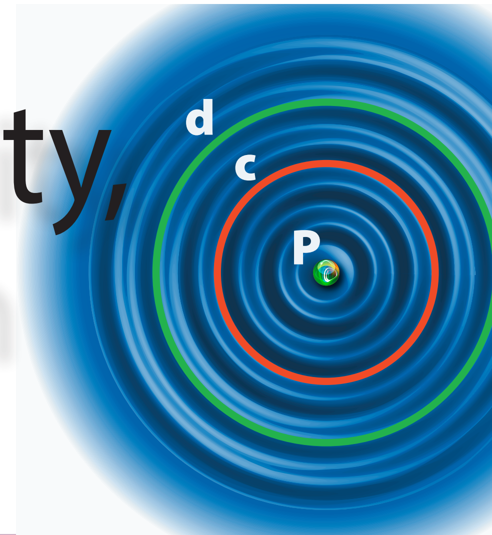
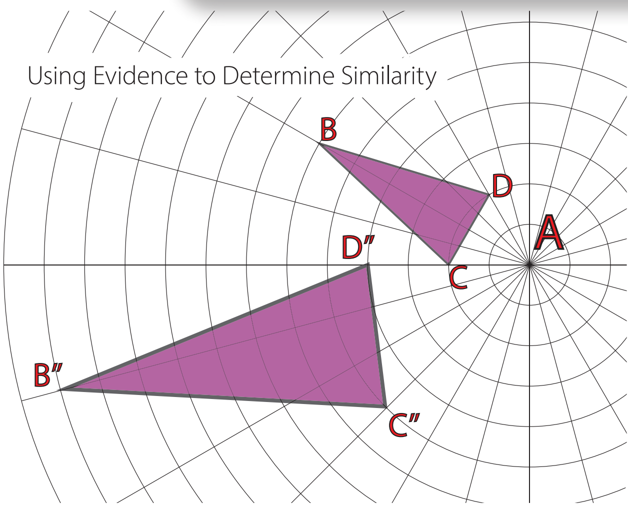
Core Knowledge® MATHEMATICS

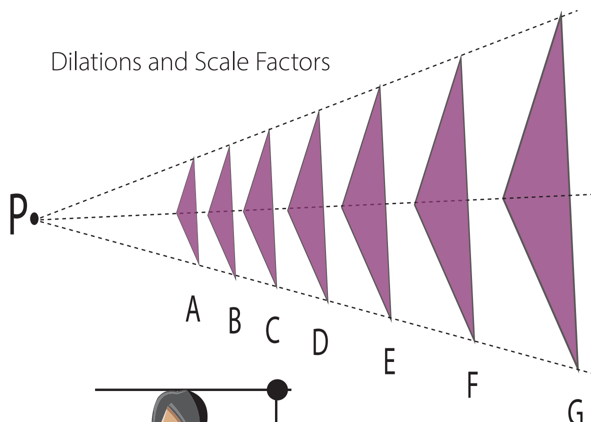
Circular Dilations

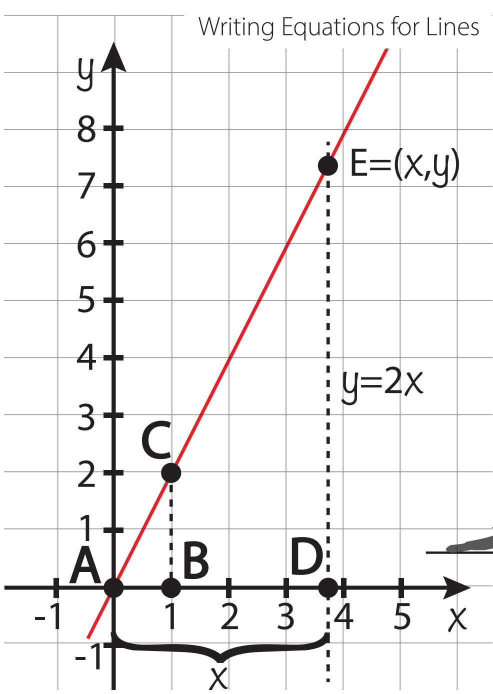


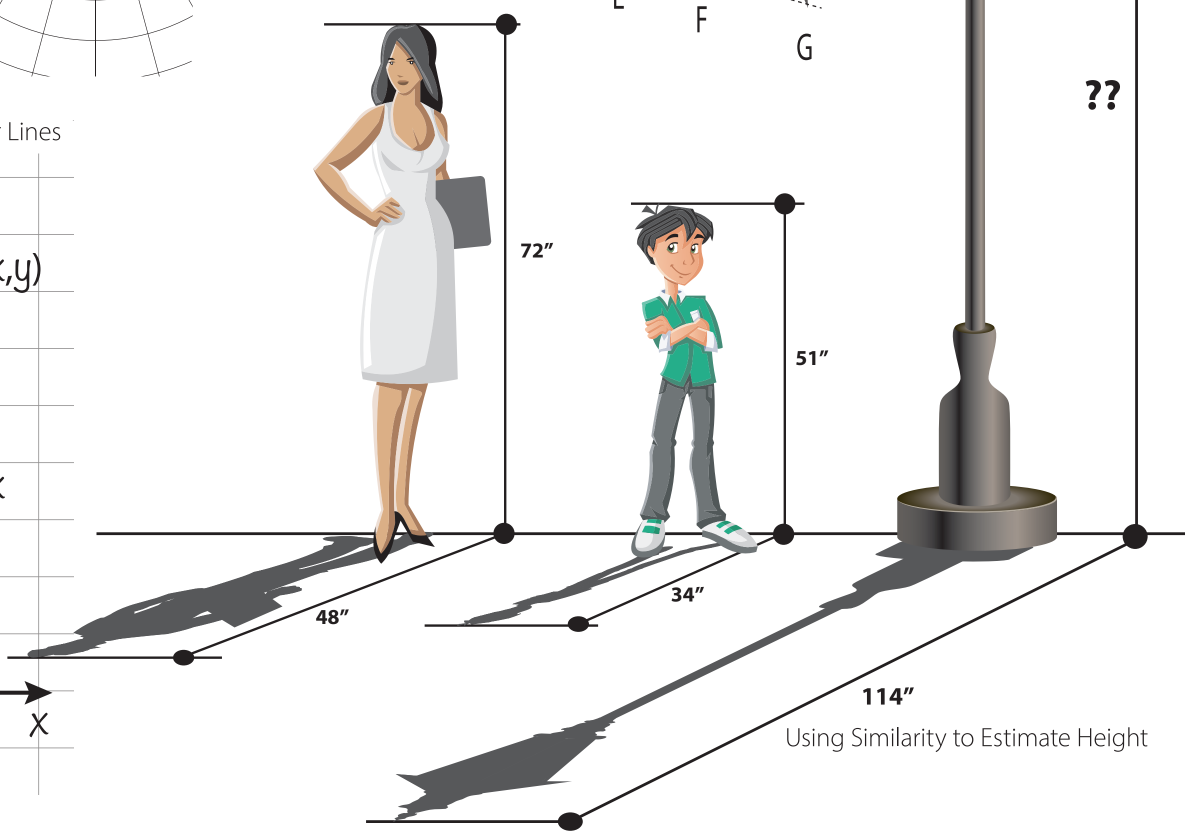
Dilations, Similarity, and Introduction to Slope

Teacher Guide









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Dilations, Similarity, & Introduction to Slope

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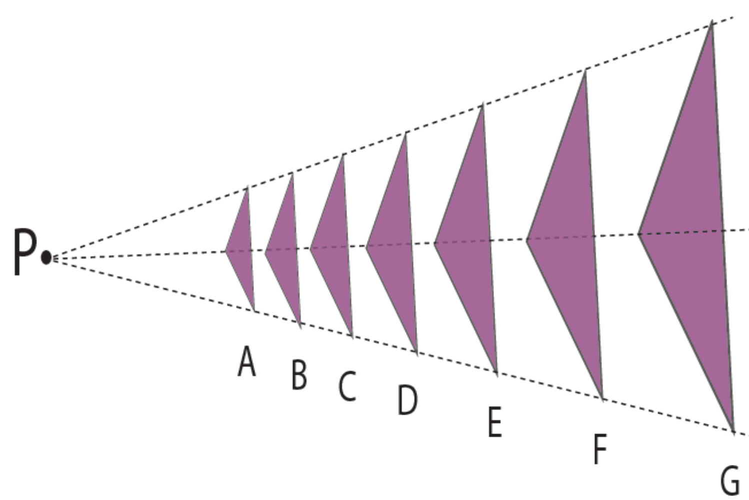


Dialtions, Similarity, and Introduction to Slope

Teacher Guide

Core Knowledge Mathematics™

Dilations, Similarity, and Introducing Slope

Unit Narrative

Work with transformations of plane figures in grade 8 builds on earlier work with geometry and geometric measurement, using students' familiarity with geometric figures, their knowledge of formulas for the areas of rectangles, parallelograms, and triangles, and their abilities to use rulers and protractors. Grade 7 work with scaled copies is especially relevant. This work was limited to pairs of figures with the same rotation and mirror orientations (i.e. that are not rotations or reflections of each other). In grade 8, students study pairs of scaled copies that have different rotation or mirror orientations, examining how one member of the pair can be transformed into the other, and describing these transformations. Initially, they view transformations as moving one figure in the plane onto another figure in the plane. As the unit progresses, they come to view transformations as moving the entire plane.

Through activities designed and sequenced to allow students to make sense of problems and persevere in solving them (MP1), students use and extend their knowledge of geometry and geometric measurement. Students begin the first lesson of the unit by looking at cut-out figures, first comparing them visually to determine if they are scaled copies of each other, then representing the figures in a diagram, and finally representing them on a circular grid with radial lines. They encounter the term "scale factor" (familiar from grade 7) and the new terms "dilation" and "center of dilation." In the next lesson, students again use a circular grid with radial lines to understand that under a dilation the image of a circle is a circle and the image of a line is a line parallel to the original. During the rest of the unit, students draw images of figures under dilations on and off square grids and the coordinate plane. In describing correspondences between a figure and its dilation, they use the terms "corresponding points," "corresponding sides," and "image." Students learn that angle measures are preserved under a dilation, but lengths in the image are multiplied by the scale factor. They learn the definition of "similar": two figures are said to be similar if there is a sequence of translations, rotations, reflections, and dilations that takes one figure to the other. They use the definition of "similar" and properties of similar figures to justify claims of similarity or non-similarity and to reason about similar figures (MP3). Using these properties, students conclude that if two triangles have two angles in common, then the triangles must be similar. Students also conclude that the quotient of a pair of side lengths in a triangle is equal to the quotient of the corresponding side lengths in a similar triangle. This conclusion is used in the lesson that follows: students learn the terms "slope" and "slope triangle," and use the similarity of slope triangles on the same line to understand that any two distinct points on a line determine the same slope (MP7). In the following lesson, students use their knowledge of slope to find an equation for a line. They will build on this initial work with slope in a subsequent grade 8 unit on linear relationships. Throughout the unit, students discuss their mathematical ideas and respond to the ideas of others (MP3, MP6).

Many of the lessons in this unit ask students to work on geometric figures that are not set in a real-world context. This design choice respects the significant intellectual work of reasoning about area. Tasks set in real-world contexts are sometimes contrived and hinder rather than help understanding. Moreover, mathematical contexts are legitimate contexts that are worthy of study. Students do have opportunities in the unit to tackle real-world applications. In the culminating activity of the unit, students examine shadows cast by objects in the Sun. This is an opportunity for them to apply what they have learned about similar triangles (MP4).

In this unit, several lesson plans suggest that each student have access to a *geometry toolkit*. Each toolkit contains tracing paper, graph paper, colored pencils, scissors, ruler, protractor, and an index card to use as a straightedge or to mark right angles, giving students opportunities to develop their abilities to select appropriate tools and use them strategically to solve problems (MP5). Note that even students in a digitally enhanced classroom should have access to such tools; apps and simulations should be considered additions to their toolkits, not replacements for physical tools.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as describing, explaining, representing, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Describe

- observations about scaled rectangles (Lesson 1)
- observations about dilated points, circles, and polygons (Lesson 2)
- sequences of transformations (Lesson 6)
- observations about side lengths in similar triangles (Lesson 9)

Explain

- how to apply dilations to find specific images (Lesson 5)
- how to determine whether triangles are congruent, similar, or neither (Lesson 8)
- strategies for finding missing side lengths (Lesson 9)
- how to apply dilations to find specific images of points (Lesson 12)
- reasoning for a conjecture (Lesson 13)

Represent

- dilations using given scale factors and coordinates (Lesson 4)
- figures using specific transformations (Lesson 6)
- graphs of lines using equations (Lesson 12)

In addition, students are expected to use language to interpret directions for dilating figures and for creating triangles; compare dilated polygons and methods for determining similarity; critique reasoning about angles, sides, and similarity; justify whether polygons are similar; and generalize about points on a line and similar triangles.

Learning Targets

Dilations, Similarity, and Introducing Slope

Lesson 1: Projecting and Scaling

- I can decide if one rectangle is a dilation of another rectangle.
- I know how to use a center and a scale factor to describe a dilation.

Lesson 2: Circular Grid

- I can apply dilations to figures on a circular grid when the center of dilation is the center of the grid.

Lesson 3: Dilations with no Grid

- I can apply a dilation to a polygon using a ruler.

Lesson 4: Dilations on a Square Grid

- I can apply dilations to figures on a rectangular grid.
- If I know the angle measures and side lengths of a polygon, I know the angles measures and side lengths of the polygon if I apply a dilation with a certain scale factor.

Lesson 5: More Dilations

- I can apply dilations to polygons on a rectangular grid if I know the coordinates of the vertices and of the center of dilation.

Lesson 6: Similarity

- I can apply a sequence of transformations to one figure to get a similar figure.
- I can use a sequence of transformations to explain why two figures are similar.

Lesson 7: Similar Polygons

- I can use angle measures and side lengths to conclude that two polygons are not similar.
- I know the relationship between angle measures and side lengths in similar polygons.

Lesson 8: Similar Triangles

- I know how to decide if two triangles are similar just by looking at their angle measures.

Lesson 9: Side Length Quotients in Similar Triangles

- I can decide if two triangles are similar by looking at quotients of lengths of corresponding sides.
- I can find missing side lengths in a pair of similar triangles using quotients of side lengths.

Lesson 10: Meet Slope

- I can draw a line on a grid with a given slope.
- I can find the slope of a line on a grid.

Lesson 11: Writing Equations for Lines

- I can decide whether a point is on a line by finding quotients of horizontal and vertical distances.

Lesson 12: Using Equations for Lines

- I can find an equation for a line and use that to decide which points are on that line.

Lesson 13: The Shadow Knows

- I can model a real-world context with similar triangles to find the height of an unknown object.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow the lesson in which it was first introduced.

lesson	new terminology	
	receptive	productive
8.2.1	**scale factor** scaled copy scaling	
8.2.2	**dilation** **center of a dilation** dilate	
8.2.4		**center of a dilation** **scale factor**
8.2.6	**similar**	dilate
8.2.7		**dilation**
8.2.9	quotient	
8.2.10	**similar**	**slope** slope triangle
8.2.11	similarity x-coordinate y-coordinate equation of a line	quotient
8.2.13	estimate approximate / approximately	

Required Materials

Blank paper
Dried linguine pasta
We specified linguine since it is flatter and less likely to roll around than spaghetti.

Four-function calculators
Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Long straightedge
Measuring tapes
Pre-printed slips, cut from copies of the blackline master
Rulers marked with inches
Scissors
Straightedges
A rigid edge that can be used for drawing line segments. Sometimes a ruler is okay to use as a straightedge, but sometimes it is preferable to use an unruled straightedge, like a blank index card.

Tape

Section: Dilations

Lesson 1: Projecting and Scaling

Goals

- Comprehend the term “dilation” as a process that produces scaled copies.
- Describe (orally) features of scaled copies of a rectangle.
- Identify rectangles that are scaled copies of one another.

Learning Targets

- I can decide if one rectangle is a dilation of another rectangle.
- I know how to use a center and a scale factor to describe a dilation.

Lesson Narrative

In grade 7, students examine scaled copies. For polygons, they identify that side lengths of scaled copies are proportional, and the constant of proportionality relating the original lengths to the corresponding lengths in the scaled copy is the scale factor. This lesson builds on this experience. In the first activity, students arrange a set of scaled copies of rectangles and observe that if the rectangles are arranged to share one angle, then the opposite vertices all lie on the same line. This motivates an informal introduction of *dilation*, a geometric process that produces scaled copies. In the context of the set of rectangles, the shared vertex is the *center of dilation* and, as students will learn in later lessons, the dilation scales the distance of all points (not just the upper right vertex of the rectangle) from the center of dilation. A second optional activity recalls explicitly work from grade 7 about scaled copies of rectangles.

Alignments

Building On

- 6.NS.A: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- 7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Addressing

- 8.G.A: Understand congruence and similarity using physical models, transparencies, or geometry software.

Building Towards

- 8.G.A: Understand congruence and similarity using physical models, transparencies, or geometry software.

Instructional Routines

- MLR2: Collect and Display
- MLR5: Co-Craft Questions
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Number Talk

Required Materials

Blank paper
Four-function calculators
Long straightedge
Rulers marked with inches
Scissors

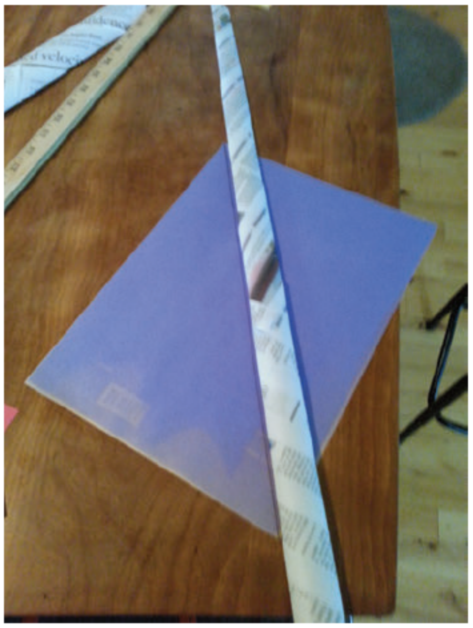
Required Preparation

For the activity Sorting Rectangles, decide whether students will create their own set of rectangles A–E or if you will create these ahead of time. If students will create their own, they need 2 sheets of copier paper and a pair of scissors. (Students do not need scissors if they are not creating the rectangles.) If you will create them ahead of time, prepare and label one set A–E for each pair of students:

- A: One full sheet, 8.5 by 11 inch
- B: One half sheet, 8.5 by 5.5
- C: One quarter sheet, 4.25 by 5.5
- D: One eighth sheet, 4.25 by 2.75
- E: One sixteenth sheet, 2.125 by 2.75

Calculators are optional. Decide whether you want students to handle the computations without a calculator or whether you will offer calculators.

Each pair of students will also need a long straightedge (at least 14 inches long). Meter or yardsticks would work, or a long straightedge can be created from newspaper, like this:



Student Learning Goals

Let's explore scaling.

1.1 Number Talk: Remembering Fraction Division

Warm Up: 10 minutes

This Number Talk gives students an opportunity to recall strategies for computation problems that will arise in the lesson. While many strategies may emerge, the focus of these problems is for students to recall and rehearse a reliable way to divide a mixed number by a whole number. Likely strategies are:

- Use the distributive property to divide each component of the mixed number separately.
- Convert the mixed number into a fraction of the form $\frac{a}{b}$, then multiply by the reciprocal of the divisor.

Three problems are given. In the limited time available, however, it may not be possible to share every possible strategy. Consider gathering only one or two different strategies per problem.

Building On

- 6.NS.A

Instructional Routines

- MLR2: Collect and Display
- MLR7: Compare and Connect
- MLR8: Discussion Supports

- Number Talk

Launch

Display one problem at a time. Give students 1 minute of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Follow with a whole-class discussion.

Support for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organization

Student Task Statement

Find each quotient. Write your answer as a fraction or a mixed number.

$6\frac{1}{4} \div 2$

$10\frac{1}{7} \div 5$

$8\frac{1}{2} \div 11$

Student Response

- $3\frac{1}{8}$ or $\frac{25}{8}$. Possible strategies:
    a. $6 \div 2 = 3$ and $\frac{1}{4} \div 2 = \frac{1}{8}$
    b. $6\frac{1}{4} = \frac{25}{4}$, and $\frac{25}{4} \div 2 = \frac{25}{4} \cdot \frac{1}{2} = \frac{25}{8}$
- $2\frac{1}{35}$ or $\frac{71}{35}$. Possible strategies:
    a. $10 \div 5 = 2$ and $\frac{1}{7} \div 5 = \frac{1}{35}$
    b. $10\frac{1}{7} = \frac{71}{7}$, and $\frac{71}{7} \div 5 = \frac{71}{7} \cdot \frac{1}{5} = \frac{71}{35}$
- $\frac{17}{22}$. Possible strategy:
    a. $8 \div 11 = \frac{8}{11}$, $\frac{1}{2} \div 11 = \frac{1}{22}$, and $\frac{8}{11} + \frac{1}{22} = \frac{17}{22}$
    b. $8\frac{1}{2} = \frac{17}{2}$, and $\frac{17}{2} \div 11 = \frac{17}{2} \cdot \frac{1}{11} = \frac{17}{22}$

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their explanations for all to see (refer to MLR 2 Collect and Display). Ask students if or how the numbers in the problem impacted their choice of strategy. Point out how the first problem differs from the third in an important way: for the first problem it is not necessary to convert the mixed number to a fraction because the whole number part of the mixed number is evenly divisible by 2.

To involve more students in the conversation, use MLR 7 (Compare and Connect) by asking probing questions and connecting students' responses, such as:

- "Who can restate ____'s reasoning using your own words?"
- "Does anyone want to add on to ____'s explanation?"
- "Do you agree or disagree ____'s reasoning? Why?"
- "How is ____'s reasoning similar to and different from ____'s reasoning?"

Support for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because . . ." or "I noticed ____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimize output (for explanation)

1.2 Sorting Rectangles

20 minutes
This activity recalls work from grade 7 on scaled copies, purposefully arranging a set of scaled copies to prepare students to understand the process of dilation. If one rectangle is a scaled copy of another, then they can be arranged so that the diagonal of the larger rectangle *contains* the diagonal of the smaller rectangle. To do this, it is sufficient to line up the rectangles so that the vertices of the right angles at their lower left match up. Students will arrange a set of rectangles into groups with shared diagonals and examine the scale factors relating the rectangles. Afterward, during the discussion, the word dilation is first used, in an informal way, as a way to make scaled copies (of the rectangle in this case). From this point of view, the shared vertex of each set of rectangles is the center of dilation and once we choose an original rectangle from each set, there is a scale factor associated to each copy, namely the scale factor needed to produce the copy from the original.

As an *optional* additional part to this activity, students may perform a visual test that helps decide whether or not two cut-out figures are scaled copies of one another. The visual test tells whether two cut-out figures are scaled copies of each other by holding each figure at a different distance from the eye and checking if it is possible to make the two figures match up exactly.

Monitor for how students sort the rectangles and how they find measurements of the new rectangles. Encourage them to use what they know about how the rectangles were created rather than measuring each new rectangle (which is likely to introduce errors). Also monitor for how they decide if one rectangle is a scaled copy of another.

Building On

- 7.G.A.1

Building Towards

- 8.G.A

Instructional Routines

- MLR2: Collect and Display

Launch

If students will perform the optional eyeball test, tell them that one way to check whether two shapes are scaled copies of each other is to use the “eyeball test.” Students will perform this test for themselves in the activity, but will watch a demonstration first.

- Hold one rectangle up in front of your face in one hand, and another rectangle farther away from your face in the other hand. (The larger rectangle should be farther away than the smaller rectangle.)
- Close one eye.
- If you can adjust your arms so that the rectangles appear to be exactly the same, then they are a match. If it is not possible to adjust your arms so that the rectangles appear to be exactly the same, then they are not a match. Explain to students that their job will be to use the eyeball test to figure out which pairs of their rectangles are matches.

Arrange students in groups of 2. Provide a set of 5 pre-cut rectangles and a long straightedge to each group and, optionally, access to calculators. (Alternatively, you could give each group two whole sheets of paper and instruct them to do the folding and cutting. This option might be attractive if your students would understand the idea of “halving” the measurements better with the concrete experience.)

If students are performing the optional “eyeball test” on pairs of rectangles, instruct them to sort the rectangles into different piles so that all of the rectangles in each pile “match” one another according to the eyeball test. Instruct them to discuss their thinking as a group to reach an agreement.

Support for Students with Disabilities

Representation: Internalize Comprehension. Provide a range of examples and counterexamples of rectangles that are scaled copies of the full sheet of paper. Show an example of a scaled copy and then a counterexample. Be sure to justify the reasoning in each example. Consider providing step-by-step directions for students to find the scale factor between rectangles and how to compare corresponding angle measures.
Supports accessibility for: Conceptual processing

Support for English Language Learners

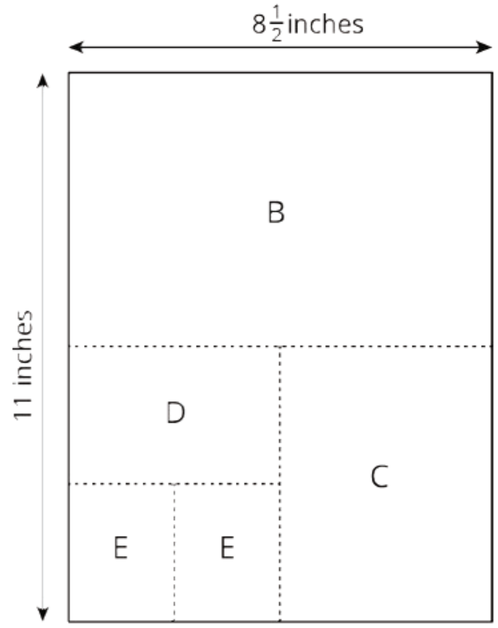
Conversing, Reading: MLR2 Collect and Display. As students work in pairs to make sense of the problem, circulate and listen to students as they determine which rectangles are scaled copies of the full sheet of paper. Write down the observations students make about the measurements of rectangles A, C, and E. As students review the language and diagrams collected in the visual display, encourage students to clarify the meaning of a word or phrase. For example, a phrase such as "rectangle E is the same as rectangle C but smaller" can be clarified by rephrasing the statement as "the side lengths of rectangle E are half of the side lengths of rectangle C." This routine will provide feedback to students in a way that supports sense-making while simultaneously increasing meta-awareness of language.
Design Principle(s): Support sense-making; Maximize meta-awareness

Anticipated Misconceptions

If students forget how to check if two rectangles are scaled copies of one another, remind them to compare the measurements to see if they have the same scale factor. Students may recall that that scaled copies have corresponding angles of the same measure, but they may not recall that equal angle measurements don't necessarily mean you have scaled copies.

Student Task Statement

Rectangles were made by cutting an $8\frac{1}{2}$-inch by 11-inch piece of paper in half, in half again, and so on, as illustrated in the diagram. Find the lengths of each rectangle and enter them in the appropriate table.



1. Some of the rectangles are scaled copies of the full sheet of paper (Rectangle A). Record the measurements of those rectangles in this table.

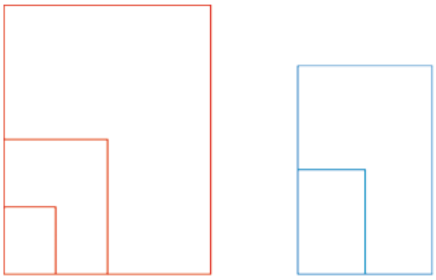
rectangle	length of short side (inches)	length of long side (inches)
A	$8\frac{1}{2}$	11

2. Some of the rectangles are *not* scaled copies of the full sheet of paper. Record the measurements of those rectangles in this table.

rectangle	length of short side (inches)	length of long side (inches)

3. Look at the measurements for the rectangles that are scaled copies of the full sheet of paper. What do you notice about the measurements of these rectangles? Look at the measurements for the rectangles that are *not* scaled copies of the full sheet. What do you notice about these measurements?

4. Stack the rectangles that are scaled copies of the full sheet so that they all line up at a corner, as shown in the diagram. Do the same with the other set of rectangles. On each stack, draw a line from the bottom left corner to the top right corner of the biggest rectangle. What do you notice?



5. Stack *all* of the rectangles from largest to smallest so that they all line up at a corner. Compare the lines that you drew. Can you tell, from the drawn lines, which set each rectangle came from?

Student Response

If students perform the eyeball test, then rectangles A, C, and E are matches. B and D also match.

1.

rectangle	length of short side (inches)	length of long side (inches)
A	$8\frac{1}{2}$	11
C	$4\frac{1}{4}$	$5\frac{1}{2}$
E	$2\frac{1}{8}$	$2\frac{3}{4}$

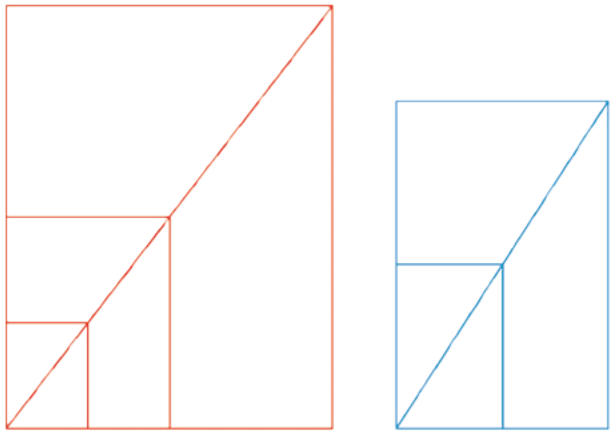
2.

rectangle	length of short side (inches)	length of long side (inches)
B	$5\frac{1}{2}$	$8\frac{1}{2}$
D	$2\frac{3}{4}$	$4\frac{1}{4}$

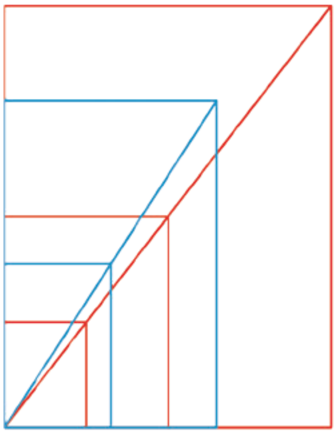
3. The lengths of the short and long sides of each set of rectangles form a set of equivalent ratios. There are different ways of expressing this idea: students might reason from column to

column and say that the measurements have the same quotient, the same unit rate, or the same constant of proportionality. For rectangles A, C, and E, the quotient of the lengths of a short side and a long side is $\frac{17}{22}$. For rectangles B and D, this quotient is $\frac{11}{17}$. They also might reason multiplicatively from row to row. For example, from A to C, you can multiply each measurement in A by $\frac{1}{2}$ to get the corresponding measurement in C.

4. The diagonal goes through two vertices of each rectangle in the pile.



5. Yes; the diagonals of the second set lie above the diagonals of the first set.



Are You Ready for More?

In many countries, the standard paper size is not 8.5 inches by 11 inches (called "letter" size), but instead 210 millimeters by 297 millimeters (called "A4" size). Are these two rectangle sizes scaled copies of one another?

Student Response

No. Converting from millimeters to inches, A4 paper is about 8.27 inches by 11.69 inches. Since it is both taller and less wide than letter paper, it could not be a scaled copy.

Activity Synthesis

Ask students how they decided that the $5\frac{1}{2}$ by $8\frac{1}{2}$ rectangle is not a scaled copy of the $8\frac{1}{2}$ by 11 rectangle. Make sure to provide a mathematical explanation since it is not easy to determine visually. For example, there is no single number that you can multiply by $5\frac{1}{2}$ to get $8\frac{1}{2}$ *and* multiply by $8\frac{1}{2}$ to get 11.

Ask students how they decided that the $5\frac{1}{2}$ by $4\frac{1}{4}$ rectangle is a scaled copy of the 11 by $8\frac{1}{2}$ rectangle, and again, emphasize mathematical explanations based on noticing equivalent ratios.

Emphasize that when all of the rectangles are aligned with the lower left angle matching, by increasing size:

- The diagonals of the rectangles fall into two sets: those that are scaled copies of the full sheet of paper and those that are scaled copies of the half sheet of paper.
- The diagonals of the rectangles that are scaled copies of one another match up.

Tell students that they are going to study a new kind of transformation (to be added to the list from previous work: translations, rotations, and reflections). This new kind of transformation makes scaled copies and is called a dilation. A dilation has a center of dilation(the common vertex for the rectangles in each stack) and a scale factor (4, for example, from Rectangle E to Rectangle A). Different choices of scale factor give scaled copies of different sizes: for example, Rectangle C uses a scale factor of 2, applied to Rectangle E.

1.3 Scaled Rectangles

Optional: 10 minutes

This activity continues to examine scaled copies of a rectangle via dividing a rectangle into smaller rectangles. In this activity, the focus is more on the scale factor and the language of scaled copies, emphasizing the link with work students did in grade 7. Unlike in the previous task, there are no given dimensions for any of the rectangles. Students need to find the scale factor using their understanding of the meaning of scale factor and the fact that the rectangles are divided evenly.

Monitor how students reason about the scale factor. They could use the diagram to see how many times as long and wide one rectangle is compared to another. They could also use what they know about how the area of rectangles changes in scaled copies. Select students who use these approaches to share during the discussion.

Building On

- 7.G.A.1

Building Towards

- 8.G.A

Instructional Routines

- MLR5: Co-Craft Questions

Launch

Tell students that they are going to examine a different set of rectangles and determine scale factors for pairs which are scaled copies of one another. Briefly, ask students to interpret what is meant by "evenly divided." (Rectangle R is cut exactly in half vertically and horizontally, and also one of its quadrants is cut exactly in thirds vertically and horizontally.) Students may want to use a ruler to validate their understanding of what "evenly divided" means.

Give students 5 minutes quiet work time followed by a whole-class discussion.

Support for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation such as: "First, I _____ because ....", "I noticed _____ so I ....", "Why did you ...?", "I agree/disagree because ...."
Supports accessibility for: Language; Social-emotional skills

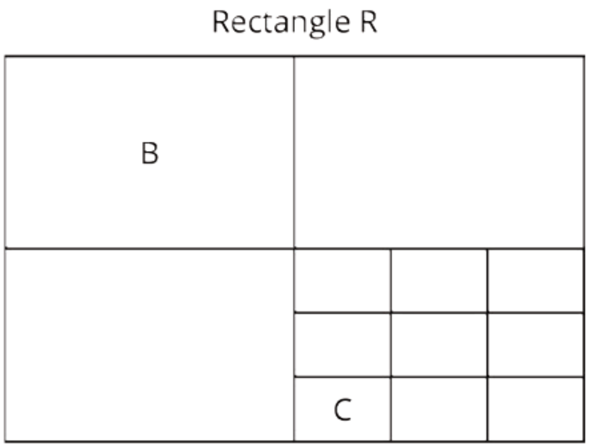
Support for English Language Learners

Conversing, Writing: MLR5 Co-Craft Questions. Before presenting the questions in this activity, display the diagram of the divided rectangles and ask students to write possible mathematical questions about the diagram. Invite students to compare the questions they generated with a partner before sharing the questions with the whole class. Listen for and amplify questions about whether two or more rectangles are scaled copies of one another. If no student asks whether two rectangles are scaled copies of one another, ask students to adapt a question to align with the learning goals of this lesson. Then reveal and ask students to work on the actual questions of the task. This routine will help develop students' meta-awareness of language as they generate questions about scaled rectangles.
Design Principle(s): Maximize meta-awareness

Student Task Statement

Here is a picture of Rectangle R, which has been evenly divided into smaller rectangles. Two of the smaller rectangles are labeled B and C.

1. Is B a scaled copy of R? If so, what is the **scale factor**?
2. Is C a scaled copy of B? If so, what is the scale factor?
3. Is C a scaled copy of R? If so, what is the scale factor?



Student Response

1. Yes, the length and width of Rectangle B are each $\frac{1}{2}$ the length and width of Rectangle R. The scale factor is $\frac{1}{2}$.
2. Yes, the length and width of Rectangle C are each $\frac{1}{3}$ the length and width of Rectangle B. The scale factor is $\frac{1}{3}$.
3. Yes, the length and width of Rectangle C are each $\frac{1}{6}$ the length and width of Rectangle R. The scale factor is $\frac{1}{6}$

Activity Synthesis

Ask selected students to share their solutions. Then ask these questions:

- "Why is rectangle B a scaled copy of rectangle R?" (The length and width in both cases have been multiplied by the same number because the rectangles are divided evenly.)
- "How are the scale factors from R to B and B to C related to the scale factor from R to C?" (The latter is the product of the former.)
- "Does the diagonal from top left to lower right of Rectangle R go through opposite vertices of one rectangle of each size?" (Yes.)

Lesson Synthesis

In previous lessons, we have studied rigid transformations, specifically translations, rotations, and reflections. When we apply a sequence of rigid transformations to a figure, we change the figure's location and orientation in the plane but not its size. In this lesson, we began to study a new "move," which makes scaled copies of figures (and hence can change their size!) This new move is called a *dilation*. We will introduce a formal definition of dilation in the next lesson and then we will investigate how figures change when dilations are allowed in addition to rigid transformations.

1.4 What is a Dilation?

Cool Down: 5 minutes

Students will learn the formal definition of a dilation in the next lesson. For now, they should just describe their current understanding of a dilation and how it works. Students may think that dilations only expand things because of its everyday meaning. In the next lesson, they will learn this is not always true.

Addressing

- 8.G.A

Launch

Ask students to think about the rectangles they worked with to explain their understanding of dilations and how they work.

Student Task Statement

In your own words, explain what a dilation is.

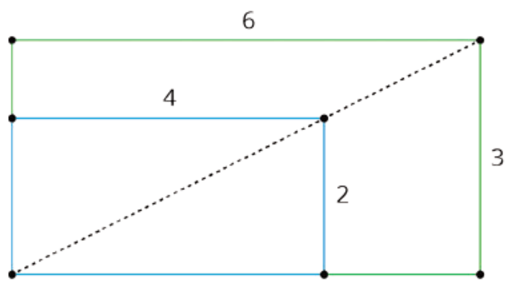
Student Response

Answers vary. Sample responses: A dilation expands things. It makes scaled copies. It blows things up. It works by pushing things outward from a center. A dilation multiplies distances from the center to change the size of the figure.

Student Lesson Summary

Scaled copies of rectangles have an interesting property. Can you see what it is?

Here, the larger rectangle is a scaled copy of the smaller one (with a scale factor of $\frac{3}{2}$). Notice how the diagonal of the large rectangle contains the diagonal of the smaller rectangle. This is the case for any two scaled copies of a rectangle if we line them up as shown. If two rectangles are *not* scaled copies of one another, then the diagonals do not match up. In this unit, we will investigate how to make scaled copies of a figure.



Glossary

- scale factor

Lesson 1 Practice Problems

Problem 1

Statement

Rectangle A measures 12 cm by 3 cm. Rectangle B is a scaled copy of Rectangle A. Select **all** of the measurement pairs that could be the dimensions of Rectangle B.

A. 6 cm by 1.5 cm

B. 10 cm by 2 cm

C. 13 cm by 4 cm

D. 18 cm by 4.5 cm

E. 80 cm by 20 cm

Solution

["A", "D", "E"]

Problem 2

Statement

Rectangle A has length 12 and width 8. Rectangle B has length 15 and width 10. Rectangle C has length 30 and width 15.

a. Is Rectangle A a scaled copy of Rectangle B? If so, what is the scale factor?

b. Is Rectangle B a scaled copy of Rectangle A? If so, what is the scale factor?

c. Explain how you know that Rectangle C is *not* a scaled copy of Rectangle B.

d. Is Rectangle A a scaled copy of Rectangle C? If so, what is the scale factor?

Solution

a. Yes, the scale factor is $\frac{4}{5}$.

b. Yes, the scale factor is $\frac{5}{4}$.

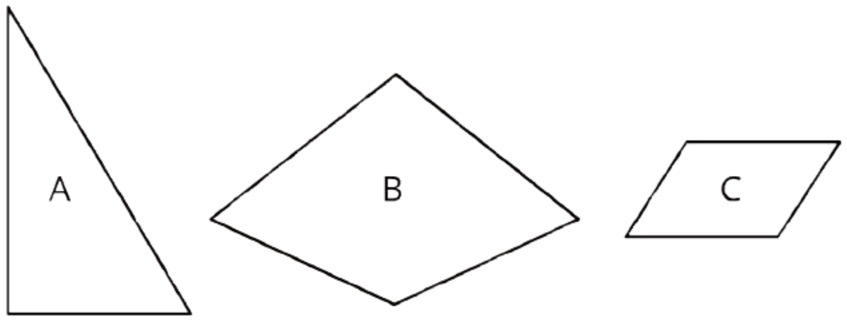
c. Rectangle C's length is double that of Rectangle B, but its width is not double.

d. No.

Problem 3

Statement

Here are three polygons.



a. Draw a scaled copy of Polygon A with scale factor $\frac{1}{2}$.

b. Draw a scaled copy of Polygon B with scale factor 2.

c. Draw a scaled copy of Polygon C with scale factor $\frac{1}{4}$.

Solution

The scaled copy of Polygon A should be a right triangle with each side half as long as the original.

The scaled copy of Polygon B should be a quadrilateral with each side twice as long as the original.

The scaled copy of Polygon C should be a parallelogram with each side one-fourth the length of the original.

Problem 4

Statement

Which of these sets of angle measures could be the three angles in a triangle?

A. $40^\circ, 50^\circ, 60^\circ$

B. $50^\circ, 60^\circ, 70^\circ$

C. $60^\circ, 70^\circ, 80^\circ$

D. $70^\circ, 80^\circ, 90^\circ$

Solution

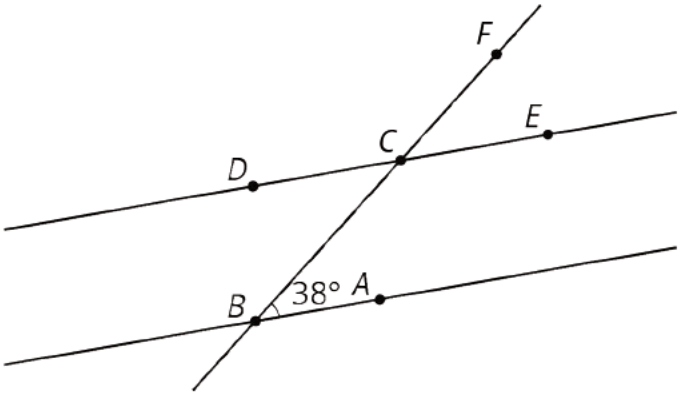
B

(From Unit 1, Lesson 15.)

Problem 5

Statement

In the picture lines AB and CD are parallel. Find the measures of the following angles. Explain your reasoning.



a. $\angle BCD$

b. $\angle ECF$

c. $\angle DCF$

Solution

a. 38 degrees. $\angle BCD$ and $\angle ABC$ are alternate interior angles for the parallel lines AB and CD cut by the transversal BC.

b. 38 degrees. $\angle ECF$ and $\angle BCD$ are a pair of vertical angles.

c. 142 degrees. $\angle DCF$ and $\angle ECF$ are supplementary angles.

(From Unit 1, Lesson 14.)

Lesson 2: Circular Grid

Goals

- Comprehend that “a point on the circle” (in written and spoken language) refers to a point that lies on the edge of the circle and not in the circle’s interior.
- Create dilations of polygons using a circular grid given a scale factor and center of dilation.
- Explain (orally) how a dilation affects the size, side lengths and angles of polygons.

Learning Targets

- I can apply dilations to figures on a circular grid when the center of dilation is the center of the grid.

Lesson Narrative

The previous lesson introduced the general idea of a dilation as a method for producing scaled copies of geometric figures. This lesson formally introduces a method for producing dilations. A **dilation** has a center and a *scale factor*. For a dilation with center P and scale factor 2, for example, the center does not move. Meanwhile each point Q stays on ray PQ but its distance from P doubles (because the scale factor is 2).

A circular grid is an effective tool for performing a dilation. A circular grid has circles with radius 1 unit, 2 units, and so on all sharing the same center. Students experiment with dilations on a circular grid, where the center of dilation is the common center of the circles. By using the structure of the grid, they make several important discoveries about the images of figures after a dilation including:

- Each grid circle maps to a grid circle.
- Line segments map to line segments and, in particular, the image of a polygon is a scaled copy of the polygon.

The next several lessons will examine dilations on a rectangular grid and with no grid, solidifying student understanding of the relationship between a polygon and its dilated image. This echoes similar work in the previous unit investigating the relationship between a figure and its image under a rigid transformation.

As with previous geometry lessons, students should have access to geometry toolkits so they can make strategic choices about which tools to use (MP5).

Alignments

Building On

- 3.G.A: Reason with shapes and their attributes.
- 4.MD.C.5: Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

Addressing

- 8.G.A: Understand congruence and similarity using physical models, transparencies, or geometry software.

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports
- Notice and Wonder

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Student Learning Goals

Let's dilate figures on circular grids.

2.1 Notice and Wonder: Concentric Circles

Warm Up: 5 minutes
The goal of this warm-up is to introduce the circular grid which students will examine in greater detail throughout this unit. The circles in the grid all have the same center and the distance between consecutive circles is the same. The circular grid is particularly useful for showing dilations where the center of dilation is the center of the grid.

Students engage in MP7 as they look for structure and relationships between the circles and lines in the picture.

Building On

- 3.G.A
- 4.MD.C.5

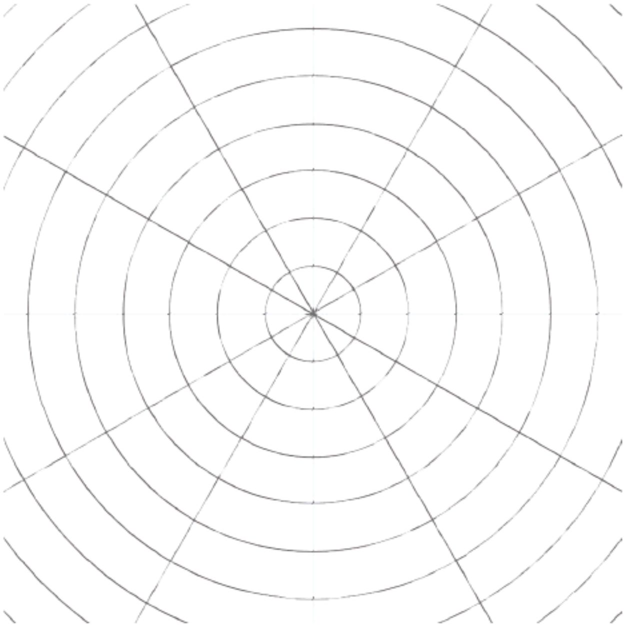
Instructional Routines

- Notice and Wonder

Launch

Arrange students in groups of 2. Tell students that they will look at an image, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the image for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Student Task Statement



What do you notice? What do you wonder?

Student Response

Answers vary. Sample responses:

- Do the circles have the same center?
- Is the center of the circles where the lines meet?
- Why are there 6 lines meeting in the center?
- Is the distance between the consecutive circles the same?
- How many pieces is each circle divided into? (This can be taken two ways depending whether you are talking about the one dimensional or two dimensional object.)

Activity Synthesis

Ask students to share their responses, highlighting these features of the picture:

- The circles share the same center
- The center of the circles is the point where the lines meet

- The distance from one circle to the next is always the same (the radius of each successive circle is one unit more than its predecessor)

Students may also notice that the angle made by successive rays from the center is always 30 degrees. Some things students may wonder include

- When is this grid useful?
- Why are the circles equally spaced?
- Why are the lines there?

2.2 A Droplet on the Surface

15 minutes (there is a digital version of this activity)
The purpose of this activity is to begin to think of a dilation with a scale factor as a rule or operation on points in the plane. Students work on a circular grid with center of dilation at the center of the grid. They examine what happens to different points on a given circle when the dilation is applied and observe that these points all map to another circle whose radius is scaled by the scale factor of the dilation. For example, if the scale factor is 3 and the points lie on a circle whose radius is 2 grid units, then the dilated points will all lie on a circle whose radius is 6 units. Students need to explain their reasoning for finding the scale factor (MP3).

Students discover that the circular grid is a powerful tool for representing dilations and they will continue to use the circular grid as they study what happens when dilations are applied to shapes other than grid circles.

In the digital activity, students encounter some new tools and the circular grid.

Addressing

- 8.G.A

Instructional Routines

- MLR8: Discussion Supports

Launch

Ask students if they have ever seen a pebble dropped in a still pond, and select students to describe what happens. (The pebble becomes the center of a sequence of circular ripples.) Display the image from the task statement, and ask students to think about how it is like a pebble dropped in a still pond. Demonstrate that distance on the circular grid is measured by counting units along one of the rays that start at the center, P. Use MLR 8 (Discussion Supports) to draw students' attention to a few important words in the task:

- "When we say 'on the circle,' we mean on the curve or on the edge. (We do not mean the circle's interior.)"

- "Remember that a ray starts at a point and goes forever in one direction. Their rays should start at P and be drawn to the edge of the grid."

If using the digital activity, you may want to demonstrate dilating a point before having students begin the task.

Support for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of these terms. Include the following terms and maintain the display for reference throughout the unit: circular grid, "on the circle," ray, scale factor, and distance. Include a circular grid on the display, give examples, and label the features.
Supports accessibility for: Conceptual processing; Language

Anticipated Misconceptions

For question 5, students might think the scale factor is 4, because the distance between the smaller and larger circle for each point increases by 4. If this happens, ask students how many grid units circle c is from the center (2) and how many grid units circle d is from the center (6). Then remind them that *scale factor* means a number you multiply by.

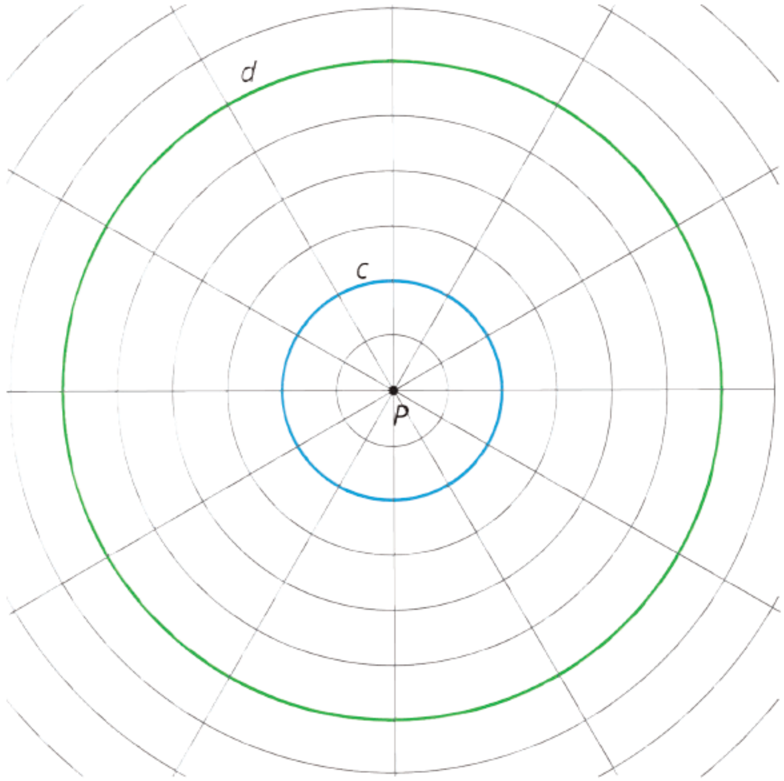
Student Task Statement

The larger Circle d is a **dilation** of the smaller Circle c. P is the **center of dilation.**

1. Draw four points *on* the smaller circle (not inside the circle!), and label them E, F, G, and H.

2. Draw the rays from P through each of those four points.

3. Label the points where the rays meet the larger circle E', F', G', and H'.



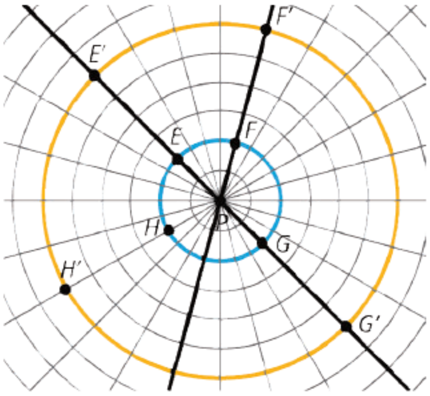
4. Complete the table. In the row labeled c, write the distance between P and the point on the smaller circle in grid units. In the row labeled d, write the distance between P and the corresponding point on the larger circle in grid units.

	E	F	G	H
c				
d				

5. The center of dilation is point P. What is the *scale factor* that takes the smaller circle to the larger circle? Explain your reasoning.

Student Response

1–3. Answers vary. Sample response:



4.

	E	F	G	H
c	2	2	2	2
d	6	6	6	6

5. The scale factor is 3 because the distance for the small circle is multiplied by 3 to find the distance in the large circle.

Activity Synthesis

Ask students if they made a strategic choice of points, such as points that lie on the grid lines coming from the center point P. Why are these points good choices for dilating?

Ask students what they think would happen if a circle were dilated about its center with a scale factor of 2 or 4. (The result would be a circle with twice the radius and 4 times the radius, respectfully, all sharing the same center.)

Two important observations coming from the lesson are:

1. The scale factor for this dilation is 3 so distances from the center of the circles triple when the dilation is applied.
2. The large circle is the dilation of the small circle, that is each point on the circle with radius 6 units is the dilated image of a point on the circle of radius 2 units. (To find which one, draw the line from the point to the center and see where it intersects the circle of radius 2 units.)

Support for English Language Learners

Speaking, Listening: MLR8 Discussion Supports. As students share the scale factor that takes the smaller circle to the larger circle, press for details in students' reasoning by asking how they know the scale factor is 3. Listen for students' explanations that reference the table with distances between the center of dilation and points on the circles. Amplify statements that use precise language such as, "The distance between point P and any point on the smaller circle is multiplied by a scale factor of 3 to get the distance between point P and the corresponding point on the larger circle." This will a support rich and inclusive discussion about how the scale factor affects the distance between the center of dilation and points on the circle.
Design Principle(s): Support sense-making

2.3 Quadrilateral on a Circular Grid

15 minutes (there is a digital version of this activity)
This activity continues studying dilations on a circular grid, this time focusing on what happens to points lying on a polygon. Students first dilate the vertices of a polygon as in the previous activity. Then they examine what happens to points on the sides of the polygon. They discover that when these points are dilated, they all lie on a side of another polygon. Just as the image of a grid circle is another circle, so the dilation of a polygon is another polygon. Moreover the dilated polygon is a scaled copy of the original polygon. These important properties of dilations are not apparent in the definition.

Monitor for students who notice that the sides of the scaled polygon $A'B'C'D'$ are parallel to the sides of $ABCD$ and that $A'B'C'D'$ is a scaled copy of $ABCD$ with scale factor 2. Also monitor for students who notice the same structure for $EFGH$ except this time the scale factor is $\frac{1}{2}$. Invite these students to share during the discussion.

Addressing

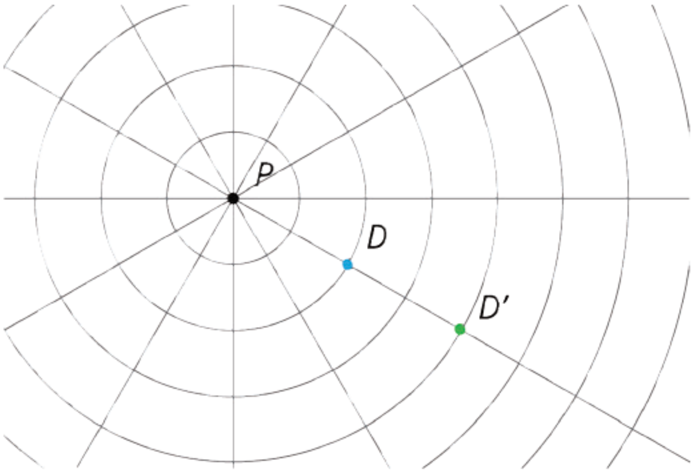
- 8.G.A

Instructional Routines

- MLR2: Collect and Display

Launch

Provide access to geometry toolkits. Tell students that they are going to dilate some points. Before they begin, demonstrate the mechanics of dilating a point using a center of dilation and a scale factor. Tell students, "In the previous activity, each point was dilated to its image using a scale factor of 3. The dilated point was three times as far from the center as the original point. When we dilate point D using P as the center of dilation and a scale factor of 2, that means we're going to take the distance from P to D and place a new point on the ray PD twice as far away from P." Display for all to see:



If using the digital activity, demonstrate the mechanics of dilating using the applet. You can also use the measurement tool to confirm.

Support for Students with Disabilities

Representation: Internalize Comprehension. Begin with a physical demonstration of the process of dilating a point using a center of dilation and a scale factor to support connections between new situations and prior understandings. Consider using these prompts: "How does this build on the previous activity in which the main task was to find distances and scale factor?" or "How does the point D' correspond to the points D and P?"
Supports accessibility for: Conceptual processing; Visual-spatial processing

Support for English Language Learners

Conversing, Reading: MLR2 Collect and Display. Circulate and listen to students as they make observations about the polygon with a scale factor of 2 and the polygon with a scale factor of $\frac{1}{2}$. Write down the words and phrases students use to compare features of the new polygons to the original polygon. As students review the language collected in the visual display, encourage students to clarify the meaning of a word or phrase. For example, a phrase such as "the new polygon is the same as the original polygon but bigger" can be clarified with the phrase "the new polygon is a scaled copy with scale factor 2 of the original polygon." A phrase such as "the polygons have the same angles" can be clarified with the phrase "each angle in the original polygon is the same as the corresponding angle in the new polygon." This routine will provide feedback to students in a way that supports sense-making while simultaneously increasing meta-awareness of language.
Design Principle(s): Support sense-making; Maximize meta-awareness

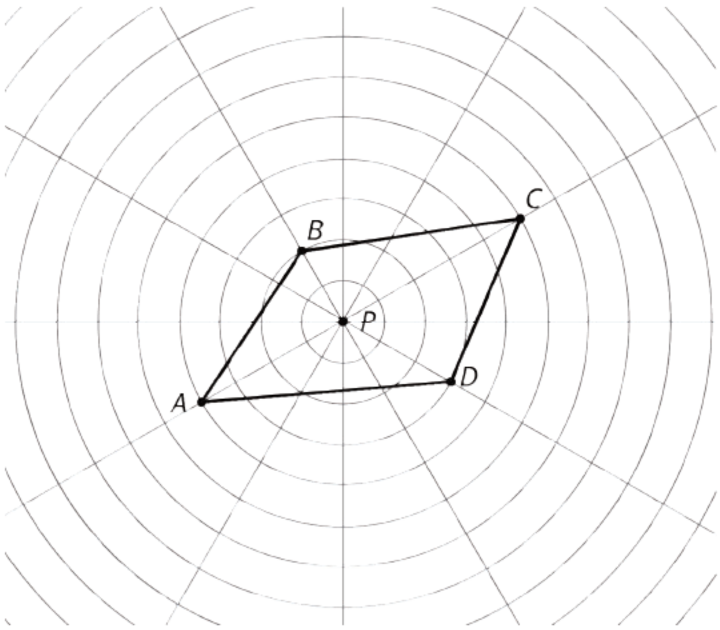
Anticipated Misconceptions

Students may think only grid points can be dilated. In fact, any point can, but they may have to measure or estimate the distances from the center. Grid points are convenient because you can measure by counting.

Student Task Statement

Here is a polygon $ABCD$.

1. Dilate each vertex of polygon $ABCD$ using P as the center of dilation and a scale factor of 2. Label the image of A as A', and label the images of the remaining three vertices as B', C', and D'.
2. Draw segments between the dilated points to create polygon $A'B'C'D'$.
3. What are some things you notice about the new polygon?



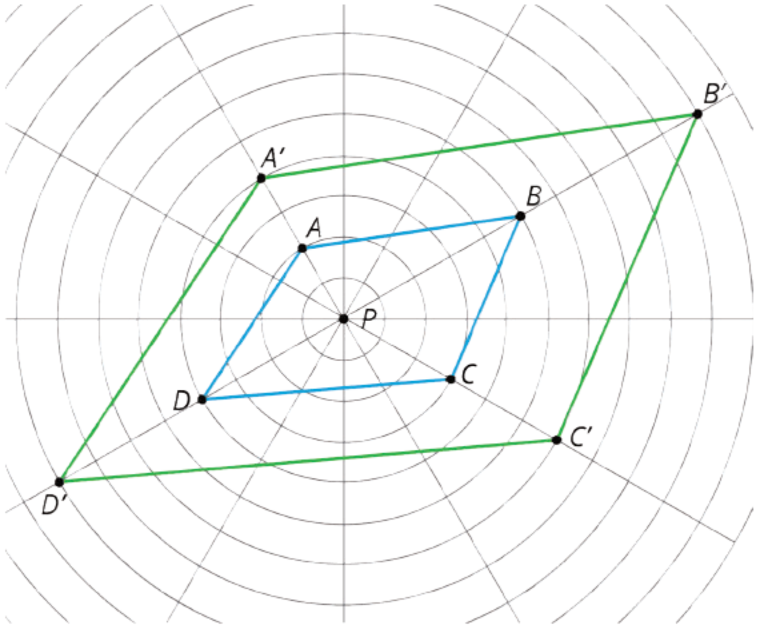
4. Choose a few more points on the sides of the original polygon and transform them using the same dilation. What do you notice?

5. Dilate each vertex of polygon $ABCD$ using P as the center of dilation and a scale factor of $\frac{1}{2}$. Label the image of A as E, the image of B as F, the image of C as G and the image of D as H.

6. What do you notice about polygon $EFGH$?

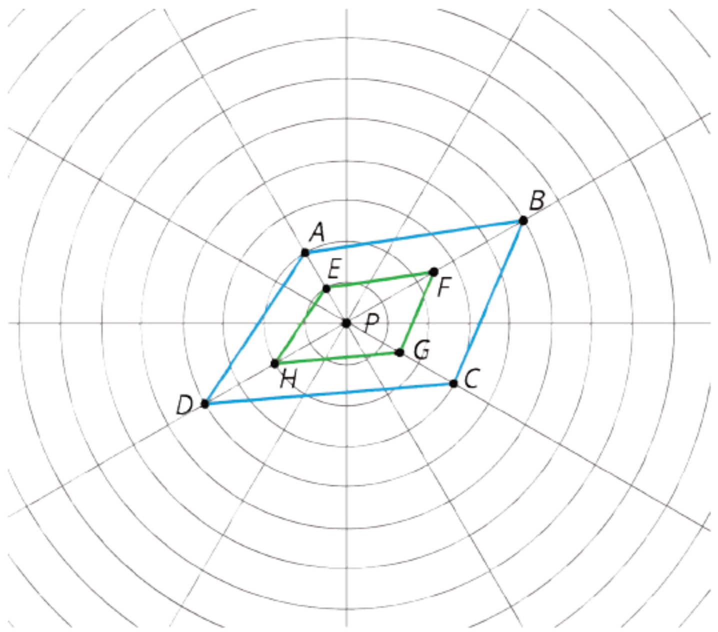
Student Response

1–5.



6. Answers vary. Possible responses:

- The new polygon is a scaled copy of the original polygon.
- Each side of the new polygon is parallel to the corresponding side on the original polygon.
- Each angle in the original figure is congruent to the corresponding angle in the dilated figure.
- Each side of the new polygon is twice the length of the corresponding side in the original polygon.



Are You Ready for More?

Suppose P is a point not on line segment $\overline{WX}$. Let $\overline{YZ}$ be the dilation of line segment $\overline{WX}$ using P as the center with scale factor 2. Experiment using a circular grid to make predictions about whether each of the following statements must be true, might be true, or must be false.

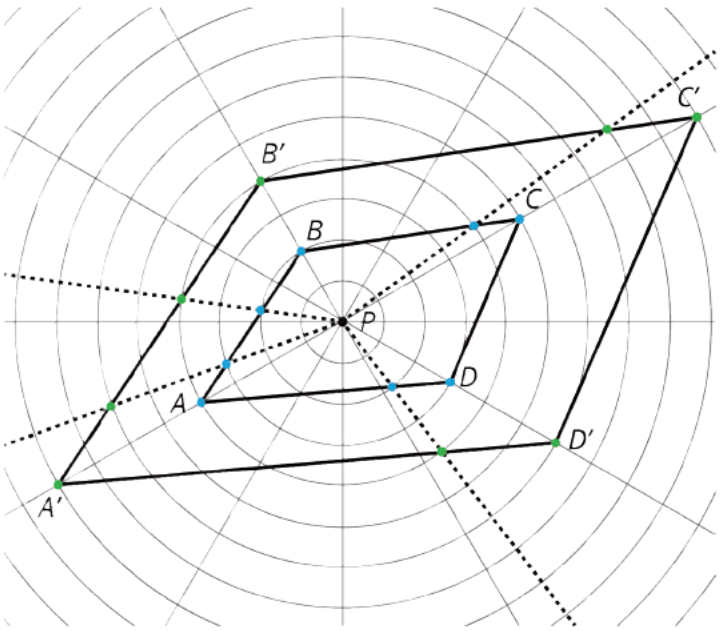
1. $\overline{YZ}$ is twice as long $\overline{WX}$.
2. $\overline{YZ}$ is five units longer than $\overline{WX}$.
3. The point P is on $\overline{YZ}$.
4. $\overline{YZ}$ and $\overline{WX}$ intersect.

Student Response

1. Must be true.
2. Might be true. (True if $\overline{WX}$ has length 5)
3. Must be false.
4. Might be true. (True, for example, if $\overline{WX}$ and $\overline{YZ}$ are both on the same line)

Activity Synthesis

Display the original figure and its image under dilation with scale factor 2 and center P.



Ask selected students to share what they notice about the new polygon. Ensure that the following observations are made. Encourage students to verify each assertion using geometry tools like tracing paper, a ruler, or a protractor.

- The new figure is a scaled copy of the original figure.
- The sides of the new figure are twice the length of the sides of the original figure.
- The corresponding segments are parallel.
- The corresponding angles are congruent.

Ask students what happened to the additional points they dilated on polygon $ABCD$. Note that a good *strategic* choice for these points are points where $ABCD$ meets one of the circles: in these cases, it is possible to double the distance from that point to the center without measuring. The additional points should have landed on a side of the dilated polygon (because of measurement error, this might not always occur exactly). The important takeaway from this observation is that dilating the polygon's vertices, and then connecting them, gives the image of the entire polygon under the dilation.

2.4 A Quadrilateral and Concentric Circles

Optional: 10 minutes (there is a digital version of this activity)
This activity continues work on dilations of polygons on a circular grid. The new twist in this activity is that the radial lines from the center of the circular grid have been removed. This means that when they dilate each point, students will need to use a ruler or other straightedge to connect that point to the center of the circular grid. If there is extra time, they can experiment dilating points other than the vertices and check that the dilation of a side of the polygon is still a line segment (though there may be small deviations due to measurement error).

Addressing

- 8.G.A

Launch

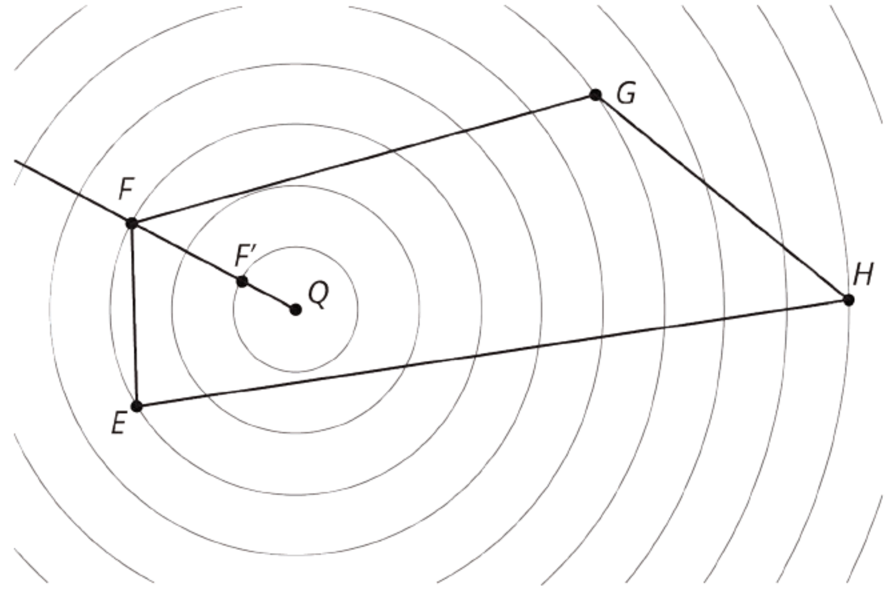
Ask students to quietly read the problem, and then ask them how this problem is alike and different from the previous one. It is alike because it shows a quadrilateral and concentric circles, and we are asked to dilate the quadrilateral using the center of the circles as the center of dilation. It is different because there is only one radial line through the center, because the scale factor is now $\frac{1}{3}$, and because one of the points is already dilated.

Tell students to study how the location of F' was determined, and then to dilate the remaining points.

Anticipated Misconceptions

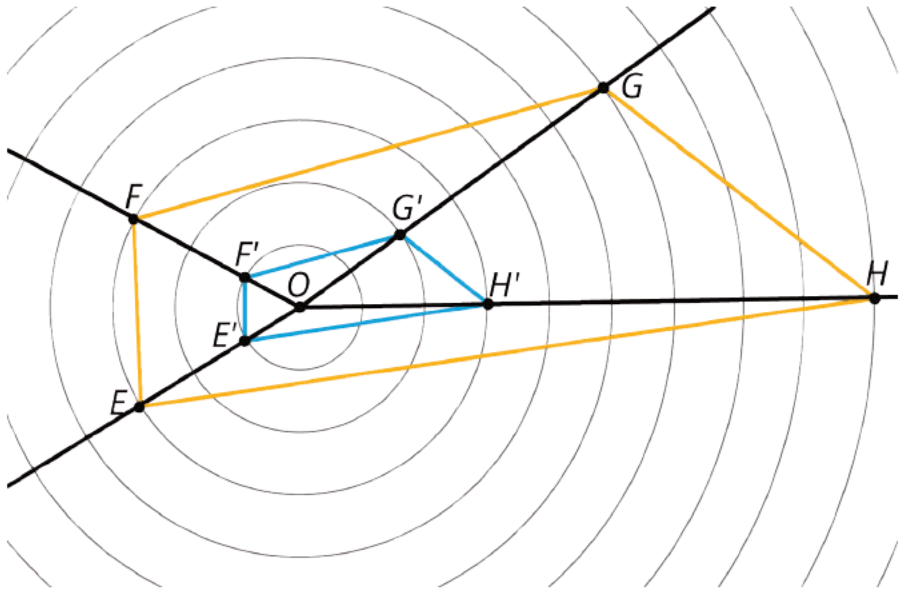
Students may be bothered because the dilated quadrilateral looks off-center and the distance between corresponding sides of the quadrilaterals depends on the side. Ensure them that the image is correct and ask them to focus on the parallel corresponding sides of the shapes or ask them if the dilated quadrilateral appears to be a scaled copy of the original (it does).

Student Task Statement



Dilate polygon $EFGH$ using Q as the center of dilation and a scale factor of $\frac{1}{3}$. The image of F is already shown on the diagram. (You may need to draw more rays from Q in order to find the images of other points.)

Student Response



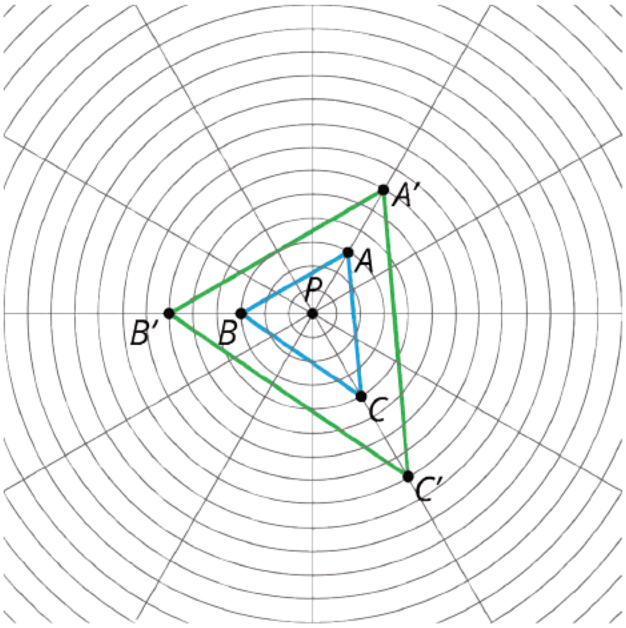
Activity Synthesis

Highlight the need to add line segments joining E, F, G, H to the center in order to find the image of those points under the dilation. Also highlight that the scale factor of $\frac{1}{3}$ resulted in an image that was smaller than the original figure instead of larger. You might ask students what scale factor would result in no change? That is, for what scale factor would the image land right on top of the original figure? They can likely name "1" as the scale factor that would accomplish this. So, scale factors that are greater than 1 result in an image larger than the original, and scale factors less than 1 result in an image smaller than the original.

Lesson Synthesis

- "What are some important properties of the circular grid?"
- "How does it help to perform dilations?"

Highlight the fact that the circular grid is mainly useful when the center of dilation is the center of the grid. When the scale factor is 3, for example, the circle with radius 1 grid unit maps to the circle with radius 3 grid units. More generally, each grid circle maps to a grid circle whose radius is three times as large.



To apply a dilation to a polygon, we can dilate the vertices and then add appropriate segments. For example, triangle $A'B'C'$ is the dilation of triangle ABC with scale factor 2 and center of dilation P:How does triangle $A'B'C'$ compare to triangle ABC? Make sure students see that it is a scaled copy with scale factor 2.

2.5 Dilating points on a circular grid

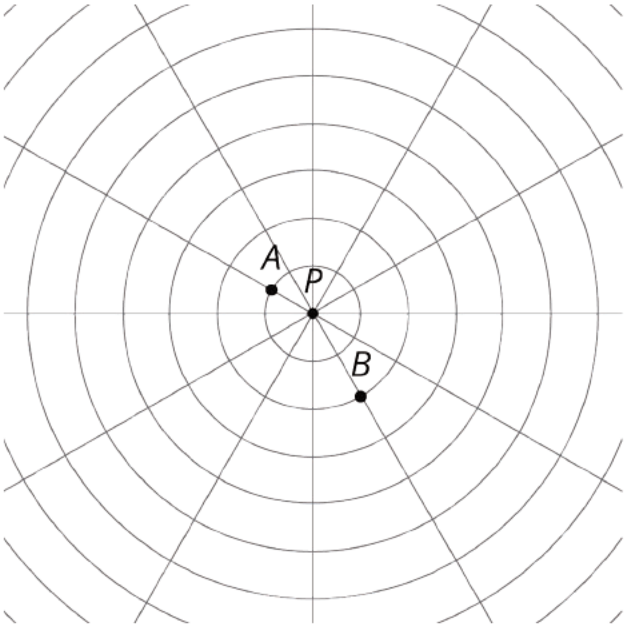
Cool Down: 5 minutes

Students apply dilations with scale factors larger than 1 to points on a circular grid that lie on radial lines.

Addressing

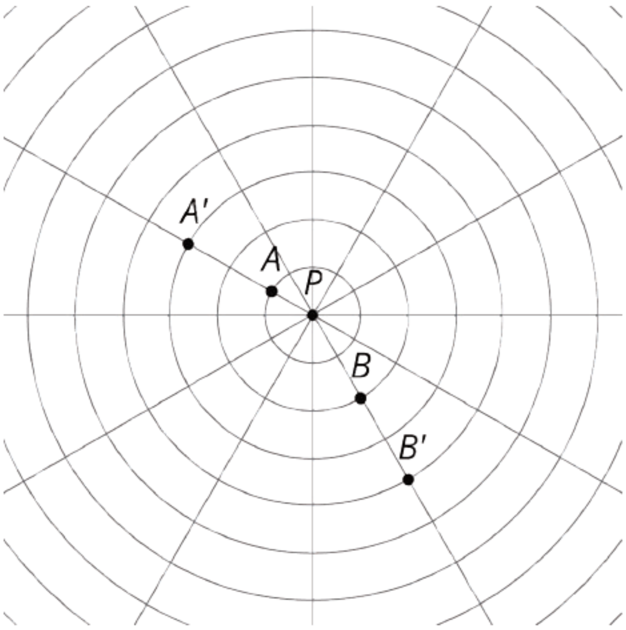
- 8.G.A

Student Task Statement



1. Dilate A using P as the center of dilation and a scale factor of 3. Label the new point A'.
2. Dilate B using P as the center of dilation and a scale factor of 2. Label the new point B'.

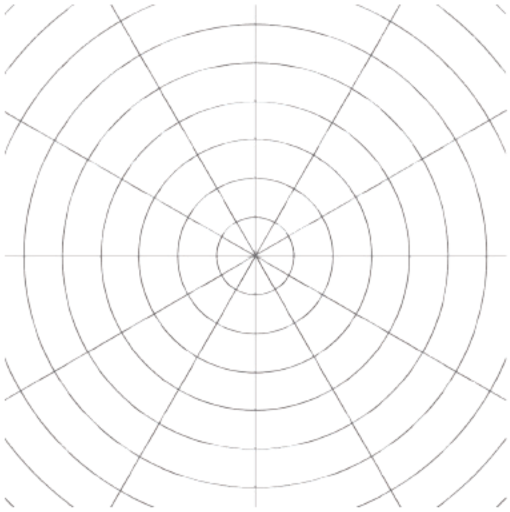
Student Response



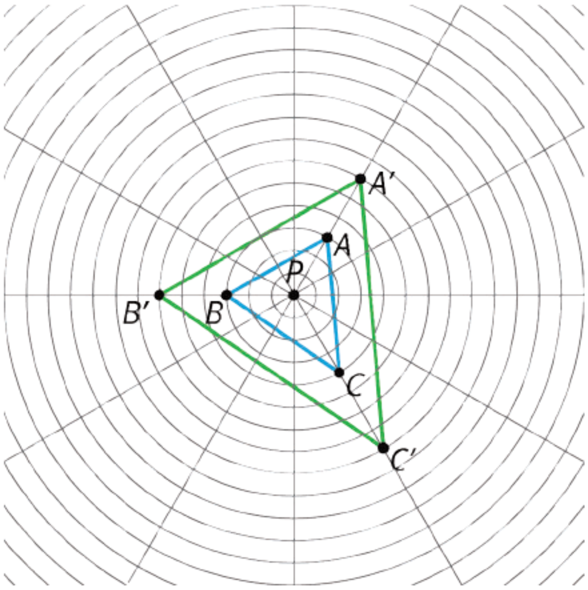
Student Lesson Summary

A circular grid like this one can be helpful for performing **dilations.**

The radius of the smallest circle is one unit, and the radius of each successive circle is one unit more than the previous one.



To perform a dilation, we need a **center of dilation**, a scale factor, and a point to dilate. In the picture, P is the center of dilation. With a scale factor of 2, each point stays on the same ray from P, but its distance from P doubles:



Since the circles on the grid are the same distance apart, segment PA' has twice the length of segment PA, and the same holds for the other points.

Glossary

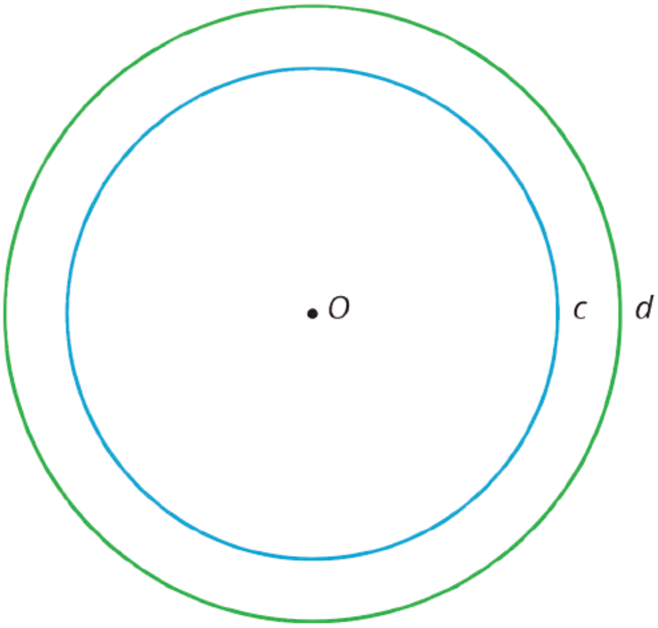
- center of a dilation
- dilation

Lesson 2 Practice Problems

Problem 1

Statement

Here are Circles c and d. Point O is the center of dilation, and the dilation takes Circle c to Circle d.



a. Plot a point on Circle c. Label the point P. Plot where P goes when the dilation is applied.

b. Plot a point on Circle d. Label the point Q. Plot a point that the dilation takes to Q.

Solution

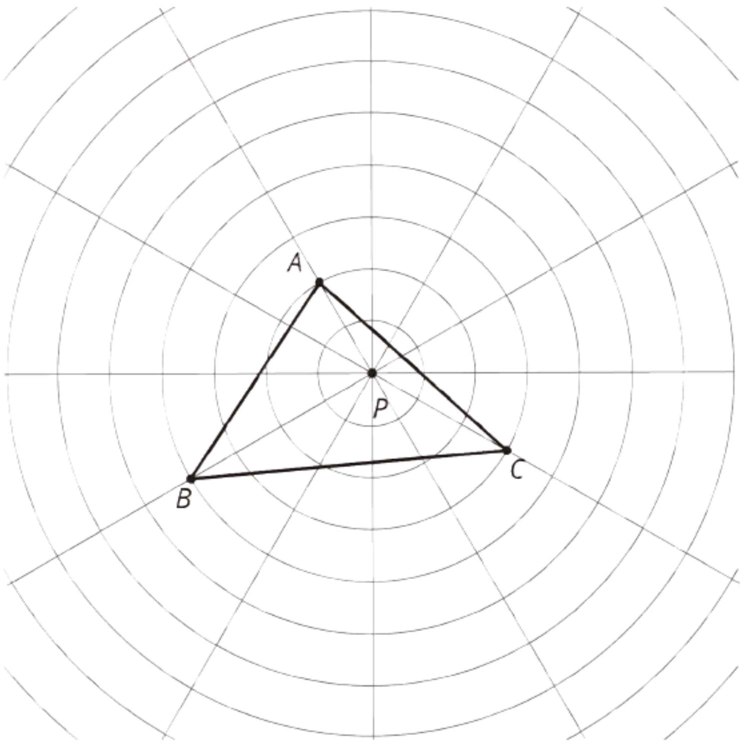
a. Plot any point P, then draw a ray from O through P. The point where this ray intersects circle d is P'.

b. Plot any point Q, then draw a ray from O through Q. The point where this ray intersects circle c is Q'.

Problem 2

Statement

Here is triangle ABC.



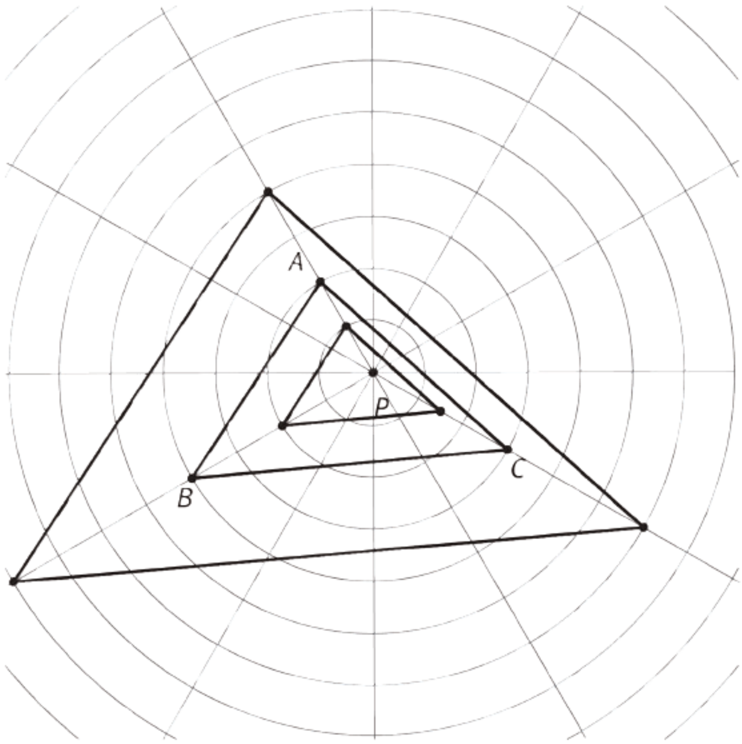
a. Dilate each vertex of triangle ABC using P as the center of dilation and a scale factor of 2. Draw the triangle connecting the three new points.

b. Dilate each vertex of triangle ABC using P as the center of dilation and a scale factor of $\frac{1}{2}$. Draw the triangle connecting the three new points.

c. Measure the longest side of each of the three triangles. What do you notice?

d. Measure the angles of each triangle. What do you notice?

Solution



a. Triangle $A'B'C'$ has each respective point at the same ray. A' is 4 units from the origin, B' is 8 units from the origin, and C' is 6 units from the origin.

b. Triangle $A''B''C''$ has each respective point at the same ray. A'' is 1 unit from the origin, B'' is 2 units from the origin, and C'' is 1.5 units from the origin.

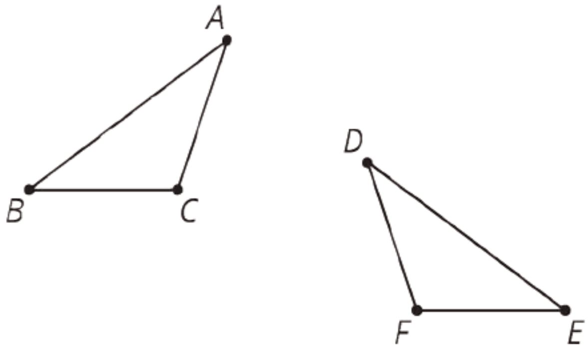
c. The longest side of the largest triangle is twice as long as the longest side of triangle ABC, which is twice as long as the smallest triangle.

d. The angles in all three triangles have the same measures.

Problem 3

Statement

Describe a rigid transformation that you could use to show the polygons are congruent.



Solution

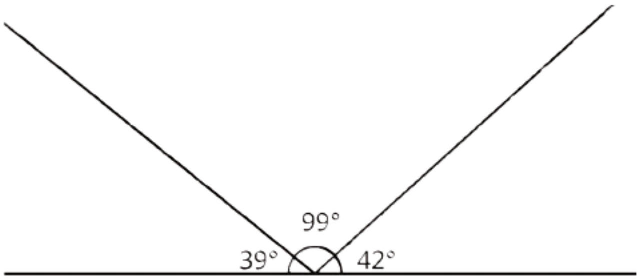
Reflect triangle ABC in a vertical line and translate so A meets D.

(From Unit 1, Lesson 12.)

Problem 4

Statement

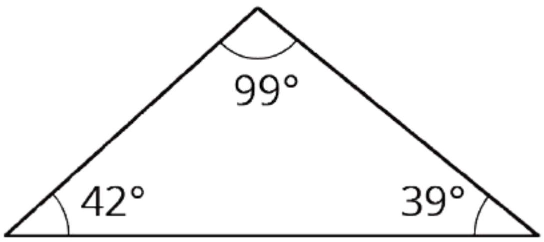
The line has been partitioned into three angles.



Is there a triangle with these three angle measures? Explain.

Solution

Yes



(From Unit 1, Lesson 15.)

Lesson 3: Dilations with no Grid

Goals

- Create a dilation of a figure given a scale factor and center of dilation.
- Explain (orally) the effect of the scale factor on the size of the image of a polygon and its distance from the center of dilation.
- Identify the center, scale factor, and image of a dilation without a circular grid.

Learning Targets

- I can apply a dilation to a polygon using a ruler.

Lesson Narrative

In the previous lesson, students applied dilations on a circular grid. The circular grid provides two levels of scaffolding:

- The radial lines give rays from the center of the grid which help find the dilated image of points on those rays.
- The circles provide a way to measure the distance of points from the center of dilation.

In this lesson, students apply dilations to points with no grid. In order to perform a dilation, three pieces of information are still needed: a center of dilation, a scale factor, and a point which is dilated. Students practice identifying centers, scale factors, and images of dilation. They also use dilations to make perspective drawings.

Performing dilations without a grid engages students in MP1 as they think about the meaning of dilation in terms of the given information (center, scale factor, point being dilated).

Alignments

Addressing

- 8.G.A: Understand congruence and similarity using physical models, transparencies, or geometry software.

Instructional Routines

- MLR7: Compare and Connect
- MLR8: Discussion Supports

Required Materials

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Required Preparation

Ensure that rulers, index cards, and colored pencils are available in the geometry toolkits.

Student Learning Goals

Let's dilate figures not on grids.

3.1 Points on a Ray

Warm Up: 5 minutes

Students apply a dilation to points on a ray. The scaffold of the circular grid has been removed but the structure of dilations is the same.

Without the grid, students will need to come up with a way to measure in order to find the point twice as far from A as B and half as far from A as B. They can use a ruler or the edge of an index card.

Monitor for these methods:

- using a ruler to measure distances
- marking off distances on an index card (for problem 1)
- folding paper in half (for problem 2)

Select students who use these methods and invite them to present.

Addressing

- 8.G.A

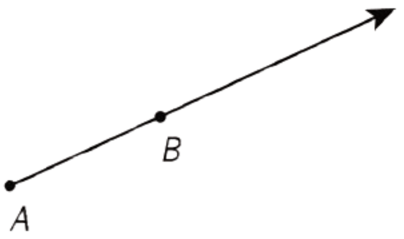
Launch

Provide access to geometry toolkits.

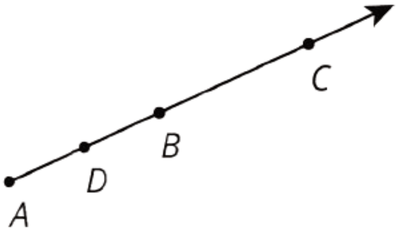
Student Task Statement

1. Find and label a point C on the ray whose distance from A is twice the distance from B to A.

2. Find and label a point D on the ray whose distance from A is half the distance from B to A.



Student Response



Activity Synthesis

Invite selected students to present their methods for finding the points which may include:

- using a ruler to measure distances
- marking off distances on an index card (for problem 1)
- folding paper in half (for problem 2)

Point out how this is similar to work with dilations on a circular grid (the points lie on the same ray at different distances) and how it is different (there are no marked distances). For the next activity, it is important for students to understand that C is the dilation of B with center A and scale factor 2. And D is the dilation of B with center A and scale factor $\frac{1}{2}$.

3.2 Dilation Obstacle Course

10 minutes (there is a digital version of this activity)

This activity investigates dilations with no grid. Students have seen these for the first time in the warm-up, which had a ray drawn between two points. That scaffold has been removed here so the teacher may need to provide guidance by suggesting that students draw appropriate rays.

Encourage students to measure distances carefully at first, since the problem statement does not state that the image of each point, after the dilation indicated, is one of the labeled points. After doing a few of the problems, the students should notice that the dilated point is always one of the labeled points and then use this observation to expedite the work. Also monitor for students who see the relationship between the scale factors used to send G to E and E to G (both with center H).

Addressing

- 8.G.A

Instructional Routines

- MLR8: Discussion Supports

Launch

Ask students to work on the first question and then pause. Demonstrate, or have a student demonstrate, drawing a ray from point A through point B (the ray goes through points H and I). Show that the length of AI is five times as long as the length of AB, either with a ruler or by marking intervals on the edge of a blank piece of paper. Or, if using the digital activity, use the measuring tool.(Click two points to measure the distance between them.)

If using the digital activity, it may be easiest for students to work with a partner, with one device used to manipulate the applet and the other device used to display the questions.

Support for Students with Disabilities

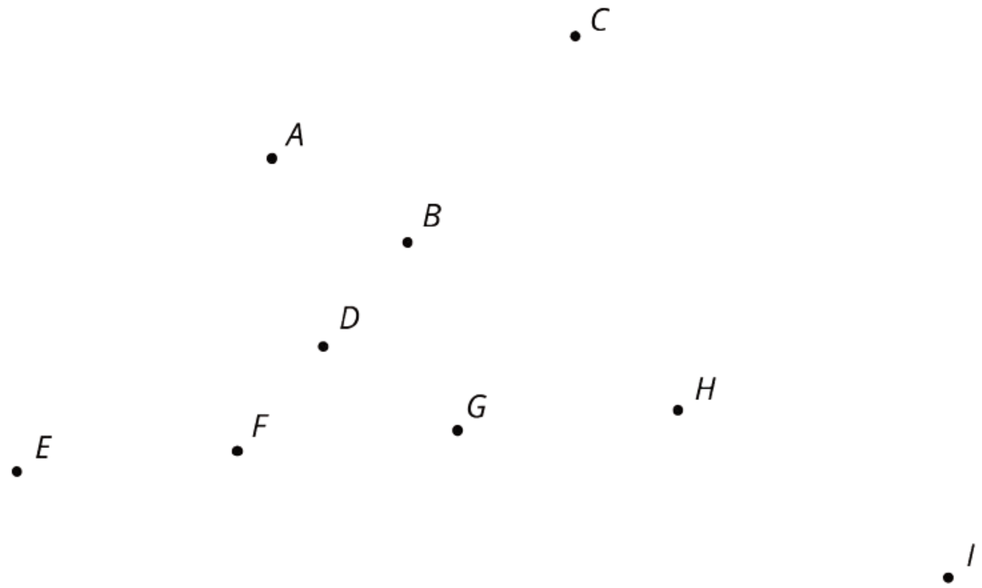
Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, allow time for students to work on the first question, then pause for a whole-class think aloud and discussion. Follow by presenting one question at a time and monitor students to ensure they are making progress throughout the activity.
Supports accessibility for: Organization; Attention

Anticipated Misconceptions

Students might need to be reminded that the image of a point under dilation must lie on the same line as the point being dilated and the center of dilation. Students might think that for a point to be a dilation of itself, the scale factor is 0. If this happens, ask them to consider multiplying the distance of the point by 0. (If they want the distance to be the same, they actually need to multiply it by 1 instead.)

Student Task Statement

Here is a diagram that shows nine points.



1. Dilate B using a scale factor of 5 and A as the center of dilation. Which point is its image?
2. Using H as the center of dilation, dilate G so that its image is E. What scale factor did you use?
3. Using H as the center of dilation, dilate E so that its image is G. What scale factor did you use?
4. To dilate F so that its image is B, what point on the diagram can you use as a center?
5. Dilate H using A as the center and a scale factor of $\frac{1}{3}$. Which point is its image?
6. Describe a dilation that uses a labeled point as its center and that would take F to H.
7. Using B as the center of dilation, dilate H so that its image is itself. What scale factor did you use?

Student Response

1. I is on the same line as B and A and segment AI is 5 times the length of segment AB.
2. 3. Segment EH is 3 times as long as segment GH.
3. $\frac{1}{3}$. Segment GH is $\frac{1}{3}$ as long as segment EH.
4. C. It needs to be on the same line as F and B, but can't be between them (if restricted to positive scale factors).
5. B. It needs to be on the same line as A and H, but closer to A than H is.
6. Use E as the center and a scale factor of 3. The center must be on the same line as F and H and not between them, so the center is E. Segment EH is 3 times as long as segment EF, so the scale factor should be 3.

7. 1. To dilate H to itself using B as the center, the distance from B to H must stay the same. That means the scale factor must be exactly 1.

Activity Synthesis

Discuss any strategies used to solve the problems. Ask selected students who noticed that the answers to all of the questions were labeled points to share their observation and how it helped them answer the questions. Next ask selected students to share their observation about the scale factors for dilating G to E and dilating E back to G. One way to reverse or "undo" a dilation is to use the same center and reciprocal scale factor.

Other important ideas to bring out include:

- The center of dilation, the point being dilated, and the image of the point after dilation must all lie on the same line.
- A scale factor of 1 does not move any points. If the scale factor is not 1, only one point does not move (the center of dilation).

Support for English Language Learners

Speaking, Listening: MLR8 Discussion Supports. As students share their strategies with the class, encourage students to use full sentences and press for details in their explanations. Ask questions such as, "How do you know this is the image of the point?", "How do you know this is the scale factor?", "How do you know this is the center of dilation?", and "Why can't that point be located somewhere else?". If necessary, rephrase "collinear" and explain that collinear means on the same line. Point out the Latin root "co," which means "together." For example, "collaborate" means to work together, and therefore "collinear" means on the same line together. This will support a rich and inclusive discussion about how to determine the scale factor, center, and image of a dilation.
Design Principle(s): Support sense-making

3.3 Getting Perspective

15 minutes (there is a digital version of this activity)
In this activity, students continue to apply dilations without a grid. Unlike in the previous activity, the dilated images of the points are not plotted. So rather than identifying the correct point, they will need to find an appropriate way to take measurements (MP5), most likely with the aid of a ruler or the edge of an index card. Different students will work with different scale factors and will produce perspective drawings of a box.

Watch for students who pick a point close to one vertex of the given rectangle. If the point is too close, it will be more difficult to visualize the box. Suggest that they move the point further away. Monitor for students who produce accurate drawings with different scale factors and invite them to share during the discussion.

Addressing

- 8.G.A

Instructional Routines

- MLR7: Compare and Connect

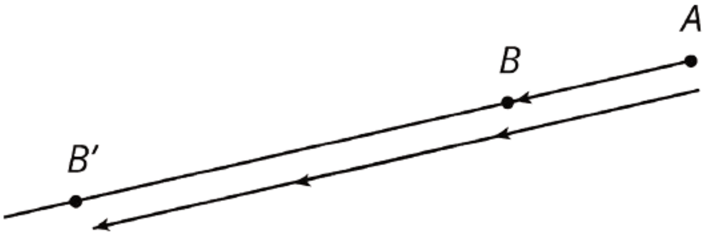
Launch

Provide access to geometry toolkits.

First, a demonstration about dilating a point on a plane with no grid.

We want to dilate point B using A as the center of dilation and a scale factor of 3.

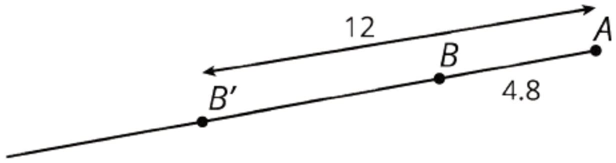
Use a straightedge to draw ray AB.



Measure the distance from B to A. Multiply the distance by 3. Draw B' so that it is 3 times as far away from A. For scale factors that are integers, an unmarked edge of an index card or a compass can also be used to transfer the distance along the ray.

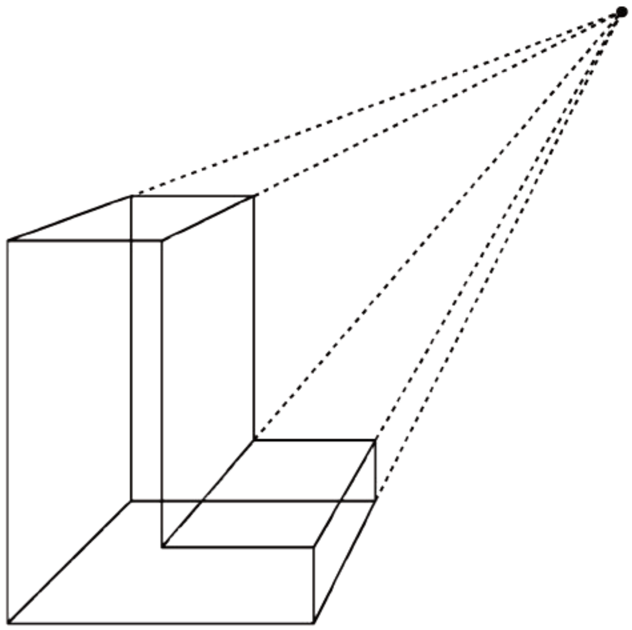
If we wanted a scale factor that is not an integer the procedure is the same. Measure the distance from A to B, multiply by the scale factor, and place B' at that new distance from A.

Let's say the distance from A to B is 4.8 cm.



If we wanted to use a scale factor of 2.5, the distance from A to the dilated point B' would be 12 cm, because $(4.8) \cdot (2.5) = 12$.

A perspective drawing is an optical illusion that makes an image printed on paper have a three-dimensional look. Display at least one example of a perspective drawing:



Students will practice some simple dilations of points, and then they will create a perspective drawing. Tell students to complete the first part of the activity dilating points P and Q. After you review their work, assign each student a scale factor to use for the second part. Appropriate scale factors include $\frac{1}{3}$, $\frac{1}{2}$, $1\frac{1}{2}$, and 2. It will work best if the center of the dilation is not too close to the rectangle the students are dilating.

Support for Students with Disabilities

Representation: Provide Access for Perception. Display or provide students with a physical copy of the Launch demonstration about dilating a point on a plane with no grid. Check for understanding by inviting students to rephrase directions for creating a dilation of points in their own words. Consider keeping the display of directions visible throughout the activity.
Supports accessibility for: Language; Memory

Anticipated Misconceptions

Students may all try to make their drawing match any example drawings shown in the launch. For example, if the center of dilation in an example is above and to the right, everyone might place their center of dilation above and to the right of the rectangle. Any point is fine as a dilation point, but the effect on what the picture looks like may vary.

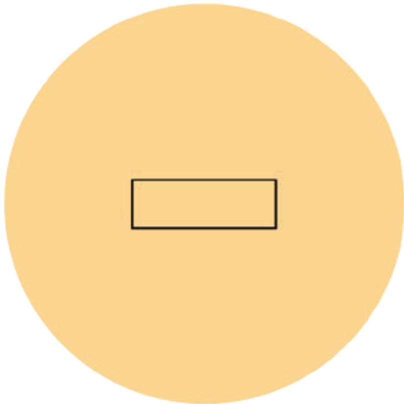
Students may not recall that to dilate a polygon, they can first dilate the vertices and then connect them in the proper order. It may be necessary to show students how to dilate one of the vertices and allow them to perform the dilation on the other three vertices.

Student Task Statement

1. Using one colored pencil, draw the images of points P and Q using C as the center of dilation and a scale factor of 4. Label the new points P' and Q'.
2. Using a different color, draw the images of points P and Q using C as the center of dilation and a scale factor of $\frac{1}{2}$. Label the new points P'' and Q''.

Pause here so your teacher can review your diagram. Your teacher will then give you a scale factor to use in the next part.

3. Now you'll make a perspective drawing. Here is a rectangle.



a. Choose a point *inside the shaded circular region* but *outside the rectangle* to use as the center of dilation. Label it C.

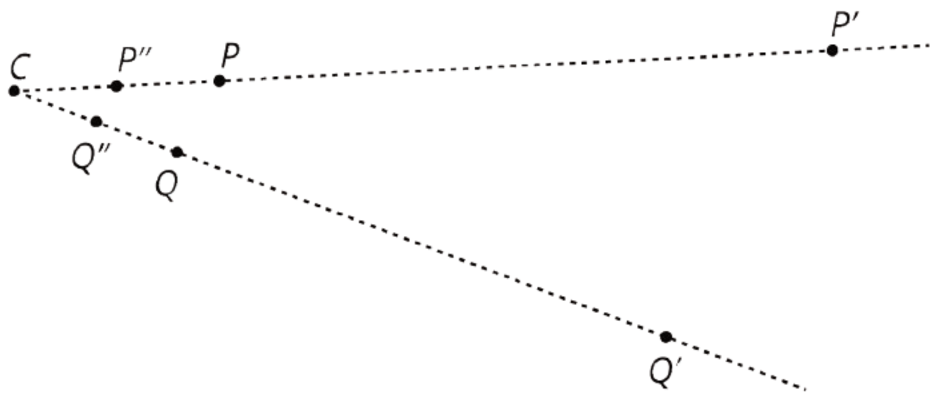
b. Using your center C and the scale factor you were given, draw the image under the dilation of each vertex of the rectangle, one at a time. Connect the dilated vertices to create the dilated rectangle.

c. Draw a segment that connects each of the original vertices with its image. This will make your diagram look like a cool three-dimensional drawing of a box! If there's time, you can shade the sides of the box to make it look more realistic.

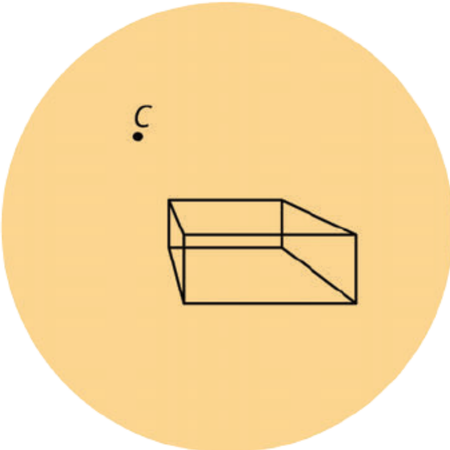
d. Compare your drawing to other people's drawings. What is the same and what is different? How do the choices you made affect the final drawing? Was your dilated rectangle closer to C than to the original rectangle, or farther away? How is that decided?

Student Response

1,2

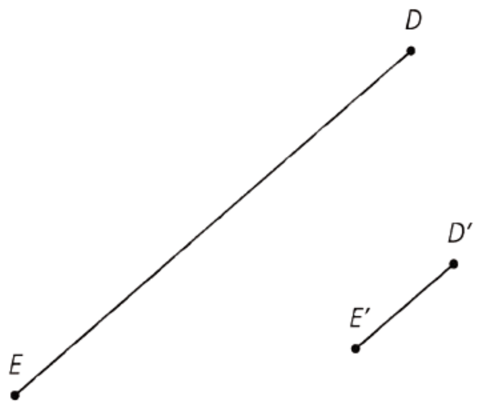


1. Answers vary. Possible response (scale factor 1.5):



Are You Ready for More?

Here is line segment DE and its image $D'E'$ under a dilation.



1. Use a ruler to find and draw the center of dilation. Label it F.
2. What is the scale factor of the dilation?

Student Response

1. Draw ray DD' and ray EE'. F is their intersection.
2. $\frac{1}{4}$

Activity Synthesis

Display the work of several students selected based on the different scale factors. Then ask students:

- “What are the effects of using a scale factor greater than 1?” (The image is larger than the original *and* farther away from the center of dilation than the original.)
- “What are the effects of using a scale factor less than 1?” (The image is smaller than the original *and* closer to the center of dilation than the original.)
- “What effect does the location of C, the center of dilation, have?” (It impacts the size and location of the dilated rectangle: if the scale factor is less than 1 then the dilated rectangle is closer to C than the original and if the scale factor is larger than 1 then the dilated rectangle is further away from C than the original.)

Time permitting, consider showing several student drawings with the same scale factor but a different location for the point C. How are they the same? How are they different? Two faces of these boxes (the original rectangle and the scaled copy) are congruent but the point of view or perspective on them is different.

Support for English Language Learners

Speaking, Listening: MLR7 Compare and Connect. As students prepare their perspective drawings, identify the drawings with scale factors greater than 1 or less than 1. As students investigate each other’s work, ask them to share what is similar about the drawings with scale factor greater than 1 (or less than 1). Listen for and amplify statements such as “a scale factor greater than 1 results in an image larger than the original” and “a scale factor less than 1 results in an image smaller than the original.” Then encourage students to make connections between the value of the scale factor and the effect on the image. Listen for and amplify language students use to describe how the scale factor affects the size of the image and its distance from the center of dilation. This will foster students’ meta-awareness and support constructive conversations as they compare perspective drawings and make connections between the value of the scale factor and the image of the original polygon.
Design Principles(s): Cultivate conversation; Maximize meta-awareness

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Lesson Synthesis

Ask students to think about how they would explain the steps for dilating a point, and either write them down or share them with a partner. Ask a few students to share their steps. Ensure that all of the important aspects are mentioned:

- “You need to know which point you want to dilate, which point is the center of dilation, and what scale factor to use.”
- “Use a straightedge to draw a ray from the center of dilation through the point you want to dilate.”

- "Measure the distance from the center of dilation through the point. Multiply this distance by the scale factor. Place the new point at this distance from the center of dilation and also on the ray you drew."
- "If the scale factor is greater than 1, the new point will be farther from the center than the original point. If the scale factor is less than 1, the new point will be closer to the center than the original point."

3.4 A Single Dilation of a Triangle

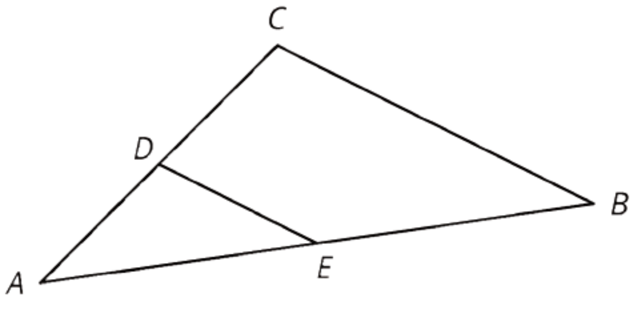
Cool Down: 5 minutes
Students identify the center of a dilation given a figure, its dilation, and the scale factor.

Addressing

- 8.G.A

Student Task Statement

Lin drew a triangle and a dilation of the triangle with scale factor $\frac{1}{2}$:



1. What is the center of the dilation? Explain how you know.
2. Which triangle is the original and which triangle is the dilation? Explain how you know.

Student Response

1. The center of dilation is A. Lines emanate from A and points lie along those lines.
2. Triangle ABC is the original and triangle AED is the dilation. Since the scale factor is less than 1, the dilation is smaller than the original figure.

Student Lesson Summary

If A is the center of dilation, how can we find which point is the dilation of B with scale factor 2?

Since the scale factor is larger than 1, the point must be farther away from A than B is, which makes C the point we are looking for. If we measure the distance between A and C, we would find that it is exactly twice the distance between A and B.

A dilation with scale factor less than 1 brings points closer. The point D is the dilation of B with center A and scale factor $\frac{1}{3}$.

Lesson 3 Practice Problems

Problem 1

Statement

Segment AB measures 3 cm. Point O is the center of dilation. How long is the image of AB after a dilation with . . .

a. Scale factor 5?

b. Scale factor 3.7?

c. Scale factor $\frac{1}{5}$?

d. Scale factor s?

Solution

a. 15 cm

b. 11.1 cm

c. $\frac{3}{5}$ cm

d. $3s$ cm

Problem 2

Statement

Here are points A and B. Plot the points for each dilation described.

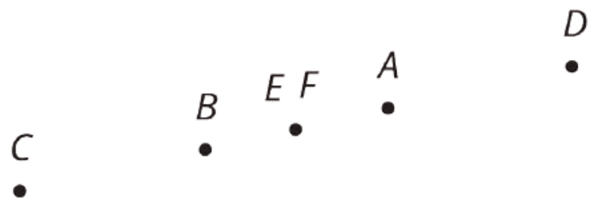
a. C is the image of B using A as the center of dilation and a scale factor of 2.

b. D is the image of A using B as the center of dilation and a scale factor of 2.

c. E is the image of B using A as the center of dilation and a scale factor of $\frac{1}{2}$.

d. F is the image of A using B as the center of dilation and a scale factor of $\frac{1}{2}$.

Solution



Problem 3

Statement

Make a perspective drawing. Include in your work the center of dilation, the shape you dilate, and the scale factor you use.

Solution

Answers vary.

Problem 4

Statement

Triangle ABC is a scaled copy of triangle DEF. Side AB measures 12 cm and is the longest side of ABC. Side DE measures 8 cm and is the longest side of DEF.

a. Triangle ABC is a scaled copy of triangle DEF with what scale factor?

b. Triangle DEF is a scaled copy of triangle ABC with what scale factor?

Solution

a. $\frac{3}{2}$

b. $\frac{2}{3}$

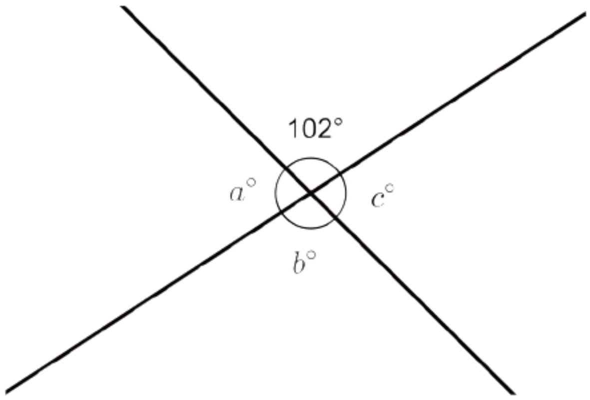
(From Unit 2, Lesson 1.)

Problem 5

Statement

The diagram shows two intersecting lines.

Find the missing angle measures.



Solution

$a = 78$

$b = 102$

$c = 78$

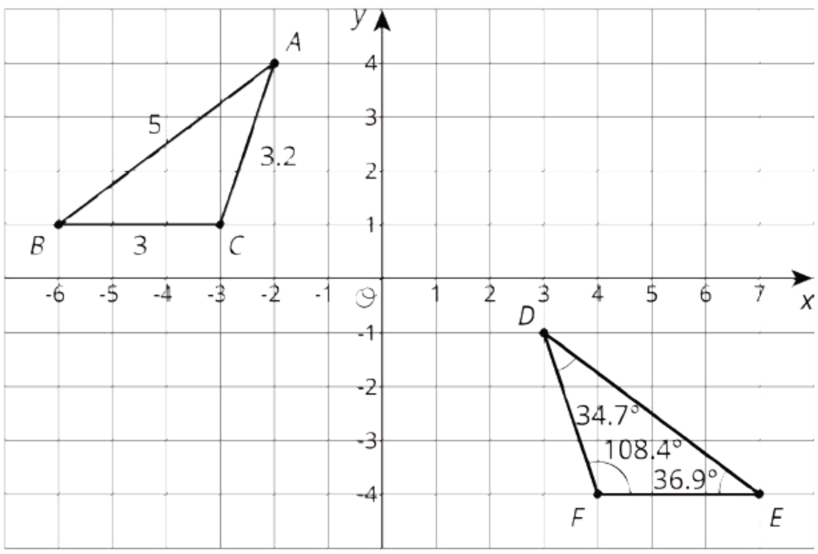
(From Unit 1, Lesson 14.)

Problem 6

Statement

a. Show that the two triangles are congruent.

b. Find the side lengths of DEF and the angle measures of ABC.



Solution

a. Reflect in the y-axis and translate until A meets D..

b. Angle ABC is 36.9 degrees. Angle BCA is 108.4 degrees. Angle CAB is 34.7 degrees. $DE = 5$. $EF = 3$. $FD = 3.2$.

(From Unit 1, Lesson 12.)

Lesson 4: Dilations on a Square Grid

Goals

- Create a dilation of a polygon on a square grid given a scale factor and center of dilation.
- Identify the image of a figure on a coordinate grid given a scale factor and center of dilation.

Learning Targets

- I can apply dilations to figures on a square grid.
- If I know the angle measures and side lengths of a polygon, I know the angles measures and side lengths of the polygon if I apply a dilation with a certain scale factor.

Lesson Narrative

In this lesson, students apply dilations to polygons on a grid, both with and without coordinates. The grid offers a way of measuring distances between points, especially points that lie at the intersection of grid lines. If point Q is three grid squares to the right and two grid squares up from P then the dilation with center P of Q with scale factor 4 can be found by counting grid squares: it will be twelve grid squares to the right of P and eight grid squares up from P. The coordinate grid gives a more concise way to describe this dilation. If the center P is $(0,0)$ then Q has coordinates $(3,2)$. The image of Q after this dilation is $(12,8)$.

Students continue to find dilations of polygons, providing additional evidence that dilations map line segments to line segments and hence polygons to polygons. The scale factor of the dilation determines the factor by which the length of those segments increases or decreases. Using coordinates to describe points in the plane helps students develop language for precisely communicating figures in the plane and their images under dilations (MP6). Strategically using coordinates to perform and describe dilations is also a good example of MP7.

Alignments

Addressing

- 8.G.A: Understand congruence and similarity using physical models, transparencies, or geometry software.
- 8.G.A.3: Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Pre-printed slips, cut from copies of the blackline master

Required Preparation

Print and cut one copy of the blackline master for each student.

Student Learning Goals

Let’s dilate figures on a square grid.

4.1 Estimating a Scale Factor

Warm Up: 5 minutes
In this warm-up, students estimate a scale factor based on a picture showing the center of the dilation, a point, and its image under the dilation.

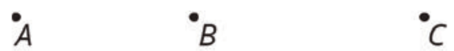
Addressing

- 8.G.A

Launch

Tell students they will estimate the scale factor for a dilation. Clarify that “estimate” doesn’t mean “guess.” Encourage students to use any tools available to make a precise estimate. Provide access to geometry toolkits.

Student Task Statement



Point C is the dilation of point B with center of dilation A and scale factor s. Estimate s. Be prepared to explain your reasoning.

Student Response

Answers vary. Sample response: about 2.3.

Activity Synthesis

Check with students to find what methods they used to compare distances. Likely methods include using a ruler and division or using an index card and marking off multiples of the distance from A to B.

Ask students:

- "Is the scale factor greater than 1?" (Yes.) "How do you know?" (the point C is further from A than B)
- "Is the scale factor greater than 2?" (Yes.) "How do you know?" (the distance from C to A is more than twice the distance from B to A)
- "Is the scale factor greater than 3?" (No.) "How do you know?" (The distance from C to A is less than 3 times the distance from B to A.)
- "Is the scale factor greater or less than 2.5?" (It is less.) "How do you know?" (The distance from C to A is less than 2.5 times the distance from B to A.)

4.2 Dilations on a Grid

10 minutes

In previous lessons, students perform dilations on a circular grid and with no grid. In this activity, they perform dilations on a square grid. A square grid is particularly helpful if the center of dilation and the points being dilated are grid points. When the extra structure of coordinates is added, as in the next activity, the grid provides an extremely convenient tool for naming points and describing the effects of dilations using coordinates. As in previous lessons, students will again see that scale factors greater than 1 produce larger copies while scale factors less than 1 produce smaller copies.

Monitor for how students find the dilated points and the language they use to describe the process. In particular:

- using a ruler or index card to measure distances along the rays emanating from the center of dilation
- taking advantage of the grid and counting how many squares to the left or right, up or down

Addressing

- 8.G.A

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

Launch

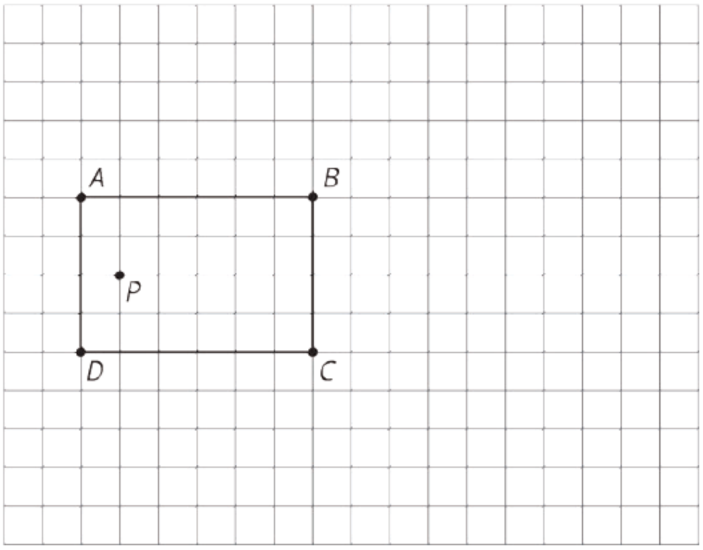
Provide access to geometry toolkits.

Support for Students with Disabilities

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. For example, reference examples from the previous lessons on methods for dilating points on a circular grid and on no grid to provide an entry point into this activity.
Supports accessibility for: Social-emotional skills; Conceptual processing

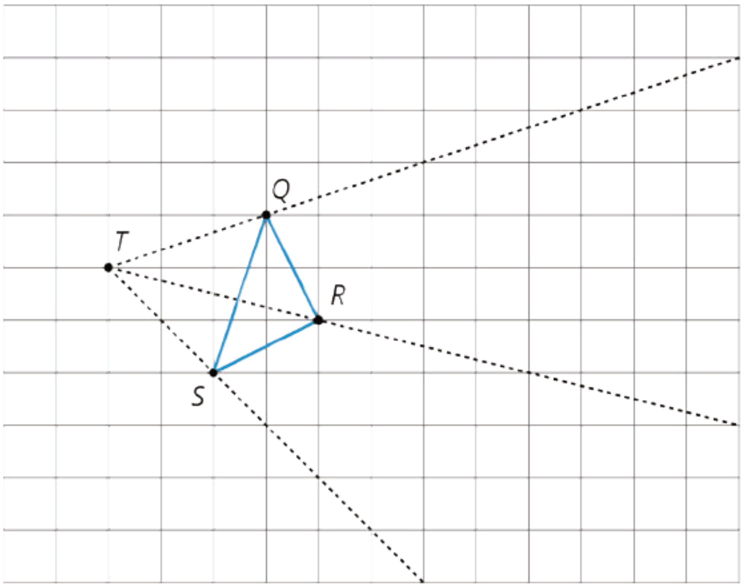
Student Task Statement

1. Find the dilation of quadrilateral $ABCD$ with center P and scale factor 2.

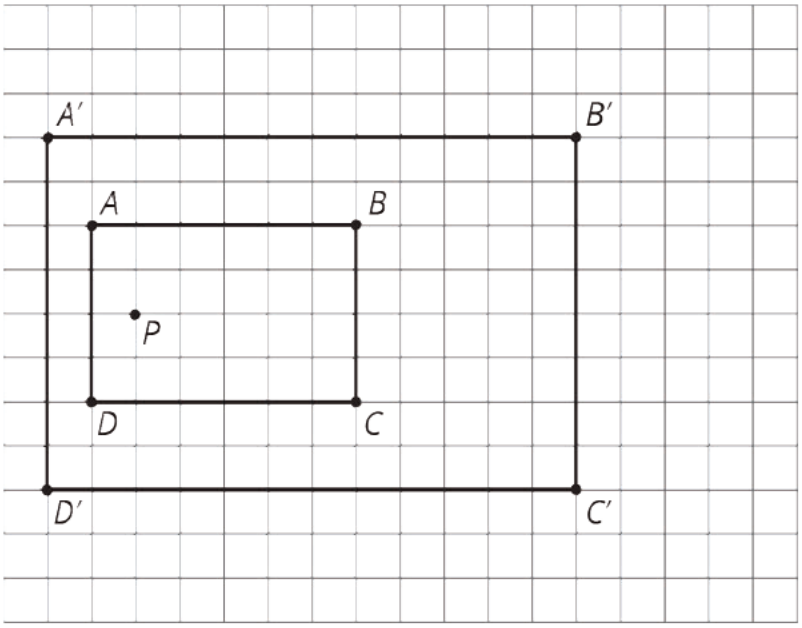


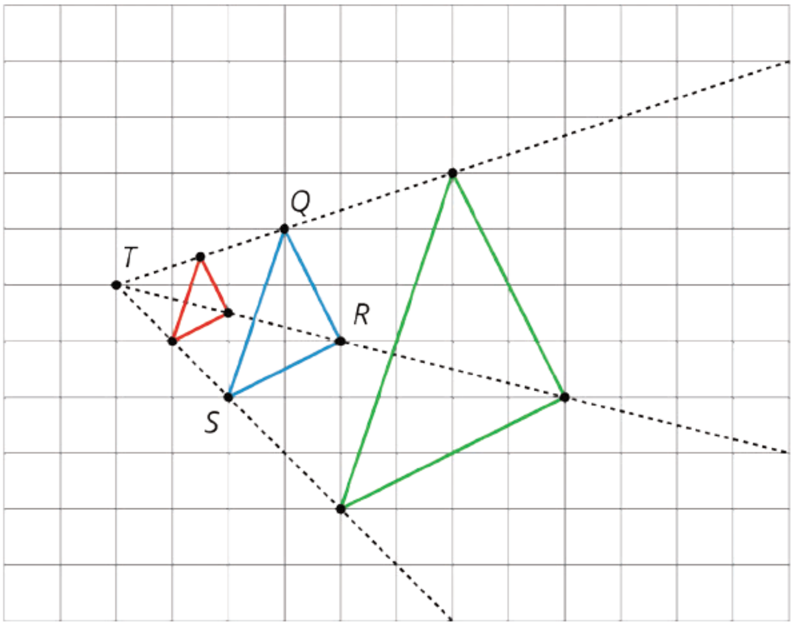
2. Find the dilation of triangle QRS with center T and scale factor 2.

3. Find the dilation of triangle QRS with center T and scale factor $\frac{1}{2}$.



Student Response





1.2-3.

Activity Synthesis

Select students to show how they found the dilations. First, select any students who used the same methods as when there was no grid, followed by students who noticed they could use the structure of the grid. Draw connections between these two methods—show that when you measure with a ruler or by making markings on an index card, the dilated point ends up in the same place as by reasoning about the grid.

Expect students to use expressions like "moving over two and up one." These measurements can be multiplied by the scale factor in order to find the location of the dilated point.

Tell students that moving forward they will do work on the grid with the added structure of coordinates. The method of performing dilations is the same. The only change is that the coordinates give a concise way to *name* points.

4.3 Card Sort: Matching Dilations on a Coordinate Grid

15 minutes
In the previous task, students worked on a square grid without coordinates. This activity adds the structure of coordinates and this extra structure plays a key role, allowing students to name points. Students match figures with their dilated images, using coordinates to describe the center of dilation and the vertices. The same strategies that were used previously in dilating images,on a circular grid and with no grid, will be useful here.

Monitor for students who identify that the dilation of a circle is a circle and similarly for triangles and quadrilaterals. This will help them eliminate certain possibilities for each match. Because there is one card that does not match, students should verify the other matches by performing the dilations. Once the card without a match has been identified, reasoning based on eliminating possibilities (without performing the dilations) is correct. Monitor for students who systematically perform the dilations to help identify a match versus those who reason by structure and elimination of possibilities. Invite both to share during the discussion.

Addressing

- 8.G.A.3

Instructional Routines

- MLR2: Collect and Display

Launch

Students practice matching an original figure and dilation description to information about the dilated images using the coordinate plane. Distribute one set (numbers 1 through 6 and letters A through F) of cards to each student.

There is one extra option that does not have a match. Students should draw the dilated image for that option themselves.

Support for Students with Disabilities

Representation: Internalize Comprehension. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. For example, give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.
Supports accessibility for: Conceptual processing; Organization

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Support for English Language Learners

Conversing, Reading: MLR2 Collect and Display. As students work in pairs on the task, circulate and listen to pairs as they decide whether two cards match. Write down the words and phrases students use to justify why an original figure card matches with a dilated figure card. As students review the language collected in the visual display, encourage students to clarify the meaning of a word or phrase. For example, a phrase such as "Card 1 matches with Card C because they are both trapezoids" can be clarified by asking students to explain why Card 1 does not match with Card A even though both are trapezoids. Listen for students who state that the scale factor and center of dilation must also be considered when matching the cards. Write down the language students use to describe how the scale factor and center of dilation affect the dilated figure. This routine will provide feedback to students in a way that supports sense-making while simultaneously increasing meta-awareness of language.
Design Principle(s): Support sense-making; Maximize meta-awareness

Anticipated Misconceptions

If students are having trouble finding accurate matches, suggest that they identify the center of dilation and consider if the dilation will result in a smaller or larger sized image.

Student Task Statement

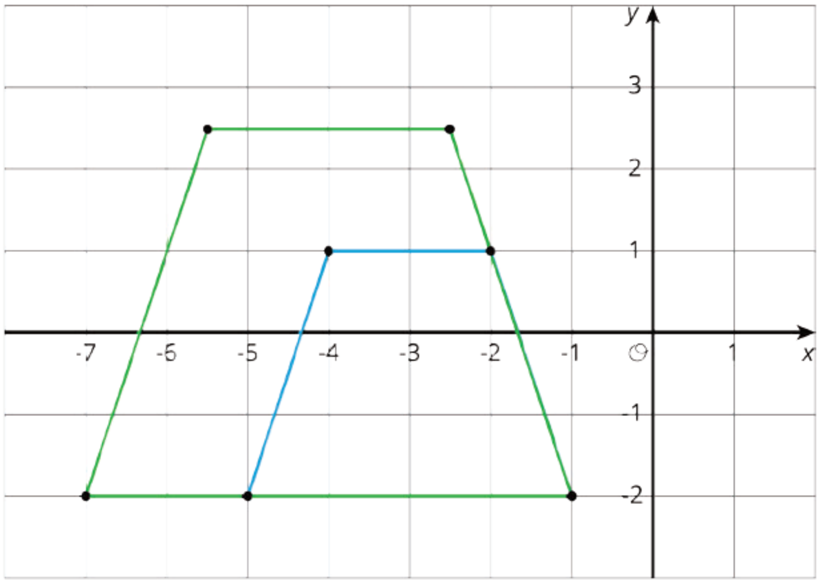
Your teacher will give you some cards. Each of Cards 1 through 6 shows a figure in the coordinate plane and describes a dilation.

Each of Cards A through E describes the image of the dilation for one of the numbered cards.

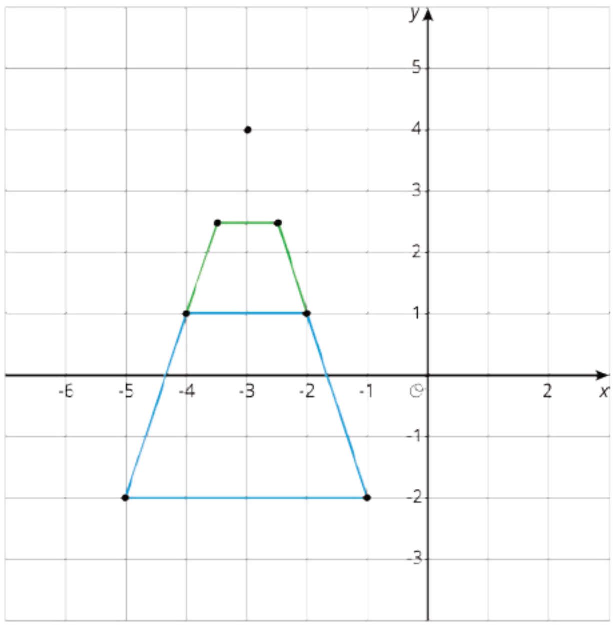
Match number cards with letter cards. One of the number cards will not have a match. For this card, you'll need to draw an image.

Student Response

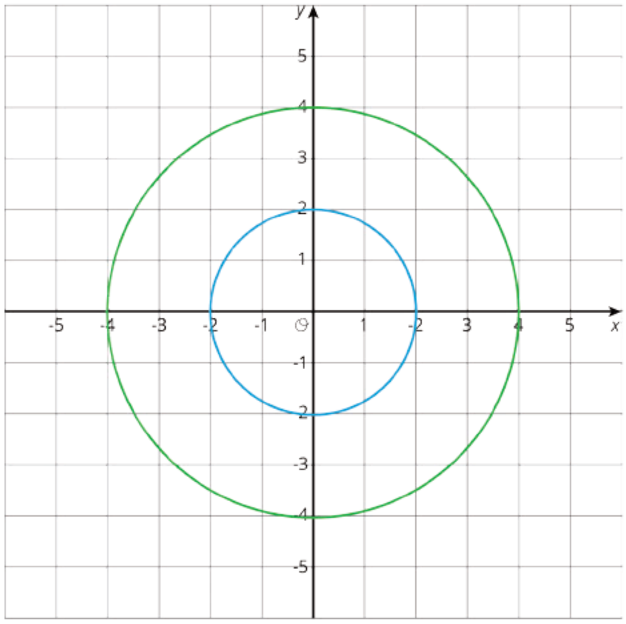
1. C. Answers vary. Sample response: The center of dilation is the point B, so the dilation also contains point B, suggesting this card. The scale factor of $\frac{3}{2}$ works for the two trapezoids which are plotted together.



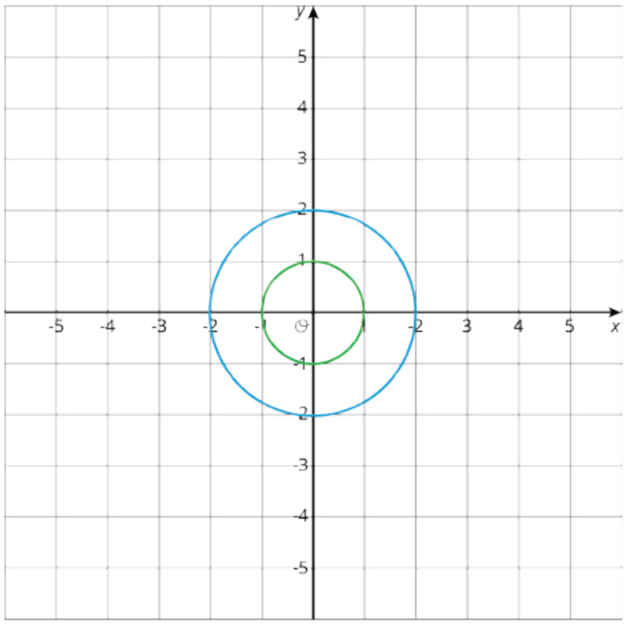
2. A. Answers vary. Sample response: The scale factor was less than one, so the dilation will be closer to the center of dilation. Card A is plotted and shows the dilation since each vertex on the green trapezoid is the midpoint between the center of dilation and the corresponding vertex on the blue trapezoid.



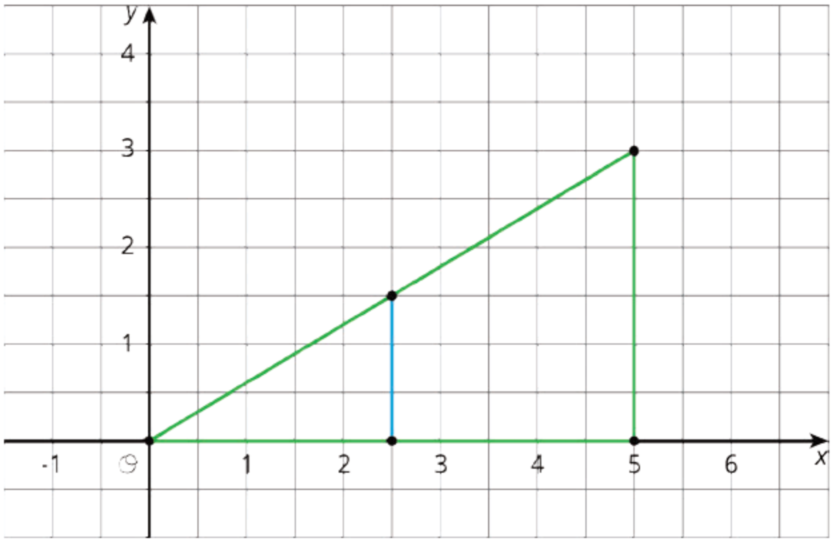
3. B. Answers vary. Sample response: The dilation scale factor was greater than one, so the dilated image will be a larger circle. The image is correct as both circles have the same center and the radius of the green circle is twice the radius of the blue circle.



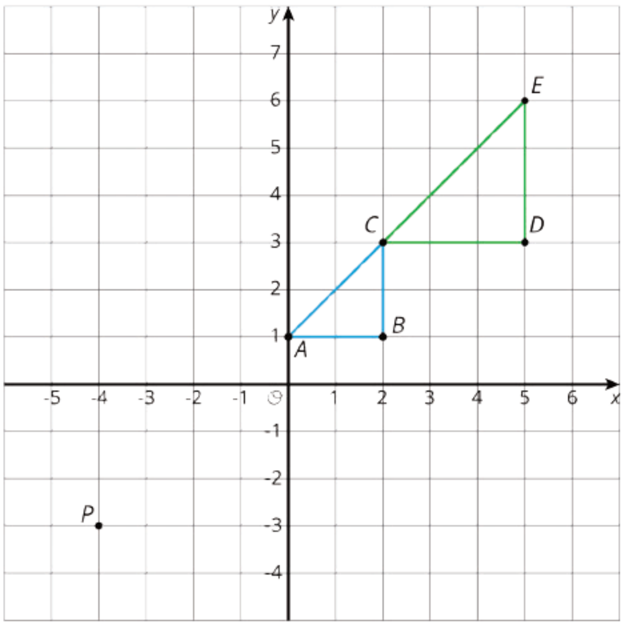
4. E. Answers vary. Sample response: This scale factor is less than one, so the image of the dilation is a circle that is smaller than the original one. The image is correct because the circles have the same center and the radius of the green circle is half the radius of the blue circle.



5. F. The center of dilation is (0,0) so the dilated image is a triangle containing (0,0). This does not match any of the lettered cards.

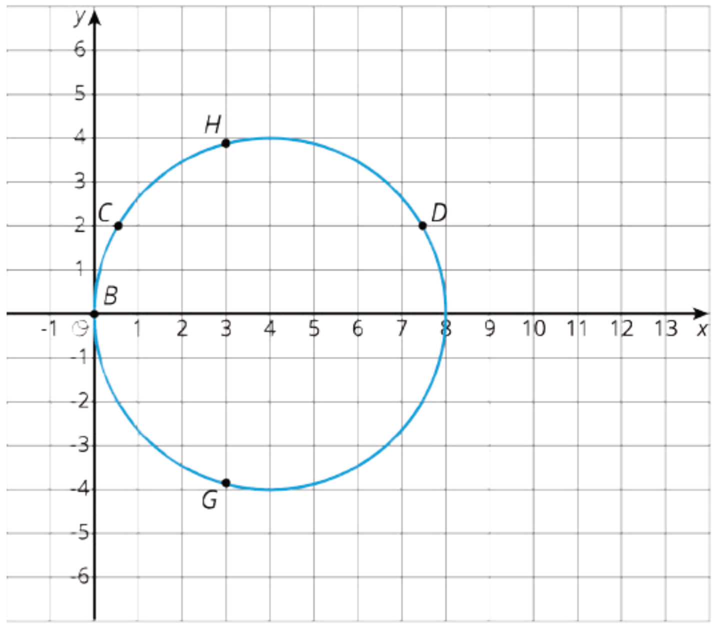


6. D. Answers vary. Sample response: the dilation of the triangle will be a triangle and it will be larger than $\triangle ABC$ since the scale factor is larger than 1. This suggests card D. The two are plotted together and $\triangle CDE$ is the dilation of $\triangle ABC$ with center P and scale factor $\frac{3}{2}$.



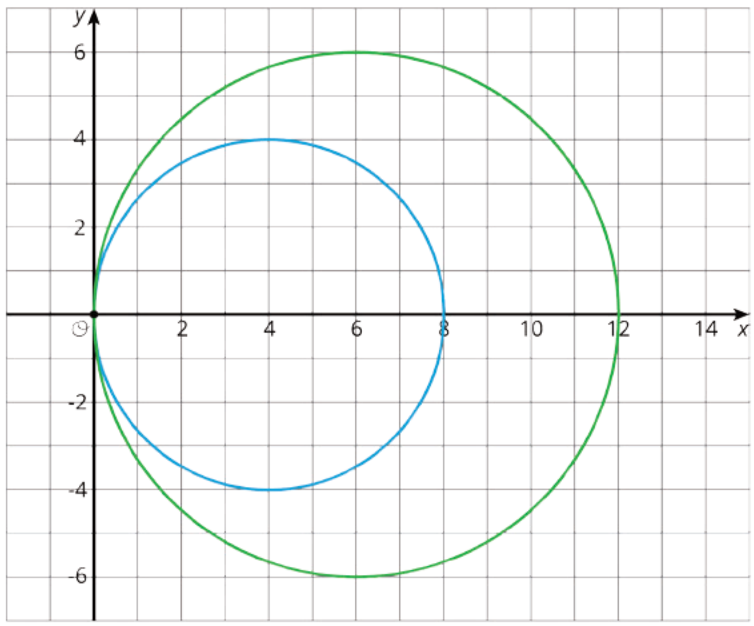
Are You Ready for More?

The image of a circle under dilation is a circle when the center of the dilation is the center of the circle. What happens if the center of dilation is a point on the circle? Using center of dilation $(0, 0)$ and scale factor 1.5, dilate the circle shown on the diagram. This diagram shows some points to try dilating.



Student Response

Original has center (4,0) and radius 4. Image has center (6,0) and radius 6.



Activity Synthesis

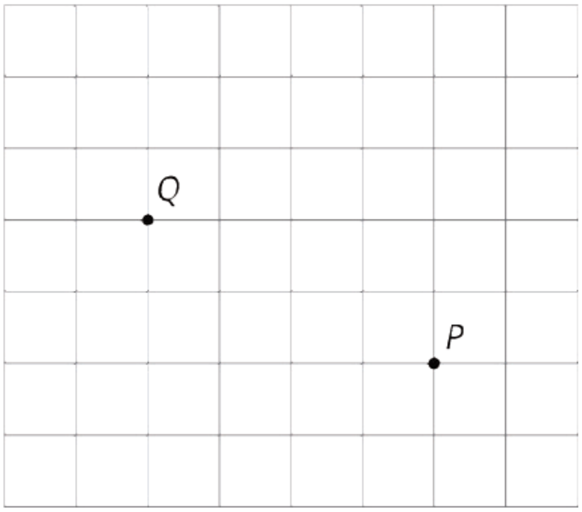
Share the correct answers and invite selected students to share the strategies they used to solve the problems. This is a matching problem, so students may not have dilated the entire image to find the correct answer among the choices. Important points to bring out include:

- A dilation maps a circle to a circle, a quadrilateral to a quadrilateral, and a triangle to a triangle.

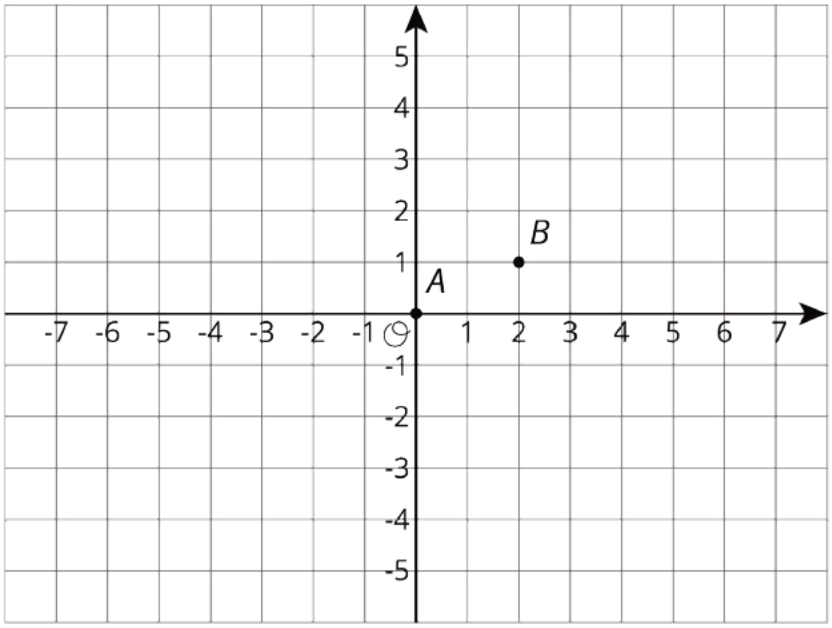
- If the center of dilation for a polygon is one of the vertices, then that vertex is on the dilated polygon.
- If the scale factor is less than 1 then the dilated image is smaller than the original figure.
- If the scale factor is larger than 1 then the dilated image is larger than the original figure.

Lesson Synthesis

- "How do we perform dilations on a square grid?"
- "How do coordinates help describe and perform dilations?"



Just like the circular grid, a square grid is useful for performing dilations. The grid lines can be used as a way to measure distance and direction between points. How can you dilate Q with center P and scale factor $\frac{1}{2}$?The image of Q will be half as many grid lines to the left and half has many grid lines up—that is, 2 grid lines to the left and 1 grid line up from P.



When the grid has coordinates, it is easier to communicate the location of new points. In the figure, we have $A = (0, 0)$ and $B = (2, 1)$.What is the dilation of B with center A and scale factor 3? To communicate the answer, we can just say $(6, 3)$. It is three times as far to the right and 3 times as far up from A as B so it is the desired point.

4.4 A Dilated Image

Cool Down: 5 minutes

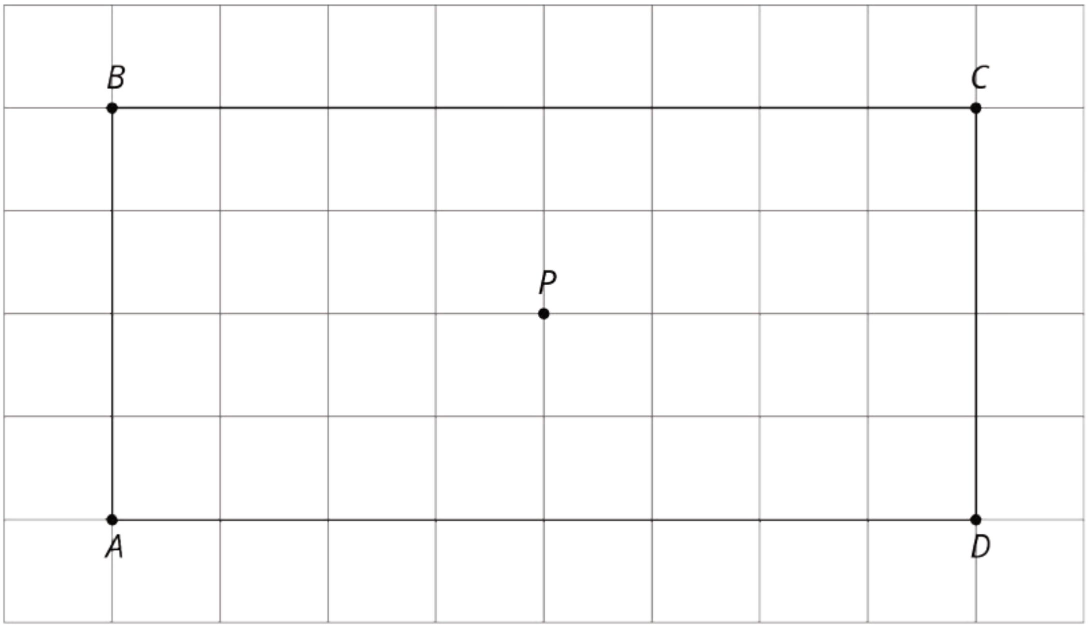
Students apply a dilation to a polygon where the center of dilation is on the interior of the figure. The polygon is on a grid without coordinates and the structure of the grid can be efficiently used to find the dilation.

Addressing

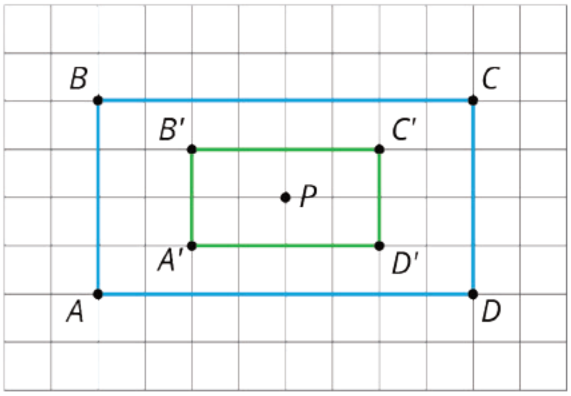
- 8.G.A

Student Task Statement

Draw the image of rectangle $ABCD$ under dilation using center P and scale factor $\frac{1}{2}$.



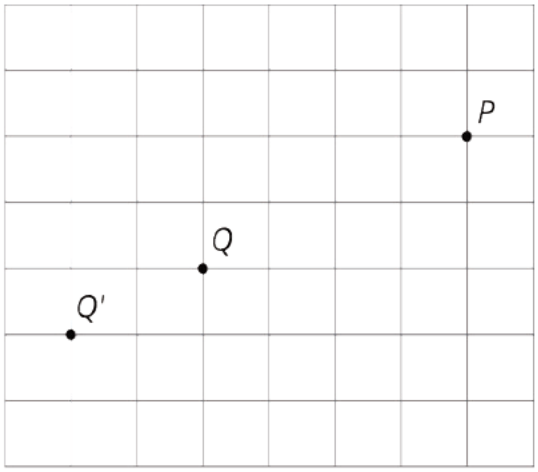
Student Response



Student Lesson Summary

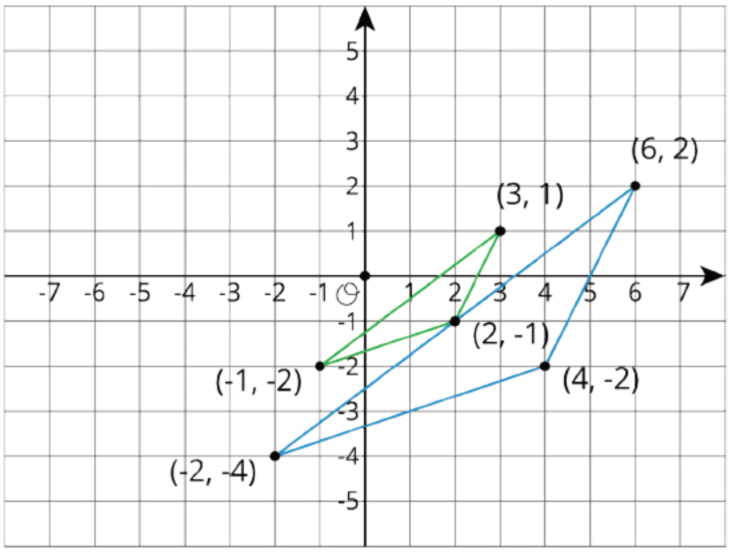
Square grids can be useful for showing dilations. The grid is helpful especially when the center of dilation and the point(s) being dilated lie at grid points. Rather than using a ruler to measure the distance between the points, we can count grid units.

For example, suppose we want to dilate point Q with center of dilation P and scale factor $\frac{3}{2}$. Since Q is 4 grid squares to the left and 2 grid squares down from P, the dilation will be 6 grid squares to the left and 3 grid squares down from P (can you see why?). The dilated image is marked as Q' in the picture.



Sometimes the square grid comes with coordinates. The coordinate grid gives us a convenient way to *name* points, and sometimes the coordinates of the image can be found with just arithmetic.

For example, to make a dilation with center $(0, 0)$ and scale factor 2 of the triangle with coordinates $(-1, -2)$, $(3, 1)$, and $(2, -1)$, we can just double the coordinates to get $(-2, -4)$, $(6, 2)$, and $(4, -2)$.



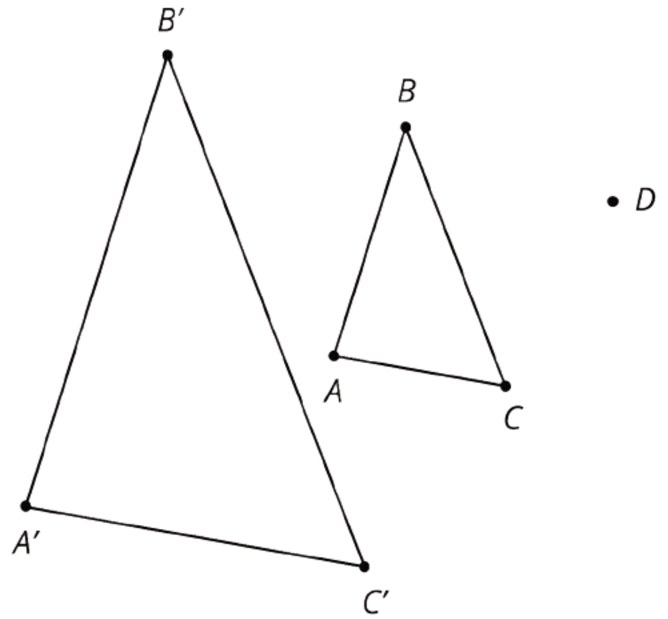
Lesson 4 Practice Problems

Problem 1

Statement

Triangle ABC is dilated using D as the center of dilation with scale factor 2.

The image is triangle $A'B'C'$. Clare says the two triangles are congruent, because their angle measures are the same. Do you agree? Explain how you know.



Solution

No. The triangles are not congruent because their side lengths are different.

Problem 2

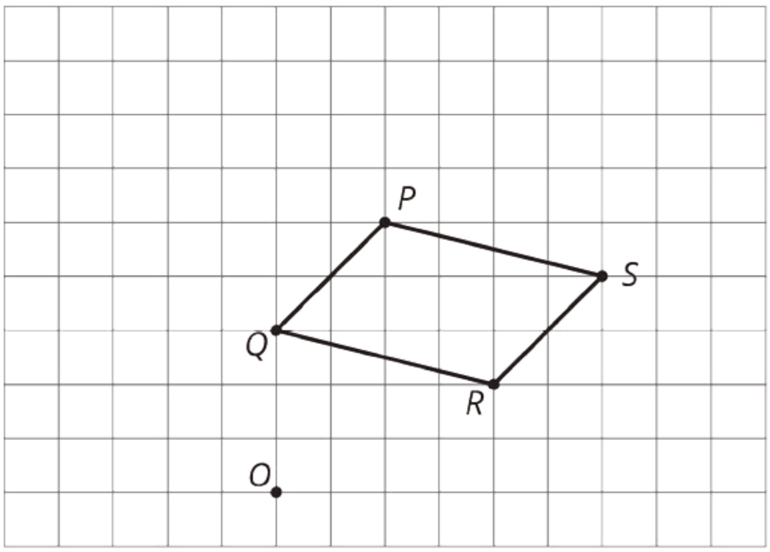
Statement

On graph paper, sketch the image of quadrilateral PQRS under the following dilations:

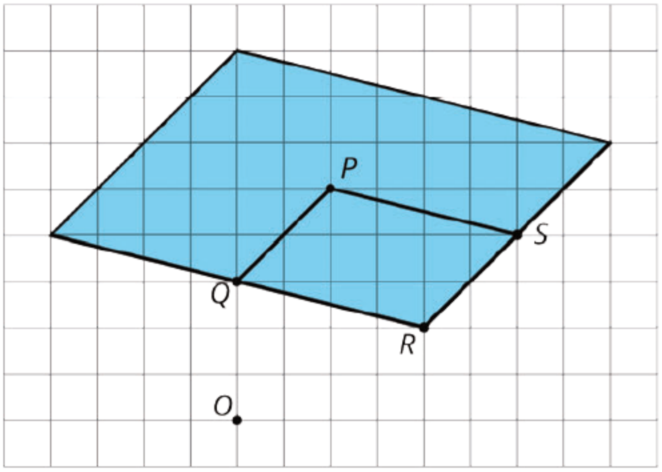
a. The dilation centered at R with scale factor 2.

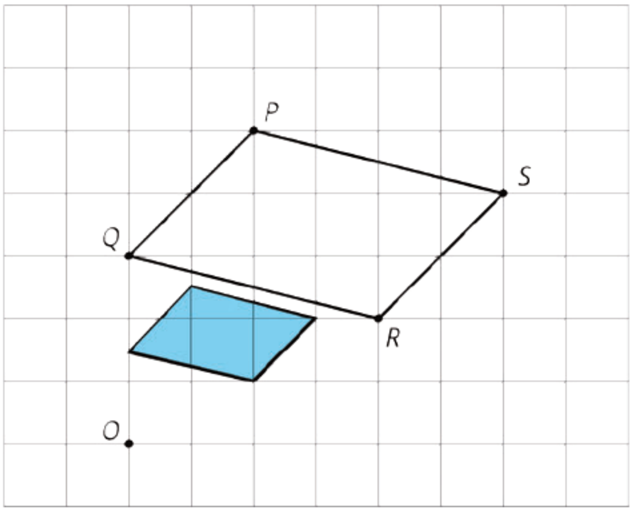
b. The dilation centered at O with scale factor $\frac{1}{2}$.

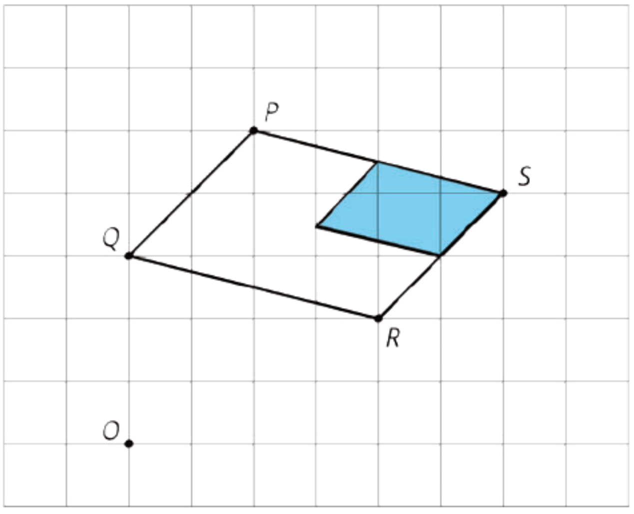
c. The dilation centered at S with scale factor $\frac{1}{2}$.



Solution



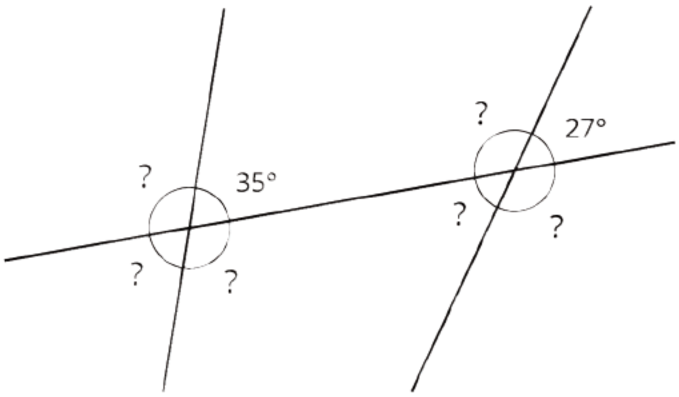




Problem 3

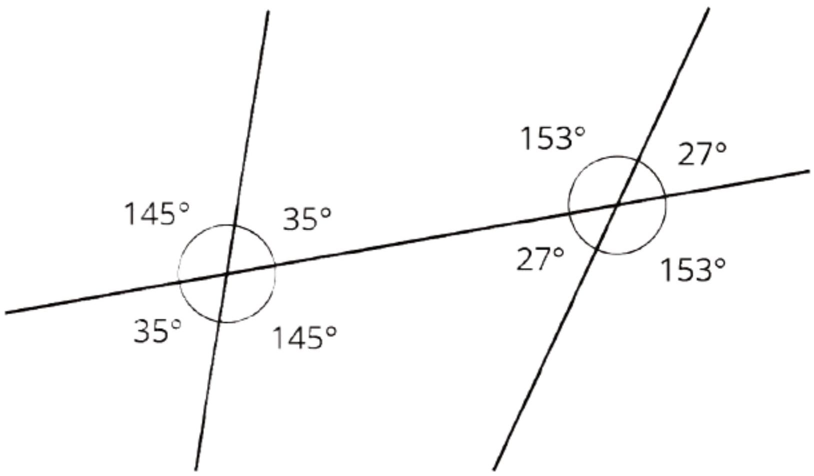
Statement

The diagram shows three lines with some marked angle measures.



Find the missing angle measures marked with question marks.

Solution

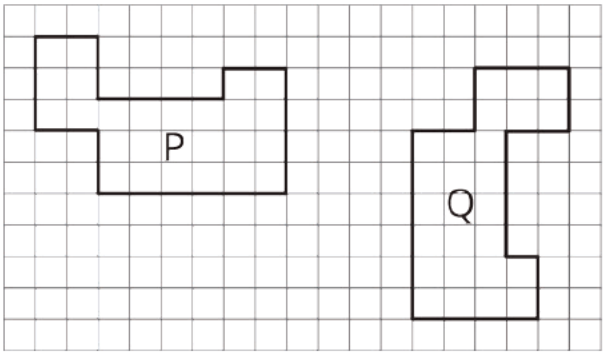


(From Unit 1, Lesson 14.)

Problem 4

Statement

Describe a sequence of translations, rotations, and reflections that takes Polygon P to Polygon Q.



Solution

Answers vary. Sample response: P is rotated 90 degrees clockwise and translated until the corresponding vertices match up.

(From Unit 1, Lesson 4.)

Problem 5

Statement

Point B has coordinates $(-2, -5)$. After a translation 4 units down, a reflection across the y-axis, and a translation 6 units up, what are the coordinates of the image?

Solution

$(2, -3)$

(From Unit 1, Lesson 6.)

Lesson 5: More Dilations

Goals

- Describe (orally) a figure on a coordinate grid and its image under a dilation, using coordinates to refer to points.
- Describe (orally) several dilations of one figure with the same center but different scale factors.
- Identify what information is needed to dilate a polygon on a coordinate grid. Ask questions to elicit that information.

Learning Targets

- I can apply dilations to polygons on a rectangular grid if I know the coordinates of the vertices and of the center of dilation.

Lesson Narrative

In previous lessons, students learned what a dilation is and practice dilating points and figures on a circular grid, on a square grid, on a coordinate grid, and with no grid. In this lesson, they work on a coordinate grid and use the coordinates to communicate precisely the information needed to perform a dilation. Students use the info gap structure. The student with the problem card needs to dilate a polygon on the coordinate grid. In order to do so, they need to request the coordinates of the polygon's vertices and the center of dilation as well as the scale factor. After obtaining all of this information from the partner with the data card, the student performs the dilation. The focus here is on deciding what information is needed and communicating clearly to request the information and explain why it is needed.

One important use of coordinates in geometry is to facilitate precise and concise communication about the location of points (MP6). This allows students to indicate where the center of the dilation is and also to communicate the vertices of the polygon that is dilated.

Alignments

Addressing

- 8.G.A: Understand congruence and similarity using physical models, transparencies, or geometry software.
- 8.G.A.3: Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Instructional Routines

- MLR4: Information Gap Cards
- Notice and Wonder

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Pre-printed slips, cut from copies of the blackline master

Required Preparation

Print and cut out copies of the blackline master for the Info Gap activity (one set of two problem cards and two data cards per pair of students).

Student Learning Goals

Let's look at dilations in the coordinate plane.

5.1 Many Dilations of a Triangle

Warm Up: 5 minutes (there is a digital version of this activity)
Students have seen many examples of scale factors that are less than and greater than 1, but the goal of this warm up is to have students focus explicitly on how the size of the scale factor impacts the dilation of a figure. Several dilations of one figure, with the same center but different scale factors, are displayed and students are asked to make observations about the shapes.

If using the digital activity, students will be able to scale a triangle by sliding the scale factor button. Students will informally make conjectures about the relationship between the original and scaled triangles. For example, the triangles are similar, the angles do not change, etc.

Addressing

- 8.G.A

Instructional Routines

- Notice and Wonder

Launch

Show students the image from the task (or demonstrate the applet from the digital activity) and ask "What do you notice? What do you wonder?" Record their responses for all to see. Students may notice and wonder many things. Relevant mathematical things students may notice:

- There are triangles that look like dilations of each other.
- The center of dilation is point P.

- The corresponding sides of the triangles are parallel to each other.
- There is a dashed line through each set of corresponding vertices.
- The corresponding angles appear to be congruent.

Things students may wonder:

- Which was the original triangle, and which are dilations of it?
- What scale factors were used?

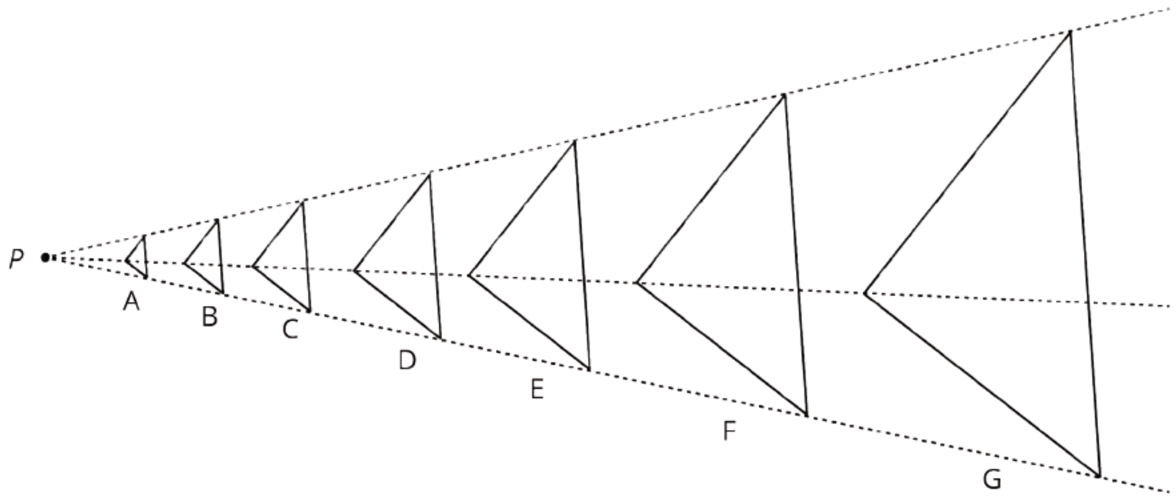
Give students 2 minutes of quiet work time followed by a whole-class discussion.

Anticipated Misconceptions

If students try to determine the actual scale factors between dilations, let them know that they only need to make an overall observation about the sizes of the dilations compared to Triangle D.

Student Task Statement

All of the triangles are dilations of Triangle D. The dilations use the same center P, but different scale factors. What do Triangles A, B, and C have in common? What do Triangles E, F, and G have in common? What does this tell us about the different scale factors used?



Student Response

Triangles A, B, and C are smaller and closer to P than Triangle D. Triangles E, F, and G are larger and farther away from P than Triangle D. The scale factor used to dilate Triangle D is less than 1 for Triangles A, B, and C. For E, F, and G, the scale factor is greater than 1.

Activity Synthesis

Important observations include

- The smaller triangles come from scale factors less than one while the bigger triangles correspond to scale factors larger than one
- The original triangle, itself, can be seen as coming from a scale factor of one
- All of the triangles have the same angles and orientation in the plane

A few other points worth discussing include

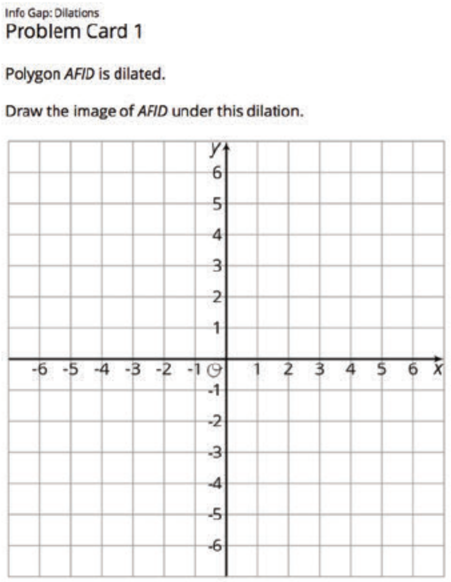
- Any of these triangles is a dilation of any other with center P.
- The lines from P contain "corresponding vertices" of the triangles (in the sense that these points map to one another via the dilations taking one triangle to another).

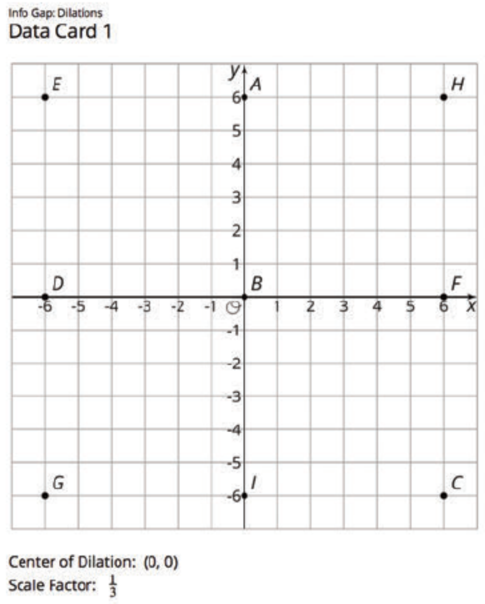
5.2 Info Gap: Dilations

20 minutes
This info gap activity gives students an opportunity to determine and request the information needed for a dilation, and to realize that using coordinates greatly simplifies talking about specific points. In order to perform a dilation, students will need to know the center of dilation (which can be communicated using the coordinate grid), the coordinates of the polygon that they are dilating (also communicated using the coordinate grid), and the scale factor. With this information, they can find the dilation as in previous activities.

The info gap structure requires students to make sense of problems by determining what information is necessary, and then to ask for the information they need to solve the problem. This may take several rounds of discussion if their first requests do not yield the information they need (MP1). It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need (MP6).

Here is the text of one of the cards for reference and planning:





Listen for how students request (and supply) information about the center of dilation and the location of the polygon that is being dilated. The coordinate grid helps name and communicate the location of points, which is essential in this activity. In addition to the location of points, listen for how students use “center of dilation” and “scale factor” in order to communicate this essential information.

Addressing

- 8.G.A.3

Instructional Routines

- MLR4: Information Gap Cards

Launch

Tell students they will continue to practice describing and drawing dilations using coordinates. Explain the Info Gap and consider demonstrating the protocol if students are unfamiliar with it. Arrange students in groups of 2. Provide access to geometry toolkits. In each group, distribute a problem card to one student and a data card to the other student. After you review their work on the first problem, give them the cards for the second problem and instruct them to switch roles.

Support for Students with Disabilities

Engagement: Develop Effort and Persistence. Display or provide students with a physical copy of the written directions. Check for understanding by inviting students to rephrase directions in their own words. Keep the display of directions visible throughout the activity.
Supports accessibility for: Memory; Organization

Support for English Language Learners

Conversing: This activity uses MLR4 Information Gap to give students a purpose for discussing information necessary to dilate a polygon on a coordinate grid. Display questions or question starters for students who need a starting point such as: "Can you tell me . . . (specific piece of information)", and "Why do you need to know . . . (that piece of information)?"
Design Principle(s): Cultivate Conversation

Student Task Statement

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

   Continue to ask questions until you have enough information to solve the problem.
4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

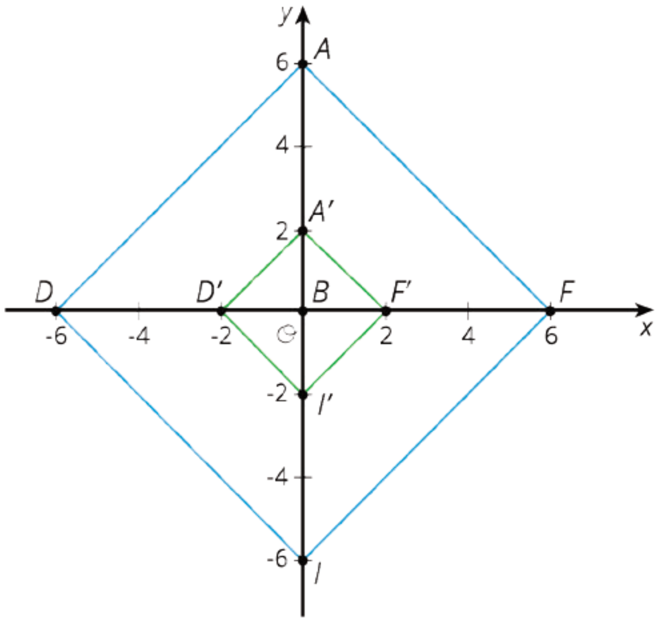
1. Silently read your card.
2. Ask your partner *"What specific information do you need?"* and wait for them to *ask* for information.

   If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don't have that information.
3. Before sharing the information, ask "*Why do you need that information?*" Listen to your partner's reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

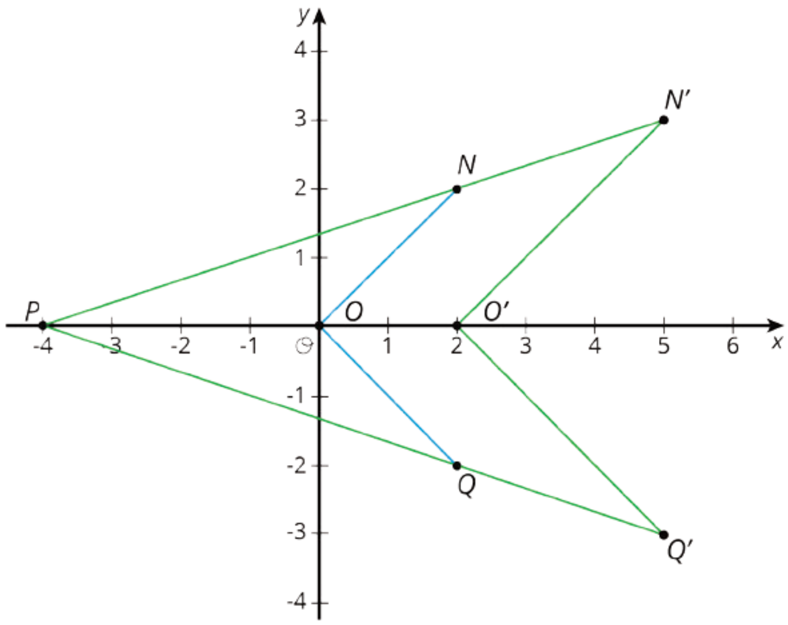
Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Student Response

Problem 1. $A' = (0,2)$, $F' = (2,0)$, $I' = (0,-2)$, $D' = (-2,0)$. Image to help visualize:

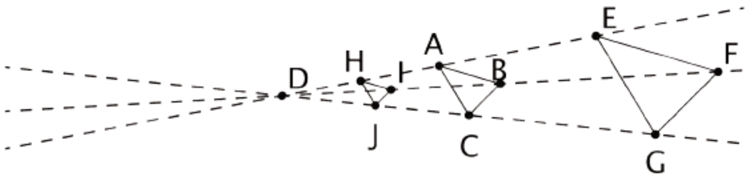


Problem 2. $O' = (2, 0)$, $N' = (5, 3)$, $P' = (-4, 0)$, $Q' = (5, -3)$. Image to help visualize:



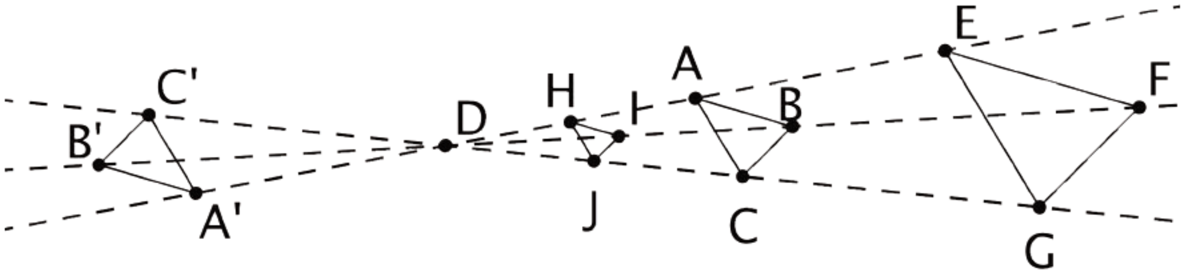
Are You Ready for More?

Triangle EFG was created by dilating triangle ABC using a scale factor of 2 and center D. Triangle HIJ was created by dilating triangle ABC using a scale factor of $\frac{1}{2}$ and center D.



1. What would the image of triangle ABC look like under a dilation with scale factor 0?
2. What would the image of the triangle look like under dilation with a scale factor of -1? If possible, draw it and label the vertices A', B', and C'. If it's not possible, explain why not.
3. If possible, describe what happens to a shape if it is dilated with a negative scale factor. If dilating with a negative scale factor is not possible, explain why not.

Student Response

1. If the distance from D to any vertex were multiplied by 0, the product would be 0. So, all three vertices of ABC would move to point D.
2. 

3. The dilated point goes to the other side of the point of dilation.

Activity Synthesis

After students have completed their work, share the correct answers and ask students to discuss the process of solving the problems. Some guiding questions:

- "Other than the answer, what information would have been nice to have?"
- "How did using coordinates help in talking about the problem?"
- "If this same problem had a figure on a grid without coordinates, how would you talk about the points?"
- "What if there had been no grid at all? Would you still have been able to request or provide the needed information to perform the transformation?"

Highlight that coordinates allow us to unambiguously provide the location of a polygon's vertices. In addition, reinforce the idea that in order to perform a dilation, we need to know the scale factor and the center of dilation. The coordinate grid again provides an efficient means to communicate the center of dilation.

Lesson Synthesis

Ask students to think about the question: "Why does anyone bother putting coordinate axes on a grid? Why are coordinates useful? What are they good for?"

Here are the points to emphasize: Coordinates are an exceptionally powerful tool for communicating the location of points in the plane. There is only one point 3 units to the left of the origin and 2 units up from the origin, the point $(3, 2)$. The location of a polygon is determined by the location and order of its vertices. On a coordinate plane, these can be communicated by giving their coordinates. When we perform a dilation, we also need to know the center of dilation (another point) and the scale factor (a number). On the coordinate plane, all of the information we need to dilate a polygon can be communicated unambiguously with some numbers!

Coordinates in the plane are much like an address in the city: they tell you where to go unambiguously. Because the plane is laid out in a grid, these "addresses" are particularly simple, consisting of two signed numbers. We will use coordinates much more in this unit and beyond, not only to describe individual points but also to describe relationships (like proportional relationships and other new types of relationships).

5.3 Identifying a Dilation

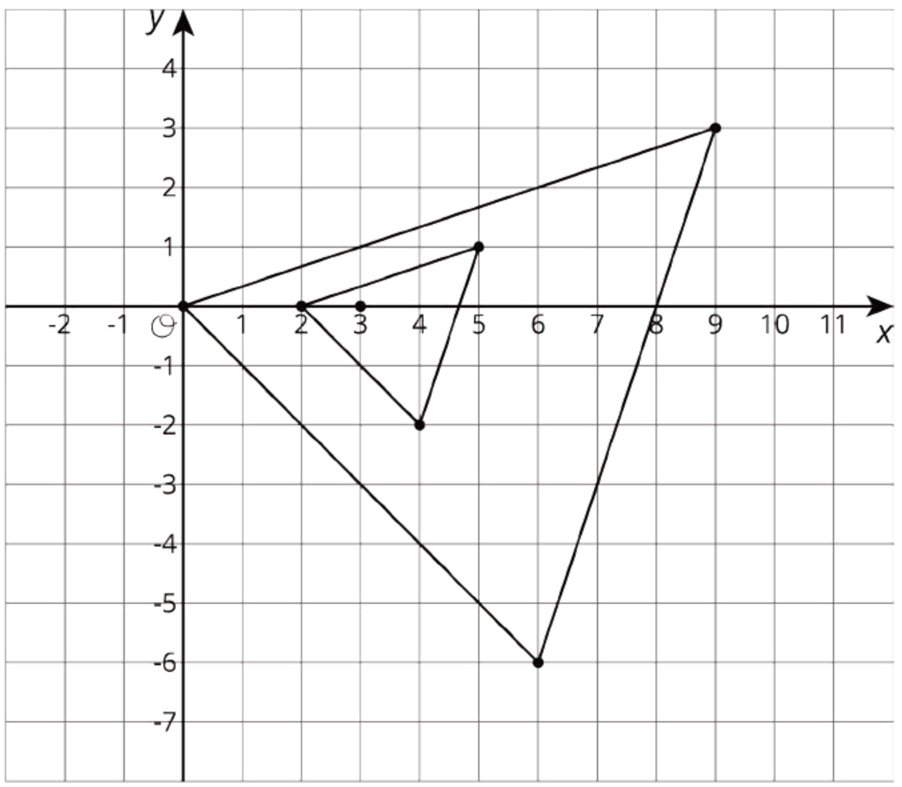
Cool Down: 5 minutes

Addressing

- 8.G.A

Student Task Statement

The smaller triangle is dilated to create the larger triangle. The center of dilation is plotted, but not labeled.



Describe this dilation. Be sure to include all of the information someone would need to perform the dilation.

Student Response

Answers vary. Information that must be included:

- The center of dilation is $(3, 0)$.
- The scale factor is 3.
- The triangle being dilated has vertices at $(2, 0)$, $(4, -2)$, and $(5, 1)$.

Student Lesson Summary

One important use of coordinates is to communicate geometric information precisely. Let's consider a quadrilateral $ABCD$ in the coordinate plane. Performing a dilation of $ABCD$ requires three vital pieces of information:

1. The coordinates of A, B, C, and D
2. The coordinates of the center of dilation, P
3. The scale factor of the dilation

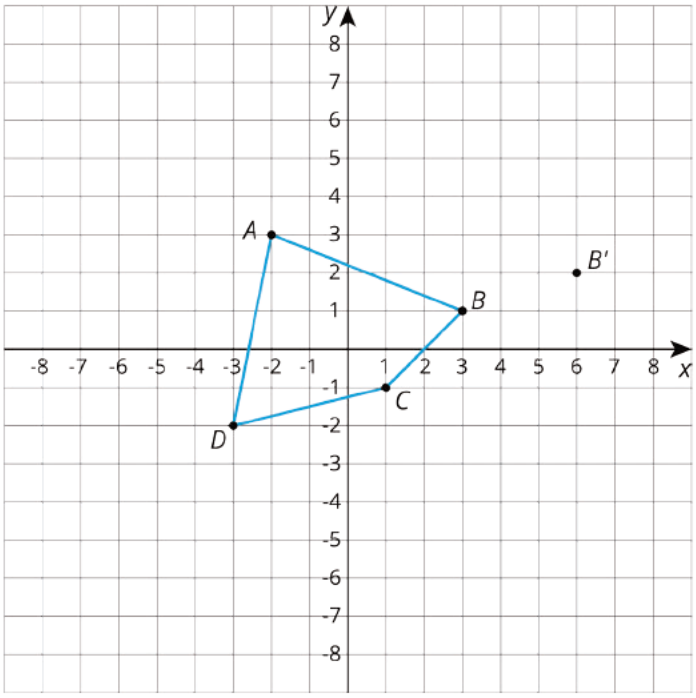
With this information, we can dilate the vertices A, B, C, and D and then draw the corresponding segments to find the dilation of $ABCD$. Without coordinates, describing the location of the new points would likely require sharing a picture of the polygon and the center of dilation.

Lesson 5 Practice Problems

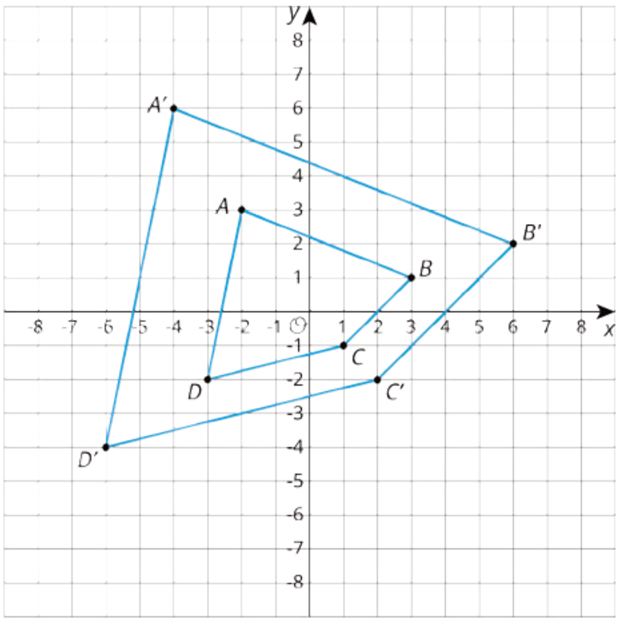
Problem 1

Statement

Quadrilateral $ABCD$ is dilated with center $(0, 0)$, taking B to B'. Draw $A'B'C'D'$.



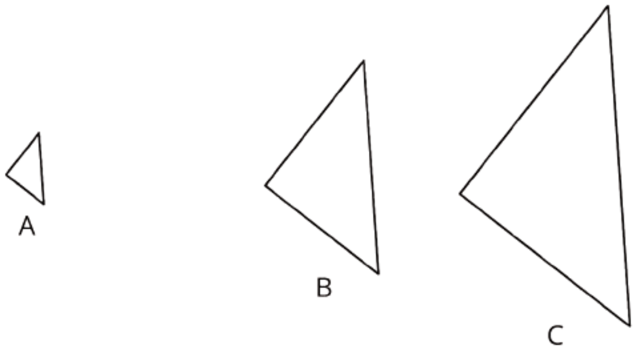
Solution



Problem 2

Statement

Triangles B and C have been built by dilating Triangle A.



a. Find the center of dilation.

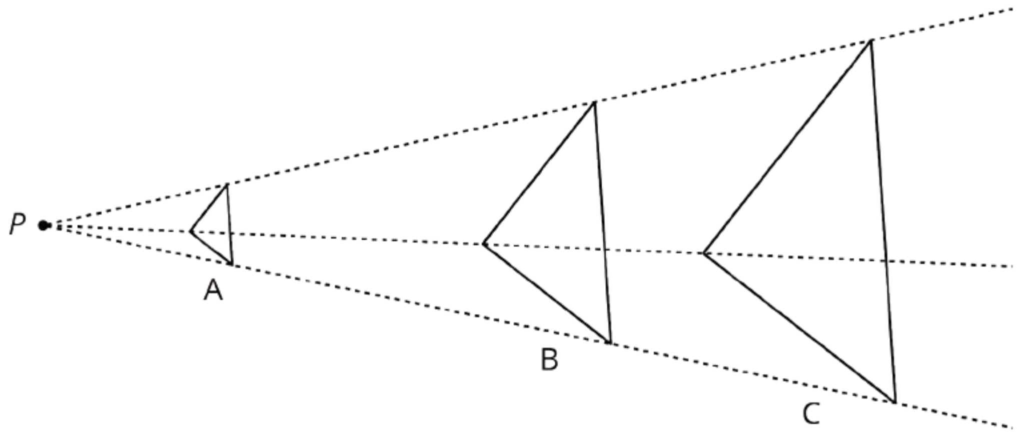
b. Triangle B is a dilation of A with approximately what scale factor?

c. Triangle A is a dilation of B with approximately what scale factor?

d. Triangle B is a dilation of C with approximately what scale factor?

Solution

a. The center of dilation is here:



b. 3

c. $\frac{1}{3}$

d. $\frac{2}{3}$

Problem 3

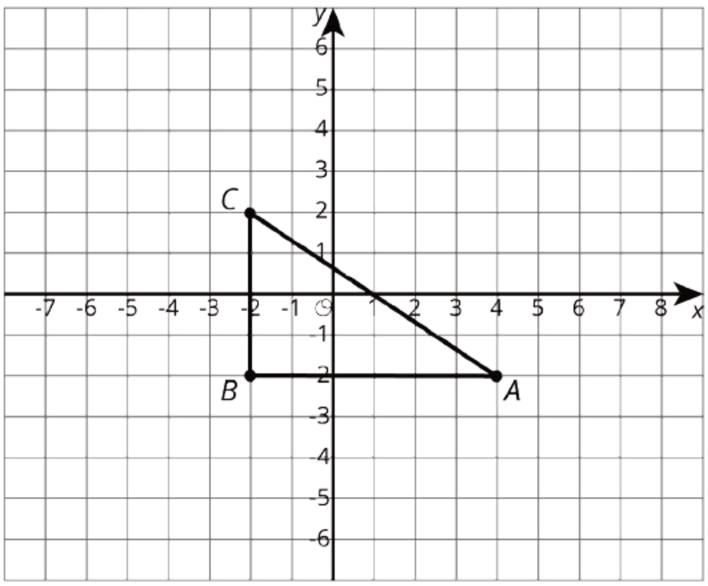
Statement

Here is a triangle.

a. Draw the dilation of triangle ABC, with center $(0, 0)$, and scale factor 2. Label this triangle $A' B' C'$.

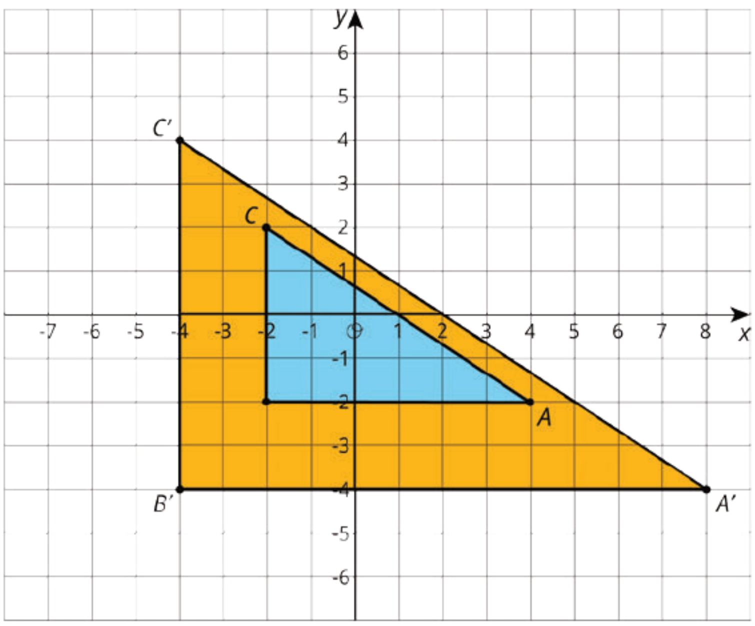
b. Draw the dilation of triangle ABC, with center $(0, 0)$, and scale factor $\frac{1}{2}$. Label this triangle $A'' B'' C''$.

c. Is $A'' B'' C''$ a dilation of triangle $A' B' C'$? If yes, what are the center of dilation and the scale factor?

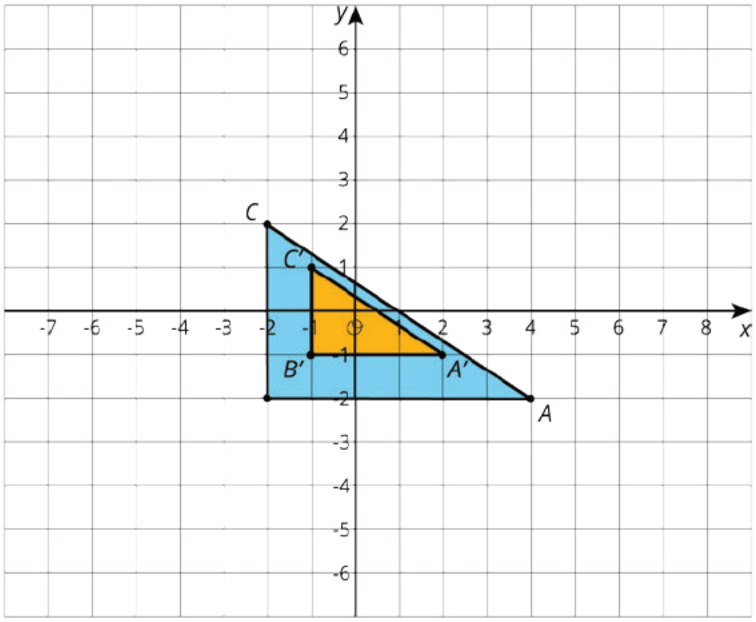


Solution

a.



b.



c. Yes, $A''B''C''$ is a dilation of $A'B'C'$ with center $(0,0)$ and scale factor $\frac{1}{4}$.

Problem 4

Statement

Triangle DEF is a right triangle, and the measure of angle D is 28°. What are the measures of the other two angles?

Solution

90° and 62°

(From Unit 1, Lesson 15.)

Section: Similarity

Lesson 6: Similarity

Goals

- Comprehend that the phrase "similar figures" (in written and spoken language) means there is a sequence of translations, rotations, reflections, and dilations that takes one figure to the other.
- Justify (orally) the similarity of two figures using a sequence of transformations that takes one figure to the other.

Learning Targets

- I can apply a sequence of transformations to one figure to get a similar figure.
- I can use a sequence of transformations to explain why two figures are similar.

Lesson Narrative

In the previous unit, students saw that two figures are congruent when there is a sequence of translations, rotations, and reflections that takes one figure to another. Now dilations, studied in previous lessons, are added to the possible set of "moves" taking one shape to another. Two figures are **similar** if there is a sequence of translations, rotations, reflections, and dilations that takes one figure to the other. When two figures are similar, there are always many different sequences that show that they are similar. One method is to apply a dilation to one figure so that the corresponding figures are congruent. Then a sequence of rigid motions will finish taking one shape to the other. Alternatively, we could translate one pair of corresponding vertices together, apply rotations and reflections to adjust the orientations, and then conclude with a dilation so that they match.

In future lessons, students will learn shortcuts for some polygons (including all triangles), but in this lesson they focus on the definition of similarity in terms of transformations. They will see that two dilations with the same scale factor but different centers differ by a translation. They will also study how transformations from polygon A to polygon B can be reversed to take polygon B to polygon A.

Alignments

Building On

- 7.NS.A: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
- 8.G.A.2: Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

Addressing

- 8.G.A.2: Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
- 8.G.A.4: Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR1: Stronger and Clearer Each Time
- MLR3: Clarify, Critique, Correct
- MLR7: Compare and Connect

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Pre-printed slips, cut from copies of the blackline master

Required Preparation

If you decide to do the optional “Methods for Translations and Dilations” activity, print and cut out 1 set of cards for every 2 students.

Student Learning Goals

Let’s explore similar figures.

6.1 Equivalent Expressions

Warm Up: 5 minutes
This warm-up prompts students to use what they know about operations and properties of operations to create related expressions. While many warm-ups encourage students to work mentally and verbally, students will write their responses to this prompt. Since many different responses are possible, the task is accessible to all students and provides an opportunity to hear how each student reasons about the operations. Some different ideas that may emerge are:

- commutative property
- distributive property
- inverse operations
- adjusting factors (for example, doubling and halving)

Examples of each are given in the student response section. Students are not expected to use these terms, but highlight the terms if students do use them.

Building On

- 7.NS.A

Launch

Arrange students in groups of 2. Tell students they are writing a list of several expressions equivalent to $10(2+3)-8 \cdot 3$. Give students 2 minutes of quiet think time followed by 1 minute to discuss their responses with a partner.

Student Task Statement

Use what you know about operations and their properties to write three expressions equivalent to the expression shown.

$$10(2+3)-8 \cdot 3$$

Student Response

Answers vary. Possible responses:

- commutative property: $10(3+2)-8 \cdot 3$ or $-8 \cdot 3+10(2+3)$
- distributive property: $10 \cdot 2+10 \cdot 3-8 \cdot 3$
- inverse operations: $10(2+3)+-8 \cdot 3$
- associative property: $10(2+3)-16 \cdot 1.5$

Activity Synthesis

Much of the discussion takes place between partners. Ask students to share any expressions that they aren't sure about, but try to resolve these and move on quickly.

6.2 Similarity Transformations (Part 1)

20 minutes (there is a digital version of this activity)
In this activity, students learn that two figures are *similar* when there is a sequence of translations, reflections, rotations and dilations that takes one figure to the other. Students practice discovering these sequences for two pairs of figures.

When two shapes are similar but not congruent, the sequence of steps showing the similarity usually has a single dilation and then the rest of the steps are rigid transformations. The dilation can come at any time. It does not matter which figure you start with. An important thing for students to notice in this activity is that there is more than one sequence of transformations that show two figures are similar. Monitor for students who insert a dilation at different places in the sequence. Also monitor for how students find the scale factor for the hexagons.

Building On

- 8.G.A.2

Addressing

- 8.G.A.4

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR1: Stronger and Clearer Each Time

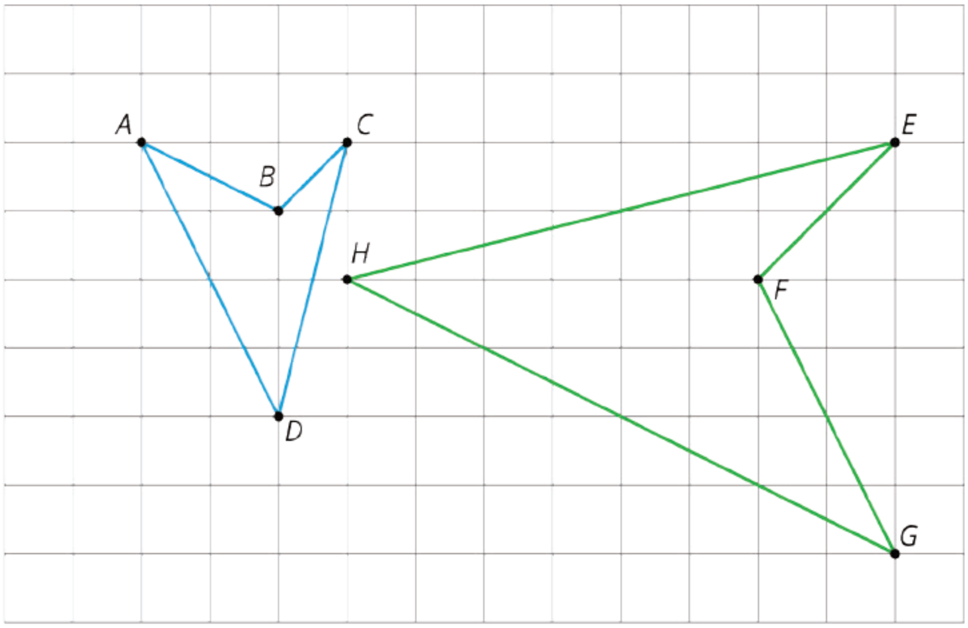
Launch

Before beginning this task, define what it means for two figures to be *similar*.

Tell students: We have talked about how one figure can be a scaled copy of another. This relationship goes in both directions. For example, if triangle DEF is a scaled copy of triangle ABC with scale factor of 2 then triangle ABC is a scaled copy of triangle DEF with scale factor $\frac{1}{2}$. We have learned that the transformation that creates scaled copies is called a dilation.

We say that triangles ABC and DEF are **similar**. The previous unit explored how translations, rotations, and reflections define congruent figures. The inclusion of dilations can change the size of the figure as well as its location and orientation.

We will start our investigation of similar figures by identifying sequences of translations, rotations, reflections, and dilations that show two figures are similar. Demonstrate using this example.



There are many methods to make this work. Explain at least two. First, identify the corresponding parts. Then come up with a plan to take one figure to the other. Ensure students understand, through demonstration with this example, that the work of showing two figures are similar requires communicating the details of each transformation in the sequence with enough precision. Some sample methods:

1. Method 1 ($ABCD$ to $GFEH$: Dilate, Translate, Rotate, Reflect)
   a. Dilate using D as the center with scale factor 2.

   b. Translate D to H

   c. Rotate using H as the center clockwise by 90 degrees

   d. Reflect using the line that contains H and F.

2. Method 2 ($ABCD$ to $GFEH$: Reflect, Translate, Rotate, Dilate)
   a. Reflect using the line that contains D and B.

   b. Translate D to H.

   c. Rotate using H as the center clockwise by 90 degrees

   d. Dilate using H as the center with a scale factor of 2.

3. Method 3 ($ABCD$ to $GFEH$: Translate, Rotate, Reflect, Dilate)
   a. Translate B to F.

   b. Rotate using F as the center clockwise by 90 degrees.

   c. Reflect using the line that contains F and H.

   d. Dilate using F as the center with scale factor 2.

These arguments can also be applied to figures that are not on a grid: the grid helps to identify directions and distances of translation, 90 degree angles of rotation, and horizontal and vertical lines of reflection.

If using the print version, provide access to geometry toolkits. If using the digital version, remind students of the meaning and functionality of each transformation tool, as necessary.

Support for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Include the following term and maintain the display for reference throughout the unit: similar. On this display, include the step-by-step instructions of at least 2 of the 3 given methods for creating similar polygons using transformations.
Supports accessibility for: Memory; Language

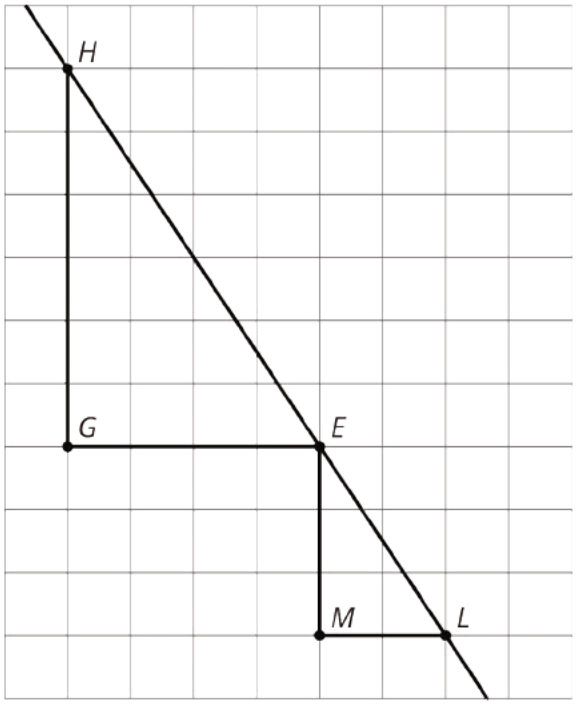
Anticipated Misconceptions

If students do not recall the three types of rigid transformations, refer them to the classroom display that provides an example of a rotation, a reflection, and a translation.

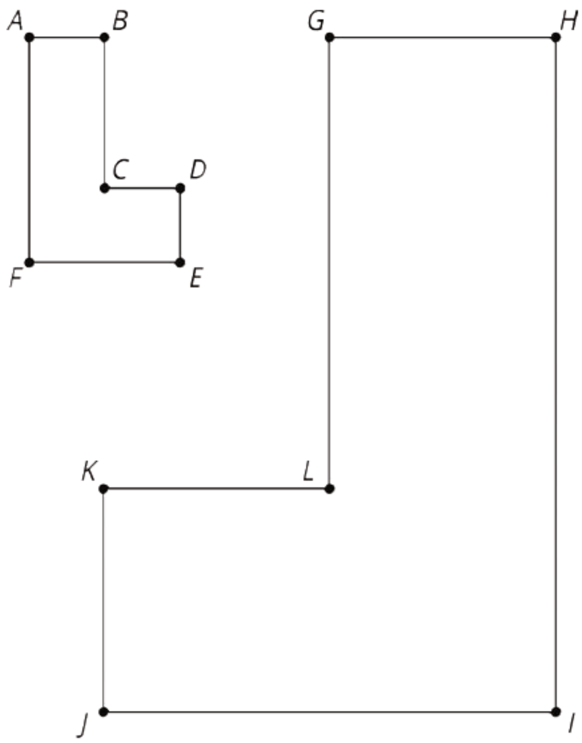
For the second problem, students may need encouragement to experiment moving the shapes (using tracing paper for example). If they get stuck finding the scale factor, tell them that they can approximate by measuring sides of the two figures.

Student Task Statement

1. Triangle EGH and triangle LME are **similar**. Find a sequence of translations, rotations, reflections, and dilations that shows this.



2. Hexagon $ABCDEF$ and hexagon $HGLKJI$ are similar. Find a sequence of translations, rotations, reflections, and dilations that shows this.



Student Response

1. Answers vary. Sample response:
   a. Begin with triangle LME

b. Translate L to E

c. Dilate using E as the center with scale factor 2

2. Answers vary. Sample response:
   a. Begin with figure $ABCDEF$

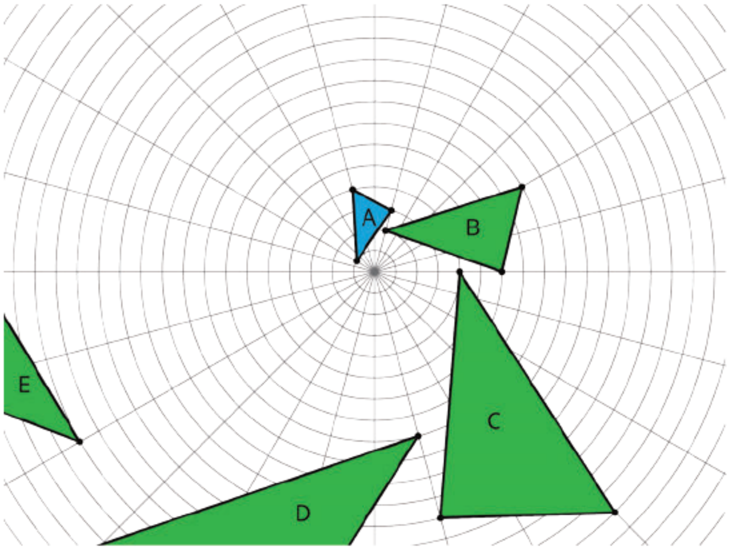
   b. Reflect using the line that contains A and F

   c. Translate F to I

   d. Dilate using I as the center with scale factor 3

Are You Ready for More?

The same sequence of transformations takes Triangle A to Triangle B, takes Triangle B to Triangle C, and so on. Describe a sequence of transformations with this property.



Student Response

Answers vary. Sample response: Dilate, from the center of the circular grid, with scale factor 2, then rotate clockwise 75 degrees.

Activity Synthesis

Select students to give a variety of solutions for the different problems. Point out that there are multiple ways to do each pair and any valid sequence is allowed. Ensure that students communicate each transformation in the sequence in sufficient detail.

Support for English Language Learners

Writing, Speaking, Listening: MLR1 Stronger and Clearer Each Time. After students have determined the sequence of transformations that shows the polygons are similar, ask students to write a detailed sequence of the transformations on their paper. Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language (e.g., "How did you know to translate point L to point E?", and "How did you know to dilate the polygon by a scale factor of 3?", etc.). Students can borrow ideas and language from each partner to refine and clarify their original explanation. This will help students refine their own explanation and learn about other ways to show polygons are similar using a sequence of transformations.
Design Principles(s): Optimize output (for explanation); Maximize meta-awareness

6.3 Similarity Transformations (Part 2)

10 minutes

This activity helps students visualize what happens to figures under different kinds of transformations. Students practice identifying which transformations might be used in the sequence of translations, rotations, reflections, and dilations in order to show figures are similar. By recognizing patterns in the image results after using certain transformations (MP8), students may be able to apply this to finding transformations for other problems. Students should pay special attention to see the connection between the orientations of the original figure and the resulting images after transformation.

Encourage students to draw rough sketches though it is also ok to use patty paper, for example, to execute the rigid motions. For the dilations, however, it could become time consuming to choose an explicit scale factor and measure carefully. Make sure, after students have worked on the first problem, to show some examples of sketches that are not exact but capture the main features of the figure.

Addressing

- 8.G.A.2
- 8.G.A.4

Instructional Routines

- MLR7: Compare and Connect

Launch

The figure in this task is intended to resemble a hand with all of the fingers together and the thumb sticking out. Encourage students to "sketch" the resulting images for this task (or make tracing paper available, indicating that the images do not need to be exact). They do not need to make the side lengths, angles, etc. perfect, but it should be clear where the corresponding parts of the image are and whether it is larger or smaller than the original. Other activities in this unit will ask students to be precise in their use of transformations, but the goal of this activity is to get an idea of how the different transformations affect a figure's image.

Select a few good examples of student work (including some that have been sketched free hand) to share with the class after problem 1, in order to clearly communicate the expectation for level of precision.

Support for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support development of organizational skills, check in with students within the first 2–3 minutes of work time. Check to make sure students are not using exact or precise measurements, as this activity only requires a sketch. After the first problem, invite a few students to think aloud and share their sketches to guide the rest of the individual work time.
Supports accessibility for: Memory; Organization

Support for English Language Learners

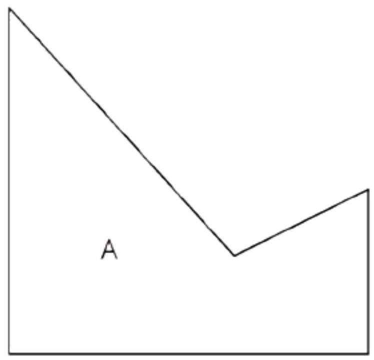
Speaking, Listening: MLR7 Compare and Connect. Ask students to prepare a visual display of their figures that are similar to Figure A. As students investigate each other's work, ask students to share what transformations are especially clear in the display of similar figures. Listen for and amplify any comments about what might make the transformations clearer in the display. Then encourage students to make connections between the words "translation," "rotation," "reflection," and "dilation" and how they affect the figure. Listen for and amplify language students use to describe what happens to figures under different kinds of transformations. This will foster students' meta-awareness and support constructive conversations as they compare images of the same figure and make connections between transformations and their effects on figures.
Design Principle(s): Cultivate conversation; Maximize meta-awareness

Anticipated Misconceptions

If students choose an exact scale factor, or measure the exact angle sizes, explain that precise measurements are not needed in this task. At this point, they are just sketching similar figures.

Student Task Statement

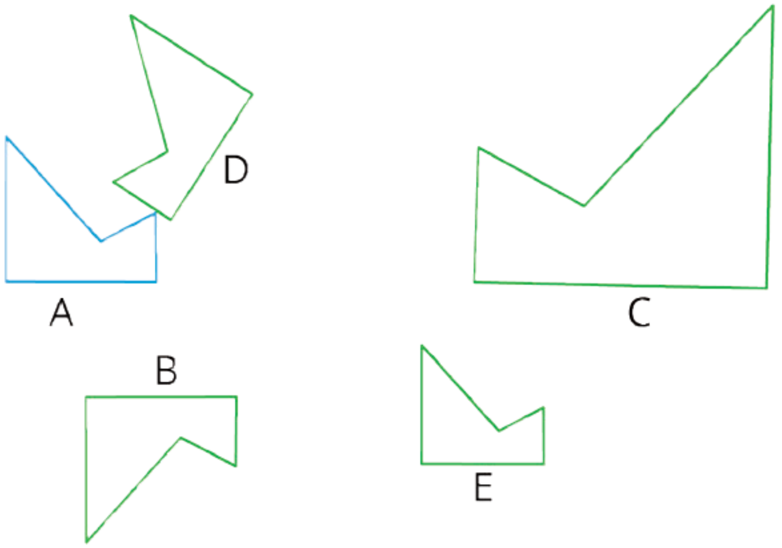
Sketch figures similar to Figure A that use only the transformations listed to show similarity.



1. A translation and a reflection. Label your sketch Figure B.
   Pause here so your teacher can review your work.
2. A reflection and a dilation with scale factor greater than 1. Label your sketch Figure C.
3. A rotation and a reflection. Label your sketch Figure D.
4. A dilation with scale factor less than 1 and a translation. Label your sketch Figure E.

Student Response

Answers vary. Sample responses:



Activity Synthesis

Select students to display their answers for each of the questions. Ask students to notice things the answers have in common so that they can make connections to the types of transformations that might be useful in showing that two figures are similar.

Assuming that the rotations are not through an angle that is a multiple of 360° and that the translations have a non-zero horizontal or vertical part, point out that:

- Dilations will create larger or smaller copies depending on the scale factor as seen in previous lessons.
- Translations will slide the figure in some direction.
- Rotations will "tilt" or "turn" the figure.
- Reflections will change the handedness so that the resulting image will look like the back of a right hand instead of the back of a left hand as in the original image.

6.4 Methods for Translations and Dilations

Optional: 10 minutes

The purpose of this task is for students to practice showing that two shapes are similar using only a few pre-determined rigid motions and dilations. Some students will start with triangle ABC and take this to triangle DEF while other start with DEF and take this to ABC. While there is flexibility in either direction, one way of getting from DEF to ABC is to "undo" the moves that take ABC to DEF.

Monitor for students who use different centers of dilation in their sequence, particularly as the first step in the sequence. Invite these students to share, highlighting the fact that two dilations with the same scale factor but different centers differ by a translation. Also monitor for students whose sequences of rigid motions and dilations are the same but in the opposite order, one set taking ABC to DEF and the opposite taking DEF back to ABC. Select these students to share this important observation during the discussion.

Addressing

- 8.G.A.4

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Again, remind students that two figures are similar if there is a sequence of translations, rotations, reflections, and dilations that takes one figure to another. Tell them that they need to find at least one way to show that triangle ABC and triangle DEF are similar using only the transformations they are given on their cards.

Arrange students in groups of 2. Give each group one complete set of cards.

Support for Students with Disabilities

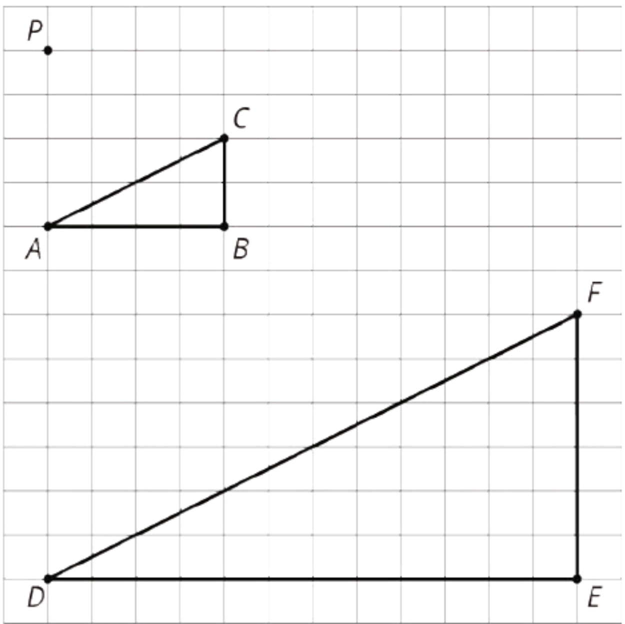
Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, “One thing that is the same is....”; “One thing that is different is....”; and “Another strategy to get the same result is....”
Supports accessibility for: Language; Social-emotional skills

Anticipated Misconceptions

Students might think that is it necessary to perform transformations in the same order or that one particular point needs to be the center. If partners choose the same methods, prompt them to try it another way that will have the same end result.

Student Task Statement

Your teacher will give you a set of five cards and your partner a different set of five cards. Using only the cards you were given, find at least one way to show that triangle ABC and triangle DEF are similar. Compare your method with your partner’s method. What is the same about your methods? What is different?



Student Response

Partner A can:

1. Dilate triangle ABC using center point P and scale factor 3.

2. Dilate using center point A and scale factor 3 followed by translating from A to D.

3. Translate from A to D followed by dilating using center D and scale factor 3.

Partner B can:

1. Dilate triangle DEF using center point P and scale factor $\frac{1}{3}$.

2. Dilate using center point D and scale factor $\frac{1}{3}$ followed by translating from D to A.

3. Translate from D to A followed by dilating using center A and scale factor $\frac{1}{3}$.

Activity Synthesis

Invite selected students to share, highlighting methods of moving ABC to DEF and DEF to ABC which are "opposite" of one another, for example dilations with center P and reciprocal scale factors.

As students share their responses, highlight these points:

- The scale factors for the dilations are reciprocals regardless of when the dilations are done in the sequence.
- If used, the translations are inverses of each other (eg "Translate A to D" instead of "Translate D to A").
- Dilations with different centers but the same scale factor produce congruent figures that differ by a translation.
- The order in which transformations are applied can influence the result.

One important conclusion (the third bullet point) is that when you are showing that two figures are similar, you can pick any point as the center of dilation if you know the scale factor, because you can always adjust the position using a translation.

Support for English Language Learners

Reading, Writing, Speaking: MLR3 Clarify, Critique, Correct. Before students share their sequence of transformations to show that triangle ABC and triangle DEF are similar, present an incorrect sequence of transformations. For example, "Translate from A to D. Then dilate using center P and scale factor 3." Ask students to identify the error, critique the reasoning, and write a correct explanation. As students discuss in partners, listen for students who clarify the meaning of the center of dilation. Prompt students to share their critiques and corrected explanations with the class. Listen for and amplify the language students use to describe what happens when P is the center of dilation and explain why D should be the center of dilation. This routine will engage students in meta-awareness as they clarify how the center of dilation affects the dilated figure.
Design Principle(s): Optimize output (for explanation); Maximize meta-awareness

Lesson Synthesis

Review the definition of similar figures and any important insights that arose during the lesson. Insights to highlight:

Two figures are similar if there is a sequence of translations, rotations, reflections, and dilations that maps one to the other. Scaled copies of figures, studied in grade 7, are all examples of similar figures. In this lesson, we found transformations that showed that two figures were similar.

We saw that there is more than one sequence of transformations that shows two figures are similar. One way to think about it is that you need to use a dilation to make corresponding side lengths the same size. The figures will be congruent after this step. Once you do that, you just need to find a sequence of rigid transformations that align the congruent figures. You can also do it the other way, by bringing the figures into alignment and then dilating one to match up with the other.

Add the term *similar* along with a definition and example to your classroom display such as a word wall or anchor chart.

6.5 Showing Similarity

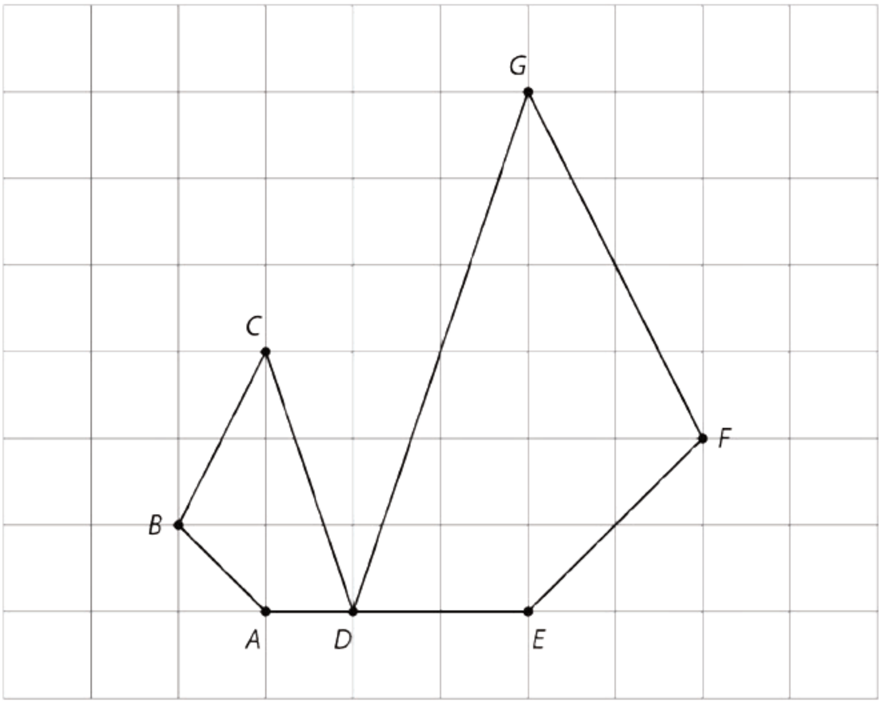
Cool Down: 5 minutes
Students analyze and correct a proposed sequence of rigid transformations and dilations to show that two figures are similar.

Addressing

- 8.G.A.4

Student Task Statement

Elena gives the following sequence of transformations to show that the two figures are similar by transforming $ABCD$ into $EFGD$.



1. Dilate using center D and scale factor 2.
2. Reflect using the line AE.

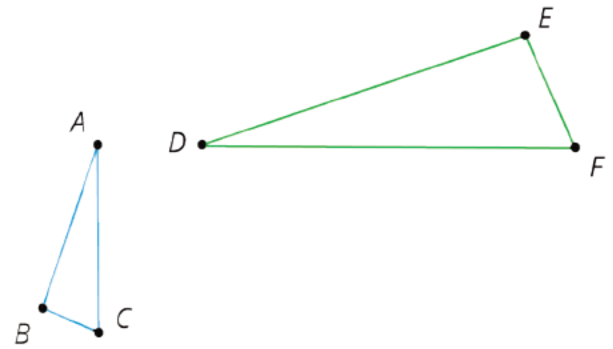
Is Elena's method correct? If not, explain how you could fix it.

Student Response

The figures are similar, but the transformations do not take $ABCD$ to $EFGD$. After dilating $ABCD$ using D as the center with a scale factor of 2, Elena can reflect over the vertical line through D rather than the horizontal line.

Student Lesson Summary

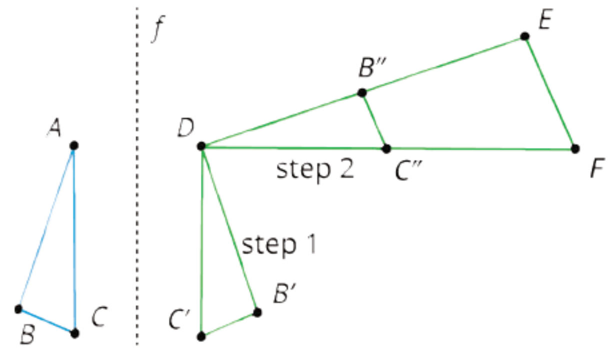
Let's show that triangle ABC is similar to triangle DEF:



Two figures are **similar** if one figure can be transformed into the other by a sequence of translations, rotations, reflections, and dilations. There are many correct sequences of transformations, but we only need to describe one to show that two figures are similar.

One way to get from ABC to DEF follows these steps:

- step 1: reflect across line f
- step 2: rotate 90° counterclockwise around D
- step 3: dilate with center D and scale factor 2



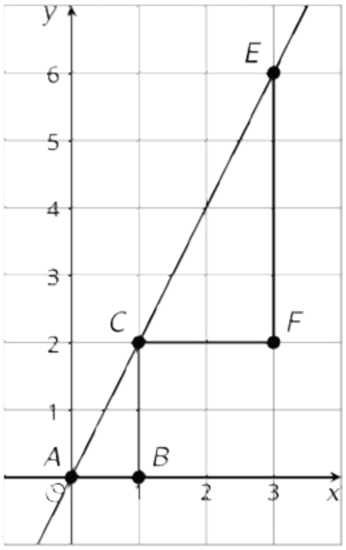
Another way would be to dilate triangle ABC by a scale factor of 2 with center of dilation A, then translate A to D, then reflect over a vertical line through D, and finally rotate it so it matches up with triangle DEF. What steps would you choose to show the two triangles are similar?

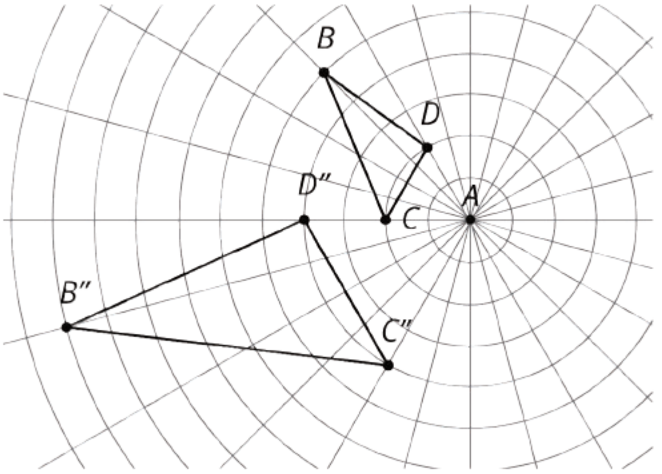
Glossary

- similar

Lesson 6 Practice Problems
Problem 1
Statement

Each diagram has a pair of figures, one larger than the other. For each pair, show that the two figures are similar by identifying a sequence of translations, rotations, reflections, and dilations that takes the smaller figure to the larger one.





Solution

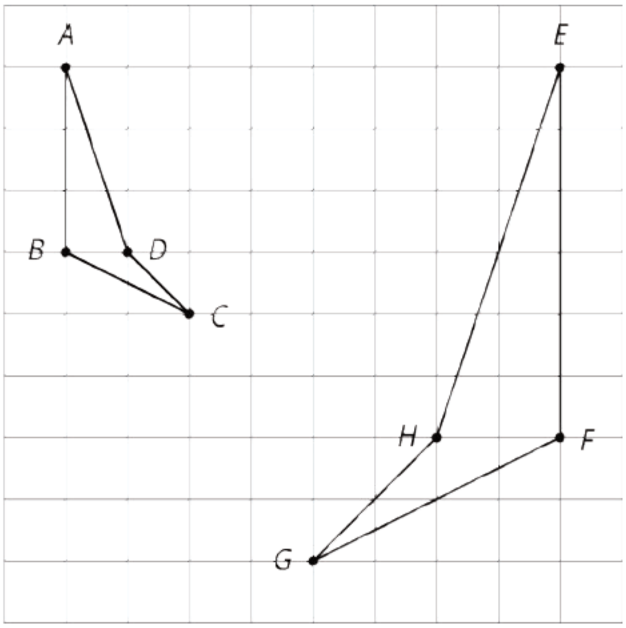
- Translate A to C, and then dilate with center A by a factor of 2.
- Rotate 60° counter-clockwise with center A, and then dilate using a scale factor of 2 centered at A.

Problem 2

Statement

Here are two similar polygons.

Measure the side lengths and angles of each polygon. What do you notice?



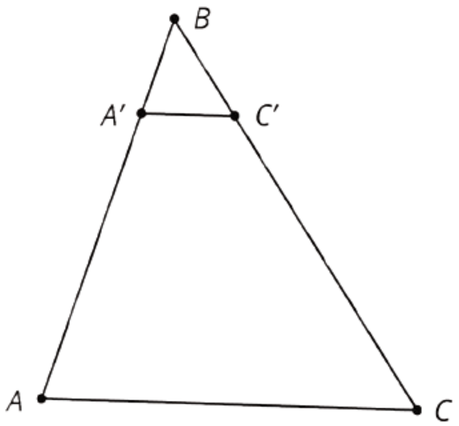
Solution

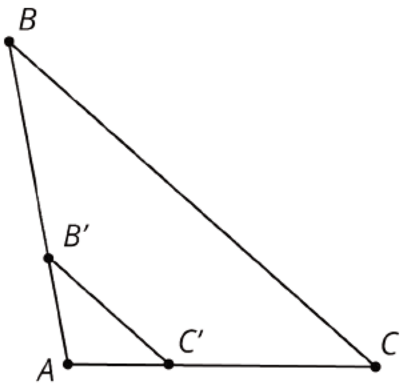
Answers vary. Sample response: Corresponding side lengths in the larger polygon are double the side lengths of the smaller polygon, while corresponding angles all have the same measure.

Problem 3

Statement

Each figure shows a pair of similar triangles, one contained in the other. For each pair, describe a point and a scale factor to use for a dilation moving the larger triangle to the smaller one. Use a measurement tool to find the scale factor.





Solution

Center of dilation: B, scale factor: $\frac{1}{4}$; center of dilation: A, scale factor: $\frac{1}{3}$

Lesson 7: Similar Polygons

Goals

- Comprehend the phrase “similar polygons” (in written and spoken language) to mean the polygons have congruent corresponding angles and proportional side lengths.
- Critique (orally) arguments that claim two polygons are similar.
- Justify (orally) the similarity of two polygons given their angle measures and side lengths.

Learning Targets

- I can use angle measures and side lengths to conclude that two polygons are not similar.
- I know the relationship between angle measures and side lengths in similar polygons.

Lesson Narrative

One of the powerful things about the definition of similarity in terms of transformations is that we can talk about whether two figures are similar even when they are not composed of straight lines. For example, we can show that all circles are similar, because we can translate one so they have the same center and then dilate one until it matches the other.

In the case of polygons, we can understand similarity by examining side lengths and angle measures. Since the transformations we have studied (translations, rotations, reflections, dilations) do not change angle measures, similar polygons have congruent corresponding angles. Only dilations change side lengths and they change them all by the *same* scale factor. This means that similar polygons have proportional corresponding side lengths. In general, both side lengths and angle measures are important to determine whether or not two polygons are similar. The next lesson will examine the special case of triangles where it turns out that congruent corresponding angles is all that is needed to conclude that two triangles are similar.

The focus in this lesson is on quadrilaterals, and students determine efficient ways to decide whether certain types of quadrilaterals are similar:

- Two rectangles are similar if the side lengths are proportional.
- Two rhombuses are similar if the angles are congruent.
- All squares are similar.

Alignments

Addressing

- 8.G.A.2: Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

- 8.G.A.4: Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR2: Collect and Display

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Pre-printed slips, cut from copies of the blackline master

Required Preparation

Make 1 copy of the blackline master for every 10 students, and cut them up ahead of time.

Student Learning Goals

Let’s look at sides and angles of similar polygons.

7.1 All, Some, None: Congruence and Similarity

Warm Up: 10 minutes
Students examine statements deciding in each case whether the statement is always true, sometimes true, or never true. Since figures are similar if one is the result of reflecting, rotating, translating, and dilating the other, and figures are congruent only as a result of the rigid transformations, congruent figures are similar but not vice versa. Though students have not studied how angles behave under dilations, students have had multiple opportunities to see that dilations influence the size of a shape but not its angles. They also know from work in grade 7 that scaled copies have corresponding angles with the same measure, and dilations have been characterized as a process that makes scaled copies.

Addressing

- 8.G.A.2
- 8.G.A.4

Launch

Display all 3 statements. Ask students to decide whether each of the statements is always, sometimes, or never true and give a signal when they have reasoning to support their decision.

Student Task Statement

Choose whether each of the statements is true in *all* cases, in *some* cases, or in *no* cases.

1. If two figures are congruent, then they are similar.
2. If two figures are similar, then they are congruent.
3. If an angle is dilated with the center of dilation at its vertex, the angle measure may change.

Student Response

1. All cases. Congruent figures can be taken from one to the other by using translations, rotations, and reflections. Similar figures use these same transformations, but also use dilations. If we don't do a dilation, the figures are still similar.
2. Some cases. Similar figures can be taken from one to the other using translations, rotations, reflections, and dilations. Congruent figures do not allow dilations. Two figures can be similar that are different sizes (a square with side length one and a square with side length two). These figures are not congruent.
3. No cases. The dilation will take the rays of the angle to themselves and so will not change the measure of the angle.

Activity Synthesis

Discuss each statement one at a time with this structure:

- Poll the class on their answer choice and display the answers.
- If everyone agrees on one answer, ask a few students to share their reasoning, recording it for all to see.
- If there is disagreement on an answer, ask students with opposing answers to explain their reasoning to come to an agreement on an answer.

7.2 Are They Similar?

10 minutes

In the previous lesson, students saw that figures are similar when there is a sequence of translations, rotations, reflections, and dilations that map one figure onto the other. This activity focuses on some common misconceptions about similar figures, and students have an opportunity to critique the reasoning of others (MP3). Two polygons with proportional side lengths but different angles are not similar and two polygons with the same angles but side lengths that are not proportional are also not similar. Reasoning through these problems will help foster student thinking about when two polygons are *not* similar.

Monitor for students who formulate these ideas for establishing that two figures are *not* similar:

- The angle measures are different (problem 1).
- The side lengths need to be multiplied by *different* scale factors in order to match up.

Select these students to share during the discussion.

Addressing

- 8.G.A.4

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Tell students they will look at some polygons that are claimed to be similar. What are some characteristics of similar polygons that are easy to recognize that can be used to confirm or deny the claims? Give 3 minutes of quiet work time followed by a whole-class discussion.

Support for Students with Disabilities

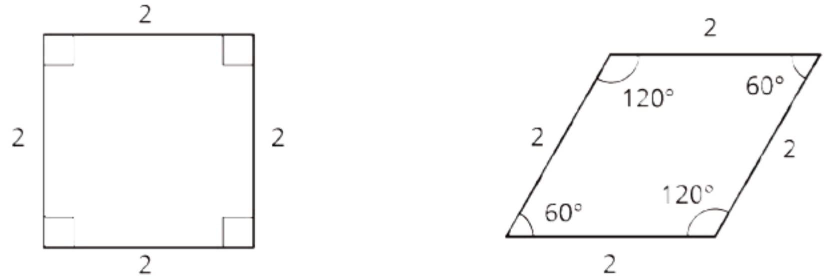
Action and Expression: Develop Expression and Communication. To help get students started, display sentence frames such as: “I agree with ___ because....” Encourage students to use the side lengths, angle measures, and scale factor in their reasoning.
Supports accessibility for: Language; Organization

Anticipated Misconceptions

Students might think that checking for angle congruence alone can determine similarity. They might also think that just checking if all side lengths have the same scaled value will be enough to determine similarity. However, both the angle sizes and the scale factors must be checked together in determining similarity.

Students might think the side lengths must be different in order for two figures to be similar, but a dilation does not need to be used in the sequence of transformations to show similarity (recall the warm-up where students saw that congruent figures are always similar).

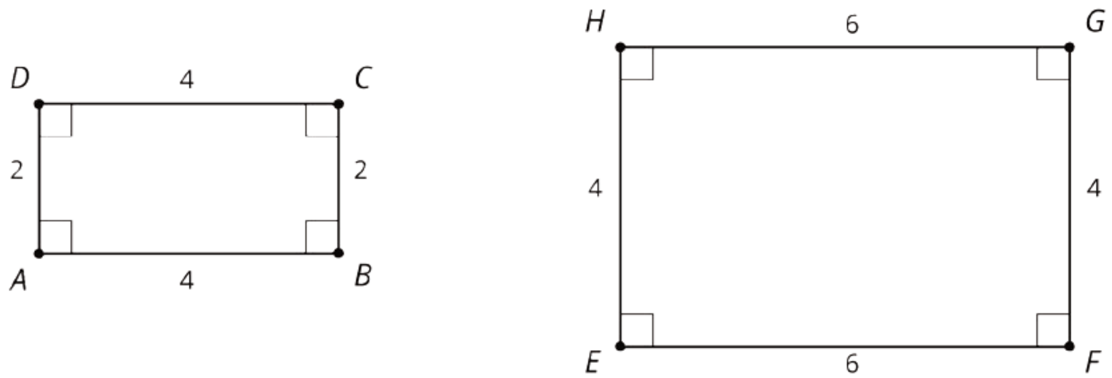
Student Task Statement



1. Let's look at a square and a rhombus.

   Priya says, "These polygons are similar because their side lengths are all the same." Clare says, "These polygons are not similar because the angles are different." Do you agree with either Priya or Clare? Explain your reasoning.

2. Now, let's look at rectangles $ABCD$ and $EFGH$.

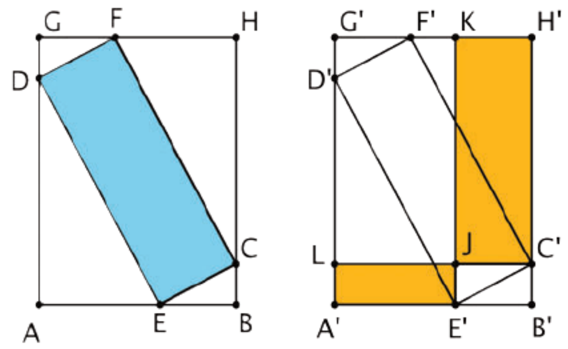


   Jada says, "These rectangles are similar because all of the side lengths differ by 2." Lin says, "These rectangles are similar. I can dilate AD and BC using a scale factor of 2 and AB and CD using a scale factor of 1.5 to make the rectangles congruent. Then I can use a translation to line up the rectangles." Do you agree with either Jada or Lin? Explain your reasoning.

Student Response

1. I agree with Clare. The angle measures in similar polygons are the same so these polygons can not be similar. Priya is not right: the side lengths of these polygons are the same but that is not enough to conclude that they are similar.
2. I disagree with both Jada and Lin. Jada is right that the side lengths all differ by 2 but the scale factor between the short sides of the rectangles and the scale factor between the long sides are not the same. This means that the two rectangles are not similar. Lin is not correct because the scale factor for dilating one set of sides can not be different than the scale factor for dilating the other set of sides.

Are You Ready for More?



Points A through H are translated to the right to create points A' through H'. All of the following are rectangles: $GHBA$, $FCED$, $KH'C'J$, and $LJE'A'$. Which is greater, the area of blue rectangle $DFCE$ or the total area of yellow rectangles $KH'C'J$ and $LJE'A'$?

Student Response

They are equal. You can show that rectangle $G'KJL$ is composed of triangles FHC and DEA, and rectangle $JC'B'E'$ is composed of triangles GFD and BEC.

Activity Synthesis

Invite selected students to share their choices and explanations, making sure that these points are highlighted:

1. If corresponding angles aren't congruent, then the figures cannot be similar (first problem).
2. If the side lengths are not all scaled by the same value, then the figures cannot be similar (second problem).

Unlike what Clare says, the same dilation (with the same scale factor) has to be performed to all parts of a figure to result in a similar figure. Another way to state the two important conclusions above is that for similar polygons:

- Corresponding angles are congruent.
- Corresponding side lengths are proportional.

Support for English Language Learners

Writing, Speaking, Listening: MLR1 Stronger and Clearer Each Time. After students have decided whether they agree with Jada or Lin, ask students to write a brief explanation of their reasoning. Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language (e.g., "What does it mean for two polygons to be similar?", "Why is Jada's or Lin's reasoning incorrect?", or "How did you know the rectangles are not similar?", etc.). Students can borrow ideas and language from each partner to refine and clarify their original explanation. This will help students revise and refine their reasoning and their verbal and written output. *Design Principles(s): Optimize output (for explanation); Maximize meta-awareness*

7.3 Find Someone Similar

15 minutes

In the previous activity, students learn that in order to be similar, two figures must have congruent corresponding angles and proportional corresponding side lengths. In this activity, students apply this knowledge. Each student has a card with a figure on it and they identify someone with a similar (but not congruent) figure.

Monitor for students using these methods to identify a partner:

- Process of elimination (figures with different angle measures cannot be similar to one another)
- Looking for the same *kind* of figure (rectangle, square, rhombus)
- Looking for a the same kind of figure with congruent corresponding angles
- Looking for a the same kind of figure with corresponding sides scaled by the same scale factor

Addressing

- 8.G.A.4

Instructional Routines

- MLR2: Collect and Display

Launch

Distribute one card to each student. Explain that the task is for each student to find someone else in the class who has a similar (but not congruent) figure to their own, and be prepared to explain how they know the two figures are similar.

If the number of students in class is not a multiple of 10, ensure that any unused cards are matching pairs of similar figures. If there is an odd number of students, one or more students can be responsible for two cards, or some students can be appointed as referees.

Support for Students with Disabilities

Representation: Internalize Comprehension. Begin by providing students with generalizations to test for similarity. Make references to the conclusions from the previous activity and consider displaying examples and counterexamples of similar polygons, focusing on distinguishable properties of the figures.
Supports accessibility for: Conceptual processing

Support for English Language Learners

Conversing, Reading: MLR2 Collect and Display. As students figure out who has a card with a figure similar to their own, circulate and listen to students as they decide whether their figures are similar but not congruent. Write down the words and phrases students use to justify why the figures are or are not similar. Listen for students who state that similar figures must have congruent corresponding angles and corresponding sides scaled by the same scale factor. As students review the language collected in the visual display, encourage students to clarify the meaning of a word or phrase. For example, a phrase such as “the figures are not similar because the sides do not match” can be clarified by restating it as “the figures are not similar because the corresponding sides are not scaled by the same scale factor.” This routine will provide feedback to students in a way that supports sense-making while simultaneously increasing meta-awareness of language.
Design Principle(s): Support sense-making; Maximize meta-awareness

Anticipated Misconceptions

If students have a hard time getting started, ask them to focus on properties of their figure that will be shared by a similar figure. For example, will a similar figure be a quadrilateral? Will a similar figure be square? A rectangle? A rhombus? What can you say about the angles in a similar figure?

Student Task Statement

Your teacher will give you a card. Find someone else in the room who has a card with a polygon that is similar but not congruent to yours. When you have found your partner, work with them to explain how you know that the two polygons are similar.

Student Response

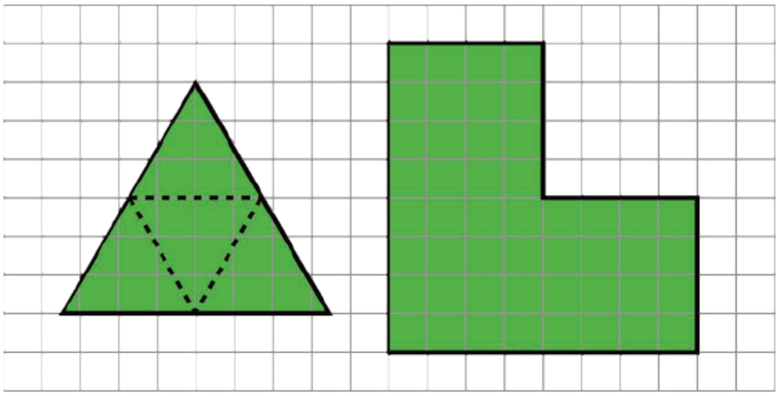
The similar quadrilaterals are:

- Rectangles with dimensions 6 by 8 and 9 by 12. Scale factor: $\frac{3}{2}$ or $\frac{2}{3}$

- Rectangles with dimensions 3 by 5 and 1.5 by 2.5. Scale factor: $\frac{1}{2}$ or 2
- Squares. Scale factor: $\frac{7}{8}$ or $\frac{8}{7}$
- Rhombuses with angles 60° and 120°. Scale factor: $\frac{9}{7}$ or $\frac{7}{9}$
- Rhombuses with angles 50° and 130°. Scale factor: $\frac{10}{9}$ or $\frac{9}{10}$

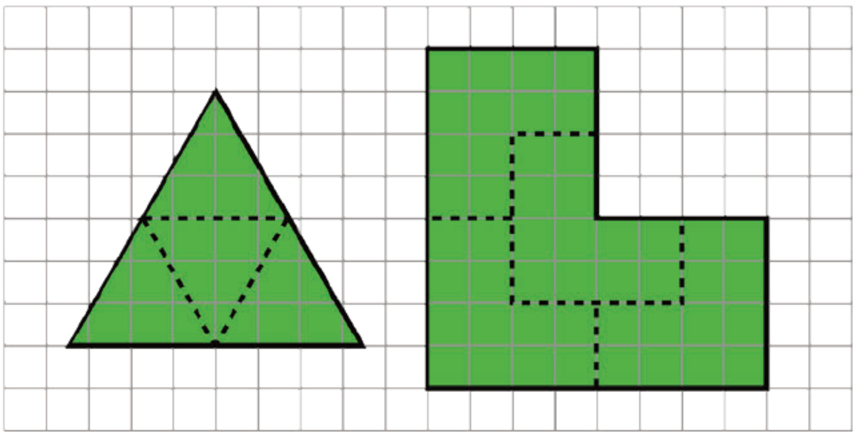
Are You Ready for More?

On the left is an equilateral triangle where dashed lines have been added, showing how you can partition an equilateral triangle into smaller similar triangles.



Find a way to do this for the figure on the right, partitioning it into smaller figures which are each similar to that original shape. What's the fewest number of pieces you can use? The most?

Student Response



The fewest you can use is 4 pieces, as in the image, each a $\frac{1}{2}$-scale dilation of the original. There is no upper limit on the number of pieces you could use. For example, you could take the four $\frac{1}{2}$-scale pieces and divide them each using four $\frac{1}{4}$-scale pieces in exactly the same pattern, then cover each of those with four $\frac{1}{8}$-scale pieces, etc.

Activity Synthesis

Invite selected students to share how they found a partner. Highlight different strategies: it is possible to proceed by process of elimination (this shape is a rectangle and mine is not so it is not a match) or by actively looking for certain features (e.g. a rhombus whose angle measures are 60 degrees and 120 degrees). Next ask how they knew their polygons were similar and which scale factor they used to show similarity.

Ask students what they looked for in searching for a partner with a similar figure. In particular ask

- "Were the side lengths important?" (For some figures, such as the rectangles, the side lengths were important.)
- "Were the angles important?" (Yes. Several of the figures had different angles and that meant they were not similar.)

Students may notice that *all* squares are similar since they have 4 right angles and proportional side lengths. On the other hand, this activity provides examples of rhombuses that are *not* similar to one another.

Lesson Synthesis

Important take aways from this lesson include:

- Similar figures have congruent corresponding angles *and* proportional corresponding side lengths.
- For some figures (like rectangles or squares), it is sufficient to focus on side lengths since corresponding angles are automatically congruent.
- For some figures (like rhombuses), it is sufficient to focus on angles since corresponding side lengths are automatically proportional.

Make sure students understand that, in general, determining whether two polygons are similar requires examining *both* side lengths *and* angles. On the other hand, it is possible to determine that two polygons are *not* similar by identifying a single pair of corresponding angles whose measures are different or a pair of corresponding side lengths with a different ratio than another pair of corresponding side lengths.

7.4 How Do You Know?

Cool Down: 5 minutes

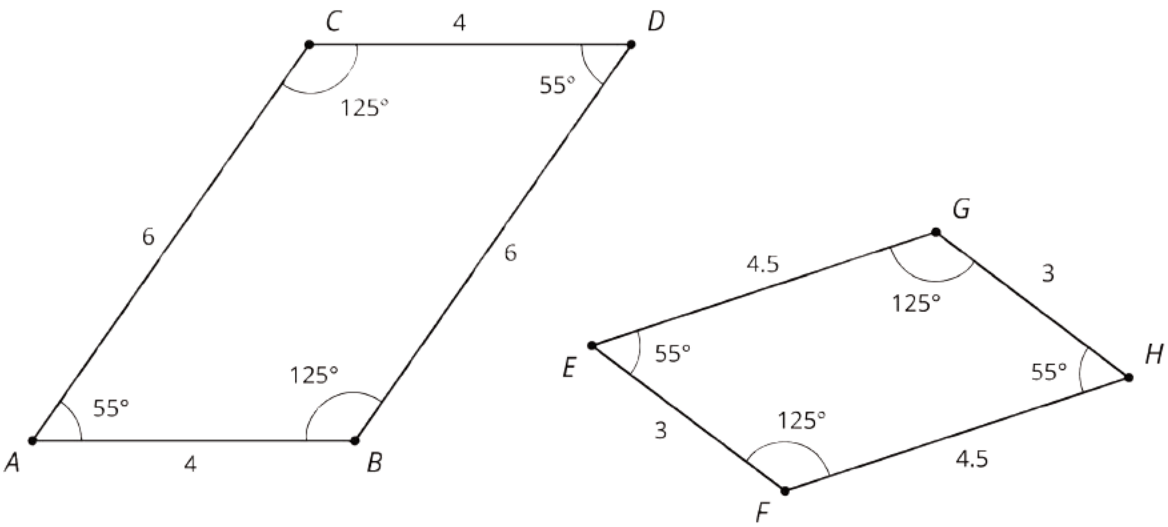
Students explain why two quadrilaterals are similar. Since they do not have criteria for when two parallelograms are similar, they will need to produce a sequence of rigid motions and dilations that take one figure to the other. They should be able to describe these motions rather than performing them.

Addressing

- 8.G.A.4

Student Task Statement

Explain how you know these two figures are similar.



Student Response

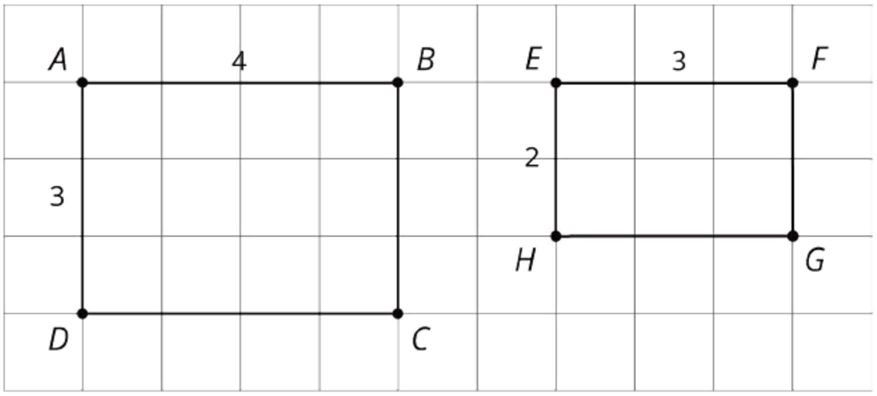
Dilating quadrilateral $ABCD$ with center A and scale factor $\frac{3}{4}$ gives a quadrilateral that is congruent to $EFGH$. This can be shown with a translation of A to E and then a rotation with center E.

Student Lesson Summary

When two polygons are similar:

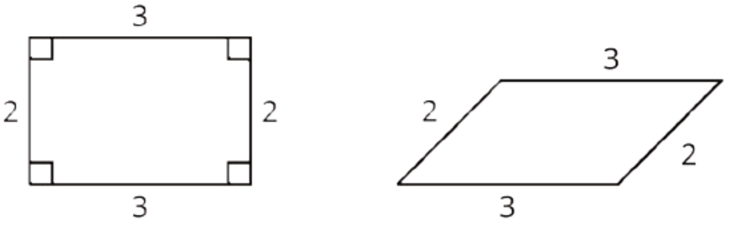
- Every angle and side in one polygon has a corresponding part in the other polygon.
- All pairs of corresponding angles have the same measure.
- Corresponding sides are related by a single scale factor. Each side length in one figure is multiplied by the scale factor to get the corresponding side length in the other figure.

Consider the two rectangles shown here. Are they similar?



It looks like rectangles $ABCD$ and $EFGH$ could be similar, if you match the long edges and match the short edges. All the corresponding angles are congruent because they are all right angles. Calculating the scale factor between the sides is where we see that "looks like" isn't enough to make them similar. To scale the long side AB to the long side EF, the scale factor must be $\frac{3}{4}$, because $4 \cdot \frac{3}{4} = 3$. But the scale factor to match AD to EH has to be $\frac{2}{3}$, because $3 \cdot \frac{2}{3} = 2$. So, the rectangles are not similar because the scale factors for all the parts must be the same.

Here is an example that shows how sides can correspond (with a scale factor of 1), but the quadrilaterals are not similar because the angles don't have the same measure:



Lesson 7 Practice Problems

Problem 1

Statement

Triangle DEF is a dilation of triangle ABC with scale factor 2. In triangle ABC, the largest angle measures 82°. What is the largest angle measure in triangle DEF?

A. 41°

B. 82°

C. 123°

D. 164°

Solution

B

Problem 2

Statement

Draw two polygons that are similar but could be mistaken for not being similar. Explain why they are similar.

Solution

Answers vary. Sample response: Two polygons with different orientations; two congruent polygons. Another sample response: Two polygons where a reflection is part of the transformation from one to the other.

Problem 3

Statement

Draw two polygons that are *not* similar but could be mistaken for being similar. Explain why they are not similar.

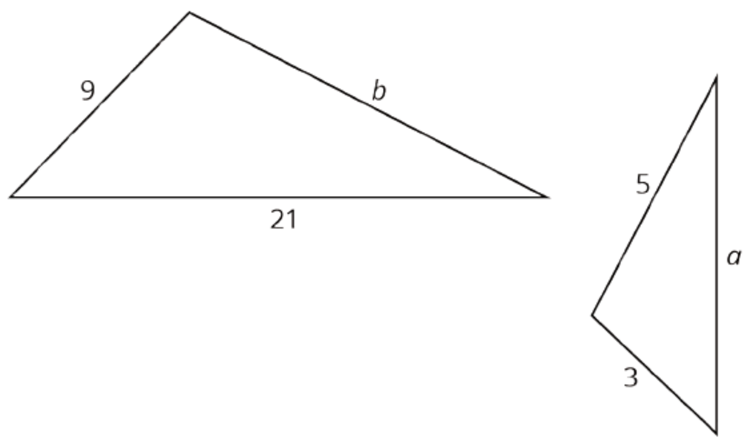
Solution

Answers vary. Sample response: Two polygons with the same angle measures but side lengths that are not proportional. Another sample response: Two polygons with proportional side lengths but incorrect angle measures.

Problem 4

Statement

These two triangles are similar. Find side lengths a and b. Note: the two figures are not drawn to scale.



Solution

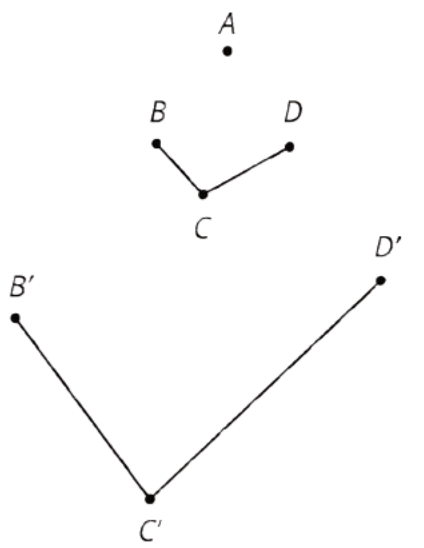
$a = 7, b = 15$

Problem 5

Statement

Jada claims that $B'C'D'$ is a dilation of BCD using A as the center of dilation.

What are some ways you can convince Jada that her claim is not true?



Solution

Answers vary. Below is a list of things which would have to be true if $B'C'D'$ is a dilation of BCD using A as the center of dilation. Any measurement which showed any of these to not hold is a complete answer.

- $m\angle BCD = m\angle B'C'D'$
- A, B, and B' should be collinear.
- A, C, and C' should be collinear.
- A, D, and D' should be collinear.
- $B'C'$ should be parallel to BC.
- $C'D'$ should be parallel to CD.

(From Unit 2, Lesson 3.)

Problem 6

Statement

a. Draw a horizontal line segment AB.

b. Rotate segment AB 90° counterclockwise around point A. Label any new points.

c. Rotate segment AB 90° clockwise around point B. Label any new points.

d. Describe a transformation on segment AB you could use to finish building a square.

Solution

a. Answers vary.

b. The segment is attached at point A and forms a right angle.

c. The segment is attached at point B and forms a right angle, parallel and in the same direction as the previous segment.

d. Answers vary. Sample response: Translate A to C.

(From Unit 1, Lesson 8.)

Lesson 8: Similar Triangles

Goals

- Generalize a process for identifying similar triangles and justify (orally) that finding two pairs of congruent angles is sufficient to show similarity.
- Justify (orally) that two triangles are similar by finding a sequence of transformations that takes one triangle to the other or checking that two pairs of corresponding angles are congruent.

Learning Targets

- I know how to decide if two triangles are similar just by looking at their angle measures.

Lesson Narrative

In the previous lesson, students found that, in order to check if two quadrilaterals are similar, it is important, in general, to check that corresponding angles are congruent and that corresponding side lengths are proportional. This lesson focuses on triangles. Triangles are special since it is possible to determine whether or not they are similar by looking *only* at the angles. If two triangles share three corresponding angle measurements, then they are similar. In fact, since the sum of the angle measures in a triangle is 180 degrees, two angle measures determine the third. Hence for triangles, all that is needed to deduce similarity is having *two* corresponding angles with equal measure.

Students deduce the criteria for similarity in terms of angle measures by experimenting with triangles built out of pasta. As a result, they will need to make sense of measurements and account for possible inaccuracies.

Students will use the similarity criterion in future lessons to understand the concept of the slope of a line. Later on in high school, they will learn that three proportional sides (but not two) is also enough to deduce that two triangles are similar.

Alignments

Building On

- 7.NS.A: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
- 7.RP.A.2.a: Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- 8.G.A.4: Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Addressing

- 8.G.A: Understand congruence and similarity using physical models, transparencies, or geometry software.
- 8.G.A.5: Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Group Presentations
- MLR2: Collect and Display
- MLR5: Co-Craft Questions

Required Materials

Blank paper
Dried linguine pasta
We specified linguine since it is flatter and less likely to roll around than spaghetti.

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Pre-printed slips, cut from copies of the blackline master
Tape

Required Preparation

Make 1 copy of the blackline master for every 4 students. Cut these into strips horizontally. Each student will receive one strip, which is a set of three angles labeled A, B, and C.

For the dried pasta that will be used to create the sides of the triangles, we recommend fettuccine or linguine so it doesn't roll off the table and is easy to break as needed.

Student Learning Goals

Let's look at similar triangles.

8.1 Equivalent Expressions

Warm Up: 5 minutes

This warm-up prompts students to use what they know about operations to create related expressions. While many warm-ups in the unit encourage students to work mentally and verbally, students will write their responses to this prompt. Since many different responses are possible, the task is accessible to all students and provides an opportunity to hear how each student reasons about the operations.

Building On

- 7.NS.A

Launch

Arrange students in groups of 2. Display problem. Give students 1 minute of quiet think time and ask them to give a signal when they have three expressions. Follow with a 1 minute partner discussion and then a whole-class discussion.

Anticipated Misconceptions

If students only rely on the associative and commutative properties, suggest that they try to include at least one expression containing parentheses.

> ### Student Task Statement
>
> Create three different expressions that are each equal to 20. Each expression should include only these three numbers: 4, -2, and 10.

Student Response

Answers vary. Possible responses:

- $-(4 \cdot 10) \div -2$
- $-(10 \div -2) \cdot 4$
- $-(4 \div -2) \cdot 10$

Activity Synthesis

Ask selected students to share their expressions. Record and display their responses for all to see. Ask students if or how the factors in the problem impacted their strategy. To involve more students in the conversation, consider asking:

- "Did anyone have a similar expression?"
- "Did anyone have a different expression related to this one?"
- "Do you agree or disagree? Why?"

8.2 Making Pasta Angles and Triangles

30 minutes

In this activity, students create triangles with given angles and compare them to classmates' triangles to see that they are not necessarily congruent, but they are similar.

Each angle measure in a triangle imposes a constraint and students are examining in this activity how many constraints there are in similar triangles. There are only two. This makes sense since we can place one side of the triangle and then the triangle is determined by the length of that side and the two angles made with that side. (Note: the length of that side can be scaled with an appropriate dilation to give any particular similar triangle with these angles.)

Watch for how students deal with measurement error. Remind students who are looking for proportional side lengths in similar triangles that they may not be exactly proportional. Also, students may find that their rounded angle measures do not add up to 180°. Remind them that the measurements are only approximate.

Building On

- 7.RP.A.2.a
- 8.G.A.4

Addressing

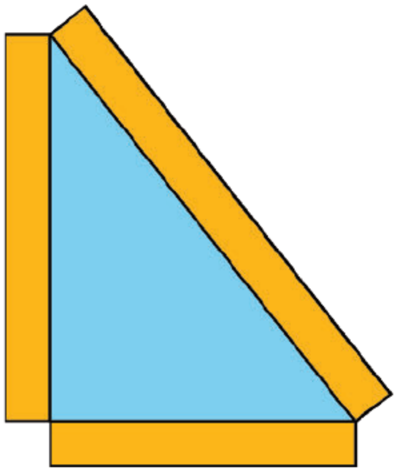
- 8.G.A.5

Instructional Routines

- Group Presentations
- MLR2: Collect and Display

Launch

Tell students that they are going to build triangles using pasta and some of their given angles. Then they will find classmates who used the same angle(s) and compare their triangles. Tell them that they will need to move around the classroom to identify partners who use the same angle(s). For large classrooms, consider asking students to find three matching partners instead of two. Demonstrate how to measure angles in a pasta triangle.



In this picture, the approximate angle measures are 90°, 50°, and 40°. Tell students that they can trace their angle(s) on the sheet of paper they are going to use to build their pasta triangle.

Provide students with 1 strip of 3 angles (A, B, and C) pre-cut from the blackline master. Each student should also have access to tape, and extra paper to tape their triangles to. Finally, each student will need a ruler (which should be available in their geometry toolkits).

Pause students after they have made and compared triangles with one angle (first problem) and three angles (second problem). Make sure that students have found some non-similar triangles on the first problem and similar triangles for the second. Consider doing a gallery walk to see how the first sets of triangles differ and how the second sets of triangles are alike. Then have students work on the third problem.

Support for Students with Disabilities

Representation: Develop Language and Symbols. Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Demonstrate how to build a triangle using pasta and how to use a protractor and ruler to measure the angles and lengths of each side. Be sure to emphasize appropriate rounding on the measurements.
Supports accessibility for: Visual-spatial processing; Conceptual processing

Support for English Language Learners

Conversing, Reading: MLR2 Collect and Display. As students find others who have the same angles in their triangles, circulate and listen to students as they decide whether their triangles are similar. Write down the words and phrases students use to justify why the triangles are or are not similar. Listen for students who state that two shared angles is enough to guarantee that two triangles are similar, but one shared angle is not enough. As students review the language collected in the visual display, encourage students to clarify the meaning of a word or phrase. For example, a phrase such as "the triangles are not similar because the angles do not match" can be clarified by restating it as "the triangles are not similar because their corresponding angles are not congruent." This routine will provide feedback to students in a way that supports sense-making while simultaneously increasing meta-awareness of language.
Design Principle(s): Support sense-making; Maximize meta-awareness

Anticipated Misconceptions

Students may need a reminder that the sum of angles in any triangle is 180 degrees.

Student Task Statement

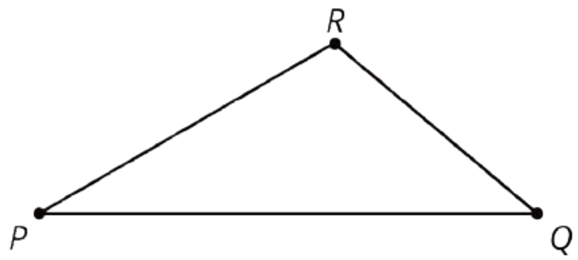
Your teacher will give you some dried pasta and a set of angles.

1. Create a triangle using three pieces of pasta and angle A. Your triangle *must* include the angle you were given, but you are otherwise free to make any triangle you like. Tape your pasta triangle to a sheet of paper so it won't move.
   a. After you have created your triangle, measure each side length with a ruler and record the length on the paper next to the side. Then measure the angles to the nearest 5 degrees using a protractor and record these measurements on your paper.
   b. Find two others in the room who have the same angle A and compare your triangles. What is the same? What is different? Are the triangles congruent? Similar?
   c. How did you decide if they were or were not congruent or similar?
2. Now use more pasta and angles A, B, and C to create another triangle. Tape this pasta triangle on a separate sheet of paper.
   a. After you have created your triangle, measure each side length with a ruler and record the length on the paper next to the side. Then measure the angles to the nearest 5 degrees using a protractor and record these measurements on your paper.

b. Find two others in the room who used your same angles and compare your triangles. What is the same? What is different? Are the triangles congruent? Similar?

c. How did you decide if they were or were not congruent or similar?

3. Here is triangle PQR. Break a new piece of pasta, different in length than segment PQ.



- Tape the piece of pasta so that it lays on top of line PQ with one end of the pasta at P (if it does not fit on the page, break it further). Label the other end of the piece of pasta S.
- Tape a full piece of pasta, with one end at S, making an angle congruent to $\angle PQR$.
- Tape a full piece of pasta on top of line PR with one end of the pasta at P. Call the point where the two full pieces of pasta meet T.

a. Is your new pasta triangle PST similar to $\triangle PQR$? Explain your reasoning.

b. If your broken piece of pasta were a different length, would the pasta triangle still be similar to $\triangle PQR$? Explain your reasoning.

Student Response

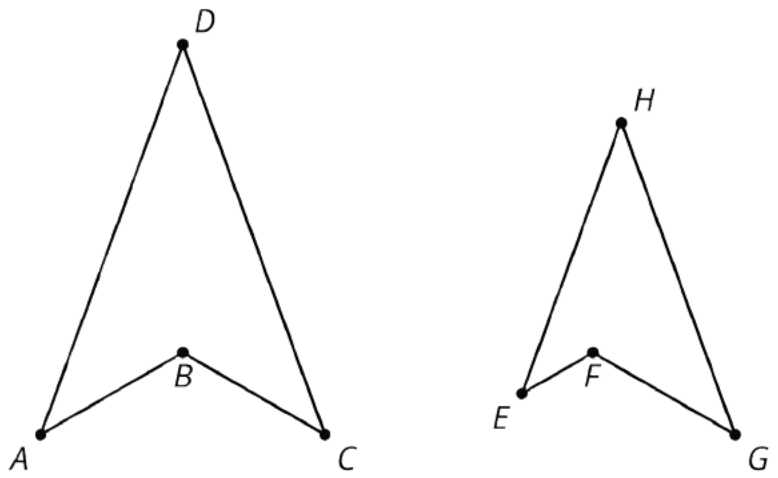
1. Measurements and answers vary. The initial one-angle triangles will often only have the one angle in common. They are not necessarily similar. One way to check that they are not similar is to see if some of the angles have different measures.
2. Answers vary. Sample response: The three-angle triangles are similar but the measured side lengths may not be exactly proportional because of possible measurement error.
3. Answers vary. The two-angle triangles are similar. The interior angles of a triangle sum to 180. So if two pair of angles are congruent, the third pair are also congruent. Again, the side lengths may not be exactly proportional due to measurement error.

Are You Ready for More?

Quadrilaterals $ABCD$ and $EFGH$ have four angles measuring 240°, 40°, 40°, and 40°. Do $ABCD$ and $EFGH$ have to be similar?

Student Response

No. We can start with a 240 degree angle ABC and then place D so that BAD and BCD are 40 degree angles.



Then angle ADC will also be 40 degrees. We can make one of these quadrilaterals no matter where A and C are placed. The figure shows another example of this construction: by placing E closer to the 240 degree angle EFG than A was to angle ABC, we do not change the angles, but we get a non-similar quadrilateral.

Activity Synthesis

Ask students if they needed all three angle measurements to build a triangle in the second problem. Some students may notice that it was not necessary: once they knew two angles, that was enough to build the pasta triangle, the third angle automatically being the right measure. Other students may find that having the third angle was helpful to get better accuracy for all three angle measures. Ask students how this relates to their findings on the last question: the last question shows that two shared angle measures would be enough to guarantee that two triangles are similar. This makes sense because the sum of the angle measures in a triangle is 180 degrees so triangles that share two pairs of congruent angles actually share three pairs of congruent angles.

Some important discussion questions include

- “Did your angles in the triangle always add up to 180 degrees?” (Answers may vary for the first triangles because the angles were rounded to the nearest 5 degrees and the rounding can influence the sum of the angles.)
- “How did you decide whether or not the sides of your triangle were proportional to other triangles?” (Answers vary, the key being that measurement error means that quotients

computed from measured side lengths may not be *exactly* equal even if the triangles are similar.)

- "How did you check whether or not your triangle was similar to another?" (Answers should include aligning an angle and then dilating when the triangles were similar, and observing that the angles were different or the sides were not proportional when they were not similar.)

A big conclusion from this activity is that if triangles share two pair of congruent angles then they are similar. For the quadrilaterals studied in the previous activity, there was another instance where a pair of congruent angles was enough to decide that quadrilaterals are similar, namely rhombuses. But the triangle result is more surprising because there are three side lengths and *no* restrictions are made on these.

8.3 Similar Figures in a Regular Pentagon

Optional: 10 minutes
This activity presents a complex figure with many triangles and asks students to find triangles similar to a given triangle. From the previous activity, students know that finding two pair of congruent angles is sufficient to show similarity. Students may also measure all three angles and check that they are congruent. In addition to using angle measures, students can describe transformations that take one triangle to another.

Monitor for students who use these methods to show that their chosen triangles are similar to triangle DJI:

- finding a sequence of translations, rotations, reflections, and dilations
- checking that two (or three) corresponding angles are congruent

Select students who use these methods and invite them to present during the discussion.

Addressing

- 8.G.A

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR5: Co-Craft Questions

Launch

Tell students that they may use anything in their geometry toolkits as well as a copy of triangle DJI to find triangles similar to it.

Support for Students with Disabilities

Representation: Internalize Comprehension. Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, use the same color to illustrate which triangles are similar to DJI. Annotate side and angle measurements in the appropriate places on the image.
Supports accessibility for: Visual-spatial processing

Support for English Language Learners

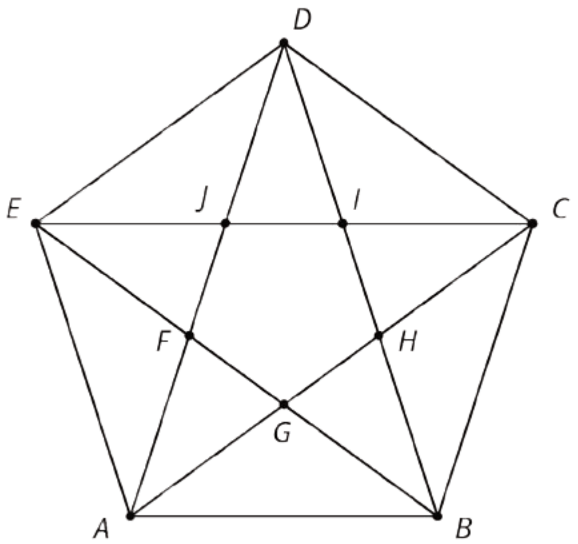
Conversing, Writing: MLR5 Co-Craft Questions. Before presenting the questions in this activity, display the diagram and ask students to write possible mathematical questions about the diagram. Ask students to compare the questions they generated with a partner before sharing questions with the whole class. Listen for and amplify questions about which triangles are similar to each other. Then reveal and ask students to work on the actual questions of the task. This routine will help develop students' meta-awareness of language as they generate questions about identifying similar triangles.
Design Principle(s): Maximize meta-awareness

Anticipated Misconceptions

If students eyeball the triangles in predicting similar triangles, make sure they justify their decisions by including specific measurements.

Student Task Statement

1. This diagram has several triangles that are similar to triangle DJI.



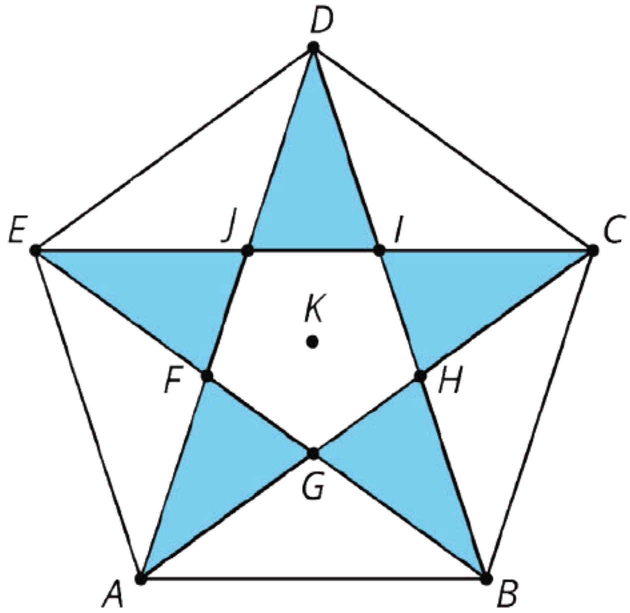
a. Three different scale factors were used to make triangles similar to DJI. In the diagram, find at least one triangle of each size that is similar to DJI.

b. Explain how you know each of these three triangles is similar to DJI.

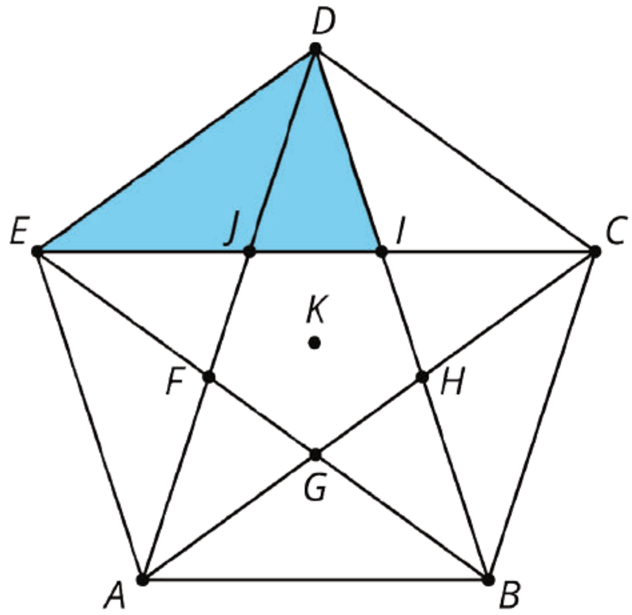
2. Find a triangle in the diagram that is not similar to DJI.

Student Response

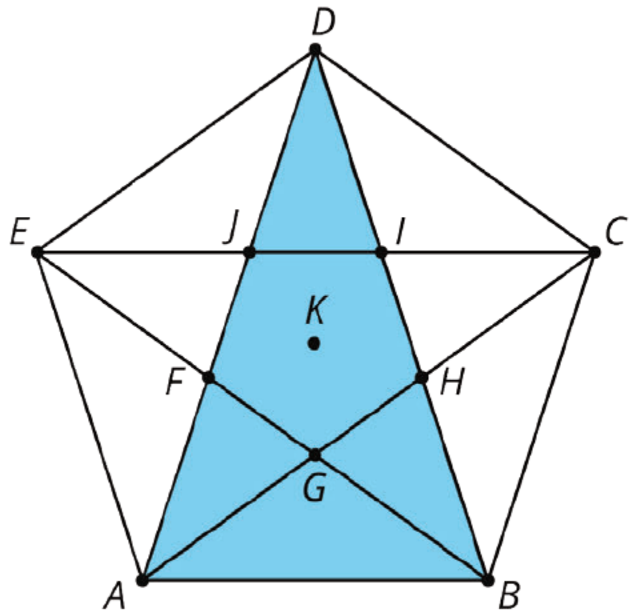
1. Answers vary.
    a. Options are (one from each)
        i. Congruent to DJI (scale factor 1): DJI, EJF, AFG, BGH, CHI



ii. Middle sized (scale factor about 1.5 from DJI): $EID, DJC, EGA, AEJ, BAF, ABH, CGB, BIC, DHC, DEF$



iii. Large sized (scale factor about 2.5 from DJI): DAB, BCE, ACD, BDE, CEA



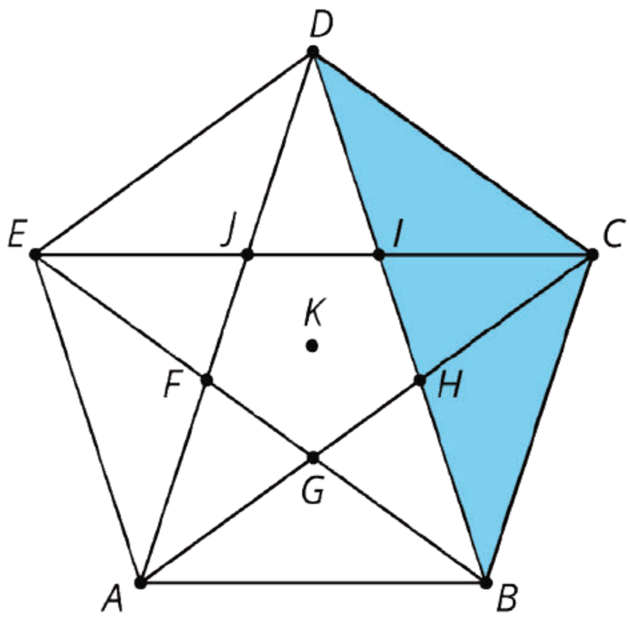
b. Answers vary. Some options:

i. Check two of the angles to be the same as DJI.

ii. Find transformations that take DJI to the other triangles.

iii. Measure the angles to be congruent and the side lengths to be proportional.

2. Answers vary. Options are:
$DEJ, EFA, ABG, BCH, CDI, BDC, CDE, DEA, ABE, ABC, DFB, EGC, DAH, BIE, ACJ$.

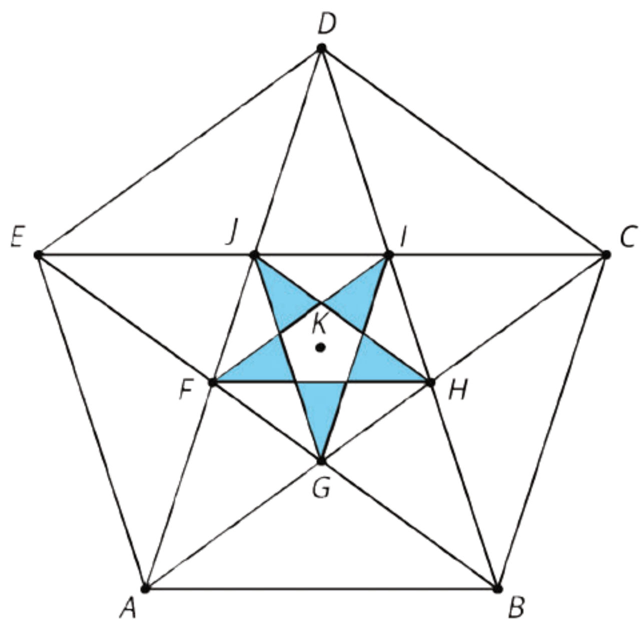


Are You Ready for More?

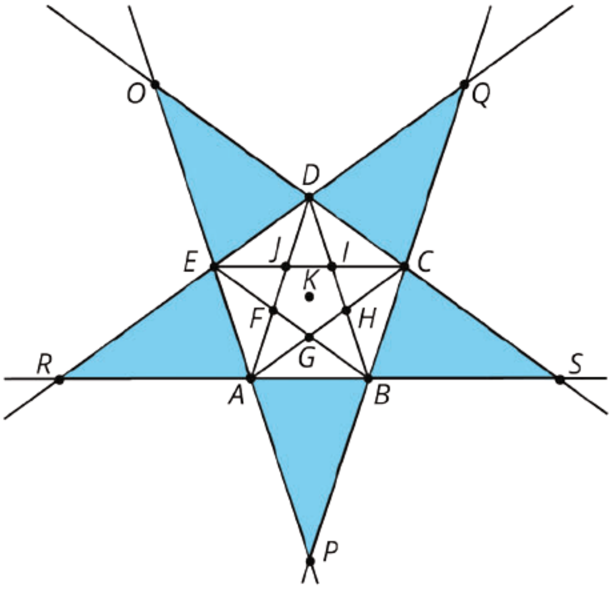
Figure out how to draw some more lines in the pentagon diagram to make more triangles similar to DJI.

Student Response

Some possibilities: Draw in the star inscribed in the inner pentagon $FGHIJ$, or just one of these segments.



Extend the sides of pentagon $ABCDE$ until the lines intersect.



Activity Synthesis

Invite selected students to share some of the triangles they found and explain how they determined the triangles to be similar. Sequence them so students who used rigid transformations and dilations go first, followed by those who use angle measures. Concluding that two triangles are similar using the angle criterion from the previous task is a quick, efficient argument. On the other hand, finding explicit transformations and dilations taking one triangle to another provides a more tactile, concrete experience. Both are important methods and students can choose based on their own personal comfort and the nature of the problem at hand.

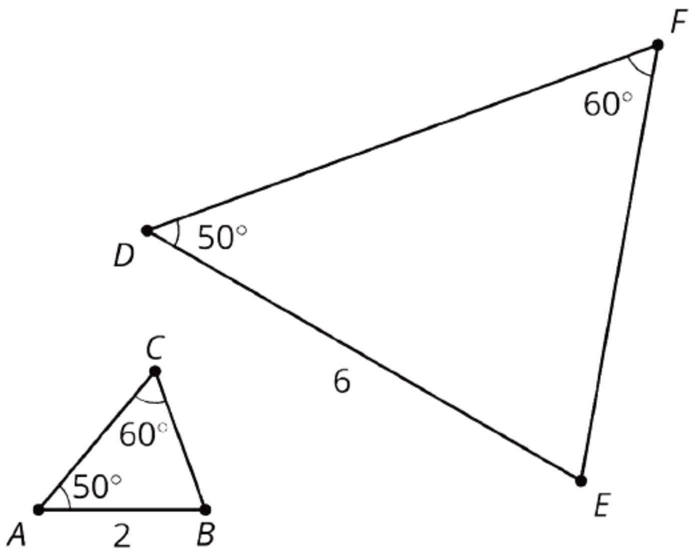
Make a list of different ways to show that two triangles are similar, including:

- using transformations (cutting out the triangle and placing it appropriately)
- finding two congruent corresponding angles in the triangles
- finding three congruent corresponding angles in the triangles

Among those triangles that are not similar to DJI, ask if some of these may be similar to each other.

Lesson Synthesis

This lesson focuses on how the angle measures influence the shape of a triangle. If a triangle has a 50 degree angle, what does that tell me about its shape? Could it be isosceles? (yes, two 50 degree angles and an 80 degree angle). Could it fail to be isosceles? (yes, one 50 degree angle, one 20 degree angle, and one 110 degree angle). Are all triangles with a 50 degree angle similar? (no, because the other two pairs of angles could be different).



If triangles have two pairs of congruent angles, then they are similar. For example, here are two triangles which each have a 50° angle and a 60° angle.If we dilate ABC with center A and scale factor 3, then AB has the same length as DE. We can apply rigid motions so that angle A matches up with angle D and angle B matches up with angle E. The vertices C and F also match up and ABC is similar to DEF.

8.4 Applying Angle-Angle Similarity

Cool Down: 5 minutes

Students explain why two triangles are similar. They have at least two good options available. They can measure angles in the triangles and use what they learned in this lesson or they can describe similarity transformations (in this case a dilation centered at $(0, 0)$) that take one triangle to the other.

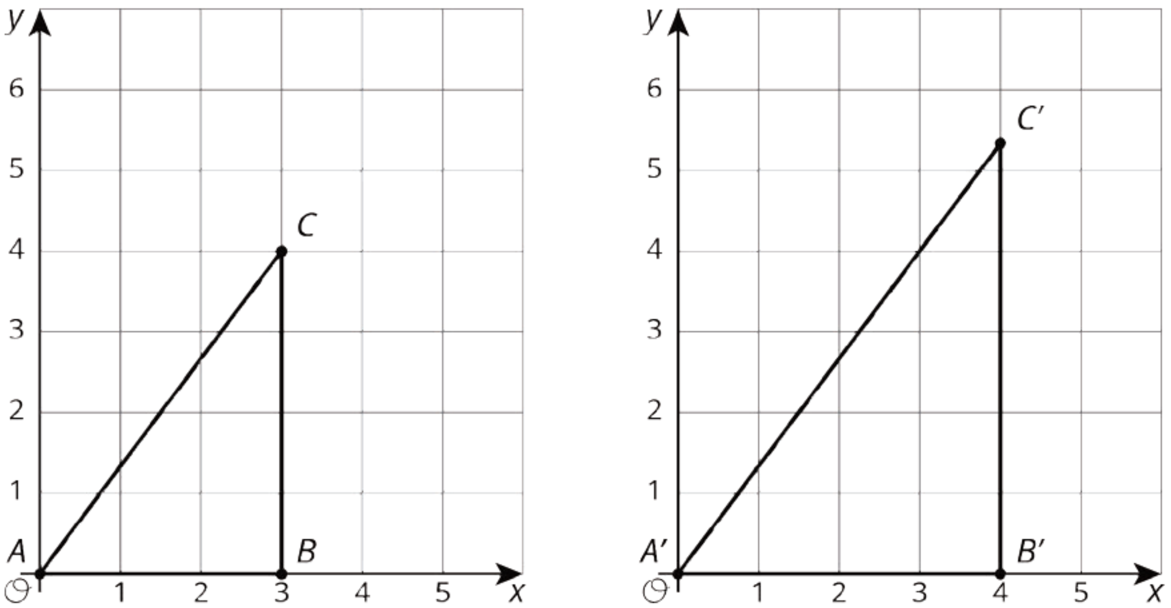
Addressing

- 8.G.A

Launch

Students should have access to Geometry toolkit, especially a protractor

> **Student Task Statement**
>
> Here are two triangles.



1. Show that the triangles are similar.

2. What is the scale factor from triangle ABC to triangle $A'B'C'$?

Student Response

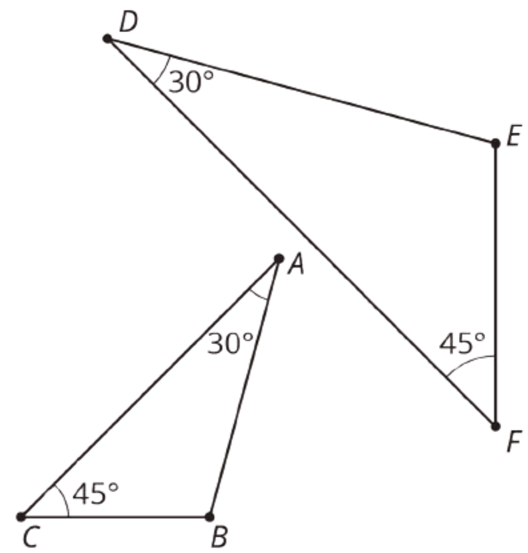
1. All three corresponding angles are congruent. Angle ABC and angle $A'B'C'$ are both right angles. Angle CAB and angle $C'A'B'$ are congruent (they both measure about 37 degrees). Angle ACB and angle $A'C'B'$ are congruent as well (they both measure about 53 degrees).

2. The scale factor is $1\frac{1}{3}$ since the length of segment AB is 3 and the length of corresponding segment $A'B'$ is 4.

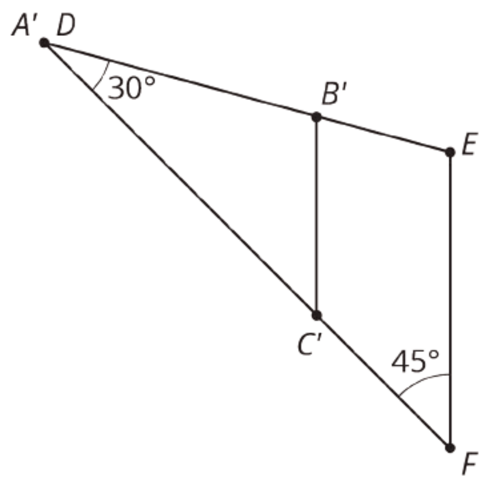
Student Lesson Summary

We learned earlier that two polygons are similar when there is a sequence of translations, rotations, reflections, and dilations taking one polygon to the other. When the polygons are triangles, we only need to check that that both triangles have two corresponding angles to show they are similar—can you tell why?

Here is an example. Triangle ABC and triangle DEF each have a 30 degree angle and a 45 degree angle.



We can translate A to D and then rotate so that the two 30 degree angles are aligned, giving this picture:



Now a dilation with center D and appropriate scale factor will move C' to F. This dilation also moves B' to E, showing that triangles ABC and DEF are similar.

Lesson 8 Practice Problems

Problem 1

Statement

In each pair, some of the angles of two triangles in degrees are given. Use the information to decide if the triangles are similar or not. Explain how you know.

- Triangle A: 53, 71, ___; Triangle B: 53, 71, ___
- Triangle C: 90, 37, ___; Triangle D: 90, 53, ___
- Triangle E: 63, 45, ____; Triangle F: 14, 71, ____

- Triangle G: 121, ___, ___; Triangle H: 70, ___, ___

Solution

- Similar: They have two pairs of angles with equal measurement.
- Similar: Since the angles in a triangle add up to 180°, the missing angle in Triangle C must be 53°. The two triangles therefore have two pairs of angles with equal measurement, so they are similar.
- Not similar: Similar triangles have equal angle measurements, and there is no way to fill in the blanks so that this is true for these two triangles.
- Not similar: Similar triangles have equal angle measurements, but no triangle can have angles which measure 121 and 70 degrees as these add up to more than 180.

Problem 2

Statement

a. Draw two equilateral triangles that are not congruent.

b. Measure the side lengths and angles of your triangles. Are the two triangles similar?

c. Do you think two equilateral triangles will be similar *always*, *sometimes*, or *never*? Explain your reasoning.

Solution

a. Answers vary.

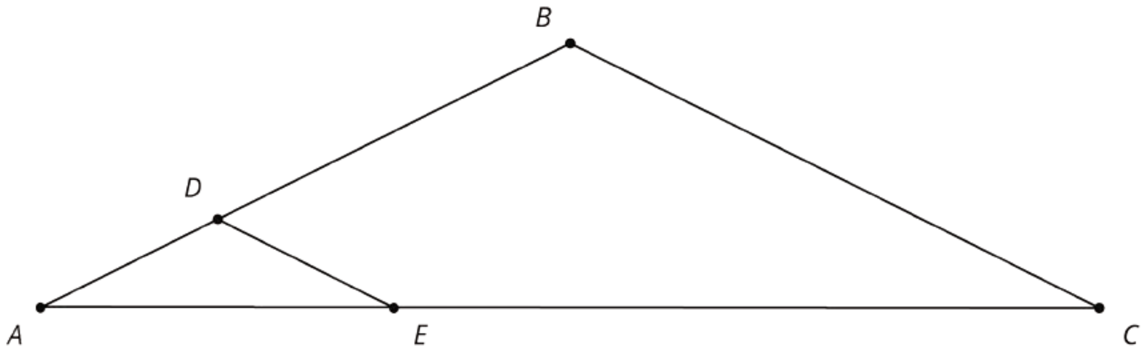
b. The side lengths in each triangle should be equal, and the angle measures should all be 60°. The triangles are similar, because the angle measures are equal.

c. Always. All equilateral triangles have the same angle measures, so they are all similar.

Problem 3

Statement

In the figure, line BC is parallel to line DE.



Explain why $\triangle ABC$ is similar to $\triangle ADE$.

Solution

Answers vary.

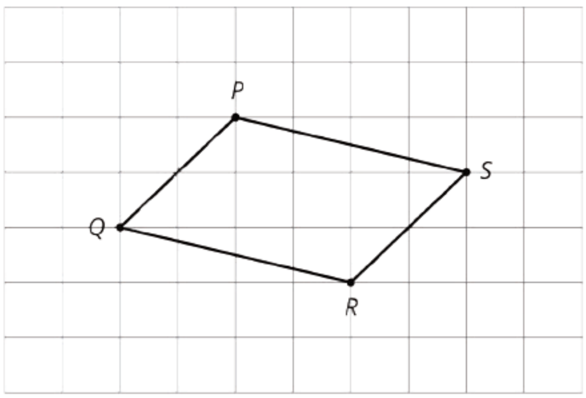
Sample Solution 1: Triangles ABC and ADE share angle A. Line AC is a transversal for parallel lines BC and DE. Therefore, angles ADE and ABC are congruent. Since they share two congruent angles, triangles ABC and ADE are similar.

Sample Solution 2: A dilation with center A and appropriate scale factor will take triangle ABC to triangle ADE. The scale factor looks like it is about $\frac{1}{3}$.

Problem 4

Statement

The quadrilateral $PQRS$ in the diagram is a parallelogram. Let $P'Q'R'S'$ be the image of $PQRS$ after applying a dilation centered at a point O (not shown) with scale factor 3.



Which of the following is true?

A. $P'Q' = PQ$

B. $P'Q' = 3PQ$

C. $PQ = 3P'Q'$

D. Cannot be determined from the information given

Solution

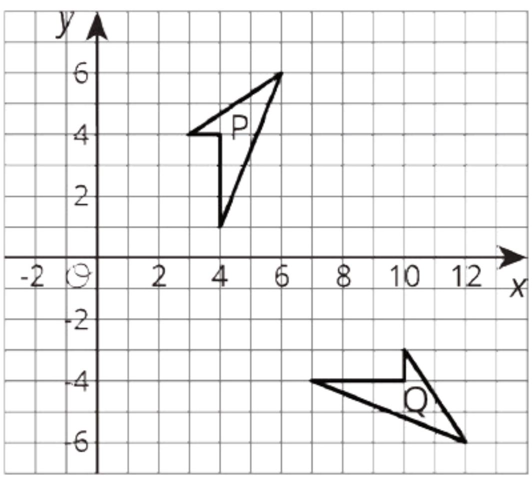
B

(From Unit 2, Lesson 4.)

Problem 5

Statement

Describe a sequence of transformations for which Quadrilateral P is the image of Quadrilateral Q.



Solution

Answers vary. Sample response: Translate Q 3 units left and 5 units up. Now, they share a point. Rotate using this point as the center, 90 degrees counterclockwise.

(From Unit 1, Lesson 6.)

Lesson 9: Side Length Quotients in Similar Triangles

Goals

- Calculate unknown side lengths in similar triangles using the ratios of side lengths within the triangles and the scale factor between similar triangles.
- Generalize (orally) that the quotients of pairs of side lengths in similar triangles are equal.

Learning Targets

- I can decide if two triangles are similar by looking at quotients of lengths of corresponding sides.
- I can find missing side lengths in a pair of similar triangles using quotients of side lengths.

Lesson Narrative

In prior lessons, students learned that similar triangles are the images of each other under a sequence of rigid transformations and dilations, and that as a result, there is a scale factor that we can use to multiply all of the side lengths in one triangle to find the corresponding side lengths in a similar triangle. In this lesson, they will discover that if you determine the quotient of a pair of side lengths in one triangle, it will be equal to the quotient of the corresponding side lengths in a similar triangle. While this fact is not limited to triangles, this lesson focuses on the particular case of triangles so that students are ready to investigate the concept of slope in upcoming lessons.

Alignments

Building On

- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

Addressing

- 8.G.A: Understand congruence and similarity using physical models, transparencies, or geometry software.
- 8.G.A.4: Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Student Learning Goals

Let's find missing side lengths in triangles.

9.1 Two-three-four and Four-five-six

Warm Up: 5 minutes
Two sets of triangle side lengths are given that do not form similar triangles. Students should recognize that there is no single scale factor that multiplies all of the side lengths in one triangle to get the side lengths in the other triangle.

Addressing

- 8.G.A

Launch

Give 2 minutes of quiet work time followed by a whole-class discussion.

Anticipated Misconceptions

Students might think that adding the same number to each side length will result in similar triangles. Drawing a picture helps students see why this is not true.

Student Task Statement

Triangle A has side lengths 2, 3, and 4. Triangle B has side lengths 4, 5, and 6. Is Triangle A similar to Triangle B?

Student Response

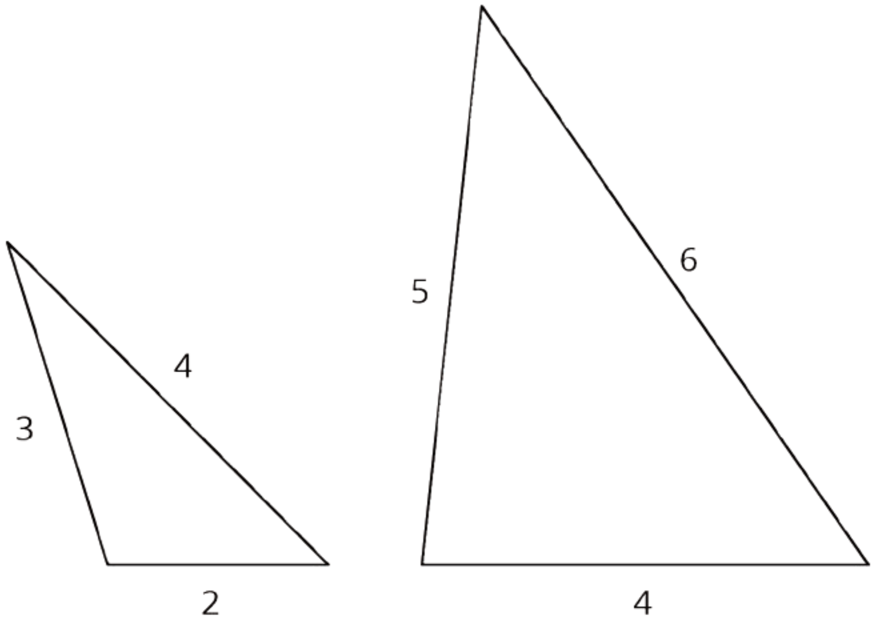
No. Sample explanations:

- If two figures are similar, then there is a single scale factor we can multiply all of the side lengths in one to get the side lengths in the other. Since doubling 2 gives 4, and doubling 3 gives 6, the third side in the second triangle would have to be 8 for the two to be similar.
- These triangles are not similar because we double the shortest side in Triangle A to get the shortest side in Triangle B, but we multiply the longest side in Triangle A by 1.5 to get the longest side in Triangle B. So the scale factor is not the same for all side lengths.

Activity Synthesis

Ask students how they can tell without drawing a diagram. Make sure students understand that the triangles can not be similar because you can't apply the same scale factor to each side of one triangle to get the corresponding sides of the other triangle.

Display diagrams of the triangles for visual confirmation.



9.2 Quotients of Sides Within Similar Triangles

15 minutes

In previous lessons, students have seen that corresponding side lengths of similar polygons are proportional. That is, the side lengths in one polygon can be calculated by multiplying corresponding side lengths in a similar polygon by the same scale factor. This activity explores ratios of side lengths *within* triangles and how these compare for similar triangles. If a and b are the side lengths of a triangle then the corresponding side lengths of a similar triangle have lengths sa and sb for some positive scale factor s. This means that the ratios $a : b$ and $sa : sb$ are equivalent.

As students work, circulate to make sure that students have the correct values in the table, and address any misconceptions with individual groups as needed. Also watch for students who look to explain why the internal ratios of corresponding side lengths of similar triangles are equivalent and invite them to share their thinking during the discussion.

Building On

- 7.RP.A.2

Addressing

- 8.G.A

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 3. Assign one of the columns in the second table to one student in each group. Tell students, "Each group is going to compare side lengths in similar triangles. Work for 5 minutes by yourself. Then compare your findings with your partners."

Support for Students with Disabilities

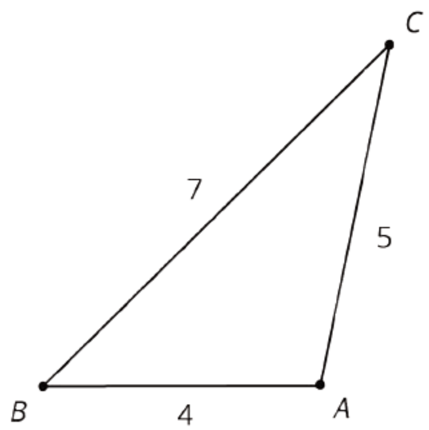
Representation: Internalize Comprehension. Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Anticipated Misconceptions

If students find quotients in fraction form, they need to recognize that the fractions are equivalent.

Student Task Statement

Triangle ABC is similar to triangles DEF, GHI, and JKL. The scale factors for the dilations that show triangle ABC is similar to each triangle are in the table.



1. Find the side lengths of triangles DEF, GHI, and JKL. Record them in the table.

triangle	scale factor	length of short side	length of medium side	length of long side
ABC	1	4	5	7
DEF	2			
GHI	3			
JKL	$\frac{1}{2}$			

2. Your teacher will assign you one of the three columns. For all four triangles, find the quotient of the triangle side lengths assigned to you and record it in the table. What do you notice about the quotients?

triangle	(long side) ÷ (short side)	(long side) ÷ (medium side)	(medium side) ÷ (short side)
ABC	$\frac{7}{4}$ or 1.75		
DEF			
GHI			
JKL			

3. Compare your results with your partners' and complete your table.

Student Response

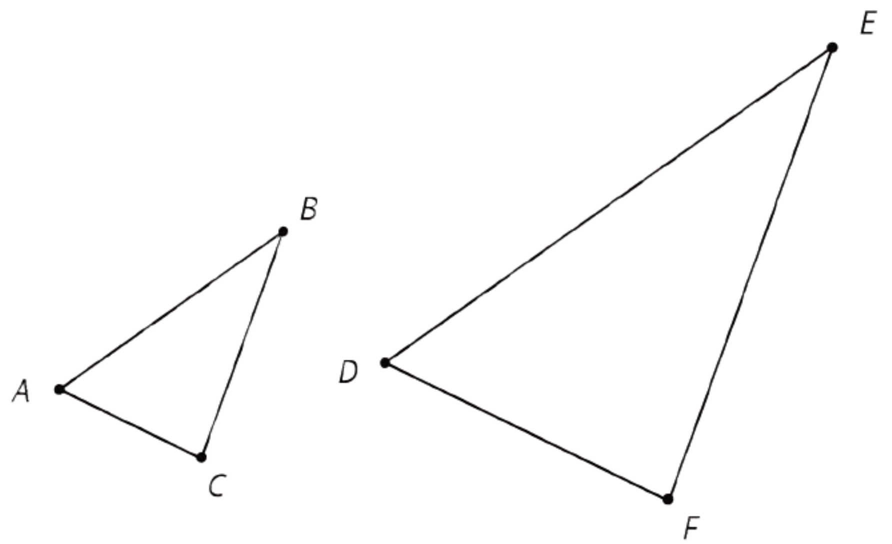
triangle	scale factor	length of short side	length of medium side	length of long side
ABC	1	4	5	7
DEF	2	8	10	14
GHI	3	12	15	21
JKL	$\frac{1}{2}$	2	2.5	3.5

triangle	(long side) ÷ (short side)	(long side) ÷ (medium side)	(medium side) ÷ (short side)
ABC	$\frac{7}{4}$ or 1.75	$\frac{7}{5}$ or 1.4	$\frac{5}{4}$ or 1.25
DEF	$\frac{14}{8}$ or 1.75	$\frac{14}{10}$ or 1.4	$\frac{10}{8}$ or 1.25
GHI	$\frac{21}{12}$ or 1.75	$\frac{21}{15}$ or 1.4	$\frac{15}{12}$ or 1.25
JKL	$\frac{3.5}{2}$ or 1.75	$\frac{3.5}{2.5}$ or 1.4	$\frac{2.5}{2}$ or 1.25

The quotients in each column are the same.

Are You Ready for More?

Triangles ABC and DEF are similar. Explain why $\frac{AB}{BC} = \frac{DE}{EF}$.



Student Response

There is a scale factor s such that $s \cdot AB = DE$ and $s \cdot BC = EF$. So $\frac{s \cdot AB}{s \cdot BC} = \frac{DE}{EF}$, and $\frac{AB}{BC} = \frac{DE}{EF}$.

Activity Synthesis

The main takeaway from this activity is that quotients of corresponding side lengths in similar triangles are equal. Ask students for the triangles examined what the value of (medium side) ÷ (long side) would be? For the original triangle, it would be $\frac{5}{7}$, and students can check that this is the same value for the other triangles.

Ask students if they think the value of (medium side) ÷ (long side) would be $\frac{5}{7}$ for *any* triangle similar to ABC. Ask them to explain why. Help them to see that a triangle similar to ABC will have side lengths $4s$, $5s$, and $7s$ for some (positive) scale factor s. The medium side divided by the long side will be $5s \div 7s = \frac{5s}{7s} = \frac{5}{7}$.

Support for English Language Learners

Speaking, Representing: MLR8 Discussion Supports. At the end of the whole-class discussion, display this prompt for all to see, "The value of (medium side) ÷ (long side) will be ____ for any triangle similar to ABC because…". Give students 2–3 minutes to write a response. Invite students to read what they wrote to a partner as a way to rehearse what they will say when they share with the whole class. Rehearsing provides students with additional opportunities to speak and clarify their thinking. Listen for and amplify statements that use both formal and informal language such as, ratio, quotient, multiple, scale factor, reduce, simplify, and common factor. This will help students to explain their reasoning using appropriate language structure. *Design Principle(s): Optimize output (for explanation)*

9.3 Using Side Quotients to Find Side Lengths of Similar Triangles

15 minutes

In this activity, students calculate side lengths of similar triangles. They can use the scale factor between the similar triangles, studied in depth in previous lessons. Or they can look at internal ratios between corresponding side lengths within the triangles, introduced in the previous lesson. Students need to think strategically about which side lengths to calculate first since there are many missing values. As they discover more side lengths, this opens up more paths for finding the remaining values.

As students work, monitor for students who:

- Use scale factors between triangles.
- Notice that the long side is twice the short side in GHI and use that to find c, d, or e.
- Notice that the long sides are equal in ABC and use that to find h, d, or e.

Select students who use different strategies to find side lengths to share during the discussion.

Addressing

- 8.G.A.4

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports

Launch

Tell students, “There are many ways to find the values of the unknown side lengths in similar triangles. Use what you have learned so far.” Give students 5 minutes of quiet work time followed by 5 minutes small group discussion and then a whole-class discussion.

Support for Students with Disabilities

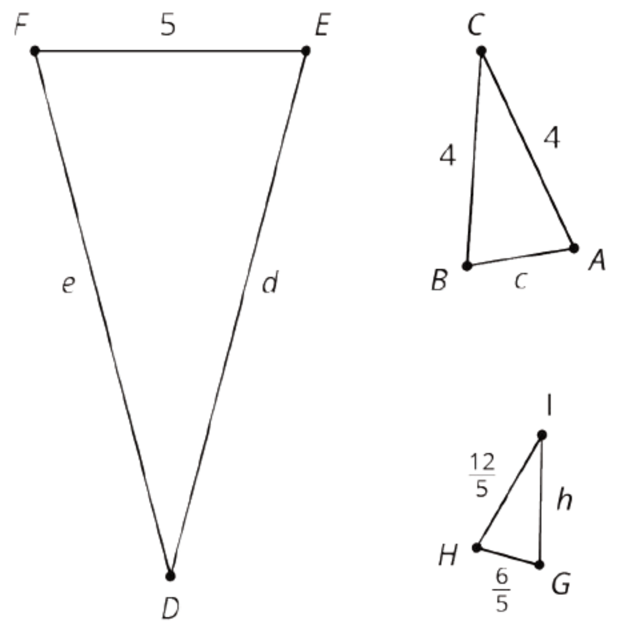
Representation: Internalize Comprehension. Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, use the same color for corresponding side lengths.
Supports accessibility for: Visual-spatial processing

Anticipated Misconceptions

If students have trouble locating corresponding sides, suggest that they use tracing paper so they can rotate and or translate them. Another technique is to color corresponding side lengths the same color. For example, they could color AB, EF, and GH all red.

Student Task Statement

Triangles ABC, EFD, and GHI are all similar. The side lengths of the triangles all have the same units. Find the unknown side lengths.



Student Response

$c = 2$, $d = 10$, $e = 10$, $h = \frac{12}{5}$

Activity Synthesis

Ask selected students to share the following strategies:

- using (external) scale factors to move from one triangle to another
- using quotients of corresponding side lengths within the triangles (internal scale factors)

Both methods are efficient and the method to use is guided by what information is missing and the numbers involved in the calculations. For example, if h is the first missing value we find, then comparing with triangle ABC and using internal scale factors is appropriate. To find c, again we can compare ABC and GHI and internal scale factors are appropriate again because $\frac{6}{5}$ is half of $\frac{12}{5}$ whereas comparing $\frac{12}{5}$ and 4 is more involved (the scale factor is $\frac{5}{3}$ from $\triangle GHI$ to $\triangle ABC$).

Ask students to articulate how they knew which sides of the similar triangles correspond. Make sure to make the following reasoning pathways explicit for all:

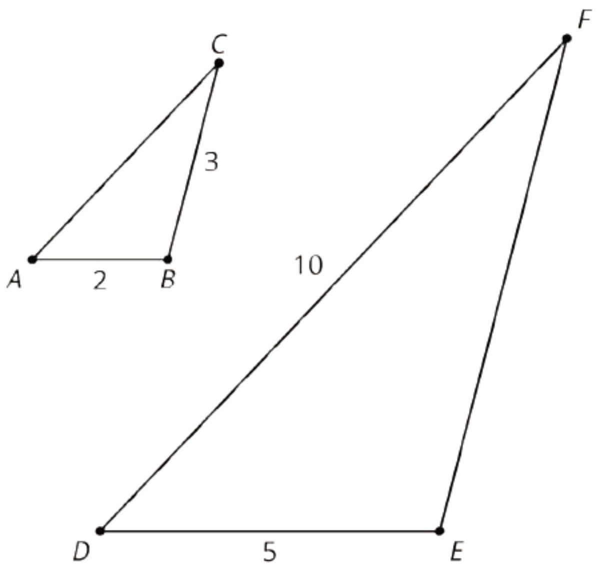
- Triangle ABC has two equal side lengths, so the other two triangles will as well. This insight is efficient for finding h.
- One side of triangle GHI is twice the length of another side, so this will be true of the other triangles as well. This insight is helpful for finding c, d, and e.

Emphasize that there are many different relationships that can be used to find side lengths of similar triangles.

Support for English Language Learners

Speaking: MLR8 Discussion Supports: To support students to articulate how they knew which sides of the similar triangles correspond, provide sentence frames such as, "For similar triangles _____ and _____ I know that sides _____ and _____ correspond because ________."
Sentence frames invite and incentivize more student participation, conversation, and meta-awareness of language. This routine should help students reason about the ratio of corresponding sides of similar triangles and communicate their understanding.
Design Principle(s): Support sense-making, Optimize output (for comparison)

Lesson Synthesis



These two triangles are similar:Ask students what the scale factor is from $\triangle ABC$ to $\triangle DEF$. It's $\frac{5}{2}$ since sides AB and DE are corresponding sides. One way to find AC would be to divide the length of DF by the scale factor $\frac{5}{2}$ giving a length of 4. A simpler arithmetic way to do this is to notice that DF is twice the length of DE. This means that AC is twice the length of AB (scaling AC and AB *both* by $\frac{5}{2}$ does not change their quotient!).

Sometimes both methods for calculating missing side lengths are equally effective. For EF, we can notice that it is $\frac{5}{2}$ the length of the corresponding side BC so that's 7.5. Or we can notice that it is $\frac{3}{2}$ the length of DE, again 7.5 ($\frac{3}{2}$ is the quotient of the corresponding sides BC and AB in $\triangle ABC$).

9.4 Similar Sides

Cool Down: 5 minutes

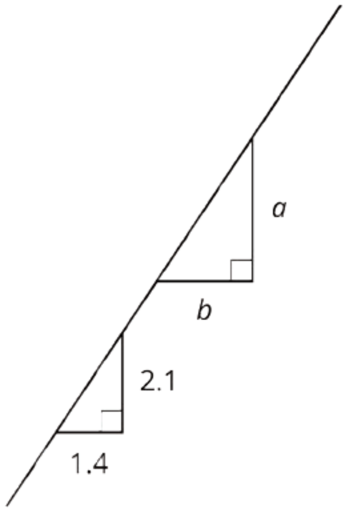
Students apply the equality of internal ratios of sides on similar triangles to find missing side lengths. These particular triangles (slope triangles) will be a focus of study in the following lessons.

Addressing

- 8.G.A

Student Task Statement

The two triangles shown are similar. Find the value of $\frac{a}{b}$.



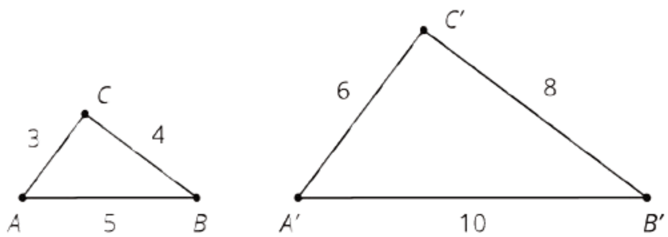
Student Response

$\frac{3}{2}$ or 1.5

Student Lesson Summary

If two polygons are similar, then the side lengths in one polygon are multiplied by the same scale factor to give the corresponding side lengths in the other polygon.

For these triangles the scale factor is 2:



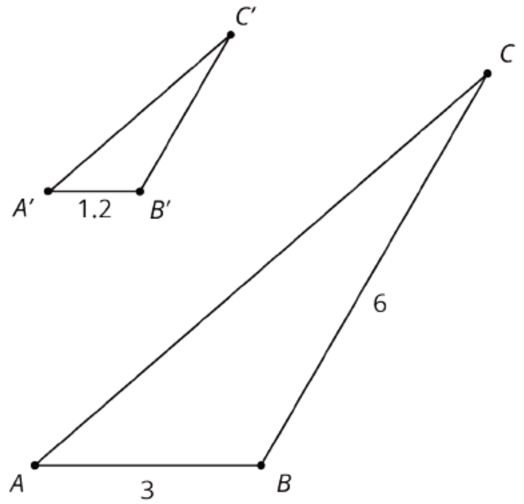
Here is a table that shows relationships between the short and medium length sides of the small and large triangle.

	small triangle	large triangle
medium side	4	8
short side	3	6
(medium side) ÷ (short side)	$\frac{4}{3}$	$\frac{8}{6} = \frac{4}{3}$

The lengths of the medium side and the short side are in a ratio of $4:3$. This means that the medium side in each triangle is $\frac{4}{3}$ as long as the short side. This is true for all similar polygons; the ratio between two sides in one polygon is the same as the ratio of the corresponding sides in a similar polygon.

We can use these facts to calculate missing lengths in similar polygons. For example, triangles $A'B'C'$ and ABC shown here are similar. Let's find the length of segment $B'C'$.

In triangle ABC, side BC is twice as long as side AB, so this must be true for any triangle that is similar to triangle ABC. Since $A'B'$ is 1.2 units long and $2 \cdot 1.2 = 2.4$, the length of side $B'C'$ is 2.4 units.

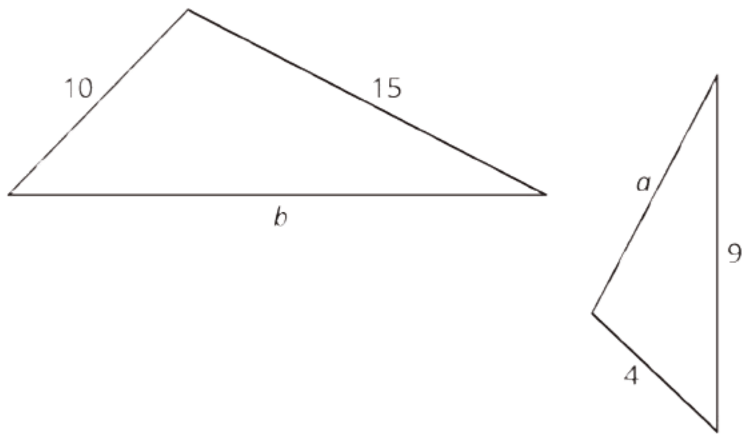


Lesson 9 Practice Problems

Problem 1

Statement

These two triangles are similar. What are a and b? Note: the two figures are not drawn to scale.



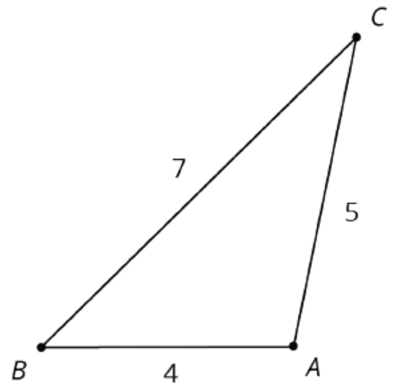
Solution

$a = 6$, $b = 22.5$ (the scale factor between the triangles is 2.5)

Problem 2

Statement

Here is triangle ABC. Triangle XYZ is similar to ABC with scale factor $\frac{1}{4}$.



a. Draw what triangle XYZ might look like.

b. How do the angle measures of triangle XYZ compare to triangle ABC? Explain how you know.

c. What are the side lengths of triangle XYZ?

d. For triangle XYZ, calculate (long side) ÷ (medium side), and compare to triangle ABC.

Solution

a. Answers vary.

b. The angle measures are the same, because in similar polygons, corresponding angles are congruent.

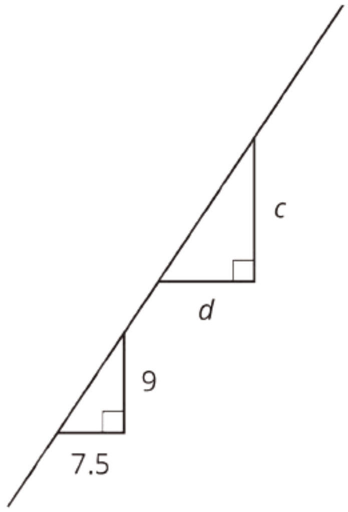
c. The side lengths are 1, $\frac{5}{4}$, and $\frac{7}{4}$.

d. The result is $\frac{7}{5}$, the same as the corresponding result for triangle ABC.

Problem 3

Statement

The two triangles shown are similar. Find the value of $\frac{d}{c}$.



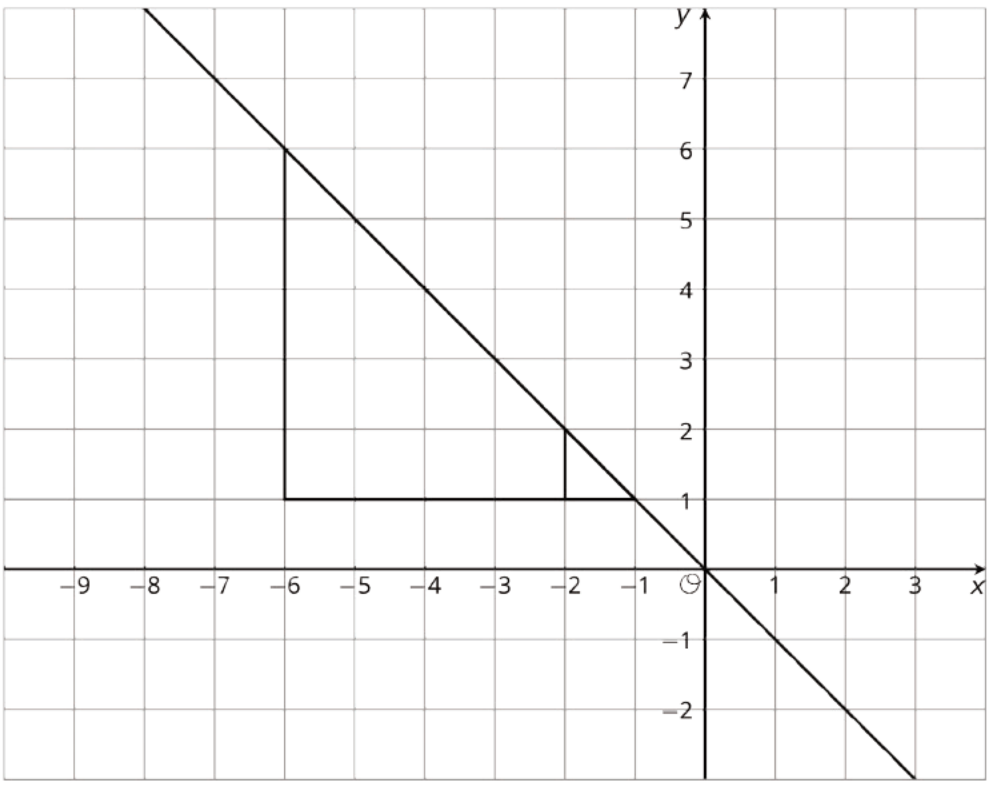
Solution

$\frac{5}{6}$ (or equivalent)

Problem 4

Statement

The diagram shows two nested triangles that share a vertex. Find a center and a scale factor for a dilation that would move the larger triangle to the smaller triangle.



Solution

Center: $(-1, 1)$, scale factor: $\frac{1}{5}$

(From Unit 2, Lesson 5.)

Section: Slope

Lesson 10: Meet Slope

Goals

- Comprehend the term “slope” to mean the quotient of the vertical distance and the horizontal distance between any two points on a line.
- Draw a line on a coordinate grid given its slope and describe (orally) observations about lines with the same slope.
- Justify (orally) that all “slope triangles” on one line are similar by using transformations or Angle-Angle Similarity.

Learning Targets

- I can draw a line on a grid with a given slope.
- I can find the slope of a line on a grid.

Lesson Narrative

A slope triangle for a line is a triangle whose longest side lies on the line and whose other two sides are vertical and horizontal. This lesson establishes the remarkable fact that the quotient of the vertical side length and the horizontal side length does not depend on the triangle: this number is called the **slope** of the line. The argument builds on many key ideas developed in this unit:

- The dilation of a slope triangle, with center of dilation on the line, is a slope triangle for the same line.
- Triangles sharing two common angle measures are similar.
- Quotients of corresponding sides in similar polygons are equal.

In future lessons, they will use slope to write equations for lines.

Alignments

Building On

- 5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
- 5.NF.B.3: Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied

by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

- 8.G.A.4: Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Addressing

- 8.EE.B.6: Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display
- MLR5: Co-Craft Questions
- MLR8: Discussion Supports

Required Materials

Straightedges
A rigid edge that can be used for drawing line segments. Sometimes a ruler is okay to use as a straightedge, but sometimes it is preferable to use an unruled straightedge, like a blank index card.

Required Preparation

If using the print version of the materials, students need a straightedge in order to draw lines. If using the digital version, an applet is made available for this purpose.

Student Learning Goals

Let's learn about the slope of a line.

10.1 Equal Quotients

Warm Up: 5 minutes
In this lesson, students will need to recognize equivalent fractions and decimals. This warm-up is designed to activate prior understanding.

Building On

- 5.NBT.B.7
- 5.NF.B.3

Launch

Ask students if they can think of some different ways to write numbers that are equal to $1 \div 2$. Some examples might be $\frac{4}{8}$ or 0.5. Tell them that in this warm-up, they will think of numbers that equal $15 \div 12$. Give students 30–60 seconds to write as many as they can, or ask them to come up with at least three or four different numbers equal to $15 \div 12$.

Anticipated Misconceptions

Students may believe that this question must have only one acceptable response. To convince them that equivalent values are acceptable, you may need to appeal to the fact that one location on a number line can be expressed many different ways, depending on how you decide to subdivide a portion of the line.

Student Task Statement

Write some numbers that are equal to $15 \div 12$.

Student Response

Answers vary. Possible responses: $\frac{15}{12}$, $\frac{5}{4}$, $\frac{10}{8}$, $\frac{50}{40}$, 1.25, 1.2500.

Activity Synthesis

Ask students to share their strategies for finding numbers equal to $15 \div 12$. Record and display their answers and strategies for all to see. To involve more students in the conversation, consider asking:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone solve the problem the same way but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to _____'s strategy?"
- "Do you agree or disagree? Why?"

10.2 Similar Triangles on the Same Line

20 minutes

In this activity, students explain why certain triangles with one side along the same line are similar. This fact about the triangles will be used to define the slope of the line. Students may show that the triangles are similar by describing a sequence of transformations and dilations or by AA (or AAA). Alternatively, they may use the fact that grid lines are parallel and use what they know about the angles where a transverse meets a pair of parallel lines.

Monitor for students who use different methods for showing that the triangles are similar. Look for these methods in particular:

- Describing a sequence of transformations
- AA (or AAA) arguments using what students know about angles made by parallel lines with a transversal

Students need to use the structure of the grid for either argument (MP7). For the similarity argument, they need to use the grid to describe transformations and dilations. For the AA argument, they need to use the fact that vertical or horizontal grid lines are parallel. In both cases, constructing a viable argument (MP3) will require care and focus.

Building On

- 8.G.A.4

Addressing

- 8.EE.B.6

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR5: Co-Craft Questions

Launch

Before starting the activity, if possible, display this link https://ggbm.at/UxvbFxVQ for all students to see. Tell students that the goal is to make one triangle match up with the other triangle. Demonstrate how the controls work. Then, invite students to describe how you should manipulate the controls to make the triangles match up. Remind students to use words associated with transformations like *translate* and *scale factor*.

Arrange students in groups of 2. Assign one partner triangles ABC and CDE and the other partner ABC and FGH. Give students 5 minutes of quiet time to construct an argument for why their two triangles are similar. Remind students that if they claim something is true, they should explain how they can be sure it is true.

After a few minutes of quiet work time, ask students to share their reasoning with their partner and listen to their partner's explanation for why their triangles are similar. Then tell them to work with their partner to finish the activity.

Support for Students with Disabilities

Action and Expression: Develop Expression and Communication. To help get students started, display sentence frames to support students when they explain their strategy. "First, I _____ because....", "I noticed _____ so I....", "Why did you...?", "I agree/disagree because...." Encourage students to use what they know about transformations, corresponding angles and sides, and scale factor in their explanations.
Supports accessibility for: Language; Organization

Support for English Language Learners

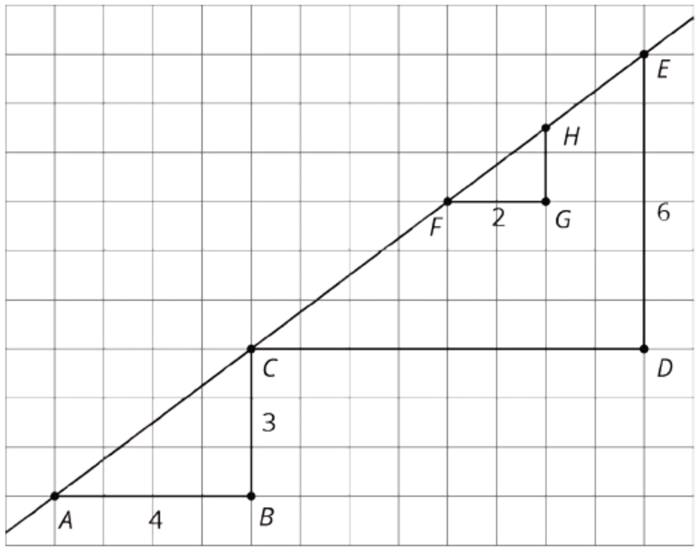
Reading, Writing: MLR5 Co-Craft Questions. To help students comprehend the diagram in this task, show students just the graph with the three slope triangles. Ask students to write down possible mathematical questions that might be asked about the situation. Students may create questions about finding the length of missing lengths of line segments, or more broadly, about which triangles are similar. Ask students to compare the questions they generated with a partner before sharing questions with the whole class. Reveal the actual question that students are to work on, which is to explain why two of these triangles are similar. Through this routine, students are able to use conversation skills to generate, choose (argue for the best one), and improve questions, as well as develop meta-awareness of the language used in mathematical questions.
Design Principle(s): Cultivate conversation; Support sense-making, Meta-awareness of the language

Anticipated Misconceptions

If students struggle getting started, ask them what it means for two triangles to be similar (there is a sequence of translations, rotations, reflections, and dilations taking one to the other).

Student Task Statement

1. The figure shows three right triangles, each with its longest side on the same line. Your teacher will assign you two triangles. Explain why the two triangles are similar.



2. Complete the table.

triangle	length of vertical side	length of horizontal side	(vertical side) ÷ (horizontal side)
ABC			
CDE			
FGH			

Student Response

1. Answers vary. Possible strategies:
   - Triangle ABC is similar to triangle CDE because you can translate triangle ABC so that A goes to C. Then perform a dilation using scale factor 2 and C as the center of dilation.
   - Triangle FGH is similar to triangle CDE by AA. We know this first because angle FGH and angle CDE are both right angles so they are congruent to each other. We also know all of the vertical sides of the triangles are along the grid so they are all parallel. Since angle GFH and angle DCE are corresponding angles for transversal $\overleftrightarrow{EA}$ (and parallel grid lines GF and DC) they are congruent. A similar argument can be made about angles CED and FHG.

2.

triangle	length of vertical side	length of horizontal side	(vertical side) ÷ (horizontal side)
ABC	3	4	$\frac{3}{4}$ or 0.75
CDE	6	8	$\frac{6}{8}$ or 0.75
FGH	1.5	2	$\frac{3}{4}$ or 0.75

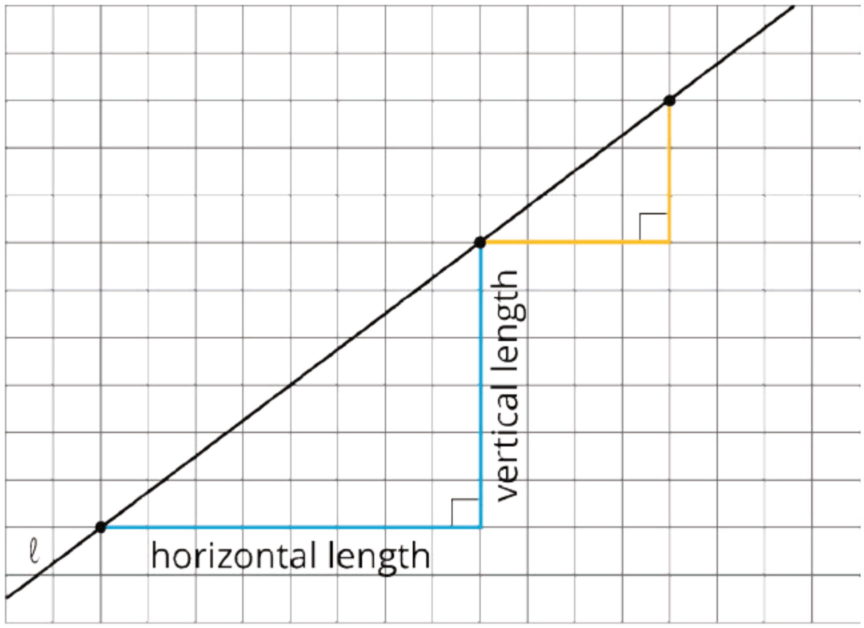
Activity Synthesis

Invite selected students to share their methods for showing that the triangles are similar. Sequence them so that a similarity transformation argument goes first, followed by an AA (or AAA) argument. For these particular triangles, describing a sequence of transformations is quick and efficient: a translation and dilation (or a dilation followed by a translation) naturally present themselves for these triangles. The AA argument requires a little extra work because no angle measures are labeled: finding angle measures uses the structure of the grid and previous results about corresponding angles created by parallel lines and a transversal. Make sure students understand that since triangles ABC and FGH are each similar to triangle CDE, they are similar to each other.

Relate the work on finding the quotients, in the second problem, back to the work in the last lesson looking at internal ratios of corresponding sides of similar triangles. Explain that whenever we have a (non vertical, non horizontal) line, we can construct triangles like these where one side is horizontal and one side is vertical, and the quotient of the length of the vertical side and the horizontal side will always be the same. This number is called the **slope** of the line. The slope of the line in this activity can be written as 0.75 or $\frac{3}{4}$ (or any value equal to these).

Make clear to students that the mathematical convention is to define slope using vertical length divided by horizontal length and not the other way around. Display the diagram below, or create a similar diagram with the same information. Post this diagram for reference for several days, well into the next unit of study, along with accompanying text:

The slope is $\text{vertical length} \div \text{horizontal length}$. The slope of line ℓ can be written as $\frac{6}{8}$, $\frac{3}{4}$, 0.75, or any equal value.



10.3 Multiple Lines with the Same Slope

10 minutes (there is a digital version of this activity)

The previous lesson introduces the ***slope*** of a line. In this activity, students practice graphing lines with a given slope. They observe two important properties of slope:

- Lines with the same slope are parallel.
- As the slope of a line increases so does its steepness (from left to right).

Look for students who draw slope triangles to help construct their lines. Other students may count off horizontal and vertical distances. Select students using each method to share during the discussion. Also watch for students who notice the facts listed above about parallel lines and the impact of the slope on the steepness of the line.

Addressing

- 8.EE.B.6

Instructional Routines

- MLR8: Discussion Supports

Launch

Tell students that they are going to apply the new idea of slope introduced in the previous activity as they draw and study properties of some lines with different slopes.

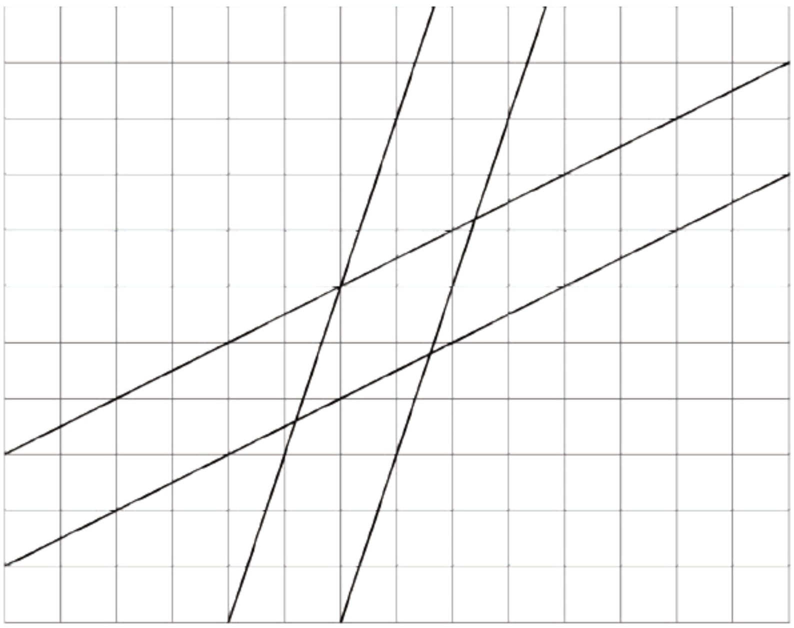
If using the print version of the materials, distribute straightedges. If using the digital version, students have access to a digital tool to draw lines.

Student Task Statement

1. Draw two lines with slope 3. What do you notice about the two lines?

2. Draw two lines with slope $\frac{1}{2}$. What do you notice about the two lines?

Student Response



The pairs of lines with the same slope are parallel.

Are You Ready for More?

As we learn more about lines, we will occasionally have to consider perfectly vertical lines as a special case and treat them differently. Think about applying what you have learned in the last couple of activities to the case of vertical lines. What is the same? What is different?

Student Response

Geometrically, vertical lines are no different than any other line. You can rotate a vertical line to a non-vertical line. Just as lines with the same slope are parallel, all vertical lines are also parallel. But some things are quite different. For example, the notion of a slope triangle doesn't make much sense since there is no "horizontal distance" to use as the base of the triangle, and trying to define a slope using

$$\text{slope} = \frac{\text{vertical distance}}{\text{horizontal distance}}$$

would have us dividing by zero. For this reason, we do not define the slope of a vertical line.

Activity Synthesis

Ask selected students to share how they found their lines with slope 3. Sequence them so that students who use slope triangles present their work first and students who count horizontal and vertical displacement (without drawing a triangle) present second. Help students see that the second method is the same as the first except that the slope triangle connecting two points on the line is only "imagined" rather than drawn. Further, that it doesn't matter if you go up 3 and over 1, or up 6 and over 2, or up 9 and over 3 . . . you get a line with the same slope. Encourage students to draw slope triangles if it helps them to see and understand the underlying structure.

If it does not come up during student comments, make sure that students notice that the two lines of slope 3 are parallel and so are the two lines with slope $\frac{1}{2}$.

Also, point out that the lines of slope 3 are "steeper" than the lines of slope $\frac{1}{2}$. Hence the definition of slope corresponds to our intuition in the sense that a larger slope corresponds to a "steeper graph" (going from left to right).

It is likely that some students will draw lines with slope -3 and $-\frac{1}{2}$. Display these alongside lines of slope 3 or slope $\frac{1}{2}$, and ask students to describe how they are alike and different. It is okay to let them know that the "uphill" lines (leaning to the right) are positive and "downhill" (leaning to the left) are negative. But you might also leave the question open, for now, about whether such lines have the same or different slopes, and how the slope would be different. Later lessons will focus on distinguishing positive from negative slopes.

Support for Students with Disabilities

Engagement: Develop Effort and Persistence. Break the class into small discussion groups and then invite a representative from each group to report back to the whole class.
Supports accessibility for: Attention; Social-emotional skills

Support for English Language Learners

Representing: MLR8 Discussion Supports. Provide sentence frames to help students to formulate statements about what they notice about two lines with the same slope, "Two lines with the same slope are____ because...," and lines with different slopes, "The lines with slopes equal to 3 are____than lines with slope 1/2, because...." This will help to encourage students to justify what they notice. Remind students to include words like "parallel" and "steep," along with informal language describing equal rates of change.
Design Principle(s): Optimize output (for explanation)

10.4 Different Slopes of Different Lines

Optional: 15 minutes
Earlier in this lesson, students have seen that the slope of a line can be calculated using any (slope) right triangle, that is a right triangle whose longest side is on the line and whose other two sides are horizontal and vertical. In this activity, students identify given lines with different slopes and draw a line with a particular slope.

For lines D and E as well as the line that students draw, encourage students to draw in slope triangles to help calculate the slope. Monitor for students who draw slope triangles of different sizes and invite them to share during the discussion to reinforce the fundamental idea for this lesson: different slope triangles whose longest side lies on the same line give the same value for slope.

Addressing

- 8.EE.B.6

Instructional Routines

- MLR2: Collect and Display

Launch

Instruct students to work individually to match each slope to a line and draw the line for the slope that does not have a match. Remind students of the previous definition of slope, referring to the display as necessary. Students still need a straightedge for drawing line F.

Support for Students with Disabilities

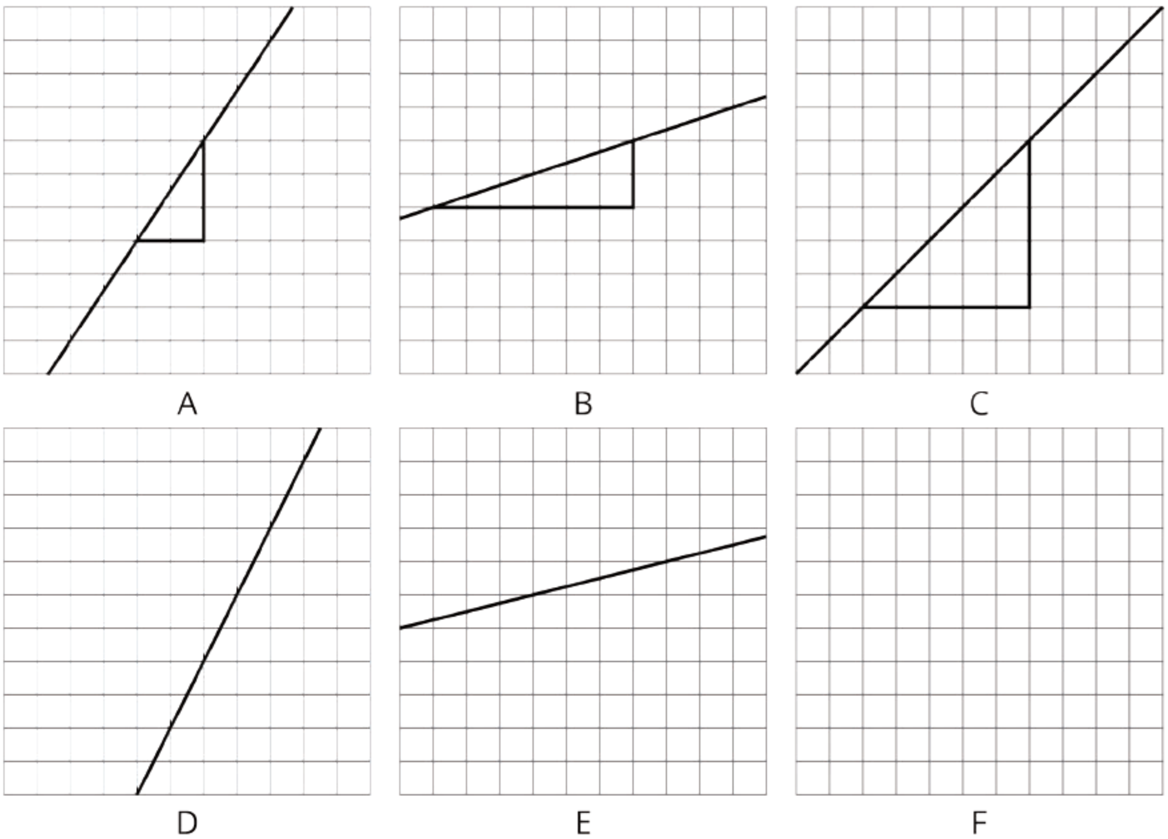
Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. During the launch, take time to review terms students will need to access for this activity. Invite students to suggest language or diagrams to include that will support their understanding of: slope, vertical distance, and horizontal distance.
Supports accessibility for: Conceptual processing; Language

Anticipated Misconceptions

For students who find it difficult to draw a triangle when one is not given, suggest that they examine two places on the line where the line crosses an intersection of grid lines.

Student Task Statement

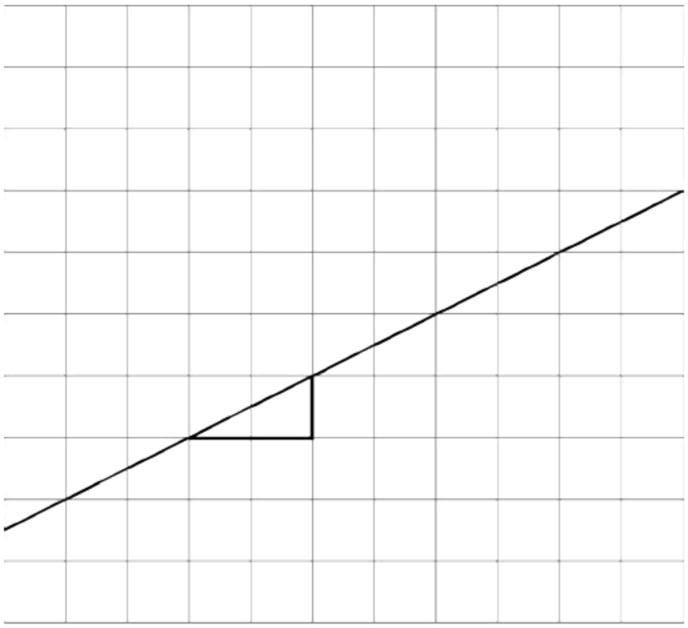
Here are several lines.



1. Match each line shown with a slope from this list: $\frac{1}{3}$, 2, 1, 0.25, $\frac{3}{2}$, $\frac{1}{2}$.
2. One of the given slopes does not have a line to match. Draw a line with this slope on the empty grid (F).

Student Response

1. $\frac{3}{2}$
2. $\frac{1}{3}$
3. 1
4. 2
5. 0.25
6. $\frac{1}{2}$ (A valid response may not include a slope triangle, or may include a slope triangle that is similar but not congruent to the one shown.)



Activity Synthesis

Two important conclusions for students to understand are:

- Given a line on a grid, they can draw a right triangle whose longest side is on the line, and then use the quotient of the vertical and horizontal sides to find the slope.
- Given a slope, they can draw a right triangle using vertical and horizontal lengths corresponding to the slope, and then extend the longest side of the right triangle to create a line with that slope.

When discussing line F, ask students to share how they drew their triangle. If possible, select students who drew their triangles correctly but at a different scale (for example, one student who used a triangle with a vertical length of 1 and a horizontal length of 2, and a different student who used a vertical length of 4 and a horizontal length of 8). Demonstrate (or get students who have drawn different triangles to do so) that the *quotient* of side lengths is the important feature, since any triangle drawn to match a given slope will be similar to any other triangle drawn to match the same slope.

Support for English Language Learners

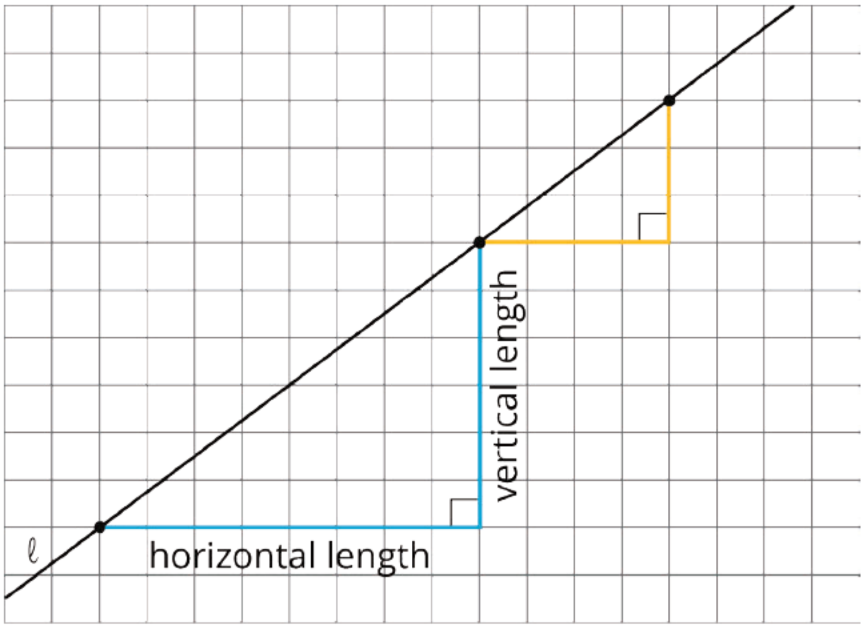
Conversing, Representing, Writing: MLR2 Collect and Display. After students have had individual work time, ask students to work in pairs to explain to each other how they matched each graph to its slope. While pairs are working, circulate and listen to student talk about how they matched each representation to another representation. Listen for how students created slope triangles where no triangle already existed, and how they used the slope to draw the line. Display the language collected visually for the whole class to use as a reference during further discussions throughout the unit. This allows students' own output to become a reference in developing mathematical language.
Design Principle(s): Support Sense-making; Maximize meta-awareness

Lesson Synthesis

In this lesson, we learned what the **slope** of a line is. Discuss:

- "What is a slope triangle for a line?" (A triangle whose long side is on the line and whose other sides are horizontal and vertical.)
- "How can you use a slope triangle to find the slope of a line?" (Divide the length of the vertical side by the length of the horizontal side.)
- "Does it matter which two points you use to create a slope triangle? Why?" (No. Any two slope triangles are similar. So the quotient of the two corresponding sides will give the same value.)
- "Why are any two slope triangles similar?" (They are right triangles whose other angles are corresponding angles for a transverse meeting parallel grid lines.)

Consider creating a permanent classroom display showing the definition of slope.



Display the diagram along with the accompanying text: The slope of the line is vertical length ÷ horizontal length. The slope of line ℓ can be written $\frac{6}{8}$, $\frac{3}{4}$, 0.75, or any equivalent value.

10.5 Finding Slope and Graphing Lines

Cool Down: 5 minutes

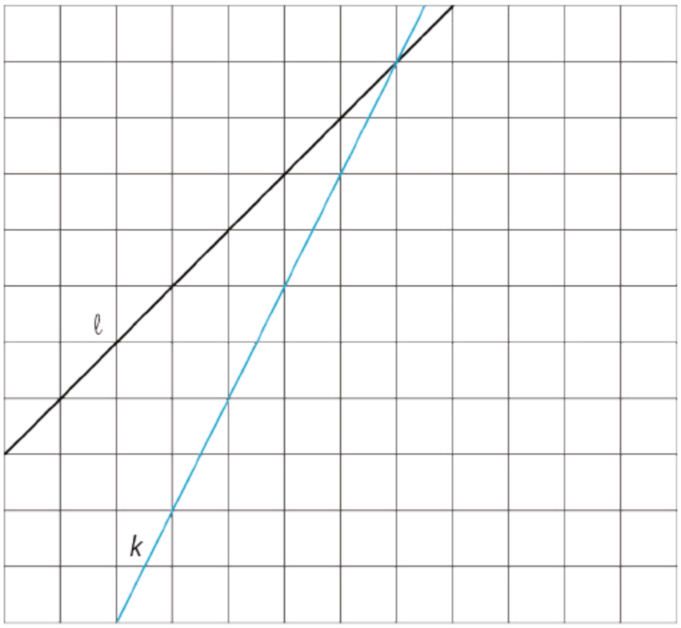
Students identify lines with different slopes and then draw a line with specified slope. They can use what they know about the meaning of slope, drawing appropriate slope triangles. They can also use the fact that a steeper line has a larger slope.

Addressing

- 8.EE.B.6

Student Task Statement

Lines ℓ and k are graphed.



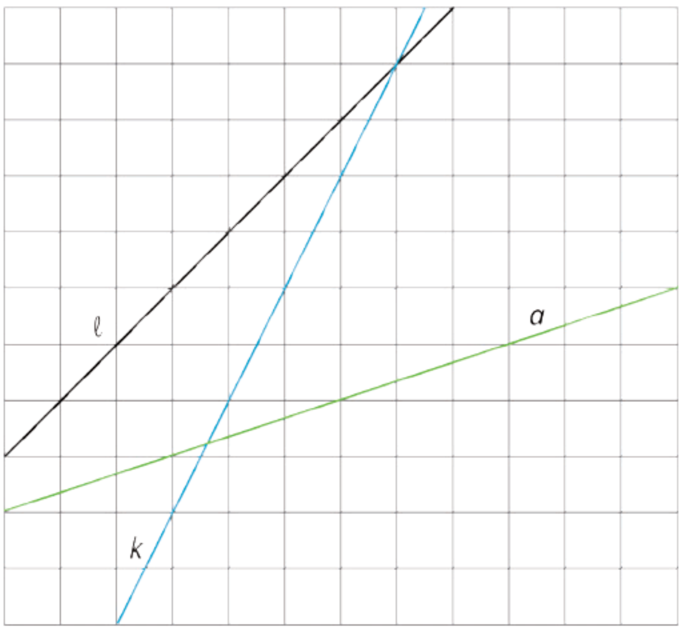
1. Which line has a slope of 1, and which has a slope of 2?

2. Use a ruler to help you graph a line whose slope is $\frac{1}{3}$. Label this line a.

Student Response

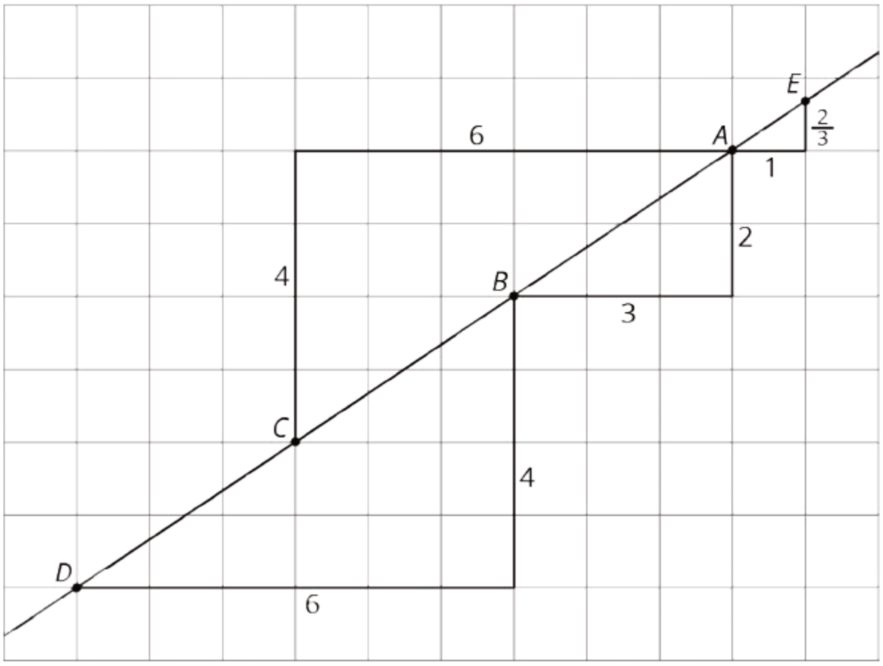
1. The slope of line ℓ is 1; the slope of line k is 2.

2. Answers vary. Possible response:



Student Lesson Summary

Here is a line drawn on a grid. There are also four right triangles drawn. Do you notice anything the triangles have in common?



These four triangles are all examples of *slope triangles*. One side of a slope triangle is on the line, one side is vertical, and another side is horizontal. The **slope** of the line is the quotient of the length of the vertical side and the length of the horizontal side of the slope triangle. This number is the same for *all* slope triangles for the same line because all slope triangles for the same line are similar.

In this example, the slope of the line is $\frac{2}{3}$, which is what all four triangles have in common. Here is how the slope is calculated using the slope triangles:

- Points A and B give $2 \div 3 = \frac{2}{3}$
- Points D and B give $4 \div 6 = \frac{2}{3}$
- Points A and C give $4 \div 6 = \frac{2}{3}$
- Points A and E give $\frac{2}{3} \div 1 = \frac{2}{3}$

Glossary

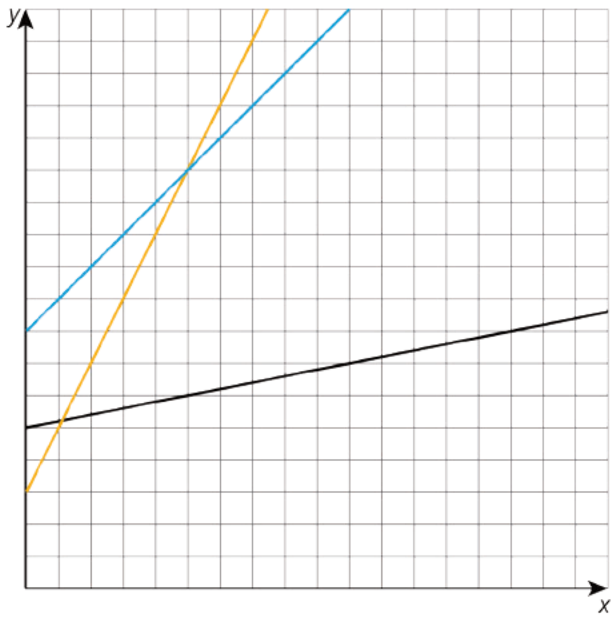
- similar
- slope

Lesson 10 Practice Problems

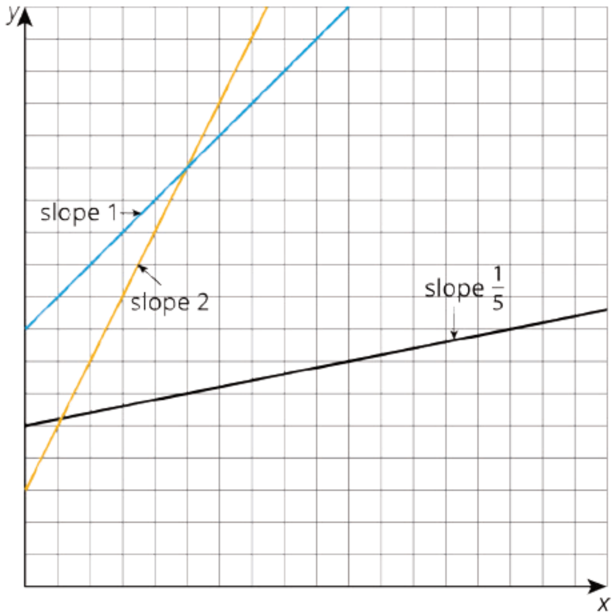
Problem 1

Statement

Of the three lines in the graph, one has slope 1, one has slope 2, and one has slope $\frac{1}{5}$. Label each line with its slope.



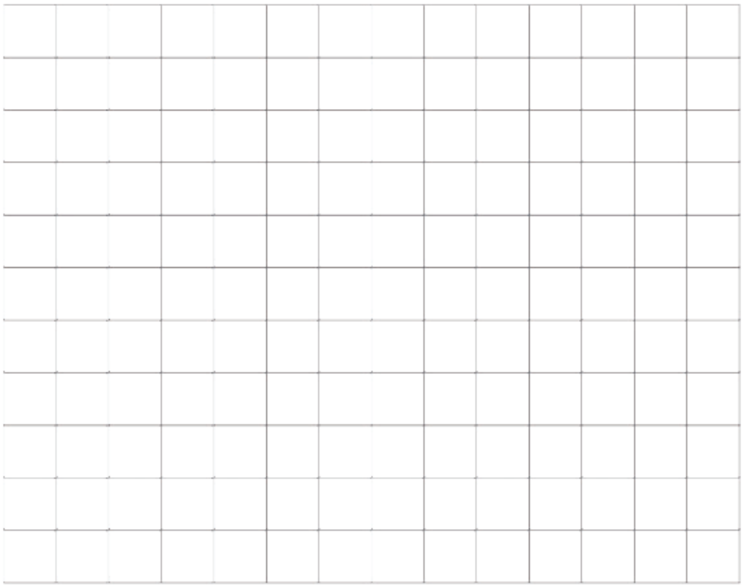
Solution



Problem 2

Statement

Draw three lines with slope 2, and three lines with slope $\frac{1}{3}$. What do you notice?



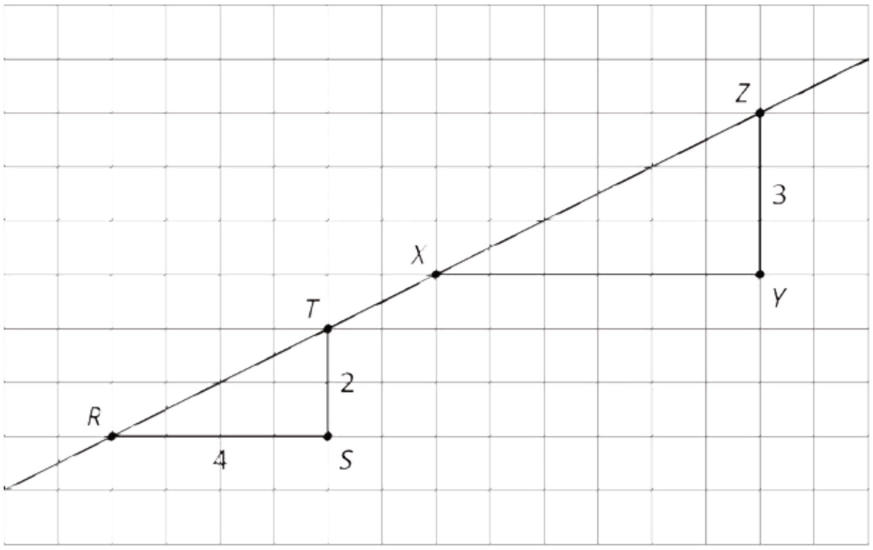
Solution

Answers vary. The three lines in each set should be parallel.

Problem 3

Statement

The figure shows two right triangles, each with its longest side on the same line.



a. Explain how you know the two triangles are similar.

b. How long is XY?

c. For each triangle, calculate (vertical side) ÷ (horizontal side).

d. What is the slope of the line? Explain how you know.

Solution

a. Explanations vary. Sample explanation: translating R to X and dilating shows there is a sequence of translations, rotations, reflections, and dilations taking one triangle to the other.

b. 6 units

c. For both triangles, the result is $\frac{1}{2}$.

d. The slope of the line is $\frac{1}{2}$. It is the quotient of the vertical side length of a slope triangle and the horizontal side length of a slope triangle. These all give the same value because the slope triangles are similar.

Problem 4

Statement

Triangle A has side lengths 3, 4, and 5. Triangle B has side lengths 6, 7, and 8.

a. Explain how you know that Triangle B is *not* similar to Triangle A.

b. Give possible side lengths for Triangle B so that it is similar to Triangle A.

Solution

a. Explanations vary. Sample explanation: the shortest side in Triangle B is twice as long, but the longest side is only 1.6 times as long. These different ratios mean the triangles cannot be similar.

b. Answers vary. Sample response: 6, 8, and 10

(From Unit 2, Lesson 9.)

Lesson 11: Writing Equations for Lines

Goals

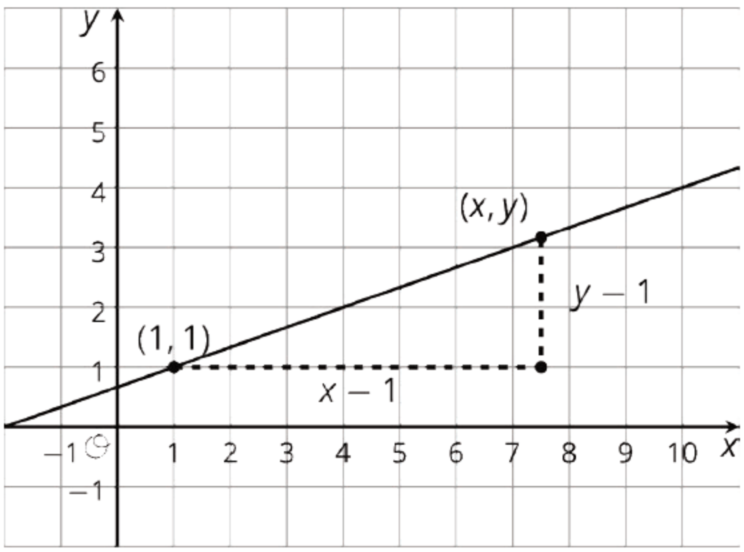
- Create an equation relating the quotient of the vertical and horizontal side lengths of a slope triangle to the slope of a line.
- Justify (orally) whether a point is on a line by finding quotients of horizontal and vertical distances.

Learning Targets

- I can decide whether a point is on a line by finding quotients of horizontal and vertical distances.

Lesson Narrative

The previous lesson introduces the idea of slope for a line. In this lesson, the slope is used to write a relationship satisfied by any point on a line. The key idea is to introduce a general or variable point on a line, that is a point with coordinates (x, y). These variables x and y can take any values as long as those values represent a point on the line. Because *all* slope triangles lead to the same value of slope, this general point can be used to write a relationship satisfied by all points on the line.



In this example, the slope of the line is $\frac{1}{3}$ since the points $(1, 1)$ and $(4, 2)$ are on the line. The slope triangle in the picture has vertical length $y - 1$ and horizontal length $x - 1$ so this gives the equation

$$\frac{y-1}{x-1} = \frac{1}{3}$$

satisfied by *any* point on the line (other than $(1, 1)$). This concise way of expressing which points lie on a line will be developed further in future units.

Alignments

Building On

- 6.G.A.3: Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
- 7.RP.A.2: Recognize and represent proportional relationships between quantities.
- 8.G.A: Understand congruence and similarity using physical models, transparencies, or geometry software.

Addressing

- 8.EE.B.6: Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.
- 8.G.A: Understand congruence and similarity using physical models, transparencies, or geometry software.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR1: Stronger and Clearer Each Time
- MLR5: Co-Craft Questions
- Think Pair Share

Student Learning Goals

Let's explore the relationship between points on a line and the slope of the line.

11.1 Coordinates and Lengths in the Coordinate Plane

Warm Up: 5 minutes
The purpose of this warm-up is to ensure students understand that

- they can infer the coordinates of a point based on knowing the coordinates of points on the same horizontal and vertical lines;
- the length of a horizontal or vertical line segment can be determined based on the coordinates of its endpoints. Each of these is important background knowledge for this lesson.

Building On

- 6.G.A.3

Addressing

- 8.G.A

Instructional Routines

- Think Pair Share

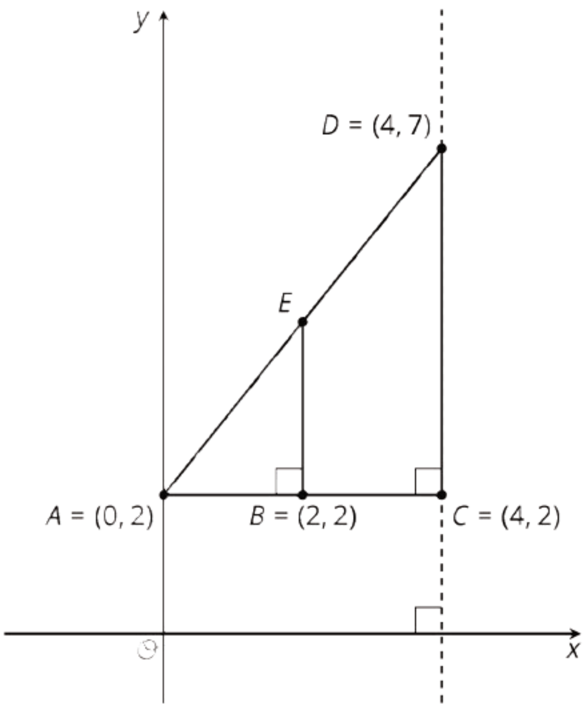
Launch

Give students 2 minutes of quiet think time. Ask them to share their reasoning with a partner followed by a whole-class discussion.

Anticipated Misconceptions

In order to find the length of segment BE, it is first necessary to find the lengths of CD, AC, and AB, and reason about side lengths in similar triangles. If students have trouble getting started, scaffold the work by suggesting they first find any *other* segment lengths in the diagram that they can find.

Student Task Statement



Find each of the following and explain your reasoning:

1. The length of segment BE.

2. The coordinates of E.

Student Response

1. 2.5 or equivalent. Triangles ABE and ACD are similar by AA, so $\frac{AC}{AB} = \frac{CD}{BE}$.

2. $(2, 4.5)$. E has the same x-coordinate as B, and its y-coordinate is $2 + 2.5$.

Activity Synthesis

There are three important things students should understand or recall as a result of working on the warm-up:

- Points on the same vertical line have the same x-coordinate and points on the same horizontal line have the same y-coordinate.
- In order to find the length of a vertical segment, you can subtract the y-coordinates of its endpoints.
- The side lengths of triangle ACD are the same as the side lengths of the similar triangle ABE multiplied by the scale factor $AC \div AB$.

Display the image from the task. For each question, invite a student to share their reasoning. The displayed image should be used as a tool for gesturing. For example, if you start at the origin and want to navigate to point D, you would move 4 units to the right. You move the same distance to the right to navigate from the origin to C. Therefore, C must have the same x-coordinate as D.

11.2 What We Mean by an Equation of a Line

10 minutes

Prior to this lesson, students have seen that right triangles with a horizontal side, a vertical side, and a long side along the same line are all similar. This activity exploits this structure to examine the coordinates of points lying on a particular line. The discussion then produces an equation for the line. In the case where the line goes through $(0, 0)$, the equation will be familiar from prior work with proportional relationships but in the next lesson similar triangles will be essential.

Monitor for different ways of answering the last question including

- With words and arithmetic: for example, divide the y-coordinate by the x-coordinate and see if it is equal to $\frac{3}{4}$.
- With words and proportional relationships: x and y are in a proportional relationship and when $x = 4$ we know $y = 3$.
- With an equation involving quotients of vertical and horizontal side lengths: for example, $\frac{y}{x} = \frac{3}{4}$.

Invite these students to share their reasoning during the discussion. If a student writes $y = \frac{3}{4}x$, this can be presented last but it is not essential that students see this now.

Building On

- 7.RP.A.2

Addressing

- 8.EE.B.6

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR1: Stronger and Clearer Each Time

Launch

Encourage students to think about what they know about slope triangles from previous work. Give 2–3 minutes of quiet work time. Then ask students to share their responses and reasoning with a partner, followed by a whole-class discussion.

Support for Students with Disabilities

Action and Expression: Develop Expression and Communication. Invite students to talk about their ideas with a partner. Display sentence frames to support students when they describe their ideas. For example: "This point is/is not on the line because…."; "First I _____ because…."; and "Our strategies are the same/different because…."
Supports accessibility for: Language; Organization

Support for English Language Learners

Writing, Speaking, Listening: MLR1 Stronger and Clearer Each Time. After students have had time to decide whether each of the three points lie on line j, ask students to write a brief explanation of their reasoning. Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language (e.g., "How did you determine the slope of line j?", "How do you know this point is on line j?", and "How do you know this point is not on line j?", etc.). Students can borrow ideas and language from each partner to refine and clarify their original explanation. This will help students revise and refine both their reasoning and their verbal and written output.
Design Principles(s): Optimize output (for explanation); Maximize meta-awareness

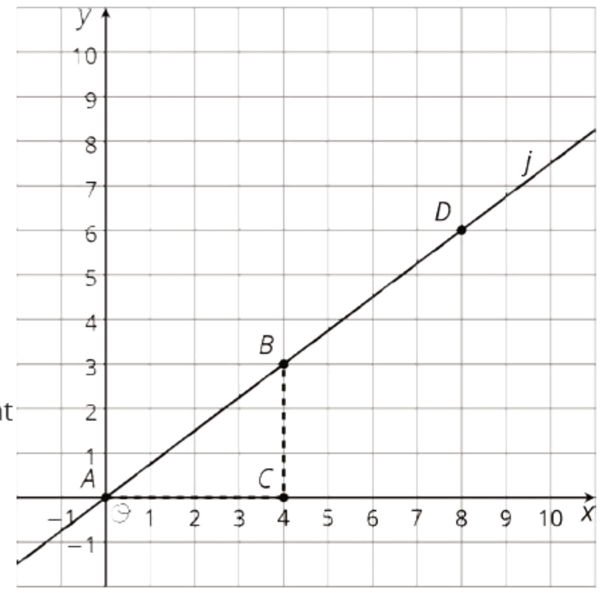
Anticipated Misconceptions

For question 5, there is a potential issue in saying all points on the line satisfy $\frac{y}{x} = \frac{3}{4}$, which is that this equation is not true for the point $(0, 0)$. Unless a student notices this, it is not necessary to bring it up at this time. If necessary, we could say that the equation describes the relationship between the x- and y-coordinate for every point on the line except $(0, 0)$.

Student Task Statement

Line j is shown in the coordinate plane.

1. What are the coordinates of B and D?
2. Is point $(20, 15)$ on line j? Explain how you know.
3. Is point $(100, 75)$ on line j? Explain how you know.
4. Is point $(90, 68)$ on line j? Explain how you know.
5. Suppose you know the x- and y-coordinates of a point. Write a rule that would allow you to test whether the point is on line j.



Student Response

1. $B = (4, 3)$ and $D = (8, 6)$.
2. Yes. Explanations vary. Possible response: $\frac{15}{20}$ is equivalent to $\frac{3}{4}$ so the slopes are the same.
3. Yes. Explanations vary. Possible response: $\frac{75}{100} = \frac{3}{4}$ so the triangles are similar.
4. No. Explanations vary. Possible response: $\frac{68}{90}$ is not equal to $\frac{3}{4}$.
5. Answers vary. Students may write a rule in words like "the quotient of the y-coordinate and x-coordinate has to be 0.75."

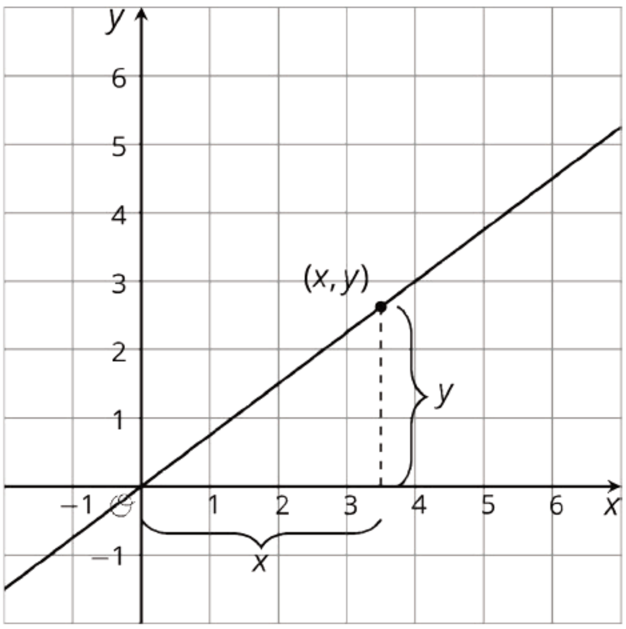
Activity Synthesis

After reviewing how students answered the first four questions, have them share their responses for question 5. Sequence responses starting with the least abstract:

- Divide the y coordinate by the x coordinate and see if the result is equal to $\frac{3}{4}$. (Ask students to explain why this works: using similar triangles or slope triangles, the key is that the quotient of the vertical length and horizontal length of these similar triangles always takes the same value.)
- The ratios $y : x$ and $3 : 4$ are equivalent (since they represent corresponding sides of similar triangles *or* since the relationship between y and x is proportional and when $y = 3$ then $x = 4$)
- $\frac{y}{x} = \frac{3}{4}$. This is a different way of stating the relationship.

This structure of coordinates for points on a line will be examined in much greater detail in upcoming lessons. Students are just starting to learn what it means to write the equation of a line—it's not intended for them to understand everything there is to know, right now.

Make sure to draw a picture showing a point (x, y) on the line and label the vertical side with its length, x, and label the horizontal side with its length, y.



Ask students how the equation $\frac{y}{x} = \frac{3}{4}$ relates to this situation. According to the reasoning in this task, the point (x, y) is on the line when $\frac{y}{x} = \frac{3}{4}$. These values are equal because they both represent the quotient of the vertical and horizontal side of slope triangles for the same line.

11.3 Writing Relationships from Slope Triangles

15 minutes

In the previous activity, students found a rule which determines whether or not a point with coordinates (x, y) lies on a certain line: there were many ways to express this rule, including an equation such as $\frac{y}{x} = \frac{3}{4}$. In this activity, students find a rule to determine if a point (x, y) lies on a line no longer containing $(0, 0)$. Proportional reasoning based on the graph of the line, which could be applied to the line in the previous activity, no longer applies here. But, proportional reasoning using similar slope triangles does still apply and gives equations that look like $\frac{y}{x} = \frac{3}{4}$ except that the quotient on the left is a little more complex. Like in the previous activity, students are using the structure of a line and properties of similar triangles to investigate these rules relating coordinates of points on the line (MP7).

Monitor for different expressions students write in the second question. For example, for line k students may write

- $\frac{y-1}{x} = \frac{3}{4}$

- $4(y-1)=3x$
- $4y=3x+4$

Invite students with different expressions to share during the discussion. Note that the rule or equation for the line is unlikely to come in the form $y=mx+b$. This is not important for now and will be addressed in future work. The important take away for this lesson is that we can write a criterion for when a point lies on a line by thinking about similar triangles and their properties.

Building On

- 8.G.A

Addressing

- 8.EE.B.6

Instructional Routines

- MLR5: Co-Craft Questions

Launch

Tell students that next, we are going to find equations satisfied by points on some more lines. These lines do not represent proportional relationships, but we can still use what we know about similar triangles to find equations.

It really helps if students write the side lengths correctly. Therefore, ask students to complete the first question and then pause. Ensure that everyone knows and understands a correct expression for each side length before proceeding. Then, instruct students to complete the second question.

Support for Students with Disabilities

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. For example, compare the graph in this activity to the the previous one in which the line passed through the origin. Review the equation of a line going through $(0,0)$ in order to provide an entry point into this activity.
Supports accessibility for: Social-emotional skills; Conceptual processing

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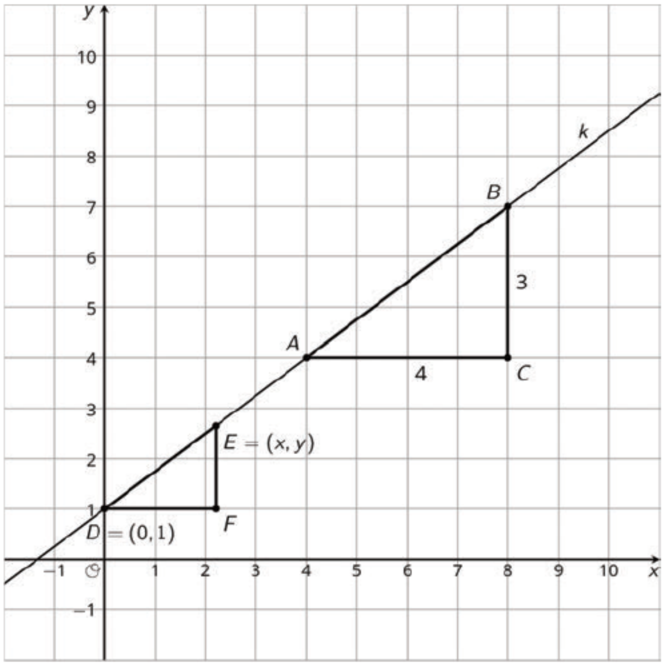
Support for English Language Learners

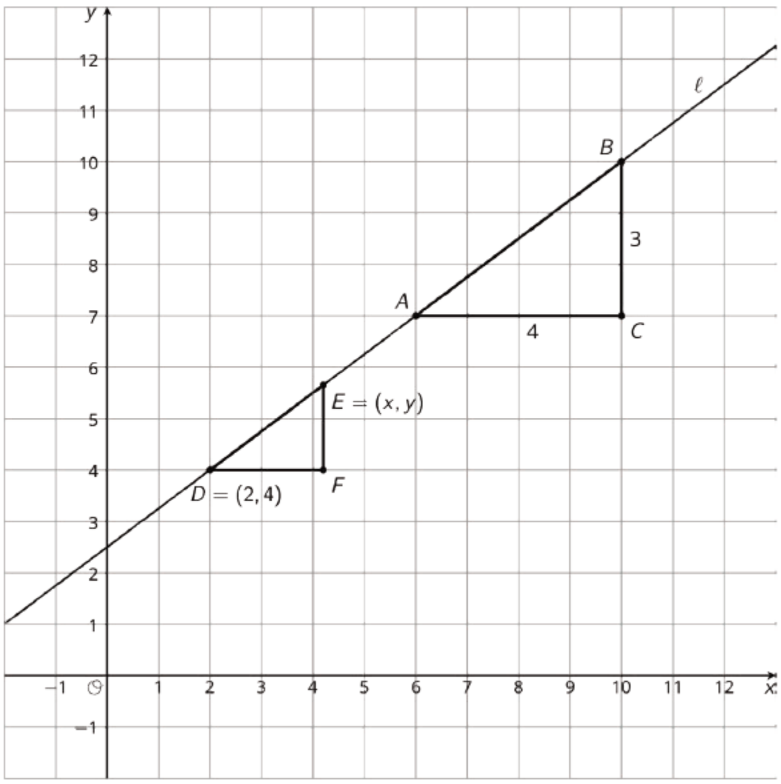
Conversing, Writing: MLR5 Co-Craft Questions. Before presenting the questions for this activity, display the diagram of the line k and have students write possible mathematical questions about the slope triangles along line k. Have students compare the questions they generated with a partner before sharing questions with the whole class. Listen for and amplify questions about creating a rule or an equation that determines whether a point with coordinates (x, y) lies on line k. Then reveal and ask students to work on the actual questions of the task. This routine will help develop students' meta-awareness of language as they generate questions about the slope triangles of a line.
Design Principle(s): Maximize meta-awareness

Student Task Statement

Here are two diagrams:

1. Complete each diagram so that all vertical and horizontal segments have expressions for their lengths.
2. Use what you know about similar triangles to find an equation for the quotient of the vertical and horizontal side lengths of $\triangle DFE$ in each diagram.





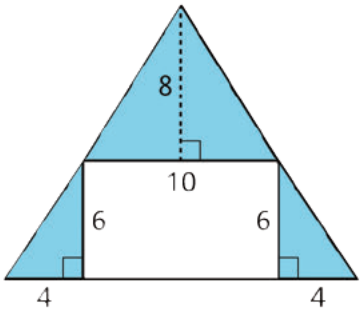
Student Response

1. On graph of line k, segment DF labeled x and segment EF labeled $y-1$. On graph of line ℓ, segment DF labeled $x-2$ and segment EF labeled $y-4$.

2. Answers vary. Possible responses: For line k, an equation satisfied by (x, y) is $\frac{y-1}{x} = \frac{3}{4}$. For line ℓ, an equation satisfied by (x, y) is $\frac{y-4}{x-2} = \frac{3}{4}$. These equations are true because the slope of a line is always the same no matter which slope triangle is used to calculate it.

Are You Ready for More?

1. Find the area of the shaded region by summing the areas of the shaded triangles.
2. Find the area of the shaded region by subtracting the area of the unshaded region from the large triangle.
3. What is going on here?



Student Response

1. 64 m^2
2. 66 m^2
3. The areas are different because the so-called "large triangle" is not actually a triangle. Each diagonal side is made up of two segments with slightly different slopes. For example, the bottom left side has a slope of $\frac{3}{2}$ while the top left side is slightly steeper, with a slope of $\frac{8}{5}$. Therefore, the second method of finding the area was not valid for this figure.

Activity Synthesis

Ask selected students what they wrote for an equation satisfied by x and y for the two graphs, making sure to note different forms. One equation is $\frac{y-1}{x} = \frac{3}{4}$. There is no reason to manipulate this, but some students might rewrite this as $4y - 4 = 3x$ or even $y = \frac{1}{4}(3x + 4)$. For the second graph, students may write $\frac{y-4}{x-2} = \frac{3}{4}$ or $4(y - 4) = 3(x - 2)$. At this point, the important thing to notice is that the coordinates for *any* point (x, y) on the line will satisfy this relationship. This is because we did not use any special properties of the point (x, y) (just that it lies on the line) to find the relationship. There is no reason to manipulate the equations $\frac{y-1}{x} = \frac{3}{4}$ or $\frac{y-4}{x-2} = \frac{3}{4}$ because these two equations contain all of the information from the similar triangles. (Once the equations are manipulated, this structure is lost, and it is this structure that is of central importance here.)

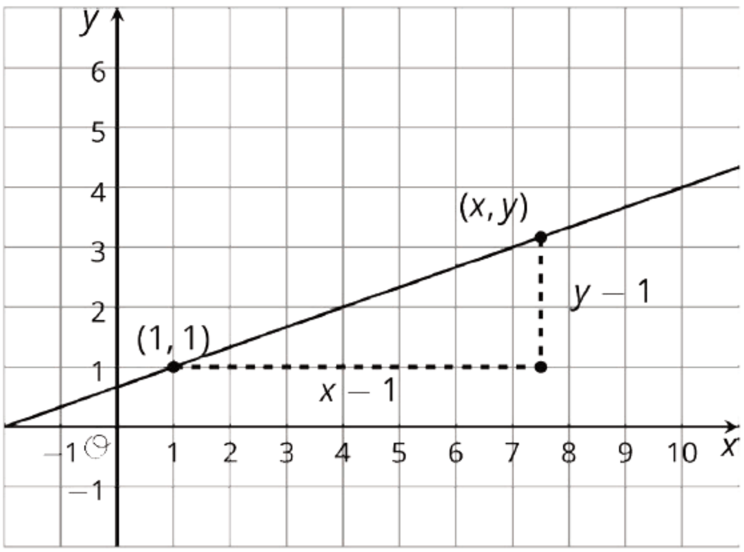
Make sure that students understand that the slope of lines k and ℓ (as well as j from the previous activity) are all $\frac{3}{4}$. The location of the lines in the plane is different, and this is reflected in the different equations for the three lines. If the equations are in the form $\frac{y}{x} = \frac{3}{4}$, $\frac{y-1}{x} = \frac{3}{4}$, and $\frac{y-4}{x-2} = \frac{3}{4}$, we can see that what is the same in each is the $\frac{3}{4}$ and that there is a y in each numerator on the left and an x in the denominator.

Note, if needed, that the equation does not make sense if the denominator is equal to zero. For example, $\frac{y-1}{x} = \frac{3}{4}$ does not make sense if $x = 0$ and $y = 1$. The reason for this is that $(0, 1)$ is the one point on the line where we can *not* build a slope triangle together with the point $(0, 1)$.

Time permitting, ask students how the equations for k, ℓ, and j are alike and how they are different. A key way they are alike is that they can all be written as some quotient involving y and x equal to $\frac{3}{4}$. The reason for the common value of $\frac{3}{4}$ is that the lines all have the same slope, namely $\frac{3}{4}$. A key difference is the values subtracted from x and y on the left hand side of the equations: these are different because each line is in a different location on the coordinate plane.

Lesson Synthesis

Today, we used slope triangles to find a relationship satisfied by the coordinates of *all* points on a line.



What is the slope of this line? It's $\frac{2}{6}$ because the points $(1, 1)$ and $(7, 3)$ are on the line. The point (x, y) lies on this line so the slope we calculate with the slope triangle for (x, y) must also be equal to $\frac{2}{6}$. The slope for this triangle is $\frac{y-1}{x-1}$ since the vertical side has length $y - 1$ and the horizontal side has length $x - 1$. This means that

$$\frac{y-1}{x-1} = \frac{2}{6}$$

This relationship is true regardless of which point (x, y) we choose on the line!

11.4 Matching Relationships to Graphs

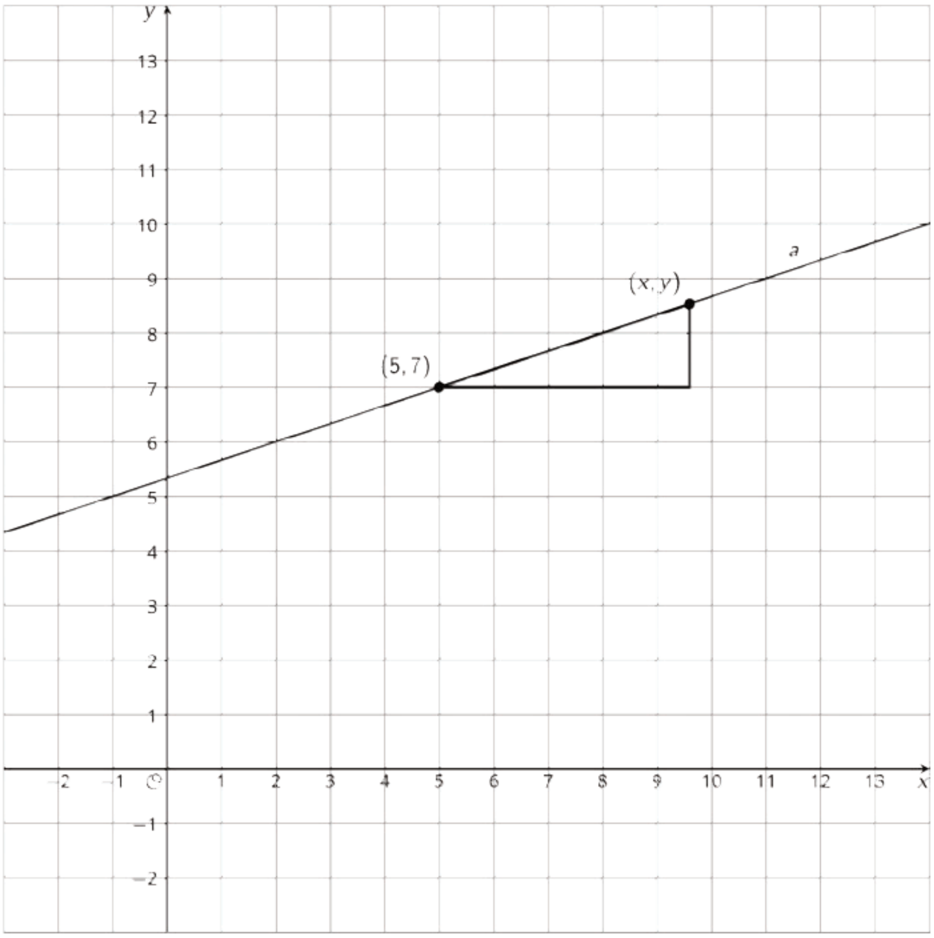
Cool Down: 5 minutes

Students use a slope triangle one of whose points has variables as coordinates in order to write an equation satisfied by the variables (and therefore satisfied by the coordinates of any point on the line). The equation comes from using similar slope triangles and identifying their slopes.

Addressing

- 8.EE.B.6

Student Task Statement



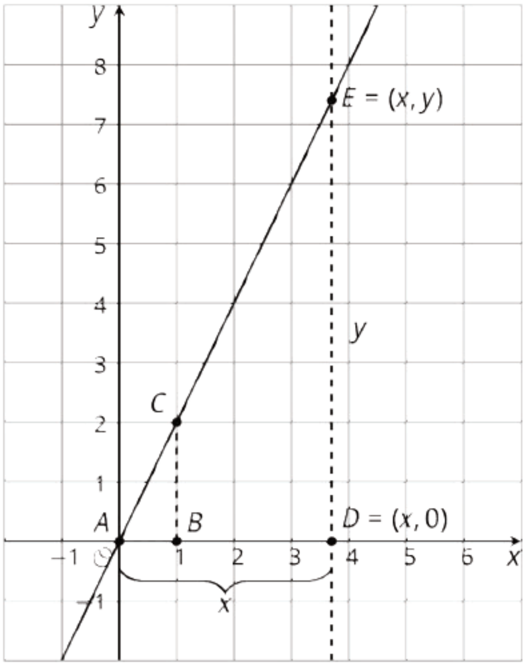
1. Explain why the slope of line a is $\frac{2}{6}$.
2. Label the horizontal and vertical sides of the triangle with expressions representing their length.
3. Explain why $\frac{y-7}{x-5} = \frac{2}{6}$.

Student Response

1. The points $(5, 7)$ and $(11, 9)$ are on the line. Using these to find the slope gives a value of $\frac{2}{6}$.
2. The vertical side has length $y - 7$, and the horizontal side has length $x - 5$.
3. The triangle in the picture is a slope triangle so the quotient of its vertical and horizontal sides has to be $\frac{2}{6}$. This means that $\frac{y-7}{x-5} = \frac{2}{6}$ is true for any point (x, y) on the line.

Student Lesson Summary

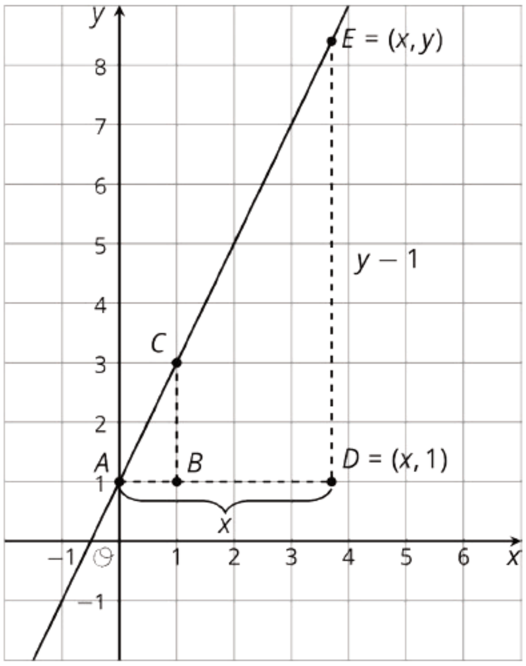
Here are the points A, C, and E on the same line. Triangles ABC and ADE are slope triangles for the line so we know they are similar triangles. Let's use their similarity to better understand the relationship between x and y, which make up the coordinates of point E.



The slope for triangle ABC is $\frac{2}{1}$ since the vertical side has length 2 and the horizontal side has length 1. The slope we find for triangle ADE is $\frac{y}{x}$ because the vertical side has length y and the horizontal side has length x. These two slopes must be equal since they are from slope triangles for the same line, and so: $\frac{2}{1} = \frac{y}{x}$.

Since $\frac{2}{1} = 2$ this means that the value of y is twice the value of x, or that $y = 2x$. This equation is true for any point (x, y) on the line!

Here are two different slope triangles. We can use the same reasoning to describe the relationship between x and y for this point E.



The slope for triangle ABC is $\frac{2}{1}$ since the vertical side has length 2 and the horizontal side has length 1. For triangle ADE, the horizontal side has length x. The vertical side has length $y-1$ because the distance from (x, y) to the x-axis is y but the vertical side of the triangle stops 1 unit short of the x-axis. So the slope we find for triangle ADE is $\frac{y-1}{x}$. The slopes for the two slope triangles are equal, meaning:

$$\frac{2}{1} = \frac{y-1}{x}$$

Since $y-1$ is twice x, another way to write this equation is $y-1=2x$. This equation is true for any point (x, y) on the line!

Lesson 11 Practice Problems

Problem 1

Statement

For each pair of points, find the slope of the line that passes through both points. If you get stuck, try plotting the points on graph paper and drawing the line through them with a ruler.

a. $(1, 1)$ and $(7, 5)$

b. $(1, 1)$ and $(5, 7)$

c. $(2, 5)$ and $(-1, 2)$

d. $(2, 5)$ and $(-7, -4)$

Solution

a. $\frac{2}{3}$

b. $\frac{3}{2}$

c. 1

d. 1

Problem 2

Statement

Line ℓ is shown in the coordinate plane.

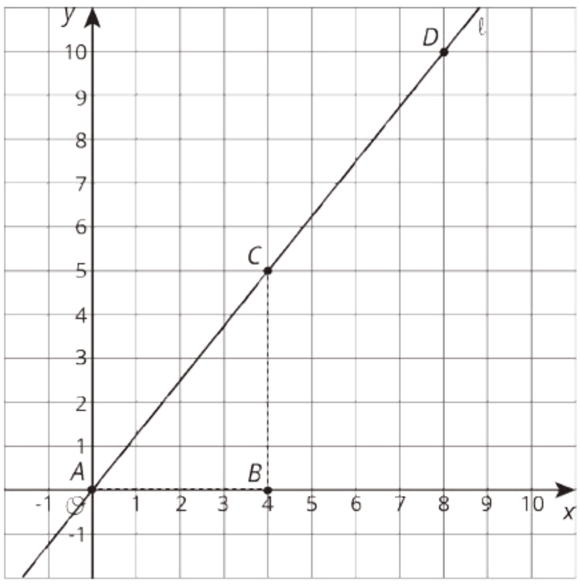
a. What are the coordinates of points B and D?

b. Is the point $(16, 20)$ on line ℓ? Explain how you know.

c. Is the point $(20, 24)$ on line ℓ? Explain how you know.

d. Is the point $(80, 100)$ on line ℓ? Explain how you know.

e. Write a rule that would allow you to test whether (x, y) is on line ℓ.



Solution

a. $B = (4, 0)$ and $D = (8, 10)$

b. Yes, because $\frac{20}{16} = \frac{5}{4}$. The slopes are the same.

c. No, because $\frac{24}{20} \neq \frac{5}{4}$.

d. Yes, because $\frac{100}{80} = \frac{5}{4}$.

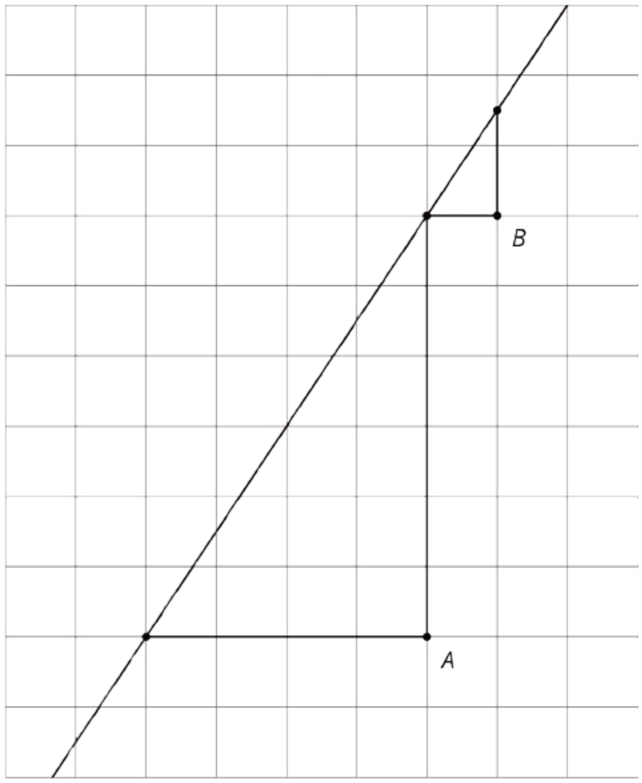
e. Answers vary. Sample response: $\frac{y}{x} = \frac{5}{4}$

Problem 3

Statement

Consider the graphed line.

Mai uses Triangle A and says the slope of this line is $\frac{6}{4}$. Elena uses Triangle B and says no, the slope of this line is 1.5. Do you agree with either of them? Explain.



Solution

They are both correct. The slope of a line can be found using any right triangle with legs parallel to the axes and longest side on the line, as any two such triangles are similar. Numerically, this checks out as $\frac{6}{4}$ and 1.5 represent the same value.

Problem 4

Statement

A rectangle has length 6 and height 4.

Which of these would tell you that quadrilateral $ABCD$ is definitely *not* similar to this rectangle? Select **all** that apply.

A. $AB = BC$

B. $m\angle ABC = 105^\circ$

C. $AB = 8$

D. $BC = 8$

E. $BC = 2 \cdot AB$

F. $2 \cdot AB = 3 \cdot BC$

Solution

["A", "B", "E"]

(From Unit 2, Lesson 7.)

Lesson 12: Using Equations for Lines

Goals

- Create an equation of a line with positive slope on a coordinate grid using knowledge of similar triangles.
- Generalize (orally) a process for dilating a slope triangle ABC on a coordinate plane with center of dilation A and scale factor s.
- Justify (orally) that a point(x, y) is on a line by verifying that the values of x and y satisfy the equation of the line.

Learning Targets

- I can find an equation for a line and use that to decide which points are on that line.

Lesson Narrative

In the previous two lessons, students saw that all slope triangles for a line give the same slope value, and this value is called the slope of the line. They also began writing relationships satisfied by all points (x, y) on a line. In this lesson, they continue to write equations but with less scaffolding, that is no similar triangles are selected so students need to figure out what to do given a line and a few points on the line.

The properties of slope triangles that make the slope of a line meaningful have to do with dilations. In particular, dilations do not change the quotient of the vertical side length and horizontal side length of a slope triangle. Students return to dilations in this lesson, applied to a single slope triangle with varying scale factor. This gives a different way of seeing how the coordinates of points on a line vary.

Both techniques, using equations and studying all of the dilations of a single slope triangle, give expressions representing points on a line.

Alignments

Addressing

- 8.EE.B.6: Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.
- 8.G.A: Understand congruence and similarity using physical models, transparencies, or geometry software.
- 8.G.A.3: Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Building Towards

- 8.EE.B.6: Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR3: Clarify, Critique, Correct
- MLR7: Compare and Connect
- Think Pair Share

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Student Learning Goals

Let's write equations for lines.

12.1 Missing center

Warm Up: 5 minutes
Given a point, the image of the point under a dilation, and the scale factor of the dilation, students must identify the center of the dilation. This requires thinking about the meaning of dilations and the fact that the center of dilation, the point dilated, and the image, are collinear.

Addressing

- 8.G.A

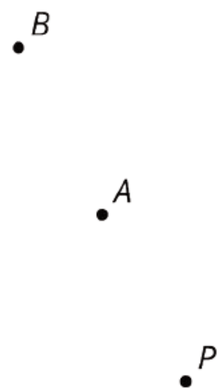
Launch

Provide access to geometry toolkits. (In particular, a ruler or index card is needed.)

Student Task Statement

A dilation with scale factor 2 sends A to B. Where is the center of the dilation?

Student Response



The point, marked as P in the figure, is on the same line as A and B, the same distance from B to A, but on the other side of A.

Activity Synthesis

Ask students what they know about the center of dilation that helps to solve this problem. The key fact is that the center of dilation lies on the same line as A and B. The scale factor is 2 so if P is the center of dilation, then the length of segment PB is twice the length of segment PA.

12.2 Writing Relationships from Two Points

10 minutes

In the previous lesson, students found an equation satisfied by the points on a line using properties of slope triangles and a general point, labeled (x, y), on the line. In this activity, they pursue this work but the scaffold of the given slope triangles has been removed. Once students draw appropriate slope triangles, this is an opportunity to practice and consolidate learning. In addition, students use the equation (satisfied by points on the line) to check whether or not specific points lie on the line.

Note that the y-intercept was intentionally left off of this diagram, so that students who may have seen some of this material are discouraged from jumping straight to $y = mx + b$ and encouraged to engage with thinking about similar triangles.

There are many slope triangles that students can draw, but the one joining (5,3) and (7,7) is the most natural for calculating the slope and then (x, y) and either $(5, 3)$ or $(7, 7)$ can be used to find an equation. Monitor for students who make these choices (and use them to decide whether or not the given points lie on the line) and invite them to present during the discussion.

Building Towards

- 8.EE.B.6

Instructional Routines

- MLR3: Clarify, Critique, Correct
- Think Pair Share

Launch

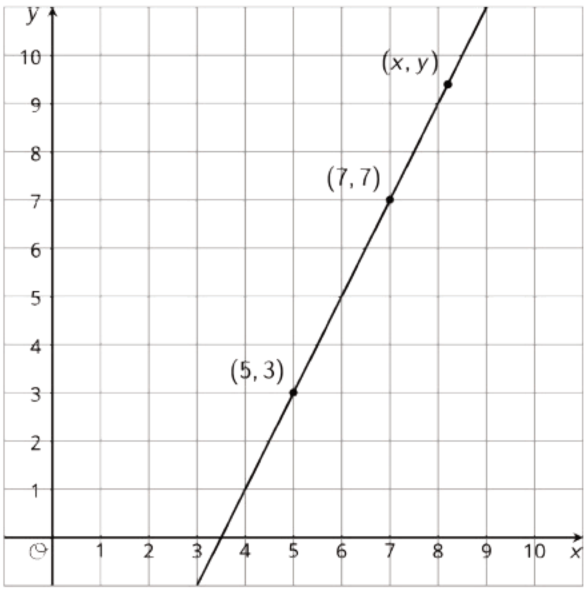
Provide access to geometry toolkits (in particular, a straightedge is helpful). Give 2–3 minutes of quiet work time. Then ask them to share their responses and reasoning with a partner, followed by a whole-class discussion.

Support for Students with Disabilities

Representation: Internalize Comprehension. Begin with a physical demonstration of using slope triangles to find the equation of a line to support connections between new situations and prior understandings. Consider using these prompts: "What does this demonstration have in common with the graph in this activity?" or "How can you apply your reasoning with slope triangles to this graph?"
Supports accessibility for: Conceptual processing; Visual-spatial processing

Student Task Statement

Here is a line.



1. Using what you know about similar triangles, find an equation for the line in the diagram.
2. What is the slope of this line? Does it appear in your equation?
3. Is $(9, 11)$ also on the line? How do you know?
4. Is $(100, 193)$ also on the line?

Student Response

1. $\frac{y-7}{x-7} = 2$ or equivalent.
2. 2. Answers vary based on equation.
3. Yes. Possible strategies:
    - For every 1 to the right, you go 2 up. I started at $(7, 7)$ and counted until I got to $(9, 11)$.
    - When y is 11 and x is 9, it makes my equation true.
4. Yes. When y is 193 and x is 100, it satisfies my equation.

Are You Ready for More?

There are many different ways to write down an equation for a line like the one in the problem. Does $\frac{y-3}{x-6} = 2$ represent the line? What about $\frac{y-6}{x-4} = 5$? What about $\frac{y+5}{x-1} = 2$? Explain your reasoning.

Student Response

No. The equation $\frac{y-3}{x-6} = 2$ represents a line of slope 2 containing the point $(6, 3)$. Since our line does not contain the point $(6, 3)$, this is not our line. The equation $\frac{y-6}{x-4} = 5$ represents a line of slope 5 containing the point $(4, 6)$. Since our line has slope 2, this is not our line. The equation $\frac{y+5}{x-1} = 2$ represents a line of slope 2 containing the point $(1, -5)$. Since our line contains the point $(1, -5)$ and has slope 2, this is another possible equation for our line.

Activity Synthesis

Invite selected students to show how they arrived at their equation. Also, ask them how the equation helps to determine whether or not the points in the last two questions lie on the line.

Emphasize that the graph, as shown, is not helpful for checking whether or not $(100, 193)$ is on the line, but the equation is true for these numbers, so this point must be on the line.

Highlight that using (x, y) and $(5, 3)$ for a slope triangle gives an equation such as $\frac{y-3}{x-5} = 2$ while using (x, y) and $(7, 7)$ gives the equation $\frac{y-7}{x-7} = 2$. These equations look different, but they both work to check whether or not a point (x, y) is on the line. Using algebra to show that these two equations are equivalent is not necessary (or appropriate in grade 8), but students can see from the picture that either equation can be used to test whether or not a point (x, y) is on the line. If students have not done so already when they share their solutions, draw and label the two slope triangles that correspond to these two equations.

Support for English Language Learners

Reading, Writing, Speaking: MLR3 Clarify, Critique, Correct. Before students share whether the point with coordinates (100, 193) is on the line, present an incorrect answer and explanation. For example, "The equation of the line is $\frac{x-5}{y-3} = 2$. When x is 100 and y is 193, it does not satisfy my equation because $\frac{100-5}{193-3} = \frac{95}{190} = \frac{1}{2}$. Since $\frac{1}{2}$ is not equal to 2, then the point (100, 193) is not on the line." Ask students to identify the error, critique the reasoning, and write a correct explanation. Prompt students to share their critiques and corrected explanations with the class. Listen for and amplify the language students use to clarify the meaning of slope and explain why the slope of the line is equivalent to $\frac{y-3}{x-5}$. This routine will engage students in meta-awareness as they critique and correct a common misconception about the slope of a line.
Design Principle(s): Optimize output (for explanation); Maximize meta-awareness

12.3 Dilations and Slope Triangles

15 minutes
This activity investigates the coordinates of points on a line from the point of view of dilations. At the beginning of this unit, students experimented with dilations and made numerous important discoveries including

- Dilations change distances between points by a scale factor s.
- Dilations preserve angles.
- Dilations take lines to lines.

Having developed the key ideas of *similar triangles* and *slope*, this activity returns to dilations, applying them systematically to a single slope triangle. All of these dilations of the triangle are similar, their long sides all lie on the same line, and the coordinates of the points on that line have a structure intimately linked with the dilations used to produce them.

For the third question, monitor for students who

- look for and express a pattern for the coordinates of the points from earlier questions (MP8); a scale factor of 1 gives $C = (2, 2)$, a scale factor of 2 gives $C = (4, 3)$, a scale factor of 2.5 gives $C = (5, 3.5)$, so the x-coordinate appears to be twice the scale factor while the y-coordinate appears to be one more than the scale factor.
- use the structure of $\triangle ABC$ and the definition of dilations.

For the final question, students can either

- reason through from scratch, looking at the x-coordinate or y-coordinate for example or
- use the coordinates they find for a scale factor of s in the third question.

Monitor for both approaches and invite students to share during the discussion.

Addressing

- 8.G.A.3

Building Towards

- 8.EE.B.6

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect

Launch

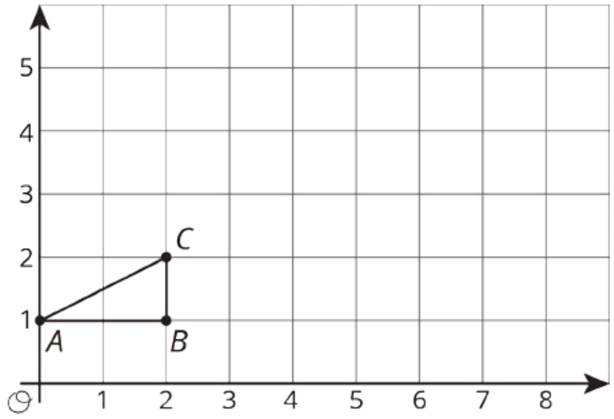
Access to geometry toolkits. Give 2–3 minutes of quiet work time, then ask students to share their reasoning with a partner. Follow with a whole-class discussion.

Support for Students with Disabilities

Representation: Develop Language and Symbols. Maintain a display of important terms and vocabulary. During the launch, take time to review terms students will need to access for this activity. Invite students to suggest language or diagrams to include that will support their understanding of: dilation and scale factor.
Supports accessibility for: Conceptual processing; Language

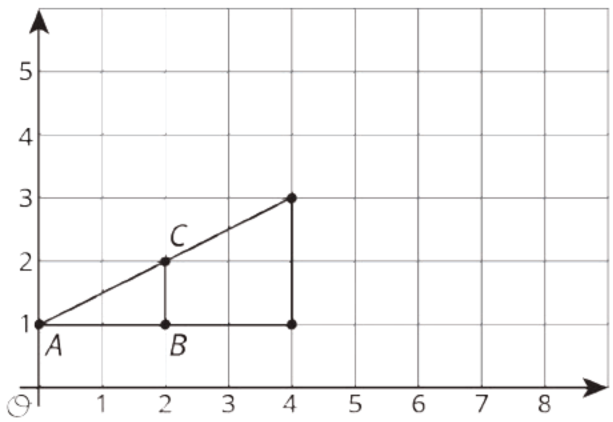
Student Task Statement

Here is triangle ABC.

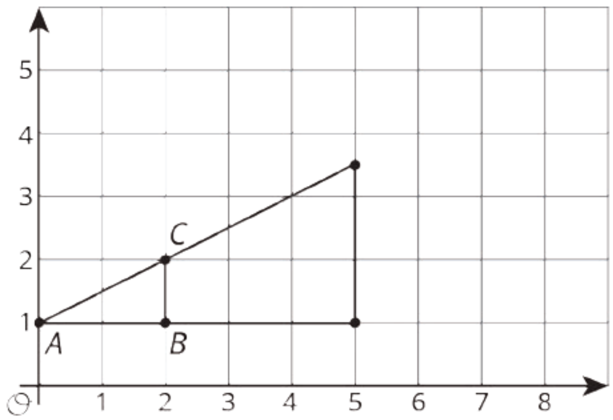


1. Draw the dilation of triangle ABC with center $(0, 1)$ and scale factor 2.
2. Draw the dilation of triangle ABC with center $(0, 1)$ and scale factor 2.5.
3. Where is C mapped by the dilation with center $(0, 1)$ and scale factor s?
4. For which scale factor does the dilation with center $(0, 1)$ send C to $(9, 5.5)$? Explain how you know.

Student Response



1.



2.

3. $(2s, s+1)$. To get from (0,1) to (2,2), we move to the right by 2 units and up 1 unit. Applying a scale factor of s will multiply each of these side lengths by s. So C will go to the point $2s$ units to the right and s units up from $(0, 1)$. That is the point $(2s, s+1)$.

4. $s = 4.5$. We want $(2s, s+1)$ to be $(9, 5.5)$. Solve either the equation $2s = 9$ or $s + 1 = 5.5$ to get $s = 4.5$.

Activity Synthesis

First, focus on the third question, inviting selected students to present, in this sequence: first those who identified a pattern from the first two questions and then those who studied the impact of a dilation with scale factor s on $\triangle ABC$. Point out that the argument looking at where C is taken by the dilation with scale factor s and center A explains *why* the x coordinate doubles and the y coordinate is 1 more than the scale factor.

Next, invite selected students to share their answers to the last question.

In previous activities, students have used similar triangles to show that the points (x, y) on the line containing the long side of triangle ABC satisfy the relationship $\frac{y-1}{x} = \frac{1}{2}$. In this activity, they find that points (x, y) on this line are of the form $(2s, s+1)$ where s is a (positive) real number. Make sure that students understand that the equation $\frac{y-1}{x} = \frac{1}{2}$ is true if we take $y - 1 = s + 1$ and $x = 2s$. Also, emphasize the key role that dilations play in these arguments.

The big take away from this lesson is that the structure of the coordinates of points on a line can be derived from properties of dilations.

Support for English Language Learners

Speaking, Listening: MLR7 Compare and Connect. When discussing where C is mapped, some students may need support to generalize that C will be mapped to $(2s, s+1)$ after a dilation with center $(0, 1)$ and scale factor s and how these distances relate to the visual image. Ask students to write a short explanation or create a visual display of how they determined where C is mapped. Provide students time to examine the different approaches or representations of their classmates. Ask students what worked well in different approaches. Next, ask students to explain how the lengths in their explanations relate to the generalized form. Ask students where they see $2s$ and $s+1$ represented in the slope triangles. Be sure to demonstrate asking questions that students can ask each other, rather than asking questions to test understanding.
Design Principle(s): Maximize meta-awareness, Cultivate conversation

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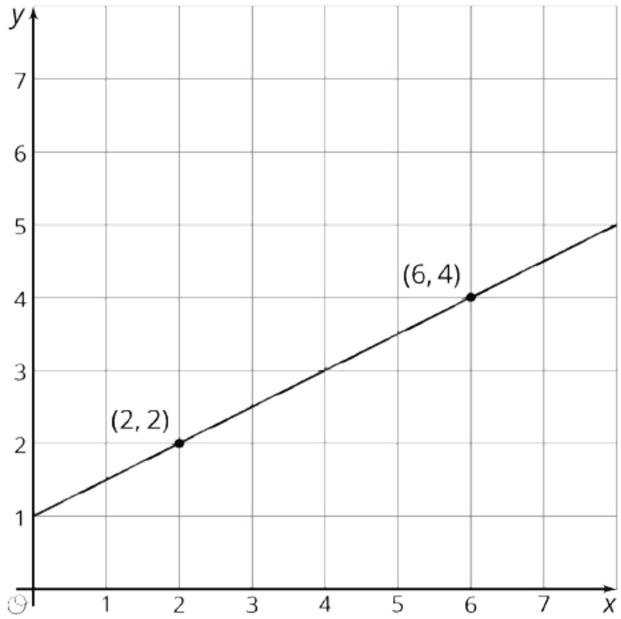
Lesson Synthesis

The coordinates of points on a line have a nice structure that is useful for checking whether or not a given point is on a line. Here is a line with a couple of labeled points.



- What is the slope of this line? It's $\frac{1}{2}$ because a slope triangle (draw in the slope triangle) for the two labeled points has horizontal side length 4 and vertical side length 2.
- What is an equation for the line? One example is $\frac{y-2}{x-2} = \frac{1}{2}$. Label a general point on the line (x, y), and draw in a slope triangle to show this relationship.

How can we find out whether or not the point $(72, 37)$ is on this line? The points on the line satisfy the equation $\frac{y-2}{x-2} = \frac{1}{2}$. Since $\frac{37-2}{72-2} = \frac{1}{2}$, the point $(72, 37)$ is on the line!

12.4 Is the Point on the Line?

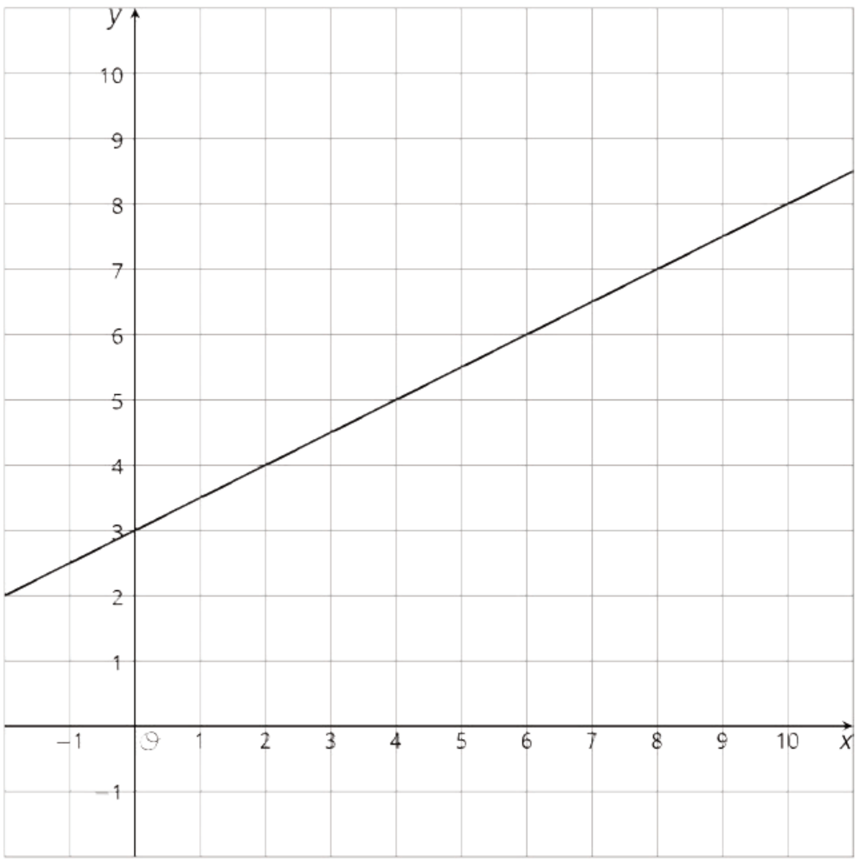
Cool Down: 5 minutes

Students determine whether or not a point with given coordinates lies on a line. The line is presented graphically with no scaffolding and the coordinates of the point are too large to check whether or not it lies on the line by examining the graph. Students are expected to write an equation satisfied by points on the line and apply this.

Addressing

- 8.EE.B.6

Student Task Statement



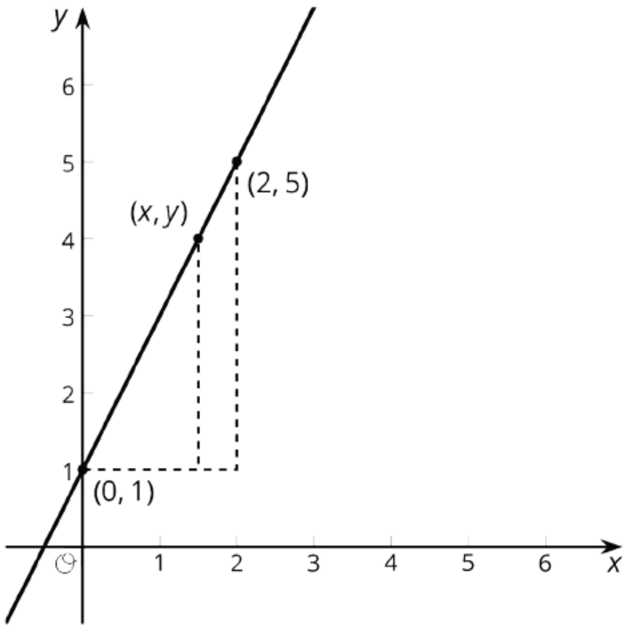
Is the point (20,13) on this line? Explain your reasoning.

Student Response

Yes. The slope of the line is $\frac{1}{2}$: moving to the right 2 units and up 1 unit gives a point on the line. Since $(0, 3)$ is on the line, moving right 20 units and up 10 units gives another point on the line. This is the point $(20, 13)$.

Student Lesson Summary

We can use what we know about slope to decide if a point lies on a line. Here is a line with a few points labeled.



The slope triangle with vertices $(0,1)$ and $(2,5)$ gives a slope of $\frac{5-1}{2-0} = 2$. The slope triangle with vertices $(0,1)$ and (x,y) gives a slope of $\frac{y-1}{x}$. Since these slopes are the same, $\frac{y-1}{x} = 2$ is an equation for the line. So, if we want to check whether or not the point $(11,23)$ lies on this line, we can check that $\frac{23-1}{11} = 2$. Since $(11,23)$ is a solution to the equation, it is on the line!

Lesson 12 Practice Problems

Problem 1

Statement

Select **all** the points that are on the line through $(0,5)$ and $(2,8)$.

A. $(4,11)$

B. $(5,10)$

C. $(6,14)$

D. $(30,50)$

E. $(40,60)$

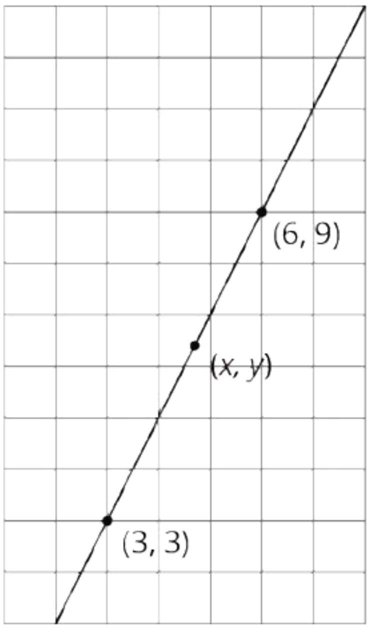
Solution

["A", "C", "D"]

Problem 2

Statement

All three points displayed are on the line. Find an equation relating x and y.



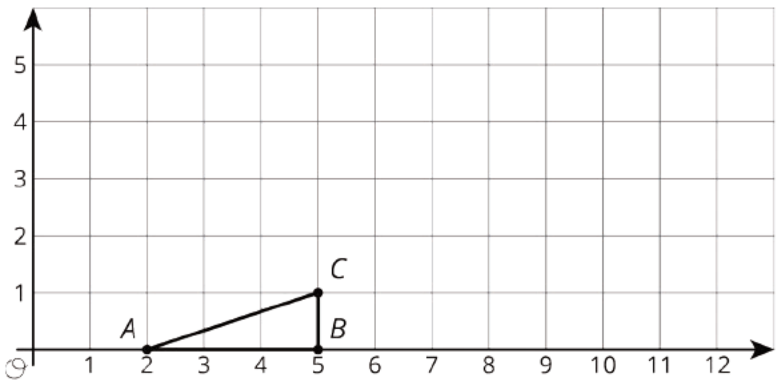
Solution

Answers vary. Sample response: $\frac{y-3}{x-3} = 2$ (or $y = 2x - 3$)

Problem 3

Statement

Here is triangle ABC.



a. Draw the dilation of triangle ABC with center $(2, 0)$ and scale factor 2.

b. Draw the dilation of triangle ABC with center $(2, 0)$ and scale factor 3.

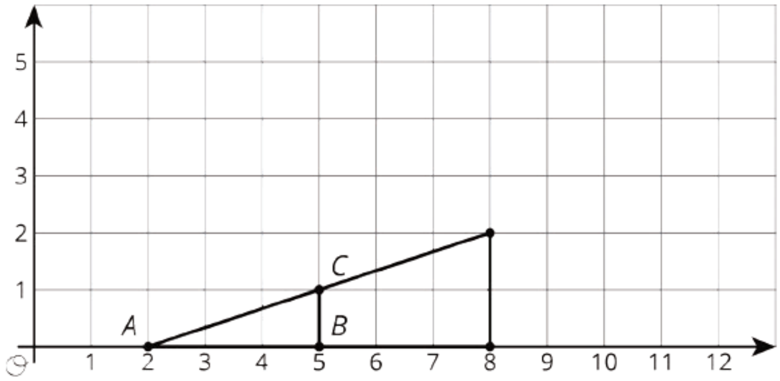
c. Draw the dilation of triangle ABC with center $(2, 0)$ and scale factor $\frac{1}{2}$.

d. What are the coordinates of the image of point C when triangle ABC is dilated with center $(2, 0)$ and scale factor s?

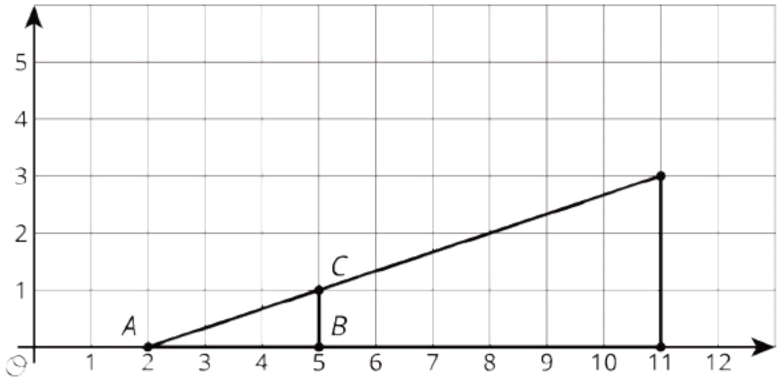
e. Write an equation for the line containing all possible images of point C.

Solution

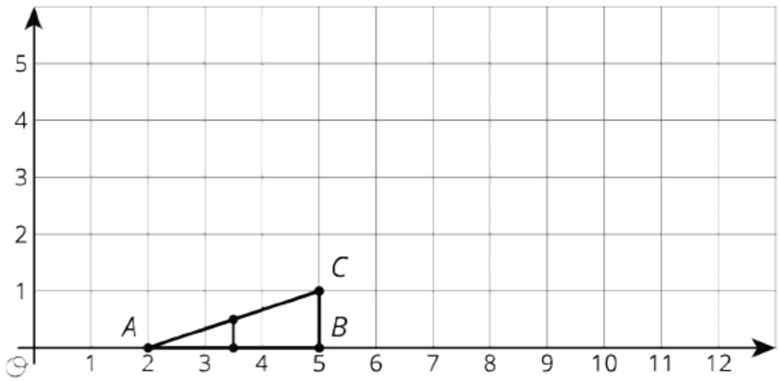
a.



b.



c.



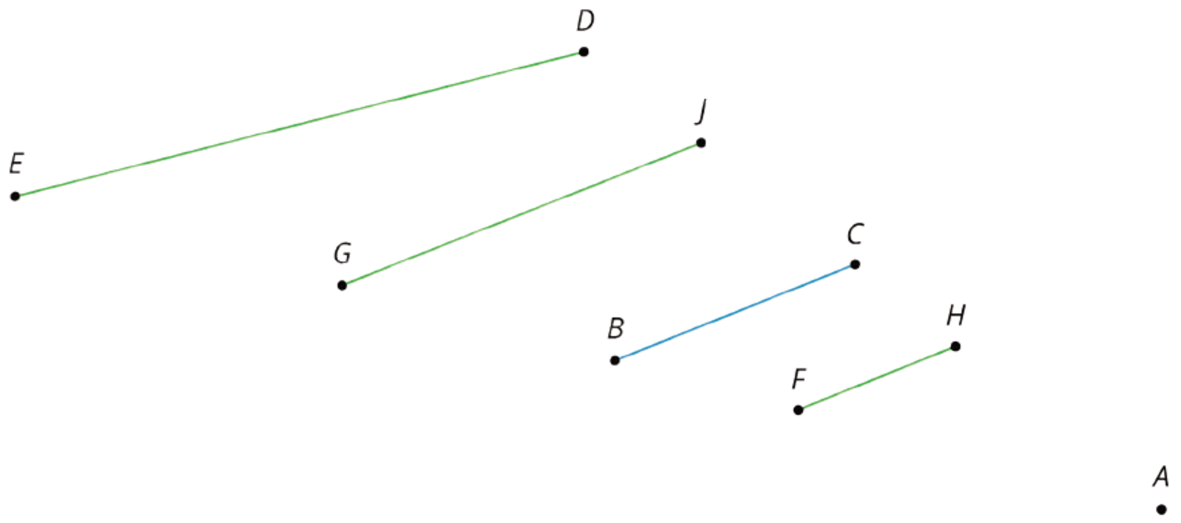
d. $(2 + 3s, s)$

e. $\frac{y}{x-2} = \frac{1}{3}$ (or equivalent)

Problem 4

Statement

Here are some line segments.



a. Which segment is a dilation of $\overline{BC}$ using A as the center of dilation and a scale factor of $\frac{2}{3}$?

b. Which segment is a dilation of $\overline{BC}$ using A as the center of dilation and a scale factor of $\frac{3}{2}$?

c. Which segment is not a dilation of $\overline{BC}$, and how do you know?

Solution

a. Segment $\overline{FH}$ (A scale factor of $\frac{2}{3}$ produces a parallel line segment with shorter length.)

b. Segment $\overline{GJ}$ (A scale factor of $\frac{3}{2}$ will produce a parallel line segment with longer length.)

c. Segment $\overline{DE}$ (Dilations take lines to parallel lines, and $\overline{DE}$ is not parallel to $\overline{FH}$.)

(From Unit 2, Lesson 4.)

Section: Let's Put It to Work

Lesson 13: The Shadow Knows

Goals

- Calculate the unknown heights of objects by using proportional reasoning and explain (orally) the solution method.
- Justify (orally) why the relationship between the height of objects and the length of their shadows cast by the sun is approximately proportional.

Learning Targets

- I can model a real-world context with similar triangles to find the height of an unknown object.

Lesson Narrative

In this lesson, students examine the length of shadows of different objects. There appears to be a proportional relationship between the height of the object and the length of the shadow. Students use this relationship to predict the height of a lamppost given the length of its shadow. In order to justify the proportional relationship, students use the hypothesis that the rays of sunlight making the shadows are parallel, together with their knowledge of similar triangles. Finally, students go outside and make their own measurements of different objects and the lengths of their shadows and use this technique to estimate the height of a tall object.

This lesson involves modeling (MP4), not only because students interpret real-world data (both the given heights and shadow lengths and the measurements that they take themselves) but also because they need to make simplifying assumptions in order to justify why the relationship is proportional.

Alignments

Building On

- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

Addressing

- 8.G.A.5: Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

Building Towards

- 8.G.A.4: Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations;

given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

- 8.G.A.5: Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR2: Collect and Display
- Notice and Wonder

Required Materials

Measuring tapes

Required Preparation

Before doing the last activity, conduct the experiment ahead of time to ensure that shadow lengths will be cooperative at the time your class takes place. Also, make preparations to take your class outside. They will need measuring devices (tape measures, yard sticks, rulers) as well as a way to record their measurements.

> **Student Learning Goals**
>
> Let's use shadows to find the heights of an object.

13.1 Notice and Wonder: Long Shadows and Short Shadows

Warm Up: 5 minutes

The purpose of this warm-up is to show what happens when shadows are cast from a lamp versus the Sun. Later in this lesson, it is important that students understand that rays of sunlight that hit Earth are essentially parallel. While students may notice and wonder many things about these images, the length of the shadows (which are different for the pens near the lamp and appear to be same for the pens in the sunshine) is the most important discussion point.

The rest of this lesson will look at the shadows of the pens (and other objects) in detail, using the idea that the rays of sunlight hitting the pens are parallel (or nearly so).

Building Towards

- 8.G.A.4

Instructional Routines

- MLR2: Collect and Display
- Notice and Wonder

Launch

Arrange students in groups of 2. 2 minutes of quiet think time then share with a partner.

Student Task Statement

What do you notice? What do you wonder?

Student Response

Things students may notice:

- It's the same pens in both photos.
- The pens have shadows in both photos.
- One photo was taken inside; one outside.
- One light source is a lamp; the other the Sun.
- In the photo where the light source is a lamp, the light is coming from the right. In the other photo, the light is coming from the left.

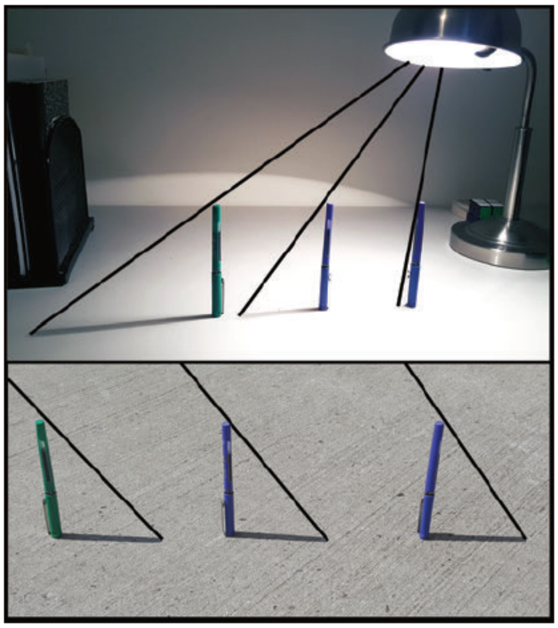
- In the photo where the light source is a lamp, the shadows are all different lengths. In the photo where the light source is the Sun, the shadows are all the same length.

Things students may wonder:

- Are there times when the pens outdoors don't leave any shadow?
- Why do the pens near the lamp have different length shadows?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. After each response, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to the images each time.



If possible, show these images where the path of the light that reaches the top of each shadow is drawn in.

Use MLR 2 (Collect and Display) to gather words and phrases students use when describing why the pens do not have the same length of shadows in the first picture but do in the second. Make explicit connections to illustrate that the Sun's light hits the pens at the same angle.

13.2 Objects and Shadows

15 minutes

In this activity, students look at a photo of three people, a lamppost, and their shadows taken on a sunny day. They notice that there is an approximately proportional relationship between the height of an object and the length of its shadow. Then, they use what they know about proportional relationships and the length of a shadow to find the height of an object that is difficult to measure directly.

In this activity, students just notice that the relationship appears to be proportional based on inspecting several corresponding lengths. In the next activity, they will create a justification for why the relationship is proportional.

The given measurements are real measurements rounded to the nearest inch. Therefore, the given values are not in a perfectly proportional relationship. The quotient of each shadow length and its corresponding object's height is around, but not exactly, $\frac{2}{3}$. Students are engaging in MP4 when they reason mathematically about real-world measurements.

Building On

- 7.RP.A.2

Building Towards

- 8.G.A.5

Instructional Routines

- MLR2: Collect and Display

Launch

Display the photo in the task, and ask students how they would go about measuring the height of each person and the lamppost. It would be straightforward to measure the height of the people using a yard stick or tape measure, but it would be difficult to measure the height of the lamppost. Tell students that even when something is too tall to measure directly, we can still figure out its height by using the length of its shadow (which, since it's on the ground, is easy to measure).

Draw students' attention to the measurements given in the table, and invite students to look for relationships in the table and use any relationships they notice to make a conjecture about the height of the lamppost.

Keep students in the same groups. 2 minutes of quiet work time and then students share thinking and continue working with a partner, followed by a whole-class discussion.

Support for Students with Disabilities

Representation: Internalize Comprehension. Represent the same information through different modalities by using a diagram. If students are unsure where to begin, suggest that they draw a diagram (a horizontal line for the shadow and a vertical line for the height for each person) and label them using the measurements provided in the table.
Supports accessibility for: Conceptual processing; Visual-spatial processing

Support for English Language Learners

Speaking: MLR2 Collect and Display. In order for students' own output to become a reference in developing mathematical language for this lesson, listen for and record the language students use to describe how they would measure the height of each person and the lamp post. As students work in pairs to determine the relationship between height and shadow length, listen for how students are comparing the quantities. Record phrases that students use, such as: "When I divide ...." and "Compared to the height, the shadow....". Over the course of this lesson, ask student to suggest revisions and updates to the display as they develop both new mathematical ideas and new ways of communicating ideas.
Design Principle(s): Support sense-making, Maximize meta-awareness

Anticipated Misconceptions

The task uses real measurements that were taken to the nearest inch. Because of the rounding, the values given are not in a perfectly proportional relationship, so students may hesitate to identify the relationship as proportional. If students struggle with this aspect of the activity, suggest that they start by coming up with a range of reasonable values for the height of the post. Also, share with them that the measurements were rounded to the nearest inch, so it's possible that the relationship is imperfect.

Some students may need help understanding the meaning of "conjecture." A simple definition to use is "a reasonable guess."

Student Task Statement

Here are some measurements that were taken when the photo was taken. It was impossible to directly measure the height of the lamppost, so that cell is blank.

	height (inches)	shadow length (inches)
younger boy	43	29
man	72	48
older boy	51	34
lamppost		114

1. What relationships do you notice between an object's height and the length of its shadow?
2. Make a conjecture about the height of the lamppost and explain your thinking.

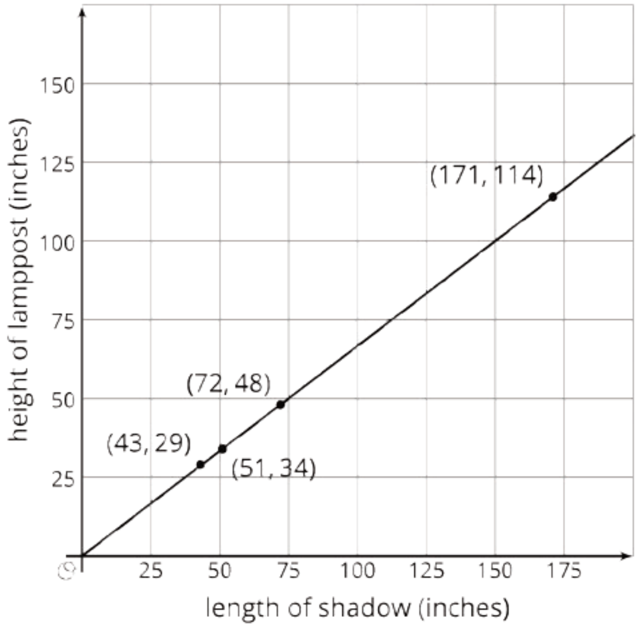
Student Response

1. There appears to be a proportional relationship between the height of an object and the length of its shadow. All of the shadow lengths are approximately $\frac{2}{3}$ the height of the object.
2. Approximately 171 inches (or 14 feet 3 inches). Possible strategies: $114 \cdot \frac{3}{2}$, $114 \div \frac{2}{3}$, 150% of 114.

Activity Synthesis

Students should notice that the relationship appears to be approximately proportional. Highlight that the height to shadow length relationship is *not* exactly proportional because, for example, $\frac{43}{29}$ and $\frac{72}{48}$ are not equal (though they are very close). This could be because of rounding error in the measurements or other factors that make the real world differ slightly from a mathematical model. It may be interesting for students to speculate on some reasons (for example, perhaps the ground is not perfectly level, or perhaps one of the people or the lamppost is at a slightly different angle to the ground), though the next activity provides ample opportunity to discuss them, too.

If any students decide to create a graph of the associated heights and shadow lengths, it looks like this (or may have the axes reversed).



13.3 Justifying the Relationship

15 minutes

The purpose of this activity is for students to write a mathematical justification for the proportional relationship between heights and shadow lengths in the photo. A version of the photo is provided with some line segments drawn to strongly suggest an argument based on similar triangles. Given the work they have done up to this point, it is likely that students will recognize that similar triangles will be part of their justification. Many students need help understanding what components are important to include in their arguments. The more that students do on their own, the more fully they are engaging in MP1 and MP3.

Because the Sun is so very far away relative to its size, the rays that reach Earth are extremely close to parallel. This is essential in the argument, and you may decide to share this information with the students or ask them to think about the shadows of the pens in the warm-up. The prompt asks to show that the relationship is *approximately proportional,* leaving room for the fact that the Sun's rays may not be exactly parallel, or one of the people is not standing precisely perpendicular to the ground, or the ground may not be perfectly level. Stating and using simplifying assumptions is a good example of MP4.

Addressing

- 8.G.A.5

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

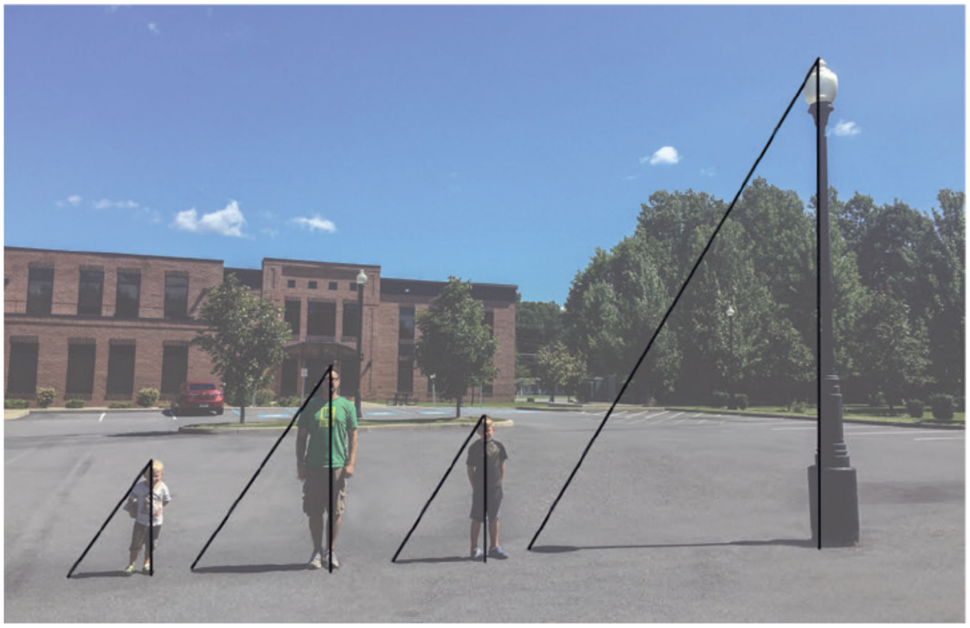
Keep students in the same groups. From the previous activity, the shadow measurements make us *suspect* there may be a proportional relationship between the object heights and the shadow lengths. Tell students that their goal here is to justify *why* the relationship is (approximately) proportional.

Anticipated Misconceptions

Students often struggle with deciding what is important to include in their explanation. A useful technique is to think about what you want to show, and then asking a series of questions. In this activity, we want to show that there is a proportional relationship between the side lengths in some triangles. Questions could include, "What types of triangles have sides that are in proportion? How do you know when triangles are similar triangles? Which pairs of angles do you know are congruent? Why are they congruent?" The answers to these questions are the building blocks of an argument.

Student Task Statement

Explain *why* the relationship between the height of these objects and the length of their shadows is approximately proportional.



Student Response

Choose to focus on one person and the lamppost. Since the Sun's rays making the shadows are approximately parallel, the angles where they strike the ground are congruent corresponding angles. Both the lamppost and the person are perpendicular to the ground, so they are both making right angles with the ground, so those angles are also congruent. The right triangles formed by the lamppost and a person are similar by AA. Therefore, their corresponding sides are proportional.

Activity Synthesis

Debrief as a class. Invite selected students to share their explanations. Ask other students to restate, support, refine, or disagree with their arguments.

Emphasize that it is often necessary to make simplifying assumptions when modeling a real-world situation. For this geometric argument to work, for example, we have to assume that the light rays coming from the Sun are parallel, that the people and the lamppost are perpendicular to the ground, and that the ground is level.

Support for Students with Disabilities

Engagement: Develop Effort and Persistence. Break the class into small discussion groups and then invite a representative from each group to report back to the whole class.
Supports accessibility for: Attention; Social-emotional skills

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Support for English Language Learners

Speaking, Listening: MLR1 Stronger and Clearer Each Time. After students respond to the prompt "Explain why the relationship between the height of these objects and the length of their shadows is approximately proportional," ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language (e.g., "How do you know that the triangles are similar?", and "Which pairs of angles are congruent?", etc.). Students can borrow ideas and language from each partner to refine and clarify their original explanation. This will help students revise and refine both their reasoning and their verbal and written output.
Design Principles(s): Optimize output (for explanation); Maximize meta-awareness

---

13.4 The Height of a Tall Object

20 minutes

This activity could be done on a sunny day. You should try it out ahead of time to ensure that the shadows created in your part of the world at the time your class takes place are cooperative! Either find a tall object outside that all students will find the height of, or let students choose a tall object (for example: a flagpole, a building, or a tree). It should be tall enough that its height can't be easily measured directly, on level ground, and perpendicular to the ground. Students head outside with tape measures (or other measuring devices) and use what they've learned in this lesson to figure out the height of the tall object.

This is a modeling task (MP4) as the goal of the task is to solve a real-world problem (find the height of some object), and students need to devise and justify a method to do this. They have developed the tools in this lesson but need to apply them appropriately.

Addressing

- 8.G.A.5

Launch

Tell students that they are going to apply what they have learned about shadow lengths of different objects to estimate the height of an object outside. Either tell them which object to use or explain the parameters for choosing an appropriate object.

Students require tape measures, yardsticks, or rulers, and a way to record their measurements. 5–10 minutes to take measurements and do calculations followed by a whole-class discussion.

Support for Students with Disabilities

Engagement: Develop Effort and Persistence. Provide prompts or checklists to increase length of on-task orientation in the face of distractions. For example, provide a checklist that chunks the various steps of the activity into a set of manageable tasks. This checklist may include necessary materials, a list of objects to choose from, steps to find the height, and an exemplar. *Supports accessibility for: Attention; Social-emotional skills*

Student Task Statement

1. Head outside. Make sure that it is a sunny day and you take a measuring device (like a tape measure or meter stick) as well as a pencil and some paper.
2. Choose an object whose height is too large to measure directly. Your teacher may assign you an object.
3. Use what you have learned to figure out the height of the object! Explain or show your reasoning.

Student Response

Answers vary.

Activity Synthesis

The main technique that students will apply to solve this problem is likely proportional reasoning. Make sure to highlight the explanation for *why* the relationship is proportional uses angles made by parallel lines cut by a transversal (studied in the previous unit) and properties of similar triangles (a focus of this unit).

Students may think of the height of the object and the length of its shadow as a pair (x, y) lying on a line whose slope is known. They know the shadow length and are looking for the corresponding height. While this line of reasoning could be considered grade 7 work, students now have the new language of slope (in addition to unit rate and constant of proportionality) to use in these calculations.

Lesson Synthesis

The activities in this lesson are a good example of using mathematics to model a real-world situation.

- The data is not always perfect (in this case, the object height to shadow length relationship was only approximately proportional).
- To explain why the object height to shadow length relationship might be proportional, simplifying assumptions were needed.
- Mathematical models can be used to make accurate guesses or predictions about quantities that are difficult or impossible to measure directly. In this case, a relationship between the height of an object and the length of its shadow was observed from easier-to-get measurements and justified by reasoning about similar triangles. Once we knew this relationship existed and why it existed, we could reasonably expect the same relationship to hold for a very tall object nearby at the same time of day, and use the length of the tall object's shadow to find its height.
- An interesting historical connection: over 2,000 years ago, the ancient Greek mathematician Eratosthenes also studied shadows closely (in a slightly different way) and used this to estimate the circumference of Earth with an error of less than 2%!

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Family Support Materials

Family Support Materials

Dilations, Similarity, and Introducing Slope

Here are the video lesson summaries for Grade 8, Unit 2: Dilations, Similarity, and Introducing Slope. Each video highlights key concepts and vocabulary that students learn across one or more lessons in the unit. The content of these video lesson summaries is based on the written Lesson Summaries found at the end of lessons in the curriculum. The goal of these videos is to support students in reviewing and checking their understanding of important concepts and vocabulary. Here are some possible ways families can use these videos:

- Keep informed on concepts and vocabulary students are learning about in class.
- Watch with their student and pause at key points to predict what comes next or think up other examples of vocabulary terms (the bolded words).
- Consider following the Connecting to Other Units links to review the math concepts that led up to this unit or to preview where the concepts in this unit lead to in future units.

Grade 8, Unit 2: Dilations, Similarity, and Introducing Slope	Vimeo	YouTube
Video 1: Dilations (Lessons 1–5)	Link	Link
Video 2: Similarity (Lesson 6–9)	Link	Link
Video 3: Slope (Lessons 10–12)	Link	Link

Video 1

Video 'VLS G8U2V1 Dilations (Lessons 1–5)' available here: https://player.vimeo.com/video/457852098.

Video 2

Video 'VLS G8U2V2 Similarity (Lesson 6–9)' available here: https://player.vimeo.com/video/457854496.

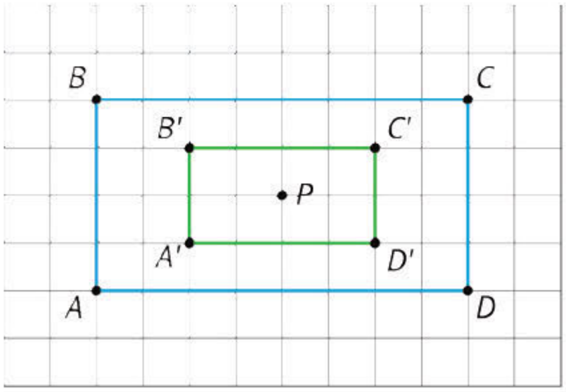
Video 3

Video 'VLS G8U2V3 Slope (Lessons 10–12)' available here: https://player.vimeo.com/video/457855739.

Dilations

Family Support Materials 1

This week your student will expand their understanding of transformations to include non-rigid transformations. Specifically, they will learn to make and describe dilations of figures. A dilation is a process to make a scaled copy of a figure, and is described using a center point and a number (the scale factor). The scale factor can be any positive number, including fractions and decimals. If the scale factor is less than 1, the dilated figure is smaller than the original, if it is greater than 1 the dilated figure is larger than the original. In this dilation, the center point P and the scale factor is $\frac{1}{2}$.



When dilating figures, the distance from the center of dilation to a point on the figure is multiplied by the scale factor to get the location of the corresponding point. In this example, the distance between center P and B multiplied by $\frac{1}{2}$ results in the distance between P and B'. Notice also how the side lengths of the dilated figure, $A'B'C'D'$ are all exactly $\frac{1}{2}$ the side lengths of the original figure, $ABCD$, while the angle measures remain the same.

Here is a task to try with your student:

Rectangle A measures 10 cm by 24 cm. Rectangle B is a scaled copy of Rectangle A.

1. If the scale factor is $\frac{1}{2}$, what are the dimensions of Rectangle B?

2. If the scale factor is 3, what are the dimensions of Rectangle B?

3. If Rectangle B has dimensions 15 cm by 36 cm, what is the scale factor?

Solution:

1. Rectangle B has dimensions 5 cm by 12 cm, since $10 \cdot \frac{1}{2} = 5$ and $24 \cdot \frac{1}{2} = 12$.

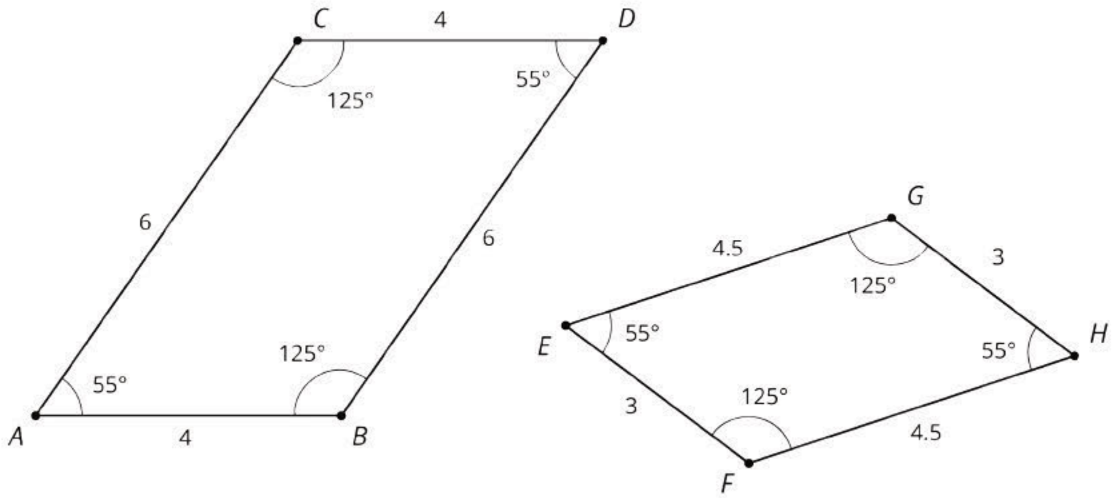
2. Rectangle B has dimensions 30 cm by 72 cm, since $10 \cdot 3 = 30$ and $24 \cdot 3 = 72$.

3. The scale factor is $\frac{3}{2}$ since $15 \div 10 = \frac{3}{2}$ and $36 \div 24 = \frac{3}{2}$.

Similarity

Family Support Materials 2

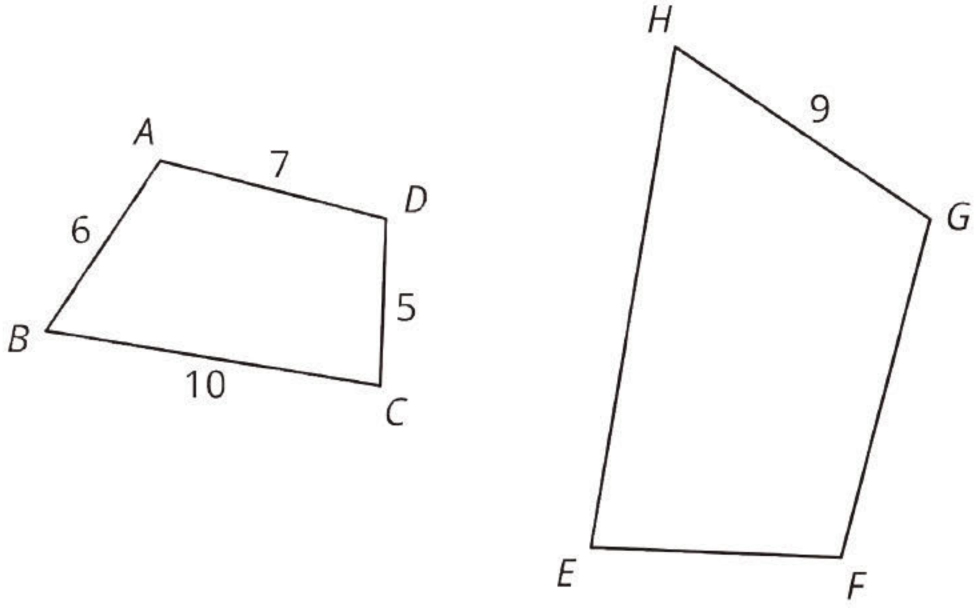
This week your student will investigate what it means for two figures to be similar. Similarity in mathematics means there is a sequence of translations, rotations, reflections, and dilations that takes one figure to the other. When two figures are similar, there are always many different sequences of transformations that can show that they are similar. Here is an example of two similar figures:



If we needed to show that these two figures are similar, we can first identify that the scale factor to go from $ABDC$ to $EFHG$ is $\frac{3}{4}$, since $3 \div 4 = 4.5 \div 6 = \frac{3}{4}$. Then, using a dilation with scale factor $\frac{3}{4}$, a translation, and a rotation, we can line up an image of $ABDC$ perfectly on top of $EFHG$.

Here is a task to try with your student:

Quadrilateral $ABCD$ is similar to quadrilateral $GHEF$.



What is the perimeter of quadrilateral $EFGH$?

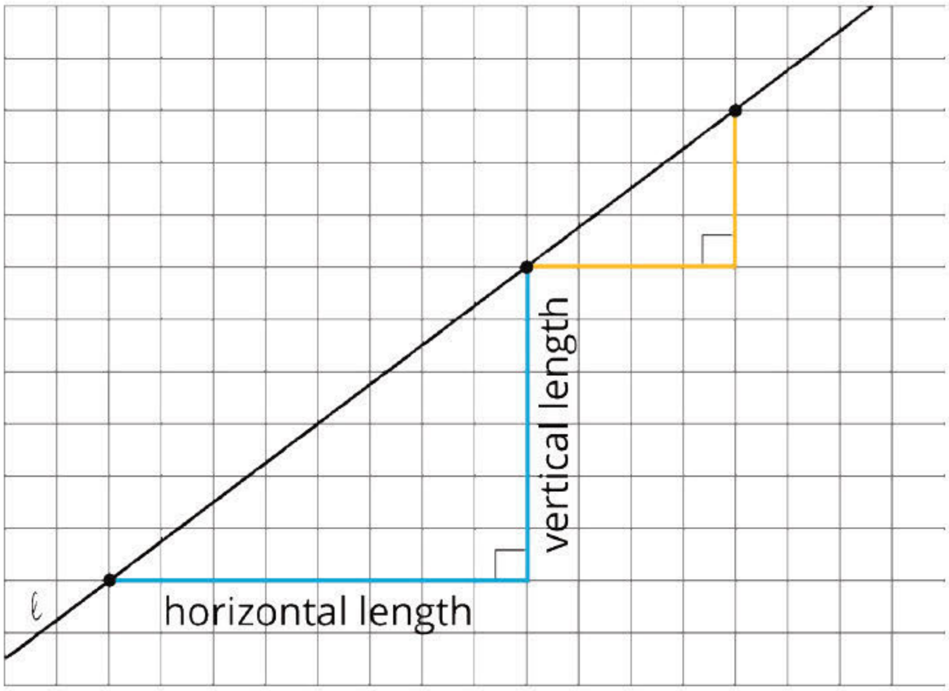
Solution:

The perimeter is 42. The scale factor is 1.5, since $9 \div 6 = 1.5$. This means the side lengths of $EFGH$ are 9, 10.5, 7.5, and 15, which are the values of the corresponding sides of $ABCD$ multiplied by 1.5. We could also just multiply the perimeter of $ABCD$, 28, by 1.5.

Slope

Family Support Materials 3

This week your student will use what they have learned about similar triangles to define the slope of a line. A slope triangle for a line is a triangle whose longest side lies on the line and whose other two sides are vertical and horizontal. Here are two slope triangles for the line ℓ:

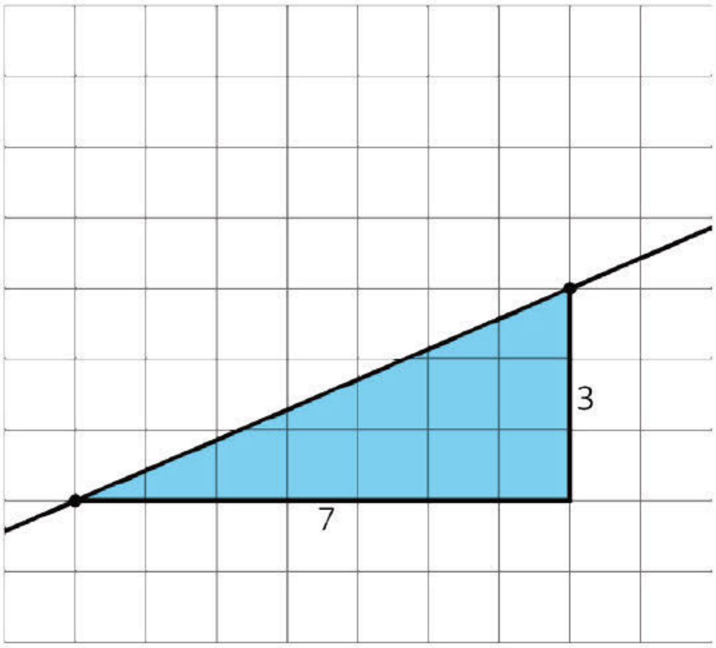


For lines, it turns out that the quotient of the vertical side length and the horizontal side length of a slope triangle does not depend on the triangle. That is, all slope triangles for a line have the same quotient between their vertical and horizontal side and this number is called the slope of the line. The slope of line ℓ shown here can be written as $\frac{6}{8}$ (from the larger triangle), $\frac{3}{4}$ (from the smaller triangle), 0.75, or any other equivalent value.

By combining what they know about the slope of a line and similar triangles, students will begin writing equations of lines—a skill they will continue to use and refine throughout the rest of the year.

Here is a task to try with your student:

Here is a line with a slope triangle already drawn in.



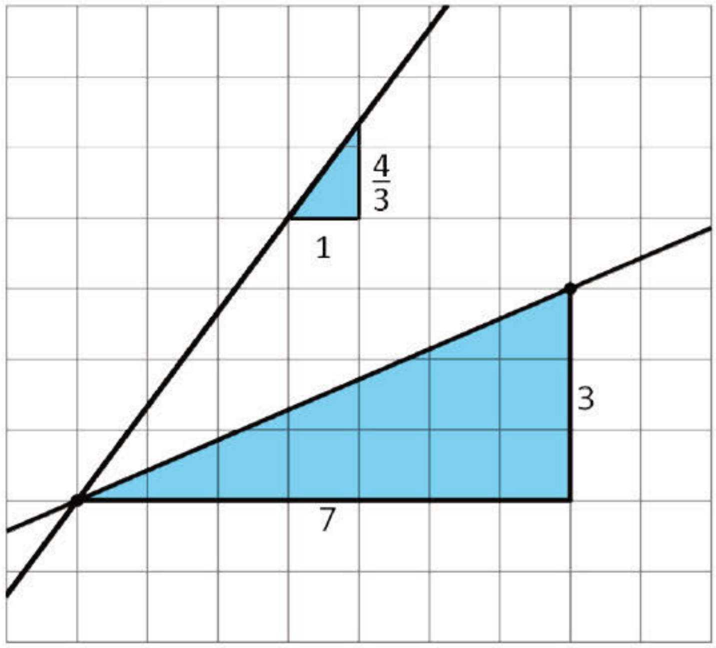
1. What is the slope of the line?

2. Draw another line with a slope of $\frac{4}{3}$ that goes through the point on the left. Include a slope triangle for the new line to show how you know this line has a slope of $\frac{4}{3}$.

Solution:

1. The slope of the line is $\frac{3}{7}$.

2.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Unit Assessments

Check Your Readiness A and B

End-of-Unit Assessment A and B

Assessments

Assessment : Check Your Readiness (A)

Problem 1

The content assessed in this problem is first encountered in Lesson 4: Dilations on a Square Grid.

In this unit, students revisit rigid transformations and also add dilations to the mix. Work with all of these transformations, especially on a grid, requires comfort with distance between a point and a line.

If most students struggle with this item, plan to use this problem and Unit 1 Lesson 5 to review distance on a coordinate grid. Students will have more opportunities to find distances on a coordinate grid in Lesson 4 Activity 3.

Statement

Which of these points is closest to the y-axis?

A. $(-6, 0)$

B. $(-2, 12)$

C. $(4, 2)$

D. $(5, 1)$

Solution

B

Aligned Standards

6.NS.C.8

Problem 2

The content assessed in this problem is first encountered in Lesson 4: Dilations on a Square Grid.

Work with dilations in this unit will involve multiplying the distance between a point and a center by a scale factor.

If most students struggle with this item, plan to launch Lesson 4 Activity 3 by reviewing this problem and the concept of distance on the coordinate plane.

Statement

Which of these points is closest to the point $(7, 1)$?

A. $(4, 1)$

B. $(7, -1)$

C. $(7, 4)$

D. $(11, 1)$

Solution

B

Aligned Standards

6.NS.C.8

Problem 3

The content assessed in this problem is first encountered in Lesson 1: Projecting and Scaling.

When students study similar triangles and slope, they will need to use proportional relationships to find unknown values.

If most students struggle with this item, plan to use this problem to review scale factors in this context of a proportional relationship. In Lesson 1 students will rely on this concept to consider scale factor with side lengths and this informal introduction to dilations.

Statement

Quantities x and y are in a proportional relationship. Complete the table.

x	y
4	16
3	
	8

Solution

x	y
4	16
3	12
2	8

Aligned Standards

7.RP.A.2

Problem 4

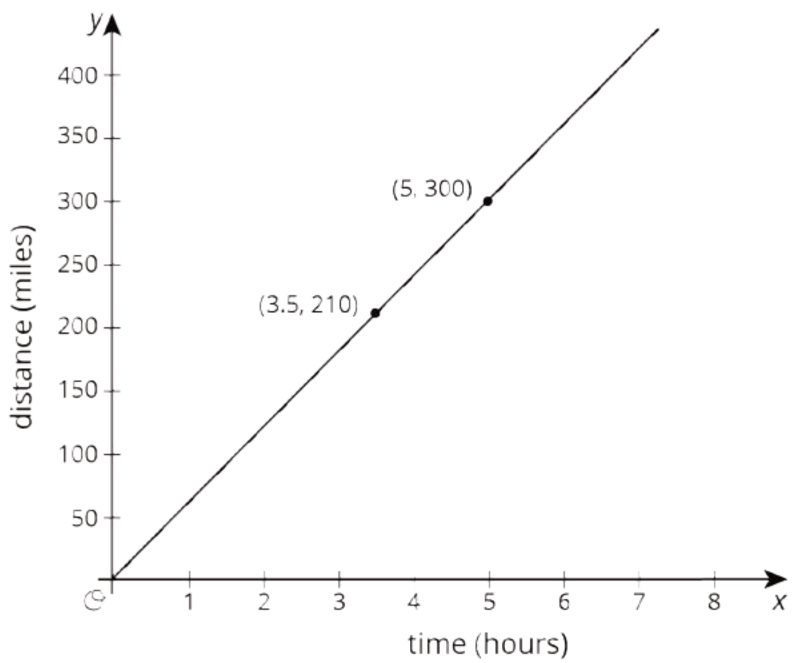
The content assessed in this problem is first encountered in Lesson 10: Meet Slope.

In grade 7, students learned that the graph of a proportional relationship is a line containing the points $(0, 0)$ and $(1, r)$, where r is the unit rate. Students will build on this understanding when they study slope in this unit.

If most students struggle with this item, plan to support this thinking in Lesson 10 Activity 2 as students investigate why two triangles sharing one side along the same line are similar. Students will have several opportunities throughout this lesson to investigate this idea. There is an optional activity in this lesson that can be used as well.

Statement

A car traveled at a constant speed. The graph shows how far the car traveled, in miles, during a given amount of time, in hours.



1. The point $(3.5, 210)$ is on the graph. Explain what this means in terms of the car.
2. Is the point $(1, 60)$ on this graph? Explain how you know.

Solution

1. It means that after 3.5 hours, the car has traveled a distance of 210 miles.
2. Yes, the car is traveling at a constant speed, and 300 miles in 5 hours means the car travels 60 miles each hour. That means the point $(1, 60)$ is on the graph.

Aligned Standards

7.RP.A.2.d

Problem 5

The content assessed in this problem is first encountered in Lesson 1: Projecting and Scaling.

This problem reviews fraction division. Students will use fraction division when they calculate unknown sides of similar triangles.

If most students struggle with this item, plan to use these problems and the three in Lesson 1 Activity 1, Number Talk, to review fraction division. Focus on the strategies described in the narrative of Lesson 1, Activity 1.

> ### Statement
>
> Evaluate each expression.
>
> 1. $4 \div \frac{1}{3}$
> 2. $\frac{3}{8} \div \frac{7}{2}$
> 3. $3\frac{1}{2} \div \frac{7}{4}$

Solution

1. 12
2. $\frac{3}{28}$ (or equivalent)
3. 2

Aligned Standards

6.NS.A.1

Problem 6

The content assessed in this problem is first encountered in Lesson 1: Projecting and Scaling.

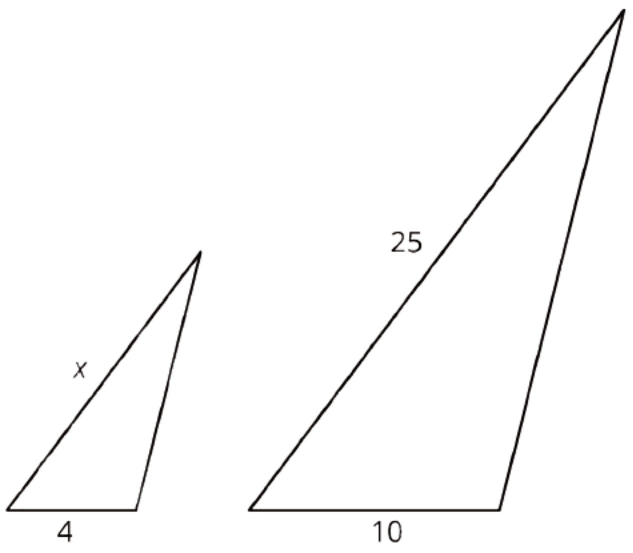
In 7th grade, students studied scaled copies and scale factors. The work with dilations and similar triangles in this unit builds on that foundation.

If most students struggle with this item, plan to do optional Lesson 1 Activity 3 allowing students an opportunity to continue working with scaled copies and finding the scale factors.

> ### Statement
>
> The two triangles displayed are scaled copies of one another.

1. Find the scale factor.

2. What is the value of x?



Solution

1. $\frac{5}{2}$ or $\frac{2}{5}$ (or equivalent)

2. 10

Aligned Standards

7.G.A.1

Problem 7

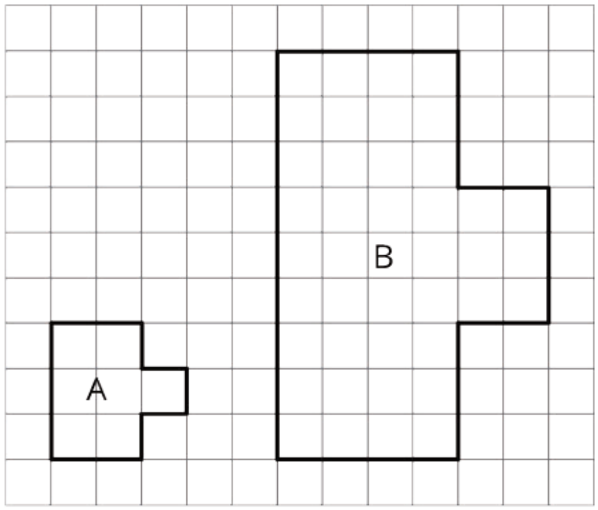
The content assessed in this problem is first encountered in Lesson 1: Projecting and Scaling.

In this unit, students will perform dilations of polygons both with and without the aid of a grid. Students will discover that side lengths of the polygons before and after the dilation have equivalent ratios, just as in their grade 7 work with scaled copies.

If most students struggle with this item, plan to spend time on Lesson 1 Activity 2, using the "eyeball test" described in the launch and emphasizing in the synthesis the relationship between equivalent ratios and scaled copies. Plan to revisit this item in the synthesis of Lesson 1 Activity 2 and ask students how they could determine whether Figure B is a scaled copy of Figure A. Emphasize strategies that take advantage of the grid in looking for equivalent ratios.

Statement

Is Figure B a scaled copy of Figure A? Explain how you know.



Solution

No, the horizontal segments in Figure B are twice as long as the corresponding segments in Figure A, and the vertical segments are three times as long.

Aligned Standards

7.G.A.1

Assessment : Check Your Readiness (B)

Problem 1

The content assessed in this problem is first encountered in Lesson 4: Dilations on a Square Grid.

In this unit, students revisit rigid transformations and also add dilations to the mix. Working with all of these transformations, especially on a grid, requires comfort with distance between a point and a line.

If most students struggle with this item, plan to use this problem and Unit 1 Lesson 5 to review distance on a coordinate grid. Students will have more opportunities to find distances on a coordinate grid in Lesson 4 Activity 3.

Statement

How far away is the point $(-5, 2)$ from the x-axis?

A. 7 units

B. 5 units

C. 3 units

D. 2 units

Solution

D

Aligned Standards

6.NS.C.8

Problem 2

The content assessed in this problem is first encountered in Lesson 4: Dilations on a Square Grid.

Work with dilations in this unit will involve multiplying the distance between a point and a center by a scale factor. Check with students who do not know how to answer this question.

If most students struggle with this item, plan to launch Lesson 4 Activity 3 by reviewing this problem and the concept of distance on the coordinate plane.

Statement

Select **all** the points that are 5 units away from $(6, 2)$.

A. $(1, 2)$

B. $(6, 5)$

C. $(11, 7)$

D. $(30, 10)$

E. $(6, -3)$

F. $(11, 2)$

Solution

["A", "E", "F"]

Aligned Standards

6.NS.C.8

Problem 3

The content assessed in this problem is first encountered in Lesson 1: Projecting and Scaling.

When students study similar triangles and slope, they will need to use proportional relationships to find other points on the graph.

If most students struggle with this item, plan to use this problem to review scale factors in this context of a proportional relationship. In Lesson 1 students will rely on this concept to consider scale factor with side lengths and this informal introduction to dilations.

Statement

The point $(5, 15)$ is on a graph representing a proportional relationship. Give the coordinates of *two* other points on the same graph.

Solution

Answers vary. Sample responses: $(1, 3), (2, 6), (3, 9), (4, 12), (10, 30)$. Any point of the form $(5a, 15a)$ where a is any number except 1.

Aligned Standards

7.RP.A.2

Problem 4

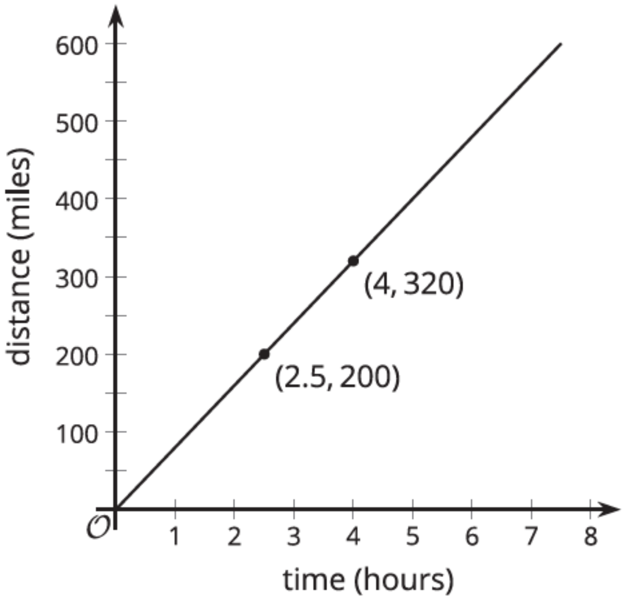
The content assessed in this problem is first encountered in Lesson 10: Meet Slope.

In grade 7, students learned that the graph of a proportional relationship is a line containing the points $(0, 0)$ and $(1, r)$, where r is the unit rate. Students will build on this understanding when they study slope in this unit.

If most students struggle with this item, plan to support this thinking in Lesson 10 Activity 2 as students investigate why two triangles sharing one side along the same line are similar. Students will have several opportunities throughout this lesson to investigate this idea. There is an optional activity in this lesson that can be used as well.

Statement

A train traveled at a constant speed. The graph shows how far the train traveled, in miles, during a given amount of time, in hours.



1. The point $(1, m)$ is on the graph. Find the value of m and explain how you know.

2. What does the value of m mean in this situation?

Solution

1. 80. Sample reasoning: 4 divided by 4 is 1 and 320 divided by 4 is 80.

2. The train is moving 80 miles per hour.

Aligned Standards

7.RP.A.2.d

Problem 5

The content assessed in this problem is first encountered in Lesson 1: Projecting and Scaling.

This problem reviews fraction division. Students will use fraction division when they calculate unknown sides of similar triangles.

If most students struggle with this item, plan to use these problems and the three in Lesson 1 Activity 1, Number Talk, to review fraction division. Focus on the strategies described in the narrative of Lesson 1, Activity 1.

Statement

Evaluate each expression.

1. $3 \div \frac{1}{8}$
2. $\frac{7}{10} \div \frac{3}{2}$
3. $3\frac{1}{3} \div \frac{5}{6}$

Solution

1. 24
2. $\frac{7}{15}$ (or equivalent)
3. 4

Aligned Standards

6.NS.A.1

Problem 6

The content assessed in this problem is first encountered in Lesson 1: Projecting and Scaling.

In 7th grade, students studied scaled copies and scale factors. The work with dilations and similar triangles in this unit builds on that foundation.

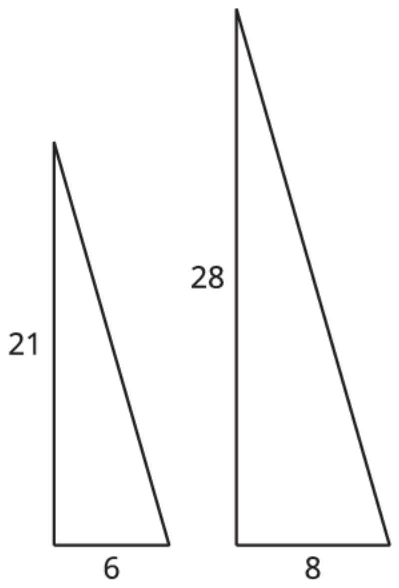
If most students struggle with this item, plan to do optional Lesson 1 Activity 3 allowing students an opportunity to continue working with scaled copies and finding the scale factors.

Statement

The two triangles displayed are scaled copies of one another.

1. Find the scale factor.

2. Sketch a new triangle that is also a scaled copy of these triangles using a different scale factor.



Solution

1. $\frac{4}{3}$ or $\frac{3}{4}$ (or equivalent)

2. Answers vary. The side lengths of the new triangle must all be k times the side lengths of one of the original triangles for some positive number k where $k \neq 1$.

Aligned Standards

7.G.A.1

Problem 7

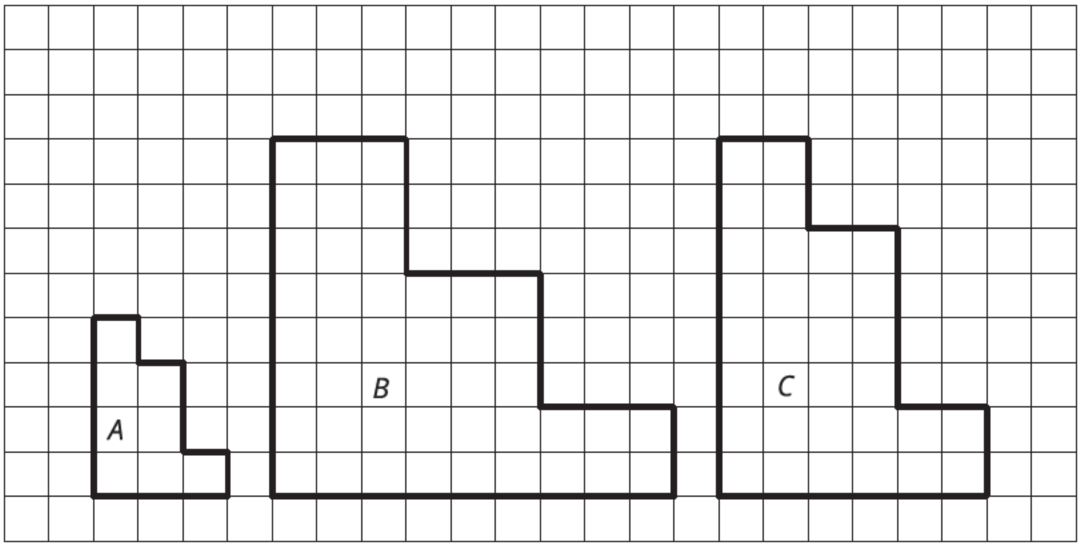
The content assessed in this problem is first encountered in Lesson 1: Projecting and Scaling.

In this unit, students will perform dilations of polygons both with and without the aid of a grid. Students will discover that side lengths of the polygons before and after the dilation have equivalent ratios, just as in their grade 7 work with scaled copies.

If most students struggle with this item, plan to spend time on Lesson 1 Activity 2, using the "eyeball test" described in the launch and emphasizing in the synthesis the relationship between equivalent ratios and scaled copies. Plan to revisit this item in the synthesis of Lesson 1 Activity 2 and ask students how they could determine whether Figure B is a scaled copy of Figure A. Emphasize strategies that take advantage of the grid in looking for equivalent ratios.

Statement

Which figure is a scaled copy of Figure A? Explain how you know.



Solution

Figure C is a scaled copy because each side of Figure C is twice as long as the corresponding side of Figure A. Figure B is not a scaled copy because the bottom side of Figure B is three times as long as the bottom side of Figure A, but the left side of Figure B is twice as long as the left side of Figure A.

Aligned Standards

7.G.A.1

Assessment : End-of-Unit Assessment (A)

Problem 1

Students selecting A may be thinking only of dilations with scale factor greater than one. Students failing to select B might not realize that “perpendicular lines to perpendicular lines” is a special case of dilations preserving angles. Students failing to select F have forgotten that similar figures are defined as figures which can be matched by a sequence of dilations and rigid transformations.

Statement

Select **all** the true statements.

A. Dilations always increase the length of line segments.

B. Dilations take perpendicular lines to perpendicular lines.

C. Dilations of an angle are congruent to the original angle.

D. Dilations increase the measure of angles.

E. Dilations of a triangle are congruent to the original triangle.

F. Dilations of a triangle are similar to the original triangle.

Solution

["B", "C", "F"]

Aligned Standards

8.G.A

Problem 2

This problem’s focus is the angle-angle criterion for similarity.

Students selecting A may believe that triangles that share only one pair of congruent angles must be similar. Students selecting B might know the angle-angle criterion, but are making a subtler mistake. If the 40° angles are both vertex angles or both base angles of their respective isosceles triangles, then the remaining angle pairs across the triangles must be the same. However, if one is a vertex angle and the other is a base angle, this reasoning falls apart. Students selecting C may have made a calculation mistake, thinking that a remaining angle in one of the triangles is a match for an angle in the other. This type of reasoning works for choice D: using the fact that the angle measures of a triangle add up to 180°, the remaining angle in Triangle 7 is 105° and the remaining angle in Triangle 8 is 25°.

Statement

Which pair of triangles **must** be similar?

A. Triangles 1 and 2 each have a 35° angle.

B. Triangles 3 and 4 are both isosceles. They each have a 40° angle.

C. Triangle 5 has a 30° angle and a 90° angle. Triangle 6 has a 30° angle and a 70° angle.

D. Triangle 7 has a 50° angle and a 25° angle. Triangle 8 has a 50° angle and a 105° angle.

Solution

D

Aligned Standards

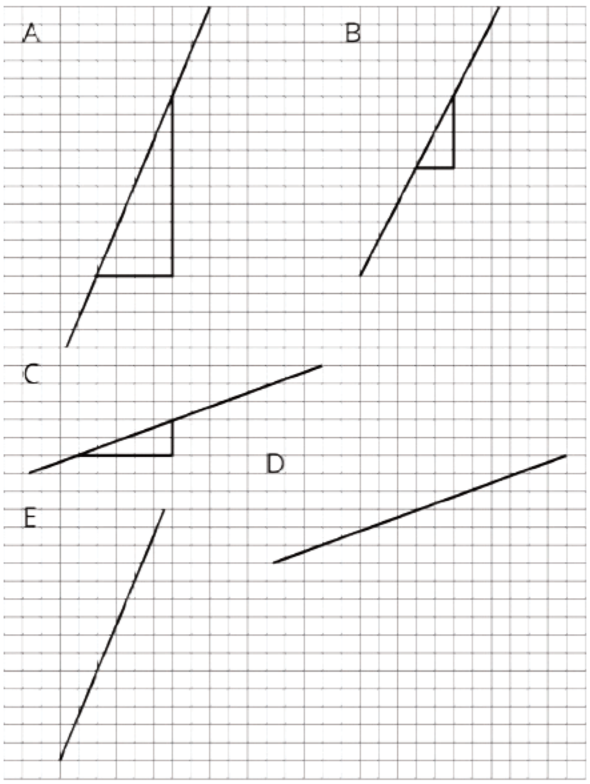
8.G.A.5

Problem 3

Students selecting B have either miscounted the vertical length of the slope triangle or are simply eyeballing—this line has slope 2. Students selecting C or D (and not selecting A or E) are dividing horizontal length by vertical length rather than the other way around.

Statement

Select **all** the lines that have a slope of $\frac{5}{2}$.



A. A

B. B

C. C

D. D

E. E

Solution

["A", "E"]

Aligned Standards

8.EE.B.6

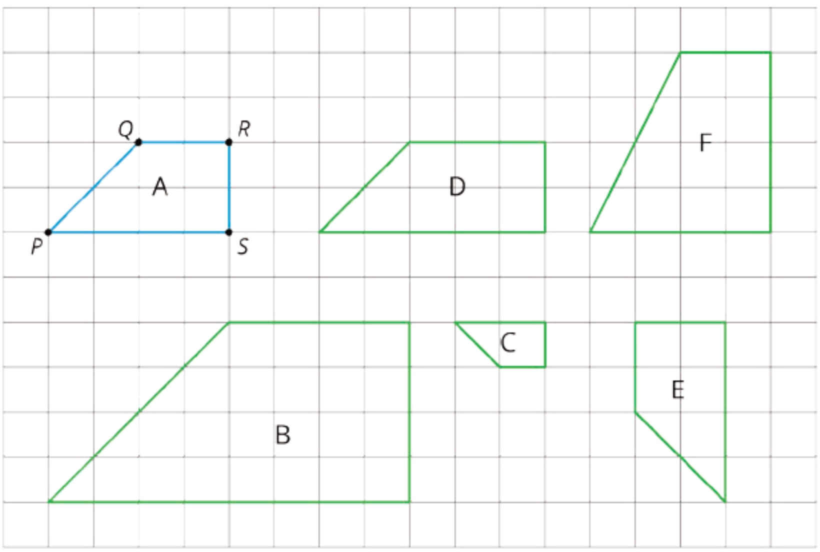
Problem 4

Students identify which polygons are similar to a given polygon on a grid. For each similar polygon, they describe similarity transformations that take one figure to another. Watch for students who do not say Polygon E is similar; they likely believe that similar polygons cannot be congruent.

Statement

Here are some polygons:

1. Which of Polygons B, C, D, E, and F are similar to Polygon A?
2. Choose *one* of the polygons that are similar to Polygon A, and describe a sequence of transformations that take Polygon A to the selected polygon.



Solution

1. Polygons B, C, and E
2. Answers vary. For B, dilate with center P and a scale factor of 2, then translate 6 squares down. For C, dilate using scale factor of $\frac{1}{2}$ and center S, and then reflect over line PS and

translate 3 squares down and $7\frac{1}{2}$ squares to the right. For E, rotate 90 degrees counterclockwise around S, and then translate 2 squares down and 11 squares to the right.

Minimal Tier 1 response:

- Work is complete and correct.
- Acceptable errors: work does not specify which polygon they are working with in part b, but the chosen polygon is clear from a correct response; use of language like “move” or “shift” instead of “translate.”
- Sample:

1. B, C, E
2. (for polygon B) Dilate with scale factor 2 from point P. Then move 6 units down.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: centers of rotations/dilations, scale factors of dilations, or lines of reflection are omitted but the meaning is clear because of intermediate drawings; one incorrect (or missing) answer in part a; instructions for the sequence of rigid motions and dilations contain a small, easily identifiable error (such as saying to translate 7 units instead of 6 units).

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: response to part b is based on an incorrect choice in part a, the sequence of rigid motions and dilations does not take Polygon A to the chosen polygon (and is not close), incorrect answer to part a.

Aligned Standards

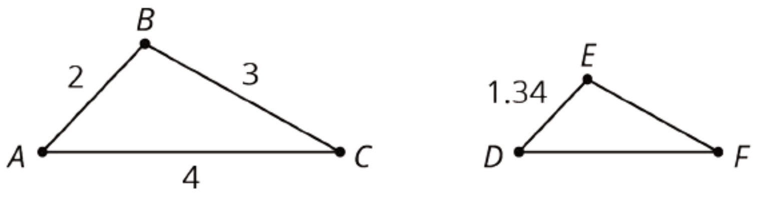
8.G.A.4

Problem 5

When two shapes are similar, a scale factor relates lengths in one figure to the corresponding lengths in the other. At the same time, ratios of lengths in one figure (e.g., length to width) are the *same* as in the other figure. In this problem, it is simpler to use this second idea when calculating side lengths of the triangles.

Statement

Triangles ABC and DEF are similar.



1. Find the length of segment DF.
2. Find the length of segment EF.

Solution

1. 2.68 units (twice as long as segment DE)
2. 2.01 units ($\frac{3}{2}$ as long as segment DE)

Aligned Standards

8.G.A.4

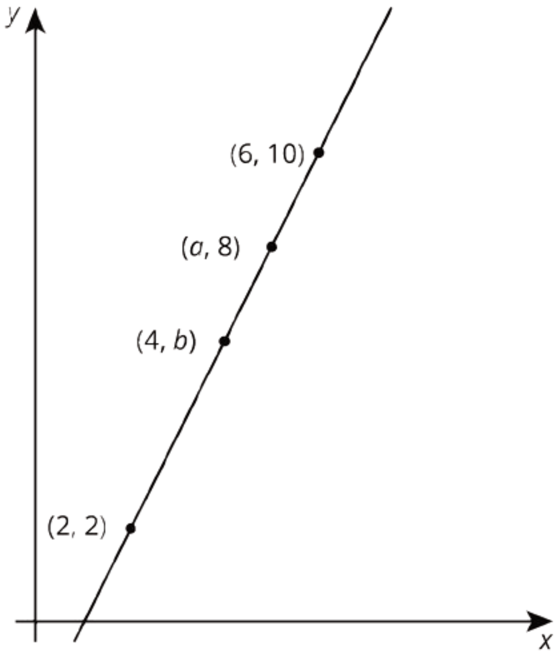
Problem 6

This problem has students use slope triangles to write an equation for a line. This equation can be used to find the unknown values later in the problem. However, students can just as easily reason about slope to find these values.

Statement

All of the points in the picture are on the same line.

1. Find the slope of the line. Explain or show your reasoning.
2. Write an equation for the line.
3. Find the values for a and b. Explain or show your reasoning.



Solution

1. 2, because $\frac{10-2}{6-2} = \frac{8}{4} = 2$.
2. $\frac{y-10}{x-6} = 2$ or equivalent
3. $a = 5$, $b = 6$. These can be found by counting “over 1, up 2” from known points or by using the equation from part b.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample:

1. $\frac{10-2}{6-2} = 2$
2. $\frac{y-10}{x-6} = 2$
3. When you go over one unit, you go up two to stay on the line. That means it goes $(2, 2)$, $(3, 4)$, $(4, 6)$, $(5, 8)$.... So $a = 5$, $b = 6$.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: incorrect answers to parts c based on small algebra errors in using the equation of the line or on miscounting when finding intermediate points; finding that the

slope is $\frac{1}{2}$ or the equation for the line is $\frac{x-2}{y-2} = 2$ or $\frac{x-2}{y-2} = \frac{1}{2}$; correct answers to two problem parts with badly incorrect answer to one problem part.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: work does not involve slope beyond part a; student answers some problems correctly by "eyeballing" but reasoning does not appeal to slope (or related concepts like, "two up, one over"); more than one incorrect answer without explanation.

Aligned Standards

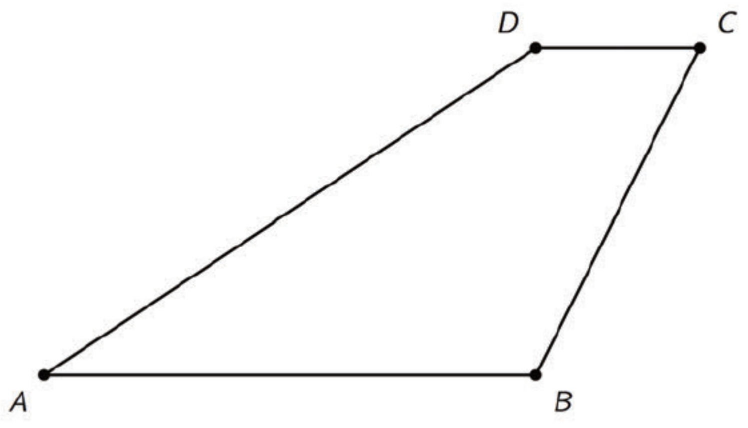
8.EE.B.6

Problem 7

Students apply dilations to a polygon off of a grid. They then reason about similarity. Because of the way the two polygons are constructed, there is a natural sequence of dilations that takes one polygon to the other, making them similar.

Statement

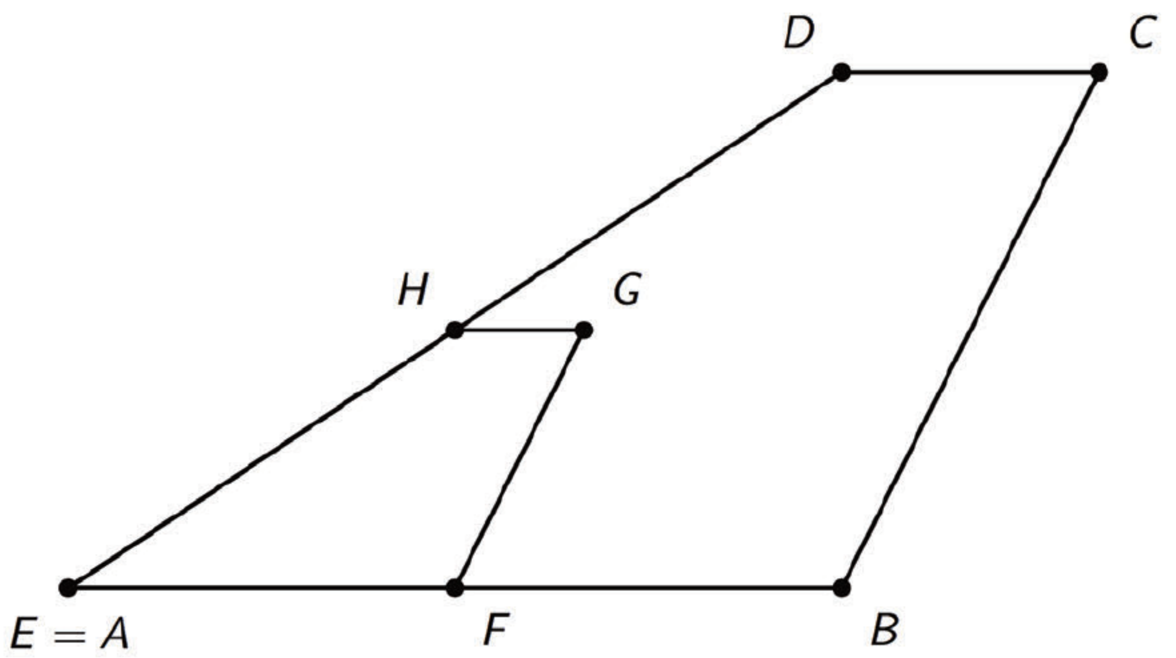
Here is a polygon:



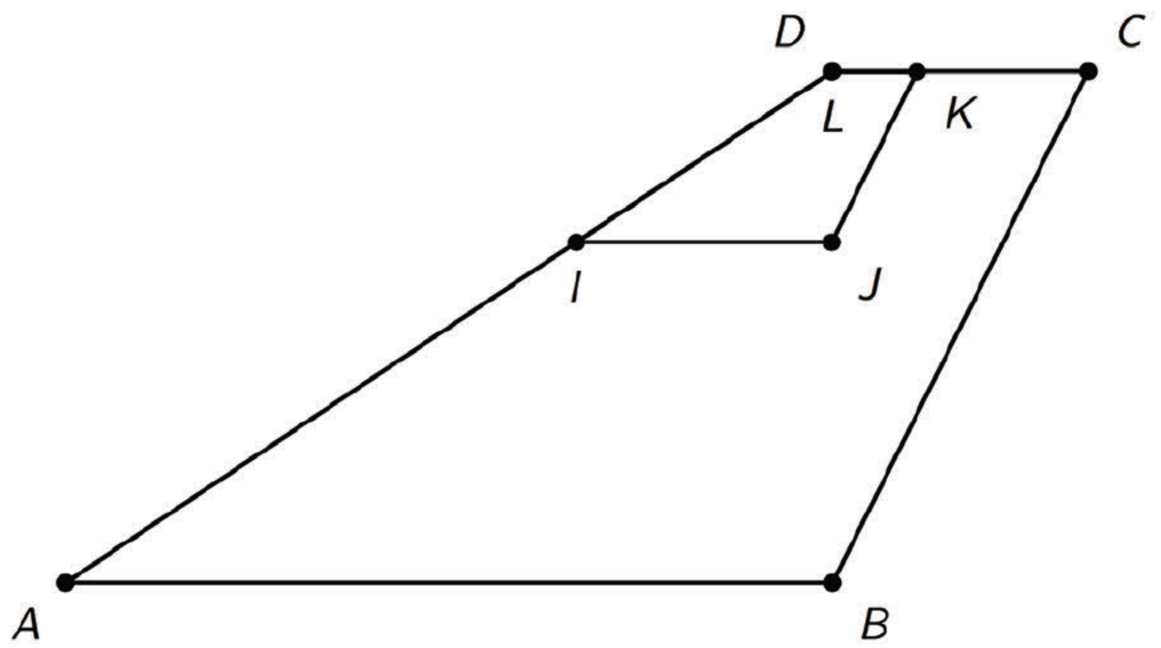
1. Draw the dilation of $ABCD$ using center A and scale factor $\frac{1}{2}$. Label the dilation $EFGH$.
2. Draw the dilation of $ABCD$ with center D and scale factor $\frac{1}{3}$. Label the dilation $IJKL$.
3. Show that $EFGH$ and $IJKL$ are similar.

Solution

1.



2\.



3. If $EFGH$ is dilated with center A and a scale factor of 2, the result is $ABCD$. If $ABCD$ is dilated with center D and a scale factor of $\frac{1}{3}$, the result is $IJKL$.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample:

1. See diagram
2. See diagram
3. $EFGH$ and $IJKL$ are both dilations of the same polygon.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: scale is correct in the dilations, but the center is incorrect; work involves a minor mistake dilating one point; response to part c is something like “$EFGH$ and $IJKL$ are dilations of each other” without a justification such as referencing $ABCD$.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: correct work for parts a and b but very weak or missing explanation in part c; work shows general understanding of dilations but a few points are placed incorrectly, dilations are performed using scale factors of 2 or 3 rather than $\frac{1}{2}$ or $\frac{1}{3}$.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: work for parts a and b does not result in anything resembling dilations.

Aligned Standards

8.G.A.4

Assessment : End-of-Unit Assessment (B)

Teacher Instructions

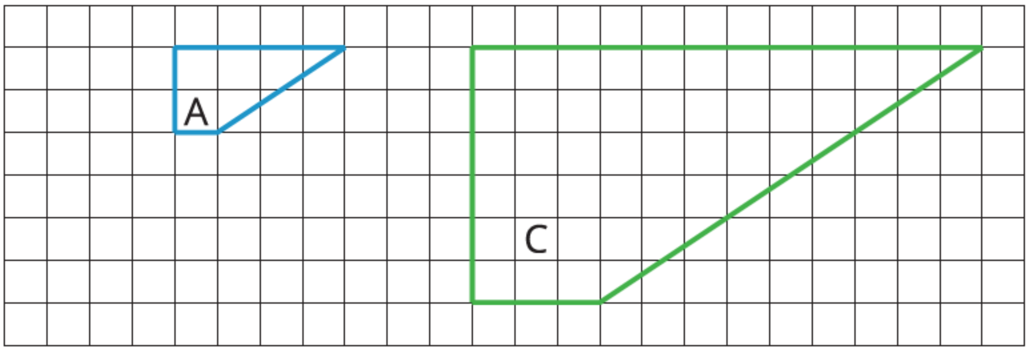
It may be helpful to make graph paper available for question 6.

Problem 1

Students identify a similarity transformation that takes one figure to another. Students that select A or B have an incomplete understanding of similar figures. Students selecting D understand that a dilation is part of a similarity transformation but forgot that they needed to include a translation to correctly explain the similarity.

Statement

Han's teacher asked him to draw a polygon similar to polygon A. Here is his work. Which explanation shows that polygons A and C are similar to each other?



A. Use a protractor to measure all four angles in polygon A and all four angles in polygon C. Since each angle in polygon A has a matching angle measure in polygon C, the polygons are similar.

B. Since each side in polygon C is three times as long as the corresponding side in polygon A, the figures are similar.

C. Dilate polygon C with center at the upper-left vertex and a scale factor of $\frac{1}{3}$. Then, translate 7 units to the left. Since polygon C can be taken to polygon A with a dilation followed by a translation, the figures are similar.

D. Dilate polygon C with center at the upper-right corner and a scale factor of 3. Since polygon C can be taken to polygon A with a dilation, the polygons are similar.

Solution

C

Aligned Standards

8.G.A.4

Problem 2

Students failing to select A may think dilations do not preserve angle measure. Students selecting B, C or D may be confusing similar and congruent or thinking that a dilation is a rigid transformation.

Statement

Select the true statement.

A. Dilations of an angle must be congruent to the original angle.

B. Dilations of a triangle must be congruent to the original triangle.

C. Dilations of a segment must be congruent to the original segment.

D. Dilations of a circle must be congruent to the original circle.

Solution

A

Aligned Standards

8.G.A

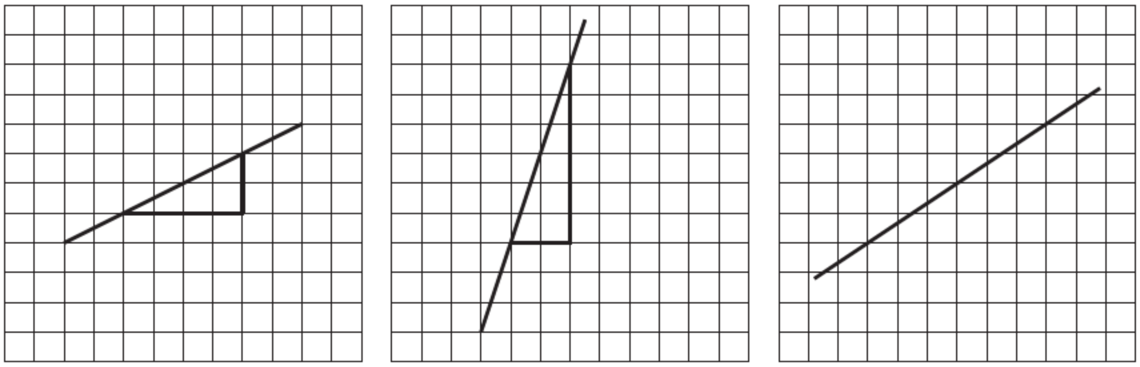
Problem 3

Students find the slope of a line on a grid without coordinate axes. Line C does not include a "slope triangle". Be mindful of students who calculate the slope as the horizontal length divided by the vertical length.

Statement

Find the slope of each line.

A **B** **C**



Solution

A. $\frac{1}{2}$ (or equivalent)

B. 3 (or equivalent)

C. $\frac{2}{3}$ (or equivalent)

Aligned Standards

8.EE.B.6

Problem 4

This problem's focus is the angle-angle criterion for similarity but students must consider if they really have enough information to use this criteria. If the 30° angles are both vertex angles or both base angles of their respective isosceles triangles, then the remaining angle pairs across the triangles must be the same. However, if one is a vertex angle and the other is a base angle, this angle-angle criterion does not apply.

Statement

Triangles 1 and 2 are both isosceles. They each have a 30° angle. Explain why these triangles do not have to be similar to each other. If you get stuck, consider drawing a diagram.

Solution

They do not have to be similar because an isosceles triangle with only one 30° angle does not have the same angle measures as a triangle with two 30° angles. If the isosceles triangle has two 30° angles then the third angle would be 120° because $180 - 30 - 30 = 120$. If the triangle has only one 30° angle, then the other two angles would be 75° because $180 - 30 = 150$ and 150 divided by 2 is 75.

Minimal Tier 1 response:

- Work is complete and correct.
- Acceptable errors: students do not need to directly mention angle-angle similarity as long as they reason correctly about needing at least two corresponding angles with equal measure.
- Sample: The triangles would not be similar if one triangle has two 30 degree angles and the other triangle only has one because similar triangles need at least two pairs of angles with the same measures.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: minor errors in computing possible angles for the triangles; draws accurate pictures but written explanation may be incomplete.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: Fails to consider that the 30° angles could be the vertex angle or the base angle. Asserts that one pair of congruent corresponding angles guarantees the triangles are similar.

Aligned Standards

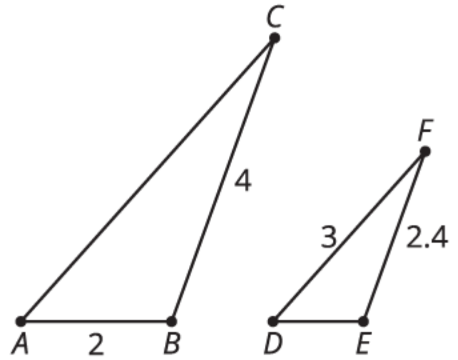
8.G.A.5

Problem 5

When two shapes are similar, a scale factor relates lengths in one figure to the corresponding lengths in the other. At the same time, ratios of lengths in one figure (e.g., length to width) are the same as in the other figure. In this problem, it is simpler to use this second idea when calculating side lengths of the triangles.

Statement

Triangles ABC and DEF are similar.



1. Find the length of segment DE.
2. Find the length of segment AC.

Solution

1. 1.2 units (half as long as segment EF)
2. 5 units ($\frac{5}{3}$ as long as segment FD)

Aligned Standards

8.G.A.4

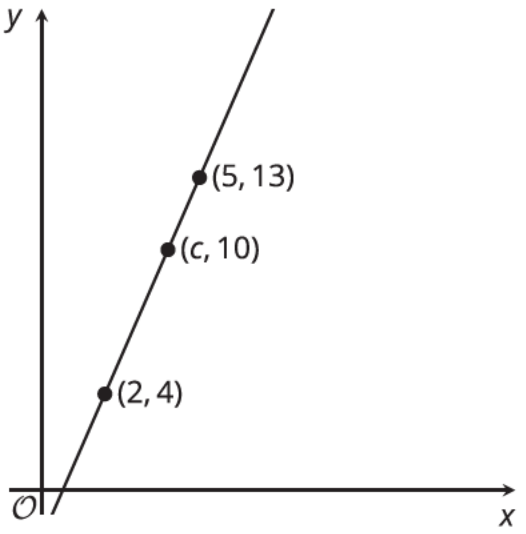
Problem 6

This problem has students use slope triangles to write an equation for a line. This equation can be used to find the unknown value later in the problem and verify a point is on the line. However, students can just as easily reason about slope to answer these questions.

Statement

All of the points in the picture are on the same line.

1. Find the slope of the line. Explain or show your reasoning.
2. Write an equation for the line.
3. What is the value of c? Explain or show your reasoning.
4. Is the point $(0, -2)$ on this line? Explain how you know.



Solution

1. 3 ($\frac{13-4}{5-2} = \frac{9}{3} = 3$)
2. $\frac{y-4}{x-2} = 3$ (or equivalent)
3. $c = 4$. This can be found by counting "over 1, up 3" from $(2, 4)$ or by using the equation from the previous question.
4. Yes. $\frac{-2-4}{0-2} = 3$ The slope of the line containing $(0, -2)$ and $(2, 4)$ is the same as the slope between the original two points. This can also be verified by counting "over 1, up 3" from one of the two given points.

Minimal Tier 1 response:

- Work is complete and correct.

Sample:

1. $\frac{13-4}{5-2} = \frac{9}{3} = 3$
2. $\frac{y-4}{x-2} = 3$
3. When you go over one unit, you go up three to stay on the line. That means it goes (2,4), (3,7), (4,10), ... So c=4.
4. Counting backwards: (2,4), (1,1), (0,-2) so the point (0,-2) is on the line.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: incorrect answers to parts c and d based on small algebra errors in using the equation of the line or on miscounting when finding intermediate points; finding that the slope is 1/3 or the equation for the line is $\frac{x-2}{y-4} = 3$; correct answers to three problem parts with badly incorrect answer to one problem part.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: work does not involve slope beyond parts b or c; student answers some problems correctly by "eyeballing" but reasoning does not appeal to slope (or related concepts like, "two up, one over"); more than one incorrect answer without explanation.

Aligned Standards

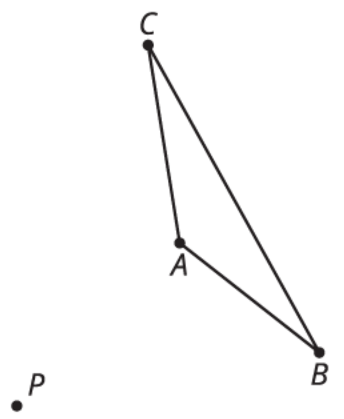
8.EE.B.6

Problem 7

Students apply dilations to a triangle off of a grid and a point not on the triangle. They then reason about similarity. Because both dilations use the same center, students can reason they triangles are similar as a result of successive dilations. They could also describe a translation and a dilation that would show the two triangles are similar.

Statement

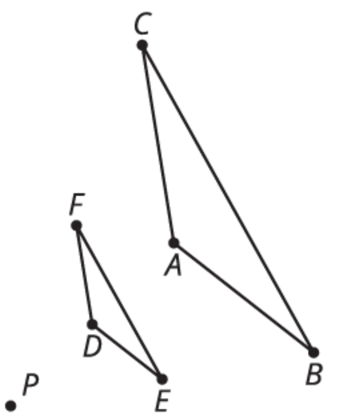
Here is triangle ABC and point P:



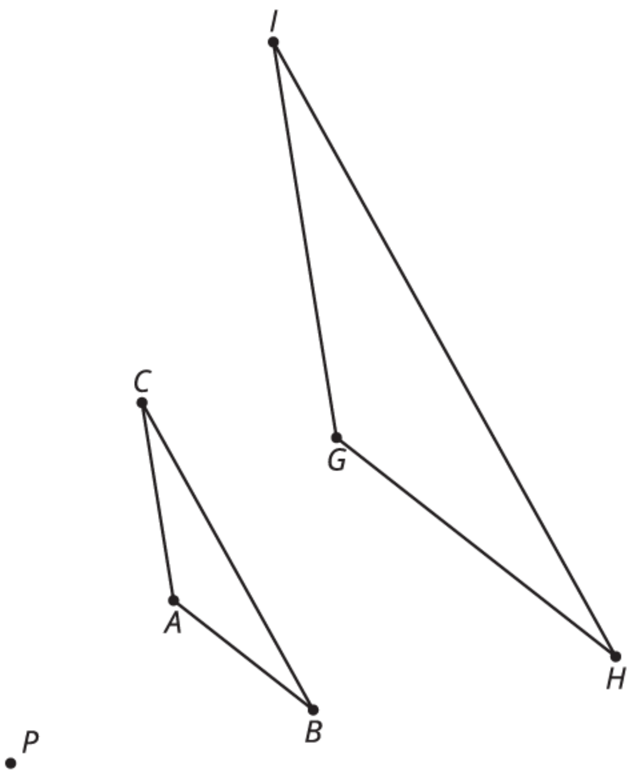
1. Draw the dilation of ABC using center P and scale factor $\frac{1}{3}$. Label the dilation DEF.
2. Draw the dilation of ABC with center P and scale factor 2. Label the dilation GHI.
3. Show that DEF and GHI are similar.

Solution

1.



2.



3. If DEF is dilated with center P and a scale factor of 3, the result is ABC. If ABC is dilated with center D and a scale factor of 2, the result is GHI. This sequence of two dilations takes triangle DEF to triangle GHI.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample:

1. See above diagram
2. See above diagram
3. DEF and GHI are both dilations of the same triangle.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: scale is correct in the dilations, but the center is incorrect; work involves a minor mistake dilating one point; response to part c is something like "DEF and GHI are dilations of each other" without a justification such as referencing ABC.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: correct work for parts a and b but very weak or missing explanation in part c; work shows general understanding of dilations but a few points are placed incorrectly, dilations are performed using scale factors of 3 or 1/2 rather than 1/3 or 2.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: work for parts a and b does not result in anything resembling dilations.

Aligned Standards

8.G.A.4

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Assessment Answer Keys

Check Your Readiness A and B End-of-Unit Assessment A and B

Dilations, Similarity, and Introducing Slope Assessment : Check Your Readiness (A)

Problem 1

The content assessed in this problem is first encountered in Lesson 4: Dilations on a Square Grid.

In this unit, students revisit rigid transformations and also add dilations to the mix. Work with all of these transformations, especially on a grid, requires comfort with distance between a point and a line.

If most students struggle with this item, plan to use this problem and Unit 1 Lesson 5 to review distance on a coordinate grid. Students will have more opportunities to find distances on a coordinate grid in Lesson 4 Activity 3.

Statement

Which of these points is closest to the y-axis?

A. $(-6, 0)$

B. $(-2, 12)$

C. $(4, 2)$

D. $(5, 1)$

Solution

B

Aligned Standards

6.NS.C.8

Problem 2

The content assessed in this problem is first encountered in Lesson 4: Dilations on a Square Grid.

Work with dilations in this unit will involve multiplying the distance between a point and a center by a scale factor.

If most students struggle with this item, plan to launch Lesson 4 Activity 3 by reviewing this problem and the concept of distance on the coordinate plane.

Statement

Which of these points is closest to the point $(7, 1)$?

A. $(4, 1)$

B. $(7, \text{-}1)$

C. $(7, 4)$

D. $(11, 1)$

Solution

B

Aligned Standards

6.NS.C.8

Problem 3

The content assessed in this problem is first encountered in Lesson 1: Projecting and Scaling.

When students study similar triangles and slope, they will need to use proportional relationships to find unknown values.

If most students struggle with this item, plan to use this problem to review scale factors in this context of a proportional relationship. In Lesson 1 students will rely on this concept to consider scale factor with side lengths and this informal introduction to dilations.

Statement

Quantities x and y are in a proportional relationship. Complete the table.

x	y
4	16
3	
	8

Solution

x	y
4	16
3	12
2	8

Aligned Standards

7.RP.A.2

Problem 4

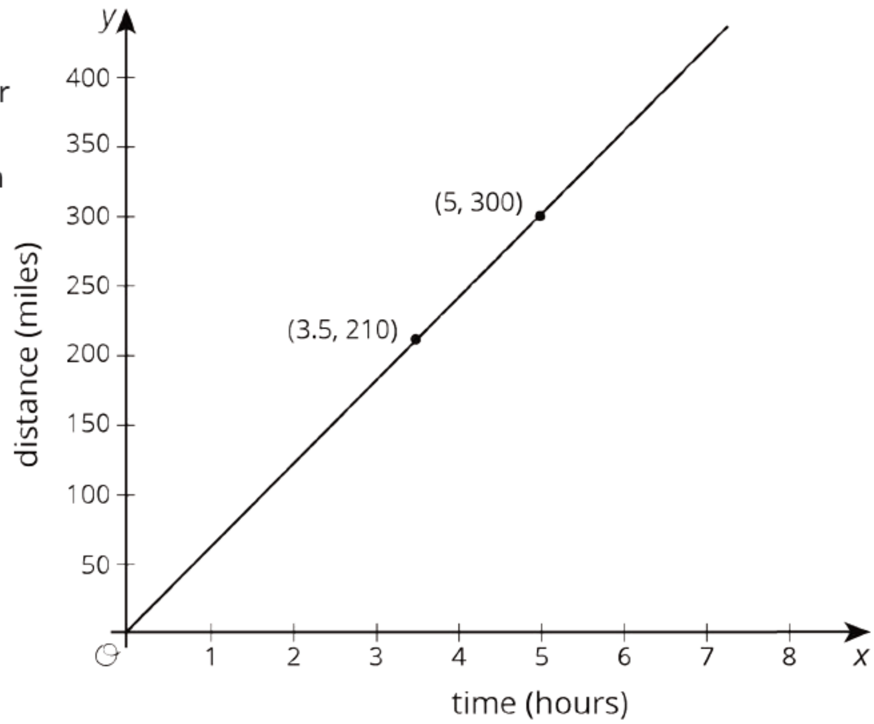
The content assessed in this problem is first encountered in Lesson 10: Meet Slope.

In grade 7, students learned that the graph of a proportional relationship is a line containing the points $(0, 0)$ and $(1, r)$, where r is the unit rate. Students will build on this understanding when they study slope in this unit.

If most students struggle with this item, plan to support this thinking in Lesson 10 Activity 2 as students investigate why two triangles sharing one side along the same line are similar. Students will have several opportunities throughout this lesson to investigate this idea. There is an optional activity in this lesson that can be used as well.

Statement

A car traveled at a constant speed. The graph shows how far the car traveled, in miles, during a given amount of time, in hours.



1. The point $(3.5, 210)$ is on the graph. Explain what this means in terms of the car.

2. Is the point $(1, 60)$ on this graph? Explain how you know.

Solution

1. It means that after 3.5 hours, the car has traveled a distance of 210 miles.

2. Yes, the car is traveling at a constant speed, and 300 miles in 5 hours means the car travels 60 miles each hour. That means the point $(1, 60)$ is on the graph.

Aligned Standards

7.RP.A.2.d

Problem 5

The content assessed in this problem is first encountered in Lesson 1: Projecting and Scaling.

This problem reviews fraction division. Students will use fraction division when they calculate unknown sides of similar triangles.

If most students struggle with this item, plan to use these problems and the three in Lesson 1 Activity 1, Number Talk, to review fraction division. Focus on the strategies described in the narrative of Lesson 1, Activity 1.

Statement

Evaluate each expression.

1. $4 \div \frac{1}{3}$
2. $\frac{3}{8} \div \frac{7}{2}$
3. $3\frac{1}{2} \div \frac{7}{4}$

Solution

1. 12
2. $\frac{3}{28}$ (or equivalent)
3. 2

Aligned Standards

6.NS.A.1

Problem 6

The content assessed in this problem is first encountered in Lesson 1: Projecting and Scaling.

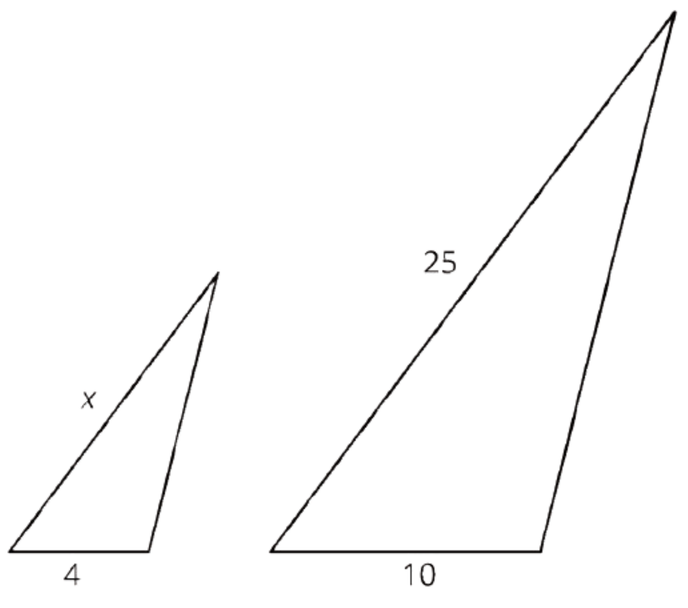
In 7th grade, students studied scaled copies and scale factors. The work with dilations and similar triangles in this unit builds on that foundation.

If most students struggle with this item, plan to do optional Lesson 1 Activity 3 allowing students an opportunity to continue working with scaled copies and finding the scale factors.

Statement

The two triangles displayed are scaled copies of one another.

1. Find the scale factor.
2. What is the value of x?



Solution

1. $\frac{5}{2}$ or $\frac{2}{5}$ (or equivalent)

2. 10

Aligned Standards

7.G.A.1

Problem 7

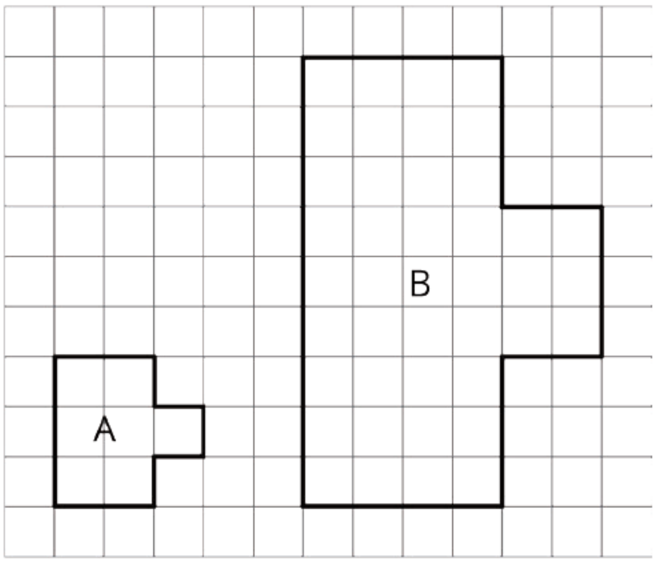
The content assessed in this problem is first encountered in Lesson 1: Projecting and Scaling.

In this unit, students will perform dilations of polygons both with and without the aid of a grid. Students will discover that side lengths of the polygons before and after the dilation have equivalent ratios, just as in their grade 7 work with scaled copies.

If most students struggle with this item, plan to spend time on Lesson 1 Activity 2, using the "eyeball test" described in the launch and emphasizing in the synthesis the relationship between equivalent ratios and scaled copies. Plan to revisit this item in the synthesis of Lesson 1 Activity 2 and ask students how they could determine whether Figure B is a scaled copy of Figure A. Emphasize strategies that take advantage of the grid in looking for equivalent ratios.

Statement

Is Figure B a scaled copy of Figure A? Explain how you know.



Solution

No, the horizontal segments in Figure B are twice as long as the corresponding segments in Figure A, and the vertical segments are three times as long.

Aligned Standards

7.G.A.1

Dilations, Similarity, and Introducing Slope Assessment : Check Your Readiness (B)

Problem 1

The content assessed in this problem is first encountered in Lesson 4: Dilations on a Square Grid.

In this unit, students revisit rigid transformations and also add dilations to the mix. Working with all of these transformations, especially on a grid, requires comfort with distance between a point and a line.

If most students struggle with this item, plan to use this problem and Unit 1 Lesson 5 to review distance on a coordinate grid. Students will have more opportunities to find distances on a coordinate grid in Lesson 4 Activity 3.

Statement

How far away is the point $(-5, 2)$ from the x-axis?

A. 7 units

B. 5 units

C. 3 units

D. 2 units

Solution

D

Aligned Standards

6.NS.C.8

Problem 2

The content assessed in this problem is first encountered in Lesson 4: Dilations on a Square Grid.

Work with dilations in this unit will involve multiplying the distance between a point and a center by a scale factor. Check with students who do not know how to answer this question.

If most students struggle with this item, plan to launch Lesson 4 Activity 3 by reviewing this problem and the concept of distance on the coordinate plane.

Statement

Select **all** the points that are 5 units away from $(6, 2)$.

A. $(1, 2)$

B. $(6, 5)$

C. $(11, 7)$

D. $(30, 10)$

E. $(6, -3)$

F. $(11, 2)$

Solution

["A", "E", "F"]

Aligned Standards

6.NS.C.8

Problem 3

The content assessed in this problem is first encountered in Lesson 1: Projecting and Scaling.

When students study similar triangles and slope, they will need to use proportional relationships to find other points on the graph.

If most students struggle with this item, plan to use this problem to review scale factors in this context of a proportional relationship. In Lesson 1 students will rely on this concept to consider scale factor with side lengths and this informal introduction to dilations.

Statement

The point $(5, 15)$ is on a graph representing a proportional relationship. Give the coordinates of *two* other points on the same graph.

Solution

Answers vary. Sample responses: $(1, 3)$, $(2, 6)$, $(3, 9)$, $(4, 12)$, $(10, 30)$. Any point of the form $(5a, 15a)$ where a is any number except 1.

Aligned Standards

7.RP.A.2

Problem 4

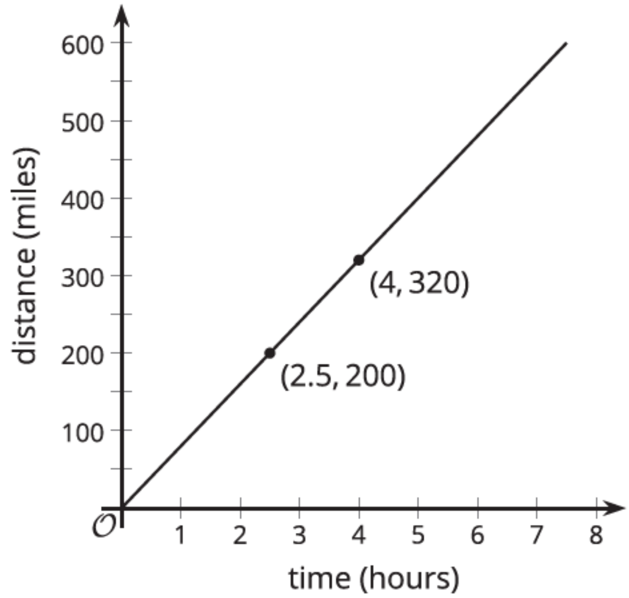
The content assessed in this problem is first encountered in Lesson 10: Meet Slope.

In grade 7, students learned that the graph of a proportional relationship is a line containing the points $(0, 0)$ and $(1, r)$, where r is the unit rate. Students will build on this understanding when they study slope in this unit.

If most students struggle with this item, plan to support this thinking in Lesson 10 Activity 2 as students investigate why two triangles sharing one side along the same line are similar. Students will have several opportunities throughout this lesson to investigate this idea. There is an optional activity in this lesson that can be used as well.

Statement

A train traveled at a constant speed. The graph shows how far the train traveled, in miles, during a given amount of time, in hours.



1. The point $(1, m)$ is on the graph. Find the value of m and explain how you know.

2. What does the value of m mean in this situation?

Solution

1. 80. Sample reasoning: 4 divided by 4 is 1 and 320 divided by 4 is 80.

2. The train is moving 80 miles per hour.

Aligned Standards

7.RP.A.2.d

Problem 5

The content assessed in this problem is first encountered in Lesson 1: Projecting and Scaling.

This problem reviews fraction division. Students will use fraction division when they calculate unknown sides of similar triangles.

If most students struggle with this item, plan to use these problems and the three in Lesson 1 Activity 1, Number Talk, to review fraction division. Focus on the strategies described in the narrative of Lesson 1, Activity 1.

Statement

Evaluate each expression.

1. $3 \div \frac{1}{8}$
2. $\frac{7}{10} \div \frac{3}{2}$
3. $3\frac{1}{3} \div \frac{5}{6}$

Solution

1. 24
2. $\frac{7}{15}$ (or equivalent)
3. 4

Aligned Standards

6.NS.A.1

Problem 6

The content assessed in this problem is first encountered in Lesson 1: Projecting and Scaling.

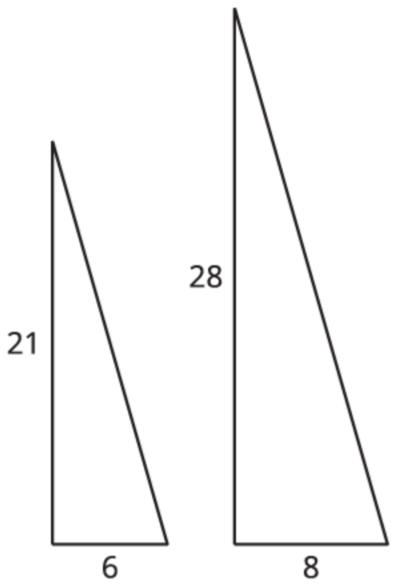
In 7th grade, students studied scaled copies and scale factors. The work with dilations and similar triangles in this unit builds on that foundation.

If most students struggle with this item, plan to do optional Lesson 1 Activity 3 allowing students an opportunity to continue working with scaled copies and finding the scale factors.

Statement

The two triangles displayed are scaled copies of one another.

1. Find the scale factor.
2. Sketch a new triangle that is also a scaled copy of these triangles using a different scale factor.



Solution

1. $\frac{4}{3}$ or $\frac{3}{4}$ (or equivalent)
2. Answers vary. The side lengths of the new triangle must all be k times the side lengths of one of the original triangles for some positive number k where $k \neq 1$.

Aligned Standards

7.G.A.1

Problem 7

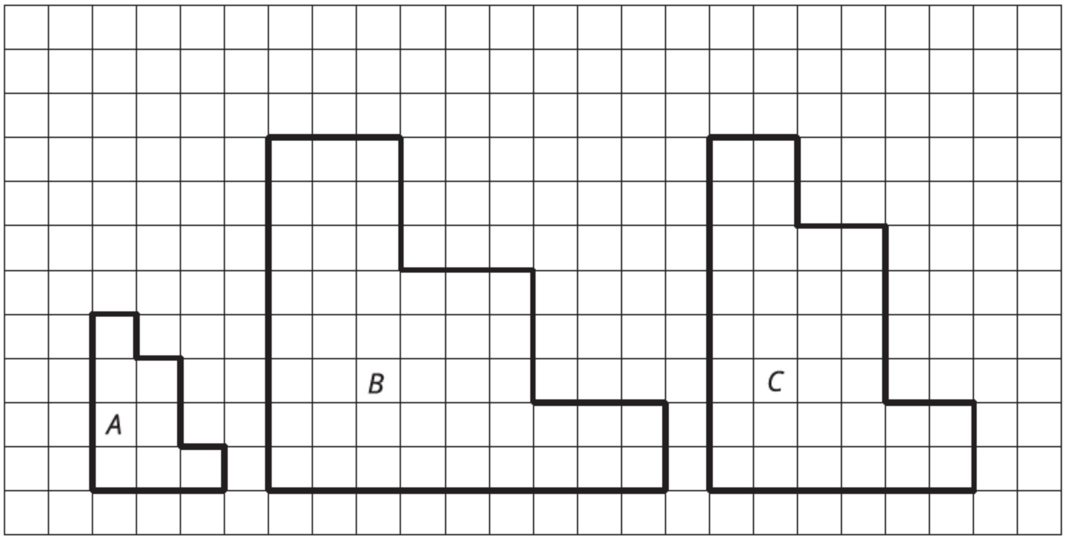
The content assessed in this problem is first encountered in Lesson 1: Projecting and Scaling.

In this unit, students will perform dilations of polygons both with and without the aid of a grid. Students will discover that side lengths of the polygons before and after the dilation have equivalent ratios, just as in their grade 7 work with scaled copies.

If most students struggle with this item, plan to spend time on Lesson 1 Activity 2, using the "eyeball test" described in the launch and emphasizing in the synthesis the relationship between equivalent ratios and scaled copies. Plan to revisit this item in the synthesis of Lesson 1 Activity 2 and ask students how they could determine whether Figure B is a scaled copy of Figure A. Emphasize strategies that take advantage of the grid in looking for equivalent ratios.

Statement

Which figure is a scaled copy of Figure A? Explain how you know.



Solution

Figure C is a scaled copy because each side of Figure C is twice as long as the corresponding side of Figure A. Figure B is not a scaled copy because the bottom side of Figure B is three times as long as the bottom side of Figure A, but the left side of Figure B is twice as long as the left side of Figure A.

Aligned Standards

7.G.A.1

Dilations, Similarity, and Introducing Slope Assessment : End-of-Unit Assessment (A)

Problem 1

Students selecting A may be thinking only of dilations with scale factor greater than one. Students failing to select B might not realize that “perpendicular lines to perpendicular lines” is a special case of dilations preserving angles. Students failing to select F have forgotten that similar figures are defined as figures which can be matched by a sequence of dilations and rigid transformations.

Statement

Select **all** the true statements.

A. Dilations always increase the length of line segments.

B. Dilations take perpendicular lines to perpendicular lines.

C. Dilations of an angle are congruent to the original angle.

D. Dilations increase the measure of angles.

E. Dilations of a triangle are congruent to the original triangle.

F. Dilations of a triangle are similar to the original triangle.

Solution

["B", "C", "F"]

Aligned Standards

8.G.A

Problem 2

This problem's focus is the angle-angle criterion for similarity.

Students selecting A may believe that triangles that share only one pair of congruent angles must be similar. Students selecting B might know the angle-angle criterion, but are making a subtler mistake. If the 40° angles are both vertex angles or both base angles of their respective isosceles triangles, then the remaining angle pairs across the triangles must be the same. However, if one is a vertex angle and the other is a base angle, this reasoning falls apart. Students selecting C may have made a calculation mistake, thinking that a remaining angle in one of the triangles is a match for an angle in the other. This type of reasoning works for choice D: using the fact that the angle measures of a triangle add up to 180°, the remaining angle in Triangle 7 is 105° and the remaining angle in Triangle 8 is 25°.

Statement

Which pair of triangles **must** be similar?

A. Triangles 1 and 2 each have a 35° angle.

B. Triangles 3 and 4 are both isosceles. They each have a 40° angle.

C. Triangle 5 has a 30° angle and a 90° angle. Triangle 6 has a 30° angle and a 70° angle.

D. Triangle 7 has a 50° angle and a 25° angle. Triangle 8 has a 50° angle and a 105° angle.

Solution

D

Aligned Standards

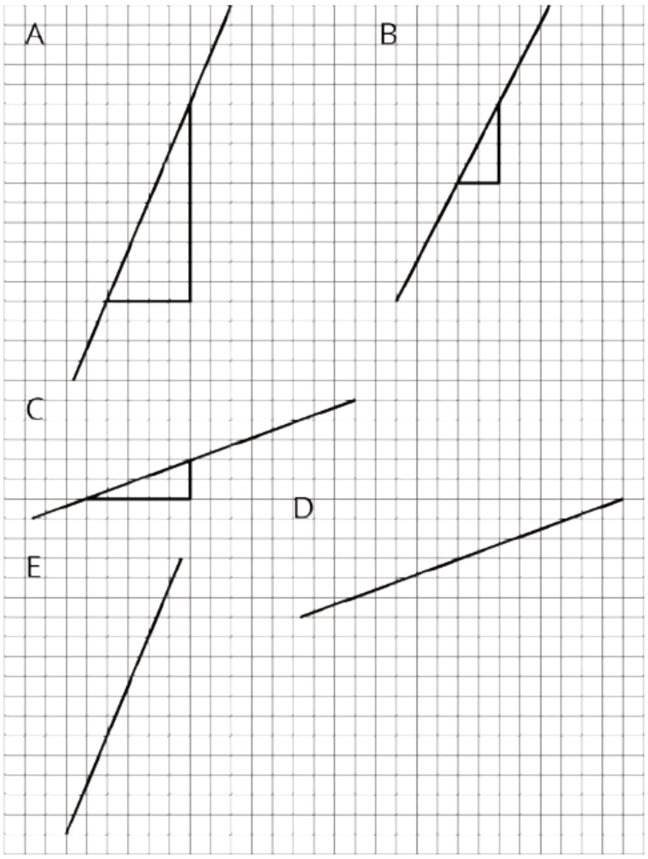
8.G.A.5

Problem 3

Students selecting B have either miscounted the vertical length of the slope triangle or are simply eyeballing—this line has slope 2. Students selecting C or D (and not selecting A or E) are dividing horizontal length by vertical length rather than the other way around.

Statement

Select **all** the lines that have a slope of $\frac{5}{2}$.



A. A

B. B

C. C

D. D

E. E

Solution

["A", "E"]

Aligned Standards

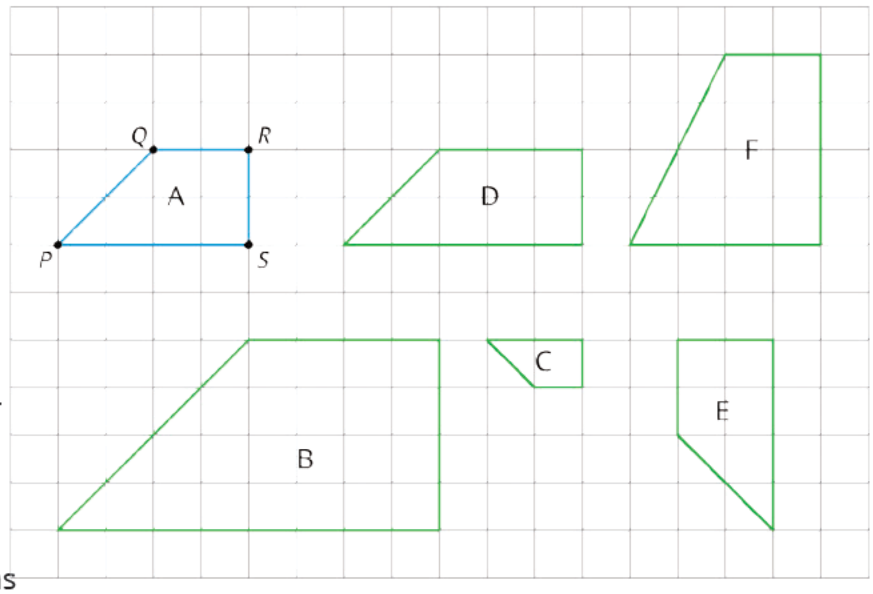
8.EE.B.6

Problem 4

Students identify which polygons are similar to a given polygon on a grid. For each similar polygon, they describe similarity transformations that take one figure to another. Watch for students who do not say Polygon E is similar; they likely believe that similar polygons cannot be congruent.

Statement

Here are some polygons:



1. Which of Polygons B, C, D, E, and F are similar to Polygon A?
2. Choose *one* of the polygons that are similar to Polygon A, and describe a sequence of transformations that take Polygon A to the selected polygon.

Solution

1. Polygons B, C, and E

2. Answers vary. For B, dilate with center P and a scale factor of 2, then translate 6 squares down. For C, dilate using scale factor of $\frac{1}{2}$ and center S, and then reflect over line PS and translate 3 squares down and $7\frac{1}{2}$ squares to the right. For E, rotate 90 degrees counterclockwise around S, and then translate 2 squares down and 11 squares to the right.

Minimal Tier 1 response:

- Work is complete and correct.
- Acceptable errors: work does not specify which polygon they are working with in part b, but the chosen polygon is clear from a correct response; use of language like “move” or “shift” instead of “translate.”
- Sample:

1. B, C, E
2. (for polygon B) Dilate with scale factor 2 from point P. Then move 6 units down.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: centers of rotations/dilations, scale factors of dilations, or lines of reflection are omitted but the meaning is clear because of intermediate drawings; one incorrect (or missing) answer in part a; instructions for the sequence of rigid motions and dilations contain a small, easily identifiable error (such as saying to translate 7 units instead of 6 units).

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: response to part b is based on an incorrect choice in part a, the sequence of rigid motions and dilations does not take Polygon A to the chosen polygon (and is not close), incorrect answer to part a.

Aligned Standards

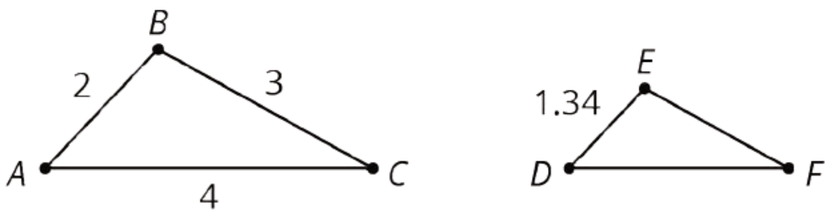
8.G.A.4

Problem 5

When two shapes are similar, a scale factor relates lengths in one figure to the corresponding lengths in the other. At the same time, ratios of lengths in one figure (e.g., length to width) are the *same* as in the other figure. In this problem, it is simpler to use this second idea when calculating side lengths of the triangles.

Statement

Triangles ABC and DEF are similar.



1. Find the length of segment DF.

2. Find the length of segment EF.

Solution

1. 2.68 units (twice as long as segment DE)
2. 2.01 units ($\frac{3}{2}$ as long as segment DE)

Aligned Standards

8.G.A.4

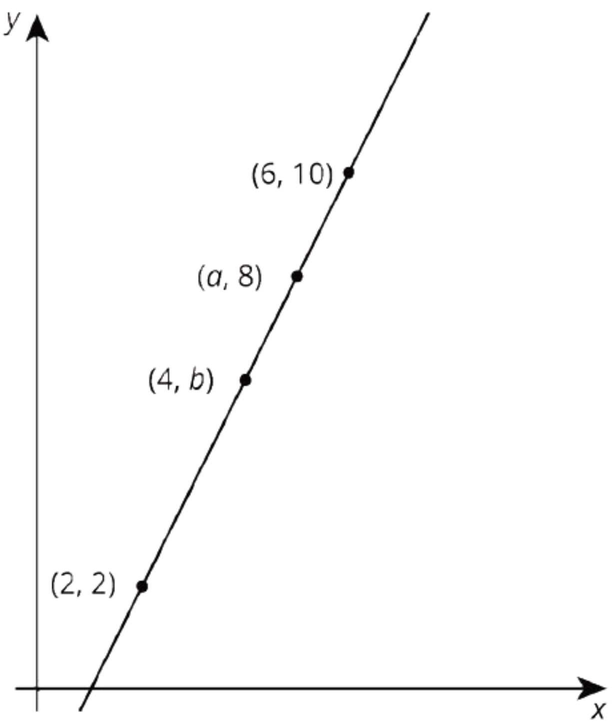
Problem 6

This problem has students use slope triangles to write an equation for a line. This equation can be used to find the unknown values later in the problem. However, students can just as easily reason about slope to find these values.

Statement

All of the points in the picture are on the same line.

1. Find the slope of the line. Explain or show your reasoning.
2. Write an equation for the line.
3. Find the values for a and b. Explain or show your reasoning.



Solution

1. 2, because $\frac{10-2}{6-2} = \frac{8}{4} = 2$.

2. $\frac{y-10}{x-6} = 2$ or equivalent

3. $a = 5$, $b = 6$. These can be found by counting "over 1, up 2" from known points or by using the equation from part b.

Minimal Tier 1 response:

- Work is complete and correct.

- Sample:

1. $\frac{10-2}{6-2} = 2$

2. $\frac{y-10}{x-6} = 2$

3. When you go over one unit, you go up two to stay on the line. That means it goes $(2, 2)$, $(3, 4)$, $(4, 6)$, $(5, 8)$.... So $a = 5$, $b = 6$.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.

- Sample errors: incorrect answers to parts c based on small algebra errors in using the equation of the line or on miscounting when finding intermediate points; finding that the slope is $\frac{1}{2}$ or the equation for the line is $\frac{x-2}{y-2} = 2$ or $\frac{x-2}{y-2} = \frac{1}{2}$; correct answers to two problem parts with badly incorrect answer to one problem part.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.

- Sample errors: work does not involve slope beyond part a; student answers some problems correctly by "eyeballing" but reasoning does not appeal to slope (or related concepts like, "two up, one over"); more than one incorrect answer without explanation.

Aligned Standards

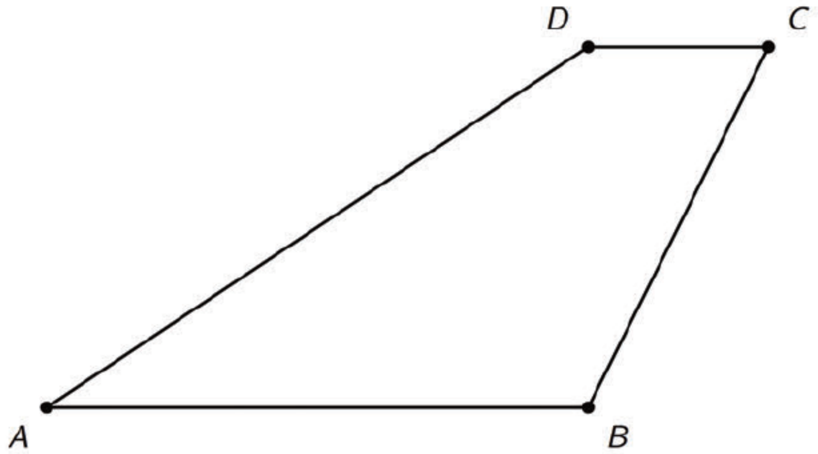
8.EE.B.6

Problem 7

Students apply dilations to a polygon off of a grid. They then reason about similarity. Because of the way the two polygons are constructed, there is a natural sequence of dilations that takes one polygon to the other, making them similar.

Statement

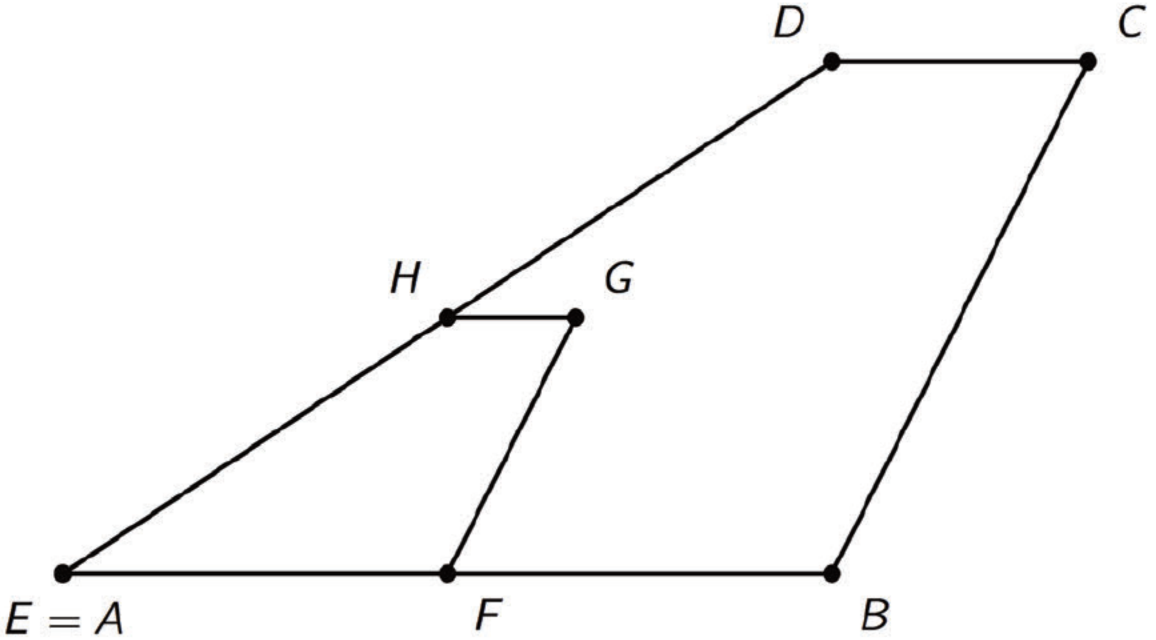
Here is a polygon:



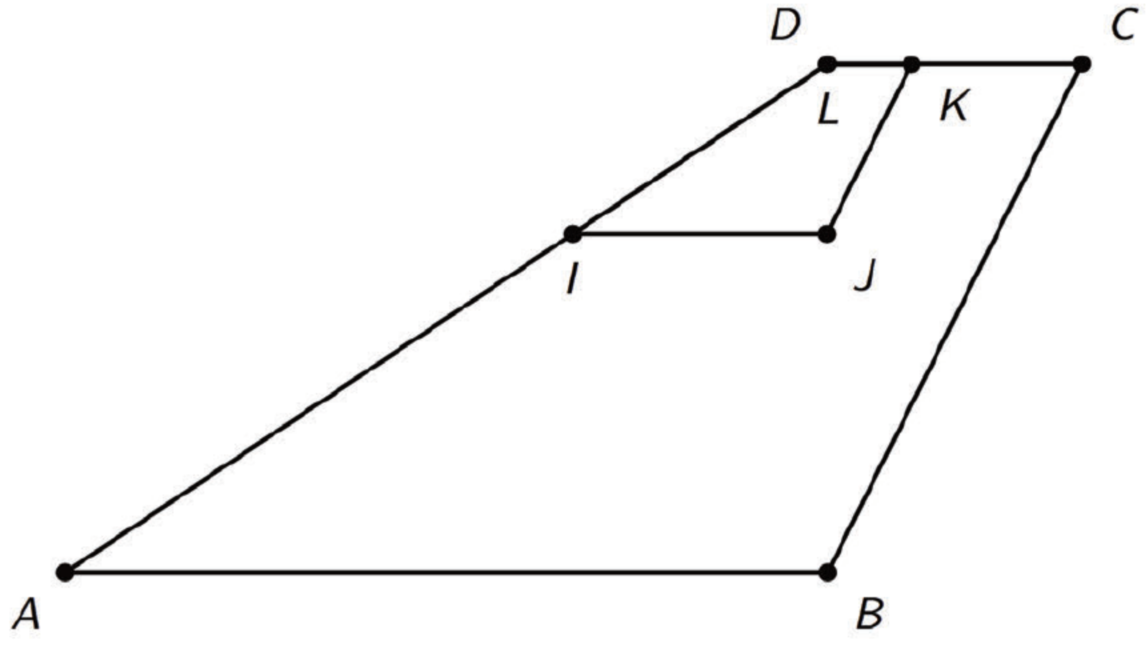
1. Draw the dilation of $ABCD$ using center A and scale factor $\frac{1}{2}$. Label the dilation $EFGH$.
2. Draw the dilation of $ABCD$ with center D and scale factor $\frac{1}{3}$. Label the dilation $IJKL$.
3. Show that $EFGH$ and $IJKL$ are similar.

Solution

1.



2.



3. If $EFGH$ is dilated with center A and a scale factor of 2, the result is $ABCD$. If $ABCD$ is dilated with center D and a scale factor of $\frac{1}{3}$, the result is $IJKL$.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample:

1. See diagram
2. See diagram
3. $EFGH$ and $IJKL$ are both dilations of the same polygon.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: scale is correct in the dilations, but the center is incorrect; work involves a minor mistake dilating one point; response to part c is something like "$EFGH$ and $IJKL$ are dilations of each other" without a justification such as referencing $ABCD$.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: correct work for parts a and b but very weak or missing explanation in part c; work shows general understanding of dilations but a few points are placed incorrectly, dilations are performed using scale factors of 2 or 3 rather than $\frac{1}{2}$ or $\frac{1}{3}$.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: work for parts a and b does not result in anything resembling dilations.

Aligned Standards

8.G.A.4

Dilations, Similarity, and Introducing Slope

Assessment : End-of-Unit Assessment (B)

Teacher Instructions

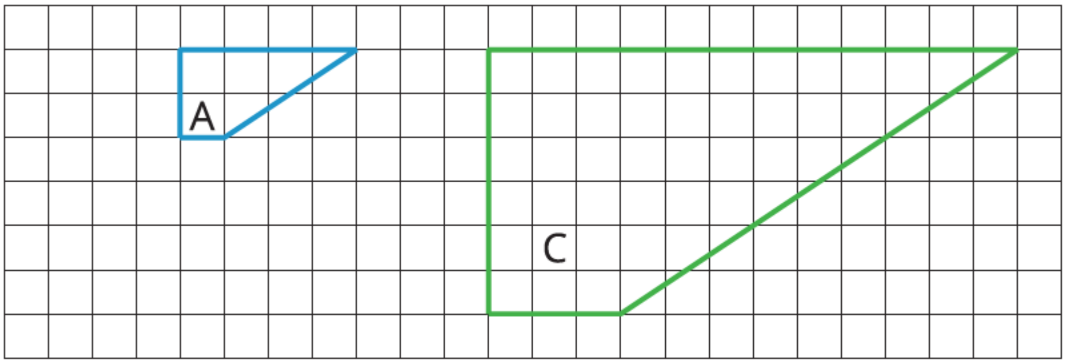
It may be helpful to make graph paper available for question 6.

Problem 1

Students identify a similarity transformation that takes one figure to another. Students that select A or B have an incomplete understanding of similar figures. Students selecting D understand that a dilation is part of a similarity transformation but forgot that they needed to include a translation to correctly explain the similarity.

Statement

Han's teacher asked him to draw a polygon similar to polygon A. Here is his work. Which explanation shows that polygons A and C are similar to each other?



A. Use a protractor to measure all four angles in polygon A and all four angles in polygon C. Since each angle in polygon A has a matching angle measure in polygon C, the polygons are similar.

B. Since each side in polygon C is three times as long as the corresponding side in polygon A, the figures are similar.

C. Dilate polygon C with center at the upper-left vertex and a scale factor of $\frac{1}{3}$. Then, translate 7 units to the left. Since polygon C can be taken to polygon A with a dilation followed by a translation, the figures are similar.

D. Dilate polygon C with center at the upper-right corner and a scale factor of 3. Since polygon C can be taken to polygon A with a dilation, the polygons are similar.

Solution

C

Aligned Standards

8.G.A.4

Problem 2

Students failing to select A may think dilations do not preserve angle measure. Students selecting B, C or D may be confusing similar and congruent or thinking that a dilation is a rigid transformation.

Statement

Select the true statement.

A. Dilations of an angle must be congruent to the original angle.

B. Dilations of a triangle must be congruent to the original triangle.

C. Dilations of a segment must be congruent to the original segment.

D. Dilations of a circle must be congruent to the original circle.

Solution

A

Aligned Standards

8.G.A

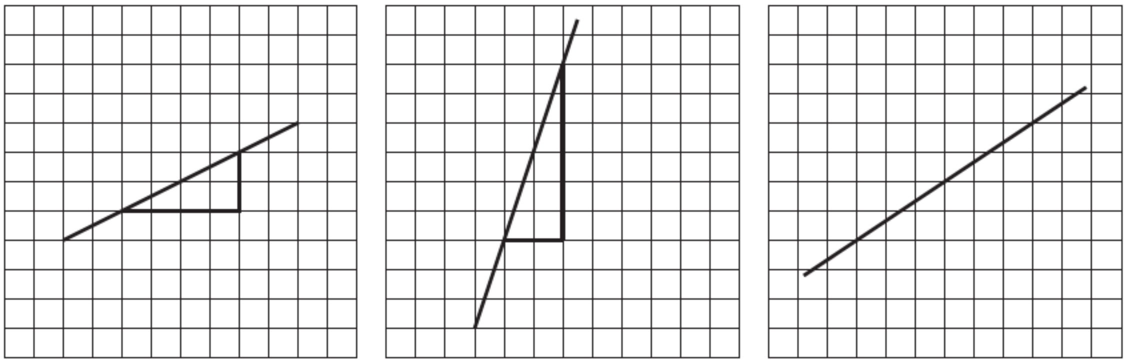
Problem 3

Students find the slope of a line on a grid without coordinate axes. Line C does not include a "slope triangle". Be mindful of students who calculate the slope as the horizontal length divided by the vertical length.

Statement

Find the slope of each line.

A **B** **C**



Solution

A. $\frac{1}{2}$ (or equivalent)

B. 3 (or equivalent)

C. $\frac{2}{3}$ (or equivalent)

Aligned Standards

8.EE.B.6

Problem 4

This problem's focus is the angle-angle criterion for similarity but students must consider if they really have enough information to use this criteria. If the 30° angles are both vertex angles or both base angles of their respective isosceles triangles, then the remaining angle pairs across the triangles must be the same. However, if one is a vertex angle and the other is a base angle, this angle-angle criterion does not apply.

Statement

Triangles 1 and 2 are both isosceles. They each have a 30° angle. Explain why these triangles do not have to be similar to each other. If you get stuck, consider drawing a diagram.

Solution

They do not have to be similar because an isosceles triangle with only one 30° angle does not have the same angle measures as a triangle with two 30° angles. If the isosceles triangle has two 30° angles then the third angle would be 120° because $180 - 30 - 30 = 120$. If the triangle has only one 30° angle, then the other two angles would be 75° because $180 - 30 = 150$ and 150 divided by 2 is 75.

Minimal Tier 1 response:

- Work is complete and correct.
- Acceptable errors: students do not need to directly mention angle-angle similarity as long as they reason correctly about needing at least two corresponding angles with equal measure.
- Sample: The triangles would not be similar if one triangle has two 30 degree angles and the other triangle only has one because similar triangles need at least two pairs of angles with the same measures.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: minor errors in computing possible angles for the triangles; draws accurate pictures but written explanation may be incomplete.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: Fails to consider that the 30° angles could be the vertex angle or the base angle. Asserts that one pair of congruent corresponding angles guarantees the triangles are similar.

Aligned Standards

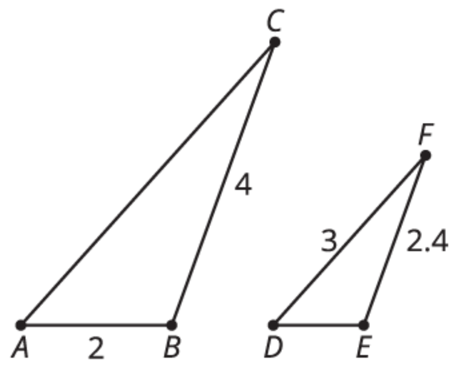
8.G.A.5

Problem 5

When two shapes are similar, a scale factor relates lengths in one figure to the corresponding lengths in the other. At the same time, ratios of lengths in one figure (e.g., length to width) are the same as in the other figure. In this problem, it is simpler to use this second idea when calculating side lengths of the triangles.

Statement

Triangles ABC and DEF are similar.



1. Find the length of segment DE.
2. Find the length of segment AC.

Solution

1. 1.2 units (half as long as segment EF)
2. 5 units ($\frac{5}{3}$ as long as segment FD)

Aligned Standards

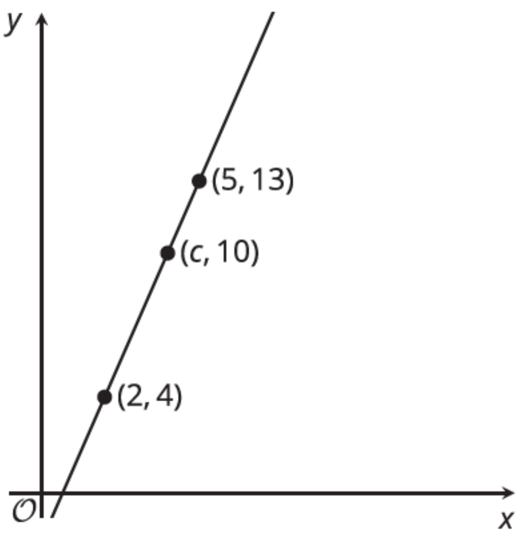
8.G.A.4

Problem 6

This problem has students use slope triangles to write an equation for a line. This equation can be used to find the unknown value later in the problem and verify a point is on the line. However, students can just as easily reason about slope to answer these questions.

Statement

All of the points in the picture are on the same line.



1. Find the slope of the line. Explain or show your reasoning.

2. Write an equation for the line.

3. What is the value of c? Explain or show your reasoning.

4. Is the point $(0, -2)$ on this line? Explain how you know.

Solution

1. 3 ($\frac{13-4}{5-2} = \frac{9}{3} = 3$)

2. $\frac{y-4}{x-2} = 3$ (or equivalent)

3. $c = 4$. This can be found by counting “over 1, up 3” from $(2,4)$ or by using the equation from the previous question.

4. Yes. $\frac{-2-4}{0-2} = 3$ The slope of the line containing $(0,-2)$ and $(2,4)$ is the same as the slope between the original two points. This can also be verified by counting “over 1, up 3” from one of the two given points.

Minimal Tier 1 response:

- Work is complete and correct.

Sample:

1. $\frac{13-4}{5-2} = \frac{9}{3} = 3$

2. $\frac{y-4}{x-2} = 3$

3. When you go over one unit, you go up three to stay on the line. That means it goes (2,4), (3,7), (4,10), ... So c=4.

4. Counting backwards: (2,4), (1,1), (0,-2) so the point (0,-2) is on the line.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: incorrect answers to parts c and d based on small algebra errors in using the equation of the line or on miscounting when finding intermediate points; finding that the slope is 1/3 or the equation for the line is $\frac{x-2}{y-4} = 3$; correct answers to three problem parts with badly incorrect answer to one problem part.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.

- Sample errors: work does not involve slope beyond parts b or c; student answers some problems correctly by “eyeballing” but reasoning does not appeal to slope (or related concepts like, “two up, one over”); more than one incorrect answer without explanation.

Aligned Standards

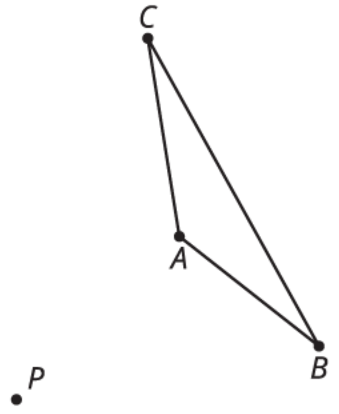
8.EE.B.6

Problem 7

Students apply dilations to a triangle off of a grid and a point not on the triangle. They then reason about similarity. Because both dilations use the same center, students can reason they triangles are similar as a result of successive dilations. They could also describe a translation and a dilation that would show the two triangles are similar.

Statement

Here is triangle ABC and point P:



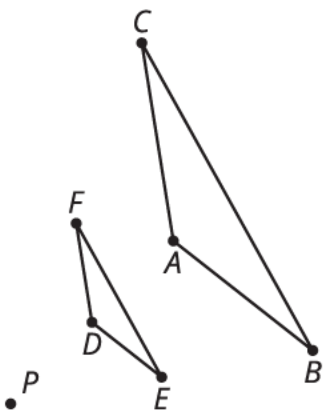
1. Draw the dilation of ABC using center P and scale factor $\frac{1}{3}$. Label the dilation DEF.

2. Draw the dilation of ABC with center P and scale factor 2. Label the dilation GHI.

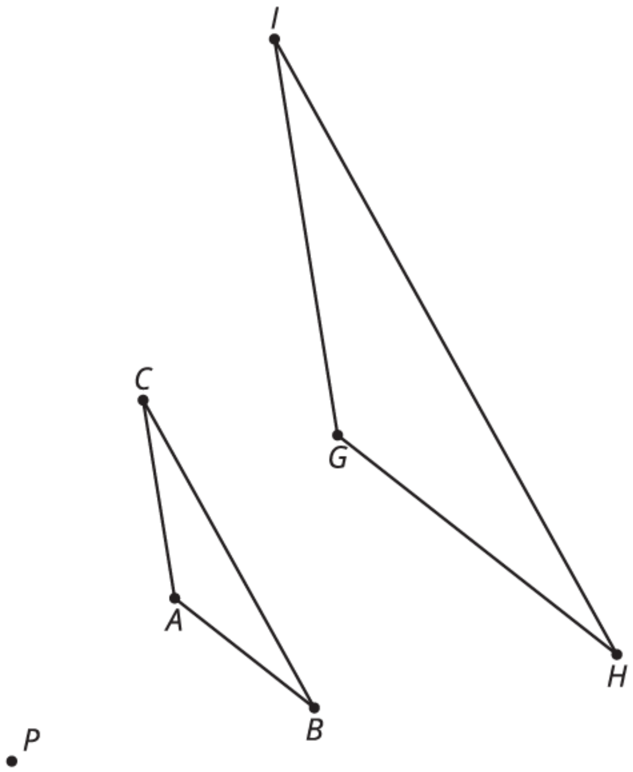
3. Show that DEF and GHI are similar.

Solution

1.



2.



3. If DEF is dilated with center P and a scale factor of 3, the result is ABC. If ABC is dilated with center D and a scale factor of 2, the result is GHI. This sequence of two dilations takes triangle DEF to triangle GHI.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample:

1. See above diagram
2. See above diagram
3. DEF and GHI are both dilations of the same triangle.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.

- Sample errors: scale is correct in the dilations, but the center is incorrect; work involves a minor mistake dilating one point; response to part c is something like “DEF and GHI are dilations of each other” without a justification such as referencing ABC.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: correct work for parts a and b but very weak or missing explanation in part c; work shows general understanding of dilations but a few points are placed incorrectly, dilations are performed using scale factors of 3 or 1/2 rather than 1/3 or 2.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: work for parts a and b does not result in anything resembling dilations.

Aligned Standards

8.G.A.4

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Lesson Cool Downs

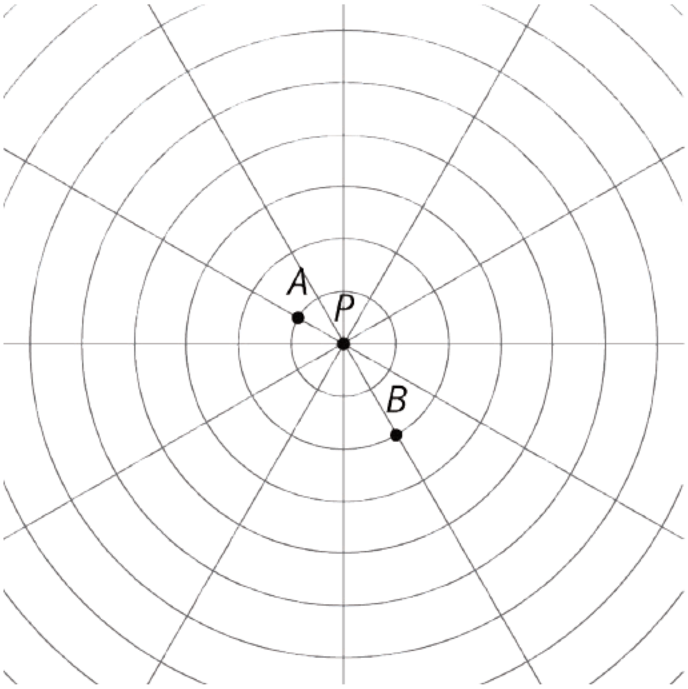
Lesson 1: Projecting and Scaling

Cool Down: What is a Dilation?

In your own words, explain what a dilation is.

Lesson 2: Circular Grid

Cool Down: Dilating points on a circular grid



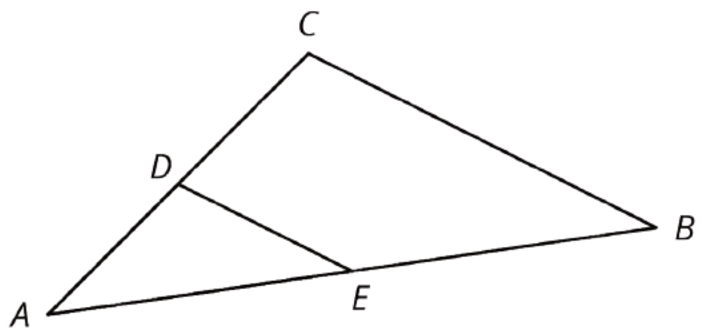
1. Dilate A using P as the center of dilation and a scale factor of 3. Label the new point A'.

2. Dilate B using P as the center of dilation and a scale factor of 2. Label the new point B'.

Lesson 3: Dilations with no Grid

Cool Down: A Single Dilation of a Triangle

Lin drew a triangle and a dilation of the triangle with scale factor $\frac{1}{2}$:



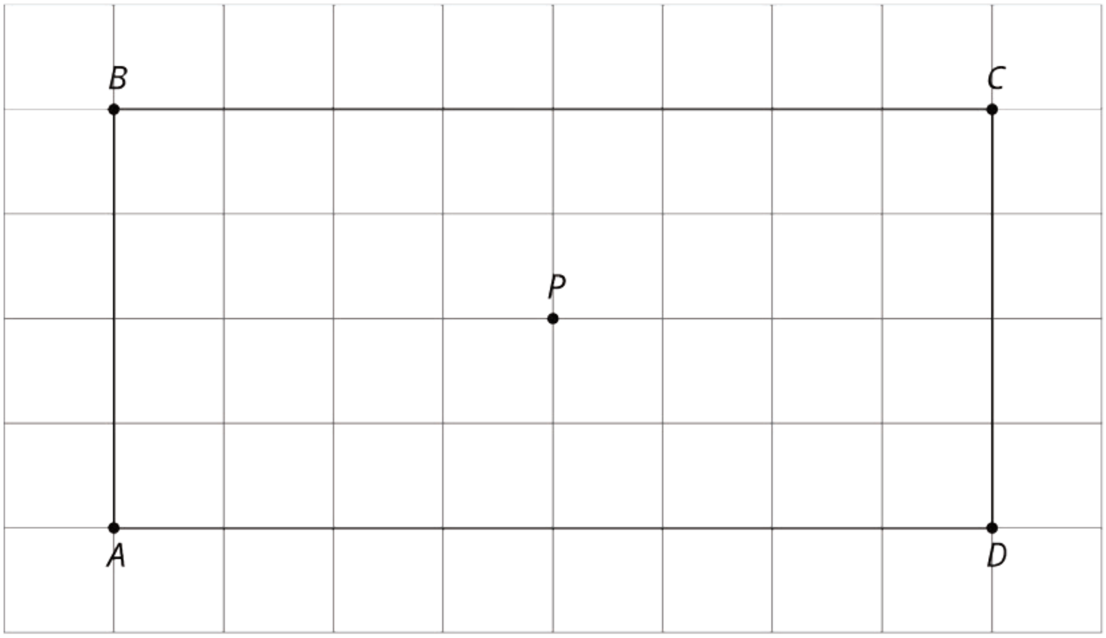
1. What is the center of the dilation? Explain how you know.

2. Which triangle is the original and which triangle is the dilation? Explain how you know.

Lesson 4: Dilations on a Square Grid

Cool Down: A Dilated Image

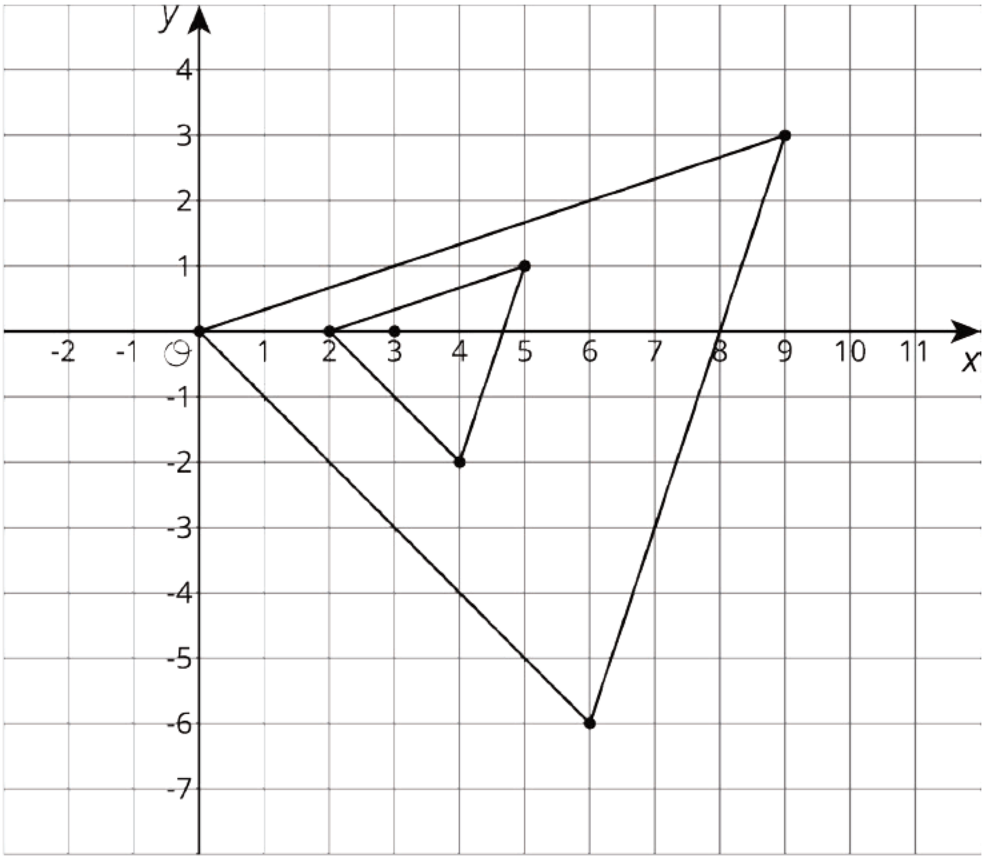
Draw the image of rectangle $ABCD$ under dilation using center P and scale factor $\frac{1}{2}$.



Lesson 5: More Dilations

Cool Down: Identifying a Dilation

The smaller triangle is dilated to create the larger triangle. The center of dilation is plotted, but not labeled.

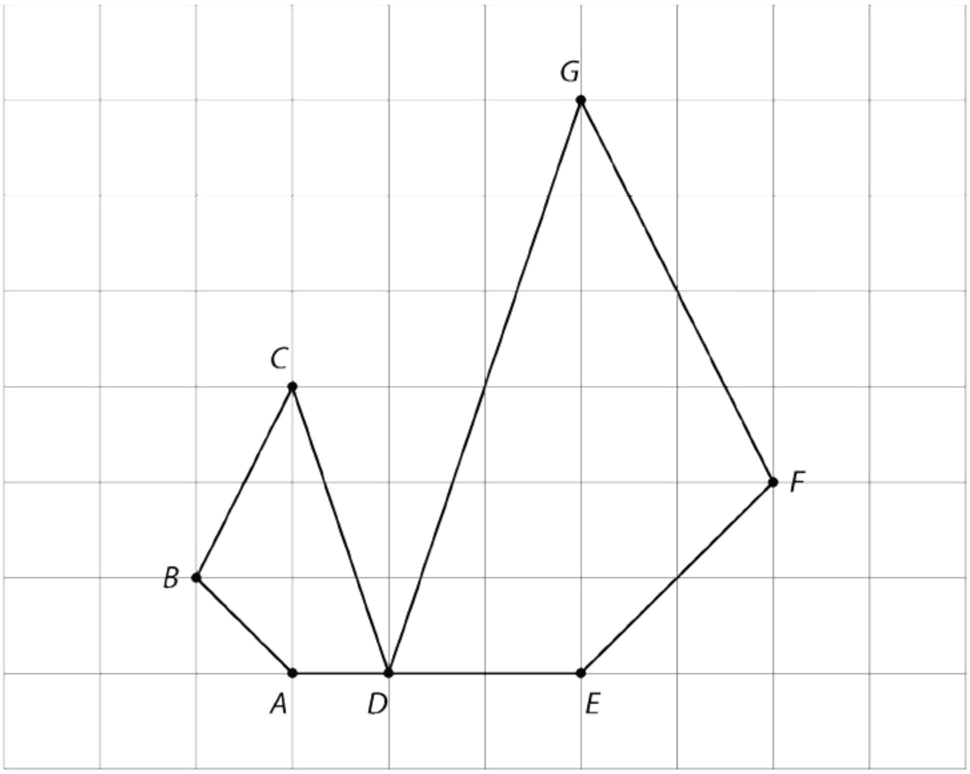


Describe this dilation. Be sure to include all of the information someone would need to perform the dilation.

Lesson 6: Similarity

Cool Down: Showing Similarity

Elena gives the following sequence of transformations to show that the two figures are similar by transforming $ABCD$ into $EFGD$.



1. Dilate using center D and scale factor 2.

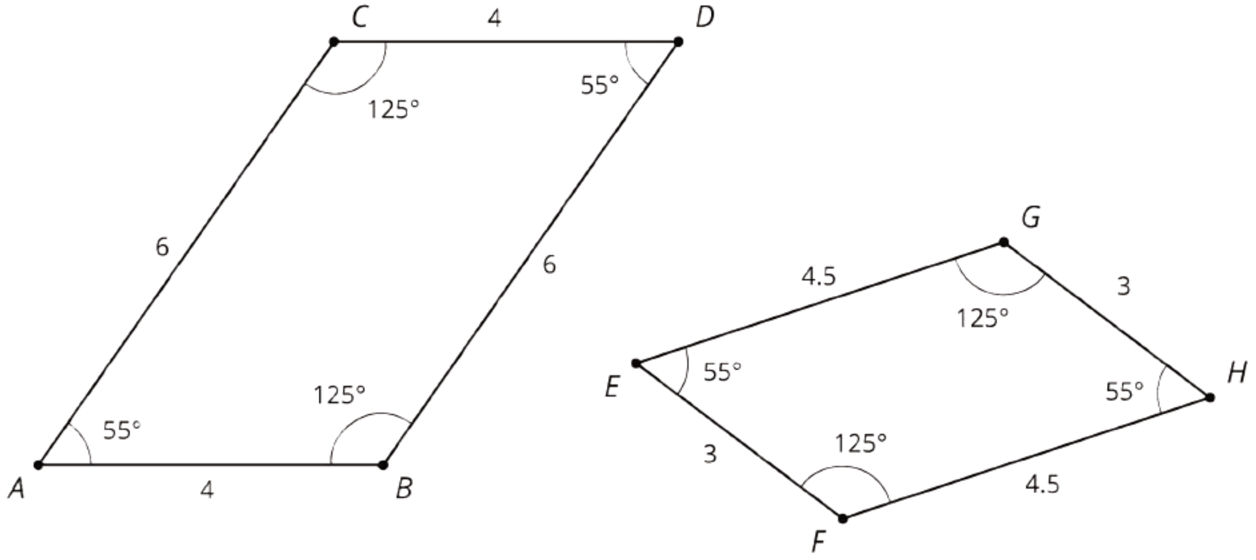
2. Reflect using the line AE.

Is Elena’s method correct? If not, explain how you could fix it.

Lesson 7: Similar Polygons

Cool Down: How Do You Know?

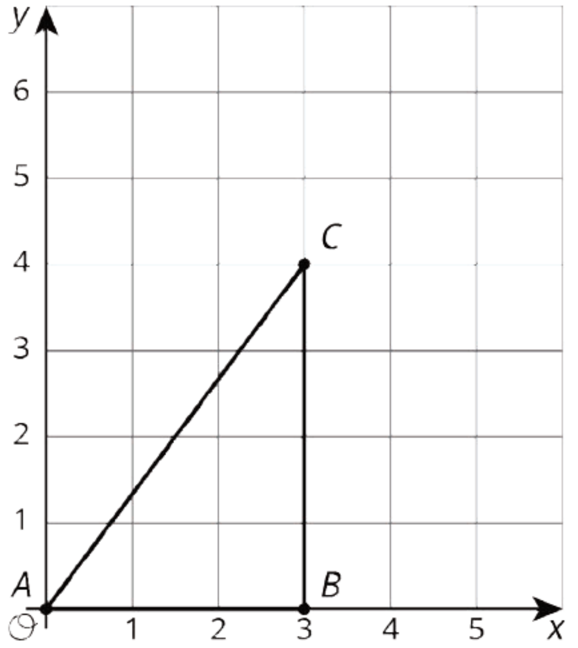
Explain how you know these two figures are similar.

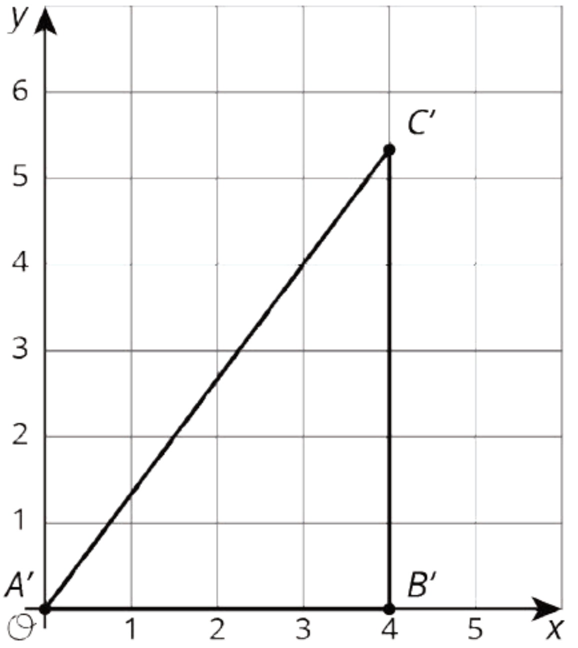


Lesson 8: Similar Triangles

Cool Down: Applying Angle-Angle Similarity

Here are two triangles.





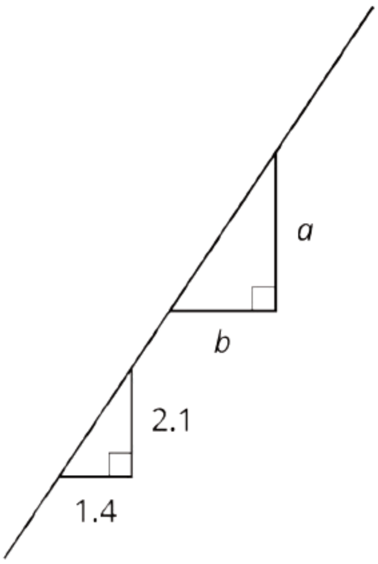
1. Show that the triangles are similar.

2. What is the scale factor from triangle ABC to triangle $A'B'C'$?

Lesson 9: Side Length Quotients in Similar Triangles

Cool Down: Similar Sides

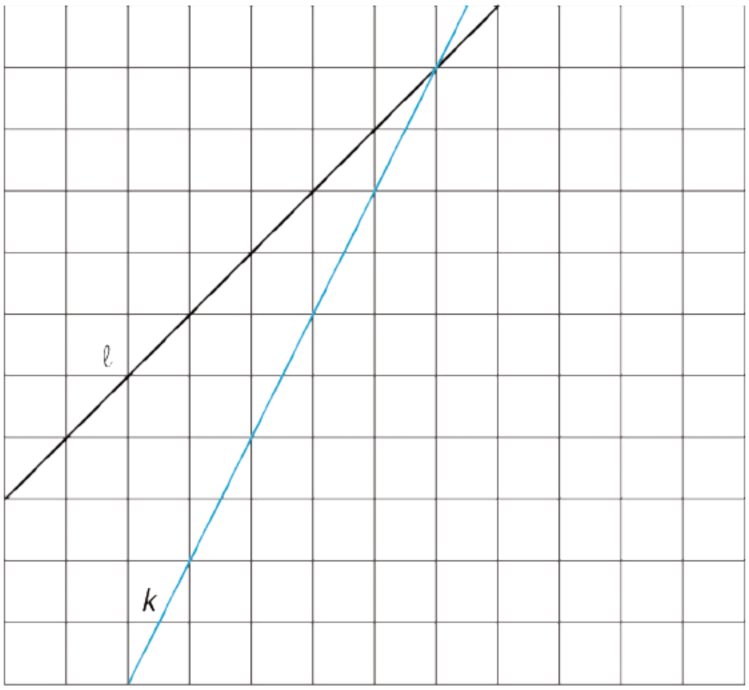
The two triangles shown are similar. Find the value of $\frac{a}{b}$.



Lesson 10: Meet Slope

Cool Down: Finding Slope and Graphing Lines

Lines ℓ and k are graphed.

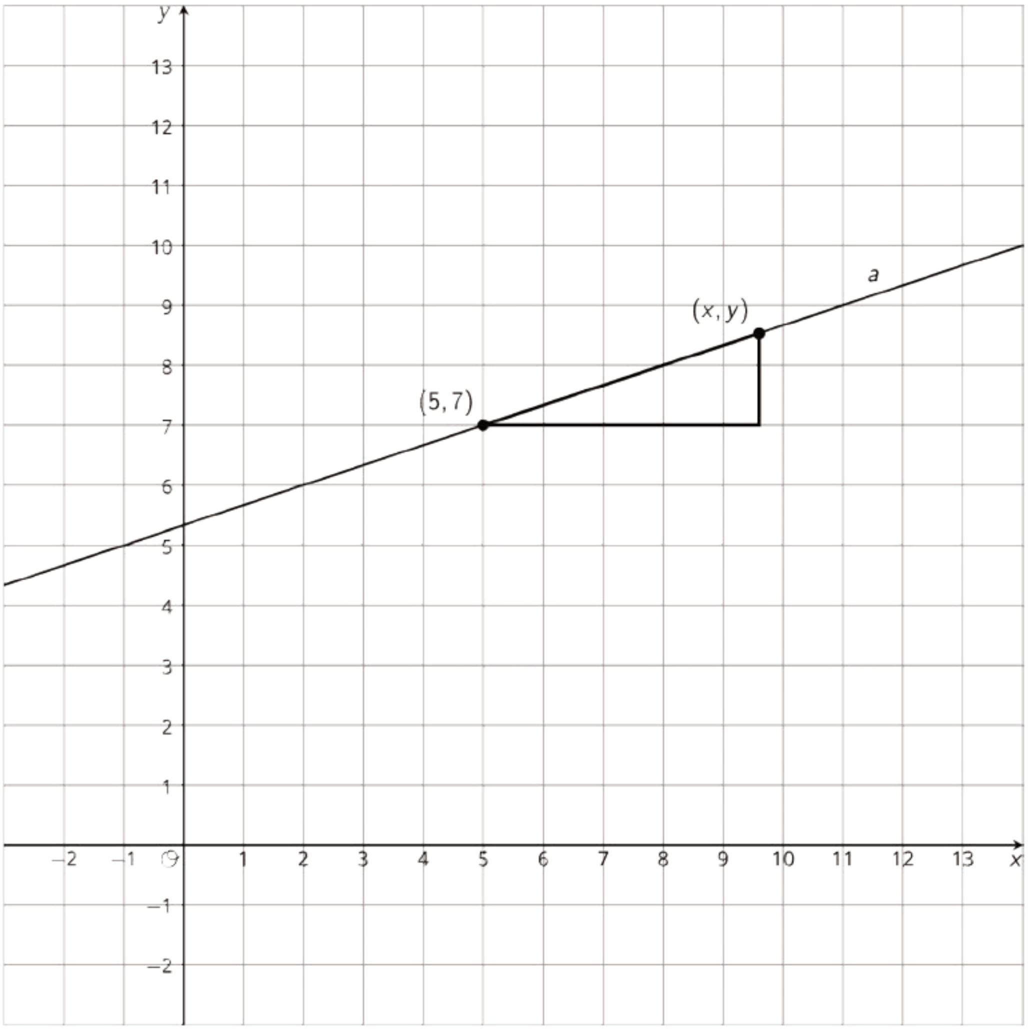


1. Which line has a slope of 1, and which has a slope of 2?

2. Use a ruler to help you graph a line whose slope is $\frac{1}{3}$. Label this line a.

Lesson 11: Writing Equations for Lines

Cool Down: Matching Relationships to Graphs



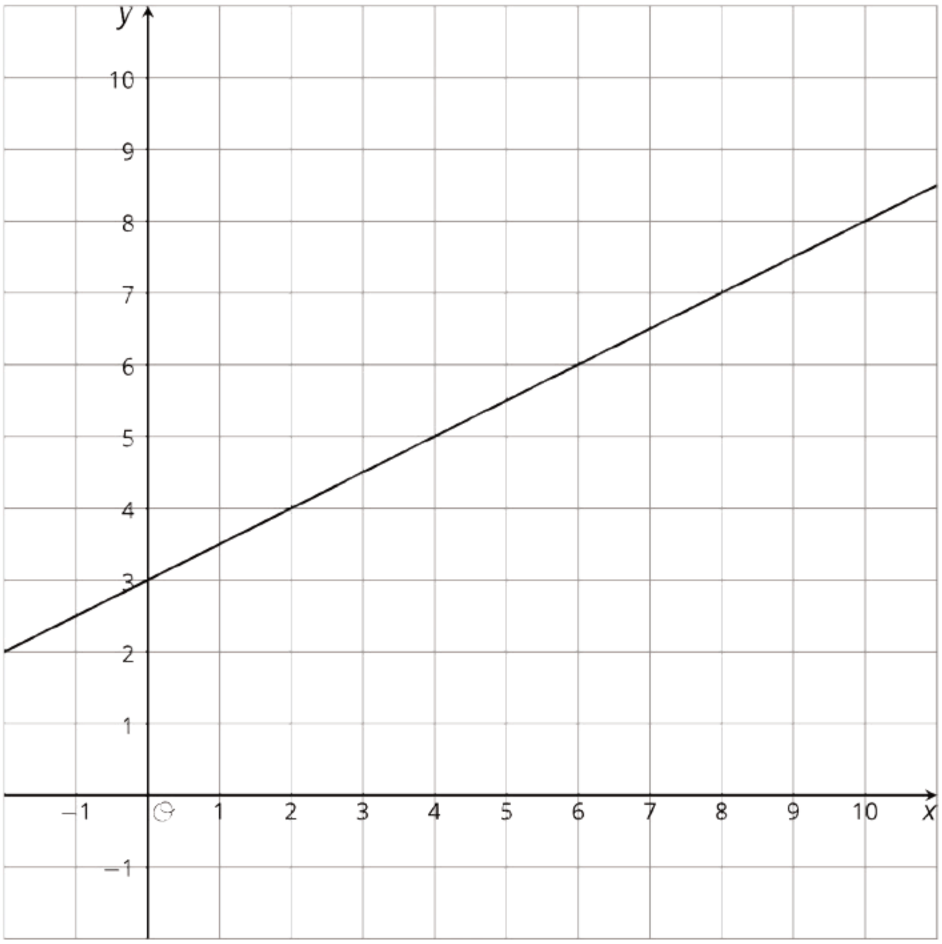
1. Explain why the slope of line a is $\frac{2}{6}$.

2. Label the horizontal and vertical sides of the triangle with expressions representing their length.

3. Explain why $\frac{y-7}{x-5} = \frac{2}{6}$.

Lesson 12: Using Equations for Lines

Cool Down: Is the Point on the Line?



Is the point (20,13) on this line? Explain your reasoning.

CKMath™

Core Knowledge MATHEMATICS™

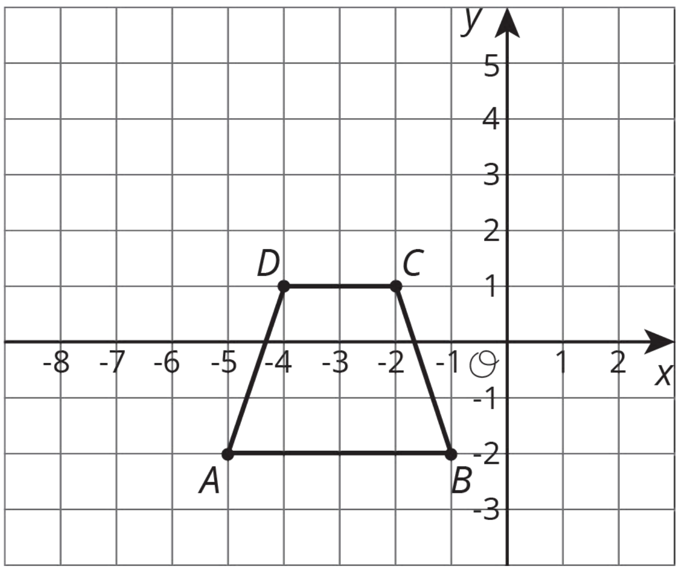
Instructional Masters

Instructional Masters for Dilations, Similarity, and Introducing Slope

address	title	students per copy	written on?	requires cutting?	card stock recommended?	color paper recommended?
Activity Grade8.2.8.2	Making Pasta Angles and Triangles	4	no	yes	no	no
Activity Grade8.2.7.3	Find Someone Similar	10	no	yes	no	no
Activity Grade8.2.5.2	Info Gap: Dilations	2	yes	yes	no	no
Activity Grade8.2.4.3	Card Sort: Matching Dilations on a Coordinate Grid	1	yes	yes	no	no
Activity Grade8.2.6.4	Methods for Translations and Dilations	2	no	yes	no	no

Card Sort: Matching Dilations on a Coordinate Grid

1.



Dilate the trapezoid using center (-1, -2) and scale factor $\frac{3}{2}$

Card Sort: Matching Dilations on a Coordinate Grid

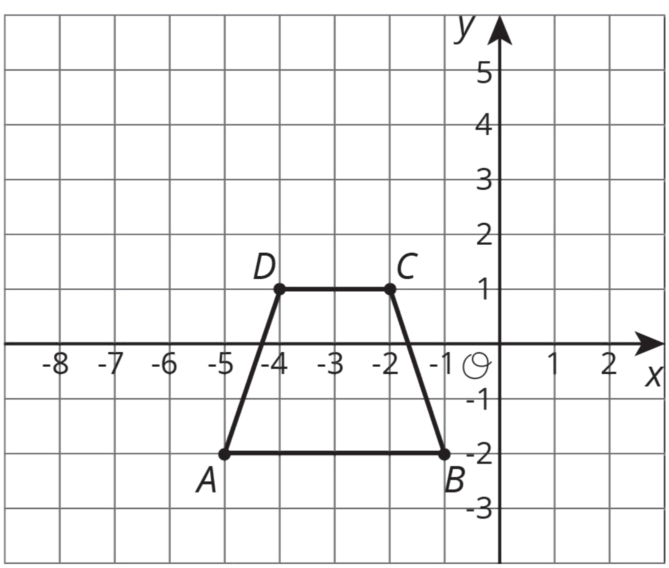
C.

The polygon with vertices at:

$A' = (\text{-}7, \text{-}2)$
$B' = (\text{-}1, \text{-}2)$
$C' = \left(\text{-}2\frac{1}{2}, 2\frac{1}{2}\right)$
$D' = \left(\text{-}5\frac{1}{2}, 2\frac{1}{2}\right)$

Card Sort: Matching Dilations on a Coordinate Grid

2.



Dilate the trapezoid using center (-3, 4) and scale factor $\frac{1}{2}$.

Card Sort: Matching Dilations on a Coordinate Grid

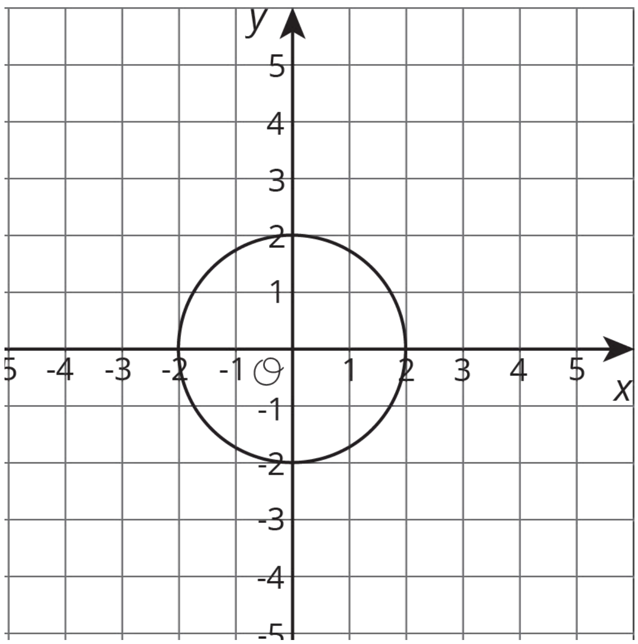
A.

The polygon with vertices at:

$A' = (\text{-}4, 1)$
$B' = (\text{-}2, 1)$
$C' = \left(\text{-}2\frac{1}{2}, 2\frac{1}{2}\right)$
$D' = \left(\text{-}3\frac{1}{2}, 2\frac{1}{2}\right)$

Card Sort: Matching Dilations on a Coordinate Grid

3.



Dilate the circle using center (0, 0) and scale factor 2.

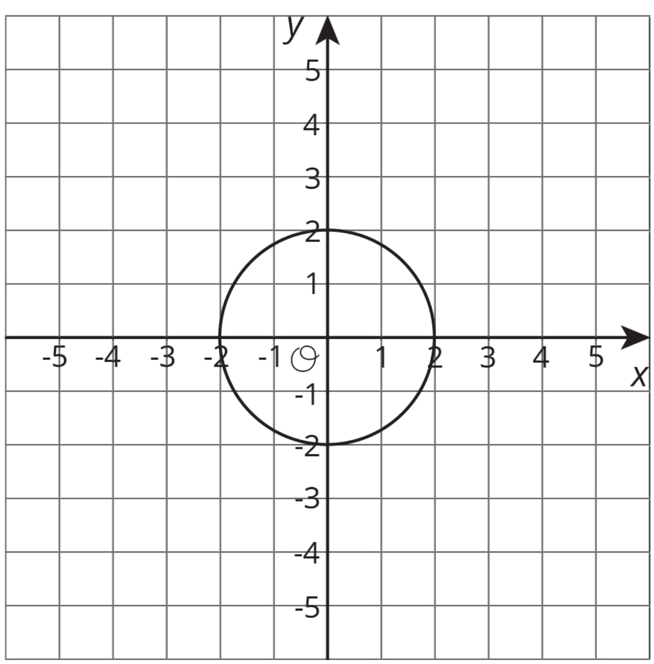
Card Sort: Matching Dilations on a Coordinate Grid

B.

The circle with center (0, 0) and radius 4.

Card Sort: Matching Dilations on a Coordinate Grid

4.



Dilate the circle using center (0,0) and scale factor $\frac{1}{2}$.

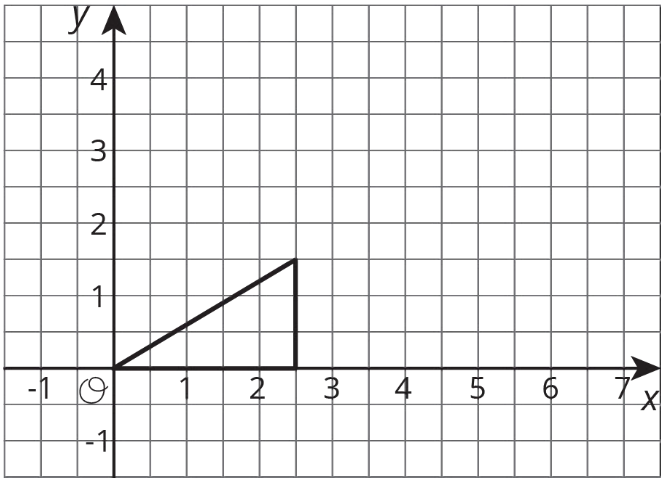
Card Sort: Matching Dilations on a Coordinate Grid

E.

The circle with center (0, 0) and radius 1.

Card Sort: Matching Dilations on a Coordinate Grid

5.

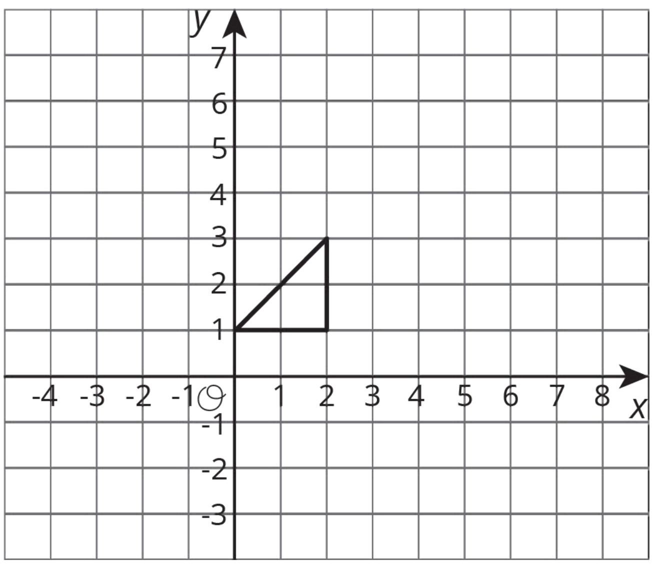


Dilate the triangle using center (0, 0) and scale factor 2.

Card Sort: Matching Dilations on a Coordinate Grid

Card Sort: Matching Dilations on a Coordinate Grid

6.



Dilate the triangle using center (-4, -3) and scale factor 1.5.

Card Sort: Matching Dilations on a Coordinate Grid

D.

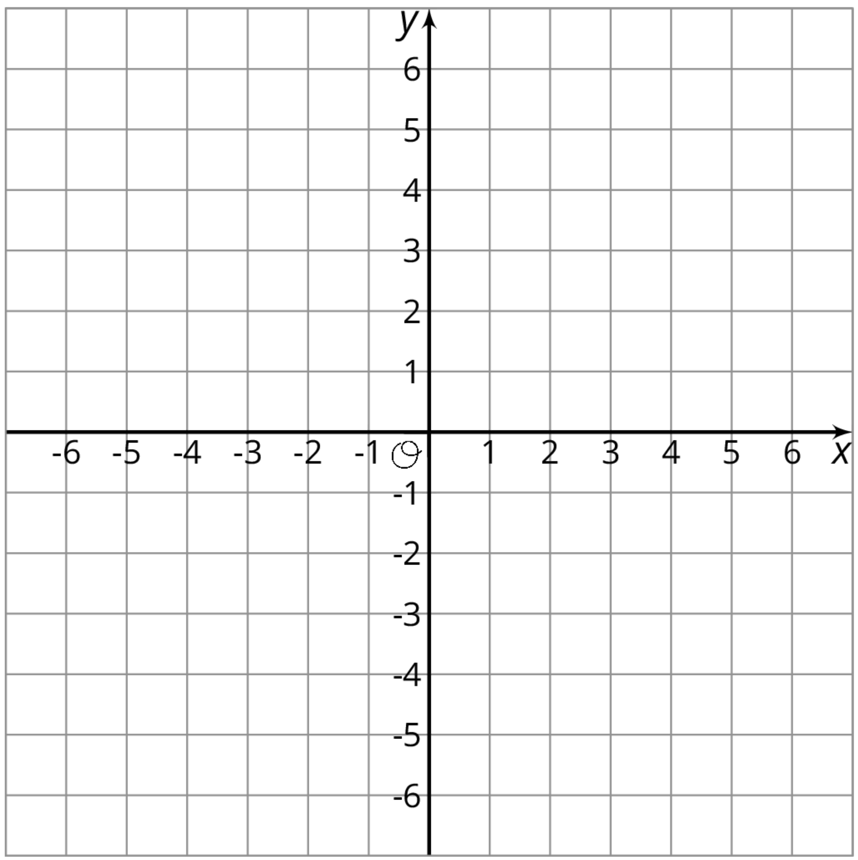
The polygon with vertices at (2, 3), (5, 3), and (5, 6).

Info Gap: Dilations

Problem Card 1

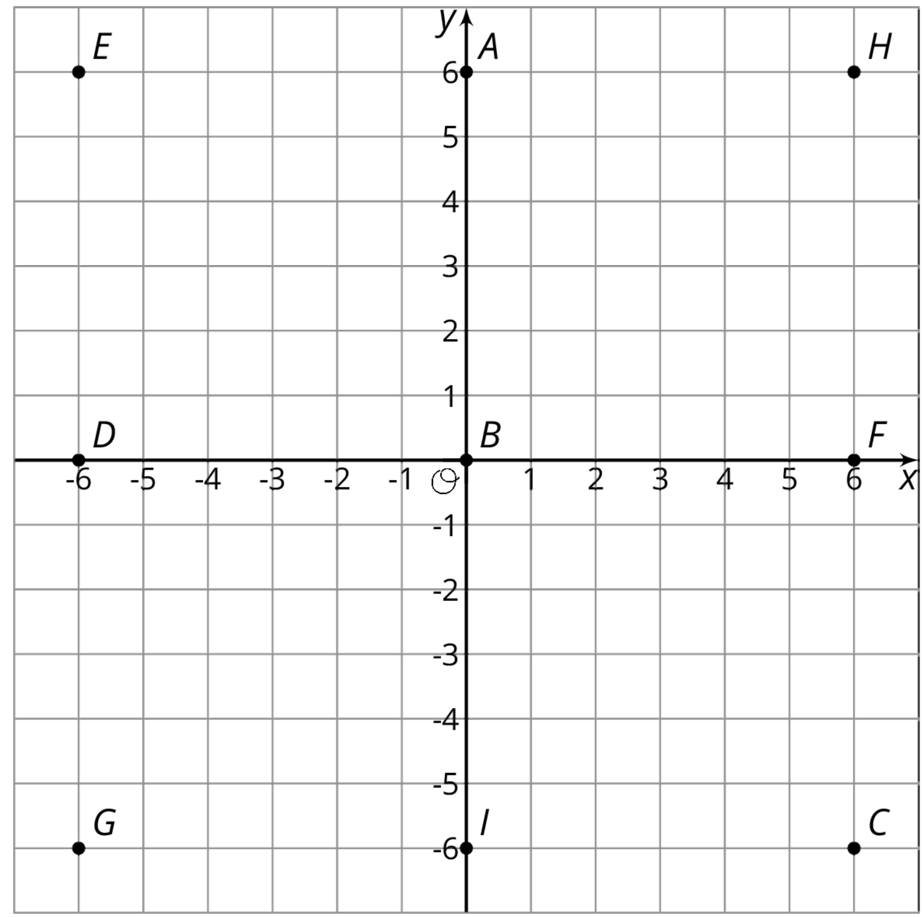
Polygon *AFID* is dilated.

Draw the image of *AFID* under this dilation.



Info Gap: Dilations

Data Card 1



Center of Dilation: (0, 0)

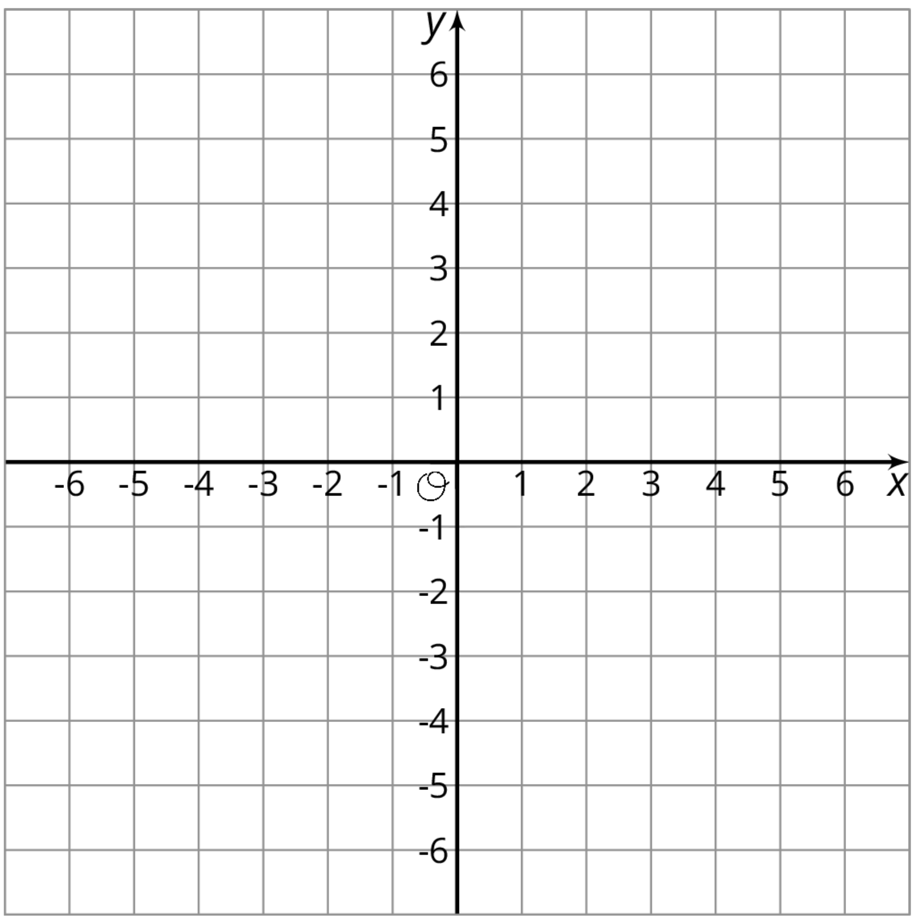
Scale Factor: $\frac{1}{3}$

Info Gap: Dilations

Problem Card 2

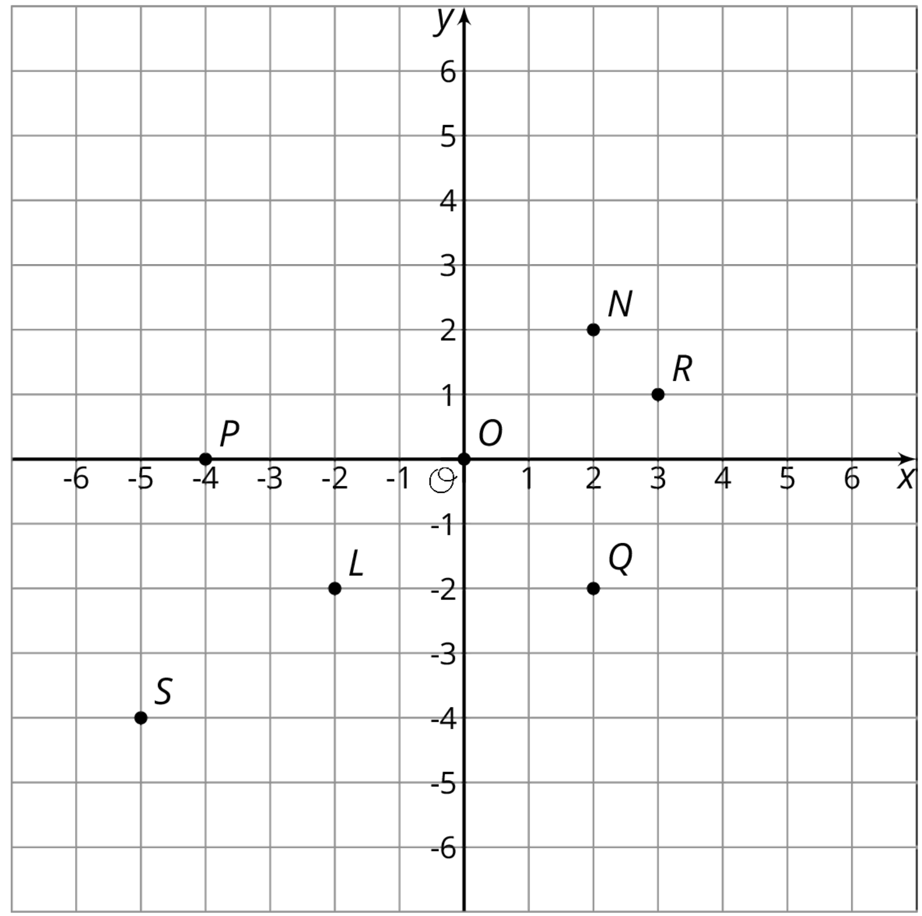
Polygon *ONPQ* is dilated.

Draw the image of *ONPQ* under this dilation.



Info Gap: Dilations

Data Card 2



Center of Dilation: (-4, 0)

Scale Factor: $\frac{3}{2}$

Methods for Translations and Dilations

Partner A

Translate from *A* to *D*

Methods for Translations and Dilations

Partner B

Translate from *D* to *A*

Methods for Translations and Dilations

Partner A

Translate from *P* to *D*

Methods for Translations and Dilations

Partner B

Translate from *D* to *P*

Methods for Translations and Dilations

Partner A

Dilate using center *A* and scale factor 3

Methods for Translations and Dilations

Partner B

Dilate using center *A* and scale factor $\frac{1}{3}$

Methods for Translations and Dilations

Partner A

Dilate using center *P* and scale factor 3

Methods for Translations and Dilations

Partner B

Dilate using center *D* and scale factor $\frac{1}{3}$

Methods for Translations and Dilations

Partner A

Dilate using center *D* and scale factor 3

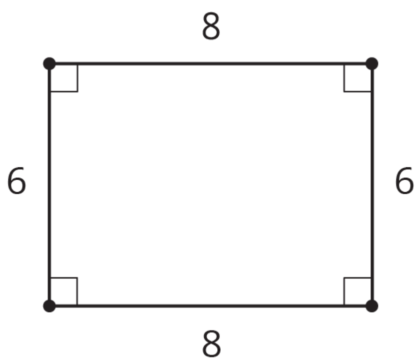
Methods for Translations and Dilations

Partner B

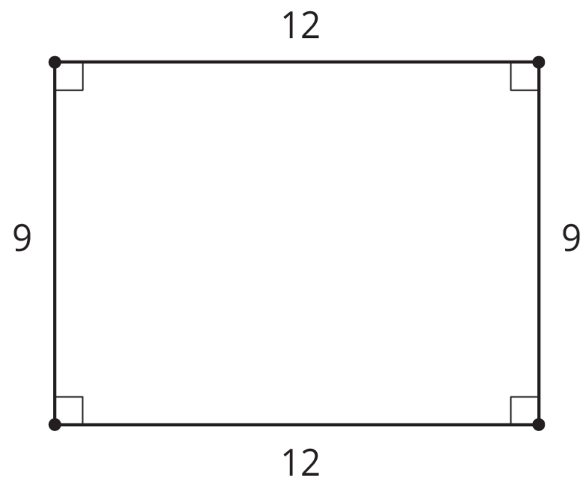
Dilate using center *P* and scale factor $\frac{1}{3}$

8.2.7.3 Find Someone Similar.

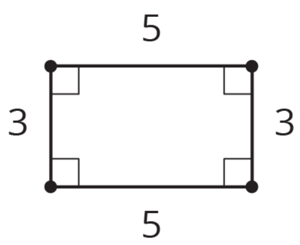
Find Someone Similar



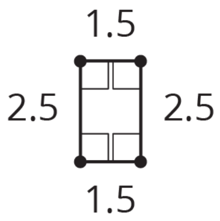
Find Someone Similar



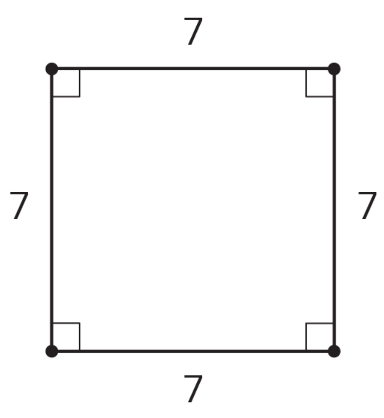
Find Someone Similar



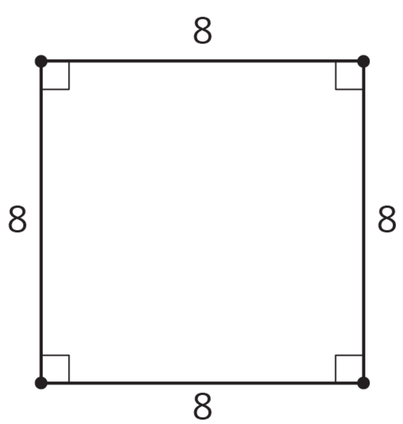
Find Someone Similar



Find Someone Similar

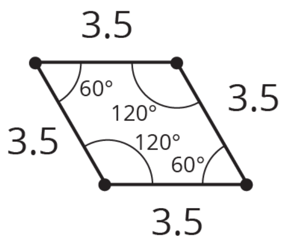


Find Someone Similar

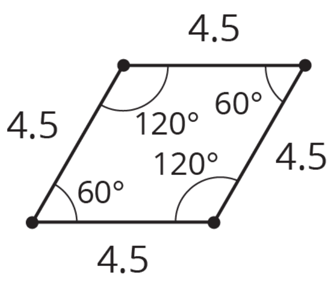


8.2.7.3 Find Someone Similar.

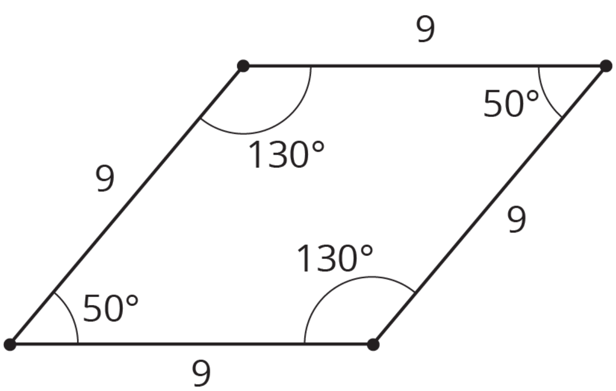
Find Someone Similar



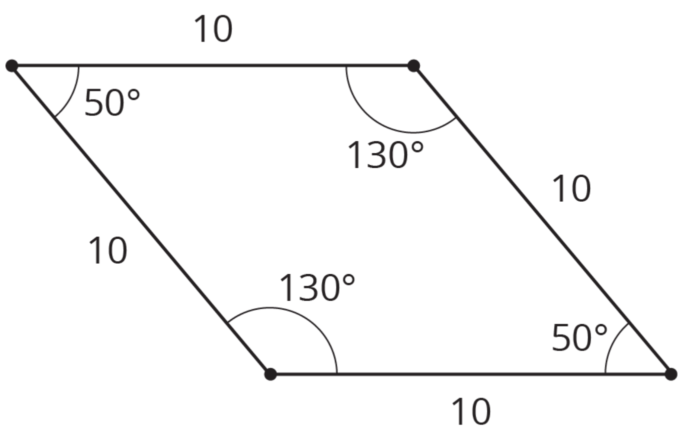
Find Someone Similar



Find Someone Similar

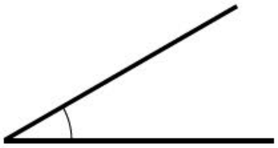


Find Someone Similar



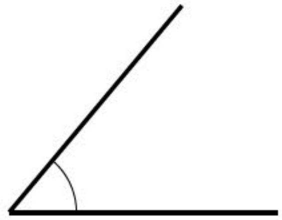
8.2.8.2 Making Pasta Angles and Triangles.

Making Pasta



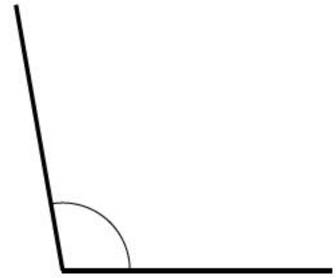
A

Making Pasta



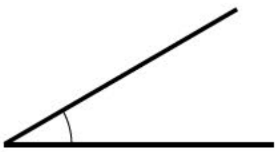
B

Making Pasta



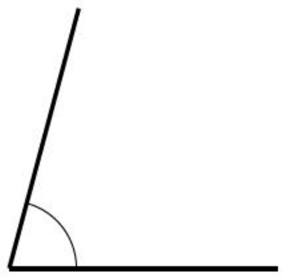
C

Making Pasta



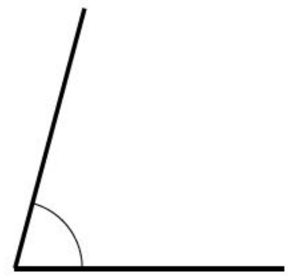
A

Making Pasta



B

Making Pasta



C

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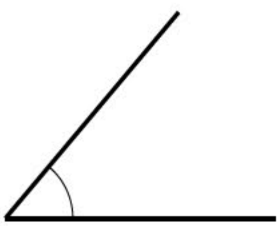
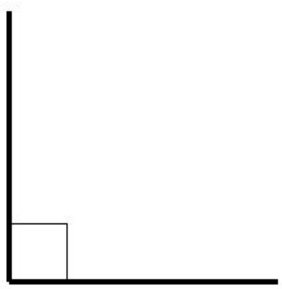


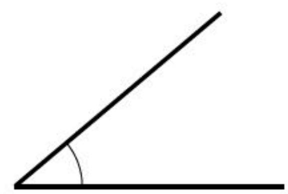
A

Making Pasta



B

Making Pasta



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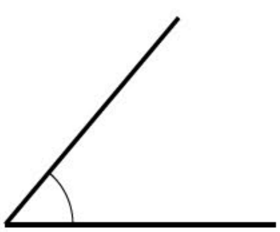
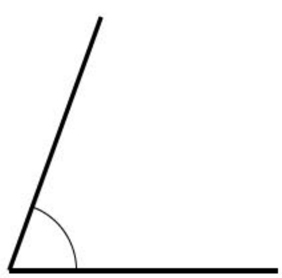


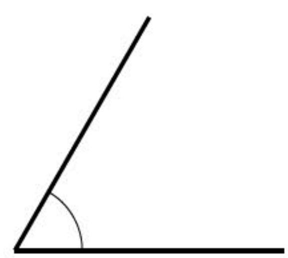
A

Making Pasta



B

Making Pasta



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