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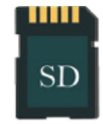

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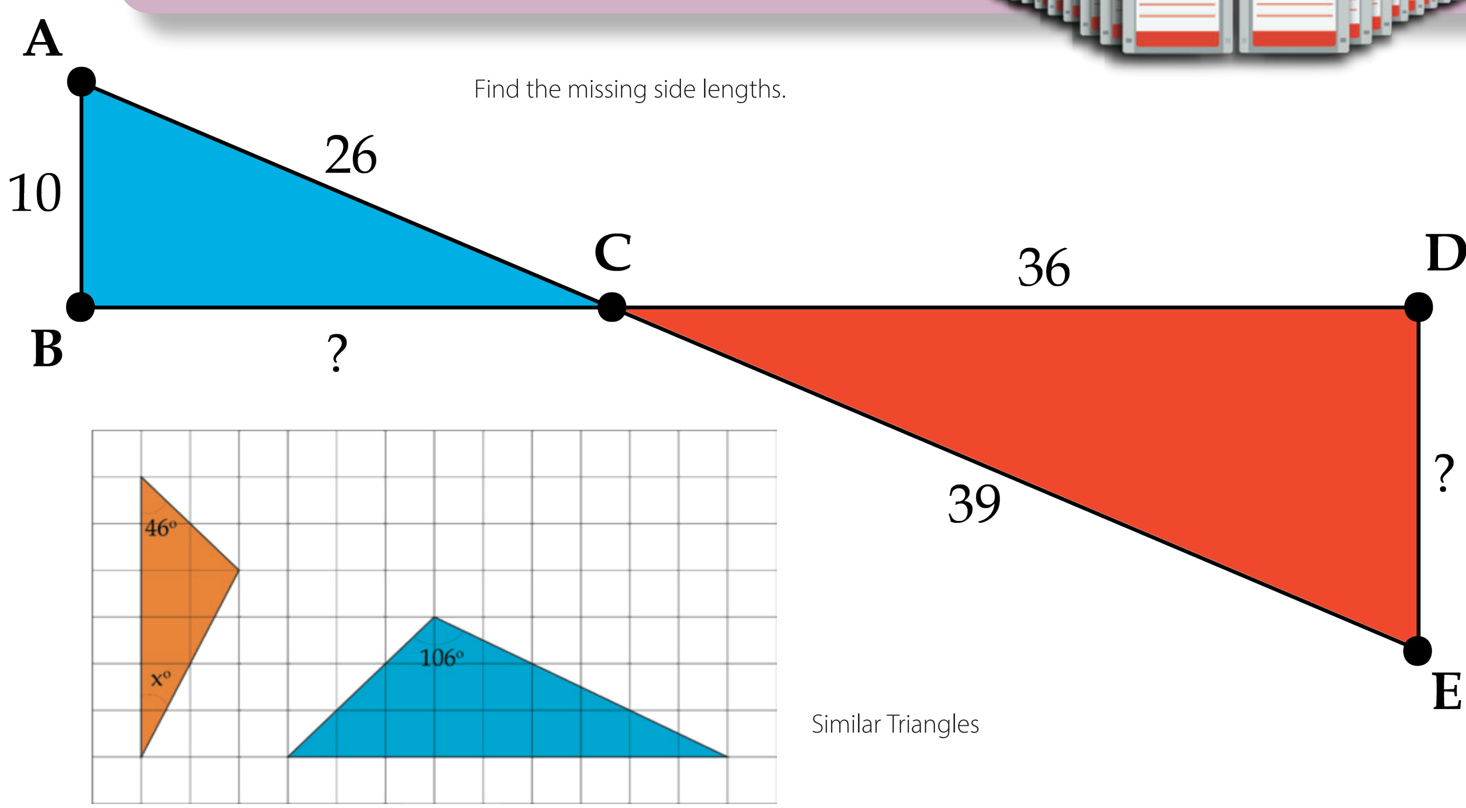
Exponents and Scientific Notation

Student Workbook



Find the missing side lengths.

Similar Triangles

expression	expanded	exponent
$5^3 \cdot 2^3$	$(5 \cdot 5 \cdot 5) \cdot (2 \cdot 2 \cdot 2) = (5 \cdot 2)(5 \cdot 2)(5 \cdot 2)$ $= 10 \cdot 10 \cdot 10$	10^3

Power of Products

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Exponents and Scientific Notation

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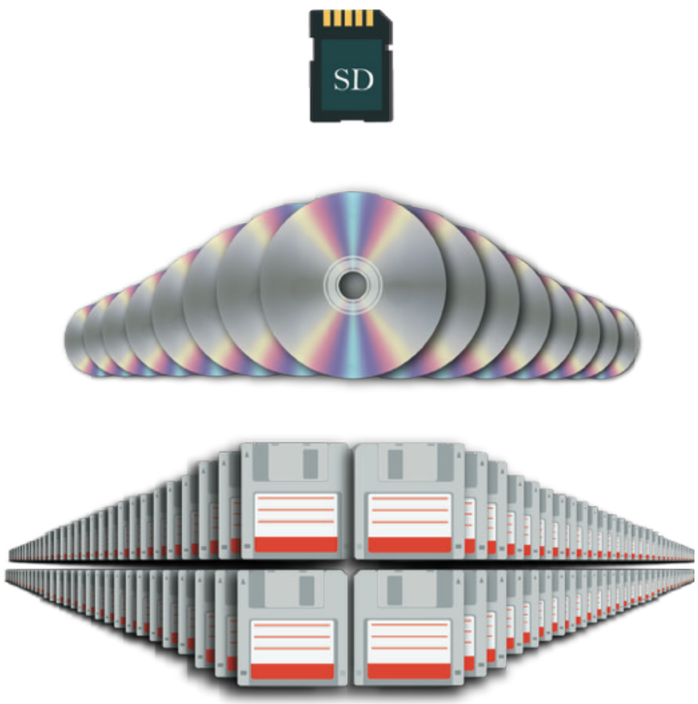


Exponents and Scientific Notation

Student Workbook

Core Knowledge Mathematics™

Lesson 1: Exponent Review

Let's review exponents.

1.1: Which One Doesn't Belong: Twos

Which expression does not belong? Be prepared to share your reasoning.

2^3	3^2
8	$2^2 \cdot 2^1$

1.2: Return of the Genie

Mai and Andre found an old, brass bottle that contained a magical genie. They freed the genie, and it offered them each a magical $1 coin as thanks.

- The magic coin turned into 2 coins on the first day.
- The 2 coins turned into 4 coins on the second day.
- The 4 coins turned into 8 coins on the third day.

This doubling pattern continued for 28 days.

Mai was trying to calculate how many coins she would have and remembered that instead of writing $1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ for the number of coins on the 6th day, she could just write 2^6.

1. The number of coins Mai had on the 28th day is very, very large. Write an expression to represent this number without computing its value.

2. Andre's coins lost their magic on the 25th day, so Mai has a lot more coins than he does. How many times more coins does Mai have than Andre?

1.3: Broken Coin

After a while, Jada picks up a coin that seems different than the others. She notices that the next day, only half of the coin is left!

- On the second day, only $\frac{1}{4}$ of the coin is left.
- On the third day, $\frac{1}{8}$ of the coin remains.

1. What fraction of the coin remains after 6 days?

2. What fraction of the coin remains after 28 days? Write an expression to describe this without computing its value.

3. Does the coin disappear completely? If so, after how many days?

Are you ready for more?

Every animal has two parents. Each of its parents also has two parents.

1. Draw a family tree showing an animal, its parents, its grandparents, and its great-grandparents.

2. We say that the animal's eight great-grandparents are "three generations back" from the animal. At which generation back would an animal have 262,144 ancestors?

Lesson 1 Summary

Exponents make it easy to show repeated multiplication. For example,

$$2^6 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2.$$

One advantage to writing 2^6 is that we can see right away that this is 2 to the *sixth* power. When this is written out using multiplication, $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$, we need to count the number of factors. Imagine writing out 2^{100} using multiplication!

Let's say you start out with one grain of rice and that each day the number of grains of rice you have doubles. So on day one, you have 2 grains, on day two, you have 4 grains, and so on. When we write 2^{25}, we can see from the expression that the rice has doubled 25 times. So this notation is not only convenient, but it also helps us see structure: in this case, we can see right away that it is on the 25th day that the number of grains of rice has doubled! That's a lot of rice (more than a cubic meter)!

Unit 7 Lesson 1 Cumulative Practice Problems

1. Write each expression using an exponent:

   a. $1 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

   b. $1 \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right) \cdot \left(\frac{4}{5}\right)$

   c. $1 \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3) \cdot (9.3)$

   d. The number of coins Jada will have on the eighth day, if Jada starts with one coin and the number of coins doubles every day. (She has two coins on the first day of the doubling.)

2. Evaluate each expression:

   a. 2^5

   b. 3^3

   c. 4^3

   a. 6^2

   b. $\left(\frac{1}{2}\right)^4$

   c. $\left(\frac{1}{3}\right)^2$

3. Clare made $160 babysitting last summer. She put the money in a savings account that pays 3% interest per year. If Clare doesn't touch the money in her account, she can find the amount she'll have the next year by multiplying her current amount by 1.03.

   a. How much money will Clare have in her account after 1 year? After 2 years?

   b. How much money will Clare have in her account after 5 years? Explain your reasoning.

   c. Write an expression for the amount of money Clare would have after 30 years if she never withdraws money from the account.

4. The equation $y = 5{,}280x$ gives the number of feet, y, in x miles. What does the number 5,280 represent in this relationship?

(From Unit 3, Lesson 1.)

5. The points $(2, 4)$ and $(6, 7)$ lie on a line. What is the slope of the line?

   A. 2

   B. 1

   C. $\frac{4}{3}$

   D. $\frac{3}{4}$

(From Unit 3, Lesson 5.)

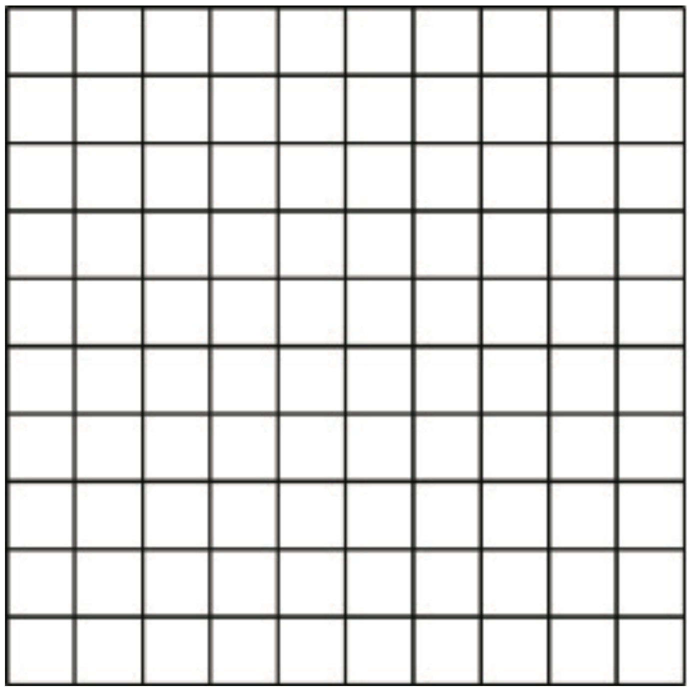
6. The diagram shows a pair of similar figures, one contained in the other. Name a point and a scale factor for a dilation that moves the larger figure to the smaller one.

(From Unit 2, Lesson 6.)

Lesson 2: Multiplying Powers of Ten

Let's explore patterns with exponents when we multiply powers of 10.

2.1: 100, 1, or $\frac{1}{100}$?



Clare said she sees 100.

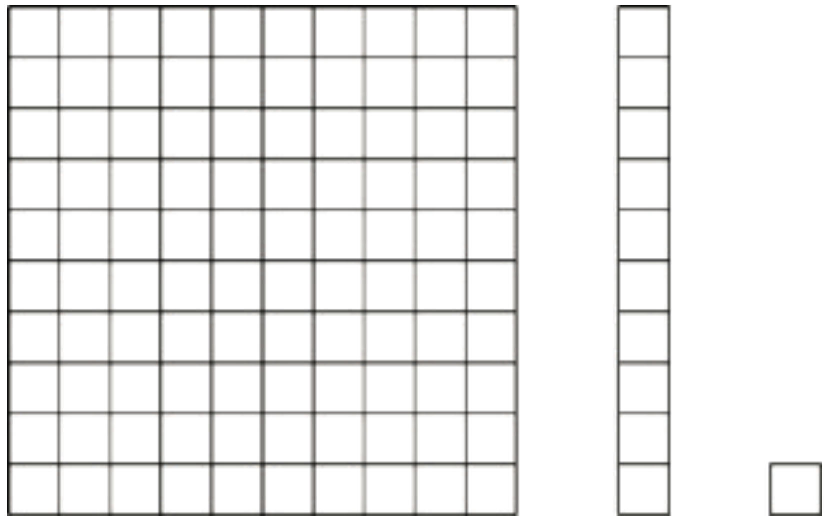
Tyler says he sees 1.

Mai says she sees $\frac{1}{100}$.

Who do you agree with?

2.2: Picture a Power of 10

In the diagram, the medium rectangle is made up of 10 small squares. The large square is made up of 10 medium rectangles.



1. How could you represent the large square as a power of 10?

2. If each small square represents 10^2, then what does the medium rectangle represent? The large square?

3. If the medium rectangle represents 10^5, then what does the large square represent? The small square?

4. If the large square represents 10^{100}, then what does the medium rectangle represent? The small square?

2.3: Multiplying Powers of Ten

1. a. Complete the table to explore patterns in the exponents when multiplying powers of 10. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power of 10
$10^2 \cdot 10^3$	$(10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^5
$10^4 \cdot 10^3$		
$10^4 \cdot 10^4$		
	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$	
$10^{18} \cdot 10^{23}$		

   b. If you chose to skip one entry in the table, which entry did you skip? Why?

2. a. Use the patterns you found in the table to rewrite $10^n \cdot 10^m$ as an equivalent expression with a single exponent, like $10^{\square}$.

   b. Use your rule to write $10^4 \cdot 10^0$ with a single exponent. What does this tell you about the value of 10^0?

3. The state of Georgia has roughly 10^7 human residents. Each human has roughly 10^{13} bacteria cells in his or her digestive tract. How many bacteria cells are there in the digestive tracts of all the humans in Georgia?

Are you ready for more?

There are four ways to make 10^4 by multiplying powers of 10 with smaller, positive exponents.

$$10^1 \cdot 10^1 \cdot 10^1 \cdot 10^1$$

$$10^1 \cdot 10^1 \cdot 10^2$$

$$10^1 \cdot 10^3$$

$$10^2 \cdot 10^2$$

(This list is complete if you don't pay attention to the order you write them in. For example, we are only counting $10^1 \cdot 10^3$ and $10^3 \cdot 10^1$ once.)

1. How many ways are there to make 10^6 by multiplying smaller powers of 10 together?

2. How about 10^7? 10^8?

Lesson 2 Summary

In this lesson, we developed a rule for multiplying powers of 10: multiplying powers of 10 corresponds to adding the exponents together. To see this, multiply 10^5 and 10^2. We know that 10^5 has five factors that are 10 and 10^2 has two factors that are 10. That means that $10^5 \cdot 10^2$ has 7 factors that are 10.

$$10^5 \cdot 10^2 = (10 \cdot 10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10) = 10^7.$$

This will work for other powers of 10 too. So $10^{14} \cdot 10^{47} = 10^{61}$.

This rule makes it easier to understand and work with expressions that have exponents.

Unit 7 Lesson 2 Cumulative Practice Problems

1. Write each expression with a single exponent:

   a. $10^3 \cdot 10^9$

   b. $10 \cdot 10^4$

   c. $10^{10} \cdot 10^7$

   d. $10^3 \cdot 10^3$

   e. $10^5 \cdot 10^{12}$

   f. $10^6 \cdot 10^6 \cdot 10^6$

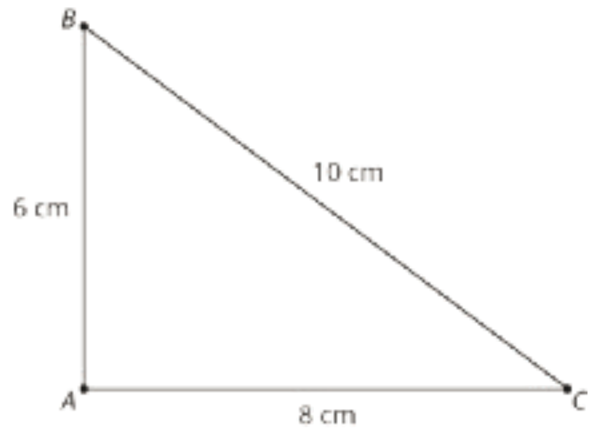
2. A large rectangular swimming pool is 1,000 feet long, 100 feet wide, and 10 feet deep. The pool is filled to the top with water.

   a. What is the area of the surface of the water in the pool?

   b. How much water does the pool hold?

   c. Express your answers to the previous two questions as powers of 10.

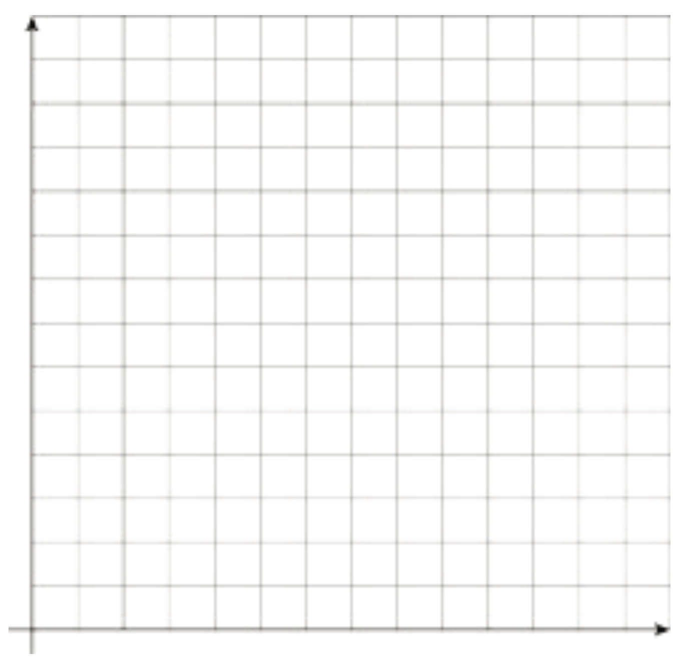
3. Here is triangle ABC. Triangle DEF is similar to triangle ABC, and the length of EF is 5 cm. What are the lengths of sides DE and DF, in centimeters?



(From Unit 2, Lesson 7.)

4. Elena and Jada distribute flyers for different advertising companies. Elena gets paid 65 cents for every 10 flyers she distributes, and Jada gets paid 75 cents for every 12 flyers she distributes.

   Draw graphs on the coordinate plane representing the total amount each of them earned, y, after distributing x flyers. Use the graph to decide who got paid more after distributing 14 flyers.



(From Unit 3, Lesson 3.)

Lesson 3: Powers of Powers of 10

Let's look at powers of powers of 10.

3.1: Big Cube

What is the volume of a giant cube that measures 10,000 km on each side?

3.2: Raising Powers of 10 to Another Power

1. a. Complete the table to explore patterns in the exponents when raising a power of 10 to a power. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power of 10
$(10^3)^2$	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^6
$(10^2)^5$	$(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)$	
	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	
$(10^4)^2$		
$(10^8)^{11}$		

   b. If you chose to skip one entry in the table, which entry did you skip? Why?

2. Use the patterns you found in the table to rewrite $(10^m)^n$ as an equivalent expression with a single exponent, like $10^{\square}$.

3. If you took the amount of oil consumed in 2 months in 2013 worldwide, you could make a cube of oil that measures 10^3 meters on each side. How many cubic meters of oil is this? Do you think this would be enough to fill a pond, a lake, or an ocean?

3.3: How Do the Rules Work?

Andre and Elena want to write $10^2 \cdot 10^2 \cdot 10^2$ with a single exponent.

- Andre says, "When you multiply powers with the same **base**, it just means you add the exponents, so $10^2 \cdot 10^2 \cdot 10^2 = 10^{2+2+2} = 10^6$."
- Elena says, "10^2 is multiplied by itself 3 times, so $10^2 \cdot 10^2 \cdot 10^2 = (10^2)^3 = 10^{2+3} = 10^5$."

Do you agree with either of them? Explain your reasoning.

Are you ready for more?

$2^{12} = 4{,}096$. How many other whole numbers can you raise to a power and get 4,096? Explain or show your reasoning.

Lesson 3 Summary

In this lesson, we developed a rule for taking a power of 10 to another power: Taking a power of 10 and raising it to another power is the same as multiplying the exponents. See what happens when raising 10^4 to the power of 3.

$$\left(10^4\right)^3 = 10^4 \cdot 10^4 \cdot 10^4 = 10^{12}$$

This works for any power of powers of 10. For example, $\left(10^6\right)^{11} = 10^{66}$. This is another rule that will make it easier to work with and make sense of expressions with exponents.

Unit 7 Lesson 3 Cumulative Practice Problems

1. Write each expression with a single exponent:

    a. $(10^7)^2$

    b. $(10^9)^3$

    c. $(10^6)^3$

    d. $(10^2)^3$

    e. $(10^3)^2$

    f. $(10^5)^7$

2. You have 1,000,000 number cubes, each measuring one inch on a side.

    a. If you stacked the cubes on top of one another to make an enormous tower, how high would they reach? Explain your reasoning.

    b. If you arranged the cubes on the floor to make a square, would the square fit in your classroom? What would its dimensions be? Explain your reasoning.

    c. If you layered the cubes to make one big cube, what would be the dimensions of the big cube? Explain your reasoning.

3. An amoeba divides to form two amoebas after one hour. One hour later, each of the two amoebas divides to form two more. Every hour, each amoeba divides to form two more.

    a. How many amoebas are there after 1 hour?

    b. How many amoebas are there after 2 hours?

    c. Write an expression for the number of amoebas after 6 hours.

    d. Write an expression for the number of amoebas after 24 hours.

    e. Why might exponential notation be preferable to answer these questions?

(From Unit 7, Lesson 1.)

4. Elena noticed that, nine years ago, her cousin Katie was twice as old as Elena was then. Then Elena said, "In four years, I'll be as old as Katie is now!" If Elena is currently e years old and Katie is k years old, which system of equations matches the story?

    A. $\begin{cases} k - 9 = 2e \\ e + 4 = k \end{cases}$

    B. $\begin{cases} 2k = e - 9 \\ e = k + 4 \end{cases}$

    C. $\begin{cases} k = 2e - 9 \\ e + 4 = k + 4 \end{cases}$

    D. $\begin{cases} k - 9 = 2(e - 9) \\ e + 4 = k \end{cases}$

(From Unit 4, Lesson 15.)

Lesson 4: Dividing Powers of 10

Let's explore patterns with exponents when we divide powers of 10.

4.1: A Surprising One

What is the value of the expression?

$$\frac{2^5 \cdot 3^4 \cdot 3^2}{2 \cdot 3^6 \cdot 2^4}$$

4.2: Dividing Powers of Ten

1. a. Complete the table to explore patterns in the exponents when dividing powers of 10. Use the "expanded" column to show why the given expression is equal to the single power of 10. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power
$10^4 \div 10^2$	$\frac{10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 = 1 \cdot 10 \cdot 10$	10^2
	$\frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 \cdot 10 = 1 \cdot 10 \cdot 10 \cdot 10$	
$10^6 \div 10^3$		
$10^{43} \div 10^{17}$		

   b. If you chose to skip one entry in the table, which entry did you skip? Why?

2. Use the patterns you found in the table to rewrite $\frac{10^n}{10^m}$ as an equivalent expression of the form $10^{\square}$.

3. It is predicted that by 2050, there will be 10^{10} people living on Earth. At that time, it is predicted there will be approximately 10^{12} trees. How many trees will there be for each person?

Are you ready for more?

expression	expanded	single power
$10^4 \div 10^6$		

4.3: Zero Exponent

So far we have looked at powers of 10 with exponents greater than 0. What would happen to our patterns if we included 0 as a possible exponent?

1. a. Write $10^{12} \cdot 10^0$ with a power of 10 with a single exponent using the appropriate exponent rule. Explain or show your reasoning.

   b. What number could you multiply 10^{12} by to get this same answer?

2. a. Write $\frac{10^8}{10^0}$ with a single power of 10 using the appropriate exponent rule. Explain or show your reasoning.

   b. What number could you divide 10^8 by to get this same answer?

3. If we want the exponent rules we found to work even when the exponent is 0, then what does the value of 10^0 have to be?

4. Noah says, "If I try to write 10^0 expanded, it should have zero factors that are 10, so it must be equal to 0." Do you agree? Discuss with your partner.

4.4: Making Millions

Write as many expressions as you can that have the same value as 10^6. Focus on using exponents, multiplication, and division. What patterns do you notice with the exponents?

Lesson 4 Summary

In an earlier lesson, we learned that when multiplying powers of 10, the exponents add together. For example, $10^6 \cdot 10^3 = 10^9$ because 6 factors that are 10 multiplied by 3 factors that are 10 makes 9 factors that are 10 all together. We can also think of this multiplication equation as division:

$$10^6 = \frac{10^9}{10^3}$$

So when dividing powers of 10, the exponent in the denominator is subtracted from the exponent in the numerator. This makes sense because

$$\frac{10^9}{10^3} = \frac{10^3 \cdot 10^6}{10^3} = \frac{10^3}{10^3} \cdot 10^6 = 1 \cdot 10^6 = 10^6$$

This rule works for other powers of 10 too. For example, $\frac{10^{56}}{10^{23}} = 10^{33}$ because 23 factors that are 10 in the numerator and in the denominator are used to make 1, leaving 33 factors remaining.

This gives us a new exponent rule:

$$\frac{10^n}{10^m} = 10^{n-m}.$$

So far, this only makes sense when n and m are positive exponents and $n > m$, but we can extend this rule to include a new power of 10, 10^0. If we look at $\frac{10^6}{10^0}$, using the exponent rule gives 10^{6-0}, which is equal to 10^6. So dividing 10^6 by 10^0 doesn't change its value. That means that if we want the rule to work when the exponent is 0, then it must be that

$$10^0 = 1$$

Unit 7 Lesson 4 Cumulative Practice Problems

1. Evaluate:

   a. 10^0

   b. $\frac{10^3}{10^3}$

   c. $10^2 + 10^1 + 10^0$

2. Write each expression as a single power of 10.

   a. $\frac{10^3 \cdot 10^4}{10^5}$

   b. $(10^4) \cdot \frac{10^{12}}{10^7}$

   c. $\left(\frac{10^5}{10^3}\right)^4$

   d. $\frac{10^4 \cdot 10^5 \cdot 10^6}{10^3 \cdot 10^7}$

   e. $\frac{(10^5)^2}{(10^2)^3}$

3. The Sun is roughly 10^2 times as wide as Earth. The star KW Sagittarii is roughly 10^5 times as wide as Earth. About how many times as wide as the Sun is KW Sagittarii? Explain how you know.

4. Bananas cost \$1.50 per pound, and guavas cost \$3.00 per pound. Kiran spends \$12 on fruit for a breakfast his family is hosting. Let b be the number of pounds of bananas Kiran buys and g be the number of pounds of guavas he buys.

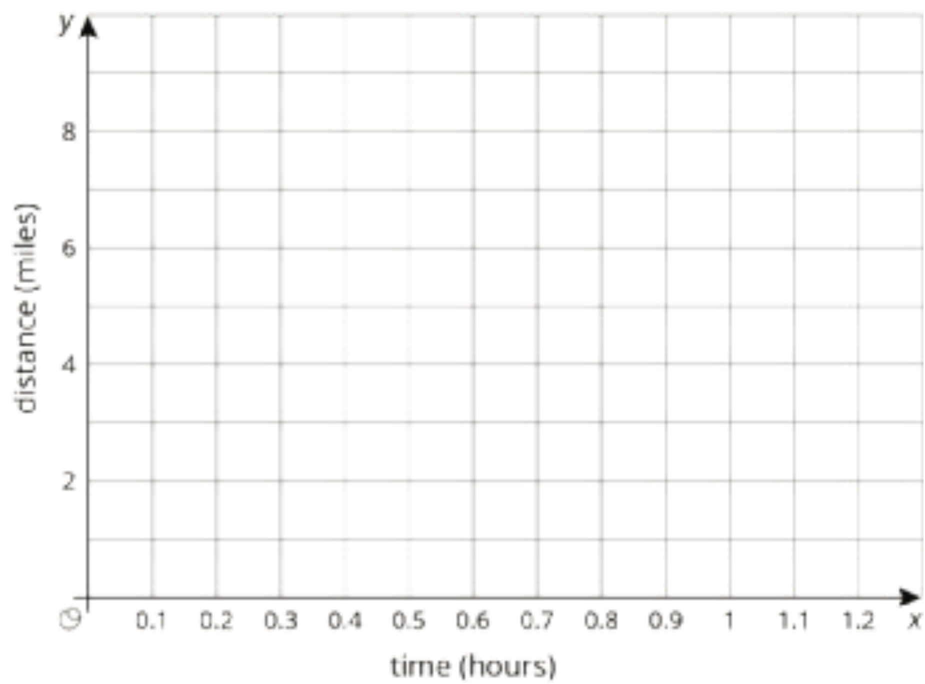
    a. Write an equation relating the two variables.

    b. Rearrange the equation so b is the independent variable.

    c. Rearrange the equation so g is the independent variable.

(From Unit 5, Lesson 3.)

5. Lin's mom bikes at a constant speed of 12 miles per hour. Lin walks at a constant speed $\frac{1}{3}$ of the speed her mom bikes. Sketch a graph of both of these relationships.



(From Unit 3, Lesson 1.)

Lesson 5: Negative Exponents with Powers of 10

Let’s see what happens when exponents are negative.

5.1: Number Talk: What's That Exponent?

Solve each equation mentally.

$\frac{100}{1} = 10^x$

$\frac{100}{x} = 10^1$

$\frac{x}{100} = 10^0$

$\frac{100}{1{,}000} = 10^x$

5.2: Negative Exponent Table

Complete the table to explore what negative exponents mean.

	$\cdot 10$	$\cdot 10$	$\cdot 10$	$\cdot 10$	$\cdot 10$	$\cdot 10$	
using exponents	10^3	10^2	10^1				
as a decimal	1000.0			1.0		0.01	
as a fraction		$\frac{100}{1}$		$\frac{1}{1}$			$\frac{1}{1000}$
	$\cdot ?$	$\cdot ?$	$\cdot ?$	$\cdot ?$	$\cdot ?$	$\cdot ?$	

1. As you move toward the left, each number is being multiplied by 10. What is the multiplier as you move right?
2. How does a multiplier of 10 affect the placement of the decimal in the product? How does the other multiplier affect the placement of the decimal in the product?
3. Use the patterns you found in the table to write 10^{-7} as a fraction.
4. Use the patterns you found in the table to write 10^{-5} as a decimal.
5. Write $\frac{1}{100,000,000}$ using a single exponent.
6. Use the patterns in the table to write 10^{-n} as a fraction.

5.3: Follow the Exponent Rules

1. a. Match each exponential expression with an equivalent multiplication expression:

$(10^2)^3$

$(10^2)^{-3}$

$(10^{-2})^3$

$(10^{-2})^{-3}$

$\frac{1}{(10 \cdot 10)} \cdot \frac{1}{(10 \cdot 10)} \cdot \frac{1}{(10 \cdot 10)}$
$\left(\frac{1}{10} \cdot \frac{1}{10}\right)\left(\frac{1}{10} \cdot \frac{1}{10}\right)\left(\frac{1}{10} \cdot \frac{1}{10}\right)$
$\frac{1}{\frac{1}{10} \cdot \frac{1}{10}} \cdot \frac{1}{\frac{1}{10} \cdot \frac{1}{10}} \cdot \frac{1}{\frac{1}{10} \cdot \frac{1}{10}}$
$(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)$

b. Write $(10^2)^{-3}$ as a power of 10 with a single exponent. Be prepared to explain your reasoning.

2. a. Match each exponential expression with an equivalent multiplication expression:

$\frac{10^2}{10^5}$

$\frac{10^2}{10^{-5}}$

$\frac{10^{-2}}{10^5}$

$\frac{10^{-2}}{10^{-5}}$

$\frac{\frac{1}{10} \cdot \frac{1}{10}}{\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}}$
$\frac{10 \cdot 10}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$
$\frac{\frac{1}{10} \cdot \frac{1}{10}}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}$
$\frac{10 \cdot 10}{\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}}$

b. Write $\frac{10^{-2}}{10^{-5}}$ as a power of 10 with a single exponent. Be prepared to explain your reasoning.

3. a. Match each exponential expression with an equivalent multiplication expression:

$10^4 \cdot 10^3$

$10^4 \cdot 10^{-3}$

$10^{-4} \cdot 10^3$

$10^{-4} \cdot 10^{-3}$

$(10 \cdot 10 \cdot 10 \cdot 10) \cdot (\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10})$
$\left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right) \cdot \left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right)$
$\left(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}\right) \cdot (10 \cdot 10 \cdot 10)$
$(10 \cdot 10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10)$

b. Write $10^{-4} \cdot 10^3$ as a power of 10 with a single exponent. Be prepared to explain your reasoning.

Are you ready for more?

Priya, Jada, Han, and Diego stand in a circle and take turns playing a game.

Priya says, SAFE. Jada, standing to Priya's left, says, OUT and leaves the circle. Han is next: he says, SAFE. Then Diego says, OUT and leaves the circle. At this point, only Priya and Han are left. They continue to alternate. Priya says, SAFE. Han says, OUT and leaves the circle. Priya is the only person left, so she is the winner.

Priya says, “I knew I’d be the only one left, since I went first.”

1. Record this game on paper a few times with different numbers of players. Does the person who starts always win?

2. Try to find as many numbers as you can where the person who starts always wins. What patterns do you notice?

Lesson 5 Summary

When we multiply a positive power of 10 by $\frac{1}{10}$, the exponent *decreases* by 1:

$$10^8 \cdot \frac{1}{10} = 10^7$$

This is true for *any* positive power of 10. We can reason in a similar way that multiplying by 2 factors that are $\frac{1}{10}$ *decreases* the exponent by 2:

$$\left(\frac{1}{10}\right)^2 \cdot 10^8 = 10^6$$

That means we can extend the rules to use negative exponents if we make $10^{-2} = \left(\frac{1}{10}\right)^2$. Just as 10^2 is two factors that are 10, we have that 10^{-2} is two factors that are $\frac{1}{10}$. More generally, the exponent rules we have developed are true for *any* integers n and m if we make

$$10^{-n} = \left(\frac{1}{10}\right)^n = \frac{1}{10^n}$$

Here is an example of extending the rule $\frac{10^n}{10^m} = 10^{n-m}$ to use negative exponents:

$$\frac{10^3}{10^5} = 10^{3-5} = 10^{-2}$$

To see why, notice that

$$\frac{10^3}{10^5} = \frac{10^3}{10^3 \cdot 10^2} = \frac{10^3}{10^3} \cdot \frac{1}{10^2} = \frac{1}{10^2}$$

which is equal to 10^{-2}.

Here is an example of extending the rule $(10^m)^n = 10^{m \cdot n}$ to use negative exponents:

$$(10^{-2})^3 = 10^{(-2)(3)} = 10^{-6}$$

To see why, notice that $10^{-2} = \frac{1}{10} \cdot \frac{1}{10}$. This means that

$$(10^{-2})^3 = \left(\frac{1}{10} \cdot \frac{1}{10}\right)^3 = \left(\frac{1}{10} \cdot \frac{1}{10}\right) \cdot \left(\frac{1}{10} \cdot \frac{1}{10}\right) \cdot \left(\frac{1}{10} \cdot \frac{1}{10}\right) = \frac{1}{10^6} = 10^{-6}$$

Unit 7 Lesson 5 Cumulative Practice Problems

1. Write with a single exponent: (ex: $\frac{1}{10} \cdot \frac{1}{10} = 10^{-2}$)

   a. $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$

   b. $\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10}$

   c. $(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10})^2$

   d. $(\frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10})^3$

   e. $(10 \cdot 10 \cdot 10)^{-2}$

2. Write each expression as a single power of 10.

   a. $10^{-3} \cdot 10^{-2}$

   b. $10^4 \cdot 10^{-1}$

   c. $\frac{10^5}{10^7}$

   d. $(10^{-4})^5$

   e. $10^{-3} \cdot 10^2$

   f. $\frac{10^{-9}}{10^5}$

3. Select **all** of the following that are equivalent to $\frac{1}{10{,}000}$:

   A. $(10{,}000)^{-1}$

   B. $(-10{,}000)$

   C. $(100)^{-2}$

   D. $(10)^{-4}$

   E. $(-10)^2$

4. Match each equation to the situation it describes. Explain what the constant of proportionality means in each equation.

Equations:

a. $y = 3x$

b. $\frac{1}{2}x = y$

c. $y = 3.5x$

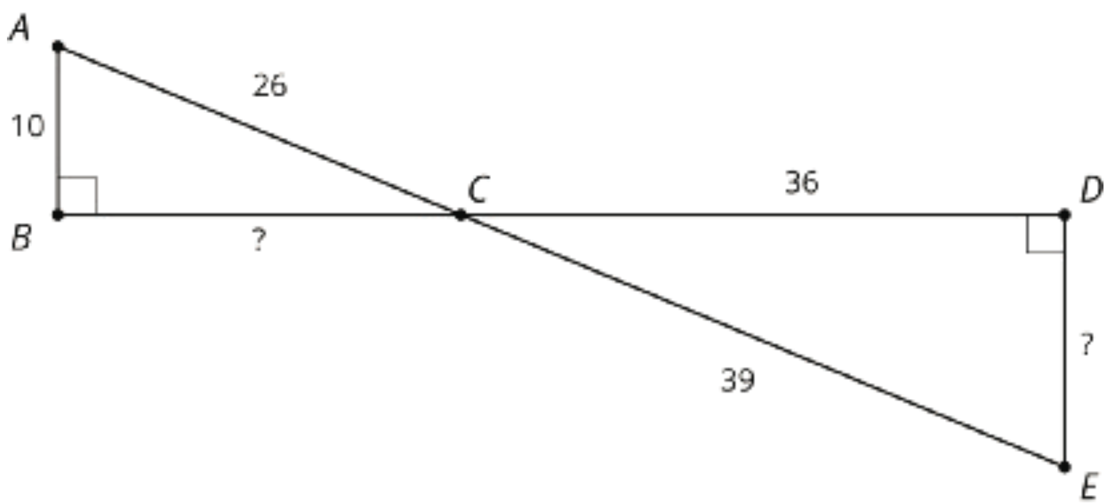
d. $y = \frac{5}{2}x$

Situations:

- A dump truck is hauling loads of dirt to a construction site. After 20 loads, there are 70 square feet of dirt.
- I am making a water and salt mixture that has 2 cups of salt for every 6 cups of water.
- A store has a “4 for $10” sale on hats.
- For every 48 cookies I bake, my students get 24.

(From Unit 3, Lesson 2.)

5. a. Explain why triangle ABC is similar to EDC.



b. Find the missing side lengths.

(From Unit 2, Lesson 8.)

Lesson 6: What about Other Bases?

Let's explore exponent patterns with bases other than 10.

6.1: True or False: Comparing Expressions with Exponents

Is each statement true or false? Be prepared to explain your reasoning.

1. $3^5 < 4^6$
2. $(-3)^2 < 3^2$
3. $(-3)^3 = 3^3$
4. $(-5)^2 > -5^2$

6.2: What Happens with Zero and Negative Exponents?

Complete the table to show what it means to have an exponent of zero or a negative exponent.

value	16					$\frac{1}{2}$			
exponent form	2^4								

1. As you move toward the left, each number is being multiplied by 2. What is the multiplier as you move toward the right?
2. Use the patterns you found in the table to write 2^{-6} as a fraction.
3. Write $\frac{1}{32}$ as a power of 2 with a single exponent.
4. What is the value of 2^0?
5. From the work you have done with negative exponents, how would you write 5^{-3} as a fraction?
6. How would you write 3^{-4} as a fraction?

Are you ready for more?

1. Find an expression equivalent to $\left(\frac{2}{3}\right)^{-3}$ but with positive exponents.

2. Find an expression equivalent to $\left(\frac{4}{5}\right)^{-8}$ but with positive exponents.

3. What patterns do you notice when you start with a fraction raised to a negative exponent and rewrite it using a single positive exponent? Show or explain your reasoning.

6.3: Exponent Rules with Bases Other than 10

Lin, Noah, Diego, and Elena decide to test each other's knowledge of exponents with bases other than 10. They each chose an expression to start with and then came up with a new list of expressions; some of which are equivalent to the original and some of which are not.

Choose 2 of the 4 lists to analyze. For each list of expressions you choose to analyze, decide which expressions are *not* equivalent to the original. Be prepared to explain your reasoning.

1. Lin's original expression is 5^{-9} and her list is:
   $(5^3)^{-3}$ $\quad$ -5^9 $\quad$ $\frac{5^{-6}}{5^3}$ $\quad$ $(5^3)^{-2}$ $\quad$ $\frac{5^{-4}}{5^{-5}}$ $\quad$ $5^{-4} \cdot 5^{-5}$

2. Noah's original expression is 3^{10} and his list is:
   $3^5 \cdot 3^2$ $\quad$ $(3^5)^2$ $\quad$ $(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)$
   $\left(\frac{1}{3}\right)^{-10}$ $\quad$ $3^7 \cdot 3^3$ $\quad$ $\frac{3^{20}}{3^{10}}$ $\quad$ $\frac{3^{20}}{3^2}$

3. Diego's original expression is x^4 and his list is:

$\frac{x^8}{x^4}$ $\quad$ $x \cdot x \cdot x \cdot x$ $\quad$ $\frac{x^{-4}}{x^{-8}}$ $\quad$ $\frac{x^{-4}}{x^8}$

$(x^2)^2$ $\quad$ $4 \cdot x$ $\quad$ $x \cdot x^3$

4. Elena's original expression is 8^0 and her list is:

1 $\quad$ 0 $\quad$ $8^3 \cdot 8^{-3}$

$\frac{8^2}{8^2}$ $\quad$ 10^0 $\quad$ 11^0

Lesson 6 Summary

Earlier we focused on powers of 10 because 10 plays a special role in the decimal number system. But the exponent rules that we developed for 10 also work for other bases. For example, if $2^0 = 1$ and $2^{-n} = \frac{1}{2^n}$, then

$$2^m \cdot 2^n = 2^{m+n}$$
$$(2^m)^n = 2^{m \cdot n}$$
$$\frac{2^m}{2^n} = 2^{m-n}.$$

These rules also work for powers of numbers less than 1. For example, $\left(\frac{1}{3}\right)^2 = \frac{1}{3} \cdot \frac{1}{3}$ and $\left(\frac{1}{3}\right)^4 = \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$. We can also check that $\left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^4 = \left(\frac{1}{3}\right)^{2+4}$.

Using a variable x helps to see this structure. Since $x^2 \cdot x^5 = x^7$ (both sides have 7 factors that are x), if we let $x = 4$, we can see that $4^2 \cdot 4^5 = 4^7$. Similarly, we could let $x = \frac{2}{3}$ or $x = 11$ or any other positive value and show that these relationships still hold.

Unit 7 Lesson 6 Cumulative Practice Problems

1. Priya says "I can figure out 5^0 by looking at other powers of 5. 5^3 is 125, 5^2 is 25, then 5^1 is 5."

   a. What pattern do you notice?

   b. If this pattern continues, what should be the value of 5^0? Explain how you know.

   c. If this pattern continues, what should be the value of 5^{-1}? Explain how you know.

2. Select **all** the expressions that are equivalent to 4^{-3}.

   A. -12

   B. 2^{-6}

   C. $\frac{1}{4^3}$

   D. $\left(\frac{1}{4}\right) \cdot \left(\frac{1}{4}\right) \cdot \left(\frac{1}{4}\right)$

   E. 12

   F. $(-4) \cdot (-4) \cdot (-4)$

   G. $\frac{8^{-1}}{2^2}$

3. Write each expression using a single exponent.

   a. $\frac{5^3}{5^6}$

   b. $(14^3)^6$

   c. $8^3 \cdot 8^6$

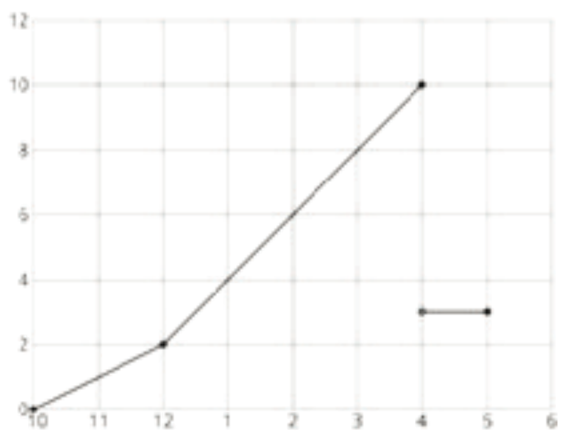
   d. $\frac{16^6}{16^3}$

   e. $(21^3)^{-6}$

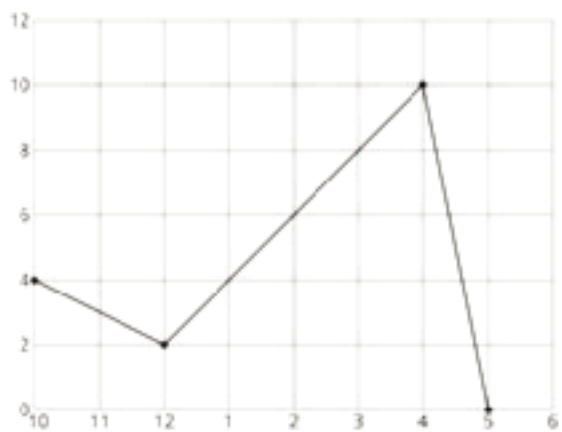
4. Andre sets up a rain gauge to measure rainfall in his back yard. On Tuesday, it rains off and on all day.

    - He starts at 10 a.m. with an empty gauge when it starts to rain.
    - Two hours later, he checks, and the gauge has 2 cm of water in it.
    - It starts raining even harder, and at 4 p.m., the rain stops, so Andre checks the rain gauge and finds it has 10 cm of water in it.
    - While checking it, he accidentally knocks the rain gauge over and spills most of the water, leaving only 3 cm of water in the rain gauge.
    - When he checks for the last time at 5 p.m., there is no change.

Graph A



Graph B



a. Which of the two graphs could represent Andre’s story? Explain your reasoning.

b. Label the axes of the correct graph with appropriate units.

c. Use the graph to determine how much total rain fell on Tuesday.

(From Unit 5, Lesson 6.)

Lesson 7: Practice with Rational Bases

Let's practice with exponents.

7.1: Which One Doesn't Belong: Exponents

Which expression doesn't belong?

$\frac{2^8}{2^5}$

$\left(\frac{3}{4}\right)^{-5} \cdot \left(\frac{3}{4}\right)^{8}$

$\left(4^{-5}\right)^{8}$

$\frac{10^8}{5^5}$

7.2: Exponent Rule Practice

1. Choose 6 of the equations to write using a single exponent:
    - $7^5 \cdot 7^6$
    - $3^{-3} \cdot 3^8$
    - $2^{-4} \cdot 2^{-3}$
    - $\left(\frac{5}{6}\right)^4 \left(\frac{5}{6}\right)^5$
    - $\frac{3^5}{3^{28}}$
    - $\frac{2^{-5}}{2^4}$
    - $\frac{6^5}{6^{-8}}$
    - $\frac{10^{-12}}{10^{-20}}$
    - $\left(7^2\right)^3$
    - $\left(4^3\right)^{-3}$
    - $\left(2^{-8}\right)^{-4}$
    - $\left(6^{-3}\right)^5$

2. Which problems did you want to skip in the previous question? Explain your thinking.

3. Choose 3 of the following to write using a single, *positive* exponent:

- 2^{-7}
- 4^{-9}
- 3^{-23}
- 2^{-32}
- 11^{-8}
- 8^{-3}

4. Choose 3 of the following to evaluate:

- $\frac{10^5}{10^5}$
- $\left(\frac{5}{4}\right)^2$
- $\left(\frac{2}{3}\right)^3$
- $\left(3^4\right)^0$
- $2^8 \cdot 2^{-8}$
- $\left(\frac{7}{2}\right)^2$

7.3: Inconsistent Bases

Mark each equation as true or false. What could you change about the false equations to make them true?

1. $\left(\frac{1}{3}\right)^2 \cdot \left(\frac{1}{3}\right)^4 = \left(\frac{1}{3}\right)^6$

2. $3^2 \cdot 5^3 = 15^5$

3. $5^4 + 5^5 = 5^9$

4. $\left(\frac{1}{2}\right)^4 \cdot 10^3 = 5^7$

5. $3^2 \cdot 5^2 = 15^2$

Are you ready for more?

Solve this equation: $3^{x-5} = 9^{x+4}$. Explain or show your reasoning.

Lesson 7 Summary

In the past few lessons, we found rules to more easily keep track of repeated factors when using exponents. We also extended these rules to make sense of negative exponents as repeated factors of the **reciprocal** of the base, as well as defining a number to the power of 0 to have a value of 1. These rules can be written symbolically as:

$$x^n \cdot x^m = x^{n+m},$$

$$(x^n)^m = x^{n \cdot m},$$

$$\frac{x^n}{x^m} = x^{n-m},$$

$$x^{-n} = \frac{1}{x^n},$$

and

$$x^0 = 1,$$

where the base x can be any positive number. In this lesson, we practiced using these exponent rules for different bases and exponents.

Unit 7 Lesson 7 Cumulative Practice Problems

1. Write with a single exponent:

   a. $\frac{7^6}{7^2}$

   b. $(11^4)^5$

   c. $4^2 \cdot 4^6$

   d. $6 \cdot 6^8$

   e. $(12^2)^7$

   f. $\frac{3^{10}}{3}$

   g. $(0.173)^9 \cdot (0.173)^2$

   h. $\frac{0.87^5}{0.87^3}$

   i. $\frac{(\frac{5}{2})^8}{(\frac{5}{2})^6}$

2. Noah says that $2^4 \cdot 3^2 = 6^6$. Tyler says that $2^4 \cdot 4^2 = 16^2$.

   a. Do you agree with Noah? Explain or show your reasoning.

   b. Do you agree with Tyler? Explain or show your reasoning.

Lesson 8: Combining Bases

Let's multiply expressions with different bases.

8.1: Same Exponent, Different Base

1. Evaluate $5^3 \cdot 2^3$
2. Evaluate 10^3

8.2: Power of Products

1. The table contains products of expressions with different bases and the same exponent. Complete the table to see how we can rewrite them. Use the "expanded" column to work out how to combine the factors into a new base.

expression	expanded	exponent
$5^3 \cdot 2^3$	$(5 \cdot 5 \cdot 5) \cdot (2 \cdot 2 \cdot 2) = (5 \cdot 2)(5 \cdot 2)(5 \cdot 2)$ $= 10 \cdot 10 \cdot 10$	10^3
$3^2 \cdot 7^2$		21^2
$2^4 \cdot 3^4$		
		15^3
		30^4
$2^4 \cdot x^4$		
$a^n \cdot b^n$		
$7^4 \cdot 2^4 \cdot 5^4$		

2. Can you write $2^3 \cdot 3^4$ with a single exponent? What happens if neither the exponents nor the bases are the same? Explain or show your reasoning.

8.3: How Many Ways Can You Make 3,600?

Your teacher will give your group tools for creating a visual display to play a game. Divide the display into 3 columns, with these headers:

$a^n \cdot a^m = a^{n+m}$ $\frac{a^n}{a^m} = a^{n-m}$ $a^n \cdot b^n = (a \cdot b)^n$

How to play:

When the time starts, you and your group will write as many expressions as you can that equal a specific number using one of the exponent rules on your board. When the time is up, compare your expressions with another group to see how many points you earn.

- Your group gets 1 point for every *unique* expression you write that is equal to the number and follows the exponent rule you claimed.
- If an expression uses negative exponents, you get 2 points instead of just 1.
- You can challenge the other group's expression if you think it is not equal to the number or if it does not follow one of the three exponent rules.

Are you ready for more?

You have probably noticed that when you square an odd number, you get another odd number, and when you square an even number, you get another even number. Here is a way to expand the concept of odd and even for the number 3. Every integer is either divisible by 3, one *more* than a multiple of 3, or one *less* than a multiple of 3.

1. Examples of numbers that are one more than a multiple of 3 are 4, 7, and 25. Give three more examples.
2. Examples of numbers that are one less than a multiple of 3 are 2, 5, and 32. Give three more examples.
3. Do you think it's true that when you square a number that is a multiple of 3, your answer will still be a multiple of 3? How about for the other two categories? Try squaring some numbers to check your guesses.

Lesson 8 Summary

Before this lesson, we made rules for multiplying and dividing expressions with exponents that only work when the expressions have the *same* base. For example,

$$10^3 \cdot 10^2 = 10^5$$

or

$$2^6 \div 2^2 = 2^4$$

In this lesson, we studied how to combine expressions with the same exponent, but *different* bases. For example, we can write $2^3 \cdot 5^3$ as $2 \cdot 2 \cdot 2 \cdot 5 \cdot 5 \cdot 5$. Regrouping this as $(2 \cdot 5) \cdot (2 \cdot 5) \cdot (2 \cdot 5)$ shows that

$$\begin{aligned} 2^3 \cdot 5^3 &= (2 \cdot 5)^3 \\ &= 10^3 \end{aligned}$$

Notice that the 2 and 5 in the previous example could be replaced with different numbers or even variables. For example, if a and b are variables then $a^3 \cdot b^3 = (a \cdot b)^3$. More generally, for a positive number n,

$$a^n \cdot b^n = (a \cdot b)^n$$

because both sides have exactly n factors that are a and n factors that are b.

Unit 7 Lesson 8 Cumulative Practice Problems

1. Select **all** the true statements:

   A. $2^8 \cdot 2^9 = 2^{17}$

   B. $8^2 \cdot 9^2 = 72^2$

   C. $8^2 \cdot 9^2 = 72^4$

   D. $2^8 \cdot 2^9 = 4^{17}$

2. Find x, y, and z if $(3 \cdot 5)^4 \cdot (2 \cdot 3)^5 \cdot (2 \cdot 5)^7 = 2^x \cdot 3^y \cdot 5^z$.

3. Han found a way to compute complicated expressions more easily. Since $2 \cdot 5 = 10$, he looks for pairings of 2s and 5s that he knows equal 10. For example, $3 \cdot 2^4 \cdot 5^5 = 3 \cdot 2^4 \cdot 5^4 \cdot 5 = (3 \cdot 5) \cdot (2 \cdot 5)^4 = 15 \cdot 10^4 = 150{,}000$. Use Han's technique to compute the following:

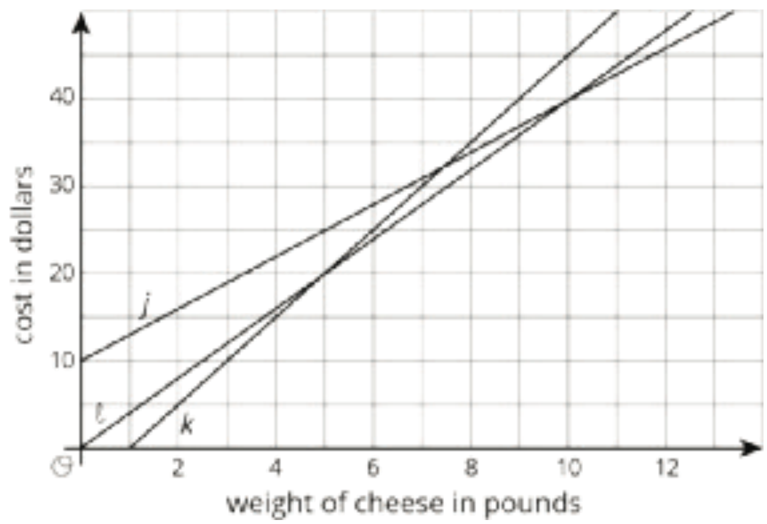
   a. $2^4 \cdot 5 \cdot (3 \cdot 5)^3$

   b. $\frac{2^3 \cdot 5^2 \cdot (2 \cdot 3)^2 \cdot (3 \cdot 5)^2}{3^2}$

4. The cost of cheese at three stores is a function of the weight of the cheese. The cheese is not prepackaged, so a customer can buy any amount of cheese.

   - Store A sells the cheese for a dollars per pound.
   - Store B sells the same cheese for b dollars per pound and a customer has a coupon for $5 off the total purchase at that store.
   - Store C is an online store, selling the same cheese at c dollar per pound, but with a $10 delivery fee.

   This graph shows the price functions for stores A, B, and C.



   a. Match Stores A, B, and C with Graphs j, k, and ℓ.

   b. How much does each store charge for the cheese per pound?

   c. How many pounds of cheese does the coupon for Store B pay for?

   d. Which store has the lowest price for a half a pound of cheese?

   e. If a customer wants to buy 5 pounds of cheese for a party, which store has the lowest price?

   f. How many pounds would a customer need to order to make Store C a good option?

(From Unit 5, Lesson 8.)

Lesson 9: Describing Large and Small Numbers Using Powers of 10

Let's find out how to use powers of 10 to write large or small numbers.

9.1: Thousand Million Billion Trillion

1. Match each expression with its corresponding value and word.

expression
10^{-3}
10^{6}
10^{9}
10^{-2}
10^{12}
10^{3}

value
1,000,000,000,000
$\frac{1}{100}$
1,000
1,000,000,000
1,000,000
$\frac{1}{1,000}$

word
billion
milli-
million
thousand
centi-
trillion

2. For each of the numbers, think of something in the world that is described by that number.

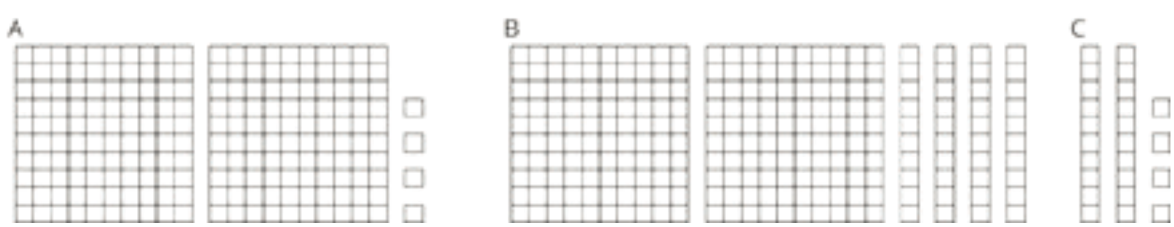
9.2: Base-ten Representations Matching

1. Match each expression to one or more diagrams that could represent it. For each match, explain what the value of a single small square would have to be.
    a. $2 \cdot 10^{-1} + 4 \cdot 10^{-2}$

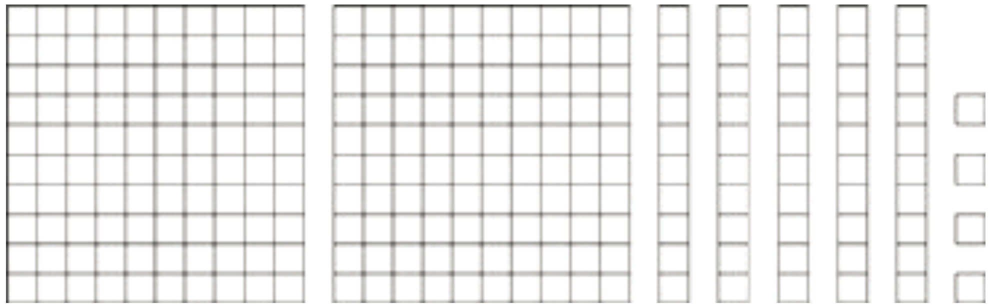
    b. $2 \cdot 10^{-1} + 4 \cdot 10^{-3}$

    c. $2 \cdot 10^{3} + 4 \cdot 10^{1}$

    d. $2 \cdot 10^{3} + 4 \cdot 10^{2}$



2. a. Write an expression to describe the base-ten diagram if each small square represents 10^{-4}. What is the value of this expression?



b. How does changing the value of the small square change the value of the expression? Explain or show your thinking.

c. Select at least two different powers of 10 for the small square, and write the corresponding expressions to describe the base-ten diagram. What is the value of each of your expressions?

9.3: Using Powers of 10 to Describe Large and Small Numbers

Your teacher will give you a card that tells you whether you are Partner A or B and gives you the information that is missing from your partner's statements. Do not show your card to your partner.

Read each statement assigned to you, ask your partner for the missing information, and write the number your partner tells you.

Partner A's statements:

1. Around the world, about ____________________ pencils are made each year.
2. The mass of a proton is ____________________ kilograms.
3. The population of Russia is about ____________________ people.
4. The diameter of a bacteria cell is about ____________________ meter.

Partner B's statements:

1. Light waves travel through space at a speed of ____________________ meters per second.
2. The population of India is about ____________________ people.
3. The wavelength of a gamma ray is _____________________ meters.
4. The tardigrade (water bear) is ____________ meters long.

Are you ready for more?

A "googol" is a name for a really big number: a 1 followed by 100 zeros.

1. If you square a googol, how many zeros will the answer have? Show your reasoning.

2. If you raise a googol to the googol power, how many zeros will the answer have? Show your reasoning.

Lesson 9 Summary

Sometimes powers of 10 are helpful for expressing quantities, especially very large or very small quantities. For example, the United States Mint has made over

500,000,000,000

pennies. In order to understand this number, we have to count all the zeros. Since there are 11 of them, this means there are 500 billion pennies. Using powers of 10, we can write this as:

$$500 \cdot 10^{9}$$

(five hundred times a billion), or even as:

$$5 \cdot 10^{11}$$

The advantage to using powers of 10 to write a large number is that they help us see right away how large the number is by looking at the exponent.

The same is true for small quantities. For example, a single atom of carbon weighs about

0.0000000000000000000000199

grams. We can write this using powers of 10 as

$$199 \cdot 10^{-25}$$

or, equivalently,

$$(1.99) \cdot 10^{-23}$$

Not only do powers of 10 make it easier to write this number, but they also help avoid errors since it would be very easy to write an extra zero or leave one out when writing out the decimal because there are so many to keep track of!

Unit 7 Lesson 9 Cumulative Practice Problems

1. Match each number to its name.

a. 1,000,000	○ One hundredth
b. 0.01	○ One thousandth
c. 1,000,000,000	○ One millionth
d. 0.000001	○ Ten thousand
e. 0.001	○ One million
f. 10,000	○ One billion

2. Write each expression as a multiple of a power of 10:

    a. 42,300

    b. 2,000

    c. 9,200,000

    d. Four thousand

    e. 80 million

    f. 32 billion

3. Each statement contains a quantity. Rewrite each quantity using a power of 10.

    a. There are about 37 trillion cells in an average human body.

    b. The Milky Way contains about 300 billion stars.

    c. A sharp knife is 23 millionths of a meter thick at its tip.

    d. The wall of a certain cell in the human body is 4 nanometers thick. (A nanometer is one billionth of a meter.)

4. A fully inflated basketball has a radius of 12 cm. Your basketball is only inflated halfway. How many more cubic centimeters of air does your ball need to fully inflate? Express your answer in terms of π. Then estimate how many cubic centimeters this is by using 3.14 to approximate π.

(From Unit 5, Lesson 20.)

5. Solve each of these equations. Explain or show your reasoning.

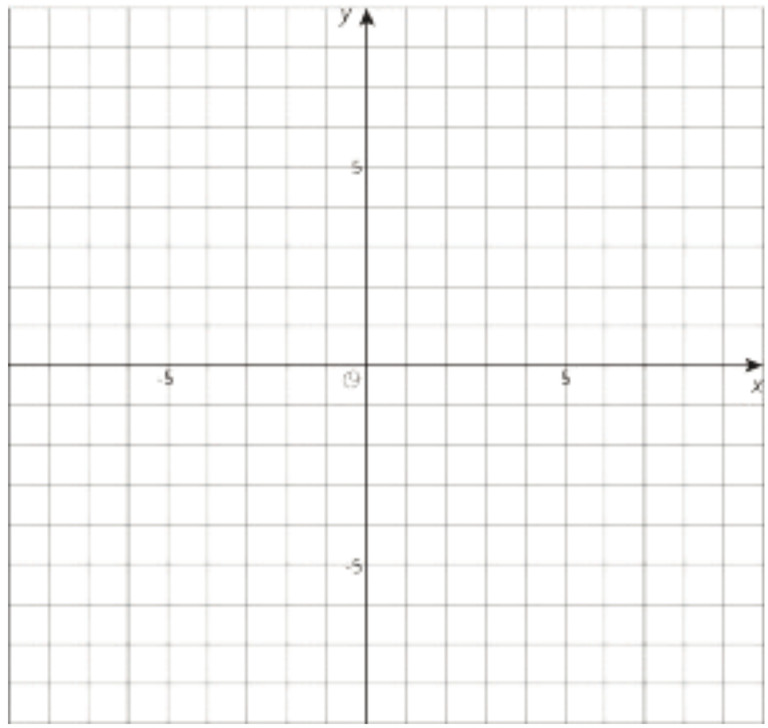
$2(3 - 2c) = 30$

$3x - 2 = 7 - 6x$

$31 = 5(b - 2)$

(From Unit 4, Lesson 5.)

6. Graph the line going through $(-6, 1)$ with a slope of $\frac{-2}{3}$ and write its equation.



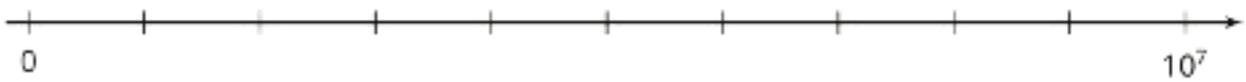
(From Unit 3, Lesson 10.)

Lesson 10: Representing Large Numbers on the Number Line

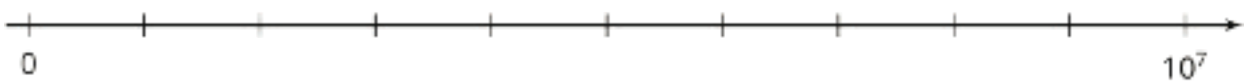
Let's visualize large numbers on the number line using powers of 10.

10.1: Labeling Tick Marks on a Number Line

Label the tick marks on the number line. Be prepared to explain your reasoning.



10.2: Comparing Large Numbers with a Number Line



1. Place the numbers on the number line. Be prepared to explain your reasoning.

    a. 4,000,000
    b. $5 \cdot 10^6$
    c. $5 \cdot 10^5$
    d. $75 \cdot 10^5$
    e. $(0.6) \cdot 10^7$

2. Trade number lines with a partner, and check each other's work. How did your partner decide how to place the numbers? If you disagree about a placement, work to reach an agreement.

3. Which is larger, 4,000,000 or $75 \cdot 10^5$? Estimate how many times larger.

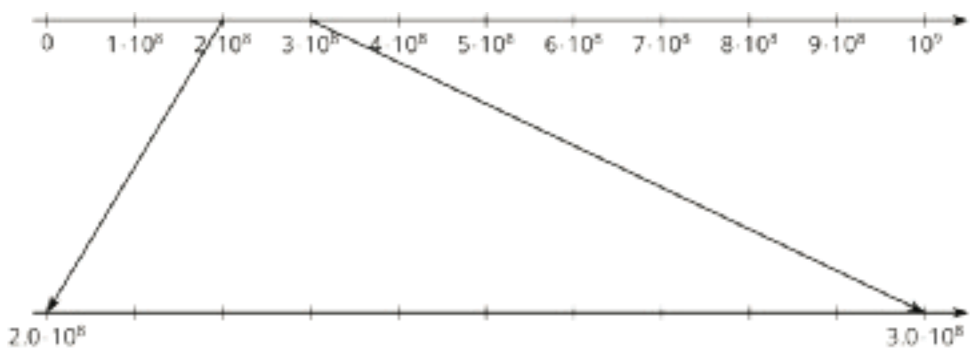
10.3: The Speeds of Light

The table shows how fast light waves or electricity can travel through different materials.

material	speed (meters per second)
space	300,000,000
water	$(2.25) \cdot 10^8$
copper wire (electricity)	280,000,000
diamond	$124 \cdot 10^6$
ice	$(2.3) \cdot 10^8$
olive oil	200,000,000

1. Which is faster, light through diamond or light through ice? How can you tell from the expressions for speed?

   Let's zoom in to highlight the values between $(2.0) \cdot 10^8$ and $(3.0) \cdot 10^8$.



2. Label the tick marks between $(2.0) \cdot 10^8$ and $(3.0) \cdot 10^8$.
3. Plot a point for each speed on both number lines, and label it with the corresponding material.
4. There is one speed that you cannot plot on the bottom number line. Which is it? Plot it on the top number line instead.
5. Which is faster, light through ice or light through diamond? How can you tell from the number line?

Are you ready for more?

Find a four-digit number using only the digits 0, 1, 2, or 3 where:

- the first digit tells you how many zeros are in the number,
- the second digit tells you how many ones are in the number,
- the third digit tells you how many twos are in the number, and
- the fourth digit tells you how many threes are in the number.

The number 2,100 is close, but doesn't quite work. The first digit is 2, and there are 2 zeros. The second digit is 1, and there is 1 one. The fourth digit is 0, and there are no threes. But the third digit, which is supposed to count the number of 2's, is zero.

1. Can you find more than one number like this?

2. How many solutions are there to this problem? Explain or show your reasoning.

Lesson 10 Summary

There are many ways to compare two quantities. Suppose we want to compare the world population, about 7.4 billion

to the number of pennies the U.S. made in 2015, about

8,900,000,000

There are many ways to do this. We could write 7.4 billion as a decimal, 7,400,000,000, and then we can tell that there were more pennies made in 2015 than there are people in the world! Or we could use powers of 10 to write these numbers:

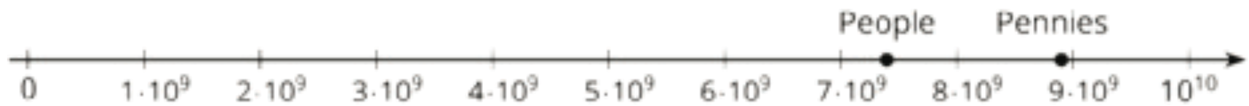
$$7.4 \cdot 10^9$$

for people in the world and

$$8.9 \cdot 10^9$$

for the number of pennies.

For a visual representation, we could plot these two numbers on a number line. We need to carefully choose our end points to make sure that the numbers can both be plotted. Since they both lie between 10^9 and 10^{10}, if we make a number line with tick marks that increase by one billion, or 10^9, we start the number line with 0 and end it with $10 \cdot 10^9$, or 10^{10}. Here is a number line with the number of pennies and world population plotted:



Unit 7 Lesson 10 Cumulative Practice Problems

1. Find three different ways to write the number 437,000 using powers of 10.

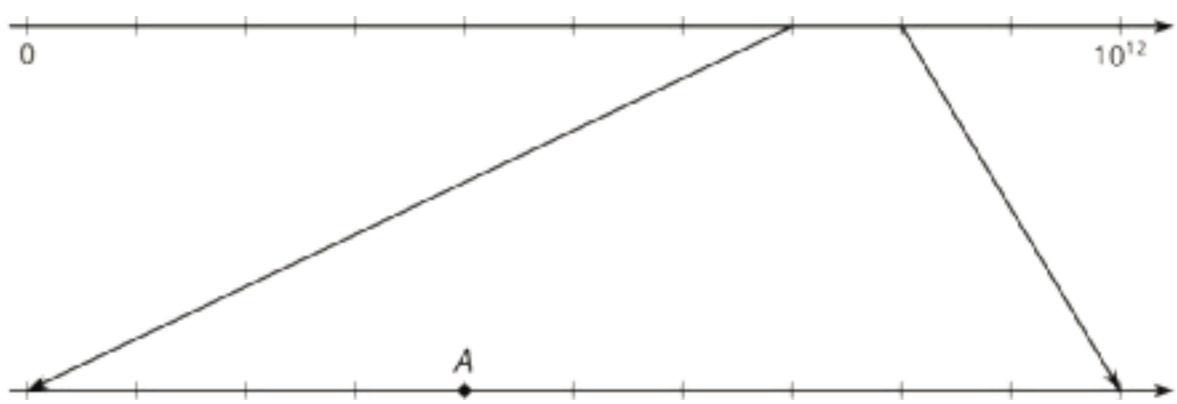
2. For each pair of numbers below, circle the number that is greater. Estimate how many times greater.

   a. $17 \cdot 10^8$ or $4 \cdot 10^8$

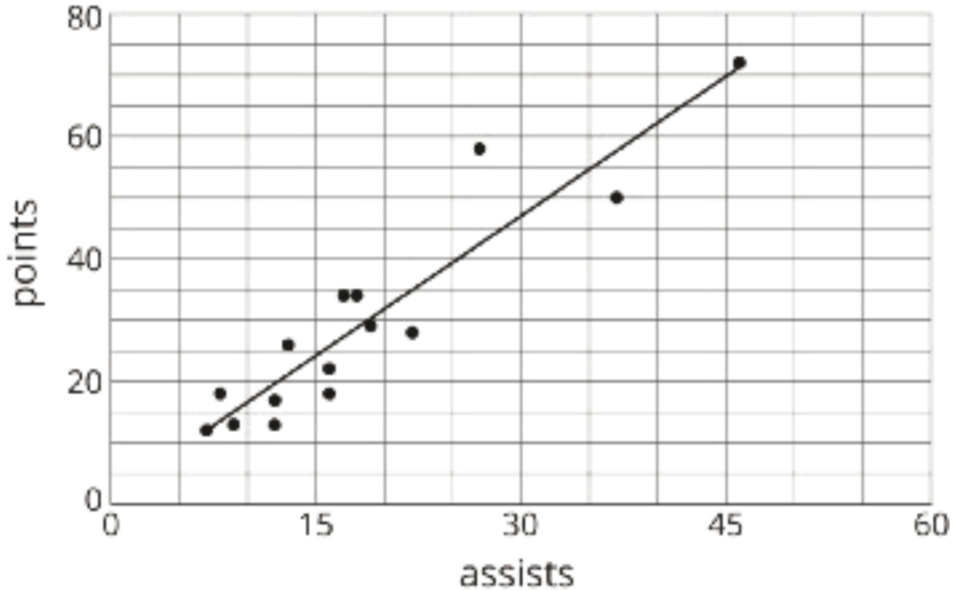
   b. $2 \cdot 10^6$ or $7.839 \cdot 10^6$

   c. $42 \cdot 10^7$ or $8.5 \cdot 10^8$

3. What number is represented by point A? Explain or show how you know.



4. Here is a scatter plot that shows the number of points and assists by a set of hockey players. Select **all** the following that describe the association in the scatter plot:



A. Linear association

B. Non-linear association

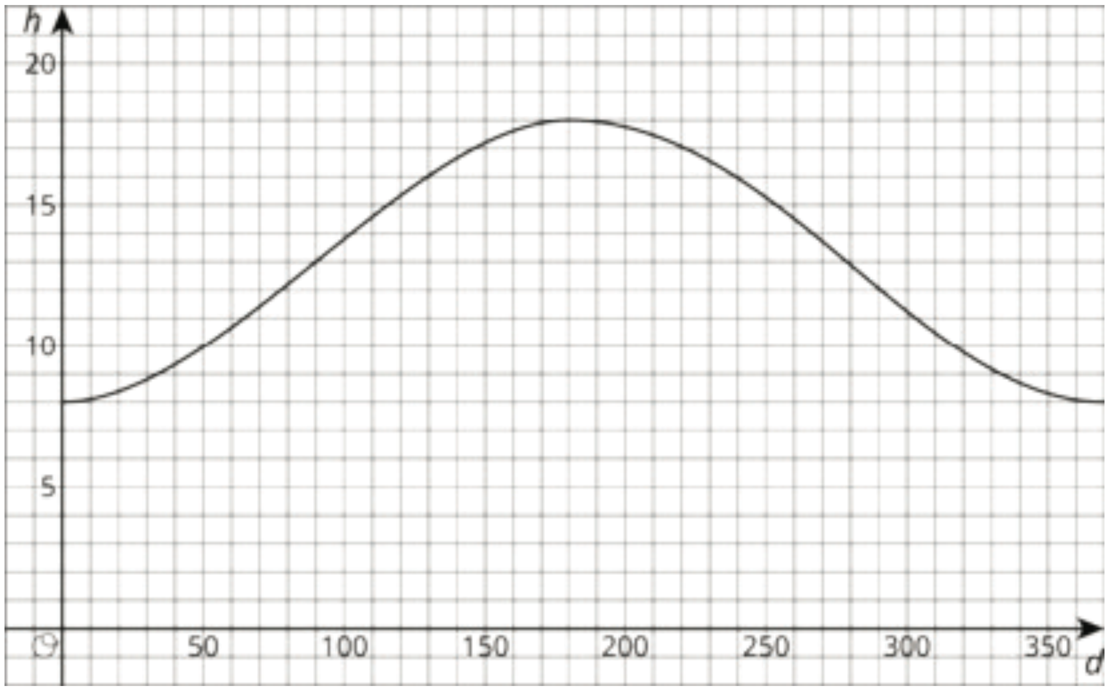
C. Positive association

D. Negative association

E. No association

(From Unit 6, Lesson 7.)

5. Here is the graph of days and the predicted number of hours of sunlight, h, on the d-th day of the year.



a. Is hours of sunlight a function of days of the year? Explain how you know.

b. For what days of the year is the number of hours of sunlight increasing? For what days of the year is the number of hours of sunlight decreasing?

c. Which day of the year has the greatest number of hours of sunlight?

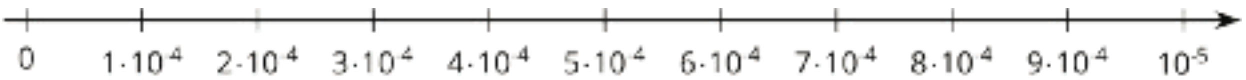
(From Unit 5, Lesson 5.)

Lesson 11: Representing Small Numbers on the Number Line

Let's visualize small numbers on the number line using powers of 10.

11.1: Small Numbers on a Number Line

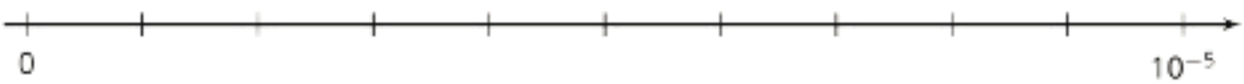
Kiran drew this number line.



Andre said, "That doesn't look right to me."

Explain why Kiran is correct or explain how he can fix the number line.

11.2: Comparing Small Numbers on a Number Line



1. Label the tick marks on the number line.
2. Plot the following numbers on the number line:

   A. $6 \cdot 10^{-6}$ B. $6 \cdot 10^{-7}$ C. $29 \cdot 10^{-7}$ D. $(0.7) \cdot 10^{-5}$

3. Which is larger, $29 \cdot 10^{-7}$ or $6 \cdot 10^{-6}$? Estimate how many times larger.
4. Which is larger, $7 \cdot 10^{-8}$ or $3 \cdot 10^{-9}$? Estimate how many times larger.

11.3: Atomic Scale

1. The radius of an electron is about 0.0000000000003 cm.

    a. Write this number as a multiple of a power of 10.

    b. Decide what power of 10 to put on the right side of this number line and label it.

    c. Label each tick mark as a multiple of a power of 10.

    d. Plot the radius of the electron in centimeters on the number line.

2. The mass of a proton is about 0.0000000000000000000000017 grams.

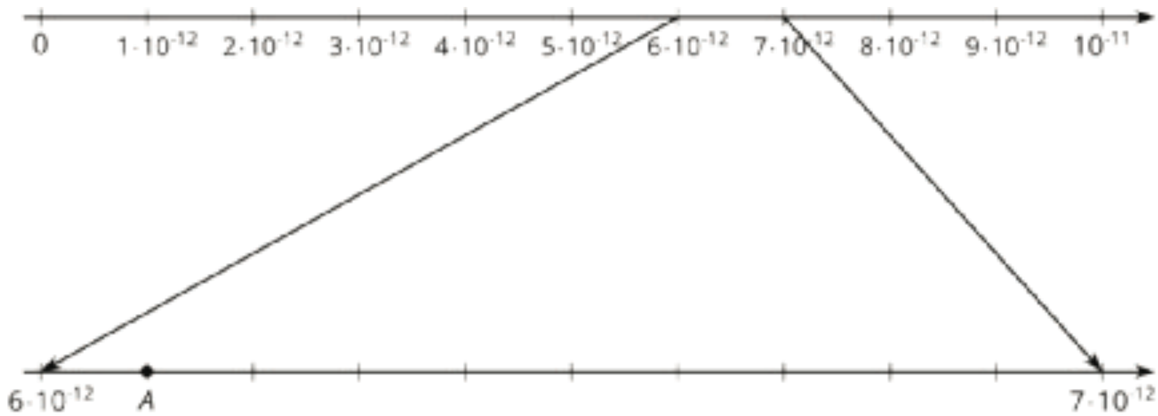
    a. Write this number as a multiple of a power of 10.

    b. Decide what power of 10 to put on the right side of this number line and label it.

    c. Label each tick mark as a multiple of a power of 10.

    d. Plot the mass of the proton in grams on the number line.

3. Point A on the zoomed-in number line describes the wavelength of a certain X-ray in meters.



a. Write the wavelength of the X-ray as a multiple of a power of 10.

b. Write the wavelength of the X-ray as a decimal.

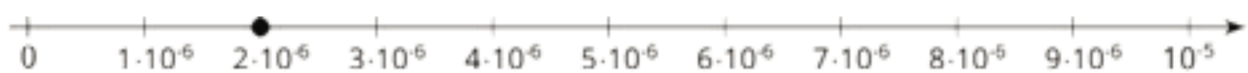
Lesson 11 Summary

The width of a bacterium cell is about

$$2 \cdot 10^{-6}$$

meters. If we want to plot this on a number line, we need to find which two powers of 10 it lies between. We can see that $2 \cdot 10^{-6}$ is a multiple of 10^{-6}. So our number line will be labeled with multiples of

$$10^{-6}$$



Note that the right side is labeled

$$10 \cdot 10^{-6} = 10^{-5}$$

The power of ten on the right side of the number line is always *greater* than the power on the left. This is true for powers with positive or negative exponents.

Unit 7 Lesson 11 Cumulative Practice Problems

1. Select **all** the expressions that are equal to $4 \cdot 10^{-3}$:

   A. $4 \cdot (\frac{1}{10}) \cdot (\frac{1}{10}) \cdot (\frac{1}{10})$

   B. $4 \cdot (-10) \cdot (-10) \cdot (-10)$

   C. $4 \cdot 0.001$

   D. $4 \cdot 0.0001$

   E. 0.004

   F. 0.0004

2. Write each expression as a multiple of a power of 10:

   a. 0.04

   b. 0.072

   c. 0.0000325

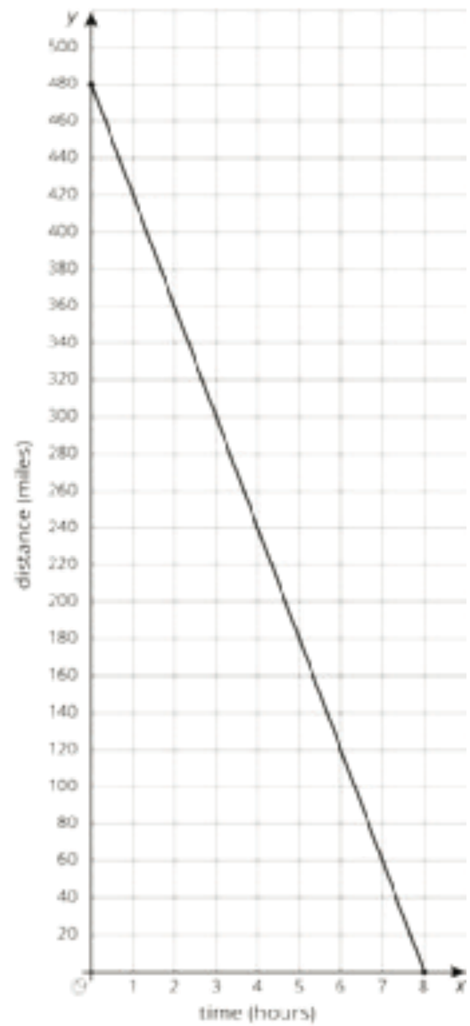
   d. Three thousandths

   e. 23 hundredths

   f. 729 thousandths

   g. 41 millionths

3. A family sets out on a road trip to visit their cousins. They travel at a steady rate. The graph shows the distance remaining to their cousins' house for each hour of the trip.



   a. How fast are they traveling?

   b. Is the slope positive or negative? Explain how you know and why that fits the situation.

   c. How far is the trip and how long did it take? Explain how you know.

(From Unit 3, Lesson 9.)

Lesson 12: Applications of Arithmetic with Powers of 10

Let's use powers of 10 to help us make calculations with large and small numbers.

12.1: What Information Do You Need?

What information would you need to answer these questions?

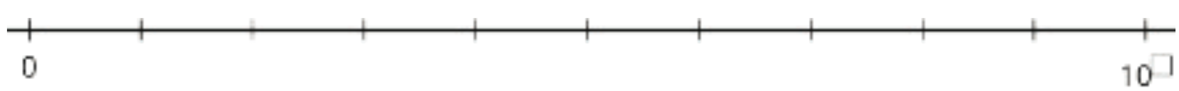
1. How many meter sticks does it take to equal the mass of the Moon?

2. If all of these meter sticks were lined up end to end, would they reach the Moon?

12.2: Meter Sticks to the Moon

1. How many meter sticks does it take to equal the mass of the Moon? Explain or show your reasoning.

2. Label the number line and plot your answer for the number of meter sticks.



3. If you took all the meter sticks from the last question and lined them up end to end, will they reach the Moon? Will they reach beyond the Moon? If yes, how many times farther will they reach? Explain your reasoning.

4. One light year is approximately 10^{16} meters. How many light years away would the meter sticks reach? Label the number line and plot your answer.

Are you ready for more?

Here is a problem that will take multiple steps to solve. You may not know all the facts you need to solve the problem. That is okay. Take a guess at reasonable answers to anything you don’t know. Your final answer will be an estimate.

If everyone alive on Earth right now stood very close together, how much area would they take up?

12.3: That’s a Tall Stack of Cash

In 2016, the Burj Khalifa was the tallest building in the world. It was very expensive to build.

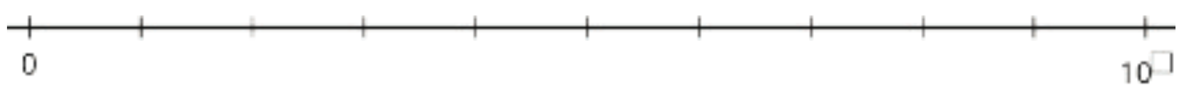
Consider the question: Which is taller, the Burj Khalifa or a stack of the money it cost to build the Burj Khalifa?

1. What information would you need to be able to solve the problem?

2. Record the information your teacher shares with the class.

3. Answer the question “Which is taller, the Burj Khalifa or a stack of the money it cost to build the Burj Khalifa?” and explain or show your reasoning.

4. Decide what power of 10 to use to label the rightmost tick mark of the number line, and plot the height of the stack of money and the height of the Burj Khalifa.



5. Which has more mass, the Burj Khalifa or the mass of the pennies it cost to build the Burj Khalifa? What information do you need to answer this?

6. Decide what power of 10 to use to label the rightmost tick mark of the number line, and plot the mass of the Burj Khalifa and the mass of the pennies it cost to build the Burj Khalifa.

Lesson 12 Summary

Powers of 10 can be helpful for making calculations with large or small numbers. For example, in 2014, the United States had

318,586,495

people who used the equivalent of

2,203,799,778,107

kilograms of oil in energy. The amount of energy per person is the total energy divided by the total number of people. We can use powers of 10 to estimate the total energy as

$$2 \cdot 10^{12}$$

and the population as

$$3 \cdot 10^{8}$$

So the amount of energy per person in the U.S. is roughly

$$(2 \cdot 10^{12}) \div (3 \cdot 10^{8})$$

That is the equivalent of

$$\frac{2}{3} \cdot 10^{4}$$

kilograms of oil in energy. That's a lot of energy—the equivalent of almost 7,000 kilograms of oil per person!

In general, when we want to perform arithmetic with very large or small quantities, estimating with powers of 10 and using exponent rules can help simplify the process. If we wanted to find the exact quotient of 2,203,799,778,107 by 318,586,495, then using powers of 10 would not simplify the calculation.

Unit 7 Lesson 12 Cumulative Practice Problems

1. Which is larger: the number of meters across the Milky Way, or the number of cells in all humans? Explain or show your reasoning.

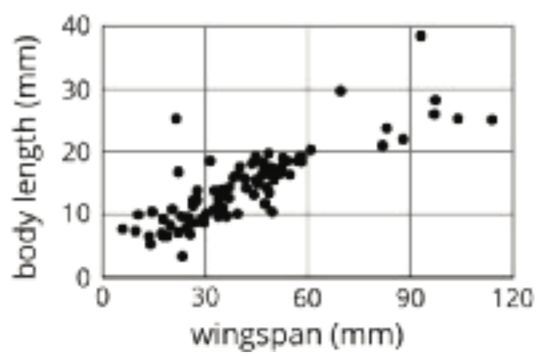
   Some useful information:

   - The Milky Way is about 100,000 light years across.
   - There are about 37 trillion cells in a human body.
   - One light year is about 10^{16} meters.
   - The world population is about 7 billion.

2. Ecologists measure the body length and wingspan of 127 butterfly specimens caught in a single field.

   a. Draw a line that you think is a good fit for the data.

   b. Write an equation for the line.



   c. What does the slope of the line tell you about the wingspans and lengths of these butterflies?

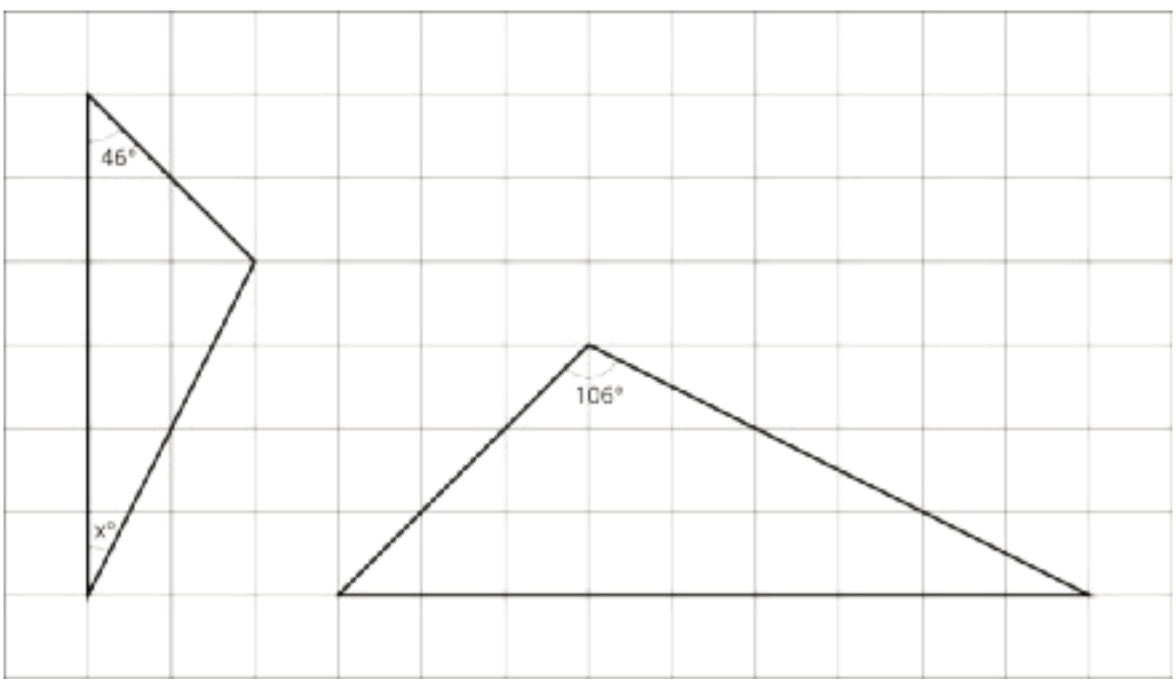
(From Unit 6, Lesson 5.)

3. Diego was solving an equation, but when he checked his answer, he saw his solution was incorrect. He knows he made a mistake, but he can't find it. Where is Diego's mistake and what is the solution to the equation?

$$\begin{aligned} -4(7-2x) &= 3(x+4) \\ -28-8x &= 3x+12 \\ -28 &= 11x+12 \\ -40 &= 11x \\ -\frac{40}{11} &= x \end{aligned}$$

(From Unit 4, Lesson 5.)

4. The two triangles are similar. Find x.



(From Unit 2, Lesson 7.)

Lesson 13: Definition of Scientific Notation

Let's use scientific notation to describe large and small numbers.

13.1: Number Talk: Multiplying by Powers of 10

Find the value of each expression mentally.

$123 \cdot 10{,}000$

$(3.4) \cdot 1{,}000$

$(0.6) \cdot 100$

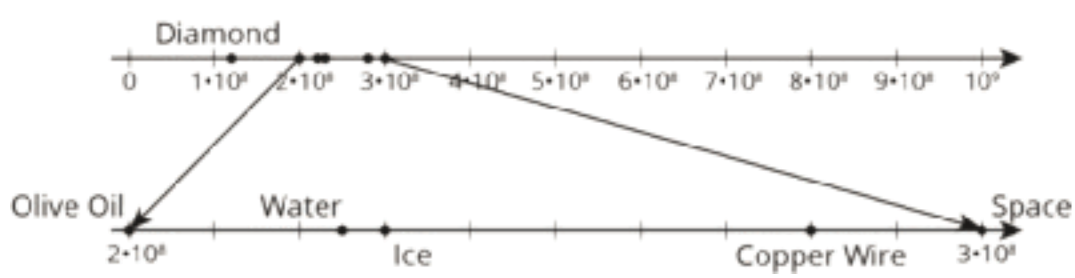
$(7.3) \cdot (0.01)$

13.2: The "Science" of Scientific Notation

The table shows the speed of light or electricity through different materials.

material	speed (meters per second)
space	300,000,000
water	2.25×10^8
copper (electricity)	280,000,000
diamond	124×10^6
ice	2.3×10^8
olive oil	0.2×10^9

Circle the speeds that are written in scientific notation. Write the others using scientific notation.



13.3: Scientific Notation Matching

Your teacher will give you and your partner a set of cards. Some of the cards show numbers in scientific notation, and other cards show numbers that are not in scientific notation.

1. Shuffle the cards and lay them facedown.
2. Players take turns trying to match cards with the same value.
3. On your turn, choose two cards to turn faceup for everyone to see. Then:
    a. If the two cards have the same value *and* one of them is written in scientific notation, whoever says "Science!" first gets to keep the cards, and it becomes that player's turn. If it's already your turn when you call "Science!", that means you get to go again. If you say "Science!" when the cards do not match or one is not in scientific notation, then your opponent gets a point.
    b. If both partners agree the two cards have the same value, then remove them from the board and keep them. You get a point for each card you keep.
    c. If the two cards do not have the same value, then set them facedown in the same position and end your turn.
4. If it is not your turn:
    a. If the two cards have the same value *and* one of them is written in scientific notation, then whoever says "Science!" first gets to keep the cards, and it becomes that player's turn. If you call "Science!" when the cards do not match or one is not in scientific notation, then your opponent gets a point.
    b. Make sure both of you agree the cards have the same value.
    If you disagree, work to reach an agreement.
5. Whoever has the most points at the end wins.

Are you ready for more?

1. What is $9 \times 10^{-1} + 9 \times 10^{-2}$? Express your answer as:
   a. A decimal

   b. A fraction

2. What is $9 \times 10^{-1} + 9 \times 10^{-2} + 9 \times 10^{-3} + 9 \times 10^{-4}$? Express your answer as:
   a. A decimal

   b. A fraction

3. The answers to the two previous questions should have been close to 1. What power of 10 would you have to go up to if you wanted your answer to be so close to 1 that it was only $\frac{1}{1,000,000}$ off?

4. What power of 10 would you have to go up to if you wanted your answer to be so close to 1 that it was only $\frac{1}{1,000,000,000}$ off? Can you keep adding numbers in this pattern to get as close to 1 as you want? Explain or show your reasoning.

5. Imagine a number line that goes from your current position (labeled 0) to the door of the room you are in (labeled 1). In order to get to the door, you will have to pass the points 0.9, 0.99, 0.999, etc. The Greek philosopher Zeno argued that you will never be able to go through the door, because you will first have to pass through an infinite number of points. What do you think? How would you reply to Zeno?

Lesson 13 Summary

The total value of all the quarters made in 2014 is 400 million dollars. There are many ways to express this using powers of 10. We could write this as $400 \cdot 10^6$ dollars, $40 \cdot 10^7$ dollars, $0.4 \cdot 10^9$ dollars, or many other ways. One special way to write this quantity is called **scientific notation**. In scientific notation,

400 million

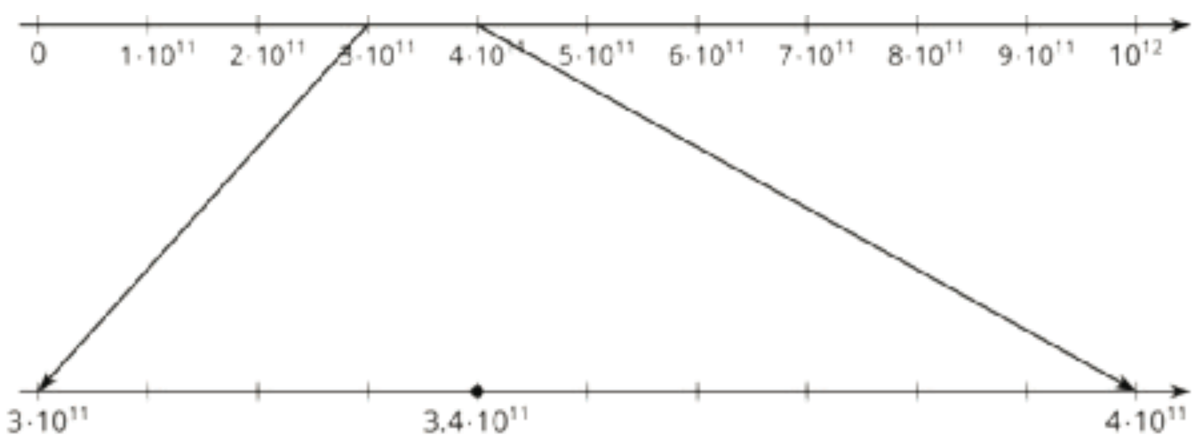
dollars would be written as

$$4 \times 10^8$$

dollars. For scientific notation, the $\times$ symbol is the standard way to show multiplication instead of the $\cdot$ symbol. Writing the number this way shows exactly where it lies between two consecutive powers of 10. The 10^8 shows us the number is between 10^8 and 10^9. The 4 shows us that the number is 4 tenths of the way to 10^9.

Some other examples of scientific notation are 1.2×10^{-8}, 9.99×10^{16}, and 7×10^{12}. The first factor is a number greater than or equal to 1, but less than 10. The second factor is an integer power of 10.

Thinking back to how we plotted these large (or small) numbers on a number line, scientific notation tells us which powers of 10 to place on the left and right of the number line. For example, if we want to plot 3.4×10^{11} on a number line, we know that the number is larger than 10^{11}, but smaller than 10^{12}. We can find this number by zooming in on the number line:



Unit 7 Lesson 13 Cumulative Practice Problems

1. Write each number in scientific notation.

    a. 14,700

    b. 0.00083

    c. 760,000,000

    d. 0.038

    e. 0.38

    f. 3.8

    g. 3,800,000,000,000

    h. 0.0000000009

2. Perform the following calculations. Express your answers in scientific notation.

    a. $(2 \times 10^5) + (6 \times 10^5)$

    b. $(4.1 \times 10^7) \cdot 2$

    c. $(1.5 \times 10^{11}) \cdot 3$

    d. $(3 \times 10^3)^2$

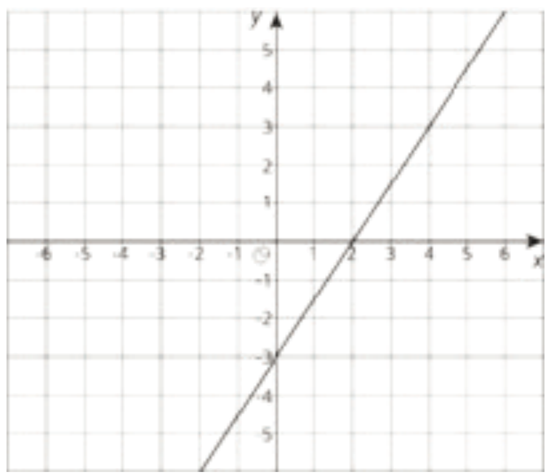
    e. $(9 \times 10^6) \cdot (3 \times 10^6)$

3. Jada is making a scale model of the solar system. The distance from Earth to the Moon is about 2.389×10^5 miles. The distance from Earth to the Sun is about 9.296×10^7 miles. She decides to put Earth on one corner of her dresser and the Moon on another corner, about a foot away. Where should she put the sun?

   - On a windowsill in the same room?
   - In her kitchen, which is down the hallway?
   - A city block away?

   Explain your reasoning.

4. Here is the graph for one equation in a system of equations.



   a. Write a second equation for the system so it has infinitely many solutions.

   b. Write a second equation whose graph goes through $(0, 2)$ so that the system has no solutions.

   c. Write a second equation whose graph goes through $(2, 2)$ so that the system has one solution at $(4, 3)$.

(From Unit 4, Lesson 12.)

Lesson 14: Multiplying, Dividing, and Estimating with Scientific Notation

Let's multiply and divide with scientific notation to answer questions about animals, careers, and planets.

14.1: True or False: Equations

Is each equation true or false? Explain your reasoning.

1. $4 \times 10^5 \times 4 \times 10^4 = 4 \times 10^{20}$

2. $\frac{7 \times 10^6}{2 \times 10^4} = (7 \div 2) \times 10^{(6-4)}$

3. $8.4 \times 10^3 \times 2 = (8.4 \times 2) \times 10^{(3 \times 2)}$

14.2: Biomass

Use the table to answer questions about different creatures on the planet. Be prepared to explain your reasoning.

creature	number	mass of one individual (kg)
humans	7.5×10^9	6.2×10^1
cows	1.3×10^9	4×10^2
sheep	1.75×10^9	6×10^1
chickens	2.4×10^{10}	2×10^0
ants	5×10^{16}	3×10^{-6}
blue whales	4.7×10^3	1.9×10^5
Antarctic krill	7.8×10^{14}	4.86×10^{-4}
zooplankton	1×10^{20}	5×10^{-8}
bacteria	5×10^{30}	1×10^{-12}

1. Which creature is least numerous? Estimate how many times more ants there are.

2. Which creature is the least massive? Estimate how many times more massive a human is.

3. Which is more massive, the total mass of all the humans or the total mass of all the ants? About how many times more massive is it?

4. Which is more massive, the total mass of all the krill or the total mass of all the blue whales? About how many times more massive is it?

14.3: Info Gap: Distances in the Solar System

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to answer the question.

2. Ask your partner for the specific information that you need.

3. Explain to your partner how you are using the information to solve the problem.

4. Solve the problem and explain your reasoning to your partner.

If your teacher gives you the *data card*:

1. Silently read the information on your card.

2. Ask your partner "What specific information do you need?" and wait for your partner to *ask* for information. *Only* give information that is on your card. (Do not figure out anything for your partner!)

3. Before telling your partner the information, ask "Why do you need that information?"

4. After your partner solves the problem, ask them to explain their reasoning and listen to their explanation.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

14.4: Professions in the United States

Use the table to answer questions about professions in the United States as of 2012.

profession	number	typical annual salary (U.S. dollars)
architect	1.074×10^5	7.3×10^4
artist	5.14×10^4	4.4×10^4
programmer	1.36×10^6	8.85×10^4
doctor	6.9×10^5	1.87×10^5
engineer	6.17×10^5	8.6×10^4
firefighter	3.07×10^5	4.5×10^4
military—enlisted	1.16×10^6	4.38×10^4
military—officer	2.5×10^5	1×10^5
nurse	3.45×10^6	6.03×10^4
police officer	7.8×10^5	5.7×10^4
college professor	1.27×10^6	6.9×10^4
retail sales	4.67×10^6	2.14×10^4
truck driver	1.7×10^6	3.82×10^4

Answer the following questions about professions in the United States. Express each answer in scientific notation.

1. Estimate how many times more nurses there are than doctors.

2. Estimate how much money all doctors make put together.

3. Estimate how much money all police officers make put together.

4. Who makes more money, all enlisted military put together or all military officers put together? Estimate how many times more.

Lesson 14 Summary

Multiplying numbers in scientific notation extends what we do when we multiply regular decimal numbers. For example, one way to find $(80)(60)$ is to view 80 as 8 tens and to view 60 as 6 tens. The product $(80)(60)$ is 48 hundreds or 4,800. Using scientific notation, we can write this calculation as

$$(8 \times 10^1)(6 \times 10^1) = 48 \times 10^2.$$

To express the product in scientific notation, we would rewrite it as 4.8×10^3.

Calculating using scientific notation is especially useful when dealing with very large or very small numbers. For example, there are about 39 million or 3.9×10^7 residents in California. Each Californian uses about 180 gallons of water a day. To find how many gallons of water Californians use in a day, we can find the product $(180)(3.9 \times 10^7) = 702 \times 10^7$, which is equal to 7.02×10^9. That's about 7 billion gallons of water each day!

Comparing very large or very small numbers by estimation also becomes easier with scientific notation. For example, how many ants are there for every human? There are 5×10^{16} ants and 7×10^9 humans. To find the number of ants per human, look at $\frac{5 \times 10^{16}}{7 \times 10^9}$. Rewriting the numerator to have the number 50 instead of 5, we get $\frac{50 \times 10^{15}}{7 \times 10^9}$. This gives us $\frac{50}{7} \times 10^6$. Since $\frac{50}{7}$ is roughly equal to 7, there are about 7×10^6 or 7 million ants per person!

Unit 7 Lesson 14 Cumulative Practice Problems

1. Evaluate each expression. Use scientific notation to express your answer.

   a. $(1.5 \times 10^2)(5 \times 10^{10})$

   b. $\dfrac{4.8 \times 10^{-8}}{3 \times 10^{-3}}$

   c. $(5 \times 10^8)(4 \times 10^3)$

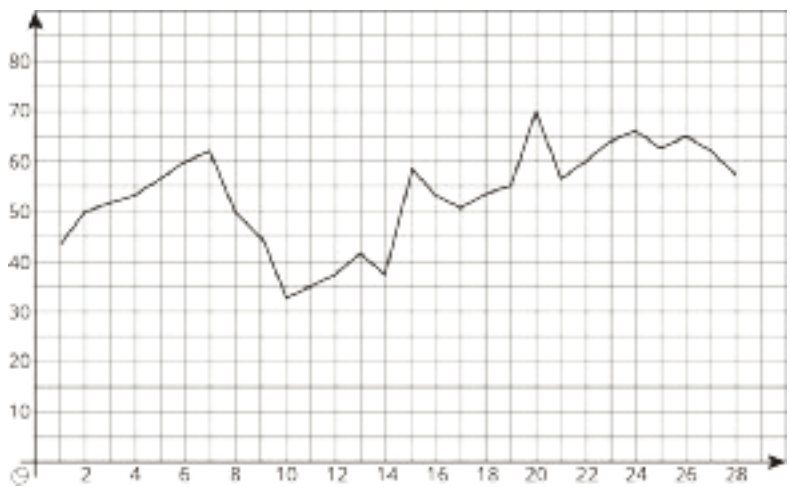
   d. $(7.2 \times 10^3) \div (1.2 \times 10^5)$

2. How many bucketloads would it take to bucket out the world's oceans? Write your answer in scientific notation.

   Some useful information:

   - The world's oceans hold roughly 1.4×10^9 cubic kilometers of water.
   - A typical bucket holds roughly 20,000 cubic centimeters of water.
   - There are 10^{15} cubic centimeters in a cubic kilometer.

3. The graph represents the closing price per share of stock for a company each day for 28 days.



   a. What variable is represented on the horizontal axis?

   b. In the first week, was the stock price generally increasing or decreasing?

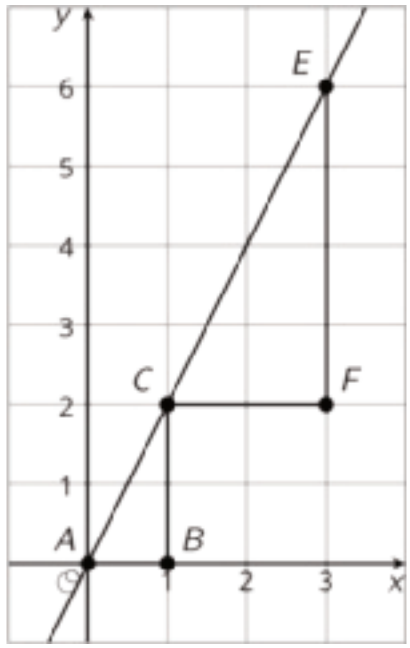
   c. During which period did the closing price of the stock decrease for at least 3 days in a row?

(From Unit 5, Lesson 5.)

4. Write an equation for the line that passes through $(-8.5, 11)$ and $(5, -2.5)$.

(From Unit 3, Lesson 11.)

5. Explain why triangle ABC is similar to triangle CFE.



(From Unit 2, Lesson 6.)

Lesson 15: Adding and Subtracting with Scientific Notation

Let's add and subtract using scientific notation to answer questions about animals and the solar system.

15.1: Number Talk: Non-zero Digits

Mentally decide how many non-zero digits each number will have.

$(3 \times 10^9)(2 \times 10^7)$

$(3 \times 10^9) \div (2 \times 10^7)$

$3 \times 10^9 + 2 \times 10^7$

$3 \times 10^9 - 2 \times 10^7$

15.2: Measuring the Planets

Diego, Kiran, and Clare were wondering:

"If Neptune and Saturn were side by side, would they be wider than Jupiter?"

1. They try to add the diameters, 4.7×10^4 km and 1.2×10^5 km. Here are the ways they approached the problem. Do you agree with any of them? Explain your reasoning.
    a. Diego says, "When we add the distances, we will get $4.7 + 1.2 = 5.9$. The exponent will be 9. So the two planets are 5.9×10^9 km side by side."

    b. Kiran wrote 4.7×10^4 as 47,000 and 1.2×10^5 as 120,000 and added them:

    $$\begin{array}{r} 120{,}000 \\ +47{,}000 \\ \hline 167{,}000 \end{array}$$

    c. Clare says, "I think you can't add unless they are the same power of 10." She adds 4.7×10^4 km and 12×10^4 to get 16.7×10^4.

2. Jupiter has a diameter of 1.43×10^5. Which is wider, Neptune and Saturn put side by side, or Jupiter?

15.3: A Celestial Dance

object	diameter (km)	distance from the Sun (km)
Sun	1.392×10^6	0×10^0
Mercury	4.878×10^3	5.79×10^7
Venus	1.21×10^4	1.08×10^8
Earth	1.28×10^4	1.47×10^8
Mars	6.785×10^3	2.28×10^8
Jupiter	1.428×10^5	7.79×10^8

1. When you add the distances of Mercury, Venus, Earth, and Mars from the Sun, would you reach as far as Jupiter?

2. Add all the diameters of all the planets except the Sun. Which is wider, all of these objects side by side, or the Sun? Draw a picture that is close to scale.

Are you ready for more?

The emcee at a carnival is ready to give away a cash prize! The winning contestant could win anywhere from \$1 to \$100. The emcee only has 7 envelopes and she wants to make sure she distributes the 100 \$1 bills among the 7 envelopes so that no matter what the contestant wins, she can pay the winner with the envelopes without redistributing the bills. For example, it's possible to divide 6 \$1 bills among 3 envelopes to get any amount from \$1 to \$6 by putting \$1 in the first envelope, \$2 in the second envelope, and \$3 in the third envelope (Go ahead and check. Can you make \$4? \$5? \$6?).

How should the emcee divide up the 100 \$1 bills among the 7 envelopes so that she can give away any amount of money, from \$1 to \$100, just by handing out the right envelopes?

15.4: Old McDonald's Massive Farm

Use the table to answer questions about different life forms on the planet.

creature	number	mass of one individual (kg)
humans	7.5×10^9	6.2×10^1
cows	1.3×10^9	4×10^2
sheep	1.75×10^9	6×10^1
chickens	2.4×10^{10}	2×10^0
ants	5×10^{16}	3×10^{-6}
blue whales	4.7×10^3	1.9×10^5
antarctic krill	7.8×10^{14}	4.86×10^{-4}
zooplankton	1×10^{20}	5×10^{-8}
bacteria	5×10^{30}	1×10^{-12}

1. On a farm there was a cow. And on the farm there were 2 sheep. There were also 3 chickens. What is the total mass of the 1 cow, the 2 sheep, the 3 chickens, and the 1 farmer on the farm?

2. Make a conjecture about how many ants might be on the farm. If you added all these ants into the previous question, how would that affect your answer for the total mass of all the animals?

3. What is the total mass of a human, a blue whale, and 6 ants all together?

4. Which is greater, the number of bacteria, or the number of all the other animals in the table put together?

Lesson 15 Summary

When we add decimal numbers, we need to pay close attention to place value. For example, when we calculate $13.25 + 6.7$, we need to make sure to add hundredths to hundredths (5 and 0), tenths to tenths (2 and 7), ones to ones (3 and 6), and tens to tens (1 and 0). The result is 19.95.

We need to take the same care when we add or subtract numbers in scientific notation. For example, suppose we want to find how much further Earth is from the Sun than Mercury. Earth is about 1.5×10^8 km from the Sun, while Mercury is about 5.8×10^7 km. In order to find

$$1.5 \times 10^8 - 5.8 \times 10^7$$

we can rewrite this as

$$1.5 \times 10^8 - 0.58 \times 10^8$$

Now that both numbers are written in terms of 10^8, we can subtract 0.58 from 1.5 to find

$$0.92 \times 10^8$$

Rewriting this in scientific notation, Earth is

$$9.2 \times 10^7$$

km further from the Sun than Mercury.

Unit 7 Lesson 15 Cumulative Practice Problems

1. Evaluate each expression, giving the answer in scientific notation:

   a. $5.3 \times 10^4 + 4.7 \times 10^4$

   b. $3.7 \times 10^6 - 3.3 \times 10^6$

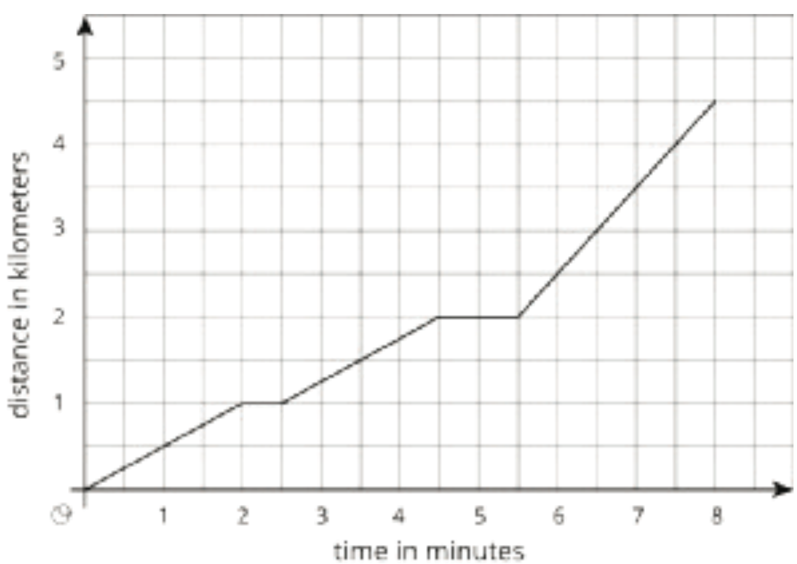
   c. $4.8 \times 10^{-3} + 6.3 \times 10^{-3}$

   d. $6.6 \times 10^{-5} - 6.1 \times 10^{-5}$

2. a. Write a scenario that describes what is happening in the graph.

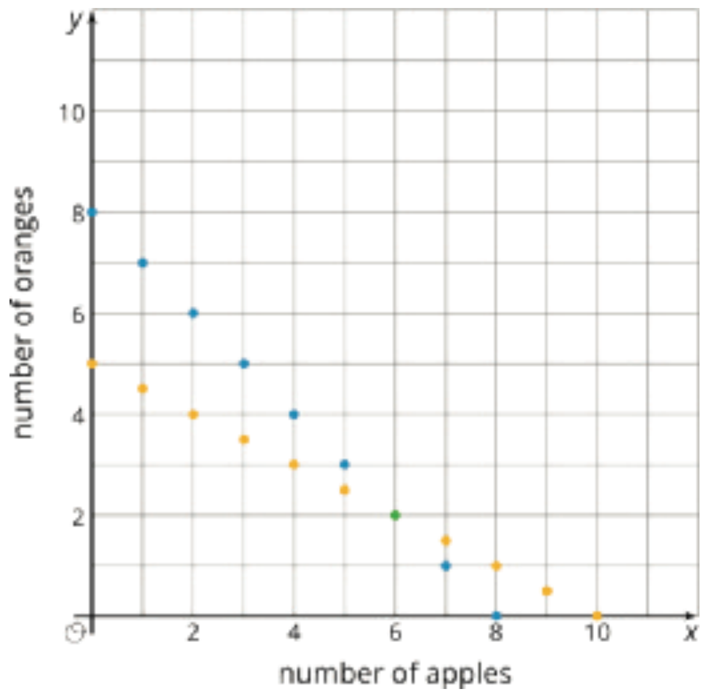
   b. What is happening at 5 minutes?

   c. What does the slope of the line between 6 and 8 minutes mean?



(From Unit 5, Lesson 10.)

3. Apples cost $1 each. Oranges cost $2 each. You have $10 and want to buy 8 pieces of fruit. One graph shows combinations of apples and oranges that total to $10. The other graph shows combinations of apples and oranges that total to 8 pieces of fruit.



   a. Name one combination of 8 fruits shown on the graph that whose cost does *not* total to $10.

   b. Name one combination of fruits shown on the graph whose cost totals to $10 that are *not* 8 fruits all together.

   c. How many apples and oranges would you need to have 8 fruits that cost $10 at the same time?

(From Unit 4, Lesson 10.)

4. Solve each equation and check your solution.

$-2(3x-4)=4(x+3)+6$

$\frac{1}{2}(z+4)-6=-2z+8$

$4w-7=6w+31$

(From Unit 4, Lesson 5.)

Lesson 16: Is a Smartphone Smart Enough to Go to the Moon?

Let's compare digital media and computer hardware using scientific notation.

16.1: Old Hardware, New Hardware

In 1966, the Apollo Guidance Computer was developed to make the calculations that would put humans on the Moon.

Your teacher will give you advertisements for different devices from 1966 to 2016. Choose one device and compare that device with the Apollo Guidance Computer. If you get stuck, consider using scientific notation to help you do your calculations.

For reference, storage is measured in bytes, processor speed is measured in hertz, and memory is measured in bytes. Kilo stands for 1,000, mega stands for 1,000,000, giga stands for 1,000,000,000, and tera stands for 1,000,000,000,000.

1. Which one can store more information? How many times more information?

2. Which one has a faster processor? How many times faster?

3. Which one has more memory? How many times more memory?

16.2: A Bit More on Bytes

For each question, think about what information you would need to figure out an answer. Your teacher may provide some of the information you ask for. Give your answers using scientific notation.

1. Mai found an 80’s computer magazine with an advertisement for a machine with hundreds of kilobytes of storage! Mai was curious and asked, “How many kilobytes would my dad’s new 2016 computer hold?”

2. The old magazine showed another ad for a 750-kilobyte floppy disk, a device used in the past to store data. How many gigabytes is this?

3. Mai and her friends are actively involved on a social media service that limits each message to 140 characters. She wonders about how the size of a message compares to other media.

   Estimate how many messages it would take for Mai to fill up a floppy disk with her 140-character messages. Explain or show your reasoning.

4. Estimate how many messages it would take for Mai to fill a floppy disk with messages that only use emojis (each message being 140 emojis). Explain or show your reasoning.

5. Mai likes to go to the movies with her friends and knows that a high-definition film takes up a lot of storage space on a computer.

   Estimate how many floppy disks it would take to store a high-definition movie. Explain or show your reasoning.

6. How many seconds of a high-definition movie would one floppy disk be able to hold?

7. If you fall asleep watching a movie streaming service and it streams movies all night while you sleep, how many floppy disks of information would that be?

Are you ready for more?

Humans tend to work with numbers using powers of 10, but computers work with numbers using powers of 2. A “binary kilobyte” is 1,024 bytes instead of 1,000, because $1{,}024 = 2^{10}$. Similarly, a “binary megabyte” is 1,024 binary kilobytes, and a “binary gigabyte” is 1,024 binary megabytes.

1. Which is bigger, a binary gigabyte or a regular gigabyte? How many more bytes is it?

2. Which is bigger, a binary terabyte or a regular terabyte? How many more bytes is it?

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