Core Knowledge® MATHEMATICS

Connecting Math to Our World: The Power of Math

Student Reader

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The Power of Math

Student Reader

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The Power of Math

Table of Contents

Let’s explore math in our world!
Math is everywhere, even in art.

It helps us understand . . . how to crack a secret code.

It helps us design . . . a community garden greenhouse.

It helps us answer questions like . . . what can ice cores tell us about past climates?

What else can math do?
Let’s find out!

Bailey Starts Banking

Chapter 1

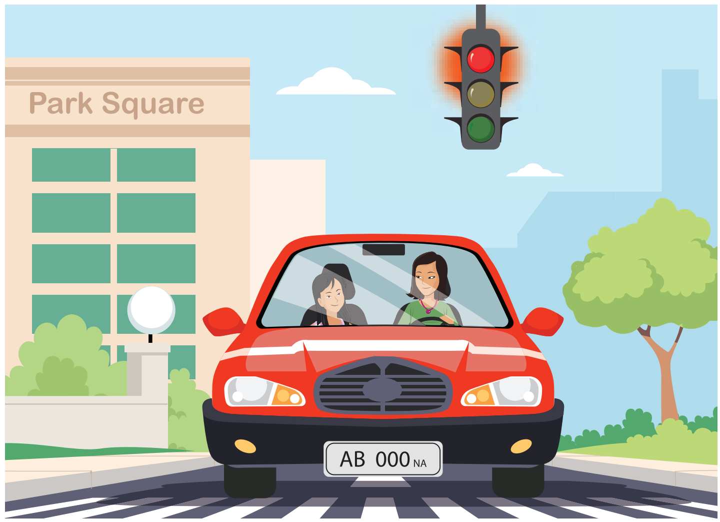
Bailey sits on her bed and opens a metal box. She looks at the money inside and smiles. Bailey keeps all her money in the box. Her grandmother likes to give her money "just because." She also earns money by doing small jobs around the house and neighborhood. She feels proud when she gets paid for work. Bailey likes to recount her money each time she opens the box.

Bailey's mom comes into her room and says, "Bailey, I think it's time for us to open a bank account for you. We can open the account in both of our names, but it will be your account. A bank account is a good way for you to keep track of your money. We will get a debit card so that you don't need to carry money all the time. People can also send payments directly to your account. Tomorrow, we'll get you a state identification card, and then we can go to the credit union to open an account."

The next day, Bailey and her mom fill out the form to get her new ID card. “What is a legal name?” asks Bailey. Her mom says, “Some people shorten their name or use a nickname in daily conversations. Your sister, Megan, likes to be called Meg. But for IDs and other legal documents, she must use her legal name, Megan.”

Bailey’s mom hands the clerk at the desk the completed form, Bailey’s birth certificate, her own ID as Bailey’s parent, and proof of where they live. The clerk looks over the documents and the form. Next Bailey signs her legal name on the electronic pad. Then she stands by the blank wall to get her picture taken. A few minutes later, her photo ID is ready to go. Then they are off to the credit union.

"What is the difference between a bank and a credit union?" asks Bailey. As she drives, her mom explains that banks and credit unions are both financial institutions. They both offer checking and savings accounts and loan out money.



Checking accounts are used for everyday spending. Most people pay for things out of their checking account online or use a debit card, but some people still write checks. Whichever method you use, the money is taken out of your account to pay someone.

Financial institutions pay you to keep your money in a savings account. They pay you interest on the money you have in your savings account. The interest may not seem like much, but it adds up. Savings accounts are good for the financial institutions, too. They use your money to loan out to other people. They make money because they charge interest on the money they loan.

A person takes out a loan when they don't have the money to buy something big—like a house or a car. Each month, they make a payment toward the loan. In the end, they will pay more money than the item cost because they will need to pay interest on the money they borrowed.

Banks and credit unions are different. Only members can have accounts at credit unions. A credit union is a nonprofit institution. It is owned by its members. The money credit unions make from interest is given back to their community. Anyone can open an account with a bank. Banks are for-profit institutions, and they are owned by shareholders. The money banks make is given to the shareholders. Both credit unions and banks are safe places to put your money.

At the credit union, Alisa the teller greets them. Bailey's mom explains they want to open Bailey's first bank account. Alisa replies, "This is wonderful! Here at Peacock Credit Union, we encourage young customers to learn about finances. Do you know what type of account you would like?"

Bailey smiles because she knows about the different accounts. "I want to open two accounts—a savings account and a checking account." Alisa says that is a great idea. She gives Bailey a form to fill out and makes a copy of Bailey's ID.

Alisa asks Bailey how much money she wants to deposit into each account. Bailey's mom suggests that Bailey keep five dollars in cash. Bailey says, "I think I should divide the rest of the money between the two accounts." She takes her money out of her money box and hands it to Alisa. Alisa counts the money and records the amount to be placed in each account.

Alisa gives Bailey her debit card and has her choose a personal identification number (PIN). "Pick a four-digit number that you will remember," says Alisa. She suggests that Bailey not use her birthday or numbers that match her address or phone number because those are easy numbers for people to guess. She cautions Bailey not to share her PIN with anyone because then they could get her money.

Bailey asks, “Can I use my credit card today?”

Alisa quickly explains, “Bailey, you can use your card today, but this is not a credit card. It’s a debit card. When someone uses a credit card, the money does not come out of their checking account. Using a credit card is the same thing as a loan. The borrowed money must be paid back with interest if the balance isn’t paid off the same month. That means the amount paid back is more than the amount that was borrowed. A debit card takes money from your account.”

Before Bailey and her mom leave the credit union, Alisa shows Bailey how to use her PIN on the automated teller machine (ATM) to deposit money into and withdraw money from her accounts. Alisa suggests that Bailey download the credit union app onto her phone. This way Bailey can use the app to see her transactions and how much money she has in her accounts.

Bailey thanks Alisa for all her help, and she and her mom head home. At home, Bailey takes her debit card and money box to her room. She likes the idea of accessing her money without carrying cash around. And using a debit card feels so grown-up!

Main Idea

Keeping track of money is an important life skill.

A Balancing Act

Chapter 2

That Saturday, Bailey decides that her first debit card purchase will be taking her mom and sister out for milkshakes. They walk to the local ice cream shop and find a table by the window. A server comes over to take their order. "I'd like a chocolate milkshake," says Meg.

"Make that two," adds her mom.

Bailey says, "I'll have a strawberry milkshake. And I'll be paying for all three."

Bailey is proud that she can pay for today's treat. When the server brings the bill to the table, Bailey looks it over to make sure it correctly shows their order. She hands the bill back to the server along with her debit card.

In a few minutes, the server is back with the two charge receipts, one for the shop and one for her to keep.

“Mom, there’s a place for a tip on the receipt. Can I use my debit card for the tip?” asks Bailey. Her mom replies, “You can put the tip on your debit card, or you can leave cash. It is up to you.”

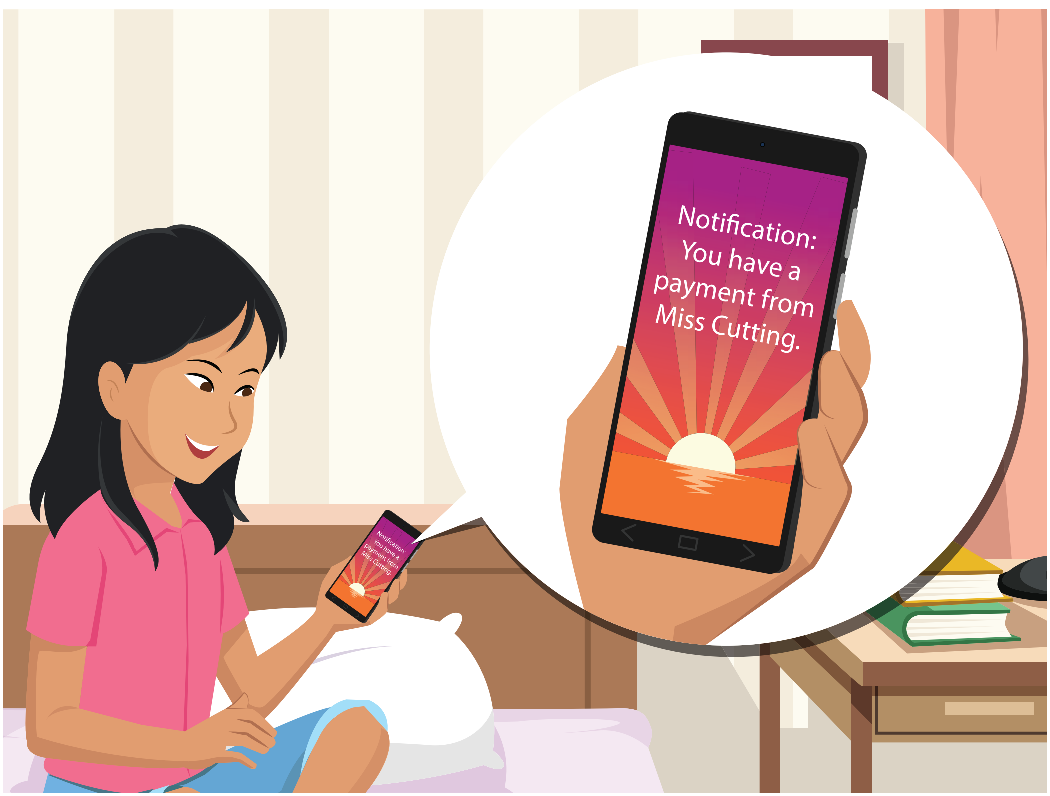
Bailey figures out 15% of the cost of the milkshakes and writes it on the receipt. She adds the amount of the sale and the tip together and records the total on the receipt. She signs her name. She fills this information onto her copy of the receipt. When she is done, she picks up her debit card and her copy of the receipt.

Bailey’s mom says, “Make sure that you keep your receipt so that you can double-check your bank statement. It doesn’t happen often, but sometimes there can be mistakes.”

As they walk home, Bailey’s mom and sister thank her for the treat.

Bailey sees that a checking account helps her manage her money. Each time she earns money, she puts some in her checking account and some in her savings account. She also keeps a little cash in case she needs it.

Bailey has learned that people can put money directly into her account from theirs. Every bank and credit union has a special number called a routing number. And every person has an account number for each account they have. If someone knows the routing number to her credit union, they can transfer money from their account to her account electronically. This helps Bailey keep track of who pays her and when they pay her.



Bailey likes to transfer money from her checking account to her savings account. Moving money to her savings account makes her less likely to spend it. Plus, she earns interest on her savings account. The more money in her savings account, the more money she will earn. She likes watching that account grow larger.

Whenever Bailey gets a receipt from a purchase or a deposit, she puts it into the metal box where she used to keep her money. "Bailey, you have some mail," calls her mom. Bailey rushes from her room and opens the envelope. It is her account statement.

"Go get your receipts," her mom says. "Let's make sure the statement agrees with your own records." The two sit at the kitchen counter. As Bailey goes through her statement and receipts, she puts checkmarks on receipts and the matching transactions on the statement. She makes sure all her deposits and payments also appear on her statement.

One day, Bailey and her mom are shopping. Bailey wants a new bike. She checks the credit union app and sees that she doesn't have enough money in her checking account to buy the bike. She wonders what happens if she doesn't check her app and tries to purchase the bike. Her mom explains that if she isn't careful, purchases can be declined at the register. The register is a computer that communicates with your bank through your account number.

"You can always transfer money from your savings account to your checking account so that you have enough money," her mom explains. Bailey thinks about whether she wants to do that or if she wants to wait until she earns more money. "I am going to wait. I hate to see my savings go down!" Bailey says.

At home, Bailey's mom shows her that her account information is also available online. It has all the same details as the mailed statement and on the app. It's just another way she can check her accounts at any time. One purchase is labeled "pending." Bailey's mom explains that sometimes it can take a few days for a purchase amount to be removed from her account. "It is always a good thing to check for pending purchases so that you don't think you have more money than you do!"

Bailey says, "I've learned a lot about finances since I opened my own checking and savings accounts. Now I know I need to check my balance before using my debit card. I must also keep track of pending transactions. I'm glad you told me to save my receipts so that I can double-check my statement. And I know how to make money by saving money!"

"Very good," says Bailey's mom. "What else have you learned?"

"I have learned that using math to keep track of my money is very rewarding!" says Bailey.

Her mom smiles and says, "Bailey, I'm proud of you. You are learning a lot about managing your money."

Main Idea

Tracking how much money you have and how much you spend helps you meet your goals.

The Mathematical Minds of Animals

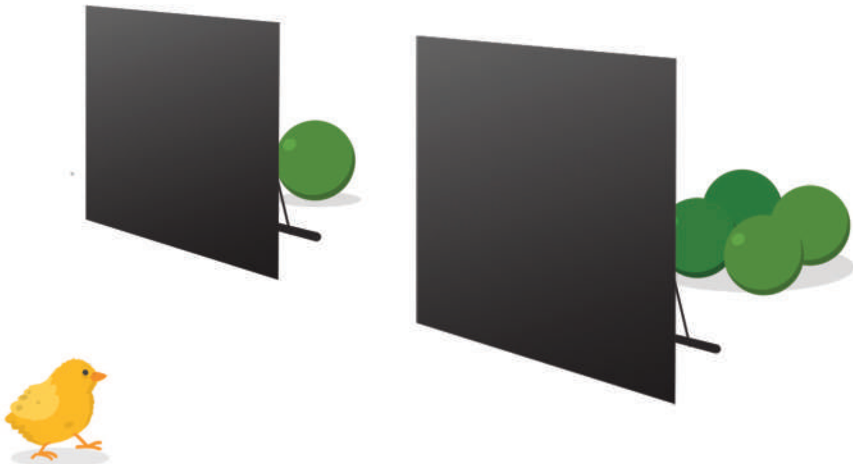
Chapter 3

Humans aren't the only ones who use math. Many mammals, fish, birds, and insects use math to make decisions. They use math to find their way home and to build places to live. They can even use math to determine which food source is bigger than another!

These creatures may not use clearly defined number systems the way we do to make their calculations, but this makes their ability to solve complex mathematical problems even more amazing!

Humans did not invent math. Instead, our math skills developed over time by observing and asking questions about nature. Today, many things we do involve using math skills.

Let's see how some animals use math!



At just three days old, chickens can determine which of two amounts is greater. Researchers who observed this wanted to know if chickens can count, so they designed an experiment. They placed small plastic balls in the nests of hatchlings. The baby chicks associated the balls with the safety of their nest. To the chicks, the balls became familiar objects, and they wanted to stay close to them.

Next, researchers moved the balls one at a time behind two screens while the chicks watched. When the chicks were released, they went to the screen with the larger group of balls.

The next step was to see if the chicks could add and subtract. After watching the researchers slowly move balls from behind one screen to another, the chicks still chose the screen that had the most balls behind it!

Many other animals can distinguish amounts. In one study, 100 chimpanzees were each given two bowls that contained chocolate pieces to see if the chimps would choose the bowls with the most chocolate. The chimps chose the bowls with the most chocolate ninety percent of the time! 90 out of 100 chimps knew which bowl contained more.

When you hear the phrase "school of fish," you may imagine a large group of fish swimming together. What you might not think about is how the fish form these large groups. Scientists observed that fish stayed in groups for protection. They then wanted to know if the fish could actually tell more from less.

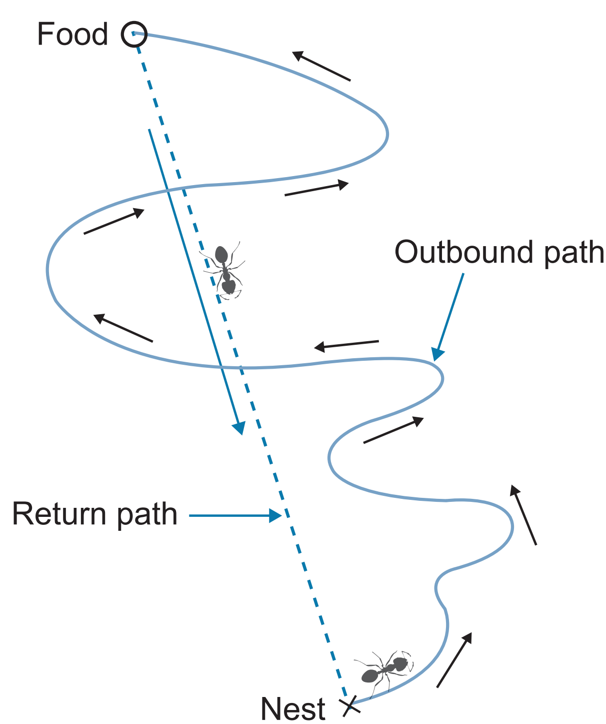
Safety in Numbers

The idea of "safety in numbers" is not unique to fish. Other animals, including humans, rely on this idea for survival. Safety in numbers is one reason humans first formed communities. Studies also show that if a human sees a threat when they are alone, they believe the threat is much closer than it actually is. When humans are in a group, they see the threat as farther away.

Studies show that newborn guppy fish can determine larger values from smaller values up to the number 4. This is the same as a newborn human baby. They know that 1 is less than 2 and that 3 is less than 4. As they age, the fish develop the ability to compare even larger numbers. Using math is important for survival. It helps them to select the largest group of fish to join so they can avoid predators.

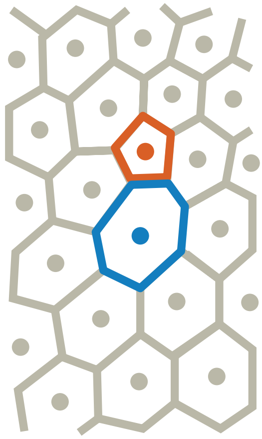
The Sahara is a challenging place to survive and navigate. The landscape is vast and has very few landmarks, which can make finding your way difficult—but not for the tiny desert ant! These insects leave their nests each day and wander through the desert in search of food, zigzagging across the sand. However, they do not retrace their steps to get back home. Instead, they return to the nest in a straight line—the shortest distance between two points!

Scientists observing the ants at the University of Zürich in Switzerland had an idea. They hypothesized that the ants count their steps and use the position of the sun and known landmarks to figure out where they are. They tested this idea by having groups of ants travel away from their nests. The scientists then attached tiny stilts to some of the ants, making their legs much longer. The ants without stilts were able to find their way straight back to the nest. The ants with stilts, however, traveled past the nest. This is because the stilts changed the length of the ants' strides. The ants with stilts still counted their steps, but each step was now much longer. The length of their legs mattered in order for their step-counting strategy to work.



Honeybees and wasps are skilled architects. Both types of insects can recognize shapes and patterns. They use hexagons to build their nests. This shape offers many benefits. Hexagons are strong and require fewer resources to build. For honeybees, this means less wax, and for wasps, less paper.

Some species of honeybees and wasps build nests using a repeating pattern of same-sized hexagons. Other species use hexagons of different sizes: smaller ones to raise larvae that grow into female workers and larger ones to raise larvae that grow into male drones and the queen. But how can smaller and larger cells fit within a structure of same-sized hexagons?



The insects solve this problem by adding and subtracting sides from the hexagons. Removing one side from a hexagon makes a smaller pentagon for the worker larvae. Adding an extra side to a hexagon forms a larger seven-sided heptagon for the drone and queen larvae. These pentagon-heptagon pairs are surrounded and supported by medium-sized hexagons.

Crows are highly intelligent animals. They can mimic human voices, play tricks on each other and other animals, and use tools to find food. Crows can also do addition and subtraction and use reasoning to make decisions based on patterns.

Researchers at the University of Tübingen in Germany tested crows' reasoning skills by giving them treats when they pecked pictures on a screen. First, the crows were rewarded with a treat each time they pecked a certain picture. Then, researchers changed how often the crows were given treats for pecking each image. For example, if Picture 1 were shown ten times and the crow pecked the photo each time, they might only get a treat three times. If Picture 2 were shown ten times and the crow pecked every time, they might get a treat six times they pecked. The crows kept track of these changes. When shown both pictures, the crows would pick the picture most likely to result in a treat!

Just like you, animals understand the importance of applying math!

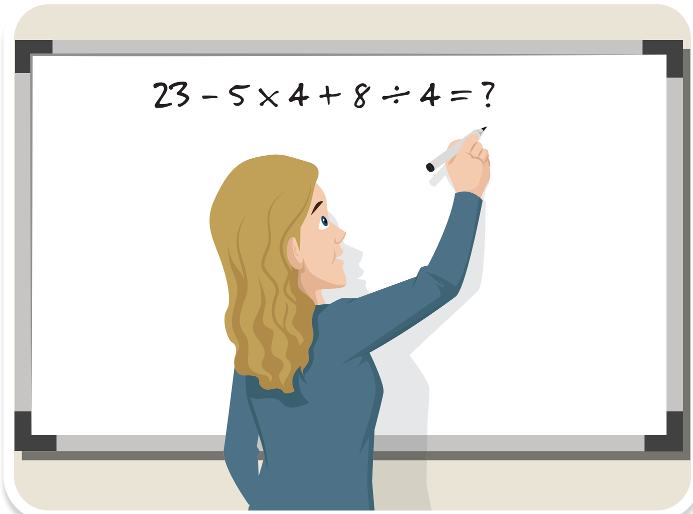
Main Idea

Some animals do things that show they make choices using math skills.

My Dear Aunt Sally

Chapter 4

One day in math class, Mrs. Cangiano writes an equation on the board.



$23 - 5 \times 4 + 8 \div 4 = ?$

She says, "Raise your hand if you know the answer to this problem."

Slowly, hands start going up around the room. "Let's see how you did! Colby, what is the answer?"

Colby answers, "20."

"Does everyone else agree?" asks Mrs. Cangiano. Most students in the class nod. "What would you say if I told you that the correct answer is 5?" The class breaks into an uproar.

Mrs. Cangiano holds up her hand. "Let me explain! Sometimes math equations have different parts. We call this a multistep equation. It is important to know what order to do the steps in to get the correct answer. This is called the order of operations. Over centuries, the order of operations changed until it became the method we use today."

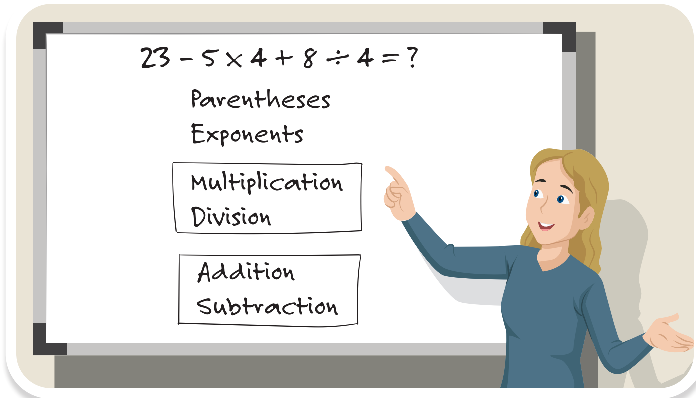
Mrs. Cangiano points to some words on the board.

Parentheses

Exponents

Multiplication Division

Addition Subtraction



She explains that when solving an equation, we follow a series of steps:

Step 1: Solve the parts that have parentheses.

Step 2: Solve the parts with exponents.

Step 3: Solve the parts that use multiplication and division. The box around the words means that these operations are solved as they appear from left to right.

Step 4: Solve the parts that use addition and subtraction as they appear from left to right.

Mrs. Cangiano directs the students to the equation on the board. "This equation doesn't have parentheses or exponents, so we move on to the next step. We solve the parts that use multiplication and division going from left to right."

"$23 - 5 \times 4 + 8 \div 4 = ?$ becomes $23 - 20 + 8 \div 4 = ?$"

"$23 - 20 + 8 \div 4 = ?$ becomes $23 - 20 + 2 = ?$"

"Then we add and subtract from left to right."

"$23 - 20 + 2 = ?$ becomes $3 + 2 = \mathbf{5}$"

Mrs. Cangiano explains that an easy way to remember the order of operations is to take the first letter from each operation and make a phrase with the letters. This is called a mnemonic. A common mnemonic used to remember the order of operations is this phrase: **Please excuse my dear Aunt Sally**. “But you can make up your own phrase if that will help you remember better!” explains Mrs. Cangiano.

Teresa raises her hand. “How about **Pink elephants mostly dance and sing**?”

Robin says, “Or **Purple earthworms must dig around soil**?”

“Those are wonderful!” says Mrs. Cangiano. “The most important thing is that the phrase helps you remember the order of operations.”

"Let me tell you a story that demonstrates why the order used to complete the steps of a math equation is important," says Mrs. Cangiano.

Please excuse my dear Aunt Sally—she really is sweet, but sometimes she gets really excited about making sure things get done in the proper order.

When I spent school break with her, she reminded me each morning to make sure I put my socks on before my shoes. Of course I put my socks on first!

One afternoon, my dear Aunt Sally took me to my favorite restaurant for lunch. I had macaroni and cheese. Then, I had an ice cream sundae for dessert. I would have had the sundae first, but I knew how Aunt Sally feels about doing things out of order!

The server brought us the check when we were done eating. “Please excuse me,” said Aunt Sally. “I need to make sure that I have enough cash, or I will need to charge our meal.”

Aunt Sally asked me to keep track as she checked the money in her wallet. She said she had one \$10 bill and three \$20 bills.

I wrote it down like this:

$$10 + 3 \times 20$$

I explained as I worked it out: “So $10 + 3 = 13$. And $13 \times 20 = 260$. You have \$260 to pay the bill.”

“I wish I did have \$260!” she exclaimed. “But I only have \$70. That’s enough for lunch.”

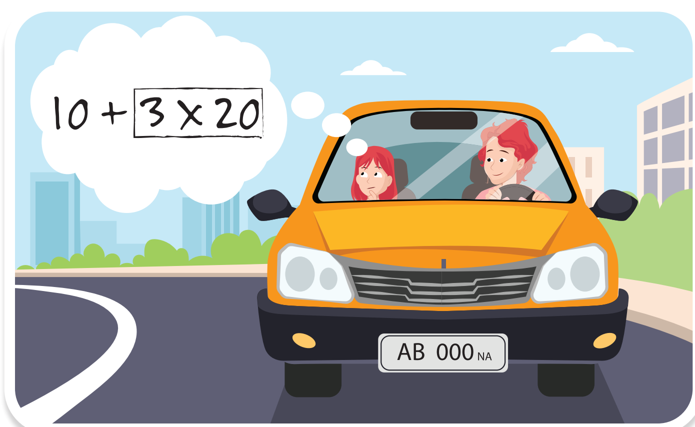
As we were driving home, I kept thinking about the math problem. I couldn't see how my answer could be wrong. I was sure my math had been correct. I knew how to add and how to multiply. I was starting to doubt my math skills. I finally asked Aunt Sally how my answer could have been so far off from the correct answer.

Aunt Sally explains, "You do know how to add and multiply. You just added and multiplied the wrong numbers! Your equation was correct, but your answer was wrong. You needed to multiply and then add to get the correct answer. I told you that I had three $20 bills. You should have multiplied the 3 and 20. That makes $60. Then you needed to add the $10 bill that I had. That makes a total of $70."

$$\mathbf{\$20 \times 3} + \$10 = \$60 + \$10 = \$70$$

Now I understood why using the correct order was important!

"The moral of the story is: When doing math, we have to follow a certain order to get the correct answer," says Mrs. Cangiano.



Main Idea

Following a set order of operations ensures that everyone finds the correct answer when solving a math problem.

Ocean-Traveling Ducks

Chapter 5

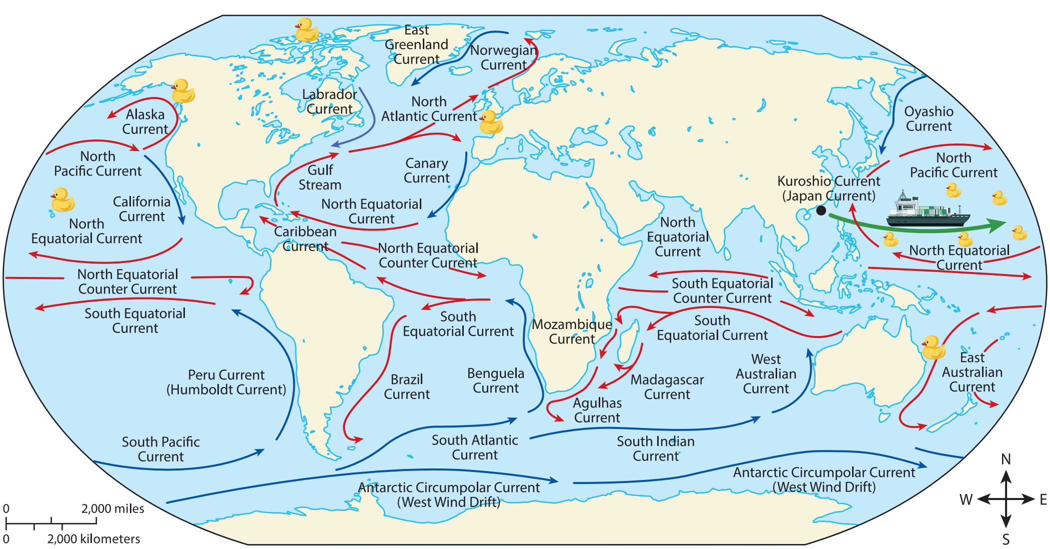
In 1992, a cargo ship left China headed to America. The ship battled rough seas in the North Pacific. During the storm, one of its shipping containers was lost. Inside that container were 28,800 rubber ducks and other plastic bath toys.

These toys were sealed in cardboard boxes, destined for homes around the world. But instead of floating in bathtubs, these ducks found themselves on an unexpected journey floating across the vast ocean.

As the container cracked open and released its colorful cargo, ocean currents carried the little ducks far and wide. Some of the escaped bath toys began washing up on the shores of Alaska at the end of 1992. Some were swept toward Hawaii, washing up on its sandy shores. Others drifted northward, getting trapped in the Arctic ice for years. Eventually, they broke free and continued their journey. In 2007, a few even traveled across the Atlantic Ocean and landed on the beaches in the United Kingdom. Scientists and beachcombers alike took notice of these wandering toys, turning the accident into a fascinating global experiment.

Oceanographers saw an opportunity. By tracking where the toys landed, they could study and model ocean currents. They also could study wind patterns and the way floating debris moves across the sea. Each toy's arrival on a distant shore provided new data. This helped to refine models that predict ocean movement.

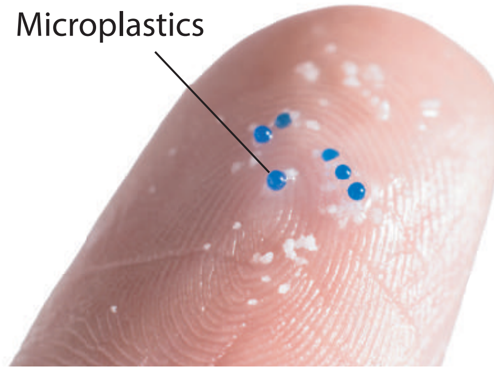
Scientists recorded information for each found rubber duck and other bath toys. The data were put into a computer. Applying what is known about ocean currents and using mathematical equations, the data allowed the computer to simulate where other bath toys might end up.



This unexpected accident provided invaluable information that scientists could not have gathered otherwise. It would be unethical for scientists to conduct a similar study. The floating bath toys did more than expand our understanding of patterns in ocean currents. They also provided information to help us study some of the biggest problems facing Earth's oceans, such as tracking pollution and the spread of marine debris.

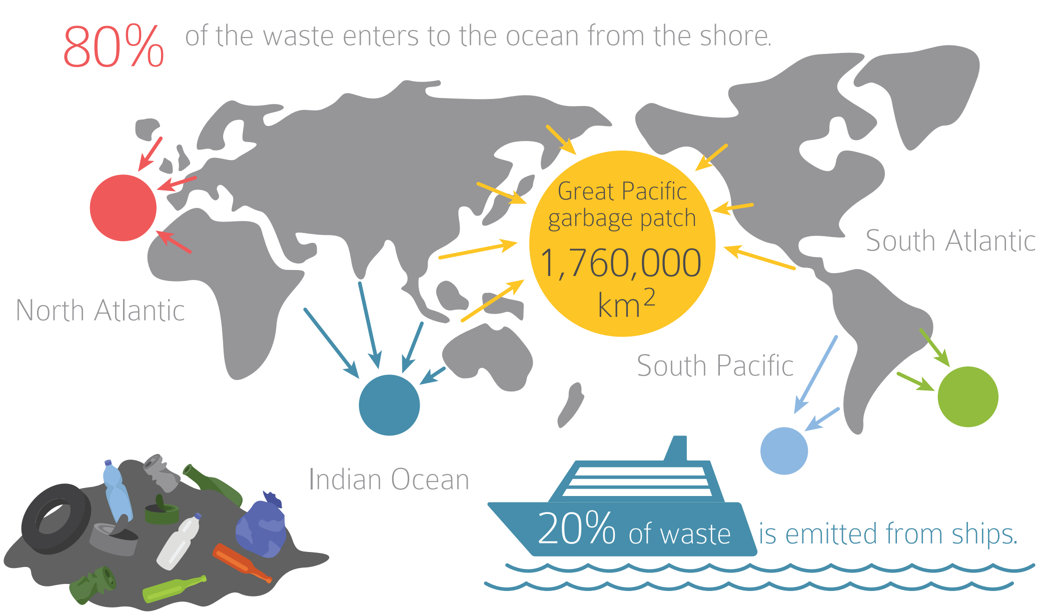
Garbage can accumulate on the ocean floor. It can also float and move with the ocean's currents. All garbage in the ocean is potentially harmful to marine life. For example, organisms can become trapped by the garbage or ingest it.

The journey of these rubber ducks highlighted a growing environmental concern—marine pollution. Their long-lasting presence in the ocean was a reminder of how plastic waste travels and lingers in the ecosystem. Scientists could see how garbage that starts in one place can end up at another far-off location. What began as an accident turned into a crucial lesson on the impact of human activity on the seas.



Ocean garbage patches form where ocean currents concentrate litter, fishing gear, and other trash. Much of the material in these patches are microplastics. Plastics do not go away. Instead, they break down into tiny pieces. Ocean animals eat these plastics as they feed.

GARBAGE PATCHES MAP



Today, the story of the lost rubber ducks remains a remarkable tale of science, chance, and the unintended consequences of global trade. Though many of the original ducks may still be floating somewhere in the ocean, their journey continues to inform research and raise awareness about protecting our planet's waters.

Main Idea

Math can be used to evaluate and model a problem by looking for patterns and using the right tools to gather information.

Cracking the Code

Chapter 6

Jenna leans forward eagerly as Nico opens an envelope. “Where did you find that?” Jenna asks.

“It was taped to my front door,” Nico replies.

Nico’s eyes dart across the page, taking in the tiny, cramped writing. “It’s from Pete,” Nico says, “but I don’t think this is a normal letter.” He hands Pete’s letter to Jenna so that she can read it for herself.

dear friends,

the Zoo is Really lovely this time of
yeaR. i like to Go often,
especially to see the ZebRas aNd
the Gentle oranGUtans.
i ate some ice cReam BY the Quail
exhibit ONE Afternoon. then i DasHed
oVer to the Pelican eXhibit.
Next Friday the 13, i pLan to Be tHere
again. Perhaps the Next visit will be
just As fun.

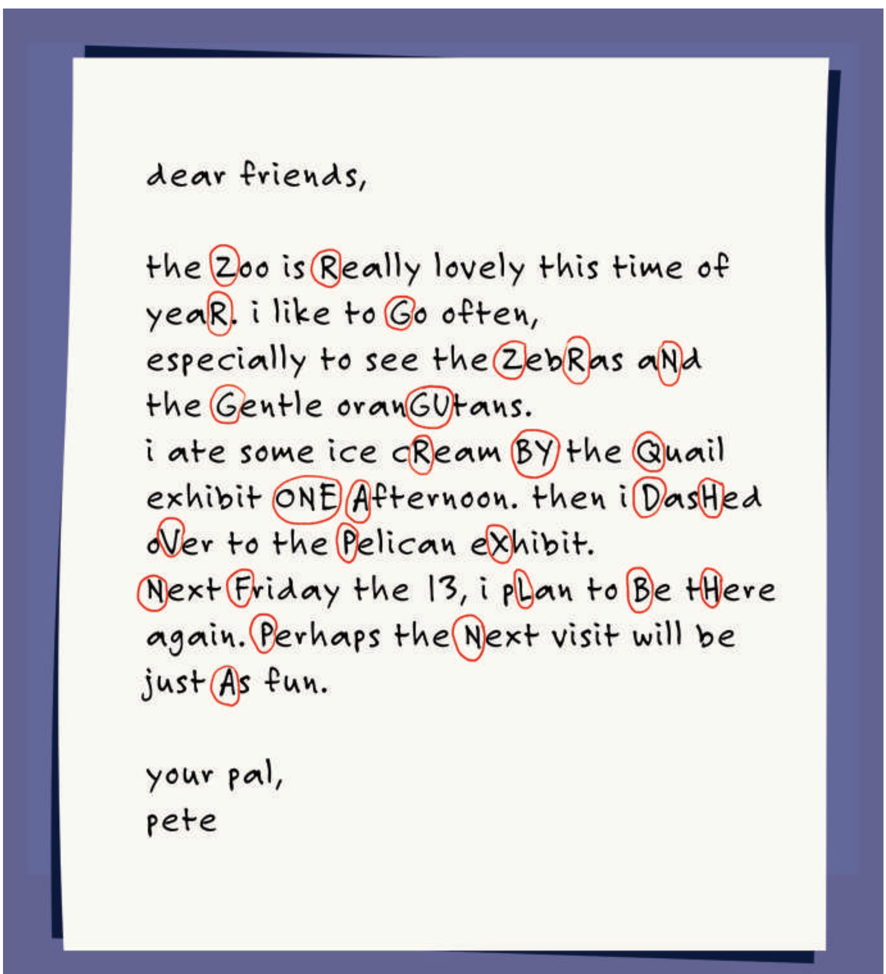
your pal,
pete

Jenna examines the letter. So many things seem strange. Pete doesn't live anywhere near a zoo. His letters are usually much longer. And why is it written in such a strange way?

Nico studies Jenna's face closely as she thinks quietly to herself. "Nico, do you have a pencil?" she asks.

Nico watches as Jenna quickly circles letters in the text. "Nico, I think Pete is sending us some sort of secret message. What do you notice about the letters?" she asks as she points to the areas she has circled.

"Some of these letters are uppercase, but others are not!" Nico exclaims.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just As fun.

your pal,
pete

Jenna says, "I think all of these capital letters make some sort of hidden message. Let's write them down and see what they spell out."

Nico glances over Jenna's shoulder as she finishes writing the final letter. "Jenna, that's not a message—that's just gibberish! Leave it to Pete to play a silly prank like this."

dear friends,

the Zoo is Really lovely this time of yeaR. i like to Go often,
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i ate some ice cReam BY the Quail
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your pal,
pete

ZRRGZRNGGURBYQONEADHVPXNFLBHPNA

Jenna shakes her head in disagreement. "Pete does love silly pranks, but he also loves codes and puzzles. I think he encoded this message using a cipher, but which one?" Nico blinks twice; what exactly is a cipher? Jenna sees the confusion on Nico's face and explains, "A cipher is a way of hiding the meaning in a message by replacing letters with other letters or symbols."

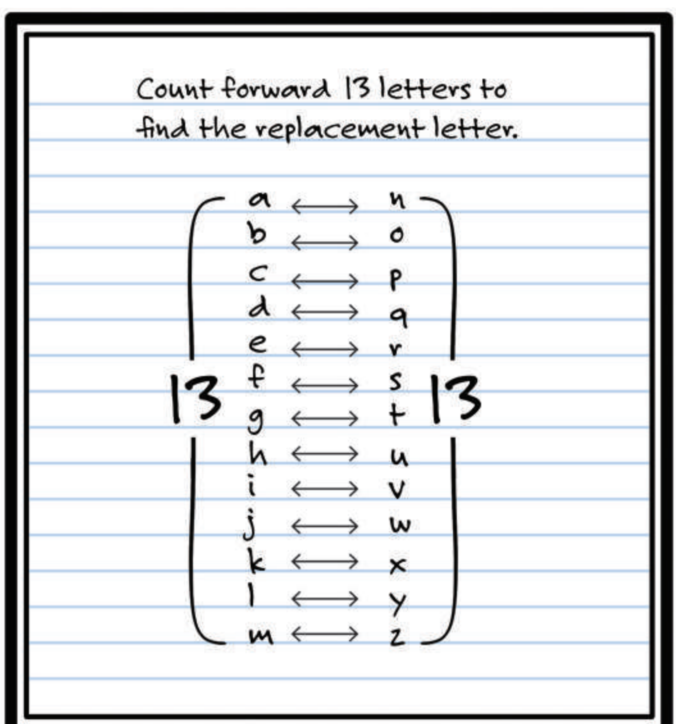
Nico then asks, "But how will we know which cipher he used?"

Codes and Ciphers

Codes and ciphers are a way of encoding, or hiding, information so that it is only understood by the intended audience. Codes swap one symbol, letter, or number for another. Ciphers use algorithms, or a set of mathematical rules or operations, to encode a message.

Jenna frowns and glances down at the letter again. Nico makes a good point. How would they know which cipher to use? Did Pete leave a clue for them in his message? She scans the text again, looking for something she had missed. She knows that trial and error is a good way to solve a problem. Try one thing, and if it doesn't work, adjust and try another.

"I think I've found it!" Jenna points to the number 13 in the text. "This is the only number that Pete mentions in the letter. I think he's telling us he used the ROT13 cipher to encode his message!" Before Nico can ask, Jenna explains. "ROT13 is a cipher that replaces each letter in the alphabet with the 13th letter after it. It's a simple way to hide information! It works because there are 26 letters in the alphabet." Jenna grabs a piece of scrap paper and begins writing letters. Nico watches in amazement as she quickly jots the ROT13 cipher for him to see.



Jenna explains, "See? Each letter stands for another one in the cipher. An 'a' in the encoded message means 'n.' And an 'n' in the message really means 'a.' It'll be faster if we work together. You read me each circled letter, and I'll find what it corresponds to in the cipher! Then you can record the deciphered letter below the coded one." As Nico reads each letter aloud, Jenna calls out the decoded letter, and Nico records it. After studying the letters, they figure out the spaces between the words. Within just a few minutes, they have cracked the code.

ZRRGZRNGGURBYQONEADHVPXNFLBHPNA

MEET ME AT THE OLD BARN QUICK AS YOU CAN

The two stare at the message for just a moment before Jenna jumps up and grabs her backpack. "The old barn is only a short bike ride away. We can be there in fifteen minutes if we leave now!"

Encoding and Decoding

Encoding keeps the information on our computers and smartphones private. Cryptographers are code makers; they develop ways to encode information using very advanced math so that it stays safe from hackers. Cryptanalysts, on the other hand, are code breakers; they decode information based on patterns.

The friends hop on their bikes and pedal furiously. They haven't seen Pete since his family moved away at the end of the last school year. Could he really be waiting for them at the old barn? They pedal onto the long driveway. As the old barn comes into view, they see something shadowy move in the large doorway. Jenna calls to Nico, "Hey, I think I see someone. Do you think that's Pete?" They don't have to wait long to find out.

Pete runs toward them from the barn, carrying something—or is it three somethings?—in his arms. Jenna squeals with delight, not only at the sight of her friend Pete but at the armful he is carrying. "My uncle invited us for the weekend," Pete shares. "His dog Maisy just had puppies!"

Nico hops off his bike to scoop up one of the adorable puppies.

Cracking the code in that message really paid off!

Main Idea

Codes often use mathematical patterns.

The Development of Number Systems

Chapter 7

Imagine you come across a cave filled with treasure. Chests overflow with hundreds, maybe even thousands, of jewels. How could you count them all? Do you count each jewel one by one? Do you make piles of jewels then count the piles? If so, how many jewels do you include in each pile—10, 12, 20, 60?

Because we use a base-10 number system, it's likely you would put 10 jewels in each pile. But your answer might be very different if you came from a civilization from the past! How exactly did the base-10 system develop? How have other people around the world organized their number systems?

Before number systems, we think people used a system of counting where 1 tally mark represented 1 item. A tally mark was easy to make because scribes were carving the marks onto clay and stone. Straight lines are easy to make with a chisel—5 marks represented 5 items, 23 marks represented 23 items, and 47 marks represented 47 items.

As you can imagine, it was easy for people to lose track of what they were counting, especially as the number of tally marks grew larger and larger. People needed a better way to count and represent numbers. Let's look at some other number systems and see how they work.

Tally Sticks

In 1937, an archaeologist named Karel Absolon found a 7-inch (18 centimeter) wolf bone in the modern-day Czech Republic. But this was no ordinary wolf bone—it was a tally stick, a piece of wood or bone that early peoples used for counting. It was covered in 2 groups of notches, suggesting the use of some kind of base system.

The Roman number system dates to the 8th century BCE. The Romans used symbols instead of tally marks to represent larger sums. This system uses 7 letters to represent 7 numbers.

I = 1   V = 5   X = 10   L = 50   C = 100   D = 500   M = 1,000

To write numbers, the Romans used these letters and their positions. For example, they wrote the number 1 as the letter I, the number 2 as II, and the number 3 as III. They did not use IIII for the number 4. Instead, they used the letter V for 5 and put I in front of it. 4 was written as IV, or 1 less than 5. This is called a subtractive notation because it represents subtracting one number from another. They wrote 6 as VI, or 1 more than 5.

There are places that we see Roman numerals today. Sometimes Roman numerals are used on clocks. Roman numerals are sometimes used to number book chapters. You can find a roman numeral at the bottom of the pyramid on the back of the dollar bill.

The Roman numeral system is better to count large numbers than the tally system, but it had some drawbacks. Because ancient Romans only assigned letters to 7 numbers, other numbers were represented by combining these 7 letters. This means some numbers were very long. Large figures could get quite confusing—and involved addition and subtraction to figure out what they meant!

The year 1999 is represented as MCMXCIX. MCMXCIX means M + CM + XC + IX, or 1,000 + 900 + 90 + 9 = 1,999.

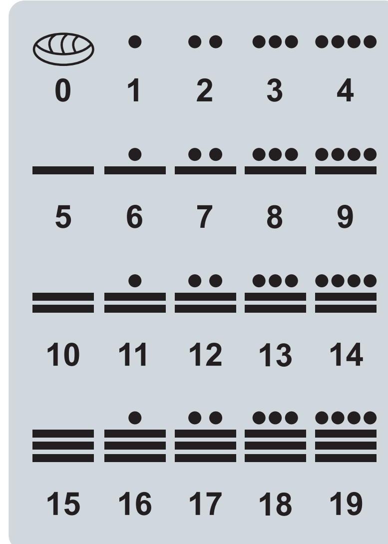
The highest number that can be written using the traditional Roman numeral system is MMMCMXCIX, or 3,999. This is because there cannot be more than 3 of the same letter together. Imagine if the largest number you could use was 3,999!

It is possible to add and subtract with the Roman numeral system, but it can be tricky. Because the Roman numeral system does not include place value, multiplication and division are challenging. Most people today would convert the numbers to today's number system to do these operations.

Over 3,000 years ago, the ancient Maya began to build an empire that grew to include what is now Mexico, Guatemala, Belize, Honduras, and El Salvador. As their civilization developed, the Maya created an impressive number system. They became skilled astronomers who used math to study the sun, moon, and planets. Using math skills, they developed an extremely accurate calendar system based on the observations they made of the sky. The Maya calendar was a system of two calendars that worked together. One of the calendars was the Haab calendar. It was circular in shape. It had 19 months—18 months that were 20 days long and 1 month that was 5 days long. Some indigenous Maya communities still use the Maya calendar.

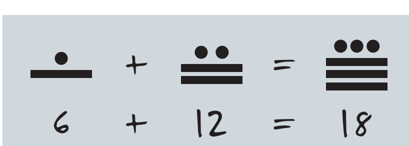
Archaeologists and math historians have studied stone tablets from this period and have discovered much about the Maya number system. We know that the Maya system is considered more advanced than the Roman numeral system. One reason is that the Maya were one of the first civilizations to use the number zero. And the Maya number system uses place value, which means very large numbers can be written. These large numbers were useful when the Maya studied astronomy and made their calendars.

The Maya number system is a base-20 number system, where the system we use today is a base-10 number system. The Maya system uses 3 symbols. Zero is represented by a shell, dots represent single digits from 1 to 4, and a horizontal bar represents 5. The chart shows how to write the numbers from 0 to 19.

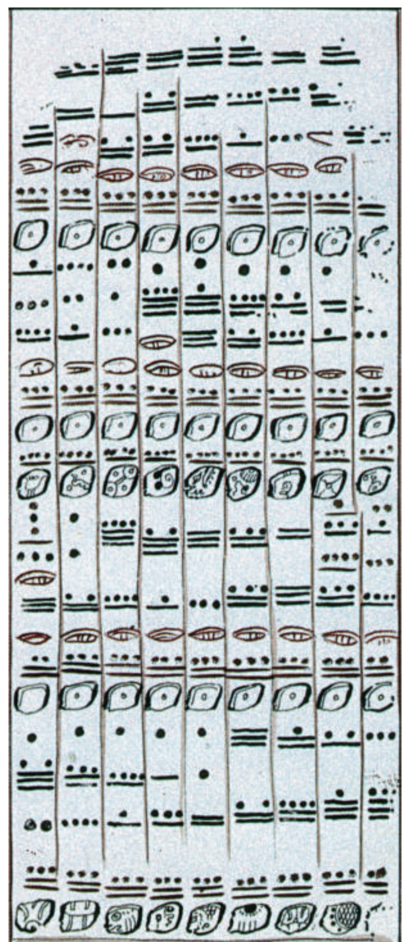


The system is positional, which means that the placement of the symbols indicates place value. The number system that we use today uses horizontal placement to show place value. The Maya number system used vertical placement to show place value. In this image, the vertical level of numbers can be seen.

Addition and subtraction are easy to do using the Maya number system.



The Maya probably used a base-20 system because most people have 10 fingers and 10 toes!

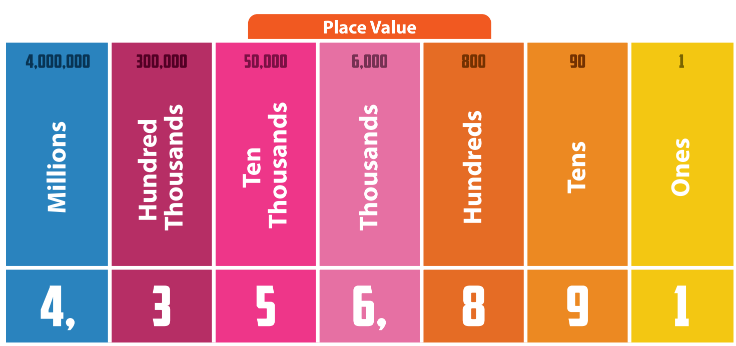


Maya numerals are represented vertically with the lowest place value at the bottom and a space separating each place value.

Scholars believe base-10 number systems developed because people have 10 fingers. The base-10 system is based on Hindu-Arabic numerals. These numerals represent values from 0 to 9. The numerals are used again and again in different combinations to show value. Using Hindu-Arabic numerals has many benefits. First of all, their value is easy to identify. We know that 2 is always greater than 1 and 3 is always less than 4. Also, the use of these numbers simplified mathematical equations. Hindu-Arabic numerals make adding, subtracting, multiplying, and dividing much easier.

Hindu-Arabic numbers were used to develop the positional system in our base-10 number system. In this system, the position of the digit tells us its value. In this image, there are 4 millions, 3 hundred thousands, 5 ten thousands, 6 thousands, 8 hundreds, 9 tens, and 1 one. The place-value system serves many different purposes. It follows a straightforward pattern that helps us quickly and easily compare numbers to determine value. It's also helpful for keeping track of extremely large and extremely small numbers.

The base-10 number system has proven to be a practical way to represent and manipulate numbers. It is a fundamental tool in mathematics and daily life.



Main Idea

Number systems have evolved throughout history based on human needs.

Binary Code

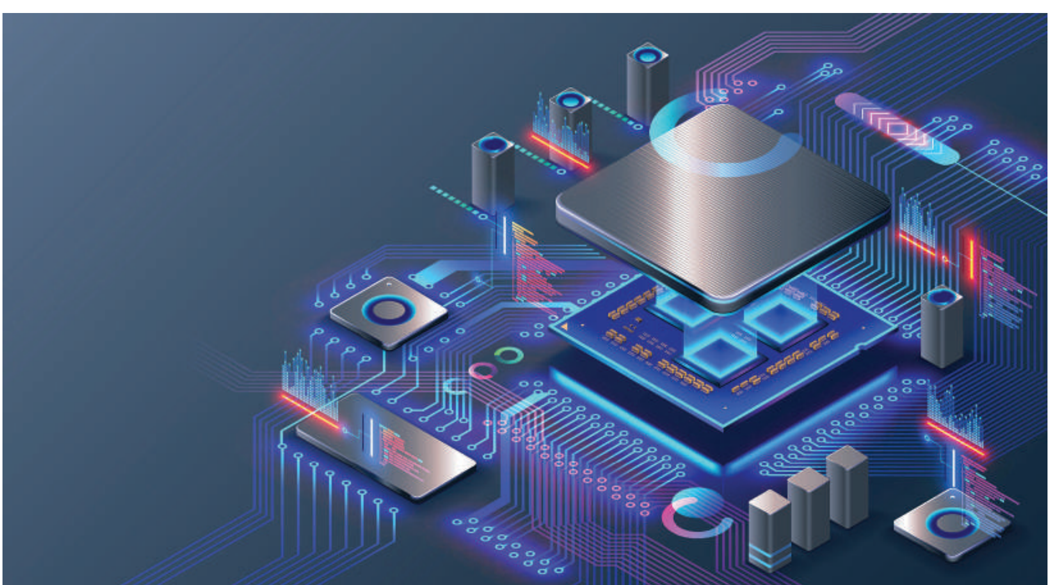
Chapter 8

Morgan types an email to her friend Pedro about a recent visit to Saguaro National Park. She selects the font and adjusts the size of the text. She bolds words to emphasize ideas. She also includes photos she took on her trip—including one of the largest cacti in the United States. She hits "send," and the email makes its way from her computer to Pedro's.

To Morgan, these actions are a keystroke or click of a button. She doesn't know that every movement she makes on her computer is controlled by a very simple base-2 system called binary code.

Morgan knows about the base-10 system. She knows a base-10 system uses 10 digits from 0 to 9. Even though Morgan may not be familiar with the base-2 system, she uses it every day. It gets its name *binary* from the Latin word *bini*, which means "two by two." If a base-10 system uses 10 digits, how many digits do you think a base-2 system uses? Just 2! All of binary code is written using just the digits 0 and 1.

Binary code is used to transmit and store data and to process information in computers, smartphones, cable TVs, satellite TVs, satellite radios, CDs, and DVDs. Anything that is computerized, such as cars, dishwashers, coffee machines, microwaves, and almost all other digital electronics, uses binary code!



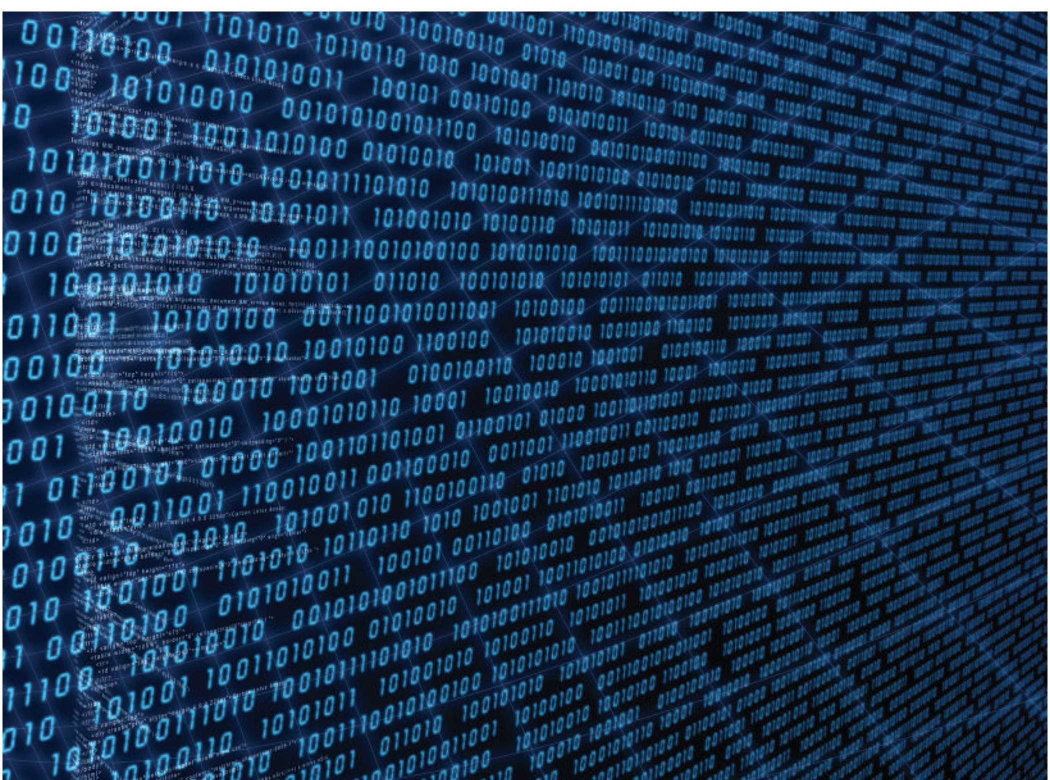
The binary code that computers use is not new. In fact, base-2 number systems have existed for hundreds of years, long before computers were invented. The binary code we use is based on the work of German mathematician Gottfried Leibniz. During the 1600s, Leibniz perfected a system based on small pieces of data called bits, short for "binary digits." In this system, a bit is the smallest unit of data. A bit is either a 0 or a 1; it can never be both, and it can never be another number.

Gottfried Leibniz

Gottfried Leibniz (1646–1716) was a German philosopher, logician, and mathematician who is known for being one of the inventors of calculus. But that is not all Leibniz accomplished. He designed clocks, submarines, and windmills, and he suggested ways of improving education. He helped create the field of geology, or the study of rocks and minerals, and he acted as a historian for a royal family in Germany.

So what exactly does binary code have to do with computers? Binary code is a coding language that is used to write instructions to electronics using bits so that the information can be interpreted quickly. Each bit, 0 and 1, communicates information to the computer through electrical signals. A 0 means the signal is off, and a 1 means the signal is on. One way to understand this idea is to think about a light switch. In one position, the switch turns the light on. In the other position, the switch turns the light off.

These 0s and 1s are the building blocks for longer strings of code. Code is a set of instructions for the computer. It tells the computer what to do. This includes what letters, numbers, or symbols to show and how to save and find files. It even tells the computer how to render, or display, pictures and images! Behind everything you see or do on a computer is code written using bits.



A byte is a group of eight bits that are arranged in a specific pattern, each with its own meaning and purpose. Think for a second about the word "hi." This is a very simple word, just two letters to type. To a computer, however, the lowercase letters *h* and *i* are changed to

A	01000001	a	01100001
B	01000010	b	01100010
C	01000011	c	01100011
D	01000100	d	01100100
E	01000101	e	01100101
F	01000110	f	01100110
G	01000111	g	01100111
H	01001000	h	01101000
I	01001001	i	01101001
J	01001010	j	01101010
K	01001011	k	01101011
L	01001100	l	01101100
M	01001101	m	01101101
N	01001110	n	01101110
O	01001111	o	01101111
P	01010000	p	01110000
Q	01010001	q	01110001
R	01010010	r	01110010
S	01010011	s	01110011
T	01010100	t	01110100
U	01010101	u	01110101
V	01010110	v	01110110
W	01010111	w	01110111
X	01011000	x	01111000
Y	01011001	y	01111001
Z	01011010	z	01111010

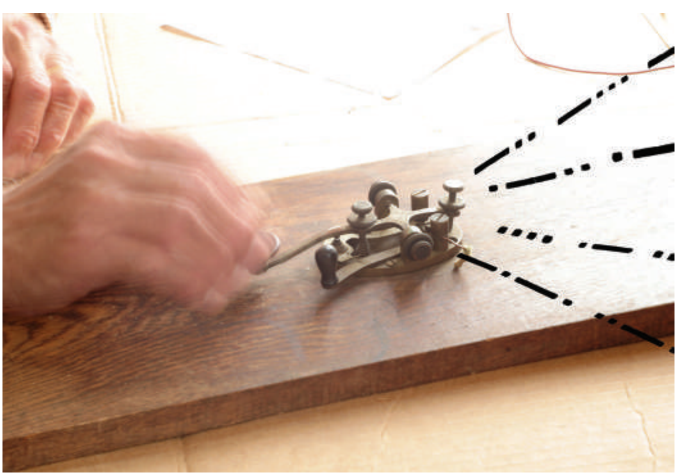
01101000 and 01101001. All letters in the alphabet have their own binary code. Uppercase letters start with 010, and lowercase letters start with 011.

Look at the chart. Can you see a pattern between the binary code and the letters in the alphabet?

The amount of information a computer is processing, including the size of documents, images, and videos, is measured in data bytes. Text document files typically have the lowest number of data bytes while videos have the highest.

Another way to understand how binary code works is to look at Morse code, invented in the 1830s by Samuel F. B. Morse and Alfred Vail. Morse developed a system of dots, dashes, and rests that could represent numbers and letters electronically. A dot is a short electrical signal, and a dash is a longer electrical signal. A rest is a period of silence between the dots and dashes. Within a letter, the rest between the dots and the dashes is shorter. Between letters and words, the rest is longer. The first telegrams used Morse code and allowed people to send and receive messages over long distances.

Morse code simplified communication by creating a shared system of meaning for users around the United States and later around the world. For example, you may be familiar with the distress signal "SOS." This sequence of letters was chosen because it's easy to send in an emergency and it's hard to mistake for another message. All across the globe, ••• — — — ••• is understood as "SOS." Binary code creates a similar system for computers.

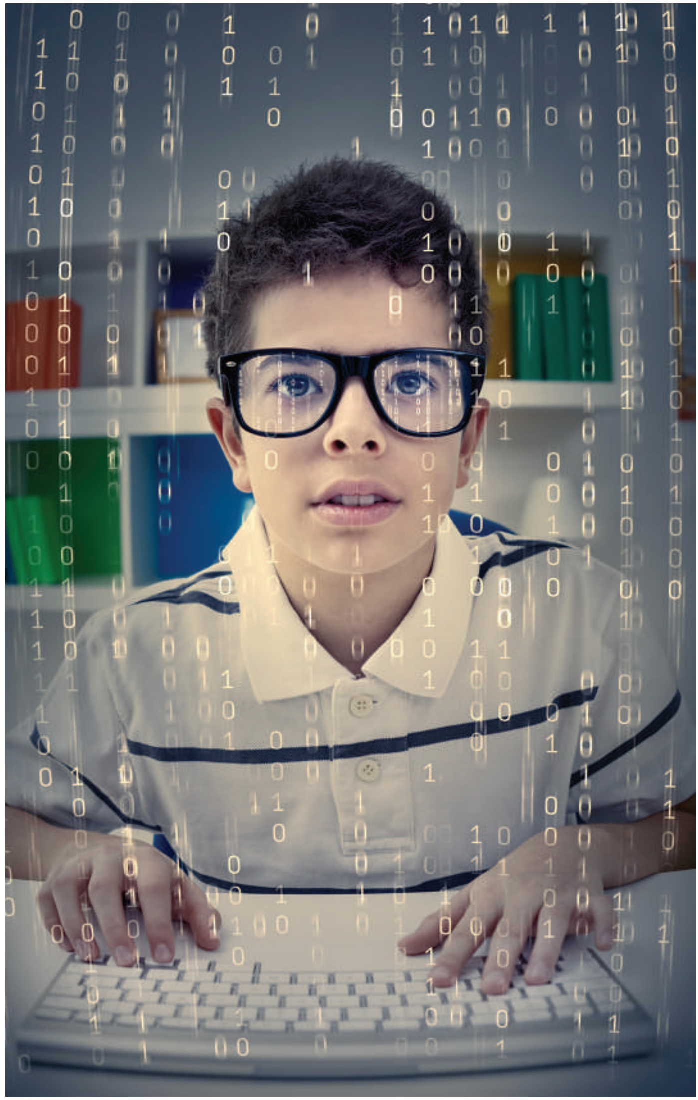


When we think about all the things our computers and smartphones can do, it seems amazing that they only really use 0s and 1s in their code. But this simplicity is what makes the system work so well. Using a base-2 system means that computers only have two options to process—0 or 1. This allows computers to decode information quickly and easily—much more quickly than using larger number systems like base 10.

As computing has gotten more complex, sometimes a byte of information—just 8 bits—is not large enough to store a piece of information. In this instance, programmers might use the hexadecimal system. This system is a base-16 system. The digits in the hexadecimal system are the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 and the letters A, B, C, D, E, and F. The hexadecimal system is useful because it can represent large numbers with fewer symbols.

The next time you use an electronic device, think about what you might be asking it to do! Is it a simple task or a complex one?

Consider all the different kinds of information that device might have to process to complete the action. Take a moment to marvel at the idea that it does it all with a system of code that uses only 0s and 1s!



You Can Try It!

Binary code has many different purposes and uses. You might want to try writing a secret message to a friend using binary code for them to decipher!

Main Idea

Computers and electronics use binary code, which is a base-2 system.

Ada and the Analytical Engine

Chapter 9

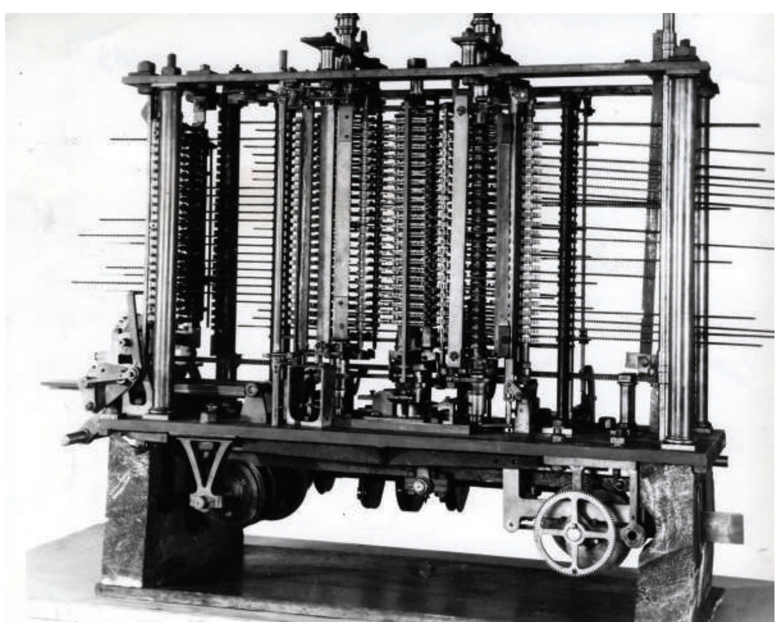

Charles Babbage

The path that led to the invention of the first computer all started with Charles Babbage. Babbage was an English mathematician and inventor. He was born during the First Industrial Revolution. This was an exciting time for England. New ideas and new inventions were changing the way people lived and worked. He saw that machines made life easier and faster. Why shouldn't they make solving math problems easier and faster, too? During the 1820s, Babbage began working on a design for a machine that could do calculations.

Next, Charles Babbage proposed another machine. He called it the Analytical Engine. He saw how punch cards were used in the making of fabrics. The punch cards instructed the weaving looms to make certain patterns on the fabric. Punches on a paper roll were also used to tell player pianos what notes to play.

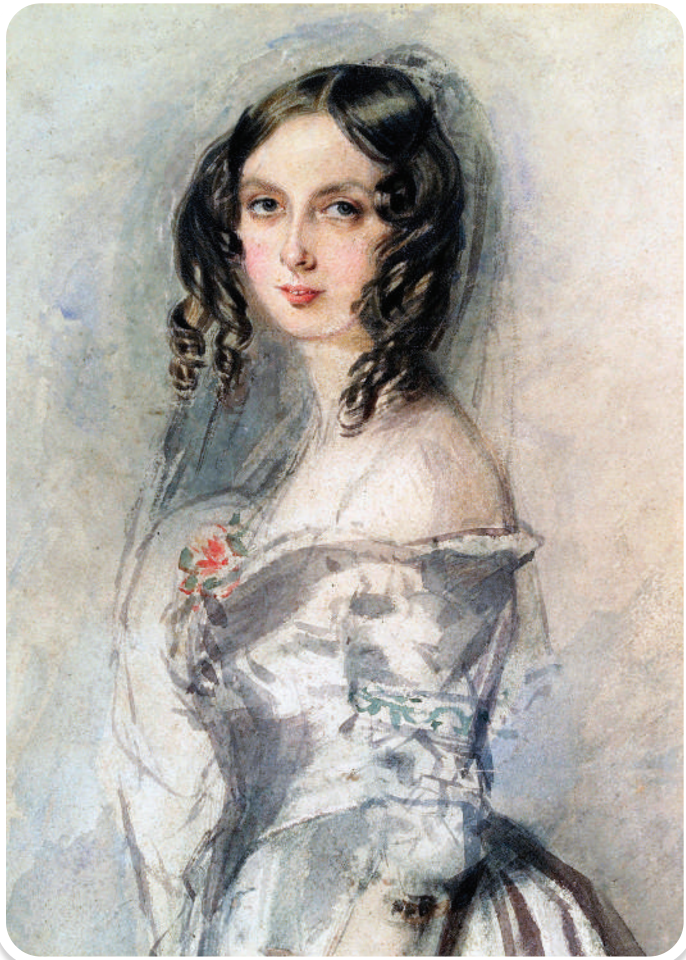
Babbage thought punch cards could be used in a similar way to tell his Analytical Engine what to do. He designed a stack of punch cards with different actions. According to his design, the engine would make a loop by running through the deck of punch cards again and again. For example, these cards would have numbers and operations to calculate, like adding and subtracting.

Babbage's design was complex and very large. He called one part of his engine the Mill, which measured 15 feet tall. If the engine had been built, the Mill would have been the control center of his engine. He called another part of his engine the Store. Twenty feet long, the Store would have been the engine's memory, or where it held information. Babbage planned for the Store to hold up to 1,000 fifty-digit numbers. This is more information than the first computers were able to hold!



A drawing of Babbage's Analytical Engine and real-life replica of Babbage's Analytical Engine

Babbage made an important friend around the time he started designing his Analytical Engine—Ada Lovelace. Lovelace, the daughter of Lord Byron and Annabella Milbanke Byron, was born in 1815. Her mother took education very seriously. She made sure her daughter was educated by the best tutors. Unlike many other women of the time, Ada Lovelace studied science and math.



Babbage became Lovelace's mentor after their first meeting in 1833. Soon Lovelace became one of the most important contributors to Babbage's work.

George Gordon Byron (1788–1824), known as Lord Byron, was an English poet whose writings were widely read throughout Europe. His poems reflected aspects of his own life, as well as his views on current events. It is interesting that someone famous in language arts would have a daughter who became a significant figure in math.

Charles Babbage continued to work on his engine. He estimated that the engine could correctly multiply two twenty-digit numbers in just three minutes. But this would take a lot of power. Of course, his engine would be steam powered, just like other engines of the time! Unfortunately, Babbage's engine was never built. His project did not have enough support in England.

Luigi Menabrea

Babbage decided to share his designs with other mathematicians and inventors. In 1840, Babbage presented the designs for the Analytical Engine in Turin, Italy. A man named Luigi Menabrea wrote in French about Babbage's ideas. Ada Lovelace was then asked to translate the paper from French to English. She was also asked to include her own ideas on Babbage's and Menabrea's work. This was her time to shine!

An old saying goes "necessity is the mother of invention." This means that when people need to do something, they find a way to do it. But what about curiosity and perseverance? Inventions like Babbage's Analytical Engine do not just spring into life—they are the result of hard work and trial and error over the course of days, weeks, months, and even years.

One such example is the light bulb. Thomas Edison and his associates tested thousands of materials before finding the right one to construct a light bulb that burned for more than just a few hours.

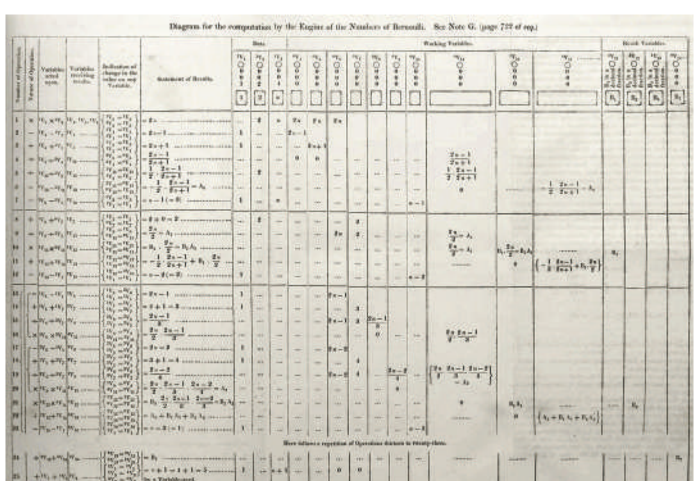
Ada Lovelace worked on the translations, adding notes of her own. The translated version was sixty-six pages long. Forty-one of those pages were Lovelace's notes that explained and supported Babbage's work. The paper was published in Taylor's Scientific Memoirs. This was considered the first published algorithm to be used with a computer. An algorithm is a set of instructions. Because of this, Lovelace is often considered the first computer programmer.

Women in STEM

Today, our society works much harder to encourage, accept, and recognize women in science, technology, engineering, and mathematics than in the past.

International Day of Women and Girls in Science is observed on February 11 each year, and Women's History Month in March recognizes the achievements and contributions of all women.

Lovelace's notes explained how the Analytical Engine would work. She explained how the engine would follow instructions from the punch cards. She also explained how the Analytical Engine would move information back and forth between the different parts of the engine. One of the most important notes was called Note G. It included tables of numbers that she had calculated by hand. She explained step-by-step how the engine would be able to calculate these same equations on its own following the punch cards.



Ada Lovelace's work was groundbreaking for the time. She understood Babbage's ideas and expanded them with her own thoughts and vision. Lovelace knew the engine could do more than math calculations. It could recognize different symbols and patterns as well. For example, she suggested the Analytical Engine could solve problems related to sound and harmony. It could even come up with its own musical pieces!

Lovelace also had views on artificial intelligence. She argued that the Analytical Engine was only as capable as the people who designed it—it would never be able to generate its own original ideas. Today, modern computers operate on many of the principles that Lovelace identified, making her a true visionary!

Main Idea

The principles of computer programming we use today are built on ideas from the early 1800s.

Lemonade for Sale

Chapter 10

Leanna loves lemonade. She makes all kinds of different flavors and loves to share them with everyone. Leanna decides to have a stand and sell her lemonade to customers at the fair.

Leanna wants her stand to be successful. Before the fair, she decides to calculate how much her ingredients will cost. This will help her decide how much to charge for each serving of lemonade. Leanna thinks about her ingredients. To make plain lemonade, she needs lemons, water, and sugar, so she decides to start with those ingredients first.

Leanna likes to make lemonade in small batches. For each batch, she uses 10 lemons and 2 cups of sugar. There are about 20 cups of sugar in a 10-pound bag, so Leanna knows she can make about 10 batches with 1 bag of sugar.

Leanna decides to use what she knows about powers of ten to help her calculate her costs. Lemons cost \$0.50 each if she buys them in bulk. So for 1 batch of lemonade, she calculates $10^1 \times \$0.50 = \$5.00$, which means she will pay \$5.00 for the 10 lemons she needs. A bag of sugar costs \$10.00, and she only needs 1 bag of sugar to make 10 batches.

Then Leanna calculates her costs for 10 batches: 10 lemons times 10 batches of lemonade is 100 lemons, or 10^2 lemons. Leanna decides to use powers of 10 again to calculate the cost of the 100 lemons. $10^2 \times \$0.50 = \$50.00$. That's a lot of lemons!

For every 10 batches of lemonade, Leanna will pay $50.00 for lemons and $10.00 for sugar, for a total cost of $60.00. With each batch of lemonade, Leanna can serve 10 customers. 10 customers times 10 batches is 100 customers!

Leanna wants to make sure that she prices each serving of lemonade to cover the costs of her expenses. She realizes she forgot to include the cost of plastic cups! She can get 1,000 plastic cups for $84. To make her calculation easier, she decides to find the cost of lemons and sugar for serving 1,000 customers.

> Making a profit means that a business needs to earn more money than it spends.

Leanna writes out her calculation:

Lemons: $10^3 \times \$0.50 = \$500$

Sugar: $10^2 \times \$10 = \$1{,}000$

Plastic Cups: $10^0 \times \$84 = \$84$

$\$500 + \$1{,}000 + \$84 = \$1{,}584$

For every 1,000 customers cost = $1,584

Leanna thinks she could charge $1.75 for each cup of lemonade. That would give her a profit because 1,000 customers paying $1.75 would be $1,750, which is more than her cost of $1,584.

> We use the word *cup* in two different ways. When used as a measurement, a cup equals a specific amount. But *cup* also means something that we drink out of. A drinking cup can hold any amount, for example 2 ounces to 32 ounces. The word *cup* used in this way does not indicate a specific amount.

But then Leanna starts to think about all the flavors of lemonade she makes. Her friends love her strawberry lemonade, which means she also has to add strawberries. *Her* favorite is blackberry mint, which means she also has to add both blackberries and fresh mint. A new flavor she tried last week was peach basil—and *that* was *also* delicious! There are so many different flavors!

How can she count the cost of all the different ingredients she uses to make all her flavors? Should each flavor of lemonade have a different price based on the cost of the ingredients?

Leanna wants the pricing of her lemonade to be simple. She decides that she will sell plain lemonade for $2.00 per serving and flavored lemonade for $2.50 per serving. She estimates that charging a little bit more for the plain lemonade and even a bit more for the flavored lemonade will help her pay for all the ingredients and still make a profit. She decides to bring enough supplies to serve 3,000 customers.

During the fair, Leanna is very busy making lemonade. She is surprised by how many customers are interested in her new flavor peach basil!

The fair lasts for 4 days. On the first day of the fair, she sells 750 servings of lemonade! She realizes that if she sells the same number of servings of lemonade on the next 3 days of the fair, she might run out!

Leanna decides to wait before she buys more supplies. On the second day of the fair, she sells 690 servings of lemonade, and on the third day, she sells 775 servings of lemonade. Leanna decides that she will most likely have enough lemonade after all.

How does Leanna determine if she will run out of lemonade? How does she use math to decide she does not need to buy additional supplies after all?

When Leanna sells her last serving of lemonade, there is just 1 hour left in the fair. Leanna is happy that she was able to sell so much lemonade!

After the fair, it's time to calculate Leanna's profit. She first calculates how much she earned from selling plain lemonade and flavored lemonade. Leanna knows that her costs for 1,000 customers are $1,584. Since she brought enough for 3,000, she multiplies her cost by 3. After the costs for the lemons, sugar, and plastic cups, Leanna spent $328 on other ingredients. After costs, she made a profit of $1,795.

Plain Lemonade: 1,250 x $2.00 = $2,500
Flavored Lemonade: 1,750 x $2.50 = $4,375
Earnings: $2,500 + $4,375 = $6,875
Costs: $1,584 x 3 = $4,752 + $328 = $5,080
Profit: $6,875 - $5,080 = $1,795

Leanna had so much fun selling her lemonade at the fair that she can't wait to do it next year, but she realizes that she did not include the cost of the booth! Next year she will think more about her prices and what else she has to pay for to sell lemonade at the fair. Until next year!

Main Idea

We can use powers of 10 and decimals to solve real-world problems about money.

Paradox in a Box

Chapter 11

Hi. I'm Zeno of Elea, but you can call me Zeno. I'm a Greek mathematician and philosopher. Most of my friends are philosophers, too. You might even know a few of them. Perhaps the name Socrates rings a bell. They're pretty famous where I'm from.

As philosophers, one of our goals is to better understand the world around us. One way is through logic. When you use logic, you think through things one step at a time. You consider all the facts before coming to a conclusion. We spend a lot of time thinking. Then we talk about and debate our thoughts.

I like to think about paradoxes. This means using logic to come up with an absurd or impossible conclusion. For example, I could use logic to conclude that true is the same as false, that 2 equals 3, or that squares are really circles. I've written about 40 paradoxes so far!

These signs represent a dichotomy. They give opposite instructions to pedestrians.

One of my most famous is the dichotomy paradox. *Dichotomy* means two ideas that disagree with or oppose each other. Examples of this type of thinking include all or nothing, good or bad, and real or imaginary. Each pair includes ideas that are opposites. To philosophers, dichotomy means to cut something in two. That is exactly what my dichotomy paradox is all about!

It all started on my daily 1-mile walk to the market. I like to walk slowly and enjoy the scenery. As I was walking, an idea occurred to me. To reach the market, I first have to get halfway to the market. I stop walking and think a bit. The distance to the halfway point of my walk is measurable, or finite. How long it takes me to reach the halfway point is also measurable. If I keep walking at the same pace, it should take me the same amount of time to travel the second half of my trip . . . right?

But then I start thinking about the second half of my trip to the market. There's another halfway point between where I'm standing and the market. When I reach that halfway point, there's another halfway point between me and the market. And another halfway point after that. And another halfway point after that! I can measure each of these distances and the amount of time it takes to travel them. But the number of halfway points is infinite. My trip gets smaller and smaller without end. And so does the amount of time it takes to reach each halfway point.

As you can imagine, I was starting to feel overwhelmed. I started thinking about my trip to the market and how math could help me clear up my confusion. I'd use fractions!

The halfway point between my house and the market is $\frac{1}{2}$ mile.

- I divided again to find the next halfway point: $\frac{1}{4}$ mile.
- Then I divided again to find the next halfway point: $\frac{1}{8}$ mile.
- I divided a fourth time to get $\frac{1}{16}$ mile.
- I divided again to get $\frac{1}{32}$ mile. On and on it goes!

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$$

I used this same math to think about the time it takes to reach the market. Yesterday, I walked for about an hour. It took me

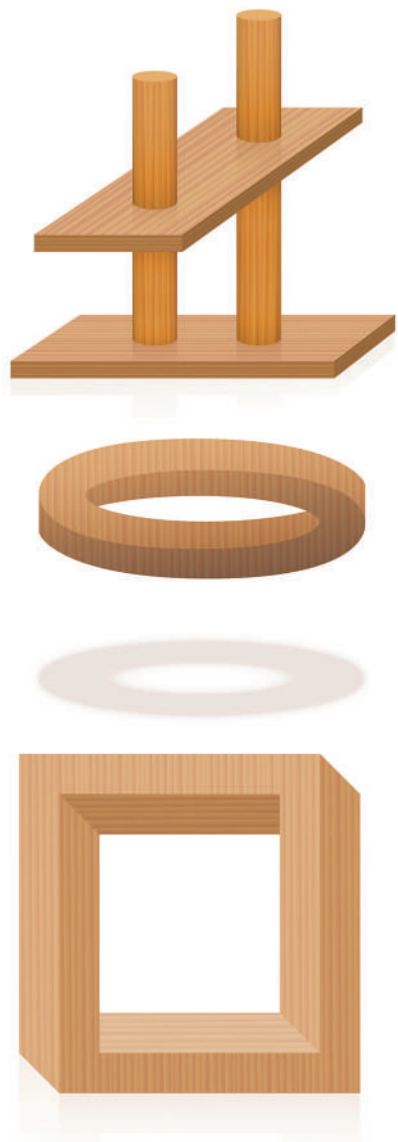
- $\frac{1}{2}$ hour to reach the first halfway point,
- $\frac{1}{4}$ hour to reach the second halfway point,
- $\frac{1}{8}$ hour to reach the third halfway point,
- $\frac{1}{16}$ hour to reach the fourth halfway point, and
- $\frac{1}{32}$ hour to reach the fifth halfway point.

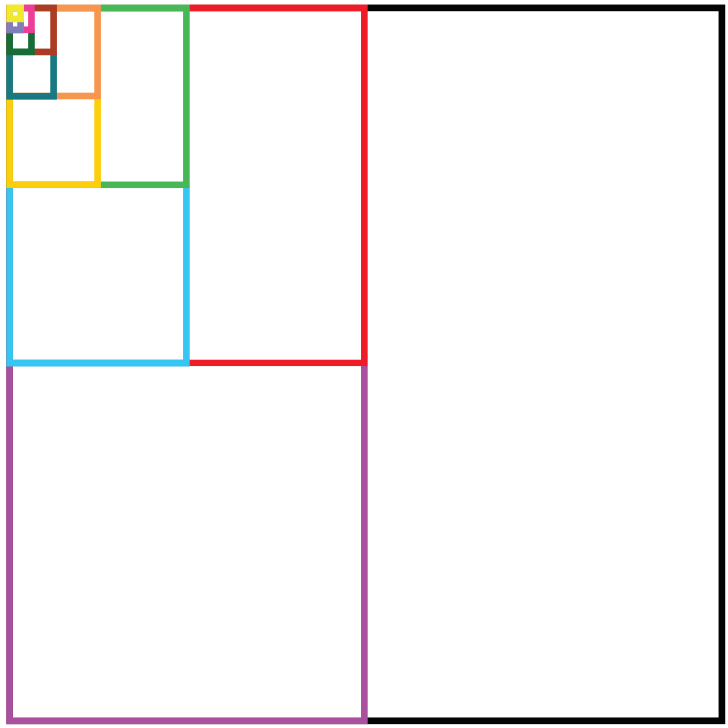
Here's where it gets really tricky. Even though we can measure the distance of each halfway point, the number of halfway points is infinite. It will never end! And even though we can measure the amount of time it takes to reach each of the halfway points, the infinite number of halfway points means that it will take an infinite amount of time to reach our destination. In other words, my trip to the market takes forever!

But that can't be right, can it? I walk to and from the market every day, and most days it takes me an hour. That's a measurable, finite amount of time. And if my trip to the market actually took forever, I wouldn't be here explaining my logic to you—I'd still be walking to buy figs at my favorite fruit stand!

Paradoxes

A paradox may seem silly, but that doesn't mean it's not important! A paradox reveals the ways we think about and view the world around us. Thinking through paradoxes and ideas that oppose each other helps us solve other problems by encouraging us to think creatively. Optical illusions can present a visual paradox—something that is impossible but possible at the same time.





My story about traveling to the market and never being able to make it all the way there is theoretical. This means it is a thinking exercise. We can use math again to explain how an impossible walk to the market is actually possible.

Let's say we have a square. We can divide a square in half and then divide that half into another half until the halves become so small we can't see them. Even though we divide it into smaller and smaller parts, the area of the whole square stays the same. That means that it always has a limit, so the distance for my trip to the market can *also* have a limit, and it cannot take an infinite amount of time. What a relief!

Main Idea

Math can be used to explain the contradictions in a paradox.

Fed Up!

Chapter 12

This is a transcript of a podcast.

Ron: You're listening to the podcast "Fed Up!"—a cooking show about avoiding kitchen catastrophes as you make tasty treats and yummy eats! I'm your host and chief tastemaker, Ron Tomatillo. If you're a longtime listener, welcome back! And if you're a first-time foodie, we hope you have a delicious time! Today, I'm joined by a special guest—Marcie Mango. Marcie, welcome to the show!

Marcie: Oh, I am absolutely delighted to be here, and I just cannot wait to share my tips and tricks with your listeners!

Ron: Why don't you share a little bit about yourself.

Marcie: Well, my love for cooking started when I was a kid. I helped my grandmother cook each weekend. Later I went on to culinary school, where I sharpened my skills. Last year I published my first cookbook about the different ways to use fruit in cooking and baking. Now I'm working on my second cookbook.

Ron: Sounds like you're quite busy! As you know, people send us emails each week asking us cooking questions. We'll work together to help solve their delicacy dilemmas! Would you like to read our first email?

Marcie: It would be my pleasure! Lydie Lima writes: Hi, Ron and guest! I need some help with my recipe for Every Bean Salad. It's hearty and healthy, and people have a hard time not having second helpings. However, this recipe makes 20 servings—that's way more beans than I need! How can I scale this recipe down for just 5 people? I've attached the recipe for your reference. All the best, Lydie!

Ron: Thanks, Lydie, for writing to us about this topic! Scaling recipes up and down is an important skill for cooks and bakers alike. We need to use our math skills. Marcie, where do you think we should start?

Marcie: First things first—let's look at the list of ingredients and their quantities. Lydie's recipe calls for the following:

8 cups mixed beans
2 cups chopped green peppers
1 cup chopped celery
$\frac{3}{4}$ cup chopped onions
$\frac{2}{3}$ cup vinegar
$\frac{1}{2}$ cup olive oil
1 tablespoon salt

Chop, Dice, Mince

Recipes use different words to let cooks know how to cut certain ingredients, including *chop*, *dice*, and *mince*. Chopping means cutting things like vegetables into the size of a nickel. Dicing means cutting things into pieces about half the size of a chop. Mincing means cutting something into the smallest pieces possible.

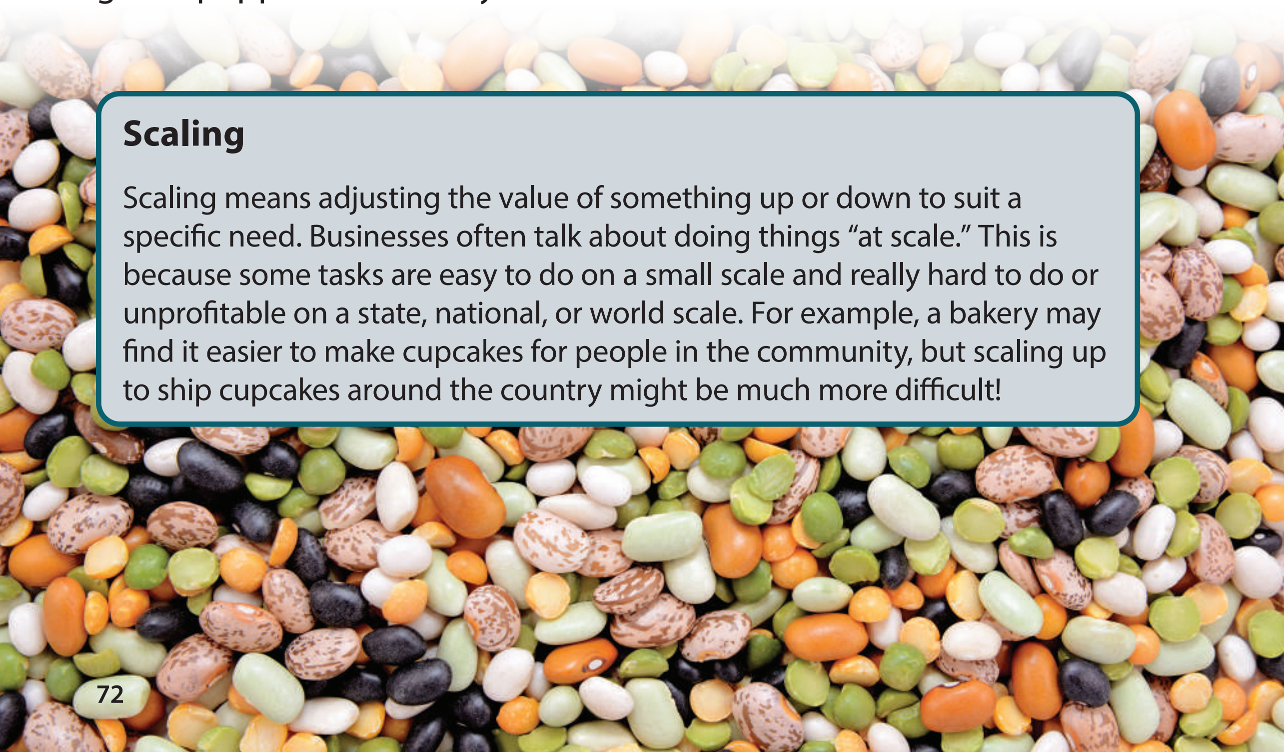
Ron: Wow, Lydie, you were right. This recipe does look good—and nutritious too! Marcie, now that we've read through the ingredients, what should we do next?

Marcie: The next step is to determine how much Lydie wants to scale her recipe down. She mentioned that the recipe makes enough for 20 people but that she only needs to feed 5. Since 20 divided by 4 is 5, we need to divide each of our ingredients by 4.

Ron: That sounds simple enough, especially for the first few ingredients! Let's start with scaling down the mixed beans.

Marcie: That's exactly right! This recipe calls for 8 cups of beans. We can divide that by 4 to get the correct amount of beans for 5 people. This calculation is straightforward because we do not have to use any fractions.

Ron: Great! Now walk me through how we would scale down the green peppers and celery.



Scaling

Scaling means adjusting the value of something up or down to suit a specific need. Businesses often talk about doing things "at scale." This is because some tasks are easy to do on a small scale and really hard to do or unprofitable on a state, national, or world scale. For example, a bakery may find it easier to make cupcakes for people in the community, but scaling up to ship cupcakes around the country might be much more difficult!

Marcie: We can repeat the same process for the peppers and celery. Let's divide by 4 to find the new amounts we need for the recipe.

Ron: The recipe calls for $\frac{3}{4}$ cup of chopped onions. How do we go about scaling a fraction?

Marcie: We'll still divide our fraction by 4, just like the other ingredients. This leaves us with $\frac{3}{16}$ cup of chopped onions.

Ron: I don't believe that's a standard measuring cup size that most people will have in their kitchens. What do you suggest Lydie should do?

Marcie: Well, we *can* actually use a measurement. There are 4 tablespoons in $\frac{1}{4}$ cup. $\frac{3}{16}$ cup is just a little less than $\frac{1}{4}$ cup, so we can say we just need a little less than $\frac{1}{4}$ cup or about 3 tablespoons!

Ron: That certainly makes it easy!

Ron: It looks like we're about halfway through our ingredient list. How will we scale down the remaining items?

Marcie: Well, as you can see, we have two more ingredients that are measured in fractions. We'll divide these by 4, just as we did with the other ingredients. We will divide $\frac{2}{3}$ cup vinegar by 4, which gives us $\frac{1}{6}$ cup of vinegar.

Ron: Ok, I know that we don't have a $\frac{1}{6}$ cup measuring cup. How can Lydie scale down the vinegar?

Marcie: In this case, we can round the ingredient up or down depending on how much Lydie likes vinegar! Rounding up to the nearest measuring cup size gives us $\frac{1}{4}$ cup. Rounding down gives us $\frac{1}{8}$ cup. There is no measuring cup that's $\frac{1}{8}$ cup, but we know that $\frac{1}{8}$ cup is equal to 2 tablespoons, because we know 4 tablespoons is $\frac{1}{4}$ cup.

Ron: And if I'm not mistaken, dividing $\frac{1}{2}$ cup of olive oil by 4 gives us $\frac{1}{8}$ cup—which is the same as 2 tablespoons!

Marcie: Correct! Now we have just one more ingredient left to scale down, Ron. The original recipe calls for 1 tablespoon of salt. Divided by 4, this gives us $\frac{1}{4}$ tablespoon. Now, I know what you're going to say: That's not a standard measurement.

Rounding in the Kitchen

Rounding up and down is common when working in the kitchen. But estimating ingredients can have consequences—especially when baking. Baking is science; different ingredients work together to cause things like a cake to rise. Too much or too little of an ingredient can be the difference between a flat cake, a perfectly springy cake, and one filled with air holes!

Ron: But it's *close* to a standard measurement! There are 3 teaspoons in a tablespoon, so we can round up to a single teaspoon, right?

Marcie: Exactly! For our listeners at home, we can also scale recipes up, or make them larger! Let's say Lydie wanted to make her Every Bean Salad recipe for 60 people. Since the original recipe serves 20, all she'd have to do is multiply the amount of each ingredient by 3.

Ron: Those are great ideas. Thanks for sharing, Marcie! Thank you for showing us how important math skills are when we cook. Everyone else, join us next week as we wash away your cooking woes!

Main Idea

Recipes can be scaled down by dividing or scaled up by multiplying the quantities of each ingredient.

The Community Garden Greenhouse

Chapter 13

Tamara is a member of her town's community garden. She loves to go to the garden, pick a bowl of vegetables, and make a healthy meal. Because it is summer, Tamara grows tomatoes,

green beans, cucumbers, and some herbs like basil, parsley, rosemary, and sage.

As summer changes to fall, Tamara will plant vegetables that grow well in cool temperatures, like lettuce, carrots, and broccoli. But there are times when she can't grow any vegetables at all. Tamara lives in New York state, where it gets cold in the wintertime. Every year, she feels sad when the plants die because of the cold. She always misses the fresh vegetables and herbs in the winter.

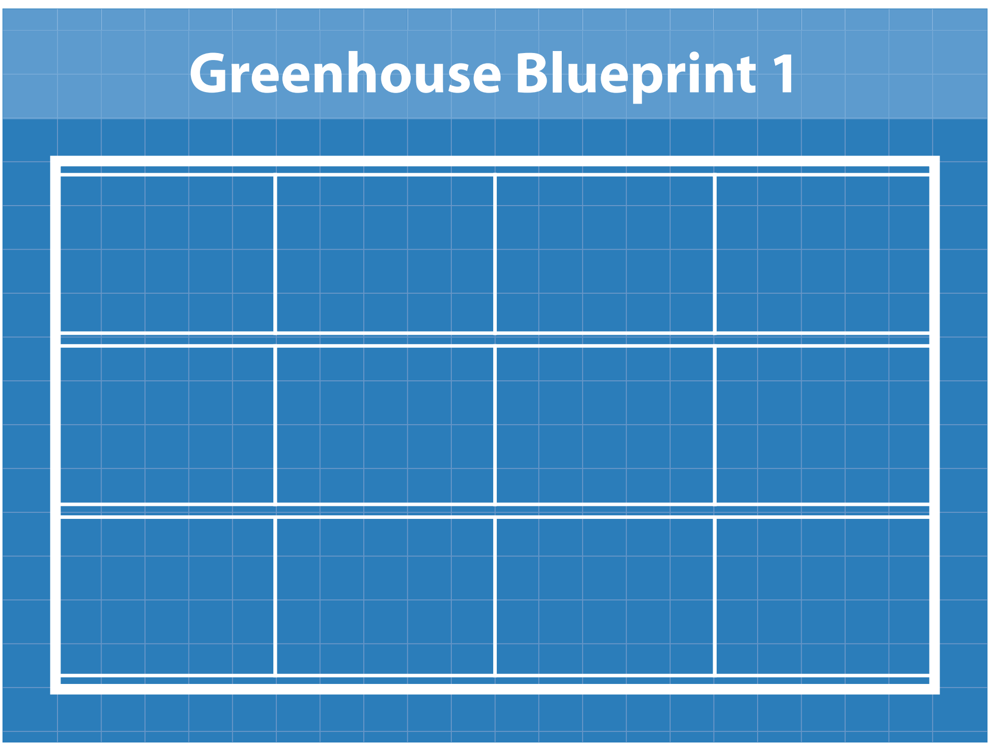
But this year, the community garden is building a greenhouse! This will allow the gardeners to extend their season or even grow some vegetables in the winter! Tamara volunteers to help plan and design the new greenhouse. She wants to maximize the growing space for the community garden members.

Tamara is on the committee in charge of planning the greenhouse. The people on the committee need to make a lot of decisions! The largest greenhouse they can build is 15 feet wide and 25 feet long. They will need to use math to decide what size planters to use and how many planters can fit in the new space.

Tamara has volunteered to figure out how many planters will fit so that the most members of the community garden can rent space and grow vegetables. She sits down to figure out which size planters are best, how she will arrange the planters, and how many planters she can fit. Tamara thinks planters that are $6\frac{1}{4}$ feet long and $4\frac{1}{2}$ feet wide would give the community gardeners a nice patch to work with.

Tamara quickly does calculations to see what will work. She realizes that $6\frac{1}{4}$ times 4 is exactly 25 and $4\frac{1}{2}$ times 3 is $13\frac{1}{2}$, which is less than 15. Tamara figures that the greenhouse can hold 3 rows of 4 planters.

Next, Tamara draws a blueprint that shows the arrangement of the planters in the greenhouse. This will help her explain her plan to the other members of the committee. In her blueprint, she neatly aligns the planters. She starts to imagine what kinds of vegetables the gardeners will plant.



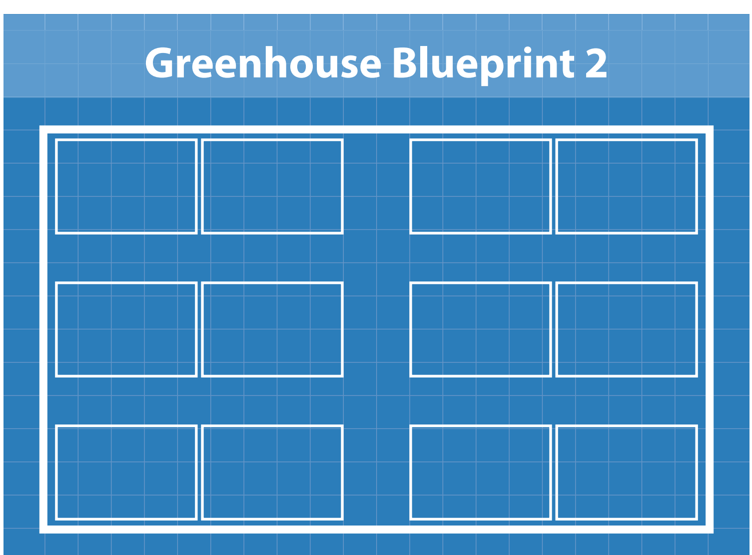
As Tamara pictures walking around the greenhouse to pick her vegetables, she looks again at her drawing and realizes that it won't work. The planters take up almost all the space in the greenhouse. She needs to leave space for the gardeners to walk and use tools like a wheelbarrow!

Tamara goes back to the drawing board. She knows that this planning stage includes trial and error and that's OK. She considers making 2 rows of 3 planters instead of 3 rows of 4. But she realizes that this will mean there is too much walking space and not enough growing space!

Tamara decides that using planters that are $6\frac{1}{4}$ feet long and $4\frac{1}{2}$ feet wide will either result in too much walking space or not enough. She decides to try planters that are $5\frac{1}{4}$ feet long and $3\frac{1}{2}$ feet wide. She thinks a walkway between the planters should be about 2 or 3 feet. This way the gardeners can move through the greenhouse comfortably. She reasons that she can place *some* of the planters close together as long as there are walkways. She is pleased with her new arrangement. She makes a new blueprint to share with the community gardeners.

Thinking in Pictures

Think about why Tamara first reduced the number of rows of planters and the number of planters in each row. What mathematical reasoning did she use to decide it was too much walking space and not enough growing space? If you were building the greenhouse, would you make the same decisions?



Finally, the time comes to build the greenhouse. First the ground must be leveled, which means made flat. Then the builders can start building. The greenhouse has a metal frame, but the walls and roof are made of polycarbonate. This material is very tough but is lighter than glass. The material lets sunlight in so the plants can grow. The sunlight also warms the air in the greenhouse. This is called the greenhouse effect! Sometimes, the sun is so strong that the greenhouse can get too hot for the plants. The greenhouse has special windows that can be opened to allow the heat to escape.

The Greenhouse Effect

Have you ever sat in a car or next to a window with the sun shining? Do you remember how the sun made it feel extra warm? This is an example of the greenhouse effect!

The community gardeners are very excited about the greenhouse. As the summer winds down and the weather cools, the greenhouse keeps the vegetables safe and warm. Late in fall, long after most gardeners are able to make meals from their gardens, the community gardeners get together for a harvest party. Each gardener makes a dish to share that features something from their planter. Tamara is in awe of how the gardeners have grown so many delicious vegetables. She is grateful for the opportunity to be part of the greenhouse community and is excited about growing vegetables and herbs through the winter.

Main Idea

We use math to design a space and decide how to best arrange objects inside it.

Ahoy, Matey!

Chapter 14

Many readers have enjoyed the pirate story *Treasure Island* by Robert Louis Stevenson. This story, like many other tales, is about pirates' adventures as they search for treasure. But here's something that may surprise you. Pirates did a lot of math! For example, they calculated distances. They also spent a lot of time trying to figure out exactly how much treasure they had. Pirates would capture ships from England, Spain, France, and other countries, and each country had its own coins!

Coins are small pieces of precious metal, such as silver or gold. Coins made by governments are stamped with official designs. These designs usually include pictures of the rulers or other important people. The design ensures that the coin is real. It guarantees that the metal in the coin is pure, unlike an unmarked coin that could be made of less valuable mixed metals.

In this passage from *Treasure Island*, a young man named Jim Hawkins helps his mother, an innkeeper, search for payment in the room of a boarder staying at the inn:

> "I'll show these rogues that I'm an honest woman," said my mother. "I'll have my dues, and not a farthing over. Hold Mrs. Crossley's bag." And she began to count over the amount of the captain's score from the sailor's bag into the one that I was holding.
>
> It was a long, difficult business, for the coins were of all countries and sizes—doubloons, and louis d'ors, and guineas, and pieces of eight, and I know not what besides, all shaken together at random. The guineas, too, were about the scarcest, and it was with these only that my mother knew how to make her count.

Jim and his mother found so many riches in the boarder's bag. He must have been a pirate! Let's look at the coins they found, starting with pieces of eight.

Pieces of eight are actually parts of a Spanish silver dollar. Everyone loved the Spanish silver dollar, not just pirates! Even American colonists used these coins. That's because Spanish silver dollars were made from pure silver. Each coin also weighed the same. The Spanish included a special pattern on the coin's edges so everyone could tell if any of the valuable silver had been shaved off the edges.

The Spanish did not make a coin smaller than the silver dollar. People would cut them into smaller pieces to spend lesser amounts. Silver is a soft metal and is relatively easy to cut. The Spanish silver dollar could be cut in half using its markings. Each half could then be cut into half, or quarters, and each of those quarters could be cut in half again, for a total of eight pieces. That's why they were called "pieces of eight"! The Spanish called these pieces reales, and the English called them bits. Robert Louis Stevenson realized just how important pieces of eight were. In fact, he made it the catchphrase of the pirate Captain Flint's parrot. That bird squawks, "Pieces of eight! Pieces of eight!"

Spanish silver dollars were not always cut into perfect eighths. People would cut off small pieces in whatever way they could to conduct business. The only way to know exact values of these was to weigh them.

Another coin that Jim and his mother found was the Spanish doubloon. A doubloon is a gold coin worth 32 reales, or bits. That means it was worth four times as much as a Spanish silver dollar! The name *doubloon* comes from the Latin word *duplus*. This means "to double." That is because the first doubloons were worth twice as much as an escudo, another value of Spanish currency.

Later, the Spanish made other gold coins worth one escudo, four escudos, and eight escudos. These coins were used all over Europe and the Americas. Some doubloons were made in Spain, and others were made in Mexico and Peru. Since there were so many Spanish galleons sailing across the Atlantic, it was easy for pirates to find ships loaded with doubloons—and help themselves to the treasure!

Jim and his mother also found louis d'or in the boarder's room. This coin was first minted in France in 1640. Named after the French king Louis XIII, the louis d'or had a lot in common with the Spanish doubloon. It had what's called uniform fineness. In other words, each coin had the same amount of gold mixed with other metals. Just like the Spanish silver dollar, each coin weighed the same amount.

Spanish coins had helped set the standard for other coins across Europe. The louis d'or was equal to half the value of a Spanish gold doubloon. Meanwhile, a double louis d'or was equal to the same value as a gold doubloon. This made it easier for France, Spain, and other countries to trade with each other. It was also great for pirates. More trade meant more ships crossing the ocean and more treasure to capture!

Remember how Jim's mother only knew how to count the guineas? This is because the Hawkins family was English and guineas were an English coin.

Currency Exchange

A currency exchange is a business that helps people change money from one country to another. In the United States, we use the dollar. But other countries use different types of currencies, each with its own value. Currency exchanges keep track of the value of each currency relative to another. One day, 1 U.S. dollar may be worth 1.2 Canadian dollars. The next day, it might be worth 1.4 Canadian dollars.

Shared Currency: The European Union (EU) is made up of countries in Europe who work together for peace, security, and economic prosperity. Many countries in the EU use the same currency, called the euro. The euro was first introduced in 1999. The euro makes trade among the countries that use it much easier.

In the old English money system, a pound was worth twenty shillings. A shilling was worth twelve pennies. A penny could be broken down into two halfpennies or four farthings—just like Spanish silver coins could be cut into reales or bits. This means that a pound was worth 240 pennies. But we can't forget about the guineas! Guineas were gold coins worth one pound plus one shilling, or twenty-one shillings total. Now do you see why being a pirate meant doing lots of math?

Main Idea

The value of old coins was based on the weight of gold and silver, and they could be cut into fractions if less money was needed.

Mars Measurement Mix-Up

Chapter 15

Groups of scientists from different organizations often work together on one project. But what happens if they don't use the same measurement system? In 1999, the National Aeronautics and Space Administration (NASA) found out the hard way!

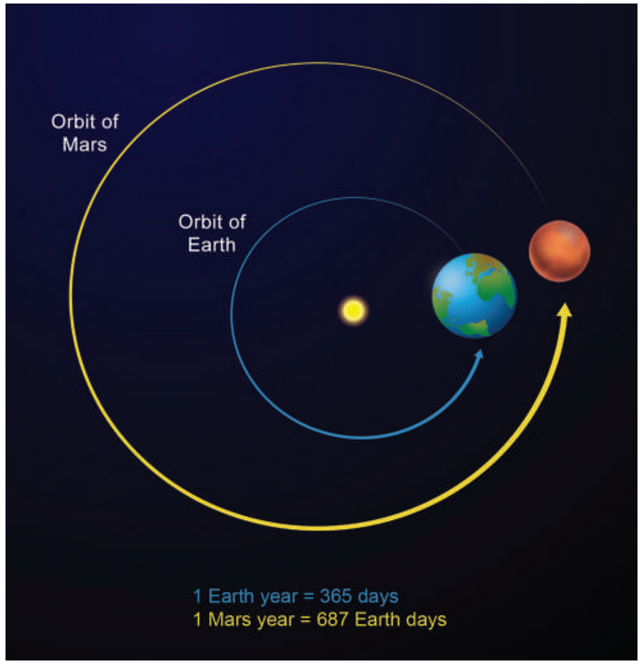
Throughout history, the night sky has caught our interest. Ancient civilizations studied the movements of Mars with their unaided eyes. In 1609, Italian astronomer Galileo got a closer look at the red planet using his telescope.

Mars is thought to be the most likely planet in our solar system to have ever hosted life. To determine if there was once life on Mars, scientists want to look for evidence of liquid water, fossils, or other elements that might suggest living things. In the late 1800s, some astronomers thought they saw rivers on Mars, leading many people to think that it might have life.

Interest in Mars and whether it had ever contained life continued into the 1900s. Throughout the 1900s, humans made strides to send rockets and then astronauts into space. The 1960s were an important time in space exploration history. In 1961, Yuri Gagarin became the first astronaut to enter space. In 1969, Neil Armstrong and Buzz Aldrin were the first astronauts to step on the moon.

During that time, there were many other space programs with different missions, including learning more about Mars. In 1964, we finally got a close look at Mars's surface when Mariner 4 made a successful flyby of the planet. It sent back pictures of Mars's cratered surface. In 1971, Mariner 9 was NASA's first spacecraft to successfully orbit Mars. In 1976, the *Viking 1* Lander was the first spacecraft to land on Mars. But not all space endeavors have been as successful.

During the 1990s, NASA began the Mars Surveyor '98 program to further study the atmosphere and surface of Mars. NASA's plan included two uncrewed spacecraft for this mission—the Mars Climate Orbiter, which launched in December 1998, and the Mars Polar Lander, which launched in January 1999.



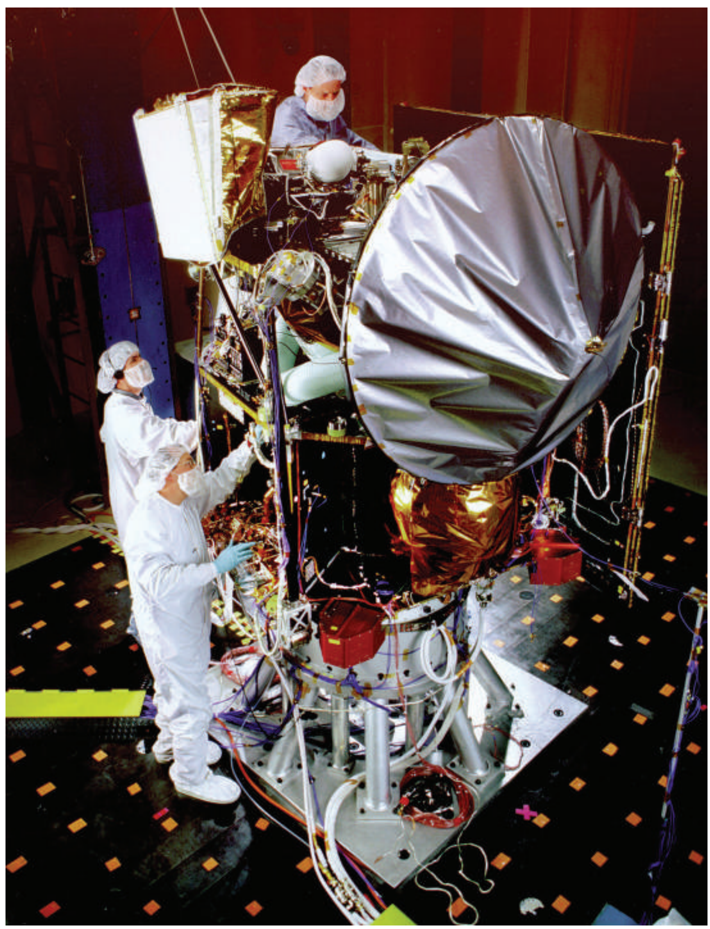
The Mars Polar Lander would land near the southern polar ice cap. It had a robotic arm that could dig for ice. It also had a camera and data-recording instruments. Two microprobes would travel to Mars on the back of the lander. Because this mission was to study the south pole of Mars, they were named "Scott" and "Amundsen" in honor of Robert Falcon Scott and Roald Amundsen, the first explorers to reach Earth's South Pole. They would be released as the lander neared the south pole of Mars. Once on Mars, the probes would penetrate the surface and collect data on the soil.

Finally, the Mars Climate Orbiter would orbit Mars for 687 days, which is the length of a Martian year. It would collect information about the weather and climate of Mars. The orbiter would also collect information from the lander and probes and relay it back to Earth.

The mission would not be easy. It would take countless scientists, engineers, astronomers, and mathematicians to design and test the spacecraft and send it to Mars.

A team at Lockheed Martin Astronautics in Colorado would build the spacecraft. The navigation team at the Jet Propulsion Laboratory (JPL) in California was tasked with overseeing the launch and navigating the spacecraft into orbit.

Sending a spacecraft to Mars takes careful planning. This is because the distance between Earth and Mars is constantly changing as the planets orbit around the sun. They can be anywhere from 40 million miles (about 64 million kilometers) to 140 million miles (about 225 million kilometers) apart! Timed correctly, it would take the Mars Climate Orbiter a little over nine months to get to Mars.



Engineers testing the Mars Climate Orbiter prior to launch

The JPL navigation team had a lot of math to do. They had to calculate how much fuel the orbiter needed to get to Mars. They also needed to plot the spacecraft's course and speed as it traveled through space.

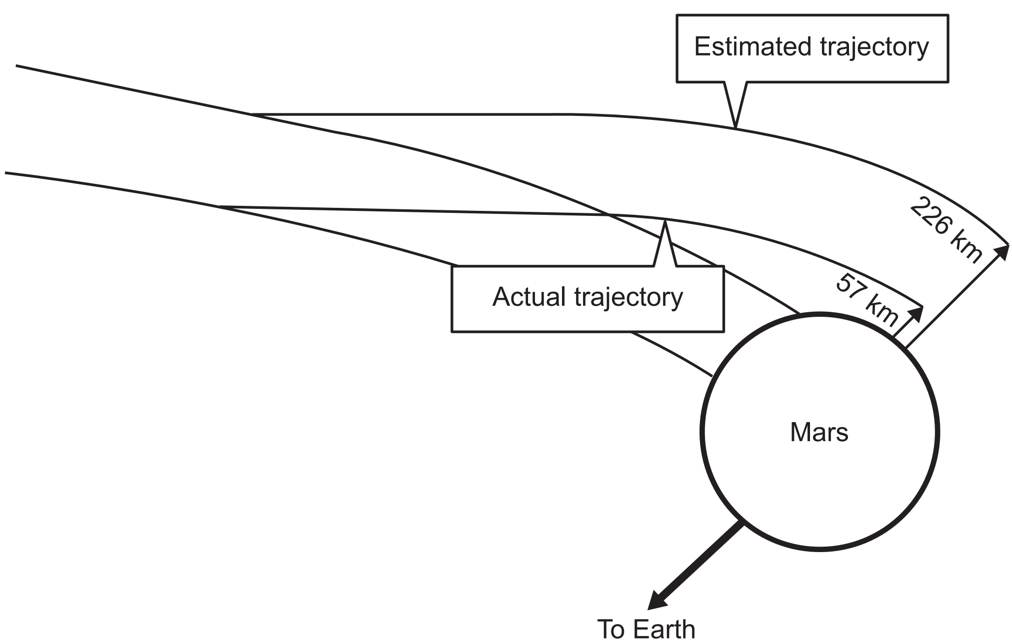
As the Lockheed Martin team built the orbiter, they sent the JPL team information about the thrusters that powered the spacecraft. The JPL team used this information to calculate and plot the orbiter's course. After years of work, the Mars Climate Orbiter was finally ready for launch. The spacecraft was sent into space on December 11, 1998, from the Cape Canaveral Air Station in Florida. The Mars Climate Orbiter was expected to reach Mars in late September 1999. The people at NASA and Lockheed Martin would finally see their hard work pay off!

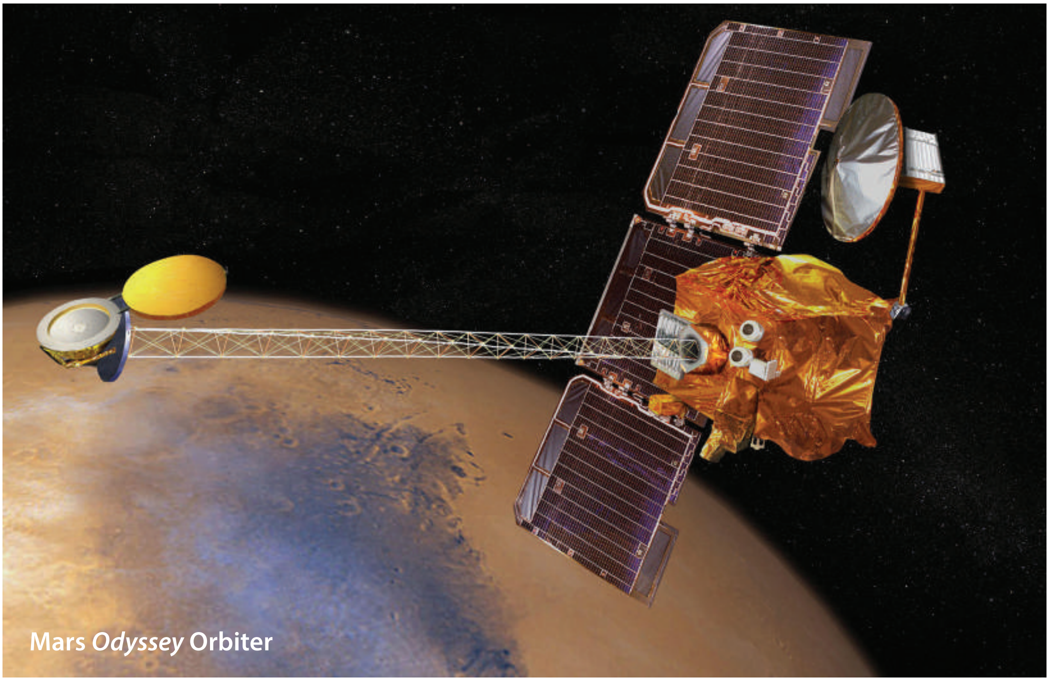
Scientists excitedly waited for the orbiter to reach its destination. Soon it would send information about Mars back to Earth. But on September 23, it was clear something was wrong. The Mars Climate Orbiter stopped sending signals back to Earth. Contact with the spacecraft was lost. A day of celebration was now a day of questioning. What went wrong with the orbiter?

Officials at NASA immediately opened an investigation to determine the cause of the mission failure. They reviewed every design and every calculation. They also checked every line of code in the computer program that ran the mission. They soon found the answer. The two teams were using different units of measurement!

The investigation showed that the information the Lockheed Martin team sent about the force of the thrusters was measured in pounds of force, instead of newtons per square meter. NASA determined that the problem was the failure to convert English units into metric units in the navigational software.

The conversion error had major consequences for the mission. The Mars Climate Orbiter approached Mars with too much force. Instead of entering the planet's orbit, it crashed into its atmosphere. The orbiter was not designed for this kind of impact. Investigations into the mission failure also determined that similar issues affected the Mars Polar Lander that had been launched after the orbiter. The seemingly minor mathematical error meant that NASA had nothing to show for its $125 million project.




Mars *Odyssey* Orbiter

There have been many other successful missions to Mars, and scientists have learned so much! In 1997, the first Mars rover, meaning a vehicle that could move itself along the surface, landed on Mars. The rover, called *Sojourner*, sent back amazing information that suggested to scientists that at one time in the past Mars was warm and wet!

In 2001, the *2001 Mars Odyssey* successfully entered Mars's orbit. This orbiter had tools to help scientists learn about what kinds of risks there might be to human explorers on Mars and to learn more about what kinds of elements and minerals were on the surface of Mars by using a special kind of mapping. This time, the mission was successful!

Main Idea

Being accurate when you measure and share data is critical for the success of projects.

Ice Cores

Chapter 16

A group of scientists huddle over a long table. Despite their heavy layers and gloves, a few still shiver. The room they work in requires extreme cold: –13° Fahrenheit (–25° Celsius) to be exact. The scientists wait patiently as a technician uses a saw to cut small sections from a large piece of ice. Then they'll take these samples back to a lab to study.

The National Science Foundation Ice Core Facility (NSF-ICF) in Colorado is just one location where these ice samples can be studied. This site has nearly 56,000 feet (17,000 meters) of ice collected from around the world. You may be wondering what exactly is so interesting about ice.

These long rods of frozen water hold the secrets of Earth's past. Scientists can learn about Earth's history by using math as they study these samples. The samples may also give us a glimpse into Earth's future!

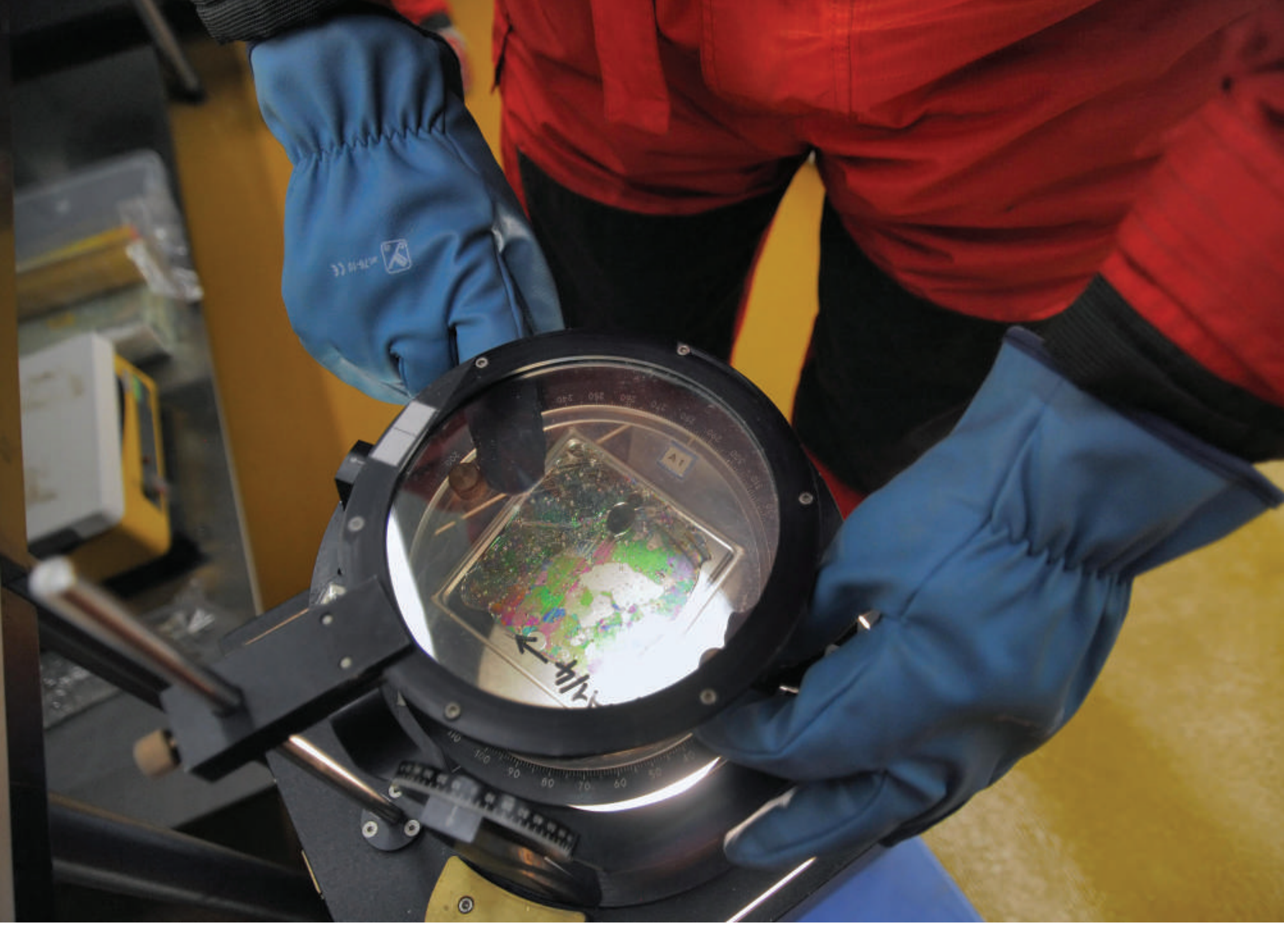
Ice sheets formed over hundreds of thousands of years from many layers of snow. Freezing temperatures meant snow did not melt. Each new snowfall formed a fresh layer on top of the previous snowfalls. This caused thick sheets of ice to form.

One way to think of the ice layers is like sedimentary rock. No two layers are the same, and each tells a story about Earth's past—especially its climate and atmosphere.

Scientists can't test the entire ice sheet, so they take samples, called cores, that they can study in a series. They collect ice cores from ice sheets in Antarctica, Greenland, and North America. The cores are cylinders of ice that are anywhere from about 3 to 20 feet (1 to 6 meters) long. They are typically 2–5 inches (50–132 millimeters) in diameter.

Which tools ice core collectors use depends on two things: how deep they want to dig and the temperature of the ice. Mechanical drills are used on ice in temperatures colder than 14° Fahrenheit (–10° Celsius). The drill is made up of a large pipe with a sharp edge at one end. As the pipe spins, it cuts through the ice to pull out an ice core. This is a very slow process! The collectors must dig a little, then collect the ice core. Then they dig a little deeper to collect the next ice core. They repeat this process until they reach their target depth.

The oldest ice is found at the bottom of the ice core samples. The youngest ice is at the top. Researchers can drill as far as 2 miles (3 kilometers) into the ice. The deeper they drill, the farther back in time they go. In Greenland, scientists can collect up to 130,000 years' worth of history from the ice. In Antarctica, they can collect up to 800,000 years' worth!



A researcher at the EuroCold laboratory in Milan, Italy, studies a sample from an ice core.

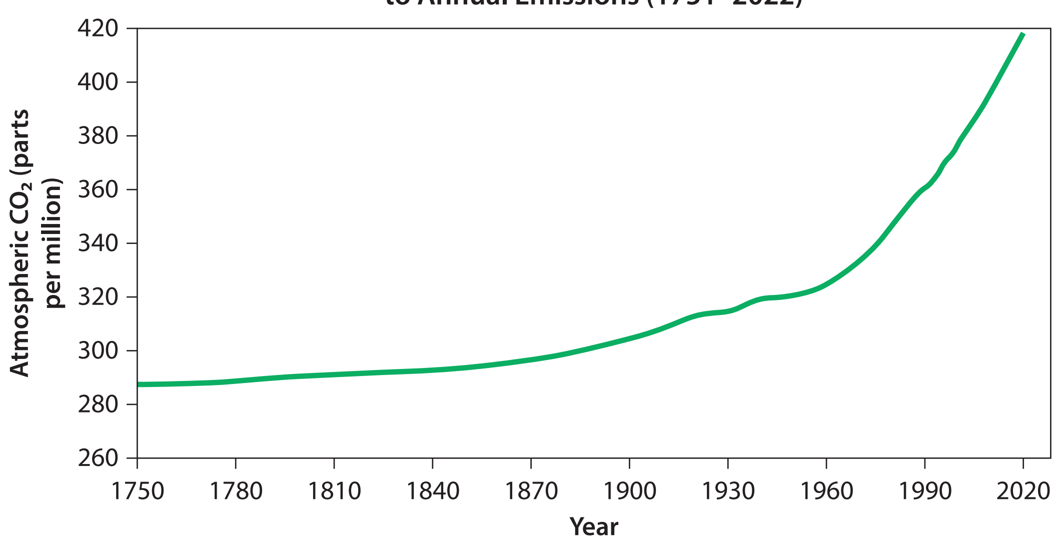
Scientists study ice cores to look for clues to Earth's past. For example, the thickness of the ice shows different seasons and temperature changes. Thicker layers may show extended periods of freezing. Thinner layers may show periods of warming.

The ice sheets contain more than just frozen water. They also include chemicals and particles from the atmosphere that were frozen in time. Some of these particles are ash from past volcanic eruptions. Chemicals and gases that were in the atmosphere long ago can be trapped in tiny frozen bubbles. Scientists are interested in these bubbles. One gas they are looking for is a gas called carbon dioxide. This gas helps traps heat inside Earth's atmosphere.

Carbon dioxide is released into the atmosphere in many ways. Sometimes it is released naturally by things like volcanic eruptions. More recently, greenhouse gas emissions come from human activity. Scientists have learned that carbon dioxide levels stayed the same for about a thousand years. They learned this by studying the bubbles in the ice.

The ice cores show that the amount of greenhouse gases started to increase in the 1800s. This was during a period called the Industrial Revolution. People were building factories and transportation systems powered by fossil fuels like coal. Burning fossil fuels is one way carbon dioxide is released into the atmosphere. Scientists can make graphs and use mathematical models to show their data. Greenhouse gas levels in the ice show there is 40% more carbon dioxide in the atmosphere now than before the Industrial Revolution.

Global Atmospheric Carbon Dioxide Compared to Annual Emissions (1751–2022)



Scientists apply math as they study the atmosphere of Earth in the past. Their calculations help the scientists identify changes. This helps them understand which changes and processes are occurring naturally. It also helps them determine the effects of human actions on the planet's atmosphere and climate.

Connecting information about the past and the present has other important benefits. For example, it helps scientists predict how the climate will continue to change.

Main Idea

Scientists measure and study the layers in ice cores to learn about Earth's climate history.

Eratosthenes, the Polymath

Chapter 17

A polymath is a person with a wide range of knowledge across many subjects. Polymaths are curious and love to learn. The internet has made it easy for today's polymaths to learn about many different subjects.

Eratosthenes was a Greek polymath born around the year 276 BCE in Cyrene, which is located in present-day Libya. He studied in Athens, which was a center for arts, learning, and philosophy. After he finished school, he settled in Alexandria, Egypt, where he became the chief librarian at the Library of Alexandria, which was one of the wonders of the ancient world. Eratosthenes collected, organized, and preserved books and helped other scholars and researchers find the information they were looking for in the library.

This 1635 painting by Bernardo Strozzi shows Eratosthenes teaching in Alexandria.

One of the subjects Eratosthenes was very interested in was geography. The study of geography involves understanding the physical features of Earth and how people and resources are affected by them. Eratosthenes decided to make a map of world.

Eratosthenes realized that he needed to know the circumference, or the distance around, the entire globe. This would help him know the correct distances between places on the map.

The most common way to measure distance in ancient Greece was to measure the distance by walking. A bematist was a person who specialized in walking long distances to measure distance in paces. A bematist was able to keep each step they took the same size. But how could Eratosthenes measure the distance all the way around Earth? It's not possible to walk all the way around the globe. He decided he could do it by observing shadows from the sun!

This well on Elephantine, an island in the Nile River, is said to be similar to the one that Eratosthenes used to help calculate Earth's circumference.

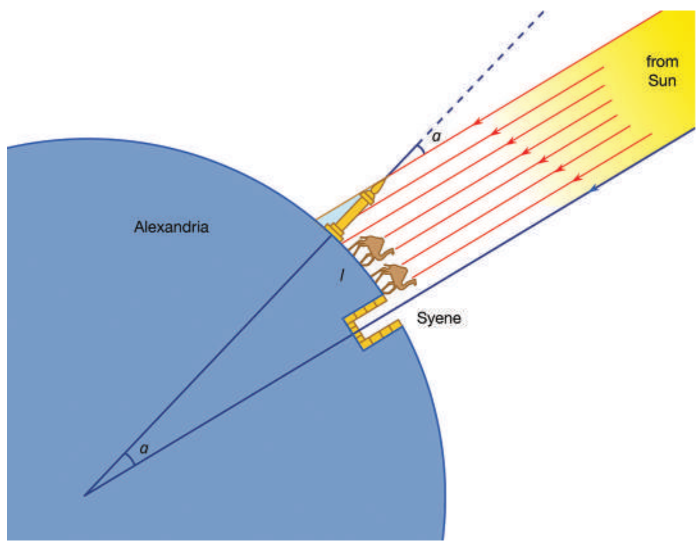
During the summer solstice, the longest day of the year, Eratosthenes knew that when the sun was at its highest in the sky, its rays reached the bottom of a deep well in Syene. At the same time, the sun did not have a shadow on the sundial near the well. On this same day at the same time in the city of Alexandria, the sun cast a shadow that covered about $\frac{1}{50}$ of the sun dial. Eratosthenes reasoned that the distance between Alexandria and Syene was $\frac{1}{50}$ of Earth's circumference.

Before We Used Degrees

The $\frac{1}{50}$ portion of Earth's circumference that Eratosthenes described equals about 7 degrees on a circle. We get this number by dividing 1 by 50, then multiplying by 360, the number of degrees in a circle: $\frac{1}{50} \times 360$. Degrees are a unit of measurement from the ancient Babylonians. This unit would not have been known to Eratosthenes, so he would not have used it. But using degrees is a helpful way for modern mathematicians and astronomers to understand Eratosthenes's logic and perform their own calculations.

Because Eratosthenes hypothesized that the distance between Alexandria and Syene was $\frac{1}{50}$ Earth's total circumference, his next step was to determine the distance between those two cities. A bematist counted the steps from Alexandria to Syene and provided Eratosthenes with the estimate that the distance between the two cities was about 5,000 stadia. A stadion was a unit of ancient measurement. One stadion was the length of the running track in a Greek stadium.

He suggested that if the 5,000-stadia distance between the two cities was $\frac{1}{50}$ the total circumference of Earth, he could multiply by 50 to calculate the full circumference! That calculation gave him 250,000 stadia.



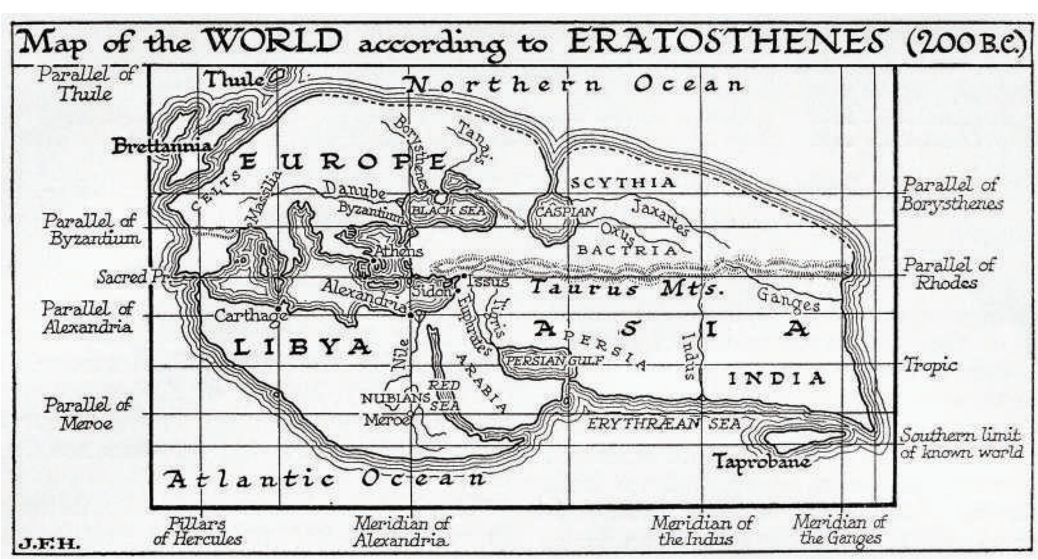
Translation, Please!

- At different times in history, a stadion had different measures. Scholars think that the stadion Eratosthenes used was somewhere between 500 and 600 feet, but it is not known for sure.
- This makes Eratosthenes's estimate of the circumference of Earth between about 24,000 and 29,000 miles (38,000 and 46,000 kilometers).
- Today, we know that Earth's circumference is about 25,000 miles (40,070 kilometers). That's pretty close!

While Eratosthenes's calculations weren't exactly correct, based on what modern scientists know now, his calculations were surprisingly close for that time. Eratosthenes used what he learned to write a three-volume book about geography. He used reports from other explorers and his own calculations to produce a map of the world, which he included in his writings. Eratosthenes was one of the first geographers to use parallel lines to show what we now call longitude and latitude. He was also one of the first geographers to represent Earth as a sphere in a map!

> Today, there are two main branches of geography. Physical geography focuses on Earth's features, such as mountains, rivers, and plains, and Earth's processes, such as weathering and erosion. Human geography focuses on human populations, culture, and activities.

Eratosthenes's original map was lost. But historians have a good idea of what it looked like based on the writings of other scholars and historians. Because of his contributions, Eratosthenes is known as the father of geography.



Eratosthenes made other amazing contributions to the fields of math and science. Earth has a slight tilt as it rotates. As he made astronomical observations to perform his calculations of Earth's circumference, Eratosthenes was able to calculate the angle of Earth's tilt to within 1 degree!

Eratosthenes also used astronomical observations to determine the length of Earth's year. He realized that one year was very slightly longer than 365 days. In order to create a calendar that worked, he had to add an extra day every four years. He was the first to discover a need for a leap year! This discovery became important as more modern calendars were developed.

He also invented a way to identify prime numbers, called the sieve of Eratosthenes. This method is still used today by researchers who study number theory.

Main Idea

Eratosthenes was an ancient polymath who used math to determine Earth's circumference.

Maps and Screens

Chapter 18

René stared up at the ceiling as he lay sick in bed. He watched as a fly landed on one ceiling tile before buzzing to another. To most, this may seem a boring way to pass the time. But for René's mathematical mind, this was an opportunity to see the world from a different perspective. As he watched the fly, a question formed in his mind: How could he use the tiles on the ceiling to describe the position of the fly?

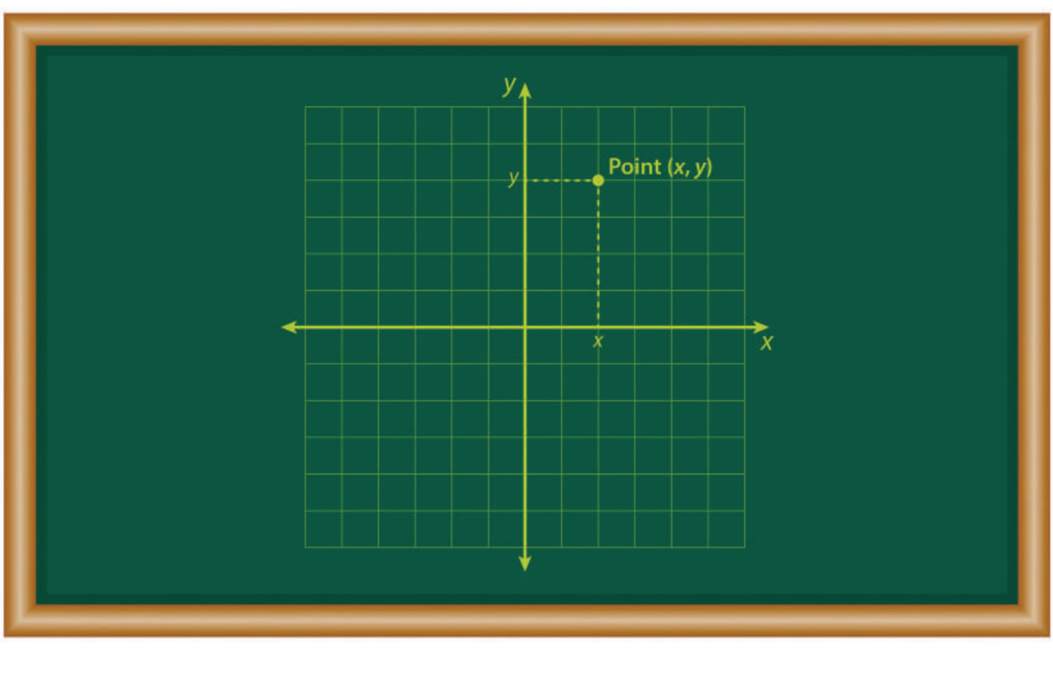
He started in one corner of the ceiling and counted the tiles from left to right. From the same corner, he counted the tiles from bottom to top. The numbers of tiles in either direction were coordinates, or two numbers that could be used to describe where the fly landed. While this story may only be a legend, René Descartes and his ideas about coordinates were very real!

René Descartes is given credit for many achievements. He's known as the "father of analytical geometry." He also helped shape how we study and understand human consciousness.

René Descartes was a French philosopher, scientist, and mathematician who lived from 1596 to 1650. He published a book called *La Géométrie* in 1637. In it, Descartes showed how to solve problems of geometry using a rectangular coordinate system.

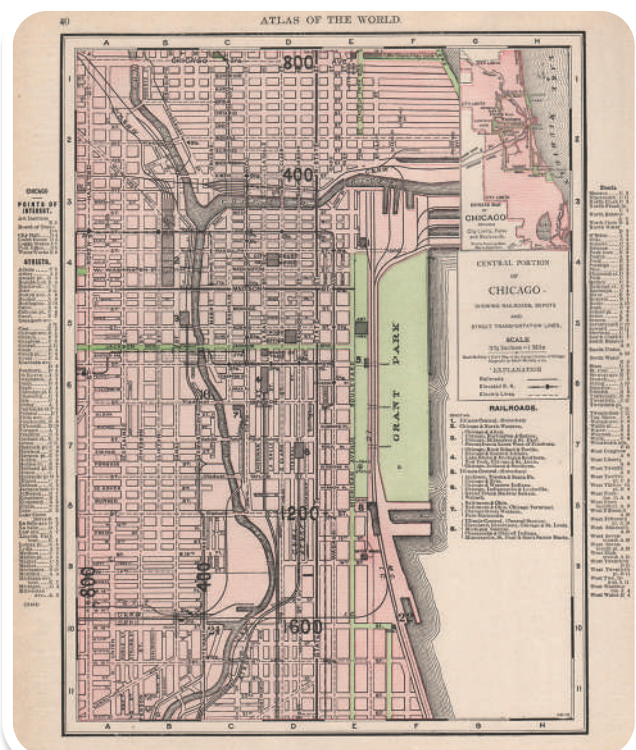
The coordinate system is made up of two perpendicular lines. The horizontal line is called the *x*-axis, and the vertical line is called the *y*-axis. The place where the two lines cross is called the origin.

Like in the story about the fly on the ceiling tiles, Descartes used the coordinate system to describe the position of different points using numbers. On a grid of equal spaces, first he counted the distance from the origin along the *x*-axis. Then, he counted the distance from the origin along the *y*-axis. Together, these numbers make up an ordered pair. Descartes was not the first person to use or describe coordinate systems, but his work led to many developments in mathematics, including many still used today.



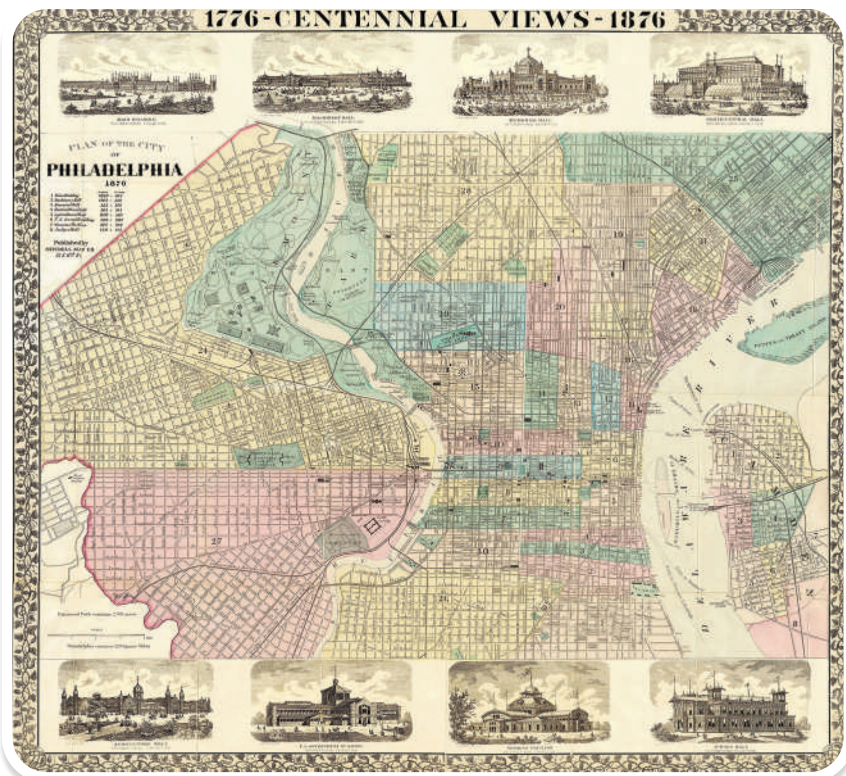
Cartesian coordinate systems—named after Descartes—are used for many things, including planning and organizing cities. Take Chicago, Illinois, for example. Chicago grew rapidly during the 1800s and early 1900s. In 1830, planners decided to organize the city like a grid. Streets ran from east to west and north to south. Over time, the city of Chicago began to annex, or add, nearby towns to the city limits. As a result, many streets in Chicago now had the same name. This made navigating the city very difficult. It also caused lots of confusion for the post office!

In 1901, a Chicagoan named Edward Brennan shared an idea with Chicago's city council. He suggested that they do two things. First, they should rename and renumber all the streets. Second, they should turn the city into a coordinate system that divides Chicago into four areas, called quadrants. After eight years, the city council was ready to put the plan into motion.



Chicago's city council made Madison Street their *x*-axis and State Street their *y*-axis. Madison Street runs east to west, and State Street runs north to south. The intersection of these two streets is the origin on this coordinate system.

The streets of Chicago were renamed or renumbered to include one of the cardinal directions: north, south, east, and west. The cardinal direction explains which way the street runs. For example, N. Broadway runs north and south. It also explains where a street is relative to the origin. This means that N. Broadway is north of Madison Street (the *x*-axis) while W. Roosevelt Street is west of State Street (the *y*-axis).



Many other cities around the world are also organized in grids. Philadelphia was one of the first grid cities in the United States. Its rectangular grid was designed by William Penn in 1682.

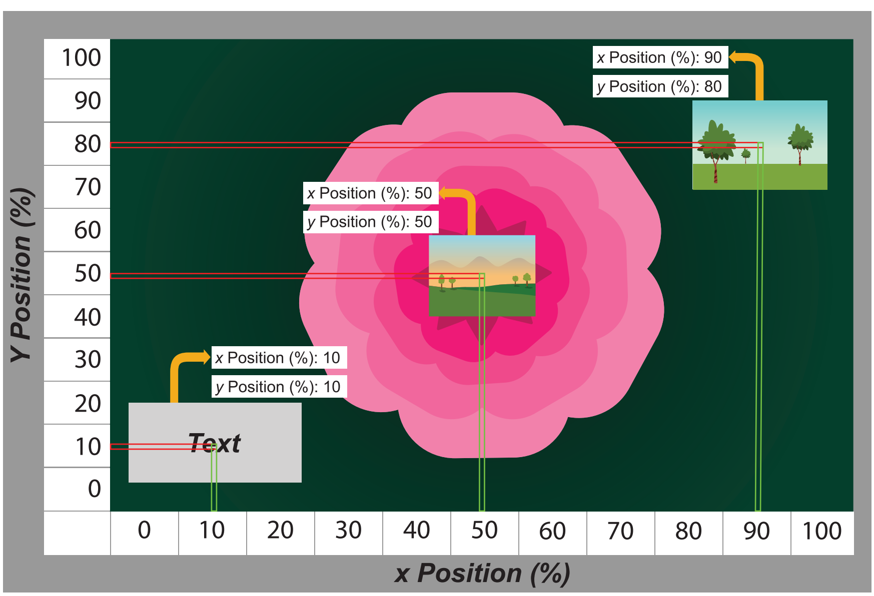
Like Chicago, Philadelphia's streets include cardinal directions. Numbered streets run north and south while named streets run east and west.

Grids are also an important part of planning efficient transportation systems. City services like buses need to service the maximum number of locations and passengers. They also need to use the most direct routes possible and keep costs low for riders. By making routes in a grid, planners can solve many of these problems. Grids help planners design routes so that they are efficient for drivers and passengers. Multiple buses may run down the same street to a centrally located stop or series of stops. Then, they go to different areas from there. This makes it easier for passengers to transfer from one bus to another.

With a grid layout, city transit systems can also have routes that run parallel to each other without overlap. Grids are also used to help identify areas with lots of traffic or places where many passengers go, like museums. They also help planners determine where to place bus stations and stops so passengers do not have to walk too far.

Every time you use a computer, you are using the Cartesian coordinate system. Computer screens use the Cartesian coordinate system to determine the position of things like icons, videos, and images. Computers recognize where these things are on the screen using coordinates on an *x*-axis and a *y*-axis.

Not all screens are divided into four parts. Some use a single quadrant with the bottom-left corner as the origin. The coordinates represent a percentage of the whole screen, regardless of its size. This means that on the *x*-axis, the origin in the lower left corner is zero, and the value of the lower right corner is 100. On the *y*-axis, the origin in the lower left corner is also zero, and the value at the top left of the screen is 100. An icon with the coordinates (50, 50) would appear in the center of the screen, halfway along the *x*-axis and halfway along the *y*-axis.



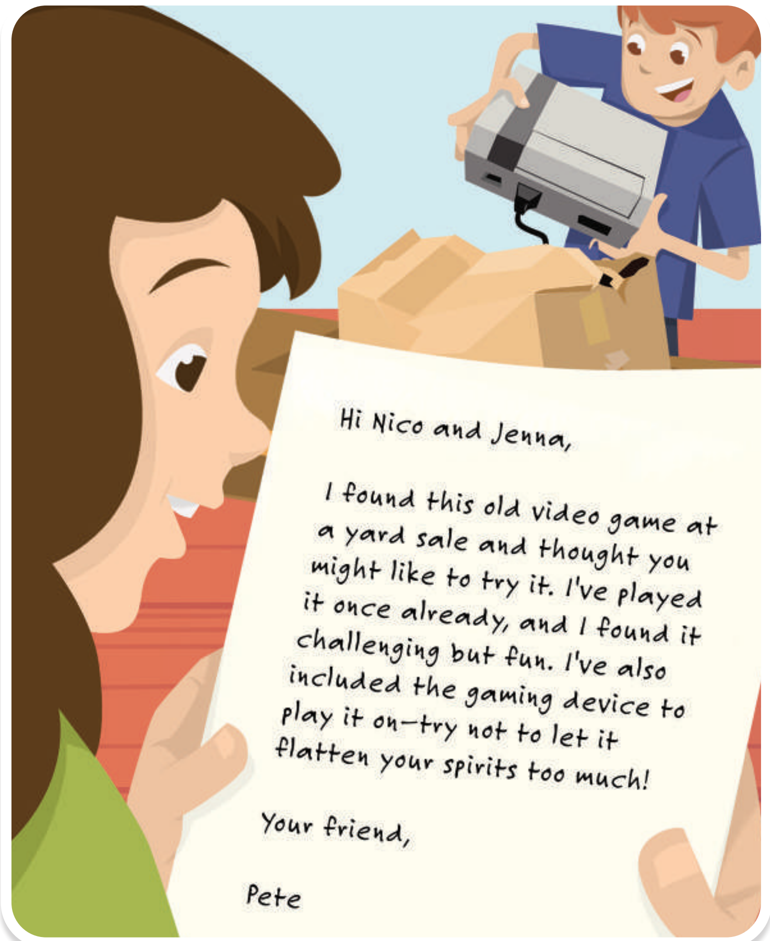
Main Idea

A Cartesian coordinate system can be used to find a specific point.

Plotting to Win

Chapter 19

Jenna opens the front door as Nico climbs the steps to the house. "Where have you been, and what are you carrying?" Jenna asks. Nico hands Jenna a note and tells her to read the note aloud as he walks past her into the living room.



Jenna and Nico decide to try playing the game. The two friends work together to set up the gaming device. "Look, there's a little red light in the corner of the device," Nico points out. "I think that means it's on!"

Jenna inserts the video game into the top of the device, and the screen lights up with a message:

> *Welcome to* Master Plotter, *a two-dimensional game for three-dimensional players! Use the A button and the arrows to drop points and create shapes to travel from one side of the grid to the other. Press the green button to start playing.*

"We'll have to use what we know about coordinate planes to play this game! The game indicates that the bottom left corner is the origin! Ready?" Nico asks Jenna.

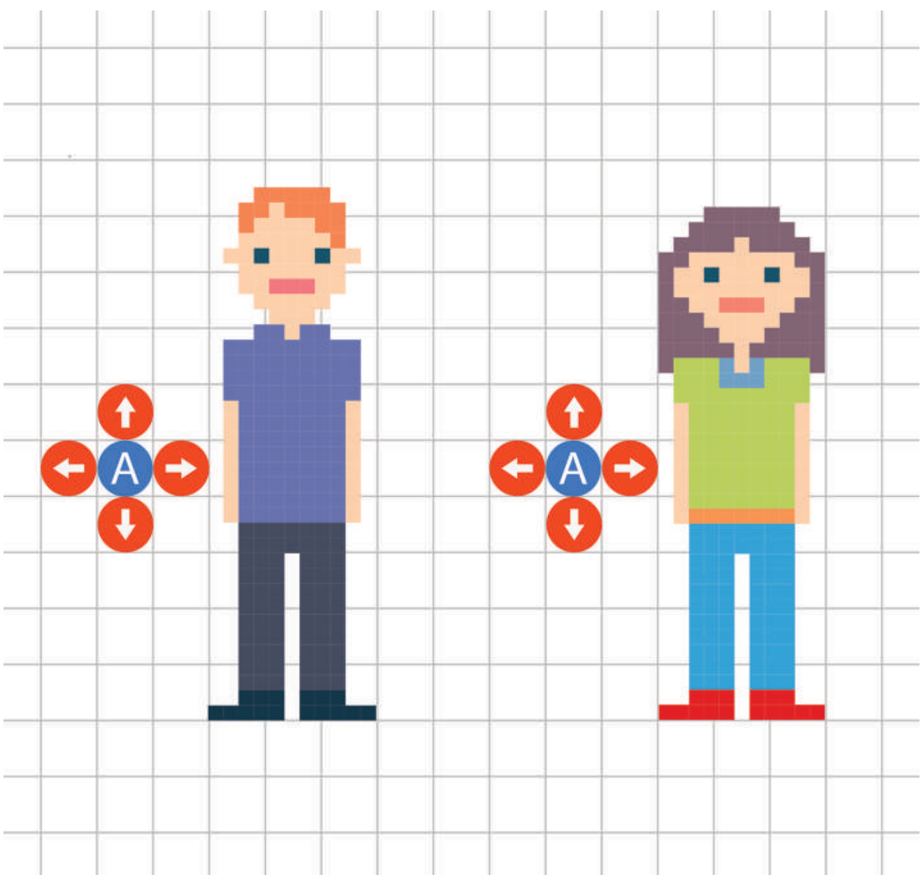
She nods and presses the green button. A blinding light fills the room, and with a loud popping noise, Jenna and Nico disappear!

Jenna blinks her eyes a few times and looks up and down and left and right. “Nico, what’s happening? Where are we right now?” She sees Nico, but he is completely flat!

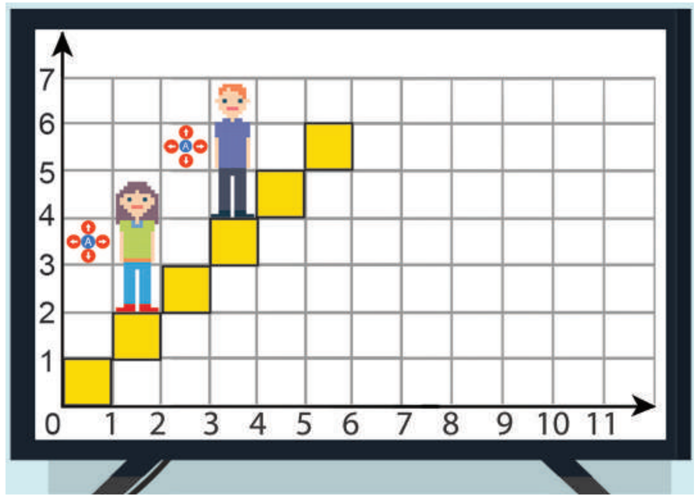
“Jenna, I think we’ve been sucked into the game!” Nico calls to her.

“Leave it to Pete to send us a video game that makes us players inside the game! How are we going to get out of here?” she calls.

Flat Nico scratches his flat chin with his flat hand then says, “I think the only way to get out is to play through. Remember: The directions said our goal is to get from one end of the grid to the other by making shapes. Look, we even have buttons to help us play!” Jenna glances down at her right hand, and sure enough—there are arrows and an *A* button!

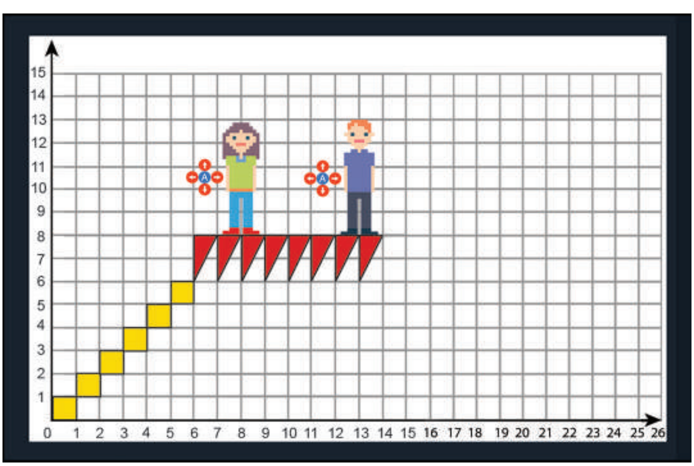


Nico hops up and down excitedly. A sign appears on the grid: **Level 1 = Square Stairs**. A pair of numbers, (6, 6), floats above them. "Jenna, to get through this round, I think we have to create squares that we can climb to reach that coordinate!" Nico says, pointing at the (6, 6). The two friends use the A buttons to drop points and their arrows to construct the first square.



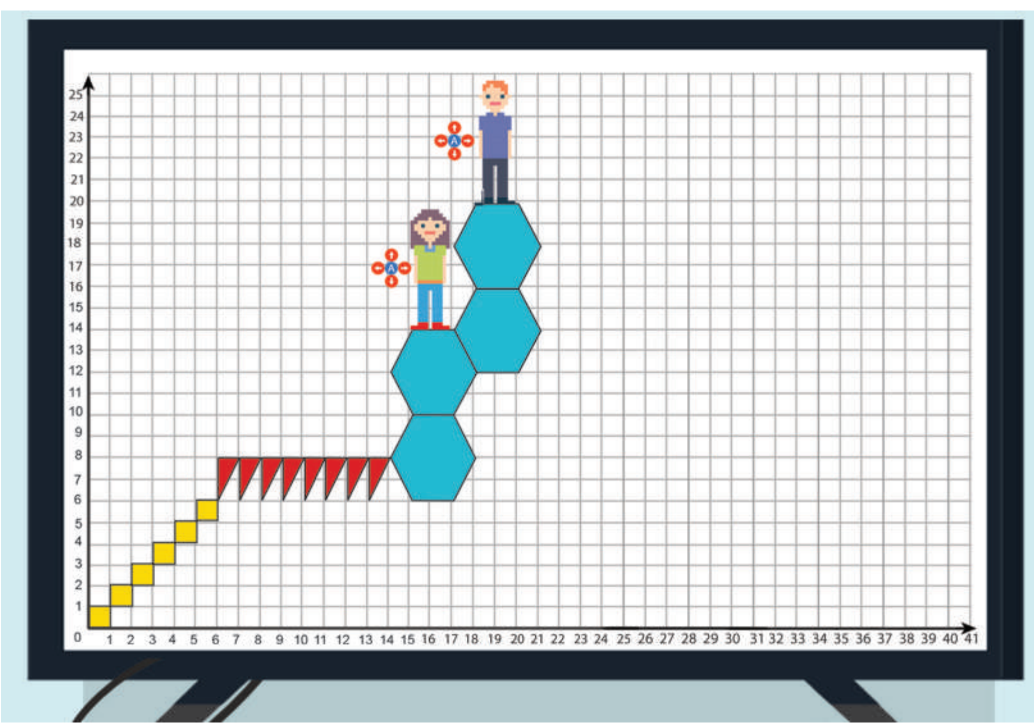
Jenna hops up onto the first square, and Nico follows. They repeat the process 5 times, making a diagonal line of 6 squares that look like a staircase.

Suddenly, a new sign and coordinate appear on the grid: **Level 2 = Triangle Terrace**; (14, 8). "Hmm," Jenna says. "I think this time we need to go across by making a walkway of triangles to reach coordinate (14, 8)!" Again, the friends use their buttons to make the shapes on the grid.



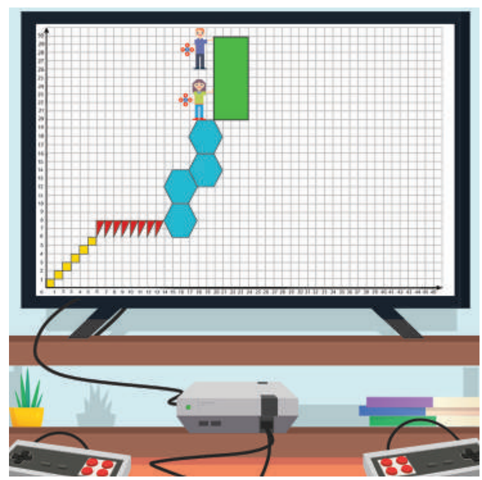
Soon, the friends reach their coordinate goal. As they give each other a high five, a third sign appears on the grid: **Level 3 = Hexagon Heap**. “Uh oh,” Nico cries. “This one might be tricky! I think we need to make a pile of hexagons and stack them up like boulders to reach that coordinate above.” Nico points to a new pair of numbers hovering on the grid: (20, 20).

Jenna pauses for a moment to think about their next steps; then inspiration strikes. “Nico, the game just said we have to make the shapes, not how big to make them. Instead of making patterns of smaller hexagons, I think we can beat this level by making 2 columns of 2 of hexagons, each 4 spaces wide from point to point and 4 spaces tall.” Nico nods and begins tapping his buttons—just 4 hexagons stand between them and Level 4!



As Jenna and Nico complete the fourth hexagon, a new message pops up: **Final Level = Cylinder Slide**. A final coordinate pops up, too: (24, 30). "Hmmm, Cylinder Slide," Jenna wonders aloud. "But cylinders aren't two-dimensional!"

Nico thinks for a second then says, "One possible cross section of a cylinder is a rectangle. We can make a rectangle to reach the coordinate, (24, 30)." Jenna and Nico tap their buttons to make the rectangle. As Jenna closes the top of the shape, the grid begins to rumble. The rectangle expands and stretches into the shape of a tube.



"Nico, I think we're supposed to hop in!" Jenna exclaims. She grabs Nico's hand, and the flat friends sail through the slide. With another flash of light, they are back in the living room in front of the television screen. "What just happened?" Jenna asks in disbelief.

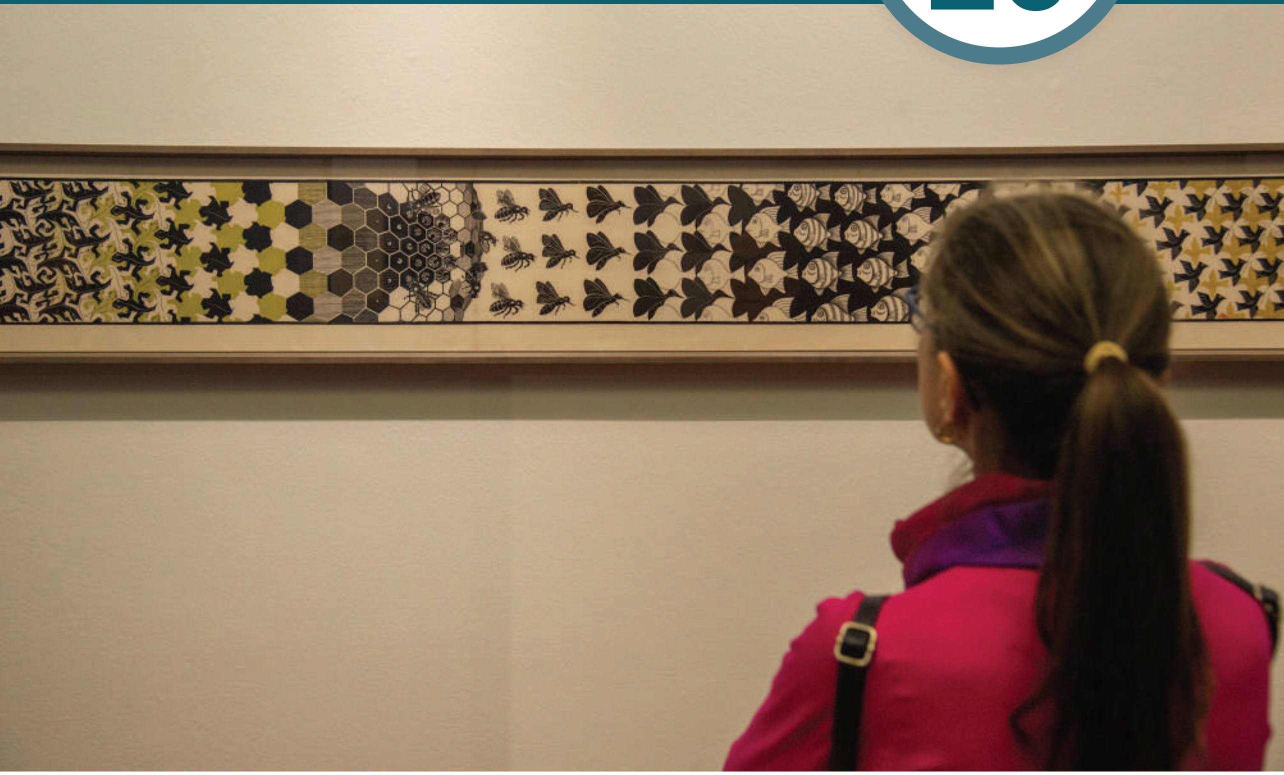
"Pete and his *Master Plotter* happened!" Nico exclaims.

Main Idea

We can use points on a coordinate plane to produce two-dimensional shapes.

M. C. Escher

Chapter 20



What comes to mind when you think of the word *art*? You might think about sculptures, paintings, or drawings. But if you have ever seen the work of Maurits Cornelis Escher—M. C., for short—you'll also be thinking about math!

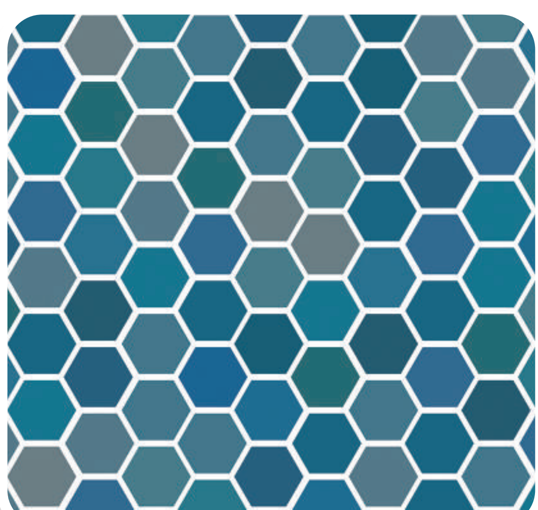
Escher was born in 1898 in the Netherlands. His parents wanted him to be an architect, like his father. He wasn't a very good student, but he *did* love to draw. He studied graphic art at the Haarlem School of Architecture and Decorative Arts.

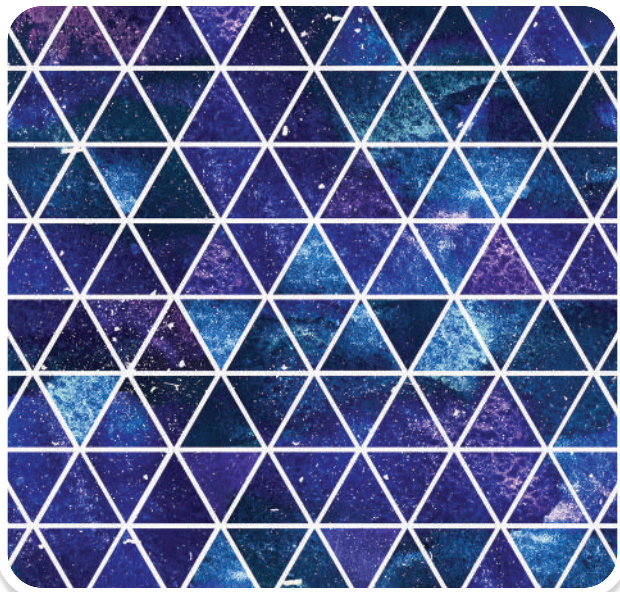
Escher was very inspired by nature and by places he visited in Spain and Italy. The place that most captured his imagination was the Alhambra in Grenada, Spain.

The Alhambra is a palace that was built by Muslim rulers from 1238 to 1358. The palace and surrounding fortress are very large and beautiful. But what interested Escher the most were the tiles that decorated it.

Notice how none of the tiles on this wall overlap and how there are no gaps between them. These beautiful patterns are called tessellations. A *tessellation* is a pattern that uses the same shapes over and over with no gaps or overlaps to produce a design. The shapes can face any direction, but they have to be the same. Sometimes the patterns are simple, but others are quite complex.

These patterns are often used on walls and floors. Where have you seen patterns like this before?





According to mathematicians, there are only certain regular polygons—shapes with equal sides and equal angles—that can be used to make tessellations. But Escher started to think about the idea of making tessellations with irregular shapes, too.

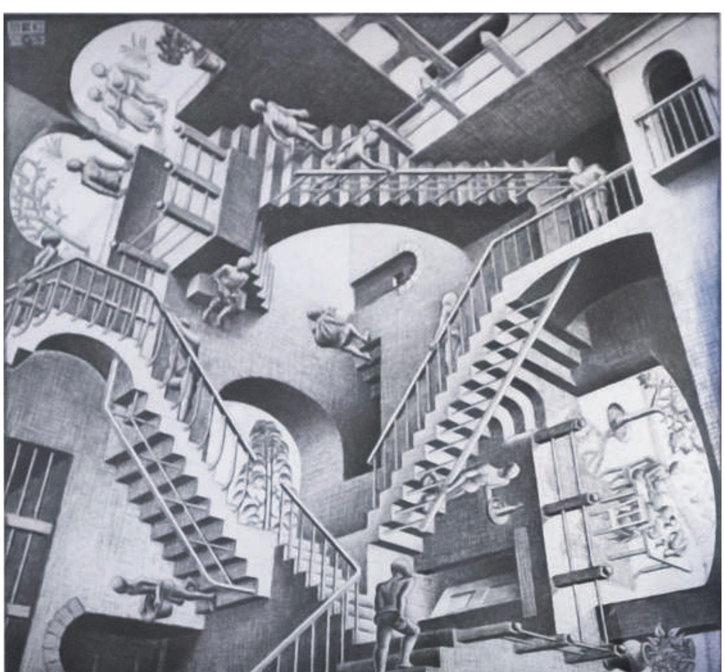
Escher included mathematical principles and his own bold ideas to create different kinds of images. To make one of his most famous pieces, he started with a simple triangle tessellation.

Then, he twisted and rotated each of the shapes until they looked like birds. The birds fit together to produce a repeating pattern with no gaps and no overlaps. He did it!

One variation of Escher's birds is displayed at a museum. Notice how some birds face right and some birds face left.

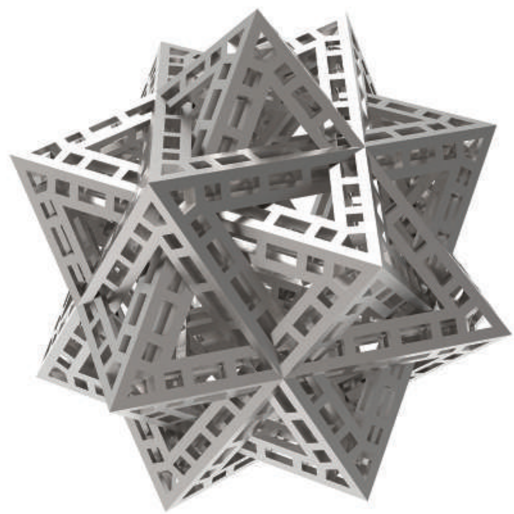
Escher was also interested in patterns that both repeated and changed to make optical illusions. An *optical illusion* is an image that looks like one thing but might actually be another. To make this type of art, he repeated some of the shapes and then changed others a little bit at a time to form new shapes. A famous work called *Metamorphosis I* shows a town transforming into a person! You can see this art at the entrance to the Escher Museum in the Netherlands today.

Escher used repeating patterns of other images to trick the brain and produce optical illusions. This is one of Escher's most famous pieces, called *Relativity*. What do you notice?



M. C. Escher explored other mathematical ideas in his works, including creating art using polyhedrons. A polyhedron is a three-dimensional shape with faces made of polygons. Escher wasn't a mathematician, but he was very interested in symmetry. He wondered if he could layer three-dimensional shapes to make symmetrical art.

To make this kind of art, Escher first created a three-dimensional model of polyhedrons layered on top of each other. His models would have looked something like this. How many three-dimensional shapes can you find?



To make two-dimensional art, he made woodcuts and then layered the colors when he printed them to make them look like they could jump off the page! He found a way to carve each shape so that the whole piece of art was symmetrical!

An important theme in M. C. Escher's art was the mathematical idea of infinity. Infinity is when something continues forever. Sometimes Escher's patterns were infinite from top to bottom or left to right.

In this piece, called *Smaller and Smaller*, he imagined what infinity would look like from *inside* the drawing.

The pattern repeats over and over, but it gets smaller as you look at the center and larger as you get to the edges! It looks like it could go on forever!

Main Idea

M. C. Escher used tessellations and other mathematical principles to create original works of art.

The Oldest Computer

Chapter 21

In 1901, a group of Greek divers put on their diving suits to look for sea sponges in the Mediterranean Sea.

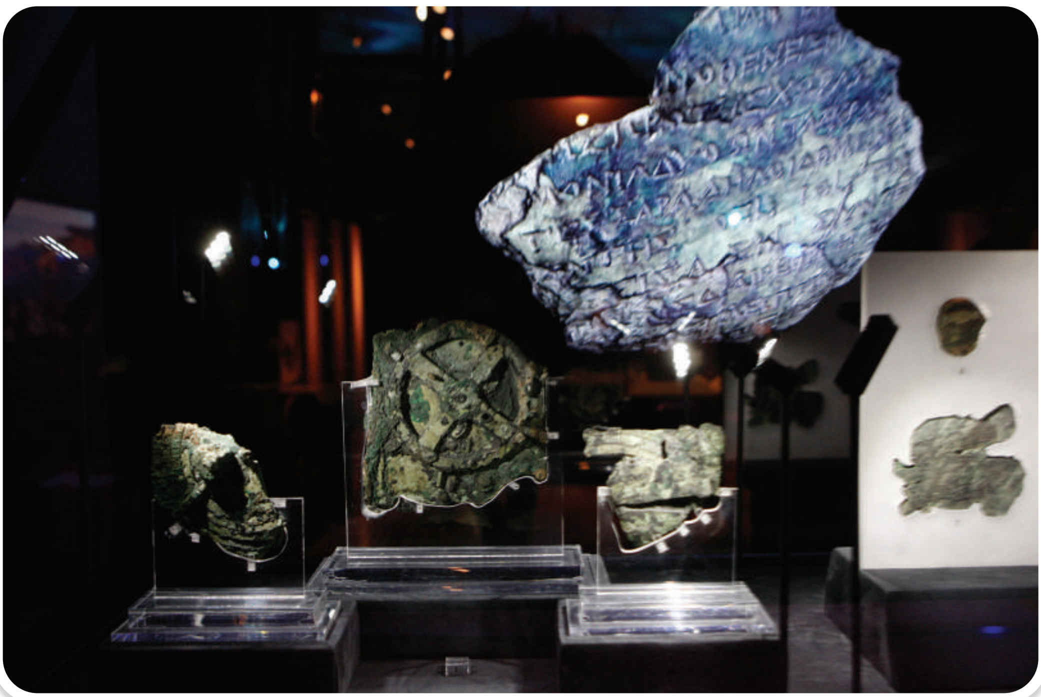
However, the first diver into the water found more than just sponges. He found a shipwreck from around the year 60 BCE! The divers knew they had found something special.

One of the items they discovered was a hunk of metal that was about the size of a large book. It was rusty and very fragile.

Scientists at the National Archeological Museum in Athens, Greece, studied the mysterious object closely and found something unexpected—gear wheels! This was surprising because there was no other evidence anywhere in the world that people living in this time period had used gear wheels. The scientists knew they had found some kind of ancient machine.

They could also see some writing, including ancient Greek words for planets and months of the year.

Scientists named the device the Antikythera mechanism for the Greek island close to where it was discovered. It was so fragile that since it was taken from the shipwreck, it has slowly broken into 82 pieces! This made it very difficult to study and understand. Scientists could not determine what the device did just by looking at it. Taking it apart could destroy it.



One of the first scientists to study the device was Albert Rehm. He discovered the numbers 19, 76, and 223 engraved on part of the device. Rehm knew that those numbers had meaning in the astronomy system invented by the ancient Babylonians, who lived about 2,000 years earlier! The device had something to do with astronomy!

A 1905 sketch showed that Rehm believed that the device was a way to show the movement of the sun, the moon, and the stars.

In 1958, science historian Derek de Solla Price got permission to study the device. Price knew a lot about the history of science and how many early scientific instruments were made. He could not see what was inside, but he agreed with Price that the Antikythera mechanism was an early computer for astronomy.

Price continued to study the Antikythera mechanism for a long time. In 1971, he had the opportunity to take x-rays of the device. X-rays allow scientists to see inside of something. Using x-rays allowed Price to see inside the device without taking it apart. The images made showed that there were 30 different gears inside!

This image shows x-rays of a bicycle gear. Scientists used x-rays of the Antikythera mechanism's gears to learn more about how it worked.

The teeth on each gear were each only about a millimeter long. The x-rays also showed Price how all the different gears in the device fit and worked together. Price used what he learned from the x-ray images and what he knew about ancient astronomy to build a model of the Antikythera mechanism.

As technology advanced, scientists and mathematicians have been able to solve most of the mysteries of the Antikythera mechanism.

The scientists and mathematicians studied the number of teeth on the gears and the inscriptions that they were able to read with the help of technology. They came to understand that one part of the device tracked the movement of the sun, another part tracked the movement of the moon, and others tracked the movements of the planets.

To work the machine, users turned a handle. They could see where each of these bodies would be in the sky on any day in the past or future! This meant they could predict what the night sky would look like and use it to predict solar and lunar eclipses. One part was also used specifically to calculate the four-year cycle of when the ancient Olympic Games should take place!

Ancient people used the Antikythera mechanism to predict eclipses, including lunar eclipses like the one shown here. A lunar eclipse takes place when Earth's shadow falls onto the moon.

The Antikythera mechanism is extremely complex. It is so complicated that its technology is often compared to devices made a thousand years later! As you can imagine, it has raised a lot of questions. Who made it? How did they make it? Why did they make it?

Nobody knows for sure, but many scholars think that the device may have been designed and built by Archimedes. Archimedes was a brilliant ancient Greek mathematician and inventor. He designed many other devices that were ahead of his time.

What makes the Antikythera mechanism so special is that it took everything that was known about astronomy at that time from different civilizations and organized it into a single device. It took an amazing level of understanding of math and science to be able to do that! It would have been an incredible tool for scientists of that time.

Archimedes of Syracuse was a brilliant inventor who designed both practical and imaginary objects. One invention people still use today is what is known as Archimedes' screw, a device that uses a large screw and gravity to move water for irrigation. One of his numerous math accomplishments was that he was the first to determine the volume of a sphere inside a cylinder. He even had this figure inscribed on his tomb.

The Antikythera mechanism changed our understanding of technology from the past. However, this device is most likely not the only example of advanced technology from ancient times. Ancient Greeks and Romans wrote about other advanced machines that were used to track the movement of the sun, moon, and planets.

Like the Antikythera mechanism, these machines may be waiting to be discovered in ancient shipwrecks buried deep in the Mediterranean Sea. In the meantime, scholars continue to unlock the secrets of the Antikythera mechanism and study other devices the ancients used to track time and objects in the night sky.

Main Idea

The Antikythera mechanism used math and science to calculate the movements of the sun, moon, and planets.

Chapter 22

Uncle Tony's Pizza Garden

Uncle Tony owns a pizzeria called Pizza Freshness. He takes a lot of pride in making sure his pizzas are always fresh and delicious. Uncle Tony makes all the dough he uses for his pizza crusts. He also grows all the vegetable toppings and herbs that he puts on his pizzas. One request he hears over and over is that customers wish there were more pepper choices for toppings.

Uncle Tony decides to start a new garden to grow different types of peppers for toppings. He wants to put the new garden in a space that is 20 feet long and 15 feet wide. He wants the garden to have the largest area possible, but he only has 65 feet of fencing that was left over from another project.

Uncle Tony asks his niece and nephews to help him decide what shape the garden should be and how much fencing he will need for the new pepper garden.

Maria tells Uncle Tony that the garden would have to be pretty small if he only has 65 feet of fencing. She suggests he make the garden in the shape of a rectangle that is 20 feet long and 3 feet wide. Uncle Tony thinks for a minute and realizes that Maria has found the area instead of the perimeter. "Remember that the fence will go around the garden, so the amount of fencing we need is the perimeter," Uncle Tony says brightly.

"If we make the garden a square that is 15 feet long and 15 feet wide, the sides will all be the same, and we'll only need 60 feet of fencing," says Marco.

Giovanni has an idea. "Uncle Tony! What if we make the garden a rectangle that is 20 feet long and 12 feet wide? We will need 64 feet of fencing to go around. That's pretty close to 65 feet!" He waits to see his uncle's response.

Uncle Tony says, "Seems we have a winner! A rectangle 20 feet by 12 feet, it shall be! Now, who wants to help me start making the garden and planting the peppers?"

Logic and Math Win the Game

At the school carnival, Malik and Phoebe see their principal behind a balloon booth. “Hi, Malik and Phoebe! Would you like a chance to find the winning ticket and choose a prize?” Principal Nelson asks.

“Sure,” they reply. “What do we have to do?”

“I give you a series of clues. Each clue costs one ticket. You use the clues to decide which balloon has the winning ticket,” Principal Nelson responds.

“Let’s do it!” says Phoebe. Malik hands Principal Nelson a ticket so they can get their first clue.

Clue 1: I am an even number.

Phoebe looks at Malik and says, “Well, that narrows it down. The number is 2, 4, 6, 8, 10, or 12. But let’s buy another clue.” Malik hands over another ticket.

Clue 2: Half of me is more than 2.

Malik says, “It can’t be 2. Half of 2 is 1. And it can’t be 4. Half of 4 is 2. The number is more than 2. That leaves 6, 8, 10, and 12. Let’s buy another clue.”

Phoebe hands Principal Nelson another ticket.

Clue 3: Half of me is an odd number.

"Okay, now we're getting somewhere," says Phoebe. "The numbers left are 6, 8, 10, and 12. Half of 6 is 3. So it could be 6."

"Half of 8 is 4, and half of 12 is 6. Both of those are even numbers," Malik says.

Phoebe responds, "And half of 10 is 5, an odd number. So we have 6 and 10 left. We have a 50/50 chance of picking the correct number and a 50/50 chance of picking the wrong number."

Malik responds, "Let's pay for one more clue to make sure." Phoebe agrees and hands over a ticket to Principal Nelson.

Clue 4: All of my multiples have a 0 in the ones place.

Malik says, "Multiples of 6 are 6, 12, 18, 24, 30, 36, and so on. Only one of those numbers has a 0 in the ones place. So 6 is definitely not the number."

At the same time, Phoebe and Malik look at Principal Nelson and say, "The winning number is 10!"

Principal Nelson takes a thumbtack and pops balloon number 10. Confetti and a silver ticket fly out of the balloon. "Congratulations!" says Principal Nelson. "Your logic and math skills helped you win the game. Now you can take this ticket to the prize booth and pick whatever prize you both want."

Fence Posts

Each summer, Ellis and Macie go to their grandparents' farm. The first thing the girls do when they arrive is feed the chickens. Both girls were there when the chickens hatched from the eggs. "I wonder how many eggs the hens will lay this time," Ellis says.

After Ellis and Macie visit the chickens, they go inside to see what Grandpa has in store to do this summer. Grandpa likes to keep busy. And he likes to have the girls help solve problems around the farm.

"Hi, girls!" Grandpa says. "Grandma says you have been making sure the chickens are fed. Those chickens sure do enjoy you coming in the summer. Come over to the corral to see our newest addition, Savannah."

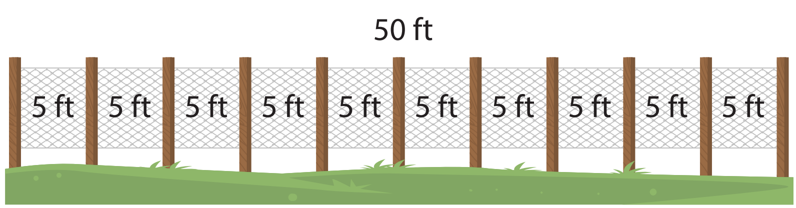
The girls rush to the corral and meet the most beautiful horse they have ever seen.

After brushing Savannah's coat, Grandpa asks the girls for some math help. "Girls, this summer we need to put a fence on the north side of the farm. Savannah likes to roam and run around in the afternoon. I want to make sure she doesn't wander off."

Grandpa explains that the straight fence will be 350 feet long with a post every 5 feet. "Think you two can figure out how many posts we'll need?" asks Grandpa with a smile.

"Macie, I have feeling this is more than an easy division problem. We know Grandpa well enough to know there is more to this problem than we see at first glance," says Ellis.

Macie agrees that there is something more to this problem. "Let's draw a picture," says Macie. They decide to figure out how many posts are needed for 50 feet of fence.



Ellis draws lines to represent posts. She draws a line at the starting point of the fence. She evenly spaces lines across the fence and labels the distance between them 5 feet. She continues writing numbers and drawing lines until she reaches 50. "Ellis, you drew 11 posts. That's one more than dividing 50 by 5! We can do that with the 350 feet too," says Macie. "350 feet divided by 5 feet is 70, and 1 more is 71! So we need 71 posts!" Grandpa nods.

Main Idea

Math ideas and operations can be used to figure out problems and puzzles.

The Mathematics of Music

Chapter 23

Have you ever wondered how musicians in a band, choir, or orchestra stay together? They use math! Math is the foundation of musical notation!

The 5 horizontal lines that make up each line of music are called a staff. Each line and space on the staff represents a specific note, or sound. Written music is organized into smaller blocks called measures. The bar lines, or vertical lines, on the staff show the musicians the beginning and end of each measure.

On the left side of the staff is a symbol showing the clef. The clef tells the pitch of each note. Different instruments have ranges that use different clefs.

Treble **Bass**

For example, a piano can have music written in both treble clef and bass clef. However, some instruments, such as a violin or a trumpet, can only produce sounds in the treble clef range, and instruments such as a double bass or a trombone produce sounds in the bass clef range.

The time signature at the beginning of a piece of music is one of the most important parts of the staff. It tells the musicians how to count the beat of the music and create the rhythm of the song! The time signature is written as a fraction. The top number tells the musicians how many beats are in each measure. The bottom number tells which kind of note has a count of 1 beat. Some pieces of music have only one time signature. Others have parts of the song where the time signature changes! Musicians have to pay close attention so that they play using the right rhythm at the right time.

Many musical pieces use a time signature of $\frac{4}{4}$. What does that mean to a musician? The top number is 4, which means there are 4 beats in each measure. The bottom number is also 4. This means that each quarter note gets a count of one beat. Musicians have to count carefully. When the measure has four quarter notes, the musician can count the beats evenly, as 1, 2, 3, 4. When other kinds of notes are used, the musician has to adjust their counting strategy.

When the time signature is $\frac{4}{4}$ . . .

A whole note 𝅝 is held for 4 beats, or the whole measure.

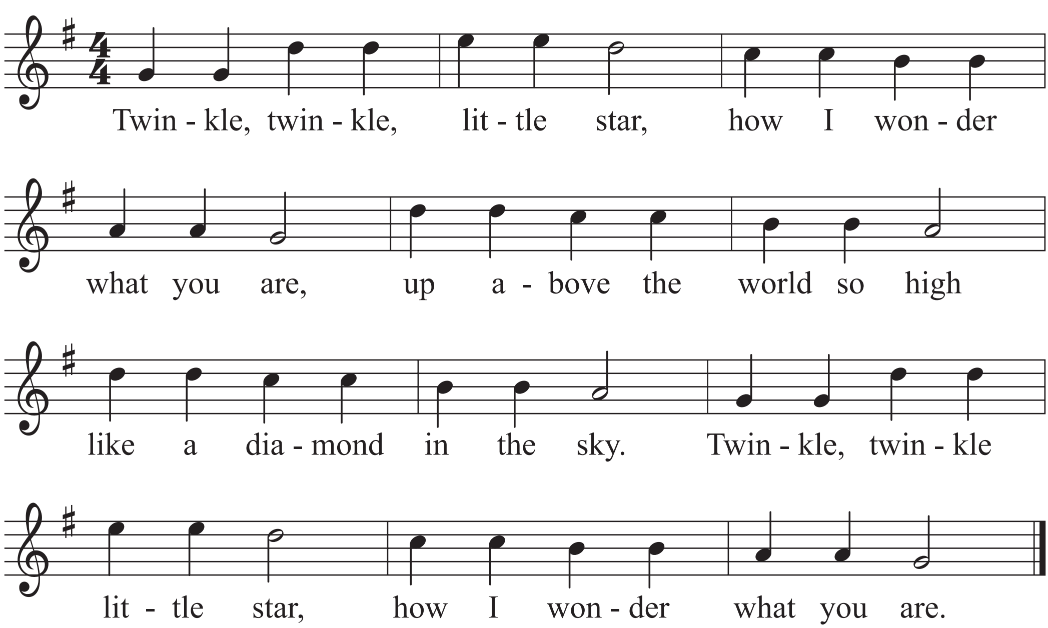
A half note 𝅗𝅥 is held for 2 beats, or half of the measure.

A quarter note ♩ is held for 1 beat, or one quarter of the measure.

The tempo of a song describes how fast the beat is. Tempos can be fast, medium, or slow. The tempo refers to how many beats there are in one minute. The tempo of a song affects a song's mood. Try singing a traditionally slow song quickly or a fast song slowly. See how it changes the mood?

Some musicians use a tool called a metronome to help them keep an even tempo. Metronomes can look very different, but they all make a ticking sound at regular intervals at whatever tempo they are set to.

"Twinkle, Twinkle, Little Star" has a time signature of $\frac{4}{4}$ and a tempo of 60 beats per minute. Try singing the song and counting the beat!

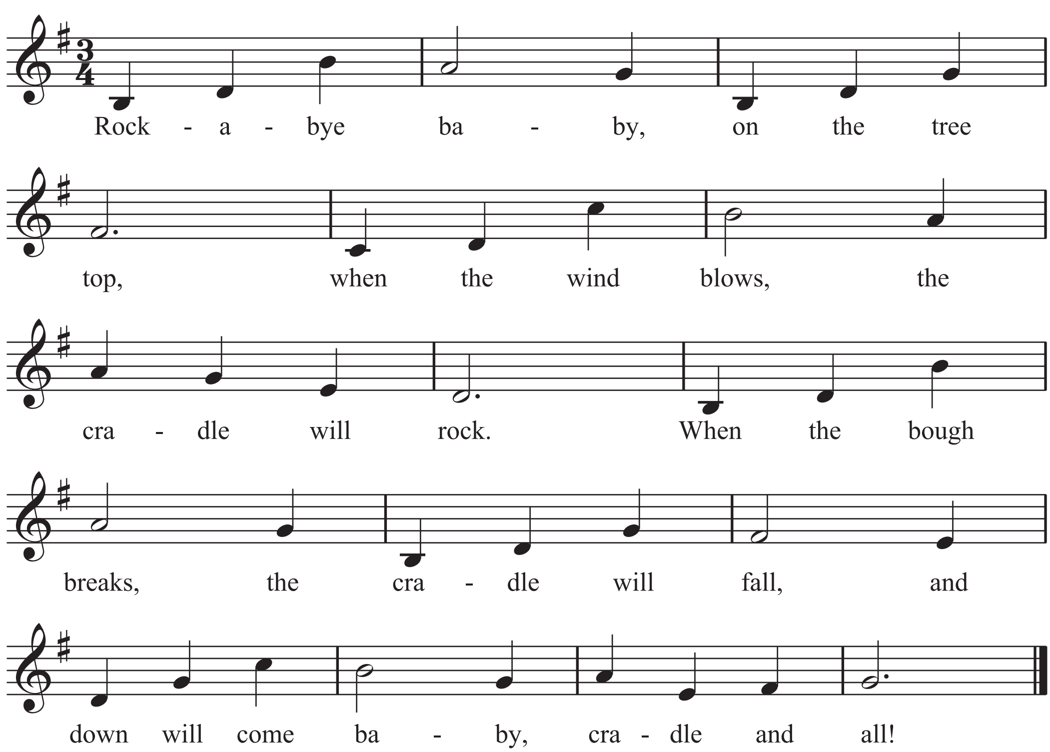


A time signature of $\frac{3}{4}$ means that there are 3 beats per measure and every quarter note gets one beat. Because there are only 3 beats in a measure, a whole note doesn't fit in one measure! The song "Rock-a-Bye Baby" is a lullaby. This means it is usually sung with a slow tempo and played very quietly.

Try singing this song with different tempos and different volumes. What do you notice about how it changes the mood of the song?

Counting Music

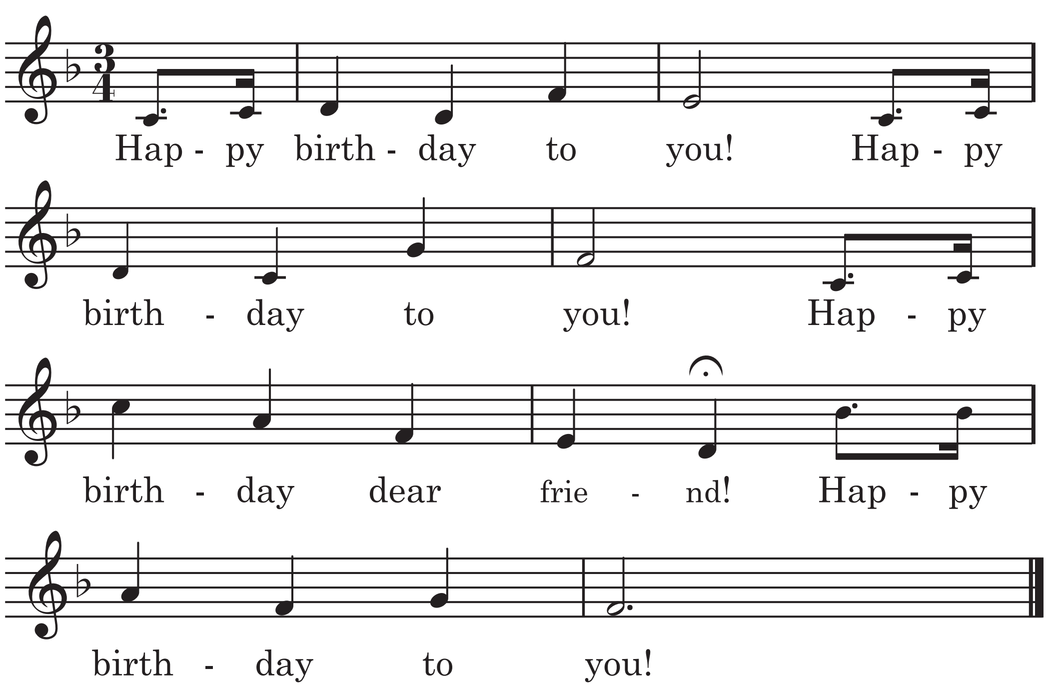
Take a look at the music for the song "Rock-a-Bye Baby." How many beats do you think a half note with a dot gets? How do you know?



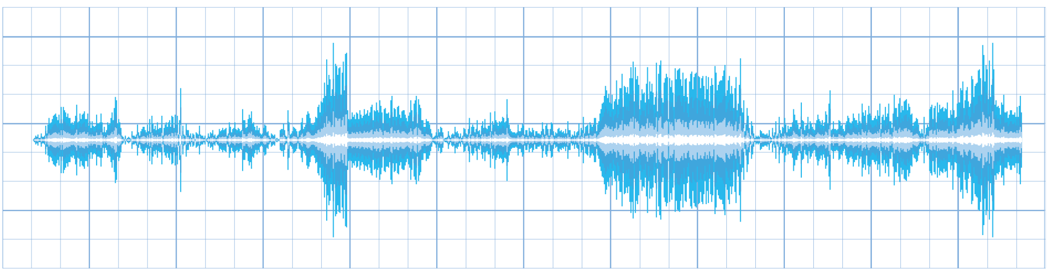
What happens when the notes need to be played more quickly to create shorter notes in a song? Think about the pattern that musical notes follow. A half note is half of a whole note. A quarter note is half of a half note. What do you think is half of a quarter note? An eighth note! And half of an eighth note is a sixteenth note! Do you see the pattern?

> An eighth note ♪ is held for $\frac{1}{2}$ of a beat.
>
> Eighth notes often come is pairs and are represented as ♫.
>
> A sixteenth note ♬ is held for $\frac{1}{4}$ of a beat.

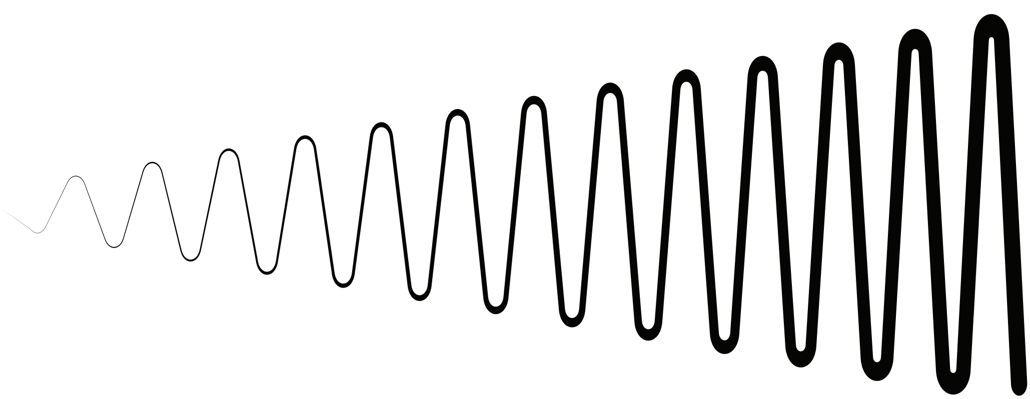
Look for the eighth notes and sixteenth notes in the song "Happy Birthday" below.



Think about what you now know about the songs "Twinkle, Twinkle, Little Star," "Rock-a-Bye Baby," and "Happy Birthday." Notice that when the sounds get higher, the notes are placed higher on the staff. The specific sound that a note has is called its pitch. The higher the line or space on the staff, the higher the pitch of the note. The lower the line or space on the staff, the lower the pitch of the note.



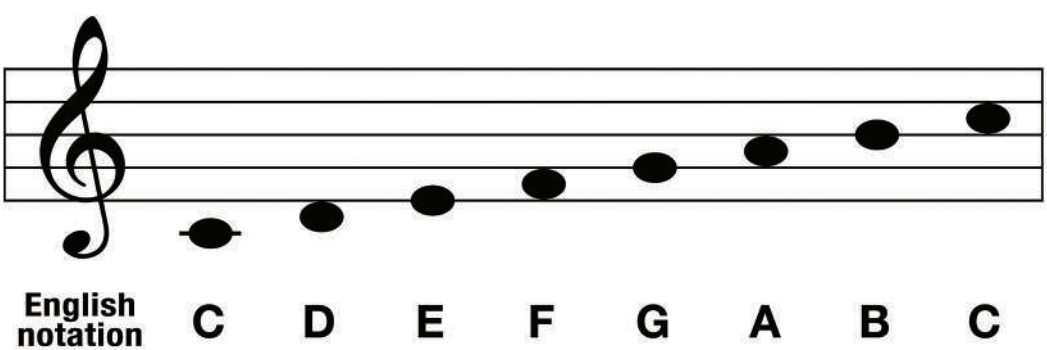
The concept of high notes and low notes has to do with their frequency. A sound with a high frequency has short wavelength, or distance between the waves. A sound with a low frequency has a longer wavelength, or a longer distance between the waves.



Singing voices can be classified by whether they are in higher or lower frequency ranges. A soprano can sing very high notes, or notes with very high frequencies. A baritone can sign very low notes, or notes with very low frequencies.

To warm up before playing, musicians sing or play scales. A scale is a series of notes, ordered by pitch, that start and end on the same note.

In 1959, Oscar Hammerstein wrote the lyrics to the song "Do Re Mi." This song explores the 8 notes in the C scale. A scale spans one octave. In an octave, the highest note is two times the frequency of the lowest note. So the high C in this scale has a frequency that is double the lower C! Scales can start and end on any note, as long as they start and end on the same note.



There are so many ways that music uses math to produce rhythms and moods!

No matter what kind of music you listen to, what time signature the music has, or how long its various notes are sustained, when you sing or play along you are subconsciously counting. You are observing the pattern of the beats, noting which parts have emphasis, and using what you notice to predict the repeating patterns of the notes. Many people find making music—or even just humming along and tapping a foot to the rhythm—to be joyful. It's the math in it that appeals to our minds!

Main Idea

Musical notation uses fractions to create a relationship between the lengths of each note.

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