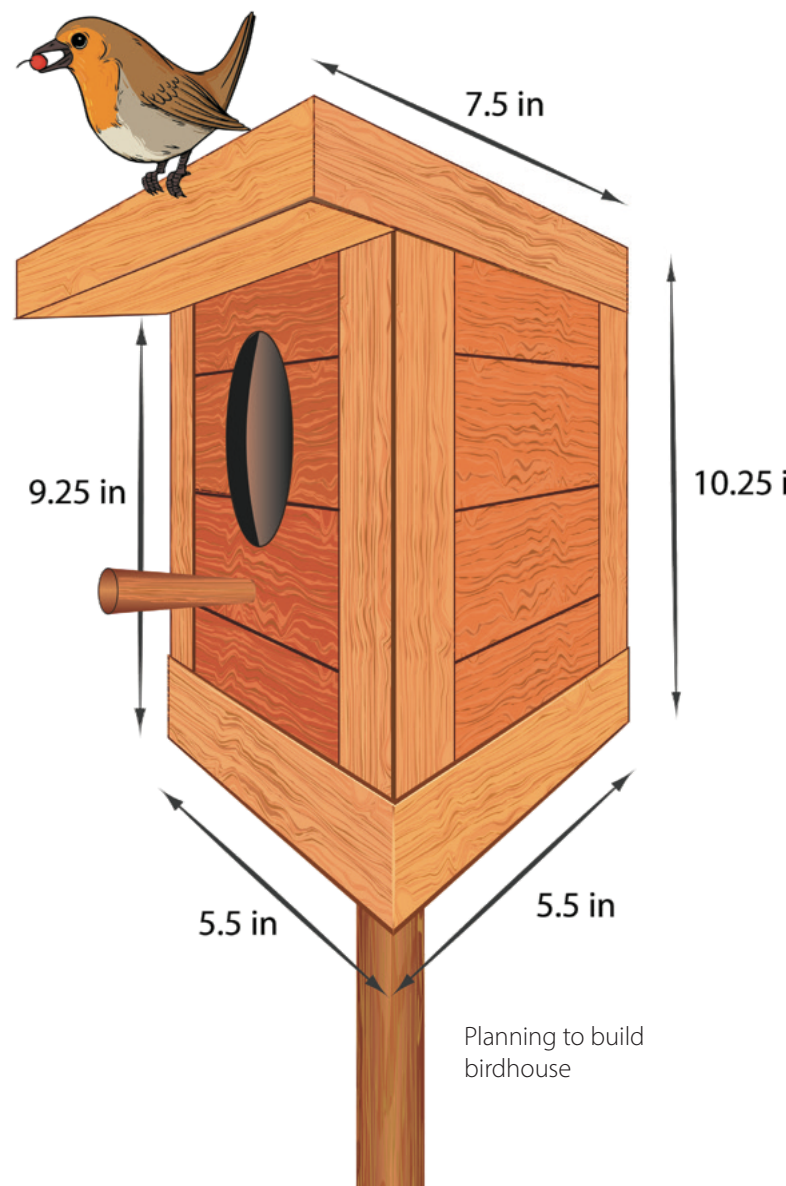
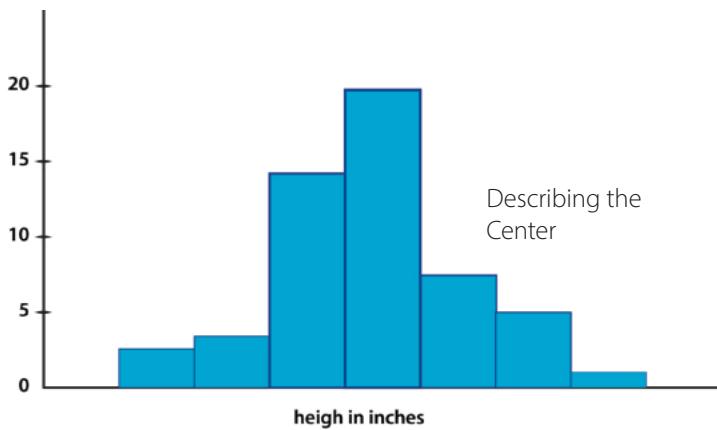
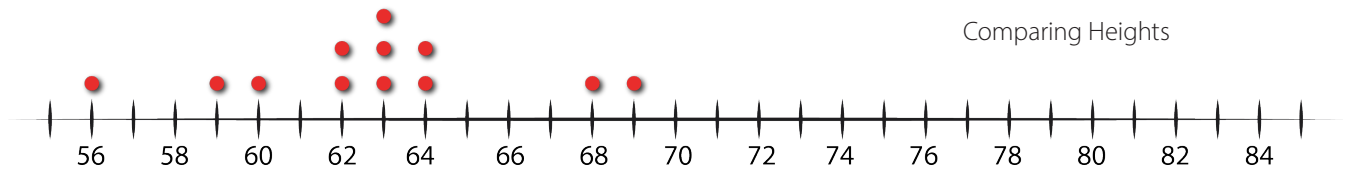




Probability and Sampling

Teacher Guide



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ISBN: 979-8-88970-869-8

Probability and Sampling

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Probability and Sampling
Teacher Guide
Core Knowledge Mathematics™

Probability and Sampling

Unit Narrative

In this unit, students understand and use the terms “event,” “sample space,” “outcome,” “chance experiment,” “probability,” “simulation,” “random,” “sample,” “random sample,” “representative sample,” “overrepresented,” “underrepresented,” “population,” and “proportion.” They design and use simulations to estimate probabilities of outcomes of chance experiments and understand the probability of an outcome as its long-run relative frequency. They represent sample spaces (that is, all possible outcomes of a chance experiment) in tables and tree diagrams and as lists. They calculate the number of outcomes in a given sample space to find the probability of a given event. They consider the strengths and weaknesses of different methods for obtaining a representative sample from a given population. They generate samples from a given population, e.g., by drawing numbered papers from a bag and recording the numbers, and examine the distributions of the samples, comparing these to the distribution of the population. They compare two populations by comparing samples from each population.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as describing, explaining, justifying, and comparing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Describe

- observations and predictions during a game (Lesson 1)
- patterns observed in repeated experiments (Lesson 4)
- chance experiments to model situations (Lessons 6 and 7)
- a simulation used to model a situation (Lesson 10)
- observations about data sets (Lessons 11 and 17)

Explain

- predictions (Lesson 2)
- how to determine which events are more likely (Lesson 3)
- possible differences in experimental and theoretical probability (Lesson 5)
- how to use simulations to estimate probability (Lesson 7)
- how to use a simulation to answer questions about the situation (Lesson 10)

Justify

- whether situations are surprising and possible (Lesson 4)
- which samples are or are not representative of a larger population (Lesson 13)

- which samples correspond with each show, which show is most appropriate for a commercial, and whether a movie is eligible for an award (Lesson 15)
- reasoning about samples and populations (Lesson 16)
- whether or not differences between samples are meaningful (Lesson 18, 19, and 20)

Compare

- sample spaces and probability of outcomes for different spinners (Lesson 5)
- methods for writing sample spaces (Lesson 8)
- heights of two groups (Lesson 11)
- measures of center with samples (Lesson 13)
- sampling methods (Lesson 14)
- populations based on samples (Lessons 18 and 20)

In addition, students are expected to critique predictions about the mean of random samples, and generalize about sample spaces, predictions, sampling, and fairness. Students also have opportunities to use language to represent data from repeated experiments, represent probabilities and sample spaces, and interpret situations involving sample spaces, probability, and populations.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear **bolded**. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

Learning Targets

Probability and Sampling

Lesson 1: Mystery Bags

- I can get an idea for the likelihood of an event by using results from previous experiments.

Lesson 2: Chance Experiments

- I can describe the likelihood of events using the words impossible, unlikely, equally likely as not, likely, or certain.
- I can tell which event is more likely when the chances of different events are expressed as fractions, decimals, or percentages.

Lesson 3: What Are Probabilities?

- I can use the sample space to calculate the probability of an event when all outcomes are equally likely.
- I can write out the sample space for a simple chance experiment.

Lesson 4: Estimating Probabilities Through Repeated Experiments

- I can estimate the probability of an event based on the results from repeating an experiment.
- I can explain whether certain results from repeated experiments would be surprising or not.

Lesson 5: More Estimating Probabilities

- I can calculate the probability of an event when the outcomes in the sample space are not equally likely.
- I can explain why results from repeating an experiment may not exactly match the expected probability for an event.

Lesson 6: Estimating Probabilities Using Simulation

- I can simulate a real-world situation using a simple experiment that reflects the probability of the actual event.

Lesson 7: Simulating Multi-step Experiments

- I can use a simulation to estimate the probability of a multi-step event.

Lesson 8: Keeping Track of All Possible Outcomes

- I can write out the sample space for a multi-step experiment, using a list, table, or tree diagram.

Lesson 9: Multi-step Experiments

- I can use the sample space to calculate the probability of an event in a multi-step experiment.

Lesson 10: Designing Simulations

- I can design a simulation to estimate the probability of a multi-step real-world situation.

Lesson 11: Comparing Groups

- I can calculate the difference between two means as a multiple of the mean absolute deviation.
- When looking at a pair of dot plots, I can determine whether the distributions are very different or have a lot of overlap.

Lesson 12: Larger Populations

- I can explain why it may be useful to gather data on a sample of a population.
- When I read or hear a statistical question, I can name the population of interest and give an example of a sample for that population.

Lesson 13: What Makes a Good Sample?

- I can determine whether a sample is representative of a population by considering the shape, center, and spread of each of them.
- I know that some samples may represent the population better than others.
- I remember that when a distribution is not symmetric, the median is a better estimate of a typical value than the mean.

Lesson 14: Sampling in a Fair Way

- I can describe ways to get a random sample from a population.
- I know that selecting a sample at random is usually a good way to get a representative sample.

Lesson 15: Estimating Population Measures of Center

- I can consider the variability of a sample to get an idea for how accurate my estimate is.
- I can estimate the mean or median of a population based on a sample of the population.

Lesson 16: Estimating Population Proportions

- I can estimate the proportion of population data that are in a certain category based on a sample.

Lesson 17: More about Sampling Variability

- I can use the means from many samples to judge how accurate an estimate for the population mean is.
- I know that as the sample size gets bigger, the sample mean is more likely to be close to the population mean.

Lesson 18: Comparing Populations Using Samples

- I can calculate the difference between two medians as a multiple of the interquartile range.
- I can determine whether there is a meaningful difference between two populations based on a sample from each population.

Lesson 19: Comparing Populations With Friends

- I can decide what information I need to know to be able to compare two populations based on a sample from each.

Lesson 20: Memory Test

- I can compare two groups by taking a random sample, calculating important measures, and determining whether the populations are meaningfully different.

lesson	new terminology	
	receptive	productive
7.8.1		more likely less likely
7.8.2	event chance experiment outcome equally likely as not	likely unlikely impossible certain
7.8.3	probability random sample space	outcome
7.8.5	simulation	probability random
7.8.7		event simulation
7.8.8	tree (diagram)	sample space
7.8.9		tree (diagram)
7.8.11	mean absolute deviation (MAD) distribution very different overlap	mean median dot plot
7.8.12	population sample survey	mean absolute deviation (MAD)
7.8.13	representative sample measure of center	distribution center (of a distribution) spread
7.8.14	random sample	
7.8.15	interquartile range (IQR) measure of variability box plot	population sample random sample symmetric

lesson	new terminology	
	receptive	productive
7.8.16	proportion	representative sample
7.8.17		interquartile range (IQR) measure of variability
7.8.18	meaningful difference	overlap measure of center
7.8.20		meaningful difference

Required Materials

Coins

any fair two-sided coin

Compasses

Copies of Instructional master

Four-function calculators

Graph paper

Number cubes

cubes with sides numbered from 1 to 6

Paper bags

Paper clips

Paper cups

Pre-printed slips, cut from copies of the Instructional master

Protractors

Clear protractors with no holes and with radial lines printed on them are recommended.

Rulers marked with inches

Scissors

Snap cubes

Sticky notes

Straightedges

A rigid edge that can be used for drawing line segments. Sometimes a ruler is okay to use as a straightedge, but sometimes it is preferable to use an unruled straightedge, like a blank index card.

Straws

lesson	new terminology	
	receptive	productive
7.8.16	proportion	representative sample
7.8.17		interquartile range (IQR) measure of variability
7.8.18	meaningful difference	overlap measure of center
7.8.20		meaningful difference

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Straws

Section: Probabilities of Single Step Events

Lesson 1: Mystery Bags

Goals

- Compare outcomes for different experiments, predict which experiment is more likely to produce a desired result, and justify (orally and in writing) the prediction.
- Describe (orally) how we can use the outcomes from previous experiments to help determine the relative likelihood of future events.

Learning Targets

- I can get an idea for the likelihood of an event by using results from previous experiments.

Lesson Narrative

To introduce the unit on probability, students play a game to collect data about what is inside bags and then make a decision based on the information they have collected. The process of using previous results from repeated trials to inform the likelihood of future events is one way to estimate probabilities that will be revisited later.

Bags that contain a certain number of colored objects will be used in this unit. To reuse materials already in the classroom, colored snap cubes are recommended, but any items of different colors that cannot be determined based on feeling will work. If there are not enough suitable items, equal sized pieces of paper can be colored or have the color written on them and used in the bags.

Alignments

Addressing

- 7.SP.C.6: Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.

Instructional Routines

- MLR8: Discussion Supports
- Poll the Class

Required Materials

Paper bags

Snap cubes

Required Preparation

Prepare enough bags of blocks so that each group of 4 students can have one bag and all groups will have had a turn with each color of bag after three rounds.

- Label one-third of the bags "green" and put 9 green blocks and 3 blocks of another color into each of these bags.
- Label one-third of the bags "blue" and put 8 blue blocks and 8 blocks of another color into each of these bags.
- Label one-third of the bags "red" and put 4 red blocks and 10 blocks of another color into each of these bags.

Student Learning Goals

Let's make predictions based on what we know.

1.1 Going Fishing

Warm Up: 5 minutes

The purpose of this warm-up is for students to use their intuition to think about the upcoming unit on probability. In particular, students guess what type of fish might be caught after knowing the results of the previous 10 fish caught. Although no answer can be given with absolute certainty, the results being heavily skewed towards one type of fish should lead students to the idea that it is more likely to be the most commonly caught fish will be caught again.

Addressing

- 7.SP.C.6

Instructional Routines

- Poll the Class

Launch

It may be helpful to explain that there are many types of fish that are caught while fishing for fun or sport. The two types listed in this warm-up, bluegill and yellow perch, are typically found in lakes and caught with the same type of bait. Both of these types of fish are suitable for eating after being caught or release back into the water.

Student Task Statement

Andre and his dad have been fishing for 2 hours. In that time, they have caught 9 bluegills and 1 yellow perch.

The next time Andre gets a bite, what kind of fish do you think it will be? Explain your reasoning.

Student Response

Answers vary. Sample response: I think Andre will pull out a bluegill. They have mostly caught bluegills, so it seems like the next one will probably be a bluegill, too.

Activity Synthesis

The purpose of the discussion is to show students that no single answer can be certain for this problem, but previous results can help inform the likelihood of future outcomes.

Poll the class regarding the type of fish they think will be caught next. Begin by asking for students who think the next fish caught will be a bluegill followed by students who think the next fish caught will be a yellow perch followed by students who think that another type of fish will be caught (or that they will not catch another fish). Display the results from the poll for all to see. Following the poll, ask at least one student representing each group with more than 1 vote for their reasoning. Tell students that we cannot know for certain what the next type of fish will be, but based on the results we have available, it is most likely that a bluegill will be caught next.

1.2 Playing the Block Game

30 minutes

Following the concept developed in the warm-up, students will continue to use the idea that outcomes from previous experiments can help inform the likelihood of an outcome when the experiment is repeated. In this activity, students play a game while collecting data. Bags of colored blocks are set up with different probabilities to win and students use random chance to collect points when the color of the block they choose matches the color written on the bag. In the discussion, students will use the data they have collected in three rounds of the game to make a decision about which bag might be most likely to produce a winning block. As an opening activity, this is meant to motivate students into thinking about likelihood of events. As such, it is not important to resolve the questions in the discussion into perfect agreement.

Students are arranged in groups of 4 and given one of 3 different bags labeled with colors. Create enough bags so that each group will have a turn with each of the different color bags after three rounds.

Addressing

- 7.SP.C.6

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 4. Explain to students how to play the game:

- There will be 3 rounds of the game. Their group will get a different bag of blocks for each round.

- They are not allowed to look into the bag or take more than one block out of the bag at a time.
- During a round, each person in the group gets 4 turns to take a block out of the bag.
- If the block matches the color that is written on the outside of the bag, the person scores (1 point during round 1, 2 points during round 2, and 3 points during round 3). When the block is any other color, they do not earn any points.
- They put the block back in the bag, shake the bag to mix up the blocks, and pass the bag to the next person in between each turn.
- At the end of a round, they record everyone's points from that round and wait for a new bag of blocks.

Distribute bags of blocks. Each group gets one of these bags:

- A bag labeled "Green" that contains 9 green blocks and 3 of another color.
- A bag labeled "Blue" that contains 8 blue blocks and 8 of another color.
- A bag labeled "Red" that contains 4 red blocks and 10 of another color.

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Begin with a small-group or whole-class demonstration of how to play the game. Check for understanding by inviting students to rephrase directions in their own words.

Supports accessibility for: Memory; Conceptual processing

Anticipated Misconceptions

Some groups may choose a bag for the bonus round based on the number of points they got in the earlier rounds. Since points are equal in the bonus round, help them see that the blocks picked out is a more important thing to consider than the points for the round.

Student Task Statement

Your teacher will give your group a bag of colored blocks.

1. Follow these instructions to play one round of the game:
 - a. Everyone in the group records the color written on the bag in the first column of the table.
 - b. Without looking in the bag, one person takes out one of the blocks and shows it to the group.
 - c. If they get a block that is the same color as the bag, they earn:

- 1 point during round 1
- 2 points during round 2
- 3 points during round 3

d. Next, they put the block back into the bag, shake the bag to mix up the blocks, and pass the bag to the next person in the group.

e. Repeat these steps until everyone in your group has had 4 turns.

2. At the end of the round, record each person's score in the table.

	What color bag?	person 1's score	person 2's score	person 3's score	person 4's score
round 1					
round 2					
round 3					

3. Pause here so your teacher can give you a new bag of blocks for the next round.

4. Repeat the previous steps to play rounds 2 and 3 of the game.

5. After you finish playing all 3 rounds, calculate the total score for each person in your group.

Student Response

Answers vary. Sample response:

	What color bag?	person 1's score	person 2's score	person 3's score	person 4's score
round 1	Green	3	2	3	3
round 2	Blue	8	10	8	6
round 3	Red	3	6	6	3

Are You Ready for More?

Tyler's class played the block game using purple, orange, and yellow bags of blocks.

- During round 1, Tyler's group picked 4 purple blocks and 12 blocks of other colors.

- During round 2, Tyler's group picked 11 orange blocks and 5 blocks of other colors.
- During round 3, Tyler forgot to record how many yellow blocks his group picked.

For a final round, Tyler's group can pick one block from any of the three bags. Tyler's group decides that picking from the orange bag would give them the best chance of winning, and that picking from the purple bag would give them the worst chance of winning. What results from the yellow bag could have lead Tyler's group to this conclusion? Explain your reasoning.

Student Response

Answers vary. Sample response: 8 yellow blocks and 8 other colors. The purple bag appears to have less pink than other colors, and the orange bag appears to have more orange than other colors. If the yellow bag has an equal number of yellow blocks and other colors, it will be more likely to win than the purple bag, but less likely to win than the orange bag.

Activity Synthesis

Tell students that there is a bonus, fourth round. In this round, a block that matches the color on the bag is worth 25 points. Each person will only have one chance to draw a bonus block. In this round, each group will get to choose which bag they would like to draw from. At least 3 people in the group must agree on the bag they will use.

Give students 5 minutes of small-group discussion to agree on which bag the group will use. Tell students that, after the bonus round, they will be asked to explain their reasoning for choosing the bag. It is ok for more than one group to select the same color bag. Identify groups that use the results from the previous rounds to determine which bag will be more likely to draw a winning block.

Allow each group to use the bag they have selected to play the bonus round and total their points.

After all students have played the bonus round, select groups to share their reasoning for choosing which bag to use. Select previously identified students to share their reasoning as well.

Ask students, "Was one bag more likely to give you a winning cube than the others? Explain your reasoning."

Open one of each color bag to reveal the contents. Ask students, "Based on what you see now, does this change your answer? Explain your reasoning."

The green bag should provide the best chance to win points, but it is not essential for the class to come to this understanding at this point.

Access for English Language Learners

Speaking, Representing: MLR8 Discussion Supports. Use this routine to support whole-class discussion as groups share which bag they would choose during the bonus round. Display sentence frames for students to use as they respond to each group: "I agree because" or "I disagree because" If necessary, revoice student ideas to demonstrate mathematical language use by restating a statement as a question in order to clarify, apply appropriate language, and involve more students.

Design Principle(s): Support sense-making

Lesson Synthesis

Ask students, "What are some examples of times you have predicted what will happen in the future based on what you have seen happen in the past?"

Tell students that in the first half of this unit they will learn about different ways to determine how likely events are to happen. Examining previous results is one of the ways.

1.3 Jada Draws Even

Cool Down: 5 minutes

Students use a different context to continue working with the idea that outcomes from previous experiments inform the likelihood of an outcome from doing the experiment additional times. In particular, when there is more than one option available, prior outcomes can help people choose the option that is most likely to provide favorable results.

Addressing

- 7.SP.C.6

Student Task Statement

A large fish tank is filled with table tennis balls with numbers written on them. Jada chooses 10 table tennis balls from the tank and writes down their numbers.

1 3 5 1 3 2 4 1 5 3

A second tank is filled with golf balls with numbers written on them. Jada chooses 10 golf balls from the tank and writes down their numbers.

1 4 5 2 6 2 2 1 4 8

To win a prize, Jada must get a ball with an even number. Should she try to win the prize using the tank of table tennis balls or the tank of golf balls? Explain your reasoning.

Student Response

Jada should use the tank of golf balls. Explanations vary. Sample explanation: From the tank of table tennis balls, Jada only got 2 even numbers out of the 10 she chose. From the tank of golf balls, she got 7 even numbers out of the 10 she chose. There seems to be a better chance of her getting even numbered balls from the tank that has golf balls.

Student Lesson Summary

One of the main ways that humans learn is by repeating experiments and observing the results. Babies learn that dropping their cup makes it hit the floor with a loud noise by repeating this action over and over. Scientists learn about nature by observing the results of repeated experiments again and again. With enough data about the results of experiments, we can begin to predict what may happen if the experiment is repeated in the future. For example, a baseball player who has gotten a hit 33 out of 100 times at bat might be expected to get a hit about 33% of his times at bat in the future as well.

In some cases, we can predict the chances of things happening based on our knowledge of the situation. For example, a coin should land heads up about 50% of the time due to the symmetry of the coin.

In other cases, there are too many unknowns to predict the chances of things happening. For example, the chances of rain tomorrow are based on similar weather conditions we have observed in the past. In these situations, we can experiment, using past results to estimate chances.

Lesson 1 Practice Problems

Problem 1

Statement

Lin is interested in how many of her classmates watch her favorite TV show, so she starts asking around at lunch. She gets the following responses:

yes	yes	yes	no	no	no	no	no	no
no	yes	no	no	no				

If she asks one more person randomly in the cafeteria, do you think they will say “yes” or “no”? Explain your reasoning.

Solution

Answers vary. Sample response: I think they will say “no,” since most people so far have said “no.”

Problem 2

Statement

An engineer tests the strength of a new material by seeing how much weight it can hold before breaking. Previous tests have held these weights in pounds:

1,200	1,400	1,300	1,500	950	1,600	1,100
-------	-------	-------	-------	-----	-------	-------

Do you think that this material will be able to hold more than 1,000 pounds in the next test? Explain your reasoning.

Solution

Answers vary. Sample response: Yes, 6 out of 7 tests did, and the one that didn’t was pretty close.

Problem 3

Statement

A company tests two new products to make sure they last for more than a year.

- Product 1 had 950 out of 1,000 test items last for more than a year.
- Product 2 had 150 out of 200 last for more than a year.

If you had to choose one of these two products to use for more than a year, which one is more likely to last? Explain your reasoning.

Solution

Product 1, since a greater proportion of the test products lasted more than 1 year.

Problem 4

Statement

Put these numbers in order from least to greatest.

$$\frac{1}{2} \quad \frac{1}{3} \quad \frac{2}{5} \quad 0.6 \quad 0.3$$

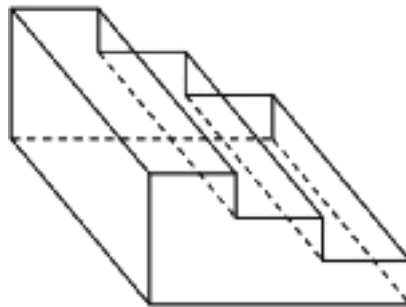
Solution

$$0.3, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, 0.6$$

Problem 5

Statement

A small staircase is made so that the horizontal piece of each step is 10 inches long and 25 inches wide. Each step is 5 inches above the previous one. What is the surface area of this staircase?



Solution

2,850 square inches

(From Unit 7, Lesson 15.)

Lesson 2: Chance Experiments

Goals

- Comprehend and use the terms “impossible,” “unlikely,” “equally likely as not,” “likely,” and “certain” (in spoken and written language) to describe the likelihood of an event.
- Interpret percentages, fractions, and decimals that represent the likelihood of events.
- Order a given set of events from least likely to most likely, and justify (orally) the reasoning.

Learning Targets

- I can describe the likelihood of events using the words impossible, unlikely, equally likely as not, likely, or certain.
- I can tell which event is more likely when the chances of different events are expressed as fractions, decimals, or percentages.

Lesson Narrative

In this lesson students investigate chance events. They use language like impossible, unlikely, equally likely as not, likely, or certain to describe a likelihood of a chance **event**. Students engage in MP1 by making sense of situations and sorting them into these categories. In some cases, a value is assigned to the likelihood of an event using a fraction, decimal, or percentage chance. By comparing loose categories early and numerical quantities later, students are attending to precision (MP6) when sorting the scenarios. Later, students will connect this language to more precise numerical values on their own.

Alignments

Addressing

- 7.SP.C.5: Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

Instructional Routines

- MLR2: Collect and Display
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Take Turns
- Think Pair Share

Required Materials

Number cubes

cubes with sides numbered from 1 to 6

Pre-printed slips, cut from copies of the Instructional master

Required Preparation

Print and cut up slips from the Card Sort: Likelihood Instructional master. One copy is needed for every 3 students. 2 standard number cubes are needed for a demonstration.

Student Learning Goals

Let's investigate chance.

2.1 Which is More Likely?

Warm Up: 5 minutes

The purpose of this warm-up is to engage students' intuition about likelihood of events. The following activities in this lesson continue to develop more formal ways of thinking about likelihood leading to the definition of probability in the next lesson.

Addressing

- 7.SP.C.5

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 2 minutes of quiet work time and time to share their response with a partner. Follow with a whole-class discussion.

Anticipated Misconceptions

Students may think that it is required to pull out a specific shoe rather than any left shoe. Ask students to visualize the problem and determine how many left shoes are in the closet.

Student Task Statement

Which is more likely to happen?

- When reaching into a dark closet and pulling out one shoe from a pile of 20 pairs of shoes, you pull out a left shoe.
- When listening to a playlist—which has 5 songs on it—in shuffle mode, the first song on the playlist plays first.

Student Response

It is more likely that you will pull out a left shoe than the first song on the playlist will be the first song played in shuffle mode. Since the shoes come in pairs, it is equally likely that you would get a

left or right shoe, so half of the time we would expect to get a right shoe. For the playlist, there are 5 different songs that could play first and only 1 of them is the first song on the list.

Activity Synthesis

The purpose of the discussion is to help students recognize their own intuition about the likelihood of an event even when prior outcomes are not available.

Have partners share responses with the class, and ask at least one student for each option for their reasoning. Give students time to discuss their reasoning until they come to an agreement.

It may be helpful to reiterate that the outcomes from these actions are not certain. It is certainly possible to do both things and draw out a right shoe and listen to the first song on the list first, but it is not very likely based on the situations.

2.2 How Likely Is It?

10 minutes

As preparation for talking about probability, students are asked to engage their intuition about the concept by loosely grouping scenarios into categories based on their likelihood by reasoning abstractly about situations in context as in MP2. Some of the categories are meant to be loosely interpreted while others (such as "certain" and "impossible") have more precise meanings.

Addressing

- 7.SP.C.5

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2.

Tell students that a “standard number cube” is an object that has the numbers 1 through 6 printed on a cube so that each face shows a different number. This item will be referenced throughout the unit.

It may help students to understand the categories of likelihood with an example of opening a book to a random page:

- Impossible: Opening a 100 page book to page -300.
- Unlikely: Opening a 100 page book to exactly page 45.
- Equally likely as not: Opening a 100 page book to a page numbered less than 51.
- Likely: Opening a 100 page book to a page numbered greater than 10.

- Certain: Opening a 100 page book to a page numbered less than 1,000.

Allow students 5--7 minutes quiet work time followed by partner and whole-class discussion.

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Leverage choice around perceived challenge. Invite students to select 5--6 of the situations to complete.

Supports accessibility for: Organization; Attention; Social-emotional skills

Access for English Language Learners

Conversing: MLR8 Discussion Supports. As students work with a partner to group the scenarios into categories, use this routine to support small group discussion. Invite students to take turns, selecting an event and explaining to their partner which category they think it belongs to (i.e., impossible, unlikely, equally likely as not, likely, or certain). Encourage students to challenge each other when they disagree. This will help students clarify their reasoning about multiplication of signed numbers.

Design Principle(s): Support sense-making; Optimize output (for explanation)


Student Task Statement

1. Label each event with one of these options:

impossible, unlikely, equally likely as not, likely, certain

- a. You will win grand prize in a raffle if you purchased 2 out of the 100 tickets.
- b. You will wait less than 10 minutes before ordering at a fast food restaurant.
- c. You will get an even number when you roll a standard number cube.
- d. A four-year-old child is over 6 feet tall.
- e. No one in your class will be late to class next week.
- f. The next baby born at a hospital will be a boy.
- g. It will snow at our school on July 1.
- h. The Sun will set today before 11:00 p.m.
- i. Spinning this spinner will result in green.
- j. Spinning this spinner will result in red.



- 
2. Discuss your answers to the previous question with your partner. If you disagree, work to reach an agreement.
 3. Invent another situation for each label, for a total of 5 more events.

Student Response

1.
 - a. unlikely
 - b. likely
 - c. equally likely as not
 - d. impossible (unlikely is acceptable)
 - e. Answers vary.
 - f. equally likely as not
 - g. impossible (or possibly unlikely depending on the location of your school)
 - h. certain
 - i. equally likely as not
 - j. impossible
2. No answer needed.
3. Answers vary. Sample response:
 - Impossible: Finding a rectangle that has 5 different sides.
 - Unlikely: A peacock being born as an albino.
 - Equally likely as not: A coin is flipped and it lands with the tails side showing.
 - Likely: A box of crayons contains the color red.
 - Certain: A triangle has 3 sides.

Activity Synthesis

The purpose of this discussion is for students to see that the loose categories can be understood in a more formal way. Some students may begin to attach numbers to the likelihood of the events and that is a good way to begin the transition to thinking about probability.

There may be some discussion about the category "equally likely as not." Are events in this category required to be at exactly 50% or would an event with 55% or 48% likelihood be placed in this category as well? At this stage, the categories are meant to be loose, so it is not necessary that everyone agree on what goes in each category.

Questions for discussion

- "Were any of the scenarios listed difficult to categorize?"
- "Which categories are the most strict about what can go in them?" ("Certain" should represent scenarios that *must* happen with 100% certainty. "Impossible" should represent scenarios that *cannot* happen.)
- "What does it mean for an event to be certain?" (That it must happen.)
- "What does it mean for an event to be likely?" (Between about 50% and 100% chance.)

2.3 Take a Chance

10 minutes (there is a digital version of this activity)

In this lesson, students begin to move towards a more quantitative understanding of likelihood by observing a game that has two rounds with different requirements for winning in each round. The game is also played multiple times to help students understand that the actual number of times an **outcome** occurs may differ from expectations based on likelihood at first, but should narrow towards the expectation in the long-run. By repeating the process many times, students engage in MP8 by recognizing a structure beginning to form with the results. Activities in later lessons will more formally show this structure forming from the repeated processes.

Addressing

- 7.SP.C.5

Instructional Routines

- MLR2: Collect and Display

Launch

Arrange students in groups of 2. Following the demonstration game, allow 5 minutes of partner work followed by a whole-class discussion.

Select two students to play this game of chance that consists of 2 rounds. Give 1 standard number cube to each student.

Round 1: One student chooses three numbers that will count as a win for them. The other student wins if any of the other numbers come up. Roll the number cube.

Round 2: Whoever lost the first round gets to choose 4 numbers that will count for a win for them while their partner gets the remaining 2 numbers. Roll the number cube.

Have the students play this game of chance 10 times by playing the two rounds each time. Display the results of each round for all to see. For example:

	person choosing number	winner	chooser wins?
game 1, round 1	Lin	Noah	
game 1, round 2	Lin	Lin	X
game 2, round 1	Noah	Noah	X
game 2, round 2	Lin	Lin	X

Tell students that the questions ask about whether the person choosing the number wins the game or not. They should not concentrate on who won more often, but how often the number chooser wins the round. In the example table, the column with the Xs is the focus.

The digital version is slightly different, with an applet generating random numbers from 1 to 6. This applet is based on the work of [pirsquared](#) in [GeoGebra](#).

Access for English Language Learners

Conversing, Reading: MLR2 Collect and Display. As students discuss the questions with a partner, write down the words and phrases students use to describe the likelihood of either player winning each round of the game. As students review the language collected in the visual display, encourage students to revise and improve how ideas are communicated. For example, a phrase such as: “The person who chooses 4 numbers has more ways to win the round” can be improved with the phrase, “The person who chooses 4 numbers has 4 out of 6 ways to win the round.” This will help students assign numbers to the likelihood of the person choosing the winning number for each round. This routine will provide feedback to students in a way that supports sense-making while simultaneously increasing meta-awareness of language.

Design Principle(s): Support sense-making; Maximize meta-awareness

Student Task Statement

Your teacher will have 2 students play a short game.

1. When the first person chose 3 numbers, did they usually win?
2. When the person chose 4 numbers, did you expect them to win? Explain your reasoning.

Student Response

Answers vary. Sample response:

1. The person who chose 3 numbers won more than lost, but not by a lot.
2. I expected the person who chose 4 numbers to usually win since they had 4 ways to win and the other person only had 2.

Are You Ready for More?

On a game show, there are 3 closed doors. One door has a prize behind it. The contestant chooses one of the doors. The host of the game show, who knows where the prize is located, opens one of the *other* doors which does not have the prize. The contestant can choose to stay with their first choice or switch to the remaining closed door.

1. Do you think it matters if the contestant switches doors or stays?
2. Practice playing the game with your partner and record your results. Whoever is the host starts each round by secretly deciding which door has the prize.
 - a. Play 20 rounds where the contestant always stays with their first choice.
 - b. Play 20 more rounds where the contestant always switches doors.
3. Did the results from playing the game change your answer to the first question? Explain.

Student Response

Answers vary. Sample response:

1. It appears like it doesn't matter: there are two doors left, one has the prize, and one doesn't.
2. No response required.
3. Switching should win more often than not switching, but other results are possible by chance.

Activity Synthesis

Define a **chance experiment**. A chance experiment is the process of making an observation of something when there is uncertainty about which of two or more possible outcomes will occur. Each time a standard number cube is thrown, it is a chance experiment. This is because we cannot be certain of which number will be the outcome of this experiment.

Some questions for discussion:

- "Who did you expect to win each time: the person choosing the number or the other person? Explain your reasoning."
- "In one round of the game it was more likely that the person choosing the numbers would win. We'll be talking a lot in this unit about how likely or probable an event is to happen and even assigning numbers to the likelihood."
- "What if we had to assign numbers to the 'likelihood' of the person choosing the numbers winning for each round? For each part of the game, what percentage or fraction would you assign to the likelihood of the person who chose the numbers winning?" (For the first round of the game, there is a 50% or $\frac{1}{2}$ chance for each person to win. For the second round of the game, the person choosing the numbers should have a 67% or $\frac{4}{6}$ or $\frac{2}{3}$ chance to win.)

- "What percentage or fraction would you assign to waiting for less than 10 minutes before your order is taken at the fast food restaurant from the previous task?" (Answers vary. Values between 80% and 100% are expected. It is not necessary to agree on a particular value at this point.)
- "How could we get more evidence to support these answers?" (Collect data from fast food restaurants to find a fraction of customers who order their food within 10 minutes.)

2.4 Card Sort: Likelihood

10 minutes

The last activity in this lesson moves one more step closer to quantifying likelihood of scenarios by ordering them individually rather than into groups. Some of the scenarios have a numerical probability expressed as a percentage, some in decimal form, some as a fraction, and some do not have a numerical probability given (MP7). This gives students the opportunity to work with probabilities expressed as percentages, decimals, and fractions.

You will need the Instructional master for this activity.

Addressing

- 7.SP.C.5

Instructional Routines

- MLR7: Compare and Connect
- Take Turns

Launch

Arrange students in groups of 3. Distribute one copy of the Instructional master for each group.

Tell students to take turns ordering the cards by beginning with one card, then adding in additional cards one at a time before, after, or between cards as needed.


Give students 5 minutes for group work followed by a whole-class discussion.

Anticipated Misconceptions

Students may have trouble understanding the Rock, Paper, Scissors context. Tell these students that a player randomly chooses one of the three items to play in each round. If students still struggle, tell them that each of the three items are expected to be played with equal likelihood.

Student Task Statement

1. Your teacher will give you some cards that describe events. Order the events from least likely to most likely.
2. After ordering the first set of cards, pause here so your teacher can review your work. Then, your teacher will give you a second set of cards.

- 
3. Add the new set of cards to the first set so that all of the cards are ordered from least likely to most likely.

Student Response

Set 1: Left handed, Rain, Cards, Flies.

Sets 1 and 2: Numbered balls, Left handed, Letter T, Rain, Rock, Toothpaste, Cards, Flies, Medical test, Pattern blocks.

Activity Synthesis

The purpose of the discussion is for students to talk about the methods they used to sort the cards and compare likelihood of different situations.

Some questions for discussion:

- "How were the numerical values of the likelihoods written?" (Some were written as percentages, some as fractions, and some as decimal values.)
- "How did you compare them when there was a mix of percentages, fractions, and decimals?"
- "Some of the cards did not have a percentage, fraction, or decimal. How did you determine where those cards would go in the order?"

Access for English Language Learners

Speaking, Listening: MLR7 Compare and Connect. Ask students to prepare a visual display of their sorted cards. Students should consider how to display their cards so that another student can interpret the reasoning behind the order they selected. Some students may wish to add notes or details to their displays to help communicate their thinking. Invite students to investigate each other's work, ask students to compare the order between displays. Listen for and amplify any comments about the use of percentages, fractions, and decimals to compare the likelihood of different situations. Then encourage students to make connections between the various ways to quantify the likelihood of a situation. Listen for and amplify language students use to explain that the percentage, fraction, and decimal used to quantify the likelihood of the same scenario are all equivalent. This will foster students' meta-awareness and support constructive conversations as they compare and connect quantities that represent the likelihood of a situation.

Design Principle(s): Cultivate conversation; Maximize meta-awareness

Lesson Synthesis

Ask students "what is a chance event?"

In the same groups of 2, have partners come up with examples of each of these types of events:

- impossible
- unlikely
- equally likely as not
- likely
- certain

Ask partners to share responses with the class.

2.5 According To

Cool Down: 5 minutes

Students rank scenarios based on likelihood expressed in different numerical forms.

Addressing

- 7.SP.C.5

Student Task Statement

Here are some scenarios:

- According to market research, a business has a 75% chance of making money in the first 3 years.
- According to lab testing, $\frac{5}{6}$ of a certain kind of experimental light bulb will work after 3 years.
- According to experts, the likelihood of a car needing major repairs in the first 3 years is 0.7.

1. Write the scenarios in order of likelihood from least to greatest after three years: the business makes money, the light bulb still works, and the car needs major repairs.
2. Name another chance experiment that has the same likelihood as one of the scenarios.

Student Response

1. The car needs major repairs, the business makes money, the light bulb still works.
2. Answers vary. Sample responses:
 - Flipping two coins and at least one not landing as heads has a 75% chance of happening.
 - Rolling a standard number cube and it lands with any number other than 1 face up.
 - That a number greater than 3 is selected when selecting a number between 1 and 10 randomly.

Student Lesson Summary

A **chance experiment** is something that happens where the outcome is unknown. For example, if we flip a coin, we don't know if the result will be a head or a tail. An **outcome** of a chance experiment is something that can happen when you do a chance experiment. For example, when you flip a coin, one possible outcome is that you will get a head. An **event** is a set of one or more outcomes.

We can describe events using these phrases: For example, if you flip a coin:

- Impossible
- Unlikely
- Equally likely as not
- Likely
- Certain
- It is *impossible* that the coin will turn into a bottle of ketchup.
- It is *unlikely* the coin will land on its edge.
- It is *equally likely as not* that you will get a tail.
- It is *likely* that you will get a head or a tail.
- It is *certain* that the coin will land somewhere.

The *probability* of an event is a measure of the likelihood that an event will occur. We will learn more about probabilities in the lessons to come.

Glossary

- chance experiment
- event
- outcome

Lesson 2 Practice Problems

Problem 1

Statement

The likelihood that Han makes a free throw in basketball is 60%. The likelihood that he makes a 3-point shot is 0.345. Which event is more likely, Han making a free throw or making a 3-point shot? Explain your reasoning.

Solution

It is more likely that Han makes a free throw. Since 0.345 is less than 0.5, making a 3-pointer is an unlikely event. Since 60% is greater than 50%, making a free throw is a more likely event.

Problem 2

Statement

Different events have the following likelihoods. Sort them from least to greatest:

60%

8 out of 10

0.37

20%

$\frac{5}{6}$

Solution

20%, 0.37, 60%, 8 out of 10, $\frac{5}{6}$

Problem 3

Statement

There are 25 prime numbers between 1 and 100. There are 46 prime numbers between 1 and 200. Which situation is more likely? Explain your reasoning.

- A computer produces a random number between 1 and 100 that is prime.
- A computer produces a random number between 1 and 200 that is prime.

Solution

A computer produces a random number between 1 and 100 that is prime. There is a 25% chance of getting a prime number from the first 100 numbers and only a 23% chance from the first 200 numbers.

Problem 4

Statement

It takes $4\frac{3}{8}$ cups of cheese, $\frac{7}{8}$ cups of olives, and $2\frac{5}{8}$ cups of sausage to make a signature pizza. How much of each ingredient is needed to make 10 pizzas? Explain or show your reasoning.

Solution

number of pizzas	cups of cheese	cups of olives	cups of sausage
1	$\frac{35}{8}$	$\frac{7}{8}$	$\frac{21}{8}$
10	$\frac{350}{8}$ or equivalent	$\frac{70}{8}$ or equivalent	$\frac{210}{8}$ or equivalent

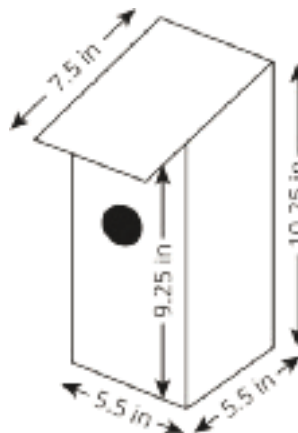
With decimals, the answers are 43.75 cups of cheese, 8.75 cups of olives, and 26.25 cups of sausage.

(From Unit 4, Lesson 2.)

Problem 5

Statement

Here is a diagram of a birdhouse Elena is planning to build. (It is a simplified diagram, since in reality, the sides will have a thickness.) About how many square inches of wood does she need to build this birdhouse?



Solution

286 square inches.

(From Unit 7, Lesson 16.)

Problem 6

Statement

Select **all** the situations where knowing the surface area of an object would be more useful than knowing its volume.

- A. Placing an order for tiles to replace the roof of a house.
- B. Estimating how long it will take to clean the windows of a greenhouse.
- C. Deciding whether leftover soup will fit in a container.
- D. Estimating how long it will take to fill a swimming pool with a garden hose.
- E. Calculating how much paper is needed to manufacture candy bar wrappers.
- F. Buying fabric to sew a couch cover.
- G. Deciding whether one muffin pan is enough to bake a muffin recipe.

Solution

["A", "B", "E", "F"]

(From Unit 7, Lesson 15.)

Lesson 3: What Are Probabilities?

Goals

- Generalize (orally) the relationship between the probability of an event and the number of possible outcomes in the sample space, for an experiment in which each outcome in the sample space is equally likely.
- List (in writing) the sample space of a simple chance experiment.
- Use the sample space to determine the probability of an event, and express it as a fraction, decimal, or percentage.

Learning Targets

- I can use the sample space to calculate the probability of an event when all outcomes are equally likely.
- I can write out the sample space for a simple chance experiment.

Lesson Narrative

In this lesson students begin to assign probabilities to chance events. They understand that the greater the probability, the more likely the event will occur. They define an *outcome* as a possible result for a chance experiment. They learn that the **sample space** is the set of all possible outcomes, and understand that a process is called **random** when the outcome of an experiment is based on chance. They reason that if there are n equally likely outcomes for a chance experiment, they construct the argument (MP3) that the **probability** of each of these outcomes is $\frac{1}{n}$.

In future lessons students will be asked to design and use simulations. Each lesson leading up to that helps prepare students by giving them hands-on experience with different types of chance experiments they could choose to use in their simulations. In this lesson students work with drawing paper slips out of a bag.

Alignments

Addressing

- 7.SP.C.5: Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
- 7.SP.C.6: Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.

- 7.SP.C.7: Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
- 7.SP.C.7.a: Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR8: Discussion Supports
- Think Pair Share

Required Materials

Paper bags

Instructional master

Pre-printed slips, cut from copies of the

Required Preparation

Print and cut up slips from the What's in the Bag? Instructional master. One copy is needed for every 4 students. Each set of slips should be put into a paper bag.

Student Learning Goals

Let's find out what's possible.

3.1 Which Game Would You Choose?

Warm Up: 5 minutes

The purpose of this warm-up is for students to choose the more likely event based on their intuition about the possible outcomes of two chance experiments. The activities in this lesson that follow define probability and give ways to compute numerical values for the probability of chance events such as these.

Addressing

- 7.SP.C.5
- 7.SP.C.7.a

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 1 minute of quiet work time followed by time to share their response with a partner. Follow with a whole-class discussion.

Anticipated Misconceptions

Some students may have trouble comparing $\frac{1}{2}$ and $\frac{2}{6}$. Review how to compare fractions with these students.

Some students may struggle with the wording of the second game. Help them understand what it means for a number to be divisible by a certain number and consider providing them with a standard number cube to examine the possible values.

Student Task Statement

Which game would you choose to play? Explain your reasoning.

Game 1: You flip a coin and win if it lands showing heads.

Game 2: You roll a standard number cube and win if it lands showing a number that is divisible by 3.

Student Response

Answers vary. Sample response: I would rather play game 1 since half of the time the coin would land so I would win. In game 2, I only have 2 out of 6 ways to win.

Activity Synthesis

Have partners share their answers and display the results for all to see. Select at least one student for each answer provided to give a reason for their choice.

If no student mentions it, explain that the number of possible outcomes that count as a win and the number of total possible outcomes are both important to determining the likelihood of an event. That is, although there are two ways to win with the standard number cube and only one way to win on the coin, the greater number of possible outcomes in the second game makes it less likely to provide a win.

3.2 What's Possible?

15 minutes

Following the warm-up discussion in which the importance of the number of possible outcomes from an experiment is discussed, students are introduced to the term sample space. Students examine experiments to determine the set of outcomes in the sample space and then use the sample spaces to think about the likelihood of the events. Following the activity, students engage in MP6 by attaching numerical values to the likelihood of events through the word probability.

In some cases, the actual sample space is unknown. For example, in the warm-up for the first lesson of this unit, all the different types of fish in the lake may not be known. In these cases, probabilities may be less precise, but can still be estimated based on the outcomes from previous experiments. Students will begin to explore this idea in the next activity and later lessons will explore the concept in more detail.

Addressing

- 7.SP.C.7.a

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Arrange students in groups of 2.

Define *random* as doing something so that the outcomes are based on chance. An example is putting the integers 1 through 20 on a spinner with each number in an equal sized section. Something that is not random might be answering a multiple choice question on a test for a subject you've studied.

Explain that, for a chance experiment, each possible result is called an *outcome*. The set of all possible outcomes is called the *sample space*. Spinning a spinner with equal sized sections marked 1 through 20 has a possible outcome of 8, but neither heads nor green is a possible outcome. The sample space is made up of all integers from 1 through 20.

Give students 10 minutes of partner work time followed by whole-class discussion.

Student Task Statement

1. For each situation, list the **sample space** and tell how many outcomes there are.

- a. Han rolls a standard number cube once.
- b. Clare spins this spinner once.



- c. Kiran selects a letter at **random** from the word "MATH."
 - d. Mai selects a letter at random from the alphabet.
 - e. Noah picks a card at random from a stack that has cards numbered 5 through 20.
2. Next, compare the likelihood of these outcomes. Be prepared to explain your reasoning.
 - a. Is Clare more likely to have the spinner stop on the red or blue section?

- b. Is Kiran or Mai more likely to get the letter T?
 - c. Is Han or Noah more likely to get a number that is greater than 5?
3. Suppose you have a spinner that is evenly divided showing all the days of the week. You also have a bag of papers that list the months of the year. Are you more likely to spin the current day of the week or pull out the paper with the current month?

Student Response

1.
 - a. Sample space: 1, 2, 3, 4, 5, 6. There are 6 outcomes.
 - b. Sample space: Red, Blue, Green, Yellow. There are 4 outcomes.
 - c. Sample space: M, A, T, H. There are 4 outcomes.
 - d. Sample space: A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z. There are 26 outcomes.
 - e. Sample space: 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20. There are 16 outcomes.
2.
 - a. Both are equally likely.
 - b. Kiran is more likely to get a T since there are fewer possibilities in the sample space.
 - c. Noah is more likely to get a number that is greater than 5, because even though he has more possibilities in the sample space, all but one of them are greater than 5.
3. It is more likely to spin the current day, since there are only 7 possible days, but 12 possible months.

Are You Ready for More?

Are there any outcomes for two people in this activity that have the same likelihood? Explain or show your reasoning.

Student Response

Answers vary. Sample response: It is equally likely that Clare will spin red and that Kiran will select an A. Since both outcomes only show up once and they both have sample spaces of 4 equally likely outcomes, these two events should be equally likely.

Activity Synthesis

Note that all of the outcomes are equally likely within each sample space. This is not always the case, but it is in these examples.

Explain that sometimes it is important to have an actual numerical value rather than a vague sense of likelihood. To answer how probable something is to happen, we assign a **probability**.

Probabilities are values between 0 and 1 and can be expressed as a fraction, decimal, or percentage. Something that has a 50% chance of happening, like a coin landing heads up, can also be described by saying, "The probability of a coin landing heads up is $\frac{1}{2}$." or "The probability of the coin landing heads up is 0.5."

When each outcome in the sample space is equally likely, we may calculate the probability of a desired event by dividing the number of outcomes for which the event occurs by the total number of outcomes in the sample space.

- "A standard number cube is rolled. What is the sample space?" (1, 2, 3, 4, 5, 6)
 - "How many outcomes are in the sample space?" (6)
- "What is the probability of rolling a 3? Explain your reasoning." ($\frac{1}{6}$ since there is a single 3 and 6 outcomes in the sample space.)
- "An experiment has one of each different possible outcome. The probability of getting one of the outcomes is $\frac{1}{30}$. How many outcomes are in the sample space?" (30)

Access for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding and memory. Include the following terms and maintain the display for reference throughout the unit: random, outcome and sample space.

Supports accessibility for: Conceptual processing; Language; Memory

Access for English Language Learners

Speaking: MLR1 Stronger and Clearer Each Time. After students decide whether it will be more likely to spin the current day of the week or the pull out the paper with the current month, ask students to write a brief explanation of their reasoning. Invite students to meet with 2–3 other partners in a row for feedback. Encourage students to ask questions such as: "How many days are there in a week?", "How many months are there in a year?", and "How did you determine the likelihood of spinning the current day of the week?" Students can borrow ideas and language from each partner to refine and clarify their original explanation. This will help students revise and refine both their reasoning and their verbal and written output.

Design Principle(s): Optimize output (for explanation)

3.3 What's in the Bag?

15 minutes

In this activity, students are introduced to the idea that not all sample spaces are obvious before actually doing the experiment. Therefore, it is not always possible to calculate the exact probabilities for events before doing or simulating the experiment. In such situations, it is important to reason abstractly about the scenario (MP2) to gain an understanding of the situation. Students refine their guesses about the sample space by repeatedly drawing items from a bag and looking for patterns in this repetition (MP8). In later lessons, students learn how to estimate probabilities from simulations.

You will need the Instructional master for this activity.

Addressing

- 7.SP.C.6
- 7.SP.C.7

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 4. Provide each group with a paper bag containing 1 set of slips cut from the Instructional master. 10 minutes partner work followed by a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, pause to check for understanding after 3–5 minutes of work time.

Supports accessibility for: Organization; Attention

Anticipated Misconceptions

Students may think that the phrase "equally likely" means there is a 50% chance of it happening. Tell students that, in this context, each outcome is equally likely if the probability does not change if you change the question to a different outcome in the sample space. For example, "What is the probability you get a letter A from this bag?" has the same answer as the question, "What is the probability you get a letter B from this bag?"

Student Task Statement

Your teacher will give your group a bag of paper slips with something printed on them. Repeat these steps until everyone in your group has had a turn.

- As a group, guess what is printed on the papers in the bag and record your guess in the table.

- Without looking in the bag, one person takes out one of the papers and shows it to the group.
- Everyone in the group records what is printed on the paper.
- The person who took out the paper puts it back into the bag, shakes the bag to mix up the papers, and passes the bag to the next person in the group.

	Guess the sample space.	What is printed on the paper?
person 1		
person 2		
person 3		
person 4		

1. How was guessing the sample space the fourth time different from the first?
2. What could you do to get a better guess of the sample space?
3. Look at all the papers in the bag. Were any of your guesses correct?
4. Are all of the possible outcomes equally likely? Explain.
5. Use the sample space to determine the **probability** that a fifth person would get the same outcome as person 1.

Student Response

1. Answers vary. Sample response: There was more information in round 4, so it narrowed the possibilities.
2. Answers vary. Sample response: Keep drawing out more papers.
3. Answers vary. Sample response: No, it wasn't the whole alphabet, just A through O.
4. Yes, there is one of each letter.
5. $\frac{1}{15}$ since there is one out of 15 possible things to choose from.

Activity Synthesis

The purpose of this discussion is for students to understand that often, in the real-world, we do not know the entire sample space before doing the experiment. They will learn in later lessons how to estimate the probabilities for such experiments.

Consider asking some of the following questions:

- "After the first paper is drawn, a group guesses, 'A bunch of letter Cs.' What might they have picked on their first paper that would lead to that guess? What could that group get on their second paper that would make them change their guess? Could they get something for the second paper that would make them sure their guess was right?" (They probably picked a letter C on their first draw. Any other letter on the second draw should make them change their guess. No, they might get another C that would make them think they are right, but with only two tries, there is still a good chance that other letters are in the bag.)
- "After the second paper is drawn, a group guesses, 'All of the consonants.' What might they have picked in their first two papers that would lead to that guess? What could that group get on their third paper that would make them change their guess? Could they get something that would make them more sure of their guess?" (They probably picked two consonants on their first two draws. If they picked a vowel, they would have to change their guess. If they picked another consonant, they might feel better about the guess, but should still not be certain of it.)
- "How did you refine your predictions with each round?"
- "If you had a new bag of papers and you took out papers 50 times and never got a 'Z,' would that mean there is no 'Z' in the bag?" (Not necessarily, but it might make me wonder if it's not in there.)

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion. Press students for details in their explanations by asking questions such as: "How many possible outcomes are there?", "How do you know that all the possible outcomes are equally likely?", and "How did you determine the probability of one of those outcomes?" Listen for and amplify the language students use to describe the sample space as one of each letter A through O for a total of 15 possible outcomes. This will support a rich and inclusive discussion about how to use the sample space to determine the probability of an outcome.

Design Principle(s): Support sense-making

Lesson Synthesis

Consider asking some of the questions:

- "If you choose one letter at random from the English alphabet, how many outcomes are in the sample space? How many outcomes are in the event that a vowel (not including Y) is chosen?" (There are 26 outcomes in the sample space. There are 5 outcomes that are vowels: A, E, I, O, and U.)
- "What is the sample space of a chance experiment? How is the number of outcomes in the sample space related to the probability of an event if the outcomes in the sample space are equally likely?" (The sample space is the list of possible outcomes for an experiment. When

there are more outcomes in the sample space, the probability of a single outcome occurring is lower.)

- "When there are 100 different outcomes in the sample space that are equally likely, what is the probability that a specific outcome will happen?" ($\frac{1}{100}$ or 1% or 0.01)

3.4 Letter of the Day

Cool Down: 5 minutes

In the cool-down, students have the opportunity to practice finding the sample space by seeing what is possible for a described experiment. They then use a sample space to write a probability for a single event. Consider having a calendar or some other resource available for students who struggle to spell the days of the week.

Addressing

- 7.SP.C.7.a

Anticipated Misconceptions

Some students may struggle to spell the days of the week. Consider having a calendar or other reference available to aid these students.

Student Task Statement

A mother decides to teach her son about a letter each day of the week. She will choose a letter from the name of the day. For example, on Saturday she might teach about the letter S or the letter U, but not the letter M.

1. What letters are possible to teach using this method? (There are 15.)
2. What are 4 letters that can't be taught using this method?
3. On TUESDAY, the mother writes the word on a piece of paper and cuts it up so that each letter is on a separate piece of paper. She mixes up the papers and picks one. What is the probability that she will choose the piece of paper with the letter Y? Explain your reasoning.

Student Response

1. A, D, E, F, H, I, M, N, O, R, S, T, U, W, Y
2. Answers vary. The complete list is: B, C, G, J, K, L, P, Q, V, X, Z
3. $\frac{1}{7}$ since there are 7 outcomes in the sample space, all outcomes are equally likely, and there is only 1 outcome that corresponds to the letter Y.

Student Lesson Summary

The **probability** of an event is a measure of the likelihood that the event will occur. Probabilities are expressed using numbers from 0 to 1.

- If the probability is 0, that means the event is impossible. For example, when you flip a coin, the probability that it will turn into a bottle of ketchup is 0. The closer the probability of some event is to 0, the less likely it is.
- If the probability is 1, that means the event is certain. For example, when you flip a coin, the probability that it will land somewhere is 1. The closer the probability of some event is to 1, the more likely it is.

If we list all of the possible outcomes for a chance experiment, we get the **sample space** for that experiment. For example, the sample space for rolling a standard number cube includes six outcomes: 1, 2, 3, 4, 5, and 6. The probability that the number cube will land showing the number 4 is $\frac{1}{6}$. In general, if all outcomes in an experiment are equally likely and there are n possible outcomes, then the probability of a single outcome is $\frac{1}{n}$.

Sometimes we have a set of possible outcomes and we want one of them to be selected at **random**. That means that we want to select an outcome in a way that each of the outcomes is *equally likely*. For example, if two people both want to read the same book, we could flip a coin to see who gets to read the book first.

Glossary

- probability
- random
- sample space

Lesson 3 Practice Problems

Problem 1

Statement

List the *sample space* for each chance experiment.

- a. Flipping a coin
- b. Selecting a random season of the year
- c. Selecting a random day of the week

Solution

- a. Heads, tails
- b. Spring, summer, fall, winter
- c. Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday

Problem 2

Statement

A computer randomly selects a letter from the alphabet.

- a. How many different outcomes are in the sample space?
- b. What is the probability the computer produces the first letter of your first name?

Solution

- a. 26
- b. $\frac{1}{26}$

Problem 3

Statement

What is the probability of selecting a random month of the year and getting a month that starts with the letter "J"? If you get stuck, consider listing the sample space.

Solution

$\frac{3}{12}$ (or equivalent)

Problem 4

Statement

E represents an object's weight on Earth and M represents that same object's weight on the Moon. The equation $M = \frac{1}{6}E$ represents the relationship between these quantities.

- What does the $\frac{1}{6}$ represent in this situation?
- Give an example of what a person might weigh on Earth and on the Moon.

Solution

- $\frac{1}{6}$ is the constant of proportionality relating an object's weight on Earth to its weight on the Moon. Something that weighs one pound on the Earth weighs $\frac{1}{6}$ of a pound on the Moon. Or, to find weight on the Moon, multiply the weight on Earth by $\frac{1}{6}$. Or for every pound of weight on Earth, something has $\frac{1}{6}$ of a pound of weight on the Moon.
- Answers vary. Sample response: A person who weighs 150 pounds on Earth weighs 25 pounds on the Moon.

(From Unit 2, Lesson 4.)

Problem 5

Statement

Here is a diagram of the base of a bird feeder which is in the shape of a pentagonal prism. Each small square on the grid is 1 square inch.

The distance between the two bases is 8 inches. What will be the volume of the completed bird feeder?



Solution

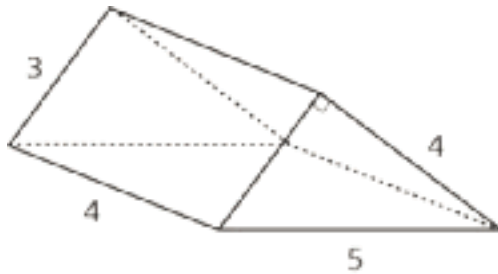
336 in^3 . The area of the base is 42 in^2 because it is composed of a rectangle with area 30 in^2 and a triangle of area 12 in^2 . $42 \cdot 8 = 336$.

(From Unit 7, Lesson 13.)

Problem 6

Statement

Find the surface area of the triangular prism.



Solution

60 square units

(From Unit 7, Lesson 14.)

Lesson 4: Estimating Probabilities Through Repeated Experiments

Goals

- Describe (orally and in writing) patterns observed on a table or graph that shows the relative frequency for a repeated experiment.
- Generalize (orally) that the cumulative relative frequency approaches the probability of the event as an experiment is repeated many times.
- Generate possible results that would or would not be surprising for a repeated experiment, and justify (orally) the reasoning.

Learning Targets

- I can estimate the probability of an event based on the results from repeating an experiment.
- I can explain whether certain results from repeated experiments would be surprising or not.

Lesson Narrative

In this lesson students roll a number cube many times and calculate the cumulative fraction of the time that an event occurs to see that in the long run this relative frequency approaches the probability of the chance event. By repeating the experiment and examining the structure of the results, students are engaging in MP8. They also see that the relative frequency of a chance event will not usually exactly match the actual probability. For example, when flipping a coin 100 times, the coin may land showing a head 46 times instead of exactly 50 times and not be considered unreasonable.

In future lessons students will be asked to design and use simulations. Each lesson leading up to that helps prepare students by giving them hands-on experience with different types of chance experiments they could choose to use in their simulations. In this lesson students work with rolling a number cube and tossing a coin.

Alignments

Building On

- 5.NBT.A.3: Read, write, and compare decimals to thousandths.
- 7.NS.A.2.d: Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Addressing

- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.

- 7.SP.C.5: Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
- 7.SP.C.6: Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
- 7.SP.C.7: Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
- 7.SP.C.7.b: Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports
- Think Pair Share

Required Materials

Graph paper

Number cubes

cubes with sides numbered from 1 to 6

Required Preparation

The In the Long Run activity requires 1 number cube for every 3 students. Access to graph paper may be useful, but is not required.

Student Learning Goals

Let's do some experimenting.

4.1 Decimals on the Number Line

Warm Up: 5 minutes

The purpose of this warm-up is for students to practice placing numbers represented with decimals on a number line and thinking about probabilities of events that involve the values of the numbers. In the following activity, students are asked to graph points involving probabilities that are represented by numbers similar to the ones in this activity.

Building On

- 5.NBT.A.3

Addressing

- 7.SP.C.7

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 2 minutes of quiet work time followed by time to share their responses with a partner. Follow with a whole-class discussion.

Student Task Statement

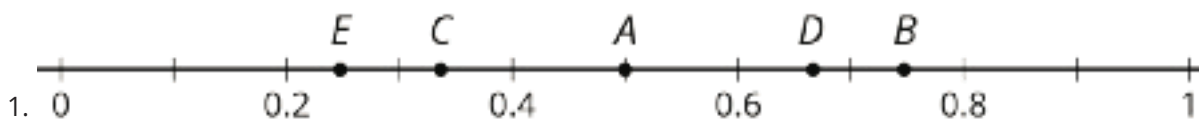
1. Locate and label these numbers on the number line.

- a. 0.5
- b. 0.75
- c. 0.33
- d. 0.67
- e. 0.25



2. Choose one of the numbers from the previous question. Describe a game in which that number represents your probability of winning.

Student Response



2. Answers vary. Sample response: Get blue when randomly selecting a color from the primary colors: red, yellow, and blue. This has a probability of 0.33.

Activity Synthesis

Select some partners to share their responses and methods for positioning the points on the number line. If time allows, select students to share a chance event for each of the values listed.

4.2 In the Long Run

20 minutes (there is a digital version of this activity)

This activity begins to answer the question brought up in the previous lesson about finding the probability when the sample space is not available. Students have the opportunity to use this

experiment for which the sample space is available to check its agreement and estimate based on repeating the experiment many times (MP8).

Students make the connection between probability and the fraction of outcomes for which the event occurs in the long-run. This activity highlights that a probability describes what happens in the the long run and that it does not guarantee that the event will occur a specific number of times after any specific number of trials. For example, an event that has probability 0.6 means that the event will occur about 60% of the time in the long run, but it does not mean that it will occur exactly 60 times when the experiment is performed 100 times.

Building On

- 7.NS.A.2.d

Addressing

- 7.SP.C.5
- 7.SP.C.6
- 7.SP.C.7

Instructional Routines

- MLR8: Discussion Supports

Launch

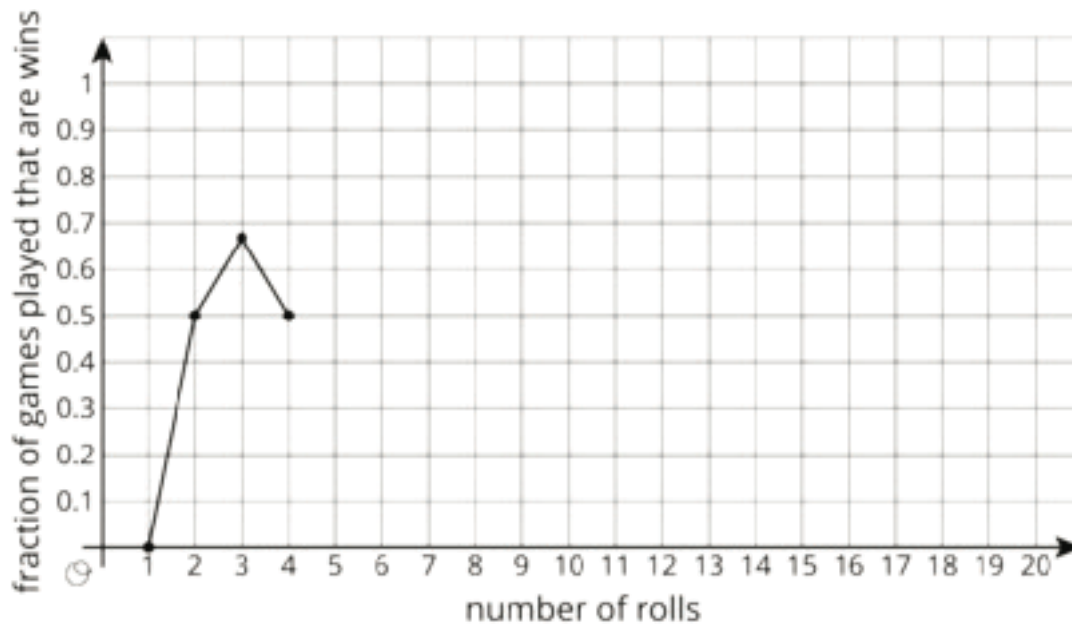
Arrange students in groups of 3. Provide one standard number cube for each group. Following the teacher demonstration, allow 10 minutes for group work, followed by a whole-class discussion.

Demonstrate how to compute and plot the current fraction of the times an event occurs.

Classes using the digital version have an applet available that automates the computation and the graphing, allowing students to focus on the probabilities.

Display the table and graph for all to see as an example of how to fill in the table and graph the results.

roll	number rolled	total number of wins for Mai	fraction of games that are wins
1	5	0	0
2	1	1	$\frac{1}{2} = 0.50$
3	2	2	$\frac{2}{3} \approx 0.67$
4	4	2	$\frac{2}{4} = 0.50$



To help students understand the graph, consider asking these questions. Ask students why the y-axis only shows 0 to 1. Ask students what the point at (3, 0.66) represents.

Anticipated Misconceptions

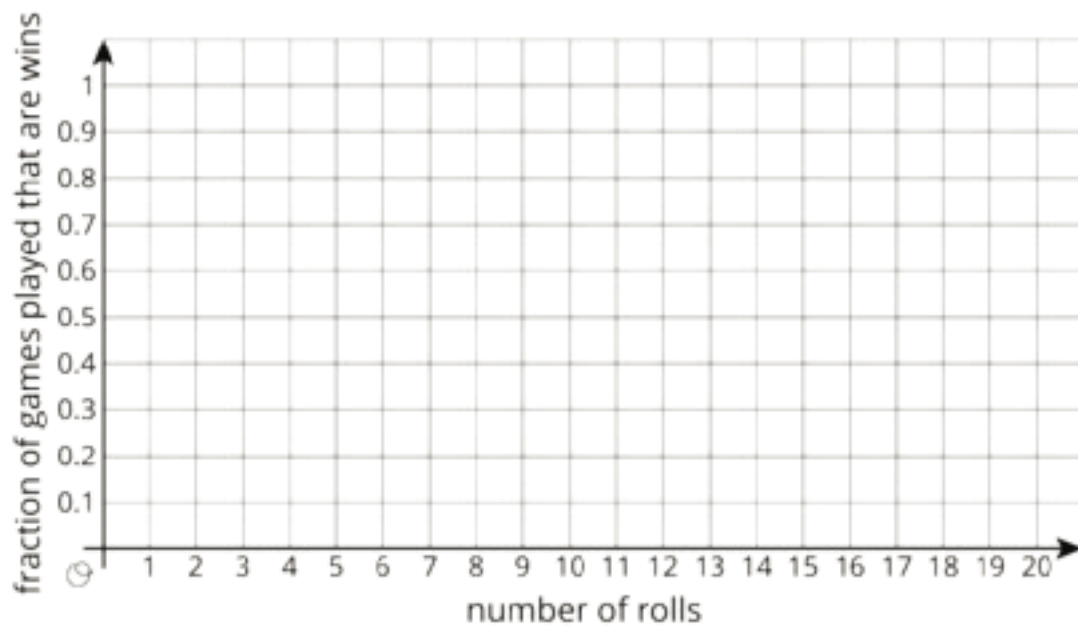
Students may not notice a pattern in the graph. Ask if they can see a pattern with the decimal values for the fraction of wins in their table. If their data does not fit the expected pattern, tell them that this is not typical and ask them to look at another group's results.

Student Task Statement

Mai plays a game in which she only wins if she rolls a 1 or a 2 with a standard number cube.

1. List the outcomes in the sample space for rolling the number cube.
2. What is the probability Mai will win the game? Explain your reasoning.
3. If Mai is given the option to flip a coin and win if it comes up heads, is that a better option for her to win?
4. With your group, follow these instructions 10 times to create the graph.
 - One person rolls the number cube. Everyone records the outcome.
 - Calculate the fraction of rolls that are a win for Mai so far. Approximate the fraction with a decimal value rounded to the hundredths place. Record both the fraction and the decimal in the last column of the table.
 - On the graph, plot the number of rolls and the fraction that were wins.
 - Pass the number cube to the next person in the group.

roll	outcome	total number of wins for Mai	fraction of games played that are wins
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			



5. What appears to be happening with the points on the graph?
6.
 - a. After 10 rolls, what fraction of the total rolls were a win?
 - b. How close is this fraction to the probability that Mai will win?

7. Roll the number cube 10 more times. Record your results in this table and on the graph from earlier.

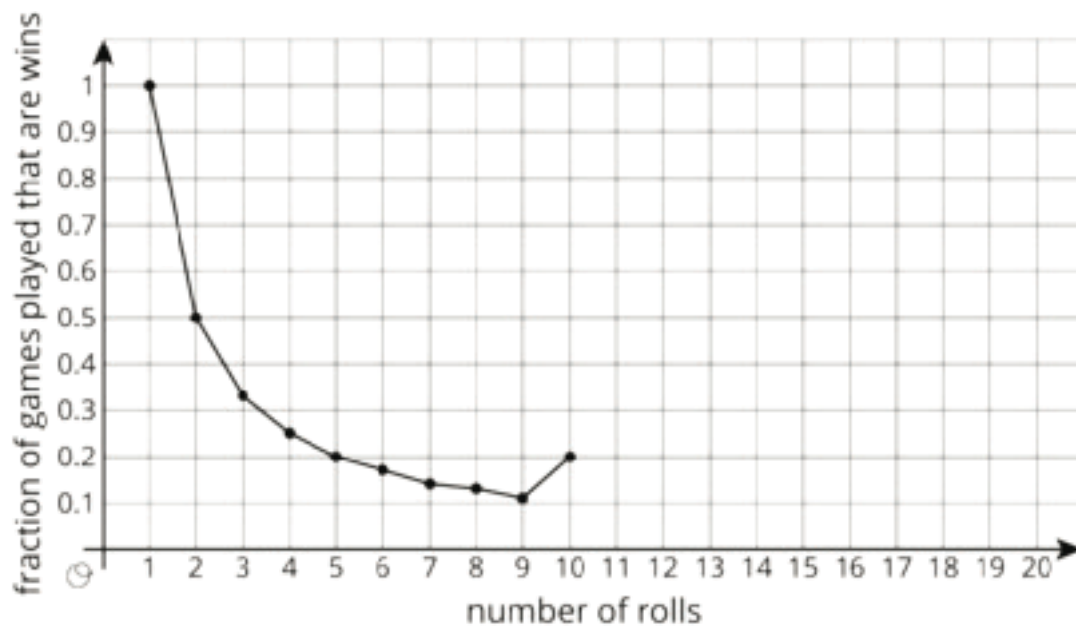
roll	outcome	total number of wins for Mai	fraction of games played that are wins
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

8. a. After 20 rolls, what fraction of the total rolls were a win?
b. How close is this fraction to the probability that Mai will win?

Student Response

1. Sample space: 1, 2, 3, 4, 5, 6.
2. Mai should win with probability $\frac{2}{6} = \frac{1}{3}$, since 2 out of the 6 numbers win.
3. Flipping the coin gives Mai a better chance of winning, since the probability of getting heads is $\frac{1}{2}$. That is greater than $\frac{1}{3}$ for the number cube.
4. Answers vary. Sample response:

roll	outcome	total number of wins for Mai	fraction of games played that are wins
1	2	1	$\frac{1}{1} = 1$
2	3	1	$\frac{1}{2} = 0.5$
3	6	1	$\frac{1}{3} \approx 0.33$
4	4	1	$\frac{1}{4} = 0.25$
5	5	1	$\frac{1}{5} = 0.2$
6	3	1	$\frac{1}{6} \approx 0.17$
7	5	1	$\frac{1}{7} \approx 0.14$
8	4	1	$\frac{1}{8} \approx 0.13$
9	5	1	$\frac{1}{9} \approx 0.11$
10	2	2	$\frac{2}{10} = 0.2$



5. The points seem to be jumping less wildly up and down.

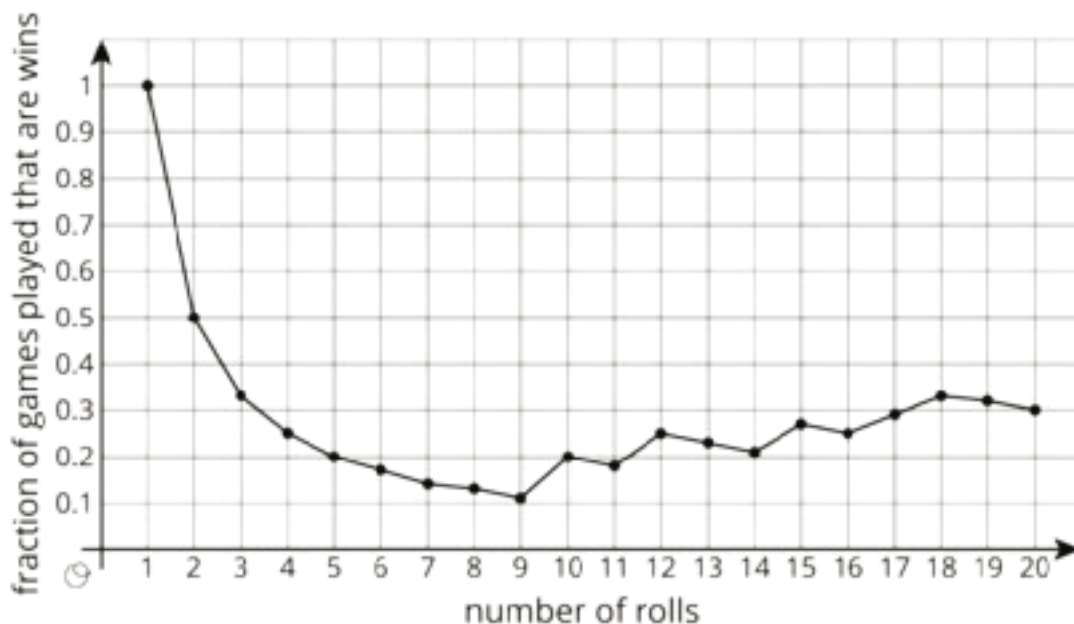
6. Answers vary. Sample response:

a. $\frac{2}{10}$

b. $\frac{2}{15} \approx 0.13$ below the expected probability since $\frac{1}{3} - \frac{2}{10} = \frac{2}{15}$

7. Answers vary. Sample response:

roll	outcome	total number of wins for Mai	fraction of games played that are wins
11	6	2	$\frac{2}{11} \approx 0.18$
12	1	3	$\frac{3}{12} = 0.25$
13	6	3	$\frac{3}{13} \approx 0.23$
14	5	3	$\frac{3}{14} \approx 0.21$
15	1	4	$\frac{4}{15} \approx 0.27$
16	6	4	$\frac{4}{16} = 0.25$
17	1	5	$\frac{5}{17} \approx 0.29$
18	2	6	$\frac{6}{18} \approx 0.33$
19	5	6	$\frac{6}{19} \approx 0.32$
20	4	6	$\frac{6}{20} = 0.3$



8. Answers vary. Sample response:

a. $\frac{6}{20}$.

b. My current fraction of rolls that are wins is $\frac{6}{20}$, but I expect the probability to be $\frac{2}{6}$, so they are about 0.03 apart.

Activity Synthesis

The purpose of this discussion is for students to understand that computing the fraction of the time an event occurs can be used to estimate the probability of the event and that more repetitions should make the estimation more accurate.

Select some students to share their answer and reasoning for the second question. If it is not mentioned by students, tell them that when there is more than one outcome that is in the desired event, then the probability of that event is the number of outcomes in the desired event divided by the number of outcomes in the sample space. In this example, there are 2 outcomes that win (a roll of 1 or 2) and 6 outcomes in the sample space, so the probability of winning is $\frac{2}{6}$ which is equivalent to $\frac{1}{3}$.

Collect the number of 1s and 2s for each group and compute the fraction for the whole class with all the data. The value should be very close to $\frac{1}{3}$.

Select students to share their thoughts on what appears to be happening with the points on their graph. (They are leveling out at 0.33.) If students struggle with noticing that the points are leveling out at a y value of 0.33, ask them to draw a horizontal line on their graphs at their answer for the probability they got in the second question.

Ask the class how many times the entire class rolled number cubes. Then ask, "Based on the probability predicted in the second question, how many times do we expect the class to have simulated a win for Mai? How does this compare to the actual number of wins the class rolled."

A probability tells you how likely an event is to occur. While it is not guaranteed to be an exact match, if the chance experiment is repeated many times, we expect the fraction of times that an event occurs to be fairly close to the calculated probability.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge about finding patterns with decimal values for the fraction of wins in the last statement. Some students may benefit from a demonstration of how to approximate fractions with decimal values to graph. Invite students to engage in the process by offering suggested directions as you demonstrate.

Supports accessibility for: Visual-spatial processing; Organization

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion. For each observation that is shared, ask students to restate and/or revoice what they heard using precise mathematical language. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. This will provide more students with an opportunity to produce language as they interpret the reasoning of others.

Design Principle(s): Support sense-making

4.3 Due For a Win

10 minutes

This activity gives students the opportunity to see that an estimate of the probability for an event should be close to what is expected from the exact probability in the long-run; however, the outcome for a chance event is not guaranteed and estimates of the probability for an event using short-term results will not usually match the actual probability exactly (MP6).

Addressing

- 7.RP.A
- 7.SP.C.6

Instructional Routines

- MLR2: Collect and Display

Launch

Tell students that the probability of a coin landing heads up after a flip is $\frac{1}{2}$.

Give students 5 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

1. For each situation, do you think the result is surprising or not? Is it possible? Be prepared to explain your reasoning.
 - a. You flip the coin once, and it lands heads up.
 - b. You flip the coin twice, and it lands heads up both times.
 - c. You flip the coin 100 times, and it lands heads up all 100 times.
2. If you flip the coin 100 times, how many times would you expect the coin to land heads up? Explain your reasoning.
3. If you flip the coin 100 times, what are some other results that would not be surprising?
4. You've flipped the coin 3 times, and it has come up heads once. The cumulative fraction of heads is currently $\frac{1}{3}$. If you flip the coin one more time, will it land heads up to make the cumulative fraction $\frac{2}{4}$?

Student Response

1.
 - a. It is not surprising and is possible.
 - b. It is a little more rare than the first one, but not very surprising. It is possible.
 - c. It is very surprising and we may suspect the coin is not fair. It is possible, though.
2. It should be heads up about 50 times out of the 100. Since the probability is $\frac{1}{2}$, there should be about the same number of heads and tails.
3. Answers vary. Possible answers should range from approximately 40 to 60.
4. Not necessarily. There's still a 50% chance it will come up tails.

Activity Synthesis

The purpose of the discussion is for students to recognize that the actual results from repeating an experiment should be close to the expected probability, but may not match exactly.

For the first problem, ask students to indicate whether or not they think each result seems surprising. For the second and third questions, select several students to provide answers and

display for all to see, then create a range of values that might not be surprising based on student responses. Ask the class if they agree with this range or to provide a reason the range is too large. It is not important for the class to get exact values, but a general agreement should arise that *some* range of values makes sense so that there does not need to be exactly 50 heads from the 100 flips.

An interesting problem in statistics is trying to define when things get "surprising." Flipping a fair coin 100 times and getting 55 heads should not be surprising, but getting either 5 or 95 heads probably is. (Although there is not a definite answer for this, a deeper study of statistics using additional concepts in high school or college can provide more information to help choose a good range of values.)

Explain that a probability represents the expected likelihood of an event occurring for a single trial of an experiment. Regardless of what has come before, each coin flip should still be equally likely to land heads up as tails up.

As another example: A basketball player who tends to make 75% of his free throw shots will probably make about $\frac{3}{4}$ of the free throws he attempts, but there is no guarantee he will make any individual shot even if he has missed a few in a row.

Access for English Language Learners

Conversing, Reading: MLR2 Collect and Display. As students share whether each result is surprising or not, write down the words and phrases students use to explain their reasoning. Listen for students who state that the actual results from repeating an experiment should be close to the expected probability. As students review the language collected in the visual display, encourage students to revise and improve how ideas are communicated. For example, a phrase such as: "If you flip a coin 100 times, it is impossible for the coin to land heads up all 100 times" can be improved with the phrase "If you flip a coin 100 times, it is very unlikely for the coin to land heads up all 100 times, but it is possible." This routine will provide feedback to students in a way that supports sense-making while simultaneously increasing meta-awareness of language.

Design Principle(s): Support sense-making; Maximize meta-awareness

Lesson Synthesis

Consider asking these questions:

- "You conduct a chance experiment many times and record the outcomes. How are these outcomes related to the probability of a certain event occurring?" (The fraction of times the event occurs after many repetitions should be fairly close to the expected probability of the event.)
- "What is the probability of rolling a 2, 3, or 4 on a standard number cube? If you roll 3 times and none of them result in a 2, 3, or 4, does the probability of getting one of those values

change with the next roll?" (The probability is 0.5 since 3 outcomes out of 6 possible are in the event. The probability should not change after 3 times. If a 2, 3, or 4 does not appear after a lot of rolls—say, 100—then we might suspect the number cube of being non-standard.)

- "The probability of getting the flu during flu season is $\frac{1}{8}$. If a family has 8 people living in the same house, is it guaranteed that one of them will get the flu? If a country has 8 million people, about how many do you expect will get the flu? Does this number have to be exact?" (No, it is very possible that none of the people in the family will get the flu and also possible that more than 1 person will get the flu. We might expect about 1 million people in the country to get the flu, but this is probably not exact.)

4.4 Fiction or Non-fiction?

Cool Down: 5 minutes

Addressing

- 7.SP.C.6
- 7.SP.C.7.b

Student Task Statement

A librarian is curious about the habits of the library's patrons. He records the type of item that the first 10 patrons check out from the library.

patron	item type
1	fiction book
2	non-fiction book
3	fiction book
4	fiction book
5	audiobook
6	non-fiction book
7	DVD
8	non-fiction book
9	fiction book
10	DVD

Based on the information from these patrons . . .

1. Estimate the probability that the next patron will check out a fiction book. Explain your reasoning.
2. Estimate the number of DVDs that will be checked out for every 100 patrons. Explain your reasoning.

Student Response

1. $\frac{4}{10}$ or equivalent since 4 of the 10 patrons in the list checked out fiction books.
2. 20 DVDs. Since 2 of the 10 patrons in the list checked out DVDs and this is $\frac{1}{5}$ of the patrons, we can expect $\frac{1}{5}$ of every 100 patrons to check out DVDs. $\frac{1}{5}$ of 100 is 20.

Student Lesson Summary

A probability for an event represents the proportion of the time we expect that event to occur in the long run. For example, the probability of a coin landing heads up after a flip is $\frac{1}{2}$, which means that if we flip a coin many times, we expect that it will land heads up about half of the time.

Even though the probability tells us what we should expect if we flip a coin many times, that doesn't mean we are more likely to get heads if we just got three tails in a row. The chances of getting heads are the same every time we flip the coin, no matter what the outcome was for past flips.

Lesson 4 Practice Problems

Problem 1

Statement

A carnival game has 160 rubber ducks floating in a pool. The person playing the game takes out one duck and looks at it.

- If there's a red mark on the bottom of the duck, the person wins a small prize.
- If there's a blue mark on the bottom of the duck, the person wins a large prize.
- Many ducks do not have a mark.

After 50 people have played the game, only 3 of them have won a small prize, and none of them have won a large prize.

Estimate the number of the 160 ducks that you think have red marks on the bottom. Then estimate the number of ducks you think have blue marks. Explain your reasoning.

Solution

Answers vary. Sample response: There are about 10 ducks with red marks on the bottom and 3 or fewer ducks with blue marks on the bottom.

- If $\frac{3}{50}$ of the people won a small prize, then the probability of getting a duck with a red mark appears to be around 0.06. Since $0.06 \cdot 160 = 9.6$, there are probably 9 or 10 ducks that have red marks out of the 160. If 9 of the ducks have a red mark, then the probability would be $\frac{9}{160} = 0.05625$. If 10 of the ducks have a red mark, then the probability would be $\frac{10}{160} = 0.0625$.
- The probability of getting a duck with a blue mark appears to be less than $\frac{1}{50}$, or 0.02. Since $0.02 \cdot 160 = 3.2$, there are probably 3 or fewer ducks that have a blue mark out of the 160. If 3 ducks have a blue mark, then the probability would be $\frac{3}{160} = 0.01875$. If 1 or 2 ducks have a blue mark, then the probability would be lower but still positive.

Problem 2

Statement

Lin wants to know if flipping a quarter really does have a probability of $\frac{1}{2}$ of landing heads up, so she flips a quarter 10 times. It lands heads up 3 times and tails up 7 times. Has she proven that the probability is not $\frac{1}{2}$? Explain your reasoning.

Solution

No. The actual results from experiments may only get close to the expected probability if they are done many, many times. Ten flips may not be enough to get close to the expected $\frac{1}{2}$ probability.

Problem 3

Statement

A spinner has four equal sections, with one letter from the word "MATH" in each section.

- You spin the spinner 20 times. About how many times do you expect it will land on A?
- You spin the spinner 80 times. About how many times do you expect it will land on something other than A? Explain your reasoning.

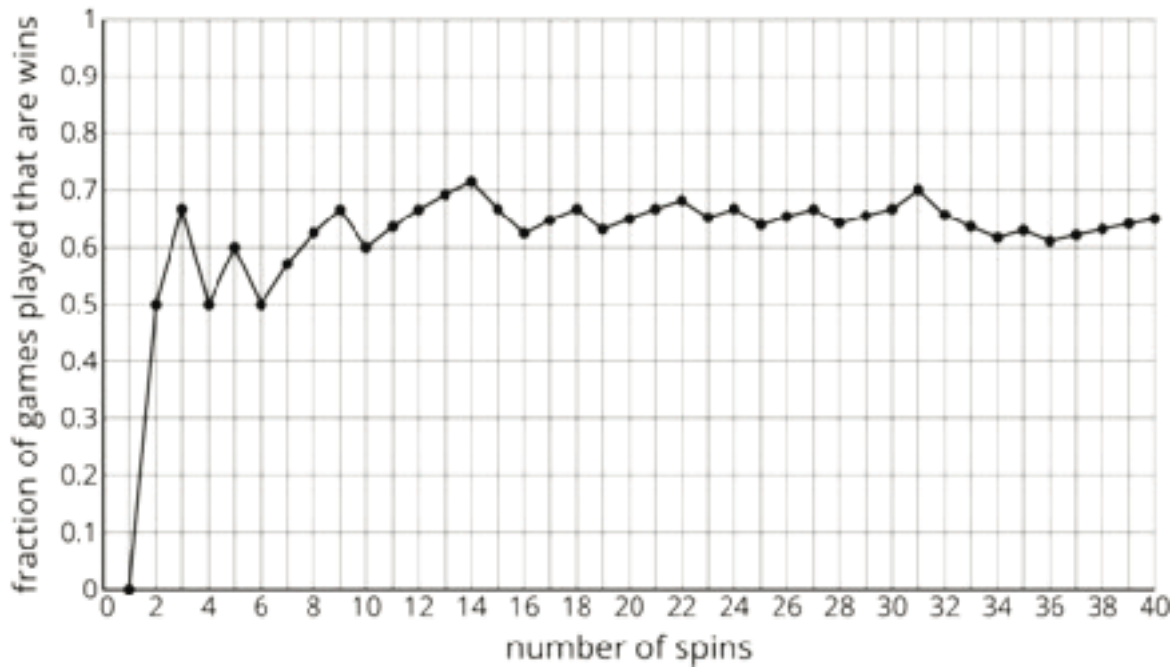
Solution

- About 5 times, because $\frac{1}{4} \cdot 20 = 5$.
- About 60 times, because $\frac{3}{4} \cdot 80 = 60$.

Problem 4

Statement

A spinner is spun 40 times for a game. Here is a graph showing the fraction of games that are wins under some conditions.



Estimate the probability of a spin winning this game based on the graph.

Solution

0.65

Problem 5

Statement

Which event is more likely: rolling a standard number cube and getting an even number, or flipping a coin and having it land heads up?

Solution

Both events are equally likely. Sample explanations:

- Each event has a 50% chance of occurring.
- Both events are as likely to happen as to not happen.

(From Unit 8, Lesson 2.)

Problem 6

Statement

Noah will select a letter at random from the word “FLUTE.” Lin will select a letter at random from the word “CLARINET.”

Which person is more likely to pick the letter “E?” Explain your reasoning.

Solution

Noah. Explanations vary. Sample response: Getting the letter “E” is more likely when selecting from the word “FLUTE” because there are fewer possible outcomes in the sample space, and each outcome is equally likely.

(From Unit 8, Lesson 3.)

Lesson 5: More Estimating Probabilities

Goals

- Describe (orally and in writing) reasons why the relative frequency from an experiment may not exactly match the actual probability of the event.
- Recognize that sometimes the outcomes in a sample space are not equally likely.
- Use the results from a repeated experiment to estimate the probability of an event, and justify (orally and in writing) the estimate.

Learning Targets

- I can calculate the probability of an event when the outcomes in the sample space are not equally likely.
- I can explain why results from repeating an experiment may not exactly match the expected probability for an event.

Lesson Narrative

In this lesson students compare the results from running actual trials of an experiment to the expected, calculated probabilities. They also use their data to see that additional trials usually produce more accurate results as minor differences even out after many trials.

In the first activity, students spin four different spinners to see that the outcomes in a sample space may not be equally likely, and they examine the spinners to construct arguments (MP3) about why some outcomes are more likely than others. In the next activity, students draw blocks out of a bag repeatedly and use the relative frequency to estimate the probability of getting a green block (MP8). This activity differs from the activity in the previous lesson where students were rolling a number cube repeatedly because in this lesson the students do not know the probability of getting a green block before they start the experiment.

In future lessons students will be asked to design and use simulations. Each lesson leading up to that helps prepare students by giving them hands-on experience with different types of chance experiments they could choose to use in their simulations. In this lesson students work with spinners and drawing blocks out of a bag.

Alignments

Addressing

- 7.SP.C.5: Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
- 7.SP.C.6: Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the

approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.

- 7.SP.C.7: Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
- 7.SP.C.7.b: Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?

Instructional Routines

- MLR5: Co-Craft Questions
- MLR8: Discussion Supports

Required Materials

Copies of Instructional
master Paper bags

Paper cups
Snap cubes

Required Preparation

Provide 1 set of 4 spinners cut from the Making My Head Spin Instructional master for every 4 students. Each student will need a pencil and paper clip to use with the spinners.

For the How Much Green activity, prepare a paper bag containing 5 snap cubes (3 green and 2 of another matching color) for every 3–4 students.

Student Learning Goals

Let's estimate some probabilities.

5.1 Is it Likely?

Warm Up: 5 minutes

The purpose of this warm-up is for students to think more deeply about probabilities and what the values actually represent. In this activity, students are asked to compare the likelihood of three events with probabilities given in different formats. In the discussion, students are also asked to think about the context of the situations to see that probabilities are not the only consideration when planning a response.

Addressing

- 7.SP.C.5

Launch

Give students 2 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

1. If the weather forecast calls for a 20% chance of light rain tomorrow, would you say that it is likely to rain tomorrow?
2. If the probability of a tornado today is $\frac{1}{10}$, would you say that there will likely be a tornado today?
3. If the probability of snow this week is 0.85, would you say that it is likely to snow this week?

Student Response

1. It is not likely to rain tomorrow, but it could happen.
2. The tornado is not likely to happen today, but it could happen.
3. It is likely that it will snow this week, but it might not happen.

Activity Synthesis

Ask students, "Which situation would you worry about the most? Is that the same situation that is the most likely?"

Note that our interpretation of the scenario influences how we feel about how likely an event is to happen. Although the likelihood of rain is higher, the implications of a tornado are much greater, so you may be more likely to worry about the tornado than the rain.

5.2 Making My Head Spin

20 minutes (there is a digital version of this activity)

In this activity, students return to calculating probabilities using the sample space, and they compare the calculated probabilities to the outcomes of their actual trials. Students have a chance to construct arguments (MP3) about why probability estimates based on carrying out the experiment many times might differ from the expected probability. Students use a spinner in this activity, which will be helpful when designing simulations in upcoming lessons.

Addressing

- 7.SP.C.6
- 7.SP.C.7.b

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 4. Provide 1 set of 4 spinners cut from the Instructional master to each group. Each student will need a pencil and paper clip. Demonstrate how to use a pencil and paper clip to spin the spinner: Unbend one end of the paper clip so that it is straight. Put the paper clip on the end of the pencil and the pencil tip at the center of the spinner. Spin the paper clip so that it rotates around the pencil and the unbent portion points to the result of the spin. If it is difficult to determine which section the end of the paper clip points to, it is okay to disregard that spin and spin again.

Classes using the digital curriculum have spinner applets to use if they choose to. These applets are based on the work of [Terry Lee Lindenmuth](#) in GeoGebra.

Following the teacher demonstration, give students 5 minutes of quiet work time, then 10 minutes of group work followed by a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Following the teacher demonstration, check for understanding by inviting students to rephrase directions in their own words.

Supports accessibility for: Memory; Conceptual processing

Anticipated Misconceptions

Students may think they need to have their probability estimates match the computed probability. Remind them of the activity "Due For a Win" to see why we might expect the estimated probability and the calculated probability to be a little different.

Student Task Statement

Your teacher will give you 4 spinners. Make sure each person in your group uses a different spinner.

1. Spin your spinner 10 times, and record your outcomes.
2. Did you get all of the different possible outcomes in your 10 spins?
3. What fraction of your 10 spins landed on 3?
4. Next, share your outcomes with your group, and record their outcomes.
 - a. Outcomes for spinner A:
 - b. Outcomes for spinner B:
 - c. Outcomes for spinner C:
 - d. Outcomes for spinner D:

5. Do any of the spinners have the same sample space? If so, do they have the same probabilities for each number to result?
6. For each spinner, what is the probability that it lands on the number 3? Explain or show your reasoning.
7. For each spinner, what is the probability that it lands on something other than the number 3? Explain or show your reasoning.
8. Noah put spinner D on top of his closed binder and spun it 10 times. It never landed on the number 1. How might you explain why this happened?
9. Han put spinner C on the floor and spun it 10 times. It never landed on the number 3, so he says that the probability of getting a 3 is 0. How might you explain why this happened?

Student Response

1. Answers vary.
2. Answers vary.
3. Answers vary.
4. Answers vary.
5. Yes, spinners A and B have the same sample space. They do not have the same probabilities for the numbers, though. For example, spinner A has two 3s out of 12 equal spaces, so the probability of spinning a 3 is $\frac{2}{12}$. Spinner B only has one 3 out of 12 equal spaces, so the probability of spinning a 3 is $\frac{1}{12}$.
6. Spinner A: $\frac{1}{6}$. Spinner B: $\frac{1}{12}$. Spinner C: $\frac{1}{12}$. Spinner D: $\frac{1}{4}$. The values for spinners A, B, and C are computed by counting the number of 3s on the spinner and dividing by the number of equal sections on the spinner. For spinner B, the section for 3 is one fourth of the circle.
7. Spinner A: $\frac{5}{6}$. Spinner B: $\frac{11}{12}$. Spinner C: $\frac{11}{12}$. Spinner D: $\frac{3}{4}$. Each of these values was computed by counting the number of things that were not 3 on the spinner and dividing by the number of sections on the spinner.
8. Since the binder is sloped, gravity may have pulled the spinner so that a 1 would not show up.
9. Han might have been holding the spinner at an angle like Noah or maybe he just did not spin enough times. Since it is possible to spin a 1, the probability should not be 0.

Are You Ready for More?

Design a spinner that has a $\frac{2}{3}$ probability of landing on the number 3. Explain how you could precisely draw this spinner.

Student Response

Answers vary. Sample response: First, I would draw a circle with a compass. Then I would divide the circle into 3 equal sections by using a protractor and measuring an angle of 120° since $360 \div 3 = 120$. I would write the number 3 in two of the sections and write the number 1 in the other section.

Activity Synthesis

The purpose of this discussion is to think about reasons why the estimate of a probability may be different from the actual probability.

Select some students to share their responses to the last 5 questions.

Ask, "How does your fraction of 3 spins compare to the probability you expect from just looking at the spinner?"

Explain that although the spinners provided were designed to have equally sized sections (except for spinner D which has the angles 180° , 45° , and 90°), sometimes it may be difficult to determine when the sections are exactly the same size. For some situations where things are not so evenly divided, some experimenting may be needed to determine that the outcomes actually follow the probability we might expect.

There are two main reasons why the fraction of the time an event occurs may differ from the actual probability:

- The simulation was designed or run poorly.
 - Maybe the spinner sections were not equal sized when they should be or maybe the spinner was tilted.
 - Maybe the items being chosen from the bag were different sizes, so you were more likely to grab one than another.
 - Maybe the coin or number cube are not evenly weighted and are more likely to land with one side up than others.
 - Maybe the computer that was programmed to return a random number has a problem with the code that returns some numbers more often than others.
- Not enough trials were run.
 - As in the previous lesson, if you flip a coin once and it comes up heads, that doesn't mean that it always will. Even if you flip it 100 times, it's not guaranteed to land heads up exactly 50 times, so some slight deviation is to be expected.

Access for English Language Learners

Speaking: MLR8 Discussion Supports. To help students think about reasons why the frequency of spinning a number may be different from the actual probability of spinning that number, provide sentence frames such as: “When we spun the spinner ten times, we noticed . . .”, “The probability of the spinner landing on the number three is ____ because . . .”, and “The result of our experiment is similar to or different than the actual probability because . . .”. This will support a rich and inclusive discussion about why it is reasonable to see outcomes that are slightly different than the actual probability of the situation.

Design Principle(s): Support sense-making; Cultivate conversation

5.3 How Much Green?

10 minutes

In the previous activity, students could see the entire spinner and compute the probability to compare with their experimental results. In many contexts, it is not possible to know the entire sample space or compute an exact probability from the situation itself. For example, the probability of rain tomorrow usually cannot be exactly estimated from available information. In this activity, students see how to approach such problems by estimating the probability of an event using the results from repeating trials (MP8). In this particular example, the exact probability can be computed when the information is revealed, so students can compare their results to this value. In situations like predicting the weather, estimates may be the best thing we have available. Students gain exposure to the process of drawing blocks from a bag, which will be useful in designing simulations in future lessons.

Addressing

- 7.SP.C.7.b

Instructional Routines

- MLR5: Co-Craft Questions

Launch

Arrange students in groups of 3–4. Distribute 1 paper bag containing 5 snap cubes (3 green cubes and 2 cubes of some other color that match each other) to each group. 5 minutes of group work followed by a whole-class discussion.

Access for English Language Learners

Conversing: MLR5 Co-Craft Questions. Before presenting the questions in this activity, provide the bag of blocks and the instructions for the experiment. Ask students to write possible mathematical questions about the situation. Then ask students to compare the questions they generated with a partner before sharing questions with the whole class. Listen for and amplify questions about estimating the probability of taking out a green block from the bag. Then reveal and ask students to work on the actual questions of the task. This will help develop students' meta-awareness of the language used to generate questions about the probability of an event.

Design Principle(s): Maximize meta-awareness

Anticipated Misconceptions

Some students may estimate a probability that is different from the fraction of times they draw a green block. Ask these students for a reason they chose a different value for their estimate.

Student Task Statement

Your teacher will give you a bag of blocks that are different colors. Do not look into the bag or take out more than 1 block at a time. Repeat these steps until everyone in your group has had 4 turns.

- Take one block out of the bag and record whether or not it is green.
 - Put the block back into the bag, and shake the bag to mix up the blocks.
 - Pass the bag to the next person in the group.
1. What do you think is the probability of taking out a green block from this bag? Explain or show your reasoning.
 2. How could you get a better estimate without opening the bag?

Student Response

1. Answers vary. Sample response: I think the probability is $\frac{7}{12}$ since we got 7 green blocks after 12 trials.
2. Answers vary. Sample response: Continuing to pick out blocks more times might get a better estimate.

Activity Synthesis

The purpose of the discussion is to show that estimating the probability of an event can be done using repeated trials and is usually improved by including more trials.

Ask each group how many green blocks they got in their trials and display the class results for all to see.

Consider asking these discussion questions:

- "How can we use the values from the class to estimate the probability of drawing out a green block?" (By using the data we have, we can estimate the fraction of blocks that are green.)
- "Based on the class data, what is the estimated probability of choosing a green block from the bag?"
- "Was the probability estimated from the class data different from the probability estimate based on the data just from your group? Why?" (Yes, since not everyone picked out the same thing each time.)
- "Some of you may have felt that there are 5 blocks in the bag. If we use that information, does that change our estimate of the probability?" (If there are only 5 blocks, it only makes sense for the probability to be $0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5},$ or $\frac{5}{5}$.)
- Allow students to open the bags and see what blocks are in there. "What is the probability based on this observation?" ($\frac{3}{5}$)
- "Does it match your group estimates? Does it match the class estimates?"
- "Was the estimate from the class data more accurate than the estimates from the groups?" (It should be since more trials are included.)

Lesson Synthesis

Consider asking these discussion questions:

- "A student repeats the process of taking blocks out of a bag and replacing them 100 times. A green block is drawn 67 times. What is a good estimate for the probability of drawing out a green block from the bag?" ($\frac{67}{100}$)
- "A chance experiment is done a few times and the fraction of outcomes in a certain event is used as an estimate for the probability of the event. If the experiments are done carefully, how could the estimate be improved?" (Usually, the more trials done for an experiment, the closer the estimate will be to a computed probability.)
- "A chance experiment is repeated many times, but the fraction of outcomes for which a certain event occurs does not match the actual probability of the event. What are some reasons this may happen?" (The experiment may not have been repeated enough times. The experiment was badly done. For example, the experiment may not have been as random as originally thought. We usually expect a little difference between the estimated probability and the actual probability.)

5.4 The Probability of Spinning B

Cool Down: 5 minutes

Addressing

- 7.SP.C.7

Student Task Statement

Jada, Diego, and Elena each use the same spinner that has four (not necessarily equal sized) sections marked A, B, C, and D.

- Jada says, "The probability of spinning B is 0.3 because I spun 10 times, and it landed on B 3 times."
- Diego says, "The probability of spinning B is 20% because I spun 5 times, and it landed on B once."
- Elena says, "The probability of spinning B is $\frac{2}{7}$ because I spun 7 times, and it landed on B twice."

1. Based on their methods, which probability estimate do you think is the most accurate? Explain your reasoning.
2. Andre measures the spinner and finds that the B section takes up $\frac{1}{4}$ of the circle. Explain why none of the methods match this probability exactly.

Student Response

Answers vary. Sample response:

1. Jada's method is probably the most accurate since she had the most attempts.
2. Since Jada spun it 10 times, she could only get estimates in 0.1 increments. Since Diego spun it 5 times, he could only get estimates in 20% increments. Since Elena spun it 7 times, she could only get estimates in $\frac{1}{7}$ increments. If they spun the spinner more times, their results would probably get closer to $\frac{1}{4}$.

Student Lesson Summary

Suppose a bag contains 5 blocks. If we select a block at random from the bag, then the probability of getting any one of the blocks is $\frac{1}{5}$.



Now suppose a bag contains 5 blocks. Some of the blocks have a star, and some have a moon. If we select a block from the bag, then we will either get a star block or a moon block. The probability of getting a star block depends on how many there are in the bag.



In this example, the probability of selecting a star block at random from the first bag is $\frac{1}{5}$, because it contains only 1 star block. (The probability of getting a moon block is $\frac{4}{5}$.) The probability of selecting a star block at random from the second bag is $\frac{3}{5}$, because it contains 3 star blocks. (The probability of getting a moon block from this bag is $\frac{2}{5}$.)

This shows that two experiments can have the same sample space, but different probabilities for each outcome.

Lesson 5 Practice Problems

Problem 1

Statement

What is the same about these two experiments? What is different?

- Selecting a letter at random from the word "ALABAMA"
- Selecting a letter at random from the word "LAMB"

Solution

Answers vary. Sample response: Both these experiments have the same sample space. Also, they are both chance experiments that have to do with selecting letters at random from words. These two experiments are different because in the word "LAMB," each letter is equally likely, but in the word "ALABAMA," the letter "A" is more likely than the other letters.

Problem 2

Statement

Andre picks a block out of a bag 60 times and notes that 43 of them were green.

- What should Andre estimate for the probability of picking out a green block from this bag?
- Mai looks in the bag and sees that there are 6 blocks in the bag. Should Andre change his estimate based on this information? If so, what should the new estimate be? If not, explain your reasoning.

Solution

- $\frac{43}{60}$
- Yes. The estimate should be changed to $\frac{4}{6}$ since the original estimate is close to $\frac{40}{60}$, which is equal to $\frac{4}{6}$, and which is actually possible with 6 blocks. Since Andre was doing an experiment, it makes sense that he would be close to, but not exactly match the calculated probability.

Problem 3

Statement

Han has a number cube that he suspects is not so standard.

- Han rolls the cube 100 times, and it lands on a six 40 times.
- Kiran rolls the cube 50 times, and it lands on a six 21 times.

- Lin rolls the cube 30 times, and it lands on a six 11 times.

Based on these results, is there evidence to help prove that this cube is not a standard number cube? Explain your reasoning.

Solution

Yes. Sample explanation: A standard number cube should land on a six about $\frac{1}{6}$ of the time. After 100 rolls, it should land on six about 16 or 17 times. All three people had it land on six more than twice as often. With this many rolls, there is strong evidence that this cube is not standard.

Problem 4

Statement

A textbook has 428 pages numbered in order starting with 1. You flip to a random page in the book in a way that it is equally likely to stop at any of the pages.

- What is the sample space for this experiment?
- What is the probability that you turn to page 45?
- What is the probability that you turn to an even numbered page?
- If you repeat this experiment 50 times, about how many times do you expect you will turn to an even numbered page?

Solution

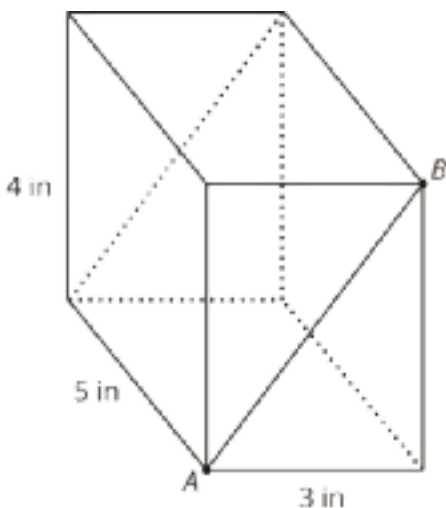
- The numbers 1 through 428
- $\frac{1}{428}$
- $\frac{214}{428}$ or $\frac{1}{2}$ (or equivalent)
- About 25 times, because $\frac{1}{2} \cdot 50 = 25$.

(From Unit 8, Lesson 3.)

Problem 5

Statement

A rectangular prism is cut along a diagonal on each face to create two triangular prisms. The distance between *A* and *B* is 5 inches.



What is the surface area of the original rectangular prism? What is the total surface area of the two triangular prisms together?

Solution

Rectangular prism: 94 square inches. Two triangular prisms together: 144 square inches

(From Unit 7, Lesson 15.)

Lesson 6: Estimating Probabilities Using Simulation

Goals

- Comprehend the term “simulation” (in written and spoken language) refers to a chance experiment used to represent a real-world situation.
- Describe (orally and in writing) a simple chance experiment that could be used to simulate a real-world event.
- Perform a simulation, and use the results to estimate the probability of a simple event in a real-world situation (using words and other representations).

Learning Targets

- I can simulate a real-world situation using a simple experiment that reflects the probability of the actual event.

Lesson Narrative

This lesson introduces the idea of simulation. Different groups of students use different chance experiments that are designed to enable you to approximate the probability of a real world event.

Students follow a process similar to what they used in previous lessons for calculating relative frequencies (the activities in which students were rolling a 1 or 2 on a number cube or drawing a green block out of a bag). The distinction in this lesson is that the outcomes students are tracking are from an experiment designed to represent the outcome of some other experiment that would be harder to study directly. Students see that a simulation depends on the experiment used in the simulation being a reasonable stand-in for the actual experiment of interest (MP4).

This lesson works with estimating the probability of simple events in preparation for students being able to estimate the probability of compound events in upcoming lessons.

Alignments

Building On

- 7.NS.A.2.d: Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Addressing

- 7.SP.C: Investigate chance processes and develop, use, and evaluate probability models.
- 7.SP.C.5: Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

- 7.SP.C.6: Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
- 7.SP.C.7.b: Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?
- 7.SP.C.8.c: Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share
- Which One Doesn't Belong?

Required Materials

Number cubes

cubes with sides numbered from 1 to 6

Paper clips

Pre-printed slips, cut from copies of the Instructional master

Paper bags

Required Preparation

Print and cut up slips and spinners from the Diego's Walk Instructional master. Provide each group of 3 supplies for 1 type of simulation: choosing a situation slip from a bag, spinning a spinner, or rolling 2 number cubes. The supplies for each simulation include:

- a paper bag containing a set of slips cut from the Instructional
- master a spinner cut from the Instructional master, a pencil and a
- paper clip 2 standard number cubes

Student Learning Goals

Let's simulate real-world situations.

6.1 Which One Doesn't Belong: Spinners

Warm Up: 5 minutes

This warm-up prompts students to compare four images. It encourages students to explain their reasoning, hold mathematical conversations, and gives you the opportunity to hear how they use terminology and talk about characteristics of the images in comparison to one another. To allow all students to access the activity, each image has one obvious reason it does not belong. Encourage students to use appropriate terminology (eg The bottom left spinner is the only one with an outcome that has a probability greater than 0.5).

During the discussion, listen for important ideas and terminology that will be helpful in upcoming work of the unit.

Addressing

- 7.SP.C.5
- 7.SP.C.7.b

Instructional Routines

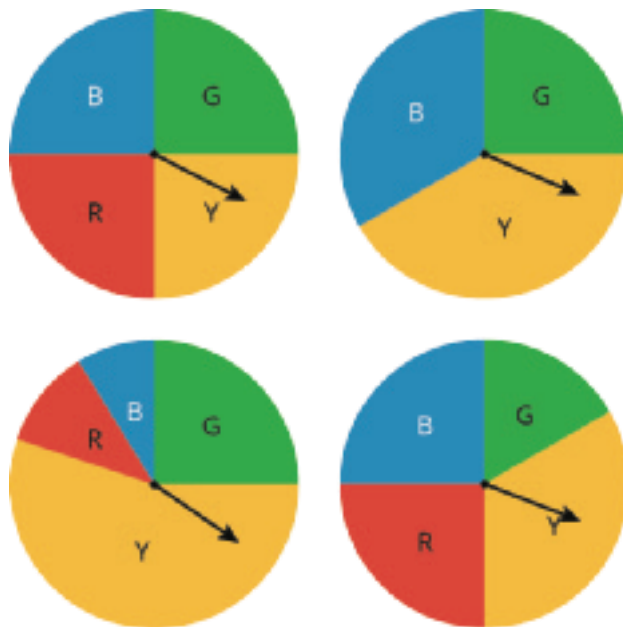
- Which One Doesn't Belong?

Launch

Arrange students in groups of 2–4. Display the image for all to see. Ask students to indicate when they have noticed which image does not belong and can explain why. Give students 2 minutes of quiet think time and then time to share their thinking with their group. After everyone has conferred in groups, ask the group to offer at least one reason each image doesn't belong. Follow with a whole-class discussion.

Student Task Statement

Which spinner doesn't belong?



Student Response

Answers vary. Sample responses:

- The top left spinner does not belong since all of the outcomes are equally likely.
- The top right spinner does not belong since it only has 3 possible outcomes.
- The bottom left spinner does not belong since it has an outcome that is more likely than the other three combined.
- The bottom right does not belong since the green outcome is not one fourth of the circle.

Activity Synthesis

Ask each group to share one reason why a particular image does not belong. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given are correct. During the discussion, ask students to explain the meaning of any terminology they use, such as probability. Also, press students on unsubstantiated claims.

6.2 Diego's Walk

20 minutes

In this activity, students estimate the probability of a real-world event by simulating the experience with a chance experiment (MP4). Students see that that multiple simulation methods can result in similar estimates for the probability of the actual event.

Building On

- 7.NS.A.2.d

Addressing

- 7.SP.C.6
- 7.SP.C.8.c

Launch

Arrange students in groups of 3. Prepare each group with supplies for 1 type of simulation: choosing a slip from a bag, spinning a spinner, or rolling 2 number cubes. The supplies for each of these simulations include:

- a bag containing a set of slips from the Instructional master
- a spinner cut from the Instructional master, a pencil and a paper
- clip 2 standard number cubes

Set up the following simulation by telling the students: Diego must cross a busy intersection at a crosswalk on his way to school. Some days he is able to cross immediately or wait only a short while. Other days, he must wait for more than 1 minute for the signal to indicate he may cross the street. We will simulate his luck at this intersection using different methods and estimate his probability of waiting more than 1 minute.

Teacher note: The bag of papers and spinner are designed to have a probability of 0.7 to wait more than 1 minute. The number cubes have a probability of approximately 0.72 to wait more than 1 minute. To the extent that the students are estimating the probabilities, these are close enough to give similar results.

Give students 15 minutes for group work followed by a whole-class discussion.

Access for Students with Disabilities

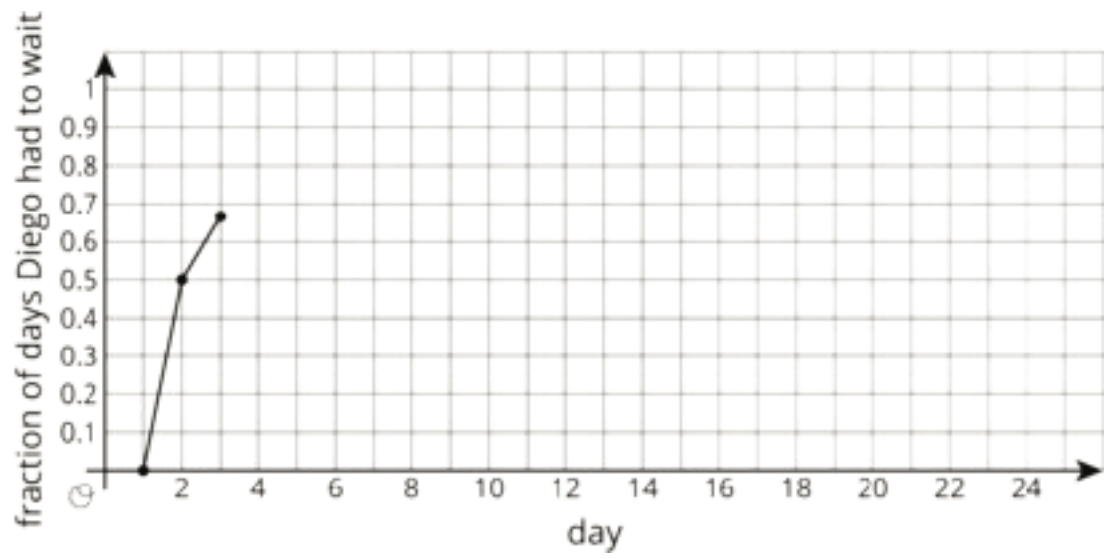
Representation: Internalize Comprehension. Check in with students after the first 3-5 minutes of work time. Check to make sure students have attended to all parts of the simulation to record one day on the graph.

Supports accessibility for: Conceptual processing; Organization

Student Task Statement

Your teacher will give your group the supplies for one of the three different simulations. Follow these instructions to simulate 15 days of Diego's walk. The first 3 days have been done for you.

- Simulate one day:
 - If your group gets a bag of papers, reach into the bag, and select one paper without looking inside.
 - If your group gets a spinner, spin the spinner, and see where it stops.
 - If your group gets two number cubes, roll both cubes, and add the numbers that land face up. A sum of 2–8 means Diego has to wait.
- Record in the table whether or not Diego had to wait more than 1 minute.
- Calculate the total number of days and the cumulative fraction of days that Diego has had to wait so far.
- On the graph, plot the number of days and the fraction that Diego has had to wait. Connect each point by a line.
- If your group has the bag of papers, put the paper back into the bag, and shake the bag to mix up the papers.
- Pass the supplies to the next person in the group.



day	Does Diego have to wait more than 1 minute?	total number of days Diego had to wait	fraction of days Diego had to wait
1	no	0	$\frac{0}{1} = 0.00$
2	yes	1	$\frac{1}{2} = 0.50$
3	yes	2	$\frac{2}{3} \approx 0.67$
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			

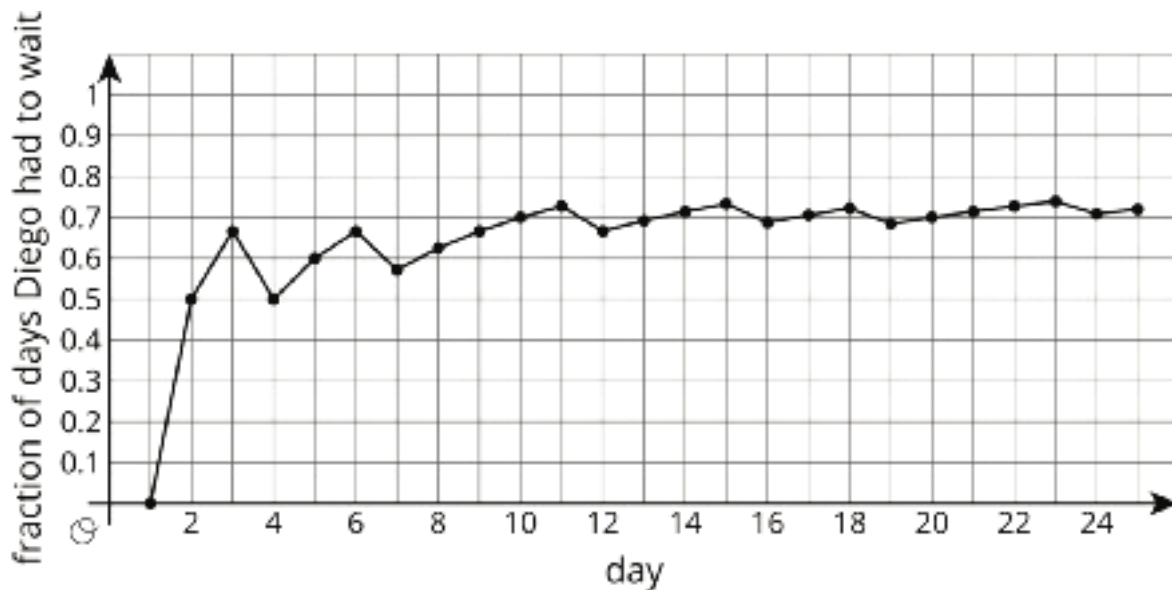
1. Based on the data you have collected, do you think the fraction of days Diego has to wait after the 16th day will be closer to 0.9 or 0.7? Explain or show your reasoning.
2. Continue the simulation for 10 more days. Record your results in this table and on the graph from earlier.

day	Does Diego have to wait more than 1 minute?	total number of days Diego had to wait	fraction of days Diego had to wait
16			
17			
18			
19			
20			
21			
22			
23			
24			
25			

- What do you notice about the graph?
- Based on the graph, estimate the probability that Diego will have to wait more than 1 minute to cross the crosswalk.

Student Response

- Answers vary. Sample response: Probably closer to 0.7 since our fraction after 16 trials was $\frac{11}{16} \approx 0.69$ which is closer to 0.7 than 0.9.
- Answers vary. Sample response:



3. Answers vary. Sample response: At the beginning, the graph jumped up and down a lot, but seems to be leveling out near the end.
4. Answers vary. Sample response: I estimate the probability to be 0.7, since the graph seems to be leveling out there.

Are You Ready for More?

Let's look at why the values tend to not change much after doing the simulation many times.

1. After doing the simulation 4 times, a group finds that Diego had to wait 3 times. What is an estimate for the probability Diego has to wait based on these results?
 - a. If this group does the simulation 1 more time, what are the two possible outcomes for the fifth simulation?
 - b. For each possibility, estimate the probability Diego has to wait.
 - c. What are the differences between the possible estimates after 5 simulations and the estimate after 4 simulations?
2. After doing the simulation 20 times, this group finds that Diego had to wait 15 times. What is an estimate for the probability Diego has to wait based on these results?
 - a. If this group does the simulation 1 more time, what are the two possible outcomes for the twenty-first simulation?
 - b. For each possibility, estimate the probability Diego has to wait.
 - c. What are the differences between the possible estimates after 21 simulations and the estimate after 20 simulations?

3. Use these results to explain why a single result after many simulations does not affect the estimate as much as a single result after only a few simulations.

Student Response

1. $\frac{3}{4}$
 - a. Either he has to wait or does not on the fifth day.
 - b. Estimates are either $\frac{4}{5}$ or $\frac{3}{5}$.
 - c. $\frac{1}{20} = 0.05$ or $\frac{3}{20} = 0.15$ since $\frac{4}{5} - \frac{3}{4} = \frac{1}{20}$ and $\frac{3}{4} - \frac{3}{5} = \frac{3}{20}$.
2. $\frac{15}{20}$, which is equal to $\frac{3}{4}$.
 - a. Either he has to wait or does not on the twenty-first day.
 - b. Estimates are either $\frac{15}{21}$ or $\frac{16}{21}$.
 - c. $\frac{1}{28} \approx 0.036$ or $\frac{1}{84} \approx 0.012$ since $\frac{15}{20} - \frac{15}{21} = \frac{1}{28}$ and $\frac{16}{21} - \frac{15}{20} = \frac{1}{84}$.
3. Explanations vary. Sample explanation: As the number of simulations grows, the denominator of each fraction grows, while the difference in the numerators remains 1. So, differences in the estimates between one simulation and the next will not change the estimate as much after many simulations.

Activity Synthesis

The purpose of this discussion is for students to understand why simulations are useful in place of actual experiments.

Select at least one group for each of the simulation methods to display the materials they used to run their simulation and explain the steps involved in using their materials.

Ask students, "Why do you think these simulations are more useful than actually doing the experiment many times?" (It would take a lot of time and work for Diego to walk to school more than usual, but it is easy to do the simulation many times quickly.)

Select students to share what they noticed about the graph of the fraction of days Diego had to wait as the simulated days went on.

6.3 Designing Experiments

10 minutes

In this activity, students have the opportunity to design their own simulations that could be used to estimate probabilities of real-life events (MP4). Students attend to precision (MP6) by assigning each possible outcome for the real-life experiment to a corresponding outcome in their simulation in such a way that the pair of outcomes have the same probability. In the discussion following the

activity, students are asked to articulate how these simulations could be used to estimate probabilities of certain events.

Addressing

- 7.SP.C

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Keep students in groups of 3. Give students 5 minutes quiet work time to design their own experiments, followed by small-group discussion to compare answers for the situations and whole-class discussion.

As students work, monitor for students who are using the same chance events for multiple scenarios (for example, always using a spinner) and encourage them to think about other ways to simulate the event.

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Leverage choice around perceived challenge. Invite students to select 2–3 of the situations to complete.

Supports accessibility for: Organization; Attention; Social-emotional skills


Anticipated Misconceptions

Students may think that the number of outcomes in the sample space must be the same in the simulation as in the real-life situation. Ask students how we could use the results from the roll of a standard number cube to represent a situation with only two equally likely outcomes. (By making use of some extra options to count as "roll again.")

Student Task Statement

For each situation, describe a chance experiment that would fairly represent it.

1. Six people are going out to lunch together. One of them will be selected at random to choose which restaurant to go to. Who gets to choose?
2. After a robot stands up, it is equally likely to step forward with its left foot or its right foot. Which foot will it use for its first step?
3. In a computer game, there are three tunnels. Each time the level loads, the computer randomly selects one of the tunnels to lead to the castle. Which tunnel is it?

- 
4. Your school is taking 4 buses of students on a field trip. Will you be assigned to the same bus that your math teacher is riding on?

Student Response

Answers vary. Sample responses:

1. Assign each person a number from 1 to 6, then roll a number cube. The person whose number is face up on the number cube is the one to choose.
2. Flip a coin. If it is heads, it should step with the left foot. If it is tails, it should step with the right.
3. Label each tunnel "left," "right," and "middle." Make a spinner that has three equal sections with one of these labels in each section. Spin the spinner and the castle is behind the one the spinner lands on.
4. Assign a color to each bus. Put one block of each color in a bag, then reach in and pull out one block. If the block is red, then you are on the same bus.

Activity Synthesis

The purpose of this discussion is for students to think more deeply about the connections between the real-life experiment and the simulation.

Select partners to share the simulations they designed for each of the situations.

Some questions for discussion:

- "How could a standard number cube be used to simulate the situation with the buses?" (Each bus is assigned a number 1 through 4. If the cube ends on 5 or 6, roll again.)
- "If one of the buses was numbered with your math teacher's favorite number and you wanted to increase the probability of that bus being selected, how could you change the simulation to do this?" (Add more of the related outcome. For example, using the standard number cube as in the previous discussion question, the bus with the favorite number could be assigned numbers 4 and 5 while the other buses are still 1 through 3.)
- Two of the tunnels in the video game lead to a swamp that ends the game. How could you use your simulation to estimate the probability of choosing one of those two tunnels? (Since all of the tunnels are equally likely to lead to the swamp, it can be assumed that "left" and "right" lead to the swamp. Spin the spinner many times and use the fraction of times it ends on "left" or "right" to estimate the probability of ending the game. It should happen $\frac{2}{3}$ or about 67% of the time.)
- "You and a friend are among the people going to lunch. How could you use the simulation you designed to estimate the probability that you or your friend will be the one to choose the restaurant?" (My friend and I will be represented by 1 and 2 on a number cube. Roll the number cube a lot of times and find the fraction of times 1 or 2 appear, then estimate the probability that we will be the ones selected.)

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion. After students share the simulations they designed, display the following sentence frames to help students respond: "I agree because" or "I disagree because" Encourage students to use mathematical language to support their response. This will support rich and inclusive discussion about how to simulate a real-world situation using a simple experiment that reflects the probability of the actual event.

Design Principle(s): Support sense-making, Cultivate conversation

Lesson Synthesis

Consider asking these discussion questions:

- "What is a simulation?"
- "Why might you want to run a simulation rather than the actual event?" (Simulations are easier and usually faster to do multiple times, so using them to get an estimate of the probability of an event is sometimes preferred.)
- "If you conduct a few trial simulations of a situation and record the the fraction of outcomes for which a particular event occurs, how might you know that you have done enough simulations to have a good estimate of the probability of that event happening?" (When the fractions seem to not be changing very much based on how accurate you want your estimate to be.)

6.4 Video Game Weather

Cool Down: 5 minutes

In this activity, students use their understanding of simulations to design a chance experiment that can be easily repeated while mimicking another situation with the same probability of a certain event.

Addressing

- 7.SP.C

Student Task Statement

In a video game, the chance of rain each day is always 30%. At the beginning of each day in the video game, the computer generates a random integer between 1 and 50. Explain how you could use this number to simulate the weather in the video game.

Student Response

Answers vary. Sample response: If the number is between 1 and 15, the video game should create a rainy day. If the number is between 16 and 50, the video game should not create a rainy day.

Student Lesson Summary

Sometimes it is easier to estimate a probability by doing a **simulation**. A simulation is an experiment that approximates a situation in the real world. Simulations are useful when it is hard or time-consuming to gather enough information to estimate the probability of some event.

For example, imagine Andre has to transfer from one bus to another on the way to his music lesson. Most of the time he makes the transfer just fine, but sometimes the first bus is late and he misses the second bus. We could set up a simulation with slips of paper in a bag. Each paper is marked with a time when the first bus arrives at the transfer point. We select slips at random from the bag. After many trials, we calculate the fraction of the times that he missed the bus to estimate the probability that he will miss the bus on a given day.

Glossary

- simulation

Lesson 6 Practice Problems

Problem 1

Statement

The weather forecast says there is a 75% chance it will rain later today.

- a. Draw a spinner you could use to simulate this probability.
- b. Describe another way you could simulate this probability.

Solution

Answers vary. Sample response:

- a. A circle with $\frac{3}{4}$ colored blue and labeled "rain" and the other $\frac{1}{4}$ left white and labeled "no rain."
- b. Put 4 marbles in a bag, three blue and one white. The blue marbles represent "rain."

Problem 2

Statement

An experiment will produce one of ten different outcomes with equal probability for each. Why would using a standard number cube to simulate the experiment be a bad choice?

Solution

A standard number cube only has 6 outcomes, so it cannot produce all 10 possibilities from the experiment.

Problem 3

Statement

An ice cream shop offers 40 different flavors. To simulate the most commonly chosen flavor, you could write the name of each flavor on a piece of paper and put it in a bag. Draw from the bag 100 times, and see which flavor is chosen the most. This simulation is not a good way to figure out the most-commonly chosen flavor. Explain why.

Solution

Answers vary. Sample response: Drawing from the bag is random, but people do not usually randomly choose ice cream flavors.

Problem 4

Statement

Each set of three numbers represents the lengths, in units, of the sides of a triangle. Which set can *not* be used to make a triangle?

- A. 7, 6, 14
- B. 4, 4, 4
- C. 6, 6, 2
- D. 7, 8, 13

Solution

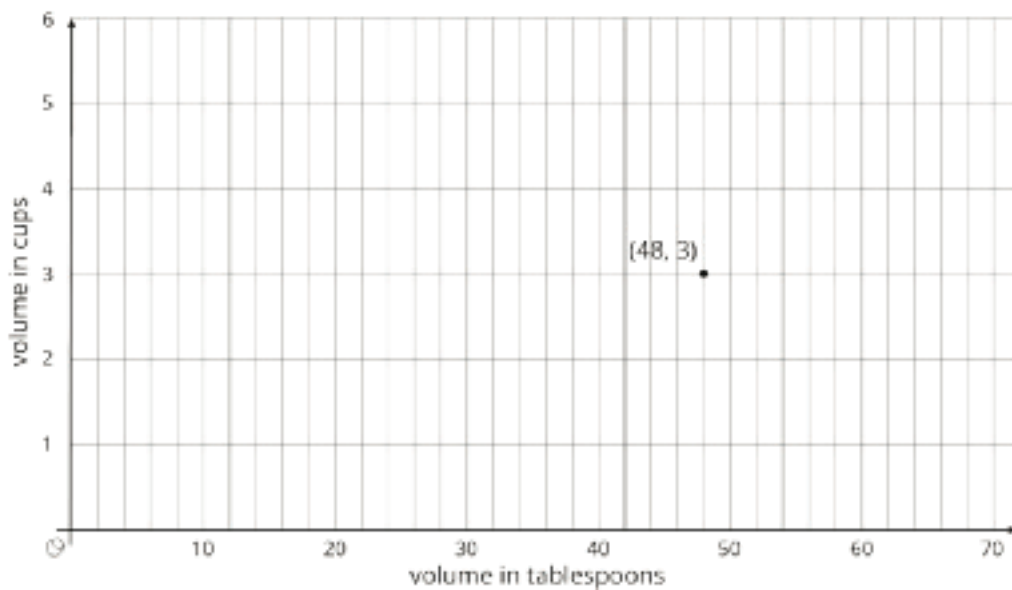
A

(From Unit 7, Lesson 7.)

Problem 5

Statement

There is a proportional relationship between a volume measured in cups and the same volume measured in tablespoons. 48 tablespoons is equivalent to 3 cups, as shown in the graph.

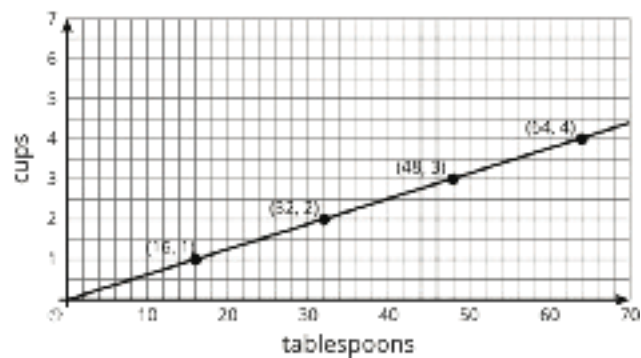


- a. Plot and label some more points that represent the relationship.
- b. Use a straightedge to draw a line that represents this proportional relationship.

- c. For which value y is $(1, y)$ on the line you just drew?
- d. What is the constant of proportionality for this relationship?
- e. Write an equation representing this relationship. Use c for cups and t for tablespoons.

Solution

- a. See below
- b. See below
- c. $\frac{1}{16}$
- d. $\frac{1}{16}$ cups per tablespoon
- e. $c = \frac{1}{16}t$ or equivalent



(From Unit 2, Lesson 14.)

Section: Probabilities of Multi-step Events

Lesson 7: Simulating Multi-step Experiments

Goals

- Coordinate (orally) a real-world situation and a chance event that could be used to simulate that situation.
- Perform a multi-step simulation, and use the results to estimate the probability of a compound event in a real-world situation (using words and other representations).

Learning Targets

- I can use a simulation to estimate the probability of a multi-step event.

Lesson Narrative

In this lesson, students see that compound events can be simulated by using multiple chance experiments. In this case, it is important to communicate precisely what represents one outcome of the simulation (MP6). For example, if we want to know the probability that a family with three children will have at least one girl, we can toss one coin to represent each child and use each set of three coin tosses to represent one family. Therefore, if we toss a coin 30 times, we will have run this simulation only 10 times.

Students continue to consider how a real-world situation can be represented using simulation (MP4).

Alignments

Addressing

- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.
- 7.SP.C.8.c: Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR5: Co-Craft Questions
- MLR8: Discussion Supports
- Notice and Wonder
- Take Turns

Required Materials

Paper bags

Paper clips

Pre-printed slips, cut from copies of the

Instructional

master Snap

cubes

Required Preparation

Print and cut up spinners from the Alpine Zoom Instructional master. One spinner for each group of 3 students.

For the Kiran's Game activity, a paper bag containing 4 snap cubes (2 black and 2 white) is needed for every 3 students.

Other simulation tools (number cubes, bags with colored snap cubes, etc.) should be available.

Student Learning Goals

Let's simulate more complicated events.

7.1 Notice and Wonder: Ski Business

Warm Up: 5 minutes

The purpose of this warm-up is to elicit ideas that will be useful in the discussions in this lesson. While students may notice and wonder many things about these images, the business side of skiing and its dependence on weather are the important discussion points.

Instructional Routines

- Notice and Wonder

Launch

Arrange students in groups of 2. Tell students that they will look at two images, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the images for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Student Task Statement

What do you notice? What do you wonder?



Student Response

Things students may notice:

- I notice that the business depends on the weather, which is not always predictable.
- I notice that they probably do not have artificial snow and rely on the weather.
- I notice there are a lot of people at this ski place.

Things students may wonder:

- I wonder what the chance of snow is.
- I wonder what they do if it doesn't snow.
- I wonder how long the skiing season is.

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After each response, ask the class whether they agree or disagree and to explain alternative ways of thinking, referring back to the images each time. If the dependence of ski businesses on weather does not come up during the conversation, ask students to discuss this idea.

7.2 Alpine Zoom

15 minutes (there is a digital version of this activity)

In this activity, students continue to model real-life situations with simulations (MP4), but now the situations have more than one part. Finding the exact probability for these situations is advanced, but simulations are not difficult to run and an estimate of the probability can be found using the long-run results from simulations (MP8). If other simulation tools are not available, you will need the Instructional master.

Addressing

- 7.SP.C.8.c

Instructional Routines

- MLR5: Co-Craft Questions

Launch

Arrange students in groups of 3. After students have had a chance to think about an experiment themselves, select groups to share their responses.

If possible, allow them to use the simulation they have suggested (rolling a number cube, papers in a bag, etc.). If the simulation is not readily available, provide each group with a spinner from the Instructional master. Give students 5 minutes for partner discussion, 5 minutes to run the simulation, then 5 minutes for a whole-class discussion.

Students using the digital version have an applet on which they can run up to 10 simulation trials.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Eliminate barriers and provide concrete manipulatives to connect symbols to concrete objects or values. Provide access to simulation tools, such as number cubes, papers in bags, and spinners.

Supports accessibility for: Visual-spatial processing; Fine-motor skills

Access for English Language Learners

Conversing: MLR5 Co-Craft Questions. To begin, display only the scenario of the Alpine Zoom ski business, without revealing the questions in this activity. Ask students to write possible mathematical questions about the situation. As pairs share their questions with the class, listen for and amplify questions about the probability that Alpine Zoom will make money during spring break. If no student asks about the probability that Alpine Zoom will make money, ask students to adapt a question to align with the learning goals of this lesson. Then reveal and ask students to work on the actual questions of the task. This routine will help develop students' meta-awareness of language as they generate questions about the probability of real-life situations.

Design Principle(s): Optimize output; Maximize meta-awareness

Anticipated Misconceptions

Students may be confused by the phrase "at least 4 days." Explain that in this context, it means 4 or more.

Student Task Statement

Alpine Zoom is a ski business. To make money over spring break, they need it to snow at least 4 out of the 10 days. The weather forecast says there is a $\frac{1}{3}$ chance it will snow each day during the break.

1. Describe a chance experiment that you could use to simulate whether it will snow on the first day of spring break.
2. How could this chance experiment be used to determine whether Alpine Zoom will make money?

Pause here so your teacher can give you the supplies for a simulation.

3. Simulate the weather for 10 days to see if Alpine Zoom will make money over spring break. Record your results in the first row of the table.

	day 1	day 2	day 3	day 4	day 5	day 6	day 7	day 8	day 9	day 10	Did they make money?
simulation 1											
simulation 2											
simulation 3											
simulation 4											
simulation 5											

4. Repeat the previous step 4 more times. Record your results in the other rows of the table.
5. Based on your group's simulations, estimate the probability that Alpine Zoom will make money.

Student Response

1. Answers vary. Sample response: Roll a number cube. If it lands on a 5 or 6, it will snow on the first day of break. If it lands on anything else, it does not snow.

2. Do the chance experiment 10 times and write down whether it snows each day. If it snows on at least 4 days, then the company will make money.

3. Answers vary.

4. Answers vary. Sample response:

	day 1	day 2	day 3	day 4	day 5	day 6	day 7	day 8	day 9	day 10	Did they make money?
simulation 1	snow	no	no	no	snow	no	snow	no	no	snow	yes
simulation 2	no	no	no	snow	snow	no	no	no	no	no	no
simulation 3	snow	no	snow	no	no	no	snow	snow	no	snow	yes
simulation 4	no	no	no	no	no	snow	no	no	no	snow	no
simulation 5	no	no	snow	no	no	no	snow	no	no	snow	no

5. Answers vary. Sample response: $\frac{2}{5}$

Activity Synthesis

The purpose of this discussion is for students to understand the connection between the results of their simulation and the real-life situation.

Ask each group for the number of times Alpine Zoom made money in their simulations.

Consider asking these discussion questions:

- “Using the class’s data, estimate the probability that Alpine Zoom will make money.” (Theoretically, this should be close to 45%.)
- “Do you anticipate Alpine Zoom will make money this spring break?” (It’s not likely, but it’s possible.)
- “Over the next 10 years, if the weather patterns continue to be the same, do you anticipate Alpine Zoom will make money over that time or not?” (This is even less likely. There is less than a 50% chance it will make money each season, so over 10 years, it will probably lose money more than make money.)

- “Is this a business you would invest in? Explain your reasoning.”
(I would not invest in it because it is unlikely to make money over the years.)

7.3 Kiran’s Game

Optional: 15 minutes

Since this activity is mainly included for practice and may take some additional time to complete, it is included as an optional task and including it is up to the teacher’s discretion.

In this activity, students practice doing many trials of multi-step situations to estimate the probability of an event. In the discussion following the activity, students construct arguments (MP3) about how changes to the game might affect the probability of winning.

Addressing

- 7.SP.C.8.c

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Arrange students in groups of 3. Provide each group with a paper bag containing 2 black blocks and 2 white blocks inside. If black and white blocks are not available, instruct students on their color equivalents. Give students 5 minutes to run the simulation, 5 minutes for partner discussion, then have a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Provide students with a graphic organizer for recording their results and calculations of probability of winning Kiran’s game.

Supports accessibility for: Language; Organization

Student Task Statement

Kiran invents a game that uses a board with alternating black and white squares. A playing piece starts on a white square and must advance 4 squares to the other side of the board within 5 turns to win the game.



For each turn, the player draws a block from a bag containing 2 black blocks and 2 white blocks. If the block color matches the color of the next square on the board, the playing piece moves onto it. If it does not match, the playing piece stays on its current square.

1. Take turns playing the game until each person in your group has played the game twice.
2. Use the results from all the games your group played to estimate the probability of winning Kiran's game.
3. Do you think your estimate of the probability of winning is a good estimate? How could it be improved?

Student Response

Answers vary. Sample response:

1. No response needed.
2. Since nobody won in all 6 games played, the probability of winning should be low. Since it's possible to win, though, I don't think the probability should be 0. The probability is probably between 0 and $\frac{1}{6}$.
3. I don't think this is a very good estimate of the probability of winning since we only played 6 times and the chances of winning are so low.
It could be improved by playing the game a lot more times.

Are You Ready for More?

How would each of these changes, on its own, affect the probability of winning the game?

1. Change the rules so that the playing piece must move 7 spaces within 8 moves.
2. Change the board so that all the spaces are black.
3. Change the blocks in the bag to 3 black blocks and 1 white block.

Student Response

1. It would be harder to win. You can still only get 1 wrong block, but now you must get it right 7 times instead of only 4.
2. This would not affect the chances of winning. There is still a probability of 0.5 to move each time.
3. This would make moving onto black squares easier, but harder to move on to white squares.
This game is slightly more difficult to win than the original.

Activity Synthesis

The purpose of the discussion is for students to think about how changing the rules of the game might change the probability of winning.

Collect the data from the class for the number of wins and display the results for all to see.

Consider asking these discussion questions:

- “Based on the class’s data, estimate the probability of winning the game.” (The theoretical probability of winning is $\frac{3}{16} \approx 0.19$.)
- “Does the game seem too easy or hard to win? If so, how could Kiran change the game slightly to make it harder or easier?” (If it is too hard, move the pawn closer to the end or allow more than 5 moves to win.)
- “The bag contained 2 black and 2 white blocks. If the bag had 4 blocks of each color, would that make it easier or harder to win?” (Neither. It would be the same difficulty since there is still an equal chance to get each color.)
- “Do you think the estimate from the class’s data is a better estimate than the one you got on your own?” (Since there is more data from the entire class, it should be a better estimate than one individual’s.)

Access for English Language Learners

Reading, Writing, Speaking: MLR3 Clarify, Critique, Correct. Before students share their estimated probability of winning Kiran’s game, present an incorrect solution based on a misconception that arises when conducting a few trial simulations. For example: “The probability of winning Kiran’s game is zero because nobody in our group won in all of the six games played.” Ask students to identify the error, critique the reasoning, and revise the original statement. As students discuss in partners, listen for students who state that estimating the probability of winning based on only six trials will not result in an accurate estimate. Therefore, more trials must be conducted in order to improve the estimate of the probability of winning. This routine will engage students in meta-awareness as they critique and correct the language used to estimate the probability of winning a game.

Design Principles(s): Cultivate conversation; Maximize meta-awareness

7.4 Simulation Nation

10 minutes

In this activity, students practice what they have learned about simulations by matching real-life scenarios to simulations. In the discussion, students are asked to explain their reasoning for their choices and think about other valid choices that could be made (MP3).

Addressing

- 7.SP.C.8.c

Instructional Routines

- MLR8: Discussion Supports
- Take Turns

Launch

Keep students in groups of 3. Give students 5 minutes of small-group time to take turns matching the items and discussing their reasoning, followed by whole-class discussion.

Access for English Language Learners

Conversing: MLR8 Discussion Supports. Invite students to take turns matching a situation to a simulation. Display sentence frames to help students explain their reasoning. For example, "Situation ___ matches with simulation ___ because . . ." Listen for the connection between the numerical quantities in the simulation and the situation. Encourage students to challenge each other when they disagree. This will help students justify how all the parts of the simulation can be used to match accordingly with the situation.

Design Principle(s): Support sense-making; Cultivate conversation

Anticipated Misconceptions

Students may not see the connection between the standard number cube and the situation with 3 doors. Remind students it is important that the probabilities match, but not necessarily the outcomes. Since the simulation matches 2 of the outcomes to one door, the probabilities will match.

Student Task Statement

Match each situation to a **simulation**.

Situations:

1. In a small lake, 25% of the fish are female. You capture a fish, record whether it is male or female, and toss the fish back into the lake. If you repeat this process 5 times, what is the probability that at least 3 of the 5 fish are female?
2. Elena makes about 80% of her free throws. Based on her past successes with free throws, what is the probability that she will make exactly 4 out of 5 free throws in her next basketball game?
3. On a game show, a contestant must pick one of three doors. In the first round, the winning door has a vacation. In the second round, the winning door has a car. What is the probability of winning a vacation and a car?
4. Your choir is singing in 4 concerts. You and one of your classmates both learned the solo. Before each concert, there is an equal chance the choir director will select you or the other student to sing the solo. What is the probability that you will be selected to sing the solo in exactly 3 of the 4 concerts?

Simulations:

1. Toss a standard number cube 2 times and record the outcomes. Repeat this process many times and find the proportion of the simulations in which a 1 or 2 appeared both times to estimate the probability.
2. Make a spinner with four equal sections labeled 1, 2, 3, and 4. Spin the spinner 5 times and record the outcomes. Repeat this process many times and find the proportion of the simulations in which a 4 appears 3 or more times to estimate the probability.
3. Toss a fair coin 4 times and record the outcomes. Repeat this process many times, and find the proportion of the simulations in which exactly 3 heads appear to estimate the probability.
4. Place 8 blue chips and 2 red chips in a bag. Shake the bag, select a chip, record its color, and then return the chip to the bag. Repeat the process 4 more times to obtain a simulated outcome. Then repeat this process many times and find the proportion of the simulations in which exactly 4 blues are selected to estimate the probability.

Student Response

1. Simulation 2
2. Simulation 4
3. Simulation 1
4. Simulation 3

Activity Synthesis

The purpose of this discussion is for students to articulate the reasons they chose to match the items they did.

For each situation, select students to explain why the simulation should go with it. Although some students may have just looked at a portion of the situation and simulation, encourage students to explain all of the parts of the simulation. Consider the problem with fish; 25% is mentioned and the spinner is the only option that also has a 25% chance associated with it. Prompt students for more details by asking,

- “Why do we need to spin the spinner 5 times?” (A fish is selected from the lake 5 times.)
- “Why does the number need to show up 3 or more times?” (We want a probability that three or more fish are female.)
- “What do the numbers 1 through 4 represent when doing a trial with the spinner?” (Each section represents a $\frac{1}{4}$ probability. The section labeled ‘4’ is the 25% chance that a fish will be female, while sections labeled 1–3 are the 75% chance that a fish will not be female.)
- “Could the spinner have 8 sections? If so, how would you label the sections? What would each label represent?” (Yes, labels vary. Sample response: Label sections 1–8, where sections 7–8 represent the 25% chance that a fish will be female, while sections labeled 1–6 are the 75% chance that a fish will not be female.)

For each of the scenarios, ask students if any part of it could be changed and still result in the simulation working. For example, there could be 4 blue chips and 1 red chip in the bag for simulation D. For simulation C, we could count the fraction of times when 3 tails appear rather than heads.

Access for Students with Disabilities

Representation: Internalize Comprehension. Use color and annotations to illustrate connections between representations. As students share their reasoning for matching situations with simulations, use color and annotations to scribe their thinking on a display of each problem so that it is visible for all students.

Supports accessibility for: Visual-spatial processing; Conceptual processing

Lesson Synthesis

Consider asking these discussion questions:

- “How are the simulations in this lesson different from the simulations in the previous lesson?” (These have multiple parts for each experiment. Also, it would be difficult to compute the exact probability, so simulations seem more necessary.)
- “The chance that it will be cloudy on a single day is simulated by rolling a standard number cube twice. How many times will the number cube need to be rolled to simulate a week?” (14 times. It is rolled twice for each day and there are 7 days in a week, so 14 rolls are needed.)

- “Each day, a student randomly reaches into a bowl of fruit and picks one for their lunch that day. To simulate the situation, he creates a spinner with 4 equal sections labeled: apple, orange, watermelon, and peach. Why might this simulation not represent the situation very well?” (Usually watermelons are much larger than the other 3 fruits listed, so there is probably not an equal chance of that being selected, so the spinner should probably have a larger wedge for watermelons.)

7.5 Battery Life

Cool Down: 5 minutes

Addressing

- 7.RP.A
- 7.SP.C.8.c

Student Task Statement

The probability of a certain brand of battery going dead within 15 hours is $\frac{1}{3}$. Noah has a toy that requires 4 of these batteries. He wants to estimate the probability that at least one battery will die before 15 hours are up.

1. Noah will simulate the situation by putting marbles in a bag. Drawing one marble from the bag will represent the outcome of one of the batteries in the toy after 15 hours. Red marbles represent a battery that dies before 15 hours are up, and green marbles represent a battery that lasts longer.

How many marbles of each color should he put in the bag? Explain your reasoning.

2. After doing the simulation 5 times, Noah has the following results. What should he use as an estimate of the probability that at least one battery will die within 15 hours?

trial	result
1	GGRG
2	GRGR
3	GGGG
4	RGGG
5	GGGR

Student Response

1. 1 red marble and 2 green marbles (or some multiple of these). Based on the probability of each battery dying, $\frac{1}{3}$ of the marbles should be red.

2. $\frac{4}{5}$ or equivalent.

Student Lesson Summary

The more complex a situation is, the harder it can be to estimate the probability of a particular event happening. Well-designed simulations are a way to estimate a probability in a complex situation, especially when it would be difficult or impossible to determine the probability from reasoning alone.

To design a good simulation, we need to know something about the situation. For example, if we want to estimate the probability that it will rain every day for the next three days, we could look up the weather forecast for the next three days. Here is a table showing a weather forecast:

	today (Tuesday)	Wednesday	Thursday	Friday
probability of rain	0.2	0.4	0.5	0.9

We can set up a simulation to estimate the probability of rain each day with three bags.

- In the first bag, we put 4 slips of paper that say "rain" and 6 that say "no rain."
- In the second bag, we put 5 slips of paper that say "rain" and 5 that say "no rain."
- In the third bag, we put 9 slips of paper that say "rain" and 1 that says "no rain."

Then we can select one slip of paper from each bag and record whether or not there was rain on all three days. If we repeat this experiment many times, we can estimate the probability that there will be rain on all three days by dividing the number of times all three slips said "rain" by the total number of times we performed the simulation.

Lesson 7 Practice Problems

Problem 1

Statement

Priya's cat is pregnant with a litter of 5 kittens. Each kitten has a 30% chance of being chocolate brown. Priya wants to know the probability that at least two of the kittens will be chocolate brown.

To simulate this, Priya put 3 white cubes and 7 green cubes in a bag. For each trial, Priya pulled out and returned a cube 5 times. Priya conducted 12 trials.

Here is a table with the results.

trial number	outcome
1	ggggg
2	gggwg
3	wgwgw
4	gwggg
5	gggwg
6	wwggg
7	gwggg
8	ggwgw
9	wwwgg
10	ggggw
11	wggwg
12	gggwg

- How many successful trials were there? Describe how you determined if a trial was a success.
- Based on this simulation, estimate the probability that *exactly* two kittens will be chocolate brown.
- Based on this simulation, estimate the probability that *at least* two kittens will be chocolate brown.
- Write and answer another question Priya could answer using this simulation.
- How could Priya increase the accuracy of the simulation?

Solution

- 5 of the 12 trials were successful. Drawing two or more white blocks (w's) counted as a success.
- $\frac{3}{12}$ (or equivalent, or nearby approximation)
- $\frac{5}{12}$ (or equivalent, or nearby approximation)

- d. Answers vary. Sample response: What is the probability that none of the kittens will be chocolate brown?
- e. Priya could conduct more trials to increase the accuracy of the simulation.

Problem 2

Statement

A team has a 75% chance to win each of the 3 games they will play this week. Clare simulates the week of games by putting 4 pieces of paper in a bag, 3 labeled “win” and 1 labeled “lose.” She draws a paper, writes down the result, then replaces the paper and repeats the process two more times. Clare gets the result: win, win, lose. What can Clare do to estimate the probability the team will win at least 2 games?

Solution

She needs to repeat the process many more times to get a good estimate of the probability. She has only done it once right now. After she has repeated the simulation of the week many times, she could count the fraction of simulated weeks that included at least 2 wins and use that as an estimate for the probability.

Problem 3

Statement

- a. List the sample space for selecting a letter a random from the word “PINEAPPLE.”
- b. A letter is randomly selected from the word “PINEAPPLE.” Which is more likely, selecting “E” or selecting “P?” Explain your reasoning.

Solution

- a. P, I, N, E, A, L (or equivalent)
- b. Selecting the letter “P” is more likely because there are 3 Ps in the word “pineapple,” but there are only 2 Es.

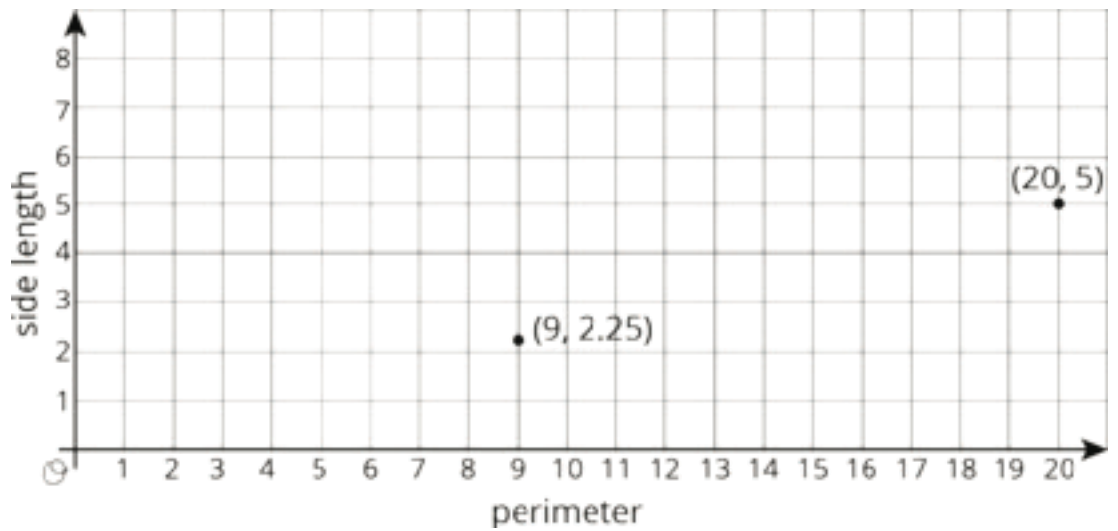
(From Unit 8, Lesson 5.)

Problem 4

Statement

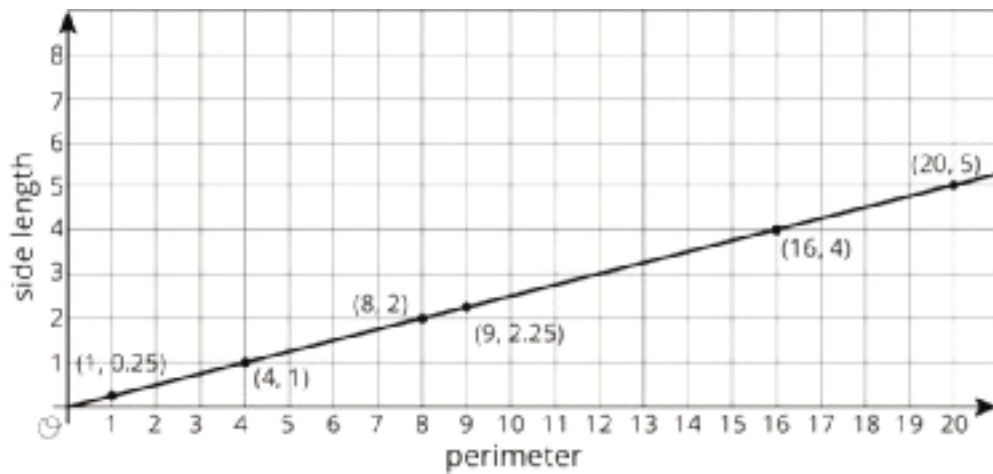
On a graph of side length of a square vs. its perimeter, a few points are plotted.

- a. Add at least two more ordered pairs to the graph.



- b. Is there a proportional relationship between the perimeter and side length? Explain how you know.

Solution



a.

- b. There is a proportional relationship between side length and perimeter. When graphed, the ordered pairs lie on a line that passes through the origin.

(From Unit 2, Lesson 11.)

Lesson 8: Keeping Track of All Possible Outcomes

Goals

- Compare and contrast (in writing) different methods for representing the sample space of a compound event, and evaluate (orally) their usefulness.
- Determine the total number of possible outcomes for a compound event, and justify the reasoning (orally, in writing, and using other representations).
- Interpret or create a list, table, or tree diagram that represents the sample space of a compound event.

Learning Targets

- I can write out the sample space for a multi-step experiment, using a list, table, or tree diagram.

Lesson Narrative

In this lesson, students practice listing the sample space for a compound event. They make use of the structure (MP7) of tree diagrams, tables, and organized lists as methods of organizing this information. Students notice that the total number of outcomes in the sample space for an experiment that can be thought of as being performed as a sequence of steps can be found by multiplying the number of possible outcomes for each step in the experiment (MP8).

In the next lesson, students will use sample spaces to calculate the probability of compound events.

Alignments

Addressing

- 7.SP.C.8.b: Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display
- MLR8: Discussion Supports
- Think Pair Share

Student Learning Goals

Let's explore sample spaces for experiments with multiple parts.

8.1 How Many Different Meals?

Warm Up: 5 minutes

The purpose of this warm-up is to elicit methods students are already using to organize their understanding of different outcomes. In this lesson, students are asked to use different structures to think about the outcomes of experiments that involve multiple steps, so this activity should give you an idea of how students are approaching the problem on their own.

Addressing

- 7.SP.C.8.b

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Tell students they should organize their work so it can be understood by others. Give students 1 minute of quiet think time, 3 minutes for partner discussion, and follow up with a whole-class discussion.

Student Task Statement

How many different meals are possible if each meal includes one main course, one side dish, and one drink?

main courses	side dishes	drinks
grilled chicken	salad	milk
turkey sandwich	applesauce	juice
pasta salad	—	water

Student Response

There are 18 different meals.

Activity Synthesis

Select several groups to share their methods for organizing their thoughts about the different meals that are possible.

Consider these questions for the discussion:

- “How did you know you counted all of the different possible meals?”
- “How did you know you didn’t repeat any meals?”

8.2 Lists, Tables, and Trees

15 minutes

In this activity, students learn 3 different methods for writing the sample spaces of multi-step experiments and explore their use in a few different situations. Since the calculated probability of an event depends on the number of outcomes in the sample space, it is important to be able to find this value in an efficient way. In the discussion, students will explore how different methods may be useful in different situations (MP1).

As students work on the second set of questions, monitor for students who:

1. Always use a list format to write out the sample space.
2. Always use a tree format to write out the sample space.
3. Change which representations they use for different questions.

Addressing

- 7.SP.C.8.b

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display

Launch

Allow 5 minutes for students to answer the first set of questions, then pause the class to discuss the methods.

Poll the class for their favorite methods in the given situation and display the results for all to see. Ask at least 1 student for each representation for their reason they believe that method is their favorite.

Following the discussion, give students another 5 minutes of quiet work time to finish the questions. Follow with a whole-class discussion.

Access for English Language Learners

Representing, Writing: MLR2 Collect and Display. As students discuss the question: “Which method do you prefer for this situation?”, write down the words and phrases students use to explain their reasoning. Listen for the language students use to describe when each method is efficient and useful for different situations. As students review the language collected in the visual display, encourage students to revise and improve how ideas are communicated. For example, a phrase such as: “I prefer the table because it is neater” can be improved with the phrase “I prefer the table because it shows all the outcomes of the sample space in an organized manner.” This routine will provide feedback to students in a way that supports sense-making while simultaneously increasing meta-awareness of language.

Design Principle(s): Support sense-making (for representation); Maximize meta-awareness

Anticipated Misconceptions

Some students may have trouble interpreting the tree diagram. Help students see that a single outcome is represented by following the “branches” from the point furthest to the left until they reach the end of a branch on the right side. It may help for students to write the full outcome on the diagram as well. In Priya’s picture, next to the uppermost 1, a student could write H1.

Student Task Statement

Consider the experiment: Flip a coin, and then roll a number cube.

Elena, Kiran, and Priya each use a different method for finding the sample space of this experiment.

- Elena carefully writes a list of all the options: Heads 1, Heads 2, Heads 3, Heads 4, Heads 5, Heads 6, Tails 1, Tails 2, Tails 3, Tails 4, Tails 5, Tails 6.
- Kiran makes a table:

	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

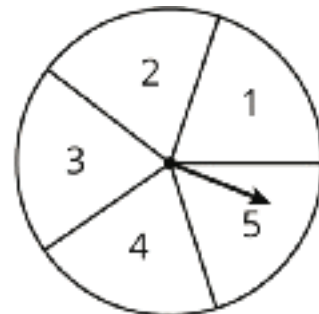
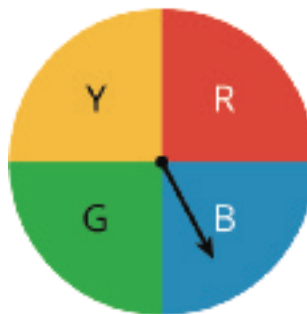
- Priya draws a tree with branches in which each pathway represents a different outcome:



- Compare the three methods. What is the same about each method? What is different? Be prepared to explain why each method produces all the different outcomes without repeating any.
- Which method do you prefer for this situation?

Pause here so your teacher can review your work.

- Find the sample space for each of these experiments using any method. Make sure you list every possible outcome without repeating any.
 - Flip a dime, then flip a nickel, and then flip a penny. Record whether each lands heads or tails up.
 - Han's closet has: a blue shirt, a gray shirt, a white shirt, blue pants, khaki pants, and black pants. He must select one shirt and one pair of pants to wear for the day.
 - Spin a color, and then spin a number.



- Spin the hour hand on an analog clock, and then choose a.m. or p.m.

Student Response

- Answers vary. Sample response: They all show the 12 possible outcomes, but Elena's method might get more difficult with more options.

2. Answers vary. Sample response: Since there are not very many outcomes in this experiment, I might use Elena's method since it is quick and I am careful.
3.
 - a. HHH, HHT, HTH, THH, HTT, THT, TTH, TTT
 - b. Blue shirt and blue pants, blue shirt and khaki pants, blue shirt and black pants, green shirt and blue pants, green shirt and khaki pants, green shirt and black pants, white shirt and blue pants, white shirt and khaki pants, white shirt and black pants.
 - c. R1, R2, R3, R4, R5, B1, B2, B3, B4, B5, G1, G2, G3, G4, G5, Y1, Y2, Y3, Y4, Y5
 - d. 1 a.m., 2 a.m., 3 a.m., 4 a.m., 5 a.m., 6 a.m., 7 a.m., 8 a.m., 9 a.m., 10 a.m., 11 a.m., 12 a.m., 1 p.m., 2 p.m., 3 p.m., 4 p.m., 5 p.m., 6 p.m., 7 p.m., 8 p.m., 9 p.m., 10 p.m., 11 p.m., 12 p.m.

Activity Synthesis

The purpose of this discussion is to think about the different methods of writing the sample space and when each might be useful. The discussion is also meant to make the connection between the methods and the number of outcomes in the sample space so that students can quickly find the number of outcomes without writing out all the possibilities.

Select previously identified students to share their group's strategies for answering the questions in the sequence identified in the Activity Narrative. For students who used the same representation throughout, ask, "Why did you choose to use this same strategy for all the questions? What are the benefits of this strategy? Did you encounter any problems using the strategy?" For students who changed strategies for different questions, ask, "How did you decide which strategy to use for each question?"

Consider these questions for the discussion:

- "What structure did you use for each situation to make sure all the different outcomes were included without duplicating any?"
- "Would each of Elena's, Kiran's, and Priya's methods work for flipping the different coins?" (A table would not work since there are three parts.)
- "Count the number of outcomes in each sample space. Is there a way to find the *number* of outcomes without writing all the possibilities? Explain or show your reasoning." (Note the connection of Kiran's table structure of "2 rows of 6 outcomes" or Priya's tree diagram of "2 groups of 6" to $2 \cdot 6$ in the experiment where they flipped a coin and rolled a number cube.)

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Provide students with a graphic organizer to support their participation during the synthesis. Invite students to describe what to look for to compare the similarities and differences between the three sample space methods, and include examples for each.

Supports accessibility for: Conceptual processing; Organization

8.3 How Many Sandwiches?

10 minutes

In this activity, students practice using their understanding of ways to calculate the number of outcomes in the sample space without writing out the entire sample space (MP7). Many situations with multiple steps have very large sample spaces for which it is not helpful to write out the entire sample space, but it is still useful to know the number of outcomes in the sample space. In this activity, students find the number of different sandwiches that can be made from available options.

Addressing

- 7.SP.C.8.b

Instructional Routines

- MLR8: Discussion Supports

Launch

Explain to students that the sandwich makers are instructed to put a certain amount of each item on the sandwich for each selection. Therefore, if a person really loves tomatoes, he should ask for tomatoes twice as his two veggie choices. Give students 5 minutes of quiet work time followed by a whole-group discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, pause to check for understanding of determining the number of outcomes after 3–5 minutes of work time.

Supports accessibility for: Organization; Attention

Anticipated Misconceptions

Some students may attempt to write out the entire sample space. Encourage them to write a few outcomes to get the idea of what is possible, but let them know that the answer for question 1 is well over 1,000, so finding a pattern or way to calculate the answer might be more efficient.

Some students may not count the “none” option for cheese as a distinct choice. Show these students that a sandwich like “Italian bread, tuna, provolone, lettuce, and tomatoes” is different than a sandwich like “Italian bread, tuna, no cheese, lettuce, and tomatoes” and should be counted separately.

Some students may notice that the order of the vegetable selection may not matter. For example, selecting lettuce then tomato would create a similar sandwich to one with tomato then lettuce selected. Some sandwich shops may offer more of the first option, so we could ask students to consider the sandwiches as different based on this idea. Otherwise, there are only 720 sandwiches possible since there are only 15 different options for 2 vegetables and $3 \cdot 4 \cdot 4 \cdot 15 = 720$.

Student Task Statement

1. A submarine sandwich shop makes sandwiches with one kind of bread, one protein, one choice of cheese, and *two* vegetables. How many different sandwiches are possible? Explain your reasoning. You do not need to write out the sample space.

- Breads: Italian, white, wheat
- Proteins: Tuna, ham, turkey, beans
- Cheese: Provolone, Swiss, American, none
- Vegetables: Lettuce, tomatoes, peppers, onions, pickles



2. Andre knows he wants a sandwich that has ham, lettuce, and tomatoes on it. He doesn't care about the type of bread or cheese. How many of the different sandwiches would make Andre happy?
3. If a sandwich is made by randomly choosing each of the options, what is the probability it will be a sandwich that Andre would be happy with?

Student Response

1. 1,200 options since $3 \cdot 4 \cdot 4 \cdot 5 \cdot 5 = 1,200$.
2. 12 sandwiches, since there are 3 options for bread and 4 options for cheese that can be made and $3 \cdot 4 = 12$.
3. $\frac{12}{1,200}$ (or equivalent) since there are 12 sandwiches that would make Andre happy out of a total of 1,200 sandwiches possible.

Are You Ready for More?

Describe a situation that involves three parts and has a total of 24 outcomes in the sample space.

Student Response

Answers vary. Sample response: Flip a coin, then select between rock, paper, scissors, then select a letter from the word MATH. This has 24 outcomes, since $2 \cdot 3 \cdot 4 = 24$.

Activity Synthesis

The purpose of the discussion is to help students understand the calculations behind the solutions of these problems.

Some questions for discussion:

- “Describe how the tree of sandwich options would look without drawing it out.” (The first column would have the 3 options for bread. Coming out from each of those options would be 4 branches for each of the proteins. From each of these there would be 4 more branches for each of the cheese options. Each of those would have 5 branches for the veggies. Finally, there would be 5 branches for the veggies again since the sandwich has 2 of them.)
- “How is the tree connected to the calculation of the size of the sample space?” (Since the first two choices [bread and protein] are 3 groups of 4 branches, there are 12 options for those two choices. When adding the cheese option, there are 12 groups of 4 or 48 options. For the first veggie, there are 48 groups of 5 things or 240 options. Finally, there are 240 groups of 5 veggies for the last option, giving a total of 1,200 outcomes.)
- “If the two veggie choices had to be different, would there be a higher or lower total number of possible sandwiches? Explain your reasoning.” (Fewer, since an item like “Italian bread, ham, Swiss, onions, and onions” was an option before, but is not an option with the new restriction.)

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion. For each observation or response that is shared, ask students to restate and/or revoice what they heard using precise mathematical language. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. This will provide more students with an opportunity to produce language as they interpret the reasoning of others.

Design Principle(s): Support sense-making

Lesson Synthesis

Consider asking these discussion questions:

- “What are some methods for writing out the sample space of a chance experiment that consists of multiple steps?” (Trees, tables, and lists.)
- “How does the tree method relate to finding the number of outcomes in a sample space?” (Each path from the start to the end of the “branches” represents one outcome in the sample space, so counting all the paths will give you the number of items in the sample space.)
- “Why is it important to know the number of outcomes in a sample space when finding probability?” (Probability can be found by $\frac{k}{n}$ where k represents the number of outcomes in the event and n represents the number of outcomes in the sample space.)

8.4 Random Points

Cool Down: 5 minutes

The cool-down asks students to count the number of outcomes in the sample space for an experiment with multiple parts. Students should be allowed to use any of the methods explored in this lesson to arrive at the answer. The size of the sample space and the number of items in a subset of the sample space will be used to find probabilities for events in subsequent lessons.

Addressing

- 7.SP.C.8.b

Student Task Statement

Andre is reviewing proportional relationships. He wants to practice using a graph that goes through a point so that each coordinate is between 1 and 10.

1. For the point, how many outcomes are in the sample space?
2. For how many outcomes are the x -coordinate and the y -coordinate the same number?

Student Response

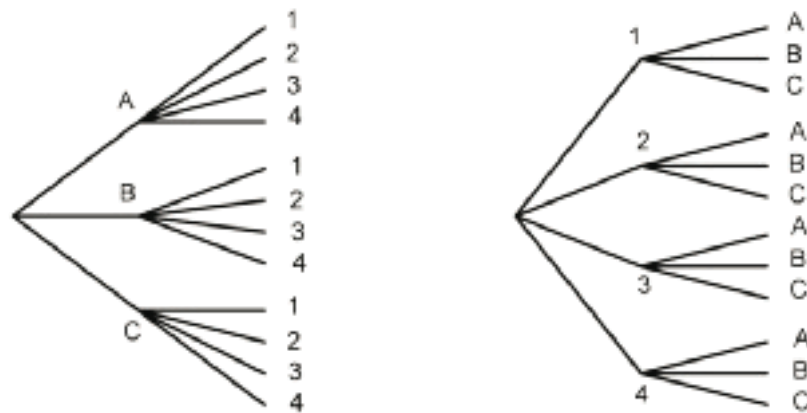
1. There are 100 outcomes in the sample space since $10 \cdot 10 = 100$.
2. 10. (1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (7,7), (8,8), (9,9), (10,10).

Student Lesson Summary

Sometimes we need a systematic way to count the number of outcomes that are possible in a given situation. For example, suppose there are 3 people (A, B, and C) who want to run for the president of a club and 4 different people (1, 2, 3, and 4) who want to run for vice president of the club. We can use a *tree*, a *table*, or an *ordered list* to count how many different combinations are possible for a president to be paired with a vice president.

With a tree, we can start with a branch for each of the people who want to be president. Then for each possible president, we add a branch for each possible vice president, for a total of

$3 \cdot 4 = 12$ possible pairs. We can also start by counting vice presidents first and then adding a branch for each possible president, for a total of $3 \cdot 4 = 12$ possible pairs.



A table can show the same result:

	1	2	3	4
A	A, 1	A, 2	A, 3	A, 4
B	B, 1	B, 2	B, 3	B, 4
C	C, 1	C, 2	C, 3	C, 4

So does this ordered list:

A1, A2, A3, A4, B1, B2, B3, B4, C1, C2, C3, C4

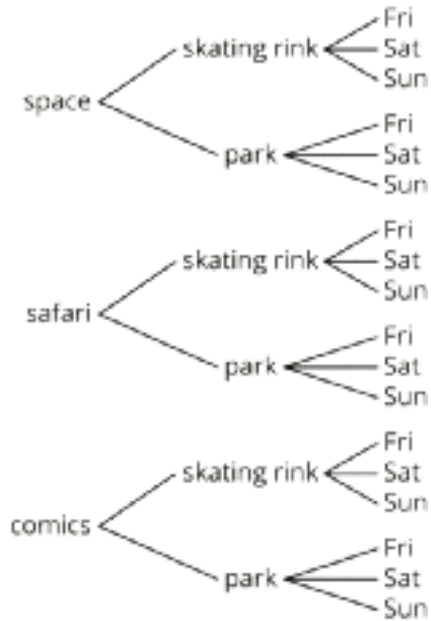
Lesson 8 Practice Problems

Problem 1

Statement

Noah is planning his birthday party. Here is a tree showing all of the possible themes, locations, and days of the week that Noah is considering.

- How many themes is Noah considering?
- How many locations is Noah considering?
- How many days of the week is Noah considering?
- One possibility that Noah is considering is a party with a space theme at the skating rink on Sunday. Write two other possible parties Noah is considering.
- How many different possible outcomes are in the sample space?



Solution

- 3 themes
- 2 locations
- 3 days
- Answers vary. Sample response: Noah is considering a comics themed party at the skating rink on Saturday or a safari themed party at the park on Friday.
- 18 outcomes (The number of outcomes is given by $3 \cdot 2 \cdot 3$ or by counting the branches in the tree diagram.)

Problem 2

Statement

For each event, write the sample space and tell how many outcomes there are.

- Lin selects one type of lettuce and one dressing to make a salad.

Lettuce types: iceberg, romaine

Dressings: ranch, Italian, French

- b. Diego chooses rock, paper, or scissors, and Jada chooses rock, paper, or scissors.
- c. Spin these 3 spinners.



Solution

- a. 6 outcomes: iceberg and Italian, iceberg and ranch, iceberg and French, romaine and Italian, romaine and ranch, romaine and French
- b. 9 outcomes: rr, rp, rs, pr, pp, ps, sr, sp, ss
- c. 12 outcomes: bat, bet, mat, met, ban, ben, man, men, bad, bed, mad, med

Problem 3

Statement

A simulation is done to represent kicking 5 field goals in a single game with a 72% probability of making each one. A 1 represents making the kick and a 0 represents missing the kick.

trial	result
1	10101
2	11010
3	00011
4	11111
5	10011

Based on these results, estimate the probability that 3 or more kicks are made.

Solution

$$\frac{4}{5}$$

(From Unit 8, Lesson 7.)

Problem 4

Statement

There is a bag of 50 marbles.

- Andre takes out a marble, records its color, and puts it back in. In 4 trials, he gets a green marble 1 time.
- Jada takes out a marble, records its color, and puts it back in. In 12 trials, she gets a green marble 5 times.
- Noah takes out a marble, records its color, and puts it back in. In 9 trials, he gets a green marble 3 times.

Estimate the probability of getting a green marble from this bag. Explain your reasoning.

Solution

Answers vary. Sample response: A good estimate of the probability of getting a green marble comes from combining Andre, Jada, and Noah's trials. They took a marble out of the bag a total of 25 times and got a green marble 9 of those times. So, the probability of getting a green marble appears to be close to $\frac{9}{25} = 0.36$. Since there are 50 marbles in the bag, it is a reasonable estimate that 18 of the 50 marbles are green, though this is not guaranteed.

(From Unit 8, Lesson 4.)

Lesson 9: Multi-step Experiments

Goals

- Choose a method for representing the sample space of a compound event, and justify (orally) the choice.
- Use the sample space to determine the probability of a compound event, and explain (orally, in writing, and using other representations) the reasoning.

Learning Targets

- I can use the sample space to calculate the probability of an event in a multi-step experiment.

Lesson Narrative

In this lesson, students continue writing out the sample spaces for chance experiments that have multiple steps and also begin using those sample spaces to calculate the probability of certain events. Students may start listing the sample space using one method and then decide to switch to a different method when they get stuck in the middle of the problem (MP1) or they might recognize certain aspects of the situation that would lead them to choose a particular method from the beginning (also MP1). For instance, a problem that involves two spinners would be easy to represent with a table, but a problem that involves three spinners may be easier to represent with a tree diagram.

Alignments

Building On

- 5.OA.A.1: Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
- 6.EE.A.2.b: Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.

Addressing

- 7.SP.C.8.a: Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
- 7.SP.C.8.b: Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.

Instructional Routines

- MLR2: Collect and Display
- MLR5: Co-Craft Questions

- MLR8: Discussion Supports
- Notice and Wonder
- Poll the Class
- Think Pair Share
- True or False

Student Learning Goals

Let's look at probabilities of experiments that have multiple steps.

9.1 True or False?

Warm Up: 5 minutes

The purpose of this warm-up is to gather strategies and understandings students have for averaging numbers. Understanding these strategies will help students develop fluency and will be useful later in this unit when students will need to be able to compute averages of values.

While 3 problems are given, it may not be possible to share every strategy for all the problems. Consider gathering only 2 or 3 different strategies per problem, saving most of the time for the final question.

Building On

- 5.OA.A.1
- 6.EE.A.2.b

Instructional Routines

- True or False

Launch

Reveal one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all previous problems displayed throughout the talk. Follow with a whole-class discussion.

Student Task Statement

Is each equation true or false? Explain your reasoning.

$$8 = (8 + 8 + 8 + 8) \div 3$$

$$(10 + 10 + 10 + 10 + 10) \div 5 = 10$$

$$(6 + 4 + 6 + 4 + 6 + 4) \div 6 = 5$$

Student Response

Explanations vary. Sample responses:

False, since $(8 + 8 + 8 + 8) = 4 \cdot 8$ and $4 \cdot 8 \div 3$ is not 8.

True, since $(10 + 10 + 10 + 10 + 10) = 5 \cdot 10$ and $5 \cdot 10 \div 5 = 10$.

True, since $(6 + 4 + 6 + 4 + 6 + 4) = (10 + 10 + 10) = 3 \cdot 10$ and $3 \cdot 10 \div 6 = 10 \div 2 = 5$.

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Did anyone have the same strategy but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to ____’s strategy?”
- “Do you agree or disagree? Why?”

9.2 Spinning a Color and Number

10 minutes

In this activity, students are reminded how to calculate probability based on the number of outcomes in the sample space, then apply that to multi-step experiments. The events are described in everyday language, so students need to reason abstractly (MP2) to identify the outcomes described. This lesson begins with students returning to a problem they have previously seen when writing out the sample space. This will save students some time if they can recall or refer back to the initial problem. In the following activities, students will work with situations for which they have not written out the sample space to practice finding probabilities using all the necessary steps.

Addressing

- 7.SP.C.8.a

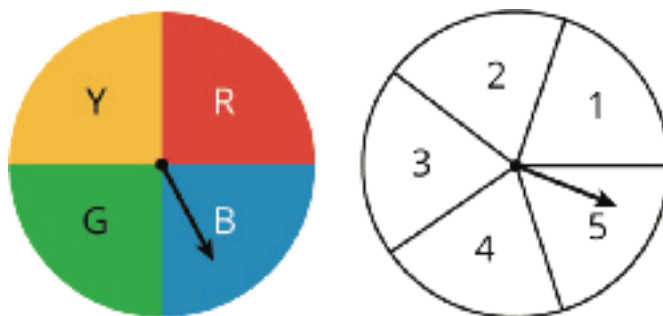
Instructional Routines

- MLR2: Collect and Display
- Notice and Wonder

Launch

Arrange students in groups of 2.

Display the two spinners for all to see. Ask students, “What do you notice? What do you wonder?”



Give students 1 minute to think about the image. Record their responses for all to see.

Students may notice:

- The number of sections in each spinner.
- The labels for the two spinners.
- Within each spinner, the sections are equally sized.

Students may wonder:

- Do you choose which one to spin or do you spin both?
- If you spin both, how many different outcomes will there be?
- Is this part of a game? If so, what is a “good” spin?

Tell students: “For sample spaces where each outcome is equally likely, recall that the probability of an event can be computed by counting the number of outcomes in the event and dividing that number by the total number of outcomes in the sample space.” For example, in the previous lesson, students found that there were 12 possible outcomes when flipping a coin and rolling a number cube. If we wanted the probability of getting heads and rolling an even number, we count that there are 3 ways to do this (H2, H4, and H6) out of the 12 outcomes in the sample space. So, the probability of getting heads and an even number should be $\frac{3}{12}$ or $\frac{1}{4}$ or 0.25.

Remind students that they have already drawn out the sample space for this chance experiment in a previous activity, and they may use that to help answer the questions.

Give students of 5 minutes quiet work time followed by partner and whole-class discussion.

Access for Students with Disabilities

Representation: Access for Perception. Provide access to concrete manipulatives. Provide spinners for students to view or manipulate. These hands-on models will help students identify characteristics or features, and support finding outcomes for calculating probabilities.

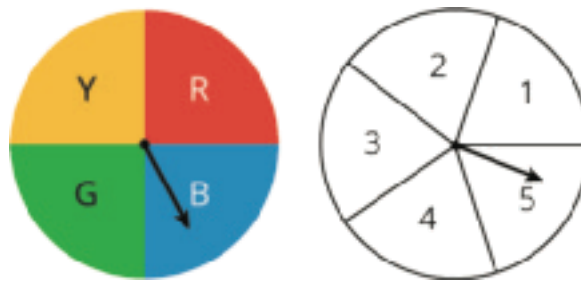
Supports accessibility for: Visual-spatial processing; Conceptual processing

Student Task Statement

The other day, you wrote the sample space for spinning each of these spinners once.

What is the probability of getting:

1. Green and 3?
2. Blue and any odd number?
3. Any color other than red and any number other than 2?



Student Response

1. $\frac{1}{20}$, since there is only 1 outcome in the event and there are 20 equally likely outcomes in the sample space.
2. $\frac{3}{20}$, since there are 3 outcomes that have blue and an odd number.
3. $\frac{12}{20}$, since there are 12 outcomes that have any color besides red and any number besides 2.

Activity Synthesis

The purpose of the discussion is for students to explain their interpretations of the questions and share methods for solving.

Some questions for discussion:

- “How did you calculate the number of outcomes in the sample space?” (Counting the items in the tree, table, or list, or using the multiplication idea from an earlier lesson.)
- “Although we had the sample space for this situation in a previous problem, how could you find the sample space if you did not know it already?” (Draw a tree, table, or list.)
- “For each problem, how many outcomes were in the event that was described? How did you count them?”

Access for English Language Learners

Representing, Speaking, Listening: MLR2 Collect and Display. As pairs discuss strategies for calculating the probability of each outcome, circulate and write down the words and phrases students use to explain their reasoning. Listen for students who reference their representation of the sample space (e.g., list, table, or tree) to determine the probability of each outcome. As students review the language collected in the visual display, encourage students to revise and improve how ideas are communicated. For example, a phrase such as: “The probability is $\frac{1}{20}$, because there are 20 outcomes” can be improved with the phrase “The probability is $\frac{1}{20}$, because there is only 1 outcome in the event, and there are 20 equally likely outcomes in the sample space.” This routine will provide feedback to students in a way that supports sense-making while simultaneously increasing meta-awareness of language.

Design Principle(s): Support sense-making; Maximize meta-awareness

9.3 Cubes and Coins

20 minutes

In this activity, students continue to compute probabilities for multi-step experiments using the number of outcomes in the sample space. The first problem involves a situation for which students have already seen the sample space. Following this problem, the class will discuss the merits of the different representations for writing out the sample space. The next two problems involve situations in which students may need to write out the sample space on their own. Students are also reminded that some events have a probability of 0, which represents an event that is impossible. In the discussion following the activity, students are asked to think about the probabilities of two events that make up the entire sample space and have no outcomes common to both events (MP2).

Addressing

- 7.SP.C.8.a

Instructional Routines

- MLR8: Discussion Supports
- Poll the Class

Launch

Keep students in groups of 2.

Assign each group a representation for writing out the sample space: a tree, a table, or a list. Tell students that they should write out the sample space for the first problem using the representation

they were assigned. (This was done for them in a previous lesson and they are allowed to use those as a guide if they wish.)

Tell students that they should work on the first problem only and then pause for a discussion before proceeding to the next problems.

Give students 2 minutes of partner work time for the first problem followed by a pause for a whole-class discussion centered around the different representations for sample space.

After all groups have completed the first question, select at least one group for each representation and have them explain how they arrived at their answer. As the groups explain, display the appropriate representations for all to see. Ask each of the groups how they counted the number of outcomes in the sample space as well as the number of outcomes in the event using their representation.

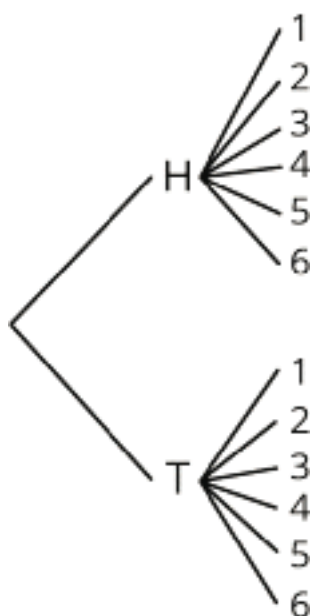
List:

Heads 1, heads 2, heads 3, heads 4, heads 5, heads 6, tails 1, tails 2, tails 3, tails 4, tails 5, tails 6

Table:

1	2	3	4	5	6
H1	H2	H3	H4	H5	H6
T1	T2	T3	T4	T5	T6

Tree:



After students have had a chance to explain how they used the representations, ask students to give some pros and cons for using each of the representations. For example, the list method may be easy to write out and interpret, but could be very long and is not the easiest method for keeping track of which outcomes have been written and which still need to be included.

Allow the groups to continue with the remaining problems, telling them they may use any method they choose to work with the sample space for these problems. Give students 10 minutes of partner work time followed by a whole-class discussion about the activity as a whole.

Anticipated Misconceptions

Some students may not recognize that rolling a 2 then a 3 is different from rolling a 3 then a 2. Ask students to imagine the number cubes are different colors to help see that there are actually 2 different ways to get these results.

Similarly, some students may think that HHT counts the same as HTH and THH. Ask the student to think about the coins being flipped one at a time rather than all tossed at once. Drawing an entire tree and seeing all the branches may further help.

Student Task Statement

The other day you looked at a list, a table, and a tree that showed the sample space for rolling a number cube and flipping a coin.

1. Your teacher will assign you one of these three structures to use to answer these questions. Be prepared to explain your reasoning.

- a. What is the probability of getting tails and a 6?
- b. What is the probability of getting heads and an odd number?

Pause here so your teacher can review your work.

2. Suppose you roll two number cubes. What is the probability of getting:

- a. Both cubes showing the same number?
- b. *Exactly* one cube showing an even number?
- c. *At least* one cube showing an even number?
- d. Two values that have a sum of 8?
- e. Two values that have a sum of 13?

3. Jada flips three quarters. What is the probability that all three will land showing the same side?

Student Response

1. a. $\frac{1}{12}$, since there is only 1 outcome in the sample space that matches the criteria.
b. $\frac{3}{12}$, since there are 3 outcomes in the sample space that have heads and an odd number.
2. a. $\frac{6}{36}$, since there are 6 outcomes where the same number is showing and the sample space contains 36 equally likely outcomes.
b. $\frac{18}{36}$, since there are 18 outcomes where exactly one of the cubes shows an even number.
c. $\frac{27}{36}$, since there are 27 outcomes where at least one of the cubes shows an even number.
d. $\frac{5}{36}$, since there are 8 outcomes where the sum is 8 (2 and 6, 3 and 5, 4 and 4, 5 and 3, 6 and 2).
e. 0, since it is impossible to get two values whose sum is 13.
3. $\frac{2}{8}$, since there are 2 ways to get the coins showing the same side (all heads or all tails) and 8 outcomes in the sample space.

Activity Synthesis

The purpose of the discussion is for students to explain their methods for solving the problems and to discuss how writing out the sample space aided in their solutions.

Poll the class on how they computed the number of outcomes in the sample space and the number of outcomes in the event for the second set of questions given these options: List, Table, Tree, Computed Outcomes Without Writing Them All Out, Another Method.

Consider these questions for discussion:

- “Which representation did you use for each of the problems?”
 - “Do you think you will always try to use the same representation, or can you think of situations when one representation might be better than another?”
- “Did you have a method for finding the number of outcomes in the sample space or event that was more efficient than just counting them?” (The number of outcomes in the sample space for the number cubes could be found using $6 \cdot 6 = 36$. To find the number of outcomes with at least 1 even number, I knew there would be 6 for each time an even was rolled first and only 3 for each time an odd number was rolled first, so I found the number of outcomes by $3 \cdot 6 + 3 \cdot 3 = 27$.)
- “One of the events had a probability of zero. What does this mean?” (It is impossible.)
- “What would be the probability of an event that was certain?” (1)

- “Jada was concerned with having all the coins show the same side. What would be the probability of having at least 1 coin *not* match the others?” ($\frac{6}{8}$, since there are 6 outcomes where at least 1 coin does not match: HHT, TTH, HTH, THT, HTT, THH.)
 - “How do the answers to Jada’s question and the one we just answered relate to one another?” (Since every outcome in the sample space has either “at least one heads” or “all tails,” and there is no outcome that applies to both events, together the sum of their probabilities must be 100% or 1.)

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion. For each response or observation that is shared, ask students to restate and/or revoice what they heard using precise mathematical language. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students’ attention to any words or phrases that helped to clarify the original statement. This will provide more students with an opportunity to produce language as they interpret the reasoning of others.

Design Principle(s): Support sense-making

9.4 Pick a Card

Optional: 15 minutes

The activity provides further practice in finding probabilities of events.

In this activity, students see an experiment that has two steps where the result of the first step influences the possibilities for the second step. Often this process is referred to as doing something “without replacement.” At this stage, students should approach these experiments in a very similar way to all of the other probability questions they have encountered, but they must be very careful about the number of outcomes in the sample space (MP6).

Addressing

- 7.SP.C.8.a
- 7.SP.C.8.b

Instructional Routines

- MLR5: Co-Craft Questions
- Think Pair Share

Launch

Keep students in groups of 2. Give students 5–7 minutes of quiet work time followed by partner and whole-class discussion.

Identify students who are not noticing that it is impossible to draw the same color twice based on the instructions. Refocus these students by asking them to imagine drawing a red card on the first pick and thinking about what's possible to get for the second card.

Access for Students with Disabilities

Representation: Access for Perception. Provide access to concrete manipulatives. Provide five different colored cards for students to view or manipulate. These hands-on models will help students identify characteristics or features, and support finding outcomes for calculating probabilities.

Supports accessibility for: Visual-spatial processing; Conceptual processing

Access for English Language Learners

Conversing: MLR5 Co-Craft Questions. Display the initial task statement that begins, "Imagine there are five cards...", before revealing the questions that follow. Ask pairs to write down possible mathematical questions that can be answered about the situation. As pairs share their questions with the class, listen for and highlight questions that ask how the "without putting the card back" part of the scenario would change the outcome of the sample space. This will help students consider the differences between this problem and previous problems and how that impacts the sample space.

Design Principle(s): Optimize output; Maximize meta-awareness

Anticipated Misconceptions

Students may misread the problem and think that they replace the card before picking the next one. Ask these students to read the problem more carefully and ask the student, "What is possible to get when you draw the second card while you already have a red card in your hand?"

Student Task Statement

Imagine there are 5 cards. They are colored red, yellow, green, white, and black. You mix up the cards and select one of them without looking. Then, without putting that card back, you mix up the remaining cards and select another one.

1. Write the sample space and tell how many possible outcomes there are.
2. What structure did you use to write all of the outcomes (list, table, tree, something else)? Explain why you chose that structure.
3. What is the probability that:
 - a. You get a white card and a red card (in either order)?

- b. You get a black card (either time)?
- c. You do not get a black card (either time)?
- d. You get a blue card?
- e. You get 2 cards of the same color?
- f. You get 2 cards of different colors?

Student Response

1. Sample space: RY, RG, RW, RB, YR, YG, YW, YB, GR, GY, GW, GB, WR, WY, WG, WB, BR, BY, BG, BW. There are 20 different outcomes.
2. Answers vary. Sample response. I used a tree, since it was easier to keep track of how the first card selected would affect what was possible for the second card.
3.
 - a. $\frac{2}{20} = \frac{1}{10}$, because there are 2 outcomes that have those 2 cards (RW and WR), and the outcomes in the sample space are equally likely.
 - b. $\frac{8}{20} = \frac{2}{5}$, because there are 8 outcomes that have a black card (RB, YB, GB, WB, BR, BY, BG, BW).
 - c. $\frac{12}{20} = \frac{3}{5}$, because there are 12 outcomes that do not have a black card.
 - d. 0, because there are no blue cards in the deck.
 - e. 0, because there is only 1 card of each color and you cannot pick the same card twice if you don't put the first one back.
 - f. 1, because all of the possible outcomes have two different colors.

Are You Ready for More?

In a game using five cards numbered 1, 2, 3, 4, and 5, you take two cards and add the values together. If the sum is 8, you win. Would you rather pick a card and put it back before picking the second card, or keep the card in your hand while you pick the second card? Explain your reasoning.

Student Response

You are more likely to win if you put the card back.

If you put it back, you can win with these outcomes: 35, 44, or 53. Since this way has 25 equally likely outcomes in the sample space, the probability of winning is $\frac{3}{25} = 0.12$. If you do not put it back, you can win with these outcomes: 35 or 53. Since this way has 20 equally likely outcomes in the sample space, the probability of winning is $\frac{2}{20} = 0.1$.

Activity Synthesis

The purpose of the discussion is for students to compare the same context with replacement and without replacement.

Consider asking these questions for discussion:

- “What would change about your calculations if the experiment required replacing the first card before picking a second card?” (There would be 25 outcomes in the sample space. The probability of getting the same color twice would be $\frac{5}{25}$. The probability of getting different colors would be $\frac{20}{25}$. The probability of getting red and white would be $\frac{2}{25}$. The probability of getting a black card would be $\frac{9}{25}$ and not getting a black card would be $\frac{16}{25}$. It would still be impossible to get a blue card, so its probability would be 0.)
- “What do you notice about the sum of the probability of getting a black card and the probability of not getting a black card?” (They have a sum of 1.)
 - “Explain why these outcomes might have probabilities with this relationship.” (Since you either get a black card or not, together their probabilities should be 1 or 100%.)

Lesson Synthesis

These discussion questions will help students reflect on their learning:

- “When the outcomes in the sample space are equally likely, how is the size of the sample space used to calculate the probability of an event?”
- “Now that you’ve have plenty of practice, do you have a favorite method for writing out the sample space?”
 - “Are there times that one strategy for writing out the sample space makes more sense than others?”

9.5 A Number Cube and 10 Cards

Cool Down: 5 minutes

Addressing

- 7.SP.C.8.a

Student Task Statement

Lin plays a game that involves a standard number cube and a deck of ten cards numbered 1 through 10. If both the cube and card have the same number, Lin gets another turn. Otherwise, play continues with the next player.

What is the probability that Lin gets another turn?

Student Response

$\frac{6}{60}$ (or equivalent), since there are 6 outcomes for which the numbers match and 60 equally likely outcomes in the sample space ($6 \cdot 10 = 60$).

Student Lesson Summary

Suppose we have two bags. One contains 1 star block and 4 moon blocks. The other contains 3 star blocks and 1 moon block.



If we select one block at random from each, what is the probability that we will get two star blocks or two moon blocks?

To answer this question, we can draw a tree diagram to see all of the possible outcomes.



There are $5 \cdot 4 = 20$ possible outcomes. Of these, 3 of them are both stars, and 4 are both moons. So the probability of getting 2 star blocks or 2 moon blocks is $\frac{7}{20}$.

In general, if all outcomes in an experiment are equally likely, then the probability of an event is the fraction of outcomes in the sample space for which the event occurs.

Lesson 9 Practice Problems

Problem 1

Statement

A vending machine has 5 colors (white, red, green, blue, and yellow) of gumballs and an equal chance of dispensing each. A second machine has 4 different animal-shaped rubber bands (lion, elephant, horse, and alligator) and an equal chance of dispensing each. If you buy one item from each machine, what is the probability of getting a yellow gumball and a lion band?

Solution

$$\frac{1}{20}$$

Problem 2

Statement

The numbers 1 through 10 are put in one bag. The numbers 5 through 14 are put in another bag. When you pick one number from each bag, what is the probability you get the same number?

Solution

$\frac{6}{100}$, since you can get 5s, 6s, 7s, 8s, 9s, or 10s, and there are 100 possible outcomes in the sample space ($10 \cdot 10$).

Problem 3

Statement

When rolling 3 standard number cubes, the probability of getting all three numbers to match is $\frac{6}{216}$. What is the probability that the three numbers *do not* all match? Explain your reasoning.

Solution

$\frac{210}{216}$, since the numbers either all match or do not match, so the rest of the options must be of this kind.

Problem 4

Statement

For each event, write the sample space and tell how many outcomes there are.

- Roll a standard number cube. Then flip a quarter.
- Select a month. Then select 2020 or 2025.

Solution

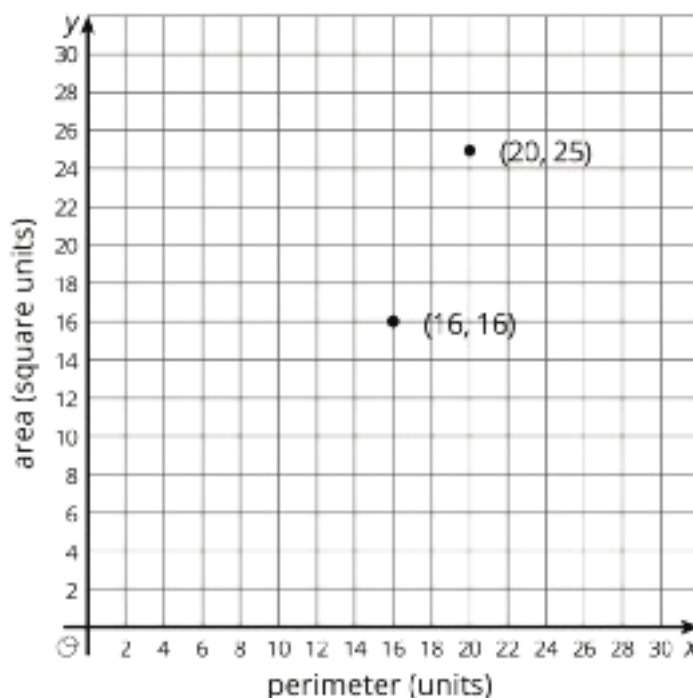
- a. 12 outcomes: 1h, 1t, 2h, 2t, 3h, 3t, 4h, 4t, 5h, 5t, 6h, 6t
- b. 24 outcomes: Jan 2020, Jan 2025, Feb 2020, Feb 2025, Mar 2020, Mar 2025, Apr 2020, Apr 2025, May 2020, May 2025, June 2020, June 2025, July 2020, July 2025, Aug 2020, Aug 2025, Sep 2020, Sep 2025, Oct 2020, Oct 2025, Nov 2020, Nov 2025, Dec 2020, Dec 2025.

(From Unit 8, Lesson 8.)

Problem 5

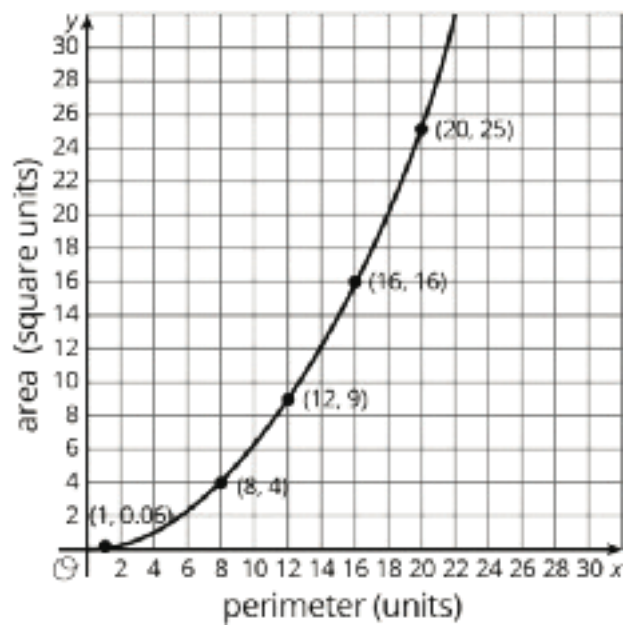
Statement

On a graph of the area of a square vs. its perimeter, a few points are plotted.



- a. Add some more ordered pairs to the graph.
- b. Is there a proportional relationship between the area and perimeter of a square?
Explain how you know.

Solution



a.

b. There is not a proportional relationship between area and perimeter. When graphed, the ordered pairs do not lie on a line that passes through the origin.

(From Unit 2, Lesson 11.)

Lesson 10: Designing Simulations

Goals

- Describe a multi-step experiment that could be used to simulate a compound event in a real-world situation, and justify (orally and in writing) that it represents the situation.
- Perform a simulation to estimate the probability of a compound event, and explain (orally and in writing) how the simulation could be improved.

Learning Targets

- I can design a simulation to estimate the probability of a multi-step real-world situation.

Lesson Narrative

In this lesson, students see that the probability of compound events can also be estimated using simulations. The last activity in this lesson is a culmination of all the work students have done with probability in this unit, as each group works to design a simulation for a different situation. Students strategically choose tools (MP5) like number cubes, spinners, blocks, etc., to represent the chance experiments in the situations they are given. This is also an opportunity for students to practice communicating precisely (MP6) about how their simulation is conducted and what their outcomes represent.

Alignments

Building On

- 5.OA.A.1: Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
- 6.SP.B.5.c: Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

Addressing

- 7.SP.C.8.c: Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Required Materials

Coins

any fair two-sided coin

Compasses

Number cubes

cubes with sides numbered from 1 to 6

Paper bags

Paper clips

Pre-printed slips, cut from copies of the Instructional master

Protractors

Clear protractors with no holes and with radial lines printed on them are recommended.

Scissors

Snap cubes

Straightedges

A rigid edge that can be used for drawing line segments. Sometimes a ruler is okay to use as a straightedge, but sometimes it is preferable to use an unruled straightedge, like a blank index card.

Required Preparation

Every 3 students need 2 coins for the Breeding Mice activity.

Print and cut up questions from the Designing Simulations Instructional master. Use one question for every 3 students. Groups will need access to number cubes, protractors, rulers, compasses, paper clips, bags, snap cubes, and scissors to simulate their scenarios.

Student Learning Goals

Let's simulate some real-life scenarios.

10.1 Number Talk: Division

Warm Up: 5 minutes

The purpose of this number talk is to elicit strategies and understandings students have for division, particularly when the quotient is the same for different expressions. These understandings will be helpful for students as they are finding mean in upcoming lessons.

While four problems are given, it may not be possible to share every strategy. Consider gathering only two or three different strategies per problem.

Building On

- 5.OA.A.1
- 6.SP.B.5.c

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Reveal one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all previous problems displayed throughout the talk. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Student Task Statement

Find the value of each expression mentally.

$$(4.2 + 3) \div 2$$

$$(4.2 + 2.6 + 4) \div 3$$

$$(4.2 + 2.6 + 4 + 3.6) \div 4$$

$$(4.2 + 2.6 + 4 + 3.6 + 3.6) \div 5$$

Student Response

- 3.6; Strategies vary. Possible strategies: $4.2 \div 2 + 3 \div 2$ or $7.2 \div 2$.
- 3.6; Strategies vary. Possible strategies: Since there are three addends with an additional 3.6 in the parentheses and the divisor is one greater than in the previous problem, the result is the same as in the first problem (or $10.8 \div 3$).
- 3.6; Strategies vary. Possible strategies: Since you are adding an additional 3.6 to the addends from the previous problem and there are now four addends with a divisor of 4, the result will be the same as in the previous problem (or $14.4 \div 4$).
- 3.6; Strategies vary. Possible strategies: Since you are adding an additional 3.6 to the addends from the previous problem and there are now five addends with a divisor of 5, the result will be the same as in the previous problem (or $18 \div 5$).

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone have the same strategy but would explain it differently?"
- "What did you notice about the answers to these questions?"

- "How could we use the number we are dividing by each time to explain why the answers are all the same?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to ____'s strategy?"
- "Do you agree or disagree? Why?"

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because . . ." or "I noticed ____ so I . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Design Principle(s): Optimize output (for explanation)

10.2 Breeding Mice

10 minutes (there is a digital version of this activity)

In this activity, students revisit the idea of simulating real-life situations with chance experiments. In the next activity, they will design and run their own simulations for situations that involve multiple steps. Here, students are asked to use their understanding of experiments that have multiple steps to simulate a single part of a larger simulation. Namely, flipping two coins represents a single offspring from a pair of mice. Since the outcome probabilities of the simulation and the real-life situation are the same, this is another option for creating simulations that represent real-life scenarios (MP4).

Addressing

- 7.SP.C.8.c

Launch

Arrange students in groups of 3. Provide 2 coins for each group.

Ask students: "When flipping two coins, what is the probability of both landing heads up?" ($\frac{1}{4}$)

For context, it might be helpful to explain that mice are often used in science experiments since they have similar genetics to humans, but are easier to maintain. Setting up a mating to work with a new generation of mice with specific combinations of genes can be costly and time consuming, so it can help to simulate some outcomes before actually beginning the experiment. The word "offspring" refers to children.

Give students 5–7 minutes for group work followed by a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, pause to check for understanding of determining the number of outcomes after 3–5 minutes of work time.
Supports accessibility for: Organization; Attention

Student Task Statement

A scientist is studying the genes that determine the color of a mouse’s fur. When two mice with brown fur breed, there is a 25% chance that each baby will have white fur. For the experiment to continue, the scientist needs at least 2 out of 5 baby mice to have white fur.

To simulate this situation, you can flip a coin twice for each baby mouse.

- If the coin lands heads up both times, it represents a mouse with white fur.
- Any other result represents a mouse with brown fur.



1. Simulate 3 litters of 5 baby mice and record your results in the table.

	mouse 1	mouse 2	mouse 3	mouse 4	mouse 5	Do at least 2 have white fur?
simulation 1						
simulation 2						
simulation 3						

2. Based on the results from everyone in your group, estimate the probability that the scientist’s experiment will be able to continue.
3. How could you improve your estimate?

Student Response

1. Answers vary.
2. Answers vary. Sample response: I estimate the probability to be $\frac{3}{9}$ since 3 of our 9 simulations had more than 2 white offspring.

3. If we did more trials, I expect the estimate to improve.

Are You Ready for More?

For a certain pair of mice, the genetics show that each offspring has a probability of $\frac{1}{16}$ that they will be albino. Describe a simulation you could use that would estimate the probability that at least 2 of the 5 offspring are albino.

Student Response

Answers vary. Sample response: Flip 4 coins. If they are all heads, that offspring is albino. Do this 5 times to represent the 5 offspring. Repeat this process many times to simulate many groups of 5 offspring and estimate the probability based on the cumulative fraction of groups that have at least 2 albino offspring.

Activity Synthesis

The purpose of the discussion is for students to articulate why the simulation is appropriate and think about other methods of simulating the same situation.

Consider asking these questions for discussion:

- “How could we get a better estimate than what you got in your group?” (Repeat the experiment many more times or combine the data from the class.)
- Collect data from the class to find a better estimate. (For reference, the actual probability is $\frac{47}{128} \approx 0.37$.)
- “Notice that we used a two-part experiment (flipping two coins) to represent a single thing (one offspring). Why was this ok to do?” (The probability of getting HH on two coins is the same as the probability of getting a single offspring with white fur.)
- “Can you think of another method that would work to simulate a single offspring?” (A spinner with 25% of the circle labeled “white” and 75% labeled “brown.” One white block and three brown blocks in a bag.)

10.3 Designing Simulations

20 minutes

In this activity, each group is assigned a situation for which they will design and perform a simulation to estimate the probability. Students will give a short presentation on the methods and results of their simulation for the class after they have designed and run the simulation. Students will need to attend to precision (MP6) as well as present arguments (MP3) for the simulation method they chose. At this stage, students have experienced a large number of simulation methods and should be able to design their own to represent the situations using the appropriate tools (MP5).

Addressing

- 7.SP.C.8.c

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 3. Assign each group a question slip from the Instructional master. Provide access to number cubes, compasses, protractors, rulers, paper bags, colored snap cubes, scissors, and coins. Give students 15 minutes for group work followed by a whole-class discussion.

Student Task Statement

Your teacher will give your group a paper describing a situation.

1. Design a simulation that you could use to estimate a probability. Show your thinking. Organize it so it can be followed by others.
2. Explain how you used the simulation to answer the questions posed in the situation.

Student Response

Answers vary. Some sample responses:

1. Flip 5 coins. Heads represents a girl and tails represents a boy. If the coins land with 4 heads and 1 tail it will match the situation. After doing this 10 times, 1 of them had this result, so the probability should be about 0.1. It is fairly unusual.
2. Spin 3 spinners that each have 75% of the circle marked as “working” and 25% as “not working.” If all three of the spinners land on “not working,” the fire is undetected. After doing this 10 times, this never happened. Since it is possible, the estimate should be between 0 and 0.1. It is not a safety problem for the factory since fires are so rare and when there is a fire, at least one of the detectors will usually work.
3. In a bag, there are 9 slips of paper that say “brakes work” and 1 that says “defective brakes.” Draw one paper out, replace it, and repeat for each of the five cars. If all 5 cars have working brakes, the dealer is ok. After doing this 10 times, this happened 6 times. The dealership should be concerned since having defective brakes is a big problem and the probability is only about 0.6 that none of the cars have this problem.
4. In a bag there are 6 slips of paper that say “eagle” and 4 that say “no eagle.” Draw one paper out, replace it, and repeat for each of the five days. If at least 2 days have an eagle, we will get to name one. After doing this 10 times, this happened 7 times. There is a probability of about 0.7 that the class will get to name an eagle.
5. In a bag there are 2 slips of paper that say “stay overnight” and 8 that say “release.” Draw one paper out, replace it, and repeat for each of the five animals. If at least 3 of the animals have to stay overnight, the hospital will have a problem. After doing this 10 times, this happened

once. There is a probability of about 0.1 that the hospital will not have enough space for the animals.

Activity Synthesis

Ask each group to share their situation, their method of simulating the situation, and their results. Students should explain why their chosen method works to simulate the situation they were given. In particular, all important outcomes should be represented with the same probability as stated in the situation.

If all groups that have the same situation use the same simulation method, ask for ideas from the class about alternate methods that could be used for the situation.

For reference, the computed probabilities for each situation are:

1. $\frac{5}{32} \approx 0.16$
2. $\frac{1}{64} \approx 0.02$
3. $\frac{59049}{100000} \approx 0.59$
4. $\frac{2072}{3125} \approx 0.66$
5. $\frac{181}{3125} \approx 0.06$

Representation: Internalize Comprehension. Use color and annotations to illustrate student thinking. As students show their representations of simulations and explain their reasoning, use color and annotations to scribe their thinking on a display of each problem so that it is visible for all students.

Supports accessibility for: Visual-spatial processing; Conceptual processing

Access for English Language Learners

Listening, Speaking: MLR8 Discussion Supports. To help students explain why their chosen method works, provide sentence frames such as: "This simulation was designed so that . . ." and "The simulation and the actual event have the same probabilities because . . ." As partners are sharing, encourage the listener to press for more explanation by asking: "Can you explain why the probabilities of the simulation and actual event match?" or "Is there another simulation that could provide the same probability?" This will help students practice justifying their simulation from their interpretation of the situation.

Design Principle(s): Maximize meta-awareness; Cultivate conversation

Lesson Synthesis

Consider these questions for discussion:

- “What are some things you had to consider when designing your simulation?” (Among other things, the probability of the actual portion of the event should match the probability of the associated simulated event.)
- “What did you learn from the simulations the other groups did?”
- “Were the results of any of the simulations surprising?”
- “Why would it make sense to design and run a simulation rather than repeat the actual experiment multiple times?” (When the actual experiment is costly in time or resources or cannot be controlled or repeated.)

10.4 The Best Power-Up

Cool Down: 5 minutes

Many modern games incorporate random numbers to simulate parts of the game. In this cool-down, students are asked to use the results from a computerized random number generator to simulate a particular result.

Addressing

- 7.SP.C.8.c

Student Task Statement

Elena is programming a video game. She needs to simulate the power-up that the player gets when they reach a certain level. The computer can run a program to return a random integer between 1 and 100. Elena wants the best power-up to be rewarded 15% of the time.

Explain how Elena could use the computer to simulate the player getting the best power-up at least 2 out of 3 times.

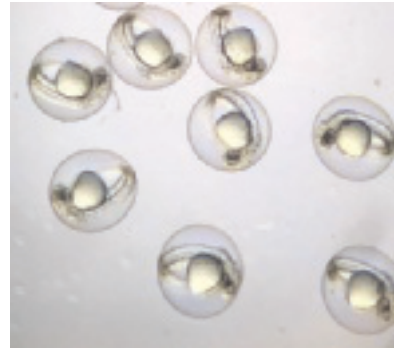
Student Response

Answers vary. Sample response: Elena could have the computer generate 3 random integers between 1 and 100. If at least 2 of the numbers are between 1 and 15, then the player got the best power-up at least twice. She could repeat this process many times and estimate the probability as the proportion of trials for which at least 2 of the numbers are between 1 and 15.

Student Lesson Summary

Many real-world situations are difficult to repeat enough times to get an estimate for a probability. If we can find probabilities for parts of the situation, we may be able to simulate the situation using a process that is easier to repeat.

For example, if we know that each egg of a fish in a science experiment has a 13% chance of having a mutation, how many eggs do we need to collect to make sure we have 10 mutated eggs? If getting these eggs is difficult or expensive, it might be helpful to have an idea about how many eggs we need before trying to collect them.



We could simulate this situation by having a computer select random numbers between 1 and 100. If the number is between 1 and 13, it counts as a mutated egg. Any other number would represent a normal egg. This matches the 13% chance of each fish egg having a mutation.

We could continue asking the computer for random numbers until we get 10 numbers that are between 1 and 13. How many times we asked the computer for a random number would give us an estimate of the number of fish eggs we would need to collect.

To improve the estimate, this entire process should be repeated many times. Because computers can perform simulations quickly, we could simulate the situation 1,000 times or more.

Lesson 10 Practice Problems

Problem 1

Statement

A rare and delicate plant will only produce flowers from 10% of the seeds planted. To see if it is worth planting 5 seeds to see any flowers, the situation is going to be simulated. Which of these options is the best simulation? For the others, explain why it is not a good simulation.

- a. Another plant can be genetically modified to produce flowers 10% of the time. Plant 30 groups of 5 seeds each and wait 6 months for the plants to grow and count the fraction of groups that produce flowers.
- b. Roll a standard number cube 5 times. Each time a 6 appears, it represents a plant producing flowers. Repeat this process 30 times and count the fraction of times at least one number 6 appears.
- c. Have a computer produce 5 random digits (0 through 9). If a 9 appears in the list of digits, it represents a plant producing flowers. Repeat this process 300 times and count the fraction of times at least one number 9 appears.
- d. Create a spinner with 10 equal sections and mark one of them “flowers.” Spin the spinner 5 times to represent the 5 seeds. Repeat this process 30 times and count the fraction of times that at least 1 “flower” was spun.

Solution

Using the computer is the best simulation. Using another plant will probably be costly and take a long time, so it is not a good simulation. Rolling the standard number cube does not match the probability for each seed, so it will not produce a good simulation. The spinner idea would work as a simulation, but it had only 30 trials instead of 300 for the computer.

Problem 2

Statement

Jada and Elena learned that 8% of students have asthma. They want to know the probability that in a team of 4 students, at least one of them has asthma. To simulate this, they put 25 slips of paper in a bag. Two of the slips say “asthma.” Next, they take four papers out of the bag and record whether at least one of them says “asthma.” They repeat this process 15 times.

- Jada says they could improve the accuracy of their simulation by using 100 slips of paper and marking 8 of them.
- Elena says they could improve the accuracy of their simulation by conducting 30 trials instead of 15.

- Do you agree with either of them? Explain your reasoning.
- Describe another method of simulating the same scenario.

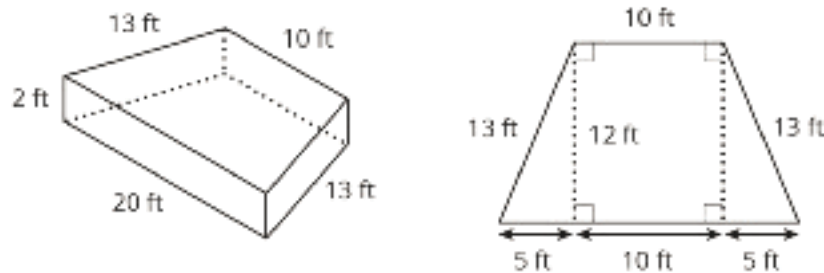
Solution

- I agree with Elena, but not with Jada. Jada's suggestion would have the same probability of a success as the original simulation, so it would work, but would not produce more accuracy. More trials help keep uncommon outcomes from having a big impact on the estimated probability.
- Answers vary. Sample response: Use a random digit list. For each trial, take 4 pairs of digits (00 through 99). Repeat the simulation many times. Use the proportion of times the pairs 01 through 08 appeared in the outcomes to estimate the probability that at least one student on the team has asthma.

Problem 3

Statement

The figure on the left is a trapezoidal prism. The figure on the right represents its base. Find the volume of this prism.



Solution

360 ft^3 . The area of the trapezoidal base is 180 ft^2 , the height is 2 ft, and $180 \cdot 2 = 360$.

(From Unit 7, Lesson 13.)

Problem 4

Statement

Match each expression in the first list with an equivalent expression from the second list.

A. $(8x + 6y) + (2x + 4y)$

B. $(8x + 6y) - (2x + 4y)$

C. $(8x + 6y) - (2x - 4y)$

D. $8x - 6y - 2x + 4y$

E. $8x - 6y + 2x - 4y$

F. $8x - (-6y - 2x + 4y)$

1. $10(x + y)$

2. $10(x - y)$

3. $6(x - \frac{1}{3}y)$

4. $8x + 6y + 2x - 4y$

5. $8x + 6y - 2x + 4y$

6. $8x - 2x + 6y - 4y$

Solution

◦ A: 1

◦ B: 6

◦ C: 5

◦ D: 3

◦ E: 2

◦ F: 4

(From Unit 6, Lesson 22.)

Section: Sampling

Lesson 11: Comparing Groups

Goals

- Calculate the mean and mean absolute deviation for a data set, and interpret (orally) these measures.
- Compare and contrast (orally and in writing) populations represented on dot plots in terms of their shape, center, spread, and visual overlap.
- Justify (in writing) whether two populations are “very different” based on the difference in their means expressed as a multiple of the mean absolute deviation.

Learning Targets

- I can calculate the difference between two means as a multiple of the mean absolute deviation.
- When looking at a pair of dot plots, I can determine whether the distributions are very different or have a lot of overlap.

Lesson Narrative

In this lesson, students review measures of center and variability from grade 6. They also work at deciding whether or not two distributions are very different from each other (MP3). This lesson introduces the idea of expressing the difference between the centers of two distributions as a multiple of a measure of variability as a way to help students make this determination (MP2). For the problems in this lesson, the populations under study are small and the data for the entire populations are known. In future lessons, students will revisit this calculation as a way to decide whether there is a meaningful difference between two populations given data from only a sample of each population.

Alignments

Addressing

- 7.SP.B: Draw informal comparative inferences about two populations.
- 7.SP.B.3: Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- MLR2: Collect and Display
- MLR8: Discussion Supports
- Notice and Wonder

Student Learning Goals

Let's compare two groups.

11.1 Notice and Wonder: Comparing Heights

Warm Up: 5 minutes

The purpose of this warm-up is to collect ideas that will be useful in the discussions in this lesson. While students may notice and wonder many things about these images, the methods of showing that the volleyball team is much taller than the gymnastic team are the important discussion points.

Addressing

- 7.SP.B

Instructional Routines

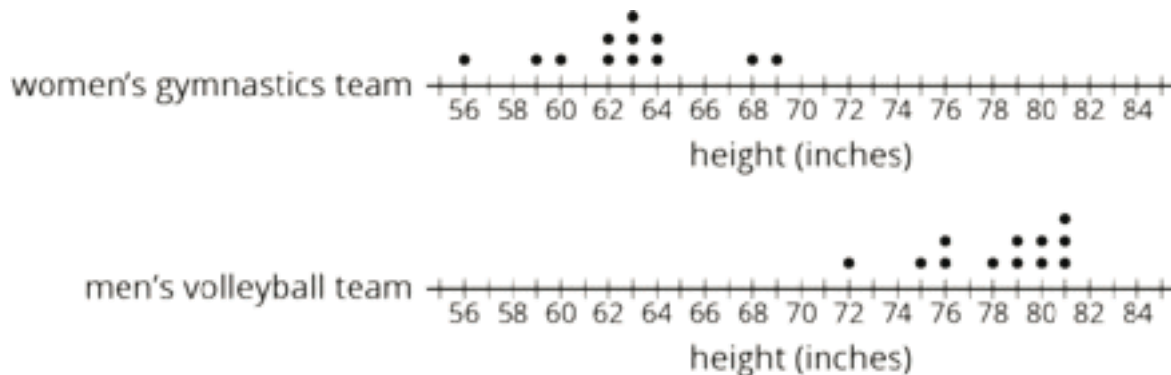
- Notice and Wonder

Launch

Arrange students in groups of 2. Tell students that they will look at the dot plots, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the dot plots for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Student Task Statement

What do you notice? What do you wonder?



Student Response

Things students may notice:

- The volleyball team is much taller than the gymnastics team.
- The gymnastics team's heights are all under 70 inches tall.
- The volleyball team's heights are all over 70 inches tall.

Things students may wonder:

- How much taller are the volleyball players?
- Are players on volleyball teams usually that much taller than gymnasts or is this just for these two teams?
- What are the heights of the female volleyball players?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After each response, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to the images each time. If the definitive difference in height does not come up during the conversation, ask students to discuss this idea.

The next activity looks more closely at comparing these data sets. It is not necessary to have students calculate anything (mean, median, MAD, IQR) yet.

11.2 More Team Heights

15 minutes

In this activity, students are asked to compare the heights of two groups of people. The wording of the questions allows for multiple interpretations and any reasonable answer should be accepted (MP1). This activity also provides an opportunity to remind students of how to analyze dot plots as well as how to calculate the measures of center and variability of the data.

Addressing

- 7.SP.B.3

Instructional Routines

- MLR8: Discussion Supports

Launch

Keep students in groups of 2.

Display the dot plots from the warm-up activity and help students see that the data sets given in their books or devices match the numbers shown in the dot plots.

For the problem addressing the tennis and badminton teams, you may suggest each student create a dot plot of one of the groups and then compare with their partner.

Allow students 10 minutes of partner work time followed by a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts. After students have solved the first problem, check-in with either select groups of students or the whole class. Invite students to share the strategies they have used so far as well as any questions they have before continuing.

Supports accessibility for: Organization; Attention

Student Task Statement

1. How much taller is the volleyball team than the gymnastics team?
 - Gymnastics team's heights (in inches) : 56, 59, 60, 62, 62, 63, 63, 63, 64, 64, 68, 69
 - Volleyball team's heights (in inches): 72, 75, 76, 76, 78, 79, 79, 80, 80, 81, 81, 81
2. Make dot plots to compare the heights of the tennis and badminton teams.
 - Tennis team's heights (in inches): 66, 67, 69, 70, 71, 73, 73, 74, 75, 75, 76
 - Badminton team's heights (in inches): 62, 62, 65, 66, 68, 71, 73

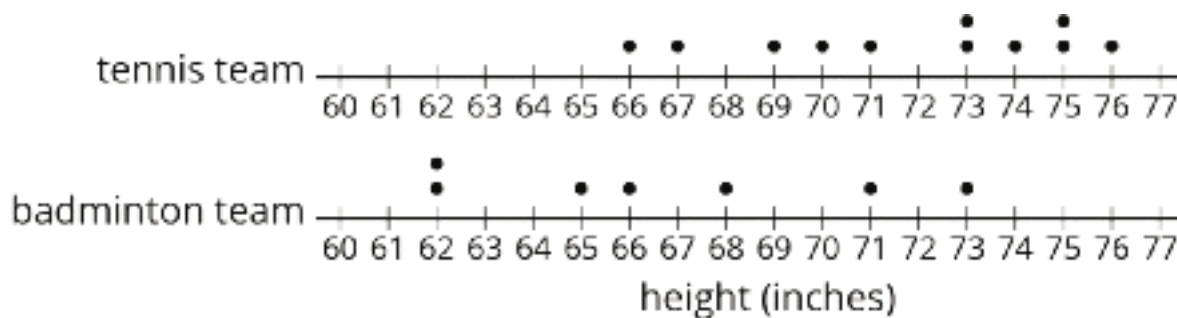
What do you notice about your dot plots?

3. Elena says the members of the tennis team were taller than the badminton team. Lin disagrees. Do you agree with either of them? Explain or show your reasoning.

Student Response

1. Answers vary. Sample responses:
 - The total height of the volleyball team is 185 inches, or about 15 feet 5 inches, greater than the total height of the gymnastics team, because I added up everyone on each team and $938 - 753 = 185$.
 - The mean height of the volleyball team is about 15.4 inches greater than the mean height of the gymnastics team, because $753 \div 12 = 62\frac{3}{4}$, $938 \div 12 = 78\frac{1}{6}$, and $78\frac{1}{6} - 62\frac{3}{4} = 15\frac{5}{12}$, or about 15.4 inches.
 - The median height of the volleyball team is 16 inches greater than the median height of the gymnastics team, and this may be more important than the difference in the means because the shape of the distribution for the volleyball team is not symmetric.

- The tallest person on the volleyball team is 12 inches taller than the tallest person on the gymnastics team.
 - The shortest person on the volleyball team is 16 inches taller than the shortest person on the gymnastics team.
 - The tallest person on the volleyball team is 25 inches taller than the shortest person on the gymnastics team.
 - The shortest person on the volleyball team is 3 inches taller than the tallest person on the gymnastics team.
2. The center of the distribution as well as the minimum and maximum heights for the tennis team are all greater than the center, minimum, and maximum for the badminton team; however, there is a lot of overlap between the two distributions. Unlike with the gymnastics and volleyball teams, the tallest person on the badminton team is taller than the shortest person on the tennis team.



- The total height of the tennis team is 322 inches, or 26 feet 10 inches, greater than the total height of the badminton team; however, it does not make sense to compare the totals, because the tennis team had more people.
- The mean height of the tennis team is about 5 inches greater than the mean height of the badminton team.
- The median height of the tennis team is 7 inches greater than the median height of the badminton team.
- The tallest person on the tennis team is only 3 inches taller than the tallest person on the badminton team.
- The shortest person on the tennis team is 4 inches taller than the shortest person on the badminton team.
- The tallest person on the tennis team is 14 inches taller than the shortest person on the badminton team.
- The shortest person on the tennis team is actually 7 inches shorter than the tallest person on the badminton team, because there is so much overlap between the two distributions.

3. Answers vary. Sample responses:

- I agree with Elena because the center of the distribution for the tennis team is greater than the center for the badminton team.
- I agree with Lin because the two distributions overlap so much. Some of the people on the badminton team were taller than some of the people on the tennis team. The difference in centers is not big enough to matter. If you mixed the people from both teams together into one group, you would not be able to determine who was a member of each team.

Activity Synthesis

The purpose of this discussion is for students to think about ways we could approach comparing two groups as well as have an opportunity to review dot plots, measures of center, and measures of variation from prior grades.

Ask, “What are some ways we can compare groups of things?” At this stage, students are only expected to informally compare the groups. Although a consistent “general rule” for comparing groups will be introduced in later lessons, this activity is about getting a general idea that some groups (like the gymnastics and volleyball teams) have a rather clear difference while others (like the tennis and badminton teams) may be more alike.

Ask students about the *distribution* of the data shown in the dot plots. Make sure to highlight the shape, center, and spread. Review how to find the **mean** as a measure of the center of a data set. Review how to calculate the **mean absolute deviation (MAD)** as a measure of the variability of a data set. Students may mention **median** and **interquartile range (IQR)** as other ways to measure center and variability. Although median and IQR are not needed in this activity, it may be useful to review how to calculate those values as well. Both measures of center and variability will be used later in the unit.

Introduce the idea of judging how much two data sets overlap.

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion. For each observation that is shared, ask students to restate what they heard using precise mathematical language. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students’ attention to any words or phrases that helped clarify the original statement. This will provide more students with an opportunity to produce language as they interpret the reasoning of others.

Design Principle(s): Support sense-making

11.3 Family Heights

15 minutes

Following the review from the previous activity, students are asked to use these calculations to compare two groups more formally. Students are shown one quantifiable method of determining whether the two groups are relatively close or relatively very different in the discussion following the activity involving describing the difference of the measures of center as a multiple of the variability (MP2). The important idea for students to grasp from this activity is that the measures of center and measures of variability of the groups work together to give an idea of how similar or different the groups are.

As students work to compare heights of the two families, monitor for students who:

1. Create dot plots of the data and look at the overlap visually.
2. Compute measures of center to compare the groups numerically.
3. Also compute measures of variability to compare the measures of center for each group accounting for the spread of values.

Addressing

- 7.SP.B.3

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display

Launch

Keep students in groups of 2.

Introduce students to the idea that both the measures of center as well as the measures of variability are important when comparing data sets by asking students about these contexts.

- “Two groups of adults have mean weights that are different by 10 pounds. Are the two groups very different in weight?”
- “Two groups of 8 year olds have weights that are different by 10 pounds. Are the two groups very different in weight?”
- “Two groups of birds have weights that are different by 3.2 ounces (0.2 pounds). Are the two groups very different in weight?”

Without knowing more about the exact numbers, it seems that the adults might not be that different. A typical adult might weigh 180 pounds and, depending on diet and activity, even one person’s weight might go up or down by 10 pounds in a few weeks. It might be hard to tell these two groups apart even with the 10 pound difference in means.

On the other hand, 8 year olds typically weigh about 50 pounds, so a 10 pound difference can mean a lot more. A group with an average weight of 50 pounds is 25% heavier than a group with an average weight of 40 pounds.

For the birds, it might be hard to say how different the groups are unless we have more information. If the two groups contain the same type of bird (e.g., parrots), we might expect the weights to be fairly consistent, so a 3.2 ounce difference could be a lot. If the two groups each contain a variety of types (from hummingbirds up to emus), the 3.2 ounce difference might not be very much.

Based on these examples, it helps to know not only the means (or medians), but also the variability of the groups. We saw this in the previous activity with the overlap in the dot plots.

For the work in this activity, tell students to try to give some values to help back up their comparison of the two groups.

Allow students 1 minute of quiet think time to examine ways of approaching the problem, followed by partner work time and a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge of measures of centers. Allow students to use calculators to ensure inclusive participation in the activity.

Supports accessibility for: Memory; Conceptual processing

Access for English Language Learners

Conversing: MLR2 Collect and Display. Use this routine to capture the language students use as they compare the heights of these two families. Circulate and listen to student talk during small-group and whole-class discussion. Record the words and phrases students use on a display for all to see. Invite students to borrow from—or add more language to—the display throughout the remainder of the lesson. This will help students read and use mathematical language during their paired and whole-group discussions.

Design Principle(s): Optimize output (for comparison); Maximize meta-awareness

Student Task Statement

Compare the heights of these two families. Explain or show your reasoning.

- The heights (in inches) of Noah's family members: 28, 39, 41, 52, 63, 66, 71
- The heights (in inches) of Jada's family members: 49, 60, 68, 70, 71, 73, 77

Student Response

The mean height of Noah's family members is about 51.43 inches. The mean height of Jada's family members is about 66.86 inches, which is greater than for Noah's family. The difference between these means is $66.86 - 51.43$, or 15.4 inches.

There is also some overlap in the heights of the two families. The tallest person in Noah's family is 22 inches taller than the shortest person in Jada's family.

Are You Ready for More?

If Jada's family adopts newborn twins who are each 18 inches tall, does this change your thinking? Explain your reasoning.

Student Response

Answers vary. Sample responses:

- Yes. After the addition of the new babies, the mean for Jada's family drops to 56 inches, which is much closer to the mean height of Noah's family (about 51 inches).
- No. With the addition of the new babies, the mean does not make sense to use as a measure of center for Jada's family anymore; we should use the median. The median of Noah's family is 52 inches, and the new median for Jada's family is 68 inches, and there is still a large difference between the two groups.

Activity Synthesis

The purpose of this discussion is for students to begin to be more formal in their comparison of data from different groups. In particular, a general rule is established that will be used in this unit.

Select students to share their approaches in the sequence outlined in the Activity Narrative.

The heights of Noah's and Jada's families overlap more than the heights of the gymnastics and volleyball teams. The difference in means for the two families is about the same as between the two teams.

- The difference between the mean heights of the volleyball and gymnastics teams is $78.17 - 62.75$, or 15.42 inches.
- The difference between the mean heights of Jada's and Noah's families is $66.86 - 51.43$, or 15.43 inches. The variability in heights for the families is greater than the variability in heights for the teams.

Explain: The difference between the means is not enough information to know whether or not the data sets are very different. One way to express the amount of overlap is to divide the difference in means by the (larger) mean absolute deviation.

Demonstrate for students how to do this calculation for the volleyball and gymnastics teams:

- The difference in means is more than 6 times the measure of variability, because $15.42 \div 2.46 \approx 6.3$.

Leave the calculation for the two teams displayed. Ask students to do the calculation for Jada's and Noah's families.

- The difference in means is a little more than 1 time the measure of variability, because $15.43 \div 13.22 \approx 1.2$.

As a general rule, we will consider it a large difference between the data sets if the difference in means is *more than twice* the mean absolute deviation. If the mean absolute deviation is different for each group, use the larger one for this calculation.

For students who ask why twice the MAD is used rather than some other value, defer the question for later in the unit. A later lesson titled *Do They Carry More?* will address this question in more detail.

11.4 Track Length

Optional: 10 minutes

In this activity, students continue to review the meaning of mean and mean absolute deviation as well as practice using the method they were shown in the previous activity to compare multiple groups based on their means and measure of variability. Students begin by matching information about a set of data to its dot plot and calculating the mean and mean absolute deviation for the remaining dot plots, then they compare the data sets pairwise using the mean and MAD values (MP3). In the discussion, students are introduced to a way to add extra information to dot plots to visualize the general rule given in the previous activity.

Addressing

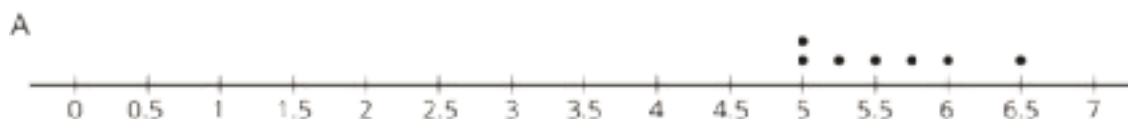
- 7.SP.B.3

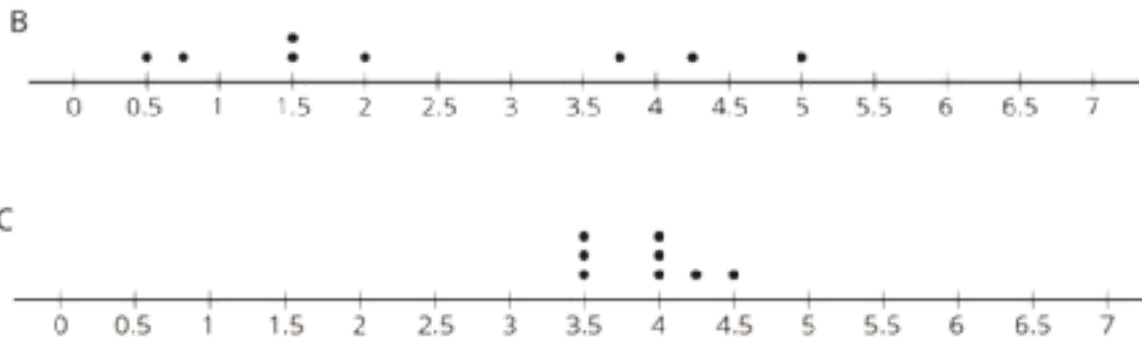
Launch

Keep students in groups of 2. Give students 5 minutes of partner work time followed by a whole-class discussion.

Student Task Statement

Here are three dot plots that represent the lengths, in minutes, of songs on different albums.





- One of these data sets has a mean of 5.57 minutes and another has a mean of 3.91 minutes.
 - Which dot plot shows each of these data sets?
 - Calculate the mean for the data set on the other dot plot.
- One of these data sets has a mean absolute deviation of 0.30 and another has a MAD of 0.44.
 - Which dot plot shows each of these data sets?
 - Calculate the MAD for the other data set.
- Do you think the three groups are very different or not? Be prepared to explain your reasoning.
- A fourth album has a mean length of 8 minutes with a mean absolute deviation of 1.2. Is this data set very different from each of the others?

Student Response

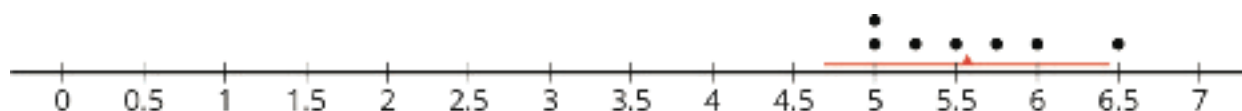
- Dot plot A has a mean of 5.57 minutes, and dot plot C has a mean of 3.91 minutes.
 - Dot plot B has a mean of 2.41 minutes.
- Dot plot C has a MAD of 0.30 minutes, and dot plot A has a MAD of 0.44 minutes.
 - Dot plot B has a MAD of 1.44 minutes.
- Since dot plot B has such a large mean absolute deviation, it is hard to say that it is very different from dot plot C, but it is very different from dot plot A. Since the MADs are much smaller for the other two albums, it is easier to say that the length of the tracks from dot plot C and dot plot A are very different.
- The large mean for the fourth album makes it very different from the others even with a large MAD.

Activity Synthesis

The purpose of the discussion is to help students visualize the calculations they performed.

Ask, “Before calculating any of the values, would you have guessed that dot plots B and C would not be very different, but A would be very different from the other two? Explain your reasoning.” (Yes, since there is some overlap between the data of dot plots B and C, but the data in A does not overlap very much with B and not at all with C.)

Demonstrate a technique for students to see the the general rule on the dot plots using dot plot A. Mark the mean of the data set on the dot plot with a triangle, then draw a line segment from the triangle so that its length is 2 MADs in each direction. On dot plot A, draw a triangle at 5.57 and a line segment from 4.69 ($5.57 - 2 \cdot 0.44$) to 6.45 ($5.57 + 2 \cdot 0.44$) as in the image here.



Tell students to draw the triangle for the mean and segment representing 2 MADs in each direction on all three dot plots. Ask students how we might see the general rule from these pictures. (If any of the means (triangles) are within 2 MADs (the line segment) of the mean of another group, then there is not a large difference between the two groups.) Ask them to compare their visual to their answers from the activity.

Lesson Synthesis

Consider asking these questions for discussion:

- “What does a dot plot tell you?”
- “What are some measures of center, and how are they calculated?”
- “Why is a measure of center useful for comparing two groups?”
- “Why is a measure of variability also needed when comparing two groups?”
- “What is the general rule we will use to determine whether two groups have a large difference or not?”

11.5 Prices of Homes

Cool Down: 5 minutes

Addressing

- 7.SP.B.3

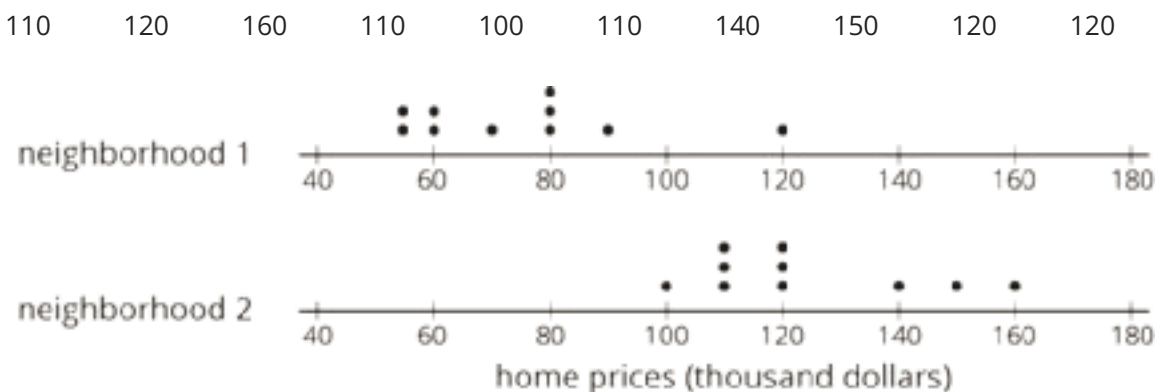
Student Task Statement

Noah's parents are interested in moving to another part of town. They look up all the prices of the homes for sale and record them in thousands of dollars.

neighborhood 1

80 55 80 120 60 90 60 80 55 70

neighborhood 2



Find the mean and MAD for each of the neighborhoods. Then decide whether the two groups are very different or not.

Student Response

Neighborhood 1: Mean = 75. MAD = 15.

Neighborhood 2: Mean = 124. MAD = 15.6.

They are very different since the difference in means is 49, which is more than 3 times the larger MAD.

Student Lesson Summary

Comparing two individuals is fairly straightforward. The question "Which dog is taller?" can be answered by measuring the heights of two dogs and comparing them directly. Comparing two groups can be more challenging. What does it mean for the basketball team to generally be taller than the soccer team?

To compare two groups, we use the distribution of values for the two groups. Most importantly, a measure of center (usually **mean** or **median**) and its associated measure of variability (usually **mean absolute deviation** or *interquartile range*) can help determine the differences between groups.

For example, if the average height of pugs in a dog show is 11 inches, and the average height of the beagles in the dog show is 15 inches, it seems that the beagles are generally taller. On the other hand, if the MAD is 3 inches, it would not be unreasonable to find a beagle that is 11 inches tall or a pug that is 14 inches tall. Therefore the heights of the two dog breeds may not be very different from one another.

Glossary

- mean
- mean absolute deviation (MAD)



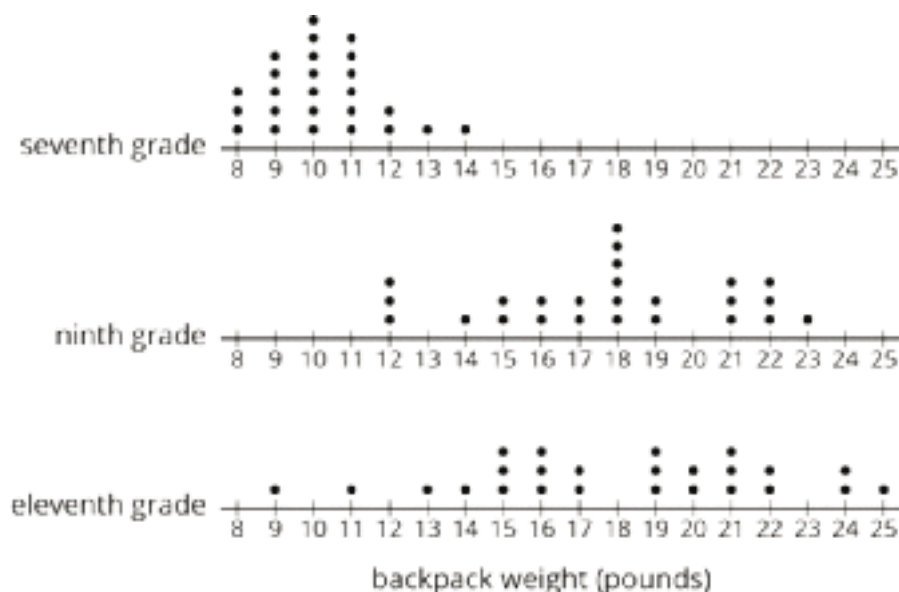
- median

Lesson 11 Practice Problems

Problem 1

Statement

Compare the weights of the backpacks for the students in these three classes.



Solution

Answers vary. Sample response:

The backpacks of the seventh graders tend to weigh less than the backpacks of the ninth graders. A typical weight for the seventh graders' packs is about 10 pounds, compared to a typical weight of about 18 pounds for the ninth graders' packs. The weights were also less variable for the seventh graders than the ninth graders. Similar things can be said when comparing seventh and eleventh graders' backpacks.

The distribution of weights for the ninth graders and the eleventh graders are similar, but the eleventh graders had a slightly larger spread. Both distributions are centered at around 18 pounds, and backpack weights varied quite a bit from student to student in both of these grades.

Problem 2

Statement

A bookstore has marked down the price for all the books in a certain series by 15%.

- How much is the discount on a book that normally costs \$18.00?
- After the discount, how much would the book cost?

Solution

a. \$2.70, because $18 \cdot 0.15 = 2.7$.

b. \$15.30, because $18 - 2.7 = 15.3$.

(From Unit 4, Lesson 11.)

Problem 3

Statement

Match each expression in the first list with an equivalent expression from the second list.

A. $6(x + 2y) - 2(y - 2x)$

1. $10(x - y)$

B. $2.5(2x + 4y) - 5(4y - x)$

2. $10(x + y)$

C. $4(5x - 3y) - 10x + 6y$

3. $10x + 6y$

D. $5.5(x + y) - 2(x + y) + 6.5(x + y)$

4. $10x - 6y$

E. $7.9(5x + 3y) - 4.2(5x + 3y) - 1.7(5x + 3y)$

Solution

- A: 2
- B: 1
- C: 4
- D: 2
- E: 3

(From Unit 6, Lesson 22.)

Problem 4

Statement

Angles C and D are complementary. The ratio of the measure of Angle C to the measure of Angle D is $2 : 3$. Find the measure of each angle. Explain or show your reasoning.

Solution

Angle C : 36° , Angle D : 54° . The two angle measures must add to 90° and since $\frac{2}{5}$ of 90° comes from angle C , it must have measure 36° . Then Angle D comes from $90 - 36 = 54$.

(From Unit 7, Lesson 2.)

Lesson 12: Larger Populations

Goals

- Comprehend the terms “population” and “sample” (in spoken and written language) to refer to the whole group and a part of the group under consideration.
- Describe (orally and in writing) a sample for a given population.
- Explain (orally) that a sample may be used when it is unreasonable to gather data about an entire population.

Learning Targets

- I can explain why it may be useful to gather data on a sample of a population.
- When I read or hear a statistical question, I can name the population of interest and give an example of a sample for that population.

Lesson Narrative

This lesson introduces the idea of using data from a *sample* of a *population* when it is impractical or impossible to gather data from every individual in the populations under study. Students consider whether the people in their class would be an adequate sample for several different questions and associated populations (MP3). While all of the answers given in this lesson are samples and may have some benefit to them, most of them are not the best way to select samples. In the next lesson, students learn about what makes some samples more representative of a population than others. In later lessons, students explore the best ways to try to obtain such samples.

Alignments

Addressing

- 7.SP.A.1: Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
- 7.SP.B: Draw informal comparative inferences about two populations.

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports
- Think Pair Share

Required Preparation

Compute the mean and MAD for the length of the preferred names (if students do not go by their first name, use their nickname, middle name, etc.). Do the same for the last names of students in the class prior to the John Jacob Jingleheimerschmidt activity.

Student Learning Goals

Let's compare larger groups.

12.1 First Name versus Last Name

Warm Up: 5 minutes

The purpose of this warm-up is for students to begin to see the need for samples of data when the population is too large. In this activity, students are asked to think about a question involving all the students at their school and compare the question to an earlier lesson in which the population was small and it was easy to obtain data for the entire population.

Addressing

- 7.SP.B

Launch

Give students 2 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

Consider the question: In general, do the students at this school have more letters in their first name or last name? How many more letters?

1. What are some ways you might get some data to answer the question?
2. The other day, we compared the heights of people on different teams and the lengths of songs on different albums. What makes this question about first and last names harder to answer than those questions?

Student Response

Answers vary. Sample responses:

1. Get a list of everyone in the school, count how many letters there are in each person's first and last names, and calculate the mean of each data set. Then subtract the means and divide this difference by the mean absolute deviation, to measure how much the data sets overlap. Alternatively, total all the letters for the first names and all the letters for the last names in the school and compare these sums. Another idea would be to survey some of the students in the school to use their data to make an informed guess about the whole school.
2. There are a lot more students in the school than there were people on the teams or songs on the albums that we looked at, so it would be a lot more work to calculate the means and mean absolute deviations for everyone's names.

Activity Synthesis

The purpose of the discussion is to highlight the methods of getting data for the school more than it is the method of computing the answer.

Select some students to share their responses.

Students who have elected to sum all the letters in the first names in the school and all the letters in the last names in the school may note that it is a simple comparison to tell whether there are more in first or last names, since you get one single large number for each group. (Comparing data sets.)

Students who have elected to calculate the mean for each group and use MAD as a method of comparison may note that while the calculations may take more time, they give you more precise information, such as knowing about how long first names and last names are, as well as a way to compare the two sets. (Using the general rule from the previous lesson.)

Students who suggest surveying a small group of students may point out that it would be easier to do the calculation with a smaller group. The information would not be as accurate, but it would take a lot less time and might give a good general idea. It would depend on how accurate you needed your answer to be. (Introduction to sampling.)

12.2 John Jacob Jingleheimerschmidt

10 minutes

In this activity, students are asked to compare two groups (length of preferred names and last names) by collecting data from the class. They are asked if the data from the class gives enough information to draw a conclusion about a larger group (MP3). In the following activities, students will be introduced to the idea of sampling. This activity gives students the first chance to experience why sampling might be needed.

Addressing

- 7.SP.A.1
- 7.SP.B

Instructional Routines

- MLR8: Discussion Supports

Launch

Compute the mean and MAD for the number of letters in each student's preferred name (if students do not go by their first name, you may use their nickname, middle name, etc.). Do the same for their last names.

Give students 1 minute of quiet work time for the first 2 questions followed by a quick display of information then 5 more minutes of quiet work time and a whole-class discussion.

If a digital solution is available, input the data for the class to find the mean and mean absolute deviation for each data set. If a digital solution is not available, this information should be calculated based on the class roster prior to this activity. After students have had a minute to work on answering the first two questions, provide students with the mean and MAD for the names in the class.

Tell students that if they have a preferred name other than their official first name (nickname, middle name, etc.) they may use this in place of the first name.

Student Task Statement

Continue to consider the question from the warm-up: In general, do the students at this school have more letters in their first name or last name? How many more letters?

1. How many letters are in your first name? In your last name?
2. Do the number of letters in your own first and last names give you enough information to make conclusions about students' names in your entire school? Explain your reasoning.
3. Your teacher will provide you with data from the class. Record the mean number of letters as well as the mean absolute deviation for each data set.
 - a. The first names of the students in your class.
 - b. The last names of the students in your class.
4. Which mean is larger? By how much? What does this difference tell you about the situation?
5. Do the mean numbers of letters in the first and last names for everyone in your class give you enough information to make conclusions about students' names in your entire school? Explain your reasoning.

Student Response

Answers vary. Sample response:

1. First name: 5. Last name: 8.
2. No, looking at just one person's name does not give enough information to answer about everyone in the school, because some students have longer or shorter names than others.
3.
 - a. Mean: 6.2. MAD: 2.1
 - b. Mean: 7.3. MAD: 2.8
4. The mean number of letters in our last names was larger, by 1.1 letters, which is about 0.4 times the MAD. This means that the last names are longer, but not by a lot. Take the difference between the means, or $7.3 - 6.2 = 1.1$. Since the MAD measures the variability of the data set, this difference divided by the MAD gives a comparison. $1.1 \div 2.8 \approx 0.4$, which is a small

number. This small number means that although the last names had a higher mean number of letters, the two groups were not very distinct. Also, their dot plots would have a lot of overlap.

5. Maybe. There are still a lot of students that we did not count.

Activity Synthesis

The purpose of the discussion is for students to see how the data they have might relate to a larger group. In particular, that a sample might give some estimate of a larger population, but the estimate should not be assumed to be exact.

Consider asking these questions for discussion:

- “Do you expect the mean length of first names for the school to be exactly the same as the mean length for the class?” (Probably not exactly the same. It may be close, though.)
- “Do you expect the mean length of first names for the school to be much larger or smaller or about the same as the mean length for the class? Explain your reasoning.” (Unless there are a few outliers in the class, it should be fairly close to the mean from the class.)

Access for English Language Learners

Speaking, Listening, Conversing: MLR8 Discussion Supports. Use this routine to support whole-class discussion. Display the sentence frames: “The mean length of first names for the school will not be exactly the same as the mean length for the class because _____. ” and “The mean length of first names for the school should be larger/smaller/about the same as the mean length for the class because _____. ” As students share their responses, press for details by asking, “Can you use an example from your name and our class data?” and “Is your answer the same for other classes and schools?” This will support rich and inclusive discussion about how the data from the sample might relate to a larger group.

Design Principle(s): Support sense-making; Cultivate conversation

12.3 Siblings and Pets

10 minutes

In this activity, students think a little more deeply about the data we would like to know and how that compares to the data we can collect easily and quickly (MP1). They are presented with a statistical question that does not have an obvious answer. Students are then asked to consider ways they might begin gathering data to answer the question, but are asked to realize that the data they could reasonably collect is not everyone addressed by the question. Following the activity, the discussion defines the terms *population* and *sample*.

Addressing

- 7.SP.A.1

Instructional Routines

- MLR2: Collect and Display

Launch

Arrange students in groups of 2.

Set up the context by asking students, “Do people who are the only child have more pets?” then to provide a possible explanation for their answer. For example, maybe only children do have more pets because the family can better afford to take care of an animal with only 1 child. Maybe they do not because smaller families may live in smaller places and not have room for a lot of pets.

Give students 5 minutes of partner work time followed by 5 minutes of whole-class discussion.

Student Task Statement

Consider the question: Do people who are the only child *have more pets*?

1. Earlier, we used information about the people in your class to answer a question about the entire school. Would surveying only the people in your class give you enough information to answer this new question? Explain your reasoning.
2. If you had to have an answer to this question by the end of class today, how would you gather data to answer the question?
3. If you could come back tomorrow with your answer to this question, how would you gather data to answer the question?
4. If someone else in the class came back tomorrow with an answer that was different than yours, what would that mean? How would you determine which answer was better?

Student Response

Answers vary. Sample responses:

1. There is not enough variation in just the class to figure out the answers for all people who are the only child compared with all people who have siblings. For example, everyone in the class is about 12 or 13 years old, so we can't tell anything about adults or people from other generations or other parts of the world.
2. I could ask my classmates if they are an only child and gather data about their pets.
3. I could also ask my neighbors if they are an only child and gather data about their pets.
4. We might get different answers because we collected data from different people. Whoever asked more people or a wider range of people would have stronger evidence.

Activity Synthesis

The purpose of the discussion is to show the difference between the data we would like to have to answer the question and the data we have available.

Some questions for discussion:

- "If we had all the time and money in the world and wanted to answer this question, who would we need to collect data from?" (Everyone in the world.)
- "What would you do with the data collected from everyone to answer the questions?" (Find the mean and MAD of the data from the two sets and compare them like we did in previous lessons.)
- "Why is it unreasonable to actually collect all the necessary data to answer the question?" (There are too many people to collect data from. There is not enough time to get to everyone in the world, and I cannot travel everywhere.)
- "Since it may be difficult to guess an answer without doing *any* research, but we cannot get all of the data we want, what data could you get that would help estimate an answer?" (It would be good to ask a few people in different parts of the world and try to get different groups represented.)

Define population and sample. A **population** is the entire pool from which data is taken. Examples include (depending on the question) "all humans in the world," "all 7th graders at our school," or "oak trees in North America." In this usage, it does not have to refer only to groups of people or animals. A **sample** is the part of the population from which data is actually collected. Examples (related to the population examples) include "5 people from each country," "the first 30 seventh graders to arrive at our school," or "8 oak trees from the forest near our school."

Ask students, "What is the population for the question about only children and their pets?" (Everyone in the world.) Note that we would need data from everyone, including those who don't have pets or do have siblings.

Ask students, "What might be a sample we could use to answer the question?" (The students in our class, my neighbors, a few people from different countries.) After getting several responses, ask, "What might be the benefits and drawbacks of each of these samples?" (Some may be more convenient, but would not represent the population as well or vice-versa.)

Explain: While it is best to have data for the entire population, there are many reasons to use a sample.

- More manageable. With very large populations, the amount of data can be hard to collect and work with, so a smaller subset may still be informative and easier to work with. Example: Find the average size of a grain of sand.
- Necessary. Sometimes it is impossible to reach the entire population, so a sample is all that is available. Example: Find the average lifespan of tuna.

- **Speed.** Sometimes a rough estimate is all that is needed and a sample of data is enough to estimate the population characteristic. Example: Out of curiosity, what is the median number of apps on smartphones.
- **Cost.** Sometimes it is very costly to obtain the data needed, so a sample can reduce the cost. Example: Find the average amount of hydrogen in moon rocks.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of: populations and sample.

Supports accessibility for: Conceptual processing; Language

Access for English Language Learners

Representing, Speaking: MLR2 Collect and Display. To help students make sense of the terms “sample” and “population”, draw a diagram of a few circles inside a larger circle on a visual display. Label the large outer circle “population” and the small inner circles “sample.” As students respond to the question “What is the population for the question about only children and their pets?”, write the population on the visual display. As students respond to the question “What might be a sample we could use to answer the question?”, write the samples in different inner circles on the visual display. Listen for and amplify words and phrases that define these terms, such as “part of” or “entire.” This will help students visualize a sample as part of a population and understand that there are multiple samples inside a population.

Design Principle(s): Support sense-making (for representation); Maximize meta-awareness

12.4 Sampling the Population

10 minutes

This activity gives students the opportunity to practice the new vocabulary of population and sample by identifying the population from a set of questions and describing a possible sample that could be used to get some information to begin answering the question. Since these words have a very specific meaning in the context of statistics that is different from the colloquial use of the words, it is important for students to work with the vocabulary in specific situations to understand their meaning (MP6).

Addressing

- 7.SP.A.1

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Allow students 3 minutes of quiet work time followed by 3 minutes of partner discussion then a whole-class discussion.

While in partner discussion, suggest students compare their answers and discuss any advantages or disadvantages for the samples they proposed.

Student Task Statement

For each question, identify the **population** and a possible **sample**.

1. What is the mean number of pages for novels that were on the best seller list in the 1990s?
2. What fraction of new cars sold between August 2010 and October 2016 were built in the United States?
3. What is the median income for teachers in North America?
4. What is the average lifespan of Tasmanian devils?

Student Response

Answers for the sample vary. Sample responses:

1. Population: All New York Times Best Seller books from the 1990s. Sample: 20 of the books on the list that I could find in our library.
2. Population: All cars sold between August 2010 and October 2016. Sample: 100 cars sold during those dates.
3. Population: All teachers in North America. Sample: The teachers at our school.
4. Population: All Tasmanian devils that ever lived. Sample: The Tasmanian devils kept at our local zoo.

Are You Ready for More?

Political parties often use samples to poll people about important issues. One common method is to call people and ask their opinions. In most places, though, they are not allowed to call cell phones. Explain how this restriction might lead to inaccurate samples of the population.

Student Response

Answers vary. Sample response: Some people, especially younger people, may only have cell phones, so they will not be included in the sample. This may lead to more information being

gathered from older people than younger people, and the information may not accurately represent everyone.

Activity Synthesis

The purpose of the discussion is to further solidify the meaning of the terms population and sample for students.

Consider asking these questions for discussion:

- “For each question, could there be another population than the one you gave?” (No. The population refers to *all* of the individuals that pertain to the question.)
- “For each question, could there be another sample than the one you gave?” (Yes. A sample refers to a few of the individuals from whom data will be collected and does not specify the number or how the individuals are selected.)
- “What are some of the advantages and disadvantages you determined for the samples you chose?” (Some are easy to work with, but might miss large sections of the population.)
- “What is a question you could ask for which the *population* would be all of the books in your house?” (For example, “What is the average number of pages in books in my house?”)
- “What is a question you could ask for which the *sample* could be all of the books in your house?” (For example, “What is the average number of pages in all the books ever written?”)

Explain that a well-phrased question should only have 1 population (a question that is not well-phrased should be reconsidered so that the purpose of the question is clear), but there are usually many ways to find samples within that population. In future lessons, we will explore some important aspects to consider while selecting a sample.

Lesson Synthesis

Consider asking these questions to reinforce the ideas from this lesson:

- “When the groups become too large, how can we obtain some data to begin answering a question about the group?”
- “What are some drawbacks of using samples instead of the entire population?” (The value for the measure of center will not be exact and some variability may be lost. Some groups may not have been included in the sample, so their input is lost.)
- “What are some reasons samples are necessary?” (More manageable, impossible to reach the entire population, speed, cost.)
- “Someone wants to know what breed of dog is most popular as a pet in the state. What is a sample that could be used?” (A few dog owners from each of the major cities in the state and a few dog owners from the rural areas.)

- “The principal of a school has access to the grades for students at the school. If we use these grades as a sample, what is a population that the data could be applied to?” (The entire school district, the state, the United States, or all students around the world.)

12.5 How Many Games?

Cool Down: 5 minutes

The cool-down checks whether students understand the meanings of the terms population and sample as well as their use in context. Additionally, students are asked for at least one reason why a sample might make sense to use rather than the entire population.

Addressing

- 7.SP.A.1

Student Task Statement

Lin wants to know how many games teenagers in the United States have on their phones.

1. What is the population for Lin's question?
2. Explain why collecting data for this population would be difficult.
3. Give an example of a sample Lin could use to help answer her question.

Student Response

1. All people who are 13 to 19 years old in the United States who have a phone.
2. There are too many people to collect data from everyone. It would take too much time, energy, and money to collect the data.
3. Answers vary. Sample response: Ask 20 teens at Lin's school how many games they have on their phones.

Student Lesson Summary

A **population** is a set of people or things that we want to study. Here are some examples of populations:

- All people in the world
- All seventh graders at a school
- All apples grown in the U.S.

A **sample** is a subset of a population. Here are some examples of samples from the listed populations:

- The leaders of each country
- The seventh graders who are in band
- The apples in the school cafeteria

When we want to know more about a population but it is not feasible to collect data from everyone in the population, we often collect data from a sample. In the lessons that follow, we will learn more about how to pick a sample that can help answer questions about the entire population.



Glossary

- population
- sample

Lesson 12 Practice Problems

Problem 1

Statement

Suppose you are interested in learning about how much time seventh grade students at your school spend outdoors on a typical school day.

Select **all** the samples that are a part of the population you are interested in.

- A. The 20 students in a seventh grade math class.
- B. The first 20 students to arrive at school on a particular day.
- C. The seventh grade students participating in a science fair put on by the four middle schools in a school district.
- D. The 10 seventh graders on the school soccer team.
- E. The students on the school debate team.

Solution

["A", "D"]

Problem 2

Statement

For each sample given, list two possible populations they could belong to.

- a. Sample: The prices for apples at two stores near your house.
- b. Sample: The days of the week the students in your math class ordered food during the past week.
- c. Sample: The daily high temperatures for the capital cities of all 50 U.S. states over the past year.

Solution

Answers vary. Sample responses:

- a. Population 1: Prices for apples at all stores in our state. Population 2: Prices for all fruit at these two stores.
- b. Population 1: The days of the week the students in your math class ordered food all year.
Population 2: The days of the week everyone in our city ordered food during the past week.

- c. Population 1: The daily high temperatures for the world over the past year. Population 2: The daily high temperatures for the capital cities of all 50 U.S. states over the past 10 years.

Problem 3

Statement

If 6 coins are flipped, find the probability that there is at least 1 heads.

Solution

$\frac{63}{64}$, since there are 64 outcomes in the sample space ($2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 64$) and there is only 1 way to not get any heads (TTTTTT), so there are 63 ways that at least 1 heads shows up.

(From Unit 8, Lesson 9.)

Problem 4

Statement

A school's art club holds a bake sale on Fridays to raise money for art supplies. Here are the number of cookies they sold each week in the fall and in the spring:

fall	20	26	25	24	29	20	19	19	24	24
spring	19	27	29	21	25	22	26	21	25	25

- Find the mean number of cookies sold in the fall and in the spring.
- The MAD for the fall data is 2.8 cookies. The MAD for the spring data is 2.6 cookies. Express the difference in means as a multiple of the larger MAD.
- Based on this data, do you think that sales were generally higher in the spring than in the fall?

Solution

- 23 in the fall, 24 in the spring
- 0.36 MADs since $1 \div 2.8 \approx 0.36$
- The mean of sales is higher for the spring, but the difference in means is not very big considering the variability in the data.

(From Unit 8, Lesson 11.)

Problem 5

Statement

A school is selling candles for a fundraiser. They keep 40% of the total sales as their commission, and they pay the rest to the candle company.

price of candle	number of candles sold
small candle: \$11	68
medium candle: \$18	45
large candle: \$25	21

How much money must the school pay to the candle company?

Solution

\$1,249.80, because the school sold \$2,083 worth of candles ($2,083 = 68 \cdot 11 + 45 \cdot 18 + 21 \cdot 25$) and 60% is paid to the company ($(0.6) \cdot 2,083 = 1,249.80$).

(From Unit 4, Lesson 11.)

Lesson 13: What Makes a Good Sample?

Goals

- Calculate the mean or median of various samples, and compare them with the mean or median of the population.
- Comprehend that the term “representative” (in spoken and written language) refers to a sample with a distribution that closely resembles the population’s shape, center, and spread.
- Given dot plots, determine whether a sample is representative of the population, and explain (orally and in writing) the reasoning.

Learning Targets

- I can determine whether a sample is representative of a population by considering the shape, center, and spread of each of them.
- I know that some samples may represent the population better than others.
- I remember that when a distribution is not symmetric, the median is a better estimate of a typical value than the mean.

Lesson Narrative

In this lesson, students examine multiple samples of the same population and learn what it means for a sample to be **representative** of the population. Students look at the structure of dot plots, attending to center, shape, and spread, to help them compare the samples and the population (MP7). Although the previous lesson pointed out the usefulness of using samples when working with large populations, the problems in this lesson use smaller populations so that students can compare each sample against the entire population.

Alignments

Building On

- 5.NBT.A.2: Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

Addressing

- 7.SP.A: Use random sampling to draw inferences about a population.
- 7.SP.A.1: Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
- 7.SP.A.2: Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the

same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

Instructional Routines

- MLR2: Collect and Display
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Notice and Wonder
- Number Talk
- Think Pair Share

Required Materials

Four-function calculators

Student Learning Goals

Let's see what makes a good sample.

13.1 Number Talk: Division by Powers of 10

Warm Up: 5 minutes

The purpose of this number talk is to gather strategies and understandings students have for dividing by powers of 10. These understandings help students develop fluency and will be helpful later in this lesson when students will need to be able to find the mean for various samples.

While four problems are given, it may not be possible to share every strategy. Consider gathering only two or three different strategies per problem, saving most of the time for the final question.

Building On

- 5.NBT.A.2

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Reveal one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all previous problems displayed throughout the talk. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Student Task Statement

Find the value of each quotient mentally.

$$34,000 \div 10$$

$$340 \div 100$$

$$34 \div 10$$

$$3.4 \div 100$$

Student Response

- 3,400 Possible Strategies: Rewriting as a fraction and reducing.
- 3.40 Possible Strategies: Long division.
- 3.4 Possible Strategies: Regrouping.
- 0.034 Possible Strategies: Moving the decimal an appropriate number of places to the left.

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone have the same strategy but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to ____'s strategy?"
- "Do you agree or disagree? Why?"

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because . . ." or "I noticed ____ so I" Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Design Principle(s): Optimize output (for explanation)

13.2 Selling Paintings

15 minutes

In this activity, students begin to see numerical evidence that different samples can produce different results and thus different estimates for population characteristics (MP2). Students look at a small population and some different collections of samples from this population. Although the data for this population is small enough that it is not necessary to use a sample, it is helpful to get an idea of how data from a sample compares to the population data.

Addressing

- 7.SP.A.2

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. In each group, one student should be assigned to work with mean as their measure of center and the other should work with median as their measure of center.

Tell students that, often in this unit, the data sets are small enough that sampling is not necessary, but it will be easier to work with small data sets so that we may compare information from the sample to the same information from the population.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge of calculating measures of center. Allow students to use calculators to ensure inclusive participation in the activity.

Supports accessibility for: Memory; Conceptual processing

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion. For each response to the discussion questions, ask students to restate and/or revoice what they heard using precise mathematical language. Ask the original speaker whether their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped clarify the original statement. This will provide more students with an opportunity to produce language as they interpret the reasoning of others.

Design Principle(s): Support sense-making

Student Task Statement

Your teacher will assign you to work with either means or medians.

1. A young artist has sold 10 paintings. Calculate the measure of center you were assigned for each of these samples:
 - a. The first two paintings she sold were for \$50 and \$350.
 - b. At a gallery show, she sold three paintings for \$250, \$400, and \$1,200.
 - c. Her oil paintings have sold for \$410, \$400, and \$375.

2. Here are the selling prices for all 10 of her paintings:

\$50 \$200 \$250 \$275 \$280 \$350 \$375 \$400 \$410 \$1,200

Calculate the measure of center you were assigned for all of the selling prices.

3. Compare your answers with your partner. Were the measures of center for any of the samples close to the same measure of center for the population?

Student Response

1.
 - a. Mean: \$200. Median: \$200.
 - b. Mean: \$616.67. Median: \$400.
 - c. Mean: \$395. Median: \$400.
2. Mean: \$379. Median: \$315.
3. Answers vary. Sample response: The mean oil paintings were close, but not exact. The other means were not very close. The sample medians were not very close for any of the samples.

Activity Synthesis

The purpose of this discussion is to show that different samples can result in different estimates for a population characteristic as well as a reminder of reasons we might choose one measure of center over another.

Some questions for discussion:

- “What is the population for this situation?” (All of the paintings sold.)
- “What are the samples used in the calculations?” (The first two paintings sold, those sold at a gallery show, and the oil paintings.)
- “Why did the different samples have different means?” (Because they used different paintings.)
- “Why were the means for the first two paintings sold and those sold at the gallery show so far off from the mean of all the paintings?” (Because they contained the cheapest one and most expensive one, respectively, with only a few other numbers to balance it out.)
- “Based on the numbers in the population, does it make more sense to use median or mean?” (Median since the \$1,200 painting is much greater than the rest of the values, so the measure of center is affected much more by the one painting when using mean.)

13.3 Sampling the Fish Market

15 minutes (there is a digital version of this activity)

In this activity, students begin to see that some samples represent the population better than others. Students compare the dot plot of a population of data with the dot plots of several samples and discuss some aspects that would make some samples better than others (MP7). In the discussion, the phrase *representative sample* is defined.

Addressing

- 7.SP.A.1
- 7.SP.A.2

Instructional Routines

- MLR2: Collect and Display
- Notice and Wonder
- Think Pair Share

Launch

Arrange students in groups of 2. Allow students 3 minutes of quiet work time followed by a partner discussion and whole-class discussion.

Student Task Statement

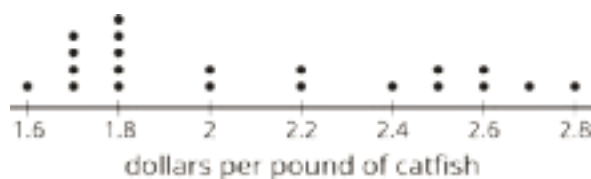
The price per pound of catfish at a fish market was recorded for 100 weeks.

1. Here are dot plots showing the population and three different samples from that population. What do you notice? What do you wonder?
2. If the goal is to have the sample represent the population, which of the samples would work best? Which wouldn't work so well? Explain your reasoning.

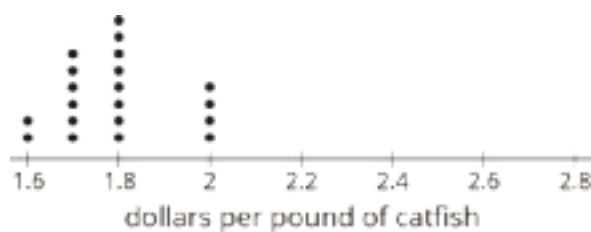
Population



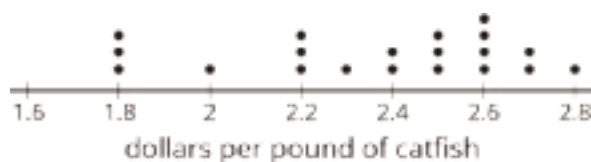
Sample 1



Sample 2



Sample 3



Student Response

1. Answers vary. Sample response: I notice that the samples each have 20 values. I notice that some of the samples look more like the population than the others. I wonder how they got these samples. I wonder if the mean for sample 1 is close to the mean for the population.

2. Answers vary. Sample response: Sample 1 represents the population fairly well since it is spread out about the same amount and has more dots in similar places as the population. Sample 3 does not represent the population very well, since most of the dots are on the right side of the graph while the population seems to have most of the dots on the left.

Are You Ready for More?

When doing a statistical study, it is important to keep the goal of the study in mind. Representative samples give us the best information about the distribution of the population as a whole, but sometimes a representative sample won't work for the goal of a study!

For example, suppose you want to study how discrimination affects people in your town. Surveying a representative sample of people in your town would give information about how the population generally feels, but might miss some smaller groups. Describe a way you might choose a sample of people to address this question.

Student Response

Answers vary. Sample response: Get a list of the different groups in the town who might experience discrimination. Select multiple people from each group to be a part of the sample.

Activity Synthesis

Ask several groups to share things they noticed and wondered about the dot plots. Record responses for all to see. If possible, display the dot plots to refer to while students share.

Consider asking these discussion questions:

- “What are some aspects that make for a good sample? Bad?” (A sample is “good” if it has a similar distribution to the population data. A sample is “bad” if the data does not have a similar distribution to the population data. For example, Sample 2 is bad because it is not centered in the same place.)
- “If you were to find a measure of center to represent a typical value for the population, would you use mean or median?” (Median since the data is not approximately symmetric.)
- “The population in this example has a mean of \$2.06 and a median of \$1.95. Sample 1 has a mean of \$2.09 and median of \$2. Sample 2 has a mean of \$1.79 and a median of \$1.80. Sample 3 has a mean of \$2.36 and a median of \$2.45. Based on this information, which seems to represent the population the best?” (Sample 1.)

Define representative sample. A **representative** sample is a sample that has a distribution that closely resembles the population distribution in center, shape, and spread.

Explain that a sample with the same mean as the population is not necessarily representative, since it may miss other important aspects of the population.

- Example 1: If the population for a question is all of the humans in the world and you use one person from each country as your sample, it may not actually be representative of the population. Larger countries, such as China are under-represented since there are actually

many Chinese people, but only 1 is included in our sample. Similarly, a smaller country like Cuba might be over-represented since it has fewer people living there, but is represented in the sample exactly the same as all of the other larger countries.

- Example 2: The average height of men in the world is approximately 70 inches. You might find two men, one who is 95 inches (7 feet 11 inches) tall and one who is 45 inches (3 feet 9 inches) tall. Their mean height may be the same as the world's, but these two certainly do not represent the heights of most men.

Explain that a representative sample is the ideal type of sample we would like to collect, but if we do not know the data for the population, it will be hard to know if a sample we collect is representative or not. If we do know the population data, then a sample is probably unnecessary. In future lessons, we will explore methods of collecting samples that are more likely to produce representative samples (although they are still not guaranteed).

Access for English Language Learners

Representing, Speaking, Listening: MLR2 Collect and Display. Create a table with column headings “good sample” and “bad sample”. As students share aspects that make for a good or bad sample, write down the words and phrases students use in the appropriate column. Listen for and amplify words that compare features of the samples such as “similar or different center,” “shape,” or “spread.” Use the words and phrases that describe a good sample to define “representative sample.” This will help students use and connect mathematical language that makes a sample representative of the population.

Design Principle(s): Support sense-making; Maximize meta-awareness

13.4 Auditing Sales

Optional: 10 minutes

This activity is additional practice for students to understand the relationship between a sample and population. It may take additional time, and so is included as an optional activity.

In this activity, students attempt to recreate the data from the population data using three given samples (MP2). It is important for students to recognize that this is difficult to do and that some samples are more representative than others. Without knowing the population data, though, it can be difficult to know which samples will be representative. Methods for selecting samples in an unbiased way are explored in future lessons.

Addressing

- 7.SP.A

Instructional Routines

- MLR7: Compare and Connect

Launch

Keep students in groups of 2.

Remind students of the activity from a previous lesson where students selected papers (labeled A through O) from the bag and guessed at the sample space. That was an example of trying to interpret information about the population given a sample of information.

Read the first sentence of the task statement: “An online company tracks the number of pieces of furniture they sell each month for a year.” And then ask the students, “How many dots should be represented in the population data for one year?” (12, one for each month of the year.)

Allow students 5 minutes of partner work time followed by a whole-class discussion.

Anticipated Misconceptions

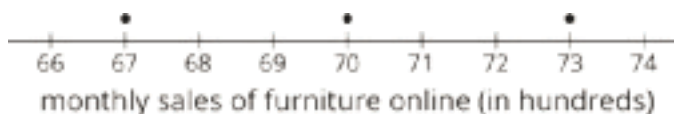
Students may consider that each of the auditors’ samples should be added together to create one larger sample, rather than considering that the auditors may have chosen the same data point in their separate samples.

Therefore, each auditor having a data point at 41,000 may mean that there is only one data point there, and each auditor included it in the sample, or it may mean that there are actually three data points there and each auditor included a different point from the population.

Student Task Statement

An online shopping company tracks how many items they sell in different categories during each month for a year. Three different auditors each take samples from that data. Use the samples to draw dot plots of what the population data might look like for the furniture and electronics categories.

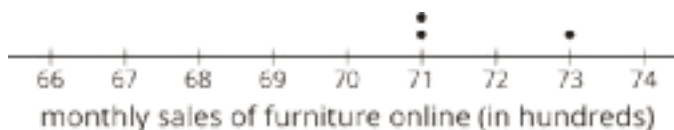
Auditor 1’s sample



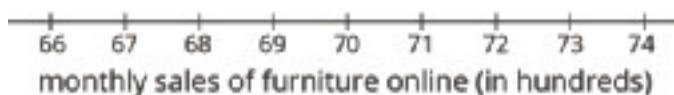
Auditor 2’s sample



Auditor 3’s sample



Population



Auditor 1's sample



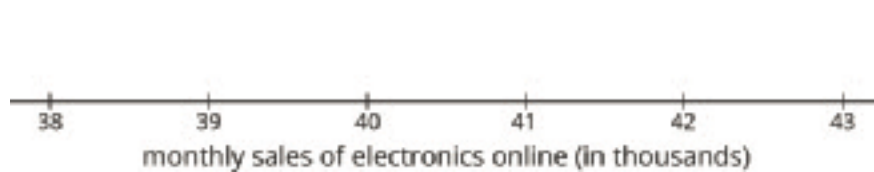
Auditor 2's sample



Auditor 3's sample



Population



Student Response

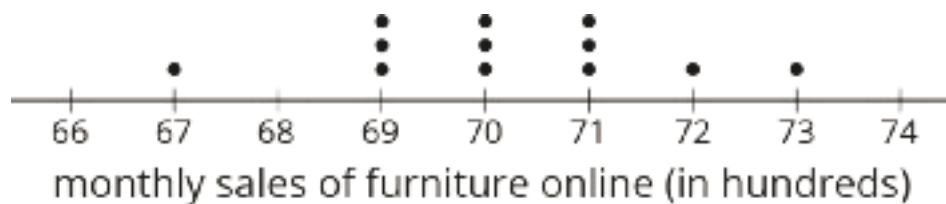
1. Answers vary. The data plot for the population will have 12 dots, and each of the auditors' samples should be able to come from it, so there are some things the population must have: at least one dot at 6,700, at least 3 dots at 7,000, at least 2 dots at 7,100, and at least 1 dot at 7,300. The other 5 data points could be anywhere on the plot.
2. Answers vary. The data plot for the population will have 12 dots, and each of the auditors' samples should be able to come from it, so there are some things the population must have: at least one dot at 39,000, at least 1 dot at 40,000, at least 1 dot at 41,000, and at least 2 dots at 43,000. The remaining 7 dots may be placed anywhere.

Activity Synthesis

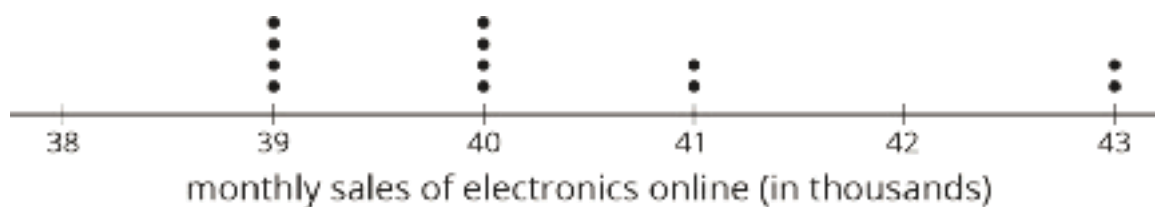
The purpose of the discussion is for students to understand that getting an understanding of the population data from a sample can be very difficult, especially when it is not known whether samples are representative of the population or not.

Display the population dot plots for all to see.

For furniture sales, the samples came from data represented in this dot plot.



For electronics sales, the samples came from data represented in this dot plot.



Ask students

- “How close was your estimate to the actual dot plot? Consider the shape, center, and spread of the data in your answer.”
- “Were any samples better at mimicking the population than others?”
- “What could the auditors have done to make their samples more representative of the population data without knowing what the population would be?” (They could include more information in their samples. They should also think about how the samples were selected. For example, if the auditors only came on months when there were large sales happening, they may be missing important data.)

We will explore how to be careful about selecting appropriate samples in future lessons.

Access for English Language Learners

Representing, Conversing: MLR7 Compare and Connect. Invite students to prepare a visual display that shows their dot plots for what the population data might look like for the furniture and electronics categories. Students should consider how to display their work so another student can interpret what is shown. Some students may wish to add notes or details to their drawings to help communicate their thinking. Invite students to investigate each other’s work and to compare representations. Listen for and amplify the language students use to describe the data plot for the population, and how they explain why it is difficult to create an accurate population dot plot, given three small samples. This will foster students’ meta-awareness and support constructive conversations as they relate sample and population data.

Design Principle(s): Optimize output (for comparison); Cultivate conversation

Lesson Synthesis

Consider asking these discussion questions:

- “What does it mean for a sample to be representative of the population?” (The sample has a similar center, shape, and spread as the population data.)
- “Why might it be important to get a representative sample rather than a more convenient sample?” (If we are going to answer questions about the entire population, it is useful if the sample looks similar to the population data. If not, we may miss some important information.)

- “Usually, a sample is used because we can’t get data for the entire population. How do we know if the sample is representative of the population if we don’t know the population?” (It is OK for students to struggle with this answer at this point. In the next lesson, we’ll explore ways to make our best attempt at getting a representative sample.)

13.5 Reviews for School Lunches

Cool Down: 5 minutes

In the cool-down, students need to understand what a representative sample means and why it might be useful to have.

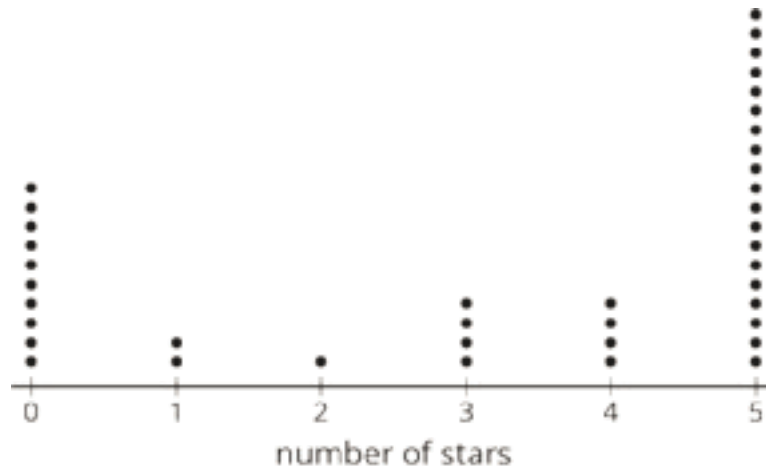
Addressing

- 7.SP.A.1

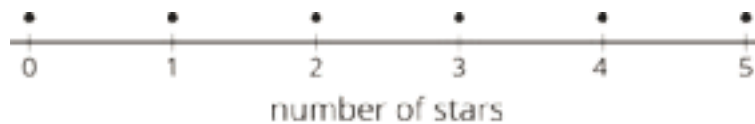
Student Task Statement

Andre is designing a website that will display reviews of school lunches. Each item on the menu is rated from 0 to 5 stars. The main display can only show 6 reviews, so Andre needs to decide how to choose which reviews to show at the top.

This is a dot plot of all 40 reviews for the lasagna.



This is a plot of the stars shown on the first page of results.



1. If each rating also has a sentence or two explaining the rating, what are some good reasons to keep this sample displayed first? What are some good reasons to change the sample that is displayed first?
2. Is the sample representative of the population?

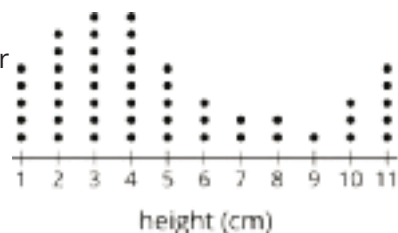
Student Response

1. Answers vary. Sample response: It might be good to keep it so that students can see the wide range of reviews possible for the lasagna. It might be good to change it because there are a lot more 0 and 5 star ratings than ones in the middle, so maybe there should be more of those ratings shown.
2. It is not representative since the shape of the distributions are not similar.

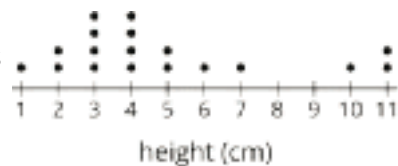
Student Lesson Summary

A sample that is **representative** of a population has a distribution that closely resembles the distribution of the population in shape, center, and spread.

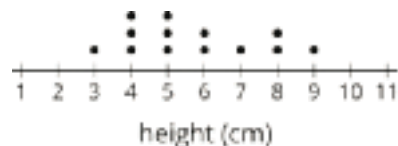
For example, consider the distribution of plant heights, in cm, for a population of plants shown in this dot plot. The mean for this population is 4.9 cm, and the MAD is 2.6 cm.



A representative sample of this population should have a larger peak on the left and a smaller one on the right, like this one. The mean for this sample is 4.9 cm, and the MAD is 2.3 cm.



Here is the distribution for another sample from the same population. This sample has a mean of 5.7 cm and a MAD of 1.5 cm. These are both very different from the population, and the distribution has a very different shape, so it is not a representative sample.



Glossary

- representative

Lesson 13 Practice Problems

Problem 1

Statement

Suppose 45% of all the students at Andre's school brought in a can of food to contribute to a canned food drive. Andre picks a representative sample of 25 students from the school and determines the sample's percentage.

He expects the percentage for this sample will be 45%. Do you agree? Explain your reasoning.

Solution

No, the percentage of students cannot be exactly 45%, since 45% of 25 is 11.25. The percentage in the sample is likely to be close to 45%, but it cannot equal the population percentage for this sample. Even if it were possible to hit 45% exactly, it is likely for there to be some variation in samples.

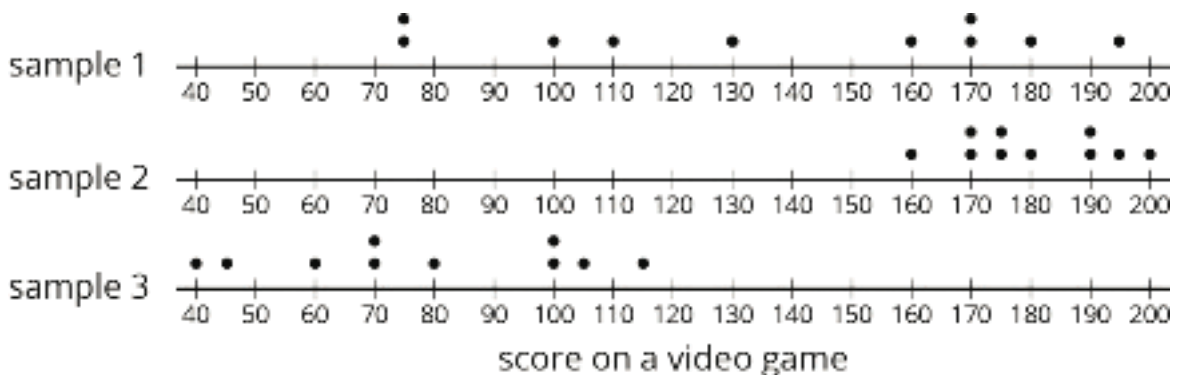
Problem 2

Statement

This is a dot plot of the scores on a video game for a population of 50 teenagers.



The three dot plots together are the scores of teenagers in three samples from this population. Which of the three samples is most representative of the population? Explain how you know.



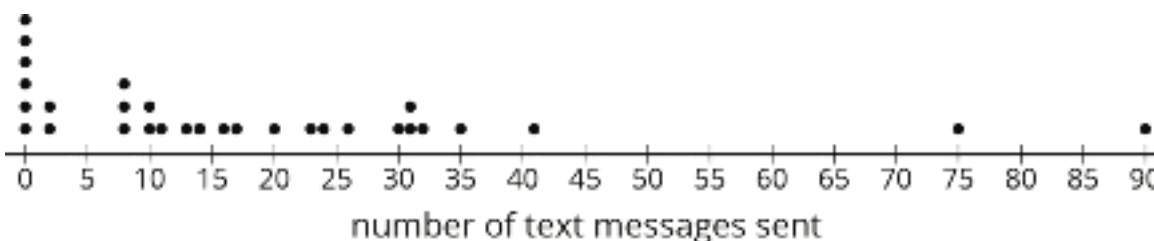
Solution

Sample 1. It is the only sample that has roughly the same center and spread of the population. Sample 2 has a very high center and very low spread. Sample 3 has a lower center and lower spread than the population.

Problem 3

Statement

This is a dot plot of the number of text messages sent one day for a sample of the students at a local high school. The sample consisted of 30 students and was selected to be representative of the population.



- What do the six values of 0 in the dot plot represent?
- Since this sample is representative of the population, describe what you think a dot plot for the entire population might look like.

Solution

- Five students in this sample didn't send any text messages.
- Answers vary. Sample response: the population dot plot should have a lot of values at 0 representing students who didn't text that day. Most of the other values would be less than 50, but there would be a few dots representing students who send a lot more text messages than the typical student.

Problem 4

Statement

A doctor suspects you might have a certain strain of flu and wants to test your blood for the presence of markers for this strain of virus. Why would it be good for the doctor to take a sample of your blood rather than use the population?

Solution

Answers vary. Sample response: To use the population, the doctor would have to test all the blood in my entire body and that is probably not possible while keeping me alive, so a smaller sample would be better.

(From Unit 8, Lesson 12.)

Problem 5

Statement

How many different outcomes are in each sample space? Explain your reasoning. (You do not need to write out the actual options, just provide the number and your reasoning.)

- a. A letter of the English alphabet is followed by a digit from 0 to 9.
- b. A baseball team's cap is selected from 3 different colors, 2 different clasps, and 4 different locations for the team logo. A decision is made to include or not to include reflective piping.
- c. A locker combination like 7-23-11 uses three numbers, each from 1 to 40. Numbers can be used more than once, like 7-23-7.

Solution

- a. 260 outcomes. There are 26 letters and 10 digits, and $26 \cdot 10 = 260$.
- b. 48 outcomes. There are 3 bill colors, 2 kinds of clasps, 4 positions for the team logo, and 2 piping options. $3 \cdot 2 \cdot 4 \cdot 2 = 48$
- c. 64,000 outcomes. $40 \cdot 40 \cdot 40 = 64,000$

(From Unit 8, Lesson 8.)

Lesson 14: Sampling in a Fair Way

Goals

- Describe (orally and in writing) methods to obtain a random sample from a population.
- Justify (orally) whether a given sampling method is fair.
- Recognize that random sampling tends to produce representative samples and support valid inferences.

Learning Targets

- I can describe ways to get a random sample from a population.
- I know that selecting a sample at random is usually a good way to get a representative sample.

Lesson Narrative

In this lesson, students consider different methods of selecting a sample. Students begin by critiquing different sampling methods for their benefits and drawbacks. In particular, students notice that some sampling methods are more biased than others. A follow-up activity shows that some methods may seem to be unbiased at first, but have a hidden bias that restricts the sample from being representative of the population. Finally, students practice recognizing when a sampling method is likely to be biased (MP3), and they see that selecting a sample at random is more likely to produce a representative sample.

Alignments

Addressing

- 7.SP.A.1: Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
- 7.SP.A.2: Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.
- 7.SP.C.7: Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

Building Towards

- 7.SP.A.1: Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports
- Poll the Class
- Think Pair Share

Required Materials

Paper bags

Straws

Rulers marked with inches

Required Preparation

For the That's the First Straw activity, prepare one paper bag containing straws cut to the specified lengths in the table for a demonstration.

$\frac{1}{2}$	1	2	3	4	5
6	6	8	6	5	4

The demonstration will also require a ruler marked with inches to measure the straw pieces chosen in a sample.

Student Learning Goals

Let's explore ways to get representative samples.

14.1 Ages of Moviegoers

Warm Up: 5 minutes

The purpose of this warm-up is for students to begin to see that different samples are more or less representative of the population from which they are drawn. Students are asked to look at a dot plot and reason about the context of the sample by matching it to their expectations about what the population should be.

Building Towards

- 7.SP.A.1

Instructional Routines

- Think Pair Share

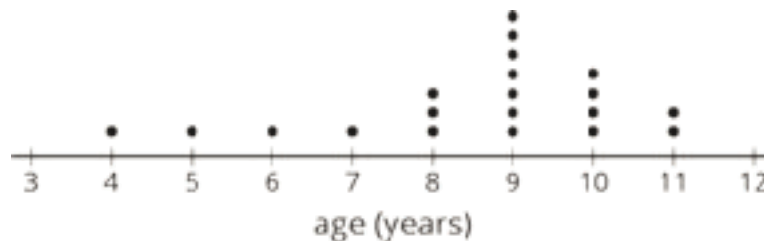
Launch

Arrange students in groups of 2. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Student Task Statement

A survey was taken at a movie theater to estimate the average age of moviegoers.

Here is a dot plot showing the ages of the first 20 people surveyed.



1. What questions do you have about the data from survey?
2. What assumptions would you make based on these results?

Student Response

Answers vary. Sample responses:

1.
 - Why is everyone so young?
 - Why is there a 4 year old at the theater?
 - Why were parents not surveyed?
2.
 - There must be at least one G-rated movie playing at the theater.
 - Maybe the survey was taken during a child's birthday party.
 - There are a lot of young people at this theater.

Activity Synthesis

The purpose of the discussion is for students to express their expectations for who would be at the movie theater and whether this group represents that expectation.

Ask several students to report any questions or assumptions they have about the information provided. If possible, display the dot plot so that students can refer to it while giving their answers.

14.2 Comparing Methods for Selecting Samples

10 minutes

In the previous lesson, students learned that it is very difficult to select representative samples when the population data is unknown. In this lesson, students learn that often the best we can do to select a representative sample is to avoid sampling methods that will be inherently biased one way or another (MP7). A randomly selected sample is not guaranteed to be representative of the population, but other methods are often biased and thus tend to produce samples that are not representative of the population.

Addressing

- 7.SP.A.1
- 7.SP.A.2

Instructional Routines

- Poll the Class

Launch

Arrange students in groups of 2. Give students 5 minutes of partner work time followed by a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts. After students have solved the first 2-3 problems, check-in with either select groups of students or the whole class. Invite students to share the strategies they have used so far as well as any questions they have before continuing.

Supports accessibility for: Organization; Attention

Student Task Statement

Take turns with your partner reading each option aloud. For each situation, discuss:

- Would the different methods for selecting a sample lead to different conclusions about the population?
- What are the benefits of each method?
- What might each method overlook?
- Which of the methods listed would be the most likely to produce samples that are representative of the population being studied?
- Can you think of a better way to select a sample for this situation?

1. Lin is running in an election to be president of the seventh grade. She wants to predict her chances of winning. She has the following ideas for surveying a sample of the students who will be voting:

- a. Ask everyone on her basketball team who they are voting for.

- b. Ask every third girl waiting in the lunch line who they are voting for.
 - c. Ask the first 15 students to arrive at school one morning who they are voting for.
2. A nutritionist wants to collect data on how much caffeine the average American drinks per day. She has the following ideas for how she could obtain a sample:
- a. Ask the first 20 adults who arrive at a grocery store after 10:00 a.m. about the average amount of caffeine they consume each day.
 - b. Every 30 minutes, ask the first adult who comes into a coffee shop about the average amount of caffeine they consume each day.

Student Response

Answers vary. Sample responses:

1. The different methods would probably lead to different conclusions. The basketball team would probably be convenient for Lin to do, but her teammates may be more likely to vote for her since they play together. The girls in the lunch line would also not be too hard to find, but would miss the opinions of any boys or those who bring their lunch from home. The first students to school might be easy to ask if Lin gets to school early, but may miss bus riders or students who get to school later for other reasons. The first students to arrive at school may be the best of these methods since they are least likely to have a direct relationship with Lin, so may represent more of the school. A better way to sample the students might be to ask a student at each different lunch table to get a wide range of students.
2. The different methods would probably lead to different conclusions. The grocery store method would probably be lower than what might be expected and the coffee shop method might be higher. Asking people at the grocery store would be a good way to get a number of responses fairly quickly, but would miss out on talking to people who have to get up early and go to work, and they might be more likely to have more caffeine in the morning. People entering a coffee shop might be more likely to know how much caffeine they have each day, but this method would not talk to people who don't buy coffee and probably have lower caffeine intakes. The grocery store method is probably the better of these two since the coffee shop method would probably produce numbers greater than expected for most people. A better way to sample people might be to ask people at the mall in the early evening since this includes a wide range of people and being at the mall at the time is probably not connected to caffeine consumption.

Activity Synthesis

The purpose of the discussion is for students to understand that some methods of sampling are better than others. Although there may be no way to guarantee that a sample is representative of the population, we can certainly avoid methods that will definitely result in some groups being over- or under-represented.

Poll the class on which of the given methods is best for each scenario. Record these answers for all to see.

Select several students to explain benefits and drawbacks of each of the sampling methods. After each method has been analyzed for a situation, ask if students have ideas for better ways to get a representative sample for the situation.

Ask students, “What are some important things to consider when getting a sample?” (Is there a group that this method will show preference for? Is there a group that will automatically be left out of my sample based on the method? If there are groups I didn’t even think about, does my method have a way of reaching them?)

Explain: People often have biases that may lead them to over- or under-represent some groups in their samples whether the biases are obvious or not. For example, if you want to send a survey out for people to respond to questions, you may not reach people who do not have email addresses. Due to the (sometimes hidden) biases, the best method for selecting samples is to remove as much of the personal selection as possible. In the rest of this lesson, we will explore methods for generating samples that avoid biases.

14.3 That’s the First Straw

10 minutes

In the previous activity, students saw that some methods for taking samples are more likely to produce samples that are not representative of the population than others. In this activity, students see an example of a hidden bias. Although the method of selecting straws by taking out the first one in the bag touched appears fair and random, it produces samples that are not representative of the population (MP1). In the next activity, students explore ways to resolve the problem by finding other methods of selecting a sample that would be fair for this same context.

Addressing

- 7.SP.A.1
- 7.SP.A.2

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2.

In an opaque bag, include straws cut into 35 pieces according to the table.

length of straw in inches	$\frac{1}{2}$	1	2	3	4	5
number of straws	6	6	8	6	5	4

Select 5 students to help with a demonstration. One at a time, each student will reach into the bag and remove the first straw piece they touch. They should measure the straw piece to the nearest

half inch and announce the value to the class for them to record. Return the straw to the bag and shake the bag. Give the bag to the next student to repeat these steps.

After the class has recorded the 5 lengths, repeat the demonstration and add the second set of 5 straw lengths to the second row of the table in this activity.

Note: Taking out the first one the student touches rather than reaching around in the bag is important for this task.

Following the demonstration, give students partner work time followed by a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge of calculating measures of center. Allow students to use calculators to ensure inclusive participation in the activity.

Supports accessibility for: Memory; Conceptual processing

Student Task Statement

Your teacher will have some students draw straws from a bag.

1. As each straw is taken out and measured, record its length (in inches) in the table.

	straw 1	straw 2	straw 3	straw 4	straw 5
sample 1					
sample 2					

2. Estimate the mean length of all the straws in the bag based on:
 - a. the mean of the first sample.
 - b. the mean of the second sample.
3. Were your two estimates the same? Did the mean length of all the straws in the bag change in between selecting the two samples? Explain your reasoning.
4. The actual mean length of all of the straws in the bag is about 2.37 inches. How do your estimates compare to this mean length?
5. If you repeated the same process again but you selected a larger sample (such as 10 or 20 straws, instead of just 5), would your estimate be more accurate? Explain your reasoning.

Student Response

Answers vary. Sample response:

1.

straw 1	straw 2	straw 3	straw 4	straw 5
5	5	4	4	2
5	5	4	3	2

2.

a. 4 inches since $(5 + 5 + 4 + 4 + 2) \div 5 = 4$

b. 3.8 inches since $(5 + 5 + 4 + 3 + 2) \div 5 = 3.8$

3. No. Since we drew out different straws the second time and used a different sample, the means were different. The same straws were in the bag each time, though, so the mean of all the straws did not change.

4. The means from the samples were much longer than the actual mean.

5. No. Since the task asks to take out the first straw touched, it is more likely to get a long straw than a little one at the bottom of the bag. Taking out more straws is not likely to change that tendency.

Activity Synthesis

Ask students:

- “What would it mean for a process of selecting straws to be ‘fair?’” (There should be an equal chance for each item to be selected.)
- “Was this selection process fair?” (No, the shorter pieces probably fell to the bottom of the bag and were less likely to be touched first.)

Reveal the contents of the bag.

- “Were your samples representative of the contents of the bag? Explain your reasoning.”
- “Did every straw in the bag have an equal chance of being selected?” (Longer straws were probably touched first so they were probably over represented in our sample.)
- “If we increased the sample size to 10, would that make the sample more representative?” (No, in fact, it may increase the over-representation of the longer straws and be even more misleading.)

Tell students that a larger sample does not help the estimate if the selection process is flawed. For example, if someone uses the heights of 40 basketball players instead of only 20 basketball players to determine average height of everyone in the United States, the larger sample probably does not represent the population any better.

Explain: Although the process may seem random since we took out as much of the human element of the choosing process as possible, the longer straws were over-represented in our samples. It is important to try to anticipate all the different ways that the selection process might be biased to avoid it as much as possible.

Access for English Language Learners

Listening, Speaking, Conversing: MLR8 Discussion Supports. To help students respond to the questions during the discussion, provide sentence frames such as: "Selecting straws from a bag would be "fair" when . . ." "I believe this selection process was/was not fair because . . ." and "I would change the selection process by . . ." This will help students discover the hidden bias involved in the activity by communicating about the process.

Design Principle(s): Support sense-making; Cultivate conversation

14.4 That's the Last Straw

10 minutes

In the previous activity, students selected straws using a method that might have seemed fair at first, but did not produce a representative sample since the method was flawed. In this activity, students determine whether alternate methods of selecting items for a sample from the same population are fair (MP3). For the methods that work, the physical objects are linked with numerical values to remove even more of the bias toward selecting certain objects more often than others.

Addressing

- 7.SP.A.1

Instructional Routines

- MLR2: Collect and Display

Launch

Tell students that the straws from the previous task are ordered and numbered with 1 representing the shortest straw and 35 representing the longest. Display the table for all to see.

straw number	length (inches)
1	0.5
2	0.5
3	0.5
4	0.5
5	0.5
6	0.5
7	1.0
8	1.0
9	1.0
10	1.0
11	1.0
12	1.0
13	2.0
14	2.0
15	2.0
16	2.0
17	2.0
18	2.0
19	2.0
20	2.0
21	3.0
22	3.0
23	3.0

straw number	length (inches)
24	3.0
25	3.0
26	3.0
27	4.0
28	4.0
29	4.0
30	4.0
31	4.0
32	5.0
33	5.0
34	5.0
35	5.0

Before beginning the task, ask students:

- “What would it mean for the sampling method to be fair?” (Each item has an equal chance of being selected.)
- “Can you think of a way of sampling the straws that would be fair?” (It is OK for students to struggle with answering this question at this stage.)

Following the discussion, allow students quiet work time followed by a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Use color and annotations to illustrate student strategies. As students describe their reasoning, use color and annotations to scribe their thinking on a display. Ask students how they knew the situation would be fair or not, and label each accordingly.

Supports accessibility for: Visual-spatial processing; Conceptual processing

Access for English Language Learners

Speaking: MLR2 Collect and Display. Create a visual display with the heading “random sample.” As students discuss whether the sampling methods are fair, write down the words and phrases students use to explain why the sample methods are fair or not fair. Listen for and amplify words that help define the term “random sample” such as “every possible sample” and “equal chances.” This will help students use appropriate mathematical language when constructing a definition for the new term.

Design Principle(s): Support sense-making; Maximize meta-awareness

Student Task Statement

There were a total of 35 straws in the bag. Suppose we put the straws in order from shortest to longest and then assigned each straw a number from 1 to 35. For each of these methods, decide whether it would be fair way to select a sample of 5 straws. Explain your reasoning.

1. Select the straws numbered 1 through 5.
2. Write the numbers 1 through 35 on pieces of paper that are all the same size. Put the papers into a bag. Without looking, select five papers from the bag. Use the straws with those numbers for your sample.
3. Using the same bag as the previous question, select one paper from the bag. Use the number on that paper to select the first straw for your sample. Then use the next 4 numbers in order to complete your sample. (For example, if you select number 17, then you also use straws 18, 19, 20, and 21 for your sample.)
4. Create a spinner with 35 sections that are all the same size, and number them 1 through 35. Spin the spinner 5 times and use the straws with those numbers for your sample.

Student Response

Answers vary. Sample response:

1. This is not a fair method since there is no chance that straws numbered 6 through 35 will get selected. Also, because the straws are numbered in order of length, it will over represent the shorter straws.
2. This is a fair method. Each straw length has an equal chance of being chosen and should represent the straws fairly well.
3. This is not a fair method since the straw lengths will tend to be grouped together rather than represent the spread of the real straw lengths.

4. This is a fair method. Each straw length has an equal chance of being chosen and should represent the straws fairly well.

Are You Ready for More?

Computers accept inputs, follow instructions, and produce outputs, so they cannot produce truly random numbers. If you knew the input, you could predict the output by following the same instructions the computer is following. When truly random numbers are needed, scientists measure natural phenomena such as radioactive decay or temperature variations. Before such measurements were possible, statisticians used random number tables, like this:

85 67 95 02 42 61 21 35 15 34 41
85 94 61 72 53 24 15 67 85 94 12
67 88 15 32 42 65 75 98 46 25 13
07 53 60 75 82 34 67 44 20 42 33
99 37 40 33 40 88 90 50 75 22 90
00 03 84 57 91 15 70 08 90 03 02
78 07 16 51 13 89 67 64 54 05 26
62 06 61 43 02 60 73 58 38 53 88
02 50 88 44 37 05 13 54 78 97 30

Use this table to select a sample of 5 straws. Pick a starting point at random in the table. If the number is between 01 and 35, include that number straw in your sample. If the number has already been selected, or is not between 01 and 35, ignore it, and move on to the next number.

Student Response

Answers vary. Sample response: The sample includes straws 24, 15, 12, 32, and 25.

Activity Synthesis

Define random sample: A *random sample* from a population is a sample that is selected in a way that gives every different possible sample of the same size an equal chance of being the sample selected.

- “Which of the four methods proposed would be a random sample?” (Putting the papers in the bag or using the spinner.)
- “Would the techniques described here work for other situations in which you wanted a sample? For example, to select 50 people in a large city to represent the views of the city residents.” (Although they would work in theory for large populations, it would be too time consuming to write over a million numbers (or names) on pieces of paper and put them in a bag. Similarly, a spinner that is divided into a million sections would be difficult to manage. Computers can be used to generate random numbers for larger populations.)
- “The most common straw in the bag was the 2 inch straw. When selecting one of the straw *numbers* (not lengths) at random, what is the probability of selecting a 2 inch straw?” ($\frac{8}{35}$ since there are 8 straws that are 2 inches long and 35 total straws.)

Explain:

- A representative sample would have more of the more common lengths, and there is also a higher probability of selecting these lengths, so a random selection should be a good way to select a representative sample.
- A random sample does not guarantee a representative sample, but it avoids methods that might over- or under-represent items of the population. Since we do not know the data for the population, a random sample usually provides the best opportunity to get a representative sample.
- While it is the most ideal method, it is not always possible to generate a random sample. For example, if you wanted to know the average size of salmon in the wild, it is impossible to know how many there are, much less identify them individually, select a few at random from the list, then capture and measure those exact individuals. In these cases, it is important to try to intentionally reduce bias as much as possible when selecting the sample.

Lesson Synthesis

Consider asking these discussion questions:

- “What makes a sample selected at random the best way to select individuals for a sample?” (It avoids biases that might be introduced using other methods.)
- “As part of an English project, you want to look at the length of lines in Shakespeare’s plays. What are some methods of selecting a random sample of lines from these plays?” (Assign each line in the plays a number and use a computer to select several random numbers that correspond to the lines.)

14.5 Sampling Spinach

Cool Down: 5 minutes

The cool-down assesses whether students understand how to select a random sample and why it would be useful to use a random sample.

Addressing

- 7.SP.A.1
- 7.SP.C.7

Student Task Statement

A public health expert is worried that a recent outbreak of a disease may be related to a batch of spinach from a certain farm. She wants to test the plants at the farm, but it will ruin the crop if she tests all of them.

1. If the farm has 5,000 spinach plants, describe a method that would produce a random sample of 10 plants.
2. Why would a random sample be useful in this situation?

Student Response

Answers vary. Sample responses:

1. She could number the plants from 1 to 5,000 and have a computer select 10 random numbers between 1 and 5,000, then test the plants that correspond to the numbers the computer generated.
2. Since it is not known where the disease may have originated, a random sample would hopefully produce a wide selection of plants that would be representative of the entire crop.

Student Lesson Summary

A sample is *selected at random* from a population if it has an equal chance of being selected as every other sample of the same size. For example, if there are 25 students in a class, then we can write each of the students' names on a slip of paper and select 5 papers from a bag to get a sample of 5 students selected at random from the class.

Other methods of selecting a sample from a population are likely to be *biased*. This means that it is less likely that the sample will be representative of the population as a whole. For example, if we select the first 5 students who walk in the door, that will not give us a random sample because students who typically come late are not likely to be selected. A sample that is selected at random may not always be a representative sample, but it is more likely to be representative than using other methods.

It is not always possible to select a sample at random. For example, if we want to know the average length of wild salmon, it is not possible to identify each one individually, select a few at random from the list, and then capture and measure those exact fish. When a sample cannot be selected at random, it is important to try to reduce bias as much as possible when selecting the sample.

Lesson 14 Practice Problems

Problem 1

Statement

The meat department manager at a grocery store is worried some of the packages of ground beef labeled as having one pound of meat may be under-filled. He decides to take a sample of 5 packages from a shipment containing 100 packages of ground beef. The packages were numbered as they were put in the box, so each one has a different number between 1 and 100.

Describe how the manager can select a fair sample of 5 packages.

Solution

Answers vary. Sample response: The manager should pick a method that will result in a random sample. One way is to write the numbers from 1 to 100 on slips of paper and put them in a bag. Mix them well, then select 5 slips of paper from the bag. Use the numbers on these slips to identify which packages in the shipment will be in the sample. The manager might also use random digits or another type of random number generator.

Problem 2

Statement

Select **all** the reasons why random samples are preferred over other methods of getting a sample.

- A. If you select a random sample, you can determine how many people you want in the sample.
- B. A random sample is always the easiest way to select a sample from a population.
- C. A random sample is likely to give you a sample that is representative of the population.
- D. A random sample is a fair way to select a sample, because each person in the population has an equal chance of being selected.
- E. If you use a random sample, the sample mean will always be the same as the population mean.

Solution

["C", "D"]

Problem 3

Statement

Jada is using a computer's random number generator to produce 6 random whole numbers between 1 and 100 so she can use a random sample. The computer produces the numbers: 1, 2, 3, 4, 5, and 6. Should she use these numbers or have the computer generate a new set of random numbers? Explain your reasoning.

Solution

Yes. Explanations vary. Sample explanation: Unless she has reason to believe the computer is messed up, she should use these numbers. To be random, every possible set of numbers should have a chance to be selected, so this set of numbers should be considered to be as random as any other set of numbers.

Problem 4

Statement

A group of 100 people is divided into 5 groups with 20 people in each. One person's name is chosen, and everyone in their group wins a prize. Noah simulates this situation by writing 100 different names on papers and putting them in a bag, then drawing one out. Kiran suggests there is a way to do it with fewer paper slips. Explain a method that would simulate this situation with fewer than 100 slips of paper.

Solution

Answers vary. Sample response: Since the entire group wins a prize, label each group 1, 2, 3, 4, or 5, and put those 5 pieces of paper in the bag to draw one. The probability of Group 1 winning is $\frac{1}{5}$, which is the same probability as the simulation using 100 slips of paper ($\frac{20}{100}$).

(From Unit 8, Lesson 6.)

Problem 5

Statement

Data collected from a survey of American teenagers aged 13 to 17 was used to estimate that 29% of teens believe in ghosts. This estimate was based on data from 510 American teenagers. What is the population that people carrying out the survey were interested in?

- A. All people in the United States.
- B. The 510 teens that were surveyed.
- C. All American teens who are between the ages of 13 and 17.
- D. The 29% of the teens surveyed who said they believe in ghosts.

Solution

C

(From Unit 8, Lesson 12.)

Problem 6

Statement

A computer simulates flipping a coin 100 times, then counts the longest string of heads in a row.

Based on these results, estimate the probability that there will be at least 15 heads in a row.

trial	most heads in a row
1	8
2	6
3	5
4	11
5	13

Solution

Answers vary. Sample response: Between 0 and $\frac{1}{5}$. It did not happen at all in these trials, but it is not impossible, so it is probably somewhere between these two values.

(From Unit 8, Lesson 7.)

Section: Using Samples

Lesson 15: Estimating Population Measures of Center

Goals

- Calculate and interpret (orally and in writing) the mean absolute deviation of a sample.
- Generalize that an estimate for the center of a population distribution is more likely to be accurate when it is based on a random sample with less variability.
- Use the mean of a random sample to make inferences about the population, and explain (orally and in writing) the reasoning.

Learning Targets

- I can consider the variability of a sample to get an idea for how accurate my estimate is.
- I can estimate the mean or median of a population based on a sample of the population.

Lesson Narrative

In this lesson students calculate measures of center and variation for samples from different populations and consider the meaning of these quantities in terms of the situation (MP2). Students see that when there is less variability in the data from different samples from a population, then there is reason to believe that the measure of center from a sample is a better estimate for the measure of center from a population than when a sample has greater variability (MP7).

Alignments

Building On

- 6.SP.B.5.d: Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Addressing

- 7.SP.A.1: Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
- 7.SP.A.2: Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

- 7.SP.B.4: Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

Building Towards

- 7.SP.A.2: Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.
- 7.SP.B.4: Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

Instructional Routines

- MLR5: Co-Craft Questions
- MLR8: Discussion Supports
- Think Pair Share

Student Learning Goals

Let's use samples to estimate measures of center for the population.

15.1 Describing the Center

Warm Up: 5 minutes

This warm-up asks students to decide whether to use the mean or median based on the distribution of the data. As students compare groups in this section, the choice of measure of center will be important.

Building On

- 6.SP.B.5.d

Building Towards

- 7.SP.A.2
- 7.SP.B.4

Instructional Routines

- Think Pair Share

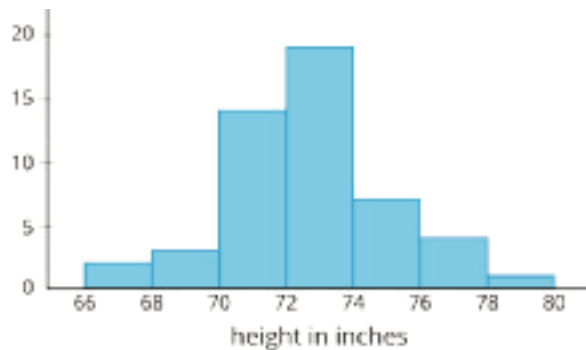
Launch

Arrange students in groups of 2. Give students 1 minute quiet work time, followed by 2 minutes to discuss their work with a partner, followed by a whole-class discussion.

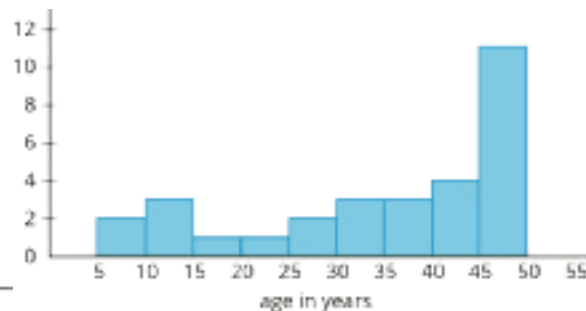
Student Task Statement

Would you use the median or mean to describe the center of each data set? Explain your reasoning.

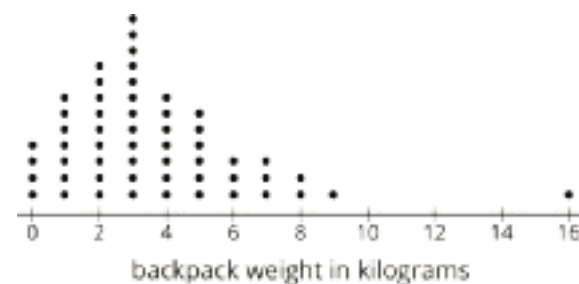
Heights of 50 basketball players



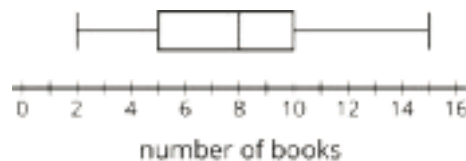
Ages of 30 people at a family dinner party



Backpack weights of sixth-grade students



How many books students read over summer break



Student Response

- Basketball players: Mean. The distribution is symmetric.
- Ages at a party: Median. The distribution is not symmetric.
- Backpacks: Median. The point at 16 would affect the mean much more than the median.
- Books: You could use the mean if you know what it is because the data set is approximately symmetric, but if you only know what's given on the box and whisker plot, then you should use the median.

Activity Synthesis

Select students to share their chosen measure of center and reasoning for their choice. Ask students what measures of variability should be used with each measure of center.

15.2 Three Different TV Shows

5 minutes

In this activity, students analyze data from samples of viewers for different TV shows. The data in this activity is used to begin the analysis as well as to get students thinking about the different shows the sample could represent. The purpose of the activity is to get students thinking about how measures of center from a sample might be used to make decisions about the population of a group.

Addressing

- 7.SP.A.1

Building Towards

- 7.SP.B.4

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 3. Tell students that each person in the group should work on a different sample then share their results with their group. Give students 1 minute quiet work time and then 1 minute to share their work with the group followed by a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge of calculating measures of center. Allow students to use calculators to ensure inclusive participation in the activity.

Supports accessibility for: Memory; Conceptual processing

Access for English Language Learners

Speaking, Representing: MLR8 Discussion Supports. Use this routine to support small-group discussion. Display the following sentence frames for students to respond to their group members' explanations: "I agree because . . ." or "I disagree because . . ." If necessary, revoice student ideas to demonstrate mathematical language use by restating a statement as a question in order to clarify, apply appropriate language, and involve more students.

Design Principle(s): Support sense-making

Student Task Statement

Here are the ages (in years) of a random sample of 10 viewers for 3 different television shows. The shows are titled, "Science Experiments YOU Can Do," "Learning to Read," and "Trivia the Game Show."

sample 1	6	6	5	4	8	5	7	8	6	6
sample 2	15	14	12	13	12	10	12	11	10	8
sample 3	43	60	50	36	58	50	73	59	69	51

1. Calculate the mean for *one* of the samples. Make sure each person in your group works with a different sample. Record the answers for all three samples.
2. Which show do you think each sample represents? Explain your reasoning.

Student Response

1.
 - Sample 1 mean: 6.1 years
 - Sample 2 mean: 11.7 years
 - Sample 3 mean: 54.9 years
2.
 - Sample 1 is probably "Learning to Read" since it is the youngest group.
 - Sample 2 is probably "Science Experiments YOU Can Do" since that sounds like a show young people would like, but should be a little older than 6 years old.
 - Sample 3 is probably "Trivia the Game Show" since that is the oldest group.

Activity Synthesis

Select students to share how they determined which shows matched with which data set. The purpose of the discussion is for students to notice that the shows are meant to appeal to different age groups.

15.3 Who's Watching What?

15 minutes

This activity continues the work begun in the previous activity for this lesson. Students compute the means for sample ages to determine what shows might be associated with each sample (MP2). They also consider the variability to assess the accuracy of population estimates. A sample from a population with less variability should provide a more accurate estimate than a sample that came from a population with more spread in the data. In the discussion, students think about why a sample is used and why an estimate of the mean is helpful, but it may miss some important aspects of the data. The discussion following the activity also asks students to think again about why different samples from the same population may produce different results.

Addressing

- 7.SP.A.2
- 7.SP.B.4

Launch

Keep students in groups of 3.

Tell students that advertisers are interested in age groups for certain television shows so that they can try to sell appropriate items to the audience. For example, it does not make sense to advertise tricycles during a nighttime crime drama show nor to show an ad for an expensive sports car during a children's cartoon.

The samples given in this activity are related to the shows mentioned in the previous activity for this lesson.

Tell students to divide samples 4 through 6 among the group members so that each person only needs to find one mean and share their answer with the group so that the group has access to all 3 answers.

Give students 3 minutes quiet work time for the first 3 problems, then pause the class after the third problem for a quick discussion and to assign items to groups before they continue.

After students have completed the first 3 problems, ask students to indicate which of the shows seem to go with each of the 6 samples (samples 1 through 3 from the previous activity as well as 4 through 6 in this activity). Discuss any disagreements until the class can agree on which samples correspond to which shows. Tell half of the groups that they will use samples 1 through 3 from the previous activity for the last 3 problems and the other half of the groups that they will use samples 4 through 6 for the last 3 problems.

Give students another 3 minutes of quiet work time to finish the activity followed by a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Use color and annotations to illustrate student strategies. As students describe how they calculated the measures of centers, use color and annotations to scribe their thinking on a display. Ask students how they matched samples and television shows, and label each representation accordingly.

Supports accessibility for: Visual-spatial processing; Conceptual processing

Student Task Statement

Here are three more samples of viewer ages collected for these same 3 television shows.

sample 4	57	71	5	54	52	13	59	65	10	71
sample 5	15	5	4	5	4	3	25	2	8	3
sample 6	6	11	9	56	1	3	11	10	11	2

1. Calculate the mean for *one* of these samples. Record all three answers.
2. Which show do you think each of these samples represents? Explain your reasoning.
3. For each show, estimate the mean age for all the show's viewers.
4. Calculate the mean absolute deviation for *one* of the shows' samples. Make sure each person in your group works with a different sample. Record all three answers.

	Learning to Read	Science Experiments YOU Can Do	Trivia the Game Show
Which sample?			
MAD			

5. What do the different values for the MAD tell you about each group?
6. An advertiser has a commercial that appeals to 15- to 16-year-olds. Based on these samples, are any of these shows a good fit for this commercial? Explain or show your reasoning.

Student Response

1. Sample 4 mean: 45.7 years. Sample 5 mean: 7.4 years. Sample 6 mean: 12 years.
2. Sample 4: "Trivia the Game Show." Sample 5: "Learning to Read." Sample 6: "Science Experiments YOU Can Do."
3. Mean age of "Learning to Read:" about 7 years. Mean age of "Science Experiments YOU Can Do:" about 12. Mean age of "Trivia the Game Show:" about 45
4.
 - MADs for "Learning to Read" Sample 1: 0.94 years. Sample 5: 5.16 years.
 - MADs for "Science Experiments YOU Can Do" Sample 2: 1.56 years. Sample 6: 8.8 years.
 - MADs for "Trivia the Game Show" Sample 3: 8.9 years. Sample 4: 21.82 years.
5. The different values tell you how spread out the values are for each sample. For example, most of the ages for "Learning to Read" are close to the mean, but for "Trivia the Game Show," the ages are spread out more.

6. It might work best with “Science Experiments YOU Can Do,” but since the average age is 11.7 years old (or 12 years old) and the MAD is 1.56 years (or 8.8 years), there may or may not not be very many 15 and 16 year olds watching this show.

Activity Synthesis

The purpose of the discussion is to understand why it might be helpful to estimate the mean of a population based on a sample.

Some questions for discussion:

- “Why do you think a sample was used in this situation rather than data from the population?” (There are probably millions of people who watch these shows and it would be difficult to collect data about their ages from all of them.)
- “How could we improve the estimate of the mean for the populations?” (Include more viewers in the sample.)
- “If a sample has a large MAD, what does that imply about the population?” (That the data in the population is very spread out.)
- “If a sample has a small MAD, what is the relationship between the data and the mean?” (Most of the data is close to the mean.)
- “Which estimate of the mean for the population do you expect to be more accurate: the mean from a sample with a large MAD or the mean from a sample with a small MAD? Explain or show your reasoning.” (The mean from a sample with a small MAD. If the data in the sample is close to the mean, then most of the data from the population is also probably close to the mean. Therefore the data in the sample is probably close to the mean of the population and will provide a good estimate.)
- “What do you notice about the different answers for the same show, but the different samples?” (The means are close, but the MADs can be different by a lot.)
- “Why were the answers different for the same show, but different samples?” (Different people were included in the samples, so the numbers may change some, but if they are representative they should be close. In these examples, even though the MADs may seem very different, the relative size compared to the other shows is similar.)

Some students may wonder why they need to calculate the mean when it might be obvious how to match the titles by just looking at the data. This example included 10 ages in each sample so that the important information could be calculated quickly. In a more realistic scenario, the sample may include hundreds of ages. A computer could still calculate the mean quickly, but scanning through all of the data may not make the connection to the correct show as obvious.

- “Notice that there is a 56 year old in sample 6. What are some reasons you think they might be watching this show?” (Maybe a grandmother is watching with her grandchild. Maybe an older person is interested in how science is shown on TV.)
- “The questions asked you to consider means, but are there any data sets for which median might be a better measure of center? Explain your reasoning.” (Yes, sample 6 has a wide range of data ranging from a 1 year old to a 56 year old, but most of the data are around 10, so median might be better to use for that sample.)
- “A lot of families might be watching ‘Learning to Read’ with their children or older people may be using the show to learn English. How might this affect the mean? How could you recognize that there are two main age groups that watch this show?” (It would bring the mean up from just the kids who watch the show. The mean would not make the two age groups obvious, so looking at a dot plot or histogram might be more helpful with this group.)

15.4 Movie Reviews

10 minutes

In this activity, students use data from a sample of movie reviews to estimate information about all the reviews for the movie. Based on the distribution of the data, students are asked to choose an appropriate measure of center and measure of variation then apply their calculations to the entire population. Finally, students gauge their trust in the measure of center they have chosen based on the associate measure of variation.

Building On

- 6.SP.B.5.d

Addressing

- 7.SP.A.1
- 7.SP.A.2

Instructional Routines

- MLR5: Co-Craft Questions

Launch

Keep students in groups of 3. Allow students 5 minutes work time in their groups followed by a whole-class discussion.

It may be helpful to use the warm-up for this lesson to review how to choose mean and median based on the distribution of data.

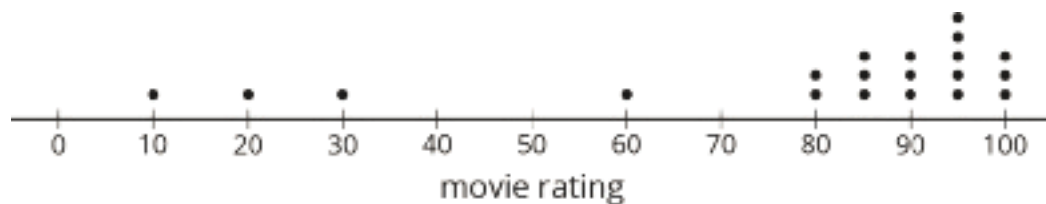
Access for English Language Learners

Writing, Conversing: MLR5 Co-Craft Questions. Display the scenario and the movie rating dot plot without revealing the questions that follow. Ask students to write down possible mathematical questions that can be asked about the situation. Invite students to share their questions with a partner before selecting 1–2 pairs to share their questions with the class. Listen for and amplify questions about how different measures of center and variability give more or less accurate information about the population. Then reveal and ask students to work on the actual questions of the task. This will help students justify their choices of using a particular measure of center to represent the population.

Design Principle(s): Optimize output (for justification); Maximize meta-awareness

Student Task Statement

A movie rating website has many people rate a new movie on a scale of 0 to 100. Here is a dot plot showing a random sample of 20 of these reviews.



1. Would the mean or median be a better measure for the center of this data? Explain your reasoning.
2. Use the sample to estimate the measure of center that you chose for *all* the reviews.
3. For this sample, the mean absolute deviation is 19.6, and the interquartile range is 15. Which of these values is associated with the measure of center that you chose?
4. Movies must have an average rating of 75 or more from all the reviews on the website to be considered for an award. Do you think this movie will be considered for the award? Use the measure of center and measure of variability that you chose to justify your answer.

Student Response

1. Median since most of the data is above 80, but there are a few points that are much lower, so the data is not symmetric.
2. 90 is the median.
3. IQR is better to use with median.

4. Answers vary. Sample response: Yes. Since the first quartile for this data set is 80, at least 75% of the ratings in the sample were 80 or higher. Therefore, it is likely that the overall rating for the entire population is 75 or higher.

Are You Ready for More?

Estimate typical temperatures in the United States today by looking up current temperatures in several places across the country. Use the data you collect to decide on the appropriate measure of center for the country, and calculate the related measure of variation for your sample.

Student Response

Answers vary.

Activity Synthesis

The purpose of the discussion is for students to review how to choose a measure of center and its associated measure of variation. Additionally, students use the measure of variation to help them think about how much to trust their population characteristic estimate.

Consider asking these discussion questions:

- “Which measure of center did you choose and why?” (Median, since the distribution is not symmetric.)
- “Based on the context, do you think other movie reviews would have non-symmetric distributions as well?” (Yes, usually a lot of people will agree whether a movie is good or bad, but a few people will have strong opinions on the other end of the scale.)
- “A random sample of 20 reviews for another movie has a median of 90 as well, but its IQR is 30. Do you think this movie is more or less likely to be considered for the award?” (Less likely since there is more variability in the sample, so it is harder to estimate the median for all of the reviews.)
- “A random sample of 20 reviews for a third movie has a median of 50 and an IQR of 20. Is it possible this third movie will be considered for an award?” (It seems unlikely, but it is possible. The random sample may have randomly selected the 20 worst reviews and all the other reviews gave it a 100 rating.)

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Provide students with a graphic organizer to support their participation during the synthesis. Invite students to describe what to look for to determine how to choose a measure of center and its associated measure of variation, and to include examples for each.

Supports accessibility for: Conceptual processing; Organization

Lesson Synthesis

Consider asking these discussion questions to review the main ideas from the lesson:

- “How do you determine which measure of center will best describe the data in a sample?” (Base it on the distribution of the data.)
- “When you have the data from a sample, how can you estimate the value of a measure of center for the population?” (If the sample is random, calculate the appropriate measure of center for the sample and use that to estimate the same characteristic for the population.)
- “What does the variability of the sample tell you about your estimate for the measure of center of the population?” (The greater the variability, the less certain I am of the estimate. If the data is spread widely in the sample, it might be spread even more widely in the population and this sample may not capture everything going on in the population, so the estimate may not be very accurate.)

15.5 More Accurate Estimate

Cool Down: 5 minutes

This cool-down assesses student understanding of how variability within the data can help gauge how far off an estimate of a population characteristic might be from its actual value.

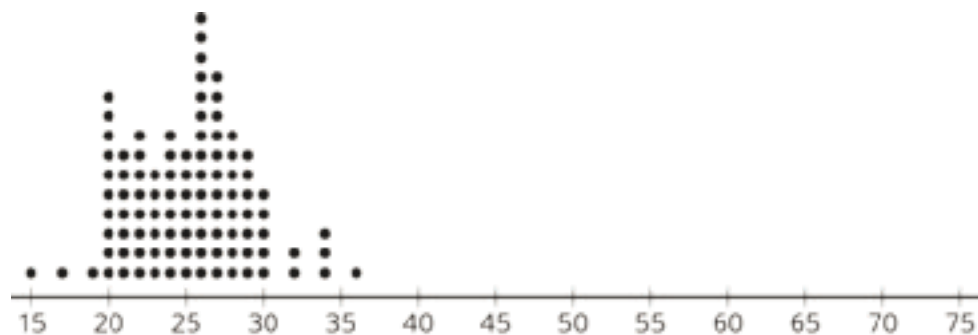
Addressing

- 7.SP.A.2

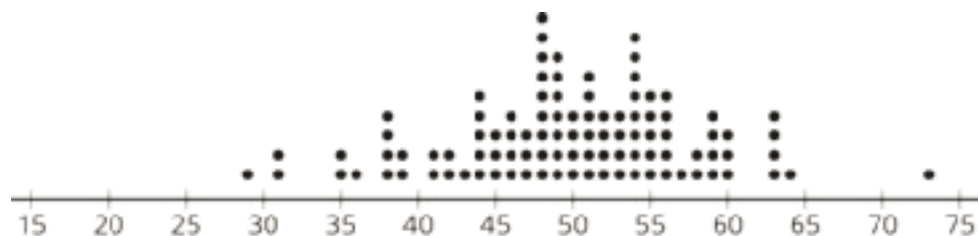
Student Task Statement

Here are dot plots that represent samples from two different populations.

Sample 1:



Sample 2:



1. Estimate the mean of each population using these samples.

2. Based on the dot plots, which estimate is more likely to be accurate? Explain your reasoning.

Student Response

1. Answers vary. Correct responses should be close to 25 and 50 respectively.
2. The estimate for sample 1 is probably more accurate since there is much less variability in the data.

Student Lesson Summary

Some populations have greater variability than others. For example, we would expect greater variability in the weights of dogs at a dog park than at a beagle meetup.

Dog park:



Mean weight: 12.8 kg MAD: 2.3 kg

Beagle meetup:



Mean weight: 10.1 kg MAD: 0.8 kg

The lower MAD indicates there is less variability in the weights of the beagles. We would expect that the mean weight from a sample that is randomly selected from a group of beagles will provide a more accurate estimate of the mean weight of all the beagles than a sample of the same size from the dogs at the dog park.

In general, a sample of a similar size from a population with *less* variability is *more likely* to have a mean that is close to the population mean.

Glossary

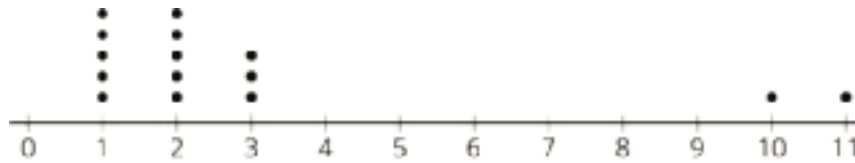
- interquartile range (IQR)

Lesson 15 Practice Problems

Problem 1

Statement

A random sample of 15 items were selected.



For this data set, is the mean or median a better measure of center? Explain your reasoning.

Solution

Median, since the data is not symmetric with a couple of values far away from most of the other numbers.

Problem 2

Statement

A video game developer wants to know how long it takes people to finish playing their new game. They surveyed a random sample of 13 players and asked how long it took them (in minutes).

1,235	952	457	1,486	1,759	1,148	548	1,037	1,864
1,245	976	866	1,431					

- Estimate the median time it will take *all* players to finish this game.
- Find the interquartile range for this sample.

Solution

Median: 1,148 minutes; IQR: 549.5 minutes

Problem 3

Statement

Han and Priya want to know the mean height of the 30 students in their dance class. They each select a random sample of 5 students.

- The mean height for Han's sample is 59 inches.
- The mean height for Priya's sample is 61 inches.

Does it surprise you that the two sample means are different? Are the population means different? Explain your reasoning.

Solution

No. Explanations vary. Sample explanation: Even though they both selected a random sample, their samples probably included different people from the population, so the two sample means would not necessarily be the same, even though there is only one population mean.

Problem 4

Statement

Clare and Priya each took a random sample of 25 students at their school.

- Clare asked each student in her sample how much time they spend doing homework each night. The sample mean was 1.2 hours and the MAD was 0.6 hours.
 - Priya asked each student in her sample how much time they spend watching TV each night. The sample mean was 2 hours and the MAD was 1.3 hours.
- a. At their school, do you think there is more variability in how much time students spend doing homework or watching TV? Explain your reasoning.
- b. Clare estimates the students at her school spend an average of 1.2 hours each night doing homework. Priya estimates the students at her school spend an average of 2 hours each night watching TV. Which of these two estimates is likely to be closer to the actual mean value for all the students at their school? Explain your reasoning.

Solution

- a. There is more variability in the times spent watching TV or playing games. The MAD is a measure of variability, and the MAD for time spent watching TV is greater than the MAD for the time spent doing homework.
- b. Clare's estimate is more likely to be closer, because the times spent doing homework don't vary as much. It is harder to get an accurate estimate of the population mean when there is a lot of variability in the population values.

Lesson 16: Estimating Population Proportions

Goals

- Compare (orally) proportions for the same category from different samples of a population.
- Comprehend that the term “proportion” refers to a number between 0 and 1 that represents the fraction of the data within a certain category.
- Use the proportion of a random sample that is within a certain category to make inferences about the population, and explain (orally and in writing) the reasoning.

Learning Targets

- I can estimate the proportion of population data that are in a certain category based on a sample.

Lesson Narrative

In the previous lesson, students used samples to estimate measures of center of a population. In this lesson, students estimate population proportions. The term **proportion** is used in statistics to refer to a number from 0 to 1 that represents the fraction of the data belonging to a given category.

Students see that if a sample is representative of the population, then we can use proportional reasoning to make predictions about the population. However, students need to understand that, due to sampling variability, these predictions are estimates, not exact answers like they get when working with actual proportional relationships (MP3).

The activity about examining a distribution of proportions from many different samples is included as an optional opportunity to deepen students' understanding of sampling variability.

Alignments

Addressing

- 7.NS.A.2.d: Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.
- 7.SP.A: Use random sampling to draw inferences about a population.
- 7.SP.A.2: Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

- 7.SP.B.4: Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

Building Towards

- 7.SP.A.1: Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
- 7.SP.A.2: Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Notice and Wonder

Required Materials

Paper bags

Instructional master

Pre-printed slips, cut from copies of the

Required Preparation

Print and cut up slips from the Reaction Times Instructional master. Prepare one set of slips in a paper bag for every 2 students.

Student Learning Goals

Let's estimate population proportions using samples.

16.1 Getting to School

Warm Up: 5 minutes

The purpose of this warm-up is for students to compute the fraction of individuals whose responses fall in a specified category. This activity gives students time to think about how to compute these fractions from categorical data.

For the second and third questions, students may debate whether to include the 10 minute times or not. According to the wording of the question asked, it does ask for *more than* 10 minute times, so maybe exactly 10 minutes should not count (since $10 \not> 10$). On the other hand, all of the values are listed as whole numbers, so a student who takes 10 minutes and 1 second to get to school may have rounded down to 10, but should have been counted. Noticing the large difference in answers for the third question, it may be worth clarifying the data in this instance, even for an estimate.

Monitor for students who include the 10 minute times for the second and third questions as well as those who do not.

Building Towards

- 7.SP.A.1
- 7.SP.A.2

Launch

Give students 2 minutes of quiet work time followed by a whole-class discussion.

Student Task Statement

A teacher asked all the students in one class how many minutes it takes them to get to school. Here is a list of their responses:

20	10	15	8	5	15	10	5	20
5	15	10	3	10	18	5	25	5
5	12	10	30	5	10			

1. What fraction of the students in this class say:
 - a. it takes them 5 minutes to get to school?
 - b. it takes them more than 10 minutes to get to school?
2. If the whole school has 720 students, can you use this data to estimate how many of them would say that it takes them more than 10 minutes to get to school?

Be prepared to explain your reasoning.

Student Response

1. a. $\frac{7}{24}$
- b. $\frac{9}{24}$ or equivalent (or $\frac{15}{24}$ or equivalent if 10 minutes is included)

2. About 270 students (or 450 students if 10 minutes is included) since $\frac{9}{24}$ of 720 is 270 (or $\frac{15}{24}$ of 720 is 450)

Activity Synthesis

Select students to share their methods for computing the solutions. Include previously identified students who did or did not include the 10 minute values in their calculations.

If it does not arise during the discussion, explain that answering the third question with the data at hand is only accurate if the sample data is representative of the school. It is possible that the class happens to contain only students who get a ride to school, but much of the school rides the bus.

16.2 Reaction Times

15 minutes

In previous lessons, students examined the estimation of the mean and median for populations using data from a sample. In this activity, students apply similar reasoning to estimating the proportion of a population that matches certain characteristics. Students collect a sample of 20 reaction times and compute the fraction of responses in their sample that are in a given range. Then, in the discussion, students compare their estimations to the known population proportion and use the class's proportions to gauge the accuracy of their estimate (MP6).

Addressing

- 7.NS.A.2.d
- 7.SP.A

Instructional Routines

- MLR7: Compare and Connect

Launch

Arrange students in groups of 2. Distribute bags of slips cut from the Instructional master.

Tell students that, in statistics, a **proportion** is a number between 0 and 1 that represents the fraction of the data that fits into the desired category. For example, with the data: {Yes, Yes, Yes, No, Maybe} the proportion of “Yes” answers is $\frac{3}{5}$ or 0.6.

Introduce the context: All 120 seniors at a high school were asked to click a button as soon as they noticed a box change color and the response time was recorded in seconds. These 120 response times represent the population for this activity. Their responses are written on the slips of paper in the bag.

When selecting a sample of 20, each value does not need to be replaced before taking the next one.

Allow students 10 minutes of partner work time followed by a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts. After students have solved the first 2-3 problems, check-in with either select groups of students or the whole class. Invite students to share the strategies they have used so far as well as any questions they have before continuing.

Supports accessibility for: Organization; Attention

Anticipated Misconceptions

The reason a proportion is between 0 and 1 is because we are specifically looking for the proportion of a desired category out of the whole group. This is a “part out of whole” fraction, which will be a number between 0 and 1.

Student Task Statement

The track coach at a high school needs a student whose reaction time is less than 0.4 seconds to help out at track meets. All the twelfth graders in the school measured their reaction times. Your teacher will give you a bag of papers that list their results.

1. Work with your partner to select a random sample of 20 reaction times, and record them in the table.

2. What **proportion** of your sample is less than 0.4 seconds?
3. Estimate the proportion of all twelfth graders at this school who have a reaction time of less than 0.4 seconds. Explain your reasoning.
4. There are 120 twelfth graders at this school. Estimate how many of them have a reaction time of less than 0.4 seconds.
5. Suppose another group in your class comes up with a different estimate than yours for the previous question.
 - a. What is another estimate that would be *reasonable*?
 - b. What is an estimate you would consider *unreasonable*?

Student Response

Answers vary. Sample response:

1. 0.5, 0.44, 0.51, 0.38, 0.42, 0.79, 0.39, 0.46, 0.34, 0.3, 0.36, 0.41, 0.31, 0.82, 0.35, 0.36, 0.48, 0.72, 0.74, 0.45
2. $\frac{8}{20} = 0.4$
3. 0.4 since the sample was chosen at random and is likely representative.
4. 48 since $0.4 \cdot 120 = 48$.
5. a. 50
 b. 100

Activity Synthesis

The purpose of the discussion is for students to see how multiple sample proportions can help revise their estimates and give an idea of how accurate the individual estimates from samples might be.

Ask the groups to share the proportion from their sample that had fast reaction times and display the results for all to see.

Some questions for discussion:

- “Using the class’s data, how accurate do you think your group’s estimate is? Explain or show your reasoning.” (Students should mention the variability of the proportions from the samples influencing the accuracy of the estimate.)
- “The actual proportion for this population is 0.5. How close was your estimate? Explain why your estimate was not exactly the same.” (Each sample might be slightly different since they do not include all of the values, but they should be close.)
- “If each group had 40 reaction times in their samples instead of 20, do you think the estimate would be more or less accurate?” (The estimate should be more accurate since there is more information available.)

Access for English Language Learners

Representing, Speaking: MLR7 Compare and Connect. Invite students to create a visual display of their work to share with other students. Displays should include a representation of the sample reaction times, the proportion of fast reaction times, and an estimate for the number of twelfth-graders with fast reaction times. As students investigate each other’s work, ask students to compare their samples and proportion of fast reaction times. Listen for and amplify the language students use to justify the accuracy of their estimate for the proportion of fast reaction times using the definition of a random sample. This will help students compare their sample proportions in order to see how accurate their population estimates might be.

Design Principle(s): Optimize output (for comparison); Cultivate conversation

16.3 A New Comic Book Hero

15 minutes

In the previous activity, students collected their own sample and computed an estimate for the population proportion based on the sample. In this activity, students use a different context to practice exploring proportions from samples and their extension to populations. The optional activity following this one continues with this context exploring sampling variability.

Addressing

- 7.NS.A.2.d
- 7.RP.A
- 7.SP.A.2
- 7.SP.B.4

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- Notice and Wonder

Launch

Keep students in groups of 2.

Tell students that three comic books, *The Adventures of Super Sam*, *Beyond Human*, and *Mysterious Planets*, are all planning to add a new superhero to their stories. A survey was sent to dedicated readers of each series to ask what type of ability the new hero should have: fly, freeze, or another power.

Display the tables from the Task Statement for all to see. Ask students, “What do you notice? What do you wonder?”

Students may notice:

- There are 4 different responses: fly, freeze, super strength, and invisibility.
- The number of responses for each of the 4 different responses.
- There are 20 responses.

Students may wonder:

- Will the decision for the new hero's power be based only on this survey?
- What proportion chose each of the different powers?
- Is this sample of 20 responses representative of the population?

Give students 5–7 minutes of partner work time followed by a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, use color to highlight connections between the same categorical data in a survey of new hero special ability.

Supports accessibility for: Visual-spatial processing

Student Task Statement

Here are the results of a survey of 20 people who read *The Adventures of Super Sam* regarding what special ability they think the new hero should have.



response	what new ability?
1	fly
2	freeze
3	freeze
4	fly
5	fly
6	freeze
7	fly
8	super strength
9	freeze
10	fly

response	what new ability?
11	freeze
12	freeze
13	fly
14	invisibility
15	freeze
16	fly
17	freeze
18	fly
19	super strength
20	freeze

1. What proportion of this sample want the new hero to have the ability to fly?

2. If there are 2,024 dedicated readers of *The Adventures of Super Sam*, estimate the number of readers who want the new hero to fly.

Two other comic books did a similar survey of their readers.

- In a survey of people who read *Beyond Human*, 42 out of 60 people want a new hero to be able to fly.
 - In a survey of people who read *Mysterious Planets*, 14 out of 40 people want a new hero to be able to fly.
3. Do you think the proportion of all readers who want a new hero that can fly are nearly the same for the three different comic books? Explain your reasoning.
 4. If you were in charge of these three comics, would you give the ability to fly to any of the new heroes? Explain your reasoning using the proportions you calculated.

Student Response

1. $\frac{8}{20} = 0.4$
2. About 810 since $0.4 \cdot 2,024 \approx 810$
3. The proportion for *Mysterious Planets* seems close to the proportion for *The Adventures of Super Sam* but not for *Beyond Human*, since the proportion for *Mysterious Planets* is 0.35 which is close to the 0.4 for *The Adventures of Super Sam* while the proportion for *Beyond Human* is 0.7.
4. Answers vary. Sample responses:
 - I would give the new heroes of all of the comics the ability to fly. Based on these results, about 40% of the readers of *Mysterious Planets* and *The Adventures of Super Sam* want the hero to have that new ability, so I think it would satisfy many of their readers. About 70% of the readers of *Beyond Human* want this new ability, so they will most likely be happy with this choice.
 - Only for the new hero of *Beyond Human*. In the data for *The Adventures of Super Sam*, more people voted for freeze than fly, so I don't think it would make sense to choose flight for that comic rather than freeze. It is hard to tell for *Mysterious Planets* without the actual data. The votes might have been split among the other choices more evenly and flight might have been the most popular choice.

Activity Synthesis

The purpose of the discussion is for students to recognize the value of a proportion and why a sample may be necessary to estimate the proportion for the population.

Some questions for discussion:

- “Explain why you think a sample was used instead of the population for this situation.” (The population is too large to ask all of the readers about their preference. Also, the authors may

want the power of the new hero to be a surprise to some readers, so they want to get some information without telling everyone about what is coming.)

- “Based on the proportions computed in each of these samples, would you suggest to the authors of any of the comic books to give their new hero the ability to fly?”
- “Although the proportion of responses from the *Mysterious Planets* sample who chose flight was only 0.35, it was the most popular choice (freeze, super strength, invisibility, and other powers split the remaining votes). Does this information change your answer to the previous question?”
- “Of the three estimates, which do you think is most accurate? Explain your reasoning.” (*Beyond Human* has the largest sample, so it may be the most accurate.)

Access for English Language Learners

Writing, Speaking, Listening: MLR1 Stronger and Clearer Each Time. Use this routine to help students improve their writing by providing them with multiple opportunities to clarify their explanations through conversation. Give students time to meet with 2–3 partners to share their response to: “Would you give the ability to fly to any of the new heroes?” Provide prompts for feedback that will help students strengthen their ideas and clarify their language. For example, “Which proportions did you consider?”, “How did you calculate each proportion?”, “Can you say more about...?”, etc. Give students 1–2 minutes to revise their writing based on the feedback they received.

Design Principle(s): Optimize output (for explanation)

16.4 Flying to the Shelves

Optional: 10 minutes

This optional activity goes beyond grade-level expectations to deepen students’ understanding of sampling variability.

This activity continues the comic book context introduced in the previous activity. There is not a measure of variability such as MAD or IQR for proportions since the data are categorical rather than quantitative, so other methods must be employed to determine the accuracy of an estimate. Students look at dot plots showing the results from multiple samples to gauge the accuracy of the estimates for population proportions (MP7). This activity will provide a foundation for work in later grades.

Addressing

- 7.SP.A.2
- 7.SP.B.4

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2.

Help students make sense of the dot plot. Each dot in the dot plot represents the proportion from a random sample of 20 readers. 50 random samples were taken and the 50 proportions are plotted on the dot plot. For example, the 0.4 proportion from the previous activity would be represented by one of the 4 dots at 0.4 on the first dot plot.

Ask, “Are any of the sample proportions greater than or equal to 0.5? What does this mean?” (Yes, 2 dots are greater than or equal to 0.5. This means that, in those samples, at least half of the people prefer the new hero to have the ability to fly.)

Give students 5–7 minutes of partner work time followed by a whole-class discussion.

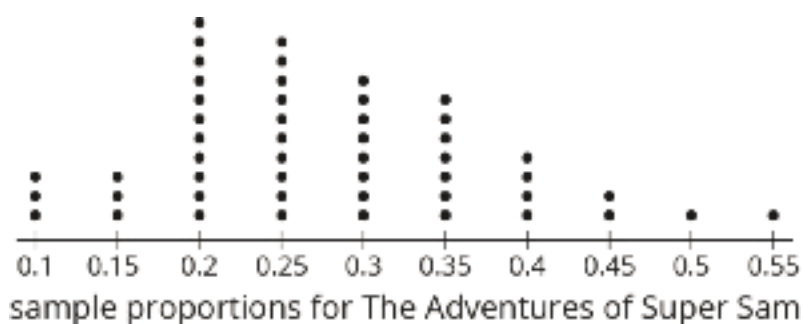
Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, pause to check for understanding after 3-5 minutes of work time.

Supports accessibility for: Organization; Attention

Student Task Statement

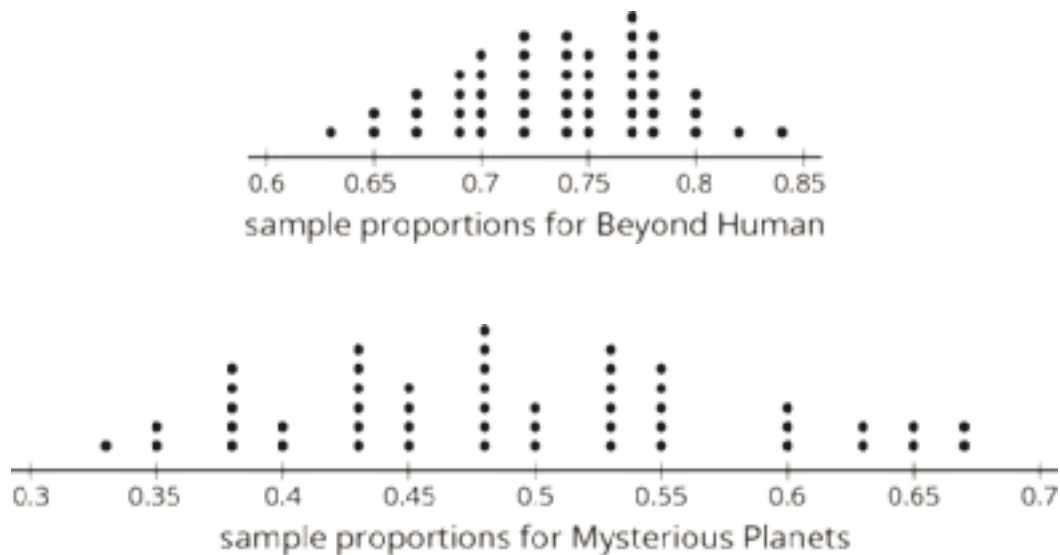
The authors of *The Adventures of Super Sam* chose 50 different random samples of readers. Each sample was of size 20. They looked at the sample proportions who prefer the new hero to fly.



1. What is a good estimate of the proportion of *all* readers who want the new hero to be able to fly?
2. Are most of the sample proportions within 0.1 of your estimate for the population proportion?

3. If the authors of *The Adventures of Super Sam* give the new hero the ability to fly, will that please most of the readers? Explain your reasoning.

The authors of the other comic book series created similar dot plots.



4. For each of these series, estimate the proportion of all readers who want the new hero to fly.
- *Beyond Human*:
 - *Mysterious Planets*:
5. Should the authors of either of these series give their new hero the ability to fly?
6. Why might it be more difficult for the authors of *Mysterious Planets* to make the decision than the authors of the other series?

Student Response

1. Answers vary. Sample response: 0.3 since the center of the distribution is near there.
2. Yes
3. No. Only about 30% of readers seem to want the new hero to fly, so there may be a more preferred super power.
4. Answers vary. Sample response:
 - *Beyond Human*: about 0.73
 - *Mysterious Planets*: about 0.5
5. *Beyond Human* should give the power to fly to its new hero, since almost $\frac{3}{4}$ voted for that option.

6. Not only do we estimate the population proportion to be about 0.5, but the values are quite varied among all the sample proportions.

Are You Ready for More?

Draw an example of a dot plot with at least 20 dots that represent the sample proportions for different random samples that would indicate that the population proportion is above 0.6, but there is a lot of uncertainty about that estimate.

Student Response

Answers vary. Responses should include a majority of the dots greater than or near 0.6, but there should be a lot of variability in the sample proportions.

Activity Synthesis

The purpose of the discussion is for students to talk about how the variability in sample proportions affects their trust in the estimates of the population proportion.

Consider these questions for discussion:

- “When estimating a population mean or median from a random sample, measures of variability from a sample can be used to help gauge the accuracy of the estimate. With proportions there is not a measure of variability in the same way. How did the information in this activity guide your thoughts about the accuracy of the population estimate?” (Many samples were taken and their proportions computed. The variability of these sample proportions showed how much to trust the population estimate.)
- “How does the distribution of values in the dot plots of sample proportions affect your trust in an estimate of population proportion?” (The more variability, the less certainty in the estimate.)
- “How would the distributions change if the number of responses in each sample were increased?” (The center should remain about the same, the variability should decrease or, in other words, the dots in the dot plot should get closer together towards the center.)

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion. For each response or observation that is shared, ask students to restate what they heard using precise mathematical language. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped clarify the original statement. This will provide more students with an opportunity to produce language as they interpret the reasoning of others.

Design Principle(s): Support sense-making

Lesson Synthesis

Consider asking these discussion questions to clarify the main ideas of the lesson:

- “When using data, what is a proportion? How is it calculated?” (A proportion is the fraction of the data that are in a certain category. It is calculated by counting the number of data values in the category and dividing by the total number of data values in the sample.)
- “In order to say that more than half of the people in a sample responded with a certain answer, what would the proportion for that answer be?” (Any value greater than 0.5)
- “A random sample indicates that a 0.45 proportion of people shopping at a certain store prefer wheat bread to white bread. The store has 3,000 customers. Estimate the number of people shopping at the store who prefer wheat bread.” (About 1,350 since $0.45 \cdot 3,000 = 1,350$)

16.5 More than 48 Grams

Cool Down: 5 minutes

This cool-down assesses student's understanding of estimating proportions for a population based on a random sample.

Addressing

- 7.SP.A.2

Student Task Statement

A chemical engineer is trying to increase the amount of the useful product in a reaction. She performs the reaction with her new equipment 10 times and gets the following amounts of the useful product in grams:

47.1 48.2 48.3 47.5 48.5 48.1 47.2 48.2 48.4 48.3

1. What proportion of the reactions were above the 48 grams threshold?
2. Other chemists typically get 65% of their reactions to produce more than 48 grams. Should the engineer say that she was able to increase the useful product when compared to the other chemists?

Student Response

1. 0.7 since 7 of the 10 reactions had more than 48 grams of the useful product.
2. Answers vary. Sample response: She could be optimistic, but her proportion does not seem far from what others have done. She should run more reactions to be more sure of the improvement. With only 10 values in her data set, 0.7 (and 0.6) is as close to 0.65 as she could get.

Student Lesson Summary

Sometimes a data set consists of information that fits into specific categories. For example, we could survey students about whether they have a pet cat or dog. The categories for these data would be {neither, dog only, cat only, both}. Suppose we surveyed 10 students. Here is a table showing possible results:

option	number of responses
neither dog nor cat	2
dog only	4
cat only	1
both dog and cat	3

In this sample, 3 of the students said they have both a dog and a cat. We can say that the **proportion** of these students who have both a dog and a cat is $\frac{3}{10}$ or 0.3. If this sample is representative of all 720 students at the school, we can predict that about $\frac{3}{10}$ of 720, or about 216 students at the school have both a dog and a cat.

In general, a proportion is a number from 0 to 1 that represents the fraction of the data that belongs to a given category.

Glossary

- proportion

Lesson 16 Practice Problems

Problem 1

Statement

Tyler wonders what proportion of students at his school would dye their hair blue, if they were allowed to. He surveyed a random sample of 10 students at his school, and 2 of them said they would. Kiran didn't think Tyler's estimate was very accurate, so he surveyed a random sample of 100 students, and 17 of them said they would.

- Based on Tyler's sample, estimate what proportion of the students would dye their hair blue.
- Based on Kiran's sample, estimate what proportion of the students would dye their hair blue.
- Whose estimate is more accurate? Explain how you know.

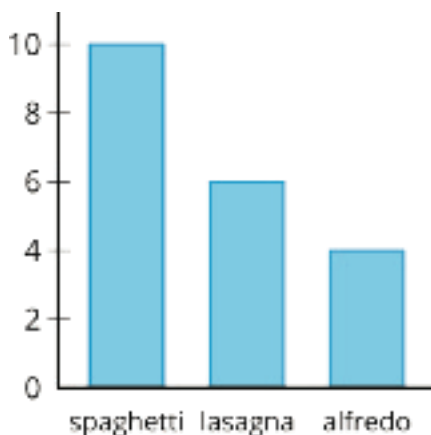
Solution

- 0.20
- 0.17
- Kiran's estimate is probably more accurate, because he used a much larger sample than Tyler. Sample proportions from larger samples tend to be more tightly clustered around that value of the population proportion. Note it is still possible for Tyler's estimate to be more accurate, coincidentally.

Problem 2

Statement

Han surveys a random sample of students about their favorite pasta dish served by the cafeteria and makes a bar graph of the results.



Estimate the proportion of the students who like lasagna as their favorite pasta dish.

Solution

0.3 (or $\frac{6}{20}$ or equivalent)

Problem 3

Statement

Elena wants to know what proportion of people have cats as pets. Describe a process she could use to estimate an answer to her question.

Solution

Answers vary. Sample response: Find a random sample of about 50 people and ask them if they have a pet cat. Once the responses are recorded, count the number of “yes” answers and divide that by 50 to get an estimate of the population proportion.

Problem 4

Statement

The science teacher gives daily homework. For a random sample of days throughout the year, the median number of problems is 5 and the IQR is 2. The Spanish teacher also gives daily homework. For a random sample of days throughout the year, the median number of problems is 10 and the IQR is 1. If you estimate the median number of science homework problems to be 5 and the median number of Spanish problems to be 10, which is more likely to be accurate? Explain your reasoning.

Solution

The Spanish estimate is more likely to be accurate. When the measure of variability (the IQR) is larger, it is hard to get a good estimate of the population.

(From Unit 8, Lesson 15.)

Problem 5

Statement

Diego wants to survey a sample of students at his school to learn about the percentage of students who are satisfied with the food in the cafeteria. He decides to go to the cafeteria on a Monday and ask the first 25 students who purchase a lunch at the cafeteria if they are satisfied with the food.

Do you think this is a good way for Diego to select his sample? Explain your reasoning.

Solution

No, this is not a good way to select a sample. Explanations vary. Sample explanation: Students who are buying lunch at the cafeteria may be choosing to buy their lunch because they like the cafeteria food. Students who bring lunch from home won't be included in the sample.

(From Unit 8, Lesson 14.)

Lesson 17: More about Sampling Variability

Goals

- Compare and contrast (orally) a distribution of sample means and the distribution of the population.
- Generalize that an estimate for the center of a population distribution is more likely to be accurate when it is based on a larger random sample.
- Interpret (orally and in writing) a dot plot that displays the means of multiple samples from the same population.

Learning Targets

- I can use the means from many samples to judge how accurate an estimate for the population mean is.
- I know that as the sample size gets bigger, the sample mean is more likely to be close to the population mean.

Lesson Narrative

This lesson is optional. It goes beyond necessary grade-level standards to examine the accuracy of estimates for population characteristics based on many samples. The lesson builds a solid foundation for future grades to build upon, but may be shortened or skipped due to time constraints.

In this lesson, students continue to look at multiple samples from the same population. Examining the structure of dot plots composed of the means from several samples, they see that different samples from the same population can have different means, but that most of these means cluster around the mean of the population (MP7). They consider how far off their estimate might be if they didn't know the mean of the population but they did know the sample mean. Additionally, students see that larger samples usually produce sample means that are less variable from one another and can more accurately estimate population means.

Alignments

Building On

- 6.SP.A.3: Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Addressing

- 7.SP.A: Use random sampling to draw inferences about a population.
- 7.SP.A.2: Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the

same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

Building Towards

- 7.SP.A.2: Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports
- Think Pair Share

Required Materials

Sticky notes

Required Preparation

For the "Reaction Population" activity, prepare a large number line for the class to use as a dot plot of their sample means from the warm-up of this lesson. Provide students with sticky notes to include as dots for this dot plot.

Student Learning Goals

Let's compare samples from the same population.

17.1 Average Reactions

Warm Up: 10 minutes

Students calculate the mean of a sample collected in a previous lesson to compare with their partners. Students experience first-hand that different samples from the same population can produce different results. In the following activities of this lesson, students will use the data they have collected here to develop a deeper understanding of sampling variability.

Building On

- 6.SP.A.3

Building Towards

- 7.SP.A.2

Launch

Arrange students in groups of 2 so that different partners are used from the ones used in the Reaction Times activity in the previous lesson.

Remind students that the numbers came from a survey of all 120 seniors from a certain school. The numbers represent their reaction time in seconds from an activity in which they clicked a button as soon as they noticed that a square changed color. These 120 values are the population for this activity.

Give students 2 minutes of quiet work time followed by partner work time and a whole-class discussion.

Student Task Statement

The other day, you worked with the reaction times of twelfth graders to see if they were fast enough to help out at the track meet. Look back at the sample you collected.

1. Calculate the mean reaction time for your sample.
2. Did you and your partner get the same sample mean? Explain why or why not.

Student Response

Answers vary. Sample responses:

1. 0.43 seconds
2. The two means were slightly different, but both close to 0.43 seconds. Since there were different samples, the means were slightly different.

Activity Synthesis

The purpose of the discussion is for students to think about how the data they collected relates to the population data.

Some questions for discussion:

- “Based on the information you currently know, estimate the mean of the population. Explain your reasoning.” (Since both my partner and I got means close to 0.4, I think the population mean will be a little greater than 0.4.)
- “If each person selected 40 reaction times for their sample instead of 20, do you think this would provide a better estimate for the population mean?” (Since the sampling method is random, and thus fair, it should produce a better estimate.)

17.2 Reaction Population

Optional: 15 minutes

In the warm-up, students computed the mean of a sample. In this activity, a dot plot is created by the class that includes all of the calculated sample means. Students then compare that display to the data from the entire population to better understand the information that can be gained from a sample mean (MP7). In the discussion that follows the activity, students look at similar displays of sample means for samples of different sizes (MP8). Students should see that larger samples tend to more accurately estimate the population means than smaller samples.

Addressing

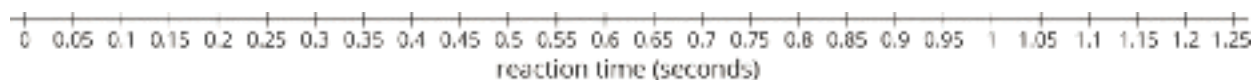
- 7.SP.A
- 7.SP.A.2

Instructional Routines

- MLR8: Discussion Supports

Launch

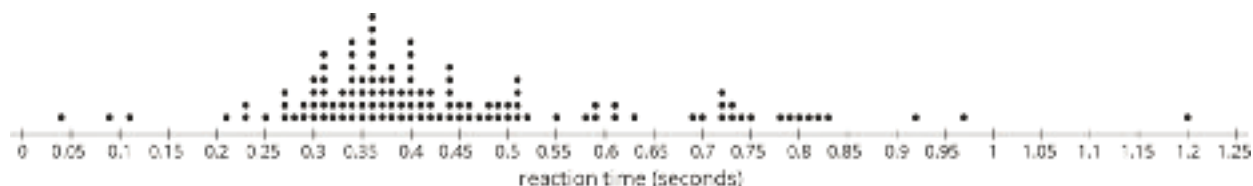
Display the basis for a dot plot of sample means for all to see.



Allow students 5 minutes to create the class dot plot and complete the first set of questions.

Ask students to include their sample means from the previous task to the display for all to see (since the samples were originated in pairs, there will be many repeats. It is ok to include the repeats or ensure only 1 of the original partners includes their point on the display). They may do this by adding a dot to the display on the board, placing a sticky note in the correct place, or passing a large sheet of paper around the class.

After students have had a chance to answer the first set of questions, pause the class to display the dot plot of the the reaction times for the entire population of 120 seniors for all to see.



Provide students 5 minutes to complete the problems. Follow with a whole-class discussion.

Anticipated Misconceptions

The center of the dot plot created by the class can be thought of as a mean of sample means. The phrase can be difficult for students to think through, so remind them what the dot they added to the dot plot represents and how they calculated that value. Consider displaying the class dot plot for the rest of the unit to help students remember this example to understand similar dot plots.

Student Task Statement

Your teacher will display a blank dot plot.

1. Plot your sample mean from the previous activity on your teacher's dot plot.
2. What do you notice about the distribution of the sample means from the class?
 - a. Where is the center?
 - b. Is there a lot of variability?
 - c. Is it approximately symmetric?
3. The population mean is 0.442 seconds. How does this value compare to the sample means from the class?

Pause here so your teacher can display a dot plot of the population of reaction times.

4. What do you notice about the distribution of the population?
 - a. Where is the center?
 - b. Is there a lot of variability?
 - c. Is it approximately symmetric?
5. Compare the two displayed dot plots.
6. Based on the distribution of sample means from the class, do you think the mean of a random sample of 20 items is likely to be:
 - a. within 0.01 seconds of the actual population mean?
 - b. within 0.1 seconds of the actual population mean?

Explain or show your reasoning.

Student Response

Answers vary. Sample responses:

1. No response required.
2.
 - a. The center of the distribution is about 0.44 seconds.
 - b. There is not a lot of variability. Most of the values are very close to 0.44 seconds.
 - c. The distribution is approximately symmetric.
3. The center of the class distribution of sample means is about the same as the population mean.

4.
 - a. The center of the population distribution is around 0.44 seconds
 - b. There is relatively large variability.
 - c. The distribution is approximately symmetric, but there are some times that are much longer than the bulk of the data.
5. The center of the two dot plots are in about the same place, but the dot plot of sample means shows much less variability.
6.
 - a. Most of the dots in our sample mean plot are not within 0.01 seconds of the center, so I do not think the mean for a random sample of 20 would be that close.
 - b. Most of the dots in our sample mean plot are within 0.1 seconds of the center, so I think a random sample mean would probably be within 0.1 seconds of the population mean.

Activity Synthesis

Display the dot plot of sample means for all to see throughout the unit. It may be helpful to refer to this display when viewing dot plots of sample means in future lessons.

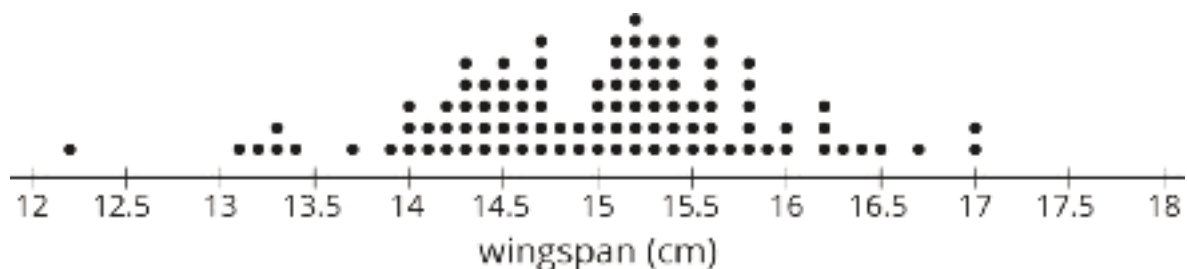
The purpose of the discussion is for students to understand that the centers of the two dot plots in the activity are close (if not the same), but the sample means have less variability and the shapes of the population distribution and the sample mean distribution are probably different. Additionally, students should use the dot plots shown here to see that larger samples have sample means that are still centered around the population mean, but have less variability than smaller samples. Therefore, larger samples should provide better estimates of the population mean than smaller samples do.

Items for further discussion:

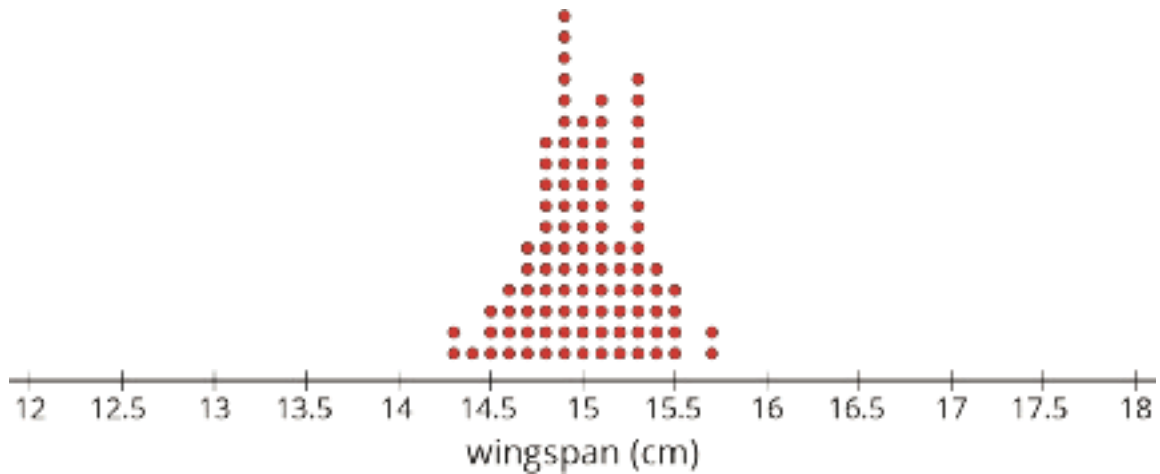
- “If the sample size were increased to 30 instead of 20, what do you think the distribution might look like?”

For a different set of data representing the wingspan in centimeters of a certain species of bird, compare the following dot plots.

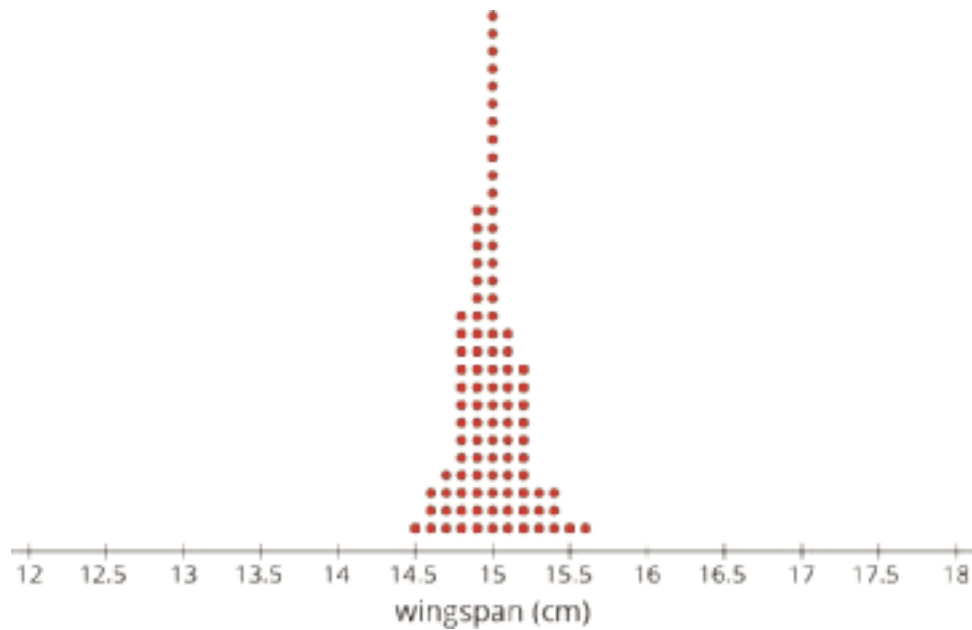
- Here is a dot plot of the population data.



- Here is a dot plot of the sample means for 100 different random samples each of size 10



- Here is a dot plot of the sample means for 100 different random samples each of size 30.



- “What do you notice about the distributions of the sample means as the sample size increases?” (The variability decreases, but the center stays in about the same place.)
- “Use the dot plots to explain why the sample mean from a random sample of size 30 would be a better estimate of the population mean than the sample mean from a random sample of size 10.” (Since the variability is less, the sample mean is probably closer to the true mean with size 30 than with size 10.)

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Provide students with a graphic organizer to support their participation during the synthesis. Invite students to describe what to look for to determine variability and shape of the population distribution and to include examples for each.

Supports accessibility for: Conceptual processing; Organization

Access for English Language Learners

Conversing: MLR8 Discussion Supports. As students compare the wingspan dot plots, encourage students to use sentence frames such as: “When the sample size is 10, I noticed that the distribution . . .”, “When the sample size is 30, I noticed that the distribution . . .”, and “It’s better to have a larger/smaller sample size to estimate the population mean because . . .” The listener should press for more detail and explanation by asking for evidence from the dot plots. This will help students compare the dot plots and make conclusions about the mean of the population through communicating with their partner.

Design Principle(s): Support sense-making; Cultivate conversation

17.3 How Much Do You Trust the Answer?

Optional: 10 minutes

This activity is a follow-up to the context used in the activities Three Different TV Shows and Who’s Watching What?, but also follows the ideas of sample means from this lesson.

In this activity, students continue to look at how variability in the sample means can be used to think about the accuracy of an estimate of a population mean. If the means from samples tend to be very spread out, then there is reason to believe that the mean from a single sample may not be especially close to the mean for the population. If the means from samples tend to be tightly grouped, then there is reason to believe that the mean from a single sample is a good estimate of the mean for the population (MP7).

Addressing

- 7.SP.A.2

Instructional Routines

- MLR2: Collect and Display
- Think Pair Share

Launch

Arrange students in groups of 2.

Tell students that, although the dot plots seem to have a similar shape, attention should be given to the scale of the horizontal axis.

Allow students 3–5 minutes of quiet work time followed by partner and whole-class discussions.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts. After students have solved the first 2-3 population mean estimate problems, check-in with either select groups of students or the whole class. Invite students to share the strategies they have used so far as well as any questions they have before continuing.

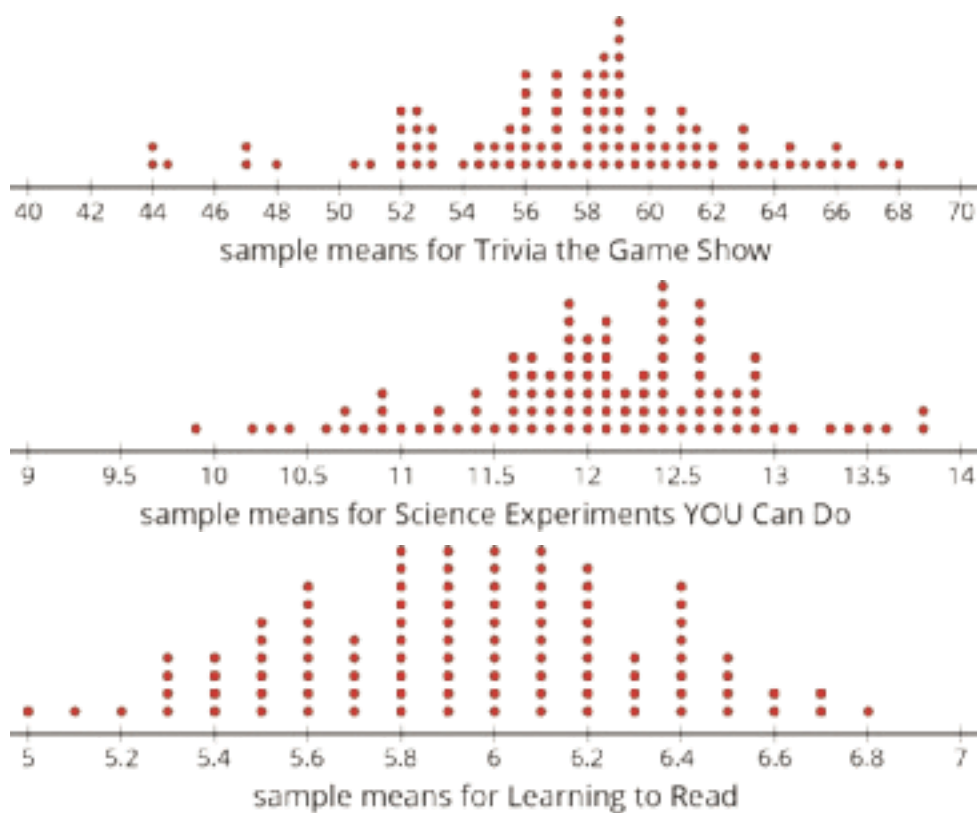
Supports accessibility for: Organization; Attention

Anticipated Misconceptions

Students may get confused about what the dot plots represent. Refer to the class dot plot of sample means from the reaction time activity to help them understand that a single dot on the dot plot represents a mean from a single sample. Each dot plot shows means from several different samples.

Student Task Statement

The other day you worked with 2 different samples of viewers from each of 3 different television shows. Each sample included 10 viewers. Here are the mean ages for 100 different samples of viewers from each show.



- For each show, use the dot plot to estimate the *population* mean.
 - Trivia the Game Show
 - Science Experiments YOU Can Do
 - Learning to Read
- For each show, are most of the sample means within 1 year of your estimated population mean?
- Suppose you take a new random sample of 10 viewers for each of the 3 shows. Which show do you expect to have the new sample mean closest to the population mean? Explain or show your reasoning.

Student Response

- "Trivia the Game Show": 58 years old. "Science Experiments YOU Can Do": 12 years old.
"Learning to Read": 6 years old.
- "Science Experiments YOU Can Do" and "Learning to Read" have most of the sample means within 1 year of the estimated population mean.
- "Learning to Read" should produce the best estimates of population mean since it has the least variability in the sample means.

Are You Ready for More?

Market research shows that advertisements for retirement plans appeal to people between the ages of 40 and 55. Younger people are usually not interested and older people often already have a plan. Is it a good idea to advertise retirement plans during any of these three shows? Explain your reasoning.

Student Response

Answers vary. Sample response: It might be worth it to advertise during “Trivia the Game Show”. Although the mean appears to be about 59, about half of the viewers are still in the desired age range of 40 to 55. It might depend on the total number of viewers to make the decision.

Activity Synthesis

The purpose of the discussion is to think about what the information in the given dot plots tells us about the accuracy of an estimate of a population mean based on the sample mean from a single sample.

Consider asking these questions for discussion:

- “In the first dot plot for ‘Trivia the Game Show,’ what does the dot at 68 represent?” (There is a sample of 10 viewers that has a mean age of 68 years.)
- “Based on the dot plot shown for ‘Learning to Read,’ do you think any 7 year olds were included in the data?” (Yes. Since one of the sample means is 6.8, there are probably a few 7 year olds and some 6 year olds in that sample.)
- “Why is this kind of dot plot useful?” (It shows how variable the means from samples can be, so it gives a good idea of how accurate a sample mean might be as an estimate of a population mean.)
- “Why might it be difficult to obtain a dot plot like this?” (Each dot represents a sample of 10. It might be hard to get 100 different samples of size 10.)

Access for English Language Learners

Representing, Speaking: MLR2 Collect and Display. Create a visual display with the heading “Dot Plots of Sample Means.” As students respond to the discussion questions, write down the words and phrases students use on the visual display. Listen for and amplify words and phrases such as “each dot represents...,” “variability of the sample means,” and “population mean.” As students review the language collected in the visual display, encourage students to clarify the meaning of a word or phrase. For example, a phrase such as “each dot represents a sample” can be clarified by restating it as “each dot represents a sample mean.” This will help students make sense of dot plots of sample means as they draw conclusions about populations.

Design Principle(s): Support sense-making: Maximize meta-awareness

Lesson Synthesis

Consider asking these discussion questions to emphasize the main ideas of the lesson:

- “Do different samples always have the same measure of center?” (No, but they should be relatively close.)
- “If multiple researchers select their own random samples and compute the mean of their sample, how can this collection of means be used to estimate the mean for the population?” (They could create a dot plot with their sample means and find the center of that distribution to estimate the true population mean.)
- “Explain why larger samples are generally better than smaller ones.” (Larger samples tend to produce more accurate estimates for the population mean since the sample means have less variability than sample means computed from smaller samples.)

17.4 How Much Mail?

Cool Down: 5 minutes

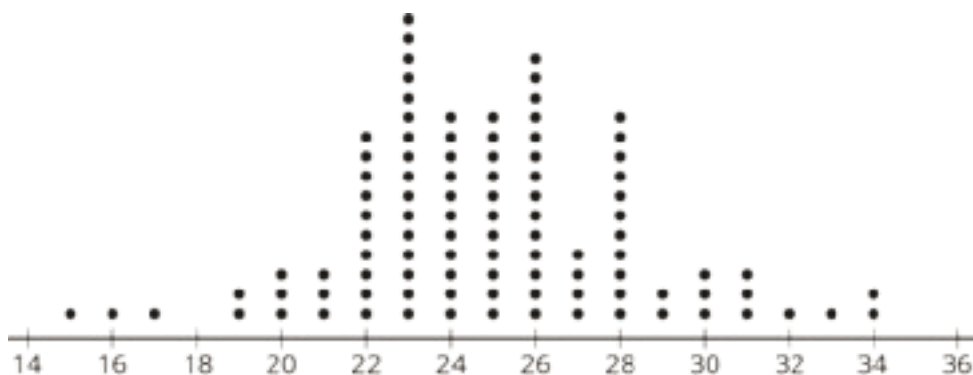
This cool-down assesses whether students understand what information can be gained from looking at a dot plot of sample means as well as the idea that larger samples tend to produce sample means that are less variable.

Addressing

- 7.SP.A
- 7.SP.A.2

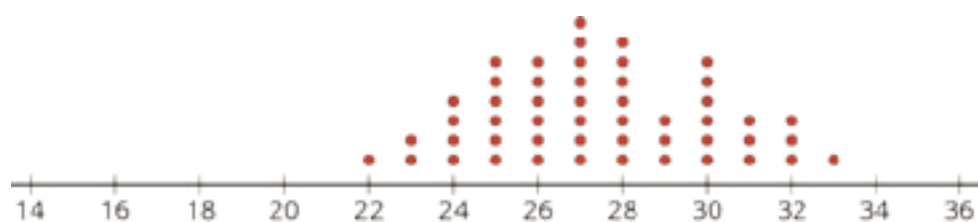
Student Task Statement

Jada collects data about the number of letters people get in the mail each week. The population distribution is shown in the dot plot.

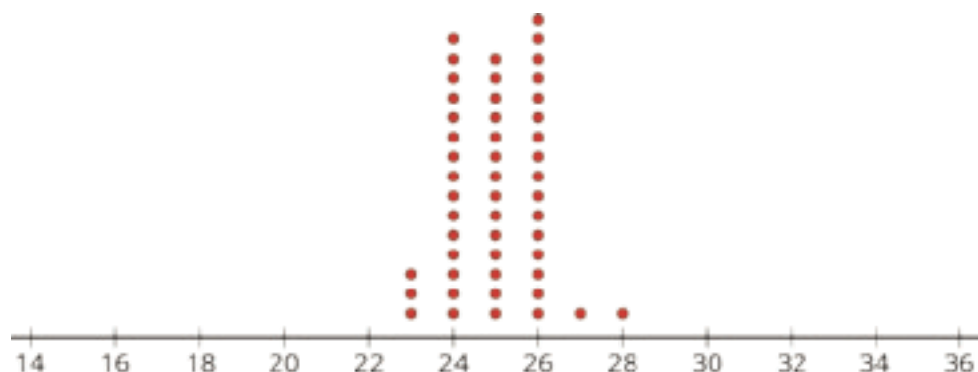


Which of the following dot plots are likely to represent the means from samples of size 10 from this population? Explain your reasoning.

Dot Plot 1



Dot Plot 2

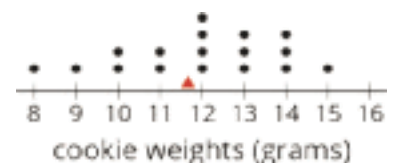


Student Response

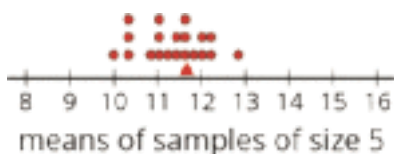
Dot plot 2 since it has the same center, but less variability than the population data. Dot plot 1 also has less variability, but has a different center than the population data, so it is probably not generated by sample means from the original population.

Student Lesson Summary

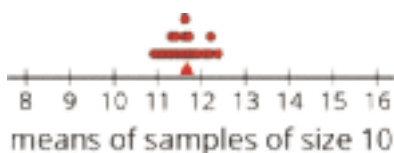
This dot plot shows the weights, in grams, of 18 cookies. The triangle indicates the mean weight, which is 11.6 grams.



This dot plot shows the *means* of 20 samples of 5 cookies, selected at random. Again, the triangle shows the mean for the *population* of cookies. Notice that most of the sample means are fairly close to the mean of the entire population.



This dot plot shows the means of 20 samples of 10 cookies, selected at random. Notice that the means for these samples are even closer to the mean for the entire population.



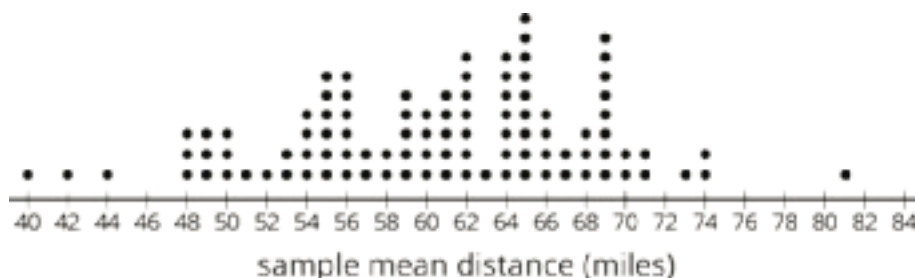
In general, as the sample size gets bigger, the mean of a sample is more likely to be closer to the mean of the population.

Lesson 17 Practice Problems

Problem 1

Statement

One thousand baseball fans were asked how far they would be willing to travel to watch a professional baseball game. From this population, 100 different samples of size 40 were selected. Here is a dot plot showing the mean of each sample.



Based on the distribution of sample means, what do you think is a reasonable estimate for the mean of the population?

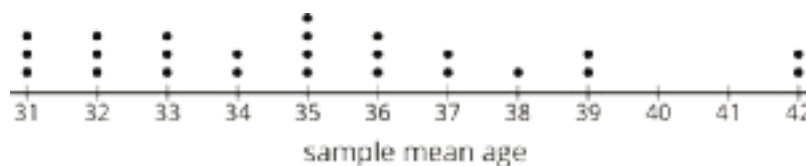
Solution

Reasonable answers are between 59 and 65.

Problem 2

Statement

Last night, everyone at the school music concert wrote their age on a slip of paper and placed it in a box. Today, each of the students in a math class selected a random sample of size 10 from the box of papers. Here is a dot plot showing their sample means, rounded to the nearest year.



- Does the number of dots on the dot plot tell you how many people were at the concert or how many students are in the math class?
- The mean age for the population was 35 years. If Elena picks a new sample of size 10 from this population, should she expect her sample mean to be within 1 year of the population mean? Explain your reasoning.
- What could Elena do to select a random sample that is more likely to have a sample mean within 1 year of the population mean?

Solution

- a. The math class
- b. No. Only 9 of the 25 sample means in the dot plot are within 1 year of the population mean. While it could happen, it is more likely Elena's sample mean will be more than 1 year away from the population mean.
- c. Select more than 10 papers for her sample

Problem 3

Statement

A random sample of people were asked which hand they prefer to write with. "l" means they prefer to use their left hand, and "r" means they prefer to use their right hand. Here are the results:

l	r	r	r	r	r	r	r	r
r	l	r	r	r	r			

Based on this sample, estimate the proportion of the population that prefers to write with their left hand.

Solution

$\frac{2}{15}$ (or about 13%)

(From Unit 8, Lesson 16.)

Problem 4

Statement

Andre would like to estimate the mean number of books the students at his school read over the summer break. He has a list of the names of all the students at the school, but he doesn't have time to ask every student how many books they read.

What should Andre do to estimate the mean number of books?

Solution

Answers vary. Sample response: Andre can estimate the mean by using the list he has to select a random sample of the students, then asking the students in the sample how many books they read over the summer. He could choose a sample size, but it would be better to take a larger sample if he can, because this probably gives him a more accurate estimate.

(From Unit 8, Lesson 15.)

Problem 5

Statement

A hockey team has a 75% chance of winning against the opposing team in each game of a playoff series. To win the series, the team must be the first to win 4 games.

- a. Design a simulation for this event.
- b. What counts as a successful outcome in your simulation?
- c. Estimate the probability using your simulation.

Solution

- a. Answers vary. Sample response: Make a spinner with 4 equal sections labeled 1, 2, 3, and 4. Spin the spinner, and record the outcomes as wins (if the spinner lands on 1, 2, or 3) or losses (when the spinner lands on 4) until one team wins the series. Repeat this process several times.
- b. Answers vary. Sample response: Count the times that 1, 2, or 3 appear as winning a game and 4 as losing. A trial is a success if 1, 2, or 3 appear four times before 4 appears four times.
- c. Answers vary. The actual probability is over 99%.

(From Unit 8, Lesson 10.)

Lesson 18: Comparing Populations Using Samples

Goals

- Calculate the difference between the mean or median of two samples from different populations, and express it as a multiple of the MAD or IQR.
- Interpret a pair of box plots, including the amount of visual overlap between the two distributions.
- Justify (orally and in writing) whether there is likely to be a meaningful difference between two populations, based on a sample from each population.

Learning Targets

- I can calculate the difference between two medians as a multiple of the interquartile range.
- I can determine whether there is a meaningful difference between two populations based on a sample from each population.

Lesson Narrative

In previous lessons, students examined the distributions of two entire populations to decide whether or not they were very different. In this lesson, students use samples to make comparative inferences about populations.

Students see that if samples of two different populations have only a small difference between their measures of center (relative to their variability), then we cannot say that there is a meaningful difference between the measures of center of the populations (MP2). Due to sampling variability, it is possible that the two populations may not be very different. However, if samples from two different populations have a large difference between their measures of center (relative to their variability), then we can say that there is likely to be a meaningful difference between the measures of center of the two populations.

Alignments

Building On

- 6.SP.B.5.c: Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

Addressing

- 7.SP.B.3: Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the

variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.

- 7.SP.B.4: Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

Building Towards

- 7.SP.B: Draw informal comparative inferences about two populations.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR5: Co-Craft Questions
- MLR7: Compare and Connect
- Think Pair Share

Student Learning Goals

Let's compare different populations using samples.

18.1 Same Mean? Same MAD?

Warm Up: 5 minutes

This warm-up reminds students of the meanings of mean and MAD by comparing two sets of data with similar but different values and asking whether they will have the same means or MADs or both.

Building On

- 6.SP.B.5.c

Building Towards

- 7.SP.B

Launch

Explain to students that the pairs of data sets are: A and B, X and Y, and P and Q.

Anticipated Misconceptions

For students who have a difficult time starting without calculating, help them to compare the values in the ones place for the first and third pairs of data.

Student Task Statement

Without calculating, tell whether each pair of data sets have the same mean and whether they have the same mean absolute deviation.

1.								
set A	1	3	3	5	6	8	10	14
set B	21	23	23	25	26	28	30	34
2.								
set X	1	2	3	4	5			
set Y	1	2	3	4	5	6		
3.								
set P	47	53	58	62				
set Q	37	43	68	72				

Student Response

1. Data sets A and B have different means, but the same MADs.
2. Data sets X and Y have different means and different MADs.
3. Data sets P and Q have the same means, but different MADs.

Activity Synthesis

The purpose of the discussion is to bring out methods students used to notice whether the pairs of data sets had the same mean or MAD or both.

Poll the class for each pair of data sets as to whether they had the same mean, MAD, both, or neither.

After students have had a chance to register their vote, ask some students to explain their reasoning for their answer.

18.2 With a Heavy Load

10 minutes

In a previous lesson, students compared heights of two teams of people when the entire populations were known. In this activity, students only have access to data from a sample of each population and are asked to determine if the populations are different based on the sample.

Students construct informal arguments to explain why the different samples come from populations that are meaningfully different or not (MP3).

Addressing

- 7.SP.B.3
- 7.SP.B.4

Instructional Routines

- MLR5: Co-Craft Questions
- Think Pair Share

Launch

Arrange students in groups of 2. Allow students 3–5 minutes quiet work time followed by partner and whole-class discussions.

Access for English Language Learners

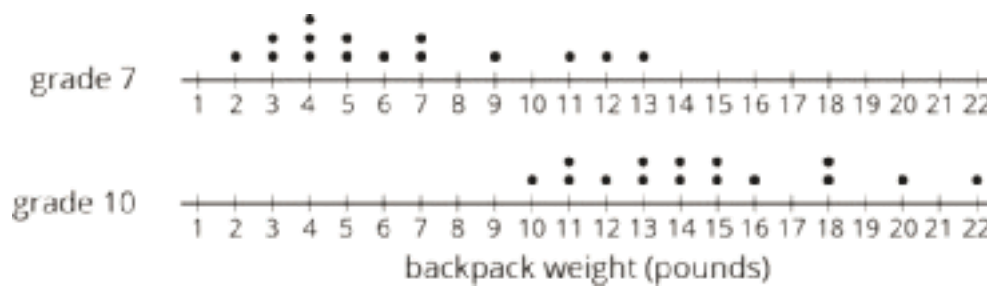
Writing, Conversing: MLR5 Co-Craft Questions. Display the initial task statement and the dot plots for grade 7 and grade 10 without revealing the questions that follow. Ask pairs to write down possible mathematical questions that can be answered about the situation. As students share their questions with the class, listen for and amplify questions that ask about the mean weight of the backpacks and whether there is a meaningful difference between the weight of all seventh-grade and tenth-grade backpacks. This will help students write and verbalize their questions about the dot plots of the samples as they attempt to draw conclusions about the populations.


Design Principle(s): Optimize output; Cultivate conversation

Student Task Statement

Consider the question: Do tenth-grade students' backpacks generally weigh more than seventh-grade students' backpacks?

Here are dot plots showing the weights of backpacks for a random sample of students from these two grades:



- 
1. Did any seventh-grade backpacks in this sample weigh more than a tenth-grade backpack?
 2. The mean weight of this sample of seventh-grade backpacks is 6.3 pounds. Do you think the mean weight of backpacks for *all* seventh-grade students is exactly 6.3 pounds?
 3. The mean weight of this sample of tenth-grade backpacks is 14.8 pounds. Do you think there is a meaningful difference between the weight of all seventh-grade and tenth-grade students' backpacks? Explain or show your reasoning.

Student Response

1. Yes, three seventh-grade backpacks weighed more than the lightest tenth-grade backpack.
2. No. 6.3 pounds is the mean of a sample, and the population mean will probably be at least a little different.
3. Answers vary. Sample response: There is still probably a meaningful difference in the mean weights since there is very little overlap.

Activity Synthesis

The purpose of the discussion is for students to think about how comparing groups by using data from samples differs from comparing groups when the population is known.

Ask partners to share their decision about whether the groups had a meaningful difference with the class.

Consider asking these questions for discussion:

- “Compare the information in this activity to the information about team heights given in an earlier lesson.” (In that lesson, we had data from the entire population and here it is only a sample.)
- “Is the overlap of the data more important when you only have a sample or when you have data from the population? Explain your reasoning.” (It is more important when you only have a sample. If there is overlap with only some of the data, it's possible there is more overlap when we include more data from the population.)
- “Is it *possible* that the data in the two samples were drawn from population data that is identical?” (It is unlikely, but possible.)

Access for Students with Disabilities

Representation: Internalize Comprehension. Use color and annotations to illustrate student thinking. As students describe their reasoning and the relationships they noticed for each sample, use color and annotations to scribe their thinking on a display of each problem so that it is visible for all students.

Supports accessibility for: Visual-spatial processing; Conceptual processing

18.3 Do They Carry More?

15 minutes

The data in the previous activity came from only one sample for each grade. This may not be enough information to make a very good determination about the entire seventh and tenth grade populations. In this activity, students look at different samples from the same population to see that their means are relatively close based on the MADs of the samples. This concept can be reversed to say that if two samples have means that are *not* very close, then the samples likely came from populations that are quite different. A general rule is given to determine whether two populations are meaningfully different based on the mean and MAD from a sample of each (MP6).

Addressing

- 7.SP.B.3

Instructional Routines

- MLR7: Compare and Connect

Launch

Keep students in groups of 2. Allow students 5 minutes of partner work time, then pause the class to assign samples and explain the general rule.

Ask students to pause after the third question in order to explain the general rule and assign a sample to each group. After all students have paused, assign each group one of the 10 samples to work with for the last 2 questions. Further, explain to students:

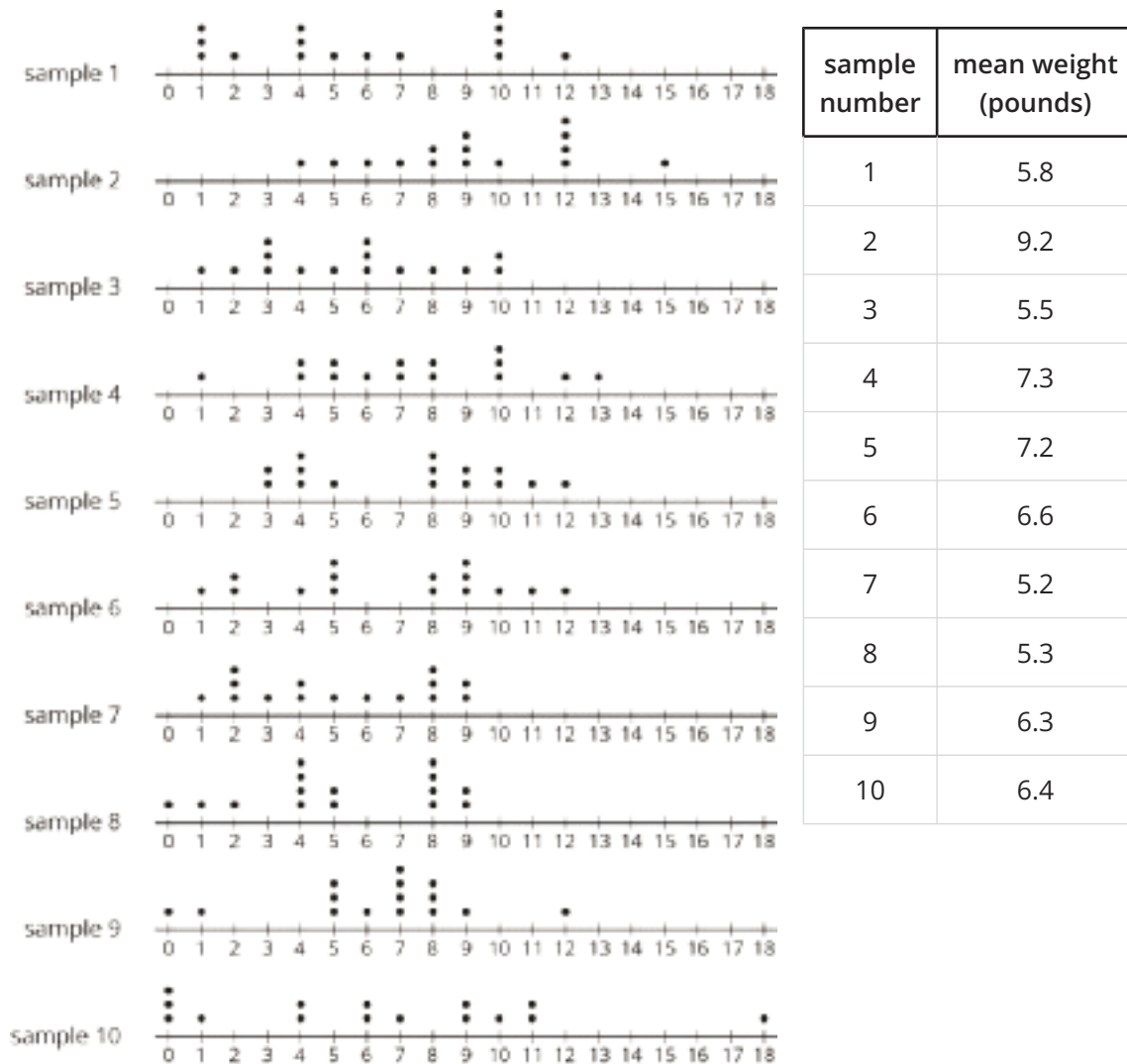
- As a general rule, if two populations have the same mean (or median) and similar variability, the sample means (or medians) should be within 2 MADs (or IQRs) of one another.
- If the sample means (or medians) are more than 2 MADs (or IQRs) apart, it is very likely that the population means (or medians) are different. We will say that there is a *meaningful difference* between the two population means (or medians).
- If the sample means (or medians) are less than or equal to 2 MADs (or IQRs) apart, it is more difficult to say that the two population means (or medians) are very different. In this case we

will say that the samples do not provide evidence that the population means (or medians) differ.

Give students 5 more minutes of partner work time followed by a whole-class discussion.

Student Task Statement

Here are 10 more random samples of seventh-grade students' backpack weights.



1.
 - a. Which sample has the highest mean weight?
 - b. Which sample has the lowest mean weight?
 - c. What is the difference between these two sample means?
2. All of the samples have a mean absolute deviation of about 2.8 pounds. Express the difference between the highest and lowest sample means as a multiple of the MAD.

3. Are these samples very different? Explain or show your reasoning.
4. Remember our sample of tenth-grade students' backpacks had a mean weight of 14.8 pounds. The MAD for this sample is 2.7 pounds. Your teacher will assign you one of the samples of seventh-grade students' backpacks to use.
 - a. What is the difference between the sample means for the the tenth-grade students' backpacks and the seventh-grade students' backpacks?
 - b. Express the difference between these two sample means as a multiple of the larger of the MADs.
5. Do you think there is a meaningful difference between the weights of all seventh-grade and tenth-grade students' backpacks? Explain or show your reasoning.

Student Response

1.
 - a. Sample 2
 - b. Sample 7
 - c. 4 pounds ($9.2 - 5.2 = 4$)
2. $4 \approx 2.8 \cdot 1.43$ since $4 \div 2.8 \approx 1.43$
3. No. All of the samples came from the same population and are all within 2 MADs of one another.
4. Answers vary. Possible responses:

sample	1	2	3	4	5	6	7	8	9	10
difference in means	9	5.6	9.3	7.5	7.6	8.2	9.6	9.5	8.5	8.4
multiplier for the MAD	3.2	2	3.3	2.7	2.7	2.9	3.4	3.4	3.0	3.0

5. Yes. The difference in the means is more than 2 MADs, so the means have a meaningful difference.

Activity Synthesis

The purpose of the discussion is for students to understand the general rule for determining if two samples suggest a meaningful difference between their populations.

Select at least one group assigned to each of the samples to share their responses to the last 2 questions and record for all to see. Note that all 10 samples from the seventh-grade students

have means that are within 2 MADs of one another, but the mean from the tenth-grade student sample is at least 2 MADs away from the mean of each of the seventh grade student samples.

Note that the general rule only has two possible outcomes: “There is a meaningful difference.” or “There is not enough information to say there is a meaningful difference.” If the means are less than 2 MADs apart, the general rule cannot say whether two samples were drawn from populations that contain identical data.

Ask students, “Based only on the dot plots for the 10 samples, would you have guessed that they all might have come from the same population? Explain your reasoning.” (Maybe. There is a lot of overlap among all of the samples.)

Access for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding. Include the following term and maintain the display for reference throughout the unit: meaningful difference.

Supports accessibility for: Language; Conceptual processing

Access for English Language Learners

Representing, Speaking, Listening: MLR7 Compare and Connect. Invite students to share their written responses to the last two questions with 2–3 other students. As students investigate each other’s work, ask students to make observations about the difference in sample means for the tenth-grade and seventh-grade students’ backpacks. Ask students how this observation helps them determine whether there is a meaningful difference between the weights of the backpacks. Listen for and amplify the language students use to describe the general rule for determining whether two samples suggest a meaningful difference between their populations. This will help students make sense of the general rule and discuss how it is used with a variety of samples.

Design Principle(s): Optimize output (for representation); Maximize meta-awareness

18.4 Steel from Different Regions

15 minutes

In previous lessons, students used sample data to estimate population means and proportions and determined if there is a meaningful difference in population means based on sample means. In this activity, students practice using the general rule developed in the previous activity by estimating the measure of center for a population and comparing populations based on those estimates as well as the associated measure of variability (MP3).

Addressing

- 7.SP.B.3
- 7.SP.B.4

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Keep students in groups of 2.

Explain to students that different regions had different raw materials and techniques for constructing metal. One way of testing ancient metal is by looking at the carbon content in the steel. In some cases, this content could determine the region where the metal was made.

Ask students how the general rule from the previous activity might be adapted to use median and interquartile range (IQR) rather than mean and MAD.

Allow students 10 minutes of partner work time followed by a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, pause to check for understanding after 3–5 minutes of work time.

Supports accessibility for: Organization; Attention

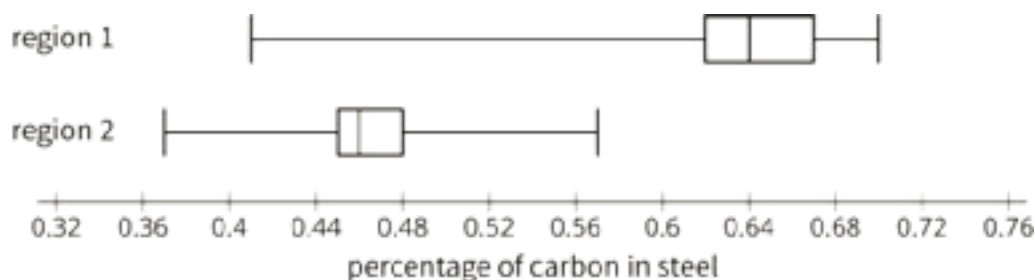
Access for English Language Learners

Writing, Conversing: MLR1 Stronger and Clearer Each Time. Ask students to write a response to the final question: “The anthropologists who conducted the study concluded there was a meaningful difference between the steel from these regions. Do you agree? Explain or show your reasoning.” Ask each student to meet with 2–3 other partners for feedback. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language. For example, “What is the general rule for determining a meaningful difference between populations?” and “How do you know there is a meaningful difference between the steel from these regions?” Students can borrow ideas and language from each partner to refine and clarify their original explanation. This will help students revise and refine their ideas about how to adapt the general rule to use median and IQR rather than mean and MAD.

Design Principle(s): Optimize output (for explanation); Cultivate conversation

Student Task Statement

When anthropologists find steel artifacts, they can test the amount of carbon in the steel to learn about the people that made the artifacts. Here are some box plots showing the percentage of carbon in samples of steel that were found in two different regions:



1. Was there any steel found in region 1 that had:
 - a. *more* carbon than some of the steel found in region 2?
 - b. *less* carbon than some of the steel found in region 2?
2. Do you think there is a meaningful difference between all the steel artifacts found in regions 1 and 2?
3. Which sample has a distribution that is *not* approximately symmetric?
4. What is the difference between the sample medians for these two regions?

	sample median (%)	IQR (%)
region 1	0.64	0.05
region 2	0.47	0.03

5. Express the difference between these two sample medians as a multiple of the larger interquartile range.
6. The anthropologists who conducted the study concluded that there was a meaningful difference between the steel from these regions. Do you agree? Explain or show your reasoning.

Student Response

1.
 - a. Yes. Most of the steel from region 1 had more carbon in it than steel from region 2.
 - b. Yes. Since the left end of the region 1 box plot overlaps with the box plot for region 2, there was at least 1 piece of steel that had less carbon in it than some of the steel from region 2.

2. Answers vary. Sample response: Based on the box plots, there is some overlap, but the boxes look so far apart that I think there will be a meaningful difference.
3. The distribution for region 1 is not symmetric with the very long segment on the left.
4. 0.17%, since $0.64 - 0.47 = 0.17$
5. $0.17 \approx 0.05 \cdot 3.4$
6. I agree with the anthropologists. There is evidence of a meaningful difference because the difference in sample medians is greater than 2 IQRs.

Activity Synthesis

The purpose of the discussion is for students to understand how to adapt the general rule for determining a meaningful difference between populations to median and IQR.

Consider asking these questions for discussion:

- “Why did this problem use median and IQR instead of mean and MAD?” (Since the distribution for region 1 is not symmetric, it makes more sense to use the median. Also the box plots will show the median and IQR, but there is not a good way to know the mean and MAD.)
- “Is there any overlap in the data from the two regions?” (Yes. The smallest percentage of carbon from the region 1 was well below the median from region 2 while the typical percentage of carbon from region 1 is much greater than from region 2.)
- “On the box plot in the activity, draw a dot two IQRs above the median for region 2. Then draw a star two IQRs below the median for region 1. How do these help you see that there is a meaningful difference in the medians?” (The dot is at 0.53 and the star is at 0.54. Since the median for region 1 is not below the dot nor is the median for region 1 above the star, there must be a meaningful difference.)
- “A piece of steel is found in a place between the two regions sampled. Would testing the percentage of carbon from this metal be useful in determining the region from which it came?” (Yes. Since there is a meaningful difference in the percentage of carbon in the steel from the two regions, it should give a good indication which region created the metal.)

Lesson Synthesis

Consider asking these discussion questions to emphasize the main ideas from this lesson:

- “When is it useful to use a median rather than a mean?” (It is useful when the distribution is not approximately symmetric.)
- “What values do you need to calculate from a sample to use the general rule for determining if the measures of center of two populations are meaningfully different?” (The measure of center and measure of variation for each sample should be calculated to compare the groups.)

- “What is the general rule used to determine if the means of two populations are meaningfully different?” (If the difference between the means for the two samples is greater than twice the greater of the MADs, then the means are meaningfully different.)

18.5 Teachers Watching Movies

Cool Down: 5 minutes

This cool-down assesses whether students understand the general rule set out to identify whether the measures of center for two groups are meaningfully different based on a sample of data from each group.

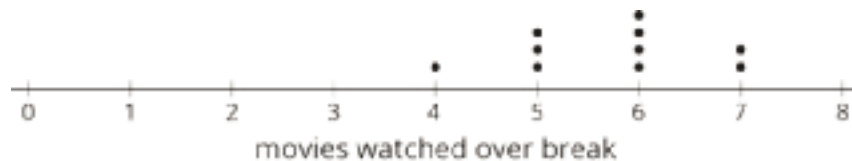
Addressing

- 7.SP.B.3

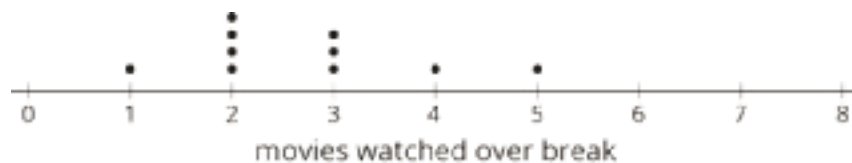
Student Task Statement

Noah is interested in comparing the number of movies watched by students and teachers over the winter break. He takes a random sample of 10 students and 10 teachers and makes a dot plot of their responses.

Students:



Teachers:



Noah then computes the measures of center and variability for each group:

- Students: Mean = 5.7 movies, MAD = 0.76 movies
- Teachers: Mean = 2.7 movies, MAD = 0.9 movies

1. Is Noah's choice of mean and MAD appropriate for the data he has? Explain your reasoning.
2. Should Noah conclude that there is a meaningful difference in the mean number of movies watched over winter break between the two groups? Explain your reasoning.

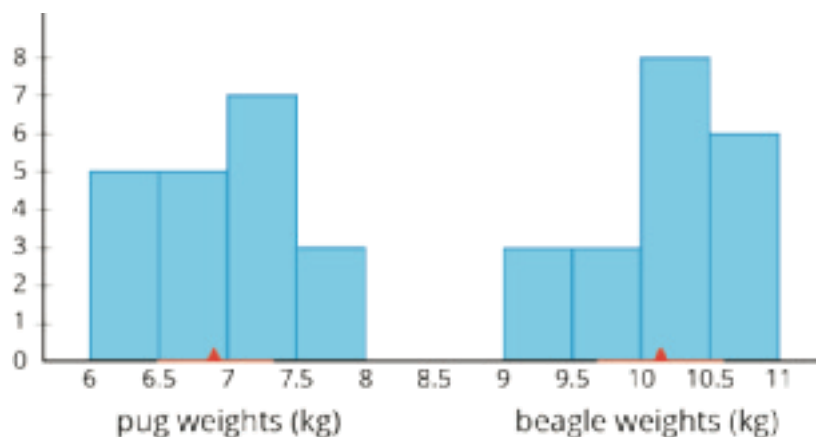
Student Response

1. Yes. Since both samples are approximately symmetric, using the mean is a good choice.

2. Yes. Since the difference in the means is greater than 2 MADs, there is a meaningful difference in the mean number of movies watched. $(5.7 - 2.7) \div 0.9 \approx 3.33$

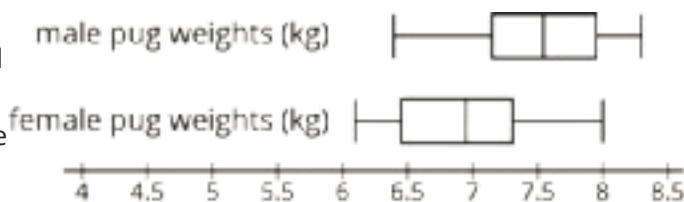
Student Lesson Summary

Sometimes we want to compare two different populations. For example, is there a meaningful difference between the weights of pugs and beagles? Here are histograms showing the weights for a sample of dogs from each of these breeds:



The red triangles show the mean weight of each sample, 6.9 kg for the pugs and 10.1 kg for the beagles. The red lines show the weights that are within 1 MAD of the mean. We can think of these as “typical” weights for the breed. These typical weights do not overlap. In fact, the distance between the means is $10.1 - 6.9$ or 3.2 kg, over 6 times the larger MAD! So we can say there *is* a meaningful difference between the weights of pugs and beagles.

Is there a meaningful difference between the weights of male pugs and female pugs? Here are box plots showing the weights for a sample of male and female pugs:



We can see that the medians are different, but the weights between the first and third quartiles overlap. Based on these samples, we would say there is *not* a meaningful difference between the weights of male pugs and female pugs.

In general, if the measures of center for two samples are at least two measures of variability apart, we say the difference in the measures of center is meaningful. Visually, this means the range of typical values does not overlap. If they are closer, then we don't consider the difference to be meaningful.

Lesson 18 Practice Problems

Problem 1

Statement

Lin wants to know if students in elementary school generally spend more time playing outdoors than students in middle school. She selects a random sample of size 20 from each population of students and asks them how many hours they played outdoors last week. Suppose that the MAD for each of her samples is about 3 hours.

Select **all** pairs of sample means for which Lin could conclude there is a meaningful difference between the two populations.

- A. elementary school: 12 hours, middle school: 10 hours
- B. elementary school: 14 hours, middle school: 9 hours
- C. elementary school: 13 hours, middle school: 6 hours
- D. elementary school: 13 hours, middle school: 10 hours
- E. elementary school: 7 hours, middle school: 15 hours

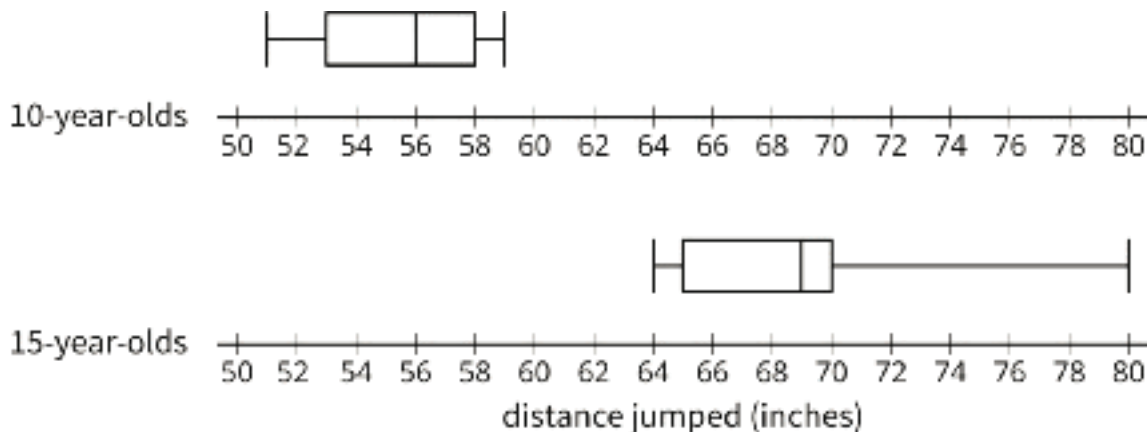
Solution

["C", "E"]

Problem 2

Statement

These two box plots show the distances of a standing jump, in inches, for a random sample of 10-year-olds and a random sample of 15-year-olds.



Is there a meaningful difference in median distance for the two populations? Explain how you know.

Solution

Yes, the difference in medians is 13 inches. This difference is more than 2 IQRs (the IQR is 5 and $13 \div 5 = 2.6$), so there is a meaningful difference in the median distances for 10-year-olds and 15-year-olds.

Problem 3

Statement

The median income for a sample of people from Chicago is about \$60,000 and the median income for a sample of people from Kansas City is about \$46,000, but researchers have determined there is not a meaningful difference in the medians. Explain why the researchers might be correct.

Solution

The medians differ by \$14,000, but if the IQR is larger than about \$7,000, there will not be a meaningful difference between the median salaries in the two cities.

Problem 4

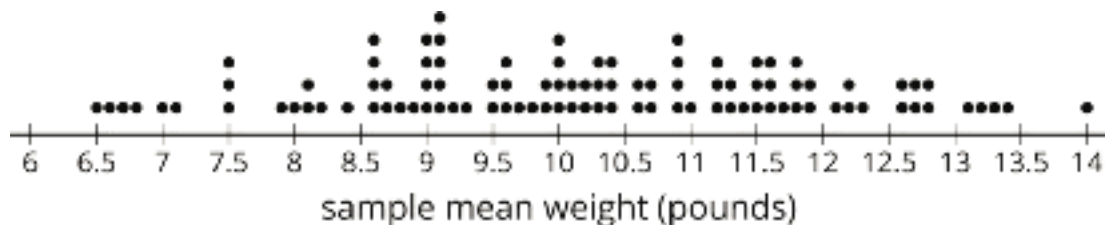
Statement

A farmer grows 5,000 pumpkins each year. The pumpkins are priced according to their weight, so the farmer would like to estimate the mean weight of the pumpkins he grew this year. He randomly selects 8 pumpkins and weighs them. Here are the weights (in pounds) of these pumpkins:


2.9 6.8 7.3 7.7 8.9 10.6 12.3 15.3

- a. Estimate the mean weight of the pumpkins the farmer grew.

This dot plot shows the mean weight of 100 samples of eight pumpkins, similar to the one above.



- b. What appears to be the mean weight of the 5,000 pumpkins?
- c. What does the dot plot of the sample means suggest about how accurate an estimate based on a single sample of 8 pumpkins might be?

- 
- d. What do you think the farmer might do to get a more accurate estimate of the population mean?

Solution

- a. 8.975 pounds
- b. About 10 pounds
- c. The sample means ranged from about 6.5 to 14 pounds. If the actual population mean is about 10 pounds, this shows that a sample mean based on a sample of size 8 might not be very close to the actual population value.
- d. Use a larger sample size.

(From Unit 8, Lesson 17.)

Lesson 19: Comparing Populations With Friends

Goals

- Apply reasoning about center and spread to determine whether two populations are likely to be meaningfully different, and explain (orally and in writing) the reasoning.
- Coordinate (orally) visual displays of data with descriptions of shape, measures of center, and measures of spread.
- Determine what information is needed to solve problems about using samples to compare populations. Ask questions to elicit that information.

Learning Targets

- I can decide what information I need to know to be able to compare two populations based on a sample from each.

Lesson Narrative

Students continue to practice comparing populations by using samples from each population. Outside of the classroom, people who wish to compare groups will not usually have all of the useful statistics presented to them in a nice package, so they will need to determine what information to gather and then work through the comparison process. In this lesson, students are paired so that one student is presented with a situation and question while the other student has information to help solve the question. They must work together to answer the question by asking their own questions and explaining how each piece of information will be useful. An optional activity is also included in which students are asked to compare data from a sample of one population to statistics from a sample of a second population.

Alignments

Building On

- 6.SP.B.4: Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

Addressing

- 7.SP.B.4: Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

Instructional Routines

- MLR4: Information Gap Cards
- MLR8: Discussion Supports

Required Materials

Pre-printed slips, cut from copies of the

Instructional master

Required Preparation

One copy of the Instructional master from Comparing Populations cut into cards for every 2 students.

Student Learning Goals

Let's ask important questions to compare groups.

19.1 Features of Graphic Representations

Warm Up: 5 minutes

In this warm-up, students review the useful information that can be gained from different graphical representations of data in preparation for comparing groups based on samples from each.

Building On

- 6.SP.B.4

Launch

For classes that may need help remembering the different representations, consider displaying an example of each type of graphical representation mentioned.

Student Task Statement

Dot plots, histograms, and box plots are different ways to represent a data set graphically.

Which of those displays would be the easiest to use to find each feature of the data?

1. the mean
2. the median
3. the mean absolute deviation
4. the interquartile range
5. the symmetry

Student Response

1. Dot plot
2. Box plot
3. Dot plot
4. Box plot
5. Dot plot or histogram

Activity Synthesis

Poll the class for their answers to each of the problems. Select at least one student to share their reasoning for each question.

19.2 Info Gap: Comparing Populations

30 minutes

In this info gap activity, students work together to compare two populations from information about samples from each of the populations. Students must pay attention to the information they need in order to solve the problem and the types of question they could ask to get to the answer.

The info gap structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need (MP1). It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need (MP6).

Here is the text of the cards for reference and planning:

<p>Info Gap: Comparing Populations Problem Card 1</p> <p>A chewing gum company records the amount of time, in minutes, it takes for a person to find that the gum has lost its flavor for two different flavors of gum. A sample of 20 people is used for mint and another sample of 20 people is used for cinnamon gum. Is there a meaningful difference in the amount of time for the different flavors? Explain your reasoning.</p>	<p>Info Gap: Comparing Populations Data Card 1</p> <ul style="list-style-type: none"> • The distributions are not symmetric • Mean for mint: 34.5 minutes • Median for mint: 65 minutes • IQR for mint: 20 minutes • MAD for mint: 20.9 minutes • Mean for cinnamon: 108.75 minutes • Median for cinnamon: 112.5 minutes • IQR for cinnamon: 22.5 minutes • MAD for cinnamon: 21.5 minutes 																				
<p>Info Gap: Comparing Populations Problem Card 2</p> <p>A grade 8 English teacher is interested in grammar scores for students coming from the two grade 7 English teachers. Is there a meaningful difference in the means for student scores on a pretest coming from each grade 7 teacher based on the samples? Explain your reasoning.</p>	<p>Info Gap: Comparing Populations Data Card 2</p> <p>Scores from Teacher A</p> <table border="1"> <tr> <td>79</td><td>75</td><td>80</td><td>80</td><td>80</td><td>85</td><td>85</td><td>85</td><td>90</td><td>95</td></tr> </table> <p>Scores from Teacher B</p> <table border="1"> <tr> <td>79</td><td>70</td><td>70</td><td>75</td><td>75</td><td>80</td><td>80</td><td>80</td><td>90</td><td>100</td></tr> </table>	79	75	80	80	80	85	85	85	90	95	79	70	70	75	75	80	80	80	90	100
79	75	80	80	80	85	85	85	90	95												
79	70	70	75	75	80	80	80	90	100												

Addressing

- 7.SP.B.4

Instructional Routines

- MLR4: Information Gap Cards

Launch

Arrange students in groups of 2. In each group, distribute a problem card to one student and a data card to the other student. After you review their work on the first problem, give them the cards for a second problem and instruct them to switch roles.

Tell students they will continue to work with comparing measures of center for populations. Explain the Info Gap structure and consider demonstrating the protocol if students are unfamiliar with it. There are step-by-step instructions in the student task statement.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Display or provide students with a physical copy of the written directions. Check for understanding by inviting students to rephrase directions in their own words. Keep the display of directions visible throughout the activity.

Supports accessibility for: Memory; Organization

Access for English Language Learners

Conversing: This activity uses *MLR4 Information Gap* to give students a purpose for discussing information necessary to use data from samples to compare two populations. Display questions or question starters for students who need a starting point such as: "Can you tell me . . . (specific piece of information)", and "Why do you need to know . . . (that piece of information)?"

Design Principle(s): Cultivate Conversation

Student Task Statement

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

Continue to ask questions until you have enough information to solve the problem.

4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.
2. Ask your partner “*What specific information do you need?*” and wait for them to *ask* for information.

If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don’t have that information.

3. Before sharing the information, ask “*Why do you need that information?*” Listen to your partner’s reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Student Response

1. There is a meaningful difference. Since the data is not symmetric, it makes sense to use the median and IQR to compare the data. The medians are more than 2 IQRs apart ($112.5 - 65 > 2 \cdot 22.5$), so there is a meaningful difference between the flavors.
2. No. The mean for teacher A is 83 and MAD is 5.4. The mean for teacher B is 79 and the MAD is 7. So, the difference in means is less than 2 MADs apart ($83 - 79 < 2 \cdot 7$).

Are You Ready for More?

Is there a meaningful difference between top sports performance in two different decades? Choose a variable from your favorite sport (for example, home runs in baseball, kills in volleyball, aces in tennis, saves in soccer, etc.) and compare the leaders for each year of two different decades. Is the performance in one decade meaningfully different from the other?

Student Response

Answers vary.

Activity Synthesis

The purpose of the discussion is to help students understand the types of questions they need to answer in order to compare groups.

Select several groups to share their answers and reasoning for each of the problems. In particular help students understand why the information “The distributions are not symmetric” was important for solving the first problem.

Consider asking these discussion questions:

- “What was the most important question you asked for the first problem? For the second problem?”
- “What are some other ways the information could have been given to solve the problems?” (Instead of the characteristics for the first question, a box plot could have been presented. The second question could have had a dot plot or characteristics like the first problem.)
- “If the distributions for the first problem had been symmetric, would the answer have been the same?” (Yes. The difference in means is 34.25 which is only 1.6 MADs apart, so there would not have been enough information to say that the two population means are meaningfully different.)

19.3 Comparing to Known Characteristics

Optional: 15 minutes

In this optional activity, students compare two populations using samples again, but this time only one sample is given. For the other sample, the characteristics have either been computed already or are the focus of the question. This type of analysis is useful when comparing two similar populations as in this activity or when comparing a group against a standard.

Addressing

- 7.SP.B.4

Instructional Routines

- MLR8: Discussion Supports

Launch

Keep students in groups of 2.

Tell students that sometimes it is useful to compare one group to a standard or another group where the important characteristics have already been computed. In these problems, a random sample from one group is given and characteristics of the second group is either given or sought.

Allow students 10 minutes partner work time followed by a whole-class discussion.

Student Task Statement

1. A college graduate is considering two different companies to apply to for a job. Acme Corp lists this sample of salaries on their website:

\$45,000 \$55,000 \$140,000 \$70,000 \$60,000 \$50,000

What typical salary would Summit Systems need to have to be meaningfully different from Acme Corp? Explain your reasoning.

2. A factory manager is wondering whether they should upgrade their equipment. The manager keeps track of how many faulty products are created each day for a week.

6 7 8 6 7 5 7

The new equipment guarantees an average of 4 or fewer faulty products per day. Is there a meaningful difference between the new and old equipment? Explain your reasoning.

Student Response

1. A median salary greater than \$97,500 (or less than \$17,500). Since the salaries for Acme Corp have a large value far from the other values, the measure of center chosen should be the median. The median salary for Acme Corp is \$57,500 and the IQR is \$20,000, so the median salary for Summit Systems must be greater than 2 IQRs above or less than 2 IQRs below Acme's median ($57,500 + 2 \cdot 20,000 = 97,500$ or $57,500 - 2 \cdot 20,000 = 17,500$).
2. Yes. The mean for the sample for the current machine is 6.57 faulty products per day and the MAD is 0.61 faulty products per day. The difference in means is 2.57 faulty products per day (since $6.57 - 4 = 2.57$) which is greater than twice the MAD, so there is a meaningful difference in the mean number of faulty products per day.

Activity Synthesis

The purpose of the discussion is to help students understand how to compare groups when one set of characteristics are known and the other group is represented by sample data.

Select some groups to share their answers and reasoning for the two problems.

Consider asking these discussion questions:

- "How did you determine what to use as a typical value for the first problem?" (Since there was one value much greater than the others, the distribution would not be symmetric, so median is a more appropriate measure of center.)
- "How did you determine what measure of center to use for the second problem?" (Since the data were all close, either value could be used, but the new equipment reported the "average" or mean, so mean should be used for the sample as well.)
- "The manufacturer for the new equipment guarantees 4 flaws *or fewer* per day with the new equipment. If the new equipment produces only 3 flaws per day does that change the answer for the second problem?" (No. There is an even greater difference between the current and new equipment, so it is even more meaningful.)

- “What other factors would the college graduate want to consider other than the meaningful difference in median salary between the two companies?” (In addition to the other factors for a job such as benefits, relationship with coworkers, type of work being done at each company, etc., the graduate should consider the salary for the type of job he will get at the company. For example, if his degree is in computer science, he may be looking at a job with computers rather than sales or some other department within the company, so he might be able to get a better comparison of salaries that way.)
- “What other factors would the factory manager want to consider other than the meaningful difference in flaws for the equipment?” (The cost of the frequent flaws as well as the cost of the new equipment will probably factor into her decision to buy new equipment. The age of the current equipment and maintenance for older equipment compared to new equipment may also be important.)

Access for Students with Disabilities

Representation: Internalize Comprehension. Use color and annotations to illustrate connections between representations. As students share their diagrams and reasoning, use color and annotations to scribe their thinking on a display of each problem so that it is visible for all students.

Supports accessibility for: Visual-spatial processing; Conceptual processing

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion. For each response or observation that is shared, ask students to restate and/or revoice what they heard using precise mathematical language. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker whether their peer was accurately able to restate their thinking. Call students’ attention to any words or phrases that helped clarify the original statement. This will provide more students with an opportunity to produce language as they interpret the reasoning of others.

Design Principle(s): Support sense-making

Lesson Synthesis

Ask students what information is important to collect when attempting to compare large groups and why each of these pieces of information is useful. Ask students if they can think of other situations in which it might be helpful to compare two large groups by generating a sample and collecting information.

19.4 A Different Box Plot

Cool Down: 5 minutes

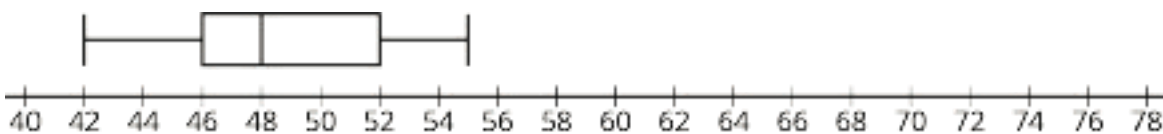
This cool-down assesses whether students understand how different groups can be compared based on data from a box plot. In particular, students need to be able to read the median and IQR from the box plot and use that information to construct an additional box plot that would be meaningfully different.

Addressing

- 7.SP.B.4

Student Task Statement

Use the box plot to answer the questions.



1. What measure of center is shown in the box plot? What measure of variability? What are the values for each of these characteristics?
2. Draw another box plot with the same measure of variability that is meaningfully different from the one shown.

Student Response

1. Median = 48. IQR = 6
2. Answers vary. Correct responses show a box plot with an IQR of 6 and median greater than or equal to 60 (or less than or equal to 36).

Student Lesson Summary

When using samples to comparing two populations, there are a lot of factors to consider.

- Are the samples representative of their populations? If the sample is biased, then it may not have the same center and variability as the population.
- Which characteristic of the populations makes sense to compare—the mean, the median, or a proportion?
- How variable is the data? If the data is very spread out, it can be more difficult to make conclusions with certainty.

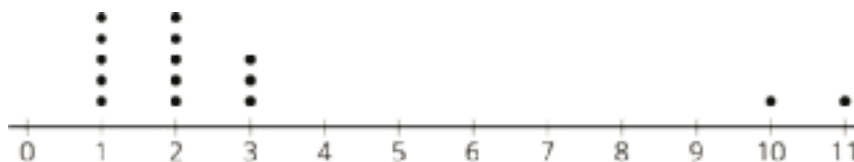
Knowing the correct questions to ask when trying to compare groups is important to correctly interpret the results.

Lesson 19 Practice Problems

Problem 1

Statement

An agent at an advertising agency asks a random sample of people how many episodes of a TV show they watch each day. The results are shown in the dot plot.



The agency currently advertises on a different show, but wants to change to this one as long as the typical number of episodes is not meaningfully less.

- What measure of center and measure of variation would the agent need to find for their current show to determine if there is a meaningful difference? Explain your reasoning.
- What are the values for these same characteristics for the data in the dot plot?
- What numbers for these characteristics would be meaningfully different if the measure of variability for the current show is similar? Explain your reasoning.

Solution

- Median and IQR, since the dot plot shows a distribution that is not symmetric, so it would make sense to compare medians.
- Median: 2 episodes; IQR: 2 episodes
- The other show would need to have a median of at least 6 episodes, since the medians would need to be at least 2 IQRs apart and $2 + 2 \cdot 2 = 6$.

Problem 2

Statement

Jada wants to know if there is a meaningful difference in the mean number of friends on social media for teens and adults. She looks at the friend count for the 10 most popular of her friends and the friend count for 10 of her parents' friends. She then computes the mean and MAD of each sample and determines there is a meaningful difference.

Jada's dad later tells her he thinks she has not come to the right conclusion. Jada checks her calculations and everything is right. Do you agree with her dad? Explain your reasoning.

Solution

Yes. She did not select her samples randomly, so they may not be representative of teens and adults.

Problem 3

Statement

The mean weight for a sample of a certain kind of ring made from platinum is 8.21 grams. The mean weight for a sample of a certain kind of ring made from gold is 8.61 grams. Is there a meaningful difference in the weights of the two types of rings? Explain your reasoning.

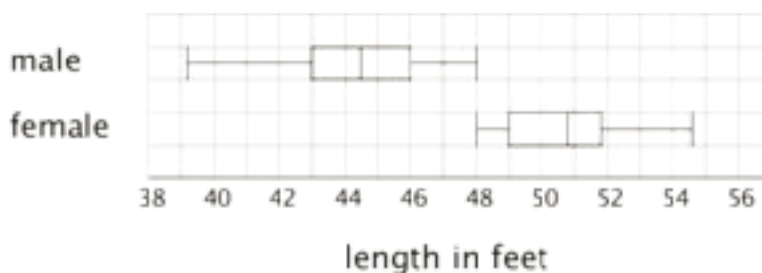
Solution

The answer is unknown with this information. For example, if the MAD for each is 0.1 grams, then there would be a meaningful difference. If the MAD is greater than 0.2 grams, then there is not a meaningful difference.

Problem 4

Statement

The lengths in feet of a random sample of 20 male and 20 female humpback whales were measured and used to create the box plot.



Estimate the median lengths of male and female humpback whales based on these samples.

Solution

Males: 44.6 feet. Females: 50.9 feet.

(From Unit 8, Lesson 15.)

Section: Let's Put it to Work

Lesson 20: Memory Test

Goals

- Describe (orally) connections between sampling and probability.
- Generate a random sample, and use it to make inferences (in writing) about the population.
- Justify (orally and in writing) whether a given method produces a random sample.

Learning Targets

- I can compare two groups by taking a random sample, calculating important measures, and determining whether the populations are meaningfully different.

Lesson Narrative

This lesson is optional. It gives students a chance to use the material they have learned in the unit with the final goal of comparing two populations, but may be shortened or skipped due to time constraints.

In this lesson, students apply what they have learned about probability, sampling, and comparing populations to analyze two data sets. Half of the class works with one data set while the other half of the class works with another. Students choose their own tools for selecting a sample at random (MP5) and calculate the mean, MAD, and a proportion to summarize their sample (MP2). Then students compare their results with a partner that had the other data set to construct an argument for whether there is a meaningful difference between the sets (MP3).

Alignments

Building On

- 6.SP.B.5.d: Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Addressing

- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.
- 7.SP.A: Use random sampling to draw inferences about a population.
- 7.SP.A.1: Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

- 7.SP.A.2: Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.
- 7.SP.B.4: Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.
- 7.SP.C.7.a: Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.

Instructional Routines

- MLR7: Compare and Connect
- MLR8: Discussion Supports

Required Materials

Copies of Instructional master
Number cubes

cubes with sides numbered from 1 to 6

Paper bags
Paper clips

Required Preparation

Print the Collecting a Sample Instructional master. Provide one data set and one spinner for each student. If the spinners are used to select a random sample during the Sample Probabilities activity, provide a paper clip and sharpened pencil to use with the spinners. If possible, provide access to other tools for selecting a random sample from a 10-by-10 grid such as a 10-sided polyhedra or colored cubes and paper bags.

Student Learning Goals

Let's put it all together.

20.1 Collecting a Sample

Optional: 5 minutes

In this activity, students review methods of obtaining samples that are fair and random (MP6).

Addressing

- 7.SP.A.1

Launch

Arrange students in groups of 2. Each group gets both sets of data from the Instructional master, one data set for each partner. Students will not need the spinners from the Instructional master for this activity, but the spinners are included for use later in the lesson. Partners may work together to answer the questions, but should not share their data set with one another until told to do so in a later activity.

Student Task Statement

You teacher will give you a paper that lists a data set with 100 numbers in it. Explain whether each method of obtaining a sample of size 20 would produce a random sample.

Option 1: A spinner has 10 equal sections on it. Spin once to get the row number and again to get the column number for each member of your sample. Repeat this 20 times.

Option 2: Since the data looks random already, use the first two rows.

Option 3: Cut up the data and put them into a bag. Shake the bag to mix up the papers, and take out 20 values.

Option 4: Close your eyes and point to one of the numbers to use as your first value in your sample. Then, keep moving one square from where your finger is to get a path of 20 values for your sample.

Student Response

1. This would produce a random sample since each row and column has an equal chance of being selected.
2. This would not produce a random sample since all of the values do not have an equal chance of being selected.
3. This would produce a random sample since all the papers are the same size and each value has an equal chance of being selected.
4. This would not produce a random sample since the path limits the values you can get in your sample. For example, the four corners could not all be in the same sample of 20.

Activity Synthesis

The purpose of the discussion is to help students solidify their understanding of methods for selecting random samples.

Consider these questions for discussion:

- “Can you think of other methods for selecting a random sample that are not listed here?” (Roll a polyhedron with 10 equal faces showing the numbers 1 through 10 to get the row and again to get the column.)

- “What do you need to look for when determining if a sample is random?” (Are all values equally likely to be included in the random sample?)

20.2 Sample Probabilities

Optional: 10 minutes

In this activity, students begin by practicing their understanding of proportions and probabilities by examining the data set they have available. In the fourth problem, students obtain a sample from the population using tools they choose (MP5) and examine the sample they selected to compare it to the expected proportions and probabilities calculated in the first 3 problems.

The problems are intended for students to use their own data set to answer. Although they are kept in pairs for the entire lesson, this activity should be done individually.

Addressing

- 7.RP.A
- 7.SP.A
- 7.SP.C.7.a

Instructional Routines

- MLR7: Compare and Connect

Launch

Keep students in the same groups of 2. Give students 5–7 minutes of quiet work time followed by a whole-class discussion.

If possible, allow students to use their chosen method of random sampling to obtain a sample of 10 for this activity. Have items such as paper clips, scissors, 10-sided polyhedra, and other materials available for student use. The Instructional master for the first activity in this lesson contains accurate spinners that could be used to select a random sample.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts. After students have solved the first 2-3 problems, check-in with either select groups of students or the whole class. Invite students to share the strategies they have used so far as well as any questions they have before continuing.

Supports accessibility for: Organization; Attention

Student Task Statement

Continue working with the data set your teacher gave you in the previous activity. The data marked with a star all came from students at Springfield Middle School.

1. When you select the first value for your random sample, what is the probability that it will be a value that came from a student at Springfield Middle School?
2. What proportion of your entire sample would you expect to be from Springfield Middle School?
3. If you take a random sample of size 10, how many scores would you expect to be from Springfield Middle School?
4. Select a random sample of size 10.
5. Did your random sample have the expected number of scores from Springfield Middle School?

Student Response

1. $\frac{20}{100} = \frac{1}{5} = 0.2$
2. The proportion I would expect is about 0.2.
3. From a random sample of 10, I expect there to be around 2 from Springfield Middle School.
4. Answers vary. Sample response: 53, 46, 63, 42, 41, 33, 71, 39, 40, 18.
5. Answers vary. Sample response: No, I only had 1 value come from Springfield Middle School.

Activity Synthesis

The purpose of this discussion is to connect the ideas of probability and random sampling from the unit.

Consider these questions for discussion:

- “How is selecting a sample at random connected to probability?” (A random sample should give each value an equal chance of being chosen. Therefore, each value has a $\frac{1}{100}$ probability of being chosen.)
- “How could we simulate the probability of getting at least 2 values in the sample of 10 from Springfield Middle School?” (Since 20% of the values come from Springfield Middle School, we could put 10 blocks in a bag with 2 colored red to represent Springfield Middle School. Draw a block from the bag, and if it is red, it represents a score from Springfield Middle School; replace the block and repeat. Get a sample of 10 and see if the sample has at least 2 red blocks. Repeat this process many times and use the fraction of times there are at least 2 red blocks as an estimate for the probability that a random sample will have at least 2 scores from Springfield Middle School.)

Access for English Language Learners

Representing, Speaking: MLR7 Compare and Connect. Invite students to create a visual display of their random sample of size 10 and response to the question: “Did your random sample have the expected number of scores from Springfield Middle School?” Invite students to investigate each other’s work and compare their responses. Listen for the language students use to describe a random sample and assign a probability of each value being chosen. This will help students connect the ideas of probability and random sampling through discussion.

Design Principle(s): Optimize output (for representation); Cultivate conversation

20.3 Estimating a Measure of Center for the Population

Optional: 10 minutes

In this activity, students practice estimating a measure of center for the population using the data from a sample. The variability is also calculated to be used in the following activity to determine if there is a meaningful difference between the measure of center for the population they used to select their sample and the measure of center for another population.

Building On

- 6.SP.B.5.d

Addressing

- 7.SP.A.2

Instructional Routines

- MLR8: Discussion Supports

Launch

Keep students in groups of 2. Students should work with their partner for the first question, then individually for the last 2 problems. Follow up with a whole-class discussion.

Student Task Statement

1. Decide which measure of center makes the most sense to use based on the distribution of your sample. Discuss your thinking with your partner. If you disagree, work to reach an agreement.
2. Estimate this measure of center for your population based on your sample.
3. Calculate the measure of variability for your sample that goes with the measure of center that you found.

Student Response

Answers vary. Sample response:

1. We chose to use mean since there were no values far from the center of the data.
2. Mean: 44.6
3. MAD: 10.92

Activity Synthesis

The purpose of the discussion is for students make clear their reasoning for choosing a particular measure of center and reiterate the importance of variability when comparing groups from samples.

Consider these questions for discussion:

- “Which measure of center did your group choose? Explain your reasoning.” (A median should be used if there are a few values far from the center that overly influence the mean in that direction. If the data is not approximately symmetric, a median should be used as well. In other cases, the mean is probably a better choice for the measure of center.)
- “Why is it important to measure variability in the data when estimating a measure of center for the population using the data from a sample?” (To use the general rule, the difference in means must be greater than 2 MADs to determine a meaningful difference. If there is small variation, then the samples may have come from a population that also has a small variation, so differences among groups may be more clearly defined.)

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion. For each response that is shared, ask students to restate and/or revoice what they heard using precise mathematical language. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker whether their peer was accurately able to restate their thinking. Call students’ attention to any words or phrases that helped clarify the original statement. This will provide more students with an opportunity to produce language as they interpret the reasoning of others.

Design Principle(s): Support sense-making

20.4 Comparing Populations

Optional: 5 minutes

In this activity, students use the values computed in the previous activity to determine if there is a meaningful difference between two populations (MP2). Following the comparison of the groups,

students are told that the populations from which they selected a sample were identical, although shuffled.

Addressing

- 7.SP.B.4

Launch

Keep students in the same groups of 2 established at the beginning of this lesson. Allow students 3 minutes of partner work time followed by a whole-class discussion.

Student Task Statement

Using only the values you computed in the previous two activities, compare your sample to your partner's.

Is it reasonable to conclude that the measures of center for each of your populations are meaningfully different? Explain or show your reasoning.

Student Response

Answers vary. Sample response: They are not meaningfully different. The difference in means is 4.9, but the larger of the two MADs is 10.9, so they are less than 2 MADs apart.

Activity Synthesis

Ask each group to share whether they found a meaningful difference.

Tell students, “With your partner, compare the starred data for the two groups. What do you notice?”

Tell students that the two populations are actually identical, but rearranged. Ask, “Did any groups get different means for your samples? Explain why that might have happened, even though the populations are the same.” (Two random samples from the population will usually not contain the same values, so different means are probably expected.)

One thing to note: The general rule is designed to say whether the two populations have a meaningful difference or if there is not enough evidence to determine if there is a meaningful difference. On its own, the general rule *cannot* determine if two populations are identical from only a sample. If the means are less than 2 MADs apart, there is still a chance that there is a difference in the populations, but there is not enough evidence in the samples to be convinced that there is a difference.

Lesson Synthesis

Key learning points:

- Probability and random samples are connected through the equal likelihood of individuals from the population being selected.

- It is important to select samples through a random process in order to compare two populations.

Consider asking these discussion questions:

- "Why was it important to select a random sample from the population data you had?" (A random sample gives us the best chance of being representative of the population.)
- "A scientist has access to data for the high temperature in London for each day of every year since 1945. Describe a process the scientist could use to compare the temperatures from 1963 and 2003." (Select a random sample of temperatures from each year. Determine the correct measure of center and variation. Use our general rule to compare the measure of center for each year based on the sample characteristics.)



Family Support Materials

Family Support Materials

Probability and Sampling

Here are the video lesson summaries for Grade 7, Unit 8: Probability and Sampling. Each video highlights key concepts and vocabulary that students learn across one or more lessons in the unit. The content of these video lesson summaries is based on the written Lesson Summaries found at the end of lessons in the curriculum. The goal of these videos is to support students in reviewing and checking their understanding of important concepts and vocabulary. Here are some possible ways families can use these videos:

- Keep informed on concepts and vocabulary students are learning about in class.
- Watch with their student and pause at key points to predict what comes next or think up other examples of vocabulary terms (the bolded words).
- Consider following the Connecting to Other Units links to review the math concepts that led up to this unit or to preview where the concepts in this unit lead to in future units.

Grade 7, Unit 8: Probability and Sampling	Vimeo	YouTube
Video 1: Estimating Probabilities (Lessons 1–5)	Link	Link
Video 2: Multi-step Experiments (Lessons 6–10)	Link	Link
Video 3: Sampling (Lessons 11–14)	Link	Link
Video 4: Using Samples (Lessons 15–19)	Link	Link

Video 1

Video 'VLS G7U8V1 Estimating Probabilities (Lessons 1–5)' available here:
<https://player.vimeo.com/video/521004741>.

Video 2

Video 'VLS G7U8V2 Multi-step Experiments (Lessons 6–10)' available here:
<https://player.vimeo.com/video/521022437>.

Video 3

Video 'VLS G7U8V3 Sampling (Lessons 11–14)' available here: <https://player.vimeo.com/video/523876549>.

Video 4

Video 'VLS G7U8V4 Using Samples (Lessons 15–19)' available here: <https://player.vimeo.com/video/523185261>.

Connecting to Other Units

- *Coming soon*

Probabilities of Single Step Events

Family Support Materials 1

This week your student will be working with probability. A **probability** is a number that represents how likely something is to happen. For example, think about flipping a coin.

- The probability that the coin lands somewhere is 1. That is certain.
- The probability that the coin lands heads up is $\frac{1}{2}$, or 0.5.
- The probability that the coin turns into a bottle of ketchup is 0. That is impossible.

Sometimes we can figure out an exact probability. For example, if we pick a random date, the chance that it is on a weekend is $\frac{2}{7}$, because 2 out of every 7 days fall on the weekend. Other times, we can estimate a probability based on what we have observed in the past.

Here is a task to try with your student:

People at a fishing contest are writing down the type of each fish they catch. Here are their results:

- Person 1: bass, catfish, catfish, bass, bass, bass
- Person 2: catfish, catfish, bass, bass, bass, bass, catfish, catfish, bass, catfish
- Person 3: bass, bass, bass, catfish, bass, bass, catfish, bass, catfish

1. Estimate the probability that the next fish that gets caught will be a bass.
2. Another person in the competition caught 5 fish. Predict how many of these fish were bass.
3. Before the competition, the lake was stocked with equal numbers of catfish and bass. Describe some possible reasons for why the results do not show a probability of $\frac{1}{2}$ for catching a bass.

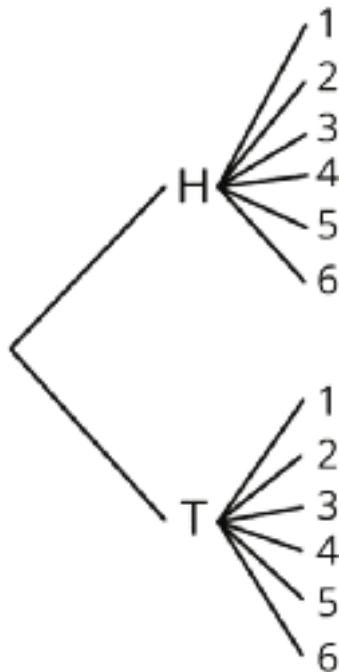
Solution:

1. About $\frac{15}{25}$, or 0.6, because of the 25 fish that have been caught, 15 of them were bass.
2. About 3 bass, because $\frac{3}{5} = 0.6$. It would also be reasonable if they caught 2 or 4 bass, out of their 5 fish.
3. There are many possible answers. For example:
 - Maybe the lures or bait they were using are more likely to catch bass.
 - With results from only 25 total fish caught, we can expect the results to vary a little from the exact probability.

Probabilities of Multi-step Events

Family Support Materials 2

To find an exact probability, it is important to know what outcomes are possible. For example, to show all the possible outcomes for flipping a coin and rolling a number cube, we can draw this tree diagram:



The branches on this tree diagram represent the 12 possible outcomes, from “heads 1” to “tails 6.” To find the probability of getting heads on the coin and an even number on the number cube, we can see that there are 3 ways this could happen (“heads 2”, “heads 4”, or “heads 6”) out of 12 possible outcomes. That means the probability is $\frac{3}{12}$, or 0.25.

Here is a task to try with your student:

A board game uses cards that say “forward” or “backward” and a spinner numbered from 1 to 5.

1. On their turn, a person picks a card and spins the spinner to find out which way and how far to move their piece. How many different outcomes are possible?
2. On their next turn, what is the probability that the person will:
 - a. get to move their piece forward 5 spaces?
 - b. have to move their piece backward some odd number of spaces?

Solution:

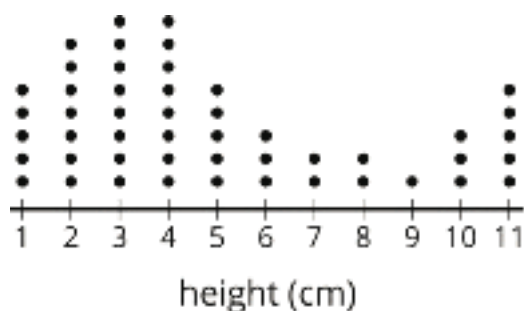
1. There are 10 possible outcomes ("forward 1", "forward 2", "forward 3", "forward 4", "forward 5", "backward 1", "backward 2", "backward 3", "backward 4", or "backward 5").
2.
 - a. $\frac{1}{10}$ or 0.1, because "forward 5" is 1 out of the 10 possibilities.
 - b. $\frac{3}{10}$ or 0.3, because there are 3 such possibilities ("backward 1", "backward 3", or "backward 5")

Sampling

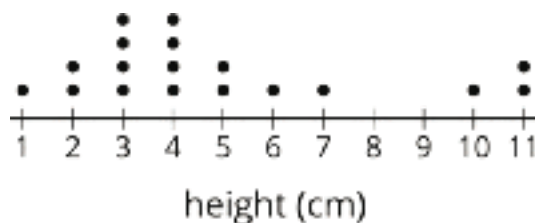
Family Support Materials 3

This week your student will be working with data. Sometimes we want to know information about a group, but the group is too large for us to be able to ask everyone. It can be useful to collect data from a **sample** (some of the group) of the **population** (the whole group). It is important for the sample to resemble the population.

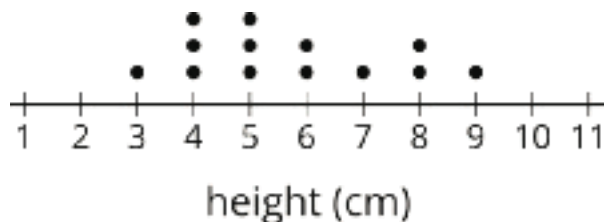
- For example, here is a dot plot showing a population: the height of 49 plants in a sprout garden.



- This sample is **representative** of the population, because it includes only a part of the data, but it still resembles the population in shape, center, and spread.



- This sample is not representative of the population. It has too many plant heights in the middle and not enough really short or really tall ones.



A sample that is selected at random is more likely to be representative of the population than a sample that was selected some other way.

Here is a task to try with your student:

A city council needs to know how many buildings in the city have lead paint, but they don't have enough time to test all 100,000 buildings in the city. They want to test a sample of buildings that will be representative of the population.

1. What would be a *bad* way to pick a sample of the buildings?

2. What would be a *good* way to pick a sample of the buildings?

Solution:

1. There are many possible answers.
 - Testing all the same type of buildings (like all the schools, or all the gas stations) would not lead to a representative sample of all the buildings in the city.
 - Testing buildings all in the same location, such as the buildings closest to city hall, would also be a bad way to get a sample.
 - Testing all the newest buildings would *bias* the sample towards buildings that don't have any lead paint.
 - Testing a small number of buildings, like 5 or 10, would also make it harder to use the sample to make predictions about the entire population.
2. To select a sample at random, they could put the addresses of all 100,000 buildings into a computer and have the computer select 50 addresses randomly from the list. Another possibility could be picking papers out a bag, but with so many buildings in the city, this method would be difficult.

Using Samples

Family Support Materials 4

We can use statistics from a sample (a part of the entire group) to estimate information about a population (the entire group). If the sample has more variability (is very spread out), we may not trust the estimate as much as we would if the numbers were closer together. For example, it would be easier to estimate the average height of all 3-year olds than all 40-year olds, because there is a wider range of adult heights.

We can also use samples to help predict whether there is a meaningful difference between two populations, or whether there is a lot of overlap in the data.

Here is a task to try with your student:

Students from seventh grade and ninth grade were selected at random to answer the question, "How many pencils do you have with you right now?" Here are the results:

how many pencils each seventh grade student had

4	1	2	5	2	1	1	2	3	3
---	---	---	---	---	---	---	---	---	---

how many pencils each ninth grade student had

9	4	1	14	6	2	0	8	2	5
---	---	---	----	---	---	---	---	---	---

1. Use the sample data to estimate the mean (average) number of pencils carried by:
 - a. all the seventh grade students in the whole school.
 - b. all the ninth grade students in the whole school.
2. Which sample had more variability? What does this tell you about your estimates in the previous question?

3. A student, who was not in the survey, has 5 pencils with them. If this is all you know, can you predict which grade they are in?

Solution:

1. Since the samples were selected at random, we predict they will represent the whole population fairly well.
 - a. About 2.4 pencils for all seventh graders, because the mean of the sample is $(4 + 1 + 2 + 5 + 2 + 1 + 1 + 2 + 3 + 3) \div 10$ or 2.4 pencils.
 - b. About 5.1 pencils for all ninth graders, because the mean of the sample is $(9 + 4 + 1 + 14 + 6 + 2 + 0 + 8 + 2 + 5) \div 10$ or 5.1 pencils.
2. The survey of ninth graders had more variability. Those numbers were more spread out, so I trust my estimate for seventh grade more than I trust my estimate for ninth grade.
3. There are many possible answers. For example:
 - Since they only asked 10 students from each grade, it is hard to predict. It would help if they could ask more students.
 - The student is probably in ninth grade, because 5 is closer to the sample mean from ninth grade than from seventh grade.
 - The student could possibly be in seventh grade, because at least one student in seventh grade has 5 pencils.

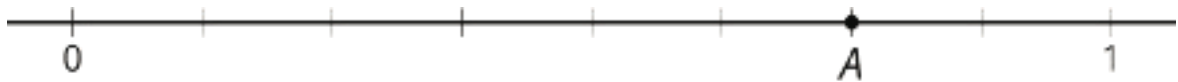
Unit Assessments

Check Your Readiness A and B
Mid-Unit Assessment A and B
End-of-Unit Assessment A and B

Probability and Sampling: Check Your Readiness (A)

Do not use a calculator.

1. Select **all** the numbers that represent point A .



- A. 0.25
 - B. 0.5
 - C. 0.75
 - D. $\frac{1}{2}$
 - E. $\frac{3}{4}$
 - F. $\frac{12}{16}$
2. Students did push-ups for a fitness test. Their goal was 20 push-ups. For each student, determine what percentage of the goal they achieved.
- a. Elena did 16 push-ups.
 - b. Jada did 42 push-ups.
 - c. Lin did 13 push-ups.
 - d. Andre did 21 push-ups.

3. Andre surveyed a random sample of 20 students and found that 13 of them were in favor of having school start one hour later.

a. The school has 250 students. Make an estimate for the number of students in the school who are in favor of having school start one hour later.

b. Do you think it would be surprising if 150 students out of 250 were in favor? Explain your reasoning.

c. Do you think it would be surprising if 100 students out of 250 were in favor? Explain your reasoning.

4. Select **all** the measures of variability or spread.

A. mean

B. IQR (interquartile range)

C. MAD (mean absolute deviation)

D. median

E. range

5. Ten students each attempted 10 free throws. This list shows how many free throws each student made.

8, 5, 6, 6, 4, 9, 7, 6, 5, 9

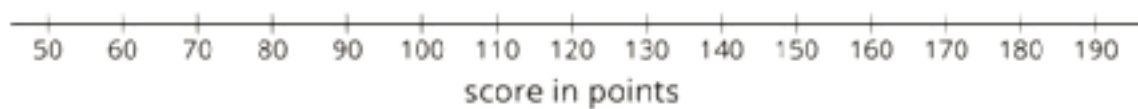
a. What is the median number of free throws made?

b. What is the IQR (interquartile range)?

6. Two groups went bowling. Here are the scores from each group.

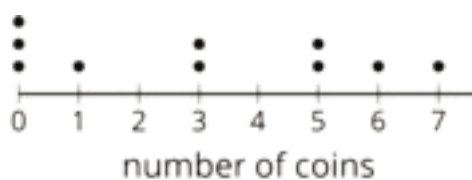
Group A: 70, 80, 90, 100, 110, 130, 190

Group B: 50, 100, 107, 110, 120, 140, 150



- Construct two box plots, one for the data in each group.
- Which group shows greater variability? Explain how you know.

7. This dot plot shows the number of coins in 10 students' pockets.



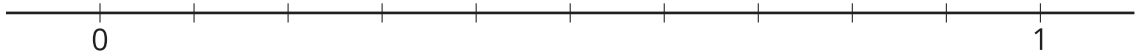
What is the mean and MAD (mean absolute deviation) of this data? Explain how you know.

Probability and Sampling: Check Your Readiness (B)

Do not use a calculator.

1. Plot and label each number on the number line.

0.75 , $\frac{1}{4}$, 0.2 , 0.5 , $\frac{8}{10}$



2. Teachers held a basketball shooting contest. Their goal was to make 60 baskets. For each teacher, determine what percentage of the goal they achieved.

a. Teacher A made 12 baskets.

b. Teacher B made 42 baskets.

c. Teacher C made 66 baskets.

d. Teacher D made 9 baskets.

3. A survey was conducted of a random sample of 15 sports team coaches in a school district, and found that 6 of them were in favor of a shorter season for their sport.
- a. The school district has 132 sports team coaches. Make an estimate for the number of coaches in the school district who are in favor of a shorter season for their sport.
 - b. Do you think it would be surprising if 99 coaches were in favor? Explain your reasoning.
 - c. Do you think it would be surprising if 60 coaches were in favor? Explain your reasoning.

4. Select **all** the measures of center.

- A. mean
- B. IQR (interquartile range)
- C. MAD (mean absolute deviation)
- D. median
- E. range

5. Jada completed eleven homework assignments. This list shows her scores for each assignment.

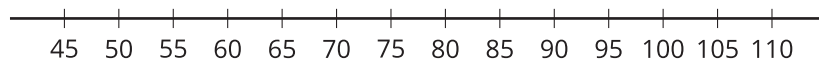
10, 9, 8, 10, 7, 9, 8, 6, 10, 10, 8

- a. What is the median score?
- b. What are the first quartile and the third quartile?
- c. What is the interquartile range?

6. Two students compared their test scores. Here are the scores for each student.

Student A: 91, 100, 82, 90, 93, 85

Student B: 80, 70, 58, 98, 75, 81



- a. Construct two box plots, one for the data for each student.
- b. Which student's scores show greater variability? Explain how you know.

7. The data set shows the number of shells that 12 people collected.

4, 5, 5, 4, 1, 6, 8, 2, 2, 2, 4, 5

- a. What is the mean number of shells collected?
- b. What is the MAD (mean absolute deviation) of this data? Explain how you know.

Probability and Sampling: Mid-Unit Assessment (A)

1. Tiles with the numbers 1 through 9 are placed in a bag. What is the probability of choosing an even number that is greater than or equal to 4?

A. $\frac{1}{9}$

B. $\frac{2}{9}$

C. $\frac{3}{9}$

D. $\frac{4}{9}$

2. Select **all** of the events that are possible, but unlikely.

A. Opening a 300-page book to exactly page 143.

B. A second grade classroom contains some books.

C. You will roll a 7 on a standard 6-sided number cube.

D. Everyone in your school will get to school on time on the next school day.

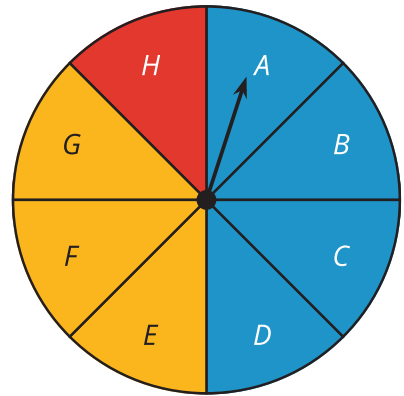
E. A triangle has 3 sides.

3. Han heard that the school's football team has a $\frac{1}{3}$ probability of winning each of their next 5 games. Select **all** the ways Han could accurately simulate the number of games the football team will win.
- A. Roll a standard number cube 5 times. Count the number of 1's and 2's.
 - B. Put the numbers 1 through 5 in a bag. Pick a number from the bag.
 - C. Roll a standard number cube 5 times. Count the number of ones.
 - D. Mark a spinner with 3 equally-sized sections. Write "win" on 2 of the sections. Spin the spinner 5 times and count the number of times it lands on "win."
 - E. Put 2 white chips and 1 red chip in a bag. Draw a chip from the bag, record its result, and put it back in the bag. Do this 5 times and count the number of red chips drawn.
4. A movie theater sells 3 different sizes of popcorn: small, medium, and large. An order of popcorn comes with 3 different topping choices: butter, cheese, or caramel. How many unique ways can you order a bag of popcorn? (You can only select one topping.)
5. Imagine that there is a spinner with 5 equal sections, and also a standard number cube. You spin the spinner and roll the cube. This is a situation that involves 2 parts (the spinner and the cube) and has a total of 30 outcomes in the sample space.

Describe a situation that involves 3 parts and has a total of 36 outcomes in the sample space.

6. Priya is about to spin a spinner like the one shown.

- a. Describe an event that is certain to happen on the next spin.
- b. Describe an event that is unlikely to happen, but possible, on the next spin.
- c. Describe an event that is impossible on the next spin.
- d. Describe an event that is likely to happen, but not certain, on the next spin.



7. A game uses a special deck of cards with 40 cards numbered from 1 to 40. You draw a card from the shuffled deck.

- a. What is the probability of drawing a card that is divisible by 2? Explain your reasoning.
- b. What is the probability of drawing a card that is divisible by 5? Explain your reasoning.
- c. What is the probability of drawing a card that is divisible by 10? Explain your reasoning.

Probability and Sampling: Mid-Unit Assessment (B)

A standard number cube has the numbers 1 through 6 on its faces.

1. A standard number cube is rolled. What is the probability of rolling a number less than or equal to 2?

A. $\frac{1}{6}$

B. $\frac{2}{6}$

C. $\frac{3}{6}$

D. $\frac{4}{6}$

2. Select **all** the events that have a probability of 0.

A. Choosing a red block from a bag with 9 white blocks and 1 red block.

B. Rolling a 6, three times in a row, on a standard number cube.

C. Rolling a 7 on a standard number cube.

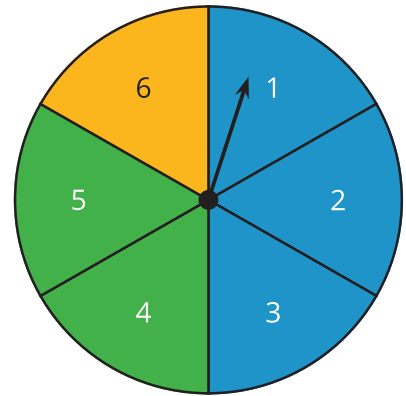
D. Opening a 400-page book to exactly page 231.

E. Opening a 400-page book a page number greater than 450.

3. The basketball coach says that based on Noah's free throws this season, Noah has a $\frac{3}{4}$ probability of making each free throw. Select **all** the ways Noah could accurately simulate the number of free throws he will make in his next 6 attempts.
- A. Flip a coin 6 times. Count the number of heads.
 - B. Make a spinner with 6 equally-sized sections, labeled with the numbers 1 through 6. Spin the spinner 6 times and count how many times the spinner lands on 3 or 4.
 - C. Put 3 red chips and 1 white chip in a bag. Draw a chip from the bag, record the result, and put it back in the bag. Do this 6 times and count the number of red chips drawn.
 - D. Make a spinner with 4 equally-sized sections. Color three sections green and one sections red. Spin the spinner 6 times and count the number of times the spinner lands on green.
 - E. Put the numbers 1 through 6 in a bag. Pick a number from the bag.
4. A frozen yogurt store sells 3 different flavors of yogurt: vanilla, chocolate, and strawberry. An order of frozen yogurt comes with 4 different topping choices: fudge, caramel, fruit, or plain. How many unique ways can you order a frozen yogurt? (You can only select one flavor of yogurt and one topping).
5. Imagine you flip a coin and roll a standard number cube. This is a situation that involves 2 parts (a coin and a cube) and has a total of 12 outcomes in the sample space. Describe a situation that involves 3 parts and has a total of 48 outcomes in the sample space.

6. Tyler is about to spin a spinner like the one shown.

- a. Describe an event that is certain to happen on the next spin.
- b. Describe an event that is impossible on the next spin.
- c. Describe an event that is likely to happen, but not certain, on the next spin.
- d. Describe an event that is unlikely to happen, but possible, on the next spin.



7. You select a card from a deck that contains 25 red cards, 12 blue cards, 8 yellow cards and 5 green cards.

- a. What is the probability of drawing a red card? Explain your reasoning.
- b. What is the probability of drawing a green card? Explain your reasoning.
- c. What is the probability of drawing a blue card? Explain your reasoning.

Probability and Sampling: End-of-Unit Assessment (A)

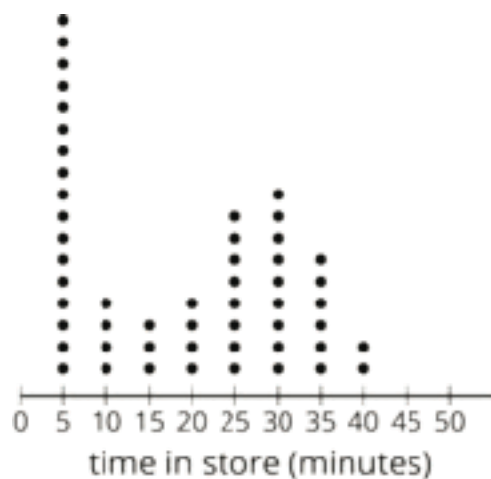
Do not use a calculator. A standard number cube has the numbers 1 through 6 on its faces.

1. Elena would like to know the average height of seventh graders in her school district. She measures the heights of everyone in a random sample of 20 students. The mean height of Elena's sample is 61 inches, and the MAD (mean absolute deviation) is 2 inches.

Select **all** the true statements.

- A. The median height of the sample must be between 59 and 63 inches.
- B. Another random sample of 20 students is likely to have a mean between 57 and 65 inches.
- C. The mean height of these 20 students is likely to be the same as the mean height of all students in the district.
- D. The mean height of these 20 students is likely to be the same as the mean height of a second random sample of 20 students.
- E. Elena would be more likely to get an accurate estimate of the mean height of the population by sampling 40 people instead of sampling 20 people.

2. Here is a dot plot showing how much time customers spent in a store, rounded to the nearest five minutes.



Which of the following is a representative sample of this population?

A



B



C



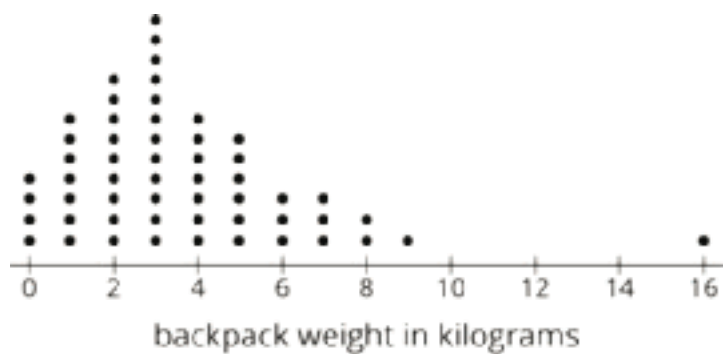
D



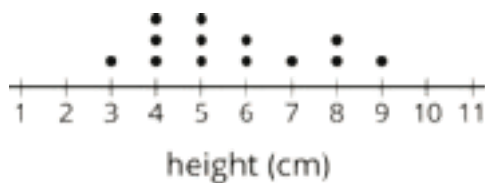
- A. A
- B. B
- C. C
- D. D

3. Select **all** of the data sets for which you would use the mean to describe the center of the data.

A.

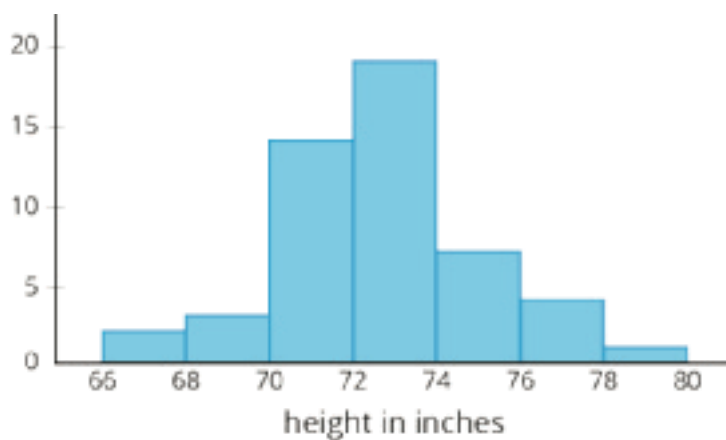


B.



C. blue, red, blue, yellow, blue

D.



E. 71, 73, 75, 72, 78, 79, 70

4. An administrator of a large middle school is installing some vending machines in the school. She wants to know what type of machine would be most popular.

a. What is the population for the administrator's question?

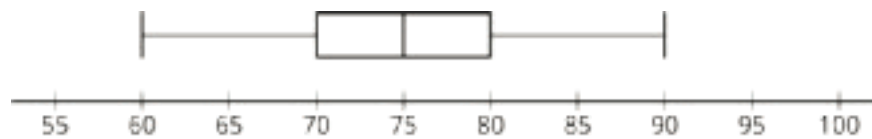
b. Give an example of a sample the administrator could use to help answer her question.

5. Two classes of students took an exam.

Here is a list of the scores in Class A:

65, 70, 70, 80, 80, 85, 85, 85, 90, 90, 100

Here is a box plot that shows the scores in Class B:



a. Which class had better overall results on the exam?

b. Which class had greater variability in the results?

6. A store owner asks each person to write “Yes” or “No” on a slip of paper as they leave, secretly writing down whether they were happy with their experience. At the end of the day, the owner selects 12 slips at random and looks at them. These were the results:

yes, yes, yes, no, yes, no, yes, yes, no, yes, no, yes

- a. Estimate the proportion of all shoppers who were happy with their experience that day.
 - b. On a different day, the owner found that 25% of the 12 selected slips were marked “Yes.” Should the store owner believe that these two results reasonably represent the overall proportion of happy shoppers, or should the owner gather more data? Explain how you know.
7. A scientist wants to know if there is a meaningful difference between two groups of gels that grow bacteria. He randomly selects six gels from each group, and counts the number of bacteria spots on each gel:

Group A: 9, 12, 13, 13, 14, 17

Group B: 8, 6, 5, 8, 13, 8

Is there a meaningful difference between the two groups? Show all calculations that lead to your answer.

Probability and Sampling: End-of-Unit Assessment (B)

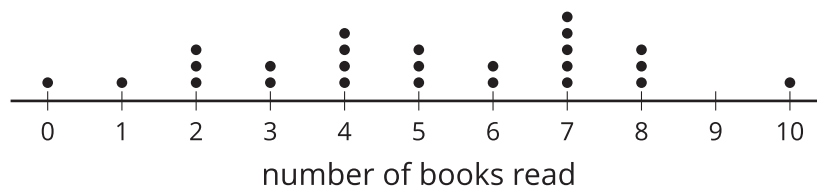
Do not use a calculator. A standard number cube has the numbers 1 through 6 on its faces.

1. Elena would like to know the average speed that people drive on her street. She tracks the speed of 30 random cars that drive through with a speedometer. The mean of Elena's sample is 30 miles per hour, and the MAD (mean absolute deviation) is 3.

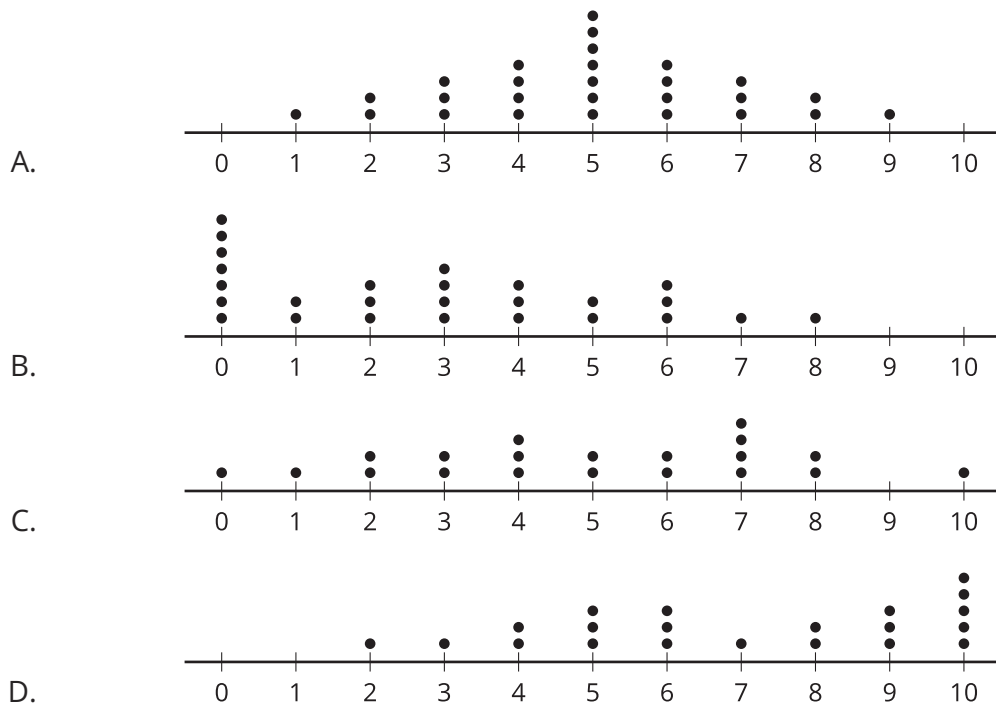
Select **all** the true statements.

- A. Elena would be more likely to get an accurate estimate of the mean speed of the population by sampling 60 cars instead of sampling 30 cars.
- B. The median speed of the sample must be between 25 and 35 miles per hour.
- C. Another random sample of 30 cars is likely to have a mean between 24 and 36 miles per hour.
- D. The mean speed of these 30 cars is likely to be the same as the mean speed of a second sample of 30 cars.
- E. The mean speed of these random 30 cars is likely to be the same as the mean of all cars that drive in a residential area.

2. Here is a dot plot showing how many books are read by each student per month.

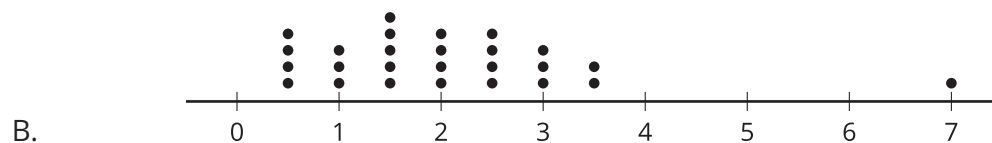


Which of the following is a representative sample of the population?

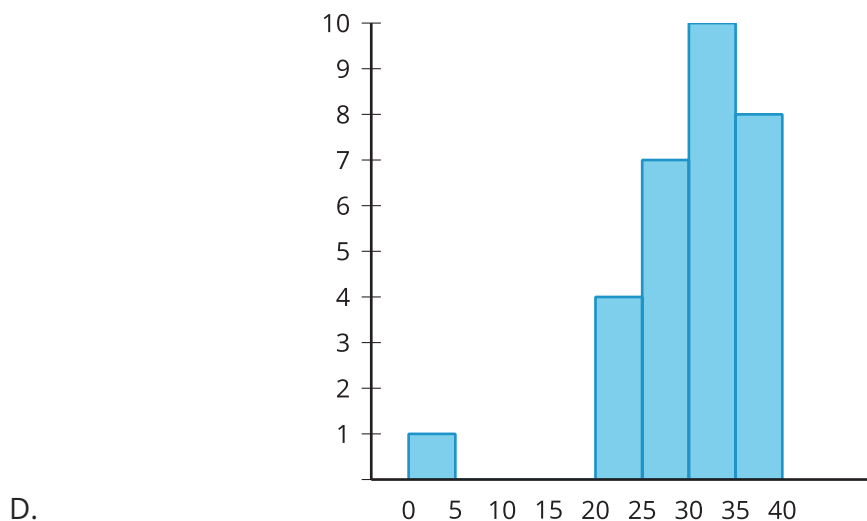


3. For which data set would the mean be a good way to describe the center of the data?

A. white, blue, white, white, red



C. 79, 80, 80, 88, 90, 92, 94

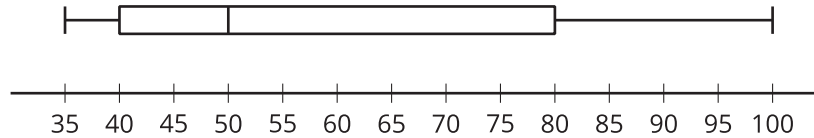


4. A school plans to start selling snacks at their basketball games. They want to know which snacks would be most popular.

a. What is the population for the school's question?

b. Give an example of a sample the school could use to help answer their question.

5. Two different classes took a survey to determine the number of minutes spent on homework one evening. Here is a box plot showing the results from Class A:



Here are the results from Class B:

10, 15, 20, 20, 25, 30, 30, 35, 40, 45, 45, 45, 60, 60, 80

- In which class did students spend less time on their homework?
 - Which class had less variability in the amount of time spent on homework?
6. The school bookstore surveyed a random sample of 10 students on whether they prefer to use a metal or plastic pen. Here were the results:
- metal, metal, metal, plastic, plastic, metal, plastic, plastic, plastic, plastic
- Estimate the proportion of all students that prefer to use a metal pen.
 - The next day, the bookstore surveyed another random sample of 10 students and found that 20% of them preferred plastic pens. Should the bookstore believe that the results from the two days reasonably represent the preferences of the student population, or should the bookstore gather more data? Explain how you know.

7. Students are conducting a class experiment to see if there is a meaningful difference between two groups of plants that have begun to sprout leaves. The teacher randomly selects 8 plants from each group and counts the number of leaves on each plant.

Group A: 2, 2, 3, 3, 5, 5, 5, 7

Group B: 10, 9, 7, 8, 10, 12, 14, 10

Is there a meaningful difference between the two groups? Show all calculations that lead to your answer.

Assessment Answer Keys

Check Your Readiness A and B
Mid-Unit Assessment A and B
End-of-Unit Assessment A and B

Assessment Answer Keys

Assessment : Check Your Readiness (A)

Teacher Instructions

Calculators should not be used, except to verify answers determined using pencil and paper.

Student Instructions

Do not use a calculator.

Problem 1

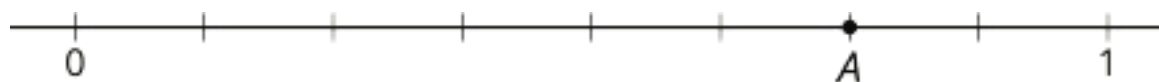
The content assessed in this problem is first encountered in Lesson 2: Chance Experiments.

Students should be fairly comfortable with a number line. This problem is intended to familiarize students with benchmark fractions and decimals between 0 and 1, which are useful in probability.

If most students struggle with this item, plan to take extra time for the launch of Activity 4. Display a blank number line and call on students to place benchmark fractions, decimals, and percents on the number line. Ask students to share their strategies for making the placement. Connect this to the questions "Which is closer to 1?" and "Which has the largest value?"

Statement

Select all the numbers that represent point A .



A. 0.25

B. 0.5

C. 0.75

D. $\frac{1}{2}$

E. $\frac{3}{4}$

F. $\frac{12}{16}$

Solution

["C", "E", "F"]

Aligned Standards

6.NS.C.6

Problem 2

The content assessed in this problem is first encountered in Lesson 2: Chance Experiments.

Calculating and making sense of percentages will come up throughout the unit as students study probability and sampling.

Watch for students struggling with the percentages that are greater than 100%. Encourage students to use fractions, benchmarks, or proportional reasoning to help. Since 1 push-up is 5% of the goal, any percentage can be determined by multiplying the number of push-ups by 5.

If most students struggle with this item, plan to emphasize the synthesis in Lesson 2, Activity 3 where percentages are used and interpreted with probability. During the unit, students have choices of reporting probabilities as fractions or percentages, but they need to interpret percentages, e.g. a 30% chance means we expect success in 3 out of 10 trials. Offer tape diagrams and double number lines as tools for helping students recall and calculate percentages. Additional practice can be found in 6th grade Unit 3, Lessons 11-16.

Statement

Students did push-ups for a fitness test. Their goal was 20 push-ups. For each student, determine what percentage of the goal they achieved.

1. Elena did 16 push-ups.
2. Jada did 42 push-ups.
3. Lin did 13 push-ups.
4. Andre did 21 push-ups.

Solution

1. 80%. 16 is 80% of 20, because $\frac{16}{20} = \frac{80}{100}$.
2. 210%. 42 is 210% of 20, because $\frac{42}{20} = \frac{210}{100}$.
3. 65%. 13 is 65% of 20, because $\frac{13}{20} = \frac{65}{100}$.
4. 105%. 21 is 105% of 20, because 1 is 5% of 20.

Aligned Standards

6.RP.A.3.c

Problem 3

The content assessed in this problem is first encountered in Lesson 3: What Are Probabilities?.

This problem mainly tests students' understanding of proportional relationships, while preparing for some of the concepts about using samples to draw inferences that will be presented in the upcoming unit.

If most students struggle with this item, it is not necessary to take any specific action. Students start off Lesson 3 with some additional examples of sample populations in this lesson, so they can start to practice this new concept. Attend to students who struggled with this problem and make sure they are able to follow the proportional reasoning they are using. More practice with this concept is found in Lesson 16.

Statement

Andre surveyed a random sample of 20 students and found that 13 of them were in favor of having school start one hour later.

1. The school has 250 students. Make an estimate for the number of students in the school who are in favor of having school start one hour later.
2. Do you think it would be surprising if 150 students out of 250 were in favor? Explain your reasoning.
3. Do you think it would be surprising if 100 students out of 250 were in favor? Explain your reasoning.

Solution

1. Estimates vary. Sample estimate: 163 students. 65% of the students surveyed were in favor, and 65% of 250 is 162.5.
2. No, this would not be surprising. 150 is 60% of 250, so to have 60% in favor instead of 65% would not be surprising.
3. Yes, this would be surprising. 100 is 40% of 250. To have 40% in favor, instead of something close to the 65% from the sample, would be surprising.

Aligned Standards

6.RP.A.3, 7.RP.A.3

Problem 4

The content assessed in this problem is first encountered in Lesson 11: Comparing Groups.

This item tests students' ability to distinguish measures of center from measures of spread. This is also an opportunity to determine whether students are familiar with these terms from their work in sixth grade. All of these terms are needed in the second half of the unit, both when using samples

to make inferences about populations and when testing whether there is a significant difference between two populations. Review of mean, MAD, and dot plots will come in this unit.

Students failing to select B or C have a significant gap in understanding about the meaning and use of IQR and MAD, or simply may not remember the terms. Students selecting A or D do not fully understand the differences between measures of center and measures of spread. Students selecting E may not know what range is, or may not recognize it as a measure of spread.

If most students struggle with this item, plan to spend additional time on the synthesis of Lesson 11, Activity 2. Review both MAD and IQR as well as mean and median. Focus on what MAD and IQR tell us about data sets, not how to calculate them. If extra practice interpreting MAD and IQR is needed, Grade 6 Unit 8 has additional work on calculating mean, median, MAD, and IQR.

Statement

Select **all** the measures of variability or spread.

- A. mean
- B. IQR (interquartile range)
- C. MAD (mean absolute deviation)
- D. median
- E. range

Solution

["B", "C", "E"]

Aligned Standards

6.SP.B.5.c

Problem 5

The content assessed in this problem is first encountered in Lesson 18: Comparing Populations Using Samples.

This question more specifically tests whether students remember how to calculate median and IQR. Watch for students attempting to answer the question without first sorting the data, and for students who have significant trouble understanding the question because they do not recognize or understand the vocabulary.

If most students struggle with this item, plan to review it, or a similar item using small data sets, during the launch of Activity 3.

Statement

Ten students each attempted 10 free throws. This list shows how many free throws each student made.

8, 5, 6, 6, 4, 9, 7, 6, 5, 9

1. What is the median number of free throws made?
2. What is the IQR (interquartile range)?

Solution

1. 6 free throws. The ordered list is 4, 5, 5, 6, 6, 6, 7, 8, 9, 9. The two middle terms in the ordered list are both 6.
2. 3 free throws. The first half of the data is 4, 5, 5, 6, 6, and its median is 5. The second half of the data is 6, 7, 8, 9, 9, and its median is 8. The IQR is 3, since $8 - 5 = 3$.

Aligned Standards

6.SP.A.3, 6.SP.B.5.c

Problem 6

The content assessed in this problem is first encountered in Lesson 18: Comparing Populations Using Samples.

This problem tests students' prior knowledge of box plots, and provides an opportunity for students to demonstrate how they are thinking about variability.

If students have difficulty, they will need a quick refresher on constructing box plots when this concept first appears in the unit. Students may argue variability by appealing to range (whether they use the term or not), but the IQR is the more important measure of spread for the work of this unit.

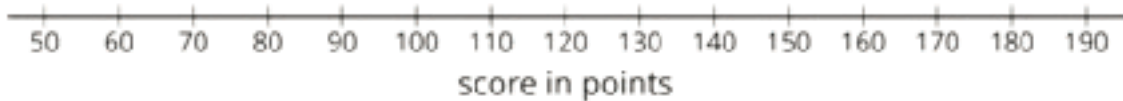
If most students struggle with this item, plan to review it during the launch of Activity 4. If students are struggling to understand how to construct box plots, Grade 6 Unit 8, Lesson 16 has activities to help students understand and construct them.

Statement

Two groups went bowling. Here are the scores from each group.

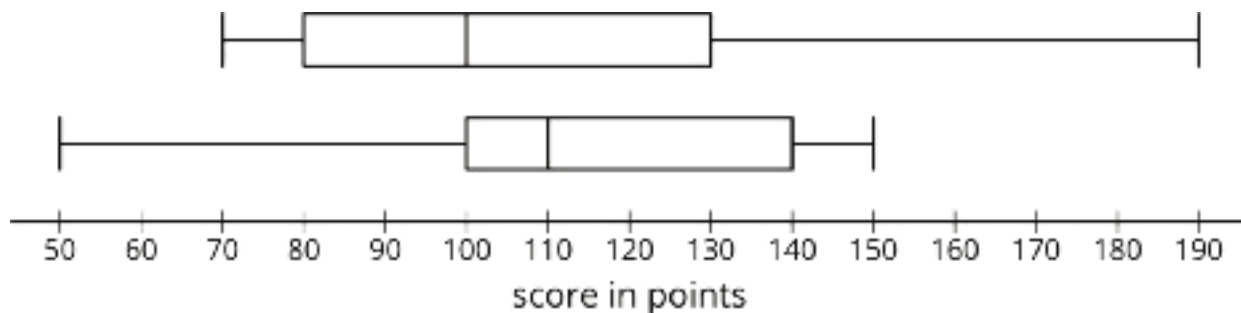
Group A: 70, 80, 90, 100, 110, 130, 190

Group B: 50, 100, 107, 110, 120, 140, 150



1. Construct two box plots, one for the data in each group.
2. Which group shows greater variability? Explain how you know.

Solution



1. See image.
2. Group A shows greater variability. It has a wider range (from 70 to 190) and a wider IQR (from 80 to 130).

Aligned Standards

6.SP.B.4, 6.SP.B.5

Problem 7

The content assessed in this problem is first encountered in Lesson 11: Comparing Groups.

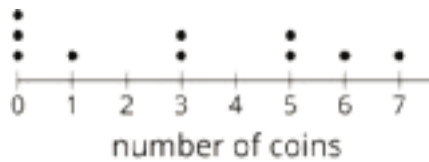
This problem tests students' fluency with mean, MAD, and dot plots.

Look at students' explanations to see which incorrect answers come from a misunderstanding of mean and MAD and which are simply arithmetic errors.

If most students struggle with this item, plan to review it during the synthesis of Lesson 11, Activity 2 to help students recall how to use the dot plot to find the mean and MAD. If students struggle with both this item and item 4, plan to use activities from 6th grade to do a more thorough review of interpreting and estimating measures of center and spread to compare data sets represented in box plots.

Statement

This dot plot shows the number of coins in 10 students' pockets.



What is the mean and MAD (mean absolute deviation) of this data? Explain how you know.

Solution

The mean is 3 coins. The MAD is 2.2 coins.

First, calculate the mean. The total number of coins is 30. The mean is 3 coins, because $\frac{30}{10} = 3$. The absolute deviations are 3, 3, 3, 2, 0, 0, 2, 2, 3, 4. The total of these is 22. The MAD is $\frac{22}{10}$.

Aligned Standards

6.SP.B.5.c

Assessment : Check Your Readiness (B)

Teacher Instructions

Calculators should not be used, except to verify answers determined using pencil and paper.

Student Instructions

Do not use a calculator.

Problem 1

The content assessed in this problem is first encountered in Lesson 2: Chance Experiments.

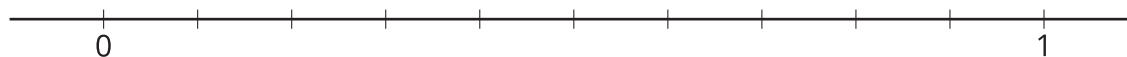
Students should be fairly comfortable with placing a list of numbers in order and on a number line. This problem is intended to familiarize students with fractions and decimals between 0 and 1, which are useful in probability.

If most students struggle with this item, plan to take extra time for the launch of Activity 4. Display a blank number line and call on students to place benchmark fractions, decimals, and percents on the number line. Ask students to share their strategies for making the placement. Connect this to the questions "Which is closer to 1?" and "Which has the largest value?"

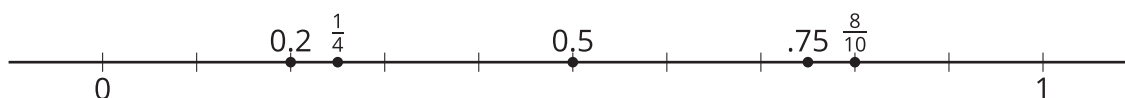
Statement

Plot and label each number on the number line.

0.75, $\frac{1}{4}$, 0.2, 0.5, $\frac{8}{10}$



Solution



Aligned Standards

6.NS.C.6

Problem 2

The content assessed in this problem is first encountered in Lesson 2: Chance Experiments.

Calculating and making sense of percentages will come up throughout the unit as students study probability and sampling.

Watch for students struggling with the percentages that are greater than 100%. Encourage students to use fractions, benchmarks, or proportional reasoning to help. Since 6 baskets are 10% of the goal, any percentage can be determined by dividing the baskets by 6 and then multiplying by 10.

If most students struggle with this item, plan to emphasize the synthesis in Lesson 2, Activity 3 where percentages are used and interpreted with probability. During the unit, students have choices of reporting probabilities as fractions or percentages, but they need to interpret percentages, e.g. a 30% chance means we expect success in 3 out of 10 trials. Offer tape diagrams and double number lines as tools for helping students recall and calculate percentages. Additional practice can be found in 6th grade Unit 3, Lessons 11-16.

Statement

Teachers held a basketball shooting contest. Their goal was to make 60 baskets. For each teacher, determine what percentage of the goal they achieved.

1. Teacher A made 12 baskets.
2. Teacher B made 42 baskets.
3. Teacher C made 66 baskets.
4. Teacher D made 9 baskets.

Solution

1. 20%. 12 is 20% of 60, because $12 \div 6 = 2$ and $2 \cdot 10 = 20$.
2. 70%. 42 is 70% of 60, because $42 \div 6 = 7$ and $7 \cdot 10 = 70$.
3. 110%. 66 is 110% of 60, because $66 \div 6 = 11$ and $11 \cdot 10 = 110$.
4. 15%. 9 is 15% of 60, because $9 \div 6 = 1.5$ and $1.5 \cdot 10 = 15$.

Aligned Standards

6.RP.A.3.c

Problem 3

The content assessed in this problem is first encountered in Lesson 3: What Are Probabilities?.

This problem mainly tests students' understanding of proportional relationships, while preparing for some of the concepts about using samples to draw inferences that will be presented in the upcoming unit.

If most students struggle with this item, it is not necessary to take any specific action. Students start off Lesson 3 with some additional examples of sample populations in this lesson, so they can start

to practice this new concept. Attend to students who struggled with this problem and make sure they are able to follow the proportional reasoning they are using. More practice with this concept is found in Lesson 16.

Statement

A survey was conducted of a random sample of 15 sports team coaches in a school district, and found that 6 of them were in favor of a shorter season for their sport.

1. The school district has 132 sports team coaches. Make an estimate for the number of coaches in the school district who are in favor of a shorter season for their sport.
2. Do you think it would be surprising if 99 coaches were in favor? Explain your reasoning.
3. Do you think it would be surprising if 60 coaches were in favor? Explain your reasoning.

Solution

1. Estimates vary. Sample estimate: 53 coaches. 40% of the coaches surveyed were in favor, and 40% of 132 is 52.8.
2. Yes, this would be surprising. 99 is 75% of 132, so to have 75% in favor instead of something close to the 40% from the sample would be surprising.
3. No, this would not be surprising. 60 is about 45% of 132. To have 45% in favor instead of 40% would not be surprising.

Aligned Standards

6.RP.A.3, 7.RP.A.3

Problem 4

The content assessed in this problem is first encountered in Lesson 11: Comparing Groups.

This item tests students' ability to distinguish measures of center from measures of spread. This is also an opportunity to determine whether students are familiar with these terms from their work in sixth grade. All of these terms are needed in the second half of the unit, both when using samples to make inferences about populations and when testing whether there is a significant difference between two populations. Review of mean, MAD, and dot plots will come in this unit.

Students failing to select A or D have a significant gap in understanding about the meaning and use of mean and median, or simply may not remember the terms. Students selecting B or C do not fully understand the differences between measures of center and measures of spread. Students selecting E may not know what range is, or may not recognize it as a measure of spread.

If most students struggle with this item, plan to spend additional time on the synthesis of Lesson 11, Activity 2. Review both MAD and IQR as well as mean and median. Focus on what MAD and IQR tell us about data sets, not how to calculate them. If extra practice interpreting MAD and IQR is needed, Grade 6 Unit 8 has additional work on calculating mean, median, MAD, and IQR.

Statement

Select **all** the measures of center.

- A. mean
- B. IQR (interquartile range)
- C. MAD (mean absolute deviation)
- D. median
- E. range

Solution

["A", "D"]

Aligned Standards

6.SP.B.5.c

Problem 5

The content assessed in this problem is first encountered in Lesson 18: Comparing Populations Using Samples.

This question more specifically tests whether students remember how to calculate range and quartiles of data. Watch for students attempting to answer the question without first sorting the data, and for students who have significant trouble understanding the question because they do not recognize or understand the vocabulary.

If most students struggle with this item, plan to review it, or a similar item using small data sets, during the launch of Activity 3.

Statement

Jada completed eleven homework assignments. This list shows her scores for each assignment.

10, 9, 8, 10, 7, 9, 8, 6, 10, 10, 8

1. What is the median score?
2. What are the first quartile and the third quartile?
3. What is the interquartile range?

Solution

1. The median is 9. The ordered list is 6,7,8,8,8,9,9,10,10,10,10.

2. Quartile 1 is 8. The first half of the data is 6,7,8,8,8, and its middle value is 8. Quartile 3 is 10. The second half of the data is 9,10,10,10,10, and its middle value is 10.
3. The IQR is 2, because $10 - 8 = 2$

Aligned Standards

6.SP.A.3, 6.SP.B.5.c

Problem 6

The content assessed in this problem is first encountered in Lesson 18: Comparing Populations Using Samples.

This problem tests students' prior knowledge of box plots, and provides an opportunity for students to demonstrate how they are thinking about variability.

If students have difficulty, they will need a quick refresher on constructing box plots when this concept first appears in the unit. Students may argue variability by appealing to range (whether they use the term or not), but the IQR is the more important measure of spread for the work of this unit.

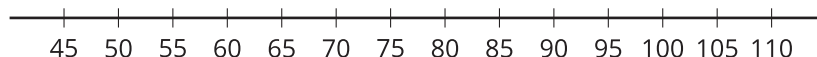
If most students struggle with this item, plan to review it during the launch of Activity 4. If students are struggling to understand how to construct box plots, Grade 6 Unit 8, Lesson 16 has activities to help students understand and construct them.

Statement

Two students compared their test scores. Here are the scores for each student.

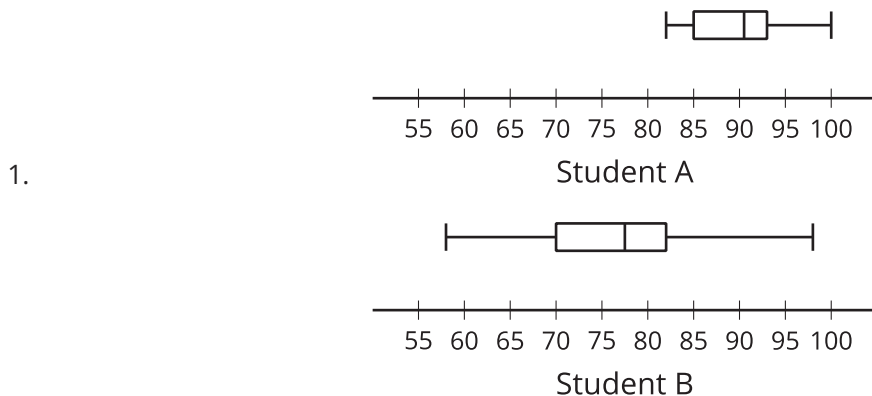
Student A: 91, 100, 82, 90, 93, 85

Student B: 80, 70, 58, 98, 75, 81



1. Construct two box plots, one for the data for each student.
2. Which student's scores show greater variability? Explain how you know.

Solution



2. Student B shows greater variability. Student B's scores have a greater IQR.

Aligned Standards

6.SP.B.4, 6.SP.B.5

Problem 7

The content assessed in this problem is first encountered in Lesson 11: Comparing Groups.

This problem tests students' fluency with a list of data, mean, and MAD. Look at students' explanations to see which incorrect answers come from a misunderstanding of mean and MAD and which are simply arithmetic errors.

If most students struggle with this item, plan to review it during the synthesis of Lesson 11, Activity 2 to help students recall how to use the dot plot to find the mean and MAD. If students struggle with both this item and item 4, plan to use activities from 6th grade to do a more thorough review of interpreting and estimating measures of center and spread to compare data sets represented in box plots.

Statement

The data set shows the number of shells that 12 people collected.

4, 5, 5, 4, 1, 6, 8, 2, 2, 2, 4, 5

1. What is the mean number of shells collected?
2. What is the MAD (mean absolute deviation) of this data? Explain how you know.

Solution

1. The mean is 4.
2. First, calculate the mean. The total number of shells is 48. The mean is 4 shells, because $48 \div 12 = 4$. The absolute deviations are 0, 1, 1, 0, 3, 2, 4, 2, 2, 2, 0, 1. The total of these is 18. The MAD is $18 \div 12 = 1.5$.

Aligned Standards

6.SP.B.5.c

Assessment : Mid-Unit Assessment (A)

Teacher Instructions

Administer this assessment after lesson 10. Allow the use of a calculator, or not, at your discretion.

Problem 1

Students choosing B may have read the problem as greater than 4. Students choosing A may have read the problem as equal to 4. Students choosing letter D may not understand probability as they just placed 4 over 9.

Statement

Tiles with the numbers 1 through 9 are placed in a bag. What is the probability of choosing an even number that is greater than or equal to 4?

- A. $\frac{1}{9}$
- B. $\frac{2}{9}$
- C. $\frac{3}{9}$
- D. $\frac{4}{9}$

Solution

C

Aligned Standards

7.SP.C.8.a

Problem 2

Students choosing B or E may not understand how to interpret “unlikely.” Students choosing C may not have noticed that they were looking for events that were possible.

Statement

Select all of the events that are possible, but unlikely.

- A. Opening a 300-page book to exactly page 143.
- B. A second grade classroom contains some books.
- C. You will roll a 7 on a standard 6-sided number cube.
- D. Everyone in your school will get to school on time on the next school day.
- E. A triangle has 3 sides.

Solution

["A", "D"]

Aligned Standards

7.SP.C.5

Problem 3

Students selecting B are missing the concept that the outcomes are not equally likely, so a choice from 1 to 5 cannot be used. Students selecting C understand the general concept but not the correct probability.

Statement

Han heard that the school's football team has a $\frac{1}{3}$ probability of winning each of their next 5 games. Select **all** the ways Han could accurately simulate the number of games the football team will win.

- A. Roll a standard number cube 5 times. Count the number of 1's and 2's.
- B. Put the numbers 1 through 5 in a bag. Pick a number from the bag.
- C. Roll a standard number cube 5 times. Count the number of ones.
- D. Mark a spinner with 3 equally-sized sections. Write "win" on 2 of the sections. Spin the spinner 5 times and count the number of times it lands on "win."
- E. Put 2 white chips and 1 red chip in a bag. Draw a chip from the bag, record its result, and put it back in the bag. Do this 5 times and count the number of red chips drawn.

Solution

["A", "E"]

Aligned Standards

7.SP.C.7, 7.SP.C.8.c

Problem 4

Students may choose to create a tree diagram or chart to represent all the choices of popcorn sizes and toppings.

Statement

A movie theater sells 3 different sizes of popcorn: small, medium, and large. An order of popcorn comes with 3 different topping choices: butter, cheese, or caramel. How many unique ways can you order a bag of popcorn? (You can only select one topping.)

Solution

9

Aligned Standards

7.SP.C.8.b

Problem 5

Statement

Imagine that there is a spinner with 5 equal sections, and also a standard number cube. You spin the spinner and roll the cube. This is a situation that involves 2 parts (the spinner and the cube) and has a total of 30 outcomes in the sample space.

Describe a situation that involves 3 parts and has a total of 36 outcomes in the sample space.

Solution

Answers vary. Sample response: flip a coin, roll a standard number cube, and choose one letter from the word CAT.

Aligned Standards

7.SP.C.8.b

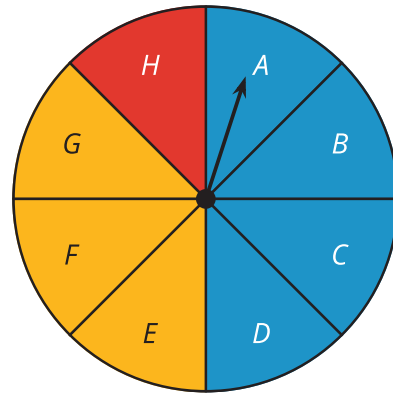
Problem 6

Use this problem to confirm that students understand how to find probability, instead of just the numbers associated with a probability.

Statement

Priya is about to spin a spinner like the one shown.

1. Describe an event that is certain to happen on the next spin.
2. Describe an event that is unlikely to happen, but possible, on the next spin.
3. Describe an event that is impossible on the next spin.
4. Describe an event that is likely to happen, but not certain, on the next spin.



Solution

Answers vary. Sample response:

1. The color will be blue, red, or yellow.
2. The color will be red.
3. The color will be purple.
4. The color will be blue or red.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample:
 - a. Priya spins a blue, red, or yellow.
 - b. Priya spins a red.
 - c. Priya spins pink.
 - d. Priya spins blue or red.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: One of the descriptions is incorrect or does not describe an event.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: two or more descriptions are incorrect or do not describe an event.

Aligned Standards

7.SP.C.5

Problem 7

Statement

A game uses a special deck of cards with 40 cards numbered from 1 to 40. You draw a card from the shuffled deck.

1. What is the probability of drawing a card that is divisible by 2? Explain your reasoning.
2. What is the probability of drawing a card that is divisible by 5? Explain your reasoning.
3. What is the probability of drawing a card that is divisible by 10? Explain your reasoning.

Solution

1. $\frac{20}{40}$ or $\frac{1}{2}$ because there are 20 cards that would be divisible by 2.
2. $\frac{8}{40}$ or $\frac{1}{5}$ because there are 8 cards that are divisible by 5.
3. $\frac{4}{40}$ or $\frac{1}{10}$ because there are 4 cards that are divisible by 10.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample:
 - $\frac{20}{40}$ or $\frac{1}{2}$ because there are 20 cards that would be divisible by 2.
 - $\frac{8}{40}$ or $\frac{1}{5}$ because there are 8 cards that are divisible by 5.
 - $\frac{4}{40}$ or $\frac{1}{10}$ because there are 4 cards that are divisible by 10.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample error: Minor visible calculation errors cause one of the probabilities to be incorrect.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: An error in counting the number of cards that are divisible by the given divisor, but the probability is computed out of 40. The probabilities $\frac{1}{40}$, $\frac{1}{40}$, and $\frac{1}{40}$ are given as answers, but with explanations that indicate the probability of drawing a 2, a 5, and a 10.

Tier 4 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: two or more probabilities are incorrect and no explanations are provided.

Aligned Standards

7.SP.C.8

Assessment : Mid-Unit Assessment (B)

Teacher Instructions

Administer this assessment after lesson 10. Allow the use of a calculator, or not, at your discretion.

Student Instructions

A standard number cube has the numbers 1 through 6 on its faces.

Problem 1

Students choosing A may have read the problem less than 2. Students choosing D may have read the problem as greater than 2. Students choosing C may not understand how to calculate probability.

Statement

A standard number cube is rolled. What is the probability of rolling a number less than or equal to 2?

- A. $\frac{1}{6}$
- B. $\frac{2}{6}$
- C. $\frac{3}{6}$
- D. $\frac{4}{6}$

Solution

B

Aligned Standards

7.SP.C.8.a

Problem 2

Students choosing A, B, or D may interpret an unlikely event as an event that is impossible.

Statement

Select **all** the events that have a probability of 0.

- A. Choosing a red block from a bag with 9 white blocks and 1 red block.
- B. Rolling a 6, three times in a row, on a standard number cube.
- C. Rolling a 7 on a standard number cube.
- D. Opening a 400-page book to exactly page 231.
- E. Opening a 400-page book a page number greater than 450.

Solution

["C", "E"]

Aligned Standards

7.SP.C.5

Problem 3

Students selecting A or B understand the general concept of simulations but not the correct probability. Students selecting E are missing the concept that the outcomes are not equally likely, so a choice from 1 to 20 cannot be used.

Statement

The basketball coach says that based on Noah's free throws this season, Noah has a $\frac{3}{4}$ probability of making each free throw. Select **all** the ways Noah could accurately simulate the number of free throws he will make in his next 6 attempts.

- A. Flip a coin 6 times. Count the number of heads.
- B. Make a spinner with 6 equally-sized sections, labeled with the numbers 1 through 6. Spin the spinner 6 times and count how many times the spinner lands on 3 or 4.
- C. Put 3 red chips and 1 white chip in a bag. Draw a chip from the bag, record the result, and put it back in the bag. Do this 6 times and count the number of red chips drawn.
- D. Make a spinner with 4 equally-sized sections. Color three sections green and one sections red. Spin the spinner 6 times and count the number of times the spinner lands on green.
- E. Put the numbers 1 through 6 in a bag. Pick a number from the bag.

Solution

["C", "D"]

Aligned Standards

7.SP.C.7, 7.SP.C.8.c

Problem 4

Students may choose to create a tree diagram or chart to represent all the choices of yogurt flavors and toppings.

Statement

A frozen yogurt store sells 3 different flavors of yogurt: vanilla, chocolate, and strawberry. An order of frozen yogurt comes with 4 different topping choices: fudge, caramel, fruit, or plain. How many unique ways can you order a frozen yogurt? (You can only select one flavor of yogurt and one topping).

Solution

12

Aligned Standards

7.SP.C.8.b

Problem 5

Statement

Imagine you flip a coin and roll a standard number cube. This is a situation that involves 2 parts (a coin and a cube) and has a total of 12 outcomes in the sample space. Describe a situation that involves 3 parts and has a total of 48 outcomes in the sample space.

Solution

Answers vary. Sample response: flip a coin, roll a standard number cube, and spin a spinner divided into 4 equally-sized sections, numbered 1 through 4.

Aligned Standards

7.SP.C.8.b

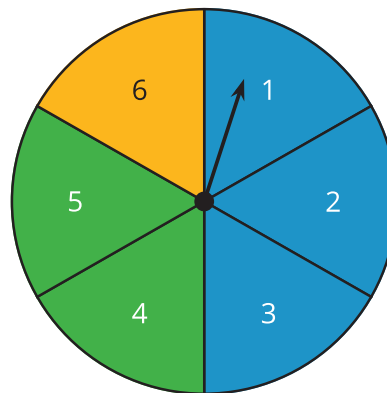
Problem 6

Use this problem to confirm that students understand how to find probability, instead of just the numbers associated with a probability.

Statement

Tyler is about to spin a spinner like the one shown.

1. Describe an event that is certain to happen on the next spin.
2. Describe an event that is impossible on the next spin.
3. Describe an event that is likely to happen, but not certain, on the next spin.
4. Describe an event that is unlikely to happen, but possible, on the next spin.



Solution

Answers vary. Sample response:

1. The color will be yellow, blue, or green.
2. The spinner lands on 7.
3. The color will be blue or green.
4. The spinner lands on 1.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample:
 - a. The color will be yellow, blue, or green.
 - b. The spinner lands on 7.
 - c. The color will be blue or green.
 - d. The spinner lands on 1.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: One of the descriptions is incorrect or does not describe the event.

Tier 3 response:

- Significant errors in work demonstrate a lack of conceptual understanding or mastery.
- Sample errors: two or more descriptions are incorrect or do not describe an event.

Aligned Standards

7.SP.C.5

Problem 7

Statement

You select a card from a deck that contains 25 red cards, 12 blue cards, 8 yellow cards and 5 green cards.

1. What is the probability of drawing a red card? Explain your reasoning.
2. What is the probability of drawing a green card? Explain your reasoning.
3. What is the probability of drawing a blue card? Explain your reasoning.

Solution

1. $\frac{25}{50}$ or $\frac{1}{2}$, because there are 25 red cards out of 50 total cards.
2. $\frac{5}{50}$ or $\frac{1}{10}$, because there are 5 green cards out of 50 total cards.
3. $\frac{12}{50}$ or $\frac{6}{25}$, because there are 12 blue cards out of 50 total cards.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample:
 - a. $\frac{25}{50}$ or $\frac{1}{2}$, because there are 25 red cards out of 50 total cards.
 - b. $\frac{5}{50}$ or $\frac{1}{10}$, because there are 5 green cards out of 50 total cards.
 - c. $\frac{12}{50}$ or $\frac{6}{25}$, because there are 12 blue cards out of 50 total cards.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample error: Minor visible calculation errors cause one of the probabilities to be incorrect.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: An error in calculating the total number of cards or students do not simplify correctly.

Tier 4 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: two or more probabilities are incorrect and no explanations are provided.

Aligned Standards

7.SP.C.8

Assessment : End-of-Unit Assessment (A)

Teacher Instructions

Calculators should not be used.

Student Instructions

Do not use a calculator. A standard number cube has the numbers 1 through 6 on its faces.

Problem 1

Students selecting A are incorrectly using the MAD and mean information applied to the median. Students failing to select B have not learned this unit's concept about twice the MAD determining expected variation. Students selecting C are making an error about the sample as it relates to the population, while students selecting D are making a similar error about the nature of multiple samples. Students failing to select E may need a refresher that the mean of a sample is more likely to be close to the mean of the population as the sample size gets bigger.

Statement

Elena would like to know the average height of seventh graders in her school district. She measures the heights of everyone in a random sample of 20 students. The mean height of Elena's sample is 61 inches, and the MAD (mean absolute deviation) is 2 inches.

Select **all** the true statements.

- A. The median height of the sample must be between 59 and 63 inches.
- B. Another random sample of 20 students is likely to have a mean between 57 and 65 inches.
- C. The mean height of these 20 students is likely to be the same as the mean height of all students in the district.
- D. The mean height of these 20 students is likely to be the same as the mean height of a second random sample of 20 students.
- E. Elena would be more likely to get an accurate estimate of the mean height of the population by sampling 40 people instead of sampling 20 people.

Solution

["B", "E"]

Aligned Standards

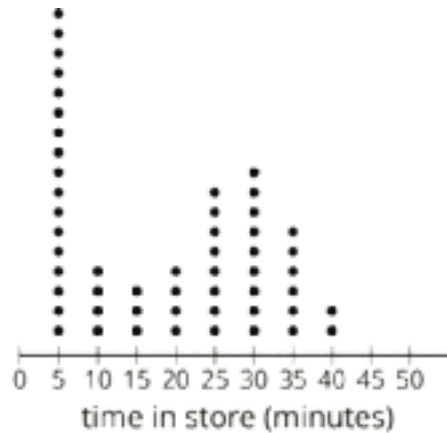
7.SP.A.1, 7.SP.A.2

Problem 2

Students selecting A noticed the overall shape of the distribution but did not take into account the large peak of data at 5 minutes. Students selecting B noticed the overall shape of the distribution but did not notice a shift in the data. Students selecting C may believe that samples must have a symmetric distribution, which is not true.

Statement

Here is a dot plot showing how much time customers spent in a store, rounded to the nearest five minutes.



Which of the following is a representative sample of this population?

A



B




C



D



- 
- A. A
 - B. B
 - C. C
 - D. D

Solution

D

Aligned Standards

7.SP.A.1

Problem 3

Students selecting A are not recognizing that the point at 16 will skew the mean of the data.

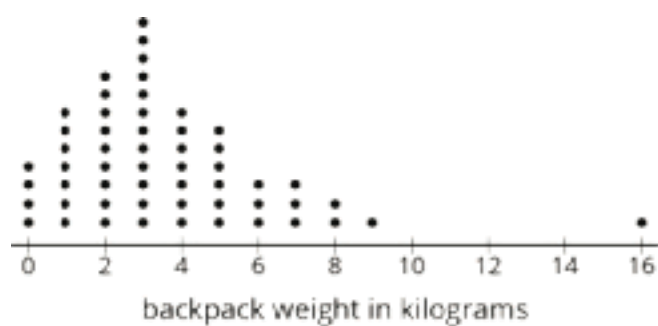
Students selecting C are not recognizing that the mode is used as the center of measure instead of the mean or they do not know what mode is. Students not selecting B, D, or E need a review of mean in graphs and data.



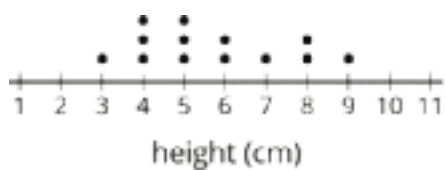
Statement

Select **all** of the data sets for which you would use the mean to describe the center of the data.

A.

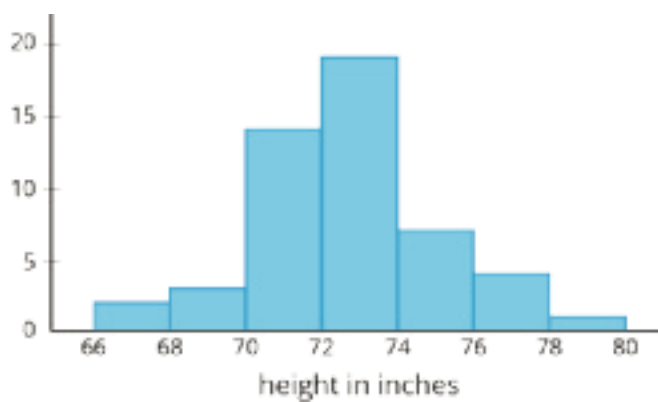


B.



C. blue, red, blue, yellow, blue

D.



E. 71, 73, 75, 72, 78, 79, 70

Solution

["B", "D", "E"]

Aligned Standards

7.SP.A.2, 7.SP.B.4

Problem 4

This item checks whether students understand the meanings of the terms “population” and “sample” as well as their use in context.

Statement

An administrator of a large middle school is installing some vending machines in the school. She wants to know what type of machine would be most popular.

1. What is the population for the administrator's question?
2. Give an example of a sample the administrator could use to help answer her question.

Solution

Answers vary. Sample response:

1. Current students, teachers, and staff of the school.
2. Ask 30 people in the school whether they would be more likely to buy water, juice, or energy drinks.

For the first question, the answer must acknowledge all the people that regularly spend time in the school, not just students. There is lots of flexibility in acceptable responses to the second question, since the item doesn't say how "large" the school is.

Aligned Standards

7.SP.A.1

Problem 5

Some students choose to make a box plot for Class A. Although this is a short answer problem, watch for students mistakenly using the mean instead of the median or failing to use measures of variability correctly.

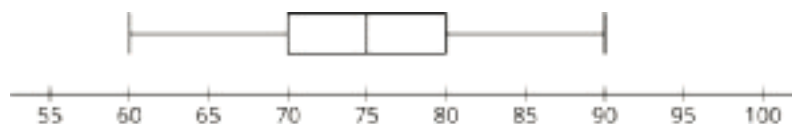
Statement

Two classes of students took an exam.

Here is a list of the scores in Class A:

65, 70, 70, 80, 80, 85, 85, 85, 90, 90, 100

Here is a box plot that shows the scores in Class B:



1. Which class had better overall results on the exam?
2. Which class had greater variability in the results?

Solution

1. Class A (The median for Class A is higher than the scores of 75% of the students in Class B.)
2. Class A (The range in Class A is larger, and the IQR in Class A is larger.)

Aligned Standards

7.SP.B.4

Problem 6

Accept a wide enough range of estimates for the population proportion, though it is likely students report the sample proportion. The large variation between the two results is the key here, recognizing that these two proportions are far away from one another. There are more formal ways to decide how much variation is considered to be significant, but those are beyond grade level.

Statement

A store owner asks each person to write “Yes” or “No” on a slip of paper as they leave, secretly writing down whether they were happy with their experience. At the end of the day, the owner selects 12 slips at random and looks at them. These were the results:

yes, yes, yes, no, yes, no, yes, yes, no, yes, no, yes

1. Estimate the proportion of all shoppers who were happy with their experience that day.
2. On a different day, the owner found that 25% of the 12 selected slips were marked “Yes.” Should the store owner believe that these two results reasonably represent the overall proportion of happy shoppers, or should the owner gather more data? Explain how you know.

Solution

1. Answers vary. Sample response: $\frac{2}{3}$. Any answer between 0.6 and 0.75 is reasonable.
2. More data is needed, specifically data from other days. There is a very high variation between these two results. It is possible that one of the results is unusual due to something that happened on that day, and data from other days would be useful in understanding what happened.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample:

1. $\frac{3}{4}$

2. The owner needs more data. The second day’s results are too different.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: proportion is close to but not in correct range; invalid explanation for why the owner should gather more data; incorrect proportion between 0 and 1 based on visible calculation error.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: proportion wildly out of range, including greater than 1 or less than 0; incorrect belief that two sets of data reasonably represent the overall proportion.

Aligned Standards

7.SP.A.2, 7.SP.C.6

Problem 7

The data is built so that students can use either the mean and MAD or the median and IQR to work the problem. Either method is worth full credit, but be certain that students are consistently applying one or the other throughout.

It is also possible students come to a conclusion visually through box plots or dot plots, without any of the associated calculations. Such a solution should not be worth full credit, since the problem asks for calculations but should still be worth most of the credit in the problem. If students build box plots to compare, note that the box plot for Group B has the same median and third quartile, which may confuse some students.

Scaffolding can be added to this problem, such as asking for specific calculations, but it should not be necessary because this is a very recent topic.

Statement

A scientist wants to know if there is a meaningful difference between two groups of gels that grow bacteria. He randomly selects six gels from each group, and counts the number of bacteria spots on each gel:

Group A: 9, 12, 13, 13, 14, 17

Group B: 8, 6, 5, 8, 13, 8

Is there a meaningful difference between the two groups? Show all calculations that lead to your answer.

Solution

Yes, there is a significant difference. Use either mean and MAD (mean absolute deviation) or median and IQR (interquartile range) to decide. Using the means, the mean of Group A is 13, and the MAD is 1.67. The mean of Group B is 8, and the MAD is 1.67. The difference in means is 3 MADs,

so it is a meaningful difference. Using the medians, the median of Group A is 13, and the IQR is 2. The median of Group B is 8, and the IQR is 2. The difference in medians is 2.5 IQRs, so it is a meaningful difference.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample: Yes. The median of Group A is 13. The median of Group B is 8. The IQR of each group is 2. The difference in medians is more than 2 IQRs.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: minor visible calculation errors cause one or two means, medians, MADs or IQRs to be incorrect. Acceptable errors: an error in calculating causes an incorrect conclusion about whether there is a meaningful difference between the two groups; an error in calculating mean or median leads to a corresponding error in calculating MAD or IQR.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: response using median and IQR does not sort Group B's data, but otherwise correctly works through the problem; conclusion about significant differences between the groups is driven only by means or medians and not MAD or IQR; incorrect conclusion about significant differences based on correct work; mixing center and spread measures, such as using mean and IQR; incorrect calculations with no work shown but a correct conclusion based on the results.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: badly incorrect algorithm for calculating mean, median, MAD, or IQR; failure to use these measures in problem work.

Aligned Standards

7.SP.B.3, 7.SP.B.4

Assessment : End-of-Unit Assessment (B)

Teacher Instructions

Calculators should not be used.

Student Instructions

Do not use a calculator. A standard number cube has the numbers 1 through 6 on its faces.

Problem 1

Students selecting B are incorrectly using the MAD and mean information applied to the median. Students failing to select C have not learned this unit's concept about how to use MAD to determine expected variation. Students selecting E are making an error about the sample as it relates to the population, while students selecting D are making a similar error about the nature of multiple samples. Students failing to select A may need a refresher that the mean of a sample is more likely to be close to the mean of the population as the sample size gets bigger.

Statement

Elena would like to know the average speed that people drive on her street. She tracks the speed of 30 random cars that drive through with a speedometer. The mean of Elena's sample is 30 miles per hour, and the MAD (mean absolute deviation) is 3.

Select **all** the true statements.

- A. Elena would be more likely to get an accurate estimate of the mean speed of the population by sampling 60 cars instead of sampling 30 cars.
- B. The median speed of the sample must be between 25 and 35 miles per hour.
- C. Another random sample of 30 cars is likely to have a mean between 24 and 36 miles per hour.
- D. The mean speed of these 30 cars is likely to be the same as the mean speed of a second sample of 30 cars.
- E. The mean speed of these random 30 cars is likely to be the same as the mean of all cars that drive in a residential area.

Solution

["A", "C"]

Aligned Standards

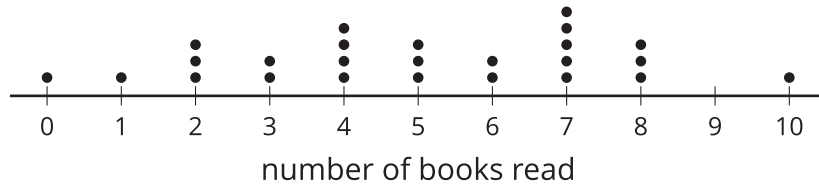
7.SP.A.1, 7.SP.A.2

Problem 2

Students selecting A may believe that samples must have a symmetric distribution, which is not true. Students selecting B and D do not understand how to show data in a sample population given the original.

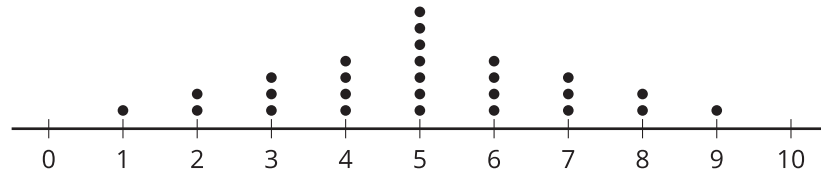
Statement

Here is a dot plot showing how many books are read by each student per month.

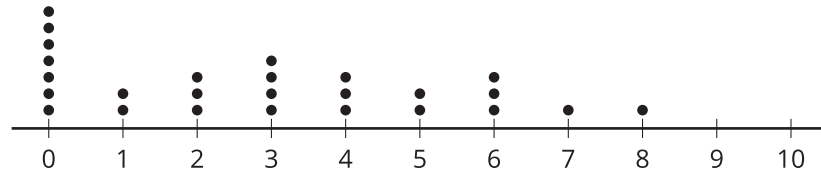


Which of the following is a representative sample of the population?

A.



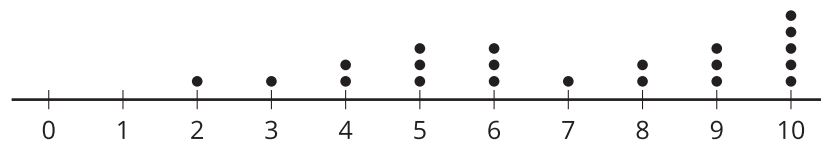
B.



C.



D.



Solution

C

Aligned Standards

7.SP.A.1

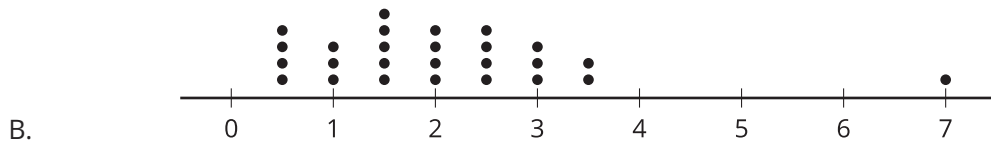
Problem 3

Students selecting A are not realizing that the mean is a numerical value that can't be computed for categorical data. Students selecting B or D are not recognizing that the presence of an extreme value makes mean a misleading way to describe the center of the data.

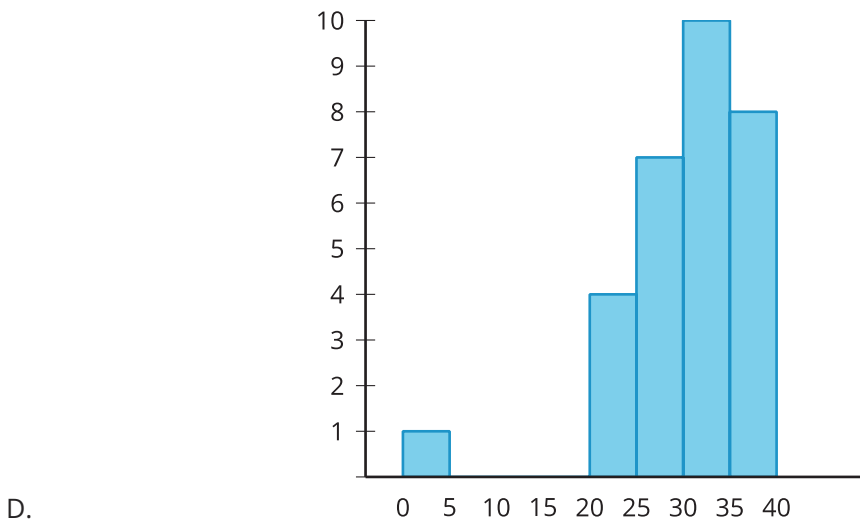
Statement

For which data set would the mean be a good way to describe the center of the data?

A. white, blue, white, white, red



C. 79, 80, 80, 88, 90, 92, 94



Solution

C

Aligned Standards

7.SP.A.2, 7.SP.B.4

Problem 4

This item checks whether students understand the meanings of the terms “population” and “sample” as well as their use in context.

Statement

A school plans to start selling snacks at their basketball games. They want to know which snacks would be most popular.

1. What is the population for the school's question?
2. Give an example of a sample the school could use to help answer their question.

Solution

Answers vary. Sample response:

1. Basketball players, coaches, parents, and other spectators.
2. Ask 30 people who attend the next basketball game whether they would be more likely to buy pretzels, carrot sticks, or string cheese.

For the first question, the answer must acknowledge all the people that attend school basketball games, not just students. There is lots of flexibility in acceptable responses to the second question, since the item doesn't say how many people usually attend the games.

Aligned Standards

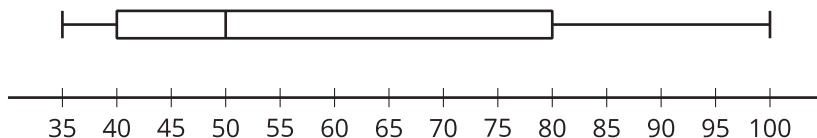
7.SP.A.1

Problem 5

Some students choose to make a box plot for Class B. Although this is a short answer problem, watch for students mistakenly using the mean instead of the median or failing to use measures of variability correctly.

Statement

Two different classes took a survey to determine the number of minutes spent on homework one evening. Here is a box plot showing the results from Class A:



Here are the results from Class B:

10, 15, 20, 20, 25, 30, 30, 35, 40, 45, 45, 45, 60, 60, 80

1. In which class did students spend less time on their homework?
2. Which class had less variability in the amount of time spent on homework?

Solution

1. Class B spent less time on their homework than Class A. (The median is 35 minutes for class B compared to 50 minutes for Class A.)
2. Class B had less variability in their time spent on homework. The IQR is for class B is 25, and for class A is 40.

Aligned Standards

7.SP.B.4

Problem 6

Accept a wide enough range of estimates for the population proportion, though it is likely students report the sample proportion. The large variation between the two results is the key here, recognizing that these two proportions are far away from one another. There are more formal ways to decide how much variation is considered to be significant, but those are beyond grade level.

Statement

The school bookstore surveyed a random sample of 10 students on whether they prefer to use a metal or plastic pen. Here were the results:

metal, metal, metal, plastic, plastic, metal, plastic, plastic, plastic, plastic

- a. Estimate the proportion of all students that prefer to use a metal pen.
- b. The next day, the bookstore surveyed another random sample of 10 students and found that 20% of them preferred plastic pens. Should the bookstore believe that the results from the two days reasonably represent the preferences of the student population, or should the bookstore gather more data? Explain how you know.

Solution

1. Answers vary. Sample response: $\frac{4}{10}$ or 0.4.
2. More data is needed, specifically data from other surveys. There is a very high variation between these two results. It is possible that one of the results is unusual due to the students that visited that day, and data from other days would be useful in understanding what happened.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample:

1. 0.4

2. The owner needs more data. The second day's results are too different.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: proportion is close to but not in correct range; invalid explanation for why the owner should gather more data; incorrect proportion between 0 and 1 based on visible calculation error.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: proportion wildly out of range, including greater than 1 or less than 0; incorrect belief that two sets of data reasonably represent the overall proportion.

Aligned Standards

7.SP.A.2, 7.SP.C.6

Problem 7

The data is built so that students can use either the mean and MAD or the median and IQR to work the problem. Either method is worth full credit, but be certain that students are consistently applying one or the other throughout.

It is also possible students come to a conclusion visually through box plots or dot plots, without any of the associated calculations. Such a solution should not be worth full credit, since the problem asks for calculations but should still be worth most of the credit in the problem. If students build box plots to compare, note that the box plot for Group B has the same median and third quartile, which may confuse some students.

Scaffolding can be added to this problem, such as asking for specific calculations, but it should not be necessary because this is a very recent topic.

Statement

Students are conducting a class experiment to see if there is a meaningful difference between two groups of plants that have begun to sprout leaves. The teacher randomly selects 8 plants from each group and counts the number of leaves on each plant.

Group A: 2, 2, 3, 3, 5, 5, 5, 7

Group B: 10, 9, 7, 8, 10, 12, 14, 10

Is there a meaningful difference between the two groups? Show all calculations that lead to your answer.

Solution

Yes, there is a significant difference. Use either mean and MAD (mean absolute deviation) or median and IQR (interquartile range) to decide. Using the means, the mean of Group A is 4, and the MAD is 1.5. The mean of Group B is 10, and the MAD is 1.5. The difference in means is 6, which is 4 MADs, so it is a meaningful difference. Using the medians, the median of Group A is 4, and the IQR is 2.5. The median of Group B is 10, and the IQR is 2.5. The difference in medians 6 which is 2.4 IQRs, so it is a meaningful difference.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample: Yes. The median of Group A is 4. The median of Group B is 10. The difference in medians is 2.4 IQRs.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: minor visible calculation errors cause one or two means, medians, MADs or IQRs to be incorrect. Acceptable errors: an error in calculating causes an incorrect conclusion about whether there is a meaningful difference between the two groups; an error in calculating mean or median leads to a corresponding error in calculating MAD or IQR.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: response using median and IQR does not sort Group B's data, but otherwise correctly works through the problem; conclusion about significant differences between the groups is driven only by means or medians and not MAD or IQR; incorrect conclusion about significant differences based on correct work; mixing center and spread measures, such as using mean and IQR; incorrect calculations with no work shown but a correct conclusion based on the results.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: badly incorrect algorithm for calculating mean, median, MAD, or IQR; failure to use these measures in problem work.

Aligned Standards

7.SP.B.3, 7.SP.B.4

Lesson Cool Downs

Lesson 1: Mystery Bags

Cool Down: Jada Draws Even

A large fish tank is filled with table tennis balls with numbers written on them. Jada chooses 10 table tennis balls from the tank and writes down their numbers.

1 3 5 1 3 2 4 1 5 3

A second tank is filled with golf balls with numbers written on them. Jada chooses 10 golf balls from the tank and writes down their numbers.

1 4 5 2 6 2 2 1 4 8

To win a prize, Jada must get a ball with an even number. Should she try to win the prize using the tank of table tennis balls or the tank of golf balls? Explain your reasoning.

Lesson 2: Chance Experiments

Cool Down: According To

Here are some scenarios:

- According to market research, a business has a 75% chance of making money in the first 3 years.
- According to lab testing, $\frac{5}{6}$ of a certain kind of experimental light bulb will work after 3 years.
- According to experts, the likelihood of a car needing major repairs in the first 3 years is 0.7.

1. Write the scenarios in order of likelihood from least to greatest after three years: the business makes money, the light bulb still works, and the car needs major repairs.
2. Name another chance experiment that has the same likelihood as one of the scenarios.

Lesson 3: What Are Probabilities?

Cool Down: Letter of the Day

A mother decides to teach her son about a letter each day of the week. She will choose a letter from the name of the day. For example, on Saturday she might teach about the letter S or the letter U, but not the letter M.

1. What letters are possible to teach using this method? (There are 15.)
2. What are 4 letters that can't be taught using this method?
3. On TUESDAY, the mother writes the word on a piece of paper and cuts it up so that each letter is on a separate piece of paper. She mixes up the papers and picks one. What is the probability that she will choose the piece of paper with the letter Y? Explain your reasoning.

Lesson 4: Estimating Probabilities Through Repeated Experiments

Cool Down: Fiction or Non-fiction?

A librarian is curious about the habits of the library's patrons. He records the type of item that the first 10 patrons check out from the library.

patron	item type
1	fiction book
2	non-fiction book
3	fiction book
4	fiction book
5	audiobook
6	non-fiction book
7	DVD
8	non-fiction book
9	fiction book
10	DVD

Based on the information from these patrons . . .

1. Estimate the probability that the next patron will check out a fiction book. Explain your reasoning.
2. Estimate the number of DVDs that will be checked out for every 100 patrons. Explain your reasoning.

Lesson 5: More Estimating Probabilities

Cool Down: The Probability of Spinning B

Jada, Diego, and Elena each use the same spinner that has four (not necessarily equal sized) sections marked A, B, C, and D.

- Jada says, "The probability of spinning B is 0.3 because I spun 10 times, and it landed on B 3 times."
- Diego says, "The probability of spinning B is 20% because I spun 5 times, and it landed on B once."
- Elena says, "The probability of spinning B is $\frac{2}{7}$ because I spun 7 times, and it landed on B twice."

1. Based on their methods, which probability estimate do you think is the most accurate? Explain your reasoning.

2. Andre measures the spinner and finds that the B section takes up $\frac{1}{4}$ of the circle. Explain why none of the methods match this probability exactly.

Lesson 6: Estimating Probabilities Using Simulation

Cool Down: Video Game Weather

In a video game, the chance of rain each day is always 30%. At the beginning of each day in the video game, the computer generates a random integer between 1 and 50. Explain how you could use this number to simulate the weather in the video game.

Lesson 7: Simulating Multi-step Experiments

Cool Down: Battery Life

The probability of a certain brand of battery going dead within 15 hours is $\frac{1}{3}$. Noah has a toy that requires 4 of these batteries. He wants to estimate the probability that at least one battery will die before 15 hours are up.

1. Noah will simulate the situation by putting marbles in a bag. Drawing one marble from the bag will represent the outcome of one of the batteries in the toy after 15 hours. Red marbles represent a battery that dies before 15 hours are up, and green marbles represent a battery that lasts longer.

How many marbles of each color should he put in the bag? Explain your reasoning.

2. After doing the simulation 5 times, Noah has the following results. What should he use as an estimate of the probability that at least one battery will die within 15 hours?

trial	result
1	GGRG
2	GRGR
3	GGGG
4	RGGG
5	GGGR

Lesson 8: Keeping Track of All Possible Outcomes

Cool Down: Random Points

Andre is reviewing proportional relationships. He wants to practice using a graph that goes through a point so that each coordinate is between 1 and 10.

1. For the point, how many outcomes are in the sample space?
2. For how many outcomes are the x -coordinate and the y -coordinate the same number?

Lesson 9: Multi-step Experiments

Cool Down: A Number Cube and 10 Cards

Lin plays a game that involves a standard number cube and a deck of ten cards numbered 1 through 10. If both the cube and card have the same number, Lin gets another turn. Otherwise, play continues with the next player.

What is the probability that Lin gets another turn?

Lesson 10: Designing Simulations

Cool Down: The Best Power-Up

Elena is programming a video game. She needs to simulate the power-up that the player gets when they reach a certain level. The computer can run a program to return a random integer between 1 and 100. Elena wants the best power-up to be rewarded 15% of the time.

Explain how Elena could use the computer to simulate the player getting the best power-up at least 2 out of 3 times.

Lesson 11: Comparing Groups

Cool Down: Prices of Homes

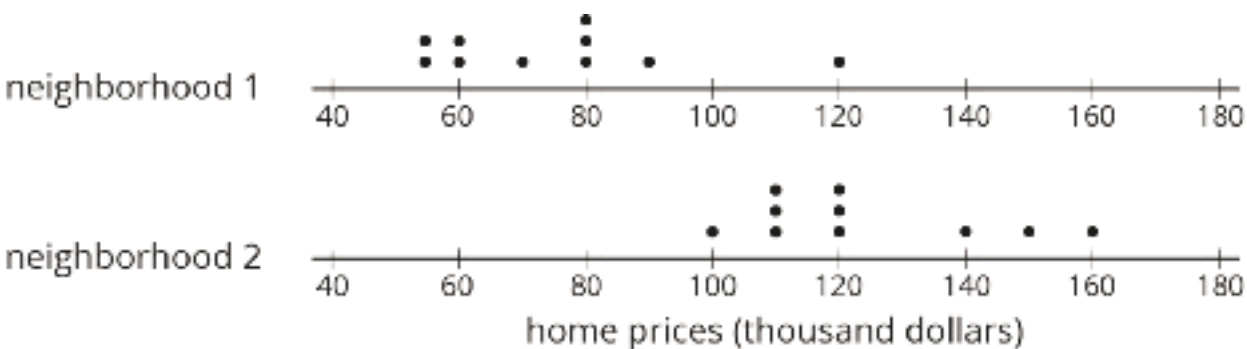
Noah's parents are interested in moving to another part of town. They look up all the prices of the homes for sale and record them in thousands of dollars.

neighborhood 1

80 55 80 120 60 90 60 80 55 70

neighborhood 2

110 120 160 110 100 110 140 150 120 120



Find the mean and MAD for each of the neighborhoods. Then decide whether the two groups are very different or not.

Lesson 12: Larger Populations

Cool Down: How Many Games?

Lin wants to know how many games teenagers in the United States have on their phones.

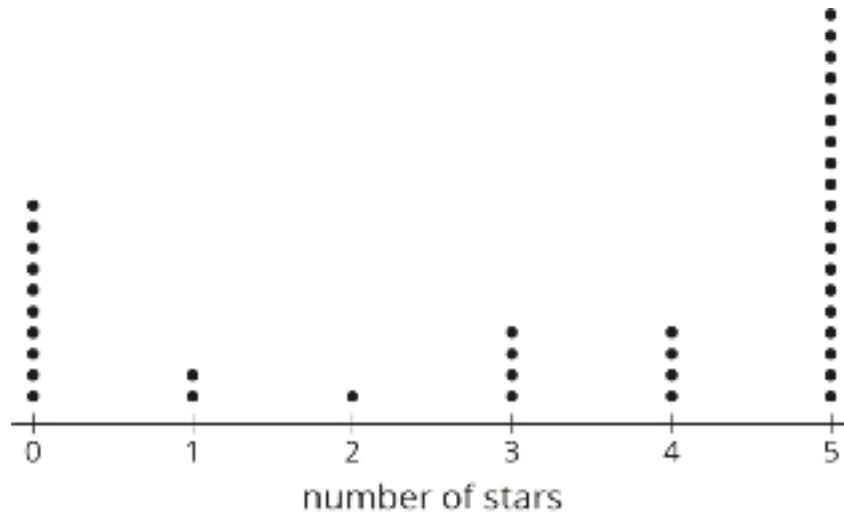
1. What is the population for Lin's question?
2. Explain why collecting data for this population would be difficult.
3. Give an example of a sample Lin could use to help answer her question.

Lesson 13: What Makes a Good Sample?

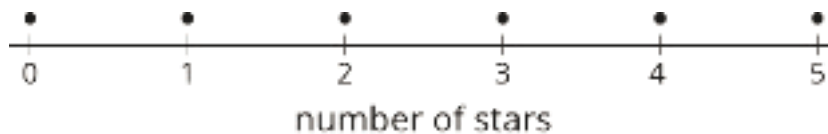
Cool Down: Reviews for School Lunches

Andre is designing a website that will display reviews of school lunches. Each item on the menu is rated from 0 to 5 stars. The main display can only show 6 reviews, so Andre needs to decide how to choose which reviews to show at the top.

This is a dot plot of all 40 reviews for the lasagna.



This is a plot of the stars shown on the first page of results.



1. If each rating also has a sentence or two explaining the rating, what are some good reasons to keep this sample displayed first? What are some good reasons to change the sample that is displayed first?
2. Is the sample representative of the population?

Lesson 14: Sampling in a Fair Way

Cool Down: Sampling Spinach

A public health expert is worried that a recent outbreak of a disease may be related to a batch of spinach from a certain farm. She wants to test the plants at the farm, but it will ruin the crop if she tests all of them.

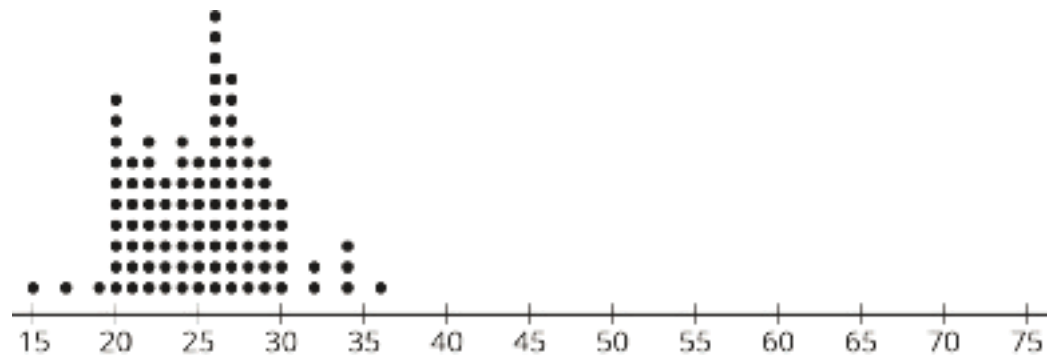
1. If the farm has 5,000 spinach plants, describe a method that would produce a random sample of 10 plants.
2. Why would a random sample be useful in this situation?

Lesson 15: Estimating Population Measures of Center

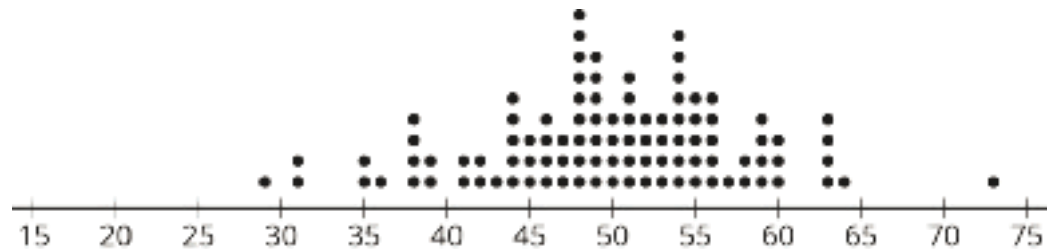
Cool Down: More Accurate Estimate

Here are dot plots that represent samples from two different populations.

Sample 1:



Sample 2:



1. Estimate the mean of each population using these samples.
2. Based on the dot plots, which estimate is more likely to be accurate? Explain your reasoning.

Lesson 16: Estimating Population Proportions

Cool Down: More than 48 Grams

A chemical engineer is trying to increase the amount of the useful product in a reaction. She performs the reaction with her new equipment 10 times and gets the following amounts of the useful product in grams:

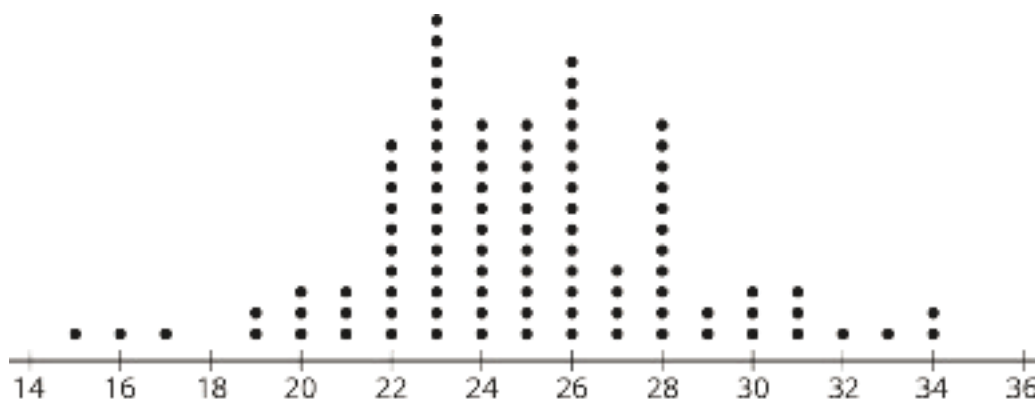
47.1 48.2 48.3 47.5 48.5 48.1 47.2 48.2 48.4 48.3

1. What proportion of the reactions were above the 48 grams threshold?
2. Other chemists typically get 65% of their reactions to produce more than 48 grams. Should the engineer say that she was able to increase the useful product when compared to the other chemists?

Lesson 17: More about Sampling Variability

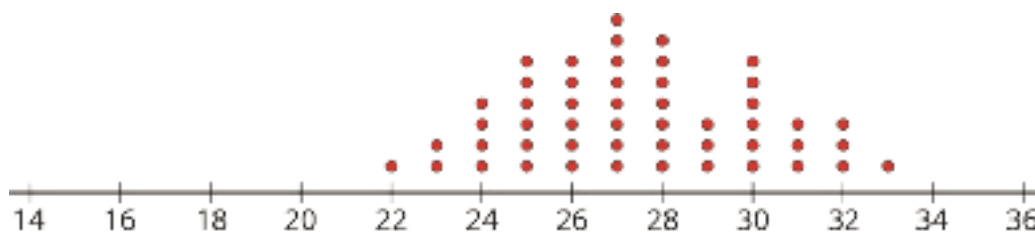
Cool Down: How Much Mail?

Jada collects data about the number of letters people get in the mail each week. The population distribution is shown in the dot plot.

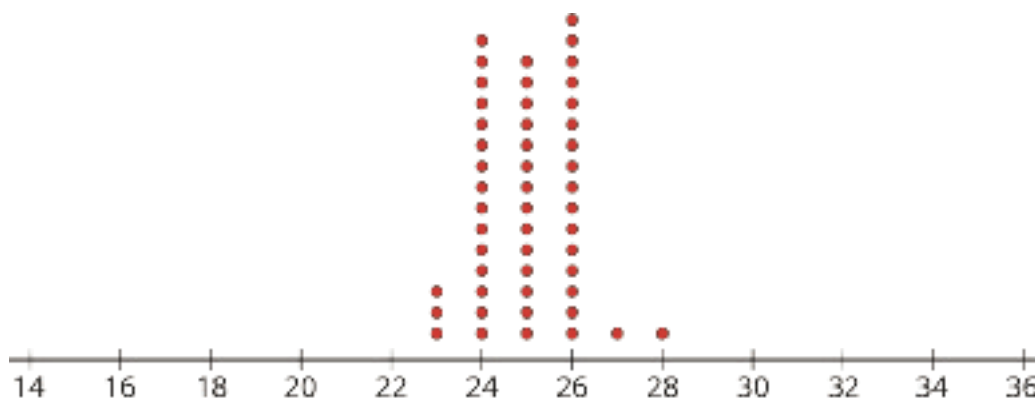


Which of the following dot plots are likely to represent the means from samples of size 10 from this population? Explain your reasoning.

Dot Plot 1



Dot Plot 2

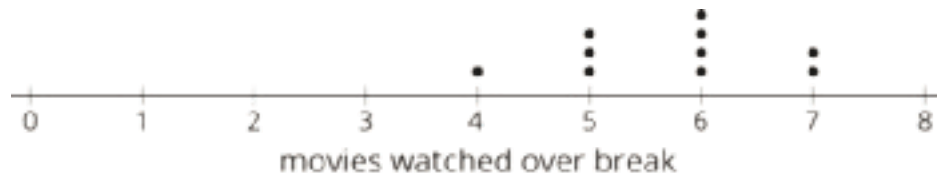


Lesson 18: Comparing Populations Using Samples

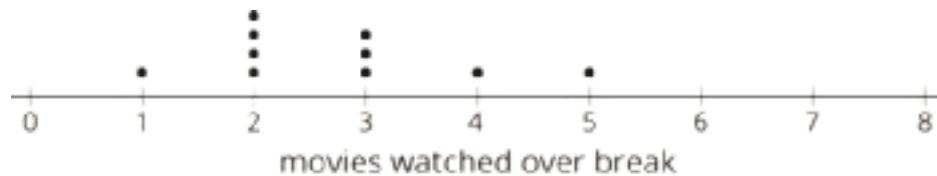
Cool Down: Teachers Watching Movies

Noah is interested in comparing the number of movies watched by students and teachers over the winter break. He takes a random sample of 10 students and 10 teachers and makes a dot plot of their responses.

Students:



Teachers:



Noah then computes the measures of center and variability for each group:

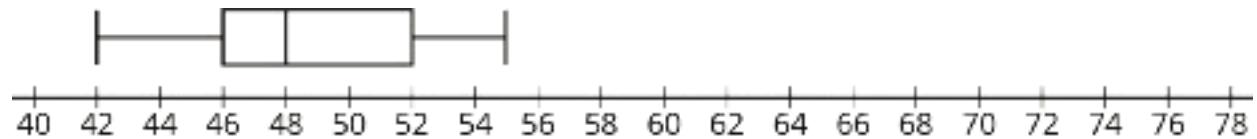
- Students: Mean = 5.7 movies, MAD = 0.76 movies
- Teachers: Mean = 2.7 movies, MAD = 0.9 movies

1. Is Noah's choice of mean and MAD appropriate for the data he has? Explain your reasoning.
2. Should Noah conclude that there is a meaningful difference in the mean number of movies watched over winter break between the two groups? Explain your reasoning.

Lesson 19: Comparing Populations With Friends

Cool Down: A Different Box Plot

Use the box plot to answer the questions.



1. What measure of center is shown in the box plot? What measure of variability? What are the values for each of these characteristics?
2. Draw another box plot with the same measure of variability that is meaningfully different from the one shown.

Instructional Masters

Instructional Masters for Probability and Sampling

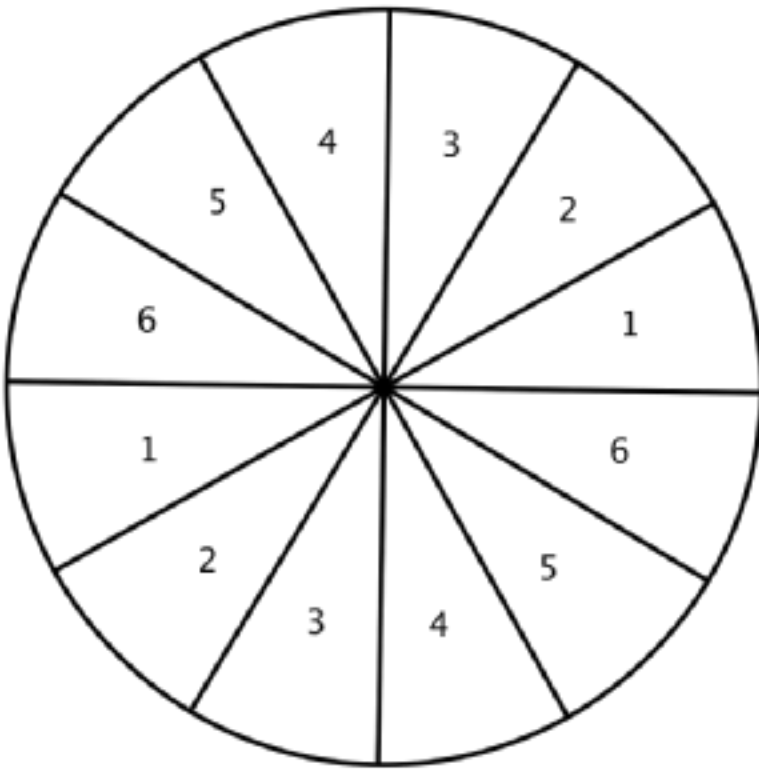
address	title	students per copy	written on?	requires cutting?	card stock recommended?	color paper recommended?
Activity Grade7.8.16.2	Reaction Times	1	no	yes	no	no
Activity Grade7.8.20.1	Collecting a Sample	1	no	yes	no	no
Activity Grade7.8.3.3	What's in the Bag?	1	no	yes	no	no
Activity Grade7.8.10.3	Designing Simulations	1	no	yes	no	no
Activity Grade7.8.19.2	Info Gap: Comparing Populations	1	no	yes	no	no
Activity Grade7.8.5.2	Making My Head Spin	1	no	yes	no	no
Activity Grade7.8.7.2	Alpine Zoom	1	no	yes	no	no
Activity Grade7.8.6.2	Diego's Walk	1	no	yes	no	no
Activity Grade7.8.2.4	Card Sort: Likelihood	1	no	yes	no	no

<p>Set 1 Likelihood</p> <p>The weather report says there is a 20% chance of rain tomorrow. The chance of rain tomorrow.</p>	<p>Set 1 Likelihood</p> <p>10% of people are left handed. The chance that a randomly chosen person is left handed.</p>
<p>Set 1 Likelihood</p> <p>The offspring of two fruit flies in a science experiment have a 75% chance of having red eyes. The chance that the first fly to hatch has red eyes.</p>	<p>Set 1 Likelihood</p> <p>Half of the cards in a deck are red and half are black. Shuffle the cards and select the first card. The chance that the card is red.</p>
<p>Set 2 Likelihood</p> <p>2 out of every 5 dentists recommend a certain brand of toothpaste. The chance that a random dentist recommends the toothpaste.</p>	<p>Set 2 Likelihood</p> <p>The chance that your opponent will play rock first in a game of paper, rock, scissors.</p>
<p>Set 2 Likelihood</p> <p>A pile contains 6 square pattern blocks and you choose one. The chance that the block you choose has 4 sides of the same length.</p>	<p>Set 2 Likelihood</p> <p>A fishbowl contains 5 balls where each one has an even number from 2 to 10 written on it and you choose one. The chance that you draw out a ball with the number 3 on it.</p>
<p>Set 2 Likelihood</p> <p>In general English usage, $\frac{4}{25}$ of words begin with the letter T. The chance that a randomly chosen word in a novel begins with the letter T.</p>	<p>Set 2 Likelihood</p> <p>The probability that a certain medical test gives the right result is 0.95. The chance that this medical test is correct for a random patient.</p>

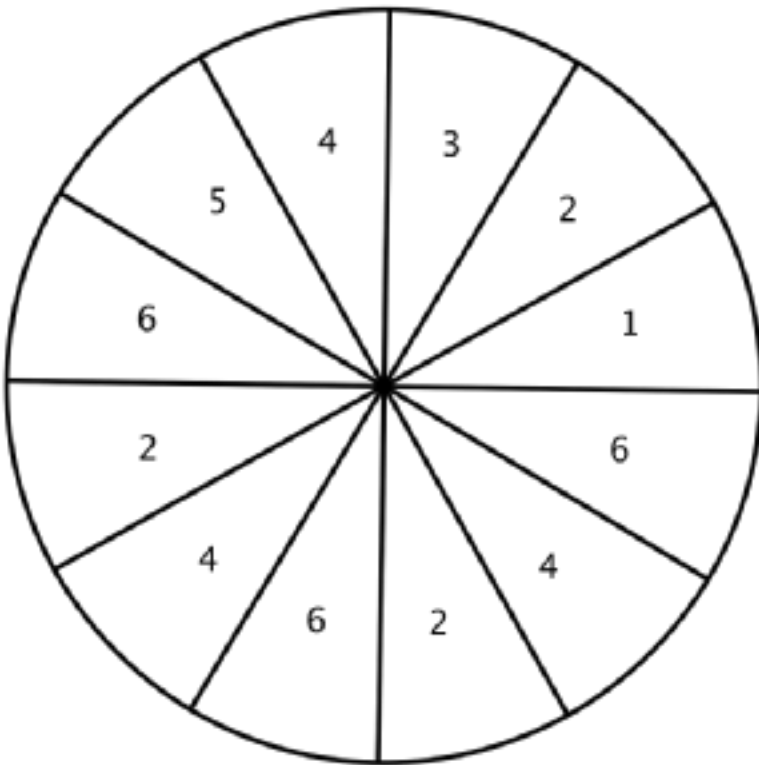
A	B	C
D	E	F
G	H	I
J	K	L
M	N	O

A	B	C
D	E	F
G	H	I
J	K	L
M	N	O

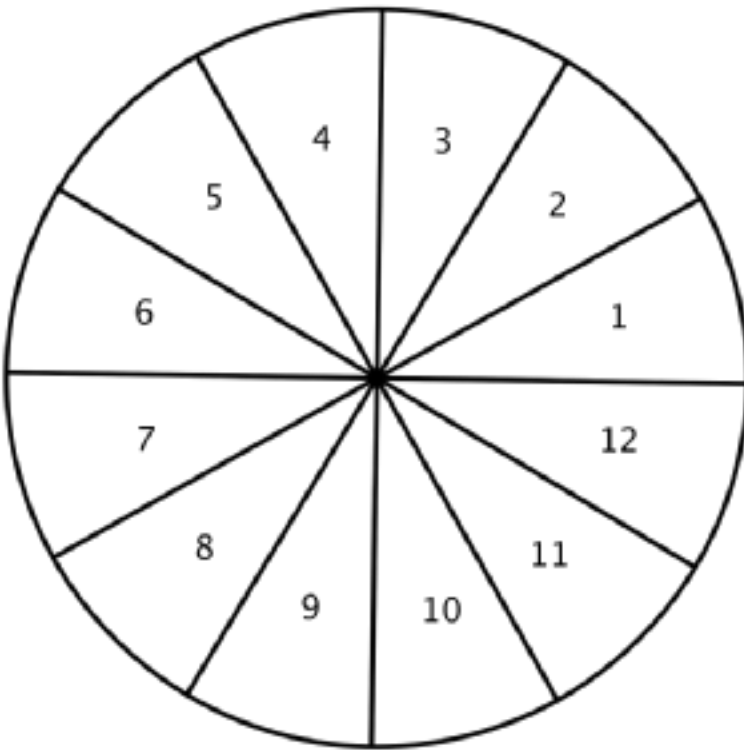
Spinner A



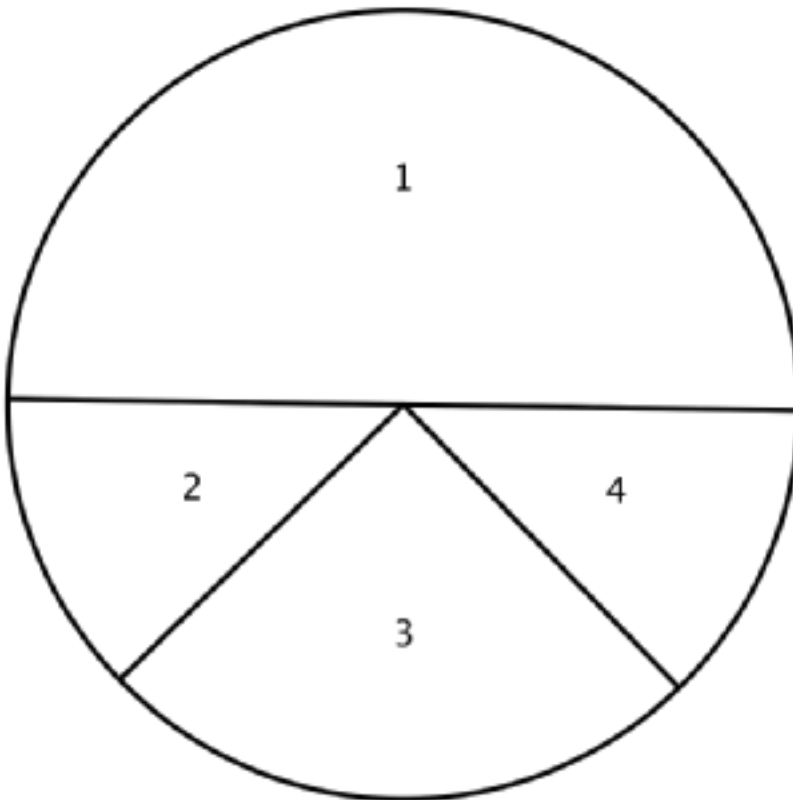
Spinner B



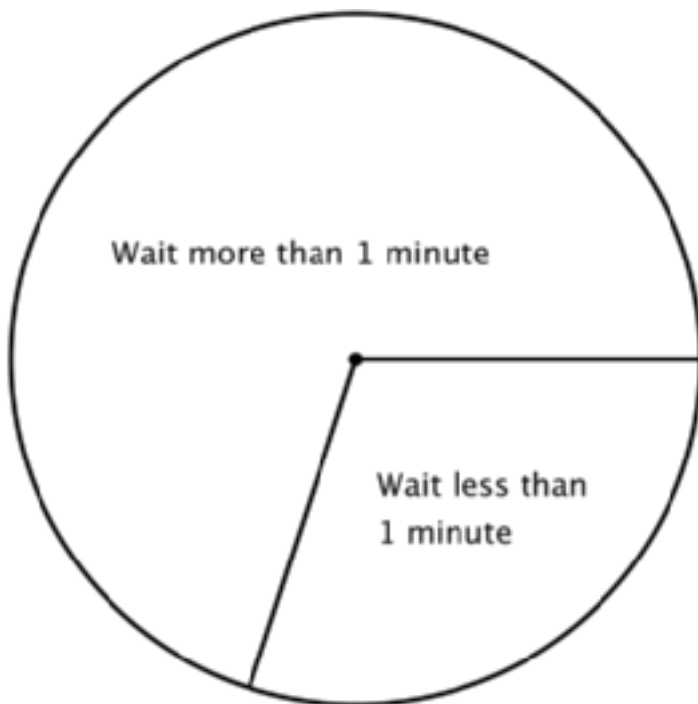
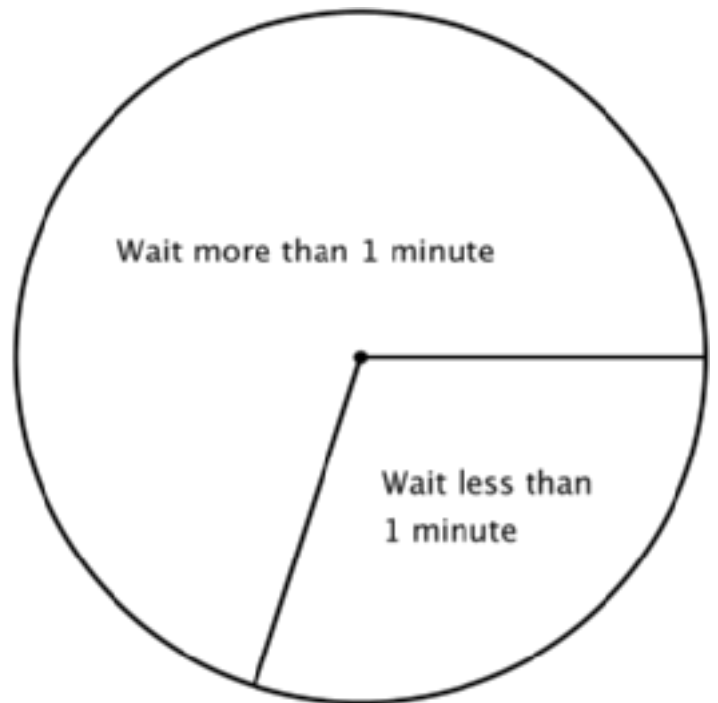
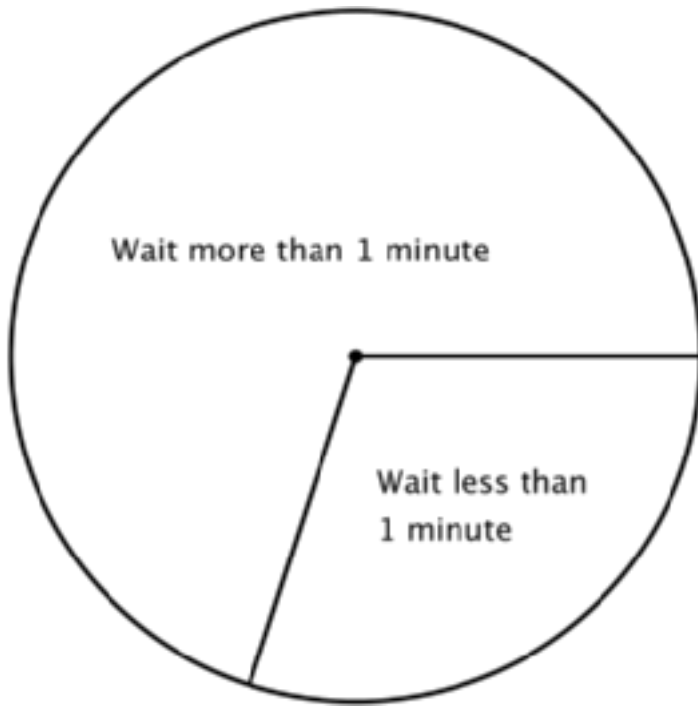
Spinner C

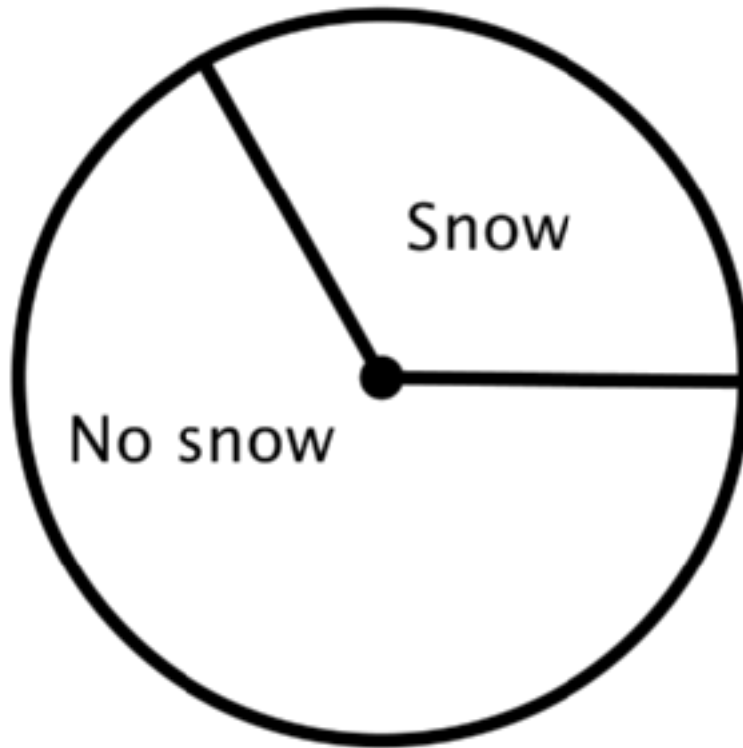


Spinner D



Diego's Walk - Set A wait <i>more</i> than 1 minute	Diego's Walk - Set A wait <i>more</i> than 1 minute	Diego's Walk - Set A wait <i>more</i> than 1 minute	Diego's Walk - Set A wait <i>more</i> than 1 minute	Diego's Walk - Set A wait <i>more</i> than 1 minute
Diego's Walk - Set A wait <i>more</i> than 1 minute	Diego's Walk - Set A wait <i>more</i> than 1 minute	Diego's Walk - Set A wait <i>less</i> than 1 minute	Diego's Walk - Set A wait <i>less</i> than 1 minute	Diego's Walk - Set A wait <i>less</i> than 1 minute
Diego's Walk - Set B wait <i>more</i> than 1 minute	Diego's Walk - Set B wait <i>more</i> than 1 minute	Diego's Walk - Set B wait <i>more</i> than 1 minute	Diego's Walk - Set B wait <i>more</i> than 1 minute	Diego's Walk - Set B wait <i>more</i> than 1 minute
Diego's Walk - Set B wait <i>more</i> than 1 minute	Diego's Walk - Set B wait <i>more</i> than 1 minute	Diego's Walk - Set B wait <i>less</i> than 1 minute	Diego's Walk - Set B wait <i>less</i> than 1 minute	Diego's Walk - Set B wait <i>less</i> than 1 minute
Diego's Walk - Set C wait <i>more</i> than 1 minute	Diego's Walk - Set C wait <i>more</i> than 1 minute	Diego's Walk - Set C wait <i>more</i> than 1 minute	Diego's Walk - Set C wait <i>more</i> than 1 minute	Diego's Walk - Set C wait <i>more</i> than 1 minute
Diego's Walk - Set C wait <i>more</i> than 1 minute	Diego's Walk - Set C wait <i>more</i> than 1 minute	Diego's Walk - Set C wait <i>less</i> than 1 minute	Diego's Walk - Set C wait <i>less</i> than 1 minute	Diego's Walk - Set C wait <i>less</i> than 1 minute





Designing Simulations

1. A man has 5 grandchildren, 4 girls and 1 boy. He thinks this is unusual. If the probability that any child born will be a girl is $\frac{1}{2}$, what is the probability that a person who has 5 grandchildren will have exactly 4 granddaughters? Is this case unusual? Explain.
-

Designing Simulations

2. To be on the safe side, three detectors were installed in a factory room to make sure that if there was a fire, at least one of them would signal a warning. The company that manufactured the smoke detectors indicated that, based on their testing, the probability that any one of the smoke detectors will work correctly is 0.75 (meaning that it works 75% of the time in the long run). This also means that there is a 25% chance that if there is smoke or a fire, the detector will not work! What is the probability that if there was smoke in the factory, none of the 3 detectors would work? Does this probability indicate a safety problem for the factory? Explain.
-

Designing Simulations

3. An automobile factory has a reputation for assembling high quality cars. However, several new cars were shipped out to dealers that had a problem with the brakes. It is estimated that approximately 10% of the cars assembled at this factory have defective brakes. Five of these cars are shipped to a dealership near your school. What is the probability that none of the 5 cars will have defective brakes? Should the dealership be concerned? Explain.
-

Designing Simulations

4. Your class is planning to collect data at a wildlife refuge center for the next 5 days. The staff at the refuge center indicated that there is a 40% chance of seeing an eagle during any one of the days of your visit. What is the probability that if your class visits the refuge for 5 days, you will see an eagle two or more days during your 5-day visit at the refuge center? Your teacher also indicated that if you see 2 or more eagles during the 5 days, your class will be able to name one of the eagles as part of a fundraiser. Do you think you have a good chance of being able to name an eagle? Explain.
-

Designing Simulations

5. At a small animal emergency hospital, there is a 20% chance that an animal brought into the hospital may need to stay overnight. The hospital only has enough room to accommodate 2 animals per night. On a particular day, five animals were brought into the hospital. What is the probability that at least 3 of the animals may need to stay overnight? If seeing five animals per day is typical for this hospital, do you think the hospital is usually able to accommodate all of the animals that might have to stay overnight? Explain.
-

0.28	0.30	1.23	0.51	0.33	0.73	0.34	0.27	0.34	0.35
0.27	0.09	0.37	0.33	0.48	0.51	0.11	0.34	0.49	0.80
0.38	0.49	0.34	0.81	0.75	0.32	0.31	0.40	0.41	0.97
0.41	0.58	0.72	0.39	0.40	0.31	0.32	0.39	0.79	0.38
0.25	0.29	0.36	0.38	0.44	0.73	0.36	0.30	0.36	0.23
0.48	0.61	0.35	0.23	0.42	0.83	0.92	0.44	0.34	0.31
0.70	0.45	0.21	0.36	0.72	0.44	0.35	0.39	0.42	0.36
0.40	0.37	0.40	0.41	0.36	0.50	0.52	0.29	0.46	0.37
0.36	0.31	0.04	0.30	0.69	0.34	0.31	0.47	0.27	0.51
0.59	0.74	0.31	0.38	0.82	0.36	0.37	0.36	0.78	0.40

0.28	0.30	1.23	0.51	0.33	0.73	0.34	0.27	0.34	0.35
0.27	0.09	0.37	0.33	0.48	0.51	0.11	0.34	0.49	0.80
0.38	0.49	0.34	0.81	0.75	0.32	0.31	0.40	0.41	0.97
0.41	0.58	0.72	0.39	0.40	0.31	0.32	0.39	0.79	0.38
0.25	0.29	0.36	0.38	0.44	0.73	0.36	0.30	0.36	0.23
0.48	0.61	0.35	0.23	0.42	0.83	0.92	0.44	0.34	0.31
0.70	0.45	0.21	0.36	0.72	0.44	0.35	0.39	0.42	0.36
0.40	0.37	0.40	0.41	0.36	0.50	0.52	0.29	0.46	0.37
0.36	0.31	0.04	0.30	0.69	0.34	0.31	0.47	0.27	0.51
0.59	0.74	0.31	0.38	0.82	0.36	0.37	0.36	0.78	0.40

Info Gap: Comparing Populations

Problem Card 1

A chewing gum company records the amount of time, in minutes, it takes for a person to find that the gum has lost its flavor for two different flavors of gum. A sample of 20 people is used for mint and another sample of 20 people is used for cinnamon gum. Is there a meaningful difference in the amount of time for the different flavors? Explain your reasoning.

Info Gap: Comparing Populations

Data Card 1

- The distributions are not symmetric
- Mean for mint: 74.5 minutes
- Median for mint: 65 minutes
- IQR for mint: 20 minutes
- MAD for mint: 20.9 minutes
- Mean for cinnamon: 108.75 minutes
- Median for cinnamon: 112.5 minutes
- IQR for cinnamon: 22.5 minutes
- MAD for cinnamon: 21.5 minutes

Info Gap: Comparing Populations

Problem Card 2

A grade 8 English teacher is interested in grammar scores for students coming from the two grade 7 English teachers. Is there a meaningful difference in the means for student scores on a pretest coming from each grade 7 teacher based on the samples? Explain your reasoning.

Info Gap: Comparing Populations

Data Card 2

Scores from Teacher A

70	75	80	80	80	85	85	85	90	95
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Scores from Teacher B

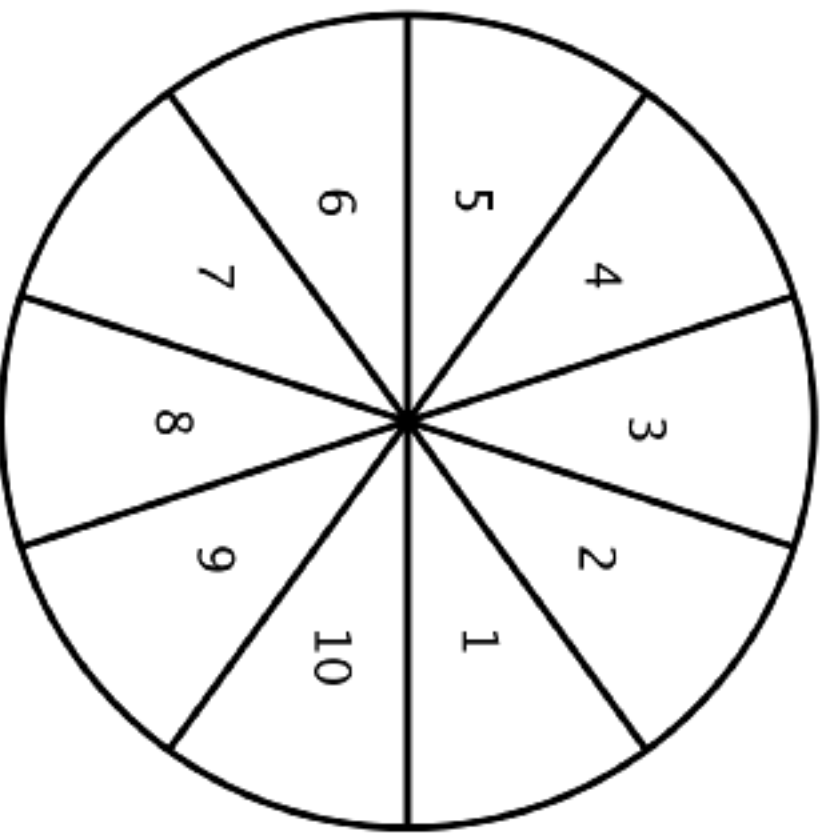
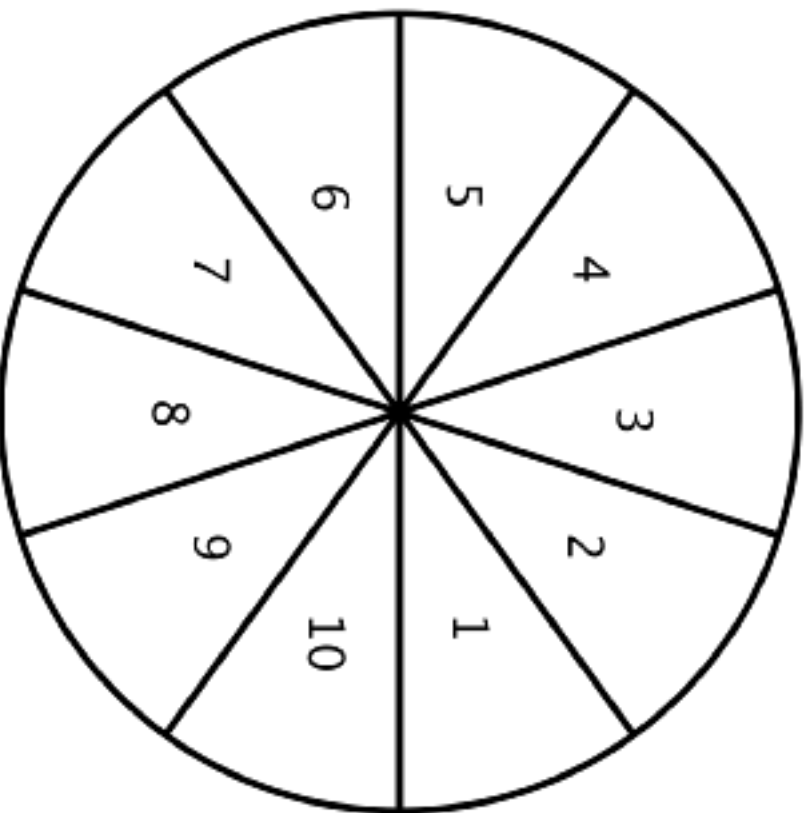
70	70	70	75	75	80	80	80	90	100
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Data Set 1

	column 1	column 2	column 3	column 4	column 5	column 6	column 7	column 8	column 9	column 10
row 1	35	47	42*	33	42	57	40	39	33	37*
row 2	58	33*	55	42	66	10	47	44	45*	54
row 3	45	59	44	51	57	52*	42	42	43	22*
row 4	63	35	37	81	37*	11*	27	43	35*	43
row 5	95*	60*	41*	70*	64	47	34	63	40	41
row 6	47	39	57	71	46	53	63	57*	62	36
row 7	53	44	45	30	73*	39	54	48	30	39
row 8	29	36	27*	69	45	55	40	58*	62	42
row 9	34	49	61	18	40	40	52	54	45	26
row 10	65	35*	63	49	76	62	46*	42*	59	32

Data Set 2

	column 1	column 2	column 3	column 4	column 5	column 6	column 7	column 8	column 9	column 10
row 1	26	39	10	42*	52	34	39	71	30	54
row 2	69	49	45	42	40	46*	47	46	30	18
row 3	22*	47	34	54	48	57*	42	58*	65	27
row 4	40	53	51	63	62	55	37*	41	59	40
row 5	33*	45	36	11*	35*	42	39	63	36	57
row 6	49	63	45	44	70*	42	29	73*	35*	59
row 7	40	32	40	47	57	42*	27*	35	64	57
row 8	54	42	45*	33	60*	63	53	43	35	33
row 9	44	43	43	62	41*	61	44	95*	52*	39
row 10	58	81	37*	45	55	37	62	76	66	47



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