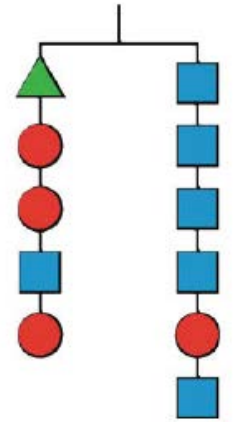
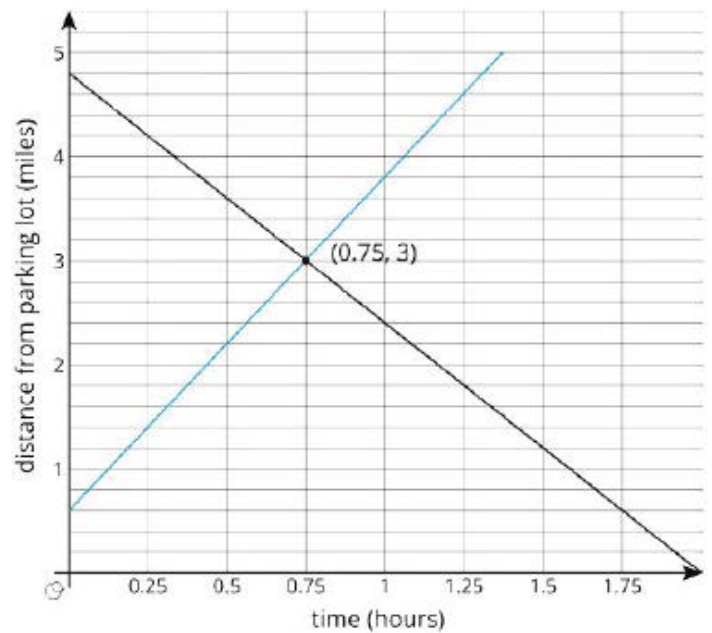




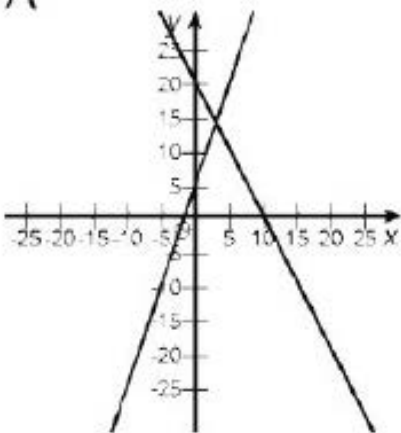
Linear Equations and Linear Systems



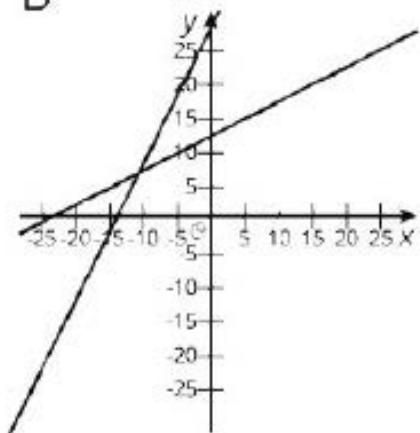
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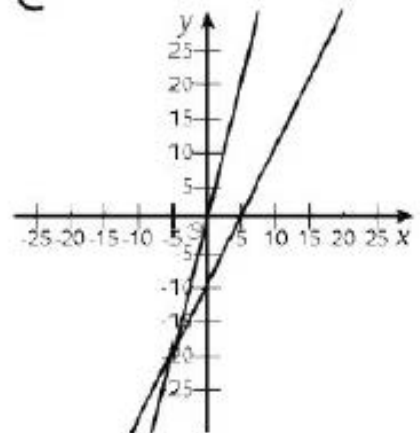
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B



C



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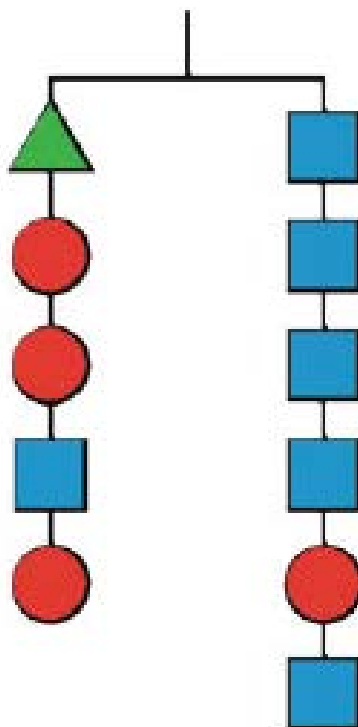
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Linear Equations and Linear Systems

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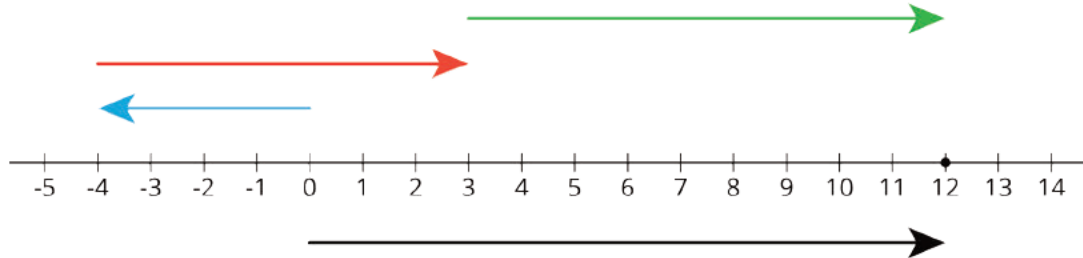


Linear Equations and Linear Systems
Student Workbook
Core Knowledge Mathematics™

Lesson 1: Number Puzzles

1.1: Notice and Wonder: A Number Line

What do you notice? What do you wonder?



1.2: Telling Temperatures

Solve each puzzle. Show your thinking. Organize it so it can be followed by others.

1. The temperature was very cold. Then the temperature doubled. Then the temperature dropped by 10 degrees. Then the temperature increased by 40 degrees. The temperature is now 16 degrees. What was the starting temperature?
2. Lin ran twice as far as Diego. Diego ran 300 m farther than Jada. Jada ran $\frac{1}{3}$ the distance that Noah ran. Noah ran 1200 m. How far did Lin run?

1.3: Making a Puzzle

Write another number puzzle with at least three steps. On a different piece of paper, write a solution to your puzzle.

Trade puzzles with your partner and solve theirs. Make sure to show your thinking.

With your partner, compare your solutions to each puzzle. Did they solve them the same way you did? Be prepared to share with the class which solution strategy you like best.

Are you ready for more?

Here is a number puzzle that uses math. Some might call it a magic trick!

1. Think of a number.
2. Double the number.
3. Add 9.
4. Subtract 3.
5. Divide by 2.
6. Subtract the number you started with.
7. The answer should be 3.

Why does this always work? Can you think of a different number puzzle that uses math (like this one) that will always result in 5?

Lesson 1 Summary

Here is an example of a puzzle problem:

Twice a number plus 4 is 18. What is the number?

There are many different ways to represent and solve puzzle problems.

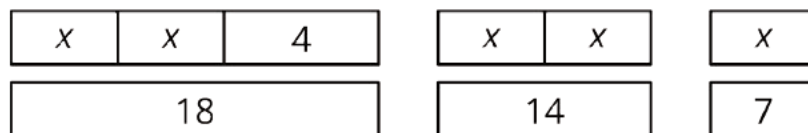
- We can reason through it.

Twice a number plus 4 is 18.

Then twice the number is $18 - 4 = 14$.

That means the number is 7.

- We can draw a diagram.



- We can write and solve an equation.

$$2x + 4 = 18$$

$$2x = 14$$

$$x = 7$$

Reasoning and diagrams help us see what is going on and why the answer is what it is. But as number puzzles and story problems get more complex, those methods get harder, and equations get more and more helpful. We will use different kinds of diagrams to help us understand problems and strategies in future lessons, but we will also see the power of writing and solving equations to answer increasingly more complex mathematical problems.

Unit 4 Lesson 1 Cumulative Practice Problems

1. Tyler reads $\frac{2}{15}$ of a book on Monday, $\frac{1}{3}$ of it on Tuesday, $\frac{2}{9}$ of it on Wednesday, and $\frac{3}{4}$ of the remainder on Thursday. If he still has 14 pages left to read on Friday, how many pages are there in the book?

2. Clare asks Andre to play the following number puzzle:

- Pick a number
- Add 2
- Multiply by 3
- Subtract 7
- Add your original number

Andre's final result is 27.

Which number did he start with?

3. In a basketball game, Elena scores twice as many points as Tyler. Tyler scores four points fewer than Noah, and Noah scores three times as many points as Mai. If Mai scores 5 points, how many points did Elena score? Explain your reasoning.

4. Select all of the given points in the coordinate plane that lie on the graph of the linear equation $4x - y = 3$.

A. $(-1, -7)$

B. $(0, 3)$

C. $(\frac{3}{4}, 0)$

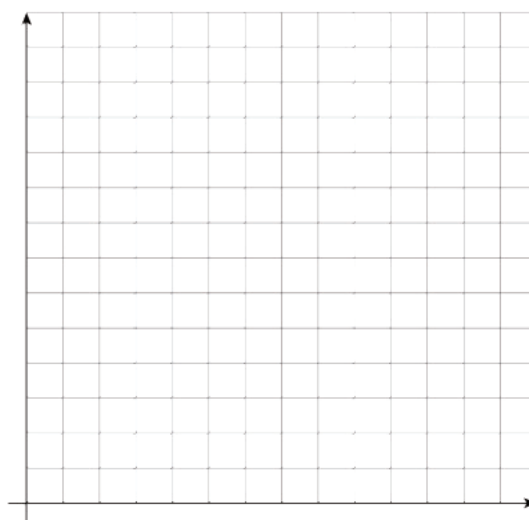
D. $(1, 1)$

E. $(2, 5)$

F. $(4, -1)$

(From Unit 3, Lesson 12.)

5. A store is designing the space for rows of nested shopping carts. Each row has a starting cart that is 4 feet long, followed by the nested carts (so 0 nested carts means there's just the starting cart). The store measured a row of 13 nested carts to be 23.5 feet long, and a row of 18 nested carts to be 31 feet long.



a. Create a graph of the situation.

b. How much does each nested cart add to the length of the row? Explain your reasoning.

c. If the store design allows for 43 feet for each row, how many total carts fit in a row?

(From Unit 3, Lesson 5.)

6. Triangle A is an isosceles triangle with two angles of measure x degrees and one angle of measure y degrees.

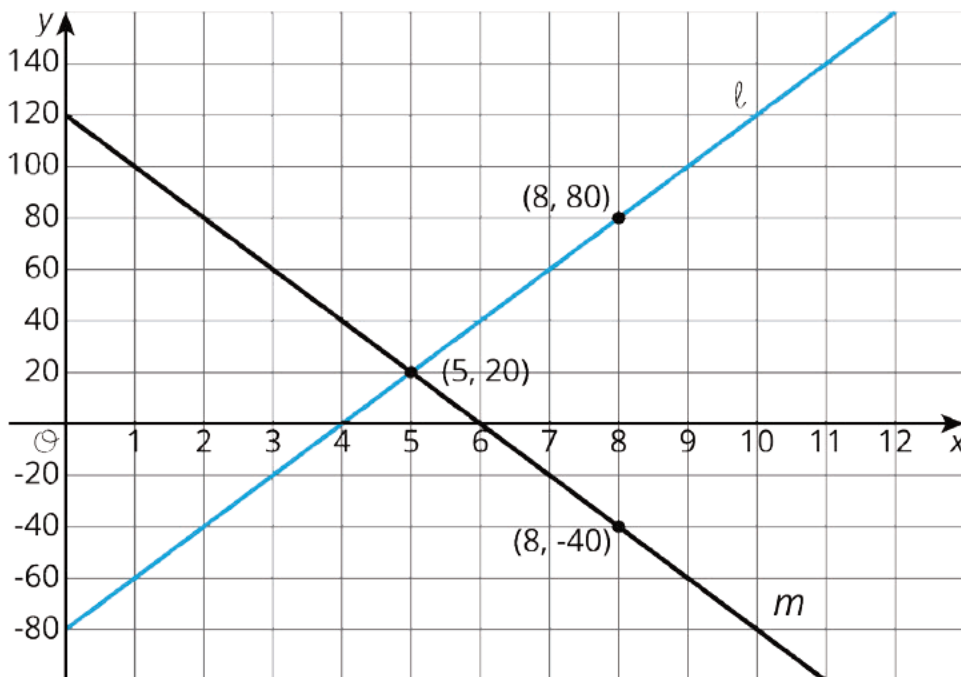
a. Find three combinations of x and y that make this sentence true.

b. Write an equation relating x and y .

c. If you were to sketch the graph of this linear equation, what would its slope be?
How can you interpret the slope in the context of the triangle?

(From Unit 3, Lesson 13.)

7. Consider the following graphs of linear equations. Decide which line has a positive slope, and which has a negative slope. Then calculate each line's exact slope.



(From Unit 3, Lesson 10.)

Lesson 2: Keeping the Equation Balanced

2.1: Notice and Wonder: Hanging Socks

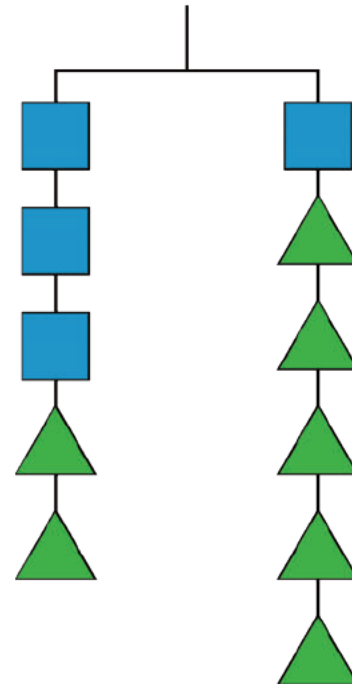
What do you notice? What do you wonder?



2.2: Hanging Blocks

This picture represents a hanger that is balanced because the weight on each side is the same.

1. Elena takes two triangles off of the left side and three triangles off of the right side. Will the hanger still be in balance, or will it tip to one side? Which side? Explain how you know.
2. If a triangle weighs 1 gram, how much does a square weigh?

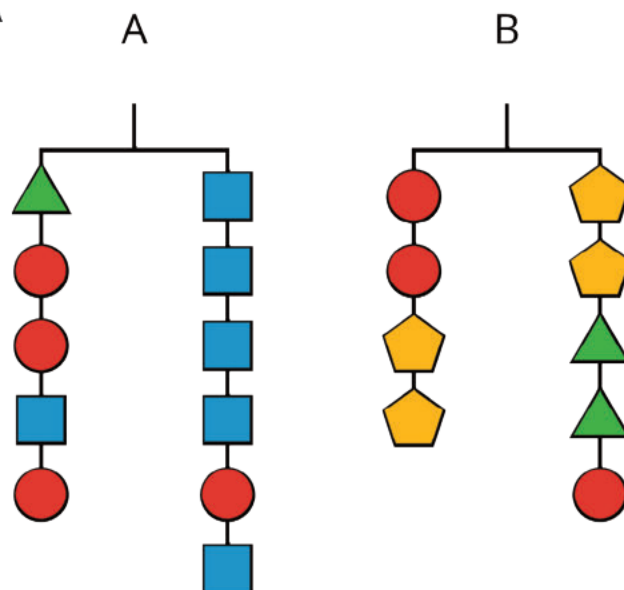


2.3: More Hanging Blocks

A triangle weighs 3 grams and a circle weighs 6 grams.

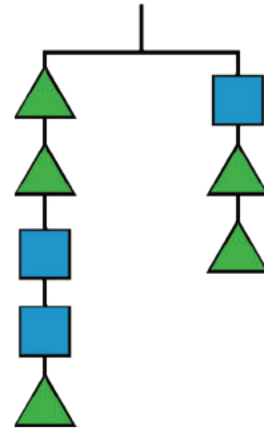
1. Find the weight of a square in Hanger A and the weight of a pentagon in Hanger B.

2. Write an equation to represent each hanger.



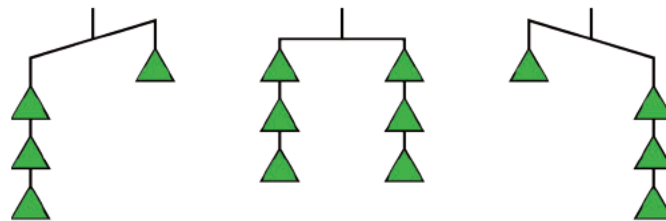
Are you ready for more?

What is the weight of a square on this hanger if a triangle weighs 3 grams?

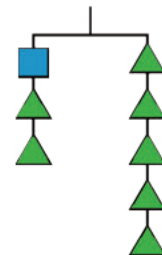


Lesson 2 Summary

If we have equal weights on the ends of a hanger, then the hanger will be in balance. If there is more weight on one side than the other, the hanger will tilt to the heavier side.

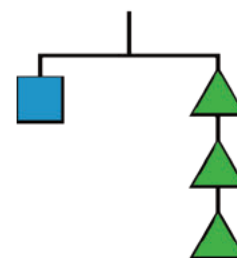


We can think of a balanced hanger as a metaphor for an equation. An equation says that the expressions on each side have equal value, just like a balanced hanger has equal weights on each side.



$$a + 2b = 5b$$

If we have a balanced hanger and add or remove the same amount of weight from each side, the result will still be in balance.



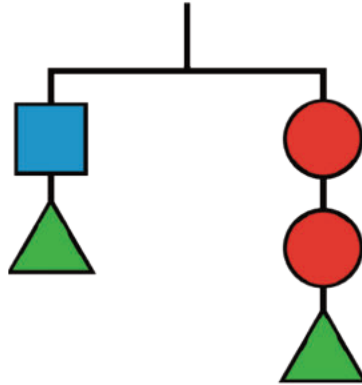
$$a = 3b$$

We can do these moves with equations as well: adding or subtracting the same amount from each side of an equation maintains the equality.

Unit 4 Lesson 2 Cumulative Practice Problems

1. Which of the changes would keep the hanger in balance?

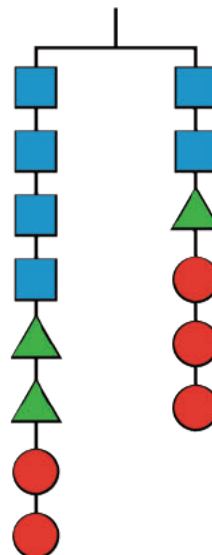
Select all that apply.



- A. Adding two circles on the left and a square on the right
- B. Adding 2 triangles to each side
- C. Adding two circles on the right and a square on the left
- D. Adding a circle on the left and a square on the right
- E. Adding a triangle on the left and a square on the right

2. Here is a balanced hanger diagram.

Each triangle weighs 2.5 pounds, each circle weighs 3 pounds, and x represents the weight of each square.
Select *all* equations that represent the hanger.



A. $x + x + x + x + 11 = x + 11.5$

B. $2x = 0.5$

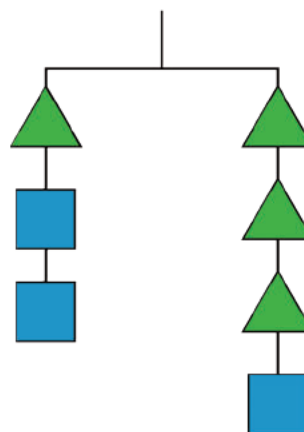
C. $4x + 5 + 6 = 2x + 2.5 + 6$

D. $2x + 2.5 = 3$

E. $4x + 2.5 + 2.5 + 3 + 3 = 2x + 2.5 + 3 + 3 + 3$

3. What is the weight of a square if a triangle weighs 4 grams?

Explain your reasoning.



4. Andre came up with the following puzzle. "I am three years younger than my brother, and I am 2 years older than my sister. My mom's age is one less than three times my brother's age. When you add all our ages, you get 87. What are our ages?"

a. Try to solve the puzzle.

b. Jada writes this equation for the sum of the ages:

$$(x) + (x + 3) + (x - 2) + 3(x + 3) - 1 = 87.$$

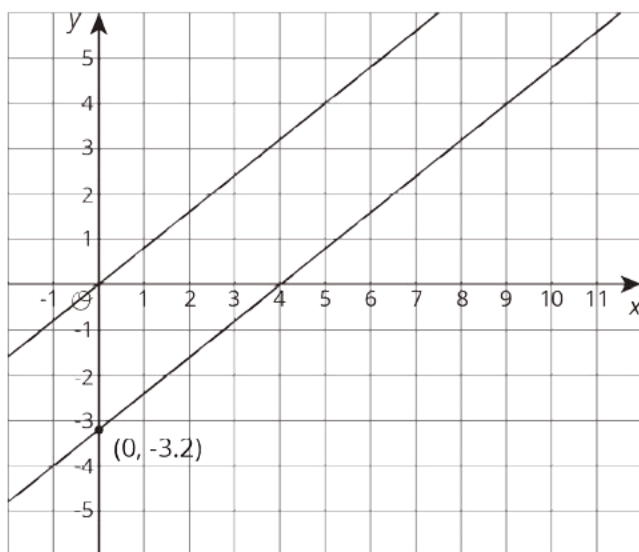
Explain the meaning of the variable and each term of the equation.

c. Write the equation with fewer terms.

d. Solve the puzzle if you haven't already.

(From Unit 4, Lesson 1.)

5. These two lines are parallel. Write an equation for each.

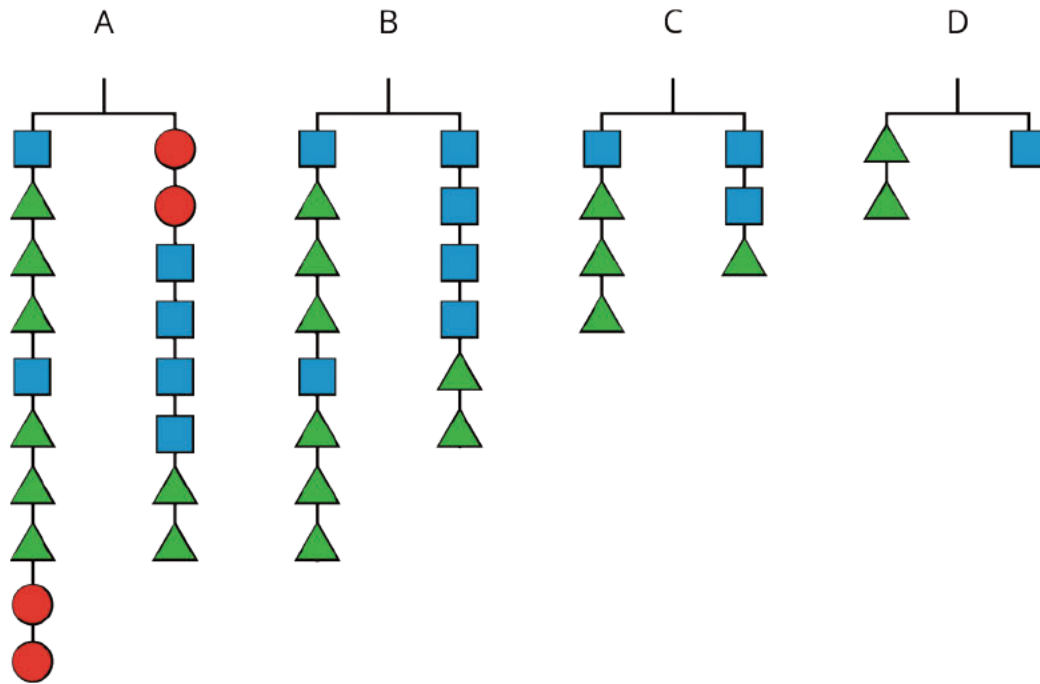


(From Unit 3, Lesson 8.)

Lesson 3: Balanced Moves

3.1: Matching Hangers

Figures A, B, C, and D show the result of simplifying the hanger in Figure A by removing equal weights from each side.



Here are some equations. Each equation represents one of the hanger diagrams.

$$2(x + 3y) = 4x + 2y$$

$$2y = x$$

$$2(x + 3y) + 2z = 2z + 4x + 2y$$

$$x + 3y = 2x + y$$

1. Write the equation that goes with each figure:

A:

B:

C:

D:

- Each variable (x , y , and z) represents the weight of one shape. Which goes with which?
- Explain what was done to each equation to create the next equation. If you get stuck, think about how the hangers changed.

3.2: Matching Equation Moves

Your teacher will give you some cards. Each of the cards 1 through 6 show two equations. Each of the cards A through E describe a move that turns one equation into another.

- Match each number card with a letter card.
- One of the letter cards will not have a match. For this card, write two equations showing the described move.

3.3: Keeping Equality

- Noah and Lin both solved the equation $14a = 2(a - 3)$.

Do you agree with either of them? Why?

Noah's solution:

Lin's solution:

$$14a = 2(a - 3)$$

$$14a = 2(a - 3)$$

$$14a = 2a - 6$$

$$7a = a - 3$$

$$12a = -6$$

$$6a = -3$$

$$a = -\frac{1}{2}$$

$$a = -\frac{1}{2}$$

- Elena is asked to solve $15 - 10x = 5(x + 9)$. What do you recommend she does to each side first?
- Diego is asked to solve $3x - 8 = 4(x + 5)$. What do you recommend he does to each side first?

Are you ready for more?

In a cryptarithmic puzzle, the digits 0–9 are represented with letters of the alphabet. Use your understanding of addition to find which digits go with the letters A, B, E, G, H, L, N, and R.

$$\text{HANGER} + \text{HANGER} + \text{HANGER} = \text{ALGEBRA}$$

Lesson 3 Summary

An equation tells us that two expressions have equal value. For example, if $4x + 9$ and $-2x - 3$ have equal value, we can write the equation

$$4x + 9 = -2x - 3$$

Earlier, we used hangers to understand that if we add the same positive number to each side of the equation, the sides will still have equal value. It also works if we add *negative numbers*! For example, we can add -9 to each side of the equation.

$$\begin{array}{ll} 4x + 9 + -9 = -2x - 3 + -9 & \text{add } -9 \text{ to each side} \\ 4x = -2x - 12 & \text{combine like terms} \end{array}$$

Since expressions represent numbers, we can also add *expressions* to each side of an equation. For example, we can add $2x$ to each side and still maintain equality.

$$\begin{array}{ll} 4x + 2x = -2x - 12 + 2x & \text{add } 2x \text{ to each side} \\ 6x = -12 & \text{combine like terms} \end{array}$$

If we multiply or divide the expressions on each side of an equation by the same number, we will also maintain the equality (so long as we do not divide by zero).

$$6x \cdot \frac{1}{6} = -12 \cdot \frac{1}{6} \quad \text{multiply each side by } \frac{1}{6}$$

or

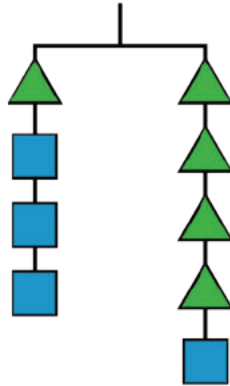
$$6x \div 6 = -12 \div 6 \quad \text{divide each side by 6}$$

Now we can see that $x = -2$ is the solution to our equation.

We will use these moves in systematic ways to solve equations in future lessons.

Unit 4 Lesson 3 Cumulative Practice Problems

1. In this hanger, the weight of the triangle is x and the weight of the square is y .



a. Write an equation using x and y to represent the hanger.

b. If x is 6, what is y ?

2. Andre and Diego were each trying to solve $2x + 6 = 3x - 8$. Describe the first step they each make to the equation.

a. The result of Andre's first step was $-x + 6 = -8$.

b. The result of Diego's first step was $6 = x - 8$.

3. a. Complete the table with values for x or y that make this equation true:
 $3x + y = 15$.

x	2		6	0	3		
y		3				0	8

- b. Create a graph, plot these points, and find the slope of the line that goes through them.



(From Unit 3, Lesson 11.)

4. Match each set of equations with the move that turned the first equation into the second.

A. $6x + 9 = 4x - 3$
 $2x + 9 = -3$

B. $-4(5x - 7) = -18$
 $5x - 7 = 4.5$

C. $8 - 10x = 7 + 5x$
 $4 - 10x = 3 + 5x$

D. $\frac{-5x}{4} = 4$
 $5x = -16$

E. $12x + 4 = 20x + 24$
 $3x + 1 = 5x + 6$

1. Multiply both sides by $\frac{-1}{4}$

2. Multiply both sides by -4

3. Multiply both sides by $\frac{1}{4}$

4. Add $-4x$ to both sides

5. Add -4 to both sides

5. Select all the situations for which only zero or positive solutions make sense.

A. Measuring temperature in degrees Celsius at an Arctic outpost each day in January.

B. The height of a candle as it burns over an hour.

C. The elevation above sea level of a hiker descending into a canyon.

D. The number of students remaining in school after 6:00 p.m.

E. A bank account balance over a year.

F. The temperature in degrees Fahrenheit of an oven used on a hot summer day.

(From Unit 3, Lesson 14.)

Lesson 4: More Balanced Moves

4.1: Different Equations?

Equation 1

$$x - 3 = 2 - 4x$$

Which of these have the same solution as Equation 1? Be prepared to explain your reasoning.

Equation A

$$2x - 6 = 4 - 8x$$

Equation B

$$x - 5 = -4x$$

Equation C

$$2(1 - 2x) = x - 3$$

Equation D

$$-3 = 2 - 5x$$

4.2: Step by Step by Step by Step

Here is an equation, and then all the steps Clare wrote to solve it:

$$\begin{aligned}14x - 2x + 3 &= 3(5x + 9) \\12x + 3 &= 3(5x + 9) \\3(4x + 1) &= 3(5x + 9) \\4x + 1 &= 5x + 9 \\1 &= x + 9 \\-8 &= x\end{aligned}$$

Here is the same equation, and the steps Lin wrote to solve it:

$$\begin{aligned}14x - 2x + 3 &= 3(5x + 9) \\12x + 3 &= 3(5x + 9) \\12x + 3 &= 15x + 27 \\12x &= 15x + 24 \\-3x &= 24 \\x &= -8\end{aligned}$$

1. Are both of their *solutions* correct? Explain your reasoning.

2. Describe some ways the steps they took are alike and different.

3. Mai and Noah also solved the equation, but some of their steps have errors. Find the incorrect step in each solution and explain why it is incorrect.

Mai:

$$14x - 2x + 3 = 3(5x + 9)$$

$$12x + 3 = 3(5x + 9)$$

$$7x + 3 = 3(9)$$

$$7x + 3 = 27$$

$$7x = 24$$

$$x = \frac{24}{7}$$

Noah:

$$14x - 2x + 3 = 3(5x + 9)$$

$$12x + 3 = 15x + 27$$

$$27x + 3 = 27$$

$$27x = 24$$

$$x = \frac{24}{27}$$

4.3: Make Your Own Steps

Solve these equations for x .

1. $\frac{12+6x}{3} = \frac{5-9}{2}$

2. $x - 4 = \frac{1}{3}(6x - 54)$

3. $-(3x - 12) = 9x - 4$

Are you ready for more?

I have 24 pencils and 3 cups. The second cup holds one more pencil than the first. The third holds one more than the second. How many pencils does each cup contain?

Lesson 4 Summary

How do we make sure the solution we find for an equation is correct? Accidentally adding when we meant to subtract, missing a negative when we distribute, forgetting to write an x from one line to the next—there are many possible mistakes to watch out for!

Fortunately, each step we take solving an equation results in a new equation with the same solution as the original. This means we can check our work by substituting the value of the solution into the original equation. For example, say we solve the following equation:

$$\begin{aligned}2x &= -3(x + 5) \\2x &= -3x + 15 \\5x &= 15 \\x &= 3\end{aligned}$$

Substituting 3 in place of x into the original equation,

$$\begin{aligned}2(3) &= -3(3 + 5) \\6 &= -3(8) \\6 &= -24\end{aligned}$$

we get a statement that isn't true! This tells us we must have made a mistake somewhere. Checking our original steps carefully, we made a mistake when distributing -3 . Fixing it, we now have

$$\begin{aligned}2x &= -3(x + 5) \\2x &= -3x - 15 \\5x &= -15 \\x &= -3\end{aligned}$$

Substituting -3 in place of x into the original equation to make sure we didn't make another mistake:

$$\begin{aligned}2(-3) &= -3(-3 + 5) \\-6 &= -3(2) \\-6 &= -6\end{aligned}$$

This equation is true, so $x = -3$ is the solution.

Unit 4 Lesson 4 Cumulative Practice Problems

1. Mai and Tyler work on the equation $\frac{2}{5}b + 1 = -11$ together. Mai's solution is $b = -25$ and Tyler's is $b = -28$. Here is their work. Do you agree with their solutions? Explain or show your reasoning.

Mai:

$$\frac{2}{5}b + 1 = -11$$

$$\frac{2}{5}b = -10$$

$$b = -10 \cdot \frac{5}{2}$$

$$b = -25$$

Tyler:

$$\frac{2}{5}b + 1 = -11$$

$$2b + 1 = -55$$

$$2b = -56$$

$$b = -28$$

2. Solve $3(x - 4) = 12x$

3. Describe what is being done in each step while solving the equation.

a. $2(-3x + 4) = 5x + 2$

b. $-6x + 8 = 5x + 2$

c. $8 = 11x + 2$

d. $6 = 11x$

e. $x = \frac{6}{11}$

4. Andre solved an equation, but when he checked his answer he saw his solution was incorrect. He knows he made a mistake, but he can't find it. Where is Andre's mistake and what is the solution to the equation?

$$-2(3x - 5) = 4(x + 3) + 8$$

$$-6x + 10 = 4x + 12 + 8$$

$$-6x + 10 = 4x + 20$$

$$10 = -2x + 20$$

$$-10 = -2x$$

$$5 = x$$

5. Choose the equation that has solutions (5, 7) and (8, 13).

A. $3x - y = 8$

B. $y = x + 2$

C. $y - x = 5$

D. $y = 2x - 3$

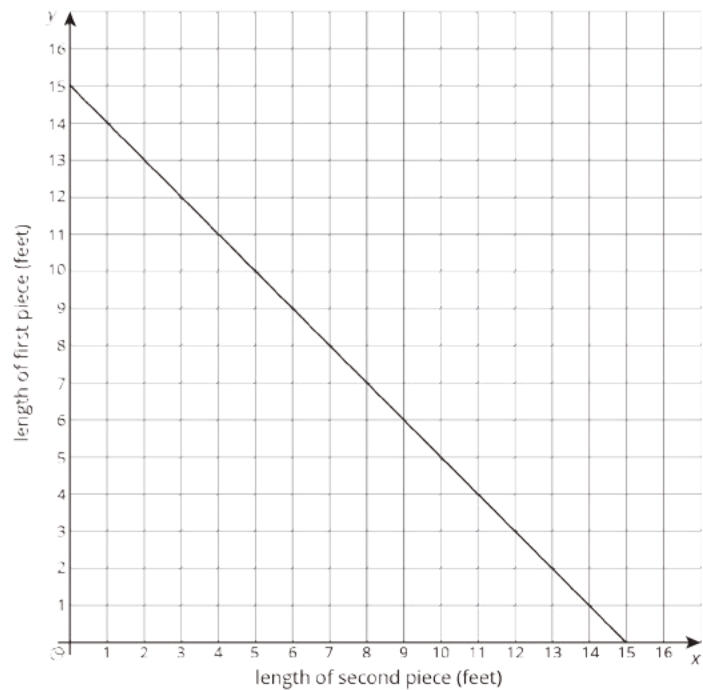
(From Unit 3, Lesson 12.)

6. A length of ribbon is cut into two pieces to use in a craft project. The graph shows the length of the second piece, x , for each length of the first piece, y .

a. How long is the ribbon? Explain how you know.

b. What is the slope of the line?

c. Explain what the slope of the line represents and why it fits the story.



(From Unit 3, Lesson 9.)

Lesson 5: Solving Any Linear Equation

5.1: Equation Talk

Solve each equation mentally.

$$5 - x = 8$$

$$-1 = x - 2$$

$$-3x = 9$$

$$-10 = -5x$$

5.2: Trading Moves

Your teacher will give you 4 cards, each with an equation.

1. With your partner, select a card and choose who will take the first turn.
2. During your turn, decide what the next move to solve the equation should be, explain your choice to your partner, and then write it down once you both agree. Switch roles for the next move. This continues until the equation is solved.
3. Choose a second equation to solve in the same way, trading the card back and forth after each move.
4. For the last two equations, choose one each to solve and then trade with your partner when you finish to check one another's work.

5.3: A Puzzling Puzzle

Tyler says he invented a number puzzle. He asks Clare to pick a number, and then asks her to do the following:

- Triple the number
- Subtract 7
- Double the result
- Subtract 22
- Divide by 6

Clare says she now has a -3. Tyler says her original number must have been a 3. How did Tyler know that? Explain or show your reasoning. Be prepared to share your reasoning with the class.

Lesson 5 Summary

When we have an equation in one variable, there are many different ways to solve it. We generally want to make moves that get us closer to an equation like

$$\text{variable} = \text{some number}.$$

For example, $x = 5$ or $t = \frac{7}{3}$. Since there are many ways to do this, it helps to choose moves that leave fewer terms or factors. If we have an equation like

$$3t + 5 = 7,$$

adding -5 to each side will leave us with fewer terms. The equation then becomes

$$3t = 2.$$

Dividing each side of this equation by 3 will leave us with t by itself on the left and that

$$t = \frac{2}{3}.$$

Or, if we have an equation like

$$4(5 - a) = 12,$$

dividing each side by 4 will leave us with fewer factors on the left,

$$5 - a = 3.$$

Some people use the following steps to solve a linear equation in one variable:

1. Use the distributive property so that all the expressions no longer have parentheses.
2. Collect like terms on each side of the equation.
3. Add or subtract an expression so that there is a variable on just one side.
4. Add or subtract an expression so that there is just a number on the other side.
5. Multiply or divide by a number so that you have an equation that looks like *variable = some number*.

For example, suppose we want to solve $9 - 2b + 6 = -3(b + 5) + 4b$.

$9 - 2b + 6 = -3b - 15 + 4b$	Use the distributive property
$15 - 2b = b - 15$	Gather like terms
$15 = 3b - 15$	Add $2b$ to each side
$30 = 3b$	Add 15 to each side
$10 = b$	Divide each side by 3

Following these steps will always work, although it may not be the most efficient method. From lots of experience, we learn when to use different approaches.

Unit 4 Lesson 5 Cumulative Practice Problems

1. Solve each of these equations. Explain or show your reasoning.

$$2(x + 5) = 3x + 1$$

$$3y - 4 = 6 - 2y$$

$$3(n + 2) = 9(6 - n)$$

2. Clare was solving an equation, but when she checked her answer she saw her solution was incorrect. She knows she made a mistake, but she can't find it. Where is Clare's mistake and what is the solution to the equation?

$$12(5 + 2y) = 4y - (5 - 9y)$$

$$72 + 24y = 4y - 5 - 9y$$

$$72 + 24y = -5y - 5$$

$$24y = -5y - 77$$

$$29y = -77$$

$$y = \frac{-77}{29}$$

3. Solve each equation, and check your solution.

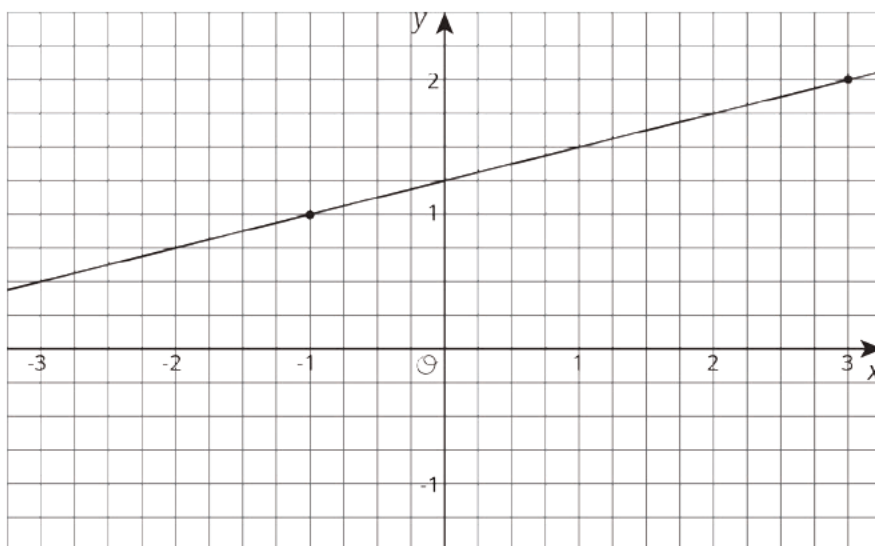
$$\frac{1}{9}(2m - 16) = \frac{1}{3}(2m + 4)$$

$$-4(r + 2) = 4(2 - 2r)$$

$$12(5 + 2y) = 4y - (6 - 9y)$$

4. Here is the graph of a linear equation.

Select all true statements about the line and its equation.



- A. One solution of the equation is $(3, 2)$.
- B. One solution of the equation is $(-1, 1)$.
- C. One solution of the equation is $(1, \frac{3}{2})$.
- D. There are 2 solutions.
- E. There are infinitely many solutions.
- F. The equation of the line is $y = \frac{1}{4}x + \frac{5}{4}$.
- G. The equation of the line is $y = \frac{5}{4}x + \frac{1}{4}$.

(From Unit 3, Lesson 13.)

5. A participant in a 21-mile walkathon walks at a steady rate of 3 miles per hour. He thinks, "The relationship between the number of miles left to walk and the number of hours I already walked can be represented by a line with slope -3." Do you agree with his claim? Explain your reasoning.

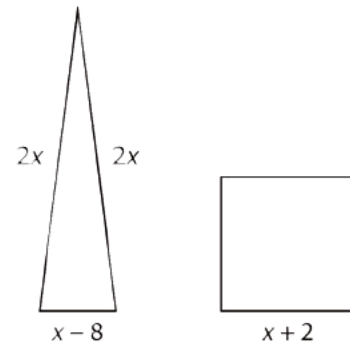
(From Unit 3, Lesson 9.)

Lesson 6: Strategic Solving

6.1: Equal Perimeters

The triangle and the square have equal perimeters.

1. Find the value of x .
2. What is the perimeter of each of the figures?



6.2: Predicting Solutions

Without solving, identify whether these equations have a solution that is positive, negative, or zero.

1. $\frac{x}{6} = \frac{3x}{4}$
2. $7x = 3.25$
3. $7x = 32.5$
4. $3x + 11 = 11$
5. $9 - 4x = 4$
6. $-8 + 5x = -20$
7. $-\frac{1}{2}(-8 + 5x) = -20$

6.3: Which Would You Rather Solve?

Here are a lot of equations:

A. $-\frac{5}{6}(8 + 5b) = 75 + \frac{5}{3}b$

F. $3(c - 1) + 2(3c + 1) = -(3c + 1)$

B. $-\frac{1}{2}(t + 3) - 10 = -6.5$

G. $\frac{4m-3}{4} = -\frac{9+4m}{8}$

C. $\frac{10-v}{4} = 2(v + 17)$

H. $p - 5(p + 4) = p - (8 - p)$

D. $2(4k + 3) - 13 = 2(18 - k) - 13$

I. $2(2q + 1.5) = 18 - q$

E. $\frac{n}{7} - 12 = 5n + 5$

J. $2r + 49 = -8(-r - 5)$

1. Without solving, identify 3 equations that you think would be least difficult to solve and 3 equations you think would be most difficult to solve. Be prepared to explain your reasoning.
2. Choose 3 equations to solve. At least one should be from your "least difficult" list and one should be from your "most difficult" list.

Are you ready for more?

Mai gave half of her brownies, and then half a brownie more, to Kiran. Then she gave half of what was left, and half a brownie more, to Tyler. That left her with one remaining brownie. How many brownies did she have to start with?

Lesson 6 Summary

Sometimes we are asked to solve equations with a lot of things going on on each side. For example,

$$x - 2(x + 5) = \frac{3(2x - 20)}{6}$$

This equation has variables on each side, parentheses, and even a fraction to think about. Before we start distributing, let's take a closer look at the fraction on the right side. The expression $2x - 20$ is being multiplied by 3 and divided by 6, which is the same as just dividing by 2, so we can re-write the equation as

$$x - 2(x + 5) = \frac{2x - 20}{2}$$

But now it's easier to see that all the terms on the numerator of right side are divisible by 2, which means we can re-write the right side again as

$$x - 2(x + 5) = x - 10$$

At this point, we could do some distribution and then collect like terms on each side of the equation. Another choice would be to use the structure of the equation. Both the left and the right side have something being subtracted from x . But, if the two sides are equal, that means the "something" being subtracted on each side must also be equal. Thinking this way, the equation can now be re-written with less terms as

$$2(x + 5) = 10$$

Only a few steps left! But what can we tell about the solution to this problem right now? Is it positive? Negative? Zero? Well, the 2 and the 5 multiplied together are 10, so that means the 2 and the x multiplied together cannot have a positive or a negative value. Finishing the steps we have:

$$\begin{array}{ll} 2(x + 5) = 10 & \\ x + 5 = 5 & \text{Divide each side by 2} \\ x = 0 & \text{Subtract 5 from each side} \end{array}$$

Neither positive nor negative. Just as predicted.

Unit 4 Lesson 6 Cumulative Practice Problems

1. Solve each of these equations. Explain or show your reasoning.

$$2b + 8 - 5b + 3 = -13 + 8b - 5$$

$$2x + 7 - 5x + 8 = 3(5 + 6x) - 12x$$

$$2c - 3 = 2(6 - c) + 7c$$

2. Solve each equation and check your solution.

$$-3w - 4 = w + 3$$

$$3(3 - 3x) = 2(x + 3) - 30$$

$$\frac{1}{3}(z + 4) - 6 = \frac{2}{3}(5 - z)$$

3. Elena said the equation $9x + 15 = 3x + 15$ has no solutions because $9x$ is greater than $3x$. Do you agree with Elena? Explain your reasoning.

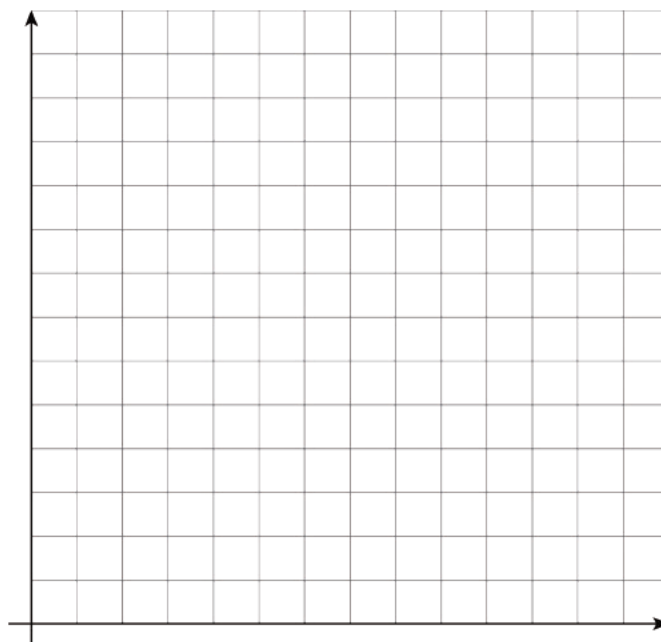
4. The table gives some sample data for two quantities, x and y , that are in a proportional relationship.

a. Complete the table.

b. Write an equation that represents the relationship between x and y shown in the table.

c. Graph the relationship. Use a scale for the axes that shows all the points in the table.

x	y
14	21
64	
	39
1	



(From Unit 3, Lesson 3.)

Lesson 7: All, Some, or No Solutions

7.1: Which One Doesn't Belong: Equations

Which one doesn't belong?

1. $5 + 7 = 7 + 5$

2. $5 \cdot 7 = 7 \cdot 5$

3. $2 = 7 - 5$

4. $5 - 7 = 7 - 5$

7.2: Thinking About Solutions

$$n = n$$

$$5 - 9 + 3x = -10 + 6 + 3x$$

$$2t + 6 = 2(t + 3)$$

$$\frac{1}{2} + x = \frac{1}{3} + x$$

$$3(n + 1) = 3n + 1$$

$$y \cdot -6 \cdot -3 = 2 \cdot y \cdot 9$$

$$\frac{1}{4}(20d + 4) = 5d$$

$$v + 2 = v - 2$$

1. Sort these equations into the two types: true for all values and true for no values.

2. Write the other side of this equation so that this equation is true for all values of u .

$$6(u - 2) + 2 =$$

3. Write the other side of this equation so that this equation is true for no values of u .

$$6(u - 2) + 2 =$$

Are you ready for more?

Consecutive numbers follow one right after the other. An example of three consecutive numbers is 17, 18, and 19. Another example is -100, -99, -98.

How many sets of two or more consecutive positive integers can be added to obtain a sum of 100?

7.3: What's the Equation?

1. Complete each equation so that it is true for all values of x .

a. $3x + 6 = 3(x + \underline{\hspace{1cm}})$

b. $x - 2 = -(\underline{\hspace{1cm}} - x)$

c. $\frac{15x-10}{5} = \underline{\hspace{1cm}} - 2$

2. Complete each equation so that it is true for no values of x .

a. $3x + 6 = 3(x + \underline{\hspace{1cm}})$

b. $x - 2 = -(\underline{\hspace{1cm}} - x)$

c. $\frac{15x-10}{5} = \underline{\hspace{1cm}} - 2$

3. Describe how you know whether an equation will be true for all values of x or true for no values of x .

Lesson 7 Summary

An equation is a statement that two expressions have an equal value. The equation

$$2x = 6$$

is a true statement if x is 3:

$$2 \cdot 3 = 6$$

It is a false statement if x is 4:

$$2 \cdot 4 = 6$$

The equation $2x = 6$ has *one and only one solution*, because there is only one number (3) that you can double to get 6.

Some equations are true no matter what the value of the variable is. For example:

$$2x = x + x$$

is always true, because if you double a number, that will always be the same as adding the number to itself. Equations like $2x = x + x$ have an *infinite number of solutions*. We say it is true for all values of x .

Some equations have *no solutions*. For example:

$$x = x + 1$$

has no solutions, because no matter what the value of x is, it can't equal one more than itself.

When we solve an equation, we are looking for the values of the variable that make the equation true. When we try to solve the equation, we make allowable moves assuming it *has* a solution. Sometimes we make allowable moves and get an equation like this:

$$8 = 7$$

This statement is false, so it must be that the original equation had no solution at all.

Unit 4 Lesson 7 Cumulative Practice Problems

1. For each equation, decide if it is always true or never true.

a. $x - 13 = x + 1$

b. $x + \frac{1}{2} = x - \frac{1}{2}$

c. $2(x + 3) = 5x + 6 - 3x$

d. $x - 3 = 2x - 3 - x$

e. $3(x - 5) = 2(x - 5) + x$

2. Mai says that the equation $2x + 2 = x + 1$ has no solution because the left hand side is double the right hand side. Do you agree with Mai? Explain your reasoning.

3. a. Write the other side of this equation so it's true for all values of x :

$$\frac{1}{2}(6x - 10) - x =$$

b. Write the other side of this equation so it's true for no values of x :

$$\frac{1}{2}(6x - 10) - x =$$

4. Here is an equation that is true for all values of x : $5(x + 2) = 5x + 10$. Elena saw this equation and says she can tell $20(x + 2) + 31 = 4(5x + 10) + 31$ is also true for any value of x . How can she tell? Explain your reasoning.

5. Elena and Lin are trying to solve $\frac{1}{2}x + 3 = \frac{7}{2}x + 5$. Describe the change they each make to each side of the equation.

a. Elena's first step is to write $3 = \frac{7}{2}x - \frac{1}{2}x + 5$.

b. Lin's first step is to write $x + 6 = 7x + 10$.

(From Unit 4, Lesson 4.)

6. Solve each equation and check your solution.

$$3x - 6 = 4(2 - 3x) - 8x \quad \frac{1}{2}z + 6 = \frac{3}{2}(z + 6) \quad 9 - 7w = 8w + 8$$

(From Unit 4, Lesson 6.)

7. The point $(-3, 6)$ is on a line with a slope of 4.

a. Find two more points on the line.

b. Write an equation for the line.

(From Unit 3, Lesson 12.)

Lesson 8: How Many Solutions?

8.1: Matching Solutions

Consider the unfinished equation $12(x - 3) + 18 = \underline{\hspace{2cm}}$. Match the following expressions with the number of solutions the equation would have with that expression on the right hand side.

- | | |
|----------------|-----------------|
| 1. $6(2x - 3)$ | • one solution |
| 2. $4(3x - 3)$ | • no solutions |
| 3. $4(2x - 3)$ | • all solutions |

8.2: Thinking About Solutions Some More

Your teacher will give you some cards.

1. With your partner, solve each equation.
2. Then, sort them into categories.
3. Describe the defining characteristics of those categories and be prepared to share your reasoning with the class.

8.3: Make Use of Structure

For each equation, determine whether it has no solutions, exactly one solution, or is true for all values of x (and has infinitely many solutions). If an equation has one solution, solve to find the value of x that makes the statement true.

1. a. $6x + 8 = 7x + 13$
 b. $6x + 8 = 2(3x + 4)$
 c. $6x + 8 = 6x + 13$

2. a. $\frac{1}{4}(12 - 4x) = 3 - x$
b. $x - 3 = 3 - x$
c. $x - 3 = 3 + x$
3. a. $-5x - 3x + 2 = -8x + 2$
b. $-5x - 3x - 4 = -8x + 2$
c. $-5x - 4x - 2 = -8x + 2$
4. a. $4(2x - 2) + 2 = 4(x - 2)$
b. $4x + 2(2x - 3) = 8(x - 1)$
c. $4x + 2(2x - 3) = 4(2x - 2) + 2$
5. a. $x - 3(2 - 3x) = 2(5x + 3)$
b. $x - 3(2 + 3x) = 2(5x - 3)$
c. $x - 3(2 - 3x) = 2(5x - 3)$
6. What do you notice about equations with one solution? How is this different from equations with no solutions and equations that are true for every x ?

Are you ready for more?

Consecutive numbers follow one right after the other. An example of three consecutive numbers is 17, 18, and 19. Another example is -100, -99, -98.

1. Choose any set of three consecutive numbers. Find their average. What do you notice?
2. Find the average of another set of three consecutive numbers. What do you notice?
3. Explain why the thing you noticed must always work, or find a counterexample.

Lesson 8 Summary

Sometimes it's possible to look at the structure of an equation and tell if it has infinitely many solutions or no solutions. For example, look at

$$2(12x + 18) + 6 = 18x + 6(x + 7).$$

Using the distributive property on the left and right sides, we get

$$24x + 36 + 6 = 18x + 6x + 42.$$

From here, collecting like terms gives us

$$24x + 42 = 24x + 42.$$

Since the left and right sides of the equation are the same, we know that this equation is true for any value of x without doing any more moves!

Similarly, we can sometimes use structure to tell if an equation has no solutions. For example, look at

$$6(6x + 5) = 12(3x + 2) + 12.$$

If we think about each move as we go, we can stop when we realize there is no solution:

$\frac{1}{6} \cdot 6(6x + 5) = \frac{1}{6} \cdot (12(3x + 2) + 12)$	Multiply each side by $\frac{1}{6}$.
$6x + 5 = 2(3x + 2) + 2$	Distribute $\frac{1}{6}$ on the right side.
$6x + 5 = 6x + 4 + 2$	Distribute 2 on the right side.

The last move makes it clear that the **constant terms** on each side, 5 and $4 + 2$, are not the same. Since adding 5 to an amount is always less than adding $4 + 2$ to that same amount, we know there are no solutions.

Doing moves to keep an equation balanced is a powerful part of solving equations, but thinking about what the structure of an equation tells us about the solutions is just as important.

Unit 4 Lesson 8 Cumulative Practice Problems

1. Lin was looking at the equation $2x - 32 + 4(3x - 2462) = 14x$. She said, "I can tell right away there are no solutions, because on the left side, you will have $2x + 12x$ and a bunch of constants, but you have just $14x$ on the right side." Do you agree with Lin? Explain your reasoning.
2. Han was looking at the equation $6x - 4 + 2(5x + 2) = 16x$. He said, "I can tell right away there are no solutions, because on the left side, you will have $6x + 10x$ and a bunch of constants, but you have just $16x$ on the right side." Do you agree with Han? Explain your reasoning.
3. Decide whether each equation is true for all, one, or no values of x .
 - a. $6x - 4 = -4 + 6x$
 - b. $4x - 6 = 4x + 3$
 - c. $-2x + 4 = -3x + 4$

4. Solve each of these equations. Explain or show your reasoning.

a. $3(x - 5) = 6$

b. $2\left(x - \frac{2}{3}\right) = 0$

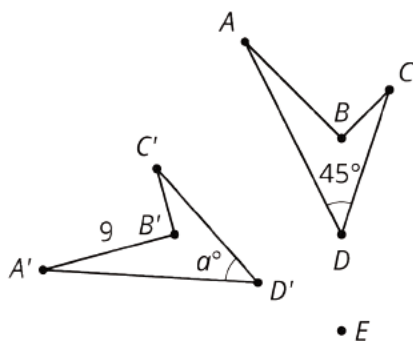
c. $4x - 5 = 2 - x$

(From Unit 4, Lesson 4.)

5. The points $(-2, 0)$ and $(0, -6)$ are each on the graph of a linear equation. Is $(2, 6)$ also on the graph of this linear equation? Explain your reasoning.

(From Unit 3, Lesson 13.)

6. In the picture triangle $A'B'C'$ is an image of triangle ABC after a rotation. The center of rotation is E .



a. What is the length of side AB ? Explain how you know.

b. What is the measure of angle D' ? Explain how you know.

(From Unit 1, Lesson 7.)

Lesson 9: When Are They the Same?

9.1: Which Would You Choose?

If you were babysitting, would you rather

- Charge \$5 for the first hour and \$8 for each additional hour?

Or

- Charge \$15 for the first hour and \$6 for each additional hour?

Explain your reasoning.

9.2: Water Tanks

The amount of water in two tanks every 5 minutes is shown in the table.

time (minutes)	tank 1 (liters)	tank 2 (liters)
0	25	1000
5	175	900
10	325	800
15	475	700
20	625	600
25	775	500
30	925	400
35	1075	300
40	1225	200
45	1375	100
50	1525	0

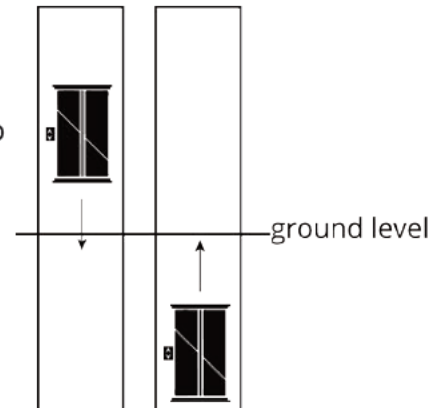
1. Describe what is happening in each tank. Either draw a picture, say it verbally, or write a few sentences.
2. Use the table to estimate when the tanks will have the same amount of water.
3. The amount of water (in liters) in tank 1 after t minutes is $30t + 25$. The amount of water (in liters) in tank 2 after t minutes is $-20t + 1000$. Find the time when the amount of water will be equal.

9.3: Elevators

A building has two elevators that both go above and below ground.

At a certain time of day, the travel time it takes elevator A to reach height h in meters is $0.8h + 16$ seconds.

The travel time it takes elevator B to reach height h in meters is $-0.8h + 12$ seconds.



1. What is the height of each elevator at this time?
2. How long would it take each elevator to reach ground level at this time?
3. If the two elevators travel toward one another, at what height do they pass each other? How long would it take?
4. If you are on an underground parking level 14 meters below ground, which elevator would reach you first?

Are you ready for more?

1. In a two-digit number, the ones digit is twice the tens digit. If the digits are reversed, the new number is 36 more than the original number. Find the number.
2. The sum of the digits of a two-digit number is 11. If the digits are reversed, the new number is 45 less than the original number. Find the number.
3. The sum of the digits in a two-digit number is 8. The value of the number is 4 less than 5 times the ones digit. Find the number.

Lesson 9 Summary

Imagine a full 1,500 liter water tank that springs a leak, losing 2 liters per minute. We could represent the number of liters left in the tank with the expression $-2x + 1,500$, where x represents the number of minutes the tank has been leaking.

Now imagine at the same time, a second tank has 300 liters and is being filled at a rate of 6 liters per minute. We could represent the amount of water in liters in this second tank with the expression $6x + 300$, where x represents the number of minutes that have passed.

Since one tank is losing water and the other is gaining water, at some point they will have the same amount of water—but when? Asking when the two tanks have the same number of liters is the same as asking when $-2x + 1,500$ (the number of liters in the first tank after x minutes) is equal to $6x + 300$ (the number of liters in the second tank after x minutes),

$$-2x + 1,500 = 6x + 300.$$

Solving for x gives us $x = 150$ minutes. So after 150 minutes, the number of liters of the first tank is equal to the number of liters of the second tank. But how much water is actually in each tank at that time? Since both tanks have the same number of liters after 150 minutes, we could substitute $x = 150$ minutes into either expression.

Using the expression for the first tank, we get $-2(150) + 1,500$ which is equal to $-300 + 1,500$, or 1,200 liters.

If we use the expression for the second tank, we get $6(150) + 300$, or just $900 + 300$, which is also 1,200 liters. That means that after 150 minutes, each tank has 1,200 liters.

Unit 4 Lesson 9 Cumulative Practice Problems

1. Cell phone Plan A costs \$70 per month and comes with a free \$500 phone. Cell phone Plan B costs \$50 per month but does not come with a phone. If you buy the \$500 phone and choose Plan B, how many months is it until your cost is the same as Plan A's?

2. Priya and Han are biking in the same direction on the same path.
 - a. Han is riding at a constant speed of 16 miles per hour. Write an expression that shows how many miles Han has gone after t hours.
 - b. Priya started riding a half hour before Han. If Han has been riding for t hours, how long has Priya been riding?
 - c. Priya is riding at a constant speed of 12 miles per hour. Write an expression that shows how many miles Priya has gone after Han has been riding for t hours.
 - d. Use your expressions to find when Han and Priya meet.

3. Which story matches the equation $-6 + 3x = 2 + 4x$?

A. At 5 p.m., the temperatures recorded at two weather stations in Antarctica are -6 degrees and 2 degrees. The temperature changes at the same constant rate, x degrees per hour, throughout the night at both locations. The temperature at the first station 3 hours after this recording is the same as the temperature at the second station 4 hours after this recording.

B. Elena and Kiran play a card game. Every time they collect a pair of matching cards, they earn x points. At one point in the game, Kiran has -6 points and Elena has 2 points. After Elena collects 3 pairs and Kiran collects 4 pairs, they have the same number of points.

4. For what value of x do the expressions $\frac{2}{3}x + 2$ and $\frac{4}{3}x - 6$ have the same value?

5. Decide whether each equation is true for all, one, or no values of x .

a. $2x + 8 = -3.5x + 19$

b. $9(x - 2) = 7x + 5$

c. $3(3x + 2) - 2x = 7x + 6$

(From Unit 4, Lesson 8.)

6. Solve each equation. Explain your reasoning.

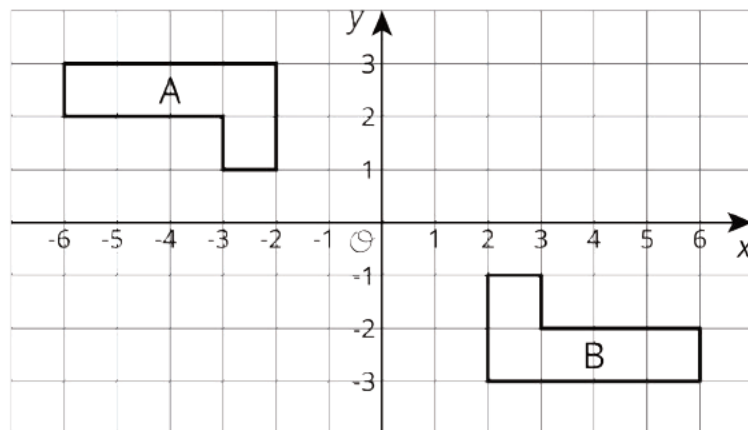
$$3d + 16 = -2(5 - 3d)$$

$$2k - 3(4 - k) = 3k + 4$$

$$\frac{3y-6}{9} = \frac{4-2y}{-3}$$

(From Unit 4, Lesson 6.)

7. Describe a rigid transformation that takes Polygon A to Polygon B.

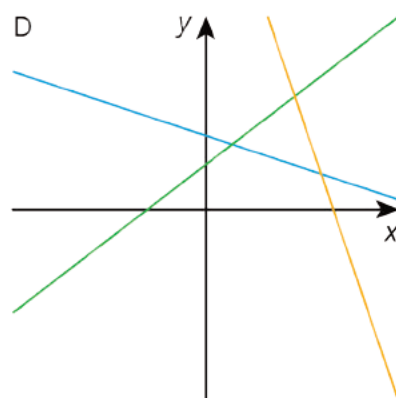
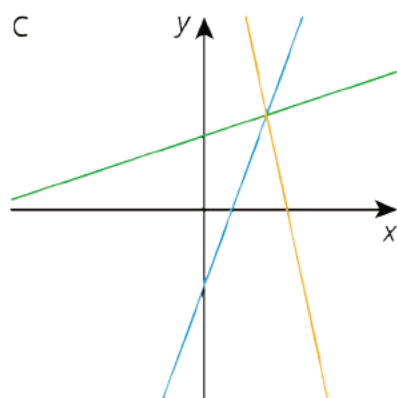
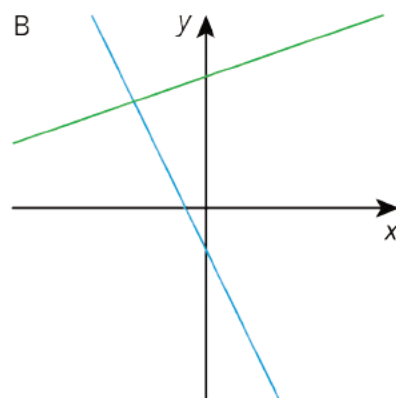
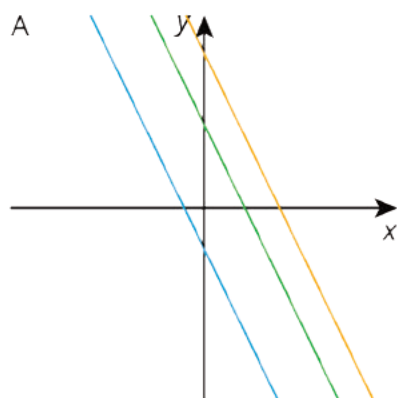


(From Unit 1, Lesson 7.)

Lesson 10: On or Off the Line?

10.1: Which One Doesn't Belong: Lines in the Plane

Which one doesn't belong? Explain your reasoning.



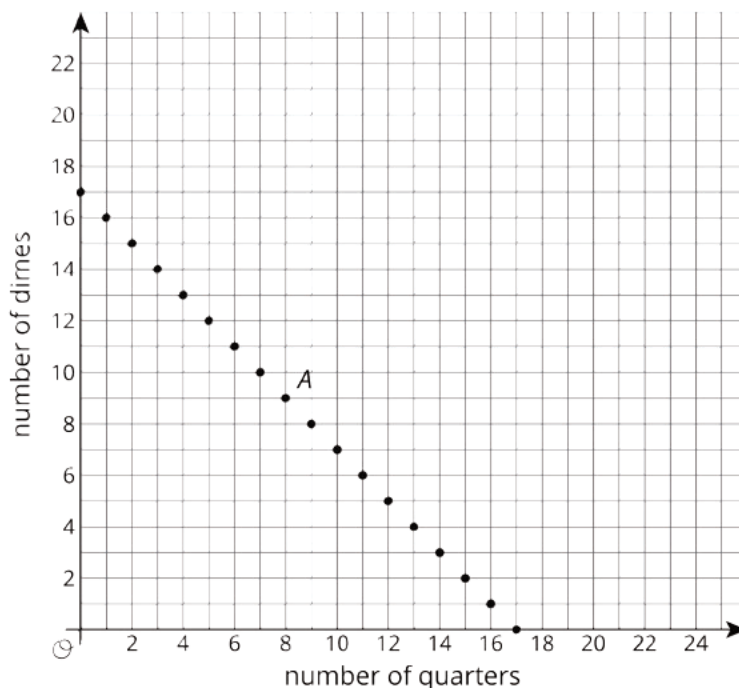
10.2: Pocket Full of Change

Jada told Noah that she has \$2 worth of quarters and dimes in her pocket and 17 coins all together. She asked him to guess how many of each type of coin she has.

- Here is a table that shows some combinations of quarters and dimes that are worth \$2. Complete the table.

number of quarters	number of dimes
0	20
4	
	0
	5

- Here is a graph of the relationship between the number of quarters and the number of dimes when there are a total of 17 coins.
 - What does Point *A* represent?
 - How much money, in dollars, is the combination represented by Point *A* worth?

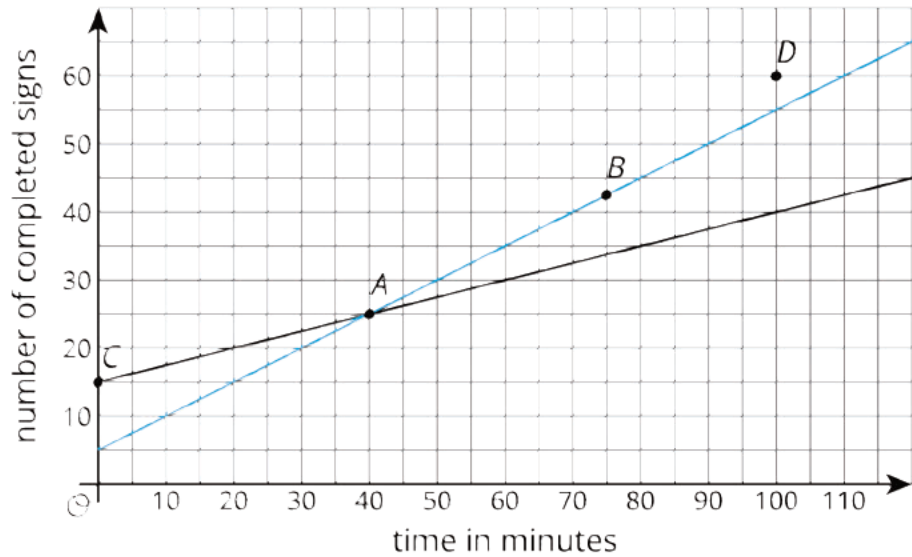


3. Is it possible for Jada to have 4 quarters and 13 dimes in her pocket? Explain how you know.

4. How many quarters and dimes must Jada have? Explain your reasoning.

10.3: Making Signs

Clare and Andre are making signs for all the lockers as part of the decorations for the upcoming spirit week. Yesterday, Andre made 15 signs and Clare made 5 signs. Today, they need to make more signs. Each person's progress today is shown in the coordinate plane.



Based on the lines, mark the statements as true or false for each person.

point	what it says	Clare	Andre
<i>A</i>	At 40 minutes, I have 25 signs completed.		
<i>B</i>	At 75 minutes, I have 42 and a half signs completed.		
<i>C</i>	At 0 minutes, I have 15 signs completed.		
<i>D</i>	At 100 minutes, I have 60 signs completed.		

Are you ready for more?

- 4 toothpicks make 1 square
- 7 toothpicks make 2 squares
- 10 toothpicks make 3 squares



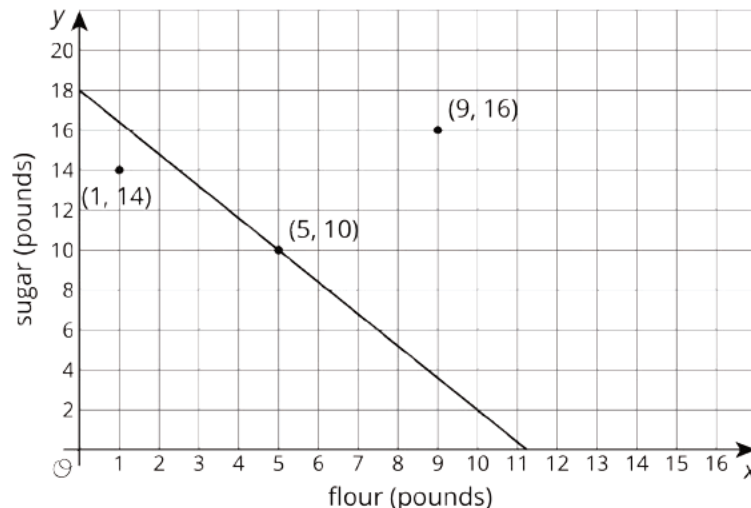
Do you see a pattern? If so, how many toothpicks would you need to make 10 squares according to your pattern? Can you represent your pattern with an expression?

Lesson 10 Summary

We studied linear relationships in an earlier unit. We learned that values of x and y that make an equation true correspond to points (x, y) on the graph. For example, if we have x pounds of flour that costs \$0.80 per pound and y pounds of sugar that costs \$0.50 per pound, and the total cost is \$9.00, then we can write an equation like this to represent the relationship between x and y :

$$0.8x + 0.5y = 9$$

Since 5 pounds of flour costs \$4.00 and 10 pounds of sugar costs \$5.00, we know that $x = 5$, $y = 10$ is a solution to the equation, and the point $(5, 10)$ is a point on the graph. The line shown is the graph of the equation:

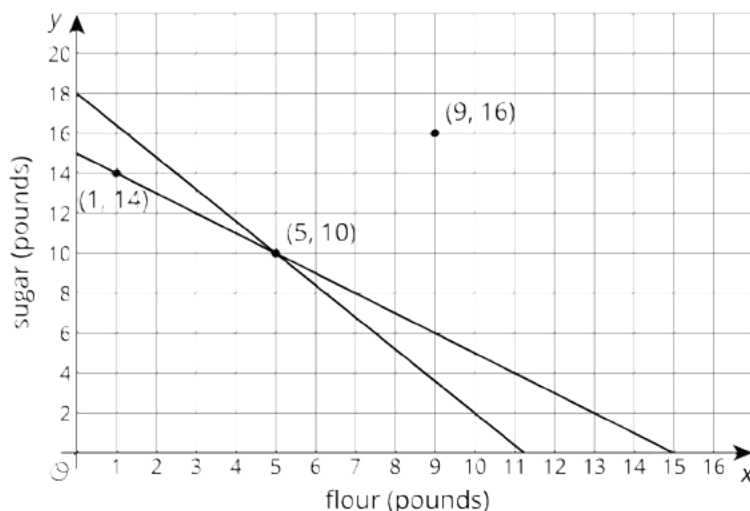


Notice that there are two points shown that are not on the line. What do they mean in the context? The point (1, 14) means that there is 1 pound of flour and 14 pounds of sugar. The total cost for this is $0.8 \cdot 1 + 0.5 \cdot 14$ or \$7.80. Since the cost is not \$9.00, this point is not on the graph. Likewise, 9 pounds of flour and 16 pounds of sugar costs $0.8 \cdot 9 + 0.5 \cdot 16$ or \$15.20, so the other point is not on the graph either.

Suppose we also know that the flour and sugar together weigh 15 pounds. That means that

$$x + y = 15$$

If we draw the graph of this equation on the same coordinate plane, we see it passes through two of the three labeled points:

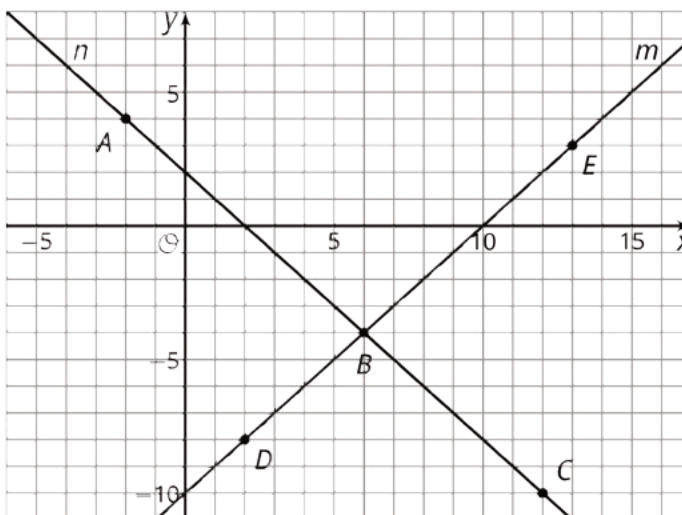


The point (1, 14) is on the graph of $x + y = 15$ because $1 + 14 = 15$. Similarly, $5 + 10 = 15$. But $9 + 16 \neq 15$, so (9, 16) is *not* on the graph of $x + y = 15$. In general, if we have two lines in the coordinate plane,

- The coordinates of a point that is on both lines makes both equations true.
- The coordinates of a point on only one line makes only one equation true.
- The coordinates of a point on neither line make both equations false.

Unit 4 Lesson 10 Cumulative Practice Problems

1. a. Match the lines m and n to the statements they represent:



- i. A set of points where the coordinates of each point have a sum of 2
 - ii. A set of points where the y -coordinate of each point is 10 less than its x -coordinate
- b. Match the labeled points on the graph to statements about their coordinates:
- i. Two numbers with a sum of 2
 - ii. Two numbers where the y -coordinate is 10 less than the x -coordinate
 - iii. Two numbers with a sum of 2 and where the y -coordinate is 10 less than the x -coordinate
2. Here is an equation: $4x - 4 = 4x + \underline{\hspace{1cm}}$. What could you write in the blank so the equation would be true for:
- a. No values of x
 - b. All values of x
 - c. One value of x

(From Unit 4, Lesson 7.)

3. Mai earns \$7 per hour mowing her neighbors' lawns. She also earned \$14 for hauling away bags of recyclables for some neighbors.

Priya babysits her neighbor's children. The table shows the amount of money m she earns in h hours. Priya and Mai have agreed to go to the movies the weekend after they have earned the *same* amount of money for the *same* number of work hours.

h	m
1	\$8.40
2	\$16.80
4	\$33.60

- How many hours do they each have to work before they go to the movies?
 - How much will each of them have earned?
 - Explain where the solution can be seen in tables of values, graphs, and equations that represent Priya's and Mai's hourly earnings.
4. For each equation, explain what you could do first to each side of the equation so that there would be no fractions. You do not have to solve the equations (unless you want more practice).

a. $\frac{3x - 4}{8} = \frac{x + 2}{3}$

a. $\frac{4p + 3}{8} = \frac{p + 2}{4}$

b. $\frac{3(2 - r)}{4} = \frac{3 + r}{6}$

b. $\frac{2(a - 7)}{15} = \frac{a + 4}{6}$

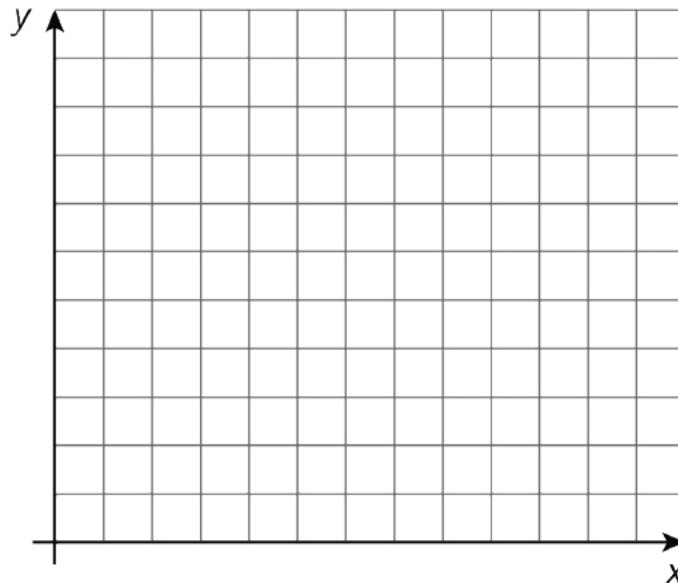
(From Unit 4, Lesson 6.)

5. The owner of a new restaurant is ordering tables and chairs. He wants to have only tables for 2 and tables for 4. The total number of people that can be seated in the restaurant is 120.

a. Describe some possible combinations of 2-seat tables and 4-seat tables that will seat 120 customers. Explain how you found them.

b. Write an equation to represent the situation. What do the variables represent?

c. Create a graph to represent the situation.



d. What does the slope tell us about the situation?

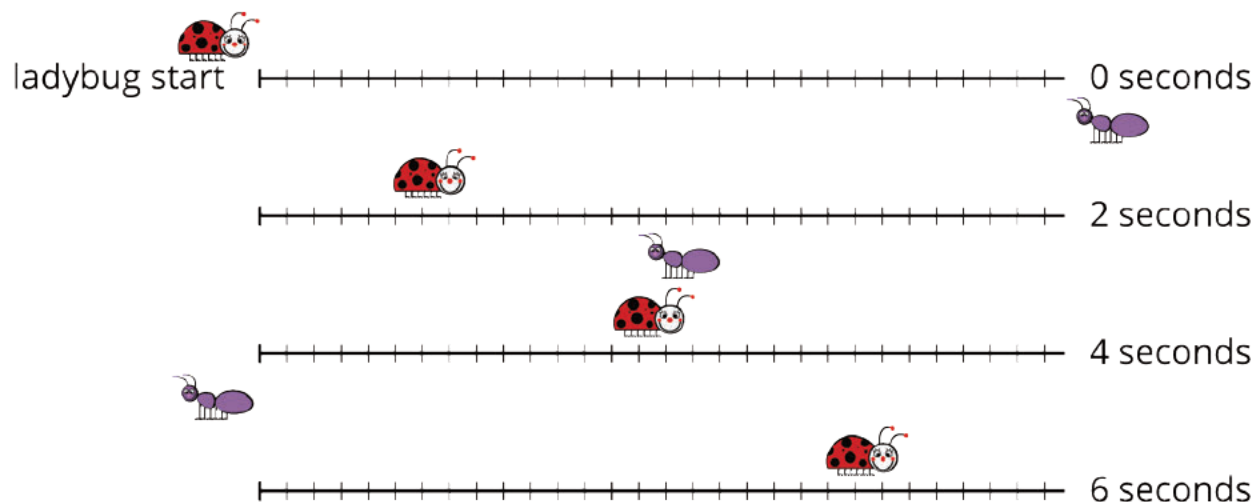
e. Interpret the x and y intercepts in the situation.

(From Unit 3, Lesson 14.)

Lesson 11: On Both of the Lines

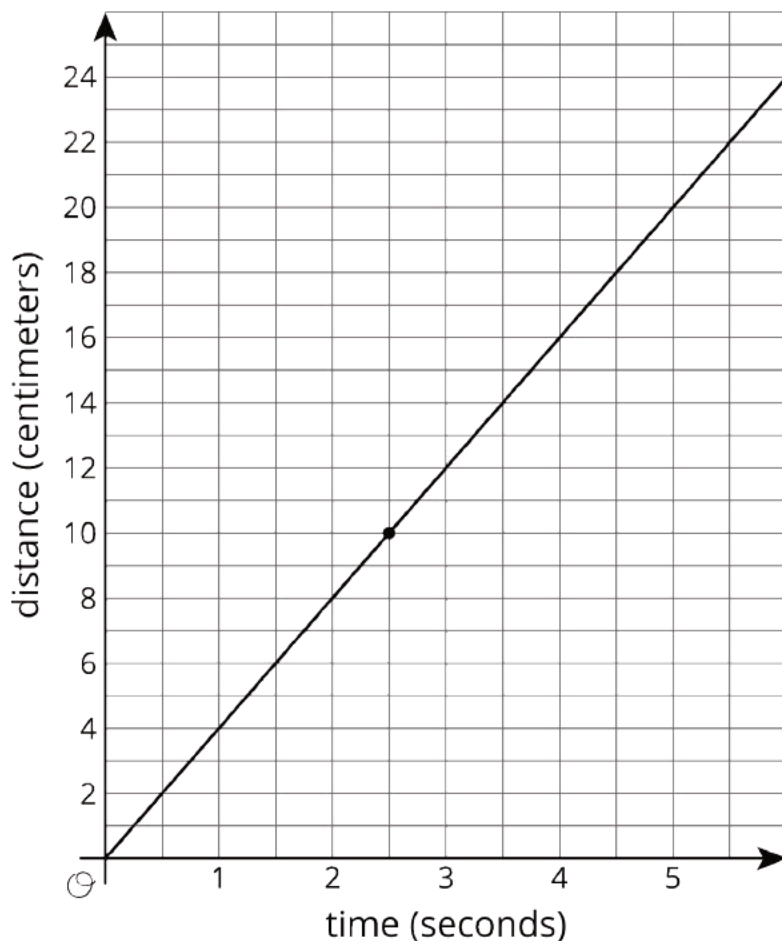
11.1: Notice and Wonder: Bugs Passing in the Night

What do you notice? What do you wonder?



11.2: Bugs Passing in the Night, Continued

A different ant and ladybug are a certain distance apart, and they start walking toward each other. The graph shows the ladybug's distance from its starting point over time and the labeled point $(2.5, 10)$ indicates when the ant and the ladybug pass each other.



The ant is walking 2 centimeters per second.

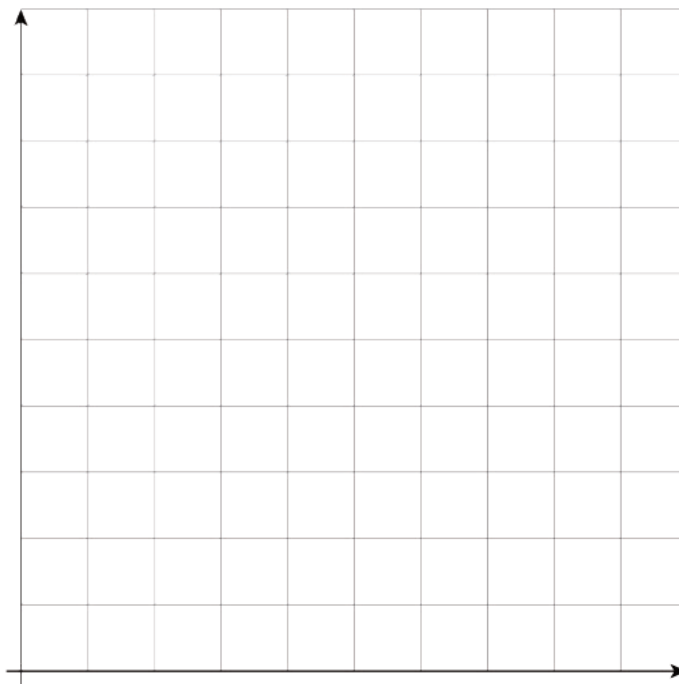
1. Write an equation representing the relationship between the ant's distance from the ladybug's starting point and the amount of time that has passed.
2. If you haven't already, draw the graph of your equation on the same coordinate plane.

11.3: A Close Race

Elena and Jada were racing 100 meters on their bikes. Both racers started at the same time and rode at constant speed. Here is a table that gives information about Jada's bike race:

time from start (seconds)	distance from start (meters)
6	36
9	54

1. Graph the relationship between distance and time for Jada's bike race. Make sure to label and scale the axes appropriately.



2. Elena traveled the entire race at a steady 6 meters per second. On the same set of axes, graph the relationship between distance and time for Elena's bike race.
3. Who won the race?

Lesson 11 Summary

The solutions to an equation correspond to points on its graph. For example, if Car A is traveling 75 miles per hour and passes a rest area when $t = 0$, then the distance in miles it has traveled from the rest area after t hours is

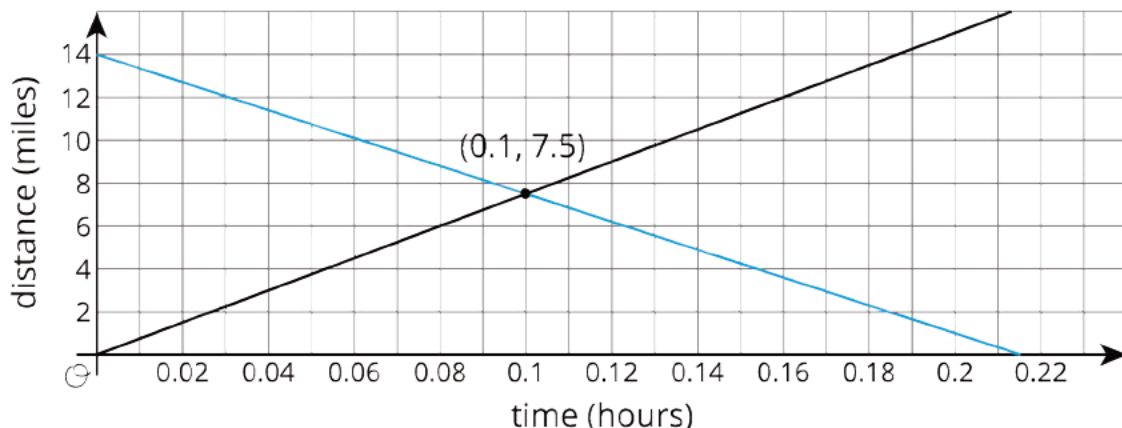
$$d = 75t$$

The point $(2, 150)$ is on the graph of this equation because $150 = 75 \cdot 2$: two hours after passing the rest area, the car has traveled 150 miles.

If you have *two* equations, you can ask whether there is an ordered pair that is a solution to *both* equations simultaneously. For example, if Car B is traveling towards the rest area and its distance from the rest area is

$$d = 14 - 65t$$

We can ask if there is ever a time when the distance of Car A from the rest area is the same as the distance of Car B from the rest area. If the answer is “yes”, then the solution will correspond to a point that is on both lines.



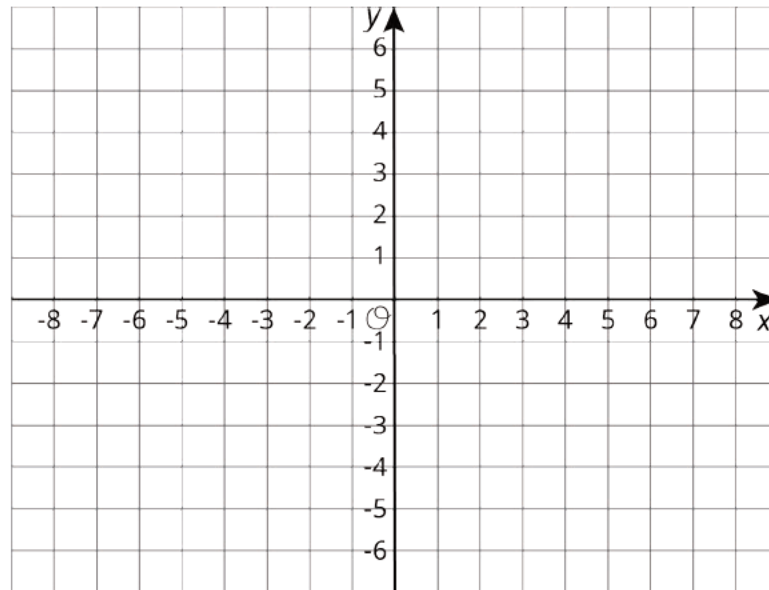
Looking at the coordinates of the intersection point, we see that Car A and Car B will both be 7.5 miles from the rest area after 0.1 hours (which is 6 minutes).

Now suppose another car, Car C, had also passed the rest stop at time $t = 0$ and traveled in the same direction as Car A, also going 75 miles per hour. Its equation would also be $d = 75t$. Any solution to the equation for Car A would also be a solution for Car C, and any solution to the equation for Car C would also be a solution for Car A. The line for Car C would land right on top of the line for Car A. In this case, every point on the graphed line is a solution to both equations, so that there are infinitely many solutions to the question “when are Car A and Car C the same distance from the rest stop?” This would mean that Car A and Car C were side by side for their whole journey.

When we have two linear equations that are equivalent to each other, like $y = 3x + 2$ and $2y = 6x + 4$, we will get two lines that are “right on top” of each other. Any solution to one equation is also solution to the other, so these two lines intersect at infinitely many points.

Unit 4 Lesson 11 Cumulative Practice Problems

1. Diego has \$11 and begins saving \$5 each week toward buying a new phone. At the same time that Diego begins saving, Lin has \$60 and begins spending \$2 per week on supplies for her art class. Is there a week when they have the same amount of money? How much do they have at that time?
2. Use a graph to find x and y values that make both $y = \frac{-2}{3}x + 3$ and $y = 2x - 5$ true.

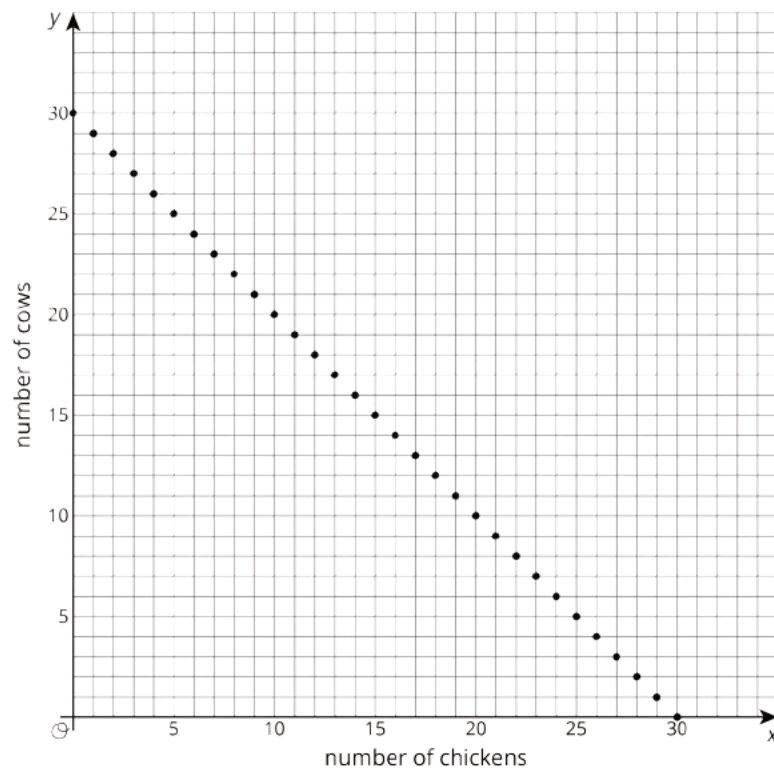


3. The point where the graphs of two equations intersect has y -coordinate 2. One equation is $y = -3x + 5$. Find the other equation if its graph has a slope of 1.

4. A farm has chickens and cows. All the cows have 4 legs and all the chickens have 2 legs. All together, there are 82 cow and chicken legs on the farm. Complete the table to show some possible combinations of chickens and cows to get 82 total legs.

number of chickens (x)	number of cows (y)
35	
7	
	10
19	
	5

Here is a graph that shows possible combinations of chickens and cows that add up to 30 animals:



If the farm has 30 chickens and cows, and there are 82 chicken and cow legs all together, then how many chickens and how many cows could the farm have?

(From Unit 4, Lesson 10.)

Lesson 12: Systems of Equations

12.1: Milkshakes

Diego and Lin are drinking milkshakes. Lin starts with 12 ounces and drinks $\frac{1}{4}$ ounce per second. Diego starts with 20 ounces and drinks $\frac{2}{3}$ ounce per second.

1. How long will it take Lin and Diego to finish their milkshakes?
2. Without graphing, explain what the graphs in this situation would look like. Think about slope, intercepts, axis labels, units, and intersection points to guide your thinking.
3. Discuss your description with your partner. If you disagree, work to reach an agreement.

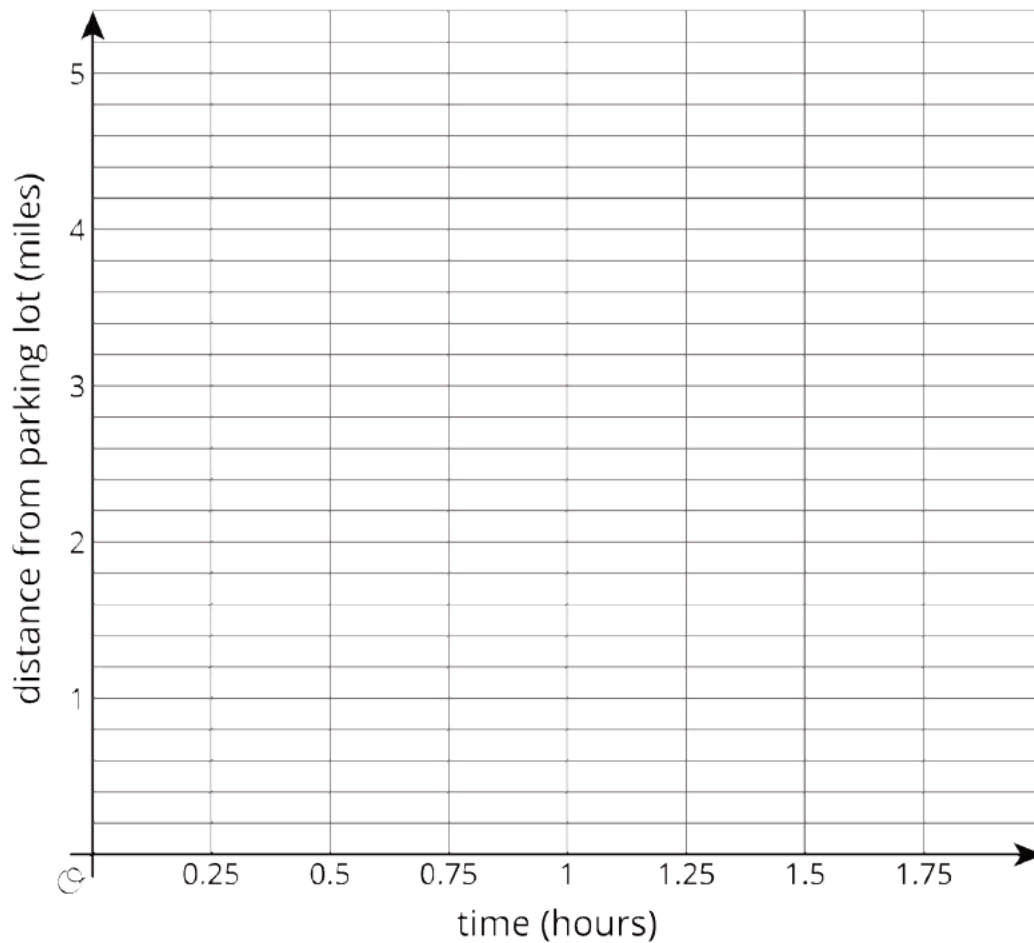
12.2: Passing on the Trail

There is a hiking trail near the town where Han and Jada live that starts at a parking lot and ends at a lake. Han and Jada both decide to hike from the parking lot to the lake and back, but they start their hikes at different times.

At the time that Han reaches the lake and starts to turn back, Jada is 0.6 miles away from the parking lot and hiking at a constant speed of 3.2 miles per hour towards the lake. Han's distance, d , from the parking lot can be expressed as $d = -2.4t + 4.8$, where t represents the time in hours since he left the lake.

1. What is an equation for Jada's distance from the parking lot as she heads toward the lake?

2. Draw both graphs: one representing Han's equation and one representing Jada's equation. It is important to be very precise! Be careful, work in pencil, and use a ruler.

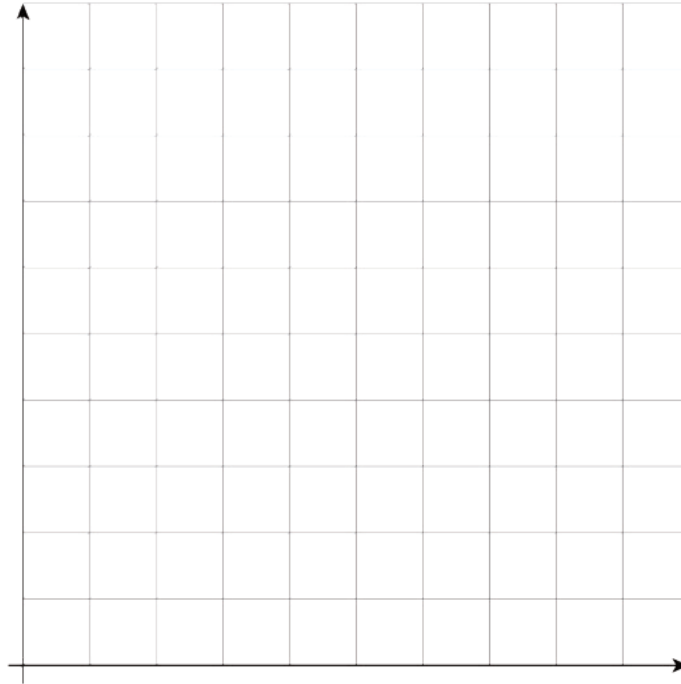


3. Find the point where the two graphs intersect each other. What are the coordinates of this point?
4. What do the coordinates mean in this situation?
5. What has to be true about the relationship between these coordinates and Jada's equation?
6. What has to be true about the relationship between these coordinates and Han's equation?

12.3: Stacks of Cups

A stack of n small cups has a height, h , in centimeters of $h = 1.5n + 6$. A stack of n large cups has a height, h , in centimeters of $h = 1.5n + 9$.

1. Graph the equations for each cup on the same set of axes. Make sure to label the axes and decide on an appropriate scale.



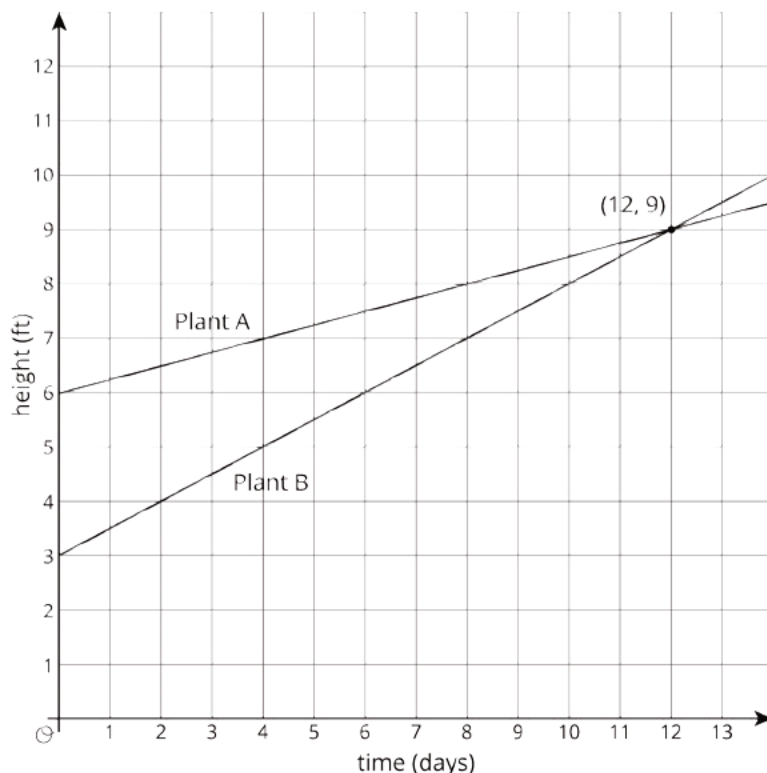
2. For what number of cups will the two stacks have the same height?

Lesson 12 Summary

A **system of equations** is a set of 2 (or more) equations where the variables represent the same unknown values. For example, suppose that two different kinds of bamboo are planted at the same time. Plant A starts at 6 ft tall and grows at a constant rate of $\frac{1}{4}$ foot each day. Plant B starts at 3 ft tall and grows at a constant rate of $\frac{1}{2}$ foot each day. We can write equations $y = \frac{1}{4}x + 6$ for Plant A and $y = \frac{1}{2}x + 3$ for Plant B, where x represents the number of days after being planted, and y represents height. We can write this system of equations.

$$\begin{cases} y = \frac{1}{4}x + 6 \\ y = \frac{1}{2}x + 3 \end{cases}$$

Solving a system of equations means to find the values of x and y that make both equations true at the same time. One way we have seen to find the solution to a system of equations is to graph both lines and find the intersection point. The intersection point represents the pair of x and y values that make both equations true. Here is a graph for the bamboo example:



The solution to this system of equations is $(12, 9)$, which means that both bamboo plants will be 9 feet tall after 12 days.

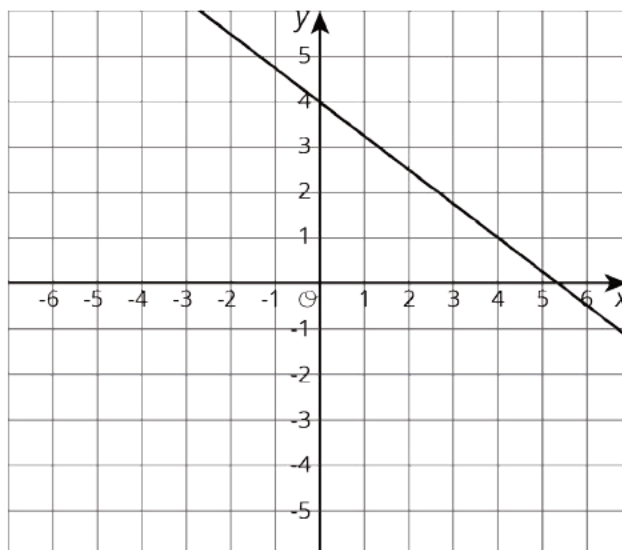
We have seen systems of equations that have no solutions, one solution, and infinitely many solutions.

- When the lines do not intersect, there is no solution. (Lines that do not intersect are *parallel*.)
- When the lines intersect once, there is one solution.
- When the lines are right on top of each other, there are infinitely many solutions.

In future lessons, we will see that some systems cannot be easily solved by graphing, but can be easily solved using algebra.

Unit 4 Lesson 12 Cumulative Practice Problems

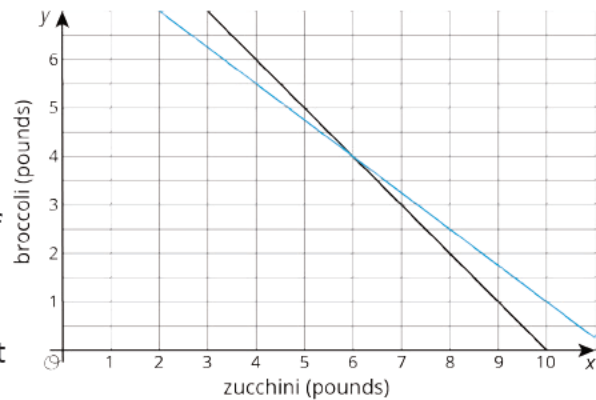
1. Here is the graph for one equation in a system of equations:



- Write a second equation for the system so it has infinitely many solutions.
 - Write a second equation whose graph goes through $(0, 1)$ so the system has no solutions.
 - Write a second equation whose graph goes through $(0, 2)$ so the system has one solution at $(4, 1)$.
2. Create a second equation so the system has no solutions.

$$\begin{cases} y = \frac{3}{4}x - 4 \end{cases}$$

3. Andre is in charge of cooking broccoli and zucchini for a large group. He has to spend all \$17 he has and can carry 10 pounds of veggies. Zucchini costs \$1.50 per pound and broccoli costs \$2 per pound. One graph shows combinations of zucchini and broccoli that weigh 10 pounds and the other shows combinations of zucchini and broccoli that cost \$17.



- Name one combination of veggies that weighs 10 pounds but does not cost \$17.
- Name one combination of veggies that costs \$17 but does not weigh 10 pounds.
- How many pounds each of zucchini and broccoli can Andre get so that he spends all \$17 and gets 10 pounds of veggies?

(From Unit 4, Lesson 10.)

4. The temperature in degrees Fahrenheit, F , is related to the temperature in degrees Celsius, C , by the equation

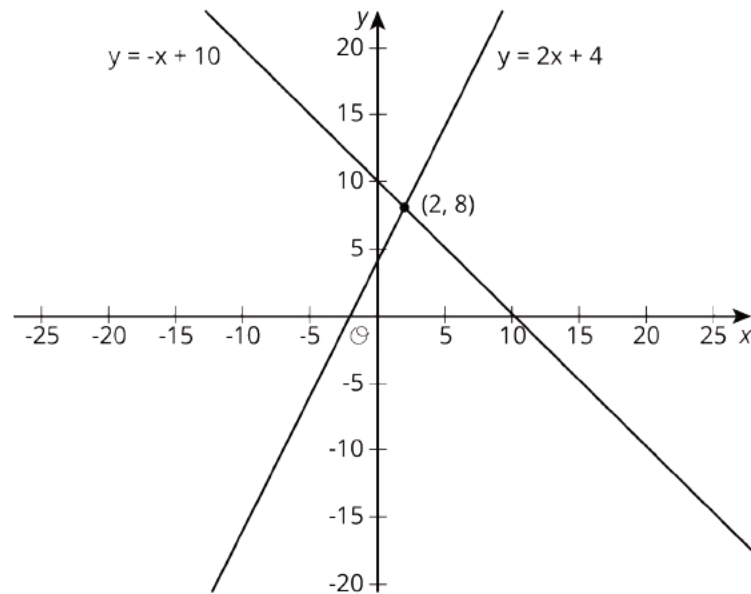
$$F = \frac{9}{5}C + 32$$

- In the Sahara desert, temperatures often reach 50 degrees Celsius. How many degrees Fahrenheit is this?
- In parts of Alaska, the temperatures can reach -60 degrees Fahrenheit. How many degrees Celsius is this?
- There is one temperature where the degrees Fahrenheit and degrees Celsius are the same, so that $C = F$. Use the expression from the equation, where F is expressed in terms of C , to solve for this temperature.

(From Unit 4, Lesson 9.)

Lesson 13: Solving Systems of Equations

13.1: True or False: Two Lines

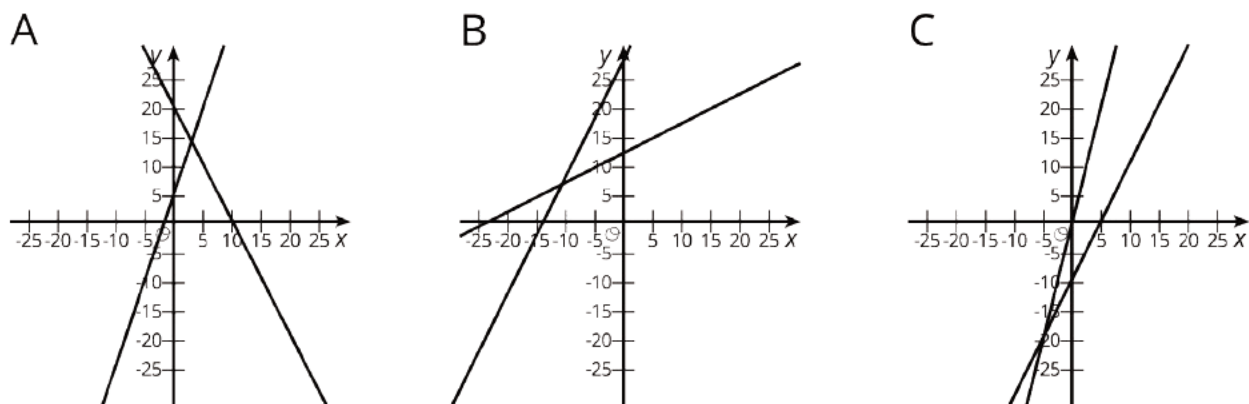


Use the lines to decide whether each statement is true or false. Be prepared to explain your reasoning using the lines.

1. A solution to $8 = -x + 10$ is 2.
2. A solution to $2 = 2x + 4$ is 8.
3. A solution to $-x + 10 = 2x + 4$ is 8.
4. A solution to $-x + 10 = 2x + 4$ is 2.
5. There are no values of x and y that make $y = -x + 10$ and $y = 2x + 4$ true at the same time.

13.2: Matching Graphs to Systems

Here are three systems of equations graphed on a coordinate plane:



1. Match each figure to one of the systems of equations shown here.

a.
$$\begin{cases} y = 3x + 5 \\ y = -2x + 20 \end{cases}$$

b.
$$\begin{cases} y = 2x - 10 \\ y = 4x - 1 \end{cases}$$

c.
$$\begin{cases} y = 0.5x + 12 \\ y = 2x + 27 \end{cases}$$

2. Find the solution to each system and check that your solution is reasonable based on the graph.

13.3: Different Types of Systems

Your teacher will give you a page with some systems of equations.

1. Graph each system of equations carefully on the provided coordinate plane.
2. Describe what the graph of a system of equations looks like when it has . . .
 - a. 1 solution
 - b. 0 solutions
 - c. infinitely many solutions

Are you ready for more?

The graphs of the equations $Ax + By = 15$ and $Ax - By = 9$ intersect at $(2, 1)$. Find A and B . Show or explain your reasoning.

Lesson 13 Summary

Sometimes it is easier to solve a system of equations without having to graph the equations and look for an intersection point. In general, whenever we are solving a system of equations written as

$$\begin{cases} y = [\text{some stuff}] \\ y = [\text{some other stuff}] \end{cases}$$

we know that we are looking for a pair of values (x, y) that makes both equations true. In particular, we know that the value for y will be the same in both equations. That means that

$$[\text{some stuff}] = [\text{some other stuff}]$$

For example, look at this system of equations:

$$\begin{cases} y = 2x + 6 \\ y = -3x - 4 \end{cases}$$

Since the y value of the solution is the same in both equations, then we know

$$2x + 6 = -3x - 4$$

We can solve this equation for x :

$$2x + 6 = -3x - 4$$

$$5x + 6 = -4$$

$$5x = -10$$

$$x = -2$$

add $3x$ to each side

subtract 6 from each side

divide each side by 5

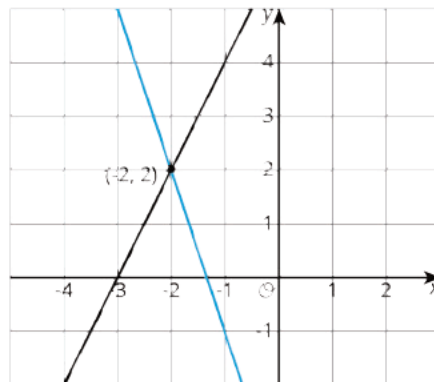
But this is only half of what we are looking for: we know the value for x , but we need the corresponding value for y . Since both equations have the same y value, we can use either equation to find the y -value:

$$y = 2(-2) + 6$$

Or

$$y = -3(-2) - 4$$

In both cases, we find that $y = 2$. So the solution to the system is $(-2, 2)$. We can verify this by graphing both equations in the coordinate plane.

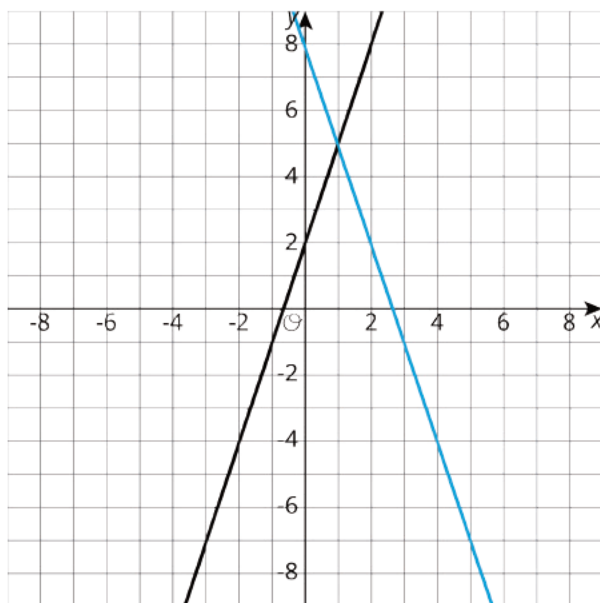


In general, a system of linear equations can have:

- No solutions. In this case, the lines that correspond to each equation never intersect.
- Exactly one solution. The lines that correspond to each equation intersect in exactly one point.
- An infinite number of solutions. The graphs of the two equations are the same line!

Unit 4 Lesson 13 Cumulative Practice Problems

1. a. Write equations for the lines shown.



- b. Describe how to find the solution to the corresponding system by looking at the graph.
- c. Describe how to find the solution to the corresponding system by using the equations.
2. The solution to a system of equations is $(5, -19)$. Choose two equations that might make up the system.
- A. $y = -3x - 6$
 - B. $y = 2x - 23$
 - C. $y = -7x + 16$
 - D. $y = x - 17$
 - E. $y = -2x - 9$

3. Solve the system of equations: $\begin{cases} y = 4x - 3 \\ y = -2x + 9 \end{cases}$

4. Solve the system of equations: $\begin{cases} y = \frac{5}{4}x - 2 \\ y = \frac{-1}{4}x + 19 \end{cases}$

5. Here is an equation: $\frac{15(x-3)}{5} = 3(2x - 3)$

a. Solve the equation by using the distributive property first.

b. Solve the equation without using the distributive property.

c. Check your solution.

(From Unit 4, Lesson 6.)

Lesson 14: Solving More Systems

14.1: Algebra Talk: Solving Systems Mentally

Solve these without writing anything down:

$$\begin{cases} x = 5 \\ y = x - 7 \end{cases}$$

$$\begin{cases} y = 4 \\ y = x + 3 \end{cases}$$

$$\begin{cases} x = 8 \\ y = -11 \end{cases}$$

14.2: Challenge Yourself

Here are a lot of systems of equations:

$$A \begin{cases} y = 4 \\ x = -5y + 6 \end{cases}$$

$$E \begin{cases} y = -3x - 5 \\ y = 4x + 30 \end{cases}$$

$$I \begin{cases} 3x + 4y = 10 \\ x = 2y \end{cases}$$

$$B \begin{cases} y = 7 \\ x = 3y - 4 \end{cases}$$

$$F \begin{cases} y = 3x - 2 \\ y = -2x + 8 \end{cases}$$

$$J \begin{cases} y = 3x + 2 \\ 2x + y = 47 \end{cases}$$

$$C \begin{cases} y = \frac{3}{2}x + 7 \\ x = -4 \end{cases}$$

$$G \begin{cases} y = 3x \\ x = -2y + 56 \end{cases}$$

$$K \begin{cases} y = -2x + 5 \\ 2x + 3y = 31 \end{cases}$$

$$D \begin{cases} y = -3x + 10 \\ y = -2x + 6 \end{cases}$$

$$H \begin{cases} x = 2y - 15 \\ y = -2x \end{cases}$$

$$L \begin{cases} x + y = 10 \\ x = 2y + 1 \end{cases}$$

1. Without solving, identify 3 systems that you think would be the least difficult to solve and 3 systems that you think would be the most difficult to solve. Be prepared to explain your reasoning.

2. Choose 4 systems to solve. At least one should be from your "least difficult" list and one should be from your "most difficult" list.

14.3: Five Does Not Equal Seven

Tyler was looking at this system of equations:

$$\begin{cases} x + y = 5 \\ x + y = 7 \end{cases}$$

He said, "Just looking at the system, I can see it has no solution. If you add two numbers, that sum can't be equal to two different numbers."

Do you agree with Tyler?

Are you ready for more?

In rectangle $ABCD$, side AB is 8 centimeters and side BC is 6 centimeters. F is a point on BC and E is a point on AB . The area of triangle DFC is 20 square centimeters, and the area of triangle DEF is 16 square centimeters. What is the area of triangle AED ?

Lesson 14 Summary

When we have a system of linear equations where one of the equations is of the form $y = [\text{stuff}]$ or $x = [\text{stuff}]$, we can solve it algebraically by using a technique called *substitution*. The basic idea is to replace a variable with an expression it is equal to (so the expression is like a substitute for the variable). For example, let's start with the system:

$$\begin{cases} y = 5x \\ 2x - y = 9 \end{cases}$$

Since we know that $y = 5x$, we can substitute $5x$ for y in the equation $2x - y = 9$,

$$2x - (5x) = 9,$$

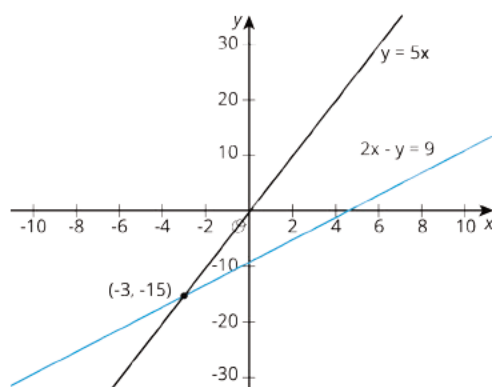
and then solve the equation for x ,

$$x = -3.$$

We can find y using either equation. Using the first one: $y = 5 \cdot -3$. So

$$(-3, -15)$$

is the solution to this system. We can verify this by looking at the graphs of the equations in the system:



Sure enough! They intersect at $(-3, -15)$.

We didn't know it at the time, but we were actually using substitution in the last lesson as well. In that lesson, we looked at the system

$$\begin{cases} y = 2x + 6 \\ y = -3x - 4 \end{cases}$$

and we substituted $2x + 6$ for y into the second equation to get $2x + 6 = -3x - 4$. Go back and check for yourself!

Unit 4 Lesson 14 Cumulative Practice Problems

1. Solve: $\begin{cases} y = 6x \\ 4x + y = 7 \end{cases}$

2. Solve: $\begin{cases} y = 3x \\ x = -2y + 70 \end{cases}$

3. Which equation, together with $y = -1.5x + 3$, makes a system with one solution?

A. $y = -1.5x + 6$

B. $y = -1.5x$

C. $2y = -3x + 6$

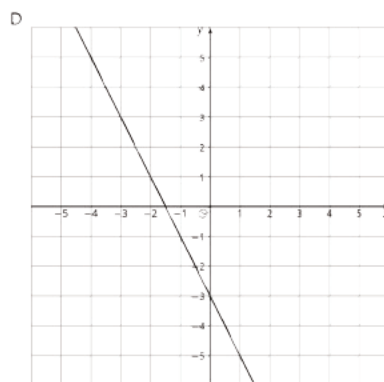
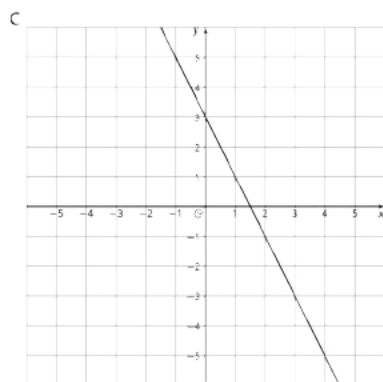
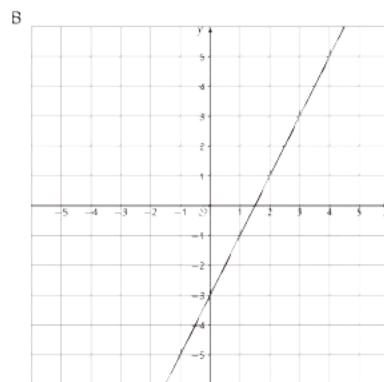
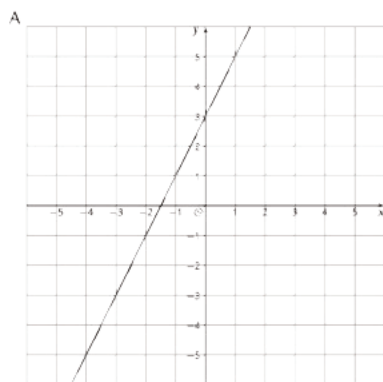
D. $2y + 3x = 6$

E. $y = -2x + 3$

4. The system $x - 6y = 4$, $3x - 18y = 4$ has no solution.
- a. Change one constant or coefficient to make a new system with one solution.

 - b. Change one constant or coefficient to make a new system with an infinite number of solutions.

5. Match each graph to its equation.



a. $y = 2x + 3$

b. $y = -2x + 3$

c. $y = 2x - 3$

d. $y = -2x - 3$

(From Unit 3, Lesson 11.)

6. Here are two points: $(-3, 4)$, $(1, 7)$. What is the slope of the line between them?

A. $\frac{4}{3}$

B. $\frac{3}{4}$

C. $\frac{1}{6}$

D. $\frac{2}{3}$

(From Unit 3, Lesson 10.)

Lesson 15: Writing Systems of Equations

15.1: How Many Solutions? Matching

Match each system of equations with the number of solutions the system has.

1. $\begin{cases} y = -\frac{4}{3}x + 4 \\ y = -\frac{4}{3}x - 1 \end{cases}$

1. No solutions

2. One solution

2. $\begin{cases} y = 4x - 5 \\ y = -2x + 7 \end{cases}$

3. Infinitely many solutions

3. $\begin{cases} 2x + 3y = 8 \\ 4x + 6y = 17 \end{cases}$

4. $\begin{cases} y = 5x - 15 \\ y = 5(x - 3) \end{cases}$

15.2: Situations and Systems

For each situation:

- Create a system of equations.
 - Then, without solving, interpret what the solution to the system would tell you about the situation.
1. Lin's family is out for a bike ride when her dad stops to take a picture of the scenery. He tells the rest of the family to keep going and that he'll catch up. Lin's dad spends 5 minutes taking the photo and then rides at 0.24 miles per minute until he meets up with the rest of the family further along the bike path. Lin and the rest were riding at 0.18 miles per minute.
 2. Noah is planning a kayaking trip. Kayak Rental A charges a base fee of \$15 plus \$4.50 per hour. Kayak Rental B charges a base fee of \$12.50 plus \$5 per hour.

3. Diego is making a large batch of pastries. The recipe calls for 3 strawberries for every apple. Diego used 52 fruits all together.
4. Flour costs \$0.80 per pound and sugar costs \$0.50 per pound. An order of flour and sugar weighs 15 pounds and costs \$9.00.

15.3: Info Gap: Racing and Play Tickets

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

Continue to ask questions until you have enough information to solve the problem.

4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.
2. Ask your partner “*What specific information do you need?*” and wait for them to *ask* for information.

If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don’t have that information.

3. Before sharing the information, ask “*Why do you need that information?*” Listen to your partner’s reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

15.4: Solving Systems Practice

Here are a lot of systems of equations:

- $\begin{cases} y = -2x + 6 \\ y = x - 3 \end{cases}$

- $\begin{cases} y = x - 6 \\ x = 6 + y \end{cases}$

- $\begin{cases} y = 5x - 4 \\ y = 4x + 12 \end{cases}$

- $\begin{cases} y = 0.24x \\ y = 0.18x + 0.9 \end{cases}$

- $\begin{cases} y = \frac{2}{3}x - 4 \\ y = -\frac{4}{3}x + 9 \end{cases}$

- $\begin{cases} y = 4.5x + 15 \\ y = 5x + 12.5 \end{cases}$

- $\begin{cases} 4y + 7x = 6 \\ 4y + 7x = -5 \end{cases}$

- $\begin{cases} y = 3x \\ x + y = 52 \end{cases}$

1. Without solving, identify 3 systems that you think would be the least difficult for you to solve and 3 systems you think would be the most difficult. Be prepared to explain your reasoning.
2. Choose 4 systems to solve. At least one should be from your "least difficult" list and one should be from your "most difficult" list.

Lesson 15 Summary

We have learned how to solve many kinds of systems of equations using algebra that would be difficult to solve by graphing. For example, look at

$$\begin{cases} y = 2x - 3 \\ x + 2y = 7 \end{cases}$$

The first equation says that $y = 2x - 3$, so wherever we see y , we can substitute the expression $2x - 3$ instead. So the second equation becomes $x + 2(2x - 3) = 7$.

We can solve for x :

$$\begin{aligned} x + 4x - 6 &= 7 && \text{distributive property} \\ 5x - 6 &= 7 && \text{combine like terms} \\ 5x &= 13 && \text{add 6 to each side} \\ x &= \frac{13}{5} && \text{multiply each side by } \frac{1}{5} \end{aligned}$$

We know that the y value for the solution is the same for either equation, so we can use either equation to solve for it. Using the first equation, we get:

$$\begin{aligned} y &= 2\left(\frac{13}{5}\right) - 3 && \text{substitute } x = \frac{13}{5} \text{ into the equation} \\ y &= \frac{26}{5} - 3 && \text{multiply } 2\left(\frac{13}{5}\right) \text{ to make } \frac{26}{5} \\ y &= \frac{26}{5} - \frac{15}{5} && \text{rewrite 3 as } \frac{15}{5} \\ y &= \frac{11}{5} \end{aligned}$$

If we substitute $x = \frac{13}{5}$ into the other equation, $x + 2y = 7$, we get the same y value. So the solution to the system is $\left(\frac{13}{5}, \frac{11}{5}\right)$.

There are many kinds of systems of equations that we will learn how to solve in future

grades, like $\begin{cases} 2x + 3y = 6 \\ -x + 2y = 3 \end{cases}$.

Or even $\begin{cases} y = x^2 + 1 \\ y = 2x + 3 \end{cases}$.

Unit 4 Lesson 15 Cumulative Practice Problems

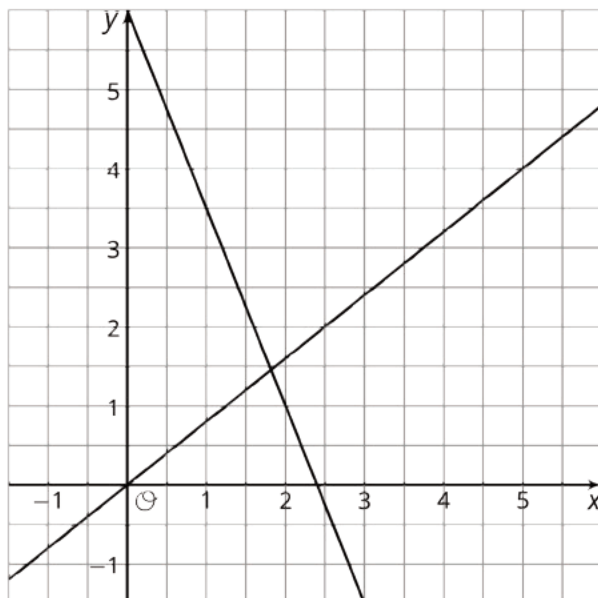
1. Kiran and his cousin work during the summer for a landscaping company. Kiran's cousin has been working for the company longer, so his pay is 30% more than Kiran's. Last week his cousin worked 27 hours, and Kiran worked 23 hours. Together, they earned \$493.85. What is Kiran's hourly pay? Explain or show your reasoning.
2. Decide which story can be represented by the system of equations $y = x + 6$ and $x + y = 100$. Explain your reasoning.
 - a. Diego's teacher writes a test worth 100 points. There are 6 more multiple choice questions than short answer questions.
 - b. Lin and her younger cousin measure their heights. They notice that Lin is 6 inches taller, and their heights add up to exactly 100 inches.

3. Clare and Noah play a game in which they earn the same number of points for each goal and lose the same number of points for each penalty. Clare makes 6 goals and 3 penalties, ending the game with 6 points. Noah earns 8 goals and 9 penalties and ends the game with -22 points.
- a. Write a system of equations that describes Clare and Noah's outcomes. Use x to represent the number of points for a goal and y to represent the number of points for a penalty.
- b. Solve the system. What does your solution mean?

4. Solve:
$$\begin{cases} y = 6x - 8 \\ y = -3x + 10 \end{cases}$$

(From Unit 4, Lesson 14.)

5. a. Estimate the coordinates of the point where the two lines meet.



- b. Choose two equations that make up the system represented by the graph.

i. $y = \frac{5}{4}x$

ii. $y = 6 - 2.5x$

iii. $y = 2.5x + 6$

iv. $y = 6 - 3x$

v. $y = 0.8x$

- c. Solve the system of equations and confirm the accuracy of your estimate.

(From Unit 4, Lesson 13.)

Lesson 16: Solving Problems with Systems of Equations

16.1: Are We There Yet?

A car is driving towards home at 0.5 miles per minute. If the car is 4 miles from home at $t = 0$, which of the following can represent the distance that the car has left to drive?

- $0.5t$
- $4 + 0.5t$
- $4 - 0.5t$
- $4 \cdot (0.5t)$

16.2: Cycling, Fundraising, Working, and ___?

Solve each problem. Explain or show your reasoning.

1. Two friends live 7 miles apart. One Saturday, the two friends set out on their bikes at 8 am and started riding towards each other. One rides at 0.2 miles per minute, and the other rides at 0.15 miles per minute. At what time will the two friends meet?

2. Students are selling grapefruits and nuts for a fundraiser. The grapefruits cost \$1 each and a bag of nuts cost \$10 each. They sold 100 items and made \$307. How many grapefruits did they sell?
3. Jada earns \$7 per hour mowing her neighbors' lawns. Andre gets paid \$5 per hour for the first hour of babysitting and \$8 per hour for any additional hours he babysits. What is the number of hours they both can work so that they get paid the same amount?
4. Pause here so your teacher can review your work. Then, invent another problem that is like one of these, but with different numbers. Solve your problem.

5. Create a visual display that includes:
 - The new problem you wrote, without the solution.
 - Enough work space for someone to show a solution.
6. Trade your display with another group, and solve each other's new problem. Make sure that you explain your solution carefully. Be prepared to share this solution with the class.
7. When the group that got the problem you invented shares their solution, check that their answer is correct.

Are you ready for more?

On a different Saturday, two friends set out on bikes at 8:00 am and met up at 8:30 am. (The same two friends who live 7 miles apart.) If one was riding at 10 miles per hour, how fast was the other riding?

Unit 4 Lesson 16 Cumulative Practice Problems

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