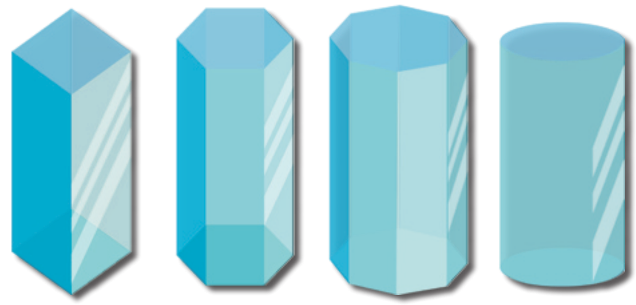
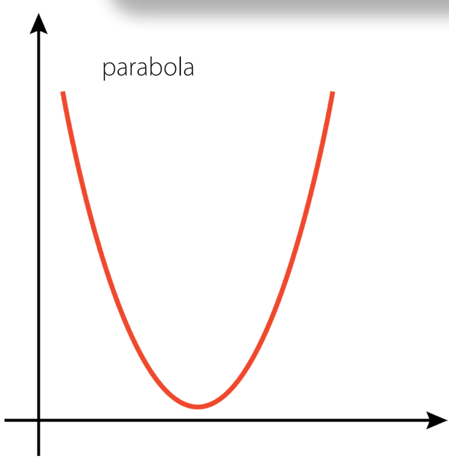
Core Knowledge® MATHEMATICS

volume



Functions and Volume

Student Workbook



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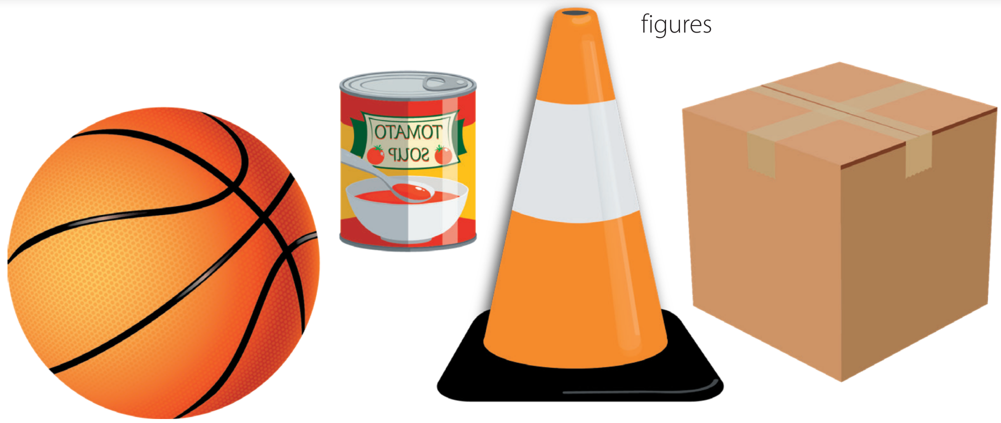
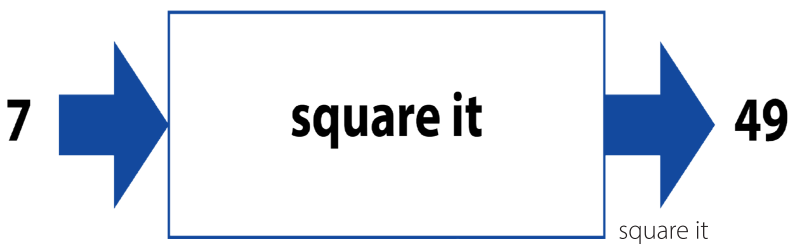
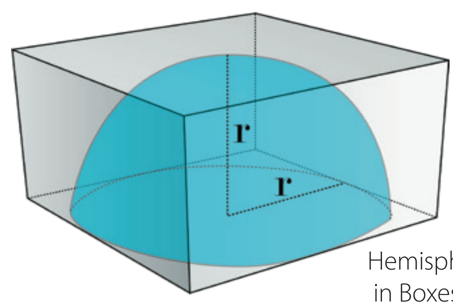




square it



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Functions and Volume

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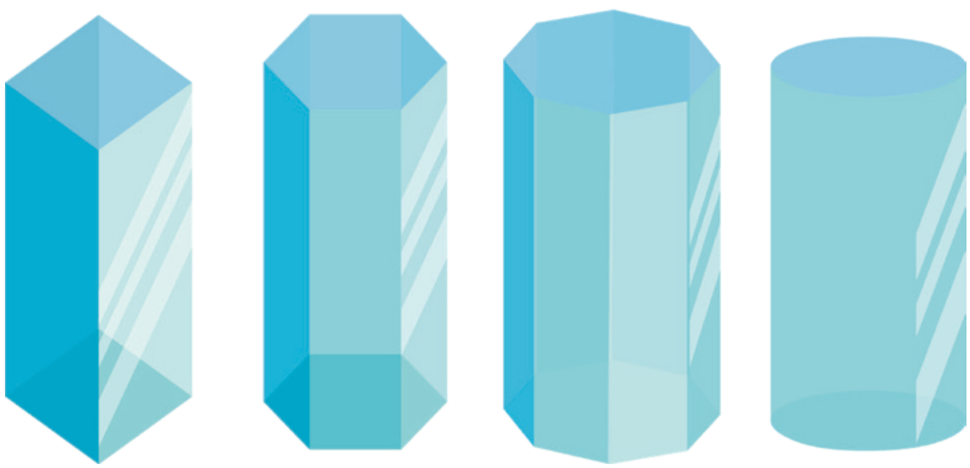


Functions and Volume
Student Workbook
Core Knowledge Mathematics™

Lesson 1: Inputs and Outputs

Let's make some rules.

1.1: Dividing by 0

Study the statements carefully.

- $12 \div 3 = 4$ because $12 = 4 \cdot 3$
- $6 \div 0 = x$ because $6 = x \cdot 0$

What value can be used in place of x to create true statements? Explain your reasoning.

1.2: Guess My Rule

Keep the rule cards face down. Decide who will go first.

1. Player 1 picks up a card and silently reads the rule without showing it to Player 2.
2. Player 2 chooses an integer and asks Player 1 for the result of applying the rule to that number.
3. Player 1 gives the result, without saying how they got it.
4. Keep going until Player 2 correctly guesses the rule.

After each round, the players switch roles.

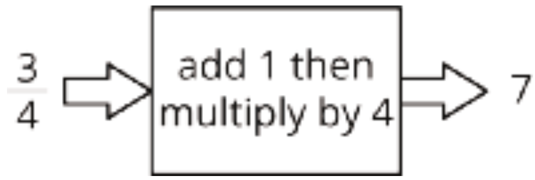
Are you ready for more?

If you have a rule, you can apply it several times in a row and look for patterns. For example, if your rule was "add 1" and you started with the number 5, then by applying that rule over and over again you would get 6, then 7, then 8, etc., forming an obvious pattern.

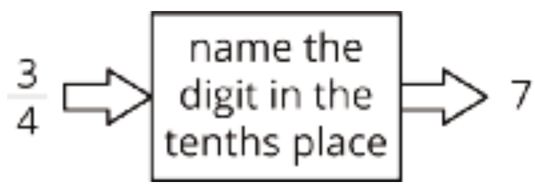
Try this for the rules in this activity. That is, start with the number 5 and apply each of the rules a few times. Do you notice any patterns? What if you start with a different starting number?

1.3: Making Tables

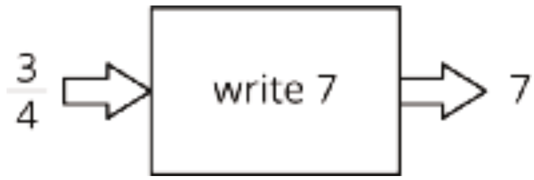
For each input-output rule, fill in the table with the outputs that go with a given input. Add two more input-output pairs to the table.

1. 

input	output
$\frac{3}{4}$	7
2.35	
42	

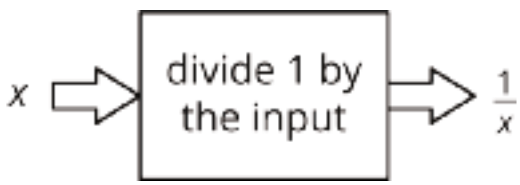
2. 

input	output
$\frac{3}{4}$	7
2.35	
42	

3. 

input	output
$\frac{3}{4}$	7
2.35	
42	

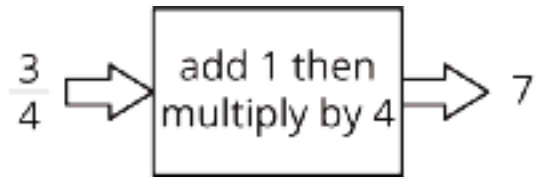
Pause here until your teacher directs you to the last rule.

4. 

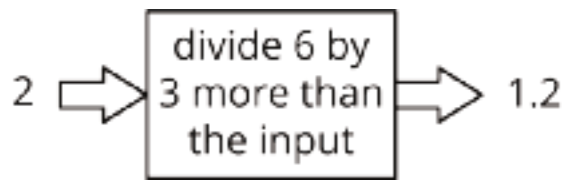
input	output
$\frac{3}{7}$	$\frac{7}{3}$
1	
0	

Lesson 1 Summary

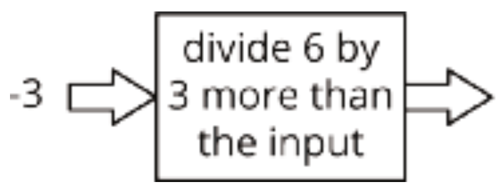
An *input-output rule* is a rule that takes an allowable input and uses it to determine an output. For example, the following diagram represents the rule that takes any number as an input, then adds 1, multiplies by 4, and gives the resulting number as an output.



In some cases, not all inputs are allowable, and the rule must specify which inputs will work. For example, this rule is fine when the input is 2:



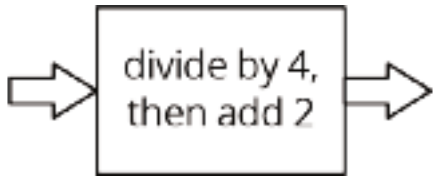
But if the input is -3, we would need to evaluate $6 \div 0$ to get the output.



So, when we say that the rule is "divide 6 by 3 more than the input," we also have to say that -3 is not allowed as an input.

Unit 5 Lesson 1 Cumulative Practice Problems

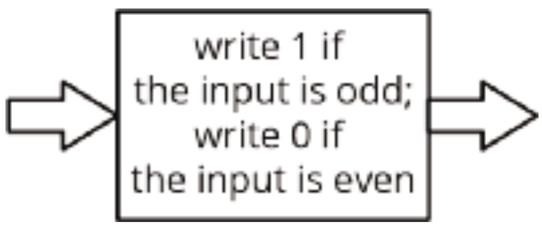
1. Given the rule:



Complete the table for the function rule for the following input values:

input	0	2	4	6	8	10
output						

2. Here is an input-output rule:



Complete the table for the input-output rule:

input	-3	-2	-1	0	1	2	3
output							

3. Andre’s school orders some new supplies for the chemistry lab. The online store shows a pack of 10 test tubes costs $4 less than a set of nested beakers. In order to fully equip the lab, the school orders 12 sets of beakers and 8 packs of test tubes.

   a. Write an equation that shows the cost of a pack of test tubes, t, in terms of the cost of a set of beakers, b.

   b. The school office receives a bill for the supplies in the amount of $348. Write an equation with t and b that describes this situation.

   c. Since t is in terms of b from the first equation, this expression can be substituted into the second equation where t appears. Write an equation that shows this substitution.

   d. Solve the equation for b.

   e. How much did the school pay for a set of beakers? For a pack of test tubes?

   (From Unit 4, Lesson 15.)

4. Solve: $\begin{cases} y = x - 4 \\ y = 6x - 10 \end{cases}$

   (From Unit 4, Lesson 14.)

5. For what value of x do the expressions $2x + 3$ and $3x - 6$ have the same value?

   (From Unit 4, Lesson 9.)

Lesson 2: Introduction to Functions

Let's learn what a function is.

2.1: Square Me

Here are some numbers in a list:

$$1, -3, -\frac{1}{2}, 3, 2, \frac{1}{4}, 0.5$$

1. How many different numbers are in the list?

2. Make a new list containing the squares of all these numbers.

3. How many different numbers are in the new list?

4. Explain why the two lists do not have the same number of different numbers.

2.2: You Know This, Do You Know That?

Say yes or no for each question. If yes, draw an input-output diagram. If no, give examples of two different outputs that are possible for the same input.

1. A person is 5.5 feet tall. Do you know their height in inches?

2. A number is 5. Do you know its square?

3. The square of a number is 16. Do you know the number?

4. A square has a perimeter of 12 cm. Do you know its area?

5. A rectangle has an area of 16 cm^2. Do you know its length?

6. You are given a number. Do you know the number that is $\frac{1}{5}$ as big?

7. You are given a number. Do you know its reciprocal?

2.3: Using Function Language

Here are the questions from the previous activity. For the ones you said yes to, write a statement like, “The height a rubber ball bounces to depends on the height it was dropped from” or “Bounce height is a **function** of drop height.” For all of the ones you said no to, write a statement like, “The day of the week does not determine the temperature that day” or “The temperature that day is not a function of the day of the week.”

1. A person is 5.5 feet tall. Do you know their height in inches?

2. A number is 5. Do you know its square?

3. The square of a number is 16. Do you know the number?

4. A square has a perimeter of 12 cm. Do you know its area?

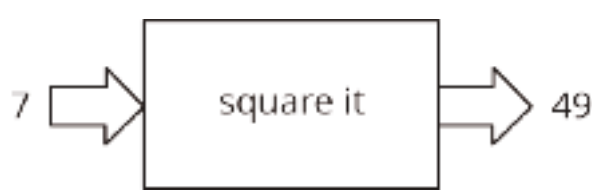
5. A rectangle has an area of 16 cm^2. Do you know its length?

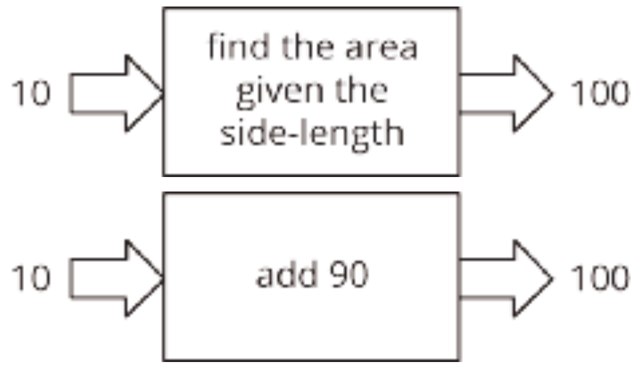
6. You are given a number. Do you know the number that is $\frac{1}{5}$ as big?

7. You are given a number. Do you know its reciprocal?

2.4: Same Function, Different Rule?

Which input-output rules could describe the same function (if any)? Be prepared to explain your reasoning.





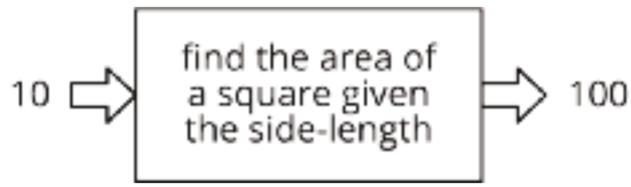
Are you ready for more?

The phrase “is a function of” gets used in non-mathematical speech as well as mathematical speech in sentences like, “The range of foods you like is a function of your upbringing.” What is that sentence trying to convey? Is it the same use of the word “function” as the mathematical one?

Lesson 2 Summary

Let's say we have an input-output rule that for each allowable input gives exactly one output. Then we say the output *depends* on the input, or the output is a **function** of the input.

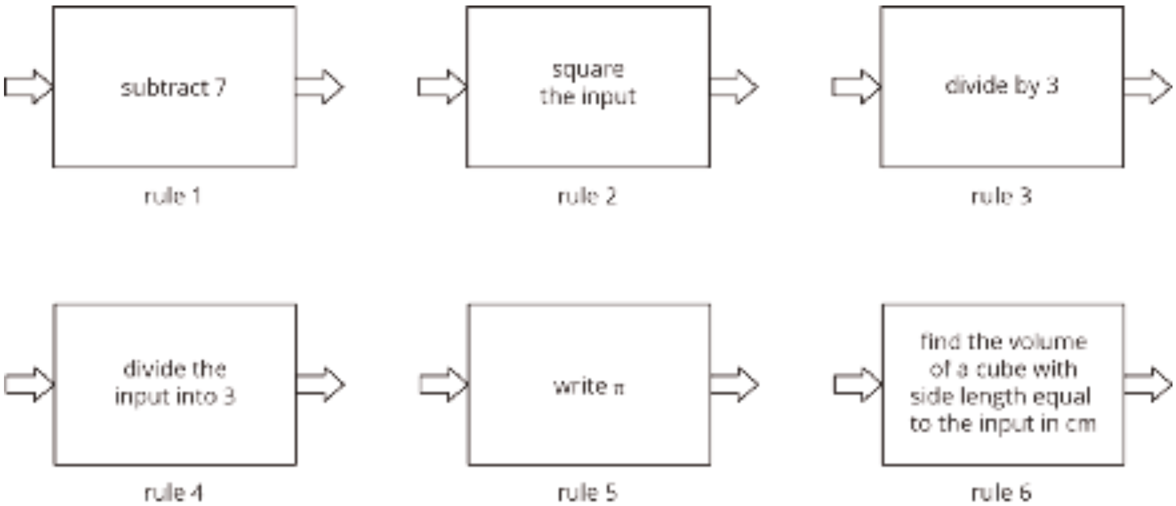
For example, the area of a square is a function of the side length, because you can find the area from the side length by squaring it. So when the input is 10 cm, the output is 100 cm^2.



Sometimes we might have two different rules that describe the same function. As long as we always get the same, single output from the same input, the rules describe the same function.

Unit 5 Lesson 2 Cumulative Practice Problems

1. Here are several function rules. Calculate the output for each rule when you use -6 as the input.



2. A group of students is timed while sprinting 100 meters. Each student's speed can be found by dividing 100 m by their time. Is each statement true or false? Explain your reasoning.

   a. Speed is a function of time.

   b. Time is a function of distance.

   c. Speed is a function of number of students racing.

   d. Time is a function of speed.

3. Diego’s history teacher writes a test for the class with 26 questions. The test is worth 123 points and has two types of questions: multiple choice worth 3 points each, and essays worth 8 points each. How many essay questions are on the test? Explain or show your reasoning.

(From Unit 4, Lesson 15.)

4. These tables correspond to inputs and outputs. Which of these input and output tables could represent a function rule, and which ones could not? Explain or show your reasoning.

Table A:

input	output
-2	4
-1	1
0	0
1	1
2	4

Table B:

input	output
4	-2
1	-1
0	0
1	1
4	2

Table C:

input	output
1	0
2	0
3	0

Table D:

input	output
0	1
0	2
0	3

Lesson 3: Equations for Functions

Let's find outputs from equations.

3.1: A Square's Area

Fill in the table of input-output pairs for the given rule. Write an algebraic expression for the rule in the box in the diagram.

input	output
8	
2.2	
$12\frac{1}{4}$	
s	

3.2: Diagrams, Equations, and Descriptions

Record your answers to these questions in the table provided.

1. Match each of these descriptions with a diagram:
    a. the circumference, C, of a circle with **radius**, r

    b. the distance in miles, d, that you would travel in t hours if you drive at 60 miles per hour

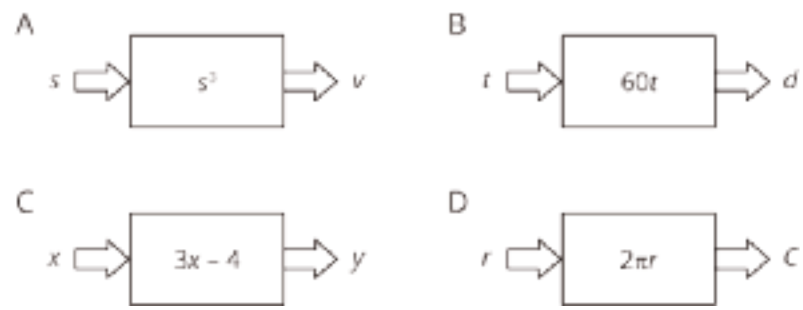
    c. the output when you triple the input and subtract 4

    d. the volume of a cube, v given its edge length, s

2. Write an equation for each description that expresses the output as a function of the input.

3. Find the output when the input is 5 for each equation.

4. Name the **independent** and **dependent variables** of each equation.



description	a	b	c	d
diagram				
equation				
input = 5 output = ?				
independent variable				
dependent variable				

Are you ready for more?

Choose a 3-digit number as an input.

Apply the following rule to it, one step at a time:

- Multiply your number by 7.
- Add one to the result.
- Multiply the result by 11.
- Subtract 5 from the result.
- Multiply the result by 13
- Subtract 78 from the result to get the output.

Can you describe a simpler way to describe this rule? Why does this work?

3.3: Dimes and Quarters

Jada had some dimes and quarters that had a total value of $12.50. The relationship between the number of dimes, d, and the number of quarters, q, can be expressed by the equation $0.1d + 0.25q = 12.5$.

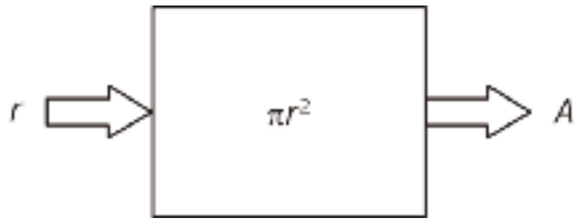
1. If Jada has 4 quarters, how many dimes does she have?
2. If Jada has 10 quarters, how many dimes does she have?
3. Is the number of dimes a function of the number of quarters? If yes, write a rule (that starts with $d =$...) that you can use to determine the output, d, from a given input, q. If no, explain why not.
4. If Jada has 25 dimes, how many quarters does she have?
5. If Jada has 30 dimes, how many quarters does she have?
6. Is the number of quarters a function of the number of dimes? If yes, write a rule (that starts with $q =$...) that you can use to determine the output, q, from a given input, d. If no, explain why not.

Lesson 3 Summary

We can sometimes represent functions with equations. For example, the area, A, of a circle is a function of the radius, r, and we can express this with an equation:

$$A = \pi r^2$$

We can also draw a diagram to represent this function:



In this case, we think of the radius, r, as the input, and the area of the circle, A, as the output. For example, if the input is a radius of 10 cm, then the output is an area of 100π cm^2, or about 314 square cm. Because this is a function, we can find the area, A, for any given radius, r.

Since it is the input, we say that r is the **independent variable** and, as the output, A is the **dependent variable**.

Sometimes when we have an equation we get to choose which variable is the independent variable. For example, if we know that

$$10A - 4B = 120$$

then we can think of A as a function of B and write

$$A = 0.4B + 12$$

or we can think of B as a function of A and write

$$B = 2.5A - 30$$

Unit 5 Lesson 3 Cumulative Practice Problems

1. Here is an equation that represents a function: $72x + 12y = 60$.

   Select **all** the different equations that describe the same function:

   A. $120y + 720x = 600$

   B. $y = 5 - 6x$

   C. $2y + 12x = 10$

   D. $y = 5 + 6x$

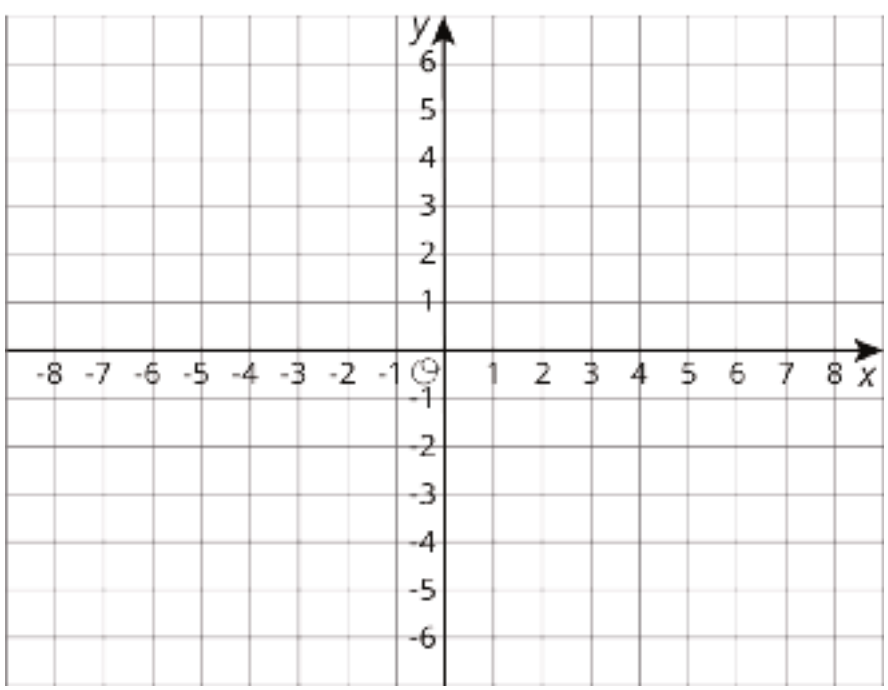
   E. $x = \frac{5}{6} - \frac{y}{6}$

   F. $7x + 2y = 6$

   G. $x = \frac{5}{6} + \frac{y}{6}$

2. a. Graph a system of linear equations with no solutions.

   b. Write an equation for each line you graph.



(From Unit 4, Lesson 13.)

3. Brown rice costs \$2 per pound, and beans cost \$1.60 per pound. Lin has \$10 to spend on these items to make a large meal of beans and rice for a potluck dinner. Let b be the number of pounds of beans Lin buys and r be the number of pounds of rice she buys when she spends all her money on this meal.

   a. Write an equation relating the two variables.

   b. Rearrange the equation so b is the independent variable.

   c. Rearrange the equation so r is the independent variable.

4. Solve each equation and check your answer.

   $2x + 4(3 - 2x) = \frac{3(2x+2)}{6} + 4$

   $4z + 5 = -3z - 8$

   $\frac{1}{2} - \frac{1}{8}q = \frac{q-1}{4}$

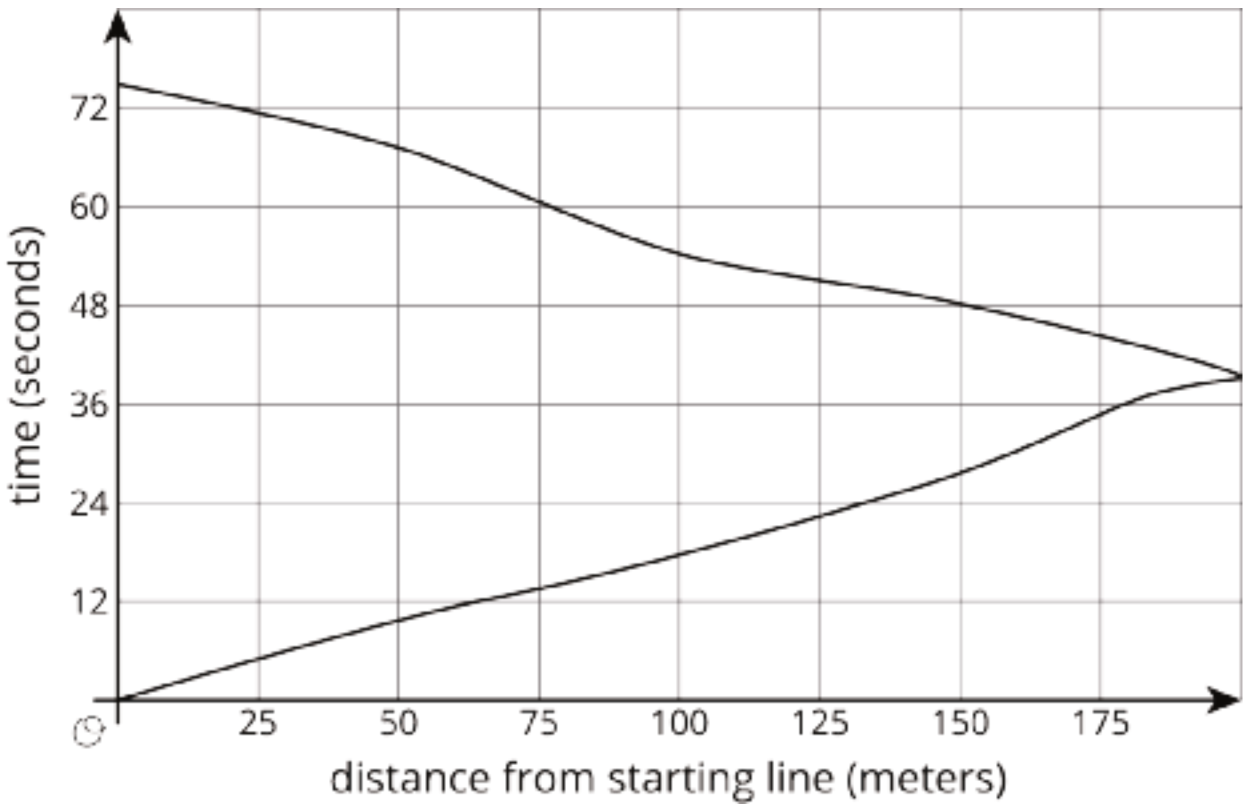
   (From Unit 4, Lesson 6.)

Lesson 4: Tables, Equations, and Graphs of Functions

Let's connect equations and graphs of functions.

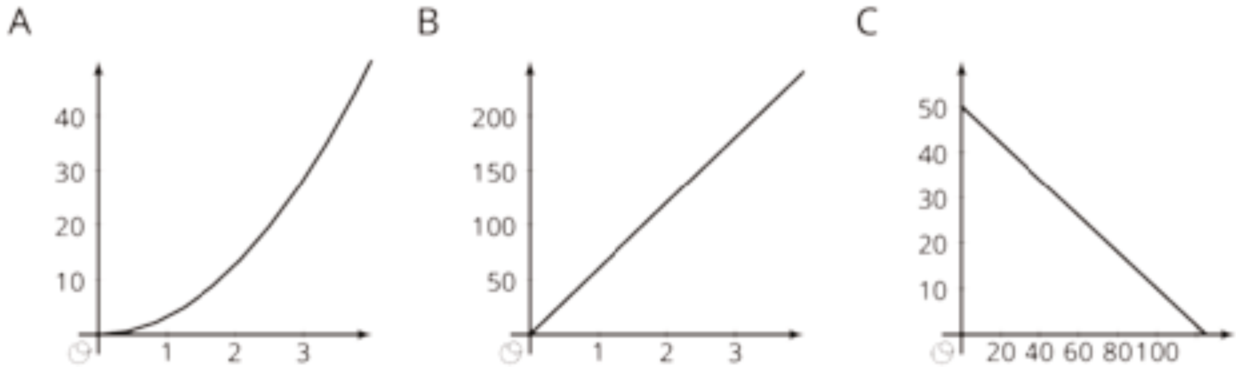
4.1: Notice and Wonder: Doubling Back

What do you notice? What do you wonder?



4.2: Equations and Graphs of Functions

The graphs of three functions are shown.



1. Match one of these equations to each of the graphs.
   a. $d = 60t$, where d is the distance in miles that you would travel in t hours if you drove at 60 miles per hour.

   b. $q = 50 - 0.4d$, where q is the number of quarters, and d is the number of dimes, in a pile of coins worth $12.50.

   c. $A = \pi r^2$, where A is the area in square centimeters of a circle with radius r centimeters.

2. Label each of the axes with the independent and dependent variables and the quantities they represent.

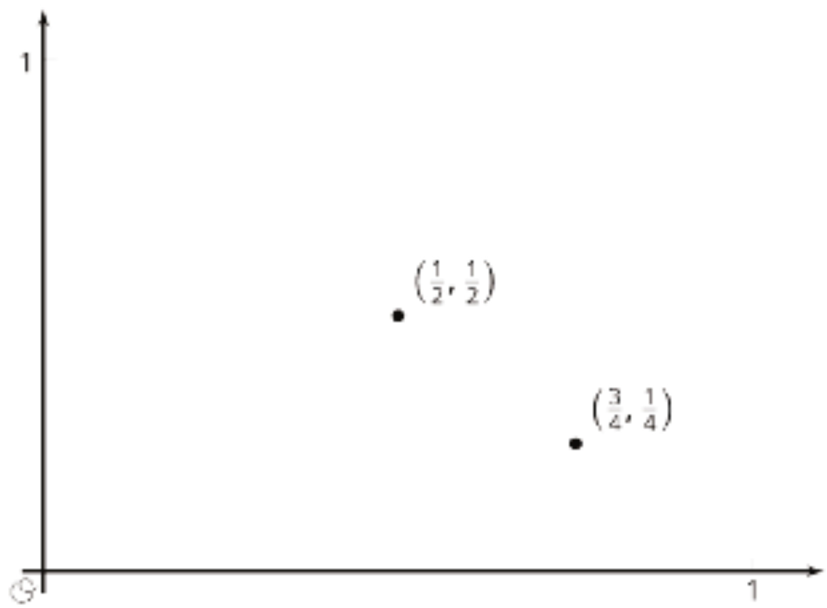
3. For each function: What is the output when the input is 1? What does this tell you about the situation? Label the corresponding point on the graph.

4. Find two more input-output pairs. What do they tell you about the situation? Label the corresponding points on the graph.

Are you ready for more?

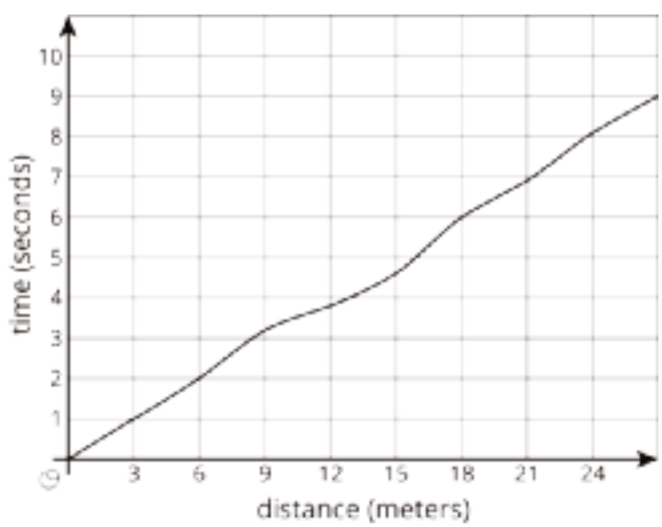
A function inputs fractions $\frac{a}{b}$ between 0 and 1 where a and b have no common factors, and outputs the fraction $\frac{1}{b}$. For example, given the input $\frac{3}{4}$ the function outputs $\frac{1}{4}$, and to the input $\frac{1}{2}$ the function outputs $\frac{1}{2}$. These two input-output pairs are shown on the graph.

Plot at least 10 more points on the graph of this function. Are most points on the graph above or below a height of 0.3? Of height 0.01?



4.3: Running around a Track

1. Kiran was running around the track. The graph shows the time, t, he took to run various distances, d. The table shows his time in seconds after every three meters.



d	0	3	6	9	12	15	18	21	24	27
t	0	1.0	2.0	3.2	3.8	4.6	6.0	6.9	8.09	9.0

a. How long did it take Kiran to run 6 meters?

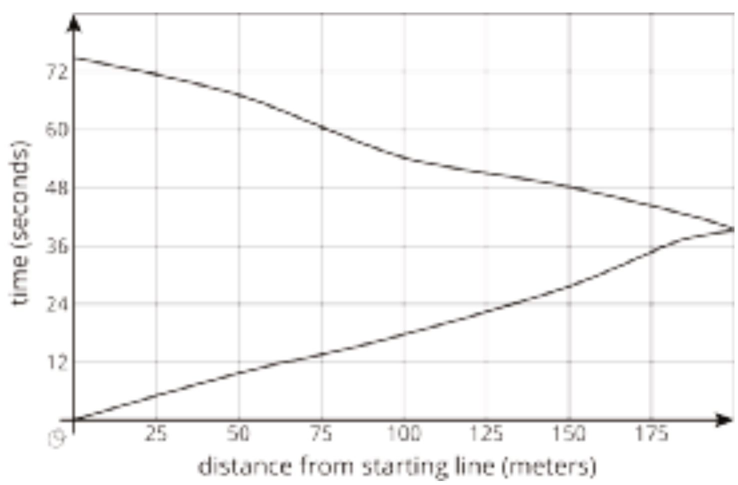
b. How far had he gone after 6 seconds?

c. Estimate when he had run 19.5 meters.

d. Estimate how far he ran in 4 seconds.

e. Is Kiran's time a function of the distance he has run? Explain how you know.

2. Priya is running once around the track. The graph shows her time given how far she is from her starting point.



a. What was her farthest distance from her starting point?

b. Estimate how long it took her to run around the track.

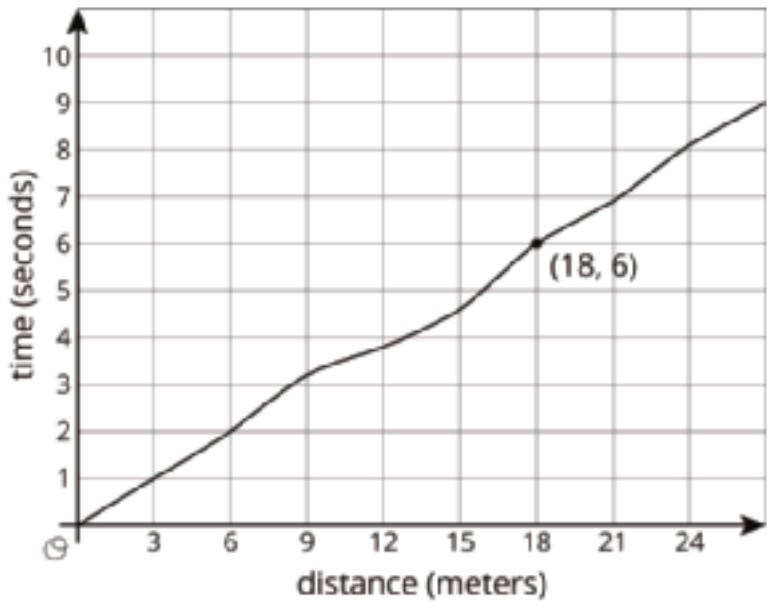
c. Estimate when she was 100 meters from her starting point.

d. Estimate how far she was from the starting line after 60 seconds.

e. Is Priya's time a function of her distance from her starting point? Explain how you know.

Lesson 4 Summary

Here is the graph showing Noah's run.

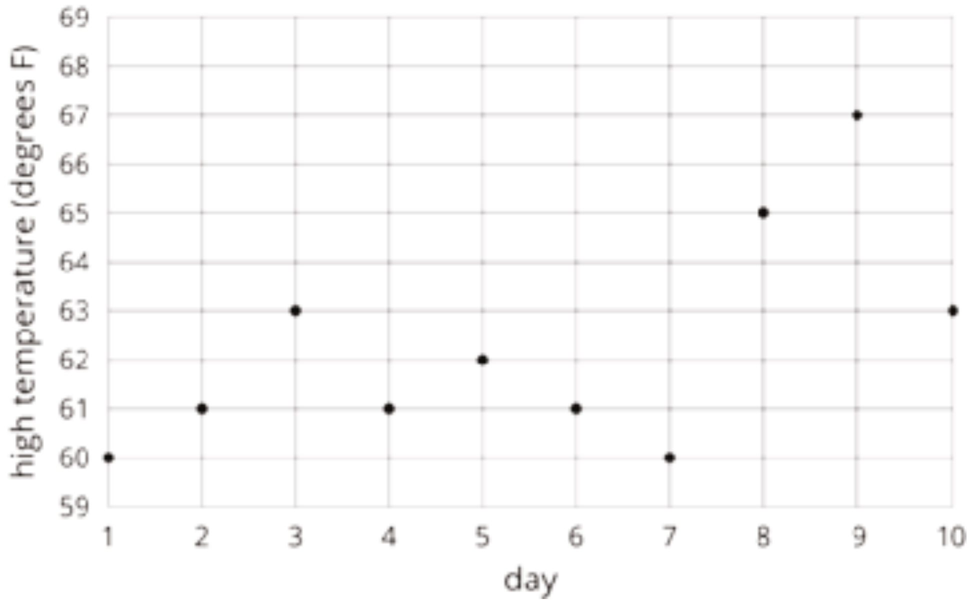


The time in seconds since he started running is a function of the distance he has run. The point (18,6) on the graph tells you that the time it takes him to run 18 meters is 6 seconds. The input is 18 and the output is 6.

The graph of a function is all the coordinate pairs, (input, output), plotted in the coordinate plane. By convention, we always put the input first, which means that the inputs are represented on the horizontal axis and the outputs, on the vertical axis.

Unit 5 Lesson 4 Cumulative Practice Problems

1. The graph and the table show the high temperatures in a city over a 10-day period.



day	1	2	3	4	5	6	7	8	9	10
temperature (degrees F)	60	61	63	61	62	61	60	65	67	63

a. What was the high temperature on Day 7?

b. On which days was the high temperature 61 degrees?

c. Is the high temperature a function of the day? Explain how you know.

d. Is the day a function of the high temperature? Explain how you know.

2. The amount Lin's sister earns at her part-time job is proportional to the number of hours she works. She earns $9.60 per hour.

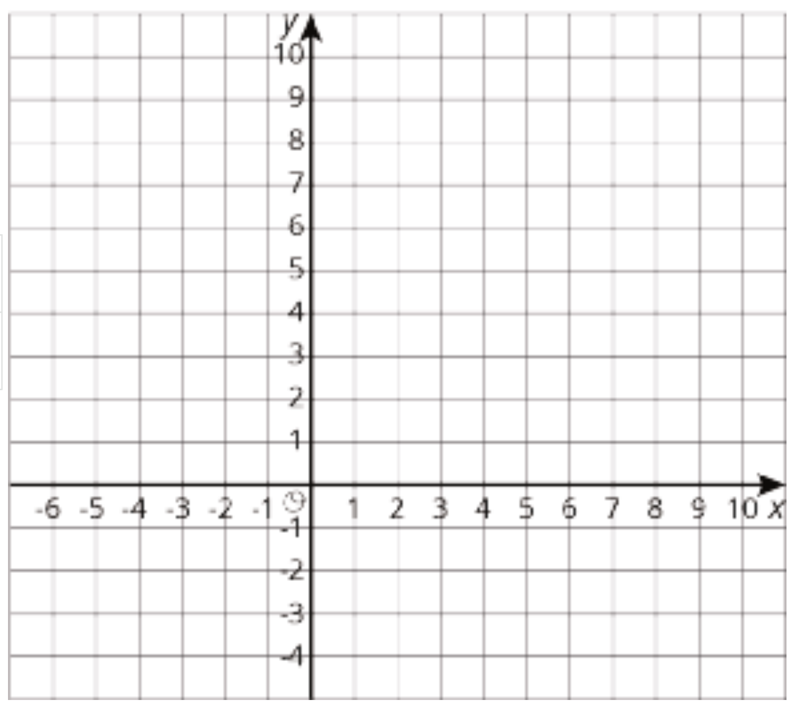
   a. Write an equation in the form $y = kx$ to describe this situation, where x represents the hours she works and y represents the dollars she earns.

   b. Is y a function of x? Explain how you know.

   c. Write an equation describing x as a function of y.

3. Use the equation $2m + 4s = 16$ to complete the table, then graph the line using s as the dependent variable.

m	0		-2	
s		3		0



4. Solve the system of equations: $\begin{cases} y = 7x + 10 \\ y = -4x - 23 \end{cases}$

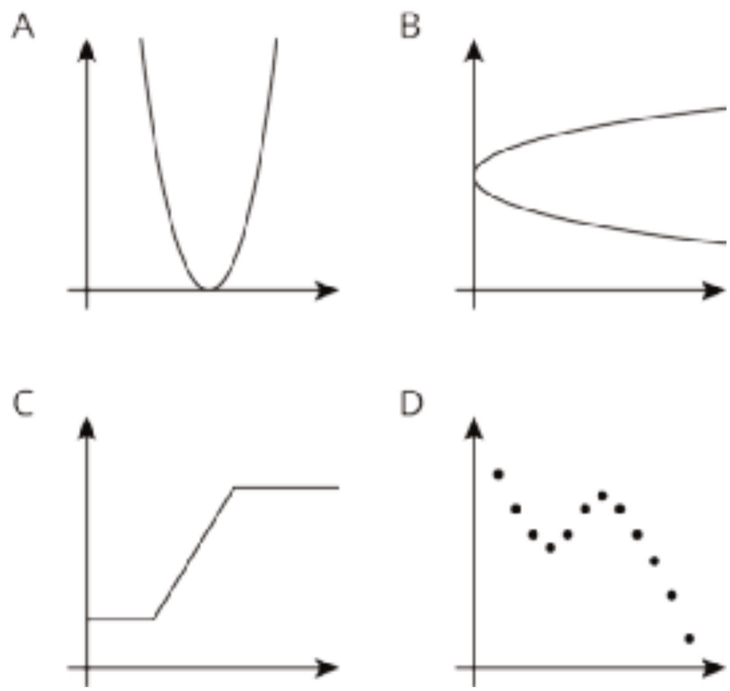
(From Unit 4, Lesson 13.)

Lesson 5: More Graphs of Functions

Let's interpret graphs of functions.

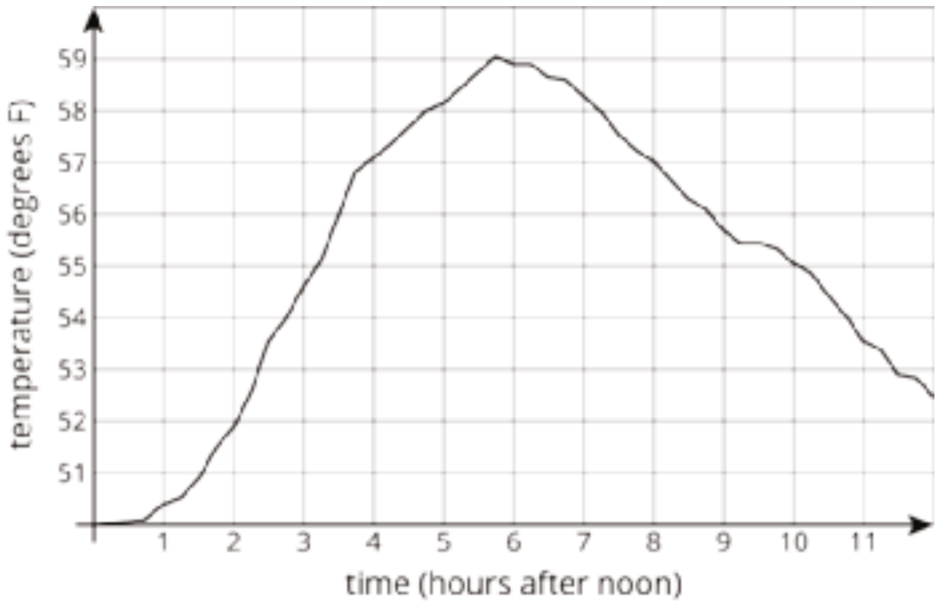
5.1: Which One Doesn't Belong: Graphs

Which graph doesn't belong?



5.2: Time and Temperature

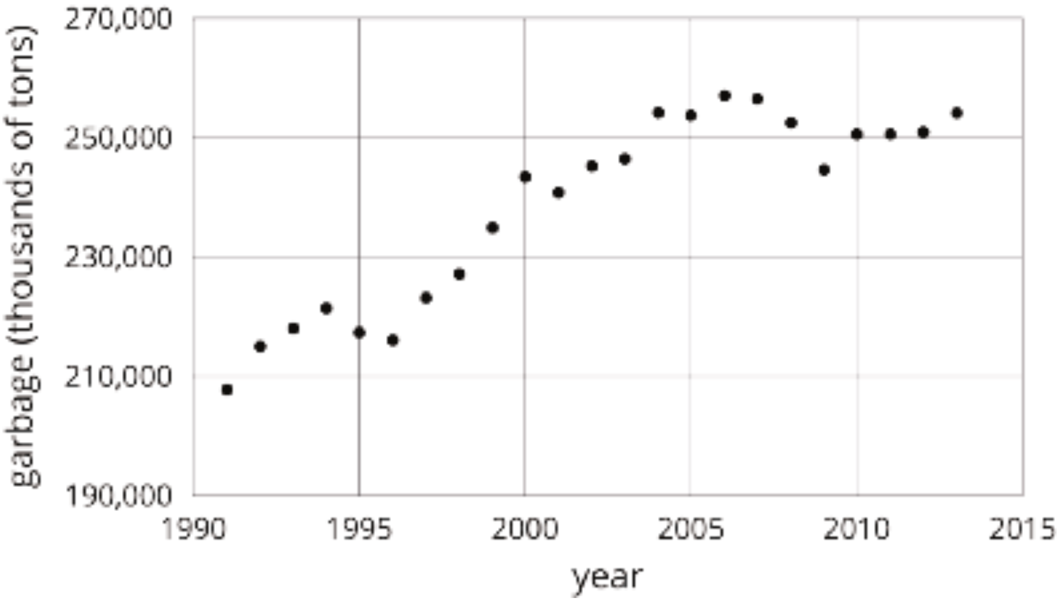
The graph shows the temperature between noon and midnight in one day in a certain city.



1. Was it warmer at 3:00 p.m. or 9:00 p.m.?
2. Approximately when was the temperature highest?
3. Find another time that the temperature was the same as it was at 4:00 p.m.
4. Did the temperature change more between 1:00 p.m. and 3:00 p.m. or between 3:00 p.m. and 5:00 p.m.?
5. Does this graph show that temperature is a function of time, or time is a function of temperature?
6. When the input for the function is 8, what is the output? What does that tell you about the time and temperature?

5.3: Garbage

1. The graph shows the amount of garbage produced in the US each year between 1991 and 2013.

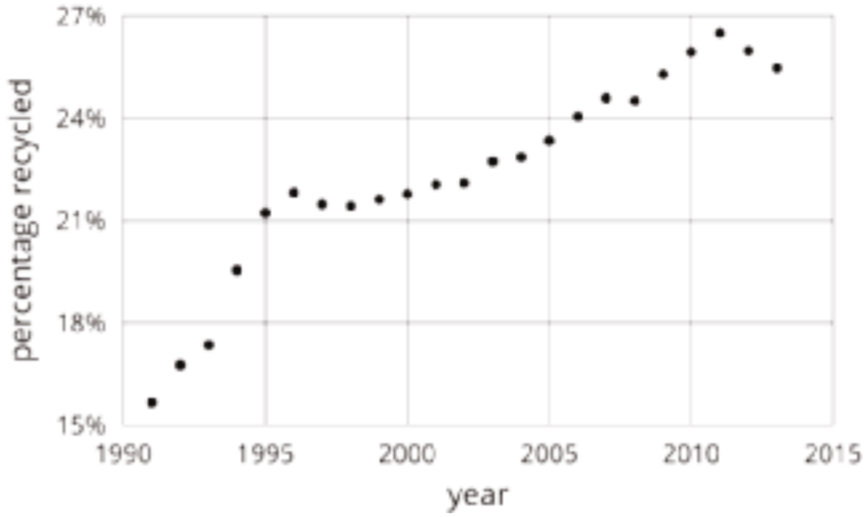


a. Did the amount of garbage increase or decrease between 1999 and 2000?

b. Did the amount of garbage increase or decrease between 2005 and 2009?

c. Between 1991 and 1995, the garbage increased for three years, and then it decreased in the fourth year. Describe how the amount of garbage changed in the years between 1995 and 2000.

2. The graph shows the percentage of garbage that was recycled between 1991 and 2013.



   a. When was it increasing?

   b. When was it decreasing?

   c. Tell the story of the change in the percentage of garbage recycled in the US over this time period.

Are you ready for more?

Refer to the graph in the first part of the activity.

1. Find a year where the amount of garbage produced increased from the previous year, but not by as much it increased the following year.

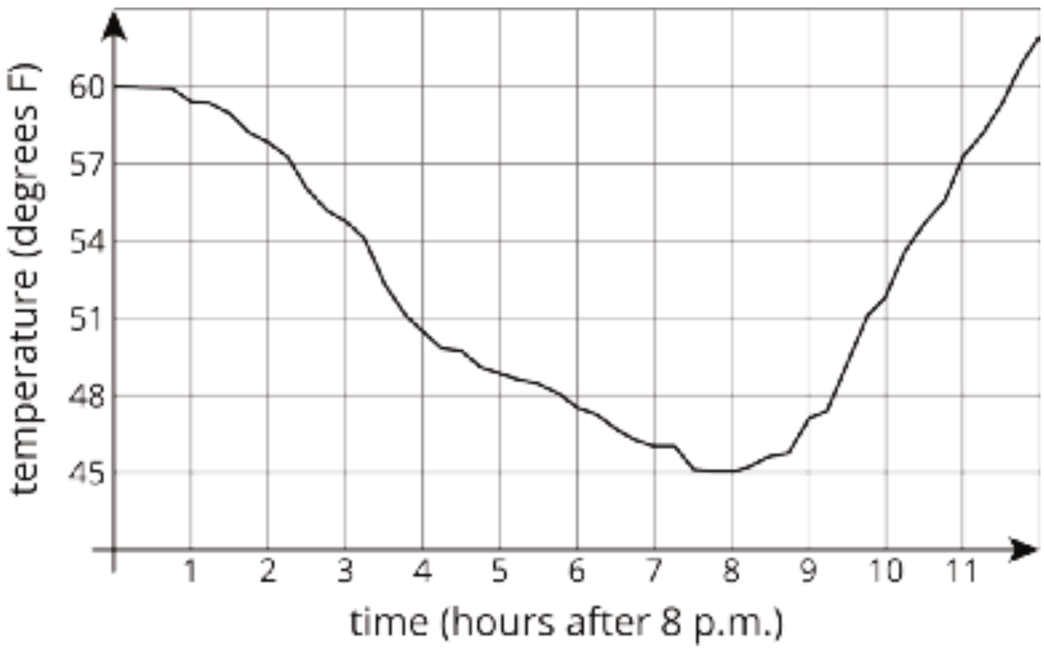
2. Find a year where the amount of garbage produced increased from the previous year, and then increased by a smaller amount the following year.

3. Find a year where the amount of garbage produced decreased from the previous year, but not by as much it decreased the following year.

4. Find a year where the amount of garbage produced decreased from the previous year, and then decreased by a smaller amount the following year.

Lesson 5 Summary

Here is a graph showing the temperature in a town as a function of time after 8:00 p.m.



The graph of a function tells us what is happening in the context the function represents. In this example, the temperature starts out at 60° F at 8:00 p.m. It decreases during the night, reaching its lowest point at 8 hours after 8:00 p.m., or 4:00 a.m. Then it starts to increase again.

Unit 5 Lesson 5 Cumulative Practice Problems

1. The solution to a system of equations is $(6, -3)$. Choose two equations that might make up the system.

   A. $y = -3x + 6$

   B. $y = 2x - 9$

   C. $y = -5x + 27$

   D. $y = 2x - 15$

   E. $y = -4x + 27$

   (From Unit 4, Lesson 13.)

2. A car is traveling on a small highway and is either going 55 miles per hour or 35 miles per hour, depending on the speed limits, until it reaches its destination 200 miles away. Letting x represent the amount of time in hours that the car is going 55 miles per hour, and y being the time in hours that the car is going 35 miles per hour, an equation describing the relationship is:

   $$55x + 35y = 200$$

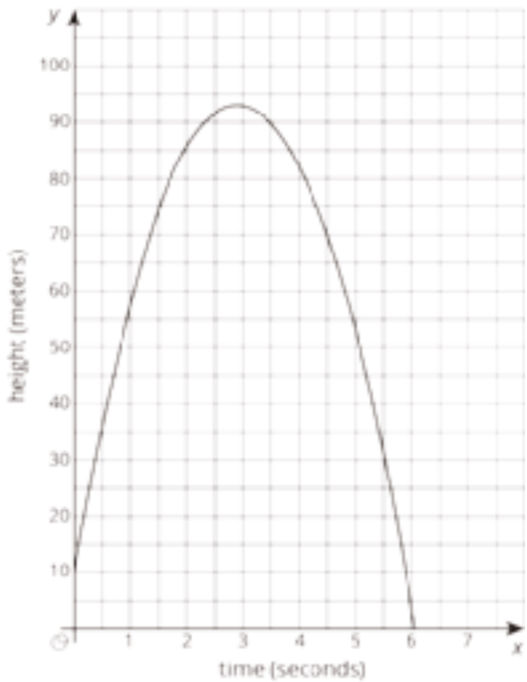
   a. If the car spends 2.5 hours going 35 miles per hour on the trip, how long does it spend going 55 miles per hour?

   b. If the car spends 3 hours going 55 miles per hour on the trip, how long does it spend going 35 miles per hour?

   c. If the car spends no time going 35 miles per hour, how long would the trip take? Explain your reasoning.

   (From Unit 5, Lesson 3.)

3. The graph represents an object that is shot upwards from a tower and then falls to the ground. The independent variable is time in seconds and the dependent variable is the object's height above the ground in meters.



a. How tall is the tower from which the object was shot?

b. When did the object hit the ground?

c. Estimate the greatest height the object reached and the time it took to reach that height. Indicate this situation on the graph.

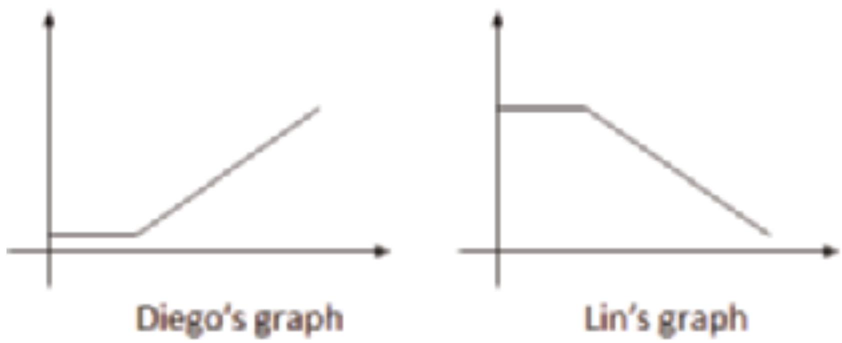
Lesson 6: Even More Graphs of Functions

Let's draw a graph from a story.

6.1: Dog Run

Here are five pictures of a dog taken at equal intervals of time.

Diego and Lin drew different graphs to represent this situation:



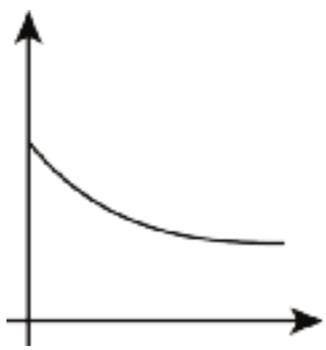
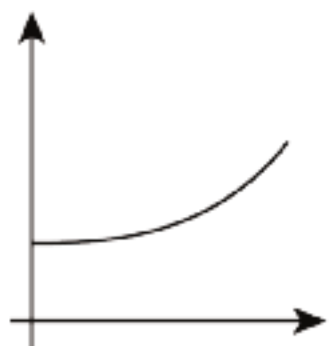
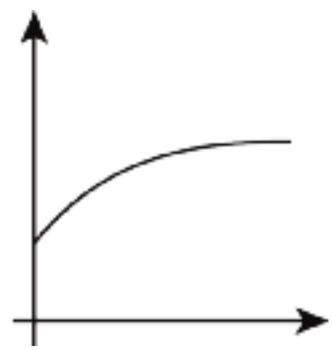
They both used time as the independent variable. What do you think each one used for the dependent variable? Explain your reasoning.

6.2: Which Graph is It?

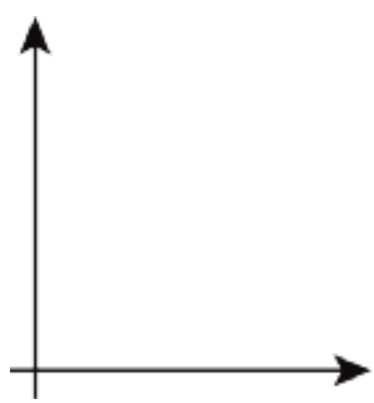
For each situation,

- name the independent and dependent variables
- pick the graph that best fits the situation, or sketch the graph if one isn't provided
- label the axes
- answer the question: which quantity is a function of which? Be prepared to explain your reasoning.

1. Jada is training for a swimming race. The more she practices, the less time it takes for her to swim one lap.

2. Andre adds some money to a jar in his room each week for 3 weeks and then takes some out in week 4.



6.3: Sketching a Story about a Boy and a Bike

Your teacher will give you tools for creating a visual display. With your group, create a display that shows your response to each question.

Here is a story: “Noah was at home. He got on his bike and rode to his friend’s house and stayed there for awhile. Then he rode home again. Then he rode to the park. Then he rode home again.”

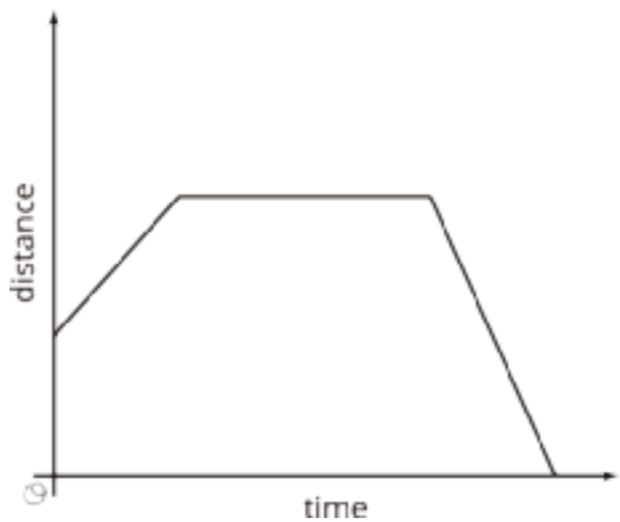
1. Create a set of axes and sketch a graph of this story.
2. What are the two quantities? Label the axes with their names and units of measure. (For example, if this were a story about pouring water into a pitcher, one of your labels might say “volume (liters).”)
3. Which quantity is a function of which? Explain your reasoning.
4. Based on your graph, is his friend’s house or the park closer to Noah's home? Explain how you know.
5. Read the story and all your responses again. Does everything make sense? If not, make changes to your work.

Are you ready for more?

It is the year 3000. Noah’s descendants are still racing around the park, but thanks to incredible technological advances, now with much more powerful gadgets at their disposal. How might their newfound access to teleportation and time-travel devices alter the graph of stories of their daily adventures? Could they affect whether or not the distance from home is a function of the time elapsed?

Lesson 6 Summary

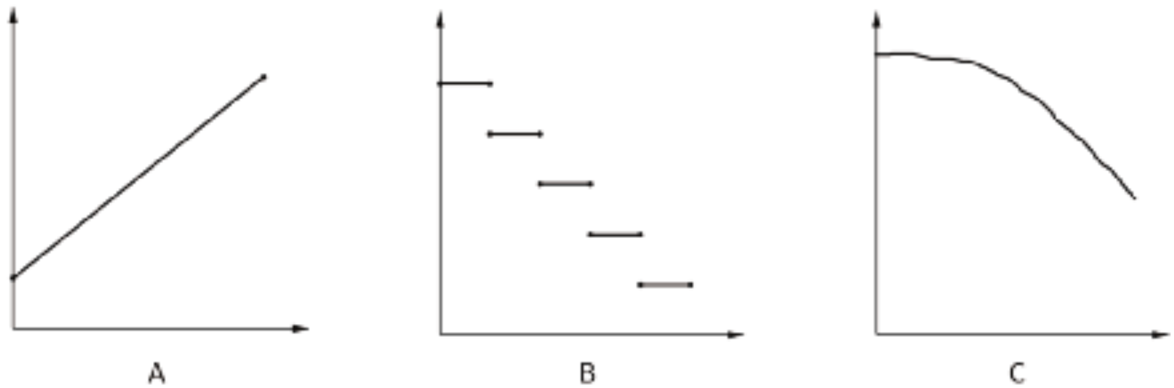
Here is a graph showing Andre's distance as a function of time.



For a graph representing a context, it is important to specify the quantities represented on each axis. For example, if this is showing distance from home, then Andre starts at some distance from home (maybe at his friend's house), moves further away (maybe to a park), then returns home. If instead the graph is showing distance from school, the story may be Andre starts out at home, moves further away (maybe to a friend's house), then goes to school. What could the story be if the graph is showing distance from a park?

Unit 5 Lesson 6 Cumulative Practice Problems

1. Match the graph to the following situations (you can use a graph multiple times). For each match, name possible independent and dependent variables and how you would label the axes.



   a. Tyler pours the same amount of milk from a bottle every morning.

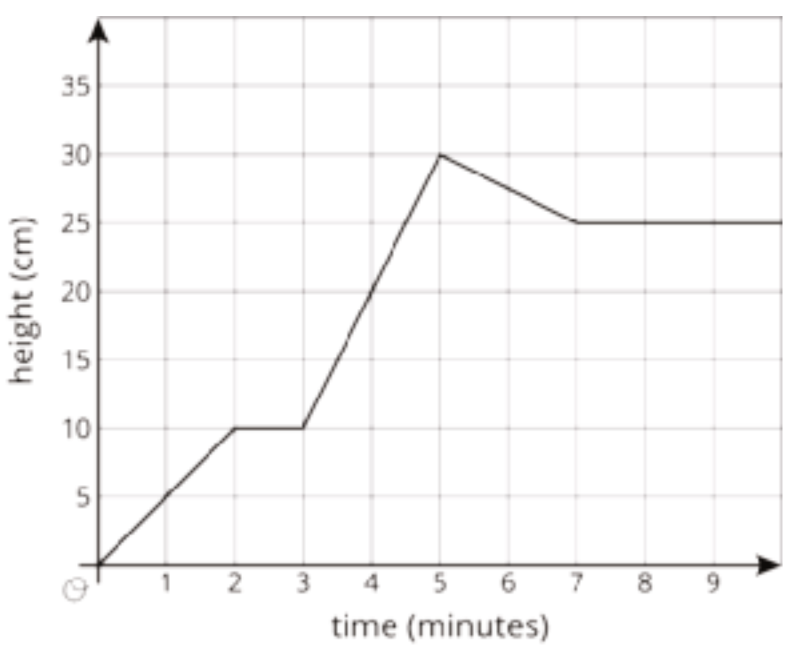
   b. A plant grows the same amount every week.

   c. The day started very warm but then it got colder.

   d. A carnival has an entry fee of $5 and tickets for rides cost $1 each.

2. Jada fills her aquarium with water.

   The graph shows the height of the water, in cm, in the aquarium as a function of time in minutes. Invent a story of how Jada fills the aquarium that fits the graph.



3. Recall the formula for area of a circle.

   a. Write an equation relating a circle's radius, r, and area, A.

   b. Is area a function of the radius? Is radius a function of the area?

   c. Fill in the missing parts of the table.

r	3		$\frac{1}{2}$	
A		16π		100π

(From Unit 5, Lesson 4.)

4. The points with coordinates $(4, 8)$, $(2, 10)$, and $(5, 7)$ all lie on the line $2x + 2y = 24$.

   a. Create a graph, plot the points, and sketch the line.

   b. What is the slope of the line you graphed?

   c. What does this slope tell you about the relationship between lengths and widths of rectangles with perimeter 24?

(From Unit 3, Lesson 11.)

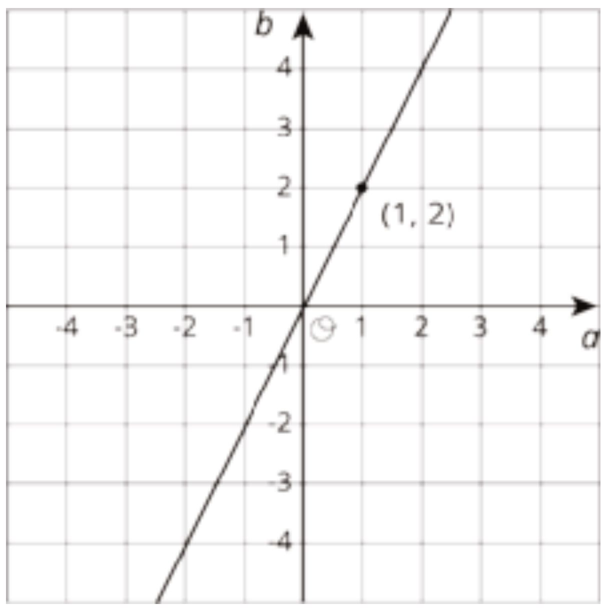
Lesson 7: Connecting Representations of Functions

Let's connect tables, equations, graphs, and stories of functions.

7.1: Which are the Same? Which are Different?

Here are three different ways of representing functions. How are they alike? How are they different?

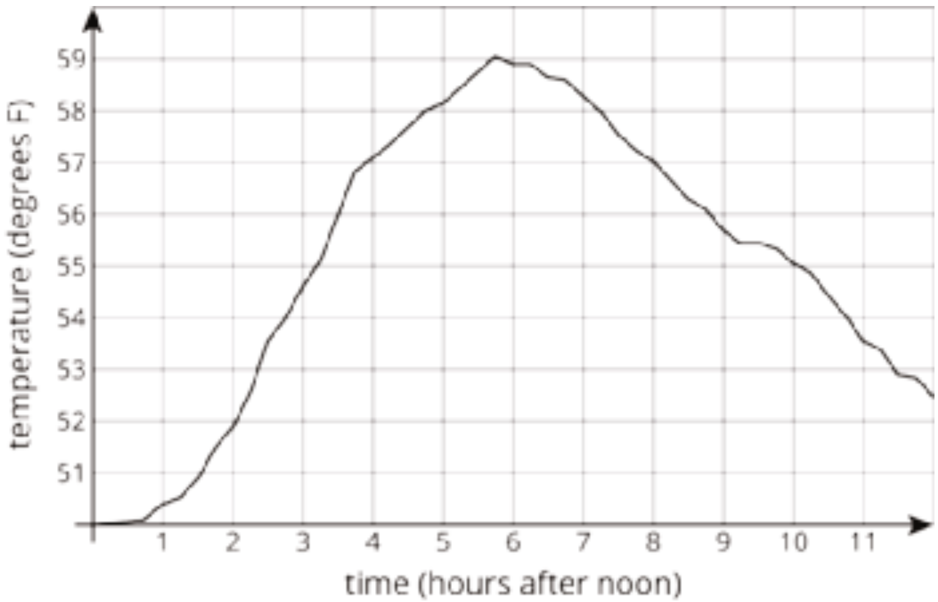
$$y = 2x$$



p	-2	-1	0	1	2	3
q	4	2	0	-2	-4	-6

7.2: Comparing Temperatures

The graph shows the temperature between noon and midnight in City A on a certain day.



The table shows the temperature, T, in degrees Fahrenheit, for h hours after noon, in City B.

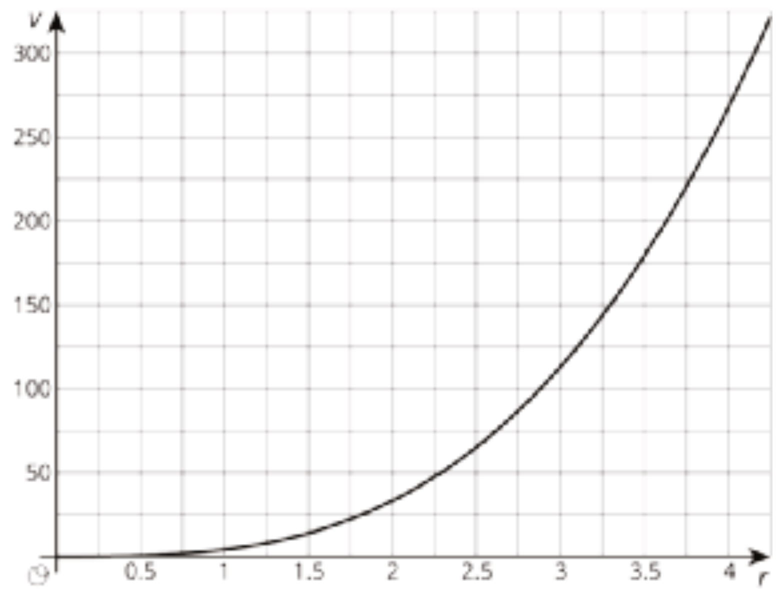
h	1	2	3	4	5	6
T	82	78	75	62	58	59

1. Which city was warmer at 4:00 p.m.?
2. Which city had a bigger change in temperature between 1:00 p.m. and 5:00 p.m.?
3. How much greater was the highest recorded temperature in City B than the highest recorded temperature in City A during this time?
4. Compare the outputs of the functions when the input is 3.

7.3: Comparing Volumes

The **volume**, V, of a cube with edge length s cm is given by the equation $V = s^3$.

The volume of a sphere is a function of its radius (in centimeters), and the graph of this relationship is shown here.



1. Is the volume of a cube with edge length $s = 3$ greater or less than the volume of a sphere with radius 3?
2. If a sphere has the same volume as a cube with edge length 5, estimate the radius of the sphere.
3. Compare the outputs of the two volume functions when the inputs are 2.

Are you ready for more?

Estimate the edge length of a cube that has the same volume as a sphere with radius 2.5.

7.4: It’s Not a Race

Elena’s family is driving on the freeway at 55 miles per hour.

Andre’s family is driving on the same freeway, but not at a constant speed. The table shows how far Andre's family has traveled, d, in miles, every minute for 10 minutes.

t	1	2	3	4	5	6	7	8	9	10
d	0.9	1.9	3.0	4.1	5.1	6.2	6.8	7.4	8	9.1

1. How many miles per minute is 55 miles per hour?

2. Who had traveled farther after 5 minutes? After 10 minutes?

3. How long did it take Elena’s family to travel as far as Andre’s family had traveled after 8 minutes?

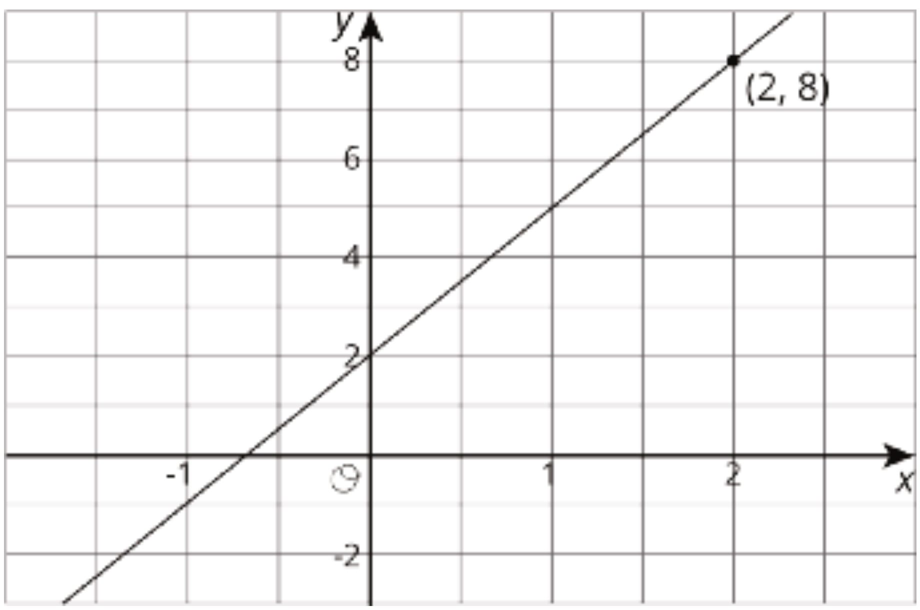
4. For both families, the distance in miles is a function of time in minutes. Compare the outputs of these functions when the input is 3.

Lesson 7 Summary

Functions are all about getting outputs from inputs. For each way of representing a function—equation, graph, table, or verbal description—we can determine the output for a given input.

Let's say we have a function represented by the equation $y = 3x + 2$ where y is the dependent variable and x is the independent variable. If we wanted to find the output that goes with 2, we can input 2 into the equation for x and finding the corresponding value of y. In this case, when x is 2, y is 8 since $3 \cdot 2 + 2 = 8$.

If we had a graph of this function instead, then the coordinates of points on the graph are the input-output pairs. So we would read the y-coordinate of the point on the graph that corresponds to a value of 2 for x. Looking at the graph of this function here, we can see the point $(2, 8)$ on it, so the output is 8 when the input is 2.



A table representing this function shows the input-output pairs directly (although only for select inputs).

x	-1	0	1	2	3
y	-1	2	5	8	11

Again, the table shows that if the input is 2, the output is 8.

Unit 5 Lesson 7 Cumulative Practice Problems

1. The equation and the tables represent two different functions. Use the equation $b = 4a - 5$ and the table to answer the questions. This table represents c as a function of a.

a	-3	0	2	5	10	12
c	-20	7	3	21	19	45

   a. When a is -3, is b or c greater?

   b. When c is 21, what is the value of a? What is the value of b that goes with this value of a?

   c. When a is 6, is b or c greater?

   d. For what values of a do we know that c is greater than b?

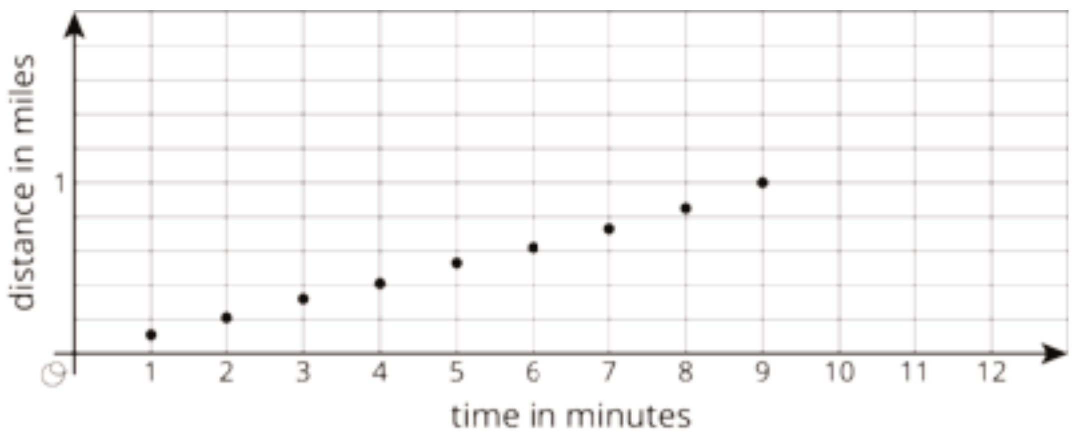
2. Elena and Lin are training for a race. Elena runs her mile at a constant speed of 7.5 miles per hour.

   Lin's total distances are recorded every minute:

time (minutes)	1	2	3	4	5	6	7	8	9
distance (miles)	0.11	0.21	0.32	0.41	0.53	0.62	0.73	0.85	1

   a. Who finished their mile first?

   b. This is a graph of Lin's progress. Draw a graph to represent Elena's mile on the same axes.



   c. For these models, is distance a function of time? Is time a function of distance? Explain how you know.

3. Match each function rule with the value that could not be a possible input for that function.

A. 3 divided by the input	1. 3
B. Add 4 to the input, then divide this value into 3	2. 4
C. Subtract 3 from the input, then divide this value into 1	3. -4
	4. 0
	5. 1

(From Unit 5, Lesson 2.)

4. Find a value of x that makes the equation true. Explain your reasoning, and check that your answer is correct.

$$-(-2x + 1) = 9 - 14x$$

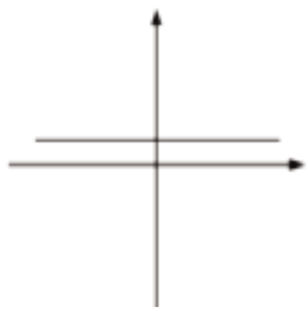
(From Unit 4, Lesson 4.)

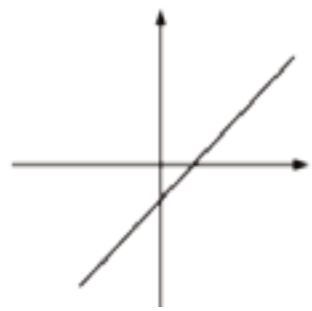
Lesson 8: Linear Functions

Let's investigate linear functions.

8.1: Bigger and Smaller

Diego said that these graphs are ordered from smallest to largest. Mai said they are ordered from largest to smallest. But these are graphs, not numbers! What do you think Diego and Mai are thinking?





8.2: Proportional Relationships Define Linear Functions

1. Jada earns $7 per hour mowing her neighbors' lawns.
   a. Name the two quantities in this situation that are in a functional relationship. Which did you choose to be the independent variable? What is the variable that depends on it?

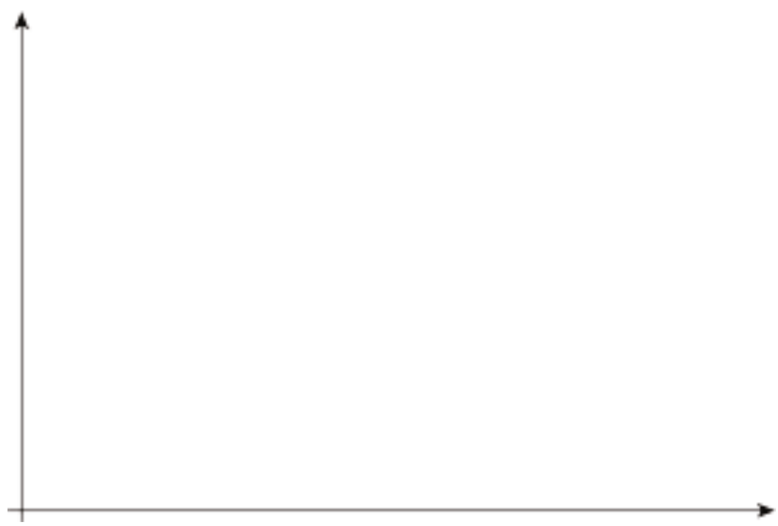
   b. Write an equation that represents the function.

   c. Here is a graph of the function. Label the axes. Label at least two points with input-output pairs.

2. To convert feet to yards, you multiply the number of feet by $\frac{1}{3}$.
   a. Name the two quantities in this situation that are in a functional relationship. Which did you choose to be the independent variable? What is the variable that depends on it?

   b. Write an equation that represents the function.

   c. Draw the graph of the function. Label at least two points with input-output pairs.



8.3: Is it Filling Up or Draining Out?

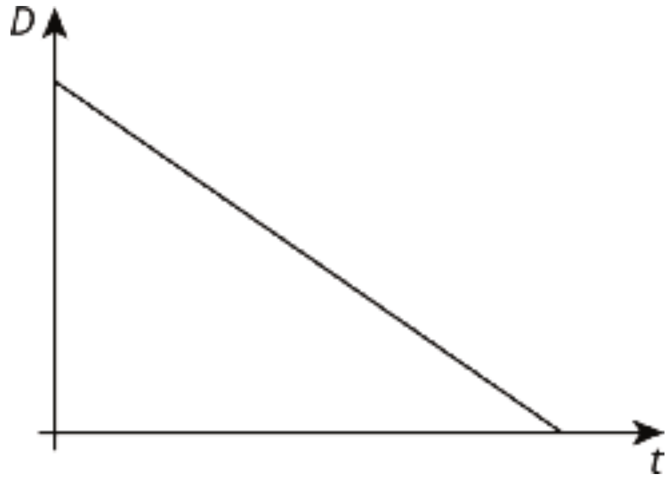
There are four tanks of water.

- The amount of water in gallons, A, in Tank A is given by the function $A = 200 + 8t$, where t is in minutes.
- The amount of water in gallons, B, in Tank B starts at 400 gallons and is decreasing at 5 gallons per minute. These functions work when $t \geq 0$ and $t \leq 80$.

1. Which tank started out with more water?
2. Write an equation representing the relationship between B and t.
3. One tank is filling up. The other is draining out. Which is which? How can you tell?

4. The amount of water in gallons, C, in Tank C is given by the function $C = 800 - 7t$. Is it filling up or draining out? Can you tell just by looking at the equation?

5. The graph of the function for the amount of water in gallons, D, in Tank D at time t is shown. Is it filling up or draining out? How do you know?



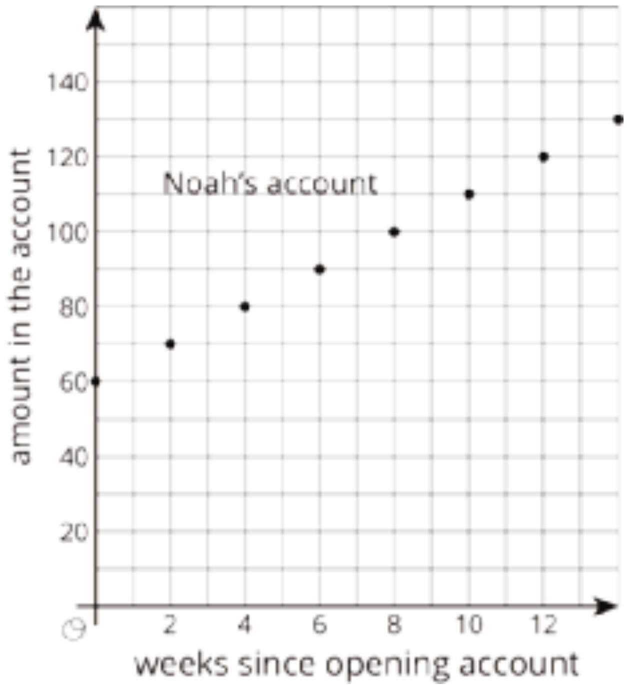
Are you ready for more?

- Pick a tank that was draining out. How long did it take for that tank to drain? What percent full was the tank when 30% of that time had elapsed? When 70% of the time had elapsed?
- What point in the plane is 30% of the way from $(0, 15)$ to $(5, 0)$? 70% of the way?
- What point in the plane is 30% of the way from $(3, 5)$ to $(8, 6)$? 70% of the way?

8.4: Which is Growing Faster?

Noah is depositing money in his account every week to save money. The graph shows the amount he has saved as a function of time since he opened his account.

Elena opened an account the same day as Noah. The amount of money E in her account is given by the function $E = 8w + 60$, where w is the number of weeks since the account was opened.



1. Who started out with more money in their account? Explain how you know.

2. Who is saving money at a faster rate? Explain how you know.

3. How much will Noah save over the course of a year if he does not make any withdrawals? How long will it take Elena to save that much?

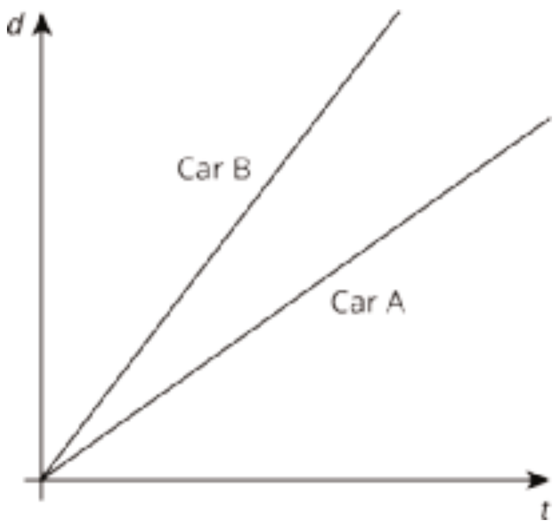
Lesson 8 Summary

Suppose a car is traveling at 30 miles per hour. The relationship between the time in hours and the distance in miles is a proportional relationship. We can represent this relationship with an equation of the form $d = 30t$, where distance is a function of time (since each input of time has exactly one output of distance). Or we could write the equation $t = \frac{1}{30}d$ instead, where time is a function of distance (since each input of distance has exactly one output of time).

More generally, if we represent a linear function with an equation like $y = mx + b$, then b is the initial value (which is 0 for proportional relationships), and m is the rate of change of the function. If m is positive, the function is increasing. If m is negative, the function is decreasing. If we represent a linear function in a different way, say with a graph, we can use what we know about graphs of lines to find the m and b values and, if needed, write an equation.

Unit 5 Lesson 8 Cumulative Practice Problems

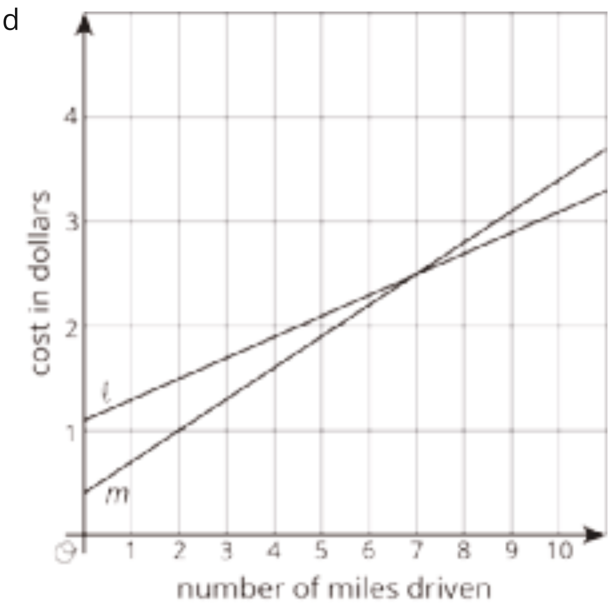
1. Two cars drive on the same highway in the same direction. The graphs show the distance, d, of each one as a function of time, t. Which car drives faster? Explain how you know.



2. Two car services offer to pick you up and take you to your destination. Service A charges 40 cents to pick you up and 30 cents for each mile of your trip. Service B charges $1.10 to pick you up and charges c cents for each mile of your trip.

   a. Match the services to the Lines ℓ and m.

   b. For Service B, is the additional charge per mile greater or less than 30 cents per mile of the trip? Explain your reasoning.



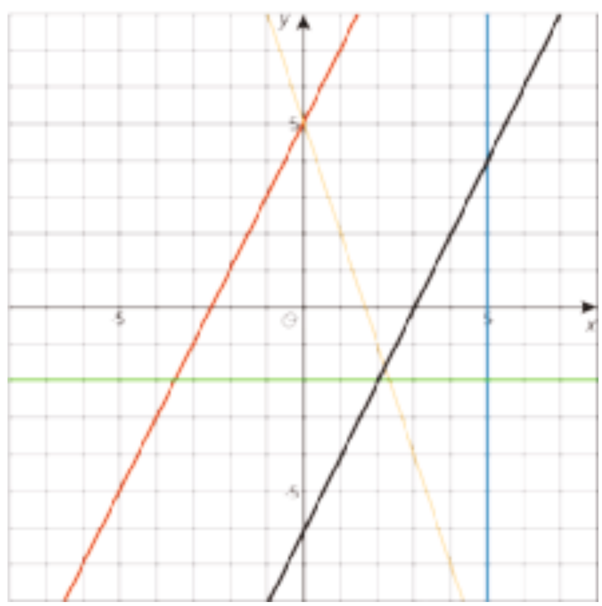
3. Kiran and Clare like to race each other home from school. They run at the same speed, but Kiran's house is slightly closer to school than Clare's house. On a graph, their distance from their homes in meters is a function of the time from when they begin the race in seconds.

    a. As you read the graphs left to right, would the lines go up or down?

    b. What is different about the lines representing Kiran's run and Clare's run?

    c. What is the same about the lines representing Kiran's run and Clare's run?

4. Write an equation for each line.



(From Unit 3, Lesson 11.)

Lesson 9: Linear Models

Let's model situations with linear functions.

9.1: Candlelight

A candle is burning. It starts out 12 inches long. After 1 hour, it is 10 inches long. After 3 hours, it is 5.5 inches long.

1. When do you think the candle will burn out completely?
2. Is the height of the candle a function of time? If yes, is it a linear function? Explain your thinking.

9.2: Shadows

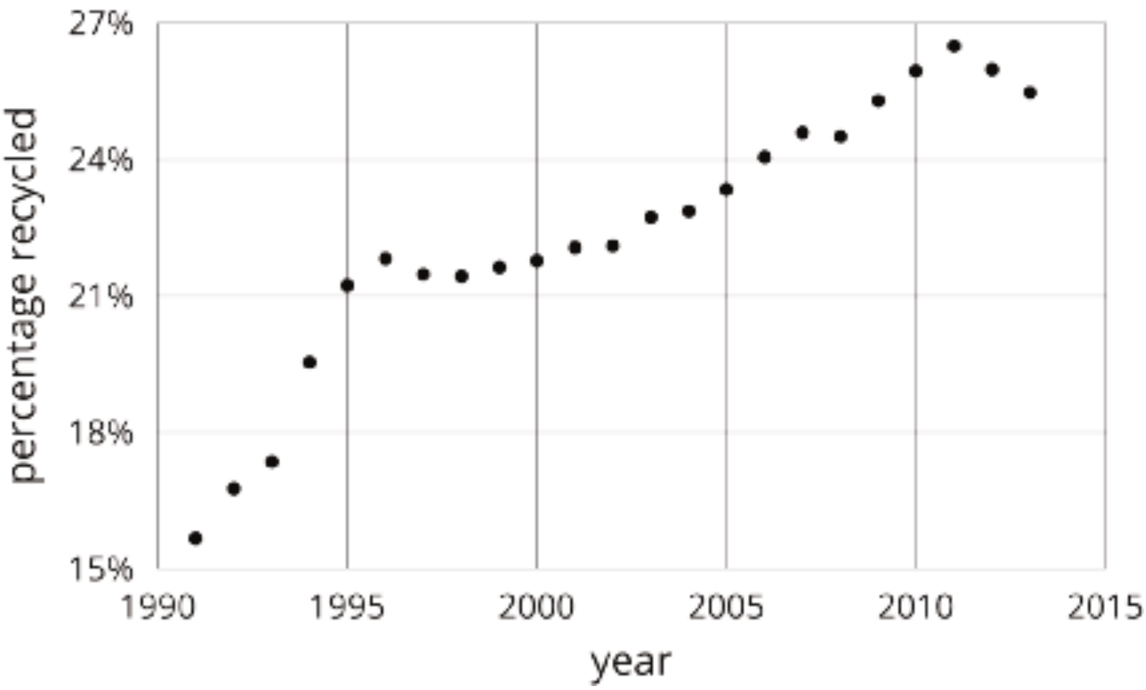
When the Sun was directly overhead, the stick had no shadow. After 20 minutes, the shadow was 10.5 cm long. After 60 minutes, it was 26 cm long.



1. Based on this information, estimate how long it will be after 95 minutes.
2. After 95 minutes, the shadow measured 38.5 cm. How does this compare to your estimate?
3. Is the length of the shadow a function of time? If so, is it linear? Explain your reasoning.

9.3: Recycling

In an earlier lesson, we saw this graph that shows the percentage of all garbage in the U.S. that was recycled between 1991 and 2013.



1. Sketch a linear function that models the change in the percentage of garbage that was recycled between 1991 and 1995. For which years is the model good at predicting the percentage of garbage that is produced? For which years is it not as good?

2. Pick another time period to model with a sketch of a linear function. For which years is the model good at making predictions? For which years is it not very good?

Lesson 9 Summary

Water has different boiling points at different elevations. At 0 m above sea level, the boiling point is 100° C. At 2,500 m above sea level, the boiling point is 91.3˚ C. If we assume the boiling point of water is a linear function of elevation, we can use these two data points to calculate the slope of the line:

$$m = \frac{91.3 - 100}{2{,}500 - 0} = \frac{-8.7}{2{,}500}$$

This slope means that for each increase of 2,500 m, the boiling point of water decreases by 8.7° C. Next, we already know the y-intercept is 100° C from the first point, so a linear equation representing the data is

$$y = \frac{-8.7}{2{,}500}x + 100$$

This equation is an example of a mathematical *model*. A mathematical model is a mathematical object like an equation, a function, or a geometric figure that we use to represent a real-life situation. Sometimes a situation can be modeled by a linear function. We have to use judgment about whether this is a reasonable thing to do based on the information we are given. We must also be aware that the model may make imprecise predictions, or may only be appropriate for certain ranges of values.

Testing our model for the boiling point of water, it accurately predicts that at an elevation of 1,000 m above sea level (when $x = 1{,}000$), water will boil at 96.5° C since $y = \frac{-8.7}{2500} \cdot 1000 + 100 = 96.5$. For higher elevations, the model is not as accurate, but it is still close. At 5,000 m above sea level, it predicts 82.6° C, which is 0.6° C off the actual value of 83.2° C. At 9,000 m above sea level, it predicts 68.7° C, which is about 3° C less than the actual value of 71.5° C. The model continues to be less accurate at even higher elevations since the relationship between the boiling point of water and elevation isn't linear, but for the elevations in which most people live, it's pretty good.

Unit 5 Lesson 9 Cumulative Practice Problems

1. On the first day after the new moon, 2% of the Moon's surface is illuminated. On the second day, 6% is illuminated.

   a. Based on this information, predict the day on which the Moon’s surface is 50% illuminated and 100% illuminated.

   b. The Moon’s surface is 100% illuminated on day 14. Does this agree with the prediction you made?

   c. Is the percentage illumination of the Moon’s surface a linear function of the day?

2. In science class, Jada uses a graduated cylinder with water in it to measure the volume of some marbles. After dropping in 4 marbles so they are all under water, the water in the cylinder is at a height of 10 milliliters. After dropping in 6 marbles so they are all under water, the water in the cylinder is at a height of 11 milliliters.

   a. What is the volume of 1 marble?

   b. How much water was in the cylinder before any marbles were dropped in?

   c. What should be the height of the water after 13 marbles are dropped in?

   d. Is the relationship between volume of water and number of marbles a linear relationship? If so, what does the slope of a line representing this relationship mean? If not, explain your reasoning.

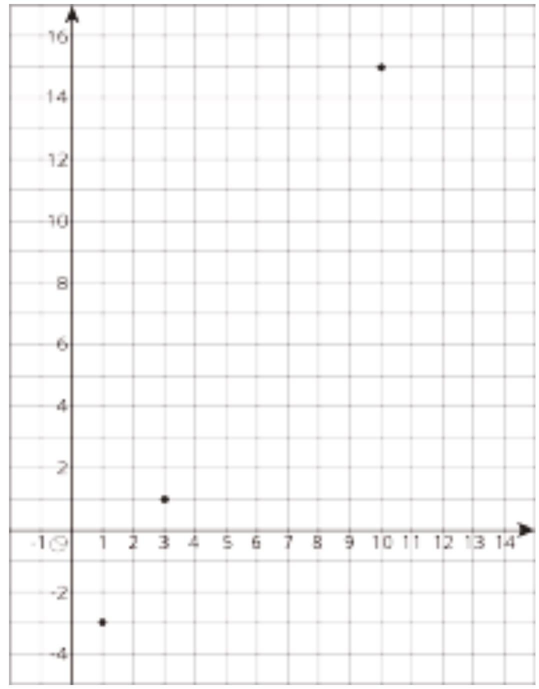
3. Solve each of these equations. Explain or show your reasoning.

   $2(3x + 2) = 2x + 28$ $\quad$ $5y + 13 = -43 - 3y$ $\quad$ $4(2a + 2) = 8(2 - 3a)$

   (From Unit 4, Lesson 5.)

4. For a certain city, the high temperatures (in degrees Celsius) are plotted against the number of days after the new year.

   Based on this information, is the high temperature in this city a linear function of the number of days after the new year?



5. The school designed their vegetable garden to have a perimeter of 32 feet with the length measuring two feet more than twice the width.

   a. Using ℓ to represent the length of the garden and w to represent its width, write and solve a system of equations that describes this situation.

   b. What are the dimensions of the garden?

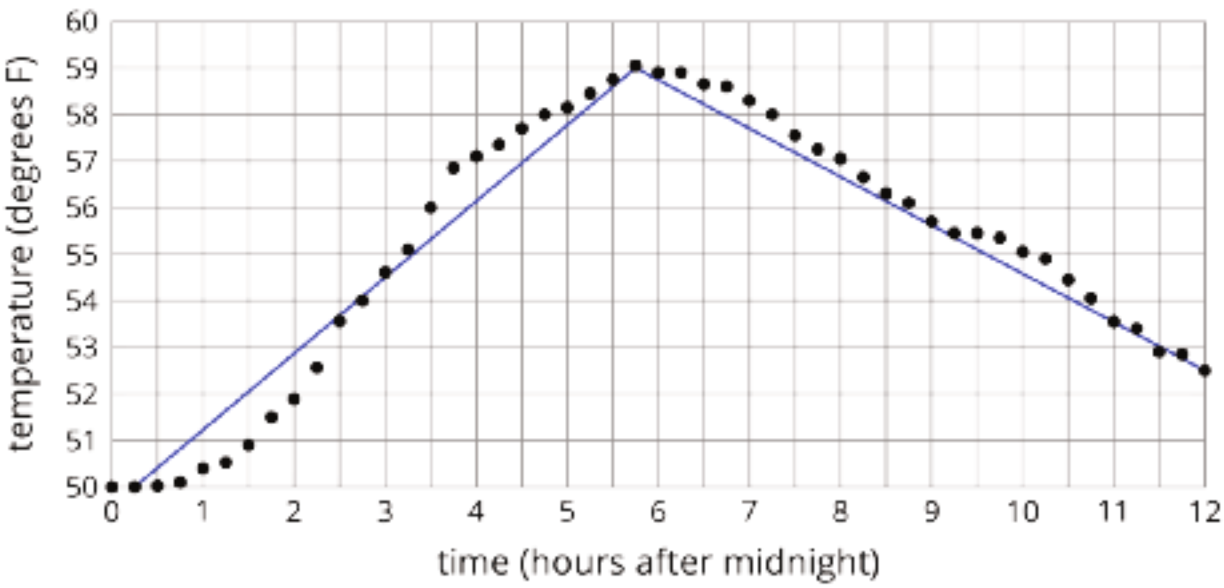
(From Unit 4, Lesson 15.)

Lesson 10: Piecewise Linear Functions

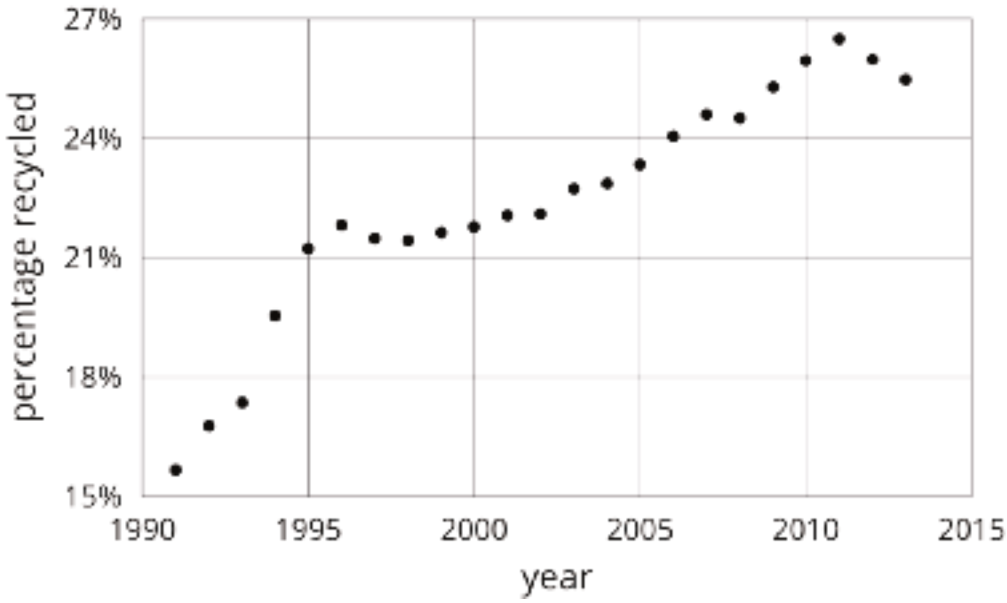
Let’s explore functions built out of linear pieces.

10.1: Notice and Wonder: Lines on Dots

What do you notice? What do you wonder?



10.2: Modeling Recycling

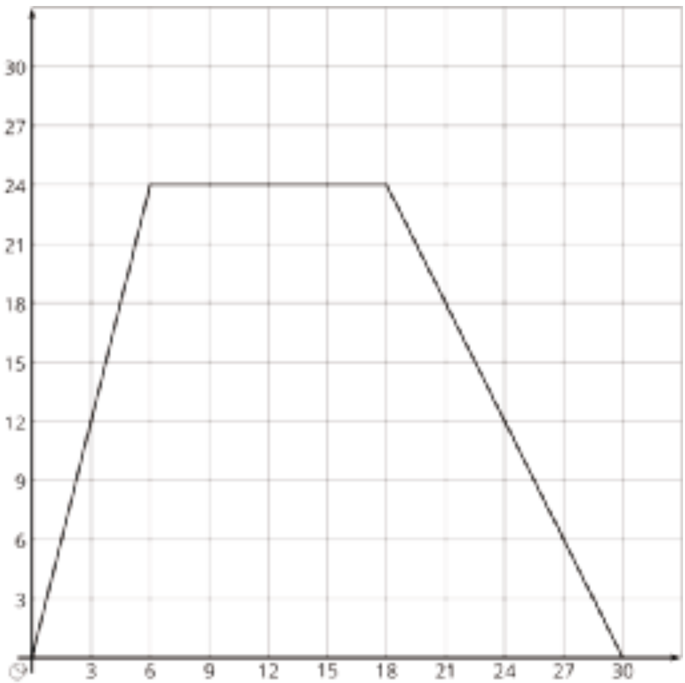


1. Approximate the percentage recycled each year with a piecewise linear function by drawing between three and five line segments to approximate the graph.

2. Find the slope for each piece. What do these slopes tell you?

10.3: Dog Bath

Elena filled up the tub and gave her dog a bath. Then she let the water out of the tub.



1. The graph shows the amount of water in the tub, in gallons, as a function of time, in minutes. Add labels to the graph to show this.

2. When did she turn off the water faucet?

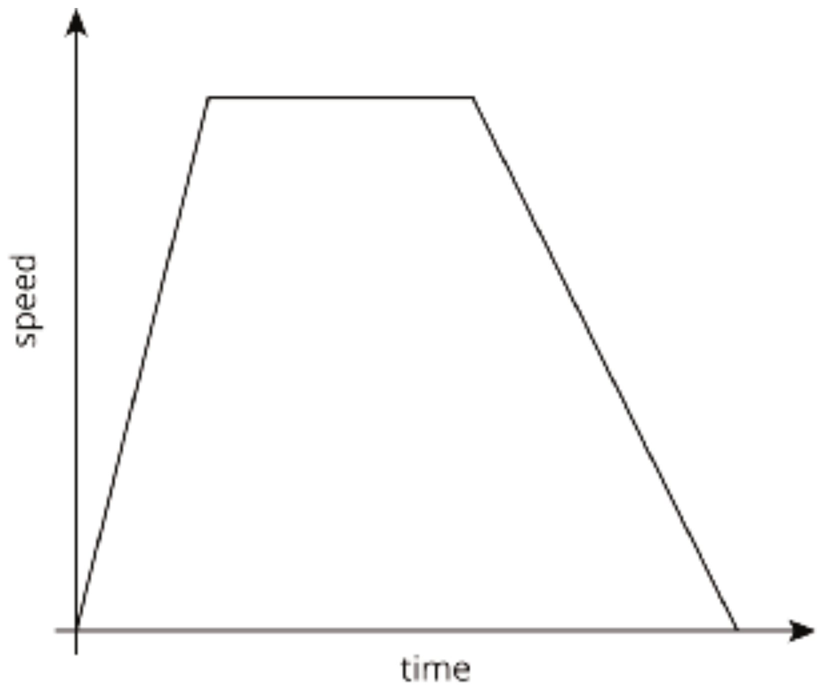
3. How much water was in the tub when she bathed her dog?

4. How long did it take for the tub to drain completely?

5. At what rate did the faucet fill the tub?

6. At what rate did the water drain from the tub?

10.4: Distance and Speed

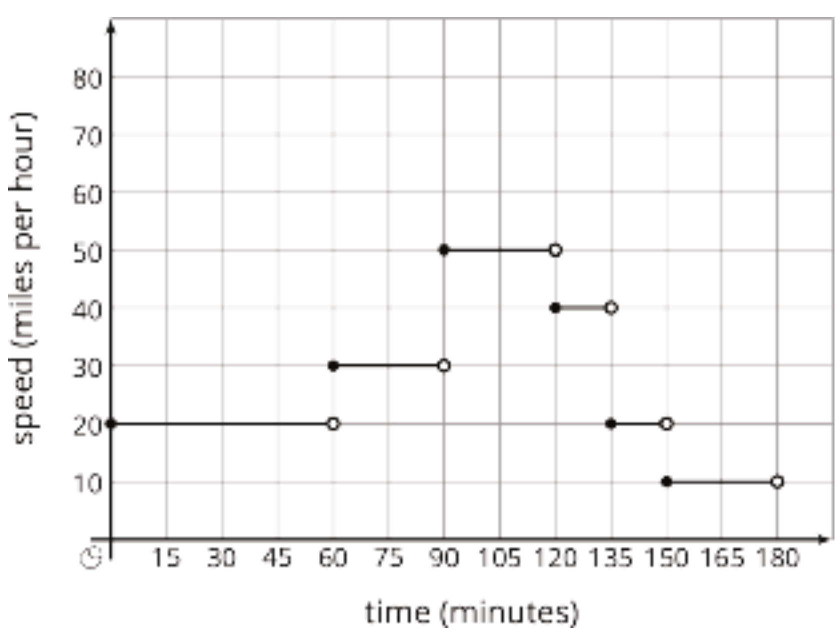


The graph shows the speed of a car as a function of time. Describe what a person watching the car would see.

Are you ready for more?

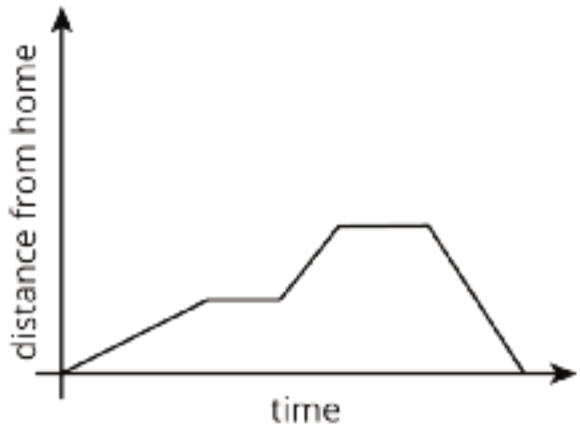
The graph models the speed of a car over a function of time during a 3-hour trip. How far did the car go over the course of the trip?

There is a nice way to visualize this quantity in terms of the graph. Can you find it?



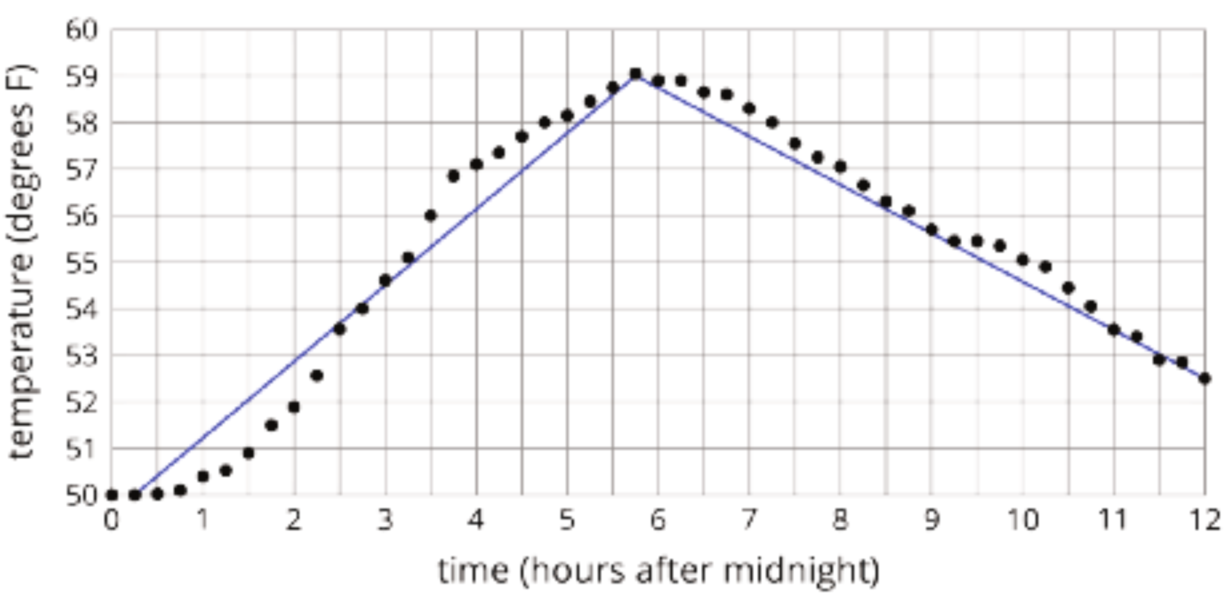
Lesson 10 Summary

This graph shows Andre biking to his friend's house where he hangs out for a while. Then they bike together to the store to buy some groceries before racing back to Andre's house for a movie night. Each line segment in the graph represents a different part of Andre's travels.



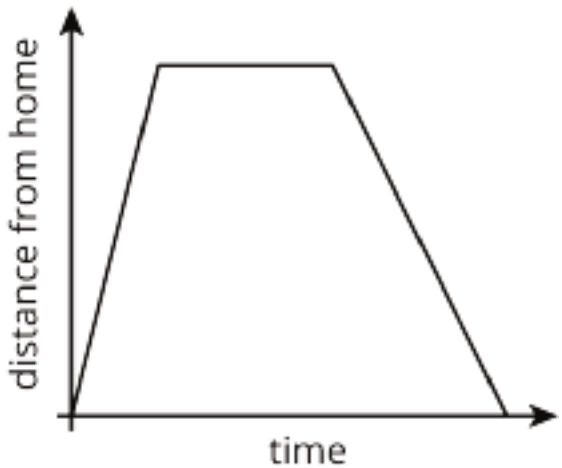
This is an example of a piecewise linear function, which is a function whose graph is pieced together out of line segments. It can be used to model situations in which a quantity changes at a constant rate for a while, then switches to a different constant rate.

We can use piecewise functions to represent stories, or we can use them to model actual data. In the second example, temperature recordings at several times throughout a day are modeled with a piecewise function made up of two line segments. Which line segment do you think does the best job of modeling the data?



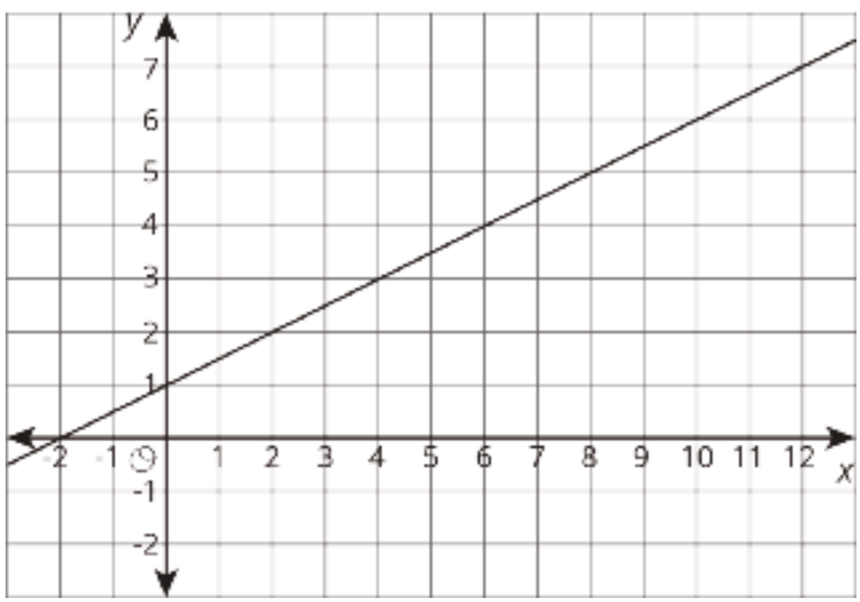
Unit 5 Lesson 10 Cumulative Practice Problems

1. The graph shows the distance of a car from home as a function of time.



Describe what a person watching the car may be seeing.

2. The equation and the graph represent two functions. Use the equation $y = 4$ and the graph to answer the questions.

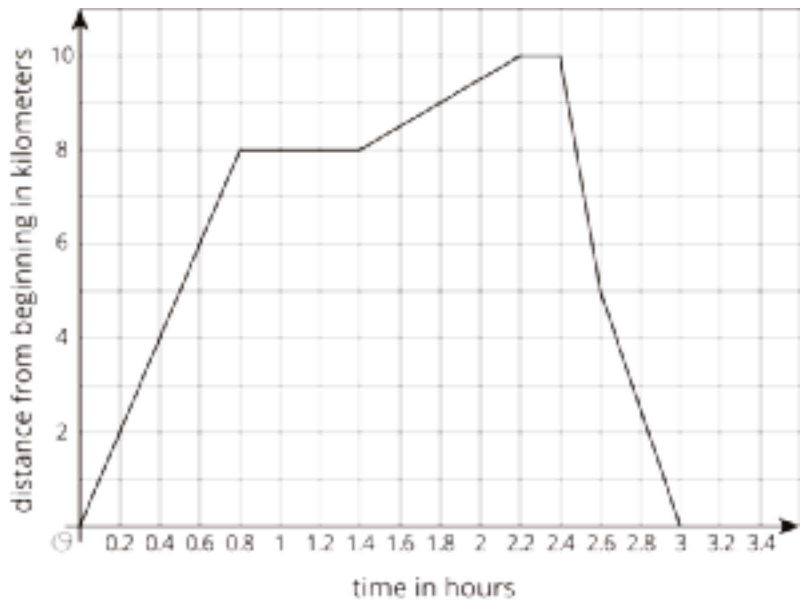


   a. When x is 4, is the output of the equation or the graph greater?

   b. What value for x produces the same output in both the graph and the equation?

(From Unit 5, Lesson 7.)

3. This graph shows a trip on a bike trail. The trail has markers every 0.5 km showing the distance from the beginning of the trail.



   a. When was the bike rider going the fastest?

   b. When was the bike rider going the slowest?

   c. During what times was the rider going away from the beginning of the trail?

   d. During what times was the rider going back towards the beginning of the trail?

   e. During what times did the rider stop?

4. The expression $-25t + 1250$ represents the volume of liquid of a container after t seconds. The expression $50t + 250$ represents the volume of liquid of another container after t seconds. What does the equation $-25t + 1250 = 50t + 250$ mean in this situation?

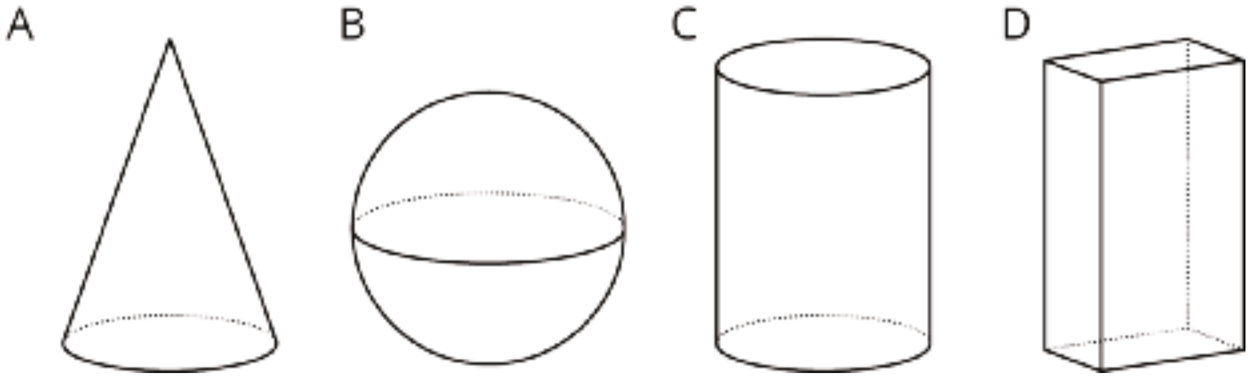
(From Unit 4, Lesson 9.)

Lesson 11: Filling Containers

Let's fill containers with water.

11.1: Which One Doesn't Belong: Solids

These are drawings of three-dimensional objects. Which one doesn't belong? Explain your reasoning.



11.2: Height and Volume

Your teacher will give you a graduated cylinder, water, and some other supplies. Your group will use these supplies to investigate the height of water in the cylinder as a function of the water volume.

1. Before you get started, make a prediction about the shape of the graph.

2. Fill the cylinder with different amounts of water and record the data in the table.

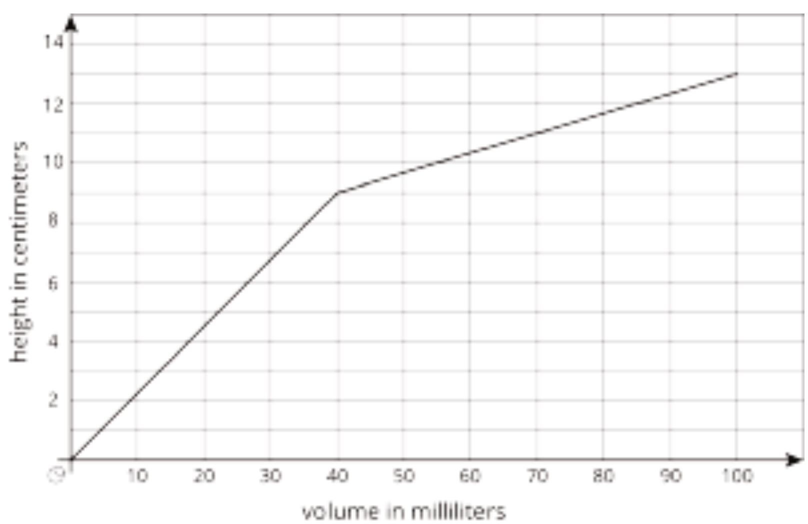
volume (ml)						
height (cm)						

3. Create a graph that shows the height of the water in the cylinder as a function of the water volume.

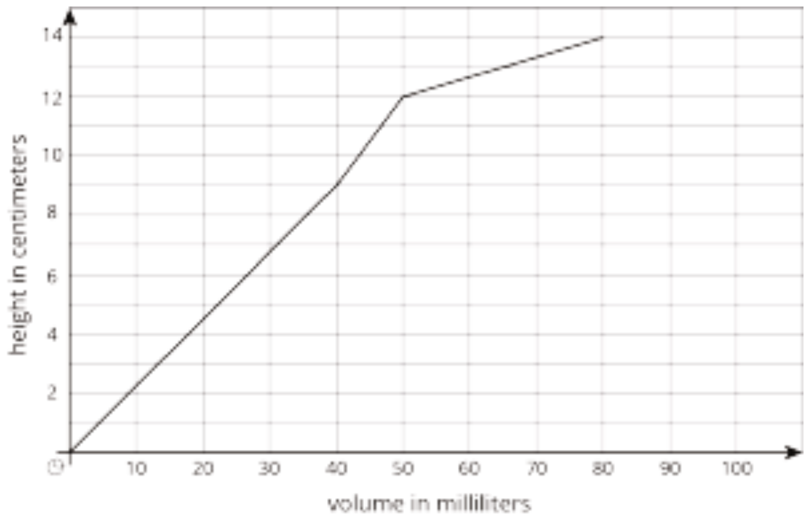
4. Choose a point on the graph and explain its meaning in the context of the situation.

11.3: What Is the Shape?

1. The graph shows the height vs. volume function of an unknown container. What shape could this container have? Explain how you know and draw a possible container.



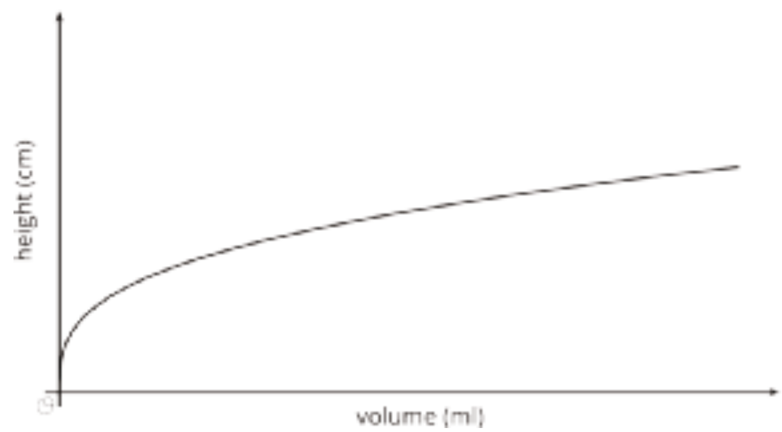
2. The graph shows the height vs. volume function of a different unknown container. What shape could this container have? Explain how you know and draw a possible container.



3. How are the two containers similar? How are they different?

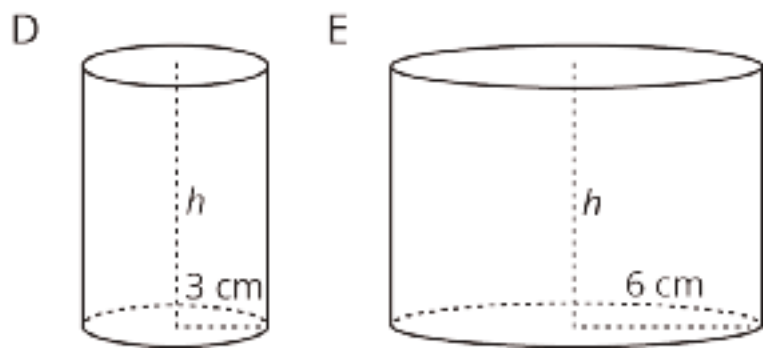
Are you ready for more?

The graph shows the height vs. volume function of an unknown container. What shape could this container have? Explain how you know and draw a possible container.



Lesson 11 Summary

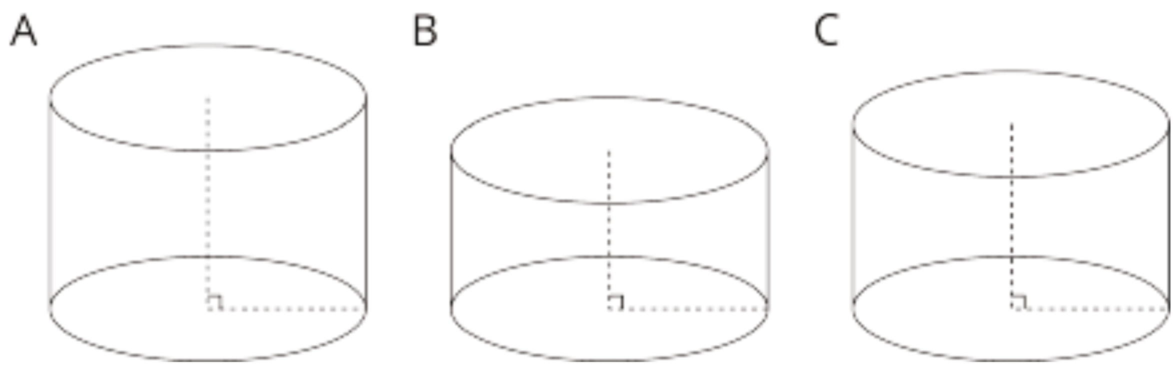
When filling a shape like a cylinder with water, we can see how the dimensions of the cylinder affect things like the changing height of the water. For example, let's say we have two cylinders, D and E, with the same height, but D has a radius of 3 cm and E has a radius of 6 cm.



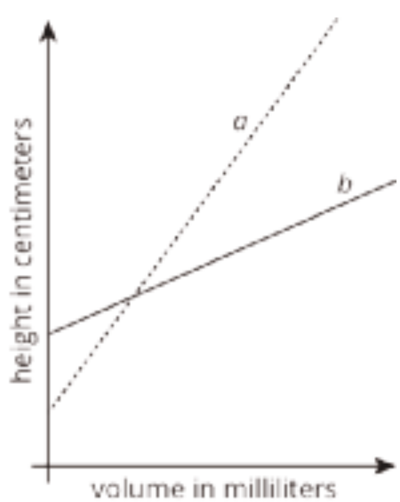
If we pour water into both cylinders at the same rate, the height of water in D will increase faster than the height of water in E due to its smaller radius. This means that if we made graphs of the height of water as a function of the volume of water for each cylinder, we would have two lines and the slope of the line for cylinder D would be greater than the slope of the line for cylinder E.

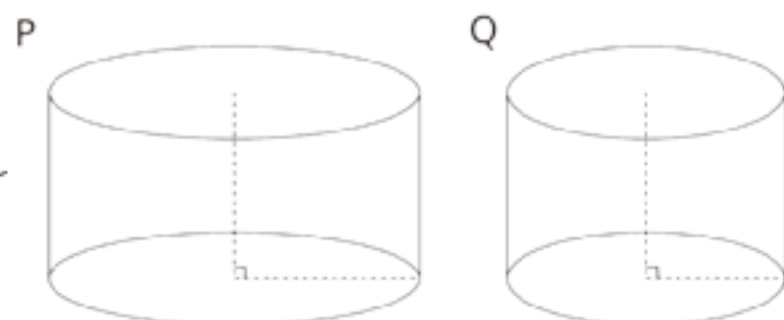
Unit 5 Lesson 11 Cumulative Practice Problems

1. Cylinder A, B, and C have the same radius but different heights. Put the cylinders in order of their volume from least to greatest.

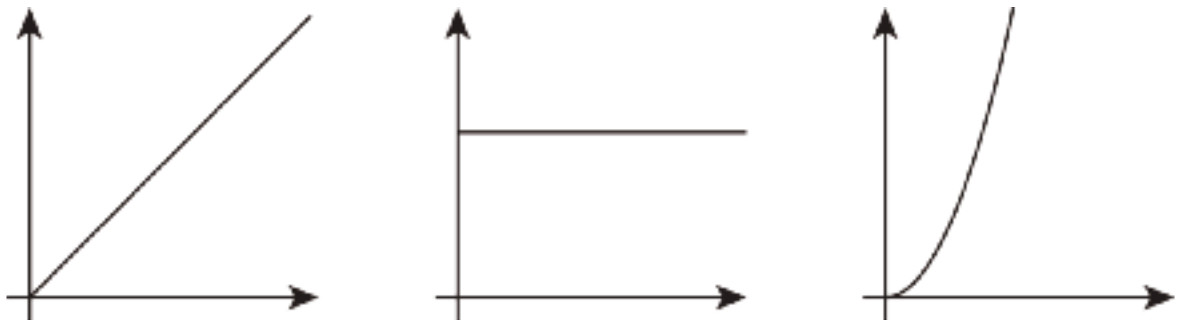


2. Two cylinders, a and b, each started with different amounts of water. The graph shows how the height of the water changed as the volume of water increased in each cylinder. Match the graphs of a and b to Cylinders P and Q. Explain your reasoning.

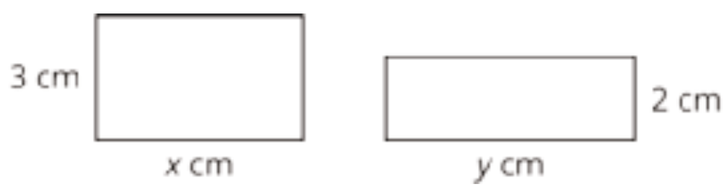




3. Which of the following graphs could represent the volume of water in a cylinder as a function of its height? Explain your reasoning.



4. Together, the areas of the rectangles sum to 30 square centimeters.



a. Write an equation showing the relationship between x and y.

b. Fill in the table with the missing values.

x	3		8		12
y		5		10	

(From Unit 5, Lesson 3.)

Lesson 12: How Much Will Fit?

Let’s reason about the volume of different shapes.

12.1: Two Containers

Your teacher will show you some containers. The small container holds 200 beans. Estimate how many beans the large jar holds.

12.2: What’s Your Estimate?

Your teacher will show you some containers.

1. If the pasta box holds 8 cups of rice, how much rice would you need for the other rectangular prisms?

2. If the pumpkin can holds 15 fluid ounces of rice, how much do the other cylinders hold?

3. If the small **cone** holds 2 fluid ounces of rice, how much does the large cone hold?

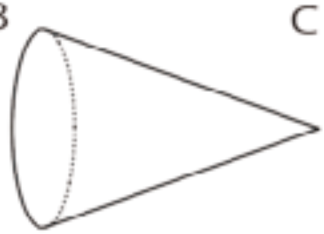
4. If the golf ball were hollow, it would hold about 0.2 cups of water. If the baseball were hollow, how much would the **sphere** hold?

12.3: Do You Know These Figures?

- What shapes are the faces of each type of object shown here? For example, all six faces of a cube are squares.

A

B



C

D

1. Which faces could be referred to as a “base” of the object?

2. Here is a method for quickly sketching a cylinder:

- Draw two ovals.
- Connect the edges.
- Which parts of your drawing would be hidden behind the cylinder? Make these parts dashed lines.

- Practice sketching some cylinders. Sketch a few different sizes, including short, tall, narrow, wide, and sideways. Label the radius r and height h on each cylinder.

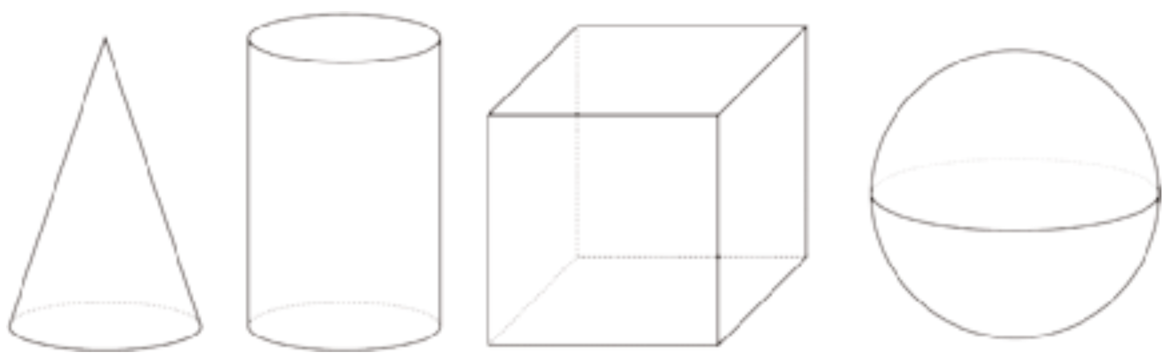
Are you ready for more?

A soccer ball is a polyhedron with 12 black pentagonal faces and 20 white hexagonal faces. How many edges in total are on this polyhedron?

Lesson 12 Summary

The volume of a three-dimensional figure, like a jar or a room, is the amount of space the shape encloses. We can measure volume by finding the number of equal-sized volume units that fill the figure without gaps or overlaps. For example, we might say that a room has a volume of 1,000 cubic feet, or that a pitcher can carry 5 gallons of water. We could even measure volume of a jar by the number of beans it could hold, though a bean count is not really a measure of the volume in the same way that a cubic centimeter is because there is space between the beans. (The number of beans that fit in the jar do depend on the volume of the jar, so it is an okay estimate when judging the relative sizes of containers.)

In earlier grades, we studied three-dimensional figures with flat faces that are polygons. We learned how to calculate the volumes of rectangular prisms. Now we will study three-dimensional figures with circular faces and curved surfaces: cones, cylinders, and spheres.



To help us see the shapes better, we can use dotted lines to represent parts that we wouldn't be able to see if a solid physical object were in front of us. For example, if we think of the cylinder in this picture as representing a tin can, the dotted arc in the bottom half of that cylinder represents the back half of the circular base of the can. What objects could the other figures in the picture represent?

Unit 5 Lesson 12 Cumulative Practice Problems

1. a. Sketch a cube and label its side length as 4 cm (this will be Cube A).

   b. Sketch a cube with sides that are twice as long as Cube A and label its side length (this will be Cube B).

   c. Find the volumes of Cube A and Cube B.

2. Two paper drink cups are shaped like cones. The small cone can hold 6 oz of water. The large cone is $\frac{4}{3}$ the height and $\frac{4}{3}$ the diameter of the small cone. Which of these could be the amount of water the large cone holds?

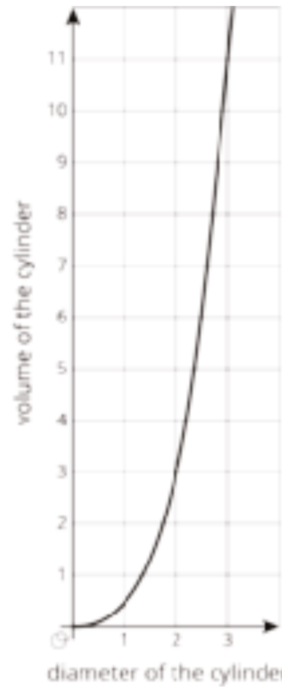
   A. 8 cm

   B. 14 oz

   C. 4.5 oz

   D. 14 cm

3. The graph represents the volume of a cylinder with a height equal to its radius.



a. When the diameter is 2 cm, what is the radius of the cylinder?

b. Express the volume of a cube of side length s as an equation.

c. Make a table for volume of the cube at $s = 0$ cm, $s = 1$ cm, $s = 2$ cm, and $s = 3$ cm.

d. Which volume is greater: the volume of the cube when $s = 3$ cm, or the volume of the cylinder when its diameter is 3 cm?

(From Unit 5, Lesson 7.)

4. Select **all** the points that are on a line with slope 2 that also contains the point $(2, \text{-}1)$.

A. $(3, 1)$

B. $(1, 1)$

C. $(1, \text{-}3)$

D. $(4, 0)$

E. $(6, 7)$

(From Unit 3, Lesson 10.)

5. Several glass aquariums of various sizes are for sale at a pet shop. They are all shaped like rectangular prisms. A 15-gallon tank is 24 inches long, 12 inches wide, and 12 inches tall. Match the dimensions of the other tanks with the volume of water they can each hold.

   A. Tank 1: 36 inches long, 18 inches wide, and 12 inches tall

   B. Tank 2: 16 inches long, 8 inches wide, and 10 inches tall

   C. Tank 3: 30 inches long, 12 inches wide, and 12 inches tall

   D. Tank 4: 20 inches long, 10 inches wide, and 12 inches tall

   1. 5 gallons
   2. 10 gallons
   3. 20 gallons
   4. 30 gallons

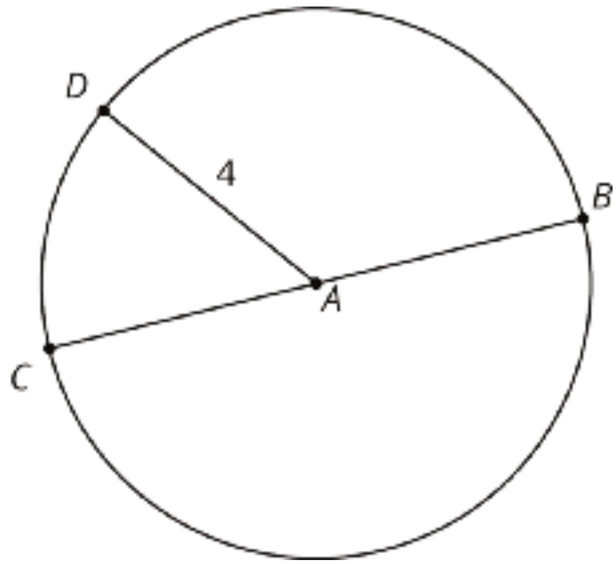
6. Solve: $\begin{cases} y = \text{-}2x - 20 \\ y = x + 4 \end{cases}$

   (From Unit 4, Lesson 14.)

Lesson 13: The Volume of a Cylinder

Let’s explore cylinders and their volumes.

13.1: A Circle's Dimensions



Here is a circle. Points A, B, C, and D are drawn, as well as Segments AD and BC.

1. What is the area of the circle, in square units? Select all that apply.
    a. 4π

    b. $\pi 8$

    c. 16π

    d. $\pi 4^2$

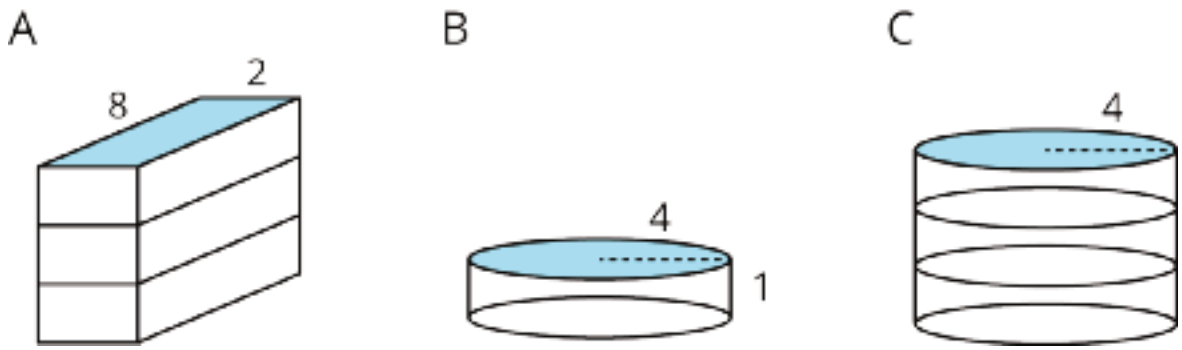
    e. approximately 25

    f. approximately 50

2. If the area of a circle is 49π square units, what is its radius? Explain your reasoning.

13.2: Circular Volumes

What is the volume of each figure, in cubic units? Even if you aren't sure, make a reasonable guess.



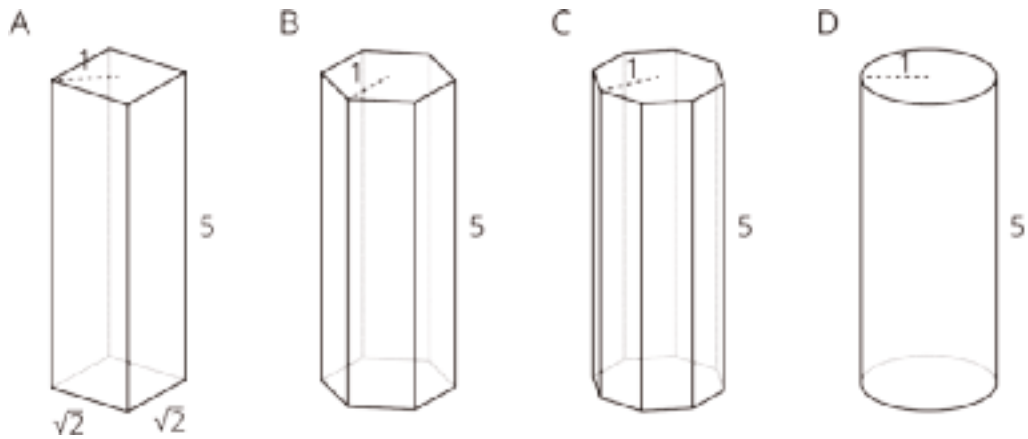
1. Figure A: A rectangular prism whose base has an area of 16 square units and whose height is 3 units.

2. Figure B: A cylinder whose base has an area of 16π square units and whose height is 1 unit.

3. Figure C: A cylinder whose base has an area of 16π square units and whose height is 3 units.

Are you ready for more?

prism	prism	prism	cylinder
base: square	base: hexagon	base: octagon	base: circle



Here are solids that are related by a common measurement. In each of these solids, the distance from the center of the base to the furthest edge of the base is 1 unit, and the height of the solid is 5 units. Use 3.14 as an approximation for π to solve these problems.

1. Find the area of the square base and the circular base.

2. Use these areas to compute the volumes of the rectangular prism and the cylinder. How do they compare?

3. Without doing any calculations, list the figures from smallest to largest by volume. Use the images and your knowledge of polygons to explain your reasoning.

4. The area of the hexagon is approximately 2.6 square units, and the area of the octagon is approximately 2.83 square units. Use these areas to compute the volumes of the prisms with the hexagon and octagon bases. How does this match your explanation to the previous question?

13.3: A Cylinder's Dimensions

1. For cylinders A–D, sketch a radius and the height. Label the radius with an r and the height with an h.

A

B

C

D

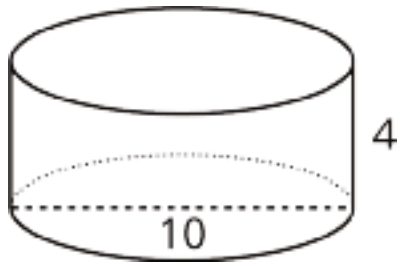
E

F

2. Earlier you learned how to sketch a cylinder. Sketch cylinders for E and F and label each one’s radius and height.

13.4: A Cylinder's Volume

1. Here is a cylinder with height 4 units and diameter 10 units.



   a. Shade the cylinder’s base.

   b. What is the area of the cylinder’s base? Express your answer in terms of π.

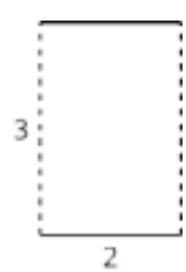
   c. What is the volume of this cylinder? Express your answer in terms of π.

2. A silo is a cylindrical container that is used on farms to hold large amounts of goods, such as grain. On a particular farm, a silo has a height of 18 feet and diameter of 6 feet. Make a sketch of this silo and label its height and radius. How many cubic feet of grain can this silo hold? Use 3.14 as an approximation for π.

Are you ready for more?

One way to construct a cylinder is to take a rectangle (for example, a piece of paper), curl two opposite edges together, and glue them in place.

Which would give the cylinder with the greater volume: Gluing the two dashed edges together, or gluing the two solid edges together?



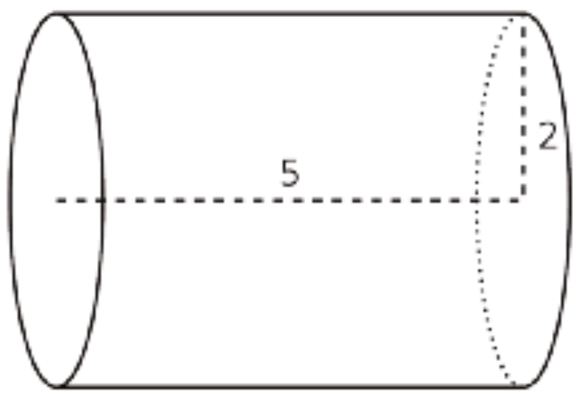
Lesson 13 Summary

We can find the volume of a cylinder with radius r and height h using two ideas we've seen before:

- The volume of a rectangular prism is a result of multiplying the area of its base by its height.
- The base of the cylinder is a circle with radius r, so the base area is πr^2.

Remember that π is the number we get when we divide the circumference of any circle by its diameter. The value of π is approximately 3.14.

Just like a rectangular prism, the volume of a cylinder is the area of the base times the height. For example, take a cylinder whose radius is 2 cm and whose height is 5 cm.



The base has an area of 4π cm^2 (since $\pi \cdot 2^2 = 4\pi$), so the volume is 20π cm^3 (since $4\pi \cdot 5 = 20\pi$). Using 3.14 as an approximation for π, we can say that the volume of the cylinder is approximately 62.8 cm^3.

In general, the base of a cylinder with radius r units has area πr^2 square units. If the height is h units, then the volume V in cubic units is

$$V = \pi r^2 h$$

Unit 5 Lesson 13 Cumulative Practice Problems

1.
    - Sketch a cylinder.
    - Label its radius 3 and its height 10.
    - Shade in one of its bases.
2. At a farm, animals are fed bales of hay and buckets of grain. Each bale of hay is in the shape a rectangular prism. The base has side lengths 2 feet and 3 feet, and the height is 5 feet. Each bucket of grain is a cylinder with a diameter of 3 feet. The height of the bucket is 5 feet, the same as the height of the bale.
    a. Which is larger in area, the rectangular base of the bale or the circular base of the bucket? Explain how you know.

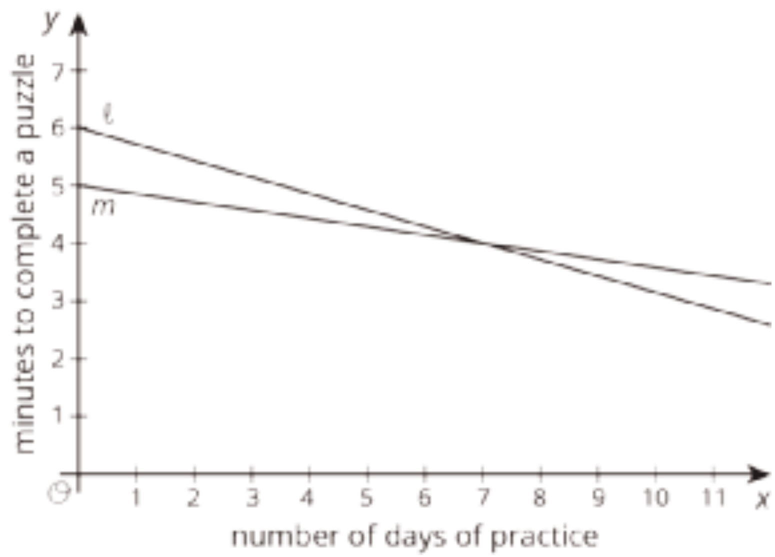
    b. Which is larger in volume, the bale or the bucket? Explain how you know.
3. Three cylinders have a height of 8 cm. Cylinder 1 has a radius of 1 cm. Cylinder 2 has a radius of 2 cm. Cylinder 3 has a radius of 3 cm. Find the volume of each cylinder.
4. A one-quart container of tomato soup is shaped like a rectangular prism. A soup bowl shaped like a hemisphere can hold 8 oz of liquid. How many bowls will the soup container fill? Recall that 1 quart is equivalent to 32 fluid ounces (oz).

    (From Unit 5, Lesson 12.)

5. Match each set of information about a circle with the area of that circle.

A. Circle A has a radius of 4 units.	1. 4π square units
B. Circle B has a radius of 10 units.	2. approximately 314 square units
C. Circle C has a diameter of 16 units.	3. 64π square units
D. Circle D has a circumference of 4π units.	4. 16π square units

6. Two students join a puzzle solving club and get faster at finishing the puzzles as they get more practice. Student A improves their times faster than Student B.



a. Match the students to the Lines ℓ and m.

b. Which student was faster at puzzle solving before practice?

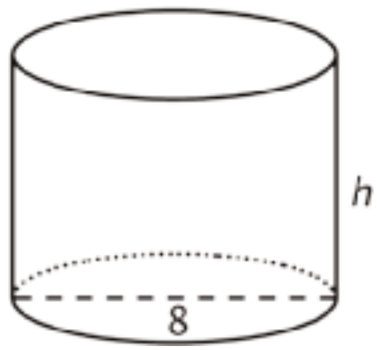
(From Unit 5, Lesson 8.)

Lesson 14: Finding Cylinder Dimensions

Let's figure out the dimensions of cylinders.

14.1: A Cylinder of Unknown Height

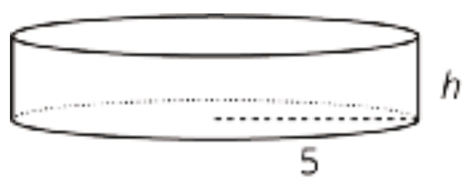
What is a possible volume for this cylinder if the diameter is 8 cm? Explain your reasoning.



14.2: What's the Dimension?

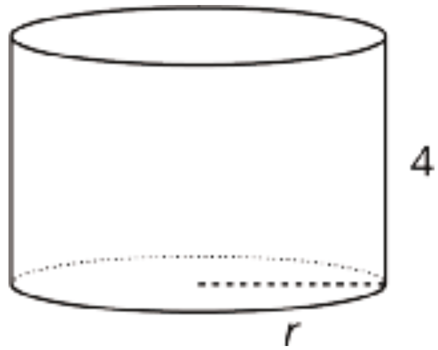
The volume V of a cylinder with radius r is given by the formula $V = \pi r^2 h$.

1. The volume of this cylinder with radius 5 units is 50π cubic units. This statement is true: $50\pi = 5^2 \pi h$



What does the height of this cylinder have to be? Explain how you know.

2. The volume of this cylinder with height 4 units is 36π cubic units. This statement is true: $36\pi = r^2 \pi 4$



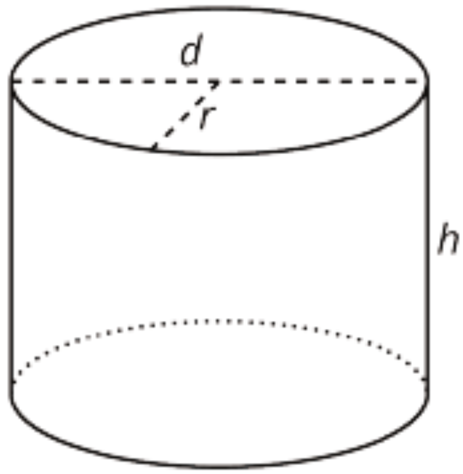
What does the radius of this cylinder have to be? Explain how you know.

Are you ready for more?

Suppose a cylinder has a volume of 36π cubic inches, but it is not the same cylinder as the one you found earlier in this activity.

1. What are some possibilities for the dimensions of the cylinder?
2. How many different cylinders can you find that have a volume of 36π cubic inches?

14.3: Cylinders with Unknown Dimensions



Each row of the table has information about a particular cylinder. Complete the table with the missing dimensions.

diameter (units)	radius (units)	area of the base (square units)	height (units)	volume (cubic units)
	3		5	
12				108π
			11	99π
8				16π
			100	16π
	10			20π
20				314
			b	$\pi \cdot b \cdot a^2$

Lesson 14 Summary

In an earlier lesson we learned that the volume, V, of a cylinder with radius r and height h is

$$V = \pi r^2 h$$

We say that the volume depends on the radius and height, and if we know the radius and height, we can find the volume. It is also true that if we know the volume and one dimension (either radius or height), we can find the other dimension.

For example, imagine a cylinder that has a volume of 500π cm^3 and a radius of 5 cm, but the height is unknown. From the volume formula we know that

$$500\pi = \pi \cdot 25 \cdot h$$

must be true. Looking at the structure of the equation, we can see that $500 = 25h$. That means that the height has to be 20 cm, since $500 \div 25 = 20$.

Now imagine another cylinder that also has a volume of 500π cm^3 with an unknown radius and a height of 5 cm. Then we know that

$$500\pi = \pi \cdot r^2 \cdot 5$$

must be true. Looking at the structure of this equation, we can see that $r^2 = 100$. So the radius must be 10 cm.

Unit 5 Lesson 14 Cumulative Practice Problems

1. Complete the table with all of the missing information about three different cylinders.

diameter of base (units)	area of base (square units)	height (units)	volume (cubic units)
4		10	
6			63π
	25π	6	

2. A cylinder has volume 45π and radius 3. What is its height?

3. Three cylinders have a volume of 2826 cm^3. Cylinder A has a height of 900 cm. Cylinder B has a height of 225 cm. Cylinder C has a height of 100 cm. Find the radius of each cylinder. Use 3.14 as an approximation for π.

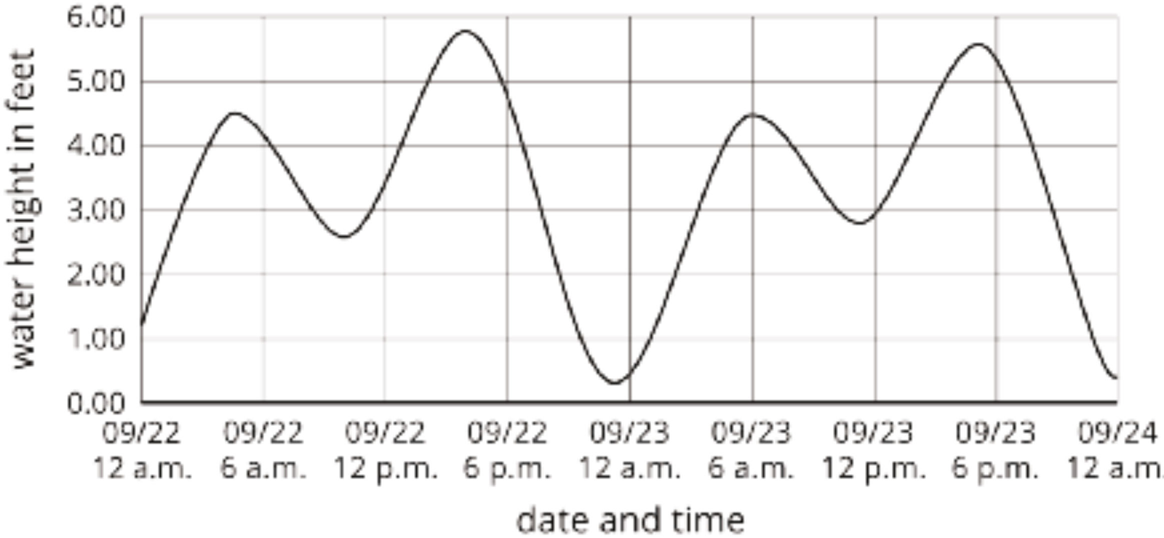
4. A gas company's delivery truck has a cylindrical tank that is 14 feet in diameter and 40 feet long.

   a. Sketch the tank, and mark the radius and the height.

   b. How much gas can fit in the tank?

(From Unit 5, Lesson 13.)

5. Here is a graph that shows the water height of the ocean between September 22 and September 24, 2016 in Bodega Bay, CA.



a. Estimate the water height at 12 p.m. on September 22.

b. How many times was the water height 5 feet? Find two times when this happens.

c. What was the lowest the water got during this time period? When does this occur?

d. Does the water ever reach a height of 6 feet?

(From Unit 5, Lesson 5.)

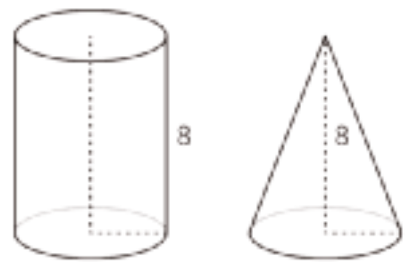
Lesson 15: The Volume of a Cone

Let's explore cones and their volumes.

15.1: Which Has a Larger Volume?

The cone and cylinder have the same height, and the radii of their bases are equal.

1. Which figure has a larger volume?
2. Do you think the volume of the smaller one is more or less than $\frac{1}{2}$ the volume of the larger one? Explain your reasoning.



3. Sketch two different sized cones. The oval doesn't have to be on the bottom! For each drawing, label the cone's radius with r and height with h.

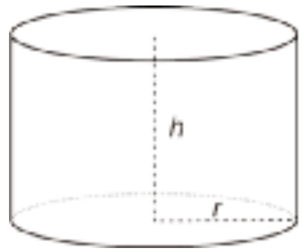
Here is a method for quickly sketching a cone:

- Draw an oval.
- Draw a point centered above the oval.
- Connect the edges of the oval to the point.
- Which parts of your drawing would be hidden behind the object? Make these parts dashed lines.

15.2: From Cylinders to Cones

A cone and cylinder have the same height and their bases are congruent circles.

1. If the volume of the cylinder is 90 cm^3, what is the volume of the cone?

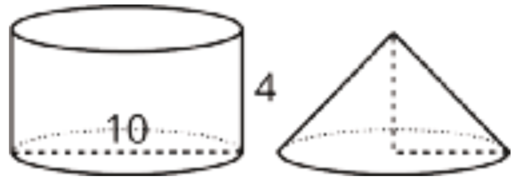


2. If the volume of the cone is 120 cm^3, what is the volume of the cylinder?

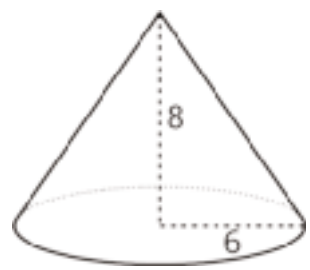
3. If the volume of the cylinder is $V = \pi r^2 h$, what is the volume of the cone? Either write an expression for the cone or explain the relationship in words.

15.3: Calculate That Cone

1. Here is a cylinder and cone that have the same height and the same base area. What is the volume of each figure? Express your answers in terms of π.



2. Here is a cone.
   a. What is the area of the base? Express your answer in terms of π.

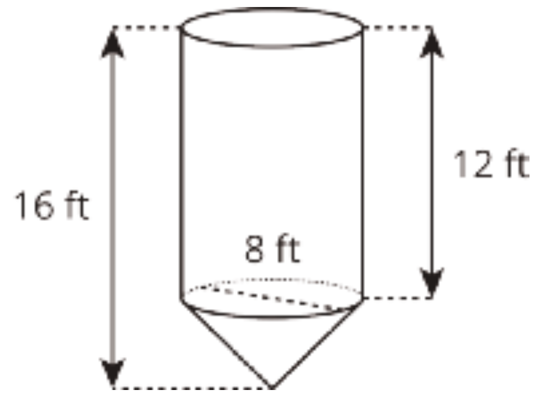


   b. What is the volume of the cone? Express your answer in terms of π.

3. A cone-shaped popcorn cup has a radius of 5 centimeters and a height of 9 centimeters. How many cubic centimeters of popcorn can the cup hold? Use 3.14 as an approximation for π, and give a numerical answer.

Are you ready for more?

A grain silo has a cone shaped spout on the bottom in order to regulate the flow of grain out of the silo. The diameter of the silo is 8 feet. The height of the cylindrical part of the silo above the cone spout is 12 feet while the height of the entire silo is 16 feet.

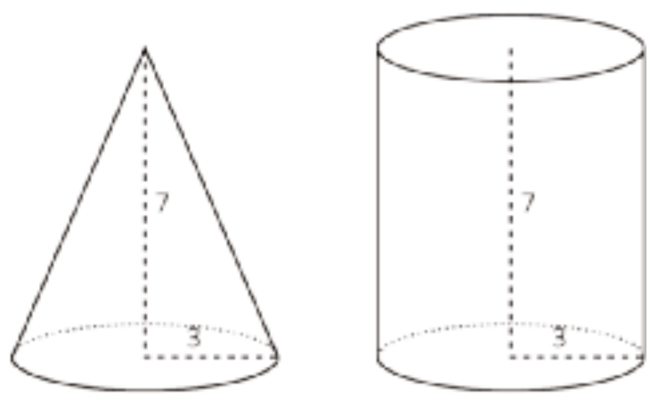


How many cubic feet of grain are held in the cone spout of the silo? How many cubic feet of grain can the entire silo hold?

Lesson 15 Summary

If a cone and a cylinder have the same base and the same height, then the volume of the cone is $\frac{1}{3}$ of the volume of the cylinder. For example, the cylinder and cone shown here both have a base with radius 3 feet and a height of 7 feet.

The cylinder has a volume of 63π cubic feet since $\pi \cdot 3^2 \cdot 7 = 63\pi$. The cone has a volume that is $\frac{1}{3}$ of that, or 21π cubic feet.



If the radius for both is r and the height for both is h, then the volume of the cylinder is $\pi r^2 h$. That means that the volume, V, of the cone is

$$V = \frac{1}{3}\pi r^2 h$$

Unit 5 Lesson 15 Cumulative Practice Problems

1. A cylinder and cone have the same height and radius. The height of each is 5 cm, and the radius is 2 cm. Calculate the volume of the cylinder and the cone.

2. The volume of this cone is 36π cubic units.

   What is the volume of a cylinder that has the same base area and the same height?

3. A cylinder has a diameter of 6 cm and a volume of 36π cm^3.

   a. Sketch the cylinder.

   b. Find its height and radius in centimeters.

   c. Label your sketch with the cylinder's height and radius.

   (From Unit 5, Lesson 14.)

4. Lin wants to get some custom T-shirts printed for her basketball team. Shirts cost $10 each if you order 10 or fewer shirts and $9 each if you order 11 or more shirts.

    a. Make a graph that shows the total cost of buying shirts, for 0 through 15 shirts.

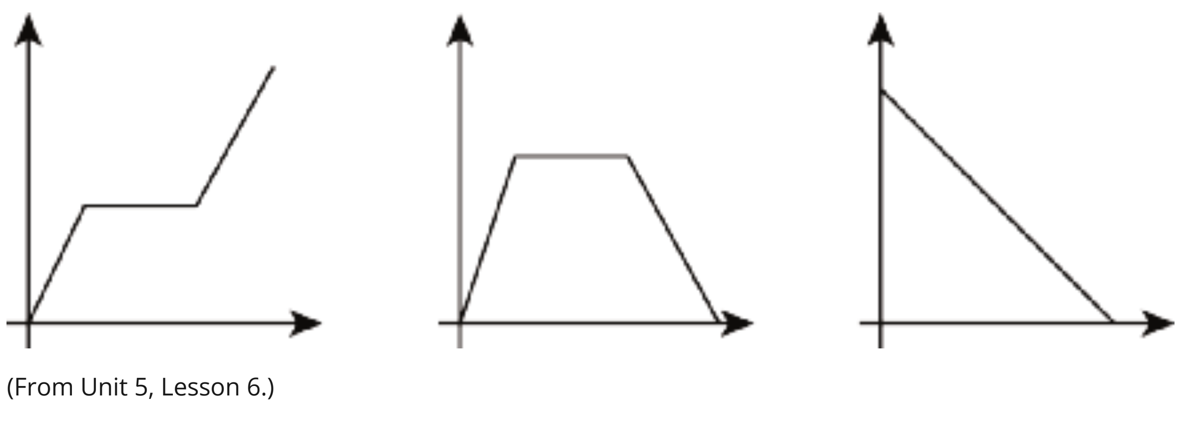
    b. There are 10 people on the team. Do they save money if they buy an extra shirt? Explain your reasoning.

    c. What is the slope of the graph between 0 and 10? What does it mean in the story?

    d. What is the slope of the graph between 11 and 15? What does it mean in the story?

(From Unit 5, Lesson 10.)

5. In the following graphs, the horizontal axis represents time and the vertical axis represents distance from school. Write a possible story for each graph.



(From Unit 5, Lesson 6.)

Lesson 16: Finding Cone Dimensions

Let’s figure out the dimensions of cones.

16.1: Number Talk: Thirds

For each equation, decide what value, if any, would make it true.

$27 = \frac{1}{3}h$

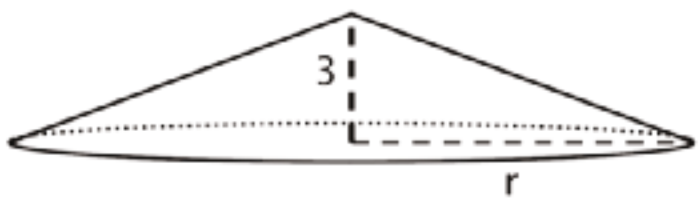
$27 = \frac{1}{3}r^2$

$12\pi = \frac{1}{3}\pi a$

$12\pi = \frac{1}{3}\pi b^2$

16.2: An Unknown Radius

The volume V of a cone with radius r is given by the formula $V = \frac{1}{3}\pi r^2 h$.

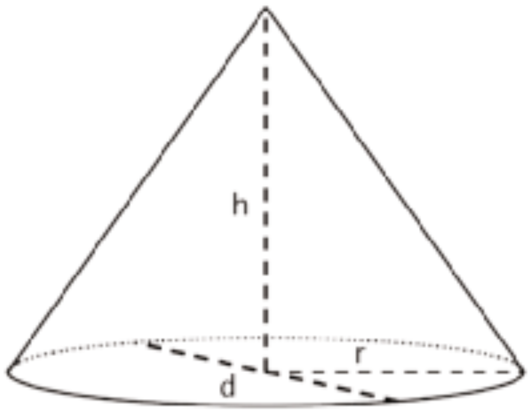


The volume of this cone with height 3 units and radius r is $V = 64\pi$ cubic units. This statement is true:

$$64\pi = \frac{1}{3}\pi r^2 \cdot 3$$

What does the radius of this cone have to be? Explain how you know.

16.3: Cones with Unknown Dimensions



Each row of the table has some information about a particular cone. Complete the table with the missing dimensions.

diameter (units)	radius (units)	area of the base (square units)	height (units)	volume of cone (cubic units)
	4		3	
	$\frac{1}{3}$		6	
		144π	$\frac{1}{4}$	
20				200π
			12	64π
			3	3.14

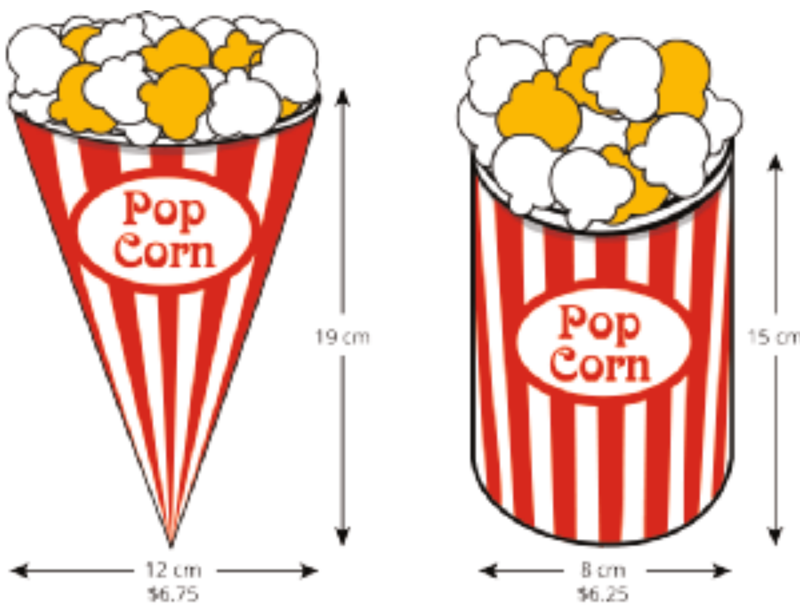
Are you ready for more?

A *frustum* is the result of taking a cone and slicing off a smaller cone using a cut parallel to the base.

Find a formula for the volume of a frustum, including deciding which quantities you are going to include in your formula.

16.4: Popcorn Deals

A movie theater offers two containers:



Which container is the better value? Use 3.14 as an approximation for π.

Lesson 16 Summary

As we saw with cylinders, the volume V of a cone depends on the radius r of the base and the height h:

$$V = \frac{1}{3}\pi r^2 h$$

If we know the radius and height, we can find the volume. If we know the volume and one of the dimensions (either radius or height), we can find the other dimension.

For example, imagine a cone with a volume of 64π cm^3, a height of 3 cm, and an unknown radius r. From the volume formula, we know that

$$64\pi = \frac{1}{3}\pi r^2 \cdot 3$$

Looking at the structure of the equation, we can see that $r^2 = 64$, so the radius must be 8 cm.

Now imagine a different cone with a volume of 18π cm^3, a radius of 3 cm, and an unknown height h. Using the formula for the volume of the cone, we know that

$$18\pi = \frac{1}{3}\pi 3^2 h$$

so the height must be 6 cm. Can you see why?

Unit 5 Lesson 16 Cumulative Practice Problems

1. The volume of this cylinder is 175π cubic units.

   What is the volume of a cone that has the same base area and the same height?

   (From Unit 5, Lesson 15.)

2. A cone has volume 12π cubic inches. Its height is 4 inches. What is its radius?

3. A cone has volume 3π.

   a. If the cone's radius is 1, what is its height?

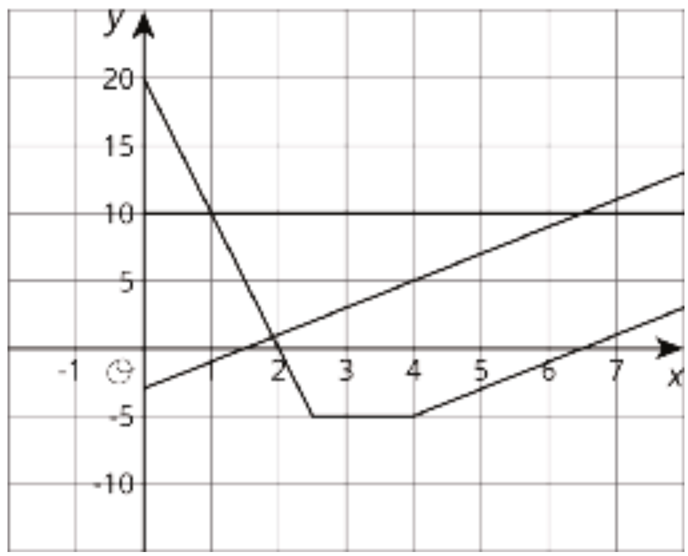
   b. If the cone's radius is 2, what is its height?

   c. If the cone's radius is 5, what is its height?

   d. If the cone's radius is $\frac{1}{2}$, what is its height?

   e. If the cone's radius in r, then what is the height?

4. Three people are playing near the water. Person A stands on the dock. Person B starts at the top of a pole and ziplines into the water, then climbs out of the water. Person C climbs out of the water and up the zipline pole. Match the people to the graphs where the horizontal axis represents time in seconds and the vertical axis represents height above the water level in feet.



(From Unit 5, Lesson 6.)

5. A room is 15 feet tall. An architect wants to include a window that is 6 feet tall. The distance between the floor and the bottom of the window is b feet. The distance between the ceiling and the top of the window is a feet. This relationship can be described by the equation

$$a = 15 - (b + 6)$$

   a. Which variable is independent based on the equation given?

   b. If the architect wants b to be 3, what does this mean? What value of a would work with the given value for b?

   c. The customer wants the window to have 5 feet of space above it. Is the customer describing a or b? What is the value of the other variable?

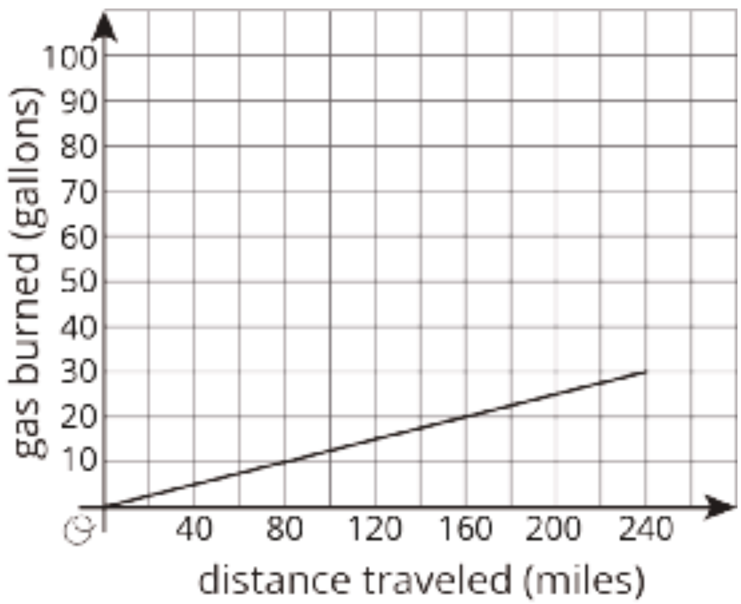
(From Unit 5, Lesson 3.)

Lesson 17: Scaling One Dimension

Let’s see how changing one dimension changes the volume of a shape.

17.1: Driving the Distance

Here is a graph of the amount of gas burned during a trip by a tractor-trailer truck as it drives at a constant speed down a highway:



1. At the end of the trip, how far did the truck drive, and how much gas did it use?

2. If a truck traveled half this distance at the same rate, how much gas would it use?

3. If a truck traveled double this distance at the same rate, how much gas would it use?

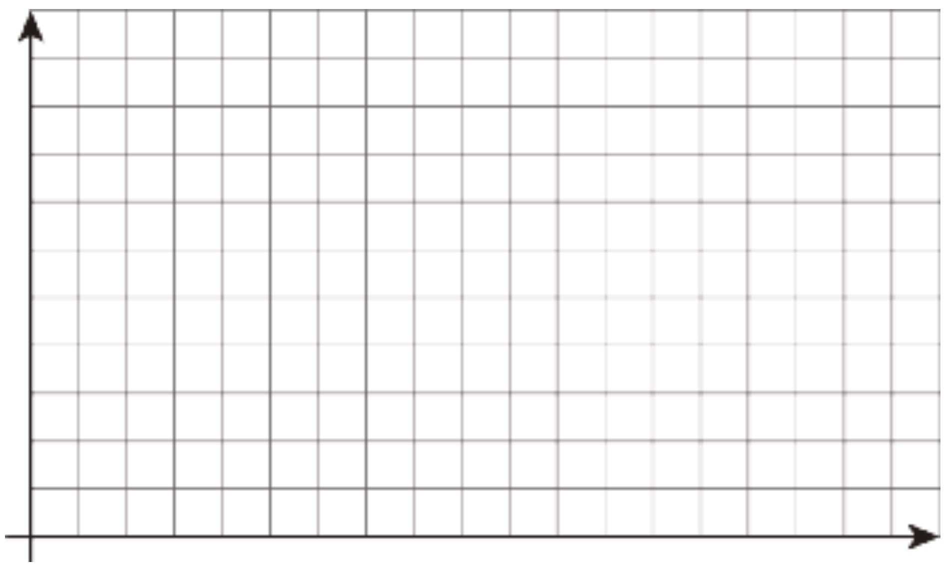
4. Complete the sentence: __________ is a function of ___________.

17.2: Double the Edge

There are many right rectangular prisms with one edge of length 5 units and another edge of length 3 units. Let s represent the length of the third edge and V represent the volume of these prisms.

1. Write an equation that represents the relationship between V and s.

2. Graph this equation and label the axes.



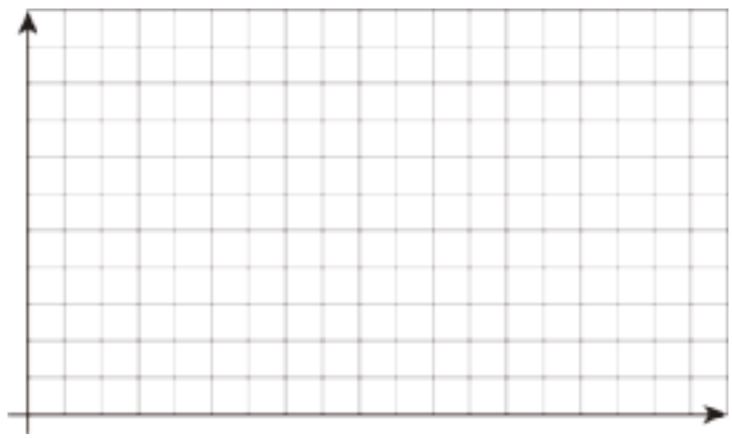
3. What happens to the volume if you double the edge length s? Where do you see this in the graph? Where do you see it algebraically?

17.3: Halve the Height

There are many cylinders with radius 5 units. Let h represent the height and V represent the volume of these cylinders.

1. Write an equation that represents the relationship between V and h. Use 3.14 as an approximation of π.

2. Graph this equation and label the axes.



3. What happens to the volume if you halve the height, h? Where can you see this in the graph? How can you see it algebraically?

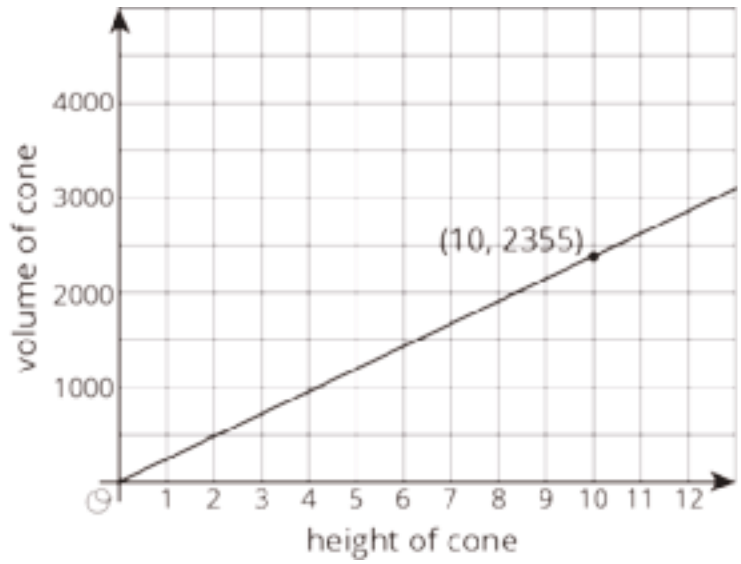
Are you ready for more?

Suppose we have a rectangular prism with dimensions 2 units by 3 units by 6 units, and we would like to make a rectangular prism of volume 216 cubic units by stretching *one* of the three dimensions.

- What are the three ways of doing this? Of these, which gives the prism with the smallest surface area?
- Repeat this process for a starting rectangular prism with dimensions 2 units by 6 units by 6 units.
- Can you give some general tips to someone who wants to make a box with a certain volume, but wants to save cost on material by having as small a surface area as possible?

17.4: Figuring Out Cone Dimensions

Here is a graph of the relationship between the height and the volume of some cones that all have the same radius:



1. What do the coordinates of the labeled point represent?

2. What is the volume of the cone with height 5? With height 30?

3. Use the labeled point to find the radius of these cones. Use 3.14 as an approximation for π.

4. Write an equation that relates the volume V and height h.

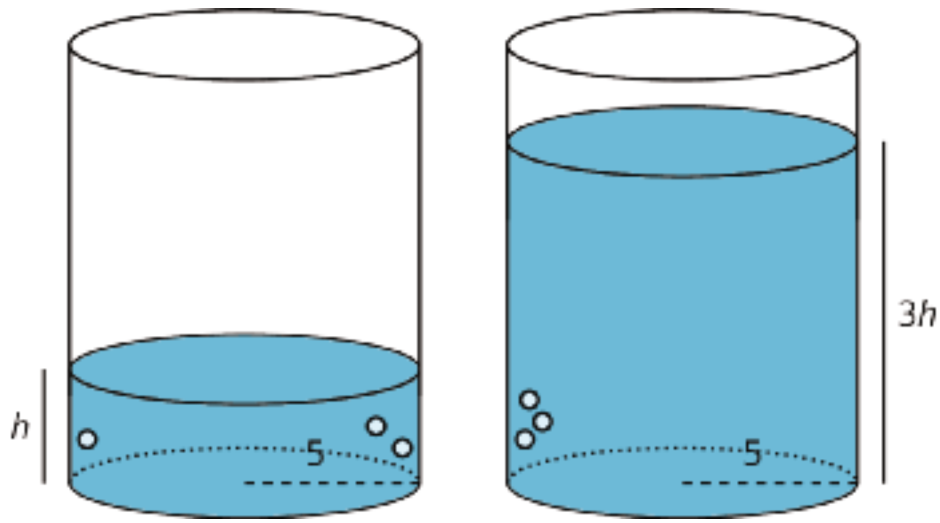
Lesson 17 Summary

Imagine a cylinder with a radius of 5 cm that is being filled with water. As the height of the water increases, the volume of water increases.

We say that the volume of the water in the cylinder, V, depends on the height of the water h. We can represent this relationship with an equation: $V = \pi \cdot 5^2 h$ or just

$$V = 25\pi h$$

This equation represents a *proportional relationship* between the height and the volume. We can use this equation to understand how the volume changes when the height is tripled.



The new volume would be $V = 25\pi(3h) = 75\pi h$, which is precisely 3 times as much as the old volume of $25\pi h$. In general, when one quantity in a proportional relationship changes by a given factor, the other quantity changes by the same factor.

Remember that proportional relationships are examples of linear relationships, which can also be thought of as functions. So in this example V, the volume of water in the cylinder, is a function of the height h of the water.

Unit 5 Lesson 17 Cumulative Practice Problems

1. A cylinder has a volume of 48π cm^3 and height h. Complete this table for volume of cylinders with the same radius but different heights.

height (cm)	volume (cm^3)
h	48π
$2h$	
$5h$	
$\frac{h}{2}$	
$\frac{h}{5}$	

2. A cylinder has a radius of 3 cm and a height of 5 cm.

   a. What is the volume of the cylinder?

   b. What is the volume of the cylinder when its height is tripled?

   c. What is the volume of the cylinder when its height is halved?

3. A graduated cylinder that is 24 cm tall can hold 1 L of water. What is the radius of the cylinder? What is the height of the 500 ml mark? The 250 ml mark? Recall that 1 liter (L) is equal to 1000 milliliters (ml), and that 1 liter (L) is equal to 1,000 cm^3.

4. An ice cream shop offers two ice cream cones. The waffle cone holds 12 ounces and is 5 inches tall. The sugar cone also holds 12 ounces and is 8 inches tall. Which cone has a larger radius?

(From Unit 5, Lesson 16.)

5. A 6 oz paper cup is shaped like a cone with a diameter of 4 inches. How many ounces of water will a plastic cylindrical cup with a diameter of 4 inches hold if it is the same height as the paper cup?

(From Unit 5, Lesson 15.)

6. Lin's smart phone was fully charged when she started school at 8:00 a.m. At 9:20 a.m., it was 90% charged, and at noon, it was 72% charged.

   a. When do you think her battery will die?

   b. Is battery life a function of time? If yes, is it a linear function? Explain your reasoning.

(From Unit 5, Lesson 9.)

Lesson 18: Scaling Two Dimensions

Let's change more dimensions of shapes.

18.1: Tripling Statements

m, n, a, b, and c all represent positive integers. Consider these two equations:

$$m = a + b + c$$
$$n = abc$$

1. Which of these statements are true? Select **all** that apply.
    a. If a is tripled, m is tripled.

    b. If a, b, and c are all tripled, then m is tripled.

    c. If a is tripled, n is tripled.

    d. If a, b, and c are all tripled, then n is tripled.

2. Create a true statement of your own about one of the equations.

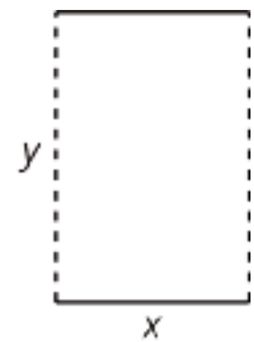
18.2: A Square Base

Clare sketches a rectangular prism with a height of 11 and a square base and labels the edges of the base s. She asks Han what he thinks will happen to the volume of the rectangular prism if she triples s.

Han says the volume will be 9 times bigger. Is he right? Explain or show your reasoning.

Are you ready for more?

A cylinder can be constructed from a piece of paper by curling it so that you can glue together two opposite edges (the dashed edges in the figure).



1. If you wanted to increase the volume inside the resulting cylinder, would it make more sense to double x, y, or does it not matter?

2. If you wanted to increase the surface area of the resulting cylinder, would it make more sense to double x, y, or does it not matter?

3. How would your answers to these questions change if we made a cylinder by gluing together the solid lines instead of the dashed lines?

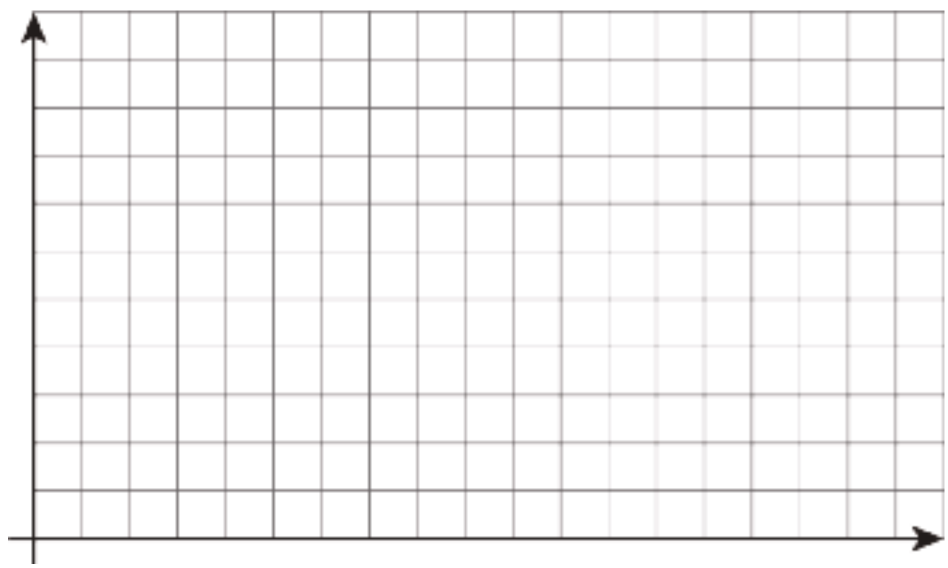
18.3: Playing with Cones

There are many cones with a height of 7 units. Let r represent the radius and V represent the volume of these cones.

1. Write an equation that expresses the relationship between V and r. Use 3.14 as an approximation for π.

2. Predict what happens to the volume if you triple the value of r.

3. Graph this equation.



4. What happens to the volume if you triple r? Where do you see this in the graph? How can you see it algebraically?

Lesson 18 Summary

There are many rectangular prisms that have a length of 4 units and width of 5 units but differing heights. If h represents the height, then the volume V of such a prism is

$$V = 20h$$

The equation shows us that the volume of a prism with a base area of 20 square units is a linear function of the height. Because this is a proportional relationship, if the height gets multiplied by a factor of a, then the volume is also multiplied by a factor of a:

$$V = 20(ah)$$

What happens if we scale *two* dimensions of a prism by a factor of a? In this case, the volume gets multiplied by a factor of a twice, or a^2.

For example, think about a prism with a length of 4 units, width of 5 units, and height of 6 units. Its volume is 120 cubic units since $4 \cdot 5 \cdot 6 = 120$. Now imagine the length and width each get scaled by a factor of a, meaning the new prism has a length of $4a$, width of $5a$, and a height of 6. The new volume is $120a^2$ cubic units since $4a \cdot 5a \cdot 6 = 120a^2$.

A similar relationship holds for cylinders. Think of a cylinder with a height of 6 and a radius of 5. The volume would be 150π cubic units since $\pi \cdot 5^2 \cdot 6 = 150\pi$. Now, imagine the radius is scaled by a factor of a. Then the new volume is $\pi \cdot (5a)^2 \cdot 6 = \pi \cdot 25a^2 \cdot 6$ or $150a^2\pi$ cubic units. So scaling the radius by a factor of a has the effect of multiplying the volume by a^2!

Why does the volume multiply by a^2 when only the radius changes? This makes sense if we imagine how scaling the radius changes the base area of the cylinder. As the radius increases, the base area gets larger in two dimensions (the circle gets wider and also taller), while the third dimension of the cylinder, height, stays the same.

Unit 5 Lesson 18 Cumulative Practice Problems

1. There are many cylinders with a height of 18 meters. Let r represent the radius in meters and V represent the volume in cubic meters.

   a. Write an equation that represents the volume V as a function of the radius r.

   b. Complete this table, giving three possible examples.

r	V
1	

   c. If the radius of a cylinder is doubled, does the volume double? Explain how you know.

   d. Is the graph of this function a line? Explain how you know.

2. As part of a competition, Diego must spin around in a circle 6 times and then run to a tree. The time he spends on each spin is represented by s and the time he spends running is r. He gets to the tree 21 seconds after he starts spinning.

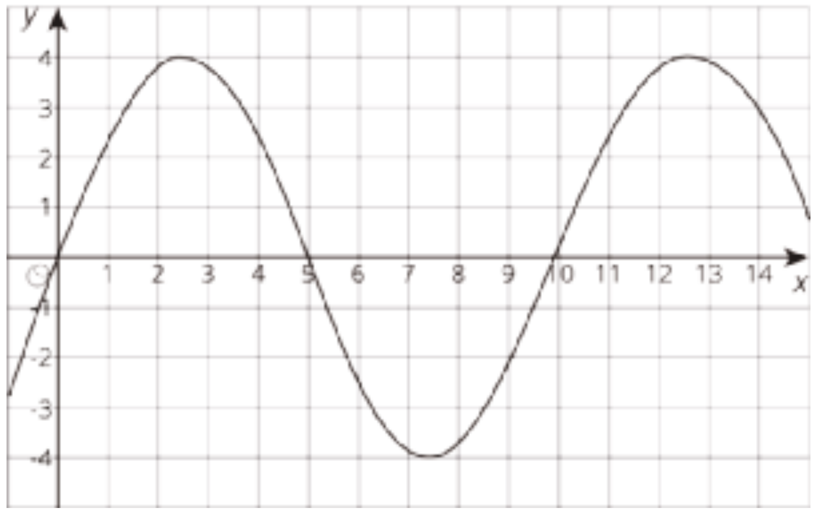
   a. Write an equation showing the relationship between s and r.

   b. Rearrange the equation so that it shows r as a function of s.

   c. If it takes Diego 1.2 seconds to spin around each time, how many seconds did he spend running?

   (From Unit 5, Lesson 3.)

3. The table and graph represent two functions. Use the table and graph to answer the questions.



x	1	2	3	4	5	6
y	3	-1	0	4	5	-1

a. For which values of x is the output from the table less than the output from the graph?

b. In the graphed function, which values of x give an output of 0?

(From Unit 5, Lesson 7.)

4. A cone has a radius of 3 units and a height of 4 units.

a. What is this volume of this cone?

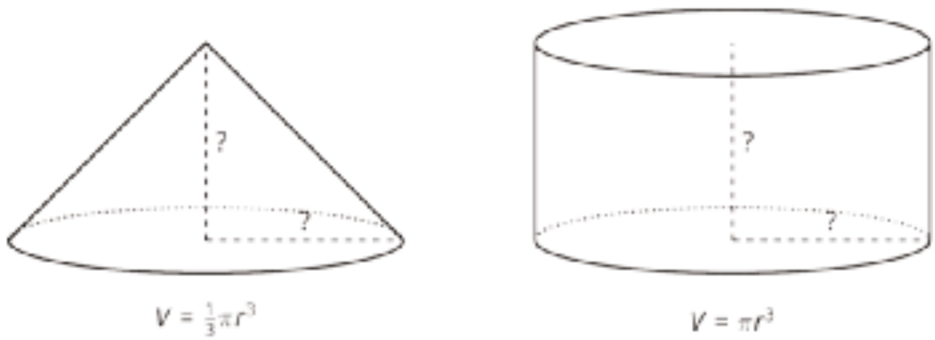
b. Another cone has quadruple the radius, and the same height. How many times larger is the new cone’s volume?

Lesson 19: Estimating a Hemisphere

Let’s estimate volume of hemispheres with figures we know.

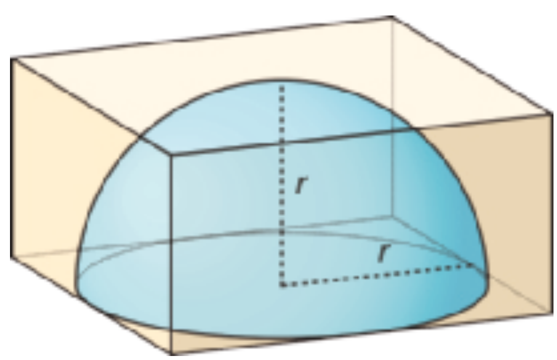
19.1: Notice and Wonder: Two Shapes

Here are two shapes.



What do you notice? What do you wonder?

19.2: Hemispheres in Boxes



1. Mai has a dome paperweight that she can use as a magnifier. The paperweight is shaped like a hemisphere made of solid glass, so she wants to design a box to keep it in so it won't get broken. Her paperweight has a radius of 3 cm.
   a. What should the dimensions of the inside of box be so the box is as small as possible?

   b. What is the volume of the box?

   c. What is a reasonable estimate for the volume of the paperweight?

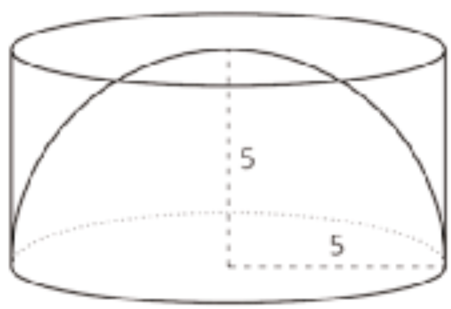
2. Tyler has a different box with side lengths that are twice as long as the sides of Mai's box. Tyler's box is just large enough to hold a different glass paperweight.
    a. What is the volume of the new box?

    b. What is a reasonable estimate for the volume of this glass paperweight?

    c. How many times bigger do you think the volume of the paperweight in this box is than the volume of Mai's paperweight? Explain your thinking.

19.3: Estimating Hemispheres

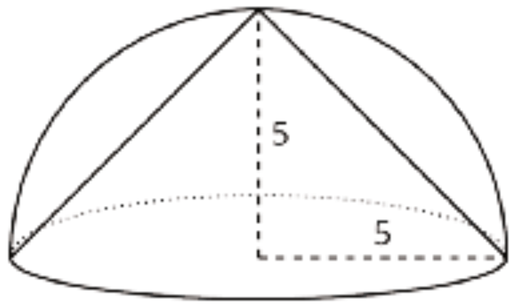
1. A hemisphere with radius 5 units fits snugly into a cylinder of the same radius and height.



    a. Calculate the volume of the cylinder.

    b. Estimate the volume of the hemisphere. Explain your reasoning.

2. A cone fits snugly inside a hemisphere, and they share a radius of 5.



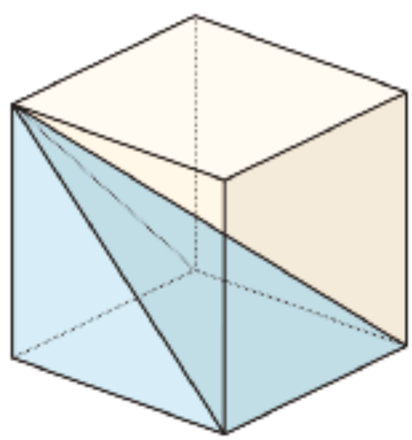
a. What is the volume of the cone?

b. Estimate the volume of the hemisphere. Explain your reasoning.

3. Compare your estimate for the hemisphere with the cone inside to your estimate of the hemisphere inside the cylinder. How do they compare to the volumes of the cylinder and the cone?

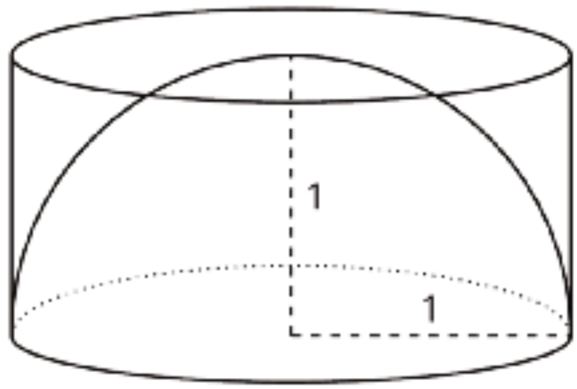
Are you ready for more?

Estimate what fraction of the volume of the cube is occupied by the pyramid that shares the base and a top vertex with the cube, as in the figure.



Lesson 19 Summary

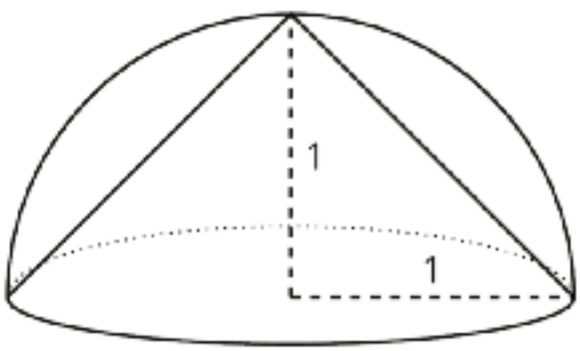
We can estimate the volume of a hemisphere by comparing it to other shapes for which we know the volume. For example, a hemisphere of radius 1 unit fits inside a cylinder with a radius of 1 unit and height of 1 unit.



Since the hemisphere is *inside* the cylinder, it must have a smaller volume than the cylinder making the cylinder's volume a reasonable over-estimate for the volume of the hemisphere.

The volume of this particular cylinder is about 3.14 units^3 since $\pi(1)^2(1) = \pi$, so we know the volume of the hemisphere is less than 3.14 cubic units.

Using similar logic, a cone of radius 1 unit and height 1 unit fits inside of the hemisphere of radius 1 unit.



Since the cone is *inside* the hemisphere, the cone must have a smaller volume than the hemisphere making the cone's volume a reasonable under-estimate for the volume of the hemisphere.

The volume of this particular cone is about 1.05 units^3 since $\frac{1}{3}\pi(1)^2(1) = \frac{1}{3}\pi \approx 1.05$, so we know the volume of the hemisphere is more than 1.05 cubic units.

Averaging the volumes of the cylinder and the cone, we can estimate the volume of the hemisphere to be about 2.10 units^3 since $\frac{3.14+1.05}{2} \approx 2.10$. And, since a hemisphere is half of a sphere, we can also estimate that a sphere with radius of 1 would be double this volume, or about 4.20 units^3.

Unit 5 Lesson 19 Cumulative Practice Problems

1. A baseball fits snugly inside a transparent display cube. The length of an edge of the cube is 2.9 inches.

   Is the baseball's volume greater than, less than, or equal to 2.9^3 cubic inches? Explain how you know.

2. There are many possible cones with a height of 18 meters. Let r represent the radius in meters and V represent the volume in cubic meters.

   a. Write an equation that represents the volume V as a function of the radius r.

   b. Complete this table for the function, giving three possible examples.

r	V
2	

   c. If you double the radius of a cone, does the volume double? Explain how you know.

   d. Is the graph of this function a line? Explain how you know.

   (From Unit 5, Lesson 18.)

3. A hemisphere fits snugly inside a cylinder with a radius of 6 cm. A cone fits snugly inside the same hemisphere.

   a. What is the volume of the cylinder?

   b. What is the volume of the cone?

   c. Estimate the volume of the hemisphere by calculating the average of the volumes of the cylinder and cone.

4. a. Find the hemisphere's diameter if its radius is 6 cm.

   b. Find the hemisphere's diameter if its radius is $\frac{1000}{3}$ m.

   c. Find the hemisphere's diameter if its radius is 9.008 ft.

   d. Find the hemisphere's radius if its diameter is 6 cm.

   e. Find the hemisphere's radius if its diameter is $\frac{1000}{3}$ m.

   f. Find the hemisphere's radius if its diameter is 9.008 ft.

5. After almost running out of space on her phone, Elena checks with a couple of friends who have the same phone to see how many pictures they have on their phones and how much memory they take up. The results are shown in the table.

number of photos	2,523	3,148	1,875
memory used in MB	8,072	10,106	6,037

   a. Could this information be reasonably modeled with a linear function? Explain your reasoning.

   b. Elena needs to delete photos to create 1,200 MB of space. Estimate the number of photos should she delete.

(From Unit 5, Lesson 9.)

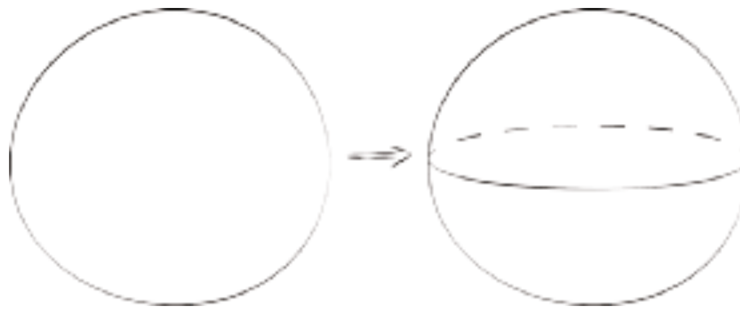
Lesson 20: The Volume of a Sphere

Let’s explore spheres and their volumes.

20.1: Sketch a Sphere

Here is a method for quickly sketching a sphere:

- Draw a circle.
- Draw an oval in the middle whose edges touch the sphere.



1. Practice sketching some spheres. Sketch a few different sizes.

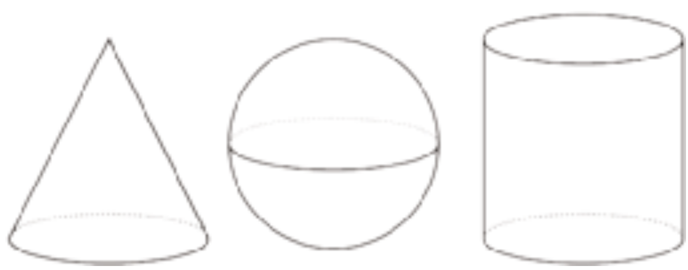
2. For each sketch, draw a radius and label it r.

20.2: A Sphere in a Cylinder

Here are a cone, a sphere, and a cylinder that all have the same radii and heights. The radius of the cylinder is 5 units. When necessary, express all answers in terms of π.

1. What is the height of the cylinder?
2. What is the volume of the cylinder?
3. What is the volume of the cone?
4. What is the volume of the sphere? Explain your reasoning.

20.3: Spheres in Cylinders



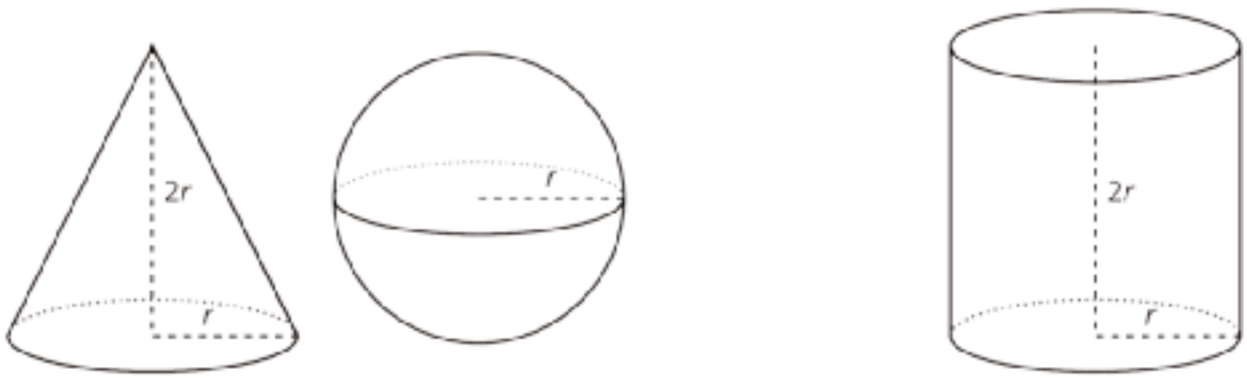
Here are a cone, a sphere, and a cylinder that all have the same radii and heights. Let the radius of the cylinder be r units. When necessary, express answers in terms of π.

1. What is the height of the cylinder in terms of r?
2. What is the volume of the cylinder in terms of r?
3. What is the volume of the cone in terms of r?
4. What is the volume of the sphere in terms of r?
5. A volume of the cone is $\frac{1}{3}$ the volume of a cylinder. The volume of the sphere is what fraction of the volume of the cylinder?

Lesson 20 Summary

Think about a sphere with radius r units that fits snugly inside a cylinder. The cylinder must then also have a radius of r units and a height of $2r$ units. Using what we have learned about volume, the cylinder has a volume of $\pi r^2 h = \pi r^2 \cdot (2r)$, which is equal to $2\pi r^3$ cubic units.

We know from an earlier lesson that the volume of a cone with the same base and height as a cylinder has $\frac{1}{3}$ of the volume. In this example, such a cone has a volume of $\frac{1}{3} \cdot \pi r^2 \cdot 2r$ or just $\frac{2}{3}\pi r^3$ cubic units.



If we filled the cone and sphere with water, and then poured that water into the cylinder, the cylinder would be completely filled. That means the volume of the sphere and the volume of the cone add up to the volume of the cylinder. In other words, if V is the volume of the sphere, then

$$V + \frac{2}{3}\pi r^3 = 2\pi r^3$$

This leads to the formula for the volume of the sphere,

$$V = \frac{4}{3}\pi r^3$$

Unit 5 Lesson 20 Cumulative Practice Problems

1. a. A cube's volume is 512 cubic units. What is the length of its edge?

   b. If a sphere fits snugly inside this cube, what is its volume?

   c. What fraction of the cube is taken up by the sphere? What percentage is this? Explain or show your reasoning.

2. Sphere A has radius 2 cm. Sphere B has radius 4 cm.

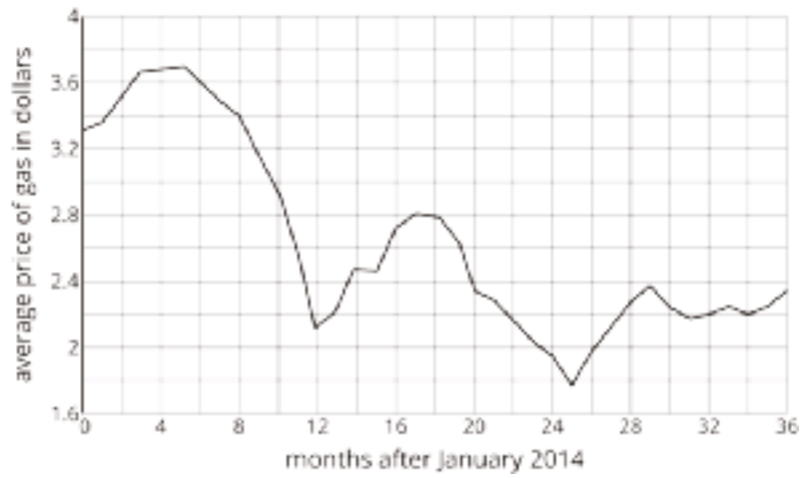
   a. Calculate the volume of each sphere.

   b. The radius of Sphere B is double that of Sphere A. How many times greater is the volume of B?

3. Three cones have a volume of 192π cm^3. Cone A has a radius of 2 cm. Cone B has a radius of 3 cm. Cone C has a radius of 4 cm. Find the height of each cone.

   (From Unit 5, Lesson 16.)

4. The graph represents the average price of regular gasoline in the United States in dollars as a function of the number of months after January 2014.



a. How many months after January 2014 was the price of gas the greatest?

b. Did the average price of gas ever get below $2?

c. Describe what happened to the average price of gas in 2014.

(From Unit 5, Lesson 5.)

5. Match the description of each sphere to its correct volume.

A. Sphere A: radius of 4 cm

B. Sphere B: diameter of 6 cm

C. Sphere C: radius of 8 cm

D. Sphere D: radius of 6 cm

1. 288π cm^3

2. $\frac{256}{3}\pi$ cm^3

3. 36π cm^3

4. $\frac{2048}{3}\pi$ cm3

6. While conducting an inventory in their bicycle shop, the owner noticed the number of bicycles is 2 fewer than 10 times the number of tricycles. They also know there are 410 wheels on all the bicycles and tricycles in the store. Write and solve a system of equations to find the number of bicycles in the store.

(From Unit 4, Lesson 15.)

Lesson 21: Cylinders, Cones, and Spheres

Let’s find the volume of shapes.

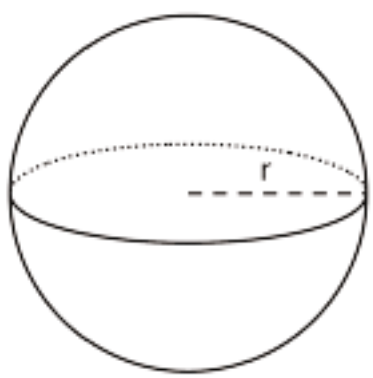
21.1: Sphere Arguments

Four students each calculated the volume of a sphere with a radius of 9 centimeters and they got four different answers.

- Han thinks it is 108 cubic centimeters.
- Jada got 108π cubic centimeters.
- Tyler calculated 972 cubic centimeters.
- Mai says it is 972π cubic centimeters.

Do you agree with any of them? Explain your reasoning.

21.2: Sphere’s Radius



The volume of this sphere with radius r is $V = 288\pi$. This statement is true:

$$288\pi = \frac{4}{3}r^3\pi.$$

What is the value of r for this sphere? Explain how you know.

21.3: Info Gap: Unknown Dimensions

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

   Continue to ask questions until you have enough information to solve the problem.
4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

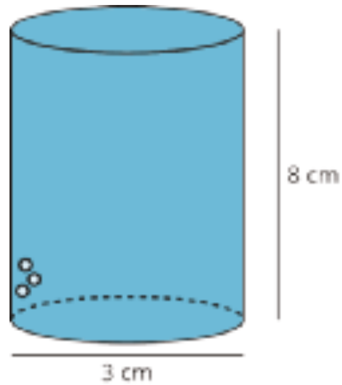
If your teacher gives you the *data card*:

1. Silently read your card.
2. Ask your partner *"What specific information do you need?"* and wait for them to *ask* for information.

   If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don't have that information.
3. Before sharing the information, ask "*Why do you need that information?*" Listen to your partner's reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

21.4: The Right Fit



A cylinder with diameter 3 centimeters and height 8 centimeters is filled with water. Decide which figures described here, if any, could hold all of the water from the cylinder. Explain your reasoning.

1. Cone with a height of 8 centimeters and a radius of 3 centimeters.

2. Cylinder with a diameter of 6 centimeters and height of 2 centimeters.

3. Rectangular prism with a length of 3 centimeters, width of 4 centimeters, and height of 8 centimeters.

4. Sphere with a radius of 2 centimeters.

Are you ready for more?

A thirsty crow wants to raise the level of water in a cylindrical container so that it can reach the water with its beak.

- The container has diameter of 2 inches and a height of 9 inches.
- The water level is currently at 6 inches.
- The crow can reach the water if it is 1 inch from the top of the container.

In order to raise the water level, the crow puts spherical pebbles in the container. If the pebbles are approximately $\frac{1}{2}$ inch in diameter, what is the fewest number of pebbles the crow needs to drop into the container in order to reach the water?

Lesson 21 Summary

The formula

$$V = \frac{4}{3}\pi r^3$$

gives the volume of a sphere with radius r. We can use the formula to find the volume of a sphere with a known radius. For example, if the radius of a sphere is 6 units, then the volume would be

$$\frac{4}{3}\pi(6)^3 = 288\pi$$

or approximately 904 cubic units. We can also use the formula to find the radius of a sphere if we only know its volume. For example, if we know the volume of a sphere is 36π cubic units but we don't know the radius, then this equation is true:

$$36\pi = \frac{4}{3}\pi r^3$$

That means that $r^3 = 27$, so the radius r has to be 3 units in order for both sides of the equation to have the same value.

Many common objects, from water bottles to buildings to balloons, are similar in shape to rectangular prisms, cylinders, cones, and spheres—or even combinations of these shapes! Using the volume formulas for these shapes allows us to compare the volume of different types of objects, sometimes with surprising results.

For example, a cube-shaped box with side length 3 centimeters holds less than a sphere with radius 2 centimeters because the volume of the cube is 27 cubic centimeters ($3^3 = 27$), and the volume of the sphere is around 33.51 cubic centimeters ($\frac{4}{3}\pi \cdot 2^3 \approx 33.51$).

Unit 5 Lesson 21 Cumulative Practice Problems

1. A scoop of ice cream has a 3-inch diameter. How tall should the ice cream cone of the same diameter be in order to contain all of the ice cream inside the cone?

2. Calculate the volume of the following shapes with the given information. For the first three questions, give each answer both in terms of π and by using 3.14 to approximate π. Make sure to include units.

   a. Sphere with a diameter of 6 inches

   b. Cylinder with a height of 6 inches and a diameter of 6 inches

   c. Cone with a height of 6 inches and a radius of 3 inches

   d. How are these three volumes related?

3. A coin-operated bouncy ball dispenser has a large glass sphere that holds many spherical balls. The large glass sphere has a radius of 9 inches. Each bouncy ball has radius of 1 inch and sits inside the dispenser.

   If there are 243 bouncy balls in the large glass sphere, what proportion of the large glass sphere's volume is taken up by bouncy balls? Explain how you know.

4. A farmer has a water tank for cows in the shape of a cylinder with radius of 7 ft and a height of 3 ft. The tank comes equipped with a sensor to alert the farmer to fill it up when the water falls to 20% capacity. What is the volume of the tank be when the sensor turns on?

   (From Unit 5, Lesson 13.)

Lesson 22: Volume As a Function of . . .

Let's compare water heights in different containers.

22.1: Missing Information?

A cylinder and sphere have the same height.

1. If the sphere has a volume of 36π cubic units, what is the height of the cylinder?
2. What is a possible volume for the cylinder? Be prepared to explain your reasoning.

22.2: Scaling Volume of a Sphere

1. Fill in the missing volumes in terms of π. Add two more radius and volume pairs of your choosing.

radius	1	2	3	$\frac{1}{2}$	$\frac{1}{3}$	100			r
volume	$\frac{4}{3}\pi$								

   a. How does the volume of a sphere with radius 2 cm compare to the volume of a sphere with radius 1 cm?

   b. How does the volume of a sphere with radius $\frac{1}{2}$ cm compare to the volume of a sphere with radius 1 cm?

2. A sphere has a radius of length r.
   a. What happens to the volume of this sphere if its radius is doubled?

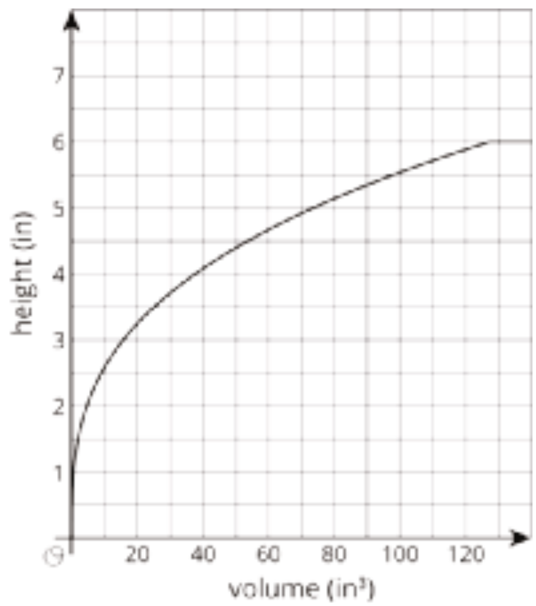
   b. What happens to the volume of this sphere if its radius is halved?

3. Sphere Q has a volume of 500 cm^3. Sphere S has a radius $\frac{1}{5}$ as large as Sphere Q. What is the volume of Sphere S?

22.3: A Cylinder, a Cone, and a Sphere

Three containers of the same height were filled with water at the same rate. One container is a cylinder, one is a cone, and one is a sphere. As they were filled, the relationship between the volume of water and the height of the water was recorded in different ways, shown here:

- Cylinder: $h = \frac{V}{4\pi}$
- Cone:



- Sphere:

volume (in^3)	height (in)
0	0
8.38	1
29.32	2
56.55	3
83.76	4
104.72	5
113.04	6
120	6
200	6

1. The maximum volume of water the cylinder can hold is 24π. What is the radius of the cylinder?

2. Graph the relationship between the volume of water poured into the cylinder and the height of water in the cylinder on the same axes as the cone. What does the slope of this line represent?

3. Which container can fit the largest volume of water? The smallest?

4. About how much water does it take for the cylinder and the sphere to have the same height? The cylinder and the cone? Explain how you know.

5. For what approximate range of volumes is the height of the water in the cylinder greater than the height of the water in the cone? Explain how you know.

6. For what approximate range of volumes is the height of the water in the sphere less than the height of the water in the cylinder? Explain how you know.

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