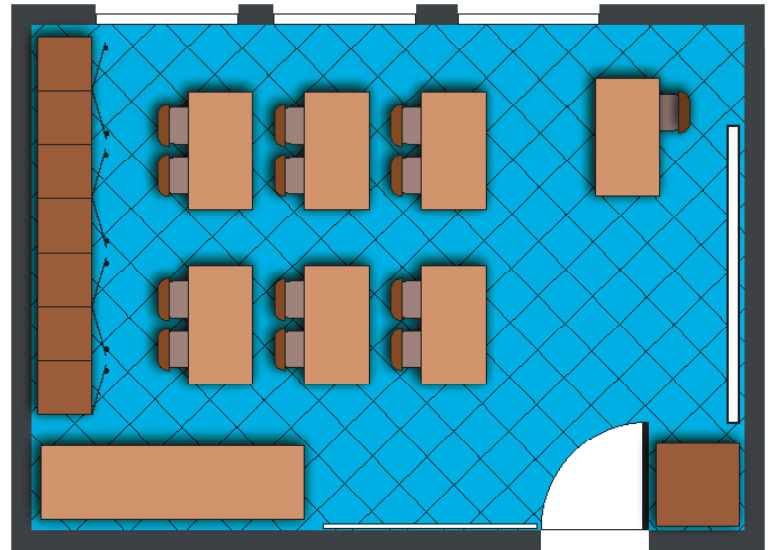




Core Knowledge[®] MATHEMATICS

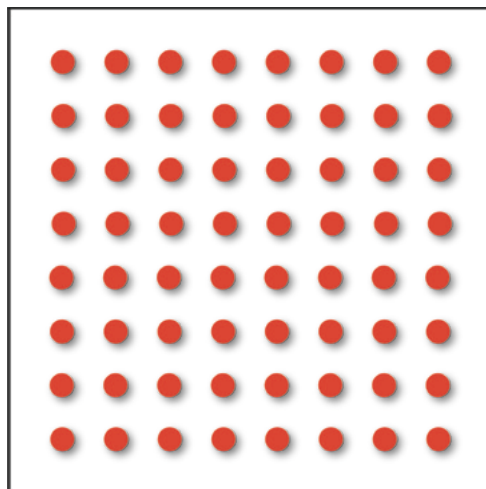
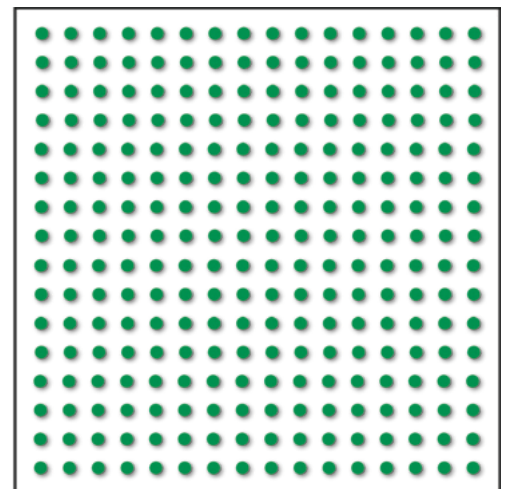
Putting It All Together

Teacher Guide



estimate pencil quantity

Classroom Floor Plan



Dot Density

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Putting it All Together

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Putting it All Together
Teacher Guide
Core Knowledge Mathematics™

Putting it All Together

Unit Narrative

In this optional unit, students use concepts and skills from previous units to solve three groups of problems. In calculating or estimating quantities associated with running a restaurant, e.g., number of calories in one serving of a recipe, expected number of customers served per day, or floor space, they use their knowledge of proportional relationships, interpreting survey findings, and scale drawings. In estimating quantities such as age in hours and minutes or number of times their hearts have beaten, they use measurement conversions and consider accuracy of their estimates. Estimation of area and volume measurements from length measurements introduces considerations of measurement error. In designing a five-kilometer race course for their school, students use their knowledge of measurement and scale drawing. They select appropriate tools and methods for measuring their school campus, build a trundle wheel and use it to make measurements, make a scale drawing of the course on a map or a satellite image of the school grounds, and describe the number of laps, start, and finish of the race.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as justifying, representing, and critiquing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Justify

- reasoning about the nutritional value of recipes (Lesson 1)
- choices and predictions in the context of running a restaurant (Lesson 2)
- reasoning about length, area, and volume in the context of a restaurant (Lesson 4)

Represent

- costs of ingredients in a spreadsheet (Lesson 2)
- situations using expressions and equations (Lesson 7)
- a map of a designed race course (Lesson 13)

Critique

- peer reasoning about calculations of age, heart beats, and hairs (Lesson 6)
- peer reasoning about percent error in length measurement (Lesson 8)
- peer reasoning about percent error in area and volume measurement (Lesson 9)
- peer methods of measuring distance (Lesson 10)

In addition, students are also expected to describe methods for measuring distance, including how to build and use a trundle wheel, and to compare advantages and disadvantages of different methods.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
7.9.1	serving	
7.9.2	spreadsheet cell formula	
7.9.3	profit expense	
7.9.5	population density	
7.9.9		percent error
7.9.11	trundle wheel	
7.9.13		trundle wheel

Required Materials

Blank paper

Compasses

Four-function calculators

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Graph paper

Index cards

Internet-enabled device

Maps or satellite images of the school grounds

Measuring tapes

Measuring tools

Metal paper fasteners

brass brads

Meter sticks

Paper plates

Recipes

Stopwatches

String

Tape

Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Trundle wheels

Yardsticks

Section: Running a Restaurant

Lesson 1: Planning Recipes

Goals

- Create a recipe that meets the requirements to be considered low calorie, low fat, or low sodium, and justify (orally) the reasoning.
- Determine whether one serving of a recipe meets the requirements to be considered low calorie, low fat, or low sodium, and explain (orally) the reasoning.
- Use proportional reasoning to calculate nutritional values of one serving of a recipe.

Lesson Narrative

This lesson is optional. In this lesson, students apply proportional reasoning to calculate nutritional values per one serving of a recipe. The second activity asks students to invent another recipe that meets nutritional requirements to be considered low calorie, low fat, or low sodium. Students likely need to perform various multi-step unit conversions to solve each problem. This context provides students with an opportunity to make sense of problems and persevere in solving them (MP1).

As with all lessons in this unit, all related standards have been addressed in prior units. This lesson provides an *optional* opportunity to go deeper and to make connections between domains. This lesson can be used as an introduction to the context of students planning their own restaurant, which continues through the next few lessons. However, it is also possible to use other lessons about this context without using this lesson as the introduction.

Alignments

Addressing

- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems.
Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR7: Compare and Connect
- MLR8: Discussion Supports

Required Materials

Four-function calculators

Recipes

Required Preparation

Students will need access to a variety of recipes to choose from for this lesson. You can tell students ahead of time to bring in two of their favorite recipes, or have a variety of recipe pages for students to look through, or give students time at the beginning of the lesson to use an internet-enabled device to search online for recipes.

Student Learning Goals

Let's choose some recipes for a restaurant.

1.1 A Recipe for Your Restaurant

Optional: 15 minutes (there is a digital version of this activity)

The purpose of this activity is for students to apply proportional reasoning to scaling down a recipe and calculating the number of calories in one serving. In the next activity (and, if desired, in the next lesson), students will continue working with the recipe that they select in this activity.

The digital version of this activity includes nutritional information about many more ingredients than were printed in students' books.

Addressing

- 7.RP.A.3

Instructional Routines

- MLR8: Discussion Supports

Launch

If desired, explain to students that they are starting a series of activities that are based on the idea of imagining they could open their own restaurant. Provide multiple recipes for students to choose from.

Access for English Language Learners

Reading: MLR8 Discussion Supports. Use this routine to support student understanding of the situation. Explain the meaning of the nutritional values students must calculate, and discuss reasons why they are important factors to consider. Review directions to ensure students understand the connection between the situation and the mathematics of the task.

Design Principle(s): Support sense-making; Cultivate conversation

Anticipated Misconceptions

When calculating the amount of calories from each ingredient, some students may struggle with converting between the units in their recipes and the units given in the tables of nutritional information. Consider displaying conversion information that your students may find helpful, for example 1 cup = 16 tablespoons.

Some students’ recipes may include an ingredient for which the nutrition information is not listed in their books. Help them research the needed information, either from the digital version of this activity or other websites.

Student Task Statement

Imagine you could open a restaurant.

- 1. Select a recipe for a main dish you would like to serve at your restaurant.
- 2. Record the amount of each ingredient from your recipe in the first two columns of the table. You may not need to use every row.

ingredient	amount	amount per serving	calories per serving

- 3. How many servings does this recipe make? Determine the amount of each ingredient in one serving, and record it in the third column of the table.
- 4. Restaurants are asked to label how many calories are in each meal on their menu.
 - a. Use the nutrition information to calculate the amount of calories from each ingredient in your meal, and record it in the last column of the table.
 - b. Next, find the total calories in one serving of your meal.
- 5. If a person wants to eat 2,000 calories per day, what percentage of their daily calorie intake would one serving of your meal be?

Grains

	mass (g)	calories	fat (g)	sodium (mg)
biscuits, refrigerated dough (1)	58	178	6.14	567
bread crumbs (1 oz)	28.35	112	1.5	208
cornmeal (1 c)	157	581	2.75	11
egg noodles (1 c)	38	146	1.69	8
hamburger or hotdog buns (1)	44	123	1.72	217
oats (1 c)	81	307	5.28	5
pasta (1 c)	91	338	1.37	5
pie crust, refrigerated (1)	229	1019	58.3	937
pita bread (1 oz)	28.35	8	0.34	152
quinoa (1 c)	170	626	10.32	8
ramen noodles (1 pkg)	81	356	14.25	1503
rice flour (1 c)	158	578	2.24	0
rice, brown (1 c)	185	679	5.92	9
rice, white (1 c)	185	675	1.22	9
saltine crackers (5)	14.9	62	1.29	140
taco shells (1)	12.9	61	2.81	42
tortillas (1)	49	146	3.71	364
wheat bread (1 slice)	29	79	1.31	137
wheat flour (1 c)	125	455	1.22	2
white bread (1 slice)	29	77	0.97	142

Vegetables

	mass (g)	calories	fat (g)	sodium (mg)
asparagus (1 c)	134	27	0.16	3
avocados (1 c)	150	240	22	10
bell peppers (1 c)	149	46	0.45	6
broccoli (1 c)	91	31	0.34	30
carrots (1 c)	128	52	0.31	88
cauliflower (1 c)	107	27	0.3	32
celery (1 c)	101	16	0.17	81
chives (1 tbsp)	3	1	0.02	0
corn (1 c)	145	125	1.96	22
cucumber (1 c)	133	16	0.21	3
green beans (1 c)	100	31	0.22	6
lettuce (1 c)	47	8	0.14	7
mushrooms (1 c)	70	15	0.24	4
onions (1 c)	160	64	0.16	6
peas, frozen (1 c)	134	103	0.54	145
potatoes ($\frac{1}{2}$ c)	75	59	0.11	14
spinach (1 c)	30	7	0.12	24
squash (1 c)	113	18	0.2	2
sweet potatoes (1 c)	133	114	0.07	73
tomatoes (1 c)	149	27	0.3	7

Fruit

	mass (g)	calories	fat (g)	sodium (mg)
apple juice (1 c)	248	114	0.32	10
apples (1 c)	110	53	0.14	0
bananas (1 c)	225	200	0.74	2
blueberries (1 c)	148	84	0.49	1
cantaloupe (1 c)	177	60	0.34	28
cherries (1 c)	138	87	0.28	0
cranberries, dried ($\frac{1}{4}$ c)	40	123	0.44	2
grapes (1 c)	151	104	0.24	3
lemon juice (1 c)	244	54	0.59	2
Mandarin oranges (1 c)	252	154	0.25	15
mangoes (1 c)	165	99	0.63	2
orange juice (1 c)	249	122	0.3	5
oranges (1 c)	180	85	0.22	0
peaches (1 c)	154	60	0.38	0
pears (1 c)	140	80	0.2	1
pineapple, canned (1 c)	181	109	0.2	2
pomegranate juice (1 c)	1249	134	0.72	22
raisins (1 c)	165	493	0.76	18
raspberries (1 c)	123	64	0.8	1
strawberries (1 c)	152	49	0.46	2

Meat

	mass (g)	calories	fat (g)	sodium (mg)
bacon (1 slice)	26	106	10.21	122
chicken thigh (1)	193	427	32.06	156
chicken, light meat (3 oz)	85	100	1.45	60
cob (3 oz)	85	61	0.17	93
crab (3 oz)	85	73	0.82	251
ground beef (4 oz)	113	375	33.9	75
ground turkey (4 oz)	113	172	9.44	80
halibut (3 oz)	85	77	1.13	58
ham (1 oz)	28.35	38	1.53	319
hot dogs (1)	51	141	12.33	498
lobster (1)	150	116	1.12	634
pepperoni (3 oz)	85	428	39.34	1345
pork sausage (1)	25	72	6.2	185
pork tenderloin (3 oz)	85	102	3	44
salmon (1 fillet)	108	373	12.34	55
shrimp (3 oz)	85	72	0.43	101
tofu ($\frac{1}{2}$ c)	126	98	5.25	15
trout (1 fillet)	79	111	4.88	40
tuna, canned (1 oz)	28.35	24	0.27	70
turkey (3 oz)	85	92	2.12	105

Nuts, Beans, and Seeds

	mass (g)	calories	fat (g)	sodium (mg)
almonds (1 c)	143	828	71.4	1
black beans (1 c)	240	218	0.7	331
cashews (1 oz)	28.35	157	12.43	3
chickpeas (1 c)	240	211	4.68	667
coconut (1 c)	80	283	26.8	16
fava beans (1 c)	256	182	0.56	1160
flaxseed (1 tbsp)	10.3	55	4.34	3
white beans (1 c)	262	299	1.02	969
kidney beans (1 c)	256	215	1.54	758
lentils (1 c)	192	676	2.04	12
lima beans (1 c)	164	216	0.72	85
macadamia nuts (1 c)	134	962	101.53	7
peanut butter (2 tbsp)	32	191	16.22	136
peanuts (1 oz)	28.35	166	14.08	116
pecans (1 c)	109	753	78.45	0
pinto beans (1 c)	240	197	1.34	643
pistachios (1 c)	123	689	55.74	1
pumpkin seeds (1 c)	129	721	63.27	9
sesame seeds (1 c)	144	825	71.52	16
sunflower seeds (1 c)	46	269	23.67	4

Dairy

	mass (g)	calories	fat (g)	sodium (mg)
almond milk (1 c)	262	39	2.88	186
blue cheese (1 oz)	28.35	100	8.15	325
butter (1 pat)	5	36	4.06	1
cheddar cheese (1 c)	132	533	43.97	862
coconut milk (1 c)	226	445	48.21	29
cream cheese (1 tbsp)	14.5	51	4.99	46
egg white (1)	33	17	0.06	55
egg yolk (1)	17	55	4.51	8
eggs (1)	50	72	4.76	71
evaporated milk (1 c)	252	270	5.04	252
whipping cream (1 c)	120	408	43.3	32
margarine (1 tbsp)	14.2	101	11.38	4
milk, skim (1 c)	245	83	0.2	103
milk, whole (1 c)	244	149	7.93	105
mozzarella cheese (1 c)	132	389	26.11	879
Parmesan cheese (1 c)	100	420	27.84	1804
sour cream (1 tbsp)	12	16	1.27	10
soy milk (1 c)	243	80	3.91	90
Swiss cheese (1 c)	132	519	40.91	247
yogurt (6 oz)	170	107	2.64	119

Sauces and Other Liquids

	mass (g)	calories	fat (g)	sodium (mg)
barbecue sauce (1 tbsp)	17	29	0.11	175
chicken broth (1 c)	249	15	0.52	924
cream of chicken soup ($\frac{1}{2}$ c)	126	113	7.27	885
gravy (1 c)	233	1	5.5	1305
honey (1 c)	339	25	0	14
Italian dressing (1 tbsp)	14.7	35	3.1	146
jams and jellies (1 tbsp)	20	56	0.01	6
ketchup (1 tbsp)	17	17	0.02	154
mayonnaise (1 tbsp)	15	103	11.67	73
mustard (1 tsp)	5	3	0.17	55
pasta sauce ($\frac{1}{2}$ c)	132	66	2.13	577
ranch dressing (1 tbsp)	15	64	6.68	135
salsa (2 tbsp)	36	10	0.06	256
soy sauce (1 tbsp)	16	8	0.09	879
vanilla extract (1 tsp)	4.2	12	0	0
vegetable broth (1 c)	221	11	0.15	654
vegetable oil (1 tbsp)	14	124	14	0
vinegar (1 tbsp)	15	3	0	0
water (1 fl oz)	29.6	0	0	1
Worcestershire sauce (1 tbsp)	17	13	0	167

Spices and Other Powders

	mass (g)	calories	fat (g)	sodium (mg)
baking powder (1 tsp)	4.6	2	0	488
baking soda (1 tsp)	4.6	0	0	1259
black pepper (1 tsp)	2.3	6	0.07	0
chicken bouillon (1 cube)	4.8	10	0.23	1152
chili powder (1 tsp)	2.7	8	0.39	77
cinnamon (1 tsp)	2.6	6	0.03	0
cocoa powder (1 c)	86	196	11.78	18
cornstarch (1 c)	128	488	0.06	12
cumin (1 tsp)	2.1	8	0.47	4
garlic (1 clove)	3	4	0.01	0.5
garlic powder (1 tsp)	3.1	10	0.02	2
onion powder (1 tsp)	2.4	8	0.02	2
onion soup mix (1 tbsp)	7.5	22	0.03	602
oregano (1 tsp)	1	3	0.04	0
paprika (1 tsp)	2.3	6	0.3	2
parsley (1 tsp)	0.5	1	0.03	2
powdered sugar (1 c)	120	467	0	2
salt (1 tsp)	6	0	0	2325
sugar (1 tsp)	2.8	11	0	0
taco seasoning (2 tsp)	5.7	18	0	411

Student Response

Answers vary. Sample responses:

1. spaghetti with meat sauce

2.

ingredient	amount	amount per serving	calories per serving
spaghetti noodles	6 c	1 c	338
ground beef	1 lb	$\frac{1}{6}$ lb	250
pasta sauce	3 c	$\frac{1}{2}$ c	66
onion	$\frac{1}{2}$ c	$\frac{1}{12}$ c	5.3
garlic	2 cloves	$\frac{1}{3}$ clove	1.5
olive oil	$1\frac{1}{2}$ tbsp	$\frac{1}{4}$ tbsp	30
basil	2 tsp	$\frac{1}{3}$ tsp	0.7
oregano	2 tsp	$\frac{1}{3}$ tsp	1

3. 6 servings

4. a. see table

b. 692.5 calories, because $338 + 250 + 66 + 5.3 + 1.5 + 30 + 0.7 + 1 = 692.5$

5. about 35%, because $692.5 \div 2000 = 0.34625$

Are You Ready for More?

The labels on packaged foods tell how much of different nutrients they contain. Here is what some different food labels say about their sodium content.

- cheese crackers, 351 mg, 14% daily value
- apple chips, 15 mg, <1% daily value
- granola bar, 82 mg, 3% daily value

Estimate the maximum recommended amount of sodium intake per day (100% daily value). Explain your reasoning.

Student Response

Answers vary. Sample response: Somewhere around 2500 mg, because $351 \div 0.14 = 2507\frac{1}{7}$.

Activity Synthesis

Ask students to trade with a partner and check each other's work. Poll the class on the amount of calories in one serving of their meal.

Consider asking the following questions:

- "What was the most difficult part of calculating the amount of calories in one serving of your meal?"
- "Did anything surprise you while you were doing your calculations?"

1.2 Health Claims

Optional: 20 minutes

The purpose of this activity is for students to apply proportional reasoning to calculate the calories, fat, and sodium content of one serving of a recipe.

Addressing

- 7.RP.A.3

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR7: Compare and Connect

Launch

Tell students they will continue using the tables of nutrition information from the previous activity. Point out that the qualifications for a food to be considered "low calorie," "low fat," or "low sodium" are all stated per 100 grams of food. Before students start working, consider giving them 30 seconds of quiet think time and then having them share their ideas on how they could solve the first problem.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.

Supports accessibility for: Memory; Conceptual processing

Anticipated Misconceptions

Some students may find the total calories, fat, and sodium in one serving of their recipe and ignore the specification about per 100 grams of food. Prompt them to tabulate the grams of each ingredient in one serving of their recipe.

Student Task Statement

For a meal to be considered:

- “low calorie”—it must have 120 calories or less per 100 grams of food.
- “low fat”—it must have 3 grams of fat or less per 100 grams of food.
- “low sodium”—it must have 140 milligrams of sodium or less per 100 grams of food.

1. Does the meal you chose in the previous activity meet the requirements to be considered:
- a. low calorie?
 - b. low fat?
 - c. low sodium?

Be prepared to explain your reasoning.

2. Select or invent another recipe you would like to serve at your restaurant that does meet the requirements to be considered either low calorie, low fat, or low sodium. Show that your recipe meets that requirement. Organize your thinking so it can be followed by others.

ingredient	amount per serving	calories per serving	fat per serving	sodium per serving

Student Response

- Answers vary. Sample response: One serving of the spaghetti with meat sauce recipe (from the previous activity) is 316.5 grams of food because $91 + 75.3 + 132 + 13.3 + 1 + 3.4 + 0.2 + 0.3 = 316.5$.
 - No. It has 218.8 calories per 100 grams of food because $692.5 \div 3.165 = 218.8$.
 - No. It has 15.4 grams of fat per 100 grams of food because $48.7 \div 3.165 = 15.4$.
 - No. It has 200 milligrams of sodium per 100 grams of food because $633 \div 3.165 = 200$.
- Answers vary. Sample response: Southwest salad meets the requirements for all 3 categories.

ingredient	amount per serving	calories per serving	fat per serving	sodium per serving	grams
lettuce	3 c	24	0.4	12	141
black beans	$\frac{1}{2}$ c	109	0.3	165	120
corn	$\frac{1}{2}$ c	63	1	11	72
tomatoes	$\frac{1}{2}$ c	14	0.1	3.5	75
avocado	$\frac{1}{8}$	30	2.8	1.3	19
shallots	$\frac{1}{2}$ tbsp	4	0	0.5	5
garlic	$\frac{1}{4}$ clove	1	0	0.1	1
lime juice	$\frac{1}{2}$ tsp	1	0.01	0.04	5
yogurt	1 tbsp	9	0.2	10	14
cilantro	$\frac{1}{8}$ c	0.5	0.01	1	2

One serving of the Southwest salad is 454 grams of food because $141 + 120 + 72 + 75 + 19 + 5 + 1 + 5 + 14 + 2 = 454$. It has only 56.3 calories per 100 grams of food because $255.5 \div 4.54 = 56.3$. It has only 1.06 grams of fat per 100 grams of food because $4.82 \div 4.54 = 1.06$. It has only 45 milligrams of sodium per 100 grams of food because $204.4 \div 4.54 = 45$.

Activity Synthesis

Ask students to take turns explaining to a partner how they know that their meal meets the requirements to be considered either “low calorie,” “low fat,” or “low sodium.” If time permits, consider using MLR 1 (Stronger and Clearer Each Time).

Consider asking:

- “What strategies did you find helpful for making sure that your meal met the requirements to be considered either ‘low calorie,’ ‘low fat,’ or ‘low sodium.’?”

Access for English Language Learners

Representing, Conversing: MLR7 Compare and Connect. After students complete the calculations for their new recipe, use this routine to help students compare recipes with the same requirements. Group students according to requirement they selected (“low calorie,” “low fat,” or “low sodium”). Invite groups to share how they completed recipe tables and ask, “What is the same and what is different?” about their strategies. This will help students connect language and reasoning when creating their menus and calculating the nutritional information.

Design Principle(s): Cultivate conversation; Maximize meta-awareness

Lesson 2: Costs of Running a Restaurant

Goals

- Comprehend the term “spreadsheet” (in written and spoken language) is a computer program in which data is arranged in the rows and columns of a grid and can be manipulated and used in calculations.
- Create formulas in a spreadsheet to perform repeated calculations.
- Use a spreadsheet to calculate the cost per serving of all the ingredients in a recipe.

Lesson Narrative

This lesson is optional. In this lesson, students gain experience using a spreadsheet program to perform repeated calculations. After exploring how spreadsheets work, students calculate the cost per serving of all the ingredients in a recipe. Students make use of structure (MP7) as they explore the syntax of how formulas are written in spreadsheet programs. Students see that spreadsheets are an appropriate tool (MP5) for organizing and performing repeated calculations. Students reason quantitatively (MP2) while deciding what formula they need for each column in their spreadsheet.

As with all lessons in this unit, all related standards have been addressed in prior units. This lesson provides an *optional* opportunity to go deeper and make connections between domains. If students have already completed the previous lesson, they can continue using the same recipes in this lesson. However, it is also possible to do this lesson without having completed the previous lesson.

Alignments

Addressing

- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems.
Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Instructional Routines

- MLR8: Discussion Supports

Required Materials

Internet-enabled device

Recipes

Required Preparation

Prepare to provide each student access to a spreadsheet program. It is recommended for them to use a program with which you have previous experience.

If students have not completed the previous lesson or do not still have access to the recipes they used for it, provide access to recipes.

Student Learning Goals

Let's explore how much the food will cost.

2.1 Introducing Spreadsheets

Optional: 15 minutes

The purpose of this activity is for students to learn about formulas in spreadsheets through hands-on experience. This exploration prepares students for using a spreadsheet to calculate the cost of recipe ingredients in the next activity.

Launch

Explain what spreadsheets are and how they work, including the following features:

- A spreadsheet is a computer program that lets you organize information in a grid of rectangles, called cells, and to do calculations. There are many different spreadsheet programs, but they all work in basically the same way. The rows are labeled with numbers and the columns are labeled with letters.
- Display this image of a sample spreadsheet.

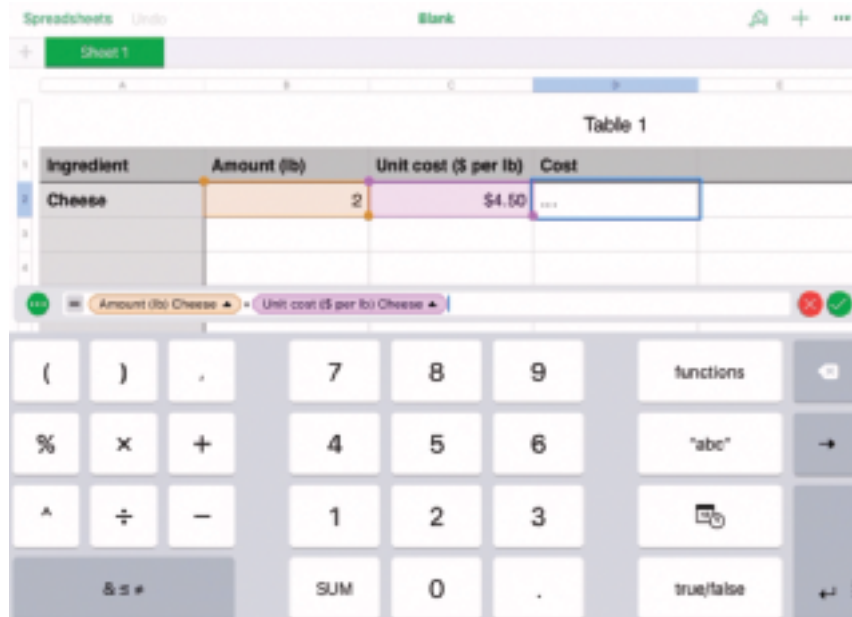
	A	B	C	D
1	Ingredient	Weight (lb)	Unit cost (\$ per lb)	Cost (\$)
2	Cheese	3.6	\$4.50	\$16.20
3	Raisins	1.6	\$2.50	\$4.00
4	Peanuts	2	\$3.00	\$6.00

Ask the students "What information is in cell B3? What does this piece of information tell us about the situation?" (1.6, how many of pounds of raisins).

- It is helpful to put headings in the first row that describe what information goes in each column (and headings in the first column to describe each row). It is also helpful to organize the information in the spreadsheet to be read from left to right and from top to bottom.
- If you type a formula correctly into any cell of a spreadsheet, the program will calculate and display the value of the expression. Formulas must start with an equal sign. If the answer is not a whole number, the spreadsheet will display the value as a decimal. It is possible to program each cell to round decimal values.

Note that spreadsheets on tablets are a bit different.

- The "enter" key on the keyboard is usually at the lower right, marked with a right-angle arrow pointing down and left.
- The Numbers spreadsheet involves less typing. Instead of typing = and a formula, tap the = button; tap a cell whose address you want in the formula; type numbers and operations. If you have typed headings, the words will appear in the formula, such as Amount (lb) Cheese.



Provide access to spreadsheets. Give students quiet work time followed by whole-class discussion.

Anticipated Misconceptions

Some students may not understand what happened when they completed the last instruction. The fact that copying a formula from one cell and pasting it in another cell updates the row and column references in the formula can be counter-intuitive. Prompt students to double click on the cell where they pasted the formula so they can see what the formula now says.

Student Task Statement

1. Type each formula into the cells of a spreadsheet program and press enter. Record what the cell displays. Make sure to type each formula exactly as it is written here.

	A	B	C	D
1	=40-32	=1.5+3.6	=14/7	=0.5*6

2.
 - a. Predict what will happen if you type the formula =A1*C1 into cell C2 of your spreadsheet.
 - b. Type in the formula, and press enter to check your prediction.
3.
 - a. Predict what will happen next if you delete the formula in cell A1 and replace it with the number 100.
 - b. Replace the formula with the number, and press enter to check your prediction.
4.
 - a. Predict what will happen if you copy cell C2 and paste it into cell D2 of your spreadsheet.

- b. Copy and paste the formula to check your prediction.

Student Response

- Cell A1 displays the number 8 because that is the answer to the subtraction problem $40 - 32$
 - Cell B1 displays the number 5.1 because that is the answer to the addition problem $1.5 + 3.6$
 - Cell C1 displays the number 2 because that is the answer to the division problem $14 \div 7$. Division is represented with the slash character in spreadsheet programs.
 - Cell D1 displays the number 3 because that is the answer to the multiplication problem $(0.5) \cdot 6$. Multiplication is represented with the asterisk symbol in spreadsheet programs.
2. Cell C2 displays the number 16 because it is multiplying the value in cell A1 times the value in cell C1 and $8 \cdot 2 = 16$.
3. Cell A1 just displays the number 100 because we entered a number, not a formula, but cell C2 updates to display 200, even though we didn't do anything to that cell, because it is still multiplying the values in cell A1 times C1, and A1 is now 100 instead of 8, so $100 \cdot 2 = 200$.
4. Cell D2 displays the number 15.3 because it is multiplying the value in cell B1 times the value in cell D1 and $(5.1) \cdot 3 = 15.3$. When the formula in C2 was copied and pasted into D2, it moved one cell to the right, so the letters in the formula automatically adjusted to refer to one cell to their right. In other words, A1 changed to B1 and C1 changed to D1. If we had pasted the formula into a different row, above or below, the numbers as well as the letters in the cell addresses would have updated.

Activity Synthesis

If possible, display a spreadsheet program and go through the steps described in the students' books or devices. Ask students to explain why each step has the result it does. (See student response for explanations.)

The most important things for students to remember from this activity are:

- Formulas in spreadsheets start with the equal sign and use $*$ and $/$ for multiplication and division, respectively.
- You can refer to the value in another cell of the spreadsheet within your formulas. For example, the formula $=A3+B2$ will display the sum of the values in cells A3 and B2, as long as those cells contain just numbers and no words.
- If you copy a formula from one cell and paste it into another cell, the program will automatically adjust any cell addresses in the formula by the number of rows and columns between the cells where the formula was copied from and pasted into.

2.2 Cost per Serving

Optional: 30 minutes

In this activity, students use a spreadsheet program to compute the cost for one serving of each meal they want to serve at their restaurant. Students have to find or estimate the cost of each ingredient. Since ingredients are sold in different units, they have to convert units from the ones they find for the cost to the ones used in the recipe. For example, olive oil is sold in 1 quart bottles, but the recipe asks for tablespoons. The calculations can be done in the spreadsheet, and entering the formulas into the cells of the spreadsheet is an important mathematical step.

Another important step in this activity is to plan the set-up of the spreadsheet.

Addressing

- 7.RP.A.3

Instructional Routines

- MLR8: Discussion Supports

Launch

If possible, have students refer back to the recipes they selected in the previous lesson. Otherwise, provide recipes for students to refer to here. Give students quiet work time followed by partner and whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge about measurement. Students may benefit from watching a quick demonstration of the set-up of the spreadsheet. Review terms such as: spreadsheet formulas and converting units.

Supports accessibility for: Memory; Conceptual processing

Access for English Language Learners

Speaking, Listening: MLR8 Discussion Supports. Use this routine to support understanding and use of the language of spreadsheets, including the terms “cell,” “column,” “row,” “formula,” “copy,” and “paste.” Invite students to use these words when stating their ideas. Ask students to chorally repeat the phrases that include these words in context of the problem. This helps students use spreadsheet terms while verbalizing calculation predictions.

Design Principle(s): Optimize output (for explanation); Maximize meta-awareness

Anticipated Misconceptions

Some students may struggle to know what formulas they should type into columns D, H, J, and K. Prompt them to write down on paper what calculation they would do for the first one or two ingredients and look for patterns that could help them figure out the other rows automatically.

Student Task Statement

1. Set up a spreadsheet with these column labels in the first row.

	A	B	C	D
1	ingredient	unit in recipe	amount in recipe	amount per serving
2				

- a. Type the information about the ingredients in your recipe into the first 3 columns of the spreadsheet.
 - b. Type a formula into cell D2 to automatically calculate the amount per serving for your first ingredient.
 - c. Copy cell D2 and paste it into the cells beneath it to calculate the amount per serving for the rest of your ingredients. Pause here so your teacher can review your work.
2. Add these column labels to your spreadsheet.

E	F	G	H
purchase price	purchase amount	purchase unit	cost per purchase unit

- a. Research the cost of each ingredient in your meal, and record the information in columns E, F, and G.
 - b. Type a formula into cell H2 to automatically calculate the cost per purchase unit for your first ingredient.
 - c. Copy cell H2, and paste it into the cells beneath it to calculate the cost per purchase unit for the rest of your ingredients.
3. Add these column labels to your spreadsheet.

I	J	K
conversion from purchase units to recipe units	cost per recipe unit	cost per serving

- Complete column I with how many of your recipe unit are in 1 of your purchase unit for each ingredient. For example, if your recipe unit was cups and your purchase unit was gallons, then your conversion would be 16 because there are 16 cups in 1 gallon.
 - Type a formula into cell J2 to calculate the cost per recipe unit for your first ingredient.
 - Type a formula into cell K2 to calculate the cost per serving for your first ingredient.
 - Compare formulas with your partner. Discuss your thinking. If you disagree, work to reach an agreement.
 - Copy cells J2 and K2, and paste them into the cells beneath them to calculate the cost per recipe unit and cost per serving for the rest of your ingredients.
- Type a formula into the first empty cell below your last ingredient in column K to calculate the total cost per serving for all of the ingredients in your recipe. Record the answer here.

Student Response

Answers vary. Sample response:

	A	B	C	D	E	F	G	H	I	J	K
1	ingredient	units in recipe	amount in recipe	amount per serving	purchase price	purchase amount	purchase units	cost per purchase unit	conversion factor from purchase units to recipe units	cost per recipe unit	cost per serving
2	spaghetti noodles	c	6	1	\$1.99	6 c		\$0.33	1	\$0.33	\$0.33
3	ground beef	lb	1	0.166667	\$3.99	1 lb		\$3.99	1	\$3.99	\$0.67
4	pasta sauce	c	3	0.5	\$1.99	4 c		\$0.50	1	\$0.50	\$0.25
5	onion	c	0.5	0.083333	\$0.99	1 onion		\$0.99	1	\$0.99	\$0.08
6	garlic	cloves	2	0.333333	\$0.50	1 head		\$0.50	12	\$0.04	\$0.01
7	olive oil	tbsp	1.5	0.25	\$3.99	1 quart		\$3.99	64	\$0.06	\$0.02
8	basil	tsp	2	0.333333	\$4.99	1 c		\$4.99	48	\$0.10	\$0.03
9	oregano	tsp	2	0.333333	\$4.99	1 c		\$4.99	48	\$0.10	\$0.03
10									Total		\$1.43
11											

Here are the formulas that were used to create this sample spreadsheet:

D	H	J	K
=C2/6	=E2/F2	=H2/I2	=J2*D2
=C3/6	=E3/F3	=H3/I3	=J3*D3
etc.	etc.	etc.	etc.
			=sum(K2:K9)

The formulas in rows 4–9 follow the same pattern as those shown for rows 2 and 3.

The formulas in column D use 6 as the divisor here because the sample recipe made 6 servings. This number needs to be adjusted to match the number of servings in the student's recipe.

If students do not know about the formula shown for cell K10 above, they could also use $=K2+K3+K4+K5+K6+K7+K8+K9$ to accomplish the calculation.

Activity Synthesis

Poll the class on the cost for one serving of their recipe.

Invite student to share their experiences using the spreadsheet:

- "What was the most difficult part of setting up your spreadsheet to do these calculations for you?"
- "What strategies did you use to help you decide on the formulas for columns D, H, J, and K?"
- "Which method do you prefer for this type of problem, using paper, pencil, and a calculator or using a spreadsheet? Why?"

Lesson 3: More Costs of Running a Restaurant

Goals

- Create an equation to represent certain expenses for a restaurant, and interpret (orally and in writing) the solution.
- Determine whether a relationship is proportional and explain (orally) the reasoning.
- Determine whether a restaurant is making a profit using estimates of ongoing expenses, number of meals sold, average price per meal, and average cost per meal.

Lesson Narrative

This lesson is optional. In this lesson, students apply expressions and signed numbers to the context of balancing projected income and expenses for a restaurant. Students decide how to model these incomes and expenses in their calculations (MP4).

As with all lessons in this unit, all related standards have been addressed in prior units. This lesson provides an *optional* opportunity to go deeper and make connections between domains.

Alignments

Addressing

- 7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
- 7.NS.A.3: Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.
- 7.RP.A.2: Recognize and represent proportional relationships between quantities.
- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.
- 7.SP.B.4: Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR8: Discussion Supports
- Notice and Wonder

Student Learning Goals

Let's explore how much it costs to run a restaurant.

3.1 Are We Making Money?

Optional: 20 minutes

The purpose of this activity is for students to choose how they can apply math concepts and strategies to a problem arising in a real-world context: predicting whether a restaurant will make a profit.

After students have estimated the monthly cost of their ongoing expenses on their list, poll the class on their total estimated monthly costs.

If students have completed the previous lesson about using spreadsheets to calculate the cost of ingredients for one serving of their recipe, they can use that as the basis for calculating the percentage of the markup. If students have not calculated the cost of ingredients in a previous lesson, then tell students that the *ingredients* for one serving of a meal typically cost about:

- \$2 for a fast-food restaurant
- \$5 for a casual sit-down restaurant
- \$10 to \$25 for a formal sit-down restaurant

Addressing

- 7.NS.A.3
- 7.RP.A.3

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Point out that restaurants have many more expenses than just the cost of the food. Give students quiet work time followed by partner discussion.

Consider having students pause their work after the first question so that you can record their answers displayed for all to see, and all students will have access to this same information for the rest of the activity.

Access for English Language Learners

Speaking, Listening, Writing: MLR1 Stronger and Clearer Each Time. Use this routine after students have assessed the profitability of Restaurant A. Ask students to share their thinking with 2–3 consecutive partners. With each share, encourage listeners to push students for clarity in mathematical language (e.g., “How do you know the restaurant is making a profit?”). With each share, students’ organization of steps should get stronger and the explanations of reasoning should get clearer. This helps students to use mathematical language as they explain their profit analysis.

Design Principle(s): Optimize output (for explanation); Cultivate conversation

Student Task Statement

1. Restaurants have many more expenses than just the cost of the food.
 - a. Make a list of other items you would have to spend money on if you were running a restaurant.
 - b. Identify which expenses on your list depend on the number of meals ordered and which are independent of the number of meals ordered.
 - c. Identify which of the expenses that are independent of the number of meals ordered only have to be paid once and which are ongoing.
 - d. Estimate the monthly cost for each of the ongoing expenses on your list. Next, calculate the total of these monthly expenses.
2. Tell whether each restaurant is making a profit or losing money if they have to pay the amount you predicted in ongoing expenses per month. Organize your thinking so it can be followed by others.
 - a. Restaurant A sells 6,000 meals in one month, at an average price of \$17 per meal and an average cost of \$4.60 per meal.
 - b. Restaurant B sells 8,500 meals in one month, at an average price of \$8 per meal and an average cost of \$2.20 per meal.
 - c. Restaurant C sells 4,800 meals in one month, at an average price of \$29 per meal and an average cost of \$6.90 per meal.
3.
 - a. Predict how many meals your restaurant would sell in one month.
 - b. How much money would you need to charge for each meal to be able to cover all the ongoing costs of running a restaurant?
4. What percentage of the cost of the ingredients is the markup on your meal?

Student Response

1. Answers vary. Sample response:

- Expenses that depend on the number of meals sold: ingredients, disposable dishes or dish washing
- One-time expenses: kitchen appliances, tables, chairs, decor, cash register, menus, uniforms, etc.
- Other ongoing expenses that don't depend on the number of meals sold:

expense	estimated cost (dollars per month)
rent	2000
maintenance	400
phone	100
utilities	500
insurance	300
payroll services	100
salaries	8000
permits	150
cleaning/laundry	500
total fixed expenses	12,050

2. Answers vary depending on the estimated monthly operating costs.

- a. Restaurant A makes a profit if their total fixed expenses are less than \$74,400 per month, because $(17 + -4.60) \cdot 6,000 = 74,400$.
- b. Restaurant B makes a profit if their total fixed expenses are less than \$49,300 per month, because $(8 + -2.20) \cdot 8,500 = 49,300$.
- c. Restaurant C makes a profit if their total fixed expenses are less than \$106,080 per month, because $(29 + -6.90) \cdot 4,800 = 106,080$.

3. Answers vary. Sample response:

- a. About 6,000 meals per month
- b. At least \$7.00, because the inequality $6,000(x + -5) > 12,050$ can represent the average menu price x for which the restaurant will make a profit.

4. Answers vary. Sample response: At least 40%, because $7 \div 5 = 1.4$, which means the menu price needs to be at least 140% of the cost of the ingredients to have enough money to cover the restaurant's other operating costs.

Activity Synthesis

Poll the class on whether they think each of the restaurants A, B, and C made money for the month. Select students to share their reasoning.

Note: The way the problem is written, there is not just one correct answer to the question. Whether or not each restaurant made money depends on how much the students estimated for the total monthly expenses (excluding food).

if students estimated the total ongoing expenses (excluding food) to be:	restaurants that would have made money	restaurants that would have lost money
below \$49,300	A, B, and C	none
between \$49,300 and \$74,400	A and C	B
between \$74,400 and \$106,080	C	A and B
above \$106,080	none	A, B, and C

Next, ask students to trade with a partner, and check their work for calculating the percentage of the mark up.

Ask students to discuss:

- "How does the amount you plan to charge for each meal compare to your partner's amount?"
- "How does your mark up percentage compare to your partner's percentage?"

Consider telling students that many restaurant owners use 300% as an estimate of a good percentage for the mark up on their meals to be able to make a profit. That means the price of the meal is 4 times what the cost of the ingredients were.

3.2 Disposable or Reusable?

Optional: 20 minutes

The purpose of this activity is for students to write and solve equations as a strategy to compare the projected costs of using reusable versus disposable plates and forks. First, students examine dot plots representing the average number of customers served per day at a sample of restaurants to make a prediction about how many customers they might serve per day. Then, students see that the cost of buying disposable plates and forks can be modeled with a proportional relationship,

while the cost of buying and washing reusable plates and forks can be modeled with an equation in the form $px + q = r$.

Addressing

- 7.EE.B.4
- 7.RP.A.2
- 7.SP.B.4

Instructional Routines

- MLR8: Discussion Supports
- Notice and Wonder

Launch

Display the dot plots about the number of customers served per day. Invite students to share what they notice and wonder.

Some things students may notice:

- Many of the fast food restaurants serve more customers per day than the full service restaurants.
- There is a lot of overlap between the two distributions, from 300 to 600 customers.

Some things students may wonder:

- Were the restaurants included in the samples selected at random?
- Is there a meaningful difference between the average number of customers served at these two types of restaurants?
- About how many customers would my restaurant serve per day?

Give students quiet work time followed by whole-class discussion.

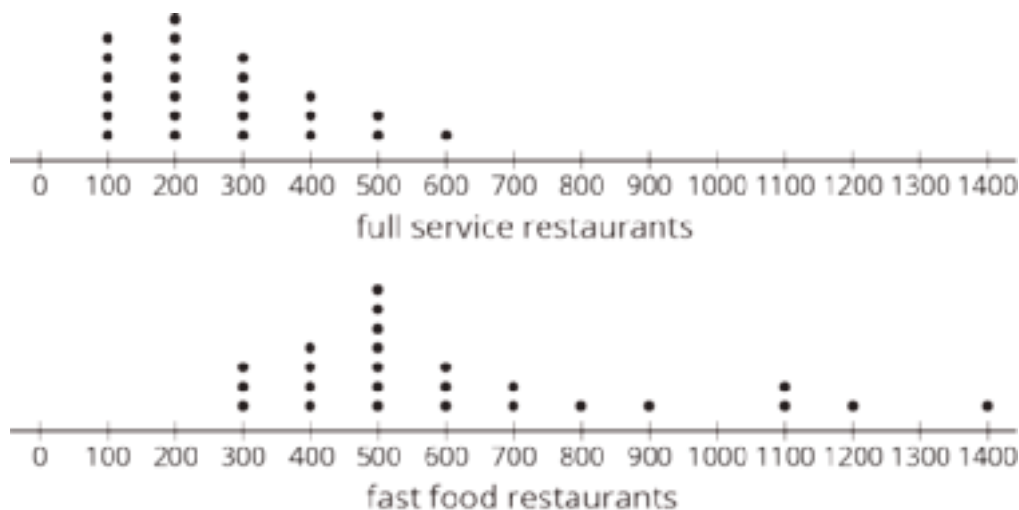
Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts. After students have solved the first 2-3 problems, check-in with either select groups of students or the whole class. Invite students to share the strategies they have used so far as well as any questions they have before continuing.

Supports accessibility for: Organization; Attention

Student Task Statement

A sample of full service restaurants and a sample of fast food restaurants were surveyed about the average number of customers they serve per day.



1. How does the average number of customers served per day at a full service restaurant generally compare to the number served at a fast food restaurant? Explain your reasoning.
2. About how many customers do you think your restaurant will serve per day? Explain your reasoning.
3. Here are prices for plates and forks:

	plates	forks
disposable	165 paper plates for \$12.50	600 plastic forks for \$10
reusable	12 ceramic plates for \$28.80	24 metal forks for \$30

- a. Using your predicted number of customers per day from the previous question, write an equation for the total cost, d , of using disposable plates and forks for every customer for n days.
 - b. Is d proportional to n ? Explain your reasoning.
 - c. Use your equation to predict the cost of using disposable plates and forks for 1 year. Explain any assumptions you make with this calculation.
4. a. How much would it cost to buy enough reusable plates and forks for your predicted number of customers per day?

- 
- b. If it costs \$10.75 a day to wash the reusable plates and forks, write an expression that represents the total cost, r , of buying and washing reusable plates and forks after n days.
 - c. Is r proportional to n ? Explain your reasoning.
 - d. How many days can you use the reusable plates and forks for the same cost that you calculated for using disposable plates and forks for 1 year?

Student Response

1. Fast food restaurants generally have more customers per day than full-service restaurants, but there is a lot of overlap. The difference in means is about 366 customers, which is 1.6 times the larger mean absolute deviation.
2. Answers vary. Sample response: 240 customers per day because 240 is close to the center of the distribution for full-service restaurants, and I think my restaurant will be a typical full-service restaurant.
3. Answers vary. Sample response for 240 customers per day:
 - a. $d = 22.18n$
 - b. The equation represents a proportional relationship because it is in the form $y = kx$, and the constant of proportionality is 22.18. However, this was using an average of 240 customers per day, and in real life the restaurant serves a different number of people each day, so it could be close to proportional, but not exactly.
 - c. Assuming the restaurant is open 365 days in the year and serves an average of 240 customers per day, it would cost \$8,095.70 to use disposable plates and forks for every customer. This also assumes that each customer uses exactly one plate and one fork.
4.
 - a. \$876 to buy 240 reusable plates and 240 reusable forks
 - b. $10.75d + 876 = r$
 - c. No, this equation does not represent a proportional relationship. It cannot be rewritten in the form $y = kx$ and if graphed, it would go through the point (0, 876) instead of (0, 0).
 - d. If $10.75d + 876 = 8095.70$, then $d = 671.6$, which is about 1 year and 10 months.

Activity Synthesis

Poll the class on how many days they can use the reusable plates and forks for the same cost as using disposable plates and forks for 1 year. Select students to share what this tells us about the situation. (If their answer is greater than the number of days they planned for their restaurant to be open during the year, then this means that buying and washing reusable plates and forks is cheaper than using disposable plates and forks.)

Select students to share their reasoning about whether there is a proportional relationship between the cost of using disposable or reusable and the number of days.

Students might share the following ideas:

- The relationship for the cost of using disposable looks like it is proportional because we wrote it in the form $y = kx$.
 - The relationship for the cost of using disposable would be close, but not exactly proportional, because the equation is assuming an average number of customers per day and in real life the restaurant serves a different number of people each day.
 - The relationship for the cost of using reusable is not proportional because:
 - there is a start-up cost of buying the reusable plates and forks.
 - the equation cannot be written in the form $y = kx$. There has to be a term that is added that represents the start-up costs.
 - if we graphed the relationship, it would not go through the origin, but would cross the y-axis at a point that represents the start-up costs.
-

Access for English Language Learners

Speaking: MLR8 Discussion Supports. As students share their reasoning about whether there is a proportional relationship between the cost of using disposable or reusable and the number of days, provide sentence frames to support their justifications, such as, "I know ___ because" Press for details in students' explanations by asking students to elaborate on an idea (e.g., "Why did you...?" and "Can you give an example of . . . ?"). This will help students communicate clearly and use more precise language.

Design Principle(s): Optimize output (for justification)

Lesson 4: Restaurant Floor Plan

Goals

- Choose an appropriate scale and create a scale drawing for a restaurant floor plan.
- Use proportional reasoning to solve problems about the area or volume of different elements of a floor plan and explain (orally) the solution method.

Lesson Narrative

This lesson is optional. In this lesson, students create a scale drawing of the floor plan for a restaurant and solve problems involving proportional reasoning about the area or volume of different elements within the floor plan.

Students can adapt an outline of their floor plan to make it easier for them to incorporate other requirements, such as the spacing between tables and the maximum distance between the tables and the food pickup area. This gives them an opportunity to make sense of the problem (MP1). If students choose to use a compass to draw a circle with a radius representing the 60-foot restriction, or if they make physical scale models of tables, they are choosing tools strategically (MP5).

As with all lessons in this unit, all related standards have been addressed in prior units. This lesson provides an *optional* opportunity to go deeper and make connections between domains.

Alignments

Addressing

- 7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
- 7.G.B.4: Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
- 7.G.B.6: Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
- 7.NS.A.2.d: Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Instructional Routines

- MLR5: Co-Craft Questions
- MLR7: Compare and Connect

Required Materials

Blank paper

Compasses

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Graph paper

Index cards

Required Preparation

Students need access to graph paper, geometry toolkits, and compasses.

Student Learning Goals

Let's design the floor plan for a restaurant.

4.1 Dining Area

Optional: 25 minutes

The purpose of this activity is for students to create a scale drawing for a restaurant floor plan. Students use proportional reasoning to consider how much space is needed per customer, both in the dining area and at specific tables. They try to find a layout for the tables in the dining area that meets restrictions both for the distance between tables and to the kitchen. Students choose their own scale for creating their scale drawing.

When trying to answer the last two questions, students might want to go back and modify the shape of their dining area from their previous answer. This is an acceptable way for students to make sense of the problem and persevere in solving it (MP1).

Addressing

- 7.G.A.1
- 7.G.B.4
- 7.NS.A.2.d
- 7.RP.A.3

Instructional Routines

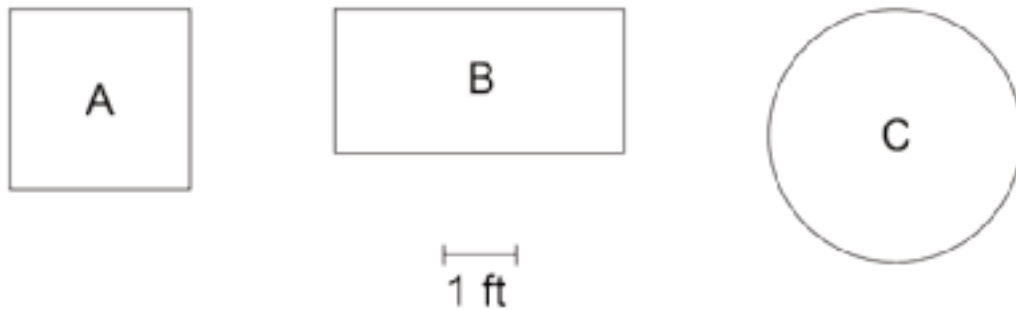
- MLR7: Compare and Connect

Launch

Provide access to graph paper, geometry toolkits, and compasses. Give students quiet work time followed by partner discussion.

Student Task Statement

1. Restaurant owners say it is good for each customer to have about 300 in^2 of space at their table. How many customers would you seat at each table?



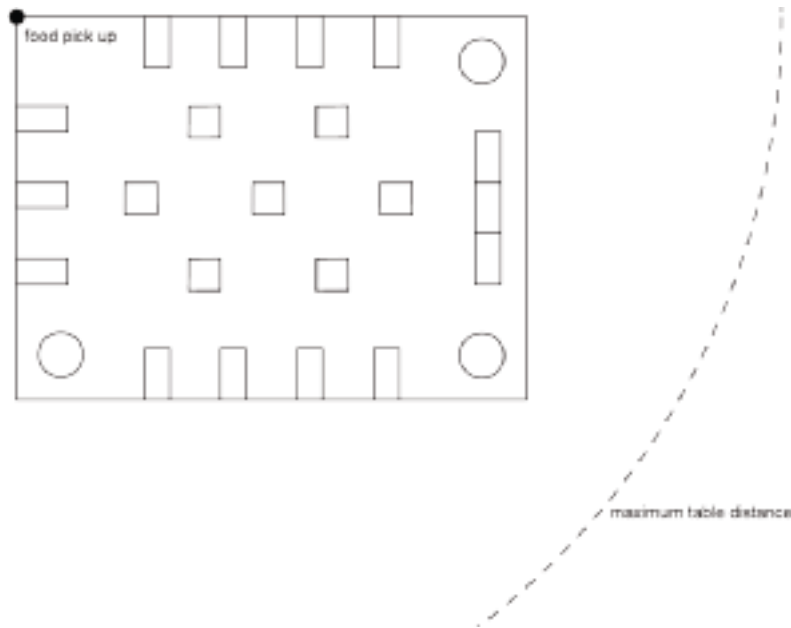
2. It is good to have about 15 ft^2 of floor space per customer in the dining area.
 - a. How many customers would you like to be able to seat at one time?
 - b. What size and shape dining area would be large enough to fit that many customers?
 - c. Select an appropriate scale, and create a scale drawing of the outline of your dining area.
3. Using the same scale, what size would each of the tables from the first question appear on your scale drawing?
4. To ensure fast service, it is good for all of the tables to be within 60 ft of the place where the servers bring the food out of the kitchen. Decide where the food pickup area will be, and draw it on your scale drawing. Next, show the limit of how far away tables can be positioned from this place.
5. It is good to have at least $1\frac{1}{2} \text{ ft}$ between each table and at least $3\frac{1}{2} \text{ ft}$ between the sides of tables where the customers will be sitting. On your scale drawing, show one way you could arrange tables in your dining area.

Student Response

Answers vary. Sample responses:

1. ○ Table A could seat 3 customers because $30 \cdot 30 = 900$ and $900 \div 300 = 3$

- Table B could seat 3 or 4 customers because $48 \cdot 24 = 1152$ and $1152 \div 300 = 3.84$
 - Table C could seat 4 or 5 customers because $\pi \cdot 21^2 \approx 1385$ and $1385 \div 300 = 4.61\bar{6}$
- 2.
- a. About 80 customers
 - b. The dining area could be a rectangle with sides 30 ft and 40 ft. This would give an area of $1,200 \text{ ft}^2$, which is enough space for 80 customers because $80 \cdot 15 = 1200$.
 - c. Using a scale of 1 cm represents 2 feet, the scale drawing would be a rectangle 15 cm wide and 20 cm long.
- 3.
- Table A would be a square with sides 1.25 cm.
 - Table B would be a rectangle with length 2 cm and width 1 cm.
 - Table C would be a circle with a diameter of 1.75 cm.
4. The food pickup area could be a point in the top left corner of the rectangular dining area. A circle centered on this point with a radius of 30 cm represents the maximum distance to a table.



Are You Ready for More?

The dining area usually takes up about 60% of the overall space of a restaurant because there also needs to be room for the kitchen, storage areas, office, and bathrooms. Given the size of your dining area, how much more space would you need for these other areas?

Student Response

Answers vary. Sample response: If the dining area is $1,200 \text{ ft}^2$, then the other areas would need about 800 ft^2 of space. We can represent the fact that the dining area takes up about 60% of the

entire restaurant area with the equation $0.6x = 1200$, where x represents the area of the entire restaurant. The entire restaurant would cover about $2,000 \text{ ft}^2$, because $x = 1200 \div 0.6 = 2000$. The other areas of the restaurant would be about 800 ft^2 because $2000 - 1200 = 800$ or $0.4 \cdot 2000 = 800$.

Activity Synthesis

Ask students to trade with a partner and check that the layout meets the requirements for spacing between tables and maximum distance between the tables and the food pickup area.

Display these questions for students to discuss with their partner:

- Is the scale drawing easy to interpret?
- Does it say somewhere what scale was used for the drawing?
- Is there anything that could be added to the drawing that would make it clearer?

Access for English Language Learners

Speaking, Listening, Conversing: MLR7 Compare and Connect. After students have prepared their scaled drawings of a floor plan, display the drawings around the room. Ask pairs to discuss “What is the same and what is different?” about the scale drawings. To help students make connections between drawings, ask, “What do you observe about our scale drawings that is easier to interpret?”. This will help students reflect on how precise and understandable their drawings are for others to interpret.

Design Principle(s): Cultivate conversation; Maximize meta-awareness

4.2 Cold Storage

Optional: 15 minutes

The purpose of this activity is for students to apply proportional reasoning in the context of area and volume to predict the cost of operating a walk-in refrigerator and freezer.

Addressing

- 7.G.B.6
- 7.RP.A.3

Instructional Routines

- MLR5: Co-Craft Questions

Launch

Arrange students in groups of 2. Give students 1 minute of quiet think time followed by time to work with their partner to solve the problem.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts for students who benefit from support with organizational skills in problem solving. Check in with students within the first 2-3 minutes of work time to ensure that they have understood the directions. If students are unsure how to begin, suggest that they consider each statement for the refrigerator first, and then for the freezer.

Supports accessibility for: Organization; Attention

Access for English Language Learners

Writing, Reading, Conversing: MLR5 Co-craft Questions. Begin by displaying only the initial text describing the context of the problem and the information about the monthly costs of standard refrigerators and freezers (i.e., withhold the scale drawing and question about the walk-in refrigerator and freezer). Ask students, “What mathematical questions can you ask about this situation?” Give groups 2–3 minutes to write down questions they have. As students share their questions, focus on questions that address how to evaluate costs in relationship to the volume of the refrigerator or freezer. This will help students understand the context and identify any assumptions they are making prior to solving the problem.

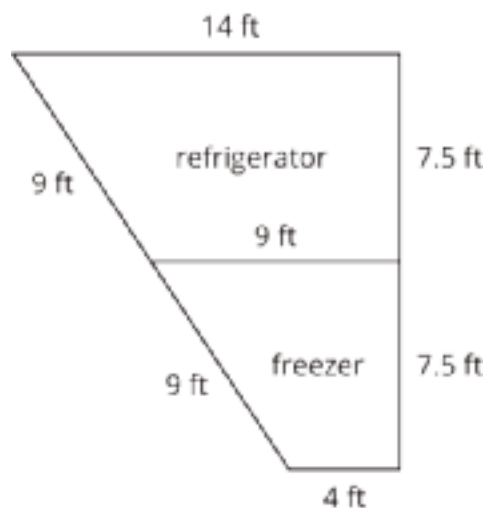
Design Principle(s): Cultivate conversation; Maximize meta-awareness

Student Task Statement

Some restaurants have very large refrigerators or freezers that are like small rooms. The energy to keep these rooms cold can be expensive.

- A standard walk-in refrigerator (rectangular, 10 feet wide, 10 feet long, and 7 feet tall) will cost about \$150 per month to keep cold.
- A standard walk-in freezer (rectangular, 8 feet wide, 10 feet long, and 7 feet tall) will cost about \$372 per month to keep cold.

Here is a scale drawing of a walk-in refrigerator and freezer. About how much would it cost to keep them both cold? Show your reasoning.



Student Response

Answers vary. Sample response: The total cost to keep both of these rooms cold would be about \$352 per month.

- The walk-in refrigerator covers an area of 86.25 ft^2 because it can be decomposed into a rectangle with an area of 67.5 ft^2 and a triangle with an area of 18.75 ft^2 .
- The walk-in freezer covers an area of 48.75 ft^2 because it can be decomposed into a rectangle with an area of 30 ft^2 and a triangle with an area of 18.75 ft^2 .
- Let's assume that the refrigerator and freezer shown in the drawings are also 7 ft tall, like the ones given in the example. That means their volumes are 603.75 ft^3 and 341.25 ft^3 , respectively, because $86.25 \cdot 7 = 603.75$ and $48.75 \cdot 7 = 341.25$.
- In the example, the refrigerator costs $150 \div (10 \cdot 10 \cdot 7)$, or about \$0.21 per cubic foot to operate for one month, and the freezer costs $372 \div (8 \cdot 10 \cdot 7)$, or about \$0.66 per cubic foot.
- Therefore, the refrigerator in the drawing would cost about \$126.79 to operate for one month because $603.75 \cdot 0.21 = 126.7875$, and the freezer in the drawing would cost about \$225.23 because $341.25 \cdot 0.66 = 225.225$.
- $126.79 + 225.23 = 352.02$.

Activity Synthesis

The goal of this discussion is for students to practice explaining the assumptions they made and the strategies they used to solve the problem.

First, poll the class on their estimates for the cost of operating the refrigerator and freezer. Discuss whether the different answers seem reasonable.

Next, select students to share their strategies for breaking the problem up into smaller parts.

Discuss what assumptions students made about proportional relationships while solving the problem. (For example, there is a proportional relationship between the volume of a walk-in refrigerator and the cost to keep it cold.)

Section: Making Connections

Lesson 5: How Crowded Is this Neighborhood?

Goals

- Compare and contrast the density of uniformly distributed dots in squares.
- Create an equation and a graph that represent the proportional relationship between the area of a square and the number of dots enclosed by the square.
- Interpret the constant of proportionality in models of housing per square kilometer or population of people per square kilometer.

Lesson Narrative

This lesson is optional. This lesson involves a sequence of four activities that prepare and introduce students to the concept of population density. The lesson can be adjusted depending on available time and teacher-identified goals from 1 to 2 class days.

Contexts involving population density are useful for helping students understand how derived units arise from a proportional relationship (MP2). Population density arises from two familiar quantities, number of people and area. The way the lesson develops helps students make sense of the somewhat abstract idea of density in very concrete terms: They start by comparing the number of dots distributed in squares and move on to houses in different neighborhoods. Finally they compare the number of people living in different cities. Unlike speed or unit pricing, density is not likely to be familiar to students, so it provides an opportunity to make sense of an unfamiliar situation by thinking about familiar quantities in a new way.

This lesson engages students in important aspects of modeling (MP4). In particular, the rates are used to model rather than represent. For example, houses may not be uniformly distributed in any given area, but rates for houses per square mile characterize differences between rural and urban areas. This lesson begins students' transition from contexts that involve constant rates to contexts that involve average rates of change. Average rate of change is a high school topic, but before high school students begin to investigate situations modeled by proportional relationships, for example, bivariate data with measurement error or quantities that change at rates that are close to constant.

As with all lessons in this unit, all related standards have been addressed in prior units. This lesson provides an *optional* opportunity to go deeper and make connections between domains.

Alignments

Building On

- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

Addressing

- 7.G.B.6: Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
- 7.RP.A.1: Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{\frac{1}{2}}{\frac{1}{4}}$ miles per hour, equivalently 2 miles per hour.
- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

Instructional Routines

- MLR5: Co-Craft Questions
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Notice and Wonder
- Think Pair Share

Required Materials

Four-function calculators

Required Preparation

If desired, prepare to display satellite images that show the housing density in different neighborhoods of your city, New York City and Los Angeles.

Provide access to four-function calculators.

Student Learning Goals

Let's see how proportional relationships apply to where people live.

5.1 Dot Density

Optional: 5 minutes

In this activity, students compare dot densities when dots are uniformly distributed. The squares are sized so that students can compare dot density in large and small squares by drawing a partition of the larger square into four smaller squares and comparing the number of dots in squares of the same size. In the next activity, the dots are not uniformly distributed, so students need to think more about the meaning of “dots per square inch.”

Addressing

- 7.G.B.6

- 7.RP.A.1

Instructional Routines

- MLR8: Discussion Supports
- Notice and Wonder

Launch

Display the image of the four squares with dots. Invite students to share what they notice and what they wonder.

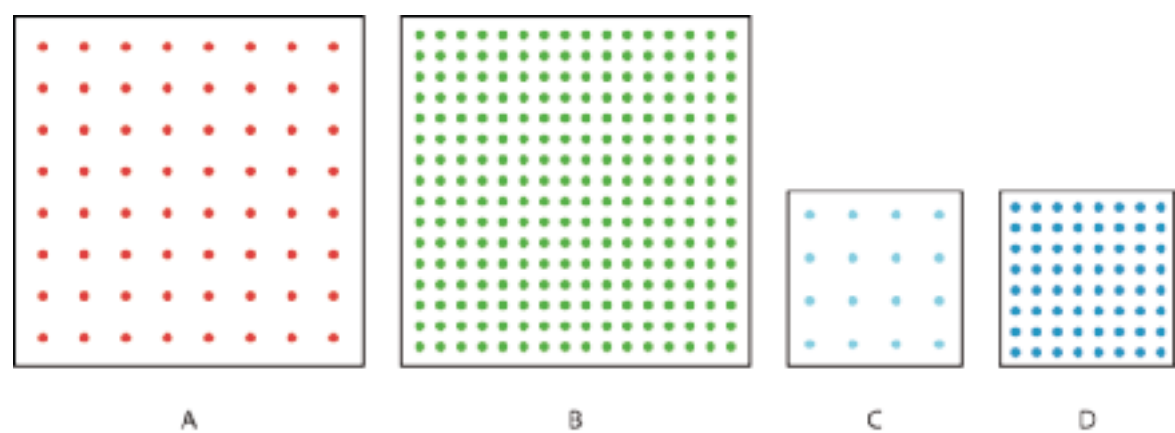
Give students 5 minutes of quiet work time followed by whole-class discussion.

Anticipated Misconceptions

Some students might not understand what the last column in the table is asking them for. Remind them that the word *per* means “for each” or “for one.”

Student Task Statement

The figure shows four squares. Each square encloses an array of dots. Squares A and B have side length 2 inches. Squares C and D have side length 1 inch.



1. Complete the table with information about each square.

square	area of the square in square inches	number of dots	number of dots per square inch
A			
B			
C			
D			

2. Compare each square to the others. What is the same and what is different?

Student Response

1. Completed table:

square	area of the square in square inches	number of dots	number of dots per square inch
A	4	64	16
B	4	256	64
C	1	16	16
D	1	64	64

- 2.
- Squares A and B have the same area. Squares C and D have the same area.
 - The number of dots in Square A is the same as the number of dots in Square D. The two other squares have different numbers of dots.
 - The number of dots per square inch is the same for Squares A and C. The number of dots per square inch is the same for Squares B and D.

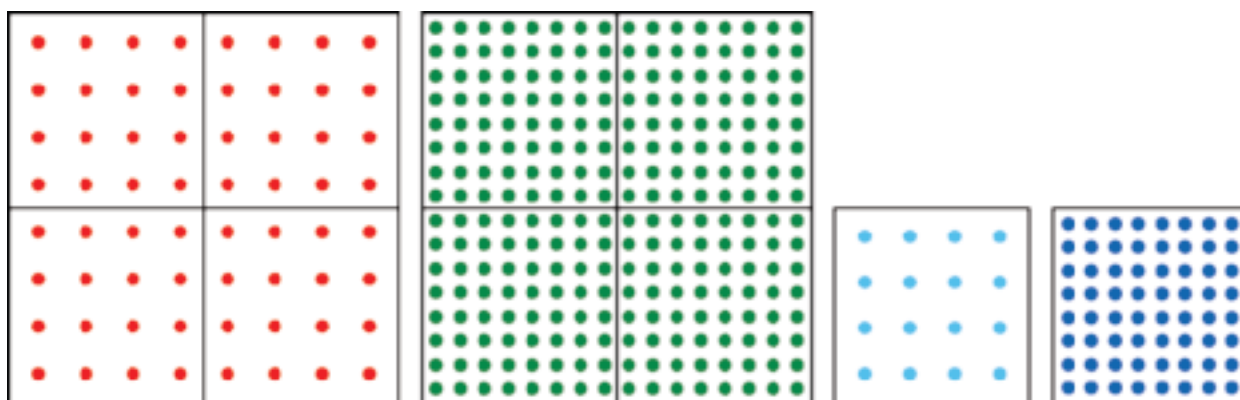
Activity Synthesis

Invite students to share what is similar and what is different about the arrays.

Define *density* as “things per square inch,” in this case dots per square inch. Demonstrate the correct use of “dense” and “density” by saying things like:

- “The green dots in Square B are more densely packed than the red dots in Square A and the blue dots in Square C.”
- “The density of the red dots in Square A and the blue dots in Square C is the same.”

If students haven’t noted it already, point out that the large square A can be partitioned into four smaller squares. Each has an array of red dots identical (except for the color) to the array of blue dots in Square C. The same is true for Squares B and D.



Access for English Language Learners

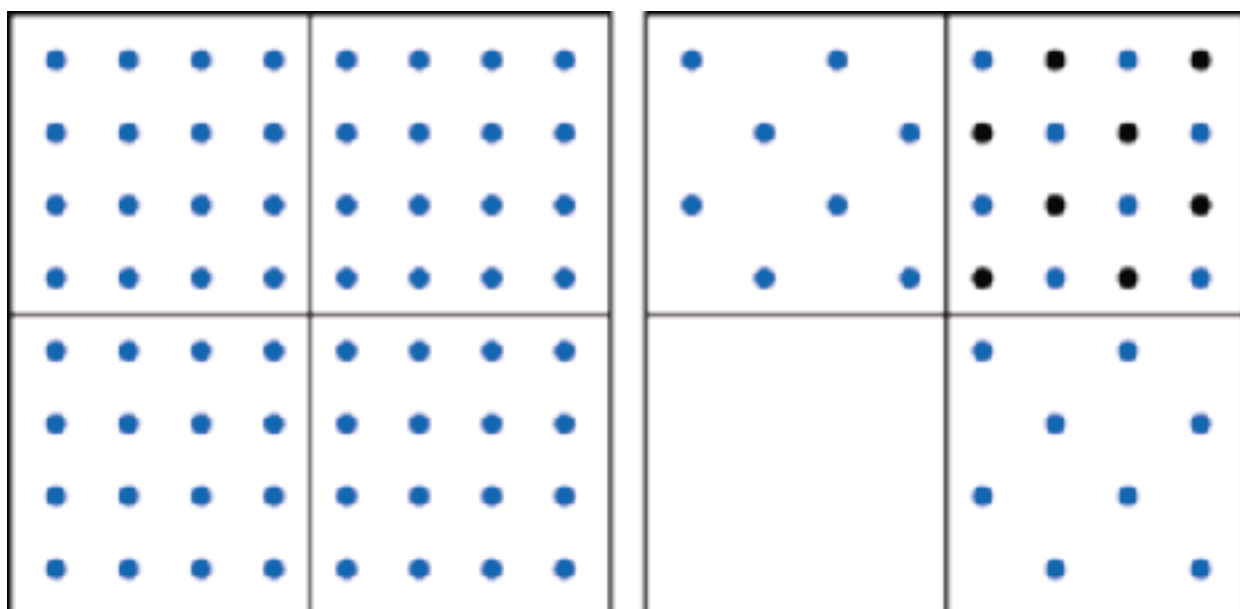
Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion. For each observation or response that is shared, ask students to restate and/or revoice what they heard using precise mathematical language. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. This will provide more students with an opportunity to produce language as they interpret the reasoning of others.

Design Principle(s): Support sense-making

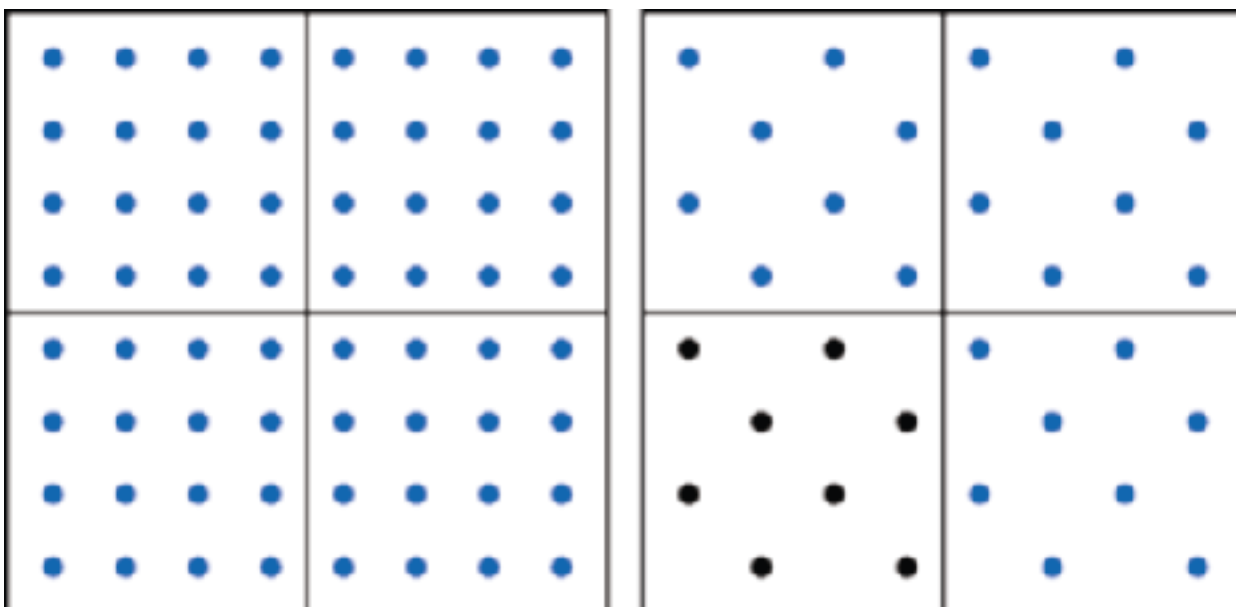
5.2 Dot Density with a Twist

Optional: 10 minutes (there is a digital version of this activity)

In this activity, the dots are distributed uniformly in the first square but not in the second square:



However, these dots are drawn so it is not too hard to see that if they were redistributed, each square inch would have 8 dots:



The fact that we have 8 dots per square inch means that if we distributed the dots uniformly throughout the square in the right way, we would see 8 dots in each square inch. This prepares students to be able to interpret the constant of proportionality in the next two activities when working with actual houses per square mile or people per square kilometer. When we speak of “500 houses per square kilometer,” we can think of this as, “If we took all of the houses in the region and spread them out uniformly, then we would see 500 houses in every square kilometer.”

Addressing

- 7.G.B.6
- 7.RP.A.2

Instructional Routines

- MLR5: Co-Craft Questions
- Think Pair Share

Launch

Display the image of the two squares with dots. Ask students to describe what is the same and what is different about these squares.

Give students 2–3 minutes of quiet work time, followed by 4–5 minutes of partner work time and whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, pause to check for understanding after the first 2–3 minutes of work time.

Supports accessibility for: Organization; Attention

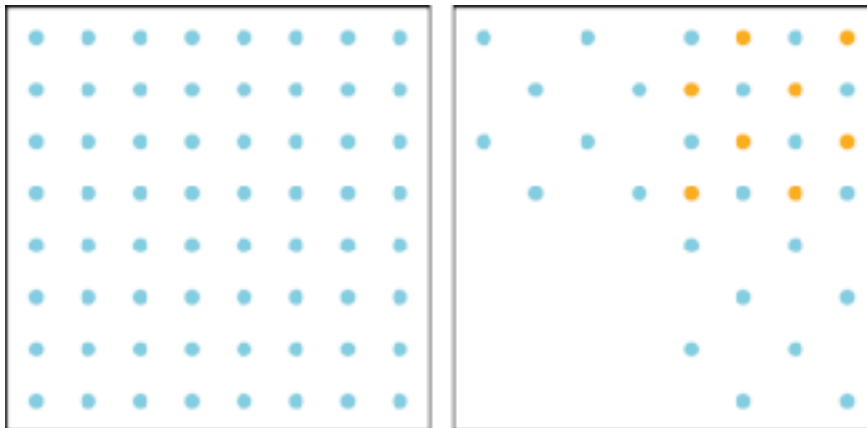
Access for English Language Learners

Writing, Conversing: MLR5 Co-craft Questions. Display only the image of the four dot arrays without revealing any of the questions that follow. Give students 1–2 minutes to write a list of possible mathematical questions they could ask about the arrays. Invite students to share their questions with a partner, and then select 2–3 students to share their questions with the whole class. Highlight any questions that refer to how dots are “distributed,” even if students do not use that particular phrase. Finally, reveal the whole problem with text so that students may begin addressing the questions. This helps amplify language related to the distribution of dots.

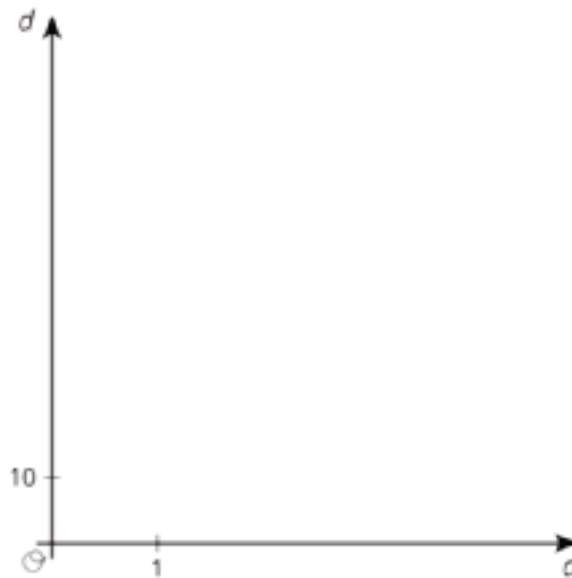
Design Principle(s): Cultivate conversation; Maximize meta-awareness

Student Task Statement

The figure shows two arrays, each enclosed by a square that is 2 inches wide.



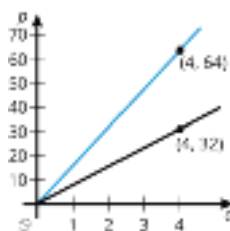
1. Let a be the area of the square and d be the number of dots enclosed by the square. For each square, plot a point that represents its values of a and d .



2. Draw lines from $(0, 0)$ to each point. For each line, write an equation that represents the proportional relationship.
3. What is the constant of proportionality for each relationship? What do the constants of proportionality tell us about the dots and squares?

Student Response

1. See figure.
2. See figure. The equations are $d = 16a$ and $d = 8a$ respectively.
3. The constants of proportionality indicate the number of dots per square inch, 16 and 8, respectively. In the first case, they tell us that if we partition the square into square inches, there will be 16 dots in each. In the second case, they tell us if we were to redistribute the dots uniformly, there would be 8 dots per square inch.



Activity Synthesis

The goal of this discussion is for students to make sense of the constant of proportionality in the case where the dots are not uniformly distributed. Invite students to share their interpretations of the constants of proportionality.

Consider asking questions like:

- “Why is one of the constants of proportionality larger than the other? How can we see this in the picture of the squares?” (There are more dots in the same area.)
- “What are the units of the constants of proportionality?” (The number of dots per square inch).

Students will have more opportunities to think about this when working on the activities that follow.

5.3 Housing Density

Optional: 15 minutes

This activity starts to transition students from arrays of dots to real-world objects distributed over the surface of Earth. This task concerns housing density, which is very similar to dot density.

The two images in the activity have the following properties:

- It is fairly easy to distinguish the houses and not too tedious to count them all.
- The scale of the images is similar, but the size of each image is not the same, nor is the number of houses in each image.
- In the first image, the houses look fairly uniformly distributed, and in the other, they look less uniformly distributed but not to the point that it is hard to interpret the image.

This task can be customized to any location, for example different neighborhoods in your city. Care should be taken in selecting the images to include noticeably different housing densities and easy-to-count houses.

Building On

- 7.RP.A.2

Addressing

- 7.G.B.6

Launch

Give students 5 minutes of quiet work time followed by partner and whole-class discussion. Provide access to four-function calculators.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge about multiplication of fractions. Allow students to use calculators to ensure inclusive participation in the activity.

Supports accessibility for: Memory; Conceptual processing

Anticipated Misconceptions

Some students may struggle interpreting the decimal numbers in the activity. A length of 0.1 km is $\frac{1}{10}$ of a kilometer. This means that 10 of these lengths will make a kilometer.

Students are accustomed to an area (of a rectangle) being *numerically* bigger than its length or width. Ask students to think about multiplication of fractions: a square that is $\frac{1}{2}$ inch by $\frac{1}{2}$ inch has area $\frac{1}{4}$ square inch because four $\frac{1}{2}$ inch by $\frac{1}{2}$ inch squares compose a square inch.

Student Task Statement

Here are pictures of two different neighborhoods.

This image depicts an area that is 0.3 kilometers long and 0.2 kilometers wide.



This image depicts an area that is 0.4 kilometers long and 0.2 kilometers wide.



For each neighborhood, find the number of houses per square kilometer.

Student Response

The first image shows 48 houses. It depicts an area that is 0.06 square kilometers, so the housing density is 800 houses per square kilometer.

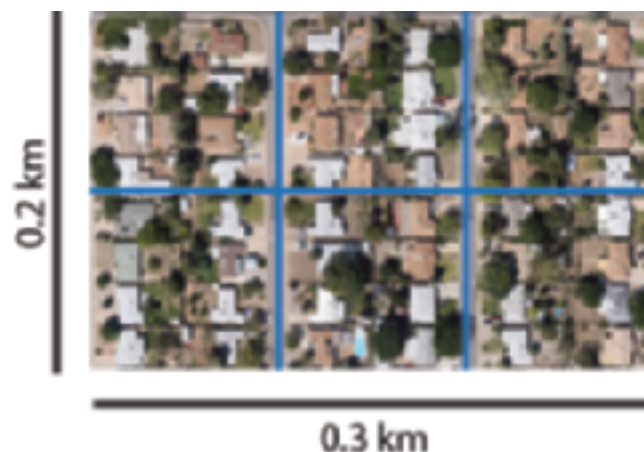
The second image shows 9 (or 10) houses. It depicts an area that is 0.08 square kilometers, so the housing density is 112.5 (or 125) houses per square kilometer.

Activity Synthesis

Poll students' answers for the two densities. Invite students to share their reasoning. Consider asking the following questions:

- One number is a lot bigger than the other. How can we see this in the images? (There are a lot more houses on the same area.)
- You only counted 48 houses in the first image but you say that there are 800 houses per square kilometer. Why is that happening? (The area in the image is just a fraction of a square kilometer. If we looked at a square kilometer with the same density, there would be 800 houses in it.)
- Some of you said that there are 112.5 houses per square kilometer in the second neighborhood. How can there be half houses? (If we had an image with an area of 2 square kilometers, there would be 225 houses in the image, or if they are uniformly distributed, the image might contain a part of a house.)

If desired, display this image to help students make sense of their answers.



The map shows a rectangle 0.3 km by 0.2 km. This means that any one of the six squares is 0.1 km by 0.1 km, which has area 0.01 square kilometer.

If you lined up 10 of these, you would have a strip 1 km long. 10 of these strips would make a square 1 km by 1 km; that is, 1 square kilometer. Now it's clear that it takes 100 of these small squares to make a square kilometer, so the small square indeed has area $\frac{1}{100}$ of a square kilometer.

In fact, there are 8 houses in this square, so if the entire square kilometer were filled the same way, there would be 800 houses.

5.4 Population Density

Optional: 15 minutes

The purpose of this activity is to introduce the concept of *population density*. One added step going from houses in a neighborhood to people in a location is the fact that people do not stay at a fixed spot but rather move around. In this activity, students make sense of what it means to say there are 42.3 people per square kilometer in some location.

This activity gives some information about New York City and Los Angeles and ultimately asks students to decide which city is more crowded. Students may benefit from a demonstration of situations that feel more crowded vs. less crowded.

In the data for this task, people are given in “blocks” of 1000 people. It’s perfectly fine to make up a new unit customized to the situation, even if it doesn’t have an official name. This is a more sophisticated use of nonstandard units. In earlier grades, nonstandard units tend to be the length of a paper clip, or the length of your shoe. When dealing with any units, it’s important to list the units: in the heading of a table, in labels for the axes of a graph, or in writing numbers.

Addressing

- 7.RP.A.2

Instructional Routines

- MLR7: Compare and Connect

Launch

Arrange students in groups of 2–4. Provide access to calculators.

Consider demonstrating situations that feel more crowded or less crowded by having a certain number of students stand in an area. “What would you have to do to feel more crowded with the same number of people?” (Stand in a smaller space, which would require people to stand closer to each other.) “Less crowded?” (Take up more space, so that people are farther apart.) Then, mark off a region on the classroom floor with tape. Ask some students to stand inside it, and then ask, “What would make the space feel more crowded?” (If more people stood in the same space.) “Less crowded?” (Fewer people.)

If desired, provide some background information about New York City and Los Angeles and display satellite images:

- New York City is the U.S. city with the largest population. The city has five parts (called boroughs): Manhattan, Brooklyn, Bronx, Queens, and Staten Island. In Manhattan, most people live in apartment buildings, many in high-rises. In the other boroughs, many people also live in single-family houses. Staten Island is quite different, almost suburban.
- Los Angeles is the U.S. city with the second largest population. Although there are some high-rise apartment buildings, many people live in single-family houses, and many of these houses are single-story.

Give students 5 minutes of quiet work time, followed by small-group and whole-class discussion.

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Activate or supply background knowledge. During the launch, take time to review the following terms from previous lessons that students will need to access for this activity: constant of proportionality, writing equations.

Supports accessibility for: Memory; Language

Anticipated Misconceptions

Be on the lookout for these areas of potential difficulty:

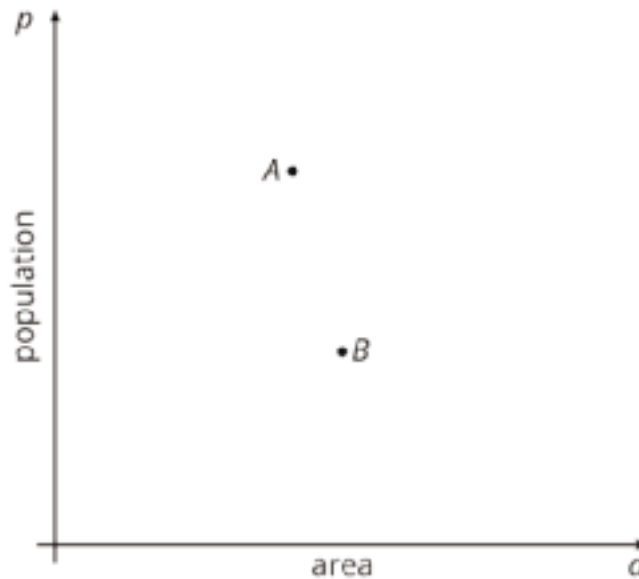
- Thinking that the numbers on the p -axis represent people, rather than thousands of people.
- Trying to find the exact values given, rather than approximating or rounding.
- Trouble adapting concepts or skills from graphing proportional relationships to this more complex situation.

If students have difficulty understanding numbers that are expressed in units of 1,000, they may need either some scaffolding or adequate time to talk about what these numbers mean.

Alternatively, the numbers can be given in terms of the more familiar millions, in which case the population densities will require some extra effort to understand.

Student Task Statement

- New York City has a population of 8,406 thousand people and covers an area of 1,214 square kilometers.
 - Los Angeles has a population of 3,884 thousand people and covers an area of 1,302 square kilometers.
1. The points labeled A and B each correspond to one of the two cities. Which is which? Label them on the graph.



- Write an equation for the line that passes through $(0, 0)$ and A . What is the constant of proportionality?
- Write an equation for the line that passes through $(0, 0)$ and B . What is the constant of proportionality?
- What do the constants of proportionality tell you about the crowdedness of these two cities?

Student Response

- New York City has a greater population and a smaller area, so it must correspond to point A .
- An equation for the line through point A is $p = 6.9a$; the constant of proportionality is about 6.9.
- An equation for the line through point B is $p = 3.0a$; the constant of proportionality is about 3.0.
- The constants of proportionality tell us that in New York City there are about 6.9 thousand people per square kilometer, or 6,900 people per square kilometer; and in LA there are about 3 thousand people per square kilometer, or 3,000 people per square kilometer.

Are You Ready for More?

- Predict where these types of regions would be shown on the graph:
 - a suburban region where houses are far apart, with big yards
 - a neighborhood in an urban area with many high-rise apartment buildings
 - a rural state with lots of open land and not many people
- Next, use this data to check your predictions:

place	description	population	area (km ²)
Chalco	a suburb of Omaha, Nebraska	10,994	7.5
Anoka County	a county in Minnesota, near Minneapolis/St. Paul	339,534	1,155
Guttenberg	a city in New Jersey	11,176	0.49
New York	a state	19,746,227	141,300
Rhode Island	a state	1,055,173	3,140
Alaska	a state	736,732	1,717,856
Tok	a community in Alaska	1,258	342.7

Student Response

Note that it's not really possible to see all the points on the same graph: the populations of Los Angeles and New York are so large, and the population of Tok so small, that if you could distinguish the point for Tok, LA and NY would be far off the paper or screen. And on the graph above showing LA and NY, Tok's population would be so small that it could not be distinguished from (0, 0).

A computer graphing program can help students understand this as it will take many steps of zooming in or out to switch between very small cities and very large cities.

Activity Synthesis

Invite some students to display their graphs and equations for all to see. Ask all students if they agree or disagree and why. Once students agree, focus on the meaning of the constants of proportionality and what they tell us about the crowdedness in the two cities.

Consider asking the following questions:

- "Why did you choose point A to represent New York City?" (New York City has more people per square kilometer than Los Angeles, so New York is more crowded than Los Angeles.)
- "What does the constant of proportionality tell us about the crowdedness in the two cities?" (Even though people aren't distributed uniformly throughout the cities, we might say that if we were to distribute everyone who has an address in the city uniformly throughout the city, then there would be about 6,900 people in every square kilometer in NYC and about 3,000 people in every square kilometer in LA.)

Access for English Language Learners

Representing: MLR7 Compare and Connect. Use this routine to help students discuss what the constant of proportionality tells them about the crowdedness of the two cities. Ask students to compare the city's equations and graphs to look for "What is the same and what is different?" between the representations. Ask students to look for where each city's constant of proportionality is represented in the equations and the graphs.

Design Principle(s): Optimize output (for comparison); Maximize meta-awareness

Lesson 6: Fermi Problems

Goals

- Calculate a rough estimate for quantities that are difficult or impossible to measure directly and explain (orally) the reasoning.
- Choose an appropriate level of accuracy when reporting estimates of quantities.
- Make simplifying assumptions to solve problems about estimating quantities.

Lesson Narrative

This lesson is optional. The activities in this lesson plan are sometimes called “Fermi problems” after the famous physicist Enrico Fermi. A Fermi problem requires students to make a rough estimate for quantities that are difficult or impossible to measure directly. Often, they use rates and require several calculations with fractions and decimals, making them well-aligned to grade 7 work. Fermi problems are examples of mathematical modeling (MP4), because one must make simplifying assumptions, estimates, research, and decisions about which quantities are important and what mathematics to use. They also encourage students to attend to precision (MP6), because one must think carefully about how to appropriately report estimates and choose words carefully to describe the quantities.

In determining your exact age, the level of accuracy depends on how exact you know your moment of birth: to the day? the minute? the second? In determining the number of heartbeats in your lifetime, it is impossible to know the exact answer because we do not have access to all of the necessary information (and even if we did, the numbers involved are too large to count). In determining the number of hairs on your head, there is no method available other than counting, yet there are no tools to do this accurately. These two scenarios are far more difficult to estimate with the same degree of accuracy as your age. You can reliably determine your age within a day: it would be very difficult to estimate heartbeats or hairs with this level of accuracy.

Any of these tasks can stand on its own. Choose those that you have time for. It is likely that it would take more than a single day to do all of the tasks in this lesson. Make sure to leave plenty of time for discussion. Important topics of discussion should include *why* the quantities in question are difficult to measure and the level of precision we should use to record our estimates.

As with all lessons in this unit, all related standards have been addressed in prior units. This lesson provides an *optional* opportunity to go deeper and make connections between domains.

Alignments

Building On

- 4.MD.A.2: Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger

unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Addressing

- 7.G.B: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
- 7.NS.A.3: Solve real-world and mathematical problems involving the four operations with rational numbers. Computations with rational numbers extend the rules for manipulating fractions to complex fractions.
- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.
- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports
- Poll the Class

Required Materials

Four-function calculators
Measuring tapes

Stopwatches
String

Required Preparation

During the A Heart Stoppingly Large Number activity, students have the option of measuring one another's pulse rate. If measuring pulse rate, students will need access to stopwatches.

During the All Hairs on Your Head activity, students have the option of measuring their head. If measuring heads, students will need access to string and measuring tape marked in centimeters.

Student Learning Goals

Let's estimate some quantities.

6.1 How Old Are You?

Optional: 20 minutes

Students attempt to calculate their *exact* age. Because this is a constantly changing quantity, they need to think carefully about how accurately to report the answer. The mathematics involved is multiple unit conversions in the context of time. In addition to the fact that our age is always growing, we may not know with great accuracy when we were born.

Building On

- 4.MD.A.2

Addressing

- 7.NS.A.3
- 7.RP.A

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students into groups of 4. Ask them to order themselves according to their age: who is the youngest? Who is the oldest? What information do you need to decide?

Tell students that they will be finding their *exact* age.

Provide access to calculators. Give students 10 minutes of group work time, followed by whole-class discussion.

Anticipated Misconceptions

Students may struggle with what is meant by *at this moment* in the prompt. Ask them how they interpret this: the question is intended to be flexible so students can interpret *at this moment* as being when they read the question, or the day of class, or in another way.

Student Task Statement

What is your *exact* age at this moment?

Student Response

Answers vary. Sample solution: I was born on April 4, 2003 at 12:09 p.m. Today is December 17, 2016, and right now it is 10:55 a.m. So I am 13 years, 256 days, 22 hours, and 46 minutes old right now. I can't get any more exact because I don't know the second I was born, and even then I wouldn't know the fraction of a second. Plus, I'm not sure if I should answer the question for when I begin to write my solution, or for when I will be done writing my solution, because I keep getting older.

Activity Synthesis

Invite students to share answers and discuss difficulties in answering the question. The discussion should include the following points:

- We can give an estimate, but the question cannot be answered because we probably do not know the exact time when we were born and “at this moment” keeps moving forward.

- How we express our answer depends on what we know about the time we were born. We probably know the day, but do we know the time? Should we answer to the nearest hour? Minute? Second?
- Did you take into account leap years?

Ask students how they usually answer the question, “How old are you?” (This will probably be in whole number of years lived.) Why? (Because this whole number communicates enough information for most purposes.) After more than six months have passed since someone’s last birthday, the person still doesn’t normally round up when they report their age, even though this is customary for reporting many other types of measurements.

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion. For each response or observation that is shared, ask students to restate and/or revoice what they heard using precise mathematical language. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students’ attention to any words or phrases that helped to clarify the original statement. This will provide more students with an opportunity to produce language as they interpret the reasoning of others.

Design Principle(s): Support sense-making

6.2 A Heart Stoppingly Large Number

Optional: 15 minutes

This activity focuses on another complex calculation: the number of times our heart has beaten in our lifetime. In addition to all of the precision issues in determining how old we are from the previous activity, there is an additional level of complexity as our heart does *not* beat at a constant rate. For this situation, we need to make a reasoned estimate, but there is no hope of getting an exact answer. This makes our lack of knowledge of the exact time of our birth (or the exact time when we are solving the problem) unimportant because a few minutes or hours will not significantly impact the answer.

Addressing

- 7.RP.A.3

Instructional Routines

- MLR2: Collect and Display
- Poll the Class

Launch

Arrange students in groups of 2. Tell students they will be investigating how many times their heart has beaten in their lifetime. Ask students to make an estimate and then poll the class. One way to accomplish this is to display a table like this and complete it with the number of students whose estimate is in each range.

estimate for number of heartbeats	number of students
around 10,000	
around 100,000	
around 1,000,000	
around 10,000,000	
around 100,000,000	
around 1,000,000,000	
around 10,000,000,000	
around 100,000,000,000	

Ask students to brainstorm what they need to know or investigate to answer this question. Provide access to stopwatches (if students will estimate their own heartbeats) and calculators. Alternatively, share with them that a normal heart beat range is 60 to 100 beats per minute with higher rates when we exercise and sometimes lower rates at rest.

Give students 10 minutes of partner work time followed by a whole-class discussion.

Access for English Language Learners

Conversing: MLR2 Collect and Display. While students work in their groups to measure heart rates and make calculations, circulate and listen to the language students use related to their estimation process (e.g., “rate,” “per,” “constant,” etc.). Record their language on a visual display and update it throughout the remainder of the lesson. Remind students to borrow language from the display as needed. This helps students produce mathematical language as they justify their estimations.

Design Principle(s): Optimize output (for justification); Cultivate conversation

Anticipated Misconceptions

Some students may struggle with simplifying the assumptions needed to solve this problem. Encourage these students by:

- Telling them to think about how old they are if they have done the previous activity.
- Telling them the normal rate of heartbeats per minute.
- Asking them how they can find out about how many times a heart beats in an hour (multiply minute rate by 60), in a day (multiply hourly rate by 24), in a year, and so on.

Student Task Statement

How many times has your heart beat in your lifetime?

Student Response

Answers vary. Sample solution: I am about 13 years old. There are 365 days per year, 24 hours per day, and 60 minutes per hour. So that is $13 \cdot 365 \cdot 24 \cdot 60$, which is about 6.8 million minutes. I just took my pulse, and my heart rate is about 75 beats per minute. Even though that changes over time, I'll use it to estimate the total number of times my heart has beaten in my lifetime so far by multiplying the number of minutes I've been alive by 75. That gives about 500 million heartbeats.

Activity Synthesis

Invite students to share answers and discuss the difficulties they encountered answering these questions:

- "How did you estimate how often your heart beats?" (By checking pulse, looking online, etc.)
- "Is your heart rate always the same?" (No, it is faster when I exercise and slower when I sleep.)
- "Did you use the information you found for your age?" (No, because I only have an estimate for my pulse *or* yes because I already had the information available.)

Time permitting, students can be encouraged to check their pulse while resting and then after a short amount of exercise (jumping jacks, running in place, push ups). This gives them an idea of the variability involved in how frequently our heart beats.

The calculation of our number of heartbeats is only an estimate. One way we can indicate this is with the way we report the final answer. An answer of 524,344,566 would not be appropriate because that makes it look like it is an exact answer. An answer of 500,000,000 makes it clear that we are only making an estimate. Alternatively, students might say that the number of heartbeats is between 400,000 and 700,000 with the estimate of 400,000 coming from the low value of 60 beats per minute and the estimate of 700,000 coming from the high value of 100 beats per minute.

6.3 All the Hairs on Your Head

Optional: 20 minutes

Students estimate how many hairs they have on their head. Although there is a definite number of hairs on each student's head, finding this number exactly is not feasible: although it is a much smaller number than the number of heartbeats in a student's lifetime, it is still too large to count and, even if we could count, we would need to cut all of those hairs to do so! An additional

geometric layer of estimation comes into play in this task as students need to estimate the surface area of part of their head. If one or more students in the class are bald or are otherwise sensitive about their hair, consider asking them to estimate the number of hairs on another student's head.

Addressing

- 7.G.B
- 7.RP.A.3

Instructional Routines

- Poll the Class

Launch

Arrange students in groups of 2. Provide access to calculators. Before starting, poll the class to guess how many hairs they have on their head. One way to accomplish this is to display a table like this, and record the number of students with each guess:

number of hairs	number of guesses
about 100	
about 1,000	
about 10,000	
about 100,000	
about 1,000,000	

Next, ask students to brainstorm the information they need to answer the question. Provide the following information below when students ask for it.

Data to keep in reserve and share with students as needed:

- The number of hairs per square cm varies from person to person, but for this analysis, we can assume approximately 150 hairs per square cm.
- Students have to be creative in their estimates for the area of their scalp. One possibility is to approximate it with a circle. 600 square centimeters is a good estimate, but students may come up with slightly different estimates.

Provide access to string, tape measures, and calculators. After taking 5 minutes to introduce the activity, give students 10 minutes of partner work time, followed by whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Provide students with a graphic organizer for recording measurements and calculations of areas of circles to represent hair on head.

Supports accessibility for: Language; Organization

Anticipated Misconceptions

Since the scalp is three dimensional, students may not think of approximating the scalp with a circle and may struggle with trying to identify the size of the circle. Consider asking these students:

- What two-dimensional shape can you use to model the part of your scalp covered with hair? (A circle)
- How can you estimate the diameter of the circle? (Measuring from front to back or from side to side)
- How do you estimate the area of the circle using the diameter? (The diameter gives the radius, and then the area can be found once the radius is known.)

Student Task Statement

How many strands of hair do you have on your head?

Student Response

Answers vary. Sample response: a piece of string around the head, from the front of the forehead to the back of the neck measures about 28 cm. A piece of string going from ear to ear also measures about 28 cm. So it is reasonable to model the scalp with a circle whose diameter is 28 cm. The radius is 14 cm, and the area of the circle is $\pi \cdot 14^2 \approx 600$ square cm. If there are 150 strands of hair per square cm, that means that there are about 90,000 strands of hair on the head.

Activity Synthesis

Invite students to share their answers and methods. Consider asking the following questions:

- “Why is it challenging to find the exact number of hairs on your head?” (There are too many to count, and they are too small to count accurately. The number changes as time goes by.)
- “How can you estimate the number of hairs on your head?” (Estimate the area of the part of the head covered by hair. Try to find how many hairs there are on a small part of the head.)
- “How did you estimate the area of your head?” (Approximated area with a circle. Covered the head with little pieces of paper (sticky notes) and added up their areas.)

This is a situation where there is a definite number of hairs on our head, but because they are so small (and because they are on our head), we cannot calculate this number exactly. Fortunately, for almost any purpose, an estimate will do.

Lesson 7: More Expressions and Equations

Goals

- Create expressions and equations to represent a linear relationship with two or more related quantities in context.
- Determine the solution to an equation of the form $px + q = r$ and explain (orally) the solution method.
- Interpret expressions, equations, and solutions that represent a linear relationship with two or more related quantities.

Lesson Narrative

This lesson is optional. The activities in this lesson can all be solved using grade-level mathematics, but are more sophisticated than earlier activities and are often left for future grades when students have access to a wider variety of algebraic tools.

Each of the situations described in the activities involve two or more unknown quantities and multiple relationships or actions. Initially students are walked through the steps of writing an expression to describe a situation, using properties to rewrite the expression with fewer terms, writing an equation to represent the situation, solving the equation, and considering the reasonableness of solutions. As they proceed through the lesson, the supports are slowly removed and students work on their own to reason through the problem (MP2).

As with all lessons in this unit, all related standards have been addressed in prior units. This lesson provides an *optional* opportunity to go more deeply into working with expressions and equations.

Alignments

Addressing

- 7.EE.A.1: Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
- 7.EE.B.4.a: Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Instructional Routines

- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Think Pair Share

Student Learning Goals

Let's solve harder problems by writing equivalent expressions.

7.1 Tickets for the School Play

Optional: 15 minutes

This activity walks students through the process of defining a variable, writing an expression, writing the expression with fewer terms, estimating a reasonable solution, computing a solution, and finally checking that the solution makes sense and is correct. Note that there are two unknown quantities (prices for student and adult tickets) and students are guided to express one in terms of the other.

Monitor for students who write the expressions different ways in the first three questions and invite them to share during the discussion. There are many possible correct answers, but some forms will lead to equations that are easier to solve (i.e. $px + q$) and some will be harder to solve.

Addressing

- 7.EE.A.1
- 7.EE.B.4.a

Launch

If your school or a nearby school has recently performed a play, consider asking if any students went to see it and have them briefly describe the experience. Alternatively, display photos from any school play, including images of the tickets or ticket booth. Invite students to share what they notice and what they wonder.

Give students 5 minutes of quiet work time followed by a whole-class discussion.

Anticipated Misconceptions

Students might write the expression for the cost of a student ticket as $2 - a$ instead of $a - 2$ because they confuse how to represent “2 less than” a quantity. Provide these students with values for the price of an adult ticket, and have them find the price of a student ticket. Help them see the structure of subtracting 2 from the cost of an adult ticket and not the other way around.

For students struggling with the last two questions, remind them that they know how to write expressions with fewer terms, and that they know how to solve equations of the forms $p(x + q) = r$ and $px + q = r$. Support them in grappling with how these concepts and skills can come together to help them solve problems.

Student Task Statement

Student tickets for the school play cost \$2 less than adult tickets.

1. If a represents the price of one adult ticket, write an expression for the price of a student ticket.
2. Write an expression that represents the amount of money they collected each night:

- a. The first night, the school sold 60 adult tickets and 94 student tickets.
 - b. The second night, the school sold 83 adult tickets and 127 student tickets.
3. Write an expression that represents the total amount of money collected from ticket sales on both nights.
4. Over these two nights, they collected a total of \$1,651 in ticket sales.
 - a. Write an equation that represents this situation.
 - b. What was the cost of each type of ticket?
5. Is your solution reasonable? Explain how you know.

Student Response

1. $a - 2$
2.
 - a. $60a + 94(a - 2)$ (or $154a - 188$)
 - b. $83a + 127(a - 2)$ (or $210a - 254$)
3. $364a - 442$ or equivalent
4.
 - a. $364a - 442 = 1651$
 - b. \$5.75 adult, \$3.75 student
5. Answers vary. Sample response: Yes, it's reasonable that tickets to the school play would cost somewhere between \$0 and \$10 and also that the adult tickets would cost most than the students tickets. I can show it is correct, because
 $5.75(60 + 83) + 3.75(94 + 127) = 822.25 + 828.75 = 1651.$

Activity Synthesis

The purpose of the discussion is for students to reflect on the problem solving process. Consider asking the following questions:

- "Why was the price of an adult ticket chosen as the variable?" (This was an arbitrary choice.)
- "Could the problem be worked by choosing the price of a student ticket as the variable? How would the expressions be different? The equation? The solution?" (Yes, instead of -2 we would have $+2$ in the expression. The solution to the equation would be 3.75 instead of 5.75.)
- "What does each term in the expressions you wrote represent?"
- "How did you find the price of a student ticket?"
- "How did you check that your solution is correct?"

7.2 A Souvenir Stand

Optional: 15 minutes

In this activity students continue to be guided through a solution process, but with fewer supports than in the previous activity. There are now three unknown quantities and students are asked to write an expression for their total, but students are not specifically guided to think about expressions for each individual quantity.

Monitor for students who compute the profit in different ways, such as:

- finding the total income and total cost, and then subtracting them
- finding the profit for each item, and then adding them together

Addressing

- 7.EE.A.1
- 7.EE.B.4.a

Instructional Routines

- MLR8: Discussion Supports

Launch

Give students 6–7 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

The souvenir stand sells hats, postcards, and magnets. They have twice as many postcards as hats, and 100 more magnets than post cards.

1. Let h represent the total number of hats. Write an expression in terms of h for the total number of items they have to sell.
2. The owner of the stand pays \$8 for each hat, \$0.10 for each post card, and \$0.50 for each magnet. Write an expression for the total cost of the items.
3. The souvenir stand sells the hats for \$11.75 each, the postcards for \$0.25 each, and the magnets for \$3.50 each. Write an expression for the total amount of money they would take in if they sold all the items.
4. Profits are calculated by subtracting costs from income. Write an expression for the profits of the souvenir stand if they sell all the items they have. Use properties to write an equivalent expression with fewer terms.
5. The souvenir stand sells all these items and makes a total profit of \$953.25.
 - a. Write an equation that represents this situation.

- b. How many of each item does the souvenir stand sell? Explain or show your reasoning.

Student Response

1. $h + 2h + (2h + 100)$
2. $8h + 0.10(2h) + 0.50(2h + 100)$ (or $9.2h + 50$)
3. $11.75h + 0.25(2h) + 3.50(2h + 100)$ (or $19.25h + 350$)
4. $10.05h + 300$
5. a. $10.05h + 300 = 953.25$
b. $h = 65$. 65 hats, 130 postcards, 230 magnets. $10.05h + 300 = 953.25$, $10.05 = 653.25$,
 $h = 65$

Activity Synthesis

Invite students to share their answers to the last two questions (and others, if they wish). Consider asking questions like the following:

- “Is it a good idea to check your solution by substituting values into the expressions and equations you wrote?” (No, you might have written or simplified them incorrectly. The best option is to go back to the original problem)
- “Why was the number of hats chosen as the unknown quantity to represent with a variable? Would you have chosen differently? How would that change the solution process?” (This is just one choice. We could make a different one.)
- “How did you compute the profit?” (Compute the total income minus total cost for all items, or add item by item.)
- “I saw that many of you wrote this expression $8h + 0.10(2h) + 0.50(2h + 100)$. What is the meaning of the different parts of the expressions in this situation?” ($8h$ is the cost for the hats, $0.10(2h)$ is the cost for the postcards, and $0.50(2h + 100)$ is the cost for the magnets.)

Access for Students with Disabilities

Representation: Internalize Comprehension. Use color and annotations to illustrate connections between representations. As students share their diagrams and reasoning, use color and annotations to scribe their thinking on a display of each problem so that it is visible for all students.

Supports accessibility for: Visual-spatial processing; Conceptual processing

Access for English Language Learners

Representing, Reading: MLR8 Discussion Supports. When students explain how they wrote their expressions, provide sentence frames such as “___ represents ___ because . . .” Encourage students to consider what details are important to share and to think about how they will explain their reasoning using mathematical language. This will help students to explicitly connect the language of the problem with the structure of the expressions that represent the context.

Design Principle(s): Maximize meta-awareness, Support sense-making

7.3 Jada Crochets a Scarf

Optional: 15 minutes

In this activity, most of the supports for the solution process are removed. Students need to think about how to choose which quantity to represent with a variable, how to represent the other two quantities with expressions in terms of the variable, and how to write an expression for a total. The only guidance offered is the reminder to write the expression with as few terms as possible.

While students are working, monitor for students who choose to have their variable represent different quantities (e.g. number of single stitches, double stitches, or triple stitches).

Addressing

- 7.EE.A.1
- 7.EE.B.4.a

Instructional Routines

- MLR7: Compare and Connect
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 6–7 minutes of quiet work time followed by partner and whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts. After students have solved the first two problems, check-in with either select groups of students or the whole class. Invite students to share the strategies they have used so far as well as any questions they have before continuing.

Supports accessibility for: Organization; Attention

Student Task Statement

Basic crochet stitches are called single, double, and triple. Jada measures her average stitch size and sees that a “double crochet” stitch is not really twice as long; it uses $\frac{1}{2}$ inch less than twice as much yarn as a single crochet stitch. Jada’s “triple crochet” stitch uses 1 inch less than three times as much yarn as a single crochet stitch.

1. Write an expression that represents the amount of yarn Jada needs to crochet a scarf that includes 800 single crochet stitches, 400 double crochet stitches, and 200 triple crochet stitches.
2. Write an equivalent expression with as few terms as possible.
3. If Jada uses 5540 inches of yarn for the entire scarf, what length of yarn does she use for a single crochet stitch?

Student Response

1. Answers vary. Sample response: Let x represent the length of yarn needed for a single crochet stitch. $800x + 400(2x - 12) + 200(3x - 1)$
2. $2,200x - 400$
3. 2.7 inches of yarn for a single crochet stitch. $2,200x - 400 = 5540$, $2,200x = 5940$, $x = 2.7$

Activity Synthesis

Display the following questions for all to see and have students discuss them with their partner:

- “How did you define your variable? Why did you make this choice?”
- “How did you come up with your expression for the total amount of yarn? Explain what each part of your expression represents.”
- “What decisions did you make while rewriting the expression with fewer terms?”
- “How did you, or could you, check your solution?”

After students have discussed with their partner, invite students to share with the class anything interesting they noticed while comparing with their partner. For example, did they both write the

same exact expression, or did they write different but equivalent expressions? Does anyone think their partner had a particularly creative strategy for solving the problem?

If any students chose their variable to represent the number of double or triple crochet stitches, invite them to explain their solution strategy to the class and discuss how it is the same and different from having the variable represent the number of single crochet stitches.

Access for English Language Learners

Representing, Writing: MLR7 Compare and Connect. As students share their solution strategies, invite pairs to look for similarities and differences between the approaches presented. Invite students to discuss “What is the same and what is different?” about how students defined their variable and wrote their expressions. Ask students to make connections between how the expressions relate to how you define the variable of the expression (e.g., “What does the variable mean in this expression compared to that expression?”). This will help students understand the importance of clearly defining the variable so their expressions accurately represent the situation.

Design Principle(s): Maximize meta-awareness; Support sense-making

Lesson 8: Measurement Error (Part 1)

Goals

- Estimate a measurement and determine the largest possible percent error of the estimate.
- Generalize (orally) a process for calculating the maximum percent error of measurements that are added together.

Lesson Narrative

This lesson is optional. The activities in this lesson and the next all address the concept of measurement error. Any of these activities can stand on its own, although activities in the next lesson are more challenging, and students would likely benefit from doing the earlier ones first. Note that the first activity in this lesson gives students an opportunity to make measurements and analyze the size of the error. The second activity provides the percent error in the measurement in an addition context. In addition to examining accuracy of measurements carefully (MP6), students will work through examples and look for patterns (MP8) in order to hypothesize, and eventually show, how percent error behaves when measurements with error are added to one another.

As with all lessons in this unit, all related standards have been addressed in prior units. This lesson provides an optional opportunity to go deeper and make connections between domains.

Alignments

Addressing

- 7.EE.B: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Required Materials

Four-function calculators

Required Preparation

Since the calculations for each of these tasks are involved, students will need access to calculators.

Student Learning Goals

Let's check how accurate our measurements are.

8.1 How Long Are These Pencils?

Optional: 20 minutes

In this activity, students measure lengths and determine possibilities for actual lengths. There are two layers of attending to precision (MP6) involved in this task:

- Deciding how accurately the pencils can be measured, probably to the nearest mm or to the nearest 2 mm, but this depends on the eyesight and confidence of the student
- Finding the possible percent error in the measurement chosen

Addressing

- 7.RP.A.3

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2. Provide access to calculators. Give students 4–5 minutes of quiet work time, followed by partner and whole-class discussion.

Access for English Language Learners

Writing, Speaking: MLR8 Discussion Supports. To support students as they respond to “How accurate are your estimates?”, provide a sentence frame such as: “My estimate is within ___ mm of the actual length because” Encourage students to consider what details are important to share and to think about how they will explain their reasoning using mathematical language. This will help students use mathematical language as they justify the accuracy of their estimates.

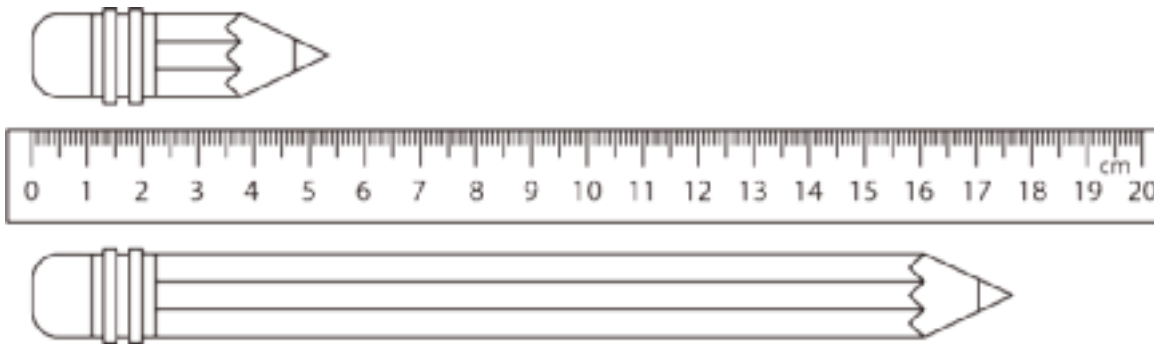
Design Principle(s): Optimize output (for explanation)

Anticipated Misconceptions

Some students may think that they can find an exact value for the length of each pencil. Because the pictures of the pencils are far enough away from the ruler, it requires a lot of care just to identify the “nearest” millimeter (or which two millimeter markings the length lies between). Prompt them to consider the error in their measurements by asking questions like, “Assuming you measured the pencil accurately to the nearest millimeter, what is the longest the actual length of the pencil could be? What is the shortest it could be?”

Some students may not remember how to calculate percent error. Ask them, “What is the biggest difference possible between the estimated and actual lengths? What percentage of the actual length would that be when the difference is as big as possible?”

Student Task Statement



1. Estimate the length of each pencil.
2. How accurate are your estimates?
3. For each estimate, what is the largest possible percent error?

Student Response

1. Answers vary. Sample response: The shorter pencil appears to be between 5.3 and 5.5 cm, perhaps 5.4 to the nearest mm, while the longer pencil appears to be 17.7 cm to the nearest mm (it is between 17.6 and 17.8 cm).
2. Answers vary. Sample response: The estimate is accurate to within 1 mm. For the short pencil, it is more than 5.3 cm and less than 5.5 cm, but it is not possible to tell which is closer. Similarly, the longer pencil is more than 17.6 cm and less than 17.8 cm.
3. For the shorter pencil, taking 5.4 cm as the measured length, the actual length x is at least 5.3 cm and at most 5.5 cm. The percent error if x is as small as possible is $\frac{0.1}{5.3} \approx 2\%$, and if x is as big as possible, then the error is $\frac{0.1}{5.5} \approx 2\%$. The first of these gives the greatest percent error, although they are close. For the longer pencil, the percent error is smaller. The biggest it can be is $\frac{0.1}{17.7} \approx 0.6\%$. This makes sense because 0.1 cm is a bigger percentage of the length of the small pencil.

Activity Synthesis

The goal of this discussion is for students to practice how they talk about precision.

Discussion questions include:

- “How did you decide how accurately you can measure the pencils?” (I looked for a value that I was certain was less than the length of the pencil and a value that I was certain was bigger. My estimate was halfway in between.)
- “Were you sure which mm measurement the length is closest to?” (Answers vary. Possible responses: Yes, I could tell that the short pencil is closest to 5.4 cm. No, the long pencil looks to be closest to 17.7 mm, but I’m not sure. I am sure it is between 17.6 cm and 17.8 cm.)

- “Were the percent errors the same for the small pencil and for the long pencil? Why or why not?” (No. I was able to measure each pencil to within 1 mm. This is a *smaller* percentage of the longer pencil length than it is of the smaller pencil length.)

Other possible topics of conversation include noting that the level of accuracy of a measurement depends on the measuring device. If the ruler were marked in sixteenths of an inch, we would only be able to measure to the nearest sixteenth of an inch. If it were only marked in cm, we would only be able to measure to the nearest cm.

8.2 How Long Are These Floor Boards?

Optional: 20 minutes

This activity examines how measurement errors behave when they are added together. In other words, if I have a measurement m with a maximum error of 1% and a measurement n with a maximum error of 1%, what percent error can $m + n$ have? In addition to examining accuracy of measurements carefully (MP6), students work through examples and look for patterns (MP8) in order to hypothesize, and eventually show, how percent error behaves when measurements with error are added to one another.

Monitor for students who look for patterns, recognize the usefulness of the distributive property, or formulate the problem abstractly with variables.

Addressing

- 7.EE.B

Launch

Read the problem out loud and ask students what information they would need to know to be able to solve the problem. Students may say that they need to know what length the boards are supposed to be, because it is likely that they haven't realized that they can solve the problem without this information. Explain that floor boards come in many possible lengths, that 18-inch and 36-inch lengths are both common, but the boards can be anywhere between 12 and 84 inches. Ask students to pick values for two actual lengths and figure out the error in that case. Then they can pick two different examples, make the calculations again, and look for patterns.

Provide access to calculators.

Access for Students with Disabilities

Representation: Internalize Comprehension. Represent the same information through different modalities by using diagrams. If students are unsure where to begin, suggest that they draw a diagram to help illustrate the information provided.

Supports accessibility for: Conceptual processing; Visual-spatial processing

Anticipated Misconceptions

Some students may pick some example lengths but then struggle with knowing what to do with them. Ask them “what would be the maximum measured lengths? The minimum? What would be the error if both measurements were maximum? What if they were both minimum?”

Some students may pick numbers that make the calculations more complicated, leading to arithmetic errors. Suggest that they choose simple, round numbers for lengths, like 50 inches or 100 centimeters.

Student Task Statement

A wood floor is made by laying multiple boards end to end. Each board is measured with a maximum percent error of 5%. What is the maximum percent error for the total length of the floor?

Student Response

The maximum percent error is 5%. Sample explanation: If x is the actual length and m is the measured length of one board, then $0.95x < m$ and $m < 1.05x$: I know this because the measurement m has a maximum error of 5%. If y is the actual length and n is the measured length of a second board, then $0.95y < n$ and $n < 1.05y$. If both boards have maximum length, the total length would be $1.05x + 1.05y = 1.05(x + y)$. If they are both minimum, the total length would be $0.95x + 0.95y = 0.95(x + y)$. So the maximum percent error would be 5%. This same argument works for any number of boards because the distributive property works for any number of addends.

Activity Synthesis

The goal of this discussion is for students to generalize from their specific examples of measurements to understand the general pattern and express it algebraically.

Poll the class on the measurements they tried and the maximum percent error they calculated. Invite students to share any patterns they noticed, especially students who recognized the usefulness of the distributive property for making sense of the general pattern.

Guide students to use variables to talk about the patterns more generally.

- If a board is supposed to have length x with a maximum percent error of 5%, then the shortest it could be is $0.95x$ and the longest it could be is $1.05x$.
- If another board is supposed to have length y , it could be between $0.95y$ and $1.05y$.
- When the boards are laid end-to-end, the shortest the total length could be is $0.95x + 0.95y$, which is equivalent to $0.95(x + y)$.
- The longest the total length could be is $1.05x + 1.05y$, or $1.05(x + y)$.
- Because of the distributive property, we can see that the maximum percent error is still 5% after the board lengths are added together.

One interesting point to make, if students have also done the previous activity about measuring pencils, is that you could *measure* the sum of the board lengths with a lower percent error than you could measure each individual board (assuming your tape measure is long enough), just like an error of 1 mm was a smaller percentage of the length of the longer pencil.

Lesson 9: Measurement Error (Part 2)

Goals

- Comprehend that percent error is greatest when the actual values are smaller than the measured values for an object.
- Generalize (orally) a process for calculating the maximum percent error for the area of a rectangle and volume of a rectangular prism.

Lesson Narrative

This lesson is optional. This is the second lesson exploring measurement error in more detail. While the previous lesson examined percent error when measurements are added together, this lesson works with percent error when measurements are multiplied. While the activities in this lesson can stand on their own, students will benefit from having done the activities in the previous lesson first.

In addition to examining accuracy of measurements carefully (MP6), students will work through examples and look for patterns (MP8) in order to hypothesize, and eventually show, how percent error behaves when measurements with error are multiplied to one another.

As with all lessons in this unit, all related standards have been addressed in prior units. This lesson provides an optional opportunity to go deeper and make connections between domains.

Alignments

Building On

- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems.
Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Addressing

- 7.G.B.6: Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports
- Think Pair Share

Required Materials

Four-function calculators

Student Learning Goals

Let's check how accurate our calculations are.

9.1 Measurement Error for Area

Optional: 20 minutes

This activity examines how measurement errors behave when quantities are multiplied. In other words, if I have a measurement m with a maximum error of 5% and a measurement n with a maximum error of 5%, what percent error can $m \cdot n$ have?

Monitor for students who use different methods to solve the problem, such as trying out sample numbers or using expressions with variables.

Building On

- 7.RP.A.3

Addressing

- 7.G.B.6

Instructional Routines

- MLR3: Clarify, Critique, Correct
- Think Pair Share

Launch

Arrange students in groups of 2. Provide access to calculators.

If desired, suggest that students try out several different sample numbers for the length and width of the rectangle, calculate the maximum percent error, and look for a pattern. Give students 4–5 minutes of quiet work time, followed by time to discuss their work with their partner and make revisions, followed by whole-class discussion.

Access for English Language Learners

Conversing, Representing: MLR3 Clarify, Critique, Correct. Present an incorrect response that reflects a possible misunderstanding from the class such as: "Since both length and width have maximum errors of 5%, I multiplied 5% by 5% to get 25%, because area is length times width." Prompt pairs to clarify and then critique the incorrect response, and then write a correct version. This provides students with an opportunity to evaluate, and improve on, the written mathematical arguments of others.

Design Principle(s): Maximize meta-awareness; Support sense-making

Anticipated Misconceptions

Students may think that the maximum error possible for the area is 5% because both the length and width are within 5% of the actual values. Encourage these students to make calculations of the biggest and smallest possible length and the biggest and smallest possible width. Then have them make calculations for the biggest and smallest possible area.

Student Task Statement

Imagine that you measure the length and width of a rectangle and you know the measurements are accurate within 5% of the actual measurements. If you use your measurements to find the area, what is the maximum percent error for the area of the rectangle?

Student Response

The maximum percent error would be 10.25%. If x is the actual length and m is the measured length, then $0.95x < m$ and $m < 1.05x$ since $0.05x$ is 5% of x . If y is the actual width and n is the measured width, then the biggest possible error is $0.05y$ so $0.95y < n$ and $n < 1.05y$. If they are both maximum, the area would be $1.05^2 xy = 1.1025xy$. If they are both minimum, the area would be $0.95^2 xy = 0.9025xy$. So the maximum percent error would be when they are both at the maximum possible error, and the percent error would be 10.25%.

Activity Synthesis

Have students trade papers with a partner and check their work.

Invite students to share their solutions, especially those who looked for a pattern or used variables. Consider discussing questions like these:

- “Did you calculate the maximum percent error for any specific sample measurements? What did you find?” (The maximum percent error for the largest and smallest possible values were not the same: 9.75% and 10.25%.)
- “How do you know that this pattern is true for any possible length and width of the rectangle?” (I used variables to express the unknown measurements.)
- “How could you use variables to help solve this problem?” (I can use variables to represent the length and width of the rectangle and write expressions in terms of these variables to represent the largest and smallest possible areas.)

9.2 Measurement Error for Volume

Optional: 25 minutes

This challenging activity examines how measurement errors behave when 3 quantities are multiplied (versus 2 quantities in the previous activity). In other words, if I have measurements a , b , and c each with a maximum error of 5%, what percent error can $a \cdot b \cdot c$ have? The arithmetic and algebraic demands of this task are high because students take a product of *three* quantities that each have a maximum percent error of 5%.

Building On

- 7.RP.A.3

Addressing

- 7.G.B.6

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. Provide access to calculators. Make sure students realize that the first question gives the measured values, not the actual values, for each dimension.

Give students 10 minutes to discuss with their partners, followed by whole-class discussion.

Access for English Language Learners

Conversing: MLR8 Discussion Supports. Before students determine the maximum measurement error for the volume of a prism, invite pairs to make a prediction and justification. Use a sentence frame such as: "In a prism, if each dimension has a 5% maximum measurement error, we predict the volume's maximum percentage error is ___ because....". This will help students use the mathematical language of justifications to begin reasoning about the measurement error for volume.

Design Principle(s): Cultivate conversation; Support sense-making

Anticipated Misconceptions

Students may think that the maximum error possible for the volume is 1% because the length, width, and height are within 1% of the actual values. Encourage these students to make calculations of the biggest and smallest possible length, width, and height. Then ask them to make calculations for the biggest and smallest possible volumes.

Student Task Statement

1. The length, width, and height of a rectangular prism were measured to be 10 cm, 12 cm, and 25 cm. Assuming that these measurements are accurate to the nearest cm, what is the largest percent error possible for:
 - a. each of the dimensions?
 - b. the volume of the prism?
2. If the length, width, and height of a right rectangular prism have a maximum percent error of 1%, what is the largest percent error possible for the volume of the prism?

Student Response

1. The actual measurements could be as much as 0.5 cm over or under the measurements given.
 - a. We know the largest percent error occurs with the *smallest* measurement, so we will only check it for the minimum possible actual lengths.
 - $0.5 \div 9.5 \approx 0.05$ (or about 5%)
 - $0.5 \div 11.5 \approx 0.04$ (or about 4%)
 - $0.5 \div 24.5 \approx 0.02$ (or about 2%)
 - b. The measured volume is 3,000 cubic cm. The smallest the actual volume could be is $9.5 \cdot 11.5 \cdot 24.5 = 2,676.625$. The percent error in the case of the smallest one is $(3,000 - 2,676.625) \div 2,676.625$ or about 12%. The biggest the actual volume could be is $10.5 \cdot 12.5 \cdot 25.5 = 3,346.875$, which gives a percent error of about 10%. Again, we find that the biggest possible error happens when the actual measurement is as small as possible.
2. If the actual dimensions are x , y , and z , then the minimum measured volume would be $(0.99)^3 xyz \approx 0.97xyz$ and the maximum measured volume would be $(1.01)^3 xyz \approx 1.03xyz$. So the largest percent error in the volume is about 3%.

Activity Synthesis

Some discussion points include:

- The first problem gives measurements and errors (but no percent error), while the second problem gives no measurements but does give the percent error. This makes the calculations notably different for the two problems.
- In the first problem, we are given measurements and the possible size of error. We need to find the greatest percent error and, as we have seen in other cases, this happens for the smallest possible value of the measurement. If we were to find the percent error of each measurement, we would find that the error for the volume is a *larger* percent error than for any of the individual measurements.
- In the second problem, we are given the maximum possible percent error but no measurements, and we need to find the largest possible error for the volume, that is, for the product of the three unknown measurements. The greatest percent error possible for the volume occurs when the measured value is as large as possible.
- A unifying feature in these two problems is that we notice that the largest percent error occurs when the actual measurements are *smaller* than the measured values, as much smaller as possible.

Access for Students with Disabilities

Representation: Internalize Comprehension. Use color and annotations to illustrate connections between representations. As students share their diagrams and reasoning, use color and annotations to scribe their thinking on a display of each problem so that it is visible for all students.

Supports accessibility for: Visual-spatial processing; Conceptual processing

Section: Designing a Course

Lesson 10: Measuring Long Distances Over Uneven Terrain

Goals

- Choose a method and appropriate measuring tools to measure the length of a path.
- Compare and contrast (orally and in writing) methods for measuring distances.
- Compare measurements of a path and represent (in writing) the difference between measurements as a percentage.

Lesson Narrative

This lesson is optional. It is the first of four lessons where students explore ways of measuring long distances. Over the course of these lessons:

- Students first think about different ways to measure distances of various lengths and in which situations different methods might work better.
- They then read about and build a trundle wheel (also known as a surveyor wheel or measuring wheel) that is commonly used to measure walking distances.
- They design a walking course for a 5K race on their school campus. (The course should be one lap of about 500 m. The actual race would go around the course multiple times.)
- They use their trundle wheel to measure the path of the walking course and make a scale drawing of the course on a map or satellite image of the school grounds.

In this first lesson, students brainstorm ideas about how to measure long distances, possibly over uneven terrain. Students work in groups to try out the accuracy and effectiveness of different methods. Some of the methods involve proportional reasoning. Students engage in many aspects of mathematical modeling (MP4) and will use appropriate tools (MP5) when they are planning and trying out methods of measurement.

As with all lessons in this unit, all related standards have been addressed in prior units. This lesson provides an optional opportunity to go deeper and make connections between domains.

Alignments

Building On

- 2.MD.A.1: Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

Addressing

- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Building Towards

- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Instructional Routines

- MLR8: Discussion Supports

Required Materials

Measuring tapes

Yardsticks

Meter sticks

Required Preparation

Choose a path outside of the classroom that students can measure the length of. The length should be between 50 and 100 meters (long enough that it cannot be measured directly with a tape measure. Ideally it should include some curves or elevation changes (but not stairs). A long hallway would also be okay, especially if it goes around corners. This is not part of the 5K course, rather it is just a path to test measuring methods.

Student Learning Goals

Let's measure long distances over uneven terrain.

10.1 How Far Is It?

Optional: 5 minutes

Students have experienced measuring short distances with a ruler or a measuring tape. In this activity, students start to think about how they can measure longer distances over uneven terrain. This activity is intended to set the stage for the upcoming activities, not to completely resolve the question. Students have an opportunity to think about the limitations of methods that may work for short distances but not for long distances. They also consider real-world situations that involve the measurement of long distances.

Building On

- 2.MD.A.1

Building Towards

- 7.RP.A

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 3–4. They will stay in these groups throughout this 4-lesson unit. Ask students how they have measured the length of objects in school (with a ruler, yardstick, or measuring tape). Where else in real life do people measure distances, especially longer ones? Brainstorm some situations together (distance driven in a car, length of a garden fence, length of a hiking trail, etc.). Give students 2–3 minutes of quiet work time, followed by small-group discussion.

Anticipated Misconceptions

Some students may get stuck thinking about classroom situations. Prompt them to think about other situations outside of school, such as driving in a car, measuring the distance between cities, measuring the length of a fence around a yard, etc.

Student Task Statement

How do people measure distances in different situations? What tools do they use? Come up with at least three different methods and situations where those methods are used.

Student Response

Answers vary. Sample responses:

- Use a yardstick, a measuring tape or ruler repeatedly, if necessary.
- When driving use the odometer to measure distance between departure and destination location.
- Count your steps and estimate how long one step is, for example, to measure the distance across the room.
- Estimate an inch with your fingers and iterate across the width of your table to measure table width.
- Count the number of ceiling tiles or the number of windows across the room, estimate the width of tiles or windows and multiply the number of objects times the width of the object.
- GPS
- Use “rate times time,” if you know your speed and how long it takes to get somewhere.
- Distance between stars.

Activity Synthesis

Invite students to share some ideas of how to measure with their group.

Access for English Language Learners

Speaking, Representing: MLR8 Discussion Supports. Give students additional time to make sure that everyone in their group can explain all three different methods and situations they created. Prompt groups to rehearse what they will say when they share with the whole class. Rehearsing provides students with additional opportunities to speak and clarify their thinking. This will also help students improve the quality of their explanations during the whole-class discussion.

Design Principle(s): Optimize output (for explanation)

10.2 Planning a 5K Course

Optional: 10 minutes

In the previous activity, students started to think about how to measure distances in different situations. The activity introduces the context of designing a course for a 5K fundraising walk. Students will continue working with this context in future lessons. In this activity, they come up with a method for measuring the walking distance of a path that is too long to measure with a measuring tape. Students will try out their method in the next activity.

Students get a chance to engage in many aspects of mathematical modeling (MP4). The modeling cycle starts with formulating the question that we want to answer, clarifying which quantities are involved, and how to measure them. In many problems, this step is done for students. Here, we are giving them the opportunity to think about how to set up the problem and what tools are appropriate to measure distances.

Building On

- 2.MD.A.1

Building Towards

- 7.RP.A

Instructional Routines

- MLR8: Discussion Supports

Launch

Keep students in the same groups. Provide access to measuring tools, such as yardsticks, meter sticks, and tape measures. Ask students if they have ever participated in or watched a walk-a-thon or race. Explain that sometimes a race is done by repeating a shorter course several times, e.g. a mile is about 4 laps around a track. For this activity, they should plan for a course that is about 500 meters long that walkers can go around multiple times.

Give students 5–6 minutes to work with their group.

Access for English Language Learners

Conversing: MLR8 Discussion Supports. When preparing a plan for measuring the course, invite students to use a sentence frame such as: “One method for measuring the course is” Encourage students to consider what details are important to share and to think about how they will explain their reasoning using mathematical language. This will help students to converse while they design the course and decide how to measure its distance.

Design Principle(s): Optimize output (for description)

Student Task Statement

The school is considering holding a 5K fundraising walk on the school grounds. Your class is supposed to design the course for the walk.

1. What will you need to do to design the course for the walk?
2. Come up with a method to measure the course. Pause here so your teacher can review your plan.

Student Response

1. Answers vary. Sample response: We need to find a course for one lap of the race, decide where the start and end is, measure it, and then figure out how often you have to go around it to complete 5 km.
2. Answers vary. Sample responses:
 - a. Use a measuring tape over and over again.
 - b. Measure your stride length, and then count the number of steps.
 - c. Find a map of the campus, and use the scale on the map to compute the length of the course.

Activity Synthesis

Students check with the teacher about their method of measurement and then move on to the next activity.

10.3 Comparing Methods

Optional: 20 minutes

In this activity, students use the method they came up with in the previous activity to measure the length of a path chosen by the teacher. Each group can begin working on this activity as soon as they have finished the previous activity and checked in with the teacher.

It is not important that students' results are very accurate. They will measure the distance again with a trundle wheel in a later lesson. The main point of this activity is to think about measurement methods and to discuss the advantages and disadvantages of different methods.

Addressing

- 7.RP.A

Launch

Keep students in the same groups. Provide access to measuring tools. Show students the path they should measure. Give students time to measure with their group.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Provide students with a graphic organizer for data collection and organizing information about methods, lengths and average between two measurements.

Supports accessibility for: Language; Organization

Anticipated Misconceptions

Some students may need to be reminded how to use the measuring tools accurately, such as starting at the 0 mark and keeping the measuring tool going in a straight line.

Student Task Statement

Let's see how close different measuring methods are to each other. Your teacher will show you a path to measure.

1. Use your method to measure the length of the path at least two times.
2. Decide what distance you will report to the class.
3. Compare your results with those of two other groups. Express the differences between the measurements in terms of percentages.
4. Discuss the advantages and disadvantages of each group's method.

Student Response

1. Answers vary.
2. Answers vary. Sample response: Report the average between the two measurements.
3. Answers vary. Sample responses: If group A's measurement is 50 m and group B's measurement is 51 m, then group B's measurement is 2% larger than group A's since $51 \div 50 = 1.02$.
4. Answers vary. For the two methods given in previous task:

- a. Use a measuring tape over and over again. Advantages: Can be very accurate.
Disadvantage: It takes two people and is quite cumbersome. If not done carefully, each time the tape moves, an error is introduced. So this is not very practical for long distances and if there are a lot of corners to go around.
- b. Measure stride length, and then count the number of strides. Advantages: Very easy to do and very quick. Disadvantage: Not all strides are equal. The longer the distance, the more chances for errors there are.

Activity Synthesis

Invite the different groups to share their solutions. Ask them to:

- Compare how close their answers are.
- Compute the approximate relative error (difference/total length).
- Discuss the advantages and disadvantages of their methods and sources of discrepancies in their measurements, and how a small error can propagate.

The takeaway should include:

- We can use proportional reasoning to find longer distances. If we know it takes 10 steps to walk 8 meters, then it will take 20 steps to walk 16 meters.
- Small errors can magnify over longer distances.
- Methods were either not very precise (prone to introduce error), or they were precise but cumbersome to implement.

Access for English Language Learners

Representing: MLR3 Clarify, Critique, Correct. Display an incorrect statement about the percentage difference between measurements such as: “One group compared their measurement of 1000 meters to another group’s measurement of 992 meters and calculated the percent difference as 8%.”. Prompt students to clarify and critique the error, and then write a correct version. This helps students evaluate, and improve on, the written mathematical arguments of others.

Design Principle(s): Cultivate conversation; Maximize meta-awareness

Lesson 11: Building a Trundle Wheel

Goals

- Create a trundle wheel and use it to calculate the length of the classroom.
- Explain (orally and in writing) how a trundle wheel is used to measure long distances.

Lesson Narrative

This lesson is optional. In the second lesson of the 4-lesson sequence, students build a trundle wheel, a device used to measure walking distances. First, they learn about a trundle wheel and discuss how such a device works (MP5). Then, students use paper plates to make a usable trundle wheel and practice using it to measure distances in the classroom.

Students can either use the same-sized paper plates or different groups can use different-sized paper plates. The use of different-sized plates allows for more mathematical discussion about how diameter and circumference of the plates affect how we report the distance being measured (MP6).

As with all lessons in this unit, all related standards have been addressed in prior units. This lesson provides an optional opportunity to go deeper and make connections between domains.

Alignments

Building On

- 7.G.B.4: Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Addressing

- 7.G.B.4: Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Instructional Routines

- MLR2: Collect and Display
- MLR7: Compare and Connect

Required Materials

Index cards

Measuring tools

Metal paper fasteners

brass brads

Paper plates

Tape

Required Preparation

To build the trundle wheel, each group of 3–4 students will need 1 paper plate, a metal paper fastener, something long object to use as the handle, an index card, and tape.

Ideally, have three different sizes of paper plates for different groups to work with (typical sizes are 6–12 inches) to help reinforce the point that the size of the plate affects how many “clicks” (rotations) it takes to measure the same distance.

There are many options for how to make the handles. A yardstick or meter stick with a hole on one end is most convenient. Alternatively, you can tape two rulers together or cut pieces of sturdy cardboard or foam core that are about 1.5 inches wide by 30 inches long, and poke a hole centered at one end.

Student Learning Goals

Let’s build a trundle wheel.

11.1 What Is a Trundle Wheel?

Optional: 10 minutes

In the previous lesson, students tried using simple methods and tools to measure long distances. In this activity, they learn that a trundle wheel is a tool used in real-world situations to measure such distances. From an image and a description of what a trundle wheel looks like, students think about how the tool works and how they could build one. Students think about the tasks for which a trundle wheel is an appropriate measuring tool (MP5). This builds on work students did in an earlier unit, when they learned about the relationship between the circumference of a wheel and the distance it travels.

Building On

- 7.G.B.4

Instructional Routines

- MLR2: Collect and Display

Launch

Keep students in the same groups of 3–4 from the previous lesson. Explain to students that a trundle wheel is a measuring device composed of a handle, a wheel, and a device that clicks each time the wheel completes on rotation. Give students 5 minutes of quiet work time followed by whole-class discussion.

Access for English Language Learners

Writing, Speaking: MLR2 Collect and Display. While groups are working on this activity, listen to student conversations and document phrases from conversations or students' written work that describe sources of error and the meaning of circumference. Display phrases for the class to consider when responding to questions posed during the synthesis. Press for student use of mathematical language during the discussion.

Design Principle(s): Support sense-making

Anticipated Misconceptions

Students may not remember the relationship between the circumference of a wheel and the distance traveled by the wheel. Remind students of this idea from previous lessons.

Student Task Statement

A tool that surveyors use to measure distances is called a trundle wheel.

1. How does a trundle wheel measure distance?
2. Why is this method of measuring distances better than the methods we used in the previous lesson?
3. How could we construct a simple trundle wheel? What materials would we need?



Student Response

Answers vary. Possible responses:

1. You push the wheel, and as it turns it keeps track of the number of rotations. If we know the circumference of the wheel, we can multiply by the number of rotations to find the distance we walked.
2. The circumference does not change, it is constant, and each rotation follows the next without any gaps. Strides can be slightly different from step to step, and if we use a measuring tape, there might be a gap between iterations.

3. We need a wheel, a handle, and a way to keep track of and count rotations.

Activity Synthesis

The goal of this discussion is for students to remember the connection between the circumference of a wheel and the distance the wheel travels during one rotation, so that they are prepared to use a trundle wheel to measure distances in the next activity. Invite students to share their ideas about how to build a trundle wheel and ask them how their design will allow them to measure distances.

Consider asking the following questions:

- “What information about the wheel do we need to know? What quantities should we measure?”
- “Why is it important to have a clicking device? What information does the device give us?”
- “If we have a wheel that has diameter 25 cm and we count 11 clicks to go across the classroom, what is the length of the room?” (Answer: $25\pi \cdot 11$ or between 8 and 9 m.)

We can measure distance by counting the rotations of the wheel and multiplying by the circumference of the wheel. The construction of the trundle wheel allows us to easily count the rotations as we walk.

11.2 Building a Trundle Wheel

Optional: 25 minutes

In this activity, students build a trundle wheel and use it to measure distances in the classroom. The trundle wheels need to be stored in the classroom for use in the next lesson. If it is not feasible to store a trundle wheel from every group of 3–4 students, have them combine to form larger groups before building the wheels. Each student should still get a chance to practice measuring with the trundle wheel.

It is suggested that students build their trundle wheels using a paper plate as the wheel, two rulers taped together end to end as the handle, a metal paper fastener, and an index card taped to the wheel to produce an audible “click” when it hits the handle. There are many other ways to build a trundle wheel. If this fits into the culture of the class, students can use other designs and materials, e.g., students could use a bike with a playing card in the spokes to count rotations.

Addressing

- 7.G.B.4
- 7.RP.A

Instructional Routines

- MLR7: Compare and Connect

Launch

Keep students in the same groups, or if necessary, combine them to form larger groups. Discuss how to build the trundle wheel and distribute the supplies. Give students 15 minutes of group work time to build and try out their trundle wheels, followed by whole-class discussion.

Anticipated Misconceptions

Students may need some trial and error in building working trundle wheels, in particular for placing the clicking device. Encourage them to try out their design and then revise it as necessary.

Student Task Statement

Your teacher will give you some supplies. Construct a trundle wheel and use it to measure the length of the classroom. Record:

1. the diameter of your trundle wheel
2. the number of clicks across classroom
3. the length of the classroom (Be prepared to explain your reasoning.)

Student Response

Answers vary. Possible solutions:

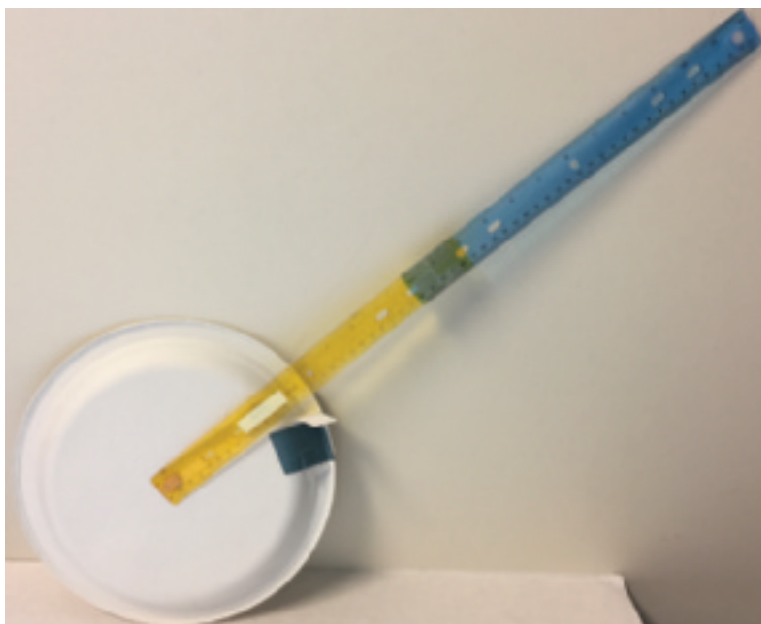


Plate radius is 13 cm. Diameter is $2\pi \cdot 13$ or 81.7 cm. 1 click corresponds to 81.7 cm. If we count 20 clicks on the path, then the distance of the path is $20 \cdot 81.7$ or about 16 m.

Activity Synthesis

The goal of this discussion is for students to double-check that their trundle wheels work correctly and their results are reasonable. Invite one group that used each size of paper plate to demonstrate how their wheel works by walking across the classroom and counting the

“clicks.” Record the data for all to see and discuss how these groups’ calculations for the length of the classroom compare, which may include the following points:

- The group with the smaller plate had a larger number of clicks and vice versa.
- Each group needs to multiply their number of clicks by the circumference of their wheel.
- The circumference of the wheel can be found by multiplying the diameter times π or the radius times 2π .
- Should we report our answer in terms of π or use an approximation?
- Are the groups’ answers for the length of the classroom pretty close or very different?
- Is it possible to count half rotations of the wheel?

Access for Students with Disabilities

Representation: Develop Language and Symbols. Display or provide charts with important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding and memory of: formula for circumference, rotation and the relationship between circumference of a wheel and the distance traveled.

Supports accessibility for: Conceptual processing; Memory

Access for English Language Learners

Representing, Conversing: MLR7 Compare and Connect. Rearrange groups of students so that they are seating with another group that used the same size paper plate to build their trundle wheel. Invite groups to share their solutions and reasoning for the length of the classroom with each other. Ask, “What is the same and what is different?” about how they used their trundle wheel to measure the classroom. This will help students to consider precision when measuring and use mathematical language as they converse.

Design Principle(s): Maximize meta-awareness; Cultivate conversation

Lesson 12: Using a Trundle Wheel to Measure Distances

Goals

- Calculate the distance of a path using the circumference and number of rotations of a trundle wheel.
- Compare measurement calculations and express differences between measurements as a percentage.
- Critique (orally) methods for measuring a long distance.

Lesson Narrative

This lesson is optional. In the third lesson of this sequence, students use their trundle wheels to re-measure the distance from the first lesson. After students have had experiences using their trundle wheels, they can connect their observations of how the wheel works with the distance computations and discuss the sources of errors in a more meaningful way. Students engage in important parts of mathematical modeling (MP4), they use appropriate tools (MP5), and attend to precision (MP6).

As with all lessons in this unit, all related standards have been addressed in prior units. This lesson provides an optional opportunity to go deeper and make connections between domains.

Alignments

Addressing

- 7.G.B.4: Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Instructional Routines

- MLR8: Discussion Supports

Required Materials

Trundle wheels

Required Preparation

Prepare to distribute the trundle wheels students built in the previous lesson. Make sure students can still get to the path (between 50 and 100 meters) that they measured the other day.

Student Learning Goals

Let's use our trundle wheels.

12.1 Measuring Distances with the Trundle Wheel

Optional: 40 minutes (there is a digital version of this activity)

In the previous lesson, students built trundle wheels using three different wheel sizes, and they tested the functionality of their wheels in the classroom. In this activity, they use their trundle wheels to measure a longer path of about 50–100 meters. This is the same path that they measured during a previous lesson with a different method. If several groups are sharing a trundle wheel, then they each measure the given path once and share their data with each other.

After students measure, they spend the remainder of the lesson on computations and sharing results. Students get a chance to connect the mathematical formulas and computations with the aspects of the hands-on experience they had in making and using the wheels. They attend to precision when they are deciding on how to report their results and when they are comparing results with other groups (MP6).

Addressing

- 7.G.B.4
- 7.RP.A

Instructional Routines

- MLR8: Discussion Supports

Launch

Keep students in the same groups from the previous lesson. Remind students of the path they should measure. Instruct them to come back to the classroom to finish their calculations as soon as they have recorded their measurements.

Give students 10–20 minutes to take turns measuring and 10 minutes of group work time to finish their calculations, followed by whole-class discussion.

There is also a digital version of a trundle wheel available. Students can be assigned a path, numbered from 1 to 5, and collect data on different-sized trundle wheels. The applet is programmed to stop automatically at one of the five distances. The students must keep track of the number of rotations the wheel makes before it stops.

Access for Students with Disabilities

Action and Expression: Provide Access for Physical Action. Support access to tools and assistive technologies. Monitor for students who may need an additional demonstration, or assistance using their trundle wheel to measure a longer path. Consider modeling a systematic way to record the number of clicks to calculate an example distance as students complete this task.

Supports accessibility for: Visual-spatial processing; Fine-motor skills

Anticipated Misconceptions

Students may lose count when counting the number of rotations along the path. Encourage them find a systematic way to record the number of clicks while their group member is walking along the path.

Student Task Statement

Earlier you made trundle wheels so that you can measure long distances. Your teacher will show you a path to measure.

1. Measure the path with your trundle wheel three times and calculate the distance. Record your results in the table.

trial number	number of clicks	computation	distance
1			
2			
3			

2. Decide what distance you will report to the class. Be prepared to explain your reasoning.
3. Compare this distance with the distance you measured the other day for this same path.
4. Compare your results with the results of two other groups. Express the differences between the measurements in terms of percentages.

Student Response

1. Answers vary. Sample response:
 - Diameter of wheel: 25 cm

trial number	number of clicks	computation	distance
1	63	$25\pi \cdot 63$	49.5 m
2	65	$25\pi \cdot 65$	51 m
3	63.5	$25\pi \cdot 63.5$	49.9

2. Answers vary. Sample response: Distance to report: 50 m
3. Answers vary.

4. Answers vary. Sample response: If group A's measurement is 50 m and group B's measurement is 51 m, then group B's measurement is 2% larger than group A's since $51 \div 50 = 1.02$.

Activity Synthesis

Ask each group to report their measurement for the length of the path and record their answers for all to see. Guide students to compare these answers by asking questions like these:

- "Do all of these answers seem reasonable? Do any of these answers seem unreasonable? Why?"
- "Why are these answers not all exactly the same? What are some sources of error?" (Not going in a straight line, the wheel wobbles, the ground is uneven, only counting number of clicks but not parts of rotations etc.)
- "What units did you use? What units would be most convenient for designing the course of a 5K walk-a-thon?" (Metric, since we are designing a 5 km course.)
- "What degree of precision is appropriate to report?" (To the closest 1 meter at most. Reporting cm or mm on such long distances with a tool like a trundle wheel would be implying a degree of precision that would not be appropriate.)

If time permits, consider asking "If you could choose your own diameter for a trundle wheel, what would it be?" (A diameter that creates a circumference of 1 m would be convenient, i.e. about 32 cm.)

Collect and store students' trundle wheels so they will have access to them again in the next lesson.

Access for English Language Learners

Speaking: MLR8 Discussion Supports. To support students' discussion of their measurements comparisons, invite them to use a sentence frame such as: "Our measurement is ____% (higher/lower) than group ____'s measurement because" Call on another student or group to restate what they heard using mathematical language. Encourage students to press each other for clear explanations and precise use of language.

Design Principle(s): Optimize output (for comparison)

Lesson 13: Designing a 5K Course

Goals

- Calculate the distance of a path using the circumference and number of rotations of a trundle wheel.
- Create a scale drawing of a 5K course and present (using words and other representations) the map and course details.
- Use proportional reasoning to calculate the number of laps of a course that is equal to 5 kilometers.

Lesson Narrative

This lesson is optional. In the final lesson of this unit, students design the 5K course and use their trundle wheels to measure distances. They draw a scale drawing of the course on a map or a satellite image of the school grounds, give instructions where the start and finish of the course should be, and decide how many laps are necessary to complete the race. In this lesson students engage in many aspects of mathematical modeling (MP4), use appropriate tools (MP5) and attend to precision (MP6). This lesson may take 2 days.

As with all lessons in this unit, all related standards have been addressed in prior units. This lesson provides an optional opportunity to go deeper and make connections between domains.

Alignments

Addressing

- 7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.
- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Instructional Routines

- Group Presentations
- MLR7: Compare and Connect
- MLR8: Discussion Supports

Required Materials

Maps or satellite images of the school grounds

Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart

paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Trundle wheels

Required Preparation

Prepare to distribute the trundle wheels students built in a previous lesson. Prepare maps or printed satellite images of the school grounds, one copy per student.

Student Learning Goals

Let's map out the 5K course.

13.1 Make a Proposal

Optional: 10 minutes

In this activity students return to the context of designing a 5K walk-a-thon that was introduced in an earlier lesson. They use a map or satellite image of the school grounds to decide where the path of the 5K course could be and estimate how many laps it would take to complete 5 kilometers. Ideally, one lap should be about 500 meters, because in the next activity, students will use their trundle wheels to measure the course they have designed.

If possible, each group chooses their own course, to help them take ownership of their work and for a greater variety of solutions. Alternatively, the whole class can come to an agreement on one path, to streamline the process if time is limited.

Addressing

- 7.G.A.1
- 7.RP.A

Instructional Routines

- MLR8: Discussion Supports

Launch

Keep students in the same groups of 3–4 from the previous lesson. Distribute maps or printed satellite images of the school grounds. Give students 10 minutes of group work time.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Provide students with a graphic organizer for data collection and organizing information about methods, lengths of estimations and revisions between the two measurements of the course.

Supports accessibility for: Language; Organization

Access for English Language Learners

Speaking: MLR8 Discussion Supports. To help students use mathematical language while reasoning about their length estimations, invite students to use a sentence frame such as: "We estimate the length of our course is ___ because . . ." Encourage students to consider what details are important to share and to think about how they will explain their reasoning using mathematical language. Invite students to consider and respond to the reasonableness of each others' estimates.

Design Principle(s): Support sense-making

Anticipated Misconceptions

Students may not have a good sense of scale and may be hesitant to estimate the length of their course. Encourage them to use the path they measured before as a guide. Tell them that whatever their estimate is, it will be wrong. They will get a chance to measure the course and to revise their plan.

Student Task Statement

Your teacher will give you a map of the school grounds.

1. On the map, draw in the path you measured earlier with your trundle wheel and label its length.
2. Invent another route for a walking course and draw it on your map. Estimate the length of the course you drew.
3. How many laps around your course must someone complete to walk 5 km?

Student Response

Answers vary. Sample response:



- One time around the course is about 500 meters.
- A person would have to go around the course 10 times to walk 5 km.

Activity Synthesis

Students check with the teacher about their proposed courses and then move on to the next activity.

13.2 Measuring and Finalizing the Course

Optional: 30 minutes

Students measure their proposed 5K courses with their trundle wheels. They compare their measurements with their estimates and make final adjustments to their proposed courses. Then they draw a finalized version of their course on the map (or a second copy of the map) including all the details necessary to organize the race: start and finish locations, walking direction, and number of laps.

Addressing

- 7.G.A.1
- 7.RP.A.3

Instructional Routines

- Group Presentations
- MLR7: Compare and Connect

Launch

Keep students in the same groups. Provide access to trundle wheels. Tell students to measure their proposed race course twice, record their measurements, and then to come back to the classroom to finish the computations and revisions.

Student Task Statement

1. Measure your proposed race course with your trundle wheel at least two times. Decide what distance you will report to the class.
2. Revise your course, if needed.
3. Create a visual display that includes:
 - A map of your final course
 - The starting and ending locations
 - The number of laps needed to walk 5 km
 - Any other information you think would be helpful to the race organizers

Student Response

Answers vary. Possible solution:



- One time around the course is actually 625 meters.
- We have to go around the course 8 times to complete the race.

Are You Ready for More?

The map your teacher gave you didn't include a scale. Create one.

Student Response

Answers vary. Sample response: 1 cm represents 12 m or 1 cm to 12 m.

Activity Synthesis

Ask students to display their maps and explain their proposed race courses. Consider doing a gallery walk. Encourage students to discuss any assumptions they made to complete their calculations as well as any revisions they made to their plan after measuring their proposed course.

Access for English Language Learners

Representing, Conversing: MLR7 Compare and Connect. Use this routine to help students consider audience when preparing a visual display of their work. Ask students to consider how to display their proposed race courses so that another student can interpret them. Groups may wish to add notes or details to their drawings to help communicate their thinking. Provide 2–3 minutes of quiet think time for students to read and interpret each other's displays before they discuss any assumptions they made to complete their calculations as well as any revisions they made to their plan after measuring their proposed course.

Design Principle(s): Cultivate conversation; Maximize meta-awareness

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