create realistic-looking images, artists use some important geometry concepts. In the context of art class, these geometry concepts are rolled into the term *perspective*.

Perspective refers to creating the illusion of a 3-dimensional image in a 2-dimensional space.

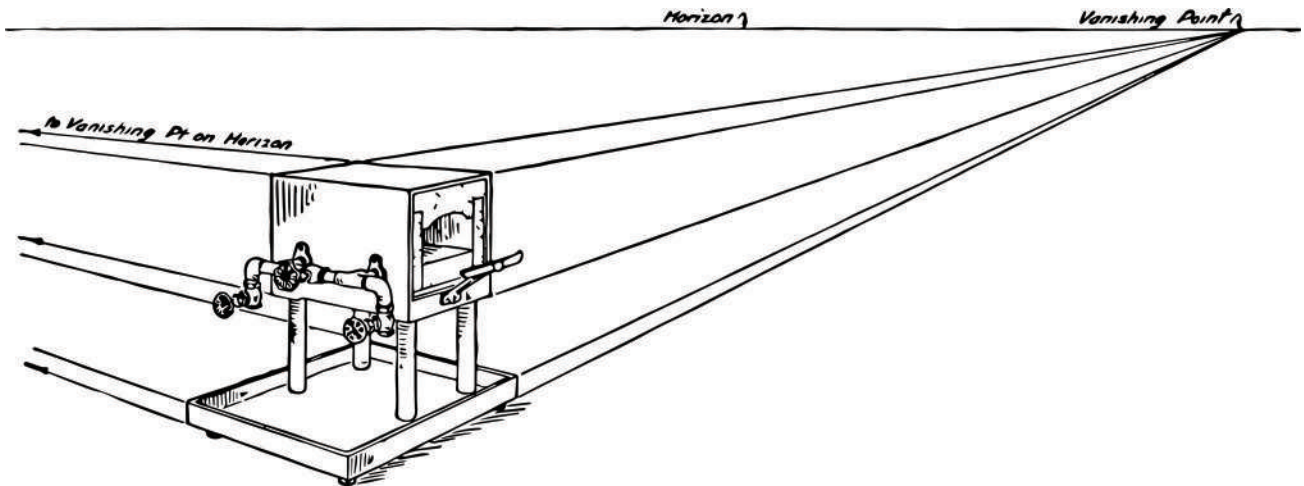
An illusion means that something is really a little bit different from the way it looks. The basic idea of perspective means using a vanishing point and a horizon line.



The vanishing point is the point where parallel lines pointing in that direction look like they would meet if they kept going. We know that two lines are parallel if they never meet, even if they were extended forever. We also know that the lines on a roof and the side of a house are parallel in the real world. But to make this image look 3-dimensional, the artist used a vanishing point to draw it in perspective. This means that in the drawing, the lines aren't really parallel but they give us the illusion that they are. A simple perspective drawing like this one has just 1 vanishing point. But sometimes drawings can use 2, 3, or even more vanishing points!

The horizon line describes where the sky meets the ground in the drawing. Depending on where the artist puts this line, it can make it look like you are looking straight at the image, up at the image, or down at the image!

But what geometric concepts are artists using to achieve this effect? Consider where you might have seen labeled diagrams like this before.



That's right! Artists use scale, dilations, and translations to achieve perspective. Remember that a dilation is a reduction or enlargement of the exact same figure. A translation is the movement of a figure across a plane.

Let's consider how this applies to this image for a moment. We can see that 1 vanishing point is marked and that the other is off the page. The artist has drawn lines to show how lines that we know are parallel on the machine would connect at the vanishing point in the drawing. This tells our eyes how close or far away the machine appears. If the artist wanted the machine to appear farther away, they could move it along the lines connecting it to the vanishing point. If the artist wanted the machine to appear closer to us, they would have to adjust their scale, which would make everything in the drawing appear larger. The machine in this drawing looks 3-dimensional because the lines that we know are parallel on a real machine are aligned correctly with the 2 vanishing points. This gives our eyes the illusion that the machine is popping off the page! So if the artist adjusts the size of the image, they still have to make sure the object aligns with the vanishing point.

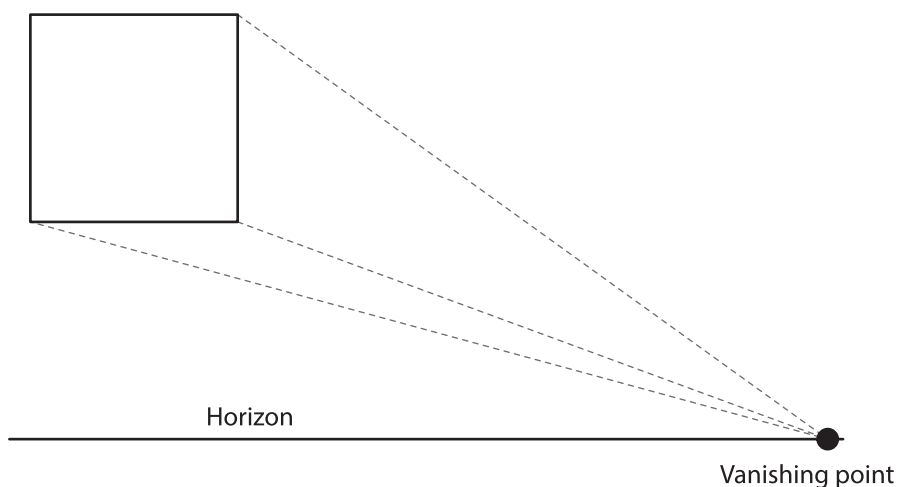
Let's create a simple drawing with a cube to test these concepts. Start by taking a piece of paper and drawing a horizon line and 1 vanishing point.

You're the artist, so it's up to you to decide whether the horizon is at the top, middle, or bottom of the page. Place the vanishing point anywhere on the line.

The next important step is to decide where you'd like to place the front of your cube. It can be above or below the horizon line. Draw a square to show the front of your cube. The position of the cube relative to the horizon affects how the cube appears. If the cube

is above the horizon line, it will look like you are looking up at it. If the cube is below the horizon line, it will look like you are looking down at it.

Remember that in a square, all the side lengths are the same.

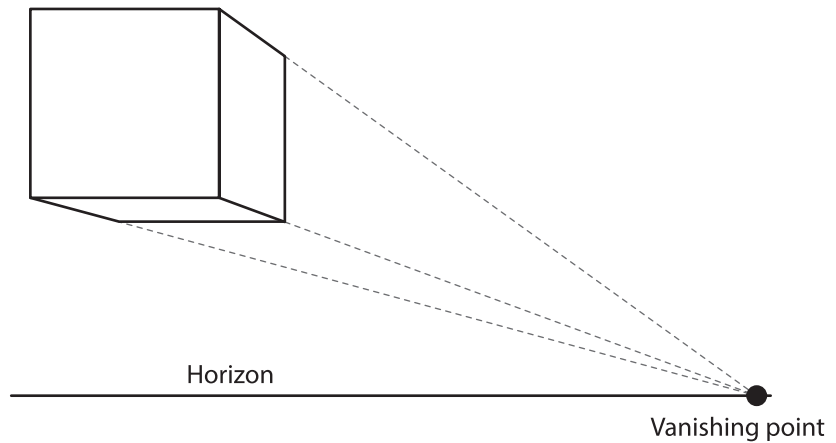


Use tools to make sure you have a perfect square and that all the corners are right angles!

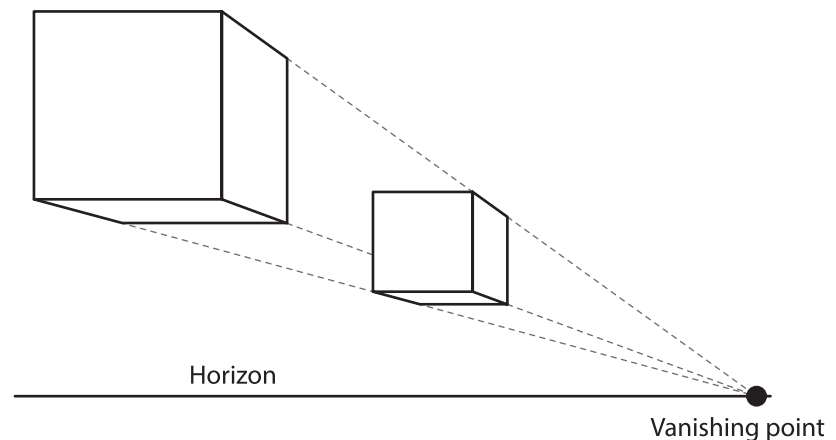
We know that in a cube, the segments of each face are parallel and perpendicular. To give this square the illusion that it's a cube popping off the page, we need to align those segments to the vanishing point. Use a ruler, and give it a try!

Because our cube is solid, we can't see through it, so we'll just connect the 3 corners with a direct path to the vanishing point. The dotted guidelines show us what angles to use to draw the new sides to make it look like it's popping off the page.

To complete the cube, we need to add a vertical line on the right and a horizontal line below and trace the dotted guidelines to the vanishing point that make up the figure. Now we have a cube drawn in perspective!



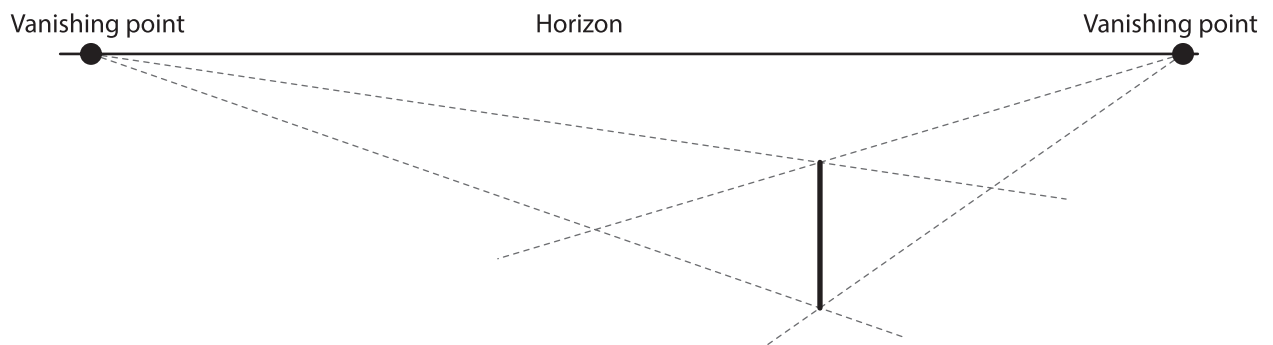
Now, let's explore how this relates to dilations. We can use the same guidelines to create another cube that is proportionally smaller than the one we just made! To do this, use the guidelines to draw a new square. The bottom edge of the square stretches between the 2 two lower guidelines, and the right edge of the square stretches to the upper guideline. Use tools to complete the figure. This represents a dilation because these two figures are proportional! It's also a translation because it's in a different place on the paper. Notice that all the horizontal lines on the square faces are parallel and that all the vertical lines are also parallel. For an artist, these ideas used together create the illusion that one figure is closer to us than the other!



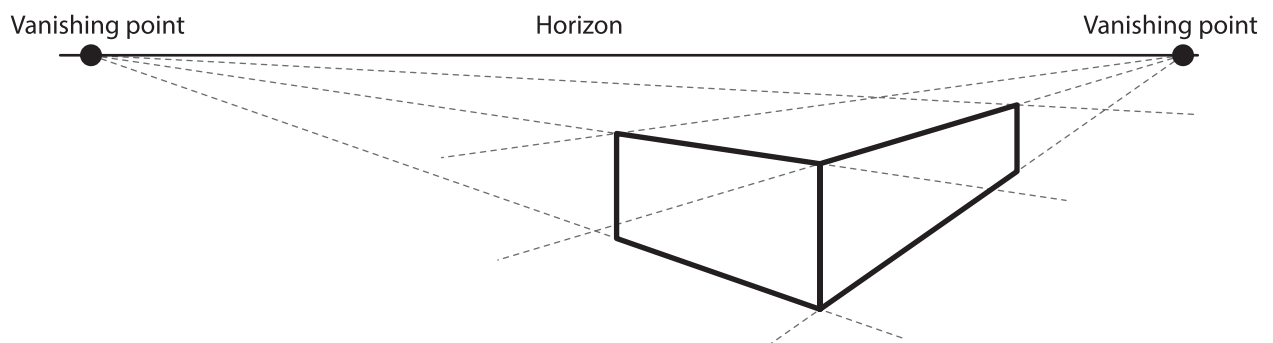
Try a few more on your own in different places on the paper with some other guidelines! Maybe even try some other figures, like a rectangular prism! Notice how the angles change as you move them around the paper.

Let's say you don't want to look directly at the front of your figure but you want to view it at an angle. In this case, you need to use 2 vanishing points. This is called drawing in 2-point perspective.

Start by drawing a new horizon line with 2 vanishing points. Then draw two sets of 2 guidelines. For the drawing to look like it's correctly drawn in perspective, you need to be able to draw a vertical line at the intersection of the 2 sets of guidelines.



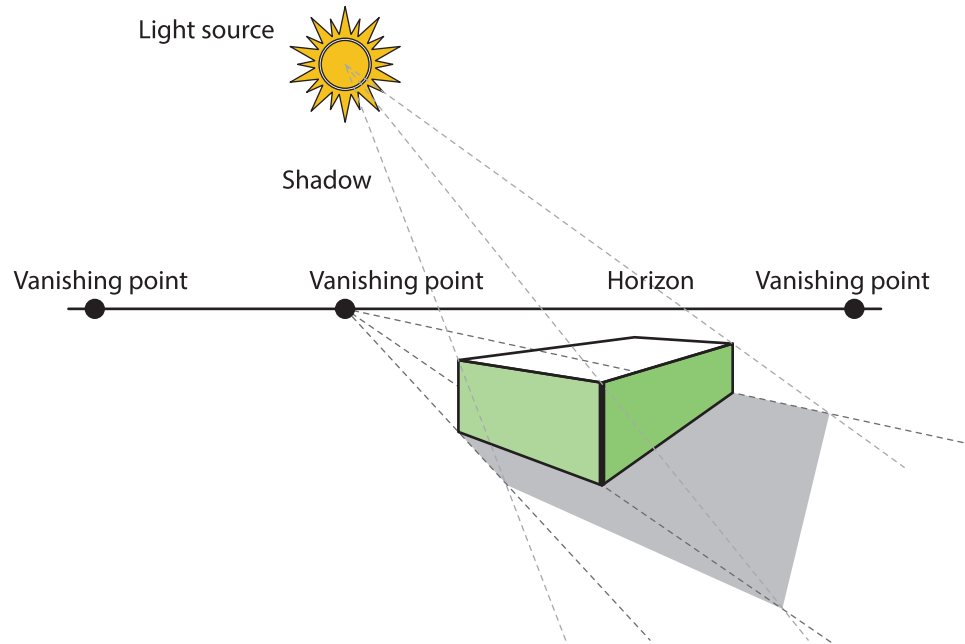
This vertical line will become the front edge of the figure. Let's make this figure a rectangular prism. Imagine that it's a large building at the intersection of 2 streets. Then trace the upper guidelines to show the length and the width of the building. From each upper guideline, extend a vertical line down to the lower guideline. Trace the lower guideline to meet it. To complete the figure, you will need to make a third set of guidelines. To make the perspective correct, draw a new guideline from each vanishing point to the opposite corner of the figure. Then trace the guidelines to complete the rectangular prism!



Have you ever noticed that when the sun is high in the sky, your shadow is short and that when the sun is lower in the sky, your shadow is long? How do artists know exactly how long or short to make those shadows? Light always travels in a straight line. So we can use a similar strategy to create shadows! They identify the position of the light source and the vanishing point for the shadow! Let's try this with our rectangular prism.

First, draw the light source for your image. Then draw a line that is vertical to the horizon.

This will be the vanishing point for the shadows your rectangular prism casts. Draw straight lines from the light source through the



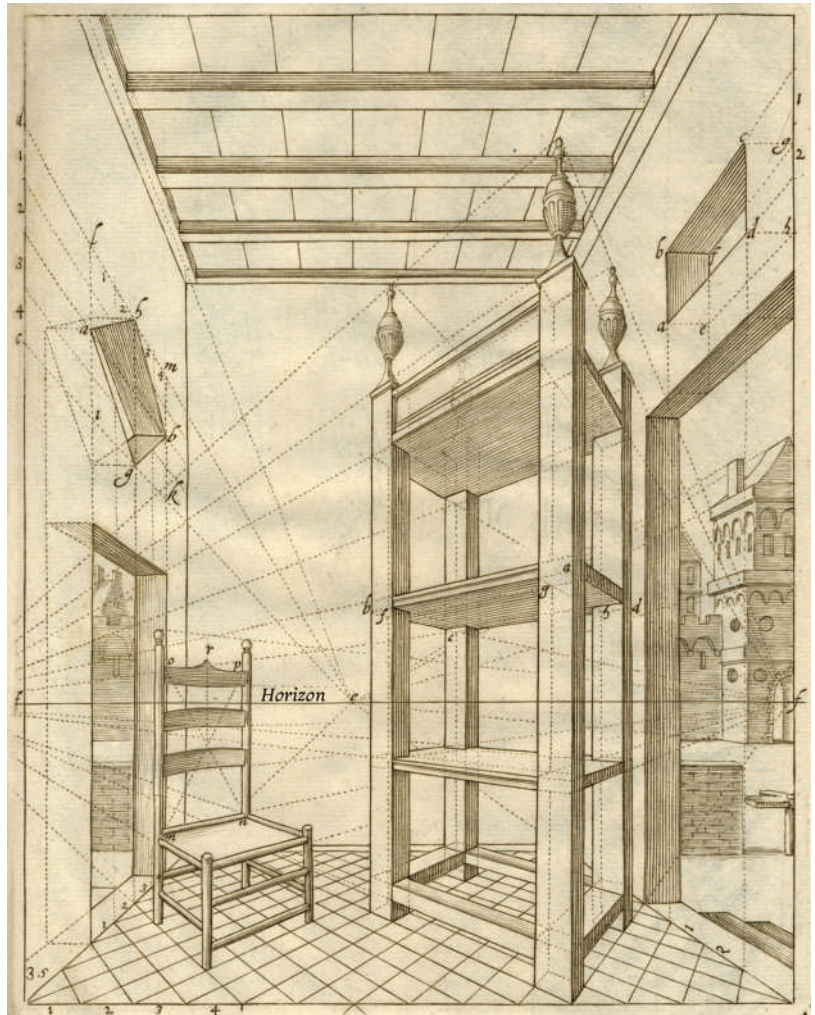
3 front top corners of the rectangular prism. The back corner won't cast a shadow. Then draw 3 straight lines from the shadow vanishing point through the bottom front corners. Use straight lines to connect the points where the lines from the light source and the shadow vanishing point intersect, and you have a shadow!

The position of the light source also tells the artist how to shade the faces of the figure. In this example, the light touches the top of the figure, so it's lighter. It touches one face just a little bit, so that face is lightly shaded, but it doesn't really touch the face of the rectangular prism that is opposite the sun, so that face is darkly shaded.

Paying close attention to just a few things can really transform a piece of art. Using the horizon line, vanishing points, and a specific source of light can make your piece of art look 3-dimensional.

In this tutorial, we covered a lot. Take a look at this drawing. What do you notice about how the artist used the horizon line? What about the vanishing points or the source of light? These things tell us so much about what the artist wants us to see and how the artist wants us to see it. Just a little change can make so much of a difference!

Try creating your own drawing using the concepts we learned today! Experiment with dilations, horizon lines, and vanishing points! Move your light source around the page, and see for yourself how much it can change the look and feel of your drawing! Join us next time for a tutorial on drawing portraits!



Main Idea

Geometric transformations like dilations and translations, along with concepts of parallel lines and perspective, create realistic 3D-looking images on flat surfaces.

The warm sand squishes between Layla's toes as she builds another tower for her sandcastle. Her younger brother, Jaxon, runs along the shoreline, chasing waves that tickle his feet before rushing back to the ocean.



The Florida coast sparkles under the bright sun as waves gently roll in and dozens of families enjoy the beautiful Saturday.

"Layla, Jaxon, come help me pack up," Mom calls from their beach umbrella. "Dad wants to check the weather report before we head home."

"Already?" Layla groans. Something about Mom's voice sounds different. She looks toward the parking lot where Dad stands next to their car, his eyes glued to the phone and his face serious. Around them, beach umbrellas are coming down. Coolers are getting packed. The ocean breeze carries the sound of quiet conversations. Layla feels it too. Something about this day is changing.

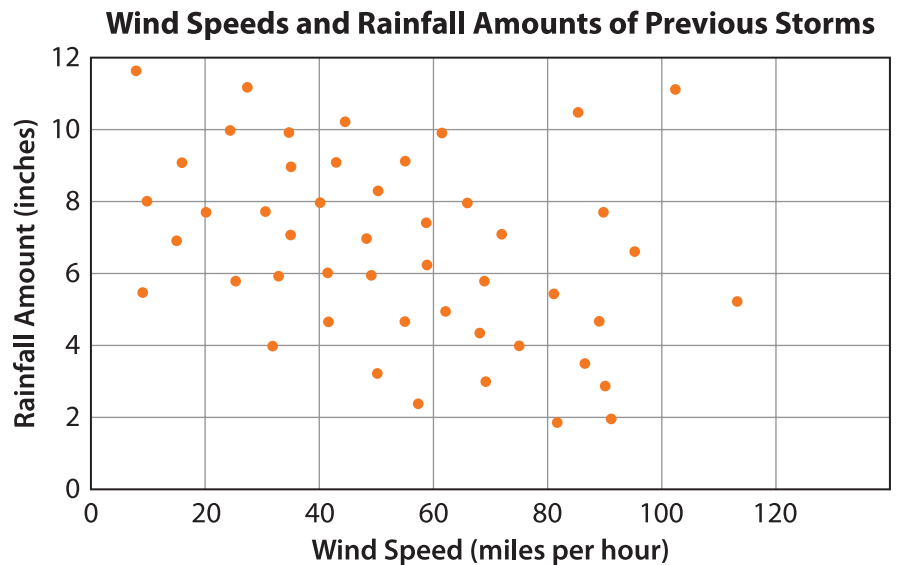
Meanwhile, 50 miles away at the National Weather Service office, meteorologist Dr. Kimberly O'Brien stares at computer screens filled with colorful maps and swirling patterns. She is studying data that shows wind speed and direction, wave heights, and rainfall amounts from weather stations across the Atlantic Ocean.

"The storm is changing course," she calls to Dr. Patel, one of her colleagues. Dr. O'Brien points to her screens that show the measurements from the weather buoys scattered across the ocean and near the shore. "We need to get this information out to the public right away so that people have time to prepare."

On the wall next to Dr. O'Brien's desk is a scatterplot of wind speeds and rainfall amounts from previous storms. Dr. O'Brien

knows that all storms are different and stronger winds don't always mean greater rainfall totals. In fact, as wind speeds increase, rainfall amounts tend to decrease slightly. Slow-moving storms can produce more rainfall because they are over an area for a longer period, allowing the rainfall to accumulate.

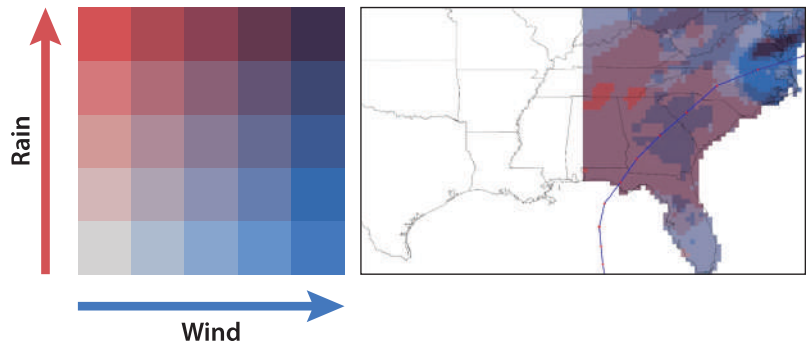
"Strong winds and heavy rainfall are the 2 main sources of danger during a hurricane," Dr. O'Brien continues. "Although hurricanes are categorized by wind speed, it is also important to consider the amount of rainfall expected. We need to look at the impact of both variables together to give people the most accurate warnings."



Dr. O'Brien creates a special map that combines both wind and rainfall data into one display. It's called a bivariate hazard map.

Bivariate data compares 2

types of data, such as wind and rain. They are examined together to see how one might affect the other. The map shows emergency planners exactly where the most dangerous combinations of wind and rain will occur. A hazard map displays the 2 variables together on a geographical map. A color key on the map shows the combined impact of both variables.



"The storm won't hit every area the same way at the same time," she says. "Some neighborhoods might get high winds early, and others could flood later on. That's why timing and location matter."

Dr. O'Brien continues, "This combination of wind and rainfall hazards shows which areas are vulnerable to wind damage, flooding, or both. Looking at both risks helps emergency planners determine evacuation zones and issue warnings ahead of the storm so families can prepare."

Back at the beach, Dad jogs across the sand toward his family. "We need to go now," he says. "The hurricane has changed course, and it's headed our way."

On the drive home, Layla stares out the window. The breeze is stronger than before, and palm trees bend with each gust. Other cars are leaving the beach.



At the National Weather Service office, Dr. O'Brien's team works quickly to update hurricane warnings based on their bivariate data analysis. "We need to get this information out to television and radio stations immediately," Dr. O'Brien says. "The hurricane will bring powerful winds and heavy rainfall. People need time to prepare for both."

The team sends the hazard map to emergency management offices throughout the region. This special map helps emergency planners see which neighborhoods face the highest risk from combinations of wind and rain damage.

Dr. O'Brien adds, "These maps also help emergency leaders decide where to send supplies, which roads to close, and which shelters to open first. The storm isn't the same strength everywhere at the same time. That's why bivariate forecasting is so useful."

On the drive home, Layla sees cars line up at gas stations to fill their tanks. Jaxon notices workers installing metal shutters to protect the storefront glass windows and doors.

"How does everyone know what to do to prepare?" Layla asks.

"The weather service gives us information about both wind speed and rainfall predictions," Mom explains.



“When we know a storm might bring 80-mile-per-hour winds and 8 inches of rain, we can get ready for both flooding and wind damage,” she continues. “The weather service says for this storm we should expect wind speeds around 85 miles per hour with 7 to 9 inches of rain.”

Layla starts to understand how the 2 weather variables work together. Strong wind alone might knock down branches. But with heavy rain, the ground softens—whole trees can fall. Together, they create more danger than either one alone.



Layla’s family arrives home to a neighborhood buzzing with activity. Mr. Gomez from next door carries plywood sheets from his garage. Mrs. Patterson tests her emergency generator. Layla’s family members immediately start

their own preparations. Mom and Layla move patio furniture into the garage while Dad and Jaxon tape large X’s on the windows.

Dad pulls a list from his emergency kit. “We have batteries for flashlights, a battery-powered radio, and a first aid kit. We need to get more food and water. We also need gasoline for the generator in case the power goes out.” At the store, the aisles bulge with families gathering supplies. Layla helps her dad load their cart with water containers and food. Everyone seems calm but focused.

The emergency planners are also busy coordinating resources. They plan evacuation routes, shelter locations, and where to position rescue equipment. Volunteers fill and stack sandbags to create a barrier against flooding.



“Neighborhoods near the coast will face 90-mile-per-hour winds with 10 inches of rain,” Dr. O’Brien tells her staff. “Inland areas will see 70-mile-per-hour winds with approximately 8 inches of rain. We’ll position emergency crews based on each zone’s risk. Areas expecting both high winds and heavy rainfall will receive priority for assistance.”

That evening, the wind picks up. Weather reporters on TV show the storm’s path and warn people to stay indoors. Thankfully, Layla’s neighborhood did not have to evacuate. She is relieved that scientists can make accurate predictions by studying how different variables act together.

Layla thinks to herself, “Now I know how those scatterplots in math class are useful. Even more than that, they are important tools to keep my family and me safe.”



Throughout the night, the storm arrives just as predicted. Wind speeds reach 87 miles per hour. The rainfall totals 8.5 inches. Thanks to emergency team preparation, teams are ready and respond quickly to help families with flooding, downed trees, and power outages.

By morning, the storm has passed. Sunlight returns. Layla looks outside and sees her neighborhood cleaning up—removing branches and pumping out water. Even with the damage, everyone is safe.

Layla says, “Wow! Look, Dad. Some trees blew down in our neighborhood!”

“The meteorologists were right,” Dad replies, examining the yard. “We got strong winds and heavy rain, just like their data predicted. Because we knew both threats were coming, we could prepare.”

She is grateful for relief workers who distribute water and supplies to families that suffered more damage than hers. As Layla helps carry waterlogged items from the garage, she thinks about what she’s learned. Meteorologists and other weather scientists do so much more than measure wind or rain—they collect and use bivariate data to see how they interact.





Dr. O'Brien and her team evaluate the accuracy of their forecast and warnings. "Our predictions were within 3 miles per hour for wind and half an inch for rainfall," she reports. "Looking at both variables gave us the clearest picture of the storm."

Her colleagues agree. Dr. O'Brien says, "The bivariate maps helped us alert the public to danger and get resources to the right places."

Bivariate maps can also be applied to floods, droughts, wildfires, and other disasters, helping communities prepare for a range of compound hazards.

Using 2 weather variables together makes forecasts more accurate and helpful because it shows how different parts of the weather work together. This helps people prepare better and stay safe!

Main Idea

Bivariate data analysis shows whether or not there is a relationship between two variables.

The courtroom is still, but tension buzzes in the air. At the center of it all is a bitter family feud—two brothers, Neal and Peter, locked in a legal battle over the land of their late mother, Laura Rivers. Neal claims the property is his and presents a will document bearing her signature. But younger



brother Peter isn't backing down. He also has a document signed by their mother, declaring him the rightful heir. The signatures look nearly identical. But were they written by the same person?

At the front of the room sits Lydia Carter, a forensic handwriting expert called to assist in settling the case. As a forensic handwriting expert, she studies signatures and handwriting to help solve crimes and explain the evidence to the court. Lydia has studied the signatures with care and is prepared to explain to the court which document is genuine.

Lydia is asked to explain to the court how she analyzed the documents. "I began by analyzing signatures from other documents signed by Ms. Rivers during her lifetime. Then I made scatterplots to study how much things change in different parts of her signature. Although people sign their names slightly differently each time, statistical analysis can show if a new signature fits within their usual style or not."

Lydia displays enlarged images of the signatures on the two will documents. She explains that because it is so difficult to immediately see the difference between the signatures on the documents, this case includes an instance of forgery called a simulation. This means that whoever created the forgery had an example of Ms. Rivers's real signature.

Signatures in Question



The image displays two cursive signatures of "Laura Rivers" stacked vertically. The top signature is the original, and the bottom one is a simulation. The simulation is a close copy but shows subtle differences in stroke thickness and slant compared to the original.

"There are many different measures that are used to analyze handwriting," Lydia explains. "Some of these measures include the slope, or slant, of the letters, the size of the letters, the space the letters take up, the pressure of the pen, and the quality of the lines. When someone is trying to copy another person's signature, they usually write more slowly. This results in slight differences in the weights of the strokes, or how thick the lines are. And sometimes, if you look with a microscope, there are places where it is clear the pen stayed a little too long or maybe the forger picked up the pen in a place where the person may not have. To analyze these, I use some complicated equations that give me something called a score-based likelihood ratio, which helps me reach conclusions. To keep it simple, we'll focus on specific data."

Lydia explains that her first step was to look at each letter in Laura Rivers's signature using as many documents as she could find that were known to have Ms. Rivers's signature.

The screen in the courtroom changes to show 4 signatures that Lydia took off documents that Laura Rivers had signed herself during her life.

Lydia explains that these are only a few examples. She had examined 25 of Laura Rivers's signatures.

"What is important to notice," she says, "is that the signatures are not identical because nobody signs their

name exactly the same way each time. However, if we examine the range of data points for some key measures, we can find some things that are very much outside of her normal style range in the forged signature. These measures that fall outside of the normal range are called outliers."

The courtroom is quiet as everyone examines the display. Lydia points out that sometimes the first *a* in Laura's first name is positioned slightly differently relative to the *L*, and that sometimes the length of *r* is a little different. She points to some variations in her last name, too. Lydia explains that sometimes the rounded part of the capital *R* is a little larger or smaller, but she notes it is always at approximately the same angle. She points out how the size of the *v* is sometimes wider or narrower but that it always connects neatly to the *e*.

Lydia explains that these are all natural variations but that the same person clearly wrote them because the range of the differences is consistent and the quality of the strokes that make up each letter suggest that the writing was quick and confident.

Known Laura Rivers Signatures

A handwritten signature in cursive script that reads "Laura Rivers". The letters are fluid and connected, with a consistent slant and height.A second handwritten signature of "Laura Rivers" in cursive, showing slight variations in stroke thickness and letter spacing compared to the first.A third handwritten signature of "Laura Rivers" in cursive, demonstrating another variation in the script.A fourth handwritten signature of "Laura Rivers" in cursive, showing further natural variation in the handwriting.

Lydia adjusts the screen to show Laura’s signatures and the two from the wills presented by Neal and Peter.

Questionable Signatures	Laura Rivers’ Signatures
	
	
	
	

Lydia reminds the court that no single comparison proves anything by itself.

“This is where math helps,” Lydia says. “When we analyze all the comparisons together, patterns start to show.”

Lydia explains that there are instances where the data are all in the same range, so it is not possible to use those data points to determine which is the real signature.

“For example,” Lydia starts, pointing back at the signatures. “Look at the letters *i* and *v* in Laura’s last name. They do not look the same in each signature, but they don’t look different enough for one signature to stand out.”

Lydia points to one of the signatures. “One of the interesting things about Laura Rivers’s name is that there are 2 two? lowercase letter *r*’s.” This provides an opportunity to compare more examples of the same letter. Since I examined 25 sample signatures from Laura Rivers, this means I had 50 lowercase *r*’s. Analyzing this letter gives me a lot if information.

"We can see in the samples that there is a lot of variability in the length of the *r*'s. However, think about the weight of the stroke, or how thick the lines are. Laura Rivers was quite consistent in how thickly she wrote letter *r*'s. But there are 2 *r*'s from 1 of the documents in question that are really different from the others. When we look at handwriting, line thickness can tell us a lot about how quickly and confidently the person wrote it. A thicker line means more hesitation. So this difference tells us that the person who wrote these letters *r*'s hesitated in a place that Laura Rivers didn't."

Lydia continues. "When analyzing signatures, there are many features we can look at. We can look at the height and width of a letter, the slant of the letters, and how much pressure a person used while writing the signature. All of these can features add information."

Lydia explains that by creating a scatterplot for each different feature, she can see a pattern emerge. The signature with the greatest number of data points outside of the range of values is the forgery.

For the next hour, Lydia shares scatterplots as well as other examples of her analysis of the signatures. When she is finished, the courtroom is quiet. The judge raises an eyebrow and looks at the lawyers and at the brothers, Neal and Peter. It is difficult to argue with math.

"Ms. Carter," the judge says to Lydia, "thank you for providing a thorough mathematical analysis of the signatures. In your expert opinion, which signature is the forgery?"

Lydia points to the second questionable signature. The judge thanks her again and tells her that she is excused.



He sits back in his chair for a moment before calling the lawyers for the brothers Neal and Peter to the bench.

The judge pushes the 2 copies of the will in front of the lawyers so they can see the different signatures. The judge asks the lawyers if they have anything else they would like to tell the court. They shake their heads.

“Ladies and gentlemen”, the judge says, “thanks to a thorough forensic handwriting statistical analysis, I believe we have identified the correct document. It appears that the documents brought by Neal contain a forged signature. Peter, you are the rightful heir of Ms. Laura Rivers’s property. I hope that there is a lesson here for everyone. Math can be used to identify many different kinds of patterns. It is nearly impossible to create a fake version of those patterns, and you are likely to be caught. Court dismissed.”

Main Idea

Using scatterplots lets us compare groups, find patterns, and make decisions based on data.

Amy is very excited today. Instead of going to school, she is riding the train to the city with her mother. Amy's mother works as a market research analyst for a major international company. Amy is looking forward to learning all about what her mother does at work.

When they walk into her mother's office, Amy notices some poster boards sitting on easels. One of the charts is a pie graph. Another is a colorful line graph. "This looks like my math class!" Amy tells her mom. "Did you make these?"

"I put the information together. Luckily, I have a team to help me prepare these boards. I will use these when I meet with my client," her mother replies. "These boards show the data we collected to help the company make some decisions about its products."

Amy's mom explains that as a market research analyst, her job is to study consumer behavior. She uses sales records, surveys, and market research to look for trends. Then, she reports back to her clients so they know what they need to do to increase the sales of their products.

"Who are your clients?" Amy asks.



"I have all sorts of clients! Right now, I am helping a fast-food chain decide which items to keep on its menu, what items to remove, and what new items it can add to its menu. People's tastes change over time."

Amy's mom explains that the first step in her research is to collect data. It is important to understand what customers want. Some companies might want to know if there are new types of food their customers want, such as international flavors for sauces. They might want to know if there are new trends in the market, such as having more vegetarian options.

Companies need to understand what their customers want. They want to make sure their customers are satisfied with their products and services. It's important to identify new trends in the market to drive product development. Companies then need to evaluate new products, get feedback from customers, and if necessary, make improvements.

Amy asks, "How do you find out all of this information?"

"First, our research team has to collect data. There are lots of ways to do that. We use many surveys. Sometimes companies put QR codes or website addresses on receipts that take customers to an online survey. Usually, companies will offer an incentive like a free food item or a chance to win a gift card to encourage customers to complete the survey," her mom explains.

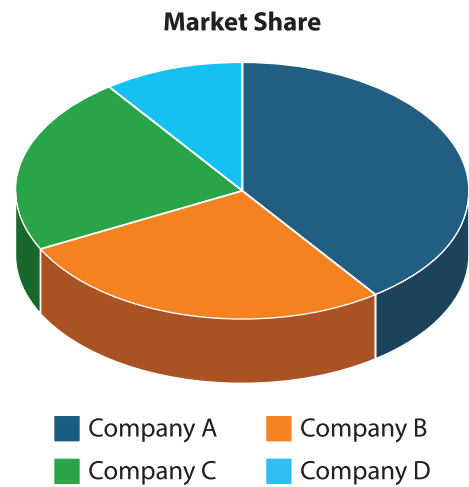


“Sometimes we post a survey to the company’s social media page or do an email campaign. Depending on what information we want to find out about, we might also monitor what people are saying in online reviews.”

Amy asks, “What do you do with all the answers from the surveys?”

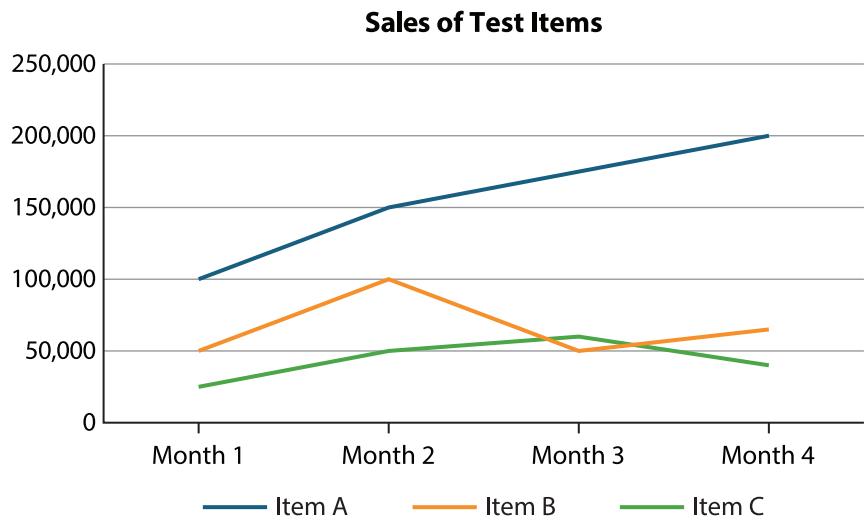
Amy’s mom smiles as she says, “That’s the fun part. At least for me it is.”

Her mom tells Amy how she uses different statistical tools to organize and interpret the data. She uses pie charts to visually compare how much one company sells compared to how much its competitors sell. Her clients can quickly see their share of the total market for a product.



“If a client’s goal is to get a bigger market share, that’s where market research comes in,” Amy’s mom explains. “Based on the information we collect, we can perform a targeted advertising campaign to boost sales.”

“Line graphs are another way to display information. Line graphs allow us to identify trends in sales over time. Companies want to know if sales are increasing or decreasing, especially when introducing new products or deciding if they need to remove a product. We usually suggest trying a new item for a limited time, and then we track the sales to see if customers continue to buy it.”



Amy says, “I have made these types of graphs in math class.” Amy points to another chart. “But I have never seen this type of graph.”

“That’s called a scatterplot. That’s my favorite,” her mother replies.

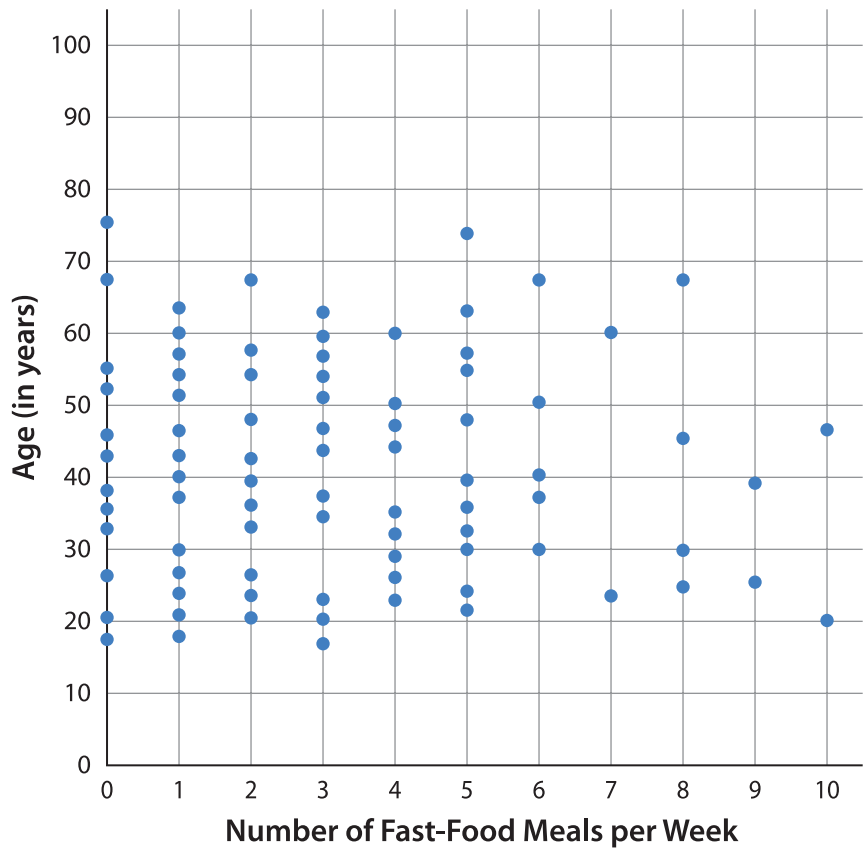
“Scatterplot display information about two variables. This makes it easier to analyze whether there is a relationship

between the variables. Scatterplots make it easier to identify patterns and trends that we might not see in other types of displays.

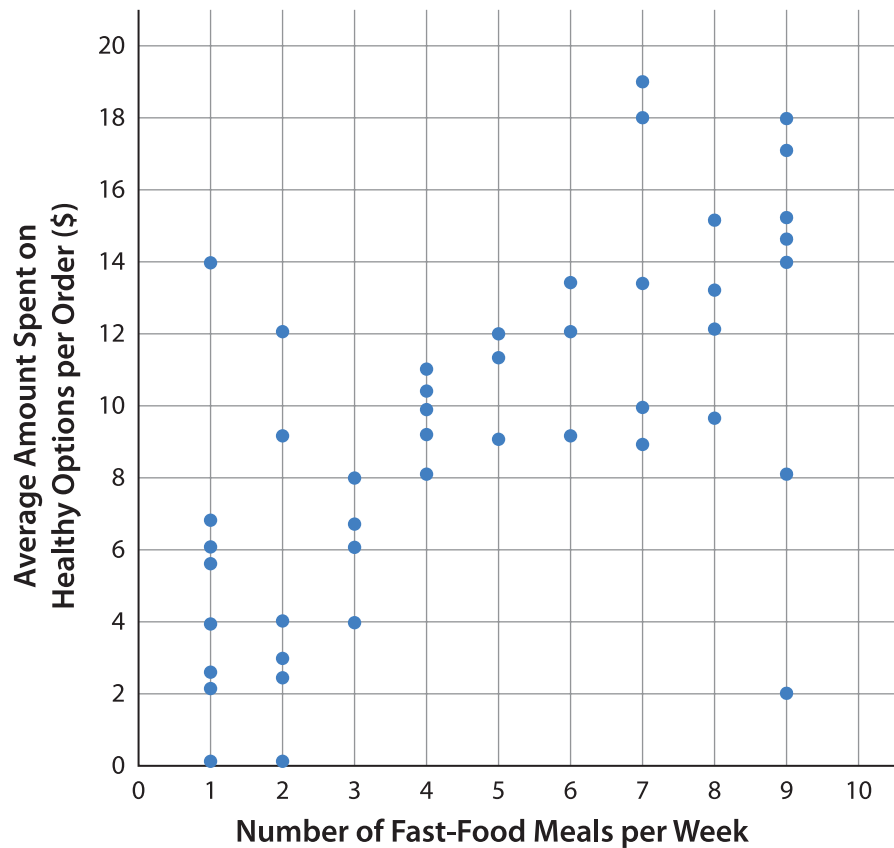
“For example, this scatterplot shows customers’ ages and how many times they eat fast food in a week. Each dot is a response from a survey. The scatterplot does not show a strong pattern in the dots. That means there is no association between the variables.”

Amy says, “So that means the number of fast-food meals a person eats in a week is not related to their age, right?”

“Right!” her mother answers. “But sometimes we can identify an association between variables that indicates they could be related.



“Look at this scatterplot. When we ask people how much money they spend on healthy options, there does seem to be a pattern. Notice that as the number of meals per week increases the amount of money spent on healthy options also increases. When both variables increase in relation to each other, we call it a positive association.”



“Why would a company want to know something like that?” Amy asks.

“Well,” her mother says, “based on this data, we suggested to the client that they should add more healthy food options to their menu. The client is going to try a new plant-based wrap with no meat and fresh vegetables.”



“I would like to try the veggie wrap,” Amy says. “So the next step is to do a survey to get people’s opinion of the wrap and then track sales to see if the sales increase over time?”

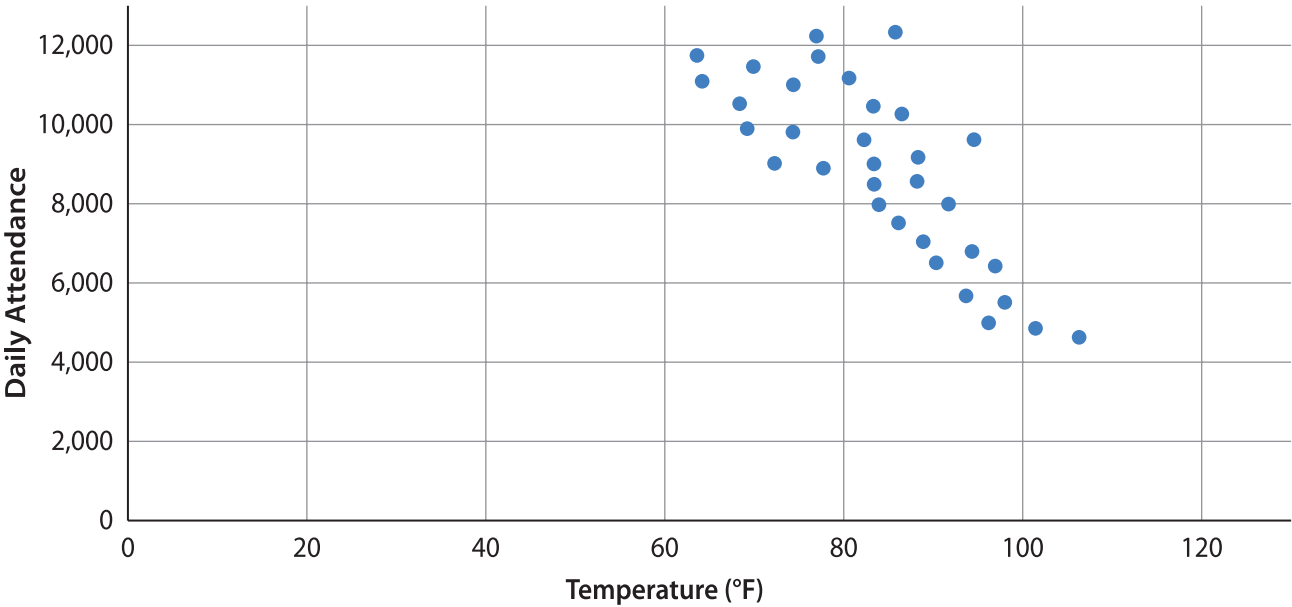
“I just might offer you a job,” her mother says, laughing.

Next, Amy’s mother explains that market analysts need to know how to interpret the data so they can make recommendations to their clients about their next steps. She says one of the other clients is the County Fairgrounds Board. The board wants to increase attendance at the fair this year.

Amy’s mother points to another chart. “We started by looking at the fair attendance over the last three years. The scatterplot shows that as temperatures increase, attendance tends to decrease, especially when it is excessively hot. This is a negative association because as one variable, temperature, increases, the other variable, attendance, decreases.”

Amy asks, “You can’t control the weather, so how can you increase the attendance?”

“Good question,” her mother replies. “The first thing we suggested was to add more shade trees and picnic tables in shaded areas. The fairgrounds are also installing water stations around the fairgrounds so people can fill up their drink containers. We also recommended that they add more events in the mornings and evenings, when the temperatures are cooler.”



Amy's mother explained that every year, her team does exit surveys as people are leaving the fair. They ask people about their reasons for coming to the fair, what exhibits they like, what zip code they live in, their age, their email address, and other information about themselves. Then the team uses that information to form the marketing strategy.



"Another thing our research found is that there is a positive correlation between attending the fair and attending other types of outdoor events like festivals and concerts. So we follow trends in music to know what musical acts to bring to the fair. We hand out flyers at local festivals to make sure people know when the fair starts."

"I bet a lot of people who go to farmers markets also go to the fair," Amy says.

Her mother's eyes light up. "That's a great suggestion! The fair attracts diverse groups of people. The fair has animal competitions, midway games, amusement rides, concessions, arts and crafts exhibits, gardening exhibits, live shows, and of course, fair food! There's something for everyone."

"I am always trying to find ways to connect products and services to people with different interests. Using scatterplots helps me identify possible relationships between different variables. If it looks like there might be a relationship, I follow up with more research to measure the strength of the relationship and make recommendations on how to focus the money spent on advertising."

"I never realized how much a company does with the results of surveys," Amy said. "Being a market analyst sounds like a fun job."



Main Idea

We use scatterplots and statistical analysis to look for relationships between variables.



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