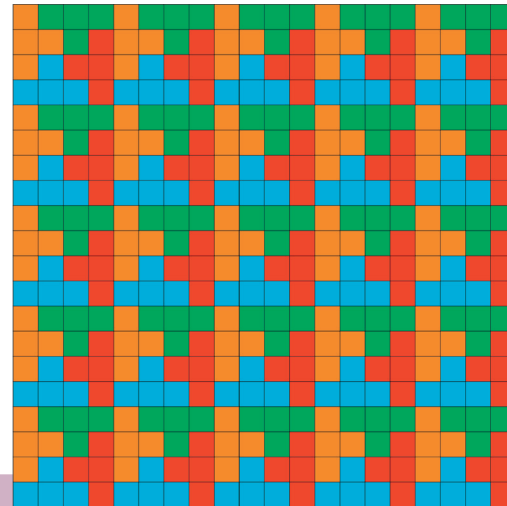
Core Knowledge® MATHEMATICS

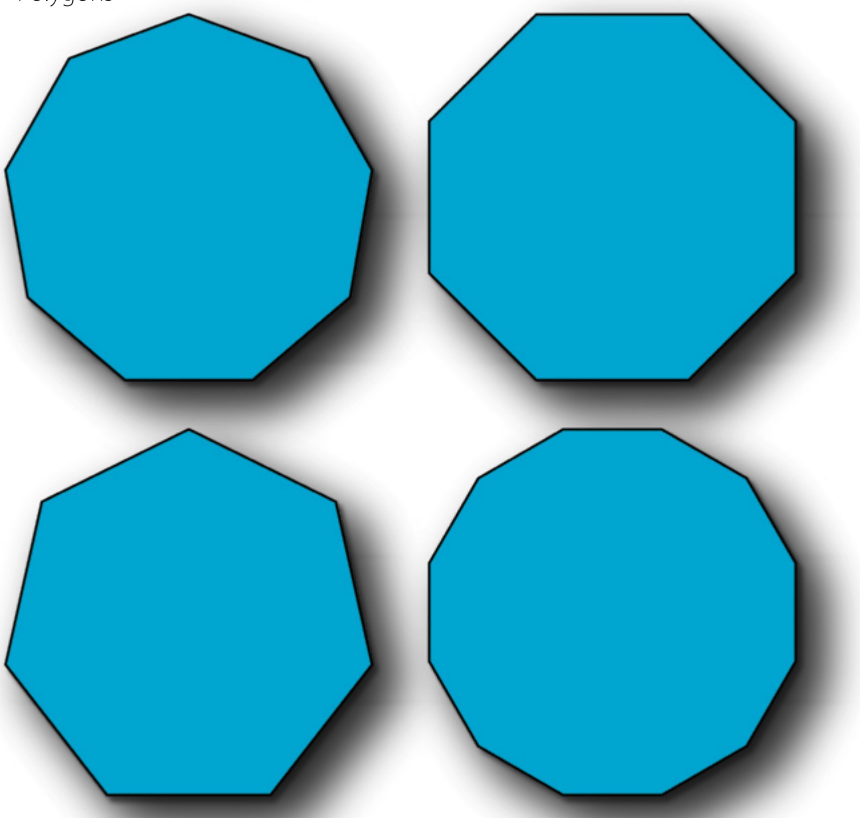
Putting It All Together



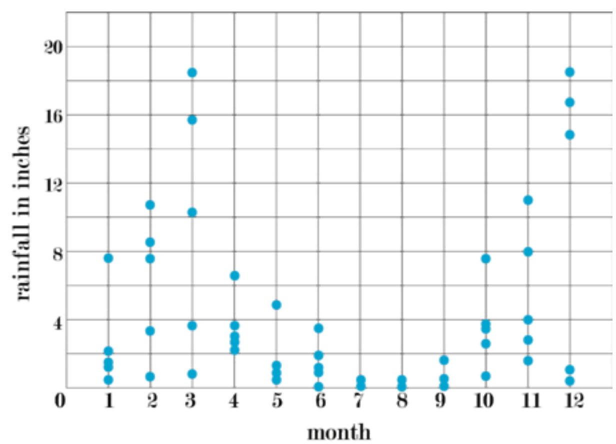
Polygon Patternss

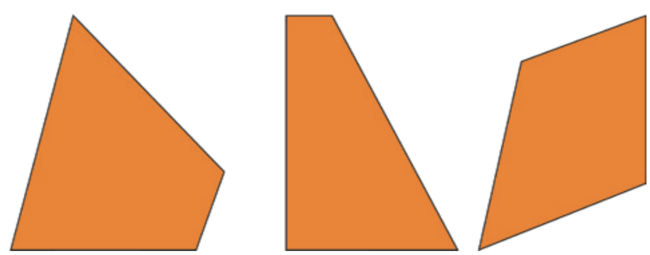
Teacher Guide

Polygons



Plotting the Weather





Tessellations

Function of Latitude

ISBN: 979-8-88970-853-7

Putting It All Together

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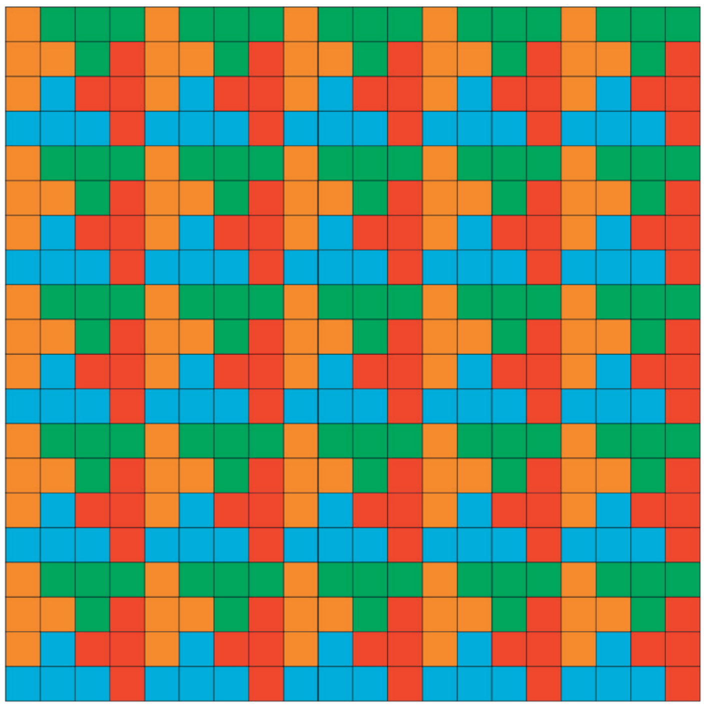

Putting It All Together

Teacher Guide

Core Knowledge Mathematics™

Putting It All Together

Unit Narrative

In these optional lessons, students solve complex problems. In the first several lessons, they consider tessellations of the plane, understanding and using the terms “tessellation” and “regular tessellation” in their work, and using properties of shapes (for example, the sum of the interior angles of a quadrilateral is 360 degrees) to make inferences about regular tessellations. These lessons need to come after unit 8.1 has been done. In the later lessons, they investigate relationships of temperature and latitude, climate, season, cloud cover, or time of day. In particular, they use scatter plots and lines of best fit to investigate the question of modeling temperature as a function of latitude. These lessons need to come after units 8.5 and 8.6 have been done.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as describing, representing, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Describe

- tessellations (Lesson 1)
- associations in bivariate data (Lesson 5)

Represent

- the relationship between latitude and weather (Lesson 5)

Justify

- claims about shapes that can and cannot be used to produce regular tessellations (Lesson 2)

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students’ use of a new term in the lessons that follow the one in which it was first introduced.

lesson	new terminology	
	receptive	productive
8.9.1		**tessellation** pattern
8.9.2	regular tessellation	regular polygon
8.9.6		mathematical model

Required Materials

Dried linguine pasta
We specified linguine since it is flatter and less likely to roll around than spaghetti.

Pre-printed slips, cut from copies of the Instructional master

Protractors
Clear protractors with no holes and with radial lines printed on them are recommended.

Tracing paper
Bundles of "patty paper" are available commercially for a very low cost. These are small sheets (about 5" by 5") of transparent paper.

lesson	new terminology	
	receptive	**productive**
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Section: Tessellations

Lesson 1: Tessellations of the Plane

Goals

- Create and describe patterns with specific polygons that fill the plane.

Lesson Narrative

This optional sequence of three lessons can be done any time after the first unit in the course. Students are introduced to tessellations of the plane. The activities in this lesson provide students a chance to go more deeply and apply grade 8 geometry concepts to a mathematical context. The activities in this sequence of three lessons build on each other, so should be done in order. It is not necessary to do all three lessons to get some benefit, although more connections are made the further one gets.

As students progress through the activities, they gain an intuition for the variety of ways some shapes can be put together to build a tessellation and the restrictions on how some shapes can be put together to build a tessellation. They start with general tessellations, and then look at regular tessellations. Finally in the third lesson, they examine tessellations using other regular polygons.

Throughout these lessons, students make use of structure when building their tessellations (MP7). They reason abstractly and quantitatively when deciding which polygons can be used in the different types of tessellations (MP2) and make viable arguments to convince each other that some polygons cannot be used to tessellate the plane.

Alignments

Building On

- 7.G.A: Draw, construct, and describe geometrical figures and describe the relationships between them.

Addressing

- 8.G.A: Understand congruence and similarity using physical models, transparencies, or geometry software.

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports
- Notice and Wonder

Required Materials

Pre-printed slips, cut from copies of the Instructional master

Tracing paper
Bundles of "patty paper" are available commercially for a very low cost. These are small sheets (about 5" by 5") of transparent paper.

Required Preparation

If using the print version of the materials, all students require tracing paper. If using the applets in the digital version of the materials, students may not need tracing paper.

Prepare 1 pre-cut copy of the Describing a Tessellation Instructional master for every 2 students. Each student needs one of the two slips showing an individual tessellation.

Student Learning Goals

Let’s explore geometric patterns!

1.1 Notice and Wonder: Polygon Patterns

Optional: 5 minutes
This activity introduces students to patterns of polygons that cover the plane. Examples include:

- patterns using a single shape
- patterns using multiple shapes

Studying these patterns and understanding how and why they repeat to fill up the plane is an example of MP8, expressing regularity in repeated reasoning. In this case, the repeated reasoning is continuing to lay out the shapes in the same pattern.

Building On

- 7.G.A

Instructional Routines

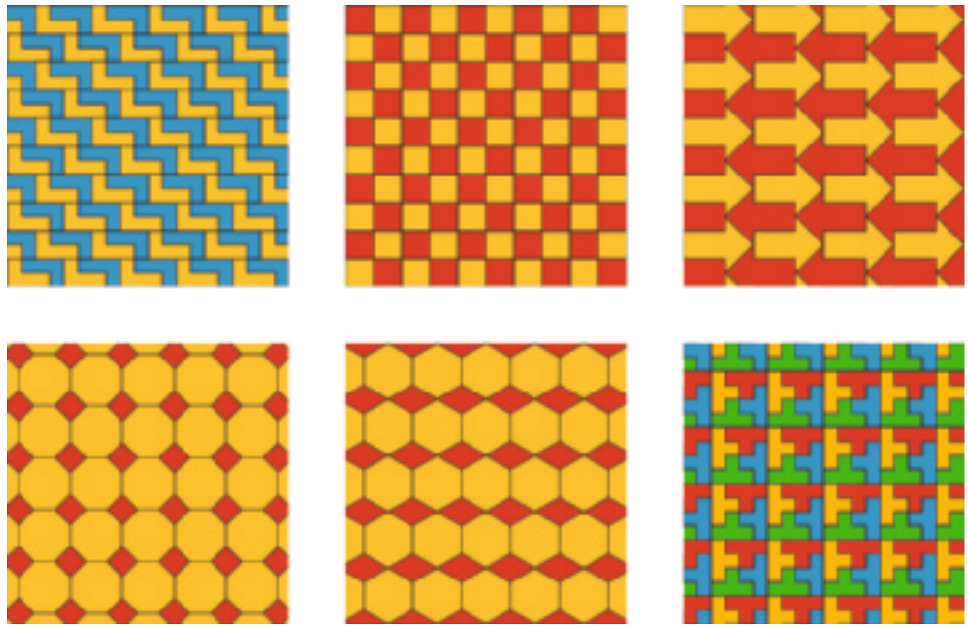
- Notice and Wonder

Launch

Arrange students in groups of 2. Tell students that they will look at a set of images, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the images for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Student Task Statement

What do you notice? What do you wonder?



Student Response

Things students may notice:

- Four of the patterns use only one shape. Two of them use two different shapes.
- The pattern could continue to the left and right or up and down.

Things students may wonder:

- Are the colors of the shapes important?
- Do the arrows count as one shape or two, since they are pointing in different directions?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the images. After each response, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to the images each time.

Important ideas that can come up are the following:

- The shapes in a tessellation cover the plane (or would if the pattern continues) without gaps.
- Sometimes there is only one shape. Sometimes there are several shapes in the tessellation.
- With the square one, all of the squares are complete. In the other patterns, the shapes on the boundary are cut off.

If time allows, ask:

- “How do I know what happens as the tessellation continues to grow off of the page?” (There is often a pattern, but we would have to indicate that the pattern continues.)
- “Are the colors of the shapes important?” (They are important for identifying patterns and making the pictures prettier, but they could be changed and the pattern would still be the same.)

1.2 Tessellations

Optional: 20 minutes (there is a digital version of this activity)
The goal of this task is to introduce the notion of a tessellation and then carefully examine how to create a tessellation with each pattern block shape (square, rhombus, equilateral triangle, isosceles trapezoid). For hexagons, there is only one way to fit them together because there will be gaps unless three hexagons meet at each vertex. The other shapes offer much more flexibility, and students have an opportunity to use their artistic creativity.

Students look for and make use of structure (MP7), both when they try to put copies of the shape together to build a tessellation and when they examine whether or not it is possible to construct a different tessellation.

Building On

- 7.G.A

Addressing

- 8.G.A

Instructional Routines

- MLR8: Discussion Supports

Launch

Introduce the definition of a **tessellation** of the plane by polygons: a tessellation covers the plane with copies of the shape with no gaps and no overlaps.

Show the students the image from the previous notice and wonder activity, and ask them if these are tessellations. (Yes.)

Demonstrate how to use tracing paper to create a tessellation.

Arrange students in groups of 2. Each pair uses the same shape for their tessellations.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Include the following term and maintain the display for reference throughout the lesson: tessellation. Consider providing step-by-step directions for students to use tracing paper to create a tessellation. Invite students to suggest language or diagrams to include that will support their understanding.
Supports accessibility for: Memory; Language

Access for English Language Learners

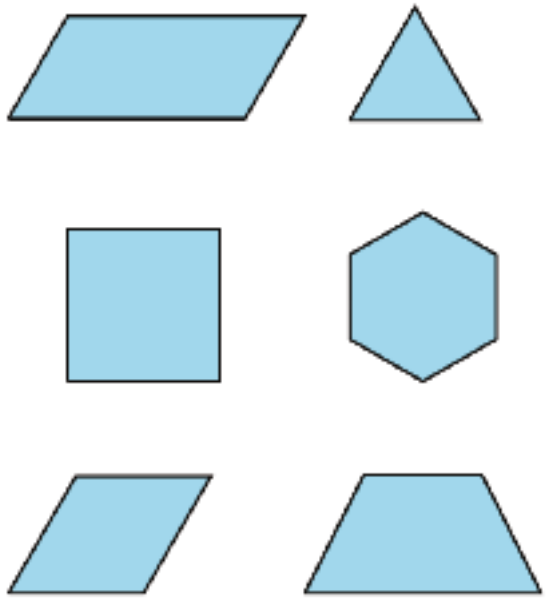
Speaking: MLR8 Discussion Supports. Amplify students' uses of mathematical language to communicate about tessellations; edge to edge, vertex, symmetrical, plane, rigid motion, etc. Encourage students to use these words, revoicing students' ideas as necessary. Ask students to chorally repeat the phrases that include these words in context.
Design Principle(s): Support sense-making; Optimize output (for explanation)

Anticipated Misconceptions

Tessellations do not need to be edge to edge. That is, pieces do not need to fit together with edges of the same lengths matching exactly.

Tessellations do not need to have symmetric, repeating patterns, though sometimes the shape forces it (as with the regular hexagons).

Student Task Statement



1. Pick one of the shapes. Create a **tessellation** by tracing copies of your shape. Make sure to use the same shape as your partner.
2. Compare your tessellation to your partner's. How are they similar? How are they different?
3. If possible, make a third tessellation of the plane with your shape (different from the ones you and your partner already created). If not possible, explain why it is not possible.

Student Response

1. Answers vary.
2. Answers vary.
3. It depends on the shape. It is not possible for the hexagon: Three hexagons have to come together at each vertex: once the first hexagon of the pattern is placed, everything else has no flexibility. It is possible for the other shapes: Triangles can be built into hexagons, or they can make rows that can be translated and stacked on top of one another. Parallelograms can also be made into hexagons or rows. Trapezoids can be made into hexagons or rows. Rhombuses can be made into rows and translated. Squares can also be made into rows and translated.

Activity Synthesis

Important questions to address include:

- "Were you able to make different tessellations with your shape?" (It depends on the shape.)
- "If not, why not?" (Three hexagons have to come together at each vertex: once the first hexagon of the pattern is placed, everything else has no flexibility.)

- “If so how?” (Triangles can be built into hexagons, or they can make rows that can be translated and stacked on top of one another. Parallelograms can also be made into hexagons or rows. Trapezoids can be made into hexagons or rows. Rhombuses can be made into rows and translated. Squares can also be made into rows and translated.)
- “What does it look like to *not* define a tessellation?” (Two octagons can be put together sharing a vertex, but there is a gap that is not large enough for a third octagon.)

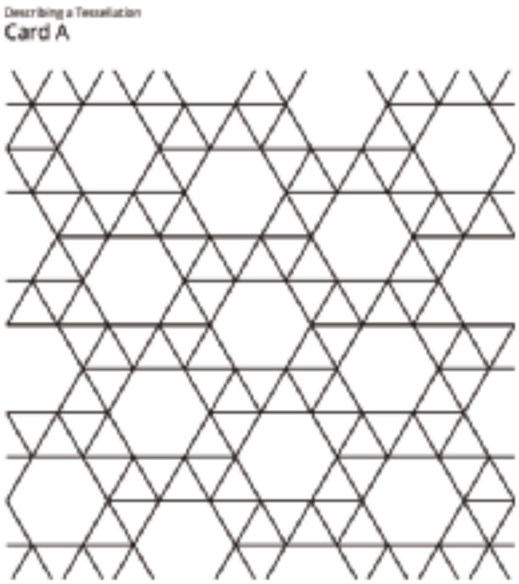
Make sure, as students share their ideas for tessellations, to use the language of rigid motions to describe the tessellations. For example, if a student has built a tessellation with parallelograms, choose two parallelograms and ask:

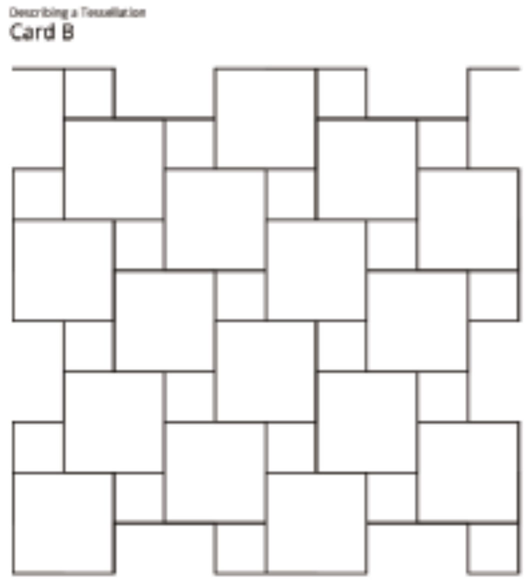
- “How can I use rigid motions to move one of these parallelograms into the position of the other?”
- “Are there other ways I could do this?”

1.3 Describing a Tessellation

Optional: 20 minutes
The goal of this activity is to accurately describe a tessellation of the plane. While students do not need to use the words translation, rotation, or reflection, their understanding of rigid motions of the plane will play a key role in explaining (and interpreting) where to place each shape in a tessellation. Communicating a geometric pattern clearly in words fully engages students in attending to precision (MP6).

As a warm-up, each student will describe a tessellation while the partner identifies a picture of the tessellation. After a brief discussion of what language was most helpful, students then take turns describing a tessellation as their partner attempts to build the tessellation. They switch roles and then reflect on any misinterpretations that happened and how they may have been related to the language used to describe the tessellations.





Building On

- 7.G.A

Addressing

- 8.G.A

Instructional Routines

- MLR2: Collect and Display

Launch

Students work in pairs. Each pair requires a set of 2 cards for Part 2, one card for each student.

Stop students after they have completed Part 1 of the task for the first part of the discussion given in the synthesis.

Access for Students with Disabilities

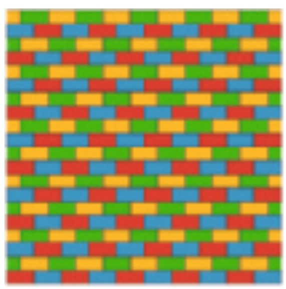
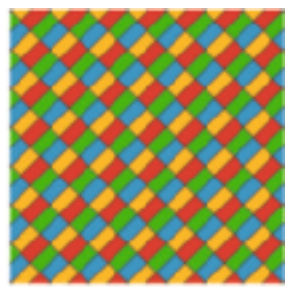
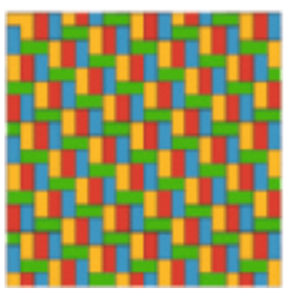
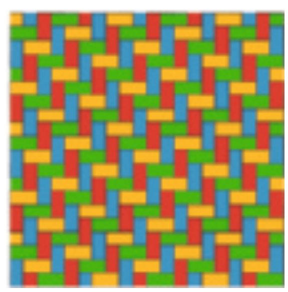
Engagement: Develop Effort and Persistence. Display or provide students with a physical copy of the written directions. Check for understanding by inviting students to rephrase directions in their own words. Keep the display of directions visible throughout the activity.
Supports accessibility for: Memory; Organization

Access for English Language Learners

MLR2 Speaking, Listening, Representing: Collect and Display. While students work with their partners during this activity, use this to record initial language used to describe their tessellations. Since students are not required to use specific language like rotation or translation yet, record the language students use as they construct and compare their tessellations. Display the recording language throughout the activity, and refer to it during the synthesis. Add or edit language as students respond to each question.
Design Principle(s): Support sense-making; Maximize meta-awareness

Student Task Statement

1. Pick one of the figures and describe the tessellation. Your partner will identify which tessellation you are describing. Then trade roles so your partner describes the tessellation and you identify the figure.

2. You and your partner each have a card with a tessellation. Describe what is on your card so that your partner can produce the tessellation (this should be done so that you cannot see your partner's work until it is complete).

3. Check together to see if your partner's tessellation agrees with your card and discuss any differences.
4. Change roles so your partner describes a tessellation, which you attempt to produce.
5. Check the accuracy of your construction and discuss any discrepancies.

Student Response

Part 1: Answers vary. Sample response (for bottom left design): This tessellation has groups of three rectangles next to one another, sharing the long sides: the long sides are vertical. In addition to these groups of three rectangles, there are single rectangles with the long side lying horizontally.

Part 2: Answers vary. Sample response (for Card A): Draw a hexagon. Use each of the hexagon's sides as one side of an equilateral triangle drawn outside the hexagon. The hexagon with its 6 triangles should look like a star. Now we make more stars, with an edge of one triangle from each new star sharing one side with a triangle on a previous star. If we draw new stars consistently on the same side of existing triangles (in this case, always on the clockwise side), we get the image in Card A. Sample response (for Card B): Draw a square with horizontal and vertical sides. Use the midpoint of the right side as the top left vertex of a second square the same size. Repeat for the other three midpoints of the original square: The midpoint of the top should be the bottom left of a new square, the midpoint of the left should be the bottom right of a new square, and the midpoint of the bottom should be the top right of a new square. Now repeat this process for each square arising to get the figure in Card B.

Activity Synthesis

For the discussion after students complete Part 1:

- "In what ways was describing the tessellation difficult?" (Finding words to communicate how the rectangles are aligned with one another.)
- "Did you use the words translate, rotate, or reflect?" (Instead of "translate" students may use words like "move." Similarly, students may describe rotations with words like "turn.")
- "What was challenging about describing or identifying the tessellation?"

For the discussion after students complete Part 2, focus on the use of the words translate, reflect, and rotate (or equivalents).

- "When you used a translation, did you specify the direction and how far?" (Maybe, but they may also use the language "put next to" or equivalent.)
- "When a rotation was involved, did you specify the number of degrees of the rotation?" (Probably not. They will more likely say "rotate until the sides match. . ." or "turn upside down.")
- "When a rotation was involved, did you specify the center of rotation?" (Answers vary. Probably not.)
- "Did you use any reflections?" (Answers vary.)

Lesson 2: Regular Tessellations

Goals

- Justify (orally and in writing) that regular triangles, squares, and hexagons are the only regular polygons that can be used to create a regular tessellation.

Lesson Narrative

In this second in a sequence of three lessons, students look at regular tessellations. Two polygons in a regular tessellation must

- not meet at all or
- share a single vertex or
- share a single side

Students show in detail that triangles, squares, and hexagons give the only possible regular tessellations.

Alignments

Building On

- 7.G.B: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Addressing

- 8.G.A: Understand congruence and similarity using physical models, transparencies, or geometry software.
- 8.G.A.5: Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

Instructional Routines

- MLR8: Discussion Supports

Required Materials

Protractors
Clear protractors with no holes and with radial lines printed on them are recommended.

Tracing paper
Bundles of "patty paper" are available commercially for a very low cost. These are small sheets (about 5" by 5") of transparent paper.

Required Preparation

If using the applet in the digital version of the activity, tracing paper and protractors are not needed.

Student Learning Goals

Let’s make some regular tessellations.

2.1 Regular Tessellations

Optional: 15 minutes (there is a digital version of this activity)

The goal of this activity is to introduce a *regular tessellation* of the plane and conjecture which shapes give regular tessellations. Students construct arguments for which shapes can and cannot be used to make a regular tessellation (MP3). The focus is on experimenting with shapes and noticing that in order for a shape to make a regular tessellation, we need to be able to put a whole number of those shapes together at a single vertex with no gaps and no overlaps. This greatly limits what angles the polygons can have and, as a result, there are only three regular tessellations of the plane. This conjecture will be demonstrated in the other two activities of this lesson.

Addressing

- 8.G.A

Launch

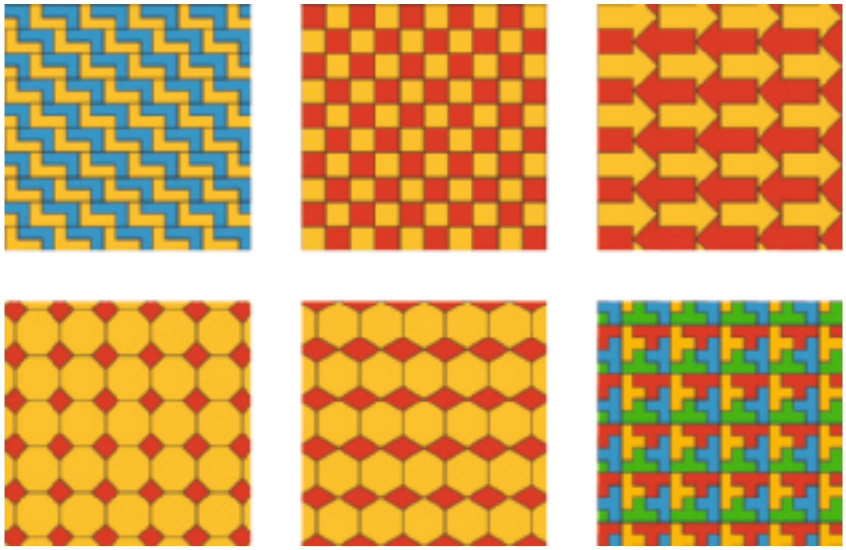
Display a table for all to see with at least two columns keeping track of which regular polygons make a tessellation and which do not. A third column could be used for extra comments (for example, about angle size of the polygon or other remarks). Here is an example of a table you might use:

shape	tessellate?	notes
octagon		
hexagon		
pentagon		
square		
triangle		

Introduce the idea of a regular tessellation:

- Only one type and size of polygon used.
- If polygons meet, they either share a single vertex or a single side.

- Show some pictures of tessellations that are not regular, and ask students to identify why they are not (e.g., several different polygons used, edges of polygons do not match up completely). Ask students which of the tessellations pictured here are regular tessellations (only the one with squares):



For the print version, make tracing paper available to all students. Tell students that they can use the tracing paper to put together several copies of the polygons.

For the digital version tell students that they can click on the shapes to make several copies.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, "The _____ can make a regular tessellation of the plane because . . ." , "I noticed _____ so I . . .", or "I agree/disagree because . . . ."
Supports accessibility for: Language; Social-emotional skills

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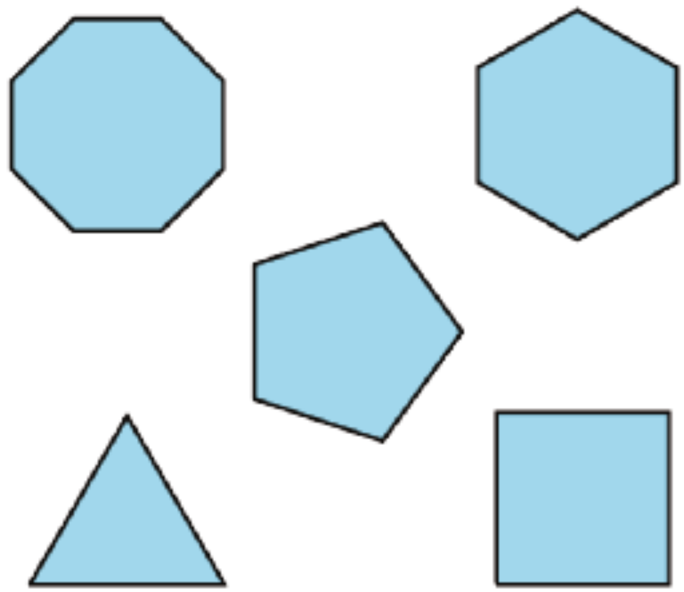
Anticipated Misconceptions

If students working with the pentagon and octagon add other shapes to make a more complicated tessellation, remind them that a regular tessellation uses copies of a single shape.

Student Task Statement

1. For each shape (triangle, square, pentagon, hexagon, and octagon), decide if you can use that shape to make a regular tessellation of the plane. Explain your reasoning.



2. For the polygons that do not work, what goes wrong? Explain your reasoning.

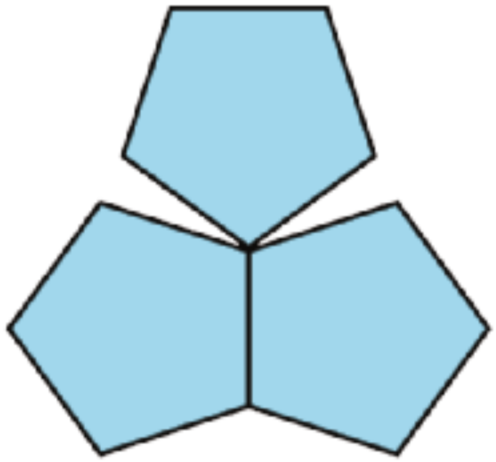
Student Response

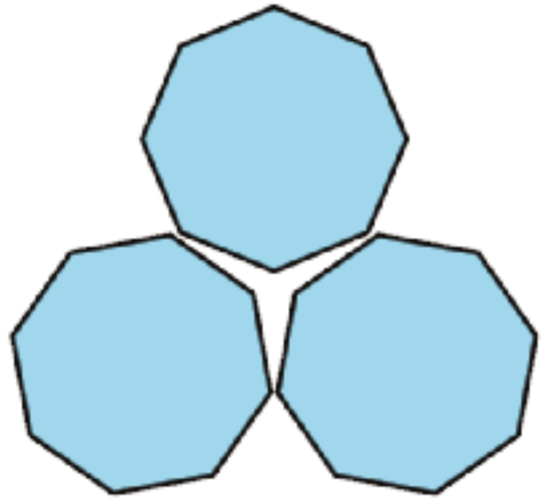
1. The triangles, squares, and hexagons all tessellate the plane. Three hexagons meet at each vertex, four squares meet at each vertex, and six triangles meet at each vertex.
2. For the pentagons, three can fit together at a vertex. There is a little extra space left, but it’s not enough to fit another pentagon. Two octagons can fit together at a vertex with space left over, but cannot tessellate the plane.

Activity Synthesis

To help students think more about what shapes do and do not tessellate and why, ask:

- “Which polygons appear to tessellate the plane?” (Square, equilateral triangle, hexagon.)
- “How did you decide?” (I put as many together as I could at one vertex and checked to see if there was extra space leftover.)
- “Why does the pentagon not work to tessellate the plane?” (3 fit together at one vertex, but there is extra space, not enough for a fourth.)
- “Why does the octagon not work?” (2 fit together, but there is not enough space for a third.)





During the discussion, fill out the table, indicating that it is possible to make a tessellation with equilateral triangles, squares, and hexagons, but not with pentagons or octagons.

2.2 Equilateral Triangle Tessellation

Optional: 15 minutes
The goal of this activity is to verify, via angle calculations, that equilateral triangles (and hence) regular hexagons can be used to make regular tessellations of the plane. Students have encountered the equilateral triangle plane tessellations earlier in grade 8 when working on an isometric grid. In order to complete their investigation of regular tessellations of the plane, it remains to be shown that no other polygons work. This will be done in the next activity.

Students are required to reason abstractly and quantitatively (MP2) in this activity. Tracing paper indicates that six equilateral triangles can be put together sharing a single vertex. Showing that this is true for abstract equilateral triangles requires careful reasoning about angle measures.

Addressing

- 8.G.A.5

Instructional Routines

- MLR8: Discussion Supports

Launch

In the previous task, equilateral triangles, squares, and hexagons appeared to make regular tessellations of the plane. Tell students that the goal of this activity is to use geometry to verify that they do.

Refer students to regular polygons printed in the previous activity for a visual representation of an equilateral triangle.

Access for Students with Disabilities

Representation: Internalize Comprehension. Represent the same information through different modalities by using physical objects to represent abstract concepts. Some students may benefit by using virtual or concrete manipulatives (geogebra, tracing paper, or equilateral triangle cut-outs) to connect abstract concepts to concrete objects.
Supports accessibility for: Conceptual processing; Visual-spatial processing

Anticipated Misconceptions

Students may know that an equilateral triangle has 60-degree angles but may not be able to explain why. Consider prompting these students for the sum of the three angles in an equilateral triangle.

Students may not see a pattern of hexagons within the triangle tessellation. Consider asking these students what shape they get when they put 6 equilateral triangles together at a single vertex.

Student Task Statement

1. What is the measure of each angle in an equilateral triangle? How do you know?
2. How many triangles can you fit together at one vertex? Explain why there is no space between the triangles.
3. Explain why you can continue the pattern of triangles to tessellate the plane.
4. How can you use your triangular tessellation of the plane to show that regular hexagons can be used to give a regular tessellation of the plane?

Student Response

1. 60 degrees. The sum of the angles is 180 degrees, and they are all congruent, so each must be a 60-degree angle.
2. 6 because $6 \cdot 60 = 360$
3. Each place where two or more triangles meet in the pattern, the rest of the six triangles at that vertex can be filled out.
4. Each set of 6 triangles meeting in a single vertex makes a regular hexagon. These hexagons tile the plane.

Activity Synthesis

Consider asking the following questions to lead the discussion of this activity:

- “How did you find the angle measures in an equilateral triangle?” (The sum of the angles is 180 degrees, and they are all congruent so each is 60 degrees.)
- “Why is there no space between six triangles meeting at a vertex?” (The angles total 360 degrees, which is a full circle.)
- “How does your tessellation with triangles relate to hexagons?” (You can group the triangles meeting at certain vertices into hexagons, which tessellate the plane.)
- “Are there other tessellations of the plane with triangles?” (Yes. You can make infinite rows of triangles that can be placed on top of one another—and displaced relative to one another.)

Consider showing students an isometric grid, used earlier in grade 8 for experimenting with transformations, and ask them how this relates to tessellations. (It shows a tessellation with equilateral triangles.)

Point out that this activity provides a mathematical justification for the “yes” in the table for triangles and hexagons.

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion. After a response to one of the discussion questions is shared with the class, invite students to restate and/or revoice what they heard using mathematical language. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. This will provide more students with an opportunity to practice using language about tessellations.
Design Principle(s): Support sense-making

2.3 Regular Tessellation for Other Polygons

Optional: 15 minutes (there is a digital version of this activity)
The goal of this activity is to show that *only* triangles, squares, and hexagons give regular tessellations of the plane. The method used is experimentation with other regular polygons. The key observation is that the angles on regular polygons get larger as we add more sides, which is a good example of observing structure (MP7). Since three is the smallest number of polygons that can meet at a vertex in a regular tessellation, this means that once we pass six sides (hexagons), we will not find any further regular tessellations. The activities in this lesson now show that there are three and only three regular tessellations of the plane: triangles, squares, and hexagons.

Building On

- 7.G.B

Instructional Routines

- MLR8: Discussion Supports

Launch

Ask students "Are there some other regular polygons, in addition to equilateral triangles, squares, and hexagons, that can be used to give regular tessellations of the plane?" Some students may suggest regular polygons with more sides than the ones they have seen already, others may think that there are no other possibilities. Tell students that for this activity, they are going to investigate polygons with 7, 8, 9, 10, and 11 sides to see if they do or do not tessellate and why.

Print version: Provide access to tracing paper and protractors and tell students that they can use these to explore their conjectures.

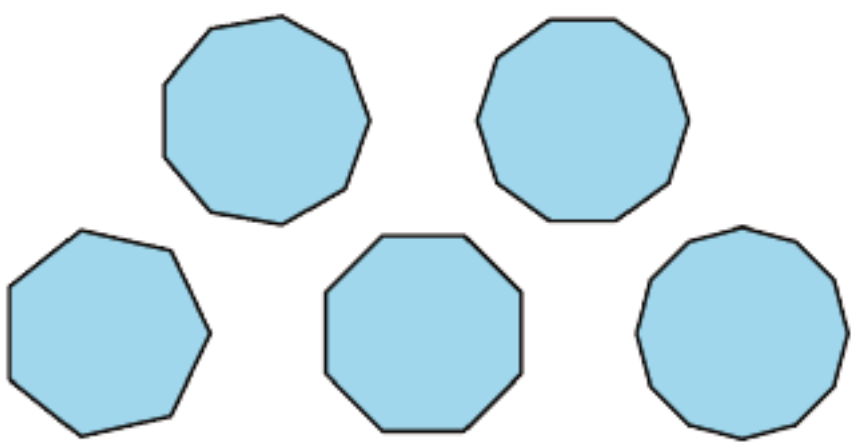
Digital version: Tell students to use the app to explore their conjectures.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.
Supports accessibility for: Organization; Attention

Student Task Statement

1. Can you make a regular tessellation of the plane using regular polygons with 7 sides? What about 9 sides? 10 sides? 11 sides? 12 sides? Explain.

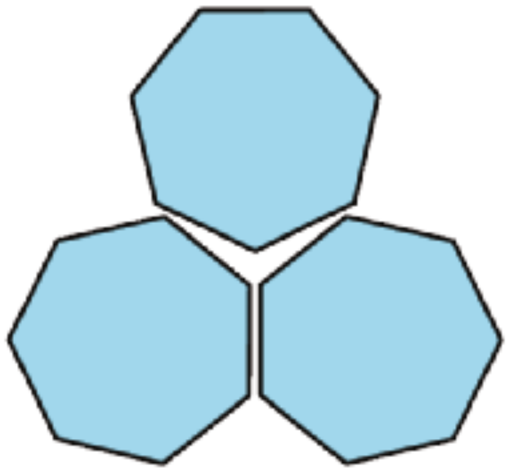


2. How does the measure of each angle in a square compare to the measure of each angle in an equilateral triangle? How does the measure of each angle in a regular 8-sided polygon compare to the measure of each angle in a regular 7-sided polygon?

3. What happens to the angles in a regular polygon as you add more sides?
4. Which polygons can be used to make regular tessellations of the plane?

Student Response

1. None of the other polygons will tessellate the plane. For each, three cannot be brought together at one vertex. Here is the attempt at doing so with the regular polygon with 7 sides.



2. The angles in a square are 90 degrees, greater than the 60-degree angles in a triangle. The angles in a regular 8-sided polygon have greater measure than the angles in a regular 7-sided polygon.
3. The angles increase in measure. Imagine opening up a polygon with 6 sides to add a 7th equal side. In order to fit the new side, all of the other sides must be spread out or opened up, increasing the measure of the angles.
4. Only the triangle, square, and hexagon. As more sides are added, the angles get greater. 120 degrees is the biggest divisor of 360 that can be the measure of an interior angle of a regular polygon.

Activity Synthesis

Consider asking the following questions:

- “How many triangles meet at each vertex in a regular tessellation with triangles?” (6)
- “What about squares?” (4)
- “Hexagons?” (3)
- “Why can’t there be any regular tessellations with polygons of more than 6 sides?” (Only two could meet at a vertex, but this isn’t possible since the angles have to add up to 360 degrees.)

There are only three regular tessellations of the plane. Ask students if they have encountered these tessellations before and if so where. For example:

- triangles (isometric grid)
- squares (checkerboard, coordinate grid, floor and ceiling tiles)
- hexagons (beehives, tiles)

Access for English Language Learners

Speaking, Listening: MLR8 Discussion Supports. During the synthesis, focus on the question about why a regular tessellation with more than six sides wouldn't work. Ask students to prove their justification to this question to their partner. They should try to convince them as if they were someone who really doesn't believe them (a skeptic). Encourage listeners to press speakers to use specific language about "angles" and "degrees". If time permits, have listeners repeat the process with a new partner, taking on the role of speaker. This will provide students with practice using specific geometric terms to justifying their reasoning.
Design Principle(s): Optimize output (for justification); Maximize meta-awareness

Lesson 3: Tessellating Polygons

Goals

- Generalize (orally) that any triangle or quadrilateral can be used to tessellate the plane.

Lesson Narrative

In this third in the sequence of three lessons, students examine tessellations using non-regular polygons. Students show that *any* triangle can be used to tessellate the plane and similarly for any quadrilateral. Pentagons do not work in general, for example, a regular pentagon cannot be used to tessellate the plane.

Tessellating the plane with a triangle uses the important idea, studied in the sixth grade, that two copies of a triangle can be put together to make a parallelogram. Tessellating the plane with a quadrilateral uses rigid motions of the plane and the fact that the sum of the angles in a quadrilateral is always 360. One example of a plane tessellation with a special pentagon also uses rotations.

Alignments

Addressing

- 8.G.A: Understand congruence and similarity using physical models, transparencies, or geometry software.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Poll the Class

Required Materials

Tracing paper
Bundles of "patty paper" are available commercially for a very low cost. These are small sheets (about 5" by 5") of transparent paper.

Required Preparation

If students are using the applets in the digital versions of the activities, tracing paper may not be needed.

> **Student Learning Goals**
>
> Let’s make tessellations with different polygons.

3.1 Triangle Tessellations

Optional: 15 minutes (there is a digital version of this activity)
In this activity, students experiment with copies of a triangle (no longer equilateral) and discover that it is always possible to build a tessellation of the plane. A key in finding a tessellation with copies of a triangle is to experiment with organizing copies of the triangle, and then reasoning that two copies of a triangle can always be arranged to form a parallelogram. Students may not remember this construction from the sixth grade, but with copies of the triangle to experiment with, they will find the parallelogram or a different method. These parallelograms can then be put together in an infinite row, and these rows can then be stacked upon one another to tessellate the plane.

Addressing

- 8.G.A

Instructional Routines

- MLR7: Compare and Connect

Launch

Assign different triangles to different students or groups of students. Provide access to tracing paper if using the print materials. If using the digital materials, the activity can be done in the applet.

If students finish early, consider asking them to work on building a different tessellation or coloring their tessellation.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support development of organizational skills, check in with students or groups of students within the first 2-3 minutes of work time. Look for students who are using two copies of the triangle to form a parallelogram in order to create a tessellation.
Supports accessibility for: Memory; Organization

Anticipated Misconceptions

Students may struggle to put together copies of their triangle in a way that can be continued to tessellate the plane.

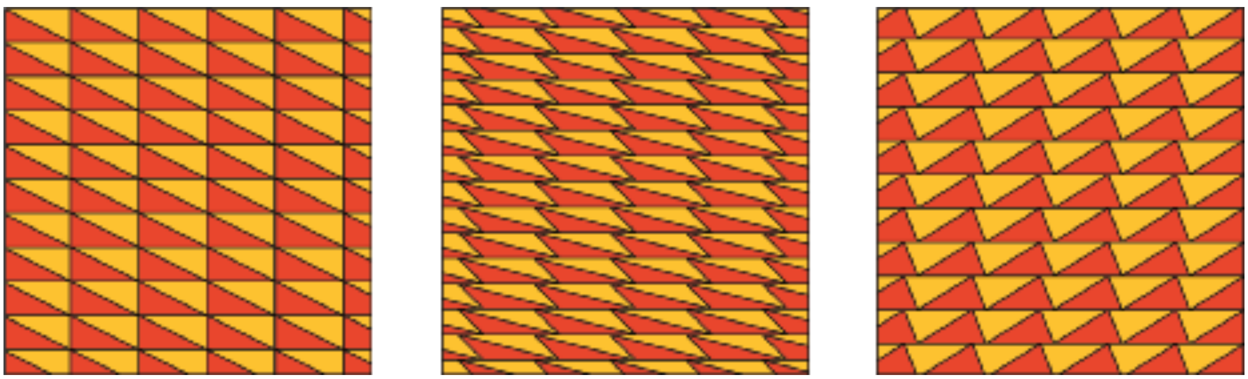
- Ask these students to put together two copies of the triangle.
- If they have made a parallelogram, ask them what kind of quadrilateral they have made.
- If they have not made a parallelogram, ask them if there is a different way they can combine two copies of the triangle.

Student Task Statement

Your teacher will assign you one of the three triangles. You can use the picture to draw copies of the triangle on tracing paper. Your goal is to find a tessellation of the plane with copies of the triangle.

Student Response

Sample response for each type of triangle:



Activity Synthesis

Invite several students to share their tessellations for all to see.

Consider asking the following questions to help summarize the lesson:

- "Were you able to make a tessellation with copies of your triangle?" (Most students should respond yes.)
- "How did you know that you could continue your pattern indefinitely to make a tessellation?" (Any parallelogram can be used to tessellate the plane as they can be placed side by side to make infinite "rows" or "columns" and then these rows or columns can be displaced to fill up the plane.)

Share some of the tessellation ideas students come up with and relate them back to previous work, that is the tessellation of the plane with rectangles and parallelograms.

Access for English Language Learners

Representing, Conversing: MLR7 Compare and Connect. After students complete their triangle tessellations, display the work around the room. Invite students to tour the room to observe and compare the displays. As students circulate, ask, "Are there any drawings that are similar in appearance?" and "Did any two drawings use the same kind of triangle, but create a different tessellation pattern?" Finish by inviting students to share the similarities and differences they discovered with the class. Look for language identifying that "two of the same triangles" placed together create a "parallelogram" no matter the type of triangle chosen. Also, amplify language that concludes that the "plane" can be filled by "repeated triangle" patterns. This helps students draw comparisons between various triangle tessellations and explain their reasoning about how they can fill the plane.
Design Principle(s): Cultivate conversation; Maximize meta-awareness

3.2 Quadrilateral Tessellations

Optional: 20 minutes (there is a digital version of this activity)
The previous activity showed how to make a tessellation with copies of a triangle. A natural question is whether or not it is possible to tessellate the plane with copies of a single quadrilateral. Students have already investigated this question for some special quadrilaterals (squares, rhombuses, regular trapezoids), but what about for an arbitrary quadrilateral? This activity gives a positive answer to this question. Pentagons are then investigated in the next lesson, and there we will find that some pentagons can tessellate the plane while others can not.

In order to show that the plane can be tessellated with copies of a quadrilateral, students will experiment with rigid motions and copies of a quadrilateral.

This activity can be made more open ended by presenting students with a polygon and asking them if it is possible to tessellate the plane with copies of the polygon.

Addressing

- 8.G.A

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- Poll the Class

Launch

Begin the activity with, "Any triangle can be used to tile the plane (some of them in many ways). Do some quadrilaterals tessellate the plane?" (Yes: squares, rectangles, rhombuses, and parallelograms.) Next, ask "Can any quadrilateral be used to tessellate the plane?" Give students a moment to ponder, and then poll the class for the number of yes and no responses. Record the responses for all to see. This question will be revisited in the Activity Synthesis.

Provide access to tracing paper.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge of quadrilaterals, as well as 180 degree rotations around the midpoint of a side. Consider demonstrating how to use tracing paper to rotate a figure 180 degrees around the midpoint of a side. Be sure to model accuracy and precision.
Supports accessibility for: Visual-spatial processing; Organization

Access for English Language Learners

Representing, Speaking, Writing: MLR1 Stronger and Clearer Each Time. Give students a few minutes to complete the first problem. Students then turn to their partner and share their tessellation and explanation. Listeners should press for details in their explanation asking whether there are "any spaces left" and whether they "can prove that the trapezoid fits the whole plane". Students will explain and show their tessellation to at least one more partner, each time adding detail to their explanation. The focus should be on explaining, even if they were not successful. Finish with students rewriting their initial explanation, adding any detail or corrections needed. This helps students explain their method for moving the trapezoid and use geometric language to prove whether it works.
Design Principle(s): Optimize output (for explanation); Maximize meta-awareness

Anticipated Misconceptions

Students may need to be reminded of the definition of a trapezoid: one pair of sides are contained in parallel lines.

If the figures are not traced accurately, it may be difficult to determine if the pattern, using 180-degree rotations, can be continued. Ask these students what they know about the sum of the three angles in a triangle and in a quadrilateral.

Student Task Statement

1. Can you make a tessellation of the plane with copies of the trapezoid? Explain.

2. Choose and trace a copy of one of the other two quadrilaterals. Next, trace images of the quadrilateral rotated 180 degrees around the midpoint of each side. What do you notice?

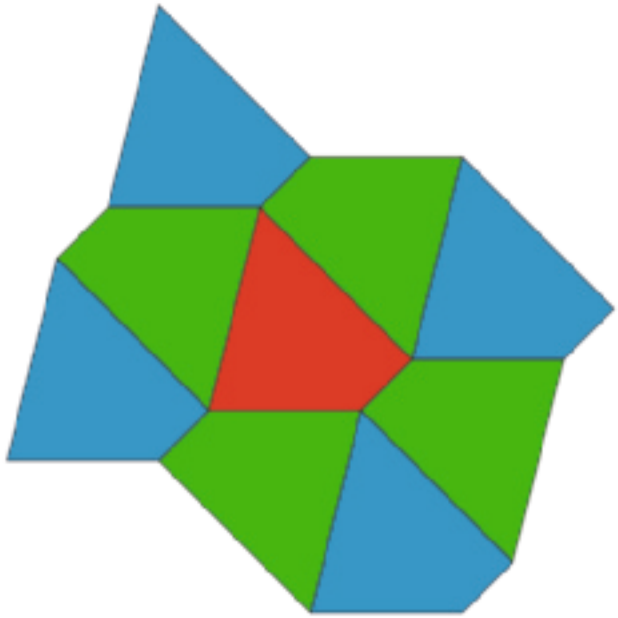
3. Can you make a tessellation of the plane with copies of the quadrilateral from the previous problem? Explain your reasoning.

Student Response

1. Yes, rotating the trapezoid 180 degrees about the slanted side to the base makes a rectangle. The plane can be tessellated with copies of this rectangle. Rotating the trapezoid 180 degrees about the other side to the base also makes a parallelogram, and then the plane can be tessellated with copies of the parallelogram.

2. It looks like there is room to fit copies of the quadrilateral in each of the wedges at the corners. When they are filled in, it will look like a checkerboard.

3. Yes, continuing these 180-degree rotations about midpoints of sides of the quadrilateral fills in the whole plane. At each vertex, the four angles of the quadrilateral meet to make a full 360-degree circle.



Activity Synthesis

Invite some students to share their tessellations.

Discussion questions include:

- “Were you able to tessellate the plane with copies of the trapezoid?” (Yes, two of them can be out together to make a parallelogram, and the plane can be tessellated with copies of this parallelogram.)
- “What did you notice about the quadrilateral and the 180-degree rotations?” (They fit together with no gaps and no overlaps and leave space for four more quadrilaterals.)

- “How do you know that there are no overlaps?” (The sum of the angles in a quadrilateral is 360 degrees. At each vertex in the tessellation, copies of the four angles of the quadrilateral come together.)

Revisit the question from the start of the activity, “Can any quadrilateral be used to tessellate the plane?” Invite students to share if their answer has changed and explain their reasoning.

3.3 Pentagonal Tessellations

Optional: 20 minutes (there is a digital version of this activity)
All triangles and all quadrilaterals give tessellations of the plane. For the quadrilaterals, this was complicated and depended on the fact that the sum of the angles in a quadrilateral is 360 degrees. Regular pentagons that do not tessellate the plane have been seen in earlier activities. The goal of this activity is to study some types of pentagons that *do* tessellate the plane. Students make use of structure (MP7) when they relate the pentagons in this activity to the hexagonal tessellation of the plane, which they have seen earlier.

This activity can be made more open ended by presenting students with a polygon and asking them if it is possible to tessellate the plane with copies of the polygon.

Addressing

- 8.G.A

Instructional Routines

- MLR8: Discussion Supports

Launch

Ask students:

- “Can you tessellate the plane with regular pentagons?” (No.)
- “Can you think of a type of pentagon that could be used to tessellate the plane?” (A square base with a 45-45-90 triangle on top, for example.)

Arrange students in groups of two.

Print version: Provide access to tracing paper.

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their ideas. For example, “First, I _____ because . . .”, “At the central vertex, I noticed that . . .”, “I agree/disagree because . . .”, or “What other details are important?”
Supports accessibility for: Language; Organization

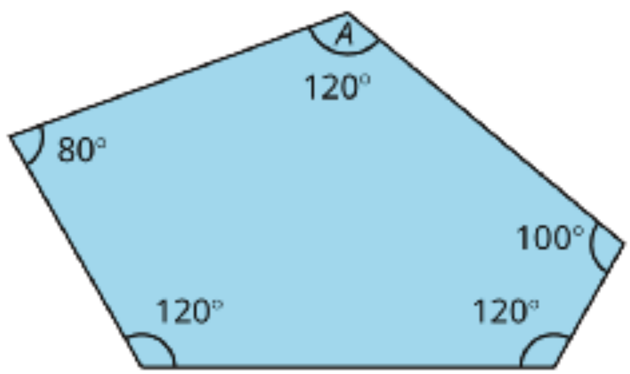
Anticipated Misconceptions

Students may struggle tracing the rotated hexagon. Ask them what happens to the segments making angle A when the hexagon is rotated about A by 120 (or 240) degrees.

Students may wonder why the hexagon that they make by putting three pentagons together is a regular hexagon. Invite these students to calculate the angles of the hexagon.

Student Task Statement

1. Can you tessellate the plane with copies of the pentagon? Explain why or why not. Note that the two sides making angle A are congruent.



   Pause your work here.

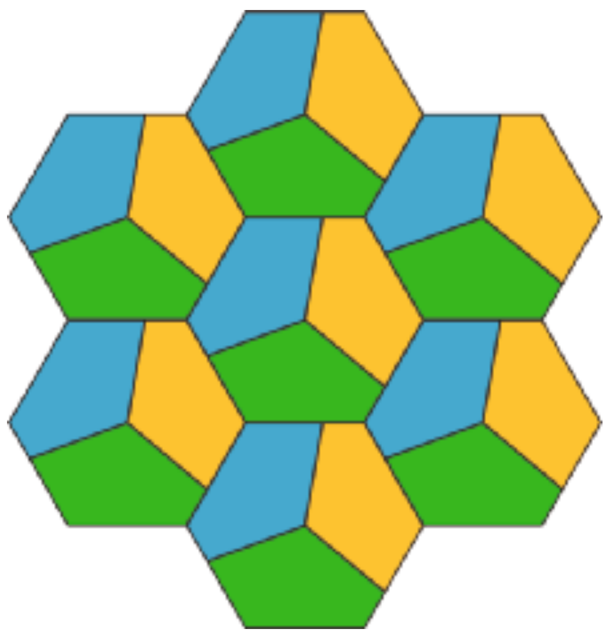
2. Take one pentagon and rotate it 120 degrees clockwise about the vertex at angle A, and trace the new pentagon. Next, rotate the pentagon 240 degrees clockwise about the vertex at angle A, and trace the new pentagon.

3. Explain why the three pentagons make a full circle at the central vertex.

4. Explain why the shape that the three pentagons make is a hexagon (that is, the sides that look like they are straight really are straight).

Student Response

1. Yes, I can make a hexagon from three pentagons and can then tessellate the plane with those hexagons.
2. See picture for item 4 showing several of the hexagons made by putting together three pentagons.
3. The angles that are joined at the central vertex each measure 120 degrees, and there are 360 degrees in the full circle.
4. Three of the six sides of the hexagon are corresponding sides of the pentagon after a rotation. For the other three sides, the angles 100 and 80 are supplementary, and so the two segments making those sides are colinear. All angles of the hexagon measure 120 degrees, so it is a regular hexagon.



Activity Synthesis

Students may be successful in building a tessellation in the first question. The following questions guide them through a method while also asking for mathematical justification. Students who are successful in the first question can verify that their tessellation uses the strategy indicated in the following, and they will still need to answer the last two questions.

Invite some students to share their tessellations.

Some questions to discuss include:

- “Does the hexagon made by three copies of the pentagon tessellate the plane?” (Yes.)

- “How do you know?” (I checked experimentally, or I noticed that all of the angles in the hexagon are 120 degrees.)
- “Why was it important that the two sides of the pentagons making the 120 angles are congruent?” (So that when I rotate my pentagon, those two sides match up with each other perfectly.)
- “What is special about this pentagon?” (Two sides are congruent, three angles measure 120 degrees. . .)

Access for English Language Learners

Representing, Conversing: MLR8 Discussion Supports. Give students a structured opportunity to revise and refine their response to the first question. After students have completed their tessellations using the hexagon, ask students to meet with 2–3 other groups for feedback. Display the questions included in the synthesis on the screen and invite each partnership to take turns asking the questions. Encourage students to press each other for detailed explanations using mathematical language. Peer questioning increases and improves student output for explanations.
Design Principle(s): Optimize output (for explanation); Cultivate conversation

Section: The Weather

Lesson 4: What Influences Temperature?

Goals

- Contrast (orally) the benefits of modeling data using functions to identify input/output pairs and using statistics to analyze bivariate data.

Lesson Narrative

This optional sequence of three lessons can be done any time after the unit 6 in the course. Students develop and use a mathematical model to predict temperature given the latitude of a location. The activities in this sequence of lessons provide students a chance to go more deeply and apply grade 8 mathematics to a real-world context. Students get a chance to engage in many aspects of mathematical modeling (MP4). The activities in this lesson build on each other, and so should be done in order. It is not necessary to do all three lessons to get some benefit, although more connections are made the further one gets.

In the first lesson, students investigate whether there is a relationship or a pattern of association between the north-south location (in North America) of a place and the temperature. This is a vague question, and the first step is to clarify the variables that we will consider for a mathematical model.

- The first activity gives students a chance to think about different factors that influence outside temperature. Some are geographical (latitude, desert or sea climate, elevation), others are time of year, cloud cover, time of day, etc. As a segue into the second activity, we ask if it is possible to vary just one factor so that we can predict how the temperature will respond. In particular, if we vary latitude, can we predict what happens to the temperature?
- In the second activity, students investigate whether the concept of a function is a good tool to model this situation: Is temperature a function of latitude? There are several issues with this question. The biggest one is that for the same latitude, we will get different temperatures at any given time. If we really want a functional relationship, then we would have to make many restrictions. For example, we could fix time and longitude. Then for each latitude as input we can report a unique temperature as output. This brings up the question of how meaningful this model would be. We will look at a variety of possibilities and discuss pros and cons.
- In the third activity, students discuss if a more meaningful model might be to look at an association between latitude and temperature, much like what they encountered in an earlier unit on bivariate data. Data like this is easy to find, and we don't have to worry about repeating "input" values.

In the next two lessons, students construct a mathematical model, analyze the model, use it to make a prediction, and discuss limitations of the model.

Throughout these lessons, students make and discuss choices, assumptions, and approximations as part of their work. While some of the choices and decisions are made as a class or through the sequencing of the classroom activities, students get a chance to grapple with all of these steps in the modeling cycle.

If it is feasible, students could be making more of the decisions themselves. For example, groups could come up with different proposals of how to investigate the relationship between temperature and latitude, present, get feedback, finalize, and then continue with different data for their model (for example, different months, different locations, different continents, or overall average temperature). For the full modeling cycle, they could then revise their assumptions and come up with a revised model.

There are many extensions possible for these activities. Students could investigate if there is a similar association between latitude and temperature in other parts of the world. They could look at different measures of temperature like yearly average, yearly average high or low temperature, or average high or low temperature in a different month.

Alignments

Addressing

- 8.F.A.1:
  Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. Function notation is not required in Grade 8.
- 8.F.B: Use functions to model relationships between quantities.
- 8.SP.A: Investigate patterns of association in bivariate data.

Instructional Routines

- MLR5: Co-Craft Questions
- MLR8: Discussion Supports

Required Preparation

For the first activity, consider finding out the latitude and average high temperature in September for your city.

> **Student Learning Goals**
>
> Let’s see if we can predict the weather.

4.1 Temperature Changes

Optional: 5 minutes
In this activity, students think about different factors that influence outside temperature. Some are geographical (latitude, desert or sea climate, elevation), others are time of year, cloud cover, time of day, etc. This is a chance to make connections with some science concepts.

It is not important that students come up with an exhaustive list. They should just get the idea that there are many factors so that they are open to the idea that we'll have to make some choices for our model and clearly define any variables we want to consider.

Addressing

- 8.F.A.1
- 8.F.B

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 3–4. Tell students that they are starting an investigation on how to predict the weather, in particular the temperature. Check that students understand the example given in the activity statement, that as the time of day changes, the temperature often changes in a predictable way. They will brainstorm other factors that also influence the temperature.

Access for English Language Learners

Speaking, Listening, Representing: MLR8 Discussion Supports. To tap into students' prior knowledge and experiences with weather and temperature, display images of various weather conditions. Include images that suggest cool or hot temperatures, showing the sun, wind, rain, snow, etc., in different locations (i.e., beach, mountains, etc.). In small groups of 3–4 students, ask each student to volunteer at least once to orally describe the weather they see and show with their body movement how the temperature feels. As each image is displayed, have a new student in the group take a turn.
Design Principle(s): Support sense-making; Cultivate conversation

Student Task Statement

What factors or variables can influence the outside temperature?

- Make a list of different factors.
- Write a sentence for each factor describing how changing it could change the temperature.

Example: One factor is time of day. Often, after sunrise, the temperature increases, reaches a peak in the early afternoon, and then decreases.

Student Response

Answers vary. Sample response:

- Time of year: It is colder in the winter and warmer in the summer.

- Location: It is colder toward the poles and warmer toward the equator.
- Altitude: The higher a location, the colder it gets.
- Cloud cover: The more clouds there are, the colder it is.
- Ocean currents: The Gulf Stream brings cold or warm water to parts of the ocean and moderates temperatures that way.
- El Niño and La Niña: Moisture in the atmosphere influences temperature.
- Global climate change: Greenhouse gases increase average temperatures.
- Volcanic eruptions: Ash in the atmosphere can lower temperature.
- Wind direction: Santa Ana winds bring warm air from inland to the coast or wind moves cold air from Canada into the central plains.

Activity Synthesis

Invite students to share some of the factors they have come up with. Note that many of them are geographical. Point out that making a model that takes into consideration all or even many of these factors is very complex (weather forecasting is really difficult!). In mathematical modeling, we often start by fixing or disregarding (or randomizing) all but one of the factors. In the next activity, we want to pick just one—latitude—and investigate how just changing the latitude changes the temperature.

4.2 Is Temperature a Function of Latitude?

Optional: 15 minutes

One mathematical concept students learned about in grade 8 that relates to variables is the idea of a function. In this activity, students discuss if this is a helpful concept to investigate latitude and temperature. To be able to use functions, variables have to be very well defined, and in this case, many restrictions are necessary to have a functional relationship where each input has exactly one output.

Addressing

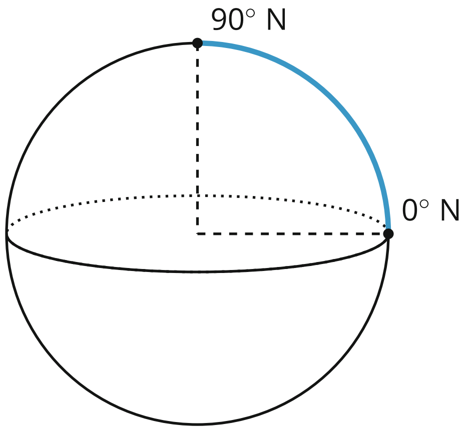
- 8.F.B

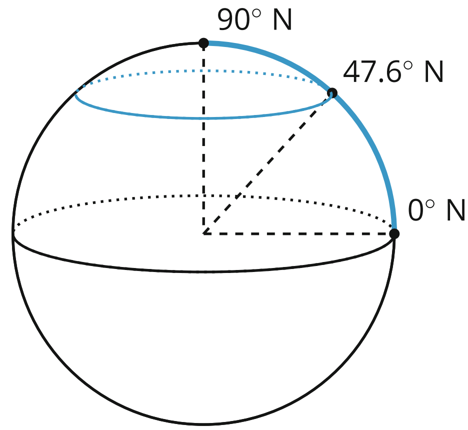
Launch

Keep students in the same groups. Ask students if they can explain exactly what latitude is. It was briefly discussed in the previous activity as one of the factors that can influence the temperature. Students should understand that latitude is a way to measure how far north or south of the equator a place is located. The unit of measurement is degrees north or degrees south.

Draw a sphere (representing Earth) for all to see and indicate the circle representing the equator. Draw a quarter circle starting somewhere on the equator and ending at the North Pole. A location on the equator is at 0 degrees north (or south), and the North Pole is at 90 degrees north. Note that the angle made by the arc is a 90-degree angle. Since North America lies entirely north of the equator, all latitudes there have units of degrees north.

In this activity, students look at locations at 47.6 degrees north. Ask students where on the highlighted quarter circle this latitude is located. Then draw in the corresponding longitude circle (parallel to the equator) to show that there are many locations that have the same latitude.





Access for Students with Disabilities

Representation: Internalize Comprehension. Provide appropriate reading accommodations and supports to ensure students access to written directions, word problems and other text-based content.
Supports accessibility for: Language; Conceptual processing

Student Task Statement

1. Andre and Lin are wondering if temperature is a function of latitude.

   Andre says, “I think it is, as long as we fix the time when we are measuring the temperature.”

   Lin says, “But what if you have two places with the same latitude? Look at this weather map for Washington State. Seattle and Spokane have the same latitude but different temperatures right now.”
   What do Andre and Lin mean?

2. Andre and Lin are discussing whether it is possible to define latitude and temperature in a way that makes sense to talk about temperature as a function of latitude. They are considering different options. What are some advantages and disadvantages of each option?

   Here are the options:
   a. Finding the temperature right now in cities with different latitudes.

   b. Finding the daily high temperature in cities that have different latitudes.

   c. Finding the average high temperature in a specific month, for example, September in cities that have different latitudes.

   d. Finding the average yearly temperature in cities that have have different latitudes.

Student Response

1. Answers vary. Sample response: Temperature varies with time, so a time must be fixed to be able to talk about temperature as a function of location. The question “What is the temperature in Seattle?” has many different answers depending on the time the temperature is measured. Similarly, Seattle is not a single location, or more generally, “latitude” is not one

single location. Even if a time is fixed, two locations that have the same latitude may have different temperatures.

2. Answers vary. Quite often cities at different latitudes have different temperatures.
    a. Answers vary. Sample response: The answers might not depict an overall pattern, as temperatures are influenced by local weather and time zones.
    b. Answers vary. This answer draws upon similar issues as in part a, except for the time zone problem.
    c. Answers vary. It is better to be able to draw conclusions about a general relationship. The issue with the "representative latitude" remains.
    d. Answers vary. The answer is similar to that in c. This might be too much of an average. Some locations might have extreme highs and lows while others have closer highs and lows.

Activity Synthesis

Discuss why it is important to clearly define the variables.

- "Temperature" is a very general idea, and we have to decide what is the most appropriate measure of temperature for our investigation. There are many different choices, but some are more appropriate than others. We want to look at an average since this will be more representative than the temperature at one point in time. It also evens out some of the other factors that could influence temperature, such as random weather events.
- Latitude is a tricky variable since each latitude value represents infinitely many locations that all have different temperatures. We can fix one line of longitude, but then how representative will our results be?

There are two main takeaways of this activity:

1. It is important to clearly define the variables of the model.
2. The function concept might be too restrictive for us to use for our model.

Statistical methods are a better tool for investigating the relationship between latitude and temperature. We'll look at these in the next activity.

4.3 Is There an Association Between Latitude and Temperature?

Optional: 10 minutes

The idea of a function is very limiting when we wish to analyze the relationship between latitude and temperature. Looking at the situation from a statistics point of view is more helpful. In this case, the question becomes: Is there an association between latitude and temperature? This activity asks students to recall the setup necessary to answer this question. As an important step of mathematical modeling, they think about what data they need to collect, how they can collect it, and what methods will help them to analyze the data. This step is often done for students to save time, but it is non-trivial and even though in the next lesson the data will be provided, it is worthwhile for students to think about this step and to come up with a plan. If it is appropriate for a class, students can collect their own data rather than use the data provided.

Addressing

- 8.F.B
- 8.SP.A

Instructional Routines

- MLR5: Co-Craft Questions

Launch

Tell students that as they saw in the previous activity, the function concept might be too restrictive in this case to create a useful model. Statistical methods are a better tool for investigating the relationship between latitude and temperature.

Brainstorm examples from earlier in the year on finding associations between variables, for example, year and price of a car, or weight of a car and fuel efficiency.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge around the term *association*. Provide students with examples of variables that have an association. Check for understanding by selecting a few students to share other possible situations that may have an association between the variables.
Supports accessibility for: Memory; Conceptual processing

Access for English Language Learners

Conversing, Writing, Reading: MLR5 Co-craft Questions. Display the following statement for all to see: "Death Valley holds the record for the hottest place on earth. In 1913, the temperature reached 134°F (56.7°C)! It can also get surprisingly cold, with temperatures below freezing." Give students 1–2 minutes to write down mathematical questions to ask about this statement. Invite students to compare their questions with a partner before selecting a few students to share with the class. Reinforce the idea that this is the same "location" with the same "latitude". Use this to reinforce the idea that other "factors" affect temperature besides latitude, and that a clear rule may not be easily generated and discussed by only looking at one "factor".
Design Principle(s): Cultivate conversation; Maximize meta-awareness

Student Task Statement

Lin and Andre decided that modeling temperature as a function of latitude doesn't really make sense. They realized that they can ask whether there is an *association* between latitude and temperature.

1. What information could they gather to determine whether temperature is related to latitude?

2. What should they do with that information to answer the question?

Student Response

Answers vary. Sample response:

1. Collect data that give the latitude and temperature at different locations in a geographical area (for example, North America). The temperature could be a yearly high temperature, an average temperature, or an average high temperature in a particular month. Latitude should have a wide enough range to show a pattern, such as examples from southern Florida to Alaska, including cities from different east-west locations.
2. Make a scatter plot to see if there is an association. If the association looks approximately linear, then find a line that best fits the data.

Activity Synthesis

To highlight some of the methods and ideas students will need, ask:

- "What are some ways we viewed associations between two variables in the past?" (We made scatterplots and fit trendlines to the data.)
- "What are some things we need to keep in mind when we collect our data?" (We should use a variety of north-south and east-west locations, or "enough" data to be able to draw conclusions.) (If not mentioned, tell students that three cities is definitely not enough, and 100 cities in the same vicinity is not necessary.)

In the next lesson, we will pick a particular measure of temperature—average high temperature in September—and analyze data to see if there is an association.

Lesson 5: Plotting the Weather

Goals

- Create a mathematical model of bivariate data using a scatter plot.
- Describe (in writing) associations in bivariate data shown in a table or scatter plot.

Lesson Narrative

In this second in a sequence of three lessons, students construct a mathematical model to investigate if there is an association between latitude and temperature.

- They make a scatter plot of latitude and average high temperature in September for cities across North America.
- They then use dried linguine pasta to find (that is, eyeball) a line that best approximates the data.
- They find an equation of the line. The data is given as part of the activities in the lesson plan. If appropriate, students could also collect their own data and use a different measure for temperature (for example, average yearly temperature, average September temperature, etc.).

Alignments

Addressing

- 8.SP.A: Investigate patterns of association in bivariate data.

Instructional Routines

- MLR8: Discussion Supports
- Notice and Wonder

Required Materials

Dried linguine pasta
We specified linguine since it is flatter and less likely to roll around than spaghetti.

> **Student Learning Goals**
>
> Let's construct a model.

5.1 California Rain

Optional: 5 minutes
This activity is a review of scatter plots and how to interpret information from a scatter plot.

Addressing

- 8.SP.A

Instructional Routines

- Notice and Wonder

Launch

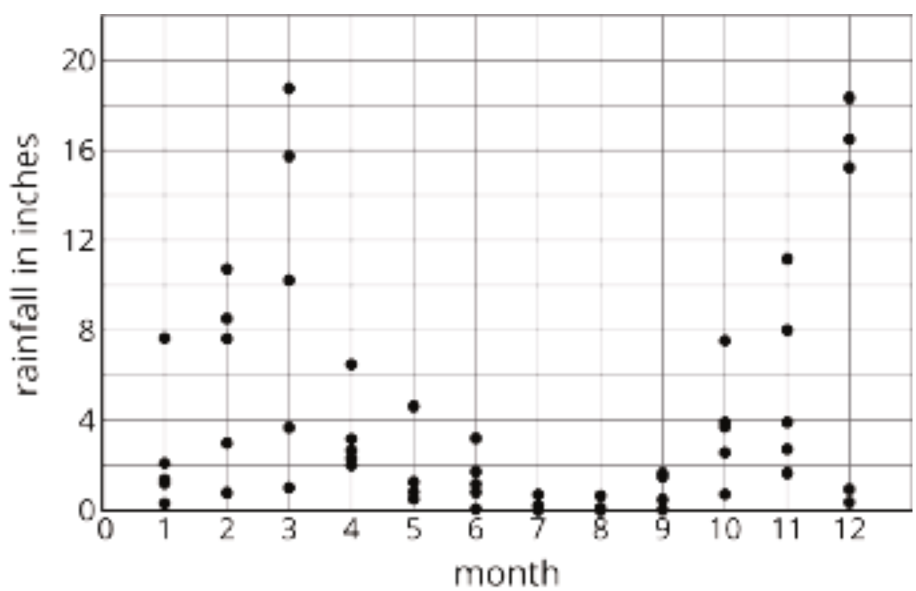
Keep students in same groups. Tell students that they will look at an image, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the image for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their group, followed by a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Provide students with a graphic organizer, such as a two-column table, to record what they notice and wonder prior to being expected to share these ideas with others.
Supports accessibility for: Language; Organization

Student Task Statement

What do you notice? What do you wonder?



Student Response

Things students may notice:

- There is hardly any rainfall in the summer.
- Most of the rainfall happens in December through March.
- The high rainfall totals happen in December and March, and they are about 19 inches for each month.
- In some months (for example, March) and in some years, it rains a lot, and in other years, it doesn't rain.

Things students may wonder:

- Why does it rain less in the summer than in the winter?
- Why are the dots so spread out in the colder months?

Activity Synthesis

Invite students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After each response, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to the images each time.

Discuss what each point in the scatter plot represents. Ask students to describe general patterns visible in the plot. Ask, "Is there a pattern of association?" (Yes, it is not linear, but it is possible to say that there is more rain in the winter and less rain in the summer.)

5.2 Data Snooping

Optional: 10 minutes
The task statement provides data students can analyze for the remainder of this lesson. It gives the average high temperature in September at different cities across North America. This is only one possible choice for data to analyze. If appropriate, students can instead collect their own data and then continue using it. If so, then the instructions are the same just with the students' data.

Addressing

- 8.SP.A

Launch

Students in same groups of 3–4.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation such as: “First, I _____ because . . .”, “I noticed _____ so I . . .”, “There is/is not an association because . . .”, or “I agree/disagree because . . .”
Supports accessibility for: Language; Social-emotional skills

Student Task Statement

The table shows the average high temperature in September for cities with different latitudes. Examine the data in the table.

city	latitude (degrees North)	temperature (degrees Fahrenheit)
Atlanta, GA	33.38	82
Portland, ME	43.38	69
Boston, MA	42.22	73
Dallas, TX	32.51	88
Denver, CO	39.46	77
Edmonton, AB	53.34	62
Fairbanks, AK	64.48	55
Juneau, AK	58.22	56
Kansas City, MO	39.16	78
Lincoln, NE	40.51	77
Miami, FL	25.45	88
Minneapolis, MN	44.53	71
New York City, NY	40.38	75
Orlando, FL	28.26	90
Philadelphia, PA	39.53	78
San Antonio, TX	29.32	89
San Francisco, CA	37.37	74
Seattle, WA	47.36	69
Tampa, FL	27.57	89
Tucson, AZ	32.13	93
Yellowknife, NT	62.27	50

1. What information does each row contain?

2. What is the range for each variable?

3. Do you see an association between the two variables? If so, describe the association.

Student Response

1. Each row lists a city, its latitude, and its average high temperature in September.
2. For latitude, the minimum value is 25.45 degrees north in Miami, FL, and the maximum is 64.48 degrees north in Fairbanks, AK. For temperature, the low is 50 degrees Fahrenheit for Yellowknife, NT, in Canada, and the maximum is 93 degrees Fahrenheit for Tucson, AZ.
3. It looks like locations with smaller latitudes have higher temperatures, and locations with higher latitudes have lower temperatures.

Activity Synthesis

Make sure students understand the information listed in the table. Invite students to share their responses to the last question, then move on to the next activity.

5.3 Temperature vs. Latitude

Optional: 15 minutes
In this activity, students use the data from the previous activity and draw a scatter plot and a line that fits the data. The given data show a clear linear association, so it is appropriate to model the data with a line. Students can use a piece of dried linguine pasta or some other rigid, slim, long object (for example, wooden skewer) to eyeball the line that best fits the data. (The line of best fit has a variance of $R^2 = 0.94$.) Even though different answers will have slightly different slopes and intercepts, they will be close to each other.

Addressing

- 8.SP.A

Instructional Routines

- MLR8: Discussion Supports

Launch

Students in same groups of 3–4. Provide access to pieces of dried linguine pasta.

Access for Students with Disabilities

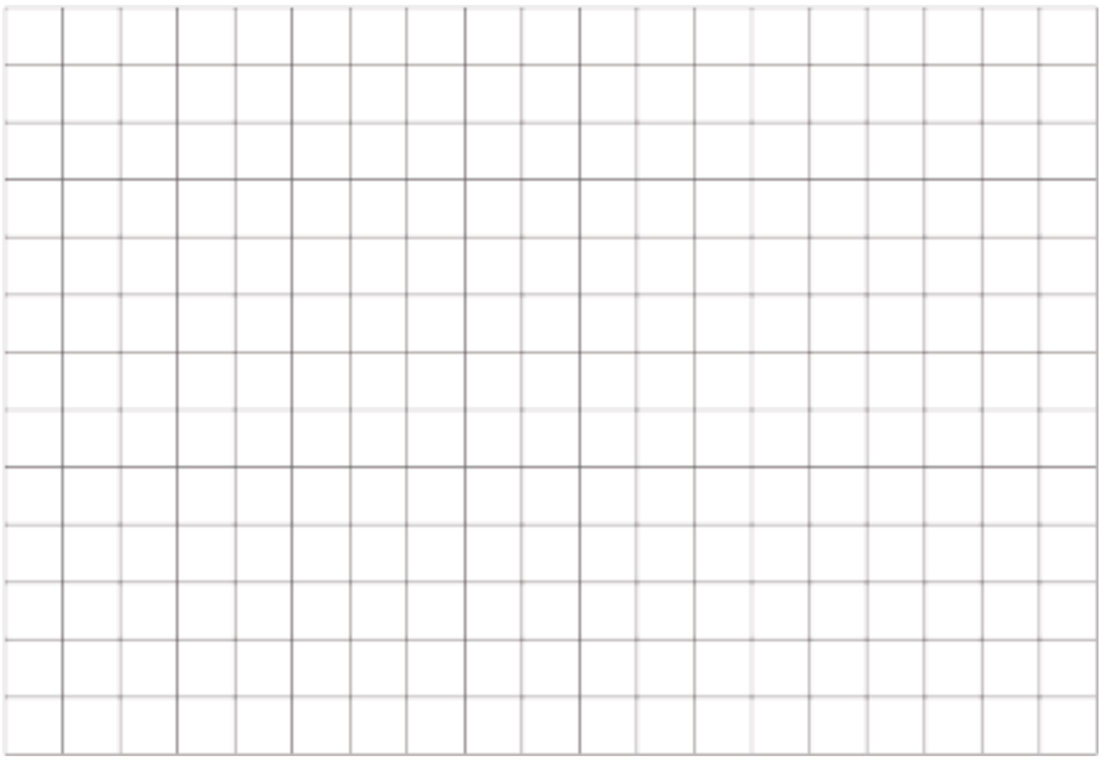
Representation: Internalize Comprehension. Provide a range of examples and counterexamples for a best fit line. Consider displaying charts and examples from previous lessons to aide in memory recall.
Supports accessibility for: Conceptual processing

Anticipated Misconceptions

If a student is stuck on making the scale on the graph, remind them to look at the range from the previous problem.

Student Task Statement

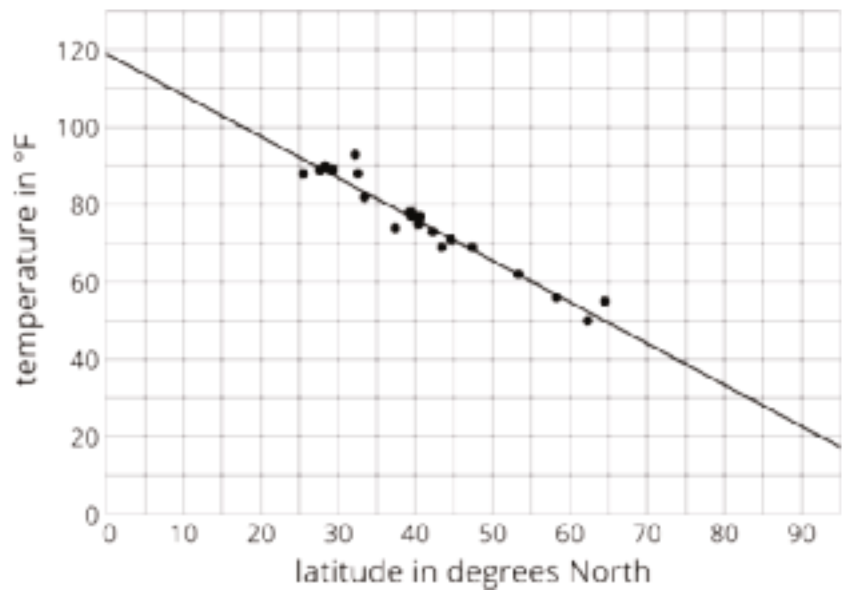
1. Make a scatter plot of the data.



2. Describe any patterns of association that you notice.

3. Draw a line that fits the data. Write an equation for this line.

Student Response

1. 

2. There seems to be a negative association. A line fits the data quite well.

3. The line is shown above. The line's equation is $y = -1.07x + 119$, where x is latitude and y is temperature. This equation was computed with software. A line obtained by eye should have a slope of approximately -1, and a y-intercept of approximately 120 degrees F.

Activity Synthesis

At the start of the discussion, make sure students agree that there seems to be a negative association that looks like a line would fit the data nicely before moving on.

Invite several groups to share the equations they came up with and how they found them. They will likely have slightly different slopes and intercepts. Ask students to explain where those differences come from. (Not everyone chose the same points to guide the trendline they drew for the data.) The differences should be small and the different models give more or less the same information. In the next activity, students will be using the model (equation and graph) to make predictions.

Access for English Language Learners

Representing, Conversing: MLR8 Discussion Supports. After students complete their scatter plots from the data, they should meet with a small group of 3–4 students to share and compare. While each student shares, circulate and encourage students to look for commonalities and to discuss their differences in displays. Tell students to generate an agreed on response for the equation and the visual display of the line even though they may have differences. Tell students that one member will share out, but they won't know who, so they all need to be prepared to share and explain. During the whole-class discussion, amplify use of mathematical language such as "slope", "intercept", "equation", "graph", and "negative association". Use this to help students develop their explanations of associations and differences that they see in how the data is interpreted.
Design Principle(s): Cultivate conversation; Maximize meta-awareness

Lesson 6: Using and Interpreting a Mathematical Model

Goals

- Interpret a scatter plot and line of fit that model temperature and latitude, and explain (orally) limitations of the model.
- Use a mathematical model of bivariate data to make predictions (in writing).

Lesson Narrative

In the last in the sequence of three lessons, students analyze the model and use it to make a prediction and to draw conclusions. They interpret mathematical features of their model (slope and intercepts of the line), and discuss limitations of the model.

Alignments

Addressing

- 8.F.B: Use functions to model relationships between quantities.
- 8.SP.A: Investigate patterns of association in bivariate data.

Instructional Routines

- MLR2: Collect and Display

Student Learning Goals

Let’s use a model to make some predictions.

6.1 Using a Mathematical Model

Optional: 15 minutes

In the previous activity, students drew a line that best fit the latitude-temperature data and found the equation of this line. The line is a mathematical model of the situation. In this lesson, they use their model to make predictions about temperatures in cities that were not included in the original data set. In the next activity, they also interpret the slope of the line and the intercepts in the context of this situation. This leads to a discussion of the limitations of the mathematical model they developed.

Addressing

- 8.F.B
- 8.SP.A

Instructional Routines

- MLR2: Collect and Display

Launch

Students in same groups of 3–4. If available, tell the students the latitude and average high temperature in September in their city.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.
Supports accessibility for: Organization; Attention

Student Task Statement

In the previous activity, you found the equation of a line to represent the association between latitude and temperature. This is a *mathematical model*.

1. Use your model to predict the average high temperature in September at the following cities that were not included in the original data set:

    a. Detroit (Lat: 42.14)

    b. Albuquerque (Lat: 35.2)

    c. Nome (Lat: 64.5)

    d. Your own city (if available)

2. Draw points that represent the predicted temperatures for each city on the scatter plot.

3. The actual average high temperature in September in these cities were:
    - Detroit: 74°F
    - Albuquerque: 82°F
    - Nome: 49°F
    - Your own city (if available):

    How well does your model predict the temperature? Compare the predicted and actual temperatures.

4. If you added the actual temperatures for these four cities to the scatter plot, would you move your line?

5. Are there any outliers in the data? What might be the explanation?

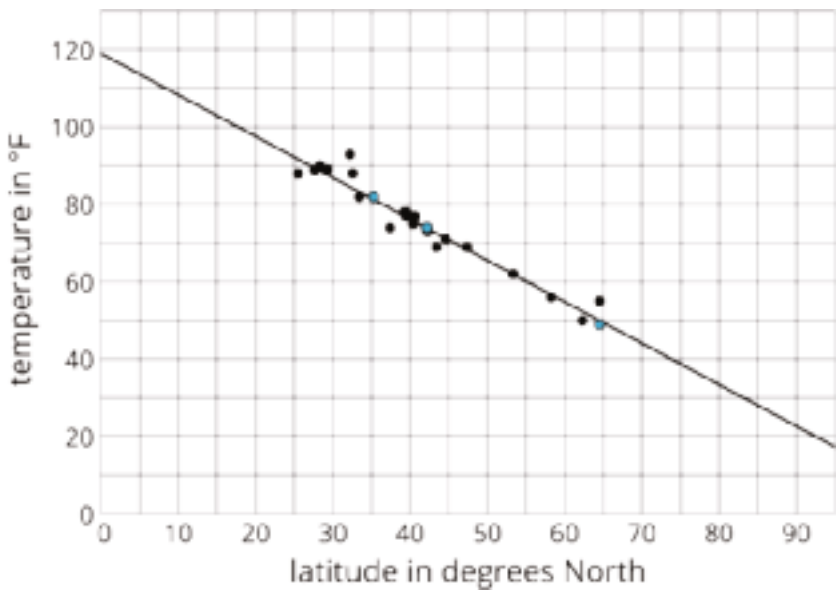
Student Response

1. Answers may vary slightly depending on the equation of the line determined by eye.
   a. Detroit: actual $74^\circ F$, prediction: $119 - 1.07(42.14)$ or $73.9^\circ F$

   b. Albuquerque: actual $82^\circ F$, prediction: $119 - 1.07(35.2)$ or $81.3^\circ F$

   c. Nome: actual $49^\circ F$, prediction: $119 - 1.07(64.5)$ or $50^\circ F$

2.



3. The predictions are very close to the actual data.

4. They would not cause a noticeable change to the line.

5. There aren't big outliers in the data. The two points that lie the farthest from the line represent Tucson and Fairbanks. This might have geographical reasons—a desert or sea climate, respectively.

Activity Synthesis

Invite some groups to share their results and compare predictions for the different cities. Ask if students think that the line is a good mathematical model to predict the temperature in a location if you know the latitude.

Access for English Language Learners

Representing, Speaking, Listening: MLR2 Collect and Display. Use this routine to collect verbal and written ideas about students' mathematical models and the limitations of them that they see. During the synthesis discussion, chart key ideas that students share out about the slope, intercept, and differences in their predictions. Remind students that we call this a "mathematical model" (i.e., the line they drew). It is one way to look at data. Ask students, "What are the limitations of this model in making predictions?" Add their language to your chart. Reference this chart in the following lesson and make revisions as needed with student input.
Design Principle(s): Support sense-making; Maximize meta-awareness

6.2 Interpreting a Mathematical Model

Optional: 15 minutes
Students interpret the slope and intercepts in the context of the situation. They also discuss limitations of the mathematical model.

This activity can be extended by having students investigate if temperature on other continents or across continents fits the same pattern that we found for North America.

Addressing

- 8.SP.A

Launch

Keep students in same groups of 3–4.

Anticipated Misconceptions

Students may want to say "For every one unit increase in x, y decreases by 1.07 units." Ask them to use the specific units and quantities in the model, latitude in degrees north and temperature in degrees Fahrenheit.

Student Task Statement

Refer to your equation for the line that models the association between latitude and temperature of the cities.

1. What does the slope mean in the context of this situation?

2. Find the vertical and horizontal intercepts and interpret them in the context of the situation.

3. Can you think of a city or a location that could not be represented using this same model? Explain your thinking.

Student Response

1. For every degree latitude moving north, the temperature decreases by 1.07°F.
2. Vertical: The temperature at 0 degrees north (that is, on the equator) is 119°F. Horizontal: A latitude where high temp is 0°F would have to be over 100 degrees north, which doesn't exist.
3. Latitudes are restricted to 0–90 degrees by the situation. Factors other than latitude influence temperature, and those factors seem to be more important close to 0 degrees and close to 90 degrees. The model only used cities in North America. It should not be used to make predictions about temperatures on other continents without checking similar data there first.

Activity Synthesis

Invite students to share their responses. Discuss the limitations and uses of the model. Consider asking the following questions:

- "What are some limitations of the model?"
- "Do limitations mean that the model is not good?" (No, it just means that we have to be aware of when we can use it and when we can't use it. Our model is pretty good for latitudes between 25 and 65 degrees north, and for locations in North America.)
- "What questions do you have about predicting temperature?"
- "How could you extend your investigation of predicting temperature or the weather?"

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Break the class into small discussion groups and then invite a representative from each group to report back to the whole class.
Supports accessibility for: Language; Social-emotional skills; Attention

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Describing a Tessellation

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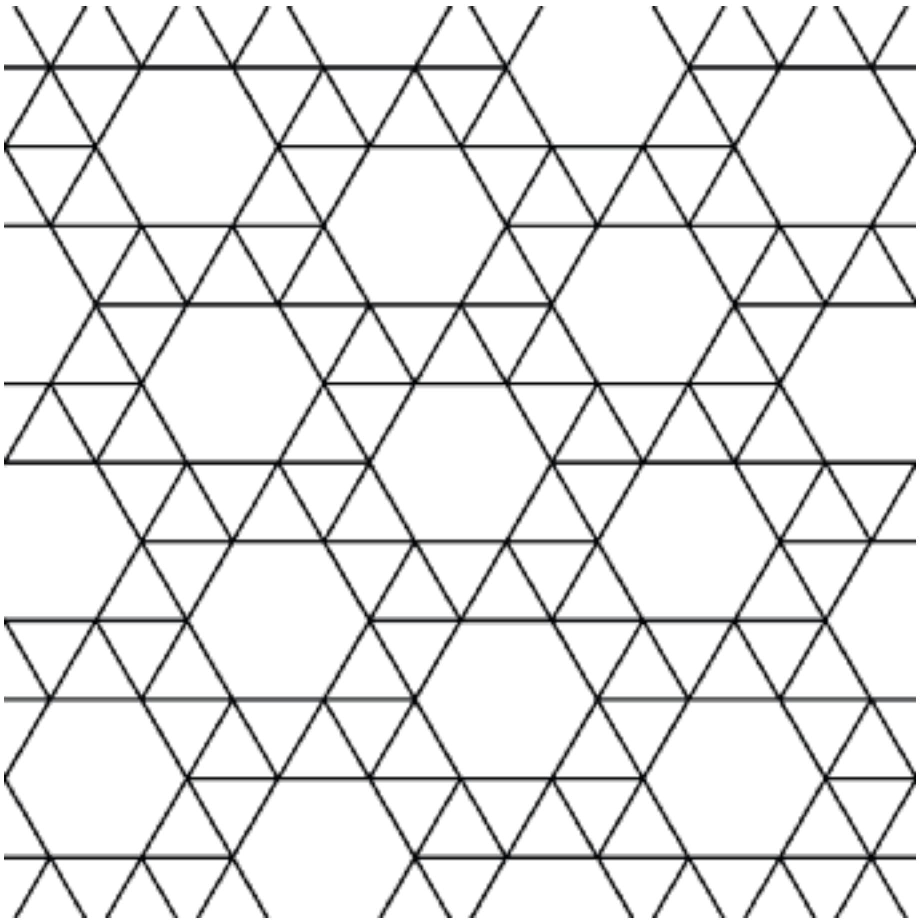
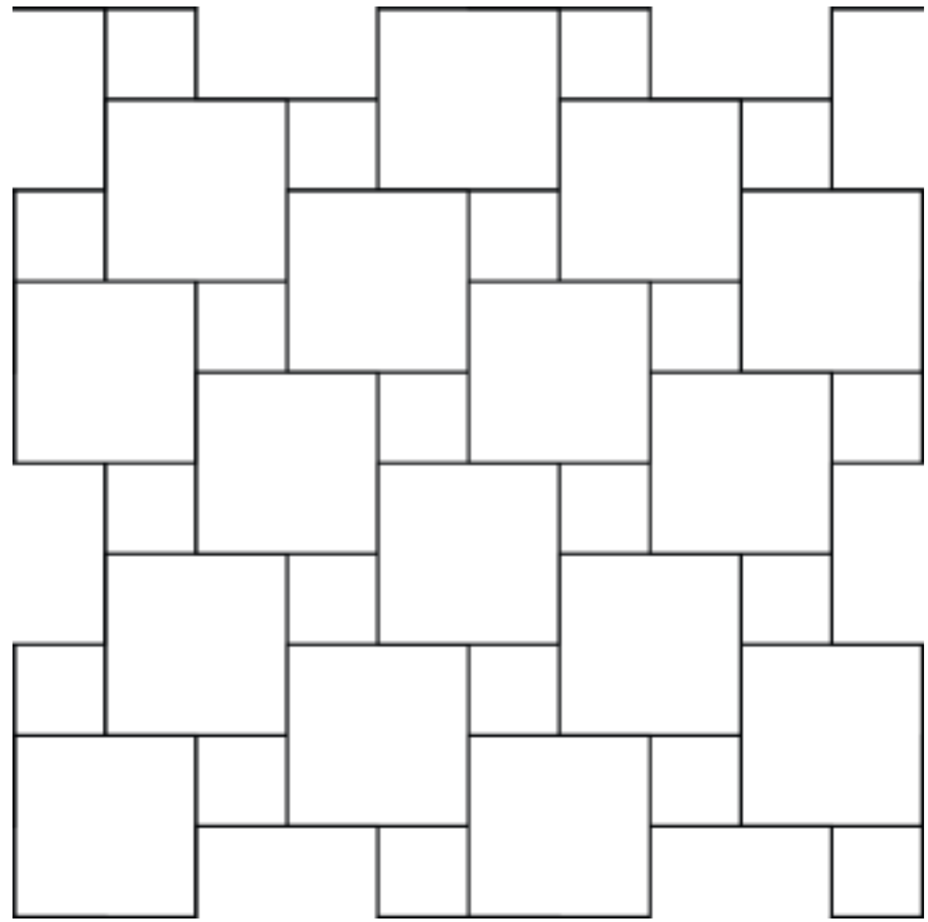


Describing a Tessellation

Card B



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