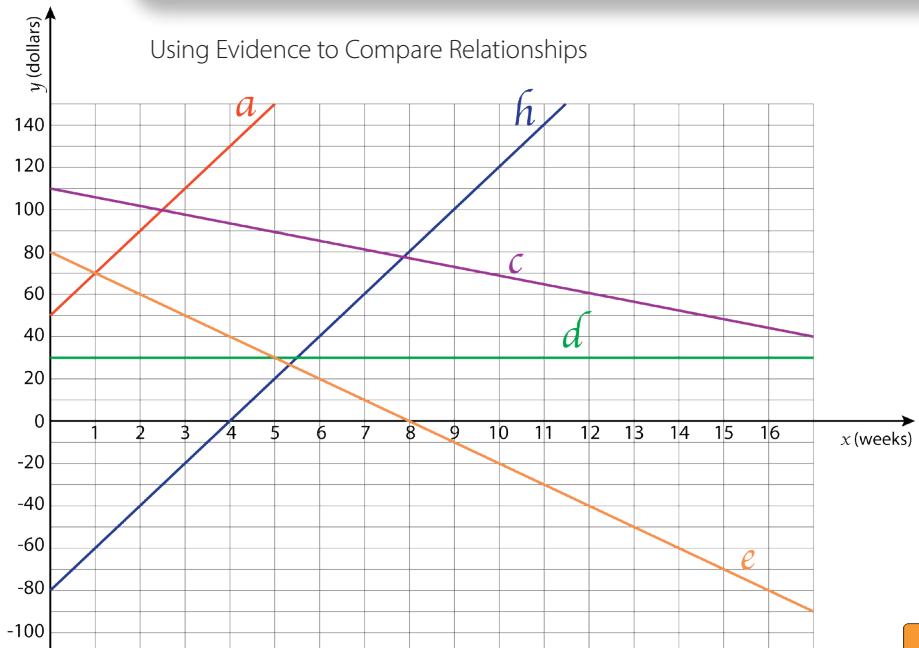
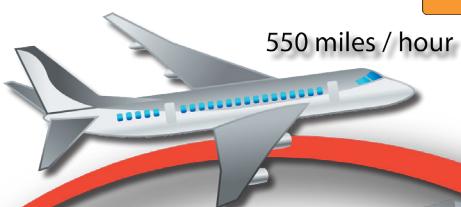


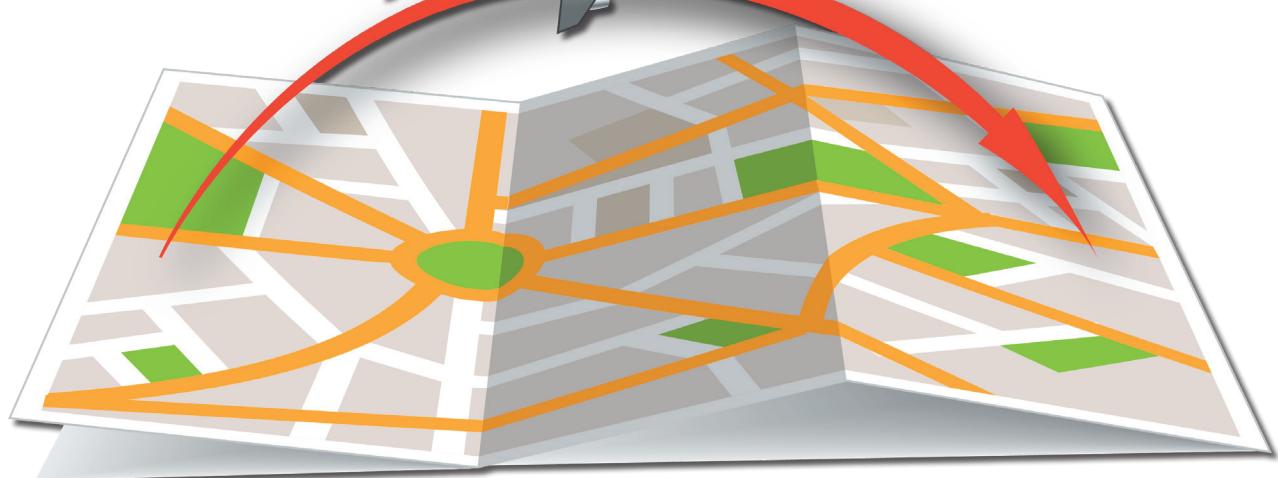
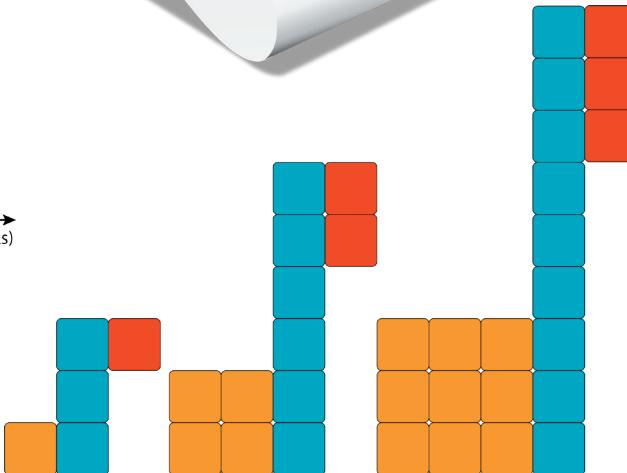
# Linear Relationships

## Teacher Guide

Using Evidence to Compare Relationships


Understanding  
Proportional Relationships


Recognizing Patterns of Change



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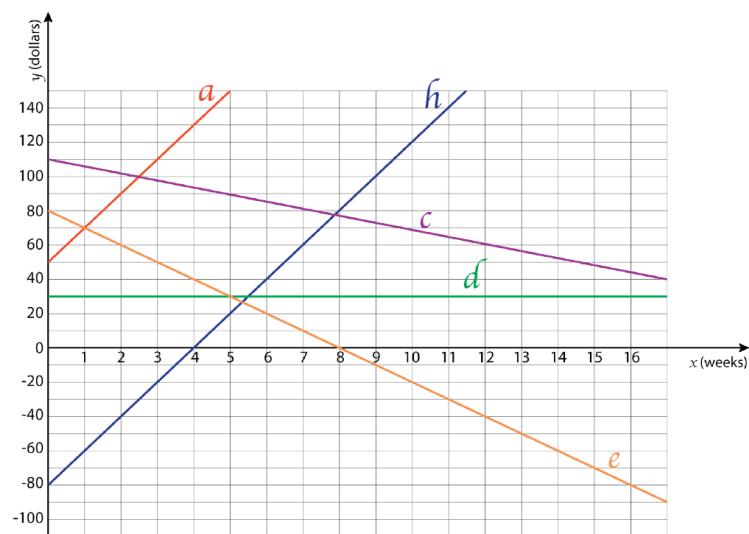
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# Linear Relationships

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## Linear Relationships

### Teacher Guide

Core Knowledge Mathematics™

# Linear Relationships

## Unit Narrative

Work with linear relationships in grade 8 builds on earlier work with rates and proportional relationships in grade 7, and grade 8 work with geometry. At the end of the previous unit on dilations, students learned the terms “slope” and “slope triangle,” used the similarity of slope triangles on the same line to understand that any two distinct points on a line determine the same slope, and found an equation for a line with a positive slope and vertical intercept. In this unit, students gain experience with linear relationships and their representations as graphs, tables, and equations through activities designed and sequenced to allow them to make sense of problems and persevere in solving them (MP1). Because of this dependency, this unit and the previous one should be done in order.

The unit begins by revisiting different representations of proportional relationships (graphs, tables, and equations), and the role of the constant of proportionality in each representation and how it may be interpreted in context (MP2).

Next, students analyze the relationship between number of cups in a given stack of cups and the height of the stack—a relationship that is linear but not proportional—in order to answer the question “How many cups are needed to get to a height of 50 cm?” They are not asked to solve this problem in a specific way, giving them an opportunity to choose and use strategically (MP5) representations that appeared earlier in this unit (table, equation, graph) or in the previous unit (equation, graph). Students are introduced to “rate of change” as a way to describe the rate per 1 in a linear relationship and note that its numerical value is the same as that of the slope of the line that represents the relationship. Students analyze another linear relationship (height of water in a cylinder vs number of cubes in the cylinder) and establish a way to compute the slope of a line from any two distinct points on the line via repeated reasoning (MP8). They learn a third way to obtain an equation for a linear relationship by viewing the graph of a line in the coordinate plane as the vertical translation of a proportional relationship (MP7).

So far, the unit has involved only lines with positive slopes and  $y$ -intercepts. Students next consider the graph of a line with a negative  $y$ -intercept and equations that might represent it. They consider situations represented by linear relationships with negative rates of change, graph these (MP4), and interpret coordinates of points on the graphs in context (MP2).

The unit concludes with two lessons that involve graphing two equations in two unknowns and finding and interpreting their solutions (MP2). Doing this involves considering correspondences among different representations (MP1), in particular, what it means for a pair of values to be a solution for an equation and the correspondence between coordinates of points on a graph and solutions of an equation.

In this unit, several lesson plans suggest that each student have access to a *geometry toolkit*. Each toolkit contains tracing paper, graph paper, colored pencils, scissors, ruler, protractor, and an index card to use as a straightedge or to mark right angles, giving students opportunities to select appropriate tools and use them strategically to solve problems (MP5). Note that even students in a digitally enhanced classroom should have access to such tools; apps and simulations should be considered additions to their toolkits, not replacements for physical tools.

*On using the terms ratio, rate, and proportion.* In these materials, a *quantity* is a measurement that is or can be specified by a number and a unit, e.g., 4 oranges, 4 centimeters, "my height in feet," or "my height" (with the understanding that a unit of measurement will need to be chosen). The term *ratio* is used to mean an association between two or more quantities and the fractions  $\frac{a}{b}$  and  $\frac{b}{a}$  are never called ratios. The fractions  $\frac{a}{b}$  and  $\frac{b}{a}$  are identified as "unit rates" for the ratio  $a : b$ . The word "per" is used with students in interpreting a unit rate in context, as in "\$3 per ounce," and "at the same rate" is used to signify a situation characterized by equivalent ratios.

In grades 6–8, students write rates without abbreviated units, for example as "3 miles per hour" or "3 miles in every 1 hour." Use of notation for derived units such as  $\frac{\text{mi}}{\text{hr}}$  waits for high school—except for the special cases of area and volume. Students have worked with area since grade 3 and volume since grade 5. Before grade 6, they have learned the meanings of such things as  $\text{sq cm}$  and  $\text{cu cm}$ . After students learn exponent notation in grade 6, they also use  $\text{cm}^2$  and  $\text{cm}^3$ .

A *proportional relationship* is a collection of equivalent ratios. In high school—after their study of ratios, rates, and proportional relationships—students discard the term "unit rate," referring to  $a$  to  $b$ ,  $a : b$ , and  $\frac{a}{b}$  as "ratios."

A proportional relationship between two quantities represented by  $a$  and  $b$  is associated with *two* constants of proportionality:  $\frac{a}{b}$  and  $\frac{b}{a}$ . Throughout the unit, the convention is if  $a$  and  $b$  are represented by columns in a table and the column for  $a$  is to the left of the column for  $b$ , then  $\frac{b}{a}$  is the constant of proportionality for the relationship represented by the table.

### Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as representing, generalizing, and explaining. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

#### Represent

- situations involving proportional relationships (Lesson 1)
- constants of proportionality in different ways (Lesson 3)
- slope using expressions (Lesson 7)
- linear relationships using graphs, tables, equations, and verbal descriptions (Lesson 8)

- situations using negative slopes and slopes of zero (Lesson 9)
- situations by graphing lines and writing equations (Lesson 12)
- situations involving linear relationships (Lesson 14)

#### Generalize

- categories for graphs (Lesson 2)
- about equations and linear relationships (Lesson 7)
- in order to make predictions about the slope of lines (Lesson 10)

#### Explain

- how to graph proportional relationships (Lesson 3)
- how to use a graph to determine information about a linear situation (Lessons 5 and 6)
- how to graph linear relationships (Lesson 10)
- how slope relates to changes in a situation (Lesson 11)

In addition, students are expected to describe observations about the equation of a translated line and describe features of an equation that could make one variable easier or harder to solve for than the other. Students will also have opportunities to use language to interpret situations involving proportional relationships, interpret graphs using different scales, interpret slopes and intercepts of linear graphs, justify reasoning about linear relationships, justify correspondences between different representations, and justify which equations correspond to graphs of horizontal and vertical lines.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow the one in which it was first introduced.

# Learning Targets

## Linear Relationships

### Lesson 1: Understanding Proportional Relationships

- I can graph a proportional relationship from a story.
- I can use the constant of proportionality to compare the pace of different animals.

### Lesson 2: Graphs of Proportional Relationships

- I can graph a proportional relationship from an equation.
- I can tell when two graphs are of the same proportional relationship even if the scales are different.

### Lesson 3: Representing Proportional Relationships

- I can scale and label a coordinate axes in order to graph a proportional relationship.

### Lesson 4: Comparing Proportional Relationships

- I can compare proportional relationships represented in different ways.

### Lesson 5: Introduction to Linear Relationships

- I can find the rate of change of a linear relationship by figuring out the slope of the line representing the relationship.

### Lesson 6: More Linear Relationships

- I can interpret the vertical intercept of a graph of a real-world situation.
- I can match graphs to the real-world situations they represent by identifying the slope and the vertical intercept.

### Lesson 7: Representations of Linear Relationships

- I can use patterns to write a linear equation to represent a situation.
- I can write an equation for the relationship between the total volume in a graduated cylinder and the number of objects added to the graduated cylinder.

### **Lesson 8: Translating to $y = mx + b$**

- I can explain where to find the slope and vertical intercept in both an equation and its graph.
- I can write equations of lines using  $y=mx+b$ .

### **Lesson 9: Slopes Don't Have to be Positive**

- I can give an example of a situation that would have a negative slope when graphed.
- I can look at a graph and tell if the slope is positive or negative and explain how I know.

### **Lesson 10: Calculating Slope**

- I can calculate positive and negative slopes given two points on the line.
- I can describe a line precisely enough that another student can draw it.

### **Lesson 11: Equations of All Kinds of Lines**

- I can write equations of lines that have a positive or a negative slope.
- I can write equations of vertical and horizontal lines.

### **Lesson 12: Solutions to Linear Equations**

- I know that the graph of an equation is a visual representation of all the solutions to the equation.
- I understand what the solution to an equation in two variables is.

### **Lesson 13: More Solutions to Linear Equations**

- I can find solutions  $(x, y)$  to linear equations given either the  $x$ - or the  $y$ -value to start from.

### **Lesson 14: Using Linear Relations to Solve Problems**

- I can write linear equations to reason about real-world situations.

lesson	new terminology	
	receptive	productive
8.3.1	represent scale label	<b>constant of proportionality</b>
8.3.2	equation	
8.3.3	<b>rate of change</b>	equation
8.3.5	<b>linear relationship</b> constant rate	<b>slope</b>
8.3.6	<b>vertical intercept</b> $y$ -intercept	
8.3.7	initial (value or amount)	constant rate
8.3.8	relate	
8.3.9	horizontal intercept $x$ -intercept	
8.3.10	intersection point	<b>rate of change</b> <b>vertical intercept</b> $y$ -intercept
8.3.11	constraint	horizontal line vertical line
8.3.12	<b>solution to an equation with two variables</b> variable combination set of solutions	

# Required Materials

## Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

## Graph paper

**Pre-printed cards, cut from copies of the Instructional master**

**Pre-printed slips, cut from copies of the Instructional master**

## Rulers

## Straightedges

A rigid edge that can be used for drawing line segments. Sometimes a ruler is okay to use as a straightedge, but sometimes it is preferable to use an unruled straightedge, like a blank index card.

## String

**Tools for creating a visual display**

Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

# Section: Proportional Relationships

## Lesson 1: Understanding Proportional Relationships

### Goals

- Comprehend that for the equation of a proportional relationship given by  $y = kx$ ,  $k$  represents the constant of proportionality.
- Create graphs and equations of proportional relationships in context, including an appropriate scale.
- Interpret diagrams or graphs of proportional relationships in context.

### Learning Targets

- I can graph a proportional relationship from a story.
- I can use the constant of proportionality to compare the pace of different animals.

### Lesson Narrative

This lesson is the first of four where students work with proportional relationships from a grade 8 perspective. Embedded alongside their work with proportional relationships, students learn about graphing from a blank set of axes. Attending to precision in labeling axes, choosing an appropriate scale, and drawing lines are skills students work with in this lesson and refine over the course of this unit and in units that follow (MP6).

The purpose of this lesson is to get students thinking about what makes a “good” graph by first considering what are the components of a graph (e.g., labels, scale) and then adding scale to graphs of the pace of two bugs. Students also graph a line based on a verbal description of a relationship and compare the newly graphed line to already graphed proportional relationships.

This lesson includes graphs with elapsed time in seconds on the vertical axis and distance traveled in centimeters on the horizontal axis. It is common for people to believe that time is always the independent variable, and should therefore always be on the horizontal axis. This is a really powerful heuristic. The problem is, it isn’t true.

In general, a context that involves a relationship between two quantities does not dictate which quantity is the independent variable and which is the dependent variable: that is a choice made by the modeler. Consider this situation: A runner is traveling one mile every 10 minutes. There is more than one way to represent this situation.

- We can say the number of miles traveled,  $d$ , depends on the number of minutes that have passed,  $t$ , and write  $d = 0.1t$ . This way of expressing the relationship might be more useful for questions like, "How far does the runner travel in 35 minutes?"
- We can also say that the number of minutes that have passed,  $t$ , depends on the number of miles traveled,  $d$ , and write  $t = 10d$ . This way of expressing the relationship might be more useful for questions like, "How long does it take the runner to travel 2 miles?"

These are both linear relationships. The rate of change in the first corresponds to speed (0.1 miles per minute), and the rate of change in the second corresponds to pace (10 minutes per mile). Both have meaning, and both could be of interest. It is up to the modeler to decide what kinds of questions she wants to answer about the context and which way of expressing the relationship will be most useful in answering those questions.

## Alignments

### Building On

- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

### Addressing

- 8.EE.B: Understand the connections between proportional relationships, lines, and linear equations.

### Building Towards

- 8.EE.B.5: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

### Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR3: Clarify, Critique, Correct
- MLR5: Co-Craft Questions
- Notice and Wonder

### Student Learning Goals

Let's study some graphs.

## 1.1 Notice and Wonder: Two Graphs

### Warm Up: 5 minutes

The purpose of this warm-up is to get a conversation started about what features a graph needs. In the following activities, students will put these ideas to use by adding scale to some axes with two proportional relationships graphed on it.

## Building Towards

- 8.EE.B.5

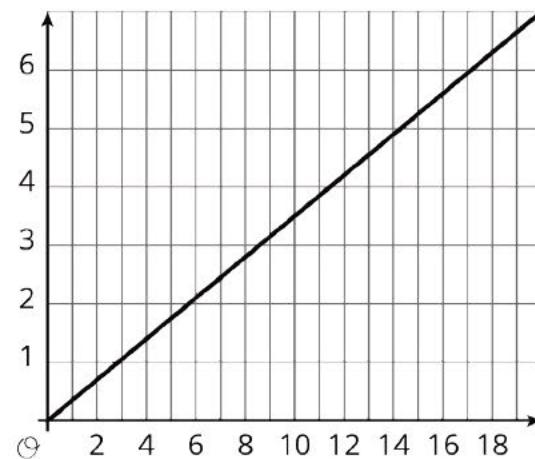
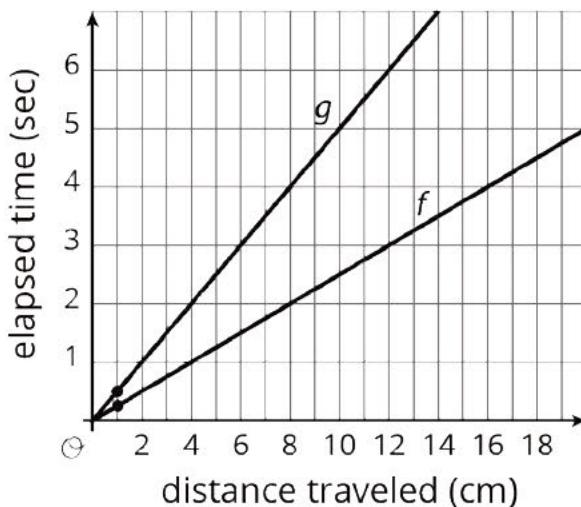
## Instructional Routines

- Notice and Wonder

### Launch

Tell students they will look at a picture, and their job is to think of at least one thing they notice and at least one thing they wonder about the picture. Display the problem for all to see and ask students to give a signal when they have noticed or wondered about something.

#### Student Task Statement



What do you notice? What do you wonder?

#### Student Response

Things students may notice:

- The second set of axes are not labeled
- If the first graph is about speed, then  $f$  is twice as fast as  $g$ .
- Graph  $g$  is something going a speed of 2 centimeters every second
- Graph  $f$  is something going a pace of about 0.25 seconds per 1 centimeter.

Things students may wonder:

- What do the two points mean?
- Why does one graph show two lines while the other only has one?
- What do  $g$  and  $f$  represent?

## Activity Synthesis

Invite students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After each response, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to the images each time. If the missing labels are not mentioned, make sure to bring them up.

# 1.2 Moving Through Representations

## 15 minutes

In this activity, students investigate the paces of two different bugs. Using the chart at the start of the activity, students answer questions about pace, decide on a scale for the axes, and mark and label the time needed to travel 1 cm for each bug (unit rate).

Identify students who use different scales on the axes to share during the Activity Synthesis. For example, some students may count by 1s on the distance axis while others may count by 0.5s.

## Addressing

- 8.EE.B

## Building Towards

- 8.EE.B.5

## Instructional Routines

- MLR5: Co-Craft Questions

## Launch

Arrange students in groups of 2. Before students start working, ensure that they understand that each bug's position is measured at the front of their head. So for example, in the second diagram, the ladybug has moved 4 centimeters and the ant has moved 6 centimeters.

Ask students to review the images and the first problem in the activity and give a signal when they have finished. Invite students to share their ideas about which bug is represented by line  $u$  and which bug is represented by line  $v$ . (The ladybug is  $u$ , the ant is  $v$ .) If not brought up in students' explanations, draw attention to how the graph shows the *pace* of the two bugs—that is, the graph shows how much time it takes to go a certain distance, which is different than a graph of speed, which shows how much distance you go for a certain amount of time.

Give students work time to complete the remaining problems with their partner followed by a whole-class discussion.

---

### Support for Students with Disabilities

*Action and Expression: Develop Expression and Communication.* Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their ideas. For example, "Based on the diagrams, line  $u$  represents \_\_\_\_\_ because...."

*Supports accessibility for: Language; Organization*

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### Support for English Language Learners

*Writing, Speaking: MLR5 Co-Craft Questions.* Use this routine to help students interpret the first image, and to increase awareness of language used to make comparisons about speed and pace. Display only the prompt and images (without the line graphs). Invite students to write possible mathematical questions about the situation. When students share their questions with the class, highlight those that wonder about distance, time and the meaning of tick marks in the diagrams. Reveal the graph and ask students to work on the questions that follow. This helps students produce the language of mathematical questions about different representations for speed.

*Design Principle(s): Optimize output; Cultivate conversation*

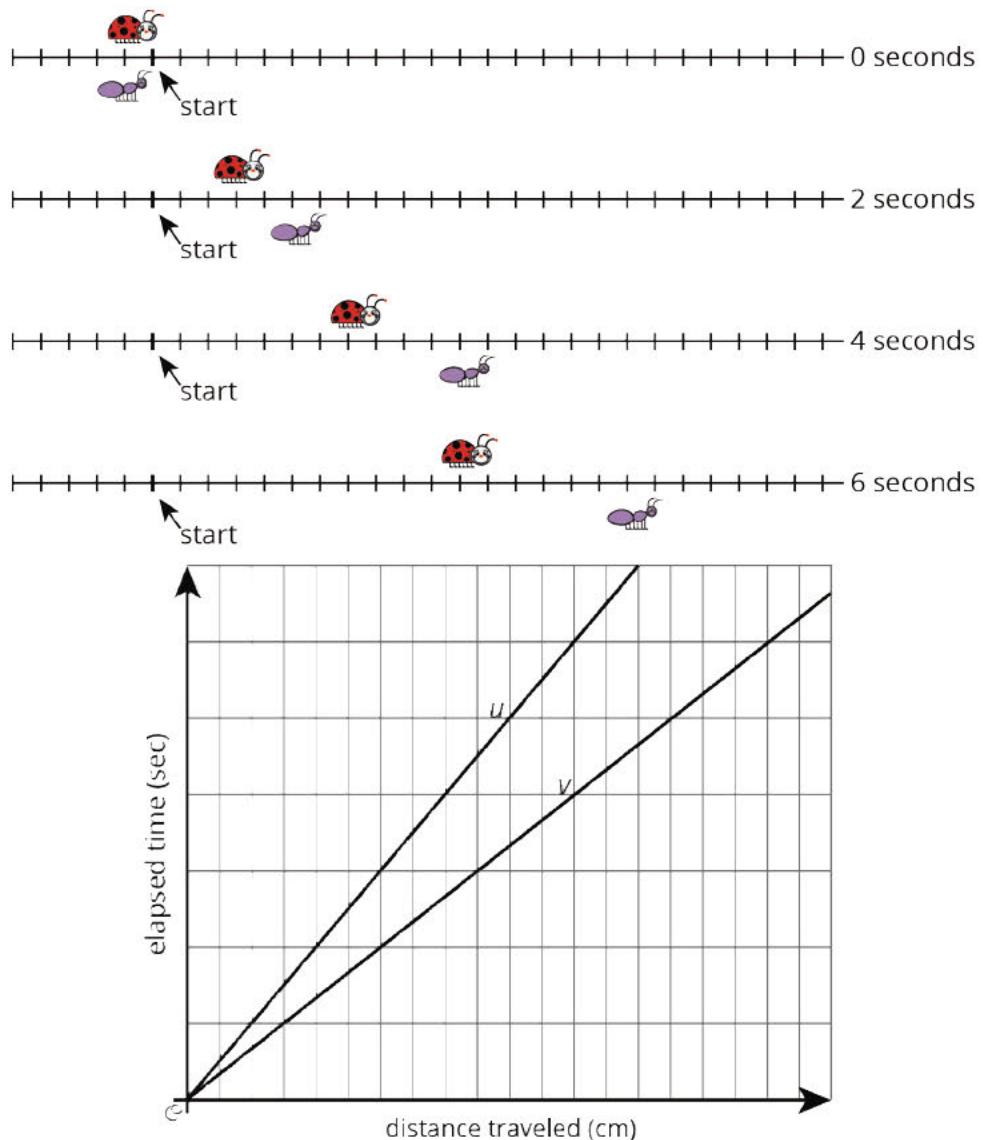
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### Anticipated Misconceptions

Students might confuse pace with speed and interpret a steeper line as meaning the ladybug is moving faster. Monitor students to ensure that they attend to the time and distance on the tick mark diagrams and plot points as (*distance, time*) with time on the *y*-axis and distance on the *x*-axis. Reinforce language of how many seconds per a given interval of distance. Make explicit that twice as fast means half the pace.

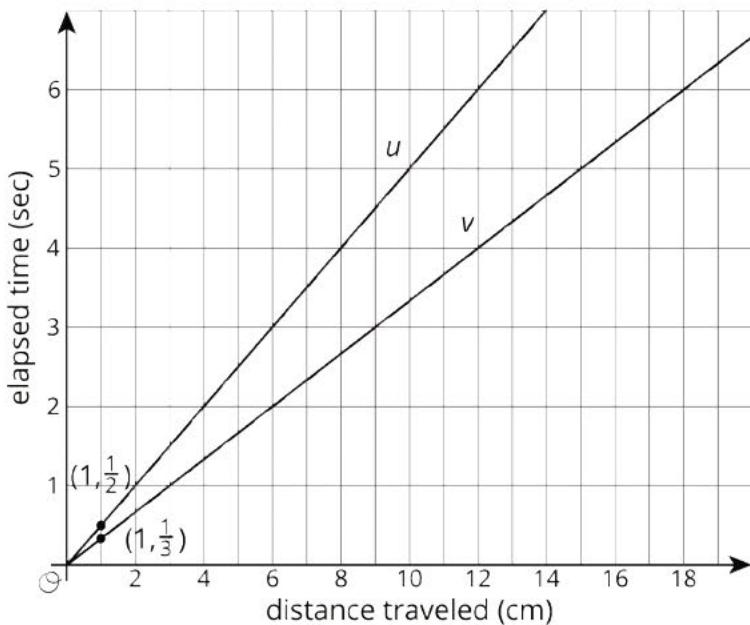
#### Student Task Statement

A ladybug and ant move at constant speeds. The diagrams with tick marks show their positions at different times. Each tick mark represents 1 centimeter.



1. Lines  $u$  and  $v$  also show the positions of the two bugs. Which line shows the ladybug's movement? Which line shows the ant's movement? Explain your reasoning.
2. How long does it take the ladybug to travel 12 cm? The ant?
3. Scale the vertical and horizontal axes by labeling each grid line with a number. You will need to use the time and distance information shown in the tick-mark diagrams.
4. Mark and label the point on line  $u$  and the point on line  $v$  that represent the time and position of each bug after traveling 1 cm.

### Student Response



1. Ladybug: line  $u$ , ant: line  $v$
2. Ladybug: 6 seconds, ant: 4 seconds
3. See graph.
4. See graph.

### Are You Ready for More?

1. How fast is each bug traveling?
2. Will there ever be a time when the ant is twice as far away from the start as the ladybug? Explain or show your reasoning.

### Student Response

1. The red bug (ladybug) is traveling at 2 cm/sec and the purple bug (ant) is traveling at 3 cm/sec.
2. No, the purple bug (ant) is always half as much again as far from the start as the red bug (ladybug).

### Activity Synthesis

Display the images from the problem for all to see. Begin the discussion by inviting students to share their solutions for how long it takes each bug to travel 12 cm. Encourage students to reference one or both of the images as they explain their thinking.

Ask previously selected students to share their graphs with added scale and how they decided on what scale to use. If possible, display these graphs for all to see. There are many correct ways to choose a scale for this situation, though some may have made it difficult for students to plot the answer to the final problem. If this happened, highlight these graphs and encourage students to

read all problems when they are making decisions about how to construct a graph. Since this activity had a problem asking for information about 1 cm, it makes sense to count by 1s (or even something smaller!) on the distance axis.

## 1.3 Moving Twice as Fast

### 15 minutes

In this activity, students use the representations from the previous activity and add a third bug that is moving twice as fast as the ladybug. Students are also asked to write equations for all three bugs. An important aspect of this activity is students making connections between the different representations.

Monitor for students using different strategies to write their equations. For example, some students may reason from the unit rates they can see on their graphs and write equations in the form of  $y = kx$ , where  $k$  is the unit rate (constant of proportionality). Others may write equations of the form  $\frac{y}{x} = \frac{b}{a}$ , where  $(a, b)$  is a point on the line. Select several of these students to share during the discussion.

### Addressing

- 8.EE.B

### Building Towards

- 8.EE.B.5

### Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR3: Clarify, Critique, Correct

### Launch

Keep students in the same groups. Give 5–7 minutes work time followed by a whole-class discussion.

---

### Support for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, show only one question at a time, pausing to check for understanding before moving on.

*Supports accessibility for: Organization; Attention*

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### Support for English Language Learners

*Speaking: MLR3 Clarify, Critique, Correct.* For the first question “Imagine a bug that is moving twice as fast as the ladybug. On each tick-mark diagram, mark the position of this bug,” display an incomplete statement like, “I looked at how far the ladybug went and made my bug go farther” or a flawed statement like “I put my bug 2 tick marks ahead of the ant.” Invite students to discuss with a partner possible ways to correct or clarify each statement. This will give students an opportunity to use language to clarify their understanding of proportionality.

*Design Principle(s): Maximize meta-awareness*

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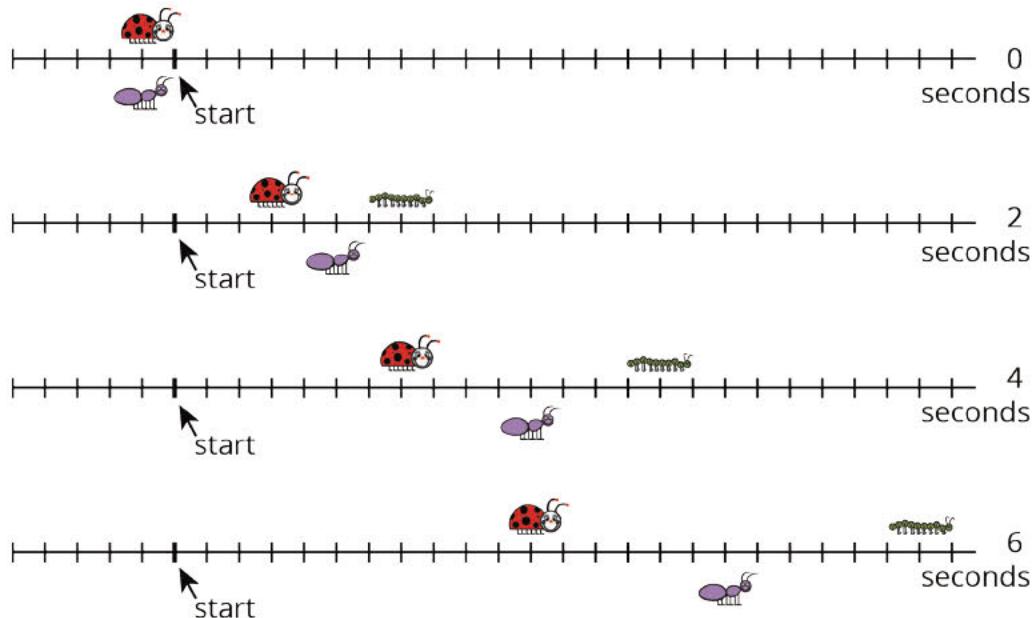
### Student Task Statement

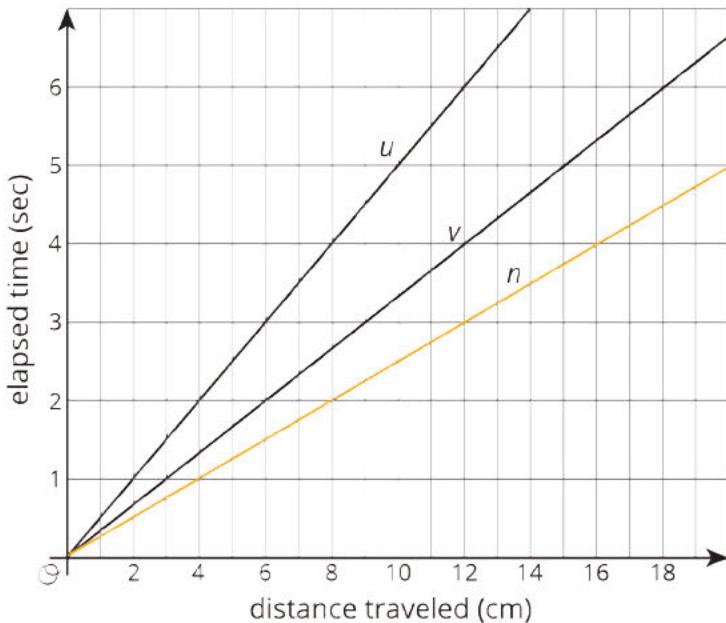
Refer to the tick-mark diagrams and graph in the earlier activity when needed.

1. Imagine a bug that is moving twice as fast as the ladybug. On each tick-mark diagram, mark the position of this bug.
2. Plot this bug’s positions on the coordinate axes with lines  $u$  and  $v$ , and connect them with a line.
3. Write an equation for each of the three lines.

### Student Response

1.





2. See graph. Line *n* represents a bug moving double the ladybug's distance in the same amount of time.

3. Answers vary. Possible response: Equations are ladybug:  $y = \frac{1}{2}x$ , ant:  $y = \frac{1}{3}x$ , new bug:  $y = \frac{1}{4}x$  (twice as fast as ladybug), where  $x$  represents the distance traveled and  $y$  represents elapsed time.

### Activity Synthesis

Display both images from the previous task for all to see. Invite previously selected students to share their equations for each bug. Sequence students so that the most common strategies are first. Record the different equations created for each bug and display these for all to see.

As students share their reasoning about the equation for the third bug, highlight strategies that support using the equation (original is  $k$  and new one is  $\frac{1}{2}k$ ) and graph (less steep, still constant proportionality, half point values). If no students write an equation of the form  $y = kx$ , do so and remind students of the usefulness of  $k$ , the constant of proportionality, when reasoning about proportional relationships.

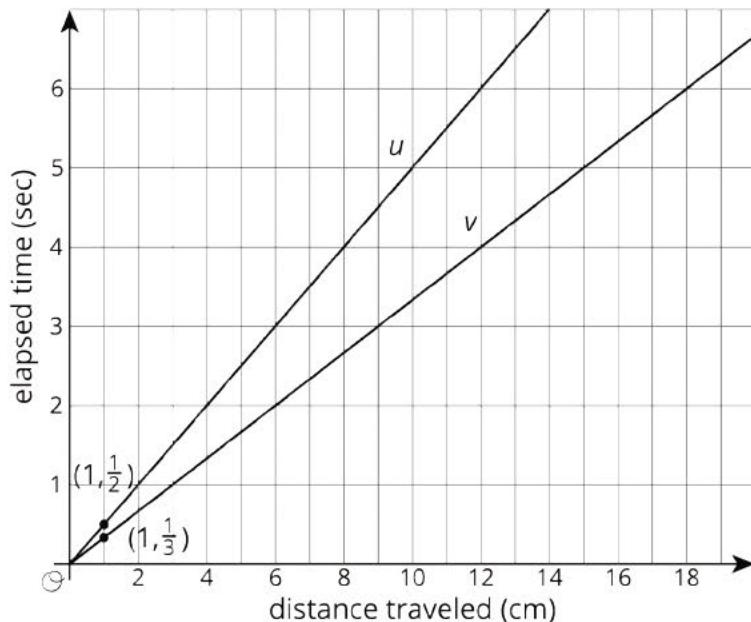
Consider asking the following questions to help students make connections between the different representations:

- “What features of the tick-mark diagrams, lines, and equations can you identify that would allow someone to figure out which bug is moving faster?” (The tick-mark diagrams give the coordinates of points that will go on the graph because they show how far each bug has gone after each amount of time. We can see the positions of the bugs on the tick-mark diagrams so we know which is faster. The graph shows how far they went for any amount of time and the slope helps to show which is faster. Both help compare the movements of the two or three bugs.)

- “The tick-mark diagrams show some of the bugs’ movements, but not all of them. How can we use the graphs of the lines to get more complete information?” (The tick-mark diagrams only show time every 2 seconds. On the graph we can see the bugs’ positions at any point in time.)
- “Are you convinced that your graph (or equation) supports the fact that the new bug is going twice as fast as the ladybug?”

## Lesson Synthesis

Display a scaled graph of the two bugs for all to see. Remind students that line  $u$  is the ladybug and that line  $v$  is the ant.



Ask students:

- “What would the graph of a bug going 3 times faster than the ant look like?” (It would go through the points  $(0,0)$ ,  $(1, \frac{1}{9})$ , and  $(9, 1)$ .)
- “What would an equation showing the relationship between the bugs’ distance and time look like?” (Since it is going 4 times faster and goes through the point  $(9, 1)$ , it has a **constant of proportionality** of  $\frac{1}{9}$ , which means one equation is  $y = \frac{1}{9}x$ .)
- “If we wanted to scale the graph so we could see how long it takes the ladybug to travel 50 cm, what numbers could we use on the vertical axis?” (The ladybug travels 50 cm in 25 seconds, so the vertical axis would need to extend to at least that value.)

## 1.4 Turtle Race

Cool Down: 5 minutes

## Building On

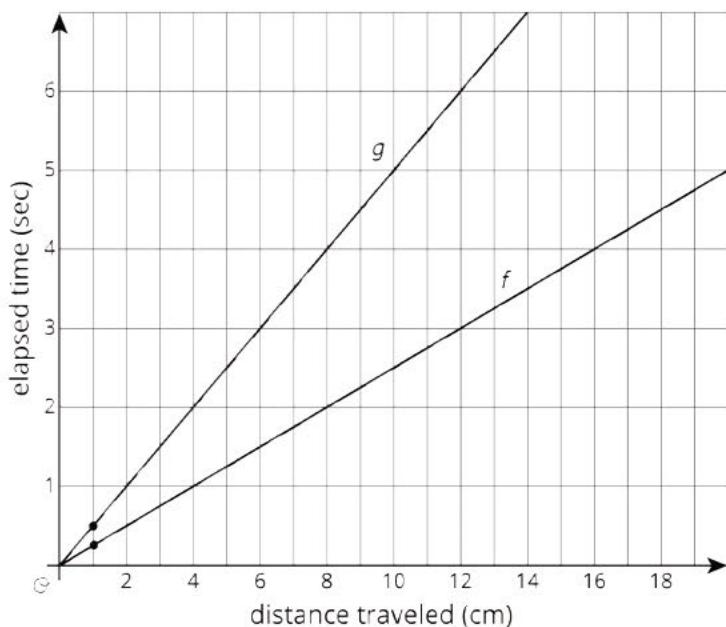
- 7.RP.A.2

## Building Towards

- 8.EE.B.5

### Student Task Statement

This graph represents the positions of two turtles in a race.



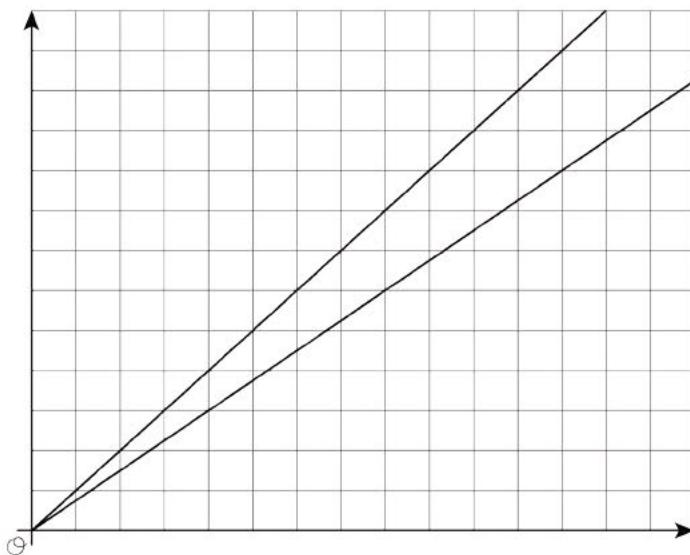
1. On the same axes, draw a line for a third turtle that is going half as fast as the turtle described by line  $g$ .
2. Explain how your line shows that the turtle is going half as fast.

### Student Response

1. A line through  $(0, 0), (1, 1), (2, 2)$ , etc.
2. Looking at the values for 2 seconds, turtle  $g$  moves 4 cm and the third turtle moves only 2 cm. This third turtle covers half the distance in the same amount of time.

### Student Lesson Summary

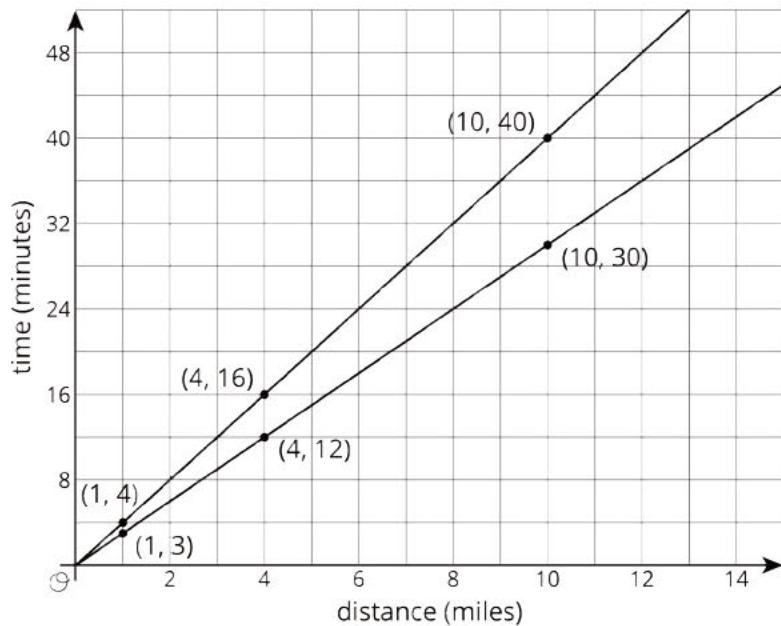
Graphing is a way to help us make sense of relationships. But the graph of a line on a coordinate axes without scale or labels isn't very helpful. For example, let's say we know that on longer bike rides Kiran can ride 4 miles every 16 minutes and Mai can ride 4 miles every 12 minutes. Here are the graphs of these relationships:



Without labels we can't even tell which line is Kiran and which is Mai! Without labels and a scale on the axes, we can't use these graphs to answer questions like:

1. Which graph goes with which rider?
2. Who rides faster?
3. If Kiran and Mai start a bike trip at the same time, how far are they after 24 minutes?
4. How long will it take each of them to reach the end of the 12 mile bike path?

Here are the same graphs, but now with labels and scale:



Revisiting the questions from earlier:

1. Which graph goes with each rider? If Kiran rides 4 miles in 16 minutes, then the point  $(4, 16)$  is on his graph. If he rides for 1 mile, it will take 4 minutes. 10 miles will take 40 minutes. So the upper graph represents Kiran's ride. Mai's points for the same distances are  $(1, 3)$ ,  $(4, 12)$ , and  $(10, 30)$ , so hers is the lower graph. (A letter next to each line would help us remember which is which!)
2. Who rides faster? Mai rides faster because she can ride the same distance as Kiran in a shorter time.
3. If Kiran and Mai start a bike trip at the same time, how far are they after 20 minutes? The points on the graphs at height 20 are 5 miles for Kiran and a little less than 7 miles for Mai.
4. How long will it take each of them to reach the end of the 12 mile bike path? The points on the graphs at a horizontal distance of 12 are 36 minutes for Mai and 48 minutes for Kiran. (Kiran's time after 12 miles is almost off the grid!)

## Glossary

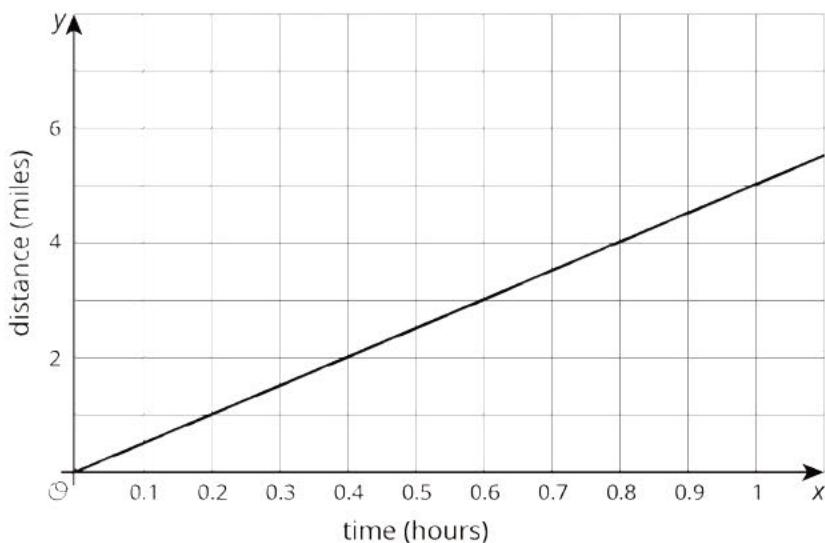
- constant of proportionality

# Lesson 1 Practice Problems

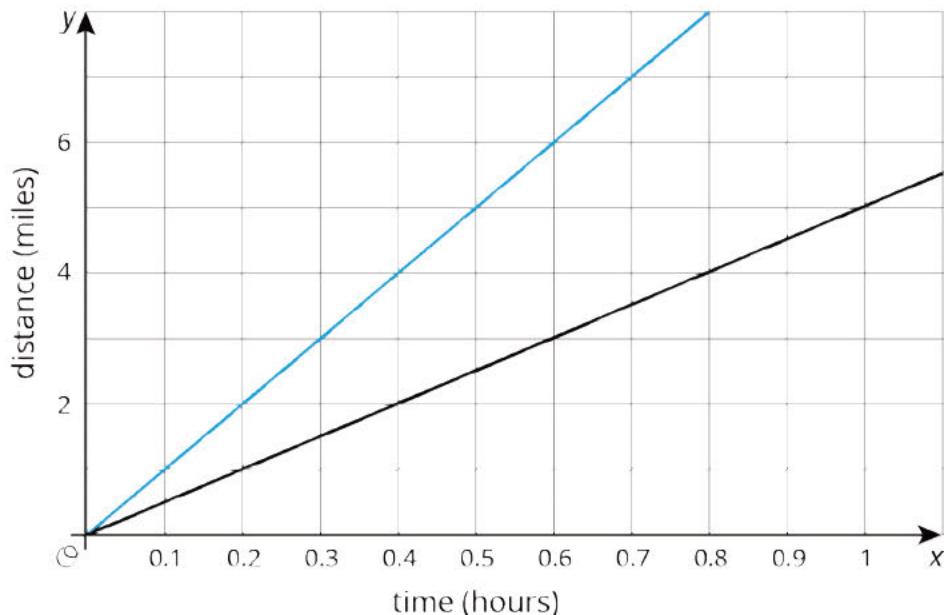
## Problem 1

### Statement

Priya jogs at a constant speed. The relationship between her distance and time is shown on the graph. Diego bikes at a constant speed twice as fast as Priya. Sketch a graph showing the relationship between Diego's distance and time.



## Solution

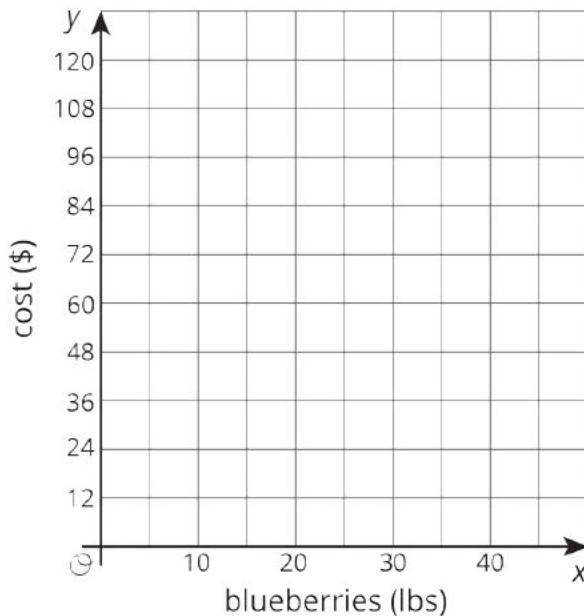


## Problem 2

### Statement

A you-pick blueberry farm offers 6 lbs of blueberries for \$16.50.

Sketch a graph of the relationship between cost and pounds of blueberries.



## Solution

A ray that passes through  $(0, 0)$  and  $(6, 16.5)$ .

## Problem 3

### Statement

A line contains the points  $(-4, 1)$  and  $(4, 6)$ . Decide whether or not each of these points is also on the line:

- a.  $(0, 3.5)$
- b.  $(12, 11)$
- c.  $(80, 50)$
- d.  $(-1, 2.875)$

### Solution

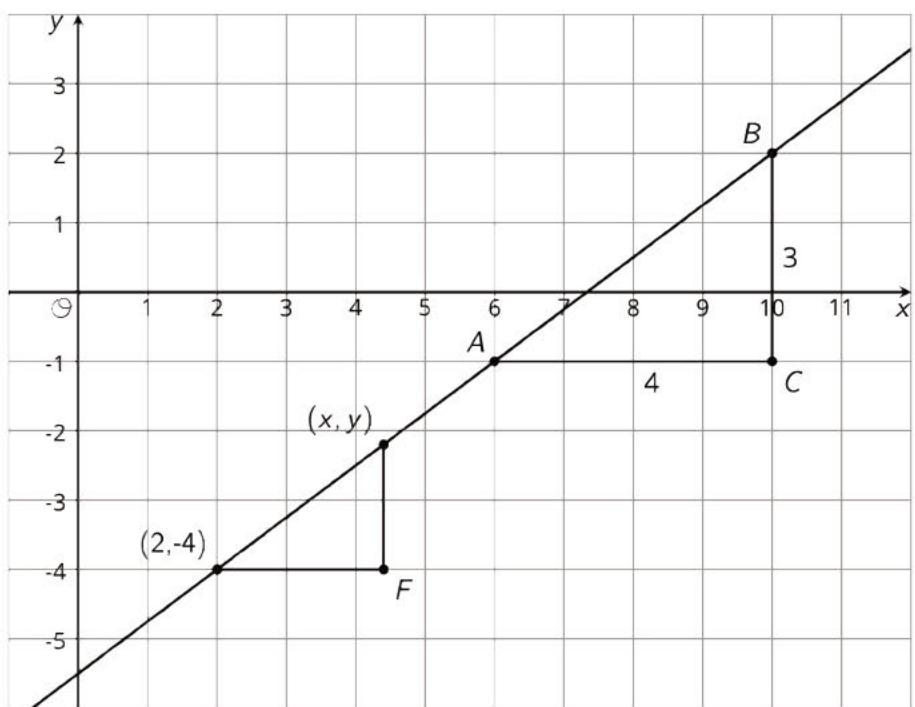
- a. On the line
- b. On the line
- c. Not on the line
- d. On the line

(From Unit 2, Lesson 12.)

## Problem 4

### Statement

The points  $(2, -4)$ ,  $(x, y)$ ,  $A$ , and  $B$  all lie on the line. Find an equation relating  $x$  and  $y$ .



## Solution

$$\frac{y+4}{x-2} = \frac{3}{4} \text{ (or equivalent)}$$

(From Unit 2, Lesson 11.)

# Lesson 2: Graphs of Proportional Relationships

## Goals

- Compare graphs that represent the same proportional relationship using differently scaled axes.
- Create graphs representing the same proportional relationship using differently scaled axes, and identify which graph to use to answer specific questions.

## Learning Targets

- I can graph a proportional relationship from an equation.
- I can tell when two graphs are of the same proportional relationship even if the scales are different.

## Lesson Narrative

The purpose of this lesson is for students to understand that there are many successful ways to set up and scale axes in order to graph a proportional relationship. Sometimes, however, we choose specific ranges for the axes in order to see specific information.

In the first activity, students sort graphs on cards based on what proportional relationship they represent. Each graph has a different scale, and some scales are purposefully quite different so students cannot use “looks like” as a way to tell the difference between the relationships. This activity presses the need for paying attention to scale and relying on mathematical definitions of steepness and not just visual ones.

In the second activity, students graph a proportional relationship representing water filling a tank on two differently scaled axes. Then they compare their graph to a graph of a non-proportional relationship and answer questions about the situation. By looking at the same two relationships graphed at different scales, students see how much effect the scale of the axes has on the information we can figure out.

## Alignments

### Building On

- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

### Addressing

- 8.EE.B: Understand the connections between proportional relationships, lines, and linear equations.
- 8.EE.B.5: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

## Building Towards

- 8.EE.B.5: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

## Instructional Routines

- MLR6: Three Reads
- MLR8: Discussion Supports
- Take Turns

## Required Materials

**Pre-printed slips, cut from copies of the blackline master**

**Straightedges**

A rigid edge that can be used for drawing line segments. Sometimes a ruler is okay to use as a

straightedge, but sometimes it is preferable to use an unruled straightedge, like a blank index card.

## Required Preparation

For Card Sort: Proportional Relationships, prepare 1 copy of the blackline master for every 2 students and cut them up ahead of time. Provide access to straightedges.

### Student Learning Goals

Let's think about scale.

## 2.1 An Unknown Situation

### Warm Up: 5 minutes

In the previous warm-up, students compared different proportional relationships on two sets of axes that were scaled the same. In this warm-up, students will work with two sets of axes scaled differently and the same proportional relationship. The purpose of this warm-up is to make explicit that the same proportional relationship can appear to have different steepness depending on the axes, which is why paying attention to scale is important when making sense of graphs or making graphs from scratch. Students learned how to graph and write equations for proportional relationships in previous grades, so this warm-up also helps refresh those skills.

Identify students using different strategies to graph the relationship on the new axes. For example, since this is a proportional relationship, some students may scale up the point  $(8, 14)$  to something like  $(40, 70)$  and then plot that point before drawing a line through it and the point  $(0, 0)$ . Other students may use the equation they wrote and one of the  $x$ -values marked on the new axes to find a point on the line and draw in the line from there.

## Building On

- 7.RP.A.2

## Building Towards

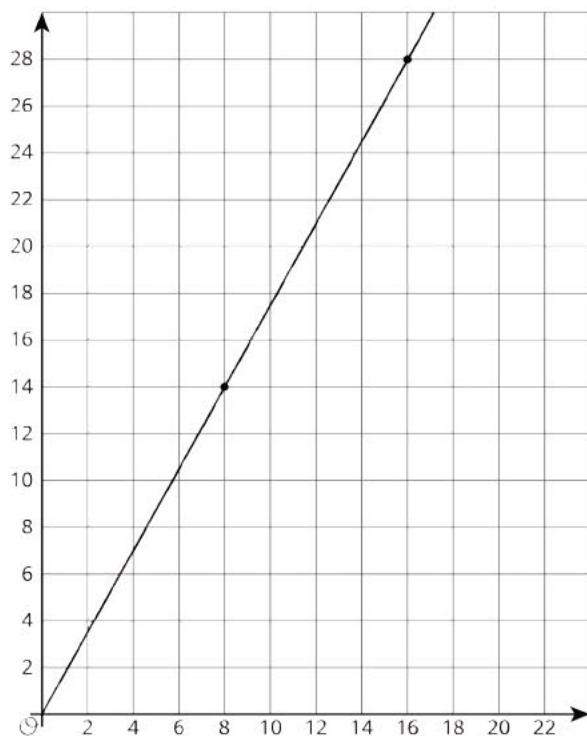
- 8.EE.B.5

## Launch

Give 2–3 minutes of quite work time followed by a whole-class discussion.

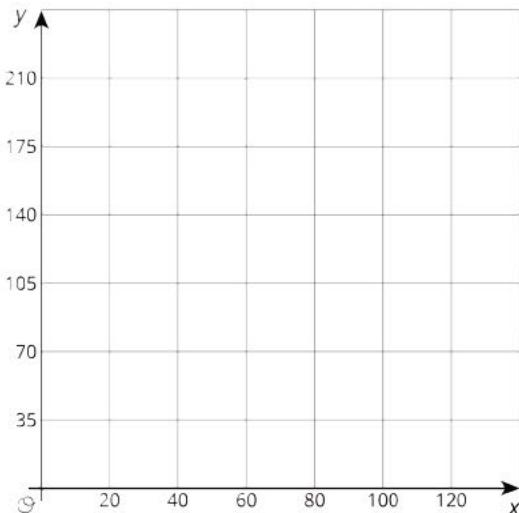
### Student Task Statement

Here is a graph that could represent a variety of different situations.



1. Write an equation for the graph.

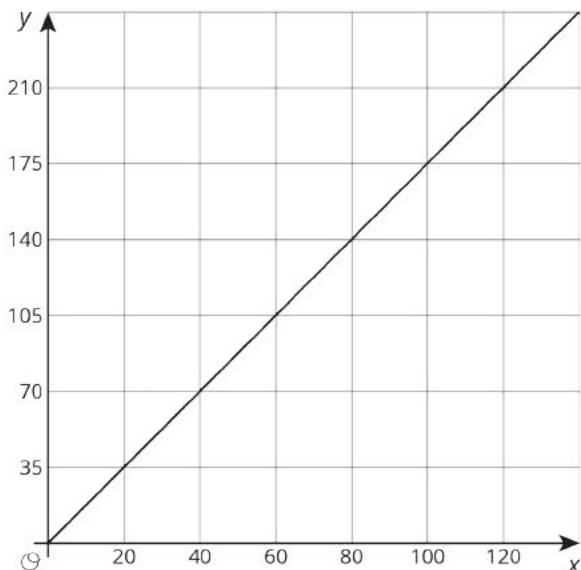
2. Sketch a new graph of this relationship.



### Student Response

1. Equation:  $y = 1\frac{3}{4}x$  or  $y = 1.75x$  or  $y = \frac{7}{4}x$  or equivalent.

2.



### Activity Synthesis

Display the two images from the activity for all to see. Invite previously identified students to share how they graphed the relationship on the new line.

Ask students, “Which graph looks steeper to you?” Students may see the first line as steeper, even though the two lines have the same slope. It is important students understand that this is one of the reasons mathematics uses numbers (slope) to talk about the steepness of lines and not “looks like.”

## 2.2 Card Sort: Proportional Relationships

**15 minutes**

The purpose of this activity is for students to identify the same proportional relationship graphed using different scales. Students will first sort the cards based on what proportional relationship they represent and then write an equation representing each relationship. Identify and select groups using different strategies to match graphs to share during the Activity Synthesis. For example, some groups may identify the unit rate for each graph in order to match while others may choose to write equations first and use those to match their graphs.

### Addressing

- 8.EE.B

### Building Towards

- 8.EE.B.5

### Instructional Routines

- MLR8: Discussion Supports

- Take Turns

## Launch

Arrange students in groups of 4. Provide each group with a set of 12 pre-cut slips.

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### Support for English Language Learners

*Speaking: MLR8 Discussion Supports.* To support students to produce statements about proportional relationships, provide sentence frames for students to use when they describe the reasoning for their matches. For example, “ \_\_\_ and \_\_\_ match/don’t match, because \_\_\_.” Encourage use of relevant vocabulary such as “constant of proportionality” and “unit rate.”

*Design Principle(s): Support sense-making; Optimize output (for explanation)*

---

## Anticipated Misconceptions

If students have trouble recalling the meaning of the *constant of proportionality*, remind them that one way to think about it is “the change in  $y$  for every unit change in  $x$ .”

### Student Task Statement

Your teacher will give you 12 graphs of proportional relationships.

1. Sort the graphs into groups based on what proportional relationship they represent.
2. Write an equation for each *different* proportional relationship you find.

### Student Response

Card sort and possible matching equation:

A:  $y = 0.25x$

B, E, H:  $y = 3x$

C, D, G, K:  $y = 3.5x$

I, L:  $y = \frac{4}{3}x$

F, J:  $y = \frac{5}{2}x$

### Activity Synthesis

As a result of this conversation, students should understand that the scale of the axes a graph is drawn on can hide the actual relationship between the two variables if you just look at the steepness of the line without paying attention to the numbers on the axes.

Ask previously selected groups to share their strategies for matching the graphs. Highlight uses of relevant vocabulary such as “constant of proportionality” or “unit rate.”

Have students look at Card A. Ask students, "Do you think this graph looks like  $y = \frac{1}{4}x$ ? Why or why not?" Possible reasons from students are:

- No, I think this graph looks like  $y = x$ .
- I would expect  $y = \frac{1}{4}x$  to be less steep since an increase in  $x$  means  $\frac{1}{4}$  as much of an increase in  $y$ .
- Since the  $x$ -scale is four times the  $y$ -scale, the line looks steeper than I expected.

Give students a moment to identify another card that they think does not "look like" the equation and select a few students to share the graph they chose and explain their thinking.

---

#### Support for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Break the class into small discussion groups and then invite a representative from each group to report back to the whole class.

*Supports accessibility for: Language; Social-emotional skills; Attention*

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## 2.3 Different Scales

**15 minutes**

Building off the work in the previous activity, students now graph a proportional relationship on two differently scaled axes and compare the proportional relationship to an already-graphed non-proportional relationship on the same axes. Students are asked to make sense of the intersections of the two graphs by reasoning about the situation and consider which scale is most helpful: the zoomed in, or the zoomed out. In this case, which graph is most helpful depends on the questions asked about the situation.

#### Addressing

- 8.EE.B.5

#### Instructional Routines

- MLR6: Three Reads

#### Launch

Arrange students in groups of 2. Provide access to straightedges.

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## Support for English Language Learners

*Reading, Writing: Math Language Routine 6 Three Reads.* This is the first time Math Language Routine 6 is suggested as a support in this course. In this routine, students are supported in reading a mathematical text, situation, or word problem three times, each with a particular focus. During the first read, students focus on comprehending the situation; during the second read, students identify quantities; during the third read, the final prompt is revealed and students brainstorm possible strategies to answer the question. The intended question is withheld until the third read so students can make sense of the whole context before rushing to a solution. The purpose of this routine is to support students' reading comprehension as they make sense of mathematical situations and information through conversation with a partner.

*Design Principle(s): Support sense-making*

### How It Happens:

1. Use this routine to support reading comprehension of this word problem without solving it for students. In the first read, students read the problem with the goal of comprehending the situation.

Invite a student to read the problem aloud while everyone else reads with them, and then ask, "What is this situation about?" Be sure to display the two graphs or ask students to reference them while reading. Allow one minute to discuss with a partner and then share with the whole class. A typical response may be: "Two large water tanks are filling with water. One of them is filling at a constant rate, while the other is not. Both graphs represent Tank A. Tank B only has an equation."

2. In the second read, students analyze the mathematical structure of the story by naming quantities.

Invite students to read the problem aloud with their partner, or select a different student to read to the class, and then prompt students by asking: "What can be counted or measured in this situation?" Give students one minute of quiet think time, followed by another minute to share with their partner. A typical written response may be: "liters of water in Tank A; liters of water in Tank B; amount of time that has passed in minutes; constant rate of  $\frac{1}{2}$  liters per minute."

3. In the third read, students brainstorm possible strategies to answer the questions.

Invite students to read the problem aloud with their partner, or select a different student to read to the class, and follow with the questions. Instruct students to think of ways to approach the questions without actually solving the problems.

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Consider using these questions to prompt students: "How would you approach this question?", "What strategy or method would you try first?", and "Can you think of a different way to solve it?".

Give students one minute of quiet think time followed by another minute to discuss with their partner. Provide these sentence frames as partners discuss: "To draw a graph for Tank B, I would....", "One way to approach the question about finding the time when the tanks have the same amount of water would be to....", and "I would use the first/second graph to find....".

4. As partners are discussing their solution strategies, select 1–2 students for each question to share their ideas with the whole class. As students are presenting their strategies to the whole class, create a display that summarizes the ideas for each question.

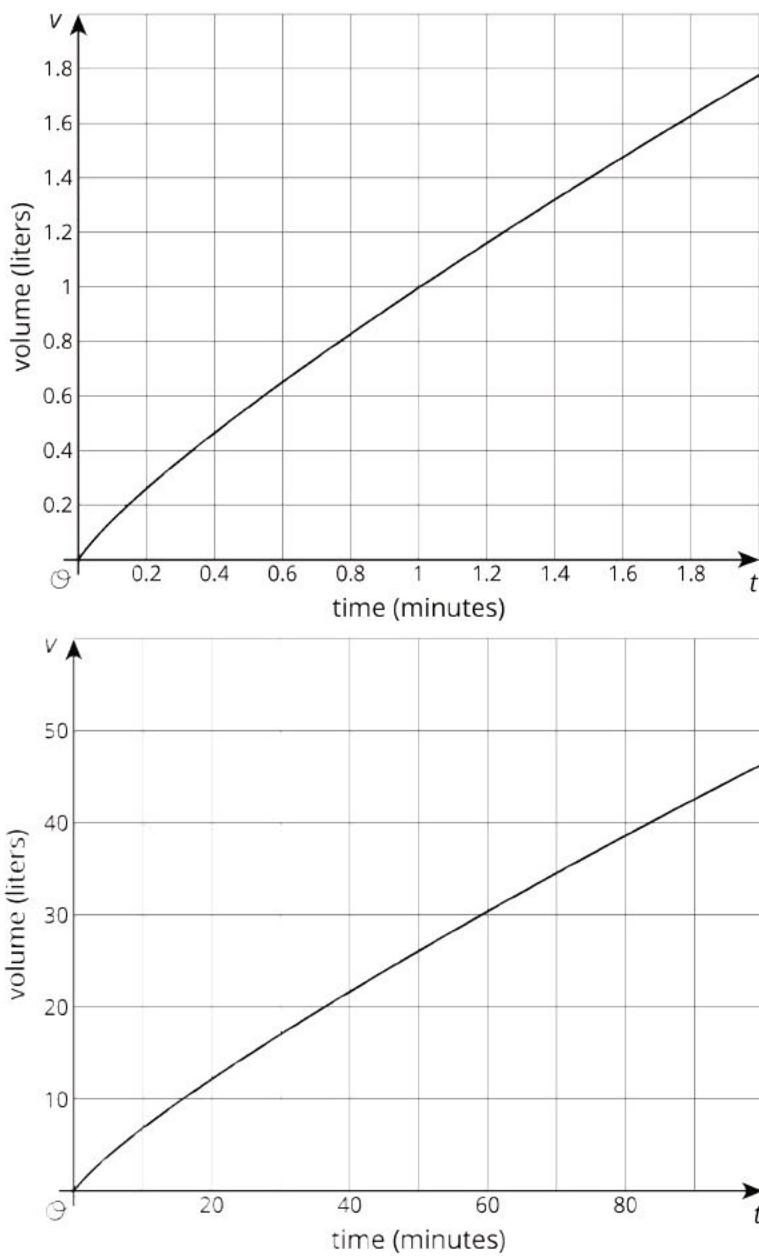
Listen for quantities that were mentioned during the second read, and take note of approaches in which the students distinguish between the graphs with differently-scaled axes.

5. Post the summary where all students can use it as a reference.

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### Student Task Statement

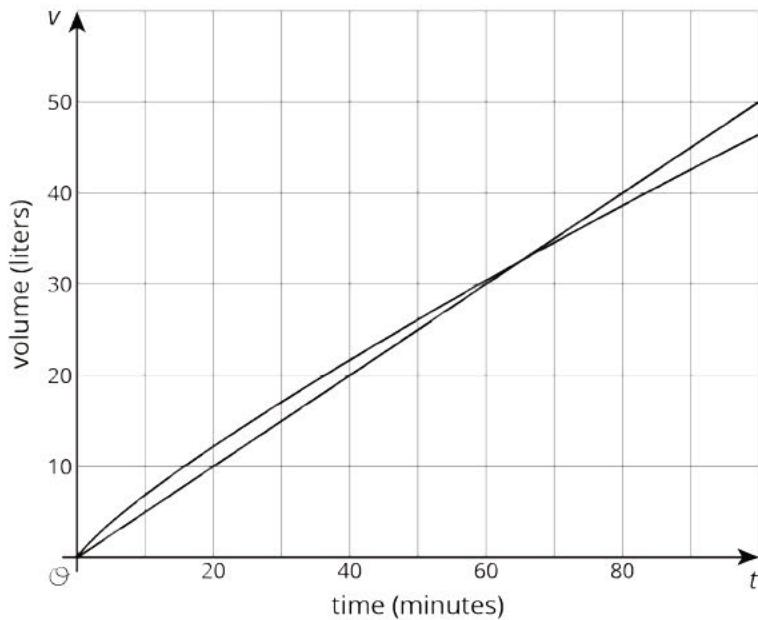
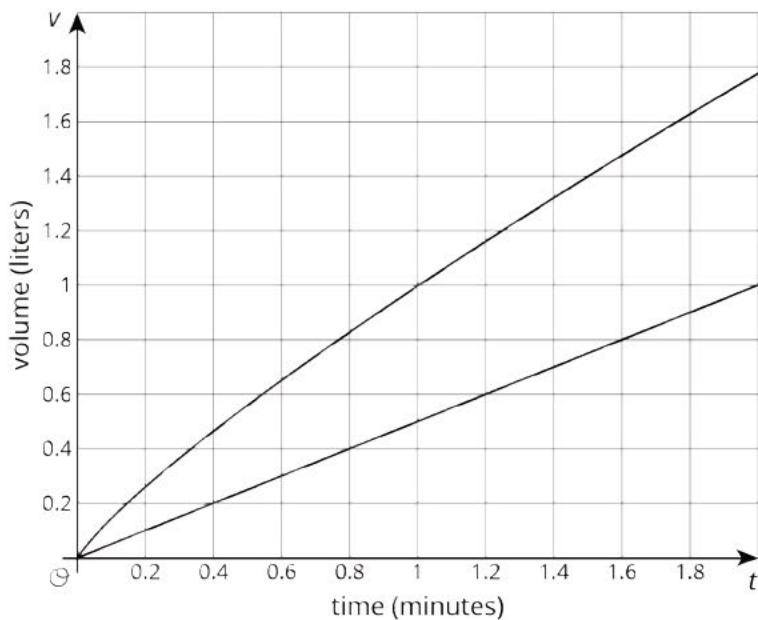
Two large water tanks are filling with water. Tank A is not filled at a constant rate, and the relationship between its volume of water and time is graphed on each set of axes. Tank B is filled at a constant rate of  $\frac{1}{2}$  liters per minute. The relationship between its volume of water and time can be described by the equation  $v = \frac{1}{2}t$ , where  $t$  is the time in minutes and  $v$  is the total volume in liters of water in the tank.



1. Sketch and label a graph of the relationship between the volume of water  $v$  and time  $t$  for Tank B on each of the axes.
2. Answer the following questions and say which graph you used to find your answer.
  - a. After 30 seconds, which tank has the most water?
  - b. At approximately what times do both tanks have the same amount of water?
  - c. At approximately what times do both tanks contain 1 liter of water? 20 liters?

### Student Response

1.



2.
  - a. Using the first graph, Tank A has more water after 30 seconds.
  - b. Using the second graph, at approximately 64 minutes both tanks have the same amount of water.
  - c. Using the first graph, Tank A has a liter of water after 1 minute. Tank B has a liter of water after 2 minutes. Using the second graph, Tank A has 20 liters of water at around 36 minutes while Tank B has 20 liters of water at 40 minutes.

### Are You Ready for More?

A giant tortoise travels at 0.17 miles per hour and an arctic hare travels at 37 miles per hour.

1. Draw separate graphs that show the relationship between time elapsed, in hours, and distance traveled, in miles, for both the tortoise and the hare.
2. Would it be helpful to try to put both graphs on the same pair of axes? Why or why not?
3. The tortoise and the hare start out together and after half an hour the hare stops to take a rest. How long does it take the tortoise to catch up?

### Student Response

1. Answers vary. Each axes should have "time elapsed (hours)" on the horizontal axis and "distance traveled (miles)" on the vertical axis. The scale for the giant tortoise graph is likely much smaller than the scale for the "arctic hare" graph.
2. Answers vary. Sample response: Because the scales on the vertical axis are so different, it is very difficult to put both graphs on the same axes without one of the graphs being squashed up very close to an axis. This makes it difficult to read coordinate values from the graph, so it is not very useful.
3. After half an hour the hare has traveled  $0.5 \cdot 37 = 18.5$  miles and the tortoise has traveled  $0.5 \cdot 0.17 = 0.085$  miles, so the hare is  $18.5 - 0.085 = 18.415$  miles ahead of the tortoise. Assuming the hare doesn't move, it will take the tortoise  $18.415/0.17 = 108.32$  hours to catch up, or about 4.5 days.

### Activity Synthesis

Begin the discussion by asking students:

- "What question can you answer using the second graph that you can't with the first?" (You can see when the two tanks have the same amount of water on the second graph.)
- "Is the first graph deceptive in any way?" (Yes, it looks like Tank A will always have more volume than Tank B.)
- "Which scale do you prefer?"

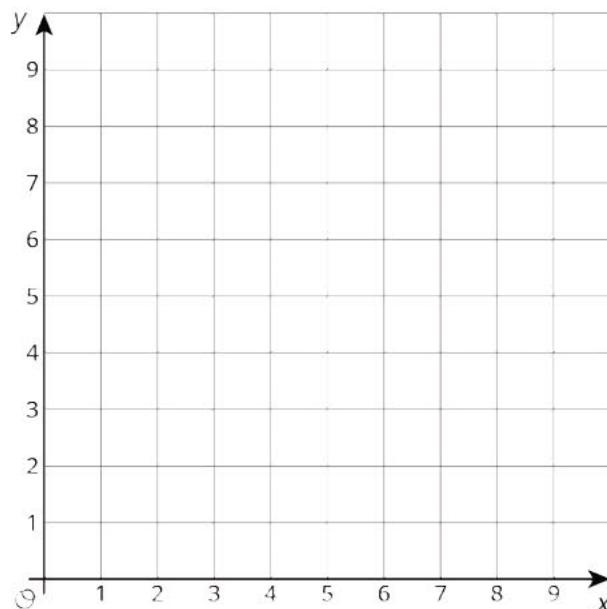
Tell students that if they had a situation where they needed to make a graph from scratch, it is important to check out what questions are asked. Some things to consider are:

- How large are the numbers in the problem? Do you need to go out to 10 or 100?
- What will you count by? 1s? 5s? 10s?
- Should both axes have the same scale?

In the next lessons, students will have to make these types of choices. Tell students that while it can seem like a lot of things to keep in your head, they should always remember that there are many good options when graphing a relationship so they shouldn't feel like they have to make the same exact graph as someone else.

## Lesson Synthesis

Display this blank graph for all to see and provide pairs of students with graph paper.



Ask pairs to draw a copy of the axis and give a signal when they have finished. (You may need to warn students to leave room on their graph paper for a second graph as sometimes students like to draw graphs that fill all the space they are given.) Invite a student to propose a proportional relationship that they consider to have a "steep" line for the class to graph on the axes.

For example, say a student proposes  $y = 6x$ . After students graph, add the line representing the equation to the graph on display. Then, ask students to make a second graph with the same horizontal scale, but with a vertical scale that makes  $y = 6x$  not look as steep when graphed. After students have made the new graph, invite students to share and explain how they decided on their new vertical scale.

Conclude by reminding students that all these graphs of  $y = 6x$  are correct since they all show a proportional relationship with a constant of proportionality equal to 6. Ask students, "Can you think of a reason we might want to graph this relationship with such a large vertical scale?" (If we needed to also graph something like  $y = 60x$ , we would need a pretty big vertical scale in order to see both lines.)

## 2.4 Different Axes

**Cool Down: 5 minutes**

## Addressing

- 8.EE.B

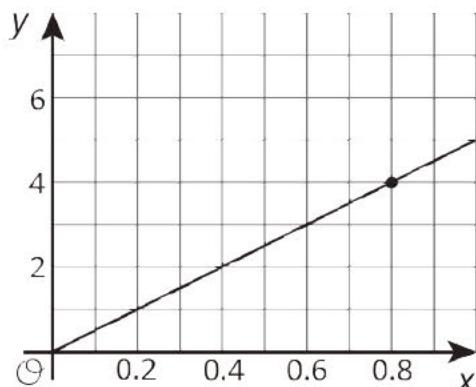
## Building Towards

- 8.EE.B.5

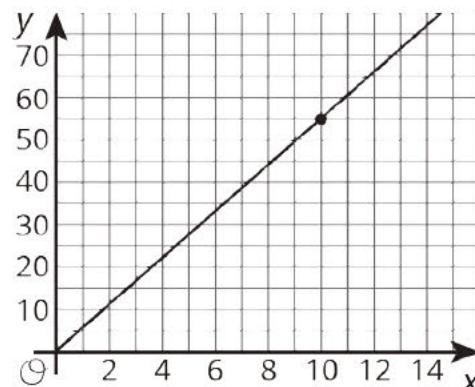
### Student Task Statement

Which one of these relationships is different than the other three? Explain how you know.

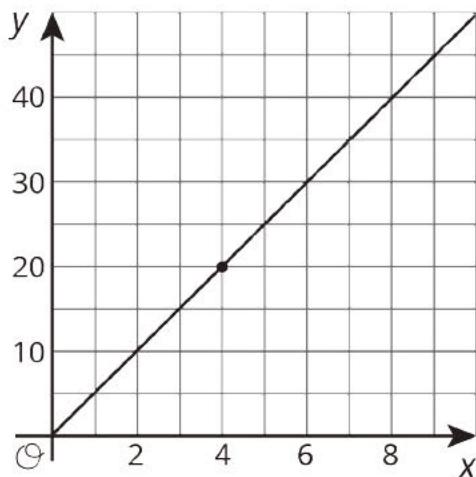
A



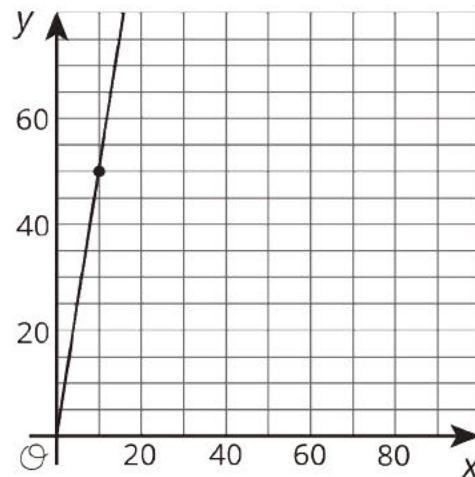
B



C



D



### Student Response

Answers vary. Sample response: Graphs A, C, and D are all representations of  $y = 5x$ . Graph B is a representation of  $y = 5.5x$ .

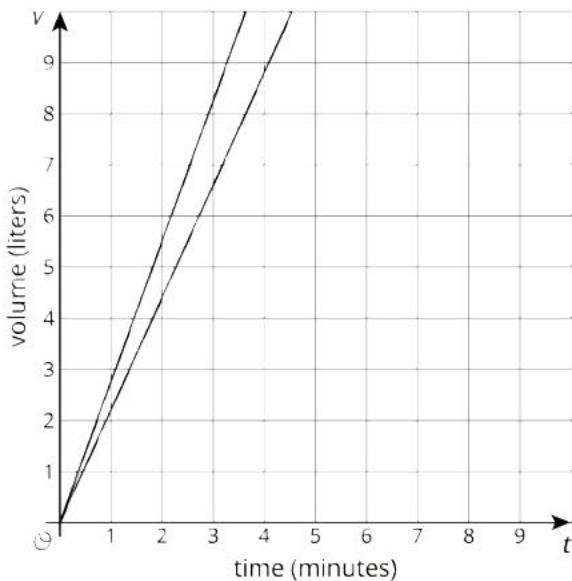
## Student Lesson Summary

The scales we choose when graphing a relationship often depend on what information we want to know. For example, say two water tanks are filled at different constant rates. The relationship between time in minutes  $t$  and volume in liters  $v$  of tank A is given by  $v = 2.2t$ .

For tank B the relationship is  $v = 2.75t$

These equations tell us that tank A is being filled at a constant rate of 2.2 liters per minute and tank B is being filled at a constant rate of 2.75 liters per minute.

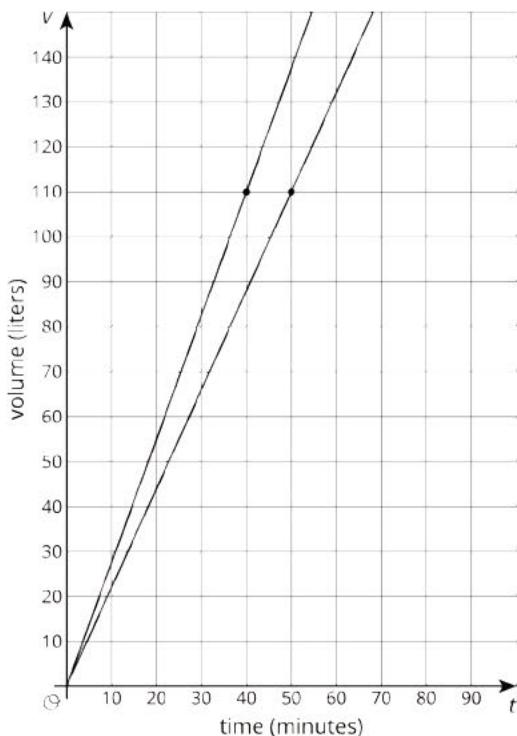
If we want to use graphs to see at what times the two tanks will have 110 liters of water, then using an axis scale from 0 to 10, as shown here, isn't very helpful.



If we use a vertical scale that goes to 150 liters, a bit beyond the 110 we are looking for, and a horizontal scale that goes to 100 minutes, we get a much more useful set of axes for answering our question.

Now we can see that the two tanks will reach 110 liters 10 minutes apart—tank B after 40 minutes of filling and tank A after 50 minutes of filling.

It is important to note that both of these graphs are correct, but one uses a range of values that helps answer the question. In order to always pick a helpful scale, we should consider the situation and the questions asked about it.



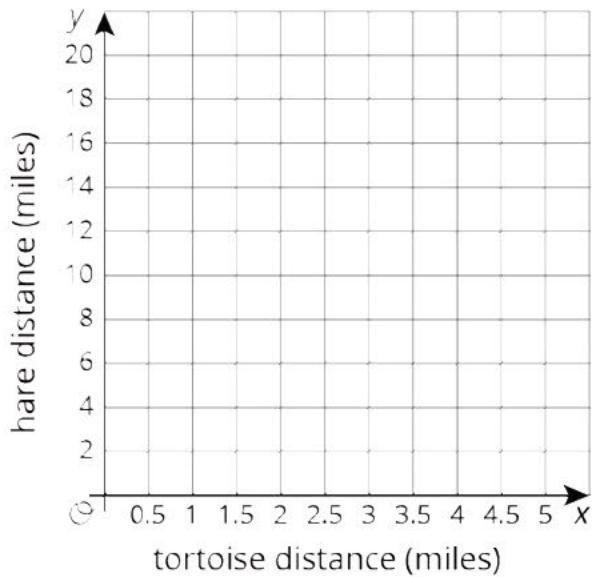
## Lesson 2 Practice Problems

### Problem 1

#### Statement

The tortoise and the hare are having a race. After the hare runs 16 miles the tortoise has only run 4 miles.

The relationship between the distance  $x$  the tortoise “runs” in miles for every  $y$  miles the hare runs is  $y = 4x$ . Graph this relationship.



#### Solution

A ray through  $(0, 0)$  and  $(2, 8)$ .

### Problem 2

#### Statement

The table shows a proportional relationship between the weight on a spring scale and the distance the spring has stretched.

- Complete the table.
- Describe the scales you could use on the  $x$  and  $y$  axes of a coordinate grid that would show all the distances and weights in the table.

distance (cm)	weight (newtons)
20	28
55	
	140
1	

## Solution

a.

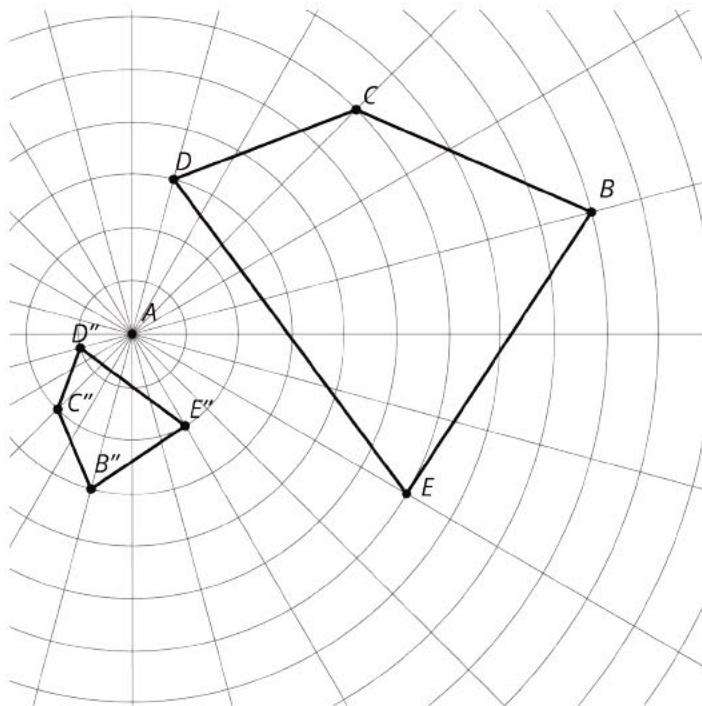
	distance (cm)	weight (newtons)
	20	28
	55	77
	100	140
	1	$\frac{7}{5}$

b. Answers vary. Typical answer: From 0 to 100 on the horizontal (distance) axis and from 0 to 140 on the vertical (weight) axis.

## Problem 3

### Statement

Find a sequence of rotations, reflections, translations, and dilations showing that one figure is similar to the other. Be specific: give the amount and direction of a translation, a line of reflection, the center and angle of a rotation, and the center and scale factor of a dilation.



## Solution

Answers vary. Sample response:

- a. Begin with figure  $BCDE$ .
- b. Dilate using  $A$  as the center of dilation with scale factor  $\frac{1}{3}$ .
- c. Rotate using  $A$  as the center clockwise 30 degrees.
- d. Reflect along the line that contains  $A$  and the image of  $E$  under the previous transformations.

(From Unit 2, Lesson 6.)

## Problem 4

### Statement

Andre said, "I found two figures that are congruent, so they can't be similar."

Diego said, "No, they are similar! The scale factor is 1."

Do you agree with either of them? Use the definition of similarity to explain your answer.

### Solution

Diego is correct. Two figures are congruent if one can be moved to the other using a sequence of rigid transformations, and they are similar if one can be moved to the other using a sequence of rigid transformations and dilations. If two figures are congruent, then they are also similar. Scalings (such as Diego's suggested scaling with a scale factor of 1) can also be applied. While scalings are allowed, they're not always *required* to show that two figures are similar.

(From Unit 2, Lesson 6.)

# Lesson 3: Representing Proportional Relationships

## Goals

- Create an equation and a graph to represent proportional relationships, including an appropriate scale and axes.
- Determine what information is needed to create graphs that represent proportional relationships. Ask questions to elicit that information.

## Learning Targets

- I can scale and label a coordinate axes in order to graph a proportional relationship.

## Lesson Narrative

Now that students have considered the scale from several perspectives, in this lesson they label and choose a scale for empty pairs of axes as part of graphing proportional relationships. In the first activity, students create representations of proportional relationships when given two to start from. For each representation, they identify key features such as the constant of proportionality and relate how they know that each representation is for the same situation. In the second activity, students use the info gap structure. The student with the problem card needs to graph a proportional relationship on an empty pair of axes that includes a specific point. In order to do so, they need to request information about the proportional relationship as well as calculate the specific point. The focus here is on the graphs students create and their decisions on how to scale the axes in an appropriate manner for the situation.

## Alignments

### Building On

- 6.EE.A.3: Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression  $3(2 + x)$  to produce the equivalent expression  $6 + 3x$ ; apply the distributive property to the expression  $24x + 18y$  to produce the equivalent expression  $6(4x + 3y)$ ; apply properties of operations to  $y + y + y$  to produce the equivalent expression  $3y$ .
- 6.NS.A.1: Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for  $(2/3) \div (3/4)$  and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that  $(2/3) \div (3/4) = 8/9$  because  $3/4$  of  $8/9$  is  $2/3$ . (In general,  $(a/b) \div (c/d) = ad/bc$ .) How much chocolate will each person get if 3 people share  $1/2$  lb of chocolate equally? How many  $3/4$ -cup servings are in  $2/3$  of a cup of yogurt? How wide is a rectangular strip of land with length  $3/4$  mi and area  $1/2$  square mi?
- 6.NS.B.3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

## Addressing

- 8.EE.B: Understand the connections between proportional relationships, lines, and linear equations.
- 8.EE.B.5: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

## Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR2: Collect and Display
- MLR4: Information Gap Cards
- MLR8: Discussion Supports
- Number Talk

## Required Materials

### Graph paper

Pre-printed cards, cut from copies of the blackline master

### Straightedges

A rigid edge that can be used for drawing line segments. Sometimes a ruler is okay to use as a

straightedge, but sometimes it is preferable to use an unruled straightedge, like a blank index card.

## Required Preparation

Print and cut up cards from the Info Gap: Proportional Relationships blackline master. Prepare 1 set of cards for every 2 students. Provide all students with access to straightedges and graph paper.

### Student Learning Goals

Let's graph proportional relationships.

## 3.1 Number Talk: Multiplication

### Warm Up: 5 minutes

This Number Talk encourages students to think about the numbers in a computation problem and rely on what they know about structure, patterns, multiplication, fractions, and decimals to mentally solve a problem. Only one problem is presented to allow students to share a variety of strategies for multiplication. Notice how students handle the multiplication by a decimal. Some students may use fraction equivalencies while others may use the decimal given in the problem. For each of those choices, ask students why they made that decision.

## Building On

- 6.EE.A.3
- 6.NS.A.1
- 6.NS.B.3

## Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports
- Number Talk

## Launch

Display one problem at a time. Give students 1 minute of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Follow with a whole-class discussion. Be sure to elicit as many strategies as possible.

---

## Support for Students with Disabilities

*Representation: Internalize Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.

*Supports accessibility for: Memory; Organization*

---

### Student Task Statement

Find the value of each product mentally.

$$15 \cdot 2$$

$$15 \cdot 0.5$$

$$15 \cdot 0.25$$

$$15 \cdot (2.25)$$

### Student Response

- 30, 7.5, 3.75, 33.75. Explanations vary. Sample explanation:  
$$15 \cdot (2.25) = 33.75$$

Possible strategies:

- Distributive property:  $(15 \cdot 2) + (15 \cdot (0.25))$  or  $(15 \cdot 2) + (15 \cdot \frac{1}{4})$
- Distributive property:  $(16 \cdot 2\frac{1}{4}) - 2.25$

- Representing 2.25 as a fraction:  $15 \cdot \frac{9}{4}$

### Activity Synthesis

Invite students to share their strategies. Use MLR 2 (Collect and Display) to record and display student explanations for all to see. Ask students to explain their choice of either using a decimal or fraction for 2.25 in their solution path. To involve more students in the conversation, consider asking:

- “Who can restate \_\_\_\_'s reasoning in a different way?”
- “Did anyone solve the problem the same way but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to \_\_\_\_'s strategy?”
- “Do you agree or disagree? Why?”

At the end of discussion, if time permits, ask a few students to share a story problem or context that  $15 \cdot (2.25)$  would represent.

---

#### Support for English Language Learners

*Speaking: MLR8 Discussion Supports.:* Display sentence frames to support students when they explain their strategy. For example, "First, I \_\_\_\_ because . . ." or "I noticed \_\_\_\_ so I . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

*Design Principle(s): Optimize output (for explanation)*

---

## 3.2 Representations of Proportional Relationships

15 minutes

The purpose of this activity is for students to graph a proportional relationship when given a blank pair of axes. They will need to label and scale the axes appropriately before adding the line representing the given relationship. In each problem, students are given two representations and asked to create two more representations so that each relationship has a description, graph, table, and equation. Then, they explain how they know these are different representations of the same situation (MP3). In the next lesson, students will use these skills to compare two proportional relationships represented in different ways.

Identify students making particularly clear graphs and using situation-appropriate scales for their axes. For example, since the second problem is about a car wash, the scale for the axis showing the number of cars does not need to extend into the thousands.

## Addressing

- 8.EE.B.5

## Instructional Routines

- MLR1: Stronger and Clearer Each Time

## Launch

Arrange students in groups of 2. Provide access to straightedges. Give 3 minutes of quiet work time for students to begin the first problem and then tell students to check in with their partners to compare tables and how they are labeling and scaling the axes. Ask students to pause their work and select a few students to share what scale they are using for the axes and why they chose it. It is important to note that the scale chosen should be reasonable based on the context. For example, using a very small scale for steps taken does not make sense.

Give partners time to finish the remaining problems and follow with a whole-class discussion.

---

## Support for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Provide students with grid or graph paper to organize their work with creating a table and graph for the situation.

*Supports accessibility for: Language; Organization*

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### Student Task Statement

1. Here are two ways to represent a situation.

Description:

Jada and Noah counted the number of steps they took to walk a set distance. To walk the same distance, Jada took 8 steps while Noah took 10 steps. Then they found that when Noah took 15 steps, Jada took 12 steps.

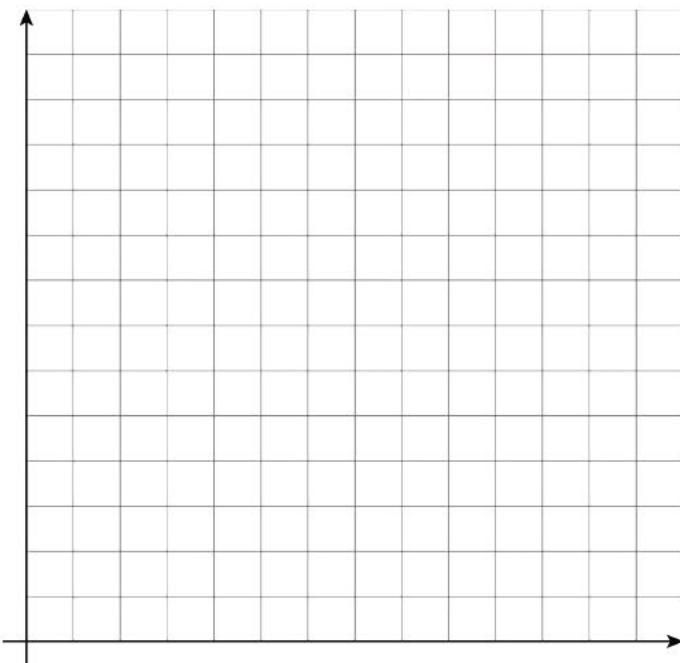
Equation:

Let  $x$  represent the number of steps Jada takes and let  $y$  represent the number of steps Noah takes.

$$y = \frac{5}{4}x$$

- a. Create a table that represents this situation with at least 3 pairs of values.

b. Graph this relationship and label the axes.



c. How can you see or calculate the constant of proportionality in each representation? What does it mean?

d. Explain how you can tell that the equation, description, graph, and table all represent the same situation.

2. Here are two ways to represent a situation.

Description:

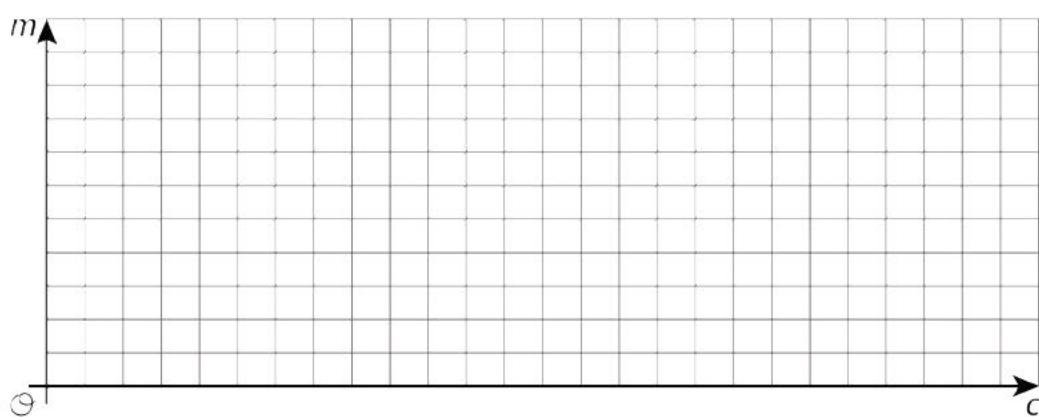
The Origami Club is doing a car wash fundraiser to raise money for a trip. They charge the same price for every car. After 11 cars, they raised a total of \$93.50. After 23 cars, they raised a total of \$195.50.

Table:

number of cars	amount raised in dollars
11	93.50
23	195.50

a. Write an equation that represents this situation. (Use  $c$  to represent number of cars and use  $m$  to represent amount raised in dollars.)

b. Create a graph that represents this situation.



c. How can you see or calculate the constant of proportionality in each representation? What does it mean?

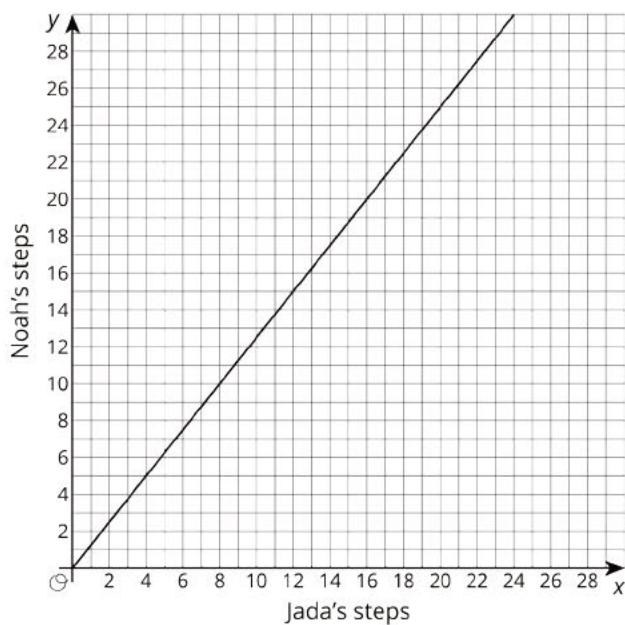
d. Explain how you can tell that the equation, description, graph, and table all represent the same situation.

**Student Response**

1. a. Answers vary. Sample response:

$x$	$y$
8	10
12	15

b. Answers vary. Sample response:



c. Table: Any  $x$  and  $y$  values have  $\frac{y}{x} = \frac{5}{4}$ . In the equation:  $k = \frac{5}{4}$ , Description: Noah's steps for each of Jada's steps ( $\frac{5}{4}$ ), Graph: every coordinate  $(x, y)$  on the graph has the relationship that  $\frac{y}{x} = \frac{5}{4}$ . For every one step Noah takes, Jada takes 1 and a quarter steps.

d. Answers vary. Sample response: All four representations have a constant of proportionality equal to  $\frac{5}{4}$ .

2. a. Equation:  $m = 8.50c$

b. Graph:



c. In the equation:  $k = 8.50$ , Table: Any  $x$  and  $y$  for which  $\frac{y}{x} = 8.50$ , Description: money raised in dollars per cars (\$), Graph: every coordinate  $(x, y)$  on the graph has the relationship that  $\frac{y}{x} = 8.50$ . \$ for each car washed.

d. Answers vary. Sample response: All four representations have a constant of proportionality of 8.5.

### Activity Synthesis

Ask previously identified students to share their graphs and how they chose the scales for their axes. If possible, display several graphs from each question for all to see as students share.

Ask students “Which representation makes it more difficult (and less difficult) to calculate the constant of proportionality? Why?” and give 1 minute of quiet think time. Invite several students to share their responses.

Tell students that the constant of proportionality can be thought of as the **rate of change** of one variable with respect to the other. In the case of Jada and Noah, the rate of change of  $y$ , the number of steps Noah takes, with respect to  $x$ , the number of steps Jada takes, is  $\frac{5}{4}$  Noah steps per Jada steps. In the case of the Origami Club’s car wash, the rate of change of  $m$ , the amount they raise in dollars, with respect to  $c$ , the number of cars they clean, is 8.50 dollars per car.

---

### Support for English Language Learners

*Writing, Conversing: MLR1 Stronger and Clearer Each Time.* Use this routine for students to respond in writing to the final question, “Explain how you can tell that the equation, description, graph, and table all represent the same situation.” Give students time to meet with 2–3 partners, to share their responses and get feedback. Encourage the listener to ask clarifying questions such as, “How did you identify the same constant of proportionality?” or “Did the scales for the axes cause any confusion?” Have the students write a final draft based on their peer feedback. This will help students to generalize the process for identifying the same constant of proportionality.

*Design Principle(s): Optimize output; Cultivate conversation*

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## 3.3 Info Gap: Proportional Relationships

**15 minutes**

This info gap activity gives students an opportunity to determine and request the information needed when working with proportional relationships. In order to graph the relationship and the requested information, students need to think carefully about how they can scale the axes.

The info gap structure requires students to make sense of problems by determining what information is necessary, and then asking for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need (MP1). It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need (MP6).

Here is the text of one of the cards for reference and planning:

Info Gap: Proportional Relationships  
Problem Card 1

Sketch a graph that shows the relationship between grams of honey and cups of flour needed for a bakery recipe. Show on the graph how much honey is needed for 17 cups of flour.



Info Gap: Proportional Relationships  
Data Card 1

salt (g)	honey (g)	flour (c)
15	27	6
25	45	10

Listen for how students request (and supply) information about the relationship between the two ingredients. Identify students using different scales for their graphs that show clearly the requested information to share during the discussion.

### Addressing

- 8.EE.B

### Instructional Routines

- MLR4: Information Gap Cards

### Launch

Tell students that they will continue their work graphing proportional relationships. Explain the Info Gap and consider demonstrating the protocol if students are unfamiliar with it. Arrange students in groups of 2. Provide access to straightedges. In each group, distribute a problem card to one student and a data card to the other student. After you review their work on the first problem, give them the cards for the second problem and instruct them to switch roles.

---

### Support for Students with Disabilities

*Representation: Provide Access for Perception.* Display or provide students with a physical copy of the written directions and read them aloud. Check for understanding by inviting students to rephrase directions in their own words. Consider keeping the display of directions visible throughout the activity.

*Supports accessibility for: Language; Memory*

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### Support for English Language Learners

*Conversing:* This activity uses MLR4 Information Gap to give students a purpose for discussing information necessary to choose a scale and graph a proportional relationship. Display questions or question starters for students who need a starting point such as: "Can you tell me . . . (specific piece of information)", and "Why do you need to know . . . (that piece of information)?"

*Design Principle(s): Cultivate Conversation*

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### Anticipated Misconceptions

Some students may be unsure how large to make their scale before they answer the question on the card. Encourage these students to answer the question on their card and then think about how to scale their graph.

#### Student Task Statement

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

Continue to ask questions until you have enough information to solve the problem.

4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.
2. Ask your partner "*What specific information do you need?*" and wait for them to *ask* for information.

If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don't have that information.

3. Before sharing the information, ask "*Why do you need that information?*" Listen to your partner's reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

### Student Response

Problem 1: 76.5 grams of honey are needed for 17 cups of flour. Graphs vary. Possible scale: 0–28 on the cups of flour axis, 0–140 on the grams of honey axis.

Problem 2: 57.5 grams of salt are needed for 23 cups of flour. Graphs vary. Possible scale: 0–28 on the cups of flour axis, 0–70 on the grams of salt axis.

### Are You Ready for More?

Ten people can dig five holes in three hours. If  $n$  people digging at the same rate dig  $m$  holes in  $d$  hours:

1. Is  $n$  proportional to  $m$  when  $d = 3$ ?
2. Is  $n$  proportional to  $d$  when  $m = 5$ ?
3. Is  $m$  proportional to  $d$  when  $n = 10$ ?

### Student Response

1. Yes. If 10 people can dig 5 holes in 3 hours, then 2 people can dig 1 hole in 3 hours, so  $n = 2m$ .
2. No. If you double the number of people then you half the time it takes to dig the holes. So  $n$  is not a constant times  $d$ .
3. Yes. If 10 people can dig 5 holes in 3 hours, then they can dig  $\frac{5}{3}$  holes in 1 hour, so  $m = \frac{5}{3}d$ .

### Activity Synthesis

After students have completed their work, ask previously identified students to share their graphs and explain how they chose their axis. Some guiding questions:

- “Other than the answer, what information would have been nice to have?”
- “How did you decide what to label the two axes?”
- “How did you decide to scale the horizontal axis? The vertical?”
- “What was the rate of change of grams of honey per cups of flour? Where can you see this on the graph you made?” (4.5 grams of honey per cup of flour.)
- “What was the rate of change of grams of salt per cups of flour? Where can you see this on the graph you made?” (2.5 grams of salt per cups of flour)

### Lesson Synthesis

Consider asking some of the following questions.

- “The proportional relationship  $y = 5.5x$  includes the point  $(18, 99)$  on its graph. How could you choose a scale for a pair of axes with a 10 by 10 grid to show this point?” (Have each grid line represent 10 or 20 units.)

- “What are some things you learned about graphing today that you are going to try to remember for later?”

## 3.4 Graph the Relationship

Cool Down: 5 minutes

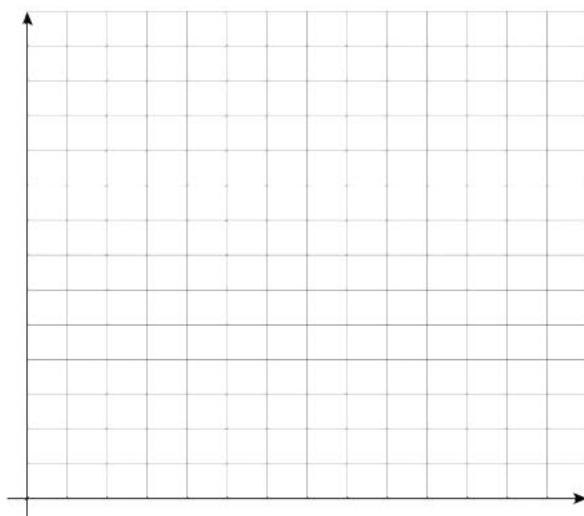
### Addressing

- 8.EE.B.5

#### Student Task Statement

Sketch a graph that shows the relationship between grams of honey and grams of salt needed for a bakery recipe. Show on the graph how much honey is needed for 70 grams of salt.

salt (g)	honey (g)	flour (c)
10	14	4
25	35	10



#### Student Response

Answers vary. Possible graph: Label each axis from 0 to 140. For grams of salt on the horizontal axis and grams of honey on the vertical, the points  $(0, 0)$ ,  $(10, 14)$ , and  $(70, 98)$ .

#### Student Lesson Summary

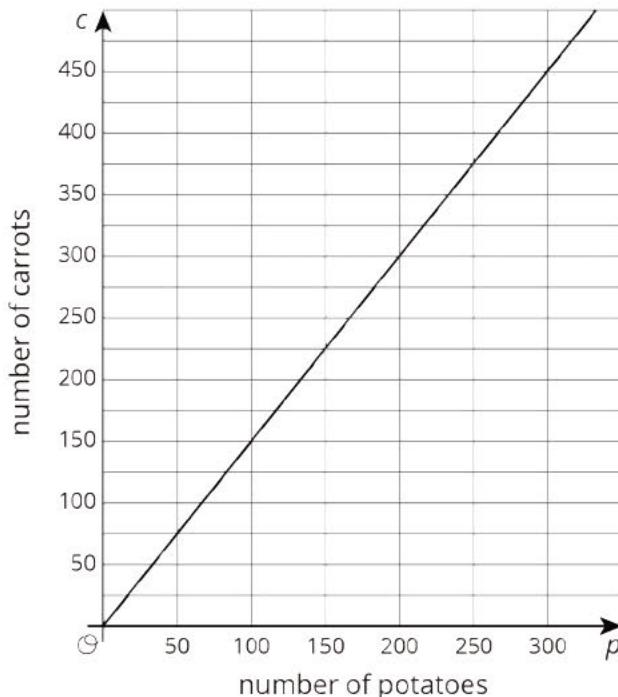
Proportional relationships can be represented in multiple ways. Which representation we choose depends on the purpose. And when we create representations we can choose helpful values by paying attention to the context. For example, a stew recipe calls for 3 carrots for every 2 potatoes. One way to represent this is using an equation. If there are  $p$  potatoes and  $c$  carrots, then  $c = \frac{3}{2}p$ .

Suppose we want to make a large batch of this recipe for a family gathering, using 150 potatoes. To find the number of carrots we could just use the equation:  $\frac{3}{2} \cdot 150 = 225$  carrots.

Now suppose the recipe is used in a restaurant that makes the stew in large batches of different sizes depending on how busy a day it is, using up to 300 potatoes at a time.

Then we might make a graph to show how many carrots are needed for different amounts of potatoes. We set up a pair of coordinate axes with a scale from 0 to 300 along the horizontal axis and 0 to 450 on the vertical axis, because  $450 = \frac{3}{2} \cdot 300$ .

Then we can read how many carrots are needed for any number of potatoes up to 300.



Or if the recipe is used in a food factory that produces very large quantities and the potatoes come in bags of 150, we might just make a table of values showing the number of carrots needed for different multiples of 150.

number of potatoes	number of carrots
150	225
300	450
450	675
600	900

No matter the representation or the scale used, the constant of proportionality,  $\frac{3}{2}$ , is evident in each. In the equation it is the number we multiply  $p$  by; in the graph, it is the slope; and in the table, it is the number we multiply values in the left column to get numbers in the right column. We can think of the constant of proportionality as a **rate of change** of  $c$  with respect to  $p$ . In this case the rate of change is  $\frac{3}{2}$  carrots per potato.

## Glossary

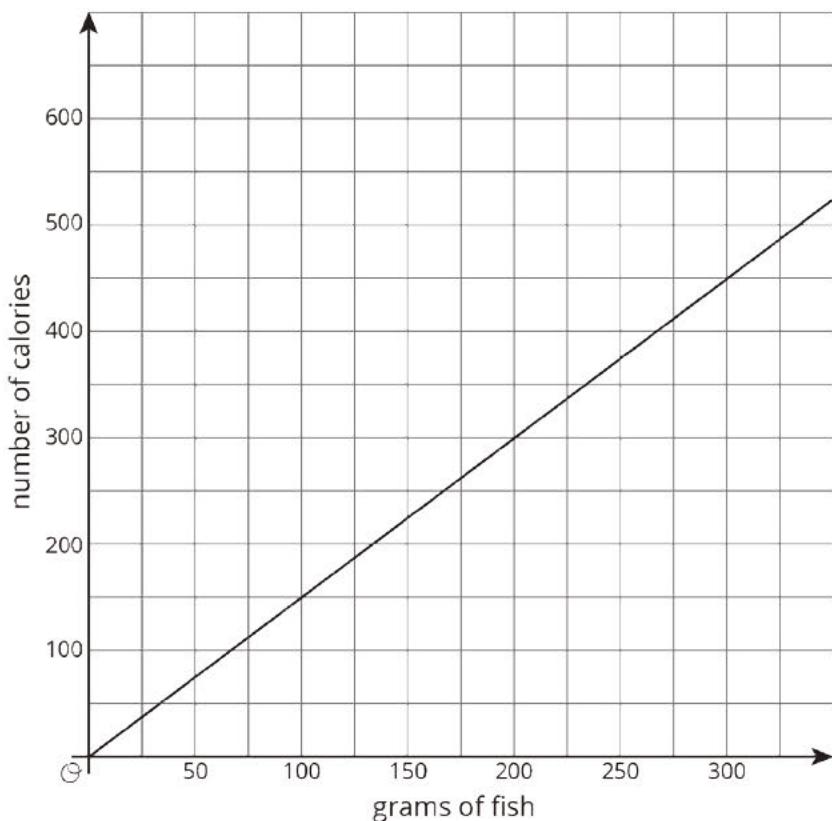
- rate of change

## Lesson 3 Practice Problems

### Problem 1

#### Statement

Here is a graph of the proportional relationship between calories and grams of fish:



- Write an equation that reflects this relationship using  $x$  to represent the amount of fish in grams and  $y$  to represent the number of calories.
- Use your equation to complete the table:

grams of fish	number of calories
1000	
	2001
1	

## Solution

a.  $y = \frac{3}{2}x$

b.

grams of fish	number of calories
1000	1500
1334	2001
1	$\frac{3}{2}$

## Problem 2

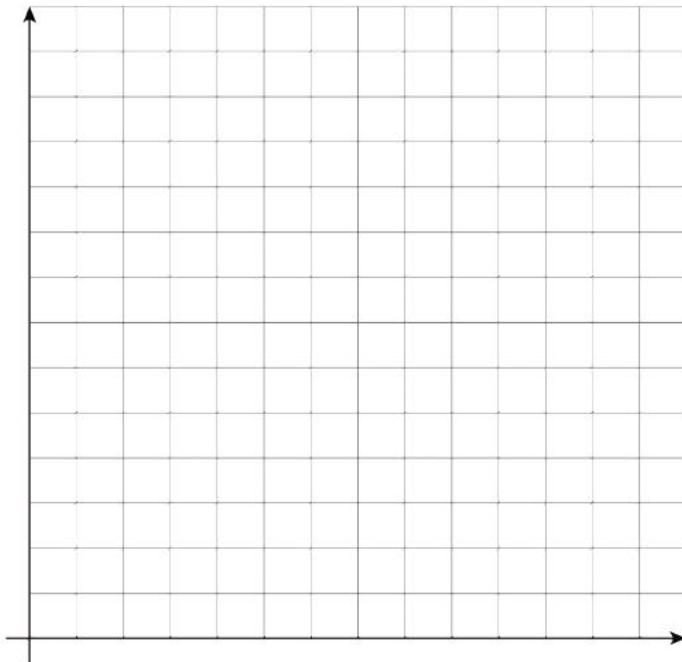
### Statement

Students are selling raffle tickets for a school fundraiser. They collect \$24 for every 10 raffle tickets they sell.

a. Suppose  $M$  is the amount of money the students collect for selling  $R$  raffle tickets.

Write an equation that reflects the relationship between  $M$  and  $R$ .

b. Label and scale the axes and graph this situation with  $M$  on the vertical axis and  $R$  on the horizontal axis. Make sure the scale is large enough to see how much they would raise if they sell 1000 tickets.



### Solution

a.  $M = \frac{12}{5}R$  (or equivalent)

b. On coordinate axes with  $R$  on the horizontal axis and  $M$  on the vertical axis, a ray through  $(0, 0)$  and  $(10, 24)$  (or equivalent)

## Problem 3

### Statement

Describe how you can tell whether a line's slope is greater than 1, equal to 1, or less than 1.

### Solution

Answers vary. Sample response: Build a slope triangle. If its vertical length is greater than its horizontal length, the slope is greater than 1. If its vertical and horizontal lengths are equal, the slope is equal to 1. If the slope triangle's vertical length is less than its horizontal length, the slope is less than 1.

(From Unit 2, Lesson 10.)

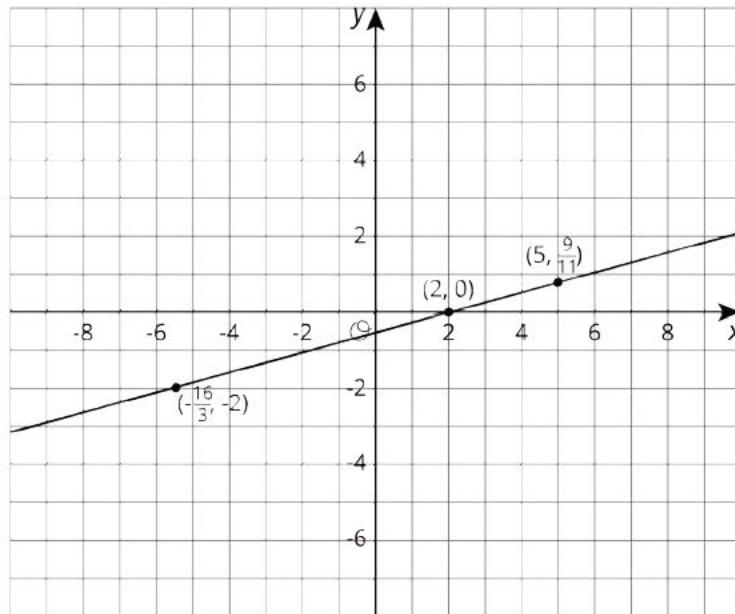
## Problem 4

### Statement

A line is represented by the equation  $\frac{y}{x-2} = \frac{3}{11}$ . What are the coordinates of some points that lie on the line? Graph the line on graph paper.

### Solution

Answers vary. Possible response:



(Note that there is one point on the line which has to be treated differently. The point  $(2, 0)$  is on the sketched line but is not a pair that can be plugged into  $\frac{y}{x-2} = \frac{3}{11}$ , as it results in a denominator of 0. The point  $(2, 0)$  does satisfy the equivalent equation  $y = \frac{3}{11}(x - 2)$ , however.)

(From Unit 2, Lesson 12.)

# Lesson 4: Comparing Proportional Relationships

## Goals

- Compare the rates of change for two proportional relationships, given multiple representations.
- Interpret multiple representations of a proportional relationship in order to answer questions (in writing), and explain the solution method.
- Present a comparison of two proportional relationships (using words and multiple other representations).

## Learning Targets

- I can compare proportional relationships represented in different ways.

## Lesson Narrative

In this fourth lesson on proportional relationships, students expand on the work of the previous lesson by comparing two situations that are represented in different ways. For example, one situation might specify a rate of change, while the other is represented by a table of values, a graph, or an equation. Students move flexibly between representations and consider how to find the information they need from each type. They respond to context-related questions that compare the two situations and solve problems with the information they've garnered from each representation.

## Alignments

### Addressing

- 8.EE.B: Understand the connections between proportional relationships, lines, and linear equations.
- 8.EE.B.5: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

## Instructional Routines

- Group Presentations
- MLR7: Compare and Connect

## Required Materials

### Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart

paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

## Student Learning Goals

Let's compare proportional relationships.

# 4.1 What's the Relationship?

### Warm Up: 10 minutes

The purpose of this warm-up is for students to create a graph and a description from an equation, building on their work in the previous lesson. Students decide on a context and then make the graph, scaling the axes appropriately to the situation. Moving between representations of a proportional relationship here is preparation for the following activity where students compare proportional relationships represented in different ways.

### Addressing

- 8.EE.B

### Launch

Arrange students in groups of 2. Give 2–3 minutes of quiet work time followed by a whole-class discussion.

### Student Task Statement

The equation  $y = 4.2x$  could represent a variety of different situations.

1. Write a description of a situation represented by this equation. Decide what quantities  $x$  and  $y$  represent in your situation.
2. Make a table and a graph that represent the situation.

### Student Response

1. Answers vary. Sample response: A frog jumps 16.8 feet in 4 seconds.
2. Graph:

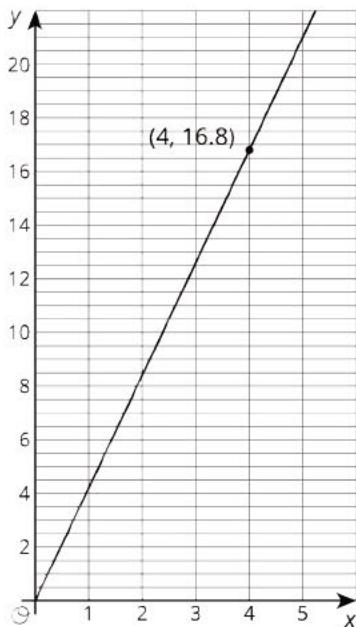


Table:

$x$	$y$
3	12.60
4	16.80

### Activity Synthesis

Invite several students to share their situations and display their graphs for all to see. Ask:

- “What does the rate of change represent in this situation?”
- “How did you decide on the scale for your axes?”

## 4.2 Comparing Two Different Representations

**25 minutes**

The purpose of this activity is for students to compare two different proportional relationships represented in different ways using the skills they have worked on over the past three lessons. Working in groups, students compare the relationships, responding to questions about their rate of change, which rate of change is higher, and one other situation-based question. Groups make a visual display for their problem set to explain each of their responses and convince others of their accuracy.

Identify groups using a variety of representations to share during the Activity Synthesis.

## **Addressing**

- 8.EE.B.5

## **Instructional Routines**

- Group Presentations
- MLR7: Compare and Connect

## **Launch**

Remind students that in previous lessons they identified representations of and created representations for a single proportional relationship. In this activity, they will consider representations of two *different* proportional relationships and make comparisons between them.

Arrange students in groups of 2–3. Assign to each group (or ask groups to choose) one of the three question sets. Tell groups that they will make a visual display for their responses to the questions. The display should clearly demonstrate their reasoning and use multiple representations in order to be convincing. Let them know that there will be a gallery walk when they finish for the rest of the class to inspect their solutions' accuracy.

If time allows, ask groups to complete all three problems and make a visual display for just one.

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## **Support for Students with Disabilities**

*Action and Expression: Internalize Executive Functions.* To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, present one question at a time.

*Supports accessibility for: Memory; Organization*

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## **Anticipated Misconceptions**

Some students may confuse the values for the rate of change of a situation. For example, Lemonade Recipe 1's equation,  $y = 4x$ , shows that the rate of change is 4 cups of water per cup of lemonade mix. Students may switch these values and think that the rate of change is 4 cups lemonade mix per cup of water. Ask students who do this to explain where in the original representations they see the rate of change. Students may need to list a few values or sketch a graph in order to see their mix-up between the two quantities.

## **Student Task Statement**

1. Elena babysits her neighbor's children. Her earnings are given by the equation  $y = 8.40x$ , where  $x$  represents the number of hours she worked and  $y$  represents the amount of money she earned.

Jada earns \$7 per hour mowing her neighbors' lawns.

a. Who makes more money after working 12 hours? How much more do they make? Explain your reasoning by creating a graph or a table.

b. What is the rate of change for each situation and what does it mean?

c. Using your graph or table, determine how long it would take each person to earn \$150.

2. Clare and Han have summer jobs stuffing envelopes for two different companies.

Han earns \$15 for every 300 envelopes he finishes.

Clare's earnings can be seen in the table.

number of envelopes	money in dollars
400	40
900	90

a. By creating a graph, show how much money each person makes after stuffing 1,500 envelopes.

b. What is the rate of change for each situation and what does it mean?

c. Using your graph, determine how much more money one person makes relative to the other after stuffing 1,500 envelopes. Explain or show your reasoning.

3. Tyler plans to start a lemonade stand and is trying to perfect his recipe for lemonade. He wants to make sure the recipe doesn't use too much lemonade mix (lemon juice and sugar) but still tastes good.

Lemonade Recipe 1 is given by the equation Lemonade Recipe 2 is given in the table.

$y = 4x$  where  $x$  represents the amount of lemonade mix in cups and  $y$  represents the amount of water in cups.

lemonade mix (cups)	water (cups)
10	50
13	65
21	105

a. If Tyler had 16 cups of lemonade mix, how many cups of water would he need for each recipe? Explain your reasoning by creating a graph or a table.

b. What is the rate of change for each situation and what does it mean?

c. Tyler has a 5-gallon jug (which holds 80 cups) to use for his lemonade stand and 16 cups of lemonade mix. Which lemonade recipe should he use? Explain or show your reasoning.

### Student Response

1. a. Elena will make \$16.80 more than Jada. Answers vary. Sample response: A graph showing hours worked along the  $x$ -axis and money earned along the  $y$ -axis. The graph scale is large enough to show the points on the lines representing each situation at 12 hours. The difference between the  $y$ -coordinates of these two points is \$16.80.  
b. Answers vary. Sample response: Elena: \$8.40 per hour, Jada: \$7 per hour. These rates of change tell us how much money Elena and Jada make for each hour they work.  
c. Answers vary. Sample response: Elena: about 18 hours, Jada: about 21.5 hours, if she gets paid for half hours or 22 hours if she is paid only in whole hour increments.
2. a. Clare earns \$150.00 and Han earns \$75.00. Answers vary. Sample response: A graph showing hours worked along the  $x$ -axis and money earned along the  $y$ -axis. The graph scale is large enough to show the points on the lines representing each situation at 1,500 envelopes.  
b. Answers vary. Sample response: Clare: \$0.10 per envelope, Han: \$0.05 per envelope. These rates of change tell us how much money they make for each envelope they finish.  
c. Clare earns \$75.00 more than Han. Explanations vary. Sample explanation: At 1,500 envelopes, the difference between the  $y$ -coordinates of these two points is \$75.00.
3. a. Recipe 1: 64 cups of water, Recipe 2: 80 cups of water. Answers vary. Sample response: A graph showing cups of lemonade mix along the  $x$ -axis and cups of water along the  $y$ -axis. The graph scale is large enough to show the points on the lines representing each situation at 16 cups of lemonade mix.  
b. Recipe 1: 4 cups of water per cup of lemonade mix, Recipe 2: 5 cups of water per cup of lemonade mix. Each rate of change tells how much water is needed per cup of lemonade mix.  
c. Answers vary. Sample response: Either recipe could be used to fill the 5 gallon jug with lemonade if there are 16 cups of mix. Recipe 1 will use all 16 cups of mix and 64 cups of water since there are 4 cups of water per cup of lemonade mix. Recipe 1 would use  $13\frac{1}{3}$  cups mix and  $16\frac{2}{3}$  cups water since there are 5 cups of water per cup of lemonade mix. We think he should use recipe 2, because then he will have  $2\frac{2}{3}$  cup lemonade mix leftover to use at his stand the next day.

### Are You Ready for More?

Han and Clare are still stuffing envelopes. Han can stuff 20 envelopes in a minute, and Clare can stuff 10 envelopes in a minute. They start working together on a pile of 1,000 envelopes.

1. How long does it take them to finish the pile?
2. Who earns more money?

### Student Response

1. Working together they can stuff 30 envelopes per minute, so it takes them  $\frac{1000}{30} = 33\frac{1}{3}$  minutes to finish the pile.
2. Han stuffs twice as many envelopes as Clare, but he only earns half as much, so they both earn the same amount of money.

### Activity Synthesis

Begin with a gallery walk for students to see how other groups answered the same set of questions they did and how students answered questions about the other two contexts.

Invite groups to share the strategies they used with the various representations. Consider asking groups the following questions:

- “What representations did you choose to answer the questions? Why did you pick them?”
- “What representation did you not use? Why?”
- “How did you decide what scale to use when you made your graph?”
- “Now that you have seen the work of other groups, is there anything about your display you would change if you could?”

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### Support for English Language Learners

*Representing and Conversing: MLR7 Compare and Connect.* During the gallery walk, invite students to discuss “what is the same and what is different?” about the representations on the posters and then share with the whole class. Look for opportunities to highlight representations that helped students answer the questions and decide which scales to use for the graph. This will help students make connections and describe the usefulness of each type of representation.

*Design Principle(s): Optimize output*

---

### Lesson Synthesis

This lesson asked students to take a single piece of information about a proportional relationship, such as an equation, and use what they know about proportional relationships, rates of change, and representing relationships to compare it with a second proportional relationship in context.

Consider asking some of the following questions. Tell students to use, if possible, examples from today's lesson when responding:

- “What do you need in order to compare two proportional relationships?”

- “What type of wording in a problem statement or description of a situation tells you that you have a rate of change?”
- “How did you decide which representation to use to solve the different types of problems?”

## 4.3 Different Salt Mixtures

Cool Down: 5 minutes

### Addressing

- 8.EE.B.5

#### Student Task Statement

Here are recipes for two mixtures of salt and water that taste different.

Information about Salt Mixture A is shown in the table.

salt (teaspoons)	water (cups)
4	5
7	$8\frac{3}{4}$
9	$11\frac{1}{4}$

Salt Mixture B is defined by the equation  $y = 2.5x$ , where  $x$  is the number of teaspoons of salt and  $y$  is the number of cups of water.

1. If you used 10 cups of water, which mixture would use more salt? How much more? Explain or show your reasoning.
2. Which mixture tastes saltier? Explain how you know.

### Student Response

1. Mixture A uses 4 more teaspoons of salt than Mixture B. Mixture A would use 8 teaspoons of salt because I can double the row with 4 and 5 to get 8 and 10. Mixture B would use 4 teaspoons of salt because considering if  $10 = 2.5x$ , the value of  $x$  must be 4.
2. Mixture A tastes saltier because it uses more salt for the same amount of water. Mixture A uses 1.25 cups of water per teaspoon of salt while Mixture B uses 2.5 cups of water per teaspoon of salt.

#### Student Lesson Summary

When two proportional relationships are represented in different ways, we compare them by finding a common piece of information.

For example, Clare's earnings are represented by the equation  $y = 14.5x$ , where  $y$  is her earnings in dollars for working  $x$  hours.

The table shows some information about Jada's pay.

time worked (hours)	earnings (dollars)
7	92.75
4.5	59.63
37	490.25

Who is paid at a higher rate per hour? How much more does that person have after 20 hours?

In Clare's equation we see that the rate of change (how many dollars she earns every hour) is 14.50.

We can calculate Jada's rate of change by dividing a value in the earnings column by the value in the same row in the time worked column. Using the last row, the rate of change for Jada is  $13.25$ , since  $490.25 \div 37 = 13.25$ . An equation representing Jada's earnings is  $y = 13.25x$ . This means she earns \$13.25 per hour.

So Clare is paid at a higher rate than Jada. Clare earns \$1.25 more per hour than Jada. After 20 hours of work, she earns \$25 more than Jada because  $20 \cdot (1.25) = 25$ .

## Lesson 4 Practice Problems

### Problem 1

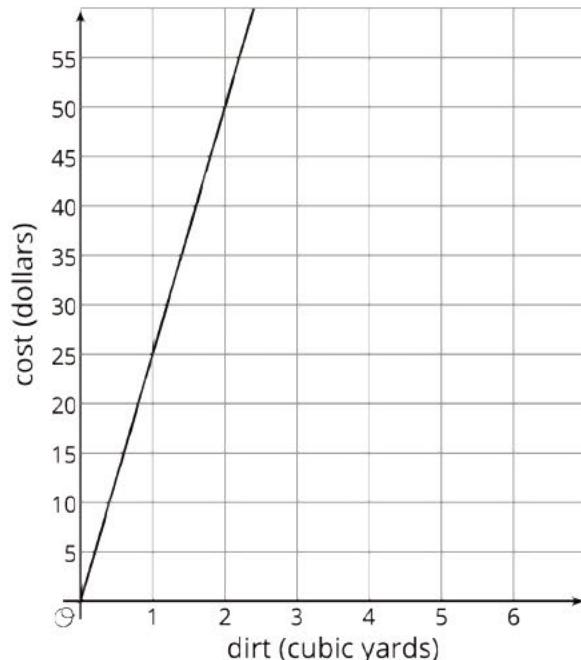
#### Statement

A contractor must haul a large amount of dirt to a work site. She collected information from two hauling companies.

EZ Excavation gives its prices in a table.

dirt (cubic yards)	cost (dollars)
8	196
20	490
26	637

Happy Hauling Service gives its prices in a graph.



- How much would each hauling company charge to haul 40 cubic yards of dirt? Explain or show your reasoning.
- Calculate the rate of change for each relationship. What do they mean for each company?
- If the contractor has 40 cubic yards of dirt to haul and a budget of \$1000, which hauling company should she hire? Explain or show your reasoning.

## Solution

- Assuming that both pricing plans are proportional relationships, EZ Excavation: \$980, Happy Hauling Service: \$1000.
- EZ Excavation: \$24.50/cu yd, Happy Hauling Service: \$25/cu yd.
- EZ Excavation. It would cost \$980 and be under budget.

## Problem 2

### Statement

Andre and Priya are tracking the number of steps they walk. Andre records that he can walk 6000 steps in 50 minutes. Priya writes the equation  $y = 118x$ , where  $y$  is the number of steps and  $x$  is the number of minutes she walks, to describe her step rate. This week, Andre and Priya each walk for a total of 5 hours. Who walks more steps? How many more?

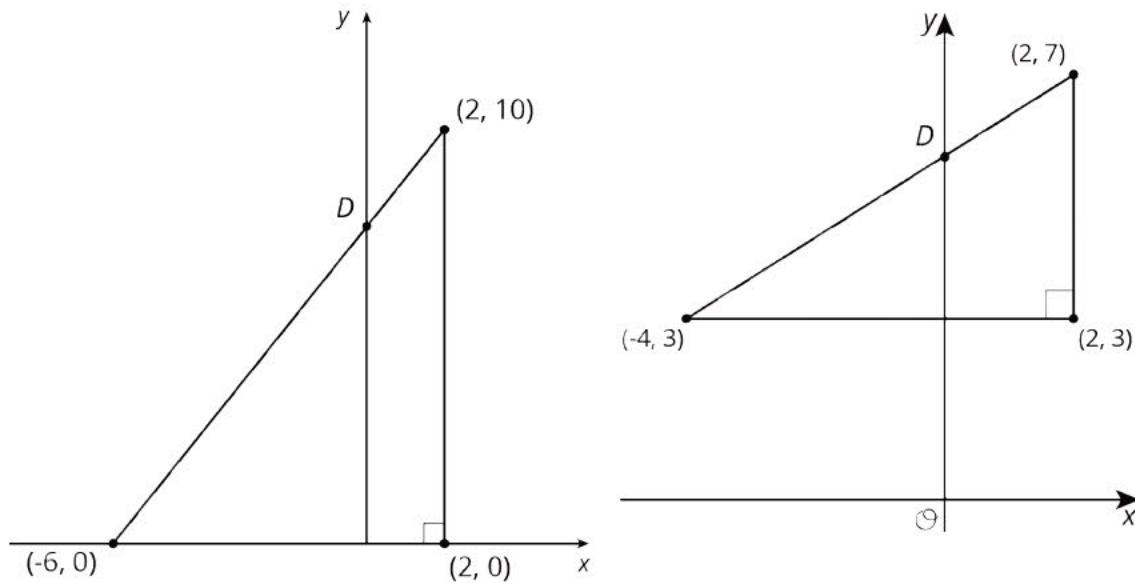
## Solution

Andre walks 600 more steps than Priya.

## Problem 3

### Statement

Find the coordinates of point  $D$  in each diagram:



### Solution

$$(0, 7\frac{1}{2}), (0, 5\frac{2}{3})$$

(From Unit 2, Lesson 11.)

## Problem 4

### Statement

Select **all** the pairs of points so that the line between those points has slope  $\frac{2}{3}$ .

- A.  $(0, 0)$  and  $(2, 3)$
- B.  $(0, 0)$  and  $(3, 2)$
- C.  $(1, 5)$  and  $(4, 7)$
- D.  $(-2, -2)$  and  $(4, 2)$
- E.  $(20, 30)$  and  $(-20, -30)$

## **Solution**

["B", "C", "D"]

(From Unit 2, Lesson 11.)

# Section: Representing Linear Relationships

## Lesson 5: Introduction to Linear Relationships

### Goals

- Compare and contrast (orally and in writing) proportional and non-proportional linear relationships.
- Interpret (orally and in writing) features of the graph (i.e., slope and  $y$ -intercept) of a non-proportional linear relationship.

### Learning Targets

- I can find the rate of change of a linear relationship by figuring out the slope of the line representing the relationship.

### Lesson Narrative

After revisiting examples of proportional relationships in the previous lessons, this lesson is the first of four lessons that moves from proportional relationships to **linear relationships** with positive rates of change. The two activities use a situation where the height of a stack of styrofoam cups is not proportional to the number of cups in the stack. Students use the same tools they learned to represent proportional relationships in this situation—graphs, tables, and equations. They see that each cup increases the height of the stack by the same amount (unit rate becomes rate of change) and that they can use this to answer questions about the height for an unknown number of cups. They investigate and describe similarities and differences between linear relationships and proportional relationships in this context. They make connections between the **rate of change** of the relationship and the slope of a line representing the relationship.

In this lesson, the focus is proportionality vs. linear relationships and rate of change. The meaning of the vertical intercept of the graph comes up briefly but will be revisited more fully in the next lesson.

The first two activities in this lesson use a particular type of cup. Photos are included of all measurements needed, so this lesson can be used without any additional preparation. However, if desired, the lesson could be modified so that students measure stacks of actual cups.

### Alignments

### Building On

- 6.NS.A: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- 7.RP.A.2.a: Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

## **Addressing**

- 8.EE.B: Understand the connections between proportional relationships, lines, and linear equations.

## **Instructional Routines**

- MLR2: Collect and Display
- MLR8: Discussion Supports
- Number Talk

## **Required Materials**

**Graph paper**

**Rulers**

## **Required Preparation**

If students will use actual cups, gather cups and perform activities ahead of time so that you know measurements for your particular type of cup.

### **Student Learning Goals**

Let's explore some relationships between two variables.

## **5.1 Number Talk: Fraction Division**

### **Warm Up: 5 minutes**

This number talk encourages students to think about the numbers in a computation problem and rely on what they know about structure, patterns, fractions, and division to mentally solve a problem. Only one problem is presented to allow students to share a variety of strategies for division. Encourage the students who solve the problem procedurally to think about the meaning of division and how that supports what they did.

### **Building On**

- 6.NS.A

## **Instructional Routines**

- MLR8: Discussion Supports
- Number Talk

## **Launch**

Display the problem for all to see. Give students 2 minutes of quiet think time and ask them to give a signal when they have an answer and a strategy. Follow with a whole-class discussion.

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### Support for Students with Disabilities

*Representation: Internalize Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.

*Supports accessibility for: Memory; Organization*

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### Student Task Statement

Find the value of  $2\frac{5}{8} \div \frac{1}{2}$ .

### Student Response

$5\frac{1}{4}$  or equivalent

Possible strategies:

- Multiply by the reciprocal:  $\frac{21}{8} \cdot 2$
- Distributive property:  $(2 \div \frac{1}{2}) + (\frac{5}{8} \div \frac{1}{2})$
- A visual solution that shows how many  $\frac{1}{2}$ 's are in  $2\frac{5}{8}$
- Long division:  $2.625 \div 0.5$

### Activity Synthesis

Invite students to share their strategies. Record and display student explanations for all to see. Ask students to explain how they thought about the division problem and, if solved procedurally, why that strategy works. To involve more students in the conversation, consider asking:

- “Who can restate \_\_\_’s reasoning in a different way?”
- “Did anyone solve the problem the same way but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to \_\_\_’s strategy?”
- “Do you agree or disagree? Why?” At the end of the discussion, if time permits, ask a few students to share a story problem for which the expression  $2\frac{5}{8} \div \frac{1}{2}$  represents the answer.

---

### Support for English Language Learners

*Speaking: MLR8 Discussion Supports.*.: Display sentence frames to support students when they explain their strategy. For example, "First, I \_\_\_\_ because ..." or "I noticed \_\_\_\_ so I ...." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

*Design Principle(s): Optimize output (for explanation)*

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## 5.2 Stacking Cups

**10 minutes**

In this task, students are presented with a situation that leads to a linear relationship that is not proportional because there is a non-zero starting amount. By trying to answer the question, "How many cups are needed to get to a height of 50 cm?" the students explore the rate of change, which is the increase per cup after the first cup. The rate of change can be seen in the graph as the slope. Students use representations and ideas from previous lessons on proportional relationships. They use tables and graphs to represent the situation and make deductions by generalizing from repeated reasoning (MP8), arguing that each additional cup increases the height of the stack by the same amount. As students are working, suggest that students make a graph or a table if they are stuck or if they have trouble explaining their reasoning. Students should be prepared to share their strategies with the class.

Students can attack this problem using any method. If they get stuck, here are some things it would be useful to figure out:

- How much height does each cup add?
- Is the first cup different from the others?
- Create a graph or table to help you reason about this problem.

As students work, identify students who use different strategies, i.e. graphs, tables, equations.

This activity and the next were written using a particular type of cup. Photos are included of all measurements needed, so these activities can be used without any additional preparation. However, if desired, the lesson could be modified so that students are measuring stacks of actual cups. The numbers then may have to be adjusted since the cups come in different sizes. Teachers who want to use real cups should measure the height of two stacks ahead of time, find the rate of change and make sure that it is approximately constant. Note that rounding error will likely play a role unless the number of cups in the stacks is chosen carefully, so some flexibility in answers may be necessary if students measure actual stacks of cups.

**Addressing**

- 8.EE.B

**Instructional Routines**

- MLR2: Collect and Display

**Launch**

Before students look at the picture with all of the given information, display just this part of the picture.



Ask students how many cups they see, and how tall the stack is. (There are 6 cups, and the stack is 15 cm tall.) Then, ask students to quietly make a prediction about the height of 12 cups and give you a silent signal when they have a response. When everyone has come up with an answer, ask a few students to share their responses and write these on the board. It is likely that some students will say 30 centimeters, and some might have an argument for why the height of 12 cups must be less than 30 centimeters. This question will be resolved as soon as students look at the photo in the task. Tell students that their job is to figure out how many cups would be needed in order to stack them to a height of 50 centimeters.

Arrange students in groups of 2–4. Give access to rulers and graph paper. Allow individual think time of 3 minutes before students work together in groups.

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### Support for Students with Disabilities

*Representation: Internalize Comprehension.* Begin the activity with concrete or familiar contexts. Set up a display of foam cups next to a ruler, as depicted. Encourage students to consider the rate of change and whether or not there is a proportional relationship.

*Supports accessibility for: Conceptual processing; Memory*

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### Support for English Language Learners

*Conversing, Representing, Writing: MLR2 Collect and Display.* As students discuss, listen for and collect the language students use to describe whether they think the relationship is proportional. Write the students' words and phrases on a visual display and update it throughout the remainder of the lesson. Remind students to borrow language from the display as needed. This will help students read and use mathematical language during their paired and whole-group discussions.

*Design Principle(s): Optimize output (for explanation); Maximize meta-awareness*

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### Anticipated Misconceptions

Students compute  $\frac{15}{6} = 2.5$  or  $\frac{23}{12} = 1.9$  and use this as the increase per cup. Measurement is approximate; students may be looking for exact, nice numbers. Make sure numbers agree approximately between students, but reassure them that they are close enough.

#### Student Task Statement

We have two stacks of styrofoam cups.

- One stack has 6 cups, and its height is 15 cm.
- The other stack has 12 cups, and its height is 23 cm.

How many cups are needed for a stack with a height of 50 cm?



### Student Response

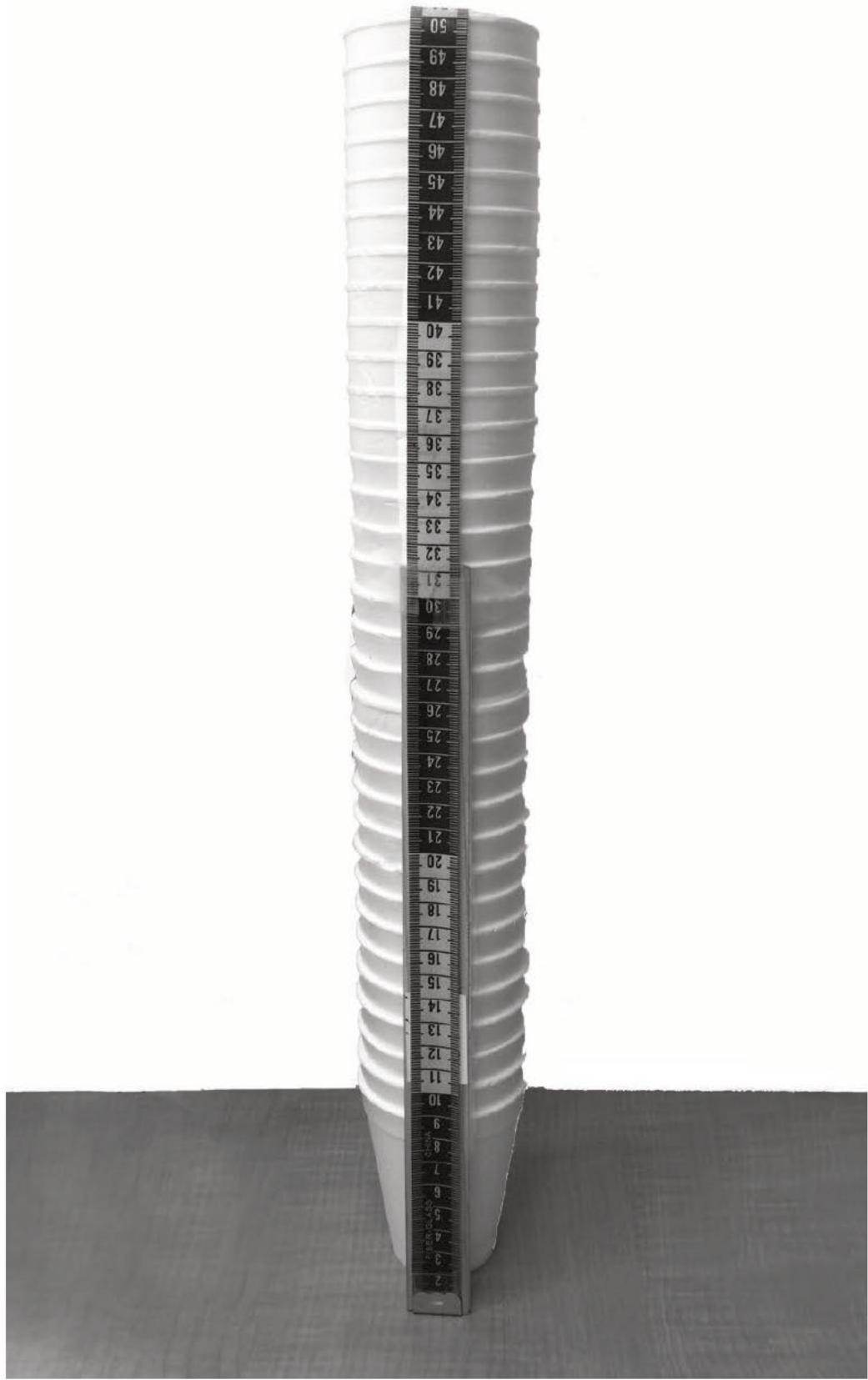
33 cups (32.25 cups get us to 50 cm height). Possible strategy:

number of cups	height in centimeters
6	15
12	23
18	31
24	39
30	47
33	51

### Activity Synthesis

Poll the class about the number of cups they came up with to reach a total height of 50 cm.

If the class used real cups, stack enough of them to get to a height of 50 cm, and let students check if they were correct. Otherwise display this photo:



Ask students who used different strategies to share their reasoning with the class.

Ask students if they think the relationship is proportional. If there is a disagreement, ask one person per opinion to share their reasons. Encourage them to come to the conclusion that it is not proportional since doubling the number of cups from 6 to 12 did not double the height. Students can see that when stacking the cups only the rim adds to the height. The first cup adds more height since there are additionally 7 cm from the bottom of the cup to the bottom of the rim.

Tell students that even though the relationship is not proportional, this relationship has things in common with proportional relationships, and that they will explore this in the next activity.

## 5.3 Connecting Slope to Rate of Change

**15 minutes**

The previous task asks students to estimate how many cups it will take to create a 50 cm stack. Students observe that the height of the stack grows regularly with the number of cups, but it is *not* a proportional relationship. In this task, they examine the relationship more closely, graph the relationship, and then interpret the graph.

Each successive cup, after the first, adds the same amount of height to the stack. This amount of height added per cup is the rate of change for the relationship between number of cups in a stack and height of the stack. It is also the slope of the line that represents this relationship. This makes sense because the slope of a line can be calculated by finding the amount of change in  $y$  (the height of the stack) when  $x$  (the number of cups in the stack) increases by 1.

Students also find where their graph intersects the  $y$ -axis and interpret this value in terms of the situation: it is the height of the bottom part of the first cup, below the rim (MP2). If this part of the cups was removed, then the graph would go through  $(0, 0)$ , and the relationship would be proportional because the stacks would just be stacks of the cup rims.

Monitor for students who recall work with slope triangles from the previous unit when they evaluate the slope of the line. A natural choice for a slope triangle in this case would be one with a horizontal side length that is a multiple of 3 and a vertical side length that is a multiple of 4 (the given data points would encourage a choice where the horizontal side length is 6 and the vertical side length is 8). As they interpret the meaning of the slope in the context, monitor for ration language, i.e. “the height added to the stack, in cm, *per* cup.”

### Building On

- 7.RP.A.2.a

### Addressing

- 8.EE.B

### Instructional Routines

- MLR8: Discussion Supports

## Launch

Keep students in the same groups. Remind students that the number of cups and the height of the stack is not a proportional relationship. Ask students how they determined the number of cups needed to make a stack 50 cm in height. A table is a likely choice and the table had some nice structure, structure which allowed them to effectively make calculations and predictions, even though it was not a ratio table. Tell them that in this task, they are to examine that structure more carefully using a graph.

Give students 8–10 minutes of group work time and follow with a whole-class discussion.

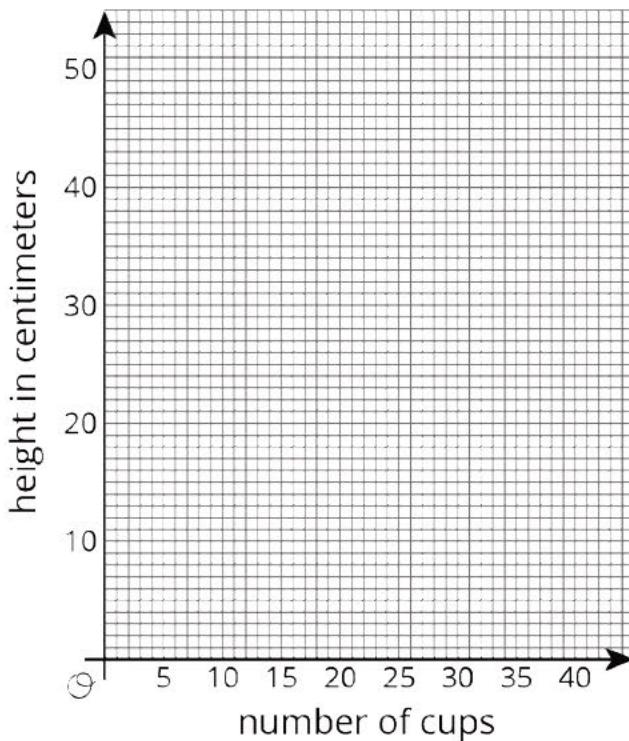
## Anticipated Misconceptions

Students may need a reminder of the meaning of slope for a line. Encourage them to begin by drawing a slope triangle.

Students may struggle with the question about the meaning of the  $y$ -intercept for the graph. Ask them to think about the cups. Ask them how much height is added with each cup after the first? What happens if you *subtract* this amount from the first cup?

### Student Task Statement

1. If you didn't create your own graph of the situation before, do so now.

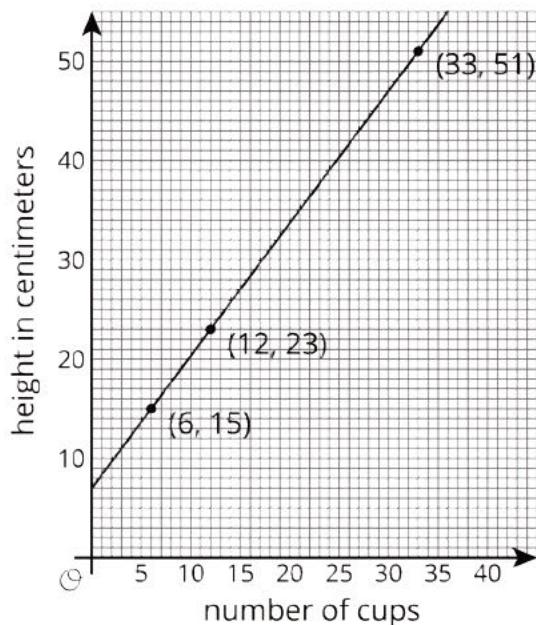


2. What are some ways you can tell that the number of cups is not proportional to the height of the stack?
3. What is the **slope** of the line in your graph? What does the slope mean in this situation?

4. At what point does your line intersect the vertical axis? What do the coordinates of this point tell you about the cups?

5. How much height does each cup after the first add to the stack?

### Student Response



1. See graph.
2. If the relationship were proportional, then the line would pass through the origin. Since the line does not pass through the origin, the relationship is not proportional. Proportional relationships have constant quotients of coordinate pairs:  $\frac{15}{6}$  is not equal to  $\frac{23}{12}$ . Or another way of looking at it, we can't scale:  $2 \cdot 6 = 12$  but  $2 \cdot 15 \neq 23$ .
3. The slope is  $\frac{4}{3}$ . Every cup (except for the first one) adds  $\frac{4}{3}$  cm to the height. Or every rim, including the first one, adds  $\frac{4}{3}$  cm to the height.
4. The line intersects the vertical axis at the point  $(0, 7)$ . If we have 0 cups, the height shows at 7 cm. This does not make sense if we think about the number of cups in the stack. If however, we notice that each cup has a rim, then 7 cm is the height from the bottom of the first cup to its rim.
5.  $\frac{4}{3}$  cm

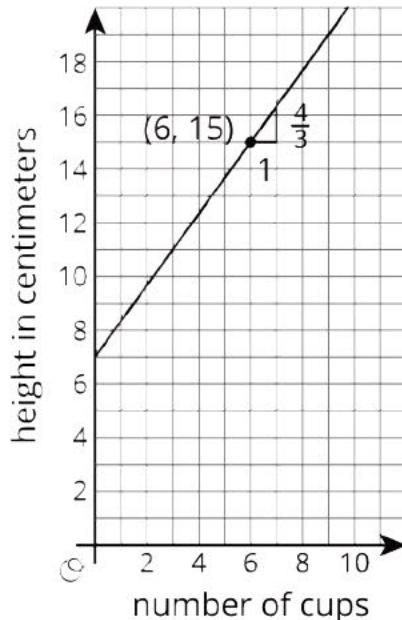
### Activity Synthesis

Begin by displaying a correct graph to support the discussion. Select students to use the graph and share:

- how they can tell the number of cups is not proportional to the height of the stack.

- how they calculated the slope of the line (on the grid, draw in any triangles used).
- what the point on the graph  $(0, 7)$  means. (It tells us the distance from the bottom to the rim of the first cup.)

Display the prompts “How can you see on the graph the amount that one cup adds to the height?” and give students 2–3 minutes of quiet work time to write a brief response. Select 3–4 students to share their responses. If not suggested by students, draw a slope triangle with a horizontal distance of 1 on the graph or display this image:



Draw a right triangle to see that each cup increases the height by  $\frac{4}{3}$  cm.

It is important to bring out here that the rate per 1,  $\frac{4}{3}$ , is not the constant of proportionality, since this is not a proportional relationship! This value is how much each cup adds to the height of the stack, and it is called the “rate of change.” The **rate of change** of  $y$  in a linear relationship between  $x$  and  $y$  is the increase in  $y$  when  $x$  increases by 1. Note that the rate of change of the relationship has the same value as the **slope** of the line representing the relationship. So asking “what is the slope?” is the same as asking “how much height does each cup add to the stack?”

Lastly, students’ graphs may consist either of discrete points corresponding to coordinate pairs (number of cups, height) or of the entire line as shown in the solution. It is understood that only the points that represent a whole number of cups have a valid interpretation in the context. This continuous graphical representation of a linear relationship, whether the context is continuous or discrete, is very common and will be seen throughout this unit.

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### Support for English Language Learners

*Speaking, Listening, Conversing: MLR8 Discussion Supports.* During the discussion, invite groups to use these sentence frames: “The number of cups is/is not proportional to the height of the stack because \_\_\_\_.” As partners share their thinking, press for details by asking questions such as: “How does the graph support your thinking?” and “Where do you see a constant rate of change?” while pressing for mathematical language. This will help students use the graph to make sense of the data and interpret the slope in context.

*Design Principle(s):* Support sense-making

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## Lesson Synthesis

The main focus of this lesson is the transition from proportional relationships to **linear relationships** that are not proportional:

- Understand that there are linear relationships that are not proportional.
- Note that the **rate of change** of the linear relationship is the same value as the **slope** of a line representing the relationship.
- Interpret the rate of change in the context of the situation.

In order to highlight this focus, ask students:

- “How can we tell if a linear relationship is proportional or not? From the graph? From a table? From the context?” (Check that when both variables are 0, this makes sense: on the graph, in the table, or in the context.)
- “What does the rate of change of a linear relationship tell us?” (The slope of the graph.)

## 5.4 Stacking More Cups

### Cool Down: 5 minutes

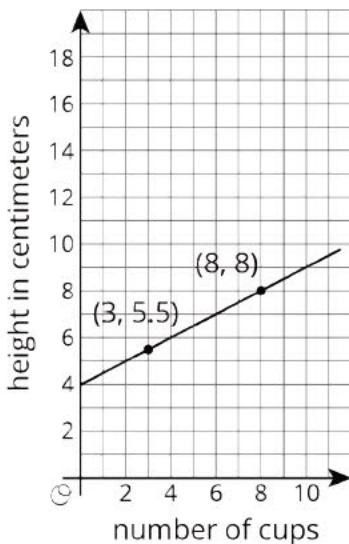
Students apply what they have learned about graphing a non-proportional relationship to another stack of cups that are different in size from those analyzed in the activities.

### Addressing

- 8.EE.B

### Student Task Statement

A shorter style of cup is stacked tall. The graph displays the height of the stack in centimeters for different numbers of cups. How much does each cup after the first add to the height of the stack? Explain how you know.

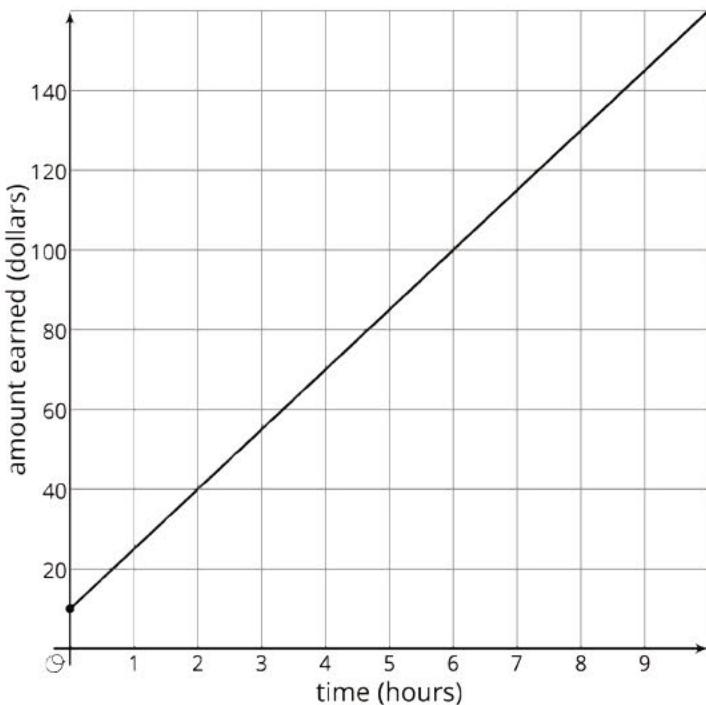


### Student Response

0.5 cm (or equivalent). Since 5 cups add 2.5 cm to the height of the stack, each cup adds 0.5 cm.

### Student Lesson Summary

Andre starts babysitting and charges \$10 for traveling to and from the job, and \$15 per hour. For every additional hour he works he charges another \$15. If we graph Andre's earnings based on how long he works, we have a line that starts at \$10 on the vertical axis and then increases by \$15 each hour. A **linear relationship** is any relationship between two quantities where one quantity has a constant **rate of change** with respect to the other.



We can figure out the rate of change using the graph. Because the rate of change is constant, we can take any two points on the graph and divide the amount of vertical change by the amount of horizontal change. For example, take the points  $(2, 40)$  and  $(6, 100)$ . They mean that Andre earns \$40 for working 2 hours and \$100 for working 6 hours. The rate of change is  $\frac{100-40}{6-2} = 15$  dollars per hour. Andre's earnings go up \$15 for each hour of babysitting.

Notice that this is the same way we calculate the **slope** of the line. That's why the graph is a line, and why we call this a linear relationship. The rate of change of a linear relationship is the same as the slope of its graph.

With proportional relationships we are used to graphs that contain the point  $(0, 0)$ . But proportional relationships are just one type of linear relationship. In the following lessons, we will continue to explore the other type of linear relationship where the quantities are not both 0 at the same time.

## Glossary

- linear relationship

## Lesson 5 Practice Problems

### Problem 1

#### Statement

A restaurant offers delivery for their pizzas. The total cost is a delivery fee added to the price of the pizzas. One customer pays \$25 to have 2 pizzas delivered. Another customer pays \$58 for 5 pizzas. How many pizzas are delivered to a customer who pays \$80?

#### Solution

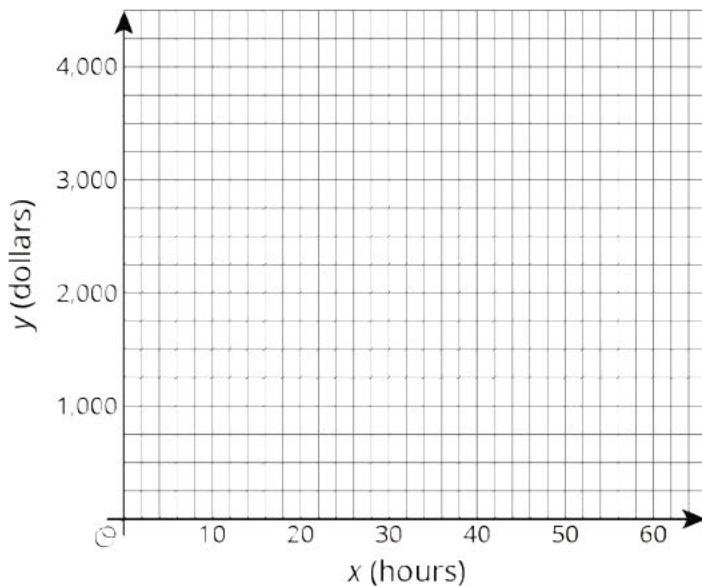
7 pizzas

## Problem 2

### Statement

To paint a house, a painting company charges a flat rate of \$500 for supplies, plus \$50 for each hour of labor.

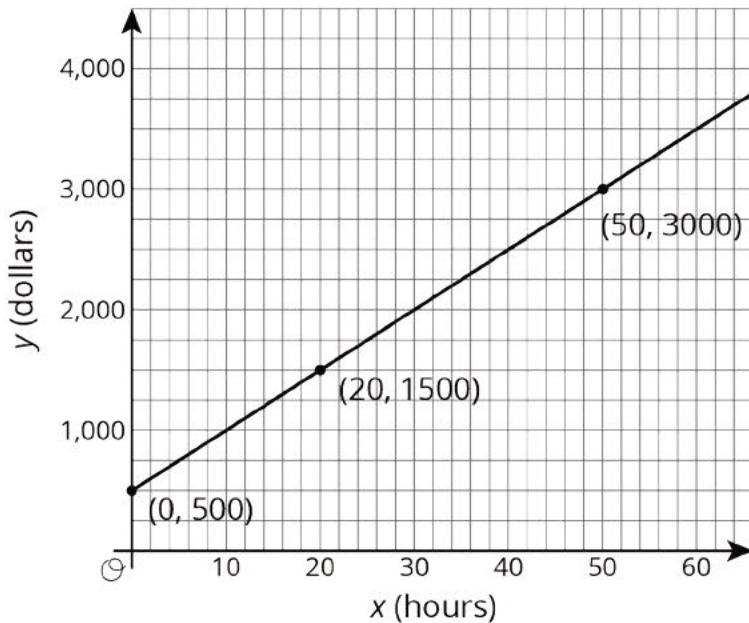
- a. How much would the painting company charge to paint a house that needs 20 hours of labor? A house that needs 50 hours?
- b. Draw a line representing the relationship between  $x$ , the number of hours it takes the painting company to finish the house, and  $y$ , the total cost of painting the house. Label the two points from the earlier question on your graph.



- c. Find the slope of the line. What is the meaning of the slope in this context?

### Solution

- a. \$1500, \$3000



b.

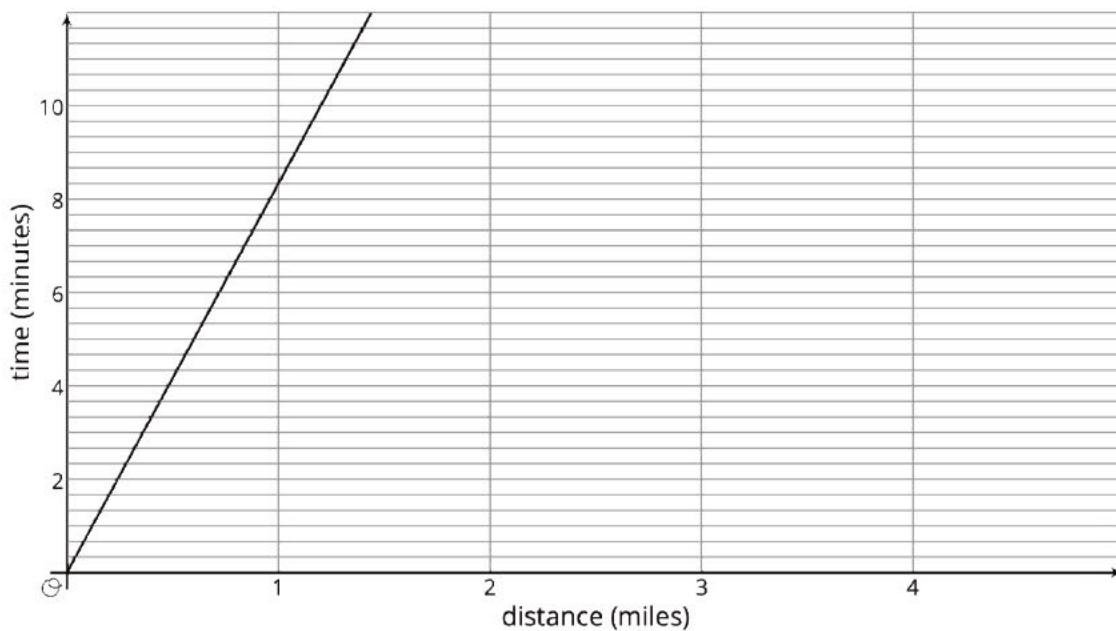
c. The slope of the line is 50, which is the same as the price per hour, in dollars, that the painting company charges for labor.

### Problem 3

#### Statement

Tyler and Elena are on the cross country team.

Tyler's distances and times for a training run are shown on the graph.



Elena's distances and times for a training run are given by the equation  $y = 8.5x$ , where  $x$  represents distance in miles and  $y$  represents time in minutes.

- a. Who ran farther in 10 minutes? How much farther? Explain how you know.
- b. Calculate each runner's pace in minutes per mile.
- c. Who ran faster during the training run? Explain or show your reasoning.

### **Solution**

- a. Tyler, who ran 2 hundredths of a mile farther.
- b. Tyler had a pace of  $8\frac{1}{3}$  minutes per mile, and Elena had a pace of 8.5 minutes per mile.
- c. Tyler ran faster because it took him fewer minutes to run a mile.

(From Unit 3, Lesson 4.)

### **Problem 4**

#### **Statement**

Write an equation for the line that passes through (2, 5) and (6, 7).

### **Solution**

$$\frac{y-5}{x-2} = \frac{1}{2} \text{ (or } y = \frac{1}{2}x + 4\text{, or equivalent)}$$

(From Unit 2, Lesson 12.)

# Lesson 6: More Linear Relationships

## Goals

- Describe (orally and in writing) how the slope and vertical intercept influence the graph of a line.
- Identify and interpret the positive vertical intercept of the graph of a linear relationship.

## Learning Targets

- I can interpret the vertical intercept of a graph of a real-world situation.
- I can match graphs to the real-world situations they represent by identifying the slope and the vertical intercept.

## Lesson Narrative

The previous lesson looked in depth at an example of a linear relationship that was not proportional and then examined an interpretation of the slope as the rate of change for a linear relationship. In this lesson, slope remains important. In addition, students learn the new term **vertical intercept** or *y*-intercept for the point where the graph of the linear relationship touches the *y*-axis.

In the first activity, students match situations to graphs and then interpret different features of the graph (slope and *y*-intercept) in terms of the situation being modeled (MP2). In the second activity, students analyze a common error, studying what happens when the slope and *y*-intercept are interchanged. This provides an opportunity to see how the *y*-intercept and slope influence the shape and location of a line: the *y*-intercept indicates where the line meets the *y*-axis while the slope determines how steep the line is.

Interpreting features of a graph or an equation in terms of a real-world context is an important component of mathematical modeling (MP4).

## Alignments

### Building On

- 5.OA.B.3: Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.
- 7.RP.A.2.a: Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

## Addressing

- 8.EE.B: Understand the connections between proportional relationships, lines, and linear equations.
- 8.EE.B.5: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

## Instructional Routines

- MLR6: Three Reads
- MLR7: Compare and Connect
- MLR8: Discussion Supports

## Required Materials

Pre-printed cards, cut from copies of the **blackline master**

## Required Preparation

Print and cut up slips from the Slopes, Vertical Intercepts, and Graphs blackline master. Prepare 1 set of cards for every 2 students.

### Student Learning Goals

Let's explore some more relationships between two variables.

## 6.1 Growing

### Warm Up: 5 minutes

This warm-up encourages students to look for regularity in how the number of tiles in the diagram are growing. This relates naturally to the work that they are doing with understanding the linear relationships as two of the three patterns students are likely to observe are linear and, in fact, proportional.

### Building On

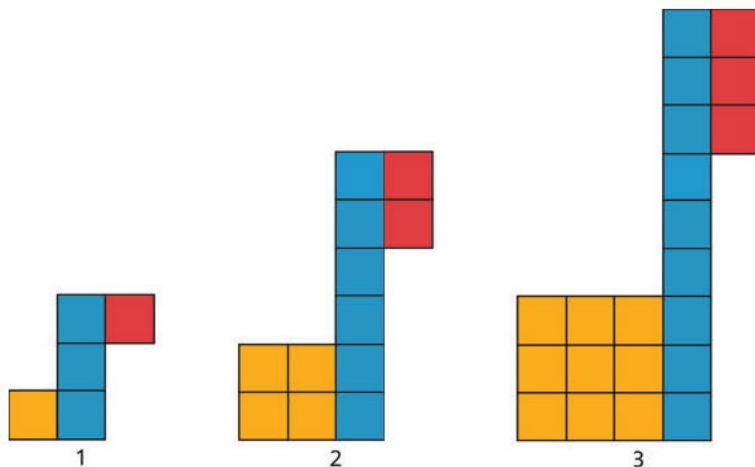
- 5.OA.B.3

### Launch

Arrange students in groups of 2. Display the image for all to see and ask students to look for a pattern in the way the collection of red, blue, and yellow tiles are growing. Ask how many tiles of each color will be in the 4th, 5th, and 10th diagrams if the diagrams keep growing in the same way. Tell students to give a signal when they have an answer and strategy. Give students 1 minute of quiet think time, and then time to discuss their responses and reasoning with their partner.

### Student Task Statement

Look for a growing pattern. Describe the pattern you see.



1. If your pattern continues growing in the same way, how many tiles of each color will be in the 4th and 5th diagram? The 10th diagram?
2. How many tiles of each color will be in the  $n$ th diagram? Be prepared to explain how your reasoning.

### Student Response

1. 4th diagram: 16 yellow, 12 blue, 4 red; 5th image: 25 yellow, 15 blue, 5 red; 10th image: 100 yellow, 30 blue, 10 red
2. Yellow:  $n^2$ , Blue:  $3n$ , Red:  $n$ .

### Activity Synthesis

Invite students to share their responses and reasoning. Record and display the different ways of thinking for all to see. If possible, record the relevant reasoning on or near the images themselves. After each explanation, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to what is happening in the images each time.

Time permitting, ask students which patterns represent a linear relationship? Which ones represent a proportional relationship? The patterns for the blue and red blocks are both proportional (hence linear) while the pattern for the yellow blocks is neither proportional nor linear.

## 6.2 Slopes, Vertical Intercepts, and Graphs

20 minutes

In the previous lesson, students analyzed the graph of a linear, non-proportional relationship (number of cups in a stack versus the height of the stack). This task focuses on interpreting the slope of a graph and where it crosses the  $y$ -axis in context. Students are given cards describing situations with a given rate of change and cards with graphs. Students match each graph with a situation it could represent, and then use the context to interpret the meaning of the slope. They

find where the line crosses the vertical axis, i.e., the **vertical intercept**, and interpret its meaning in each situation. They also decide if the two quantities in each situation are in a proportional relationship.

Make sure that students draw the triangle they use to compute the slope. There are strategic choices that can be made to make the computation easier and more precise. Watch for students who use different triangles for the same slope computation, and ask them to share their reasoning during the whole-group discussion.

In the whole-group discussion at the end of the task, discuss and emphasize the meaning of the terms slope and vertical intercept or  $y$ -intercept (in situations where the name of the variable graphed on the vertical axis is  $y$ ).

You will need the Slopes and Graphs blackline master for this lesson.

### **Building On**

- 7.RP.A.2.a

### **Addressing**

- 8.EE.B

### **Instructional Routines**

- MLR7: Compare and Connect

### **Launch**

Tell students that they will match a set of cards describing different relationships with a set of cards showing graphs of lines. The axes on the graphs are not labeled (since this could be used as an aid in the matching). Instruct students to add labels to the axes as they make their matches.

Arrange students in groups of 2. Distribute a set of 12 cards to each group. 10 minutes of group work and then whole-class discussion.

#### **Student Task Statement**

Your teacher will give you 6 cards describing different situations and 6 cards with graphs.

1. Match each situation to a graph.
2. Pick one proportional relationship and one non-proportional relationship and answer the following questions about them.
  - a. How can you find the slope from the graph? Explain or show your reasoning.
  - b. Explain what the slope means in the situation.
  - c. Find the point where the line crosses the vertical axis. What does that point tell you about the situation?

### Student Response

A:

1. Graph 2.
2.  $y$  increases by 10 when  $x$  increases by 1.
3. The slope is the cost per month of the streaming service.
4. The line crosses the vertical axis at  $(0, 40)$ . The tablet costs \$40.
5. Not proportional.

B:

1. Graph 6.
2. The graph passes through the points  $(0, 0)$  and  $(5, 20)$ .
3. Every increase of 1 in side length adds 4 to the perimeter.
4. Line crosses vertical axis at  $(0, 0)$ . The perimeter of a square of side length 0 is 0.
5. Proportional.

C:

1. Graph 1.
2.  $y$  increases by 5 when  $x$  increases by 1.
3. Diego puts in \$5 each week.
4. Line crosses vertical axis at  $(0, 10)$ . There initially was \$10 in the piggy bank.
5. Not proportional.

D:

1. Graph 3.
2.  $y$  increases by 15 when  $x$  increases by 1.
3. Noah adds \$15 each week.
4. Line crosses vertical axis at  $(0, 40)$ . He started with \$40 in the piggy bank.
5. Not proportional.

E:

1. Graph 5.
2.  $y$  increases by 0.25 when  $x$  increases by 1.

3. The amount of money she added per day was \$0.25.
4. Line crosses vertical axis at  $(0, 9)$ . There were initially \$9 in the piggy bank.
5. Not proportional.

F:

1. Graph 4.
2.  $y$  increases by 40 when  $x$  increases by 1.
3. Lin's mom pays \$40 each month for internet service.
4. Line crosses vertical axis at  $(0, 0)$ . Lin's mom paid no money before the contract started.
5. Proportional.

### Activity Synthesis

The slopes of the 6 lines given on the situation card are all different, so the matching part of the task can be accomplished by examining the slopes of the different lines. Invite students who have made strategic choices of slope triangles for calculating the slopes (for example, Graph 6 contains the points  $(0, 0)$  and  $(5, 20)$ , which give a value of  $\frac{20}{5}$  for the slope), and ask them to share.

Next, focus the discussion on the interpretation of the point where the line crosses (or touches) the  $y$ -axis. For some situations, the contextual meaning of this point is abstract. For example, in Situation B, a square with side length 0 is just a point that has no perimeter and so it "makes sense" that  $(0, 0)$  is on the graph. Some students may argue that a point is not a square at all but if we consider it to be a square then it definitely has 0 perimeter. In other situations, the point where the line touches the  $y$ -axis has a very natural meaning. For example, in Situation A, it is the amount Lin's dad spent on the tablet and 0 months of service: so this is the cost of the tablet.

Define the **vertical intercept** or  $y$ -intercept as the point where a line crosses the  $y$ -axis. Note that sometimes "y-intercept" refers to the numerical value of the  $y$ -coordinate where the line crosses the  $y$ -axis. Go over each of the situations and ask students for the meaning of the vertical intercept in the situation (A: cost of the device, B: perimeter of a square with 0 side length, C: amount of money in Diego's piggy bank before he started adding \$5 each week, D: money Noah saved helping his neighbor, E: amount of money in Elena's piggy bank before she started adding money, F: amount Lin's mom has paid for internet service before her service begins).

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### Support for Students with Disabilities

*Representation: Develop Language and Symbols.* Create a display of important terms and vocabulary. Include the following terms and maintain the display for reference throughout the unit: vertical intercept,  $y$ -intercept. Invite students to suggest language or diagrams to include on the display that will support their understanding of these terms.

*Supports accessibility for: Memory; Language*

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### Support for English Language Learners

*Representing, Conversing: MLR7 Compare and Connect.* As students share their matches with the class, call students' attention to the different ways the vertical intercept is represented graphically and within the context of each situation. Take a close look at Graphs 2 and 3 to distinguish what the 40 represents in each corresponding situation. Wherever possible, amplify student words and actions that describe the correspondence between specific features of one mathematical representation and a specific feature of another representation.

*Design Principle(s): Maximize meta-awareness; Support sense-making*

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## 6.3 Summer Reading

**10 minutes**

Students have just learned the meaning of the  $y$ -intercept for a line and have been interpreting slope in context. In this activity, they investigate the  $y$ -intercept and slope together and investigate what happens when their values are switched.

In contexts like this one, the  $y$ -intercept and the slope come with natural units and understanding this can help graph accurately. Specifically, the  $y$ -intercept is the number of pages Lin read before she starts gathering data for the graph. The slope, on the other hand, is a rate: it's the number of pages Lin reads *per day*.

Watch for students who understand the source of Lin's error (confusing the  $y$ -intercept with the slope) and invite them to share this observation during the discussion.

### Addressing

- 8.EE.B

### Instructional Routines

- MLR6: Three Reads
- MLR8: Discussion Supports

## Launch

Work time followed by whole-class discussion.

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### Support for Students with Disabilities

*Representation: Internalize Comprehension.* Use MLR6 Three Reads to support reading comprehension of the word problem. Use the first read to orient students to the situation. Ask students to describe what the situation is about without using numbers (Lin's reading assignment). Use the second read to identify quantities and relationships. Ask students what can be counted or measured without focusing on the values. Listen for and amplify the important quantities that vary in relation to each other in this situation: first 30 pages and 40 pages each day. After the third read, ask students to brainstorm possible strategies to answer the question, "What does the vertical intercept mean in this situation?"

*Supports accessibility for: Language; Conceptual processing*

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### Support for English Language Learners

*Speaking: MLR8 Discussion Supports.* Use this to amplify mathematical uses of language to communicate about vertical intercepts, slope, and constant rate. Invite students to use these words when describing their ideas. Ask students to chorally repeat phrases that include these words in context.

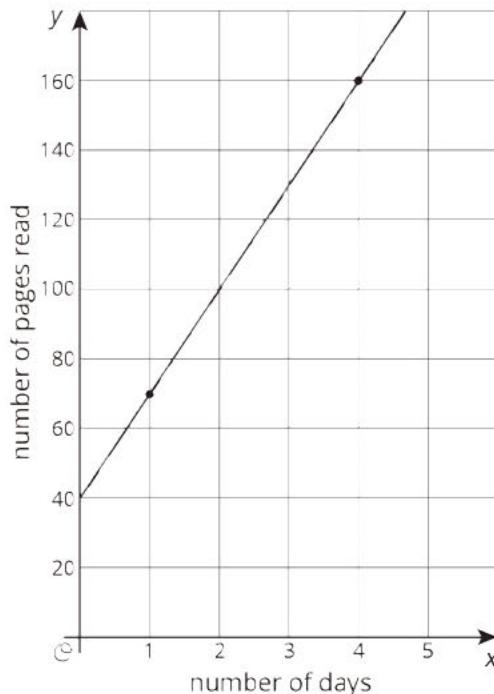
*Design Principle(s): Support sense-making, Optimize output (for explanation)*

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### Student Task Statement

Lin has a summer reading assignment. After reading the first 30 pages of the book, she plans to read 40 pages each day until she finishes. Lin makes the graph shown here to track how many total pages she'll read over the next few days.

After day 1, Lin reaches page 70, which matches the point  $(1, 70)$  she made on her graph. After day 4, Lin reaches page 190, which does not match the point  $(4, 160)$  she made on her graph. Lin is not sure what went wrong since she knows she followed her reading plan.



1. Sketch a line showing Lin's original plan on the axes.
2. What does the **vertical intercept** mean in this situation? How do the vertical intercepts of the two lines compare?
3. What does the slope mean in this situation? How do the slopes of the two lines compare?

### Student Response

1. The graph should start at  $(0, 30)$  since Lin read 30 pages before monitoring her progress each day. It should go through  $(1, 70)$ , because she reads 40 pages each day after the first. So the graph will also contain  $(2, 110)$ ,  $(3, 150)$ , etc.
2. The vertical intercept is the number of pages Lin has read before she monitors her progress. Lin's graph shows 40 pages but this is not accurate. She had only read 30 pages before beginning to track her progress.
3. The slope is the number of pages Lin reads per day (after she starts to record her progress). The slope of the line Lin drew is 30. This is not correct since she is reading 40 pages per day. The correct slope is 40.

### Are You Ready for More?

Jada's grandparents started a savings account for her in 2010. The table shows the amount in the account each year.

If this relationship is graphed with the year on the horizontal axis and the amount in dollars on the vertical axis, what is the vertical intercept? What does it mean in this context?

year	amount in dollars
2010	600
2012	750
2014	900
2016	1050

### Student Response

The vertical intercept corresponds to the amount in the year 0. The amount goes up by \$150 every two years, which is the same as \$75 per year. If you extend the graph backwards to the year zero, it goes down by  $2010 \times 75 = 150,750$  from its value in 2010, so the vertical intercept would be 600, the amount in 2010, minus 150,750, which is  $-150,150$ . You could think of this as a balance of the account in the year zero, but it doesn't really make sense to extend the graph back that far because the account was not open then. The intercept has no useful meaning in this context.

### Activity Synthesis

Ask students:

- "How did your graph compare to Lin's?" (It is steeper but starts off at 30 instead of 40 pages.)
- "Which point does your graph have in common with Lin's?" (The point  $(1, 70)$ , 70 pages read after one day.)

Invite selected students to share what's the likely source of Lin's error (confusing the  $y$ -intercept with the slope). In this context, the  $y$ -intercept is the number of pages Lin read before she starts counting the days (30), and the slope is the number of pages Lin reads per day (40).

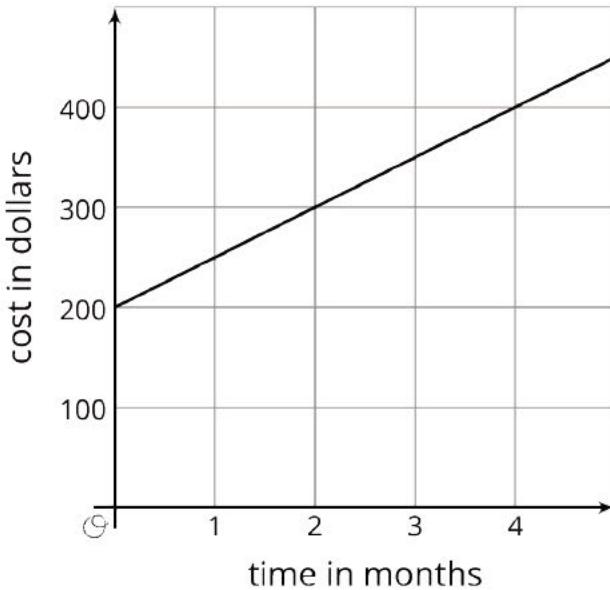
Emphasize how the  $y$ -intercept and slope influence the graph of a line.

- The  $y$ -intercept indicates where the line touches or crosses the  $y$ -axis.
- The slope indicates how steep the line is.

### Lesson Synthesis

Lines have a slope and **vertical intercept**. The vertical intercept indicates where the line meets the  $y$ -axis. For example, a line represents a proportional relationship when the vertical intercept is 0.

Here is a graph of a line showing the amount of money paid for a new cell phone and monthly plan:



- "What is the vertical intercept for the graph?" ((0, 200))
- "What does it mean?" (There was an initial cost of \$200 for the phone.)

The slope of the line is 50 (draw a slope triangle connecting the points such as (0, 200) and (2, 300)). This means that the phone service costs \$50 per month in addition to the initial \$200 for the phone.

## 6.4 Savings

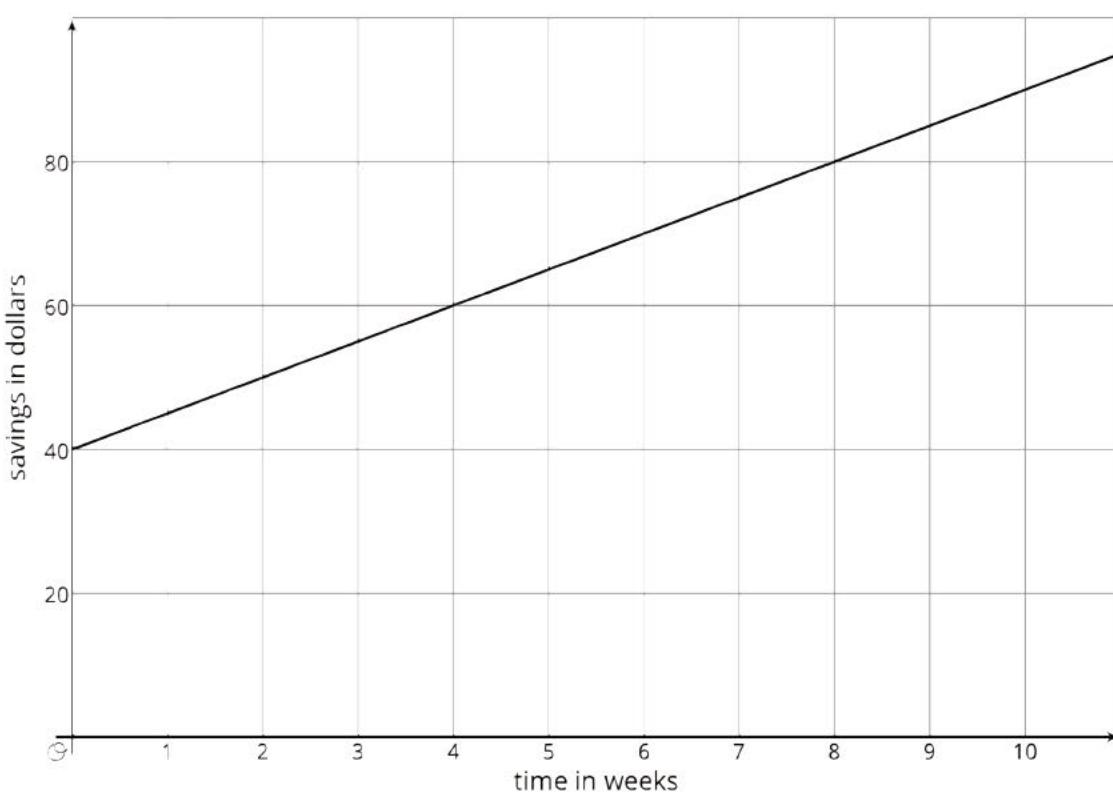
Cool Down: 5 minutes

### Addressing

- 8.EE.B.5

#### Student Task Statement

The graph shows the savings in Andre's bank account.



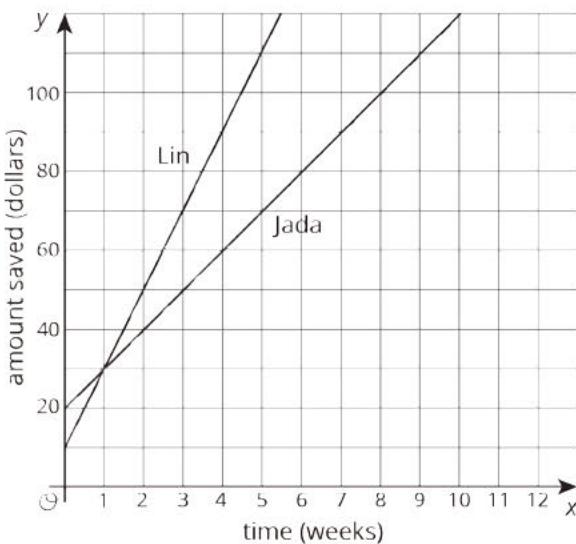
1. Explain what the slope represents in this situation.
2. Explain what the vertical intercept represents in this situation.

#### **Student Response**

1. The slope is 5 in this situation. That means that Andre saves 5 dollars every week.
2. The vertical intercept is 40. That means that Andre initially has \$40 in his bank account.

## Student Lesson Summary

At the start of summer break, Jada and Lin decide to save some of the money they earn helping out their neighbors to use during the school year. Jada starts by putting \$20 into a savings jar in her room and plans to save \$10 a week. Lin starts by putting \$10 into a savings jar in her room and plans to save \$20 a week. Here are graphs of how much money they will save after 10 weeks if they each follow their plans:



The value where a line intersects the vertical axis is called the **vertical intercept**. When the vertical axis is labeled with a variable like  $y$ , this value is also often called the  $y$ -*intercept*. Jada's graph has a vertical intercept of \$20 while Lin's graph has a vertical intercept of \$10. These values reflect the amount of money they each started with. At 1 week they will have saved the same amount, \$30. But after week 1, Lin is saving more money per week (so she has a larger rate of change), so she will end up saving more money over the summer if they each follow their plans.

## Glossary

- vertical intercept

## Lesson 6 Practice Problems

### Problem 1

#### Statement

Explain what the slope and intercept mean in each situation.

- A graph represents the perimeter,  $y$ , in units, for an equilateral triangle with side length  $x$  units. The slope of the line is 3 and the  $y$ -intercept is 0.
- The amount of money,  $y$ , in a cash box after  $x$  tickets are purchased for carnival games. The slope of the line is  $\frac{1}{4}$  and the  $y$ -intercept is 8.
- The number of chapters read,  $y$ , after  $x$  days. The slope of the line is  $\frac{5}{4}$  and the  $y$ -intercept is 2.

d. The graph shows the cost in dollars,  $y$ , of a muffin delivery and the number of muffins,  $x$ , ordered. The slope of the line is 2 and the  $y$ -intercept is 3.

## Solution

Answers vary. Sample responses:

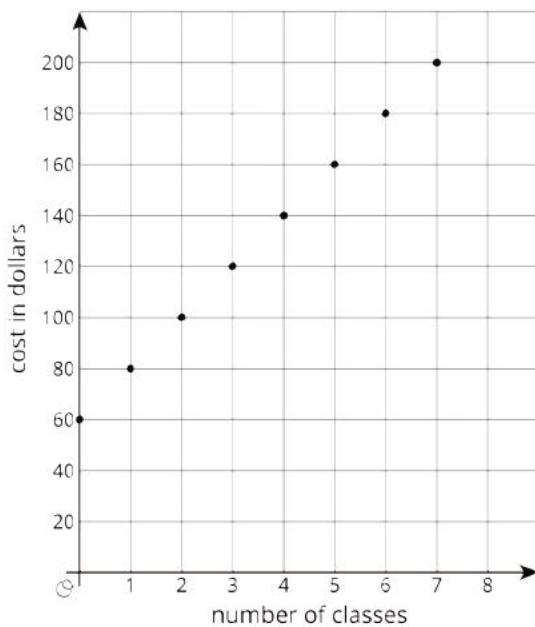
- The slope of 3 shows that the triangle has 3 sides. For each increase of 1 unit of the side length, the perimeter increases by 3 units. The intercept of 0 shows that the relationship is proportional—a triangle with sides of length 0 has a perimeter of length 0.
- The slope of  $\frac{1}{4}$  means that each ticket is \$0.25. The intercept of 8 represents the \$8 already in the cash box.
- The slope of  $\frac{5}{4}$  shows that 5 chapters are read every 4 days. The intercept might show that 2 chapters were read before beginning to read 5 chapters every 4 days, or it shows that an additional 2 chapters were read on the first day.
- The slope shows that \$2 are added for each muffin ordered. The intercept of 3 probably represents a \$3 delivery fee or tip for the order.

## Problem 2

### Statement

Customers at the gym pay a membership fee to join and then a fee for each class they attend. Here is a graph that represents the situation.

- What does the slope of the line shown by the points mean in this situation?
- What does the vertical intercept mean in this situation?



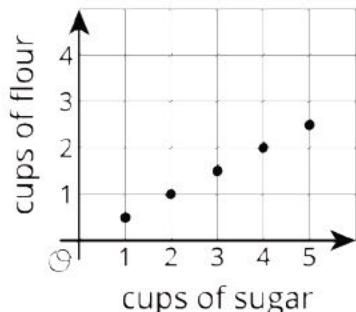
## Solution

- The cost for each class, which is \$20
- The membership fee to join, which is \$60

## Problem 3

### Statement

The graph shows the relationship between the number of cups of flour and the number of cups of sugar in Lin's favorite brownie recipe.



The table shows the amounts of flour and sugar needed for Noah's favorite brownie recipe.

cups of sugar	cups of flour
$\frac{3}{2}$	1
3	2
$4\frac{1}{2}$	3

- Noah and Lin buy a 12-cup bag of sugar and divide it evenly to make their recipes. If they each use all their sugar, how much flour do they each need?
- Noah and Lin buy a 10-cup bag of flour and divide it evenly to make their recipes. If they each use all their flour, how much sugar do they each need?

## Solution

- Lin: 3 cups, Noah: 4 cups
- Lin: 10 cups, Noah:  $7\frac{1}{2}$  cups

(From Unit 3, Lesson 4.)

# Lesson 7: Representations of Linear Relationships

## Goals

- Create an equation that represents a linear relationship.
- Generalize (orally and in writing) a method for calculating slope based on coordinates of two points.
- Interpret the slope and  $y$ -intercept of the graph of a line in context.

## Learning Targets

- I can use patterns to write a linear equation to represent a situation.
- I can write an equation for the relationship between the total volume in a graduated cylinder and the number of objects added to the graduated cylinder.

## Lesson Narrative

In this lesson, students develop an equation for a linear relationship by expressing regularity in repeated calculations (MP8). In an activity, students measure the volume of water in a graduated cylinder, repeatedly adding objects to the cylinder. Each additional object increases the volume of water by the same amount. They graph the relationship and interpret the initial water volume as the vertical intercept; they also interpret the slope as the rate of change, that is the amount by which the volume increases when one object is added.

In the second activity, students explicitly formulate a procedure to compute the slope of a line from *any* two points that lie on the line, including two different general points. They have been doing this since the previous unit, in order to calculate and make sense of slope as well as to find an equation satisfied by all points on a line. In this unit, students have continued to graph lines, draw slope triangles, and calculate slope; this time with an emphasis on understanding the slope as a rate of change: how much vertical displacement is there per unit of horizontal displacement? This lesson continues to focus on positive slopes; in future lessons, students will start to investigate non-positive slope values.

## Alignments

### Building On

- 5.MD.C.3: Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

### Addressing

- 8.EE.B: Understand the connections between proportional relationships, lines, and linear equations.

- 8.EE.B.6: Use similar triangles to explain why the slope  $m$  is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation  $y = mx$  for a line through the origin and the equation  $y = mx + b$  for a line intercepting the vertical axis at  $b$ .

### Building Towards

- 8.EE.B.6: Use similar triangles to explain why the slope  $m$  is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation  $y = mx$  for a line through the origin and the equation  $y = mx + b$  for a line intercepting the vertical axis at  $b$ .

### Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR2: Collect and Display

### Required Materials

#### Rulers

### Required Preparation

Students will work in groups of 2–3.

If doing the video presentation of spheres being added to cylinder, prepare the video for presentation.

If doing the 20 minute version of the water level task with an initial demonstration, you will need one graduated cylinder partially filled with water and 10 to 15 identical solid objects that fit into the cylinder and don't float (marbles, dice, cubes, or hardware items such as nuts or bolts).

If doing the 45 minute version of the water level task, each group will need one graduated cylinder partially filled with water. Each group will also need about 15 identical solid objects that fit into the cylinder and don't float. Determine a good initial water level and the approximate volume of each type of equally sized object.

### Student Learning Goals

Let's write equations from real situations.

## 7.1 Estimation: Which Holds More?

#### Warm Up: 5 minutes

This warm-up prompts students to estimate the volume of different glasses by reasoning about characteristics of their shape. As students discuss their reasoning with a partner, monitor the discussions and identify students who identified important characteristics of each of the glasses in their response.

### Building On

- 5.MD.C.3

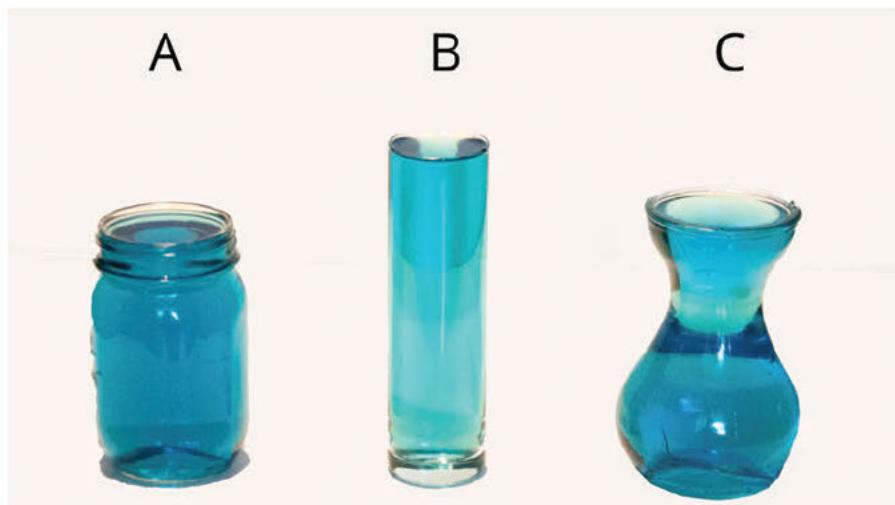
## Launch

Arrange students in groups of 2. Tell students they will be estimating which glass would hold the most liquid. Ask students to give a signal when they have an estimate and reasoning. Give students 1 minute of quiet think time followed by 2 minutes to discuss their estimates with a partner. Ask them to discuss the following questions, displayed for all to see:

- "What was important to you in the image when making your decision?"
- "What information would be helpful in finding the answer to this question?"

## Student Task Statement

Which glass will hold the most water? The least?



## Student Response

Answers vary.

## Activity Synthesis

For each glass, ask students to indicate if they think it holds the most or least amount of water. Invite a few students to share their reasoning and the characteristics of the object that were important in making their decision. After each explanation, solicit from the class questions that could help the student clarify his or her reasoning. Record the characteristics, and display them for all to see. If possible, record these characteristics on the images themselves during the discussion.

If there is time, discuss the information they would need to find the answer to the question accurately. Note that no measurements need to be taken to answer the question; to compare the volume of two containers, it is enough to pour the liquid from one into the other.

It turns out that B holds the least and A and C hold the same. If the right technology is available, consider showing the answer video.

Video 'Glasses' available here: <https://player.vimeo.com/video/304138068>.

## 7.2 Rising Water Levels

**20 minutes (there is a digital version of this activity)**

The goal of this task is to analyze a linear relationship for data gathered in context. Students examine data gathered by successively submerging equal-size objects in a graduated cylinder, partially filled with water, and then measuring the level of the water. Here and in the sample solutions, we use a 100 ml graduated cylinder, 60 ml as the initial amount of water, and number cubes whose volume is approximately 3.7 ml each. Your measurements may be different. You will want to make sure to leave enough space so many number cubes can be added before the water reaches the top (but not too much space as the number cubes need to all be submerged in the water).

After gathering the data, students plot it and notice that it lies on a line, up to measurement error. They estimate the slope using slope triangles and then interpret the meaning of the  $y$ -intercept and the slope in terms of the context. With  $x$  number cubes and an initial amount of 60 ml in the cylinder,  $60 + 3.7x$  is an expression giving the level of the water:

- The number 60 represents the initial amount of water in the cylinder.
- 3.7 is the volume per number cube in ml (and the rate of change in water level).
- $x$  represents how many number cubes have been added to the cylinder.

There are two versions of this task, one for a shorter 20 minute time and one for a longer time (40–45 minutes). The difference is in how the data is gathered. For the shorter time, the teacher performs a demonstration, adding dice to the cylinder, and then the whole class works with this data. For the second, longer version, students gather their own data.

Note that because of measurement error the data do not lie *exactly* on a line, and different slope triangles may lead to slightly different values for the slope. Monitor for students who get slightly different values, and invite them to share during the discussion.

### Addressing

- 8.EE.B

### Instructional Routines

- MLR2: Collect and Display

### Launch

Tell students, "Have you ever noticed that when you put ice cubes in your drink, the level of the liquid goes up? Today, we want to investigate what happens when we drop objects into a container with water."

For the 20 minute version, begin with a demonstration, either using the applet with the digital version of the task or a physical demonstration. For a physical demonstration, consider measuring the volume after putting in 1, 2, 5, 8, and 10 dice. Record the measurements for all to see or choose students to do so. After students have the information for the table, they work in small groups to complete the activity.

For the 45 minute version, distribute materials to each group:

- 1 graduated cylinder.
- 15 identical solid objects that fit into the cylinder and have a higher density than water and don't float (marbles, dice, cubes, hardware items such as nuts or bolts, etc.). Tell students how much water to put initially into their cylinders. Groups spend 10 minutes conducting the experiment. The analysis can be done as a combination of class discussion and student work.

For classrooms without materials to create the lab, there is a digital activity to recreate the experience. The applet might also be useful to show classes before they begin the lab and after they have predictions. Adapted from an applet made in GeoGebra by [John Golden](#).

Arrange students in groups of 2-3.

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### Support for Students with Disabilities

*Action and Expression: Provide Access for Physical Action.* Provide access to tools and assistive technologies. If possible, allow students to use the applet for this activity to facilitate observing the impact of adding marbles to the cylinder.

*Supports accessibility for: Visual-spatial processing; Conceptual processing; Organization*

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### Support for English Language Learners

*Conversing, Representing, Writing: MLR2 Collect and Display.* Use of the longer version of this task, where students gather their own data, will increase opportunities for student discourse. As students collect their data, listen for and collect vocabulary and phrases students use to describe what happens to the water level as they add items. Amplify phrases that relate to volume, rate, slope, and vertical intercepts. Remind students to borrow language from the display as needed as they complete the follow-up questions. This will help students use mathematical language during their paired discussions and in their written work.

*Design Principle(s): Optimize output (for justification); Maximize meta-awareness*

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### Anticipated Misconceptions

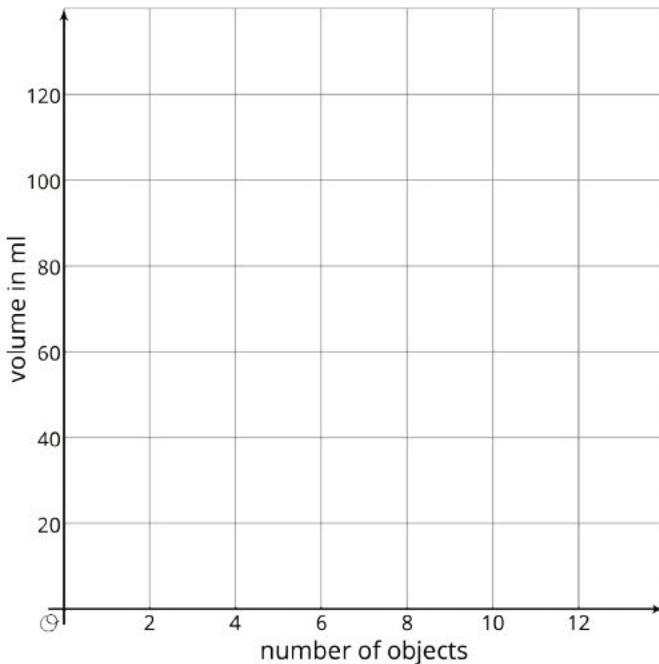
Students may think the marks on the cylinder indicate the height of the water. Milliliters are a measure of volume. However, in a cylinder, the height is proportional to the volume, so it does

make sense to measure the height. But then the objects will be measured by what height of water they displace, not volume.

### Student Task Statement

1. Record data from your teacher's demonstration in the table. (You may not need all the rows.)
2. What is the volume,  $V$ , in the cylinder after you add  $x$  objects? Explain your reasoning.
3. If you wanted to make the water reach the highest mark on the cylinder, how many objects would you need?
4. Plot and label points that show your measurements from the experiment.
5. The points should fall on a line. Use a ruler to graph this line.
6. Compute the slope of the line. What does the slope mean in this situation?
7. What is the vertical intercept? What does vertical intercept mean in this situation?

number of objects	volume in ml



### Student Response

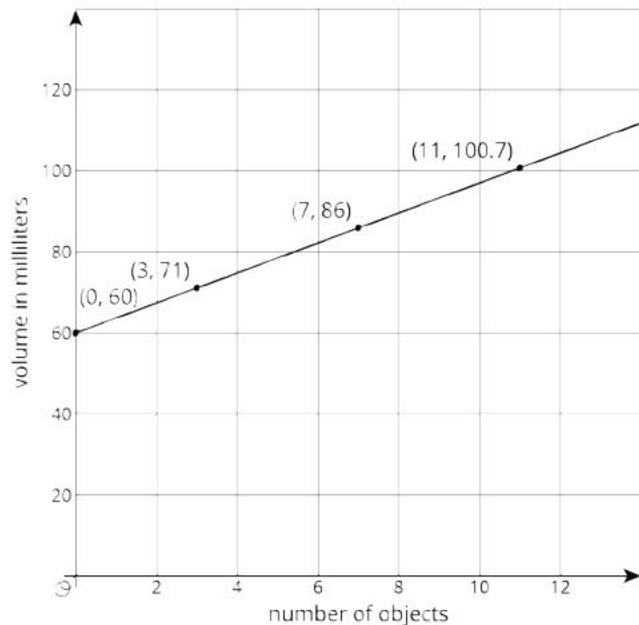
1. This narrative uses a 100 ml graduated cylinder, 60 ml as the initial amount of water, and dice whose volume was approximately 3.7 ml each. Your measurements may be different.

number of objects	volume in ml
3	71
7	86
11	100.7

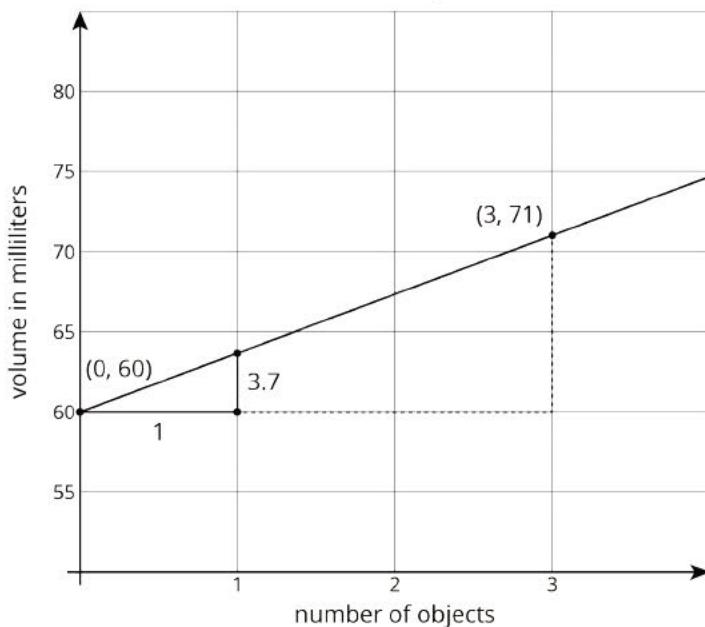
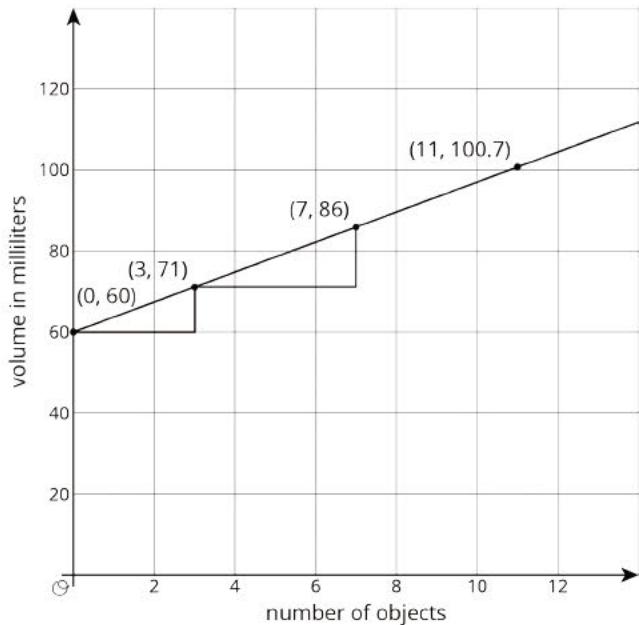
2.  $60 + (3.7) \cdot x$  ml. Since each number cube adds about 3.7 ml of volume, start with 60 and add 3.7 ml for each object.

3. With 11 number cubes, the water overflows a little bit. With 10 number cubes, it is not yet to the top.

4, 5 Graph:



We can use any two points on the graph to compute the slope since all slope triangles are similar and therefore the quotient (vertical side length)/(horizontal side length) is the same for all of them. In particular, for a triangle with horizontal side length of 1, the vertical side length is the rate of change. Each additional object added will increase the volume by the same amount.



The vertical intercept is 60. It means that there were 60 ml of water in the container before any objects were added.

### Are You Ready for More?

A situation is represented by the equation  $y = 5 + \frac{1}{2}x$ .

1. Invent a story for this situation.
2. Graph the equation.
3. What do the  $\frac{1}{2}$  and the 5 represent in your situation?

4. Where do you see the  $\frac{1}{2}$  and 5 on the graph?

### Student Response

Answers vary. Possible response:

1. When you plant a tree in your backyard, it is 5 ft tall. It grows  $\frac{1}{2}$  foot every year after that. The total height of the tree is  $y$  ft after  $x$  years since the tree was planted.
2. A graph showing  $(x, y)$  with a vertical intercept of 5, slope of 0.5.
3. The  $\frac{1}{2}$  is the rate of growth of the tree in ft per year, the tree grows  $\frac{1}{2}$  ft every year. The 5 is the initial height of the tree; when it was planted, the tree was 5 ft tall.
4. For every one unit increase in  $x$ , the line rises by  $\frac{1}{2}$  unit in  $y$ , so the slope of the line is  $\frac{1}{2}$ . The vertical intercept is 5. The line crosses the vertical axis at  $(0, 5)$ , when the tree was planted, i.e., at 0 years, the height was 5 ft.

### Activity Synthesis

Display the equation  $V = 3.7x + 60$  for all to see. Note: if you used a different amount of water or objects with a different volume, display the equation corresponding to your class data. Elicit understanding of each part of this equation by asking these questions:

- "How does this equation represent the situation we saw today?" (This is the equation for the line we graphed of the cylinder with the number cubes being added.)
- "What do the variables represent?" ( $V$  is the total volume in the cylinder in ml.  $x$  is the number of objects added to the cylinder.)
- "Where do you see rate of change in this equation? What does it mean in this situation?" (3.7 is the volume of each object in ml.)
- "What does the number 60 represent?" (60 is the initial amount of water in the cylinder in ml.)

In other words, the equation  $V = 3.7x + 60$  can be interpreted as saying

$$\text{total volume} = (\text{3.7 ml per object}) * (\text{number of objects}) + \text{initial volume}$$

In terms of the graph, 3.7 is the slope and 60 is the vertical intercept.

It might be necessary to discuss the precision of the computed slope. It makes sense that every object increases the volume by the same amount (because the dice, for example, all have the same size). This amount is approximately 3.7 ml for the dice shown in the pictures. For smaller objects or graduated cylinders with bigger diameters, it may be difficult to accurately measure the change for 3 objects. If so, have students add objects in larger sets. The slope calculation will tell the amount of change for one object.

Ignoring precision issues, we can use any two points on the graph to compute the slope since all slope triangles are similar and therefore (vertical side length)/(horizontal side length) is the same for all of them. In particular, for a triangle with horizontal side length of 1, the vertical side length is the

rate of change (3.7 ml per object for our example)—each additional object added will increase the volume by this same amount.

## 7.3 Calculate the Slope

**10 minutes**

The purpose of this task is to develop a quick method or formula to calculate the slope of a line based on the coordinates of two points. It is not critical or even recommended to use the traditional formula invoking subscripts. The goal is for students to realize that they can calculate the vertical and horizontal side lengths of the slope triangle without a grid (eventually without even drawing the slope triangle) and that the slope is the quotient of these side lengths.

After students compute the slopes of three specific lines they generalize the procedure. Students are asked to express regularity in repeated reasoning (MP8) to both describe a procedure in words and then as an algebraic expression. It is more important that students know a technique or way of thinking about it that works for them than it is that they memorize a particular way to express a formula.

### Building Towards

- 8.EE.B.6

### Instructional Routines

- MLR1: Stronger and Clearer Each Time

### Launch

Quiet work time followed by a class discussion.

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### Support for Students with Disabilities

*Representation: Internalize Comprehension.* Encourage and support opportunities for peer interactions. After students complete the table, invite them to compare solutions as well as their procedure for finding slope with a partner. Once students agree on procedure, they can write it down and continue with the task.

*Supports accessibility for: Conceptual processing; Social-emotional skills*

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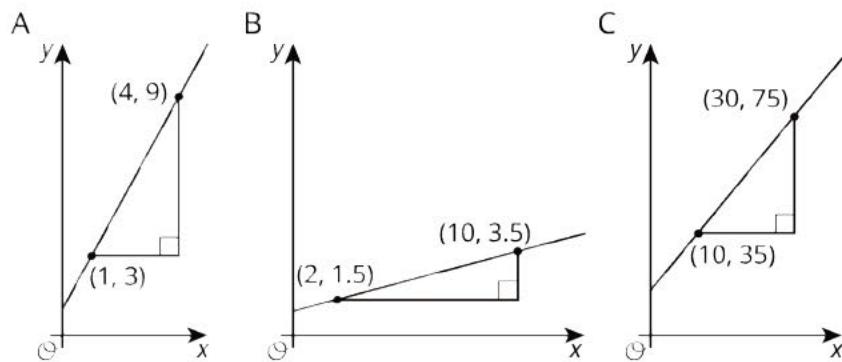
### Support for English Language Learners

*Writing, Speaking: MLR1 Stronger and Clearer Each Time.* Use this routine to give students a structured opportunity to revise and refine their response to “Describe a procedure for finding the slope between any two points on a line.” Ask each student to meet with 2–3 other partners in a row for feedback. Provide listeners with prompts for feedback that will help students strengthen their ideas and clarify their language (e.g., “Can you explain how...?” and “Will your procedure always work?”, etc.). Students can borrow ideas and language from each partner to strengthen the final product. This will help students use more precise language and solidify their process for finding the slope between any two points on a line.

*Design Principle(s): Optimize output (for generalization)*

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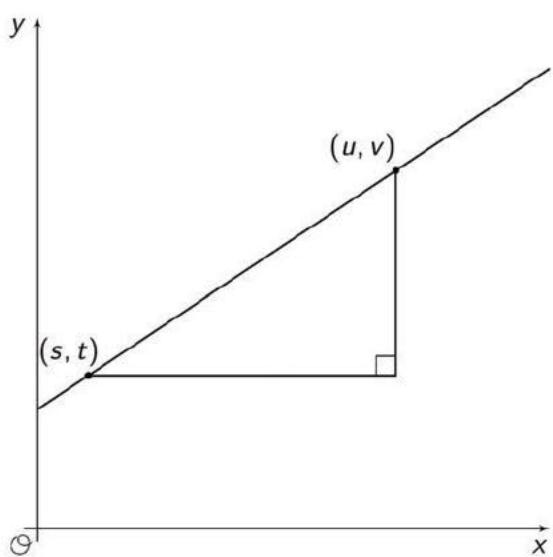
#### Student Task Statement



1. For each graph, record:

vertical change	horizontal change	slope

2. Describe a procedure for finding the slope between any two points on a line.
3. Write an expression for the slope of the line in the graph using the letters  $u$ ,  $v$ ,  $s$ , and  $t$ .



### Student Response

1.

	vertical change	horizontal change	slope
	6	3	2
	2	8	$\frac{1}{4}$
	40	20	2

2. Divide the vertical change between the two points by the horizontal change between the two points.

3. In general:

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{v - t}{u - s}$$

### Activity Synthesis

Ask students to share solutions to the first three problems, and ensure that everyone understands why the correct answers are correct. There is no requirement that students simplify fractions; a student who comes up with  $\frac{40}{20}$  for graph C is correct. The convention of simplifying a fraction can be especially helpful if further calculations need to be made (or in this case, to give an immediate sense of the size of the number).

Invite students to share the procedure they came up with. Ideally, they will share several versions of "Subtract the  $y$ -coordinates, subtract the  $x$ -coordinates, and then divide the difference in  $y$ 's by the difference in  $x$ 's." Note that it is important to subtract the  $x$ -coordinates for the two points *in the same order* as the  $y$ -coordinates, that is  $\frac{9-3}{4-1}$  for graph A, not  $\frac{9-3}{1-4}$ .

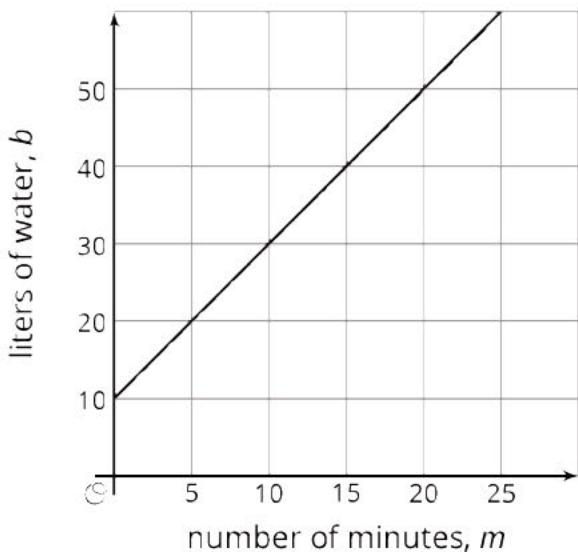
For the last question, invite students to share the expressions they came up with. Acknowledge any response that is equivalent to the correct answer, but be on the lookout for expressions like  $v - t \div u - s$ , or  $(v - t) \div (s - u)$ , which are incorrect for different reasons.

## Lesson Synthesis

Ask students to create and interpret an equation for a situation with linear growth. For example, imagine you have a bucket of water that already contains 10 liters of water and you turn on the water faucet, which adds 2 liters of water every minute. Ask students:

- “Can we use a linear equation to represent this situation?” (Yes.)
- “Why or why not?” (Each minute, 2 more liters of water are added to the bucket; the rate of change is constant.)
- “What is the equation?” ( $b = 10 + 2m$  where  $b$  is the number of liters of water in the bucket and  $m$  is the number of minutes since you turned on the faucet.)

Sketch a graph of the line  $b = 10 + 2m$ .



Ask students the meaning of 10 from the equation and where they see it on the graph. (It's the  $y$ -intercept, and it is how many liters of water were in the bucket at the beginning.) Ask the meaning of 2 from the equation and where they see it on the graph. (It's the slope. A slope triangle with horizontal side length 1 will have vertical side length 2.)

We can find the slope of a line using *any* two points on the line. We use the coordinates of the two points to find the vertical and horizontal side lengths of the slope triangle. The slope is the quotient of the vertical and horizontal lengths. Label two general points  $(a, b)$  and  $(x, y)$  on the triangle. The vertical side has length  $y - b$ , and the horizontal side has length  $x - a$  so  $\frac{y-b}{x-a} = 2$ .

## 7.4 Graphing a Line

Cool Down: 5 minutes

### Addressing

- 8.EE.B.6

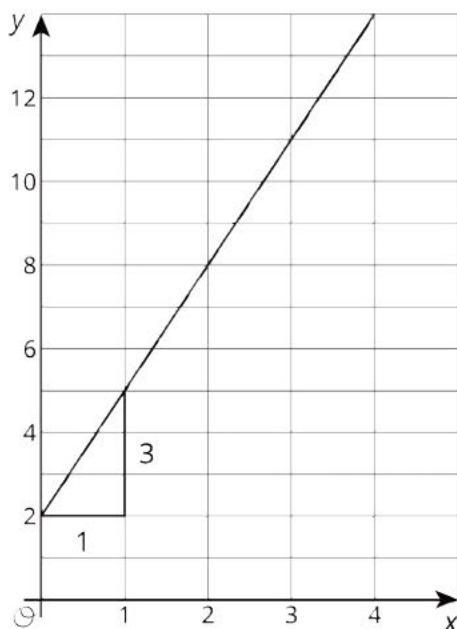
### Student Task Statement

Make a sketch of a linear relationship with slope of 3 that is not a proportional relationship.

Show how you know that the slope is 3. Write an equation for the line.

### Student Response

Answers vary. Sample response:



The slope triangle has vertical side length 3 and horizontal side length 1, so the slope of the line is 3.

$$y = 3x + 2$$

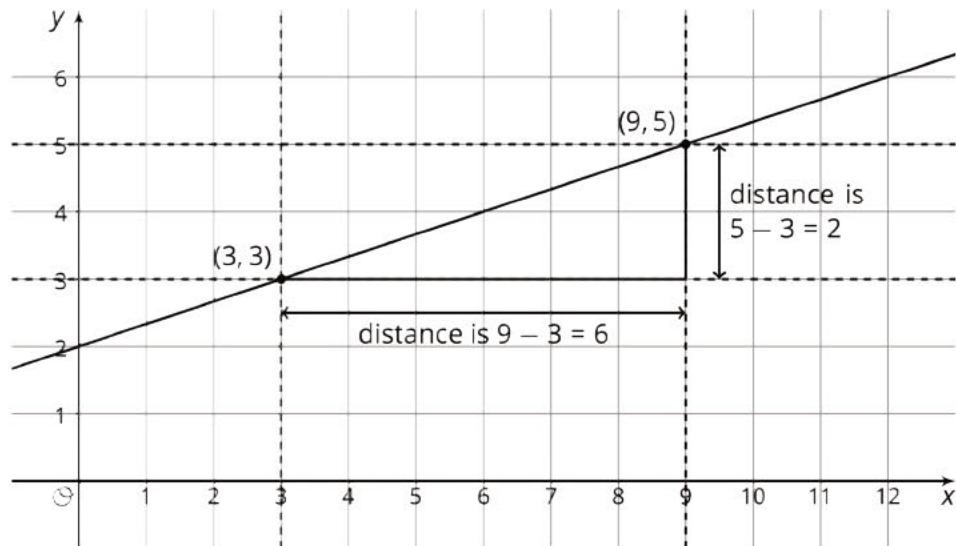
### Student Lesson Summary

Let's say we have a glass cylinder filled with 50 ml of water and a bunch of marbles that are 3 ml in volume. If we drop marbles into the cylinder one at a time, we can watch the height of the water increase by the same amount, 3 ml, for each one added. This constant rate of change means there is a linear relationship between the number of marbles and the height of the water. Add one marble, the water height goes up 3 ml. Add 2 marbles, the water height goes up 6 ml. Add  $x$  marbles, the water height goes up  $3x$  ml.

Reasoning this way, we can calculate that the height,  $y$ , of the water for  $x$  marbles is  $y = 3x + 50$ . Any linear relationships can be expressed in the form  $y = mx + b$  using just the

rate of change,  $m$ , and the initial amount,  $b$ . The 3 represents the rate of change, or slope of the graph, and the 50 represents the initial amount, or vertical intercept of the graph. We'll learn about some more ways to think about this equation in future lessons.

Now what if we didn't have a description to use to figure out the slope and the vertical intercept? That's okay so long as we can find some points on the line! For the line graphed here, two of the points on the line are  $(3, 3)$  and  $(9, 5)$  and we can use these points to draw in a slope triangle as shown:



The slope of this line is the quotient of the length of the vertical side of the slope triangle and the length of the horizontal side of the slope triangle. So the slope,  $m$ , is

$$\frac{\text{vertical change}}{\text{horizontal change}} = \frac{2}{6} = \frac{1}{3}$$
. We can also see from the graph that the vertical intercept,  $b$ , is 2.

Putting these together, we can say that the equation for this line is  $y = \frac{1}{3}x + 2$ .

## Lesson 7 Practice Problems

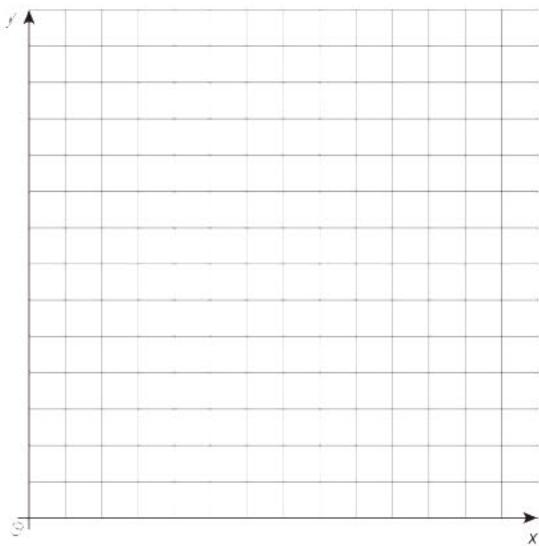
### Problem 1

#### Statement

Create a graph that shows three linear relationships with different  $y$ -intercepts using the following slopes, and write an equation for each line.

Slopes:

- $\frac{1}{5}$
- $\frac{3}{5}$
- $\frac{6}{5}$



## Solution

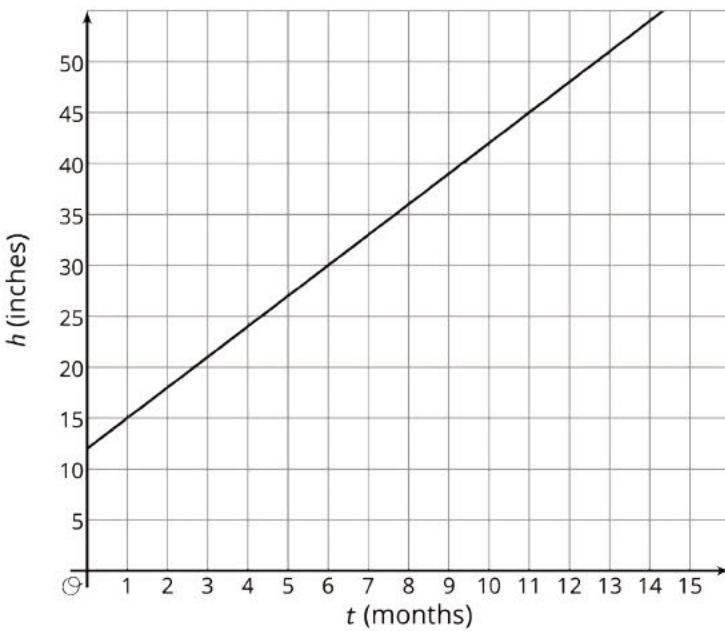
Answers vary. Sample response: Three graphs, one a line through  $(0, 2)$  and  $(5, 3)$ , one a line through  $(0, 3)$  and  $(5, 6)$ , and one a line through  $(0, 4)$  and  $(5, 10)$ .

$$y = \frac{1}{5}x + 2, y = \frac{3}{5}x + 3, y = \frac{6}{5}x + 4$$

## Problem 2

### Statement

The graph shows the height in inches,  $h$ , of a bamboo plant  $t$  months after it has been planted.



a. Write an equation that describes the relationship between  $h$  and  $t$ .

b. After how many months will the bamboo plant be 66 inches tall? Explain or show your reasoning.

## Solution

a.  $h = 3t + 12$

b. 18 months. Explanations vary. Sample response: Substitute  $h = 66$ , and solve the equation  $66 = 3t + 12$ .  $3t = 54$ ,  $t = 18$ .

## Problem 3

### Statement

Here are recipes for two different banana cakes. Information for the first recipe is shown in the table.

sugar (cups)	flour (cups)
$\frac{1}{2}$	$\frac{3}{4}$
$2\frac{1}{2}$	$3\frac{3}{4}$
3	$4\frac{1}{2}$

The relationship between cups of flour  $y$  and cups of sugar  $x$  in the second recipe is  $y = \frac{7}{4}x$

a. If you used 4 cups of sugar, how much flour does each recipe need?

b. What is the constant of proportionality for each situation and what does it mean?

## Solution

a. First: 6 cups, second: 7 cups

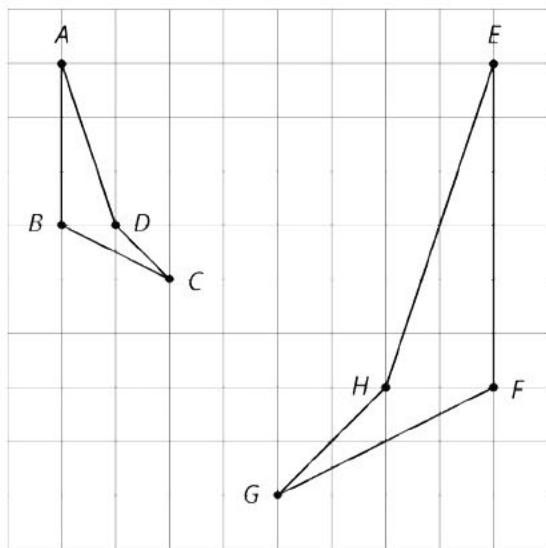
b. First:  $1\frac{1}{2}$  cups of flour per cup of sugar, second:  $1\frac{3}{4}$  cups of flour per cup of sugar

(From Unit 3, Lesson 4.)

## Problem 4

### Statement

Show that the two figures are similar by identifying a sequence of translations, rotations, reflections, and dilations that takes the larger figure to the smaller one.



### Solution

Translate  $H$  to  $D$ , reflect across a vertical line through  $D$ , and then dilate using a scale factor of  $\frac{1}{2}$  centered at  $D$ .

(From Unit 2, Lesson 6.)

# Lesson 8: Translating to $y = mx + b$

## Goals

- Coordinate (orally) features of the equation  $y = b + mx$  to the graph, including lines with a negative  $y$ -intercept.
- Create and compare (orally and in writing) graphs that represent linear relationships with the same rate of change but different initial values.

## Learning Targets

- I can explain where to find the slope and vertical intercept in both an equation and its graph.
- I can write equations of lines using  $y=mx+b$ .

## Lesson Narrative

This lesson develops a third way to understand an equation for a line in the coordinate plane. In previous lessons, students wrote an equation of a line by generalizing from repeated calculations using their understanding of similar triangles and slope (MP8). They have also written an equation of a linear relationship by reasoning about initial values and rates of change and have graphed the equation as a line in the plane. This lesson introduces the idea that any line in the plane can be considered a vertical translation of a line through the origin.

In the previous lesson, the terms in the expression are more likely to be arranged  $b + mx$  because the situation involves a starting amount and then adding on a multiple. In this lesson,  $mx + b$  is more likely because the situation involves starting with a relationship that includes  $(0, 0)$  and shifting up or down. Students continue to only consider lines with positive slopes, but in this lesson, the notion of a negative  $y$ -intercept (not in a context) is introduced.

In addition, students match lines presented in many different forms: equation, graph, description, table. This combines much of what they have learned about lines in this unit, including slope and vertical intercept.

## Alignments

### Addressing

- 8.EE.B: Understand the connections between proportional relationships, lines, and linear equations.
- 8.G.A.1: Verify experimentally the properties of rotations, reflections, and translations:

## Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect
- MLR8: Discussion Supports

## Required Materials

### Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

### Pre-printed cards, cut from copies of the blackline master

## Required Preparation

Print and cut up slips from the Translating a Line blackline master. Prepare 1 set of cards for every 2 students (this is not needed if doing the digital version).

### Student Learning Goals

Let's see what happens to the equations of translated lines.

## 8.1 Lines that Are Translations

### Warm Up: 5 minutes

The purpose of this warm-up is to remind students that the translation of a line is parallel to the original line, and to plant the seed that a line can be taken to a parallel line by translating it. They inspect several lines to decide which could be translations of a given line. Then they describe the translations by specifying the number of units and the direction the original line should be translated.

### Addressing

- 8.G.A.1

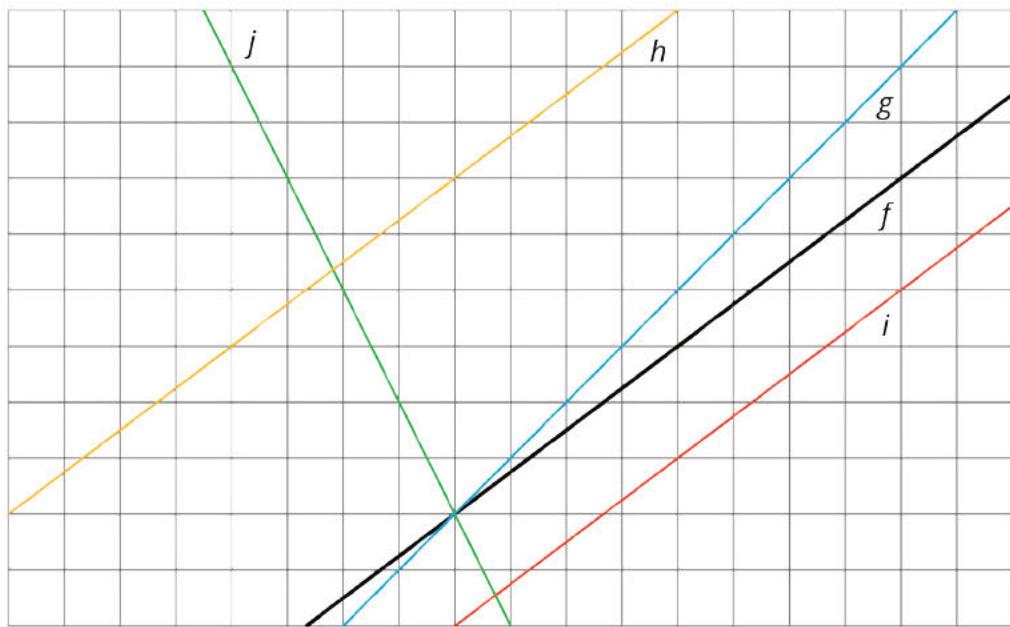
### Launch

Arrange students in groups of 2. Give students 2 minutes of quiet think time and access to geometry toolkits. Ask them to share their responses with a partner afterwards.

### Anticipated Misconceptions

Students may think that lines  $i$  and  $h$  can't be images of line  $f$  because the part of  $i$  and  $h$  we can see is shorter than the part of  $f$  we can see. Tell these students that all of the lines go on indefinitely in both directions.

### Student Task Statement

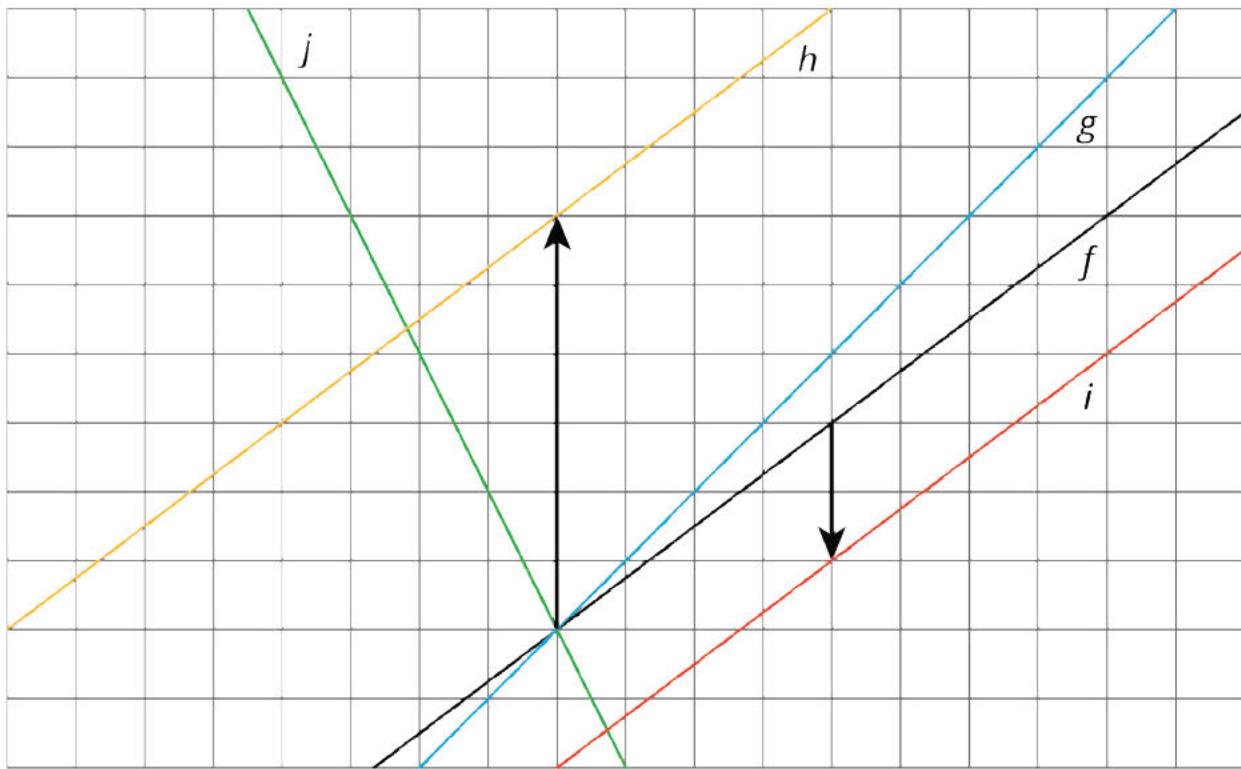


The diagram shows several lines. You can only see part of the lines, but they actually continue forever in both directions.

1. Which lines are images of line  $f$  under a translation?
2. For each line that is a translation of  $f$ , draw an arrow on the grid that shows the vertical translation distance.

### Student Response

1.  $h$  and  $i$ . They appear to be parallel to  $f$ , and translated lines are parallel to the original.
2.  $h$  is  $f$  translated up 6 units.  $i$  is  $f$  translated down 2 units.



### Activity Synthesis

Invite students to share how they know that lines  $h$  and  $i$  are translations of  $f$ . The main reason to bring out is that they are parallel to  $f$ . It might be worth demonstrating with a transparency that we can translate  $f$  to match up with  $h$  and  $i$ , but to match up with the other lines, we would need to rotate (or reflect) the transparency. If using a transparency to demonstrate the number of units to translate  $f$  up or down, it is helpful to draw a dot on a specific point on both the underlying graph and on the transparency.

## 8.2 Increased Savings

15 minutes (there is a digital version of this activity)

The goal of this activity is to get students to think about translations of lines in a context. Students examine two scenarios. In the first, there is a proportional relationship between the number of hours Diego works and his total earnings. In the second, Diego starts with \$30 saved and then saves all of his earnings. Graphically, the two lines showing these relationships are parallel (the second is a vertical translation of the first for the extra \$30 Diego has already saved). The lines have the same slope (Diego's hourly rate of pay is the same) but different  $y$ -intercepts (one has a  $y$ -intercept of 0 while the other has a  $y$ -intercept of 30). Students will observe this structure in the equations that they write for the two lines.

Even though babysitters are most often paid for increments of 1 hour or  $\frac{1}{2}$  hour and in increments of \$1, this uses a continuous line to represent this relationship. If a student brings up the idea that it would be better to represent the relationship using discrete points than a line, acknowledge the

observation but suggest that a continuous line is an acceptable representation. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation (MP4).

Monitor for students who use these approaches as they graph the two earnings scenarios:

- Making a table (showing earnings for different numbers of hours worked) and then graphing
- Plotting points directly
- Writing an equation that represents the situation and then graphing the solutions to the equation

Select students using these approaches and invite them to present during the discussion.

### **Addressing**

- 8.EE.B

### **Instructional Routines**

- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect

### **Launch**

Give students 5 minutes of quiet work time and tell them to pause after the first 3 questions. After they have discussed their responses with a partner, ensure that students have graphed both situations correctly and can articulate that the second graph can be viewed as a vertical translation of the first graph. Then, instruct students to complete the remaining question either independently or with a partner.

If using the digital activity, the mathematical thinking and reasoning remains the same. The applet, however, will assist students in creating graphs to model the babysitting scenarios. Furthermore, some students may start exploring translations in general by creating multiple rays. The structure for the lesson will remain the same for print and digital.

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### **Support for Students with Disabilities**

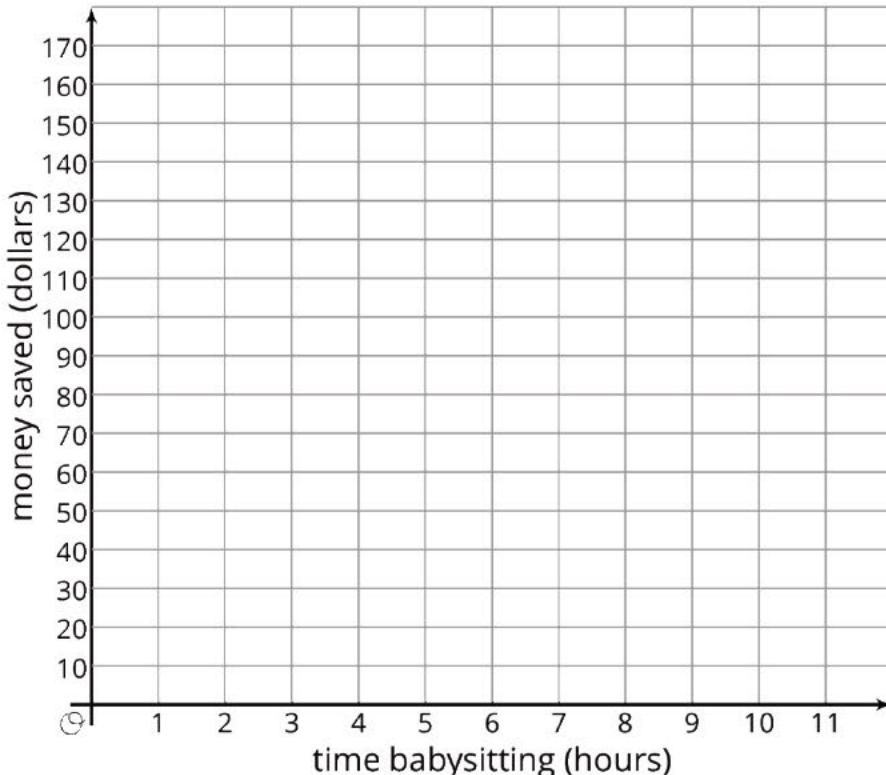
*Engagement: Develop Effort and Persistence.* Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, “First I \_\_\_ because....” or “One thing that is different about the two lines is ....”

*Supports accessibility for: Language; Social-emotional skills*

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### Student Task Statement

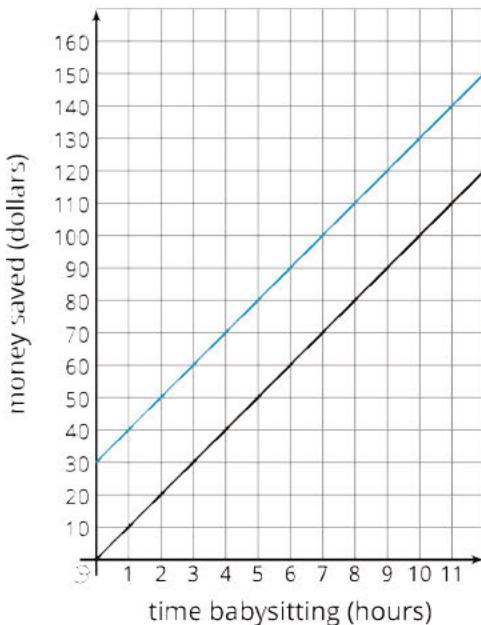
1. Diego earns \$10 per hour babysitting. Assume that he has no money saved before he starts babysitting and plans to save all of his earnings. Graph how much money,  $y$ , he has after  $x$  hours of babysitting.



2. Now imagine that Diego started with \$30 saved before he starts babysitting. On the same set of axes, graph how much money,  $y$ , he would have after  $x$  hours of babysitting.
3. Compare the second line with the first line. How much *more* money does Diego have after 1 hour of babysitting? 2 hours? 5 hours?  $x$  hours?
4. Write an equation for each line.

### Student Response

1 and 2. The line through  $(0, 0)$  is the solution to first question, and the line through  $(0, 30)$  is the solution to the second question:



3. He has \$30 more in the second case, no matter how many hours he babysits. Can be seen as a vertical translation up by 30 dollars.

4.  $y = 10x$  for the first question, and  $s = 30 + 10x$  or equivalent for the second question.

### Activity Synthesis

Invite selected students to share their methods for graphing Diego's earnings, sequenced in this order:

- Making a table (showing earnings for different numbers of hours worked) and then graphing
- Plotting points directly
- Writing an equation that represents the situation and then graphing the solutions to the equation

Notice that making a table has the advantage of revealing the *arithmetic* relationship between Diego's earnings in the two situations. No matter how many hours, Diego works, he has \$30 more in the second situation. Plotting the points directly shows geometrically that the vertical distance is always the same between pairs of points for the same number of hours worked (this vertical distance represents the \$30). The equations are the most abstract and contain all of the arithmetic and geometric information once we can interpret them. If  $h$  is the number of hours Diego works and  $m$  is how much money he has saved, then the two equations can be written as  $m = 10h$  and  $m = 30 + 10h$ . The graphs of these lines have slope 10 (and they are parallel!), but the second equation has a  $y$ -intercept of 30 while the first has a  $y$ -intercept of 0. The graph of  $m = 30 + 10h$  is the same as the graph of  $m = 10h$  except that every point is moved up by the same amount (this explains why they are parallel).

Have students connect the equations and situations they have graphed in the activity by asking students what the graph of  $m = 20 + 10h$  would look like (it would lie between the graphs of  $m = 30 + 10h$  and  $m = 10h$ ). What situation does this equation represent? (Diego started with \$20 saved and then earned \$10 for each hour of baby sitting.)

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### Support for English Language Learners

*Representing, Conversing: MLR7 Compare and Connect.* To begin the whole-class discussion, arrange students in groups of 2, and invite students to compare the methods they used to graph Diego's earnings. Ask students to discuss what is the same and what is different about their approaches. Once the class has discussed the various methods, spend some time focusing on the connections between the equations and other representations. Ask students, "Where does the '10x' in each equation come from?" "How is it represented in each representation?" Listen for connections that relate to the differences in methods students discussed prior to starting the synthesis. This will help students better understand the connection between the multiple representations.

*Design Principle(s): Optimize output (for comparison); Cultivate conversation*

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## 8.3 Translating a Line

15 minutes (there is a digital version of this activity)

The previous activity examined parallel lines and equations that define them, focusing on their common attributes (slope) and their different attributes ( $y$ -intercept). This activity further examines parallel lines, including situations where the  $y$ -intercept is negative. In addition, students match lines represented in many different ways including

- graphically
- verbal description
- table of values
- equations

You will need the Translating a Line blackline master for this activity.

### Addressing

- 8.EE.B

### Instructional Routines

- MLR8: Discussion Supports

## Launch

Arrange students in groups of 2. Students first identify equations that describe a line with a negative  $y$ -intercept, then they complete a matching activity involving graphs, equations, and tables.

Display the image in the activity. Ask students how the pair of lines is the same and different from the lines in the previous activity. Instead of being translated up, line  $a$  was translated down. Ask students to explain how to change  $y = \frac{1}{4}x$ , an equation representing line  $a$ , to represent line  $h$ . They should readily come up with  $y = \frac{1}{4}x - 5$ . Ask students for another way to write the same equation, for example,  $\frac{1}{4}x - 5 = y$ . The first part of the activity is about finding different ways to write this equation.

Tell students to complete the first part of the activity and then pause for discussion. Ensure that all understand why the incorrect equations are incorrect, then distribute the cards to sort. Tell students to take turns identifying cards that go together, and for each choice they make, explain to their partner why they think they go together. Once they have sorted them into groups, they write an equation on the blank card to represent the line that does not have an equation.

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### Support for Students with Disabilities

*Representation: Internalize Comprehension.* Demonstrate and encourage students to use color coding and annotations to highlight connections between graphs, equations and tables to justify their matches. For example, use the same color to highlight connections between line  $h$  and the matching equation.

*Supports accessibility for: Visual-spatial processing*

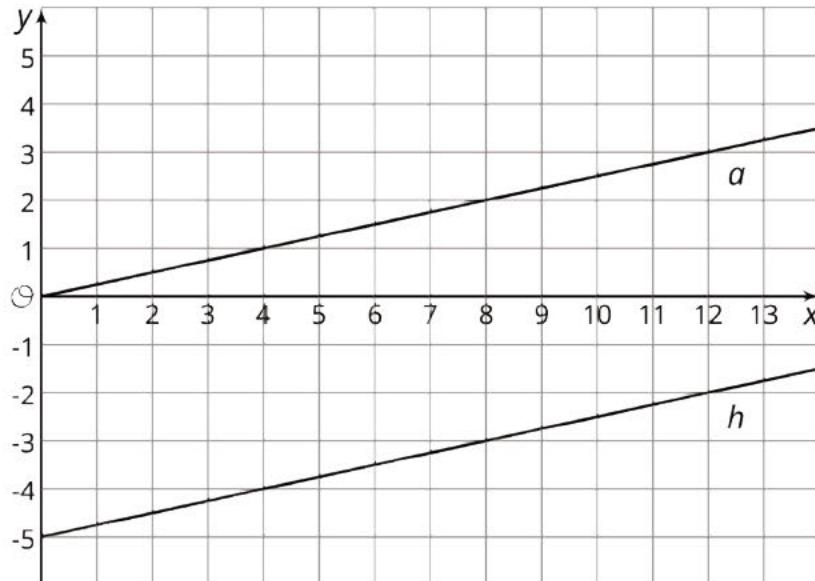
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### Student Task Statement

This graph shows two lines.

Line  $a$  goes through the origin  $(0, 0)$ .

Line  $h$  is the image of line  $a$  under a translation.



1. Select all of the equations whose graph is line  $h$ .

a.  $y = \frac{1}{4}x - 5$

b.  $y = \frac{1}{4}x + 5$

c.  $\frac{1}{4}x - 5 = y$

d.  $y = -5 + \frac{1}{4}x$

e.  $-5 + \frac{1}{4}x = y$

f.  $y = 5 - \frac{1}{4}x$

2. Your teacher will give you 12 cards. There are 4 pairs of lines, A-D, showing the graph,  $a$ , of a proportional relationship and the image,  $h$ , of  $a$  under a translation. Match each line  $h$  with an equation and either a table or description. For the line with no matching equation, write one on the blank card.

### Student Response

1. Equations A, C, D, E describe line  $h$ .
2. The blackline master shows the correct matching. The equation students must write is  $y = \frac{1}{2}x - 4$  or equivalent.

### Are You Ready for More?

A student says that the graph of the equation  $y = 3(x + 8)$  is the same as the graph of  $y = 3x$ , only translated upwards by 8 units. Do you agree? Why or why not?

### Student Response

Disagree. Using the distributive property on  $3(x + 8)$  gives  $3x + 24$ , so the equation is  $y = 3x + 24$ . This is the graph of  $y = 3x$  translated up by 24 units.

### Activity Synthesis

Once all groups have completed the matching, discuss the following:

- "Were any of the matches more difficult than others? What made them difficult?"
- "Did any groups have to adjust an initial guess that turned out to be wrong? What adjustments were made and why?"
- "What clues did you look for to see which equation went with a graph?"

Solicit all versions of the missing equation. Ensure that students understand that  $y = \frac{1}{2}x - 4$  and  $y = -4 + \frac{1}{2}x$  are equivalent.

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### Support for English Language Learners

*Speaking: MLR8 Discussion Supports.* Use this routine to support students in producing statements to describe the reasons for their matches. Provide sentence frames such as: "The equation \_\_\_\_\_ matches line  $h$  on this card because...." or "This description matches line  $h$  on this card because...." Encourage students to press for details from their partners by challenging their explanations and/or encouraging mathematical vocabulary. These exchanges strengthen students' use of mathematical language related to representations of translations.

*Design Principle(s): Support sense-making, Optimize output (for explanation)*

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## Lesson Synthesis

Display a graph of two lines on the same set of axes: one of the form  $y = mx$  and the other of the form  $y = mx + b$ . Discuss:

- "How can we think of one of these lines as a transformation of the other?"
- "What is the equation of the line that goes through the origin?" (Discuss how you need to figure out the slope.)
- "How is the equation of the line that does not go through the origin different?" (Make sure to bring out that the  $b$  in  $mx + b$  gives the vertical translation to get from the graph of  $y = mx$  to the graph of  $y = mx + b$ ; the translation is up when  $b > 0$  and down when  $b < 0$ .)

## 8.4 Similarities and Differences in Two Lines

Cool Down: 5 minutes

### Addressing

- 8.EE.B

#### Student Task Statement

Describe how the graph of  $y = 2x$  is the same and different from the graph of  $y = 2x - 7$ . Explain or show your reasoning.

#### Student Response

Answers vary. Students may or may not sketch graphs as part of their solution.

Possible responses to how they are the same:

- They have the same slope.
- They both have a slope of 2.
- They are parallel to each other.

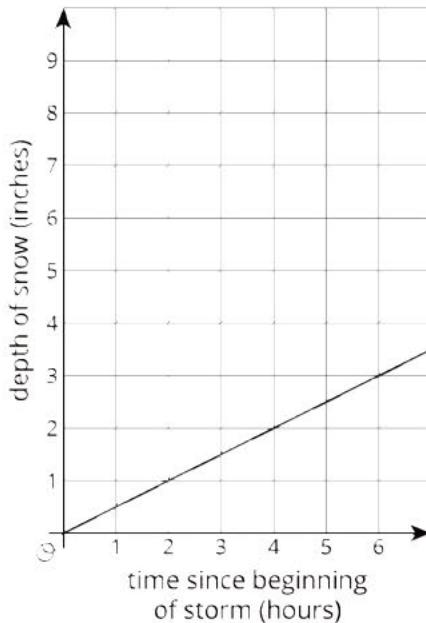
Possible responses to how they are different:

- They are in a different location.
- One is a translation of the other.
- They cross the  $y$ -axis (or  $x$ -axis) at a different point.
- They don't go through any of the same points.

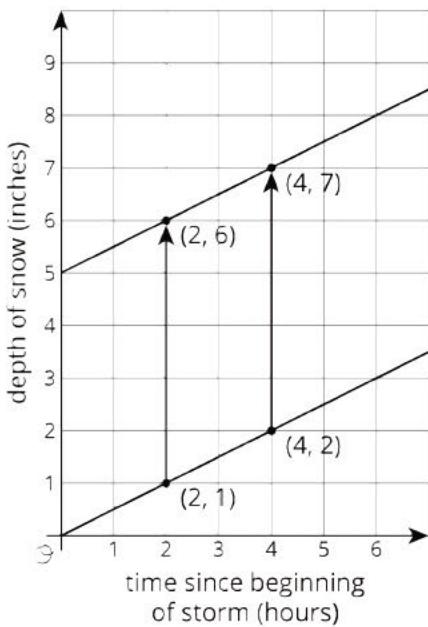
## Student Lesson Summary

During an early winter storm, the snow fell at a rate of  $\frac{1}{2}$  inch per hour. We can see the rate of change,  $\frac{1}{2}$ , in both the equation that represents this storm,  $y = \frac{1}{2}x$ , and in the slope of the line representing this storm.

In addition to being a linear relationship between the time since the beginning of the storm and the depth of the snow, we can also call this as a proportional relationship since the depth of snow was 0 at the beginning of the storm.



During a mid-winter storm, the snow again fell at a rate of  $\frac{1}{2}$  inch per hour, but this time there was already 5 inches of snow on the ground.



We can graph this storm on the same axes as the first storm by taking all the points on the graph of the first storm and translating them up 5 inches.

Two hours after each storm begins, 1 inch of new snow has fallen. For the first storm, this means there is now 1 inch of snow on the ground. For the second storm, this means there are now 6 inches of snow on the ground.

Unlike the first storm, the second is not a proportional relationship since the line representing the second storm has a vertical intercept of 5. The equation representing the storm,  $y = \frac{1}{2}x + 5$ , is of the form  $y = mx + b$ , where  $m$  is the rate of change, also the slope of the graph, and  $b$  is the initial amount, also the vertical intercept of the graph.

## Lesson 8 Practice Problems

### Problem 1

#### Statement

Select **all** the equations that have graphs with the same  $y$ -intercept.

- A.  $y = 3x - 8$
- B.  $y = 3x - 9$
- C.  $y = 3x + 8$
- D.  $y = 5x - 8$
- E.  $y = 2x - 8$
- F.  $y = \frac{1}{3}x - 8$

#### Solution

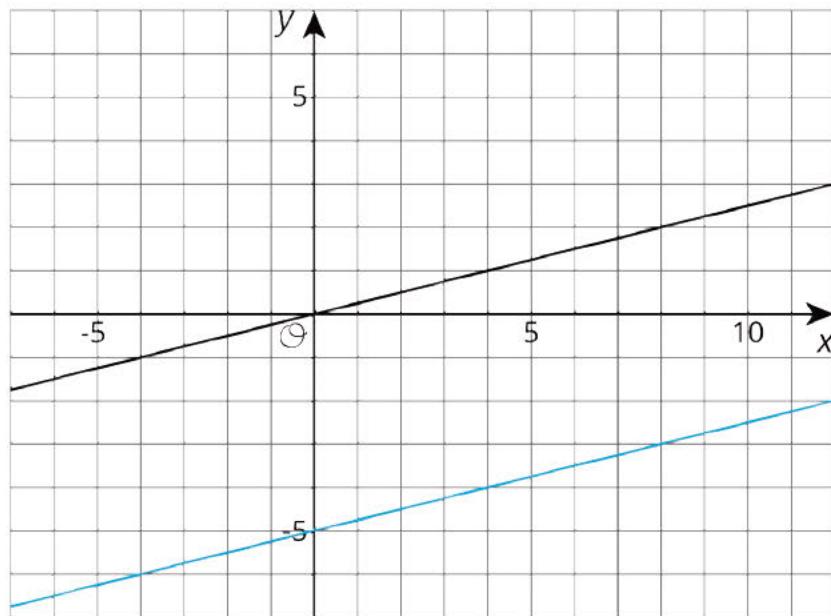
["A", "D", "E", "F"]

### Problem 2

#### Statement

Create a graph showing the equations  $y = \frac{1}{4}x$  and  $y = \frac{1}{4}x - 5$ . Explain how the graphs are the same and how they are different.

## Solution



Answers vary. Sample response: The graphs have the same slope of  $\frac{1}{4}$  but different  $y$ -intercepts. The first is 0, and the second is -5. Another sample response: Each  $(x, y)$  on the first graph is translated down by 5 to get a corresponding point on the second graph.

## Problem 3

### Statement

A cable company charges \$70 per month for cable service to existing customers.

- Find a linear equation representing the relationship between  $x$ , the number of months of service, and  $y$ , the total amount paid in dollars by an existing customer.
- For new customers, there is an additional one-time \$100 service fee. Repeat the previous problem for new customers.
- When the two equations are graphed in the coordinate plane, how are they related to each other geometrically?

## Solution

- $y = 70x$
- $y = 70x + 100$
- The two lines are parallel, with the second line being the first line translated vertically 100 units upwards.

## Problem 4

### Statement

A mountain road is 5 miles long and gains elevation at a constant rate. After 2 miles, the elevation is 5500 feet above sea level. After 4 miles, the elevation is 6200 feet above sea level.

- Find the elevation of the road at the point where the road begins.
- Describe where you would see the point in part (a) on a graph where  $y$  represents the elevation in feet and  $x$  represents the distance along the road in miles.

### Solution

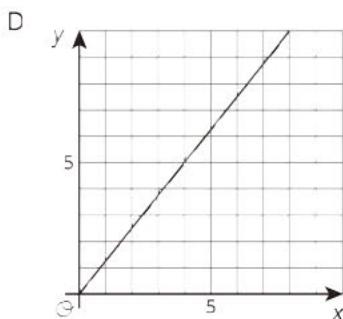
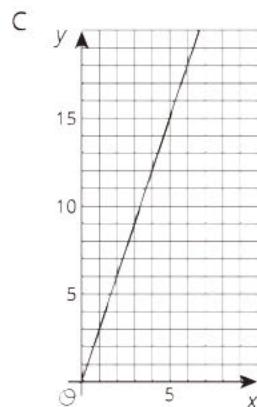
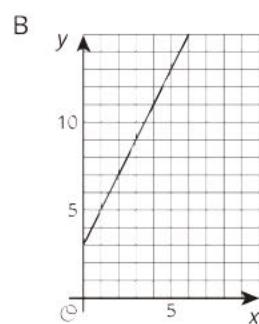
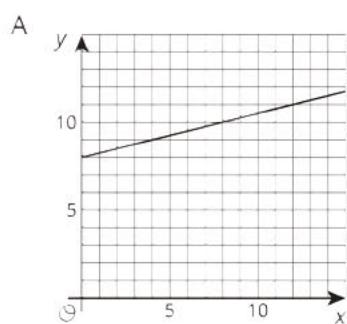
- 4800 feet above sea level
- The point would be  $(0, 4800)$  located on the  $y$ -axis.

(From Unit 3, Lesson 6.)

## Problem 5

### Statement

Match each graph to a situation.



- A. Graph A
- B. Graph B
- C. Graph C
- D. Graph D

1. The graph represents the perimeter,  $y$ , in units, for an equilateral triangle with side length of  $x$  units. The slope of the line is 3.
2. The amount of money,  $y$ , in a cash box after  $x$  tickets are purchased for carnival games. The slope of the line is  $\frac{1}{4}$ .
3. The number of chapters read,  $y$ , after  $x$  days. The slope of the line is  $\frac{5}{4}$ .
4. The graph shows the cost in dollars,  $y$ , of a muffin delivery and the number of muffins,  $x$ , ordered. The slope of the line is 2.

## Solution

- A: 2
- B: 4
- C: 1
- D: 3

(From Unit 3, Lesson 6.)

# Section: Finding Slopes

## Lesson 9: Slopes Don't Have to be Positive

### Goals

- Create a graph of a line representing a linear relationship with a non-positive rate of change.
- Interpret the slope of a non-increasing line in context.

### Learning Targets

- I can give an example of a situation that would have a negative slope when graphed.
- I can look at a graph and tell if the slope is positive or negative and explain how I know.

### Lesson Narrative

In previous lessons, students have arrived at an equation for a line in three ways:

- By reasoning about similarity of slope triangles on a line
- By reasoning about starting values and rates of change in a linear relationship
- By reasoning about vertical translations of lines through the origin

Students encountered linear relationships with positive rates of change and either positive or negative vertical intercepts. The graphs of these relationships all had an uphill appearance.

In this lesson, students get their first glimpse of lines that visually slope downhill as well as a “flat” line or line with 0 slope. After reflecting on commonalities and differences between lines that slope in different directions, students explore a situation in which one quantity decreases at a constant rate in relation to a second quantity. They interpret a graph of the situation and reason that it makes sense for the slope to be negative in terms of the context. The scenario is then extended to consider a quantity that does not change with respect to another, and students realize that a flat graph has a slope of zero.

### Alignments

#### Addressing

- 8.EE.B: Understand the connections between proportional relationships, lines, and linear equations.

### Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR5: Co-Craft Questions
- MLR6: Three Reads

- Notice and Wonder
- Which One Doesn't Belong?

## Required Materials

### Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

### Student Learning Goals

Let's find out what a negative slope means.

## 9.1 Which One Doesn't Belong: Odd Line Out

### Warm Up: 5 minutes

This warm-up prompts students to compare four lines. It invites students to explain their reasoning and hold mathematical conversations, and allows you to hear how they use terminology and talk about lines and their properties. To allow all students to access the activity, each figure has one obvious reason it does not belong. Encourage students to move past the obvious reasons (e.g., line  $t$  has a different color) and find reasons based on geometric properties (e.g., a slope triangle of line  $u$  is not similar to the slope triangles of the other three lines).

So far, we have only considered lines with positive slopes. The purpose of this warm-up is to suggest similarities (same vertical and horizontal lengths of slope triangles) and differences (since they are not parallel, there is something fundamentally different going on here) between lines whose slopes have the same absolute value but opposite signs.

As students share their responses, listen for important ideas and terminology that will be helpful in the work of this lesson. Students may:

- Identify lines that have the same slope.
- Identify points where lines intersect.
- Distinguish between lines with positive and negative values for slope and be able to articulate the difference clearly.

### Addressing

- 8.EE.B

## Instructional Routines

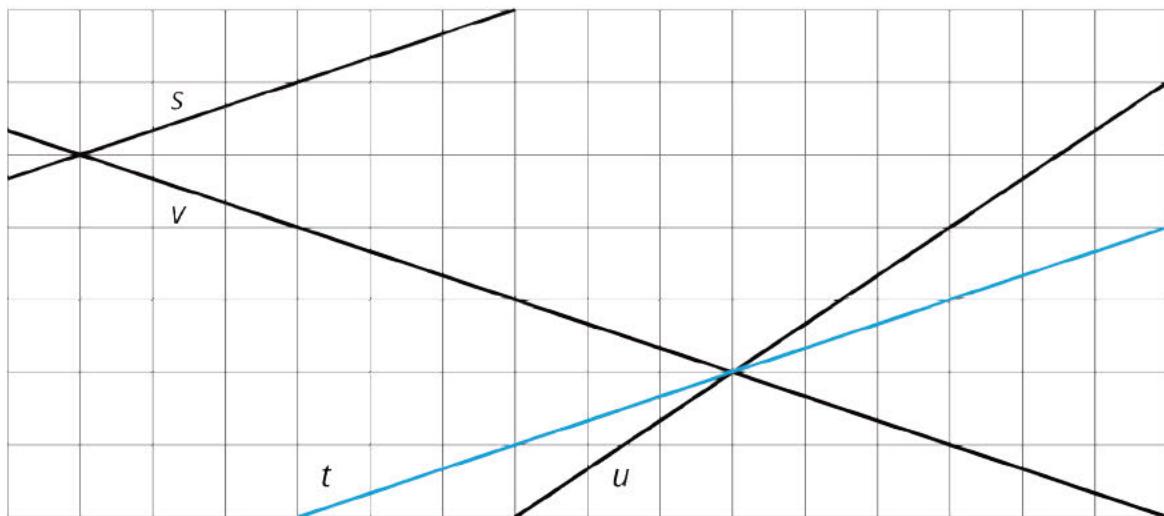
- Which One Doesn't Belong?

### Launch

Arrange students in groups of 2–4 and provide access to geometry toolkits. Display the image of the four lines for all to see. Ask students to indicate when they have noticed one line that does not belong and can explain why. Give students 2 minutes of quiet think time and then time to share their thinking with their group. After everyone has conferred in groups, ask each group to offer at least one reason a particular line doesn't belong.

### Student Task Statement

Which line doesn't belong?



### Student Response

Answers vary. Possible responses:

- $s$  doesn't belong because it doesn't go through that one point the rest of them do.
- $t$  does not belong because it is parallel to  $s$  (and it is a different color).
- $u$  doesn't belong because its slope triangle isn't similar to a triangle whose vertical side has length of 1 and whose horizontal side has length 3.
- $v$  doesn't belong because it "leans to the left" instead of the right, or slopes down instead of up.

### Activity Synthesis

After students have conferred in groups, invite each group to share one reason why a particular line might not belong. Record and display the responses for all to see. After each response, ask the rest

of the class if they agree or disagree. Since there is no single correct answer to the question asking which shape does not belong, attend to students' explanations and ensure the reasons given are correct.

During the discussion, prompt students to use mathematical terminology (parallel, intersect, slope) correctly. Also, press students on unsubstantiated claims. For example, a student may claim that  $u$  does not belong because it has a different slope. Ask how they know for sure that its slope is different from the other lines. Demonstrate drawing a slope triangle and computing slope.

Based on the work done up to this point in the unit, students are likely to assume that the slope of  $v$  is  $\frac{1}{3}$ . In the discussion, solicit the idea that there is something fundamentally different about line  $v$  compared to the others. You could use informal language like "uphill" and "downhill," or "tilt direction." The expressions positive and negative slope do not need to be introduced at this time.

## 9.2 Stand Clear of the Closing Doors, Please

**15 minutes (there is a digital version of this activity)**

In previous activities with linear relationships, when  $x$  increases the  $y$  value increases as well; adding objects to a cylinder *increases* the water level and adding money to a bank account *increases* the balance. The slope of the lines that represent these relationships were positive. In this activity, students see negative slopes for the first time.

In this activity, students answer questions about a public transportation fare card context. After computing the amount left on the card after 0, 1, and 2 rides, they express regularity in repeated reasoning (MP8) to represent the amount remaining on the card after  $x$  rides. They are told that the slope of this line is  $-2.5$ , and are prompted to explain why a negative value makes sense.

While the language is not introduced in the task statement, the value of  $x$  for which the money on the card is 0 is called the  $x$ -intercept or horizontal intercept. Unlike the  $y$ -intercept, which can be seen in the equation  $y = 40 - 2.5x$ , the  $x$  intercept has to be calculated: it is the value of  $x$  for which  $0 = 40 - 2.5x$ .

### Addressing

- 8.EE.B

### Instructional Routines

- MLR1: Stronger and Clearer Each Time

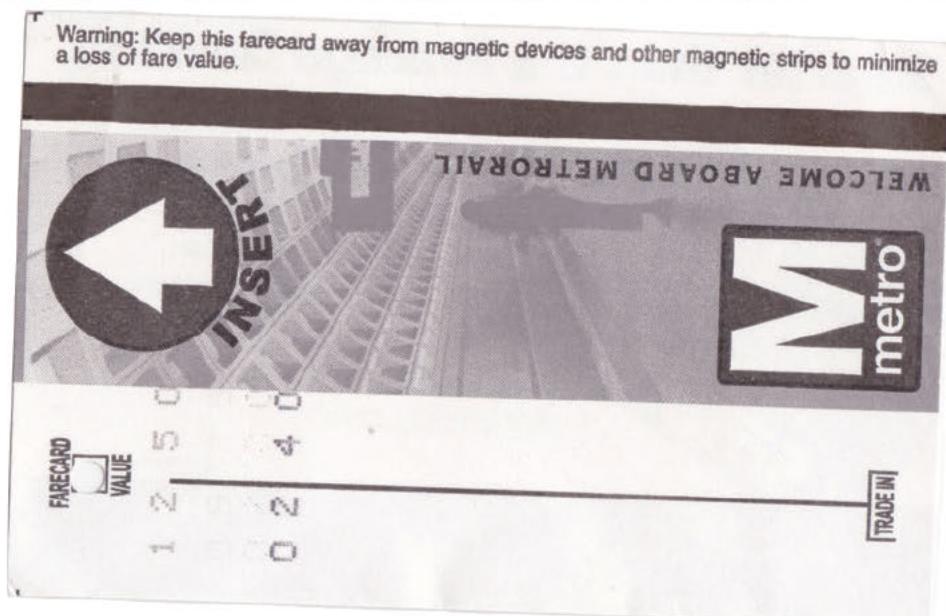
### Launch

If your students are unlikely to be familiar with public transportation, you may need to give them some quick information about how a fare card works. If possible, prepare some photos related to purchasing and using a fare card. (Some example images are provided, here.)

Explain to students that someone who wants to ride the bus or subway in a city often uses a card like this. The rider pays money which a computer system associates with the card. Every time the

rider wants to ride, they swipe the card, and the cost of the ride is subtracted in the computer system from the balance on the card. Eventually, the amount available on the card runs out, and the rider must spend more money to increase the amount available on the card.

Arrange students in groups of 2 and give them 5 minutes of quiet work time. Provide access to rulers. After they have discussed their responses with a partner, discuss why a negative value for the slope makes sense in the context and ways to visually tell whether a line has a positive or negative slope.



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### Support for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, present one question at a time.

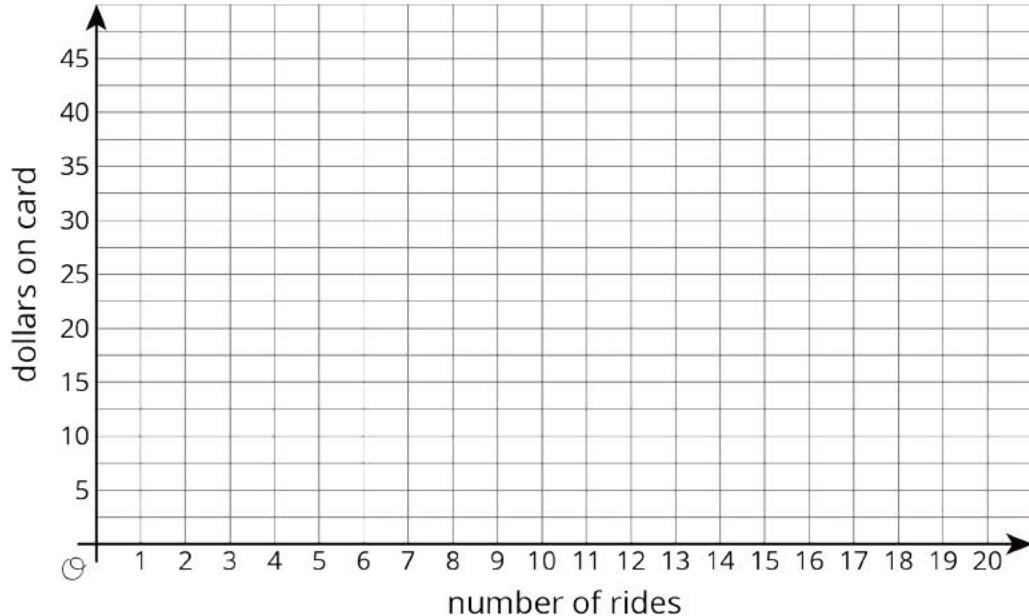
*Supports accessibility for: Organization; Attention*

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### Student Task Statement

Noah put \$40 on his fare card. Every time he rides public transportation, \$2.50 is subtracted from the amount available on his card.

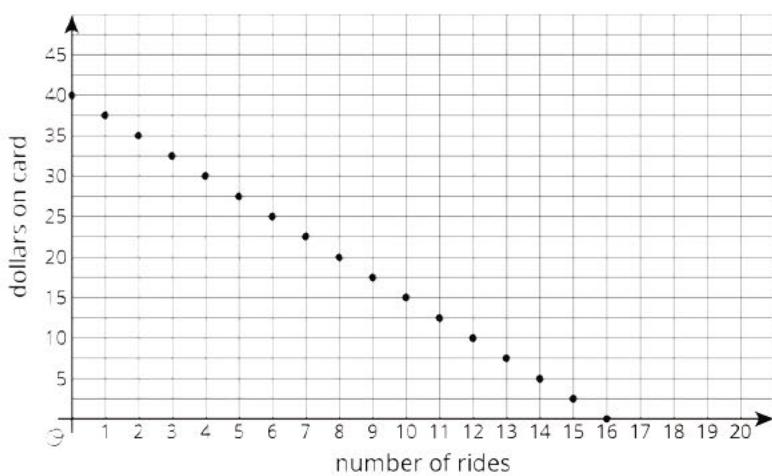
1. How much money, in dollars, is available on his card after he takes
  - a. 0 rides?
  - b. 1 ride?
  - c. 2 rides?
  - d.  $x$  rides?
2. Graph the relationship between amount of money on the card and number of rides.



3. How many rides can Noah take before the card runs out of money? Where do you see this number of rides on your graph?

### Student Response

1. Money stored on card in dollars:
  - a. 40
  - b. 37.5
  - c. 35
  - d.  $40 - 2.5x$  (or equivalent)
2. See graph. A graph that shows only points with integer values of number of rides is also acceptable.
3. 16 rides. The point  $(16, 0)$  is on the graph.



### Activity Synthesis

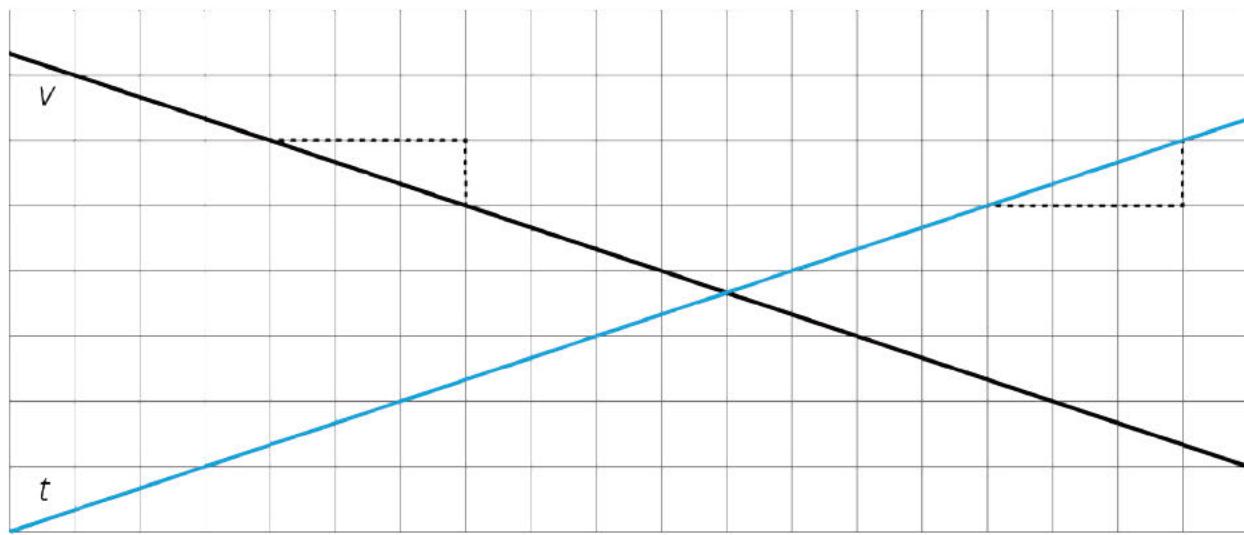
Ask students, "Why does it make sense to say the slope of this graph is  $-2.5$  rather than  $2.5$ ?" We can say that the rate of change of the amount on the card is  $-2.5$  dollars per ride (for each ride, the amount on the card *decreases* by  $2.5$ ). We can also say the slope of the graph representing the relationship is  $-2.5$  (when  $x$  increases by 1,  $y$  decreases by  $2.5$ ). Ensure that students can articulate some version of "For every ride, the amount on the card *decreased* by \$2.50, and this is why it makes sense that the slope is  $-2.5$ ."

Ask students, "If we let  $y$  represent the amount of money on the card, in dollars, and  $x$  the number of rides, the linear relationship in this activity has the equation  $y = 40 - 2.5x$ . When will there be no money left on the card?" (After 16 rides, the card has no money left, so when  $x = 16$ ,  $y = 0$ .) Explain that just like the point  $(0, 40)$  on the graph is called the vertical intercept, the point  $(16, 0)$  is called the horizontal intercept. In this situation, the vertical intercept tells us how much money Noah put on the card and the horizontal intercept tells us how many rides Noah can take before the card has no more money on it.

Some students may connect the plotted points with a line, or only draw the line and not each point. If this comes up, acknowledge that it's a common practice in mathematical modeling to connect points showing a line to make a relationship easier to see. Ask if a point on the line such as

(1.5, 36.25) makes sense in this situation. We understand that the line helps us to understand the situation, but we also recognize that only integer values for the number of rides make sense in this situation.

Display once again lines  $v$  and  $t$  from the warm-up. Explain that even though they both have slope triangles similar to a triangle with vertical length of 1 and horizontal length of 3, they don't have the same slope. Ask students to articulate which line has a slope of  $\frac{1}{3}$  and which line has a slope of  $-\frac{1}{3}$ , and why. Validate students' use of informal language to describe the differences in their own words. For example, they might say that if you put a pencil on a line and move it along the line from left to right, line  $t$  goes "uphill" but line  $v$  goes "downhill."



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#### Support for English Language Learners

*Writing, Conversing: MLR1 Stronger and Clearer Each Time.* Use this routine to support students to explain their reasoning for the question, "Why does it make sense to say the slope of this graph is  $-2.5$  rather than  $2.5$ ?" Give students time to meet with 2–3 partners, to share and get feedback on their responses. Encourage listeners to ask their partner clarifying questions such as, "Can you demonstrate your thinking by using the graph?" and "How does this negative value make sense for this situation?" Allow students to write a revised draft that reflects ideas and language from their shares. This will help students refine their thinking around mean through conversing with their partners.

*Design Principle(s): Optimize output (for explanation); Cultivate conversation*

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## 9.3 Travel Habits in July

10 minutes (there is a digital version of this activity)

The previous activity examines the meaning of a negative slope with the context of money on a transportation card. This activity takes advantage of familiarity with the same context, introducing

the idea of 0 slope. In the previous activity, the slope of -2.5 meant that for every ride, the amount on the card decreased by \$2.50. In this activity, the amount on the card is graphed with respect to days in July. For every new day, the amount on the card does not change; it does not go up or down at all. The purpose is for students to understand the meaning of a slope of 0 in this context.

### **Addressing**

- 8.EE.B

### **Instructional Routines**

- MLR5: Co-Craft Questions
- Notice and Wonder

### **Launch**

Show the image and ask students “What do you notice? What do you wonder?” Expect students to notice that the line is horizontal (or the amount of money on the card does not change). They may wonder why the line is horizontal or what its slope is.

Keep students in the same groups. 5 minutes of quiet think time, followed by partner and class discussion. Before students begin the task, ensure that they understand that the  $x$ -axis no longer represents number of rides, but rather, different days in July.

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### **Support for Students with Disabilities**

*Action and Expression: Internalize Executive Functions.* Provide students with a graphic organizer to record what they notice and wonder prior to being expected to share these ideas with others.  
*Supports accessibility for: Language; Organization*

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### **Support for English Language Learners**

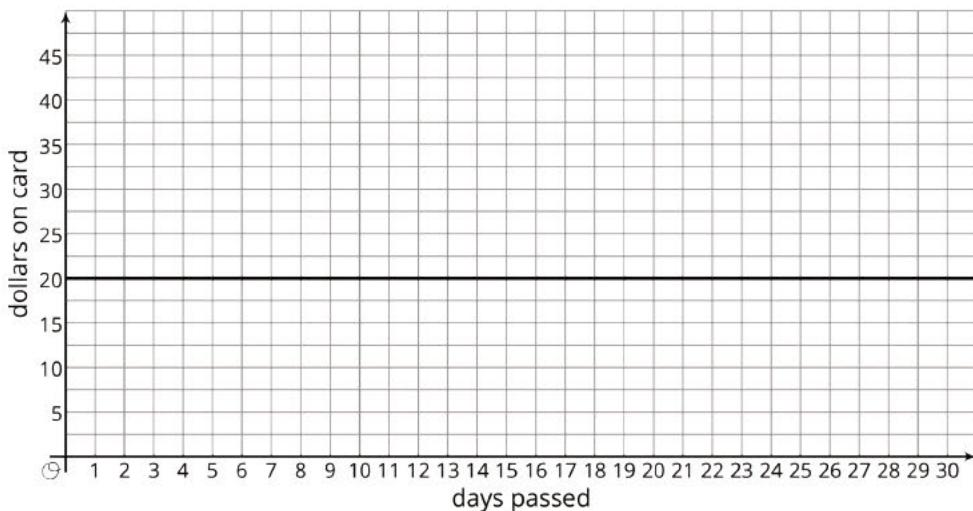
*Writing, Speaking: MLR5 Co-Craft Questions.* Present the graph that shows the amount on Han’s fare card for every day of last July, without revealing the questions that follow. Give students time to write down possible mathematical questions that can be asked about the situation, and then invite them to share their questions with a partner. Listen for and amplify questions that wonder about the slope of a horizontal line. This will help students make sense of a graph with a “flat line,” or a line having a zero slope, by generating questions using mathematical language.

*Design Principle(s): Cultivate conversation; Maximize meta-awareness*

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### **Student Task Statement**

Here is a graph that shows the amount on Han’s fare card for every day of last July.



1. Describe what happened with the amount on Han's fare card in July.
2. Plot and label 3 different points on the line.
3. Write an equation that represents the amount on the card in July,  $y$ , after  $x$  days.
4. What value makes sense for the slope of the line that represents the amounts on Han's fare card in July?

### Student Response

1. The amount on the card was \$20 every day. The amount on the card did not change.
2. Answers vary. Possible points:  $(0, 20)$ ,  $(2, 20)$ , and  $(30, 20)$ .
3.  $y = 20$
4. The slope of the graph is 0. The rate of change is \$0 per day. Also, the line is neither "uphill" nor "downhill," so it makes sense that the slope is neither positive nor negative.

### Are You Ready for More?

Let's say you have taken out a loan and are paying it back. Which of the following graphs have positive slope and which have negative slope?

1. Amount paid on the vertical axis and time since payments started on the horizontal axis.
2. Amount owed on the vertical axis and time remaining until the loan is paid off on the horizontal axis.
3. Amount paid on the vertical axis and time remaining until the loan is paid off on the horizontal axis.

### Student Response

1. Positive slope. At the beginning, the amount paid is nothing and the time that has passed is zero. Later, the amount paid is more and the time that has passed is more, so the graph slopes upward.
2. Positive slope. If the time remaining is zero, then you are at the end of the loan, and so the amount owed is also zero. So you are at the origin on the graph. If time remaining is greater than zero then something is owed, so you are at a point with both coordinates positive. So the graph has to slope upward.
3. Negative slope. If the time remaining is zero, then you are at the end of the loan, and the amount paid is the whole loan. So you are at a point on the positive vertical axis. If the time remaining is the entire period of the loan, then you are at the beginning of the loan, and the amount paid is zero. So you are at a point on the positive horizontal axis. So the graph has to slope down.

### Activity Synthesis

Display the graph for all to see, and ask students to articulate what they think the slope of the graph is, and why. The goal is to understand that a slope of 0 makes sense because no money is added or subtracted each day. If students have been thinking in terms of uphill and downhill lines, they might describe this line as “flat,” indicating that the slope can’t be positive or negative, so 0 makes sense. Thinking of slope as the quotient of horizontal displacement by vertical displacement for two points on a line is very effective here: the vertical displacement is 0 for any two points on this line, and so the quotient or slope is also 0.

If no one brings it up, ask students what would happen if we tried to create a slope triangle for this line. They might claim that it would be impossible, but suggest that we can think of a slope “triangle” where the vertical segment has length 0. In other words, it is possible to imagine a horizontal line segment as a triangle whose base is that segment and whose height is 0. If possible, display and demonstrate with the following: <https://ggbm.at/vvQPuta>.

## 9.4 Payback Plan

### Optional: 10 minutes

This activity gives students an opportunity to interpret the graph of a line in context, including the meaning of a negative slope and the meaning of the horizontal and vertical intercepts.

### Addressing

- 8.EE.B

### Instructional Routines

- MLR6: Three Reads

### Launch

3 minutes of quiet work time, followed by 2 minutes to confer in small groups to verify answers.

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### Support for English Language Learners

*Reading: MLR6 Three Reads.* Use this routine for Elena's situation to support reading comprehension for students. In the first read, students read the problem with the goal of comprehending the situation (e.g., Elena is paying back her brother every week.). In the second read, ask students to look for quantities represented in the graph (e.g., at week 0, Elena owes him \$18; at 6 weeks, she has paid him back completely). In the third read, ask students to brainstorm possible strategies to answer the question: "What is the slope of the line and what does it represent?" This will help students reflect on the situation and interpret the slope in the given context.

*Design Principle(s): Support sense-making; Maximize meta-awareness*

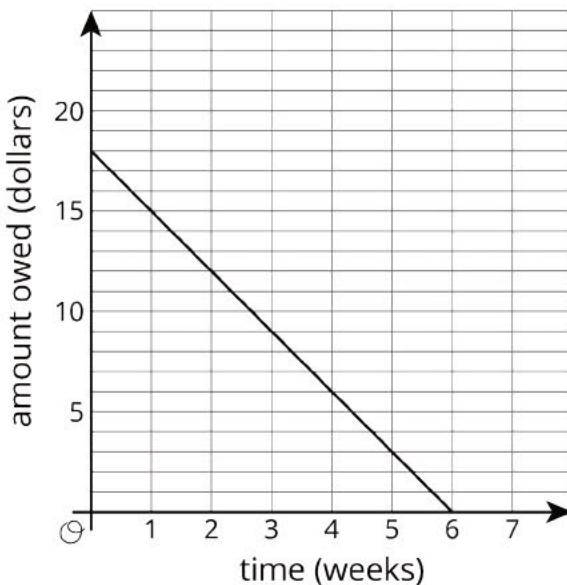
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### Anticipated Misconceptions

Students may interpret money owed as negative. In this setup, the axis is labeled so that money owed is treated as positive.

#### Student Task Statement

Elena borrowed some money from her brother. She pays him back by giving him the same amount every week. The graph shows how much she owes after each week.



Answer and explain your reasoning for each question.

1. What is the slope of the line?
2. Explain how you know whether the slope is positive or negative.
3. What does the slope represent in this situation?

4. How much did Elena borrow?
5. How much time will it take for Elena to pay back all the money she borrowed?

### Student Response

1. The slope is -3.
2. Answers vary. Sample response: Negative, because the line slopes down. Negative, because the amount owed is decreasing as time increases.
3. Answers vary. Sample response: The slope represents the amount that she pays back each week. The slope represents how much less she owes every time a week goes by.
4. She borrowed \$18, which we see from the point  $(0, 18)$  on the graph.
5. It took her 6 weeks to pay back the \$18, which we can see from the point  $(6, 0)$  on the graph.

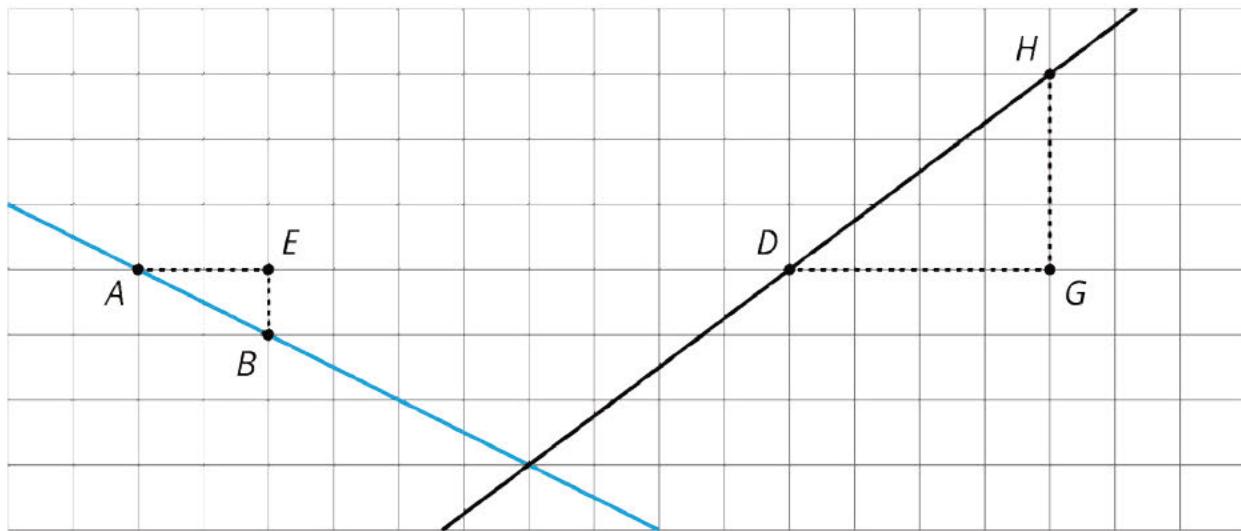
### Activity Synthesis

Ask students to share answers to each question and indicate how to use the graph to find the answers. For example, draw a slope triangle for the first question: the slope is -3 rather than 3, because the number of dollars owed is decreasing over time. Label the vertical intercept for the amount Elena borrowed and the horizontal intercept for the time it took her to pay back the loan.

### Lesson Synthesis

In this lesson, students learned that the slope of a line can be a negative value or 0. They saw some linear relationships with a negative slope and some with 0 slope. Students learned about cues to identify whether a graphed line has a positive slope, a negative slope, or 0 slope.

Display the graph for all to see. Ask students to pretend that their partner has been absent from class for a few days. Their job is to explain, verbally or in writing, how someone would figure out the slope of one of the graphed lines. Then, switch roles and listen to their partner explain how to figure out the slope of the other line.



## 9.5 The Slopes of Graphs

Cool Down: 5 minutes

### Addressing

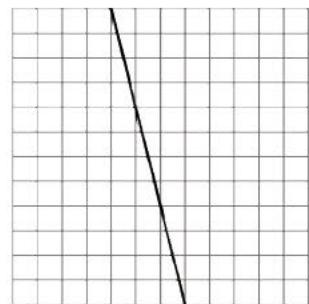
- 8.EE.B

#### Student Task Statement

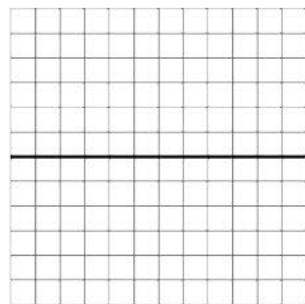
Each square on a grid represents 1 unit on each side.

- Calculate the slope of graph D. Explain or show your reasoning.
- Calculate the slope of graph E. What situation could the graph represent?
- On the blank grid F, draw a line that passes through the indicated point and has slope -2.

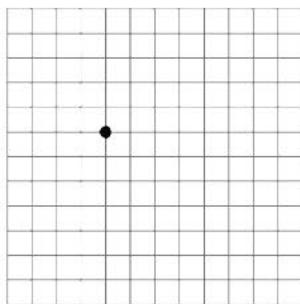
D



E



F



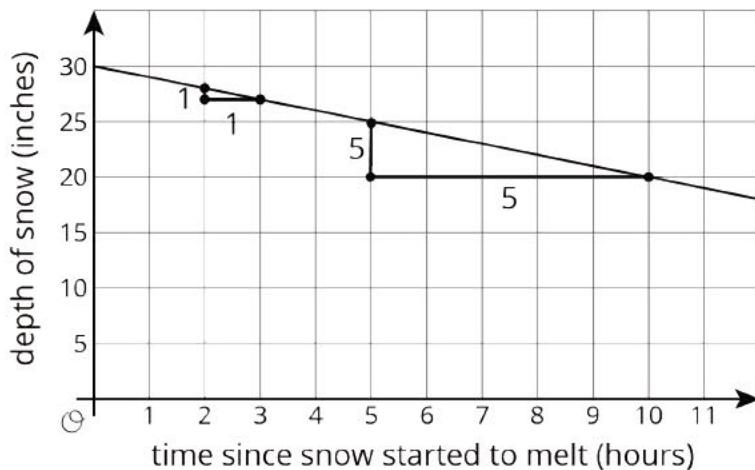
#### Student Response

- 4. Explanations vary. Sample explanation: for each horizontal change of 1, the vertical change is -4, so the slope of the line is -4.
0. Responses vary. Sample response: The amount of rainfall on a day with no rain.

3. A graph through the indicated point with a slope of -2.

## Student Lesson Summary

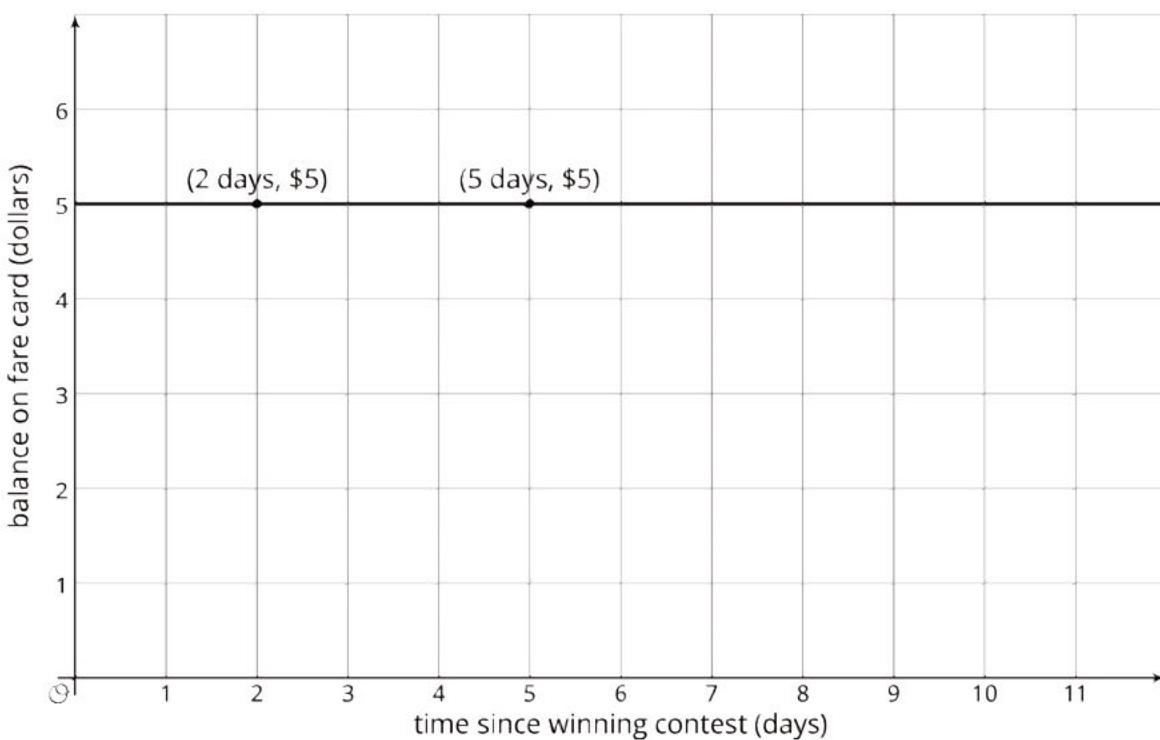
At the end of winter in Maine, the snow on the ground was 30 inches deep. Then there was a particularly warm day and the snow melted at the rate of 1 inch per hour. The graph shows the relationship between the time since the snow started to melt and the depth of the snow.



The slope of the graph is -1 since the rate of change is -1 inch per hour. That is, the depth goes *down* 1 inch per hour. The vertical intercept is 30 since the snow was 30 inches deep when the warmth started to melt the snow. The two slope triangles show how the rate of change is constant. It just also happens to be negative in this case since after each hour that passes, there is 1 inch *less* snow.

Graphs with negative slope often describe situations where some quantity is decreasing over time, like the depth of snow on warm days or the amount of money on a fare card being used to take rides on buses.

Slopes can be positive, negative, or even zero! A slope of 0 means there is no change in the  $y$ -value even though the  $x$ -value may be changing. For example, Elena won a contest where the prize was a special pass that gives her free bus rides for a year. Her fare card had \$5 on it when she won the prize. Here is a graph of the amount of money on her fare card after winning the prize:



The vertical intercept is 5, since the graph starts when she has \$5 on her fare card. The slope of the graph is 0 since she doesn't use her fare card for the next year, meaning the amount on her fare card doesn't change for a year. In fact, all graphs of linear relationships with slopes equal to 0 are horizontal—a rate of change of 0 means that, from one point to the next, the  $y$ -values remain the same.

## Lesson 9 Practice Problems

### Problem 1

#### Statement

Suppose that during its flight, the elevation  $e$  (in feet) of a certain airplane and its time  $t$ , in minutes since takeoff, are related by a linear equation. Consider the graph of this equation, with time represented on the horizontal axis and elevation on the vertical axis. For each situation, decide if the slope is positive, zero, or negative.

- The plane is cruising at an altitude of 37,000 feet above sea level.
- The plane is descending at rate of 1000 feet per minute.
- The plane is ascending at a rate of 2000 feet per minute.

#### Solution

- Zero
- Negative

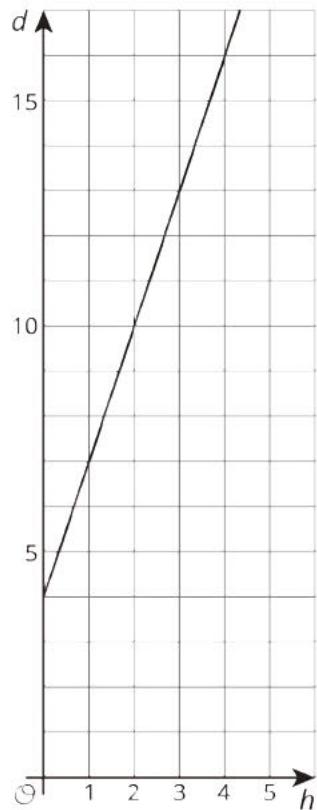
c. Positive

## Problem 2

### Statement

A group of hikers park their car at a trail head and walk into the forest to a campsite. The next morning, they head out on a hike from their campsite walking at a steady rate. The graph shows their distance in miles,  $d$ , from the car after  $h$  hours of hiking.

- How far is the campsite from their car? Explain how you know.
- Write an equation that describes the relationship between  $d$  and  $h$ .
- After how many hours of hiking will they be 16 miles from their car? Explain or show your reasoning.



### Solution

- 4 miles. The  $y$ -intercept represents this initial distance before the start of the hike.
- $d = 4 + 3h$
- 4 hours. Explanations vary. Sample response: On the graph,  $d = 16$  when  $h = 4$ . The equation can be used to solve for  $h$  when  $d = 16$ :  $16 = 4 + 3h$ ,  $12 = 3h$  and  $h = 4$ .

(From Unit 3, Lesson 7.)

## Problem 3

### Statement

Elena's aunt pays her \$1 for each call she makes to let people know about her aunt's new business.

The table shows how much money Diego receives for washing windows for his neighbors.

number of windows	number of dollars
27	30
45	50
81	90

Select **all** the statements about the situation that are true.

- A. Elena makes more money for making 10 calls than Diego makes for washing 10 windows.
- B. Diego makes more money for washing each window than Elena makes for making each call.
- C. Elena makes the same amount of money for 20 calls as Diego makes for 18 windows.
- D. Diego needs to wash 35 windows to make as much money as Elena makes for 40 calls.
- E. The equation  $y = \frac{9}{10}x$ , where  $y$  is number of dollars and  $x$  is number of windows, represents Diego's situation.
- F. The equation  $y = x$ , where  $y$  is the number of dollars and  $x$  is the number of calls, represents Elena's situation.

### Solution

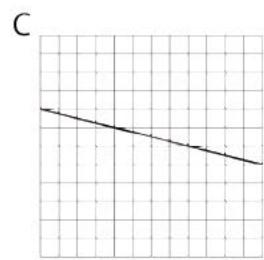
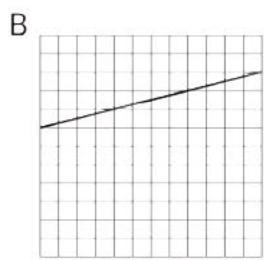
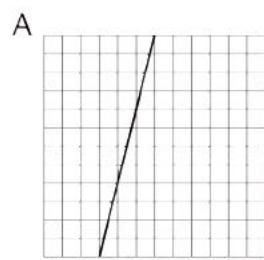
["B", "C", "F"]

(From Unit 3, Lesson 4.)

## Problem 4

### Statement

Each square on a grid represents 1 unit on each side. Match the graphs with the slopes of the lines.



- $-\frac{1}{4}$
- $\frac{1}{4}$
- 4

### Solution

A has slope 4, B has slope  $\frac{1}{4}$ , C has slope  $-\frac{1}{4}$

# Lesson 10: Calculating Slope

## Goals

- Create a graph of a line using a verbal description of its features.
- Describe (orally) the graph of a line using formal or informal language precisely enough to identify a unique line.
- Generate a method to find slope values given two points on the line.

## Learning Targets

- I can calculate positive and negative slopes given two points on the line.
- I can describe a line precisely enough that another student can draw it.

## Lesson Narrative

Students extend their work with slope triangles to develop a method for finding the slope of any line given the coordinates of two points on the line. They practice finding slopes this way and use a graph in order to check their answer (especially the sign).

Then students consider what information is sufficient to define (and accurately communicate) the position of a line in the coordinate plane. Lines with positive and negative slope are examined as students move flexibly between coordinates of points on a line, the slope of the line, and the graph showing the “uphill” or “downhill” orientation of the line. In order to communicate the location of the lines clearly, students engage in MP6. Many methods for describing the location of the lines are available, but students need to calculate carefully and use the coordinate grid in order to communicate the positions of the line clearly.

## Alignments

### Building On

- 7.NS.A: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

### Addressing

- 8.EE.B: Understand the connections between proportional relationships, lines, and linear equations.
- 8.EE.B.6: Use similar triangles to explain why the slope  $m$  is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation  $y = mx$  for a line through the origin and the equation  $y = mx + b$  for a line intercepting the vertical axis at  $b$ .

### Instructional Routines

- MLR4: Information Gap Cards
- MLR7: Compare and Connect

- Number Talk

### Required Materials

#### Graph paper

**Pre-printed slips, cut from copies of the blackline master**

#### Straightedges

A rigid edge that can be used for drawing line segments. Sometimes a ruler is okay to use as a

straightedge, but sometimes it is preferable to use an unruled straightedge, like a blank index card.

### Required Preparation

Print and cut up slips from the Making Designs blackline master. Prepare 1 copy for every 2 students.

#### Student Learning Goals

Let's calculate slope from two points.

## 10.1 Number Talk: Integer Operations

#### Warm Up: 5 minutes

Only three problems are given to allow time to discuss different values of  $a$  and  $b$  for each problem. It may not be possible to share every student's responses, given limited time. Consider gathering only two or three different sets of values per problem. Each problem was chosen to elicit a different understanding of integer operations, so as students share theirs, ask how the information in the equation impacted their choices for  $a$  and  $b$ .

This warm-up is intended as an opportunity to review operations on positive and negative numbers.

#### Building On

- 7.NS.A

#### Instructional Routines

- Number Talk

#### Launch

Display the problems all at once. Give students 1 minute of quiet think time per problem and ask them to give a signal when they have at least one set of values for each question. Follow with a whole-class discussion.

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### Support for Students with Disabilities

*Representation: Internalize Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.

*Supports accessibility for: Memory; Organization*

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### Anticipated Misconceptions

Students may have forgotten that the quotient of two negative numbers is positive.

#### Student Task Statement

Find values for  $a$  and  $b$  that make each side have the same value.

$$\frac{a}{b} = -2$$

$$\frac{a}{b} = 2$$

$$a - b = -2$$

#### Student Response

- Answers vary. Sample responses: Any combination of two numbers with different signs for which, in absolute value,  $a$  is twice  $b$ . Examples:  $6 \div -3$ ,  $-6 \div 3$
- Answers vary. Sample responses: Any combination of two numbers with the same sign in which, in absolute value,  $a$  is twice  $b$ . Examples:  $-6 \div -3$ ,  $6 \div 3$
- Answers vary. Sample response: Any combination of two numbers in which  $b$  is two more than  $a$ . Examples:  $6 - 8$ ,  $-6 - (-4)$ ,  $0 - 2$ ,  $-2 - 0$ ,  $-1 - 1$

#### Activity Synthesis

Ask students to share their values for  $a$  and  $b$  for each problem. Include at least one set of values for each problem where  $a$  or  $b$  (or both) are negative. Record and display their values for  $a$  and  $b$  for all to see. Ask students how they decided on their values based on the information given in the equation. To involve more students in the conversation, consider asking:

- “Did anyone choose the same values?”
- “Who can restate \_\_\_'s reasoning in a different way?”
- “Did anyone choose different values?”
- “Does anyone want to add on to \_\_\_'s reasoning?”
- “Do you agree or disagree? Why?”

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### Support for English Language Learners

*Speaking: MLR8 Discussion Supports.*: Display sentence frames to support students when they explain their strategy. For example, "First, I \_\_\_\_ because . . ." or "I noticed \_\_\_\_ so I . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

*Design Principle(s): Optimize output (for explanation)*

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## 10.2 Toward a More General Slope Formula

**15 minutes**

The purpose of this activity is to compute the slopes of different lines to get familiar with the formula "subtract  $y$ -coordinates, subtract  $x$ -coordinates, then divide." Students first compute slopes for some lines with positive slopes, and then special attention is drawn to the fact that a line has a negative slope. Students attend to what makes the same computations have a negative result instead of a positive result.

### Addressing

- 8.EE.B

### Instructional Routines

- MLR7: Compare and Connect

### Launch

Have partners figure out the slope of the line that passes through each pair of points.

1. (12, 4) and (7, 1) (answer:  $\frac{3}{5}$ )
2. (4, -11) and (7, -8) (answer: 1)
3. (1, 2) and (600, 3) (answer:  $\frac{1}{599}$ )
4. (37, 40) and (30, 33) (answer: 1)

Ask students to share their results and how they did it. Students may say they just found the difference between the numbers; making this more precise is part of the goal of the discussion for this activity. Ask students to complete the questions in the task and share their responses with a partner before class discussion. Provide access to graph paper and rulers.

---

### Support for Students with Disabilities

*Representation: Develop Language and Symbols.* Create a display of important terms and vocabulary. Include the following terms and maintain the display for reference throughout the unit: slope. Invite students to suggest language or diagrams to include on the display that will support their understanding of this term.

*Supports accessibility for: Memory; Language*

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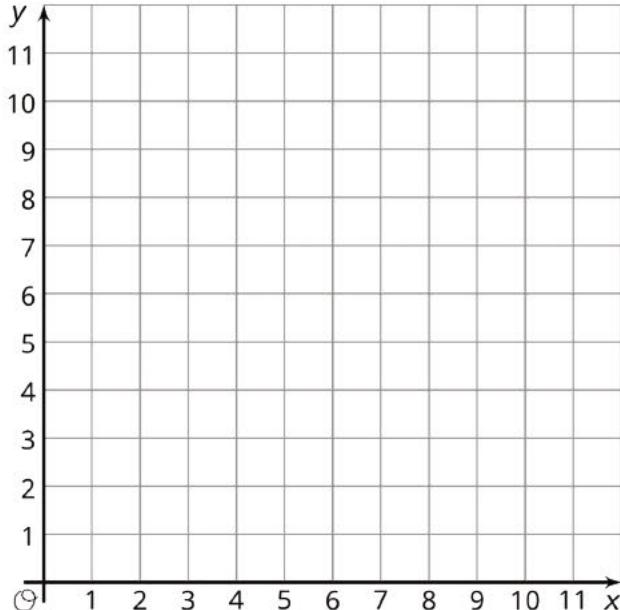
### Anticipated Misconceptions

Students may struggle with operations on negative numbers and keeping everything straight if they try to take a purely algorithmic approach. Encourage them to plot both points and reason about the length of the vertical and horizontal portions of the slope triangle and the sign of the slope of the line, one step at a time. Sketching a graph of the line is also useful for verifying the sign of the line's slope.

It is common for students to calculate a slope and leave it in a form such as  $\frac{-3}{5}$ . Remind students that this fraction is representing a division operation, and the quotient would be a positive value.

#### Student Task Statement

1. Plot the points  $(1, 11)$  and  $(8, 2)$ , and use a ruler to draw the line that passes through them.
2. Without calculating, do you expect the slope of the line through  $(1, 11)$  and  $(8, 2)$  to be positive or negative? How can you tell?
3. Calculate the slope of this line.



#### Student Response

1. A line is graphed that passes through  $(1, 11)$  and  $(8, 2)$ .
2. Negative. Explanations vary. Sample responses: Because it's a downhill line; because  $y$  is getting smaller as  $x$  gets bigger.

3.  $\frac{-9}{7}$ , because  $11 - 2 = 9$  and  $1 - 8 = -7$ , and  $9 \div -7 = \frac{-9}{7}$ .

### Are You Ready for More?

Find the value of  $k$  so that the line passing through each pair of points has the given slope.

1.  $(k, 2)$  and  $(11, 14)$ , slope = 2
2.  $(1, k)$  and  $(4, 1)$ , slope = -2
3.  $(3, 5)$  and  $(k, 9)$ , slope =  $\frac{1}{2}$
4.  $(-1, 4)$  and  $(-3, k)$ , slope =  $\frac{-1}{2}$
5.  $(\frac{-15}{2}, \frac{3}{16})$  and  $(\frac{-13}{22}, k)$ , slope = 0

### Student Response

1. 5
2. 7
3. 11
4. 5
5.  $\frac{3}{16}$

### Activity Synthesis

When using two points to calculate the slope of a line, care needs to be taken to subtract the  $x$  and  $y$  values in the same order. Using the first pair as an example, the slope could be calculated either of these ways:

$$\frac{4 - 1}{12 - 7} = \frac{3}{5}$$
$$\frac{1 - 4}{7 - 12} = \frac{-3}{-5} = \frac{3}{5}$$

But if one of the orders were reversed, this would yield a negative value for the slope when we know the slope should have a positive value.

It is worth demonstrating, or having a student demonstrate an algorithmic approach to finding the negative slope in the last part of the task. Draw attention to the fact that keeping the coordinates “in the same order” results in the numerator and denominator having opposite signs (one positive and one negative), so that their quotient must be negative. It might look like:

$$\frac{11 - 2}{1 - 8} = \frac{9}{-7} = -\frac{9}{7}$$

Ask students how sketching a graph of the line will tell them whether the slope is positive or negative. They should recognize that when it goes up, from left to right, the slope is positive, and when it goes down, then the slope is negative. Using a sketch of the graph can also be helpful to judge whether the magnitude of the (positive or negative) slope is reasonable.

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### Support for English Language Learners

*Representing, Speaking: MLR7 Compare and Connect.* Use this support as students calculate the slope of the line between the points  $(1, 11)$  and  $(8, 2)$ . Invite students to demonstrate their strategy using a visual or numerical representation. In pairs or groups, ask students to compare their strategies. Ask students to discuss how the strategies are the same and/or different, and then share with the whole class. This will help students connect how different approaches led to the same result of a negative slope by keeping coordinates “in the same order.”

*Design Principle(s): Optimize output (for comparison); Maximize meta-awareness*

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## 10.3 Making Designs

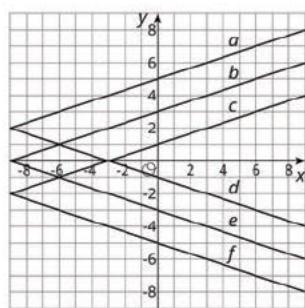
**20 minutes**

The goal of this activity is for students to recognize information that determines the location of a line in the coordinate plane, and to practice distinguishing between positive and negative slopes. In this activity, one partner has a design that they verbally describe to their partner, who then tries to draw it. The purpose of this activity is to provide an environment where students have to describe or interpret the slope and locations of several lines. (Students are not expected to communicate by saying the equations of the lines, though there is nothing stopping them from doing so.) Students take turns describing and interpreting by doing this two times with two different designs.

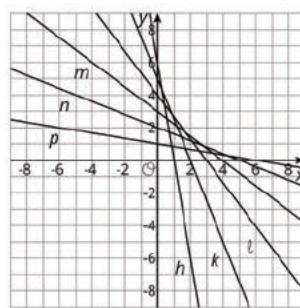
Monitor for students who use language of slope and vertical or horizontal intercepts to communicate the location of each line. Invite these students to share during the discussion. There are many other ways students might communicate the location of each line, but the recent emphasis on studying slope and intercepts should make these choices natural.

The two designs in the blackline masters look like this:

Making Designs  
Design 1



Making Designs  
Design 2



You will need the Info Gap: Making Designs blackline master for this activity.

Thanks to [Henri Picciotto](#) for permission to use these designs.

## Addressing

- 8.EE.B

## Instructional Routines

- MLR4: Information Gap Cards

## Launch

Tell students they will describe some lines to a partner to try and get them to recreate a design. The protocol is described in the student task statement. Consider asking a student to serve as your partner to demonstrate the protocol to the class before distributing the designs and blank graphs.

Arrange students in groups of 2. Provide access to straightedges.

From the blackline master that you have copied and cut up ahead of time, give one partner the design, and the other partner a blank graph. Arrange the room to ensure that the partner drawing the design cannot peek at the design anywhere in the room. Once the first design has been successfully created, provide the second design and a blank graph to the other student in each partnership.

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## Support for Students with Disabilities

*Representation: Provide Access for Perception.* Display or provide students with a physical copy of the written directions and read them aloud. Check for understanding by inviting students to rephrase directions in their own words. Keep directions visible throughout the activity.

*Supports accessibility for: Language; Memory*

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## Support for English Language Learners

*Conversing:* Use this modified version of *MLR4 Information Gap* to give students an opportunity to describe (orally) the graph of a line using formal or informal language. Circulate and listen for common words or phrases students use to describe the designs. Record this language and display for all to see. Encourage students to borrow words or phrases from the display as needed.

*Design Principle(s): Cultivate conversation*

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## Student Task Statement

Your teacher will give you either a design or a blank graph. Do not show your card to your partner.

If your teacher gives you the design:

1. Look at the design silently and think about how you could communicate what your partner should draw. Think about ways that you can describe what a line looks like, such as its slope or points that it goes through.
2. Describe each line, one at a time, and give your partner time to draw them.
3. Once your partner thinks they have drawn all the lines you described, only then should you show them the design.

When finished, place the drawing next to the card with the design so that you and your partner can both see them. How is the drawing the same as the design? How is it different? Discuss any miscommunication that might have caused the drawing to look different from the design.

Pause here so your teacher can review your work. When your teacher gives you a new set of cards, switch roles for the second problem.

### Student Response

Student designs should match those in the blackline master.

### Activity Synthesis

After students have completed their work, ask students to discuss the process of communicating how to draw a line. Some guiding questions:

- "What details were important to pay attention to?"
- "How did you use coordinates to help communicate where the line is?"
- "How did you use slope to communicate how to draw the line?"
- "Were there any cases where your partner did not give enough information to know where to draw the line? What more information did you need?"

Students might notice that the lines with the same slope can be described in terms of translations (for example, line  $b$  is a vertical translation of line  $a$  down two units). This is an appropriate use of rigid motion language which recalls work done earlier in this unit. Students might also describe  $b$  as parallel to  $a$  and containing the point  $(0, 3)$ . Finally, some students might use equations to communicate the location of the lines. All of these methods are appropriate. Keep the discussion focused on describing each line using slope and coordinates for each individual line on its own.

If your teacher gives you the blank graph:

1. Listen carefully as your partner describes each line, and draw each line based on their description.
2. You are not allowed to ask for more information about a line than what your partner tells you.
3. Do not show your drawing to your partner until you have finished drawing all the lines they describe.

## Lesson Synthesis

In this lesson, students explored the interplay between the coordinates of points on a line and the slope of that line, where the slope could be positive or negative.

Ask students, “What information do you need to know exactly where a line is?” Valid responses might be the coordinates of two points on the line or the coordinates of one point and the line’s slope. Demonstrate why knowing one point is not enough information (the line goes through it, but could have any slope), and why only knowing the slope is not enough information (you know at what slope to draw the line, but it could be located anywhere—it could be any of a set of parallel lines). It can be helpful to use a yardstick to represent “the line” in this situation as you move it around a coordinate plane on the board.

Ask students, “If you know the coordinates of two points on a line, how can you tell if it has a positive or negative slope?” Responses might include sketching a graph of the line to see if it’s “uphill” or “downhill,” or an algorithm involving subtraction and division, attending to keeping coordinates in “the same order” and performing operations correctly.

## 10.4 Different Slopes

### Cool Down: 5 minutes

Students calculate the slope of the line through two points. They are only given the coordinates of the points and are specifically directed not to graph the line. By now, students should have internalized an efficient method for finding slope using the coordinates of two points on the line.

### Addressing

- 8.EE.B.6

#### Student Task Statement

Without graphing, find the slope of the line that goes through

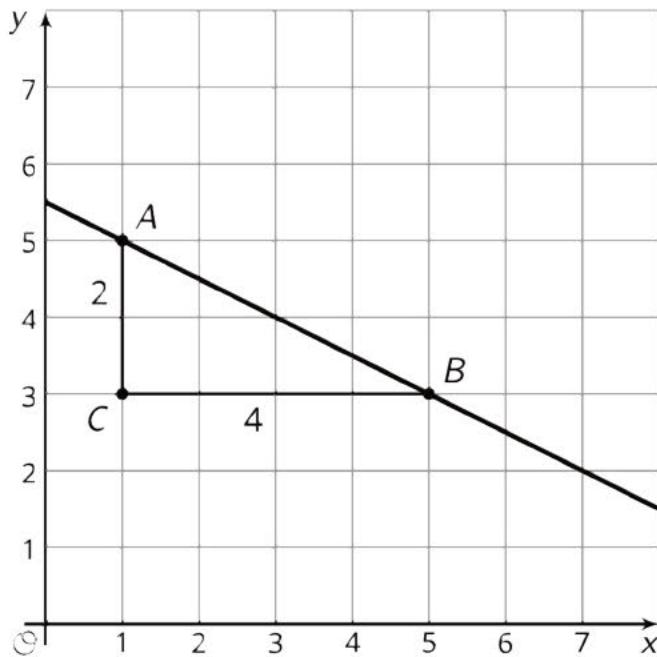
1. (0, 5) and (8, 2).
2. (2, -1) and (6, 1).
3. (-3, -2) and (-1, -5).

#### Student Response

1.  $\frac{-3}{8}$
2.  $\frac{1}{2}$
3.  $\frac{-3}{2}$

## Student Lesson Summary

We learned earlier that one way to find the slope of a line is by drawing a slope triangle. For example, using the slope triangle shown here, the slope of the line is  $-\frac{2}{4}$ , or  $-\frac{1}{2}$  (we know the slope is negative because the line is decreasing from left to right).



But slope triangles are only one way to calculate the slope of a line. Let's compute the slope of this line a different way using just the points  $A = (1, 5)$  and  $B = (5, 3)$ . Since we know the slope is the vertical change divided by the horizontal change, we can calculate the change in the  $y$ -values and then the change in the  $x$ -values. Between points  $A$  and  $B$ , the  $y$ -value change is  $3 - 5 = -2$  and the  $x$ -value change is  $5 - 1 = 4$ . This means the slope is  $-\frac{2}{4}$ , or  $-\frac{1}{2}$ , which is the same as what we found using the slope triangle.

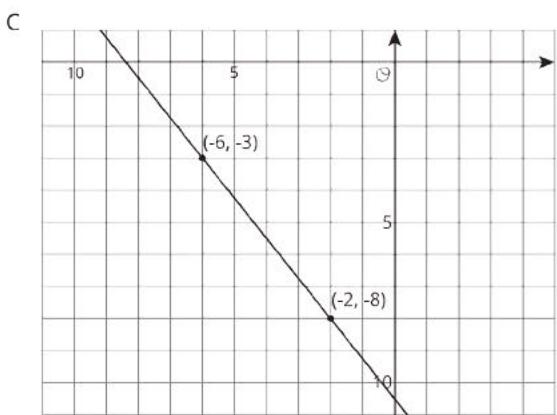
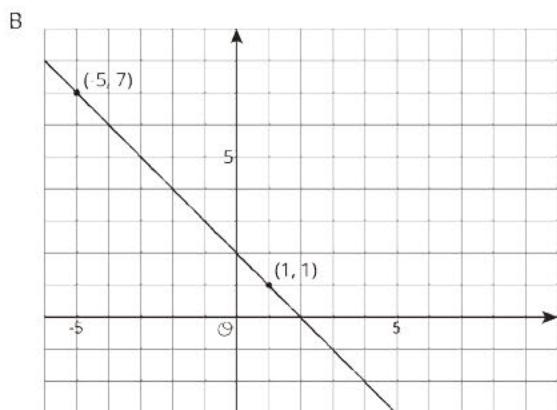
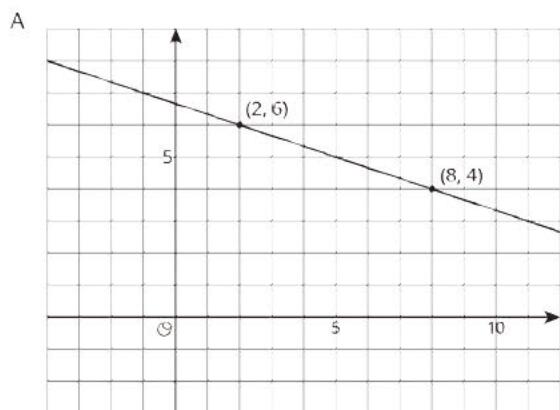
Notice that in each of the calculations, we subtracted the value from point  $A$  from the value from point  $B$ . If we had done it the other way around, then the  $y$ -value change would have been  $5 - 3 = 2$  and the  $x$ -value change would have been  $1 - 5 = -4$ , which still gives us a slope of  $-\frac{1}{2}$ . But what if we were to mix up the orders? If that had happened, we would think the slope of the line is *positive*  $\frac{1}{2}$  since we would either have calculated  $\frac{-2}{4}$  or  $\frac{2}{4}$ . Since we already have a graph of the line and can see it has a negative slope, this is clearly incorrect. If we don't have a graph to check our calculation, we could think about how the point on the left,  $(1, 5)$ , is higher than the point on the right,  $(5, 3)$ , meaning the slope of the line must be negative.

## Lesson 10 Practice Problems

### Problem 1

#### Statement

For each graph, calculate the slope of the line.



## Solution

A:  $-\frac{2}{6}$ , B:  $-1$ , C:  $\frac{-5}{4}$

## Problem 2

### Statement

Match each pair of points to the slope of the line that joins them.

A. $(9, 10)$ and $(7, 2)$	1. $4$
B. $(-8, -11)$ and $(-1, -5)$	2. $-3$
C. $(5, -6)$ and $(2, 3)$	3. $-\frac{5}{2}$
D. $(6, 3)$ and $(5, -1)$	4. $\frac{6}{7}$
E. $(4, 7)$ and $(6, 2)$	

## Solution

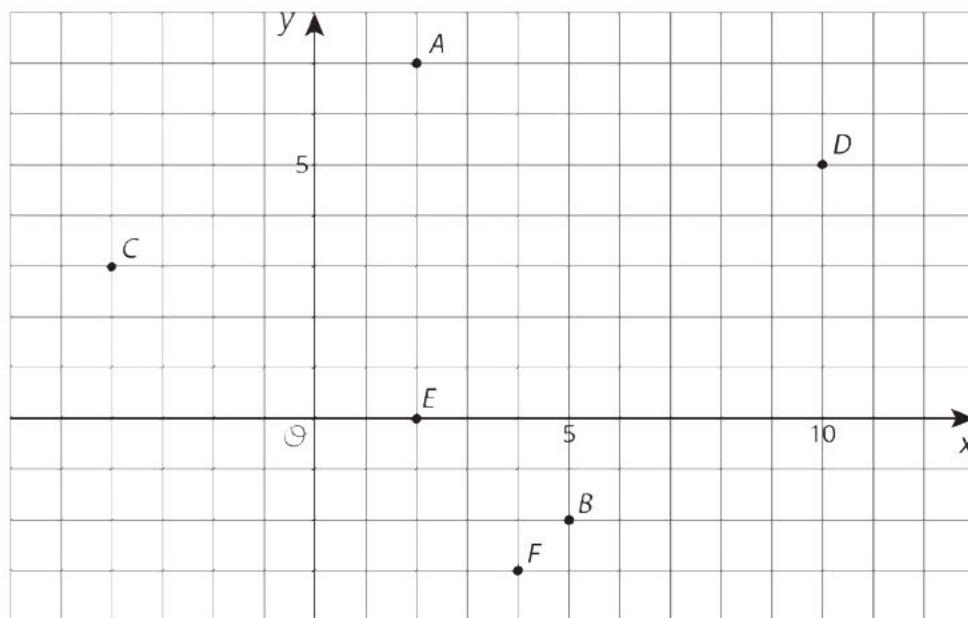
- A: 1

- B: 4
- C: 2
- D: 1
- E: 3

### Problem 3

#### Statement

Draw a line with the given slope through the given point. What other point lies on that line?



- a. Point A, slope = -3
- b. Point A, slope =  $-\frac{1}{4}$
- c. Point C, slope =  $-\frac{1}{2}$
- d. Point E, slope =  $-\frac{2}{3}$

#### Solution

- a. Point B
- b. Point D
- c. Point E
- d. Point B

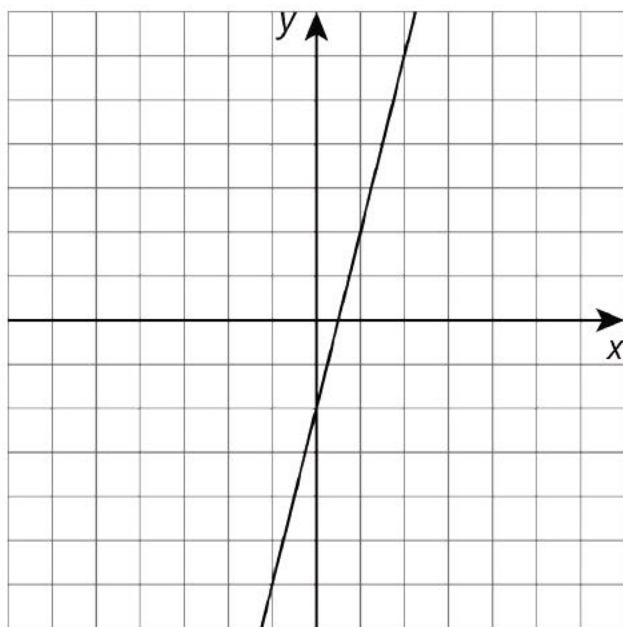
## Problem 4

### Statement

Make a sketch of a linear relationship with a slope of 4 and a negative  $y$ -intercept. Show how you know the slope is 4 and write an equation for the line.

### Solution

Answers vary. Sample response:



The equation is  $y = 4x - 2$ . I can tell the slope is 4 by looking at the points  $(0, -2)$  and  $(1, 2)$  since  $\frac{2-(-2)}{1-0} = \frac{4}{1} = 4$ .

(From Unit 3, Lesson 8.)

# Lesson 11: Equations of All Kinds of Lines

## Goals

- Comprehend that for the graph of a vertical or horizontal line, one variable does not vary, while the other can take any value.
- Create multiple representations of linear relationship, including a graph, equation, and table.
- Generalize (in writing) that a set of points of the form  $(x, b)$  satisfy the equation  $y = b$  and that a set of points of the form  $(a, y)$  satisfy the equation  $x = a$ .

## Learning Targets

- I can write equations of lines that have a positive or a negative slope.
- I can write equations of vertical and horizontal lines.

## Lesson Narrative

In previous lessons, students have studied lines with positive and negative slope and have learned to write equations for them, usually in the form  $y = mx + b$ . In this lesson, students extend their previous work to include equations for horizontal and vertical lines. Horizontal lines can still be written in the form  $y = mx + b$  but because  $m = 0$  in this case, the equation simplifies to  $y = b$ . Students interpret this to mean that, for a horizontal line, the  $y$  value does not change, but  $x$  can take *any* value. This structure is identical for vertical lines except that now the equation has the form  $x = a$  and it is  $x$  that is determined while  $y$  can take any value.

Note that the equation of a vertical line cannot be written in the form  $y = mx + b$ . It can, however, be written in the form  $Ax + By = C$  (with  $B = 0$ ). This type of linear equation will be studied in greater detail in upcoming lessons. In this lesson, students encounter a context where this form arises naturally: if a rectangle has length  $\ell$  and width  $w$  and its perimeter is 50, this means that  $2\ell + 2w = 50$ .

## Alignments

### Building On

- 7.G.A: Draw, construct, and describe geometrical figures and describe the relationships between them.

### Addressing

- 8.EE.B: Understand the connections between proportional relationships, lines, and linear equations.
- 8.EE.B.6: Use similar triangles to explain why the slope  $m$  is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation  $y = mx$  for a line through the origin and the equation  $y = mx + b$  for a line intercepting the vertical axis at  $b$ .

## Instructional Routines

- MLR2: Collect and Display
- MLR7: Compare and Connect
- Which One Doesn't Belong?

## Required Materials

String

## Required Preparation

Take a piece of string 50 centimeters long and tie the ends together to be used as demonstration in the third activity.

### Student Learning Goals

Let's write equations for vertical and horizontal lines.

## 11.1 Which One Doesn't Belong: Pairs of Lines

### Warm Up: 5 minutes

This warm-up prompts students to compare four pairs of lines. It invites students to explain their reasoning and hold mathematical conversations, and allows you to hear how they use terminology and talk about lines. To allow all students to access the activity, each figure has one obvious reason it does not belong. Encourage students to find reasons based on geometric properties (e.g., only one set of lines are not parallel, only one set of lines have negative slope).

### Building On

- 7.G.A

## Instructional Routines

- Which One Doesn't Belong?

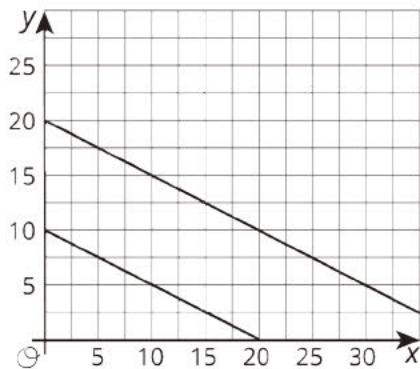
### Launch

Arrange students in groups of 2–4. Display the image of the four graphs for all to see. Ask students to indicate when they have noticed one graph that does not belong and can explain why. Give students 2 minutes of quiet think time and then time to share their thinking with their group. After everyone has conferred in groups, ask the group to offer at least one reason *each* graph doesn't belong.

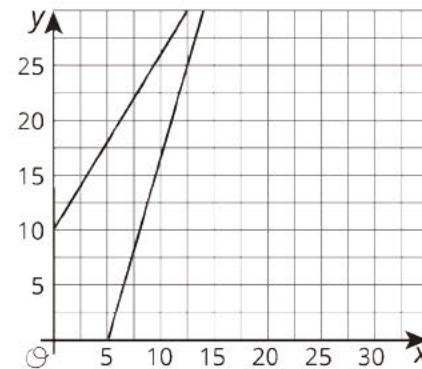
### Student Task Statement

Which one doesn't belong?

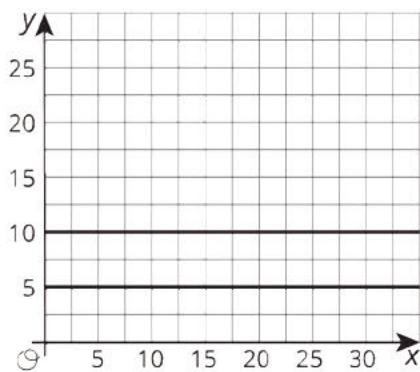
A



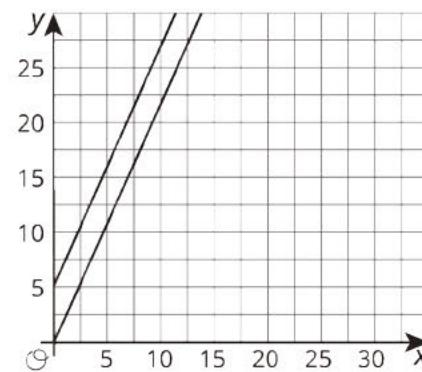
B



C



D



### Student Response

Answers vary. Some sample responses:

- A: The lines have a negative slope.
- B: The lines are not parallel.
- C: The lines have 0 slope.
- D: Neither line goes through the point  $(0, 10)$ .

### Activity Synthesis

After students have conferred in groups, invite each group to share one reason why a particular pair of lines might not belong. Record and display the responses for all to see. After each response, ask the rest of the class if they agree or disagree. Since there is no single correct answer to the question asking which shape does not belong, attend to students' explanations and ensure the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology they use, such as parallel, intersect, origin, coordinate, ordered pair, quadrant or slope. Also, press students on

claims of lines being parallel to one another. Ask students how they know they are parallel and highlight ideas about slope.

## 11.2 All the Same

15 minutes (there is a digital version of this activity)

In previous lessons, students have studied lines with positive slope, negative slope, and 0 slope and have written equations for lines with positive and negative slope. In this activity, they write equations for horizontal lines (lines of slope 0) and vertical lines and they graph horizontal and vertical lines from equations. Students explain their reasoning (MP3).

Horizontal lines can be thought of as being described by equations of the form  $y = mx + b$  where  $m = 0$ . In other words, a horizontal line can be thought of as a line with slope 0. Vertical lines, on the other hand, cannot be described by an equation of the form  $y = mx + b$ .

### Addressing

- 8.EE.B.6

### Instructional Routines

- MLR2: Collect and Display

### Launch

Allow students quiet think time. Instruct them to pause their work after question 2 and discuss which equation makes sense and why. Tell students to resume working and pause again after question 4 for discussion. Discuss why the equations only contain one variable and what this means about the relationship between the quantities represented by  $x$  and  $y$ . After students complete questions 5 and 6, ask if they can think of some real-world situations that can be represented by vertical and horizontal lines.

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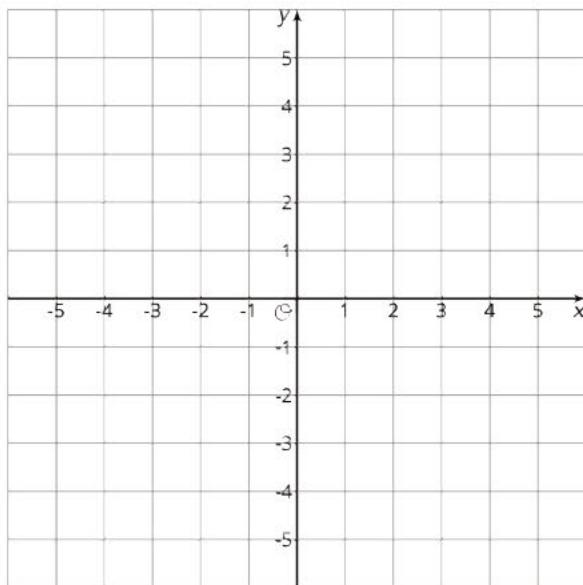
### Support for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Chunk this task into more manageable parts to support students who benefit from support with organizational skills. For example, present one question at a time.

*Supports accessibility for: Organization; Attention*

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### Student Task Statement



1. Plot at least 10 points whose  $y$ -coordinate is  $-4$ . What do you notice about them?
2. Which equation makes the most sense to represent all of the points with  $y$ -coordinate  $-4$ ? Explain how you know.

$$x = -4$$

$$y = -4x$$

$$y = -4$$

$$x + y = -4$$

3. Plot at least 10 points whose  $x$ -coordinate is  $3$ . What do you notice about them?
4. Which equation makes the most sense to represent all of the points with  $x$ -coordinate  $3$ ? Explain how you know.

$$x = 3$$

$$y = 3x$$

$$y = 3$$

$$x + y = 3$$

5. Graph the equation  $x = -2$ .
6. Graph the equation  $y = 5$ .

### Student Response

1. Answers vary. Sample responses: Points all lie on a horizontal line that crosses the  $y$ -axis at  $-4$ . Points all lie on a line parallel to and 4 units down from  $x$ -axis.
2.  $y = -4$  is the only equation that is true for every point we graphed and all points for which the  $y$ -coordinate is  $-4$ .
3. Answers vary. Sample responses: Points all lie on a vertical line that crosses the  $x$ -axis at  $3$ . Points all lie on a line parallel to and 3 units to the right of the  $y$ -axis.
4.  $x = 3$ .

5. A vertical line through  $(-2, 0)$ .
6. A horizontal line through  $(0, 5)$ .

### Are You Ready for More?

1. Draw the rectangle with vertices  $(2, 1), (5, 1), (5, 3), (2, 3)$ .
2. For each of the four sides of the rectangle, write an equation for a line containing the side.
3. A rectangle has sides on the graphs of  $x = -1, x = 3, y = -1, y = 1$ . Find the coordinates of each vertex.

### Student Response

1. Figure: Graph showing first quadrant, axes marked in integers from 1 to 6 horizontally and 1 to 4 vertically, points marked and labeled at  $(2, 1), (5, 1), (5, 3), (2, 3)$ , points connected by horizontal and vertical lines to form a rectangle.
2.  $x = 2, x = 5, y = 1, y = 3$ .
3.  $(-1, -1), (3, -1), (3, 1), (-1, 1)$

### Activity Synthesis

In order to highlight the structure of equations of vertical and horizontal lines, ask students:

- "Why does the equation for the points with  $y$ -coordinate  $-4$  not contain the variable  $x$ ?" ( $x$  can take any value while  $y$  is always  $-4$ . The only constraint is on  $y$  and there is no dependence of  $x$  on  $y$ .)
- "Why does the equation for the points with  $x$ -coordinate  $3$  not contain the variable  $y$ ?" ( $y$  can take any value while  $x$  is always  $3$ . The only constraint is on  $x$  and there is no dependence of  $y$  on  $x$ .)
- "What does this say about the relationship between the quantities represented by  $x$  and  $y$  in these situations?" (Changes in one do not affect the other. One is not dependent on the other. They don't change together according to a formula.)
- "What would be some real-world examples of situations that could be represented by these types of equations?" (Examples: You pay the same fee regardless of your age; bus tickets cost the same no matter how far you travel; you remain the same distance from home as the hours pass during the school day.)

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### Support for English Language Learners

*Representing, Speaking, Listening: MLR2 Collect and Display.* As students discuss which equation makes sense and why with a partner, create a 2-column table with the headings “horizontal lines” and “vertical lines”. Circulate through the groups and record student language in the appropriate column. Look for phrases such as “ $x$  (or  $y$ ) is always the same,” “ $x$  (or  $y$ ) is always changing,” and “the slope is 0.” This will help students make sense of the structure of equations for horizontal and vertical lines.

*Design Principle(s): Support sense-making; Maximize meta-awareness*

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## 11.3 Same Perimeter

**15 minutes (there is a digital version of this activity)**

In this activity, students analyze a line and an equation defining the line in a geometric context. Students find pairs of numbers for the width and length of rectangles that all have the same perimeter. Next, they draw some of the rectangles with a vertex at the origin in the coordinate plane, and discover that the set of opposite vertices lie on a line. Finally, they write an equation for the line and consider how the slope of the line relates to the changing lengths and widths of the rectangles.

Identify students who come up with different equations for the line. An equation of the form  $2\ell + 2w = 50$ , where  $\ell$  is the length of the rectangle and  $w$  is its width, is natural if they are thinking about the context, namely, that the perimeter of the rectangle is 50 units. In fact, students have likely seen this equation in this context in grade 6. On the other hand, students who follow the steps in the task and draw the line connecting the rectangle vertices are likely to write an equation in the form  $w = 25 - \ell$  (or perhaps  $2w = 50 - 2\ell$ ) because the  $w$ -intercept of the graph is 25 and its slope is -1. Invite students who wrote these different forms for an equation to present during the discussion.

### Addressing

- 8.EE.B

### Instructional Routines

- MLR7: Compare and Connect

### Launch

Ask students to sketch a rectangle whose perimeter is 50 units and label the lengths of its sides. After giving them a minute to come up with their rectangle, ask them to share some of the lengths and widths they found. Examples might be 10 and 15, 5 and 20, or 1 and 24. Then demonstrate with a 50-cm long string with its ends tied together that a given perimeter can yield several different rectangles by varying the width and length. Ensure that everyone understands that rectangles have

4 sides, that rectangles have two pairs of congruent sides, and that there is more than one rectangle whose perimeter is 50 units.

### Support for Students with Disabilities

*Action and Expression: Develop Expression and Communication.* Maintain a display of important terms and vocabulary. During the launch take time to review the following terms from previous lessons that students will need to access for this activity: width, length, perimeter, rectangle, intercept, and slope.

*Supports accessibility for: Memory; Language*

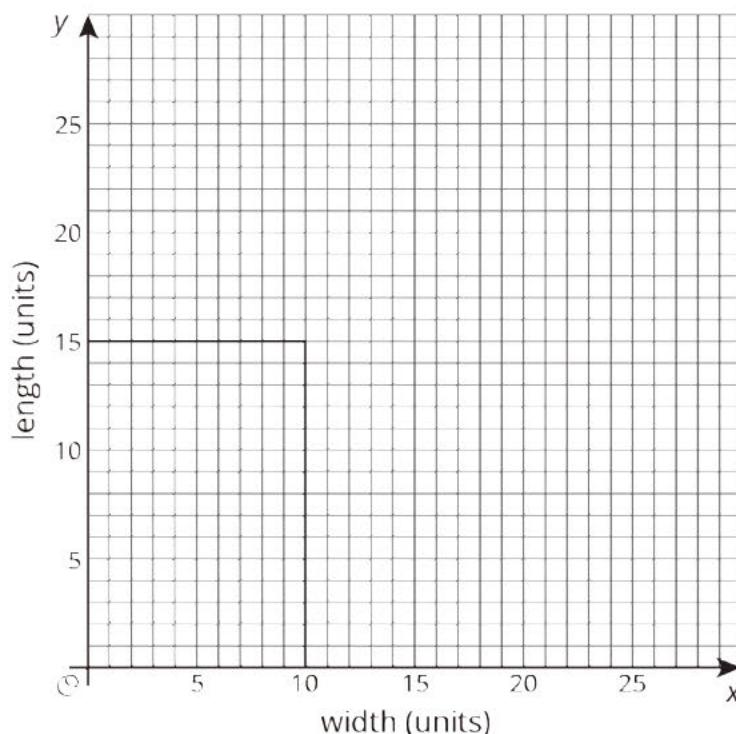
### Student Task Statement

1. There are many possible rectangles whose perimeter is 50 units. Complete the table with lengths,  $\ell$ , and widths,  $w$ , of at least 10 such rectangles.

$\ell$											
$w$											

2. The graph shows one rectangle whose perimeter is 50 units, and has its lower left vertex at the origin and two sides on the axes.

On the same graph, draw more rectangles with perimeter 50 units using the values from your table. Make sure that each rectangle has a lower left vertex at the origin and two sides on the axes.



3. Each rectangle has a vertex that lies in the first quadrant. These vertices lie on a line. Draw in this line and write an equation for it.

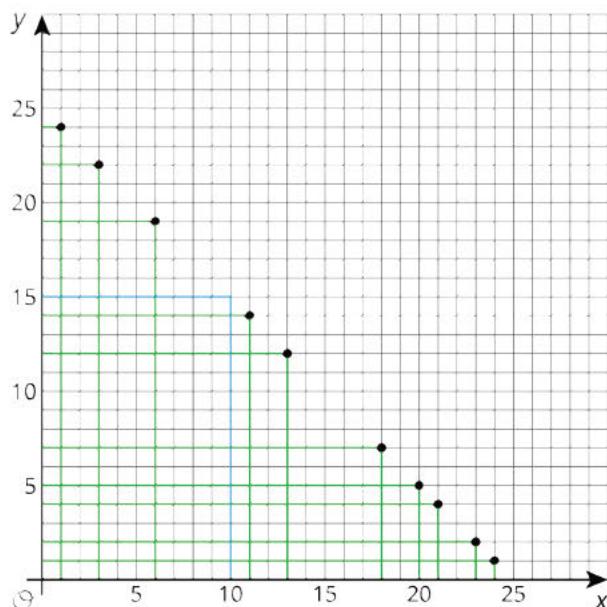
4. What is the the slope of this line? How does the slope describe how the width changes as the length changes (or vice versa)?

### Student Response

1. Answers vary. Sample response:

$\ell$	1	4	7	12	24	19	2	5	14	22
$w$	24	21	18	13	1	6	23	20	11	3

2. Answers vary. Sample response:



3. Answers vary. Some possibilities:  $2x + 2y = 50$ ,  $x + y = 25$ ,  $y = 25 - x$ ,  $y = -x + 25$

4. The slope is  $-1$ . Answers vary. Sample response: If you take away one from the width you have to add one to the length.

### Activity Synthesis

Invite students who write  $2\ell + 2w = 50$  for an equation of the line to share their response (if no one has written this equation for the third question, ask students if it is correct). This equation has the advantage of directly modeling the context: the perimeter of a rectangle is  $2\ell + 2w$ , so if the perimeter is 50, then  $2\ell + 2w = 50$ . An additional advantage to this form of the equation is that *every* line, including horizontal and vertical lines, can be written in this form:  $\ell = 5$  is an equation for a horizontal line while  $w = 3$  is an equation for a vertical line.

Invite students who write  $\ell = 25 - w$  (or some variant) to share. This equation is not apparent from the scenario, but it reveals a few interesting aspects of the problem:

- The rectangle has to have length less than 25 (since the width has to be positive)
- For each unit the length increases, the width decreases by one unit (in order to balance out when the sides of the rectangle are added to get the perimeter)

Ask students if the lengths and widths need to be whole numbers. In the next two lessons, students will encounter equations where the contexts determine what values the variables can take on. If students agree that the lengths and widths can take on any measurable value, ask how many different rectangles can be drawn. Practically speaking, the number is limited by what we can measure and draw with reasonable precision, while in theory, there are an infinite number of rectangles with perimeter 50.

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### Support for English Language Learners

*Representing, Conversing: MLR7 Compare and Connect.* As the selected students show and describe their equations for the line connecting the vertices of the rectangles, invite pairs to discuss "What is the same and what is different?" about their own equation and reasoning. Highlight connections between the equations by amplifying use of the targeted language (i.e., horizontal and vertical lines, perimeter, slope, length is less than, or width is less than). This will help students to better understand that different forms of the equation represent the same geometrical context.

*Design Principle(s): Maximize meta-awareness; Support sense-making*

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## Lesson Synthesis

Students have spent considerable time in the 7th and 8th grades solving problems with proportional relationships and non-proportional relationships that can be represented by equations and graphs with positive slopes. Ask students to now consider real-world situations where slopes are not positive.

Ask students, "how can you tell from a real-world situation that the graph of the equation that represents it will be a horizontal line? Be a vertical line? Have a negative slope?"

For horizontal and vertical lines, the key feature is that one of the two variables does *not* vary while the other one can take *any* value. In the  $x$ - $y$  plane, when the variable  $x$  can take any value, it is a vertical line, and when the variable  $y$  can take any value, it is a horizontal line.

## 11.4 Line Design

### Cool Down: 5 minutes

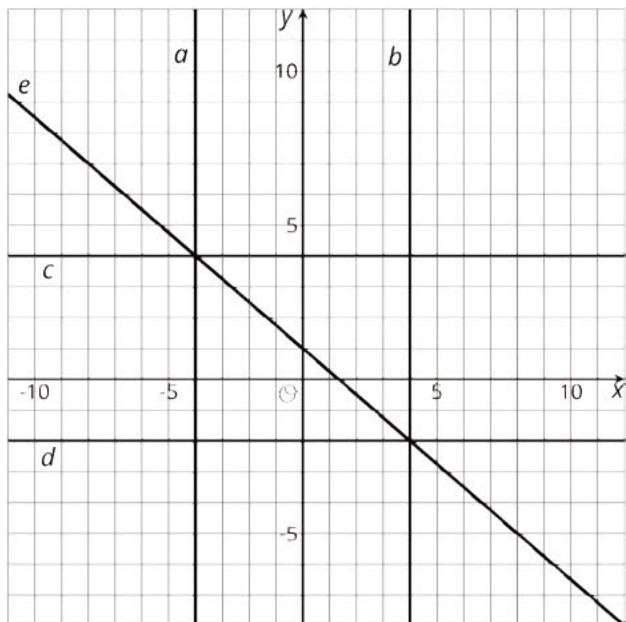
Students write equations for lines that are horizontal, vertical, or have negative slope.

## Addressing

- 8.EE.B.6

### Student Task Statement

Here are 5 lines on a coordinate grid:



Write equations for lines *a*, *b*, *c*, *d*, and *e*.

### Student Response

a:  $x = -4$  b:  $x = 4$  c:  $y = 4$  d:  $y = -2$  e:  $y = -\frac{3}{4}x + 1$  or  $\frac{y-1}{x} = -\frac{3}{4}$  (or equivalent equation)

### Student Lesson Summary

Horizontal lines in the coordinate plane represent situations where the  $y$  value doesn't change at all while the  $x$  value changes. For example, the horizontal line that goes through the point  $(0, 13)$  can be described in words as "for all points on the line, the  $y$  value is always 13." An equation that says the same thing is  $y = 13$ .

Vertical lines represent situations where the  $x$  value doesn't change at all while the  $y$  value changes. The equation  $x = -4$  describes a vertical line through the point  $(-4, 0)$ .

## Lesson 11 Practice Problems

### Problem 1

#### Statement

Suppose you wanted to graph the equation  $y = -4x - 1$ .

- a. Describe the steps you would take to draw the graph.
- b. How would you check that the graph you drew is correct?

## Solution

Answers vary. Sample response:

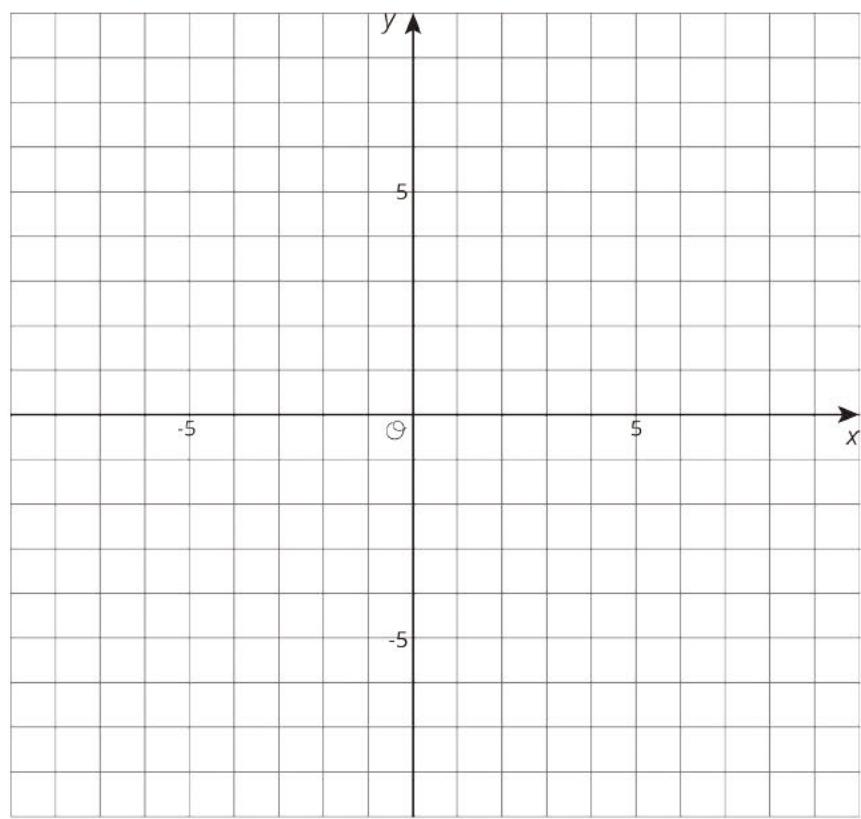
- a. Start with the intercept  $(0, -1)$ , and use the slope of  $-4$  to move down 4 and 1 to the right (or up 4 and 1 to the left) to find other points. Or, find two or more solutions to the equation and graph the points whose coordinates are the ordered pairs of the solutions, then draw a line connecting the points.
- b. Check the intercept and slope. Identify the coordinates of some points on the line, and substitute them into the equation to make sure they make the equation true.

## Problem 2

### Statement

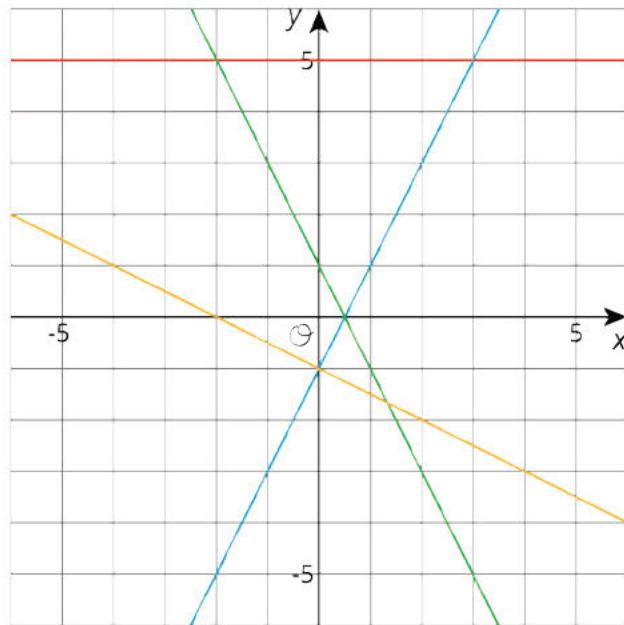
Draw the following lines and then write an equation for each.

- a. Slope is 0,  $y$ -intercept is 5
- b. Slope is 2,  $y$ -intercept is -1
- c. Slope is -2,  $y$ -intercept is 1
- d. Slope is  $-\frac{1}{2}$ ,  $y$ -intercept is -1



## Solution

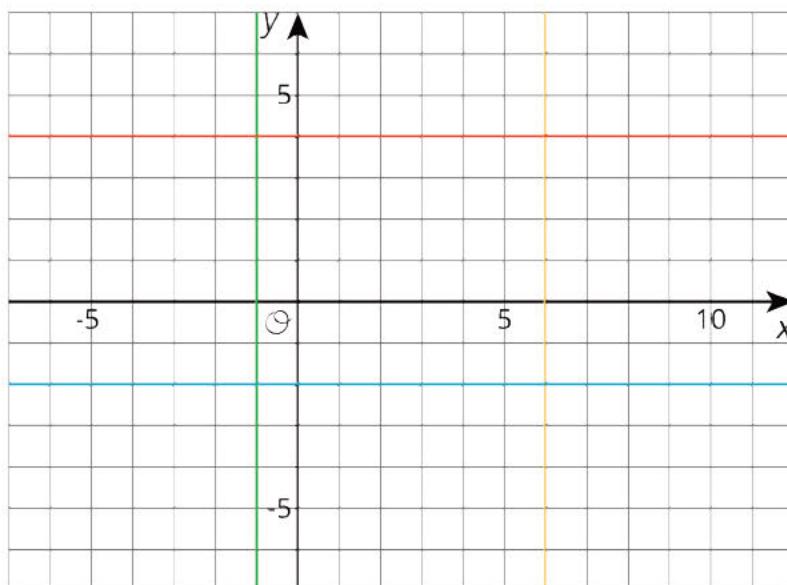
- a. Red line  $y = 5$
- b. Blue line  $y = 2x - 1$  or equivalent
- c. Green line  $y = -2x + 1$  or equivalent
- d. Yellow line  $y = -\frac{1}{2}x - 1$  or equivalent



### Problem 3

#### Statement

Write an equation for each line.



#### Solution

Green line:  $x = -1$ , yellow line:  $x = 6$ , red line:  $y = 4$ , blue line:  $y = -2$

## Problem 4

### Statement

A publisher wants to figure out how thick their new book will be. The book has a front cover and a back cover, each of which have a thickness of  $\frac{1}{4}$  of an inch. They have a choice of which type of paper to print the book on.

- a. Bond paper has a thickness of  $\frac{1}{4}$  inch per one hundred pages. Write an equation for the width of the book,  $y$ , if it has  $x$  hundred pages, printed on bond paper.
- b. Ledger paper has a thickness of  $\frac{2}{5}$  inch per one hundred pages. Write an equation for the width of the book,  $y$ , if it has  $x$  hundred pages, printed on ledger paper.
- c. If they instead chose front and back covers of thickness  $\frac{1}{3}$  of an inch, how would this change the equations in the previous two parts?

### Solution

- a.  $y = \frac{1}{2} + \frac{1}{4}x$
- b.  $y = \frac{1}{2} + \frac{2}{5}x$
- c.  $y = \frac{2}{3} + \frac{1}{4}x$  and  $y = \frac{2}{3} + \frac{2}{5}x$ , respectively

(From Unit 3, Lesson 7.)

# Section: Linear Equations

## Lesson 12: Solutions to Linear Equations

### Goals

- Comprehend that the points that lie on the graph of an equation represent exactly the solution set of the equation of the line (i.e., that every point on the line is a solution, and any point not on the line is not a solution).
- Create a graph and an equation in the form  $Ax + By = C$  that represent a linear relationship.
- Determine pairs of values that satisfy or do not satisfy a linear relationship using an equation or graph.

### Learning Targets

- I know that the graph of an equation is a visual representation of all the solutions to the equation.
- I understand what the solution to an equation in two variables is.

### Lesson Narrative

The goal of this lesson and the next is to start getting students to think about linear equations in two variables in a different way in preparation for their work on systems of linear equations in the next unit. Until now, students have mostly been working with contexts where one variable depends on another, for example, distance depending on time. The linear equation representing such a situation is often written in the form  $y = mx + b$ . In this lesson, they look at contexts where both variables have to satisfy a constraint, and a natural way to write the constraint is with an equation of the form  $Ax + By = C$ . For example, the first activity gets students to think about different ways of spending a fixed sum of money on two differently priced items, and the second activity gets them to write an equation expressing a numerical constraint on two numbers (twice the first number plus the second number adds up to 10).

Pairs of numbers that make the equation true are **solutions to the equation (with two variables)**; they are the coordinates of points that lie on the graph. Students also consider pairs of numbers that do not make the equation true, and notice that they do not lie on the graph. This insight is developed in the next lesson, where students look for points that are simultaneously the solution to two different equations.

### Alignments

#### Building On

- 6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

## Addressing

- 8.EE.B: Understand the connections between proportional relationships, lines, and linear equations.
- 8.EE.C: Analyze and solve linear equations and pairs of simultaneous linear equations.

## Building Towards

- 8.EE.C: Analyze and solve linear equations and pairs of simultaneous linear equations.

## Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR1: Stronger and Clearer Each Time
- MLR6: Three Reads
- Think Pair Share

## Required Materials

### Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

## Student Learning Goals

Let's think about what it means to be a solution to a linear equation with two variables in it.

# 12.1 Estimate Area

### Warm Up: 5 minutes

This warm-up prompts students to reason about the area of triangles and quadrilaterals. If there is time after sharing the estimates and reasoning, ask the students for the information they would need to get an exact answer to this question and how they would use that information.

## Building On

- 6.G.A.1

## Launch

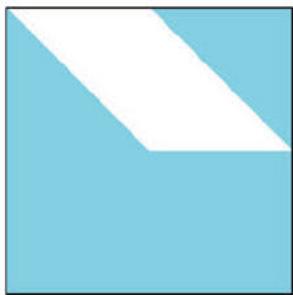
Ensure students know what is meant by the shaded region. Display the figures. Give students 1 minute of quiet think time and ask them to give a signal when they have an answer and reasoning.

Encourage students to have a reasoning to defend their choice beyond, “It looks like more.” Follow with a whole-class discussion.

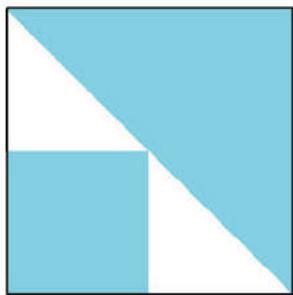
### Student Task Statement

Which figure has the largest shaded region?

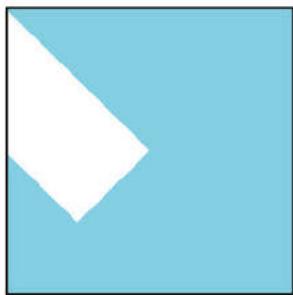
A



B



C



### Student Response

Figure C. Figures A and B have the same amount shaded,  $\frac{3}{4}$  of the whole square, and Figure C has  $\frac{3}{4} + \frac{1}{16}$  or  $\frac{13}{16}$  of the whole square shaded. These calculations assume that all distances that appear equal are equal (which can be verified by folding).

### Activity Synthesis

Invite students to share how they visualized the shaded region of each figure. Record and display their explanations for all to see. Solicit from the class alternative ways of quantifying the shaded portion and alternative ways of naming the size of the shaded portion (to elicit representations as fractions, decimals, or percentages). To involve more students in the conversation, consider asking:

- “Who can restate \_\_\_’s reasoning in a different way?”
- “Did anyone solve the shaded portion the same way but would explain it differently?”
- “Did anyone solve the shaded portion in a different way?”
- “Does anyone want to add on to \_\_\_’s strategy?”
- “Do you agree or disagree? Why?”

## 12.2 Apples and Oranges

15 minutes

In previous lessons in this unit, students have analyzed multiple situations that lead to equations of the form  $y = mx + b$ , including positive, negative, and 0 slope. In the previous lesson, they saw that vertical lines cannot be described by equations of this form and saw a geometric situation that could be represented by an equation of the form  $Ax + By = C$ . This form of a linear equation is

examined in greater detail here as students consider combinations of numbers that keep a total cost constant and write an equation that will be used again in the next activity. In this problem, the solutions are limited to non-negative integers and the set of all solutions is finite. In the next activity, the solutions comprise all real numbers and the set of solutions is infinite.

There are many equations students could use to describe the cost of apples in dollars,  $a$ , and the cost of oranges in dollars,  $r$ . Identify students who write  $a + 2r = 10$ , and ask them to share during the discussion. There is no reason to solve for one variable in terms of the other, in this case, because a graph has not been requested and neither variable is a “natural” candidate for dependent versus independent. In this activity, students need to explain their reasoning (MP3).

### Addressing

- 8.EE.B

### Instructional Routines

- MLR6: Three Reads
- Think Pair Share

### Launch

Arrange students in groups of 2. Give students 3–5 minutes of quiet think time to answer the first question and think about the others. Have partners compare solutions and discuss the remaining questions. Follow with a whole-class discussion.

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### Support for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, “First I \_\_\_ because . . .” or “That could/couldn’t be true because . . .”

*Supports accessibility for: Language; Social-emotional skills*

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### Support for English Language Learners

*Reading: MLR6 Three Reads.* To support reading comprehension for students, use this routine with the corner produce market problem. In the first read, students read the problem with the goal of comprehending the situation (e.g., finding the cost of different combinations of apples and oranges, determining how many apples and oranges can be bought with \$10, and writing an equation). In the second read, ask students to look for quantities that can be used or measured (e.g., apples cost \$1, oranges cost \$2, Noah has \$10). In the third read, ask students to brainstorm possible strategies to answer the question, “What combinations of apples and oranges can Noah buy if he spends all his \$10? Use two variables to write an equation that represents \$10-combinations of apples and oranges.” This will help students make sense of the problem by looking for patterns and strategies to come up with an equation before being asked to do so.

*Design Principle(s): Support sense-making; Maximize meta-awareness*

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### Anticipated Misconceptions

Students may write an equation like  $a + r = 10$  where  $a$  is the *cost* of the apples and  $r$  is the *cost* of the oranges. While this is technically correct, it leaves extra work figuring out whether or not a solution to this corresponds to some actual number of apples and oranges. Ask these students what they are trying to figure out (numbers of apples and numbers of oranges that cost \$10). Then suggest that they use their variables to represent the number of apples and the number of oranges rather than their cost.

### Student Task Statement

At the corner produce market, apples cost \$1 each and oranges cost \$2 each.

1. Find the cost of:
  - a. 6 apples and 3 oranges
  - b. 4 apples and 4 oranges
  - c. 5 apples and 4 oranges
  - d. 8 apples and 2 oranges
2. Noah has \$10 to spend at the produce market. Can he buy 7 apples and 2 oranges?  
Explain or show your reasoning.
3. What combinations of apples and oranges can Noah buy if he spends all of his \$10?
4. Use two variables to write an equation that represents \$10-combinations of apples and oranges. Be sure to say what each variable means.

5. What are 3 combinations of apples and oranges that make your equation true? What are three combinations of apples and oranges that make it false?

### Student Response

1. \$12, \$12, \$13, \$12.
2. No. 7 apples and 2 oranges would cost \$11.
3. 0 apples and 5 oranges, 2 apples and 4 oranges, 4 apples and 3 oranges, 6 apples and 2 oranges, 8 apples and 1 orange, 10 apples and 0 oranges.
4. Answers vary. Sample response:  $a + 2r = 10$  where  $a$  represents the number of apples and  $r$  represents the number of oranges.
5. The combinations that add up to \$10 make the equation true. The combinations that add up to other amounts make it false.

### Are You Ready for More?

1. Graph the equation you wrote relating the number of apples and the number of oranges.
2. What is the slope of the graph? What is the meaning of the slope in terms of the context?
3. Suppose Noah has \$20 to spend. Graph the equation describing this situation. What do you notice about the relationship between this graph and the earlier one?

### Student Response

1. The graph of the equation  $x + 2y = 10$  should show horizontal and vertical intercepts, first quadrant only, with the horizontal axis labeled “number of apples” and the vertical axis labeled “number of oranges”. Note that a graph with axes labels reversed is also correct.
2. The slope is  $-0.5$ . (-2 is a possible answer if the axes are swapped.) This is the number of oranges per apple. Since oranges cost \$2 and apples cost \$1, you have to give up 0.5 oranges to buy an apple.
3. The graph is the same as in the first question except that the vertical and horizontal intercepts are at 20 and 10 instead of 10 and 5. The new line has the same slope, because the tradeoff between apples and oranges is the same.

### Activity Synthesis

The equation  $a + 2r = 10$  (or some equivalent) is an example of a naturally arising equation of the form  $Ax + By = C$ , introduced in the previous lesson. In a graphical representation of this situation, either variable (oranges or apples) could be plotted on either axis. If students mention

slope, make sure to ask them how they are plotting the variables. This will be taken up in greater detail in the next activity where the graphical representation will be central.

Invite students from different groups to share what they discovered with their partners by asking:

- “How many combinations that cost \$10 did you find in total? How do you know you have found them all?” (6 combinations, these are all the possibilities of whole numbers that would make the sum 10.)
- “Did you notice any patterns that helped you find the combination?” (There are many patterns; the one we would like students to notice is that buying 1 less orange means you can buy 2 more apples.)
- “What would the graph of the equation you wrote look like?” (Some may realize that the graph would contain the six points that represent the six combinations for \$10. Help them to see that these points would all lie on a line, given by the equation  $a + 2r = 10$ . The non-integer points on the line would represent a fractional number of pieces of each fruit.)
- “According to the equation you wrote, if you bought  $\frac{1}{2}$  orange and 9 apples you would spend \$10. Do you think this situation is realistic when buying fruit in a store?” (Probably not, you would only buy whole pieces of fruit.)

## 12.3 Solutions and Everything Else

**15 minutes**

Students write an equation representing a stated relationship between two quantities, and use the equation to find pairs of numbers that make it true and pairs of numbers for which it is not true. By graphing both sets of points, students see that the graph of a linear equation is the set of its solutions, that is, the points whose coordinates make the equation true.

Students should be encouraged to use rational number coordinates and points in all four quadrants and on the two axes in their graphs, and to see that solutions exist all along the line, between and beyond the pairs they explicitly choose, making for an infinite number of solutions.

Some students might notice that the points where  $x = 0$  and  $y = 0$  are easy to quickly find and that connecting these two points yields the line containing all the other solutions. They may then try to read solutions from the graph instead of using the equation to more accurately calculate the coordinates of solutions. While this is a valuable observation in terms of quickly drawing the graph of the linear equation, students should be guided to realize that the equation enables a level of precision that reading from the graph often lacks. Precision can be improved through the use of technology in the form of graphing calculators and applications.

As students find points, both satisfying the relationship from the first question and not satisfying the relationship, monitor for these choices and invite these students to share during the discussion.

- Points in the first quadrant

- Points on the axes (there are only two of these!)
- Points in the first and/or fourth quadrants
- Points with non-integer coordinate values

### **Building Towards**

- 8.EE.C

### **Instructional Routines**

- Anticipate, Monitor, Select, Sequence, Connect
- MLR1: Stronger and Clearer Each Time

### **Launch**

Students should have access to geometry toolkits, especially rulers and graph paper.

Display for all to see the following statement: “You have two numbers. If you double the first number and add it to the second number, the sum is 10.”

Poll the class for “predict how many pairs of numbers make this statement true?” Display the answers for all to see.

Instruct students to pause their work after creating their graph and then to check their graph with you. Identify students who struggle with writing the equation or with finding and graphing solutions. Check that the graph and the equation match since some students might write the equation as  $x + 2y = 10$  instead of  $2x + y = 10$ . Also check that students are not restricting  $x$  and  $y$  values to non-negative integers.

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### **Support for Students with Disabilities**

*Representation: Internalize Comprehension.* Display and discuss a range of examples and counterexamples of pairs of numbers that satisfy the given statement. For example, the pair (1, 8) makes the statement true, whereas (2, 4) does not.

*Supports accessibility for: Conceptual processing*

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### **Anticipated Misconceptions**

Students might write  $x + 2y = 10$ . Ask these students what  $x$  and  $y$  represent and to pick a couple of values of  $x$  and  $y$  so that the double of  $x$  and  $y$  sum to 10. Ask them to see if these are solutions to their equation. They most likely will not be. Suggest that these students reconsider their equation.

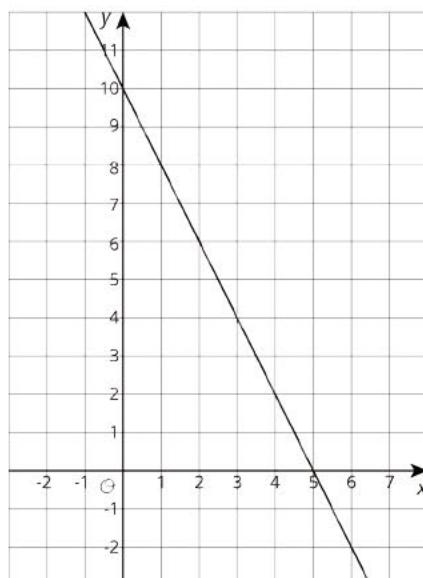
### **Student Task Statement**

You have two numbers. If you double the first number and add it to the second number, the sum is 10.

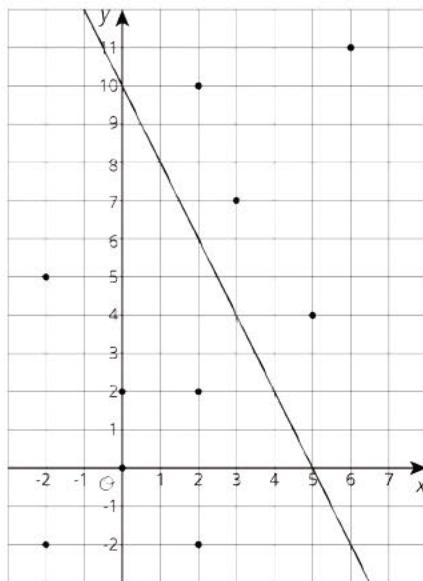
1. Let  $x$  represent the first number and let  $y$  represent the second number. Write an equation showing the relationship between  $x$ ,  $y$ , and 10.
2. Draw and label a set of  $x$ - and  $y$ -axes. Plot at least five points on this coordinate plane that make the statement and your equation true. What do you notice about the points you have plotted?
3. List ten points that do *not* make the statement true. Using a different color, plot each point in the same coordinate plane. What do you notice about these points compared to your first set of points?

### Student Response

1.  $2x + y = 10$ .
2. Answers vary. Sample response: All the points lie on a line.



3. Answers vary. Sample responses: These points all lie off the line formed by the set of points that make the statement true.



### Activity Synthesis

Select students to share their examples of points in the coordinate plane that make their equation true, sequenced in the following order:

- Points in the first quadrant
- Points on the axes (there are only two of these!)
- Points in the first and/or fourth quadrants
- Points with non-integer coordinate values

If it does not come up in student work, ask them if the equation could be true if  $x = -3$  (yes,  $y = 16$ ) or if  $y = -4$  (yes,  $x = 7$ ). What if  $x = 3\frac{1}{2}$ ? (Yes,  $y = 3$ .) Through the discussion questions, bring out that there are an infinite number of solutions to this equation and that, taken collectively and plotted in the plane, they make up the line of all solutions to the equation  $2x + y = 10$ .

Connect student solutions to the graphical representation of the scenario by asking:

- “For an equation with two variables, a pair of numbers that makes the equation true is called a *solution* of the equation. Where in your graph do you see solutions of the equation that you wrote? Are all possible solutions on your graph?” Bring out here that the set of *all* solutions to the equation is a line with  $y$ -intercept 10 and slope -2. So an alternative way to write its equation, other than  $2x + y = 10$ , is  $y = -2x + 10$ .
- “Where in your graph do you see pairs that are *not* solutions of the equation you wrote?” (All points not on the line.)
- “Based on your observations, what is the relationship between the solutions of an equation and its graph?” (For this equation, the set of solutions is a line.)

- “What does the graph tell you about the number of solutions of your equation?” (There are an infinite number.)
- “Does the line contain all possible solutions to your equation? How do you know?” (Yes, for each value of  $x$ , there is exactly one value of  $y$  giving a solution.)

In summary, articulate the idea that the solutions are precisely the points on the line, and no more.

Ask students to compare solutions to the apple and orange activity with solutions in this activity. “Would a graph of all solutions to the apples and oranges problem look the same as the graph in this activity?” It would not because you can only purchase non-negative whole number values of apples and oranges. Make sure students understand that in this activity it makes sense to draw a line connecting all points because the coordinates of all the points on the line are solutions to the equation.

A **solution to an equation with two variables** is an ordered pair of values that makes an equation true, and the graph of the equation is the set of *all* solution pairs  $(x, y)$  plotted as points in coordinate plane: for  $2x + y = 10$ , this set of solutions is a line with slope -2 and  $y$ -intercept 10.

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#### Support for English Language Learners

*Writing, Conversing: MLR1 Stronger and Clearer Each Time.* Use this routine for students to respond in writing to the prompt: “What does a graph tell you about the solutions to an equation with two variables?” Give students time to meet with 2–3 partners, to share and get feedback on their responses. Encourage the listener to press for supporting details and evidence by asking, “Could you give an example from your graph?” or “Could you make a generalization about the solutions to an equation from the specific case you mentioned?” Have the students write a second draft based on their peer feedback. This will help students articulate their understanding of the solution to an equation and clearly define it using a graph.

*Design Principle(s): Optimize output ; Cultivate conversation*

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## Lesson Synthesis

Students explored several big ideas in this lesson:

1. A solution to a linear equation is a pair of values that makes the equation true.
2. Solutions can be found by substituting a value for one of the variables and solving the equation for the other.
3. The set of all the solutions to a linear equation can be shown visually in the coordinate plane and is called the graph of the equation.
4. The graph of a linear equation is a line.

5. Any points in the coordinate plane that do not lie on the line that is on the graph of the linear equation are not solutions to the equation.
6. The number of solutions might be limited in a real-world situation even though the equation has an infinite number of solutions.

Invite students to describe how they found solutions to linear equations and to explain how they knew they had found a solution.

Ask what difference there may be between reading solutions from a graph and calculating them using the equation. Listen for students to identify a possible lack of precision when reading from a graph. If you are using graphing software this will be less of an issue.

Finally, ask if points that are not on the line can be solutions to the equation represented by the line. Listen for students to understand that the line represents *all* pairs that make the equation true so a point not on the line cannot be a solution.

## 12.4 Identify the Points

**Cool Down: 5 minutes**

Students verify whether or not certain points in the  $x$ - $y$  plane make a linear equation true.

### Addressing

- 8.EE.C

#### Student Task Statement

Which of the following coordinate pairs make the equation  $x - 9y = 12$  true?

1.  $(12, 0)$
2.  $(0, 12)$
3.  $(3, -1)$
4.  $(0, -\frac{4}{3})$

#### Student Response

1. Yes.
2. No.
3. Yes.
4. Yes.

## Student Lesson Summary

Think of all the rectangles whose perimeters are 8 units. If  $x$  represents the width and  $y$  represents the length, then

$$2x + 2y = 8$$

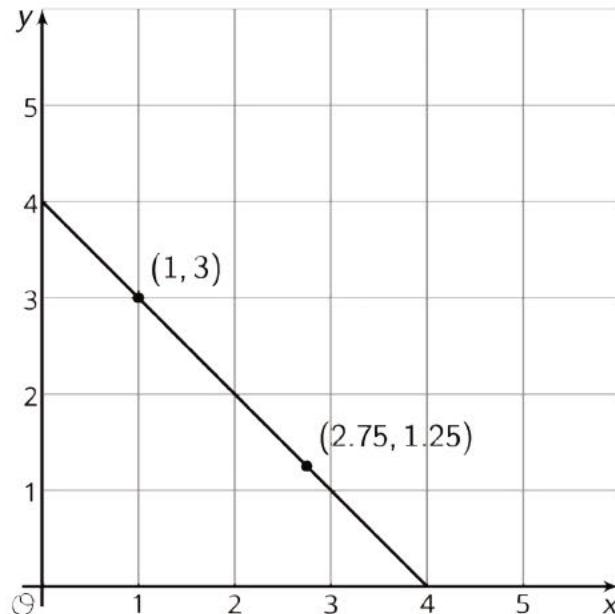
expresses the relationship between the width and length for all such rectangles.

For example, the width and length could be 1 and 3, since  $2 \cdot 1 + 2 \cdot 3 = 8$  or the width and length could be 2.75 and 1.25, since  $2 \cdot (2.75) + 2 \cdot (1.25) = 8$ .

We could find many other possible pairs of width and length,  $(x, y)$ , that make the equation true—that is, pairs  $(x, y)$  that when substituted into the equation make the left side and the right side equal.

A **solution to an equation with two variables** is any pair of values  $(x, y)$  that make the equation true.

We can think of the pairs of numbers that are solutions of an equation as points on the coordinate plane. Here is a line created by all the points  $(x, y)$  that are solutions to  $2x + 2y = 8$ . Every point on the line represents a rectangle whose perimeter is 8 units. All points not on the line are not solutions to  $2x + 2y = 8$ .



## Glossary

- solution to an equation with two variables

## Lesson 12 Practice Problems

### Problem 1

#### Statement

Select **all** of the ordered pairs  $(x, y)$  that are solutions to the linear equation  $2x + 3y = 6$ .

- A.  $(0, 2)$
- B.  $(0, 6)$
- C.  $(2, 3)$
- D.  $(3, -2)$
- E.  $(3, 0)$
- F.  $(6, -2)$

## Solution

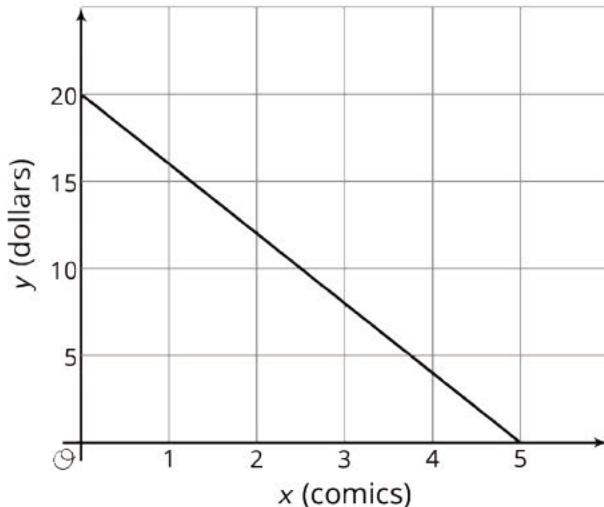
["A", "E", "F"]

## Problem 2

### Statement

The graph shows a linear relationship between  $x$  and  $y$ .

$x$  represents the number of comic books Priya buys at the store, all at the same price, and  $y$  represents the amount of money (in dollars) Priya has after buying the comic books.



- a. Find and interpret the  $x$ - and  $y$ -intercepts of this line.
- b. Find and interpret the slope of this line.
- c. Find an equation for this line.
- d. If Priya buys 3 comics, how much money will she have remaining?

## Solution

- a. Priya has \$20 before buying any comics, and if she buys 5 comics, Priya will have no money left.

b. The slope is  $-4$ . The amount of money left goes down by  $4$  with each comic book; each comic book costs  $\$4$ .

c.  $y = 20 - 4x$

d.  $\$8$

### Problem 3

#### Statement

Match each equation with its three solutions.

A.  $y = 1.5x$

1.  $(14, 21), (2, 3), (8, 12)$

B.  $2x + 3y = 7$

2.  $(-3, -7), (0, -4), (-1, -5)$

C.  $x - y = 4$

3.  $(\frac{1}{8}, \frac{7}{8}), (\frac{1}{2}, \frac{1}{2}), (\frac{1}{4}, \frac{3}{4})$

D.  $3x = \frac{y}{2}$

4.  $(1, 1\frac{2}{3}), (-1, 3), (0, 2\frac{1}{3})$

E.  $y = -x + 1$

5.  $(0.5, 3), (1, 6), (1.2, 7.2)$

#### Solution

- A: 1
- B: 4
- C: 2
- D: 5
- E: 3

### Problem 4

#### Statement

A container of fuel dispenses fuel at the rate of  $5$  gallons per second. If  $y$  represents the amount of fuel remaining in the container, and  $x$  represents the number of seconds that have passed since the fuel started dispensing, then  $x$  and  $y$  satisfy a linear relationship.

In the coordinate plane, will the slope of the line representing that relationship have a positive, negative, or zero slope? Explain how you know.

#### Solution

Negative because the amount of fuel in the tank is decreasing.

(From Unit 3, Lesson 10.)

## Problem 5

### Statement

A sandwich store charges a delivery fee to bring lunch to an office building. One office pays \$33 for 4 turkey sandwiches. Another office pays \$61 for 8 turkey sandwiches. How much does each turkey sandwich add to the cost of the delivery? Explain how you know.

### Solution

\$7. Explanations vary. Sample response: The second office pays  $61 - 33$ , or 28 dollars more, for  $8 - 4$ , or 4, more sandwiches. So each sandwich adds  $28 \div 4$ , or 7 dollars, to the cost.

(From Unit 3, Lesson 5.)

# Lesson 13: More Solutions to Linear Equations

## Goals

- Calculate the solution to a linear equation given one variable, and explain (orally) the solution method.
- Determine whether a point is a solution to an equation of a line using a graph of the line.

## Learning Targets

- I can find solutions  $(x, y)$  to linear equations given either the  $x$ - or the  $y$ -value to start from.

## Lesson Narrative

The previous lesson focused on the relationship between a linear equation in two variables, its solution set, and its graph. These themes continue to develop in this lesson. In the first activity after the warm-up, students analyze statements about a collection of three graphs, deciding whether or not certain ordered pairs are solutions to the equations defining the lines. In particular, students realize that values  $x = a$  and  $y = b$  satisfy two different linear equations simultaneously when the point  $(a, b)$  lies on both lines represented by the equations. This is important preparation for thinking about what it means to be a solution to a system of equations in the next unit.

In the second activity, students consider equations given in many different forms, ask their partner for either the  $x$ - or  $y$ -coordinate of a solution to the equation, and then give the other coordinate. This activity prepares students for finding solutions to systems of equations, because it gets them to look at the structure of an equation and decide whether it would be easier to solve for  $y$  given  $x$ , or to solve for  $x$  given  $y$  (MP7).

## Alignments

### Addressing

- 8.EE.C: Analyze and solve linear equations and pairs of simultaneous linear equations.
- 8.EE.C.8.a: Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

## Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports

## Required Materials

Pre-printed slips, cut from copies of the **blackline master**

## Required Preparation

One copy of the I'll Take an X Please blackline master for every pair of students.

### Student Learning Goals

Let's find solutions to more linear equations.

## 13.1 Coordinate Pairs

### Warm Up: 5 minutes

The purpose of this warm-up is for students to practice solving an equation for an unknown value while thinking about a coordinate pair,  $(x, y)$ , that makes the equation true. While the steps to solve the equation are the same regardless of which value of  $x$  students choose, there are strategic choices that make solving the resulting equation simpler. This should be highlighted in the discussion.

### Addressing

- 8.EE.C

### Launch

Encourage students to not pick 0 for  $x$  each time.

### Student Task Statement

For each equation choose a value for  $x$  and then solve to find the corresponding  $y$  value that makes that equation true.

1.  $6x = 7y$
2.  $5x + 3y = 9$
3.  $y + 5 - \frac{1}{3}x = 7$

### Student Response

Answers vary. Sample responses:

1.  $x = 7, y = 6$
2.  $x = 3, y = -2$
3.  $x = 3, y = 3$

### Activity Synthesis

Collect the pairs of  $x$ 's and  $y$ 's students calculated and graph them on a set of axes. For each equation, they form a different line. Have students share how they picked their  $x$  values. For example:

- For the first problem, choosing  $x$  to be a multiple of 7 makes  $y$  an integer.
- For the last problem, picking  $x$  to be a multiple of 3 makes  $y$  an integer.

## 13.2 True or False: Solutions in the Coordinate Plane

15 minutes

In the previous lesson, students studied the set of solutions to a linear equation, the set of all values of  $x$  and  $y$  that make the linear equation true. They identified that this was a line in the coordinate plane. In this activity, they are given graphs of lines and then are asked whether or not different  $x$ - $y$  coordinate pairs are solutions to equations that define the lines. This helps students solidify their understanding of the relationship between a linear equation and its graph in the coordinate plane.

Consider asking students to work on the first 5 problems only if time is an issue. Since this activity largely reinforces the material of the previous lesson, it is not essential to do all 8 problems.

### Addressing

- 8.EE.C.8.a

### Instructional Routines

- MLR3: Clarify, Critique, Correct

### Launch

Arrange students in groups of 2. Students work through the eight statements individually and then compare and discuss their answers with their partner. Tell students that if they disagree, they should work to come to an agreement.

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### Support for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Check for understanding by inviting students to rephrase directions in their own words. Provide the following sentence frame to support student explanations: “Statement \_\_\_ is true/false because . . .”

*Supports accessibility for: Organization; Attention*

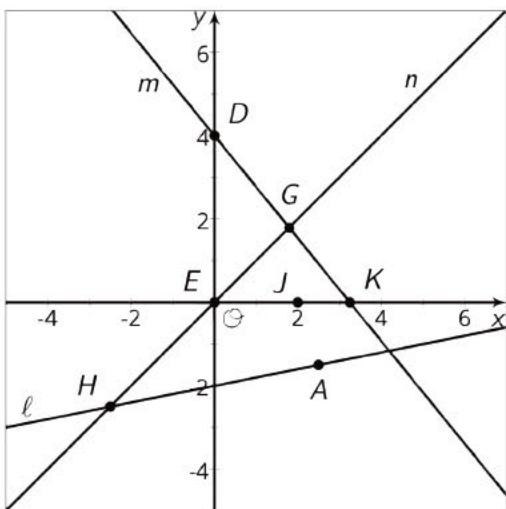
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### Anticipated Misconceptions

Some students may try to find equations for the lines. There is not enough information to accurately find these equations, and it is not necessary since the questions only require understanding that a coordinate pair lies on a line when it gives a solution to the corresponding linear equation. Ask these students if they can answer the questions without finding equations for the lines.

#### Student Task Statement

Here are graphs representing three linear relationships. These relationships could also be represented with equations.



For each statement below, decide if it is true or false. Explain your reasoning.

1.  $(4, 0)$  is a solution of the equation for line  $m$ .
2. The coordinates of the point  $G$  make both the equation for line  $m$  and the equation for line  $n$  true.
3.  $x = 0$  is a solution of the equation for line  $n$ .
4.  $(2, 0)$  makes both the equation for line  $m$  and the equation for line  $n$  true.
5. There is no solution for the equation for line  $\ell$  that has  $y = 0$ .
6. The coordinates of point  $H$  are solutions to the equation for line  $\ell$ .
7. There are exactly two solutions of the equation for line  $\ell$ .
8. There is a point whose coordinates make the equations of all three lines true.

After you finish discussing the eight statements, find another group and check your answers against theirs. Discuss any disagreements.

### Student Response

1. False. The point  $(4, 0)$  does not lie on line  $m$ , so it is not a solution to the equation for line  $m$ . The point  $(0, 4)$  does lie on the line and is a solution to the equation for line  $m$ .
2. True. Since point  $G$  lies on both lines, its coordinates are a solution to both equations.
3. False. Since the equation has two variables, a solution must be a pair of numbers, or both coordinates of a point on the line.  $x = 0, y = 0$  is a solution to the equation for line  $n$ .
4. False.  $(2, 0)$  does not lie on either line and is therefore not a solution to either equation.
5. False. We don't see the solution here but can see that line  $\ell$  and the  $x$ -axis (where  $y = 0$ ) will meet in a point when the lines are extended.

6. True, because point  $H$  lies on line  $\ell$ .
7. False. There are infinitely many solutions for line  $\ell$ : the coordinates of every point on the line.
8. False. There is no point that lies on all three lines: the picture shows all intersection points of these lines.

### Activity Synthesis

Display the correct answer to each question, and give students a few minutes to discuss any discrepancies with their partner. For the third question, some students might say yes because there is a solution to the equation for line  $n$ , which has  $x = 0$ , namely if  $y = 0$  as well. For the fifth question, make sure students understand that the line  $\ell$  meets the  $x$ -axis even if that point is not shown on the graph.

Some key points to highlight, reinforcing conclusions from the previous lesson as well as this activity:

- A solution of an equation in two variables is an ordered pair of numbers.
- Solutions of an equation lie on the graph of the equation.

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### Support for English Language Learners

*Reading, Conversing: MLR3 Clarify, Critique, Correct.* After students have had time to make decisions about whether the statements are true or false, offer an incorrect response such as: “The statement ‘ $x = 0$  is a solution of the equation for line  $n$ ’ is true because I see the line go through the  $x$  value at 0.” Invite students to work with a partner to clarify the meaning of this incorrect response and then critique it. Invite pairs to offer a correct response by asking, “What language might you add or change to make this statement more accurate?” Listen for key phrases such as “ $x = 0$  is a line, not a point.” This will help students to solidify their understanding that the solution to an equation with two variables is both coordinates of the point on the line.

*Design Principle(s): Cultivate conversation; Maximize meta-awareness*

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## 13.3 I'll Take an X, Please

**15 minutes**

Students are given linear equations—some of which represent proportional relationships—in various forms, and are also given solutions to their partner’s equations in the form of coordinates of a point. The student with the equation decides which quantity they would like to know,  $x$  or  $y$ , and requests this information from their partner. They then solve for the other quantity. The activity reinforces the concept that solutions to equations with two variables are a pair of numbers, and that knowing one can give you the other by using the value you know and solving the equation.

Students also have a chance to think about the most efficient way to find solutions for equations in different forms.

You will need the I'll Take An  $x$  Please blackline master for this activity.

### **Addressing**

- 8.EE.C.8.a

### **Instructional Routines**

- MLR8: Discussion Supports

### **Launch**

Consider demonstrating the first step with a student. Write the equation  $y = 5x - 11$  on one slip of paper and the point  $(1, -6)$  on another slip of paper. Give the slip with the coordinate pair to the student. You can ask for the  $x$  or  $y$  coordinate of a point on the graph and then need to find the other coordinate. Ask the student helper for either  $x$  or  $y$ . (In this case, asking for the  $x$  coordinate is wise because then you can just plug it into the equation to find the corresponding  $y$ -coordinate for the point on the graph.) Display your equation for all to see and demonstrate substituting the value in and solving for the other variable. Alternatively, ask students how they might use the information given (one of the values for  $x$  or  $y$  to find the other given your equation).

Arrange students in groups of 2. One partner receives Cards A through F from the left side of the blackline master and the other receives Cards a through f from the right side. Students take turns asking for either  $x$  or  $y$  then solving their equation for the other, and giving their partner the information requested.

Students play three rounds, where each round consists of both partners having a turn to ask for a value and to solve their equation. Follow with a whole-class discussion.

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### **Support for Students with Disabilities**

*Representation: Provide Access for Perception.* Display or provide students with a physical copy of the written directions and read them aloud. Check for understanding by inviting students to rephrase directions in their own words. Display directions throughout the activity.

*Supports accessibility for: Language; Memory.*

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### **Student Task Statement**

One partner has 6 cards labeled A through F and one partner has 6 cards labeled a through f. In each pair of cards (for example, Cards A and a), there is an equation on one card and a coordinate pair,  $(x, y)$ , that makes the equation true on the other card.

1. The partner with the equation asks the partner with a solution for either the  $x$ -value or the  $y$ -value and explains why they chose the one they did.

2. The partner with the equation uses this value to find the other value, explaining each step as they go.
3. The partner with the coordinate pair then tells the partner with the equation if they are right or wrong. If they are wrong, both partners should look through the steps to find and correct any errors. If they are right, both partners move onto the next set of cards.
4. Keep playing until you have finished Cards A through F.

### Student Response

The A through F cards have both coordinates for each point.

### Are You Ready for More?

Consider the equation  $ax + by = c$ , where  $a$ ,  $b$ , and  $c$  are positive numbers.

1. Find the coordinates of the  $x$ - and  $y$ -intercepts of the graph of the equation.
2. Find the slope of the graph.

### Student Response

1.  $(\frac{c}{a}, 0)$  and  $(0, \frac{c}{b})$ . Putting  $y = 0$  in the equation we get  $ax = c$ , so  $x = \frac{c}{a}$ . So the  $x$ -intercept is  $(\frac{c}{a}, 0)$ . Putting  $x = 0$  in the equation we get  $by = c$ , so  $y = \frac{c}{b}$ . So the  $y$ -intercept is  $(0, \frac{c}{b})$ .
2. Using the two intercepts to calculate the slope, we get slope =  $\frac{\frac{c}{b} - 0}{0 - \frac{c}{a}} = -\frac{\frac{c}{b}}{\frac{c}{a}} = -\frac{a}{b}$

### Activity Synthesis

The discussion should focus on using the given information to efficiently find a solution for the equation. Consider asking students:

- “How did you decide whether you wanted the value of  $x$  or the value of  $y$ ?” (One might be more efficient to solve for: for example, with card B asking for  $x$  makes sense while with card d the arithmetic to perform is similar whether asking for  $x$  or for  $y$ .)
- “Which equations represent proportional relationships? How do you know? Which do not?” (C and F are proportional because they can be written as  $y = kx$ , although this is hidden at first glance in C.)
- “Once you have identified one solution to your equation, what are some ways you could find others?” (Use the constant rate of change to add/subtract to the solution you know, solve the equation for  $x$  or  $y$ , choose values for one variable and solve for the other, for proportional relationships you could find equivalent ratios.)

Point out that all of the equations in this activity are linear. They are given in many different forms, not just  $y = mx + b$  or  $Ax + By = C$ .

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### Support for English Language Learners

*Conversing: MLR8 Discussion Supports.* Use this routine to support student discussions. Provide the following sentence frames, and invite the partner who had the equation to begin with: “I decided to ask for the value of  $x$  (or  $y$ ) because.... The steps I took to find the other value were....” Encourage the listener to ask clarifying questions such as: “What would have happened if you chose the other variable?” or “Could you think of a more efficient way to solve?” This will help students justify their reasoning for choosing a certain value to start and explain the steps they took to obtain the other coordinate.

*Design Principle(s): Support sense-making; Cultivate conversation (for explanation)*

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## Lesson Synthesis

In order to highlight student thinking about different strategies for finding a solution to a linear equation, ask students:

- “What are different ways to find a solution to the linear equation  $3y + x = 12$ ?” (Substitute in a value for one variable and solve for the other; graph the equation and find points that lie on the line; rearrange the equation so that one variable is written in terms of the other variable.)
- “How do you know when you have found a solution to the equation  $3y + x = 12$ ?” (The coordinates of the point will make the statement true.)
- “What are some easy values to substitute into the equation?” (In this case, a good strategic choice is  $x = 0$ , which gives  $y = 4$ , and  $y = 0$ , which gives  $x = 12$ . This says that the  $y$ -intercept of the graph of the equation is  $(0, 4)$ . Similarly, the  $x$ -intercept of the equation’s graph is  $(12, 0)$ .)
- “How can you find the slope of the line?” (Graphing the line shows that the slope is negative, and we can verify this by rewriting the equation as  $y = -\frac{1}{3}x + 4$ .)

## 13.4 Intercepted

### Cool Down:

Students show their understanding of solutions to linear equations in two variables and connections to the graph of the equation.

### Addressing

- 8.EE.C.8.a

### Student Task Statement

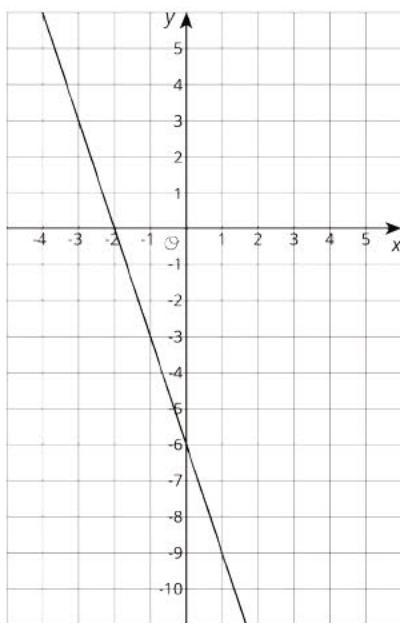
A graph of a linear equation passes through  $(-2, 0)$  and  $(0, -6)$ .

1. Use the two points to sketch the graph of the equation.

2. Is  $3x - y = -6$  an equation for this graph? Explain how you know.

### Student Response

1.



2. No. Answers vary. Sample response: Test the two given pairs:  $3(-2) - 0 = -6 - 0 = -6$  so the coordinates of this point represent a solution to the equation.  $3(0) - (-6) = 0 + 6 = 6$ , not  $-6$ , so the coordinates of this point do not represent a solution to the equation. The graph of a linear equation contains only ordered pairs whose coordinates are solutions to the equation, so the equation is not represented by the line with the two given points.

### Student Lesson Summary

Let's think about the linear equation  $2x - 4y = 12$ . If we know  $(0, -3)$  is a solution to the equation, then we also know  $(0, -3)$  is a point on the graph of the equation. Since this point is on the  $y$ -axis, we also know that it is the vertical intercept of the graph. But what about the coordinate of the horizontal intercept, when  $y = 0$ ? Well, we can use the equation to figure it out.

$$\begin{aligned}2x - 4y &= 12 \\2x - 4(0) &= 12 \\2x &= 12 \\x &= 6\end{aligned}$$

Since  $x = 6$  when  $y = 0$ , we know the point  $(6, 0)$  is on the graph of the line. No matter the form a linear equation comes in, we can always find solutions to the equation by starting with one value and then solving for the other value.

## Lesson 13 Practice Problems

### Problem 1

#### Statement

For each equation, find  $y$  when  $x = -3$ . Then find  $x$  when  $y = 2$

- a.  $y = 6x + 8$
- b.  $y = \frac{2}{3}x$
- c.  $y = -x + 5$
- d.  $y = \frac{3}{4}x - 2\frac{1}{2}$
- e.  $y = 1.5x + 11$

#### Solution

- a.  $y = -10, x = -1$
- b.  $y = -2, x = 3$
- c.  $y = 8, x = 3$
- d.  $y = \frac{-19}{4}, x = 6$
- e.  $y = 6.5, x = -6$

### Problem 2

#### Statement

True or false: The points  $(6, 13)$ ,  $(21, 33)$ , and  $(99, 137)$  all lie on the same line. The equation of the line is  $y = \frac{4}{3}x + 5$ . Explain or show your reasoning.

#### Solution

True, all three points make the equation true.

### Problem 3

#### Statement

Here is a linear equation:  $y = \frac{1}{4}x + \frac{5}{4}$

- a. Are  $(1, 1.5)$  and  $(12, 4)$  solutions to the equation? Explain or show your reasoning.
- b. Find the  $x$ -intercept of the graph of the equation. Explain or show your reasoning.

## Solution

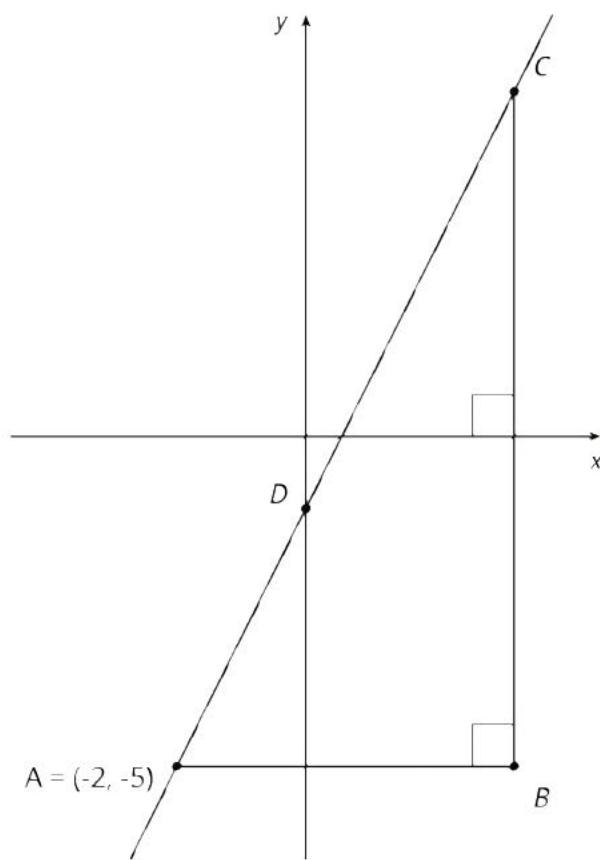
a. (1, 1.5): Yes, check it with the equation; (12, 4): No, when  $x = 12$ ,  $y$  would be 4.25, not 4.

b. (-5, 0) Explanations vary. Sample response: Set  $y = 0$  in the equation.

### Problem 4

## Statement

Find the coordinates of  $B$ ,  $C$ , and  $D$  given that  $AB = 5$  and  $BC = 10$ .



### Solution

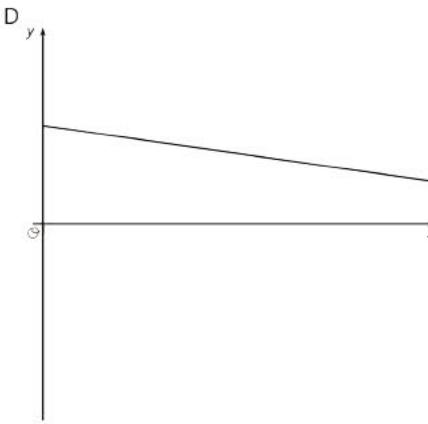
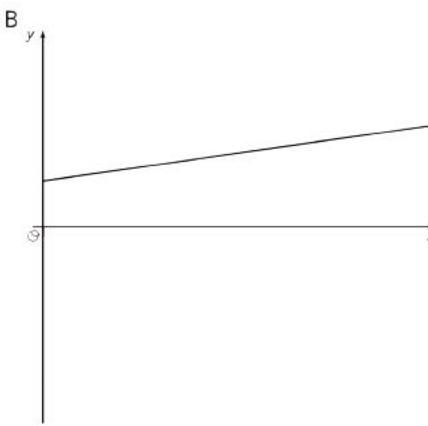
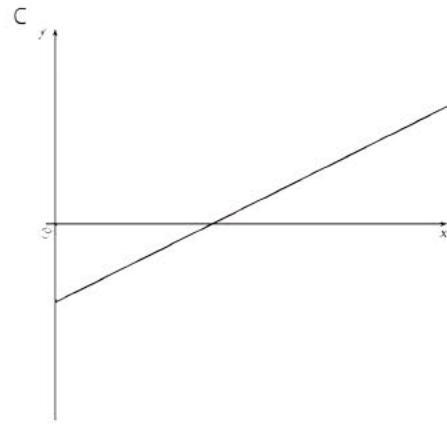
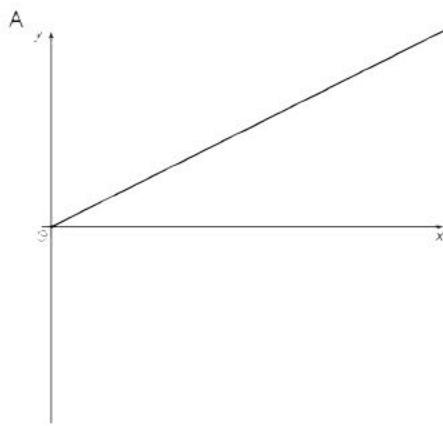
$$B = (3, -5), C = (3, 5), D = (0, -1)$$

(From Unit 2, Lesson 11.)

### Problem 5

## Statement

Match each graph of a linear relationship to a situation that most reasonably reflects its context.



A. Graph A

B. Graph B

C. Graph C

D. Graph D

1.  $y$  is the weight of a kitten  $x$  days after birth.
2.  $y$  is the distance left to go in a car ride after  $x$  hours of driving at a constant rate toward its destination.
3.  $y$  is the temperature, in degrees C, of a gas being warmed in a laboratory experiment.
4.  $y$  is the amount of calories consumed eating  $x$  crackers.

## Solution

- A: 4
- B: 1
- C: 3
- D: 2

(From Unit 3, Lesson 9.)

## Section: Let's Put it to Work

### Lesson 14: Using Linear Relations to Solve Problems

#### Goals

- Describe (orally) limitations of a graphical representation of a situation based on real-world constraints on the quantities.
- Interpret the graph of a linear equation in context, including slope, intercept, and solution, in contexts using multiple representations of non-proportional linear relationships.

#### Learning Targets

- I can write linear equations to reason about real-world situations.

#### Lesson Narrative

In this culminating lesson for the unit, students put what they have learned to work in solving real-world problems, using all the different forms of equations they have studied (MP4).

#### Alignments

#### Addressing

- 8.EE.B.6: Use similar triangles to explain why the slope  $m$  is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation  $y = mx$  for a line through the origin and the equation  $y = mx + b$  for a line intercepting the vertical axis at  $b$ .
- 8.EE.C.8.a: Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

#### Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports
- Think Pair Share

#### Student Learning Goals

Let's write equations for real-world situations and think about their solutions.

## 14.1 Buying Fruit

#### Warm Up: 5 minutes

Students write expressions and equations representing total cost. The purpose of this activity is to support students in writing the equation for the “Ordering Fish” activity.

## Addressing

- 8.EE.B.6

## Instructional Routines

- Think Pair Share

## Launch

Arrange students in groups of 2. Display questions for all to see. Give 2 minutes quiet think time, followed by 2 minutes partner discussion then whole-class discussion.

## Anticipated Misconceptions

Some students may not be sure how to approach writing the scenario as an equation. For these students, suggest that they make a table of possible prices based on the amount of fruit purchased.

### Student Task Statement

For each relationship described, write an equation to represent the relationship.

1. Grapes cost \$2.39 per pound. Bananas cost \$0.59 per pound. You have \$15 to spend on  $g$  pounds of grapes and  $b$  pounds of bananas.
2. A savings account has \$50 in it at the start of the year and \$20 is deposited each week. After  $x$  weeks, there are  $y$  dollars in the account.

## Student Response

1.  $2.39g + 0.59b = 15$
2.  $y = 20x + 50$

## Activity Synthesis

The purpose of this discussion is to have students explain strategies for writing equations for real-world scenarios. Ask students to share what they discussed with their partners by asking:

- “What did each of the variables mean in the situations?” (Since we want a price based on the number of pounds of fruit,  $b$  and  $g$  represent the amount of bananas and grapes purchased. For the savings account, the  $x$  was the number of weeks, and the  $y$  was number of dollars.)
- “Was the slope for each of these equations positive or negative? Why does that make sense with the scenario?” (For the fruit, the slope was negative, which makes sense because if you buy more of one fruit, you have to buy less of the other. For the savings account, the slope is positive, which makes sense because the more weeks go by, the more money will be in the account.)

## 14.2 Five Savings Accounts

25 minutes

Given a graph with five lines representing changes in bank account balance over time, students write equations and interpret how points represent solutions. The activity also connects to and contextualizes students' prior understanding of slope and intercepts, and lays the foundation for the coming unit on systems of equations by considering what points of intersection of lines and non-intersecting lines represent.

### **Addressing**

- 8.EE.B.6

### **Instructional Routines**

- MLR8: Discussion Supports

### **Launch**

Display the image from the lesson for all to see and ask the students to consider line  $a$ . Invite 2–3 students to describe in words what line  $a$  shows. If no students bring it up, tell students that they saw this line before in the warm-up, and they wrote an equation for it. Instruct students that for #1, they should not choose line  $a$ .

Arrange students in groups of 3–4. Groups work for about 10 minutes, followed by a whole-class discussion.

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### **Support for Students with Disabilities**

*Representation: Internalize Comprehension.* Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.

*Supports accessibility for: Memory; Conceptual processing*

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### **Support for English Language Learners**

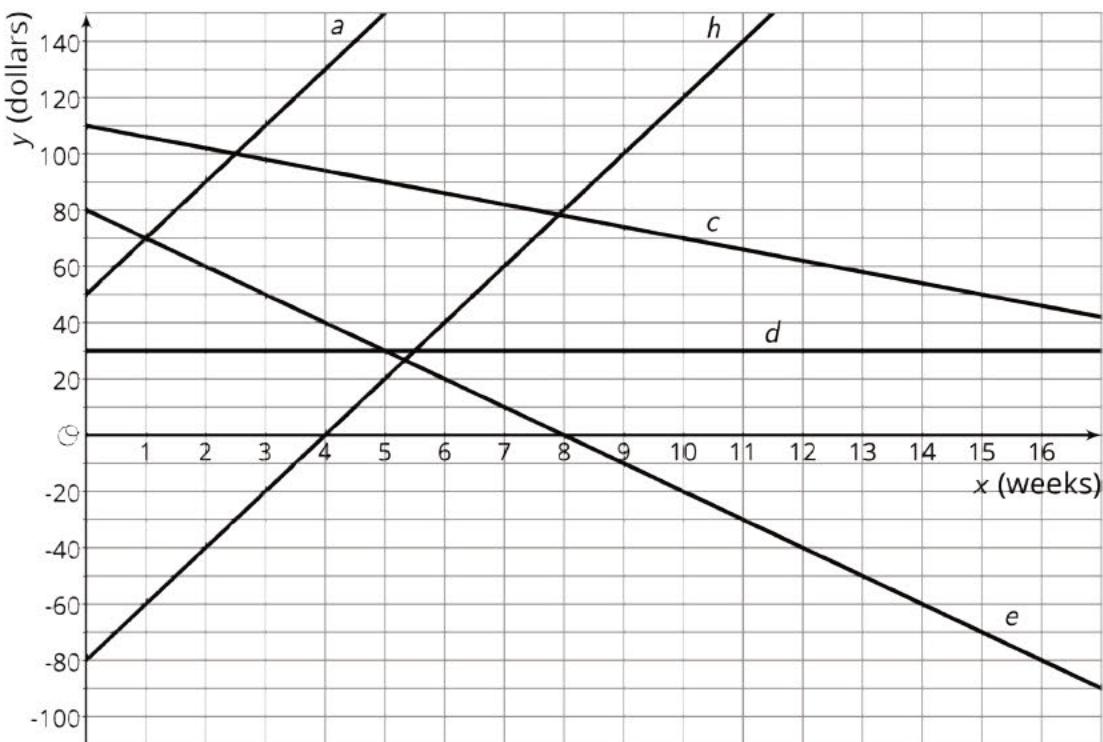
*Conversing: MLR8 Discussion Supports.* Use this routine to support discussion about the question, "What can we say about the points where two lines cross?" Invite students work with a partner to select a few of the intersection points to discuss (between lines  $a$  and  $c$ ,  $a$  and  $e$ ,  $h$  and  $c$ ,  $h$  and  $d$ ,  $h$  and  $e$ , and  $d$  and  $e$ ). Consider providing these sentence frames for pairs to use: "Lines \_\_\_\_\_ and \_\_\_\_\_ cross at the point \_\_\_\_\_ and this tells me that . . . . I know this because . . . ." Encourage students to make explicit references to the number of weeks and dollar amounts that are represented. This will help students communicate about the point where two lines intersect in the context of a real-world situation.

*Design Principle(s): Support sense-making; Cultivate conversation*

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### **Student Task Statement**

Each line represents one person's weekly savings account balance from the start of the year.



1. Choose one line and write a description of what happens to that person's account over the first 17 weeks of the year. Do not tell your group which line you chose.
2. Share your story with your group and see if anyone can guess your line.
3. Write an equation for each line on the graph. What do the slope,  $m$ , and vertical intercept,  $b$ , in each equation mean in the situation?
4. For which equation is  $(1, 70)$  a solution? Interpret this solution in terms of your story.
5. Predict the balance in each account after 20 weeks.

### Student Response

1. Answers vary. Sample responses: Person *a* starts with \$50 and is saving money at the rate of \$20 per week. Person *h* owed \$80 and is paying it back at the rate of \$20 per week, then saving once the debt is paid off. Person *c* starts with \$110 and is spending money at the rate of \$20 every 5 weeks, or \$4 per week. Person *d* has \$30 and is neither saving or spending. Person *e* starts with \$80 and spends at the rate of \$10 per week.
2. Responses vary.
3. *a*:  $y = 20x + 50$ ; *h*:  $y = 20x - 80$ ; *c*:  $y = -4x + 110$ ; *d*:  $y = 30$ ; *e*:  $y = -10x + 80$ ; For each equation, the slope tells the rate of change of saving (positive) or spending (negative). The value of  $b$  indicates the amount of money they started with, positive represents a saved balance, negative represents money they owe. Person *d* shows a slope of zero—neither saving or spending, so that they remain over time with the same amount that they start with.

4. We can see from the graph that lines  $a$  and  $e$  share a common point, or solution, at  $x = 1$  week, where \$ for both. Sample explanation: at 1 week, each of these people had \$70 in their accounts.
5. Person  $a$  will have \$. Person  $h$  will have \$. Person  $c$  will have \$. Person  $d$  will have \$30. Person  $e$  will have \$.

### Activity Synthesis

Students should understand that points on a line show solutions to the equation of the line. Discuss with students:

- “What can we say about the points where two lines cross?” (The accounts had the same amount of money at the same time.)
- “How do the slopes of the lines help to tell the story from the graph?” (The slope tells us whether a person is spending or saving each week.)
- “What does your answer to question 3 tell us about their rates of saving?” (By knowing the value of the slope, we can compare who is spending or saving more quickly or more slowly.)

## 14.3 Fabulous Fish

### 20 minutes

Students represent a scenario with an equation and use the equation to find solutions. They create a graph (either with a table of values or by using two intercepts), interpret points on the graph, and interpret points not on the graph (MP2).

### Addressing

- 8.EE.C.8.a

### Instructional Routines

- MLR2: Collect and Display

### Launch

Allow about 10 minutes quiet think time for questions 1 through 4, then have students work with a partner to discuss questions 4 and 5. Look for students who define the variables or label the axes differently. This can be an opportunity to discuss the importance of defining what quantities your variables represent and that different graphs can represent the same information.

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### Support for English Language Learners

*Speaking, Listening: MLR2 Collect and Display.* Listen for and record the language students use to discuss the statement “List two ways that you can tell if a pair of numbers is a solution to an equation.” Organize and group similar strategies in the display for students to refer back to throughout the lesson. For example, group strategies that reference the equation in one area and strategies that reference the graph in another area. This will help students solidify their understanding about solutions to an equation.

*Design Principle(s): Support sense-making; Maximize meta-awareness*

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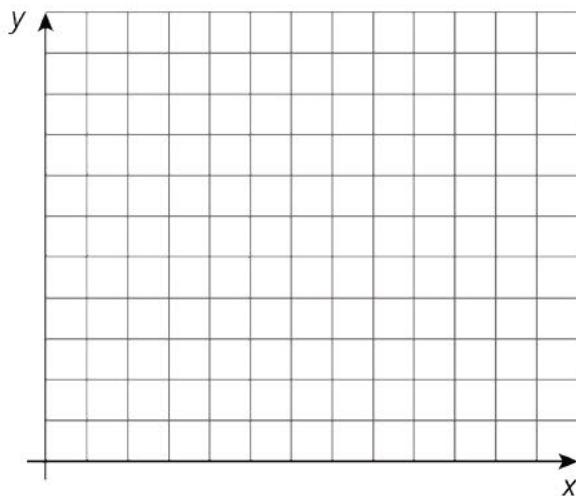
### Anticipated Misconceptions

If students let  $x$  be pounds of salmon, then the equation would be  $5x + 3y = 210$  and the coordinates would be reversed. The intercepts of the graph would be  $(0, 70)$  and  $(42, 0)$ . This is a good place to mention the importance of defining what quantities your variables represent.

### Student Task Statement

The Fabulous Fish Market orders tilapia, which costs \$3 per pound, and salmon, which costs \$5 per pound. The market budgets \$210 to spend on this order each day.

1. What are five different combinations of salmon and tilapia that the market can order?
2. Define variables and write an equation representing the relationship between the amount of each fish bought and how much the market spends.
3. Sketch a graph of the relationship. Label your axes.



4. On your graph, plot and label the combinations A—F.

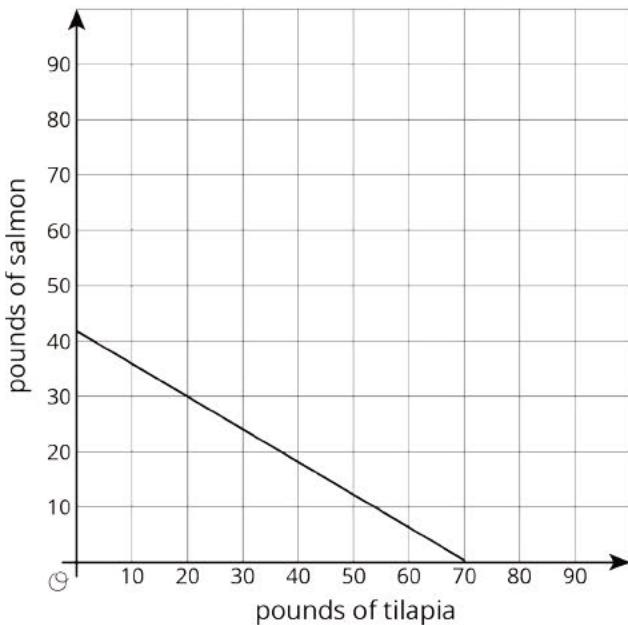
	A	B	C	D	E	F
pounds of tilapia	5	19	27	25	65	55
pounds of salmon	36	30.6	25	27	6	4

5. Which of these combinations can the market order? Explain or show your reasoning.

6. List two ways you can tell if a pair of numbers is a solution to an equation.

### Student Response

1. Answers vary. Sample responses: (0, 42), (10, 36), (20, 30), (30, 24), (50, 12), (70, 0)
2. Answers vary. Sample response: Let  $x$  be number of pounds of tilapia, let  $y$  be number of pounds of salmon:  $3x + 5y = 210$ .
3. Descriptions vary. Sample response: graph is a line that begins at (0, 42) and slopes downward until reaching (70, 0). It will not continue on indefinitely because negative pounds of fish does not make sense in the situation.



4. A does not work because  $3(5) + 5(36)$  is 195, not 210.  
**B** works because  $3(19) + 5(30.6)$  is 210.  
C does not work because  $3(27) + 5(25)$  is 206, not 210.  
D works because  $3(25) + 5(27)$  is 210.  
E does not work because  $3(65) + 5(6)$  is 225, not 210.  
F does not work because  $3(55) + 5(4)$  is 185, not 210.
5. Responses vary. Sample response: Solutions make the equation true and can be found on the graph of the equation.

## Activity Synthesis

Ask students to share some strategies for graphing and features of their graphs. Invite 2–3 students to display their graphs. Consider asking:

- "Was the slope of the line positive or negative? Why does that make sense in this situation?" (In this relationship, as one quantity increases the other must decrease in order to keep the sum  $3x + 5y$  constant.)
- "What was your strategy for graphing the relationship?" (Plotting points from question 1, using the table, figuring out the intercepts and connecting the line)
- "Why does it make sense for the graph to be only in quadrant I?" (You cannot purchase a negative amount of fish, so the  $x$  and  $y$  values cannot be negative.)
- "How is this situation different from the apples and oranges problem in a previous lesson?" (Buying  $\frac{1}{2}$  pound of fish is reasonable while buying  $\frac{1}{2}$  of an apple probably is not.)

Briefly reiterate key concepts: If a point is not on the graph of the equation then it is not a solution. The ordered pairs that are solutions to the equation all make the equation true and are all found on the line that is the graph of the equation.

A discussion could also include the detail that orders that are less than \$210 can also be considered to work, because there is money left over. That gives an opportunity to discuss the shape of the graph of  $3x + 5y \leq 210$ .

## Lesson Synthesis

Ask students to consider the real-world situations described in this lesson. Discuss:

- "Give an example of a solution to an equation that doesn't make sense in the context it represents." (Some values might not make sense in the context, like negative values. A length cannot be negative, for example.)
- "If some values make sense in the equation but not in the context, how could this impact the graph?" (We might only draw part of the line or draw some of the points that lie on the line.)





# Family Support Materials

# Family Support Materials

## Linear Relationships

Here are the video lesson summaries for Grade 8, Unit 3: Linear Relationships. Each video highlights key concepts and vocabulary that students learn across one or more lessons in the unit. The content of these video lesson summaries is based on the written Lesson Summaries found at the end of lessons in the curriculum. The goal of these videos is to support students in reviewing and checking their understanding of important concepts and vocabulary. Here are some possible ways families can use these videos:

- Keep informed on concepts and vocabulary students are learning about in class.
- Watch with their student and pause at key points to predict what comes next or think up other examples of vocabulary terms (the bolded words).
- Consider following the Connecting to Other Units links to review the math concepts that led up to this unit or to preview where the concepts in this unit lead to in future units.

Grade 8, Unit 3: Linear Relationships	Vimeo	YouTube
Video 1: Representing Proportional Relationships (Lessons 1–4)	<a href="#">Link</a>	<a href="#">Link</a>
Video 2: Representing Linear Relationships (Lessons 5–8)	<a href="#">Link</a>	<a href="#">Link</a>
Video 3: Finding Slopes (Lessons 9–10)	<a href="#">Link</a>	<a href="#">Link</a>
Video 4: Linear Equations (Lessons 11–13)	<a href="#">Link</a>	<a href="#">Link</a>

### Video 1

Video 'VLS G8U3V1 Representing Proportional Relationships (Lessons 1–4)' available here: <https://player.vimeo.com/video/469396489>.

### Video 2

Video 'VLS G8U3V2 Representing Linear Relationships (Lessons 5–8)' available here: <https://player.vimeo.com/video/470710599>.

### **Video 3**

Video 'VLS G8U3V3 Finding Slopes (Lessons 9–10)' available here:  
<https://player.vimeo.com/video/469397707>.

### **Video 4**

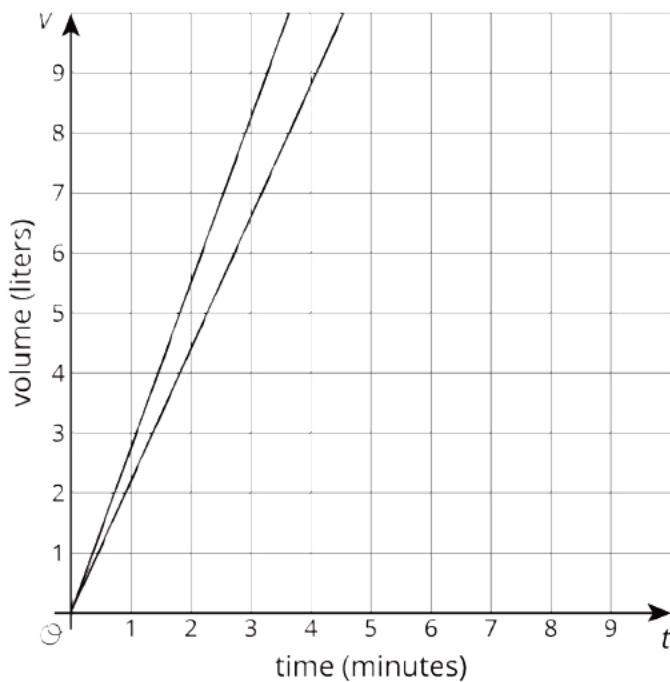
Video 'VLS G8U3V4 Linear Equations (Lessons 11–13)' available here:  
<https://player.vimeo.com/video/470020696>.

# Proportional Relationships

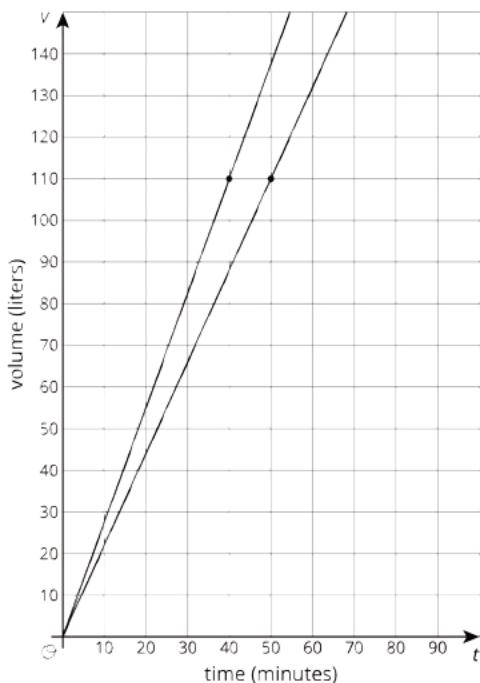
## Family Support Materials 1

This week your student will consider what it means to make a useful graph that represents a situation and use graphs, equations, tables, and descriptions to compare two different situations.

There are many successful ways to set up and add scale to a pair of axes in preparation for making a graph of a situation. Sometimes we choose specific ranges for the axes in order to see specific information. For example, if two large, cylindrical water tanks are being filled at a constant rate, we could show the amount of water in them using a graph like this:



While this graph is accurate, it only shows up to 10 liters, which isn't that much water. Let's say we wanted to know how long it would take each tank to have 110 liters. With 110 as a guide, we could set up our axes like this:



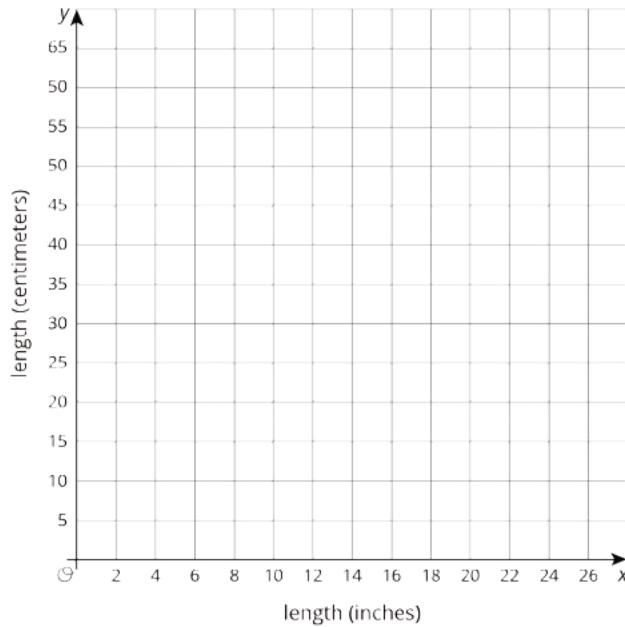
Notice how the vertical scale goes beyond the value we are interested in. Also notice how each axis has values that increase by 10, which, along with numbers like 1, 2, 5, 25, is a friendly number to count by.

Here is a task to try with your student:

This table shows some lengths measured in inches and the equivalent length in centimeters.

length (inches)	length (centimeters)
1	2.54
2	
10	
	50.8

1. Complete the table.
2. Sketch a graph of the relationships between inches and centimeters. Scale the axis so that all the values in the table can be seen on the graph.

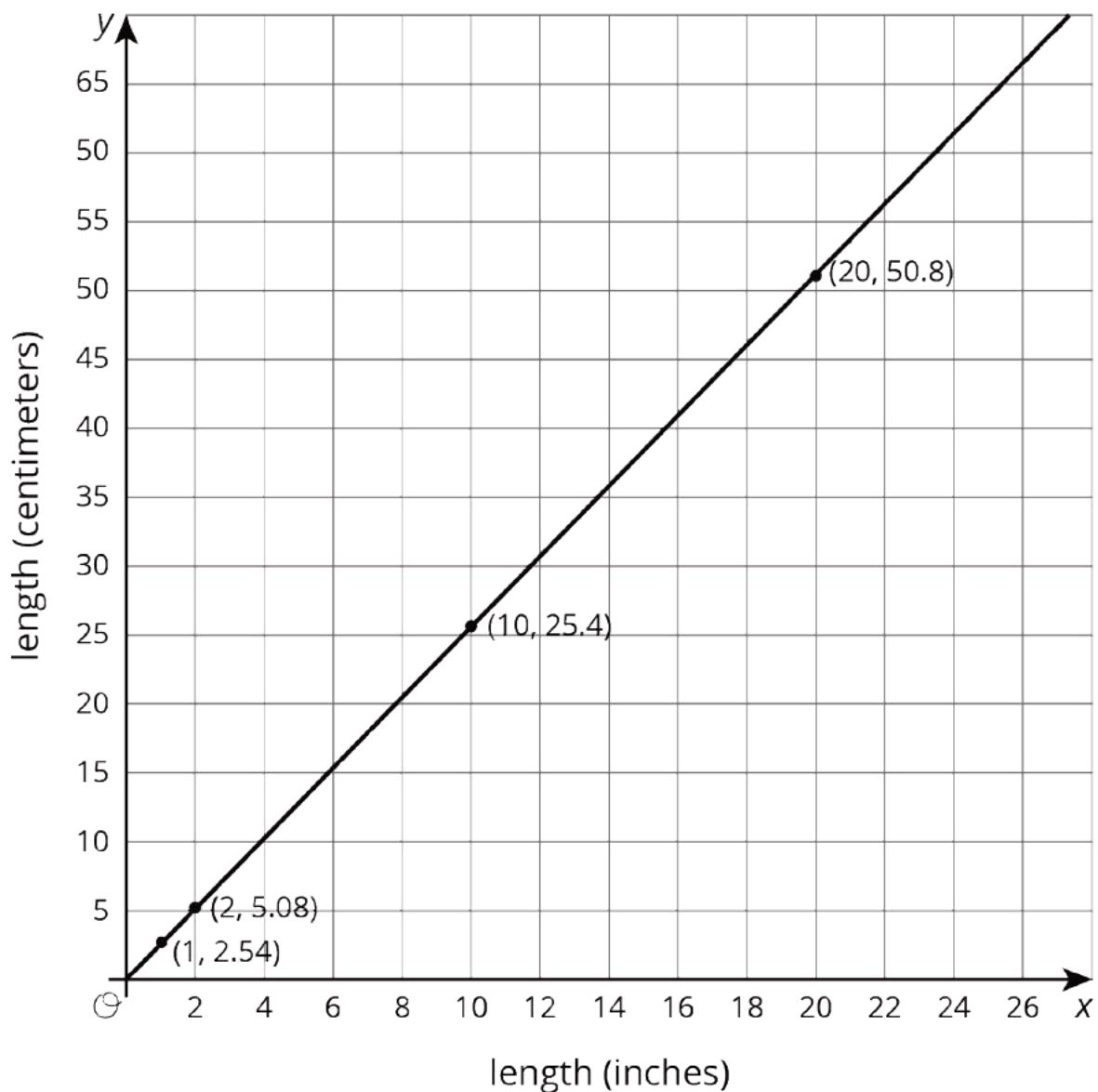


Solution:

- 1.

length (inches)	length (centimeters)
1	2.54
2	5.08
10	25.4
20	50.8

2.



# Representing Linear Relationships

## Family Support Materials 2

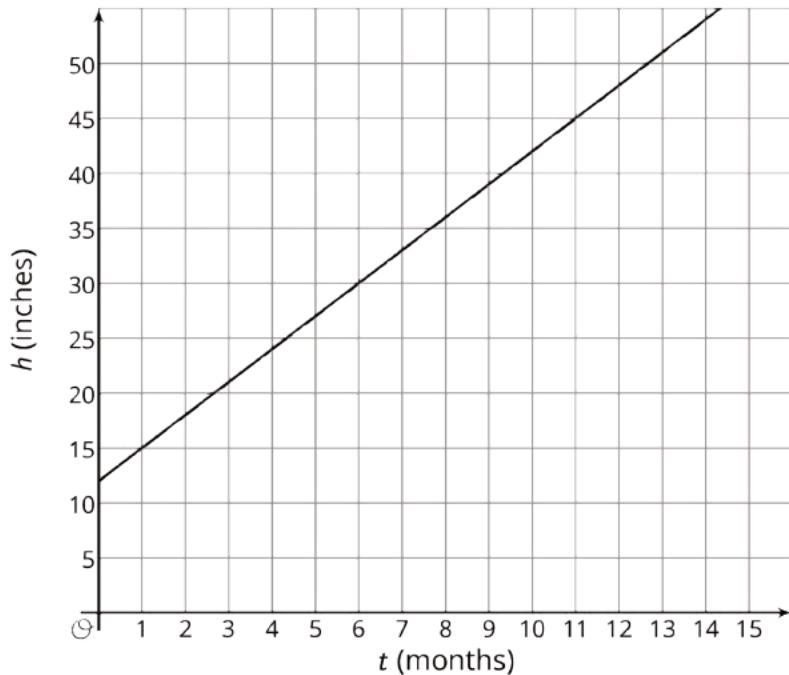
This week your student will learn how to write equations representing linear relationships. A linear relationship exists between two quantities where one quantity has a constant rate of change with respect to the other. The relationship is called linear because its graph is a line.

For example, say we are 5 miles into a hike heading toward a lake at the end of the trail. If we walk at a speed of 2.5 miles per hour, then for each hour that passes we are 2.5 miles further along the trail. After 1 hour we would be 7.5 miles from the start. After 2 hours we would be 10 miles from the start (assuming no stops). This means there is a linear relationship between miles traveled and hours walked. A graph representing this situation is a line with a slope of 2.5 and a vertical intercept of 5.

Here is a task to try with your student:

The graph shows the height in inches,  $h$ , of a bamboo plant  $t$  months after it has been planted.

1. What is the slope of this line? What does that value mean in this context?



2. At what point does the line intersect the  $h$ -axis? What does that value mean in this context?

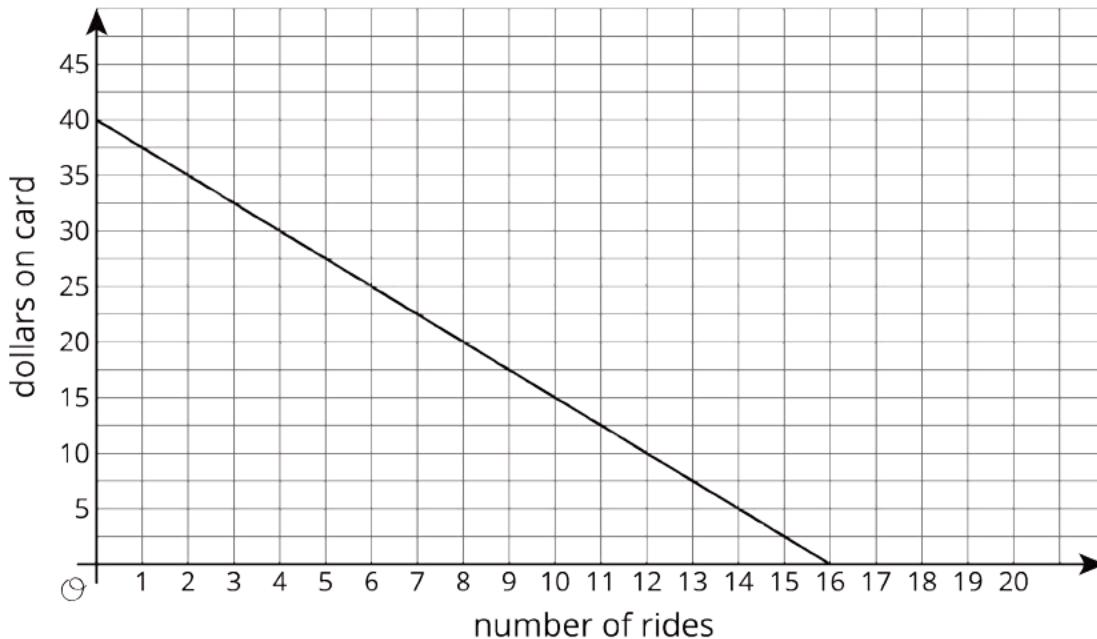
Solution:

1. 3. Every month that passes, the bamboo plant grows an additional 3 inches.
2. (0, 12). This bamboo plant was planted when it was 12 inches tall.

# Finding Slopes

## Family Support Materials 3

This week your student will investigate linear relationships with slopes that are not positive. Here is an example of a line with negative slope that represents the amount of money on a public transit fare card based on the number of rides you take:

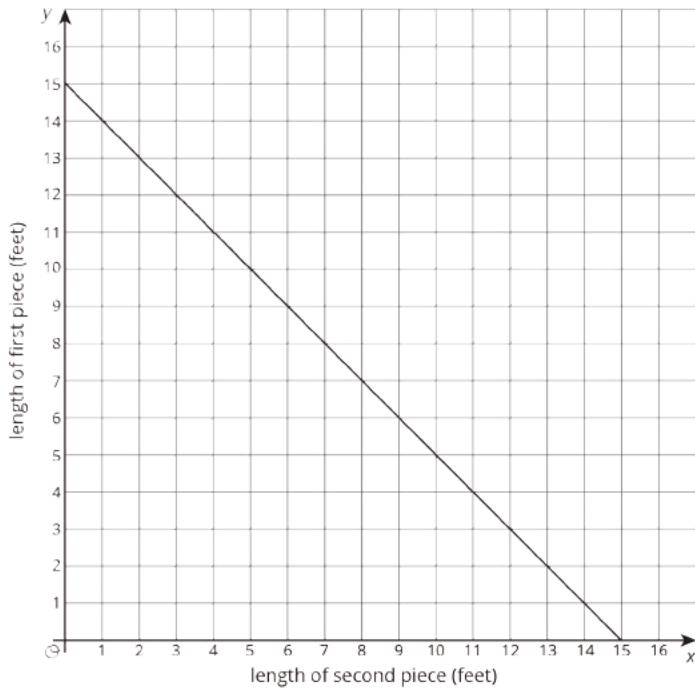


The slope of the line graphed here is  $-2.5$  since  $\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{40}{16} = -2.5$ . This corresponds to the cost of 1 ride. The vertical intercept is 40, which means the card started out with \$40 on it.

One possible equation for this line is  $y = -2.5x + 40$ . It is important for students to understand that every pair of numbers  $(x, y)$  that is a solution to the equation representing the situation is also a point on the graph representing the situation. (We can also say that every point  $(x, y)$  on the graph of the situation is a solution to the equation representing the situation.)

Here is a task to try with your student:

A length of ribbon is cut into two pieces. The graph shows the length of the second piece,  $x$ , for each length of the first piece,  $y$ .



1. How long is the original ribbon? Explain how you know.
2. What is the slope of the line? What does it represent?
3. List three possible pairs of lengths for the two pieces and explain what they mean.

Solution:

1. 15 feet. When the second piece is 0 feet long, the first is 15 feet long, so that is the length of the ribbon.
2. -1. For each length the second piece increases by, the first piece must decrease by the same length. For example, if we want the second piece to be 1 foot longer, then the first piece must be 1 foot shorter.
3. Three possible pairs: (14.5, 0.5), which means the second piece is 14.5 feet long so the first piece is only a half foot long. (7.5, 7.5), which means each piece is 7.5 feet long, so the original ribbon was cut in half. (0, 15), which means the original ribbon was not cut at all to make a second piece, so the first piece is 15 feet long.



# Unit Assessments

Check Your Readiness A and B  
End-of-Unit Assessment A and B

# Linear Relationships: Check Your Readiness

## (A)

You will need a straight edge for this assessment.

1. Select all the tables that could represent proportional relationships.

A.

x	y
2	3
5	7.5
10	15

B.

x	y
0	0
3	7
6	14

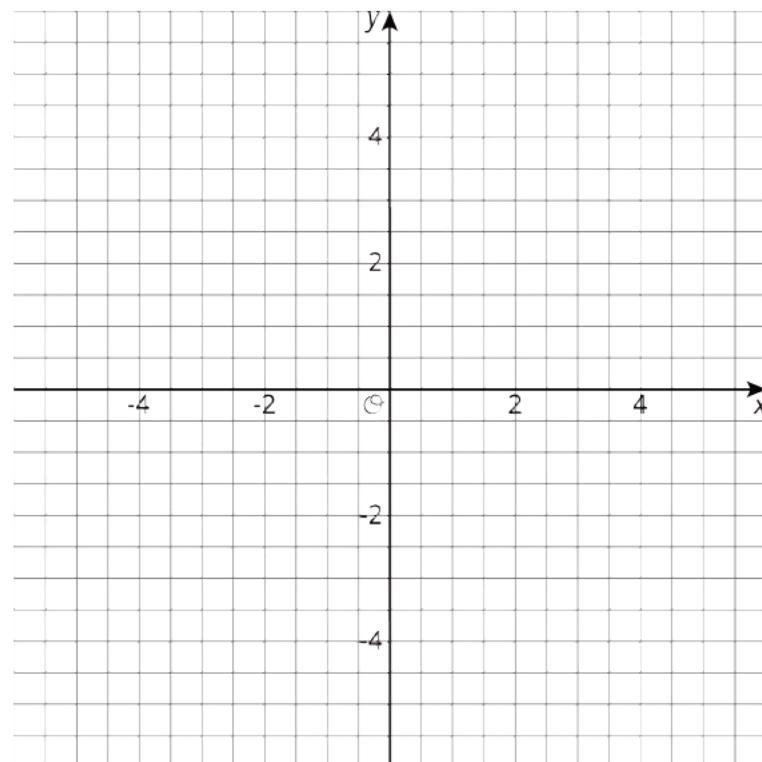
C.

x	y
0	2
2	4
4	6

2. To mix a particular shade of purple paint, red paint and blue paint are mixed in the ratio 5 : 3. To make 20 gallons of this shade of purple paint, how many gallons of red and blue paint should be used?

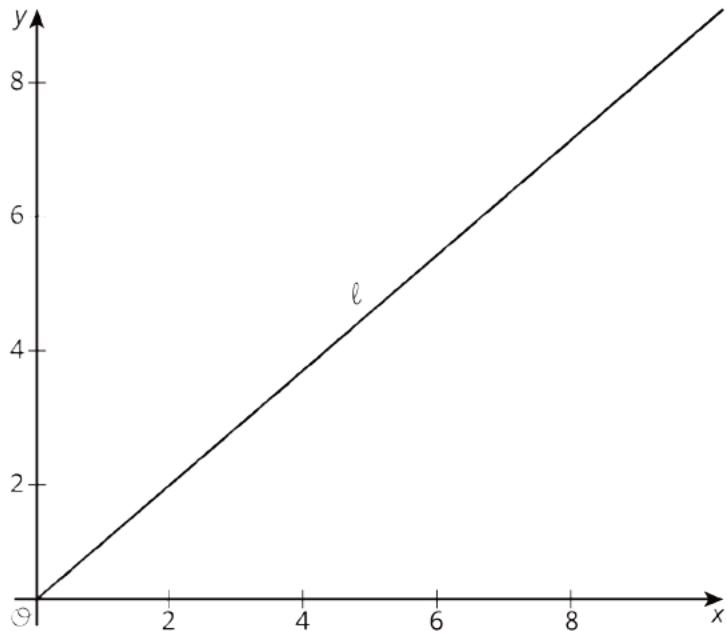
3. At one gas station, gas costs \$2.75 per gallon. Write an equation that relates the total cost,  $C$ , to the number of gallons of gas purchased,  $g$ .

4. a. Plot and label 3 different points with  $x$ -coordinate 3.  
b. Sketch or describe all points in the plane with  $x$ -coordinate 3.



5. On the coordinate plane, draw:

- A line  $m$  that is a translation of line  $\ell$ .
- A line  $n$  that is a rotation of line  $\ell$ , using the origin as the center of rotation.



6. A store sells ice cream with assorted toppings. They charge \$3.00 for an ice cream, plus 50 cents per ounce of toppings.

- How much does an ice cream cost with 4 ounces of toppings?
- How much does an ice cream cost with 11 ounces of toppings?
- If Elena's ice cream cost \$1.50 more than Jada's ice cream, how much more did Elena's toppings weigh?

# Linear Relationships: Check Your Readiness

## (B)

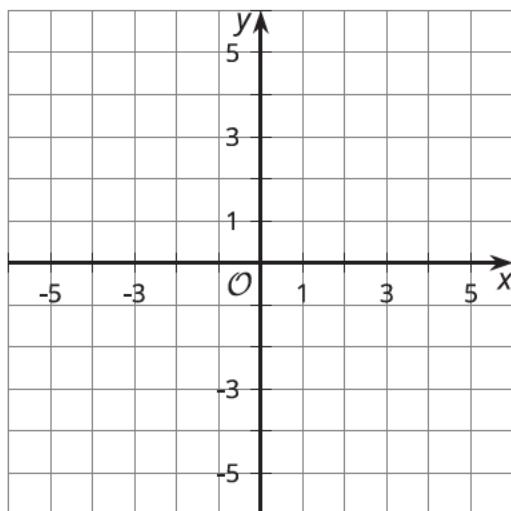
You will need a straight edge for this assessment.

1. There is a proportional relationship between  $x$  and  $y$ . Complete the table with the missing values.

$x$	$y$
4.5	3
18	12
	30
15	

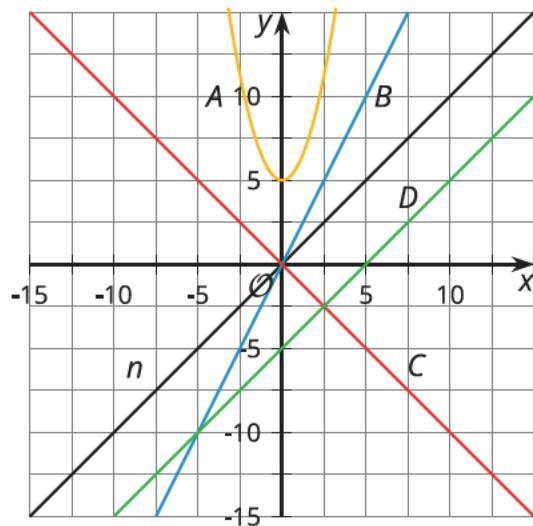
2. To mix a particular shade of pink paint, white paint and red paint are mixed in the ratio 4 : 3. If 12 gallons of red paint are mixed with some white paint to make this same shade of pink, how many total gallons of pink paint result?
  - 48
  - 36
  - 28
  - 16
3. At a grocery store, 2 gallons of milk cost \$7.20, and 5 gallons of milk cost \$18. Which equation relates the total cost,  $t$ , to the gallons of milk purchased,  $m$ ?
  - $m = 7.2t$
  - $t = 7.2m$
  - $m = 3.6t$
  - $t = 3.6m$

4. a. Plot and label 3 different points with  $y$ -coordinate -4.



b. Sketch or describe all points in the plane with  $y$ -coordinate -4.

5. Which graph is a translation of line  $n$ ?



6. At a sandwich shop, any sandwich costs \$4.50, plus \$0.25 for each extra topping.

a. How much does a sandwich cost with 3 extra toppings?

b. How much does a sandwich cost with 13 extra toppings?

c. If Andre's sandwich cost \$2.00 more than Clare's sandwich, how many more toppings did Andre add to his sandwich?

# Linear Relationships: End-of-Unit Assessment

## (A)

1. Select all the points that are on the graph of the line  $2x + 4y = 20$ .

A. (0, 5)

B. (0, 10)

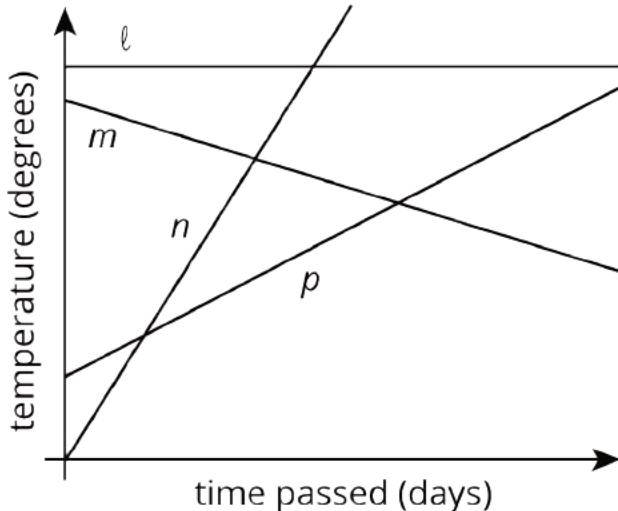
C. (1, 2)

D. (1, 4)

E. (5, 0)

F. (10, 0)

2. For two weeks, the highest temperature each day was recorded in four different cities. Lines  $\ell$ ,  $m$ ,  $n$ , and  $p$  are graphs of the temperature over time in Lubbock, Memphis, New Orleans, and Phoenix. Which statement is true?



A. The high temperature in Lubbock increased as time passed.

B. The high temperature in Memphis decreased steadily.

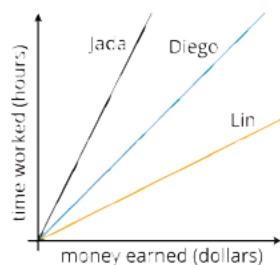
C. Initially, the high temperature was warmer in Phoenix than in Memphis.

D. The high temperature in Phoenix rose faster than the temperature in New Orleans.

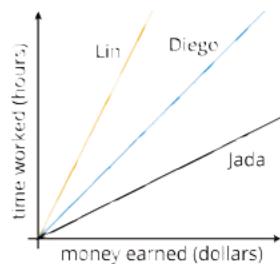
3. Jada earns twice as much money per hour as Diego. Diego earns twice as much money per hour as Lin.

Select **all** the graphs that could represent how much Jada, Diego, and Lin earn for different amounts of time worked.

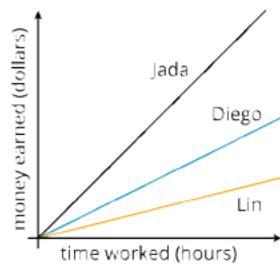
A.



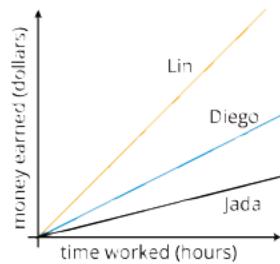
B.



C.



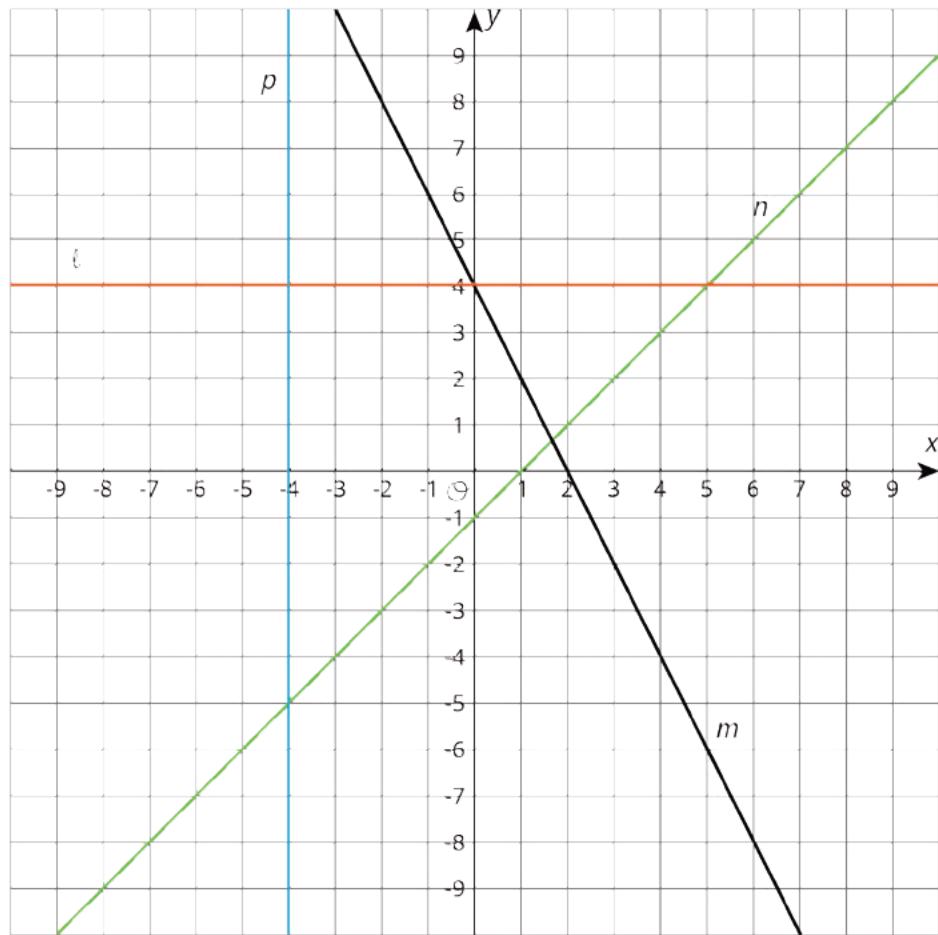
D.



E.

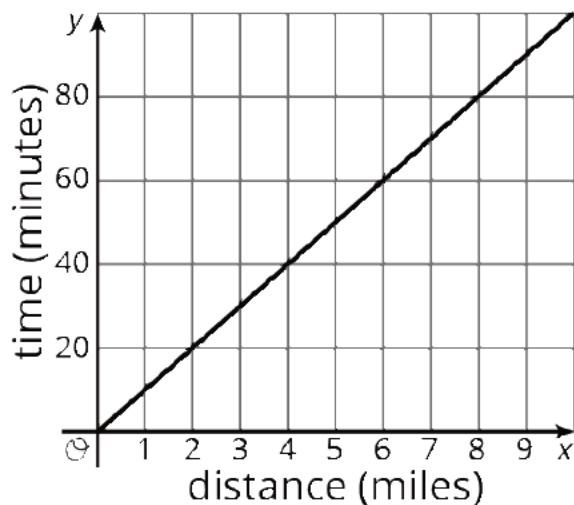


4. Write an equation for each line.



5. Three runners are training for a marathon. One day, they all run about ten miles, each at their own constant speed.

- This graph shows how long, in minutes, it takes Runner #1 to run  $d$  miles.



- The equation that relates Runner #2's distance (in miles) with time (in minutes) is  $t = 8.5d$ .
- Runner #3's information is in the table:

distance (miles)	time (minutes)
2	18
4	36
6	54
8	72
10	90

Which of the three runners has the fastest pace? Explain how you know.

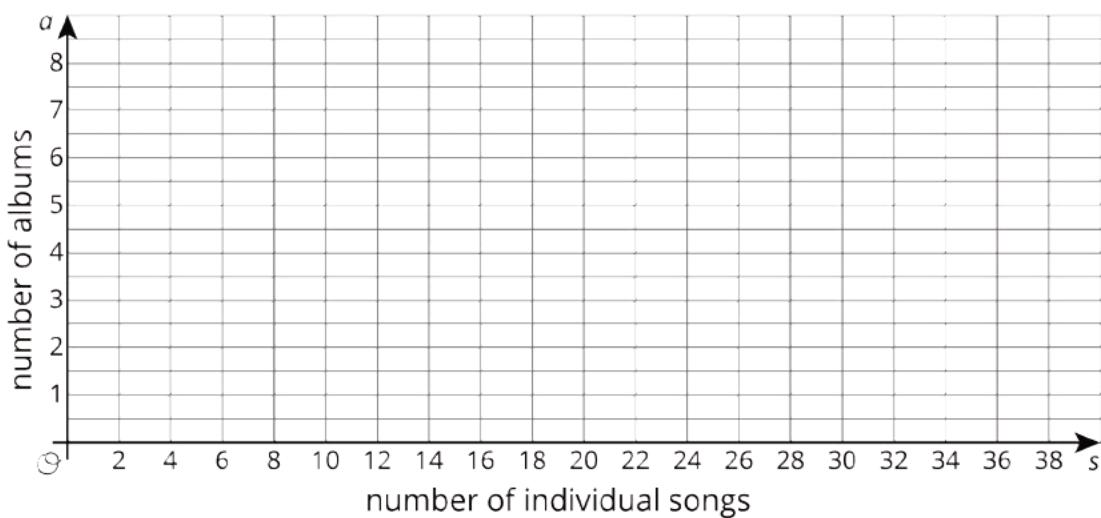
6. It costs \$0.50 to download an individual song and \$4 to download an album. Jada has \$15 to spend downloading music.

a. Complete the table showing three ways Jada can spend \$15 downloading individual songs and albums.

number of individual songs, $s$	number of albums, $a$
6	
	1
30	

b. Write an equation relating the the number of individual songs  $s$  and the number of albums  $a$  Jada can download.

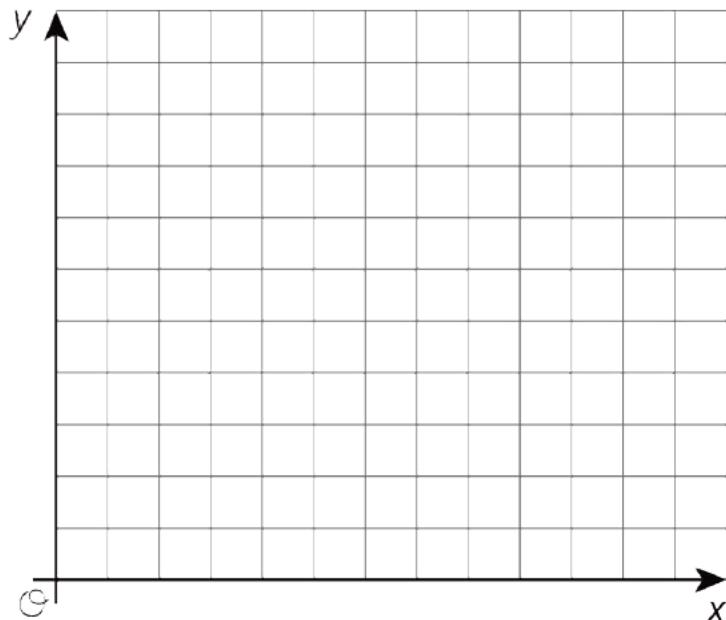
c. Sketch a graph of the solutions to your equation.



7. Han's cell phone plan costs \$200 to start. Then there is a \$50 charge each month.

a. What is the total cost (start up fee and monthly charge) to use the cell phone plan for 1 month?

b. What is the total cost for  $x$  months?



c. Graph the cost of the cell phone plan over a period of two years, using months as the units of time. Be sure to label your axes and scale them by labeling each gridline with a number.

d. Is there a proportional relationship between time and the cost of the cell phone plan? Explain how you know.

e. Tyler's cell phone plan costs \$350 to start, then there is a \$50 charge each month. On the same grid as Han's plan, graph the cost of Tyler's cell phone plan over a period of two years. Describe how the two graphs are the same and how they are different.

## Linear Relationships: End-of-Unit Assessment (B)

1. Select all the equations on which the point  $(10, 0)$  lies.

A.  $5x + 2y = 15$

B.  $2x + 4y = 20$

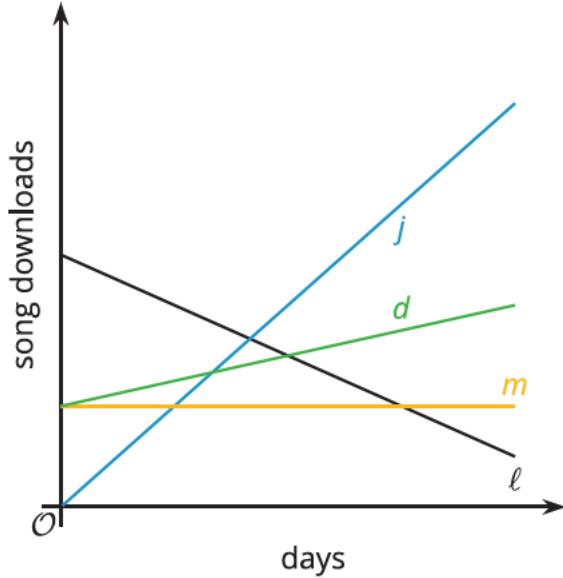
C.  $x + 6y = 10$

D.  $3x + 3y = 13$

E.  $4x + 2y = 20$

F.  $6x + y = 50$

2. A successful music app tracked the number of song downloads each day for a month for 4 music artists, represented by lines  $\ell$ ,  $j$ ,  $m$ , and  $d$  over the course of a month. Which line represents an artist whose downloads remained constant over the month?



A.  $\ell$

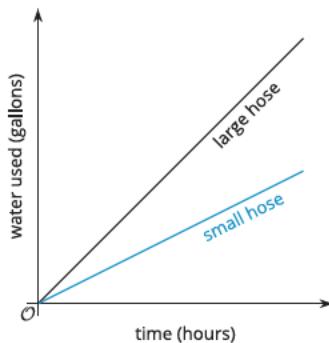
B.  $j$

C.  $m$

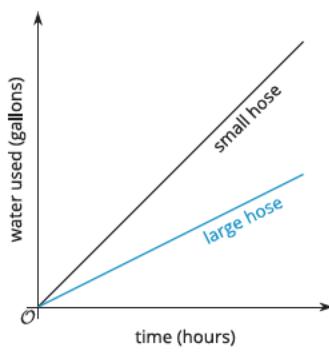
D.  $d$

3. A pool holds 19,900 gallons of water. There are two hoses that can be used to fill the pool, a large one and a small one. On an average day, it takes 5 hours to fill the pool with the large hose and 12 hours with the small hose. Which graph best represents this scenario?

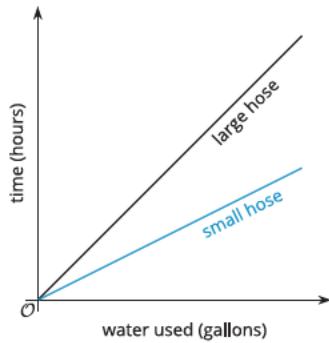
A.



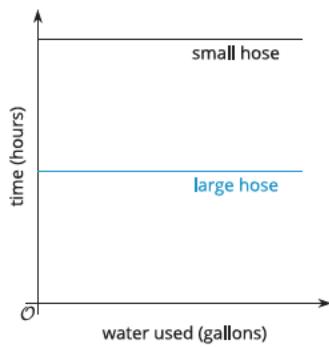
B.



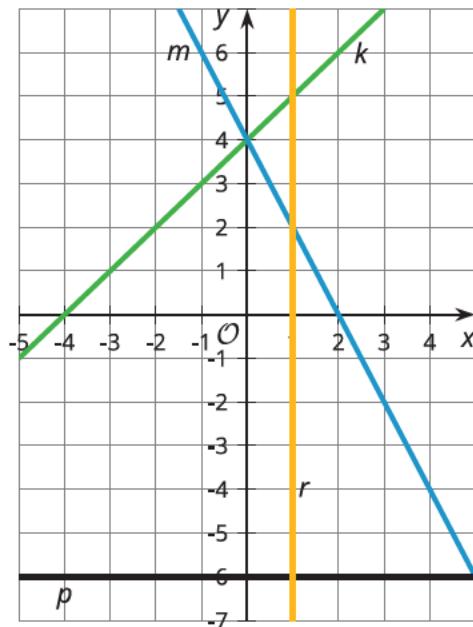
C.



D.



4. Write an equation for each line.

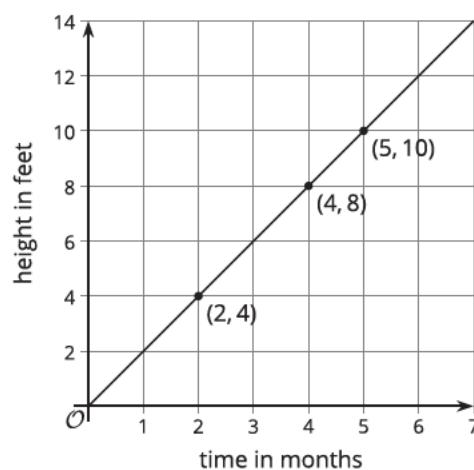


5. Noah is growing three different types of trees. He is keeping track of the height of each tree over time.

- This equation represents the height (in feet) of the first tree over  $m$  months.  

$$h = 3.5m$$
- The second tree's information is in the table:
- This graph shows how long, in months, it takes the third tree to grow  $h$  feet.

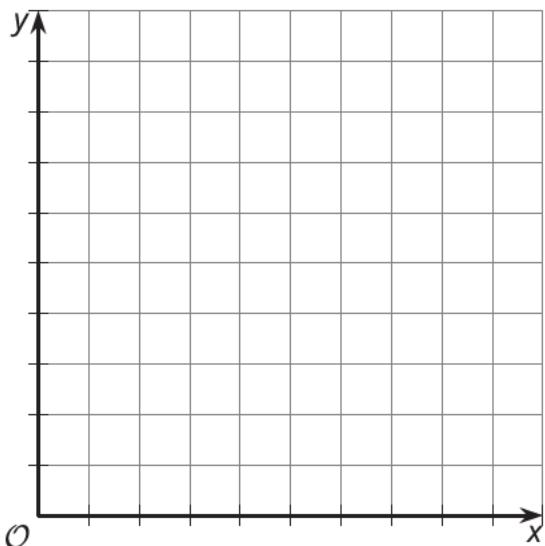
time (months)	height of tree (feet)
2	5
4	10
5	12.5
8	20



Which tree is growing the slowest? Explain how you know.

6. A sandwich store charges a \$10 delivery fee, and \$4.50 for each sandwich.

- What is the total cost (sandwiches and delivery charge) if an office orders 6 sandwiches?
- What is the total cost for  $x$  sandwiches?
- Graph the total cost of sandwiches and delivery based on number of sandwiches ordered. Be sure to label your axes and scale them by labeling each gridline with a number.

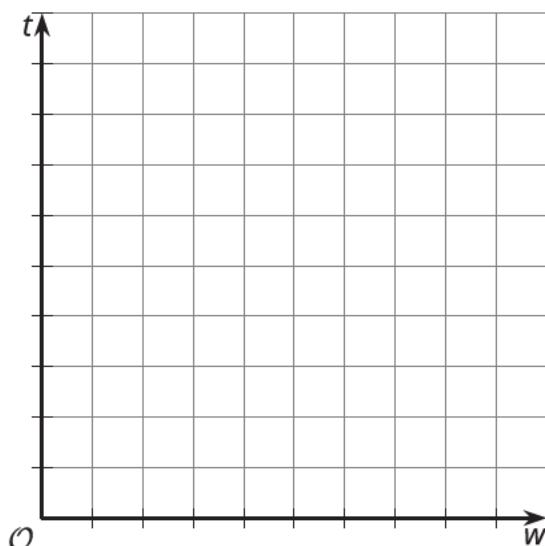


- Is there a proportional relationship between number of sandwiches and the cost of the order? Explain how you know.
- At a different sandwich shop, there is a \$5 delivery fee, and each sandwich costs \$4.50. On the same grid, graph the total cost of sandwiches and delivery based on number of sandwiches ordered for this new shop. Describe how the two graphs are the same and how they are different.

7. A truck is shipping jugs of drinking water and cases of paper towels. A jug of drinking water weighs 40 pounds and a case of paper towels weighs 16 pounds. The truck can carry 2,000 pounds of cargo altogether.

jugs of drinking water, $w$	cases of paper towels, $t$
10	
	50

- Complete the table showing three ways the truck could be packed with jugs of water and cases of paper towels.
- Write an equation relating the number of jugs of water and the number of cases of paper towels the truck can carry.
- Sketch a graph of the solutions to your equation.





# Assessment Answer Keys

Check Your Readiness A and B  
End-of-Unit Assessment A and B

# Assessments

## Assessment : Check Your Readiness (A)

### Teacher Instructions

Students will need a straight edge for this assessment.

### Student Instructions

You will need a straight edge for this assessment.

### Problem 1

The content assessed in this problem is first encountered in Lesson 3: Representing Proportional Relationships.

In this unit, students review their work with proportional relationships as a lead-in to linear equations.

If most students struggle with this item, plan to use this problem or a similar one as an additional warm-up activity. While they are working use MLR2: Collect and Display as a way to gather and show the student discourse. Note any words or phrases that can be added to a visual display for students to use throughout the unit. In addition, during Lessons 1 and 2 plan to stress multiple ways we can tell a relationship is proportional, such as finding a constant of proportionality, and how we can do that using coordinates of points on the graph.

#### Statement

Select **all** the tables that could represent proportional relationships.

A.

x	y
2	3
5	7.5
10	15

B.

x	y
0	0
3	7
6	14

C.

x	y
0	2
2	4
4	6

## Solution

["A", "B"]

### Aligned Standards

7.RP.A.2.a

### Problem 2

The content assessed in this problem is first encountered in Lesson 1: Understanding Proportional Relationships.

Students move from scale factors to genuine proportional relationships to prepare for linear relationships. If most students struggle with this item before beginning Lesson 1, do Grade 6 Unit 3 Lesson 7 Activity 3, Making Bracelets, to practice the concept of generating equivalent ratios.

### Statement

To mix a particular shade of purple paint, red paint and blue paint are mixed in the ratio 5 : 3. To make 20 gallons of this shade of purple paint, how many gallons of red and blue paint should be used?

## Solution

12.5 gallons red, 7.5 gallons blue

## Aligned Standards

6.RP.A.3

### Problem 3

The content assessed in this problem is first encountered in Lesson 1: Understanding Proportional Relationships.

In 7th grade, students learned to write equations to describe proportional relationships. The graphs of these equations are lines through the origin. In this unit, students will write equations for proportional relationships as well as other linear relationships.

If most students struggle with this item, plan to do Lesson 1 Activity 3, Moving Twice as Fast. During the activity synthesis spend some extra time discussing the third question and sharing their equations.

### Statement

At one gas station, gas costs \$2.75 per gallon. Write an equation that relates the total cost,  $C$ , to the number of gallons of gas purchased,  $g$ .

## Solution

$C = 2.75g$  (or equivalent)

## Aligned Standards

7.RP.A.2.c

### Problem 4

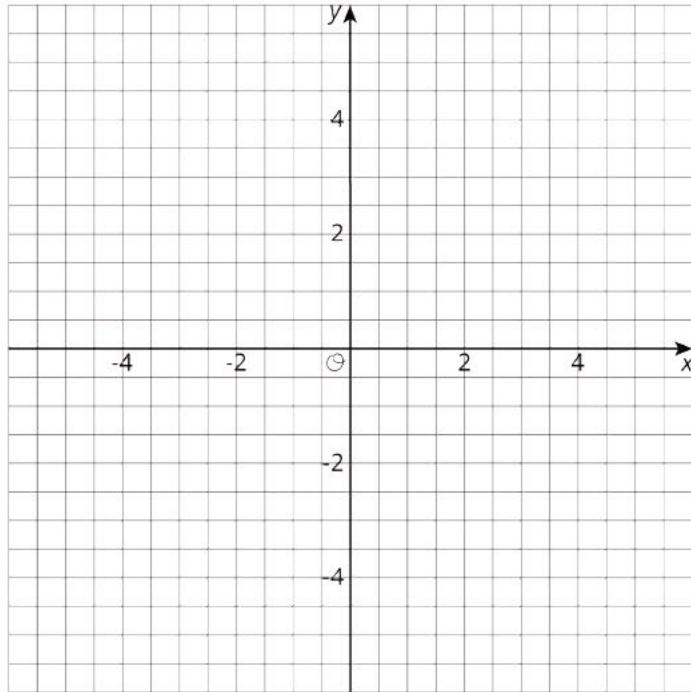
The content assessed in this problem is first encountered in Lesson 1: Understanding Proportional Relationships.

Students will need to be familiar with the coordinate plane for their work with graphing lines. Do not be concerned if students do not come up with an equation for the set of all points with  $x$ -coordinate 3. That is something they will learn to do in this unit.

If most students struggle with this item, plan to pause students as they are working on Lesson 1 Activity 2 Question 4 to ensure that they can plot and mark points once they have identified the bug's location at the given time. If students need additional practice, refer to 6th Grade, Unit 7, Lesson 11, Activity 1. If students struggle with describing the location of all points with  $x$ -coordinate 3, they will have additional opportunities to learn this concept beginning in Lesson 11. Follow the suggestions in the launch of Activity 2.

## Statement

1. Plot and label 3 different points with  $x$ -coordinate 3.
2. Sketch or describe all points in the plane with  $x$ -coordinate 3.



## Solution

1. Answers vary. At least three points are plotted and labeled. Sample responses:  $(3, 0), (3, -2), (3, 4)$ .
2. Answers vary. Sample responses: The line  $x = 3$  is drawn, or the equation  $x = 3$  is written.

## Aligned Standards

8.EE.B

### Problem 5

The content assessed in this problem is first encountered in Lesson 8: Translating to  $y = mx + b$ .

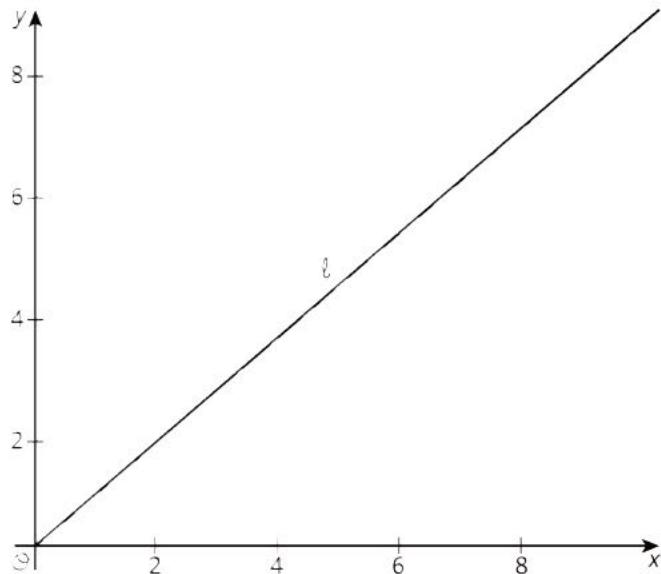
In this unit, students are presented with various forms of linear equations and various ways of thinking about those forms. One interpretation of the form  $y = mx + b$  is to consider it a vertical translation of the line  $y = mx$ .

If most students struggle with this item, plan to use Activity 1 in Lesson 8 to review translations. If students need additional practice recalling translations, especially translations of lines, refer to Unit 1 Lesson 9, Moves in Parallel.

## Statement

On the coordinate plane, draw:

1. A line  $m$  that is a translation of line  $\ell$ .
2. A line  $n$  that is a rotation of line  $\ell$ , using the origin as the center of rotation.

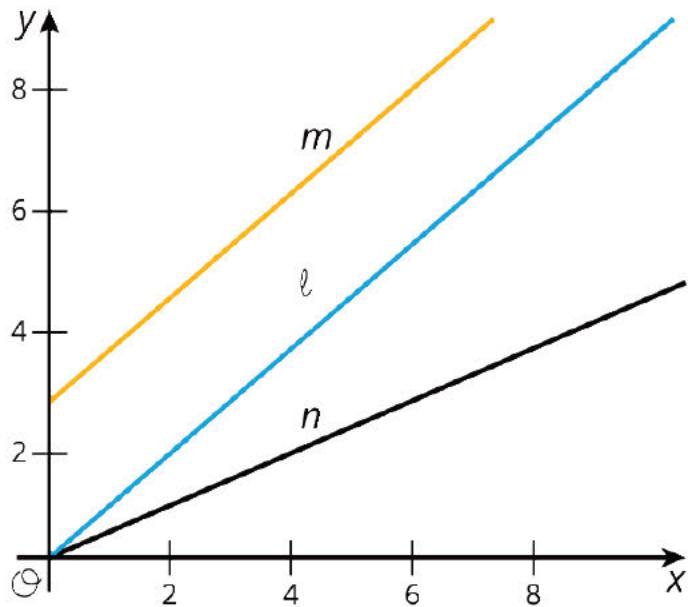


## Solution

Answers vary.

1. Line  $m$  can be any line parallel to  $\ell$ .
2. Line  $n$  can be any line through the origin.

Sample response:



## Aligned Standards

8.G.A.1, 8.G.A.1.c

### Problem 6

The content assessed in this problem is first encountered in Lesson 2: Graphs of Proportional Relationships.

Another interpretation of the form  $y = mx + b$  is to start with a given amount and thereafter increase the amount at a constant rate. Students are asked to engage in repeated reasoning in anticipation of this way of thinking.

If most students struggle with this item, plan to review it with students before beginning Lesson 2 Activity 2. Be sure to amplify vocabulary such as "constant of proportionality" and "unit rate" throughout this lesson.

#### Statement

A store sells ice cream with assorted toppings. They charge \$3.00 for an ice cream, plus 50 cents per ounce of toppings.

1. How much does an ice cream cost with 4 ounces of toppings?
2. How much does an ice cream cost with 11 ounces of toppings?
3. If Elena's ice cream cost \$1.50 more than Jada's ice cream, how much more did Elena's toppings weigh?

## **Solution**

1. \$5
2. \$8.50
3. 3 ounces

## **Aligned Standards**

7.EE.B.3

# Assessment : Check Your Readiness (B)

## Teacher Instructions

Students will need a straight edge for this assessment.

## Student Instructions

You will need a straight edge for this assessment.

### Problem 1

The content assessed in this problem is first encountered in Lesson 3: Representing Proportional Relationships.

In this unit, students review their work with proportional relationships as a lead-in to linear equations.

If most students struggle with this item, plan to use this problem or a similar one as an additional warm-up activity. While they are working use MLR2: Collect and Display as a way to gather and show the student discourse. Note any words or phrases that can be added to a visual display for students to use throughout the unit. In addition, during Lessons 1 and 2 plan to stress multiple ways we can tell a relationship is proportional, such as finding a constant of proportionality, and how we can do that using coordinates of points on the graph.

### Statement

There is a proportional relationship between  $x$  and  $y$ . Complete the table with the missing values.

$x$	$y$
4.5	3
18	12
	30
15	

## Solution

$x$	$y$
4.5	3
18	12
<b>45</b>	30
15	<b>10</b>

## Aligned Standards

7.RP.A.2.a

### Problem 2

The content assessed in this problem is first encountered in Lesson 1: Understanding Proportional Relationships.

Students move from scale factors to genuine proportional relationships to prepare for linear relationships. If most students struggle with this item before beginning Lesson 1, do Grade 6 Unit 3 Lesson 7 Activity 3, Making Bracelets, to practice the concept of generating equivalent ratios.

#### Statement

To mix a particular shade of pink paint, white paint and red paint are mixed in the ratio 4 : 3. If 12 gallons of red paint are mixed with some white paint to make this same shade of pink, how many total gallons of pink paint result?

- A. 48
- B. 36
- C. 28
- D. 16

## Solution

C

## Aligned Standards

7.RP.A.3

### Problem 3

The content assessed in this problem is first encountered in Lesson 1: Understanding Proportional Relationships.

In grade 7, students learned to write equations to describe proportional relationships. The graphs of these equations are lines through the origin. In this unit, students will write equations for proportional relationships as well as other linear relationships.

If most students struggle with this item, plan to do Lesson 1 Activity 3, Moving Twice as Fast. During the activity synthesis spend some extra time discussing the third question and sharing their equations.

## Statement

At a grocery store, 2 gallons of milk cost \$7.20, and 5 gallons of milk cost \$18. Which equation relates the total cost,  $t$ , to the gallons of milk purchased,  $m$ ?

- A.  $m = 7.2t$
- B.  $t = 7.2m$
- C.  $m = 3.6t$
- D.  $t = 3.6m$

## Solution

D

## Aligned Standards

7.RP.A.2.c

## Problem 4

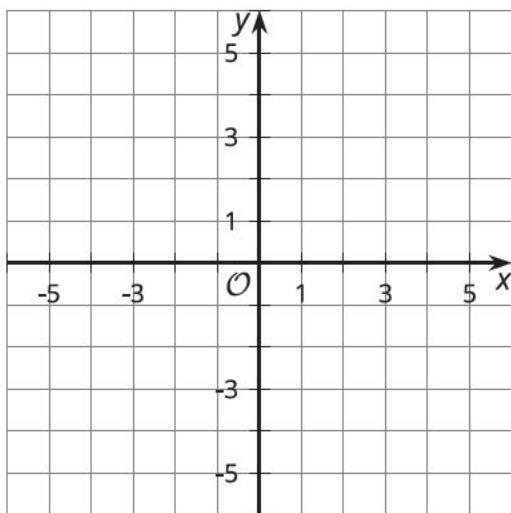
The content assessed in this problem is first encountered in Lesson 1: Understanding Proportional Relationships.

Students will need to be familiar with the coordinate plane for their work with graphing lines. Do not be concerned if students do not come up with an equation for the set of all points with  $y$ -coordinate -4. That is something they will learn to do in this unit.

If most students struggle with this item, plan to pause students as they are working on Lesson 1 Activity 2 Question 4 to ensure that they can plot and mark points once they have identified the bug's location at the given time. If students need additional practice, refer to 6th Grade, Unit 7, Lesson 11, Activity 1. If students struggle with describing the location of all points with  $y$ -coordinate 3, they will have additional opportunities to learn this concept beginning in Lesson 11. Follow the suggestions in the launch of Activity 2.

## Statement

1. Plot and label 3 different points with  $y$ -coordinate -4.



2. Sketch or describe all points in the plane with  $y$ -coordinate  $-4$ .

## Solution

1. Answers vary. At least three points are plotted and labeled, such as  $(0, -4)$ ,  $(-2, -4)$ , and  $(1, -4)$ .
2. Answers vary. Sample responses: The line  $y = -4$  is drawn, or the equation  $y = -4$  is written.

## Aligned Standards

8.EE.B

### Problem 5

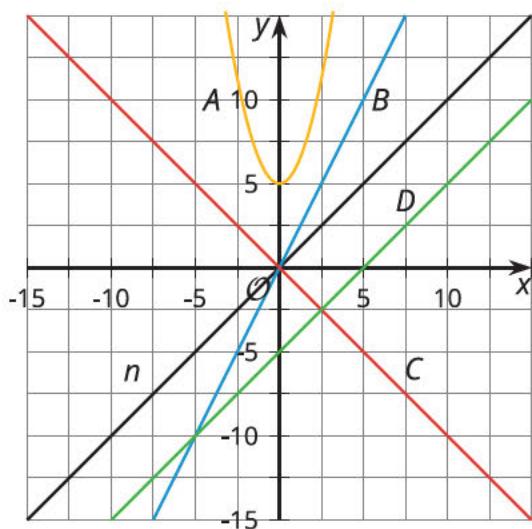
The content assessed in this problem is first encountered in Lesson 8: Translating to  $y = mx + b$ .

In this unit, students are presented with various forms of linear equations and various ways of thinking about those forms. One interpretation of the form  $y = mx + b$  is to consider it a vertical translation of the line  $y = mx$ .

If most students struggle with this item, plan to use Activity 1 in Lesson 8 to review translations. If students need additional practice recalling translations, especially translations of lines, refer to Unit 1 Lesson 9, Moves in Parallel.

### Statement

Which graph is a translation of line  $n$ ?



## Solution

Line D

## Aligned Standards

8.G.A.1, 8.G.A.1.c

## Problem 6

The content assessed in this problem is first encountered in Lesson 2: Graphs of Proportional Relationships.

Another interpretation of the form  $y = mx + b$  is to start with a given amount and thereafter increase the amount at a constant rate. Students are asked to engage in repeated reasoning in anticipation of this way of thinking.

If most students struggle with this item, plan to review it with students before beginning Lesson 2 Activity 2. Be sure to amplify vocabulary such as "constant of proportionality" and "unit rate" throughout this lesson.

## Statement

At a sandwich shop, any sandwich costs \$4.50, plus \$0.25 for each extra topping.

1. How much does a sandwich cost with 3 extra toppings?
2. How much does a sandwich cost with 13 extra toppings?
3. If Andre's sandwich cost \$2.00 more than Clare's sandwich, how many more toppings did Andre add to his sandwich?

## **Solution**

1. \$5.25
2. \$7.75
3. 8 toppings

## **Aligned Standards**

7.EE.B.3

# Assessment : End-of-Unit Assessment (A)

## Problem 1

Students failing to select A or F may not know that the graph of a line is the set of all solutions to the corresponding equation. Students selecting B or E may be reversing the  $x$ - and  $y$ -coordinates. Students selecting C may be misled by the fact that the coefficients of  $x$  and  $y$ , 2 and 4, are in a 1:2 ratio. Students selecting D may be misled by having  $2x + 4y$  equal 10, half the desired value of 20.

### Statement

Select **all** the points that are on the graph of the line  $2x + 4y = 20$ .

- A. (0, 5)
- B. (0, 10)
- C. (1, 2)
- D. (1, 4)
- E. (5, 0)
- F. (10, 0)

### Solution

["A", "F"]

### Aligned Standards

8.EE.B.5

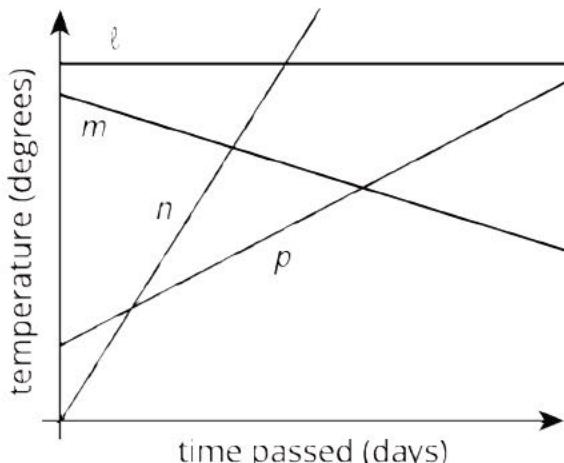
## Problem 2

Students identify descriptions that fit various graphs. The descriptions indicate the rate of change or slope of the linear graph. They deal with positive, zero, and negative slope. They also compare two positive slopes.

Students selecting A have not mastered the interpretation of a horizontal line as "no change." Students failing to select B do not understand the interpretation of negative slope as "decreasing." Students selecting C are having trouble interpreting slope as a unit rate. Students selecting D are not making the connection between the vertical intercept and an initial amount.

## Statement

For two weeks, the highest temperature each day was recorded in four different cities. Lines  $\ell$ ,  $m$ ,  $n$ , and  $p$  are graphs of the temperature over time in Lubbock, Memphis, New Orleans, and Phoenix. Which statement is true?



- A. The high temperature in Lubbock increased as time passed.
- B. The high temperature in Memphis decreased steadily.
- C. Initially, the high temperature was warmer in Phoenix than in Memphis.
- D. The high temperature in Phoenix rose faster than the temperature in New Orleans.

## Solution

B

## Aligned Standards

8.EE.B, 8.F.B.4

## Problem 3

Students interpret proportional relationships from given lines. They must not only identify the slope of the lines as the unit rate but also quantitatively compare these unit rates in the absence of a given scale on the axes.

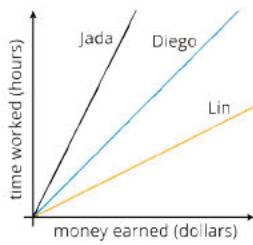
Students selecting A instead of B may be associating the slope with the pay rate, but the axes do not reflect this. Students selecting D instead of C may not have noticed the change in axes from the earlier parts. Students selecting E have a misunderstanding about the relative value of slopes.

## Statement

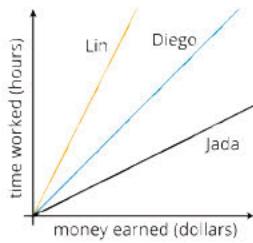
Jada earns twice as much money per hour as Diego. Diego earns twice as much money per hour as Lin.

Select **all** the graphs that could represent how much Jada, Diego, and Lin earn for different amounts of time worked.

A.



B.



C.



D.



E.



## Solution

["B", "C"]

## Aligned Standards

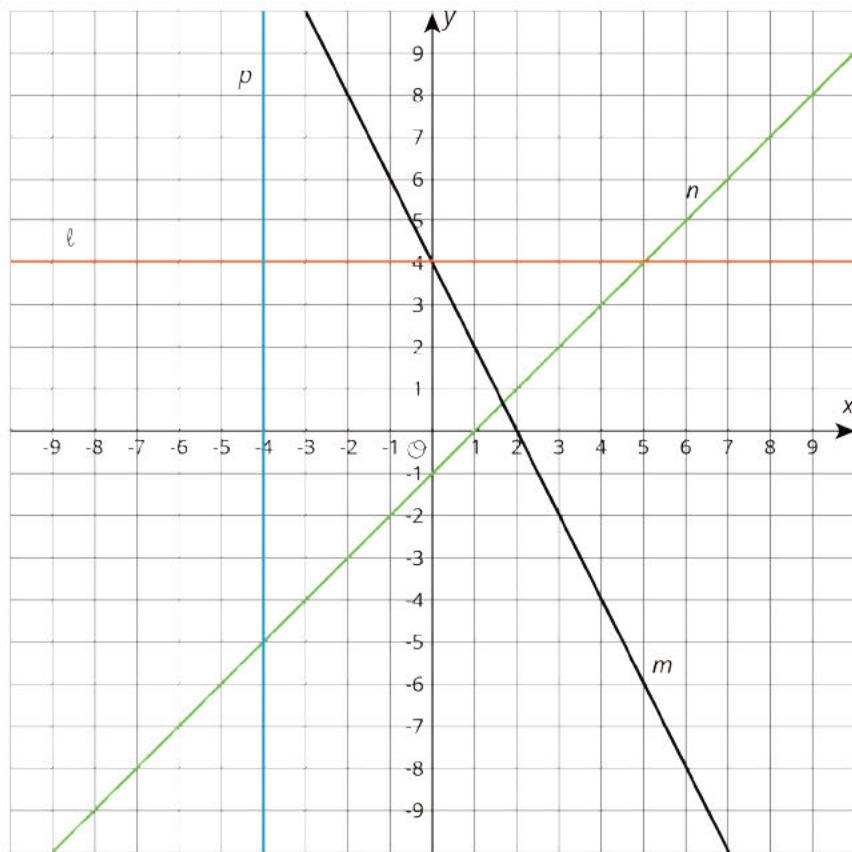
8.EE.B.5

## Problem 4

Students write equations of four lines given their graphs. One line is vertical, one is horizontal, one has positive slope, and one has negative slope.

## Statement

Write an equation for each line.



## Solution

$$\ell: y = 4, m: y = 4 - 2x, n: y = x - 1, p: x = -4$$

## Aligned Standards

8.EE.B

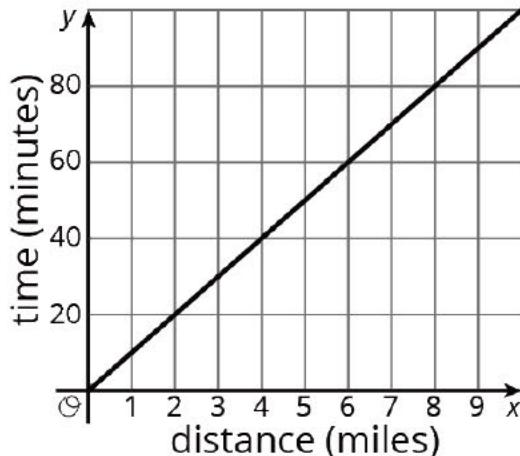
### Problem 5

Students compare the pace of three different runners. The proportional relationship between time and distance is represented in three different ways. There is more than one way to do this problem correctly. For example, students could determine how long it takes each runner to run 5 miles to determine the fastest runner, or could determine each runner's speed in miles per minute or miles per hour.

## Statement

Three runners are training for a marathon. One day, they all run about ten miles, each at their own constant speed.

- This graph shows how long, in minutes, it takes **Runner #1** to run  $d$  miles.



- The equation that relates **Runner #2**'s distance (in miles) with time (in minutes) is  $t = 8.5d$ .
- Runner #3**'s information is in the table:

distance (miles)	time (minutes)
2	18
4	36
6	54
8	72
10	90

Which of the three runners has the fastest pace? Explain how you know.

## Solution

Runner #2 is the fastest runner. Sample explanation: The point  $(2, 20)$  is on Runner #1's graph, so Runner #1's pace is 10 minutes per mile. Runner #2's equation shows that their pace is 8.5 minutes per mile. Calculate the unit rate for Runner #3 from the table:  $18 \div 2 = 9$ . Runner #3's pace is 9 minutes per mile.

Runner #2 takes the least time to travel 1 mile, so Runner #2 is the fastest.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: Runner #1 goes at 10 minutes per mile, Runner #2 goes at 8.5 minutes per mile, and Runner #3 goes at 9 minutes per mile. Runner #2 is fastest, because they take the least time to run one mile.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: Work contains correct unit rates for all three runners but concludes that runner #1 or #3 is the fastest or does not name a fastest runner; one unit rate is incorrect (possibly with an incorrect fastest runner identified as a consequence); insufficient explanation of work.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: two or more incorrect unit rates; the correct runner is identified but with no justification; response to the question is not based on unit rates or on similar methods such as calculating which runner has gone the farthest after 10 miles.

## Aligned Standards

8.EE.B.5

### Problem 6

A linear equation is described in terms of a constraint on the total cost of two different types of music a person can download with a given budget. The constraint gives an equation that students produce. Students then graph the solutions to the equation.

#### Statement

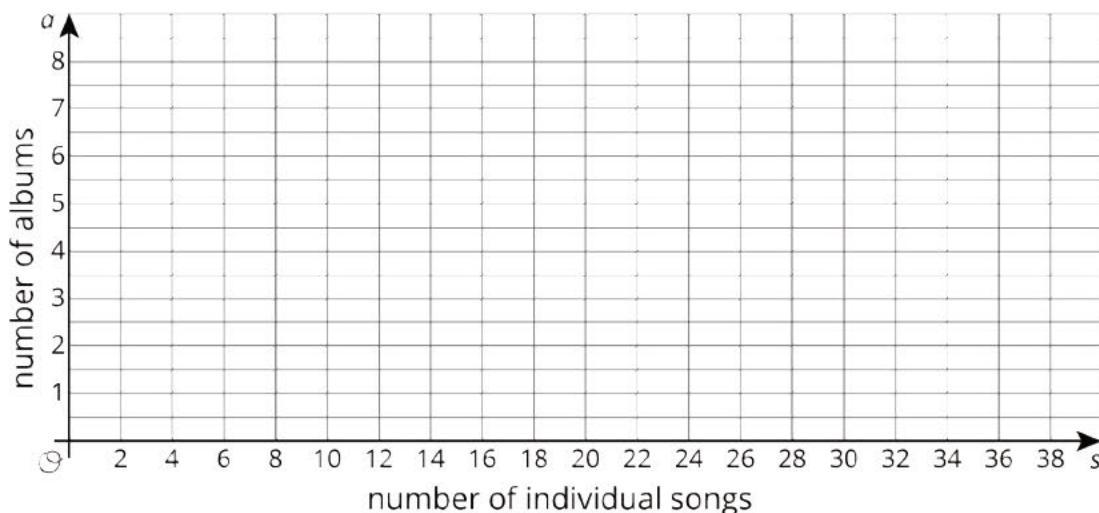
It costs \$0.50 to download an individual song and \$4 to download an album. Jada has \$15 to spend downloading music.

1. Complete the table showing three ways Jada can spend \$15 downloading individual songs and albums.

number of individual songs, $s$	number of albums, $a$
6	
	1
30	

2. Write an equation relating the the number of individual songs  $s$  and the number of albums  $a$  Jada can download.

3. Sketch a graph of the solutions to your equation.



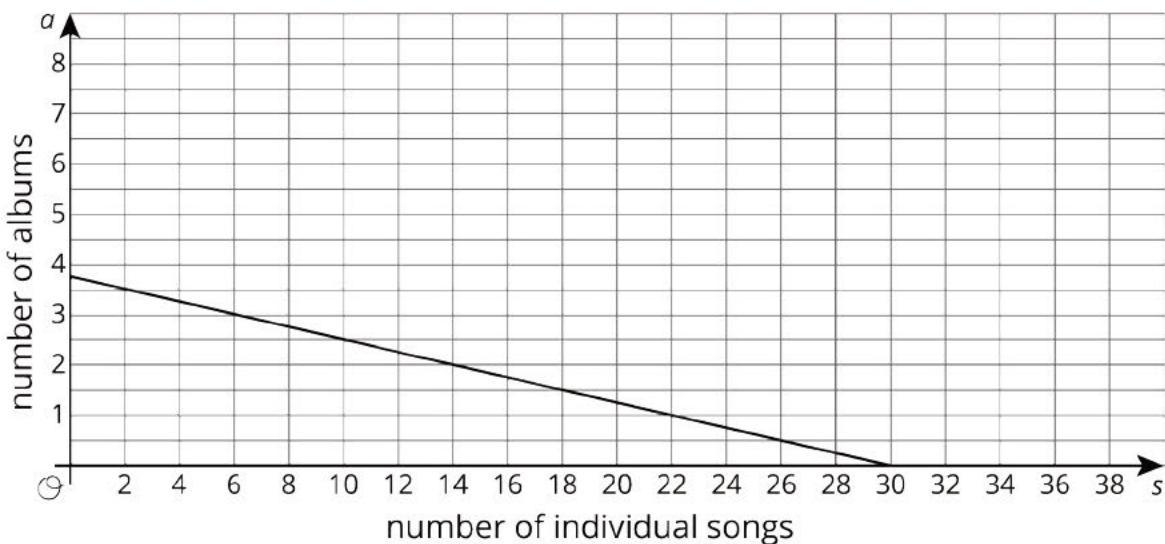
## Solution

1.

individual songs	albums
6	3
22	1
30	0

2.  $0.5s + 4a = 15$

3.



Minimal Tier 1 response:

- Work is complete and correct.
- The graph may be a continuous line, or it may consist only of points representing whole numbers of songs and albums.
- Sample: See solution above.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: graph contains only the three points from the table; equation is incorrect, but plausible.
- Acceptable errors: work for parts b and c is correctly based on an incorrect proportional relationship in part a; work in part c is correctly based on an incorrect equation in part b.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: work does not factor in the \$15 constraint; equation is badly off.

## Aligned Standards

8.EE.B

### Problem 7

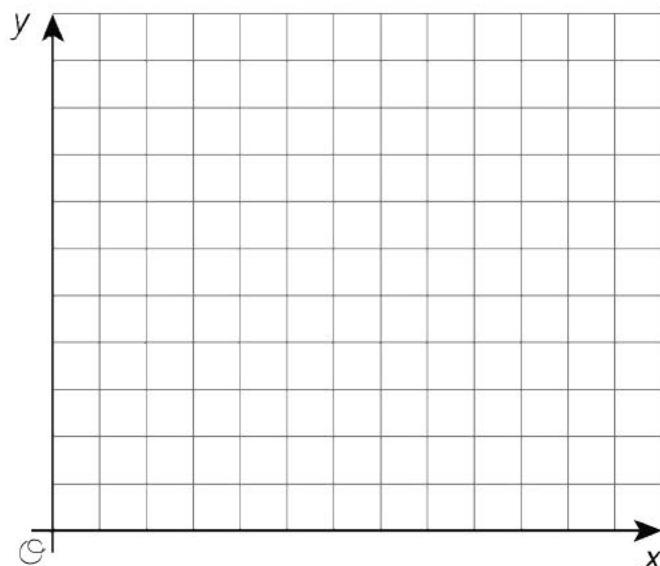
Students analyze a nonproportional linear relationship. The relationship is given via a description, and from this the students determine an equation, provide a graph, and determine whether the relationship is proportional or not. The major work of making the graph is choosing an appropriate

scale for each axis. The last question requires that students understand similarities and differences in graphs when the rate of change stays the same but the intial value changes.

## Statement

Han's cell phone plan costs \$200 to start. Then there is a \$50 charge each month.

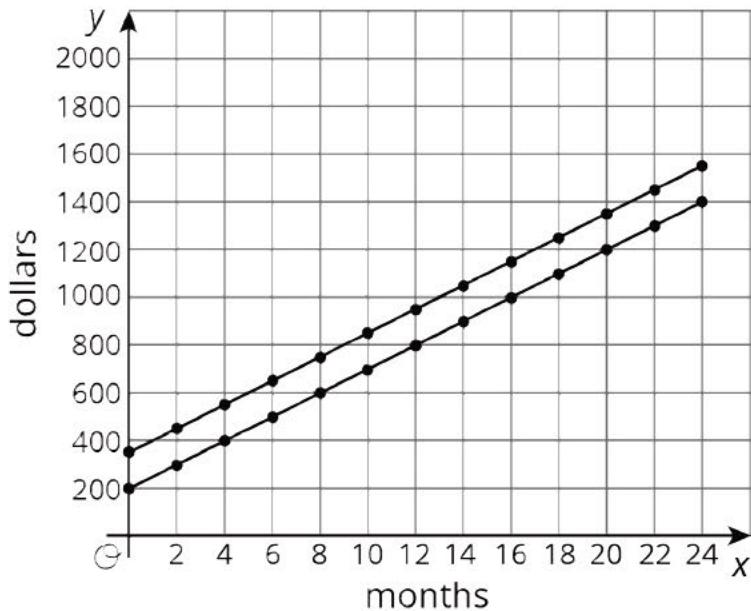
1. What is the total cost (start up fee and monthly charge) to use the cell phone plan for 1 month?
2. What is the total cost for  $x$  months?



3. Graph the cost of the cell phone plan over a period of two years, using months as the units of time. Be sure to label your axes and scale them by labeling each gridline with a number.
4. Is there a proportional relationship between time and the cost of the cell phone plan? Explain how you know.
5. Tyler's cell phone plan costs \$350 to start, then there is a \$50 charge each month. On the same grid as Han's plan, graph the cost of Tyler's cell phone plan over a period of two years. Describe how the two graphs are the same and how they are different.

## Solution

1. \$250
2.  $200 + 50x$  dollars
3. Solution for parts 3 and 5



4. The relationship between cost and time is not proportional. Sample explanation: the graph is a line, but it does not go through  $(0, 0)$ .

5. Graph is shown in part 3. Answers vary. Possible responses to how they are the same: They have the same slope. They both have a slope of 50. Possible responses to how they are different: They have different vertical intercepts. They are parallel to each other. One is a translation of the other.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.

- Acceptable errors: omission of units (dollars).

- Sample:

1. \$250

2.  $200 + 50x$

3. See graph

4. No, because  $250 \div 1 \neq 300 \div 2$ .

5. See graph, one similarity and one difference.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.

- Sample errors: minor mistakes in the positioning of a few points on the graph; the graph does show the data over a two-year period but the scale is chosen strangely, such as putting the 24-month mark halfway across the horizontal axis instead of near the end; reasonable work in parts c, d, and e is based on an incorrect equation in part b; an otherwise correct answer to part e is based on an incorrectly drawn line; Omission of or very poor work in parts a, d, or e.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: an incorrect equation followed by poor analysis in subsequent problem parts; the scale of the graph is chosen so as not to show all of the relevant points; omission of or very poor work in parts b or c; omission of or very poor work in two problem parts.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: Failure to come up with an equation or graph means that the rest of the problem cannot be tackled; omission of or very poor work in three or more problem parts.

## Aligned Standards

8.EE.B.5

## Assessment : End-of-Unit Assessment (B)

### Problem 1

Students failing to select B or C may not know that the graph of a line is the set of all solutions to the corresponding equation. Students selecting E may be reversing the  $x$ - and  $y$ -coordinates. Students selecting A or D may not realize that the coefficient of  $x$  is multiplied and not added.

#### Statement

Select **all** the equations on which the point  $(10, 0)$  lies.

- A.  $5x + 2y = 15$
- B.  $2x + 4y = 20$
- C.  $x + 6y = 10$
- D.  $3x + 3y = 13$
- E.  $4x + 2y = 20$
- F.  $6x + y = 50$

#### Solution

["B", "C"]

#### Aligned Standards

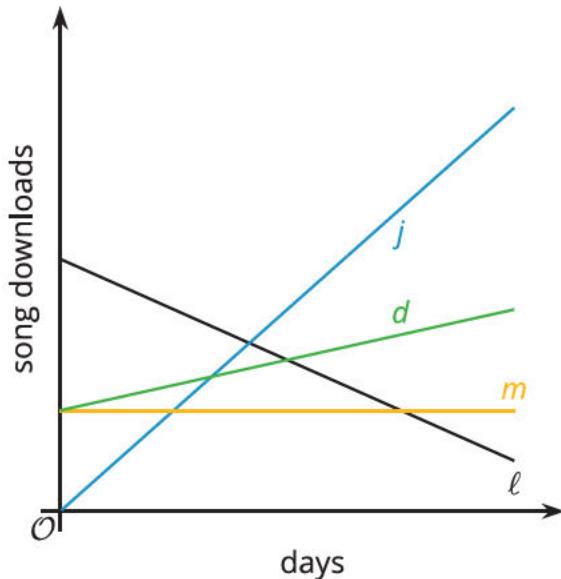
8.EE.B.5

### Problem 2

Students identify descriptions that fit various graphs. The descriptions indicate the rate of change or slope of the linear graph. Students deal with positive, zero, and negative slope. They also compare two slopes. Students selecting A do not understand the interpretation of negative slope as "decreasing". Students failing to select C have not mastered the interpretation of a horizontal line as "no change."

## Statement

A successful music app tracked the number of song downloads each day for a month for 4 music artists, represented by lines  $\ell$ ,  $j$ ,  $m$ , and  $d$  over the course of a month. Which line represents an artist whose downloads remained constant over the month?



- A.  $\ell$
- B.  $j$
- C.  $m$
- D.  $d$

## Solution

C

## Aligned Standards

8.EE.B, 8.F.B.4

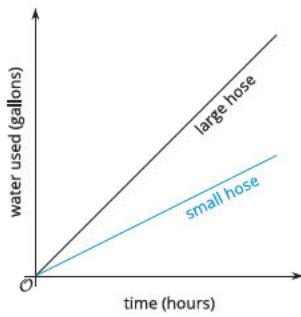
## Problem 3

Students interpret proportional relationships from given lines. They must not only identify the slope of the lines as the unit rate but also quantitatively compare these unit rates in the absence of a given scale on the axes. Students selecting C instead of A may be associating the slope with the gallons of water, but the axes do not reflect this. Students selecting B have misinterpreted the meaning of the slope of each line. Students selecting D may not understand the horizontal line represents no change over time.

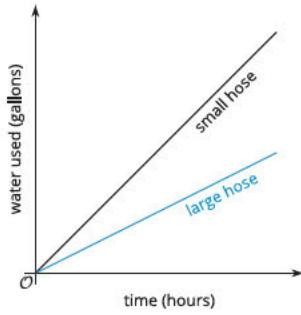
## Statement

A pool holds 19,900 gallons of water. There are two hoses that can be used to fill the pool, a large one and a small one. On an average day, it takes 5 hours to fill the pool with the large hose and 12 hours with the small hose. Which graph best represents this scenario?

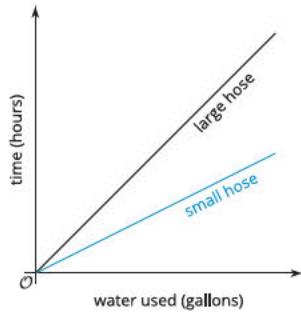
A.



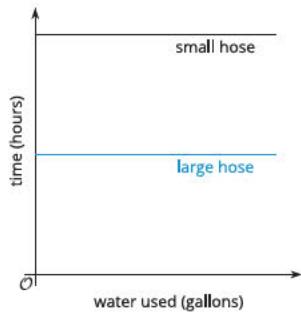
B.



C.



D.



## Solution

A

### Aligned Standards

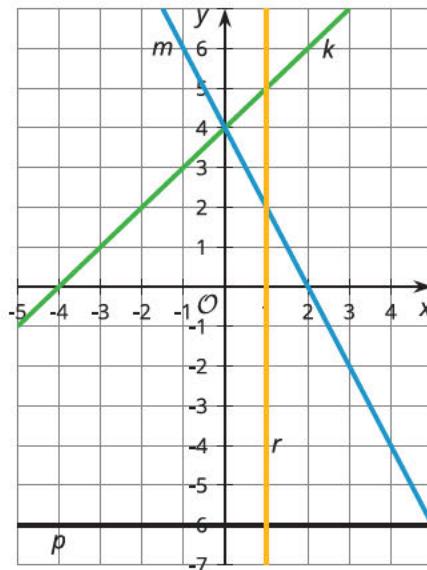
8.EE.B.5

### Problem 4

Students write equations to represent lines. One line is vertical, one is horizontal, one has positive slope, and one has negative slope.

## Statement

Write an equation for each line.



## Solution

Line  $k$ :  $y = x + 4$ , Line  $m$ :  $y = -2x + 4$ , Line  $p$ :  $y = -6$ , Line  $r$ :  $x = 1$

## Aligned Standards

8.EE.B.5

## Problem 5

Students compare the growth rate of three different trees. The proportional relationship between time and height is represented in three different ways. There is more than one way to do this problem correctly. For example, students could determine how long it takes each tree to grow 10 feet to determine the fastest growth rate, or could determine each tree's speed in feet per month.

## Statement

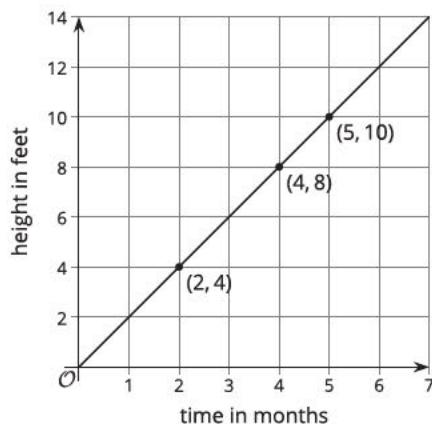
Noah is growing three different types of trees. He is keeping track of the height of each tree over time.

- This equation represents the height (in feet) of the first tree over  $m$  months.  $h = 3.5m$ .

- The second tree's information is in the table:

time (months)	height of tree (feet)
2	5
4	10
5	12.5
8	20

- This graph shows how long, in months, it takes the third tree to grow  $h$  feet.



Which tree is growing the slowest? Explain how you know.

## Solution

The third tree has the slowest growth rate. Sample explanation: The unit rate of the first tree is 3.5 feet per month. Within the table, the rate is 2.5 feet per month, because  $5 \div 2 = 2.5$ . Using the point  $(2, 4)$  on the graph, the unit rate is 2 feet per month. The third tree growing 2 feet per month is growing the slowest.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: Tree 1 grows 3.5 feet per month. Tree 2 grows 2.5 feet per month. Tree 3 grows 2 feet per month. Tree 3 grows the slowest, because it after 2 months it is the shortest tree.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: Work contains correct unit rates for all three trees but concludes that tree #1 or #2 has the slowest growth or does not name the slowest growing tree; one unit rate is incorrect (possibly with an incorrect slowest growth identified as a consequence); insufficient explanation of work.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: two or more incorrect unit rates; the correct tree is identified but with no justification; response to the question is not based on unit rates or on similar methods.

## Aligned Standards

8.EE.B.5

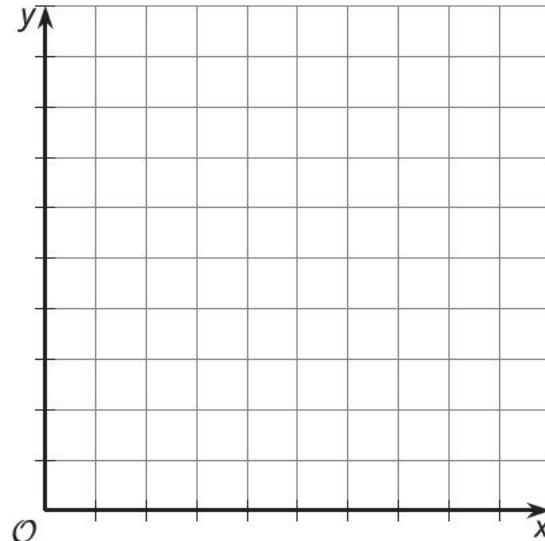
### Problem 6

Students analyze a nonproportional linear relationship. The relationship is given via a description, and from this the students determine an equation, provide a graph, and determine whether the relationship is proportional or not. The major work of making the graph is choosing an appropriate scale for each axis. The last question requires that students understand the vertical intercept of a graph as an initial value, and the slope of a graph as the rate of change.

#### Statement

A sandwich store charges a \$10 delivery fee, and \$4.50 for each sandwich.

1. What is the total cost (sandwiches and delivery charge) if an office orders 6 sandwiches?
2. What is the total cost for  $x$  sandwiches?
3. Graph the total cost of sandwiches and delivery based on number of sandwiches ordered. Be sure to label your axes and scale them by labeling each gridline with a number.



4. Is there a proportional relationship between number of sandwiches and the cost of the order? Explain how you know.
5. At a different sandwich shop, there is a \$5 delivery fee, and each sandwich costs \$4.50. On the same grid, graph the total cost of sandwiches and delivery based on number of sandwiches ordered for this new shop. Describe how the two graphs are the same and how they are different.

#### Solution

1. \$37

2.  $10 + 4.5x$  or equivalent.
3.  $x$ -axis labeled "number of sandwiches" and  $y$ -axis labeled "cost of order." A line passes through the points  $(0, 10)$ ,  $(1, 14.5)$ , and  $(2, 19)$ .
4. There is not a proportional relationship between the number of sandwiches and cost of order.  
Sample reasoning: When graphed, the line does not pass through the origin.
5. Possible responses: They have different vertical intercepts. They are parallel to each other.  
One is a translation of the other.

Minimal Tier 1 response:

- Work is complete and correct.
- The graph may be a continuous line, or it may consist only of points representing whole numbers of number of sandwiches and the total cost.
- Sample: See solution above.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: graph contains only a subset of points that would fit within the chose scale; equation is incorrect, but plausible.
- Acceptable errors: work for parts a and b is correctly based on an incorrect proportional relationship.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: work does not include delivery fee of \$10; equation is badly off.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: failure to come up with an equation or graph means that the rest of the problem cannot be tackled; omission of or very poor work in three or more problem parts.

## Aligned Standards

8.EE.B.6

## Problem 7

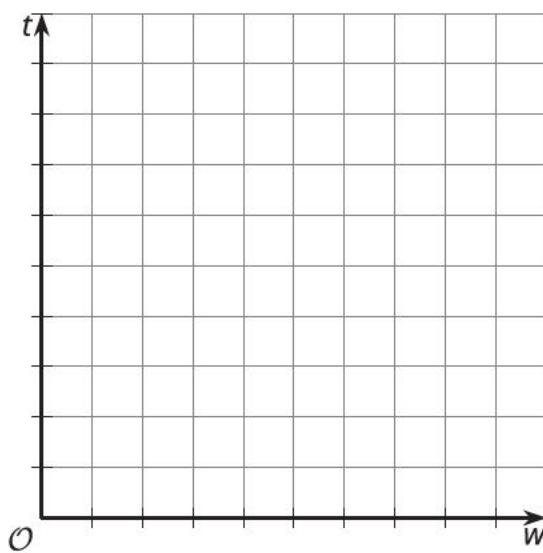
A linear equation is described in terms of a constraint on the total weight of two different types of freight. The constraint gives an equation that students produce. Students then graph the solutions to the equation.

### Statement

A truck is shipping jugs of drinking water and cases of paper towels. A jug of drinking water weighs 40 pounds and a case of paper towels weighs 16 pounds. The truck can carry 2,000 pounds of cargo altogether.

jugs of drinking water, $w$	cases of paper towels, $t$
10	
	50

1. Complete the table showing three ways the truck could be packed with jugs of water and cases of paper towels.
2. Write an equation relating the number of jugs of water and the number of cases of paper towels the truck can carry.
3. Sketch a graph of the solutions to your equation.



### Solution

1. Answers may vary for third entry.

jugs of drinking water, $w$	cases of paper towels, $t$
10	100
30	50
50	0

2.  $40w + 16t = 2000$

3. A line which passes through the points  $(10, 100)$ ,  $(30, 50)$ , and  $(50, 0)$ .

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Acceptable errors: omission of labels on the graph.

1. Answers may vary for third entry. See table above.

2.  $40w + 16t = 2000$

3. A line which passes through the points  $(10, 100)$ ,  $(30, 50)$ , and  $(50, 0)$ .

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: minor mistakes in the positioning of a few points on the graph; the graph does show the data of the relationship between pounds of jugs of water and pounds of paper towels but the scale is chosen strangely; reasonable work in part a but results in an incorrect equation in part b.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: errors within table for part a followed by an incorrect equation in part b; the scale of the graph is chosen so as not to show all of the relevant points; omission of or very poor work in parts a or c; omission of or very poor work in two problem parts.

## Aligned Standards

8.EE.B.5

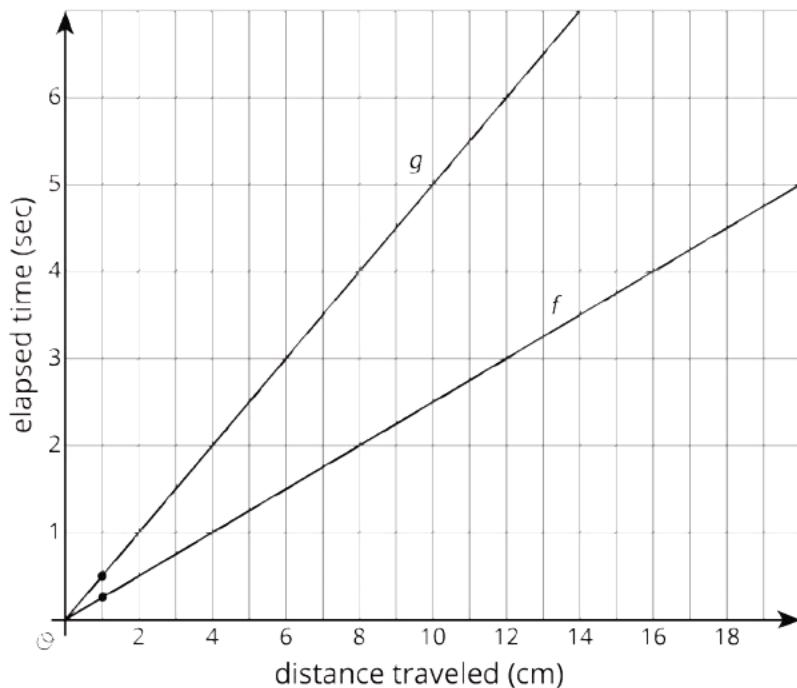


# Lesson Cool Downs

# Lesson 1: Understanding Proportional Relationships

## Cool Down: Turtle Race

This graph represents the positions of two turtles in a race.



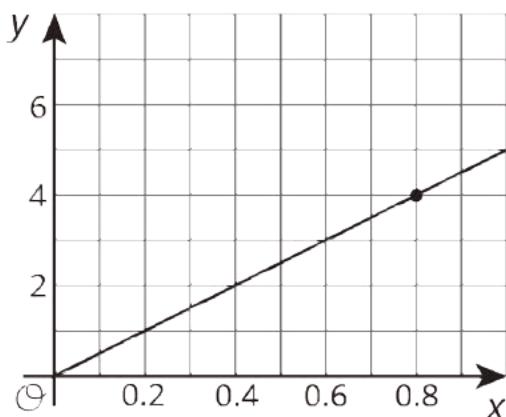
1. On the same axes, draw a line for a third turtle that is going half as fast as the turtle described by line  $g$ .
2. Explain how your line shows that the turtle is going half as fast.

# Lesson 2: Graphs of Proportional Relationships

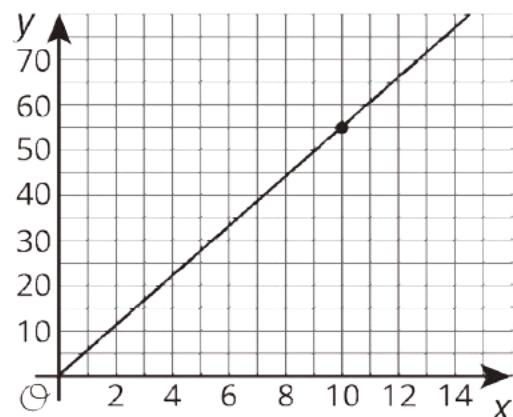
## Cool Down: Different Axes

Which one of these relationships is different than the other three? Explain how you know.

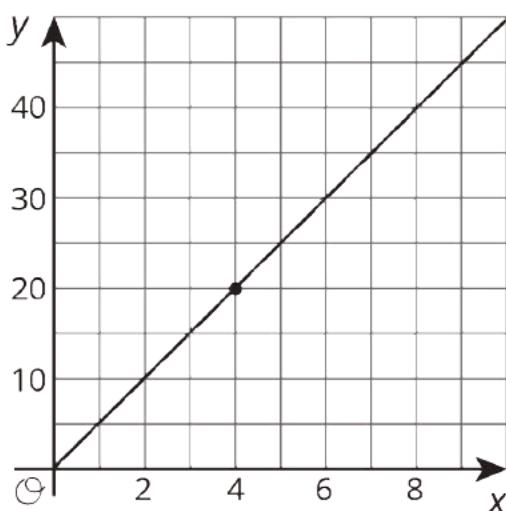
A



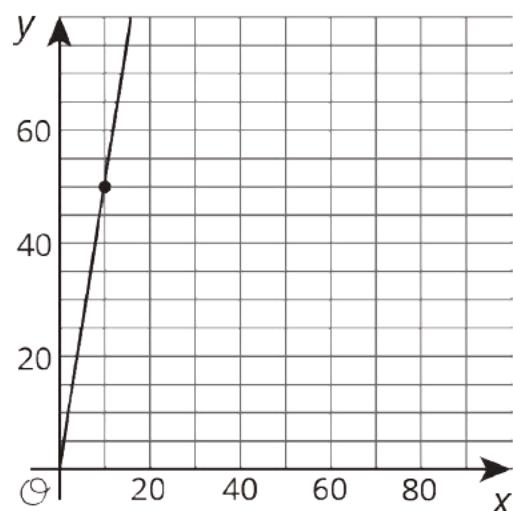
B



C



D

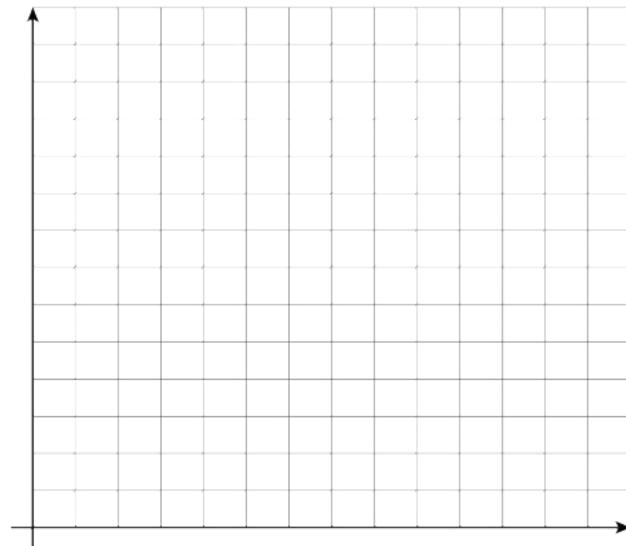


# Lesson 3: Representing Proportional Relationships

## Cool Down: Graph the Relationship

Sketch a graph that shows the relationship between grams of honey and grams of salt needed for a bakery recipe. Show on the graph how much honey is needed for 70 grams of salt.

salt (g)	honey (g)	flour (c)
10	14	4
25	35	10



# Lesson 4: Comparing Proportional Relationships

## Cool Down: Different Salt Mixtures

Here are recipes for two mixtures of salt and water that taste different.

Information about Salt Mixture A is shown in the table.

salt (teaspoons)	water (cups)
4	5
7	$8\frac{3}{4}$
9	$11\frac{1}{4}$

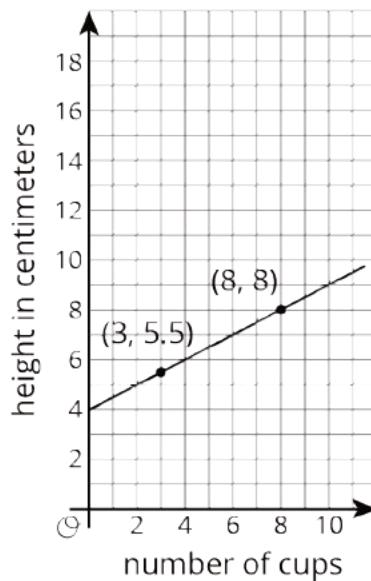
Salt Mixture B is defined by the equation  $y = 2.5x$ , where  $x$  is the number of teaspoons of salt and  $y$  is the number of cups of water.

1. If you used 10 cups of water, which mixture would use more salt? How much more? Explain or show your reasoning.
2. Which mixture tastes saltier? Explain how you know.

# Lesson 5: Introduction to Linear Relationships

## Cool Down: Stacking More Cups

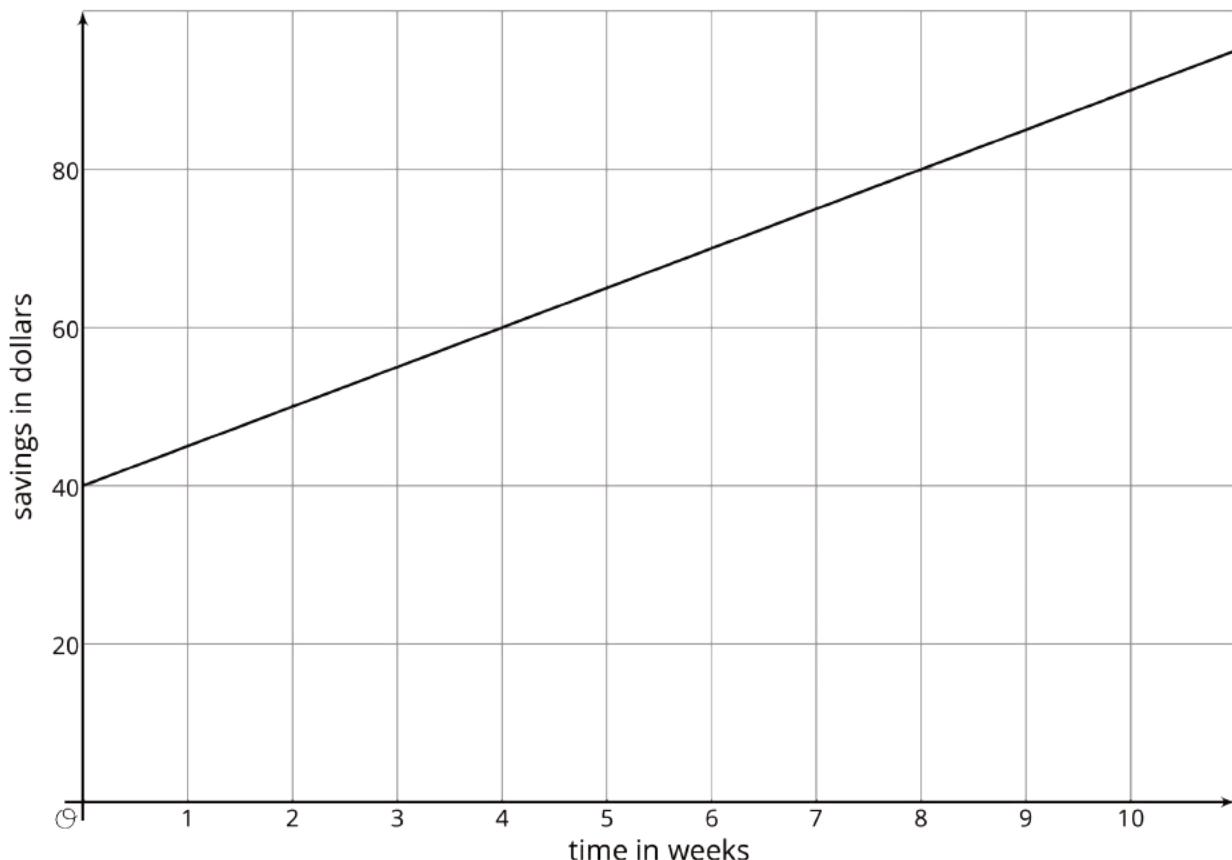
A shorter style of cup is stacked tall. The graph displays the height of the stack in centimeters for different numbers of cups. How much does each cup after the first add to the height of the stack? Explain how you know.



# Lesson 6: More Linear Relationships

## Cool Down: Savings

The graph shows the savings in Andre's bank account.



1. Explain what the slope represents in this situation.
2. Explain what the vertical intercept represents in this situation.

# Lesson 7: Representations of Linear Relationships

## Cool Down: Graphing a Line

Make a sketch of a linear relationship with slope of 3 that is not a proportional relationship. Show how you know that the slope is 3. Write an equation for the line.

## Lesson 8: Translating to $y = mx + b$

### Cool Down: Similarities and Differences in Two Lines

Describe how the graph of  $y = 2x$  is the same and different from the graph of  $y = 2x - 7$ . Explain or show your reasoning.

# Lesson 9: Slopes Don't Have to be Positive

## Cool Down: The Slopes of Graphs

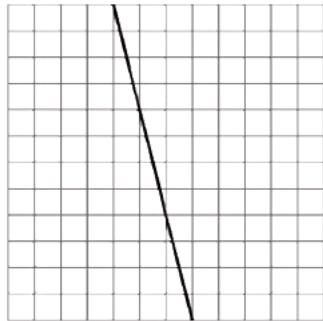
Each square on a grid represents 1 unit on each side.

1. Calculate the slope of graph D. Explain or show your reasoning.

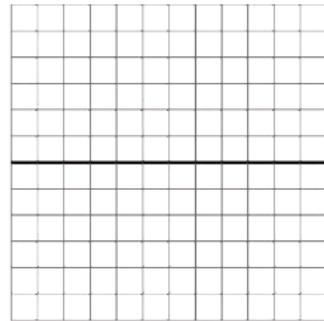
2. Calculate the slope of graph E. What situation could the graph represent?

3. On the blank grid F, draw a line that passes through the indicated point and has slope -2.

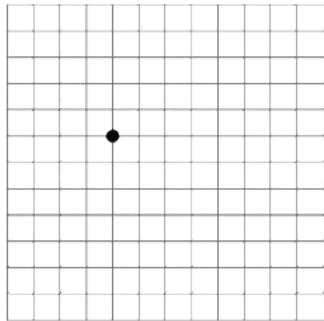
D



E



F



# Lesson 10: Calculating Slope

## Cool Down: Different Slopes

Without graphing, find the slope of the line that goes through

1.  $(0, 5)$  and  $(8, 2)$ .

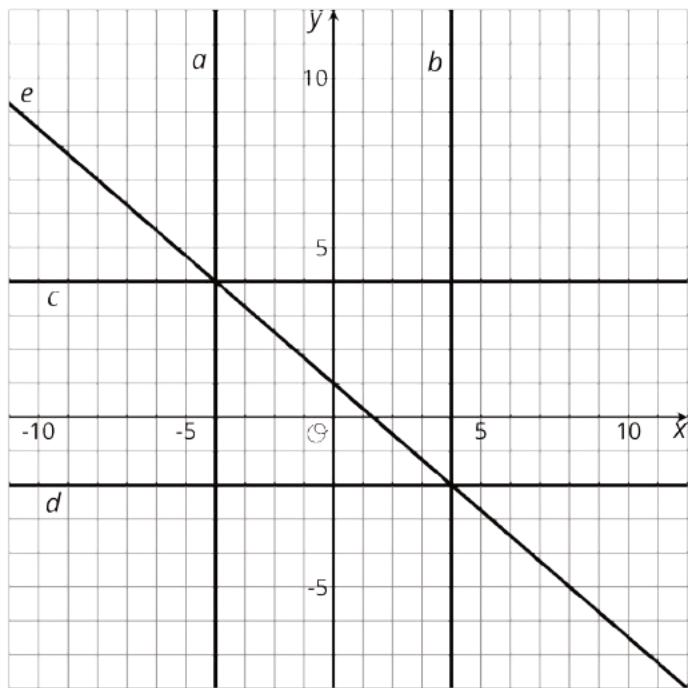
2.  $(2, -1)$  and  $(6, 1)$ .

3.  $(-3, -2)$  and  $(-1, -5)$ .

# Lesson 11: Equations of All Kinds of Lines

## Cool Down: Line Design

Here are 5 lines on a coordinate grid:



Write equations for lines *a*, *b*, *c*, *d*, and *e*.

# Lesson 12: Solutions to Linear Equations

## Cool Down: Identify the Points

Which of the following coordinate pairs make the equation  $x - 9y = 12$  true?

1. (12, 0)
2. (0, 12)
3. (3, -1)
4.  $(0, -\frac{4}{3})$

# Lesson 13: More Solutions to Linear Equations

## Cool Down: Intercepted

A graph of a linear equation passes through  $(-2, 0)$  and  $(0, -6)$ .

1. Use the two points to sketch the graph of the equation.

2. Is  $3x - y = -6$  an equation for this graph? Explain how you know.



# Instructional Masters

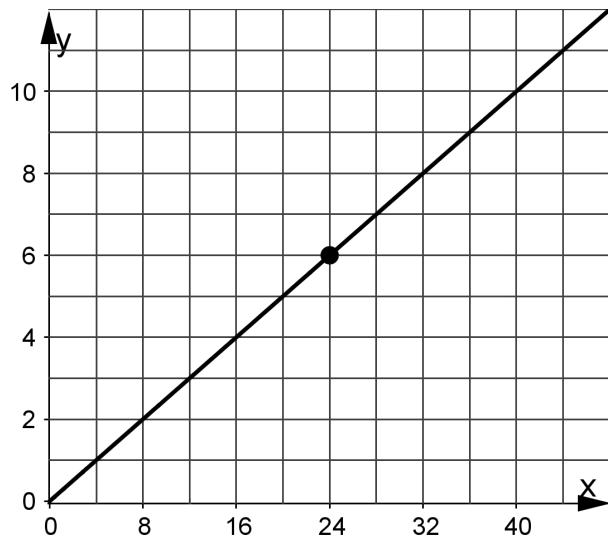
# Instructional Masters for Linear Relationships

address	title	students per copy	written on?	requires cutting?	card stock recommended?	color paper recommended?
Activity Grade8.3.13.3	I'll Take an X, Please	2	no	yes	no	yes
Activity Grade8.3.3.3	Info Gap: Proportional Relationships	2	yes	yes	no	no
Activity Grade8.3.10.3	Making Designs	2	no	yes	no	no
Activity Grade8.3.6.2	Slopes, Vertical Intercepts, and Graphs	2	no	yes	no	yes
Activity Grade8.3.2.2	Card Sort: Proportional Relationships	4	no	yes	no	no
Activity Grade8.3.8.3	Translating a Line	2	no	yes	no	no

8.8.2.2 Card Sort: Proportional Relationships.

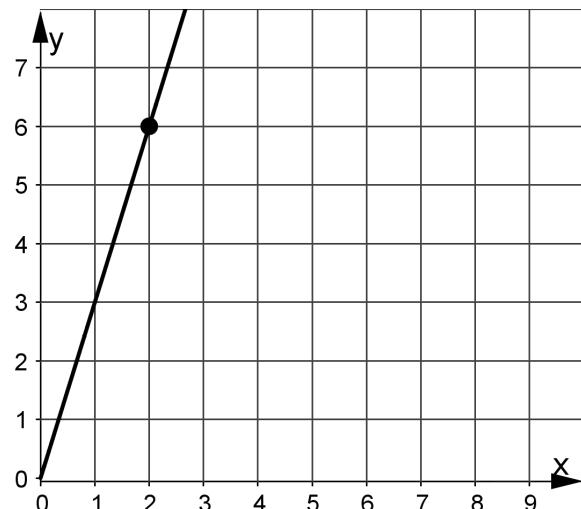
Proportional Relationships

**Card A**



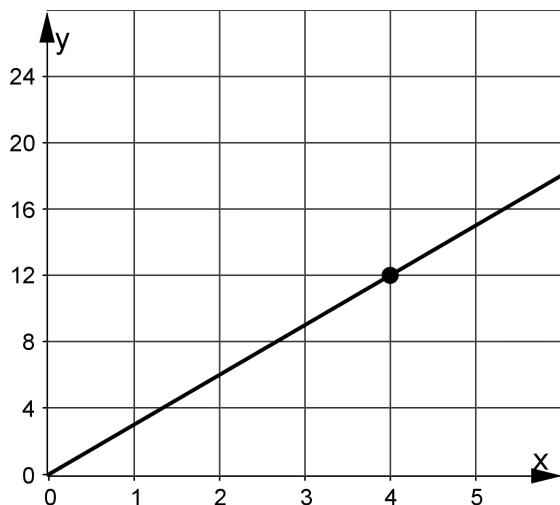
Proportional Relationships

**Card B**



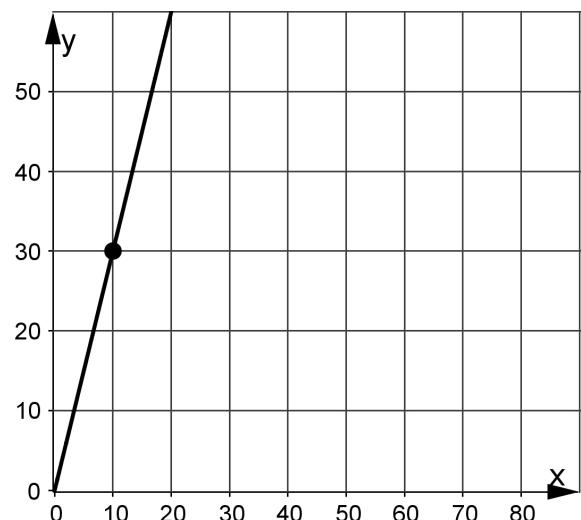
Proportional Relationships

**Card E**



Proportional Relationships

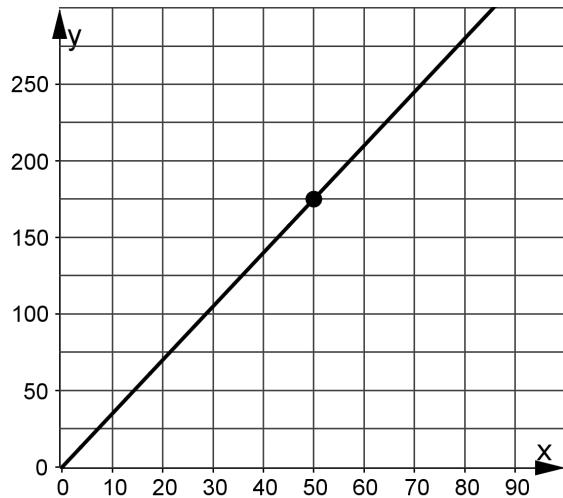
**Card H**



### 8.3.2.2 Card Sort: Proportional Relationships.

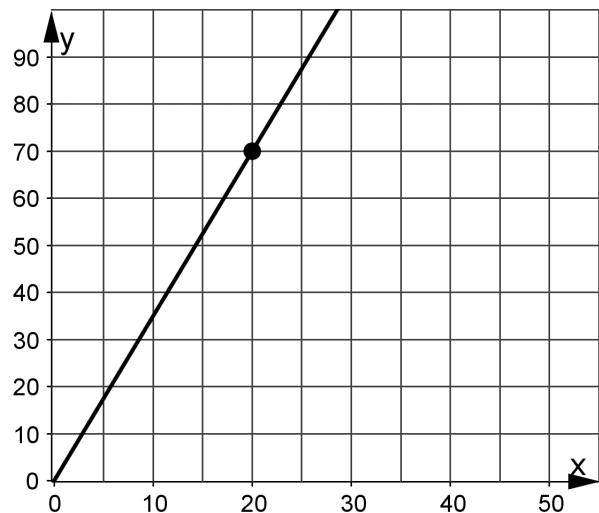
Proportional Relationships

**Card C**



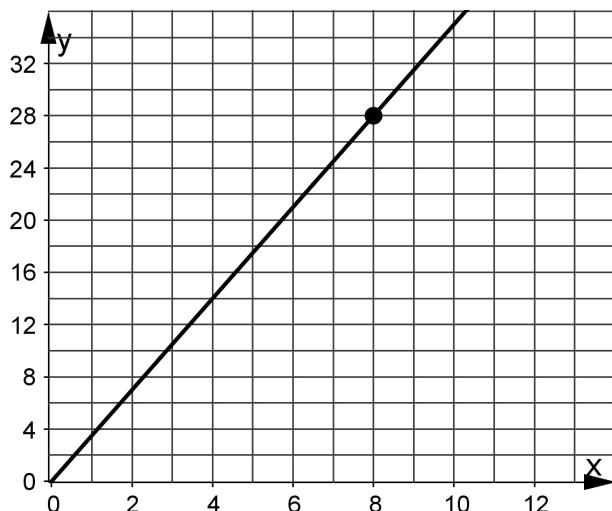
Proportional Relationships

**Card D**



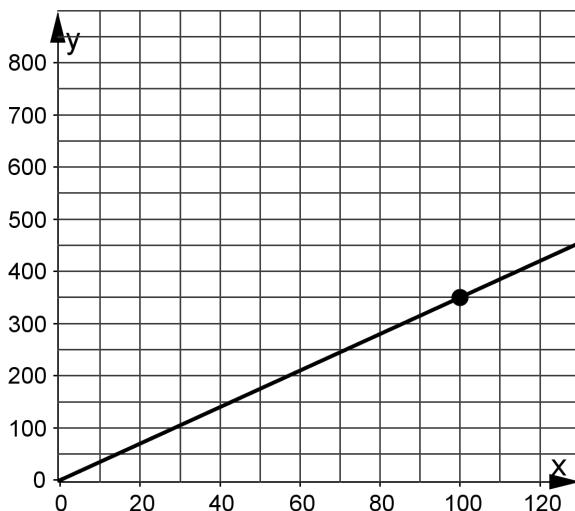
Proportional Relationships

**Card G**



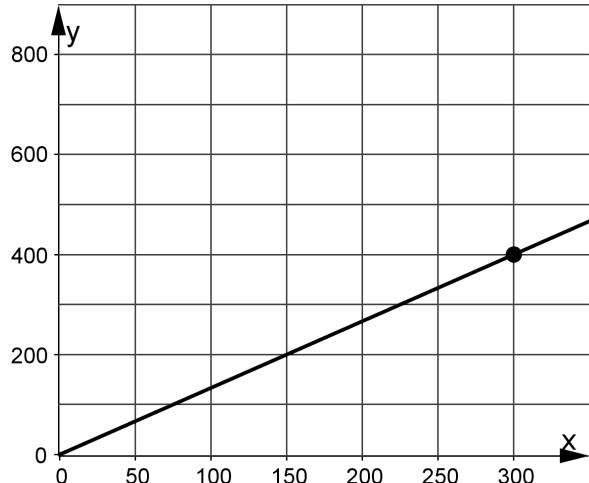
Proportional Relationships

**Card K**

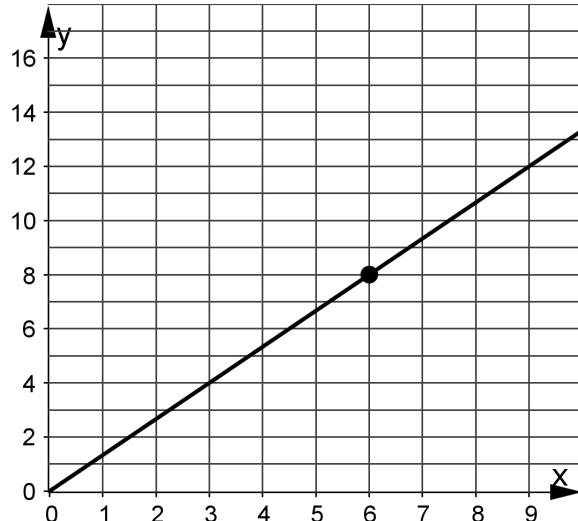


### 8.3.2.2 Card Sort: Proportional Relationships.

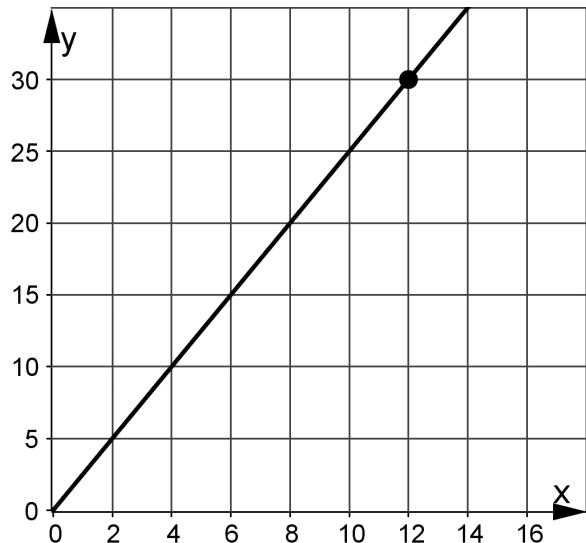
Proportional Relationships  
**Card I**



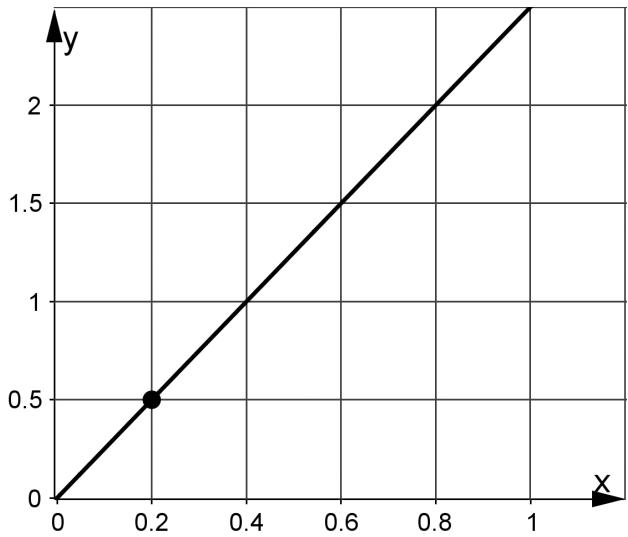
Proportional Relationships  
**Card L**



Proportional Relationships  
**Card F**



Proportional Relationships  
**Card J**

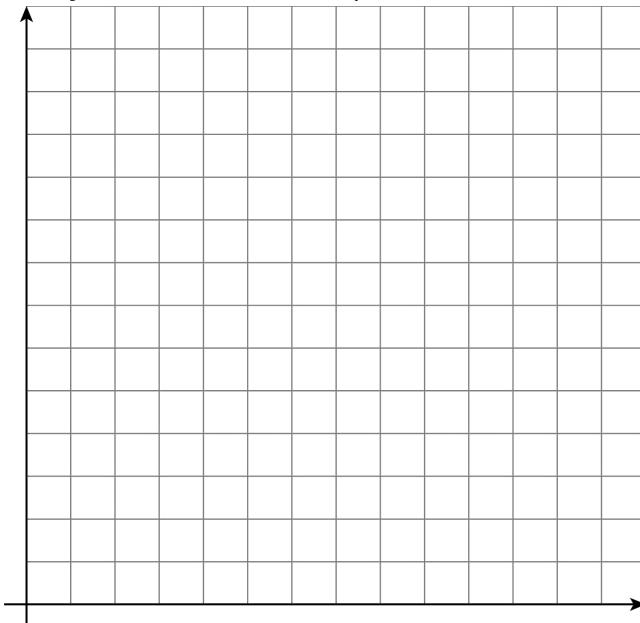


### 8.3.3.3 Info Gap: Proportional Relationships.

#### Info Gap: Proportional Relationships

#### Problem Card 1

Sketch a graph that shows the relationship between grams of honey and cups of flour needed for a bakery recipe. Show on the graph how much honey is needed for 17 cups of flour.



#### Info Gap: Proportional Relationships

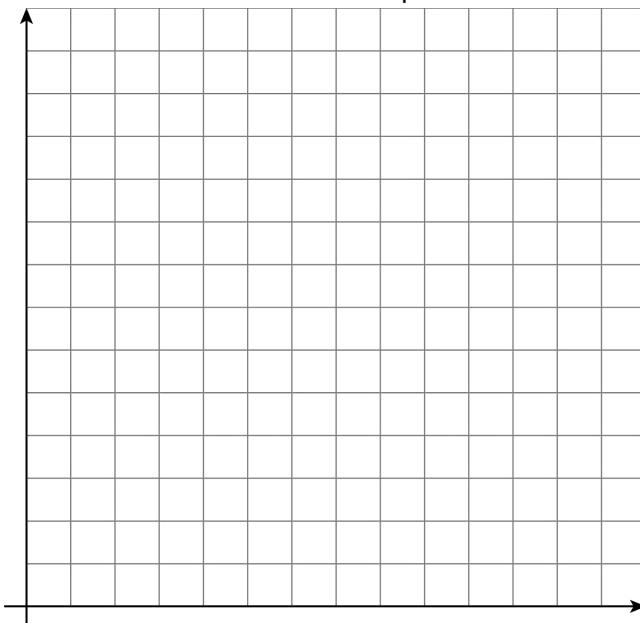
#### Data Card 1

salt (g)	honey (g)	flour (c)
15	27	6
25	45	10

#### Info Gap: Proportional Relationships

#### Problem Card 2

Sketch a graph that shows the relationship between grams of salt and cups of flour needed for a bakery recipe. Then show on the graph how much salt is needed for 23 cups of flour.



#### Info Gap: Proportional Relationships

#### Data Card 2

salt (g)	honey (g)	flour (c)
10	14	4
25	35	10

### 8.3.6.2 Slopes, Vertical Intercepts, and Graphs.

Slopes, Vertical Intercepts, and Graphs

#### Situation A

Lin's dad bought a tablet. He pays the same amount each month for a subscription to a movie streaming service. The graph represents how much money he spent,  $y$ , for the tablet and  $x$  months of the service. The slope of the line is 10.

Slopes, Vertical Intercepts, and Graphs

#### Situation C

Diego puts his piggy bank on his desk. Each week he adds the same amount of money to his bank. The graph represents the amount in the piggy bank,  $y$ , after  $x$  weeks. The slope of the line is 5.

Slopes, Vertical Intercepts, and Graphs

#### Situation E

Elena adds a quarter to her piggy bank every day. The graph represents the number of dollars,  $y$ , in her piggy bank  $x$  days after she put the piggy bank in the closet. The slope of the line is 0.25.

Slopes, Vertical Intercepts, and Graphs

#### Situation B

The graph represents the perimeter,  $y$ , of a square whose side length is  $x$ . The slope of the line is 4.

Slopes, Vertical Intercepts, and Graphs

#### Situation D

Noah starts his piggy bank off with money he's saved helping a neighbor out. Each month he adds the same amount of money to his bank. The graph represents the amount in the piggy bank,  $y$ , after  $x$  months. The slope of the line is 15.

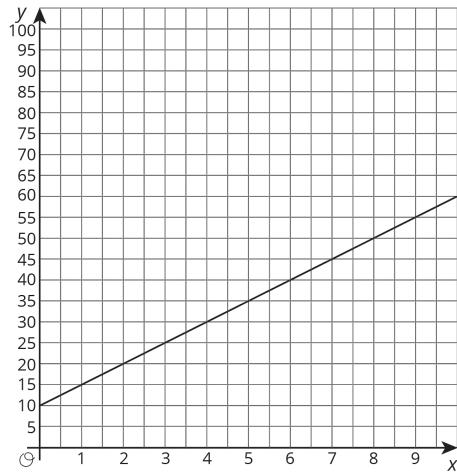
Slopes, Vertical Intercepts, and Graphs

#### Situation F

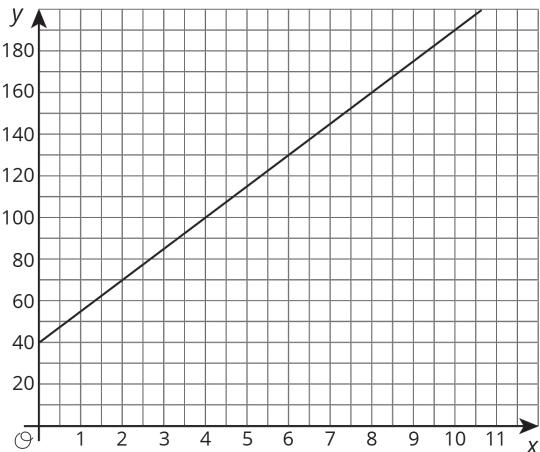
Lin's mom pays the same amount each month for internet service for her business tablet. The graph represents how much money she spent,  $y$ , for  $x$  months of service. The slope of the line is 40.

### 8.3.6.2 Slopes, Vertical Intercepts, and Graphs.

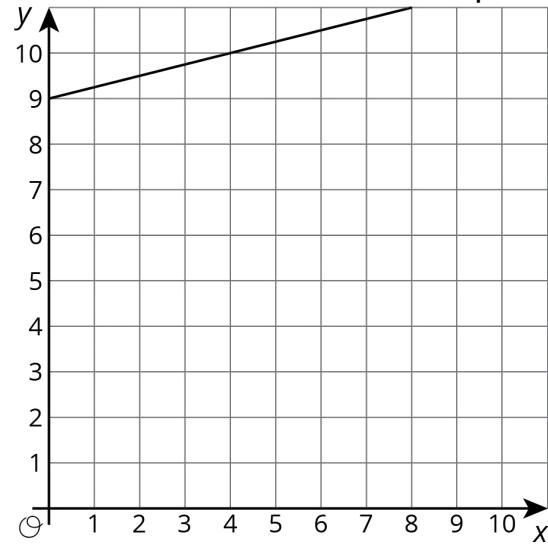
Slopes, Vertical Intercepts, and Graphs - Graph 1



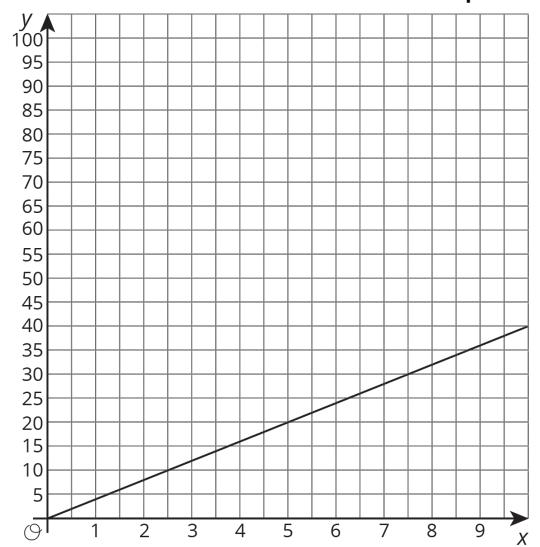
Slopes, Vertical Intercepts, and Graphs - Graph 3



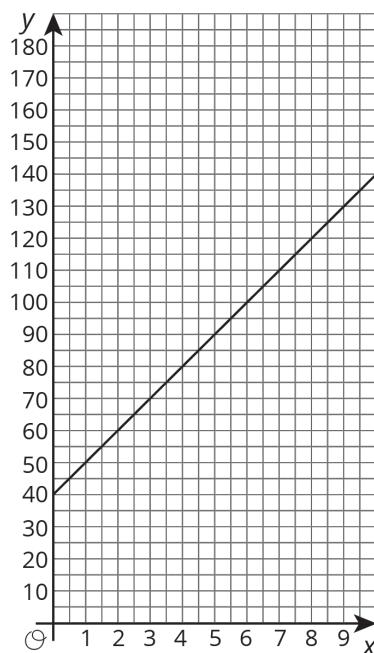
Slopes, Vertical Intercepts, and Graphs - Graph 5



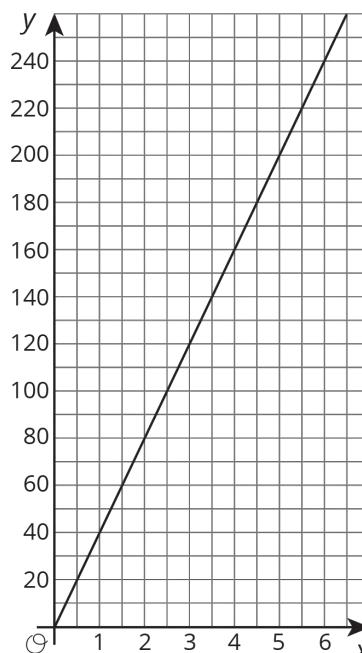
Slopes, Vertical Intercepts, and Graphs - Graph 6



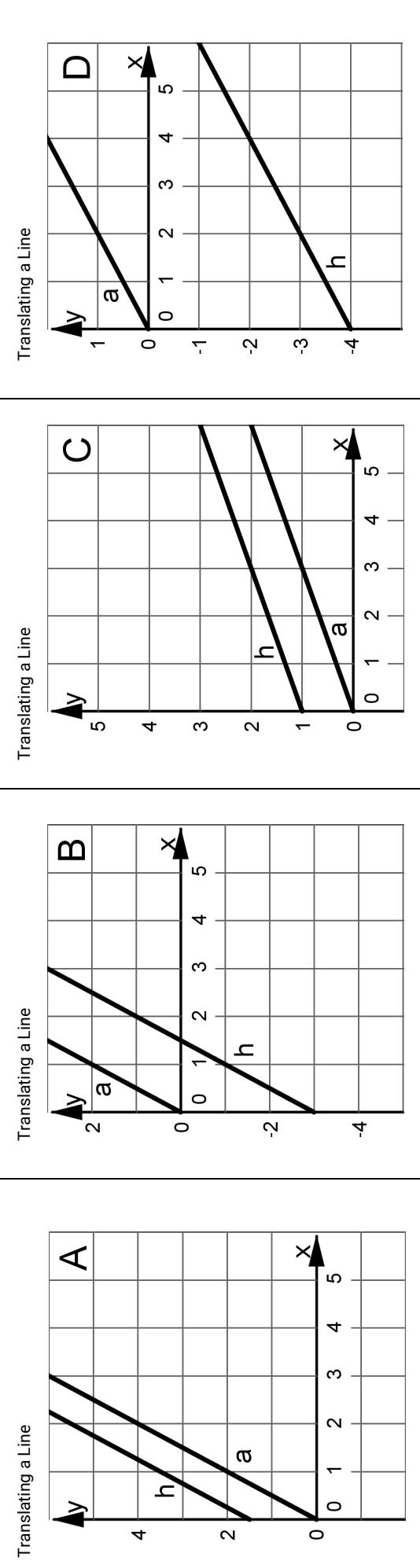
Slopes, Vertical Intercepts, and Graphs - Graph 2



Slopes, Vertical Intercepts, and Graphs - Graph 4



Translating a Line	$y = \frac{3}{2} + 2x$	Translating a Line	$2x - 3 = y$	Translating a Line	$y = \frac{1}{3}x + 1$
--------------------	------------------------	--------------------	--------------	--------------------	------------------------



Translating a Line

The y-coordinate is one more than a third of the x-coordinate.

Translating a Line

The y-coordinate is three less than double the x-coordinate.

Translating a Line

The y-coordinate is one more than a third of the x-coordinate.

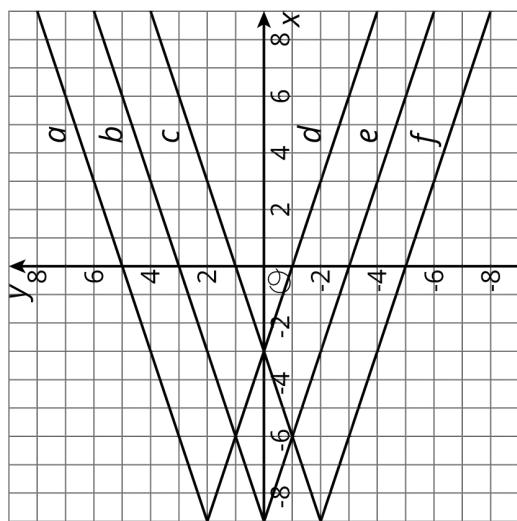
Translating a Line

Translating a Line

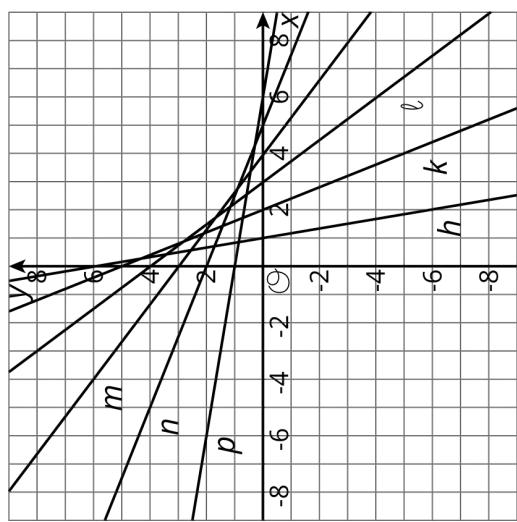
$x$	$y$
0	1.5
1	3.5
1.5	4.5
2.75	7

$x$	$y$
0	-4
2	-3
4	-2
8	0

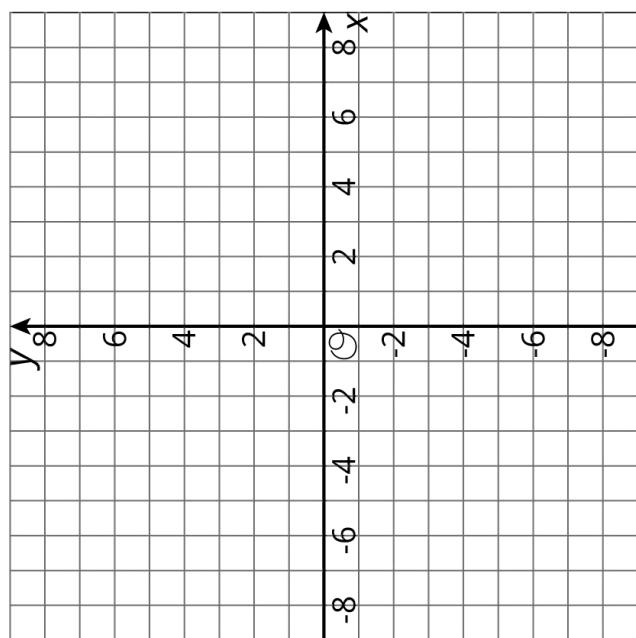
Making Designs  
**Design 1**



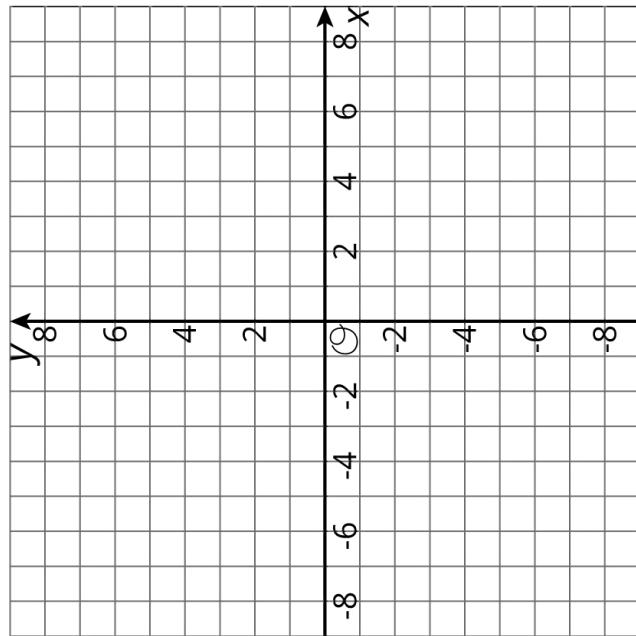
Making Designs  
**Design 2**



Making Designs  
**Design 1**



Making Designs  
**Design 2**



## 8.3.13.3 I'll Take an X, Please

I'll Take an x, Please

A

$$(10.5, 7)$$

I'll Take an x, Please

a

$$2x - y = 14$$

I'll Take an x, Please

B

$$y = -2x - \frac{1}{2}$$

I'll Take an x, Please

b

$$\left(0, -\frac{1}{2}\right)$$

I'll Take an x, Please

C

$$7 + 5x = y + 7$$

I'll Take an x, Please

c

$$(3, 15)$$

I'll Take an x, Please

D

$$(2, 15)$$

I'll Take an x, Please

d

$$3x + 2y = 36$$

I'll Take an x, Please

E

$$6 - x = 4y$$

I'll Take an x, Please

e

$$\left(7, -\frac{1}{4}\right)$$

I'll Take an x, Please

F

$$(4, 18)$$

I'll Take an x, Please

f

$$9x = 2y$$

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**Editorial Director**  
Richard B. Talbot

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