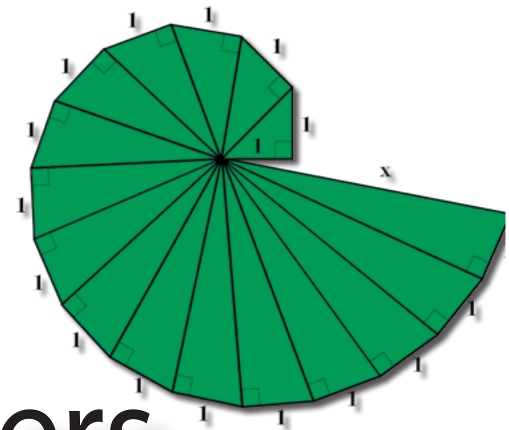
Core Knowledge® MATHEMATICS

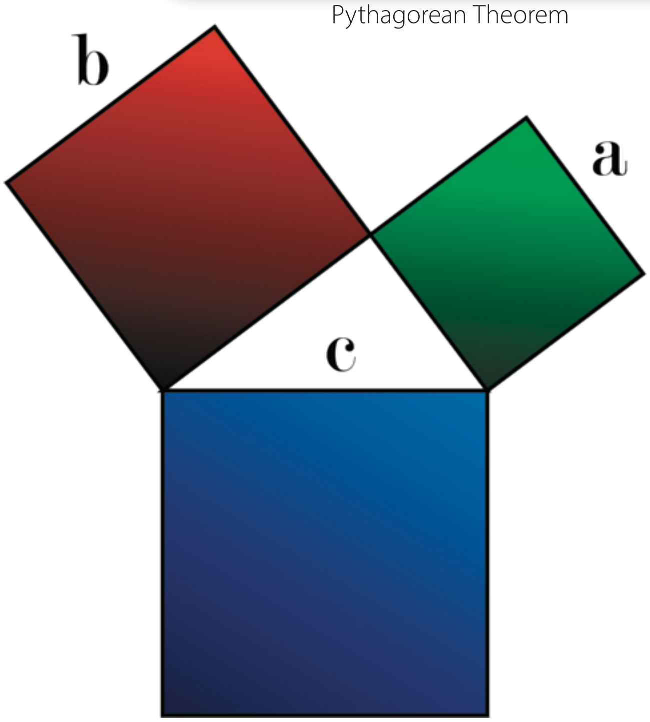
Pythagorean Theorem and Irrational Numbers

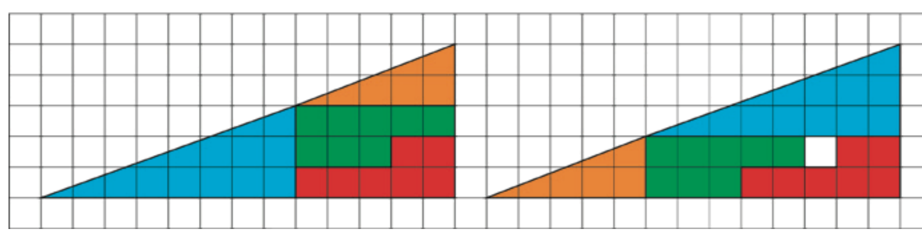


The spiral

Student Workbook

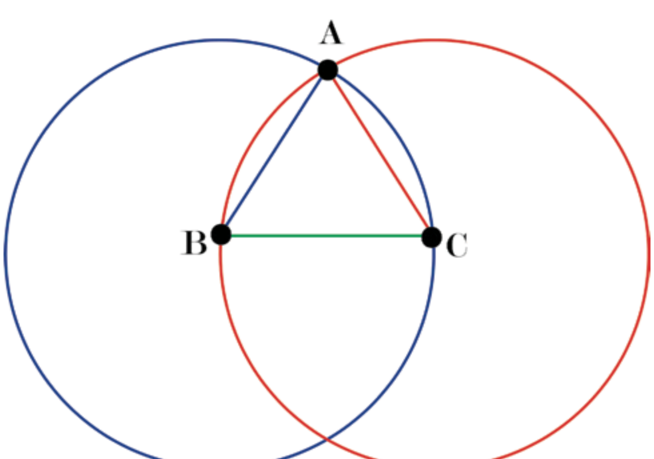
Pythagorean Theorem





Find the area of each triangle

The Hands of a Clock



Intersecting Circles

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Pythagorean Theorem and Irrational Numbers

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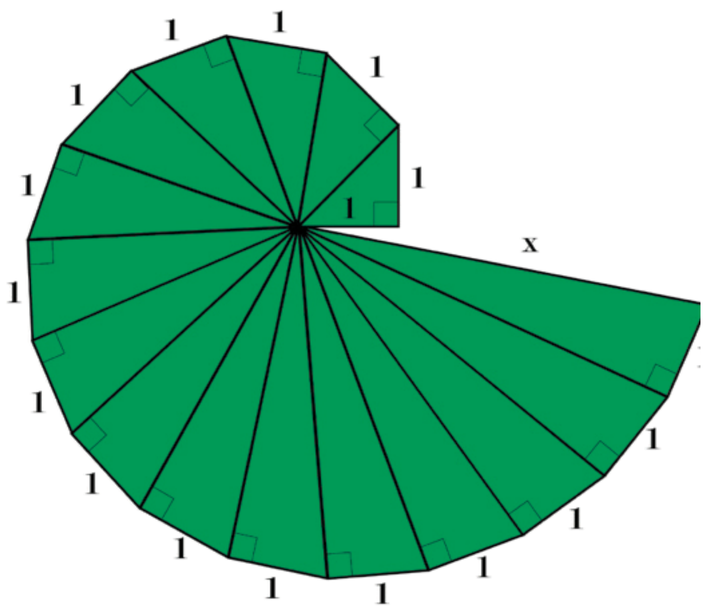


Pythagorean Theorem and Irrational Numbers

Student Workbook

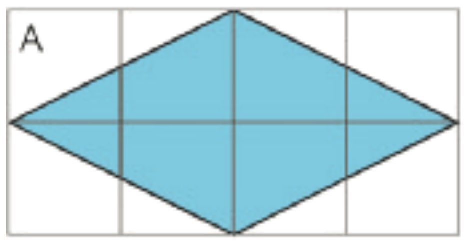
Core Knowledge Mathematics™

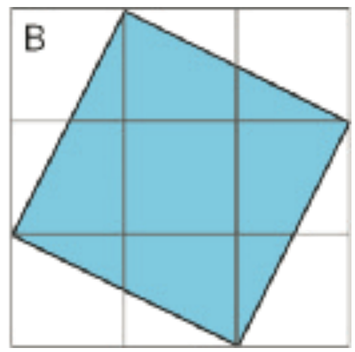
Lesson 1: The Areas of Squares and Their Side Lengths

Let's investigate the squares and their side lengths.

1.1: Two Regions

Which shaded region is larger? Explain your reasoning.

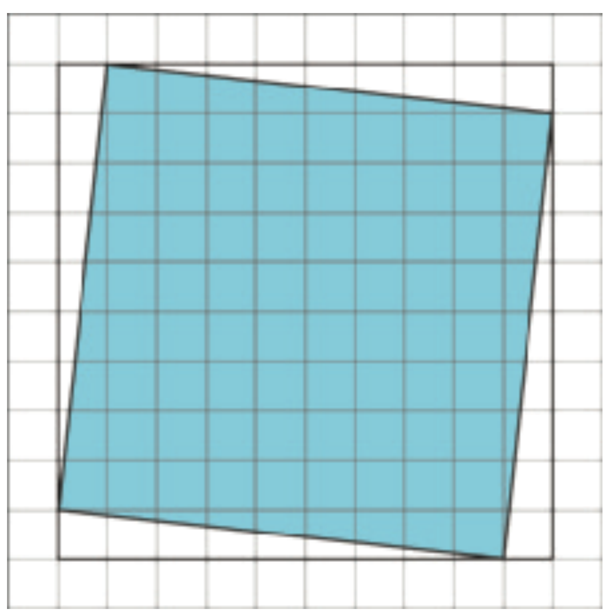




1.2: Decomposing to Find Area

Find the area of each shaded square (in square units).

1.



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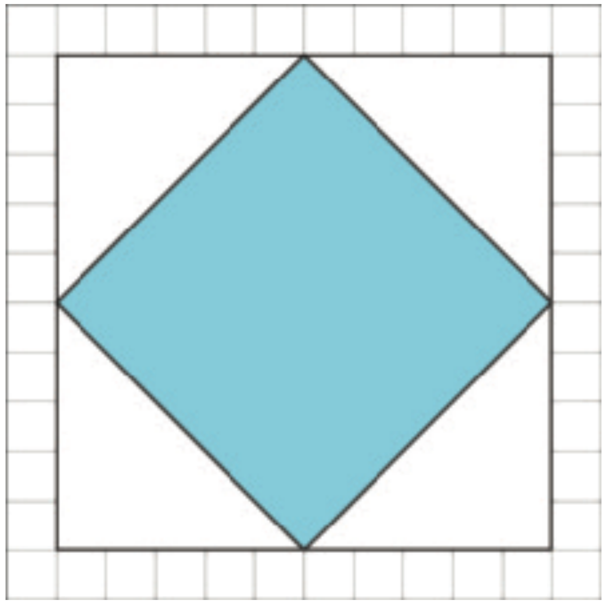

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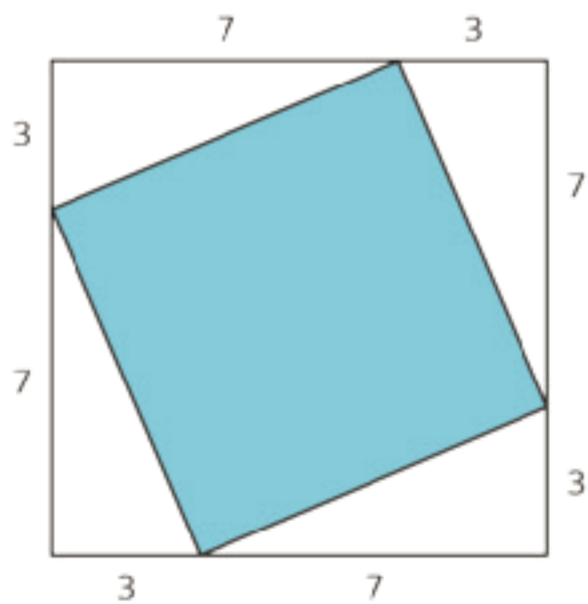
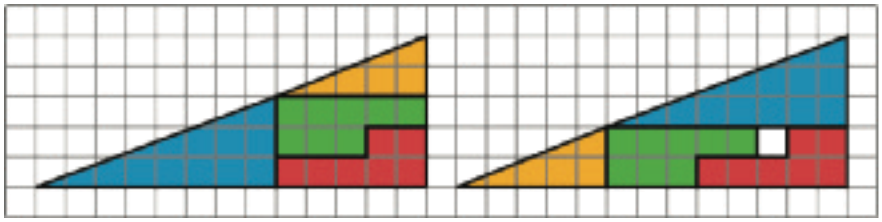


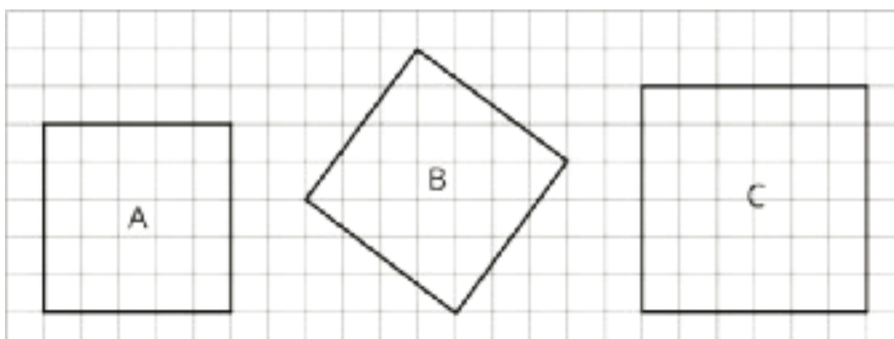
Are you ready for more?

Any triangle with a base of 13 and a height of 5 has an area of $\frac{65}{2}$.



Both shapes in the figure have been partitioned into the same four pieces. Find the area of each of the pieces, and verify the corresponding parts are the same in each picture. There appears to be one extra square unit of area in the right figure. If all of the pieces have the same area, how is this possible?

1.3: Estimating Side Lengths from Areas

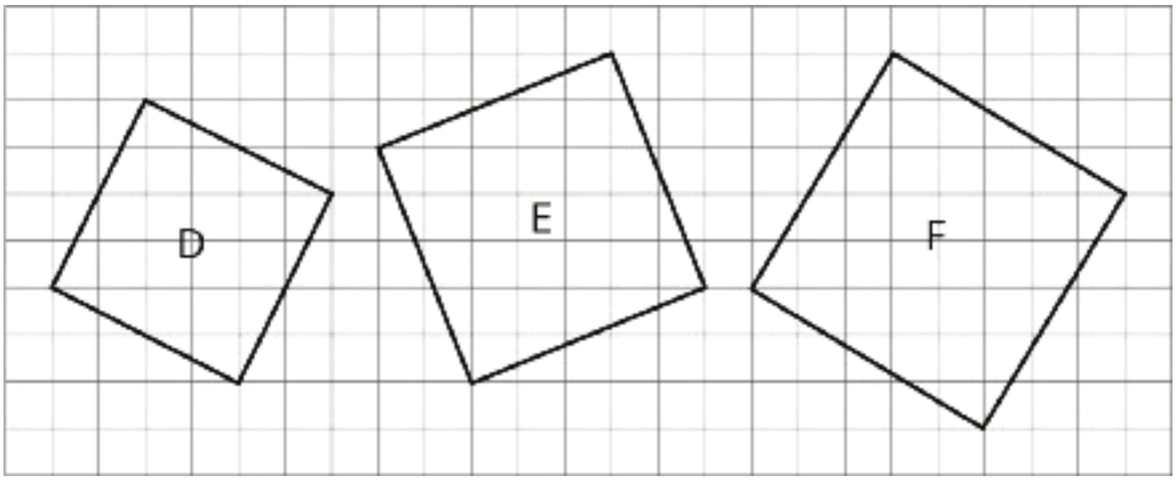


1. What is the side length of square A? What is its area?

2. What is the side length of square C? What is its area?

3. What is the area of square B? What is its side length? (Use tracing paper to check your answer to this.)

4. Find the areas of squares D, E, and F. Which of these squares must have a side length that is greater than 5 but less than 6? Explain how you know.



1.4: Making Squares

Your teacher will give your group a sheet with three squares and 5 cut out shapes labeled D, E, F, G, and H. Use the squares to find the total area of the five shapes. Assume each small square is equal to 1 square unit.

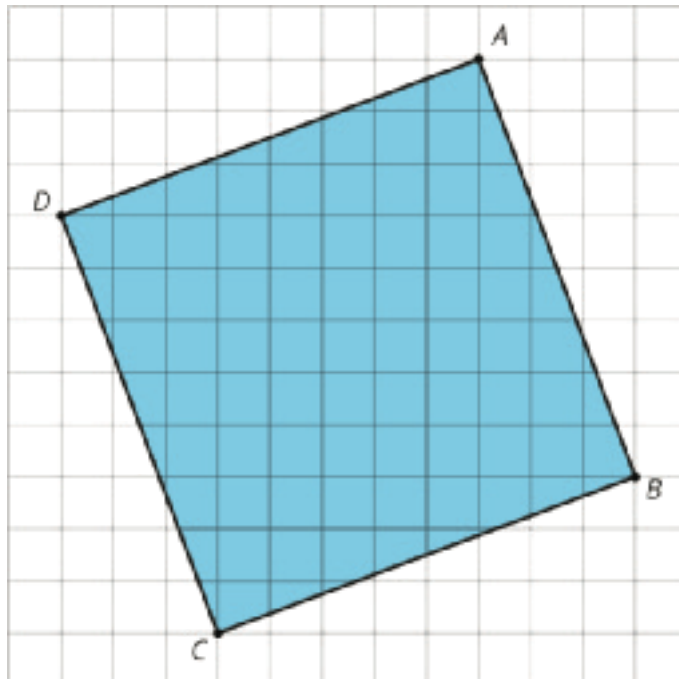
Lesson 1 Summary

The area of a square with side length 12 units is 12^2 or 144 units^2.

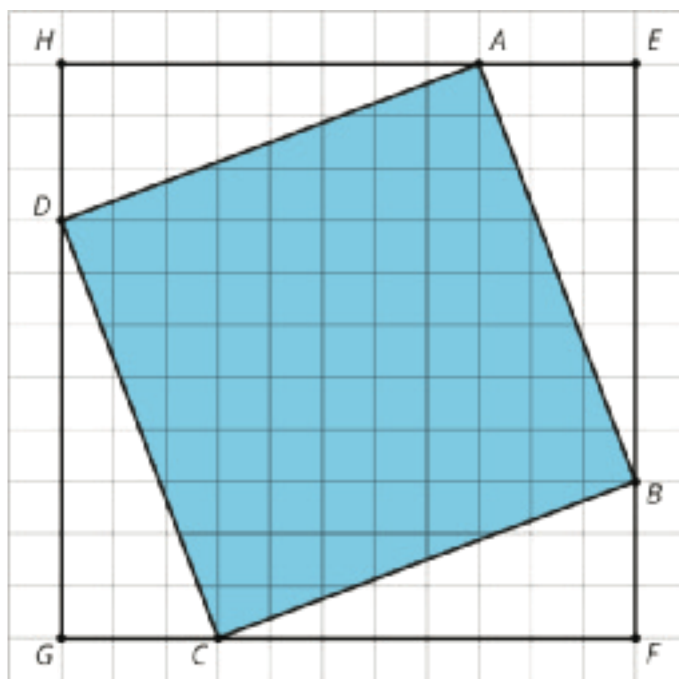
The side length of a square with area 900 units^2 is 30 units because $30^2 = 900$.

Sometimes we want to find the area of a square but we don't know the side length. For example, how can we find the area of square $ABCD$?

One way is to enclose it in a square whose side lengths we do know.

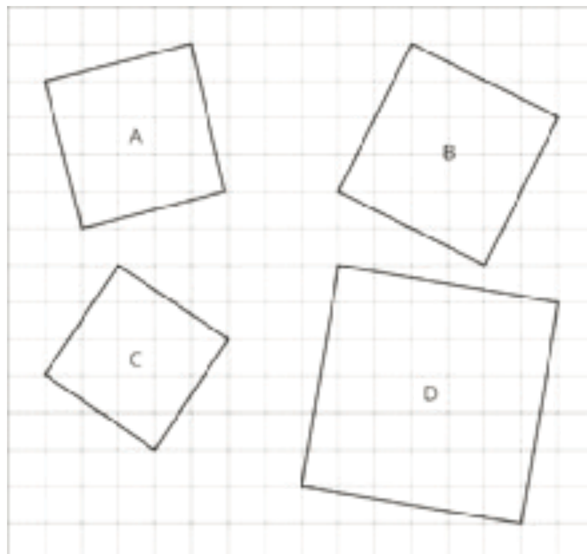


The outside square $EFGH$ has side lengths of 11 units, so its area is 121 units^2. The area of each of the four triangles is $\frac{1}{2} \cdot 8 \cdot 3 = 12$, so the area of all four together is $4 \cdot 12 = 48$ units^2. To get the area of the shaded square, we can take the area of the outside square and subtract the areas of the 4 triangles. So the area of the shaded square $ABCD$ is $121 - 48 = 73$ or 73 units^2.



Unit 8 Lesson 1 Cumulative Practice Problems

1. Find the area of each square. Each grid square represents 1 square unit.



2. Find the length of a side of a square if its area is:

   a. 81 square inches

   b. $\frac{4}{25}$ cm^2

   c. 0.49 square units

   d. m^2 square units

3. Find the area of a square if its side length is:

a. 3 inches

b. 7 units

c. 100 cm

d. 40 inches

e. x units

4. Evaluate $(3.1 \times 10^4) \cdot (2 \times 10^6)$. Choose the correct answer:

A. 5.1×10^{10}

B. 5.1×10^{24}

C. 6.2×10^{10}

D. 6.2×10^{24}

(From Unit 7, Lesson 14.)

5. Noah reads the problem, "Evaluate each expression, giving the answer in scientific notation." The first problem part is: $5.4 \times 10^5 + 2.3 \times 10^4$.

   Noah says, "I can rewrite 5.4×10^5 as 54×10^4. Now I can add the numbers: $54 \times 10^4 + 2.3 \times 10^4 = 57.3 \times 10^4$."

   Do you agree with Noah's solution to the problem? Explain your reasoning.

   (From Unit 7, Lesson 15.)

6. Select **all** the expressions that are equivalent to 3^8.

   A. $(3^2)^4$

   B. 8^3

   C. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

   D. $(3^4)^2$

   E. $\frac{3^6}{3^{-2}}$

   F. $3^6 \cdot 10^2$

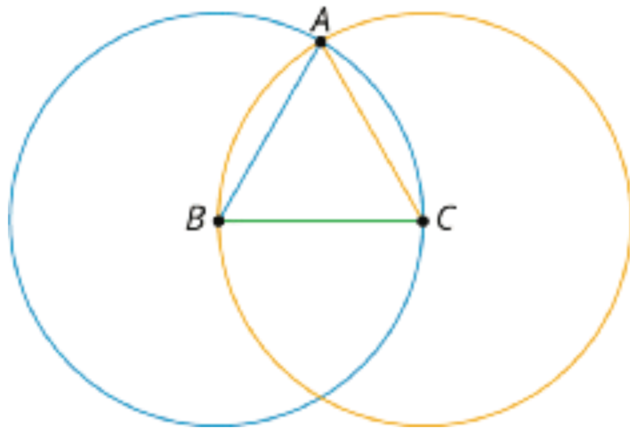
   (From Unit 7, Lesson 6.)

Lesson 2: Side Lengths and Areas

Let's investigate some more squares.

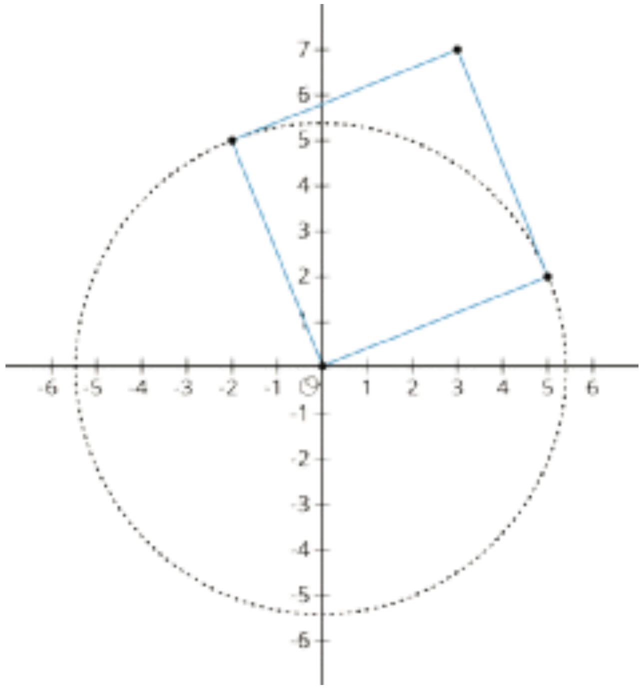
2.1: Notice and Wonder: Intersecting Circles

What do you notice? What do you wonder?

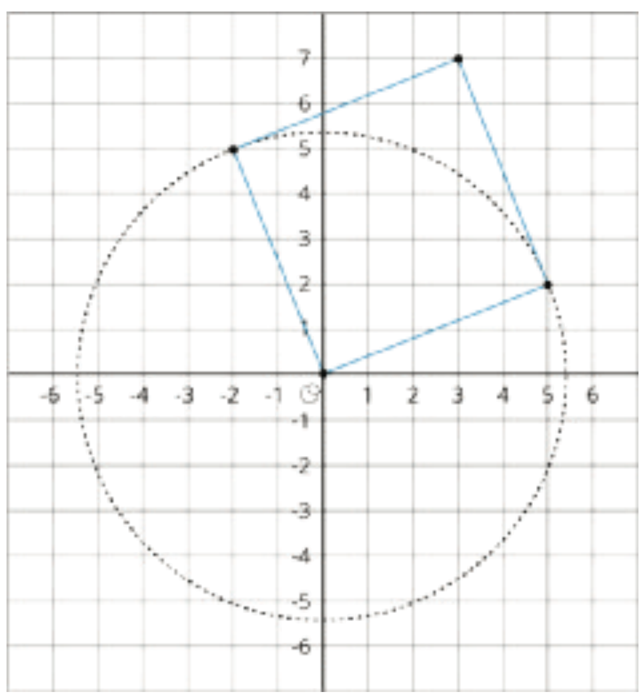


2.2: One Square

1. Use the circle to estimate the area of the square shown here:

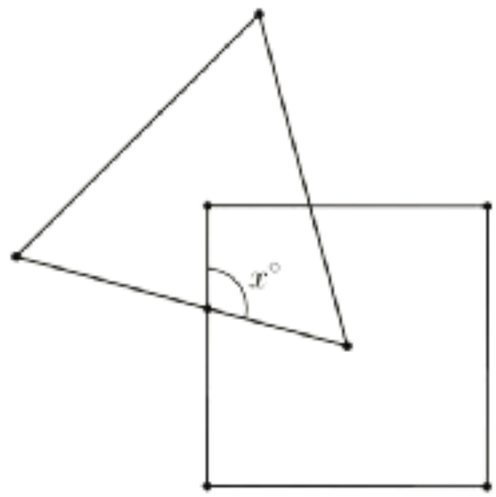


2. Use the grid to check your answer to the first problem.



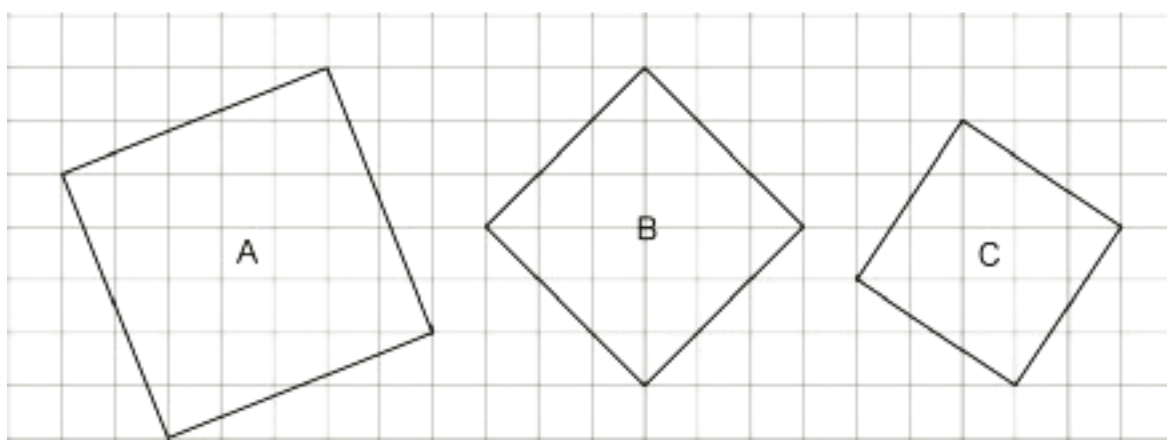
Are you ready for more?

One vertex of the equilateral triangle is in the center of the square, and one vertex of the square is in the center of the equilateral triangle. What is x?



2.3: The Sides and Areas of Tilted Squares

1. Find the area of each square and estimate the side lengths using your geometry toolkit. Then write the exact lengths for the sides of each square.

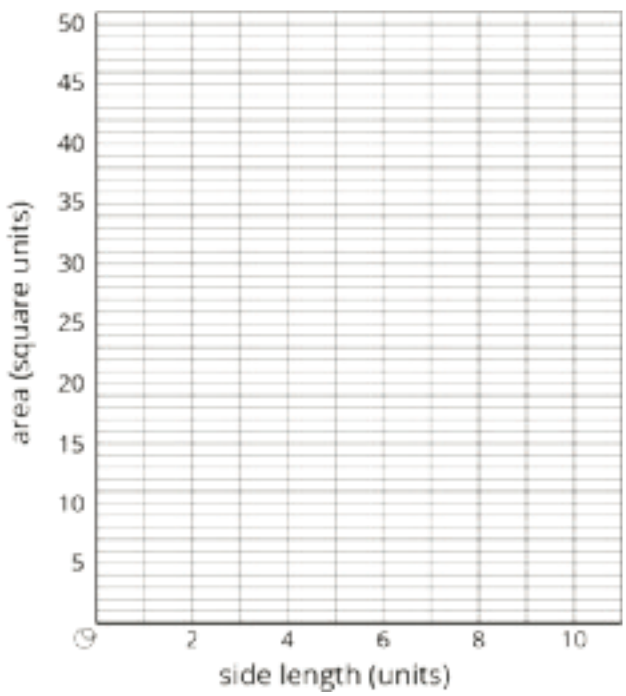


2. Complete the tables with the missing side lengths and areas.

side length, s	0.5		1.5		2.5		3.5	
area, a		1		4		9		16

side length, s	4.5		5.5		6.5		7.5	
area, a		25		36		49		64

3. Plot the points, (s, a), on the coordinate plane shown here.

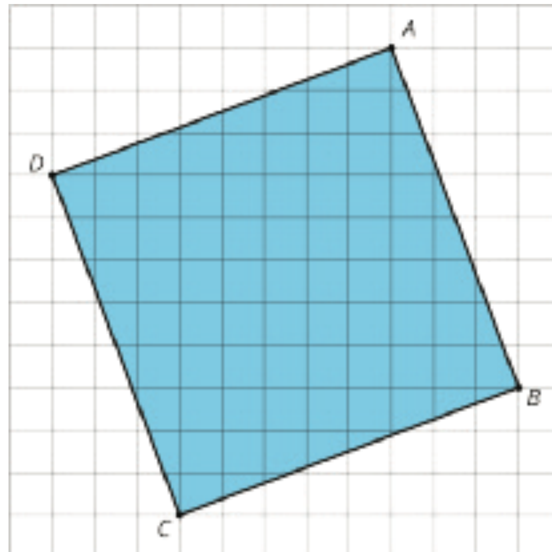


4. Use this graph to estimate the side lengths of the squares in the first question. How do your estimates from the graph compare to the estimates you made initially using your geometry toolkit?

5. Use the graph to approximate $\sqrt{45}$.

Lesson 2 Summary

We saw earlier that the area of square ABCD is 73 units^2.

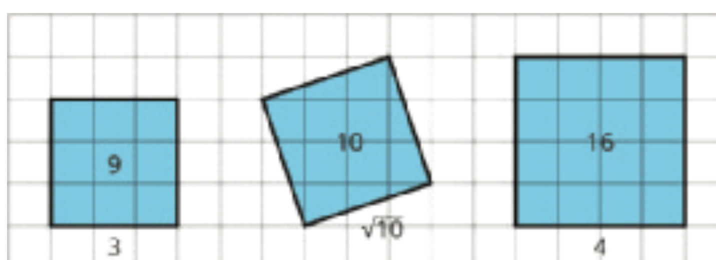


What is the side length? The area is between $8^2 = 64$ and $9^2 = 81$, so the side length must be between 8 units and 9 units. We can also use tracing paper to trace a side length and compare it to the grid, which also shows the side length is between 8 units and 9 units. But we want to be able to talk about its *exact* length. In order to write "the side length of a square whose area is 73 square units," we use the **square root** symbol. "The square root of 73" is written $\sqrt{73}$, and it means "the length of a side of a square whose area is 73 square units."

We say the side length of a square with area 73 units^2 is $\sqrt{73}$ units. This means that

$$\left(\sqrt{73}\right)^2 = 73$$

All of these statements are also true:



$\sqrt{9} = 3$ because $3^2 = 9$

$\sqrt{16} = 4$ because $4^2 = 16$

$\sqrt{10}$ units is the side length of a square whose area is 10 units^2, and $\left(\sqrt{10}\right)^2 = 10$

Unit 8 Lesson 2 Cumulative Practice Problems

1. A square has an area of 81 square feet. Select **all** the expressions that equal the side length of this square, in feet.

   A. $\frac{81}{2}$

   B. $\sqrt{81}$

   C. 9

   D. $\sqrt{9}$

   E. 3

2. Write the exact value of the side length, in units, of a square whose area in square units is:

   a. 36

   b. 37

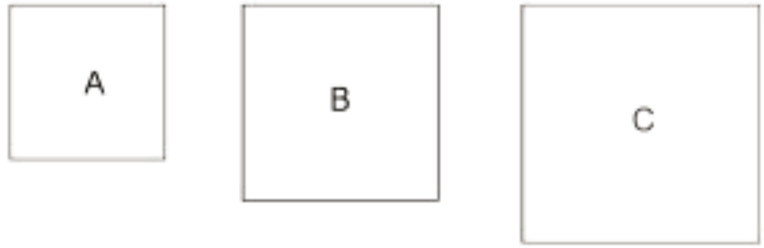
   c. $\frac{100}{9}$

   d. $\frac{2}{5}$

   e. 0.0001

   f. 0.11

3. Square A is smaller than Square B. Square B is smaller than Square C.



The three squares' side lengths are $\sqrt{26}$, 4.2, and $\sqrt{11}$.

What is the side length of Square A? Square B? Square C? Explain how you know.

4. Find the area of a square if its side length is:

a. $\frac{1}{5}$ cm

b. $\frac{3}{7}$ units

c. $\frac{11}{8}$ inches

d. 0.1 meters

e. 3.5 cm

(From Unit 8, Lesson 1.)

5. Here is a table showing the areas of the seven largest countries.

    a. How much larger is Russia than Canada?

    b. The Asian countries on this list are Russia, China, and India. The American countries are Canada, the United States, and Brazil. Which has the greater total area: the three Asian countries, or the three American countries?

country	area (in km^2)
Russia	1.71×10^7
Canada	9.98×10^6
China	9.60×10^6
United States	9.53×10^6
Brazil	8.52×10^6
Australia	6.79×10^6
India	3.29×10^6

(From Unit 7, Lesson 15.)

6. Select **all** the expressions that are equivalent to 10^{-6}.

    A. $\frac{1}{1000000}$

    B. $\frac{-1}{1000000}$

    C. $\frac{1}{10^6}$

    D. $10^8 \cdot 10^{-2}$

    E. $\left(\frac{1}{10}\right)^6$

    F. $\frac{1}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$

(From Unit 7, Lesson 5.)

Lesson 3: Rational and Irrational Numbers

Let’s learn about irrational numbers.

3.1: Algebra Talk: Positive Solutions

Find a positive solution to each equation:

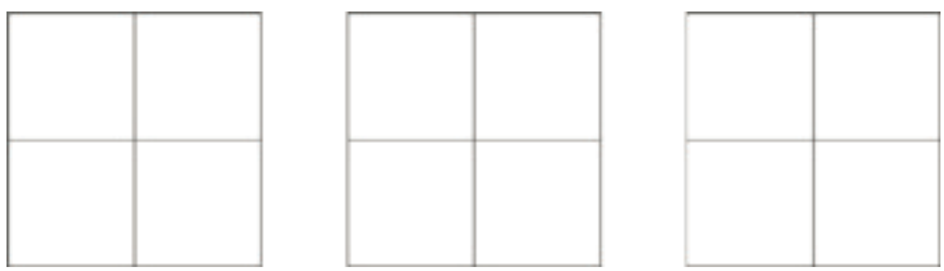
$x^2 = 36$

$x^2 = \frac{9}{4}$

$x^2 = \frac{1}{4}$

$x^2 = \frac{49}{25}$

3.2: Three Squares



1. Draw 3 squares of different sizes with vertices aligned to the vertices of the grid.
2. For each square:
    a. Label the area.
    b. Label the side length.
    c. Write an equation that shows the relationship between the side length and the area.

3.3: Looking for a Solution

Are any of these numbers a solution to the equation $x^2 = 2$? Explain your reasoning.

- 1
- $\frac{1}{2}$
- $\frac{3}{2}$
- $\frac{7}{5}$

3.4: Looking for $\sqrt{2}$

A **rational number** is a fraction or its opposite (or any number equivalent to a fraction or its opposite).

1. Find some more rational numbers that are close to $\sqrt{2}$.
2. Can you find a rational number that is exactly $\sqrt{2}$?

Are you ready for more?

If you have an older calculator evaluate the expression

$$\left(\frac{577}{408}\right)^2$$

and it will tell you that the answer is 2, which might lead you to think that $\sqrt{2} = \frac{577}{408}$.

1. Explain why you might be suspicious of the calculator's result.

2. Find an explanation for why $408^2 \cdot 2$ could not possibly equal 577^2. How does this show that $\left(\frac{577}{408}\right)^2$ could not equal 2?

3. Repeat these questions for

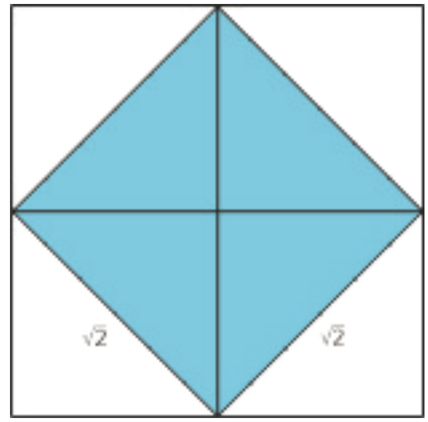
$$\left(\frac{1414213562375}{1000000000000}\right)^2 \neq 2,$$

an equation that even many modern calculators and computers will get wrong.

Lesson 3 Summary

In an earlier activity, we learned that square root notation is used to write the length of a side of a square given its area. For example, a square whose area is 2 square units has a side length of $\sqrt{2}$ units, which means that

$$\sqrt{2} \cdot \sqrt{2} = 2.$$



A square whose area is 25 square units has a side length of $\sqrt{25}$ units, which means that

$$\sqrt{25} \cdot \sqrt{25} = 25.$$

Since $5 \cdot 5 = 25$, we know that

$$\sqrt{25} = 5.$$

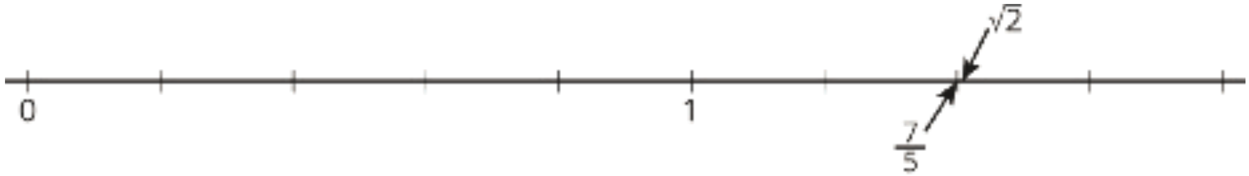
$\sqrt{25}$ is an example of a rational number. A **rational number** is a fraction or its opposite. Remember that a fraction $\frac{a}{b}$ is a point on the number line found by dividing the segment from 0 to 1 into b equal intervals and going a of those intervals to the right of 0. We can always write a fraction in the form $\frac{a}{b}$ where a and b are whole numbers (and b is not 0), but there are other ways to write them. For example, we can write $\sqrt{25} = \frac{5}{1}$. You first learned about fractions in earlier grades, and at that time, you probably didn't know about negative numbers. Rational numbers are fractions, but they can be positive or negative. So, -5 is also a rational number. Because fractions and ratios are closely related ideas, fractions and their opposites are called RATIOnal numbers.

Here are some examples of rational numbers:

$$\frac{7}{4},\ 0, \frac{6}{3}, 0.2, -\frac{1}{3}, -5, \sqrt{9},\ -\frac{\sqrt{16}}{\sqrt{100}}$$

Can you see why they are each examples of “a fraction or its opposite?”

An **irrational number** is a number that is not rational. That is, it is a number that is not a fraction or its opposite. $\sqrt{2}$ is an example of an irrational number. It has a location on the number line, and its location can be approximated by rational numbers (it's a tiny bit to the right of $\frac{7}{5}$), but $\sqrt{2}$ can not be found on a number line by dividing the segment from 0 to 1 into b equal parts and going a of those parts away from 0 (if a and b are whole numbers).



$\frac{17}{12}$ is also close to $\sqrt{2}$, because $\left(\frac{17}{12}\right)^2 = \frac{289}{144}$. $\frac{289}{144}$ is very close to 2, since $\frac{288}{144} = 2$. But we could keep looking forever for solutions to $x^2 = 2$ that are rational numbers, and we would not find any. $\sqrt{2}$ is not a rational number! It is irrational.

In your future studies, you may have opportunities to understand or write a proof that $\sqrt{2}$ is irrational, but for now, we just take it as a fact that $\sqrt{2}$ is irrational. Similarly, the square root of any whole number is either a whole number ($\sqrt{36} = 6$, $\sqrt{64} = 8$, etc.) or irrational ($\sqrt{17}$, $\sqrt{65}$, etc.). Here are some other examples of irrational numbers:

$$\sqrt{10},\ \text{-}\sqrt{3},\ \frac{\sqrt{5}}{2},\ \pi$$

Unit 8 Lesson 3 Cumulative Practice Problems

1. Decide whether each number in this list is *rational* or *irrational*.

$$\frac{-13}{3},\ 0.1234,\ \sqrt{37},\ -77,\ -\sqrt{100},\ -\sqrt{12}$$

2. Which value is an exact solution of the equation $m^2 = 14$?

   A. 7

   B. $\sqrt{14}$

   C. 3.74

   D. $\sqrt{3.74}$

3. A square has vertices $(0, 0)$, $(5, 2)$, $(3, 7)$, and $(-2, 5)$. Which of these statements is true?

   A. The square's side length is 5.

   B. The square's side length is between 5 and 6.

   C. The square's side length is between 6 and 7.

   D. The square's side length is 7.

   (From Unit 8, Lesson 2.)

4. Rewrite each expression in an equivalent form that uses a single exponent.

   a. $(10^2)^{-3}$

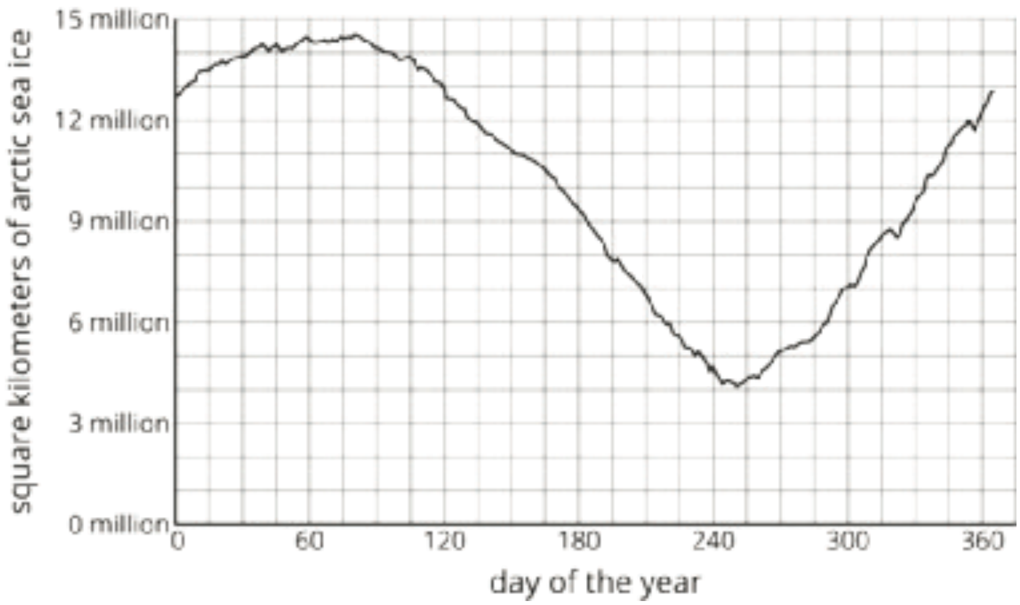
   b. $(3^{-3})^2$

   c. $3^{-5} \cdot 4^{-5}$

   d. $2^5 \cdot 3^{-5}$

   (From Unit 7, Lesson 8.)

5. The graph represents the area of arctic sea ice in square kilometers as a function of the day of the year in 2016.



a. Give an approximate interval of days when the area of arctic sea ice was decreasing.

b. On which days was the area of arctic sea ice 12 million square kilometers?

(From Unit 5, Lesson 5.)

6. The high school is hosting an event for seniors but will also allow some juniors to attend. The principal approved the event for 200 students and decided the number of juniors should be 25% of the number of seniors. How many juniors will be allowed to attend? If you get stuck, try writing two equations that each represent the number of juniors and seniors at the event.

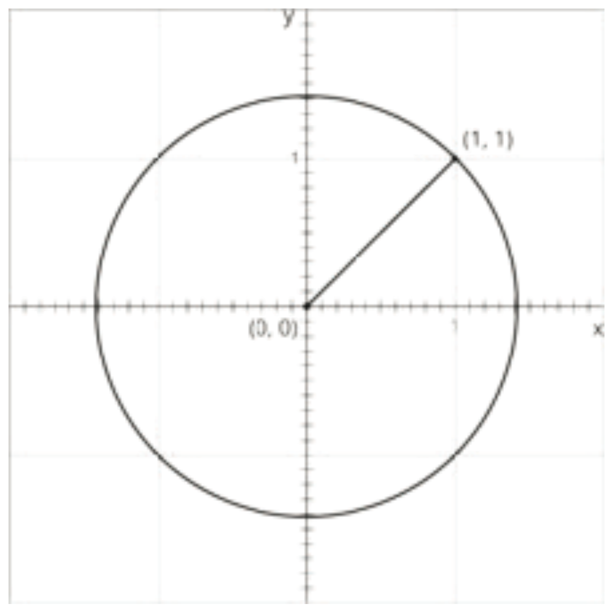
(From Unit 4, Lesson 14.)

Lesson 4: Square Roots on the Number Line

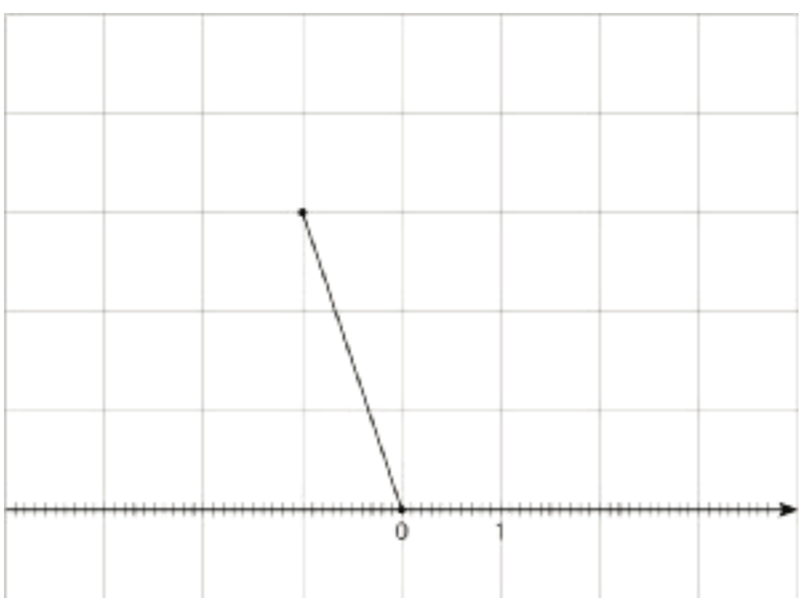
Let's explore square roots.

4.1: Notice and Wonder: Diagonals

What do you notice? What do you wonder?



4.2: Squaring Lines

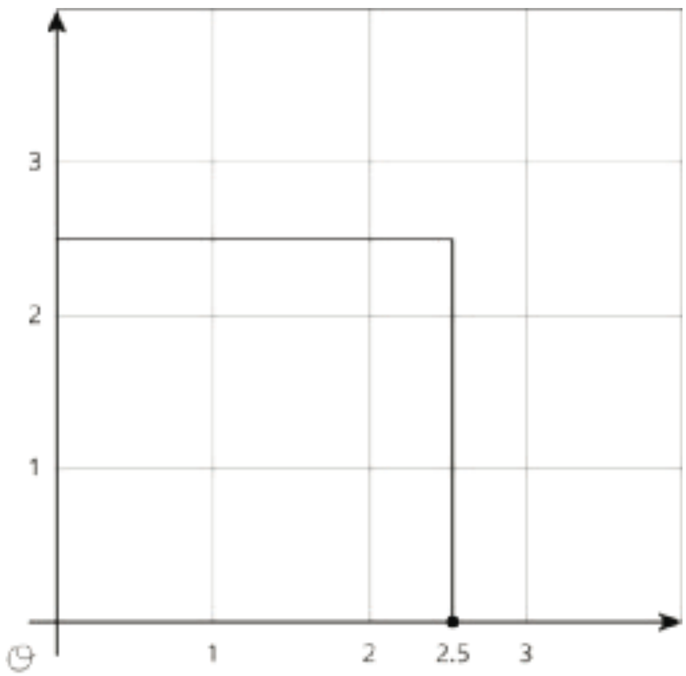


1. Estimate the length of the line segment to the nearest tenth of a unit (each grid square is 1 square unit).

2. Find the exact length of the segment.

4.3: Square Root of 3

Diego said that he thinks that $\sqrt{3} \approx 2.5$.



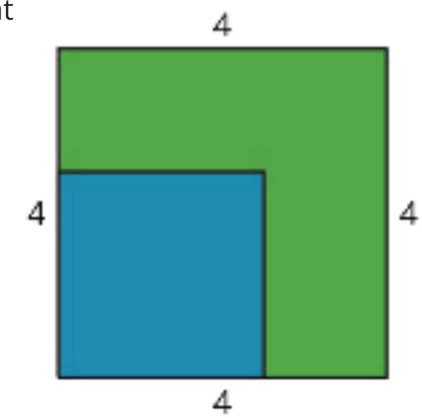
1. Use the square to explain why 2.5 is not a very good approximation for $\sqrt{3}$. Find a point on the number line that is closer to $\sqrt{3}$. Draw a new square on the axes and use it to explain how you know the point you plotted is a good approximation for $\sqrt{3}$.

2. Use the fact that $\sqrt{3}$ is a solution to the equation $x^2 = 3$ to find a decimal approximation of $\sqrt{3}$ whose square is between 2.9 and 3.1.

Are you ready for more?

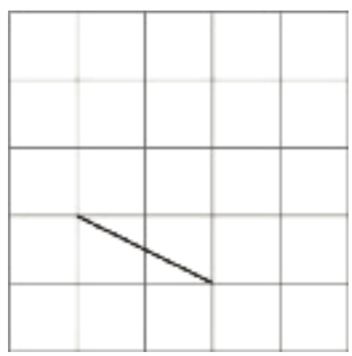
A farmer has a grassy patch of land enclosed by a fence in the shape of a square with a side length of 4 meters. To make it a suitable home for some animals, the farmer would like to carve out a smaller square to be filled with water, as in the figure.

What should the side length of the smaller square be so that half of the area is grass and half is water?

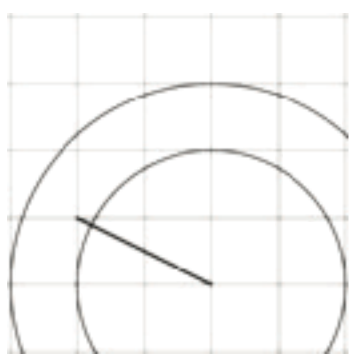


Lesson 4 Summary

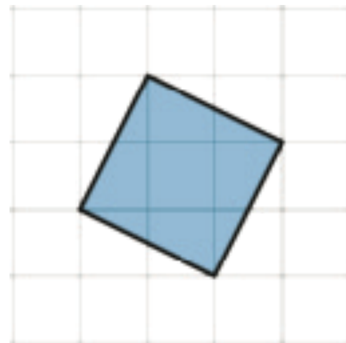
Here is a line segment on a grid. What is the length of this line segment?



By drawing some circles, we can tell that it’s longer than 2 units, but shorter than 3 units.



To find an exact value for the length of the segment, we can build a square on it, using the segment as one of the sides of the square.



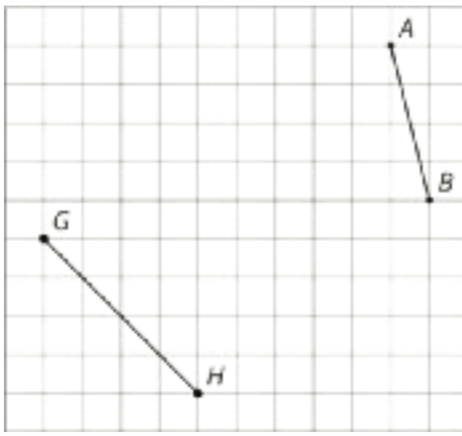
The area of this square is 5 square units. (Can you see why?) That means the exact value of the length of its side is $\sqrt{5}$ units.

Notice that 5 is greater than 4, but less than 9. That means that $\sqrt{5}$ is greater than 2, but less than 3. This makes sense because we already saw that the length of the segment is in between 2 and 3.

With some arithmetic, we can get an even more precise idea of where $\sqrt{5}$ is on the number line. The image with the circles shows that $\sqrt{5}$ is closer to 2 than 3, so let's find the value of 2.1^2 and 2.2^2 and see how close they are to 5. It turns out that $2.1^2 = 4.41$ and $2.2^2 = 4.84$, so we need to try a larger number. If we increase our search by a tenth, we find that $2.3^2 = 5.29$. This means that $\sqrt{5}$ is greater than 2.2, but less than 2.3. If we wanted to keep going, we could try 2.25^2 and eventually narrow the value of $\sqrt{5}$ to the hundredths place. Calculators do this same process to many decimal places, giving an approximation like $\sqrt{5} \approx 2.2360679775$. Even though this is a lot of decimal places, it is still not exact because $\sqrt{5}$ is irrational.

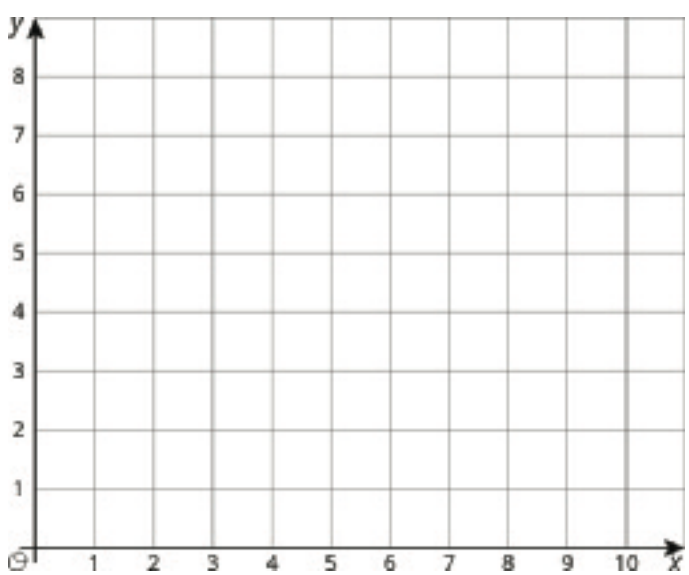
Unit 8 Lesson 4 Cumulative Practice Problems

1. a. Find the exact length of each line segment.



b. Estimate the length of each line segment to the nearest tenth of a unit. Explain your reasoning.

2. Plot each number on the x-axis: $\sqrt{16}$, $\sqrt{35}$, $\sqrt{66}$. Consider using the grid to help.

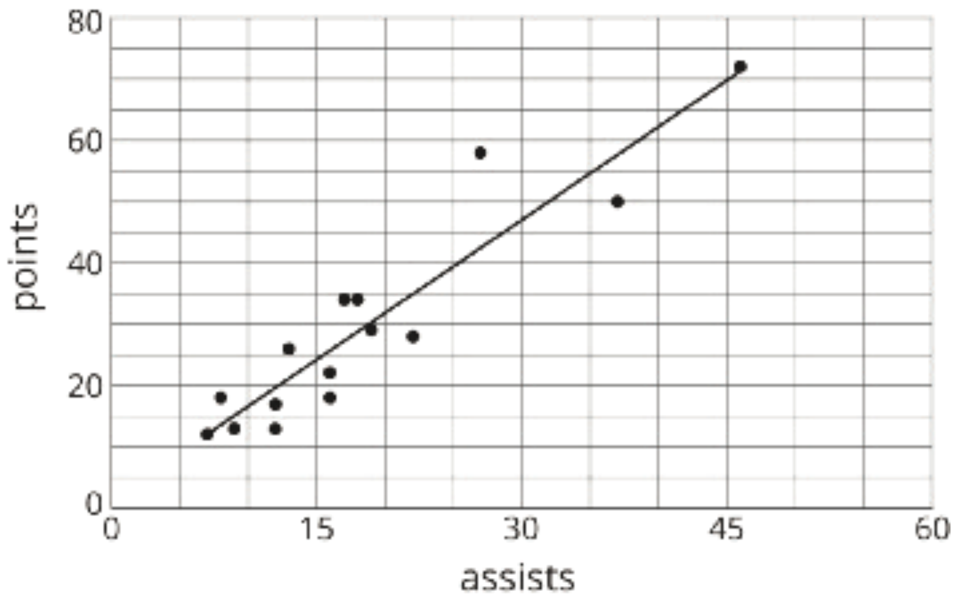


3. Use the fact that $\sqrt{7}$ is a solution to the equation $x^2 = 7$ to find a decimal approximation of $\sqrt{7}$ whose square is between 6.9 and 7.1.

4. Graphite is made up of layers of graphene. Each layer of graphene is about 200 picometers, or 200×10^{-12} meters, thick. How many layers of graphene are there in a 1.6-mm-thick piece of graphite? Express your answer in scientific notation.

(From Unit 7, Lesson 14.)

5. Here is a scatter plot that shows the number of assists and points for a group of hockey players. The model, represented by $y = 1.5x + 1.2$, is graphed with the scatter plot.



a. What does the slope mean in this situation?

b. Based on the model, how many points will a player have if he has 30 assists?

(From Unit 6, Lesson 6.)

6. The points $(12, 23)$ and $(14, 45)$ lie on a line. What is the slope of the line?

(From Unit 3, Lesson 5.)

Lesson 5: Reasoning About Square Roots

Let's approximate square roots.

5.1: True or False: Squared

Decide if each statement is true or false.

$\left(\sqrt{5}\right)^2 = 5$

$\left(\sqrt{10}\right)^2 = 100$

$\left(\sqrt{9}\right)^2 = 3$

$\left(\sqrt{16}\right) = 2^2$

$7 = \left(\sqrt{7}\right)^2$

5.2: Square Root Values

What two whole numbers does each square root lie between? Be prepared to explain your reasoning.

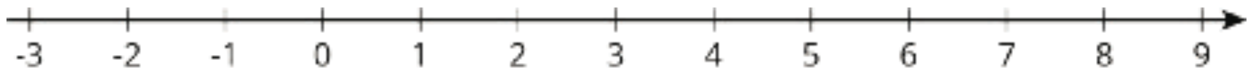
1. $\sqrt{7}$
2. $\sqrt{23}$
3. $\sqrt{50}$
4. $\sqrt{98}$

Are you ready for more?

Can we do any better than "between 3 and 4" for $\sqrt{12}$? Explain a way to figure out if the value is closer to 3.1 or closer to 3.9.

5.3: Solutions on a Number Line

The numbers x, y, and z are positive, and $x^2 = 3$, $y^2 = 16$, and $z^2 = 30$.



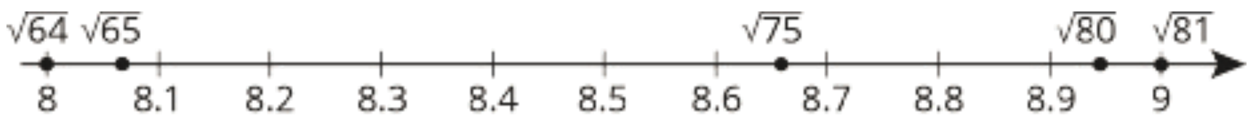
1. Plot x, y, and z on the number line. Be prepared to share your reasoning with the class.

2. Plot -$\sqrt{2}$ on the number line.

Lesson 5 Summary

In general, we can approximate the values of square roots by observing the whole numbers around it, and remembering the relationship between square roots and squares. Here are some examples:

- $\sqrt{65}$ is a little more than 8, because $\sqrt{65}$ is a little more than $\sqrt{64}$ and $\sqrt{64} = 8$.
- $\sqrt{80}$ is a little less than 9, because $\sqrt{80}$ is a little less than $\sqrt{81}$ and $\sqrt{81} = 9$.
- $\sqrt{75}$ is between 8 and 9 (it's 8 point something), because 75 is between 64 and 81.
- $\sqrt{75}$ is approximately 8.67, because $8.67^2 = 75.1689$.



If we want to find a square root between two whole numbers, we can work in the other direction. For example, since $22^2 = 484$ and $23^2 = 529$, then we know that $\sqrt{500}$ (to pick one possibility) is between 22 and 23.

Many calculators have a square root command, which makes it simple to find an approximate value of a square root.

Unit 8 Lesson 5 Cumulative Practice Problems

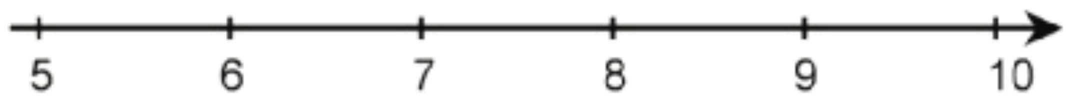
1. a. Explain how you know that $\sqrt{37}$ is a little more than 6.

   b. Explain how you know that $\sqrt{95}$ is a little less than 10.

   c. Explain how you know that $\sqrt{30}$ is between 5 and 6.

2. Plot each number on the number line:

$$6, \sqrt{83}, \sqrt{40}, \sqrt{64}, 7.5$$



3. The equation $x^2 = 25$ has *two* solutions. This is because both $5 \cdot 5 = 25$, and also $-5 \cdot -5 = 25$. So, 5 is a solution, and also -5 is a solution.

   Select **all** the equations that have a solution of -4:

   A. $10 + x = 6$

   B. $10 - x = 6$

   C. $-3x = -12$

   D. $-3x = 12$

   E. $8 = x^2$

   F. $x^2 = 16$

4. Find all the solutions to each equation.

   a. $x^2 = 81$

   b. $x^2 = 100$

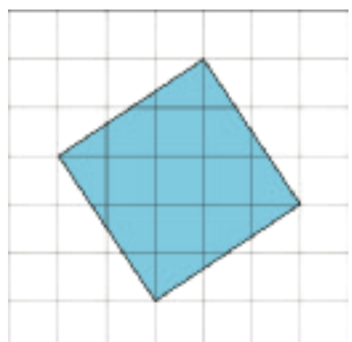
   c. $\sqrt{x} = 12$

5. Select all the irrational numbers in the list.

$$\frac{2}{3}, \frac{-123}{45}, \sqrt{14}, \sqrt{64}, \sqrt{\frac{9}{1}}, -\sqrt{99}, -\sqrt{100}$$

(From Unit 8, Lesson 3.)

6. Each grid square represents 1 square unit. What is the exact side length of the shaded square?



(From Unit 8, Lesson 2.)

7. For each pair of numbers, which of the two numbers is larger? Estimate how many times larger.

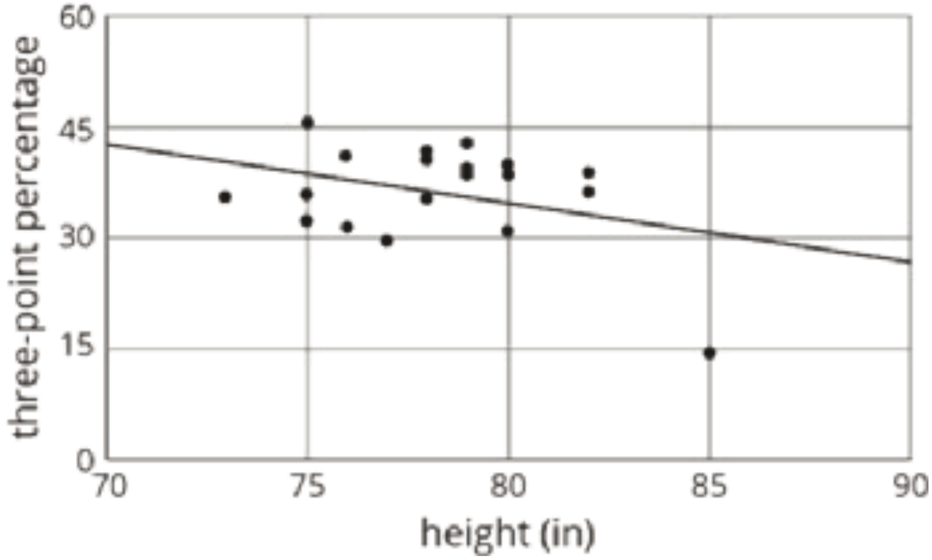
   a. $0.37 \cdot 10^6$ and $700 \cdot 10^4$

   b. $4.87 \cdot 10^4$ and $15 \cdot 10^5$

   c. $500,000$ and $2.3 \cdot 10^8$

(From Unit 7, Lesson 10.)

8. The scatter plot shows the heights (in inches) and three-point percentages for different basketball players last season.



a. Circle any data points that appear to be outliers.

b. Compare any outliers to the values predicted by the model.

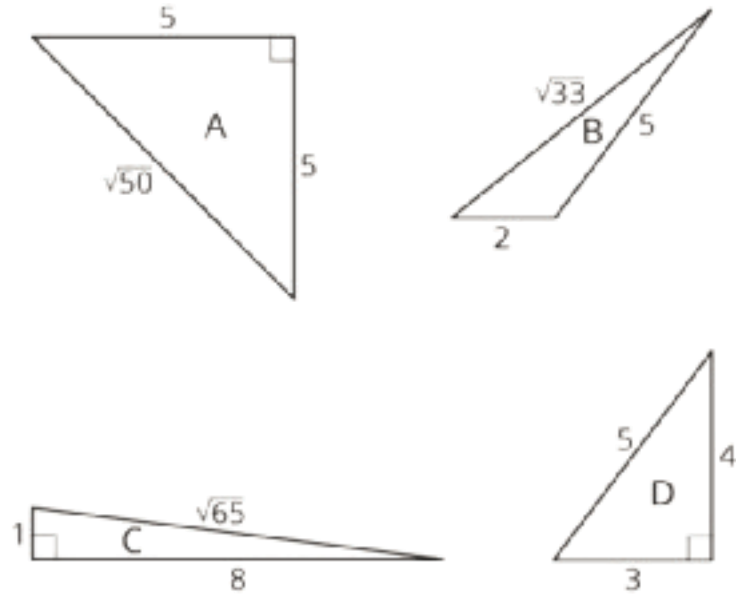
(From Unit 6, Lesson 4.)

Lesson 6: Finding Side Lengths of Triangles

Let's find triangle side lengths.

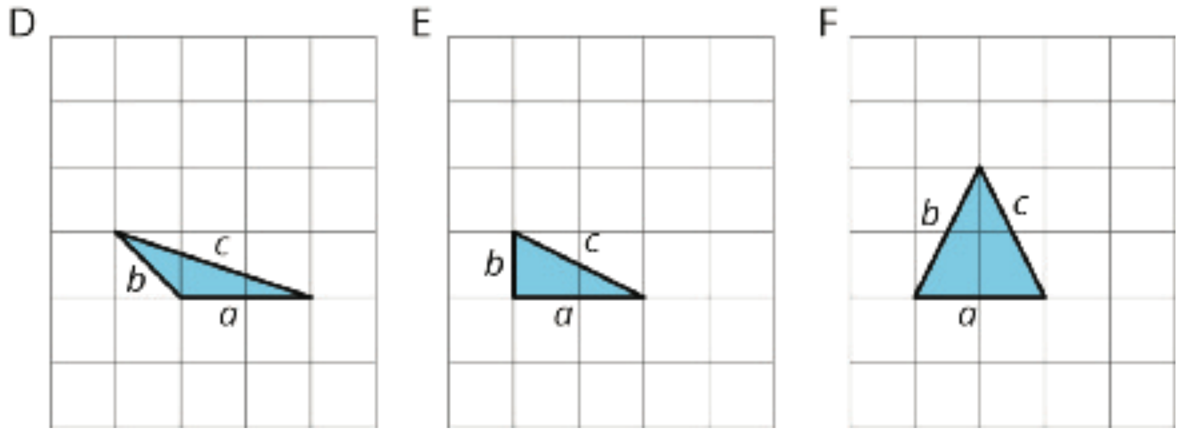
6.1: Which One Doesn't Belong: Triangles

Which triangle doesn't belong?



6.2: A Table of Triangles

1. Complete the tables for these three triangles:

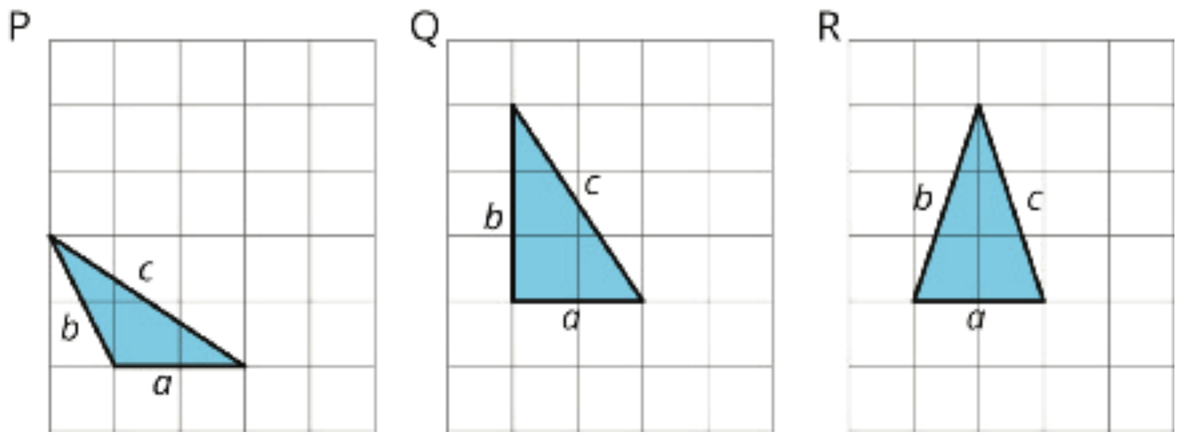


triangle	a	b	c
D			
E			
F			

triangle	a^2	b^2	c^2
D			
E			
F			

2. What do you notice about the values in the table for Triangle E but not for Triangles D and F?

3. Complete the tables for these three more triangles:



triangle	a	b	c
P			
Q			
R			

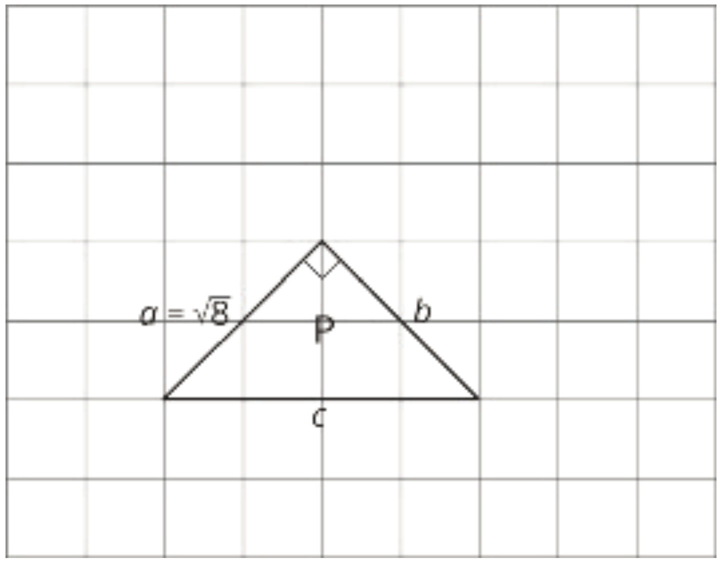
triangle	a^2	b^2	c^2
P			
Q			
R			

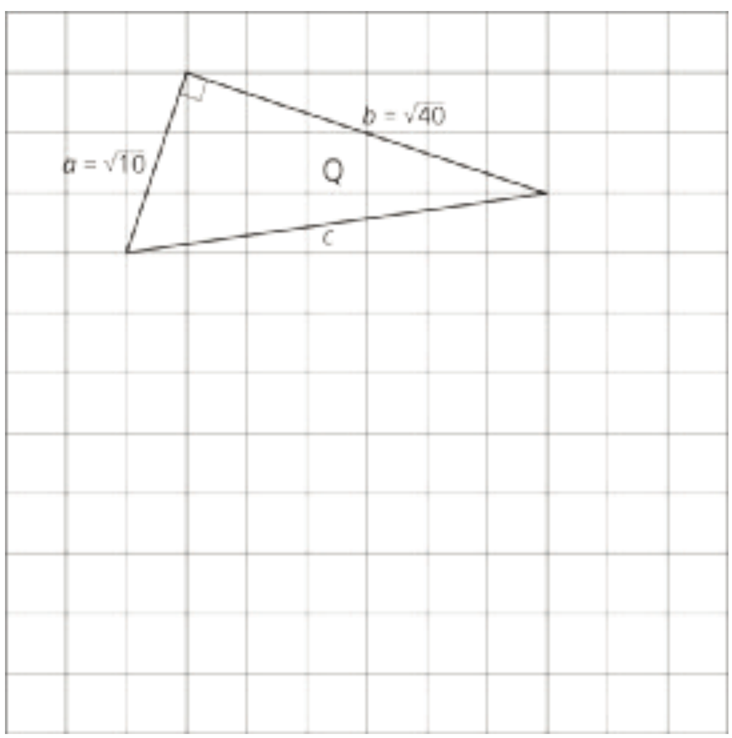
4. What do you notice about the values in the table for Triangle Q but not for Triangles P and R?

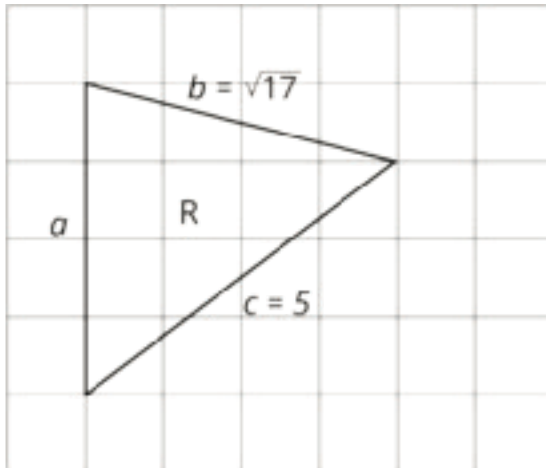
5. What do Triangle E and Triangle Q have in common?

6.3: Meet the Pythagorean Theorem

1. Find the missing side lengths. Be prepared to explain your reasoning.
2. For which triangles does $a^2 + b^2 = c^2$?

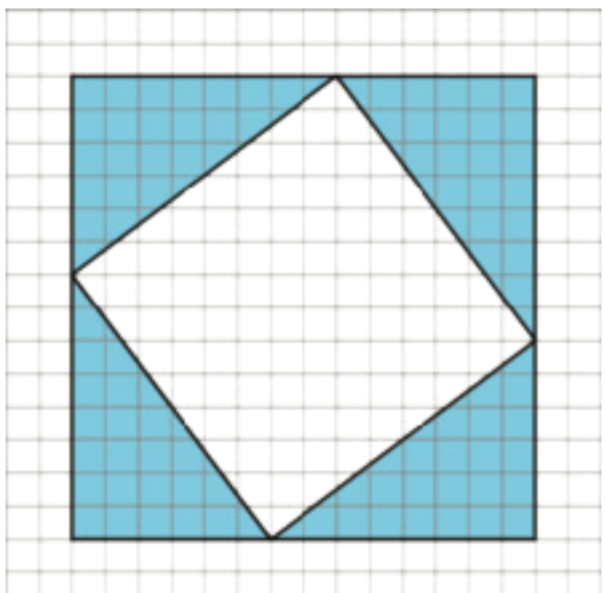






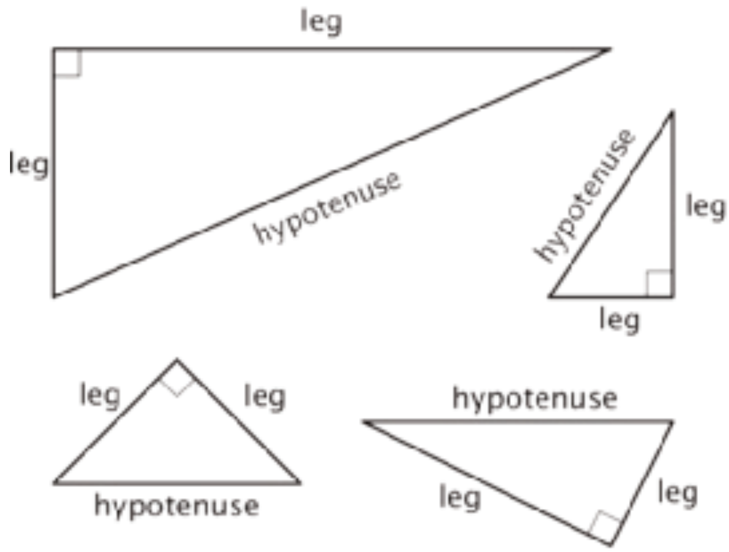
Are you ready for more?

If the four shaded triangles in the figure are congruent right triangles, does the inner quadrilateral have to be a square? Explain how you know.

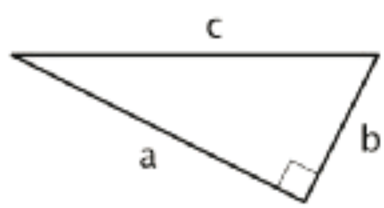


Lesson 6 Summary

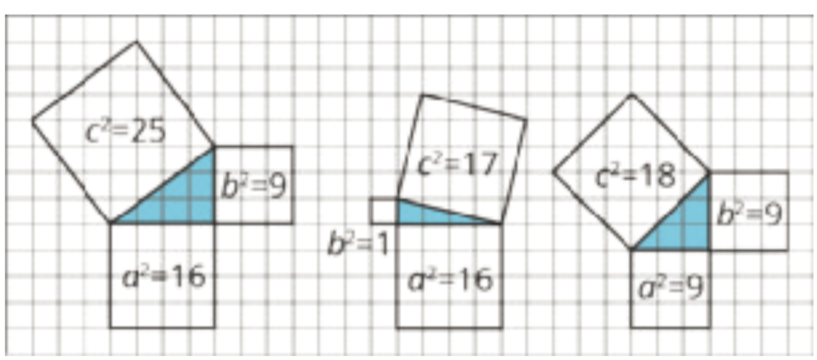
A *right triangle* is a triangle with a right angle. In a right triangle, the side opposite the right angle is called the **hypotenuse**, and the two other sides are called its **legs**. Here are some right triangles with the hypotenuse and legs labeled:



We often use the letters a and b to represent the lengths of the shorter sides of a triangle and c to represent the length of the longest side of a right triangle. If the triangle is a right triangle, then a and b are used to represent the lengths of the legs, and c is used to represent the length of the hypotenuse (since the hypotenuse is always the longest side of a right triangle). For example, in this right triangle, $a = \sqrt{20}$, $b = \sqrt{5}$, and $c = 5$.



Here are some right triangles:

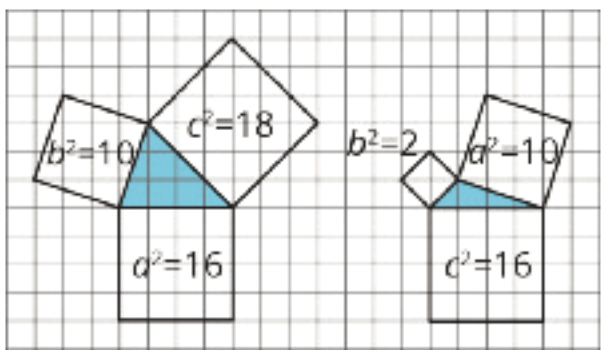


Notice that for these examples of right triangles, the square of the hypotenuse is equal to the sum of the squares of the legs. In the first right triangle in the diagram, $9 + 16 = 25$, in the second, $1 + 16 = 17$, and in the third, $9 + 9 = 18$. Expressed another way, we have

$$a^2 + b^2 = c^2$$

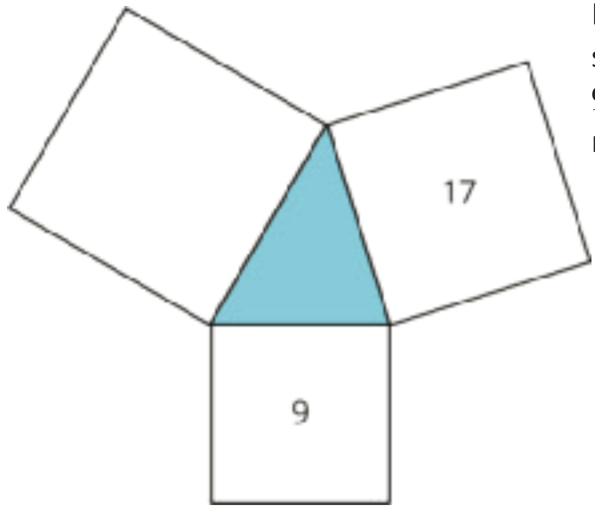
This is a property of all right triangles, not just these examples, and is often known as the **Pythagorean Theorem**. The name comes from a mathematician named Pythagoras who lived in ancient Greece around 2,500 BCE, but this property of right triangles was also discovered independently by mathematicians in other ancient cultures including Babylon, India, and China. In China, a name for the same relationship is the Shang Gao Theorem. In future lessons, you will learn some ways to explain why the Pythagorean Theorem is true for *any* right triangle.

It is important to note that this relationship does not hold for *all* triangles. Here are some triangles that are not right triangles, and notice that the lengths of their sides do not have the special relationship $a^2 + b^2 = c^2$. That is, $16 + 10$ does not equal 18, and $2 + 10$ does not equal 16.



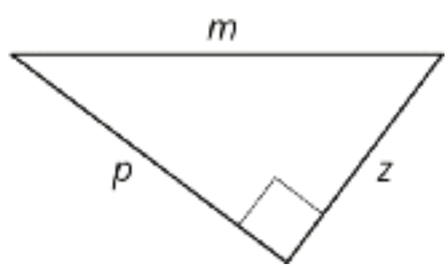
Unit 8 Lesson 6 Cumulative Practice Problems

1. Here is a diagram of an acute triangle and three squares.



Priya says the area of the large unmarked square is 26 square units because $9 + 17 = 26$. Do you agree? Explain your reasoning.

2. m, p, and z represent the lengths of the three sides of this right triangle.



Select **all** the equations that represent the relationship between m, p, and z.

A. $m^2 + p^2 = z^2$

B. $m^2 = p^2 + z^2$

C. $m^2 = z^2 + p^2$

D. $p^2 + m^2 = z^2$

E. $z^2 + p^2 = m^2$

F. $p^2 + z^2 = m^2$

3. The lengths of the three sides are given for several right triangles. For each, write an equation that expresses the relationship between the lengths of the three sides.

   a. 10, 6, 8

   b. $\sqrt{5}, \sqrt{3}, \sqrt{8}$

   c. 5, $\sqrt{5}, \sqrt{30}$

   d. 1, $\sqrt{37}$, 6

   e. 3, $\sqrt{2}, \sqrt{7}$

4. Order the following expressions from least to greatest.

   $25 \div 10$ $\quad$ $250{,}000 \div 1{,}000$ $\quad$ $2.5 \div 1{,}000$ $\quad$ $0.025 \div 1$

   (From Unit 4, Lesson 1.)

5. Which is the best explanation for why $-\sqrt{10}$ is irrational?

A. $-\sqrt{10}$ is irrational because it is not rational.

B. $-\sqrt{10}$ is irrational because it is less than zero.

C. $-\sqrt{10}$ is irrational because it is not a whole number.

D. $-\sqrt{10}$ is irrational because if I put $-\sqrt{10}$ into a calculator, I get -3.16227766, which does not make a repeating pattern.

(From Unit 8, Lesson 3.)

6. A teacher tells her students she is just over 1 and $\frac{1}{2}$ billion seconds old.

a. Write her age in seconds using scientific notation.

b. What is a more reasonable unit of measurement for this situation?

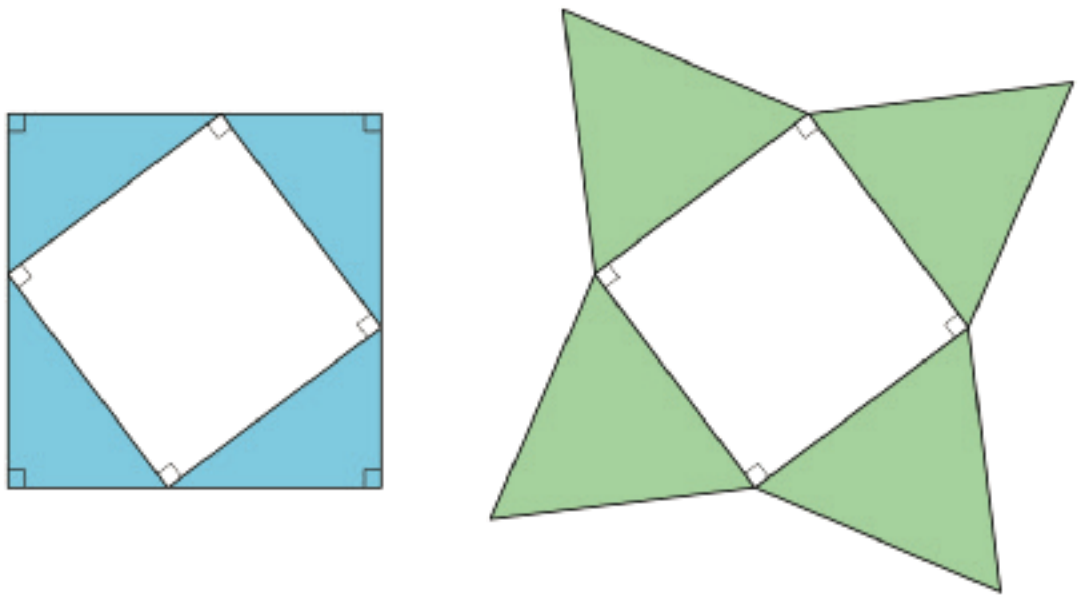
c. How old is she when you use a more reasonable unit of measurement?

(From Unit 7, Lesson 15.)

Lesson 7: A Proof of the Pythagorean Theorem

Let’s prove the Pythagorean Theorem.

7.1: Notice and Wonder: A Square and Four Triangles

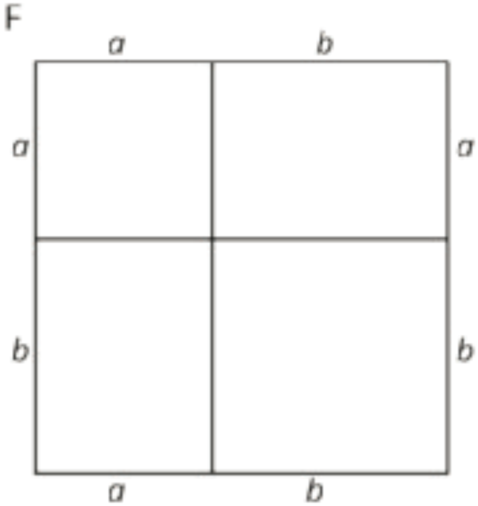


What do you notice? What do you wonder?

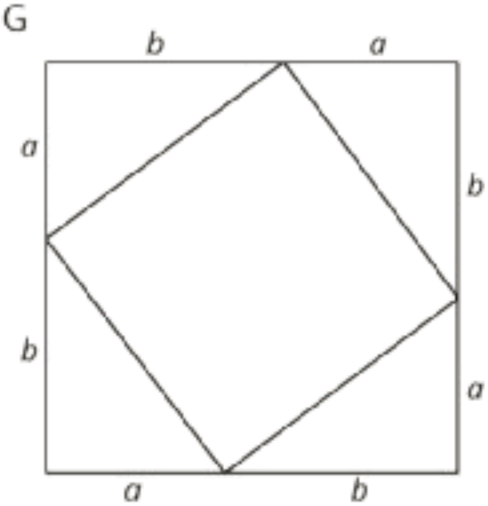
7.2: Adding Up Areas

Both figures shown here are squares with a side length of $a + b$. Notice that the first figure is divided into two squares and two rectangles. The second figure is divided into a square and four right triangles with legs of lengths a and b. Let's call the hypotenuse of these triangles c.

F



G



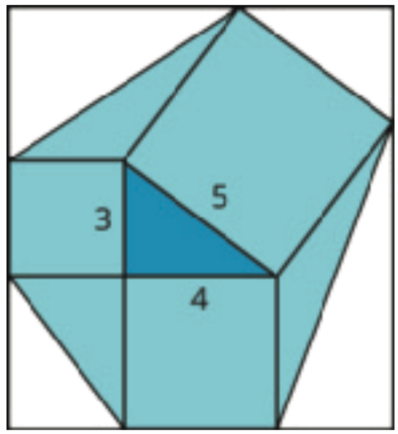
1. What is the total area of each figure?

2. Find the area of each of the 9 smaller regions shown the figures and label them.

3. Add up the area of the four regions in Figure F and set this expression equal to the sum of the areas of the five regions in Figure G. If you rewrite this equation using as few terms as possible, what do you have?

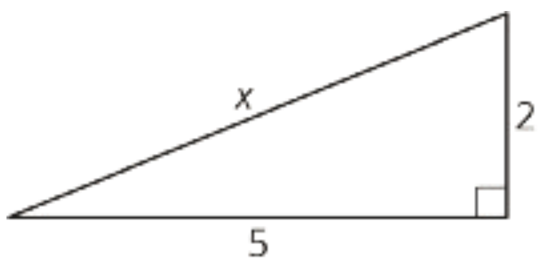
Are you ready for more?

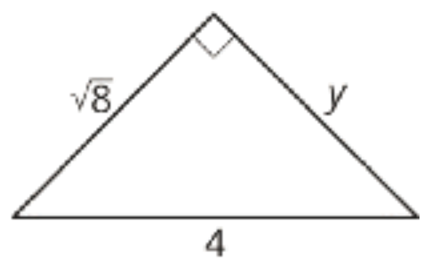
Take a 3-4-5 right triangle, add on the squares of the side lengths, and form a hexagon by connecting vertices of the squares as in the image. What is the area of this hexagon?



7.3: Let's Take it for a Spin

Find the unknown side lengths in these right triangles.





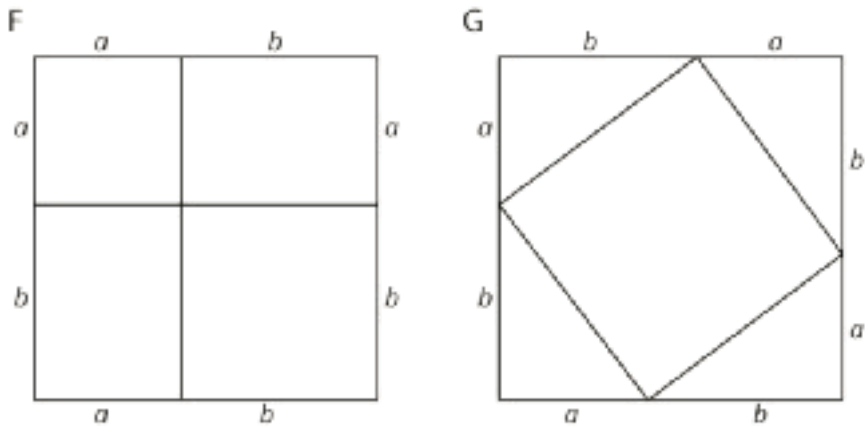
7.4: A Transformational Proof

Your teacher will give your group a sheet with 4 figures and a set of 5 cut out shapes labeled D, E, F, G, and H.

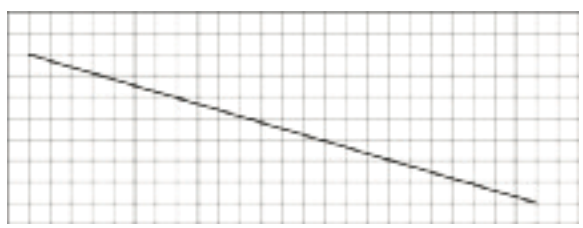
1. Arrange the 5 cut out shapes to fit inside Figure 1. Check to see that the pieces also fit in the two smaller squares in Figure 4.
2. Explain how you can transform the pieces arranged in Figure 1 to make an exact copy of Figure 2.
3. Explain how you can transform the pieces arranged in Figure 2 to make an exact copy of Figure 3.
4. Check to see that Figure 3 is congruent to the large square in Figure 4.
5. If the right triangle in Figure 4 has legs a and b and hypotenuse c, what have you just demonstrated to be true?

Lesson 7 Summary

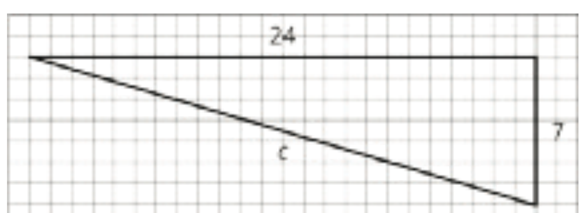
The figures shown here can be used to see why the Pythagorean Theorem is true. Both large squares have the same area, but they are broken up in different ways. (Can you see where the triangles in Square G are located in Square F? What does that mean about the smaller squares in F and H?) When the sum of the four areas in Square F are set equal to the sum of the 5 areas in Square G, the result is $a^2 + b^2 = c^2$, where c is the hypotenuse of the triangles in Square G and also the side length of the square in the middle. Give it a try!



This is true for any right triangle. If the legs are a and b and the hypotenuse is c, then $a^2 + b^2 = c^2$. This property can be used any time we can make a right triangle. For example, to find the length of this line segment:



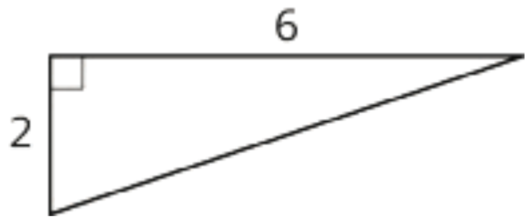
The grid can be used to create a right triangle, where the line segment is the hypotenuse and the legs measure 24 units and 7 units:

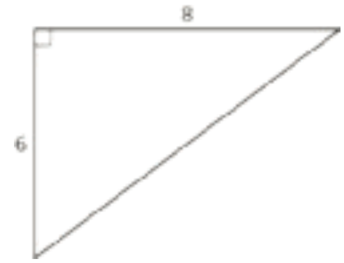


Since this is a right triangle, $24^2 + 7^2 = c^2$. The solution to this equation (and the length of the line segment) is $c = 25$.

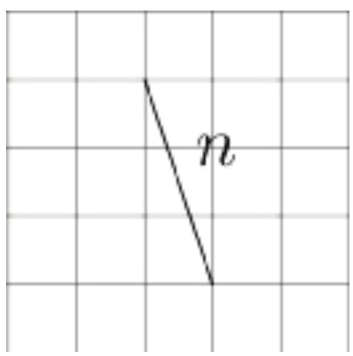
Unit 8 Lesson 7 Cumulative Practice Problems

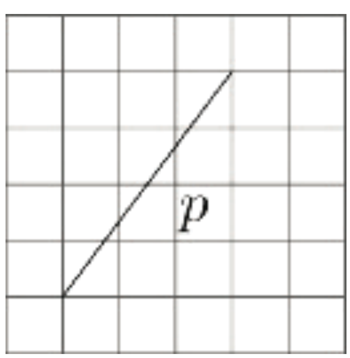
1. a. Find the lengths of the unlabeled sides.



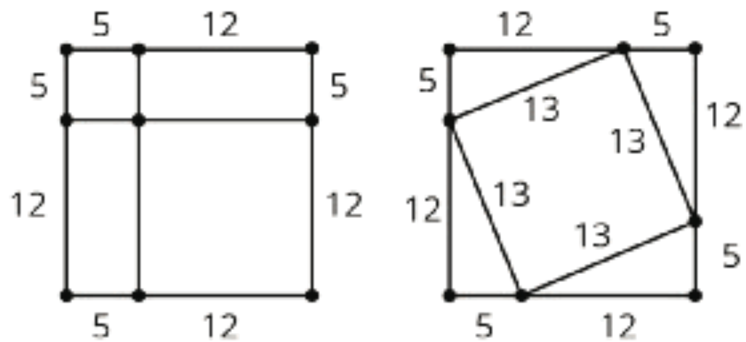


b. One segment is n units long and the other is p units long. Find the value of n and p. (Each small grid square is 1 square unit.)





2. Use the areas of the two identical squares to explain why $5^2 + 12^2 = 13^2$ without doing any calculations.



3. Each number is between which two consecutive integers?

a. $\sqrt{10}$

b. $\sqrt{54}$

c. $\sqrt{18}$

d. $\sqrt{99}$

e. $\sqrt{41}$

(From Unit 8, Lesson 5.)

4. a. Give an example of a rational number, and explain how you know it is rational.

   b. Give three examples of irrational numbers.

(From Unit 8, Lesson 3.)

5. Write each expression as a single power of 10.

   a. $10^5 \cdot 10^0$

   b. $\frac{10^9}{10^0}$

(From Unit 7, Lesson 4.)

6. Andre is ordering ribbon to make decorations for a school event. He needs a total of exactly 50.25 meters of blue and green ribbon. Andre needs 50% more blue ribbon than green ribbon for the basic design, plus an extra 6.5 meters of blue ribbon for accents. How much of each color of ribbon does Andre need to order?

(From Unit 4, Lesson 15.)

Lesson 8: Finding Unknown Side Lengths

Let's find missing side lengths of right triangles.

8.1: Which One Doesn't Belong: Equations

Which one doesn't belong?

$3^2 + b^2 = 5^2$

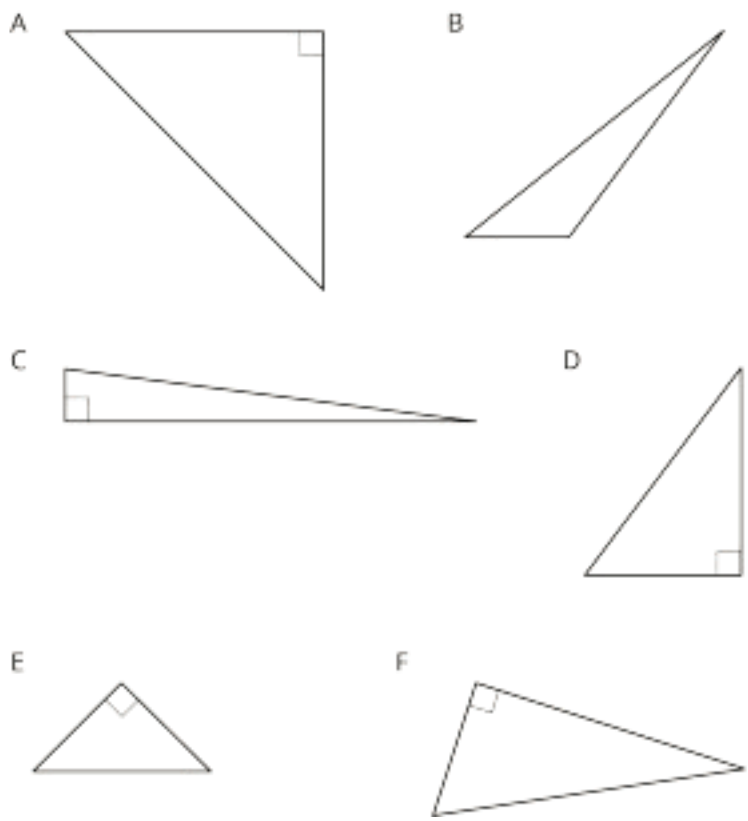
$b^2 = 5^2 - 3^2$

$3^2 + 5^2 = b^2$

$3^2 + 4^2 = 5^2$

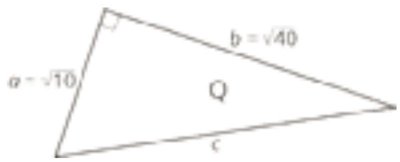
8.2: Which One Is the Hypotenuse?

Label all the hypotenuses with c.

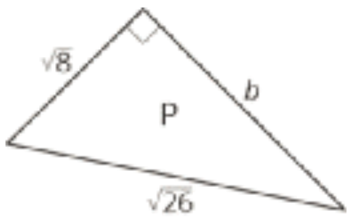


8.3: Find the Missing Side Lengths

1. Find c.



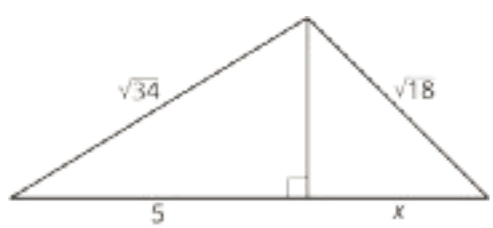
2. Find b.



3. A right triangle has sides of length 2.4 cm and 6.5 cm. What is the length of the hypotenuse?

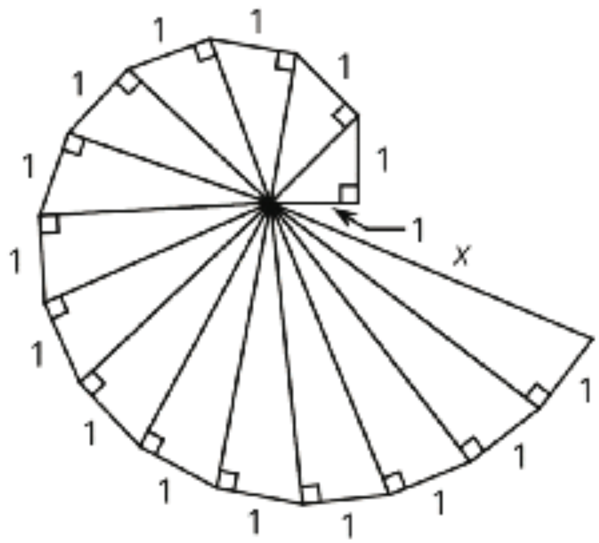
4. A right triangle has a side of length $\frac{1}{4}$ and a hypotenuse of length $\frac{1}{3}$. What is the length of the other side?

5. Find the value of x in the figure.



Are you ready for more?

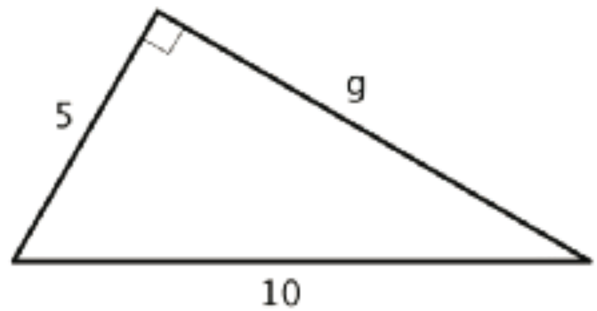
The spiral in the figure is made by starting with a right triangle with both legs measuring one unit each. Then a second right triangle is built with one leg measuring one unit, and the other leg being the hypotenuse of the first triangle. A third right triangle is built on the second triangle's hypotenuse, again with the other leg measuring one unit, and so on.



Find the length, x, of the hypotenuse of the last triangle constructed in the figure.

Lesson 8 Summary

There are many examples where the lengths of two legs of a right triangle are known and can be used to find the length of the hypotenuse with the Pythagorean Theorem. The Pythagorean Theorem can also be used if the length of the hypotenuse and one leg is known, and we want to find the length of the other leg. Here is a right triangle, where one leg has a length of 5 units, the hypotenuse has a length of 10 units, and the length of the other leg is represented by g.



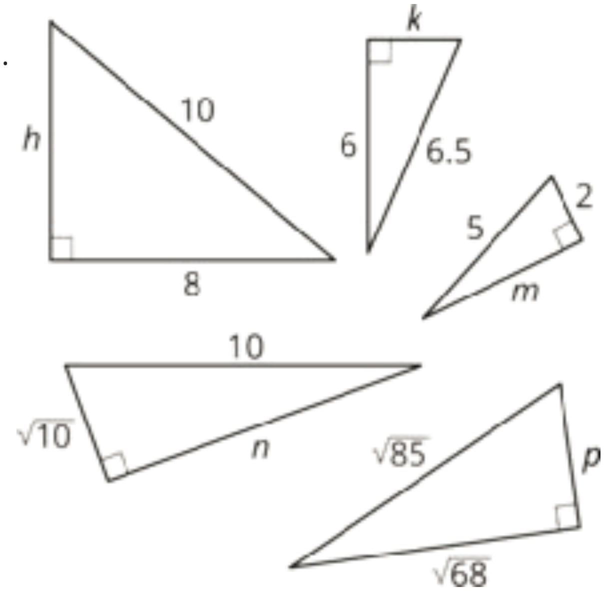
Start with $a^2 + b^2 = c^2$, make substitutions, and solve for the unknown value. Remember that c represents the hypotenuse: the side opposite the right angle. For this triangle, the hypotenuse is 10.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 5^2 + g^2 &= 10^2 \\ g^2 &= 10^2 - 5^2 \\ g^2 &= 100 - 25 \\ g^2 &= 75 \\ g &= \sqrt{75} \end{aligned}$$

Use estimation strategies to know that the length of the other leg is between 8 and 9 units, since 75 is between 64 and 81. A calculator with a square root function gives $\sqrt{75} \approx 8.66$.

Unit 8 Lesson 8 Cumulative Practice Problems

1. Find the exact value of each variable that represents a side length in a right triangle.

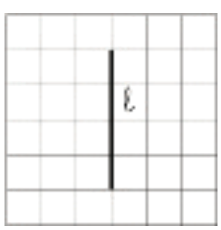


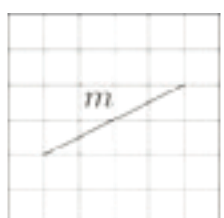
2. A right triangle has side lengths of a, b, and c units. The longest side has a length of c units. Complete each equation to show three relations among a, b, and c.

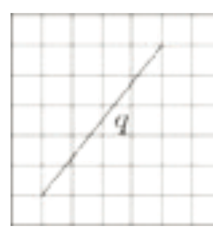
   - $c^2 =$
   - $a^2 =$
   - $b^2 =$

   (From Unit 8, Lesson 7.)

3. What is the exact length of each line segment? Explain or show your reasoning. (Each grid square represents 1 square unit.)

   a. 

   b. 

   c. 

   (From Unit 8, Lesson 7.)

4. In 2015, there were roughly 1×10^6 high school football players and 2×10^3 professional football players in the United States. About how many times more high school football players are there? Explain how you know.

(From Unit 7, Lesson 15.)

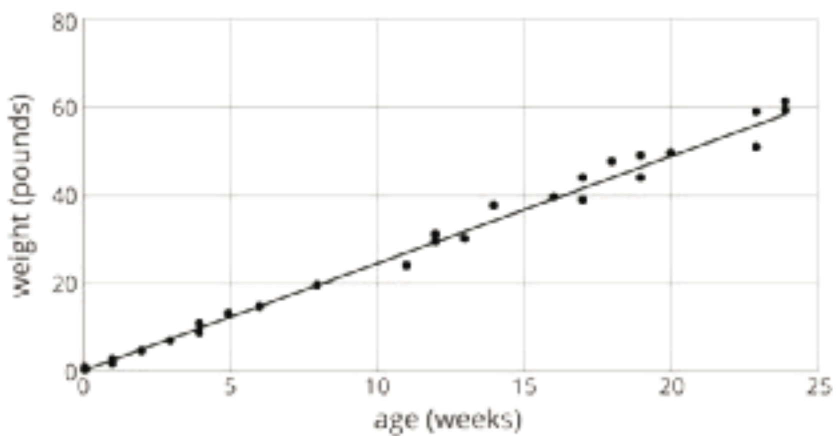
5. Evaluate:

   a. $\left(\frac{1}{2}\right)^3$

   b. $\left(\frac{1}{2}\right)^{-3}$

(From Unit 7, Lesson 6.)

6. Here is a scatter plot of weight vs. age for different Dobermans. The model, represented by $y = 2.45x + 1.22$, is graphed with the scatter plot. Here, x represents age in weeks, and y represents weight in pounds.



   a. What does the slope mean in this situation?

   b. Based on this model, how heavy would you expect a newborn Doberman to be?

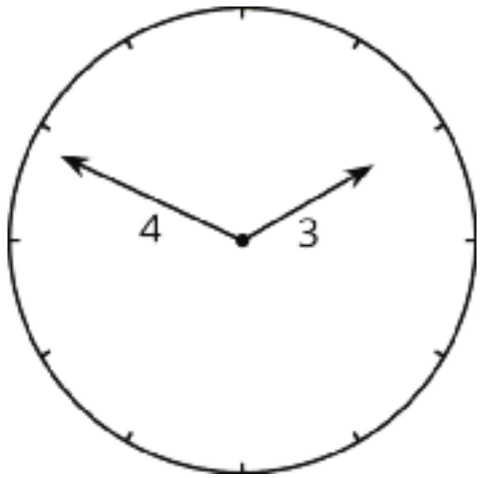
(From Unit 6, Lesson 6.)

Lesson 9: The Converse

Let's figure out if a triangle is a right triangle.

9.1: The Hands of a Clock

Consider the tips of the hands of an analog clock that has an hour hand that is 3 centimeters long and a minute hand that is 4 centimeters long.



Over the course of a day:

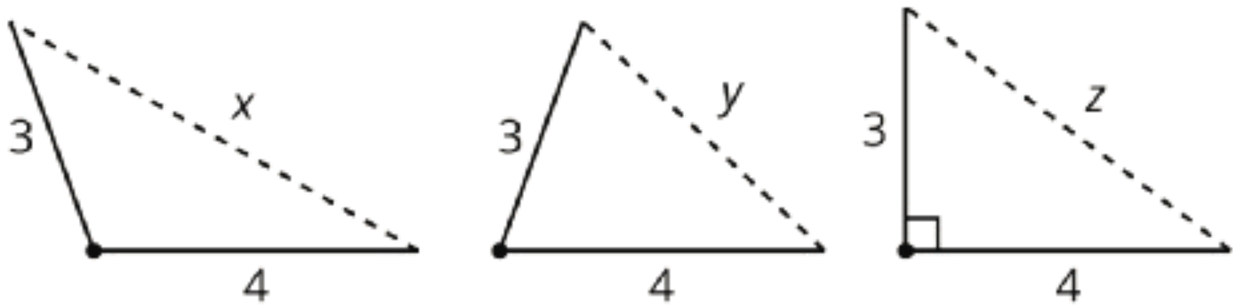
1. What is the farthest apart the two tips get?

2. What is the closest the two tips get?

3. Are the two tips ever exactly five centimeters apart?

9.2: Proving the Converse

Here are three triangles with two side lengths measuring 3 and 4 units, and the third side of unknown length.



Sort the following six numbers from smallest to largest. Put an equal sign between any you know to be equal. Be ready to explain your reasoning.

$$1 \qquad 5 \qquad 7 \qquad x \qquad y \qquad z$$

Are you ready for more?

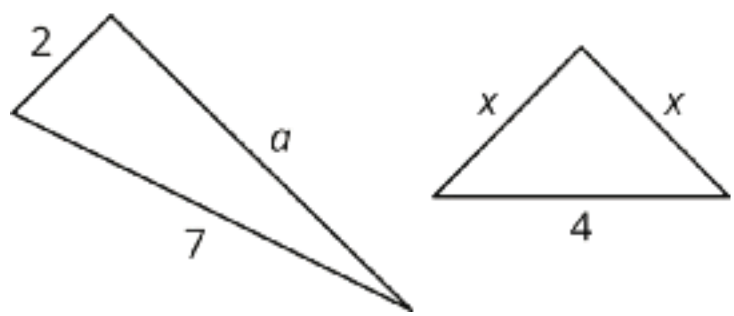
A related argument also lets us distinguish acute from obtuse triangles using only their side lengths.

Decide if triangles with the following side lengths are acute, right, or obtuse. In right or obtuse triangles, identify which side length is opposite the right or obtuse angle.

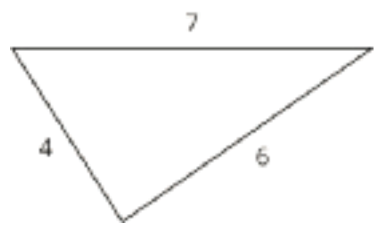
- $x = 15$, $y = 20$, $z = 8$
- $x = 8$, $y = 15$, $z = 13$
- $x = 17$, $y = 8$, $z = 15$

9.3: Calculating Legs of Right Triangles

1. Given the information provided for the right triangles shown here, find the unknown leg lengths to the nearest tenth.

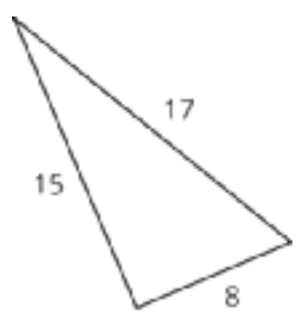


2. The triangle shown here is not a right triangle. What are two different ways you change *one* of the values so it would be a right triangle? Sketch these new right triangles, and clearly label the right angle.



Lesson 9 Summary

What if it isn't clear whether a triangle is a right triangle or not? Here is a triangle:



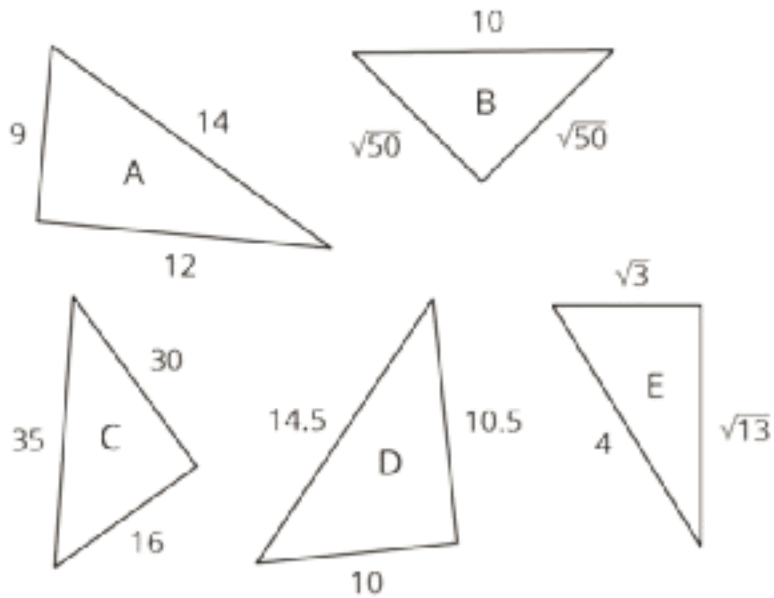
Is it a right triangle? It's hard to tell just by looking, and it may be that the sides aren't drawn to scale.

If we have a triangle with side lengths a, b, and c, with c being the longest of the three, then the *converse* of the Pythagorean Theorem tells us that any time we have $a^2 + b^2 = c^2$, we *must* have a right triangle. Since $8^2 + 15^2 = 64 + 225 = 289 = 17^2$, any triangle with side lengths 8, 15, and 17 *must* be a right triangle.

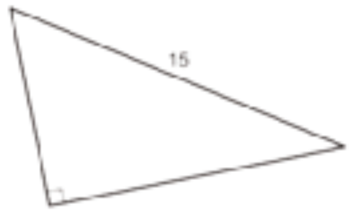
Together, the Pythagorean Theorem and its converse provide a one-step test for checking to see if a triangle is a right triangle just using its side lengths. If $a^2 + b^2 = c^2$, it is a right triangle. If $a^2 + b^2 \neq c^2$, it is not a right triangle.

Unit 8 Lesson 9 Cumulative Practice Problems

1. Which of these triangles are definitely right triangles? Explain how you know. (Note that not all triangles are drawn to scale.)



2. A right triangle has a hypotenuse of 15 cm. What are possible lengths for the two legs of the triangle? Explain your reasoning.



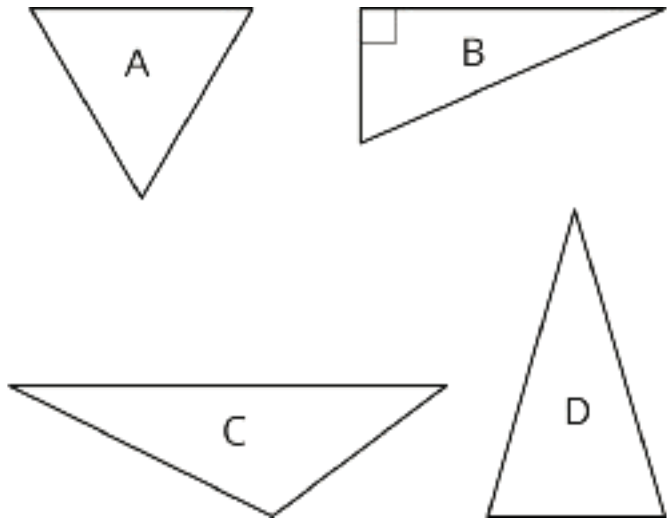
3. In each part, a and b represent the length of a leg of a right triangle, and c represents the length of its hypotenuse. Find the missing length, given the other two lengths.

    a. $a = 12, b = 5, c = ?$

    b. $a = ?, b = 21, c = 29$

(From Unit 8, Lesson 8.)

4. For which triangle does the Pythagorean Theorem express the relationship between the lengths of its three sides?



(From Unit 8, Lesson 6.)

5. Andre makes a trip to Mexico. He exchanges some dollars for pesos at a rate of 20 pesos per dollar. While in Mexico, he spends 9000 pesos. When he returns, he exchanges his pesos for dollars (still at 20 pesos per dollar). He gets back $\frac{1}{10}$ the amount he started with. Find how many dollars Andre exchanged for pesos and explain your reasoning. If you get stuck, try writing an equation representing Andre's trip using a variable for the number of dollars he exchanged.

(From Unit 4, Lesson 5.)

Lesson 10: Applications of the Pythagorean Theorem

Let's explore some applications of the Pythagorean Theorem.

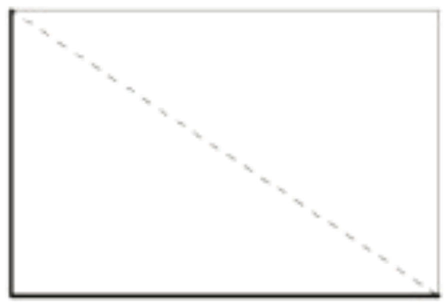
10.1: Closest Estimate: Square Roots

Which estimate is closest to the actual value of the expression? Explain your reasoning.

1. $\sqrt{24}$
    - 4
    - 4.5
    - 5
2. $\sqrt{7}$
    - 2
    - 2.5
    - 3
3. $\sqrt{42}$
    - 6
    - 6.5
    - 7
4. $\sqrt{10} + \sqrt{97}$
    - 13
    - 13.25
    - 13.5

10.2: Cutting Corners

Mai and Tyler were standing at one corner of a large rectangular field and decided to race to the opposite corner. Since Mai had a bike and Tyler did not, they thought it would be a fairer race if Mai rode along the sidewalk that surrounds the field while Tyler ran the shorter distance directly across the field. The field is 100 meters long and 80 meters wide. Tyler can run at around 5 meters per second, and Mai can ride her bike at around 7.5 meters per second.



1. Before making any calculations, who do you think will win? By how much? Explain your thinking.

2. Who wins? Show your reasoning.

Are you ready for more?

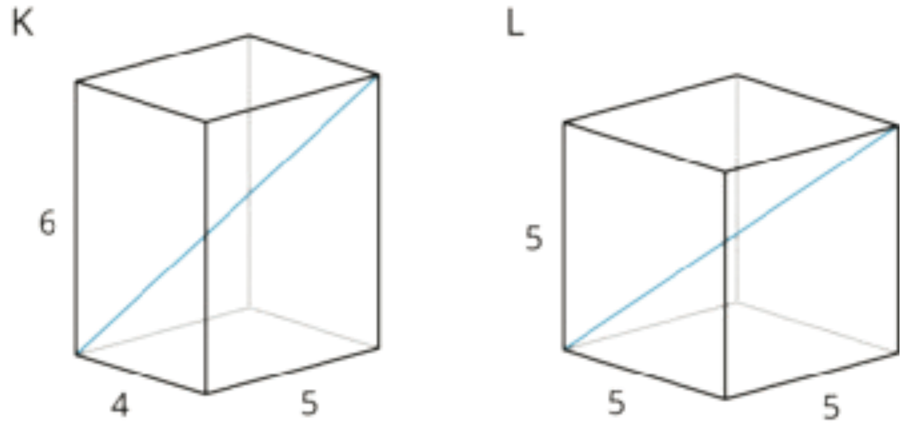
A calculator may be necessary to answer the following questions. Round answers to the nearest hundredth.

1. If you could give the loser of the race a head start, how much time would they need in order for both people to arrive at the same time?

2. If you could make the winner go slower, how slow would they need to go in order for both people to arrive at the same time?

10.3: Internal Dimensions

Here are two rectangular prisms:



1. Which figure do you think has the longer diagonal? Note that the figures are not drawn to scale.

2. Calculate the lengths of both diagonals. Which one is actually longer?

Lesson 10 Summary

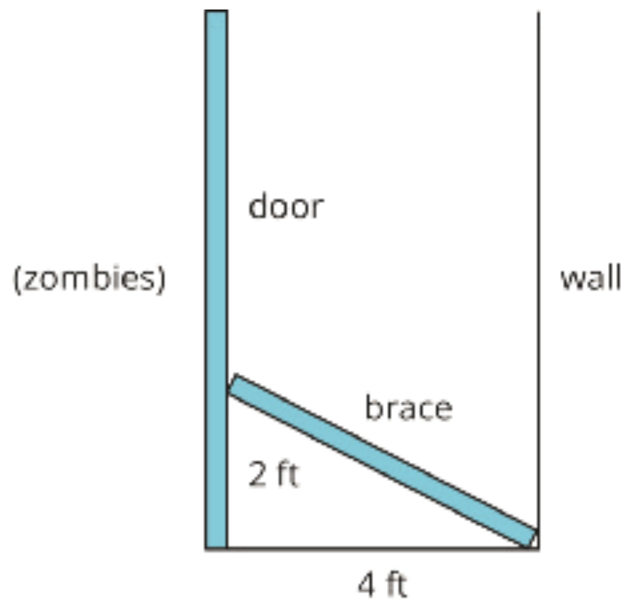
The Pythagorean Theorem can be used to solve any problem that can be modeled with a right triangle where the lengths of two sides are known and the length of the other side needs to be found. For example, let's say a cable is being placed on level ground to support a tower. It's a 17-foot cable, and the cable should be connected 15 feet up the tower. How far away from the bottom of the tower should the other end of the cable connect to the ground?

It is often very helpful to draw a diagram of a situation, such as the one shown here:

It's assumed that the tower makes a right angle with the ground. Since this is a right triangle, the relationship between its sides is $a^2 + b^2 = c^2$, where c represents the length of the hypotenuse and a and b represent the lengths of the other two sides. The hypotenuse is the side opposite the right angle. Making substitutions gives $a^2 + 15^2 = 17^2$. Solving this for a gives $a = 8$. So, the other end of the cable should connect to the ground 8 feet away from the bottom of the tower.

Unit 8 Lesson 10 Cumulative Practice Problems

1. A man is trying to zombie-proof his house. He wants to cut a length of wood that will brace a door against a wall. The wall is 4 feet away from the door, and he wants the brace to rest 2 feet up the door. About how long should he cut the brace?



2. At a restaurant, a trash can's opening is rectangular and measures 7 inches by 9 inches. The restaurant serves food on trays that measure 12 inches by 16 inches. Jada says it is impossible for the tray to accidentally fall through the trash can opening because the shortest side of the tray is longer than either edge of the opening. Do you agree or disagree with Jada's explanation? Explain your reasoning.

3. Select **all** the sets that are the three side lengths of right triangles.

A. 8, 7, 15

B. 4, 10, $\sqrt{84}$

C. $\sqrt{8}$, 11, $\sqrt{129}$

D. $\sqrt{1}$, 2, $\sqrt{3}$

(From Unit 8, Lesson 9.)

4. For each pair of numbers, which of the two numbers is larger? How many times larger?

a. $12 \cdot 10^9$ and $4 \cdot 10^9$

b. $1.5 \cdot 10^{12}$ and $3 \cdot 10^{12}$

c. $20 \cdot 10^4$ and $6 \cdot 10^5$

(From Unit 7, Lesson 10.)

5. A line contains the point $(3, 5)$. If the line has negative slope, which of these points could also be on the line?

   A. $(2, 0)$

   B. $(4, 7)$

   C. $(5, 4)$

   D. $(6, 5)$

   (From Unit 3, Lesson 10.)

6. Noah and Han are preparing for a jump rope contest. Noah can jump 40 times in 0.5 minutes. Han can jump y times in x minutes, where $y = 78x$. If they both jump for 2 minutes, who jumps more times? How many more?

   (From Unit 3, Lesson 4.)

Lesson 11: Finding Distances in the Coordinate Plane

Let's find distances in the coordinate plane.

11.1: Closest Distance

1. Order the following pairs of coordinates from closest to farthest apart. Be prepared to explain your reasoning.
   a. $(2, 4)$ and $(2, 10)$

   b. $(-3, 6)$ and $(5, 6)$

   c. $(-12, -12)$ and $(-12, -1)$

   d. $(7, 0)$ and $(7, -9)$

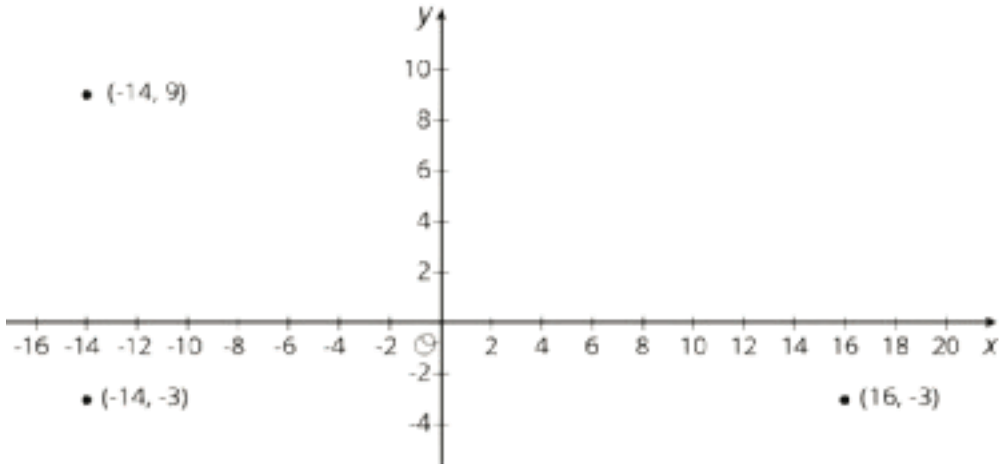
   e. $(1, -10)$ and $(-4, -10)$

2. Name another pair of coordinates that would be closer together than the first pair on your list.

3. Name another pair of coordinates that would be farther apart than the last pair on your list.

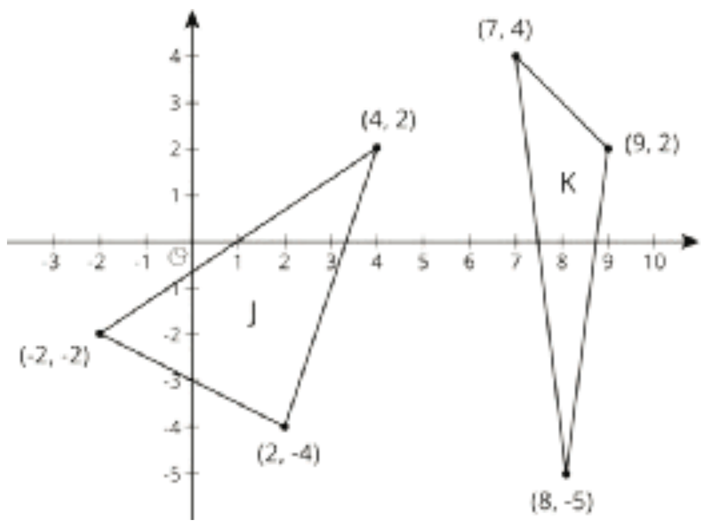
11.2: How Far Apart?

Find the distances between the three points shown.



11.3: Perimeters with Pythagoras

1. Which figure do you think has the longer perimeter?



2. Select one figure and calculate its perimeter. Your partner will calculate the perimeter of the other. Were you correct about which figure had the longer perimeter?

Are you ready for more?

Quadrilateral $ABCD$ has vertices at $A = (-5, 1)$, $B = (-4, 4)$, $C = (2, 2)$, and $D = (1, -1)$.

1. Use the Pythagorean Theorem to find the lengths of sides AB, BC, CD, and AD.

2. Use the Pythagorean Theorem to find the lengths of the two diagonals, AC and BD.

3. Explain why quadrilateral $ABCD$ is a rectangle.

11.4: Finding the Right Distance

Have each person in your group select one of the sets of coordinate pairs shown here. Then calculate the length of the line segment between those two coordinates. Once the values are calculated, have each person in the group briefly share how they did their calculations.

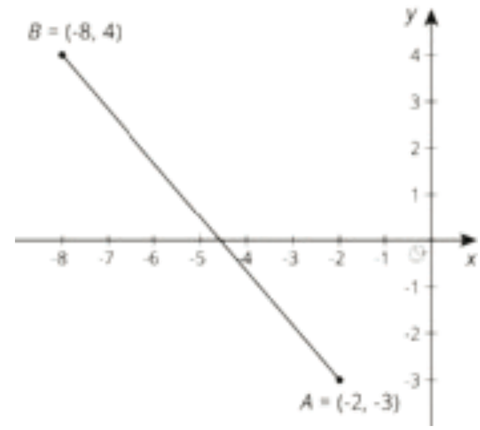
- $(-3, 1)$ and $(5, 7)$
- $(-1, -6)$ and $(5, 2)$
- $(-1, 2)$ and $(5, -6)$
- $(-2, -5)$ and $(6, 1)$

1. How does the value you found compare to the rest of your group?

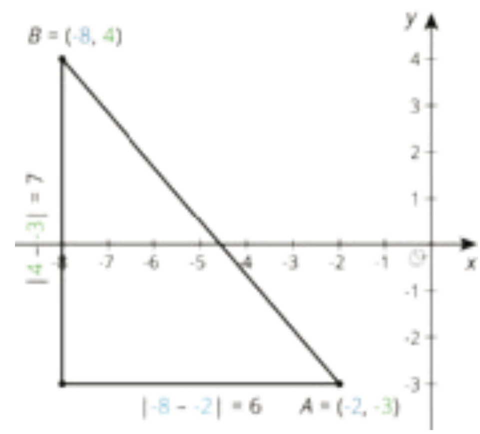
2. In your own words, write an explanation to another student for how to find the distance between any two coordinate pairs.

Lesson 11 Summary

We can use the Pythagorean Theorem to find the distance between any two points on the coordinate plane. For example, if the coordinates of point A are $(-2, -3)$, and the coordinates of point B are $(-8, 4)$, let's find the distance between them. This distance is also the length of line segment AB. It is a good idea to plot the points first.



Think of the distance between A and B, or the length of segment AB, as the hypotenuse of a right triangle. The lengths of the legs can be deduced from the coordinates of the points.



The length of the horizontal leg is 6, which can be seen in the diagram, but it is also the distance between the x-coordinates of A and B since $|-8 - -2| = 6$. The length of the vertical leg is 7, which can be seen in the diagram, but this is also the distance between the y-coordinates of A and B since $|4 - -3| = 7$.

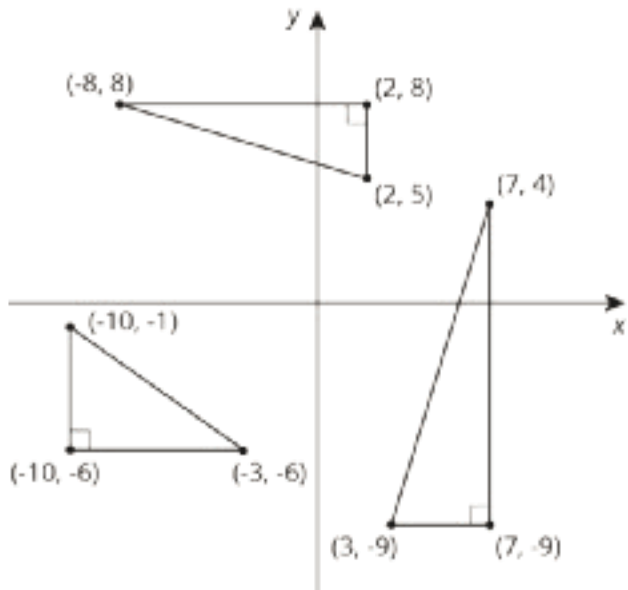
Once the lengths of the legs are known, we use the Pythagorean Theorem to find the length of the hypotenuse, AB, which we can represent with c. Since c is a positive number, there is only one value it can take:

$$\begin{aligned} 6^2 + 7^2 &= c^2 \\ 36 + 49 &= c^2 \\ 85 &= c^2 \\ \sqrt{85} &= c \end{aligned}$$

This length is a little longer than 9, since 85 is a little longer than 81. Using a calculator gives a more precise answer, $\sqrt{85} \approx 9.22$.

Unit 8 Lesson 11 Cumulative Practice Problems

1. The right triangles are drawn in the coordinate plane, and the coordinates of their vertices are labeled. For each right triangle, label each leg with its length.



2. Find the distance between each pair of points. If you get stuck, try plotting the points on graph paper.

   a. $M = (0, -11)$ and $P = (0, 2)$

   b. $A = (0, 0)$ and $B = (-3, -4)$

   c. $C = (8, 0)$ and $D = (0, -6)$

3. a. Find an object that contains a right angle. This can be something in nature or something that was made by humans or machines.

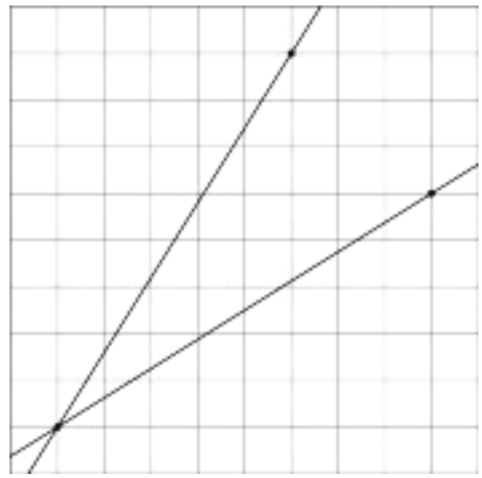
   b. Measure the two sides that make the right angle. Then measure the distance from the end of one side to the end of the other.

   c. Draw a diagram of the object, including the measurements.

   d. Use the Pythagorean Theorem to show that your object really does have a right angle.

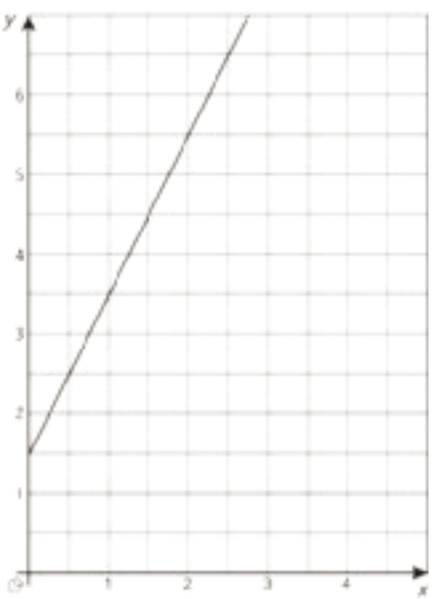
(From Unit 8, Lesson 9.)

4. Which line has a slope of 0.625, and which line has a slope of 1.6? Explain why the slopes of these lines are 0.625 and 1.6.



(From Unit 2, Lesson 10.)

5. Write an equation for the graph.



(From Unit 3, Lesson 7.)

Lesson 12: Edge Lengths and Volumes

Let’s explore the relationship between volume and edge lengths of cubes.

12.1: Ordering Squares and Cubes

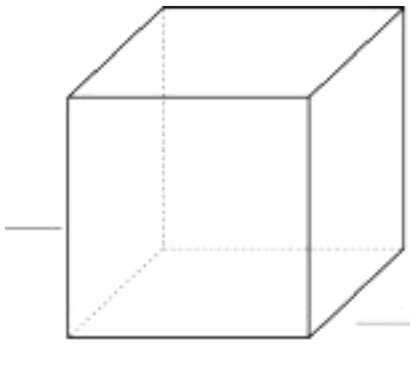
Let a, b, c, d, e, and f be positive numbers.

Given these equations, arrange a, b, c, d, e, and f from least to greatest. Explain your reasoning.

- $a^2 = 9$
- $b^3 = 8$
- $c^2 = 10$
- $d^3 = 9$
- $e^2 = 8$
- $f^3 = 7$

12.2: Name That Edge Length!

Fill in the missing values using the information provided:



sides	volume	volume equation
	27 in^3	
$\sqrt[3]{5}$		
		$(\sqrt[3]{16})^3 = 16$

Are you ready for more?

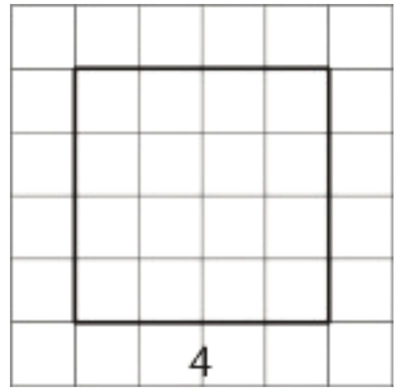
A cube has a volume of 8 cubic centimeters. A square has the same value for its area as the value for the surface area of the cube. How long is each side of the square?

12.3: Card Sort: Rooted in the Number Line

Your teacher will give your group a set of cards. For each card with a letter and value, find the two other cards that match. One shows the location on a number line where the value exists, and the other shows an equation that the value satisfies. Be prepared to explain your reasoning.

Lesson 12 Summary

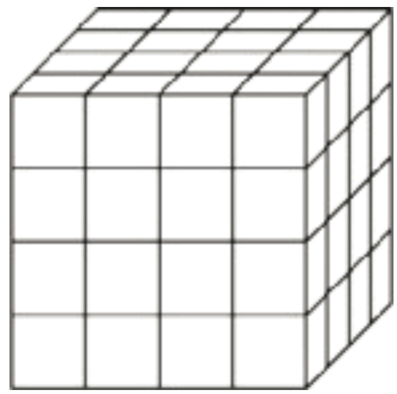
To review, the side length of the square is the square root of its area. In this diagram, the square has an area of 16 units and a side length of 4 units.



These equations are both true:

$$4^2 = 16$$
$$\sqrt{16} = 4$$

Now think about a solid cube. The cube has a volume, and the edge length of the cube is called the **cube root** of its volume. In this diagram, the cube has a volume of 64 units and an edge length of 4 units:



These equations are both true:

$$4^3 = 64$$

$$\sqrt[3]{64} = 4$$

$\sqrt[3]{64}$ is pronounced "The cube root of 64." Here are some other values of cube roots:

$\sqrt[3]{8} = 2$, because $2^3 = 8$

$\sqrt[3]{27} = 3$, because $3^3 = 27$

$\sqrt[3]{125} = 5$, because $5^3 = 125$

Unit 8 Lesson 12 Cumulative Practice Problems

1. a. What is the volume of a cube with a side length of

   i. 4 centimeters?

   ii. $\sqrt[3]{11}$ feet?

   iii. s units?

   b. What is the side length of a cube with a volume of

   i. 1,000 cubic centimeters?

   ii. 23 cubic inches?

   iii. v cubic units?

2. Write an equivalent expression that doesn't use a cube root symbol.

   a. $\sqrt[3]{1}$

   b. $\sqrt[3]{216}$

   c. $\sqrt[3]{8000}$

   d. $\sqrt[3]{\frac{1}{64}}$

   e. $\sqrt[3]{\frac{27}{125}}$

   f. $\sqrt[3]{0.027}$

   g. $\sqrt[3]{0.000125}$

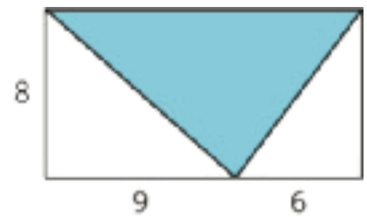
3. Find the distance between each pair of points. If you get stuck, try plotting the points on graph paper.

   a. $X = (5, 0)$ and $Y = (-4, 0)$

   b. $K = (-21, -29)$ and $L = (0, 0)$

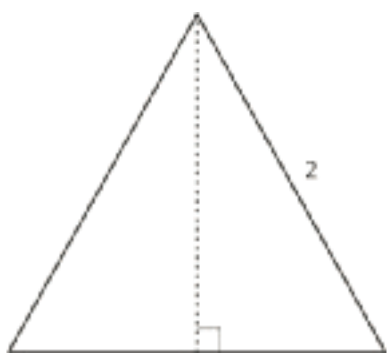
   (From Unit 8, Lesson 11.)

4. Here is a 15-by-8 rectangle divided into triangles. Is the shaded triangle a right triangle? Explain or show your reasoning.



(From Unit 8, Lesson 9.)

5. Here is an equilateral triangle. The length of each side is 2 units. A height is drawn. In an equilateral triangle, the height divides the opposite side into two pieces of equal length.



   a. Find the exact height.

   b. Find the area of the equilateral triangle.

   c. (Challenge) Using x for the length of each side in an equilateral triangle, express its area in terms of x.

(From Unit 8, Lesson 10.)

Lesson 13: Cube Roots

Let’s compare cube roots.

13.1: True or False: Cubed

Decide if each statement is true or false.

$\left(\sqrt[3]{5}\right)^3 = 5$

$\left(\sqrt[3]{27}\right)^3 = 3$

$7 = \left(\sqrt[3]{7}\right)^3$

$\left(\sqrt[3]{10}\right)^3 = 1{,}000$

$\left(\sqrt[3]{64}\right) = 2^3$

13.2: Cube Root Values

What two whole numbers does each cube root lie between? Be prepared to explain your reasoning.

1. $\sqrt[3]{5}$

2. $\sqrt[3]{23}$

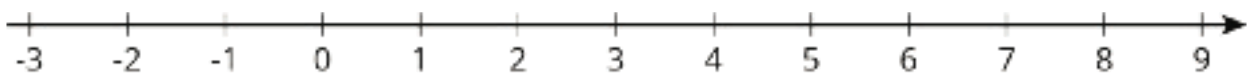
3. $\sqrt[3]{81}$

4. $\sqrt[3]{999}$

13.3: Solutions on a Number Line

The numbers x, y, and z are positive, and:

$$x^3 = 5 \qquad y^3 = 27 \qquad z^3 = 700$$



1. Plot x, y, and z on the number line. Be prepared to share your reasoning with the class.

2. Plot $-\sqrt[3]{2}$ on the number line.

Are you ready for more?

Diego knows that $8^2 = 64$ and that $4^3 = 64$. He says that this means the following are all true:

- $\sqrt{64} = 8$
- $\sqrt[3]{64} = 4$
- $\sqrt{-64} = -8$
- $\sqrt[3]{-64} = -4$

Is he correct? Explain how you know.

Lesson 13 Summary

Remember that square roots of whole numbers are defined as side lengths of squares. For example, $\sqrt{17}$ is the side length of a square whose area is 17. We define cube roots similarly, but using cubes instead of squares. The number $\sqrt[3]{17}$, pronounced "the cube root of 17," is the edge length of a cube which has a volume of 17.

We can approximate the values of cube roots by observing the whole numbers around it and remembering the relationship between cube roots and cubes. For example, $\sqrt[3]{20}$ is between 2 and 3 since $2^3 = 8$ and $3^3 = 27$, and 20 is between 8 and 27. Similarly, since 100 is between 4^3 and 5^3, we know $\sqrt[3]{100}$ is between 4 and 5. Many calculators have a cube root function which can be used to approximate the value of a cube root more precisely. Using our numbers from before, a calculator will show that $\sqrt[3]{20} \approx 2.7144$ and that $\sqrt[3]{100} \approx 4.6416$.

Also like square roots, most cube roots of whole numbers are irrational. The only time the cube root of a number is a whole number is when the original number is a perfect cube.

Unit 8 Lesson 13 Cumulative Practice Problems

1. Find the positive solution to each equation. If the solution is irrational, write the solution using square root or cube root notation.

   a. $t^3 = 216$

   b. $a^2 = 15$

   c. $m^3 = 8$

   d. $c^3 = 343$

   e. $f^3 = 181$

2. For each cube root, find the two whole numbers that it lies between.

   a. $\sqrt[3]{11}$

   b. $\sqrt[3]{80}$

   c. $\sqrt[3]{120}$

   d. $\sqrt[3]{250}$

3. Order the following values from least to greatest:

$$\sqrt[3]{530},\ \sqrt{48},\ \pi,\ \sqrt{121},\ \sqrt[3]{27},\ \frac{19}{2}$$

4. Select **all** the equations that have a solution of $\frac{2}{7}$:

A. $x^2 = \frac{2}{7}$

B. $x^2 = \frac{4}{14}$

C. $x^2 = \frac{4}{49}$

D. $x^3 = \frac{6}{21}$

E. $x^3 = \frac{8}{343}$

F. $x^3 = \frac{6}{7}$

5. The equation $x^2 = 25$ has *two* solutions. This is because both $5 \cdot 5 = 25$, and also $-5 \cdot -5 = 25$. So, 5 is a solution, and also -5 is a solution. But! The equation $x^3 = 125$ only has one solution, which is 5. This is because $5 \cdot 5 \cdot 5 = 125$, and there are no other numbers you can cube to make 125. (Think about why -5 is not a solution!)

Find all the solutions to each equation.

a. $x^3 = 8$

b. $\sqrt[3]{x} = 3$

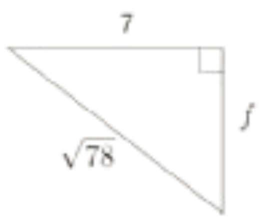
c. $x^2 = 49$

d. $x^3 = \frac{64}{125}$

6. Find the value of each variable, to the nearest tenth.

   a.

   b.



   c.

(From Unit 8, Lesson 8.)

7. A standard city block in Manhattan is a rectangle measuring 80 m by 270 m. A resident wants to get from one corner of a block to the opposite corner of a block that contains a park. She wonders about the difference between cutting across the diagonal through the park compared to going around the park, along the streets. How much shorter would her walk be going through the park? Round your answer to the nearest meter.

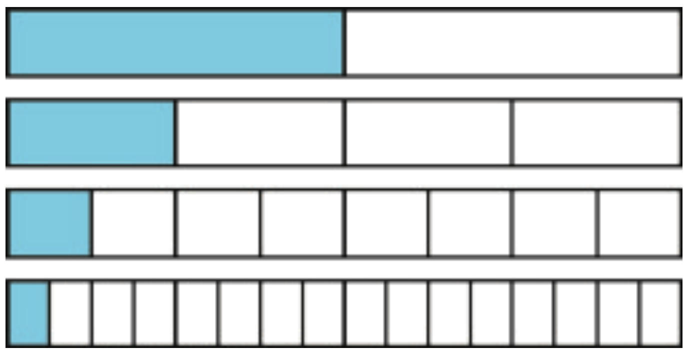
(From Unit 8, Lesson 10.)

Lesson 14: Decimal Representations of Rational Numbers

Let’s learn more about how rational numbers can be represented.

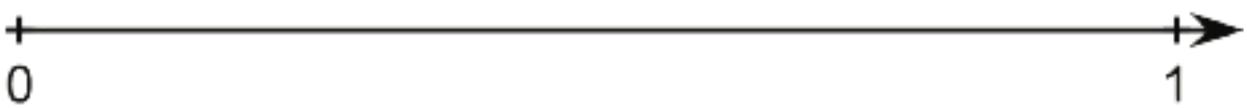
14.1: Notice and Wonder: Shaded Bars

What do you notice? What do you wonder?



14.2: Halving the Length

Here is a number line from 0 to 1.



1. Mark the midpoint between 0 and 1. What is the decimal representation of that number?
2. Mark the midpoint between 0 and the newest point. What is the decimal representation of that number?
3. Repeat step two. How did you find the value of this number?
4. Describe how the value of the midpoints you have added to the number line keep changing as you find more. How do the decimal representations change?

14.3: Recalculating Rational Numbers

1. Rational numbers are fractions and their opposites. All of these numbers are rational numbers. Show that they are rational by writing them in the form $\frac{a}{b}$ or $-\frac{a}{b}$.

   a. 0.2

   b. $-\sqrt{4}$

   c. 0.333

   d. $\sqrt[3]{1000}$

   e. -1.000001

   f. $\sqrt{\frac{1}{9}}$

2. All rational numbers have decimal representations, too. Find the decimal representation of each of these rational numbers.

   a. $\frac{3}{8}$

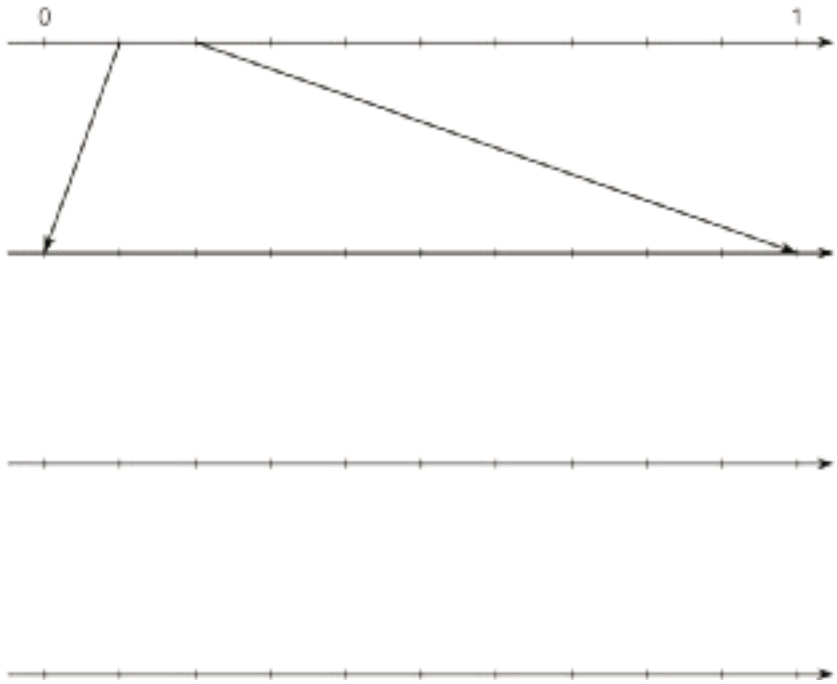
   b. $\frac{7}{5}$

   c. $\frac{999}{1000}$

   d. $\frac{111}{2}$

e. $\sqrt[3]{\frac{1}{8}}$

14.4: Zooming In On $\frac{2}{11}$



1. On the topmost number line, label the tick marks. Next, find the first decimal place of $\frac{2}{11}$ using long division and estimate where $\frac{2}{11}$ should be placed on the top number line.

2. Label the tick marks of the second number line. Find the next decimal place of $\frac{2}{11}$ by continuing the long division and estimate where $\frac{2}{11}$ should be placed on the second number line. Add arrows from the second to the third number line to zoom in on the location of $\frac{2}{11}$.

3. Repeat the earlier step for the remaining number lines.

4. What do you think the decimal expansion of $\frac{2}{11}$ is?

Are you ready for more?

Let $x = \frac{25}{11} = 2.272727...$ and $y = \frac{58}{33} = 1.75757575...$

For each of the following questions, first decide whether the fraction or decimal representations of the numbers are more helpful to answer the question, and then find the answer.

- Which of x or y is closer to 2?
- Find x^2.

Lesson 14 Summary

We learned earlier that rational numbers are a fraction or the opposite of a fraction. For example, $\frac{3}{4}$ and $-\frac{5}{2}$ are both rational numbers. A complicated-looking numerical expression can also be a rational number as long as the value of the expression is a positive or negative fraction. For example, $\sqrt{64}$ and $-\sqrt[3]{\frac{1}{8}}$ are rational numbers because $\sqrt{64} = 8$ and $-\sqrt[3]{\frac{1}{8}} = -\frac{1}{2}$.

Rational numbers can also be written using decimal notation. Some have finite decimal expansions, like 0.75, -2.5, or -0.5. Other rational numbers have infinite decimal expansions, like 0.7434343 . . . where the 43s repeat forever. To avoid writing the **repeating** part over and over, we use the notation $0.7\overline{43}$ for this number. The bar over part of the expansion tells us the part which is to repeat forever.

A decimal expansion of a number helps us plot it accurately on a number line divided into tenths. For example, $0.7\overline{43}$ should be between 0.7 and 0.8. Each further decimal digit increases the accuracy of our plotting. For example, the number $0.7\overline{43}$ is between 0.743 and 0.744.

Unit 8 Lesson 14 Cumulative Practice Problems

1. Andre and Jada are discussing how to write $\frac{17}{20}$ as a decimal.

   Andre says he can use long division to divide 17 by 20 to get the decimal.

   Jada says she can write an equivalent fraction with a denominator of 100 by multiplying by $\frac{5}{5}$, then writing the number of hundredths as a decimal.

   a. Do both of these strategies work?

   b. Which strategy do you prefer? Explain your reasoning.

   c. Write $\frac{17}{20}$ as a decimal. Explain or show your reasoning.

2. Write each fraction as a decimal.

   a. $\sqrt{\frac{9}{100}}$

   b. $\frac{99}{100}$

   c. $\sqrt{\frac{9}{16}}$

   d. $\frac{23}{10}$

3. Write each decimal as a fraction.

a. $\sqrt{0.81}$

b. 0.0276

c. $\sqrt{0.04}$

d. 10.01

4. Find the positive solution to each equation. If the solution is irrational, write the solution using square root or cube root notation.

a. $x^2 = 90$

b. $p^3 = 90$

c. $z^2 = 1$

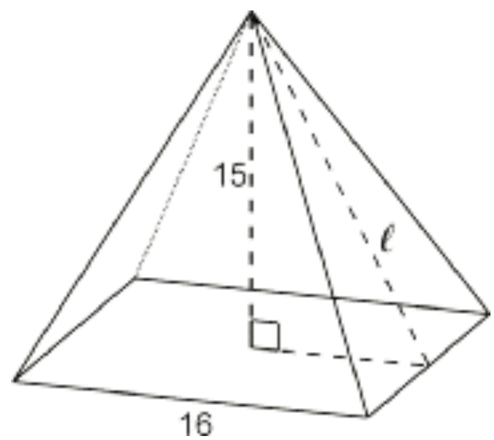
d. $y^3 = 1$

e. $w^2 = 36$

f. $h^3 = 64$

(From Unit 8, Lesson 13.)

5. Here is a right square pyramid.



a. What is the measurement of the slant height ℓ of the triangular face of the pyramid? If you get stuck, use a cross section of the pyramid.

b. What is the surface area of the pyramid?

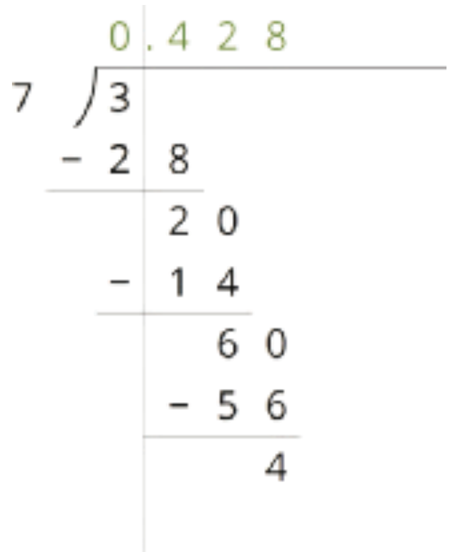
(From Unit 8, Lesson 10.)

Lesson 15: Infinite Decimal Expansions

Let's think about infinite decimals.

15.1: Searching for Digits

The first 3 digits after the decimal for the decimal expansion of $\frac{3}{7}$ have been calculated. Find the next 4 digits.



15.2: Some Numbers Are Rational

Your teacher will give your group a set of cards. Each card will have a calculations side and an explanation side.

1. The cards show Noah's work calculating the fraction representation of $0.4\overline{85}$. Arrange these in order to see how he figured out that $0.4\overline{85} = \frac{481}{990}$ without needing a calculator.

2. Use Noah's method to calculate the fraction representation of:

   a. $0.1\overline{86}$

   b. $0.7\overline{88}$

Are you ready for more?

Use this technique to find fractional representations for $0.\overline{3}$ and $0.\overline{9}$.

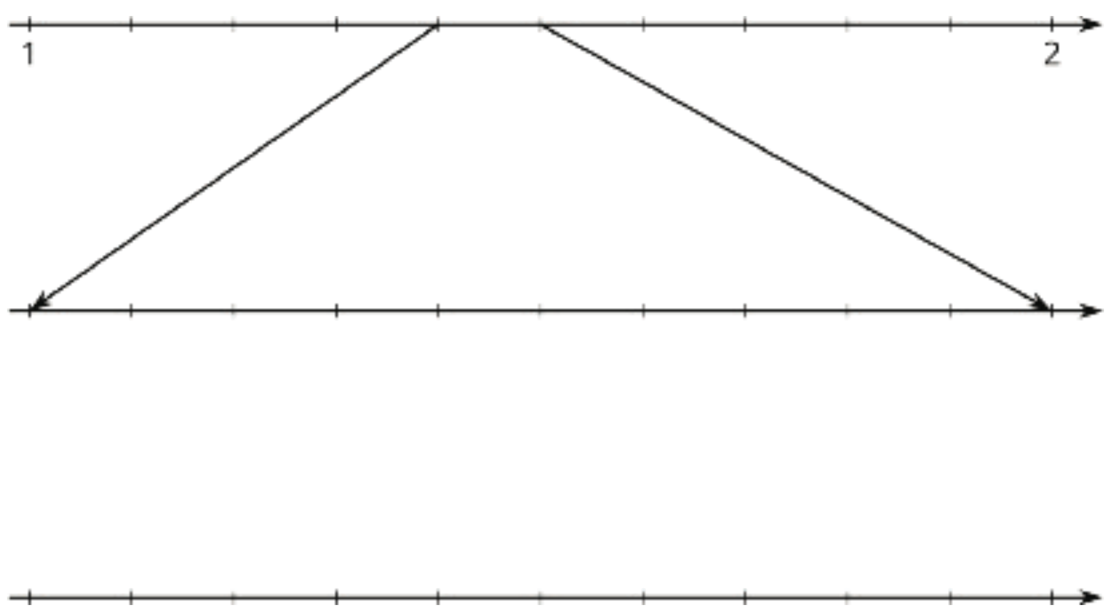
15.3: Some Numbers Are Not Rational

1. a. Why is $\sqrt{2}$ between 1 and 2 on the number line?

   b. Why is $\sqrt{2}$ between 1.4 and 1.5 on the number line?

   c. How can you figure out an approximation for $\sqrt{2}$ accurate to 3 decimal places?

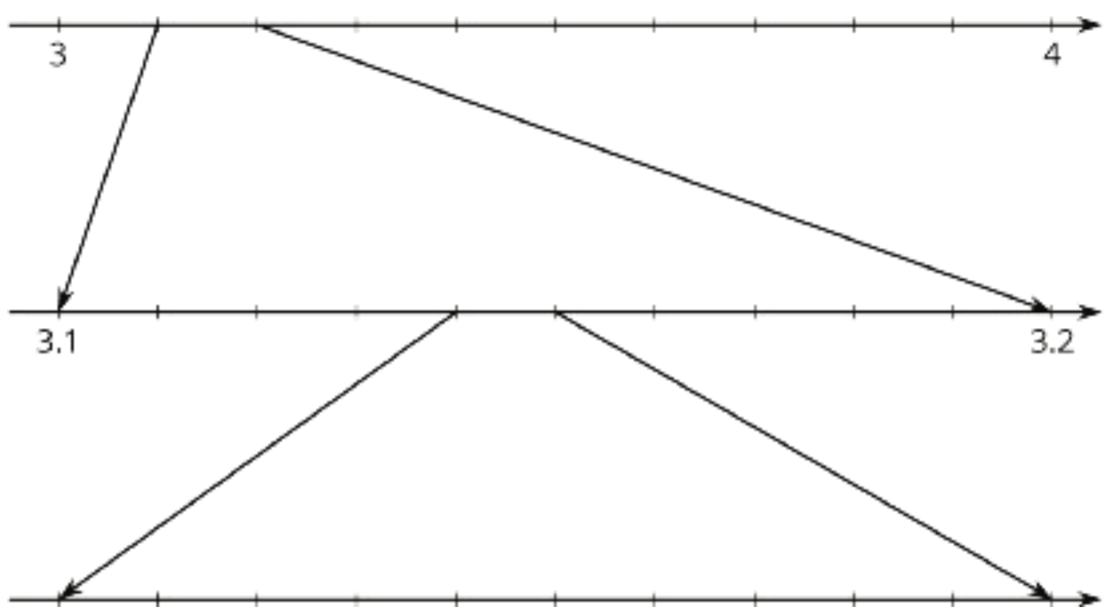
   d. Label all of the tick marks. Plot $\sqrt{2}$ on all three number lines. Make sure to add arrows from the second to the third number lines.



2. a. Elena notices a beaker in science class says it has a diameter of 9 cm and measures its circumference to be 28.3 cm. What value do you get for π using these values and the equation for circumference, $C = 2\pi r$?

b. Diego learned that one of the space shuttle fuel tanks had a diameter of 840 cm and a circumference of 2,639 cm. What value do you get for π using these values and the equation for circumference, $C = 2\pi r$?

c. Label all of the tick marks on the number lines. Use a calculator to get a very accurate approximation of π and plot that number on all three number lines.



d. How can you explain the differences between these calculations of π?

Lesson 15 Summary

Not every number is rational. Earlier we tried to find a fraction whose square is equal to 2. That turns out to be impossible, although we can get pretty close (try squaring $\frac{7}{5}$). Since there is no fraction equal to $\sqrt{2}$ it is not a rational number, which is why we call it an irrational number. Another well-known irrational number is π.

Any number, rational or irrational, has a decimal expansion. Sometimes it goes on forever. For example, the rational number $\frac{2}{11}$ has the decimal expansion $0.181818...$ with the 18s repeating forever. Every rational number has a decimal expansion that either stops at some point or ends up in a repeating pattern like $\frac{2}{11}$. Irrational numbers also have infinite decimal expansions, but they don't end up in a repeating pattern. From the decimal point of view we can see that rational numbers are pretty special. Most numbers are irrational, even though the numbers we use on a daily basis are more frequently rational.

Unit 8 Lesson 15 Cumulative Practice Problems

1. Elena and Han are discussing how to write the repeating decimal $x = 0.13\overline{7}$ as a fraction. Han says that $0.13\overline{7}$ equals $\frac{13764}{99900}$. "I calculated $1000x = 137.77\overline{7}$ because the decimal begins repeating after 3 digits. Then I subtracted to get $999x = 137.64$. Then I multiplied by 100 to get rid of the decimal: $99900x = 13764$. And finally I divided to get $x = \frac{13764}{99900}$." Elena says that $0.13\overline{7}$ equals $\frac{124}{900}$. "I calculated $10x = 1.37\overline{7}$ because one digit repeats. Then I subtracted to get $9x = 1.24$. Then I did what Han did to get $900x = 124$ and $x = \frac{124}{900}$."

   Do you agree with either of them? Explain your reasoning.

2. How are the numbers 0.444 and $0.\overline{4}$ the same? How are they different?

3. a. Write each fraction as a decimal.
      i. $\frac{2}{3}$
      ii. $\frac{126}{37}$

   b. Write each decimal as a fraction.
      i. $0.\overline{75}$
      ii. $0.\overline{3}$

4. Write each fraction as a decimal.

a. $\frac{5}{9}$

b. $\frac{5}{4}$

c. $\frac{48}{99}$

d. $\frac{5}{99}$

e. $\frac{7}{100}$

f. $\frac{53}{90}$

5. Write each decimal as a fraction.

a. $0.\overline{7}$

b. $0.\overline{2}$

c. $0.1\overline{3}$

d. $0.\overline{14}$

e. $0.\overline{03}$

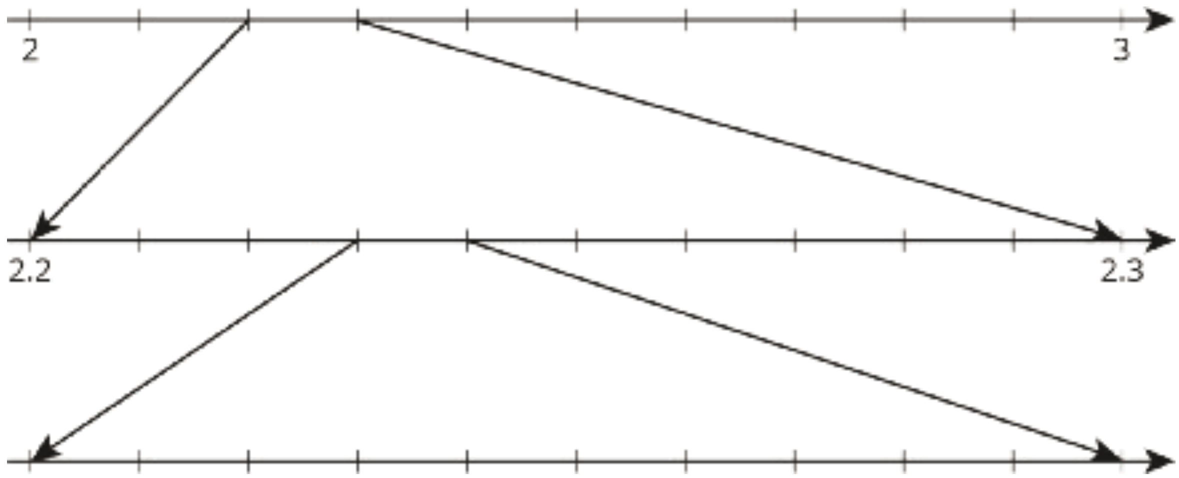
f. $0.6\overline{38}$

g. $0.52\overline{4}$

h. $0.1\overline{5}$

6. $2.2^2 = 4.84$ and $2.3^2 = 5.29$. This gives some information about $\sqrt{5}$.

Without directly calculating the square root, plot $\sqrt{5}$ on all three number lines using successive approximation.

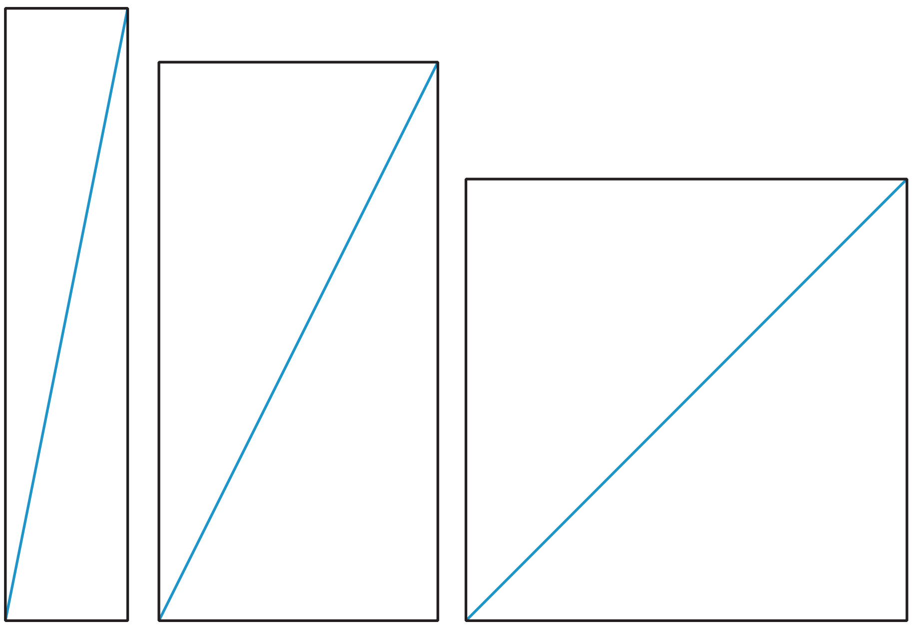


Lesson 16: When Is the Same Size Not the Same Size?

- Let's figure out how aspect ratio affects screen area.

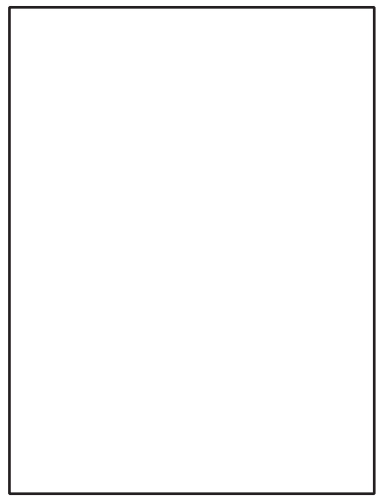
16.1: Three Figures

How are these shapes the same? How are they different?



16.2: A $4 : 3$ Rectangle

A typical aspect ratio for photos is $4 : 3$. Here's a rectangle with a $4 : 3$ aspect ratio.



1. What does it mean that the aspect ratio is $4 : 3$? Mark up the diagram to show what that means.

2. If the shorter side of the rectangle measures 15 inches:
    a. What is the length of the longer side?

    b. What is the length of the rectangle's diagonal?

3. If the diagonal of the $4 : 3$ rectangle measures 10 inches, how long are its sides?

4. If the diagonal of the $4 : 3$ rectangle measures 6 inches, how long are its sides?

16.3: The Screen Is the Same Size . . . Or Is It?

Before 2017, a smart phone manufacturer’s phones had a diagonal length of 5.8 inches and an aspect ratio of $16 : 9$. In 2017, they released a new phone that also had a 5.8-inch diagonal length, but an aspect ratio of $18.5 : 9$. Some customers complained that the new phones had a smaller screen. Were they correct? If so, how much smaller was the new screen compared to the old screen?

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