



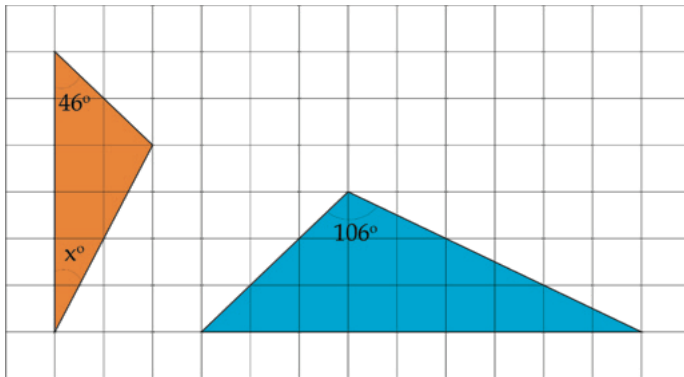
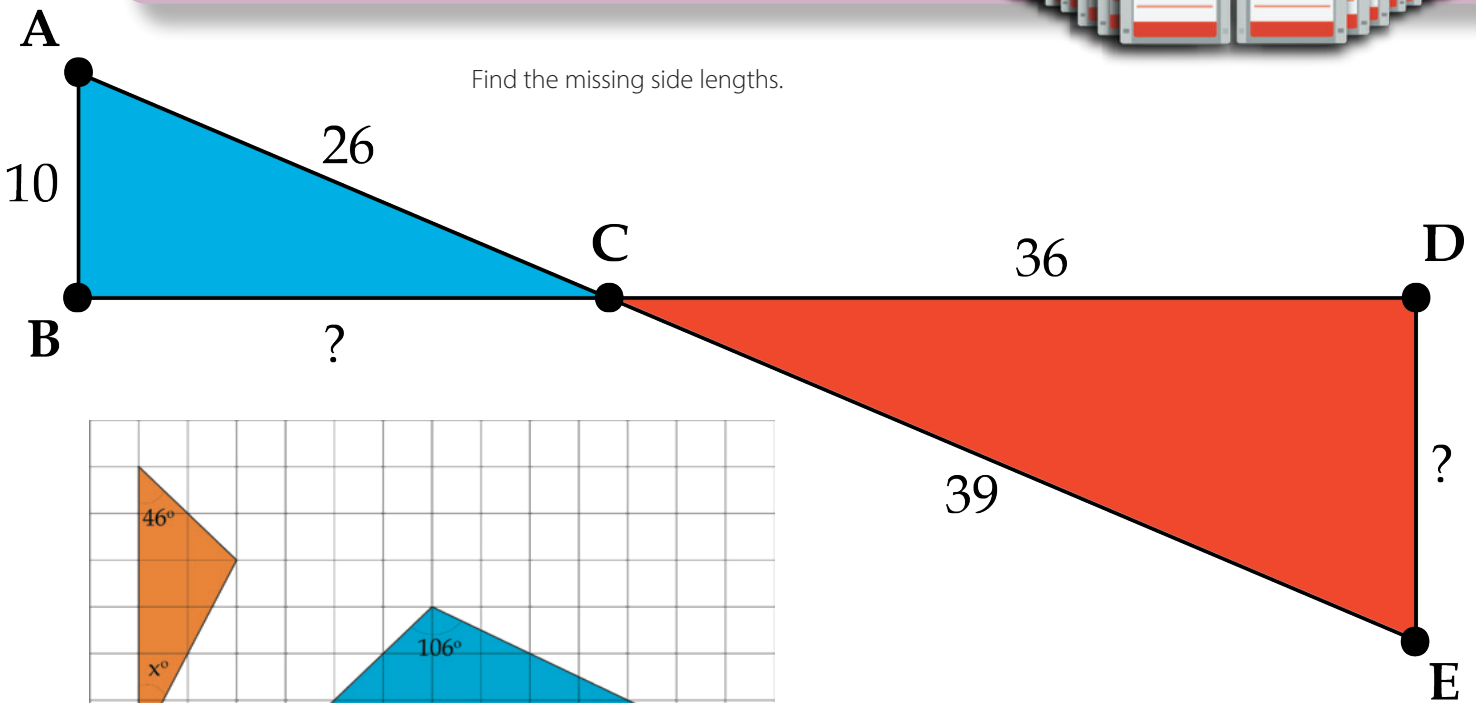
SD, CD,
and Floppy Disc

Exponents and Scientific Notation



Teacher Guide

Find the missing side lengths.



Similar Triangles

expression	expanded	exponent
$5^3 \cdot 2^3$	$(5 \cdot 5 \cdot 5) \cdot (2 \cdot 2 \cdot 2) = (5 \cdot 2)(5 \cdot 2)(5 \cdot 2) = 10 \cdot 10 \cdot 10$	10^3

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Exponents and Scientific Notation

Table of Contents

Introduction: Unit Narrative	1
Student Learning Targets	3
Terminology	5
Required Materials	6
Lessons Plans and Student Task Statements:	
Section 1: Lesson 1 Exponent Review	7
Section 2: Lessons 2–8 Exponent Rules	19
Section 3: Lessons 9–15 Scientific Notation	110
Section 4: Lesson 16 Let’s Put It to Work	214
Teacher Resources	222
Family Support Materials	
Unit Assessments	
Assessment Answer Keys	
Cool Downs (Lesson-Level Assessments)	
Instructional Masters	



Exponents and Scientific Notation Teacher Guide

Core Knowledge Mathematics™

Exponents and Scientific Notation

Unit Narrative

Students were introduced to exponent notation in grade 6. They worked with expressions that included parentheses and positive whole-number exponents with whole-number, fraction, decimal, or variable bases, using properties of exponents strategically, but did not formulate rules for use of exponents.

In this unit, students build on their grade 6 work. The first section of the unit begins with a lesson that reviews exponential expressions, including work with exponential expressions with bases 2 and $\frac{1}{2}$. In the next two lessons, students examine powers of 10, formulating the rules $10^n \cdot 10^m = 10^{n+m}$, $(10^n)^m = 10^{n \cdot m}$, and, for $n > m$, $\frac{10^n}{10^m} = 10^{n-m}$ where n and m are positive integers. After working with these powers of 10, they consider what the value of 10^0 should be and define 10^0 to be 1. In the next lesson, students consider what happens when the exponent rules are used on exponential expressions with base 10 and negative integer exponents and define 10^{-n} to be $\frac{1}{10^n}$. In the next two lessons, they expand their work to numerical bases other than 10, using exponent rules with products of exponentials with the same base and contrasting it with products of exponentials with different bases. They note numerical instances of $a^n \cdot b^n = (a \cdot b)^n$.

The third section of the unit returns to powers of 10 as a prelude to the introduction of scientific notation. Students consider differences in magnitude of powers of 10 and use powers of 10 and multiples of powers of 10 to describe magnitudes of quantities, e.g., the distance from Earth to the Sun or the population of Russia. Initially, they work with large quantities, locating powers of 10 and positive-integer multiples of powers of 10 on the number line. Most of these multiples are products of single-digit numbers and powers of 10. The remainder are products of two- or three-digit numbers and powers of 10, allowing students to notice that these numbers may be expressed in different ways, e.g., $75 \cdot 10^5$ can be written $7.5 \cdot 10^6$, and that some forms may be more helpful in finding locations on the number line. In the next lesson, students do similar work with small quantities.

In the remaining five lessons, students write estimates of quantities in terms of integer or non-integer multiples of powers of 10 and use their knowledge of exponential expressions to solve problems, e.g., How many meter sticks does it take to equal the mass of the Moon? They are introduced to the term “scientific notation,” practice distinguishing scientific from other notation, and use scientific notation (with no more than three significant figures) in order to make additive and multiplicative comparisons of pairs of quantities. They compute sums, differences, products, and quotients of numbers written in scientific notation (some with as many as four significant figures), using such calculations to estimate quantities. They make measurement conversions that involve powers of ten, e.g., converting bytes to kilobytes or gigabytes, choose appropriate units for measurements and express them in scientific notation.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as critiquing, representing, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Critique

- reasoning about powers of powers (Lesson 3)
- reasoning about zero exponents (Lesson 4)
- applications of exponent rules (Lesson 7)
- reasoning about scientific notation (Lesson 15)

Represent

- situations using exponents (Lesson 1)
- large and small numbers using number lines, exponents, and decimals (Lesson 9–11)
- situations comparing quantities expressed in scientific notation (Lesson 14)

Justify

- reasoning about multiplying powers of 10 (Lesson 2)
- reasoning about powers of powers (Lesson 3)
- reasoning about dividing powers of 10 (Lesson 4)
- whether or not expressions are equivalent to exponential expressions (Lesson 6)
- reasoning about situations comparing powers of 10 (Lesson 12)

In addition, students are expected to use language to generalize reasoning about repeated multiplication and generalize about patterns when multiplying different bases and exponents; describe how negative powers of 10 affect placement of decimals; and interpret situations comparing quantities expressed in scientific notation. Students also have opportunities to compare correspondences between exponential expressions and base-ten diagrams; compare expressions in scientific notation to other expressions; explain how to simplify expressions with negative powers of 10; and explain how to place and order large numbers on a number line.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow the one in which it was first introduced.

Learning Targets

Exponents and Scientific Notation

Lesson 1: Exponent Review

- I can use exponents to describe repeated multiplication.
- I understand the meaning of a term with an exponent.

Lesson 2: Multiplying Powers of Ten

- I can explain and use a rule for multiplying powers of 10.

Lesson 3: Powers of Powers of 10

- I can explain and use a rule for raising a power of 10 to a power.

Lesson 4: Dividing Powers of 10

- I can evaluate 10^0 and explain why it makes sense.
- I can explain and use a rule for dividing powers of 10.

Lesson 5: Negative Exponents with Powers of 10

- I can use the exponent rules with negative exponents.
- I know what it means if 10 is raised to a negative power.

Lesson 6: What about Other Bases?

- I can use the exponent rules for bases other than 10.

Lesson 7: Practice with Rational Bases

- I can change an expression with a negative exponent into an equivalent expression with a positive exponent.
- I can choose an appropriate exponent rule to rewrite an expression to have a single exponent.

Lesson 8: Combining Bases

- I can use and explain a rule for multiplying terms that have different bases but the same exponent.

Lesson 9: Describing Large and Small Numbers Using Powers of 10

- Given a very large or small number, I can write an expression equal to it using a power of 10.

Lesson 10: Representing Large Numbers on the Number Line

- I can plot a multiple of a power of 10 on such a number line.
- I can subdivide and label a number line between 0 and a power of 10 with a positive exponent into 10 equal intervals.
- I can write a large number as a multiple of a power of 10.

Lesson 11: Representing Small Numbers on the Number Line

- I can plot a multiple of a power of 10 on such a number line.
- I can subdivide and label a number line between 0 and a power of 10 with a negative exponent into 10 equal intervals.
- I can write a small number as a multiple of a power of 10.

Lesson 12: Applications of Arithmetic with Powers of 10

- I can apply what I learned about powers of 10 to answer questions about real-world situations.

Lesson 13: Definition of Scientific Notation

- I can tell whether or not a number is written in scientific notation.

Lesson 14: Multiplying, Dividing, and Estimating with Scientific Notation

- I can multiply and divide numbers given in scientific notation.
- I can use scientific notation and estimation to compare very large or very small numbers.

Lesson 15: Adding and Subtracting with Scientific Notation

- I can add and subtract numbers given in scientific notation.

Lesson 16: Is a Smartphone Smart Enough to Go to the Moon?

- I can use scientific notation to compare different amounts and answer questions about real-world situations.

lesson	new terminology	
	receptive	productive
8.7.1	exponent power factor reciprocal	repeated multiplication
8.7.2	powers of 10	
8.7.3	base (of an exponent) power of powers	
8.7.4	expanded positive exponent zero exponent	
8.7.5	negative exponent	positive exponent
8.7.6		exponent base (of an exponent) power zero exponent
8.7.7	reciprocal evaluate	factor power of powers negative exponent
8.7.8	square (of a number)	
8.7.9	billion trillion multiple of	
8.7.10	integer	
8.7.12		multiple of
8.7.13	scientific notation	integer
8.7.14		powers of 10 billion trillion
8.7.15		scientific notation

Required Materials

Copies of Instructional master

Pre-printed cards, cut from copies of the Instructional master

Pre-printed slips, cut from copies of the Instructional master

Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Section: Exponent Review

Lesson 1: Exponent Review

Goals

- Comprehend that repeated division by 2 is equivalent to repeated multiplication by one-half.
- Create an expression that represents repeated multiplication, and explain (orally) how the structure of the expression helps compare quantities.

Learning Targets

- I can use exponents to describe repeated multiplication.
- I understand the meaning of a term with an exponent.

Lesson Narrative

In grade 6, students worked with whole number exponents. This lesson reviews those concepts and subtly introduces the idea that repeated division by a number can be thought of as repeated multiplication by the reciprocal of that number, which plays a key role in later work on negative exponents.

Alignments

Building On

- 6.EE.A.1: Write and evaluate numerical expressions involving whole-number exponents.

Building Towards

- 8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR7: Compare and Connect
- Think Pair Share
- Which One Doesn't Belong?

Student Learning Goals

Let's review exponents.

1.1 Which One Doesn't Belong: Twos

Warm Up: 5 minutes

This warm-up prompts students to compare four expressions with exponents. It encourages students to explain their reasoning, hold mathematical conversations, and gives you the opportunity to hear how they use terminology and talk about characteristics of the expressions in comparison to one another. To allow all students to access the activity, each expression except for 2^3 has one obvious reason it does not belong. Don't let students dwell on trying to explain why 2^3 doesn't belong. During the discussion, listen for important ideas and terminology that will be helpful in upcoming work of the unit.

Building On

- 6.EE.A.1

Building Towards

- 8.EE.A.1

Instructional Routines

- Which One Doesn't Belong?

Launch

Arrange students in groups of 2–4. Before introducing the warm-up, display the expression 7^4 for all to see. Ask students if they recognize this notation and to explain what it means. Then display the expressions in the warm-up for all to see. Ask students to indicate when they have noticed one expression that does not belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their small group. In their small groups, tell each student to share their reasoning why a particular expression does not belong and together, *try* to find at least one reason each expression doesn't belong.

Student Task Statement

Which expression does not belong? Be prepared to share your reasoning.

$$2^3$$

$$3^2$$

$$8$$

$$2^2 \cdot 2^1$$

Student Response

Answers vary. Sample responses:

- 8 does not belong because it is the only one with no exponent.
- 3^2 does not belong because it is the only one that does not equal 8.
- $2^2 \cdot 2^1$ does not belong because it is the only one that is multiplying two exponential terms.

There is not an obvious reason 2^3 doesn't belong since the other expressions have little in common.

Activity Synthesis

Ask each group to share one reason why a particular expression does not belong. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given are correct. During the discussion, ask students to explain the meaning of any terminology they use, such as "base" or "exponent." Also, press students on unsubstantiated claims.

1.2 Return of the Genie

15 minutes (there is a digital version of this activity)

This activity uses the context of a genie who gives a magic coin that doubles in number each day. This context reminds students about the need for exponential notation in thinking about problems involving repeated multiplication. For the sake of simplicity, the problem was written so that the exponent is equal to the number of days.

Building On

- 6.EE.A.1

Building Towards

- 8.EE.A.1

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Invite a student to read the first paragraph for the class. Make sure students understand how the magic coin works before moving on to the problem statement, perhaps by drawing a picture of a coin that doubles each day. Allow 10 minutes of work time before a whole-class discussion.

For classes using the digital version, there is an applet to help visualize the growth. If projection is available, teachers using the print version can display it from this link. <https://ggbm.at/xQP9xNDm>

Access for Students with Disabilities

Representation: Internalize Comprehension. Represent the same information through different modalities. If students are unsure where to begin, suggest that they draw a diagram to help organize the information provided.

Supports accessibility for: Conceptual processing; Visual-spatial processing

Anticipated Misconceptions

There may be some confusion about what time of day the coin doubles or how the exponent connects to the number of days. Clarify that the number of days is equal to the number of doublings.

Some students may misinterpret “How many times more coins does Mai have than Andre” as “How many more coins does Mai have than Andre.” Others may think they need to know exactly how many coins Mai and Andre have in order to answer this question. Suggest to students who are stuck that they first figure out how many times more coins Mai had on the 8th day than on the 5th day.

Student Task Statement

Mai and Andre found an old, brass bottle that contained a magical genie. They freed the genie, and it offered them each a magical \$1 coin as thanks.

- The magic coin turned into 2 coins on the first day.
- The 2 coins turned into 4 coins on the second day.
- The 4 coins turned into 8 coins on the third day.



This doubling pattern continued for 28 days.

Mai was trying to calculate how many coins she would have and remembered that instead of writing $1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ for the number of coins on the 6th day, she could just write 2^6 .

1. The number of coins Mai had on the 28th day is very, very large. Write an expression to represent this number without computing its value.
2. Andre’s coins lost their magic on the 25th day, so Mai has a lot more coins than he does. How many times more coins does Mai have than Andre?

Student Response

1. 2^{28} , because the number of coins has doubled 28 times.
2. Mai has $2 \cdot 2 \cdot 2 = 8$ times as many coins as Andre because her coins doubled 3 times after his stopped.

Activity Synthesis

The goal is for students to understand exponential notation and use it to reason about a situation that involves repeated multiplication. Display the table for all to see. Tell students that exponents allow us to perform operations and reason about numbers that are too large to calculate by hand. Explain that the “expanded” column shows the factors being multiplied, the “exponent” column shows how to write the repeated multiplication more succinctly with exponents, and the “value” column shows the decimal value. Consider asking, “How many times larger is 2^6 than 2^4 ? How does expanding into factors help you see this?”

expanded	exponent	value
2	2^1	2
$2 \cdot 2 \cdot 2 \cdot 2$	2^4	16
$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$	2^6	64

Access for English Language Learners

Reading, Writing: MLR3 Critique, Correct and Clarify. Display the incorrect statement: “Mai has 6 times more coins than Andre because she had 3 more doublings, and $3 \cdot 2 = 6$.” Ask students to critique the response by asking, “Do you agree with the author’s reasoning? Why or why not?” Give students 2–3 minutes of quiet think time to write feedback to the author that identifies how to improve the solution and expand on his/her work. Invite students to share written feedback with a partner before selecting 2–3 students to share with the whole class. Listen for students who refer to repeated multiplication and use the language of exponents in their feedback to the author. This helps students evaluate, and improve on, the written mathematical arguments of others.

Design Principle(s): Maximize meta-awareness, Optimize output (for explanation)

1.3 Broken Coin

15 minutes (there is a digital version of this activity)

The broken coin prompts students to think about repeated division, laying the foundation for later work on negative exponents. Understanding repeated division by 2 as being equivalent to repeated multiplication by $\frac{1}{2}$ will later allow students to make sense of negative exponents.

Look for students who express their answers to question 2 as $(\frac{1}{2})^{28}$ and those who write $\frac{1}{2^{28}}$. Ask them to share their responses later.

Building On

- 6.EE.A.1

Building Towards

- 8.EE.A.1

Instructional Routines

- MLR7: Compare and Connect
- Think Pair Share

Launch

Arrange students in groups of 2. Allow 10 minutes of work time, 2 minutes for partner discussion, and follow with a brief whole-class discussion. It is expected that some students will multiply 6 times for question 1. For the second question, give students time to realize that they need to use a more efficient method to describe the number.

For students using the digital activity, there is an applet to help visualize the coin halving each day.

Access for Students with Disabilities

Representation: Internalize Comprehension. Begin with a physical demonstration of the actions that occur in the situation to support connections between new situations and prior understandings. Remind students that repeated multiplication by $\frac{1}{2}$ is equivalent to repeated division by 2.

Supports accessibility for: Conceptual processing; Visual-spatial processing

Anticipated Misconceptions

If students spend more than several minutes on trying to multiply $(\frac{1}{2})^{28}$, remind them of the more efficient exponential notation.

If students are wondering how to represent dividing repeatedly, ask if they can think of division by 2 as multiplication, perhaps by another value.

Student Task Statement

After a while, Jada picks up a coin that seems different than the others. She notices that the next day, only half of the coin is left!

- On the second day, only $\frac{1}{4}$ of the coin is left.
 - On the third day, $\frac{1}{8}$ of the coin remains.
1. What fraction of the coin remains after 6 days?
 2. What fraction of the coin remains after 28 days? Write an expression to describe this without computing its value.
 3. Does the coin disappear completely? If so, after how many days?

Student Response

1. $\frac{1}{64}$, $((\frac{1}{2})^6 = \frac{1}{64})$
2. $\frac{1}{2^{28}}$, or $(\frac{1}{2})^{28}$ ($\frac{1}{2}$ is multiplied by itself 28 times).

3. The coin never completely disappears because each time that half of the coin disappears, there is still half left.

Are You Ready for More?

Every animal has two parents. Each of its parents also has two parents.

1. Draw a family tree showing an animal, its parents, its grandparents, and its great-grandparents.
2. We say that the animal's eight great-grandparents are "three generations back" from the animal. At which generation back would an animal have 262,144 ancestors?

Student Response

1. A diagram with a tree structure that shows the animal on one level (0 generations back), its two parents on the next, its four grandparents on the next, and its eight great-grandparents last (3 generations back).
2. 18 generations, because $2^{18} = 262,144$.

Activity Synthesis

The goal is for students to understand that dividing by 2 repeatedly corresponds to multiplying by $\frac{1}{2}$ repeatedly. Ask students to discuss their responses with their partner. Select previously identified students to share their responses to the second question, highlighting the difference between $(\frac{1}{2})^{28}$ and $\frac{1}{2^{28}}$. Ask students whether they agree or disagree with either of those responses. Students should come away with the idea that repeatedly dividing by 2 is the same as repeatedly multiplying by $\frac{1}{2}$. Here are some questions to consider:

- "Why are exponents useful when thinking about the coin after many days?" (It is shorter to write than a lot of 2s.)
- "What does your partner think about the last question? Do you agree? Why or why not?"

Access for English Language Learners

Speaking, Listening: MLR7 Compare and Connect. As students share different approaches for representing the fraction of the coin remaining after 6 days (or 28 days), ask students to identify how the approaches are alike, and how they are different. Invite students to connect the approaches by asking, "Where is the number of days represented in each approach?" Ask students to describe what worked well with their approach. These exchanges strengthen students' mathematical language use and reasoning based on exponents.

Design Principle(s): Maximize meta-awareness, Support sense-making

Lesson Synthesis

The goal of the discussion is to check whether students understand that exponents indicate repeated multiplication. Consider recording and displaying student responses for all to see during the discussion.

Here are some questions to consider for discussion:

- “What does it mean when we write 2^{42} ?” (2^{42} means that 2 has been repeatedly multiplied 42 times. To expand this into factors would show 42 factors that are 2.)
- “How many times larger is 2^{45} than 2^{42} ?” (2^{45} is 8 times larger than 2^{42} because it has 3 more factors that are 2, so it has been multiplied by 2 an extra 3 times.)
- “What does it mean when I write $(\frac{1}{2})^{42}$?” ($(\frac{1}{2})^{42}$ means that $\frac{1}{2}$ has been repeatedly multiplied 42 times. To expand this into factors would show 42 factors that are $\frac{1}{2}$.)
- “Which is greater, $(\frac{1}{2})^{42}$ or $(\frac{1}{2})^{45}$? Why?” ($(\frac{1}{2})^{42}$ is greater since multiplying by $\frac{1}{2}$ results in a value closer to 0 and $(\frac{1}{2})^{45}$ has been multiplied by $\frac{1}{2}$ three extra times.)

1.4 Exponent Check

Cool Down: 5 minutes

Building On

- 6.EE.A.1

Building Towards

- 8.EE.A.1

Student Task Statement

1. What is the value of 3^4 ?
2. How many times bigger is 3^{15} compared to 3^{12} ?

Student Response

1. 81, because $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 9 \cdot 3 \cdot 3 = 27 \cdot 3 = 81$.
2. 3^{15} is 27 times larger than 3^{12} , because 3^{15} has 3 more factors that are 3 and $3^3 = 27$.

Student Lesson Summary

Exponents make it easy to show repeated multiplication. For example,

$$2^6 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2.$$

One advantage to writing 2^6 is that we can see right away that this is 2 to the *sixth* power. When this is written out using multiplication, $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$, we need to count the number of factors. Imagine writing out 2^{100} using multiplication!

Let's say you start out with one grain of rice and that each day the number of grains of rice you have doubles. So on day one, you have 2 grains, on day two, you have 4 grains, and so on. When we write 2^{25} , we can see from the expression that the rice has doubled 25 times. So this notation is not only convenient, but it also helps us see structure: in this case, we can see right away that it is on the 25th day that the number of grains of rice has doubled! That's a lot of rice (more than a cubic meter)!

Glossary

- exponent

Lesson 1 Practice Problems

Problem 1

Statement

Write each expression using an exponent:

a. $1 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

b. $1 \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right)$

c. $1 \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3)$

d. The number of coins Jada will have on the eighth day, if Jada starts with one coin and the number of coins doubles every day. (She has two coins on the first day of the doubling.)

Solution

a. 7^5

b. $\left(\frac{4}{5}\right)^5$

c. $(9.3)^8$

d. 2^8

Problem 2

Statement

Evaluate each expression:

a. 2^5

b. 3^3

c. 4^3

a. 6^2

b. $\left(\frac{1}{2}\right)^4$

c. $\left(\frac{1}{3}\right)^2$

Solution

a. 32

b. 27

c. 64

d. 36

e. $\frac{1}{16}$

f. $\frac{1}{9}$

Problem 3

Statement

Clare made \$160 babysitting last summer. She put the money in a savings account that pays 3% interest per year. If Clare doesn't touch the money in her account, she can find the amount she'll have the next year by multiplying her current amount by 1.03.

- How much money will Clare have in her account after 1 year? After 2 years?
- How much money will Clare have in her account after 5 years? Explain your reasoning.
- Write an expression for the amount of money Clare would have after 30 years if she never withdraws money from the account.

Solution

- \$164.80, \$169.74
- \$185.48. Reasoning varies. Sample reasoning: $160 \cdot 1.03^5$ (or multiply 160 by 1.03 five times)
- $160 \cdot 1.03^{30}$

Problem 4

Statement

The equation $y = 5,280x$ gives the number of feet, y , in x miles. What does the number 5,280 represent in this relationship?

Solution

There are 5,280 feet in every mile. For example, each additional mile that someone travels is equivalent to traveling an additional 5,280 feet.

(From Unit 3, Lesson 1.)

Problem 5

Statement

The points (2, 4) and (6, 7) lie on a line. What is the slope of the line?

A. 2

B. 1

C. $\frac{4}{3}$

D. $\frac{3}{4}$

Solution

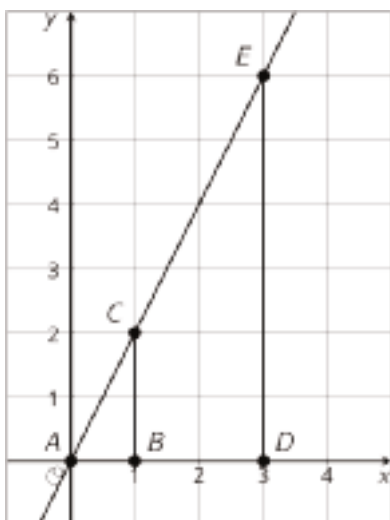
D

(From Unit 3, Lesson 5.)

Problem 6

Statement

The diagram shows a pair of similar figures, one contained in the other. Name a point and a scale factor for a dilation that moves the larger figure to the smaller one.



Solution

Center: A , scale factor: $\frac{1}{3}$

(From Unit 2, Lesson 6.)

Section: Exponent Rules

Lesson 2: Multiplying Powers of Ten

Goals

- Generalize a process for multiplying exponential expressions with the same base, and justify (orally and in writing) that $10^n \cdot 10^m = 10^{n+m}$.

Learning Targets

- I can explain and use a rule for multiplying powers of 10.

Lesson Narrative

Students make use of repeated reasoning to discover the exponent rule $10^n \cdot 10^m = 10^{n+m}$ (MP8). At this time, students develop rules for positive exponents. In subsequent lessons, students will extend the exponent rules to cases where the exponents are zero or negative. Students reason abstractly and quantitatively when applying exponent rules, pausing to consider the meaning of quantities, not just how to compute them (MP2).

Alignments

Building On

- 5.NBT.A.3.a: Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g.,
 $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.

Addressing

- 8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Building Towards

- 8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
- 8.EE.A.3: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9 , and determine that the world population is more than 20 times larger.
- 8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Instructional Routines

- MLR2: Collect and Display
- MLR3: Clarify, Critique, Correct
- Think Pair Share

Required Materials

Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart

paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Required Preparation

Create visual displays of the exponent rule $10^n \cdot 10^m = 10^{n+m}$ to be displayed for all to see throughout the unit. Here is a sample visual display:

Rule	Example showing how it works
$10^n \cdot 10^m = 10^{n+m}$	$10^2 \cdot 10^3 = (10 \cdot 10) \cdot (10 \cdot 10 \cdot 10) = 10^5$ two factors that are ten + three factors that are ten = five factors that are ten

Student Learning Goals

Let's explore patterns with exponents when we multiply powers of 10.

2.1 100, 1, or $\frac{1}{100}$?

Warm Up: 5 minutes

This warm-up gives students a chance to think about different numbers that a diagram might represent. In later activities, students thinking about diagrams that represent different powers of 10.

Building On

- 5.NBT.A.3.a

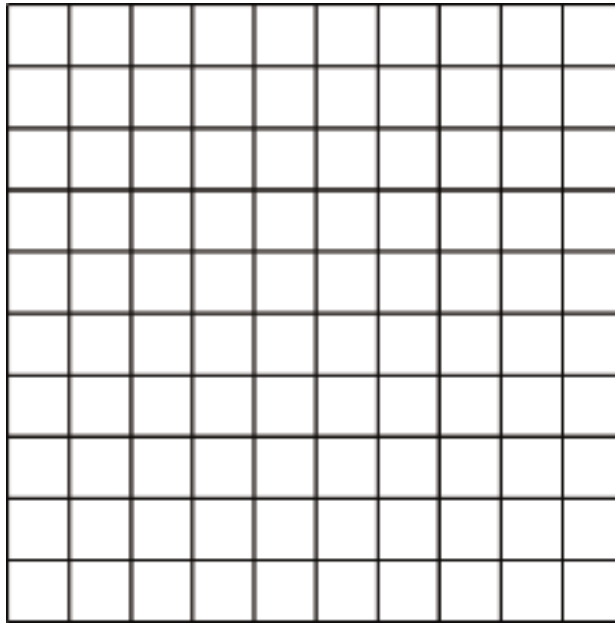
Building Towards

- 8.EE.A.1
- 8.EE.A.3

Launch

Give students 1 minute of quiet think time followed by 2 minutes of partner discussion.

Student Task Statement



Clare said she sees 100.

Tyler says he sees 1.

Mai says she sees $\frac{1}{100}$.

Who do you agree with?

Student Response

Answers vary. Sample response: I agree with all of them. There are 100 small squares and 1 big square. Each small square is $\frac{1}{100}$ of the large square.

Activity Synthesis

Poll the class to see who agrees with each person in turn. Then ask someone to explain in each case.

2.2 Picture a Power of 10

15 minutes

The purpose of this activity is for students to develop a sense of visual scale between powers of 10. Students should understand that multiplying by 10 corresponds to increasing the exponent by 1. Even though the notation for 10^{100} does not appear to be much different than 10^{98} , it is 100

times larger. Small changes in the exponent can result in large changes in the value of the expression.

Building Towards

- 8.EE.A.1
- 8.EE.A.3
- 8.EE.A.4

Instructional Routines

- MLR2: Collect and Display
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 5 minutes of quiet work time followed by 5 minutes to share their responses with their partner and a whole-class discussion.

Access for Students with Disabilities

Representation: Provide Access for Perception. Provide students with additional copies of the representations to draw on or highlight. Students may benefit from being able to mark each small square with the different powers of ten to develop a sense of visual scale between powers of 10.

Supports accessibility for: Conceptual processing; Visual-spatial processing

Access for English Language Learners

Conversing, Representing, Writing: MLR2 Collect and Display. Circulate and listen to students talk about the different powers of 10 representations during pair work or group work, and jot notes about important words or phrases (e.g., power of 10, multiply by 10) and expressions (e.g., 10^2 vs. $10 \cdot 10$), together with helpful sketches or diagrams of the respective rectangle and squares. Scribe students' words and sketches on a visual display to refer back to during whole-class discussions throughout the lesson and unit. This will help students read and use mathematical language during their paired and whole-group discussions.

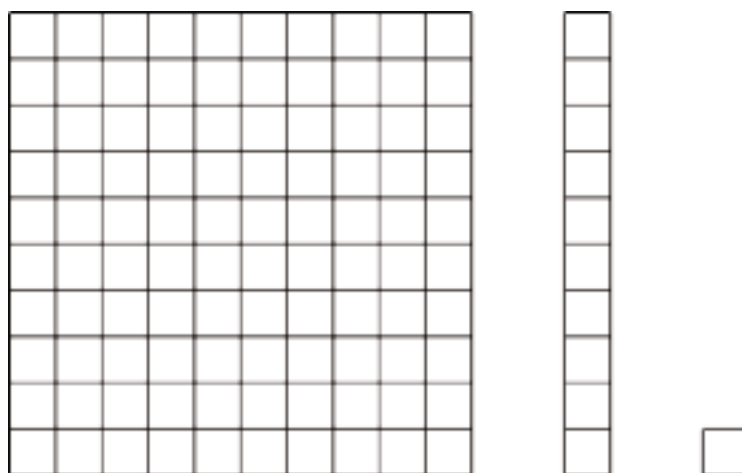
Design Principle(s): Support sense-making

Anticipated Misconceptions

Students may multiply the exponent by 10, for example 10^2 , 10^{20} , 10^{200} . Ask these students to write out what 10^{20} would mean and whether that matches their intention.

Student Task Statement

In the diagram, the medium rectangle is made up of 10 small squares. The large square is made up of 10 medium rectangles.



1. How could you represent the large square as a power of 10?
2. If each small square represents 10^2 , then what does the medium rectangle represent?
The large square?
3. If the medium rectangle represents 10^5 , then what does the large square represent?
The small square?
4. If the large square represents 10^{100} , then what does the medium rectangle represent?
The small square?

Student Response

1. The large square can be seen as 10^2 because there are 100 small squares.
2. The medium rectangle represents 10^3 because $10^2 \cdot 10 = 10^3$. The large square represents 10^4 because $10^2 \cdot 10^2 = 10^4$.
3. The small square represents 10^4 because $10^5 \div 10 = 10^4$. The large square represents 10^6 because $10^5 \cdot 10 = 10^6$.
4. The medium rectangle represents 10^{99} because $10^{100} \div 10 = 10^{99}$ and the small square represents 10^{98} because $10^{100} \div 10^2 = 10^{98}$.

Activity Synthesis

The key takeaway is that increasing the exponent of a power of 10 by 1 corresponds to multiplying or dividing by 10, and decreasing the exponent by 1 corresponds to dividing by 10.

Ask students to share their responses for the last several questions. Record and display their responses for all to see. If possible, reference the image to highlight the magnitude of change when the power of 10 increases or decreases by 1 or 2. If a student gives an answer such as, "the area of

the medium rectangle is $10^2 + 10^2 + \dots + 10^2$, ask for volunteers to help write the expression using a single power of 10.

If projection is available, consider sharing this applet, which illustrates animals measured in units that vary by powers of ten. <https://ggbm.at/NhpASDzz>

2.3 Multiplying Powers of Ten

15 minutes

The goal of this activity is to help students flexibly transition between different notations for powers of 10 and introduce the property of multiplication of values with the same base. Students observe that $10^n \cdot 10^m = 10^{n+m}$ for values of n and m that are positive integers. The second question hints at the reasoning that will extend exponent rules to include zero exponents, but students will investigate that more deeply in a later lesson.

Notice students who need help writing the general rule in terms of n and m . You might ask, “What patterns did you notice with the exponents in the table? So if the exponents are n and m , how do you write what you did with the exponents?”

Addressing

- 8.EE.A.1

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Give students 1 minute of quiet think time to complete the first unfinished row in the table before asking 1–2 students to share and explain their answers. When it is clear that students know how to complete the table, explain to them that they can skip one entry in the table, but they have to be able to explain why they skipped it. Give students 7–8 minutes to work before a brief whole-class discussion.

Student Task Statement

1. a. Complete the table to explore patterns in the exponents when multiplying powers of 10. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power of 10
$10^2 \cdot 10^3$	$(10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^5
$10^4 \cdot 10^3$		
$10^4 \cdot 10^4$		
	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$	
$10^{18} \cdot 10^{23}$		

- b. If you chose to skip one entry in the table, which entry did you skip? Why?
2. a. Use the patterns you found in the table to rewrite $10^n \cdot 10^m$ as an equivalent expression with a single exponent, like 10^{\square} .
- b. Use your rule to write $10^4 \cdot 10^0$ with a single exponent. What does this tell you about the value of 10^0 ?
3. The state of Georgia has roughly 10^7 human residents. Each human has roughly 10^{13} bacteria cells in his or her digestive tract. How many bacteria cells are there in the digestive tracts of all the humans in Georgia?

Student Response

1. a.

expression	expanded	single power of 10
$10^2 \cdot 10^3$	$(10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^5
$10^4 \cdot 10^3$	$(10 \cdot 10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^7
$10^4 \cdot 10^4$	$(10 \cdot 10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10)$	10^8
$10^3 \cdot 10^5$	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$	10^8
$10^{18} \cdot 10^{23}$	skip	10^{41}

- b. I chose to skip the expanded column of $10^{18} \cdot 10^{23}$ because there are too many factors that are 10 and they won't fit in the table.
2. a. $10^n \cdot 10^m = 10^{n+m}$ because multiplying n factors that are 10 with m factors that are 10 results in $n + m$ factors that are 10.
- b. 10^4 because $10^4 \cdot 10^0 = 10^{4+0}$. That means 10^0 must equal 1 for the rule to work.

3. There are 10^{20} bacteria because 10^7 people times 10^{13} bacteria per person is equal to 10^{20} total bacteria.

Are You Ready for More?

There are four ways to make 10^4 by multiplying powers of 10 with smaller, positive exponents.

$$10^1 \cdot 10^1 \cdot 10^1 \cdot 10^1$$

$$10^1 \cdot 10^1 \cdot 10^2$$

$$10^1 \cdot 10^3$$

$$10^2 \cdot 10^2$$

(This list is complete if you don't pay attention to the order you write them in. For example, we are only counting $10^1 \cdot 10^3$ and $10^3 \cdot 10^1$ once.)

1. How many ways are there to make 10^6 by multiplying smaller powers of 10 together?
2. How about 10^7 ? 10^8 ?

Student Response

1. This question is equivalent to the question, "How many ways are there to write the number 6 as the sum of smaller positive whole numbers?" There are ten ways to do this.
2. 14 ways, 22 ways

Activity Synthesis

Create and post visual displays showing the exponent rules for reference throughout the unit, with one visual display for each rule. The visual display could include an example to illustrate how the rule works, along with visual aids and use of color. Here is a sample visual display:

Rule	Example for Why it Works
$10^n \cdot 10^m = 10^{n+m}$	$10^2 \cdot 10^3 = (10 \cdot 10) \cdot (10 \cdot 10 \cdot 10) = 10^5$ <p style="text-align: center;"> two factors that are ten + three factors that are ten = five factors that are ten </p>

Explain the visual display to students and display it for all to see throughout the unit.

Access for English Language Learners

Writing: MLR3 Clarify, Critique, Correct. Present a hypothetical student statement that represents a misunderstanding of the exponent rule, such as “ $10^2 \cdot 10^3 = 10^6$ because $2 \cdot 3 = 6$.” Prompt discussion by asking, “Do you agree with the statement? Why or why not?” Then, ask students to individually write an improved statement. Improved responses should include connections between the representations of a single expression, expanded form, and the single power of 10. This will help students evaluate, and improve on, the written mathematical arguments of others and highlight why $10^n \cdot 10^m = 10^{n+m}$.

Design Principle(s): Maximize meta-awareness.

Lesson Synthesis

The purpose of the discussion is to check whether students understand why $10^n \cdot 10^m = 10^{n+m}$. Consider recording student responses and displaying them for all to see.

Here are some questions for discussion:

- “How could you write $10^{15} \cdot 10^5$ using a single exponent without expanding all of the factors?” (The first part is 15 factors that are 10 and the second is 5 factors that are 10. This makes a total of 20 factors that are 10.)
- “In general, what is a rule for multiplying two powers of 10 together into a single power of 10?” (The exponents of the two powers of 10 are added together.)

2.4 That's a Lot of Dough, Though!

Cool Down: 5 minutes

Addressing

- 8.EE.A.1

Student Task Statement

1. Rewrite $10^{32} \cdot 10^6$ using a single exponent.
2. Each year, roughly 10^6 computer programmers each make about $\$10^5$. How much money is this all together? Express your answer both as a power of 10 and as a dollar amount.

Student Response

1. 10^{38} , because $10^{32} \cdot 10^6 = 10^{32+6} = 10^{38}$.
2. This is \$100,000,000,000 (one hundred billion dollars). Each programmer makes 10^5 dollars, and there are 10^6 programmers. So we multiply $10^5 \cdot 10^6 = 10^{5+6}$, which is 10^{11} .

Student Lesson Summary

In this lesson, we developed a rule for multiplying powers of 10: multiplying powers of 10 corresponds to adding the exponents together. To see this, multiply 10^5 and 10^2 . We know that 10^5 has five factors that are 10 and 10^2 has two factors that are 10. That means that $10^5 \cdot 10^2$ has 7 factors that are 10.

$$10^5 \cdot 10^2 = (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10) = 10^7.$$

This will work for other powers of 10 too. So $10^{14} \cdot 10^{47} = 10^{61}$.

This rule makes it easier to understand and work with expressions that have exponents.

Lesson 2 Practice Problems

Problem 1

Statement

Write each expression with a single exponent:

a. $10^3 \cdot 10^9$

b. $10 \cdot 10^4$

c. $10^{10} \cdot 10^7$

d. $10^3 \cdot 10^3$

e. $10^5 \cdot 10^{12}$

f. $10^6 \cdot 10^6 \cdot 10^6$

Solution

a. 10^{12}

b. 10^5

c. 10^{17}

d. 10^6

e. 10^{17}

f. 10^{18}

Problem 2

Statement

A large rectangular swimming pool is 1,000 feet long, 100 feet wide, and 10 feet deep. The pool is filled to the top with water.

a. What is the area of the surface of the water in the pool?

b. How much water does the pool hold?

c. Express your answers to the previous two questions as powers of 10.

Solution

a. 100,000 square feet

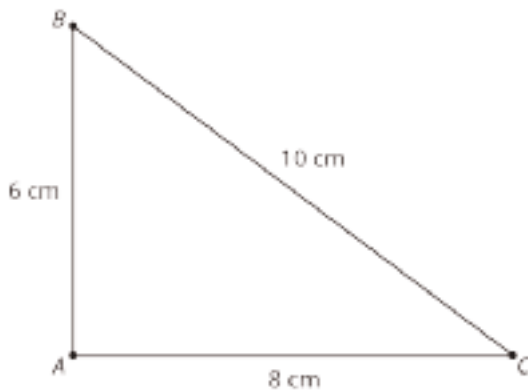
b. 1,000,000 cubic feet

c. 10^5 square feet, 10^6 cubic feet

Problem 3

Statement

Here is triangle ABC . Triangle DEF is similar to triangle ABC , and the length of EF is 5 cm. What are the lengths of sides DE and DF , in centimeters?



Solution

$DE = 3$ and $DF = 4$ (The scale factor is $\frac{1}{2}$, so each side length is half the corresponding side length of ABC .)

(From Unit 2, Lesson 7.)

Problem 4

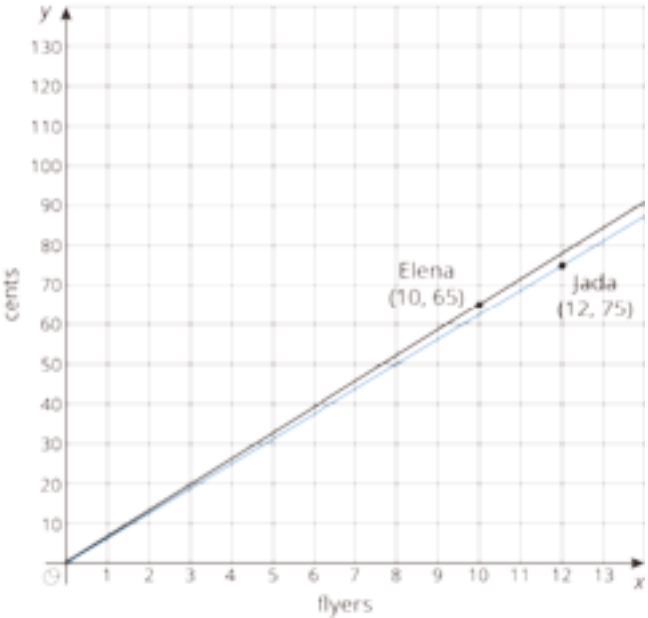
Statement

Elena and Jada distribute flyers for different advertising companies. Elena gets paid 65 cents for every 10 flyers she distributes, and Jada gets paid 75 cents for every 12 flyers she distributes.

Draw graphs on the coordinate plane representing the total amount each of them earned, y , after distributing x flyers. Use the graph to decide who got paid more after distributing 14 flyers.



Solution



Elena is paid more money after distributing 14 flyers because her graph is steeper than Jada's. For any number of flyers, the point on Elena's graph is higher than the point on Jada's graph.

(From Unit 3, Lesson 3.)

Lesson 3: Powers of Powers of 10

Goals

- Generalize a process for finding a power raised to a power, and justify (orally and in writing) that $(10^n)^m = 10^{n \cdot m}$.

Learning Targets

- I can explain and use a rule for raising a power of 10 to a power.

Lesson Narrative

Students make use of repeated reasoning to discover the exponent rule $(10^n)^m = 10^{n \cdot m}$ (MP8). At this time, students develop rules for positive exponents. In subsequent lessons, students will extend the exponent rules to cases where the exponents are zero or negative. Students reason abstractly and quantitatively when applying exponent rules, pausing to consider the meaning of quantities, not just how to compute them (MP2).

Alignments

Addressing

- 8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Building Towards

- 8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR8: Discussion Supports
- Think Pair Share

Required Preparation

Create a visual display for the rule $(10^m)^n = 10^{m \cdot n}$ to display for all to see throughout the unit. The display will be introduced during the discussion of "Taking Powers of Powers of 10" activity. For an example of how the rule works, consider showing $(10^2)^3 = (10 \cdot 10)(10 \cdot 10)(10 \cdot 10) = 10^6$ using colors or other visual aids to highlight that the result is 10^6 because there are three groups of 10^2 . For example,

Rule

$$(10^n)^m = 10^{n \cdot m}$$

Example for Why it Works

$$(10^2)^3 = \underbrace{(10 \cdot 10)}_{\text{two factors that are ten}} \cdot \underbrace{(10 \cdot 10)}_{\text{two factors that are ten}} \cdot \underbrace{(10 \cdot 10)}_{\text{two factors that are ten}} = 10^6$$

three groups of two factors that are ten = six factors that are ten

Student Learning Goals

Let's look at powers of powers of 10.

3.1 Big Cube

Warm Up: 5 minutes

The purpose of this warm-up is to introduce students the idea of raising a value with an exponent to another power. Computing the volume of a cube whose side lengths are themselves powers of 10 introduces the basic structure of a power to a power, which will lead to a general exponent rule during later activities.

Look out for different strategies used to compute $10,000^3$ so they can be discussed during whole-class discussion. Some students may count zeros to keep track of place value, while other students will write 10,000 as 10^4 and use exponent rules.

Addressing

- 8.EE.A.1

Building Towards

- 8.EE.A.4

Launch

Give students 3 minutes of quiet work time followed by a brief whole-class discussion.

Anticipated Misconceptions

Some students may want to compute by multiplying $(10,000) \cdot (10,000) \cdot (10,000)$. Ask these students whether they can use powers of 10 and the exponent rule from the last lesson to make their calculations easier.

Student Task Statement

What is the volume of a giant cube that measures 10,000 km on each side?

Student Response

1,000,000,000,000 km^3 , because $10,000^3 = (10^4)^3 = 10^4 \cdot 10^4 \cdot 10^4$. Using the exponent rule from the previous lesson, this is equal to 10^{4+4+4} or just 10^{12} . The side length is given in km, so the volume will be in units of km^3 .

Activity Synthesis

The purpose of discussion is to highlight the fact that $(10^4)^3$ is equal to 10^{12} as a way to transition to the next activity, where this pattern is generalized. Select previously identified students to share their strategies for computing $10,000^3$. Ask students what patterns they notice between $(10^4)^3$ and 10^{12} . If students mention the strategy of counting zeros to multiply powers of 10, connect it to the process of adding exponents. In other words, the total volume in km^3 is a 1 followed by the number of zeros in $(10,000) \cdot (10,000) \cdot (10,000)$ because $10^4 \cdot 10^4 \cdot 10^4 = 10^{4+4+4}$.

3.2 Raising Powers of 10 to Another Power

15 minutes

Students explore patterns to discover the property $(10^m)^n = 10^{m \cdot n}$ for values of m and n that are positive integers. Non-positive integer exponents will be explored in a subsequent lesson. Notice students who try to rush to complete the table without recognizing the patterns moving between columns in the table. As students work, ask them if they can explain what patterns they are finding. Select students who can explain the patterns they see to share during whole-class discussion.

Addressing

- 8.EE.A.1

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Give students 1 minute of quiet think time to complete the first unfinished row in the table, then select 1–2 students to share and explain their solutions. When it is clear that students know how to complete the table, explain to them that they can skip one entry in the table, but they have to be able to explain why they skipped it. Give students 10–12 minutes to work before a brief whole-class discussion.

Student Task Statement

1. a. Complete the table to explore patterns in the exponents when raising a power of 10 to a power. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power of 10
$(10^3)^2$	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^6
$(10^2)^5$	$(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)$	
	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	
$(10^4)^2$		
$(10^8)^{11}$		

- b. If you chose to skip one entry in the table, which entry did you skip? Why?
2. Use the patterns you found in the table to rewrite $(10^m)^n$ as an equivalent expression with a single exponent, like 10^{\square} .
3. If you took the amount of oil consumed in 2 months in 2013 worldwide, you could make a cube of oil that measures 10^3 meters on each side. How many cubic meters of oil is this? Do you think this would be enough to fill a pond, a lake, or an ocean?

Student Response

1. a.

expression	expanded	single power of 10
$(10^3)^2$	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^6
$(10^2)^5$	$(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)$	10^{10}
$(10^3)^4$	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^{12}
$(10^4)^2$	$(10 \cdot 10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10)$	10^8
$(10^8)^{11}$	skip	10^{88}

- b. Skip the expanded column of $(10^8)^6$ because the table cannot fit 88 factors that are 10.
2. $(10^m)^n = 10^{m \cdot n}$ because there are n groups of m factors that are 10.
3. This is 10^9 cubic meters of oil because $(10^3)^3 = 10^9$. Answers vary. Sample response: This is closest to a lakeful of oil.

Activity Synthesis

Select students who can explain the patterns they noticed to share in a whole-class discussion. Create a visual display for the rule $(10^m)^n = 10^{m \cdot n}$ to display for all to see throughout the unit. For an example of how the rule works, consider showing $(10^2)^3 = (10 \cdot 10)(10 \cdot 10)(10 \cdot 10) = 10^6$ using colors or other visual aids to highlight that the result is 10^6 because there are three groups of 10^2 . For example,

Rule	Example for Why it Works
$(10^m)^n = 10^{m \cdot n}$	$(10^2)^3 = \underbrace{(10 \cdot 10)}_{\text{two factors that are ten}} \cdot \underbrace{(10 \cdot 10)}_{\text{two factors that are ten}} \cdot \underbrace{(10 \cdot 10)}_{\text{two factors that are ten}} = 10^6$ three groups of two factors that are ten = six factors that are ten

Access for English Language Learners

Writing, Speaking: MLR1 Stronger and Clearer Each Time. Use this routine to give students a structured opportunity to revise and refine their explanation of the patterns they noticed in the task. Ask each student to meet with 2–3 other partners in a row for feedback. Provide student with prompts for feedback that will help students strengthen their ideas and clarify their language (e.g., “Can you give an example?”, “Can you say that another way?”, “How do you know...?”, etc.). Students can borrow ideas and language from each partner to strengthen their final explanation.

Design Principle(s): Optimize output (for explanation)

3.3 How Do the Rules Work?

10 minutes

This activity presents two valid ways to write $10^2 \cdot 10^2 \cdot 10^2$ with a single exponent, but where the execution of one of the ideas has a mistake. Thinking through this problem will reveal whether students understand the two exponent rules discussed in this lesson, providing opportunity for formative assessment.

Notice students who point out what was correct in both Andre and Elena’s responses. The eventual goal of learning exponent rules is to avoid expanding factors, but expect students at this stage to expand exponential expressions to test whether or not each step is correct.

Addressing

- 8.EE.A.1

Building Towards

- 8.EE.A.4

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2. Give 6–7 minutes to work followed by partner and whole-class discussions.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Continue to display, or provide a physical copy of the visual display for the rule $(10^n)^m = 10^{n \cdot m}$ from the previous activity.

Supports accessibility for: Memory; Conceptual processing

Student Task Statement

Andre and Elena want to write $10^2 \cdot 10^2 \cdot 10^2$ with a single exponent.

- Andre says, "When you multiply powers with the same **base**, it just means you add the exponents, so $10^2 \cdot 10^2 \cdot 10^2 = 10^{2+2+2} = 10^6$."
- Elena says, " 10^2 is multiplied by itself 3 times, so $10^2 \cdot 10^2 \cdot 10^2 = (10^2)^3 = 10^{2+3} = 10^5$."

Do you agree with either of them? Explain your reasoning.

Student Response

Answers vary. Sample response: Andre uses the rule for multiplying values with the same base correctly. Elena is partially correct because $10^2 \cdot 10^2 \cdot 10^2 = (10^2)^3$, but she did not use the correct exponent rule. She should have written $(10^2)^3 = 10^{2 \cdot 3}$ which is also equal to 10^6 .

Are You Ready for More?

$2^{12} = 4,096$. How many other whole numbers can you raise to a power and get 4,096? Explain or show your reasoning.

Student Response

Since $4,096 = 2^{12}$, it can be broken down into other representations of the form $(2^m)^n$ so that $m \cdot n = 12$. For example, $(2^2)^6 = 4^6$, $(2^3)^4 = 8^4$, $(2^4)^3 = 16^3$, $(2^6)^2 = 64^2$, and $(2^{12})^1 = 4,096^1$.

Activity Synthesis

It is important to note in the discussion that Elena is not completely wrong. She recognizes that $10^2 \cdot 10^2 \cdot 10^2$ is equivalent to $(10^2)^3$, which shows good conceptual understanding of exponents as repeated multiplication. Select students to share any correct and incorrect steps that they and their partner noticed in Andre and Elena's work. Ask students:

- "Andre wrote $10^2 \cdot 10^2 \cdot 10^2 = 10^{2+2+2}$ and Elena wrote $10^2 \cdot 10^2 \cdot 10^2 = (10^2)^3$. How are these ways of thinking different? How are they the same?" (Both methods are combining three powers using properties of exponents without writing out all of the factors. Andre's method only considers the bases of the three powers to combine them while Elena's method notices that each power is actually the same.)
- "What is one way you could avoid making the kinds of mistake that happened in this problem?" (Even if I don't write out all the factors, I could think about how many there will be if I were to write them out.)

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this to amplify mathematical uses of language to communicate about multiplying powers, bases, and exponents. Invite students to use these words when stating their ideas, and restating the ideas of others. Ask students to chorally repeat the phrases that include these words in context.

Design Principle(s): Support sense-making; Optimize output (for explanation)

Lesson Synthesis

The purpose of the discussion is to check whether students understand why $(10^n)^m = 10^{n \cdot m}$. Consider recording student responses and displaying them for all to see.

Here are some questions for discussion:

- "We looked at repeated multiplication of powers of 10. How would you write $10^4 \cdot 10^4 \cdot 10^4$ with exponents instead of repeated multiplication?" ($(10^4)^3$)
- "Then how would you write $(10^4)^3$ using a single exponent?" (10^{12})
- "In general, why do you multiply the exponents when you write a power to a power with a single exponent? Give an example to show your reasoning." (You multiply the exponents because the inner exponent shows how many factors are in each group and the outer exponent shows how many groups of factors there are. For example, $(10^4)^3$ means that there are 3 groups of factors, and each group has 4 factors that are 10. So there are a total of $3 \cdot 4 = 12$ factors that are 10 altogether.)

3.4 Making a Million

Cool Down: 5 minutes

There are many ways to express a given power of 10 using the exponent sum and product rules. Generating many ways of expressing the same value encourages the student to think more deeply about the rules and how they work.

Addressing

- 8.EE.A.1

Student Task Statement

Here are some equivalent ways of writing 10^4 :

- 10,000
- $10 \cdot 10^3$
- $(10^2)^2$

Write as many expressions as you can that have the same value as 10^6 . Focus on using exponents and multiplication.

Student Response

Answers vary. Sample responses:

- 1,000,000
- A million
- $10^2 \cdot 10^4$
- $(10^3)^2$

Student Lesson Summary

In this lesson, we developed a rule for taking a power of 10 to another power: Taking a power of 10 and raising it to another power is the same as multiplying the exponents. See what happens when raising 10^4 to the power of 3.

$$(10^4)^3 = 10^4 \cdot 10^4 \cdot 10^4 = 10^{12}$$

This works for any power of powers of 10. For example, $(10^6)^{11} = 10^{66}$. This is another rule that will make it easier to work with and make sense of expressions with exponents.

Glossary

- base (of an exponent)

Lesson 3 Practice Problems

Problem 1

Statement

Write each expression with a single exponent:

- a. $(10^7)^2$
- b. $(10^9)^3$
- c. $(10^6)^3$
- d. $(10^2)^3$
- e. $(10^3)^2$
- f. $(10^5)^7$

Solution

- a. 10^{14}
- b. 10^{27}
- c. 10^{18}
- d. 10^6
- e. 10^6
- f. 10^{35}

Problem 2

Statement

You have 1,000,000 number cubes, each measuring one inch on a side.

- a. If you stacked the cubes on top of one another to make an enormous tower, how high would they reach? Explain your reasoning.
- b. If you arranged the cubes on the floor to make a square, would the square fit in your classroom? What would its dimensions be? Explain your reasoning.
- c. If you layered the cubes to make one big cube, what would be the dimensions of the big cube? Explain your reasoning.

Solution

- a. The height of the tower would be 1,000,000 inches. That is about 83,333 feet ($1,000,000 \div 12 \approx 83,333$), which is almost 16 miles ($83,333 \div 5,280 \approx 15.78$).

- b. The square would be 1,000 inches on a side, because $1,000 \cdot 1,000 = 1,000,000$. 1,000 inches is about 83 feet. This probably wouldn't fit in a classroom.
- c. The cube would be 100 inches on a side because $100 \cdot 100 \cdot 100 = 1,000,000$. This is $8\frac{1}{3}$ feet.

Problem 3

Statement

An amoeba divides to form two amoebas after one hour. One hour later, each of the two amoebas divides to form two more. Every hour, each amoeba divides to form two more.

- How many amoebas are there after 1 hour?
- How many amoebas are there after 2 hours?
- Write an expression for the number of amoebas after 6 hours.
- Write an expression for the number of amoebas after 24 hours.
- Why might exponential notation be preferable to answer these questions?

Solution


- 2
- 4
- 2^6 (or 64)
- 2^{24} (or 16,777,216)
- Exponential notation is simpler to write than very large or small numbers, and the expression 2^{24} visibly includes the information that the amoebas have divided 24 times.

(From Unit 7, Lesson 1.)

Problem 4

Statement

Elena noticed that, nine years ago, her cousin Katie was twice as old as Elena was then. Then Elena said, "In four years, I'll be as old as Katie is now!" If Elena is currently e years old and Katie is k years old, which system of equations matches the story?



A. $\begin{cases} k - 9 = 2e \\ e + 4 = k \end{cases}$

B. $\begin{cases} 2k = e - 9 \\ e = k + 4 \end{cases}$

C. $\begin{cases} k = 2e - 9 \\ e + 4 = k + 4 \end{cases}$

D. $\begin{cases} k - 9 = 2(e - 9) \\ e + 4 = k \end{cases}$

Solution

D

(From Unit 4, Lesson 15.)

Lesson 4: Dividing Powers of 10

Goals

- Generalize a process for dividing powers of 10, and justify (orally and in writing) that $\frac{10^n}{10^m} = 10^{n-m}$.
- Use exponent rules to multiply and divide with 10^0 , and justify (orally) that 10^0 is 1.

Learning Targets

- I can evaluate 10^0 and explain why it makes sense.
- I can explain and use a rule for dividing powers of 10.

Lesson Narrative

Students continue to use repeated reasoning to discover the exponent rule $\frac{10^n}{10^m} = 10^{n-m}$ (MP8). For now, students work with expressions where n and m are positive integers and $n > m$. In the last activity, students extend to the case where $n = m$ to make sense of why 10^0 is defined to be equal to 1 and critique a faulty argument that it should be defined to be equal to 0 (MP3). Students make sense of this rule when they recognize that separating the same number of factors from the numerator and denominator, then dividing has the effect of multiplying by 1. This essentially means that m factors are subtracted from the n factors in the numerator. For example $\frac{10^3}{10^2} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10$ which is the same as $1 \cdot 10 = 10$ or the same as $10^{3-2} = 10^1$. In a subsequent lesson, students will extend this rule to include situations where $n < m$.

Alignments

Building On

- 5.NF.B.5.b: Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

Addressing

- 8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Building Towards

- 8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Instructional Routines

- MLR1: Stronger and Clearer Each Time

- MLR8: Discussion Supports
- Notice and Wonder
- Think Pair Share

Required Materials

Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart

paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Required Preparation

Create a visual display for the rule $\frac{10^n}{10^m} = 10^{n-m}$. For a guiding example, consider

$$\frac{10^5}{10^2} = \frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 \cdot 10 = 1 \cdot 10^3 = 10^3.$$

Create a visual display for the rule $10^0 = 1$. For an example, you can show

$$\frac{10^6}{1} = \frac{10^6}{10^0} = 10^{6-0} = 10^6. \text{ Another possibility is to write } 10^5 \cdot 1 = 10^5 \cdot 10^0 = 10^{5+0} = 10^5 \text{ and}$$

use visual aids to highlight that each of these examples implies $10^0 = 1$.

Student Learning Goals

Let's explore patterns with exponents when we divide powers of 10.

4.1 A Surprising One

Warm Up: 10 minutes

In this activity, students investigate fractions that are equal to 1. This is an important concept that helps students make sense of the exponent division rule explored later in this lesson. It is expected that students will try to compute the numerator and denominator of the fraction directly in the first problem. Notice any students who instead make use of structure to show multiplication by 1.

Building On

- 5.NF.B.5.b

Building Towards


- 8.EE.A.1

Launch

Give students 5 minutes of quiet work time. Expect students to attempt to work out all of the multiplication without using exponent rules. Follow with a brief whole-class discussion.

Student Task Statement

What is the value of the expression?


$$\frac{2^5 \cdot 3^4 \cdot 3^2}{2 \cdot 3^6 \cdot 2^4}$$

Student Response

$\frac{2^5 \cdot 3^4 \cdot 3^2}{2 \cdot 3^6 \cdot 2^4}$ is equal to 1. Strategies vary. Sample strategies:

- Compute the numerator and denominator and then realize that they are equal ($\frac{23,328}{23,328}$), giving an overall value of 1.
- Notice that there are 5 factors that are 2 and 6 factors that are 3 in both numerator and denominator, thus making the entire fraction equal to 1.

Activity Synthesis

The key takeaway is that a fraction is often easier to analyze when dividing matching factors from the numerator and denominator to show multiplication by 1. Select any students who made use of structure in this way. If no students did this, provide the following example:

$$\frac{2 \cdot 3 \cdot 7 \cdot 11}{5 \cdot 3 \cdot 7 \cdot 11} = \frac{3 \cdot 7 \cdot 11}{3 \cdot 7 \cdot 11} \cdot \frac{2}{5} = 1 \cdot \frac{2}{5} = \frac{2}{5}.$$

If time allows, consider the following questions for discussion:

- “What has to be true about a fraction for it to equal 1?” (The numerator and denominator must be the same value and something other than 0.)
- “Create your own fraction that is equivalent to 1 that has several bases and several exponents.”

4.2 Dividing Powers of Ten

10 minutes

Explore division of powers of 10 to derive the rule $\frac{10^n}{10^m} = 10^{n-m}$. At this point, students will only be working in cases where $n > m$ and will later extend the rule to include $n = m$. The case of $n < m$ is left for the next lesson. The rule arises from the fact that $\frac{10^n}{10^m} = \frac{10^m}{10^m} \cdot 10^{n-m}$ which is equivalent to 10^{n-m} . These problems are also meant to underscore the connection between fractions and division, and students should be expected to use both fractions and division interchangeably as they work.

Addressing

- 8.EE.A.1

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- Notice and Wonder

Launch

Before students begin working, ask students to notice and wonder about the “expanded” column for the expression $10^4 \div 10^2$.

It is important for students to understand that the “expanded” column shows each power of 10 expanded into factors, the division written as a fraction, and a certain number of factors in the numerator and denominator being grouped because their quotient is 1.

Give students 5–7 minutes quiet work time followed by a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support use of structure. For example, check in with students within the first 2-3 minutes of work time. Ask students to share how they made sense of the first row of the table. Use color or annotation to highlight connections between the “expanded” column and the single power.

Supports accessibility for: Memory; Organization

Student Task Statement

- a. Complete the table to explore patterns in the exponents when dividing powers of 10. Use the “expanded” column to show why the given expression is equal to the single power of 10. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power
$10^4 \div 10^2$	$\frac{10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 = 1 \cdot 10 \cdot 10$	10^2
	$\frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 \cdot 10 = 1 \cdot 10 \cdot 10 \cdot 10$	
$10^6 \div 10^3$		
$10^{43} \div 10^{17}$		

- b. If you chose to skip one entry in the table, which entry did you skip? Why?
- Use the patterns you found in the table to rewrite $\frac{10^n}{10^m}$ as an equivalent expression of the form 10^{\square} .

3. It is predicted that by 2050, there will be 10^{10} people living on Earth. At that time, it is predicted there will be approximately 10^{12} trees. How many trees will there be for each person?

Student Response

expression	expanded	single power
$10^4 \div 10^2$	$\frac{10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 = 1 \cdot 10 \cdot 10$	10^2
$10^5 \div 10^2$	$\frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 \cdot 10 = 1 \cdot 10 \cdot 10 \cdot 10$	10^3
$10^6 \div 10^3$	$\frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10} = \frac{10 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10} \cdot 10 \cdot 10 \cdot 10 = 1 \cdot 10 \cdot 10 \cdot 10$	10^3
$10^{43} \div 10^{17}$	skip	10^{26}

b. I chose to skip the expanded column of $10^{43} \div 10^{17}$ because there is not enough space in the table for all of the factors.

2. $\frac{10^n}{10^m} = 10^{n-m}$ because m factors in the numerator and denominator are divided to make 1, leaving $n - m$ factors remaining.
3. There are roughly 100 trees per person because 10^{12} trees divided equally among 10^{10} people is $10^{12-10} = 10^2$ trees per person.

Are You Ready for More?

expression	expanded	single power
$10^4 \div 10^6$		

Student Response

expression	expanded	single power
$10^4 \div 10^6$	$\frac{10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10} = \frac{10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10 \cdot 10} \cdot \frac{1}{10} \cdot \frac{1}{10} = 1 \cdot \frac{1}{10} \cdot \frac{1}{10}$	$\frac{1}{10^2}$

Activity Synthesis

The key idea for writing $\frac{10^n}{10^m}$ with one exponent is that m factors in the numerator and denominator can be divided to make 1, leaving $n - m$ factors in the numerator. Here are some sample questions to guide students to this idea:

- “Say we want to write $10^6 \div 10^3$ with a single power of 10. What happens to the 3 factors that are 10 in the denominator? How many factors that are 10 are left in the numerator?” (The 3 factors that are 10 in the denominator are matched with 3 of the factors that are 10 in the numerator, then divided to make 1. Since 3 factors that are 10 from the numerator are used to make 1, there are still 3 left.)
- “If you wanted to write $10^{80} \div 10^{20}$, what would happen with the 20 factors that are 10 in the denominator? How many factors that are 10 would still be left in the numerator?” (To make this connection explicit, you might show the calculation $\frac{10^{80}}{10^{20}} = \frac{10^{20} \cdot 10^{60}}{10^{20}} = \frac{10^{20}}{10^{20}} \cdot 10^{60}$. Compare this to $10^{80} \div 10^{20} = 10^{80-20} = 10^{60}$.)

Make another “Exponent Rule” poster for the rule $\frac{10^n}{10^m} = 10^{n-m}$. An example to illustrate the rule could be $\frac{10^5}{10^3} = \frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10} = \frac{10 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10} \cdot 10 \cdot 10 = 1 \cdot 10 \cdot 10 = 10^2$. Use colors and other visual aids to highlight the fact that the exponents are subtracted because m factors that are 10 in the numerator and denominator are used to make a factor of 1, leaving $n - m$ factors that are 10.

Access for English Language Learners

Writing, Speaking: MLR1 Stronger and Clearer Each Time. Use this routine to give students a structured opportunity to revise and refine their explanation of the patterns they noticed in the task. Ask each student to meet with 2–3 other partners in a row for feedback. Provide student with prompts for feedback that will help students strengthen their ideas and clarify their language (e.g., “Can you give an example?”, “Can you say that another way?”, “How do you know...?”, etc.). Students can borrow ideas and language from each partner to strengthen their final explanation.

Design Principle(s): Optimize output (for explanation)

4.3 Zero Exponent

15 minutes

Students extend exponent rules to discover why it makes sense to define 10^0 as 1. Students create viable arguments and critique the reasoning of others when discussing Noah’s argument that 10^0 should equal 0 (MP3).

Addressing

- 8.EE.A.1

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 5 minutes to answer the first 3 problems and a few minutes to discuss the last problem with a partner before a whole-class discussion.

Student Task Statement

So far we have looked at powers of 10 with exponents greater than 0. What would happen to our patterns if we included 0 as a possible exponent?

- Write $10^{12} \cdot 10^0$ with a power of 10 with a single exponent using the appropriate exponent rule. Explain or show your reasoning.
 - What number could you multiply 10^{12} by to get this same answer?
- Write $\frac{10^8}{10^0}$ with a single power of 10 using the appropriate exponent rule. Explain or show your reasoning.
 - What number could you divide 10^8 by to get this same answer?
- If we want the exponent rules we found to work even when the exponent is 0, then what does the value of 10^0 have to be?
- Noah says, "If I try to write 10^0 expanded, it should have zero factors that are 10, so it must be equal to 0." Do you agree? Discuss with your partner.

Student Response

- 10^{12} because $10^{12} \cdot 10^0 = 10^{12+0} = 10^{12}$.
 - 1 because $10^{12} \cdot 1 = 10^{12}$.
- 10^8 because $\frac{10^8}{10^0} = 10^{8-0} = 10^8$.
 - 1 because $\frac{10^8}{1} = 10^8$.
- The value of 10^0 has to be 1 in order for the exponent rules to work when the exponent is 0.
- Answers vary. Sample response: Noah's answer appears to make sense with how we first defined exponents, but if we want to be able to use the exponent rules when the exponents are not positive, we have to define it differently.

Activity Synthesis

The important concept is that $10^0 = 1$ is a convenient definition that extends the usefulness of the exponent rules to a wider range of numbers. This idea is developed in the next lesson to further extend exponent rules to include negative exponents.

Ask students to share their thinking about what 10^0 means. Noah's argument raises a valid concern that 10^0 doesn't fit the definition that we use when the exponent is positive. One way to address this concern is to allow alternate definitions and see what happens to the exponent rules. For example, if we want to define 10^0 as 0, then we can choose to do that. However, when we look at an example like $10^3 \cdot 10^0$, the result would be zero. This just means 10^0 does not match the pattern of the exponent rule, which says the result should be 10^{3+0} which is equal to 10^3 .

Introduce the visual display for the rule $10^0 = 1$. Display for all to see throughout the unit. To illustrate the rule, consider displaying the example $\frac{10^6}{10^0} = 10^{6-0} = 10^6$. For this to be true, the denominator 10^0 must be equal to 1. Another possibility is to write $10^5 \cdot 10^0 = 10^{5+0} = 10^5$ which can only be true if $10^0 = 1$.

Access for English Language Learners

Speaking: MLR8 Discussion Supports. To scaffold students in explaining the patterns they noticed when 10^0 , provide sentence frames for students to use when they are working with their partner. For example, "I think ___ because ____ ." or "I (agree/disagree) because ____ ."

Design Principle(s): Support sense-making; Optimize output for (explanation)

4.4 Making Millions

Optional: 10 minutes

This activity expands on a previous cool-down as students generate different representations of the same number to solidify what they have learned about exponent sum, product, and subtraction rules. Notice students who use the exponent rules they have learned in different ways to achieve an exponent of 6, especially those who combine different exponent rules together. It is not expected that students will make an exponent of 6 using negative exponents, but do not discourage it if they do. Explain to these students that, while the rules still work when using negative exponents, it is not yet clear what the value of, say, 10^{-2} is.

Addressing

- 8.EE.A.1

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. Give 5–7 minutes of quiet work time before asking students to share their response with their partner. Follow with a whole-class discussion.

Student Task Statement

Write as many expressions as you can that have the same value as 10^6 . Focus on using exponents, multiplication, and division. What patterns do you notice with the exponents?

Student Response

Answers vary. Sample responses:

- 1,000,000
- A million
- $10^4 \cdot 10^2$
- $(10^2)^3$
- $10^8 \div 10^2$
- $\frac{10^{80}}{10^{74}}$ For multiplying powers of 10, the exponents must always have a sum of 6. For dividing powers of 10, the exponents must always have a difference of 6. When raising powers of 10 to another power, the exponents must have a product that is 6.

Activity Synthesis

Tell students to compare their response with their partner's to discuss what differences there might be. In a whole-class discussion, select students to share a variety of responses. Look especially for examples that show creativity or that combine multiple rules together. The main idea is for students to show flexibility with the exponent rules they have learned so far. To include more students in the discussion, consider asking: "How can you build on __'s method to come up with another expression that makes a million?" If time allows, ask whether it's possible to make an exponent of 6 using negative numbers and indicate that a future lesson will explore negative exponents in detail.

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion. For each observation or response that is shared, ask students to restate and/or revoice what they heard using mathematical language. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. This will provide more students with an opportunity to produce language as they interpret the reasoning of others.

Design Principle(s): Support sense-making

Lesson Synthesis

The goal of the discussion is to check that students can explain why the exponents are subtracted when rewriting a quotient of powers of 10 with a single exponent, and why it makes sense to define 10^0 as equal to 1. Consider recording student responses and displaying them for all to see.

Here are possible discussion questions:

- “How can you write $\frac{10^{36}}{10^{12}}$ using a single exponent?” (10^{24})
- “It’s a common mistake for students to try to divide the exponents. Why do we subtract the exponents instead of divide them?” (The exponents subtract because we are counting the number of factors that are 10 that survive division. In this case, we have $\frac{10^{36}}{10^{12}} = \frac{10^{12} \cdot 10^{24}}{10^{12}} = 10^{24}$.)
- “Why did we define 10^0 to be equal to 1?” (10^0 is defined to be equal to 1 so that it fits with the exponent rules we have discovered for positive exponents. It is a logical extension of the rules. For example, the rules indicate $10^4 \cdot 10^0$ should be equal to 10^{4+0} , which is just 10^4 . So 10^0 is a number that doesn’t change the value of other numbers when it is multiplied. The only number with this property is 1, so it only makes sense to define 10^0 as 1.)

4.5 Why Subtract?

Cool Down: 5 minutes

Addressing

- 8.EE.A.1

Student Task Statement

Why is $\frac{10^{15}}{10^4}$ equal to 10^{11} ? Explain or show your thinking.

Student Response

Answers vary. Sample response: $\frac{10^{15}}{10^4} = 10^{11}$ because 4 factors that are 10 in the numerator and denominator are used to make 1, leaving 11 remaining factors that are 10. In other words,

$$\frac{10^{15}}{10^4} = \frac{10^4 \cdot 10^{11}}{10^4} = 10^{11}.$$

Student Lesson Summary

In an earlier lesson, we learned that when multiplying powers of 10, the exponents add together. For example, $10^6 \cdot 10^3 = 10^9$ because 6 factors that are 10 multiplied by 3 factors that are 10 makes 9 factors that are 10 all together. We can also think of this multiplication equation as division:

$$10^6 = \frac{10^9}{10^3}$$

So when dividing powers of 10, the exponent in the denominator is subtracted from the exponent in the numerator. This makes sense because

$$\frac{10^9}{10^3} = \frac{10^3 \cdot 10^6}{10^3} = \frac{10^3}{10^3} \cdot 10^6 = 1 \cdot 10^6 = 10^6$$

This rule works for other powers of 10 too. For example, $\frac{10^{56}}{10^{23}} = 10^{33}$ because 23 factors that are 10 in the numerator and in the denominator are used to make 1, leaving 33 factors remaining.

This gives us a new exponent rule:

$$\frac{10^n}{10^m} = 10^{n-m}.$$

So far, this only makes sense when n and m are positive exponents and $n > m$, but we can extend this rule to include a new power of 10, 10^0 . If we look at $\frac{10^6}{10^0}$, using the exponent rule gives 10^{6-0} , which is equal to 10^6 . So dividing 10^6 by 10^0 doesn't change its value. That means that if we want the rule to work when the exponent is 0, then it must be that

$$10^0 = 1$$

Lesson 4 Practice Problems

Problem 1

Statement

Evaluate:

a. 10^0

b. $\frac{10^3}{10^3}$

c. $10^2 + 10^1 + 10^0$

Solution

a. 1

b. 1

c. 111

Problem 2

Statement

Write each expression as a single power of 10.

a. $\frac{10^3 \cdot 10^4}{10^5}$

b. $(10^4) \cdot \frac{10^{12}}{10^7}$

c. $\left(\frac{10^5}{10^3}\right)^4$

d. $\frac{10^4 \cdot 10^5 \cdot 10^6}{10^3 \cdot 10^7}$

e. $\frac{(10^5)^2}{(10^2)^3}$

Solution

a. 10^2

b. 10^9

c. 10^8

d. 10^5

e. 10^4

Problem 3

Statement

The Sun is roughly 10^2 times as wide as Earth. The star KW Sagittarii is roughly 10^5 times as wide as Earth. About how many times as wide as the Sun is KW Sagittarii? Explain how you know.

Solution

10^3 (or 1,000). This can be determined by calculating $\frac{10^5}{10^2}$, since both the Sun and KW Sagittarii's widths can be compared to the width of Earth.

Problem 4

Statement

Bananas cost \$1.50 per pound, and guavas cost \$3.00 per pound. Kiran spends \$12 on fruit for a breakfast his family is hosting. Let b be the number of pounds of bananas Kiran buys and g be the number of pounds of guavas he buys.

- Write an equation relating the two variables.
- Rearrange the equation so b is the independent variable.
- Rearrange the equation so g is the independent variable.

Solution

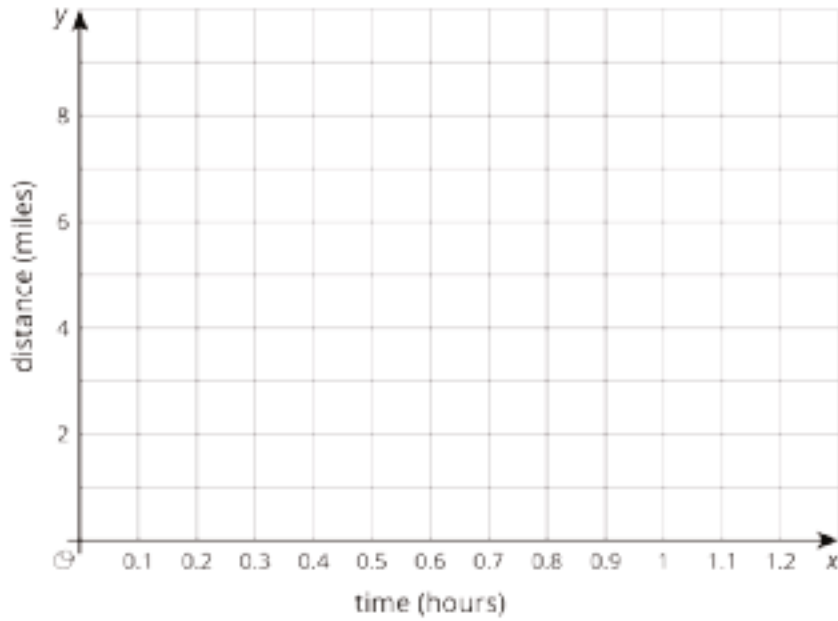
- $1.5b + 3g = 12$
- $g = 4 - \frac{1}{2}b$
- $b = 8 - 2g$

(From Unit 5, Lesson 3.)

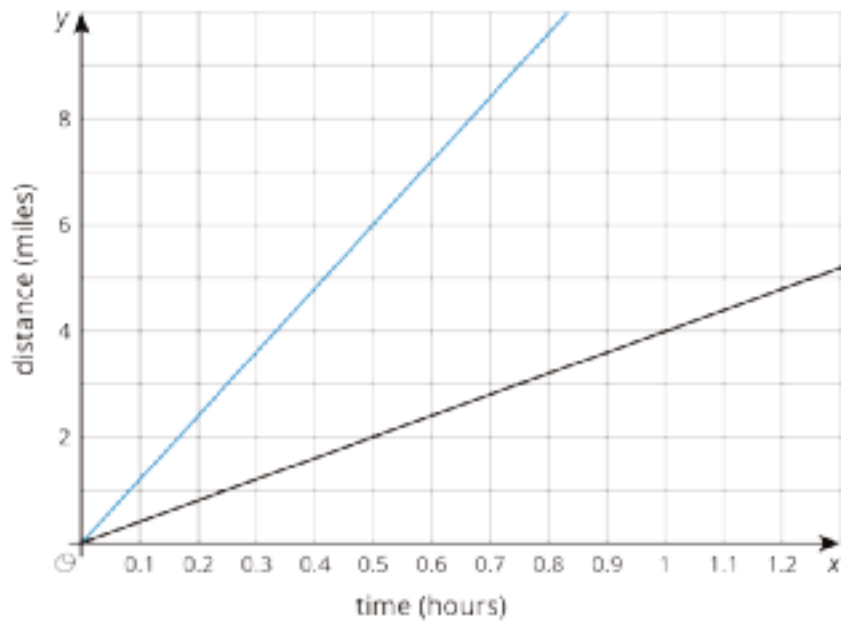
Problem 5

Statement

Lin's mom bikes at a constant speed of 12 miles per hour. Lin walks at a constant speed $\frac{1}{3}$ of the speed her mom bikes. Sketch a graph of both of these relationships.



Solution



(From Unit 3, Lesson 1.)

Lesson 5: Negative Exponents with Powers of 10

Goals

- Describe (orally and in writing) how exponent rules extend to expressions involving negative exponents.
- Describe patterns in repeated multiplication and division with 10 and $\frac{1}{10}$, and justify (orally and in writing) that $10^{-n} = \frac{1}{10^n}$.

Learning Targets

- I can use the exponent rules with negative exponents.
- I know what it means if 10 is raised to a negative power.

Lesson Narrative

Sometimes in mathematics, extending existing theories to areas outside of the original definition leads to new insights and new ways of thinking. Students practice this here by extending the rules they have developed for working with powers to a new situation with negative exponents. The challenge then becomes to make sense of what negative exponents might mean. This type of reasoning appears again in high school when students extend the rules of exponents to make sense of exponents that are not integers.

In analogy to positive powers of 10 that describe repeated multiplication by 10, this lesson presents negative powers of 10 as repeated multiplication by $\frac{1}{10}$, leading ultimately to the rule $10^{-n} = \frac{1}{10^n}$. Students use repeated reasoning to generalize about negative exponents (MP8). Students create viable arguments and critique the reasoning of others when comparing and contrasting, for example, $(10^{-2})^3$ and $(10^2)^{-3}$ (MP3). With this understanding of negative exponents, all of the exponent rules created so far are seen to be valid for any integer exponents.

Alignments

Building On

- 5.NBT.A.2: Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Addressing

- 8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports

- Number Talk

Required Materials

Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart

paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Required Preparation

Create a visual display for $10^{-n} = \frac{1}{10^n}$. For an example of how the rule works, consider showing $10^{-3} = \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{10^3}$.

Student Learning Goals

Let's see what happens when exponents are negative.

5.1 Number Talk: What's That Exponent?

Warm Up: 10 minutes

The purpose of this Number Talk is to elicit strategies and understandings students have for dividing powers. These understandings help students develop fluency and will be helpful later in this lesson when students investigate negative exponents. While four problems are given, it may not be possible to share every strategy. Consider gathering only two or three different strategies per problem. It is expected that students won't know how to approach the final question. Encourage them to make their best guess based on patterns they notice.

Addressing

- 8.EE.A.1

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 1 minute of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Allow students to share their answers with a partner and note any discrepancies.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Student Task Statement

Solve each equation mentally.

$$\frac{100}{1} = 10^x$$

$$\frac{100}{x} = 10^1$$

$$\frac{x}{100} = 10^0$$

$$\frac{100}{1000} = 10^x$$

Student Response

- $x = 2$. Possible strategies: Dividing 100 by 1 gives 100, and 100 is equal to 10^2 .
- $x = 10$. Possible strategies: 10^1 is equal to 10, and 100 divided by 10 is also equal to 10.
- $x = 100$. Possible strategies: 10^0 is equal to 1, so the left side of the equation must be $\frac{100}{100}$.
- $x = -1$. Possible strategies: Looking on the pattern on the right sides of the equations suggests the exponent may be -1. When the numerator is larger than the denominator, the exponent is positive. When the numerator is the same size as the denominator, the exponent is 0. In this case, the numerator is smaller than the denominator, so it appears the exponent should be negative.

Activity Synthesis

Ask students to share what they noticed about the first three problems. Record the equations with the solutions written in place:

$$\frac{100}{1} = 10^2$$

$$\frac{100}{10} = 10^1$$

$$\frac{100}{100} = 10^0$$

Ask students to describe any patterns they see, and how they would continue the patterns for the last problem:

$$\frac{100}{1000} = 10^x$$

Tell them that in this lesson, we will explore negative exponents.

Access for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because . . ." or "I noticed ____ so I" Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Design Principle(s): Optimize output (for explanation)

5.2 Negative Exponent Table

10 minutes

Students extend their understanding of exponents to include negative exponents and explain patterns in the placements of the decimal point when a decimal is multiplied by 10 or $\frac{1}{10}$. Students use repeated reasoning to recognize that negative powers of 10 represent repeated multiplication by $\frac{1}{10}$ and generalize to the rule $10^{-n} = \frac{1}{10^n}$ (MP8).

A table is used to show different representations of decimals, fractions, and exponents. The table is horizontal to mimic the structure of decimals. As students work, notice the strategies they use to go between the different representations of the given powers of 10. Ask students who use contrasting strategies to share later.

Building On

- 5.NBT.A.2

Addressing

- 8.EE.A.1

Instructional Routines

- MLR2: Collect and Display

Launch

Tell students to complete the table one row at a time to see the patterns most clearly. Ask a student to read the first question aloud. Select a student to explain the idea of a “multiplier” in this context. Give students 5–7 minutes to work. Follow with a whole-class discussion.

Anticipated Misconceptions

Some students may think that, for example, $\frac{1}{1,000,000} = 10^{-7}$ because the number 1,000,000 has 7 digits. Ask these students if it is true that $\frac{1}{10} = 10^{-2}$.

Student Task Statement

Complete the table to explore what negative exponents mean.

		$\overset{-10}{\curvearrowright}$	$\overset{-10}{\curvearrowright}$	$\overset{-10}{\curvearrowright}$	$\overset{-10}{\curvearrowright}$	$\overset{-10}{\curvearrowright}$	$\overset{-10}{\curvearrowright}$
using exponents	10^3	10^2	10^1				
as a decimal	1000.0			1.0		0.01	
as a fraction		$\frac{100}{1}$		$\frac{1}{1}$			$\frac{1}{1000}$
		$\underset{-7}{\curvearrowleft}$	$\underset{-7}{\curvearrowleft}$	$\underset{-7}{\curvearrowleft}$	$\underset{-7}{\curvearrowleft}$	$\underset{-7}{\curvearrowleft}$	$\underset{-7}{\curvearrowleft}$

- As you move toward the left, each number is being multiplied by 10. What is the multiplier as you move right?
- How does a multiplier of 10 affect the placement of the decimal in the product? How does the other multiplier affect the placement of the decimal in the product?
- Use the patterns you found in the table to write 10^{-7} as a fraction.
- Use the patterns you found in the table to write 10^{-5} as a decimal.
- Write $\frac{1}{100,000,000}$ using a single exponent.
- Use the patterns in the table to write 10^{-n} as a fraction.

Student Response

using exponents	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
as a decimal	1,000.0	100.0	10.0	1.0	0.1	0.01	0.001
as a fraction	$\frac{1,000}{1}$	$\frac{100}{1}$	$\frac{10}{1}$	$\frac{1}{1}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1,000}$

- As you move towards the right, the multiplier is $\frac{1}{10}$
- Multiplying by 10 results in the decimal point shifting one place to the right. Multiplying by $\frac{1}{10}$ results in the decimal point shifting one place to the left.

$$3. 10^{-7} = \frac{1}{10,000,000} \text{ or } \frac{1}{10^7}$$

$$4. 10^{-5} = 0.00001$$

$$5. \frac{1}{100,000,000} = 10^{-8}$$

$$6. 10^{-n} = \frac{1}{10^n}$$

Activity Synthesis

One important idea is that multiplying by 10 increases the exponent, thus multiplying by $\frac{1}{10}$ decreases the exponent. So negative exponents can be thought of as repeated multiplication by $\frac{1}{10}$, whereas positive exponents can be thought of as repeated multiplication by 10. Another key point is the effect that multiplying by 10 or $\frac{1}{10}$ has on the placement of the decimal.

Ask students to share how they converted between fractions, decimals, and exponents. Record their reasoning for all to see. Here are some possible questions to consider for whole-class discussion:

- “Do you agree or disagree? Why?”
- “Did anyone think of this a different way?”
- “In your own words, what does 10^{-7} mean? How is it different from 10^7 ?”

Introduce the visual display for $10^{-n} = \frac{1}{10^n}$ and display it for all to see throughout the unit. For an example that illustrates the rule, consider displaying $10^{-3} = \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{10^3}$.

Access for Students with Disabilities

Representation: Internalize Comprehension. Demonstrate and encourage students to use color coding and annotations to highlight connections. For example, annotate the patterns students notice to help synthesize how negative exponents can be thought of as repeated multiplication by $\frac{1}{10}$, whereas positive exponents can be thought of as repeated multiplication by 10.

Supports accessibility for: Visual-spatial processing

Access for English Language Learners

Conversing, Representing, Writing: MLR2 Collect and Display. During the whole-class discussion, capture the vocabulary and phrases students use to describe their strategies for converting between fractions, decimals, and exponents. Listen for students who make connections between repeated multiplication and the placement of the decimal point. Record this language onto a visual display that can be referenced in future discussions. This routine provides feedback to students that supports sense-making and increases meta-awareness of mathematical language.

Design Principle(s): Support sense-making; Maximize meta-awareness

5.3 Follow the Exponent Rules

20 minutes

This activity requires students to make sense of negative powers of 10 as repeated multiplication by $\frac{1}{10}$ in order to distinguish between equivalent exponential expressions. If students have time, instruct them to write the other expressions in each table as a power of 10 with a single exponent as well. Look for students who have productive debate regarding the interpretation of, for example, $(10^2)^{-3}$ versus $(10^{-2})^3$ and ask them to share their reasoning later (MP3).

Addressing

- 8.EE.A.1

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. Give students 15 minutes of partner work time followed by a whole-class discussion. Ask students to explain their reasoning to their partner as they work. If there is disagreement, tell students to work to reach an agreement.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, present one question at a time.

Supports accessibility for: Organization; Attention

Anticipated Misconceptions

Some students may struggle to distinguish between $(10^{-2})^3$ and $(10^2)^{-3}$. Breaking down each expression into parts that emphasize repeated multiplication will help to illustrate the difference. For the first expression, $10^{-2} = \frac{1}{10} \cdot \frac{1}{10}$. The outer exponent of 3 means that $\frac{1}{10} \cdot \frac{1}{10}$ is multiplied repeatedly 3 times. Similarly for the second expression, $10^2 = 10 \cdot 10$ and the outer exponent of -3 means that the *reciprocal* of $10 \cdot 10$ is multiplied repeatedly 3 times. In the end, both expressions are equal to 10^{-6} .

Student Task Statement

1. a. Match each exponential expression with an equivalent multiplication expression:

$$(10^2)^3$$

$$\frac{1}{(10 \cdot 10)} \cdot \frac{1}{(10 \cdot 10)} \cdot \frac{1}{(10 \cdot 10)}$$

$$(10^2)^{-3}$$

$$\left(\frac{1}{10} \cdot \frac{1}{10}\right) \left(\frac{1}{10} \cdot \frac{1}{10}\right) \left(\frac{1}{10} \cdot \frac{1}{10}\right)$$

$$(10^{-2})^3$$

$$\frac{1}{\frac{1}{10} \cdot \frac{1}{10}} \cdot \frac{1}{\frac{1}{10} \cdot \frac{1}{10}} \cdot \frac{1}{\frac{1}{10} \cdot \frac{1}{10}}$$

$$(10^{-2})^{-3}$$

$$(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)$$

- b. Write $(10^2)^{-3}$ as a power of 10 with a single exponent. Be prepared to explain your reasoning.

2. a. Match each exponential expression with an equivalent multiplication expression:

$$\frac{10^2}{10^5}$$

$$\frac{\frac{1}{10} \cdot \frac{1}{10}}{\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}}$$

$$\frac{10^2}{10^{-5}}$$

$$\frac{10 \cdot 10}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$$

$$\frac{10^{-2}}{10^5}$$

$$\frac{\frac{1}{10} \cdot \frac{1}{10}}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$$

$$\frac{10^{-2}}{10^{-5}}$$

$$\frac{10 \cdot 10}{\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}}$$

- b. Write $\frac{10^{-2}}{10^{-5}}$ as a power of 10 with a single exponent. Be prepared to explain your reasoning.

3. a. Match each exponential expression with an equivalent multiplication expression:

$10^4 \cdot 10^3$

$(10 \cdot 10 \cdot 10 \cdot 10) \cdot \left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right)$

$10^4 \cdot 10^{-3}$

$\left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right) \cdot \left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right)$

$10^{-4} \cdot 10^3$

$\left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right) \cdot (10 \cdot 10 \cdot 10)$

$10^{-4} \cdot 10^{-3}$

$(10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10)$

b. Write $10^{-4} \cdot 10^3$ as a power of 10 with a single exponent. Be prepared to explain your reasoning.

Student Response

1. a.

$(10^2)^3$	$(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)$
$(10^2)^{-3}$	$\frac{1}{(10 \cdot 10)} \cdot \frac{1}{(10 \cdot 10)} \cdot \frac{1}{(10 \cdot 10)}$
$(10^{-2})^3$	$\left(\frac{1}{10} \cdot \frac{1}{10}\right) \left(\frac{1}{10} \cdot \frac{1}{10}\right) \left(\frac{1}{10} \cdot \frac{1}{10}\right)$
$(10^{-2})^{-3}$	$\frac{1}{\frac{1}{10} \cdot \frac{1}{10}} \cdot \frac{1}{\frac{1}{10} \cdot \frac{1}{10}} \cdot \frac{1}{\frac{1}{10} \cdot \frac{1}{10}}$

b. $(10^2)^{-3} = 10^{2 \cdot (-3)} = 10^{-6}$ or $(10^2)^{-3} = \frac{1}{(10 \cdot 10)} \cdot \frac{1}{(10 \cdot 10)} \cdot \frac{1}{(10 \cdot 10)} = \frac{1}{10^6} = 10^{-6}$.

2. a.

$\frac{10^2}{10^5}$	$\frac{10 \cdot 10}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$
$\frac{10^2}{10^{-5}}$	$\frac{10 \cdot 10}{\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}}$
$\frac{10^{-2}}{10^5}$	$\frac{\frac{1}{10} \cdot \frac{1}{10}}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$
$\frac{10^{-2}}{10^{-5}}$	$\frac{\frac{1}{10} \cdot \frac{1}{10}}{\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}}$

b. $\frac{10^{-2}}{10^{-5}} = 10^{-2 - (-5)} = 10^{-2+5} = 10^3$ or

$$\frac{10^{-2}}{10^{-5}} = \frac{\frac{1}{10} \cdot \frac{1}{10}}{\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}} = \frac{\frac{1}{10^2}}{\frac{1}{10^5}} = \frac{1}{10^2} \cdot \frac{10^5}{1} = \frac{10^5}{10^2} = 10^3.$$

3. a.

$10^4 \cdot 10^3$	$(10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10)$
$10^4 \cdot 10^{-3}$	$(10 \cdot 10 \cdot 10 \cdot 10) \cdot (\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10})$
$10^{-4} \cdot 10^3$	$(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}) \cdot (10 \cdot 10 \cdot 10)$
$10^{-4} \cdot 10^{-3}$	$(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}) \cdot (\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10})$

b. $10^{-4} \cdot 10^3 = 10^{-4+3} = 10^{-1}$ or

$$10^{-4} \cdot 10^3 = (\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}) \cdot (10 \cdot 10 \cdot 10) = \frac{10^3}{10^4} = \frac{1}{10} = 10^{-1}.$$

Are You Ready for More?

Priya, Jada, Han, and Diego stand in a circle and take turns playing a game.

Priya says, SAFE. Jada, standing to Priya's left, says, OUT and leaves the circle. Han is next: he says, SAFE. Then Diego says, OUT and leaves the circle. At this point, only Priya and Han are left. They continue to alternate. Priya says, SAFE. Han says, OUT and leaves the circle. Priya is the only person left, so she is the winner.

Priya says, "I knew I'd be the only one left, since I went first."

1. Record this game on paper a few times with different numbers of players. Does the person who starts always win?
2. Try to find as many numbers as you can where the person who starts always wins. What patterns do you notice?

Student Response

1. No. If you play with five players, for example, the fifth person will win.
2. The person who starts will win if the number of people is a power of two. Otherwise, someone else will win.

Activity Synthesis

Select students who had disagreements during the activity to share what they disagreed about and how they came to an agreement. Consider asking:

- "How is $(10^{-2})^3$ different from $(10^2)^{-3}$? How are they the same?"
- "Do you agree or disagree? Why?"
- "Could you restate __'s reasoning in a different way?"

It is important for students to understand that the exponent rules work even with negative exponents. To see why, the whole class discussion must make a clear connection between the

exponent rules and the process of multiplying repeated factors that are 10 and $\frac{1}{10}$. Contrast the expanded version of $(10^{-2})^3$ and $(10^2)^{-3}$. For $(10^{-2})^3$, there are 3 factors that are 10^{-2} , where 10^{-2} is two factors that are $\frac{1}{10}$, so

$$(10^{-2})^3 = \left(\frac{1}{10} \cdot \frac{1}{10}\right) \left(\frac{1}{10} \cdot \frac{1}{10}\right) \left(\frac{1}{10} \cdot \frac{1}{10}\right) = \frac{1}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10} = 10^{-6}.$$

For $(10^2)^{-3}$, there are 3 factors that are $\frac{1}{10^2}$, so

$$(10^2)^{-3} = \left(\frac{1}{10 \cdot 10}\right) \left(\frac{1}{10 \cdot 10}\right) \left(\frac{1}{10 \cdot 10}\right) = \frac{1}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10} = 10^{-6}.$$

Both $(10^{-2})^3$ and $(10^2)^{-3}$ are equal to 10^{-6} in the same way that $-2 \cdot 3 = 2 \cdot -3 = -6$.

Access for English Language Learners

Listening, Conversing: MLR8 Discussion Supports. As students share the disagreements they had with their partner, press for details in students' statements by requesting that students elaborate on an idea or give an example of their process of reconciliation. For example, ask students, "How did you explain your reasoning to your partner in a way that convinced them?" In this discussion, it may be useful to revoice student ideas to model mathematical language use by restating a statement in order to clarify, apply appropriate language, or involve more students.

Design Principle(s): Maximize meta-awareness

Lesson Synthesis

The purpose of the discussion is to take a step back in order to see that negative exponents are not something new and different, but rather a natural part of the decimal place value system that we have been exploring for years.

Remind students that in elementary school, we learned about our place value system and saw that it was possible to write very large numbers in a very small space because of positional notation. We learned that the value of a number is the sum of the numbers of each base 10 unit (ones, tens, hundreds, and so forth). We sometimes wrote things like $456 = 4 \cdot 100 + 5 \cdot 10 + 6 \cdot 1$. We can express this with exponents as $456 = 4 \cdot 10^2 + 5 \cdot 10^1 + 6 \cdot 10^0$. We now extend the discussion of place value by considering ways to write very small numbers in a manner consistent with what we did with large numbers. Ask students:

- "How would you write 2,796 as a sum with powers of 10?"
($2 \cdot 10^3 + 7 \cdot 10^2 + 9 \cdot 10^1 + 6 \cdot 10^0$)
- "What about small numbers? How do you write the place value units of 0.1, 0.01, and 0.001 with powers of 10?" (10^{-1} , 10^{-2} , 10^{-3})

- “Then how would you write 0.2796 as a sum with powers of 10?”
($2 \cdot 10^{-1} + 7 \cdot 10^{-2} + 9 \cdot 10^{-3} + 6 \cdot 10^{-4}$)
- “Think about the meaning of exponents. How is 10^3 related to 10^{-3} ?” (Exponents tell us to repeatedly multiply by a base. Whether the base is 10 or $\frac{1}{10}$, the structure of repeated multiplication is the same. 10^3 is multiplication by 10 repeated 3 times and 10^{-3} is multiplication by $\frac{1}{10}$ repeated 3 times.)
- “Who would need to work with very large numbers? Who would need to work with very small numbers?” (Astronomers might need to work with very large numbers. Biologists, physicists, engineers and others might need to work with very small numbers.)

5.4 Negative Exponent True or False

Cool Down: 5 minutes

Addressing

- 8.EE.A.1

Student Task Statement

Mark each of the following equations as true or false. Explain or show your reasoning.

1. $10^{-5} = -10^5$

2. $(10^2)^{-3} = (10^{-2})^3$

3. $\frac{10^3}{10^{14}} = 10^{-11}$

Student Response

1. False, because $10^{-5} = \frac{1}{100,000}$, whereas $-10^5 = -100,000$.
2. True, because both $(10^2)^{-3}$ and $(10^{-2})^3$ are equal to 10^{-6} .
3. True, because $\frac{10^3}{10^{14}} = 10^{3-14} = 10^{-11}$.

Student Lesson Summary

When we multiply a positive power of 10 by $\frac{1}{10}$, the exponent *decreases* by 1:

$$10^8 \cdot \frac{1}{10} = 10^7$$

This is true for *any* positive power of 10. We can reason in a similar way that multiplying by 2 factors that are $\frac{1}{10}$ *decreases* the exponent by 2:

$$\left(\frac{1}{10}\right)^2 \cdot 10^8 = 10^6$$

That means we can extend the rules to use negative exponents if we make $10^{-2} = \left(\frac{1}{10}\right)^2$. Just as 10^2 is two factors that are 10, we have that 10^{-2} is two factors that are $\frac{1}{10}$. More generally, the exponent rules we have developed are true for *any* integers n and m if we make

$$10^{-n} = \left(\frac{1}{10}\right)^n = \frac{1}{10^n}$$

Here is an example of extending the rule $\frac{10^n}{10^m} = 10^{n-m}$ to use negative exponents:

$$\frac{10^3}{10^5} = 10^{3-5} = 10^{-2}$$

To see why, notice that

$$\frac{10^3}{10^5} = \frac{10^3}{10^3 \cdot 10^2} = \frac{10^3}{10^3} \cdot \frac{1}{10^2} = \frac{1}{10^2}$$

which is equal to 10^{-2} .

Here is an example of extending the rule $(10^m)^n = 10^{m \cdot n}$ to use negative exponents:

$$(10^{-2})^3 = 10^{(-2)(3)} = 10^{-6}$$

To see why, notice that $10^{-2} = \frac{1}{10} \cdot \frac{1}{10}$. This means that

$$(10^{-2})^3 = \left(\frac{1}{10} \cdot \frac{1}{10}\right)^3 = \left(\frac{1}{10} \cdot \frac{1}{10}\right) \cdot \left(\frac{1}{10} \cdot \frac{1}{10}\right) \cdot \left(\frac{1}{10} \cdot \frac{1}{10}\right) = \frac{1}{10^6} = 10^{-6}$$

Lesson 5 Practice Problems

Problem 1

Statement

Write with a single exponent: (ex: $\frac{1}{10} \cdot \frac{1}{10} = 10^{-2}$)

a. $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$

b. $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$

c. $(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10})^2$

d. $(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10})^3$

e. $(10 \cdot 10 \cdot 10)^{-2}$

Solution

a. 10^{-3}

b. 10^{-7}

c. 10^{-8}

d. 10^{-9}

e. 10^{-6}

Problem 2

Statement

Write each expression as a single power of 10.

a. $10^{-3} \cdot 10^{-2}$

b. $10^4 \cdot 10^{-1}$

c. $\frac{10^5}{10^7}$

d. $(10^{-4})^5$

e. $10^{-3} \cdot 10^2$

f. $\frac{10^{-9}}{10^5}$

Solution

a. 10^{-5}

- b. 10^3
- c. 10^{-2}
- d. 10^{-20}
- e. 10^{-1}
- f. 10^{-14}

Problem 3

Statement

Select all of the following that are equivalent to $\frac{1}{10,000}$:

- A. $(10,000)^{-1}$
- B. $(-10,000)$
- C. $(100)^{-2}$
- D. $(10)^{-4}$
- E. $(-10)^2$

Solution

["A", "C", "D"]

Problem 4

Statement

Match each equation to the situation it describes. Explain what the constant of proportionality means in each equation.

Equations:

- a. $y = 3x$
- b. $\frac{1}{2}x = y$
- c. $y = 3.5x$
- d. $y = \frac{5}{2}x$

Situations:

- A dump truck is hauling loads of dirt to a construction site. After 20 loads, there are 70 square feet of dirt.
- I am making a water and salt mixture that has 2 cups of salt for every 6 cups of water.
- A store has a "4 for \$10" sale on hats.
- For every 48 cookies I bake, my students get 24.

Solution

Explanations vary. Sample responses:

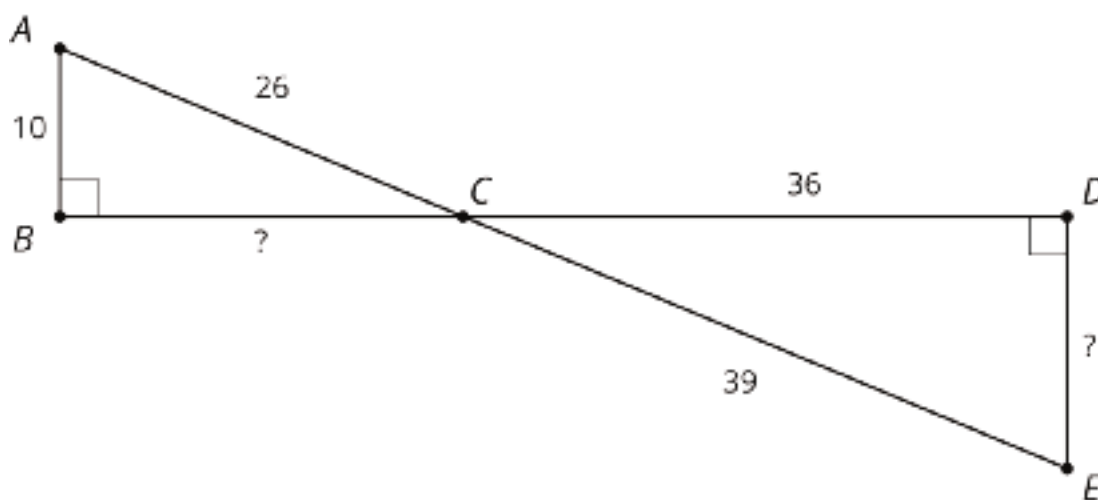
- a. Water and salt. For each cup of salt, there are 3 cups of water.
- b. Cookies. For every cookie I bake, my students get half.
- c. Dump truck. The dump truck hauls 3.5 square feet of dirt in each load.
- d. Sale on hats. Each hat costs \$2.50.

(From Unit 3, Lesson 2.)

Problem 5

Statement

- a. Explain why triangle ABC is similar to EDC .



- b. Find the missing side lengths.

Solution

- a. Explanations vary. Sample explanation: Both triangles contain a right angle, and angles ACB and ECD are vertical angles. The triangles are similar because two pairs of corresponding angles are congruent.
- b. Side BC measures 24, and side DE measures 15. (The scale factor is $\frac{39}{26}$ or 1.5.)

(From Unit 2, Lesson 8.)

Lesson 6: What about Other Bases?

Goals

- Generalize exponent rules for nonzero bases, including bases other than 10.
- Use exponent rules to identify (in writing) equivalent exponential expressions, and explain (orally) the reasoning.

Learning Targets

- I can use the exponent rules for bases other than 10.

Lesson Narrative

In this lesson, students use their understanding of exponent rules with powers of 10 to make sense of exponent rules for powers of other bases. Students work through problems analogous to the ones used to develop exponent rules for powers of 10. The same underlying patterns emerge, revealing the fact that the exponent rules are the same for bases other than 10. For simplicity, students do not develop general exponent rules for negative bases.

Alignments

Addressing

- 8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports
- Think Pair Share
- True or False

Required Materials

Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart

paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Required Preparation

Create a new set of visual displays for exponent rules that replace the base of 10 with other bases. For example, the visual display for the rule $10^n \cdot 10^m = 10^{n+m}$ should be updated to a new visual display with $b^n \cdot b^m = b^{n+m}$ with a guiding example that uses a base other than 10.

Student Learning Goals

Let's explore exponent patterns with bases other than 10.

6.1 True or False: Comparing Expressions with Exponents

Warm Up: 5 minutes

The purpose of this warm-up is for students to reason about powers of positive and negative numbers. While some students may find the numeric value of each expression to make a comparison, it is not always necessary. Encourage students to reason about the meaning of the integers, bases, and exponents rather than finding the numeric value of each expression.

Addressing

- 8.EE.A.1

Instructional Routines

- True or False

Launch

Display one problem at a time. Give students up to 1 minute of quiet think time per problem and tell them to give a signal when they have an answer and a strategy. Follow with a whole-class discussion.

There may be some confusion about how negative numbers interact with exponents. Tell students that exponents work the same way for both positive and negative bases. In other words, $(-3)^2 = -3 \cdot -3$.

Students may also struggle to interpret the notation difference between two expressions such as $(-4)^2$ and -4^2 . Tell students that the interpretation of the first expression is $(-4)^2 = -4 \cdot -4$ and the interpretation of the second expression is $-4^2 = -1 \cdot 4 \cdot 4$.

Student Task Statement

Is each statement true or false? Be prepared to explain your reasoning.

1. $3^5 < 4^6$
2. $(-3)^2 < 3^2$
3. $(-3)^3 = 3^3$
4. $(-5)^2 > -5^2$

Student Response

1. True. Sample response: The base and exponent are both smaller on the left.
2. False. Sample response: When you square a negative, the result is positive.
3. False. Sample response: When you cube a negative, the result is negative, and a negative is always less than a positive.

4. True. Sample response: A positive is always greater than a negative.

Activity Synthesis

Poll students on their response for each problem. Record and display their responses for all to see. If all students agree, ask 1 or 2 students to share their reasoning. If there is disagreement, ask students to share their reasoning until an agreement is reached. After the final problem, ask students if the same strategies would have worked if all of the bases were 10 to highlight the idea that powers with a base of 10 behave the same as those with bases other than 10.

To involve more students in the conversation, consider asking:

- “Do you agree or disagree? Why?”
- “Who can restate ___’s reasoning in a different way?”
- “Does anyone want to add on to ____’s reasoning?”

6.2 What Happens with Zero and Negative Exponents?

15 minutes

Extend the definition of non-positive exponents to other bases. The work of this activity is parallel to the work with the powers of 10 table in the previous lesson. Notice students who transfer their understanding of powers of 10, especially non-positive exponents, to exponential expressions with bases other than 10.

Addressing

- 8.EE.A.1

Instructional Routines

- MLR2: Collect and Display
- Think Pair Share

Launch

Arrange students in groups of 2. Tell students to work on their own and then share their reasoning with their partner after each has had a chance to complete the problem. Encourage students to look for similarities to work they have already done with base 10. Give students 10 minutes of work time, followed by a whole class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Check in with students within the first 2-3 minutes of work time. Look for students who did not notice direction of the arrows on the table.

Supports accessibility for: Visual-spatial processing; Organization

Access for English Language Learners

Conversing, Representing, Writing: MLR2 Collect and Display. Use this routine to support small-group and whole-class discussion. Circulate and listen to students as they work, capture the vocabulary and phrases students use to describe the patterns they noticed and how these are related to exponent rules. Listen for students who make connections between repeated multiplication, reciprocals, and negative and positive exponents. Record their language and any relevant diagrams onto a visual display that can be referenced in future discussions. This will help students produce and make sense of the language needed to communicate their reasoning of exponent rules for powers of positive bases other than 10.

Design Principle(s): Support sense-making; Maximize meta-awareness

Student Task Statement

Complete the table to show what it means to have an exponent of zero or a negative exponent.

value	16				$\frac{1}{2}$			
exponent form	2^4							

1. As you move toward the left, each number is being multiplied by 2. What is the multiplier as you move toward the right?
2. Use the patterns you found in the table to write 2^{-6} as a fraction.
3. Write $\frac{1}{32}$ as a power of 2 with a single exponent.
4. What is the value of 2^0 ?
5. From the work you have done with negative exponents, how would you write 5^{-3} as a fraction?
6. How would you write 3^{-4} as a fraction?

Student Response

value	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$
exponent form	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}

1. As you move right, each number is multiplied by $\frac{1}{2}$.
2. $2^{-6} = \frac{1}{2^6}$ or $\frac{1}{64}$
3. $\frac{1}{32} = \frac{1}{2^5} = 2^{-5}$
4. $2^0 = 1$.
5. $5^{-3} = \frac{1}{5^3}$ or $\frac{1}{125}$
6. $3^{-4} = \frac{1}{3^4}$ or $\frac{1}{81}$

Are You Ready for More?

1. Find an expression equivalent to $\left(\frac{2}{3}\right)^{-3}$ but with positive exponents.
2. Find an expression equivalent to $\left(\frac{4}{5}\right)^{-8}$ but with positive exponents.
3. What patterns do you notice when you start with a fraction raised to a negative exponent and rewrite it using a single positive exponent? Show or explain your reasoning.

Student Response

1. $\left(\frac{3}{2}\right)^3$, because the opposite of repeatedly multiplying by $\frac{2}{3}$ is repeatedly multiplying by $\frac{3}{2}$.
2. $\left(\frac{5}{4}\right)^8$, because the opposite of repeatedly multiplying by $\frac{4}{5}$ is repeatedly multiplying by $\frac{5}{4}$.
3. To write $\left(\frac{a}{b}\right)^{-n}$ with a positive exponent, notice that the opposite of repeatedly multiplying by $\frac{a}{b}$ is repeatedly multiplying by $\frac{b}{a}$. The general rule is that a fraction to a negative exponent is equal to the reciprocal to the positive exponent. So $\left(\frac{b}{a}\right)^n$

Activity Synthesis

The important idea for this activity is that the same logic that we used to analyze powers of 10 applies to other bases as well. Students will work mostly with bases that are whole numbers and positive fractions.

Here are some sample questions for whole-class discussion:

- “How does working with powers of 2 compare to working with powers of 10?” (The properties apply in the same way, but computing the value of the exponents is more difficult.)
- “What happens when you use exponent rules when one of the exponents is 0?” (The rules apply in the same way. $2^0 = 1$ in the same way that $10^0 = 1$.)
- “How does 10^{-3} compare to 2^{-3} ?” (Both values are positive and less than 1. Since 10^{-3} has greater numbers in the denominator, it is less than 2^{-3} .)

Introduce new (or update current) visual displays to illustrate exponent rules for positive, rational bases. Display them for all to see throughout the unit. An example would be to update the rule $10^n \cdot 10^m = 10^{n+m}$ with $x^n \cdot x^m = x^{n+m}$ with a guiding example that uses a base other than 10.

6.3 Exponent Rules with Bases Other than 10

15 minutes

This activity gives students a chance to practice using exponent rules to analyze expressions and identify equivalent ones. Five lists of expressions are given. Students choose two lists to analyze. As students work, notice the different strategies used to analyze the expressions in each list. Ask students using contrasting strategies to share later.

Addressing

- 8.EE.A.1

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. Tell students to take turns selecting an expression, determining whether the expression matches the original, and explaining their reasoning. Give students 10 minutes to work followed by a whole-class discussion.

Anticipated Misconceptions

Some students may struggle with comparing -5^9 to 5^{-9} . Ask these students to explain the difference between 10^3 and 10^{-3} , and relate that to the difference between 5^9 and 5^{-9} . If needed, explain that -5^9 is a large negative number, whereas 5^{-9} is a very small positive number.

Student Task Statement

Lin, Noah, Diego, and Elena decide to test each other’s knowledge of exponents with bases other than 10. They each chose an expression to start with and then came up with a new list of expressions; some of which are equivalent to the original and some of which are not.

Choose 2 of the 4 lists to analyze. For each list of expressions you choose to analyze, decide which expressions are *not* equivalent to the original. Be prepared to explain your reasoning.

1. Lin's original expression is 5^{-9} and her list is:

$$(5^3)^{-3} \quad -5^9 \quad \frac{5^{-6}}{5^3} \quad (5^3)^{-2} \quad \frac{5^{-4}}{5^{-5}} \quad 5^{-4} \cdot 5^{-5}$$

2. Noah's original expression is 3^{10} and his list is:

$$3^5 \cdot 3^2 \quad (3^5)^2 \quad (3 \cdot 3)(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)$$

$$\left(\frac{1}{3}\right)^{-10} \quad 3^7 \cdot 3^3 \quad \frac{3^{20}}{3^{10}} \quad \frac{3^{20}}{3^2}$$

3. Diego's original expression is x^4 and his list is:

$$\frac{x^8}{x^4} \quad x \cdot x \cdot x \cdot x \quad \frac{x^{-4}}{x^{-8}} \quad \frac{x^{-4}}{x^8}$$

$$(x^2)^2 \quad 4 \cdot x \quad x \cdot x^3$$

4. Elena's original expression is 8^0 and her list is:

$$1 \quad 0 \quad 8^3 \cdot 8^{-3}$$

$$\frac{8^2}{8^2} \quad 10^0 \quad 11^0$$

Student Response

1. The following are not equivalent to 5^{-9} :

- -5^9 because it is negative and 5^{-9} is positive.
- $(5^3)^{-2}$ because it is equal to 5^{-6} .
- $\frac{5^{-4}}{5^{-5}}$ because $\frac{5^{-4}}{5^{-5}} = 5^{-4-(-5)} = 5^{-4+5} = 5^1$.

2. The following are not equivalent to 3^{10} :

- $3^5 \cdot 3^2$ because $3^5 \cdot 3^2 = 3^{5+2} = 3^7$.
- $\frac{3^{20}}{3^2}$ because $\frac{3^{20}}{3^2} = 3^{20-2} = 3^{18}$.

3. The following are not equivalent to x^4 :

- $\frac{x^{-4}}{x^8}$ because $\frac{x^{-4}}{x^8} = x^{-4-8} = x^{-12}$.
- $4 \cdot x$. Sample reasoning: If $x = 10$, then $4 \cdot 10 = 40$ and $10^4 = 10000$.

4. The only expression not equivalent to 8^0 in Elena's list is 0, because all of the rest are defined to be equal to 1.

Activity Synthesis

The key idea is that the exponent rules are valid for nonzero bases other than 10. Select students to share which expressions did not match the original in the lists they chose to study. Consider focusing on questions 3 and 4 to assess how students have generalized in terms of variables and whether students have internalized the 0 exponent. Select previously identified students to share the different strategies used to analyze the same list.

Here are some questions to consider for discussion:

- “What mistakes might lead to the expressions that are not equivalent to the original expression?”
- “How do you know that expression is not equivalent to the original?”
- “Did anyone think about that expression in a different way?”

Access for English Language Learners

Speaking: MLR8 Discussion Supports. To support students in explaining why identified expressions are or are not equivalent, provide sentence frames for students to use when they are working with their partner. For example, “___ and ___ are (equivalent/not equivalent), because ___.” or “I (agree/disagree) because ___.”

Design Principle(s): Support sense-making; Optimize output for (explanation)

Lesson Synthesis

The purpose of the discussion is to check whether students understand that the rules they have developed for doing arithmetic with powers of 10 are valid for any other non-zero rational bases as well. This is also a good opportunity to address possible misconceptions students have, especially regarding non-positive exponents.

Consider using these discussion questions to emphasize the important concepts in this lesson:

- “We saw before that 10^3 shows the repeated multiplication $10 \cdot 10 \cdot 10$ and 10^{-3} shows the repeated multiplication $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$. How do 2^3 and 2^{-3} compare to 10^3 and 10^{-3} ?” (When the base is 2 rather than 10, the repeated multiplication works the same way. 2^3 is $2 \cdot 2 \cdot 2$ and 2^{-3} is $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$.)
- “How do 5^3 and 5^{-3} compare to 10^3 and 10^{-3} ?” (The exponent rules work in exactly the same way. The base is just 5 instead of 10.)
- “How is $(-3)^4$ different than -3^4 ? If you get stuck, think about what base is being repeatedly multiplied.” ($(-3)^4$ is $(-3)(-3)(-3)(-3)$ whereas -3^4 is $-(3 \cdot 3 \cdot 3 \cdot 3)$.)

- “Do the other exponent rules work the same way that they did for base 10? Give some examples with different bases.” (Yes, the other exponents rules work the same way. With base 7 for example, we have that $\frac{7^5}{7^2} = 7^3$ and $7^3 \cdot 7^6 = 7^9$.)
- “What about combining bases? For example, do you agree that $3^5 \cdot 4^3 = 12^8$?” (No, you cannot combine bases in this way. To see this, expand the factors of both sides. The left side has 5 factors that are 3 and 3 factors that are 4. One could combine some of these factors to make 12, but the right side has 8 factors that are 12. They are not equal.)

Close by telling students that the exponent rules with other bases work exactly the same way as they did with 10 and they can always check this by expanding the factors of an expression with exponents (unless the exponent is very large and it would take too long to expand).

6.4 Spot the Mistake

Cool Down: 5 minutes

Students examine common conceptual errors to reveal their current understanding.

Addressing

- 8.EE.A.1

Student Task Statement

1. Diego was trying to write $2^3 \cdot 2^2$ with a single exponent and wrote $2^3 \cdot 2^2 = 2^{3 \cdot 2} = 2^6$. Explain to Diego what his mistake was and what the answer should be.
2. Andre was trying to write $\frac{7^4}{7^{-3}}$ with a single exponent and wrote $\frac{7^4}{7^{-3}} = 7^{4-3} = 7^1$. Explain to Andre what his mistake was and what the answer should be.

Student Response

1. 2^5 . Diego multiplied the exponents when he should have added them. To see this, he could have expanded the expressions: $2^3 \cdot 2^2 = (2 \cdot 2 \cdot 2)(2 \cdot 2) = 2^{3+2} = 2^5$.
2. 7^7 . Andre did 7^{4-3} when he should have done $7^{4-(-3)}$.

Student Lesson Summary

Earlier we focused on powers of 10 because 10 plays a special role in the decimal number system. But the exponent rules that we developed for 10 also work for other bases. For example, if $2^0 = 1$ and $2^{-n} = \frac{1}{2^n}$, then

$$\begin{aligned} 2^m \cdot 2^n &= 2^{m+n} \\ (2^m)^n &= 2^{m \cdot n} \\ \frac{2^m}{2^n} &= 2^{m-n}. \end{aligned}$$

These rules also work for powers of numbers less than 1. For example, $(\frac{1}{3})^2 = \frac{1}{3} \cdot \frac{1}{3}$ and $(\frac{1}{3})^4 = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$. We can also check that $(\frac{1}{3})^2 \cdot (\frac{1}{3})^4 = (\frac{1}{3})^{2+4}$.

Using a variable x helps to see this structure. Since $x^2 \cdot x^5 = x^7$ (both sides have 7 factors that are x), if we let $x = 4$, we can see that $4^2 \cdot 4^5 = 4^7$. Similarly, we could let $x = \frac{2}{3}$ or $x = 11$ or any other positive value and show that these relationships still hold.

Lesson 6 Practice Problems

Problem 1

Statement

Priya says "I can figure out 5^0 by looking at other powers of 5. 5^3 is 125, 5^2 is 25, then 5^1 is 5."

- What pattern do you notice?
- If this pattern continues, what should be the value of 5^0 ? Explain how you know.
- If this pattern continues, what should be the value of 5^{-1} ? Explain how you know.

Solution

- When the power of 5 drops by one, the value is divided by 5.
1. The value of 5^0 should be the value of 5^1 divided by 5.
- $\frac{1}{5}$. The value of 5^{-1} should be the value of 5^0 divided by 5.

Problem 2

Statement

Select all the expressions that are equivalent to 4^{-3} .

- 12
- 2^{-6}
- $\frac{1}{4^3}$
- $(\frac{1}{4}) \cdot (\frac{1}{4}) \cdot (\frac{1}{4})$
- 12
- $(-4) \cdot (-4) \cdot (-4)$
- $\frac{8^{-1}}{2^2}$

Solution

["B", "C", "D"]

Problem 3

Statement

Write each expression using a single exponent.

a. $\frac{5^3}{5^6}$

b. $(14^3)^6$

c. $8^3 \cdot 8^6$

d. $\frac{16^6}{16^3}$

e. $(21^3)^{-6}$

Solution

a. 5^{-3}

b. 14^{18}

c. 8^9

d. 16^3

e. 21^{-18}

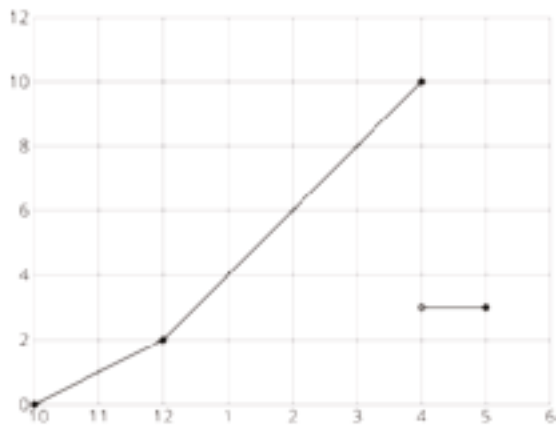
Problem 4

Statement

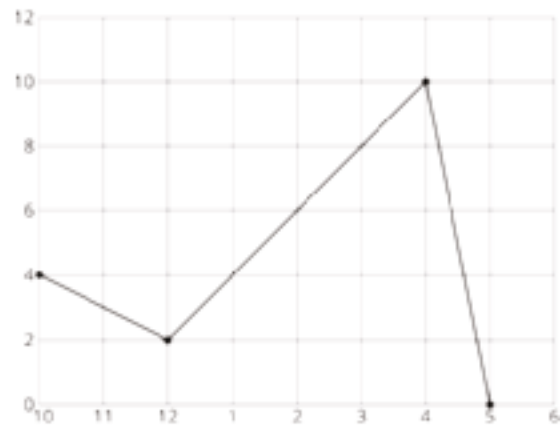
Andre sets up a rain gauge to measure rainfall in his back yard. On Tuesday, it rains off and on all day.

- He starts at 10 a.m. with an empty gauge when it starts to rain.
- Two hours later, he checks, and the gauge has 2 cm of water in it.
- It starts raining even harder, and at 4 p.m., the rain stops, so Andre checks the rain gauge and finds it has 10 cm of water in it.
- While checking it, he accidentally knocks the rain gauge over and spills most of the water, leaving only 3 cm of water in the rain gauge.
- When he checks for the last time at 5 p.m., there is no change.

Graph A



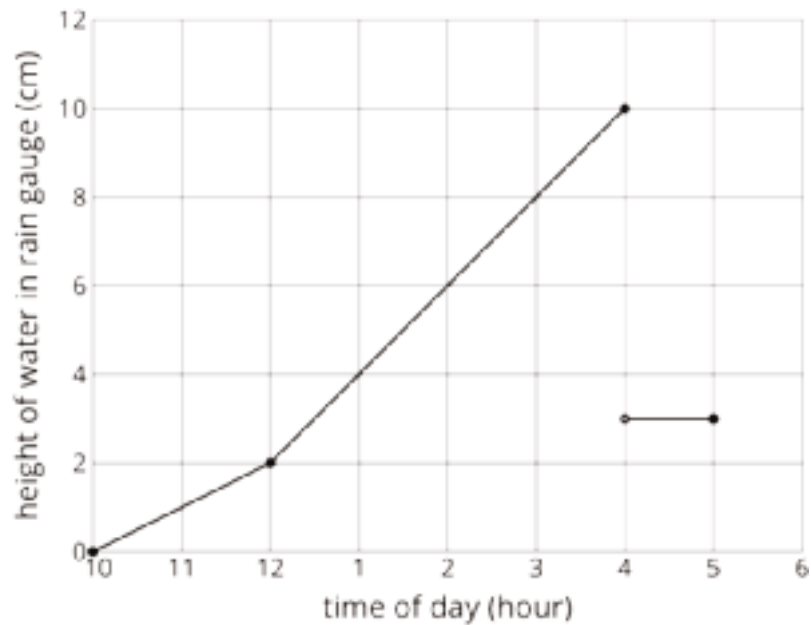
Graph B



- Which of the two graphs could represent Andre's story? Explain your reasoning.
- Label the axes of the correct graph with appropriate units.
- Use the graph to determine how much total rain fell on Tuesday.

Solution

- Graph A



-
-
- 10 cm of rain fell in total on Tuesday. No rain fell after Andre spilled the rain gauge.

(From Unit 5, Lesson 6.)

Lesson 7: Practice with Rational Bases

Goals

- Identify (orally) misapplications of exponent rules to expressions with multiple bases (orally and in writing).
- Use exponent rules to rewrite exponential equations involving negative exponents to have a single positive exponent, and explain (orally) the strategy.

Learning Targets

- I can change an expression with a negative exponent into an equivalent expression with a positive exponent.
- I can choose an appropriate exponent rule to rewrite an expression to have a single exponent.

Lesson Narrative

In this lesson, students practice all of the exponent rules they have learned so far and begin to look at expressions with multiple bases. The first activity asks students to reflect on their own conceptual understanding and procedural fluency with the exponent rules they have learned so far. The second activity asks students to analyze the structure of exponents to make sense of expressions with multiple bases, paving the way towards the rule $a^n \cdot b^n = (a \cdot b)^n$ in the next lesson (MP7).

Alignments

Addressing

- 8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports
- Which One Doesn't Belong?

Student Learning Goals

Let's practice with exponents.

7.1 Which One Doesn't Belong: Exponents

Warm Up: 5 minutes

This warm-up prompts students to compare four exponential expressions. It encourages students to explain their reasoning, hold mathematical conversations, and gives you the opportunity to hear how they use terminology and talk about characteristics of the expressions in comparison to one another. To allow all students to access the activity, each expression has one obvious reason it does

not belong. Encourage students to move past the obvious reasons and find reasons based on mathematical properties. During the discussion, listen for important ideas and terminology that will be helpful in the upcoming work of the unit.

Addressing

- 8.EE.A.1

Instructional Routines

- Which One Doesn't Belong?

Launch

Arrange students in groups of 2–4. Display the expression for all to see. Ask students to indicate when they have noticed one expression that doesn't belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their small group. In their small groups, tell each student to share their reasoning why a particular expression doesn't belong and together find at least one reason each expression doesn't belong.

Anticipated Misconceptions

In the last question, students may think $\frac{10^8}{5^5}$ is equal to 10^3 or $\left(\frac{10}{5}\right)^3$. Encourage these students to think about expanding the exponents into their repeated factors.

Student Task Statement

Which expression doesn't belong?

$$\begin{array}{cc} \frac{2^8}{2^5} & \left(\frac{3}{4}\right)^{-5} \cdot \left(\frac{3}{4}\right)^8 \\ (4^{-5})^8 & \frac{10^8}{5^5} \end{array}$$

Student Response

Answers vary. Sample responses:

- $\frac{2^8}{2^5}$ doesn't belong because it is the only one where the bases are the same and the exponents are both positive.
- $(4^{-5})^8$ doesn't belong because it is the only one where it is a single power of a power.
- $\left(\frac{3}{4}\right)^{-5} \cdot \left(\frac{3}{4}\right)^8$ doesn't belong because it is the only one that has fractions in the base.
- $\frac{10^8}{5^5}$ doesn't belong because it is the only one with different bases.

Activity Synthesis

Ask each group to share one reason why a particular expression does not belong. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given are correct.

During the discussion, ask students to explain the meaning of any terminology they use, such as "base" or "exponent." Also, press students on unsubstantiated claims. For example: " $(4^{-5})^8$ is a really large number." (It isn't, because of the negative exponent.) Or, " $\frac{2^8}{2^5}$ and $\frac{10^8}{5^5}$ are whole numbers." (They are, but how do we know?)

7.2 Exponent Rule Practice

15 minutes

This activity develops procedural fluency with exponent rules and encourages students to think about their own learning. Students choose 6 of 12 possible problems to solve, thereby identifying problems that they consider more difficult versus less difficult. Notice which problems students choose more than others, and which problems are skipped more than others. The first set of problems checks whether students can apply the exponent rules procedurally. The next set of problems checks whether students understand what negative exponents mean. The last set of problems asks students to evaluate exponents to check whether they understand the meaning of the zero exponent and the definition of exponents as repeated multiplication (by the base, or by the reciprocal of the base in the case of negative exponents).

Addressing

- 8.EE.A.1

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. Encourage students to work together with their partners. Encourage partners to choose mostly the same problems, but if they differ, partners should check one another's work. Encourage students to explain their reasoning by referencing the visual displays for the exponent rules. Give students 10–12 minutes to work followed by a brief whole-class discussion. Problems that many students chose to skip can be assigned as additional practice.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge about exponents. Maintain a display of important terms and vocabulary. During the launch take time to review the visual display of rules for exponents. Some students may benefit from their own physical copy of the display.

Supports accessibility for: Memory; Language

Access for English Language Learners

Speaking: MLR8 Discussion Supports. To support small-group discussion, invite students to explain how they used exponent rules to solve at least 3 of the problems. Display sentence frames for students to use such as: "First I ____, then I ____." and "I used ____ rule, because ____."

Design Principle(s): Optimize output for (explanation)

Student Task Statement

1. Choose 6 of the equations to write using a single exponent:

- | | | |
|---|---|---------------------------------------|
| <input type="radio"/> $7^5 \cdot 7^6$ | <input type="radio"/> $\frac{3^5}{3^{28}}$ | <input type="radio"/> $(7^2)^3$ |
| <input type="radio"/> $3^{-3} \cdot 3^8$ | <input type="radio"/> $\frac{2^{-5}}{2^4}$ | <input type="radio"/> $(4^3)^{-3}$ |
| <input type="radio"/> $2^{-4} \cdot 2^{-3}$ | <input type="radio"/> $\frac{6^5}{6^{-8}}$ | <input type="radio"/> $(2^{-8})^{-4}$ |
| <input type="radio"/> $\left(\frac{5}{6}\right)^4 \left(\frac{5}{6}\right)^5$ | <input type="radio"/> $\frac{10^{-12}}{10^{-20}}$ | <input type="radio"/> $(6^{-3})^5$ |

2. Which problems did you want to skip in the previous question? Explain your thinking.

3. Choose 3 of the following to write using a single, *positive* exponent:

- | | |
|---------------------------------|---------------------------------|
| <input type="radio"/> 2^{-7} | <input type="radio"/> 4^{-9} |
| <input type="radio"/> 3^{-23} | <input type="radio"/> 2^{-32} |
| <input type="radio"/> 11^{-8} | <input type="radio"/> 8^{-3} |

4. Choose 3 of the following to evaluate:



○ $\frac{10^5}{10^5}$

○ $\left(\frac{2}{3}\right)^3$

○ $2^8 \cdot 2^{-8}$

○ $\left(\frac{5}{4}\right)^2$

○ $(3^4)^0$

○ $\left(\frac{7}{2}\right)^2$

Student Response

1.

a. 7^{11}

a. 3^{-23}

a. 7^6

b. 3^5

b. 2^{-9}

b. 4^{-9}

c. 2^{-7}

c. 6^{13}

c. 2^{32}

d. $\left(\frac{5}{6}\right)^9$

d. 10^8

d. 6^{-15}

2. Answers vary. Sample response: I wanted to skip questions with negative exponents because I'm least comfortable with those.

3.

a. $\frac{1}{2^7}$

a. $\frac{1}{4^9}$

b. $\frac{1}{3^{23}}$

b. $\frac{1}{2^{32}}$

c. $\frac{1}{11^8}$

c. $\frac{1}{8^3}$

4.

a. 1

a. $\frac{25}{16}$

b. $\frac{8}{27}$

b. 1

c. 1

c. $\frac{49}{4}$

Activity Synthesis

The goal of the discussion is to get a general sense of how fluent students have become with the exponent rules. Poll the class about how successful they felt while working on the problems. Here are some questions for discussion:

- “Which problems would you assign to your best friend? Why?”
- “Which problems really made you think? Why?”
- “What are some resources you could use to get more comfortable with the problems you are uncomfortable with?”

7.3 Inconsistent Bases

15 minutes

In this activity, students analyze powers that involve different bases. The goal is for them to recognize that exponents can be added (or subtracted) only when the powers being multiplied (or divided) have the same base. It is expected that students compute the value of the expressions on the left and right sides of the equation to show they are not actually equal. The last problem alludes to the rule $a^n \cdot b^n = (a \cdot b)^n$ which will be explored further in the next lesson.

As students work, notice those who check for equality by computing the value on either side of each equation or by expanding each power into its factors. Invite them to share later.

Addressing

- 8.EE.A.1

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Display the false equation $2^3 \cdot 5^2 = 10^{3+2} = 10^5$ for all to see. Ask students whether they think the equation is true or false, and choose a few students to explain their reasoning. If not mentioned by students, expand $2^3 \cdot 5^2$ and 10^5 to show their repeated factors. Give students 10 minutes to work followed by a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge of multiplication and repeated factors. Allow students to use calculators to ensure inclusive participation in the activity.

Supports accessibility for: Memory; Conceptual processing

Student Task Statement

Mark each equation as true or false. What could you change about the false equations to make them true?

1. $\left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^4 = \left(\frac{1}{3}\right)^6$

2. $3^2 \cdot 5^3 = 15^5$

3. $5^4 + 5^5 = 5^9$

4. $\left(\frac{1}{2}\right)^4 \cdot 10^3 = 5^7$

5. $3^2 \cdot 5^2 = 15^2$

Student Response

1. True because there are 6 factors that are $\frac{1}{3}$ on each side of the equation.
2. False because $3 \cdot 3 \cdot 5 \cdot 5 \cdot 5 < 15 \cdot 15 \cdot 15 \cdot 15 \cdot 15$. Sample change: $3^5 \cdot 5^5 = 15^5$.
3. False because $5^4 + 5^5 < 5^9$. Sample change: $5^4 \cdot 5^5 = 5^9$.
4. False because $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot 10 \cdot 10 \cdot 10 < 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$. Sample change: $\left(\frac{1}{2}\right)^3 \cdot 10^3 = 5^3$.
5. True because $15^2 = 15 \cdot 15 = (3 \cdot 5)(3 \cdot 5) = 3 \cdot 3 \cdot 5 \cdot 5 = 3^2 \cdot 5^2$.

Are You Ready for More?

Solve this equation: $3^{x-5} = 9^{x+4}$. Explain or show your reasoning.

Student Response

$x = -13$. Explanations vary. Sample explanation: Since $9 = 3^2$, the right side of the equation becomes $(3^2)^{x+4}$, or 3^{2x+8} . This means that $x - 5 = 2x + 8$, so $x = -13$.

Activity Synthesis

The important take-away from this activity is that the exponent rules work because they capture patterns of repeated multiplication of a *single* base. The equation in the launch erroneously applies an exponent rule to a situation that involves multiple bases. This fails because with multiple bases, there are not the same patterns of repeated multiplication. Ask students to share their responses and display them for all to see. For the final question, ask students whether they think it is a coincidence that the equation is true, or if there is another, more general explanation. It is not necessary to dwell on this point since it will be addressed more fully in the next lesson. Consider involving more students in a whole-class discussion with the following questions:

- “Who can restate ___’s reasoning in a different way?”
- “Why do the exponent rules we have looked at so far only work when looking at one particular base rather than mixing different bases together?” ($3^2 \cdot 3^3 = 3^5$ because there are 5 factors that are 3 on the left side, but $3^2 \cdot 4^3$ isn’t 12^5 because there are not 5 factors that are 12.)

Access for English Language Learners

Writing: MLR3 Clarify, Critique, Correct. Display an incorrect statement that represents a common misunderstanding of the exponent rule. For example, “ $3^2 \cdot 5^3 = 15^5$ because $3 \cdot 2 = 15$ and $2 + 3 = 5$.” Ask pairs of students to critique the reasoning by asking, “Do you agree? Why or why not?” Invite students to work with a partner to identify any errors and write an improved statement. Listen for students who notice the relationships between exponents, expanded form and exponent rules. This helps students evaluate, and improve on, the written mathematical arguments of others.

Design Principle(s): Maximize meta-awareness, Optimize output (for explanation)

Lesson Synthesis

In this lesson, students honed their skills working with exponent rules and discovered where the rules break down when looking at expressions with mismatching bases. Here are some questions for discussion:

- “Why is the equation $2^5 \cdot 2^3 = 2^{15}$ false?” (Multiplying 5 factors that are 2 by 3 factors that are 2 results in a total of 8 factors that are 2. Multiplying the exponents doesn’t make sense in this case.)
- “Why is the equation $\frac{3^5}{3^2} = 3^3$ true?” (Expanding the left side, we get $\frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3}$, which is equal to $1 \cdot 3 \cdot 3 \cdot 3$ or just 3^3 .)
- “Why is the equation $\frac{6^5}{3^2} = 2^3$ false? Why might someone make this mistake?” (Expanding the left side, we get $\frac{6 \cdot 6 \cdot 6 \cdot 6 \cdot 6}{3 \cdot 3}$, which is equal to $2 \cdot 2 \cdot 6 \cdot 6 \cdot 6$. Someone might make this mistake because they divide 6 by 3 and use the exponent rule for division to subtract the exponent in the denominator from the exponent in the numerator.)

7.4 Working with Exponents

Cool Down: 5 minutes

Addressing

- 8.EE.A.1

Student Task Statement

1. Rewrite each expression using a single, positive exponent:

a. $\frac{9^3}{9^9}$

b. $14^{-3} \cdot 14^{12}$

2. Diego wrote $6^4 \cdot 8^3 = 48^7$. Explain what Diego's mistake was and how you know the equation is not true.

Student Response

- $\frac{1}{9^6}$
 - 14^9
- Explanations vary. Sample explanation: Diego multiplied the bases and added their exponents. The equation is not true because 4 repeated factors that are 6 multiplied by 3 repeated factors that are 8 is much smaller than 7 repeated factors that are 48.

Student Lesson Summary

In the past few lessons, we found rules to more easily keep track of repeated factors when using exponents. We also extended these rules to make sense of negative exponents as repeated factors of the **reciprocal** of the base, as well as defining a number to the power of 0 to have a value of 1. These rules can be written symbolically as:

$$x^n \cdot x^m = x^{n+m},$$

$$(x^n)^m = x^{n \cdot m},$$

$$\frac{x^n}{x^m} = x^{n-m},$$

$$x^{-n} = \frac{1}{x^n},$$

and

$$x^0 = 1,$$

where the base x can be any positive number. In this lesson, we practiced using these exponent rules for different bases and exponents.

Glossary

- reciprocal

Lesson 7 Practice Problems

Problem 1

Statement

Write with a single exponent:

a. $\frac{7^6}{7^2}$

b. $(11^4)^5$

c. $4^2 \cdot 4^6$

d. $6 \cdot 6^8$

e. $(12^2)^7$

f. $\frac{3^{10}}{3}$

g. $(0.173)^9 \cdot (0.173)^2$

h. $\frac{0.87^5}{0.87^3}$

i. $\frac{(\frac{5}{2})^8}{(\frac{5}{2})^6}$

Solution

a. 7^4

b. 11^{20}

c. 4^8

d. 6^9

e. 12^{14}

f. 3^9

g. 0.173^{11}


h. 0.87^2

i. $(\frac{5}{2})^2$

Problem 2

Statement

Noah says that $2^4 \cdot 3^2 = 6^6$. Tyler says that $2^4 \cdot 4^2 = 16^2$.

- 
- a. Do you agree with Noah? Explain or show your reasoning.
 - b. Do you agree with Tyler? Explain or show your reasoning.

Solution

- a. Disagree. Reasoning varies. Sample reasoning: $2^4 \cdot 3^2 = 16 \cdot 9 = 144$, but 6^6 is much bigger than 144.
- b. Agree. Reasoning varies. Sample reasoning: $2^4 = 16$ and $4^2 = 16$, so $2^4 \cdot 4^2$ should equal $16 \cdot 16$ or 16^2 .

Lesson 8: Combining Bases

Goals

- Generalize a process for multiplying expressions with different bases having the same exponent, and justify (orally and in writing) that $(ab)^n = a^n \cdot b^n$.

Learning Targets

- I can use and explain a rule for multiplying terms that have different bases but the same exponent.

Lesson Narrative

Previously, students saw that the exponent rules thus far only apply when the bases are the same. In this lesson, students explore what happens when bases are different. This leads to the rule $a^n b^n = (a \cdot b)^n$. Students make use of structure when decomposing numbers into their constituent factors and regrouping them (MP7). Students create viable arguments and critique the reasoning of others when they generate expressions equivalent to 3,600 and $\frac{1}{200}$ using exponent rules and determine the validity of other teams' expressions (MP3).

Alignments

Addressing

- 8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR8: Discussion Supports

Required Materials

Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart

paper and markers, whiteboard space and

markers, shared online drawing tool, access to a document camera.

Required Preparation

Create a visual display for the rule $(a \cdot b)^n = a^n \cdot b^n$. As a guiding example, consider $2^3 \cdot 5^3 = 2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5 = (2 \cdot 5) \cdot (2 \cdot 5) \cdot (2 \cdot 5) = 10 \cdot 10 \cdot 10 = 10^3$.

Student Learning Goals

Let's multiply expressions with different bases.

8.1 Same Exponent, Different Base

Warm Up: 5 minutes

The purpose of this warm-up is to encourage students to relate expressions of the form $a^n \cdot b^n$ to $(a \cdot b)^n$ by exploring the structure of the factors (MP7). Students should notice that the factors in the expanded form of $5^3 \cdot 2^3$ can be rearranged and multiplied to show the factors in the expanded form of 10^3 . Evaluating and expanding expressions with exponents helps prepare students for the next activity in which they more generally explore products of bases with the same exponent.

Addressing

- 8.EE.A.1

Launch

Give students 2 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

1. Evaluate $5^3 \cdot 2^3$
2. Evaluate 10^3

Student Response

1. $5^3 \cdot 2^3 = 125 \cdot 8 = 1,000$.
2. $10^3 = 1,000$.

Activity Synthesis

Consider asking some of the following questions to focus the conversation on the common exponents:

- “What connections do you see between the two expressions?” (The product of the bases in the first expression is equal to the base in the second expression: $2 \cdot 5 = 10$. The exponents are the same in both expressions.)
- “Is there a way to tell just by looking at the expressions that they would be equal? How?” (Since there are 3 factors that are 5 and 3 factors that are 2, group the 2s and 5s together to get 3 factors that are 10.)

Highlight student explanations that clearly show the connection between $2^3 \cdot 5^3$ and 10^3 by inspecting their factors.

8.2 Power of Products

15 minutes

Students use repeated reasoning to discover the rule $(a \cdot b)^n = a^n \cdot b^n$ (MP8).

Addressing

- 8.EE.A.1

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Arrange students in groups of 2. Encourage students to share their reasoning with their partner as they work to complete the table. Give students 10–12 minutes of work time followed by a whole-class discussion.

Access for Students with Disabilities

Engagement: Internalize Self Regulation. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. For example, check in with select students after they have completed the first 2-3 rows.

Supports accessibility for: Memory; Organization

Anticipated Misconceptions

Some students may write $2x^4$ instead of $(2x)^4$. Similarly, students may write $a \cdot b^n$ instead of $(a \cdot b)^n$. Ask these students to explain the difference between $3 \cdot 4^2$ and $(3 \cdot 4)^2$.

Student Task Statement

1. The table contains products of expressions with different bases and the same exponent. Complete the table to see how we can rewrite them. Use the “expanded” column to work out how to combine the factors into a new base.

expression	expanded	exponent
$5^3 \cdot 2^3$	$(5 \cdot 5 \cdot 5) \cdot (2 \cdot 2 \cdot 2) = (5 \cdot 2)(5 \cdot 2)(5 \cdot 2)$ $= 10 \cdot 10 \cdot 10$	10^3
$3^2 \cdot 7^2$		21^2
$2^4 \cdot 3^4$		
		15^3
		30^4
$2^4 \cdot x^4$		
$a^n \cdot b^n$		
$7^4 \cdot 2^4 \cdot 5^4$		

2. Can you write $2^3 \cdot 3^4$ with a single exponent? What happens if neither the exponents nor the bases are the same? Explain or show your reasoning.

Student Response

1.	expression	expanded	exponent
	$5^3 \cdot 2^3$	$(5 \cdot 5 \cdot 5) \cdot (2 \cdot 2 \cdot 2) = (5 \cdot 2)(5 \cdot 2)(5 \cdot 2)$ $= 10 \cdot 10 \cdot 10$	10^3
	$3^2 \cdot 7^2$	$(3 \cdot 3) \cdot (7 \cdot 7) = (3 \cdot 7)(3 \cdot 7) = 21 \cdot 21$	21^2
	$2^4 \cdot 3^4$	$(2 \cdot 2 \cdot 2 \cdot 2) \cdot (3 \cdot 3 \cdot 3 \cdot 3)$ $= (2 \cdot 3)(2 \cdot 3)(2 \cdot 3)(2 \cdot 3)$ $= 6 \cdot 6 \cdot 6 \cdot 6$	6^4
	$3^3 \cdot 5^3$	$15 \cdot 15 \cdot 15 = (3 \cdot 5)(3 \cdot 5)(3 \cdot 5)$ $= (3 \cdot 3 \cdot 3)(5 \cdot 5 \cdot 5)$	15^3
	Answers vary. Sample: $3^4 \cdot 10^4$	Answers vary. Sample: $30 \cdot 30 \cdot 30 \cdot 30$ $= (3 \cdot 10)(3 \cdot 10)(3 \cdot 10)(3 \cdot 10)$ $= (3 \cdot 3 \cdot 3 \cdot 3)(10 \cdot 10 \cdot 10 \cdot 10)$	30^4
	$2^4 \cdot x^4$	$(2 \cdot 2 \cdot 2 \cdot 2) \cdot (x \cdot x \cdot x \cdot x)$ $= (2 \cdot x)(2 \cdot x)(2 \cdot x)(2 \cdot x)$ $= 2x \cdot 2x \cdot 2x \cdot 2x$	$(2x)^4$
	$a^n \cdot b^n$	$(a \cdot \dots \cdot a)(b \cdot \dots \cdot b) = (a \cdot b) \cdot \dots \cdot (a \cdot b)$	$(a \cdot b)^n$
	$7^4 \cdot 2^4 \cdot 5^4$	$(7 \cdot 7 \cdot 7 \cdot 7)(2 \cdot 2 \cdot 2 \cdot 2)(5 \cdot 5 \cdot 5 \cdot 5)$ $= (7 \cdot 2 \cdot 5)(7 \cdot 2 \cdot 5)(7 \cdot 2 \cdot 5)(7 \cdot 2 \cdot 5)$ $= 70 \cdot 70 \cdot 70 \cdot 70$	70^4

2. Answers vary. Sample response: No, it is not possible to write $2^3 \cdot 3^4$ with a single exponent because regrouping it into factors that are 6 will leave an extra factor of 3. If the exponents are not the same, the factors cannot be grouped together evenly.

Activity Synthesis

Ask students to share their reasoning about whether $2^3 \cdot 3^4$ can be written with a single exponent. The key takeaway for the discussion should be that the exponents need to be the same to combine the bases into a single base with that exponent.

Introduce a visual display for the rule $(a \cdot b)^n = a^n \cdot b^n$. Display it for all to see throughout the unit.

Access for English Language Learners

Writing, Speaking: MLR1 Stronger and Clearer Each Time. Use this routine to give students a structured opportunity to revise and refine their response to “What happens if neither the exponents nor the bases are the same?” Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them to strengthen their ideas and clarify their language (e.g., “What exponent rule are you using?”, “Can you give an example?”, “How do you know...?”, etc.). Students can borrow ideas and language from each partner to strengthen their final version.

Design Principle(s): Optimize output (for generalization)

8.3 How Many Ways Can You Make 3,600?

Optional: 15 minutes

At this point, students have worked with many different patterns involving exponents. This activity gives students an opportunity to deepen their thinking by generating different equivalent expressions using the rules of exponents. The process of generating different expressions requires students to understand the numerous ways numbers can be broken into factors and how to combine those factors and express the result using exponents.

Addressing

- 8.EE.A.1

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2–3. Provide students with tools for creating a visual display. There will be several rounds in which students will have to generate multiple expressions equivalent to a specific number. For the first round, students have to generate expressions that are equal to 3,600. Show students how to set up their display with an example, such as the one shown here. As a whole class, come up with an expression equivalent to 3,600 using each of the three rules. If time allows, discuss using prime factorization as a strategy. For an example using the first rule:

$3,600 = (600 \cdot 6) = (2^3 \cdot 3^1 \cdot 5^2)(2^1 \cdot 3^1) = 2^{3+1} \cdot 3^{1+1} \cdot 5^2 = 2^4 \cdot 3^2 \cdot 5^2$. Some simpler examples include:

$a^n \cdot a^m = a^{n+m}$	$\frac{a^n}{a^m} = a^{n-m}$	$a^n \cdot b^n = (a \cdot b)^n$
$60^1 \cdot 60^1 = 60^2$	$\frac{6^5}{6^3} = 60^2$	$6^2 \cdot 10^2 = 60^2$

Display these examples for all to see while students are working to generate their own expressions equivalent to 3,600. Set a timer for 1 minute (or other amount, depending on time available) and let students work.

When time is up, pair each group with another group for scoring. It would be beneficial to choose 2 groups to use as examples and demonstrate this process for the whole class:

- A group gets 1 point for every *unique* expression they found that is equivalent to 3,600. (If the two groups found the same expression, neither group gets a point for it.)
- 2 points for every unique expression that uses negative exponents.
- Students can challenge the other group's expressions if they think they don't really equal 3,600, or if the group didn't use any of the three rules.

In the second round, shift the students' attention to the number $\frac{1}{200}$ and have them create another visual display (perhaps on the back of their first visual display). Again, the group gets 1 point for each unique expression equivalent to $\frac{1}{200}$ except if it uses negative exponents, in which case it gets 2 points.

Play as many rounds of this game as time allows. In subsequent rounds, groups pair up with a different opponent. Consider using the following numbers in different rounds if time permits: 810,000; $\frac{1}{64}$; 3,375. Leave a few minutes for a brief whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Begin with a whole-class think aloud to demonstrate the steps of the game. Keep the worked-out calculations on display for students to reference as they work.

Supports accessibility for: Memory; Conceptual processing

Access for English Language Learners

Representing, Conversing: MLR8 Discussion Supports. Demonstrate how to play the game. To do this, select a group of students (and include yourself) to begin the game while the rest of the class observes. This will help clarify the expectations of the task, invite more student participation, and facilitate meta-awareness of the language involving exponent rules.

Design Principle(s): Support sense-making; Maximize meta-awareness.

Student Task Statement

Your teacher will give your group tools for creating a visual display to play a game. Divide the display into 3 columns, with these headers:

$$a^n \cdot a^m = a^{n+m}$$

$$\frac{a^n}{a^m} = a^{n-m}$$

$$a^n \cdot b^n = (a \cdot b)^n$$

How to play:

When the time starts, you and your group will write as many expressions as you can that equal a specific number using one of the exponent rules on your board. When the time is up, compare your expressions with another group to see how many points you earn.

- Your group gets 1 point for every *unique* expression you write that is equal to the number and follows the exponent rule you claimed.
- If an expression uses negative exponents, you get 2 points instead of just 1.
- You can challenge the other group's expression if you think it is not equal to the number or if it does not follow one of the three exponent rules.

Student Response

- 3,600: Answers vary. Sample responses: 60^2 , $6^2 \cdot 10^2$, $6^{-3} \cdot 6^5 \cdot \frac{10^8}{10^6}$.
- $\frac{1}{200}$: Answers vary. Sample responses: $2^{-3} \cdot 5^{-2}$, $\frac{2^6}{2^9} \cdot \frac{5^4}{5^6}$.

Are You Ready for More?

You have probably noticed that when you square an odd number, you get another odd number, and when you square an even number, you get another even number. Here is a way to expand the concept of odd and even for the number 3. Every integer is either divisible by 3, one *more* than a multiple of 3, or one *less* than a multiple of 3.

1. Examples of numbers that are one more than a multiple of 3 are 4, 7, and 25. Give three more examples.
2. Examples of numbers that are one less than a multiple of 3 are 2, 5, and 32. Give three more examples.
3. Do you think it's true that when you square a number that is a multiple of 3, your answer will still be a multiple of 3? How about for the other two categories? Try squaring some numbers to check your guesses.

Student Response

1. Answers vary. Sample answers: 10, 13, 16
2. Answers vary. Sample answers: 8, 11, 14
3. This is true for multiples of 3 and numbers that are one more than a multiple of 3, but not for numbers that are one less than a multiple of 3.

Activity Synthesis

In a whole-class discussion, ask “Explain in your own words: What did you learn about exponents from this activity?”

The main point is that numbers can be broken down into their factors in many ways and the exponent rules can be used to express the same value in many ways.

Lesson Synthesis

In this lesson, students saw that it is possible to combine bases together as long as the exponent is the same. Students also practiced using all of the rules they know to write many equivalent exponential expressions. The goal of the discussion is mainly to check that students understand why the exponent rule $a^n \cdot b^n = (a \cdot b)^n$ works. If there is time and interest, students can also share their observations about what they learned by trying to generate as many equivalent exponential expressions as they can.

Here are questions for discussion:

- “Is it possible to write $4^5 \cdot 5^5$ using a single exponent?” (Yes, $4^5 \cdot 5^5 = 20^5$.)
- “What about $4^3 \cdot 5^5$?” (No. You could combine 3 factors that are 4 and 3 factors that are 5 to make 3 factors that are 20, but there are still 2 factors that are 5 left over.)
- “When is it possible to combine bases together in a single exponent?” (It is only possible when both bases have the same exponent.)
- “What are some patterns or strategies you saw in the ‘How Many Ways?’ game?”

To close the discussion, it may help to mention that looking for patterns in the factors of numbers has a long tradition in mathematics, with many applications for building better computers and devices. Many ways that computers have been programmed to think are based on patterns of factors. Both hacking and protecting computer networks from hackers are rooted in patterns of factors at a fundamental level. This might be a topic that interested students can further explore.

8.4 Help an Absent Student

Cool Down: 5 minutes

Addressing

- 8.EE.A.1

Student Task Statement

Using words and equations, explain what you learned about exponents in this lesson so that someone who was absent could read what you wrote and understand the lesson. Consider using an example like $2^4 \cdot 3^4 = 6^4$.

Student Response

Answers vary. Sample response: Today we learned about how to multiply numbers with exponents together. For example, when you multiply $2^4 \cdot 3^4$, you can rearrange the factors to get 6^4 . To see this, notice that $2^4 \cdot 3^4 = (2 \cdot 2 \cdot 2 \cdot 2) \cdot (3 \cdot 3 \cdot 3 \cdot 3) = (2 \cdot 3)(2 \cdot 3)(2 \cdot 3)(2 \cdot 3) = 6^4$. The bases only pair up like this if the exponents are the same. If the exponents aren't the same, then there will be some unpaired factors. For example, $2^3 \cdot 3^4 = (2 \cdot 3) \cdot (2 \cdot 3) \cdot (2 \cdot 3) \cdot 3 = 6^3 \cdot 3$. There is an extra factor that is 3 in this case.

Student Lesson Summary

Before this lesson, we made rules for multiplying and dividing expressions with exponents that only work when the expressions have the *same* base. For example,

$$10^3 \cdot 10^2 = 10^5$$

or

$$2^6 \div 2^2 = 2^4$$

In this lesson, we studied how to combine expressions with the same exponent, but *different* bases. For example, we can write $2^3 \cdot 5^3$ as $2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$. Regrouping this as $(2 \cdot 5) \cdot (2 \cdot 5) \cdot (2 \cdot 5)$ shows that

$$\begin{aligned} 2^3 \cdot 5^3 &= (2 \cdot 5)^3 \\ &= 10^3 \end{aligned}$$

Notice that the 2 and 5 in the previous example could be replaced with different numbers or even variables. For example, if a and b are variables then $a^3 \cdot b^3 = (a \cdot b)^3$. More generally, for a positive number n ,

$$a^n \cdot b^n = (a \cdot b)^n$$

because both sides have exactly n factors that are a and n factors that are b .

Lesson 8 Practice Problems

Problem 1

Statement

Select all the true statements:

A. $2^8 \cdot 2^9 = 2^{17}$

B. $8^2 \cdot 9^2 = 72^2$

C. $8^2 \cdot 9^2 = 72^4$

D. $2^8 \cdot 2^9 = 4^{17}$

Solution

["A", "B"]

Problem 2

Statement

Find x , y , and z if $(3 \cdot 5)^4 \cdot (2 \cdot 3)^5 \cdot (2 \cdot 5)^7 = 2^x \cdot 3^y \cdot 5^z$.

Solution

$x = 12$, $y = 9$, $z = 11$

Problem 3

Statement

Han found a way to compute complicated expressions more easily. Since $2 \cdot 5 = 10$, he looks for pairings of 2s and 5s that he knows equal 10. For example,

$3 \cdot 2^4 \cdot 5^5 = 3 \cdot 2^4 \cdot 5^4 \cdot 5 = (3 \cdot 5) \cdot (2 \cdot 5)^4 = 15 \cdot 10^4 = 150,000$. Use Han's technique to compute the following:

a. $2^4 \cdot 5 \cdot (3 \cdot 5)^3$

b. $\frac{2^3 \cdot 5^2 \cdot (2 \cdot 3)^2 \cdot (3 \cdot 5)^2}{3^2}$

Solution

a. 270,000

b. 180,000

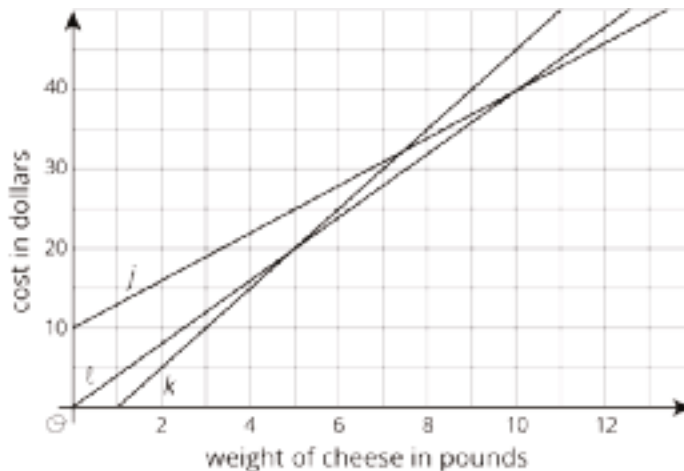
Problem 4

Statement

The cost of cheese at three stores is a function of the weight of the cheese. The cheese is not prepackaged, so a customer can buy any amount of cheese.

- Store A sells the cheese for a dollars per pound.
- Store B sells the same cheese for b dollars per pound and a customer has a coupon for \$5 off the total purchase at that store.
- Store C is an online store, selling the same cheese at c dollar per pound, but with a \$10 delivery fee.

This graph shows the price functions for stores A, B, and C.



- Match Stores A, B, and C with Graphs j , k , and l .
- How much does each store charge for the cheese per pound?
- How many pounds of cheese does the coupon for Store B pay for?
- Which store has the lowest price for a half a pound of cheese?
- If a customer wants to buy 5 pounds of cheese for a party, which store has the lowest price?
- How many pounds would a customer need to order to make Store C a good option?

Solution

- Store A: Graph l
Store B: Graph k
Store C: Graph j

- b. Store A charges \$4 per pound, Store B charges \$5 per pound, and Store C charges \$3 per pound.
- c. 1 pound of cheese.
- d. Store B
- e. Store A or Store B would both charge the same amount for 5 lbs of cheese.
- f. If a customer orders more than 10 pounds of cheese, Store C has the lowest price.

(From Unit 5, Lesson 8.)

Section: Scientific Notation

Lesson 9: Describing Large and Small Numbers Using Powers of 10

Goals

- Describe (orally and in writing) large and small numbers as multiples of powers of 10.
- Interpret a diagram for base-ten units, and explain (orally) how the small squares, long rectangles, and large squares relate to each other.

Learning Targets

- Given a very large or small number, I can write an expression equal to it using a power of 10.

Lesson Narrative

This lesson serves as a prelude to scientific notation and builds on work students have done in previous grades with numbers in base ten. Students use base-ten diagrams to represent different powers of 10 and review how multiplying and dividing by 10 affect the decimal representation of numbers. They use their understanding of base-ten structure as they express very large and very small numbers using exponents.

Students also practice communicating—describing and writing—very large and small numbers in an activity, which requires attending to precision (MP6). This leads to a discussion about how powers of 10 can be used to more easily communicate such numbers.

Alignments

Building On

- 5.NBT.A.2: Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
- 5.NBT.A.3.a: Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g.,
 $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.

Addressing

- 8.EE.A.3: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9 , and determine that the world population is more than 20 times larger.

Building Towards

- 8.EE.A.3: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9 , and determine that the world population is more than 20 times larger.
- 8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Instructional Routines

- MLR2: Collect and Display
- MLR7: Compare and Connect
- Think Pair Share

Required Materials

Pre-printed cards, cut from copies of the Instructional master

Required Preparation

Print and cut up cards from the included Instructional master. Prepare 1 set of cards for every 2 students.

Student Learning Goals

Let's find out how to use powers of 10 to write large or small numbers.

9.1 Thousand Million Billion Trillion

Warm Up: 5 minutes

In this warm-up, students connect thousand, million, billion, and trillion to their respective powers of ten— 10^3 , 10^6 , 10^9 , and 10^{12} . Understanding powers of 10 associated with these denominations will help students reason about quantities in real-world contexts such as the number of cells in a human body (trillions), world population (billions), etc.

Building On

- 5.NBT.A.2

Building Towards

- 8.EE.A.3
- 8.EE.A.4

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 2 minutes of quiet work time and then 1 minute to compare their responses with their partner. Given the limited time, it may not be possible for students to create examples for each of the values in the second question. Tell students to try to at least find 1 or 2 examples and then to find others as time allows. Follow with a whole-class discussion.

Anticipated Misconceptions

Some students may think that, for example, $1,000,000 = 10^7$ because the number 1,000,000 has 7 digits. Ask these students if it is true that $10 = 10^2$.

Some students may confuse the prefix “milli-” with the word “million.” The word “million” literally means “a big thousand,” and so both “million” and “mille” are related to the Latin “mille,” meaning “thousand.” While “milli-” is talking about a thousand parts (thousandths), “million” is talking about a thousand thousands.

Student Task Statement

1. Match each expression with its corresponding value and word.

expression	value	word
10^{-3}	1,000,000,000,000	billion
10^6	$\frac{1}{100}$	milli-
10^9	1,000	million
10^{-2}	1,000,000,000	thousand
10^{12}	1,000,000	centi-
10^3	$\frac{1}{1,000}$	trillion

2. For each of the numbers, think of something in the world that is described by that number.

Student Response

1.

expression	value	word
10^3	1,000	thousand
10^6	1,000,000	million
10^9	1,000,000,000	billion
10^{12}	1,000,000,000,000	trillion
10^{-2}	$\frac{1}{100}$	centi-
10^{-3}	$\frac{1}{1,000}$	milli-

2. Answers vary. Sample response: 10^3 (thousand): number of students in a school. 10^6 (million): population of a state. 10^9 (billion): population of China. 10^{12} (trillion): number of stars in a large galaxy. 10^{-2} (hundredth): There are 100 centimeters in a meter. 10^{-3} (thousandth): There are 1,000 milliliters in a liter.

Activity Synthesis

Ask students to share the corresponding expressions, words, and values. Record and display their responses for all to see. If time is limited, consider displaying the completed table for all to see and discussing any questions or disagreements. Then, invite students to share their examples for the final question. After each student shares, ask the class whether they agree that the given example could be described by that value.

If students struggle to find something that could be described by each value, consider sharing some of the following examples:

- Thousand (10^3)
 - Number of students in a school
 - Population of an endangered species
 - Cost of a car that barely runs
 - Number of brain cells of a jellyfish
- Million (10^6)
 - Population of a state
 - Cost of the most expensive car in the world
 - Number of brain cells of a cockroach
- Billion (10^9)

- Population of India
- Population of China
- Number of students in the world
- Number of brain cells of a monkey
- Number of trees in the U.S.
- Trillion (10^{12})
 - Amount of wealth produced by a developed country in a year in U.S. dollars
 - Number of stars in a large galaxy
 - Number of brain cells of 10 students

9.2 Base-ten Representations Matching

20 minutes

In this activity, students use their understanding of decimal place value and base-ten diagrams to practice working with the structure of scientific notation before it is formally introduced. They express numbers as sums of terms, each term being multiples of powers of 10. For example, 254 can be written as $2 \cdot 10^2 + 5 \cdot 10^1 + 4 \cdot 10^0$.

Notice students who choose different, yet correct, diagrams for the first and last problems. Ask them to share their reasoning in the discussion later.

Building On

- 5.NBT.A.3.a

Building Towards

- 8.EE.A.3

Instructional Routines

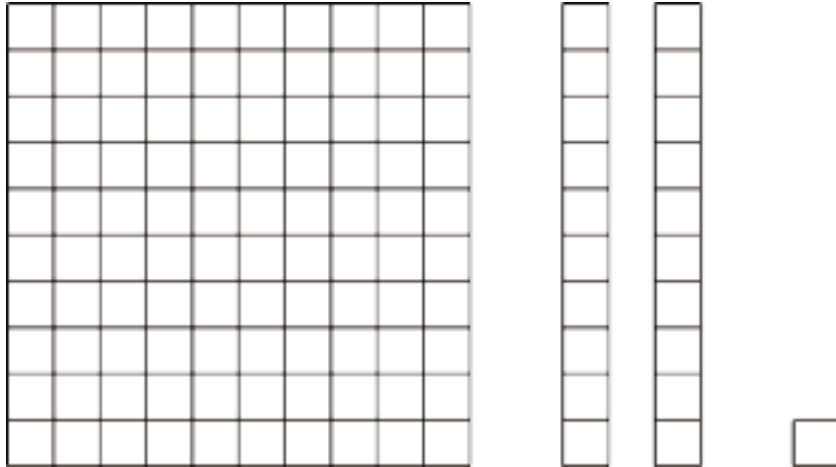
- MLR7: Compare and Connect

Launch

Display the following diagram for all to see. Pause for quiet think time after asking each question about the diagram. Ask students to explain their thinking. Here are some questions to consider:

- “If each small square represents 1 tree, what does the whole diagram represent?” (121 trees)
- “If each small square represents 10 books, what does the whole diagram represent?” (1,210 books)
- “If each small square represents 1,000,000 stars, what does the whole diagram represent?” (121,000,000 stars)

- “If each small square represents 0.1 seconds, what does the whole diagram represent?” (12.1 seconds)
- “If each small square represents 10^3 people, what does the whole diagram represent?” (121,000 people)



Give students 10 minutes to work on the task, followed by a brief whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. For example, display only the first expression and ask students what they notice before inviting them to select one or more diagrams that could represent it. Ask 1-2 students to explain their match before revealing the remaining expressions.

Supports accessibility for: Conceptual processing; Organization

Anticipated Misconceptions

Some students may think that the small square must always represent one unit. Explain to these students that, as in the launch, the small square might represent 10 units, 0.1 units, or any other power of 10.

Some students may have trouble writing the value of expressions that involve powers of 10, especially if they involve negative exponents. As needed, suggest that they expand the expression into factors that are 10, and remind them that 10^{-1} corresponds to the tenths place, 10^{-2} corresponds to the hundredths place, etc.

Student Task Statement

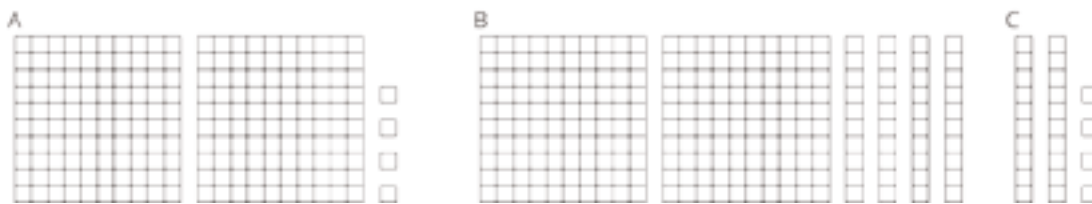
1. Match each expression to one or more diagrams that could represent it. For each match, explain what the value of a single small square would have to be.

a. $2 \cdot 10^{-1} + 4 \cdot 10^{-2}$

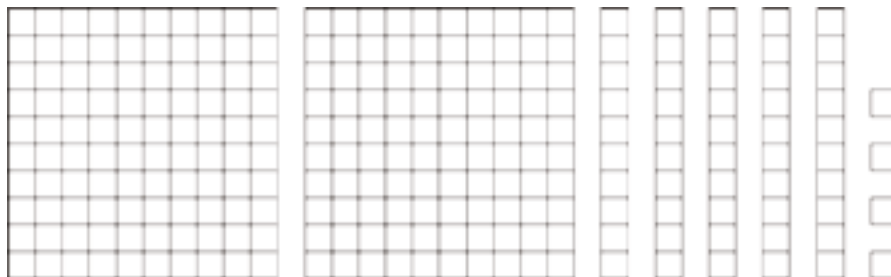
b. $2 \cdot 10^{-1} + 4 \cdot 10^{-3}$

c. $2 \cdot 10^3 + 4 \cdot 10^1$

d. $2 \cdot 10^3 + 4 \cdot 10^2$



2. a. Write an expression to describe the base-ten diagram if each small square represents 10^{-4} . What is the value of this expression?



- b. How does changing the value of the small square change the value of the expression? Explain or show your thinking.
- c. Select at least two different powers of 10 for the small square, and write the corresponding expressions to describe the base-ten diagram. What is the value of each of your expressions?

Student Response

1. a. C if a small square is 10^{-2} or B if a small square is 10^{-3}
- b. A if a small square is 10^{-3}
- c. A if a small square is 10^1
- d. C if a small square is 10^2 or B if a small square is 10^1

2. a. $4 \cdot 10^{-4} + 5 \cdot 10^{-3} + 2 \cdot 10^{-2}$ which is 0.0254.
- b. Answers vary. Sample response: Changing the value of the small square changes the powers of 10. The long rectangle always has an exponent that is 1 higher than for the small square, and the large square always has an exponent that is 2 higher than for the long rectangle.
- c. Answers vary. Sample response: If the small square has a value of 10^6 , then the expression is $4 \cdot 10^6 + 5 \cdot 10^7 + 2 \cdot 10^8$, which is 254,000,000. If the small square has a value of 10^{-1} , then the expression is $4 \cdot 10^{-1} + 5 \cdot 10^0 + 2 \cdot 10^1$, which is 25.4.

Activity Synthesis

Select students to share their responses to the first and last problems. Bring attention to the fact that diagrams B or C could be used depending on the choice of the value of the small square.

The main goal of the discussion is to make sure students see the connection between decimal place value to sums of terms that are multiples of powers of 10. To highlight this connection explicitly, consider discussing the following questions:

- “How are the diagrams related to our base-ten numbers and place value system?” (In base-ten numbers, each place value is ten times larger than the one to its right; so every 1 unit of a place value can be composed of 10 units of the next place value to its right. The diagrams work the same way: each shape representing a base-ten unit can be composed of 10 that represent another unit that is one tenth of its value.)
- “How are the diagrams related to numbers written using powers of 10?” (We can think of each place value as a power of ten. So a ten would be 10^1 , a hundred would be 10^2 , a tenth would be 10^{-1} , and so on.)
- “If each large square represents 10^2 , what do 2 large squares and 4 long rectangles represent?” (A long rectangle is a tenth of the large square, so we know the long rectangle represents 10^1 . This means 2 large squares and 4 long rectangles represent $2 \cdot 10^2 + 4 \cdot 10^1$.)
- “Why is it possible for one base-ten diagram to represent many different numbers?” (Because of the structure of our place value system—where every group of 10 of a base-ten unit composes 1 of the next higher unit—is consistent across all place values.)

Access for English Language Learners

Speaking, Listening: MLR7 Compare and Connect. As students prepare to share their responses to the first and last problems, look for those using different strategies for matching base-ten representations. During the discussion, ask students to share what worked well in a particular approach. Listen for and amplify any comments that describe each representation (i.e., place value, base-ten unit, powers of 10). Then encourage students to make the connection between decimal place value to sums of terms that are multiples of powers of 10. This will foster students' meta-awareness and support constructive conversations as they compare strategies for describing large and small quantities.

Design Principle(s): Cultivate conversation; Maximize meta-awareness

9.3 Using Powers of 10 to Describe Large and Small Numbers

15 minutes

This activity motivates students to find easier ways to communicate about very large and very small numbers, using powers of 10 and working toward using scientific notation. Students take turns reading aloud and writing down quantities that involve long strings of digits, noticing the challenges of expressing such numbers.

As students work, monitor the different ways students communicate the number of zeros precisely to their partners. Some might use standard vocabulary (billion, ten-thousandth, etc.), or some may communicate the number of zeros after the decimal point or after the significant digits. Select students using different strategies to share later.

Building On

- 5.NBT.A.2

Addressing

- 8.EE.A.3

Instructional Routines

- MLR2: Collect and Display

Launch

Arrange students in groups of 2. Distribute a pair of cards (one for Partner A and one for Partner B) from the Instructional master to each group. Ask partners not to show their card to each other.

Tell students that one partner should read an incomplete statement in the materials and the other partner should read aloud the missing information on the card. The goal is for each partner to write

down the missing quantity correctly. Partners should take turns reading and writing until all four statements for each person are completed.

Consider explaining (either up front or as needed during work time) that students who have the numbers can describe or name them in any way that they think convey the quantities fully. Likewise, those writing the numbers can write in any way that captures the quantities accurately.

Give students 10 minutes to work. Leave a few minutes for a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Begin with a whole-class think aloud to demonstrate how students should work together.

Supports accessibility for: Memory; Conceptual processing

Student Task Statement

Your teacher will give you a card that tells you whether you are Partner A or B and gives you the information that is missing from your partner's statements. Do not show your card to your partner.

Read each statement assigned to you, ask your partner for the missing information, and write the number your partner tells you.

Partner A's statements:

1. Around the world, about _____ pencils are made each year.
2. The mass of a proton is _____ kilograms.
3. The population of Russia is about _____ people.
4. The diameter of a bacteria cell is about _____ meter.

Partner B's statements:

1. Light waves travel through space at a speed of _____ meters per second.
2. The population of India is about _____ people.
3. The wavelength of a gamma ray is _____ meters.
4. The tardigrade (water bear) is _____ meters long.

Student Response

Partner A's statements:

1. Around the world, about 14,000,000,000 pencils are made each year.

quantities	using powers of 10
150,000,000,000 meters	$150 \cdot 10^9$ meters
300,000,000 meters per second	$300 \cdot 10^6$ meters per second
0.00000000000048 meters	$48 \cdot 10^{-13}$ meters
0.00000000000000000000000000000167 kilogram	$167 \cdot 10^{-29}$ kilogram

Access for English Language Learners

Conversing, Representing: MLR2 Collect and Display. While pairs are working, circulate and listen to students talk about very large and very small numbers. Write down common or important phrases you hear the ways students communicate the number of zeros to their partners. Listen for students who use standard vocabulary (billion, ten-thousandth, etc.), provide the number of zeros after the decimal point or after the significant digits, or refer to powers of 10. Display their representations together with the students' language onto a visual display. This will help students use mathematical language when communicating about very small and very large numbers during their paired and whole-group discussions.

Design Principle(s): Maximize meta-awareness; Support sense-making

Lesson Synthesis

The focus of the discussion is the structure of our place value system and the rationale and usefulness of describing large and small numbers in different ways. Consider asking some of the following questions:

- “How do base-ten diagrams help us make sense of (or explain) the exponents in powers of 10?” (When using diagrams, grouping 10 of the next smaller unit means multiplying by 10. When dealing with powers of 10, multiplying by 10 increases the exponent by 1. Likewise, decomposing a base-ten unit into 10 of the next smaller unit means multiplying by $\frac{1}{10}$, so the exponent in the power of 10 goes down by 1.)
- “How does using powers of 10 make it easier to communicate about very large or very small numbers?” (We can write in a smaller space. It’s also faster to read and easier to understand the size of a number and to compare numbers. Using powers of 10 helps us avoid errors of missing zeros or extra zeros.)
- “What are some different ways to describe a large number like 123 billion?” (123 • (1,000,000,000 or $123 \cdot 10^9$.)

- “What are some different ways to describe a small number like 0.0000000789?” (789 ten-billionths, $789 \cdot \frac{1}{10,000,000,000}$, or $789 \cdot 10^{-10}$.)

9.4 Better with Powers of 10

Cool Down: 5 minutes

Addressing

- 8.EE.A.3

Student Task Statement

1. Write 0.000000123 as a multiple of a power of 10.
2. Write 123,000,000 as a multiple of a power of 10.

Student Response

1. Answers vary since students don't yet know the standard of scientific notation. Sample response: $(1.23) \cdot 10^{-7}$.
2. Answers vary. Sample response: $(1.23) \cdot 10^8$.

Student Lesson Summary

Sometimes powers of 10 are helpful for expressing quantities, especially very large or very small quantities. For example, the United States Mint has made over

500,000,000,000

pennies. In order to understand this number, we have to count all the zeros. Since there are 11 of them, this means there are 500 billion pennies. Using powers of 10, we can write this as:

$$500 \cdot 10^9$$

(five hundred times a billion), or even as:

$$5 \cdot 10^{11}$$

The advantage to using powers of 10 to write a large number is that they help us see right away how large the number is by looking at the exponent.

The same is true for small quantities. For example, a single atom of carbon weighs about


0.000000000000000000000000199

grams. We can write this using powers of 10 as

$$199 \cdot 10^{-25}$$

or, equivalently,

$$(1.99) \cdot 10^{-23}$$



Not only do powers of 10 make it easier to write this number, but they also help avoid errors since it would be very easy to write an extra zero or leave one out when writing out the decimal because there are so many to keep track of!

Lesson 9 Practice Problems

Problem 1

Statement

Match each number to its name.

- | | |
|------------------|--------------------------------------|
| a. 1,000,000 | <input type="radio"/> One hundredth |
| b. 0.01 | <input type="radio"/> One thousandth |
| c. 1,000,000,000 | <input type="radio"/> One millionth |
| d. 0.000001 | <input type="radio"/> Ten thousand |
| e. 0.001 | <input type="radio"/> One million |
| f. 10,000 | <input type="radio"/> One billion |

Solution

- a. one million
- b. one hundredth
- c. one billion
- d. one millionth
- e. one thousandth
- f. ten thousand

Problem 2

Statement

Write each expression as a multiple of a power of 10:

- a. 42,300
- b. 2,000
- c. 9,200,000
- d. Four thousand
- e. 80 million
- f. 32 billion

Solution

- a. Answers vary. Sample responses: $423 \cdot 10^2$, $4.23 \cdot 10^4$

- b. Answers vary. Sample response: $2 \cdot 10^3$
- c. Answers vary. Sample responses: $92 \cdot 10^5$, $9.2 \cdot 10^6$
- d. Answers vary. Sample response: $4 \cdot 10^3$
- e. Answers vary. Sample response: $8 \cdot 10^7$
- f. Answers vary. Sample responses: $32 \cdot 10^9$, $3.2 \cdot 10^{10}$

Problem 3

Statement

Each statement contains a quantity. Rewrite each quantity using a power of 10.

- a. There are about 37 trillion cells in an average human body.
- b. The Milky Way contains about 300 billion stars.
- c. A sharp knife is 23 millionths of a meter thick at its tip.
- d. The wall of a certain cell in the human body is 4 nanometers thick. (A nanometer is one billionth of a meter.)

Solution

- a. $37 \cdot 10^{12}$ (or equivalent)
- b. $300 \cdot 10^9$ (or equivalent)
- c. $23 \cdot 10^{-6}$ (or equivalent)
- d. $4 \cdot 10^{-9}$ (or equivalent)

Problem 4

Statement

A fully inflated basketball has a radius of 12 cm. Your basketball is only inflated halfway. How many more cubic centimeters of air does your ball need to fully inflate? Express your answer in terms of π . Then estimate how many cubic centimeters this is by using 3.14 to approximate π .

Solution

$1,152\pi$ cubic cm, 3,617.28 cubic cm

(From Unit 5, Lesson 20.)

Problem 5

Statement

Solve each of these equations. Explain or show your reasoning.

$$2(3 - 2c) = 30$$

$$3x - 2 = 7 - 6x$$

$$31 = 5(b - 2)$$

Solution

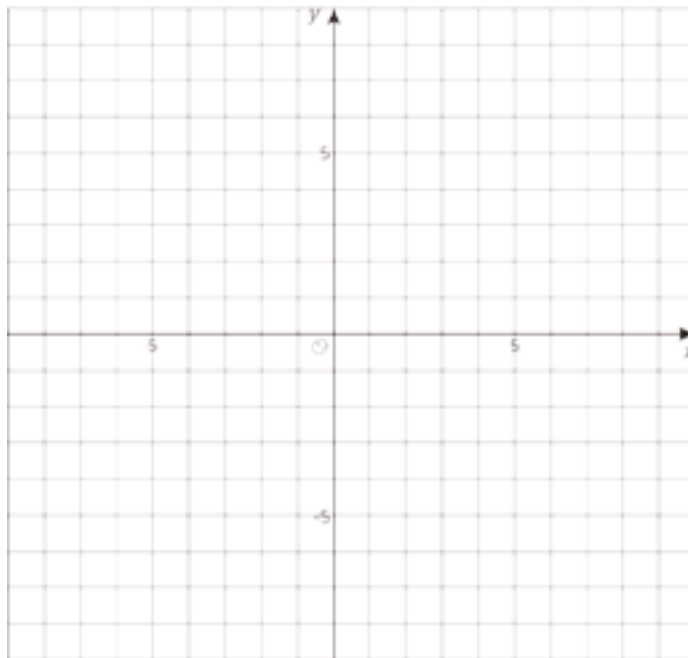
- $c = -6$. Responses vary. Sample response: Divide each side by 2, then subtract 3 from each side, then divide each side by -2.
- $x = 1$. Responses vary. Sample response: Add 2 to each side, then add $6x$ to each side, then divide each side by 9.
- $b = \frac{41}{5}$. Responses vary. Sample response: Distribute 5 on the right side, add 10 to each side, then divide each side by 5.

(From Unit 4, Lesson 5.)

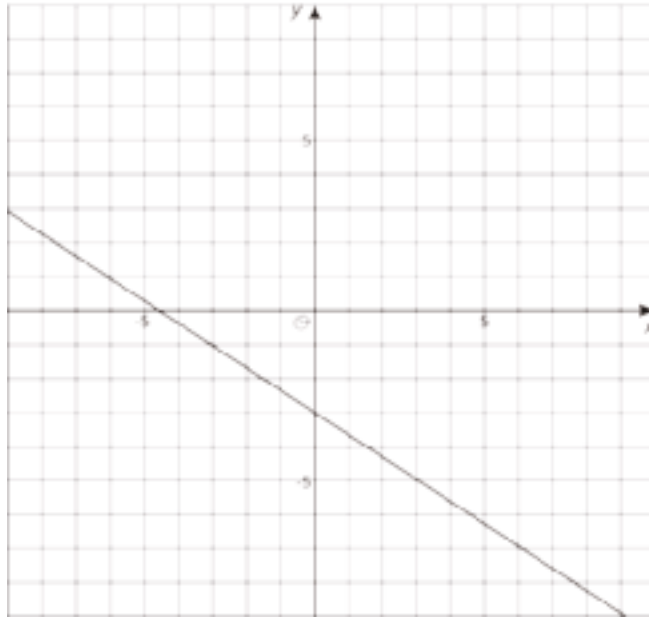
Problem 6

Statement

Graph the line going through $(-6, 1)$ with a slope of $\frac{-2}{3}$ and write its equation.



Solution



$$y = -\frac{2}{3}x - 3$$

(From Unit 3, Lesson 10.)

Lesson 10: Representing Large Numbers on the Number Line

Goals

- Compare large numbers using powers of 10, and explain (orally) the solution method.
- Use number lines to represent (orally and in writing) large numbers as multiples of powers of 10.

Learning Targets

- I can plot a multiple of a power of 10 on such a number line.
- I can subdivide and label a number line between 0 and a power of 10 with a positive exponent into 10 equal intervals.
- I can write a large number as a multiple of a power of 10.

Lesson Narrative

In this lesson, students use number lines to visualize powers of 10, compare very large numbers, and make sense of orders of magnitude (MP2). They use the structure of a number line that is subdivided into 10 equal intervals to express large numbers as multiples of a power of 10, which naturally leads to the idea of scientific notation, which will be introduced in subsequent lessons (MP7).

In these materials, “multiple of a power of 10” does not necessarily mean an *integer* multiple of a power of 10. Students explore numbers of the form $b \cdot 10^n$, where b is some decimal number. Eventually, when students are formally introduced to scientific notation, b is restricted to values between 1 and 10.

Alignments

Addressing

- 8.EE.A.3: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9 , and determine that the world population is more than 20 times larger.
- 8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Building Towards

- 8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR5: Co-Craft Questions
- Think Pair Share

Student Learning Goals

Let's visualize large numbers on the number line using powers of 10.

10.1 Labeling Tick Marks on a Number Line

Warm Up: 5 minutes

This warm-up prompts students to reason about values on a number line that end in a power of 10. It enables them to visualize and make sense of numbers expressed as a product of a single digit and a power of 10, which prepares them to begin working with scientific notation.

Expect student responses to include a variety of incorrect or partially-correct ideas. It is not important that all students understand the correct notation at this point, so it is not necessary to extend the time for this reason.

During the partner discussions, identify and select students who have partially-correct responses to share during the whole-class discussion.

Addressing

- 8.EE.A.3

Instructional Routines

- Think Pair Share

Launch

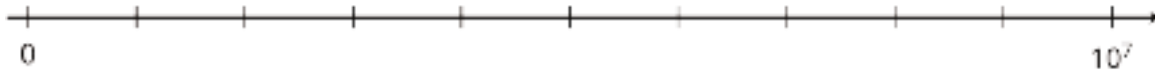
Arrange students in groups of 2. Give students 1 minute of quiet work time and then 2 minutes to compare their number line with their partner. Tell the partners to try and come to an agreement on the values on the number line. Follow with a whole-class discussion.

Anticipated Misconceptions

- Some students may count the tick marks instead of the segments and think 10^7 is being divided into 9 or 11 parts. Consider asking these students to mark each segment with a highlighter to count them.
- Some students may try to label the tick marks 10^1 , 10^2 , 10^3 , etc., and as a result, they may say that the 10^7 is shown in the wrong place. Ask these students how many equal parts 10^7 is divided into, and how they would write that as a division problem or with a fraction.
- Some students may label the tick marks as 10^6 , 20^6 , 30^6 , etc. Ask these students to expand 20^6 and 10^6 into their repeated factors and compare them so they see that 20^6 is not twice as much as 10^6 .
- Some students may work with the idea of $\frac{1}{2}$ and label the middle tick mark as $10^{3.5}$. Explain that this tick mark would have a value of $\frac{1}{2} \cdot 10^7$ and ask these students to use what they know about powers of 10 to find the value of the first tick mark after 0.

Student Task Statement

Label the tick marks on the number line. Be prepared to explain your reasoning.



Student Response

The tick marks should be labeled:

$0, 1 \cdot 10^6, 2 \cdot 10^6, 3 \cdot 10^6, 4 \cdot 10^6, 5 \cdot 10^6, 6 \cdot 10^6, 7 \cdot 10^6, 8 \cdot 10^6, 9 \cdot 10^6, 10^7$.

Activity Synthesis

Ask selected students to explain how they labeled the number line. Record and display their responses on the number line for all to see. As students share, use their responses, correct or incorrect, to guide students to the idea that the first tick mark is $1 \cdot 10^6$, the second is $2 \cdot 10^6$, etc. This gives students the opportunity to connect the number line representation with the computational rules they developed in previous lessons. For example, if a student claims that the second tick mark is 20^6 , they can check whether 20^6 is equal to $2 \cdot 10^6$ by expanding both expressions.

If not uncovered in students' explanations, ask the following questions to make sure students see how to label the number line correctly:

- "How many equal parts is 10^7 being divided into?" (10)
- "If the number at the end of this number line were 20, how would we find the value of each tick mark?" (Divide 20 by 10)

- “Can we use the same reasoning with 10^7 at the end?” (Yes)
- “What is $10^7 \div 10^6$?” (10^6) “What does this number represent?” (The distance between two tick marks)
- “Can we write 10^6 as $1 \cdot 10^6$?” (Yes).
- “If the first tick mark is $1 \cdot 10^6$, then what is the second tick mark?” ($2 \cdot 10^6$) “

10.2 Comparing Large Numbers with a Number Line

10 minutes (there is a digital version of this activity)

This activity encourages students to use the number line to make sense of powers of 10 and think about how to rewrite expressions in the form $b \cdot 10^n$, where b is between 1 and 10 (as in the case of scientific notation). It prompts students to use the structure of the number line to compare numbers, and to extend their use to estimate relative sizes of other numbers when no number lines are given.

As students work, notice the ways in which they compare expressions that are not written as multiples of 10^6 . Highlight some of these methods in the discussion.

Addressing

- 8.EE.A.3

Building Towards

- 8.EE.A.4

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Arrange students in groups of 2. Give students 4 minutes of quiet time to work on the first problem, followed by 1–2 minutes to exchange and discuss their work with their partner (second problem).

Then, tell students that representing numbers as a single digit times a power of 10 is useful for making rough comparisons. Give an example: $9 \cdot 10^{11}$ is roughly 200 times as large as $4 \cdot 10^9$, because 10^{11} is 100 times as much as 10^9 , and 9 is roughly twice as much as 4. Give students the remaining time to answer the last question. Follow with a brief whole-class discussion.

Classes using the digital version have an interactive applet. Students need to drag the points, marked with open circles and their coordinates, to their proper places on the number line. When all five points are on the line, feedback is available. Note: labels are placed above or below the points only to avoid crowding on the number line.

Access for Students with Disabilities

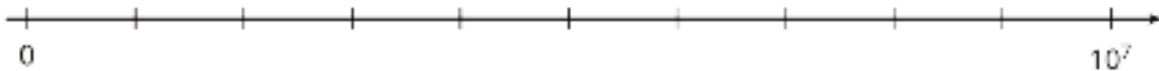
Representation: Internalize Comprehension. Activate or supply background knowledge about number lines. Keep the display from the warm-up visible for students to reference.

Supports accessibility for: Memory; Conceptual processing

Anticipated Misconceptions

Some students may misplace expressions like $(0.6) \cdot 10^7$ or $75 \cdot 10^5$ on the number line. For $75 \cdot 10^5$, point out that 75 is the same as $(7.5) \cdot 10$, so $75 \cdot 10^5$ is equivalent to $(7.5) \cdot 10 \cdot 10^5$. For $(0.6) \cdot 10^7$, point out that 0.6 is the same as $6 \cdot 10^{-1}$, so $(0.6) \cdot 10^7$ is equivalent to $6 \cdot 10^{-1} \cdot 10^7$. Alternatively, tell the student to think of $(0.6) \cdot 10^7$ as between 0 and 10^7 in the same way that 0.6 is between 0 and 1 to guide them to the correct placement on the number line.

Student Task Statement



1. Place the numbers on the number line. Be prepared to explain your reasoning.
 - a. 4,000,000
 - b. $5 \cdot 10^6$
 - c. $5 \cdot 10^5$
 - d. $75 \cdot 10^5$
 - e. $(0.6) \cdot 10^7$
2. Trade number lines with a partner, and check each other's work. How did your partner decide how to place the numbers? If you disagree about a placement, work to reach an agreement.
3. Which is larger, 4,000,000 or $75 \cdot 10^5$? Estimate how many times larger.

Student Response

1.
 - a. 4,000,000 is on the 4th tick mark because it's equal to $4 \cdot 10^6$.
 - b. $5 \cdot 10^6$ is on the 5th tick mark.
 - c. $5 \cdot 10^5$ is between 0 and the 1st tick mark because it is equal to $(0.5) \cdot 10^6$.
 - d. $75 \cdot 10^5$ is between the 7th and 8th tick marks because it is equal to $(7.5) \cdot 10^6$.
 - e. $0.6 \cdot 10^7$ is on the 6th tick mark because it is equal to $6 \cdot 10^6$.
2. No answer required.

3. $75 \cdot 10^5$ (or $(7.5) \cdot 10^6$) is about twice as large as 4,000,000 (or $4 \cdot 10^6$), because 7.5 is roughly twice as large as 4.

Activity Synthesis

Ask students, “How could you change 4,000,000, $75 \cdot 10^5$, and $(0.6) \cdot 10^7$ so that all the expressions have the same power of 10?” Highlight the main idea that it’s always possible to rewrite an expression that is a multiple of a power of 10 so that the leading factor is between 1 and 10. For example, 75 can be written as $(7.5) \cdot 10$, and 0.6 can be written as $6 \cdot 10^{-1}$.

If time allows, consider presenting a problem that allows students to use powers of 10 to estimate the relative sizes of large numbers and use them to answer a question in context:

- The population of the United States is roughly $3 \cdot 10^8$ people. The global population is roughly $7 \cdot 10^9$ people. Estimate how many times larger the global population is than the U.S. population. ($7 \cdot 10^9$ is roughly 20 times as large as $3 \cdot 10^8$, because 7 is roughly twice as large as 3, and 10^9 is 10 times as large as 10^8 .)

Access for English Language Learners

Writing, Speaking: MLR1 Stronger and Clearer Each Time. Use this routine to give students a structured opportunity to revise and refine their response to the last question. Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language (e.g., “Why do you think ___ is larger?”, “How did you compare the two terms?”, “Can you represent your thinking in another way?”, etc.). Students can borrow ideas and language from each partner to strengthen their final version.

Design Principle(s): Optimize output (for explanation)

10.3 The Speeds of Light

20 minutes (there is a digital version of this activity)

This activity guides students to thinking in terms of scientific notation while investigating the properties of light. A number line that shows a power of 10 partitioned into 10 equal intervals is again used to illustrate the base-ten structure. Plotting numbers along it gives a clearer meaning to expressions that are a product of a single digit and a power of 10.

To distinguish more easily between the different speeds of light through various materials, the interval between $2 \cdot 10^8$ and $3 \cdot 10^8$ is magnified on the number line. This illustrates numbers with more decimal places and allows students to see how they are expressed in scientific notation.

Once a number line is labeled with powers of 10 and its structure is understood, numbers given in scientific notation can be placed on the number line fairly straightforwardly. This encourages students to look for ways to write the other numbers in scientific notation.

Addressing

- 8.EE.A.3
- 8.EE.A.4

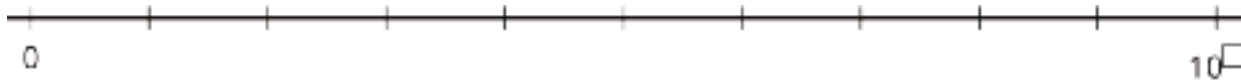
Instructional Routines

- MLR5: Co-Craft Questions

Launch

Display the following number line for all to see. Explain to students that as light moves through different materials, it slows down. The speed of light through empty space, with nothing in its way, is roughly 300,000,000 meters per second. The speed of light through olive oil is much slower at roughly 200,000,000 meters per second.

Ask students to decide what the power of 10 to use for the label of the rightmost tick mark on the number line so that the speed of light through space and through olive oil can be plotted. Give 1 minute of quiet think time before asking 1–2 students to share their responses. Make sure students see that 10^9 is appropriate because for 200,000,000 (which is $2 \cdot 10^8$) to be plotted between 0 and the last tick mark, the last power of 10 needs to be greater than 10^8 .



Next, give students 10–12 minutes to work followed by a whole-class discussion.

Students using the digital materials can use the applet to plot the numbers. The magnifying glass allows them to zoom into any interval between two tick marks and plot numbers to an additional decimal place.

Access for English Language Learners

Writing, Conversing: MLR5 Co-Craft Questions. Display only the table and ask pairs of students to write possible questions that could be answered by the data in the table. Select 2–3 groups to share their questions with the class. Highlight questions that ask students to compare quantities. Next, reveal the questions of the activity. This routine allows students to produce the language of mathematical questions and talk about the quantities in this task that are represented in different ways (i.e., powers of 10, expanded form) prior to being asked to solve questions based on the values.

Design Principle(s): Maximize meta-awareness; Support sense-making

Anticipated Misconceptions

Students may struggle to rewrite a number written using one power of 10 as a number with a different power, for example writing $125 \cdot 10^6$ using 10^8 . Some may know the relationship between the powers of 10 (say, between 10^6 and 10^8), but may not know how expressing one in terms of the

other affects the other factor. Help students make sense of the rewriting process with a series of questions such as these:

- “What do we need to multiply 10^6 by to get 10^8 ?” (100 or 10^2)
- “If we multiply 10^6 by 10^2 , what must we also do to maintain the value of the expression? (Divide the other factor in the expression by 10^2 .)
- “What is the resulting expression?” ($(125 \div 10^2) \cdot (10^6 \cdot 10^2) = (1.25) \cdot 10^8$)
- “How do we know that the two expressions are equivalent?” (We multiplied the expression by 10^2 and then divided it by 10^2 , which is equal to multiplying it by 1.)

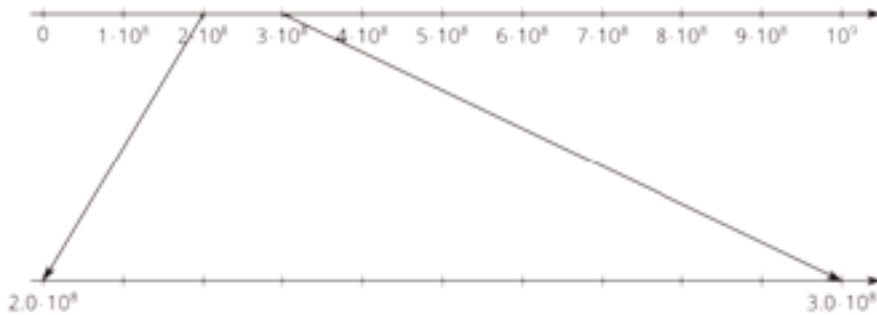
Student Task Statement

The table shows how fast light waves or electricity can travel through different materials.

material	speed (meters per second)
space	300,000,000
water	$(2.25) \cdot 10^8$
copper wire (electricity)	280,000,000
diamond	$124 \cdot 10^6$
ice	$(2.3) \cdot 10^8$
olive oil	200,000,000

1. Which is faster, light through diamond or light through ice? How can you tell from the expressions for speed?

Let’s zoom in to highlight the values between $(2.0) \cdot 10^8$ and $(3.0) \cdot 10^8$.

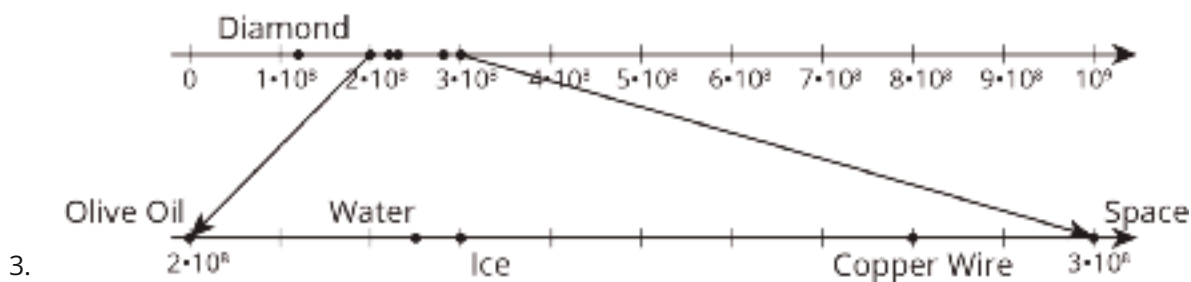


2. Label the tick marks between $(2.0) \cdot 10^8$ and $(3.0) \cdot 10^8$.

- Plot a point for each speed on both number lines, and label it with the corresponding material.
- There is one speed that you cannot plot on the bottom number line. Which is it? Plot it on the top number line instead.
- Which is faster, light through ice or light through diamond? How can you tell from the number line?

Student Response

- Light is faster through ice than through diamond. You can tell because the speed of light through diamond is $(1.24) \cdot 10^8$ meters per second compared to through ice which is $(2.3) \cdot 10^8$ meters per second.
- The zoomed-in part of the number line should be labeled $(2.1) \cdot 10^8$ through $(2.9) \cdot 10^8$.



- Diamond. See number lines in problem 3.
- Light through ice is faster because it is further to the right on the number line.

Are You Ready for More?

Find a four-digit number using only the digits 0, 1, 2, or 3 where:

- the first digit tells you how many zeros are in the number,
- the second digit tells you how many ones are in the number,
- the third digit tells you how many twos are in the number, and
- the fourth digit tells you how many threes are in the number.

The number 2,100 is close, but doesn't quite work. The first digit is 2, and there are 2 zeros. The second digit is 1, and there is 1 one. The fourth digit is 0, and there are no threes. But the third digit, which is supposed to count the number of 2's, is zero.

- Can you find more than one number like this?
- How many solutions are there to this problem? Explain or show your reasoning.

Student Response

The two possible solutions are 1,210 and 2,020. Explanations vary. Sample explanation: Since this is a four-digit number and the digits of this number count how many occurrences of each digit there

are, the sum of the digits must be four. There cannot be any 3's in the number, because that would mean some number needs to occur three times. But 3,000 doesn't work, nor do 2,322 or 1,131. Numbers of the form $_333$, 3_33 , or 33_3 can't work, either, because the sum of the digits is more than four. Therefore, we are looking for combinations of the numbers 0, 1, and 2 that add up to 4, knowing that the last digit must be 0. At this point, there are not many choices left, and we can test them all.

Activity Synthesis

Ask students to explain how they were able to compare the speeds of light. Students do not yet know the definition of scientific notation, but this activity should help them see that expressing values in this format allows us to more easily compare them. Consider using the applet to further illustrate how each number could be plotted on the number line.

Tell students, "We saw that the speed of light through ice can be written as $(2.3) \cdot 10^8$ meters per second, and the speed of electricity can be written as $(2.8) \cdot 10^8$ meters per second. When you write them both the same way like this, it makes it much easier to compare them."

Lesson Synthesis

The purpose of the discussion is to check that students understand how to express a large number as a multiple of a power of 10, find the value of a given multiple of a power of 10, and compare different large numbers by expressing them as multiples of the same power of 10.

It is important students understand that "multiple of a power of 10" does not mean *integer* multiple, necessarily. Tell students that they will be asked to express numbers as "multiples of a power of 10," which might mean writing 52,000 as $(5.2) \cdot 10^4$, for example.

Here are some questions for discussion:

- "What are some ways you came up with to write 230,000,000 using powers of 10?" (Since the value of 230,000,000 will stay the same if it is multiplied by 10 and then divided by 10, we can think of it as:
 $230,000,000 = 23,000,000 \cdot 10 = 2,300,000 \cdot 10^2 = \dots = 23 \cdot 10^7 = (2.3) \cdot 10^8$.)
- "What are some of the ways you came up with to find the value of $(5.4) \cdot 10^5$?" (The value of $(5.4) \cdot 10^5$ is 540,000 because $(5.4) \cdot 10^5 = 54 \cdot 10^4 = 540 \cdot 10^3 = \dots = 540,000$.)
- "How did you compare which was faster—the speed of light through diamond and the speed of light through ice?" (The speed of light through diamond was given as $124 \cdot 10^6$ meters per second, and the speed of light through ice was given as $(2.3) \cdot 10^8$ meters per second. The speed through diamond could be rewritten as $(1.24) \cdot 10^8$ meters per second, which makes it clear that it is slower than the speed of light through ice because $1.24 < 2.3$.)

10.4 Describe the Point

Cool Down: 5 minutes

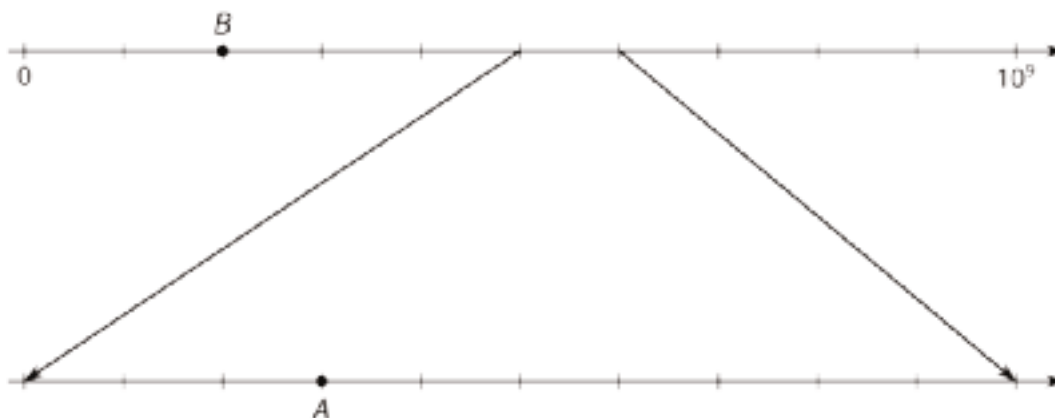
This cool-down measures whether students are able to conceptualize numbers written as a multiple of a positive power of 10 on the number line.

Addressing

- 8.EE.A.3
- 8.EE.A.4

Student Task Statement

We described numbers in this lesson using both powers of 10 and using standard decimal value. For example, the speed of light through ice can be written as a multiple of a power of 10, such as $(2.3) \cdot 10^8$ meters per second, or as a value, such as 230,000,000 meters per second. Use the number line to answer questions about points *A* and *B*.



1. Describe point *B* as:
 - a. A multiple of a power of 10
 - b. A value
2. Describe point *A* as:
 - a. A multiple of a power of 10
 - b. A value
3. Plot a point *C* that is greater than *B* and less than *A*. Describe point *C* as:
 - a. A multiple of a power of 10
 - b. A value

Student Response

1.
 - a. Point *B* represents $2 \cdot 10^8$.
 - b. Point *B* has a value of 200,000,000.

2. a. Point A represents $(5.3) \cdot 10^8$.
 b. Point A has a value of 530,000,000.
3. Answers vary.

Student Lesson Summary

There are many ways to compare two quantities. Suppose we want to compare the world population, about 7.4 billion

to the number of pennies the U.S. made in 2015, about

8,900,000,000

There are many ways to do this. We could write 7.4 billion as a decimal, 7,400,000,000, and then we can tell that there were more pennies made in 2015 than there are people in the world! Or we could use powers of 10 to write these numbers:

$$7.4 \cdot 10^9$$

for people in the world and

$$8.9 \cdot 10^9$$

for the number of pennies.

For a visual representation, we could plot these two numbers on a number line. We need to carefully choose our end points to make sure that the numbers can both be plotted. Since they both lie between 10^9 and 10^{10} , if we make a number line with tick marks that increase by one billion, or 10^9 , we start the number line with 0 and end it with $10 \cdot 10^9$, or 10^{10} . Here is a number line with the number of pennies and world population plotted:



Lesson 10 Practice Problems

Problem 1

Statement

Find three different ways to write the number 437,000 using powers of 10.

Solution

Answers vary. Possible answers: $4.37 \cdot 10^5$, $43.7 \cdot 10^4$, $437 \cdot 10^3$

Problem 2

Statement

For each pair of numbers below, circle the number that is greater. Estimate how many times greater.

a. $17 \cdot 10^8$ or $4 \cdot 10^8$

b. $2 \cdot 10^6$ or $7.839 \cdot 10^6$

c. $42 \cdot 10^7$ or $8.5 \cdot 10^8$

Solution

a. $17 \cdot 10^8$, about 4 times larger

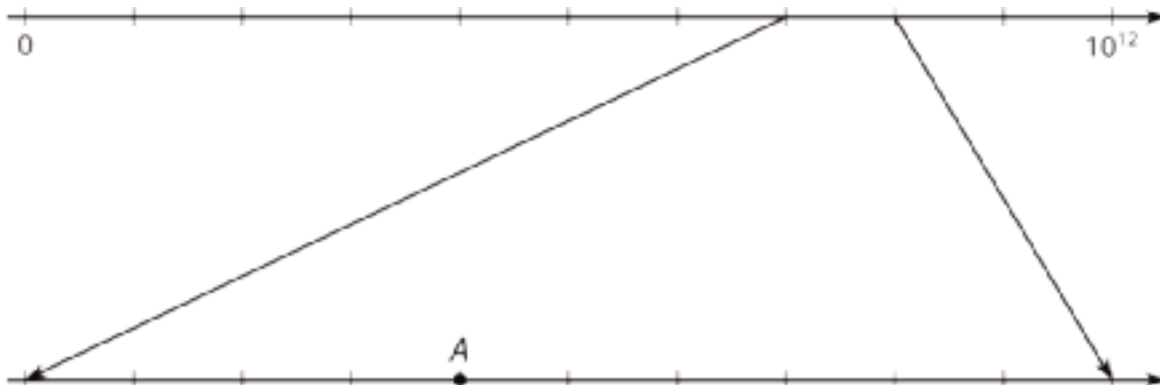
b. $7.839 \cdot 10^6$, about 4 times larger

c. $8.5 \cdot 10^8$, about 2 times larger

Problem 3

Statement

What number is represented by point A ? Explain or show how you know.



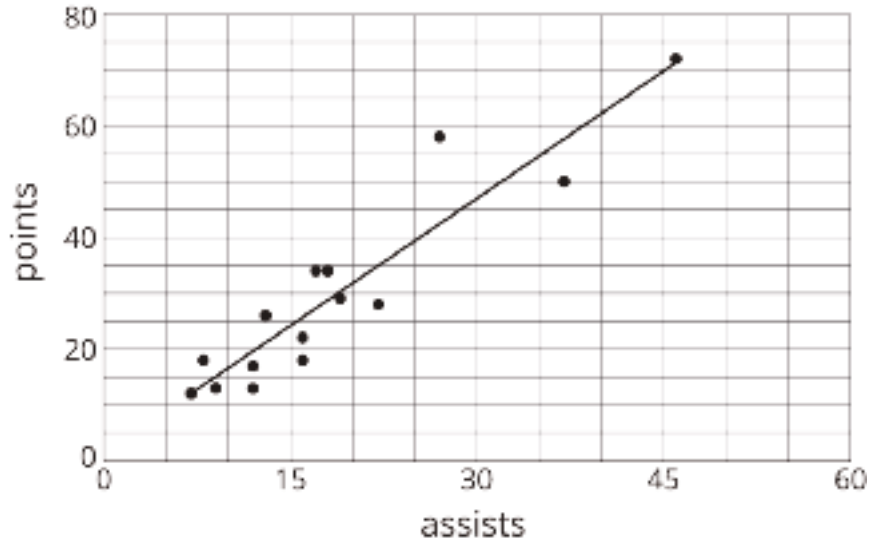
Solution

Answers vary. Sample response: $7.4 \cdot 10^{11}$. Point A lies between $7 \cdot 10^{11}$ and $8 \cdot 10^{11}$. It is $7.4 \cdot 10^{11}$ because it is four tick marks from $7.0 \cdot 10^{11}$.

Problem 4

Statement

Here is a scatter plot that shows the number of points and assists by a set of hockey players. Select **all** the following that describe the association in the scatter plot:



- A. Linear association
- B. Non-linear association
- C. Positive association
- D. Negative association
- E. No association

Solution

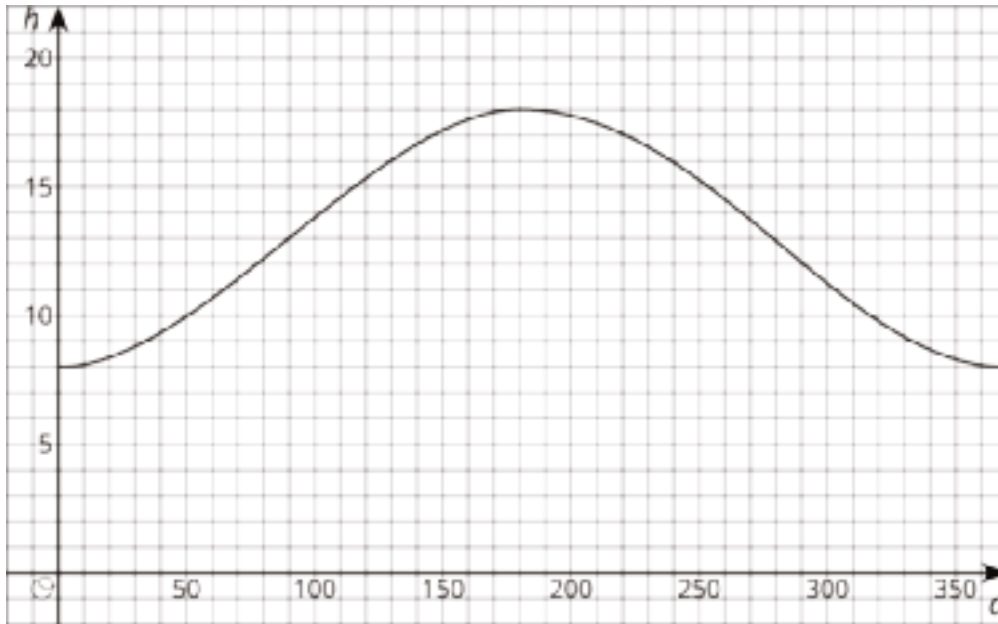
["A", "C"]

(From Unit 6, Lesson 7.)

Problem 5

Statement

Here is the graph of days and the predicted number of hours of sunlight, h , on the d -th day of the year.



- Is hours of sunlight a function of days of the year? Explain how you know.
- For what days of the year is the number of hours of sunlight increasing? For what days of the year is the number of hours of sunlight decreasing?
- Which day of the year has the greatest number of hours of sunlight?

Solution

- h is a function of d . For every d there is one and only one value of h .
- From day 0 to day 180, the hours of sunlight are increasing. From day 180 to day 365, the hours of sunlight are decreasing.
- The day with the greatest number of hours of sunlight is day 180.

(From Unit 5, Lesson 5.)

Lesson 11: Representing Small Numbers on the Number Line

Goals

- Coordinate (orally and in writing) decimals and multiples of powers of 10 representing the same small number.
- Use number lines to represent (orally and in writing) small numbers as multiples of powers of 10 with negative exponents.

Learning Targets

- I can plot a multiple of a power of 10 on such a number line.
- I can subdivide and label a number line between 0 and a power of 10 with a negative exponent into 10 equal intervals.
- I can write a small number as a multiple of a power of 10.

Lesson Narrative

Previously, students used the number line and positive exponents to explore very large numbers. In this lesson, they use the number line and negative exponents to explore very small numbers. Students create viable arguments and critique the reasoning of others when discussing how to represent powers of 10 with negative exponents on a number line (MP3). They attend to precision when deciding how to label the powers of 10 on the number line and how to plot numbers correctly (MP6).

Alignments

Addressing

- 8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
- 8.EE.A.3: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9 , and determine that the world population is more than 20 times larger.
- 8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports

Student Learning Goals

Let's visualize small numbers on the number line using powers of 10.

11.1 Small Numbers on a Number Line

Warm Up: 5 minutes

The purpose of this warm-up is for students to reason about expressions with negative exponents on a number line. Students explore a common misunderstanding about negative exponents that is helpful to address before scientific notation is used to describe very small numbers.

For students' reference, consider displaying a number line from a previous lesson that shows powers of 10 on a number line.

Addressing

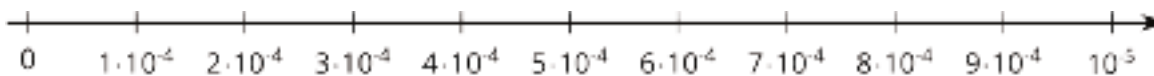
- 8.EE.A.3

Launch

Give students 2 minutes of quiet work time, followed by a whole-class discussion.

Student Task Statement

Kiran drew this number line.



Andre said, "That doesn't look right to me."

Explain why Kiran is correct or explain how he can fix the number line.

Student Response

Answers vary. Sample response: Change each instance of 10^{-4} into 10^{-6} or change 10^{-5} to 10^{-3} .

Activity Synthesis

The important idea to highlight during the discussion is that the larger the size (or absolute value) of a negative exponent, the closer the value of the expression is to zero. This is because the negative exponent indicates the number of factors that are $\frac{1}{10}$. For example, 10^{-5} represents 5 factors that are $\frac{1}{10}$ and 10^{-6} represents 6 factors that are $\frac{1}{10}$, so 10^{-6} is 10 times smaller than 10^{-5} .

Ask one or more students to explain whether they think the number line is correct and ask for their reasoning. Record and display their reasoning for all to see, preferably on the number line. If possible, show at least two correct ways the number line can be fixed.

11.2 Comparing Small Numbers on a Number Line

10 minutes

This task is analogous to a previous activity with positive exponents. The number line strongly encourages students to think about how to change expressions so they all take the form $b \cdot 10^k$, where b is between 1 and 10, as in the case of scientific notation. The number line also is a useful representation to show that, for example, $29 \cdot 10^{-7}$ is about half as much as $6 \cdot 10^{-6}$. The last two questions take such a comparison a step further, asking students to estimate relative sizes using numbers expressed with powers of 10.

As students work, look for different strategies they use to compare expressions that are not written as a product of a number and 10^k . Also look for students who can explain how they estimated in the last two problems. Select them to share their strategies later.

Addressing

- 8.EE.A.3

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. Give students 5 minutes of quiet work time, followed by partner discussion and whole-class discussion. During partner discussion, ask students to share their responses for the first two questions and reach an agreement about where the numbers should be placed on the number line.

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Maintain a display of important terms and vocabulary. During the launch take time to review the visual display of rules for exponents.

Supports accessibility for: Memory; Language

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Display sentence frames to support students as they describe how they compared and estimated the difference in magnitude between the pairs of values $29 \cdot 10^{-7}$ and $6 \cdot 10^{-6}$ or $7 \cdot 10^{-8}$ and $3 \cdot 10^{-9}$. For example, "First I ____, then I ____." or "____ is (bigger/smaller) because ____."

Design Principle(s): Optimize output (for explanation)

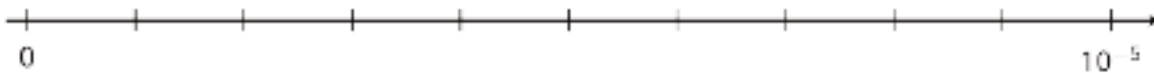
Anticipated Misconceptions

Students may have trouble comparing negative powers of 10. Remind these students that, for example, 10^{-5} is 5 factors that are $\frac{1}{10}$ and 10^{-6} is 6 factors that are $\frac{1}{10}$, so 10^{-5} is 10 times larger than 10^{-6} .

Students may also have trouble estimating how many times one larger expression is than another. Offer these students an example to illustrate how representing numbers as a single digit times a power of 10 is useful for making rough estimations. We have that $9 \cdot 10^{-12}$ is roughly 50 times as much as $2 \cdot 10^{-13}$ because 10^{-12} is 10 times as much as 10^{-13} and 9 is roughly 5 times as much as 2. In other words,

$$\frac{9 \cdot 10^{-12}}{2 \cdot 10^{-13}} \approx \frac{10 \cdot 10^{-12}}{2 \cdot 10^{-13}} = 5 \cdot 10^{-12-(-13)} = 5 \cdot 10^1 = 50$$

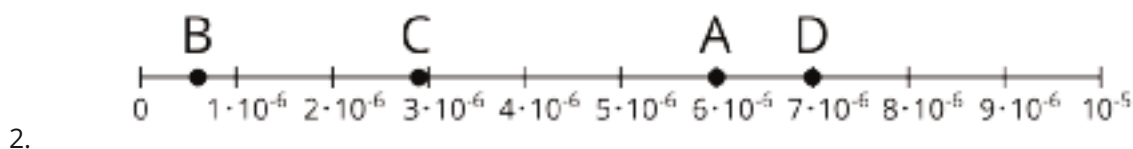
Student Task Statement



1. Label the tick marks on the number line.
2. Plot the following numbers on the number line:
A. $6 \cdot 10^{-6}$ B. $6 \cdot 10^{-7}$ C. $29 \cdot 10^{-7}$ D. $(0.7) \cdot 10^{-5}$
3. Which is larger, $29 \cdot 10^{-7}$ or $6 \cdot 10^{-6}$? Estimate how many times larger.
4. Which is larger, $7 \cdot 10^{-8}$ or $3 \cdot 10^{-9}$? Estimate how many times larger.

Student Response

1. The tick marks should be labeled as increasing multiples of 10^{-6} .



3. $6 \cdot 10^{-6}$ is about twice as large as $29 \cdot 10^{-7}$ because $29 \cdot 10^{-7} = (2.9) \cdot 10^{-6}$, which is roughly $3 \cdot 10^{-6}$.

4. $7 \cdot 10^{-8}$ is roughly 20 times as large as $3 \cdot 10^{-9}$ because 7 is roughly twice as much as 3, and 10^{-8} is 10 times as much as 10^{-9} .

Activity Synthesis

Select previously identified students to explain how they used the number line and powers of 10 to compare the numbers in the last two problems.

One important concept is that it's always possible to change an expression that is a multiple of a power of 10 so that the leading factor is between 1 and 10. For example, we can think of $29 \cdot 10^{-7}$ as $(2.9) \cdot 10 \cdot 10^{-7}$ or $(2.9) \cdot 10^{-6}$. Another important concept is that powers of 10 can be used to make rough estimates. Make sure these ideas are uncovered during discussion.

11.3 Atomic Scale

20 minutes

Students convert a decimal to a multiple of a power of 10 and plot it on a number line. The first problem leads to a product of an integer and a power of 10, and the second leads to a product of a decimal and a power of 10. It is difficult to fit the numbers on the number line without using scientific notation. Again, students build experience with scientific notation before the term is formally introduced.

As students work, notice those who connect the number of decimal places to negative powers of 10. For example, they might notice that counting decimal places to the right of the decimal point corresponds to multiplying by $\frac{1}{10}$ a certain number of times.

Addressing

- 8.EE.A.1
- 8.EE.A.3
- 8.EE.A.4

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

This is the first time students convert small decimals into a multiple of a power of 10 with a negative exponent. Before students begin the activity, review the idea that a decimal can be thought of as a product of a number and $\frac{1}{10}$. Explain, for example, that 0.3 is $3 \cdot \frac{1}{10}$ (3 tenths), 0.03 is $3 \cdot \frac{1}{10} \cdot \frac{1}{10}$ (three hundredths). Similarly, 0.0003 is 3 multiplied by $\frac{1}{10}$ 4 times (3 ten-thousandths). So $(0.0003) = 3 \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} = 3 \cdot 10^{-4}$.

Arrange students in groups of 2. Give students 10 minutes to work, followed by whole-class discussion. Encourage students to share their reasoning with a partner and work to reach an agreement during the task.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support use of structure. For example, check in with students within the first 2-3 minutes of work time. Ask students to share how they decide what power of 10 to put on the right side of this number line.

Supports accessibility for: Visual-spatial processing; Organization

Access for English Language Learners

Writing: MLR3 Clarify, Critique, Correct. Display the incomplete statement: "I just count how many places and write the number in the exponent." Prompt discussion by asking, "What is unclear?" or "What do you think the author is trying to say?" Then, ask students to write a more precise version to explain the strategy of converting. Improved statements should include reference to the relationship between the number of decimal places to the right of the decimal point and repeated multiplication of $\frac{1}{10}$. This helps students evaluate, and improve on, the written mathematical arguments of others.

Design Principle(s): Optimize output (for explanation); Support sense-making

Anticipated Misconceptions

For the mass of the proton, students might find that it is equal to $17 \cdot 10^{-25}$, but 17 does not fit on the number line because there are not 17 tick marks. Ask students whether 1.7 would fit on the number line

(it does). Follow up by asking how to replace 17 with 1.7 times something ($17 = (1.7) \cdot 10$). So $17 \cdot 10^{-25} = (1.7) \cdot 10 \cdot 10^{-25} = (1.7) \cdot 10^{-24}$.

Student Task Statement

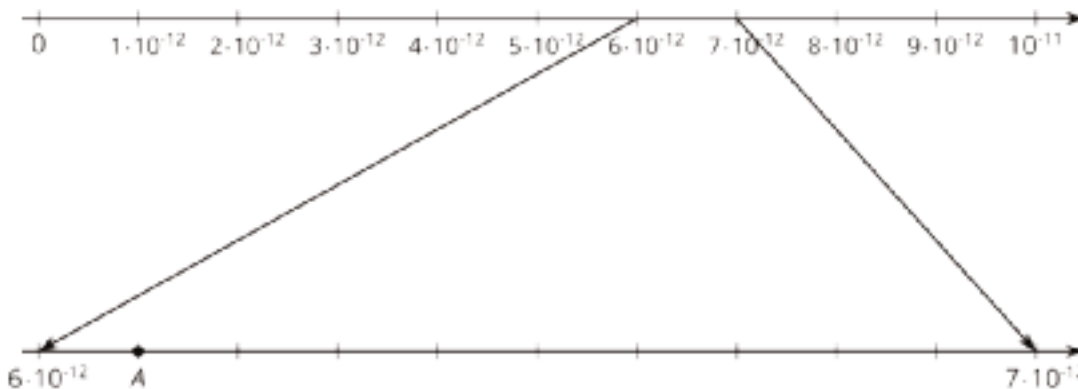
1. The radius of an electron is about 0.000000000000003 cm.
 - a. Write this number as a multiple of a power of 10.
 - b. Decide what power of 10 to put on the right side of this number line and label it.
 - c. Label each tick mark as a multiple of a power of 10.



- d. Plot the radius of the electron in centimeters on the number line.
2. The mass of a proton is about 0.000000000000000000000017 grams.
- Write this number as a multiple of a power of 10.
 - Decide what power of 10 to put on the right side of this number line and label it.
 - Label each tick mark as a multiple of a power of 10.



- d. Plot the mass of the proton in grams on the number line.
3. Point *A* on the zoomed-in number line describes the wavelength of a certain X-ray in meters.



- Write the wavelength of the X-ray as a multiple of a power of 10.
- Write the wavelength of the X-ray as a decimal.

Student Response

- Radius of electron:
 - Using powers of 10, the radius of the electron is $3 \cdot 10^{-13}$ cm.
 - The power on the right side should be 10^{-12} , because $3 \cdot 10^{-13}$ is greater than 10^{-13} but less than 10^{-12} .
 - The tick marks should be labeled in multiples of 10^{-13} .
 - The radius of the electron in cm should be placed at the 3rd tick mark.
- Mass of proton:
 - Using powers of 10, the mass of the proton is $(1.7) \cdot 10^{-24}$ grams.
 - The power on the right side should be 10^{-23} , because $(1.7) \cdot 10^{-24}$ is greater than 10^{-24} but less than 10^{-23} .

- c. The tick marks should be labeled in multiples of 10^{-24} .
 - d. The mass of the proton in grams should be placed between the 1st and 2nd tick marks, closer to the 2nd than the 1st.
3. a. The length of the X-ray's wavelength is $(6.1) \cdot 10^{-12}$ meters.
- b. The length of the X-ray's wavelength is 0.0000000000061 meters.

Activity Synthesis

One key idea is for students to convert small decimals to scientific notation in the process of placing them on a number line. Select a student to summarize how they wrote the mass of the proton as a multiple of a power of 10. Poll the class on whether they agree or disagree and why. The discussion should lead to one or more methods to rewrite a decimal as a multiple of a power of 10. For example, students might count the decimal places to the right of the decimal point and recognize that number as the number of factors that are $\frac{1}{10}$. With this method, 0.0000003, for example, would equal $3 \cdot 10^{-7}$ because 3 has been multiplied by $\frac{1}{10}$ seven times.

Lesson Synthesis

The purpose of the discussion is to check that students know how to convert between decimal numbers and numbers expressed as multiples of powers of 10, and that they understand the order of numbers with negative exponents on the number line.

Some questions for discussion:

- “As we move to the right on the number line, what happens to the value of the numbers we encounter?” (They get larger.)
- “Would 10^{-5} appear to the left or to the right of 10^{-4} on a number line? Explain.” (10^{-5} is smaller than 10^{-4} , so it would be to the left.)
- “How does zooming in on the number line help express numbers between the tick marks?” (Zooming in allows us to subdivide the distance between two tick marks into 10 equal intervals, which allows us to describe a number to an additional decimal place.)
- “Describe how to convert a number such as 0.000278 into a multiple of a power of 10.” (The number is equivalent to $278 \cdot (0.000001)$ or $278 \cdot \frac{1}{100,000}$. The fraction $\frac{1}{100,000}$ is $\frac{1}{10^6}$ or 10^{-6} , so 0.000278 can be written as $278 \cdot 10^{-6}$.)

If time allows, give students other small numbers that are written as decimals and ask them to write them as multiples of powers of 10, and vice versa.

11.4 Describing Very Small Numbers

Cool Down: 5 minutes

Addressing

- 8.EE.A.4

Student Task Statement

1. Write 0.00034 as a multiple of a power of 10.
2. Write $(5.64) \cdot 10^{-7}$ as a decimal.

Student Response

1. $(3.4) \cdot 10^{-4}$, $34 \cdot 10^{-5}$, or equivalent.
2. 0.000000564

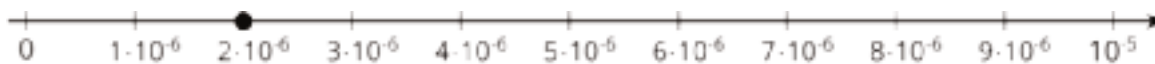
Student Lesson Summary

The width of a bacterium cell is about

$$2 \cdot 10^{-6}$$

meters. If we want to plot this on a number line, we need to find which two powers of 10 it lies between. We can see that $2 \cdot 10^{-6}$ is a multiple of 10^{-6} . So our number line will be labeled with multiples of

$$10^{-6}$$



Note that the right side is labeled

$$10 \cdot 10^{-6} = 10^{-5}$$

The power of ten on the right side of the number line is always *greater* than the power on the left. This is true for powers with positive or negative exponents.

Lesson 11 Practice Problems

Problem 1

Statement

Select all the expressions that are equal to $4 \cdot 10^{-3}$:

- A. $4 \cdot \left(\frac{1}{10}\right) \cdot \left(\frac{1}{10}\right) \cdot \left(\frac{1}{10}\right)$
- B. $4 \cdot (-10) \cdot (-10) \cdot (-10)$
- C. $4 \cdot 0.001$
- D. $4 \cdot 0.0001$
- E. 0.004
- F. 0.0004

Solution

["A", "C", "E"]

Problem 2

Statement

Write each expression as a multiple of a power of 10:

- a. 0.04
- b. 0.072
- c. 0.0000325
- d. Three thousandths
- e. 23 hundredths
- f. 729 thousandths
- g. 41 millionths

Solution

- a. Answers vary. Sample response: $4 \cdot 10^{-2}$
- b. Answers vary. Sample response: $7.2 \cdot 10^{-2}$, $72 \cdot 10^{-3}$
- c. Answers vary. Sample responses: $3.25 \cdot 10^{-5}$, $325 \cdot 10^{-7}$

d. Answers vary. Sample response: $3 \cdot 10^{-3}$

e. Answers vary. Sample responses: $2.3 \cdot 10^{-1}$, $23 \cdot 10^{-2}$

f. Answers vary. Sample responses: $7.29 \cdot 10^{-1}$, $729 \cdot 10^{-3}$

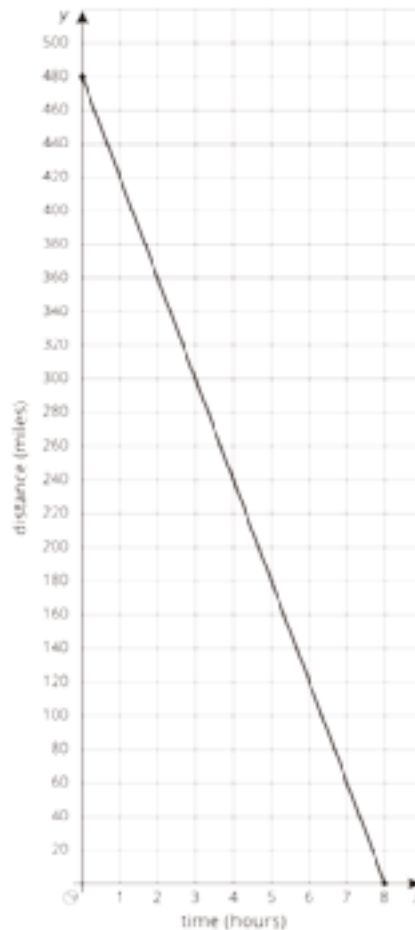
g. Answers vary. Sample responses: $4.1 \cdot 10^{-5}$, $41 \cdot 10^{-6}$

Problem 3

Statement

A family sets out on a road trip to visit their cousins. They travel at a steady rate. The graph shows the distance remaining to their cousins' house for each hour of the trip.

- How fast are they traveling?
- Is the slope positive or negative? Explain how you know and why that fits the situation.
- How far is the trip and how long did it take? Explain how you know.



Solution

- 60 miles per hour
- Negative. Explanations vary. Sample explanation: The slope is negative because the line moves down toward the right. It shows the change in remaining miles for each hour. There are 60 fewer miles remaining each hour, which means the car is traveling at a steady rate of 60 miles each hour.

c. 480 miles and 8 hours. Explanations vary. Sample explanation: The trip is 480 miles because the remaining distance was 480 miles when they started out (after 0 hours). The trip took 8 hours because after 8 hours, there were 0 miles remaining.

(From Unit 3, Lesson 9.)

Lesson 12: Applications of Arithmetic with Powers of 10

Goals

- Determine what information is needed to answer a question about large numbers, and explain (orally) how that information would help solve the problem.
- Use exponent rules and powers of 10 to solve problems in context, and explain (orally) the steps used to organize thinking.

Learning Targets

- I can apply what I learned about powers of 10 to answer questions about real-world situations.

Lesson Narrative

Students apply what they have learned about working with exponents (especially powers of ten) to solve rich problems in context. The style of questioning requires students to identify essential features of the problem and persevere to calculate and interpret the solutions in context (MP1, MP2, MP4). Students must attend to precision when considering appropriate units of measurement (MP6).

Alignments

Addressing

- 8.EE.A.3: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9 , and determine that the world population is more than 20 times larger.
- 8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Building Towards

- 8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Instructional Routines

- MLR1: Stronger and Clearer Each Time

- MLR8: Discussion Supports

Student Learning Goals

Let's use powers of 10 to help us make calculations with large and small numbers.

12.1 What Information Do You Need?

Warm Up: 5 minutes

The purpose of this warm-up is for students to reason about a real-world situation and consider the essential information required to solve problems (MP4).

Building Towards

- 8.EE.A.4

Launch

Arrange students in groups of 2. Give students 1 minute of quiet think time, followed by 1 minute to share their responses with a partner. Follow with a whole-class discussion.

Student Task Statement

What information would you need to answer these questions?

1. How many meter sticks does it take to equal the mass of the Moon?
2. If all of these meter sticks were lined up end to end, would they reach the Moon?

Student Response

1. The mass of the Moon and the mass of a meter stick.
2. Distance to the Moon.

Activity Synthesis

Ask students to share their responses for each question. Record and display the responses for all to see.

Consider asking questions like these to encourage students to reason further about each question:

- "Why do you need that piece of information?"
- "How would you use that piece of information in finding the solution?"
- "Where would you look to find that piece of information?"

If there is time, ask students for predictions for each of the questions. Record and display their responses for all to see.

12.2 Meter Sticks to the Moon

20 minutes

The large quantities involved in these questions lend themselves to arithmetic with powers of 10, giving students the opportunity to make use of scientific notation before it is formally introduced. This activity was designed so students could practice modeling skills such as identifying essential features of the problem and gathering the required information (MP4). Students use powers of 10 and the number line as tools to make it easier to calculate and interpret results.

Notice the ways in which students use relevant information to answer the questions. Identify students who can explain why they are calculating with one operation rather than another. Speed is not as important as carefully thinking through each problem.

Addressing

- 8.EE.A.3
- 8.EE.A.4

Instructional Routines

- MLR8: Discussion Supports

Launch

From the warm-up, students have decided what information they need to solve the problem. Invite students to ask for the information they need. Provide students with only the information they request. Display the information for students to see throughout the activity. If students find they need more information later, provide it to the whole class then.

Here is information students might ask for in order to solve the problems:

- The mass of an average classroom meter stick is roughly 0.2 kg.
- The length of an average classroom meter stick is 1 meter.
- The mass of the Moon is approximately $7 \cdot 10^{22}$ kg.
- The Moon is roughly $(3.8) \cdot 10^8$ meters away from Earth.
- The distances to various astronomical bodies the students might recognize, in light years, as points of reference for their last answer. (Consider researching other distances in advance or, if desired, encouraging interested students to do so.)

Arrange students in groups of 2–4 so they can discuss how to use the information to solve the problem. Give students 15 minutes of work time.

Student Task Statement

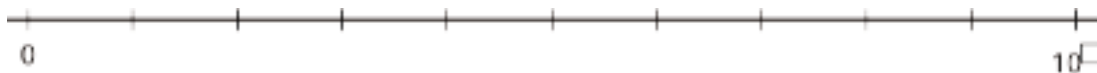
1. How many meter sticks does it take to equal the mass of the Moon? Explain or show your reasoning.

2. Label the number line and plot your answer for the number of meter sticks.



3. If you took all the meter sticks from the last question and lined them up end to end, will they reach the Moon? Will they reach beyond the Moon? If yes, how many times farther will they reach? Explain your reasoning.

4. One light year is approximately 10^{16} meters. How many light years away would the meter sticks reach? Label the number line and plot your answer.



Student Response

1. $(3.5) \cdot 10^{23}$ meter sticks, because the mass of the Moon divided by the mass of a meter stick is

$$\frac{7 \cdot 10^{22}}{2 \cdot 10^{-1}} = \frac{7}{2} \cdot 10^{22-(-1)} = (3.5) \cdot 10^{23}$$

2. The right side of the number line should be labeled with 10^{24} with the tick marks labeled as multiples of 10^{23} . The number of meter sticks should be placed between the 3rd and 4th tick marks.

3. About 10^{15} , or a thousand trillion times as far as the Moon, because $4 \cdot 10^{23}$ is 10^{15} times as much as $4 \cdot 10^8$.

4. $(3.5) \cdot 10^7$, or 35 million light years away. This should be plotted on a number line with 10^8 as the rightmost tick mark, labeled with multiples of 10^7 . The number of light years should be placed between the 3rd and 4th tick marks.

Are You Ready for More?

Here is a problem that will take multiple steps to solve. You may not know all the facts you need to solve the problem. That is okay. Take a guess at reasonable answers to anything you don't know. Your final answer will be an estimate.

If everyone alive on Earth right now stood very close together, how much area would they take up?

Student Response

Answers vary (and are likely to vary wildly). Sample response: There are between 7 billion and 8 billion people on Earth right now, so let's say 8 billion. Most teenagers and adults can fit into a rectangle of about 1 meter by half a meter when standing, so half a square meter. Smaller children would take up less space—maybe about half the space of an adult, so say children take up a quarter of a square meter. Let's guess that about a quarter of the people on Earth are small

children. The space the adults will take up should be 6 billion times half a square meter, which is 3 billion square meters. The children will take up 2 billion times a quarter of a square meter, which is half a billion square meters. In total, the people will take up about 3.5 billion square meters.

Students may want to convert this answer to kilometers (or feet to miles, etc). To do this requires the tricky realization that, even though there are 1,000 meters in a kilometer, there are $1,000^2$ square meters in a square kilometer. Knowing this, we can divide $(3.5) \cdot 10^9$ by $1 \cdot 10^6$ to get $(3.5) \cdot 10^3$, or 35,000 square kilometers.

Activity Synthesis

Select previously identified students to share how they organized their relevant information and how they planned to use the information to answer the questions. The important idea students should walk away with is that powers of 10 are a great tool to tackle challenging, real-world problems that involve very large numbers.

It might be illuminating to put 35 million light years into some context. It is over a thousand trillion times as far as the distance to the Moon, or about the size of a supercluster of galaxies. The Sun is less than 1.6×10^{-5} light year away from Earth.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation such as: "First, I ____ because . . .", "I noticed ____ so I . . .", "Why did you . . .?", "I agree/disagree because . . ."

Supports accessibility for: Language; Social-emotional skills

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Give students additional time to make sure everyone in their group can explain their response to the first question. Then, vary who is called on to represent the ideas of each group. This routine will prepare students for the role of group representative and to support each other to take on that role.

Design Principle(s): Optimize output (for explanation); Cultivate conversation

12.3 That's a Tall Stack of Cash

Optional: 20 minutes

This activity also illustrates the utility of using powers of 10 to work with and interpret very large quantities. Students practice modeling skills, such as identifying essential features of a problem and

gathering the required information (MP4). Students use numbers and exponents flexibly and interpret their results in context (MP2).

As students work, look for students who use powers of 10 and the number line as tools to make it easier to calculate and interpret their results.

Addressing

- 8.EE.A.4

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Ask the class to predict which is taller, the Burj Khalifa or a stack of money it cost to build the Burj Khalifa. Push them further by asking them to predict how high they think the stack would go. Record some of these predictions. Students will ask for the information they need to solve these problems:

- A 1-meter stack of 100-dollar bills is about 1,000,000 dollars. The Burj Khalifa is 830 meters tall and cost 1.5 billion dollars.
- The Burj Khalifa weighs 450,000,000 kg. A penny weighs $(2.5) \cdot 10^{-3}$ kg. There are 100 pennies in a dollar.

Arrange students in groups of 2–4. Give students 10–15 minutes to work. As students work to finish the fourth problem (plotting the heights on a number line), tell the class that their next step is to read the fifth problem and think about what additional information they would need to know to solve the problem. When many students have finished problem 4, pause to allow the class to ask these questions before proceeding.

Access for Students with Disabilities

Representation: Internalize Comprehension. Begin the activity with concrete or familiar contexts. Review an image or video of the Burj Khalifa to activate prior knowledge of the context of the problem.

Supports accessibility for: Conceptual processing; Memory

Access for English Language Learners

Writing, Speaking: MLR1 Stronger and Clearer Each Time. Use this routine to give students a structured opportunity to revise and refine their response to the question: “Which is taller, the Burj Khalifa, or the stack of the money it cost to build the Burj Khalifa?” Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them to strengthen their ideas and clarify their language (e.g., “What did you do first?”, “How do you know...?”, “How did you compare the two values?”, etc.). Students can borrow ideas and language from each partner to strengthen their final version.

Design Principle(s): Optimize output (for explanation)

Anticipated Misconceptions

Students may overlook the fact that there are 100 pennies in a dollar. Remind these students of this fact and ask, “If you use pennies instead of dollars, would there be more coins or fewer coins? How many times more? If 1.5 billion dollars is $(1.5) \cdot 10^9$, then how would you find the number of pennies?”

Student Task Statement

In 2016, the Burj Khalifa was the tallest building in the world. It was very expensive to build.

Consider the question: Which is taller, the Burj Khalifa or a stack of the money it cost to build the Burj Khalifa?

1. What information would you need to be able to solve the problem?
2. Record the information your teacher shares with the class.
3. Answer the question “Which is taller, the Burj Khalifa or a stack of the money it cost to build the Burj Khalifa?” and explain or show your reasoning.
4. Decide what power of 10 to use to label the rightmost tick mark of the number line, and plot the height of the stack of money and the height of the Burj Khalifa.



5. Which has more mass, the Burj Khalifa or the mass of the pennies it cost to build the Burj Khalifa? What information do you need to answer this?

6. Decide what power of 10 to use to label the rightmost tick mark of the number line, and plot the mass of the Burj Khalifa and the mass of the pennies it cost to build the Burj Khalifa.



Student Response

1. The height of the Burj Khalifa, the cost to build it, and the height of a stack of a million dollars.
2. The Burj Khalifa is 830 meters tall and cost 1.5 billion dollars to build. A 1 meter stack of \$100 bills is about 1,000,000 dollars.
3. The stack of cash is $(1.5) \cdot 10^3$ or 1,500 meters tall, about twice as tall as the Burj Khalifa. This is because

$$\frac{(1.5) \cdot 10^9}{10^6} = (1.5) \cdot 10^3$$

4. 10^4 for the rightmost tick mark and the first 9 integer multiples of 10^3 for tick marks leading up to it. The height of the stack of cash should be placed between the first and second tick marks. The height of the building should be placed between 0 and the first tick mark, but closer to the first tick mark than 0.
5. The Burj Khalifa has a mass of $(4.5) \cdot 10^8$ kg, and a penny has a mass of $(2.5) \cdot 10^{-3}$ kg. Since there are 100 pennies in each dollar, the number of pennies in 1.5 billion dollars is $(1.5) \cdot 10^9 \cdot 10^2 = (1.5) \cdot 10^{11}$ pennies. Since each penny has a mass of $(2.5) \cdot 10^{-3}$ kg, then the total mass of the pennies is $(2.5) \cdot 10^{-3} \cdot (1.5) \cdot 10^{11} = (3.75) \cdot 10^8$ kg, which is not quite as massive as the Burj Khalifa.
6. 10^9 for the rightmost tick mark and the first 9 integer multiples of 10^8 for tick marks leading up to it. The mass of the pennies should be placed $\frac{3}{4}$ of the way between the 3rd and 4th tick marks, and the mass of the Burj Khalifa should be placed between the 4th and 5th tick marks.

Activity Synthesis

If time allows, return to some of the recorded predictions. Acknowledge predictions that were accurate and discuss how powers of 10 made this problem much more approachable.

Lesson Synthesis

To prompt students to reflect on the modeling process and on using exponents to solve problems, consider asking some of these questions:

- “To solve the problems in this lesson you had to determine what information was needed. Did you find that to be fairly straightforward or challenging? What made it straightforward or challenging?”
- “Describe your thinking as you planned a solution path for the problems. For example, did you ask for information first and then decide what to do with it, or did you decide what needs to be done first before asking for certain information?”
- “Once you had the information you needed, what were some difficulties you encountered? How did you work through them?”
- “How did exponent rules and powers of 10 make the calculations easier?” (Powers of 10 make the numbers easier to express and interpret. The rules of exponents were handy for comparing how many times as large or as small one number is as another number.)
- “Would an estimate be an acceptable answer for problems like these? Why or why not? When might we need more precise solutions?” (It depends on the questions and how the answers would be used. For example, if we were planning a space exploration, we would likely need a high level of precision to ensure that we hit our targets. But if the answers are for comparison or general information, estimates are likely adequate.)

12.4 Reflecting on Using Powers of 10

Cool Down: 5 minutes

Addressing

- 8.EE.A.4

Student Task Statement

What is a mistake you would expect to see others make when doing problems like the ones in this lesson? Give an example of what such a mistake looks like.

Student Response

Answers vary. Sample response: I would expect to see others make mistakes using the exponent rules if they are not careful. For example, when multiplying two powers of 10 together, the exponents are added together. Someone might try to multiply the exponents instead. In the “Tall Stack of Cash” activity, someone might try to calculate how much an 830-meter stack of money would be and write $(8.3) \cdot 10^2$ meters times 10^6 dollars per meter and get $(8.3) \cdot 10^{12}$ dollars.

Student Lesson Summary

Powers of 10 can be helpful for making calculations with large or small numbers. For example, in 2014, the United States had

318,586,495

people who used the equivalent of

2,203,799,778,107

kilograms of oil in energy. The amount of energy per person is the total energy divided by the total number of people. We can use powers of 10 to estimate the total energy as

$$2 \cdot 10^{12}$$

and the population as

$$3 \cdot 10^8$$

So the amount of energy per person in the U.S. is roughly

$$(2 \cdot 10^{12}) \div (3 \cdot 10^8)$$

That is the equivalent of

$$\frac{2}{3} \cdot 10^4$$

kilograms of oil in energy. That's a lot of energy—the equivalent of almost 7,000 kilograms of oil per person!

In general, when we want to perform arithmetic with very large or small quantities, estimating with powers of 10 and using exponent rules can help simplify the process. If we wanted to find the exact quotient of 2,203,799,778,107 by 318,586,495, then using powers of 10 would not simplify the calculation.

Lesson 12 Practice Problems

Problem 1

Statement

Which is larger: the number of meters across the Milky Way, or the number of cells in all humans? Explain or show your reasoning.

Some useful information:

- The Milky Way is about 100,000 light years across.
- There are about 37 trillion cells in a human body.
- One light year is about 10^{16} meters.
- The world population is about 7 billion.

Solution

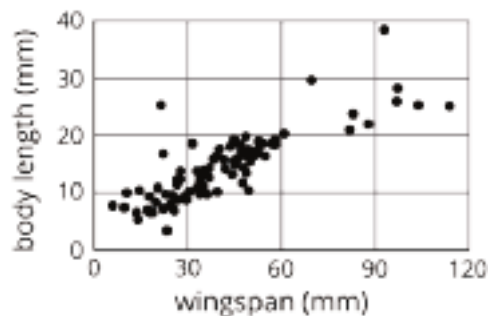
There are more human cells than there are meters across the Milky Way. Since 100,000 is 10^5 , it is about $10^5 \cdot 10^{16}$ or 10^{21} meters across the Milky Way. Notice that 37 trillion is $(3.7) \cdot 10^{13}$ and 7 billion is $7 \cdot 10^9$, so the total number of cells of all humans is $(3.7) \cdot 10^{13} \cdot 7 \cdot 10^9$. This gives $(25.9) \cdot 10^{22}$ human cells. This is about 260 times larger than 10^{21} , the approximate number of meters across the Milky Way. Using more precise values for population and the number of meters in a light year will yield slightly different results.

Problem 2

Statement

Ecologists measure the body length and wingspan of 127 butterfly specimens caught in a single field.

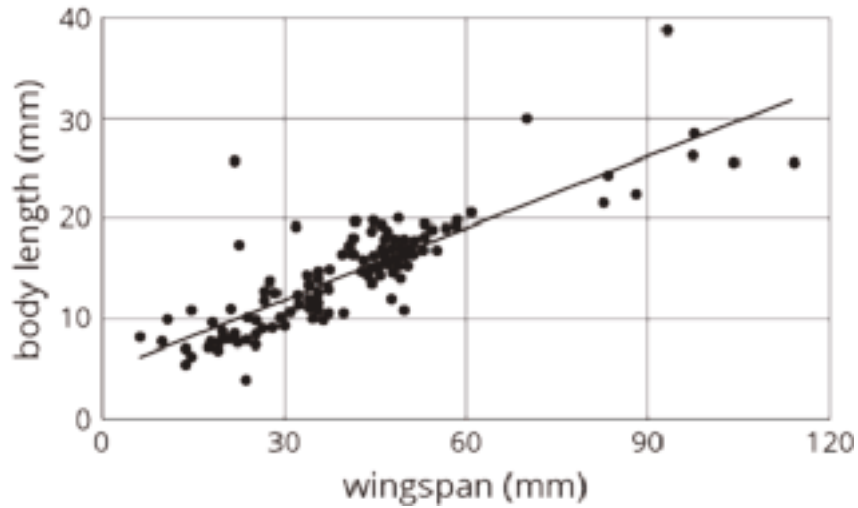
- Draw a line that you think is a good fit for the data.
- Write an equation for the line.
- What does the slope of the line tell you about the wingspans and lengths of these butterflies?



Solution

Answers vary. Sample response:

a.



b. $y = \frac{1}{4}x + 5$

c. For every 4 millimeters the length of the wingspan increases, the body length increases 1 millimeter.

(From Unit 6, Lesson 5.)

Problem 3

Statement

Diego was solving an equation, but when he checked his answer, he saw his solution was incorrect. He knows he made a mistake, but he can't find it. Where is Diego's mistake and what is the solution to the equation?

$$\begin{aligned} -4(7 - 2x) &= 3(x + 4) \\ -28 - 8x &= 3x + 12 \\ -28 &= 11x + 12 \\ -40 &= 11x \\ \frac{40}{11} &= x \end{aligned}$$

Solution

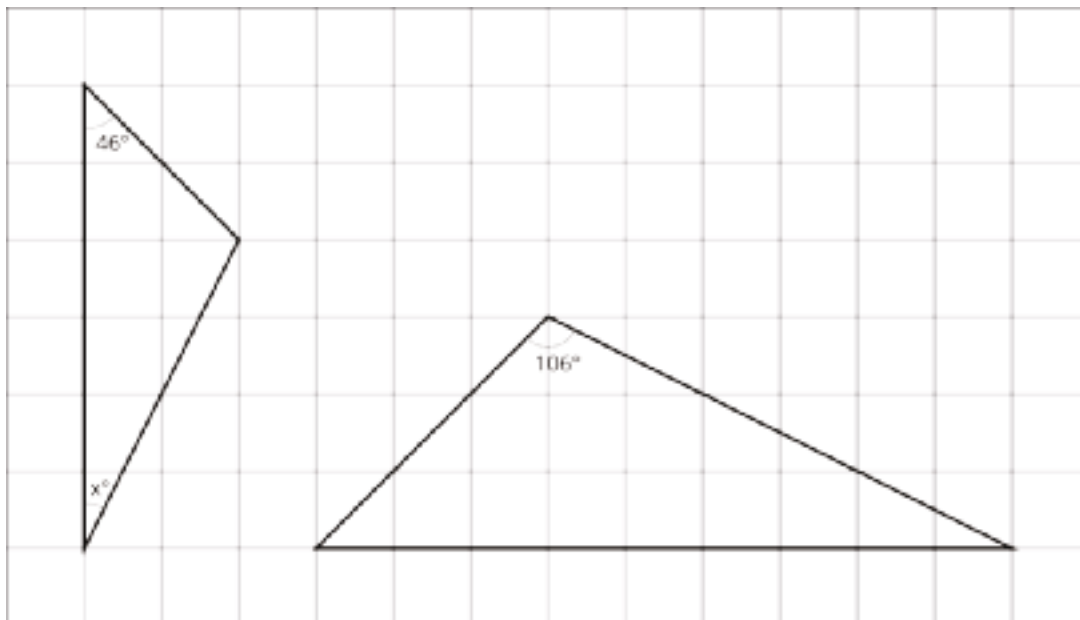
Diego's mistake occurred in the transition from the first line to the second line. The distributive property with $-4(7 - 2x)$ should give $-28 + 8x$. The correct solution is $x = 8$.

(From Unit 4, Lesson 5.)

Problem 4

Statement

The two triangles are similar. Find x .



Solution

$x = 28$ (The obtuse angle in both triangles measures 106° because they are similar. The sum of the three angles in a triangle is 180° .)

(From Unit 2, Lesson 7.)

Lesson 13: Definition of Scientific Notation

Goals

- Identify (in writing) numbers written in scientific notation, and describe (orally) the features of an expression in scientific notation.

Learning Targets

- I can tell whether or not a number is written in scientific notation.

Lesson Narrative

In the previous few lessons, students have built familiarity with arithmetic involving powers of 10 to solve problems with very large and very small quantities. This lesson formalizes what they have learned by introducing the definition of **scientific notation**. A number is said to be in scientific notation if it is written as a product of two factors: the first factor is a number greater than or equal to 1, but less than 10; and the second factor is an integer power of 10. This definition does not include negative numbers for simplicity. Students must attend to precision as they decide whether or not numbers are in scientific notation and convert to scientific notation (MP6).

Alignments

Building On

- 5.NBT.A.2: Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Addressing

- 8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Building Towards

- 8.EE.A.3: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9 , and determine that the world population is more than 20 times larger.

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports

- Number Talk

Required Materials

Pre-printed slips, cut from copies of the **Instructional master**

Required Preparation

The Instructional master for Scientific Notation Matching has three sets of cards. Set A is for the teacher to demonstrate the process, so only one copy of set A is needed. Cut out one set of cards (either set B or set C) for every 2 students. If possible, copy each complete set on a different color of paper, so that a stray slip can quickly be put back.

Student Learning Goals

Let's use scientific notation to describe large and small numbers.

13.1 Number Talk: Multiplying by Powers of 10

Warm Up: 5 minutes

The purpose of this Number Talk is to elicit strategies and understandings students have for multiplying by a power of 10. These understandings help students develop fluency and will be helpful later in this lesson when students will need to be able to work with numbers in scientific notation. While four problems are given, it may not be possible to share every strategy. Consider gathering only two or three different strategies per problem, saving most of the time for the final question.

Building On

- 5.NBT.A.2

Building Towards

- 8.EE.A.3

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all previous problems displayed throughout the talk. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Student Task Statement

Find the value of each expression mentally.

$$123 \cdot 10,000$$

$$(3.4) \cdot 1,000$$

$$(0.6) \cdot 100$$

$$(7.3) \cdot (0.01)$$

Student Response

Explanations vary. Sample responses:

- $123 \cdot 10^4 = 1,230,000$ because multiplying by 10^4 puts 4 more decimal places left of the decimal point.
- $(3.4) \cdot 1,000 = 3,400$ because multiplying by 1,000 puts 3 more decimal places left of the decimal point.
- $(0.6) \cdot 100 = 60$ because multiplying by 100 puts 2 more decimal places left of the decimal point.
- $(7.3) \cdot (0.01) = 0.073$ because multiplying by 0.01 puts 2 more decimal places right of the decimal point.

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. After the last problem, ask students, "How could we rewrite each expression as a product of a number and a power of 10?" Record and display their responses next to each of the original expressions for all to see.

To involve more students in the conversation, consider asking:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone have the same strategy but would explain it differently?"
- "Did anyone solve the problem in a different way?"

- “Does anyone want to add on to ____’s strategy?”
- “Do you agree or disagree? Why?”

Access for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, “First, I ____ because . . .” or “I noticed ____ so I . . .” Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Design Principle(s): Optimize output (for explanation)

13.2 The “Science” of Scientific Notation

15 minutes

Students learn the definition of scientific notation and practice using it. Students attend to precision when determining whether or not a number is in scientific notation and converting numbers into scientific notation (MP6).

Throughout the activity, students use the usual \cdot symbol to indicate multiplication, but the discussion establishes the standard way to show multiplication in scientific notation with the \times symbol. Although these materials tend to avoid the \times symbol because it is easy to confuse with x , the ubiquitous use of \times for scientific notation outside of these materials necessitates its use here.

Addressing

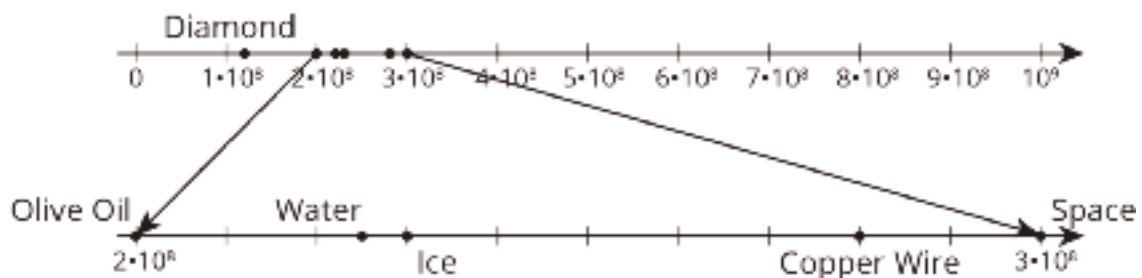
- 8.EE.A.4

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Tell students, “Earlier, we examined the speed of light through different materials. We zoomed into the number line to focus on the interval between 2.0×10^8 meters per second and 3.0×10^8 meters per second as shown in the figure.” Display the following image notation for all to see.



Tell students, "We saw that the speed of light through ice was 2.3×10^8 meters per second. This way of writing the number is called *scientific notation*. Scientific notation is useful for understanding very large and very small numbers."

Display and explain the following definition of scientific notation for all to see.

A number is said to be in scientific notation when it is written as a product of two factors:

- The first factor is a number greater than or equal to 1, but less than 10, for example 1.2, 8, 6.35, or 2.008.
- The second factor is an integer power of 10, for example 10^8 , 10^{-4} , or 10^{22} .

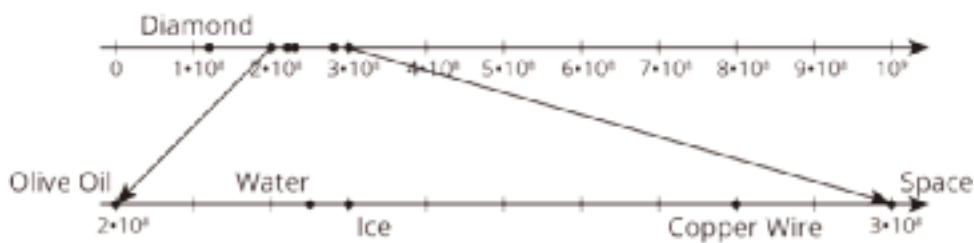
Carefully consider the first question and go through the list of numbers as a class, frequently referring to the definition to decide whether the number is written in scientific notation. When all numbers written in scientific notation have been circled, consider demonstrating or discussing how a number that was not circled could be written in scientific notation. Then, ask students to complete the second question (representing the other numbers in scientific notation). Leave 3–4 minutes for a whole-class discussion.

Student Task Statement

The table shows the speed of light or electricity through different materials.

material	speed (meters per second)
space	300,000,000
water	2.25×10^8
copper (electricity)	280,000,000
diamond	124×10^6
ice	2.3×10^8
olive oil	0.2×10^9

Circle the speeds that are written in scientific notation. Write the others using scientific notation.



Student Response

The speeds of light through water and ice are given in scientific notation.

The following are speeds of light through the material (meters per second):

- Space: 3×10^8
- Copper: 2.8×10^8
- Diamond: 1.24×10^8
- Olive oil: 2×10^8

Activity Synthesis

Tell students that almost all books and information about scientific notation use the \times symbol to indicate multiplication between the two factors, so from now on, these materials will use the \times symbol in this same way. Display $(2.8) \cdot 10^8$ for all to see, and then rewrite it as 2.8×10^8 . Emphasize that using \cdot is not incorrect, but that \times is the most common usage.

Ask students to come up with at least two examples of numbers that are *not* in scientific notation. Select responses that highlight the fact that the first factor must be between 1 and 10 and other responses that highlight that one of the factors must be an integer power of 10. Make sure students recognize what does and does not count as scientific notation.

Also make sure students understand how to write an expression that may use a power of 10 but is not in scientific notation as one that is in scientific notation. Consider using the speed of light through diamond as an example. Ask a series of questions such as:

- “In 124×10^6 , how must we write the first factor for the expression to be in scientific notation?” (A number between 1 and 10, so 1.24 in this case)
- “How can we rewrite 124 as an expression that has 1.24?” (Write it as 1.24×100 or 1.24×10^2)
- “What is the equivalent expression in scientific notation?” ($(1.24 \times 10^2) \times 10^6$, which is 1.24×10^8)

Access for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Include the following term and maintain the display for reference throughout the unit: scientific notation. Invite students to suggest language or diagrams to include on the display that will support their understanding of this term.

Supports accessibility for: Memory; Language

Access for English Language Learners

Writing: MLR3 Clarify, Critique, Correct. Display a hypothetical student statement that represents a misunderstanding about how to write values in scientific notation, such as: “To write the speed of light through a diamond is 12.4×10^7 .” Ask pairs of students to critique the response by asking, “Do you agree with the author? Why or why not?” Invite students to write feedback to the author that identifies the reasoning error and how to improve the statement. Listen for students who include in their feedback a need for the first factor to be between 1 and 10. This helps students evaluate, and improve on, the written mathematical arguments of others.

Design Principle(s): Maximize meta-awareness; Support sense-making

13.3 Scientific Notation Matching

15 minutes

In this activity, students match cards written in scientific notation with their decimal values. The game grants advantage to students who distinguish between numbers written in scientific notation from numbers that superficially resemble scientific notation (e.g. 0.43×10^5).

Addressing

- 8.EE.A.4

Instructional Routines

- MLR8: Discussion Supports

Launch

The Instructional master has three sets of cards: set A, set B, and set C. Set A is meant for demonstration purposes, so only a single copy of set A is necessary.

Arrange students in groups of 2. Consider giving students a minute of quiet time to read the directions. Then, use set A to demonstrate a round of the game for the class. Explain to students that a match can be made by pairing any two cards that have the same value, but it is favorable to be able to tell the difference between numbers in scientific notation and numbers that simply look like they are in scientific notation.

When students indicate that they understand how to play, distribute a set of cards (either set B or set C) to each group. Save a few minutes for a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Begin with a whole-class think aloud to demonstrate the steps of the game. Consider providing some groups with cards that contain more accessible values to begin with.

Supports accessibility for: Memory; Conceptual processing

Access for English Language Learners

Representing, Conversing: MLR8 Discussion Supports. Demonstrate the steps of how to play the game. To do this, select a student to play the game with you while the rest of the class observes. This will help clarify the expectations of the task, invite more student participation, and facilitate meta-awareness of the language involving scientific notation.

Design Principle(s): Support sense-making; Maximize meta-awareness

Student Task Statement

Your teacher will give you and your partner a set of cards. Some of the cards show numbers in scientific notation, and other cards show numbers that are not in scientific notation.

1. Shuffle the cards and lay them facedown.
2. Players take turns trying to match cards with the same value.
3. On your turn, choose two cards to turn faceup for everyone to see. Then:
 - a. If the two cards have the same value *and* one of them is written in scientific notation, whoever says "Science!" first gets to keep the cards, and it becomes that player's turn. If it's already your turn when you call "Science!", that means you get to go again. If you say "Science!" when the cards do not match or one is not in scientific notation, then your opponent gets a point.
 - b. If both partners agree the two cards have the same value, then remove them from the board and keep them. You get a point for each card you keep.
 - c. If the two cards do not have the same value, then set them facedown in the same position and end your turn.
4. If it is not your turn:
 - a. If the two cards have the same value *and* one of them is written in scientific notation, then whoever says "Science!" first gets to keep the cards, and it becomes that player's turn. If you call "Science!" when the cards do not match or one is not in scientific notation, then your opponent gets a point.

- b. Make sure both of you agree the cards have the same value.
If you disagree, work to reach an agreement.

5. Whoever has the most points at the end wins.

Student Response

No response required. Sample pairs of cards in scientific notation and decimal:

Set B: 4.3×10^{-7} and 0.00000043; 4.3×10^4 and 43,000; 4.3×10^7 and 43,000,000; 4.3×10^{-4} and 0.00043; 4.3×10^5 and 430,000; 4.3×10^{-5} and 0.000043; 4.3×10^2 and 430; 4.3×10^{-2} and 0.043;

Not in scientific notation: 43×10^4 ; 0.43×10^3 ; 0.43×10^{-4} ; 43×10^{-3}

Set C: 6.3×10^4 and 63,000; 6.3×10^5 and 630,000; 6.3×10^{-4} and 0.00063; 6.3×10^{-5} and 0.000063; 6.3×10^3 and 6,300; 6.3×10^6 and 6,300,000; 6.3×10^{-3} and 0.0063; 6.3×10^{-6} and 0.0000063

Not in scientific notation: 63×10^5 ; 0.63×10^4 ; 0.63×10^{-5} ; 63×10^{-4}

Are You Ready for More?

- What is $9 \times 10^{-1} + 9 \times 10^{-2}$? Express your answer as:
 - A decimal
 - A fraction
- What is $9 \times 10^{-1} + 9 \times 10^{-2} + 9 \times 10^{-3} + 9 \times 10^{-4}$? Express your answer as:
 - A decimal
 - A fraction
- The answers to the two previous questions should have been close to 1. What power of 10 would you have to go up to if you wanted your answer to be so close to 1 that it was only $\frac{1}{1,000,000}$ off?
- What power of 10 would you have to go up to if you wanted your answer to be so close to 1 that it was only $\frac{1}{1,000,000,000}$ off? Can you keep adding numbers in this pattern to get as close to 1 as you want? Explain or show your reasoning.
- Imagine a number line that goes from your current position (labeled 0) to the door of the room you are in (labeled 1). In order to get to the door, you will have to pass the points 0.9, 0.99, 0.999, etc. The Greek philosopher Zeno argued that you will never be able to go through the door, because you will first have to pass through an infinite number of points. What do you think? How would you reply to Zeno?

Student Response

1. $0.99, \frac{99}{100}$
2. $0.9999, \frac{9,999}{10,000}$
3. 10^{-6}
4. 10^{-9}
5. Yes. In the previous example, adding $9 \times 10^{-1} + \dots + 9 \times 10^{-9}$ gave us a number that was 10^{-9} away from 1. In general, adding $9 \times 10^{-1} + \dots + 9 \times 10^{-n}$ will be 10^{-n} away from 1, and we can choose n to make this distance as small as we want.
6. Answers vary. The goal is for students to think about and discuss the problem rather than coming to a substantive conclusion. Sample response: the points 0.9, 0.99, 0.999 get much closer together the farther we go in the sequence, and so the time it takes to pass each one will shrink accordingly.

Activity Synthesis

The main idea is for students to practice using the definition of scientific notation and flexibly convert numbers to scientific notation. Consider selecting students to explain how they could tell whether two cards had the same value and whether they were written in scientific notation.

Lesson Synthesis

The purpose of the discussion is to make sure that students understand the definition of scientific notation. Consider displaying student responses for all to see.

- “What are some examples of expressions that are in scientific notation? How can you tell they are in scientific notation?”
- “What are some examples of expressions that are *not* in scientific notation? Try to come up with examples that would test whether someone knows what scientific notation is.”
- “How would you write a very small number like 0.000021 in scientific notation?” (2.1×10^{-5})
- “How would you write a very large number like 21,000,000 in scientific notation?” (2.1×10^7)
- “Why might scientific notation be useful?”
- “Can you think of information in the real world that might be easier to work with in scientific notation?”

If time allows, arrange students in groups of 2 and ask students to create a small decimal or large number for a partner to rewrite with scientific notation.

13.4 Scientific Notation Check

Cool Down: 5 minutes

Students convert numbers to scientific notation.

Addressing

- 8.EE.A.4

Student Task Statement

State whether each of the following is in scientific notation. If not, write it in scientific notation.

1. 5.23×10^8
2. 48,200
3. 0.00099
4. 36×10^5
5. 8.7×10^{-12}
6. 0.78×10^{-3}

Student Response

1. Already in scientific notation
2. 4.82×10^4
3. 9.9×10^{-4}
4. 3.6×10^6
5. Already in scientific notation
6. 7.8×10^{-4}

Student Lesson Summary

The total value of all the quarters made in 2014 is 400 million dollars. There are many ways to express this using powers of 10. We could write this as $400 \cdot 10^6$ dollars, $40 \cdot 10^7$ dollars, $0.4 \cdot 10^9$ dollars, or many other ways. One special way to write this quantity is called **scientific notation**. In scientific notation,

400 million

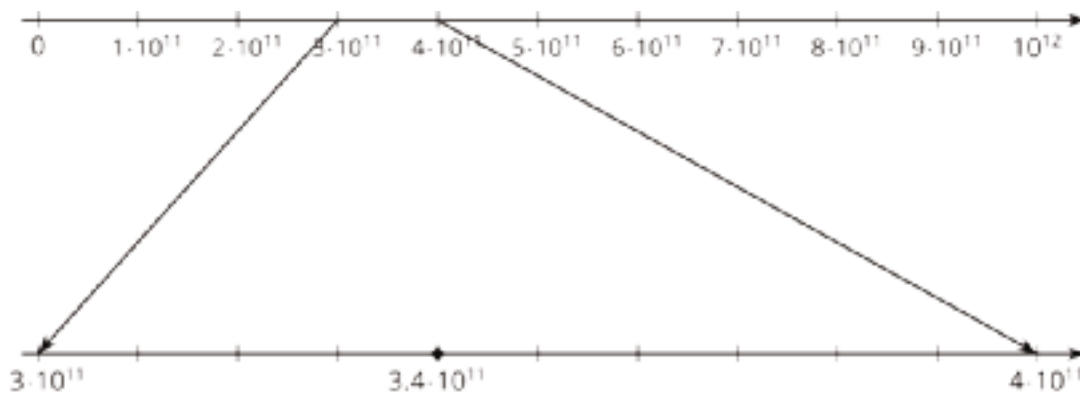
dollars would be written as

4×10^8

dollars. For scientific notation, the \times symbol is the standard way to show multiplication instead of the \cdot symbol. Writing the number this way shows exactly where it lies between two consecutive powers of 10. The 10^8 shows us the number is between 10^8 and 10^9 . The 4 shows us that the number is 4 tenths of the way to 10^9 .

Some other examples of scientific notation are 1.2×10^{-8} , 9.99×10^{16} , and 7×10^{12} . The first factor is a number greater than or equal to 1, but less than 10. The second factor is an integer power of 10.

Thinking back to how we plotted these large (or small) numbers on a number line, scientific notation tells us which powers of 10 to place on the left and right of the number line. For example, if we want to plot 3.4×10^{11} on a number line, we know that the number is larger than 10^{11} , but smaller than 10^{12} . We can find this number by zooming in on the number line:



Glossary

- scientific notation

Lesson 13 Practice Problems

Problem 1

Statement

Write each number in scientific notation.

- a. 14,700
- b. 0.00083
- c. 760,000,000
- d. 0.038
- e. 0.38
- f. 3.8
- g. 3,800,000,000,000
- h. 0.0000000009

Solution

- a. 1.47×10^4
- b. 8.3×10^{-4}
- c. 7.6×10^8
- d. 3.8×10^{-2}
- e. 3.8×10^{-1}
- f. 3.8×10^0
- g. 3.8×10^{12}
- h. 9×10^{-10}

Problem 2

Statement

Perform the following calculations. Express your answers in scientific notation.

- a. $(2 \times 10^5) + (6 \times 10^5)$
- b. $(4.1 \times 10^7) \cdot 2$

c. $(1.5 \times 10^{11}) \cdot 3$

d. $(3 \times 10^3)^2$

e. $(9 \times 10^6) \cdot (3 \times 10^6)$

Solution

a. 8×10^5

b. 8.2×10^7

c. 4.5×10^{11}

d. 9×10^6

e. 2.7×10^{13}

Problem 3

Statement

Jada is making a scale model of the solar system. The distance from Earth to the Moon is about 2.389×10^5 miles. The distance from Earth to the Sun is about 9.296×10^7 miles. She decides to put Earth on one corner of her dresser and the Moon on another corner, about a foot away. Where should she put the sun?

- On a windowsill in the same room?
- In her kitchen, which is down the hallway?
- A city block away?

Explain your reasoning.

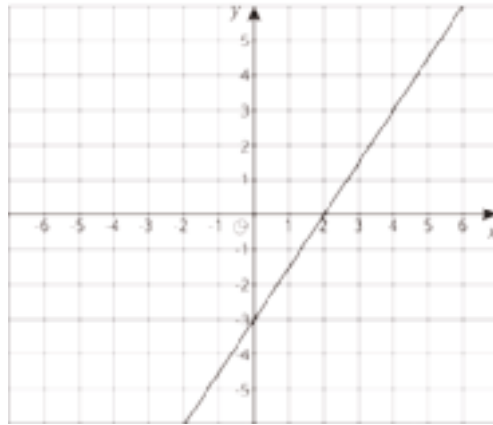
Solution

The model Sun should go down the block. Explanations vary. The distance from Earth to the Sun is about 4×10^2 or 400 times the distance from the Earth to the Moon. Since Jada's dresser is about a foot long, this means that her model Sun should be about 400 feet away from the dresser. Jada's house or apartment is probably not 400 feet long; a block away is about right.

Problem 4

Statement

Here is the graph for one equation in a system of equations.



- Write a second equation for the system so it has infinitely many solutions.
- Write a second equation whose graph goes through $(0, 2)$ so that the system has no solutions.
- Write a second equation whose graph goes through $(2, 2)$ so that the system has one solution at $(4, 3)$.

Solution

a. $y = \frac{3}{2}x - 3$

b. $y = \frac{3}{2}x + 2$

c. $y = \frac{1}{2}x + 1$

(From Unit 4, Lesson 12.)

Lesson 14: Multiplying, Dividing, and Estimating with Scientific Notation

Goals

- Generalize (orally and in writing) a process of multiplying and dividing numbers in scientific notation.
- Use scientific notation and estimation to compare quantities and interpret (orally and in writing) results in context.

Learning Targets

- I can multiply and divide numbers given in scientific notation.
- I can use scientific notation and estimation to compare very large or very small numbers.

Lesson Narrative

Students perform operations with numbers expressed in scientific notation, use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and express how many times as much one quantity is than the other. Students interpret their results in context (MP2).

Alignments

Addressing

- 8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.
- 8.EE.A.3: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9 , and determine that the world population is more than 20 times larger.
- 8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Building Towards

- 8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g.,

use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR4: Information Gap Cards
- MLR5: Co-Craft Questions
- Notice and Wonder
- True or False

Required Materials

Pre-printed slips, cut from copies of the Instructional master

Required Preparation

Consider finding an attention-grabbing image of cats and people for the launch of the "Biomass" activity to get students thinking about how the global human population compares to the global cat population.

Print and cut up slips from the Info Gap: Distances in the Solar System Instructional master. One copy of the Instructional master is needed for every 4 students. A class set could be re-used if you have more than one class.

Student Learning Goals

Let's multiply and divide with scientific notation to answer questions about animals, careers, and planets.

14.1 True or False: Equations

Warm Up: 5 minutes

The purpose of this warm-up is to encourage students to apply the properties of integer exponents to reason about equivalent expressions. While students may evaluate each side of the equation to determine if it is true or false, encourage students to reason about the properties of exponents and operations in their solution.

Addressing

- 8.EE.A.1

Building Towards

- 8.EE.A.4

Instructional Routines

- True or False

Launch

Display one problem at a time. Tell students to give a signal when they have decided if the equation is true or false. Give students 2 minutes of quiet think time followed by a whole-class discussion.

Student Task Statement

Is each equation true or false? Explain your reasoning.

1. $4 \times 10^5 \times 4 \times 10^4 = 4 \times 10^{20}$

2. $\frac{7 \times 10^6}{2 \times 10^4} = (7 \div 2) \times 10^{(6-4)}$

3. $8.4 \times 10^3 \times 2 = (8.4 \times 2) \times 10^{(3 \times 2)}$

Student Response

1. False. Sample explanation: The exponents were multiplied when they should have been added.
2. True. Sample explanation: When dividing, the exponent in the denominator is subtracted from the numerator.
3. False. Sample explanation: Multiplication can be done in any order, but the multiplication in this problem doesn't affect the power of 10.

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- "Do you agree or disagree? Why?"
- "Who can restate ___'s reasoning in a different way?"
- "Does anyone want to add on to ___'s reasoning?"

After each false equation, ask students how the problem could be changed to make the equation true.

14.2 Biomass

15 minutes

In this activity, students answer questions about quantities in context. They use scientific notation as a tool for working with small and large numbers—to describe quantities, make estimates, and make comparisons (e.g., to express how many times as much one is as the other).

Addressing

- 8.EE.A.3
- 8.EE.A.4

Instructional Routines

- MLR5: Co-Craft Questions

Launch

If possible, display an attention-grabbing image of cats and people for all to see. Ask students, “How many humans do you think there are for each cat in the world?” Ask for estimates that are too high, too low, and as reasonable as possible.

Explain that large numbers like populations are often estimated using scientific notation. There are an estimated 7.5×10^9 humans and 6×10^8 cats in the world. Guide students through the example $\frac{7.5 \times 10^9}{6 \times 10^8} = \frac{7.5}{6} \times 10^{9-8} = 1.25 \times 10^1 = 12.5$. So there are roughly 12.5 humans for each cat.

Arrange students in groups of 2 to allow partner discussions as they work. Select a student to read the first paragraph before telling students to answer the questions. Tell students that estimation will help answer these questions much more easily, so if they get stuck computing, they should try to make reasonable estimates. For example, estimating the number of humans for each cat could have looked like $\frac{7.5 \times 10^9}{6 \times 10^8} \approx \frac{6 \times 10^9}{6 \times 10^8} = 1 \times 10^1 = 10$. In this case, the final estimate is within 20% of the original calculation (not very accurate, but within a power of 10).

Give groups 10–12 minutes to work, followed by a brief whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge of multiplication and repeated factors. Allow students to use calculators to ensure inclusive participation in the activity.

Supports accessibility for: Memory; Conceptual processing

Access for English Language Learners

Writing, Conversing: MLR5 Co-Craft Questions. Display only the table, and invite groups to write a list of mathematical questions that could be answered using the data in the table. Select 2–3 groups to share their questions with the class. Look for questions that ask students to compare quantities. Next, reveal the questions of the activity. This routine allows students to produce the language of mathematical questions and talk about the quantities in this task that are represented in scientific notation prior to being asked to solve questions based on the values.

Design Principle(s): Maximize meta-awareness; Support sense-making

Anticipated Misconceptions

Students may need help remembering how to estimate using a single digit times a power of 10. Remind these students that 1.9×10^5 , for example, could be estimated as 2×10^5 to make calculation easier.

Student Task Statement

Use the table to answer questions about different creatures on the planet. Be prepared to explain your reasoning.

creature	number	mass of one individual (kg)
humans	7.5×10^9	6.2×10^1
cows	1.3×10^9	4×10^2
sheep	1.75×10^9	6×10^1
chickens	2.4×10^{10}	2×10^0
ants	5×10^{16}	3×10^{-6}
blue whales	4.7×10^3	1.9×10^5
Antarctic krill	7.8×10^{14}	4.86×10^{-4}
zooplankton	1×10^{20}	5×10^{-8}
bacteria	5×10^{30}	1×10^{-12}

1. Which creature is least numerous? Estimate how many times more ants there are.
2. Which creature is the least massive? Estimate how many times more massive a human is.

3. Which is more massive, the total mass of all the humans or the total mass of all the ants? About how many times more massive is it?
4. Which is more massive, the total mass of all the krill or the total mass of all the blue whales? About how many times more massive is it?

Student Response

1. Blue whales are the least numerous. There are about 10^{13} , or 10 trillion, times as many ants as blue whales because $\frac{5 \times 10^{16}}{5 \times 10^3}$. This uses 5×10^3 to estimate the number of blue whales.
2. Bacteria is the least massive. A human is about 60 trillion times more massive because $\frac{6.2 \times 10^1}{1 \times 10^{-12}} = 6.2 \times 10^{13}$.
3. The total human mass is about 3 times as massive as the total ant mass. The total human mass is 7.5×10^9 times 6.2×10^1 kg per human, which is approximately 45×10^{10} kg. The total ant mass is 5×10^{16} times 3×10^{-6} , which is 15×10^{10} kg.
4. The total mass of krill is about 400 times the total mass of blue whales. The total krill mass is 7.8×10^{14} times 4.86×10^{-4} , which is approximately 8×10^{14} times 5×10^{-4} or 400 billion kg. The total mass of blue whales is 4.7×10^3 times 1.9×10^5 , which is approximately 5×10^3 times 2×10^5 or 1 billion kg.

Activity Synthesis

Select students to explain how they used scientific notation and estimation to compare quantities. Record and display their reasoning for all to see. After each student presents, ask others if they reasoned the same way and if there are other approaches.

Students should see that being able to multiply and divide quantities in scientific notation is particularly helpful for reasoning and estimating about very large or very small quantities, which would be challenging to work with in their decimal representations.

14.3 Info Gap: Distances in the Solar System

15 minutes (there is a digital version of this activity)

In this info gap activity, students continue to use scientific notation as a tool for working with small and large numbers—to describe quantities, make estimates, and make comparisons (e.g., to express how many times as much one is as the other).

The info gap structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need (MP1). It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need (MP6).

As students work, identify those who use slightly different estimations to compare during whole-class discussion. While students may have slightly different results based on their estimations, the results should be relatively close in value compared to the power of 10.

Here is the text of the cards for reference and planning:

<p>Info Gap: Distances in the Solar System</p> <p>Problem Card 1</p> <p>Estimate:</p> <ol style="list-style-type: none">1. How many Earths side by side would have the same width as the Sun?2. How many Earths would it take to equal the mass of the Sun?	<p>Info Gap: Distances in the Solar System</p> <p>Data Card 1</p> <ul style="list-style-type: none">● The distance from Earth to the Sun is approximately 1.496×10^8 km.● The diameter of the Sun is 1.392×10^6 km.● The diameter of Earth is 1.28×10^4 km.● The mass of the Sun is 1.989×10^{30} kg.● The mass of Earth is 5.98×10^{24} kg.
<p>Info Gap: Distances in the Solar System</p> <p>Problem Card 2</p> <p>Estimate:</p> <ol style="list-style-type: none">1. How many times as far away from Earth is the planet Neptune compared to Venus?2. How many copies of the planet Mercury would it take to equal the mass of Neptune?	<p>Info Gap: Distances in the Solar System</p> <p>Data Card 2</p> <ul style="list-style-type: none">● The average distance from Earth . . .<ul style="list-style-type: none">○ to Mercury is 7.73×10^7 km.○ to Venus is 4×10^7 km.○ to Neptune is 4.3×10^9 km.● The mass of Mercury is 3.3×10^{23} kg.● The mass of Venus is 4.87×10^{24} kg.● The mass of Neptune is 1.024×10^{26} kg.

Addressing

- 8.EE.A.3
- 8.EE.A.4

Instructional Routines

- MLR4: Information Gap Cards

Launch

Arrange students in groups of 2. In each group, distribute a problem card to one student and a data card to the other student.

The digital version of this activity includes an extension that challenges students to create their own scale drawing of the planets. They need to calculate values for the radii and follow the simple steps in the activity.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Display or provide students with a physical copy of the written directions. Check for understanding by inviting students to rephrase directions in their own words. Keep the display of directions visible throughout the activity.

Supports accessibility for: Memory; Organization

Access for English Language Learners

Conversing: This activity uses MLR4 Information Gap to give students a purpose for discussing information necessary to solve problems involving scientific notation. Display questions or question starters for students who need a starting point such as: “Can you tell me . . . (specific piece of information)”, and “Why do you need to know . . . (that piece of information)?”

Design Principle(s): Cultivate Conversation

Anticipated Misconceptions

Some students may not recognize that “width” and “diameter” refer to the same measurement in this context. If needed, prompt students with Data Card 1 to give the information about the diameter when their partner asks for the width.

Student Task Statement

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain to your partner how you are using the information to solve the problem.
4. Solve the problem and explain your reasoning to your partner.

If your teacher gives you the *data card*:

1. Silently read the information on your card.
2. Ask your partner "What specific information do you need?" and wait for your partner to *ask* for information. *Only* give information that is on your card. (Do not figure out anything for your partner!)
3. Before telling your partner the information, ask "Why do you need that information?"
4. After your partner solves the problem, ask them to explain their reasoning and listen to their explanation.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Student Response

Card 1

1. The Sun is about 100 times the width of Earth because $\frac{1.392 \times 10^6}{1.28 \times 10^4} \approx \frac{10^6}{10^4} = 10^2$.
2. The Sun is about 300,000 times as massive as Earth because $\frac{1.989 \times 10^{30}}{5.98 \times 10^{24}} \approx \frac{2 \times 10^{30}}{6 \times 10^{24}} = \frac{1}{3} \times 10^6$, which is 333,333. $\bar{3}$.

Card 2

1. Neptune is about 100 times as far from Earth as Venus because $\frac{4.3 \times 10^9}{4 \times 10^7} \approx \frac{10^9}{10^7} = 10^2$.
2. It would take around 300 Mercuries to equal the mass of Neptune because $\frac{1.024 \times 10^{26}}{3.3 \times 10^{23}} \approx \frac{1 \times 10^{26}}{3 \times 10^{23}} = \frac{1}{3} \times 10^3$, which is 333. $\bar{3}$.

Activity Synthesis

Select students to explain how they used scientific notation and estimation to compare the sizes of objects in the solar system. Poll the class on whether they estimated the same way. Ask previously identified students (who had slightly different ways of estimating) to compare their estimates and to show that any reasonable estimate will differ by an amount that is much smaller than the given quantities.

If time allows, discuss how scientific notation was useful in answering these questions.

14.4 Professions in the United States

Optional: 15 minutes

This activity gives students additional practice using scientific notation to work with small and large numbers and answering questions about quantities in context. Students express numbers in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, as well as to determine how many times as much one is than the other.

Addressing

- 8.EE.A.3
- 8.EE.A.4

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- Notice and Wonder

Launch

Give students a minute of quiet time to observe the values in the table. Ask them to be prepared to share at least one thing they notice and one thing they wonder. Then, invite students to share their observations and questions. Consider recording their responses for all to see and using students' observations or questions about scientific notation, order of magnitude, and place values to orient students to the work in the activity.

Tell students that estimation will make it much easier to answer these questions. Give students 10–12 minutes to work, followed by a brief whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge of working with small and large numbers. Allow students to use calculators to ensure inclusive participation in the activity.

Supports accessibility for: Memory; Conceptual processing

Access for English Language Learners

Writing, Speaking: MLR1 Stronger and Clearer Each Time. Use this routine to give students a structured opportunity to revise and refine their response to the last question. Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language (e.g., “What did you do first?”, “How do you know...?”, “How did you compare the two values?”, etc.).

Students can borrow ideas and language from each partner to strengthen their final version.

Design Principle(s): Optimize output (for explanation)

Student Task Statement

Use the table to answer questions about professions in the United States as of 2012.

profession	number	typical annual salary (U.S. dollars)
architect	1.074×10^5	7.3×10^4
artist	5.14×10^4	4.4×10^4
programmer	1.36×10^6	8.85×10^4
doctor	6.9×10^5	1.87×10^5
engineer	6.17×10^5	8.6×10^4
firefighter	3.07×10^5	4.5×10^4
military—enlisted	1.16×10^6	4.38×10^4
military—officer	2.5×10^5	1×10^5
nurse	3.45×10^6	6.03×10^4
police officer	7.8×10^5	5.7×10^4
college professor	1.27×10^6	6.9×10^4
retail sales	4.67×10^6	2.14×10^4
truck driver	1.7×10^6	3.82×10^4

Answer the following questions about professions in the United States. Express each answer in scientific notation.

1. Estimate how many times more nurses there are than doctors.
2. Estimate how much money all doctors make put together.
3. Estimate how much money all police officers make put together.
4. Who makes more money, all enlisted military put together or all military officers put together? Estimate how many times more.

Student Response

1. There are about 5 times as many nurses as doctors because $\frac{3.45 \times 10^6}{6.9 \times 10^5} \approx \frac{3.5 \times 10^6}{7 \times 10^5} = \frac{1}{2} \times 10^1 = 5$.
2. Doctors make about \$140 billion put together, because 6.9×10^5 individuals times 1.87×10^5 per doctor is roughly equal to $7 \times 10^5 \times 2 \times 10^5$, which is equal to 14×10^{10} .
3. Police make about \$48 billion put together, because 7.8×10^5 individuals times 5.7×10^4 per police is roughly equal to $8 \times 10^5 \times 6 \times 10^4$, which is equal to 48×10^9 .
4. All enlisted military put together make about twice as much as all officers put together. All enlisted officers together make 1.16×10^6 times 4.38×10^4 , which is roughly 5×10^{10} . All officers make 2.5×10^5 times 1×10^5 , which is 2.5×10^{10} .

Activity Synthesis

Select students who use slightly different estimations and discuss how their answers are different, but very close compared to the size of the numbers they are working with. The main idea is that students can use a single digit times a power of 10 to estimate numbers and use what they know about exponent arithmetic to compare how many times larger one value is than another.

Lesson Synthesis

The discussion should focus on why scientific notation is helpful in making multiplicative comparisons of numbers.

- “What did you notice about the numbers in the tables you saw today?” (They were all written in scientific notation.)
- “A number is expressed in scientific notation if it is the product of an integer power of 10 and a number greater than or equal to 1 but less than 10. How does the convention about the numeric factor help us quickly get an idea about the size of a number? How does it help us compare numbers?” (Since the factor is always less than 10, the exponent gives most of the information about size of the number. Numbers can be compared by quickly examining the exponents and rounding the numerical factor.)
- “Suppose we had a table where the numbers were not in scientific notation. Would you be able to take a quick look at the table and have a feel for the relative sizes of the numbers? Describe how the process of answering ‘how many times as great’ questions would be different for such a table than the work you did with today’s problems.” (If the numbers were

written using different powers of 10, we would need to look at both the first factor and the power of 10 to gauge the size of each number. If the numbers were written as decimals, it would be even more challenging, as it would require counting zeros or decimal places. Multiplying and dividing these numbers—to answer ‘how many times as great’ questions—would also be much more cumbersome and prone to error.)

14.5 Estimating with Scientific Notation

Cool Down: 5 minutes

This cool-down checks how students make decisions about estimating with scientific notation.

Addressing

- 8.EE.A.3
- 8.EE.A.4

Student Task Statement

1. Estimate how many times larger 6.1×10^7 is than 2.1×10^{-4} .
2. Estimate how many times larger 1.9×10^{-8} is than 4.2×10^{-13} .

Student Response

1. 6.1×10^7 is about 300 billion times larger than 2.1×10^{-4} because

$$\frac{6.1 \times 10^7}{2.1 \times 10^{-4}} \approx \frac{6 \times 10^7}{2 \times 10^{-4}} = 3 \times 10^{7-(-4)} = 3 \times 10^{11}.$$

2. 1.9×10^{-8} is about 50,000 times larger than 4.2×10^{-13} because

$$\frac{1.9 \times 10^{-8}}{4.2 \times 10^{-13}} \approx \frac{2 \times 10^{-8}}{4 \times 10^{-13}} = 0.5 \times 10^5 = 5 \times 10^4.$$

Student Lesson Summary

Multiplying numbers in scientific notation extends what we do when we multiply regular decimal numbers. For example, one way to find $(80)(60)$ is to view 80 as 8 tens and to view 60 as 6 tens. The product $(80)(60)$ is 48 hundreds or 4,800. Using scientific notation, we can write this calculation as

$$(8 \times 10^1)(6 \times 10^1) = 48 \times 10^2.$$

To express the product in scientific notation, we would rewrite it as 4.8×10^3 .

Calculating using scientific notation is especially useful when dealing with very large or very small numbers. For example, there are about 39 million or 3.9×10^7 residents in California. Each Californian uses about 180 gallons of water a day. To find how many gallons of water Californians use in a day, we can find the product $(180)(3.9 \times 10^7) = 702 \times 10^7$, which is equal to 7.02×10^9 . That’s about 7 billion gallons of water each day!

Comparing very large or very small numbers by estimation also becomes easier with scientific notation. For example, how many ants are there for every human? There are 5×10^{16} ants and 7×10^9 humans. To find the number of ants per human, look at $\frac{5 \times 10^{16}}{7 \times 10^9}$. Rewriting the numerator to have the number 50 instead of 5, we get $\frac{50 \times 10^{15}}{7 \times 10^9}$. This gives us $\frac{50}{7} \times 10^6$. Since $\frac{50}{7}$ is roughly equal to 7, there are about 7×10^6 or 7 million ants per person!

Lesson 14 Practice Problems

Problem 1

Statement

Evaluate each expression. Use scientific notation to express your answer.

a. $(1.5 \times 10^2)(5 \times 10^{10})$

b. $\frac{4.8 \times 10^{-8}}{3 \times 10^{-3}}$

c. $(5 \times 10^8)(4 \times 10^3)$

d. $(7.2 \times 10^3) \div (1.2 \times 10^5)$

Solution

a. 7.5×10^{12}

b. 1.6×10^{-5}

c. 2×10^{12}

d. 6×10^{-2}

Problem 2

Statement

How many bucketloads would it take to bucket out the world's oceans? Write your answer in scientific notation.

Some useful information:

- The world's oceans hold roughly 1.4×10^9 cubic kilometers of water.
- A typical bucket holds roughly 20,000 cubic centimeters of water.
- There are 10^{15} cubic centimeters in a cubic kilometer.

Solution

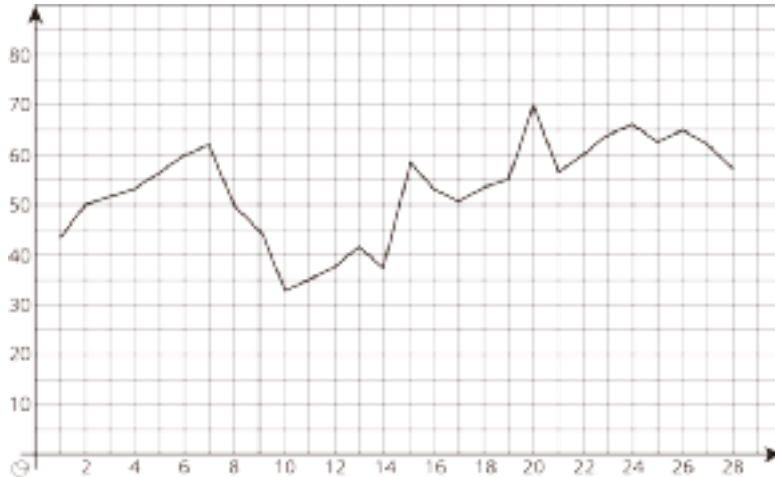
7×10^{19} . The world's oceans hold 1.4×10^9 cubic kilometers of water, found by multiplying 1.4×10^9 by 10^{15} . Then divide by 2×10^4 to get 0.7×10^{20} . In scientific notation, this quotient is 7×10^{19} .

Problem 3

Statement

The graph represents the closing price per share of stock for a company each day for 28 days.

- What variable is represented on the horizontal axis?
- In the first week, was the stock price generally increasing or decreasing?
- During which period did the closing price of the stock decrease for at least 3 days in a row?



Solution

- The day
- Increasing
- Days 7 to 10

(From Unit 5, Lesson 5.)

Problem 4

Statement

Write an equation for the line that passes through $(-8.5, 11)$ and $(5, -2.5)$.

Solution

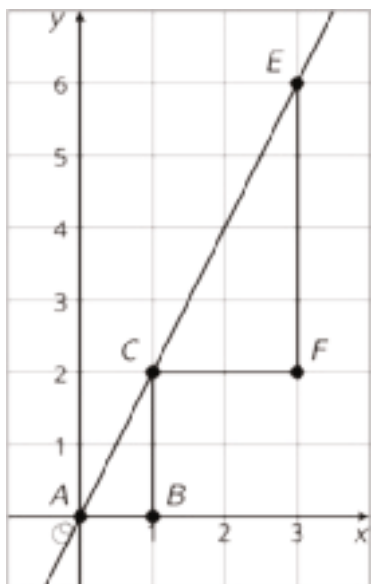
$$y = -x + 2.5$$

(From Unit 3, Lesson 11.)

Problem 5

Statement

Explain why triangle ABC is similar to triangle CFE .



Solution

Answers vary. Sample responses:

- Translate C to A , and then dilate with center A by a factor of $\frac{1}{2}$.
- Dilate with center A by a factor of 2, then translate A to C .

(From Unit 2, Lesson 6.)

Lesson 15: Adding and Subtracting with Scientific Notation

Goals

- Generalize (orally and in writing) a process of adding and subtracting numbers in scientific notation and interpret results in context.

Learning Targets

- I can add and subtract numbers given in scientific notation.

Lesson Narrative

Students add and subtract with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Students must make sense and use quantitative reasoning when making comparisons, for example, when comparing whether 5 planets side by side are wider than the Sun (MP1, MP2).

Alignments

Addressing

- 8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Instructional Routines

- MLR5: Co-Craft Questions
- MLR8: Discussion Supports
- Number Talk

Student Learning Goals

Let's add and subtract using scientific notation to answer questions about animals and the solar system.

15.1 Number Talk: Non-zero Digits

Warm Up: 10 minutes

The purpose of this Number Talk is to elicit strategies and understandings students have for addition, subtraction, multiplication, and division. These understandings help students develop fluency and will be helpful later in this lesson when students compute with numbers in scientific notation. While four problems are given, it may not be possible to share every strategy. Consider

gathering only two or three different strategies per problem, saving most of the time for the final question.

Addressing

- 8.EE.A.4

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 1 minute of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Anticipated Misconceptions

Students may write $3 \times 10^9 - 2 \times 10^7 = 1 \times 10^2$, or something similar. Ask these students to evaluate each product first before subtracting.

Student Task Statement

Mentally decide how many non-zero digits each number will have.

$$(3 \times 10^9)(2 \times 10^7)$$

$$(3 \times 10^9) \div (2 \times 10^7)$$

$$3 \times 10^9 + 2 \times 10^7$$

$$3 \times 10^9 - 2 \times 10^7$$

Student Response

- One non-zero digit. Multiplying 3 and 2 gives us 6, and the rest will just add a bunch of zeros.
- Two non-zero digits. Dividing 3 by 2 gives us 1.5, and the rest will just move the decimal place and add a bunch of zeros.
- Two non-zero digits. The first digit on the left will be a 3 and the third digit will be a 2.

- Three non-zero digits. The first digit on the left will be a 2, the second digit will be a 9, and the third will be an 8.

Activity Synthesis

Ask the class how the first two questions are different than the second two questions. Note that we have to pay more attention to place value to answer the second two than the first two because when we add or subtract, we can only add or subtract digits that correspond to the same powers of 10. Consider asking:

- “Which problem was easier? Why?”
- “Of the four operations, which operations are easier to do with scientific notation? Which are harder?” (Multiplying and dividing is easier with scientific notation, because the exponent rules involve multiplication and division. Addition and subtraction are harder, because the exponent rules don’t involve those operations.)

Access for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, “First, I ____ because . . .” or “I noticed ____ so I . . .” Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Design Principle(s): Optimize output (for explanation)

15.2 Measuring the Planets

15 minutes

Students attend to precision when adding numbers in scientific notation, taking care that the numbers are first written as a decimal or with powers of 10 with the same exponent (MP6). Students critique the reasoning of Diego, Clare, and Kiran as they make sense of adding numbers in scientific notation (MP3).

Addressing

- 8.EE.A.4

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. Give 10–12 minutes to work followed by a whole-class discussion. Encourage students to share their thinking with a partner as they work.

Access for Students with Disabilities

Representation: Access for Perception. Read the student task statement aloud. Students who both listen to and read the information will benefit from extra processing time.

Supports accessibility for: Language

Anticipated Misconceptions

Students may make various mistakes that show a misunderstanding of exponent rules, for example multiplying the exponents like $4.7 \times 10^4 + 1.2 \times 10^5 = 5.9 \times 10^{20}$ or adding exponents like $4.7 \times 10^4 + 1.2 \times 10^5 = 1.67 \times 10^9$. For these students, emphasize that the terms are being added, not multiplied, so the factors that are 10 are not being grouped in the same way.

Student Task Statement

Diego, Kiran, and Clare were wondering:

“If Neptune and Saturn were side by side, would they be wider than Jupiter?”

1. They try to add the diameters, 4.7×10^4 km and 1.2×10^5 km. Here are the ways they approached the problem. Do you agree with any of them? Explain your reasoning.
 - a. Diego says, “When we add the distances, we will get $4.7 + 1.2 = 5.9$. The exponent will be 9. So the two planets are 5.9×10^9 km side by side.”
 - b. Kiran wrote 4.7×10^4 as 47,000 and 1.2×10^5 as 120,000 and added them:
$$\begin{array}{r} 120,000 \\ +47,000 \\ \hline 167,000 \end{array}$$
 - c. Clare says, “I think you can’t add unless they are the same power of 10.” She adds 4.7×10^4 km and 12×10^4 to get 16.7×10^4 .
2. Jupiter has a diameter of 1.43×10^5 . Which is wider, Neptune and Saturn put side by side, or Jupiter?

Student Response

1.
 - a. Diego is incorrect, because their decimal place values do not match. It would be analogous to writing $47 + 1.2 = 59$.
 - b. Kiran correctly added the two distances, but his answer is hard to compare to the width of Jupiter because it is not in scientific notation.

c. Clare also correctly added the two distances, and her answer is also hard to compare to the width of Jupiter because it is not in scientific notation.

2. Neptune and Saturn side by side are wider than Jupiter, because $1.67 \times 10^5 > 1.43 \times 10^5$.

Activity Synthesis

Select students to share their reasoning about how to add 4.7×10^4 km and 1.2×10^5 km. It is important that students understand that 4.7×10^4 and 1.2×10^5 are off by roughly a factor of 10. In order to compare them, they either have to be written as decimal numbers or with the same power of 10.

Discuss some of the following questions:

- “How are Clare’s and Kiran’s approaches alike?” (They both reached the same sum by attending to place value.)
- “How are their approaches different?” (Kiran wrote the numbers as decimals and added them. Clare wrote the numbers with the same power of 10 and added them. Kiran’s method might not work very well if the numbers are very large or very small. The decimal form of those number would be unwieldy.)
- “Why must the terms have the same power of 10 to be added?” (We can only add digits that are of the same place value. If the powers of 10 are different, the place values of the digits in the first factors of the two expressions would be different. For example, the 4 in 4.7×10^4 means 4 ten-thousands and the 1 in 1.2×10^5 means 1 hundred-thousand, so we cannot add 4.7 and 1.2.)
- “How might Clare have reasoned that 1.2×10^5 can be written as 12×10^4 ?” (One way is to see that changing 1.2 into 12 requires multiplying by 10. To keep the value of the expression the same, we must divide it by 10, which decreases the exponent by 1, from 10^5 to 10^4 .)
- “How did you compare Clare and Kiran’s results to the width of Jupiter?” (Converting the widths to scientific notation makes it easy to compare.)

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion. For each observation or response that is shared, ask students to restate and/or revoice what they heard using mathematical language. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students’ attention to any words or phrases that helped to clarify the original statement. This will provide more students with an opportunity to produce language as they interpret the reasoning of others.

Design Principle(s): Support sense-making

15.3 A Celestial Dance

15 minutes

In this activity, students add quantities written in scientific notation in order to answer questions in context. To add numbers in scientific notation, students must attend to precision by aligning place value (MP6).

As students work, notice the different strategies used to align place value. One strategy would be to convert all the distances to decimal, align the place values vertically, and then add in the usual way. Another example would be to rewrite all the addends to use the same power of 10 before adding.

Addressing

- 8.EE.A.4

Instructional Routines

- MLR5: Co-Craft Questions

Launch

Arrange students in groups of 2. Tell students to discuss their thinking with a partner and work to reach agreement. Give students 12 minutes to work, followed by a brief whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge of working with very large numbers. Allow students to use calculators to ensure inclusive participation in the activity. *Supports accessibility for: Memory; Conceptual processing*

Access for English Language Learners

Writing, Conversing: MLR5 Co-Craft Questions. Display only the table, and ask pairs of students to write possible questions that could be answered by the data in the table. Invite pairs to share their questions with the class. Highlight questions that involve adding or subtracting values. Next, reveal the questions of the activity. This routine allows students to produce the language of mathematical questions and talk about the quantities in this task that are represented in scientific notation prior to being asked to solve questions based on the values.

Design Principle(s): Maximize meta-awareness; Support sense-making

Student Task Statement

object	diameter (km)	distance from the Sun (km)
Sun	1.392×10^6	0×10^0
Mercury	4.878×10^3	5.79×10^7
Venus	1.21×10^4	1.08×10^8
Earth	1.28×10^4	1.47×10^8
Mars	6.785×10^3	2.28×10^8
Jupiter	1.428×10^5	7.79×10^8

1. When you add the distances of Mercury, Venus, Earth, and Mars from the Sun, would you reach as far as Jupiter?
2. Add all the diameters of all the planets except the Sun. Which is wider, all of these objects side by side, or the Sun? Draw a picture that is close to scale.

Student Response

1. The sums of the distances is not enough to reach Jupiter because
 $5.79 \times 10^7 + 1.08 \times 10^8 + 1.47 \times 10^8 + 2.28 \times 10^8 = (0.579 + 1.08 + 1.47 + 2.28) \times 10^8$
 $= 5.409 \times 10^8$ km, which is less than 7.79×10^8 km.
2. The first 5 planets side by side are not as wide as the Sun (about $\frac{1}{7}$ th of the width) because
 $4.878 \times 10^3 + 1.21 \times 10^4 + 1.28 \times 10^4 + 6.785 \times 10^3 + 1.428 \times 10^5$
 $= (0.4878 + 1.21 + 1.28 + 0.6785 + 14.28) \times 10^4 = 17.9363 \times 10^4 = 1.79363 \times 10^5$ km,
which is less than 1.392×10^6 km. The scale drawing should show a large circle (the Sun) with 5 smaller planet circles (Jupiter much larger than the others) that reach about $\frac{1}{10}$ of the way across the large circle.

Are You Ready for More?

The emcee at a carnival is ready to give away a cash prize! The winning contestant could win anywhere from \$1 to \$100. The emcee only has 7 envelopes and she wants to make sure she distributes the 100 \$1 bills among the 7 envelopes so that no matter what the contestant wins, she can pay the winner with the envelopes without redistributing the bills. For example, it's possible to divide 6 \$1 bills among 3 envelopes to get any amount from \$1 to \$6 by putting \$1 in the first envelope, \$2 in the second envelope, and \$3 in the third envelope (Go ahead and check. Can you make \$4? \$5? \$6?).

How should the emcee divide up the 100 \$1 bills among the 7 envelopes so that she can give away any amount of money, from \$1 to \$100, just by handing out the right envelopes?

Student Response

\$1 in the first envelope, \$2 in the second, \$4 in the third, and increasing powers of two up through \$32 in the sixth envelope. At this point, there are \$63 total in the six envelopes, so put the remaining \$37 in the seventh envelope.

Activity Synthesis

The main point to highlight is that values given in scientific notation can be added by carefully aligning the place values of all of the addends. Select students who show different ways of aligning the place values. Record their strategies and display them for all to see. Ask students to explain how they decided to scale the objects in their drawing.

15.4 Old McDonald's Massive Farm

Optional: 15 minutes

Consider taking the time to engage with this activity if students need more experience with negative exponents and additional practice with adding quantities expressed in scientific notation. Students work with positive and negative exponents simultaneously.

Addressing

- 8.EE.A.4

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. Tell students to explain their thinking to their partner and work to reach agreement. Give students 10-12 minutes to work followed by a whole-class discussion.

Student Task Statement

Use the table to answer questions about different life forms on the planet.

creature	number	mass of one individual (kg)
humans	7.5×10^9	6.2×10^1
cows	1.3×10^9	4×10^2
sheep	1.75×10^9	6×10^1
chickens	2.4×10^{10}	2×10^0
ants	5×10^{16}	3×10^{-6}
blue whales	4.7×10^3	1.9×10^5
antarctic krill	7.8×10^{14}	4.86×10^{-4}
zooplankton	1×10^{20}	5×10^{-8}
bacteria	5×10^{30}	1×10^{-12}

1. On a farm there was a cow. And on the farm there were 2 sheep. There were also 3 chickens. What is the total mass of the 1 cow, the 2 sheep, the 3 chickens, and the 1 farmer on the farm?
2. Make a conjecture about how many ants might be on the farm. If you added all these ants into the previous question, how would that affect your answer for the total mass of all the animals?
3. What is the total mass of a human, a blue whale, and 6 ants all together?
4. Which is greater, the number of bacteria, or the number of all the other animals in the table put together?

Student Response

1. 588 kg, because the sum of the masses of 1 cow, 2 sheep, 3 chickens, and 1 farmer is $4 \times 10^2 + 2(6 \times 10^1) + 3(2 \times 10^0) + 6.2 \times 10^1 = 400 + 120 + 6 + 62 = 588$ kg.
2. Answers vary. Sample response: Suppose there are roughly 10^6 ants on the farm. The mass of these ants is 3×10^{-6} kg per ant times 10^6 ants, which is equal to 3 kg. The new total mass would be 591 kg.
3. 190,062.000018 kg, because the sum of the masses of 1 human, 1 blue whale, and 6 ants is $6.2 \times 10^1 + 1.9 \times 10^5 + 6(3 \times 10^{-6}) = 62 + 190,000 + 18 \times 10^{-6} = 190,062.000018$ kg.
4. The number of bacteria is greater. Strategies vary. Sample strategies:

- There are 8 entries in the list that are not bacteria, and the second-highest number of individuals is 1×10^{20} , so the sum of all the individuals must be less than 8×10^{20} , which is still much less than the number of bacteria.
- Adding all the non-bacteria individuals gives 1.0005078×10^{20} . There are roughly 50 billion times as many bacteria.

Activity Synthesis

In a whole-class discussion, talk about what the difference between multiplying or dividing numbers in scientific notation and adding or subtracting numbers. Consider asking: “Which is easier? What do you have to be careful about?”

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this to amplify mathematical uses of language to communicate about multiplying and dividing values in scientific notation. As students share their strategies for multiplying or dividing values in scientific notation, press for details by requesting that students challenge an idea, elaborate on an idea, or give an example of their process. Revoice student ideas to model mathematical language use in order to clarify, apply appropriate language, and involve more students.

Design Principle(s): Support sense-making; Optimize output (for explanation)

Lesson Synthesis

The purpose of this discussion is to check that students understand how to add and subtract numbers given in scientific notation.

Here are some questions for discussion:

- “In the first activity, which method did you prefer to make sense of adding two numbers in scientific notation?”
- “How is adding and subtracting with scientific notation different from multiplying and dividing? Which is easier? Why do you think that is?” (Multiplying and dividing with scientific notation is easier, because it is possible to use exponent rules to help do calculations.)
- “Is there anything you found surprising or interesting in the problems you did?”

15.5 Adding with Scientific Notation

Cool Down: 5 minutes

Students must attend to precision to make sure that all orders of magnitude are the same when adding numbers in scientific notation.

Addressing

- 8.EE.A.4

Student Task Statement

Elena wants to add $(2.3 \times 10^5) + (3.6 \times 10^6)$ and writes $(2.3 \times 10^5) + (3.6 \times 10^6) = 5.9 \times 10^6$.

Explain to Elena what her mistake was and what the correct solution is.

Student Response

3.83×10^6 . Sample response: Elena added 2.3 and 3.6 without realizing 3.6×10^6 is over 10 times as large as 2.3×10^5 . Instead, she should have done $36 \times 10^5 + 2.3 \times 10^5 = (36 + 2.3) \times 10^5 = 38.3 \times 10^5 = 3.83 \times 10^6$.

Student Lesson Summary

When we add decimal numbers, we need to pay close attention to place value. For example, when we calculate $13.25 + 6.7$, we need to make sure to add hundredths to hundredths (5 and 0), tenths to tenths (2 and 7), ones to ones (3 and 6), and tens to tens (1 and 0). The result is 19.95.

We need to take the same care when we add or subtract numbers in scientific notation. For example, suppose we want to find how much further Earth is from the Sun than Mercury. Earth is about 1.5×10^8 km from the Sun, while Mercury is about 5.8×10^7 km. In order to find

$$1.5 \times 10^8 - 5.8 \times 10^7$$

we can rewrite this as

$$1.5 \times 10^8 - 0.58 \times 10^8$$

Now that both numbers are written in terms of 10^8 , we can subtract 0.58 from 1.5 to find

$$0.92 \times 10^8$$

Rewriting this in scientific notation, Earth is

$$9.2 \times 10^7$$

km further from the Sun than Mercury.

Lesson 15 Practice Problems

Problem 1

Statement

Evaluate each expression, giving the answer in scientific notation:

- a. $5.3 \times 10^4 + 4.7 \times 10^4$
- b. $3.7 \times 10^6 - 3.3 \times 10^6$
- c. $4.8 \times 10^{-3} + 6.3 \times 10^{-3}$
- d. $6.6 \times 10^{-5} - 6.1 \times 10^{-5}$

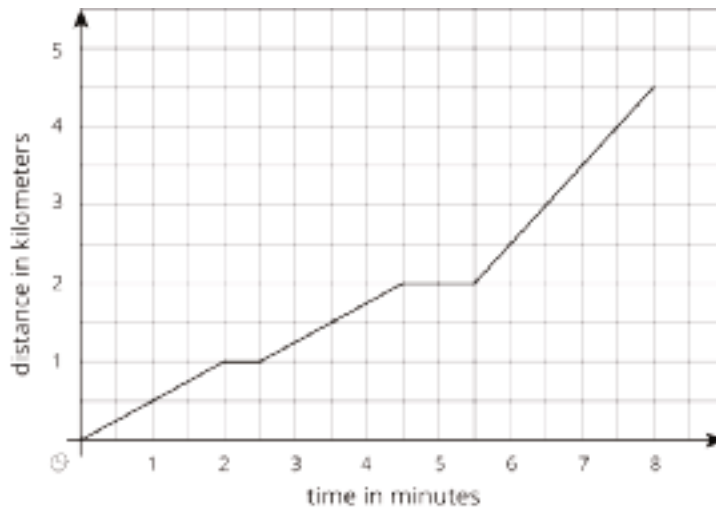
Solution

- a. 1×10^5
- b. 4×10^5
- c. 1.11×10^{-2}
- d. 5×10^{-6}

Problem 2

Statement

- a. Write a scenario that describes what is happening in the graph.
- b. What is happening at 5 minutes?
- c. What does the slope of the line between 6 and 8 minutes mean?



Solution

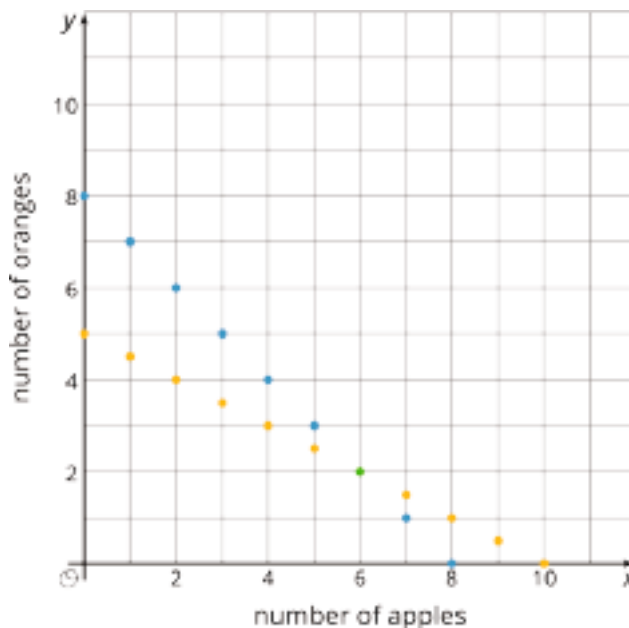
- Answers vary. Sample response: A person is driving. The distance measures distance away from their house.
- Answers vary. Sample response: The person is stopped 2 km from home.
- Answers vary. Sample response: The slope between 6 and 8 minutes indicates the speed the person is driving (1 km per minute), which is faster than any of their speeds between 0 and 6 minutes.

(From Unit 5, Lesson 10.)

Problem 3

Statement

Apples cost \$1 each. Oranges cost \$2 each. You have \$10 and want to buy 8 pieces of fruit. One graph shows combinations of apples and oranges that total to \$10. The other graph shows combinations of apples and oranges that total to 8 pieces of fruit.



- Name one combination of 8 fruits shown on the graph that whose cost does *not* total to \$10.
- Name one combination of fruits shown on the graph whose cost totals to \$10 that are *not* 8 fruits all together.
- How many apples and oranges would you need to have 8 fruits that cost \$10 at the same time?

Solution

- Answers vary. Sample response: 4 apples, 4 oranges
- Answers vary. Sample response: 2 apples, 4 oranges
- 6 apples and 2 oranges

(From Unit 4, Lesson 10.)

Problem 4

Statement

Solve each equation and check your solution.

$$-2(3x - 4) = 4(x + 3) + 6$$

$$\frac{1}{2}(z + 4) - 6 = -2z + 8$$

$$4w - 7 = 6w + 31$$

Solution

a. $x = -1$

b. $z = \frac{24}{5}$

c. $w = -19$

(From Unit 4, Lesson 5.)

Section: Let's Put It to Work

Lesson 16: Is a Smartphone Smart Enough to Go to the Moon?

Goals

- Use scientific notation to compare quantities in context, and describe (orally) how using scientific notation helps with making comparisons between very large and very small quantities.

Learning Targets

- I can use scientific notation to compare different amounts and answer questions about real-world situations.

Lesson Narrative

In this culminating lesson, students use scientific notation as a tool for making comparisons. Students compare old hardware to new hardware using various digital media as a form of measurement. For example, students compare floppy drives to modern technology by measuring how many floppy drives it would take to store a high-definition film. Students must identify the essential features of the questions and reason qualitatively and abstractly in order to answer them in context (MP4, MP2).

Alignments

Addressing

- 8.EE.A.3: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9 , and determine that the world population is more than 20 times larger.
- 8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Instructional Routines

- MLR7: Compare and Connect
- MLR8: Discussion Supports

Required Materials

Copies of Instructional master

Required Preparation

Print the Old Hardware, New Hardware Instructional master. Prepare 1 copy for every 2 students.

Student Learning Goals

Let's compare digital media and computer hardware using scientific notation.

16.1 Old Hardware, New Hardware

20 minutes

Students perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Students use scientific notation and choose units of appropriate size for measurements of very large or very small quantities.

As students work, look for those who use scientific notation to make their calculations and estimations easier. Consider asking them to share their work later.

Addressing

- 8.EE.A.3
- 8.EE.A.4

Instructional Routines

- MLR7: Compare and Connect

Launch

Arrange students in groups of 2. Display or distribute the included Instructional master containing computer hardware specifications over time for all to see throughout the activity. Give students 15–20 minutes to work before a brief whole-class discussion.

Student Task Statement

In 1966, the Apollo Guidance Computer was developed to make the calculations that would put humans on the Moon.

Your teacher will give you advertisements for different devices from 1966 to 2016. Choose one device and compare that device with the Apollo Guidance Computer. If you get stuck, consider using scientific notation to help you do your calculations.

For reference, storage is measured in bytes, processor speed is measured in hertz, and memory is measured in bytes. Kilo stands for 1,000, mega stands for 1,000,000, giga stands for 1,000,000,000, and tera stands for 1,000,000,000,000.



1. Which one can store more information? How many times more information?
2. Which one has a faster processor? How many times faster?
3. Which one has more memory? How many times more memory?

Student Response

1977 Desktop:

- Storage is $\frac{28}{15}$ times as much as Apollo
- Processing is $\frac{1}{2}$ as much as Apollo
- Memory is the same as Apollo

2001 Desktop:

- Storage is $2.\bar{6} \times 10^5$ times as much as Apollo
- Processing is 550 times as much as Apollo
- Memory is 32,000 times as much as Apollo

2007 Desktop:

- Storage is $6.\bar{6} \times 10^6$ times as much as Apollo
- Processing is 2,000 times as much as Apollo
- Memory is 10^6 times as much as Apollo

2007 Smartphone:

- Storage is $5.\bar{3} \times 10^4$ times as much as Apollo

- Processing is 200 times as much as Apollo
- Memory is 32,000 times as much as Apollo

2016 Smartphone:

- Storage is $4.2\bar{6} \times 10^5$ times as much as Apollo
- Processing is 4,400 times as much as Apollo
- Memory is 7.5×10^5 times as much as Apollo

2016 Desktop:

- Storage is $1.\bar{3} \times 10^7$ times as much as Apollo
- Processing is 6,000 times as much as Apollo
- Memory is 2×10^6 times as much as Apollo

Activity Synthesis

Select students who chose various devices to share their results. A key insight to take away would be how rapidly technology improves and how modern smartphones are much, much more sophisticated than the computer that put people on the Moon.

Access for English Language Learners

Speaking, Listening: MLR7 Compare and Connect. Ask students to create a visual display of their strategy and result for comparing the Apollo Guidance Computer and the device they selected. Invite students to take a tour of the displays and identify “what is the same and what is different about each approach”. Draw students’ attention to the ways the values were compared using different strategies (e.g., using estimation, calculating differences using scientific notation versus expanded form). In this discussion, emphasize the mathematical language used to make sense of the different strategies to compare the values. These exchanges strengthen students’ mathematical language use and reasoning when comparing large and small quantities.

Design Principle(s): Maximize meta-awareness

16.2 A Bit More on Bytes

25 minutes

Students use scientific notation as a tool to understand the relative scale of different units (MP2). They practice modeling skills by identifying essential elements of the problems and gathering relevant information before computing (MP4).

Addressing

- 8.EE.A.3
- 8.EE.A.4

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. Instruct students to first read through the problems and decide on what information they need to solve each problem. Record relevant information for all students to see. Only record information when students have asked for it. Possible information students will ask for include:

- Mai’s dad’s computer holds 500 gigabytes of storage space.
- A kilobyte is 1,000 bytes, a megabyte is 1,000,000 bytes, and a gigabyte is 1,000,000,000 bytes.
- 1 character is roughly 1 byte.
- An emoji is roughly 4 bytes.
- A full-length, high-definition film is around 8 gigabytes and runs 2 hours.
- A person sleeps about 8 hours in a night.

Give 15–20 minutes of work time before a brief whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge of working with very large numbers. Allow students to use calculators to ensure inclusive participation in the activity.

Supports accessibility for: Memory; Conceptual processing

Student Task Statement

For each question, think about what information you would need to figure out an answer. Your teacher may provide some of the information you ask for. Give your answers using scientific notation.

1. Mai found an 80’s computer magazine with an advertisement for a machine with hundreds of kilobytes of storage! Mai was curious and asked, “How many kilobytes would my dad’s new 2016 computer hold?”
2. The old magazine showed another ad for a 750-kilobyte floppy disk, a device used in the past to store data. How many gigabytes is this?

3. Mai and her friends are actively involved on a social media service that limits each message to 140 characters. She wonders about how the size of a message compares to other media.

Estimate how many messages it would take for Mai to fill up a floppy disk with her 140-character messages. Explain or show your reasoning.

4. Estimate how many messages it would take for Mai to fill a floppy disk with messages that only use emojis (each message being 140 emojis). Explain or show your reasoning.
5. Mai likes to go to the movies with her friends and knows that a high-definition film takes up a lot of storage space on a computer.

Estimate how many floppy disks it would take to store a high-definition movie. Explain or show your reasoning.

6. How many seconds of a high-definition movie would one floppy disk be able to hold?
7. If you fall asleep watching a movie streaming service and it streams movies all night while you sleep, how many floppy disks of information would that be?

Student Response

1. 500 million kilobytes. Mai's dad's computer can hold 500 gigabytes, which is 500×10^9 bytes. A kilobyte is 10^3 bytes, so his computer holds $\frac{500 \times 10^9}{10^3}$ or 500×10^6 kilobytes.

2. 0.00075 gigabytes. A floppy drive holds 750 kilobytes, which is 750×10^3 bytes. As a fraction of a gigabyte (10^9 bytes), divide $\frac{750 \times 10^3}{10^9} = 750 \times 10^{-6} = 0.00075$.

3. About 5,000 messages, because:

$$\frac{7.5 \times 10^5 \text{ bytes per floppy}}{1.4 \times 10^2 \text{ bytes per message}} \approx 5 \times 10^3 \text{ messages.}$$

4. About 1,250 messages. Emojis take up 4 times as much storage as a character, so there will be 4 times fewer messages. $\frac{5,000}{4} = 1250$.

5. About 10,000 floppy disks, because:

$$\frac{8 \times 10^9 \text{ bytes per movie}}{7.5 \times 10^5 \text{ bytes per floppy}} \approx 1 \times 10^4 \text{ floppy disks.}$$

6. 0.72 seconds. A 2-hour film is 120 minutes, which is 7,200 seconds. Therefore,

$$\frac{7.2 \times 10^3 \text{ seconds per movie}}{10^4 \text{ floppy disks per movie}} = 7.2 \times 10^{-1} = 0.72 \text{ seconds.}$$

7. About 40,000 floppy disks. Eight hours is equivalent to 4 movies, which is 40,000 floppy disks.

Are You Ready for More?

Humans tend to work with numbers using powers of 10, but computers work with numbers using powers of 2. A “binary kilobyte” is 1,024 bytes instead of 1,000, because $1,024 = 2^{10}$. Similarly, a “binary megabyte” is 1,024 binary kilobytes, and a “binary gigabyte” is 1,024 binary megabytes.

1. Which is bigger, a binary gigabyte or a regular gigabyte? How many more bytes is it?
2. Which is bigger, a binary terabyte or a regular terabyte? How many more bytes is it?

Student Response

1. A binary gigabyte is about 74 million more bytes (74 megabytes) than a regular gigabyte. A binary gigabyte is equal to 1,024 binary megabytes, which is equal to 1,024 binary kilobytes, which is equal to 1,024 bytes. So, a binary gigabyte is $1,024^3$ (or 1,073,741,824) bytes.
2. A binary terabyte is about 100 billion more bytes (100 gigabytes) than a regular terabyte. A binary terabyte is 1,024 times a binary gigabyte, so a binary terabyte would be $1,024^4$ (or 1,099,511,627,776) bytes.

Activity Synthesis

In a whole-class discussion, ask students what they might have found surprising or interesting when comparing different digital media and different hardware. If time permits, discuss how scientific notation helps to make those comparisons.

Access for English Language Learners

Speaking: MLR8 Discussion Supports. As students explain what they noticed about the differences in digital media and hardware, press for details in students’ ideas by requesting that students challenge an idea, elaborate on an idea, or make explicit their process for calculating or comparing the values. Revoice student ideas to model mathematical language use in order to clarify, apply appropriate language, and involve more students. This will help students produce and make sense of the language needed to communicate their own ideas.

Design Principle(s): Support sense-making; Optimize output (for explanation)

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Family Support
Materials

Family Support Materials

Exponents and Scientific Notation

Here are the video lesson summaries for Grade 8, Unit 7: Exponents and Scientific Notation. Each video highlights key concepts and vocabulary that students learn across one or more lessons in the unit. The content of these video lesson summaries is based on the written Lesson Summaries found at the end of lessons in the curriculum. The goal of these videos is to support students in reviewing and checking their understanding of important concepts and vocabulary. Here are some possible ways families can use these videos:

- Keep informed on concepts and vocabulary students are learning about in class.
- Watch with their student and pause at key points to predict what comes next or think up other examples of vocabulary terms (the bolded words).
- Consider following the Connecting to Other Units links to review the math concepts that led up to this unit or to preview where the concepts in this unit lead to in future units.

Grade 8, Unit 7: Exponents and Scientific Notation	Vimeo	YouTube
Video 1: Exponent Rules (Lessons 1–4)	Link	Link
Video 2: More Exponent Rules (Lesson 5–8)	Link	Link
Video 3: Powers of 10 (Lessons 9–12)	Link	Link
Video 4: Scientific Notation (Lessons 13–15)	Link	Link

Video 1

Video 'VLS G8U7V1 Exponent Rules (Lessons 1–4)' available here:
<https://player.vimeo.com/video/514770006>.

Video 2

Video 'VLS G8U7V2 More Exponent Rules (Lesson 5–8)' available here:
<https://player.vimeo.com/video/514774451>.

Video 3

Video 'VLS G8U7V3 Powers of 10 (Lessons 9–12)' available here: <https://player.vimeo.com/video/514773112>.

Video 4

Video 'VLS G8U7V4 Scientific Notation (Lessons 13–15)' available here:
<https://player.vimeo.com/video/514792288>.

Connecting to Other Units

- *Coming soon*

Exponent Review

Family Support Materials 1

This week your student will learn the rules for multiplying and dividing expressions with exponents. Exponents are a way of keeping track of how many times a number has been repeatedly multiplied. For example, instead of writing $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$, we can write 8^7 instead. The number repeatedly multiplied is called the base, which in this example is 8. The 7 here is called the exponent.

Using our understanding of repeated multiplication, we'll figure out several "rules" for exponents. For example, suppose we want to understand the expression $10^3 \cdot 10^4$. Rewriting this to show all the factors, we get $(10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10)$. Since this is really 7 10s multiplied together, we can write $10^3 \cdot 10^4 = 10^7$. By counting the repeated factors that are 10, we've added the exponents together (there are 3 of them, and then 4 more). This leads us to understanding a more general rule about exponents; when multiplying powers of the same base, we add the exponents together:

$$x^n \cdot x^m = x^{n+m}$$

Using similar reasoning, we can figure out that when working with powers of powers, we multiply the exponents together:

$$(x^n)^m = x^{n \cdot m}$$

These patterns will lead to other discoveries later on.

Here is a task to try with your student:

1. Jada and Noah were trying to understand the expression $10^4 \cdot 10^5$. Noah said, "since we are multiplying, we will get 10^{20} ." Jada said, "But I don't think you can get 20 10s multiplied together from that." Do you agree with either of them?
2. Next, Jada and Noah were thinking about a similar expression, $(10^4)^5$. Noah said, "Ok this one will be 10^{20} because you will have 5 groups of 4." Jada said, "I agree it will be 10^{20} , but it's because there will be 4 groups of 5." Do you agree with either of them?

Solution:

1. Jada is correct. Rewriting $10^4 \cdot 10^5$ to show all the factors looks like $(10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$. We can see that there are a total of 9 10s

being multiplied. This helps us understand what's going on when we use the rule to write $10^4 \cdot 10^5 = 10^{4+5} = 10^9$.

2. This time, Noah is correct. When we look at $(10^4)^5$, the outside exponent of 5 tells us that there are 5 10^4 s being multiplied together. So

$(10^4)^5 = 10^4 \cdot 10^4 \cdot 10^4 \cdot 10^4 \cdot 10^4$. This means there are 5 groups of 4 10s being multiplied together. We could write this out the long way as

$(10^4)^5 = (10 \cdot 10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10)$.

This helps us understand what's going on when we use the rule to write

$(10^4)^5 = 10^{4 \cdot 5} = 10^{20}$.

Scientific Notation

Family Support Materials 2

This week your student will use powers of 10 to work with very large or very small numbers. For example, the United States mint has made over 500,000,000,000 pennies. In order to understand this number, we have to count all of the zeros. Since there are 11 of them this means there are 500 billion pennies. Using powers of 10, we can write this as $5 \cdot 10^{11}$. The advantage to this way of writing the number is that we can see right away how many zeros there are (11), and more efficiently compare numbers when they are both written in this form. The same is true for small quantities. For example, a single atom of carbon weighs about 0.00000000000000000000000199 grams. If we write this using powers of 10, it becomes $(1.99) \cdot 10^{-23}$.

Not only do powers of 10 make it easier to write this number, but they also help avoid errors since it would be very easy to add or take away a zero when writing out the decimal without realizing! Writing numbers in this way is called scientific notation. We can use the exponent rules learned earlier to estimate and solve problems with scientific notation.

Here is a task to try with your student:

This table shows the top speeds of different vehicles.

vehicle	speed (kilometers per hour)
sports car	$(4.15) \cdot 10^2$
Apollo Command/Service Module	$(3.99) \cdot 10^4$
jet boat	$(5.1) \cdot 10^2$
autonomous drone	$(2.1) \cdot 10^4$

1. Order the vehicles from fastest to slowest.
2. The top speed of a rocket sled is 10,326 kilometers per hour. Is this faster or slower than the autonomous drone?
3. Estimate how many times as fast the Apollo Command/Service Module is than the sports car.

Solution:

1. The order is: Apollo CSM, autonomous drone, jet boat, sports car. Since all of these values are in scientific notation, we can look at the power of 10 to compare. The speeds of the Apollo CSM and autonomous drone both have the highest power of 10 (10^4), so they are fastest. The Apollo CSM is faster than the drone because 3.99 is greater than 2.1. Similarly, the jet boat is faster than the sports car because their speeds both have the same power of 10 (10^2) but 5.1 is greater than 4.15.
2. The autonomous drone is faster than the rocket sled. In scientific notation, the rocket sled's speed is $1.0326 \cdot 10^4$, and the drone's speed is $2.1 \cdot 10^4$ and 2.1 is greater than 1.0326.
3. To find how many times as fast the Apollo CSM is than the sports car, we are trying to find out what number times $4.15 \cdot 10^2$ equals $3.99 \cdot 10^4$. So we are trying to compute $\frac{3.99 \cdot 10^4}{4.15 \cdot 10^2}$. Since we are estimating, we can simplify the calculation to $\frac{4 \cdot 10^4}{4 \cdot 10^2}$. Using exponent rules and our understanding of fractions, we have $\frac{4 \cdot 10^4}{4 \cdot 10^2} = 1 \cdot 10^{4-2} = 10^2$, so the Apollo CSM is about 100 times as fast as the sports car!

Unit Assessments

Check Your Readiness A and B
End-of-Unit Assessment A and B

Exponents and Scientific Notation: Check Your Readiness (A)

Do not use a calculator.

1. Select **all** the expressions that are equal to $761 \div 5$.

A. $7,610 \div 50$

B. $762 \div 6$

C. $76.1 \div 0.05$

D. $7.61 \div 0.5$

E. $0.761 \div 0.005$

2. Which is closest to the quotient $2,967 \div 0.003$?

A. 1,000

B. 10,000

C. 100,000

D. 1,000,000

3. Select **all** the expressions that equal $3^4 \cdot 3^2$.

A. 3^6

B. 3^8

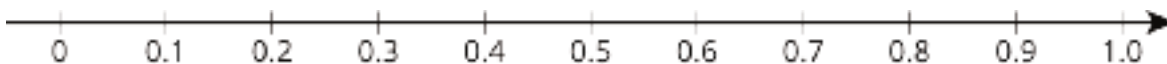
C. 9^3

D. 9^6

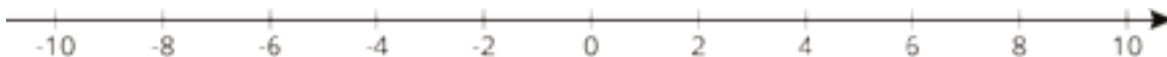
E. 9^8

4. A new phone costs \$450. There is a 40% discount on the price of the phone and an 8% sales tax on the discount price. What is the final cost of the phone after the discount and the sales tax?

5. Plot and label these numbers on the same number line:
0.8, 0.65, 0.27, 0.52, 0.052



6. Plot and label these numbers on the same number line:
 $(-2)^1, (-2)^2, (-2)^3, \left(\frac{1}{3}\right)^2, \left(\frac{1}{3}\right)^3$



7. Write three other fractions that are equivalent to $\frac{16}{128}$. Explain or show your reasoning.

Exponents and Scientific Notation: Check Your Readiness (B)

Do not use a calculator.

1. Select **all** the expressions that are equal to $362 \cdot 5.9$.

- A. $361 \cdot 6.9$
- B. $3.62 \cdot 5,900$
- C. $36.2 \cdot 59$
- D. $36,200 \cdot 0.059$
- E. $0.362 \cdot 0.0059$

2. Which is closest to the product $19,898 \cdot 0.002$?

- A. 4,000
- B. 400
- C. 40
- D. 4

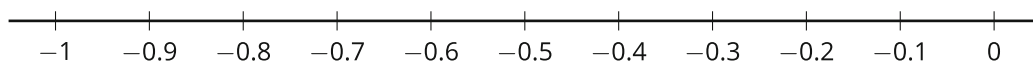
3. Select **all** the expressions that equal $2^8 \cdot 2^4$.

- A. 2^4
- B. 2^{12}
- C. 4^6
- D. 2^{32}
- E. 4^{32}

4. A population of red ants was 640. After a season of heavy rainfall, the ant population decreased by 50%. In the following dry season, the population increased by 5%. What is the ant population after the increase?

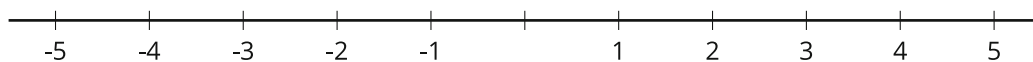
5. Plot and label these numbers on the same number line:

-0.23, -0.023, -0.352, -0.58, -0.7



6. Plot and label these numbers on the same number line:

2^1 , 2^2 , $\left(-\frac{1}{2}\right)^2$, $\left(-\frac{1}{2}\right)^3$, $\left(-\frac{1}{2}\right)^4$



7. Write three other fractions that are equivalent to $\frac{12}{72}$. Explain or show your reasoning.

Exponents and Scientific Notation: End-of-Unit Assessment (A)

Do not use a calculator.

1. Select **all** the expressions that equal 4×10^6 .

A. $(2 \times 10^8)(2 \times 10^{-2})$

B. 40×10^5

C. 40^6

D. 400,000

E. $\frac{1.2 \times 10^9}{3 \times 10^2}$

2. Select **all** the expressions that equal 6^{-10} .

A. $6^{-5} \cdot 6^2$

B. $\left(\frac{1}{6^2}\right)^5$

C. $(6^{-5})^2$

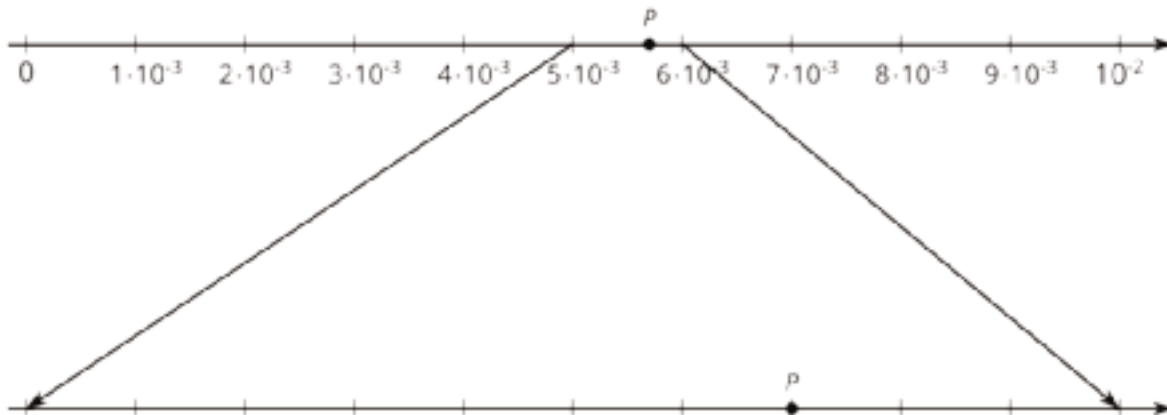
D. $\frac{6^{-3}}{6^7}$

E. $\frac{6^5 \cdot 6^{-3}}{6^{-8}}$

3. About 3.9×10^7 people live in California. About 1.3×10^6 people live in Maine. About how many more people live in California than live in Maine?

- A. 2.6×10^6
- B. 2.6×10^7
- C. 3.77×10^6
- D. 3.77×10^7

4. What number is represented by point P ?



5. In 2015, there were about 22 million teenagers (aged 13–17) in the United States. They each sent an average of 900 text messages per month. About how many text messages did all of the teenagers in the United States send each month? Express your answer using scientific notation.

6. Place a number in each box so that each equation is true and each equation has at least one negative number.

a. $2^{\square} \cdot 2^{\square} = 2^0$

b. $\frac{2^3}{2^{\square}} = 2^{\square}$

c. $2^{-3} \cdot \square^{-3} = 10^{\square}$

7. Here are the approximate populations of three countries, expressed in scientific notation: Panama: 4×10^6 ; Peru: 3.2×10^7 ; Thailand: 7×10^7 .

a. Lin says that more than 20 times as many people live in Thailand than in Panama. Is this correct? Explain how you know.

b. Decide what power of 10 to put on the label for the rightmost tick mark of this number line so that all three countries' populations can be distinguished.



c. Label each tick mark as a multiple of a power of 10.

d. Plot and label the three countries' populations on the number line.

Exponents and Scientific Notation: End-of-Unit Assessment (B)

Do not use a calculator.

1. Select **all** the expressions that equal 6×10^5 .

A. $(3 \times 10^8)(2 \times 10^{-2})$

B. $600,000,000 \times 10^{-3}$

C. $\frac{1.2 \times 10^7}{2 \times 10^2}$

D. 600,000

E. 60^5

2. Select **all** the expressions that equal 7^8 .

A. $7^{-2} \cdot 7^{10}$

B. $(7^3)^5$

C. $\frac{(7^3)^4}{7^{-4}}$

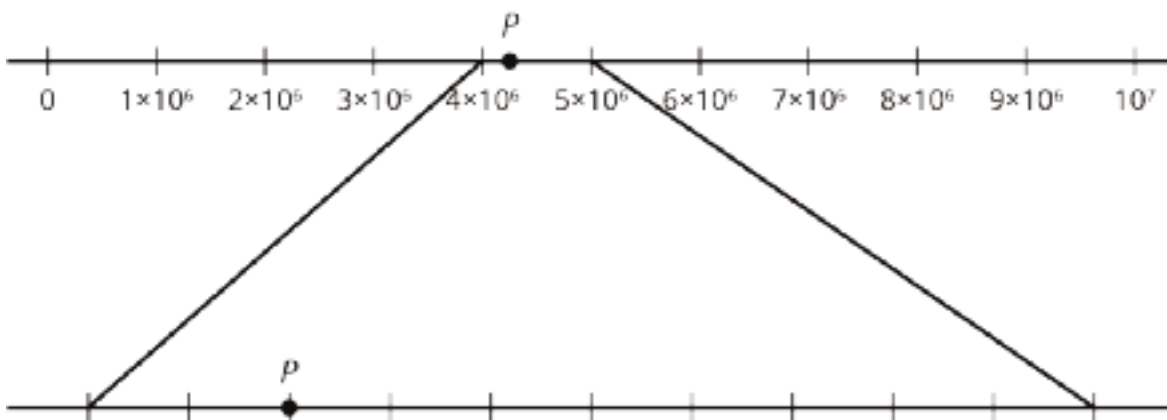
D. $(7^4)^{-2}$

E. $\frac{7^6}{7^{-2}}$

3. About 3.2×10^8 people live in the United States. About 3.9×10^7 people live in Canada. And about 1.1×10^8 people live in Mexico. About how many people live in all three countries altogether?

- A. 4.69×10^7
- B. 4.69×10^8
- C. 8.2×10^7
- D. 8.2×10^8

4. What number is represented by point P ?



5. There were approximately 3×10^5 firefighters in the United States in 2012. The average salary of a firefighter was \$45,000.

About how much did all firefighters in the United States earn from their salary altogether in 2012? Express your answer using scientific notation.

6. Place a number in each box so that each equation is true and each equation has at least one negative number.

a. $5^{\square} \cdot 5^3 = 5^{\square}$

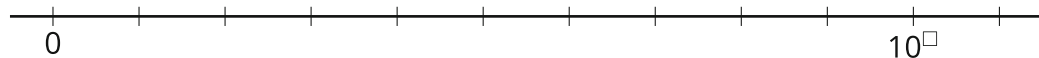
b. $\frac{5^{\square}}{5^{\square}} = 5^0$

c. $(5^{\square})^{\square} = 5^{-20}$

7. Here are the approximate populations of three cities in the United States, expressed in scientific notation: San Jose: 1.1×10^6 ; Washington: 7×10^5 ; Atlanta: 4.8×10^5 .

a. Lin says that about 6.2×10^5 more people live in San Jose than in Atlanta. Do you agree with her? Explain your reasoning.

b. Decide what power of 10 to put on the labeled tick mark on this number line so that all three countries' populations can be distinguished.



c. Label each tick mark as a multiple of a power of 10.

d. Plot and label the three cities' populations on the number line.

CKMath™
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Assessment Answer Keys

Check Your Readiness A and B
End-of-Unit Assessment A and B

Assessment Answer Keys

Assessment : Check Your Readiness (A)

Teacher Instructions

Calculators should not be used unless as a means of checking work. Many of the problems are testing fluency in calculation, and several problems can be answered directly through use of calculators.

Student Instructions

Do not use a calculator.

Problem 1

The content assessed in this problem is first encountered in Lesson 12: Applications of Arithmetic with Powers of 10.

Students examine different quotients of decimals and identify those that have the same value. The key here is thinking about multiplying both numbers of the quotient by the same power of 10, which does not change the quotient. For answers C and D, the two powers of 10 are not the same. B is incorrect, based on adding the same amount (1) to both numbers.

If most students struggle with this item, plan to use this item or a similar question before Activity 1. Prompt students to think about what they know about writing numbers using powers of 10 to make their decisions.

Statement

Select **all** the expressions that are equal to $761 \div 5$.

- A. $7,610 \div 50$
- B. $762 \div 6$
- C. $76.1 \div 0.05$
- D. $7.61 \div 0.5$
- E. $0.761 \div 0.005$

Solution

["A", "E"]

Aligned Standards

6.NS.B.3

Problem 2

The content assessed in this problem is first encountered in Lesson 12: Applications of Arithmetic with Powers of 10.

Students estimate a quotient of decimals. While students could find the answer through long division, the numbers are not friendly. The expectation is that they estimate using their knowledge of place value. The quotient $2,967 \div 0.003$ has the same value as $2,967,000 \div 3$, which they can estimate more readily.

If most students struggle with this item, plan to use this item or a similar question before Activity 1. Prompt students to think about what they know about writing numbers using powers of 10 to make their estimate.

Statement

Which is closest to the quotient $2,967 \div 0.003$?

- A. 1,000
- B. 10,000
- C. 100,000
- D. 1,000,000

Solution

D

Aligned Standards

6.NS.B.2

Problem 3

The content assessed in this problem is first encountered in Lesson 6: What about Other Bases?.

Watch for students mistakenly selecting either B or E; these are the most common errors made in working with whole-number exponents. Students who select A but not C may not recognize that more than one exponential expression can evaluate to the same number, using $9 = 3^2$ to convert from one to the other. That is a common concept in the upcoming unit for expressions like $1,000^2 = 10^6$.

If most students struggle with this item, plan to revisit it in Activity 3 or include it in the Cool-Down. If most students do well with this item, it may be possible to skip Lesson 2 Activity 3 and to move faster through Lesson 2 in general.

Statement

Select all the expressions that equal $3^4 \cdot 3^2$.

- A. 3^6
- B. 3^8
- C. 9^3
- D. 9^6
- E. 9^8

Solution

["A", "C"]

Aligned Standards

6.EE.A.1

Problem 4

The content assessed in this problem is first encountered in Lesson 12: Applications of Arithmetic with Powers of 10.

Use this problem to test students' fluency with decimal arithmetic and their work with percentages.

If most students struggle with this item, plan to connect writing numbers as the product of a number and a power of 10 to placing the decimal point in the product of decimals. You may choose to bring in Grade 6 Unit 5 Lesson 8, which helps students understand how multiplication with decimals works.

Statement

A new phone costs \$450. There is a 40% discount on the price of the phone and an 8% sales tax on the discount price. What is the final cost of the phone after the discount and the sales tax?

Solution

\$291.60 (After the discount, the phone costs \$270 because $(0.6) \cdot 450 = 270$. The sales tax is 8% of \$270, which is \$21.60. The total cost including the sales tax is \$291.60 since $270 + 21.60 = 291.60$.)

Aligned Standards

7.EE.B.3, 7.RP.A.3

Problem 5

The content assessed in this problem is first encountered in Lesson 10: Representing Large Numbers on the Number Line.

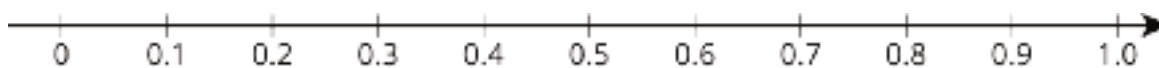
In this unit, students will plot decimals on the number line. Use this problem also to judge that students know the relative size of decimals.

If most students struggle with this item, plan to revisit it as part of Activity 1. You may show the number line labeled only with 0 and 1.0. Ask students to label the tick marks and then to plot and label the given numbers. Be sure to discuss the size of the intervals.

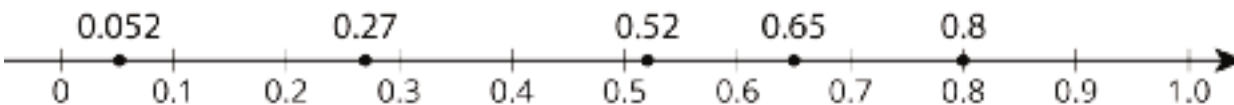
Statement

Plot and label these numbers on the same number line:

0.8, 0.65, 0.27, 0.52, 0.052



Solution



Aligned Standards

6.NS.C.6

Problem 6

The content assessed in this problem is first encountered in Lesson 10: Representing Large Numbers on the Number Line.

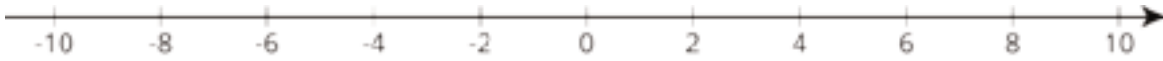
In this unit, students plot especially large and small numbers on number lines. This problem provides some practice while checking to see if students recognize that negative numbers raised to even exponents are positive.

If most students struggle with this item, plan to revisit it as part of Activity 1. You may show the number line labeled only with -10 and 10. Ask students to label the tick marks and then to plot and label the given numbers. Be sure to discuss the size of the intervals.

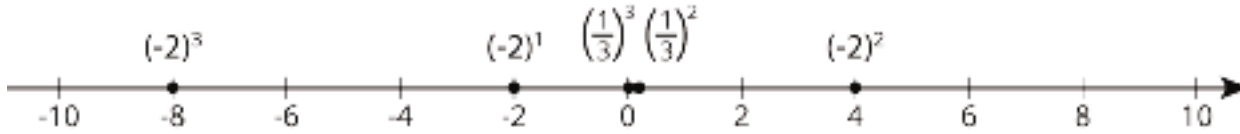
Statement

Plot and label these numbers on the same number line:

$$(-2)^1, (-2)^2, (-2)^3, \left(\frac{1}{3}\right)^2, \left(\frac{1}{3}\right)^3$$



Solution



Aligned Standards

6.NS.C.6

Problem 7

The content assessed in this problem is first encountered in Lesson 7: Practice with Rational Bases.

Some of the work in this unit involves equivalent fractions (notably, fractions that are equivalent to integers or reciprocals of integers).

If most students struggle with this item, plan to revisit it at the beginning of Activity 2, and ask students to try writing the numbers in factored form. Students should notice that it's possible to write the numbers using exponents, and they can use the rules they've learned to write fractions equivalent to the given one. Additionally, Number Talks focused on generating equivalent fractions, and Which One Doesn't Belong activities involving equivalent fractions written in factored form, standard form, and with and without exponents, could be added to Warm-Ups prior to Lesson 7 if students need additional fluency practice.

Statement

Write three other fractions that are equivalent to $\frac{16}{128}$. Explain or show your reasoning.

Solution

Answers vary. Sample response: $\frac{1}{8}$, $\frac{2}{16}$, $\frac{3}{24}$. The fraction $\frac{16}{128}$ can be written as $\frac{16 \cdot 1}{16 \cdot 8}$. This equals $\frac{1}{8}$ because it can be written as $\frac{16}{16} \cdot \frac{1}{8}$.

Aligned Standards

4.NF.A.1, 5.NF.B.5.b

Assessment : Check Your Readiness (B)

Teacher Instructions

Calculators should not be used unless as a means of checking work. Many of the problems are testing fluency in calculation, and several problems can be answered directly through use of calculators.

Student Instructions

Do not use a calculator.

Problem 1

The content assessed in this problem is first encountered in Lesson 12: Applications of Arithmetic with Powers of 10.

Students examine different products of decimals and identify those that have the same value. The key here is balancing the changes to each number by the powers of 10. Choice A is incorrect: subtracting a value from one number and adding that same value to a second number will result in a different product of the two numbers. In choice B, the value of 362 has been divided by 10^2 but 5.9 has been multiplied by 10^3 . In choice E, the value of 362 has been divided by 10^3 but 5.9 is has also been divided by 10^3 .

If most students struggle with this item, plan to use this item or a similar question before Activity 1. Prompt students to think about what they know about writing numbers using powers of 10 to make their decisions.

Statement

Select all the expressions that are equal to $362 \cdot 5.9$.

- A. $361 \cdot 6.9$
- B. $3.62 \cdot 5,900$
- C. $36.2 \cdot 59$
- D. $36,200 \cdot 0.059$
- E. $0.362 \cdot 0.0059$

Solution

["C", "D"]

Aligned Standards

6.NS.B.3

Problem 2

The content assessed in this problem is first encountered in Lesson 12: Applications of Arithmetic with Powers of 10.

Students estimate a product of decimals. While students could find the answer through multiplication by hand, the numbers are not friendly. The expectation is that they estimate, using their knowledge of place value. The product of $19,898 \cdot 0.002$ has the same value as $19.898 \cdot 2$, which they can estimate more readily as $20 \cdot 2$.

If most students struggle with this item, plan to use this item or a similar question before Activity 1. Prompt students to think about what they know about writing numbers using powers of 10 to make their estimate.

Statement

Which is closest to the product $19,898 \cdot 0.002$?

- A. 4,000
- B. 400
- C. 40
- D. 4

Solution

C

Aligned Standards

6.NS.B.2

Problem 3

The content assessed in this problem is first encountered in Lesson 6: What about Other Bases?.

Watch for students mistakenly selecting either D or E; these are the most common errors made in working with whole-number exponents. Students who select B but not C may not recognize that more than one exponential expression can evaluate to the same number. That understanding will be developed further in the upcoming unit for expressions like $1,000^2 = 10^6$.

If most students struggle with this item, plan to revisit it in Activity 3 or include it in the Cool-Down. If most students do well with this item, it may be possible to skip Lesson 2 Activity 3 and to move faster through Lesson 2 in general.

Statement

Select **all** the expressions that equal $2^8 \cdot 2^4$.

- A. 2^4
- B. 2^{12}
- C. 4^6
- D. 2^{32}
- E. 4^{32}

Solution

["B", "C"]

Aligned Standards

6.EE.A.1

Problem 4

The content assessed in this problem is first encountered in Lesson 12: Applications of Arithmetic with Powers of 10.

Use this problem to test students' fluency with decimal arithmetic and their work with percentages.

If most students struggle with this item, plan to connect writing numbers as the product of a number and a power of 10 to placing the decimal point in the product of decimals. You may choose to bring in Grade 6 Unit 5 Lesson 8, which helps students understand how multiplication with decimals works.

Statement

A population of red ants was 640. After a season of heavy rainfall, the ant population decreased by 50%. In the following dry season, the population increased by 5%. What is the ant population after the increase?

Solution

336 ants. After the decrease, the population was 320 because $(0.5) \cdot 640 = 320$. The increase in population is 5% of 320, which is 16. The ant population after the increase is 336 since $320 + 16 = 336$.

Aligned Standards

7.EE.B.3, 7.RP.A.3

Problem 5

The content assessed in this problem is first encountered in Lesson 10: Representing Large Numbers on the Number Line.

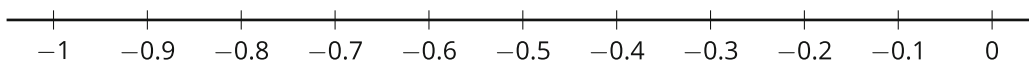
In this unit, students will plot decimals on the number line. Use this problem to assess students' knowledge of the relative size of decimals.

If most students struggle with this item, plan to revisit it as part of Activity 1. You may show the number line labeled only with 0 and 1.0. Ask students to label the tick marks and then to plot and label the given numbers. Be sure to discuss the size of the intervals.

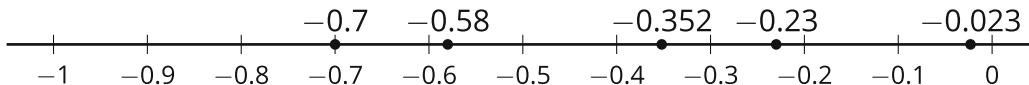
Statement

Plot and label these numbers on the same number line:

$-0.23, -0.023, -0.352, -0.58, -0.7$



Solution



Aligned Standards

6.NS.C.6

Problem 6

The content assessed in this problem is first encountered in Lesson 10: Representing Large Numbers on the Number Line.

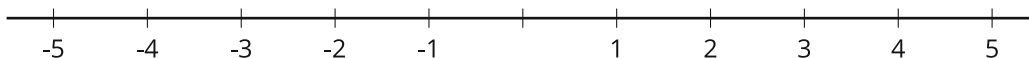
In this unit, students plot especially large and small numbers on number lines. This problem provides some practice while checking to see if students recognize that negative numbers raised to even exponents are positive.

If most students struggle with this item, plan to revisit it as part of Activity 1. You may show the number line labeled only with -10 and 10. Ask students to label the tick marks and then to plot and label the given numbers. Be sure to discuss the size of the intervals.

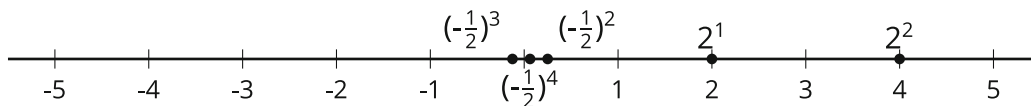
Statement

Plot and label these numbers on the same number line:

$2^1, 2^2, \left(-\frac{1}{2}\right)^2, \left(-\frac{1}{2}\right)^3, \left(-\frac{1}{2}\right)^4$



Solution



Aligned Standards

6.NS.C.6

Problem 7

The content assessed in this problem is first encountered in Lesson 7: Practice with Rational Bases.

Some of the work in this unit involves equivalent fractions (notably, fractions that are equivalent to integers or reciprocals of integers).

If most students struggle with this item, plan to revisit it at the beginning of Activity 2, and ask students to try writing the numbers in factored form. Students should notice that it's possible to write the numbers using exponents, and they can use the rules they've learned to write fractions equivalent to the given one. Additionally, Number Talks focused on generating equivalent fractions, and Which One Doesn't Belong activities involving equivalent fractions written in factored form, standard form, and with and without exponents, could be added to Warm-Ups prior to Lesson 7 if students need additional fluency practice.

Statement

Write three other fractions that are equivalent to $\frac{12}{72}$. Explain or show your reasoning.

Solution

Answers vary. Sample response: $\frac{1}{6}$, $\frac{2}{12}$, $\frac{3}{18}$. The fraction $\frac{12}{72}$ can be written as $\frac{12 \cdot 1}{12 \cdot 6}$. This equals $\frac{1}{6}$ because it can be written as $\frac{12}{12} \cdot \frac{1}{6}$.

Aligned Standards

4.NF.A.1, 5.NF.B.5.b

Assessment : End-of-Unit Assessment (A)

Teacher Instructions

Calculators should not be used. Many of the problems test fluency in calculation, and several problems can be answered directly through the use of calculators.

Student Instructions

Do not use a calculator.

Problem 1

Students failing to select A may not understand the rules for integer exponents. Students failing to select B did not rewrite 40 as 4×10 or recognize this as a way to add 1 to the exponent. Students selecting C made an error in the order of operations, combining 4 and 10 incorrectly. Students selecting D are off by a factor of 10 and may be reading 10^6 as a six-digit number. Students failing to select E may have written the expression as 0.4×10^7 but did not recognize that this is equal to 4×10^6 ; these students may need a review on scientific notation.

Statement

Select **all** the expressions that equal 4×10^6 .

A. $(2 \times 10^8)(2 \times 10^{-2})$

B. 40×10^5

C. 40^6

D. 400,000

E. $\frac{1.2 \times 10^9}{3 \times 10^2}$

Solution

["A", "B", "E"]

Aligned Standards

8.EE.A.1, 8.EE.A.4

Problem 2

Students selecting A have made the mistake of multiplying exponents. Students failing to select B may not understand the rule that $\frac{1}{a^n} = a^{-n}$. Students selecting C and D, and not A and E, have a good understanding of working with negative exponents. Students selecting E may have made an error in subtracting exponents, since this expression equals 6^{10} .

Statement

Select all the expressions that equal 6^{-10} .

A. $6^{-5} \cdot 6^2$

B. $\left(\frac{1}{6^2}\right)^5$

C. $(6^{-5})^2$

D. $\frac{6^{-3}}{6^7}$

E. $\frac{6^5 \cdot 6^{-3}}{6^{-8}}$

Solution

["B", "C", "D"]

Aligned Standards

8.EE.A.1

Problem 3

Students selecting A or B are ignoring the exponents and the need to change one of the exponents before subtracting. Students selecting C have made an error regarding the exponents, perhaps thinking the answer must be in terms of 10^6 after subtracting $39 \times 10^6 - 1.3 \times 10^6$.

Statement

About 3.9×10^7 people live in California. About 1.3×10^6 people live in Maine. About how many more people live in California than live in Maine?

A. 2.6×10^6

B. 2.6×10^7

C. 3.77×10^6

D. 3.77×10^7

Solution

D

Aligned Standards

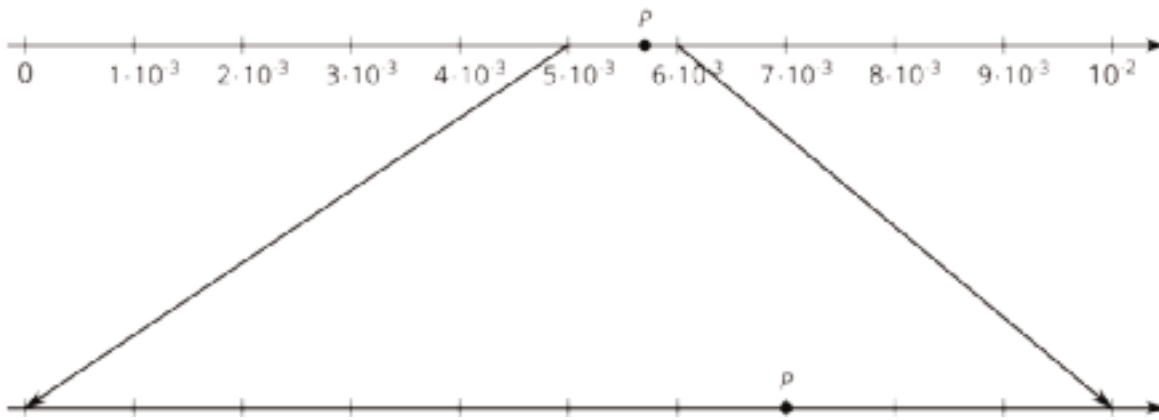
8.EE.A.4

Problem 4

Some students may think that the second number line should use even smaller powers of 10. Also, watch for students making errors in magnitude, typically failing to recognize the tick marks in the first number line are multiples of 10^{-3} .

Statement

What number is represented by point P ?



Solution

5.7×10^{-3} or 0.0057 (or equivalent) (Point P lies between 5×10^{-3} and 6×10^{-3} . In the zoomed-in number line, point P lies on the seventh of ten tick marks, so it represents 5.7×10^{-3} .)

Aligned Standards

8.EE.A.4

Problem 5

Because approximate values are used, it is reasonable but not necessary for students to approximate the answer to 2×10^{10} .

Statement

In 2015, there were about 22 million teenagers (aged 13–17) in the United States. They each sent an average of 900 text messages per month. About how many text messages did all of the teenagers in the United States send each month? Express your answer using scientific notation.

Solution

1.98×10^{10} or 2×10^{10} (Multiply $(22 \times 10^6) \cdot (9 \times 10^2) = 198 \times 10^8$. Then express in scientific notation, 1.98×10^{10} . Because the number of teenagers and the number of texts per teenager is approximate, the total number of texts can also be approximated to 2×10^{10} .)

Aligned Standards

8.EE.A.3, 8.EE.A.4

Problem 6

In part a, students may use the fact that $2^0 = 1$ to help, or use the exponent rule. Students who have difficulty with part c need a review of $a^n \cdot b^n = (ab)^n$.

Statement

Place a number in each box so that each equation is true and each equation has at least one negative number.

1. $2^{\square} \cdot 2^{\square} = 2^0$

2. $\frac{2^3}{2^{\square}} = 2^{\square}$

3. $2^{-3} \cdot \square^{-3} = 10^{\square}$

Solution

1. Answers vary. Sample response: $2^5 \cdot 2^{-5} = 2^0$

2. Answers vary. Sample response: $\frac{2^3}{2^5} = 2^{-2}$

3. $2^{-3} \cdot 5^{-3} = 10^{-3}$

Minimal Tier 1 response:

- Work is complete and correct.
- Sample:

1. $2^{-1} \cdot 2^1 = 2^0$

2. $\frac{2^3}{2^{-1}} = 2^4$

3. $2^{-3} \cdot 5^{-3} = 10^{-3}$

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: one of the three equations produced is incomplete or incorrect; all three equations are correct but some do not include negative numbers.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: two or more of the three equations produced are incomplete or incorrect.

Aligned Standards

8.EE.A.1

Problem 7

Watch for students accidentally placing the points in the order Peru, Panama, Thailand, not recognizing that Panama is at a different order of magnitude. Students may also make this mistake if they do not understand that Panama can still be placed on the number line at 0.4×10^7 .

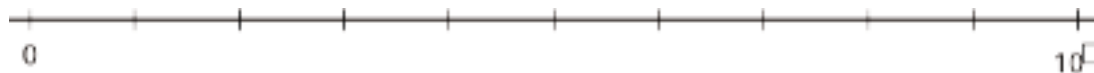
Students selecting 10^7 as their maximum tick mark might be thinking to use the highest exponent in the data: watch for students still labeling the tick marks in terms of 10^7 .

Students' answer to the first question should help determine whether or not they recognize the different order of magnitude for Panama.

Statement

Here are the approximate populations of three countries, expressed in scientific notation: Panama: 4×10^6 ; Peru: 3.2×10^7 ; Thailand: 7×10^7 .

1. Lin says that more than 20 times as many people live in Thailand than in Panama. Is this correct? Explain how you know.
2. Decide what power of 10 to put on the label for the rightmost tick mark of this number line so that all three countries' populations can be distinguished.

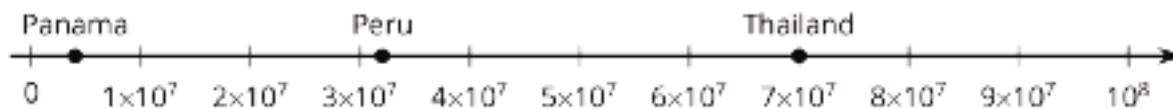


3. Label each tick mark as a multiple of a power of 10.
4. Plot and label the three countries' populations on the number line.

Solution

1. No. Explanations vary. Sample explanation: If more than 20 times as many people lived in Thailand as in Panama, their population would be more than 80 million, but it is actually 70 million.

2. The right side should be 10^8 .
3. The labels on the tick marks are 1×10^7 through 9×10^7 .
- 4.



Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample:
 1. No, if 20 times as many people lived in Thailand, that would be 8×10^7 .
 2. 10^8
 3. Number line is marked as multiples of 10^7 .
 4. All three countries plotted and labeled.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: one value significantly misplaced on number line; invalid or incorrect explanation for why there aren't more than 20 times as many people in Thailand; points plotted correctly on number line but mislabeled.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: two or more error types from Tier 2 response; incorrectly stating there are more than 20 times as many people in Thailand; using incorrect power of 10 when labeling number line. Acceptable errors: countries and values misplaced on number line as result of using incorrect power of 10, if at least one placement is correct based on the power of 10 used.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: two or more error types from Tier 3 response; labeling number line using successive powers of 10.

Aligned Standards

8.EE.A.3, 8.EE.A.4

Assessment : End-of-Unit Assessment (B)

Teacher Instructions

Calculators should not be used unless as a means of checking work. Many of the problems are testing fluency in calculation, and several problems can be answered directly through use of calculators.

Student Instructions

Do not use a calculator.

Problem 1

Students selecting A may not understand the rules for integer exponents. Students failing to select B did not rewrite 600,000,000 as 6×10^8 or may not understand the rules for integer exponents. Students selecting C may have written the expression as 0.6×10^5 but did not recognize that this is equal to 6×10^4 . Students failing to select D may be off by a factor of 10 and may be reading 10^5 as a five-digit number. Students selecting E made an error in the order of operations, combining 6 and 10 incorrectly.

Statement

Select all the expressions that equal 6×10^5 .

A. $(3 \times 10^8)(2 \times 10^{-2})$

B. $600,000,000 \times 10^{-3}$

C. $\frac{1.2 \times 10^7}{2 \times 10^2}$

D. 600,000

E. 60^5

Solution

["B", "D"]

Aligned Standards

8.EE.A.1, 8.EE.A.4

Problem 2

Students selecting B and not A may not understand the rules for integer exponents. Students selecting E, and not C or D, have a good understanding of working with negative exponents. Students who select D may not understand the rule that $\frac{1}{a^n} = a^{-n}$.

Statement

Select all the expressions that equal 7^8 .

A. $7^{-2} \cdot 7^{10}$

B. $(7^3)^5$

C. $\frac{(7^3)^4}{7^{-4}}$

D. $(7^4)^{-2}$

E. $\frac{7^6}{7^{-2}}$

Solution

["A", "E"]

Aligned Standards

8.EE.A.1

Problem 3

Students selecting C or D are ignoring the exponents and the need to change one or more of the exponents before adding. Students selecting A have made an error regarding the exponents, perhaps thinking the answer must be in terms of 10^7 after adding values together.

Statement

About 3.2×10^8 people live in the United States. About 3.9×10^7 people live in Canada. And about 1.1×10^8 people live in Mexico. About how many people live in all three countries altogether?

A. 4.69×10^7

B. 4.69×10^8

C. 8.2×10^7

D. 8.2×10^8

Solution

B

Aligned Standards

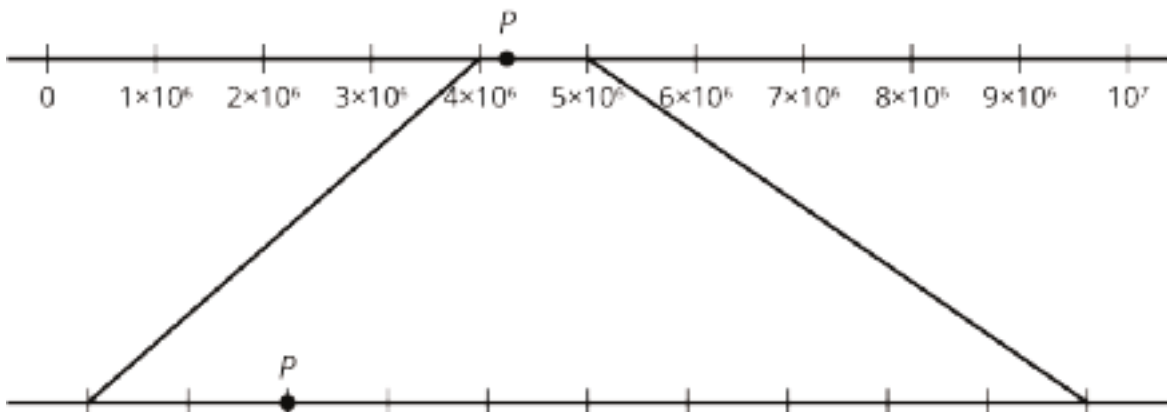
8.EE.A.4

Problem 4

Some students may think that the second number line should use even smaller powers of 10. Also, watch for students making errors in magnitude, typically failing to recognize the tick marks in the first number line are multiples of 10^6 .

Statement

What number is represented by point P ?



Solution

4.2×10^6 or 4,200,000 (or equivalent). Point P lies between 4×10^6 and 5×10^6 . In the zoomed-in number line, point P lies on the second of ten tick marks, so it represents 4.2×10^6 .

Aligned Standards

8.EE.A.4

Problem 5

Because approximate values are used, it is reasonable but not necessary for students to approximate the answer.

Statement

There were approximately 3×10^5 firefighters in the United States in 2012. The average salary of a firefighter was \$45,000.

About how much did all firefighters in the United States earn from their salary altogether in 2012? Express your answer using scientific notation.

Solution

1.35×10^{10} . (Multiply $(3 \times 10^5) \cdot (4.5 \times 10^4) = 13.5 \times 10^9$. Then express in scientific notation, 1.35×10^{10} .)

Aligned Standards

8.EE.A.3, 8.EE.A.4

Problem 6

In part 2, students may use the fact that $5^0 = 1$ to help, or use the exponent rule. Students who have difficulty with part 3 need a review of $(5^a)^b = 5^{(a \cdot b)}$.

Statement

Place a number in each box so that each equation is true and each equation has at least one negative number.

1. $5^{\square} \cdot 5^3 = 5^{\square}$

2. $\frac{5^{\square}}{5^{\square}} = 5^0$

3. $(5^{\square})^{\square} = 5^{-20}$

Solution

1. Answers vary. Sample response: $5^{-1} \cdot 5^3 = 5^2$

2. Answers vary. Sample response: $\frac{5^{-7}}{5^{-7}} = 5^0$

3. Answers vary. Sample response: $(5^{-4})^5 = 5^{-20}$

Minimal Tier 1 response:

- Work is complete and correct.
- Sample:

1. $5^{-2} \cdot 5^3 = 5^1$

2. $\frac{5^{-3}}{5^{-3}} = 5^0$

3. $(5^{-2})^{10} = 5^{-20}$

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: one of the three equations produced is incomplete or incorrect; all three equations are correct but some do not include negative numbers.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: two or more of the three equations produced are incomplete or incorrect.

Aligned Standards

8.EE.A.1

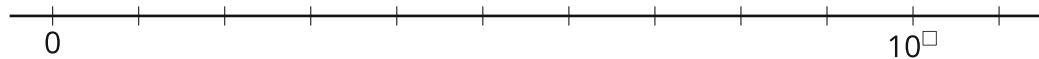
Problem 7

Watch for students accidentally placing the points in the order San Jose, Washington, Atlanta, not recognizing that San Jose is at a different order of magnitude. Students may also make this mistake if they do not understand that San Jose can still be placed on the number line to the right of 10^6 . Watch for students still labeling the tick marks in terms of 10^6 instead of 10^5 . Students' answer to the first question should help determine whether or not they recognize the different order of magnitude for San Jose.

Statement

Here are the approximate populations of three cities in the United States, expressed in scientific notation: San Jose: 1.1×10^6 ; Washington: 7×10^5 ; Atlanta: 4.8×10^5 .

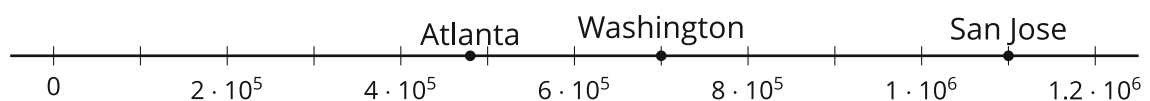
1. Lin says that about 6.2×10^5 more people live in San Jose than in Atlanta. Do you agree with her? Explain your reasoning.
2. Decide what power of 10 to put on the labeled tick mark on this number line so that all three countries' populations can be distinguished.



3. Label each tick mark as a multiple of a power of 10.
4. Plot and label the three cities' populations on the number line.

Solution

1. Yes. Explanations vary. Sample explanation: $(4.8 \times 10^5) + (6.2 \times 10^5) = 1.1 \times 10^6$.
2. The labeled tick mark should be 10^6 .
3. The labels on the tick marks are 1×10^5 through 9×10^5 .
4. The number line is marked in even multiples of 10 to save space; student solutions should include odd multiples of 10 as well.



Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Acceptable errors: Ticks greater than 10^6 are marked 11×10^5 and 12×10^5
- Sample:

1. Yes. $(4.8 \times 10^5) + (6.2 \times 10^5) = 1.1 \times 10^6$.

2. 10^6

3. See number line.

4. See number line.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: one value significantly misplaced on number line; tick marks greater than 10^6 are mislabeled; a small computational error results in a conclusion that Lin is incorrect; points plotted correctly on number line but mislabeled.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: two or more error types from Tier 2 response; an error involving scientific notation in the explanation for part a; using incorrect power of 10 when labeling number line.
- Acceptable errors: cities and values misplaced on number line as result of using incorrect power of 10, if at least one placement is correct based on the power of 10 used.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: two or more error types from Tier 3 response; labeling number line using successive powers of 10.

Aligned Standards

8.EE.A.3, 8.EE.A.4

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Lesson
Cool Downs

Lesson 1: Exponent Review

Cool Down: Exponent Check

1. What is the value of 3^4 ?
2. How many times bigger is 3^{15} compared to 3^{12} ?

Lesson 2: Multiplying Powers of Ten

Cool Down: That's a Lot of Dough, Though!

1. Rewrite $10^{32} \cdot 10^6$ using a single exponent.
2. Each year, roughly 10^6 computer programmers each make about $\$10^5$. How much money is this all together? Express your answer both as a power of 10 and as a dollar amount.

Lesson 3: Powers of Powers of 10

Cool Down: Making a Million

Here are some equivalent ways of writing 10^4 :

- 10,000
- $10 \cdot 10^3$
- $(10^2)^2$

Write as many expressions as you can that have the same value as 10^6 . Focus on using exponents and multiplication.

Lesson 4: Dividing Powers of 10

Cool Down: Why Subtract?

Why is $\frac{10^{15}}{10^4}$ equal to 10^{11} ? Explain or show your thinking.

Lesson 5: Negative Exponents with Powers of 10

Cool Down: Negative Exponent True or False

Mark each of the following equations as true or false. Explain or show your reasoning.

1. $10^{-5} = -10^5$

2. $(10^2)^{-3} = (10^{-2})^3$

3. $\frac{10^3}{10^{14}} = 10^{-11}$

Lesson 6: What about Other Bases?

Cool Down: Spot the Mistake

1. Diego was trying to write $2^3 \cdot 2^2$ with a single exponent and wrote $2^3 \cdot 2^2 = 2^{3 \cdot 2} = 2^6$. Explain to Diego what his mistake was and what the answer should be.

2. Andre was trying to write $\frac{7^4}{7^{-3}}$ with a single exponent and wrote $\frac{7^4}{7^{-3}} = 7^{4-3} = 7^1$.

Explain to Andre what his mistake was and what the answer should be.

Lesson 7: Practice with Rational Bases

Cool Down: Working with Exponents

1. Rewrite each expression using a single, positive exponent:

a. $\frac{9^3}{9^9}$

b. $14^{-3} \cdot 14^{12}$

2. Diego wrote $6^4 \cdot 8^3 = 48^7$. Explain what Diego's mistake was and how you know the equation is not true.

Lesson 8: Combining Bases

Cool Down: Help an Absent Student

Using words and equations, explain what you learned about exponents in this lesson so that someone who was absent could read what you wrote and understand the lesson. Consider using an example like $2^4 \cdot 3^4 = 6^4$.

Lesson 9: Describing Large and Small Numbers Using Powers of 10

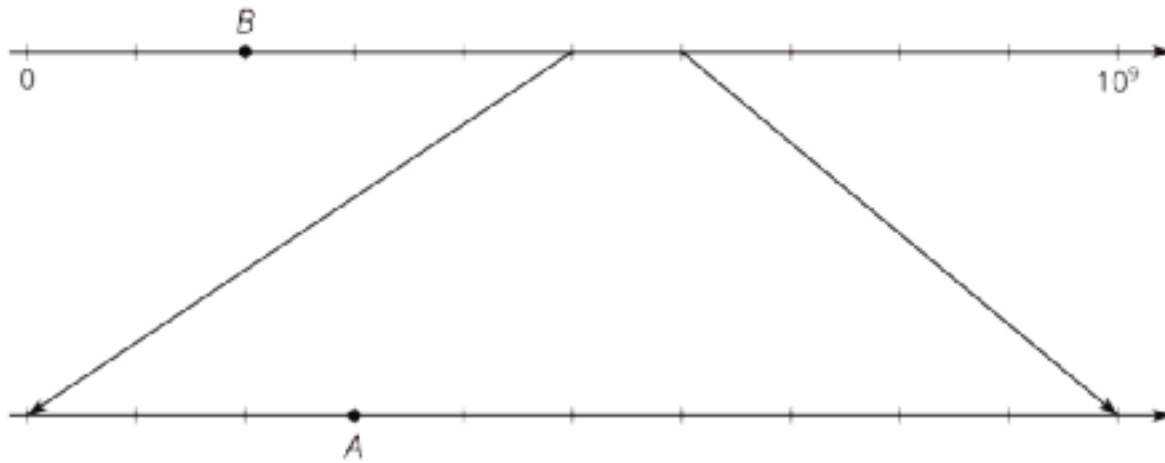
Cool Down: Better with Powers of 10

1. Write 0.000000123 as a multiple of a power of 10.
2. Write 123,000,000 as a multiple of a power of 10.

Lesson 10: Representing Large Numbers on the Number Line

Cool Down: Describe the Point

We described numbers in this lesson using both powers of 10 and using standard decimal value. For example, the speed of light through ice can be written as a multiple of a power of 10, such as $(2.3) \cdot 10^8$ meters per second, or as a value, such as 230,000,000 meters per second. Use the number line to answer questions about points *A* and *B*.



1. Describe point *B* as:
 - a. A multiple of a power of 10
 - b. A value
2. Describe point *A* as:
 - a. A multiple of a power of 10
 - b. A value
3. Plot a point *C* that is greater than *B* and less than *A*. Describe point *C* as:
 - a. A multiple of a power of 10
 - b. A value

Lesson 11: Representing Small Numbers on the Number Line

Cool Down: Describing Very Small Numbers

1. Write 0.00034 as a multiple of a power of 10.

2. Write $(5.64) \cdot 10^{-7}$ as a decimal.

Lesson 12: Applications of Arithmetic with Powers of 10

Cool Down: Reflecting on Using Powers of 10

What is a mistake you would expect to see others make when doing problems like the ones in this lesson? Give an example of what such a mistake looks like.

Lesson 13: Definition of Scientific Notation

Cool Down: Scientific Notation Check

State whether each of the following is in scientific notation. If not, write it in scientific notation.

1. 5.23×10^8

2. 48,200

3. 0.00099

4. 36×10^5

5. 8.7×10^{-12}

6. 0.78×10^{-3}

Lesson 14: Multiplying, Dividing, and Estimating with Scientific Notation

Cool Down: Estimating with Scientific Notation

1. Estimate how many times larger 6.1×10^7 is than 2.1×10^{-4} .

2. Estimate how many times larger 1.9×10^{-8} is than 4.2×10^{-13} .

Lesson 15: Adding and Subtracting with Scientific Notation

Cool Down: Adding with Scientific Notation

Elena wants to add $(2.3 \times 10^5) + (3.6 \times 10^6)$ and writes $(2.3 \times 10^5) + (3.6 \times 10^6) = 5.9 \times 10^6$.

Explain to Elena what her mistake was and what the correct solution is.

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Instructional Masters

Instructional Masters for Exponents and Scientific Notation

address	title	students per copy	written on?	requires cutting?	card stock recommended?	color paper recommended?
Activity Grade8.7.14.3	8.7.C14.BlacklineMaster	1	no	yes	no	no
Activity Grade8.7.9.3	Using Powers of 10 to Describe Large and Small Numbers	1	no	yes	no	no
Activity Grade8.7.13.3	Scientific Notation Matching	1	no	yes	no	yes
Activity Grade8.7.16.1	Old Hardware, New Hardware	1	no	no	no	no

Using Powers of 10 to Describe Large and Small Numbers

Partner A Card

You are Partner A.

Here are the answers to Partner B's questions:

- Light waves travel through space at a speed of 3 hundred million meters per second.
- The population of India is about 1,300,000,000 people.
- The wavelength of a gamma ray is 0.0000000000048 meters.
- The tardigrade (water bear) is 5 ten-thousandths of a meter long.

Using Powers of 10 to Describe Large and Small Numbers

Partner B Card

You are Partner B.

Here are the answers to Partner A's questions:

- Around the world, about 14,000,000,000 pencils are made each year.
- The mass of a proton is 0.0000000000000000000000000000167 kilograms.
- The population of Russia is about 144 million people.
- The diameter of a bacteria cell is about 2 ten-millionths of a meter.

Using Powers of 10 to Describe Large and Small Numbers

Partner A Card

You are Partner A.

Here are the answers to Partner B's questions:

- Light waves travel through space at a speed of 3 hundred million meters per second.
- The population of India is about 1,300,000,000 people.
- The wavelength of a gamma ray is 0.0000000000048 meters.
- The tardigrade (water bear) is 5 ten-thousandths of a meter long.

Using Powers of 10 to Describe Large and Small Numbers

Partner B Card

You are Partner B.

Here are the answers to Partner A's questions:

- Around the world, about 14,000,000,000 pencils are made each year.
- The mass of a proton is 0.0000000000000000000000000000167 kilograms.
- The population of Russia is about 144 million people.
- The diameter of a bacteria cell is about 2 ten-millionths of a meter.

Using Powers of 10 to Describe Large and Small Numbers

Partner A Card

You are Partner A.

Here are the answers to Partner B's questions:

- Light waves travel through space at a speed of 3 hundred million meters per second.
- The population of India is about 1,300,000,000 people.
- The wavelength of a gamma ray is 0.0000000000048 meters.
- The tardigrade (water bear) is 5 ten-thousandths of a meter long.

Using Powers of 10 to Describe Large and Small Numbers

Partner B Card

You are Partner B.

Here are the answers to Partner A's questions:

- Around the world, about 14,000,000,000 pencils are made each year.
- The mass of a proton is 0.0000000000000000000000000000167 kilograms.
- The population of Russia is about 144 million people.
- The diameter of a bacteria cell is about 2 ten-millionths of a meter.

8.7.13.3 Scientific Notation Matching.

Scientific Notation Matching - Set A 5×10^5	Scientific Notation Matching - Set A 5×10^6	Scientific Notation Matching - Set A 5×10^{-5}	Scientific Notation Matching - Set A 5×10^{-6}
Scientific Notation Matching - Set A 0.5×10^6	Scientific Notation Matching - Set A 50×10^5	Scientific Notation Matching - Set A 0.5×10^{-4}	Scientific Notation Matching - Set A 50×10^{-7}
Scientific Notation Matching - Set A 500,000	Scientific Notation Matching - Set A 5,000,000	Scientific Notation Matching - Set A 0.000005	Scientific Notation Matching - Set A 0.0000005
Scientific Notation Matching - Set A 500,000	Scientific Notation Matching - Set A 5,000,000	Scientific Notation Matching - Set A 0.000005	Scientific Notation Matching - Set A 0.0000005

8.7.13.3 Scientific Notation Matching.
Scientific Notation Matching - Set B

$$4.3 \times 10^7$$

Scientific Notation Matching - Set B

$$4.3 \times 10^4$$

Scientific Notation Matching - Set B

$$0.43 \times 10^{-1}$$

Scientific Notation Matching - Set B

$$4.3 \times 10^{-7}$$

Scientific Notation Matching - Set B

$$4.3 \times 10^5$$

Scientific Notation Matching - Set B

$$4.3 \times 10^2$$

Scientific Notation Matching - Set B

$$4.3 \times 10^{-5}$$

Scientific Notation Matching - Set B

$$4.3 \times 10^{-4}$$

Scientific Notation Matching - Set B

$$43 \times 10^4$$

Scientific Notation Matching - Set B

$$0.43 \times 10^3$$

Scientific Notation Matching - Set B

$$0.43 \times 10^{-4}$$

Scientific Notation Matching - Set B

$$43 \times 10^{-3}$$

Scientific Notation Matching - Set B

$$43,000$$

Scientific Notation Matching - Set B

$$43,000,000$$

Scientific Notation Matching - Set B

$$0.00000043$$

Scientific Notation Matching - Set B

$$0.00043$$

Scientific Notation Matching - Set B

$$430,000$$

Scientific Notation Matching - Set B

$$430$$

Scientific Notation Matching - Set B

$$0.000043$$

Scientific Notation Matching - Set B

$$0.043$$

Scientific Notation Matching - Set B

$$430,000$$

Scientific Notation Matching - Set B

$$430$$

Scientific Notation Matching - Set B

$$0.000043$$

Scientific Notation Matching - Set B

$$0.043$$

8.7.13.3 Scientific Notation Matching.
Scientific Notation Matching - Set C

$$6.3 \times 10^4$$

Scientific Notation Matching - Set C

$$6.3 \times 10^5$$

Scientific Notation Matching - Set C

$$6.3 \times 10^{-4}$$

Scientific Notation Matching - Set C

$$6.3 \times 10^{-5}$$

Scientific Notation Matching - Set C

$$6.3 \times 10^3$$

Scientific Notation Matching - Set C

$$6.3 \times 10^6$$

Scientific Notation Matching - Set C

$$6.3 \times 10^{-3}$$

Scientific Notation Matching - Set C

$$6.3 \times 10^{-6}$$

Scientific Notation Matching - Set C

$$0.63 \times 10^4$$

Scientific Notation Matching - Set C

$$63 \times 10^5$$

Scientific Notation Matching - Set C

$$0.63 \times 10^{-5}$$

Scientific Notation Matching - Set C

$$63 \times 10^{-4}$$

Scientific Notation Matching - Set C

$$63,000$$

Scientific Notation Matching - Set C

$$630,000$$

Scientific Notation Matching - Set C

$$0.00063$$

Scientific Notation Matching - Set C

$$0.000063$$

Scientific Notation Matching - Set C

$$6,300$$

Scientific Notation Matching - Set C

$$6,300,000$$

Scientific Notation Matching - Set C

$$0.0063$$

Scientific Notation Matching - Set C

$$0.0000063$$

Scientific Notation Matching - Set C

$$6,300$$

Scientific Notation Matching - Set C

$$6,300,000$$

Scientific Notation Matching - Set C

$$0.0063$$

Scientific Notation Matching - Set C

$$0.0000063$$

Info Gap: Distances in the Solar System

Problem Card 1

Estimate:

1. How many Earths side by side would have the same width as the Sun?
2. How many Earths would it take to equal the mass of the Sun?

Info Gap: Distances in the Solar System

Data Card 1

- The distance from Earth to the Sun is approximately 1.496×10^8 km.
- The diameter of the Sun is 1.392×10^6 km.
- The diameter of Earth is 1.28×10^4 km.
- The mass of the Sun is 1.989×10^{30} kg.
- The mass of Earth is 5.98×10^{24} kg.

Info Gap: Distances in the Solar System

Problem Card 2

Estimate:

1. How many times as far away from Earth is the planet Neptune compared to Venus?
2. How many copies of the planet Mercury would it take to equal the mass of Neptune?

Info Gap: Distances in the Solar System

Data Card 2

- The average distance from Earth . . .
 - to Mercury is 7.73×10^7 km.
 - to Venus is 4×10^7 km.
 - to Neptune is 4.3×10^9 km.
- The mass of Mercury is 3.3×10^{23} kg.
- The mass of Venus is 4.87×10^{24} kg.
- The mass of Neptune is 1.024×10^{26} kg.

Info Gap: Distances in the Solar System

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Info Gap: Distances in the Solar System

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Info Gap: Distances in the Solar System

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Info Gap: Distances in the Solar System

Data Card 2

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1966 Apollo Guidance Computer	Storage	75 kilobytes
	Processing Speed	2 megahertz
	Memory	4 kilobytes

2007 Smartphone	Storage	4 gigabytes
	Processing Speed	400 megahertz
	Memory	128 megabytes

1977 Desktop Computer	Storage	140 kilobytes
	Processing Speed	1 megahertz
	Memory	4 kilobytes

2016 Smartphone	Storage	32 gigabytes
	Processing Speed	4 processors, 2.2 gigahertz each
	Memory	3 gigabytes

2001 Desktop Computer	Storage	20 gigabytes
	Processing Speed	1.1 gigahertz
	Memory	128 megabytes

2016 Desktop Computer	Storage	1 terabyte
	Processing Speed	4 processors, 3 gigahertz each
	Memory	8 gigabytes

2007 Desktop Computer	Storage	500 gigabytes
	Processing Speed	2 processors, 2 gigahertz each
	Memory	4 gigabytes

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