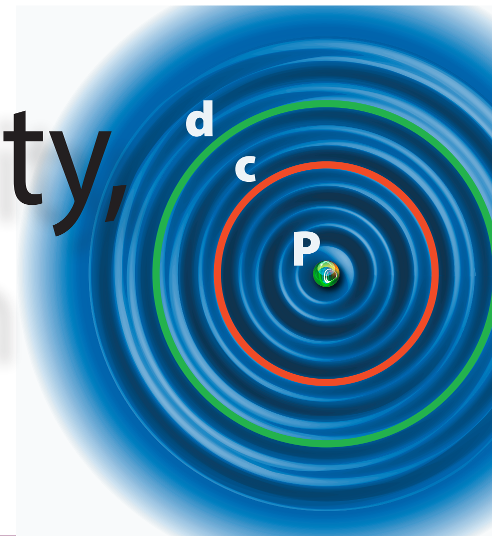
# Dilations, Similarity, and Introduction to Slope

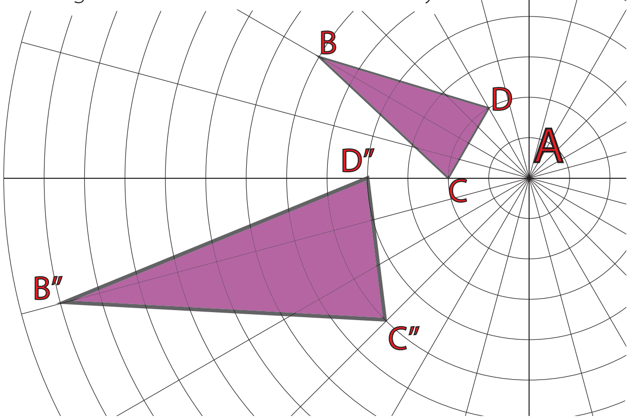
Core Knowledge® MATHEMATICS

## Student Workbook

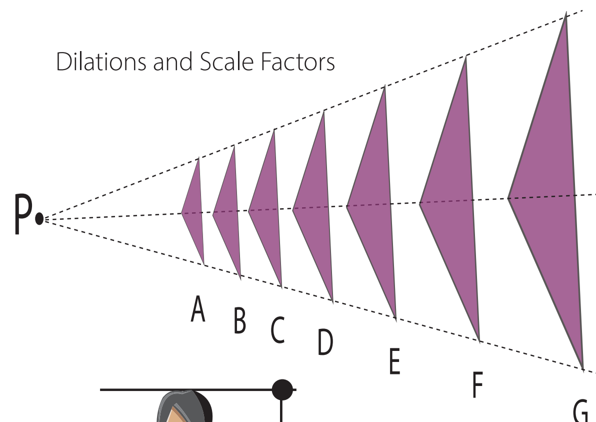
Circular Dilations



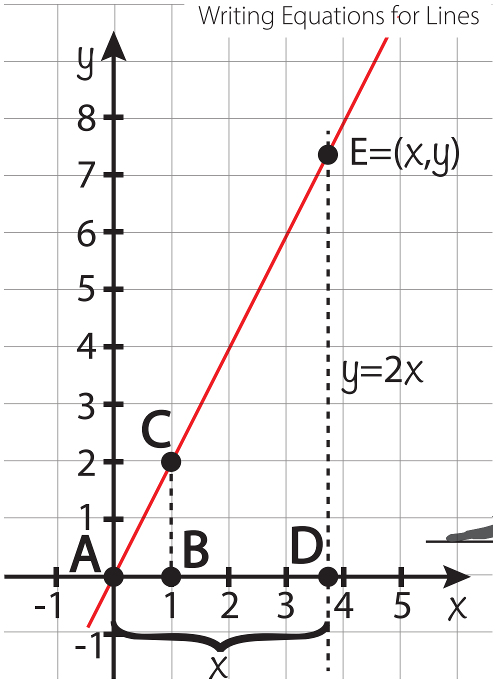

Using Evidence to Determine Similarity

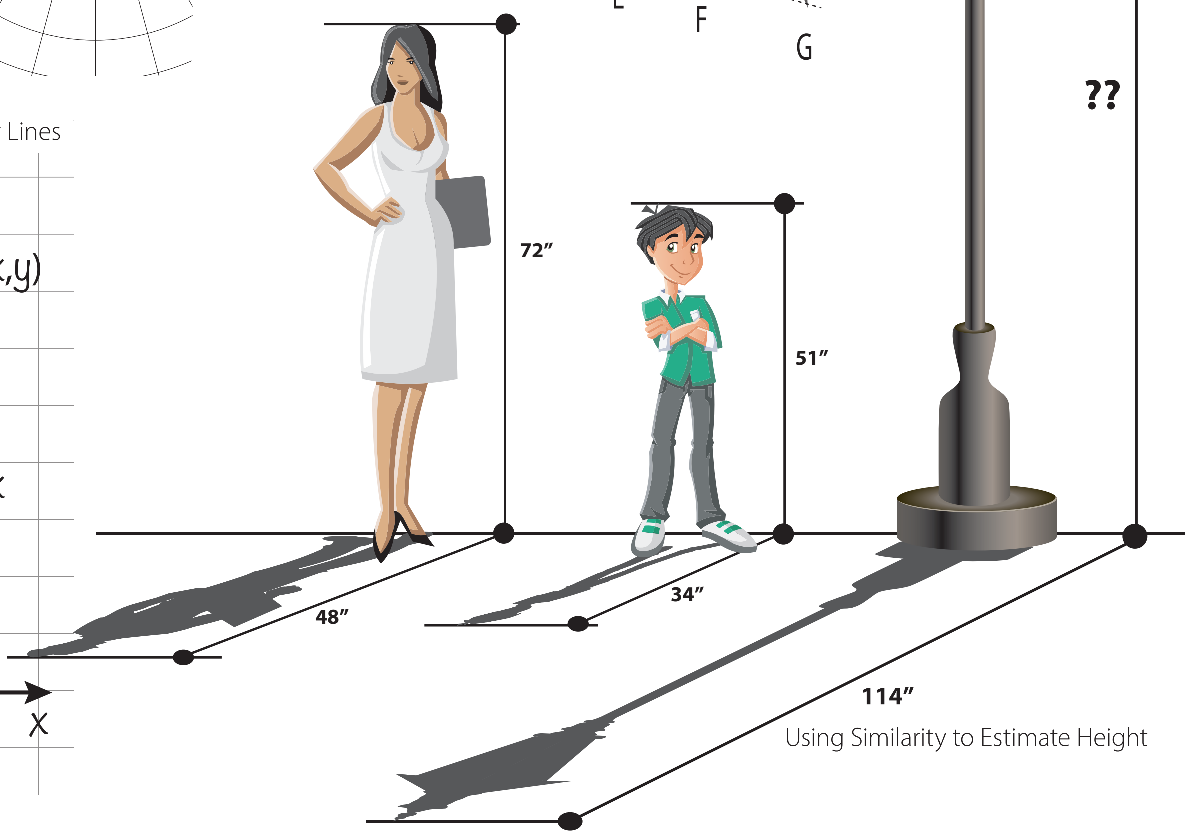



Dilations and Scale Factors




Writing Equations for Lines




Using Similarity to Estimate Height

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# Dilations, Similarity, & Introduction to Slope

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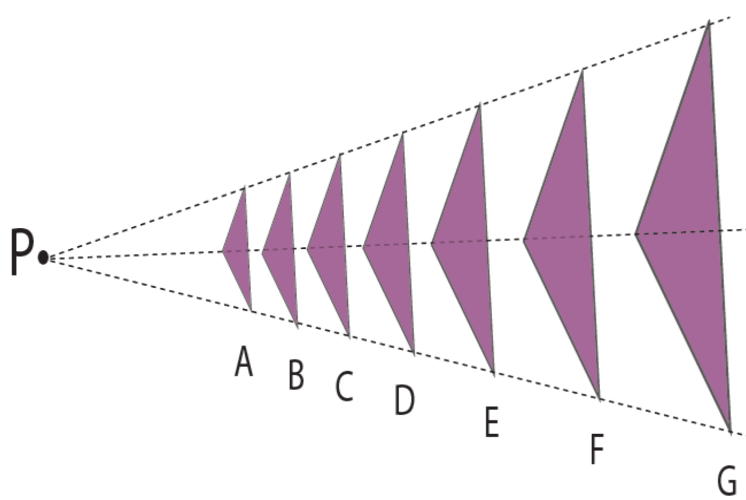



# Dilations, Similarity, and Introduction to Slope

**Student Workbook**

Core Knowledge Mathematics™

# Lesson 1: Projecting and Scaling

## 1.1: Number Talk: Remembering Fraction Division

Find each quotient. Write your answer as a fraction or a mixed number.

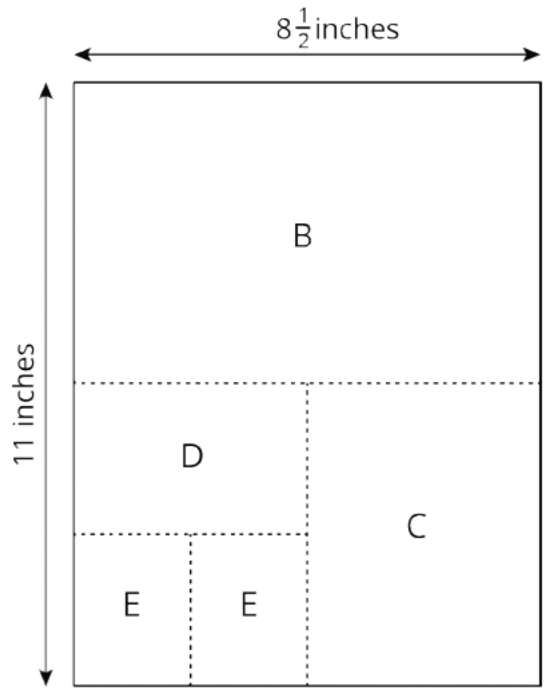
$6\frac{1}{4} \div 2$

$10\frac{1}{7} \div 5$

$8\frac{1}{2} \div 11$

## 1.2: Sorting Rectangles

Rectangles were made by cutting an $8\frac{1}{2}$-inch by 11-inch piece of paper in half, in half again, and so on, as illustrated in the diagram. Find the lengths of each rectangle and enter them in the appropriate table.



1. Some of the rectangles are scaled copies of the full sheet of paper (Rectangle A). Record the measurements of those rectangles in this table.

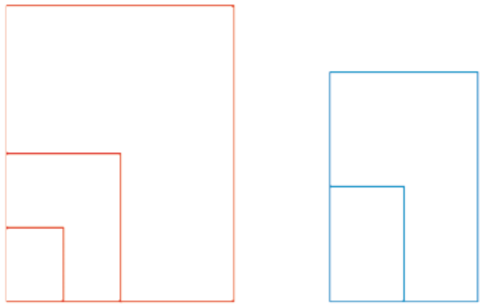
| rectangle | length of short side (inches) | length of long side (inches) |
|---|---|---|
| A | $8\frac{1}{2}$ | 11 |
| | | |
| | | |
| | | |

2. Some of the rectangles are *not* scaled copies of the full sheet of paper. Record the measurements of those rectangles in this table.

| rectangle | length of short side (inches) | length of long side (inches) |
|---|---|---|
| | | |
| | | |
| | | |

3. Look at the measurements for the rectangles that are scaled copies of the full sheet of paper. What do you notice about the measurements of these rectangles? Look at the measurements for the rectangles that are *not* scaled copies of the full sheet. What do you notice about these measurements?

4. Stack the rectangles that are scaled copies of the full sheet so that they all line up at a corner, as shown in the diagram. Do the same with the other set of rectangles. On each stack, draw a line from the bottom left corner to the top right corner of the biggest rectangle. What do you notice?



5. Stack *all* of the rectangles from largest to smallest so that they all line up at a corner. Compare the lines that you drew. Can you tell, from the drawn lines, which set each rectangle came from?

### Are you ready for more?

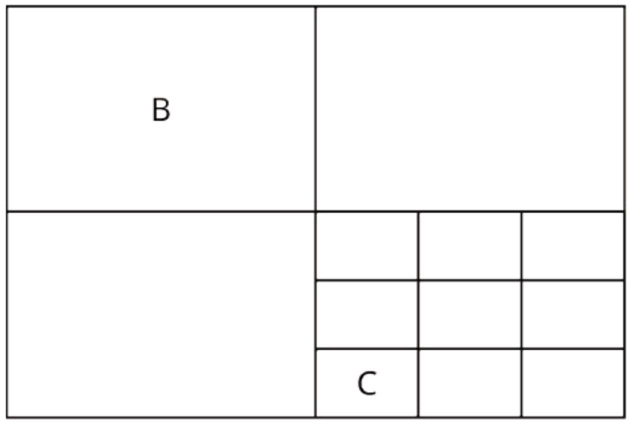
In many countries, the standard paper size is not 8.5 inches by 11 inches (called “letter” size), but instead 210 millimeters by 297 millimeters (called “A4” size). Are these two rectangle sizes scaled copies of one another?

## 1.3: Scaled Rectangles

Here is a picture of Rectangle R, which has been evenly divided into smaller rectangles. Two of the smaller rectangles are labeled B and C.

1. Is $B$ a scaled copy of $R$? If so, what is the **scale factor**?
2. Is $C$ a scaled copy of $B$? If so, what is the scale factor?
3. Is $C$ a scaled copy of $R$? If so, what is the scale factor?

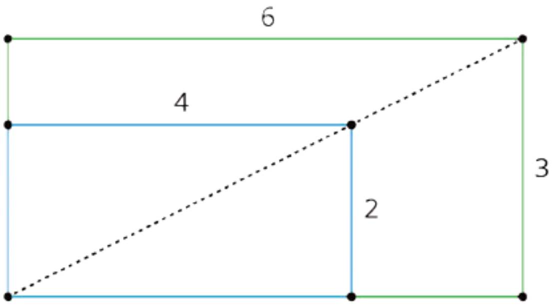
Rectangle R




### Lesson 1 Summary

Scaled copies of rectangles have an interesting property. Can you see what it is?

Here, the larger rectangle is a scaled copy of the smaller one (with a scale factor of $\frac{3}{2}$). Notice how the diagonal of the large rectangle contains the diagonal of the smaller rectangle. This is the case for any two scaled copies of a rectangle if we line them up as shown. If two rectangles are *not* scaled copies of one another, then the diagonals do not match up. In this unit, we will investigate how to make scaled copies of a figure.



# Unit 2 Lesson 1 Cumulative Practice Problems

1. Rectangle $A$ measures 12 cm by 3 cm. Rectangle $B$ is a scaled copy of Rectangle $A$. Select **all** of the measurement pairs that could be the dimensions of Rectangle $B$.

   A. 6 cm by 1.5 cm

   B. 10 cm by 2 cm

   C. 13 cm by 4 cm

   D. 18 cm by 4.5 cm

   E. 80 cm by 20 cm

2. Rectangle $A$ has length 12 and width 8. Rectangle $B$ has length 15 and width 10. Rectangle $C$ has length 30 and width 15.

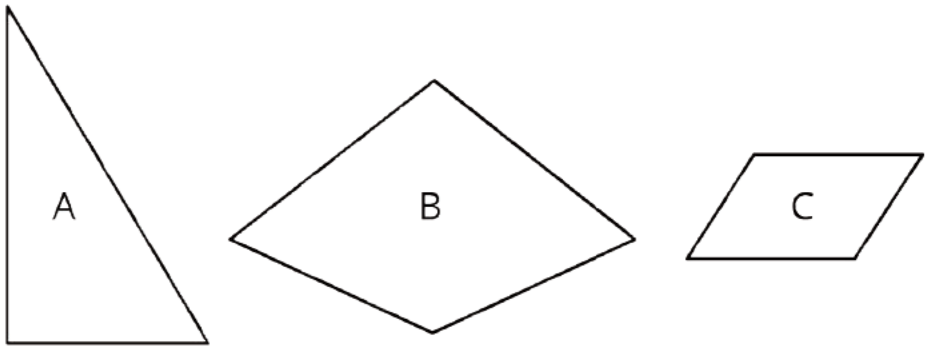
   a. Is Rectangle $A$ a scaled copy of Rectangle $B$? If so, what is the scale factor?

   b. Is Rectangle $B$ a scaled copy of Rectangle $A$? If so, what is the scale factor?

   c. Explain how you know that Rectangle $C$ is *not* a scaled copy of Rectangle $B$.

   d. Is Rectangle $A$ a scaled copy of Rectangle $C$? If so, what is the scale factor?

3. Here are three polygons.




a. Draw a scaled copy of Polygon A with scale factor $\frac{1}{2}$.

b. Draw a scaled copy of Polygon B with scale factor 2.

c. Draw a scaled copy of Polygon C with scale factor $\frac{1}{4}$.

4. Which of these sets of angle measures could be the three angles in a triangle?

   A. $40^\circ, 50^\circ, 60^\circ$

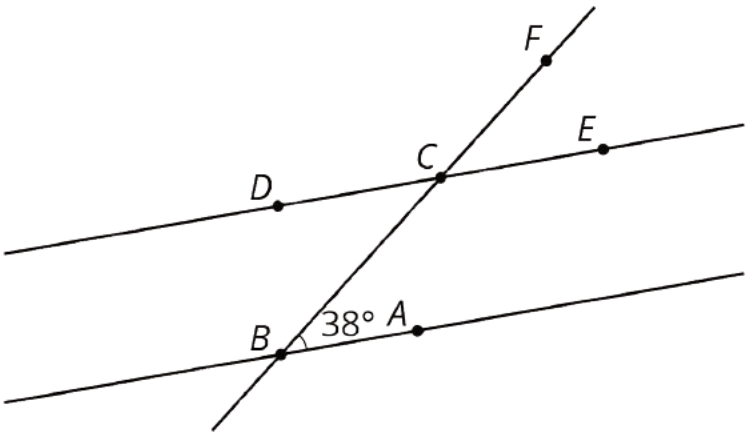
   B. $50^\circ, 60^\circ, 70^\circ$

   C. $60^\circ, 70^\circ, 80^\circ$

   D. $70^\circ, 80^\circ, 90^\circ$

(From Unit 1, Lesson 15.)

5. In the picture lines $AB$ and $CD$ are parallel. Find the measures of the following angles. Explain your reasoning.




   a. $\angle BCD$

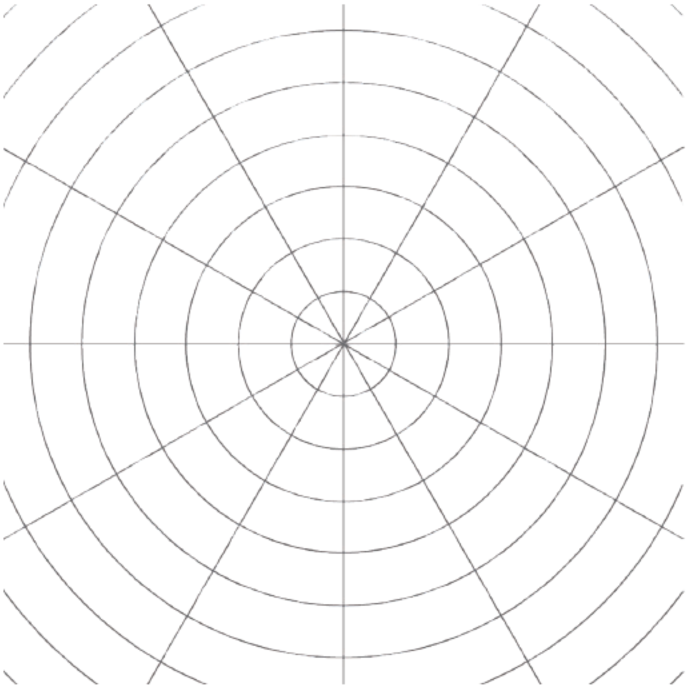
   b. $\angle ECF$

   c. $\angle DCF$

(From Unit 1, Lesson 14.)

# Lesson 2: Circular Grid

## 2.1: Notice and Wonder: Concentric Circles



What do you notice? What do you wonder?

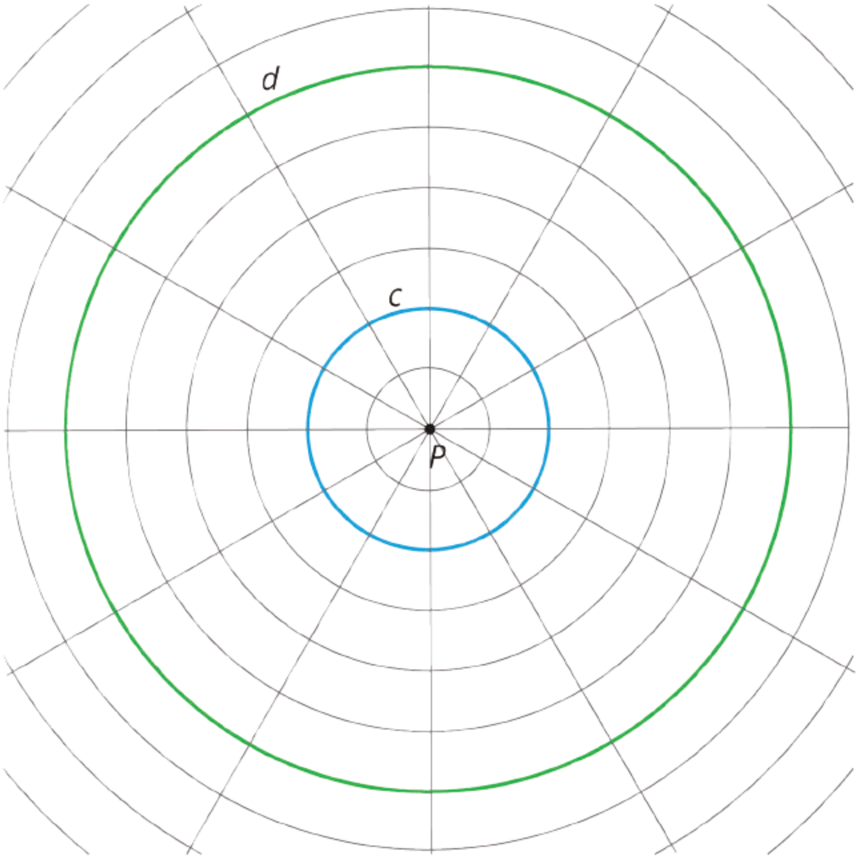
## 2.2: A Droplet on the Surface

The larger Circle d is a **dilation** of the smaller Circle c. $P$ is the **center of dilation**.

1. Draw four points *on* the smaller circle (not inside the circle!), and label them $E$, $F$, $G$, and $H$.

2. Draw the rays from $P$ through each of those four points.

3. Label the points where the rays meet the larger circle $E'$, $F'$, $G'$, and $H'$.



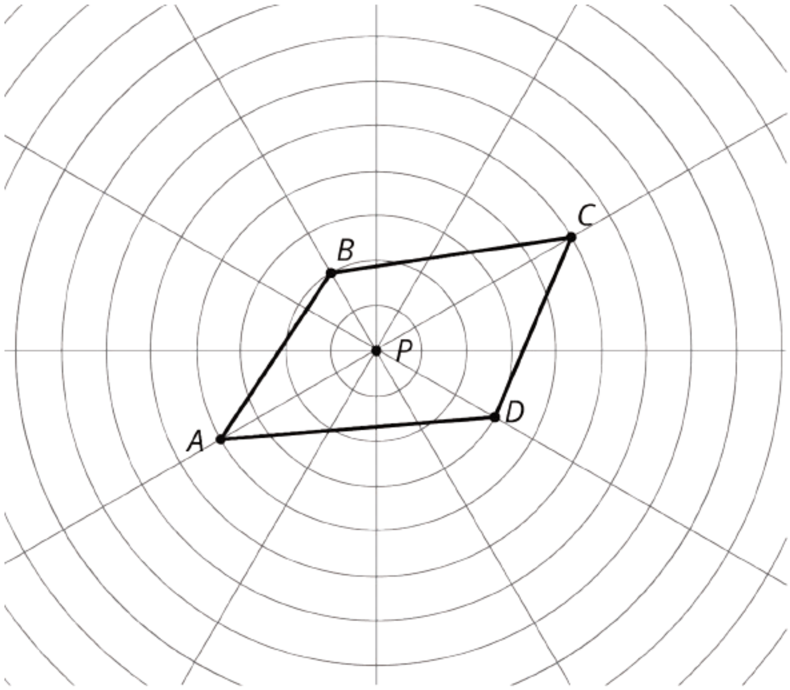

4. Complete the table. In the row labeled c, write the distance between $P$ and the point on the smaller circle in grid units. In the row labeled d, write the distance between $P$ and the corresponding point on the larger circle in grid units.

| | $E$ | $F$ | $G$ | $H$ |
|---|---|---|---|---|
| c | | | | |
| d | | | | |

5. The center of dilation is point $P$. What is the *scale factor* that takes the smaller circle to the larger circle? Explain your reasoning.

## 2.3: Quadrilateral on a Circular Grid

Here is a polygon $ABCD$.




1. Dilate each vertex of polygon $ABCD$ using $P$ as the center of dilation and a scale factor of 2. Label the image of $A$ as $A'$, and label the images of the remaining three vertices as $B'$, $C'$, and $D'$.

2. Draw segments between the dilated points to create polygon $A'B'C'D'$.

3. What are some things you notice about the new polygon?

4. Choose a few more points on the sides of the original polygon and transform them using the same dilation. What do you notice?

5. Dilate each vertex of polygon $ABCD$ using $P$ as the center of dilation and a scale factor of $\frac{1}{2}$. Label the image of $A$ as $E$, the image of $B$ as $F$, the image of $C$ as $G$ and the image of $D$ as $H$.

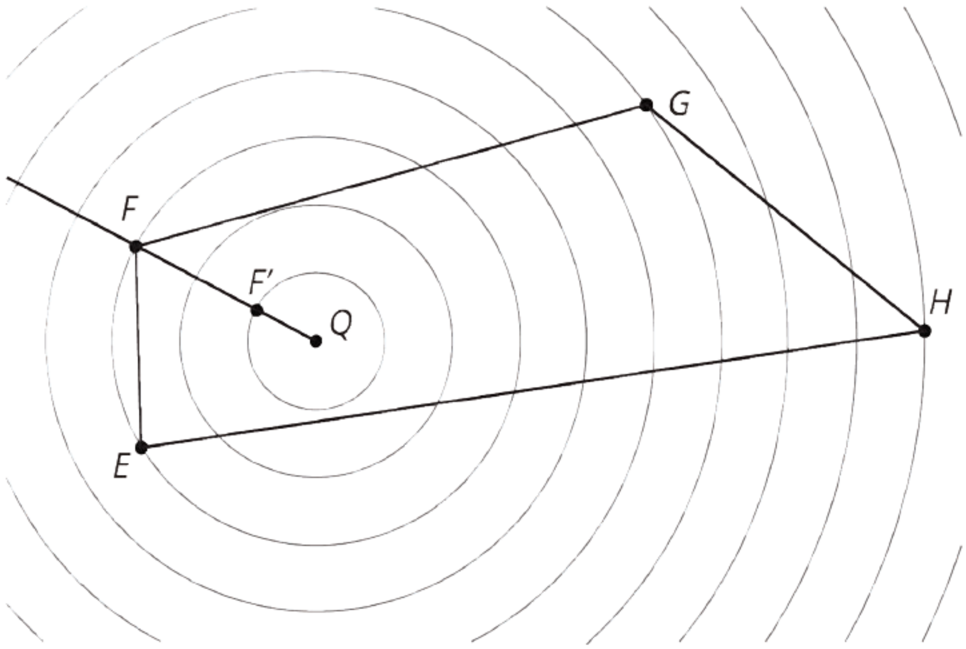
6. What do you notice about polygon $EFGH$?

**Are you ready for more?**

Suppose $P$ is a point not on line segment $\overline{WX}$. Let $\overline{YZ}$ be the dilation of line segment $\overline{WX}$ using $P$ as the center with scale factor 2. Experiment using a circular grid to make predictions about whether each of the following statements must be true, might be true, or must be false.

1. $\overline{YZ}$ is twice as long $\overline{WX}$.
2. $\overline{YZ}$ is five units longer than $\overline{WX}$.
3. The point $P$ is on $\overline{YZ}$.
4. $\overline{YZ}$ and $\overline{WX}$ intersect.

## 2.4: A Quadrilateral and Concentric Circles

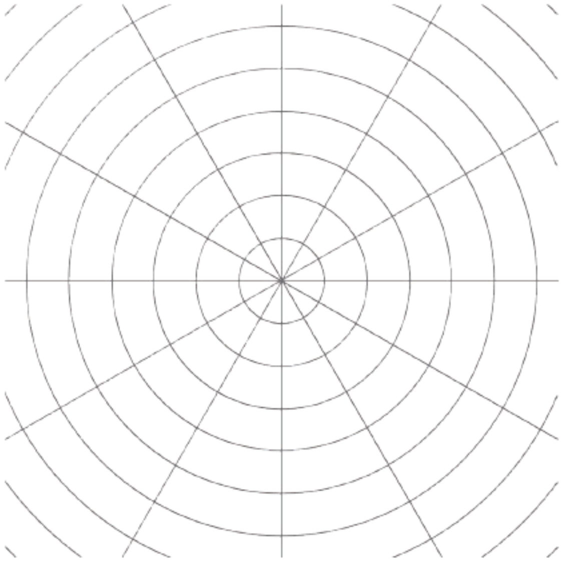



Dilate polygon $EFGH$ using $Q$ as the center of dilation and a scale factor of $\frac{1}{3}$. The image of $F$ is already shown on the diagram. (You may need to draw more rays from $Q$ in order to find the images of other points.)

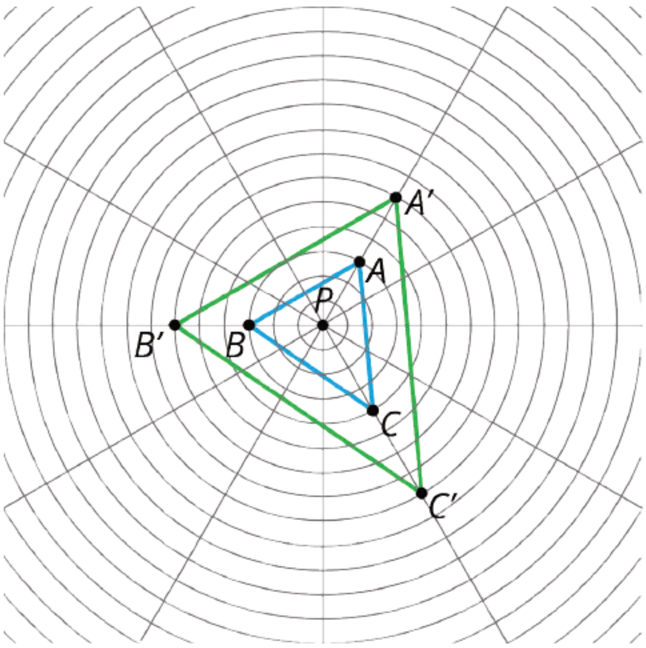
## Lesson 2 Summary

A circular grid like this one can be helpful for performing **dilations**.

The radius of the smallest circle is one unit, and the radius of each successive circle is one unit more than the previous one.



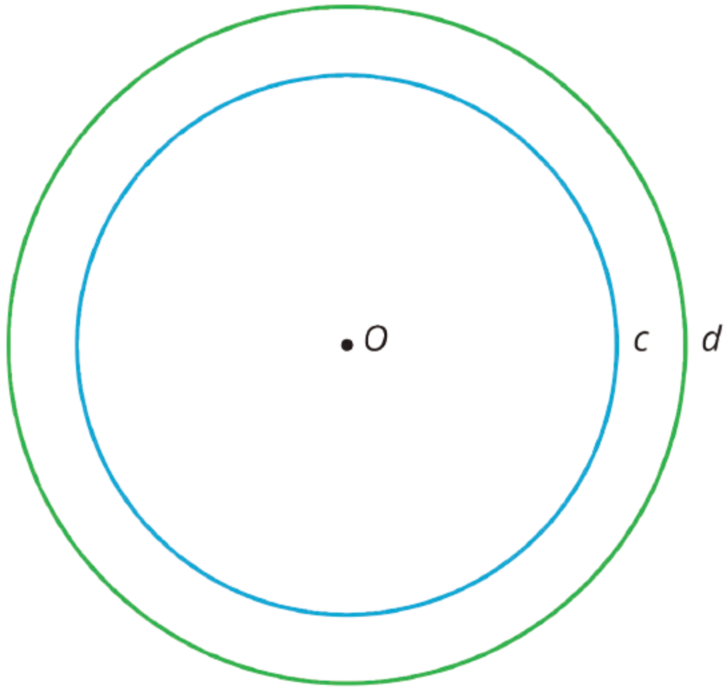
To perform a dilation, we need a **center of dilation**, a scale factor, and a point to dilate. In the picture, $P$ is the center of dilation. With a scale factor of 2, each point stays on the same ray from $P$, but its distance from $P$ doubles:




Since the circles on the grid are the same distance apart, segment $PA'$ has twice the length of segment $PA$, and the same holds for the other points.

# Unit 2 Lesson 2 Cumulative Practice Problems

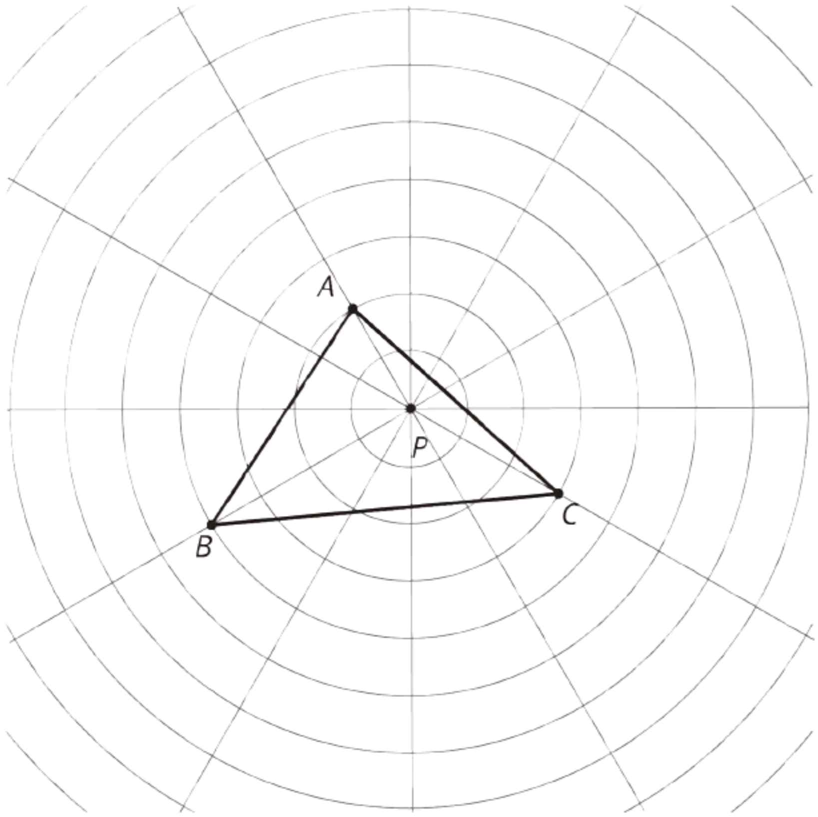
1. Here are Circles $c$ and $d$. Point $O$ is the center of dilation, and the dilation takes Circle $c$ to Circle $d$.




   a. Plot a point on Circle $c$. Label the point $P$. Plot where $P$ goes when the dilation is applied.

   b. Plot a point on Circle $d$. Label the point $Q$. Plot a point that the dilation takes to $Q$.

2. Here is triangle $ABC$.



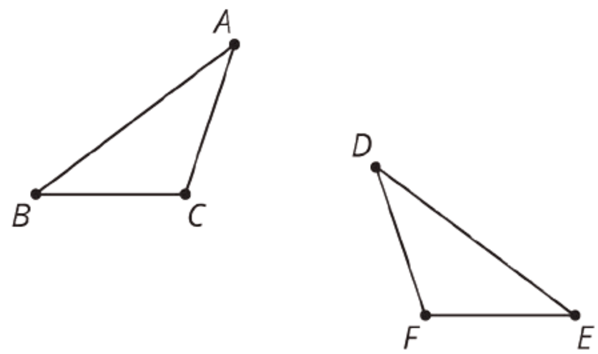

a. Dilate each vertex of triangle $ABC$ using $P$ as the center of dilation and a scale factor of 2. Draw the triangle connecting the three new points.

b. Dilate each vertex of triangle $ABC$ using $P$ as the center of dilation and a scale factor of $\frac{1}{2}$. Draw the triangle connecting the three new points.

c. Measure the longest side of each of the three triangles. What do you notice?

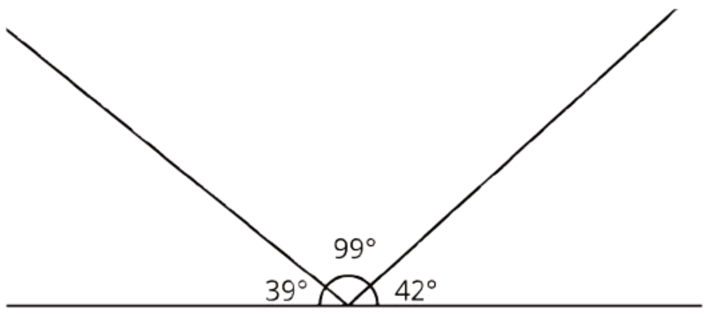
d. Measure the angles of each triangle. What do you notice?

3. Describe a rigid transformation that you could use to show the polygons are congruent.




(From Unit 1, Lesson 12.)

4. The line has been partitioned into three angles.




Is there a triangle with these three angle measures? Explain.

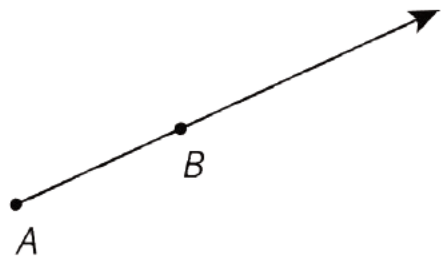
(From Unit 1, Lesson 15.)

# Lesson 3: Dilations with no Grid

## 3.1: Points on a Ray

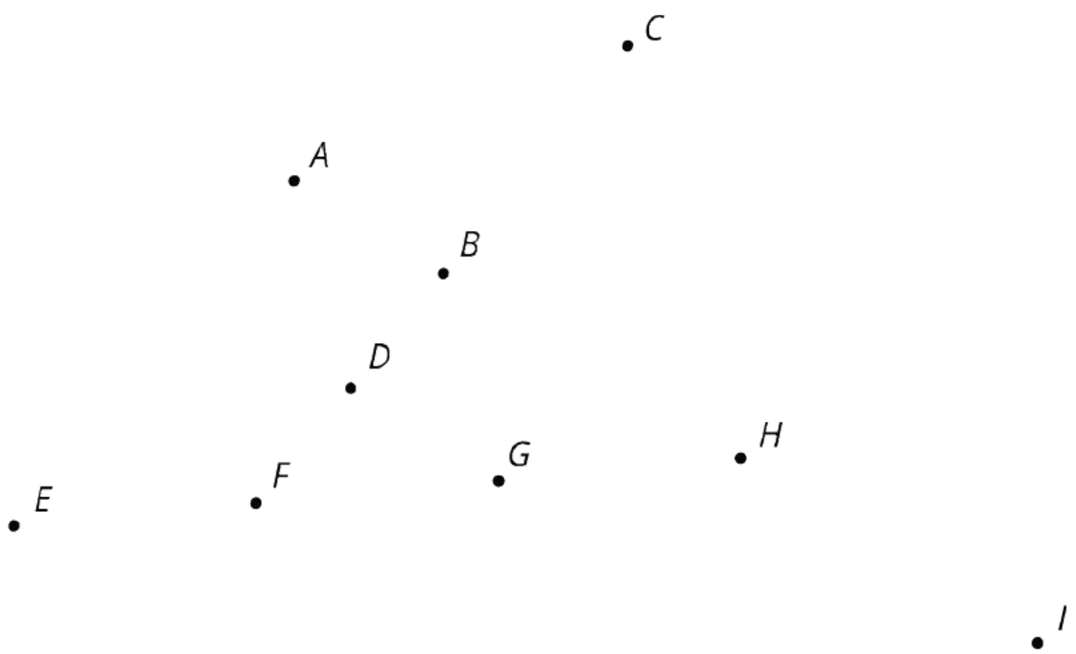
1. Find and label a point $C$ on the ray whose distance from $A$ is twice the distance from $B$ to $A$.

2. Find and label a point $D$ on the ray whose distance from $A$ is half the distance from $B$ to $A$.




## 3.2: Dilation Obstacle Course

Here is a diagram that shows nine points.




1. Dilate $B$ using a scale factor of 5 and $A$ as the center of dilation. Which point is its image?

2. Using $H$ as the center of dilation, dilate $G$ so that its image is $E$. What scale factor did you use?

3. Using $H$ as the center of dilation, dilate $E$ so that its image is $G$. What scale factor did you use?

4. To dilate $F$ so that its image is $B$, what point on the diagram can you use as a center?

5. Dilate $H$ using $A$ as the center and a scale factor of $\frac{1}{3}$. Which point is its image?

6. Describe a dilation that uses a labeled point as its center and that would take $F$ to $H$.

7. Using $B$ as the center of dilation, dilate $H$ so that its image is itself. What scale factor did you use?

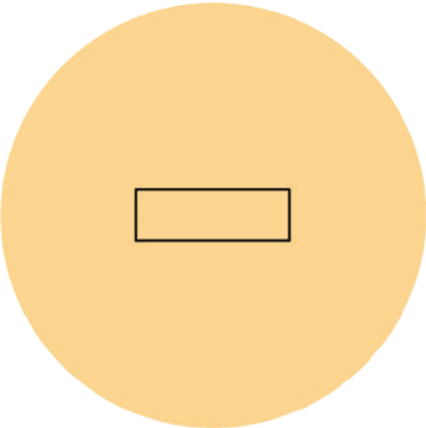
## 3.3: Getting Perspective

1. Using one colored pencil, draw the images of points $P$ and $Q$ using $C$ as the center of dilation and a scale factor of 4. Label the new points $P'$ and $Q'$.

2. Using a different color, draw the images of points $P$ and $Q$ using $C$ as the center of dilation and a scale factor of $\frac{1}{2}$. Label the new points $P''$ and $Q''$.


Pause here so your teacher can review your diagram. Your teacher will then give you a scale factor to use in the next part.

3. Now you'll make a perspective drawing. Here is a rectangle.



a. Choose a point *inside the shaded circular region* but *outside the rectangle* to use as the center of dilation. Label it $C$.

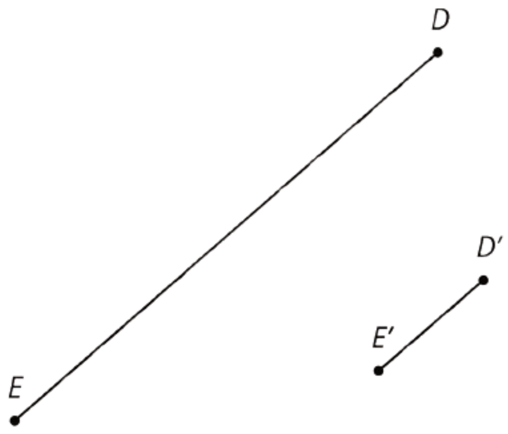
b. Using your center $C$ and the scale factor you were given, draw the image under the dilation of each vertex of the rectangle, one at a time. Connect the dilated vertices to create the dilated rectangle.

c. Draw a segment that connects each of the original vertices with its image. This will make your diagram look like a cool three-dimensional drawing of a box! If there's time, you can shade the sides of the box to make it look more realistic.

d. Compare your drawing to other people's drawings. What is the same and what is different? How do the choices you made affect the final drawing? Was your dilated rectangle closer to $C$ than to the original rectangle, or farther away? How is that decided?

### Are you ready for more?

Here is line segment $DE$ and its image $D'E'$ under a dilation.




1. Use a ruler to find and draw the center of dilation. Label it $F$.

2. What is the scale factor of the dilation?

### Lesson 3 Summary

If $A$ is the center of dilation, how can we find which point is the dilation of $B$ with scale factor 2?


Since the scale factor is larger than 1, the point must be farther away from $A$ than $B$ is, which makes $C$ the point we are looking for. If we measure the distance between $A$ and $C$, we would find that it is exactly twice the distance between $A$ and $B$.

A dilation with scale factor less than 1 brings points closer. The point $D$ is the dilation of $B$ with center $A$ and scale factor $\frac{1}{3}$.

# Unit 2 Lesson 3 Cumulative Practice Problems

1. Segment $AB$ measures 3 cm. Point $O$ is the center of dilation. How long is the image of $AB$ after a dilation with . . .

   a. Scale factor 5?

   b. Scale factor 3.7?

   c. Scale factor $\frac{1}{5}$?

   d. Scale factor $s$?

2. Here are points $A$ and $B$. Plot the points for each dilation described.


   a. $C$ is the image of $B$ using $A$ as the center of dilation and a scale factor of 2.

   b. $D$ is the image of $A$ using $B$ as the center of dilation and a scale factor of 2.

   c. $E$ is the image of $B$ using $A$ as the center of dilation and a scale factor of $\frac{1}{2}$.

   d. $F$ is the image of $A$ using $B$ as the center of dilation and a scale factor of $\frac{1}{2}$.

3. Make a perspective drawing. Include in your work the center of dilation, the shape you dilate, and the scale factor you use.

4. Triangle $ABC$ is a scaled copy of triangle $DEF$. Side $AB$ measures 12 cm and is the longest side of $ABC$. Side $DE$ measures 8 cm and is the longest side of $DEF$.

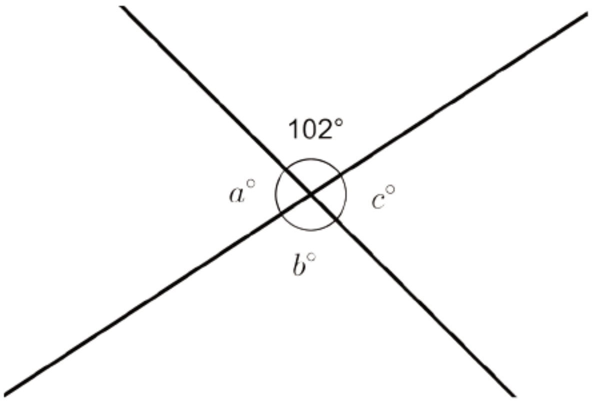
   a. Triangle $ABC$ is a scaled copy of triangle $DEF$ with what scale factor?

   b. Triangle $DEF$ is a scaled copy of triangle $ABC$ with what scale factor?

(From Unit 2, Lesson 1.)

5. The diagram shows two intersecting lines.

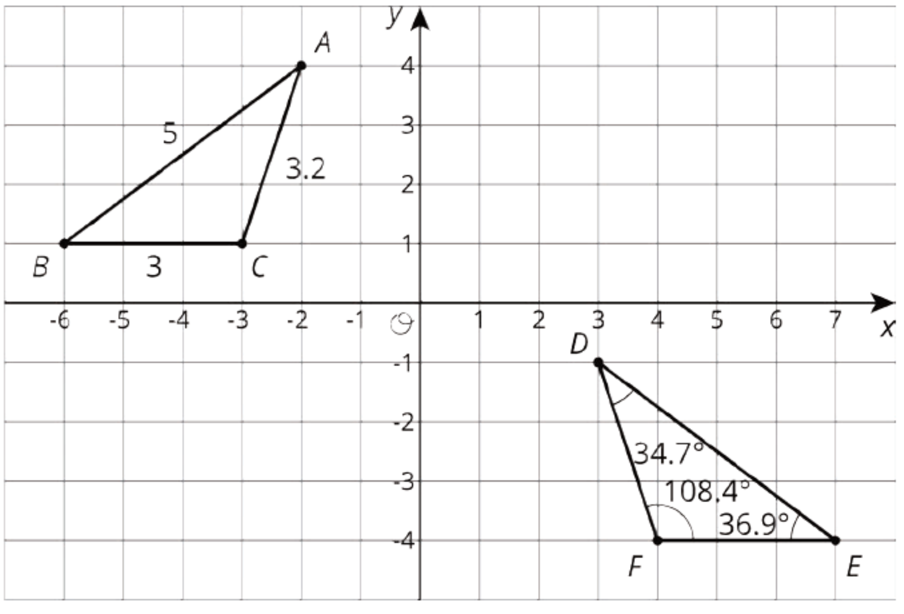
   Find the missing angle measures.




(From Unit 1, Lesson 14.)

6. a. Show that the two triangles are congruent.

   b. Find the side lengths of $DEF$ and the angle measures of $ABC$.




(From Unit 1, Lesson 12.)

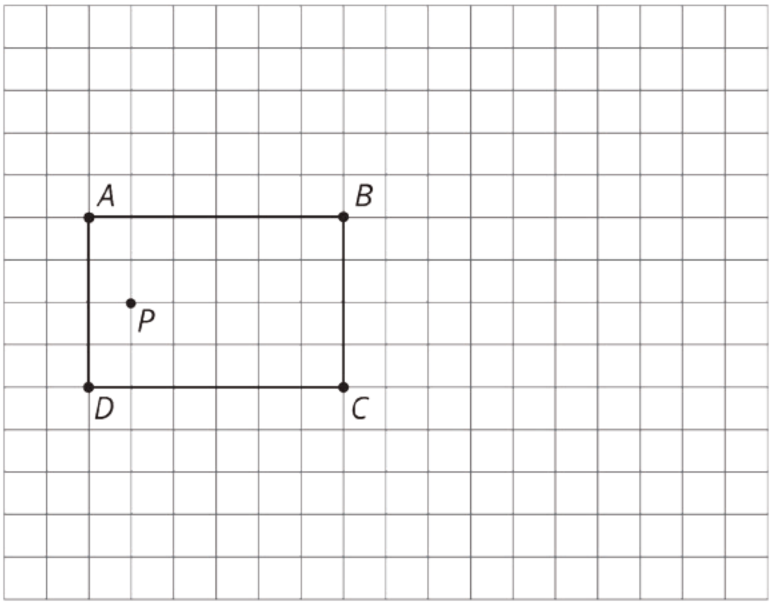
# Lesson 4: Dilations on a Square Grid

## 4.1: Estimating a Scale Factor


Point $C$ is the dilation of point $B$ with center of dilation $A$ and scale factor $s$. Estimate $s$. Be prepared to explain your reasoning.

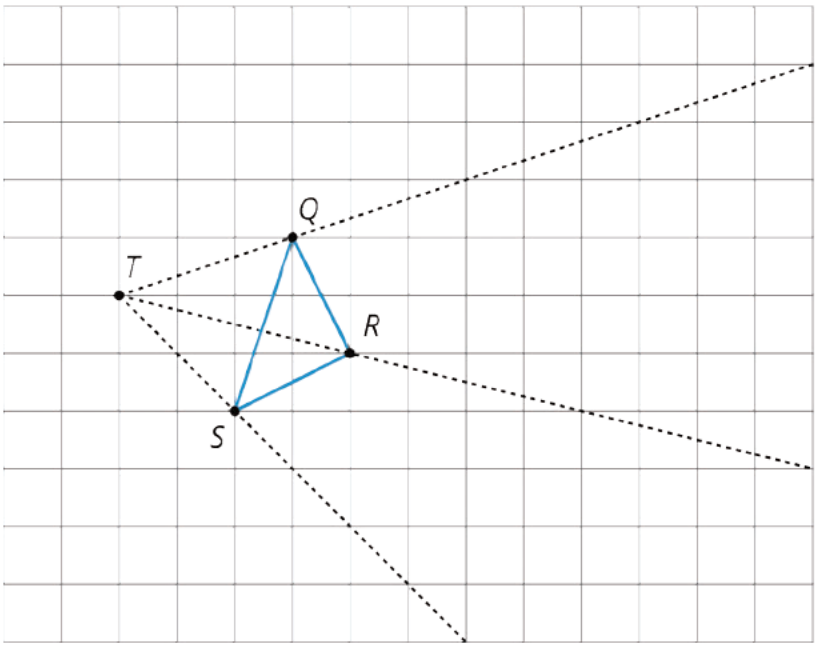
## 4.2: Dilations on a Grid

1. Find the dilation of quadrilateral $ABCD$ with center $P$ and scale factor 2.




2. Find the dilation of triangle $QRS$ with center $T$ and scale factor 2.

3. Find the dilation of triangle $QRS$ with center $T$ and scale factor $\frac{1}{2}$.



## 4.3: Card Sort: Matching Dilations on a Coordinate Grid

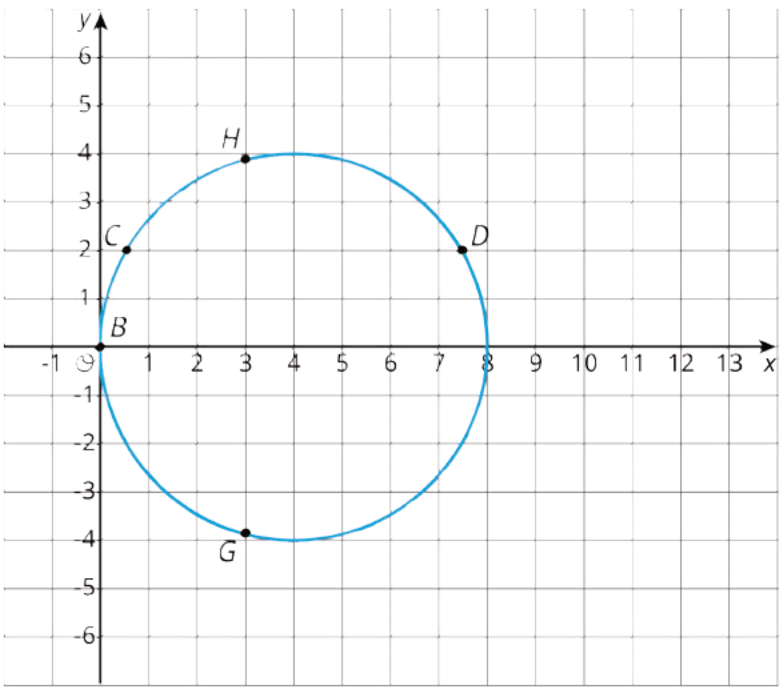
Your teacher will give you some cards. Each of Cards 1 through 6 shows a figure in the coordinate plane and describes a dilation.

Each of Cards A through E describes the image of the dilation for one of the numbered cards.

Match number cards with letter cards. One of the number cards will not have a match. For this card, you'll need to draw an image.

### Are you ready for more?

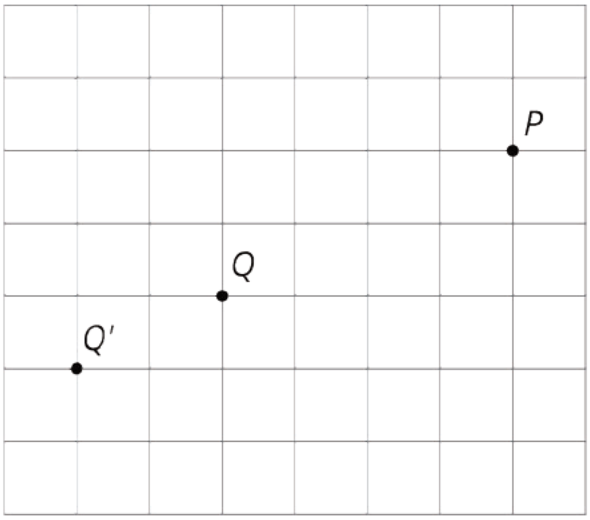
The image of a circle under dilation is a circle when the center of the dilation is the center of the circle. What happens if the center of dilation is a point on the circle? Using center of dilation $(0, 0)$ and scale factor 1.5, dilate the circle shown on the diagram. This diagram shows some points to try dilating.



## Lesson 4 Summary

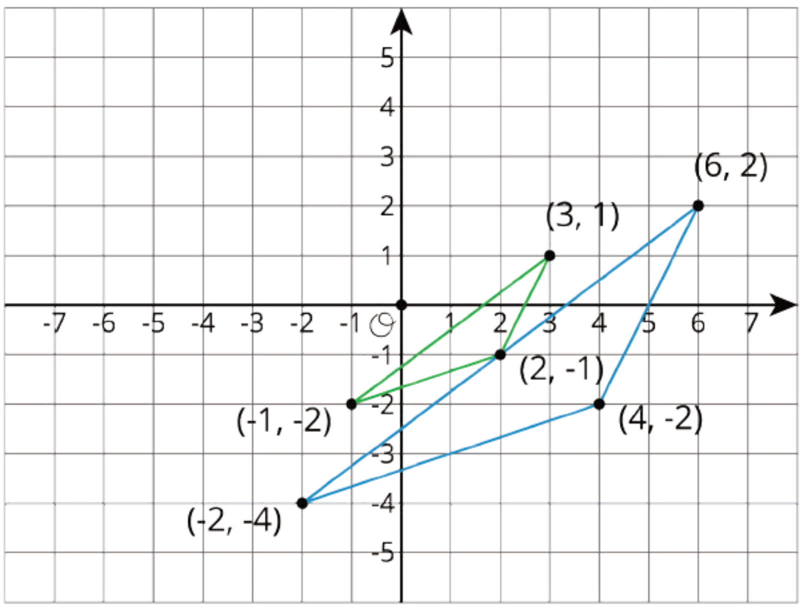
Square grids can be useful for showing dilations. The grid is helpful especially when the center of dilation and the point(s) being dilated lie at grid points. Rather than using a ruler to measure the distance between the points, we can count grid units.

For example, suppose we want to dilate point $Q$ with center of dilation $P$ and scale factor $\frac{3}{2}$. Since $Q$ is 4 grid squares to the left and 2 grid squares down from $P$, the dilation will be 6 grid squares to the left and 3 grid squares down from $P$ (can you see why?). The dilated image is marked as $Q'$ in the picture.




Sometimes the square grid comes with coordinates. The coordinate grid gives us a convenient way to *name* points, and sometimes the coordinates of the image can be found with just arithmetic.

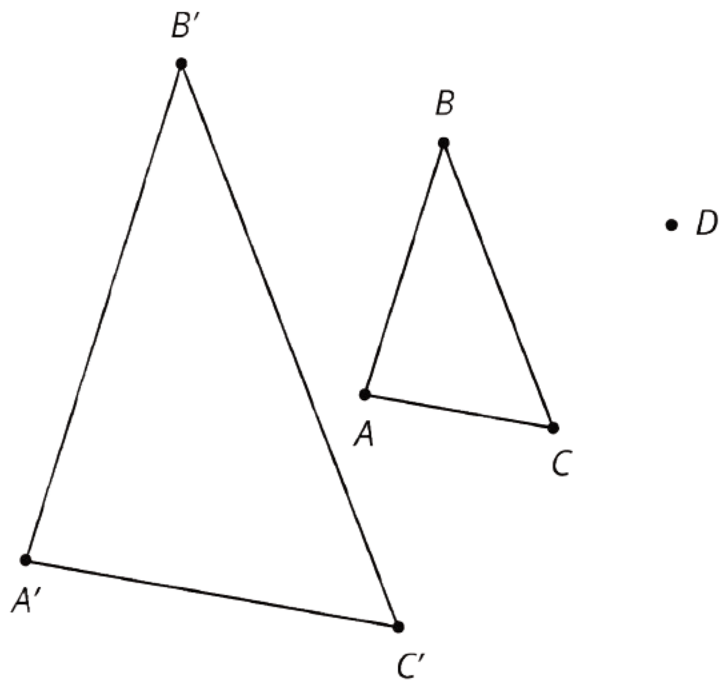
For example, to make a dilation with center $(0, 0)$ and scale factor 2 of the triangle with coordinates $(-1, -2)$, $(3, 1)$, and $(2, -1)$, we can just double the coordinates to get $(-2, -4)$, $(6, 2)$, and $(4, -2)$.



# Unit 2 Lesson 4 Cumulative Practice Problems

1. Triangle $ABC$ is dilated using $D$ as the center of dilation with scale factor 2.

   The image is triangle $A'B'C'$. Clare says the two triangles are congruent, because their angle measures are the same. Do you agree? Explain how you know.

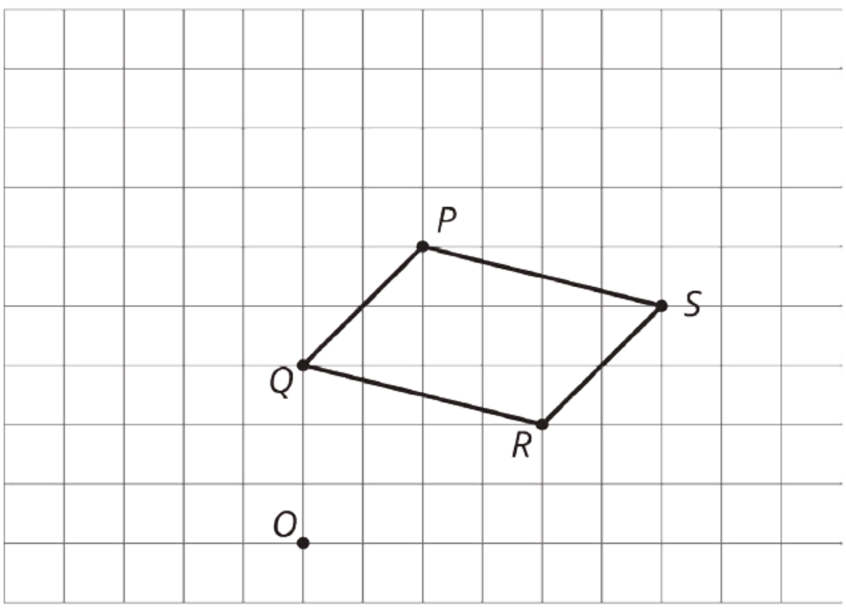



2. On graph paper, sketch the image of quadrilateral PQRS under the following dilations:

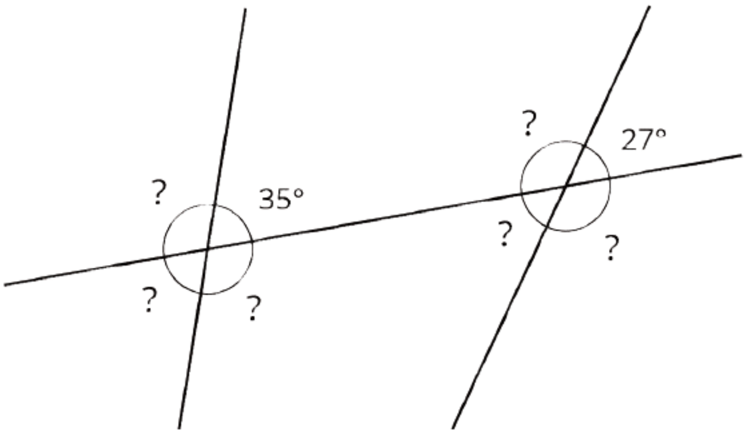
   a. The dilation centered at $R$ with scale factor 2.

   b. The dilation centered at $O$ with scale factor $\frac{1}{2}$.

   c. The dilation centered at $S$ with scale factor $\frac{1}{2}$.



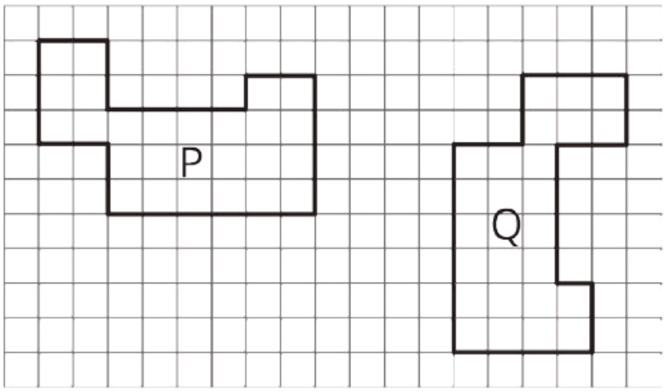
3. The diagram shows three lines with some marked angle measures.




Find the missing angle measures marked with question marks.

(From Unit 1, Lesson 14.)

4. Describe a sequence of translations, rotations, and reflections that takes Polygon P to Polygon Q.




(From Unit 1, Lesson 4.)

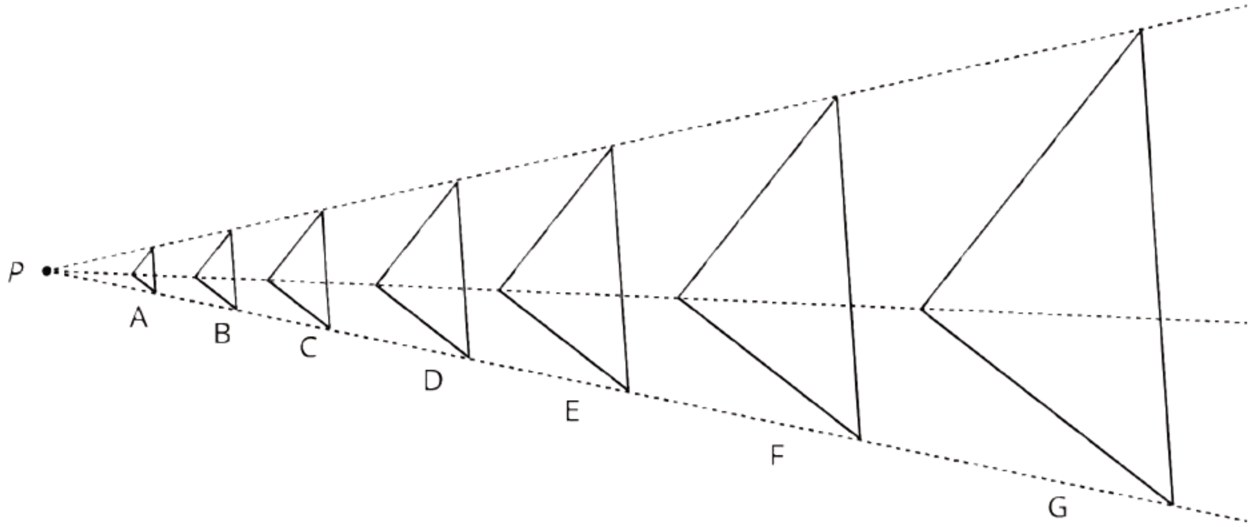
5. Point $B$ has coordinates $(-2, -5)$. After a translation 4 units down, a reflection across the $y$-axis, and a translation 6 units up, what are the coordinates of the image?

(From Unit 1, Lesson 6.)

# Lesson 5: More Dilations

## 5.1: Many Dilations of a Triangle

All of the triangles are dilations of Triangle D. The dilations use the same center $P$, but different scale factors. What do Triangles A, B, and C have in common? What do Triangles E, F, and G have in common? What does this tell us about the different scale factors used?



## 5.2: Info Gap: Dilations

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

   Continue to ask questions until you have enough information to solve the problem.
4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

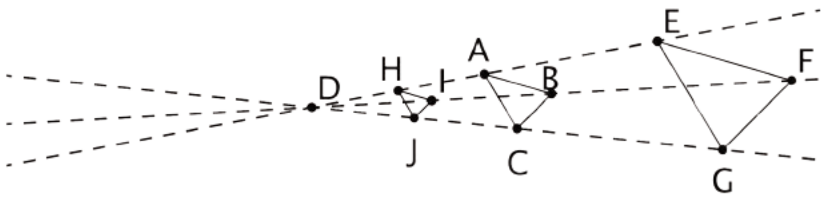
1. Silently read your card.
2. Ask your partner *"What specific information do you need?"* and wait for them to *ask* for information.

   If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don't have that information.
3. Before sharing the information, ask *"Why do you need that information?"* Listen to your partner's reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

### Are you ready for more?

Triangle $EFG$ was created by dilating triangle $ABC$ using a scale factor of 2 and center $D$. Triangle $HIJ$ was created by dilating triangle $ABC$ using a scale factor of $\frac{1}{2}$ and center $D$.




1. What would the image of triangle $ABC$ look like under a dilation with scale factor 0?

2. What would the image of the triangle look like under dilation with a scale factor of -1? If possible, draw it and label the vertices $A'$, $B'$, and $C'$. If it's not possible, explain why not.

3. If possible, describe what happens to a shape if it is dilated with a negative scale factor. If dilating with a negative scale factor is not possible, explain why not.

### Lesson 5 Summary

One important use of coordinates is to communicate geometric information precisely. Let's consider a quadrilateral $ABCD$ in the coordinate plane. Performing a dilation of $ABCD$ requires three vital pieces of information:

1. The coordinates of $A$, $B$, $C$, and $D$

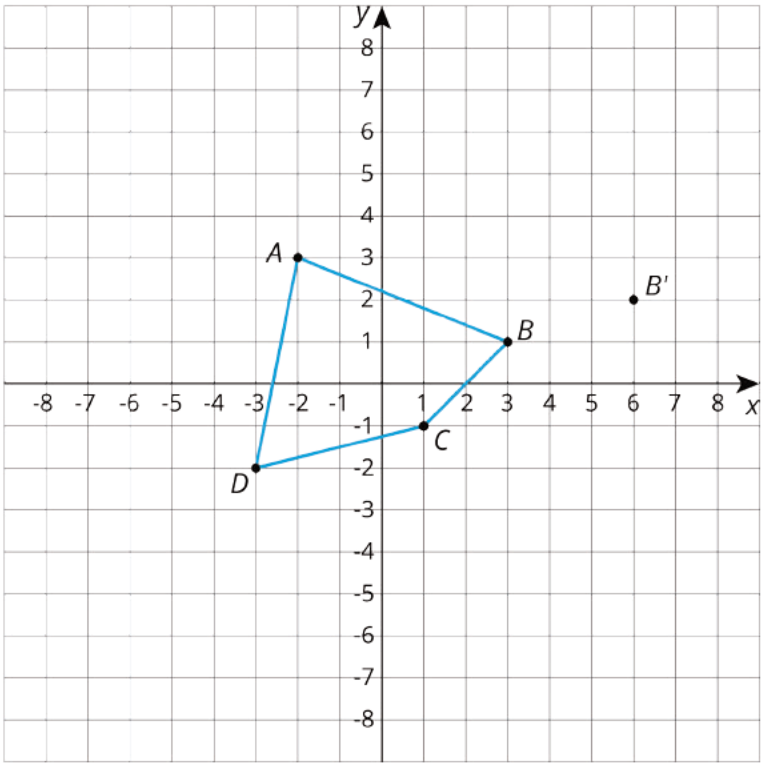
2. The coordinates of the center of dilation, $P$

3. The scale factor of the dilation

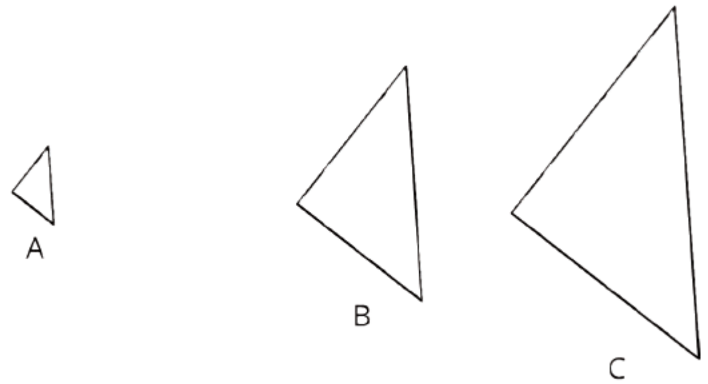
With this information, we can dilate the vertices $A$, $B$, $C$, and $D$ and then draw the corresponding segments to find the dilation of $ABCD$. Without coordinates, describing the location of the new points would likely require sharing a picture of the polygon and the center of dilation.

# Unit 2 Lesson 5 Cumulative Practice Problems

1. Quadrilateral $ABCD$ is dilated with center $(0, 0)$, taking $B$ to $B'$. Draw $A'B'C'D'$.



2. Triangles $B$ and $C$ have been built by dilating Triangle $A$.




a. Find the center of dilation.

b. Triangle $B$ is a dilation of $A$ with approximately what scale factor?

c. Triangle $A$ is a dilation of $B$ with approximately what scale factor?

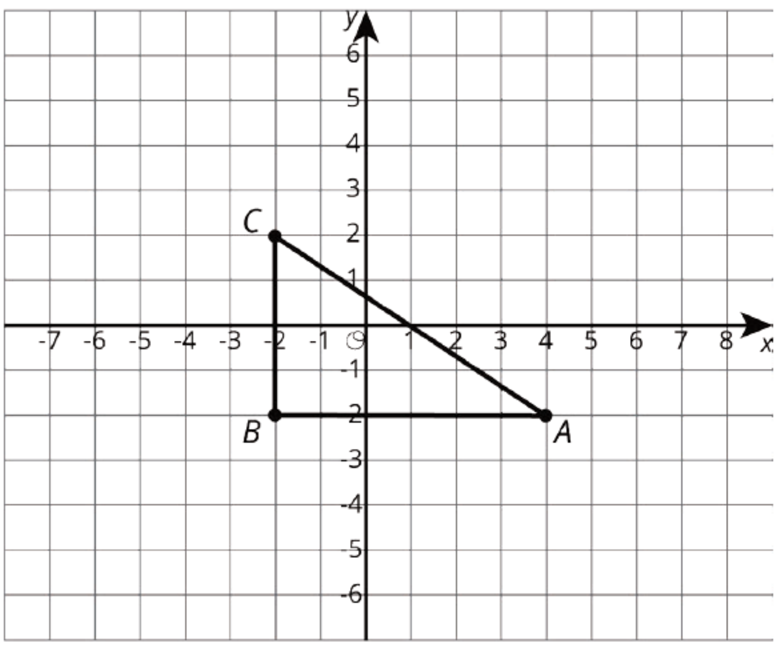
d. Triangle $B$ is a dilation of $C$ with approximately what scale factor?

3. Here is a triangle.

a. Draw the dilation of triangle $ABC$, with center $(0, 0)$, and scale factor 2. Label this triangle $A'B'C'$.

b. Draw the dilation of triangle $ABC$, with center $(0, 0)$, and scale factor $\frac{1}{2}$. Label this triangle $A''B''C''$.

c. Is $A''B''C''$ a dilation of triangle $A'B'C'$? If yes, what are the center of dilation and the scale factor?



4. Triangle $DEF$ is a right triangle, and the measure of angle $D$ is $28^\circ$. What are the measures of the other two angles?

   (From Unit 1, Lesson 15.)

# Lesson 6: Similarity

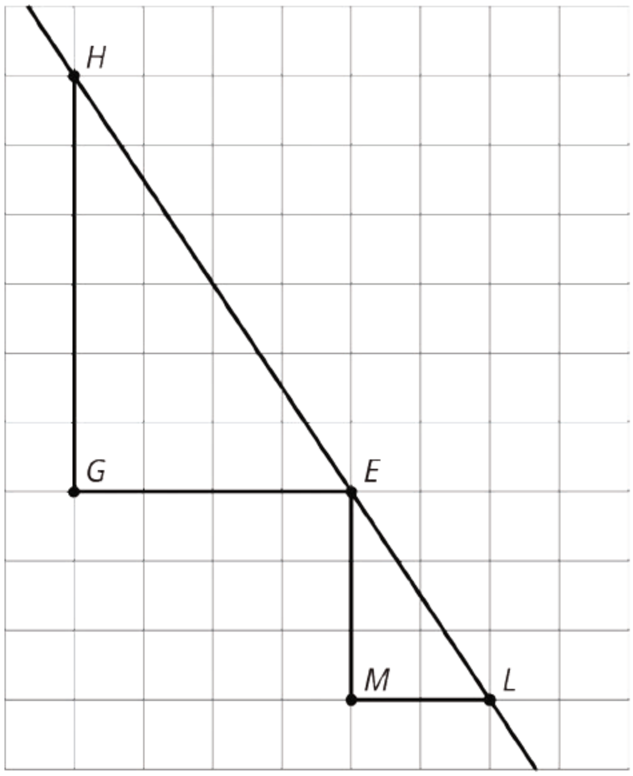
## 6.1: Equivalent Expressions

Use what you know about operations and their properties to write three expressions equivalent to the expression shown.

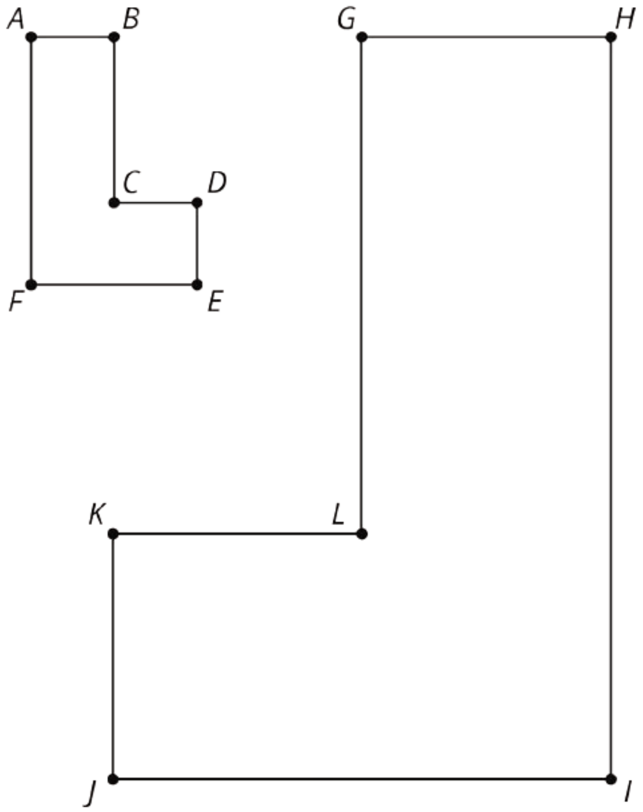
$$10(2+3) - 8 \cdot 3$$

## 6.2: Similarity Transformations (Part 1)

1. Triangle $EGH$ and triangle $LME$ are **similar**. Find a sequence of translations, rotations, reflections, and dilations that shows this.

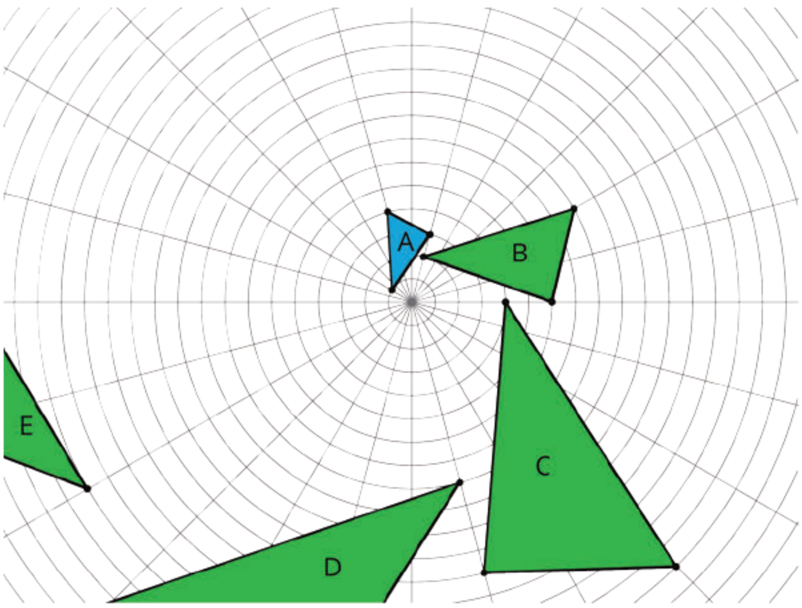


2. Hexagon $ABCDEF$ and hexagon $HGLKJI$ are similar. Find a sequence of translations, rotations, reflections, and dilations that shows this.



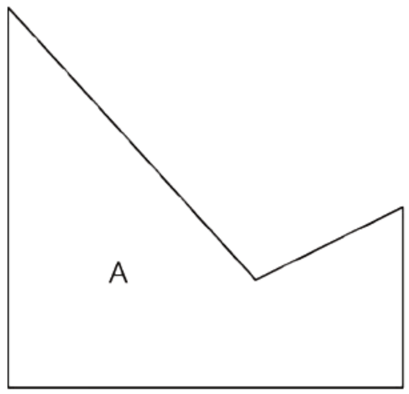

**Are you ready for more?**

The same sequence of transformations takes Triangle A to Triangle B, takes Triangle B to Triangle C, and so on. Describe a sequence of transformations with this property.



## 6.3: Similarity Transformations (Part 2)

Sketch figures similar to Figure A that use only the transformations listed to show similarity.




1. A translation and a reflection. Label your sketch Figure B.
   Pause here so your teacher can review your work.

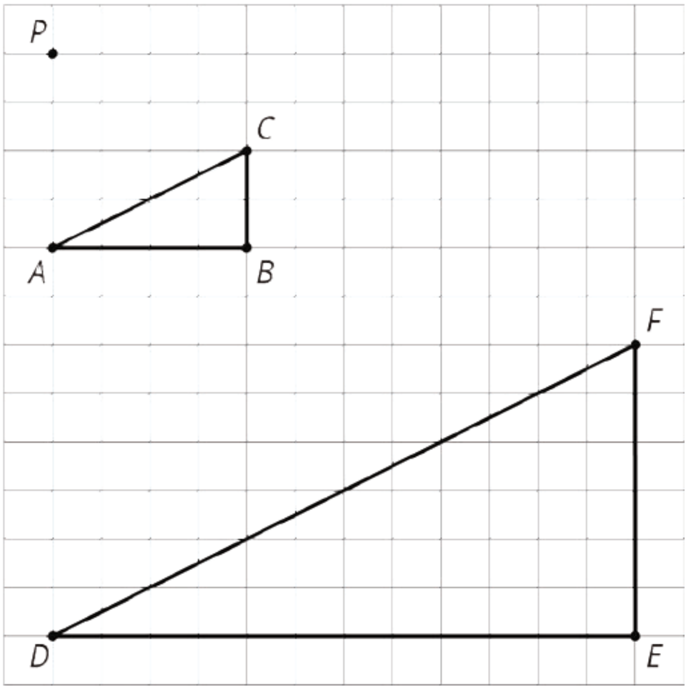
2. A reflection and a dilation with scale factor greater than 1. Label your sketch Figure C.

3. A rotation and a reflection. Label your sketch Figure D.

4. A dilation with scale factor less than 1 and a translation. Label your sketch Figure E.

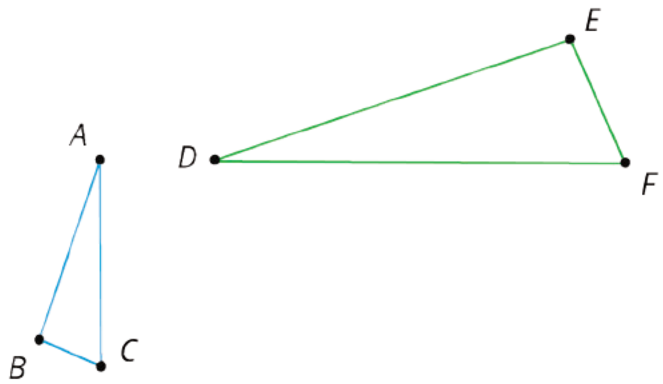
## 6.4: Methods for Translations and Dilations

Your teacher will give you a set of five cards and your partner a different set of five cards. Using only the cards you were given, find at least one way to show that triangle $ABC$ and triangle $DEF$ are similar. Compare your method with your partner's method. What is the same about your methods? What is different?



## Lesson 6 Summary

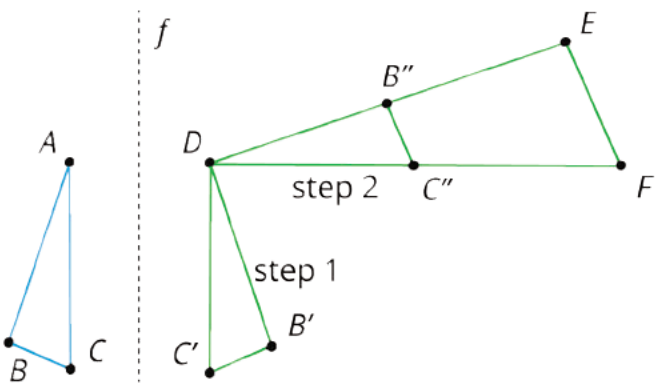
Let's show that triangle $ABC$ is similar to triangle $DEF$:




Two figures are **similar** if one figure can be transformed into the other by a sequence of translations, rotations, reflections, and dilations. There are many correct sequences of transformations, but we only need to describe one to show that two figures are similar.

One way to get from $ABC$ to $DEF$ follows these steps:

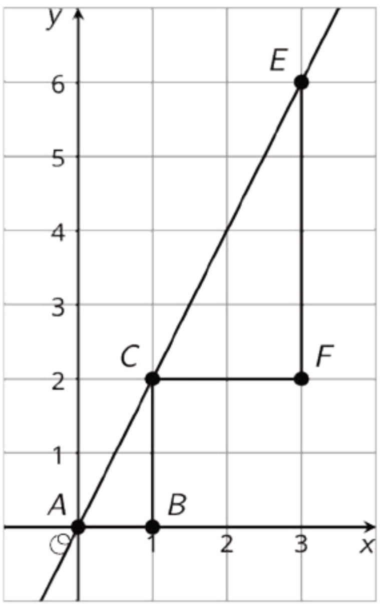
- step 1: reflect across line $f$
- step 2: rotate $90^\circ$ counterclockwise around $D$
- step 3: dilate with center $D$ and scale factor 2

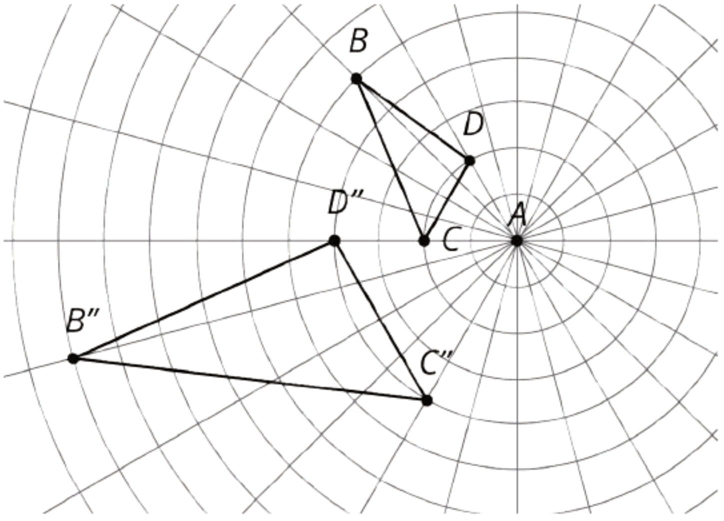



Another way would be to dilate triangle $ABC$ by a scale factor of 2 with center of dilation $A$, then translate $A$ to $D$, then reflect over a vertical line through $D$, and finally rotate it so it matches up with triangle $DEF$. What steps would you choose to show the two triangles are similar?

# Unit 2 Lesson 6 Cumulative Practice Problems

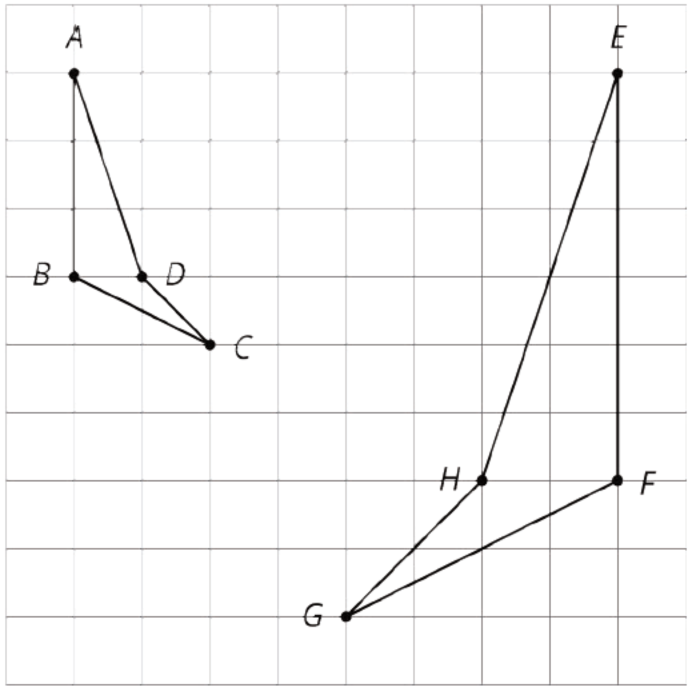
1. Each diagram has a pair of figures, one larger than the other. For each pair, show that the two figures are similar by identifying a sequence of translations, rotations, reflections, and dilations that takes the smaller figure to the larger one.



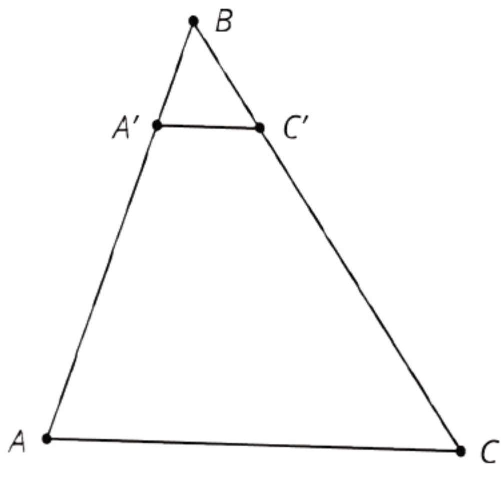



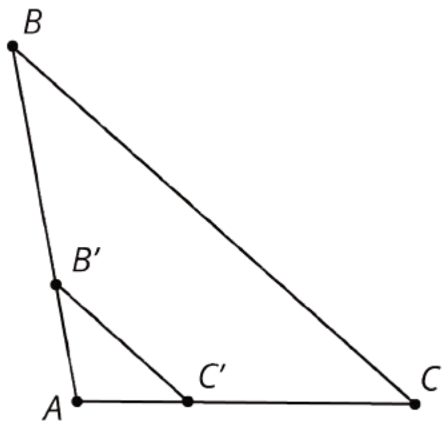
2. Here are two similar polygons.

   Measure the side lengths and angles of each polygon. What do you notice?




3. Each figure shows a pair of similar triangles, one contained in the other. For each pair, describe a point and a scale factor to use for a dilation moving the larger triangle to the smaller one. Use a measurement tool to find the scale factor.



# Lesson 7: Similar Polygons

## 7.1: All, Some, None: Congruence and Similarity

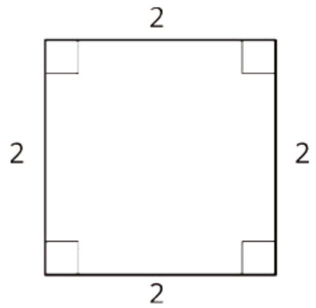
Choose whether each of the statements is true in *all* cases, in *some* cases, or in *no* cases.

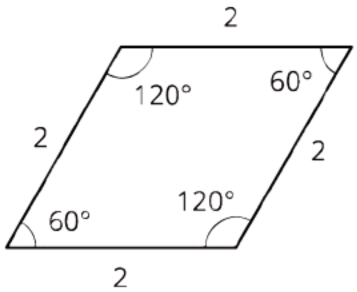
1. If two figures are congruent, then they are similar.

2. If two figures are similar, then they are congruent.

3. If an angle is dilated with the center of dilation at its vertex, the angle measure may change.

## 7.2: Are They Similar?

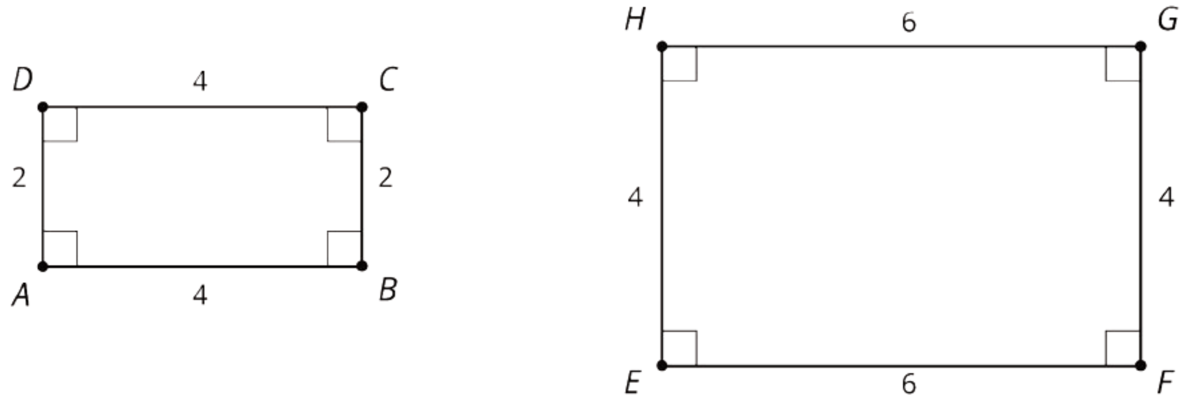



1. Let’s look at a square and a rhombus.

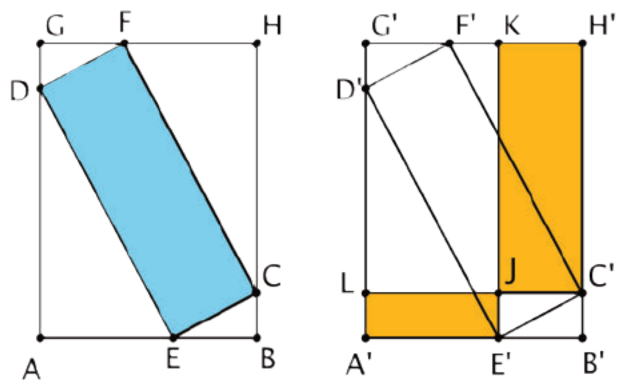
   Priya says, “These polygons are similar because their side lengths are all the same.” Clare says, “These polygons are not similar because the angles are different.” Do you agree with either Priya or Clare? Explain your reasoning.

2. Now, let’s look at rectangles $ABCD$ and $EFGH$.




Jada says, “These rectangles are similar because all of the side lengths differ by 2.” Lin says, “These rectangles are similar. I can dilate $AD$ and $BC$ using a scale factor of 2 and $AB$ and $CD$ using a scale factor of 1.5 to make the rectangles congruent. Then I can use a translation to line up the rectangles.” Do you agree with either Jada or Lin? Explain your reasoning.

**Are you ready for more?**



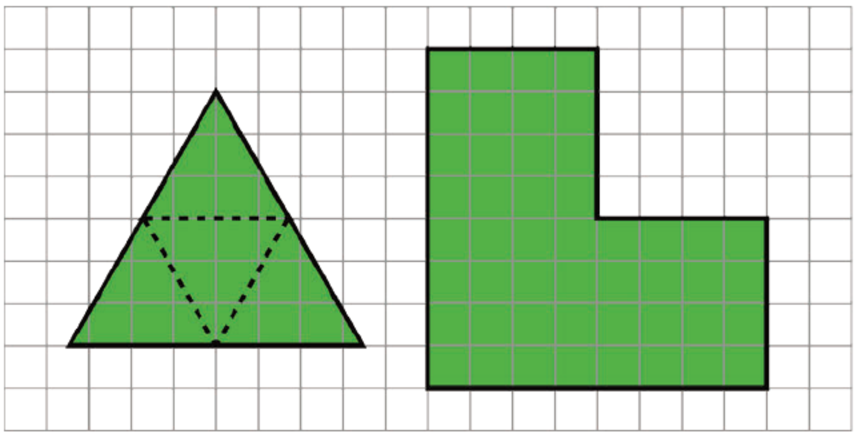

Points $A$ through $H$ are translated to the right to create points $A'$ through $H'$. All of the following are rectangles: $GHBA$, $FCED$, $KH'C'J$, and $LJE'A'$. Which is greater, the area of blue rectangle $DFCE$ or the total area of yellow rectangles $KH'C'J$ and $LJE'A'$?

## 7.3: Find Someone Similar

Your teacher will give you a card. Find someone else in the room who has a card with a polygon that is similar but not congruent to yours. When you have found your partner, work with them to explain how you know that the two polygons are similar.

### Are you ready for more?

On the left is an equilateral triangle where dashed lines have been added, showing how you can partition an equilateral triangle into smaller similar triangles.



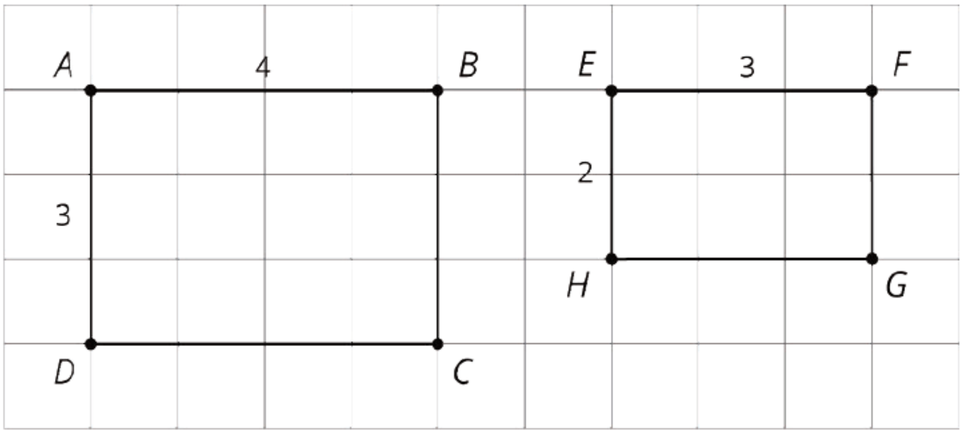
Find a way to do this for the figure on the right, partitioning it into smaller figures which are each similar to that original shape. What's the fewest number of pieces you can use? The most?

## Lesson 7 Summary

When two polygons are similar:

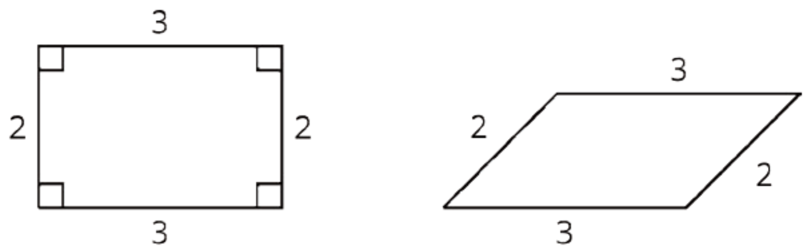
- Every angle and side in one polygon has a corresponding part in the other polygon.
- All pairs of corresponding angles have the same measure.
- Corresponding sides are related by a single scale factor. Each side length in one figure is multiplied by the scale factor to get the corresponding side length in the other figure.

Consider the two rectangles shown here. Are they similar?




It looks like rectangles $ABCD$ and $EFGH$ could be similar, if you match the long edges and match the short edges. All the corresponding angles are congruent because they are all right angles. Calculating the scale factor between the sides is where we see that "looks like" isn't enough to make them similar. To scale the long side $AB$ to the long side $EF$, the scale factor must be $\frac{3}{4}$, because $4 \cdot \frac{3}{4} = 3$. But the scale factor to match $AD$ to $EH$ has to be $\frac{2}{3}$, because $3 \cdot \frac{2}{3} = 2$. So, the rectangles are not similar because the scale factors for all the parts must be the same.

Here is an example that shows how sides can correspond (with a scale factor of 1), but the quadrilaterals are not similar because the angles don't have the same measure:



# Unit 2 Lesson 7 Cumulative Practice Problems

1. Triangle $DEF$ is a dilation of triangle $ABC$ with scale factor 2. In triangle $ABC$, the largest angle measures $82^\circ$. What is the largest angle measure in triangle $DEF$?

   A. $41^\circ$

   B. $82^\circ$

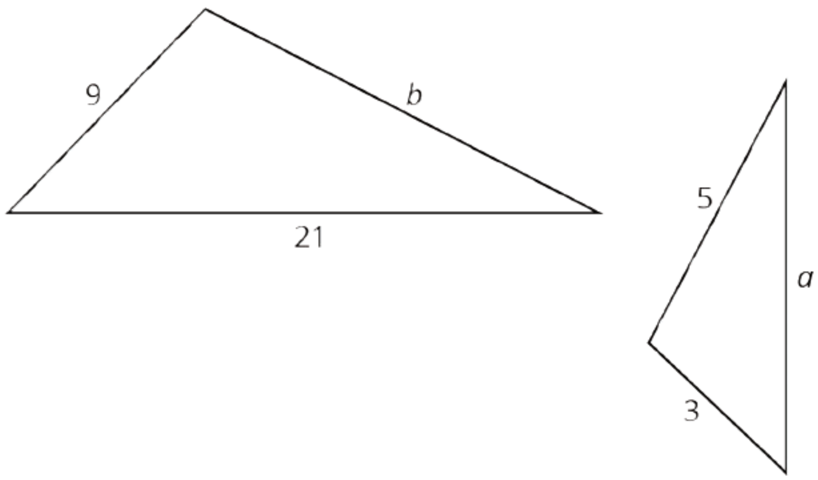
   C. $123^\circ$

   D. $164^\circ$

2. Draw two polygons that are similar but could be mistaken for not being similar. Explain why they are similar.

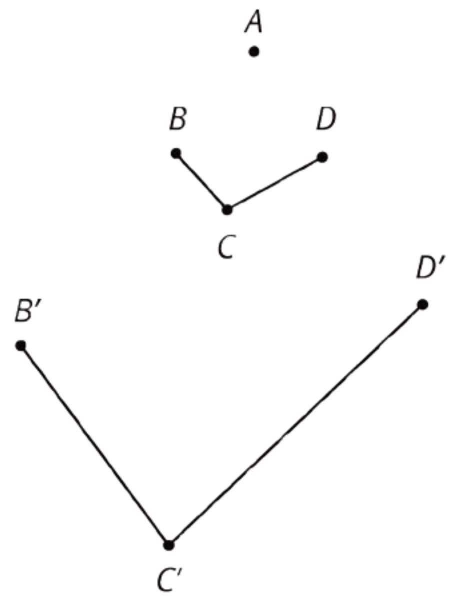
3. Draw two polygons that are *not* similar but could be mistaken for being similar. Explain why they are not similar.

4. These two triangles are similar. Find side lengths $a$ and $b$. Note: the two figures are not drawn to scale.



5. Jada claims that $B'C'D'$ is a dilation of $BCD$ using $A$ as the center of dilation.

What are some ways you can convince Jada that her claim is not true?




(From Unit 2, Lesson 3.)

6. a. Draw a horizontal line segment $AB$.

b. Rotate segment $AB$ $90^\circ$ counterclockwise around point $A$. Label any new points.

c. Rotate segment $AB$ $90^\circ$ clockwise around point $B$. Label any new points.

d. Describe a transformation on segment $AB$ you could use to finish building a square.

(From Unit 1, Lesson 8.)

# Lesson 8: Similar Triangles

## 8.1: Equivalent Expressions

Create three different expressions that are each equal to 20. Each expression should include only these three numbers: 4, -2, and 10.

## 8.2: Making Pasta Angles and Triangles

Your teacher will give you some dried pasta and a set of angles.

1. Create a triangle using three pieces of pasta and angle $A$. Your triangle *must* include the angle you were given, but you are otherwise free to make any triangle you like. Tape your pasta triangle to a sheet of paper so it won't move.
    a. After you have created your triangle, measure each side length with a ruler and record the length on the paper next to the side. Then measure the angles to the nearest 5 degrees using a protractor and record these measurements on your paper.

    b. Find two others in the room who have the same angle $A$ and compare your triangles. What is the same? What is different? Are the triangles congruent? Similar?

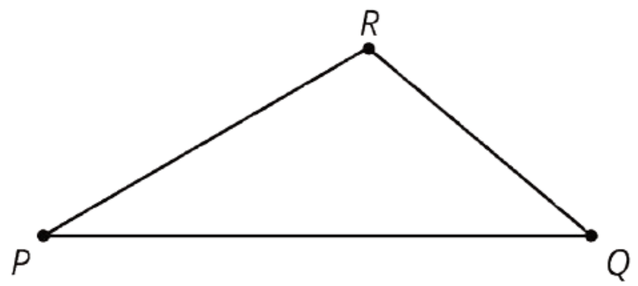
    c. How did you decide if they were or were not congruent or similar?

2. Now use more pasta and angles $A$, $B$, and $C$ to create another triangle. Tape this pasta triangle on a separate sheet of paper.
    a. After you have created your triangle, measure each side length with a ruler and record the length on the paper next to the side. Then measure the angles to the nearest 5 degrees using a protractor and record these measurements on your paper.

b. Find two others in the room who used your same angles and compare your triangles. What is the same? What is different? Are the triangles congruent? Similar?

c. How did you decide if they were or were not congruent or similar?

3. Here is triangle $PQR$. Break a new piece of pasta, different in length than segment $PQ$.




- Tape the piece of pasta so that it lays on top of line $PQ$ with one end of the pasta at $P$ (if it does not fit on the page, break it further). Label the other end of the piece of pasta $S$.
- Tape a full piece of pasta, with one end at $S$, making an angle congruent to $\angle PQR$.
- Tape a full piece of pasta on top of line $PR$ with one end of the pasta at $P$. Call the point where the two full pieces of pasta meet $T$.

a. Is your new pasta triangle $PST$ similar to $\triangle PQR$? Explain your reasoning.

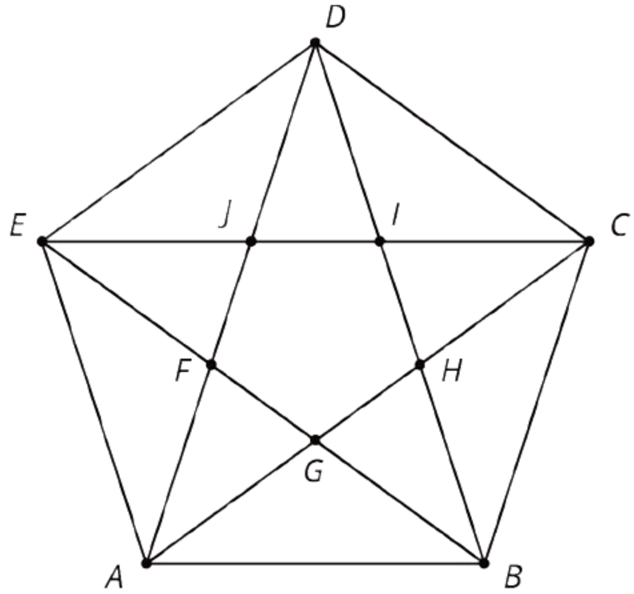
b. If your broken piece of pasta were a different length, would the pasta triangle still be similar to $\triangle PQR$? Explain your reasoning.

**Are you ready for more?**

Quadrilaterals $ABCD$ and $EFGH$ have four angles measuring $240^\circ$, $40^\circ$, $40^\circ$, and $40^\circ$. Do $ABCD$ and $EFGH$ have to be similar?

## 8.3: Similar Figures in a Regular Pentagon

1. This diagram has several triangles that are similar to triangle $DJI$.




   a. Three different scale factors were used to make triangles similar to $DJI$. In the diagram, find at least one triangle of each size that is similar to $DJI$.

   b. Explain how you know each of these three triangles is similar to $DJI$.

2. Find a triangle in the diagram that is not similar to $DJI$.

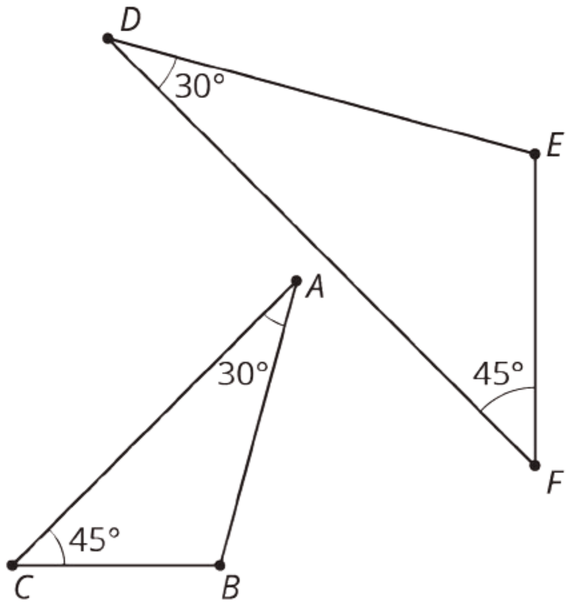
**Are you ready for more?**

Figure out how to draw some more lines in the pentagon diagram to make more triangles similar to $DJI$.

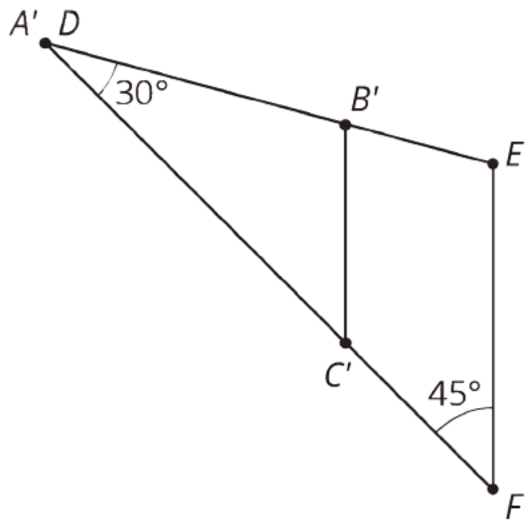
## Lesson 8 Summary

We learned earlier that two polygons are similar when there is a sequence of translations, rotations, reflections, and dilations taking one polygon to the other. When the polygons are triangles, we only need to check that that both triangles have two corresponding angles to show they are similar—can you tell why?

Here is an example. Triangle $ABC$ and triangle $DEF$ each have a 30 degree angle and a 45 degree angle.




We can translate $A$ to $D$ and then rotate so that the two 30 degree angles are aligned, giving this picture:




Now a dilation with center $D$ and appropriate scale factor will move $C'$ to $F$. This dilation also moves $B'$ to $E$, showing that triangles $ABC$ and $DEF$ are similar.

# Unit 2 Lesson 8 Cumulative Practice Problems

1. In each pair, some of the angles of two triangles in degrees are given. Use the information to decide if the triangles are similar or not. Explain how you know.

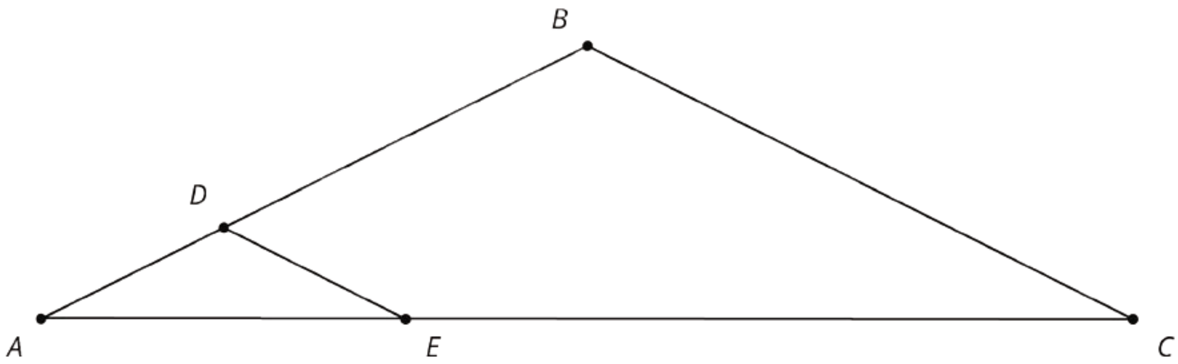
    - Triangle A: 53, 71, ___; Triangle B: 53, 71, ___
    - Triangle C: 90, 37, ___; Triangle D: 90, 53, ___
    - Triangle E: 63, 45, ____; Triangle F: 14, 71, ____
    - Triangle G: 121, ___, ___; Triangle H: 70, ___, ___

2. a. Draw two equilateral triangles that are not congruent.

    b. Measure the side lengths and angles of your triangles. Are the two triangles similar?

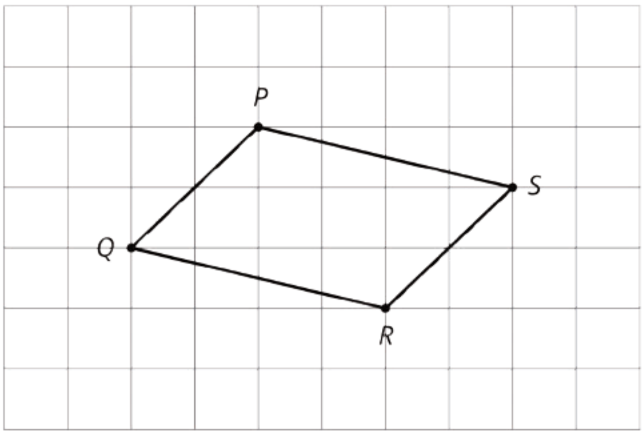
    c. Do you think two equilateral triangles will be similar *always*, *sometimes*, or *never*? Explain your reasoning.

3. In the figure, line $BC$ is parallel to line $DE$.




Explain why $\triangle ABC$ is similar to $\triangle ADE$.

4. The quadrilateral $PQRS$ in the diagram is a parallelogram. Let $P'Q'R'S'$ be the image of $PQRS$ after applying a dilation centered at a point O (not shown) with scale factor 3.




Which of the following is true?

A. $P'Q' = PQ$

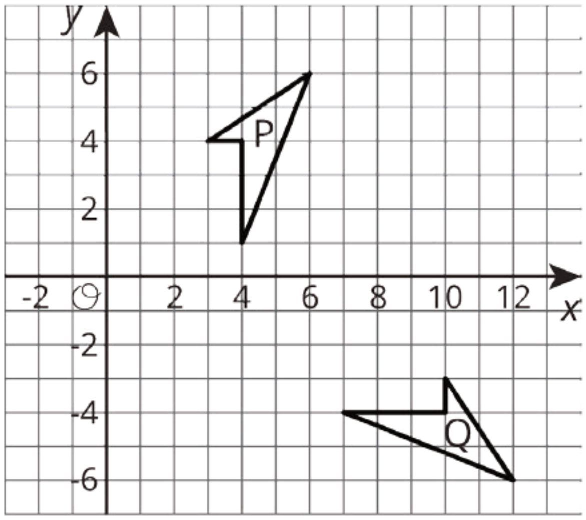
B. $P'Q' = 3PQ$

C. $PQ = 3P'Q'$

D. Cannot be determined from the information given

(From Unit 2, Lesson 4.)

5. Describe a sequence of transformations for which Quadrilateral P is the image of Quadrilateral Q.




(From Unit 1, Lesson 6.)

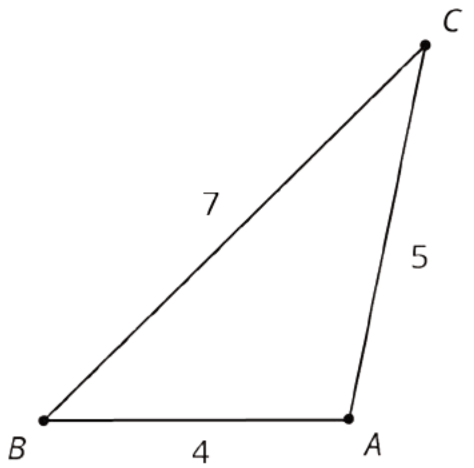
# Lesson 9: Side Length Quotients in Similar Triangles

## 9.1: Two-three-four and Four-five-six

Triangle $A$ has side lengths 2, 3, and 4. Triangle $B$ has side lengths 4, 5, and 6. Is Triangle $A$ similar to Triangle $B$?

## 9.2: Quotients of Sides Within Similar Triangles

Triangle $ABC$ is similar to triangles $DEF$, $GHI$, and $JKL$. The scale factors for the dilations that show triangle $ABC$ is similar to each triangle are in the table.




1. Find the side lengths of triangles $DEF$, $GHI$, and $JKL$. Record them in the table.

| triangle | scale factor | length of short side | length of medium side | length of long side |
|---|---|---|---|---|
| $ABC$ | 1 | 4 | 5 | 7 |
| $DEF$ | 2 | | | |
| $GHI$ | 3 | | | |
| $JKL$ | $\frac{1}{2}$ | | | |

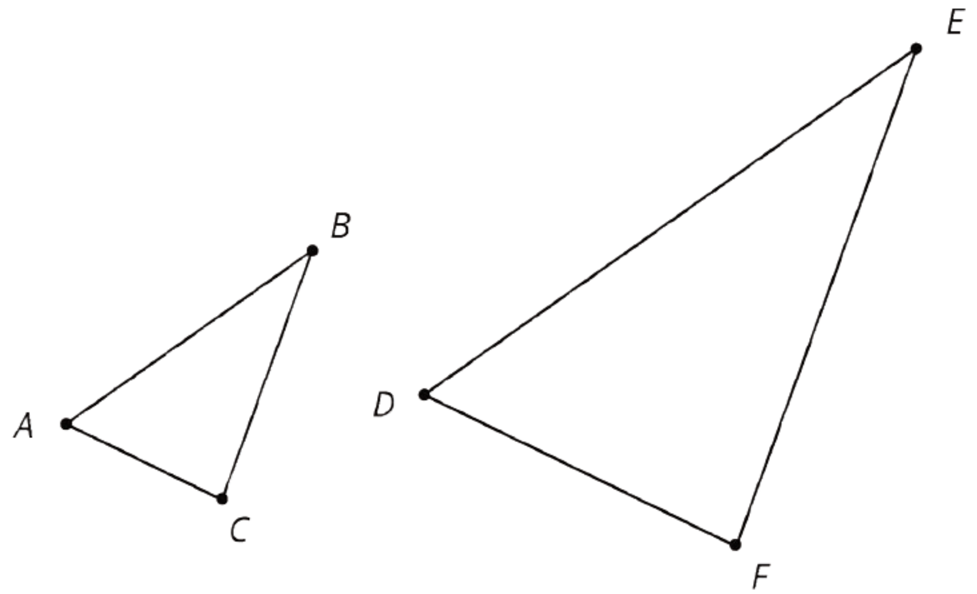
2. Your teacher will assign you one of the three columns. For all four triangles, find the quotient of the triangle side lengths assigned to you and record it in the table. What do you notice about the quotients?

| triangle | (long side) ÷ (short side) | (long side) ÷ (medium side) | (medium side) ÷ (short side) |
|---|---|---|---|
| $ABC$ | $\frac{7}{4}$ or 1.75 | | |
| $DEF$ | | | |
| $GHI$ | | | |
| $JKL$ | | | |

3. Compare your results with your partners' and complete your table.

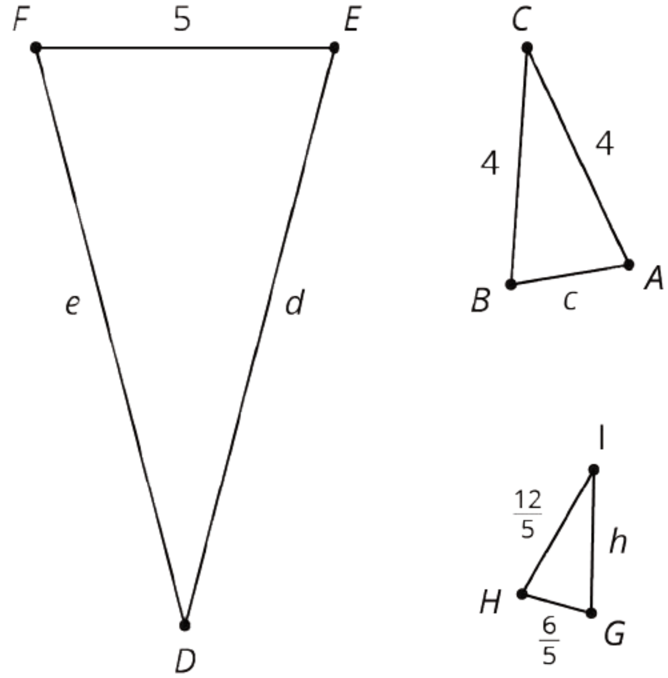
**Are you ready for more?**

Triangles $ABC$ and $DEF$ are similar. Explain why $\frac{AB}{BC} = \frac{DE}{EF}$.



## 9.3: Using Side Quotients to Find Side Lengths of Similar Triangles

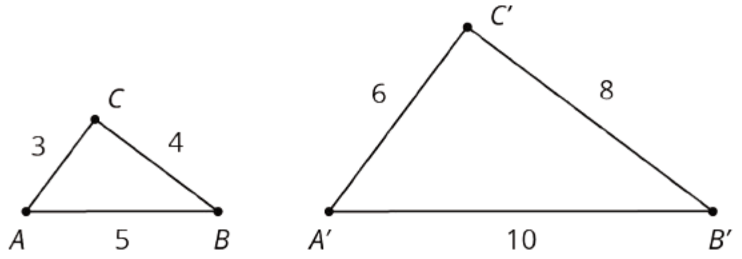
Triangles $ABC$, $EFD$, and $GHI$ are all similar. The side lengths of the triangles all have the same units. Find the unknown side lengths.



## Lesson 9 Summary

If two polygons are similar, then the side lengths in one polygon are multiplied by the same scale factor to give the corresponding side lengths in the other polygon.

For these triangles the scale factor is 2:



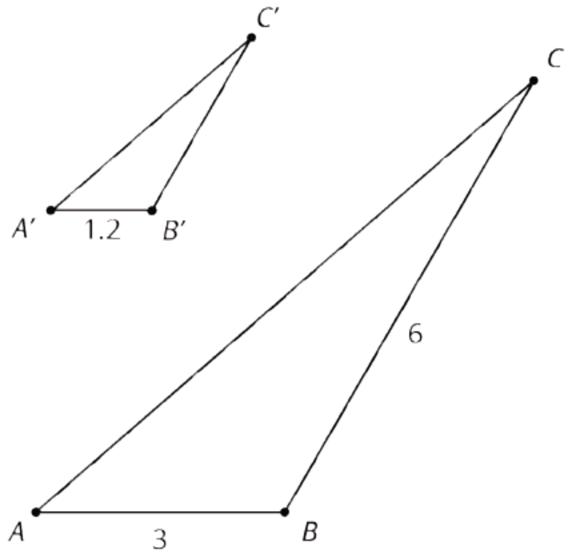

Here is a table that shows relationships between the short and medium length sides of the small and large triangle.

| | small triangle | large triangle |
|---|---|---|
| **medium side** | 4 | 8 |
| **short side** | 3 | 6 |
| **(medium side) ÷ (short side)** | $\frac{4}{3}$ | $\frac{8}{6} = \frac{4}{3}$ |

The lengths of the medium side and the short side are in a ratio of $4 : 3$. This means that the medium side in each triangle is $\frac{4}{3}$ as long as the short side. This is true for all similar polygons; the ratio between two sides in one polygon is the same as the ratio of the corresponding sides in a similar polygon.

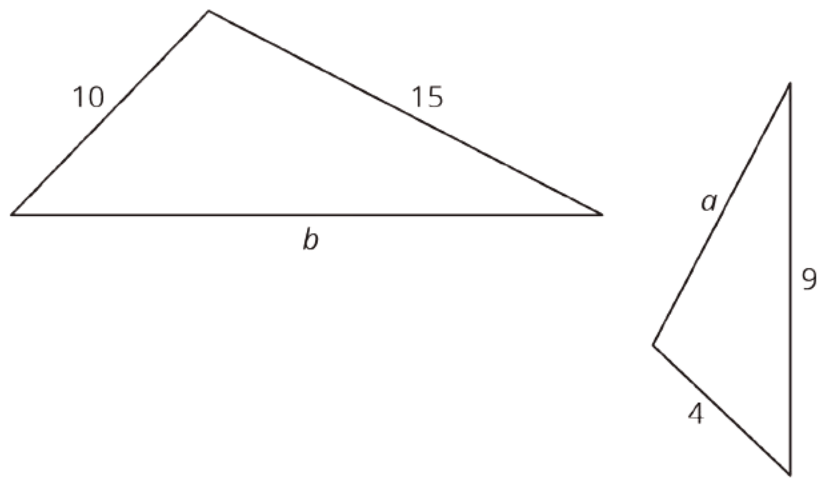
We can use these facts to calculate missing lengths in similar polygons. For example, triangles $A'B'C'$ and $ABC$ shown here are similar. Let's find the length of segment $B'C'$.

In triangle $ABC$, side $BC$ is twice as long as side $AB$, so this must be true for any triangle that is similar to triangle $ABC$. Since $A'B'$ is 1.2 units long and $2 \cdot 1.2 = 2.4$, the length of side $B'C'$ is 2.4 units.

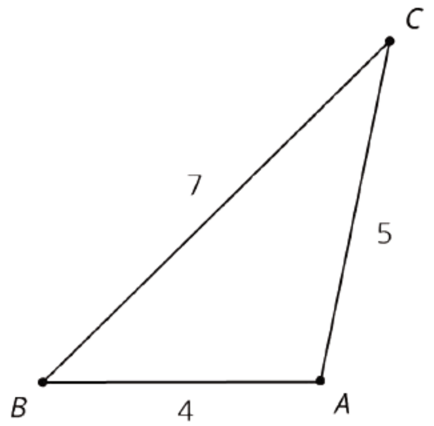


# Unit 2 Lesson 9 Cumulative Practice Problems

1. These two triangles are similar. What are $a$ and $b$? Note: the two figures are not drawn to scale.



2. Here is triangle $ABC$. Triangle $XYZ$ is similar to $ABC$ with scale factor $\frac{1}{4}$.



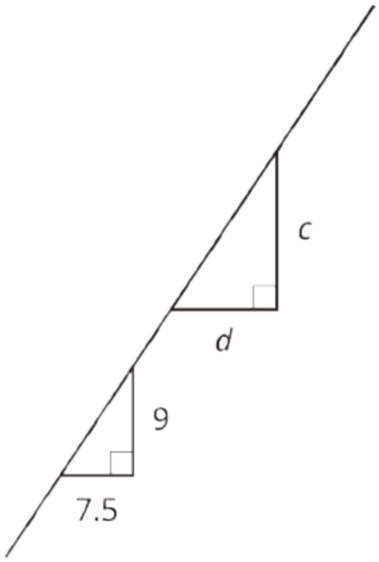

a. Draw what triangle $XYZ$ might look like.

b. How do the angle measures of triangle $XYZ$ compare to triangle $ABC$? Explain how you know.

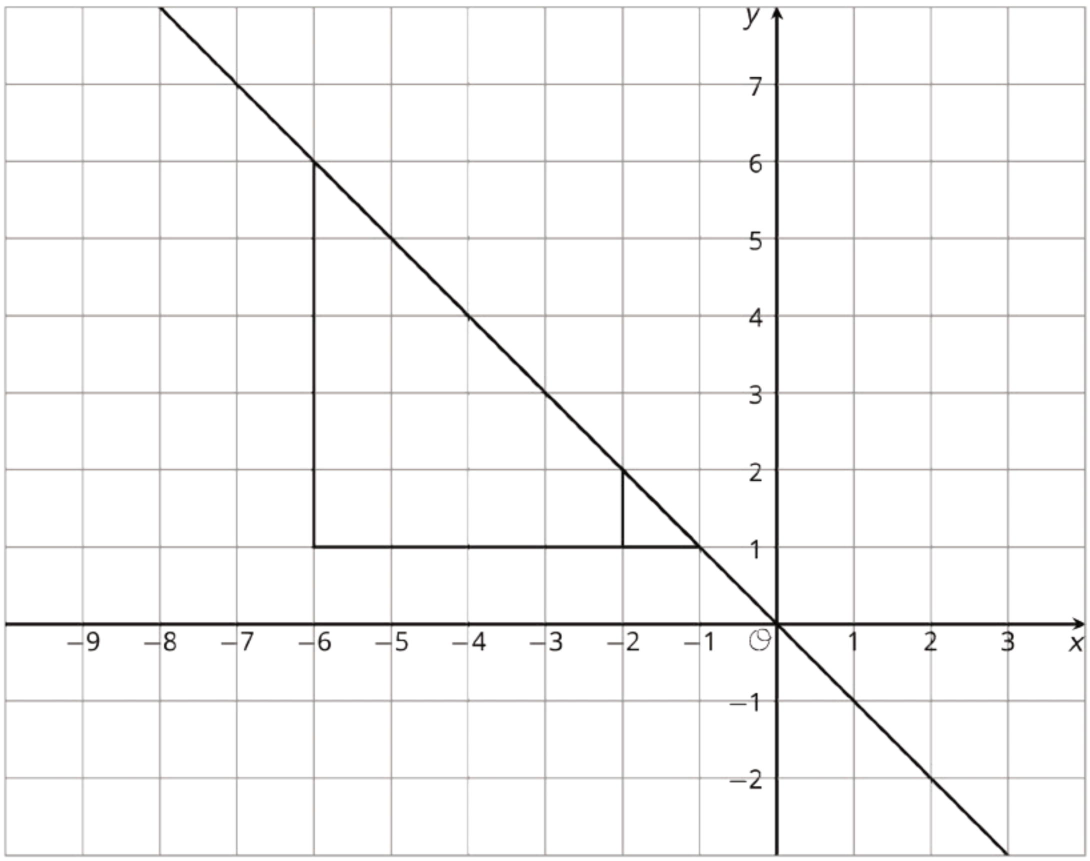
c. What are the side lengths of triangle $XYZ$?

d. For triangle $XYZ$, calculate (long side) ÷ (medium side), and compare to triangle $ABC$.

3. The two triangles shown are similar. Find the value of $\frac{d}{c}$.




4. The diagram shows two nested triangles that share a vertex. Find a center and a scale factor for a dilation that would move the larger triangle to the smaller triangle.




(From Unit 2, Lesson 5.)

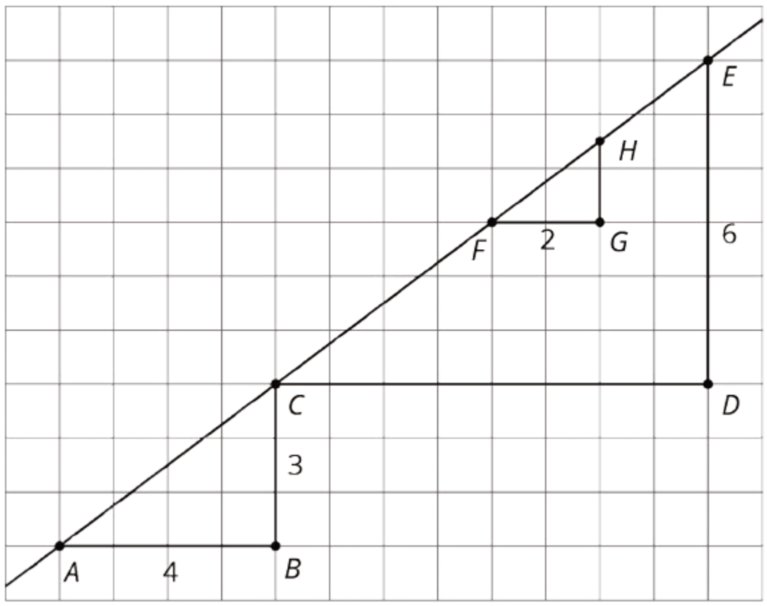
# Lesson 10: Meet Slope

## 10.1: Equal Quotients

Write some numbers that are equal to $15 \div 12$.

## 10.2: Similar Triangles on the Same Line

1. The figure shows three right triangles, each with its longest side on the same line. Your teacher will assign you two triangles. Explain why the two triangles are similar.




2. Complete the table.

| triangle | length of vertical side | length of horizontal side | (vertical side) ÷ (horizontal side) |
|---|---|---|---|
| $ABC$ | | | |
| $CDE$ | | | |
| $FGH$ | | | |

## 10.3: Multiple Lines with the Same Slope

1. Draw two lines with slope 3. What do you notice about the two lines?

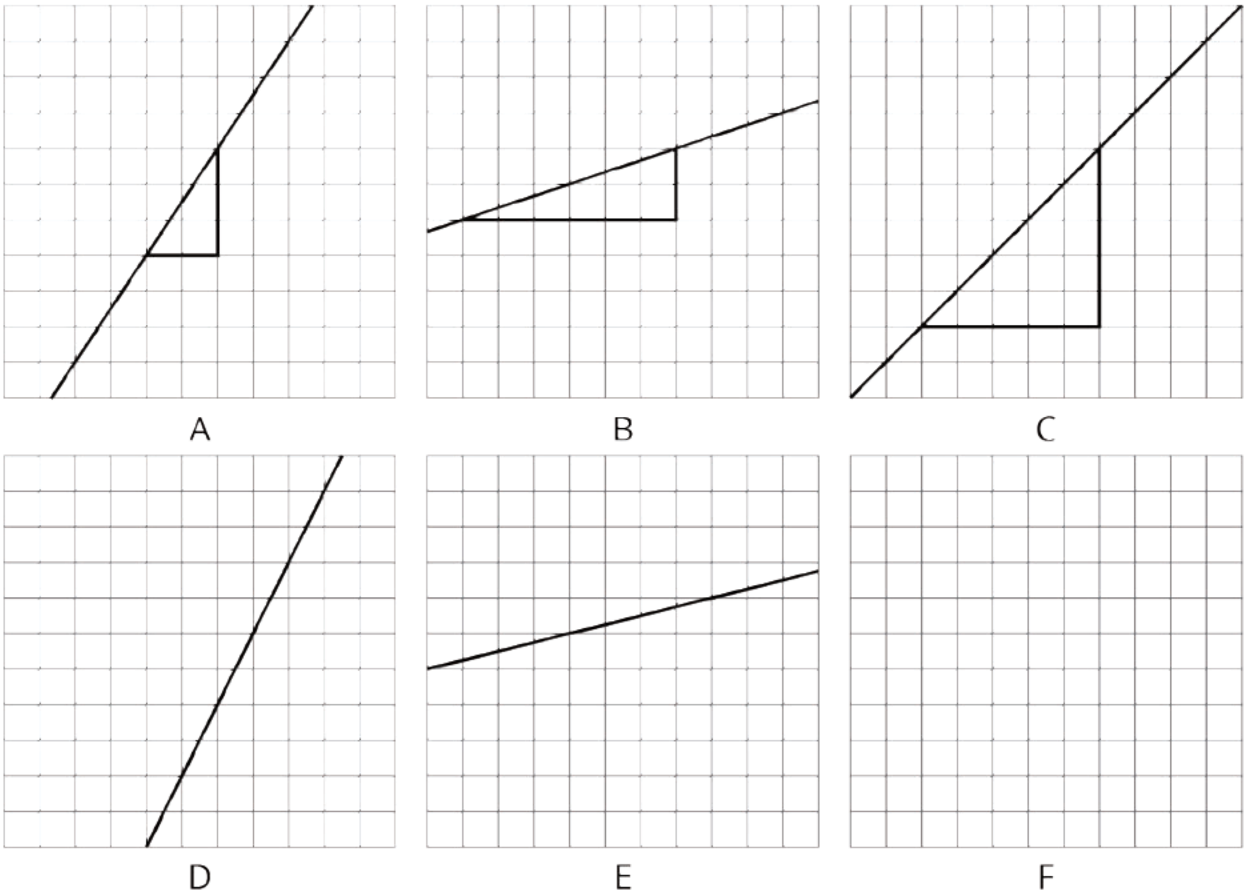
2. Draw two lines with slope $\frac{1}{2}$. What do you notice about the two lines?

### Are you ready for more?

As we learn more about lines, we will occasionally have to consider perfectly vertical lines as a special case and treat them differently. Think about applying what you have learned in the last couple of activities to the case of vertical lines. What is the same? What is different?

## 10.4: Different Slopes of Different Lines

Here are several lines.

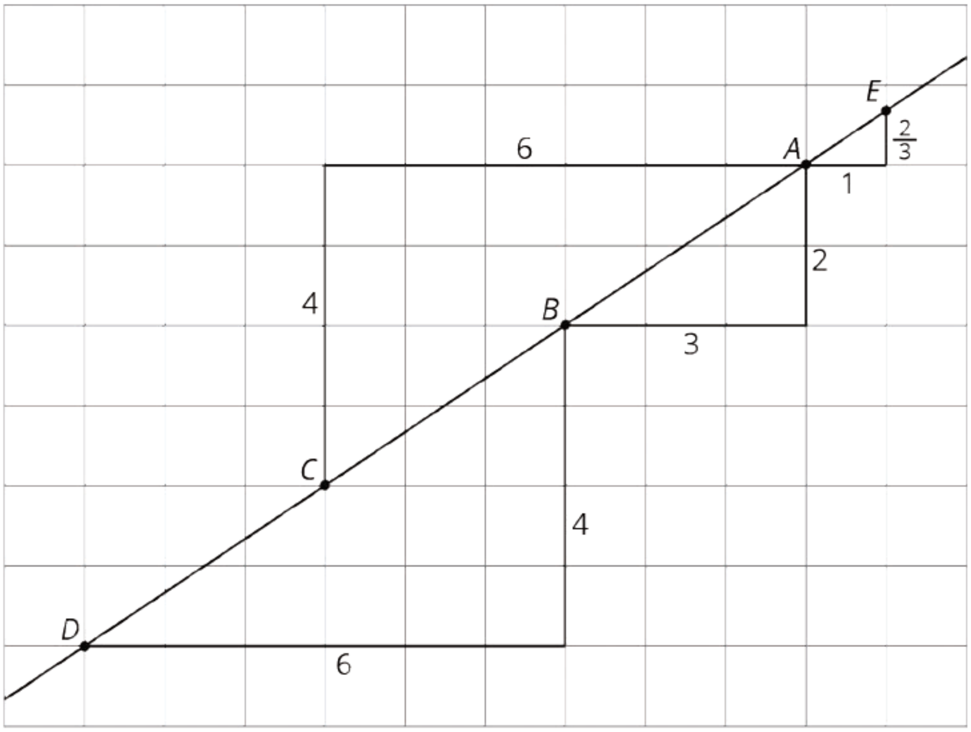



1. Match each line shown with a slope from this list: $\frac{1}{3}$, 2, 1, 0.25, $\frac{3}{2}$, $\frac{1}{2}$.

2. One of the given slopes does not have a line to match. Draw a line with this slope on the empty grid (F).

## Lesson 10 Summary

Here is a line drawn on a grid. There are also four right triangles drawn. Do you notice anything the triangles have in common?



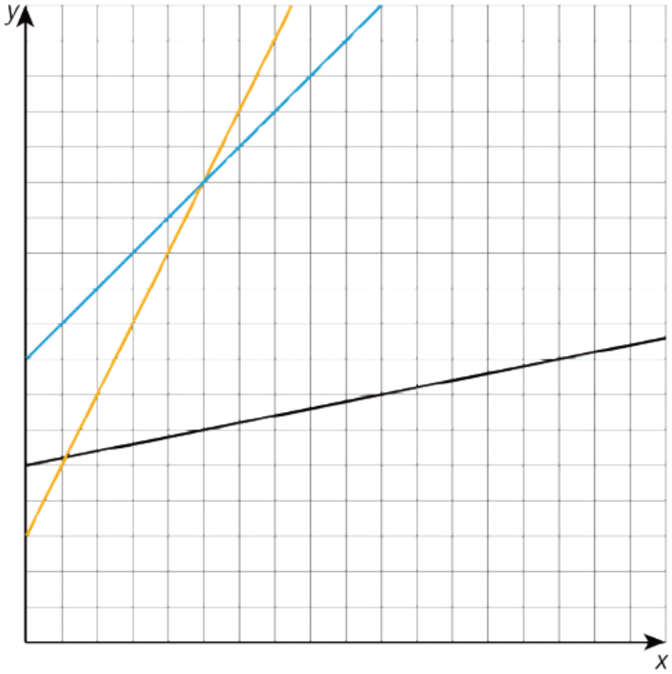

These four triangles are all examples of *slope triangles*. One side of a slope triangle is on the line, one side is vertical, and another side is horizontal. The **slope** of the line is the quotient of the length of the vertical side and the length of the horizontal side of the slope triangle. This number is the same for *all* slope triangles for the same line because all slope triangles for the same line are similar.

In this example, the slope of the line is $\frac{2}{3}$, which is what all four triangles have in common. Here is how the slope is calculated using the slope triangles:

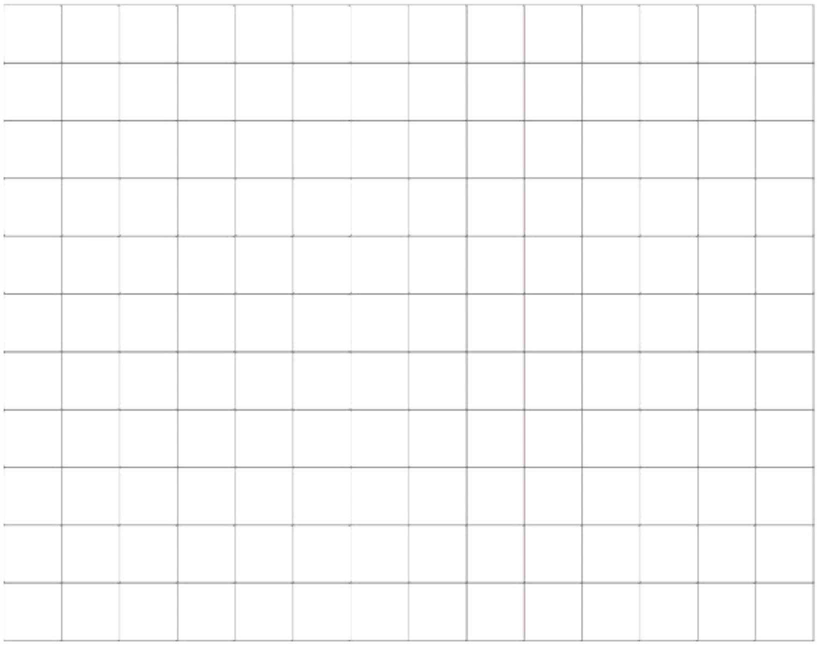
- Points $A$ and $B$ give $2 \div 3 = \frac{2}{3}$
- Points $D$ and $B$ give $4 \div 6 = \frac{2}{3}$
- Points $A$ and $C$ give $4 \div 6 = \frac{2}{3}$
- Points $A$ and $E$ give $\frac{2}{3} \div 1 = \frac{2}{3}$

# Unit 2 Lesson 10 Cumulative Practice Problems

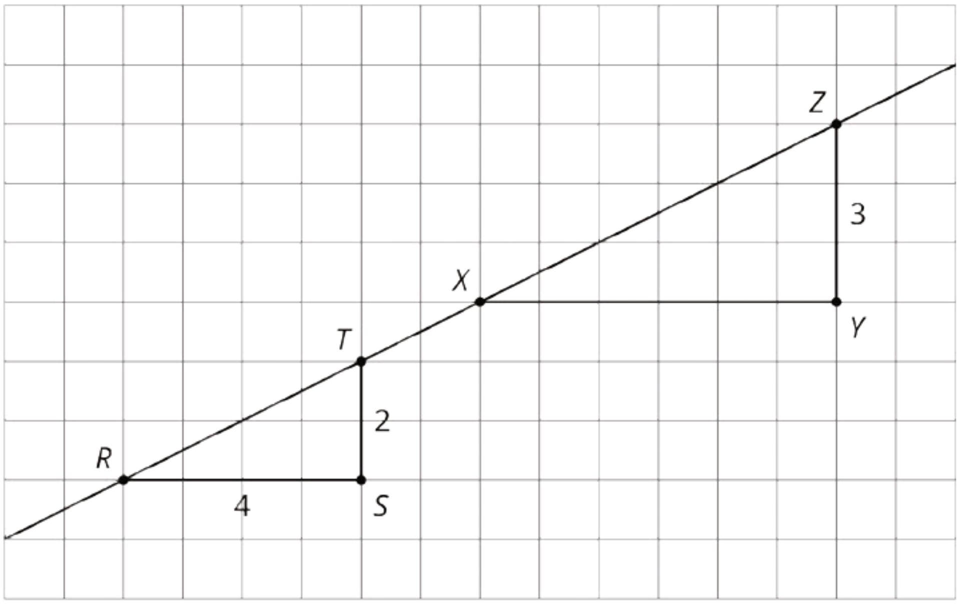
1. Of the three lines in the graph, one has slope 1, one has slope 2, and one has slope $\frac{1}{5}$. Label each line with its slope.



2. Draw three lines with slope 2, and three lines with slope $\frac{1}{3}$. What do you notice?



3. The figure shows two right triangles, each with its longest side on the same line.




a. Explain how you know the two triangles are similar.

b. How long is $XY$?

c. For each triangle, calculate (vertical side) ÷ (horizontal side).

d. What is the slope of the line? Explain how you know.

4. Triangle $A$ has side lengths 3, 4, and 5. Triangle $B$ has side lengths 6, 7, and 8.

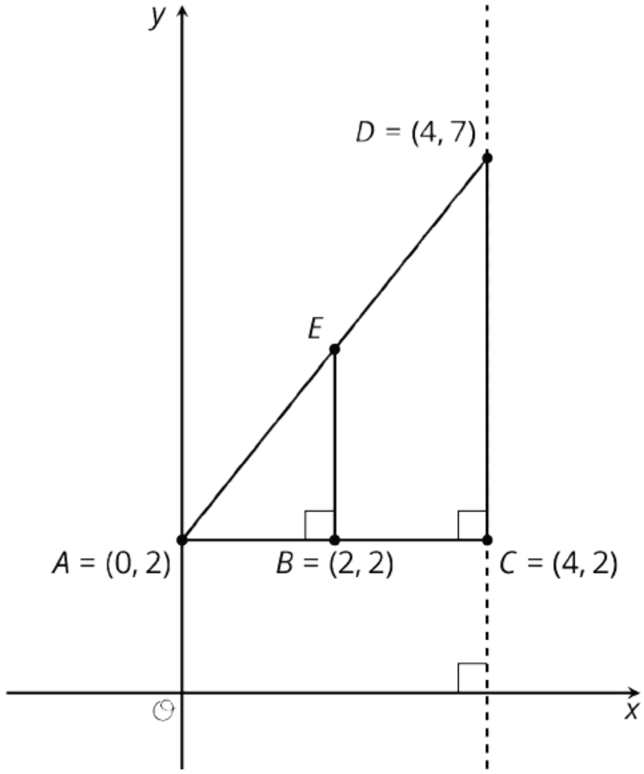
a. Explain how you know that Triangle $B$ is *not* similar to Triangle $A$.

b. Give possible side lengths for Triangle $B$ so that it is similar to Triangle $A$.

(From Unit 2, Lesson 9.)

# Lesson 11: Writing Equations for Lines

## 11.1: Coordinates and Lengths in the Coordinate Plane



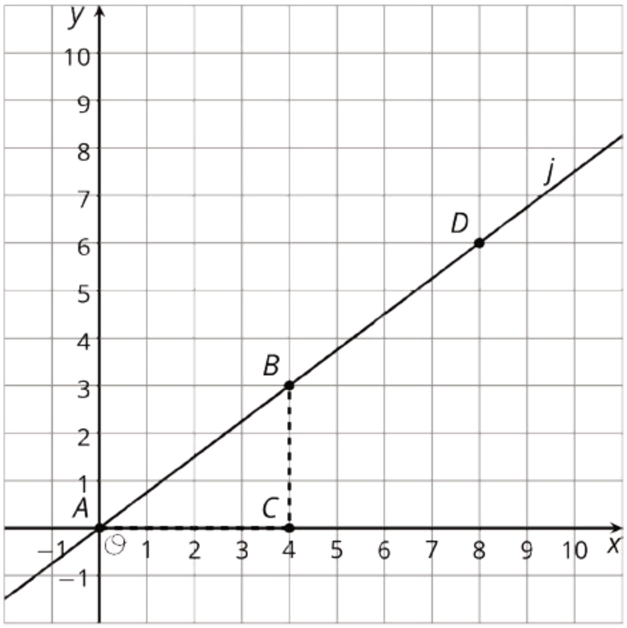

Find each of the following and explain your reasoning:

1. The length of segment $BE$.

2. The coordinates of $E$.

## 11.2: What We Mean by an Equation of a Line

Line $j$ is shown in the coordinate plane.




1. What are the coordinates of $B$ and $D$?

2. Is point $(20, 15)$ on line $j$? Explain how you know.

3. Is point $(100, 75)$ on line $j$? Explain how you know.

4. Is point $(90, 68)$ on line $j$? Explain how you know.

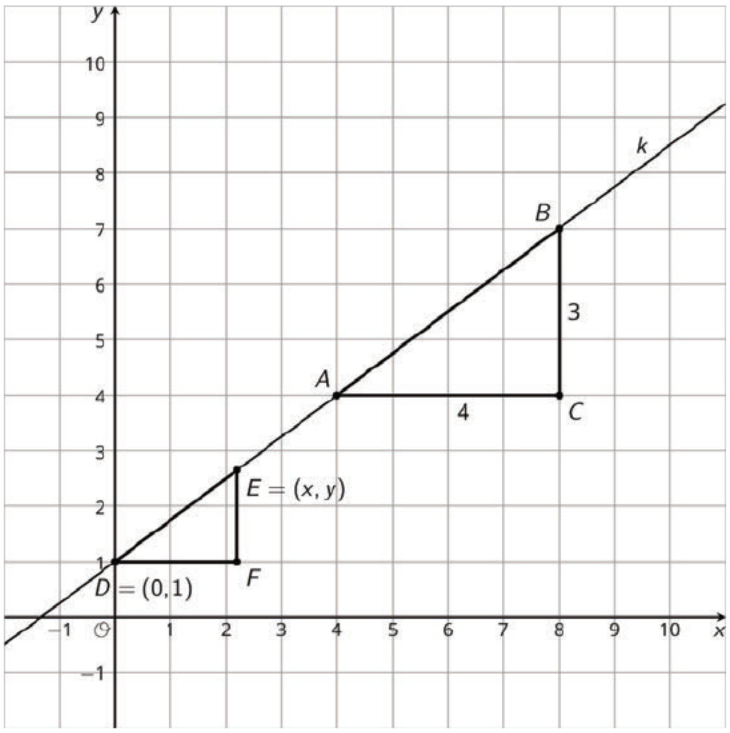
5. Suppose you know the $x$- and $y$-coordinates of a point. Write a rule that would allow you to test whether the point is on line $j$.

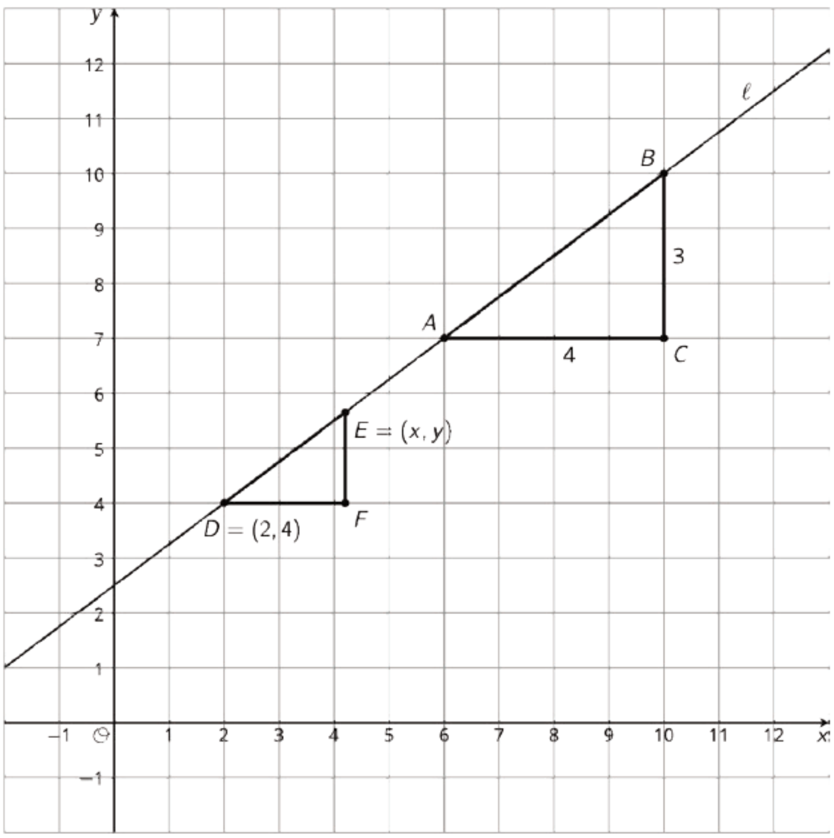
## 11.3: Writing Relationships from Slope Triangles

Here are two diagrams:

1. Complete each diagram so that all vertical and horizontal segments have expressions for their lengths.

2. Use what you know about similar triangles to find an equation for the quotient of the vertical and horizontal side lengths of $\triangle DFE$ in each diagram.



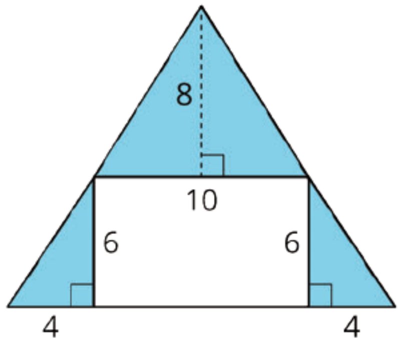



### Are you ready for more?

1. Find the area of the shaded region by summing the areas of the shaded triangles.

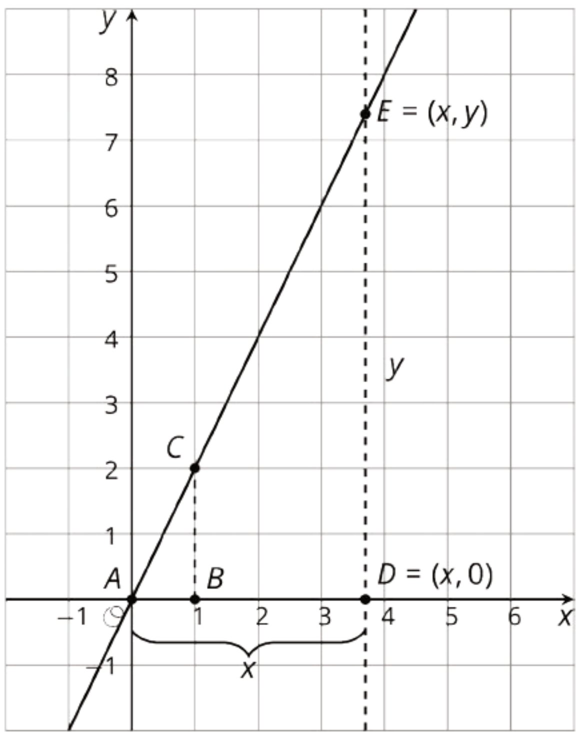
2. Find the area of the shaded region by subtracting the area of the unshaded region from the large triangle.

3. What is going on here?




### Lesson 11 Summary

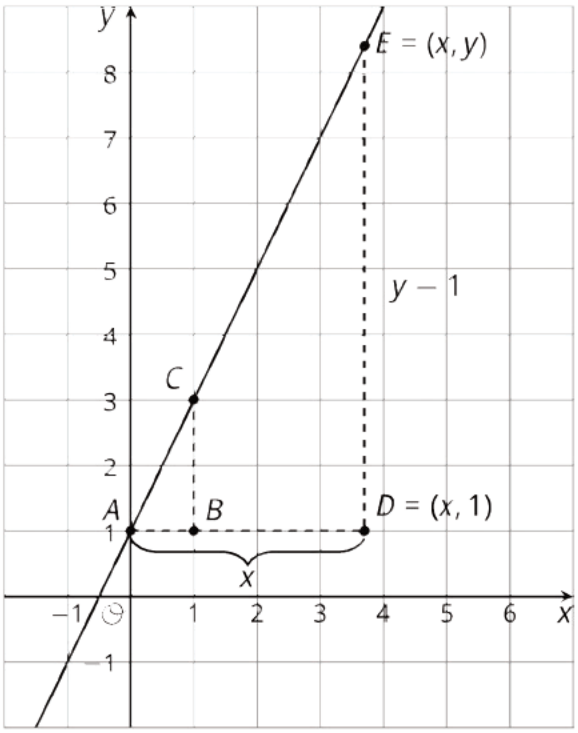
Here are the points $A$, $C$, and $E$ on the same line. Triangles $ABC$ and $ADE$ are slope triangles for the line so we know they are similar triangles. Let's use their similarity to better understand the relationship between $x$ and $y$, which make up the coordinates of point $E$.



The slope for triangle $ABC$ is $\frac{2}{1}$ since the vertical side has length 2 and the horizontal side has length 1. The slope we find for triangle $ADE$ is $\frac{y}{x}$ because the vertical side has length $y$ and the horizontal side has length $x$. These two slopes must be equal since they are from slope triangles for the same line, and so: $\frac{2}{1} = \frac{y}{x}$.

Since $\frac{2}{1} = 2$ this means that the value of $y$ is twice the value of $x$, or that $y = 2x$. This equation is true for any point $(x, y)$ on the line!

Here are two different slope triangles. We can use the same reasoning to describe the relationship between $x$ and $y$ for this point $E$.




The slope for triangle $ABC$ is $\frac{2}{1}$ since the vertical side has length 2 and the horizontal side has length 1. For triangle $ADE$, the horizontal side has length $x$. The vertical side has length $y - 1$ because the distance from $(x, y)$ to the $x$-axis is $y$ but the vertical side of the triangle stops 1 unit short of the $x$-axis. So the slope we find for triangle $ADE$ is $\frac{y-1}{x}$. The slopes for the two slope triangles are equal, meaning:

$$\frac{2}{1} = \frac{y-1}{x}$$

Since $y - 1$ is twice $x$, another way to write this equation is $y - 1 = 2x$. This equation is true for any point $(x, y)$ on the line!

# Unit 2 Lesson 11 Cumulative Practice Problems

1. For each pair of points, find the slope of the line that passes through both points. If you get stuck, try plotting the points on graph paper and drawing the line through them with a ruler.

   a. $(1, 1)$ and $(7, 5)$

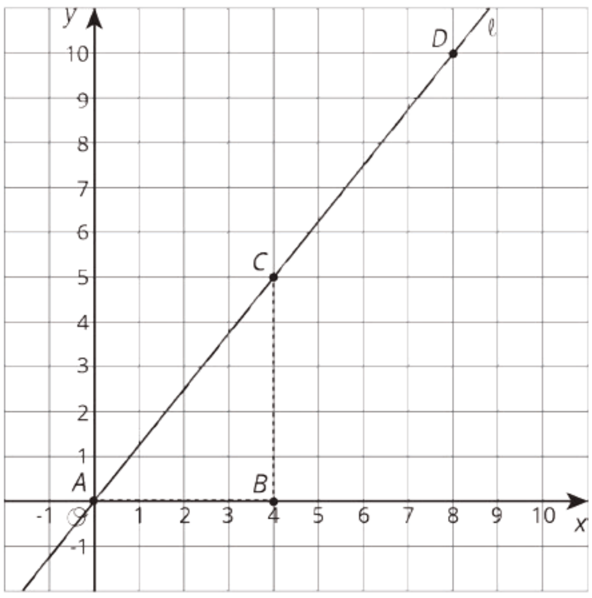
   b. $(1, 1)$ and $(5, 7)$

   c. $(2, 5)$ and $(-1, 2)$

   d. $(2, 5)$ and $(-7, -4)$

2. Line $\ell$ is shown in the coordinate plane.

   a. What are the coordinates of points $B$ and $D$?




   b. Is the point $(16, 20)$ on line $\ell$? Explain how you know.

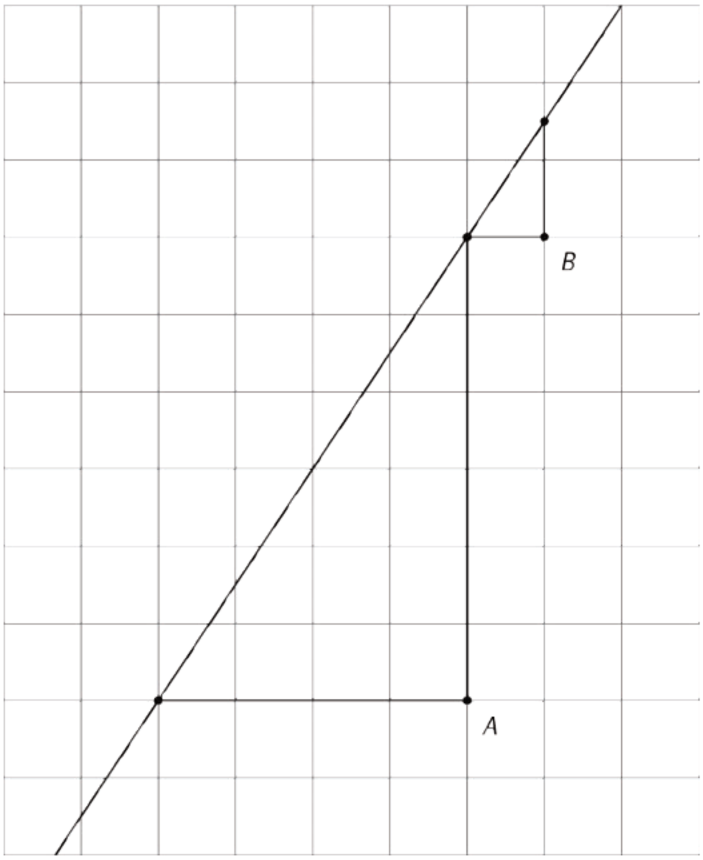
   c. Is the point $(20, 24)$ on line $\ell$? Explain how you know.

   d. Is the point $(80, 100)$ on line $\ell$? Explain how you know.

   e. Write a rule that would allow you to test whether $(x, y)$ is on line $\ell$.

3. Consider the graphed line.

Mai uses Triangle A and says the slope of this line is $\frac{6}{4}$. Elena uses Triangle B and says no, the slope of this line is 1.5. Do you agree with either of them? Explain.




4. A rectangle has length 6 and height 4.

Which of these would tell you that quadrilateral $ABCD$ is definitely *not* similar to this rectangle? Select **all** that apply.

A. $AB = BC$

B. $m\angle ABC = 105^\circ$

C. $AB = 8$

D. $BC = 8$

E. $BC = 2 \cdot AB$

F. $2 \cdot AB = 3 \cdot BC$

(From Unit 2, Lesson 7.)

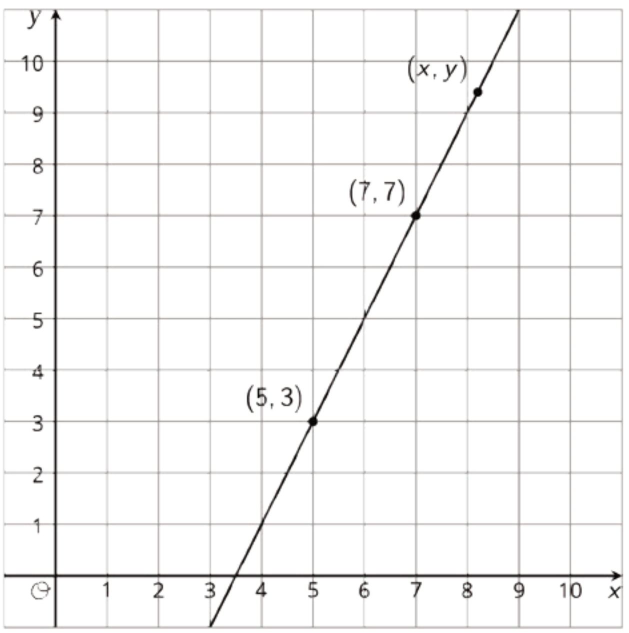
# Lesson 12: Using Equations for Lines

## 12.1: Missing center

A dilation with scale factor 2 sends $A$ to $B$. Where is the center of the dilation?


## 12.2: Writing Relationships from Two Points

Here is a line.




1. Using what you know about similar triangles, find an equation for the line in the diagram.

2. What is the slope of this line? Does it appear in your equation?

3. Is $(9, 11)$ also on the line? How do you know?

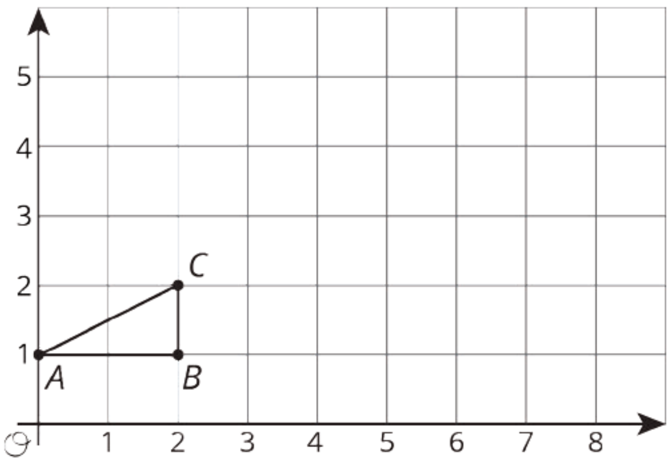
4. Is $(100, 193)$ also on the line?

**Are you ready for more?**

There are many different ways to write down an equation for a line like the one in the problem. Does $\frac{y-3}{x-6}=2$ represent the line? What about $\frac{y-6}{x-4}=5$? What about $\frac{y+5}{x-1}=2$? Explain your reasoning.

## 12.3: Dilations and Slope Triangles

Here is triangle $ABC$.




1. Draw the dilation of triangle $ABC$ with center $(0,1)$ and scale factor 2.

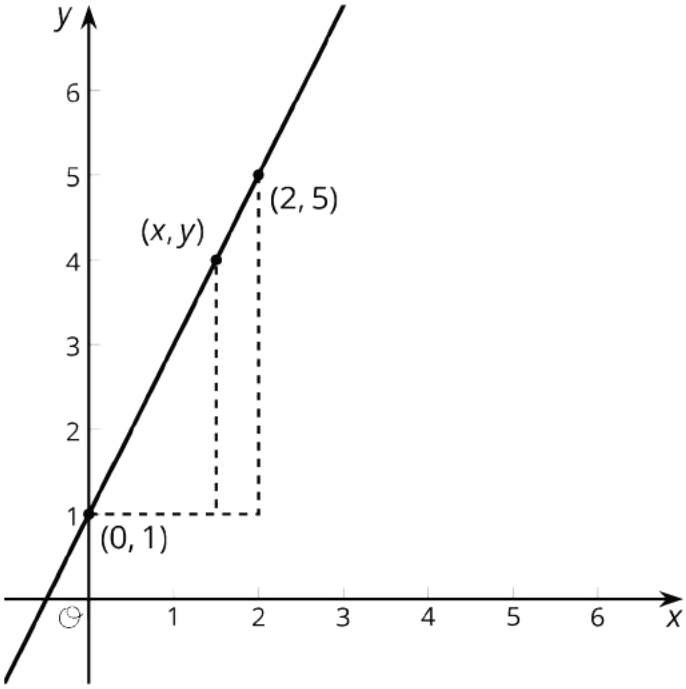
2. Draw the dilation of triangle $ABC$ with center $(0,1)$ and scale factor 2.5.

3. Where is $C$ mapped by the dilation with center $(0,1)$ and scale factor $s$?

4. For which scale factor does the dilation with center $(0,1)$ send $C$ to $(9,5.5)$? Explain how you know.

## Lesson 12 Summary

We can use what we know about slope to decide if a point lies on a line. Here is a line with a few points labeled.




The slope triangle with vertices $(0, 1)$ and $(2, 5)$ gives a slope of $\frac{5-1}{2-0} = 2$. The slope triangle with vertices $(0, 1)$ and $(x, y)$ gives a slope of $\frac{y-1}{x}$. Since these slopes are the same, $\frac{y-1}{x} = 2$ is an equation for the line. So, if we want to check whether or not the point $(11, 23)$ lies on this line, we can check that $\frac{23-1}{11} = 2$. Since $(11, 23)$ is a solution to the equation, it is on the line!

# Unit 2 Lesson 12 Cumulative Practice Problems

1. Select **all** the points that are on the line through $(0, 5)$ and $(2, 8)$.

    A. $(4, 11)$

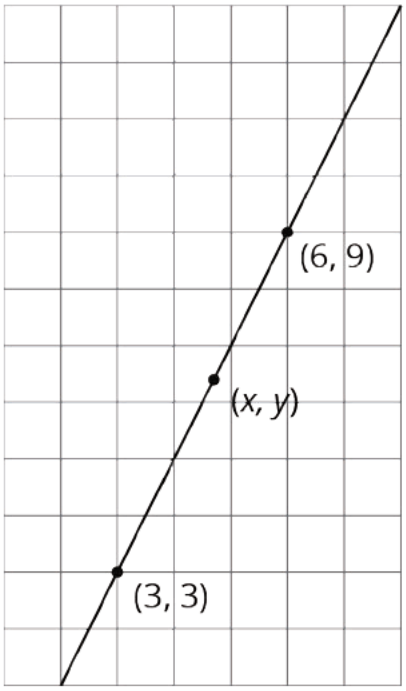
    B. $(5, 10)$

    C. $(6, 14)$

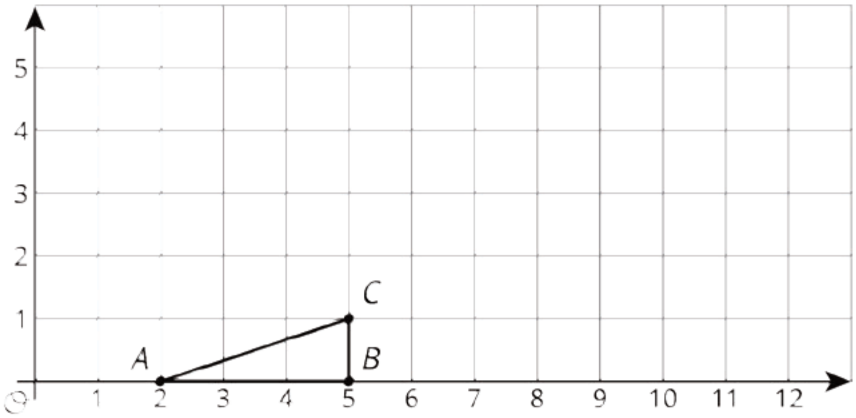
    D. $(30, 50)$

    E. $(40, 60)$

2. All three points displayed are on the line. Find an equation relating $x$ and $y$.



3. Here is triangle $ABC$.




a. Draw the dilation of triangle $ABC$ with center $(2, 0)$ and scale factor 2.

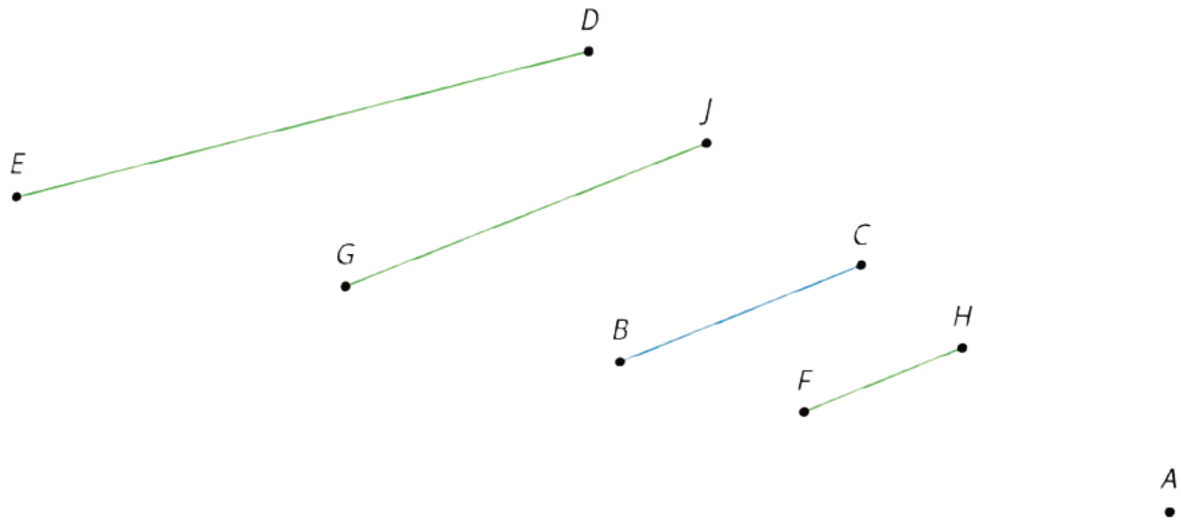
b. Draw the dilation of triangle $ABC$ with center $(2, 0)$ and scale factor 3.

c. Draw the dilation of triangle $ABC$ with center $(2, 0)$ and scale factor $\frac{1}{2}$.

d. What are the coordinates of the image of point $C$ when triangle $ABC$ is dilated with center $(2, 0)$ and scale factor $s$?

e. Write an equation for the line containing all possible images of point $C$.

4. Here are some line segments.




a. Which segment is a dilation of $\overline{BC}$ using $A$ as the center of dilation and a scale factor of $\frac{2}{3}$?

b. Which segment is a dilation of $\overline{BC}$ using $A$ as the center of dilation and a scale factor of $\frac{3}{2}$?

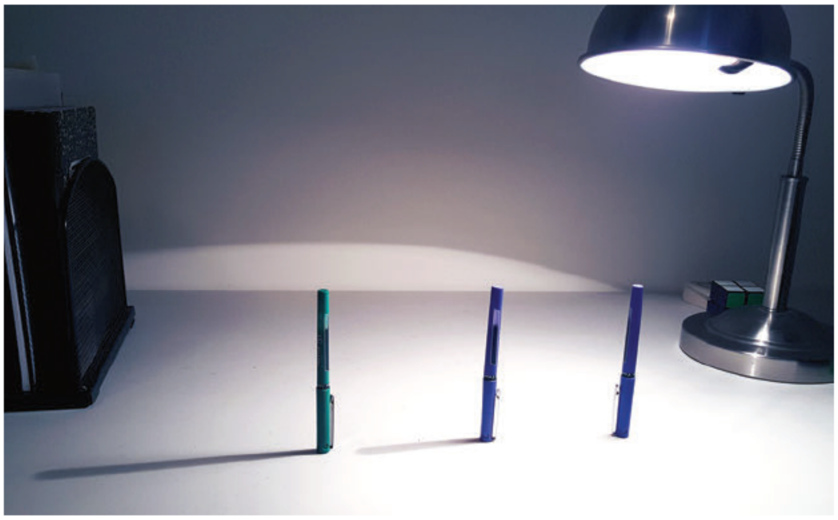
c. Which segment is not a dilation of $\overline{BC}$, and how do you know?

(From Unit 2, Lesson 4.)

# Lesson 13: The Shadow Knows

## 13.1: Notice and Wonder: Long Shadows and Short Shadows

What do you notice? What do you wonder?



## 13.2: Objects and Shadows

Here are some measurements that were taken when the photo was taken. It was impossible to directly measure the height of the lamppost, so that cell is blank.

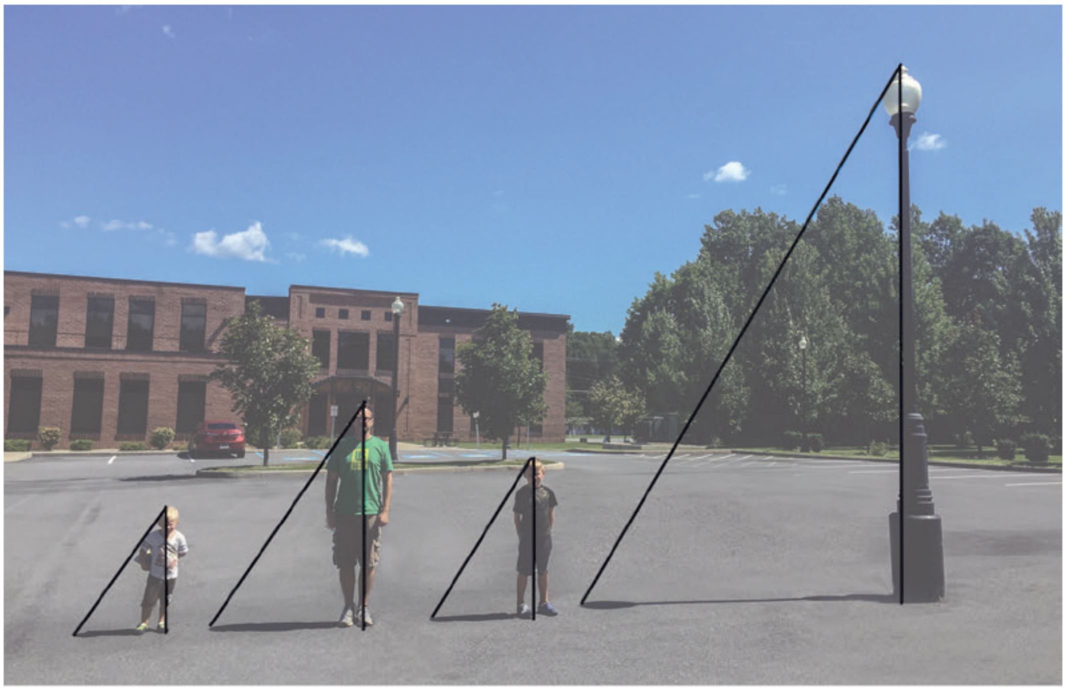
| | height (inches) | shadow length (inches) |
|---|---|---|
| younger boy | 43 | 29 |
| man | 72 | 48 |
| older boy | 51 | 34 |
| lamppost | | 114 |

1. What relationships do you notice between an object's height and the length of its shadow?

2. Make a conjecture about the height of the lamppost and explain your thinking.

## 13.3: Justifying the Relationship

Explain *why* the relationship between the height of these objects and the length of their shadows is approximately proportional.



## 13.4: The Height of a Tall Object

1. Head outside. Make sure that it is a sunny day and you take a measuring device (like a tape measure or meter stick) as well as a pencil and some paper.
2. Choose an object whose height is too large to measure directly. Your teacher may assign you an object.
3. Use what you have learned to figure out the height of the object! Explain or show your reasoning.

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