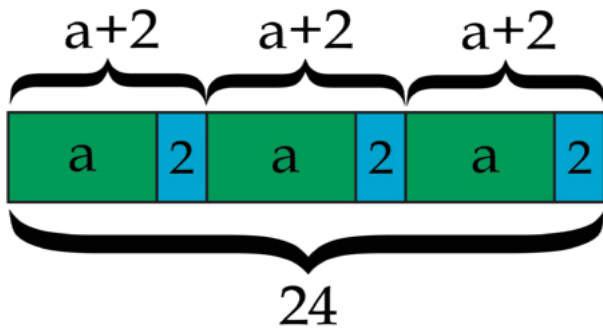


# Expressions, Equations, and Inequalities

## Teacher Guide

Tape Diagram



Make a Koch snowflake



Which Cupon to use?



Calculating Height



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# Expressions, Equations, and Inequalities

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**Expressions, Equations, and Inequalities**  
**Teacher Guide**

Core Knowledge Mathematics™

# Expressions, Equations, and Inequalities

## Unit Narrative

In this unit, students solve equations of the forms  $px + q = r$  and  $p(x + q) = r$ , and solve related inequalities, e.g., those of the form  $px + q > r$  and  $px + q \geq r$ , where  $p$ ,  $q$ , and  $r$  are rational numbers.

In the first section of the unit, students represent relationships of two quantities with tape diagrams and with equations, and explain correspondences between the two types of representations (MP1). They begin by examining correspondences between descriptions of situations and tape diagrams, then draw tape diagrams to represent situations in which the variable representing the unknown is specified. Next, they examine correspondences between equations and tape diagrams, then draw tape diagrams to represent equations, noticing that one tape diagram can be described by different (but related) equations. At the end of the section, they draw tape diagrams to represent situations in which the variable representing the unknown is not specified, then match the diagrams with equations. The section concludes with an example of the two main types of situations examined, characterized in terms of whether or not they involve equal parts of an amount or equal *and* unequal parts of an amount, and as represented by equations of different forms, e.g.,  $6(x + 8) = 72$  and  $6x + 8 = 72$ . This initiates a focus on seeing two types of structure in the situations, diagrams, and equations of the unit (MP7).

In the second section of the unit, students solve equations of the forms  $px + q = r$  and  $p(x + q) = r$ , then solve problems that can be represented by such equations (MP2). They begin by considering balanced and unbalanced “hanger diagrams,” matching hanger diagrams with equations, and using the diagrams to understand two algebraic steps in solving equations of the form  $px + q = r$ : subtract the same number from both sides, then divide both sides by the same number. Like a tape diagram, the same balanced hanger diagram can be described by different (but related) equations, e.g.,  $3x + 6 = 18$  and  $3(x + 2) = 18$ . The second form suggests using the same two algebraic steps to solve the equation, but in reverse order: divide both sides by the same number, then subtract the same number from both sides. Each of these algebraic steps and the associated structure of the equation is illustrated by hanger diagrams (MP1, MP7).

So far, the situations in the section have been described by equations in which  $p$  is a whole number, and  $q$  and  $r$  are positive (and frequently whole numbers). In the remainder of the section, students use the algebraic methods that they have learned to solve equations of the forms  $px + q = r$  and  $p(x + q) = r$  in which  $p$ ,  $q$ , and  $r$  are rational numbers. They use the distributive property to transform an equation of one form into the other (MP7) and note how such transformations can be used strategically in solving an equation (MP5). They write equations in order to solve problems involving percent increase and decrease (MP2).

In the third section of the unit, students work with inequalities. They begin by examining values that make an inequality true or false, and using the number line to represent values that make an inequality true. They solve equations, examine values to the left and right of a solution, and use those values in considering the solution of a related inequality. In the last two lessons of the section, students solve inequalities that represent real-world situations (MP2).

In the last section of the unit, students work with equivalent linear expressions, using properties of operations to explain equivalence (MP3). They represent expressions with area diagrams, and use the distributive property to justify factoring or expanding an expression.

### **Progression of Disciplinary Language**

In this unit, teachers can anticipate students using language for mathematical purposes such as comparing, explaining, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

#### **Compare**

- stories with corresponding tape diagrams (Lesson 2)
- tape diagrams with corresponding equations (Lesson 3)
- hanger diagrams and equations (Lesson 7)
- solution pathways (especially Lesson 10)
- descriptions of situations with corresponding inequalities (Lesson 16)

#### **Explain**

- strategies for using hanger diagrams to solve equations (Lesson 8)
- different strategies for solving equations (Lesson 9) and inequalities (Lesson 14)
- reasoning about situations, tape diagrams, and equations (Lesson 12)
- strategies for identifying and writing equivalent expressions (Lesson 22)

#### **Justify**

- reasoning about inequalities (Lesson 13)
- reasoning about solutions to inequalities (Lesson 15)
- the need for specific information in order to write and solve inequalities (Lesson 17)
- reasoning about the distributive property (Lesson 19)
- whether different sequences of calculations give the same result (Lesson 23)

In addition, students are expected to interpret solutions to equations, interpret and represent non-proportional situations with constant rates of change, represent non-proportional situations using tape diagrams, describe the structure of equations and tape diagrams, critique reasoning of peers about expressions and corresponding diagrams, critique reasoning about solving equations, critique reasoning about equivalent expressions, and generalize about solving equations and about when expressions are equivalent.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear

bolded. Teachers should continue to support students' use of a new term in the lessons that follow were it was first introduced.

# Learning Targets

## Expressions, Equations, and Inequalities

### Lesson 1: Relationships between Quantities

- I can think of ways to solve some more complicated word problems.

### Lesson 2: Reasoning about Contexts with Tape Diagrams

- I can explain how a tape diagram represents parts of a situation and relationships between them.
- I can use a tape diagram to find an unknown amount in a situation.

### Lesson 3: Reasoning about Equations with Tape Diagrams

- I can match equations and tape diagrams that represent the same situation.
- If I have an equation, I can draw a tape diagram that shows the same relationship.

### Lesson 4: Reasoning about Equations and Tape Diagrams (Part 1)

- I can draw a tape diagram to represent a situation where there is a known amount and several copies of an unknown amount and explain what the parts of the diagram represent.
- I can find a solution to an equation by reasoning about a tape diagram or about what value would make the equation true.

### Lesson 5: Reasoning about Equations and Tape Diagrams (Part 2)

- I can draw a tape diagram to represent a situation where there is more than one copy of the same sum and explain what the parts of the diagram represent.
- I can find a solution to an equation by reasoning about a tape diagram or about what value would make the equation true.

### Lesson 6: Distinguishing between Two Types of Situations

- I understand the similarities and differences between the two main types of equations we are studying in this unit.
- When I have a situation or a tape diagram, I can represent it with an equation.

### **Lesson 7: Reasoning about Solving Equations (Part 1)**

- I can explain how a balanced hanger and an equation represent the same situation.
- I can find an unknown weight on a hanger diagram and solve an equation that represents the diagram.
- I can write an equation that describes the weights on a balanced hanger.

### **Lesson 8: Reasoning about Solving Equations (Part 2)**

- I can explain how a balanced hanger and an equation represent the same situation.
- I can explain why some balanced hangers can be described by two different equations, one with parentheses and one without.
- I can find an unknown weight on a hanger diagram and solve an equation that represents the diagram.
- I can write an equation that describes the weights on a balanced hanger.

### **Lesson 9: Dealing with Negative Numbers**

- I can use the idea of doing the same to each side to solve equations that have negative numbers or solutions.

### **Lesson 10: Different Options for Solving One Equation**

- For an equation like  $3(x + 2) = 15$ , I can solve it in two different ways: by first dividing each side by 3, or by first rewriting  $3(x + 2)$  using the distributive property.
- For equations with more than one way to solve, I can choose the easier way depending on the numbers in the equation.

### **Lesson 11: Using Equations to Solve Problems**

- I can solve story problems by drawing and reasoning about a tape diagram or by writing and solving an equation.

### **Lesson 12: Solving Problems about Percent Increase or Decrease**

- I can solve story problems about percent increase or decrease by drawing and reasoning about a tape diagram or by writing and solving an equation.

### **Lesson 13: Reintroducing Inequalities**

- I can explain what the symbols  $\leq$  and  $\geq$  mean.
- I can represent an inequality on a number line.
- I understand what it means for a number to make an inequality true.

### **Lesson 14: Finding Solutions to Inequalities in Context**

- I can describe the solutions to an inequality by solving a related equation and then reasoning about values that make the inequality true.
- I can write an inequality to represent a situation.

### **Lesson 15: Efficiently Solving Inequalities**

- I can graph the solutions to an inequality on a number line.
- I can solve inequalities by solving a related equation and then checking which values are solutions to the original inequality.

### **Lesson 16: Interpreting Inequalities**

- I can match an inequality to a situation it represents, solve it, and then explain what the solution means in the situation.
- If I have a situation and an inequality that represents it, I can explain what the parts of the inequality mean in the situation.

### **Lesson 17: Modeling with Inequalities**

- I can use what I know about inequalities to solve real-world problems.

### **Lesson 18: Subtraction in Equivalent Expressions**

- I can organize my work when I use the distributive property.
- I can re-write subtraction as adding the opposite and then rearrange terms in an expression.

### **Lesson 19: Expanding and Factoring**

- I can organize my work when I use the distributive property.
- I can use the distributive property to rewrite expressions with positive and negative numbers.
- I understand that factoring and expanding are words used to describe using the distributive property to write equivalent expressions.

### **Lesson 20: Combining Like Terms (Part 1)**

- I can figure out whether two expressions are equivalent to each other.
- When possible, I can write an equivalent expression that has fewer terms.

### **Lesson 21: Combining Like Terms (Part 2)**

- I am aware of some common pitfalls when writing equivalent expressions, and I can avoid them.
- When possible, I can write an equivalent expression that has fewer terms.

### **Lesson 22: Combining Like Terms (Part 3)**

- Given an expression, I can use various strategies to write an equivalent expression.
- When I look at an expression, I can notice if some parts have common factors and make the expression shorter by combining those parts.

### **Lesson 23: Applications of Expressions**

- I can write algebraic expressions to understand and justify a choice between two options.

lesson	new terminology	
	receptive	productive
7.6.2	unknown amount	
7.6.3	<b>equivalent expressions</b> commutative (property)	
7.6.4		unknown amount relationship
7.6.6		<b>variable</b>
7.6.7	balanced hanger each side (of an equation)	
7.6.8		<b>equivalent expression</b> each side (of an equation)
7.6.9		operation solve
7.6.10	distribute substitute	
7.6.13	inequality less than or equal to greater than or equal to open / closed circle	less than greater than
7.6.14	<b>solution to an inequality</b> boundary direction (of an inequality)	less than or equal to greater than or equal to substitute
7.6.15		open / closed circle
7.6.16		<b>solution to an inequality</b>
7.6.17		inequality
7.6.18	<b>term</b>	
7.6.19	<b>factor (an expression)</b> <b>expand (an expression)</b>	

lesson	new terminology	
	receptive	productive
7.6.20	combine like terms	<b>term</b> commutative (property)
7.6.21		distribute
7.6.22	associative property	<b>factor (an expression)</b> <b>expand (an expression)</b>

## Required Materials

**Index cards**

**Pre-printed slips, cut from copies of the**

**Instructional master**

**Sticky notes**

**Tools for creating a visual display**

Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

## Section: Representing Situations of the Form

$$px + q = r \text{ and } p(x + q) = r$$

### Lesson 1: Relationships between Quantities

#### Goals

- Determine unknown values in a relationship that is not proportional, and explain (orally and in writing) the solution method.
- Interpret and describe (orally and in writing) relationships that are predictable, but not proportional.
- Justify (orally) that a given relationship is not proportional.

#### Learning Targets

- I can think of ways to solve some more complicated word problems.

#### Lesson Narrative

In this introductory lesson, students encounter some engaging contexts characterized by relationships that are not proportional. The goal is simply to see that we need some new strategies—it is the work of the upcoming unit to develop strategies for efficiently solving problems about contexts like some of the ones in this lesson. In this lesson, is not expected that students write expressions or equations, or use any specific representation. Students may choose to make diagrams or tables or reason in some other way.

#### Alignments

##### Building On

- 7.RP.A.2: Recognize and represent proportional relationships between quantities.
- 7.RP.A.2.a: Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

##### Building Towards

- 7.EE.B: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
- 7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
- 7.EE.B.4.a: Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently.

Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

### **Instructional Routines**

- MLR6: Three Reads
- MLR7: Compare and Connect
- Think Pair Share

### **Student Learning Goals**

Let's try to solve some new kinds of problems.

## **1.1 Pricing Theater Popcorn**

### **Warm Up: 10 minutes**

A context is described, and students generate two sets of values. The purpose of this warm-up is to remind students of some characteristics that make a relationship proportional or not proportional, so that later in the lesson, they are better equipped to recognize that a relationship is not proportional and explain why.

The numbers were deliberately chosen to encourage different ways of viewing a proportional relationship. For 20 ounces and 35 ounces, students might move from row to row and think in terms of scale factors. This approach is less straightforward for 48 ounces, and some students may shift to thinking in terms of unit rates.

There are many possible rationales for choosing numbers so that size is not proportional to price. As long as the numbers are different from those in the "proportional" column, the relationship between size and price is guaranteed to be not proportional. Look for students who have a reasonable way to explain why their set of numbers is not proportional, like "the unit price is different for each size," or "each size costs a different amount per ounce."

### **Building On**

- 7.RP.A.2

### **Building Towards**

- 7.EE.B

### **Instructional Routines**

- Think Pair Share

### **Launch**

Ask students to remember the last time they went to the movies. What do they know about the popcorn for sale? What sizes does it come in? About how much does it cost? Tell students that in this activity, they will come up with prices for different sizes of popcorn—one set of prices in which

the price is in proportion to the size, and another set of prices in which the price is not in proportion to the size, but is still reasonable. Ask students to be ready to explain the reasons they chose the numbers they did.

Arrange students in groups of 2. Give 2 minutes of quiet work time and then invite students to share their sentences with their partner, followed by whole-class discussion.

### Student Task Statement

A movie theater sells popcorn in bags of different sizes. The table shows the volume of popcorn and the price of the bag.

Complete one column of the table with prices where popcorn is priced at a constant rate. That is, the amount of popcorn is proportional to the price of the bag. Then complete the other column with realistic example prices where the amount of popcorn and price of the bag are not in proportion.

volume of popcorn (ounces)	price of bag, proportional (\$)	price of bag, not proportional (\$)
10	6	6
20		
35		
48		

### Student Response

Answers vary for the rightmost column. Sample response:

volume of popcorn (ounces)	price of bag, proportional (\$)	price of bag, not proportional (\$)
10	6	6
20	12	11
35	21	20
48	28.8	25

### Activity Synthesis

Invite a student to share their prices for the proportional relationship and how they decided on those numbers. Ask if any students thought of it in a different way.

Then, invite a student to share their prices for the relationship that is not proportional and record these for all to see. Ask students to explain ways you can tell that the relationship is not proportional.

## 1.2 Entrance Fees

### 10 minutes

This context was used in an earlier unit about proportional relationships as an example of a relationship that is not proportional. However, a different rule for determining the entrance fee is used here.

Watch for students who organize the given information in a table or another visual representation, and for unique, correct approaches to the first two questions.

### Building On

- 7.RP.A.2.a

### Building Towards

- 7.EE.B.4
- 7.EE.B.4.a

### Instructional Routines

- MLR6: Three Reads
- Think Pair Share

### Launch

Tell students that unlike in the previous activity where they could make up any numbers, this activity has a relationship where there *is* a pattern, and part of the work is to figure out the pattern. This activity has to do with an entrance fee to a park, where the fee is based on the number of people in the vehicle.

Keep students in the same groups. Give 2 minutes of quiet work time and then invite students to share their sentences with their partner, followed by whole-class discussion.

---

### Access for Students with Disabilities

*Representation: Internalize Comprehension.* Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, use the same color to highlight the number of people with its corresponding entrance fee, making annotations of how the fee was calculated.

*Supports accessibility for: Visual-spatial processing*

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## Access for English Language Learners

*Reading, Representing: MLR6 Three Reads.* Use this routine to help students understand the question and to represent the relationships between quantities. Use the first read to orient students to the situation. Ask students to describe what the situation is about without using numbers (the park charges an entrance fee that includes the number of people in the car). After the second read, ask students what can be counted or measured in this situation. Listen for, and amplify, the quantities that vary in relation to each other: number of people in a vehicle; entrance fee amount, in dollars. After the third read, ask students to organize the information (using a list, table, or diagram) and brainstorm ideas for how much the park charges for each person in the car.

*Design Principle(s): Support sense-making*

---

## Anticipated Misconceptions

Students may misunderstand that the first two questions require noticing and extending a pattern, and (because of the warm-up) think that any reasonable number is acceptable. Encourage them to organize the given information and think about what rule the park might use to determine the entrance fee based on the number of people in the vehicle.

Students may come up with “rules” that aren’t supported by the context or the given information. For example, they may notice that each additional person costs \$3, but then reason that 30 people must cost \$90. Whatever their rule, ask them to check that whether it works for all of the information given. For example, since 2 people cost \$14, we can tell that “\$3 per person” is not the rule.

## Student Task Statement

A state park charges an entrance fee based on the number of people in a vehicle. A car containing 2 people is charged \$14, a car containing 4 people is charged \$20, and a van containing 8 people is charged \$32.

1. How much do you think a bus containing 30 people would be charged?
2. If a bus is charged \$122, how many people do you think it contains?
3. What rule do you think the state park uses to decide the entrance fee for a vehicle?

## Student Response

1. \$98. Sample reasoning: From 2 people to 4 people, there are 2 additional people that cost 6 additional dollars. From 4 people to 8 people, there are 4 additional people that cost 12 additional dollars. It seems like each additional person costs 3 additional dollars. From 8 people to 30 people is 22 additional people, so they should cost 66 additional dollars, and  $32 + 66 = 98$ .

2. 38 people. Sample reasoning: \$122 is \$24 more than \$98. An additional \$24 is 8 additional people, and  $30 + 8 = 38$ .
3. \$8 for the vehicle plus \$3 for each passenger. Sample reasoning: 2 people cost \$14, so if each person is charged \$3, that leaves \$8 for the vehicle.

### Activity Synthesis

Invite a student who organized the given information in a table to share. If no students did this, display this table for all to see:

number of people	entrance fee in dollars
2	14
4	20
8	32
30	
	122

Ask: "What are some ways that you can tell that this relationship is not proportional?" Possible responses:

- 2 people to 4 people is double, but 14 to 20 is not double.
- $14 \div 2 = 7$ , but  $20 \div 4 = 5$ . If the entrance fee were in proportion to the number of people, each quotient would be equal.
- You can't describe the situation with an equation like  $px = q$ .

Invite students who had different strategies for answering the first two questions to share their responses. Ask them to share as many unique strategies as time allows. Ask each student who responds to state their rule that the park uses to decide the entrance fee. Record all unique, correct rules for all to see so students can see different ways of expressing the same idea. For example, the rule might be expressed:

- 8 dollars for the vehicle plus 3 dollars per person
- 3 dollars for every person and an additional \$8
- 3 times the number of people plus 8
- $8 + 3 \cdot \text{people}$

Note: We have the entire rest of the unit to systematically develop relationships like these. There is no need to formalize or generalize anything yet!

# 1.3 Making Toast

**Optional: 10 minutes**

In this activity, students are presented with a different relationship that is not proportional and also doesn't fit a pattern that can be characterized by an equation in the form  $y = px + q$  (like the previous activity could be). This optional activity is a good opportunity for students to interpret another context and describe a relationship, but it can be safely skipped if the previous activity takes too much time.

## Building Towards

- 7.EE.B

## Instructional Routines

- MLR7: Compare and Connect
- Think Pair Share

## Launch

Keep students in the same groups. Give 2 minutes of quiet work time and then invite students to share their sentences with their partner, followed by whole-class discussion.

---

### Access for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Provide students with a two column table for processing and organizing information. Invite students to share their column labels (for example, number of slices and number of seconds) and how they organized the given information.

*Supports accessibility for: Language; Organization*

---

### Student Task Statement

A toaster has 4 slots for bread. Once the toaster is warmed up, it takes 35 seconds to make 4 slices of toast, 70 seconds to make 8 slices, and 105 seconds to make 12 slices.

1. How long do you think it will take to make 20 slices?
2. If someone makes as many slices of toast as possible in 4 minutes and 40 seconds, how many slices do think they can make?

### Student Response

1. 175 seconds
2. 32 slices

Sample table:

number of slices	seconds it would take to make that number of slices
1	35
2	35
3	35
4	35
5	70
6	70
7	70
8	70
9	105
10	105
11	105
12	105

### Are You Ready for More?

What is the smallest number that has a remainder of 1, 2, and 3 when divided by 2, 3, and 4, respectively? Are there more numbers that have this property?

### Student Response

11; yes

### Activity Synthesis

Invite students to share their responses and their reasoning. Select as many unique approaches as time allows.

---

### Access for English Language Learners

*Reading, Speaking: MLR7 Compare and Connect.* During the whole-class discussion, invite students to look for what is the same and what is different between the various approaches to solving the problem. Display and discuss differences in the tables and diagrams. Invite students to make connections by looking for the same quantity (e.g., 20 slices) in each representation. These exchanges strengthen students' mathematical language use and reasoning based on ratios.

*Design Principle(s): Maximize meta-awareness*

---

## Lesson Synthesis

The goal of this lesson is to recognize that there are situations in the world that are more complicated than what we have studied until this point, and to let students know this unit is about developing tools to solve some more sophisticated problems. Questions for discussion:

- “Describe some rules we encountered in this lesson for how one quantity was related to another quantity.”
- “What made these situations more complicated than relationships we have seen in the past?”
- “What were some tools or strategies we used that were particularly helpful?”

## 1.4 Movie Theater Popcorn, Revisited

Cool Down: 5 minutes

### Student Task Statement

A movie theater sells popcorn in bags of different sizes. The table shows the volume of popcorn and the price of the bag.

volume of popcorn (ounces)	price of bag (\$)
10	6
20	8
35	11
48	13.6

If the theater wanted to offer a 60-ounce bag of popcorn, what would be a good price? Explain your reasoning.

## Student Response

Answers vary. Sample responses:

- \$16, because there is a pattern of \$4 plus \$0.20 per ounce.
- \$15, because there should be a discount for buying a larger bag of popcorn.

## Student Lesson Summary

In much of our previous work that involved relationships between two quantities, we were often able to describe amounts as being so much more than another, or so many times as much as another. We wrote equations like  $x + 3 = 8$  and  $4x = 20$  and solved for unknown amounts.

In this unit, we will see situations where relationships between amounts involve more operations. For example, a pizza store might charge the amounts shown in the table for delivering pies.

number of pies	total cost in dollars
1	13
2	23
3	33
5	53

We can see that each additional pie adds \$10 to the total cost, and that each total includes a \$3 additional cost, maybe representing a delivery fee. In this situation, 8 pies will cost  $8 \cdot 10 + 3$  and a total cost of \$63 means 6 pies were ordered.

In this unit, we will see many situations like this one, and will learn how to use diagrams and equations to answer questions about unknown amounts.

# Lesson 1 Practice Problems

## Problem 1

### Statement

Lin and Tyler are drawing circles. Tyler's circle has twice the diameter of Lin's circle. Tyler thinks that his circle will have twice the area of Lin's circle as well. Do you agree with Tyler?

### Solution

No, radius and area are not proportional. The area of Tyler's circle will be 4 times as big as the area of Lin's circle.

(From Unit 3, Lesson 7.)

## Problem 2

### Statement

Jada and Priya are trying to solve the equation  $\frac{2}{3} + x = 4$ .

- Jada says, "I think we should multiply each side by  $\frac{3}{2}$  because that is the reciprocal of  $\frac{2}{3}$ ."
- Priya says, "I think we should add  $-\frac{2}{3}$  to each side because that is the opposite of  $\frac{2}{3}$ ."

- a. Which person's strategy should they use? Why?
- b. Write an equation that can be solved using the other person's strategy.

### Solution

- a. Priya is correct. The operation in the expression  $\frac{2}{3} + x$  is addition. Adding the additive inverse of  $\frac{2}{3}$  to both sides of the equation will change the equation to the form " $x = \dots$ "
- b. Answers vary. Sample response:  $\frac{2}{3}x = 4$ .

(From Unit 5, Lesson 15.)

## Problem 3

### Statement

What are the missing operations?

- a.  $48 ? (-8) = (-6)$
- b.  $(-40) ? 8 = (-5)$
- c.  $12 ? (-2) = 14$

- d.  $18 \div (-12) = 6$
- e.  $18 \div (-20) = -2$
- f.  $22 \div (-0.5) = -11$

## Solution

- a. Divide
- b. Divide
- c. Subtract
- d. Add
- e. Add
- f. Multiply

(From Unit 5, Lesson 13.)

## Problem 4

### Statement

In football, the team that has the ball has four chances to gain at least ten yards. If they don't gain at least ten yards, the other team gets the ball. Positive numbers represent a gain and negative numbers represent a loss. Select **all** of the sequences of four plays that result in the team getting to keep the ball.

- A. 8, -3, 4, 21
- B. 30, -7, -8, -12
- C. 2, 16, -5, -3
- D. 5, -2, 20, -1
- E. 20, -3, -13, 2

## Solution

["A", "C", "D"]

(From Unit 5, Lesson 14.)

## Problem 5

### Statement

A sandwich store charges \$20 to have 3 turkey subs delivered and \$26 to have 4 delivered.

- a. Is the relationship between number of turkey subs delivered and amount charged proportional? Explain how you know.
- b. How much does the store charge for 1 additional turkey sub?
- c. Describe a rule for determining how much the store charges based on the number of turkey subs delivered.

## Solution

- a. No. Sample reasoning: If they deliver 3 turkey subs, they charge \$6.67 per sub, but for 4 subs, they charge \$6.50 per sub.
- b. \$6
- c. The rule could be \$6 per sub plus a \$2 delivery fee. 6 times 3 is 18, but they charged \$2 more than that for 3 subs. 6 times 4 is 24, but they charged \$2 more than that for 4 subs.

## Problem 6

### Statement

Which question cannot be answered by the solution to the equation  $3x = 27$ ?

- A. Elena read three times as many pages as Noah. She read 27 pages. How many pages did Noah read?
- B. Lin has 27 stickers. She gives 3 stickers to each of her friends. With how many friends did Lin share her stickers?
- C. Diego paid \$27 to have 3 pizzas delivered and \$35 to have 4 pizzas delivered. What is the price of one pizza?
- D. The coach splits a team of 27 students into 3 groups to practice skills. How many students are in each group?

### Solution

C

# Lesson 2: Reasoning about Contexts with Tape Diagrams

## Goals

- Draw and label a tape diagram to represent relationships between quantities in a situation.
- Explain (orally and in writing) how to use a tape diagram to determine the value of an unknown quantity in a situation.
- Interpret a tape diagram that represents a relationship of the form  $px + q = r$  or  $p(x + q) = r$ .

## Learning Targets

- I can explain how a tape diagram represents parts of a situation and relationships between them.
- I can use a tape diagram to find an unknown amount in a situation.

## Lesson Narrative

In this lesson, students represent and reason about contexts using tape diagrams. Students may have had experience with tape diagrams in earlier grades, and have seen some examples of their use in prior units. For example, tape diagrams were used to represent percent increase and decrease situations. First, they interpret some given tape diagrams. Then, they interpret a story and create tape diagrams. While the contexts lead to equations of the forms  $p(x + q) = r$  and  $px + q = r$ , this lesson is not about writing equations. Likewise, students are asked to find an unknown value in several story problems, but the intention is for them to use any reasoning that makes sense to them. It is not expected that they write and solve equations, or that any particular method is stressed.

## Alignments

### Addressing

- 7.EE.B.3: Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. \$

### Building Towards

- 7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

### Instructional Routines

- MLR3: Clarify, Critique, Correct

- Notice and Wonder

### Student Learning Goals

Let's use tape diagrams to make sense of different kinds of stories.

## 2.1 Notice and Wonder: Remembering Tape Diagrams

### Warm Up: 5 minutes

The purpose of this warm-up is to re-introduce students to these diagrams as a representation of relationships between quantities. As students use tape diagrams as a tool for reasoning, they understand that the length of a piece of the "tape" carries meaning. Two pieces drawn to be the same length are understood to represent the same value. These pieces can be labeled with values to clarify what is known about the diagram, so two pieces labeled with the same letter indicate that they have the same value, even if that value is not known. These diagrams will be helpful for reasoning about situations in activities in this lesson. When students choose to use a tape diagram to represent a relationship between values and reason about a problem, they are using appropriate tools strategically (MP5). Tasks like this one ensure that students understand how such a tool works so that they are more likely to choose to use it correctly and appropriately.

### Building Towards

- 7.EE.B.4

### Instructional Routines

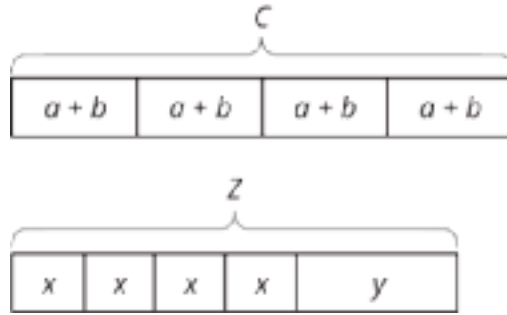
- Notice and Wonder

### Launch

Arrange students in groups of 2. Tell students that they will look at an image, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the image for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Give students 1 additional minute of quiet work time to complete the second question followed by a whole-class discussion.

## Student Task Statement



1. What do you notice? What do you wonder?
2. What are some possible values for  $a$ ,  $b$ , and  $c$  in the first diagram?

For  $x$ ,  $y$ , and  $z$  in the second diagram? How did you decide on those values?

## Student Response

Answers vary. Sample responses:

1. Things students may notice or wonder:
  - There are two diagrams of rectangles with pieces labeled  $a$ ,  $b$ ,  $c$ ,  $x$ ,  $y$ , and  $z$ .
  - The  $c$  and  $z$  appear at the top of the diagrams.
  - Each diagrams consist of a large rectangle and they appear to be the same length as each other.
  - In the first diagram, the rectangle contains 4  $(a + b)$ 's.
  - In the second diagram, the rectangle contains 4  $x$ 's and 1  $y$ .
  - What do the diagrams represent?
  - What do the pieces of the diagrams represent?
  - Do all of the  $x$ 's represent the same value? All the  $y$ 's? All the  $z$ 's?
  - Are longer pieces "worth" more?
2. In the first diagram, if  $a = 1$  and  $b = 4$ , and we assume that  $c$  is the total, then  $c = (1 + 4) \cdot 4 = 20$ . In the second diagram, if  $x = 2$  and  $y = 1$ , and we assume that  $z$  is the total, then  $z = 4 \cdot 2 + 1 = 9$ .

## Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After each response, ask the class whether they agree or disagree and to explain alternative ways of thinking, referring back to the images each time.

Ask students to share possible values for the variables in each diagram. Record and display their responses for all to see. If possible, record the values on the displayed diagram. If the idea that pieces labeled with the same variable represent the same value does not arise in the discussion, make that idea explicit. For example, students should assume that all the pieces labeled with  $y$  in one diagram have the same value. When they make tape diagrams, they know to draw rectangles of the same length to show the same value, but since quick diagrams are sometimes sloppy, it's also important to label pieces with numbers or letters to show known and relative values.

## 2.2 Every Picture Tells a Story

### 15 minutes

In this activity, students explain how a tape diagram represents a situation. They also use the tape diagram to reason about the value of the unknown quantity. Students are not expected to write and solve equations here; any method they can explain for finding values for  $x$  and  $y$  is acceptable. While some students might come up with equations to describe the diagram and solve for the unknown, there is no need to focus on developing those ideas at this time.

### Addressing

- 7.EE.B.3

### Building Towards

- 7.EE.B.4

### Instructional Routines

- MLR3: Clarify, Critique, Correct

### Launch

Arrange students in groups of 3. (Some groups of 2 are okay, if needed.)

Ask students if they know what a “flyer” is. If any students do not know, explain or ask a student to explain. If possible, reference some examples of flyers hanging in school.

Ensure students understand they should take turns speaking and listening, and that there are two things to do for each diagram: explain why it represents the story, and also figure out any unknown values in the story.

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### Access for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. Consider pausing after the first question for a brief class discussion before moving on.  
*Supports accessibility for: Organization; Attention*

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## Anticipated Misconceptions

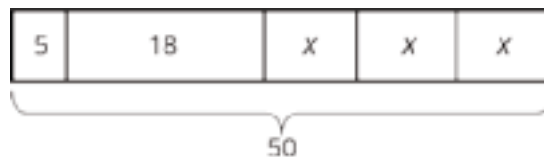
Students may not realize that when a variable is assigned to represent a quantity in a situation, it has the same value each time it appears. Revisit what  $x$  and  $y$  represent in these problems and why each occurrence of a variable must represent the same value.

In the second situation, students might argue that a more accurate representation would be 5 boxes with  $y$  to show the first distribution of stickers, and then five boxes with 2 to show the second distribution. Tell students that such a representation would indeed correctly describe the actions in the situation, but that the work of the task is to understand *this* diagram to set us up for success later.

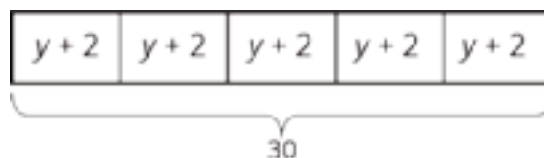
### Student Task Statement

Here are three stories with a diagram that represents it. With your group, decide who will go first. That person explains why the diagram represents the story. Work together to find any unknown amounts in the story. Then, switch roles for the second diagram and switch again for the third.

1. Mai made 50 flyers for five volunteers in her club to hang up around school. She gave 5 flyers to the first volunteer, 18 flyers to the second volunteer, and divided the remaining flyers equally among the three remaining volunteers.



2. To thank her five volunteers, Mai gave each of them the same number of stickers. Then she gave them each two more stickers. Altogether, she gave them a total of 30 stickers.



3. Mai distributed another group of flyers equally among the five volunteers. Then she remembered that she needed some flyers to give to teachers, so she took 2 flyers from each volunteer. Then, the volunteers had a total of 40 flyers to hang up.



## Student Response

1. Answers vary. Unknown amounts that students may find include the remaining number of flyers (27) and the number of flyers given to each of the 3 remaining volunteers (9). The whole rectangle represents the 50 flyers that Mai made. She split them up into five parts: 5, 18, and 3 equal parts for the rest. The 3 equal parts are shown by 3 same-sized boxes.  $x$  represents the number of flyers for each of the 3 remaining volunteers.  $3 \cdot x$  is the number of flyers remaining after Mai gave out 5 and 18. That part has to represent 27 flyers, since 23 of them ( $5 + 18$ ) have already been given out. So each  $x$  represents 9 flyers.
2. Answers vary. Unknown amounts that students may find include the total number of stickers each student receives (6) and the number they received at first (4). The whole rectangle represents the 30 stickers. They are divided into 5 equal parts since the 5 volunteers each got the same number of stickers. They each got some ( $y$ ) and then each got 2 more, so each one got  $y + 2$  stickers. We can find  $y$  by thinking that 30 divides into 5 groups of 6. If each volunteer received 6 stickers in total, they got 4 before the extra 2 were added. Another way to think about  $y$  is to first take away the 10 extra stickers that were given out. Then 5 groups of 4 would make up the remaining 20 stickers. So  $y$  represents 4 stickers.
3. Answers vary. Unknown amounts that students may find include the number of flyers each student has in the end (8), the number they received at first (10), and the total number of flyers (50). The whole rectangle represents 40 flyers. They are divided into 5 equal parts since the 5 volunteers each got the same number of flyers. They each got some ( $w$ ) and then each gave back 2, so each one has  $w - 2$  flyers. We can find  $w$  by thinking that 40 divides into 5 groups of 8. If each volunteer has 8 flyers, they got 10 before the 2 were taken away. So there were originally 50 flyers, which is the 40 that the volunteers have, plus the 10 that Mai took back.

## Activity Synthesis

Tape diagrams represent relationships between quantities in stories. The goals here are to make sure students understand how parts of the diagram match the information about the story, and for them to begin to reason about how the diagrams connect to the operations that can help find unknown amounts.

Invite one group to provide an explanation for each diagram—both how the diagram represents the story, and how they reasoned about the unknown amounts. After each, ask the class if anyone thought about it a different way. (One additional line of reasoning for each diagram is probably sufficient.)

Here are some questions you might ask to encourage students to be more specific:

- “What question could you ask about the story?”
- “Where in the diagrams do you see equal parts? How do you know they are equal?”
- “What quantity does the variable represent in the story? How do you know?”
- “In the first story, where in the diagram do we see the ‘remaining flyers?’”

- “Why don’t we see the number 3 in the first diagram to show the 3 remaining volunteers?”
- “In the second diagram, where are the five volunteers represented?”
- “How did the diagrams help you find the value of the unknown quantities?”

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### Access for English Language Learners

*Conversing: MLR3 Clarify, Critique, Correct.* Present an incorrect statement for the second situation that reflects a possible misunderstanding from the class. For example, “Mai gave 6 stickers to each of the volunteers because 30 divided by 5 is 6. So  $y$  is 6.” Prompt students to identify the error, and then write a correct statement. This helps students evaluate, and improve on, the written mathematical arguments of others and to understand the importance of defining the variable in context of the situation.

*Design Principle(s): Maximize meta-awareness*

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## 2.3 Every Story Needs a Picture

15 minutes (there is a digital version of this activity)

In the previous activity, students interpreted given tape diagrams and explained how they represented a story. Here, they have a chance to draw tape diagrams to represent a story. The first story is a bit more scaffolded because it specifies what  $x$  represents. In the other two stories, students need to decide which quantity to represent with a variable and choose a letter to use. As with all activities in this lesson, students are *not* expected to write and solve an equation. This preliminary work supports the understanding needed to be able to represent such situations with equations.

### Addressing

- 7.EE.B.3

### Building Towards

- 7.EE.B.4

### Launch

Keep students in the same groups. You might have *each* student draw *all three* diagrams and compare them with their groups, working together to resolve any discrepancies. Or if time is short, you might assign *each* student in the group *a different story*—ask each student to explain their diagram to their group to see if their group members agree with their interpretation.

For classrooms using the digital version of the materials, take a minute to demonstrate how the controls work in the applet. Some students may prefer to draw the diagrams in their notebooks or on scratch paper.

## Access for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Provide students with a blank template of a tape diagram to represent each story.

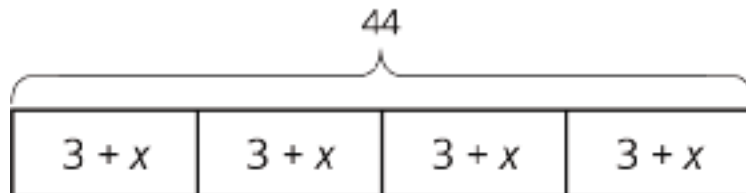
*Supports accessibility for: Language; Organization*

### Student Task Statement

Here are three more stories. Draw a tape diagram to represent each story. Then describe how you would find any unknown amounts in the stories.

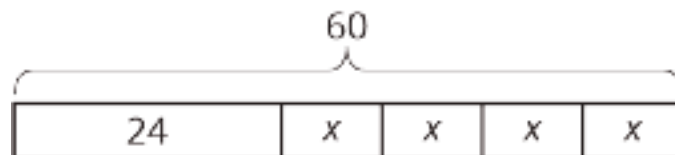
1. Noah and his sister are making gift bags for a birthday party. Noah puts 3 pencil erasers in each bag. His sister puts  $x$  stickers in each bag. After filling 4 bags, they have used a total of 44 items.
2. Noah's family also wants to blow up a total of 60 balloons for the party. Yesterday they blew up 24 balloons. Today they want to split the remaining balloons equally between four family members.
3. Noah's family bought some fruit bars to put in the gift bags. They bought one box each of four flavors: apple, strawberry, blueberry, and peach. The boxes all had the same number of bars. Noah wanted to taste the flavors and ate one bar from each box. There were 28 bars left for the gift bags.

### Student Response



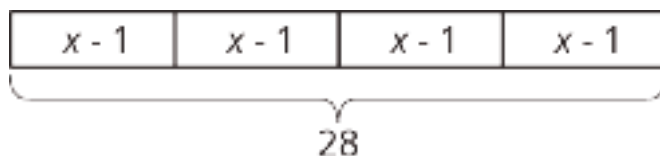
1. or equivalent.

Answers vary. Unknown amounts that students may find include the number of stickers in each bag (8) and the total number of items in each bag (11). Dividing 44 into 4 equal parts gives 11 items for each bag, which means 3 erasers and 8 stickers in each. Another way is to subtract the 12 erasers from 44, giving 32 items left. 32 stickers split evenly among 4 bags is 8 in each bag.



2. or equivalent.

Answers vary. Unknown amounts that students may find include the number of balloons they need to blow up today (36) and the number that each of the four family members blows up (9). The number of balloons left to blow up is found by  $60 - 24$  or 36. Splitting those up equally among four people is  $36 \div 4$  or 9 each.



3. or equivalent.

Answers vary. Unknown amounts that students may find include the number of bars left in each box (7), the number of bars originally in each box (8), and the total number of bars there were in the four boxes (32). Dividing 28 into 4 equal parts gives 7 bars left in each box. Adding 1 to each gives 8 in each box originally for a total of  $8 \cdot 4$  or 32 bars.

### Are You Ready for More?

Design a tiling that uses a repeating pattern consisting of 2 kinds of shapes (e.g., 1 hexagon with 3 triangles forming a triangle). How many times did you repeat the pattern in your picture? How many individual shapes did you use?

### Student Response

Answers vary.

### Activity Synthesis

Much of the discussion will take place in groups. Here are some ideas for synthesizing students' learning about creating tape diagrams:

- Ask students if they had any disagreements in their groups and how they resolved them.
- Ask students how they decided which unknown quantity to find in the story. The first story specifies  $x$  stickers, but the other stories do not define a variable.
- Display one diagram for each story and ask students to explain how they are alike and how they are different.

### Lesson Synthesis

Display one or more of the tape diagrams students encountered or created during the lesson. Ask, "What are some ways that tape diagrams give information about a story?" Responses to highlight:

- A total amount is indicated.
- Pieces that represent equal amounts are the same length (or roughly the same length, if sketching by hand).

- Pieces that represent different amounts are not the same length.
- Pieces are labeled with either their amounts, a variable representing an unknown amount, or an expression like  $x + 1$  to mean "1 more than the unknown amount."

## 2.4 Red and Yellow Apples

Cool Down: 5 minutes

### Addressing

- 7.EE.B.3

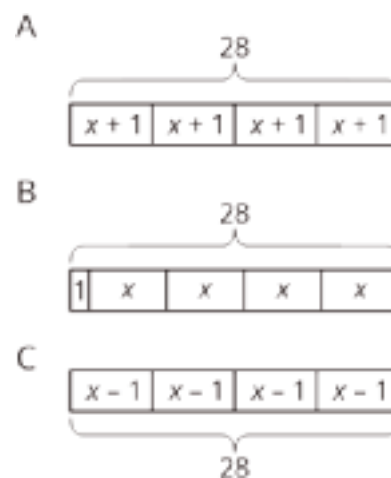
### Building Towards

- 7.EE.B.4

#### Student Task Statement

Here is a story: Lin bought 4 bags of apples. Each bag had the same number of apples. After eating 1 apple from each bag, she had 28 apples left.

1. Which diagram best represents the story?  
Explain why the diagram represents it.



2. What part of the story does  $x$  represent?
3. Describe how you would find the unknown amount in the story.

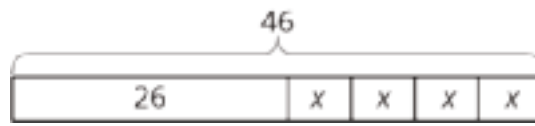
#### Student Response

1. C. When she ate 1 apple from each bag, there were  $x - 1$  apples left in each bag.
2.  $x$  represents the number of apples in 1 bag before Lin ate any apples.
3. Each of the 4 pieces of the diagram represents 7 apples, because  $28 \div 4 = 7$ . If  $x - 1 = 7$ , then  $x$  is 8.

## Student Lesson Summary

Tape diagrams are useful for representing how quantities are related and can help us answer questions about a situation.

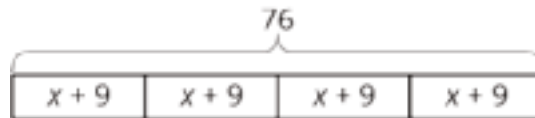
Suppose a school receives 46 copies of a popular book. The library takes 26 copies and the remainder are split evenly among 4 teachers. How many books does each teacher receive? This situation involves 4 equal parts and one other part. We can represent the situation with a rectangle labeled 26 (books given to the library) along with 4 equal-sized parts (books split among 4 teachers). We label the total, 46, to show how many the rectangle represents in all. We use a letter to show the unknown amount, which represents the number of books each teacher receives. Using the same letter,  $x$ , means that the same number is represented four times.



Some situations have parts that are all equal, but each part has been increased from an original amount:

A company manufactures a special type of sensor, and packs them in boxes of 4 for shipment. Then a new design increases the weight of each sensor by 9 grams. The new package of 4 sensors weighs 76 grams. How much did each sensor weigh originally?

We can describe this situation with a rectangle representing a total of 76 split into 4 equal parts. Each part shows that the new weight,  $x + 9$ , is 9 more than the original weight,  $x$ .



# Lesson 2 Practice Problems

## Problem 1

### Statement

The table shows the number of apples and the total weight of the apples.

number of apples	weight of apples (grams)
2	511
5	1200
8	2016

Estimate the weight of 6 apples.

### Solution

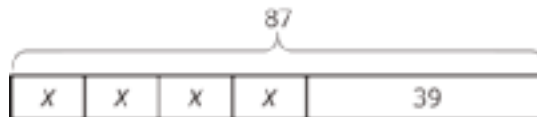
About 1500 grams.

(From Unit 3, Lesson 1.)

## Problem 2

### Statement

Select all stories that the tape diagram can represent.



- A. There are 87 children and 39 adults at a show. The seating in the theater is split into 4 equal sections.
- B. There are 87 first graders in after-care. After 39 students are picked up, the teacher put the remaining students into 4 groups for an activity.
- C. Lin buys a pack of 87 pencils. She gives 39 to her teacher and shared the remaining pencils between herself and 3 friends.
- D. Andre buys 4 packs of paper clips with 39 paper clips in each. Then he gives 87 paper clips to his teacher.
- E. Diego's family spends \$87 on 4 tickets to the fair and a \$39 dinner.

## Solution

["B", "C", "E"]

### Problem 3

#### Statement

Andre wants to save \$40 to buy a gift for his dad. Andre's neighbor will pay him weekly to mow the lawn, but Andre always gives a \$2 donation to the food bank in weeks when he earns money. Andre calculates that it will take him 5 weeks to earn the money for his dad's gift. He draws a tape diagram to represent the situation.



- Explain how the parts of the tape diagram represent the story.
- How much does Andre's neighbor pay him each week to mow the lawn?

## Solution

- Answers vary. Sample response: The 5 equal parts represent the 5 weeks. In each week, Andre will earn  $x$  dollars for mowing his neighbor's lawn and give \$2 to the food bank, so he will save  $x - 2$  dollars. In five weeks, he will save a total of \$40.
- \$10

### Problem 4

#### Statement

Without evaluating each expression, determine which value is the greatest. Explain how you know.

- $7\frac{5}{6} - 9\frac{3}{4}$
- $(-7\frac{5}{6}) + (-9\frac{3}{4})$
- $(-7\frac{5}{6}) \cdot 9\frac{3}{4}$
- $(-7\frac{5}{6}) \div (-9\frac{3}{4})$

## Solution

$(-7\frac{5}{6}) \div (-9\frac{3}{4})$  is the greatest because it is the only expression with a positive value.

(From Unit 5, Lesson 13.)

## Problem 5

### Statement

Solve each equation.

a.  $(8.5) \cdot (-3) = a$

b.  $(-7) + b = (-11)$

c.  $c - (-3) = 15$

d.  $d \cdot (-4) = 32$

### Solution

a. -25.5

b. -4

c. 12

d. -8

(From Unit 5, Lesson 15.)

# Lesson 3: Reasoning about Equations with Tape Diagrams

## Goals

- Coordinate tape diagrams and equations of the form  $px + q = r$  or  $p(x + q) = r$ .
- Create a tape diagram to represent an equation of the form  $px + q = r$  or  $p(x + q) = r$ , and use it to solve the equation.
- Identify equivalent equations, and justify (using words and other representations) that they are equivalent.

## Learning Targets

- I can match equations and tape diagrams that represent the same situation.
- If I have an equation, I can draw a tape diagram that shows the same relationship.

## Lesson Narrative

The purpose of this lesson is to make connections between a tape diagram and an equation of the form  $px + q = r$  or  $p(x + q) = r$ . Students match tape diagrams to corresponding equations and sort them into categories, and then they draw tape diagrams to represent equations. They use the tape diagram and the equation to reason about a solution, but it is expected that students reason using any method that makes sense to them. It's not yet time to teach particular methods for solving particular types of equations.

## Alignments

### Building On

- 6.EE.A.4: Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions  $y + y + y$  and  $3y$  are equivalent because they name the same number regardless of which number  $y$  stands for.

### Addressing

- 7.EE.B.3: Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. \$

### Building Towards

- 7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

## Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR1: Stronger and Clearer Each Time
- MLR2: Collect and Display
- Take Turns
- Think Pair Share

## Student Learning Goals

Let's see how equations can describe tape diagrams.

# 3.1 Find Equivalent Expressions

### Warm Up: 10 minutes

Students learned all about the distributive property in grade 6, including how to use the distributive property to rewrite expressions like  $6(2x - 3)$  and  $12a + 8a$ . This is an opportunity for students to remember what the distributive property is all about before they will be expected to use it in the process of solving equations of the form  $p(x + q) = r$  later in this unit. If this activity indicates that students remember little of the distributive property from grade 6, heavier interventions may be needed.

Look for students who:

- Rule out expressions by testing values
- Use the term *distributive property*

## Building On

- 6.EE.A.4

## Instructional Routines

- Think Pair Share

## Launch

Ask students to think of anything they know about *equivalent expressions*. Ask if they can:

- Explain why  $2x$  and  $2 + x$  are not equivalent. (These expressions are equal when  $x$  is 2, but not equal for other values of  $x$ . Multiplying 2 by a number usually gives a different result than adding that number to 2.)
- Explain why  $3 + x$  and  $x + 3$  are equivalent. (These expressions are equal no matter the value of  $x$ . Also, addition is commutative.)
- Think of another example of two equivalent expressions. (Examples:  $2x$  and  $x \cdot 2$ ,  $a + a + a$ , and  $3a$ .)

- Explain what this term means. (Equivalent expressions are equal no matter the value assigned to the variable.)
- Describe ways to decide whether expressions are equivalent. (Test some values, draw diagrams for different values, analyze them for properties of the operations involved.)

Arrange students in groups of 2. Give 3 minutes of quiet work time and then invite students to share their responses with their partner, followed by a whole-class discussion.

### Student Task Statement

Select **all** the expressions that are **equivalent** to  $7(2 - 3n)$ . Explain how you know each expression you select is equivalent.

1.  $9 - 10n$
2.  $14 - 3n$
3.  $14 - 21n$
4.  $(2 - 3n) \cdot 7$
5.  $7 \cdot 2 \cdot (-3n)$

### Student Response

$14 - 21n$  is equivalent because of the distributive property.  $(2 - 3n) \cdot 7$  is equivalent because multiplication is commutative.

### Activity Synthesis

Select a student who tested values to explain how they know two expressions are not equivalent. For example,  $9 - 10n$  is not equivalent to  $7(2 - 3n)$ , because if we use 0 in place of  $n$ ,  $9 - 10 \cdot 0$  is 9 but  $7(2 - 3 \cdot 0)$  is 14. If no one brings this up, demonstrate an example.

Select a student who used the term *distributive property* to explain why  $7(2 - 3n)$  is equivalent to  $14 - 21n$  to explain what they mean by that term. In general, an expression of the form  $a(b + c)$  is equivalent to  $ab + ac$ .

## 3.2 Matching Equations to Tape Diagrams

### 15 minutes

In this activity, the tape diagrams and equations use the same numbers, so students must attend to the meaning of the operations in the equations and to the structure of the tape diagrams.

Look for students who have sensible ways to distinguish diagram A from diagram C, and also diagram D from diagram E. These correspond to using categories to sort the equations like “multiply by 2” vs. “multiply by 5.” Also look for students who categorize equations by “parentheses” vs. “no parentheses.” These categories correspond to distinguishing A, B, and C from D and E.

## Building Towards

- 7.EE.B.4

## Instructional Routines

- MLR2: Collect and Display
- Take Turns

## Launch

Keep students in the same groups. Tell them that, in this activity, they will match some diagrams, like the ones they saw in previous lessons, to corresponding equations. Then, they sort the list of equations into categories of their choosing. When they sort the equations, they should work with their partner to come up with categories, and then take turns sorting each equation into one of their categories, explaining why they are doing so. If necessary, demonstrate this protocol before students start working.

Give students 5 minutes to work with their partner followed by a whole-class discussion.

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### Access for Students with Disabilities

*Representation: Internalize Comprehension.* Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, use the same color to highlight the variable in the tape diagram to the same variable in the matching equation. Give students time to compare their color coding and annotations during their partner discussion.

*Supports accessibility for: Visual-spatial processing*

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### Access for English Language Learners

*Speaking, Representing: MLR2 Collect and Display.* As students explain how the equation matches the diagram, listen for and collect students' descriptions of the equation (e.g., "two groups of  $x + 5$  equal 19"). Display collected language next to the corresponding tape diagram and equation for all to see. Invite students to borrow language from the displayed examples while sorting into categories, after the matching is complete. This will help students make connection between language, diagrams, and equations.

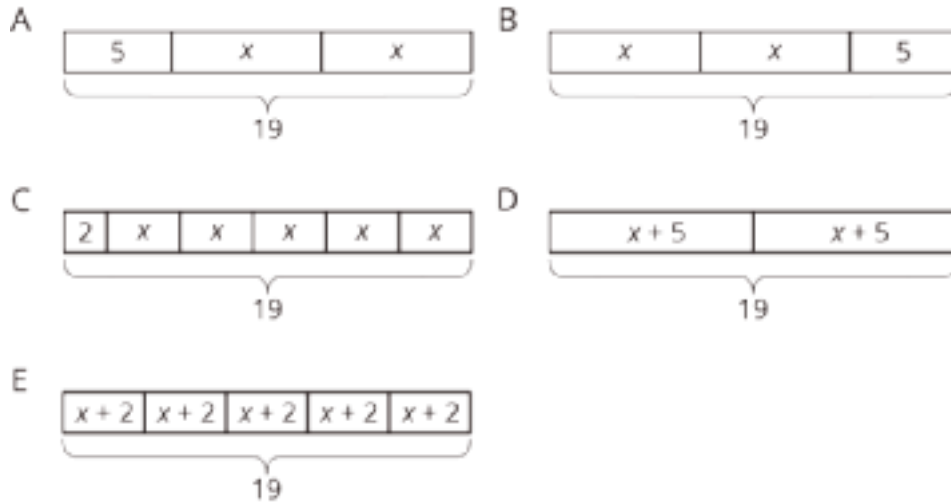
*Design Principle(s): Support sense-making; Maximize meta-awareness*

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## Anticipated Misconceptions

If students don't know where to begin, encourage them to describe the diagrams and equations in words. For example, diagram E could be described "two groups of  $x + 5$  equal 19," and so could the equation  $2(x + 5) = 19$ .

## Student Task Statement



- Match each equation to one of the tape diagrams. Be prepared to explain how the equation matches the diagram.
  - $2x + 5 = 19$
  - $2 + 5x = 19$
  - $2(x + 5) = 19$
  - $5(x + 2) = 19$
- Sort the equations into categories of your choosing. Explain the criteria for each category.
  - $19 = 5 + 2x$
  - $(x + 5) \cdot 2 = 19$
  - $19 = (x + 2) \cdot 5$
  - $19 \div 2 = x + 5$
  - $19 - 2 = 5x$

## Student Response

- B or A  $2x + 5 = 19$
- C  $2 + 5x = 19$
- D  $2(x + 5) = 19$
- E  $5(x + 2) = 19$
- A or B  $19 = 5 + 2x$
- D  $(x + 5) \cdot 2 = 19$
- E  $19 = (x + 2) \cdot 5$
- D  $19 \div 2 = x + 5$

- C  $19 - 2 = 5x$

For the second question, answers vary. Likely categories include:

- Equation contains parentheses vs. no parentheses
- 19 on the left side vs. the right side of the =
- Multiply by 2 vs. multiply by 5

### Activity Synthesis

Select a student who used the categories “multiply by 2” vs. “multiply by 5” to share their reasoning. Ask them to explain how we can see these categories in the corresponding diagrams. How did they categorize  $19 \div 2 = x + 5$  which contains no multiplication?

Select a student to share their reasoning who used the categories “parentheses” vs. “no parentheses.” Did they have any misgivings about  $19 \div 2 = x + 5$ ? It contains no parentheses, but the corresponding diagram D also matches  $2(x + 5) = 19$ , which has parentheses.

If students express uncertainty about  $2x + 5 = 19$ , spend some time here. Some students are likely to match it to exactly one diagram and some students match it to both A and B. The point isn't that one of these is right; it is to have the conversation about the idea of expressions or equations being identical vs. equivalent. Equivalent expressions or equivalent equations can have different literal interpretations, but when matching equations to tape diagrams, all that matters for the purposes of solving is that the equations are equivalent.

## 3.3 Drawing Tape Diagrams to Represent Equations

10 minutes (there is a digital version of this activity)

This activity is parallel to one in the previous lesson, except that students are creating a tape diagram after interpreting an equation rather than interpreting a story. The intention is for students to reason in any way that makes sense to them about the equations and diagrams to figure out the solution to each equation. Do not demonstrate any equation solving procedures yet.

For each equation, monitor for a student who used their diagram to reason about a solution and a student who used the structure of the equation to reason about a solution.

### Addressing

- 7.EE.B.3

### Building Towards

- 7.EE.B.4

### Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- MLR1: Stronger and Clearer Each Time

## Launch

Give students 5 minutes of quiet work time followed by a whole-class discussion.

### Access for Students with Disabilities

*Representation: Develop Language and Symbols.* Display or provide charts with symbols and meanings. For example, display a blank template of a tape diagram labeling the different parts with generalizations for what content will go inside. In addition, consider using a previous example situation or equation to make connections to the blank template.

*Supports accessibility for: Conceptual processing; Memory*

### Access for English Language Learners

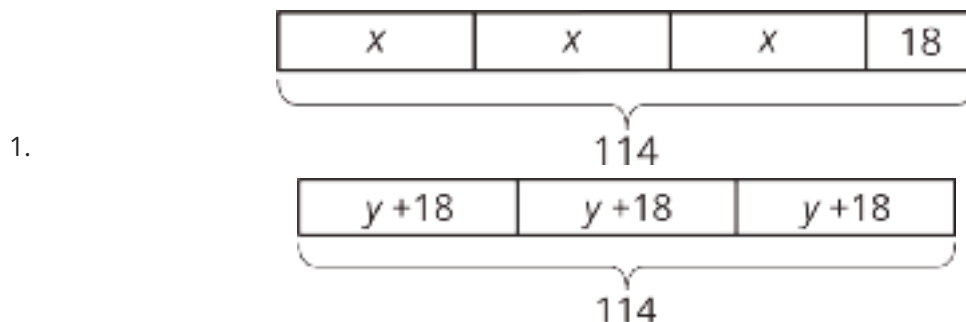
*Speaking, Representing: MLR1 Stronger and Clearer Each Time.* Use this routine to give students a structured opportunity to refine their tape diagrams. Ask students to meet with 2–3 partners to get feedback on their diagram of one or both equations. Listeners should press for details from the equations (e.g., “Where is the \_\_\_ from the equation in your diagram?”). This will help students use language to describe connections between the diagrams and the equations.

*Design Principle(s): Support sense-making*

## Student Task Statement

- $114 = 3x + 18$
  - $114 = 3(y + 18)$
1. Draw a tape diagram to match each equation.
  2. Use any method to find values for  $x$  and  $y$  that make the equations true.

## Student Response



2.  $x = 32$  and  $y = 20$  Strategies vary.

## Are You Ready for More?

To make a Koch snowflake:

- Start with an equilateral triangle that has side lengths of 1. This is step 1.
- Replace the middle third of each line segment with a small equilateral triangle with the middle third of the segment forming the base. This is step 2.
- Do the same to each of the line segments. This is step 3.
- Keep repeating this process.



1. What is the perimeter after step 2? Step 3?
2. What happens to the perimeter, or the length of line traced along the outside of the figure, as the process continues?

### Student Response

1. 4,  $5\frac{1}{3}$
2. The perimeter increases as the process continues.

### Activity Synthesis

For each equation, select a student who used their diagram to reason about a solution and a student who used the structure of the equation to reason about a solution. Ask these students to explain how they arrived at a solution. Display the diagram and the equation side by side as students are explaining, and draw connections between the two representations.

Do not demonstrate any equation-solving procedures yet.

### Lesson Synthesis

Display one or more tape diagrams students encountered or created during the lesson, along with their corresponding equations. Ask, "What are some ways that tape diagrams represent equations?" Responses to highlight:

- Multiplication in the equation is represented with multiple copies of the same piece in the diagram.
- The total amount is shown in both the equation and the diagram.

- An unknown amount is represented with a variable.
- Either the equation or the diagram can be used to reason about a solution to the equation.

## 3.4 Three of These Equations Belong Together

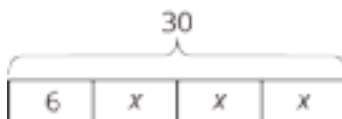
Cool Down: 5 minutes

### Building Towards

- 7.EE.B.4

#### Student Task Statement

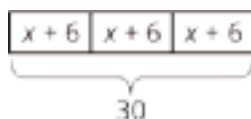
Here is a diagram.



1. Circle the equation that the diagram does *not* match.
  - $6 + 3x = 30$
  - $3(x + 6) = 30$
  - $3x = 30 - 6$
  - $30 = 3x + 6$
2. Draw a diagram that matches the equation you circled.

#### Student Response

1.  $3(x + 6) = 30$  does not match.
2. Sample diagram:



#### Student Lesson Summary

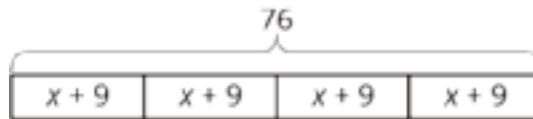
We have seen how tape diagrams represent relationships between quantities. Because of the meaning and properties of addition and multiplication, more than one equation can often be used to represent a single tape diagram.

Let's take a look at two tape diagrams.



We can describe this diagram with several different equations. Here are some of them:

- $26 + 4x = 46$ , because the parts add up to the whole.
- $4x + 26 = 46$ , because addition is commutative.
- $46 = 4x + 26$ , because if two quantities are equal, it doesn't matter how we arrange them around the equal sign.
- $4x = 46 - 26$ , because one part (the part made up of 4  $x$ 's) is the difference between the whole and the other part.



For this diagram:

- $4(x + 9) = 76$ , because multiplication means having multiple groups of the same size.
- $(x + 9) \cdot 4 = 76$ , because multiplication is commutative.
- $76 \div 4 = x + 9$ , because division tells us the size of each equal part.

## Glossary

- equivalent expressions

# Lesson 3 Practice Problems

## Problem 1

### Statement

Solve each equation mentally.

a.  $2x = 10$

b.  $-3x = 21$

c.  $\frac{1}{3}x = 6$

d.  $-\frac{1}{2}x = -7$

### Solution

a. 5

b. -7

c. 18

d. 14

(From Unit 5, Lesson 15.)

## Problem 2

### Statement

Complete the magic squares so that the sum of each row, each column, and each diagonal in a grid are all equal.

0	7	2
	3	

1		
	3	-2
		5

4	2	0
-1		

## Solution

0	7	2
5	3	1
4	-1	6

1	2	6
8	3	-2
0	4	5

3	-2	5
4	2	0
-1	6	1

(From Unit 5, Lesson 3.)

## Problem 3

### Statement

Draw a tape diagram to match each equation.

a.  $5(x + 1) = 20$

b.  $5x + 1 = 20$

## Solution

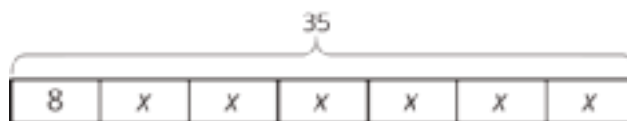
a. A diagram showing 5 equal parts of  $x + 1$  for a total of 20

b. A diagram showing 5 equal parts of  $x$  and one part of 1 for a total of 20

## Problem 4

### Statement

Select all the equations that match the tape diagram.



A.  $35 = 8 + x + x + x + x + x + x$

B.  $35 = 8 + 6x$

C.  $6 + 8x = 35$

D.  $6x + 8 = 35$

E.  $6x + 8x = 35x$

F.  $35 - 8 = 6x$

## Solution

["A", "B", "D", "F"]

## Problem 5

### Statement

Each car is traveling at a constant speed. Find the number of miles each car travels in 1 hour at the given rate.

a. 135 miles in 3 hours

b. 22 miles in  $\frac{1}{2}$  hour

c. 7.5 miles in  $\frac{1}{4}$  hour

d.  $\frac{100}{3}$  miles in  $\frac{2}{3}$  hour

e.  $97\frac{1}{2}$  miles in  $\frac{3}{2}$  hour

## Solution

a. 45 miles

b. 44 miles

c. 30 miles

d. 50 miles

e. 65 miles

(From Unit 4, Lesson 2.)

# Lesson 4: Reasoning about Equations and Tape Diagrams (Part 1)

## Goals

- Coordinate tape diagrams, equations of the form  $px + q = r$ , and verbal descriptions of the situations.
- Explain (orally and in writing) how to use a tape diagram to determine the value of an unknown quantity in an equation of the form  $px + q = r$ .
- Interpret (in writing) the solution to an equation in the context of the situation it represents.

## Learning Targets

- I can draw a tape diagram to represent a situation where there is a known amount and several copies of an unknown amount and explain what the parts of the diagram represent.
- I can find a solution to an equation by reasoning about a tape diagram or about what value would make the equation true.

## Lesson Narrative

The focus of this lesson is situations that lead to equations of the form  $px + q = r$ . Tape diagrams are used to help students understand why these situations can be represented with equations of this form, and to help them reason about solving equations of this form. Students also attend to the meaning of the equation's solution in the context (MP2). Note that we are not generalizing solution methods yet; just using diagrams as a tool to reason about solving equations.

## Alignments

### Building On

- 6.EE.B.5: Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

### Addressing

- 7.EE.B.3: Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. \$
- 7.EE.B.4.a: Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the

operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

### Building Towards

- 7.EE.B.4.a: Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

### Instructional Routines

- Algebra Talk
- MLR3: Clarify, Critique, Correct
- MLR7: Compare and Connect
- MLR8: Discussion Supports

### Student Learning Goals

Let's see how tape diagrams can help us answer questions about unknown amounts in stories.

## 4.1 Algebra Talk: Seeing Structure

### Warm Up: 10 minutes

The purpose of this Algebra Talk is to elicit strategies and understandings students have for solving equations. These understandings help students develop fluency and will be helpful later in this unit when students will need to be able to come up with ways to solve equations of this form. While four equations are given, it may not be possible to share every strategy. Consider gathering only two or three different strategies per problem, saving most of the time for the final question.

Students should understand the meaning of *solution to an equation* from grade 6 work as well as from work earlier in this unit, but this is a good opportunity to re-emphasize the idea.

In this string of equations, each equation has the same solution. Digging into why this is the case requires noticing and using the structure of the equations (MP7). Noticing and using the structure of an equation is an important part of fluency at solving equations.

### Building On

- 6.EE.B.5

### Building Towards

- 7.EE.B.4.a

## Instructional Routines

- Algebra Talk
- MLR8: Discussion Supports

## Launch

Display one equation at a time. Give students 30 seconds of quiet think time for each equation and ask them to give a signal when they have an answer and a strategy. Keep all equations displayed throughout the talk. Follow with a whole-class discussion.

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### Access for Students with Disabilities

*Representation: Internalize Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.

*Supports accessibility for: Memory; Organization*

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### Student Task Statement

Find a solution to each equation without writing anything down.

$$x + 1 = 5$$

$$2(x + 1) = 10$$

$$3(x + 1) = 15$$

$$500 = 100(x + 1)$$

### Student Response

- 4 is the solution because  $4 + 1 = 5$ .
- 4. Possible strategies: Trial and error to arrive at  $2(4 + 1) = 10$ , noticing that the equation is 2 times something is 10, so the something must be a 5, applying the distributive property to get  $2x + 2 = 10$ , and reasoning from there.
- 4
- 4

$x = 4$  is a solution to each equation because  $x + 1$  has to equal 5.

### Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- "Who can restate \_\_\_'s reasoning in a different way?"

- “Did anyone have the same strategy but would explain it differently?”
- “Did anyone solve the equation in a different way?”
- “Does anyone want to add on to \_\_\_\_’s strategy?”
- “Do you agree or disagree? Why?”

An important idea to highlight is the meaning of a solution to an equation; a solution is a value that makes the equation true.

For the second and third equations, some students may first think about applying the distributive property before reasoning about the solution. As students see the third and fourth equations, they are likely to notice commonalities among the equations that can support solving them. One likely observation to highlight is that each equation has the same solution. It is worth asking why each equation has the same solution. A satisfying answer to this question requires attending to the structure of the equations.

### Access for English Language Learners

*Speaking: MLR8 Discussion Supports.*: Display sentence frames to support students when they explain their strategy. For example, “First, I \_\_\_\_ because . . .” or “I noticed \_\_\_\_ so I . . .” Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

*Design Principle(s): Optimize output (for explanation)*

## 4.2 Situations and Diagrams

15 minutes (there is a digital version of this activity)

The purpose of this activity is to work toward showing students that some situations can be represented by an equation of the form  $px + q = r$  (or equivalent). In this activity, students are simply tasked with drawing a tape diagram to represent each situation. In the following activity, they will work with corresponding equations.

The last question is tough to represent with a tape diagram, because you would have to divide the diagram into 30 equal pieces. This is intentional, and can be used to make the point that we are trying to develop more efficient ways of solving problems than drawing a diagram every time.

For each question, monitor for one student with a correct diagram. Press students to explain what any variables used to label the diagram represent in the situation.

### Building Towards

- 7.EE.B.4.a

## Instructional Routines

- MLR7: Compare and Connect

## Launch

Ensure students understand that the work of this task is to draw a tape diagram to represent each situation. There is no requirement to write an equation or solve a problem yet.

Arrange students in groups of 2. Give 5–10 minutes to work individually or with their partner, followed by a whole-class discussion.

---

### Access for Students with Disabilities

*Engagement: Internalize Self Regulation.* Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. Invite students to choose and respond to 3 out of the 5 situations. Require students to choose the last situation as it is different from the others.


*Supports accessibility for: Organization; Attention*

---

### Student Task Statement

Draw a tape diagram to represent each situation. For some of the situations, you need to decide what to represent with a variable.

1. Diego has 7 packs of markers. Each pack has  $x$  markers in it. After Lin gives him 9 more markers, he has a total of 30 markers.
2. Elena is cutting a 30-foot piece of ribbon for a craft project. She cuts off 7 feet, and then cuts the remaining piece into 9 equal lengths of  $x$  feet each.
3. A construction manager weighs a bundle of 9 identical bricks and a 7-pound concrete block. The bundle weighs 30 pounds.
  
4. A skating rink charges a group rate of \$9 plus a fee to rent each pair of skates. A family rents 7 pairs of skates and pays a total of \$30.

- 
5. Andre bakes 9 pans of brownies. He donates 7 pans to the school bake sale and keeps the rest to divide equally among his class of 30 students.

### Student Response

Answers vary. Sample responses are rectangles with the following features:

1. 7 same-sized boxes each marked  $x$ , one box marked 9, bracket showing total is 30
2. 9 same-sized boxes each marked  $x$ , one box marked 7, bracket showing total is 30
3. 9 same-sized boxes each marked  $x$ , one box marked 7, bracket showing total is 30
4. 7 same-sized boxes each marked  $x$ , one box marked 9, bracket showing total is 30
5. 30 same-sized boxes each marked  $x$  (or one marked  $30x$  or equivalent), one box marked 7, bracket showing total is 9

### Activity Synthesis

Select one student for each situation to present their correct diagram. Ensure that students explain the meaning of any variables used to label their diagram. Possible questions for discussion:

- “For the situations with no  $x$ , how did you decide what quantity to represent with a variable?” (Think about which amount is unknown but has a relationship to one or more other amounts in the story.)
- “Did any situations have the same diagrams? How can you tell from the story that the diagrams would be the same?” (Same number of equal parts, same amount for unequal parts, same amount for the total.)
- “How is the last situation different from the others?” (It’s the only one where 30 is the coefficient of  $x$  rather than the total.)
- “Why was it tough to draw a diagram for the last question?” (You would have to divide the diagram into 30 equal pieces.)

---

### Access for English Language Learners

*Conversing, Representing: MLR7 Compare and Connect.* Use this routine to support student understanding of the connections between tape diagrams and the situations they represent. Because all situations in this activity share the same quantities of 7, 9, and 30, students can compare how each situation affects how quantities appear in a tape diagram. Invite students to compare their diagrams with their partner. Display the following sentence frames: “One thing our diagrams have in common is ...” and “One thing that is different about our diagrams is ...”. This routine will help students identify, explain, and verbally respond to correspondences between context and mathematical representations.

*Design Principle(s): Optimize output (for comparison); Cultivate conversation*

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## 4.3 Situations, Diagrams, and Equations

### 10 minutes

This activity is a continuation of the previous one. Students match each situation from the previous activity with an equation, solve the equation by any method that makes sense to them, and interpret the meaning of the solution. Students are still using any method that makes sense to them to reason about a solution. In later lessons, a hanger diagram representation will be used to justify more efficient methods for solving. For example, when they are using tape diagrams, they could just say “I subtracted the 9 extra markers and then divided the remaining 21 markers by 7.” Later, when working with hanger diagrams, we can press them to say “I can subtract 9 from each side and then divide each side by 7.”

For each equation, monitor for a student using their diagram to reason about the solution and a student using the structure of the equation to reason about the solution.

### Addressing

- 7.EE.B.3

### Building Towards

- 7.EE.B.4.a

### Instructional Routines

- MLR3: Clarify, Critique, Correct

### Launch

Keep students in the same groups. 5 minutes to work individually or with a partner, followed by a whole-class discussion.

---

## Access for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation such as “To find the solution, first I \_\_\_ because...”, “I made this match because I noticed...”, “Why did you...?”, or “I agree/disagree because...”

*Supports accessibility for: Language; Social-emotional skills*

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### Student Task Statement

Each situation in the previous activity is represented by one of the equations.

- $7x + 9 = 30$
  - $30 = 9x + 7$
  - $30x + 7 = 9$
1. Match each situation to an equation.
  2. Find the solution to each equation. Use your diagrams to help you reason.
  3. What does each solution tell you about its situation?

### Student Response

1.
  - $7x + 9 = 30$ : Situations 1 (markers) and 4 (skating rink)
  - $9x + 7 = 30$ : Situations 2 (ribbons) and 3 (bricks)
  - $30x + 7 = 9$ : Situation 5 (pans of brownies)
2.
  - $7x + 9 = 30$ : 3
  - $9x + 7 = 30$ :  $2\frac{5}{9}$  or about 2.6
  - $30x + 7 = 9$ :  $\frac{1}{15}$
3.
  - a. There are 3 markers in each pack.
  - b. Each of the 9 pieces is about 2.6 feet long.
  - c. Each brick weighs about 2.6 pounds.
  - d. It costs \$3 to rent a pair of skates.
  - e. Each student receives  $\frac{1}{15}$  of a pan of brownies.

### Are You Ready for More?

While in New York City, is it a better deal for a group of friends to take a taxi or the subway to get from the Empire State Building to the Metropolitan Museum of Art? Explain your reasoning.

## Student Response

Answers vary. Sample response: If there are 4 people in the group of friends, then they should take the subway. The subway fare for 4 people is  $4(2.75) = 11$ . The taxi fare is \$2.5 initially and \$2.5 for each mile they drive. The distance between the 2 landmarks is 2.9 miles, so the total taxi fare is  $2.5 + 2.9 \cdot 2.5 = 9.75$ . After paying a tip to the taxi driver, it costs more than the subway.

## Activity Synthesis

For each equation, ask one student who reasoned with the diagram and one who reasoned only about the equation to explain their solutions. Display the diagram and the equation side by side, drawing connections between the two representations. If no students bring up one or both of these approaches, demonstrate maneuvers on a diagram side by side with maneuvers on the corresponding equation. For example, "I subtracted the 9 extra markers and then divided the remaining 21 markers by 7," can be shown on a tape diagram and on a corresponding equation. It is not necessary to invoke the more abstract language of "doing the same thing to each side" of an equation yet.

---

### Access for English Language Learners

*Speaking, Representing: MLR3 Clarify, Critique, Correct.* Present an incorrect match of an equation with a situation and/or diagram (e.g.,  $30 = 9x + 7$  with Situation 1). Invite students to clarify and then critique the error (e.g., there are 7 packs of an unknown amount of markers in Situation 1; not 9 as illustrated in the equation). Press for details, if needed. For example, students can reference "markers" in Situation 1, not "x", as the unknown value represent by the variable  $x$ . This will help students reflect on the quantity of the unknown values that would be a reasonable solution and improve on their reasoning when matching an equation to a situation.

*Design Principle(s): Optimize output (for justification)*

---

## Lesson Synthesis

Display one of the situations from the lesson and its corresponding equation. Ask students to explain:

- "What does each number and letter in the equation represent in the situation?"
- "What is the reason for each operation (multiplication or addition) used in the equation?"
- "What is the solution to the equation? What does it mean to be a solution to an equation? What does the solution represent in the situation?"

## 4.4 Finding Solutions

Cool Down: 5 minutes

## Addressing

- 7.EE.B.3
- 7.EE.B.4.a

### Student Task Statement

Here is a diagram and its corresponding equation. Find the solution to the equation and explain your reasoning.



$$4x + 17 = 23$$

### Student Response

$x = 1\frac{1}{2}$ . Explanations vary.

## Student Lesson Summary

Many situations can be represented by equations. Writing an equation to represent a situation can help us express how quantities in the situation are related to each other, and can help us reason about unknown quantities whose value we want to know. Here are three situations:

1. An architect is drafting plans for a new supermarket. There will be a space 144 inches long for rows of nested shopping carts. The first cart is 34 inches long and each nested cart adds another 10 inches. The architect wants to know how many shopping carts will fit in each row.
2. A bakery buys a large bag of sugar that has 34 cups. They use 10 cups to make some cookies. Then they use the rest of the bag to make 144 giant muffins. Their customers want to know how much sugar is in each muffin.
3. Kiran is trying to save \$144 to buy a new guitar. He has \$34 and is going to save \$10 a week from money he earns mowing lawns. He wants to know how many weeks it will take him to have enough money to buy the guitar.

We see the same three numbers in the situations: 10, 34, and 144. How could we represent each situation with an equation?

In the first situation, there is one shopping cart with length 34 and then an unknown number of carts with length 10. Similarly, Kiran has 34 dollars saved and then will save 10 each week for an unknown number of weeks. Both situations have one part of 34 and then equal parts of size 10 that all add together to 144. Their equation is  $34 + 10x = 144$ .

Since it takes 11 groups of 10 to get from 34 to 144, the value of  $x$  in these two situations is  $(144 - 34) \div 10$  or 11. There will be 11 nested shopping carts in each row, and it will take Kiran 11 weeks to raise the money for the guitar.

In the bakery situation, there is one part of 10 and then 144 equal parts of unknown size that all add together to 34. The equation is  $10 + 144x = 34$ . Since 24 is needed to get from 10 to 34, the value of  $x$  is  $(34 - 10) \div 144$  or  $\frac{1}{6}$ . There is  $\frac{1}{6}$  cup of sugar in each giant muffin.

# Lesson 4 Practice Problems

## Problem 1

### Statement

Draw a square with side length 7 cm.

- Predict the perimeter and the length of the diagonal of the square.
- Measure the perimeter and the length of the diagonal of the square.
- Describe how close the predictions and measurements are.

### Solution

- Perimeter: 28 cm. Length of diagonal: Approximately 9.9 cm.
- Answers vary.
- Answers vary.

(From Unit 3, Lesson 1.)

## Problem 2

### Statement

Find the products.

- $(100) \cdot (-0.09)$
- $(-7) \cdot (-1.1)$
- $(-7.3) \cdot (5)$
- $(-0.2) \cdot (-0.3)$

### Solution

- 9
- 7.7
- 36.5
- 0.06

(From Unit 5, Lesson 9.)

### Problem 3

#### Statement

Here are three stories:

- A family buys 6 tickets to a show. They also pay a \$3 parking fee. They spend \$27 to see the show.
- Diego has 27 ounces of juice. He pours equal amounts for each of his 3 friends and has 6 ounces left for himself.
- Jada works for 6 hours preparing for the art fair. She spends 3 hours on a sculpture and then paints 27 picture frames.

Here are three equations:

- $3x + 6 = 27$
- $6x + 3 = 27$
- $27x + 3 = 6$

- Decide which equation represents each story. What does  $x$  represent in each equation?
- Find the solution to each equation. Explain or show your reasoning.
- What does each solution tell you about its situation?

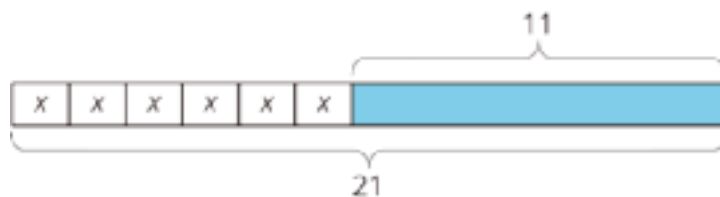
#### Solution

- Tickets to the show:  $6x + 3 = 27$ ,  $x$  represents the cost of a ticket. Diego's juice:  $3x + 6 = 27$ ,  $x$  represents the number of ounces of juice he gave each friend. The art fair:  $27x + 3 = 6$ ,  $x$  represents the number of hours spent on each picture frame.
- $6x + 3 = 27$ :  $x = 4$ .  $3x + 6 = 27$ :  $x = 7$ .  $27x + 3 = 6$ :  $x = \frac{1}{9}$ . Explanations vary.
- Each ticket to the show cost \$4. Diego gave each friend 7 ounces of juice. Jada spent  $\frac{1}{9}$  of an hour painting each picture frame.

### Problem 4

#### Statement

Here is a diagram and its corresponding equation. Find the solution to the equation and explain your reasoning.



$$6x + 11 = 21$$

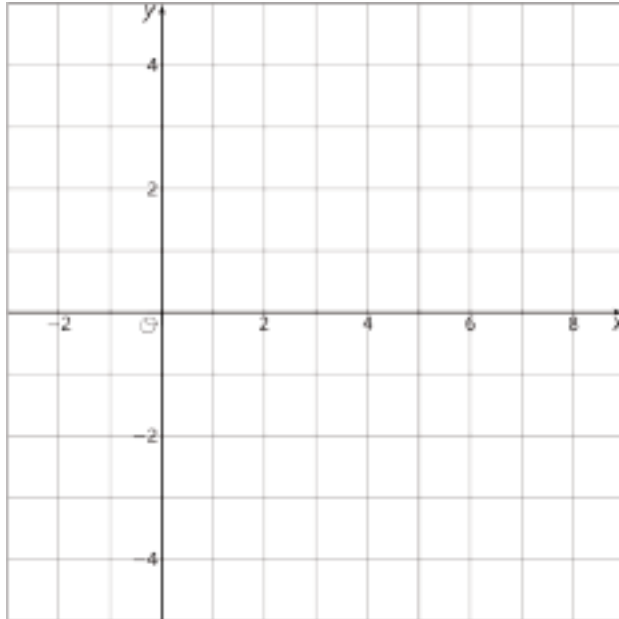
#### Solution

$x = \frac{10}{6}$  (or equivalent). Explanations vary.

## Problem 5

### Statement

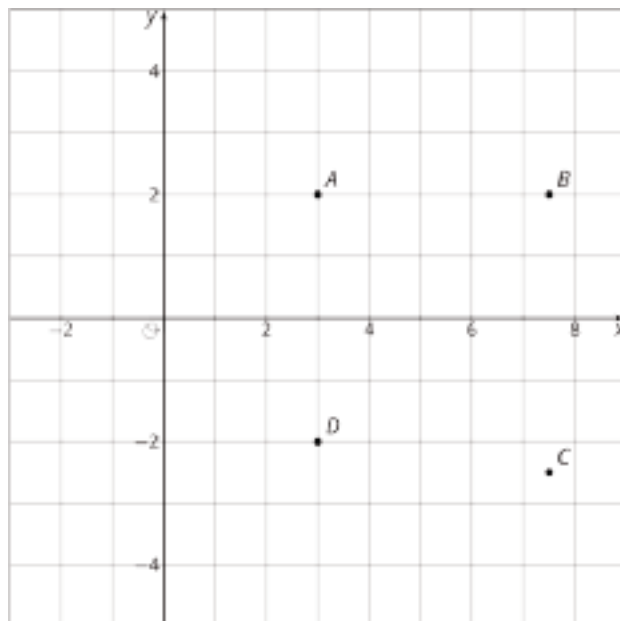
- a. Plot these points on the coordinate plane:  
 $A = (3, 2)$ ,  $B = (7.5, 2)$ ,  $C = (7.5, -2.5)$ ,  $D = (3, -2)$



- b. What is the vertical difference between  $D$  and  $A$ ?
- c. Write an expression that represents the vertical distance between  $B$  and  $C$ .

### Solution

a.



b. The vertical difference between  $D$  and  $A$  is -4 units.

c. An expression for the vertical distance between  $B$  and  $C$  is  $|2 - (-2.5)|$ .

(From Unit 5, Lesson 7.)

# Lesson 5: Reasoning about Equations and Tape Diagrams (Part 2)

## Goals

- Coordinate tape diagrams, equations of the form  $p(x + q) = r$ , and verbal descriptions of the situations.
- Explain (orally and in writing) how to use a tape diagram to determine the value of an unknown quantity in an equation of the form  $p(x + q) = r$ .
- Interpret (in writing) the solution to an equation in the context of the situation it represents.

## Learning Targets

- I can draw a tape diagram to represent a situation where there is more than one copy of the same sum and explain what the parts of the diagram represent.
- I can find a solution to an equation by reasoning about a tape diagram or about what value would make the equation true.

## Lesson Narrative

This lesson parallels the previous one, except the focus is on situations that lead to equations of the form  $p(x + q) = r$ . Tape diagrams are used to help students understand why these situations can be represented with equations of this form, and to help them reason about solving equations of this form. Students also attend to the meaning of the equation's solution in the context (MP2). Note that we are not generalizing solution methods yet; just using diagrams as a tool to reason about solving equations.

## Alignments

### Addressing

- 7.EE.B.3: Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. \$
- 7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
- 7.EE.B.4.a: Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

## Building Towards

- 7.EE.B.4.a: Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

## Instructional Routines

- Algebra Talk
- MLR1: Stronger and Clearer Each Time
- MLR2: Collect and Display
- MLR8: Discussion Supports

## Student Learning Goals

Let's use tape diagrams to help answer questions about situations where the equation has parentheses.

# 5.1 Algebra Talk: Seeing Structure

## Warm Up: 10 minutes

This warm-up parallels the one in the previous lesson. The purpose of this Algebra Talk is to elicit strategies and understandings students have for solving equations. These understandings help students develop fluency and will be helpful later in this unit when students will need to be able to come up with ways to solve equations of this form. While four equations are given, it may not be possible to share every strategy. Consider gathering only two or three different strategies per problem, saving most of the time for the final question.

Students should understand the meaning of *solution to an equation* from grade 6 work as well as from work earlier in this unit, but this is a good opportunity to re-emphasize the idea.

In this string of equations, each equation has the same solution. Digging into why this is the case requires noticing and using the structure of the equations (MP7). Noticing and using the structure of an equation is an important part of fluency in solving equations.

## Addressing

- 7.EE.B.4
- 7.EE.B.4.a

## Instructional Routines

- Algebra Talk
- MLR8: Discussion Supports

## Launch

Display one equation at a time. Give students 30 seconds of quiet think time for each equation and ask them to give a signal when they have an answer and a strategy. Keep all equations displayed throughout the talk. Follow with a whole-class discussion.

---

### Access for Students with Disabilities

*Representation: Internalize Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.

*Supports accessibility for: Memory; Organization*

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### Student Task Statement

Solve each equation mentally.

$$x - 1 = 5$$

$$2(x - 1) = 10$$

$$3(x - 1) = 15$$

$$500 = 100(x - 1)$$

### Student Response

- 6 is the solution because  $6 - 1 = 5$ .
- 6. Possible strategies: Trial and error to arrive at  $2(6 - 1) = 10$ , noticing that the equation is 2 times something is 10, so the something must be a 5, applying the distributive property to get  $2x - 2 = 10$ , and reasoning from there.
- 6
- 6

$x = 6$  is a solution to each equation because in each one,  $x - 1$  has to equal 5.

### Activity Synthesis

This discussion may go quickly, because students are likely to recognize similarities between this equation string and the one in the previous day's warm-up.

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- "Who can restate \_\_\_'s reasoning in a different way?"
- "Did anyone have the same strategy but would explain it differently?"

- “Did anyone solve the equation in a different way?”
- “Does anyone want to add on to \_\_\_\_’s strategy?”
- “Do you agree or disagree? Why?”

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### Access for English Language Learners

*Speaking: MLR8 Discussion Supports.*: Display sentence frames to support students when they explain their strategy. For example, “First, I \_\_\_\_ because . . .” or “I noticed \_\_\_\_ so I . . .” Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

*Design Principle(s): Optimize output (for explanation)*

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## 5.2 More Situations and Diagrams

15 minutes (there is a digital version of this activity)

The purpose of this activity is to work toward showing students that some situations can be represented by an equation of the form  $p(x + q) = r$  (or equivalent). In this activity, students are simply tasked with drawing a tape diagram to represent each situation. In the following activity, they will work with corresponding equations.

For each question, monitor for one student with a correct diagram. Press students to explain what any variables used to label the diagram represent in the situation.

### Building Towards

- 7.EE.B.4.a

### Instructional Routines

- MLR1: Stronger and Clearer Each Time

### Launch

Ensure students understand that the work of this task is to draw a tape diagram to represent each situation. There is no requirement to write an equation or solve a problem yet.

Arrange students in groups of 2. Give 5–10 minutes to work individually or with their partner, followed by a whole-class discussion.

---

## Access for Students with Disabilities

*Action and Expression: Develop Expression and Communication.* Maintain a display of important terms, vocabulary, and examples. During the launch, take time to review examples of drawing a tape diagram based on situations from previous lessons that students will need to access for this activity. Consider providing step-by-step directions that generalize the process using student input and ideas.

*Supports accessibility for: Memory; Language*

---

### Student Task Statement

Draw a tape diagram to represent each situation. For some of the situations, you need to decide what to represent with a variable.

1. Each of 5 gift bags contains  $x$  pencils. Tyler adds 3 more pencils to each bag. Altogether, the gift bags contain 20 pencils.
2. Noah drew an equilateral triangle with sides of length 5 inches. He wants to increase the length of each side by  $x$  inches so the triangle is still equilateral and has a perimeter of 20 inches.
3. An art class charges each student \$3 to attend plus a fee for supplies. Today, \$20 was collected for the 5 students attending the class.
4. Elena ran 20 miles this week, which was three times as far as Clare ran this week. Clare ran 5 more miles this week than she did last week.

### Student Response

Answers vary. Sample diagrams:

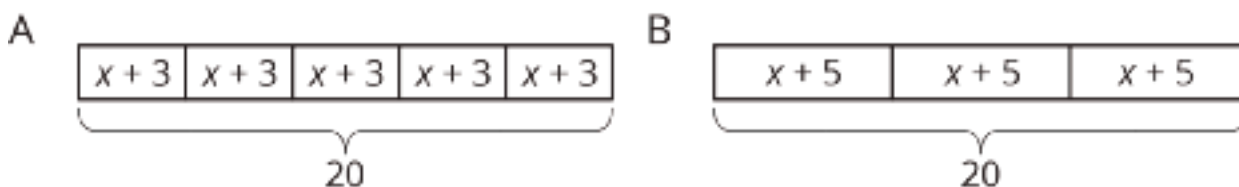


Diagram A corresponds to situations 1 and 3. Diagram B corresponds to situations 2 and 4.

### Activity Synthesis

Select one student for each situation to present their correct diagram. Ensure that students explain the meaning of any variables used to label their diagram. Possible questions for discussion:

- “For the situations with no  $x$ , how did you decide what quantity to represent with variable?” (Think about which amount is unknown but has a relationship to one or more other amounts in the story.)

- “What does the variable you used to label the diagram represent in the story?”
  - “Did any situations have the same diagrams? How can you tell from the story that the diagrams would be the same?” (Same number of equal parts, same amount for the total.)
- 

### Access for English Language Learners

*Speaking, Representing, Reading: MLR1 Stronger and Clearer Each Time.* Ask students to explain to a partner how they created the tape diagram to represent the situation “An art class charges each student \$3 to attend plus a fee for supplies. Today, \$20 was collected for the 5 students attending the class.” Ask listeners to press for details in the arrangement of the grouped quantities (e.g., “Explain how you chose what values go in each box.”). When roles are switched, listeners can press for details in what “ $x$ ” represents in the diagram. Allow students to revise their diagrams, if necessary, based on the feedback they received from their partner. Once their revision is complete, invite students to turn to a new partner to explain their revised diagram. This will help students productively engage in discussion as they make connections between written situations and visual diagrams.

*Design Principle(s): Optimize output (for explanation); Cultivate conversation*

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## 5.3 More Situations, Diagrams, and Equations

### 10 minutes

This activity is a continuation of the previous one. Students match each situation from the previous activity with an equation, solve the equation by any method that makes sense to them, and interpret the meaning of the solution. Students are still using any method that makes sense to them to reason about a solution. In later lessons, a hanger diagram representation will be used to justify more efficient methods for solving.

For each equation, monitor for a student using their diagram to reason about the solution and a student using the structure of the equation to reason about the solution.

### Addressing

- 7.EE.B.3

### Building Towards

- 7.EE.B.4.a

### Instructional Routines

- MLR2: Collect and Display

### Launch

Keep students in the same groups. 5 minutes to work individually or with a partner, followed by a whole-class discussion.

---

## Access for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation such as “To find the solution, first, I \_\_\_\_ because...”, “I made this match because I noticed...”, “Why did you...?”, or “I agree/disagree because...”

*Supports accessibility for: Language; Social-emotional skills*

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## Access for English Language Learners

*Speaking, Representing: MLR2 Collect and Display.* As students share their ideas about how the equations match the situations, listen for and collect students’ description of the situation (e.g., “5 gift bags,  $x$  pencils, adds 3 more, 20 pencils”) with the corresponding equation. Remind students to borrow language from the displayed examples while describing what each solution tells about the situation, after the matching is complete. This will help students make connection between language, diagrams, and equations.

*Design Principle(s): Support sense-making; Maximize meta-awareness*

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### Student Task Statement

Each situation in the previous activity is represented by one of the equations.

- $(x + 3) \cdot 5 = 20$

- $3(x + 5) = 20$

1. Match each situation to an equation.
2. Find the solution to each equation. Use your diagrams to help you reason.
3. What does each solution tell you about its situation?

### Student Response

1.
  - $(x + 3) \cdot 5 = 20$ : Situations 1 (gift bags) and 3 (art class)
  - $3(x + 5) = 20$ : Situations 2 (triangle perimeter) and 4 (miles run)
2.
  - $(x + 3) \cdot 5 = 20$ :  $x = 1$
  - $3(x + 5) = 20$ :  $x = 1\frac{2}{3}$
3.
  - a. There was originally one pencil in each bag.
  - b. Noah increased the length of each side by  $1\frac{2}{3}$  inches.

c. The fee for supplies is \$1.

d. Clare ran  $1\frac{2}{3}$  miles last week.

### Are You Ready for More?

Han, his sister, his dad, and his grandmother step onto a crowded bus with only 3 open seats for a 42-minute ride. They decide Han's grandmother should sit for the entire ride. Han, his sister, and his dad take turns sitting in the remaining two seats, and Han's dad sits 1.5 times as long as both Han and his sister. How many minutes did each one spend sitting?

### Student Response

Han's grandmother: 42, Han's dad: 36, Han: 24, Han's sister: 24

### Activity Synthesis

For each equation, ask one student who reasoned with the diagram and one who reasoned only about the equation to explain their solutions. Display the diagram and the equation side by side, drawing connections between the two representations. If no students bring up one or both of these approaches, demonstrate the maneuvers on a diagram side by side with the maneuvers on the corresponding equation. For example, "I divided the number of gift bags by 5, leaving me with 4 pencils per gift bag. Since Tyler added 3 pencils to each gift bag, there must have been 1 pencil in each gift bag to start," can be shown on a tape diagram and on a corresponding equation. It is not necessary to invoke the more abstract language of "doing the same thing to each side" of an equation yet.

## Lesson Synthesis

Display one of the situations from the lesson and its corresponding equation. Ask students to explain:

- "What does each number and letter in the equation represent in the situation?"
- "What is the reason for each operation (multiplication or addition) used in the equation?"
- "What is the solution to the equation? What does it mean to be a solution to an equation? What does the solution represent in the situation?"

## 5.4 More Finding Solutions

Cool Down: 5 minutes

### Addressing

- 7.EE.B.3

### Building Towards

- 7.EE.B.4.a

## Student Task Statement

Here is a diagram and its corresponding equation. Find the solution to the equation and explain your reasoning.



$$4(x + 7) = 38$$

## Student Response

$x = 2\frac{1}{2}$ . Sample reasoning: the tape diagram is in 4 equal pieces, each of which represents  $\frac{38}{4}$  (or  $9\frac{1}{2}$ ).  $x + 7 = 9\frac{1}{2}$ , so  $x$  must be  $2\frac{1}{2}$ .

## Student Lesson Summary

Equations with parentheses can represent a variety of situations.

1. Lin volunteers at a hospital and is preparing toy baskets for children who are patients. She adds 2 items to each basket, after which the supervisor's list shows that 140 toys have been packed into a group of 10 baskets. Lin wants to know how many toys were in each basket before she added the items.
2. A large store has the same number of workers on each of 2 teams to handle different shifts. They decide to add 10 workers to each team, bringing the total number of workers to 140. An executive at the company that runs this chain of stores wants to know how many employees were in each team before the increase.

Each bag in the first story has an unknown number of toys,  $x$ , that is increased by 2. Then ten groups of  $x + 2$  give a total of 140 toys. An equation representing this situation is  $10(x + 2) = 140$ . Since 10 times a number is 140, that number is 14, which is the total number of items in each bag. Before Lin added the 2 items there were  $14 - 2$  or 12 toys in each bag.

The executive in the second story knows that the size of each team of  $y$  employees has been increased by 10. There are now 2 teams of  $y + 10$  each. An equation representing this situation is  $2(y + 10) = 140$ . Since 2 times an amount is 140, that amount is 70, which is the new size of each team. The value of  $y$  is  $70 - 10$  or 60. There were 60 employees on each team before the increase.

# Lesson 5 Practice Problems

## Problem 1

### Statement

Here are some prices customers paid for different items at a farmer's market. Find the cost for 1 pound of each item.

- a. \$5 for 4 pounds of apples
- b. \$3.50 for  $\frac{1}{2}$  pound of cheese
- c. \$8.25 for  $1\frac{1}{2}$  pounds of coffee beans
- d. \$6.75 for  $\frac{3}{4}$  pounds of fudge
- e. \$5.50 for a  $6\frac{1}{4}$  pound pumpkin

### Solution

- a. \$1.25
- b. \$7
- c. \$5.50
- d. \$9
- e. \$0.88

(From Unit 4, Lesson 2.)

## Problem 2

### Statement

Find the products.

- a.  $\frac{2}{3} \cdot \left(\frac{-4}{5}\right)$
- b.  $\left(\frac{-5}{7}\right) \cdot \left(\frac{7}{5}\right)$
- c.  $\left(\frac{-2}{39}\right) \cdot 39$
- d.  $\left(\frac{2}{5}\right) \cdot \left(\frac{-3}{4}\right)$

### Solution

- a.  $\frac{-8}{15}$

- b. -1
- c. -2
- d.  $\frac{-3}{10}$  or equivalent

(From Unit 5, Lesson 9.)

## Problem 3

### Statement

Here are two stories:

- A family buys 6 tickets to a show. They also *each* spend \$3 on a snack. They spend \$24 on the show.
- Diego has 24 ounces of juice. He pours equal amounts for each of his 3 friends, and then adds 6 more ounces for each.

Here are two equations:

- $3(x + 6) = 24$
- $6(x + 3) = 24$

- a. Which equation represents which story?
- b. What does  $x$  represent in each equation?
- c. Find the solution to each equation. Explain or show your reasoning.
- d. What does each solution tell you about its situation?

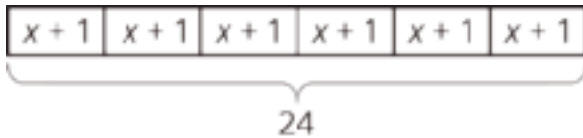
### Solution

- a. Family at the show:  $6(x + 3) = 24$ , Diego's juice:  $3(x + 6) = 24$
- b. Family at the show:  $x$  represents the cost of a ticket. Diego's juice:  $x$  represents the number of ounces of juice Diego originally poured for each friend.
- c.  $6(x + 3) = 24$ :  $x = 1$ ,  $3(x + 6) = 24$ :  $x = 2$
- d. Tickets to the show cost \$1. Diego originally poured 2 ounces of juice.

## Problem 4

### Statement

Here is a diagram and its corresponding equation. Find the solution to the equation and explain your reasoning.



$$6(x + 1) = 24$$

## Solution

3. Sample response: in the tape diagram, there are six units of  $x + 1$  that make 24, so  $x + 1$  must be  $24 \div 6$ , which is 4. Since  $x + 1 = 4$ ,  $x = 3$ .

## Problem 5

### Statement

Below is a set of data about temperatures. The *range* of a set of data is the distance between the lowest and highest value in the set. What is the range of these temperatures?

$9^{\circ}\text{C}$ ,  $-3^{\circ}\text{C}$ ,  $22^{\circ}\text{C}$ ,  $-5^{\circ}\text{C}$ ,  $11^{\circ}\text{C}$ ,  $15^{\circ}\text{C}$

## Solution

27

(From Unit 5, Lesson 7.)

## Problem 6

### Statement

A store is having a 25% off sale on all shirts. Show two different ways to calculate the sale price for a shirt that normally costs \$24.

## Solution

Answers vary. Possible strategies:

- $(0.25) \cdot 24 = 6$ , and  $24 - 6 = 18$  (find 25% of \$24 and subtract this from \$24)
- $1 - 0.25 = 0.75$ , and  $(0.75) \cdot 24 = 18$  (find 75% of \$24)
- $24 \div 4 = 6$ , and  $3 \cdot 6 = 18$  (find 25% of \$24 and multiply this by 3)

(From Unit 4, Lesson 11.)

# Lesson 6: Distinguishing between Two Types of Situations

## Goals

- Categorize equations of the forms  $px + q = r$  and  $p(x + q) = r$ , and describe (orally) the categories.
- Interpret a verbal description of a situation (in written language), and write an equation of the form  $px + q = r$  or  $p(x + q) = r$  to represent it.

## Learning Targets

- I understand the similarities and differences between the two main types of equations we are studying in this unit.
- When I have a situation or a tape diagram, I can represent it with an equation.

## Lesson Narrative

The purpose of this lesson is to distinguish equations of the form  $px + q = r$  and  $p(x + q) = r$ . Corresponding tape diagrams are used as tools in this work, along with situations that these equations can represent. First, students sort equations into categories of their choosing. The main categories to highlight distinguish between the two main types of equations being studied. Then, students consider two stories and corresponding diagrams and write equations to represent them. They use these representations to find an unknown value in the story.

## Alignments

### Addressing

- 7.EE.B.3: Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. \$

### Building Towards

- 7.EE.B.4.a: Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

### Instructional Routines

- MLR6: Three Reads
- MLR8: Discussion Supports

- Take Turns
- Think Pair Share
- Which One Doesn't Belong?

### Required Materials

Pre-printed slips, cut from copies of the **Instructional master**

### Required Preparation

Print and cut up copies of the Instructional master ahead of time. You will need 1 set for every 2 students. If possible, copy each complete set on a different color of paper, so that a stray slip can quickly be put back.

### Student Learning Goals

Let's think about equations with and without parentheses and the kinds of situations they describe.

## 6.1 Which One Doesn't Belong: Seeing Structure

### Warm Up: 5 minutes

This warm-up prompts students to compare four equations. It encourages students to explain their reasoning, hold mathematical conversations, and gives you the opportunity to hear how they use terminology and talk about characteristics of the equations in comparison to one another. To allow all students to access the activity, each equation has one obvious reason it does not belong. Encourage students to find reasons based on the structure of the equation (MP7). During the discussion, listen for important ideas and terminology.

### Building Towards

- 7.EE.B.4.a

### Instructional Routines

- Which One Doesn't Belong?

### Launch

Arrange students in groups of 2–4. Display the equations for all to see. Ask students to indicate when they have noticed one that does not belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their group. In their groups, tell each student to share their reasoning about why a particular question does not belong, and together, find at least one reason each question doesn't belong.

### Student Task Statement

Which equation doesn't belong?

$$4(x + 3) = 9$$

$$4 + 3x = 9$$

$$4 \cdot x + 12 = 9$$

$$9 = 12 + 4x$$

### Student Response

1.  $4(x + 3) = 9$  (only one with a side that is only the product of two expressions)
2.  $4 \cdot x + 12 = 9$  (only one that uses a dot instead of next-to for multiplication)
3.  $4 + 3x = 9$  (only one not equivalent to the others, possibly notice it's the only one with a positive solution)
4.  $9 = 12 + 4x$  (only one with only a number on the left side)

### Activity Synthesis

Ask each group to share one reason why a particular equation does not belong. Record and display the responses for all to see. After each response, ask the class whether they agree or disagree. Since there is no single correct answer to the question asking which one does not belong, attend to students' explanations and ensure the reasons given are correct. During the discussion, ask students to explain the meaning of any terminology they use, such as *coefficient* or *solution*. Also, press students on unsubstantiated claims.

## 6.2 Card Sort: Categories of Equations

15 minutes

The goal of this activity is for students to notice the structure of equations. Any way of sorting is fine, but the discussion should land on explaining how equations involving an expression like  $p(x + q)$  are different from ones that have an expression like  $px + q$ . Monitor for different ways groups choose to categorize the equations, but especially for categories that distinguish between these two types of expressions. As students work, encourage them to refine their descriptions of equations using more precise language and mathematical terms (MP6).

### Building Towards

- 7.EE.B.4.a

### Instructional Routines

- MLR8: Discussion Supports
- Take Turns

### Launch

Arrange students in groups of 2. Tell them that in this activity, they will sort some cards into categories of their choosing. When they sort the equations, they should work with their partner to come up with categories, and then take turns sorting each equation into one of their categories, explaining why they are doing so. If necessary, demonstrate this protocol before students start working.

Distribute one set of cards to each group of students. Give students 5 minutes to work with their partner, followed by a whole-class discussion.

### Student Task Statement

Your teacher will give you a set of cards that show equations. Sort the cards into 2 categories of your choosing. Be prepared to explain the meaning of your categories. Then, sort the cards into 2 categories in a different way. Be prepared to explain the meaning of your new categories.

### Student Response

Categories vary.

### Activity Synthesis

Select groups of students to share their categories and how they sorted their equations. You can choose as many different types of categories as time allows, but ensure that one set of categories distinguishes between equations that involved expressions of the form  $px + q$  vs  $p(x + q)$ . Attend to the language that students use to describe their categories and equations, giving them opportunities to describe their equations more precisely. Highlight the use of terms like *coefficient*, *sum*, *product*, *variable*, and *solution*.

---

### Access for English Language Learners

*Speaking: MLR8 Discussion Supports.* Use this routine to support whole-class discussion when students share the categories they created to sort their equations. After groups describe the set of categories that distinguish between equations that involved expressions of the form  $px + q$  vs.  $p(x + q)$ , call on students to restate and/or revoice their peers' descriptions using mathematical terms (e.g., *coefficient*, *sum*, *product*, *variable*, and *solution*). This will provide more students with an opportunity to produce language that describes the form and structure of the two equations.

*Design Principle(s): Support sense-making; Maximize meta-awareness*

---

## 6.3 Even More Situations, Diagrams, and Equations

15 minutes

This activity is an opportunity to put together the learning of the past several lessons: correspondences between tape diagrams, equations, and stories, and using representations to reason about a solution. The focus of this activity is still contrasting the two main types of equations that students encounter in this unit.

### Addressing

- 7.EE.B.3

## Building Towards

- 7.EE.B.4.a

## Instructional Routines

- MLR6: Three Reads
- Think Pair Share

## Launch

Keep students in the same groups. 5 minutes of quiet work time followed by sharing with a partner and a whole-class discussion.

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### Access for Students with Disabilities

*Representation: Internalize Comprehension.* Provide appropriate reading accommodations and supports to ensure students access to written directions, word problems, and other text-based content.

*Supports accessibility for: Language; Conceptual processing*

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### Access for English Language Learners

*Reading, Representing: MLR6 Three Reads.* Use this routine with the first story to support students' understanding of the situation and how to represent it with a tape diagram or equation. Use the first read to orient students to the situation (Lin and volunteers are hanging flyers at school). Use the second read to identify the important quantities (number of volunteers, number of flyers each). After the third read, ask students to brainstorm how the situation can be represented in a tape diagram or equation. This will help students make connections between equations and diagrams and understand the language of a situation that will distinguish which form of an equation would be an appropriate representation.

*Design Principle(s): Support sense-making; Maximize meta-awareness*

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## Student Task Statement



Story 1: Lin had 90 flyers to hang up around the school. She gave 12 flyers to each of three volunteers. Then she took the remaining flyers and divided them up equally between the three volunteers.

Story 2: Lin had 90 flyers to hang up around the school. After giving the same number of flyers to each of three volunteers, she had 12 left to hang up by herself.

1. Which diagram goes with which story? Be prepared to explain your reasoning.
2. In each diagram, what part of the story does the variable represent?
3. Write an equation corresponding to each story. If you get stuck, use the diagram.
4. Find the value of the variable in the story.

### Student Response

1. Diagram A goes with story 2 and diagram B goes with story 1.
2. In diagram A,  $x$  represents the number of flyers she gave to each volunteer. In diagram B,  $y$  represents the remaining flyers she gave to each volunteer (after giving each of them 12 to start).
3. Story 1:  $3(y + 12) = 90$  or equivalent. Story 2:  $3x + 12 = 90$  or equivalent.
4. Story 1:  $y = 18$ . Story 2:  $x = 26$ .

### Are You Ready for More?

A tutor is starting a business. In the first year, they start with 5 clients and charge \$10 per week for an hour of tutoring with each client. For each year following, they double the number of clients and the number of hours each week. Each new client will be charged 150% of the charges of the clients from the previous year.

1. Organize the weekly earnings for each year in a table.
2. Assuming a full-time week is 40 hours per week, how many years will it take to reach full time and how many new clients will be taken on that year?
3. After reaching full time, what is the tutor's annual salary if they take 2 weeks of vacation?
4. Is there another business model you'd recommend for the tutor? Explain your reasoning.

## Student Response

1.

Year	Existing Charges ( $q$ )	Rate on New Clients ( $p$ )	New Clients ( $x$ )	Weekly Rate ( $px + q$ )
1	0	10	5	50
2	50	15	10	200
3	200	22.5	20	700
4	700	33.75	5	868.75

2. 4 years, 5 clients

3. 43,437.50

4. Answers vary. Sample response: The tutor could raise rates on existing clients by a little bit each year.

## Activity Synthesis

For each story, select 1 or more groups to present the matching diagram, their equation, and their solution method. Possible questions to ask:

- “How were the diagrams alike? How were they different?” (They have the same numbers and a letter. One has 3 equal groups and an extra bit, the other just has 3 equal groups, but each group is a sum.)
- “How were the stories alike? How were they different?” (They were both about distributing 90 flyers. In one story, Lin makes a series of moves to each volunteer. In the other story, she gives each volunteer the same amount, but then there are some left over.)
- “What parts of the story made you think that one diagram represented it?”
- “Explain how you reasoned about the story, diagram, or equation to find the value of the variable.”

## Lesson Synthesis

Display the two equations from the last activity for all to see:

$$3x + 12 = 90$$

$$3(y + 12) = 90$$

Tell students, “These equations have lots of things in common. They each have a 3, a letter, a 12, a 90, an equal sign, multiplication, and addition. Explain how these equations are *different*.” Ask

students to think about this question quietly for a moment and share with a partner, then ask a few students to share with the whole class.

Highlight any responses that speak in general terms about the structure of the equations. For example, one equation is the sum of a product and a number and the other is the product of a number and a sum. Alternatively, if we evaluated one expression for a value of the variable, we would multiply it by 3 first and then add 12. For the other, we would add 12 first and then multiply by 3. One has three equal groups and an extra bit, and the other just has 3 equal groups, but the groups are each the result of adding 12 to an unknown.

## 6.4 After School Tutoring

Cool Down: 5 minutes

### Addressing

- 7.EE.B.3

### Building Towards

- 7.EE.B.4.a

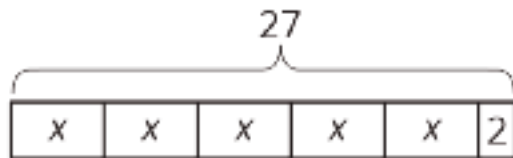
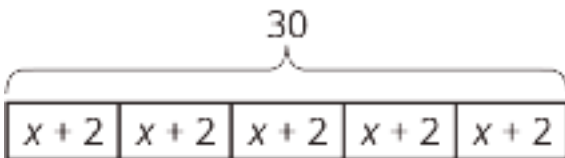
### Student Task Statement

Write an equation for each story. Then, find the number of problems originally assigned by each teacher. If you get stuck, try drawing a diagram to represent the story.

1. Five students came for after-school tutoring. Lin's teacher assigned each of them the same number of problems to complete. Then he assigned each student 2 more problems. 30 problems were assigned in all.
2. Five students came for after-school tutoring. Priya's teacher assigned each of them the same number of problems to complete. Then she assigned 2 more problems to one of the students. 27 problems were assigned in all.

### Student Response

1.  $5(x + 2) = 30$  (or equivalent), solution: 4
2.  $5x + 2 = 27$  (or equivalent), solution: 5



### Student Lesson Summary

In this unit, we encounter two main types of situations that can be represented with an equation. Here is an example of each type:

1. After adding 8 students to each of 6 same-sized teams, there were 72 students altogether.
2. After adding an 8-pound box of tennis rackets to a crate with 6 identical boxes of ping pong paddles, the crate weighed 72 pounds.

The first situation has all equal parts, since additions are made to *each* team. An equation that represents this situation is  $6(x + 8) = 72$ , where  $x$  represents the original number of students on each team. Eight students were added to each group, there are 6 groups, and there are a total of 72 students.

In the second situation, there are 6 equal parts added to one other part. An equation that represents this situation is  $6x + 8 = 72$ , where  $x$  represents the weight of a box of ping pong paddles, there are 6 boxes of ping pong paddles, there is an additional box that weighs 8 pounds, and the crate weighs 72 pounds altogether.

In the first situation, there were 6 equal groups, and 8 students added to each group.  
 $6(x + 8) = 72$ .

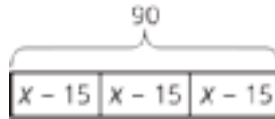
In the second situation, there were 6 equal groups, but 8 more pounds in addition to that.  
 $6x + 8 = 72$ .

# Lesson 6 Practice Problems

## Problem 1

### Statement

A school ordered 3 large boxes of board markers. After giving 15 markers to each of 3 teachers, there were 90 markers left. The diagram represents the situation. How many markers were originally in each box?



### Solution

45

(From Unit 6, Lesson 2.)

## Problem 2

### Statement

The diagram can be represented by the equation  $25 = 2 + 6x$ . Explain where you can see the 6 in the diagram.



### Solution

There are 6 equal parts labeled  $x$ .

(From Unit 6, Lesson 3.)

## Problem 3

### Statement

Match each equation to a story. (Two of the stories match the same equation.)

a.  $3(x + 5) = 17$

b.  $3x + 5 = 17$

c.  $5(x + 3) = 17$

d.  $5x + 3 = 17$

- a. Jada's teacher fills a travel bag with 5 copies of a textbook. The weight of the bag and books is 17 pounds. The empty travel bag weighs 3 pounds. How much does each book weigh?
- b. A piece of scenery for the school play is in the shape of a 5-foot-long rectangle. The designer decides to increase the length. There will be 3 identical rectangles with a total length of 17 feet. By how much did the designer increase the length of each rectangle?
- c. Elena spends \$17 and buys a \$3 book and a bookmark for each of her 5 cousins. How much does each bookmark cost?
- d. Noah packs up bags at the food pantry to deliver to families. He packs 5 bags that weigh a total of 17 pounds. Each bag contains 3 pounds of groceries and a packet of papers with health-related information. How much does each packet of papers weigh?
- e. Andre has 3 times as many pencils as Noah and 5 pens. He has 17 pens and pencils all together. How many pencils does Noah have?

## Solution

- a. D  
b. A  
c. C  
d. C  
e. B

## Problem 4

### Statement

Elena walked 20 minutes more than Lin. Jada walked twice as long as Elena. Jada walked for 90 minutes. The equation  $2(x + 20) = 90$  describes this situation. Match each expression with the statement in the story with the expression it represents.



A.  $x$

B.  $x + 20$

C.  $2(x + 20)$

D. 90

### Solution

○ A: 3

○ B: 2

○ C: 1

○ D: 1

1. The number of minutes that Jada walked

2. The number of minutes that Elena walked

3. The number of minutes that Lin walked

# Section: Solving Equations of the Form $px + q = r$ and $p(x + q) = r$ and Problems That Lead to Those Equations

## Lesson 7: Reasoning about Solving Equations (Part 1)

### Goals

- Compare and contrast (orally) different strategies for solving an equation of the form  $px + q = r$ .
- Explain (orally and in writing) how to use a balanced hanger diagram to solve an equation of the form  $px + q = r$ .
- Interpret a balanced hanger diagram, and write an equation of the form  $px + q = r$  to represent the relationship shown.

### Learning Targets

- I can explain how a balanced hanger and an equation represent the same situation.
- I can find an unknown weight on a hanger diagram and solve an equation that represents the diagram.
- I can write an equation that describes the weights on a balanced hanger.

### Lesson Narrative

The goal of this lesson is for students to understand that we can generally approach equations of the form  $px + q = r$  by subtracting  $q$  from each side and dividing each side by  $p$  (or multiplying by  $\frac{1}{p}$ ). Students only work with examples where  $p$ ,  $q$ , and  $r$  are specific numbers, not represented by letters. This is accomplished by considering what can be done to a hanger to keep it balanced.

Students are solving equations in this lesson in a different way than they did in the previous lessons. They are reasoning about things one could “do” to hangers while keeping them balanced alongside an equation that represents a hanger, so they are thinking about “doing” things to each side of an equation, rather than simply thinking “what value would make this equation true” or reasoning with situations or diagrams.

### Alignments

#### Addressing

- 7.EE.B.4.a: Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the

operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

### Building Towards

- 7.EE.B.4.a: Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

### Instructional Routines

- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Notice and Wonder
- Think Pair Share

### Student Learning Goals

Let's see how a balanced hanger is like an equation and how moving its weights is like solving the equation.

## 7.1 Hanger Diagrams

### Warm Up: 10 minutes

Students encounter and reason about a concrete situation, hangers with equal and unequal weights on each side. They then see diagrams of balanced and unbalanced hangers and think about what must be true and false about the situations. In subsequent activities, students will use the hanger diagrams to develop general strategies for solving equations.

### Building Towards

- 7.EE.B.4.a

### Instructional Routines

- Notice and Wonder

### Launch

Display the photo of socks and ask students, "What do you notice? What do you wonder?"



Give students 1 minute to think about the picture. Record their responses for all to see.

Things students may notice:

- There are two pink socks and two blue socks.
- The socks are clipped to either ends of two clothes hangers. The hangers are hanging from a rod.
- The hanger holding the pink socks is level; the hanger holding the blue socks is not level.

Things students may wonder:

- Why is the hanger holding the blue socks not level?
- Is something inside one of the blue socks to make it heavier than the other sock?
- What does this picture have to do with math?

Use the word “balanced” to describe the hanger on the left and “unbalanced” to describe the hanger on the right. Tell students that the hanger on the left is balanced because the two pink socks have an equal weight, and the hanger on the right is unbalanced because one blue sock is heavier than the other. Tell students that they will look at a diagram that is like the photo of socks, except with more abstract shapes, and they will reason about the weights of the shapes.

Give students 3 minutes of quiet work time followed by a whole-class discussion.

### **Student Task Statement**

In the two diagrams, all the triangles weigh the same and all the squares weigh the same.

For each diagram, come up with . . .

1. One thing that *must* be true
2. One thing that *could* be true
3. One thing that *cannot possibly* be true



### Student Response

Answers vary. Possible responses:

1. Triangle is heavier than square; 1 triangle weighs same as 3 squares and a circle.
2. Triangle weighs 32 ounces, square weighs 10 ounces, and circle weighs 2 ounces.
3. Triangle and square weigh the same.

### Activity Synthesis

Ask students to share some things that must be true, could be true, and cannot possibly be true about the diagrams. Ask them to explain their reasoning. The purpose of this discussion is to understand how the hanger diagrams work. When the diagram is balanced, there is equal weight on each side. For example, since diagram B is balanced, we know that one triangle weighs the same as three squares. When the diagram is unbalanced, one side is heavier than the other. For example, since diagram A is unbalanced, we know that one triangle is heavier than one square.

## 7.2 Hanger and Equation Matching

15 minutes

Students are presented with four hanger diagrams and are asked to match an equation to each hanger. They analyze relationships and find correspondences between the two representations. Then students use the diagrams and equations to find the unknown value in each diagram. This value is a solution of the equation.

### Building Towards

- 7.EE.B.4.a

## Instructional Routines

- MLR7: Compare and Connect
- Think Pair Share

## Launch

Display the diagrams and explain that each square labeled with a 1 weighs 1 unit, and each shape labeled with a letter has an unknown weight. Shapes labeled with the same letter have the same weight.

Arrange students in groups of 2. Give 5–10 minutes of quiet work time and time to share their responses with a partner, followed by a whole-class discussion.

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### Access for Students with Disabilities

*Representation: Internalize Comprehension.* Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, use the same color to highlight the variables in the hanger with the same variables in its corresponding equation.

*Supports accessibility for: Visual-spatial processing*

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### Student Task Statement

On each balanced hanger, figures with the same letter have the same weight.

1. Match each hanger to an equation. Complete the equation by writing  $x$ ,  $y$ ,  $z$ , or  $w$  in the empty box.

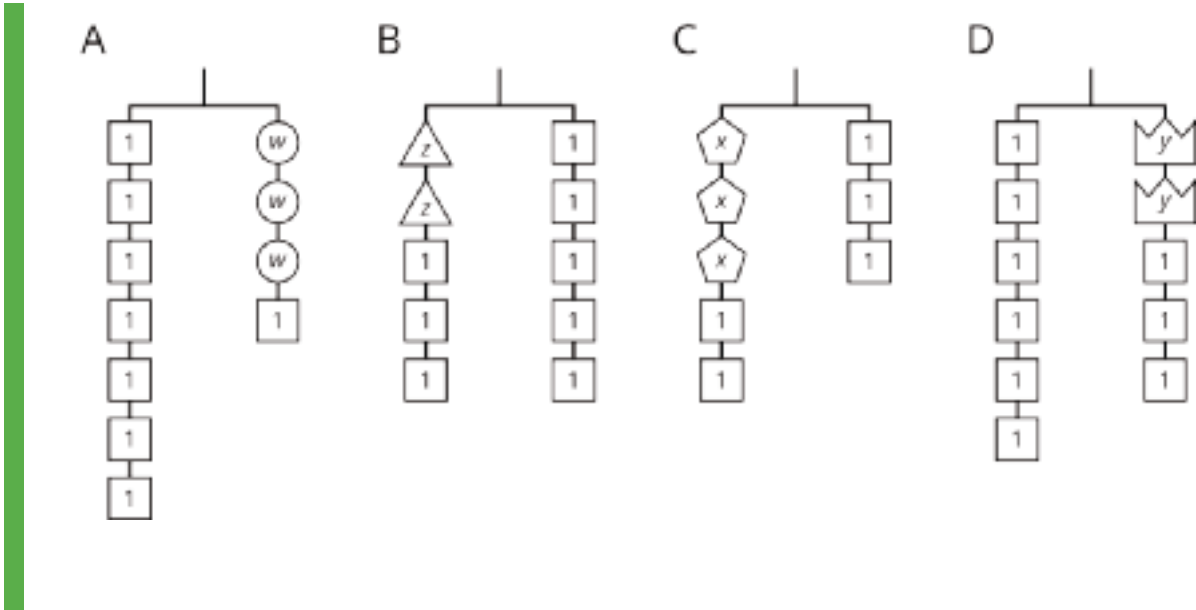
◦  $2\boxed{\phantom{x}} + 3 = 5$

◦  $3\boxed{\phantom{x}} + 2 = 3$

◦  $6 = 2\boxed{\phantom{x}} + 3$

◦  $7 = 3\boxed{\phantom{x}} + 1$

2. Find the solution to each equation. Use the hanger to explain what the solution means.



**Student Response**

1.
  - a.  $7 = 3w + 1$
  - b.  $2z + 3 = 5$
  - c.  $3x + 2 = 3$
  - d.  $6 = 2y + 3$
  
2.
  - a.  $w = 2$ , because 1 circle weighs the same as 2 squares.
  - b.  $z = 1$ , because 1 triangle weighs the same as 1 square.
  - c.  $x = \frac{1}{3}$ , because 3 pentagons weigh the same as 1 square.
  - d.  $y = \frac{3}{2}$ , because 2 crowns weigh the same as 3 squares.

**Activity Synthesis**

Demonstrate one of the hangers alongside its equation, removing the same number from each side, and then dividing each side by the same thing. Show how these moves correspond to doing the same thing to each side of the equation. (See the student lesson summary for an example of this.)

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### Access for English Language Learners

*Representing, Speaking: MLR7 Compare and Connect.* After students have discussed what the solutions to the four equations mean, invite students to compare approaches to finding unknown values through different representations (e.g., visual hanger, equation). Help students make connections between the representations by asking questions such as, “Where do you see division in both the hanger diagram and the equation?” This will help students reason about the ways to find unknown values in balanced hangers and to explain the meaning of a solution to an equation.

*Design Principle(s): Maximize meta-awareness; Cultivate conversation*

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## 7.3 Use Hangers to Understand Equation Solving

15 minutes

This activity continues the work of using a balanced hanger to develop strategies for solving equations. Students are presented with balanced hangers and are asked to write equations that represent them. They are then asked to explain how to use the diagrams, and then the equations, to reason about a solution. Students notice the structure of equations and diagrams and find correspondences between them and between solution strategies.

### Addressing

- 7.EE.B.4.a

### Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

### Launch

Draw students’ attention to the diagrams in the task statement. Ensure they notice that the hangers are balanced and that each object is labeled with its weight. Some weights are labeled with numbers but some are unknown, so they are labeled with a variable.

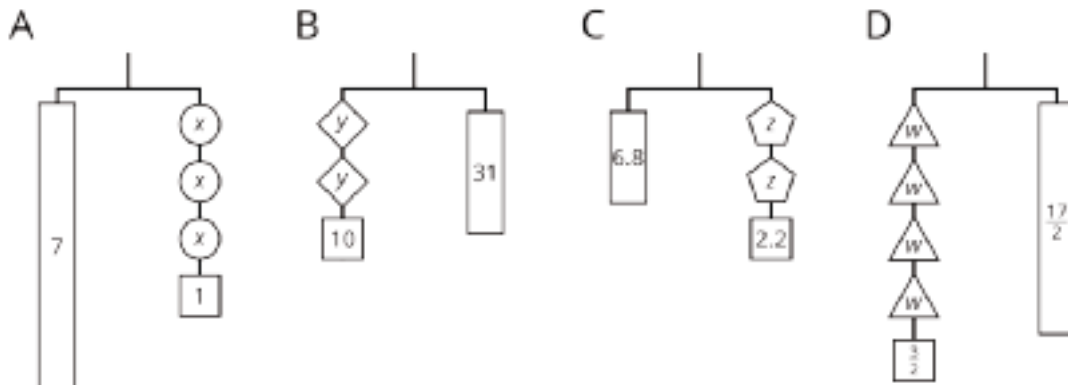
Keep students in the same groups. Give 5–10 minutes of quiet work time and time to share their responses with a partner, followed by a whole-class discussion.

### Student Task Statement

Here are some balanced hangers where each piece is labeled with its weight. For each diagram:

1. Write an equation.

- Explain how to figure out the weight of a piece labeled with a letter by reasoning about the diagram.
- Explain how to figure out the weight of a piece labeled with a letter by reasoning about the equation.



### Student Response

- A:  $7 = 3x + 1$  B:  $2y + 10 = 31$  C:  $6.8 = 2z + 2.2$  D:  $4w + \frac{3}{2} = \frac{17}{2}$
- Sample reasoning for diagram A: remove 1 unit of weight from each side of the hanger, leaving 6 units on the left and 3  $x$ 's on the right. Split each side into three equal groups, showing that  $x = 2$ .
- Sample reasoning for  $7 = 3x + 1$ : Subtract 1 from each side, leaving  $6 = 3x$ . Divide each side by 3, leaving  $2 = x$ .

### Activity Synthesis

Invite students to demonstrate, side by side, how they reasoned with both the diagram and the equation. For example, diagram A can be shown next to the equation  $7 = 3x + 1$ . Cross out a piece representing 1 from each side, and write  $7 - 1 = 3x + 1 - 1$ , followed by  $6 = 3x$ . Encircle 3 equal groups on each side, and write  $6 \div 3 = 3x \div 3$ , followed by  $2 = x$ . Repeat for as many diagrams as time allows. If diagrams A and B did not present much of a challenge for students, spend most of the time on diagrams C and D.

We want students to walk away with two things:

- An instant recognition of the structure of equations of the form  $px + q = r$  where  $p$ ,  $q$ , and  $r$  are specific, given numbers.
- A visual representation in their minds that can be used to support understanding of why for equations of this type, you can subtract  $q$  from each side and then divide each side by  $p$  to find the solution.

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### Access for Students with Disabilities

*Representation: Develop Language and Symbols.* Create a display of important terms and vocabulary. Include the following terms and maintain the display for reference throughout the unit: hanger diagram. For example, display an example of a balanced hanger. With class participation, create step-by-step instructions on how to write and solve an equation based on the hanger.

*Supports accessibility for: Memory; Language*

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### Access for English Language Learners

*Representing, Speaking: MLR8 Discussion Supports.* To invite participation in the whole-class discussion, use sentence frames to support students' explanations for how they found the weights by reasoning about the hangers and equations. For example, provide the frame "First, I \_\_\_ because . . .", "Then I \_\_\_ because . . ." Be sure to verbalize and amplify mathematical language in the students' explanations (e.g., "subtracting the constant", and "dividing by the coefficient"). This will help students explain their reasoning with the diagram and the equation.

*Design Principle(s): Optimize output (for explanation); Cultivate conversation*

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## Lesson Synthesis

Display the equation  $4x + 6 = 9.2$ . Ask students to work with their partner to draw a corresponding hanger diagram. Then, one partner solves by reasoning about the equation, the other solves by reasoning about the diagram. Ask students to compare the two strategies and discuss how they are alike and how they are different.

## 7.4 Solve the Equation

Cool Down: 5 minutes

### Addressing

- 7.EE.B.4.a

### Student Task Statement

Solve the equation. If you get stuck, try using a diagram.

$$5x + \frac{1}{4} = \frac{61}{4}$$

### Student Response

$$x = 3$$

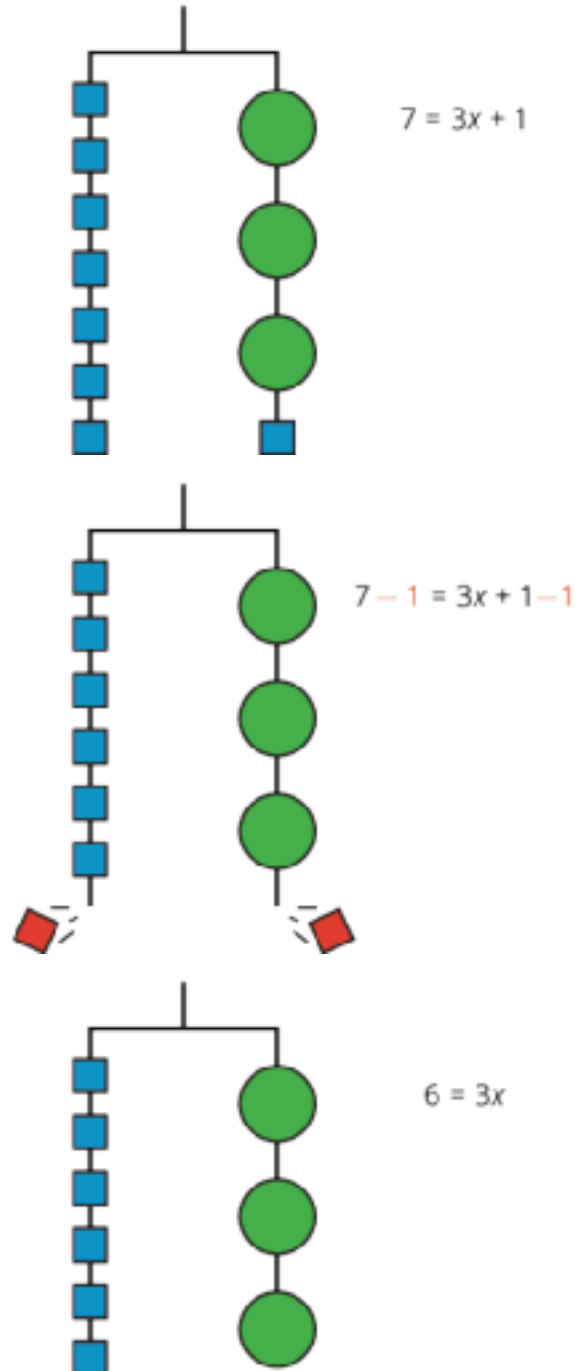
## Student Lesson Summary

In this lesson, we worked with two ways to show that two amounts are equal: a balanced hanger and an equation. We can use a balanced hanger to think about steps to finding an unknown amount in an associated equation.

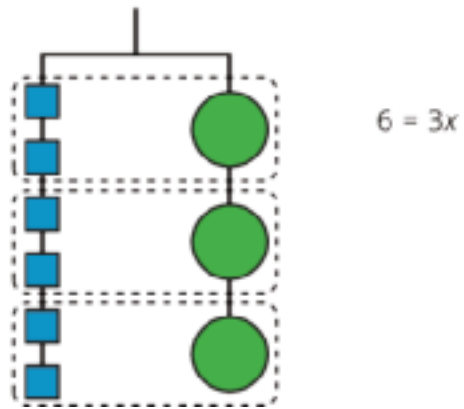
The hanger shows a total weight of 7 units on one side that is balanced with 3 equal, unknown weights and a 1-unit weight on the other. An equation that represents the relationship is  $7 = 3x + 1$ .

We can remove a weight of 1 unit from each side and the hanger will stay balanced. This is the same as subtracting 1 from each side of the equation.

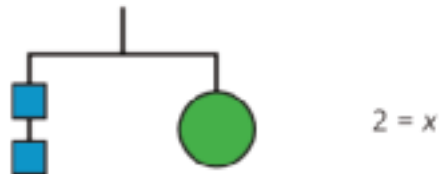
An equation for the new balanced hanger is  $6 = 3x$ .



So the hanger will balance with  $\frac{1}{3}$  of the weight on each side:  $\frac{1}{3} \cdot 6 = \frac{1}{3} \cdot 3x$ .



The two sides of the hanger balance with these weights: 6 1-unit weights on one side and 3 weights of unknown size on the other side.



Here is a concise way to write the steps above:

$$7 = 3x + 1$$

$$6 = 3x \quad \text{after subtracting 1 from each side}$$

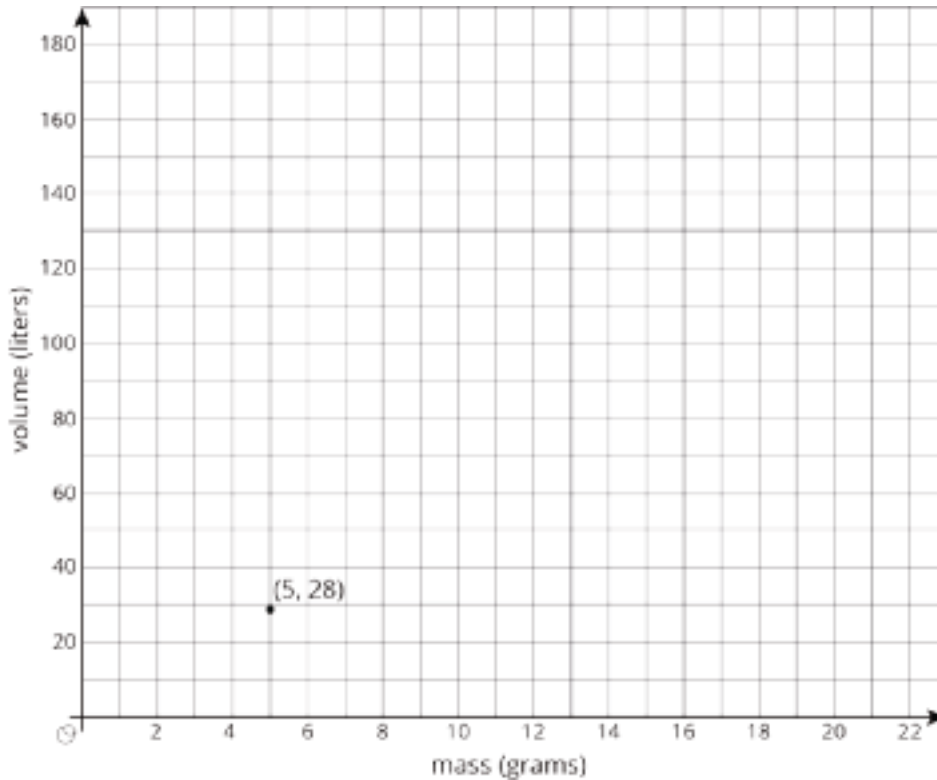
$$2 = x \quad \text{after multiplying each side by } \frac{1}{3}$$

# Lesson 7 Practice Problems

## Problem 1

### Statement

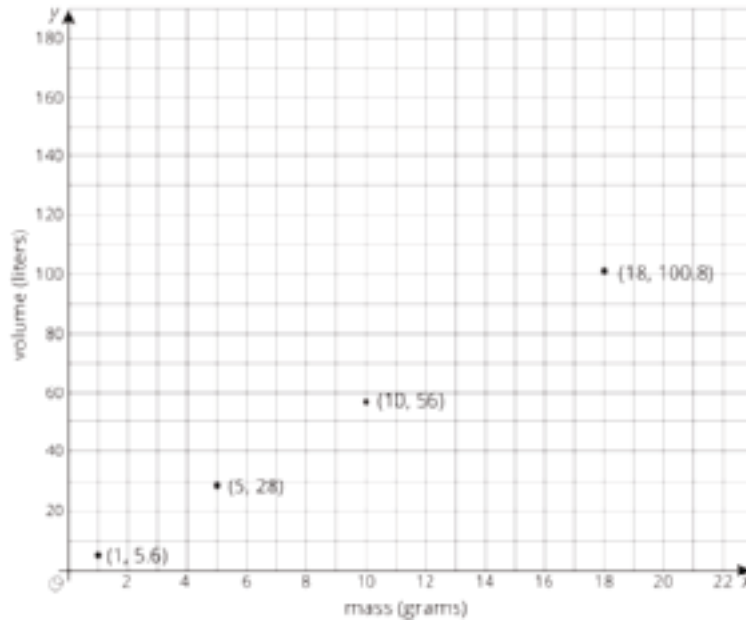
There is a proportional relationship between the volume of a sample of helium in liters and the mass of that sample in grams. If the mass of a sample is 5 grams, its volume is 28 liters.  $(5, 28)$  is shown on the graph below.



- What is the constant of proportionality in this relationship?
- In this situation, what is the meaning of the number you found in part a?
- Add at least three more points to the graph above, and label with their coordinates.
- Write an equation that shows the relationship between the mass of a sample of helium and its volume. Use  $m$  for mass and  $v$  for volume.

### Solution

- 5.6 liters per gram
- The volume of 1 gram of helium is 5.6 liters.
- Answers vary. Sample answer:



d.  $v = 5.6m$

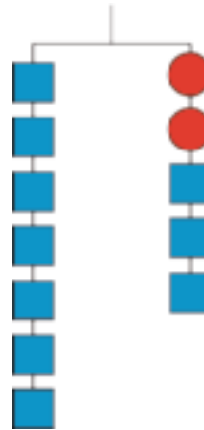
(From Unit 2, Lesson 11.)

## Problem 2

### Statement

Explain how the parts of the balanced hanger compare to the parts of the equation.

$$7 = 2x + 3$$



### Solution

Responses vary. Sample response: The fact that the hanger is balanced (equal weights on each side) matches the equal sign in the equation (equal expressions on each side). On the left of the hanger there are 7 equal weights. The equation shows 7 on the left side, so we can assume that each square represents 1 unit. The right side of the hanger has 2 circles of unknown weight, which matches the  $2x$  in the equation - twice an unknown amount. The right side of the hanger also has 3 squares of unit weight, which matches the 3 on the right side of the equation. The weight of the 2

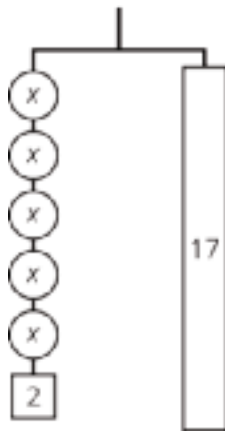
circles and 3 squares added together (the plus sign in the equation) is the same as (equal sign) the weight of the 7 squares.

### Problem 3

#### Statement

For the hanger below:

- Write an equation to represent the hanger.
- Draw more hangers to show each step you would take to find  $x$ . Explain your reasoning.
- Write an equation to describe each hanger you drew. Describe how each equation matches its hanger.



#### Solution

- $5x + 2 = 17$
- Subtract 2 from each side to get a hanger with 5 circles on the left and a rectangle labeled 15 on the right. Then divide both sides by 5 to get a hanger with one circle on the left and a rectangle labeled 3 on the right.
- $5x = 15$ ,  $x = 3$

# Lesson 8: Reasoning about Solving Equations (Part 2)

## Goals

- Compare and contrast (orally) different strategies for solving an equation of the form  $p(x + q) = r$ .
- Explain (orally and in writing) how to use a balanced hanger diagram to solve an equation of the form  $p(x + q) = r$ .
- Interpret a balanced hanger diagram with multiple groups, and justify (in writing) that there is more than one way to write an equation that represents the relationship shown.

## Learning Targets

- I can explain how a balanced hanger and an equation represent the same situation.
- I can explain why some balanced hangers can be described by two different equations, one with parentheses and one without.
- I can find an unknown weight on a hanger diagram and solve an equation that represents the diagram.
- I can write an equation that describes the weights on a balanced hanger.

## Lesson Narrative

This lesson continues the work of developing efficient equation solving strategies, justified by working with hanger diagrams. The goal of this lesson is for students to understand two different ways to solve an equation of the form  $p(x + q) = r$  efficiently. After a warm-up to revisit the distributive property, the first activity asks students to explain why either of two equations could represent a diagram and reason about a solution. The next activity presents four diagrams, asks students to match equations and then solve them. The goal is for students to see and understand two approaches to solving this type of equation.

## Alignments

### Building On

- 6.EE.A.4: Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions  $y + y + y$  and  $3y$  are equivalent because they name the same number regardless of which number  $y$  stands for.

### Addressing

- 7.EE.B.4.a: Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the

operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

### Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR7: Compare and Connect
- Think Pair Share

### Student Learning Goals

Let's use hangers to understand two different ways of solving equations with parentheses.

## 8.1 Equivalent to $2(x + 3)$

### Warm Up: 5 minutes

Students worked with the distributive property with variables in grade 6 and with numbers in earlier grades. In order to understand the two ways of solving an equation of the form  $p(x + q) = r$  in the upcoming lessons, it is helpful to have some fluency with the distributive property.

### Building On

- 6.EE.A.4

### Instructional Routines

- Think Pair Share

### Launch

Arrange students in groups of 2. Give 3 minutes of quiet work time and then invite students to share their responses with their partner, followed by a whole-class discussion.

### Student Task Statement

Select all the expressions equivalent to  $2(x + 3)$ .

1.  $2 \cdot (x + 3)$
2.  $(x + 3)^2$
3.  $2 \cdot x + 2 \cdot 3$
4.  $2 \cdot x + 3$
5.  $(2 \cdot x) + 3$
6.  $(2 + x)^3$

### Student Response

1, 2, 3

## Activity Synthesis

Focus specifically on why 1 and 3 are equivalent to lead into the next activity. You may also recall the warm-ups from prior lessons and ask if  $2(x + 3)$  is equal to 10, or other numbers, how much is  $x + 3$ ?

Ask students to write another expression that is equivalent to  $2(x + 3)$  (look for  $2x + 6$ ).

## 8.2 Either Or

### 15 minutes

This activity continues the work of using a balanced hanger to develop strategies for solving equations. Students are presented with a balanced hanger and are asked to explain why each of two different equations could represent it. They are then asked to find the unknown weight. Note that no particular solution method is prescribed. Give students a chance to come up with a reasonable approach, and then use the synthesis to draw connections between the diagram and each of the two equations. Students notice the structure of equations and diagrams and find correspondences between them and between solution strategies.

### Addressing

- 7.EE.B.4.a

### Instructional Routines

- MLR1: Stronger and Clearer Each Time
- Think Pair Share

### Launch

Keep students in the same groups. Give 5–10 minutes of quiet work time and time to share their responses with a partner, followed by a whole-class discussion.

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### Access for Students with Disabilities

*Representation: Develop Language and Symbols.* Use virtual or concrete manipulatives to connect symbols to concrete objects or values. For example, create a balanced hanger using concrete objects. Be sure to use individual pieces for each part of the diagram. Demonstrate moving pieces off of the hanger to create an equation. Invite students to show different ways to create the same equation.

*Supports accessibility for: Visual-spatial processing; Conceptual processing*

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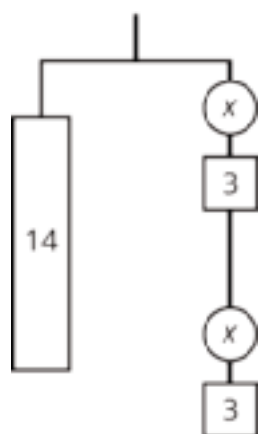
## Access for English Language Learners

*Conversing, Representing: MLR1 Stronger and Clearer Each Time.* Use this routine to help students improve their written response to the first question, by providing them with multiple opportunities to clarify their explanations through conversation. Give students time to meet with 2–3 partners, to share and get feedback on their responses. Provide listeners with prompts for feedback that will help their partners strengthen their ideas and clarify their language. For example, students can ask their partner, "How is  $2(x + 3)$  represented in the hanger?" or "Can you say more about..." After both students have shared and received feedback, provide students with 3–4 minutes to revise their initial draft, including ideas and language from their partner. This will help students communicate why the same hanger can be represented with equations in either form.

*Design Principle(s): Optimize output (for explanation); Cultivate conversation*

### Student Task Statement

1. Explain why either of these equations could represent this hanger:



$$14 = 2(x + 3) \text{ or } 14 = 2x + 6$$

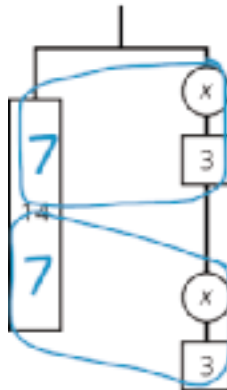
2. Find the weight of one circle. Be prepared to explain your reasoning.

### Student Response

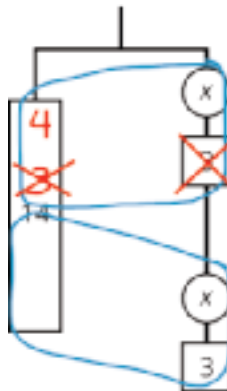
1. Answers vary. The diagram shows 14 balanced with 2 groups of  $x + 3$ , and this corresponds to  $14 = 2(x + 3)$ . The diagram also shows 14 balanced with 2  $x$ 's and another 6 units of weight, which corresponds to  $14 = 2x + 6$ .
2. 4 units. Sample reasoning:
  - Since 2 groups of  $x + 3$  weighs 14 units, 1 group must weigh 7 units. If  $x + 3 = 7$ , then  $x = 4$ .
  - Remove 6 units from each side, leaving  $8 = 2x$ . Therefore,  $x = 4$ .

## Activity Synthesis

Have one student present who did  $7 = x + 3$  first, and another student present who subtracted 6 first. If no one mentions one of these approaches, demonstrate it. Show how the hanger supports either approach. The finished work might look like this for the first equation:



$$14 = 2(x + 3)$$
$$7 = x + 3$$



$$7 = x + 3$$
$$4 = x$$

For the second equation, rearrange the right side of the hanger, first, so that 2  $x$ 's are on the top and 6 units of weight are on the bottom. Then, cross off 6 from each side and divide each side by 2. Show this side by side with "doing the same thing to each side" of the equation.

## 8.3 Use Hangers to Understand Equation Solving, Again

15 minutes

The first question is straightforward since each diagram uses a different letter, but it's there to make sure students start with  $p(x + q) = r$ . If some want to rewrite as  $px + pq = r$  first, that's great. We want some to do that but others to divide both sides by  $p$  first. Monitor for students who take each approach.

### Addressing

- 7.EE.B.4.a

## Instructional Routines

- MLR7: Compare and Connect
- Think Pair Share

## Launch

Keep students in the same groups. Give 5–10 minutes of quiet work time and time to share their responses with a partner, followed by a whole-class discussion.

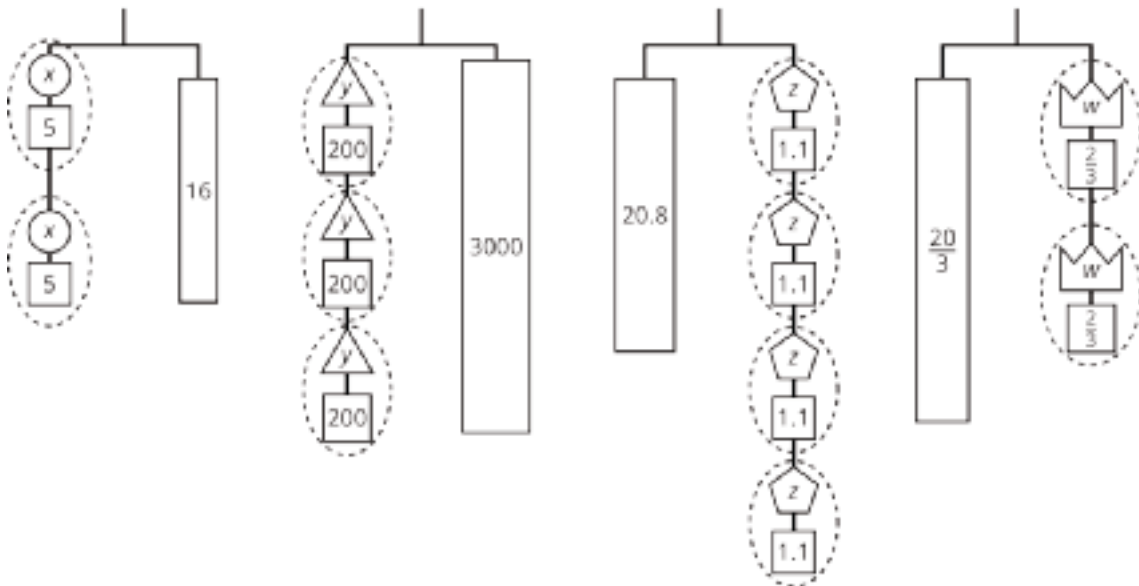
### Access for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, show only 2 hangers and 2 equations. If students finish early, assign the remaining hangers and equations.

*Supports accessibility for: Memory; Organization*

### Student Task Statement

Here are some balanced hangers. Each piece is labeled with its weight.



For each diagram:

1. Assign one of these equations to each hanger:

$$2(x + 5) = 16$$

$$3(y + 200) = 3,000$$

$$20.8 = 4(z + 1.1)$$

$$\frac{20}{3} = 2\left(w + \frac{2}{3}\right)$$

2. Explain how to figure out the weight of a piece labeled with a letter by reasoning about the diagram.
3. Explain how to figure out the weight of a piece labeled with a letter by reasoning about the equation.

### Student Response

1. Each equation corresponds to the diagram with the variable that matches.
2. Sample reasoning for the first diagram:
  - a. Split each side into two groups with  $x + 5$  on the left and 8 on the right. From one of these groups, remove 5 units from each side. This shows that  $x = 3$ .
  - b. Rearrange the left side so that there are 2  $x$ 's on top and 10 units on the bottom. Remove 10 units of weight from each side, leaving 2  $x$ 's on the left and 6 on the right. Each  $x$  must weigh 3 units for the hanger to be in balance.
3. Sample reasoning for  $2(x + 5) = 16$ :
  - a. Divide each side by 2, leaving  $x + 5 = 8$ . Subtract 5 from each side, leaving  $x = 3$ .
  - b. Use the distributive property to write  $2x + 10 = 16$ . Subtract 10 from each side leaving  $2x = 6$ . Divide each side by 2 leaving  $x = 3$ .

### Activity Synthesis

Select one hanger for which one student divided by  $p$  first and another student distributed  $p$  first. Display the two solution methods side by side, along with the hanger.

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#### Access for English Language Learners

*Representing, Conversing: MLR7 Compare and Connect.* Use this routine to prepare students for the whole-class discussion. Give students quiet think time to consider what is the same and what is different about the two solution methods. Next, ask students to discuss what they noticed with a partner. Listen for and amplify mathematical language students use to describe how each solution method can be represented by the hanger.

*Design Principle(s): Cultivate conversation*

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### Lesson Synthesis

Display the equation  $4(x + 7) = 40$ . Ask one partner to solve by dividing first and the other to solve by distributing first. Then, check that they got the same solution and that it makes the equation true. If they get stuck, encourage them to draw a diagram to represent the equation.

## 8.4 Solve Another Equation

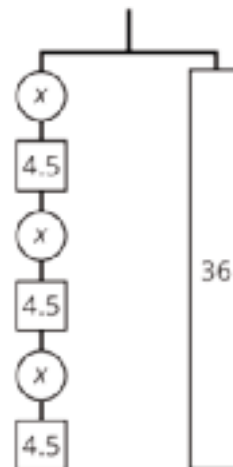
Cool Down: 5 minutes

## Addressing

- 7.EE.B.4.a

### Student Task Statement

Solve the equation  $3(x + 4.5) = 36$ . If you get stuck, use the diagram.



### Student Response

7.5. Sample reasoning:

- Divide each side by 3 leaving  $x + 4.5 = 12$  then subtract 4.5 from each side.
- The distributive property gives  $3x + 13.5 = 36$ . Subtract 13.5 from each side leaving  $3x = 22.5$ . Divide each side by 3.

### Student Lesson Summary

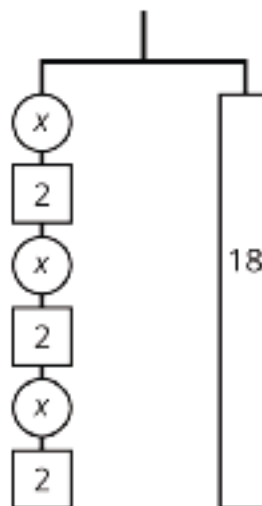
The balanced hanger shows 3 equal, unknown weights and 3 2-unit weights on the left and an 18-unit weight on the right.

There are 3 unknown weights plus 6 units of weight on the left. We could represent this balanced hanger with an equation and solve the equation the same way we did before.

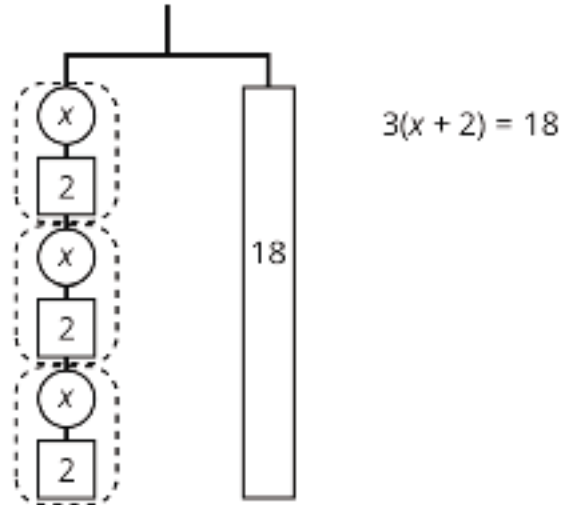
$$3x + 6 = 18$$

$$3x = 12$$

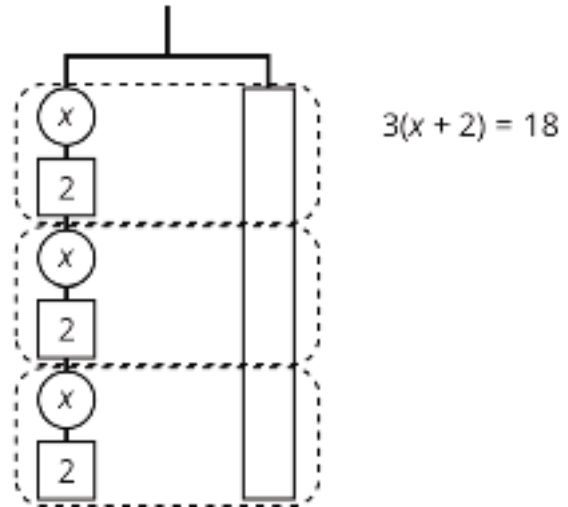
$$x = 4$$



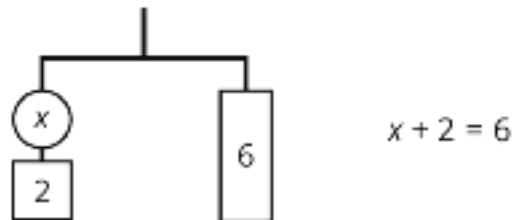
Since there are 3 groups of  $x + 2$  on the left, we could represent this hanger with a different equation:  $3(x + 2) = 18$ .



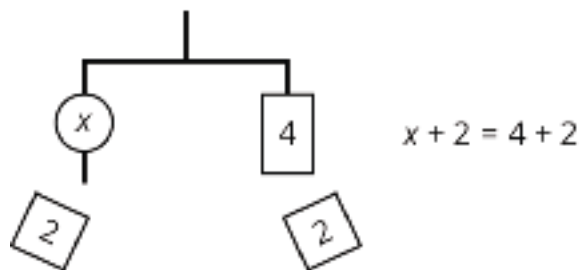
The two sides of the hanger balance with these weights: 3 groups of  $x + 2$  on one side, and 18, or 3 groups of 6, on the other side.



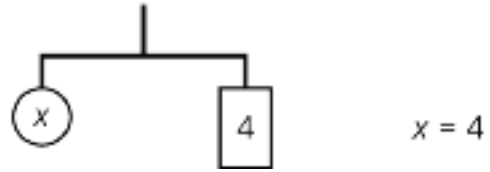
The two sides of the hanger will balance with  $\frac{1}{3}$  of the weight on each side:  
 $\frac{1}{3} \cdot 3(x + 2) = \frac{1}{3} \cdot 18$ .



We can remove 2 units of weight from each side, and the hanger will stay balanced. This is the same as subtracting 2 from each side of the equation.



An equation for the new balanced hanger is  $x = 4$ . This gives the solution to the original equation.



Here is a concise way to write the steps above:

$$3(x + 2) = 18$$

$$x + 2 = 6 \quad \text{after multiplying each side by } \frac{1}{3}$$

$$x = 4 \quad \text{after subtracting 2 from each side}$$

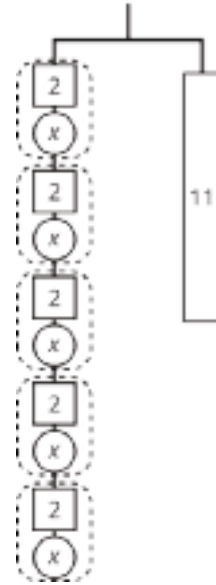
# Lesson 8 Practice Problems

## Problem 1

### Statement

Here is a hanger:

- Write an equation to represent the hanger.
- Solve the equation by reasoning about the equation or the hanger. Explain your reasoning.



### Solution

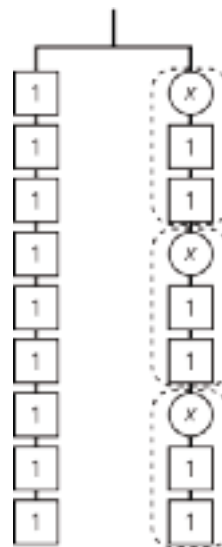
- $5(x + 2) = 11$
- $x + 2 = 2.2$ ,  $x = 0.2$ . Explanations vary. Sample explanation: Divide both sides by 5 to get a circle labeled  $x$  and a rectangle labeled 2 on the left, and a rectangle labeled 2.2 on the right. Then subtract 2 from each side to get a circle on the left and rectangle with 0.2 on the right.

## Problem 2

### Statement

Explain how each part of the equation  $9 = 3(x + 2)$  is represented in the hanger.

- $x$
- 9
- 3
- $x + 2$
- $3(x + 2)$
- the equal sign



## Solution

Answers vary. Sample response:

- The circle has an unknown weight, so use  $x$  to represent it.
- The left side has 9 squares, each weighing 1 unit.
- There are 3 identical groups on the right side.
- Each group on the right side is made up of one circle with weight  $x$  units and 2 squares of weight 1 unit each.
- The total weight of those 3 identical groups is the total weight of the right side.
- The equal sign is seen in the hanger being balanced.

## Problem 3

### Statement

Select the word from the following list that best describes each situation.

- |  |   |
|--|---|
| <p>A. You deposit money in a savings account, and every year the amount of money in the account increases by 2.5%.</p> <p>B. For every car sold, a car salesman is paid 6% of the car's price.</p> <p>C. Someone who eats at a restaurant pays an extra 20% of the food price. This extra money is kept by the person who served the food.</p> <p>D. An antique furniture store pays \$200 for a chair, adds 50% of that amount, and sells the chair for \$300.</p> <p>E. The normal price of a mattress is \$600, but it is on sale for 10% off.</p> <p>F. For any item you purchase in Texas, you pay an additional 6.25% of the item's price to the state government.</p> | <p>1. Tax</p> <p>2. Commission</p> <p>3. Discount</p> <p>4. Markup</p> <p>5. Tip or gratuity</p> <p>6. Interest</p> |
|--|---|

## Solution

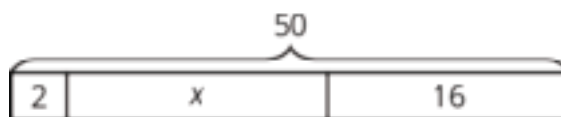
- A: 6
- B: 2
- C: 5
- D: 4
- E: 3
- F: 1


(From Unit 4, Lesson 11.)

## Problem 4

### Statement

Clare drew this diagram to match the equation  $2x + 16 = 50$ , but she got the wrong solution as a result of using this diagram.



- 
- a. What value for  $x$  can be found using the diagram?
  - b. Show how to fix Clare's diagram to correctly match the equation.
  - c. Use the new diagram to find a correct value for  $x$ .
  - d. Explain the mistake Clare made when she drew her diagram.

## Solution

- a. 32.  $x$  can be found by subtracting 2 and 16 from 50 since the three parts 2,  $x$ , and 16 sum to 50 in the diagram.
- b. The diagram correctly represents the equation if the first block is changed from 2 to  $x$ . Then the three parts of the diagram are  $x$ ,  $x$ , and 16, for a total of  $2x + 16$ .
- c. Since the corrected diagram shows that the number 50 is divided into parts of size  $x$ ,  $x$  and 16, the two  $x$ 's must together equal 16 less than 50, which is 34. This means that one  $x$  is 17.
- d. Sample explanation: Clare showed  $2 + x$  instead of  $2 \cdot x$ . She might not understand that  $2x$  means 2 multiplied by  $x$ , or she might not understand that the tape diagram shows parts adding up to a whole.

(From Unit 6, Lesson 3.)

# Lesson 9: Dealing with Negative Numbers

## Goals

- Generalize (orally) that doing the same thing to each side of an equation generates an equivalent equation.
- Solve equations of the form  $px + q = r$  or  $p(x + q) = r$  that involve negative numbers, and explain (orally and in writing) the solution method.

## Learning Targets

- I can use the idea of doing the same to each side to solve equations that have negative numbers or solutions.

## Lesson Narrative

In the previous lessons, we used hangers to reason about ways to approach equations of the form  $px + q = r$  or  $p(x + q) = r$  (which can be summed up as “do the same thing to each side until the unknown equals a number”). Since the things we do to each side of an equation are just arithmetic operations, and the properties of operations extend to negative numbers, this method of solving equations also works when there are negative numbers, even though it doesn’t make physical sense to think about weights on hangers representing negative numbers. After a warm-up designed to remind students about operating on rational numbers, students are asked to solve some straightforward equations involving negative numbers. “Doing the same thing to each side” is presented as a valid method, even though negative numbers are involved. In the last activity, students do the same thing to each side of an equation and their partner tries to guess what they did. The purpose of this is to communicate that doing the same thing to each side maintains equality even when the moves aren’t intended to lead to the equation's solution.

## Alignments

### Building On

- 7.NS.A: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

### Addressing

- 7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
- 7.EE.B.4.a: Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

## Building Towards

- 7.EE.B.4.a: Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

## Instructional Routines

- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Which One Doesn't Belong?

## Student Learning Goals

Let's show that doing the same to each side works for negative numbers too.

# 9.1 Which One Doesn't Belong: Rational Number Arithmetic

## Warm Up: 10 minutes

This warm-up prompts students to compare four equations. It encourages students to explain their reasoning, hold mathematical conversations, and gives you the opportunity to hear what they recall of arithmetic on signed numbers. To allow all students to access the activity, each equation has one obvious reason it does not belong. Encourage students to move past the obvious reasons and find reasons based on mathematical properties.

During the discussion, listen for strategies for evaluating expressions with rational numbers that will be helpful in the work of this lesson.

## Building On

- 7.NS.A

## Instructional Routines

- Which One Doesn't Belong?

## Launch

Arrange students in groups of 2–4. Display the equations for all to see. Ask students to indicate when they have noticed one that does not belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their small group. In their small groups, tell each student to share their reasoning why a particular equation does not belong and together find at least one reason each question doesn't belong.

### Student Task Statement

Which equation doesn't belong?

$$15 = -5 \cdot -3$$

$$4 - -2 = 6$$

$$2 + -5 = -3$$

$$-3 \cdot -4 = -12$$

### Student Response

1.  $15 = -5 \cdot -3$  doesn't belong because it's the only one with a number on the left and an operation on the right.
2.  $2 + -5 = -3$  doesn't belong because it's the only one with addition.
3.  $4 - -2 = 6$  doesn't belong because it's the only one with subtraction.
4.  $-3 \cdot -4 = -12$  doesn't belong because it's the only one that is not true.

### Activity Synthesis

Ask each group to share one reason why a particular equation does not belong. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given are correct. During the discussion, highlight any strategies for adding, subtracting, or multiplying signed numbers.

## 9.2 Old and New Ways to Solve

15 minutes

These are all solvable by thinking "what value would make the equation true." So, it's straightforward to figure out what the solution would be, but these equations present an opportunity to demonstrate that "doing the same thing to each side" still works when there are negative numbers. Monitor for students who reason about what value would make the equation true and those who reason by doing the same thing to each side.

### Addressing

- 7.EE.B.4.a

### Instructional Routines

- MLR7: Compare and Connect

### Launch

Give 5–10 minutes of quiet work time followed by a whole-class discussion.

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### Access for Students with Disabilities

*Representation: Internalize Comprehension.* Activate or supply background knowledge about strategies for performing operations on signed numbers, as well as different representations of equations. This will help students make connections between new problems and prior work.  
*Supports accessibility for: Memory; Conceptual processing*

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### Anticipated Misconceptions

Some students may need some additional support remembering and applying strategies for performing operations on signed numbers. Draw their attention to any anchor charts or notes that are available from the previous unit.

### Student Task Statement

Solve each equation. Be prepared to explain your reasoning.

1.  $x + 6 = 4$

2.  $x - 4 = -6$

3.  $2(x - 1) = -200$

4.  $2x + -3 = -23$

### Student Response

1.  $x = -2$

2.  $x = -10$

3.  $x = -99$

4.  $x = -10$

### Activity Synthesis

For each equation, ask one student to explain how they know their solution is correct. If no students mention this approach, demonstrate solving each equation by doing the same thing to each side.

The purpose of this discussion is to make the claim that “do the same thing to each side” also works when subtraction or negative numbers are involved. Tell students that even though it doesn’t make sense to represent negative numbers using the hanger metaphor, we are going to take it as a fact that we can still do the same thing to each side of an equation even when we are working with negative numbers.

---

### Access for English Language Learners

*Representing: MLR7 Compare and Connect.* Use this routine when students explain how they solved their equations. Ask students, “What is the same and what is different?” about the strategies. Draw students’ attention to the connection between the approaches of ‘finding the value that makes the equation true’ and ‘doing the same to each side.’ These exchanges strengthen students’ mathematical language use and reasoning based on ways to solve equations that involve negative numbers.

*Design Principle(s): Support sense-making; Maximize meta-awareness*

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## 9.3 Keeping It True

15 minutes

The purpose of this activity is to note that doing the same thing to each side of an equation keeps it in balance, even if those moves don’t get us closer to solving the equation. Students first explain how each equation in a sequence follows logically from the previous one. Then, they start with the equation  $-5 = x$  and repeatedly do the same thing to each side to create a new equation. Their partner tries to guess which moves they made.

### Building Towards

- 7.EE.B.4.a

### Instructional Routines

- MLR8: Discussion Supports

### Launch

Arrange students in groups of 2. Give students 2 minutes of quiet work time on the first question, pause for a discussion, and then time to complete the task with their partner.

After students have a chance to work on the first question, pause for a discussion. Ask students what different types of moves could we do to  $x = -6$ ? List the different kinds of things in the board, so when students do their own, you can say you have to use different combinations of things on the list. The purpose of this is to prevent students from going wild and generating equations that are far out of the scope of the work in this unit.

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### Access for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. Consider pausing after the first question for a brief class discussion before moving on.  
*Supports accessibility for: Organization; Attention*

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### Access for English Language Learners

*Speaking: MLR8 Discussion Supports.* To support students to produce statements about how their partner transformed the equation  $-5 = x$ , ask students to revisit the moves that were used earlier in the lesson to transform  $x = -6$ . Students should use different combinations of the moves on the list in conjunction with sentence frames such as "I noticed \_\_\_ so I . . ." or "I know \_\_\_ because . . ." This will help students explain their thinking. *Design Principle(s): Optimize output (for explanation)*

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### Anticipated Misconceptions

Some students may need some additional support remembering and applying strategies for performing operations on signed numbers. Draw their attention to any anchor charts or notes that are available from the previous unit.

#### Student Task Statement

Here are some equations that all have the same solution.

$$\begin{aligned}x &= -6 \\x - 3 &= -9 \\-9 &= x - 3 \\900 &= -100(x - 3) \\900 &= (x - 3) \cdot (-100) \\900 &= -100x + 300\end{aligned}$$

1. Explain how you know that each equation has the same solution as the previous equation. Pause for discussion before moving to the next question.
2. Keep your work secret from your partner. Start with the equation  $-5 = x$ . Do the same thing to each side at least three times to create an equation that has the same solution as the starting equation. Write the equation you ended up with on a slip of paper, and trade equations with your partner.
3. See if you can figure out what steps they used to transform  $-5 = x$  into their equation. When you think you know, check with them to see if you are right.

## Student Response

1. Answers vary. Sample responses:
  - subtract 3 from each side
  - swap the two sides of the equation
  - multiply each side by -100
  - swap the factors -100 and  $x - 3$  (the commutative property of multiplication)
  - apply the distributive property
2. Answers vary.
3. Answers vary.

## Activity Synthesis

Much of the discussion will take place in small groups. Questions for discussion:

- “Did you have any disagreements, and how did you resolve them?”
- “Did anything surprise you? Explain.”
- “What are some important things to keep in mind when working with negative numbers?”

## Lesson Synthesis

Ask students to think of one or two important things they learned in this lesson, and share them with a partner. Points to highlight include:

- Doing the same thing to each side of an equation still keeps the equation balanced, even when there are negative numbers.
- Doing the same thing to each side of an equation still keeps the equation balanced, even when the moves don't get you closer to a solution.

# 9.4 Solve Two More Equations

Cool Down: 5 minutes

### Addressing

- 7.EE.B.4

### Student Task Statement

Solve each equation. Show your work, or explain your reasoning.

1.  $-3x - 5 = 16$

2.  $-4(y - 2) = 12$

### Student Response

1.  $x = -7$

$$2. y = -1$$

## Student Lesson Summary

When we want to find a solution to an equation, sometimes we just think about what value in place of the variable would make the equation true. Sometimes we perform the same operation on each side (for example, subtract the same amount from each side). The balanced hangers helped us to understand that doing the same thing to each side of an equation keeps the equation true.

Since negative numbers are just numbers, then doing the same thing to each side of an equation works for negative numbers as well. Here are some examples of equations that have negative numbers and steps you could take to solve them.

Example:

$$\begin{aligned} 2(x - 5) &= -6 \\ \frac{1}{2} \cdot 2(x - 5) &= \frac{1}{2} \cdot (-6) && \text{multiply each side by } \frac{1}{2} \\ x - 5 &= -3 \\ x - 5 + 5 &= -3 + 5 && \text{add 5 to each side} \\ x &= 2 \end{aligned}$$

Example:

$$\begin{aligned} -2x + -5 &= 6 \\ -2x + -5 - -5 &= 6 - -5 && \text{subtract -5 from each side} \\ -2x &= 11 \\ -2x \div -2 &= 11 \div -2 && \text{divide each side by -2} \\ x &= -\frac{11}{2} \end{aligned}$$

Doing the same thing to each side maintains equality even if it is not helpful to solving for the unknown amount. For example, we could take the equation  $-3x + 7 = -8$  and add  $-2$  to each side:

$$\begin{aligned} -3x + 7 &= -8 \\ -3x + 7 + -2 &= -8 + -2 && \text{add -2 to each side} \\ -3x + 5 &= -10 \end{aligned}$$

If  $-3x + 7 = -8$  is true then  $-3x + 5 = -10$  is also true, but we are no closer to a solution than we were before adding  $-2$ . We can use moves that maintain equality to make new equations that all have the same solution. Helpful combinations of moves will eventually lead to an

equation like  $x = 5$ , which gives the solution to the original equation (and every equation we wrote in the process of solving).

# Lesson 9 Practice Problems

## Problem 1

### Statement

Solve each equation.

a.  $4x = -28$

b.  $x - 6 = -2$

c.  $-x + 4 = -9$

d.  $-3x + 7 = 1$

e.  $25x + -11 = -86$

### Solution

a. -7

b. -8

c. 13

d. 2

e. -3

## Problem 2

### Statement

Here is an equation  $2x + 9 = -15$ . Write three different equations that have the same solution as  $2x + 9 = -15$ . Show or explain how you found them.

### Solution

Equations vary. Sample equations:  $24 + 2x = 0$ ,  $4x + 10 = -38$ ,  $85 = 2x + 109$

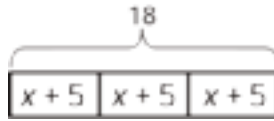
Sample explanation:

- Start with:  $2x + 9 = -15$ .
- Add 20 to each side:  $2x + 29 = 5$ .
- Use the commutative property of addition:  $29 + 2x = 5$ .
- Subtract 5 from each side:  $24 + 2x = 0$ .

### Problem 3

#### Statement

Select all the equations that match the diagram.



- A.  $x + 5 = 18$
- B.  $18 \div 3 = x + 5$
- C.  $3(x + 5) = 18$
- D.  $x + 5 = \frac{1}{3} \cdot 18$
- E.  $3x + 5 = 18$

#### Solution

["B", "C", "D"]

(From Unit 6, Lesson 3.)

### Problem 4

#### Statement

There are 88 seats in a theater. The seating in the theater is split into 4 identical sections. Each section has 14 red seats and some blue seats.

- a. Draw a tape diagram to represent the situation.
- b. What unknown amounts can be found by using the diagram or reasoning about the situation?

#### Solution

Answers vary. Sample responses:

- a. A tape diagram with 4 equal parts, each labeled  $x + 14$ , for a total of 88.
- b. Each section has 22 seats, of which 8 are blue. There are 32 blue seats and 56 red seats in the theater.

(From Unit 6, Lesson 2.)

## Problem 5

### Statement

Match each story to an equation.

A. A stack of nested paper cups is 8 inches tall. The first cup is 4 inches tall and each of the rest of the cups in the stack adds  $\frac{1}{4}$  inch to the height of the stack.

B. A baker uses 4 cups of flour. She uses  $\frac{1}{4}$  cup to flour the counters and the rest to make 8 identical muffins.

C. Elena has an 8-foot piece of ribbon. She cuts off a piece that is  $\frac{1}{4}$  of a foot long and cuts the remainder into four pieces of equal length.

1.  $\frac{1}{4} + 4x = 8$

2.  $4 + \frac{1}{4}x = 8$

3.  $8x + \frac{1}{4} = 4$

### Solution

- A: 2
- B: 3
- C: 1

(From Unit 6, Lesson 4.)

# Lesson 10: Different Options for Solving One Equation

## Goals

- Critique (orally and in writing) a given solution method for an equation of the form  $p(x + q) = r$ .
- Evaluate (orally) the usefulness of different approaches for solving a given equation of the form  $p(x + q) = r$ .
- Recognize that there are two common approaches for solving an equation of the form  $p(x + q) = r$ , i.e., expanding using the distributive property or dividing each side by  $p$ .

## Learning Targets

- For an equation like  $3(x + 2) = 15$ , I can solve it in two different ways: by first dividing each side by 3, or by first rewriting  $3(x + 2)$  using the distributive property.
- For equations with more than one way to solve, I can choose the easier way depending on the numbers in the equation.

## Lesson Narrative

The purpose of this lesson is to practice solving equations of the form  $p(x + q) = r$ , and to notice that one of the two ways of solving may be more efficient depending on the numbers in the equation.

## Alignments

### Addressing

- 7.EE.B.4.a: Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

### Building Towards

- 7.EE.B.4.a: Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

## Instructional Routines

- Algebra Talk

- MLR8: Discussion Supports
- Think Pair Share

### Student Learning Goals

Let's think about which way is easier when we solve equations with parentheses.

## 10.1 Algebra Talk: Solve Each Equation

### Warm Up: 5 minutes

The purpose of this Algebra Talk is to promote seeing structure in equations of the form  $p(x + q) = r$ . The goal is for students to see  $(x - 3)$  as a chunk of the equation. For each equation, the equation is true if  $(x - 3)$  is 10. These understandings help students develop fluency. While four problems are given, it may not be possible to share every strategy. Consider gathering only two or three different strategies per problem, saving most of the time for the final question.

### Building Towards

- 7.EE.B.4.a

### Instructional Routines

- Algebra Talk
- MLR8: Discussion Supports

### Launch

Display one equation at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

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### Access for Students with Disabilities

*Representation: Internalize Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.

*Supports accessibility for: Memory; Organization*

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### Student Task Statement

$$100(x - 3) = 1,000$$

$$500(x - 3) = 5,000$$

$$0.03(x - 3) = 0.3$$

$$0.72(x + 2) = 7.2$$

## Student Response

- 13
- 13
- 13
- 8

## Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- “Who can restate \_\_\_’s reasoning in a different way?”
- “Did anyone have the same strategy but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to \_\_\_\_’s strategy?”
- “Do you agree or disagree? Why?”

It may help to display an equation like  $100(x - 3) = 1,000$  but cover the  $(x - 3)$  with your hand or with an eraser. “100 times something is 1,000. What is the something?”

---

### Access for English Language Learners

*Speaking: MLR8 Discussion Supports.*: Display sentence frames to support students when they explain their strategy. For example, “First, I \_\_\_\_ because . . .” or “I noticed \_\_\_\_ so I . . .” Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

*Design Principle(s): Optimize output (for explanation)*

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## 10.2 Analyzing Solution Methods

15 minutes

From previous work in the unit, students should already understand that distributing first is a valid solution method, though this activity reinforces that understanding. The purpose of this activity is to make explicit a common pitfall (in Noah’s method). Monitor for students with different, valid reasons for agreeing or disagreeing. For example, disagree with Noah because . . .

- his answer  $\frac{19}{2}$  doesn’t make the original equation true.
- a tape diagram shows that adding 9 to each side does not result in a diagram that can be represented with  $2x = 19$ .

- when you add 9 to the left side, you are adding it to  $2x - 18$  by the distributive property, which doesn't result in  $2x$ .

### Building Towards

- 7.EE.B.4.a

### Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

### Launch

Arrange students in groups of 2. Give 5–10 minutes quiet work time and time to share their reasoning with their partner, followed by a whole-class discussion.

Explain to students that their job is to analyze three solution methods for errors. They should share with their partner whether they agree or disagree with each method, and explain why.

### Access for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, “I agree/disagree with \_\_\_\_ because...” or “Instead of \_\_\_\_, he/she should have...”

*Supports accessibility for: Language; Social-emotional skills*

### Anticipated Misconceptions

If students aren't sure how to begin analyzing Noah's method, ask them to explain what it means for a number to be a solution of an equation. Alternatively, suggest that they draw a tape diagram of  $2(x - 9) = 10$ .

### Student Task Statement

Three students each attempted to solve the equation  $2(x - 9) = 10$ , but got different solutions. Here are their methods. Do you agree with any of their methods, and why?

Noah's method:

$$\begin{array}{l}
 2(x - 9) = 10 \\
 2(x - 9) + 9 = 10 + 9 \quad \text{add 9 to each side} \\
 2x = 19 \\
 2x \div 2 = 19 \div 2 \quad \text{divide each side by 2} \\
 x = \frac{19}{2}
 \end{array}$$

Elena's method:

$$\begin{aligned}2(x - 9) &= 10 \\2x - 18 &= 10 && \text{apply the distributive property} \\2x - 18 - 18 &= 10 - 18 && \text{subtract 18 from each side} \\2x &= -8 \\2x \div 2 &= -8 \div 2 && \text{divide each side by 2} \\x &= -4\end{aligned}$$

Andre's method:

$$\begin{aligned}2(x - 9) &= 10 \\2x - 18 &= 10 && \text{apply the distributive property} \\2x - 18 + 18 &= 10 + 18 && \text{add 18 to each side} \\2x &= 28 \\2x \div 2 &= 28 \div 2 && \text{divide each side by 2} \\x &= 14\end{aligned}$$

## Student Response

Answers vary. Sample responses:

1. I disagree with Noah's method, because  $2(x - 9) + 9$  is not  $2x$ . Noah should distribute the 2 before adding a number to each side.
2. I disagree with Elena's method, because  $2x - 18 - 18$  is  $2x - 36$ , not  $2x$ . Instead of subtracting 18, it would be better to add 18.
3. I agree with Andre's method, because all of his moves are valid, and 14 makes the original equation true when substituted for  $x$ .

## Activity Synthesis

Invite students to share as many unique reasons they agree or disagree with each method as time allows. See the Activity Narrative for anticipated approaches. Pay particular attention to Noah's method, since this represents a common error.

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### Access for English Language Learners

*Speaking: MLR8 Discussion Supports.* To help students produce statements that explain why they agree (or disagree) with the different solution methods, provide sentence frames such as "I notice \_\_\_ so I ...," "That could be true because..." and "This method works because..." This will provide students with a structure to communicate their reasoning while promoting access to both content and language.

*Design Principle(s): Optimize output (for explanation)*

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## 10.3 Solution Pathways

15 minutes

The purpose of this activity is to practice solving equations of the form  $p(x + q) = r$ , recognizing that there are two valid approaches, and making judgments about which one is more sensible for a given equation.

### Addressing

- 7.EE.B.4.a

### Instructional Routines

- MLR8: Discussion Supports

### Launch

Display this equation and a hanger diagram to match:  $3(x + 2) = 21$ . Tell students, “Any time you want to solve an equation in this form, you have a choice to make about how to proceed. You can either divide each side by 3 or you can distribute the 3.” Demonstrate each solution method side by side, while appealing to reasoning about the hanger diagram.

Keep students in the same groups. 5–10 minutes of quiet or partner work time followed by a whole-class discussion.

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### Access for Students with Disabilities

*Representation: Internalize Comprehension.* Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.

*Supports accessibility for: Memory; Conceptual processing*

---

### Student Task Statement

For each equation, try to solve the equation using each method (dividing each side first, or applying the distributive property first). Some equations are easier to solve by one method than the other. When that is the case, stop doing the harder method and write down the reason you stopped.

1.  $2,000(x - 0.03) = 6,000$

2.  $2(x + 1.25) = 3.5$

3.  $\frac{1}{4}(4 + x) = \frac{4}{3}$

4.  $-10(x - 1.7) = -3$

5.  $5.4 = 0.3(x + 8)$

### Student Response

1. 3.03
2. 0.5
3.  $\frac{4}{3}$  or equivalent
4. 2
5. 10

### Activity Synthesis

Reveal the solution to each equation and give students a few minutes to resolve any discrepancies with their partner.

Display the list of equations in the task, and ask students to help you label them with which solution method was easier, either “divide first” or “distribute first.” Discuss any disagreements and the reasons one method is easier than the other. (There is really no right or wrong answer here. Some people might prefer moves that eliminate fractions and decimals as early as possible. Some might want to minimize the number of computations.)

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### Access for English Language Learners

*Speaking: MLR8 Discussion Supports.* Use this routine to help students produce mathematical language to communicate about which method is more efficient. Give groups of students 3–4 minutes to discuss which method would be easiest for each problem. Next, select groups to share how their solution methods minimize the number of computations needed, or address eliminating fractions and decimals. Consider providing sentence frames such as: “Dividing first was easier because \_\_\_\_\_,” or “Distributing first was easier because\_\_\_\_\_.” Call on students to restate and/or revoice their peers' descriptions using mathematical language.

*Design Principle(s): Optimize output (for justification)*

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### Lesson Synthesis

Possible questions for discussion:

- “What are the two main ways we can approach solving equations like the ones we saw today?” (divide first or distribute first)
- “What kinds of things do we look for to decide which approach is better?” (powers of ten, operations that result in whole numbers, moves that will eliminate fractions or decimals)

- “How can we check if our answer is a solution to the original equation?” (Substitute our answer for the variable and see if it makes the equation true.)

## 10.4 Solve Two Equations

Cool Down: 5 minutes

### Addressing

- 7.EE.B.4.a

### Student Task Statement

Solve each equation. Show or explain your method.

$$1. 8.88 = 4.44(x - 7)$$

$$2. 5 \left( y + \frac{2}{5} \right) = -13$$

### Student Response

$$1. x = 9$$

$$2. y = -3$$

### Student Lesson Summary

Equations can be solved in many ways. In this lesson, we focused on equations with a specific structure, and two specific ways to solve them.

Suppose we are trying to solve the equation  $\frac{4}{5}(x + 27) = 16$ . Two useful approaches are:

- divide each side by  $\frac{4}{5}$
- apply the distributive property

In order to decide which approach is better, we can look at the numbers and think about which would be easier to compute. We notice that  $\frac{4}{5} \cdot 27$  will be hard, because 27 isn't divisible by 5. But  $16 \div \frac{4}{5}$  gives us  $16 \cdot \frac{5}{4}$ , and 16 is divisible by 4. Dividing each side by  $\frac{4}{5}$  gives:

$$\begin{aligned} \frac{4}{5}(x + 27) &= 16 \\ \frac{5}{4} \cdot \frac{4}{5}(x + 27) &= 16 \cdot \frac{5}{4} \\ x + 27 &= 20 \\ x &= -7 \end{aligned}$$

Sometimes the calculations are simpler if we first use the distributive property. Let's look at the equation  $100(x + 0.06) = 21$ . If we first divide each side by 100, we get  $\frac{21}{100}$  or 0.21 on the right side of the equation. But if we use the distributive property first, we get an equation that only contains whole numbers.



$$\begin{aligned}100(x + 0.06) &= 21 \\100x + 6 &= 21 \\100x &= 15 \\x &= \frac{15}{100}\end{aligned}$$

# Lesson 10 Practice Problems

## Problem 1

### Statement

Andre wants to buy a backpack. The normal price of the backpack is \$40. He notices that a store that sells the backpack is having a 30% off sale. What is the sale price of the backpack?

### Solution

\$28

(From Unit 4, Lesson 11.)

## Problem 2

### Statement

On the first math exam, 16 students received an A grade. On the second math exam, 12 students received an A grade. What percentage decrease is that?

### Solution

25% ( $4 \div 16 = 0.25$ )

(From Unit 4, Lesson 12.)

## Problem 3

### Statement

Solve each equation.

a.  $2(x - 3) = 14$

b.  $-5(x - 1) = 40$

c.  $12(x + 10) = 24$

d.  $\frac{1}{6}(x + 6) = 11$

e.  $\frac{5}{7}(x - 9) = 25$

### Solution

a. 10

b. -7

c. -8

d. 60

e. 44

## Problem 4

### Statement

Select **all** expressions that represent a correct solution to the equation  $6(x + 4) = 20$ .

A.  $(20 - 4) \div 6$

B.  $\frac{1}{6}(20 - 4)$

C.  $20 - 6 - 4$

D.  $20 \div 6 - 4$

E.  $\frac{1}{6}(20 - 24)$

F.  $(20 - 24) \div 6$

### Solution

["D", "E", "F"]

## Problem 5

### Statement

Lin and Noah are solving the equation  $7(x + 2) = 91$ .

Lin starts by using the distributive property. Noah starts by dividing each side by 7.

a. Show what Lin's and Noah's full solution methods might look like.

b. What is the same and what is different about their methods?

### Solution

Answers vary. Sample response:

a. ■ Lin's solution method:  $7x + 14 = 91$ ,  $7x = 77$ ,  $x = 11$

■ Noah's solution method:  $x + 2 = 13$ ,  $x = 11$

b. Both methods involve dividing by 7, but Noah does the division first, while Lin does the division last. Also, Lin's method involves subtracting 14, while Noah's method involves

subtracting 2. Both solutions are correct and valid. Noah's solution could be considered more efficient for this example, because it takes fewer steps and has equally complicated arithmetic work.

# Lesson 11: Using Equations to Solve Problems

## Goals

- Interpret and coordinate tape diagrams, equations, and verbal descriptions for situations involving signed numbers.
- Solve an equation of the form  $px + q = r$  or  $p(x + q) = r$  to determine an unknown quantity in a situation, and present the solution method (orally, in writing, and through other representations).
- Write an equation of the form  $px + q = r$  or  $p(x + q) = r$  to represent a situation involving signed numbers.

## Learning Targets

- I can solve story problems by drawing and reasoning about a tape diagram or by writing and solving an equation.

## Lesson Narrative

This lesson brings together the skills and concepts that have been studied in the unit so far. Students solve problems that can be represented by equations of the form  $p(x + q) = r$  and  $px + q = r$ . A bit of scaffolding is offered in the first activity to reactivate their understanding of tape diagrams, but after that no scaffolding is offered so that students can make sense of problems (MP1) and choose representations to use (MP5).

## Alignments

### Addressing

- 7.EE.B.3: Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. \$
- 7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
- 7.EE.B.4.a: Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

## Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- Group Presentations
- MLR7: Compare and Connect
- Think Pair Share

## Required Materials

### Sticky notes

### Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart

paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

## Required Preparation

Decide if students will conduct group presentations or a gallery walk for the last activity. If so, prepare tools for creating a visual display and around 3 sticky notes per student. If not, these materials are not necessary.

### Student Learning Goals

Let's use tape diagrams, equations, and reasoning to solve problems.

# 11.1 Remember Tape Diagrams

### Warm Up: 5 minutes

The purpose of this warm-up is to reactivate students' understanding of tape diagrams to make it more likely that tape diagrams are accessible as a tool for them to choose in this lesson. The diagram was deliberately constructed to encourage some students to write an equation like  $24 = 3(a + 2)$  and others like  $24 = 3a + 6$ . Monitor for one student who writes each type of equation.

### Addressing

- 7.EE.B.4.a

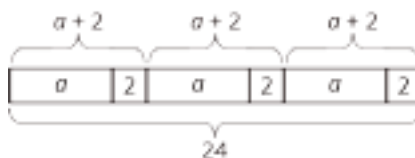
### Instructional Routines

- Think Pair Share

### Launch

Arrange students in groups of 2. Give 5 minutes of quiet think time and time to share their work with a partner followed by a whole-class discussion.

### Student Task Statement



1. Write a story that could be represented by this tape diagram.

2. Write an equation that could be represented by this tape diagram.

### Student Response

1. Answers vary. Sample story: A baker put  $a$  cookies in each of three boxes. Then, he put 2 more cookies in each box, and there were 24 total cookies in the 3 boxes.
2.  $24 = 3(a + 2)$  or  $24 = 3a + 6$  or equivalent

### Activity Synthesis

After students have had a chance to share their work with their partner, select a few students to share their stories. Then, select one student to share each type of equation and explain its structure:  $3(a + 2) = 24$  and  $3a + 6 = 24$ .

## 11.2 At the Fair

### 15 minutes

In this activity, students use a tape diagram to help them reason about a situation, write an equation that represents it, and solve the equation. Students can use both the diagram and the solution strategy of doing the same to each side and undoing that they saw in the past few lessons. The first two questions provide more scaffolding and the last question provides none.

When students work on the last question, monitor for students who

- reason numerically without any diagrams or representations.
- create a tape diagram and use it to reason numerically.
- write an equation like  $6(x - 1.5) = 46.5$  and solve it by using the distributive property to find the total amount saved,  $6 \cdot 1.50$ .
- write an equation and solve it by first dividing by 6 to find the cost of each discounted ticket.

### Addressing

- 7.EE.B.3
- 7.EE.B.4.a

### Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect

### Launch

Keep students in the same groups. Give students 5–10 minutes of quiet work time and partner discussions followed by a whole-class discussion.

## Access for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.

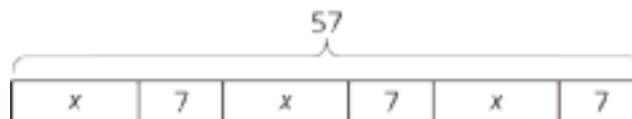
*Supports accessibility for: Organization; Attention*

### Student Task Statement

1. Tyler is making invitations to the fair. He has already made some of the invitations, and he wants to finish the rest of them within a week. He is trying to spread out the remaining work, to make the same number of invitations each day. Tyler draws a diagram to represent the situation.



- a. Explain how each part of the situation is represented in Tyler's diagram:
- How many total invitations Tyler is trying to make.
  - How many invitations he has made already.
  - How many days he has to finish the invitations.
- b. How many invitations should Tyler make each day to finish his goal within a week? Explain or show your reasoning.
- c. Use Tyler's diagram to write an equation that represents the situation. Explain how each part of the situation is represented in your equation.
- d. Show how to solve your equation.
2. Noah and his sister are making prize bags for a game at the fair. Noah is putting 7 pencil erasers in each bag. His sister is putting in some number of stickers. After filling 3 of the bags, they have used a total of 57 items.



- a. Explain how the diagram represents the situation.
- b. Noah writes the equation  $3(x + 7) = 57$  to represent the situation. Do you agree with him? Explain your reasoning.

- c. How many stickers is Noah's sister putting in each prize bag? Explain or show your reasoning.
3. A family of 6 is going to the fair. They have a coupon for \$1.50 off each ticket. If they pay \$46.50 for all their tickets, how much does a ticket cost without the coupon? Explain or show your reasoning. If you get stuck, consider drawing a diagram or writing an equation.

### Student Response

- 122 total invitations. 66 have been made already. 7 days to finish because there are 7 boxes with an equal, unknown amount  $x$ .
  8. Sample reasoning: subtract 66 from 122 and divide the result, 56, by 7.
  - $7x + 66 = 122$ . He makes the same amount  $x$  each of 7 days, so  $7x$  represents the number made in 7 days. He already made 66, so add those on, for a total of 122.
  - $7x + 66 = 122$ ,  $7x + 66 + (-66) = 122 + (-66)$ ,  $7x = 56$ ,  $7x \div 7 = 56 \div 7$ ,  $x = 8$ .
- Answers vary. Sample response: There are 3 groups of  $x$  stickers and 3 groups of 7 erasers. All together there are 57 items.
  - Answers vary. Sample response: Yes, Noah's equation says that 3 groups of  $x + 7$  gives a total of 57 items.
  12. Explanations vary. Sample responses: Each group of  $x + 7$  represents  $57 \cdot \frac{1}{3}$  or 19 items, so  $x = 12$ . Another way to find  $x$  is to subtract 21 from 57 and then  $3x = 57 - 21 = 36$  so  $x = 12$ . The first strategy represents first multiplying each side of the equation by  $\frac{1}{3}$ , and the second represents using the distributive property to write  $3(x + 7)$  as  $3x + 21$ .
- \$9.25. Explanations vary. Sample response: Divide  $46.50 \div 6 = 7.75$  to find what they paid for each ticket and then add \$1.50. Another way is to reason that they saved a total of  $6 \cdot 1.50 = 9$ , so add the 9 back to 46.50 to find the price of 6 tickets without the coupon, and then divide by 6. These two strategies connect to writing the equation  $6(t - 1.50) = 46.50$ , and solving it either by dividing by 6 first, or by using the distributive property to write  $6t - 9 = 46.50$ ,  $6t = 55.50$ ,  $t = 9.25$ .

### Activity Synthesis

Invite selected students to share their strategies for the last problem, following the sequence of approaches in the Activity Narrative. As students present, display the different approaches side by side, and ask students to explain the meaning of the numbers they find.

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### Access for English Language Learners

*Speaking, Representing: MLR7 Compare and Connect.* Use this routine during the whole-group discussion as students compare approaches for solving the last problem. Comparisons should focus on different representations of the situation. Listen for phrases like: “I used the tape diagram because....” to highlight students’ justifications for the representations they choose. Then ask students, “What is the same and what is different?” between the approaches. Amplify language that connect quantities between representations (e.g., “How is the ‘family of 6’ represented in your equation? In your diagram?”). This will help students produce language that describes their thinking about the connections between representations and approaches.

*Design Principle(s): Maximize meta-awareness; Support sense-making*

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## 11.3 Running Around

15 minutes

This activity offers four word problems. Depending on time constraints, you may have all students complete all four problems or assign a different problem to each group. The problems increase in difficulty. It is suggested that students create a visual display of one of the problems and do a gallery walk or presentation, but if time is short, you may choose to just have students work in their workbooks or devices.

### Addressing

- 7.EE.B.3
- 7.EE.B.4
- 7.EE.B.4.a

### Instructional Routines

- Group Presentations

### Launch

Keep students in the same groups. Either instruct students to complete all four problems or assign one problem to each group. If opting to have students do presentations or a gallery walk, distribute tools for making a visual display.

Give students 5–6 minutes quiet work time and a partner discussion followed by a whole-class discussion or gallery walk.

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## Access for Students with Disabilities

*Representation: Internalize Comprehension.* Provide appropriate reading accommodations and supports to ensure students access to written directions, word problems, and other text-based content.

*Supports accessibility for: Language; Conceptual processing*

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## Anticipated Misconceptions

The phrases “9 times as far” and “9 times as many” may lead students to think about multiplying by 9 instead of dividing (or multiplying by  $\frac{1}{9}$ ). Encourage students to act out the situations or draw diagrams to help reason about the relationship between the quantities. Remind them to pay careful attention to what or who a comparison refers to.

## Student Task Statement

Priya, Han, and Elena, are members of the running club at school.

1. Priya was busy studying this week and ran 7 fewer miles than last week. She ran 9 times as far as Elena ran this week. Elena only had time to run 4 miles this week.
  - a. How many miles did Priya run last week?
  - b. Elena wrote the equation  $\frac{1}{9}(x - 7) = 4$  to describe the situation. She solved the equation by multiplying each side by 9 and then adding 7 to each side. How does her solution compare to the way you found Priya's miles?
2. One day last week, 6 teachers joined  $\frac{5}{7}$  of the members of the running club in an after-school run. Priya counted a total of 31 people running that day. How many members does the running club have?
3. Priya and Han plan a fundraiser for the running club. They begin with a balance of -80 because of expenses. In the first hour of the fundraiser they collect equal donations from 9 family members, which brings their balance to -44. How much did each parent give?
4. The running club uses the money they raised to pay for a trip to a canyon. At one point during a run in the canyon, the students are at an elevation of 128 feet. After descending at a rate of 50 feet per minute, they reach an elevation of -472 feet. How long did the descent take?

## Student Response

1. a. 43 miles

b. Answers vary. Sample response: Since Priya ran 9 times as far as Elena, I multiplied 9 by 4 to get 36 miles. This is the number of miles Priya ran this week, so 36 is 7 less than what she ran last week. I added 36 and 7 to get 43 miles. Elena solved an equation and I worked backwards with the information in the problem (or used a diagram). But we both took the same steps, multiply by 9 and add 7.

2. 35 members (one way is to solve the equation  $\frac{5}{7}x + 6 = 31$ )

3. \$4 (one way is to solve  $-80 + 9x = -44$ )

4. 12 minutes (one way is to solve  $128 - 50x = -472$ )

### Are You Ready for More?

A musician performed at three local fairs. At the first he doubled his money and spent \$30. At the second he tripled his money and spent \$54. At the third, he quadrupled his money and spent \$72. In the end he had \$48 left. How much did he have before performing at the fairs?

### Student Response

\$29.

Arithmetic solution: Work backwards:  $48 + 72 = 120$ ,  $120 \div 4 = 30$ ,  $30 + 54 = 84$ ,  $84 \div 3 = 28$ ,  $28 + 30 = 58$ ,  $58 \div 2 = 29$ .

Algebraic solution: Let  $m$  represent original amount of money. After first fair:  $2x - 30$ . After second fair:  $3(2x - 30) - 54 = 6x - 144$ . After third fair,  $4(6x - 144) - 72 = 24x - 648$ . Write equation  $24x - 648 = 48$  to show how much money is left. Solution is  $696 \div 24 = 29$ .

### Activity Synthesis

If students created a visual display and you opt to conduct a gallery walk, ask students to post their solutions. Distribute sticky notes and ask students to read others' solutions, using the sticky notes to leave questions or comments. Give students a moment to review any questions or comments left on their display.

Invite any students who chose to draw a diagram to share; have the class agree or disagree with their diagrams and suggest any revisions. Next, invite students who did not try to draw a diagram to share strategies. Ask students about any difficulties they had creating the expressions and equations. Did the phrase "9 times as many" suggest an incorrect expression? If yes, how did they catch and correct for this error?

### Lesson Synthesis

Ask students to reflect on the work done in this unit so far. What strategies have they learned? What kinds of problems can they solve that they weren't able to, previously? Ask them to write down or share with a partner one new thing they have learned and one thing they still have questions or confusion about.

## 11.4 The Basketball Game

Cool Down: 5 minutes

### Addressing

- 7.EE.B.3
- 7.EE.B.4.a

### Student Task Statement

Diego scored 9 points less than Andre in the basketball game. Noah scored twice as many points as Diego. If Noah scored 10 points, how many points did Andre score?

### Student Response

14 points. Explanations vary. Sample responses:

- Equation:  $2(x - 9) = 10$ , where  $x$  is the number of points scored by Andre.  $x - 9 = 5$ ,  $x = 14$ .
- Reasoning: Diego scored half as many points as Noah, so he scored 5 points. Andre scored 9 points more than Diego, or 14 points.
- Diagram: One possibility is two boxes each with  $x - 9$  showing a total of 10. Each box represents 5 points, so  $x$  is 14.

### Student Lesson Summary

Many problems can be solved by writing and solving an equation. Here is an example:

Clare ran 4 miles on Monday. Then for the next six days, she ran the same distance each day. She ran a total of 22 miles during the week. How many miles did she run on each of the 6 days?

One way to solve the problem is to represent the situation with an equation,  $4 + 6x = 22$ , where  $x$  represents the distance, in miles, she ran on each of the 6 days. Solving the equation gives the solution to this problem.

$$\begin{aligned}4 + 6x &= 22 \\6x &= 18 \\x &= 3\end{aligned}$$

# Lesson 11 Practice Problems

## Problem 1

### Statement

Find the value of each variable.

a.  $a \cdot 3 = -30$

b.  $-9 \cdot b = 45$

c.  $-89 \cdot 12 = c$

d.  $d \cdot 88 = -88,000$

### Solution

a.  $a = -10$

b.  $b = -5$

c.  $c = -1,068$

d.  $d = -1,000$

(From Unit 5, Lesson 9.)

## Problem 2

### Statement

Match each equation to its solution and to the story it describes.

Equations:

a.  $5x - 7 = 3$

b.  $7 = 3(5 + x)$

c.  $3x + 5 = -7$

d.  $\frac{1}{3}(x + 7) = 5$

Solutions:

a.  $-4$

b.  $\frac{-8}{3}$

c.  $2$

d.  $8$

Stories:

- The temperature is  $-7$ . Since midnight the temperature tripled and then rose 5 degrees. What was temperature at midnight?
- Jada has 7 pink roses and some white roses. She gives all of them away: 5 roses to each of her 3 favorite teachers. How many white roses did she give away?

- A musical instrument company reduced the time it takes for a worker to build a guitar. Before the reduction it took 5 hours. Now in 7 hours they can build 3 guitars. By how much did they reduce the time it takes to build each guitar?
- A club puts its members into 5 groups for an activity. After 7 students have to leave early, there are only 3 students left to finish the activity. How many students were in each group?

## Solution

- a. 3, club activity story
- b. 2, building guitars story
- c. 1, temperature story
- d. 4, roses story

## Problem 3

### Statement

The baby giraffe weighed 132 pounds at birth. He gained weight at a steady rate for the first 7 months until his weight reached 538 pounds. How much did he gain each month?

## Solution

58 pounds. He gained  $538 - 132$ , or 406 pounds, over 7 months.  $406 \div 7 = 58$ . (Or solve  $132 + 7x = 538$ .)

## Problem 4

### Statement

Six teams are out on the field playing soccer. The teams all have the same number of players. The head coach asks for 2 players from each team to come help him move some equipment. Now there are 78 players on the field. Write and solve an equation whose solution is the number of players on each team.

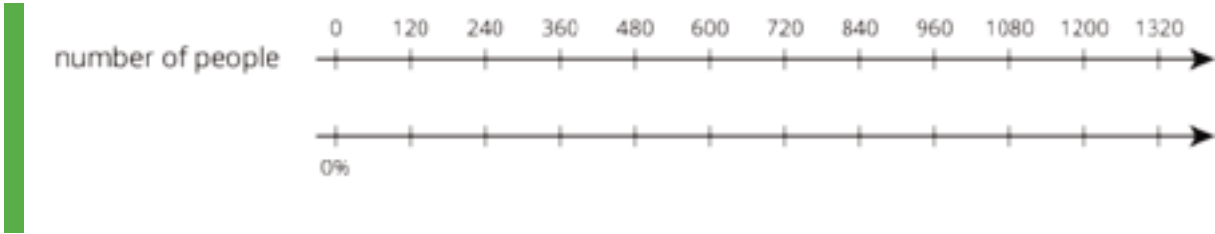
## Solution

$$6(x - 2) = 78 \text{ (or } 6x - 12 = 78), x = 15$$

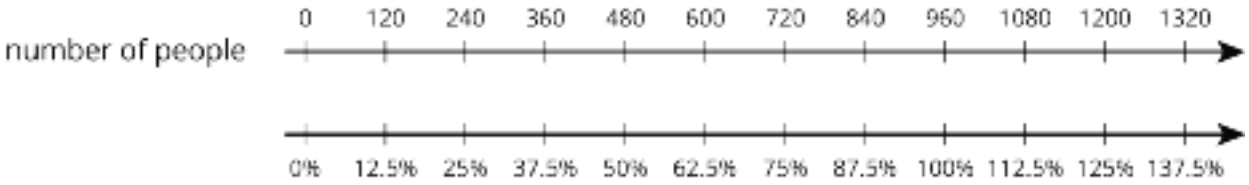
## Problem 5

### Statement

A small town had a population of 960 people last year. The population grew to 1200 people this year. By what percentage did the population grow?



**Solution**



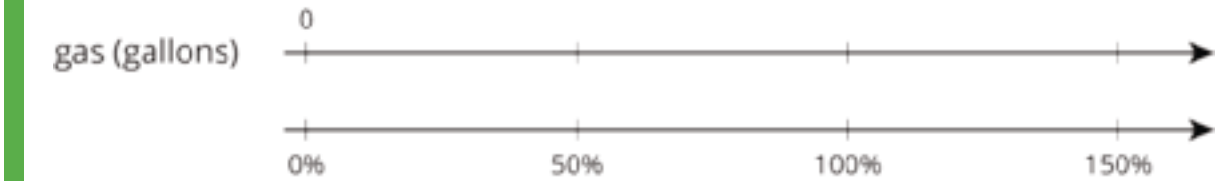
The town has grown by 25%.

(From Unit 4, Lesson 7.)

**Problem 6**

**Statement**

The gas tank of a truck holds 30 gallons. The gas tank of a passenger car holds 50% less. How many gallons does it hold?



**Solution**

15 gallons because 50% less than 30 is 15. (If the double number line is used, the tick marks on the top are labeled 0, 15, 30, 45.)

(From Unit 4, Lesson 7.)

# Lesson 12: Solving Problems about Percent Increase or Decrease

## Goals

- Solve word problems leading to equations of the form  $px + q = r$  or  $p(x + q) = r$

## Learning Targets

- I can solve story problems about percent increase or decrease by drawing and reasoning about a tape diagram or by writing and solving an equation.

## Lesson Narrative

This lesson is an opportunity for students to revisit percentages of and percentage change to solve word problems. Minimal scaffolding is provided, so students will need to make sense of the problems and perhaps attempt different solution pathways (MP1). Now, they can choose to use their deeper understanding of tape diagrams and writing and solving equations (MP5).

## Alignments

### Addressing

- 7.EE.A.2: Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example,  $a + 0.05a = 1.05a$  means that “increase by 5%” is the same as “multiply by 1.05.”
- 7.EE.B.3: Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. \$
- 7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
- 7.EE.B.4.a: Solve word problems leading to equations of the form  $px + q = r$  and  $p(x + q) = r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

## Instructional Routines

- Group Presentations
- MLR6: Three Reads
- MLR8: Discussion Supports

- Think Pair Share

## Required Materials

### Sticky notes

paper and markers, whiteboard space and

### Tools for creating a visual display

markers, shared online drawing tool, access to a

Any way for students to create work that can be easily displayed to the class. Examples: chart

document camera.

## Required Preparation

Decide if students will conduct group presentations or a gallery walk for the last activity. If so, prepare tools for creating a visual display and around 3 sticky notes per student. If not, these materials are not necessary.

## Student Learning Goals

Let's use tape diagrams, equations, and reasoning to solve problems with negatives and percents.

# 12.1 20% Off

## Warm Up: 10 minutes

The purpose of this warm-up is for students to review how to solve for percentage change and represent these situations with expressions. Analyzing the structure of equivalent expressions for the same situation helps students see how the quantities in it are related. Since there are many equivalent expressions to represent how to find percentage change in a situation, encourage students to look for relationships between the expressions.

## Addressing

- 7.EE.A.2
- 7.EE.B.4

## Launch

Arrange students in groups of 2. Give students 1 minute of quiet work time followed by 2 minutes to compare their responses with their partner. During the partner discussion, tell students to discuss the expressions they have in common, ones they don't and then try to come to an agreement on the correct expressions that represent the price of the item after the discount. Follow with a whole-class discussion.

## Anticipated Misconceptions

Some students may choose expressions that represent the discount itself instead of the price of the item after the discount. Ask those students to refer back to the situation to identify which piece of the problem the expression they chose finds. If students are still unclear, it may be helpful to give students a price for  $x$  such as \$10 and ask students if 20% of \$10 makes sense as the new price of the item after the discount and then what piece of the problem they found.

### Student Task Statement

An item costs  $x$  dollars and then a 20% discount is applied. Select **all** the expressions that could represent the price of the item after the discount.

1.  $\frac{20}{100}x$
2.  $x - \frac{20}{100}x$
3.  $(1 - 0.20)x$
4.  $\frac{100-20}{100}x$
5.  $0.80x$
6.  $(100 - 20)x$

### Student Response

Expressions 2, 3, 4, and 5 represent the price of the item after the discount.

### Activity Synthesis

Ask students to indicate whether each expression represents the price of the item after the discount. If all students agree on an expression, ask 1 or 2 students to explain their reasoning and move to the next expression. Record and display their responses for all to see. If there is a disagreement on an expression, ask students to explain their reasoning for both choices and come to an agreement.

After the class had agreed on the four expressions that represent the price of the item after the discount, record them as a list and display them all to see. Ask students to discuss any connections they see between the expressions to show they are equivalent.

## 12.2 Walking More Each Day

### 10 minutes

The first three questions help students recall how tape diagrams can be used to represent a percentage increase situation and draw connections to an equation representing percent increase. Monitor for different approaches in the first three questions (see solutions for different approaches).

The last question is review of previous work in this unit. This fourth question can be used for additional practice, but it can be safely skipped if time is short.

### Addressing

- 7.EE.B.3
- 7.EE.B.4.a

## Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

## Launch

Keep students in the same groups. Tell students to work on the first three questions and pause for discussion. Give 5 minutes of quiet work time and time to share their responses with a partner, followed by a whole-class discussion. If time permits, the last question can be used as more practice on work from earlier in the unit.

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### Access for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* To support development of organizational skills, check in with students within the first 2–3 minutes of work time. Look for students who use equivalent fractions and percents in explaining their understanding of the diagram.

*Supports accessibility for: Memory; Organization*

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### Access for English Language Learners

*Representing, Speaking: MLR8 Discussion Supports.* Use this routine to help students understand how tape diagrams can be used to represent a percentage increase situation. Invite students to label their tape diagrams to show what each section represents for Day 1, 2, and 3. Arrange students in groups of 2. Ask groups to compare how they used the tape diagrams to solve each question. Listen for common language students use to describe different approaches.

*Design Principle(s): Support sense-making; Maximize meta-awareness*

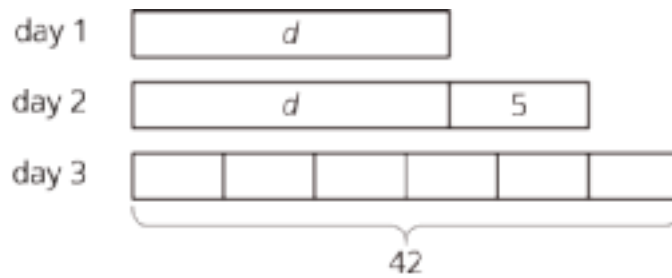
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## Anticipated Misconceptions

If students bring up that the diagram represents 120% or  $\frac{6}{5}$ , or if they refer to each equal part as 20% or  $\frac{1}{5}$ , ask what whole the fraction or percent refers to. They should understand that the whole is the amount from Day 2,  $d + 5$ .

### Student Task Statement

1. Mai started a new exercise program. On the second day, she walked 5 minutes more than on the first day. On the third day, she increased her walking time from day 2 by 20% and walked for 42 minutes. Mai drew a diagram to show her progress.



Explain how the diagram represents the situation.

- Noah said the equation  $1.20(d + 5) = 42$  also represents the situation. Do you agree with Noah? Explain your reasoning.
- Find the number of minutes Mai walked on the first day. Did you use the diagram, the equation, or another strategy? Explain or show your reasoning.
- Mai has been walking indoors because of cold temperatures. On Day 4 at noon, Mai hears a report that the temperature is only 9 degrees Fahrenheit. She remembers the morning news reporting that the temperature had doubled since midnight and was expected to rise 15 degrees by noon. Mai is pretty sure she can draw a diagram to represent this situation but isn't sure if the equation is  $9 = 15 + 2t$  or  $2(t + 15) = 9$ . What would you tell Mai about the diagram and the equation and how they might be useful to find the temperature,  $t$ , at midnight?

### Student Response

- Answers vary. Sample response: The last day is day 2 plus  $\frac{1}{5}$  (20%) of day 2. Day 2 is 5 more than Day 1.
- Answers vary. Sample responses: Yes, she walked 42 minutes on Day 3, which is the same as (equal to) 20% more than (1.20 times) 5 more than Day 1 ( $d + 5$ ). No, I wrote the equation  $\frac{6}{5}(d + 5) = 42$  because the diagram shows that 42 is  $\frac{1}{5}$  more than  $d + 5$ .
- 30 minutes. Answers vary. Sample responses:
  - From the diagram,  $42 \div 6 = 7$ ,  $7 \cdot 5 = 35$ ,  $35 - 5 = 30$
  - From the equation,  $1.2(d + 5) = 42$ ,  $d + 5 = 42 \div 1.2$ ,  $d + 5 = 35$ ,  $d = 30$
  - From another version of the equation:  $\frac{6}{5}(d + 5) = 42$ ,  $d + 5 = \frac{5}{6} \cdot 42$ ,  $d + 5 = 35$ ,  $d = 30$
- Answers vary. Sample response: Since the temperature doubled and then increased by 15, the diagram would show two equal parts and another part of 15, all with a total of 9. The equation would be  $2t + 15 = 9$  (or equivalent). The temperature at midnight can be found with the equation or the diagram by subtracting 15 from 9 to get -6 and dividing by 2 to get -3. The temperature,  $t$ , at midnight was -3 degrees Fahrenheit.

### Activity Synthesis

Select groups with different approaches to share their responses to the first three questions.

## 12.3 A Sale on Shoes

15 minutes

This activity offers four word problems. Depending on time constraints, you may have all students complete all four problems or assign a different problem to each group. The problems increase in difficulty. It is suggested that students create a visual display of one of the problems and do a gallery walk or presentation, but if time is short you may choose to just have students work in their workbooks or devices.

### Addressing

- 7.EE.A.2
- 7.EE.B.3
- 7.EE.B.4.a

### Instructional Routines

- Group Presentations
- MLR6: Three Reads

### Launch

Keep students in the same groups. Either instruct students to complete all four problems or assign one problem to each group. If opting to have students do presentations or a gallery walk, distribute tools for making a visual display.

Give students 5–6 minutes quiet work time and partner discussion followed by a whole-class discussion or gallery walk.

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### Access for Students with Disabilities

*Representation: Internalize Comprehension.* Represent the same information through different modalities by using diagrams, equations, or other drawings to depict the situation. If students are unsure where to begin, suggest that they draw a diagram to help organize the information provided. During the Synthesis, annotate drawings to illustrate connections between representations.

*Supports accessibility for: Conceptual processing; Visual-spatial processing*

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## Access for English Language Learners

*Reading, Representing: MLR6 Three Reads.* Use this routine to support reading comprehension of the first problem without solving it for students. For the first read, read the situation to students, without revealing the final question. Ask students "What is this question about?" (A store is having a sale. Diego is buying shoes with a coupon). In the second read, ask students to name the important quantities (e.g., discount of 20%, coupon for \$3 off of the regular price, Diego pays \$18.40), and then create a diagram to represent the relationships among these quantities. After the third read, ask students to brainstorm possible strategies to answer the question, "What was the original price before the sale and without the coupon?" This will help students connect the language in the word problem and reasoning needed to solve percent problems.

*Design Principle(s): Support sense-making; Maximize meta-awareness*

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### Student Task Statement

1. A store is having a sale where all shoes are discounted by 20%. Diego has a coupon for \$3 off of the regular price for one pair of shoes. The store first applies the coupon and then takes 20% off of the reduced price. If Diego pays \$18.40 for a pair of shoes, what was their original price before the sale and without the coupon?
2. Before the sale, the store had 100 pairs of flip flops in stock. After selling some, they notice that  $\frac{3}{5}$  of the flip flops they have left are blue. If the store has 39 pairs of blue flip flops, how many pairs of flip flops (any color) have they sold?
3. When the store had sold  $\frac{2}{9}$  of the boots that were on display, they brought out another 34 pairs from the stock room. If that gave them 174 pairs of boots out, how many pairs were on display originally?
4. On the morning of the sale, the store donated 50 pairs of shoes to a homeless shelter. Then they sold 64% of their remaining inventory during the sale. If the store had 288 pairs after the donation and the sale, how many pairs of shoes did they have at the start?

## Student Response

1. \$26. Explanations vary. Sample response: I wrote the equation  $0.8(x - 3) = 18.40$  to show that Diego paid 80% (0.8) of the original price  $x$  less the \$3 coupon ( $x - 3$ ), which came to a discounted price of \$18.40.
2. 35 pairs of flip flops.  $\frac{3}{5}(100 - x) = 39$ ,  $x = 35$ .
3. 180 pairs of boots.  $\frac{7}{9}x + 34 = 174$ ,  $x = 180$ .
4. 850 pairs of shoes.  $0.36(x - 50) = 288$ ,  $x = 850$ .

### Are You Ready for More?

A coffee shop offers a special: 33% extra free or 33% off the regular price. Which offer is a better deal? Explain your reasoning.

## Student Response

Answers vary. Sample response: 33% off the price is a better deal. Suppose you buy 1 cup of coffee at price  $p$ . 33% off means you pay  $0.67p$  for one cup. 33% extra free means you pay  $p$  for 1.33 cups of coffee, or  $\frac{p}{1.33}$  for 1 cup, which is about  $0.75p$ . The unit price for 1 cup of coffee is less with 33% off the price.

## Activity Synthesis

If students created a visual display and you opt to conduct a gallery walk, ask students to post their solutions. Distribute sticky notes and ask students to read others' solutions, using the sticky notes to leave questions or comments. Give students a moment to review any questions or comments left on their display.

Invite any students who chose to draw a diagram to share; have the class agree or disagree with their diagrams and suggest any revisions. Next, invite students who did not try to draw a diagram to share strategies. Ask students about any difficulties they had creating the expressions and equations. Highlight equivalent expressions that represent the same quantity and different strategies for solving equations.

## Lesson Synthesis

Ask students to reflect on the work done in this unit so far. What strategies have they learned? What kinds of problems can they solve that they weren't able to, previously? Ask them to write down or share with a partner one new thing they have learned and one thing they still have questions or confusion about.

# 12.4 Timing the Relay Race

Cool Down: 5 minutes

## Addressing

- 7.EE.A.2
- 7.EE.B.3
- 7.EE.B.4.a

### Student Task Statement

The track team is trying to reduce their time for a relay race. First they reduce their time by 2.1 minutes. Then they are able to reduce that time by  $\frac{1}{10}$ . If their final time is 3.96 minutes, what was their beginning time? Show or explain your reasoning.

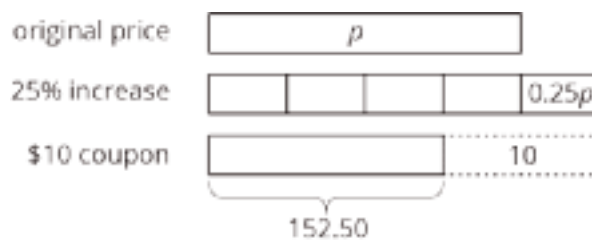
### Student Response

6.5 minutes. Explanations vary. Sample responses:

- With equation:  $.9(x - 2.1) = 3.96$ ,  $x - 2.1 = 4.4$ ,  $x = 6.5$ .
- Reasoning with or without a diagram: 9 out of 10 parts represent 3.96 minutes, so the  $\frac{1}{10}$  reduction was  $3.96 \div 9$  or 0.44 minutes. That makes the time before the 2.1 minute reduction  $3.96 + 0.44$  or 4.4 minutes. The original time was  $4.4 + 2.1$  or 6.5 minutes.

### Student Lesson Summary

We can solve problems where there is a percent increase or decrease by using what we know about equations. For example, a camping store increases the price of a tent by 25%. A customer then uses a \$10 coupon for the tent and pays \$152.50. We can draw a diagram that shows first the 25% increase and then the \$10 coupon.



The price after the 25% increase is  $p + 0.25p$  or  $1.25p$ . An equation that represents the situation could be  $1.25p - 10 = 152.50$ . To find the original price before the increase and discount, we can add 10 to each side and divide each side by 1.25, resulting in  $p = 130$ . The original price of the tent was \$130.

# Lesson 12 Practice Problems

## Problem 1

### Statement

A backpack normally costs \$25 but it is on sale for \$21. What percentage is the discount?

### Solution

16%

(From Unit 4, Lesson 12.)

## Problem 2

### Statement

Find each product.

a.  $\frac{2}{5} \cdot (-10)$

b.  $-8 \cdot \left(\frac{-3}{2}\right)$

c.  $\frac{10}{6} \cdot 0.6$

d.  $\left(\frac{-100}{37}\right) \cdot (-0.37)$

### Solution

a. -4

b. 12

c. 1

d. 1

(From Unit 5, Lesson 9.)

## Problem 3

### Statement

Select all expressions that show  $x$  increased by 35%.

- A.  $1.35x$
- B.  $\frac{35}{100}x$
- C.  $x + \frac{35}{100}x$
- D.  $(1 + 0.35)x$
- E.  $\frac{100+35}{100}x$
- F.  $(100 + 35)x$

## Solution

["A", "C", "D", "E"]

## Problem 4

### Statement

Complete each sentence with the word *discount*, *deposit*, or *withdrawal*.

- a. Clare took \$20 out of her bank account. She made a \_\_\_\_.
- b. Kiran used a coupon when he bought a pair of shoes. He got a \_\_\_\_.
- c. Priya put \$20 into her bank account. She made a \_\_\_\_.
- d. Lin paid less than usual for a pack of gum because it was on sale. She got a \_\_\_\_.

## Solution

- a. withdrawal
- b. discount
- c. deposit
- d. discount

(From Unit 4, Lesson 11.)

## Problem 5

### Statement

Here are two stories:

- The initial freshman class at a college is 10% smaller than last year's class. But then during the first week of classes, 20 more students enroll. There are then 830 students in the freshman class.

- A store reduces the price of a computer by \$20. Then during a 10% off sale, a customer pays \$830.

Here are two equations:

- $0.9x + 20 = 830$

- $0.9(x - 20) = 830$

- Decide which equation represents each story.
- Explain why one equation has parentheses and the other doesn't.
- Solve each equation, and explain what the solution means in the situation.

## Solution

Answers vary. Sample responses:

- The freshman class:  $0.9x + 20 = 830$ , computer:  $0.9(x - 20) = 830$
- It depends on which came first, the additive increase or decrease (parentheses needed) or the percent decrease (no parentheses needed, since the convention is to multiply before adding when there are no parentheses).
- The freshman class:  $x = 900$ , which is the size of last year's freshman class. Computer:  $x = 942.22$  (rounding to the nearest cent), which is the original price of the computer.

## Section: Inequalities

### Lesson 13: Reintroducing Inequalities

#### Goals

- Comprehend the terms “less than or equal to” and “greater than or equal to” (in spoken and written language) and the symbols  $\leq$  and  $\geq$  (in written language).
- Recognize that more than one value for a variable makes the same inequality true.
- Use substitution to determine whether a given value for a variable makes an inequality true, and justify (orally) the answer.

#### Learning Targets

- I can explain what the symbols  $\leq$  and  $\geq$  mean.
- I can represent an inequality on a number line.
- I understand what it means for a number to make an inequality true.

#### Lesson Narrative

In grade 6, students learned how to write and interpret inequalities of the form  $x < c$  and  $x > c$ . In this lesson, students begin to investigate inequalities of the form  $px < q$  and  $x + p < q$ .

First, they are reintroduced to the notation  $<$  and  $>$  and reminded how inequalities can be expressed algebraically and graphically on a number line. A context is used to help students make sense of inequalities. The symbols  $\leq$  and  $\geq$  are introduced, which are the relevant symbols to use in many of the modeling problems they will see later on. Then they use substitution to check whether given values of  $x$  satisfy inequalities.

#### Alignments

##### Building On

- 6.EE.B.5: Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

##### Addressing

- 7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

## Building Towards

- 7.EE.B.4.b: Solve word problems leading to inequalities of the form  $px + q > r$  or  $px + q < r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. \$

## Instructional Routines

- MLR5: Co-Craft Questions
- MLR8: Discussion Supports
- Think Pair Share

## Student Learning Goals

Let's work with inequalities.

# 13.1 Greater Than One

## Warm Up: 5 minutes

The purpose of this activity is for students to substitute values into an inequality and check to see if each value satisfies the inequality based on a number line representation. This is the first opportunity (of many to come) to practice this type of substitution with inequalities.

## Building On

- 6.EE.B.5

## Building Towards

- 7.EE.B.4.b

## Instructional Routines

- Think Pair Share

## Launch

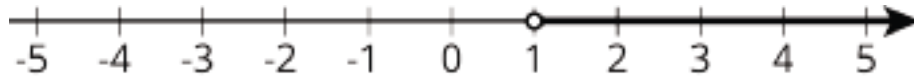
Arrange students in groups of 2. Since it may have been a while since students encountered this notation, remind students that  $x > 1$  is read “ $x$  is greater than 1.” Give students 2 minutes of quiet work time followed by 1 minute to share their responses with a partner. During the partner discussion, tell students to compare their answers for the first question and see if they agree with their partner’s chosen values for the second question. Follow with a whole-class discussion.

## Anticipated Misconceptions

Some students may think 700 is not a solution to  $x > 1$ . Tell students that since there is an arrow at the end of the dark line, it includes all values that would fall on that line, even the ones not shown.

## Student Task Statement

The number line shows values of  $x$  that make the inequality  $x > 1$  true.



1. Select **all** the values of  $x$  from this list that make the inequality  $x > 1$  true.

- a. 3
- b. -3
- c. 1
- d. 700
- e. 1.05

2. Name two more values of  $x$  that are solutions to the inequality.

### Student Response

1. a (3), d (700), and e (1.05)
2. Answers vary. Sample response: 4 and 10.

### Activity Synthesis

Ask a few students to share their responses for the last question. After each student shares, ask the class whether they agree or disagree. If students focus solely on the number line representation, be sure to tell them that to test whether a value makes an inequality true, you can substitute the value for the variable. Highlight the fact that “greater than 1” does not include 1.

Here are some questions for discussion:

- What does the open circle at 1 mean? (It means 1 is not included.)
- Is 1 a solution to the inequality  $x > 1$ ?

## 13.2 The Roller Coaster

### 15 minutes

The purpose of this activity is to remind students that the symbol  $<$  is read “is less than” and the symbol  $>$  is read “is greater than.” Also, remind students of the use of an open circle or closed circle to indicate that the boundary point is included. Then, the symbols  $\leq$  and  $\geq$  are introduced.

Monitor for students who express the answer to the last question using words or using symbols. The responses to the last question will be used to introduce the new notation.

### Building Towards

- 7.EE.B.4.b

## Instructional Routines

- MLR5: Co-Craft Questions
- Think Pair Share

## Launch

Allow students 5–10 minutes quiet work time and time to share their responses with a partner, followed by a whole-class discussion.

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### Access for Students with Disabilities

*Representation: Access for Perception.* Read all problems aloud. Students who both listen to and read the information will benefit from extra processing time.

*Supports accessibility for: Language*

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### Access for English Language Learners

*Conversing, Writing: MLR5 Co-craft Questions.* Use this routine to help students consider the context of the first problem and to increase awareness about language used to describe situations involving inequalities. Begin by displaying only the initial text and photo of the roller coaster, without revealing the follow-up questions. In groups of 2, invite students to write down mathematical questions they have about this situation. Ask pairs to share their questions with the whole class. Amplify questions that highlight the mathematical language of “at least.”

*Design Principle(s): Cultivate conversation; Maximize meta-awareness*

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## Anticipated Misconceptions

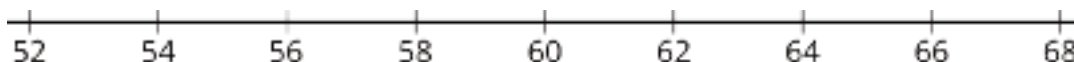
If students are having trouble interpreting the first three questions or articulating their responses, encourage them to make use of the number line that appears in question 4.

### Student Task Statement

A sign next to a roller coaster at an amusement park says, “You must be at least 60 inches tall to ride.” Noah is happy to know that he is tall enough to ride.



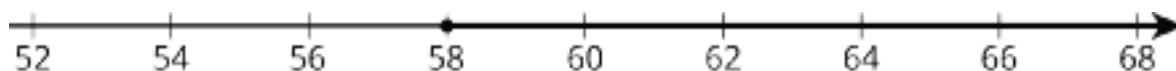
1. Noah is  $x$  inches tall. Which of the following can be true:  $x > 60$ ,  $x = 60$ , or  $x < 60$ ? Explain how you know.
2. Noah's friend is 2 inches shorter than Noah. Can you tell if Noah's friend is tall enough to go on the ride? Explain or show your reasoning.
3. List one possible height for Noah that means that his friend is tall enough to go on the ride, and another that means that his friend is too short for the ride.
4. On the number line below, show all the possible heights that Noah's friend could be.



5. Noah's friend is  $y$  inches tall. Use  $y$  and any of the symbols  $<$ ,  $=$ ,  $>$  to express this height.

### Student Response

1.  $x > 60$  (or  $x = 60$ ). Explanations vary. Sample response: "At least" means that Noah must be 60 inches or taller.
2. No, we don't know if Noah's friend is tall enough to go on the ride. Explanations vary. Sample response: Since we don't know whether Noah's height is exactly 60, within 2 inches of 60, or more than 2 inches above 60, we can't know if 2 less than his height is at least 60.
3. Answers vary. Sample response: Noah could be 63 inches tall, which means his friend is 61 inches tall and can ride. Noah can be 61 inches tall, which means his friend is 59 inches tall and cannot ride.
- 4.



5.  $y > 58$  (or  $y = 58$ ).

### Activity Synthesis

Ask selected students to share their response to the last question. They are likely to write something like " $y = 58$  or  $y > 58$ ." The  $<$  and  $>$  symbols are not enough to capture what we need here with a single mathematical statement. In grade 6, students saw graphs of inequalities using open and closed circles. Now, we can introduce new symbols  $\leq$  and  $\geq$ , that mean "less than or equal to" and "greater than or equal to."

## 13.3 Is the Inequality True or False?

15 minutes

The purpose of this task is for students to interpret the notation  $<$ ,  $>$ ,  $\leq$ , and  $\geq$  to evaluate whether different values make an inequality true. It is not expected that students will solve the inequalities

generally. The work in this activity sets students up for success in later lessons when they test points to determine the direction of the inequality in the process of solving.

Notice students who use the following approaches:

- Substituting a value in for  $x$ , evaluating the expression, thinking about whether the statement is true. For example, “Is  $-300$  less than  $75$ ? Yes, so this is true.”
- Drawing a number line for each inequality and using it to reason about different values.

### Building Towards

- 7.EE.B.4.b

### Instructional Routines

- MLR8: Discussion Supports

### Launch

Keep students in the same groups. Give 5–10 minutes quiet and partner work time followed by a whole-class discussion.

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#### Access for Students with Disabilities

*Engagement: Internalize Self Regulation.* Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. After students have completed the first 1–2 rows of the table, consider pausing for a brief class discussion before moving on.

*Supports accessibility for: Organization; Attention*

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### Anticipated Misconceptions

Students who try to apply what they know about solving equations to solve the inequalities algebraically may come up with incorrect solutions. For instance,  $100 < 4x$  may at first glance look equivalent to  $x < 25$ , since the “less than” sign appears. Students may incorrectly think that  $-3x > -75$  is equivalent to  $x > 25$ . Ask these students, for example, what the solution to  $100 = 4x$  means ( $25$  is the value of  $x$  that makes  $4x$  equal to  $100$ ). Then encourage these students to test values like  $24$  and  $26$  to see whether they are solutions to  $100 < 4x$ . This will be covered in greater detail in a later lesson, so this understanding does not need to be solidified at this time.

### Student Task Statement

The table shows four inequalities and four possible values for  $x$ . Decide whether each value makes each inequality true, and complete the table with “true” or “false.” Discuss your thinking with your partner. If you disagree, work to reach an agreement.

$x$	0	100	-100	25
$x \leq 25$				
$100 < 4x$				
$-3x > -75$				
$10 \geq 35 - x$				

### Student Response

$x$	0	100	-100	25
$x \leq 25$	true	false	true	true
$100 < 4x$	false	true	false	false
$-3x > -75$	true	false	true	false
$10 \geq 35 - x$	false	true	false	true

### Are You Ready for More?

Find an example of an inequality used in the real world and describe it using a number line.

### Student Response

Answers vary.

### Activity Synthesis

The purpose of the discussion is to note the consequences of an inequality using  $\leq$  versus  $<$ , and  $\geq$  versus  $>$ . Direct students' attention to  $100 < 4x$  and  $10 \geq 35 - x$ . Substituting 25 for  $x$  in each of these inequalities gives  $100 < 100$ , which is false, and  $10 \geq 10$ , which is true. The key distinction is that  $\geq$  and  $\leq$  inequalities are considered true when both sides are equal, whereas  $<$  and  $>$  inequalities are considered false when both sides are equal. Ask students to present different strategies for determining whether a value makes an inequality true. Ask whether students were surprised by or initially incorrect about any of the answers. Emphasize that substituting a value in for  $x$ , and thinking about whether the resulting inequality is saying something true, is the most direct way to check whether the value is a solution.

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### Access for English Language Learners

*Speaking, Conversing: MLR8 Discussion Supports.* Provide students with sentence frames that will help them define what each inequality symbol means. For example, “\_\_\_ is greater than or equal to \_\_\_.”, “\_\_\_ is less than or equal to \_\_\_.”, “\_\_\_ is greater than \_\_\_.” and “\_\_\_ is less than \_\_\_.” Create a chart with the matching symbols for each frame and encourage students to refer to this chart when they explain whether they think each inequality is “true” or “false.”

*Design Principle(s): Support sense-making*

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## Lesson Synthesis

By the end of this lesson, students should be able to interpret the symbols  $\geq$  and  $\leq$  and be able to test whether a given value makes an inequality true. Ask students to write an equality to which  $-5$  is a solution, then trade with their partner to see if their partner agrees.

## 13.4 Some Values, All Values

**Cool Down: 5 minutes**

The purpose of this activity is for students to use any of the strategies they developed in this lesson to graph the solution to an inequality and contrast  $<$  with  $\leq$ .

### Addressing

- 7.EE.B.4

### Anticipated Misconceptions

Students may divide both sides of the inequality by  $-2$  to arrive at the incorrect solution  $x > -5$ .

### Student Task Statement

Here is an inequality:  $-2x > 10$ .

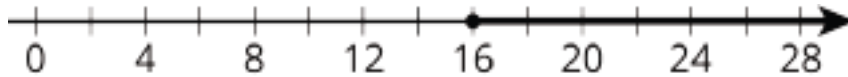
1. List some values for  $x$  that would make this inequality true.
2. How are the solutions to the inequality  $-2x \geq 10$  different from the solutions to  $-2x > 10$ ? Explain your reasoning.

### Student Response

1. Any number less than  $-5$  is a solution.
2. Responses vary. Sample response: The solutions to  $-2x > 10$  and  $-2x \geq 10$  are the same, except when  $x$  is  $-5$ . The number  $-5$  would be included as a solution to  $10 \leq -2x$  because  $10 \leq -2(-5)$  is a true statement.

## Student Lesson Summary

We use inequalities to describe a range of numbers. In many places, you are allowed to get a driver's license when you are at least 16 years old. When checking if someone is old enough to get a license, we want to know if their age is at least 16. If  $h$  is the age of a person, then we can check if they are allowed to get a driver's license by checking if their age makes the inequality  $h > 16$  (they are older than 16) or the equation  $h = 16$  (they are 16) true. The symbol  $\geq$ , pronounced "greater than or equal to," combines these two cases and we can just check if  $h \geq 16$  (their age is greater than or equal to 16). The inequality  $h \geq 16$  can be represented on a number line:



# Lesson 13 Practice Problems

## Problem 1

### Statement

For each inequality, find two values for  $x$  that make the inequality true and two values that make it false.

- a.  $x + 3 > 70$
- b.  $x + 3 < 70$
- c.  $-5x < 2$
- d.  $5x < 2$

### Solution

Answers vary. Sample response:

- a. True:  $x = 70$  and  $x = 100$ , false:  $x = 0$  and  $x = -10$
- b. True:  $x = 60$  and  $x = 0$ , false:  $x = 70$  and  $x = 100$
- c. True:  $x = 1$  and  $x = 2$ , false:  $x = -1$  and  $x = -2$
- d. True:  $x = 0$  and  $x = -1$ , false:  $x = 1$  and  $x = 100$

## Problem 2

### Statement

Here is an inequality:  $-3x > 18$ .

- a. List some values for  $x$  that would make this inequality true.
- b. How are the solutions to the inequality  $-3x \geq 18$  different from the solutions to  $-3x > 18$ ? Explain your reasoning.

### Solution

- a. Any value less than  $-6$
- b. The inequalities have almost the same solutions, but the first includes  $-6$  and the second does not.

### Problem 3

#### Statement

Here are the prices for cheese pizza at a certain pizzeria:

pizza size	price in dollars
small	11.60
medium	
large	16.25

- You had a coupon that made the price of a large pizza \$13.00. For what percent off was the coupon?
- Your friend purchased a medium pizza for \$10.31 with a 30% off coupon. What is the price of a medium pizza without a coupon?
- Your friend has a 15% off coupon and \$10. What is the largest pizza that your friend can afford, and how much money will be left over after the purchase?

#### Solution

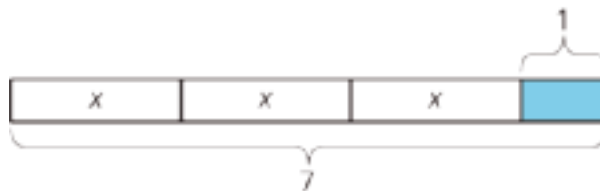
- 20%
- \$14.73
- Small, \$0.14

(From Unit 4, Lesson 12.)

### Problem 4

#### Statement

Select **all** the stories that can be represented by the diagram.



- Andre studies 7 hours this week for end-of-year exams. He spends 1 hour on English and an equal number of hours each on math, science, and history.
- Lin spends \$3 on 7 markers and a \$1 pen.
- Diego spends \$1 on 7 stickers and 3 marbles.
- Noah shares 7 grapes with 3 friends. He eats 1 and gives each friend the same number of grapes.
- Elena spends \$7 on 3 notebooks and a \$1 pen.

## **Solution**

["A", "D", "E"]

(From Unit 6, Lesson 4.)

# Lesson 14: Finding Solutions to Inequalities in Context

## Goals

- Interpret inequalities that represent situations with a constraint.
- Solve an equation of the form  $px + q = r$  to determine the boundary point for an inequality of the form  $px + q > r$  or  $px + q < r$ .
- Use substitution or reasoning about the context to justify (orally and in writing) whether the values that make an inequality true are greater than or less than the boundary point.

## Learning Targets

- I can describe the solutions to an inequality by solving a related equation and then reasoning about values that make the inequality true.
- I can write an inequality to represent a situation.

## Lesson Narrative

In this lesson, students see more examples of inequalities in a context. This time, many inequalities involve negative coefficients. This illustrates the idea that solving an inequality is not as simple as solving the corresponding equation. After students find the boundary point, they must do some extra work to figure out the direction of inequality. This might involve reasoning about the context, substituting in values on either side of the boundary point, or reasoning about number lines. All of these techniques exemplify MP1: making the problem more concrete and visual and asking “Does this make sense?” In this lesson, the emphasis is on reasoning about the context.

It is important to understand that the goal is not to have students learn and practice an algorithm for solving inequalities like “whenever you multiply or divide by a negative, flip the inequality.” Rather, we want students to understand that solving a related equation tells you the lower or upper bound of an inequality. To know whether values greater than or less than the boundary number make the inequality true, it's best to either think about the context or test some values that are above and below the boundary number. This way of reasoning about inequalities will serve students well long into their future studies, whereas students are very likely to forget a procedure memorized for a special case.

## Alignments

### Addressing

- 7.EE.B.4.b: Solve word problems leading to inequalities of the form  $px + q > r$  or  $px + q < r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. \$

## Building Towards

- 7.EE.B.4.b: Solve word problems leading to inequalities of the form  $px + q > r$  or  $px + q < r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. \$

## Instructional Routines

- MLR5: Co-Craft Questions
- MLR8: Discussion Supports
- Think Pair Share

## Required Preparation

Several activities suggest providing students with blank number lines to use for scratch work. One way to accomplish this is to print a line with unlabeled, evenly-spaced tick marks, and place these into sheet protectors. Students can write on these with dry erase markers and wipe them off.

### Student Learning Goals

Let's solve more complicated inequalities.

# 14.1 Solutions to Equations and Solutions to Inequalities

## Warm Up: 10 minutes

This warm-up highlights the link between an inequality and its associated equation. This will be solidified throughout the lesson as students solve the associated equation and reason in context to determine the direction of inequality. Notice students who use the value -10 as a boundary as they test values to find solutions to the inequalities.

## Building Towards

- 7.EE.B.4.b

## Launch

Give students 5 minutes of quiet work time followed by a whole-class discussion. Optionally, provide students with blank number lines for scratch work.

### Student Task Statement

1. Solve  $-x = 10$
2. Find 2 solutions to  $-x > 10$
3. Solve  $2x = -20$
4. Find 2 solutions to  $2x > -20$

## Student Response

1. -10
2. Answers vary. Possible responses: -12, -28.7, -209. (Any value that is less than -10 works.)
3. -10
4. Answers vary. Possible responses: -9, 0,  $82\frac{3}{4}$ . (Any value that is greater than -10 works.)

## Activity Synthesis

Display two number lines for all to see that each include -10 and some integral values to its left and right. Ask a few students to share their responses to the first two questions, recording their responses on one number line and gauging the class for agreement. Ask a few students to share their responses to the last two questions, recording their responses on the other number line and gauging the class for agreement.

Highlight the fact that  $-x = 10$  and  $2x = -20$  have the same solution (-10), but the inequalities  $-x > 10$  and  $2x > -20$  don't have the same solutions. Select students to share strategies they had for finding solutions. If not mentioned by students, discuss the fact that since -10 makes the sides equal, the neighborhood of values around -10 is a good place to start looking for solutions.

# 14.2 Earning Money for Soccer Stuff

15 minutes

Previously in this unit, students wrote expressions and equations that are similar to the ones in this activity. Here, they are prompted in a scaffolded way to notice that they can express not just that an outcome can be *equal* to a value, but that an outcome can be *at least as much as* a value by using the new notation  $\geq$ .

## Addressing

- 7.EE.B.4.b

## Instructional Routines

- MLR5: Co-Craft Questions
- Think Pair Share

## Launch

Optionally, provide access to blank number lines to use for scratch work.

Arrange students in groups of 2. Allow 10 minutes of quiet work time and partner discussion followed by a whole-class discussion. Depending on the needs of your class, you may decide to ask students to pause after the first question for the whole-class discussion before tackling the second question.

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### Access for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* To support development of organizational skills, check in with students within the first 2–3 minutes of work time. Look for students who correctly write an expression but struggle in writing an equation with the correct inequality sign. Consider pausing for a brief whole-class discussion inviting students to share strategies for determining the correct sign.

*Supports accessibility for: Memory; Organization*

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### Access for English Language Learners

*Conversing: MLR5 Co-craft Questions.* Reveal only the context for Andre’s summer job, without revealing the questions that follow. Ask students to create mathematical questions about this situation. Give students 1–2 minutes to write down mathematical questions that could be asked about the situation. Invite students to share their questions with the class, before revealing the rest of the activity. Listen for an amplify questions that contain the phrase “at least.”

*Design Principle(s): Maximize meta-awareness; Support sense-making*

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### Student Task Statement

1. Andre has a summer job selling magazine subscriptions. He earns \$25 per week plus \$3 for every subscription he sells. Andre hopes to make at least enough money this week to buy a new pair of soccer cleats.
  - a. Let  $n$  represent the number of magazine subscriptions Andre sells this week. Write an expression for the amount of money he makes this week.
  - b. The least expensive pair of cleats Andre wants costs \$68. Write and solve an equation to find out how many magazine subscriptions Andre needs to sell to buy the cleats.
  - c. If Andre sold 16 magazine subscriptions this week, would he reach his goal? Explain your reasoning.
  - d. What are some other numbers of magazine subscriptions Andre could have sold and still reached his goal?
  - e. Write an *inequality* expressing that Andre wants to make at least \$68.
  - f. Write an inequality to describe the number of subscriptions Andre must sell to reach his goal.

2. Diego has budgeted \$35 from his summer job earnings to buy shorts and socks for soccer. He needs 5 pairs of socks and a pair of shorts. The socks cost different amounts in different stores. The shorts he wants cost \$19.95.
- Let  $x$  represent the price of one pair of socks. Write an expression for the total cost of the socks and shorts.
  - Write and solve an equation that says that Diego spent exactly \$35 on the socks and shorts.
  - List some other possible prices for the socks that would still allow Diego to stay within his budget.
  - Write an inequality to represent the amount Diego can spend on a single pair of socks.

### Student Response

- $3n + 25$
  - $3n + 25 = 68, n = 14\frac{1}{3}$
  - Yes.  $16 > 14\frac{1}{3}$ . He made \$73, which is more than enough to buy the cleats.
  - Answers vary. Sample responses: 15, 17, 100. (Any whole number greater than 14 would make sense.)
  - $3n + 25 \geq 68$
  - $n \geq 14\frac{1}{3}$
- $5x + 19.95$
  - $5x + 19.95 = 35, x = 3.01$ . In this situation, Diego paid \$3.01 for each pair of socks.
  - Answers will vary. Any price under \$3.01 is an acceptable response.
  - $x \leq 3.01$ .

### Activity Synthesis

Here is what we want students to understand as a result of this activity:

In order to find the solution to an inequality like  $3n + 25 \geq 68$ , we can solve an equation to find the point where  $3n + 25 = 68$ . This is the point that separates numbers that are solutions to the inequality from numbers that are not solutions. To find whether the solution to the inequality is  $n \geq 14\frac{1}{3}$  or  $n \leq 14\frac{1}{3}$ , we can substitute some values of  $n$  that are greater than  $14\frac{1}{3}$  and some that are less than  $14\frac{1}{3}$  to check. Alternatively, we can think about the context: If Andre wants to make more money, he needs to sell more magazines, not fewer. If Diego wants to spend less than \$35, he needs to spend less for socks, not more. Ask students:

- How does solving the equation help us solve an inequality? What does the solution tell us about solutions to the inequality?
- What are some ways we can determine whether the solution to an inequality should use less than or greater than?
- How can we check whether a value is a solution to the inequality?
- Could Andre sell exactly  $14\frac{1}{3}$  subscriptions?
- Can Diego pay exactly \$3.01 for each pair of socks?
- How can we tell if there are restrictions on the solutions of the inequality, such as only positive numbers or only whole numbers?

## 14.3 Granola Bars and Savings

15 minutes

The purpose of this activity is for students to interact with contexts in which the direction of inequality is the opposite of what they might expect if they try to solve like they would with an equation. For example, in the second problem, the original inequality is  $9(7 - x) \leq 36$ , but the solution to the inequality is  $x \geq 3$ .

Some students might solve the associated equation and then test values of  $x$  to determine the direction of inequality. That method will be introduced in more generality in the next lesson. This activity emphasizes thinking about the context in deciding the direction of inequality.

### Addressing

- 7.EE.B.4.b

### Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

### Launch

Keep students in the same groups. Give 5–10 minutes of quiet work time and partner discussion followed by a whole-class discussion.

### Student Task Statement

1. Kiran has \$100 saved in a bank account. (The account doesn't earn interest.) He asked Clare to help him figure out how much he could take out each month if he needs to have at least \$25 in the account a year from now.
  - a. Clare wrote the inequality  $-12x + 100 \geq 25$ , where  $x$  represents the amount Kiran takes out each month. What does  $-12x$  represent?

- b. Find some values of  $x$  that would work for Kiran.
- c. We could express *all* the values that would work using either  $x \leq \underline{\hspace{1cm}}$  or  $x \geq \underline{\hspace{1cm}}$ . Which one should we use?
- d. Write the answer to Kiran's question using mathematical notation.
2. A teacher wants to buy 9 boxes of granola bars for a school trip. Each box usually costs \$7, but many grocery stores are having a sale on granola bars this week. Different stores are selling boxes of granola bars at different discounts.
- a. If  $x$  represents the dollar amount of the discount, then the amount the teacher will pay can be expressed as  $9(7 - x)$ . In this expression, what does the quantity  $7 - x$  represent?
- b. The teacher has \$36 to spend on the granola bars. The equation  $9(7 - x) = 36$  represents a situation where she spends all \$36. Solve this equation.
- c. What does the solution mean in this situation?
- d. The teacher does not have to spend all \$36. Write an inequality relating 36 and  $9(7 - x)$  representing this situation.
- e. The solution to this inequality must either look like  $x \geq 3$  or  $x \leq 3$ . Which do you think it is? Explain your reasoning.

### Student Response

1. a. The difference in Kiran's account balance after one year (because there are 12 months in a year).
- b. Answers vary. Sample responses: 1, 2, 6. (Any value less than or equal to 6.25 will work.)
- c.  $x \leq \underline{\hspace{1cm}}$ . Kiran must draw less than a certain amount each month in order to end up with \$25 in the account at the end of the year.
- d.  $x \leq 6.25$ . Possible strategy: Solve the equation  $-12x + 100 = 25$  and then use the reasoning in the previous problem part (or test values of  $x$ ) to decide between  $\geq$  and  $\leq$ .
2. a. The price of 1 box after the discount.
- b.  $x = 3$ . Possible strategy: divide each side by 9 resulting in  $7 - x = 4$ , then notice that  $7 - 3$  is 4.
- c. If the discount is \$3, then the teacher will pay exactly \$36 for the granola bars.
- d.  $9(7 - x) \leq 36$  or  $36 \geq 9(7 - x)$
- e.  $x \geq 3$ , because a discount higher than \$3 per box will mean that the teacher will pay a lower price for granola bars.

### Are You Ready for More?

Jada and Diego baked a large batch of cookies.

- They selected  $\frac{1}{4}$  of the cookies to give to their teachers.
- Next, they threw away one burnt cookie.
- They delivered  $\frac{2}{5}$  of the remaining cookies to a local nursing home.
- Next, they gave 3 cookies to some neighborhood kids.
- They wrapped up  $\frac{2}{3}$  of the remaining cookies to save for their friends.

After all this, they had 15 cookies left. How many cookies did they bake?

### Student Response

108 cookies. Possible strategy: Draw a diagram to represent the situation:



Next, work backwards:

$$15 \cdot 3 = 45, 45 + 3 = 48$$

$$48 \div 3 = 16, 16 \cdot 5 = 80, 80 + 1 = 81$$

$$81 \div 3 = 27, 27 \cdot 4 = 108$$

An equation that represents the number of cookies remaining would be  $\frac{1}{3}(\frac{3}{5}(\frac{3}{4}x - 1) - 3) = 15$ . To solve this equation, we would multiply by 3, add 3, multiply by  $\frac{5}{3}$ , add 1, and then multiply by  $\frac{4}{3}$ . Compare these steps to the steps we took to solve with the diagram to see they are the same (note that divide by 3, then multiply by 5 is the same as multiply by  $\frac{5}{3}$ ).

### Activity Synthesis

The purpose of the discussion is to let students voice their reasoning about the direction of the inequality by reasoning about the context. Ask students to share their reasons for choosing the direction of inequality in their solutions. Some students may notice that the algebra in both problems involves multiplying or dividing by a negative number. Honor this observation, but again, the goal is not to turn this observation into a rule for students to memorize and follow. Interpreting the meaning of the solution in the context should remain at the forefront.

As students model real-world situations, questions about the interpretation of the mathematical solution should continue to come up in the conversation. For instance, the amount of the granola bar discount cannot be \$3.5923, even though this is a solution to the inequality  $x \geq 3$ . The value -10 is a solution to Kiran's inequality, even though he can't withdraw -10 dollars. Students can argue

that negative values for  $x$  simply don't make sense in this context. Some may argue that we should interpret  $x = -10$  to mean that Kiran deposits \$10 every month.

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### Access for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Break the class into small discussion groups and then invite a representative from each group to report back to the whole class.

*Supports accessibility for: Language; Social-emotional skills; Attention*

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### Access for English Language Learners

*Representing, Writing, Conversing: MLR8 Discussion Supports.* Use this routine to support whole-class discussion. After a student shares their reasons for the direction of the inequality in their solution, ask students to restate and/or revoice what they heard using mathematical language. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. This will help students understand how context determines the direction of inequality.

*Design Principle(s): Support sense-making*

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## Lesson Synthesis

By the time students have finished this lesson, they have reasoned about solutions to several inequalities, all of which involve some kind of final decision about the direction of inequality. Return to the ideas in the warm-up for the previous lesson. Draw the number line showing solutions to  $x > 1$  on the board. Ask students to name some values of  $x$  that satisfy the inequality. For each of those values of  $x$ , plot the value of  $-x$  on the number line together (perhaps in a different color). What inequality did we just graph?

## 14.4 Colder and colder

Cool Down: 5 minutes

### Addressing

- 7.EE.B.4.b

### Student Task Statement

It is currently 0 degrees outside, and the temperature is dropping 4 degrees every hour. The temperature after  $h$  hours is  $-4h$ .

1. Explain what the equation  $-4h = -14$  represents.
2. What value of  $h$  makes the equation true?

3. Explain what the inequality  $-4h \leq -14$  represents.
4. What values of  $h$  make the inequality true?

### Student Response

1. After  $h$  hours, the temperature has dropped to -14 degrees.
2.  $h = 3.5$ . It takes 3.5 hours for the temperatures to drop to -14 degrees.
3. After  $h$  hours, the temperature is less than or equal to -14 degrees.
4.  $h \geq 3.5$ . After reaching a temperature of -14 degrees at 3.5 hours, the temperature will continue to decrease.

### Student Lesson Summary

Suppose Elena has \$5 and sells pens for \$1.50 each. Her goal is to save \$20. We could solve the equation  $1.5x + 5 = 20$  to find the number of pens,  $x$ , that Elena needs to sell in order to save *exactly* \$20. Adding -5 to both sides of the equation gives us  $1.5x = 15$ , and then dividing both sides by 1.5 gives the solution  $x = 10$  pens.

What if Elena wants to have some money left over? The inequality  $1.5x + 5 > 20$  tells us that the amount of money Elena makes needs to be *greater* than \$20. The solution to the previous equation will help us understand what the solutions to the inequality will be. We know that if she sells 10 pens, she will make \$20. Since each pen gives her more money, she needs to sell *more* than 10 pens to make more than \$20. So the **solution to the inequality** is  $x > 10$ .

### Glossary

- solution to an inequality

## Lesson 14 Practice Problems

### Problem 1

#### Statement

The solution to  $5 - 3x > 35$  is either  $x > -10$  or  $-10 > x$ . Which solution is correct? Explain how you know.

#### Solution

$x < -10$ . Sample reasoning: If I try  $-100$  in place of  $x$ , I get  $305 > 35$ , which is true. Any value of  $x$  that is less than  $-10$  makes the inequality true.  $-10 > x$  refers to all values of  $x$  that are less than  $-10$ .

### Problem 2

#### Statement

The school band director determined from past experience that if they charge  $t$  dollars for a ticket to the concert, they can expect attendance of  $1000 - 50t$ . The director used this model to figure out that the ticket price needs to be \$8 or greater in order for at least 600 to attend. Do you agree with this claim? Why or why not?

#### Solution

No. Explanations vary. Sample response: If ticket prices are higher, fewer people will attend (this can be seen by trying some different values of  $t$  in  $1000 - 50t$ ). 8 is the solution to  $1000 - 50t = 600$ , but they need to charge \$8 or less if they want 600 people or more to attend.

### Problem 3

#### Statement

Which inequality is true when the value of  $x$  is  $-3$ ?

- A.  $-x - 6 < -3.5$
- B.  $-x - 6 > 3.5$
- C.  $-x - 6 > -3.5$
- D.  $x - 6 > -3.5$

#### Solution

C

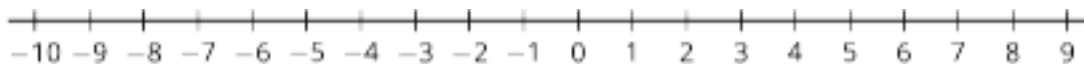
(From Unit 6, Lesson 13.)

## Problem 4

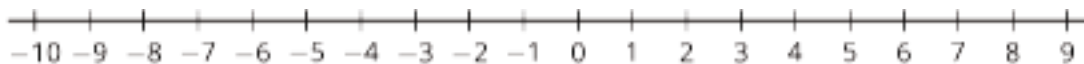
### Statement

Draw the solution set for each of the following inequalities.

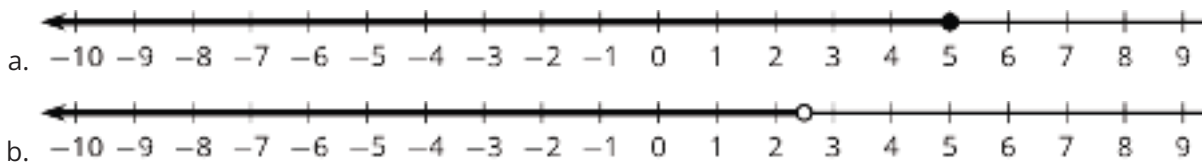
a.  $x \leq 5$



b.  $x < \frac{5}{2}$



### Solution



(From Unit 6, Lesson 13.)

## Problem 5

### Statement

Write three different equations that match the tape diagram.



### Solution

Answers vary. Sample responses:

a.  $7x + 19 = 40$

b.  $40 = 7x + 19$

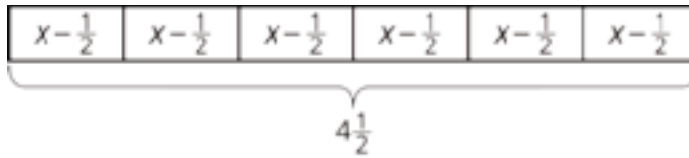
c.  $7x = 40 - 19$

(From Unit 6, Lesson 3.)

## Problem 6

### Statement

A baker wants to reduce the amount of sugar in his cake recipes. He decides to reduce the amount used in 1 cake by  $\frac{1}{2}$  cup. He then uses  $4\frac{1}{2}$  cups of sugar to bake 6 cakes.



- a. Describe how the tape diagram represents the story.
- b. How much sugar was originally in each cake recipe?

### Solution

a. Answers vary. Sample response: The six equal parts of the diagram represent the 6 cakes the baker bakes. The label  $x - \frac{1}{2}$  in each part represents the amount of sugar, measured in number of cups, that the baker used in each cake.  $x$  represents the original amount of sugar used in each cake and  $x - \frac{1}{2}$  represents the original number of cups reduced by  $\frac{1}{2}$  cup.  $4\frac{1}{2}$  is the total amount of sugar, measured in cups, used for the 6 cakes.

b.  $1\frac{1}{4}$  cups

(From Unit 6, Lesson 2.)

## Problem 7

### Statement

One year ago, Clare was 4 feet 6 inches tall. Now Clare is 4 feet 10 inches tall. By what percentage did Clare's height increase in the last year?

### Solution

About 7% (4 feet 6 inches is 54 inches and she grew 4 inches:  $\frac{4}{54} \approx 0.07$ )

(From Unit 4, Lesson 12.)

# Lesson 15: Efficiently Solving Inequalities

## Goals

- Compare and contrast (orally) solutions to equations and solutions to inequalities.
- Draw and label a graph on the number line that represents all the solutions to an inequality.
- Generalize (orally) that you can solve an inequality of the form  $px + q > r$  or  $px + q < r$  by solving the equation  $px + q = r$  and then testing a value to determine the direction of the inequality in the solution.

## Learning Targets

- I can graph the solutions to an inequality on a number line.
- I can solve inequalities by solving a related equation and then checking which values are solutions to the original inequality.

## Lesson Narrative

In this lesson, students see more examples of inequalities. This time, many inequalities involve negative coefficients. This reinforces the point that solving an inequality is not as simple as solving the corresponding equation. After students find the boundary point, they must do some extra work to figure out the direction of inequality. This might involve reasoning about the context, substituting in values on either side of the boundary point, and reasoning about number lines. All of these techniques exemplify MP1: making the problem more concrete and visual and asking, “Does this make sense?”

It is important to understand that the goal is not to have students learn and practice an algorithm for solving inequalities like “whenever you multiply or divide by a negative, flip the inequality.” Rather, we want students to understand that solving a related equation tells you the lower or upper bound of an inequality. To know whether values greater than or less than the boundary number make the inequality true, it's best to test some values that are above and below the boundary number. This way of reasoning about inequalities will serve students well long into their future studies, whereas students are very likely to forget a procedure memorized for a special case.

## Alignments

### Addressing

- 7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

### Building Towards

- 7.EE.B.4.b: Solve word problems leading to inequalities of the form  $px + q > r$  or  $px + q < r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. \$

## Instructional Routines

- MLR7: Compare and Connect
- Think Pair Share

## Student Learning Goals

Let's solve more complicated inequalities.

# 15.1 Lots of Negatives

## Warm Up: 5 minutes

This warm-up primes students for inequalities that include variables with negative coefficients without context. Students first predict a solution set, and then are given some values to test so that the solution emerges. Do *not* formalize a procedure for “flipping the inequality” when multiplying by a negative. Look for students who predict solution sets that are incorrect because of the sign.

## Building Towards

- 7.EE.B.4.b

## Launch

Give students 3 minutes of quiet work time followed by a whole-class discussion.

## Student Task Statement

Here is an inequality:  $-x \geq -4$ .

1. Predict what you think the solutions on the number line will look like.
2. Select **all** the values that are solutions to  $-x \geq -4$ :
  - a. 3
  - b. -3
  - c. 4
  - d. -4
  - e. 4.001
  - f. -4.001
3. Graph the solutions to the inequality on the number line:



## Student Response

1. Answers vary.
2. a, b, c, d, f

3. A filled-in circle at 4 and all points to its left are graphed. The same graph that one would draw for  $x \leq 4$ .

### Activity Synthesis

The purpose of the discussion is to highlight how negatives in the inequality sometimes make it hard to predict what the solutions will be. (It is important to reason carefully by first determining the value for which both sides are equal and then testing points to determine which on side of that value the solutions lie.) Select students to share how their predictions differed from their final solutions. Consider asking how the solutions to  $-x \geq -4$  are different from the solutions to  $x \geq 4$ . (The solutions go in the opposite direction on the number line.)

## 15.2 Inequalities with Tables

### 15 minutes

The purpose of this activity is for students to get a visual feel for the relationship between an unsimplified inequality ( $x - 3 > -2$ ) and its solution ( $x > 1$ ). The tables suggest a way for students to see why the values of  $x$  that satisfy  $x - 3 > -2$  should also satisfy  $x > 1$ . Graphing solutions on the number line reinforces this connection. The second and third questions, taken together, demonstrate how a negative coefficient can make the solutions to an inequality go “the other way.” The work in this activity suggests a procedure for solving inequalities: solve the corresponding equation, then test a number on either side. But the purpose of this activity is not to teach students a procedure, but rather to provide underlying knowledge and experience.

### Building Towards

- 7.EE.B.4.b

### Launch

A potentially challenging aspect of this task is that students must consider the two rows of a table at different times and relate the values in the table to solutions of an inequality. Consider displaying a table like this for all to see, and then asking some questions about it:

$x$	-4	-3	-2	-1	0	1	2	3	4
$x + 2$	-2	-1	0	1	2	3	4	5	6

After students have had a chance to look at the table, ask them some familiarizing questions:

- How are the numbers in the top row and bottom row related?
- Think about the equation  $x + 2 = -2$ . What value of  $x$  makes this true? Where do you see that in the table?
- Think about the inequality  $x + 2 > 3$ . What values of  $x$  make this true? Where do you see that in the table?

Give 5–10 minutes of quiet work time to complete the tables and questions followed by a whole-class discussion. Depending on the needs of your class, you might instruct students to pause after each question for discussion before continuing with the next question.

### Access for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity. Consider pausing after the first question for a brief class discussion before moving on.

*Supports accessibility for: Organization; Attention*

### Anticipated Misconceptions

Some students may answer  $x > 2$  for the first question, since that is the place where the value of  $x - 3$  first surpasses the number  $-2$ . Remind these students that there are values between 1 and 2. Ask them whether 1.1 is a solution, for example.

Some students may graph only whole-number solutions. Ask these students to think about whether values in between whole numbers are also solutions.

### Student Task Statement

- Let's investigate the inequality  $x - 3 > -2$ .

$x$	-4	-3	-2	-1	0	1	2	3	4
$x - 3$	-7		-5				-1		1

- Complete the table.
- For which values of  $x$  is it true that  $x - 3 = -2$ ?
- For which values of  $x$  is it true that  $x - 3 > -2$ ?
- Graph the solutions to  $x - 3 > -2$  on the number line:



- Here is an inequality:  $2x < 6$ .

- Predict which values of  $x$  will make the inequality  $2x < 6$  true.
- Complete the table. Does it match your prediction?

$x$	-4	-3	-2	-1	0	1	2	3	4
$2x$									

c. Graph the solutions to  $2x < 6$  on the number line:



3. Here is an inequality:  $-2x < 6$ .

a. Predict which values of  $x$  will make the inequality  $-2x < 6$  true.

b. Complete the table. Does it match your prediction?

$x$	-4	-3	-2	-1	0	1	2	3	4
$-2x$									

c. Graph the solutions to  $-2x < 6$  on the number line:



d. How are the solutions to  $2x < 6$  different from the solutions to  $-2x < 6$ ?

### Student Response

1. For  $x - 3 > -2$

a. Table:

$x$	-4	-3	-2	-1	0	1	2	3	4
$x - 3$	-7	-6	-5	-4	-3	-2	-1	0	1

b.  $x = 1$

c.  $x > 1$

d. The graph should have an open circle at  $x = 1$  with all values greater than 1 shaded.

2. For  $2x < 6$

a. Answers vary. Any value less than 3 will work.

b. Table:

$x$	-4	-3	-2	-1	0	1	2	3	4
$2x$	-8	-6	-4	-2	0	2	4	6	8

c. The graph should have an open circle at  $x = 3$  with all values less than 3 shaded.

3. For  $-2x < 6$

a. Answers vary. Sample response: Based on the solution to  $2x < 6$ , I predict that for  $-2x < 6$ , the solutions will be values less than -3.

b. The table may or may not match the prediction.

$x$	-4	-3	-2	-1	0	1	2	3	4
$-2x$	8	6	4	2	0	-2	-4	-6	-8

c. The graph should have an open circle at  $x = -3$ , with all values greater than -3 shaded.

d. The solution to  $2x < 6$  is all values less than 3, but the solution to  $-2x < 6$  is all values greater than -3.

### Activity Synthesis

The main take-away is that solving the associated equation to an inequality gives the value that is the boundary between solutions and non-solutions. In this activity, students have a table to check on which side of the boundary are solutions and which side are not solutions. In order to transition to the next activity, ask students whether they need to complete an *entire* table to test on which side of the boundary the solutions are. The goal is to get students to understand that they only need to test one number. If that number is a solution, then all points on the same side of the boundary are solutions. If the point is not a solution, then the solutions are all the points on the other side of the boundary. The next activity will give students an opportunity to apply this insight and start to articulate such a procedure.

Resist the temptation to summarize the last two problems into a procedure like “whenever you multiply or divide by a negative, flip the inequality.”

## 15.3 Which Side are the Solutions?

15 minutes

In the previous activity, students saw that solving the equation associated with an inequality gives a boundary point that separates values that make the inequality true from values that make the inequality false. This activity builds on that understanding to solidify a process for solving inequalities: first solve the associated equation to find the boundary point, then test a value to determine on which side of that boundary the solutions lie. The first two questions offer more

scaffolding, and the last two questions simply give an inequality and ask students to solve and graph.

### Building Towards

- 7.EE.B.4.b

### Instructional Routines

- MLR7: Compare and Connect
- Think Pair Share

### Launch

Arrange students in groups of 2. Give 5–10 minutes of quiet work time, time to share their responses and reasoning with a partner, and follow with a whole-class discussion.

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### Access for Students with Disabilities

*Representation: Internalize Comprehension.* Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.

*Supports accessibility for: Memory; Conceptual processing*

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### Student Task Statement

1. Let's investigate  $-4x + 5 \geq 25$ .
  - a. Solve  $-4x + 5 = 25$ .
  - b. Is  $-4x + 5 \geq 25$  true when  $x$  is 0? What about when  $x$  is 7? What about when  $x$  is -7?
  - c. Graph the solutions to  $-4x + 5 \geq 25$  on the number line.

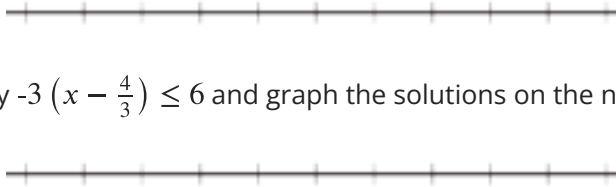


2. Let's investigate  $\frac{4}{3}x + 3 < \frac{23}{3}$ .
  - a. Solve  $\frac{4}{3}x + 3 = \frac{23}{3}$ .
  - b. Is  $\frac{4}{3}x + 3 < \frac{23}{3}$  true when  $x$  is 0?
  - c. Graph the solutions to  $\frac{4}{3}x + 3 < \frac{23}{3}$  on the number line.



3. Solve the inequality  $3(x + 4) > 17.4$  and graph the solutions on the number line.

4. Solve the inequality  $-3\left(x - \frac{4}{3}\right) \leq 6$  and graph the solutions on the number line.



### Student Response

- For  $-4x \geq 25$ :
  - 5
  - No, no, yes
  - A closed circle at -5 and all values to the left shaded.
- For  $\frac{4}{3}x + 3 < \frac{23}{3}$ :
  - $\frac{7}{2}$  (or equivalent)
  - Yes
  - An open circle at  $\frac{7}{2}$  and all values to the left shaded.
- The solution is  $x > 1.8$ . An open circle at 1.8 and all values to the right shaded.
- The solution is  $x \geq -\frac{2}{3}$ . A closed circle at  $-\frac{2}{3}$  and all values to the right shaded.

### Are You Ready for More?

Write at least three different inequalities whose solution is  $x > -10$ . Find one with  $x$  on the left side that uses a  $<$ .

### Student Response

Answers vary. Possible responses:  $2x > -20$ ,  $x + 50 > 40$ . Responses that involve  $x <$ :  $-5x < 50$ ,  $\frac{x}{-6} < 60$ .

### Activity Synthesis

For each question, ask one student to demonstrate and explain their process for solving the inequality. For each, highlight the moment when they find the boundary value (the solution to the related equation) and then when they test one or more numbers on either side to decide which side has values that make the inequality true.

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### Access for English Language Learners

*Representing, Conversing: MLR7 Compare and Connect.* During the whole-class discussion, ask students to discuss with their partner, what is the same and what is different between the demonstrated processes for solving each inequality. To call students' attention to the different ways the boundary values were determined, ask students, "Which numbers can be used to check the direction of the solution?" This will help students produce language, including symbols, common to inequalities as they reason about procedures for solving inequalities.

*Design Principle(s): Maximize meta-awareness; Cultivate conversation*

---

## Lesson Synthesis

By the time students have finished this lesson, they should have a variety of methods for solving inequalities, all of which involve some kind of final decision about the direction of inequality. Students will have had some summative practice with this in the final activity and the cool-down.

Ask students to consider, "What if someone asked for your help with how to solve inequalities? What would you tell them? How would you describe to someone how to solve any inequality?" Ask them to either write this down or share their thoughts with their partner. Consider creating a persistent display showing the procedure using language the class develops, along with an example.

## 15.4 Testing for Solutions

**Cool Down: 5 minutes**

The purpose of this cool-down is to check whether students can determine the direction of inequality. The questions involve using algebra to find boundary points, then testing values of  $x$ . Since the boundary points are given, some students may skip directly to testing points.

### Addressing

- 7.EE.B.4

### Student Task Statement

For each inequality, decide whether the solution is represented by  $x < 2.5$  or  $x > 2.5$ .

1.  $-4x + 5 > -5$

2.  $-25 > -5(x + 2.5)$

### Student Response

1.  $x < 2.5$

2.  $x > 2.5$

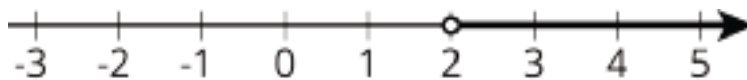
## Student Lesson Summary

Here is an inequality:  $3(10 - 2x) < 18$ . The solution to this inequality is all the values you could use in place of  $x$  to make the inequality true.

In order to solve this, we can first solve the related equation  $3(10 - 2x) = 18$  to get the solution  $x = 2$ . That means 2 is the boundary between values of  $x$  that make the inequality true and values that make the inequality false.

To solve the inequality, we can check numbers greater than 2 and less than 2 and see which ones make the inequality true.

Let's check a number that is greater than 2:  $x = 5$ . Replacing  $x$  with 5 in the inequality, we get  $3(10 - 2 \cdot 5) < 18$  or just  $0 < 18$ . This is true, so  $x = 5$  is a solution. This means that all values greater than 2 make the inequality true. We can write the solutions as  $x > 2$  and also represent the solutions on a number line:



Notice that 2 itself is not a solution because it's the value of  $x$  that makes  $3(10 - 2x)$  *equal* to 18, and so it does not make  $3(10 - 2x) < 18$  true.

For confirmation that we found the correct solution, we can also test a value that is less than 2. If we test  $x = 0$ , we get  $3(10 - 2 \cdot 0) < 18$  or just  $30 < 18$ . This is false, so  $x = 0$  and all values of  $x$  that are less than 2 are not solutions.

# Lesson 15 Practice Problems

## Problem 1

### Statement

a. Consider the inequality  $-1 \leq \frac{x}{2}$ .

i. Predict which values of  $x$  will make the inequality true.

ii. Complete the table to check your prediction.

$x$	-4	-3	-2	-1	0	1	2	3	4
$\frac{x}{2}$									

b. Consider the inequality  $1 \leq \frac{-x}{2}$ .

i. Predict which values of  $x$  will make it true.

ii. Complete the table to check your prediction.

$x$	-4	-3	-2	-1	0	1	2	3	4
$\frac{-x}{2}$									

### Solution

a. i.  $x \geq -2$

ii.

$x$	-4	-3	-2	-1	0	1	2	3	4
$\frac{x}{2}$	-2	-1.5 (or $-\frac{3}{2}$ )	-1	-0.5 (or $-\frac{1}{2}$ )	0	0.5 (or $\frac{1}{2}$ )	1	1.5 (or $\frac{3}{2}$ )	2

b. i.  $x \leq -2$

ii.

$x$	-4	-3	-2	-1	0	1	2	3	4
$\frac{x}{2}$	2	1.5 (or $\frac{3}{2}$ )	1	0.5 (or $\frac{1}{2}$ )	0	-0.5 (or $-\frac{1}{2}$ )	-1	-1.5 (or $-\frac{3}{2}$ )	-2

## Problem 2

### Statement

Diego is solving the inequality  $100 - 3x \geq -50$ . He solves the equation  $100 - 3x = -50$  and gets  $x = 50$ . What is the solution to the inequality?

- A.  $x < 50$
- B.  $x \leq 50$
- C.  $x > 50$
- D.  $x \geq 50$

### Solution

B

## Problem 3

### Statement

Solve the inequality  $-5(x - 1) > -40$ , and graph the solution on a number line.

### Solution

- a.  $x < 9$
- b. A number line with an open circle at 9 and the arrow going to the left

## Problem 4

### Statement

Select all values of  $x$  that make the inequality  $-x + 6 \geq 10$  true.

- A. -3.9
- B. 4
- C. -4.01
- D. -4
- E. 4.01
- F. 3.9
- G. 0
- H. -7

## Solution

["C", "D", "H"]

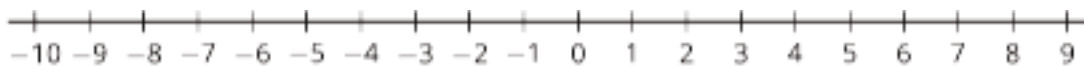
(From Unit 6, Lesson 13.)

## Problem 5

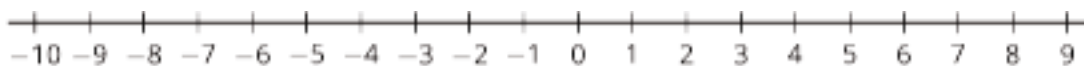
### Statement

Draw the solution set for each of the following inequalities.

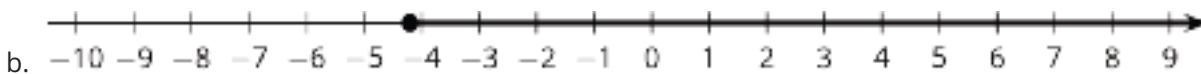
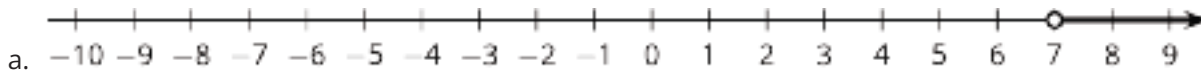
a.  $x > 7$



b.  $x \geq -4.2$



## Solution



(From Unit 6, Lesson 13.)

## Problem 6

### Statement

The price of a pair of earrings is \$22 but Priya buys them on sale for \$13.20.

a. By how much was the price discounted?

b. What was the percentage of the discount?

## Solution

a. \$8.80

b. 40%

(From Unit 4, Lesson 12.)

# Lesson 16: Interpreting Inequalities

## Goals

- Critique (orally) a solution method for a problem involving an inequality.
- Identify the inequality that represents a situation, and justify (in writing) the choice.
- Present (orally, in writing, and using other representations) the solution method for a problem involving an inequality, and interpret the solution.

## Learning Targets

- I can match an inequality to a situation it represents, solve it, and then explain what the solution means in the situation.
- If I have a situation and an inequality that represents it, I can explain what the parts of the inequality mean in the situation.

## Lesson Narrative

In this lesson and the next, we move on to applying inequalities to solve problems. The warm-up is a review of the work in the previous lesson about solving inequalities when no context is given. Then students interpret and solve inequalities that represent real-life situations, making sense of quantities and their relationships in the problem (MP2).

## Alignments

### Addressing

- 7.EE.B.4.b: Solve word problems leading to inequalities of the form  $px + q > r$  or  $px + q < r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. \$

### Building Towards

- 7.EE.B.4.b: Solve word problems leading to inequalities of the form  $px + q > r$  or  $px + q < r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. \$

## Instructional Routines

- Group Presentations
- MLR2: Collect and Display
- MLR7: Compare and Connect

## Required Materials

### Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart

paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

### Student Learning Goals

Let's write inequalities.

## 16.1 Solve Some Inequalities!

### Warm Up: 5 minutes

This warm-up is an opportunity for students to recall understandings and techniques from the previous lesson.

### Building Towards

- 7.EE.B.4.b

### Launch

Optionally, provide access to blank number lines.

### Anticipated Misconceptions

If students express the solution in words or by graphing on a number line, applaud their use of these representations. Encourage them to attempt to express the solution using the efficient notation, as well. Direct their attention to any anchor charts or notes that remind them of the meaning of the symbols involved.

### Student Task Statement

For each inequality, find the value or values of  $x$  that make it true.

1.  $8x + 21 \leq 56$

2.  $56 < 7(7 - x)$

### Student Response

1.  $x \leq 4.375$

2.  $x < -1$

### Activity Synthesis

Ask one student to share their process for reasoning about a solution to each problem. Address and resolve any discrepancies that arise.

## 16.2 Club Activities Matching

10 minutes

In this activity, students analyze four situations and select the inequality that best represents the situation. (In the activity that follows, students will work in small groups to create a visual display showing the solution for one of these situations.)

### Addressing

- 7.EE.B.4.b

### Instructional Routines

- MLR2: Collect and Display

### Launch

Tell students that their job in this activity is to read four situations carefully and decide which inequality best represents the situation. In the next activity, they will be responsible for writing a solution for one of these situations. Give 5–10 minutes of quiet work time.

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#### Access for Students with Disabilities

*Representation: Internalize Comprehension.* Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, use the same color to highlight key words or phrases in the situation with its corresponding inequality sign in the matching inequality.

*Supports accessibility for: Visual-spatial processing*

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#### Access for English Language Learners

*Reading, Representing: MLR2 Collect and Display.* As students work, circulate and collect examples of words and phrases students use in their written response to "Explain your reasoning" for each question. Look for different ways students describe what the variable represents, how they know which number is the constant term, how they know which number should be multiplied by the variable, and the direction of the inequality symbol that makes sense for each context. Organize the phrases for each of these considerations and display for all to see. This will help students to focus on all of the important elements of the inequality they are assigned in the next activity, with language they can use in small group discussions.

*Design Principle(s): Support sense-making; Maximize meta-awareness*

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### Student Task Statement

Choose the inequality that best matches each given situation. Explain your reasoning.

1. The Garden Club is planting fruit trees in their school's garden. There is one large tree that needs 5 pounds of fertilizer. The rest are newly planted trees that need  $\frac{1}{2}$  pound fertilizer each.

a.  $25x + 5 \leq \frac{1}{2}$

b.  $\frac{1}{2}x + 5 \leq 25$

c.  $\frac{1}{2}x + 25 \leq 5$

d.  $5x + \frac{1}{2} \leq 25$

2. The Chemistry Club is experimenting with different mixtures of water with a certain chemical (sodium polyacrylate) to make fake snow.

To make each mixture, the students start with some amount of water, and then add  $\frac{1}{7}$  of that amount of the chemical, and then 9 more grams of the chemical. The chemical is expensive, so there can't be more than a certain number of grams of the chemical in any one mixture.

a.  $\frac{1}{7}x + 9 \leq 26.25$

b.  $9x + \frac{1}{7} \leq 26.25$

c.  $26.25x + 9 \leq \frac{1}{7}$

d.  $\frac{1}{7}x + 26.25 \leq 9$

3. The Hiking Club is on a hike down a cliff. They begin at an elevation of 12 feet and descend at the rate of 3 feet per minute.

a.  $37x - 3 \geq 12$

b.  $3x - 37 \geq 12$

c.  $12 - 3x \geq -37$

d.  $12x - 37 \geq -3$

4. The Science Club is researching boiling points. They learn that at high altitudes, water boils at lower temperatures. At sea level, water boils at 212°F. With each increase of 500 feet in elevation, the boiling point of water is lowered by about 1°F.

a.  $212 - \frac{1}{500}e < 195$

b.  $\frac{1}{500}e - 195 < 212$

c.  $195 - 212e < \frac{1}{500}$

d.  $212 - 195e < \frac{1}{500}$

## Student Response

1. b, because  $\frac{1}{2}$  a pound for each of an unknown number of trees, plus 5 pounds of fertilizer, is less than or equal to 25, which is likely the maximum amount of fertilizer available.
2. a, because  $\frac{1}{7}$  of the amount of water plus 9 grams is the amount of the chemical used. This total must be less than 26.25 grams, which is likely the maximum amount of the chemical that can be used in a mixture.
3. c, because they start at 12 feet and then lose 3 feet per minute. If  $x$  is the number of minutes they hike, then  $3x$  is the change in elevation. Their elevation must be above -37 feet; perhaps this is the bottom of the cliff.
4. a, because the boiling point is 212 at sea level, and decreases  $\frac{1}{500}$  of a degree for every foot of elevation. The solution will tell us for which elevations the temperature is below 195 degrees.

## Activity Synthesis

At this time, consider *not* validating which inequalities are correct. When students get into groups for the next activity, they can compare their responses with the members of their groups and resolve any discrepancies at that time.

# 16.3 Club Activities Display

## 20 minutes

In this activity, students interpret parts of an inequality in context, term by term; for example, what quantity must  $\frac{1}{2}x$  represent? Then they make sense of the entire inequality by thinking about what question would be answered by the solution to the inequality. Notice groups that create displays that communicate their mathematical thinking clearly, contain an error that would be instructive to discuss, or organize the information in a way that is useful for all to see. At this point, there is very little scaffolding for the solving of the inequality itself.

## Addressing

- 7.EE.B.4.b

## Instructional Routines

- Group Presentations
- MLR7: Compare and Connect

## Launch

Arrange students in groups of 2–3 and provide tools for making a visual display. Assign one situation to each group. Note that the level of difficulty increases for the situations, so this is an opportunity to differentiate by assigning more or less challenging situations to different groups.

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## Access for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Provide prompts, reminders, guides, rubrics, or checklists that focus on increasing the length of on-task orientation in the face of distractions. For example, create an exemplar display including all required components, highlighting different ways to communicate mathematical thinking clearly.

*Supports accessibility for: Attention; Social-emotional skills*

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### Student Task Statement

Your teacher will assign your group *one* of the situations from the last task. Create a visual display about your situation. In your display:

- Explain what the variable and each part of the inequality represent
- Write a question that can be answered by the solution to the inequality
- Show how you solved the inequality
- Explain what the solution means in terms of the situation

### Student Response

- $x$  represents the number of small trees that can be fertilized with the remaining fertilizer. 5 is the number of pounds of fertilizer for the large tree.  $\frac{1}{2}x$  represents the number of pounds of fertilizer needed to grow  $x$  small trees. 25 is the total weight of fertilizer available in pounds.
  - How many small trees can be planted with the available fertilizer?
  - $\frac{1}{2}x + 5 \leq 25$ ,  $\frac{1}{2}x \leq 20$ ,  $x \leq 40$
  - Up to 40 small trees can be planted with the fertilizer available.
- $x$  represents the amount (in grams) of water used in a given mixture.  $\frac{x}{7} + 9$  represents the amount of chemical (in grams) that is added to the water.
  - How much water can you start with so that you don't use up too much of the chemical?
  - $\frac{x}{7} + 9 \leq 26.25$ ,  $\frac{x}{7} \leq 17.25$ ,  $x \leq 120.75$ .
  - In order to make a mixture that doesn't use too much of the chemical, you have to start with 120.75 grams of water or less.
- -3 represents the elevation lost each minute.  $x$  is the number of minutes the students have been hiking.  $-3x$  is the amount of elevation loss after  $x$  minutes. 12 is the initial elevation.  $12 - 3x$  represents the students' elevation after hiking for  $x$  minutes. -37 represents the elevation at the bottom of the cliff: 37 feet below sea level.

- What are the times during which the students are hiking toward the bottom of the cliff?
  - $12 - 3x \geq -37, -3x \geq -49, x \leq 16\frac{1}{3}$
  - The students hike for a time period of  $16\frac{1}{3}$  minutes, at which point they come to the bottom of the cliff.
- 4.
- $\frac{1}{500}e$  represents the change in the boiling point of water after a 1-foot increase in elevation.  $212 - \frac{1}{500}e$  is the boiling point of water at elevation  $e$ .
  - At which elevations is the boiling point of water below 195 degrees?
  - $212 - \frac{1}{500}e < 195, -\frac{1}{500}e < -17, e > 8500$
  - At elevations greater than 8500 feet, the boiling point of water is less than 195 degrees.

### Are You Ready for More?

$\{3, 4, 5, 6\}$  is a set of four consecutive integers whose sum is 18.

1. How many sets of three consecutive integers are there whose sum is between 51 and 60? Can you be sure you've found them all? Explain or show your reasoning.
  
2. How many sets of four consecutive integers are there whose sum is between 59 and 82? Can you be sure you've found them all? Explain or show your reasoning.

### Student Response

Both of these problems can be solved by intelligent guess-and-check, or other more conceptual strategies, and by using the first answer one finds to generate the others. If students use these strategies, help them to crystalize their reasoning: how do they know they have all of the sets? Also encourage students to see if they can write inequalities in addition (not instead of!) whatever strategies they use.

1. 4 sets:  $\{16, 17, 18\}, \{17, 18, 19\}, \{18, 19, 20\}, \{19, 20, 21\}$   
 $x + (x + 1) + (x + 2) \geq 51$  and  $x + (x + 1) + (x + 2) \leq 60, 16 \leq x \leq 19$ .
2. 5 sets:  $\{14, 15, 16, 17\}, \{15, 16, 17, 18\}, \{16, 17, 18, 19\}, \{17, 18, 19, 20\}, \{18, 19, 20, 21\}$ .  
 $x + (x + 1) + (x + 2) + (x + 3) \geq 59$  and  $x + (x + 1) + (x + 2) + (x + 3) \leq 82$ ,  
 $13.25 \leq x \leq 18.5$ .

### Activity Synthesis

Select groups to share their visual displays. Encourage students to ask questions about the mathematical thinking or design approach that went into creating the display. Here are questions for discussion, if not already mentioned by students:

- How did you figure out what the  $\frac{x}{7}$  term represents?
- How did you decide on the direction of the inequality for the solutions?
- Did anyone with the same problem do one of the steps differently? Share what you did differently so we can learn from what happened.
- How do you know there are 25 pounds of fertilizer available?

Alternatively, have students do a “gallery walk” in which they leave written feedback on sticky notes for the other groups. Here is guidance for the kind of feedback students should aim to give each other:

- What is one thing that group did that would have made your project better if you had done it?
- What is one thing your group did that would have improved their project if they did it too?
- How did the group decide the direction of inequality for the solutions?
- Does their answer make sense in the situation?
- Is their mathematics clear and correct?
- If there was a mistake, what could they be more careful about in similar problems?

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### Access for English Language Learners

*Representing, Conversing: MLR7 Compare and Connect.* During the launch, make sure at least two groups are assigned to each situation (assign fewer contexts if there are fewer than 8 groups). Assign groups who worked on the same situation to review each other's display. Ask groups to look closely at how the inequality was solved, then to identify and discuss what is the same and what is different, compared to their own display. If the other group's solution is the same, students should compare the strategies used. If the solution is different, students should look for any errors in reasoning, either in their own or the other group's method. Ask each group to leave a comment on a sticky note that describes the comparison they discussed. This will help students make sense of the reasoning of others by interpreting work that is similar to their own.

*Design Principle(s): Maximize meta-awareness; Support sense-making*

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## Lesson Synthesis

In this lesson, we saw how inequalities can be applied to real-world situations. Some questions to bring this work together:

- Suppose your friend asks you to write some practice problems for solving inequalities. You want to write an inequality that has a solution of  $x \leq -8\frac{2}{3}$ . Describe how to write such an inequality.

- Think about an after-school activity in which you are involved. Write an inequality that represents a situation related to that activity. Be prepared to share the inequality and an explanation of its terms with the class.

If time allows, have students solve their inequalities.

## 16.4 Party Decorations

Cool Down: 5 minutes

### Addressing

- 7.EE.B.4.b

#### Student Task Statement

Andre is making paper cranes to decorate for a party. He plans to make one large paper crane for a centerpiece and several smaller paper cranes to put around the table. It takes Andre 10 minutes to make the centerpiece and 3 minutes to make each small crane. He will only have 30 minutes to make the paper cranes once he gets home.

1. Andre wrote the inequality  $3x + 10 \leq 30$  to plan his time. Describe what  $x$ ,  $3x$ , 10, and 30 represent in this inequality.
2. Solve Andre's inequality and explain what the solution means.

#### Student Response

1. The variable  $x$  represents the number of small paper cranes Andre will make. 10 is the number of minutes it takes to make the centerpiece.  
 $3x$  is the amount of time it takes to make  $x$  small cranes (it takes 3 minutes to make one crane). 30 is Andre's time limit in minutes.
2.  $x \leq 6\frac{2}{3}$ . Andre can make 6 or fewer small cranes.

#### Student Lesson Summary

We can represent and solve many real-world problems with inequalities. Writing the inequalities is very similar to writing equations to represent a situation. The expressions that make up the inequalities are the same as the ones we have seen in earlier lessons for equations. For inequalities, we also have to think about how expressions compare to each other, which one is bigger, and which one is smaller. Can they also be equal?

For example, a school fundraiser has a minimum target of \$500. Faculty have donated \$100 and there are 12 student clubs that are participating with different activities. How much money should each club raise to meet the fundraising goal? If  $n$  is the amount of money that each club raises, then the solution to  $100 + 12n = 500$  is the minimum amount each club has to raise to meet the goal. It is more realistic, though, to use the inequality  $100 + 12n \geq 500$  since the more money we raise, the more successful the fundraiser will be. There are many

■ solutions because there are many different amounts of money the clubs could raise that would get us above our minimum goal of \$500.

# Lesson 16 Practice Problems

## Problem 1

### Statement

Priya looks at the inequality  $12 - x > 5$  and says “I subtract a number from 12 and want a result that is bigger than 5. That means that the solutions should be values of  $x$  that are smaller than something.”

Do you agree with Priya? Explain your reasoning and include solutions to the inequality in your explanation.

### Solution

Yes, Priya is correct. Explanations vary. Sample response: Try subtracting different numbers from 12. For example,  $12 - 3$  is larger than  $12 - 8$  because subtracting 3 is subtracting less. When  $x = 7$ , the inequality is not true anymore, but for anything smaller than 7, it is still true. The solution to the inequality is  $x < 7$ .

## Problem 2

### Statement

When a store had sold  $\frac{2}{5}$  of the shirts that were on display, they brought out another 30 from the stockroom. The store likes to keep at least 150 shirts on display. The manager wrote the inequality  $\frac{3}{5}x + 30 \geq 150$  to describe the situation.

- Explain what  $\frac{3}{5}$  means in the inequality.
- Solve the inequality.
- Explain what the solution means in the situation.

### Solution

Answers vary. Sample responses:

- Since  $\frac{2}{5}$  of the original shirts were sold,  $\frac{3}{5}$  of the original shirts remain on display.
- $\frac{3}{5}x + 30 \geq 150$ ,  $\frac{3}{5}x \geq 120$ ,  $x \geq 200$
- There were 200 or more shirts originally on display. At least 120 were left when they brought out 30 more.

### Problem 3

#### Statement

You know  $x$  is a number less than 4. Select **all** the inequalities that *must* be true.

- A.  $x < 2$
- B.  $x + 6 < 10$
- C.  $5x < 20$
- D.  $x - 2 > 2$
- E.  $x < 8$

#### Solution

["B", "C", "E"]

(From Unit 6, Lesson 13.)

### Problem 4

#### Statement

Here is an unbalanced hanger.



- a. If you knew each circle weighed 6 grams, what would that tell you about the weight of each triangle? Explain your reasoning.
- b. If you knew each triangle weighed 3 grams, what would that tell you about the weight of each circle? Explain your reasoning.

#### Solution

- a. The triangles would weigh more than 4 grams each. The 3 triangles weigh *more* than 2 circles. The 2 circles weigh 12 grams, so that means each triangle would weigh more than 4 grams.
- b. The circles would weigh less than 4.5 grams each. The 3 triangles weigh 9 grams and this is *more* than 2 circles. So each circle weighs less than 4.5 grams.

(From Unit 6, Lesson 13.)

## Problem 5

### Statement

Match each sentence with the inequality that could represent the situation.

- |   |                        |
|---|------------------------|
| A. Han got \$2 from Clare, but still has less than \$20.                        | 1. $x - 2 < 20$        |
| B. Mai spent \$2 and has less than \$20.  | 2. $2x < 20$           |
| C. If Tyler had twice the amount of money he has, he would have less than \$20. | 3. $x + 2 < 20$        |
| D. If Priya had half the money she has, she would have less than \$20.          | 4. $\frac{1}{2}x < 20$ |

### Solution

- A: 3
- B: 1
- C: 2
- D: 4

(From Unit 6, Lesson 13.)

## Problem 6

### Statement

At a skateboard shop:

- The price tag on a shirt says \$12.58. Sales tax is 7.5% of the price. How much will you pay for the shirt?
- The store buys a helmet for \$19.00 and sells it for \$31.50. What percentage was the markup?
- The shop pays workers \$14.25 per hour plus 5.5% commission. If someone works 18 hours and sells \$250 worth of merchandise, what is the total amount of their paycheck for this pay period? Explain or show your reasoning.

### Solution

- \$13.52

b. 65.8% or 66%

c. \$270.25, because  $18 \cdot (14.25) + (0.055) \cdot 250 = 270.25$ .

(From Unit 4, Lesson 12.)

# Lesson 17: Modeling with Inequalities

## Goals

- Critique (orally) the solution to an inequality, including whether fractional or negative values are reasonable.
- Determine what information is needed to solve a problem involving a quantity constrained by a maximum or minimum acceptable value. Ask questions to elicit that information.
- Write and solve an inequality of the form  $px + q > r$  or  $px + q < r$  to answer a question about a situation with a constraint.

## Learning Targets

- I can use what I know about inequalities to solve real-world problems.

## Lesson Narrative

By now, students have had plenty of experience writing and solving inequalities. This lesson focuses on the modeling process (MP4), in which students start with a question they want to answer and decide on their own how they will represent the situation mathematically.

As students apply inequalities in context, they must think about how to interpret their solutions. For instance, if they find that  $x < 7$ , but  $x$  represents the number of students who can go on a trip, then they should realize that  $x$  cannot be 3.25, nor can  $x$  be -2.

## Alignments

### Addressing

- 7.EE.B.4.b: Solve word problems leading to inequalities of the form  $px + q > r$  or  $px + q < r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. \$

### Building Towards

- 7.EE.B.4.b: Solve word problems leading to inequalities of the form  $px + q > r$  or  $px + q < r$ , where  $p$ ,  $q$ , and  $r$  are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. \$

## Instructional Routines

- MLR4: Information Gap Cards
- MLR6: Three Reads
- Think Pair Share

## Required Materials

Pre-printed slips, cut from copies of the

Instructional master

## Required Preparation

Print and cut up copies of the Instructional master for the Giving Advice activity. You will need one set of cards for every 4 students.

### Student Learning Goals

Let's look at solutions to inequalities.

# 17.1 Possible Values

## Warm Up: 5 minutes

The purpose of this warm-up is for students to interpret an inequality in a real-world situation and reason about the quantities in its solution. Some of the statements involve reasoning about the how a sandwich shop sells its sandwiches, however, the focus of the discussion should be on the meaning of the solution to the inequality. Students should reason that they cannot order anything more than 13.86 sandwiches, but can order any amount less than 13.86 sandwiches.

## Building Towards

- 7.EE.B.4.b

## Instructional Routines

- Think Pair Share

## Launch

Arrange students in groups of 2. Give students 2 minutes of quiet work time followed by 1 minute to compare their responses with a partner. Follow with a whole-class discussion.

## Anticipated Misconceptions

Some students may think of 13.86 sandwiches as 14 whole sandwiches because it rounds to that number, and 13.86 doesn't make sense to them in the context of sandwiches. It may be helpful for these students to use a calculator to find the cost of 14 sandwiches to see that is not a solution to the inequality. Tell these students that, although sandwich shops may not sell sandwiches in fractional pieces, the maximum amount that can be ordered is 13.86.

### Student Task Statement

The stage manager of the school musical is trying to figure out how many sandwiches he can order with the \$83 he collected from the cast and crew. Sandwiches cost \$5.99 each, so he lets  $x$  represent the number of sandwiches he will order and writes  $5.99x \leq 83$ . He solves this to 2 decimal places, getting  $x \leq 13.86$ .

Which of these are valid statements about this situation? (Select **all** that apply.)

1. He can call the sandwich shop and order exactly 13.86 sandwiches.
2. He can round up and order 14 sandwiches.
3. He can order 12 sandwiches.
4. He can order 9.5 sandwiches.
5. He can order 2 sandwiches.
6. He can order -4 sandwiches.

### Student Response

He can order 12, 2, and maybe 9.5 sandwiches.

13.86: Probably not: it is unlikely a sandwich shop would sell precisely .86 of a sandwich.

14: No. The solution of  $x \leq 13.86$  means that 14 sandwiches would cost more than the \$83 the group can spend.

12: Yes. The solution of  $x \leq 13.86$  means that 12 sandwiches will cost less than (or equal to) \$83.

9.5: Possibly. The sandwich shop may sell half sandwiches for half the price of a whole sandwich.

2: Yes. At a glance, 2 sandwiches will cost much less than \$83.

-4: No. Though  $-4 \leq 13.86$  and -4 is a numerical solution to the inequality, it does not make sense to order -4 sandwiches.

### Activity Synthesis

Poll the class about whether they think each statement is valid. Ask a student to explain why the invalid statements don't work. Record and display their responses for all to see.

For each statement, students should mention the following ideas:

1. Even though 13.86 makes the inequality true, most sandwich shops would not let you order 13.86 sandwiches.
2. He doesn't have enough money to order 14 sandwiches. He has to order a number of sandwiches that is less than or equal to 13.86.
3. Works.
4. Might be okay if the shop allows you to order sandwiches in  $\frac{1}{2}$ -sandwich increments.
5. Works.
6. Even though -4 makes the inequality true, that value doesn't make sense in this context.

## 17.2 Elevator

15 minutes

This problem is an introduction to the series of modeling problems in the next activity. Here, students read a question and are prompted to think about what extra information they would need to solve it (MP4). Then they write and solve inequalities to answer the question.

The context in this problem provides an opportunity for students to think about aspects of mathematical modeling like discrete versus continuous solutions and rounding. Make sure to touch on these topics in discussion before moving on to the next activity.

### Addressing

- 7.EE.B.4.b

### Instructional Routines

- MLR6: Three Reads

### Launch

Ask students to close their books or devices or to leave them closed. Present this scenario verbally or display for all to see:

“A mover is loading an elevator with identical boxes. He wants to take all the boxes up the elevator at once, but he is worried about overloading the elevator. What are all the possibilities for the number of boxes the mover can take on the elevator at once?”

Give students a few minutes of quiet think time to brainstorm what information they would need to answer this question, followed by 1–2 minutes to discuss with a partner. Ask a few students to share their questions with the class and record them for all to see.

Then, ask students to open their books or devices to this activity and use the given information to help solve the problem.

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### Access for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation such as “I chose to use the \_\_\_\_ inequality sign because...”, “Why did you...?”, “I agree/disagree because...”, or “The solution means that the mover...”

*Supports accessibility for: Language; Social-emotional skills*

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## Access for English Language Learners

*Reading; Representing: MLR6 Three Reads.* Use this routine to support reading comprehension of this word problem, without solving it for students. In the first read, have students read a display of the description of the scenario posed in the Activity Launch. Ask students, "What is this situation about?" (A mover is loading boxes in an elevator.). In the second read, ask students to brainstorm the important quantities by identifying what can be counted or measured in this situation. Sample quantities include: number of boxes, size(s) of the boxes, size of the elevator, weight of each box, weight limit of the elevator, weight of the mover. In the third read, ask students to read the actual problems and work with a partner to brainstorm strategies to write an inequality that can represent the relationship among the number of boxes, the total weight of the boxes and the mover, and the weight limit of the elevator. Invite students to sketch a diagram of these quantities. This helps students connect the language in the word problem and the reasoning needed to write an inequality for this situation.

*Design Principle(s): Support sense-making*

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### Student Task Statement

A mover is loading an elevator with many identical 48-pound boxes.  
The mover weighs 185 pounds. The elevator can carry at most 2000 pounds.

1. Write an inequality that says that the mover will not overload the elevator on a particular ride. Check your inequality with your partner.
2. Solve your inequality and explain what the solution means.
3. Graph the solution to your inequality on a number line.
4. If the mover asked, "How many boxes can I load on this elevator at a time?" what would you tell them?

### Student Response

1.  $48b + 185 \leq 2,000$ , where  $b$  is the number of identical boxes.
2.  $b \leq 37.8125$  so the mover can put 37 or fewer boxes on the elevator.
3. Number line shows whole numbers 37 or less.
4. Answers vary. Sample response: 37 or fewer boxes.

### Activity Synthesis

Many issues will come up in the discussion of this problem that will recur throughout the lesson. Some examples:

- “How can we represent the solution on a number line? Is 5.5 a solution?” (Not in the context of this problem; you can’t have a half a box.)
- “Do we want to change the number line somehow to show this?” (We could plot discrete points, or we could simply leave it as is, but just know that for a problem with this context, we’re only going to use integer solutions.)
- “Which type of inequality would you use to describe answers using *no more than* or *no less than*?” ( $\leq$  and  $\geq$ , respectively.)
- “How did you know which way to round?” (Round down, otherwise you’ve gone over the weight limit.)
- “What other limitations do the contexts place on the solutions?” (You must have a positive number of boxes.)

## 17.3 Info Gap: Giving Advice

15 minutes

In this activity, students set up and solve inequalities that represent real-life situations. Students will think about how to interpret their mathematical solutions. For example, if they use  $w$  to represent width in centimeters and find  $w < 25.5$ , does that mean  $w = -10$  is a solution to the inequality?

### Addressing

- 7.EE.B.4.b

### Instructional Routines

- MLR4: Information Gap Cards

### Launch

Tell students they will practice using their knowledge of inequalities to think about specific situations and interpret what their solutions mean in those situations. Ask students to represent their solution using words, an inequality, and a graph. Arrange students in groups of 2. If necessary, demonstrate the protocol for an Info Gap activity. In each group, distribute a problem card to one student and a data card to the other student. Give students who finish early a different pair of cards and ask them to switch roles.

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### Access for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Display or provide students with a physical copy of the written directions. Check for understanding by inviting students to rephrase directions in their own words. Keep the display of directions visible throughout the activity.

*Supports accessibility for: Memory; Organization*

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## Access for English Language Learners

*Conversing:* This activity uses *MLR4 Information Gap* to give students a purpose for discussing information necessary for solving problems involving inequalities. Display questions or question starters for students who need a starting point such as: “Can you tell me . . . (specific piece of information)”, and “Why do you need to know . . . (that piece of information)?”

*Design Principle(s):* Cultivate conversation

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## Anticipated Misconceptions

If students do not know where to start, suggest that they first identify the quantity that should be variable and choose a letter to represent it.

In Elena’s problem, it may help to remind students that they know how to write a formula for the area of a rectangle.

### Student Task Statement

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

Continue to ask questions until you have enough information to solve the problem.

4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.
2. Ask your partner “*What specific information do you need?*” and wait for them to *ask* for information.

If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don’t have that information.

3. Before sharing the information, ask “*Why do you need that information?*”

Listen to your partner’s reasoning and ask clarifying questions.

4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

## Student Response

1.
  - a.  $-1.65x + 50 \geq 15$ ,  $x \leq 21\frac{7}{33}$ .
  - b. Number lines may vary. Some may have a closed (or open?) circle at a point representing  $21\frac{7}{33}$ , with an arrow extending to the left. Some may have closed dots on each integer less than 22.
  - c. Noah's family can wash 21 or fewer loads of laundry before having to add more money to the card (assuming they don't also use the card to pay for drying the clothes).
2.
  - a.  $14 + 2w \leq 65$ ,  $w \leq 25.5$ .
  - b. All number lines will have a closed circle at  $w = 25.5$ . Some may have arrows extending indefinitely to the left. Others may extend an arrow to the left, but stop at zero.
  - c. Elena can choose widths between very close to zero and 25.5 centimeters.

### Are You Ready for More?

In a day care group, nine babies are five months old and 12 babies are seven months old. How many full months from now will the average age of the 21 babies first surpass 20 months old?

## Student Response

14 months. Right now, the sum of all their ages is 129, because  $9 \cdot 5 + 12 \cdot 7 = 129$ . After  $x$  months, the sum of all the babies' ages will have increased by  $21x$ . For 21 babies to have an average age of 20 months, the sum of all their ages would need to be 420 months, because  $\frac{420}{21} = 20$ . Solving the inequality  $129 + 21x > 420$  we get  $x > 13.86$ .

## Activity Synthesis

As the groups report on their work, encourage other students to think and ask questions about whether the answers are plausible. If students do not naturally raise these questions, consider asking:

- In Noah's problem, is 1.5 loads of laundry a solution to the inequality?
- In Elena's problem, can the width of the frame be -10 centimeters? Can the width of the frame be 0 centimeters? How about 0.1 centimeters?

Other questions for discussion:

- Which situations are discrete (have only whole-number solutions)?
- In Noah's problem, should we round up or down?

## Lesson Synthesis

In the last few lessons, students have seen a variety of situations in which inequalities described situations. Ask students to think about a career they might be interested in pursuing and have them write a few sentences about the usefulness of inequalities in the work of that profession, including at least one example. Ask them to think about whether inequalities are sometimes more helpful than equations, and if so, why.

## 17.4 Movies on a Hard Drive

### Cool Down: 5 minutes

This cool-down checks through error analysis to determine whether students can interpret solutions to an inequality in context.

### Addressing

- 7.EE.B.4.b

### Student Task Statement

Elena is trying to figure out how many movies she can download to her hard drive. The hard drive is supposed to hold 500 gigabytes of data, but 58 gigabytes are already taken up by other files. Each movie is 8 gigabytes. Elena wrote the inequality  $8x + 58 \geq 500$  and solved it to find the solution  $x \geq 55.25$ .

1. Explain how you know Elena made a mistake based on her solution.
2. Fix Elena's inequality and explain what each part of the inequality means.

### Student Response

1.  $x \geq 55.25$  means Elena has to put more than 55 movies on her hard drive. This doesn't make sense because there should be a maximum limit on movies rather than a minimum limit.
2. The correct inequality is  $8x + 58 \leq 500$ . The number 8 represents the size of each movie. The variable  $x$  represents the number of movies that Elena downloads. The  $\leq 500$  represents that the number of gigabytes can't exceed 500.

### Student Lesson Summary

We can represent and solve many real-world problems with inequalities. Whenever we write an inequality, it is important to decide what quantity we are representing with a variable. After we make that decision, we can connect the quantities in the situation to write an expression, and finally, the whole inequality.

As we are solving the inequality or equation to answer a question, it is important to keep the meaning of each quantity in mind. This helps us to decide if the final answer makes sense in the context of the situation.

For example: Han has 50 centimeters of wire and wants to make a square picture frame with a loop to hang it that uses 3 centimeters for the loop. This situation can be represented by  $3 + 4s = 50$ , where  $s$  is the length of each side (if we want to use all the wire). We can also use  $3 + 4s \leq 50$  if we want to allow for solutions that don't use all the wire. In this case, any positive number that is less or equal to 11.75 cm is a solution to the inequality. Each solution represents a possible side length for the picture frame since Han can bend the wire at any point. In other situations, the variable may represent a quantity that increases by whole numbers, such as with numbers of magazines, loads of laundry, or students. In those cases, only whole-number solutions make sense.

# Lesson 17 Practice Problems

## Problem 1

### Statement

28 students travel on a field trip. They bring a van that can seat 12 students. Elena and Kiran's teacher asks other adults to drive cars that seat 3 children each to transport the rest of the students.

Elena wonders if she should use the inequality  $12 + 3n > 28$  or  $12 + 3n \geq 28$  to figure out how many cars are needed. Kiran doesn't think it matters in this case. Do you agree with Kiran? Explain your reasoning.

### Solution

Sample explanation: Yes, it doesn't matter. In this case  $n$  represents a number of cars, so only whole number values of  $n$  make sense for the situation, and there can't be fractions of cars.  $12 + 3n = 28$  has the solution  $n = \frac{16}{3}$ , so the number of cars needed is 6.

## Problem 2

### Statement

- In the cafeteria, there is one large 10-seat table and many smaller 4-seat tables. There are enough tables to fit 200 students. Write an inequality whose solution is the possible number of 4-seat tables in the cafeteria.
- 5 barrels catch rainwater in the schoolyard. Four barrels are the same size, and the fifth barrel holds 10 liters of water. Combined, the 5 barrels can hold at least 200 liters of water. Write an inequality whose solution is the possible size of each of the 4 barrels.
- How are these two problems similar? How are they different?

### Solution

- $10 + 4n \geq 200$
- $10 + 4n \geq 200$
- Solutions to the first inequality must be whole numbers greater or equal to 47.5 because a solution represents a number of tables. Solutions to the second inequality can be any number greater or equal to 47.5 because a solution represents the volume of a bucket, which can be a whole number or not.

## Problem 3

### Statement

Solve each equation.

a.  $5(n - 4) = -60$

b.  $-3t + -8 = 25$

c.  $7p - 8 = -22$

d.  $\frac{2}{5}(j + 40) = -4$

e.  $4(w + 1) = -6$

## Solution

a.  $n = -8$

b.  $t = -11$

c.  $p = -2$

d.  $j = -50$

e.  $w = \frac{-10}{4}$  (or equivalent)

(From Unit 6, Lesson 9.)

## Problem 4

### Statement

Select all the inequalities that have the same graph as  $x < 4$ .

A.  $x < 2$

B.  $x + 6 < 10$

C.  $5x < 20$

D.  $x - 2 > 2$

E.  $x < 8$

## Solution


["B", "C"]

(From Unit 6, Lesson 13.)

## Problem 5

### Statement

A 200 pound person weighs 33 pounds on the Moon.

- 
- a. How much did the person's weight decrease?
  - b. By what percentage did the person's weight decrease?

## **Solution**

a. 167 pounds

b. About 84% ( $167 \div 200 = 0.835$ )

(From Unit 4, Lesson 12.)

## Section: Writing Equivalent Expressions

### Lesson 18: Subtraction in Equivalent Expressions

#### Goals

- Explain (orally, in writing, and using other representations) how the distributive and commutative properties apply to expressions with negative coefficients.
- Justify (orally and in writing) whether expressions are equivalent, including rewriting subtraction as adding the opposite.

#### Learning Targets

- I can organize my work when I use the distributive property.
- I can re-write subtraction as adding the opposite and then rearrange terms in an expression.

#### Lesson Narrative

Previously in this unit, students solved equations of the form  $px + q = r$  and  $p(x + q) = r$ . Sometimes, work has to be done on a more complicated expression to get an equation into one of these forms. And sometimes, it is desirable to rewrite an expression in an equivalent form to understand how the quantities it represents are related. This work has some pitfalls when the expression has negative numbers or subtraction. For example, it is common for people to rewrite  $6x - 5 + 2x$  as  $4x + 5$  by reading “6x minus” and so subtracting the  $2x$  from the  $6x$ . Another example is rewriting an expression like  $5x - 2(x + 3)$  as  $5x - 2x + 6$ . Students do not see expressions as complicated as these in this lesson (they are coming in the next few lessons), but this lesson is meant to inoculate students against errors like these by reminding them that while subtraction is not commutative, addition is, and subtraction can be rewritten as adding the opposite. So in our example,  $6x - 5 + 2x$  can be rewritten  $6x + -5 + 2x$  and then rearranged  $6x + 2x + -5$ . Likewise,  $5x - 2(x + 3)$  can be rewritten  $5x + -2(x + 3)$  before distributing  $-2$ .

#### Alignments

##### Building On

- 7.NS.A.1.c: Understand subtraction of rational numbers as adding the additive inverse,  $p - q = p + (-q)$ . Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

##### Addressing

- 7.EE.A.1: Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
- 7.NS.A.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

- 7.NS.A.1.c: Understand subtraction of rational numbers as adding the additive inverse,  $p - q = p + (-q)$ . Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

### Building Towards

- 7.EE.A.1: Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

### Instructional Routines

- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Number Talk

### Student Learning Goals

Let's find ways to work with subtraction in expressions.

## 18.1 Number Talk: Additive Inverses

### Warm Up: 5 minutes

The purpose of this Number Talk is to elicit strategies and understandings that students have for adding and subtracting signed numbers. These understandings help students develop fluency and will be helpful later in this lesson when students will need to be able to rewrite subtraction as adding the opposite. While four problems are given, it may not be possible to share every strategy. Consider gathering only two or three different strategies per problem, saving most of the time for the final question.

### Addressing

- 7.NS.A.1

### Instructional Routines

- MLR8: Discussion Supports
- Number Talk

### Launch

Display one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

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## Access for Students with Disabilities

*Representation: Internalize Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.

*Supports accessibility for: Memory; Organization*

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### Student Task Statement

Find each sum or difference mentally.

$$-30 + -10$$

$$-10 + -30$$

$$-30 - 10$$

$$10 - -30$$

### Student Response

Answers vary. Possible responses:

- $-30 + -10$  is  $-40$ , because I can represent  $-30$  as an arrow pointing left from 0 to  $-30$  on the number line. Adding  $-10$  tacks on an additional 10 to the left, arriving at  $-40$ .
- $-10 + -30$  is  $-40$ , because addition is commutative.
- $-30 - 10$  is  $-40$ , because subtracting 10 is the same as adding  $-10$ , and  $-30 + -10 = -40$ .
- $10 - -30$  is  $40$ , because subtracting  $-30$  is the same as adding 30, and  $10 + 30 = 40$ .

### Activity Synthesis

When it comes up, emphasize that “subtract 10” can be rewritten “add negative 10.” Also that addition is commutative but subtraction is not. Mention these points even if students do not bring them up.

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- “Who can restate \_\_\_’s reasoning in a different way?”
- “Did anyone have the same strategy but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to \_\_\_\_’s strategy?”
- “Do you agree or disagree? Why?”

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### Access for English Language Learners

*Speaking: MLR8 Discussion Supports.*: Display sentence frames to support students when they explain their strategy. For example, "First, I \_\_\_\_ because . . ." or "I noticed \_\_\_\_ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

*Design Principle(s): Optimize output (for explanation)*

---

## 18.2 A Helpful Observation

10 minutes

Students recall that subtracting a number (or expression) is the same as adding its additive inverse. This concept is applied to get students used to the idea that the subtraction sign has to stay with the term it is in front of. Making this concept explicit through a numeric example will help students see its usefulness and help them avoid common errors in working with expressions that involve subtraction.

### Building On

- 7.NS.A.1.c

### Building Towards

- 7.EE.A.1

### Instructional Routines

- MLR8: Discussion Supports

### Launch

Display the expression  $7\frac{3}{4} + 3\frac{5}{6} - 1\frac{3}{4}$  and ask students to evaluate. After they have had a chance to think about the expression, read through the task statement together before setting students to work.

---

### Access for Students with Disabilities

*Representation: Access for Perception.* Read the dialogue between Lin and Kiran aloud. Students who both listen to and read the information will benefit from extra processing time. Consider having pairs of students role play the scenario together and repeat it as necessary in order to comprehend the situation.

*Supports accessibility for: Language*

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## Access for English Language Learners

*Conversing, Speaking: MLR8 Discussion Supports.* Provide sentence frames to help students produce explanations about equivalent expressions. For example, “I agree/disagree that \_\_\_ is equivalent to  $7\frac{3}{4} + 3\frac{5}{6} - 1\frac{3}{4}$  because . . .” This will help students use the language of justification for comparing equivalent expressions related to the communicative property of addition.

*Design Principle(s): Optimize output (for justification)*

---

### Student Task Statement

Lin and Kiran are trying to calculate  $7\frac{3}{4} + 3\frac{5}{6} - 1\frac{3}{4}$ . Here is their conversation:

Lin: “I plan to first add  $7\frac{3}{4}$  and  $3\frac{5}{6}$ , so I will have to start by finding equivalent fractions with a common denominator.”

Kiran: “It would be a lot easier if we could start by working with the  $1\frac{3}{4}$  and  $7\frac{3}{4}$ . Can we rewrite it like  $7\frac{3}{4} + 1\frac{3}{4} - 3\frac{5}{6}$ ?”

Lin: “You can’t switch the order of numbers in a subtraction problem like you can with addition;  $2 - 3$  is not equal to  $3 - 2$ .”

Kiran: “That’s true, but do you remember what we learned about rewriting subtraction expressions using addition?  $2 - 3$  is equal to  $2 + (-3)$ .”

1. Write an expression that is equivalent to  $7\frac{3}{4} + 3\frac{5}{6} - 1\frac{3}{4}$  that uses addition instead of subtraction.
2. If you wrote the **terms** of your new expression in a different order, would it still be equivalent? Explain your reasoning.

### Student Response

1.  $7\frac{3}{4} + 3\frac{5}{6} + (-1\frac{3}{4})$
2. Answers vary. Sample response: It works as long as the subtraction or negative sign is moved along with the number that follows. What doesn’t work is moving the numbers but leaving the subtraction sign in the same place.

### Activity Synthesis

Ensure everyone agrees that  $7\frac{3}{4} + 3\frac{5}{6} - 1\frac{3}{4}$  is equivalent to  $7\frac{3}{4} + 3\frac{5}{6} + (-1\frac{3}{4})$  is equivalent to  $7\frac{3}{4} + (-1\frac{3}{4}) + 3\frac{5}{6}$ . Use the language “commutative property of addition.”

## 18.3 Organizing Work

15 minutes

Students learn that we can still organize our work with the distributive property in a familiar way, even with negative numbers where thinking in terms of area breaks down.

### Building On

- 7.NS.A.1.c

### Addressing

- 7.EE.A.1

### Instructional Routines

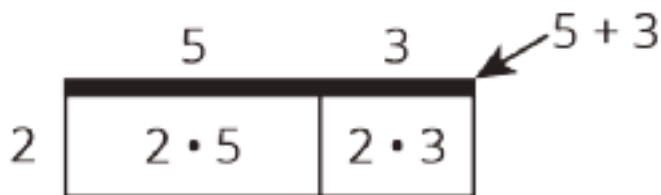
- MLR7: Compare and Connect

### Launch

Display the image and ask students to write an expression for the area of the big rectangle in at least 3 different ways.



Collect responses. If students simply say “16,” ask them to explain how they calculated 16 and record these processes for all to see. Remind students that thinking about area gives us a way to understand the distributive property. This diagram can be used to show that  $2 \cdot 5 + 2 \cdot 3 = 2(5 + 3)$ . Be sure that students see you write the partial products in the diagram, and that they see every piece of the associated identity  $2 \cdot 5 + 2 \cdot 3 = 2(5 + 3)$ .



Tell students that when we are working with negative numbers, thinking about area doesn't work so well, but the distributive property still holds when there are negative numbers. The expressions involved still have the same structure, and we can still organize our work the same way.

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### Access for Students with Disabilities

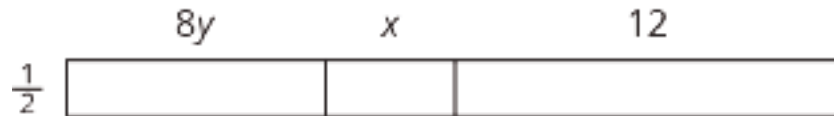
*Representation: Internalize Comprehension.* Differentiate the degree of difficulty or complexity by beginning with an example with more accessible values, such as the one given. Highlight connections between representations by recording the calculations in the boxes. In addition, consider creating a display showing a general example using only variables and keeping it as a reference throughout the remainder of the unit.

*Supports accessibility for: Conceptual processing*

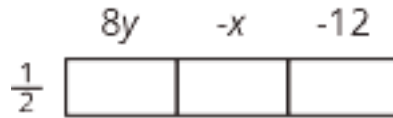
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#### Student Task Statement

1. Write two expressions for the area of the big rectangle.



2. Use the distributive property to write an expression that is equivalent to  $\frac{1}{2}(8y + -x + -12)$ . The boxes can help you organize your work.



3. Use the distributive property to write an expression that is equivalent to  $\frac{1}{2}(8y - x - 12)$ .

#### Student Response

1.  $\frac{1}{2}(8y + x + 12)$  and  $4y + \frac{1}{2}x + 6$
2.  $4y + -\frac{1}{2}x + -6$
3.  $4y - \frac{1}{2}x - 6$

## Are You Ready for More?

Here is a calendar for April 2017.

April 2017						
Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30						

Let's choose a date: the 10th. Look at the numbers above, below, and to either side of the 10th: 3, 17, 9, 11.

1. Average these four numbers. What do you notice?
2. Choose a different date that is in a location where it has a date above, below, and to either side. Average these four numbers. What do you notice?
3. Explain why the same thing will happen for any date in a location where it has a date above, below, and to either side.

### Student Response

1. The average of the four numbers is 10, because  $(3 + 17 + 9 + 11) \div 4 = 40 \div 4 = 10$ . The average of the four surrounding numbers equals the original date chosen.
2. Answers vary. Sample response: Let's choose 21. The four numbers are 14, 28, 20, 22. The average of these is 21, because  $(14 + 28 + 20 + 22) \div 4 = 84 \div 4 = 21$ . The average of the four surrounding numbers equals the original date chosen.
3. Answers vary. Sample response using algebra. If the original date chosen is represented by  $x$ , then the date above is  $x - 7$  because it must be 7 days prior. The date below is  $x + 7$  because it must be 7 days after. The date to the left is  $x - 1$  and the date to the right is  $x + 1$ . The sum of these four dates is  $x - 7 + x + 7 + x - 1 + x + 1$  which equals  $4x$ . To find the average, I would divide this by 4, giving the original date chosen,  $x$ .

### Activity Synthesis

Solicit responses to the second question and demonstrate thinking about one product at a time:

$$\frac{1}{2} \begin{array}{|c|c|c|} \hline 8y & -x & -12 \\ \hline 4y & -\frac{1}{2}x & -6 \\ \hline \end{array}$$

Then ask students to share how they approached the last question. Highlight responses where students noticed that  $\frac{1}{2}(8y - x - 12)$  can be rewritten like  $\frac{1}{2}(8y + -x - 12)$  (because of what they talked about in the warm-up). So the two questions have the same answer.

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### Access for English Language Learners

*Speaking, Representing: MLR7 Compare and Connect.* Use this routine when students present their expressions. Ask students “What is the same and what is different?” about the approaches and representations involving subtraction with the distributive property. Help students connect how the expressions that have a subtraction operation are equivalent to expressions that add its additive inverse. These exchanges strengthen students’ mathematical language use and reasoning with the distributive property and the subtraction operation.

*Design Principle(s): Maximize meta-awareness*

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## Lesson Synthesis

Display two expressions like  $x + 2 - 3x - 10$  and  $x + 3x - 2 - 10$ . Ask students to think about why these expressions are *not* equivalent and explain to a partner. Two explanations should be highlighted:

- Subtraction isn't commutative.  $2 - 3x$  and  $3x - 2$  are not equivalent; you can't just switch terms around a subtraction sign.
- Since  $-3x$  is the same as  $+(-3x)$ , the negative sign needs to stay with the  $3x$  when terms are rearranged.

Ask students how they could fix the second expression to make it equivalent to the first. Ensure that everyone agrees and understands why  $x + -3x + 2 + -10$  and  $x - 3x + 2 - 10$  are equivalent to the first expression.

## 18.4 Equivalent to $4 - x$

Cool Down: 5 minutes

### Addressing

- 7.EE.A.1
- 7.NS.A.1.c

### Student Task Statement

1. Select **all** the expressions that are equivalent to  $4 - x$ .

- a.  $x - 4$
- b.  $4 + -x$
- c.  $-x + 4$
- d.  $-4 + x$

e.  $4 + x$

2. Use the distributive property to write an expression that is equivalent to  $5(-2x - 3)$ . If you get stuck, use the boxes to help organize your work.



### Student Response

1. b, c
2.  $-10x - 15$  or equivalent

### Student Lesson Summary

Working with subtraction and signed numbers can sometimes get tricky. We can apply what we know about the relationship between addition and subtraction—that subtracting a number gives the same result as adding its opposite—to our work with expressions. Then, we can make use of the properties of addition that allow us to add and group in any order. This can make calculations simpler. For example:

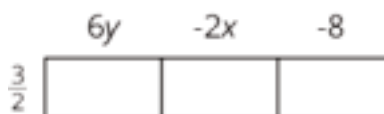
$$\frac{5}{8} - \frac{2}{3} - \frac{1}{8}$$

$$\frac{5}{8} + -\frac{2}{3} + -\frac{1}{8}$$

$$\frac{5}{8} + -\frac{1}{8} + -\frac{2}{3}$$

$$\frac{4}{8} + -\frac{2}{3}$$

We can also organize the work of multiplying signed numbers in expressions. The product  $\frac{3}{2}(6y - 2x - 8)$  can be found by drawing a rectangle with the first factor,  $\frac{3}{2}$ , on one side, and the three terms inside the parentheses on the other side:



Multiply  $\frac{3}{2}$  by each term across the top and perform the multiplications:

$$\frac{3}{2} \begin{array}{|c|c|c|} \hline 6y & -2x & -8 \\ \hline \frac{3}{2} \cdot 6y & \frac{3}{2} \cdot -2x & \frac{3}{2} \cdot -8 \\ \hline \end{array}$$

$$\frac{3}{2} \begin{array}{|c|c|c|} \hline 6y & -2x & -8 \\ \hline 9y & -3x & -12 \\ \hline \end{array}$$

Reassemble the parts to get the expanded version of the original expression:

$$\frac{3}{2}(6y - 2x - 8) = 9y - 3x - 12$$

## Glossary

- term

# Lesson 18 Practice Problems

## Problem 1

### Statement

For each expression, write an equivalent expression that uses only addition.

a.  $20 - 9 + 8 - 7$

b.  $4x - 7y - 5z + 6$

c.  $-3x - 8y - 4 - \frac{8}{7}z$

### Solution

a.  $20 + -9 + 8 + -7$

b.  $4x + -7y + -5z + 6$

c.  $-3x + -8y + -4 + -\frac{8}{7}z$

## Problem 2

### Statement

Use the distributive property to write an expression that is equivalent to each expression. If you get stuck, consider drawing boxes to help organize your work.

a.  $9(4x - 3y - \frac{2}{3})$

b.  $-2(-6x + 3y - 1)$

c.  $\frac{1}{5}(20y - 4x - 13)$

d.  $8(-x - \frac{1}{2})$

e.  $-8(-x - \frac{3}{4}y + \frac{7}{2})$

### Solution

a.  $36x - 27y - 6$

b.  $12x - 6y + 2$

c.  $4y - \frac{4}{5}x - \frac{13}{5}$

d.  $-8x - 4$

e.  $8x + 6y - 28$

### Problem 3

#### Statement

Kiran wrote the expression  $x - 10$  for this number puzzle: "Pick a number, add -2, and multiply by 5."

Lin thinks Kiran made a mistake.

- How can she convince Kiran he made a mistake?
- What would be a correct expression for this number puzzle?

#### Solution

- Answers vary. Sample response: for  $x = 1$  the number puzzle should result in  $-5$ . But Kiran's expression gives  $1 - 10 = -9$ .
- $(x - 2) \cdot 5$  (or  $5x - 10$ )

### Problem 4

#### Statement

The output from a coal power plant is shown in the table:

energy in megawatts	number of days
1,200	2.4
1,800	3.6
4,000	8
10,000	20

Similarly, the output from a solar power plant is shown in the table:

energy in megawatts	number of days
100	1
650	4
1,200	7
1,750	10

Based on the tables, is the energy output in proportion to the number of days for either plant? If so, write an equation showing the relationship. If not, explain your reasoning.

## Solution

The coal power plant could be a proportional relationship. Its equation would be  $E = 500 \cdot d$  where  $E$  is the energy output in megawatts and  $d$  is the number of days. The solar power plant would not be a proportional relationship since the ratio between the number of days and the energy output is not constant.

(From Unit 2, Lesson 7.)

# Lesson 19: Expanding and Factoring

## Goals

- Apply the distributive property to expand or factor an expression that includes negative coefficients, and explain (orally and using other representations) the reasoning.
- Comprehend the terms “expand” and “factor” (in spoken and written language) in relation to the distributive property.

## Learning Targets

- I can organize my work when I use the distributive property.
- I can use the distributive property to rewrite expressions with positive and negative numbers.
- I understand that factoring and expanding are words used to describe using the distributive property to write equivalent expressions.

## Lesson Narrative

In grade 6, students worked extensively with the distributive property involving both addition and subtraction, but only with positive coefficients. In the previous lesson, students learned to rewrite subtraction as “adding the opposite” to avoid common pitfalls. In this lesson, students practice using the distributive property to write equivalent expressions when there are rational coefficients. Some of the expressions they will work with are in preparation for understanding combining like terms in terms of the distributive property, coming up in the next lesson. (For example,  $17a - 13a$  can be rewritten  $a(17 - 13)$  using the distributive property, so it is equivalent to  $a \cdot 4$  or  $4a$ .)

## Alignments

### Building On

- 7.NS.A: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

### Addressing

- 7.EE.A.1: Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

### Building Towards

- 7.EE.A.1: Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

## Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports

- Number Talk

### Student Learning Goals

Let's use the distributive property to write expressions in different ways.

## 19.1 Number Talk: Parentheses

### Warm Up: 10 minutes

The purpose of this number talk is to remind students that when we evaluate expressions, we multiply before we add or subtract. Parentheses are used to indicate that the order should be different. Remembering how this works will be important for an activity in this lesson.

### Building On

- 7.NS.A

### Building Towards

- 7.EE.A.1

### Instructional Routines

- MLR8: Discussion Supports
- Number Talk

### Launch

Display one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

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### Access for Students with Disabilities

*Representation: Internalize Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.

*Supports accessibility for: Memory; Organization*

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### Student Task Statement

Find the value of each expression mentally.

$$2 + 3 \cdot 4$$

$$(2 + 3)(4)$$

$$2 - 3 \cdot 4$$

$$2 - (3 + 4)$$

## Student Response

Strategies vary. Possible responses:

- 14, because absent parentheses, I know to evaluate multiplication before addition. So  $2 + 3 \cdot 4 = 2 + 12 = 14$ .
- 20, because parentheses indicate their contents should be evaluated first, and next-to means multiply. So  $(2 + 3)(4) = 5 \cdot 4 = 20$
- -10, because multiply before subtract, so  $2 - 3 \cdot 4 = 2 - 12 = -10$ .
- -5, because parentheses first, so  $2 - (3 + 4) = 2 - 7 = -5$ .

## Activity Synthesis

The second question is an opportunity to remind students that “next to” implies “multiply.” The second expression could be rewritten  $(2 + 3) \cdot 4$ . Point out that we can also know that the second expression is 20 by using the distributive property:  $2 \cdot 4 + 3 \cdot 4$ .

The fourth expression could also be rewritten  $2 - 3 - 4$ .

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- “Who can restate \_\_\_’s reasoning in a different way?”
- “Did anyone have the same strategy but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to \_\_\_’s strategy?”
- “Do you agree or disagree? Why?”

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### Access for English Language Learners

*Speaking: MLR8 Discussion Supports.*: Display sentence frames to support students when they explain their strategy. For example, “First, I \_\_\_ because . . .” or “I noticed \_\_\_ so I . . .” Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

*Design Principle(s): Optimize output (for explanation)*

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## 19.2 Factoring and Expanding with Negative Numbers

20 minutes

This activity is an opportunity for students to practice rewriting expressions using the distributive property. It is a step up from the same type of work in grade 6 because arithmetic with signed numbers is required.

The row with  $6a - 2b$  is designed to allow students to figure out how to factor by reasoning based on structure they already understand, instead of learning how to factor based on a procedure that a teacher demonstrates first. The rows with  $k(4 - 17)$  and  $10a - 13a$  are designed to prepare students for combining like terms in the next lesson.

## Addressing

- 7.EE.A.1

## Instructional Routines

- MLR3: Clarify, Critique, Correct

## Launch

Draw students' attention to the organizers that appear above the table, and tell them that these correspond to the first three rows in the table. Let them know that they are encouraged to draw more organizers like this for other rows as needed.

Arrange students in groups of 2. Instruct them to take turns writing an equivalent expression for each row. One partner writes the equivalent expression and explains their reasoning while the other listens. If the partner disagrees, they work to resolve the discrepancy before moving to the next row.

---

### Access for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Provide students with blank diagrams to organize their work for the factored and expanded expressions. Include example diagrams labeled "expanded" and "factored."

*Supports accessibility for: Language; Organization*

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## Anticipated Misconceptions

If students are unsure how to proceed, remind them of tools and understandings they have seen recently that would be helpful. For example, "Draw an organizer and think about how the organizer represents terms in the expression." Also, "Rewrite subtraction as adding the opposite."

## Student Task Statement

In each row, write the equivalent expression. If you get stuck, use a diagram to organize your work. The first row is provided as an example. Diagrams are provided for the first three rows.



factored	expanded
$-3(5 - 2y)$	$-15 + 6y$
$5(a - 6)$	
	$6a - 2b$
$-4(2w - 5z)$	
$-(2x - 3y)$	
	$20x - 10y + 15z$
$k(4 - 17)$	
	$10a - 13a$
$-2x(3y - z)$	
	$ab - bc - 3bd$
$-x(3y - z + 4w)$	

### Student Response

Expressions equivalent to these are also acceptable. For example, instead of  $a(10 - 13)$ , one could write  $(10 - 13) \cdot a$  or  $-3a$ .

factored	expanded
$-3(5 - 2y)$	$-15 + 6y$
$5(a - 6)$	$5a - 30$
$2(3a - b)$	$6a - 2b$
$-4(2w - 5z)$	$-8w + 20z$
$-(2x - 3y)$	$-2x + 3y$
$5(4x - 2y + 3z)$	$20x - 10y + 15z$
$k(4 - 17)$	$4k - 17k$
$a(10 - 13)$	$10a - 13a$
$-2x(3y - z)$	$-6xy + 2xz$
$b(a - c - 3d)$	$ab - bc - 3bd$
$-x(3y - z + 4w)$	$-3xy + xz - 4xw$

### Are You Ready for More?

Expand to create an equivalent expression that uses the fewest number of terms:  $\left(\left(\left(\left(x + 1\right) \frac{1}{2}\right) + 1\right) \frac{1}{2}\right) + 1$ . If we wrote a new expression following the same pattern so that there were 20 sets of parentheses, how could it be expanded into an equivalent expression that uses the fewest number of terms?

### Student Response

$$\frac{1}{4}(x + 7), \frac{1}{2^{10}}(x + 2^{11} - 1)$$

### Activity Synthesis

Much of the discussion will take place in small groups. Display the correct equivalent expressions and work to resolve any discrepancies. Expanding the term  $-(2x - 3y)$  may require particular care. One way to interpret it is to rewrite as  $-1 \cdot (2x - 3y)$ . If any confusion about handling subtraction arises, encourage students to employ the strategy of rewriting subtraction as adding the opposite.

To wrap up the activity, ask:

- “Which rows did you and your partner disagree about? How did you resolve the disagreement?”
- “Which rows are you the most unsure about?”

- “Describe a process or procedure for taking a factored expression and writing its corresponding expanded expression.”
- “Describe a process or procedure for taking an expanded expression and writing its corresponding factored expression.”

---

### Access for English Language Learners

*Writing: MLR3 Clarify, Critique, Correct.* Present an incorrect response for one of the expanded expressions in the table. For example, “An equivalent expression for  $6a - 2b$  is  $4(a - b)$  because  $6 - 2$  is 4 and  $(a - b)$  is left on its own.” Prompt students to clarify any of the language and reasoning in the incorrect response and then to identify the error(s). Invite students to work with a partner to write a correct response. This helps students evaluate and improve on the written mathematical arguments of others.

*Design Principle(s): Maximize meta-awareness*

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## Lesson Synthesis

- To write an equivalent expression by factoring means to use the distributive property to write a sum as a product.
- To write an equivalent expression by expanding means to use the distributive property to write a product as a sum.

Ask students to give an example of each.

## 19.3 Equivalent Expressions

Cool Down: 5 minutes

### Addressing

- 7.EE.A.1

### Student Task Statement

1. Expand to write an equivalent expression:  $-\frac{1}{2}(-2x + 4y)$
2. Factor to write an equivalent expression:  $26a - 10$

If you get stuck, use a diagram to organize your work.

### Student Response

1.  $x - 2y$
2.  $2(13a - 5)$

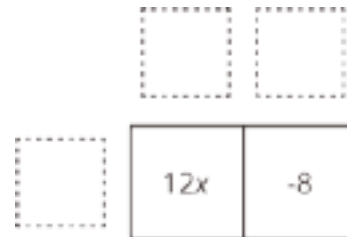
Expressions equivalent to these are also acceptable, like  $(13a - 5) \cdot 2$ .

## Student Lesson Summary

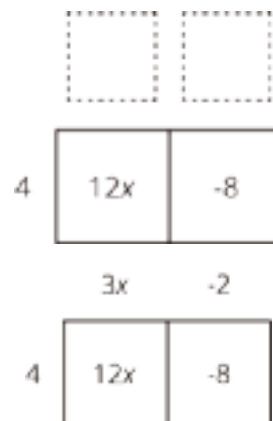
We can use properties of operations in different ways to rewrite expressions and create equivalent expressions. We have already seen that we can use the distributive property to **expand** an expression, for example  $3(x + 5) = 3x + 15$ . We can also use the distributive property in the other direction and **factor** an expression, for example  $8x + 12 = 4(2x + 3)$ .

We can organize the work of using distributive property to rewrite the expression  $12x - 8$ . In this case we know the product and need to find the factors.

The terms of the product go inside:



We look at the expressions and think about a factor they have in common.  $12x$  and  $-8$  each have a factor of 4. We place the common factor on one side of the large rectangle:



Now we think: "4 times *what* is  $12x$ ?" "4 times *what* is  $-8$ ?" and write the other factors on the other side of the rectangle:



So,  $12x - 8$  is equivalent to  $4(3x - 2)$ .

## Glossary

- expand
- factor (an expression)

# Lesson 19 Practice Problems

## Problem 1

### Statement

- Expand to write an equivalent expression:  $\frac{-1}{4}(-8x + 12y)$
- Factor to write an equivalent expression:  $36a - 16$

### Solution

- $2x - 3y$
- $4(9a - 4)$  (or  $2(18a - 8)$ )

## Problem 2

### Statement

Lin missed math class on the day they worked on expanding and factoring. Kiran is helping Lin catch up.

- Lin understands that expanding is using the distributive property, but she doesn't understand what factoring is or why it works. How can Kiran explain factoring to Lin?
- Lin asks Kiran how the diagrams with boxes help with factoring. What should Kiran tell Lin about the boxes?
- Lin asks Kiran to help her factor the expression  $-4xy - 12xz + 20xw$ . How can Kiran use this example to Lin understand factoring?

### Solution

- Answers vary. Sample response: Factoring is the distributive property in the other direction. Instead of expanding a product to a sum of terms, factoring takes a sum of terms and makes it into a product by looking for common factors in the terms that can be written outside the parentheses.
- Answers vary. Sample response: The expression in each box is the product of {the expression to the left of the big rectangle} and {the expression above the box}, just as the area of a rectangle is length times width. Together, the boxes form a long rectangle, so it is still true that {the expression to the left of the box} times {the expression above the long rectangle} equals the sum of all the terms in the boxes. If you want to factor an expression, look for a common factor in each box, and place it to the left of the rectangle. To decide what to write above each box, think, "What times that common factor equals what is in the box?"
- Answers vary and should describe the box or steps. Sample response: First, find the common factor, which is  $4x$ . Write " $4x(\dots)$ ." We are going to decide what needs to go in the

parentheses to make an expression equivalent to  $-4xy - 12xz + 20xw$ . To get  $-4xy$ , we need to multiply by  $-y$ . Using similar reasoning, we can fill in the rest:  $4x(-y - 3z + 5w)$ .

### Problem 3

#### Statement

Complete the equation with numbers that makes the expression on the right side of the equal sign equivalent to the expression on the left side.

$$75a + 25b = \underline{\quad}(\underline{\quad}a + b)$$

#### Solution

$$25(3a + b)$$

### Problem 4

#### Statement

Elena makes her favorite shade of purple paint by mixing 3 cups of blue paint,  $1\frac{1}{2}$  cups of red paint, and  $\frac{1}{2}$  of a cup of white paint. Elena has  $\frac{2}{3}$  of a cup of white paint.

- Assuming she has enough red paint and blue paint, how much purple paint can Elena make?
- How much blue paint and red paint will Elena need to use with the  $\frac{2}{3}$  of a cup of white paint?

#### Solution

- $\frac{20}{3}$  cups. One batch of purple paint makes 5 cups. Elena can make  $\frac{2}{3} \div \frac{1}{2} = \frac{4}{3}$  batches so that's  $\frac{20}{3}$  cups.
- 4 cups of blue paint and 2 cups of red paint.

(From Unit 4, Lesson 3.)

### Problem 5

#### Statement

Solve each equation.

- $\frac{-1}{8}d - 4 = \frac{-3}{8}$
- $\frac{-1}{4}m + 5 = 16$
- $10b + -45 = -43$

d.  $-8(y - 1.25) = 4$

e.  $3.2(s + 10) = 32$

## Solution

a.  $d = -29$

b.  $m = -44$

c.  $b = \frac{1}{5}$  (or equivalent)

d.  $y = 0.75$  (or equivalent)

e.  $s = 0$

(From Unit 6, Lesson 9.)

## Problem 6

### Statement

Select all the inequalities that have the same solutions as  $-4x < 20$ .

A.  $-x < 5$

B.  $4x > -20$

C.  $4x < -20$

D.  $x < -5$

E.  $x > 5$

F.  $x > -5$

## Solution

["A", "B", "F"]

(From Unit 6, Lesson 13.)

# Lesson 20: Combining Like Terms (Part 1)

## Goals

- Apply properties of operations to justify (orally and in writing) that expressions are equivalent.
- Generate an expression that is equivalent to a given expression with fewer terms.
- Interpret different methods for determining whether expressions are equivalent, and evaluate (orally) their usefulness.

## Learning Targets

- I can figure out whether two expressions are equivalent to each other.
- When possible, I can write an equivalent expression that has fewer terms.

## Lesson Narrative

In this lesson, students have a chance to recall one way of understanding equivalent expressions, that is, the expressions have the same value for any number substituted for a variable. Then they use properties they have studied over the past several lessons to understand how to properly write an equivalent expression using fewer terms. We are gently building up to students being able to fluently *combine like terms*, though that language is not used with students yet.

## Alignments

### Building On

- 6.EE.A.4: Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions  $y + y + y$  and  $3y$  are equivalent because they name the same number regardless of which number  $y$  stands for.

### Addressing

- 7.EE.A.1: Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

### Building Towards

- 7.EE.A.1: Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

## Instructional Routines

- MLR8: Discussion Supports

### Student Learning Goals

Let's see how we can tell that expressions are equivalent.

## 20.1 Why is it True?

### Warm Up: 10 minutes

The purpose of this warm-up is to remind students about a few algebraic moves they have studied in the past several lessons by prompting them to explain the reason the moves are allowed. These moves are important to understand as students work toward fluency in writing expressions with fewer terms. Although this activity isn't properly a Number Talk, a similar routine can be followed.

### Building On

- 6.EE.A.4

### Building Towards

- 7.EE.A.1

### Launch

Display one statement at a time. Give students 30 seconds of quiet think time for each statement and ask them to give a signal when they have an explanation. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

### Student Task Statement

Explain why each statement is true.

1.  $5 + 2 + 3 = 5 + (2 + 3)$
2.  $9a$  is equivalent to  $11a - 2a$ .
3.  $7a + 4 - 2a$  is equivalent to  $7a + -2a + 4$ .
4.  $8a - (8a - 8)$  is equivalent to 8.

### Student Response

Answers vary. Sample responses:

1. Associative property: The convention is to add left to right so  $5 + 2$  is added first, but the associative property says grouping differently with addition gives the same result.
2. Distributive property:  $11a - 2a = (11 - 2)a = 9a$
3. Subtraction can be written as adding the opposite, and then the order can be switched with the commutative property:  $7a + 4 - 2a = 7a + 4 + -2a = 7a + -2a + 4$ .
4. Subtracting a negative is the same as adding its opposite, and then the distributive property means  $8a - (8a - 8) = 8a - (8a + -8) = 8a - 8a - -8 = 8$ .

## Activity Synthesis

Ask students to share their reasons why each statement is true. Record and display their responses for all to see. Highlight correct use of precise, mathematical language and give students opportunities to revise their response to be more precise.

To involve more students in the conversation, consider asking:

- “Who can restate \_\_\_’s reasoning in a different way?”
- “Did anyone have the same reason but would explain it differently?”
- “Does anyone want to add on to \_\_\_\_’s reason?” “Do you agree or disagree? Why?”

## 20.2 A’s and B’s

10 minutes

In this activity students see an example of why applying properties is the only reliable way to decide whether two expressions are equivalent. They begin by substituting a value of the variable into expressions believed to be equivalent, and discover that the expressions are equal for that value. They then substitute other values and find that one of the expressions has a different value than the others. Students follow up by expanding the terms of the expression to consider each instance of the variables individually, and uncover the properties applied in each step of writing the expression with fewer terms.

### Building Towards

- 7.EE.A.1

### Instructional Routines

- MLR8: Discussion Supports

### Launch

Display the first part of the task statement for all to see:

Diego and Jada are both trying to write an expression with fewer terms that is equivalent to

$$7a + 5b - 3a + 4b$$

- Jada thinks  $10a + 1b$  is equivalent to the original expression.
- Diego thinks  $4a + 9b$  is equivalent to the original expression.

Remind students that we can tell whether the expressions are equivalent by substituting some different values for  $a$  and  $b$  and evaluating the expressions.

First, ask students to substitute the values  $a = 4$  and  $b = 3$  and evaluate the original expression, Jada’s expression, and Diego’s expression. Both expressions come out to 43. Perhaps these are both equivalent to the original expression?

Then, ask students to choose some different values for  $a$  and  $b$  and evaluate: the original expression, Jada's expression, and Diego's expression. For any other values of  $a$  and  $b$ , Jada and Diego's expressions do not evaluate to the same thing. For example, for  $a = 1$  and  $b = 1$ , the original is 13, Jada's expression is 11, and Diego's is 13.

The outcome of Diego's expression will match the original expressions, and Jada's will not.

Tell students that experimenting with numbers can tell us that two expressions are *not* equivalent, but can't prove that two expressions are equivalent. For example, Jada and Diego's expressions yielded the same outcome for  $a = 4$  and  $b = 3$ , but aren't equivalent. For that, we need to reason about the expressions using the properties that we know.

Arrange students in groups of 2. Allow 6–7 minutes quiet work time and partner discussions followed by a whole class discussion.

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### Access for Students with Disabilities

*Representation: Internalize Comprehension.* Begin with a physical demonstration of expanding terms and substituting values into an expression to support connections between new situations and prior understandings. For example, demonstrate how to expand " $7a$ " by writing out 7 " $a$ 's" with the addition sign between them and then evaluate it using  $a = 2$ . Ask students "How are these expressions equivalent?" and "How does substituting values help determine when expressions are equivalent?"

*Supports accessibility for: Conceptual processing; Visual-spatial processing*

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### Access for English Language Learners

*Speaking: MLR8 Discussion Supports.* Use sentence frames to support students in producing explanations about why expressions are equivalent. For example, "This row is equivalent to the last row because . . . ."

*Design Principle(s): Support sense-making; Optimize output (for explanation)*

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### Anticipated Misconceptions

Students may think the expressions are equivalent after finding them equal for  $a = 4$  and  $b = 3$ . Remind them that equivalent expressions must be equal for every possible value of the variable.

Students might have trouble describing the moves in the last two questions and justifying that the expressions are equivalent. Encourage students to closely examine the changes from row to row and consider why they do not change the value of the expression.

### Student Task Statement

Diego and Jada are both trying to write an expression with fewer terms that is equivalent to

$$7a + 5b - 3a + 4b$$

- Jada thinks  $10a + 1b$  is equivalent to the original expression.
- Diego thinks  $4a + 9b$  is equivalent to the original expression.

1. We can show expressions are equivalent by writing out all the variables. Explain why the expression on each row (after the first row) is equivalent to the expression on the row before it.

$$\begin{array}{c}
 7a + 5b - 3a + 4b \\
 (a + a + a + a + a + a + a) + (b + b + b + b + b) - (a + a + a) + (b + b + b + b) \\
 (a + a + a + a) + (a + a + a) + (b + b + b + b + b) - (a + a + a) + (b + b + b + b) \\
 (a + a + a + a) + (b + b + b + b + b) + (a + a + a) - (a + a + a) + (b + b + b + b) \\
 (a + a + a + a) + (b + b + b + b + b) + (b + b + b + b) \\
 (a + a + a + a) + (b + b + b + b + b + b + b + b + b + b) \\
 4a + 9b
 \end{array}$$

2. Here is another way we can rewrite the expressions. Explain why the expression on each row (after the first row) is equivalent to the expression on the row before it.

$$\begin{array}{c}
 7a + 5b - 3a + 4b \\
 7a + 5b + (-3a) + 4b \\
 7a + (-3a) + 5b + 4b \\
 (7 + -3)a + (5 + 4)b \\
 4a + 9b
 \end{array}$$

## Student Response

1. Answers vary. Sample responses:

- First row: Write products as sums (distributive property).
- Second row: Group first set of  $a$ 's differently (associative property).
- Third row: Switch second and third groups of addends (commutative property).
- Fourth row: Subtract an addend from itself to get 0:  $a + a + a - (a + a + a)$ .
- Fifth row: Group all the  $b$ 's together (associative property).
- Sixth row: Write sums as products (distributive property).

2. Answers vary. Sample responses:

- First row: Write subtraction as addition.
- Second row: Switch 2nd and 3rd terms (commutative property).
- Third Row: Write sums as products (distributive property).
- Fourth row: Evaluate numerical expressions.

### Are You Ready for More?

Follow the instructions for a number puzzle:

- Take the number formed by the first 3 digits of your phone number and multiply it by 40
  - Add 1 to the result
  - Multiply by 500
  - Add the number formed by the last 4 digits of your phone number, and then add it again
  - Subtract 500
  - Multiply by  $\frac{1}{2}$
1. What is the final number?
  2. How does this number puzzle work?
  3. Can you invent a new number puzzle that gives a surprising result?

### Student Response

Explanations vary. Sample response:

- Let  $x$  represent the 3-digit number, so  $40x$
- $40x + 1$
- $500(40x + 1)$
- Let  $y$  represent the 4-digit number, so  $500(40x + 1) + y + y$  or  $500(40x + 1) + 2y$
- $500(40x + 1) + 2y - 500 = 20,000x + 500 + 2y - 500 = 20,000x + 2y$
- $\frac{20000x+2y}{2} = 10,000x + y$   $10,000x + y$  means the 3-digit number,  $x$  was moved 4 place values to the left followed 4 zeros and then the 4-digit number was added to the zeros, forming the phone number.

### Activity Synthesis

Invite students to justify that the steps taken by Diego do not change the value of the expressions. Emphasize places where he used the distributive property and the commutative property.

Ask students which method they prefer (substituting values or using the properties of operations) for telling whether expressions are equivalent. Explain that while checking values can give us useful information, there is usually no way to check *all* possible values. That is why it is important to have some algebraic methods to rely on.

## 20.3 Making Sides Equal

15 minutes

In this activity, students use what they have learned so far to find a missing term that makes two expressions equivalent. They have many tools at their disposal to reason about the missing term.

For example, for the first problem  $6x + ? = 10x$ , they might reason as in the last activity and write out the sum of 6  $x$ 's and ? on one side, and a string of 10  $x$ 's on the other side, and reason that 4  $x$ 's are needed to make the sides equivalent. Alternatively, they might reason with the distributive property, and rewrite the left side as  $x(6 + ?) = 10x$ . These alternative ways of reasoning about equivalent expressions should be highlighted in the discussion.

## Addressing

- 7.EE.A.1

## Instructional Routines

- MLR8: Discussion Supports

## Launch

Arrange students in groups of 2. Explain that they will use what they have learned so far to find a missing term that will make two expressions equivalent. Draw their attention to the instructions, which instruct students to complete the first set of problems, check in with their partner, and then proceed. If desired, you might ask students to pause after the first set for whole-class discussion.

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### Access for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Encourage and support opportunities for peer interactions. Display sentence frames to support student conversation such as "To find the missing term, first, I \_\_\_\_ because...", "Why did you...?", "Can you explain or show that another way?" or "I agree/disagree because..."

*Supports accessibility for: Language; Social-emotional skills*

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### Student Task Statement


Replace each ? with an expression that will make the left side of the equation equivalent to the right side.

Set A

1.  $6x + ? = 10x$

2.  $6x + ? = 2x$

3.  $6x + ? = -10x$



4.  $6x + ? = 0$

5.  $6x + ? = 10$

Check your results with your partner and resolve any disagreements. Next move on to Set B.

Set B

1.  $6x - ? = 2x$

2.  $6x - ? = 10x$

3.  $6x - ? = x$

4.  $6x - ? = 6$

5.  $6x - ? = 4x - 10$

### Student Response

Set A

1.  $6x + 4x = 10x$

2.  $6x + -4x = 2x$

3.  $6x + -16x = -10x$

$$4. 6x + -6x = 0$$

$$5. 6x + (10 - 6x) = 10$$

Set B

$$1. 6x - 4x = 2x$$

$$2. 6x - (-4x) = 10x$$

$$3. 6x - 5x = x$$

$$4. 6x - (6x - 6) = 6$$

$$5. 6x - (2x + 10) = 4x - 10$$

### Activity Synthesis

Ask students to share their expressions for each problem. Record and display their responses for all to see. After each student shares, ask the class if they agree or disagree.

The following questions, when applicable, can be used as students share:

- “Why didn't you combine  $x$  terms and numbers?” (Rewriting expressions using the properties of multiplication or the distributive property shows why this doesn't result in an equivalent expression.)
- “How did you decide on the components of the missing term?”
- “Did you use the commutative property?”
- “Did you use the distributive property?”

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### Access for English Language Learners

*Speaking, Representing: MLR8 Discussion Supports.* Use this routine to support whole-class discussion. After each student shares their expressions, provide the class with the following sentence frames to help them respond: “I agree because ....” or “I disagree because ....” Encourage students to name a property as part of their explanation for why they agree or disagree. This will help students to connect the properties with the processes they used to confirm equivalent expressions.

*Design Principle(s): Optimize output (for explanation)*

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### Lesson Synthesis

Consider asking students to choose one of these questions, think about it quietly for a few minutes, and then explain it to their partner either verbally or in writing. Their partner listens or reads carefully, and asks any clarifying questions if they don't fully understand.

- “What are some ways we can tell that  $7x + 2$  is *not* equivalent to  $9x$ ?”
- “Someone is doubtful that  $3b - 8b$  is equivalent to  $-5b$ , but they do understand the distributive property. How could you convince them that these expressions are equivalent?”
- “What are some ways we could rearrange the terms in the expression  $-2x + 6y - 6x + 15y$  and create an equivalent expression?”

## 20.4 Fewer Terms

Cool Down: 5 minutes

### Addressing

- 7.EE.A.1

### Student Task Statement

Write each expression with fewer terms. Show your work or explain your reasoning.

1.  $10x - 2x$
2.  $10x - 3y + 2x$

### Student Response

1.  $8x$
2.  $12x - 3y$

### Student Lesson Summary

There are many ways to write equivalent expressions that may look very different from each other. We have several tools to find out if two expressions are equivalent.

- Two expressions are definitely not equivalent if they have different values when we substitute the same number for the variable. For example,  $2(-3 + x) + 8$  and  $2x + 5$  are not equivalent because when  $x$  is 1, the first expression equals 4 and the second expression equals 7.
- If two expressions are equal for many different values we substitute for the variable, then the expressions *may* be equivalent, but we don't know for sure. It is impossible to compare the two expressions for all values. To know for sure, we use properties of operations. For example,  $2(-3 + x) + 8$  is equivalent to  $2x + 2$  because:

$$\begin{aligned}
 &2(-3 + x) + 8 \\
 &-6 + 2x + 8 && \text{by the distributive property} \\
 &2x + -6 + 8 && \text{by the commutative property} \\
 &2x + (-6 + 8) && \text{by the associative property} \\
 &2x + 2
 \end{aligned}$$

## Lesson 20 Practice Problems

### Problem 1

#### Statement

Andre says that  $10x + 6$  and  $5x + 11$  are equivalent because they both equal 16 when  $x$  is 1. Do you agree with Andre? Explain your reasoning.

#### Solution

No, equivalent expressions are equal for any value of their variable. When  $x$  is 0, they are not equal.

### Problem 2

#### Statement

Select all expressions that can be subtracted from  $9x$  to result in the expression  $3x + 5$ .

- A.  $-5 + 6x$
- B.  $5 - 6x$
- C.  $6x + 5$
- D.  $6x - 5$
- E.  $-6x + 5$

#### Solution

["A", "D"]

### Problem 3

#### Statement

Select all the statements that are true for any value of  $x$ .

- A.  $7x + (2x + 7) = 9x + 7$
- B.  $7x + (2x - 1) = 9x + 1$
- C.  $\frac{1}{2}x + (3 - \frac{1}{2}x) = 3$
- D.  $5x - (8 - 6x) = -x - 8$
- E.  $0.4x - (0.2x + 8) = 0.2x - 8$
- F.  $6x - (2x - 4) = 4x + 4$

## Solution

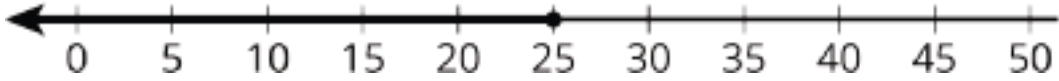
["A", "C", "E", "F"]

### Problem 4

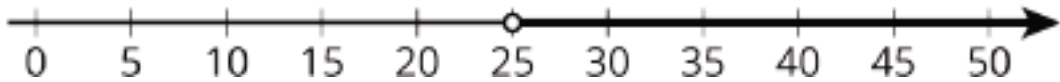
#### Statement

For each situation, would you describe it with  $x < 25$ ,  $x > 25$ ,  $x \leq 25$ , or  $x \geq 25$ ?

- The library is having a party for any student who read at least 25 books over the summer. Priya read  $x$  books and was invited to the party.
- Kiran read  $x$  books over the summer but was not invited to the party.
- 



d.



## Solution

- $x \geq 25$
- $x < 25$
- $x \leq 25$
- $x > 25$

(From Unit 6, Lesson 13.)

### Problem 5

#### Statement

Consider the problem: A water bucket is being filled with water from a water faucet at a constant rate. When will the bucket be full? What information would you need to be able to solve the problem?

## Solution

Answers vary. Possible response:

- How big is the bucket?
- What is the rate of water flow?

c. How high is the bucket?

d. How high is the water in the bucket after 1 minute?

(From Unit 2, Lesson 9.)

# Lesson 21: Combining Like Terms (Part 2)

## Goals

- Critique (in writing) methods for generating equivalent expressions with fewer terms.
- Generate expressions that are not equivalent, but differ only in the placement of parentheses, and justify (orally) that they are not equivalent.
- Write expressions with fewer terms that are equivalent to a given expression that includes negative coefficients and parentheses.

## Learning Targets

- I am aware of some common pitfalls when writing equivalent expressions, and I can avoid them.
- When possible, I can write an equivalent expression that has fewer terms.

## Lesson Narrative

In this lesson, students are still working toward gaining fluency in writing equivalent expressions. The goal of this lesson is to highlight a particular common error: mishandling the subtraction in an expression like  $8 - 3(4 + 9x)$ . To this end, students first analyze and explain the error in several incorrect ways of rewriting this expression. Then, they consider the effect of inserting parentheses in different places in an expression with four terms.

## Alignments

### Building On

- 6.EE.A.2.c: Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas  $V = s^3$  and  $A = 6s^2$  to find the volume and surface area of a cube with sides of length  $s = 1/2$ .
- 6.EE.A.4: Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions  $y + y + y$  and  $3y$  are equivalent because they name the same number regardless of which number  $y$  stands for.

### Addressing

- 7.EE.A.1: Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

## Building Towards

- 7.EE.A.1: Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

## Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports
- Think Pair Share
- True or False

## Required Materials

### Index cards

## Required Preparation

Access to index cards is suggested for students to need help isolating one expression at a time. They can use the index card to cover up nearby expressions.

## Student Learning Goals

Let's see how to use properties correctly to write equivalent expressions.

# 21.1 True or False?

## Warm Up: 10 minutes

In this warm-up, students consider some correct and incorrect numerical examples of distributing when the expression is being subtracted. A main goal of this lesson is to help students to understand how to write equivalent expressions that contain variables. By first looking at numbers only, students have a way to tell if the expressions are equivalent by evaluating each side.

Look for students who evaluate each side and students who reason about operations and properties.

## Building On

- 6.EE.A.2.c
- 6.EE.A.4

## Building Towards

- 7.EE.A.1

## Instructional Routines

- Think Pair Share
- True or False

## Launch

Tell students that their job is to consider four equations and decide which of them are true.

The amount of numbers and symbols presented relatively close together might present a challenge. It is important that students think of how the different equations compare to each other, but they also need to consider them one at a time. Provide access to index cards, so that, for example, students can cover up questions 2, 3, and 4 while considering question 1.

Arrange students in groups of 2. Give them 3 minutes of quiet work time and time to share their thoughts with a partner, followed by whole-class discussion.

### Student Task Statement

Select **all** the statements that are true. Be prepared to explain your reasoning.

1.  $4 - 2(3 + 7) = 4 - 2 \cdot 3 - 2 \cdot 7$
2.  $4 - 2(3 + 7) = 4 + -2 \cdot 3 + -2 \cdot 7$
3.  $4 - 2(3 + 7) = 4 - 2 \cdot 3 + 2 \cdot 7$
4.  $4 - 2(3 + 7) = 4 - (2 \cdot 3 + 2 \cdot 7)$

### Student Response

Sample reasoning:

1. true, because the 2 must be multiplied by both 3 and 7 because of the distributive property, and each product must be subtracted since the product of 2 and  $(3 + 7)$  is subtracted
2. true, because subtracting 2 is the same as adding -2, and then the distributive property is applied
3. false, because  $2 \cdot 7$  needs to be subtracted
4. true, because the 2 is distributed but the parentheses indicate that the result of  $2(3 + 7)$  is subtracted

### Activity Synthesis

Ask students to explain why the true statements are true. Select students who reason by evaluating each side, and also students who reason using properties. For example, statement 2 is true because subtracting 2 is the same as adding negative 2, and then the distributive property is applied.

Then, spend some time on why statement 3 is false. First, we can tell it's false because when each side is evaluated, we get  $-16 = 12$ . The order of operations is just a convention, but we need to all follow one convention so that we can communicate mathematically. When the order of operations is followed on the left side, the result of  $2(3 + 7)$  is subtracted from 4. However on the right side of statement 3, only the  $2 \cdot 3$  is being subtracted, and the  $2 \cdot 7$  is being added.

## 21.2 Seeing it Differently

10 minutes

In this activity, students encounter typical errors with signed numbers, operations, and properties. They are tasked with identifying which strategies are correct and for those that are not, describing the error that was made.

### Addressing

- 7.EE.A.1

### Instructional Routines

- MLR3: Clarify, Critique, Correct

### Launch

Ensure students understand the task: first they decide whether they agree with each person's strategy, but they also need to describe the errors that were made. Give 5 minutes quiet work time followed by a whole-class discussion.

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#### Access for Students with Disabilities

*Representation: Internalize Comprehension.* Begin by providing students with rules for adding, subtracting, and multiplying signed numbers. Invite students to share their prior knowledge and ideas using simple examples.

*Supports accessibility for: Conceptual processing*

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#### Access for English Language Learners

*Speaking, Writing: MLR3 Clarify, Critique, Correct.* This activity provides students with the opportunity to improve upon the written work of another by correcting errors and clarifying meaning. Ask students to select one of the errors they notice, and to produce a written explanation, intended for the student who made the error (Noah, Lin, Jada, or Andre), that describes the error that was made, and how to fix it. Give students 3–5 minutes to complete a first draft before they read their writing to a partner. Provide students with prompts they can use to give each other feedback such as “Can you say that another way?” or “Can you try to explain this using an example?” This will provide students with an additional opportunity to produce language related to writing equivalent expressions.

*Design Principle(s): Optimize output (for explanation)*

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### Student Task Statement

Some students are trying to write an expression with fewer terms that is equivalent to  $8 - 3(4 - 9x)$ .

Noah says, "I worked the problem from left to right and ended up with  $20 - 45x$ ."

$$8 - 3(4 - 9x)$$

$$5(4 - 9x)$$

$$20 - 45x$$

Lin says, "I started inside the parentheses and ended up with  $23x$ ."

$$8 - 3(4 - 9x)$$

$$8 - 3(-5x)$$

$$8 + 15x$$

$$23x$$

Jada says, "I used the distributive property and ended up with  $27x - 4$ ."

$$8 - 3(4 - 9x)$$

$$8 - (12 - 27x)$$

$$8 - 12 - (-27x)$$

$$27x - 4$$

Andre says, "I also used the distributive property, but I ended up with  $-4 - 27x$ ."

$$8 - 3(4 - 9x)$$

$$8 - 12 - 27x$$

$$-4 - 27x$$

1. Do you agree with any of them? Explain your reasoning.
2. For each strategy that you disagree with, find and describe the errors.

### Student Response

1. Answers vary. Sample response: I agree with Jada because I tried some values of  $x$  and Jada's expression always evaluates to the same number as the original expression.
2. Answers vary. Sample response: Noah subtracted before multiplying. Lin combined 4 and  $9x$ . Andre multiplied  $-3$  and  $-9$  and got  $-27$ .

### Are You Ready for More?

1. Jada's neighbor said, "My age is the difference between twice my age in 4 years and twice my age 4 years ago." How old is Jada's neighbor?
2. Another neighbor said, "My age is the difference between twice my age in 5 years and twice my age 5 years ago." How old is this neighbor?
3. A third neighbor had the same claim for 17 years from now and 17 years ago, and a fourth for 21 years. Determine those neighbors' ages.

### Student Response

1. 16. An expression for the neighbor's age is  $2(a + 4) - 2(a - 4)$  or  $2a + 8 - 2a + 8$  which is 16.
2. 20. An expression for this neighbor's age is  $2(a + 5) - 2(a - 5)$  or  $2a + 10 - 2a + 10 = 20$ .

3. 68 and 84. The expression is always twice the number of years + twice the number of years, or  $4y$  where  $y$  is the number of years:  $4(17) = 68$ ,  $4(21) = 84$ .

### Activity Synthesis

Ask students, “Which way do you see it?” In grade 6, students learned that equivalent expressions meant two expressions had to be equal for any value of the variable. Since each student’s work contains at least two steps, the steps can be used to identify where the error occurs—where the expressions are no longer equal for a value of the variable.

A handy approach is to rewrite subtraction operations as adding the opposite. If desired, demonstrate such an approach for this expression along with a box to organize the work of multiplying  $-3(4 - 9x)$ :

	4	-9x
-3		

	4	-9x
-3	$-3 \cdot 4$	$-3 \cdot -9x$

	4	-9x
-3	-12	$27x$

$$\begin{aligned}
 &8 - 3(4 - 9x) \\
 &8 + (-3)(4 + (-9x)) \\
 &8 + (-3)(4) + (-3)(-9x) \\
 &8 + (-12) + 27x \\
 &-4 + 27x
 \end{aligned}$$

## 21.3 Grouping Differently

15 minutes

In this activity students continue the work of generating equivalent expressions as they decide where to place a set of parentheses and explore how that placement affects the expressions.

### Addressing

- 7.EE.A.1

## Instructional Routines

- MLR8: Discussion Supports

### Launch

Arrange students in groups of 2. Tell students to first complete both questions independently. Then, trade one of their expressions with their partner. The partner's job is to decide whether the new expression is equivalent to the original or not, and explain how they know.

### Anticipated Misconceptions

Students may not realize that they can break up a term and place a parentheses, for example, between the 8 and  $x$  in the term  $8x$ . Clarify that they may place the parentheses anywhere in the expression.

### Student Task Statement

Diego was taking a math quiz. There was a question on the quiz that had the expression  $8x - 9 - 12x + 5$ . Diego's teacher told the class there was a typo and the expression was supposed to have one set of parentheses in it.

1. Where could you put parentheses in  $8x - 9 - 12x + 5$  to make a new expression that is still equivalent to the original expression? How do you know that your new expression is equivalent?
2. Where could you put parentheses in  $8x - 9 - 12x + 5$  to make a new expression that is not equivalent to the original expression? List as many different answers as you can.

### Student Response

Answers vary. Sample responses:

1.  $(8x - 9 - 12x + 5)$ ,  $(8x - 9) - 12x + 5$
2.  $8x - 9 - (12x + 5) = -4x - 14$ ,  $8x - (9 - 12x) + 5 = 20x - 4$ ,  
 $8x - (9 - 12x + 5) = 20x - 14$ ,  $8(x - 9 - 12x + 5) = -88x - 32$ ,  
 $8(x - 9 - 12x) + 5 = -88x - 67$ ,  $8(x - 9) - 12x + 5 = -4x - 67$

### Activity Synthesis

Questions for discussion:

- "How can you write the original expression in different ways with fewer terms?" ( $-4x - 4$ ,  $-4(x + 1)$ ,  $4(-x - 1)$ )
- "Where did you place the parentheses to create an equivalent expression?"
- "Share other ways you placed parentheses and the resulting expression with fewer terms."

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### Access for Students with Disabilities

*Representation: Develop Language and Symbols.* Create a display of important terms and vocabulary. Include the following terms and maintain the display for reference throughout the unit: parenthesis. The display should include examples of how to create equivalent expressions using parentheses by breaking up the components. Be sure to emphasize how the placement of the parentheses affects the expression.

*Supports accessibility for: Memory; Language*

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### Access for English Language Learners

*Speaking, Writing: MLR8 Discussion Supports.* When students interpret the new expression they receive, ask students to press their partner for more details in their explanations by challenging an idea, or asking for an example. Listen for, and amplify language students use to describe how the parentheses influence the resulting expression with fewer terms. This will help call students' attention to the types of details and language to look for to determine if an expression is equivalent to another or not.

*Design Principle(s): Maximize meta-awareness; Support sense-making*

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## Lesson Synthesis

Display the expression  $5 - 2(3x - x)$ . Ask students to think of a mistake someone would be likely to make when trying to write an expression that is equivalent to this one. Select a student to share, and then ask if anyone can think of a different likely mistake. Continue until each of these common errors arises:

- Subtracting 2 from 5 first, resulting in  $3(3x - x)$
- Distributing positive 2, resulting in  $5 - 6x - 2x$
- Thinking that  $3x - x$  is 3, resulting in  $5 - 2(3)$

Then, ask students for strategies for preventing these errors. Reliable properties to use are: rewriting subtraction as adding the opposite, the commutative property of multiplication and addition, and the distributive property. Suggest this way of rewriting this example:

$$5 - 2(3x - x)$$

$$5 + -2(3x + -x)$$

$$5 + -2 \cdot 3x + -2 \cdot -x$$

$$5 + -6x + 2x$$

$$5 + x(-6 + 2)$$

$$5 + -4x \text{ or } 5 - 4x$$

## 21.4 How Many Are Equivalent?

Cool Down: 5 minutes

### Addressing

- 7.EE.A.1

### Student Task Statement

Select all the expressions that are equivalent to  $16x - 12 - 24x + 4$ . Explain or show your reasoning.

1.  $4 + 16x - 12(1 + 2x)$
2.  $40x - 16$
3.  $16x - 24x - 4 + 12$
4.  $-8x - 8$
5.  $10(1.6x - 1.2 - 2.4x + 4)$

### Student Response

1, 4

### Student Lesson Summary

Combining like terms allows us to write expressions more simply with fewer terms. But it can sometimes be tricky with long expressions, parentheses, and negatives. It is helpful to think about some common errors that we can be aware of and try to avoid:

- $6x - x$  is not equivalent to 6. While it might be tempting to think that subtracting  $x$  makes the  $x$  disappear, the expression is really saying take 1  $x$  away from 6  $x$ 's, and the distributive property tells us that  $6x - x$  is equivalent to  $(6 - 1)x$ .
- $7 - 2x$  is not equivalent to  $5x$ . The expression  $7 - 2x$  tells us to double an unknown amount and subtract it from 7. This is not always the same as taking 5 copies of the unknown.
- $7 - 4(x + 2)$  is not equivalent to  $3(x + 2)$ . The expression tells us to subtract 4 copies of an amount from 7, not to take  $(7 - 4)$  copies of the amount.

If we think about the meaning and properties of operations when we take steps to rewrite expressions, we can be sure we are getting equivalent expressions and are not changing their value in the process.

# Lesson 21 Practice Problems

## Problem 1

### Statement

- Noah says that  $9x - 2x + 4x$  is equivalent to  $3x$ , because the subtraction sign tells us to subtract everything that comes after  $9x$ .
- Elena says that  $9x - 2x + 4x$  is equivalent to  $11x$ , because the subtraction only applies to  $2x$ .

Do you agree with either of them? Explain your reasoning.

### Solution

Elena is correct. Rewriting addition as subtraction gives us  $9x + -2x + 4x$ , which shows that the subtraction symbol in front of the  $2x$  applies only to the  $2x$  and not to the terms that come after it.

## Problem 2

### Statement

Identify the error in generating an expression equivalent to  $4 + 2x - \frac{1}{2}(10 - 4x)$ . Then correct the error.

$$\begin{aligned}4 + 2x + \frac{-1}{2}(10 + -4x) \\4 + 2x + -5 + 2x \\4 + 2x - 5 + 2x \\-1\end{aligned}$$

### Solution

The error is in the last step. The second  $2x$  was subtracted instead of being added. This would be correct if there were parentheses around  $5 + 2x$ . The last step should be  $4x - 1$ .

## Problem 3

### Statement

Select all expressions that are equivalent to  $5x - 15 - 20x + 10$ .

A.  $5x - (15 + 20x) + 10$

B.  $5x + -15 + -20x + 10$

C.  $5(x - 3 - 4x + 2)$

D.  $-5(-x + 3 + 4x + -2)$

E.  $-15x - 5$

F.  $-5(3x + 1)$

G.  $-15(x - \frac{1}{3})$

## Solution

["A", "B", "C", "D", "E", "F"]

## Problem 4

### Statement

The school marching band has a budget of up to \$750 to cover 15 new uniforms and competition fees that total \$300. How much can they spend for one uniform?

- Write an inequality to represent this situation.
- Solve the inequality and describe what it means in the situation.

## Solution

- $15x + 300 \leq 750$
- $x \leq 30$ . They can spend at most \$30 on each uniform.

(From Unit 6, Lesson 14.)

## Problem 5

### Statement

Solve the inequality that represents each story. Then interpret what the solution means in the story.

- For every \$9 that Elena earns, she gives  $x$  dollars to charity. This happens 7 times this month. Elena wants to be sure she keeps at least \$42 from this month's earnings.  $7(9 - x) \geq 42$
- Lin buys a candle that is 9 inches tall and burns down  $x$  inches per minute. She wants to let the candle burn for 7 minutes until it is less than 6 inches tall.  $9 - 7x < 6$

## Solution

a.  $x \leq 3$ . Elena can give \$3 or less to charity for every \$9 she earns.

b.  $x > \frac{3}{7}$ . The candle needs to burn down more than  $\frac{3}{7}$  inch each minute.

(From Unit 6, Lesson 16.)

## Problem 6

### Statement

A certain shade of blue paint is made by mixing  $1\frac{1}{2}$  quarts of blue paint with 5 quarts of white paint. If you need a total of 16.25 gallons of this shade of blue paint, how much of each color should you mix?

### Solution

You should mix  $3\frac{3}{4}$  gallons of blue paint with  $12\frac{1}{2}$  gallons of white paint.

(From Unit 4, Lesson 3.)

# Lesson 22: Combining Like Terms (Part 3)

## Goals

- Explain (orally and in writing) how to write an equivalent expression with fewer terms.
- Generalize (orally) about what strategies are useful and what mistakes are common when writing equivalent expressions with fewer terms.
- Identify equivalent expressions, and justify (orally and in writing) that they are equivalent.

## Learning Targets

- Given an expression, I can use various strategies to write an equivalent expression.
- When I look at an expression, I can notice if some parts have common factors and make the expression shorter by combining those parts.

## Lesson Narrative

In this lesson, students have an opportunity to demonstrate fluency in combining like terms and look for and make use of structure (MP7) to apply the distributive property in more sophisticated ways.

## Alignments

### Building On

- 6.EE.A.4: Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions  $y + y + y$  and  $3y$  are equivalent because they name the same number regardless of which number  $y$  stands for.

### Addressing

- 7.EE.A.1: Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

### Building Towards

- 7.EE.A.1: Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

## Instructional Routines

- MLR8: Discussion Supports
- Take Turns
- Think Pair Share

## Student Learning Goals

Let's see how we can combine terms in an expression to write it with less terms.

# 22.1 Are They Equal?

## Warm Up: 5 minutes

The purpose of this activity is to remind students of things they learned in the previous lesson using numerical examples. Look for students who evaluate each expression and students who use reasoning about operations and properties.

## Building On

- 6.EE.A.4

## Building Towards

- 7.EE.A.1

## Launch

Remind students that working with subtraction can be tricky, and to think of some strategies they have learned in this unit. Encourage students to reason about the expressions without evaluating them.

Give students 2 minutes of quiet think time followed by whole-class discussion.

## Anticipated Misconceptions

Students who selected  $8 - 6 - 12 + 4$  or  $8 - 12 + (6 + 4)$  might not understand that the subtraction sign outside the parentheses applies to the 4 and that 4 is always to be subtracted in any equivalent expression.

Students who selected  $8 - 12 + (6 + 4)$  might think the subtraction sign in front of 12 also applies to  $(6 + 4)$  and that the two subtractions become addition.

## Student Task Statement

Select **all** expressions that are equal to  $8 - 12 - (6 + 4)$ .

1.  $8 - 6 - 12 + 4$
2.  $8 - 12 - 6 - 4$
3.  $8 - 12 + (6 + 4)$
4.  $8 - 12 - 6 + 4$
5.  $8 - 4 - 12 - 6$

## Student Response

2, 5

## Activity Synthesis

For each expression, poll the class for whether the expression is equal to the given expression, or not. For each expression, select a student to explain why it is equal to the given expression or not. If the first student reasoned by evaluating each expression, ask if anyone reasoned without evaluating each expression.

## 22.2 X's and Y's

15 minutes

In this activity students take turns with a partner and work to make sense of writing expressions in equivalent ways. This activity is a step up from the previous lesson because there are more negatives for students to deal with, and each expression contains more than one variable.

### Addressing

- 7.EE.A.1

### Instructional Routines

- MLR8: Discussion Supports
- Take Turns

### Launch

Arrange students in groups of 2. Tell students that for each expression in column A, one partner finds an equivalent expression in column B and explains why they think it is equivalent. The partner's job is to listen and make sure they agree. If they don't agree, the partners discuss until they come to an agreement. For the next expression in column A, the students swap roles. If necessary, demonstrate this protocol before students start working.

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### Access for Students with Disabilities

*Representation: Internalize Comprehension.* Differentiate the degree of difficulty or complexity by beginning with an example with more accessible values. For example, start with an expression with three terms such as " $6x - (2x + 8)$ " and show different forms of equivalent expressions. Highlight connections between expressions by using the same color on equivalent parts of the expression.

*Supports accessibility for: Conceptual processing*

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## Access for English Language Learners

*Listening, Speaking: MLR8 Discussion Supports.* Display sentence frames for students to use to describe the reasons for their matches. For example, “I matched expression \_\_\_ with expression \_\_\_ because . . .” or “I used the \_\_\_ property to help me match expression \_\_\_ with expression \_\_\_.” Provide a sentence frame for the partner to respond with, such as: “I agree/disagree with this match because . . .” These sentence frames provide students with language structures that help them to produce explanations, and also to critique their partner’s reasoning.

*Design Principle(s): Maximize meta-awareness; Support sense-making*

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## Anticipated Misconceptions

For the second and third rows, some students may not understand that the subtraction sign in front of the parentheses applies to both terms inside that set of parentheses. Some students may get the second row correct, but not realize how the third row relates to the fact that the product of two negative numbers is a positive number. For the last three rows, some students may not recognize the importance of the subtraction sign in front of  $7y$ . Prompt them to rewrite the expressions replacing subtraction with adding the inverse.

Students might write an expression with fewer terms but not recognize an equivalent form because the distributive property has been used to write a sum as a product. For example,  $9x - 7y + 3x - 5y$  can be written as  $9x + 3x - 7y - 5y$  or  $12x - 12y$ , which is equivalent to the expression  $12(x - y)$  in column B. Encourage students to think about writing the column B expressions in a different form and to recall that the distributive property can be applied to either factor or expand an expression.

### Student Task Statement

Match each expression in column A with an equivalent expression from column B. Be prepared to explain your reasoning.

A

1.  $(9x + 5y) + (3x + 7y)$

2.  $(9x + 5y) - (3x + 7y)$

3.  $(9x + 5y) - (3x - 7y)$

4.  $9x - 7y + 3x + 5y$

5.  $9x - 7y + 3x - 5y$

6.  $9x - 7y - 3x - 5y$

B

1.  $12(x + y)$

2.  $12(x - y)$

3.  $6(x - 2y)$

4.  $9x + 5y + 3x - 7y$

5.  $9x + 5y - 3x + 7y$

6.  $9x - 3x + 5y - 7y$

## Student Response

The correct pairings:

A	B
$(9x + 5y) + (3x + 7y)$	$12(x + y)$
$(9x + 5y) - (3x + 7y)$	$9x - 3x + 5y - 7y$
$(9x + 5y) - (3x - 7y)$	$9x + 5y - 3x + 7y$
$9x - 7y + 3x + 5y$	$9x + 5y + 3x - 7y$
$9x - 7y + 3x - 5y$	$12(x - y)$
$9x - 7y - 3x - 5y$	$6(x - 2y)$

## Activity Synthesis

Much discussion takes place between partners. Invite students to share how they used properties to generate equivalent expressions and find matches.

- “Which term(s) does the subtraction sign apply to in each expression? How do you know?”
- “Were there any expressions from column A that you wrote with fewer terms but were unable to find a match for in column B? If yes, why do you think this happened?”
- “What were some ways you handled subtraction with parentheses? Without parentheses?”
- “Describe any difficulties you experienced and how you resolved them.”

## 22.3 Seeing Structure and Factoring

10 minutes

This activity is an opportunity to notice and make use of structure (MP7) in order to apply the distributive property in more sophisticated ways.

### Addressing

- 7.EE.A.1

### Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

## Launch

Display the expression  $18 - 45 + 27$  and ask students to calculate as quickly as they can. Invite students to explain their strategies. If no student brings it up, ask if the three numbers have anything in common (they are all multiples of 9). One way to quickly compute would be to notice that  $18 - 45 + 27$  can be written as  $2 \cdot 9 - 5 \cdot 9 + 3 \cdot 9$  or  $(2 - 5 + 3) \cdot 9$  which can be quickly calculated as 0. Tell students that noticing common factors in expressions can help us write them with fewer terms or more simply.

Keep students in the same groups. Give them 5 minutes of quiet work time and time to share their expressions with their partner, followed by a whole-class discussion.

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### Access for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* To support development of organizational skills, check in with students within the first 2–3 minutes of work time. Look for students who identify common factors or rearrange terms to write the expressions with fewer terms.

*Supports accessibility for: Memory; Organization*

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### Student Task Statement

Write each expression with fewer terms. Show or explain your reasoning.

1.  $3 \cdot 15 + 4 \cdot 15 - 5 \cdot 15$

2.  $3x + 4x - 5x$

3.  $3(x - 2) + 4(x - 2) - 5(x - 2)$

4.  $3\left(\frac{5}{2}x + 6\frac{1}{2}\right) + 4\left(\frac{5}{2}x + 6\frac{1}{2}\right) - 5\left(\frac{5}{2}x + 6\frac{1}{2}\right)$

### Student Response

- $2 \cdot 15$  or 30. Explanations vary. Sample response: Move the common factor 15 out of each term and combine the other factors:  $3 \cdot 15 + 4 \cdot 15 - 5 \cdot 15 = (3 + 4 - 5) \cdot 15 = 2 \cdot 15 = 30$ .
- $2x$ . Explanations vary. Sample response: Move the common factor  $x$  out of each term and combine the other factors:  $3x + 4x - 5x = (3 + 4 - 5)x = 2x$ .
- $2(x - 2)$  or  $2x - 4$ . Explanations vary. Sample response: Move the common factor  $(x - 2)$  out of each term and combine the other factors:  
 $3(x - 2) + 4(x - 2) - 5(x - 2) = (3 + 4 - 5)(x - 2) = 2(x - 2)$ .
- $2\left(\frac{5}{2}x + 6\frac{1}{2}\right)$  or  $5x + 13$ . Explanations vary. Sample response: Move the common factor  $\left(\frac{5}{2}x + 6\frac{1}{2}\right)$  out of each term and combine the other factors:  
 $3\left(\frac{5}{2}x + 6\frac{1}{2}\right) + 4\left(\frac{5}{2}x + 6\frac{1}{2}\right) - 5\left(\frac{5}{2}x + 6\frac{1}{2}\right) = (3 + 4 - 5)\left(\frac{5}{2}x + 6\frac{1}{2}\right) = 2\left(\frac{5}{2}x + 6\frac{1}{2}\right) = 5x + 13$ .

## Activity Synthesis

For each expression, invite a student to share their process for writing it with fewer terms. Highlight the use of the distributive property.

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### Access for English Language Learners

*Speaking: MLR8 Discussion Supports.* Provide sentence frames to help students explain their strategies. For example, "I noticed that \_\_\_\_\_, so I \_\_\_\_\_." or "First, I \_\_\_\_\_ because \_\_\_\_\_." When students share their answers with a partner, prompt them to rehearse what they will say when they share with the full group. Rehearsing provides students with additional opportunities to clarify their thinking.

*Design Principle(s): Optimize output (for explanation)*

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## Lesson Synthesis

Ask students to reflect on their work in this unit. They can share their response to one or more of these prompts either in writing or verbally with a partner.

- "Describe something that you found confusing at first that you now understand well."
- "Think of a story problem that you would not have been able to solve before this unit that you can solve now."
- "What is a tool or strategy that you learned in this lesson that was particularly useful?"
- "Describe a common mistake that people make when using the ideas we studied in this unit and how they can avoid that mistake."
- "Which is your favorite, and why? The distributive property, rewriting subtraction as adding the opposite, or the commutative property."

## 22.4 R's and T's

Cool Down: 5 minutes

### Addressing

- 7.EE.A.1

### Student Task Statement

Match each expression in column A with an equivalent expression from column B. Show or explain your reasoning.

### Column A

1.  $12r + t - 4r + 6t$
2.  $(12r + 2t) - (4r + 10t)$
3.  $12(r + t) - 6(r + t) + 4(r + t) - 2(r + t)$

### Column B

- $8(r - t)$
- $8(r + t)$
- $8r + 7t$

### Student Response

1.  $8r + 7t$
2.  $8(r - t)$
3.  $8(r + t)$

## Student Lesson Summary

Combining like terms is a useful strategy that we will see again and again in our future work with mathematical expressions. It is helpful to review the things we have learned about this important concept.

- Combining like terms is an application of the distributive property. For example:

$$\begin{aligned} &2x + 9x \\ &(2 + 9) \cdot x \\ &11x \end{aligned}$$


- It often also involves the commutative and associative properties to change the order or grouping of addition. For example:

$$\begin{aligned} &2a + 3b + 4a + 5b \\ &2a + 4a + 3b + 5b \\ &(2a + 4a) + (3b + 5b) \\ &6a + 8b \end{aligned}$$

- We can't change order or grouping when subtracting; so in order to apply the commutative or associative properties to expressions with subtraction, we need to rewrite subtraction as addition. For example:

$$\begin{aligned} &2a - 3b - 4a - 5b \\ &2a + -3b + -4a + -5b \\ &2a + -4a + -3b + -5b \\ &-2a + -8b \\ &-2a - 8b \end{aligned}$$

- Since combining like terms uses properties of operations, it results in expressions that are equivalent.

- 
- The like terms that are combined do not have to be a single number or variable; they may be longer expressions as well. Terms can be combined in any sum where there is a common factor in all the terms. For example, each term in the expression  $5(x + 3) - 0.5(x + 3) + 2(x + 3)$  has a factor of  $(x + 3)$ . We can rewrite the expression with fewer terms by using the distributive property:

$$\begin{aligned}5(x + 3) - 0.5(x + 3) + 2(x + 3) \\(5 - 0.5 + 2)(x + 3) \\6.5(x + 3)\end{aligned}$$

## Lesson 22 Practice Problems

### Problem 1

#### Statement

Jada says, "I can tell that  $\frac{-2}{3}(x + 5) + 4(x + 5) - \frac{10}{3}(x + 5)$  equals 0 just by looking at it." Is

Jada correct? Explain how you know.

#### Solution

Yes. Explanations vary. Sample response: Factor out  $x + 5$ :

$$(x + 5)\left(\frac{-2}{3} + 4 + \frac{-10}{3}\right) = (x + 5)\left(\frac{-12}{3} + 4\right) = (x + 5)(0) = 0.$$

### Problem 2

#### Statement

In each row, decide whether the expression in column A is equivalent to the expression in column B. If they are not equivalent, show how to change one expression to make them equivalent.

A

a.  $3x - 2x + 0.5x$

b.  $3(x + 4) - 2(x + 4)$

c.  $6(x + 4) - 2(x + 5)$

d.  $3(x + 4) - 2(x + 4) + 0.5(x + 4)$

e.  $20\left(\frac{2}{5}x + \frac{3}{4}y - \frac{1}{2}\right)$

B

a.  $1.5x$

b.  $x + 3$

c.  $2(2x + 7)$

d.  $1.5$

e.  $\frac{1}{2}(16x + 30y - 20)$

#### Solution

a. Equivalent

b. Not equivalent. Answers vary. Sample responses: Change the column B entry to  $x + 4$ , change the column A entry to  $3(x + 4) - 2(x + 4) - 1$

c. Equivalent

d. Not equivalent. Answers vary. Sample response: Change the column B entry to  $1.5(x + 4)$ .

e. Equivalent

## Problem 3

### Statement

For each situation, write an expression for the new balance using as few terms as possible.

- A checking account has a balance of  $-\$126.89$ . A customer makes two deposits, one  $3\frac{1}{2}$  times the other, and then withdraws  $\$25$ .
- A checking account has a balance of  $\$350$ . A customer makes two withdrawals, one  $\$50$  more than the other. Then he makes a deposit of  $\$75$ .

### Solution

- $126.89 + x + 3.5x - 25 = -151.89 + 4.5x$
- $350 - x - (x + 50) + 75 = 375 - 2x$

(From Unit 6, Lesson 20.)

## Problem 4

### Statement

Tyler is using the distributive property on the expression  $9 - 4(5x - 6)$ . Here is his work:

$$\begin{aligned} &9 - 4(5x - 6) \\ &9 + (-4)(5x + -6) \\ &9 + -20x + -6 \\ &3 - 20x \end{aligned}$$

Mai thinks Tyler's answer is incorrect. She says, "If expressions are equivalent then they are equal for any value of the variable. Why don't you try to substitute the same value for  $x$  in all the equations and see where they are not equal?"

- Find the step where Tyler made an error.
- Explain what he did wrong.
- Correct Tyler's work.

### Solution

Answers vary. Sample response:

- Try 1:
  - $9 - 4(5 \cdot 1 - 6) = 9 - 4(-1) = 9 + 4 = 13$
  - $9 + (-4)(5 \cdot 1 - 6) = 9 + (-4)(-1) = 9 + 4 = 13$
  - $9 + (-20)(1) + -6 = 9 + -20 + -6 = -17$

■  $3 - 20(1) = 3 - 20 = -17$

The value of the expression switched in the third step, so that's where the error is.

b. Tyler forgot to distribute  $-4$  to the  $-6$  term in the parentheses.

c. Starting at step 3:

$$9 + -20x + 24$$

$$33 - 20x$$

(From Unit 6, Lesson 21.)

## Problem 5

### Statement

- If  $(11 + x)$  is positive, but  $(4 + x)$  is negative, what is one number that  $x$  could be?
- If  $(-3 + y)$  is positive, but  $(-9 + y)$  is negative, what is one number that  $y$  could be?
- If  $(-5 + z)$  is positive, but  $(-6 + z)$  is negative, what is one number that  $z$  could be?

### Solution

- Answers vary.  $x$  can be any number in between  $-11$  and  $-4$ .
- Answers vary.  $y$  can be any number in between  $3$  and  $9$ .
- Answers vary.  $z$  can be any number in between  $5$  and  $6$ , for example,  $5\frac{1}{2}$ .

(From Unit 6, Lesson 13.)

## Section: Let's Put it to Work

### Lesson 23: Applications of Expressions

#### Goals

- Determine which order for applying multiple coupons gives the better discount and explain (orally and in writing) the reasoning.
- Justify (orally, in writing, and using other representations) that two different sequences of calculations give the same result.

#### Learning Targets

- I can write algebraic expressions to understand and justify a choice between two options.

#### Lesson Narrative

In this culminating lesson, students look at several real-world situations that can be represented by an expression with a variable. In the warm-up, students decide whether each of four expressions is equivalent to a given expression, recalling what it means for expressions to be equivalent and relevant terminology. In the following activity, students write expressions corresponding to two ways of doing a real-world calculation, and explain why the two ways are equivalent. Finally, students are presented with two coupons to a store (a 20% off coupon and a \$30 off coupon), and use their skills to decide in which order the coupons should be applied to save more money on a purchase. In this lesson, students write expressions to represent calculation methods, which allows them to use familiar properties to decide whether two methods are equivalent. This is an example of decontextualizing and recontextualizing (MP2) and creating a mathematical model to understand a situation (MP4).

#### Alignments

##### Building On

- 7.EE.A.1: Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
- 7.EE.B: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

##### Instructional Routines

- Algebra Talk
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Notice and Wonder

## Student Learning Goals

- Let's use expressions to solve problems.

# 23.1 Algebra Talk: Equivalent to $0.75t - 21$

## Warm Up: 5 minutes

The purpose of this algebra talk is to remind students that expressions can be written in different, equivalent ways, and to give them an opportunity to recall relevant terminology like “distributive property.”

## Building On

- 7.EE.A.1

## Instructional Routines

- Algebra Talk
- MLR8: Discussion Supports

## Launch

Display one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

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### Access for Students with Disabilities

*Representation: Internalize Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.

*Supports accessibility for: Memory; Organization*

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## Student Task Statement

Decide whether each expression is equivalent to  $0.75t - 21$ . Be prepared to explain how you know.

$$\frac{3}{4}t - 21$$

$$\frac{3}{4}(t - 21)$$

$$0.75(t - 28)$$

$$t - 0.25t - 21$$

## Student Response

- $\frac{3}{4}t - 21$  is equivalent because  $0.75 = \frac{3}{4}$ .

- $\frac{3}{4}(t - 21)$  is not equivalent because by the distributive property, it is equivalent to  $\frac{3}{4}t - 15.75$ .
- $0.75(t - 28)$  is equivalent because of the distributive property.
- $t - 0.25t - 21$  is equivalent because if you combine  $t - 0.25t$ , you get  $0.75t$ .

### Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- “Who can restate \_\_\_\_’s reasoning in a different way?”
- “Did anyone have the same strategy but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to \_\_\_\_’s strategy?”
- “Do you agree or disagree? Why?”

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### Access for English Language Learners

*Speaking: MLR8 Discussion Supports.*: Display sentence frames to support students when they explain their strategy. For example, “First, I \_\_\_\_ because . . .” or “I noticed \_\_\_\_ so I . . .” Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

*Design Principle(s): Optimize output (for explanation)*

---

## 23.2 Two Ways to Calculate

15 minutes

The purpose of this task is to encourage students to represent each situation using an expression with a variable, so that the calculation methods can be shown to be equivalent in a straightforward way using familiar properties. This approach will be useful for tackling the next activity. Students may find other ways of explaining why the calculation methods are equivalent, but the synthesis should highlight explaining why two expressions that use a variable are equivalent.

### Building On

- 7.EE.A.1
- 7.EE.B

### Instructional Routines

- MLR7: Compare and Connect

## Launch

Depending on the time available, ask each student to respond to 1, 2, or all of the situations. Note that the last situation is a bit more difficult than the first two.

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### Access for Students with Disabilities

*Representation: Internalize Comprehension.* Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.

*Supports accessibility for: Memory; Conceptual processing*

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## Anticipated Misconceptions

For students who have trouble getting started, ask them to first calculate using a few specific values. For example, calculate the temperature in Fahrenheit if the temperature in Celsius is 0 degrees, 5 degrees, 10 degrees, and then use the same operations to write an expression for  $x$  degrees. Another option would be to provide a bank of expressions for students to choose from, rather than asking them to generate the expressions.

### Student Task Statement

Usually when you want to calculate something, there is more than one way to do it. For one or more of these situations, show how the two different ways of calculating are equivalent to each other.

1. Estimating the temperature in Fahrenheit when you know the temperature in Celsius
  - a. Double the temperature in Celsius, then add 30.
  - b. Add 15 to the temperature in Celsius, then double the result.
2. Calculating a 15% tip on a restaurant bill
  - a. Take 10% of the bill amount, take 5% of the bill amount, and add those two values together.
  - b. Multiply the bill amount by 3, divide the result by 2, and then take  $\frac{1}{10}$  of that result.
3. Changing a distance in miles to a distance in kilometers
  - a. Take the number of miles, double it, then decrease the result by 20%.
  - b. Divide the number of miles by 5, then multiply the result by 8.

## Student Response

1. Celsius to Fahrenheit:
  - a. If  $c$  represents the temperature in Celsius, this way of calculating can be expressed with  $2c + 30$ .
  - b. This way of calculating can be represented with  $2(c + 15)$ . These are equivalent because of the distributive property.

2. 15% tip:

- a. If  $b$  represents the bill amount, this way of calculating can be expressed with  $0.1b + 0.05b$ , which is equivalent to  $0.15b$  by combining like terms.
- b. This way of calculating can be represented with  $\frac{3b}{2} \cdot \frac{1}{10}$ . This is equivalent to  $1.5b \cdot 0.1$  because  $\frac{3}{2} = 1.5$  and  $\frac{1}{10} = 0.1$ . This is equivalent to  $0.1 \cdot 1.5b$  because multiplication is commutative, and  $0.15b$  because  $0.1 \cdot 1.5 = 0.15$ .

3. miles to kilometers:

- a. If  $m$  represents the distance in miles, this way of calculating can be expressed with  $0.8(2m)$ , and then  $1.6m$  after multiplying. Alternatively, it can be expressed with  $2m - 0.2(2m)$ , which is equivalent to  $1.6m$  after multiplying to get  $2m - 0.4m$  and then combining like terms.
- b. This way of calculating can be expressed with  $\frac{m}{5} \cdot 8$ , which is equivalent to  $0.2m \cdot 8$  because  $\frac{1}{5} = 0.2$ . After rearranging and multiplying, this is also equivalent to  $1.6m$ .

### Activity Synthesis

For each situation, invite at least one student to share their reasoning for why the two calculation methods are equivalent. Be sure to include, for each situation, an approach that involves writing expressions that contain a variable and using properties of operations to show that they are equivalent.

Spend a little time on the last situation, emphasizing that 20% off an amount is the same as 80% of the amount. That is, if you want to compute  $x$  decreased by 20%, you can write  $0.8x$ , because  $x - 0.2x$  is equivalent to  $0.8x$ . This insight will help students write less-complicated expressions in the next activity.

---

#### Access for English Language Learners

*Representing, Conversing: MLR 7 Compare and Connect.* Use this routine to prepare students for the whole-class discussion. Give students quiet think time to consider what is the same and what is different about the two different ways of calculating the last situation. Next, ask students to share what they noticed with a partner. Listen for and amplify mathematical language students use to explain why the two ways are equivalent.

*Design Principle(s): Cultivate conversation*

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## 23.3 Which Way?

15 minutes

The purpose of this activity is to use expressions with a variable to represent applying coupons in a different order. By analyzing these expressions, you learn that no matter how much you spend, you always pay \$6 less if the 20% off coupon is applied first.

Monitor for how students are approaching the problem. If, once students have an answer, they have not written expressions with a variable to show the best way to apply the coupons, consider asking them to try writing an expression using a variable to represent the purchase amount.

Students should be familiar with the idea of coupons and discounts from their work in an earlier unit. If this is not the case, more time may be needed for the launch to familiarize students with the context.

Students may notice that if you spend less than \$30, the store probably won't let you take \$30 off. This is a possible constraint, and students who include this constraint are engaging in aspects of mathematical modeling (MP4).

### **Building On**

- 7.EE.B

### **Instructional Routines**

- Notice and Wonder

### **Launch**

Display the image of two coupons for all to see. Ask students to think of some things they notice and some things they wonder. Things students might notice:

- There are two coupons to the same store.
- One coupon is for 20% off and one coupon is for \$30 off.

Things students might wonder:

- Can you use both coupons on the same purchase?
- Do the coupons expire?
- Is there a minimum or maximum amount you have to spend?

If no students wonder this, ask, "What if you could use both coupons on the same purchase? Should you deduct 20%, and then \$30 from the result? Or the other way around? Does it matter?"

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### **Access for Students with Disabilities**

*Representation: Internalize Comprehension.* Provide appropriate reading accommodations and supports to ensure students access to written directions, word problems, and other text-based content.

*Supports accessibility for: Language; Conceptual processing*

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## Anticipated Misconceptions

If students have trouble getting started, suggest that they calculate which order is better for some specific purchase amounts.

### Student Task Statement



You have two coupons to the same store: one for 20% off and one for \$30 off. The cashier will let you use them both, and will let you decide in which order to use them.

- Mai says that it doesn't matter in which order you use them. You will get the same discount either way.
- Jada says that you should apply the 20% off coupon first, and then the \$30 off coupon.
- Han says that you should apply the \$30 off coupon first, and then the 20% off coupon.
- Kiran says that it depends on how much you are spending.

Do you agree with any of them? Explain your reasoning.

### Student Response

It is always better to use the 20% off coupon first.

- Let  $x$  represent the amount of the purchase.
- 20% off is  $0.8x$  or equivalent. \$30 off that is  $0.8x - 30$ .
- \$30 off is  $x - 30$ . 20% off that is  $0.8(x - 30)$ . By the distributive property, this is equivalent to  $0.8x - 24$ .
- Comparing  $0.8x - 30$  to  $0.8x - 24$ , your resulting bill will always be \$6 less if you use the 20% off coupon first.

### Activity Synthesis

If any students only computed their resulting bill using a specific dollar amount, ask them to present their solution first. For example, on a \$100 purchase, this might look like:

- 20% off is \$80. Then \$30 off of \$80 is \$50.
- \$30 off is \$70. Then 20% off of \$70 is \$56.

So if your purchase was \$100, it's better to apply the 20% off coupon first. What about other purchase amounts? (If students tried other purchase amounts, consider also having them demonstrate. It is helpful if students see a few different examples that always result in a \$6 difference.)

Select a student to present who wrote an expression using a variable for the purchase amount. If no students did so, demonstrate this approach.

- Let  $x$  represent the amount of the purchase.
- 20% off is  $0.8x$  or equivalent. \$30 off that is  $0.8x - 30$ .
- \$30 off is  $x - 30$ . 20% off that is  $0.8(x - 30)$ . By the distributive property, this is equivalent to  $0.8x - 24$ .
- Comparing  $0.8x - 30$  to  $0.8x - 24$ , your resulting bill will always be \$6 less if you use the 20% off coupon first.



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Family Support  
Materials

# Family Support Materials

## Expressions, Equations, and Inequalities

Here are the video lesson summaries for Grade 7, Unit 6: Expressions, Equations, and Inequalities. Each video highlights key concepts and vocabulary that students learn across one or more lessons in the unit. The content of these video lesson summaries is based on the written Lesson Summaries found at the end of lessons in the curriculum. The goal of these videos is to support students in reviewing and checking their understanding of important concepts and vocabulary. Here are some possible ways families can use these videos:

- Keep informed on concepts and vocabulary students are learning about in class.
- Watch with their student and pause at key points to predict what comes next or think up other examples of vocabulary terms (the bolded words).
- Consider following the Connecting to Other Units links to review the math concepts that led up to this unit or to preview where the concepts in this unit lead to in future units.

Grade 7, Unit 6: Expressions, Equations, and Inequalities	Vimeo	YouTube
Video 1: Representing Two Types of Situations (Lessons 1–6)	<a href="#">Link</a>	<a href="#">Link</a>
Video 2: Reasoning About Solving Equations (Lessons 7–9)	<a href="#">Link</a>	<a href="#">Link</a>
Video 3: Using Equations to Solve Problems (Lessons 10–12)	<a href="#">Link</a>	<a href="#">Link</a>
Video 4: Solving Inequalities (Lessons 14–17)	<a href="#">Link</a>	<a href="#">Link</a>
Video 5: Writing Equivalent Expressions (Lessons 18–22)	<a href="#">Link</a>	<a href="#">Link</a>

### Video 1

Video 'VLS G7U6V1 Representing Two Types of Situations (Lessons 1–6)' available here: <https://player.vimeo.com/video/513963265>.

## Video 2

Video 'VLS G7U6V2 Reasoning About Solving Equations (Lessons 7–9)' available here:  
<https://player.vimeo.com/video/513024045>.

## Video 3

Video 'VLS G7U6V3 Using Equations to Solve Problems (Lessons 10–12)' available here:  
<https://player.vimeo.com/video/514745993>.

## Video 4

Video 'VLS G7U6V4 Solving Inequalities (Lessons 14–17)' available here:  
<https://player.vimeo.com/video/533191590>.

## Video 5

Video 'VLS G7U6V5 Writing Equivalent Expressions (Lessons 18–22)' available here:  
<https://player.vimeo.com/video/521623062>.

## Connecting to Other Units

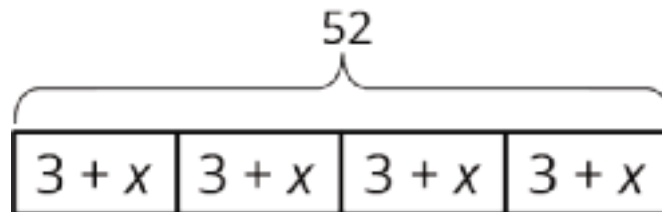
- *Coming soon*

## Representing Situations of the Form $px + q = r$ and $p(x + q) = r$

### Family Support Materials 1

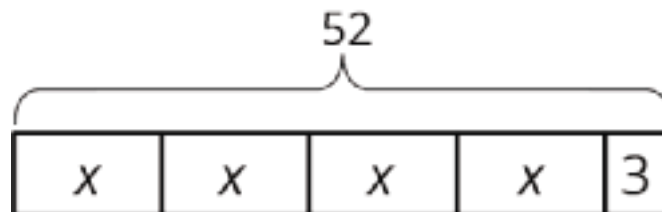
In this unit, your student will be representing situations with diagrams and equations. There are two main categories of situations with associated diagrams and equations.

Here is an example of the first type: A standard deck of playing cards has four suits. In each suit, there are 3 face cards and  $x$  other cards. There are 52 total cards in the deck. A diagram we might use to represent this situation is:



and its associated equation could be  $52 = 4(3 + x)$ . There are 4 groups of cards, each group contains  $x + 3$  cards, and there are 52 cards in all.

Here is an example of the second type: A chef makes 52 pints of spaghetti sauce. She reserves 3 pints to take home to her family, and divides the remaining sauce equally into 4 containers. A diagram we might use to represent this situation is:



and its associated equation could be  $52 = 4x + 3$ . From the 52 pints of sauce, 3 were set aside, and each of 4 containers holds  $x$  pints of sauce.

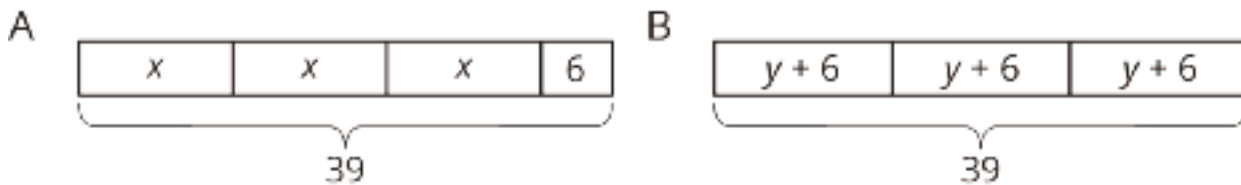
Here is a task to try with your student:

1. Draw a diagram to represent the equation  $3x + 6 = 39$
2. Draw a diagram to represent the equation  $39 = 3(y + 6)$
3. Decide which story goes with which equation-diagram pair:

- Three friends went cherry picking and each picked the same amount of cherries, in pounds. Before they left the cherry farm, someone gave them an additional 6 pounds of cherries. Altogether, they had 39 pounds of cherries.
- One of the friends made three cherry tarts. She put the same number of cherries in each tart, and then added 6 more cherries to each tart. Altogether, the three tarts contained 39 cherries.

Solution:

Diagram A represents  $3x + 6 = 39$  and the story about cherry picking. Diagram B represents  $3(y + 6) = 39$  and the story about making cherry tarts.



# Solving Equations of the Form $px + q = r$ and $p(x + q) = r$ and Problems That Lead to Those Equations

## Family Support Materials 2

Your student is studying efficient methods to solve equations and working to understand why these methods work. Sometimes to solve an equation, we can just think of a number that would make the equation true. For example, the solution to  $12 - c = 10$  is 2, because we know that  $12 - 2 = 10$ . For more complicated equations that may include decimals, fractions, and negative numbers, the solution may not be so obvious.

An important method for solving equations is *doing the same thing to each side*. For example, let's show how we might solve  $-4(x - 1) = 20$  by doing the same thing to each side.

$$\begin{aligned} -4(x - 1) &= 24 \\ -\frac{1}{4} \cdot -4(x - 1) &= -\frac{1}{4} \cdot 24 && \text{multiply each side by } -\frac{1}{4} \\ x - 1 &= -6 \\ x - 1 + 1 &= -6 + 1 && \text{add 1 to each side} \\ x &= -5 \end{aligned}$$

Another helpful tool for solving equations is to apply the distributive property. In the example above, instead of multiplying each side by  $-\frac{1}{4}$ , you could apply the distributive property to  $-4(x - 1)$  and replace it with  $-4x + 4$ . Your solution would look like this:

$$\begin{aligned} -4(x - 1) &= 24 \\ -4x + 4 &= 24 && \text{apply the distributive property} \\ -4x + 4 - 4 &= 24 - 4 && \text{subtract 4 from each side} \\ -4x &= 20 \\ -4x \div -4 &= 20 \div -4 && \text{divide each side by -4} \\ x &= -5 \end{aligned}$$

Here is a task to try with your student:

Elena picks a number, adds 45 to it, and then multiplies by  $\frac{1}{2}$ . The result is 29. Elena says that you can find her number by solving the equation  $29 = \frac{1}{2}(x + 45)$ .

Find Elena's number. Describe the steps you used.

Solution:

Elena's number was 13. There are many different ways to solve her equation. Here is one example:

$$29 = \frac{1}{2}(x + 45)$$

$$2 \cdot 29 = 2 \cdot \frac{1}{2}(x + 45) \quad \text{multiply each side by 2}$$

$$58 = x + 45$$

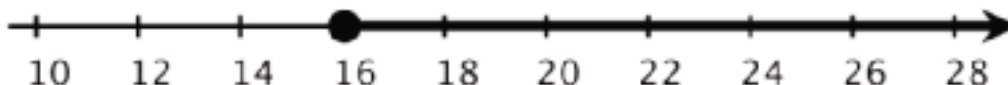
$$58 - 45 = x + 45 - 45 \quad \text{subtract 45 from each side}$$

$$13 = x$$

# Inequalities

## Family Support Materials 3

This week your student will be working with inequalities (expressions with  $>$  or  $<$  instead of  $=$ ). We use inequalities to describe a range of numbers. For example, in many places you need to be at least 16 years old to be allowed to drive. We can represent this situation with the inequality  $a \geq 16$ . We can show all the solutions to this inequality on the number line.



Here is a task to try with your student:

Noah already has \$10.50, and he earns \$3 each time he runs an errand for his neighbor. Noah wants to know how many errands he needs to run to have at least \$30, so he writes this inequality:

$$3e + 10.50 \geq 30$$

We can test this inequality for different values of  $e$ . For example, 4 errands is not enough for Noah to reach his goal, because  $3 \cdot 4 + 10.50 = 22.5$ , and \$22.50 is less than \$30.

1. Will Noah reach his goal if he runs:
  - a. 8 errands?
  - b. 9 errands?
2. What value of  $e$  makes the equation  $3e + 10.50 = 30$  true?
3. What does this tell you about all the solutions to the inequality  $3e + 10.50 \geq 30$ ?
4. What does this mean for Noah's situation?

Solution:

1.
  - a. Yes, if Noah runs 8 errands, he will have  $3 \cdot 8 + 10.50$ , or \$34.50.
  - b. Yes, since 9 is more than 8, and 8 errands was enough, so 9 will also be enough.
2. The equation is true when  $e = 6.5$ . We can rewrite the equation as  $3e = 30 - 10.50$ , or  $3e = 19.50$ . Then we can rewrite this as  $e = 19.50 \div 3$ , or  $e = 6.5$ .
3. This means that when  $e \geq 6.5$  then Noah's inequality is true.
4. Noah can't really run 6.5 errands, but he could run 7 or more errands, and then he would have more than \$30.

# Writing Equivalent Expressions

## Family Support Materials 4

This week your student will be working with equivalent expressions (expressions that are always equal, for any value of the variable). For example,  $2x + 7 + 4x$  and  $6x + 10 - 3$  are equivalent expressions. We can see that these expressions are equal when we try different values for  $x$ .

	$2x + 7 + 4x$	$6x + 10 - 3$
when $x$ is 5	$2 \cdot 5 + 7 + 4 \cdot 5$ $10 + 7 + 20$ 37	$6 \cdot 5 + 10 - 3$ $30 + 10 - 3$ 37
when $x$ is -1	$2 \cdot -1 + 7 + 4 \cdot -1$ $-2 + 7 + -4$ 1	$6 \cdot -1 + 10 - 3$ $-6 + 10 - 3$ 1

We can also use properties of operations to see why these expressions have to be equivalent—they are each equivalent to the expression  $6x + 7$ .

Here is a task to try with your student:

Match each expression with an equivalent expression from the list below. One expression in the list will be left over.

1.  $5x + 8 - 2x + 1$

2.  $6(4x - 3)$

3.  $(5x + 8) - (2x + 1)$

4.  $-12x + 9$

List:

•  $3x + 7$

•  $3x + 9$

•  $-3(4x - 3)$

•  $24x + 3$

•  $24x - 18$

Solution:

1.  $3x + 9$  is equivalent to  $5x + 8 - 2x + 1$ , because  $5x + -2x = 3x$  and  $8 + 1 = 9$ .

2.  $24x - 18$  is equivalent to  $6(4x - 3)$ , because  $6 \cdot 4x = 24x$  and  $6 \cdot -3 = -18$ .

3.  $3x + 7$  is equivalent to  $(5x + 8) - (2x + 1)$ , because  $5x - 2x = 3x$  and  $8 - 1 = 7$ .

4.  $-3(4x - 3)$  is equivalent to  $-12x + 9$ , because  $-3 \cdot 4x = -12x$  and  $-3 \cdot -3 = 9$ .

# Unit Assessments

Check Your Readiness A and B  
Mid-Unit Assessment A and B  
End-of-Unit Assessment A and B

# Expressions, Equations, and Inequalities: Check Your Readiness (A)

Do not use a calculator.

1. Jada is collecting stickers. After getting 15 more stickers, she has 60 stickers in total.

Select **all** the equations Jada can solve to find  $x$ , the number of stickers she started with.

A.  $x + 15 = 60$

B.  $x - 15 = 60$

C.  $x = 60 + 15$

D.  $x = 60 - 15$

E.  $15x = 60$

F.  $x = 60 \cdot 15$

G.  $x = \frac{60}{15}$

2. Solve each equation.

$$p + 12 = 17$$

$$\frac{7}{3} = q + \frac{2}{3}$$

$$90 = 20r$$

$$\frac{1}{3}s = 7$$

$$15 = 1.5t$$

$$79 + u = 65$$

$$6v = -9$$

3. Lin is selling boxes of cookies. Each box costs \$3.75. Lin's goal is to earn more than \$30 selling cookies.

a. If Lin sells 6 boxes of cookies, will she make her goal?

b. If Lin sells 20 boxes of cookies, will she make her goal?

c. If Lin sells  $b$  boxes of cookies, write an inequality (using the symbol  $<$  or  $>$ ) that will be true whenever Lin makes her goal, and false whenever she does not.

d. Graph your inequality on a number line.



4. Select **all** the equations that are true when  $x$  is  $-4$ .

A.  $-8 = 2x$

B.  $-12 = x \cdot -3$

C.  $-12 = x + x + x$

D.  $\frac{x}{4} = -1$

E.  $x + 4 = -8$

F.  $x^2 = -16$

5. Which expression is equivalent to  $2(3x - 4)$ ?

A.  $3x - 4$

B.  $5x - 6$

C.  $6x - 4$

D.  $6x - 8$

6. Next to each equation, write *A*, *B*, or *neither*, to indicate whether it matches diagram A, diagram B, or neither diagram.

A



a.  $7 = 3 + 4$

b.  $4 - 3 = 7$

c.  $7 - 4 = 3$

d.  $4 \cdot 3 = 7$

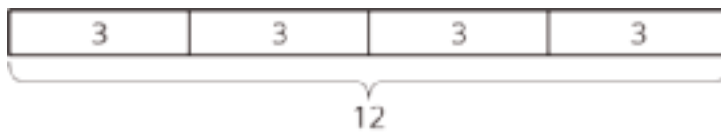
e.  $3 + 3 + 3 + 3 = 12$

f.  $12 = 4 \cdot 3$

g.  $12 \div 4 = 3$

h.  $3 \cdot 3 \cdot 3 \cdot 3 = 12$

B



# Expressions, Equations, and Inequalities: Check Your Readiness (B)

Do not use a calculator.

1. Lin is collecting coins. After giving away 13 coins, she has 75 coins remaining. Select **all** the equations Lin can solve to find  $x$ , the number of coins she started with.

A.  $x + 13 = 75$

B.  $x - 13 = 75$

C.  $x = 75 - 13$

D.  $13x = 75$

E.  $x = 75 + 13$

F.  $x = 75 \cdot 13$

G.  $x = \frac{75}{13}$

2. Solve each equation.

$$\frac{9}{5} = p + \frac{3}{5}$$

$$\frac{1}{4}q = 7$$

$$r + 18 = 24$$

$$35 = 3.5s$$

$$80 = 30t$$

$$98 + u = 37$$

$$8v = -18$$

3. Noah is selling boxes of greeting cards. Each box costs \$4.25. Noah's goal is to earn more than \$50 selling cards.

a. If Noah sells 20 boxes of cards, will he make his goal?

b. If Noah sells 10 boxes of cards, will he make his goal?

c. If Noah sells  $b$  boxes of cards, write an inequality (using the symbol  $<$  or  $>$ ) that will be true whenever Noah makes his goal, and false whenever he does not.

d. Graph your inequality on the number line.



4. Select **all** the equations that are true when  $x$  is -6.

A.  $18 = x + x + x$

B.  $4 = x + 10$

C.  $-12 = 2x$

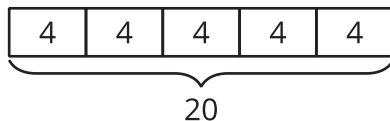
D.  $9 - x = 3$

E.  $\frac{x}{2} = -3$

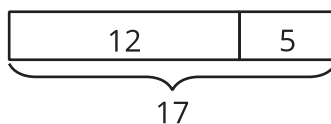
F.  $-18 = x \cdot -3$

5. Next to each equation, write *A*, *B*, or *neither*, to indicate whether it matches diagram A, diagram B, or neither diagram.

**A**



**B**



- a.  $17 = 5 + 12$
- b.  $17 - 12 = 5$
- c.  $12 - 5 = 17$
- d.  $4 + 4 + 4 + 4 + 4 = 20$
- e.  $20 = 5 \cdot 4$
- f.  $12 \cdot 5 = 17$
- g.  $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 20$
- h.  $20 \div 4 = 5$

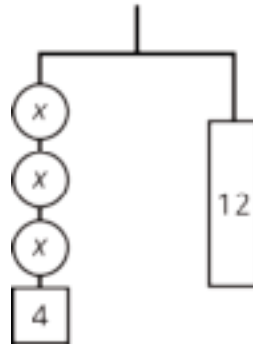
6. Select **all** expressions that are equivalent to  $5n - 30$ .

- A.  $n + n + n + n + n - 30$
- B.  $5(n - 30)$
- C.  $(n - 6) \cdot 5$
- D.  $5n - 30n$
- E.  $5n + -30$

# Expressions, Equations, and Inequalities: Mid-Unit Assessment (A)

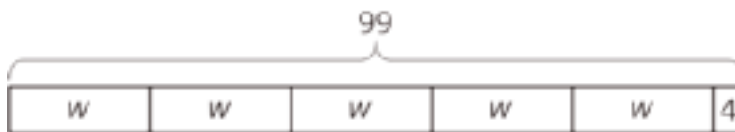
You may use a four-function or scientific calculator, but not a graphing calculator.

1. This hanger is in balance. There are two labeled weights of 4 grams and 12 grams. The three circles each have the same weight. What is the weight of each circle, in grams?



- A.  $\frac{3}{8}$
- B. 1
- C.  $\frac{8}{3}$
- D. 8

2. Select **all** the situations that can be represented by the tape diagram.



- A. Clare buys 4 bouquets, each with the same number of flowers. The florist puts an extra flower in each bouquet before she leaves. She leaves with a total of 99 flowers.
  - B. Andre babysat 5 times this past month and earned the same amount each time. To thank him, the family gave him an extra \$4 at the end of the month. Andre earned \$99 from babysitting.
  - C. A family of 5 drove to a concert. They paid \$4 for parking, and all of their tickets were the same price. They paid \$99 in total.
  - D. 5 bags of marbles each contain 4 large marbles and the same number of small marbles. Altogether, the bags contain 99 marbles.
  - E. Han is baking five batches of muffins. Each batch needs the same amount of sugar in the muffins, and each batch needs four extra teaspoons of sugar for the topping. Han uses 99 total teaspoons of sugar.
3. At practice, Diego does twice as many push-ups as Noah, and also 40 jumping jacks. He does 62 exercises in total. The equation  $2x + 40 = 62$  describes this situation. What does the variable  $x$  represent?
- A. The number of jumping jacks Diego does
  - B. The number of push-ups Diego does
  - C. The number of jumping jacks Noah does
  - D. The number of push-ups Noah does

4. Solve each equation.

a.  $25 - 3x = 40$

b.  $\frac{1}{3}(x - 10) = -4$

5. Andre tried to solve the equation  $\frac{1}{4}(x + 12) = 2$ . What was his mistake?

$$\frac{1}{4}(x + 12) = 2$$

$$\frac{1}{4}x = -10$$

$$x = -40$$

6. A food pantry is making packages. Each package weighs 64 pounds.

Here are two situations. For each situation, write an equation to represent the situation. If you get stuck, consider drawing a diagram.

a. Each package contains 4 boxes. Each box contains a 7-pound bag of beans and a bag of rice. The bags of rice are all identical.

b. Each package contains 4 identical bags of rice and a 7-pound bag of beans.

7. For a school fundraiser, Elena will make T-shirts. She will order T-shirts and print graphics on them. Elena must spend \$349 on a printing machine and \$4.80 per shirt for the blank shirts, ink, and other supplies.

a. Complete the table giving the total cost Elena will spend to make each specific number of T-shirts.

number of shirts	cost in dollars
20	
40	
60	

b. Write an expression for the cost of making  $n$  T-shirts.

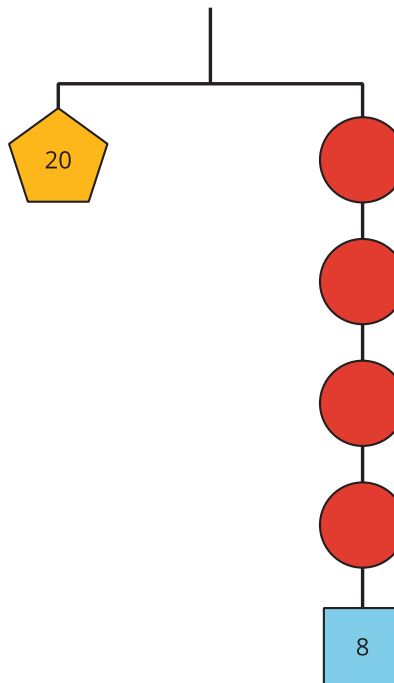
c. What is the maximum number of T-shirts Elena can make with a budget of \$1,000?

# Expressions, Equations, and Inequalities: Mid-Unit Assessment (B)

You may use a four-function or scientific calculator, but not a graphing calculator.

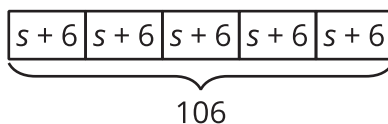
1. This hanger is in balance. There are two labeled weights of 8 grams and 20 grams. The four circles each have the same weight.

What is the weight of each circle, in grams?



- A. 3
- B. 7
- C. 5
- D.  $\frac{20}{12}$

2. Select **all** the situations that can be represented by the tape diagram.



- A. Noah walked a family's dog 5 times this past month and earned the same amount each time. To thank him, the family gave him an extra \$6 at the end of the month. Noah earned \$106 from dog walking.
  - B. A family of 5 drove to a soccer game. They paid \$6 for parking, and all of their tickets were the same price. They paid \$106 in total.
  - C. 5 bags of coins each contain 6 nickels and the same number of pennies. Altogether, the bags contain 106 coins.
  - D. Kiran is baking 5 batches of muffins. Each batch needs the same amount of sugar in the muffins, and each batch needs six extra teaspoons of sugar for the topping. Kiran uses 106 total teaspoons of sugar.
  - E. Priya buys 5 cases of water, each with the same number of bottles. The store clerk gives her an extra 6 bottles before she leaves. She leaves with a total of 106 bottles.
3. Over the weekend, Jada made three times as much money babysitting than Han did mowing lawns. Jada made an additional \$27 selling lemonade. From babysitting and selling lemonade, she has a total of \$96. If the equation  $3x + 27 = 96$  represents this situation, what does the variable  $x$  represent?

4. Solve each equation.

a.  $10 - \frac{1}{4}x = 7$

b.  $3(x + 8) = 21$

5. Diego tried to solve the equation  $\frac{1}{3}(x + 15) = 3$ . What was his mistake?

$$\frac{1}{3}(x + 15) = 3$$

$$\frac{1}{3}x = -12$$

$$x = -4$$

6. A church is packing Thanksgiving baskets. Each basket weighs 30 pounds.

Here are two situations. For each situation, write an equation to represent the situation. If you get stuck, consider drawing a diagram.

a. Each basket contains 3 identical bags of stuffing and a 6-pound bag of rice.

b. Each basket contains 3 boxes. Each box contains a 6-pound bag of rice and a bag of stuffing. The bags of stuffing are all identical.

7. For a school fundraiser, Mai will make school spirit bracelets. She will order bead wiring and alphabet beads to create the school name on the bracelets. Mai must spend \$250 on wire and \$5.30 per bracelet for beads.

a. Complete the table giving the total cost Mai will spend to make each specific number of bracelets.

number of bracelets	cost in dollars
20	
40	
60	

b. Write an expression for the cost of making  $n$  bracelets.

c. What is the maximum number of bracelets Mai can make with a budget of \$1,500?

# Expressions, Equations, and Inequalities: End-of-Unit Assessment (A)

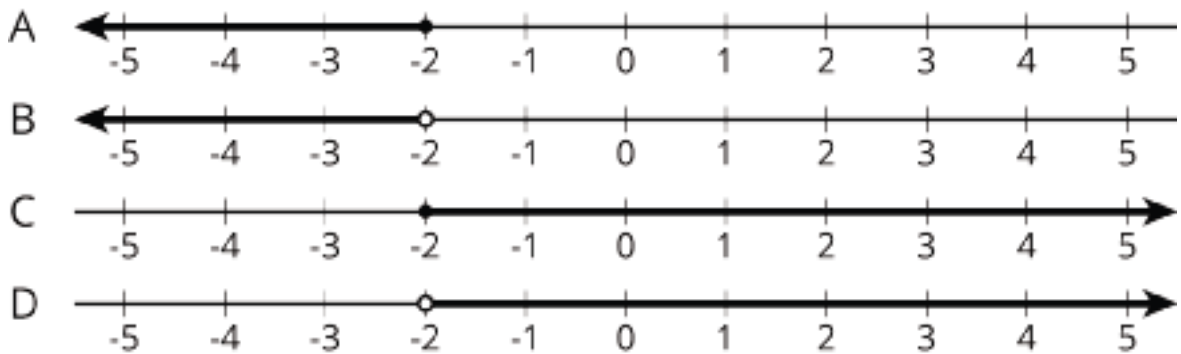
You may use a four-function or scientific calculator, but not a graphing calculator.

1. Lin got a \$50 gift card to an online music store. She uses the gift card to buy an album for \$9.99. She also wants to use the gift card to buy some songs. Each song costs \$1.29.

Which of these inequalities describes this situation, where  $n$  is the number of songs Lin wants to buy?

- A.  $9.99 + 1.29n \geq 50$
- B.  $9.99 + 1.29n \leq 50$
- C.  $9.99 - 1.29n \geq 50$
- D.  $9.99 - 1.29n \leq 50$

2. Which number line shows all the values of  $x$  that make the inequality  $-3x + 1 < 7$  true?



- A. A
- B. B
- C. C
- D. D

3. Select **all** expressions that are equivalent to  $6x + 1 - (3x - 1)$ .

A.  $6x + 1 - 3x - 1$

B.  $6x + -3x + 1 + 1$

C.  $3x + 2$

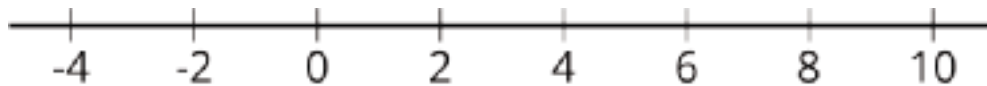
D.  $6x - 3x + 1 - 1$

E.  $6x + 1 + -3x - -1$

4. At midnight, the temperature in a city was 5 degrees Celsius. The temperature was dropping at a steady rate of 2 degrees Celsius per hour.

a. Write an inequality that represents  $t$ , the number of hours past midnight, when the temperature was colder than -4 degrees Celsius. Explain or show your reasoning.

b. On the number line, show all the values of  $t$  that make your inequality true.



5. a. Expand to write an equivalent expression:

$$-\frac{1}{4}(-8x + 12y)$$

- b. Factor to write an equivalent expression:

$$36a - 16$$

6. Tyler is simplifying the expression  $6 - 2x + 5 + 4x$ . Here is his work:

$$6 - 2x + 5 + 4x$$

$$(6 - 2)x + (5 + 4)x$$

$$4x + 9x$$

$$13x$$

- a. Tyler's work is incorrect. Explain the error he made.

- b. Write an expression equivalent to  $6 - 2x + 5 + 4x$  that only has two terms.

7. Diego's family car holds 14 gallons of fuel. Each day the car uses 0.6 gallons of fuel. A warning light comes on when the remaining fuel is 1.5 gallons or less.
- Starting from a full tank, can Diego's family drive the car for 14 days without the warning light coming on? Explain or show your reasoning.
  - Starting from a full tank, can Diego's family drive the car for 25 days without the warning light coming on? Explain or show your reasoning.
  - Diego says the expression  $14 - 0.6t$  helps him understand this situation. In this situation, what does this expression represent, and what does the variable  $t$  stand for?
  - Write and solve an equation to determine the number of days Diego's father can drive the car without the warning light coming on.
  - Write and solve an inequality that represents this situation. Explain clearly what the solution to the inequality means in the context of this situation.

# Expressions, Equations, and Inequalities: End-of-Unit Assessment (B)

You may use a four-function or scientific calculator, but not a graphing calculator.

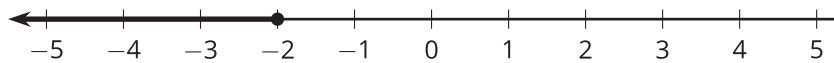
1. Tyler has run 15 miles this month. For the rest of the month, he plans to run the same number of miles each day. There are 12 days left in the month. Before the end of the month, Tyler needs to run at least 35 miles to meet his goal.

Which of these inequalities describes this situation, where  $m$  is the number of miles Tyler runs each day to meet his goal?

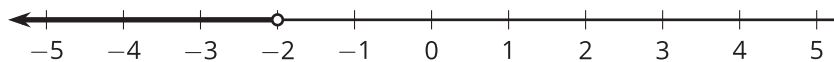
- A.  $12m - 15 \geq 35$
- B.  $12m - 15 \leq 35$
- C.  $35 \geq 15 + 12m$
- D.  $35 \leq 15 + 12m$

2. Which number line shows all the values of  $y$  that make the inequality  $-6y - 2 \geq 10$  true?

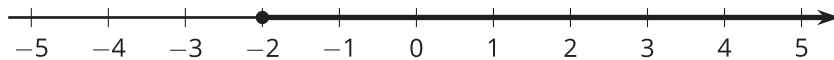
A. **A**



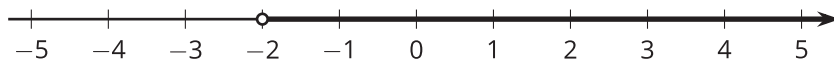
B. **B**



C. **C**



D. **D**



3. Select **all** expressions that are equivalent to  $-4(x + 2) - 2x + 4$ .

A.  $-6x - 4$

B.  $-4x + 2 - 2x + 4$

C.  $-10x$

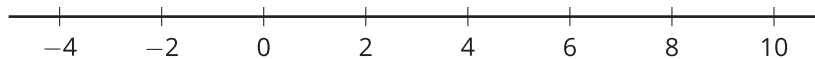
D.  $-4x + -8 - 2x + 4$

E.  $-4x - 2x - 8 + 4$

4. At the start of a hike, a hiker was at an elevation of -50 feet (where 0 represents sea level). The hiker climbs at a rate of 15 feet per minute.

a. Write an inequality that represents  $t$ , the number of minutes after the start of the hike, when the hiker's elevation was higher than 5 feet above sea level. Explain or show your reasoning.

b. On the number line, show all the values of  $t$  that make your inequality true.



5. a. Factor to write an equivalent expression:

$$-9x + 15y$$

- b. Expand to write an equivalent expression:

$$-5(7m - 2)$$

6. Priya is simplifying the expression  $-2(4 - 2x) - 3 - 2x$ . Here is her work:

$$-2(4 - 2x) - 3 - 2x$$

$$-8 - 2x - 3 - 2x$$

$$-11 - 4x$$

- a. Priya's work is incorrect. Explain the error she made.

- b. Write an expression equivalent to  $-4(x + 3) - 12x + 5$  that only has two terms.

7. A community center rents their hall for special events. They charge a fixed fee of \$200 plus an hourly fee of \$22.50. Lin has \$300 to spend on renting the hall for a fundraiser.
- Using only the money she has, can Lin pay for a 6 hour event? Explain or show your reasoning.
  - Using only the money she has, can Lin pay for a 3 hour event? Explain or show your reasoning.
  - If  $h$  represents the number of hours, what does  $22.50h + 200$  represent?
  - Write and solve an equation to determine the number of hours Lin can rent the community center.
  - Write and solve an inequality that represents this situation. Explain what the solution to the inequality means in this situation.

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# Assessment Answer Keys

Check Your Readiness A and B  
Mid-Unit Assessment A and B  
End-of-Unit Assessment A and B

# Assessment Answer Keys

## Assessment : Check Your Readiness (A)

### Teacher Instructions

Calculators should not be used.

### Student Instructions

Do not use a calculator.

### Problem 1

The content assessed in this problem is first encountered in Lesson 4: Reasoning about Equations and Tape Diagrams (Part 1).

Students will solve more advanced equations in this unit, building from the equation types they have worked with in sixth grade.

If most students struggle with this item, plan to revisit it before Activity 2. Ask students to draw a tape diagram to represent the situation, and then try matching equations again. You may also choose to revisit the stories in Lesson 2 and ask students to write equations to match the tape diagrams and stories, understanding that the story connects to both an addition and subtraction equation.

### Statement

Jada is collecting stickers. After getting 15 more stickers, she has 60 stickers in total.

Select **all** the equations Jada can solve to find  $x$ , the number of stickers she started with.

A.  $x + 15 = 60$

B.  $x - 15 = 60$

C.  $x = 60 + 15$

D.  $x = 60 - 15$

E.  $15x = 60$

F.  $x = 60 \cdot 15$

G.  $x = \frac{60}{15}$

## Solution

["A", "D"]

## Aligned Standards

6.EE.B.7

### Problem 2

The content assessed in this problem is first encountered in Lesson 9: Dealing with Negative Numbers.

Students should have experience solving these types of equations for non-negative rational numbers. The last two extend students' understanding to equations involving negative numbers.

If most students struggle with this item, plan to incorporate practice solving equations with one operation into earlier lessons. You may choose to use the equations from the item for practice. Algebra Talks can be used to give students practice reasoning about equations of the form  $x+p=q$  and  $px=q$ .

### Statement

Solve each equation.

$$p + 12 = 17$$

$$\frac{7}{3} = q + \frac{2}{3}$$

$$90 = 20r$$

$$\frac{1}{3}s = 7$$

$$15 = 1.5t$$

$$79 + u = 65$$

$$6v = -9$$

### Solution

1.  $p = 5$

2.  $q = \frac{5}{3}$

3.  $r = \frac{9}{2}$  (or equivalent)

4.  $s = 21$

5.  $t = 10$

6.  $u = -14$

7.  $v = \frac{-9}{6}$  (or equivalent)

## Aligned Standards

6.EE.B.7, 7.EE.B.4

### Problem 3

The content assessed in this problem is first encountered in Lesson 13: Reintroducing Inequalities.

The first two parts ask students to test individual values, encouraging the strategy of testing when an inequality is true or false. That key concept will be developed further in this unit to help students solve and graph more complicated inequalities.

Check to see if students recall the “open circle” concept for graphing inequalities. It is unlikely that students will graph the solution as a set of points, but technically the number of boxes must be an integer.

If most students do well with this item, it may be possible to move more quickly through Activity 1 and Activity 2.

#### Statement

Lin is selling boxes of cookies. Each box costs \$3.75. Lin’s goal is to earn more than \$30 selling cookies.

1. If Lin sells 6 boxes of cookies, will she make her goal?
2. If Lin sells 20 boxes of cookies, will she make her goal?
3. If Lin sells  $b$  boxes of cookies, write an inequality (using the symbol  $<$  or  $>$ ) that will be true whenever Lin makes her goal, and false whenever she does not.
4. Graph your inequality on a number line.



#### Solution

1. No, she only earns \$22.50.
2. Yes, she earns \$75.
3.  $b > 8$ , because  $30 \div 3.75 = 8$
4. The solution is the graph of  $b > 8$ . Alternately, the solution is a set of dots at the whole numbers from  $b = 9$  and above because the number of boxes must be a whole number.

#### Aligned Standards

6.EE.B.5, 6.EE.B.8

### Problem 4

The content assessed in this problem is first encountered in Lesson 9: Dealing with Negative

Numbers.

The work with negative number arithmetic previews work that will come up when solving equations in this unit. Students' general understanding that a number is a solution to an equation when using that value for the variable makes the equation true is crucial. Watch for errors in students' arithmetic work. Students selecting B or F may need to be reminded about the properties of multiplication by negatives throughout the unit.

If most students struggle with this item, plan to use an Algebra Talk or "True or False" routine to address students' needs before this lesson. Note whether the struggle is a result of arithmetic needs or understanding how to determine if a value for the variable makes the equation true.

### Statement

Select **all** the equations that are true when  $x$  is  $-4$ .

A.  $-8 = 2x$

B.  $-12 = x \cdot -3$

C.  $-12 = x + x + x$

D.  $\frac{x}{4} = -1$

E.  $x + 4 = -8$

F.  $x^2 = -16$

### Solution

["A", "C", "D"]

### Aligned Standards

6.EE.B.5, 7.NS.A.1, 7.NS.A.2

### Problem 5

The content assessed in this problem is first encountered in Lesson 3: Reasoning about Equations with Tape Diagrams.

If most students struggle with this item, plan to spend additional time on the warm-up of Lesson 3. Use area models and tape diagrams to help students recall what they learned about the distributive property in 6th grade. If students need additional practice with the distributive property, Unit 6 of the 6th grade material, in particular lessons 9-11, focuses extensively on the distributive property.

### Statement

Which expression is equivalent to  $2(3x - 4)$ ?

- A.  $3x - 4$
- B.  $5x - 6$
- C.  $6x - 4$
- D.  $6x - 8$

## Solution

D

## Aligned Standards

6.EE.A.3

## Problem 6

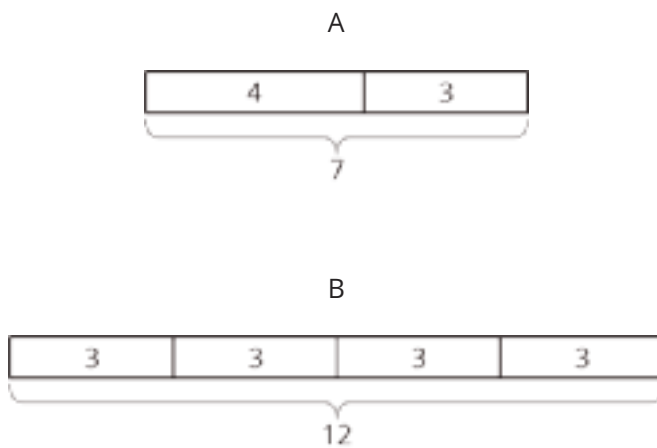
The content assessed in this problem is first encountered in Lesson 2: Reasoning about Contexts with Tape Diagrams.

In this unit, students use tape diagrams to represent the structures  $px + q = r$  and  $p(x + q) = r$ . They will match tape diagrams to situations and use the diagrams as tools to decide how a situation should be represented algebraically. Note that not all the equations listed are true, which is fine.

If most students struggle with this item, plan to revisit it as part of the warm-up. You might show the students the most popular answers from the item and ask if they agree or disagree with their classmates' choices. Students have several chances to work with tape diagrams in the first section of the unit.

## Statement

Next to each equation, write *A*, *B*, or *neither*, to indicate whether it matches diagram A, diagram B, or neither diagram.



1.  $7 = 3 + 4$

2.  $4 - 3 = 7$

3.  $7 - 4 = 3$

4.  $4 \cdot 3 = 7$

5.  $3 + 3 + 3 + 3 = 12$

6.  $12 = 4 \cdot 3$

7.  $12 \div 4 = 3$

8.  $3 \cdot 3 \cdot 3 \cdot 3 = 12$

## **Solution**

1. A
2. Neither
3. A
4. Neither
5. B
6. B
7. B
8. Neither

## **Aligned Standards**

2.OA.A, 3.OA.A

# Assessment : Check Your Readiness (B)

## Teacher Instructions

Calculators should not be used.

## Student Instructions

Do not use a calculator.

## Problem 1

The content assessed in this problem is first encountered in Lesson 4: Reasoning about Equations and Tape Diagrams (Part 1).

Students will solve more advanced equations in this unit, building from the equation types students have worked with in sixth grade.

If most students struggle with this item, plan to revisit it before Activity 2. Ask students to draw a tape diagram to represent the situation, and then try matching equations again. You may also choose to revisit the stories in Lesson 2 and ask students to write equations to match the tape diagrams and stories, understanding that the story connects to both an addition and subtraction equation.

## Statement

Lin is collecting coins. After giving away 13 coins, she has 75 coins remaining. Select **all** the equations Lin can solve to find  $x$ , the number of coins she started with.

A.  $x + 13 = 75$

B.  $x - 13 = 75$

C.  $x = 75 - 13$

D.  $13x = 75$

E.  $x = 75 + 13$

F.  $x = 75 \cdot 13$

G.  $x = \frac{75}{13}$

## Solution

["B", "E"]

## Aligned Standards

6.EE.B.7

### Problem 2

The content assessed in this problem is first encountered in Lesson 9: Dealing with Negative Numbers.

Students should have experience solving these types of equations for non-negative rational numbers. The last two extend students' understanding to equations involving negative numbers.

If most students struggle with this item, plan to incorporate practice solving equations with one operation into earlier lessons. You may choose to use the equations from the item for practice. Algebra Talks can be used to give students practice reasoning about equations of the form  $x+p=q$  and  $px=q$ .

### Statement

Solve each equation.

$$\frac{9}{5} = p + \frac{3}{5}$$

$$\frac{1}{4}q = 7$$

$$r + 18 = 24$$

$$35 = 3.5s$$

$$80 = 30t$$

$$98 + u = 37$$

$$8v = -18$$

### Solution

1.  $p = \frac{6}{5}$

2.  $q = 28$

3.  $r = 6$

4.  $s = 10$

5.  $t = \frac{8}{3}$  (or equivalent)

6.  $u = -61$

7.  $v = -\frac{18}{8}$  (or equivalent)

## Aligned Standards

6.EE.B.7, 7.EE.B.4

### Problem 3

The content assessed in this problem is first encountered in Lesson 13: Reintroducing Inequalities.

The first two parts ask students to test individual values, encouraging the strategy of testing when an inequality is true or false. That key concept will be developed further in this unit to help students solve and graph more complicated inequalities.

Check to see if students recall the “open circle” concept for graphing inequalities. It is unlikely that students will graph the solution as a set of points, but technically the number of boxes must be an integer.

## Statement

Noah is selling boxes of greeting cards. Each box costs \$4.25. Noah’s goal is to earn more than \$50 selling cards.

1. If Noah sells 20 boxes of cards, will he make his goal?
2. If Noah sells 10 boxes of cards, will he make his goal?
3. If Noah sells  $b$  boxes of cards, write an inequality (using the symbol  $<$  or  $>$ ) that will be true whenever Noah makes his goal, and false whenever he does not.
4. Graph your inequality on the number line.



## Solution

1. Yes, he earns \$85
2. No, he only earns \$42.50.
3.  $b > 11$ , because  $50 \div 4.25$  is between 11 and 12.
4. The solution is the graph of  $b > 11$ . Alternately, the solution is a set of dots at the whole numbers from  $b = 12$  and greater, because the number of boxes must be a whole number.

## Aligned Standards

6.EE.B.5, 6.EE.B.8

### Problem 4

The content assessed in this problem is first encountered in Lesson 9: Dealing with Negative Numbers.

The work with negative number arithmetic previews work that will come up when solving equations in this unit. Students’ general understanding that a number is a solution to an equation when using that value for the variable makes the equation true is crucial. Watch for errors in students’

arithmetic work. Students selecting E or F may need to be reminded about the properties of multiplication by negative numbers throughout the unit.

If most students struggle with this item, plan to use an Algebra Talk or "True or False" routine to address students' needs before this lesson. Note whether the struggle is a result of arithmetic needs or understanding how to determine if a value for the variable makes the equation true.

## Statement

Select **all** the equations that are true when  $x$  is  $-6$ .

A.  $18 = x + x + x$

B.  $4 = x + 10$

C.  $-12 = 2x$

D.  $9 - x = 3$

E.  $\frac{x}{2} = -3$

F.  $-18 = x \cdot -3$

## Solution

["B", "C", "E"]

## Aligned Standards

6.EE.B.5, 7.NS.A.1, 7.NS.A.2

## Problem 5

The content assessed in this problem is first encountered in Lesson 2: Reasoning about Contexts with Tape Diagrams.

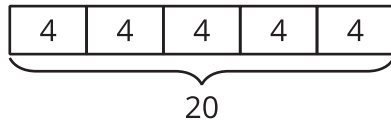
In this unit, students use tape diagrams to represent the structures  $px + q = r$  and  $p(x + q) = r$ . They will match tape diagrams to situations and use the diagrams as tools to decide how a situation should be represented algebraically. Note that not all the equations listed are true, which is fine.

If most students struggle with this item, plan to revisit it as part of the warm-up. You might show the students the most popular answers from the item and ask if they agree or disagree with their classmates' choices. Students have several chances to work with tape diagrams in the first section of the unit.

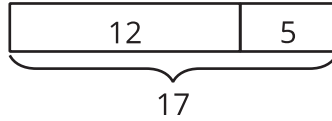
## Statement

Next to each equation, write *A*, *B*, or *neither*, to indicate whether it matches diagram A, diagram B, or neither diagram.

A



B



1.  $17 = 5 + 12$
2.  $17 - 12 = 5$
3.  $12 - 5 = 17$
4.  $4 + 4 + 4 + 4 + 4 = 20$
5.  $20 = 5 \cdot 4$
6.  $12 \cdot 5 = 17$
7.  $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 20$
8.  $20 \div 4 = 5$

## Solution

1. B
2. B
3. Neither
4. A
5. A
6. Neither
7. Neither
8. A

## Aligned Standards

2.OA.A, 3.OA.A

## Problem 6

The content assessed in this problem is first encountered in Lesson 3: Reasoning about Equations with Tape Diagrams.

This is another problem reminding students of the distributive property. If most students struggle with this item, plan to spend additional time on the warm-up of Lesson 3. Use area models and tape diagrams to help students recall what they learned about the distributive property in 6th grade. If students need additional practice with the distributive property, Unit 6 of the 6th grade material, in particular lessons 9-11, focuses extensively on the distributive property.

### Statement

Select all expressions that are equivalent to  $5n - 30$ .

- A.  $n + n + n + n + n - 30$
- B.  $5(n - 30)$
- C.  $(n - 6) \cdot 5$
- D.  $5n - 30n$
- E.  $5n + -30$

### Solution

["A", "C", "E"]

### Aligned Standards

6.EE.A.4

# Assessment : Mid-Unit Assessment (A)

## Teacher Instructions

Use of a four-function or scientific calculator is encouraged but not required. Do not allow use of more advanced calculators that may include functions to solve equations directly.

This assessment is designed to be administered after lesson 12.

## Student Instructions

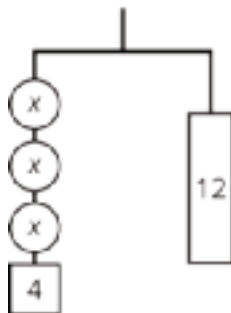
You may use a four-function or scientific calculator, but not a graphing calculator.

## Problem 1

Many students will solve this problem by writing the equation  $3x + 4 = 12$ , though reasoning purely about the weights on the hanger is also fine. Students selecting A likely made a division mistake in the last step of their algebra. Students selecting B probably divided by 4 instead of subtracting. Students selecting D may have stopped their equation solving at  $3x = 8$ , or they may have looked at the hanger and imagined isolating all the weights labeled  $x$ , without dividing by 3.

### Statement

This hanger is in balance. There are two labeled weights of 4 grams and 12 grams. The three circles each have the same weight. What is the weight of each circle, in grams?



- A.  $\frac{3}{8}$
- B. 1
- C.  $\frac{8}{3}$
- D. 8

### Solution

C

## Aligned Standards

7.EE.B.4

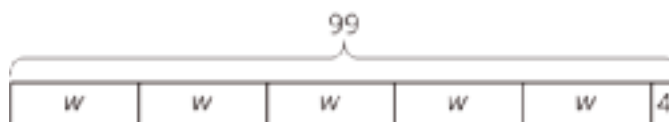
### Problem 2

The tape diagram represents the equation  $5w + 4 = 99$ .

Choice A would be correct if there were only four blocks of length  $w$  in the tape diagram: students making this choice may have miscounted, since the four extra flowers fit with the block of length 4 at the end. Students failing to select B or C may be interpreting those situations in ways that can be described using the equation  $5(w + 4) = 99$ , rather than  $5w + 4 = 99$ . Likewise, students selecting D or E may incorrectly believe that those situations are of a type that can be represented using the equation  $5w + 4 = 99$ .

### Statement

Select **all** the situations that can be represented by the tape diagram.



- A. Clare buys 4 bouquets, each with the same number of flowers. The florist puts an extra flower in each bouquet before she leaves. She leaves with a total of 99 flowers.
- B. Andre babysat 5 times this past month and earned the same amount each time. To thank him, the family gave him an extra \$4 at the end of the month. Andre earned \$99 from babysitting.
- C. A family of 5 drove to a concert. They paid \$4 for parking, and all of their tickets were the same price. They paid \$99 in total.
- D. 5 bags of marbles each contain 4 large marbles and the same number of small marbles. Altogether, the bags contain 99 marbles.
- E. Han is baking five batches of muffins. Each batch needs the same amount of sugar in the muffins, and each batch needs four extra teaspoons of sugar for the topping. Han uses 99 total teaspoons of sugar.

### Solution

["B", "C"]

## Aligned Standards

7.EE.B.4

### Problem 3

Students selecting B may be confused specifically about how to represent the statement "Diego does twice as many push-ups as Noah." Students selecting A or C have made a mistake reading the

problem: we already know Diego does 40 jumping jacks, and we do not know whether Noah does any push-ups at all.

## Statement

At practice, Diego does twice as many push-ups as Noah, and also 40 jumping jacks. He does 62 exercises in total. The equation  $2x + 40 = 62$  describes this situation. What does the variable  $x$  represent?

- A. The number of jumping jacks Diego does
- B. The number of push-ups Diego does
- C. The number of jumping jacks Noah does
- D. The number of push-ups Noah does

## Solution

D

## Aligned Standards

7.EE.B.4

## Problem 4

Some students may struggle with the form of the equation in part a: after subtracting 25, is it  $3x$  or  $-3x$  that remains? The most likely error in part b is forgetting to properly distribute, though some students may multiply each side by 3 instead to get  $x - 10 = -12$ .

## Statement

Solve each equation.

1.  $25 - 3x = 40$
2.  $\frac{1}{3}(x - 10) = -4$

## Solution

1.  $x = -5$
2.  $x = -2$

## Aligned Standards

7.EE.B.4.a

## Problem 5

This problem points to a common error in solving equations of the form  $p(x + q) = r$ .

## Statement

Andre tried to solve the equation  $\frac{1}{4}(x + 12) = 2$ . What was his mistake?

$$\frac{1}{4}(x + 12) = 2$$

$$\frac{1}{4}x = -10$$

$$x = -40$$

## Solution

Sample response: he tried to subtract 12 from each side, but on the left side of the original equation, the 12 is being multiplied by  $\frac{1}{4}$ . He needed to either first distribute the  $\frac{1}{4}$  or first divide each side by  $\frac{1}{4}$ .

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: Andre subtracted 12 from each side, but that's wrong because the 12 is in the parentheses. He should have done  $12 \cdot \frac{1}{4}$  and then subtracted.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: the error is correctly identified, but the explanation is either incorrect or vague; the problem step is identified by substituting  $x = -40$  at each step, but no algebra mistake is identified; work involves solving the equation correctly but does not identify Andre's error.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: a different step is incorrectly identified as the problem step; work points to a different "error" in the problem step.

## Aligned Standards

7.EE.B.4.a

### Problem 6

Students need to distinguish a situation leading to an equation of the form  $p(x + q) = r$  from a situation leading to an equation of the form  $px + q = r$ . The instructions encourage using a tape diagram, but the diagram is not required.

## Statement

A food pantry is making packages. Each package weighs 64 pounds.

Here are two situations. For each situation, write an equation to represent the situation. If you get stuck, consider drawing a diagram.

1. Each package contains 4 boxes. Each box contains a 7-pound bag of beans and a bag of rice. The bags of rice are all identical.
2. Each package contains 4 identical bags of rice and a 7-pound bag of beans.

## Solution

Sample response:

1.  $4(x + 7) = 64$  (diagram shows 4 equal parts of  $x + 7$  with a total of 64)
2.  $4x + 7 = 64$  (diagram shows 4 equal boxes labeled  $x$  and one box labeled 7, with a total of 64)

Minimal Tier 1 response:

- Work is complete and correct.
- Sample:
  1.  $4(x + 7) = 64$  (diagram shows 4 equal parts of  $x + 7$  with a total of 64)
  2.  $4x + 7 = 64$  (diagram shows 4 equal boxes labeled  $x$  and one box labeled 7, with a total of 64)

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: one of the equations has been written correctly with errors in the other, diagram for one situation is drawn correctly but the other has errors.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: both equations have been written incorrectly or both diagrams have been drawn incorrectly, both responses are flawed in some way.

## Aligned Standards

7.EE.B.4.a

### Problem 7

The expectation is for students to fill in the table using numeric evaluation, but some students may write the expression  $4.8x + 350$  right away and use it. Similarly, some students will solve the

equation  $4.8x + 350 = 1,000$  in the last part, while others will backtrack from the given information.

Watch for students answering 135.6 or 136 instead of 135. These students may not be taking the time to contextualize (MP2), failing to take the mathematical solution to the equation and apply it to the context.

## Statement

For a school fundraiser, Elena will make T-shirts. She will order T-shirts and print graphics on them. Elena must spend \$349 on a printing machine and \$4.80 per shirt for the blank shirts, ink, and other supplies.

1. Complete the table giving the total cost Elena will spend to make each specific number of T-shirts.

number of shirts	cost in dollars
20	
40	
60	

2. Write an expression for the cost of making  $n$  T-shirts.
3. What is the maximum number of T-shirts Elena can make with a budget of \$1,000?

## Solution

1.

number of shirts	cost in dollars
20	445
40	541
60	637

2.  $4.8n + 349$  (or equivalent)
3. 135 T-shirts. Explanations vary. Sample explanation: Since the printing machine costs \$349, Elena will have \$651 left to spend on the shirts. Each shirt costs \$4.80 to produce. Since  $\frac{651}{4.8}$  is about 135.6, Elena can make a maximum of 135 T-shirts. She can't make 136.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.

- Sample:

1. See table.

2.  $4.8n + 349$

3. The equation is  $4.8n + 349 = 1000$ .  $4.8n = 651$ , so  $n = 135.625$ . She can make 135 shirts.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: one incorrect entry in the table; work for part c involves good reasoning but contains a calculation error; work for part c involves a correctly written equation but contains an arithmetic error; response of 135.6 or 136 to part c.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: more than one incorrect entry in the table; an incorrect equation in part b that is more than a transcription error; work for part c involves a correctly written equation, but the work to solve that equation contains an algebraic error.
- Acceptable errors: work for parts b and c is based on incorrect table entries or on a misunderstanding of the situation that nonetheless leads to an equation of the form  $px + q = r$  or  $p(x + q) = r$ .

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: work does not involve a consistent relationship between number of shirts and cost (including part a); work involves a consistent relationship between these things but the relationship does not fit  $px + q = r$  or  $p(x + q) = r$ ; three or more error types under Tier 3 response.

## Aligned Standards

7.EE.B.4.a

# Assessment : Mid-Unit Assessment (B)

## Teacher Instructions

Use of a four-function or scientific calculator is encouraged but not required. Do not allow use of more advanced calculators that may include functions to solve equations directly. This assessment is designed to be administered after lesson 12.

## Student Instructions

You may use a four-function or scientific calculator, but not a graphing calculator.

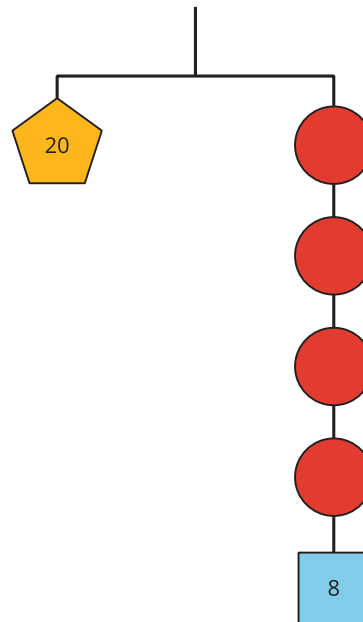
### Problem 1

Many students will solve this problem by writing the equation  $4x + 8 = 20$ , though reasoning purely about the weights on the hanger is also fine. Students selecting B likely added 8 to 20 instead of subtracting. Students selecting C probably imagined isolating all the weights labeled  $x$ , without accounting for the 8. Students selecting D may have combined  $4x$  and 8 to end of with  $12x = 20$ , then divided by 12.

#### Statement

This hanger is in balance. There are two labeled weights of 8 grams and 20 grams. The four circles each have the same weight.

What is the weight of each circle, in grams?



- A. 3
- B. 7
- C. 5
- D.  $\frac{20}{12}$

## Solution

A

## Aligned Standards

7.EE.B.4

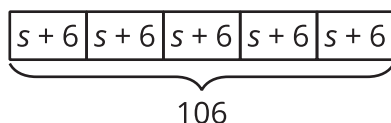
## Problem 2

The tape diagram represents the equation  $5(s + 6) = 106$ .

Students failing to select C or D may be interpreting those situations in ways that can be described using the equation  $5s + 6 = 106$ , rather than  $5(s + 6) = 106$ . Situations A and B describe 5 groups of  $n$  with a constant of 6 which is not equivalent to  $5s + 30$ .

## Statement

Select **all** the situations that can be represented by the tape diagram.



- A. Noah walked a family's dog 5 times this past month and earned the same amount each time. To thank him, the family gave him an extra \$6 at the end of the month. Noah earned \$106 from dog walking.
- B. A family of 5 drove to a soccer game. They paid \$6 for parking, and all of their tickets were the same price. They paid \$106 in total.
- C. 5 bags of coins each contain 6 nickels and the same number of pennies. Altogether, the bags contain 106 coins.
- D. Kiran is baking 5 batches of muffins. Each batch needs the same amount of sugar in the muffins, and each batch needs six extra teaspoons of sugar for the topping. Kiran uses 106 total teaspoons of sugar.
- E. Priya buys 5 cases of water, each with the same number of bottles. The store clerk gives her an extra 6 bottles before she leaves. She leaves with a total of 106 bottles.

## Solution

["C", "D"]

## Aligned Standards

7.EE.B.4

## Problem 3

## Statement

Over the weekend, Jada made three times as much money babysitting than Han did mowing lawns. Jada made an additional \$27 selling lemonade. From babysitting and selling lemonade, she has a total of \$96. If the equation  $3x + 27 = 96$  represents this situation, what does the variable  $x$  represent?

## Solution

The amount (in dollars) that Han made mowing lawns over the weekend.

## Aligned Standards

7.EE.B.4

### Problem 4

Some students may struggle with the form of the equation in part a: after subtracting 10, is it  $\frac{1}{4}$  or  $-\frac{1}{4}x$  that remains? The most likely error in part b is forgetting to properly distribute, though some students may multiply each side by  $\frac{1}{3}$  instead to get  $x + 8 = 7$ .

## Statement

Solve each equation.

1.  $10 - \frac{1}{4}x = 7$

2.  $3(x + 8) = 21$

## Solution

1.  $x = 12$

2.  $x = -1$

## Aligned Standards

7.EE.B.4.a

### Problem 5

This problem points to a common error in solving equations of the form  $p(x + q) = r$ .

## Statement

Diego tried to solve the equation  $\frac{1}{3}(x + 15) = 3$ . What was his mistake?

$$\frac{1}{3}(x + 15) = 3$$

$$\frac{1}{3}x = -12$$

$$x = -4$$

## Solution

Sample response: He tried to subtract 15 from each side, but on the left side of the original equation, the 15 is being multiplied by  $\frac{1}{3}$ . He needed to either first distribute the  $\frac{1}{3}$  or first divide each side by  $\frac{1}{3}$ .

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: Diego subtracted 15 from each side, but that's wrong because the 15 is in the parentheses. He should have done  $\frac{1}{3} \cdot x$  and  $\frac{1}{3} \cdot 15$  before subtracting.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: the error is correctly identified, but the explanation is either incorrect or vague; the problem step is identified by substituting  $x = -6$  at each step, but no algebra mistake is identified; work involves solving the equation correctly but does not identify Diego's error.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: a different step is incorrectly identified as the problem step; work points to a different "error" in the problem step.

## Aligned Standards

7.EE.B.4.a

### Problem 6

Students need to distinguish a situation leading to an equation of the form  $p(x + q) = r$  from a situation leading to an equation of the form  $px + q = r$ . The instructions encourage using a tape diagram, but the diagram is not required.

#### Statement

A church is packing Thanksgiving baskets. Each basket weighs 30 pounds.

Here are two situations. For each situation, write an equation to represent the situation. If you get stuck, consider drawing a diagram.

1. Each basket contains 3 identical bags of stuffing and a 6-pound bag of rice.
2. Each basket contains 3 boxes. Each box contains a 6-pound bag of rice and a bag of stuffing. The bags of stuffing are all identical.

## Solution

1.  $3x + 6 = 30$  (diagram shows 3 equal boxes labeled  $x$  and one box labeled 6, with a total of 30).
2.  $3(x + 6) = 30$  (diagram shows 3 equal parts of  $x + 6$  with a total of 30).

Minimal Tier 1 response:

- Work is complete and correct.
- Sample:
- $3x + 6 = 30$  (diagram shows 3 equal boxes labeled  $x$  and one box labeled 6, with a total of 30).
- $3(x + 6) = 30$  (diagram shows 3 equal parts of  $x + 6$  with a total of 30).

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: one of the equations has been written correctly with errors in the other, diagram for one situation is drawn correctly but the other has errors.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: both equations have been written incorrectly or both diagrams have been drawn incorrectly, both responses are flawed in some way.

## Aligned Standards

7.EE.B.4.a

### Problem 7

The expectation is for students to complete the table using numeric evaluation, but some students may write the expression  $5.3x + 250$  right away and use it. Similarly, some students will solve the equation  $5.3x + 250 = 1,500$  in the last part, while others will backtrack from the given information. Watch for students answering 235.8 or 236 instead of 235. These students may not be taking the time to contextualize (MP2), failing to take the mathematical solution to the equation and apply it to the context.

### Statement

For a school fundraiser, Mai will make school spirit bracelets. She will order bead wiring and alphabet beads to create the school name on the bracelets. Mai must spend \$250 on wire and \$5.30 per bracelet for beads.

1. Complete the table giving the total cost Mai will spend to make each specific number of bracelets.

number of bracelets	cost in dollars
20	
40	
60	

2. Write an expression for the cost of making  $n$  bracelets.
3. What is the maximum number of bracelets Mai can make with a budget of \$1,500?

## Solution

1.

number of bracelets	cost in dollars
20	\$356
40	\$462
60	\$568

2.  $5.3n + 250$  (or equivalent)
3. 235 bracelets. Explanations vary. Sample explanation: Since the wire costs \$250, Mai will have \$1,250 left to spend on the bracelets. Each bracelet uses \$5.30 of beads. Since  $\frac{1250}{5.3}$  is about 235.8, Mai can make a maximum of 235 bracelets. She can't make 236.
- Minimal Tier 1 response:
    - Work is complete and correct, with complete explanation or justification.
    - Sample:
    - See table.
    - $5.3n + 250$
    - The equation is  $5.3n + 250 = 1500$ .  $5.3n = 1250$ , so  $n = 235.849$ . She can make 235 bracelets.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.

- Sample errors: one incorrect entry in the table; work for part c involves good reasoning but contains a calculation error; work for part c involves a correctly written equation but contains an arithmetic error; response of 235.8 or 236 to part c.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: more than one incorrect entry in the table; an incorrect equation in part b that is more than a transcription error; work for part c involves a correctly written equation, but the work to solve that equation contains an algebraic error.
- Acceptable errors: work for parts b and c is based on incorrect table entries or on a misunderstanding of the situation that nonetheless leads to an equation of the form  $px + q = r$  or  $p(x + q) = r$ .

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: work does not involve a consistent relationship between number of bracelets and cost (including part a); work involves a consistent relationship between these things but the relationship does not fit  $px + q = r$  or  $p(x + q) = r$ ; three or more error types under Tier 3 response.

## Aligned Standards

7.EE.B.4.a

# Assessment : End-of-Unit Assessment (A)

## Teacher Instructions

Use of a four-function or scientific calculator is encouraged but not required. Do not allow use of more advanced calculators that may include functions to solve equations directly.

## Student Instructions

You may use a four-function or scientific calculator, but not a graphing calculator.

### Problem 1

Students selecting C or D may be thinking that, because each song costs \$1.29, the amount should be subtracted. However, the \$1.29 per song is added to the amount being spent on the gift card. Students selecting A or C have made a direction error in the inequality and may need a reminder about how to check the correct direction of a linear inequality.

#### Statement

Lin got a \$50 gift card to an online music store. She uses the gift card to buy an album for \$9.99. She also wants to use the gift card to buy some songs. Each song costs \$1.29.

Which of these inequalities describes this situation, where  $n$  is the number of songs Lin wants to buy?

- A.  $9.99 + 1.29n \geq 50$
- B.  $9.99 + 1.29n \leq 50$
- C.  $9.99 - 1.29n \geq 50$
- D.  $9.99 - 1.29n \leq 50$

#### Solution

B

#### Aligned Standards

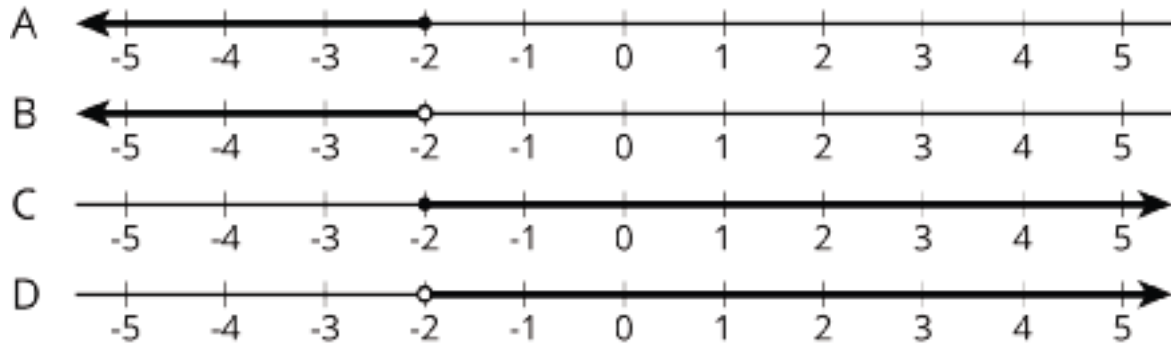
7.EE.B.4.b

### Problem 2

Students selecting A or B instead of D should be reminded of the recent work on checking numbers to determine the solution of an inequality. For example,  $x = 0$  makes the inequality true, so it should be part of the graph. Students selecting A or C need to re-learn the meaning of open and closed circles for inequalities.

## Statement

Which number line shows all the values of  $x$  that make the inequality  $-3x + 1 < 7$  true?



- A. A
- B. B
- C. C
- D. D

## Solution

D

## Aligned Standards

7.EE.B.4.b

## Problem 3

Students selecting A have not distributed the negative sign. Students failing to select B might be thrown by the different ordering of the terms, or might not be distributing the negative sign correctly. Students failing to select C could have made a variety of algebra mistakes on their way to simplifying the expression. Students selecting D have also fallen victim to incorrectly distributing the negative sign. Choice E shows the process for distributing the negative sign explicitly. A student failing to select E may need some extra coaching with the distributive property.

## Statement

Select **all** expressions that are equivalent to  $6x + 1 - (3x - 1)$ .

- A.  $6x + 1 - 3x - 1$
- B.  $6x + -3x + 1 + 1$
- C.  $3x + 2$
- D.  $6x - 3x + 1 - 1$
- E.  $6x + 1 + -3x - -1$

## Solution

["B", "C", "E"]

### Aligned Standards

7.EE.A.1

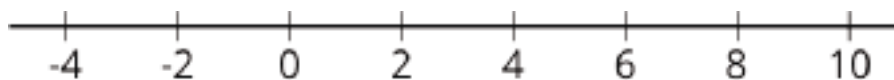
### Problem 4

Students are not required to solve the inequality in part a, but many will anyway, perhaps by first finding  $t = \frac{9}{2}$  as the time when the temperature was exactly -4 degrees Celsius.

### Statement

At midnight, the temperature in a city was 5 degrees Celsius. The temperature was dropping at a steady rate of 2 degrees Celsius per hour.

1. Write an inequality that represents  $t$ , the number of hours past midnight, when the temperature was colder than -4 degrees Celsius. Explain or show your reasoning.
2. On the number line, show all the values of  $t$  that make your inequality true.



### Solution

1.  $5 - 2t < -4$  or  $t > \frac{9}{2}$  (or equivalent). Explanations vary. Sample explanations:
  - $5 - 2t$  shows the temperature starting at 5 degrees Celsius and decreasing by 2 degrees every hour. Because we want this quantity to be less than -4 degrees Celsius, write  $5 - 2t < -4$ .
  - The temperature will reach -4 degrees Celsius after  $\frac{9}{2}$  hours. Since the temperature needs to be colder,  $t$  must be larger than  $\frac{9}{2}$ .
2. The graph shows an open circle at  $4\frac{1}{2}$  with shading to the right.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample:
  1.  $5 - 2t < -4$ . The temperature starts at 5 degrees, then it goes down 2 degrees every hour. The temperature has to be less than -4 degrees.
  2. The graph shows an open circle at  $4\frac{1}{2}$  with shading to the right.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Acceptable errors: a graph in part b based on an incorrect inequality in part a.
- Sample errors: no explanation for a correct inequality; inequality is close but has one mistake, like  $5 + 2t < -4$ ; graph does not use “open circle” notation; direction of inequality is incorrect in parts a, b, or both.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: inequality in part a is not close to correct; graph in part b is omitted or simply incorrect.

## Aligned Standards

7.EE.B.4.b

### Problem 5

Look for sign errors in part a. There are multiple ways to factor the expression in part b.

#### Statement

1. Expand to write an equivalent expression:

$$-\frac{1}{4}(-8x + 12y)$$

2. Factor to write an equivalent expression:

$$36a - 16$$

#### Solution

1.  $2x - 3y$
2.  $4(9a - 4)$  or  $2(18a - 8)$

## Aligned Standards

7.EE.A.1

### Problem 6

Students who are very comfortable with algebra can jump straight to the answer in part b without showing reasoning. However, an incorrect answer with no reasoning earns an automatic Tier 3 rating.

#### Statement

Tyler is simplifying the expression  $6 - 2x + 5 + 4x$ . Here is his work:

$$6 - 2x + 5 + 4x$$

$$(6 - 2)x + (5 + 4)x$$

$$4x + 9x$$

$$13x$$

1. Tyler's work is incorrect. Explain the error he made.
2. Write an expression equivalent to  $6 - 2x + 5 + 4x$  that only has two terms.

## Solution

1. Tyler's second step,  $(6 - 2)x + (5 + 4)x$ , is not equivalent to the original. Explanations vary.

Sample explanations:

- $(6 - 2)x$  is not equivalent to  $6 - 2x$ . I can show this because when  $x$  is 0, one expression equals 0 but the other equals 6.
- Tyler did not use the distributive property correctly.  $(6 - 2)x = 6x - 2x$ , not  $6 - 2x$ .

2. Strategies vary. Sample reasoning:

$$6 - 2x + 5 + 4x$$

$$6 + -2x + 5 + 4x$$

$$-2x + 4x + 6 + 5$$

$$(-2 + 4)x + 11$$

$$2x + 11$$

Minimal Tier 1 response:

- Work is complete and correct.
- Sample:
  1.  $6 - 2x$  does not equal  $(6 - 2)x$ .
  2.  $2x + 11$

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: Tyler's error is correctly identified, but the explanation is either incorrect or vague; the problem step is identified by substituting a value like  $x = 0$  at each step, but no algebra mistake is identified; an algebra mistake in part b with otherwise correct work shown.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: failure to identify the error in Tyler's reasoning; incorrect answer to part b with no work shown.

## Aligned Standards

7.EE.A.1

### Problem 7

The first two parts test students' understanding of the situation numerically and help them discover or recognize the meaning of the expression  $14 - 0.6t$ .

There are multiple other possible equations and inequalities, including  $0.6t = 12.5$  and  $14 - 0.6t \leq 1.5$ .

### Statement

Diego's family car holds 14 gallons of fuel. Each day the car uses 0.6 gallons of fuel. A warning light comes on when the remaining fuel is 1.5 gallons or less.

1. Starting from a full tank, can Diego's family drive the car for 14 days without the warning light coming on? Explain or show your reasoning.
2. Starting from a full tank, can Diego's family drive the car for 25 days without the warning light coming on? Explain or show your reasoning.
3. Diego says the expression  $14 - 0.6t$  helps him understand this situation. In this situation, what does this expression represent, and what does the variable  $t$  stand for?
4. Write and solve an equation to determine the number of days Diego's father can drive the car without the warning light coming on.
5. Write and solve an inequality that represents this situation. Explain clearly what the solution to the inequality means in the context of this situation.

### Solution

1. Yes, in 14 days the car uses 8.4 gallons of gas, since  $14 \cdot (0.6) = 8.4$ . 5.6 gallons remain, so the warning light is off.
2. No, in 25 days the car uses 15 gallons of gas, since  $25 \cdot (0.6) = 15$ . This is not possible, since the car only holds 14 gallons of gas. The warning light would come on, and the tank would run out of gas before then.
3.  $14 - 0.6t$  is the amount of gas in the tank, in gallons, after  $t$  days of driving.

4. Answers vary. Sample response: The equation is  $14 - 0.6t = 1.5$ . Subtract 14 from each side to get  $-0.6t = -12.5$ . Divide each side by  $-0.6$  to get  $t \approx 20.83$ . The car can drive for 20 days, and the light will come on near the end of the 21st day.
5. Answers vary. Sample response: The inequality  $14 - 0.6t > 1.5$  represents the times when the warning light is off. The solution to this inequality is  $t < 20.83$ , so the warning light is off for all times until near the end of the 21st day.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Acceptable errors: saying that Diego's father can drive for 21 days if it is specified that the light will come on during the 21st day.
- Sample:
  1. Yes, because  $14 - 14 \cdot 0.6 = 5.6$ .
  2. No, because  $14 - 25 \cdot 0.6 = -1$ .
  3. The expression is how much gas is left.  $t$  is the number of days.
  4.  $14 - 0.6t = 1.5$ ,  $-0.6t = -12.5$ ,  $t \approx 20.83$ . He can drive for 20 days.
  5.  $14 - 0.6t > 1.5$ .  $t < 20$  because 20.83 is when the light comes on and before that the light would be off. This means that Diego's father can drive for 20 days before the warning light comes on.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: any arithmetic errors with work shown; poor explanations (especially in part c) with otherwise correct work; assertion that since  $t \approx 20.83$  is a solution to the equation, Diego's father can drive for 21 full days; failure to justify the direction of inequality in the solution to part e (where justification can involve algebra, testing points, or appealing to the context).

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: a misinterpretation of the situation leads to incorrect answers for a and b; equation in part d is incorrect; reversed inequality sign in either the original inequality or the solution to part e; omission of real-world interpretation in parts d and e.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: little progress on most problem parts; three or more error types under Tier 3 response.

## **Aligned Standards**

7.EE.B.3, 7.EE.B.4

# Assessment : End-of-Unit Assessment (B)

## Teacher Instructions

Use of a four-function or scientific calculator is encouraged but not required. Do not allow use of more advanced calculators that may include functions to solve equations directly.

## Student Instructions

You may use a four-function or scientific calculator, but not a graphing calculator.

### Problem 1

#### Statement

Tyler has run 15 miles this month. For the rest of the month, he plans to run the same number of miles each day. There are 12 days left in the month. Before the end of the month, Tyler needs to run at least 35 miles to meet his goal.

Which of these inequalities describes this situation, where  $m$  is the number of miles Tyler runs each day to meet his goal?

- A.  $12m - 15 \geq 35$
- B.  $12m - 15 \leq 35$
- C.  $35 \geq 15 + 12m$
- D.  $35 \leq 15 + 12m$

#### Solution

D

#### Aligned Standards

7.EE.B.4.b

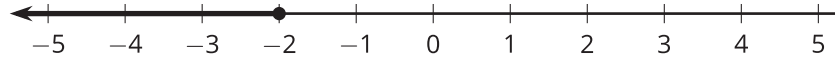
### Problem 2

Students selecting C or D instead of A should be reminded of the recent work on checking numbers to determine the solution of an inequality. For example,  $y = 0$  makes the inequality false, so it should not be part of the graph. Students selecting B or D need to re-learn the meaning of open and closed circles for inequalities.

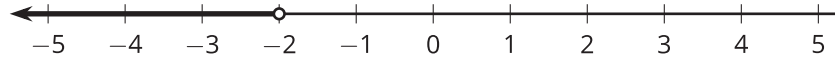
#### Statement

Which number line shows all the values of  $y$  that make the inequality  $-6y - 2 \geq 10$  true?

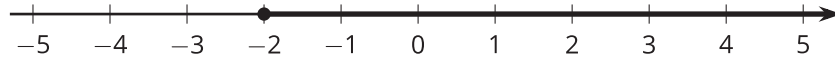
A. **A**



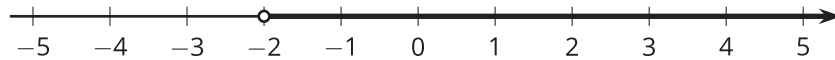
B. **B**



C. **c**



D. **D**



## Solution

A

### Aligned Standards

7.EE.B.4.b

### Problem 3

Students selecting B have not distributed the negative sign to both terms within the parentheses. Students failing to select E might be thrown by the different ordering of the terms, or might not be distributing the negative sign correctly. Students failing to select A could have made a variety of algebra mistakes on their way to simplifying the expression. Choice D shows the process for distributing the negative sign explicitly. A student failing to select D may need some extra coaching with the distributive property.

## Statement

Select all expressions that are equivalent to  $-4(x + 2) - 2x + 4$ .

A.  $-6x - 4$

B.  $-4x + 2 - 2x + 4$

C.  $-10x$

D.  $-4x + -8 - 2x + 4$

E.  $-4x - 2x - 8 + 4$

## Solution

["A", "D", "E"]

### Aligned Standards

7.EE.A.1

#### Problem 4

Students will need to denote the initial elevation by showing -50. Students are not required to solve the inequality, but many will anyway, perhaps by first finding  $t > \frac{11}{3}$  as the number of minutes it would take to be higher than 5 feet above sea level.

#### Statement

At the start of a hike, a hiker was at an elevation of -50 feet (where 0 represents sea level). The hiker climbs at a rate of 15 feet per minute.

1. Write an inequality that represents  $t$ , the number of minutes after the start of the hike, when the hiker's elevation was higher than 5 feet above sea level. Explain or show your reasoning.
2. On the number line, show all the values of  $t$  that make your inequality true.



#### Solution

1.  $-50 + 15t > 5$  or  $t > \frac{11}{3}$  (or equivalent). Explanations vary. Sample explanations:
  - $-50 + 15t$  shows the elevation of the hiker after  $t$  minutes of hiking. Because we want this quantity to be greater than 5, write  $-50 + 15t > 5$ .
  - After  $3\frac{2}{3}$  minutes of hiking, the hiker will be at an elevation higher than 5 feet above sea level.
2. The graph shows an open circle at approximately  $3\frac{2}{3}$  with shading to the right.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample:
  - a.  $-50 + 15t > 5$ . The hiker begins at an elevation of -50 and climbs 15 feet per  $t$  minutes to an elevation greater than 5 feet above sea level.
  - b. The graph shows an open circle at approximately  $3\frac{2}{3}$  with shading to the right.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Acceptable errors: a correct graph based on an incorrect inequality.
- Sample errors: no explanation for a correct inequality; inequality is close but has one mistake, like  $-50 - 15t > 5$ ; graph does not use “open circle” notation; direction of inequality is incorrect in either or both parts. Gives an answer of 3 minutes.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: inequality is not close to correct; graph is omitted or simply incorrect.

## Aligned Standards

7.EE.B.4.b

### Problem 5

Look for sign errors.

#### Statement

1. Factor to write an equivalent expression:

$$-9x + 15y$$

2. Expand to write an equivalent expression:

$$-5(7m - 2)$$

#### Solution

1.  $3(-3x + 5y)$  or  $-3(3x - 5y)$
2.  $-35m + 10$

## Aligned Standards

7.EE.A.1

### Problem 6

Students who are very comfortable with algebra can jump straight to the equivalent expression that has only two terms. However, an incorrect answer with no reasoning earns an automatic Tier 3 rating.

#### Statement

Priya is simplifying the expression  $-2(4 - 2x) - 3 - 2x$ . Here is her work:

$$-2(4 - 2x) - 3 - 2x$$

$$-8 - 2x - 3 - 2x$$

$$-11 - 4x$$

1. Priya's work is incorrect. Explain the error she made.
2. Write an expression equivalent to  $-4(x + 3) - 12x + 5$  that only has two terms.

## Solution

1. Her second step,  $-8 - 2x - 3 - 2x$ , is not equivalent to the original. Explanations vary. Sample explanations:

- $-8 - 2x$  is not equivalent to  $-2(4 - 2x)$ . I can show this because when  $x$  is 1, one expression equals -1 and the other equals -4.
- Priya didn't use the distributive property correctly.  $-2(4 - 2x) = -8 + 4x$ , not  $-8 - 2x$ .

2. Strategies vary. Sample reasoning:

$$-4(x + 3) - 12x + 5$$

$$-4x - 12 - 12x + 5$$

$$-16x - 7$$

Minimal Tier 1 response:

- Work is complete and correct.
- Sample:
  - a. She did not properly distribute -2.
  - b. Distribute -4 to  $(x + 3)$ , then combine the like terms.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: Priya's error is correctly identified, but the explanation is either incorrect or vague; the problem step is identified by substituting a value like  $x = 0$  at each step, but no algebra mistake is identified; an algebra mistake when writing the equivalent expression with otherwise correct work shown.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: failure to identify the error in Priya's reasoning; incorrect answer when writing the equivalent expression.

## Aligned Standards

7.EE.A.1

### Problem 7

The first two parts test students' understanding of the situation numerically and help them discover or recognize the meaning of the expression  $200 + 22.5h$ . There are multiple other possible equations and inequalities, including  $22.5h = 135$  and  $300 \geq 22.5h + 200$ .

#### Statement

A community center rents their hall for special events. They charge a fixed fee of \$200 plus an hourly fee of \$22.50. Lin has \$300 to spend on renting the hall for a fundraiser.

1. Using only the money she has, can Lin pay for a 6 hour event? Explain or show your reasoning.
2. Using only the money she has, can Lin pay for a 3 hour event? Explain or show your reasoning.
3. If  $h$  represents the number of hours, what does  $22.50h + 200$  represent?
4. Write and solve an equation to determine the number of hours Lin can rent the community center.
5. Write and solve an inequality that represents this situation. Explain what the solution to the inequality means in this situation.

#### Solution

1. No, in 6 hours the total cost of the rental would be \$335 since  $22.50(6) + 200 = 335$ . Lin would not have enough money to pay to rental the community center that long.
2. Yes, in 3 hours the total cost of the rental would be \$267.50 which is less than the amount Lin has to spend.
3.  $22.50h + 200$  represents the total amount Lin would pay to rent the community center for  $h$  hours.
4. Answers vary. Sample response: The equation is  $22.50h + 200 = 300$ . Subtract 200 from each side to get  $22.50h = 100$ . Divide each side by 22.50 to get 4.4. Lin can rent the community center for 4 hours before she runs out of money.
5. Answers vary. Sample response: The inequality  $22.50h + 200 \leq 300$  represents the amount of money Lin would pay to rent the community center for a given number of hours. The solution to this inequality is  $h \leq 4$ , so Lin can rent the community center for no more than 4 hours.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.

- Sample:
  - a. No, because  $22.50 \cdot 6 + 200 = 335$ .
  - b. Yes, because  $22.50 \cdot 3 + 200 = 267.50$ .
  - c. The expression represents the amount of money Lin would spend to rent the community center.
  - d.  $22.50h + 200 = 300$ .  $h = 4.4$ . Lin can rent the community center for 4 hours (assuming they only rent increments of whole hours).
  - e.  $22.50h + 200 \leq 300$ .  $h \leq 4$ . Lin can rent the community center for no more than 4 hours (assuming they only rent increments of whole hours).

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: any arithmetic errors with work shown; poor explanations (especially the meaning of  $22.50h + 200$ ) with otherwise correct work.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: a misinterpretation of the situation leads to incorrect answers to the first two parts; equation is incorrect; reversed inequality sign in either the original inequality or the solution; omission of real-world interpretation.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: little progress on most problem parts; three or more error types under Tier 3 response.

## Aligned Standards

7.EE.B.3, 7.EE.B.4

**CKMath™**  
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Lesson  
Cool Downs

# Lesson 1: Relationships between Quantities

## Cool Down: Movie Theater Popcorn, Revisited

A movie theater sells popcorn in bags of different sizes. The table shows the volume of popcorn and the price of the bag.

volume of popcorn (ounces)	price of bag (\$)
10	6
20	8
35	11
48	13.6

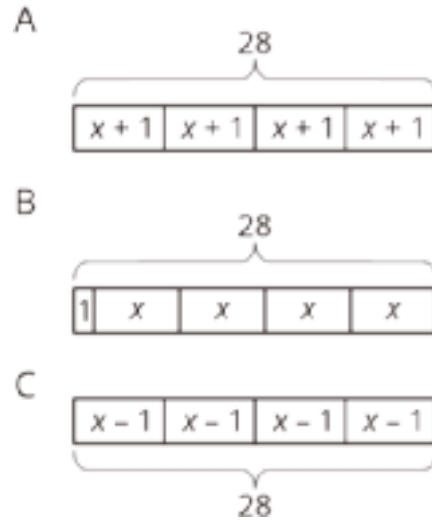
If the theater wanted to offer a 60-ounce bag of popcorn, what would be a good price? Explain your reasoning.

# Lesson 2: Reasoning about Contexts with Tape Diagrams

## Cool Down: Red and Yellow Apples

Here is a story: Lin bought 4 bags of apples. Each bag had the same number of apples. After eating 1 apple from each bag, she had 28 apples left.

1. Which diagram best represents the story?  
Explain why the diagram represents it.



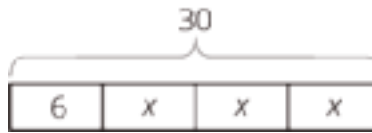
2. What part of the story does  $x$  represent?

3. Describe how you would find the unknown amount in the story.

# Lesson 3: Reasoning about Equations with Tape Diagrams

## Cool Down: Three of These Equations Belong Together

Here is a diagram.



1. Circle the equation that the diagram does *not* match.

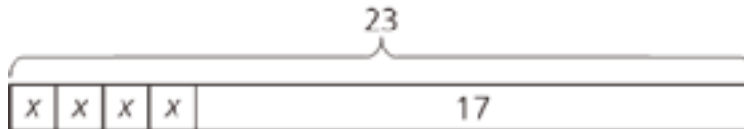
- $6 + 3x = 30$
- $3(x + 6) = 30$
- $3x = 30 - 6$
- $30 = 3x + 6$

2. Draw a diagram that matches the equation you circled.

# Lesson 4: Reasoning about Equations and Tape Diagrams (Part 1)

## Cool Down: Finding Solutions

Here is a diagram and its corresponding equation. Find the solution to the equation and explain your reasoning.

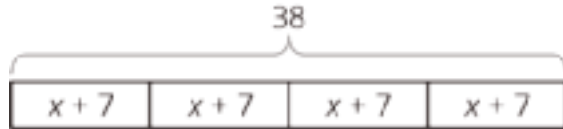


$$4x + 17 = 23$$

# Lesson 5: Reasoning about Equations and Tape Diagrams (Part 2)

## Cool Down: More Finding Solutions

Here is a diagram and its corresponding equation. Find the solution to the equation and explain your reasoning.



$$4(x + 7) = 38$$



# Lesson 7: Reasoning about Solving Equations (Part 1)

## Cool Down: Solve the Equation

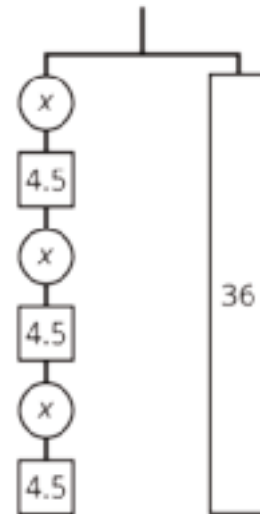
Solve the equation. If you get stuck, try using a diagram.

$$5x + \frac{1}{4} = \frac{61}{4}$$

# Lesson 8: Reasoning about Solving Equations (Part 2)

## Cool Down: Solve Another Equation

Solve the equation  $3(x + 4.5) = 36$ . If you get stuck, use the diagram.



# Lesson 9: Dealing with Negative Numbers

## Cool Down: Solve Two More Equations

Solve each equation. Show your work, or explain your reasoning.

1.  $-3x - 5 = 16$

2.  $-4(y - 2) = 12$

# Lesson 10: Different Options for Solving One Equation

## Cool Down: Solve Two Equations

Solve each equation. Show or explain your method.

1.  $8.88 = 4.44(x - 7)$

2.  $5\left(y + \frac{2}{5}\right) = -13$

# Lesson 11: Using Equations to Solve Problems

## Cool Down: The Basketball Game

Diego scored 9 points less than Andre in the basketball game. Noah scored twice as many points as Diego. If Noah scored 10 points, how many points did Andre score?

# Lesson 12: Solving Problems about Percent Increase or Decrease

## Cool Down: Timing the Relay Race

The track team is trying to reduce their time for a relay race. First they reduce their time by 2.1 minutes. Then they are able to reduce that time by  $\frac{1}{10}$ . If their final time is 3.96 minutes, what was their beginning time? Show or explain your reasoning.



# Lesson 14: Finding Solutions to Inequalities in Context

## Cool Down: Colder and colder

It is currently 0 degrees outside, and the temperature is dropping 4 degrees every hour. The temperature after  $h$  hours is  $-4h$ .

1. Explain what the equation  $-4h = -14$  represents.
2. What value of  $h$  makes the equation true?
3. Explain what the inequality  $-4h \leq -14$  represents.
4. What values of  $h$  make the inequality true?

# Lesson 15: Efficiently Solving Inequalities

## Cool Down: Testing for Solutions

For each inequality, decide whether the solution is represented by  $x < 2.5$  or  $x > 2.5$ .

1.  $-4x + 5 > -5$

2.  $-25 > -5(x + 2.5)$

# Lesson 16: Interpreting Inequalities

## Cool Down: Party Decorations

Andre is making paper cranes to decorate for a party. He plans to make one large paper crane for a centerpiece and several smaller paper cranes to put around the table. It takes Andre 10 minutes to make the centerpiece and 3 minutes to make each small crane. He will only have 30 minutes to make the paper cranes once he gets home.

1. Andre wrote the inequality  $3x + 10 \leq 30$  to plan his time. Describe what  $x$ ,  $3x$ , 10, and 30 represent in this inequality.

2. Solve Andre's inequality and explain what the solution means.



# Lesson 18: Subtraction in Equivalent Expressions

## Cool Down: Equivalent to $4 - x$

1. Select all the expressions that are equivalent to  $4 - x$ .

a.  $x - 4$

b.  $4 + -x$

c.  $-x + 4$

d.  $-4 + x$

e.  $4 + x$

2. Use the distributive property to write an expression that is equivalent to  $5(-2x - 3)$ . If you get stuck, use the boxes to help organize your work.

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# Lesson 19: Expanding and Factoring

## Cool Down: Equivalent Expressions

1. Expand to write an equivalent expression:  $-\frac{1}{2}(-2x + 4y)$

2. Factor to write an equivalent expression:  $26a - 10$

If you get stuck, use a diagram to organize your work.

# Lesson 20: Combining Like Terms (Part 1)

## Cool Down: Fewer Terms

Write each expression with fewer terms. Show your work or explain your reasoning.

1.  $10x - 2x$

2.  $10x - 3y + 2x$

# Lesson 21: Combining Like Terms (Part 2)

## Cool Down: How Many Are Equivalent?

Select **all** the expressions that are equivalent to  $16x - 12 - 24x + 4$ . Explain or show your reasoning.

1.  $4 + 16x - 12(1 + 2x)$

2.  $40x - 16$

3.  $16x - 24x - 4 + 12$

4.  $-8x - 8$

5.  $10(1.6x - 1.2 - 2.4x + 4)$

# Lesson 22: Combining Like Terms (Part 3)

## Cool Down: R's and T's

Match each expression in column A with an equivalent expression from column B. Show or explain your reasoning.

### Column A

1.  $12r + t - 4r + 6t$

2.  $(12r + 2t) - (4r + 10t)$

3.  $12(r + t) - 6(r + t) + 4(r + t) - 2(r + t)$

### Column B

•  $8(r - t)$

•  $8(r + t)$

•  $8r + 7t$

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address	title	students per copy	written on?	requires cutting?	card stock recommended?	color paper recommended?
Activity Grade7.6.6.2	Categories of Equations Card Sort	1	no	yes	no	yes
Activity Grade7.6.17.3	Giving Advice	1	no	yes	no	no

Card Sort: Categories of Equations

$$100 = 8(x + 9)$$

Card Sort: Categories of Equations

$$9(7 + x) = 100$$

Card Sort: Categories of Equations

$$8(9 + x) = 100$$

Card Sort: Categories of Equations

$$100 = (x + 7) \cdot 9$$

Card Sort: Categories of Equations

$$63 + 9x = 100$$

Card Sort: Categories of Equations

$$100 = 8x + 72$$

Card Sort: Categories of Equations

$$9x + 63 = 100$$

Card Sort: Categories of Equations

$$72 + 8x = 100$$

Card Sort: Categories of Equations

$$100 = 9x + 63$$

Card Sort: Categories of Equations

$$100 = (x + 9) \cdot 8$$

Giving Advice

**Problem Card 1**

Noah's apartment building has a washing machine that uses a card for payment. His family likes to keep a minimum balance on the card. How many loads of laundry can Noah's family do before needing to add money to the card?

Giving Advice

**Data Card 1**

- Each load of laundry costs \$1.65.
- Noah's family likes to make sure that the balance on the card is at least \$15, at which point they add money to the card.
- There is \$50 on the card right now.

Giving Advice

**Problem Card 2**

Elena is designing a rectangular picture frame with a lace border. What widths can Elena choose for the frame?

Giving Advice

**Data Card 2**

- In order to fit nicely on her desk, the frame needs to be 7 centimeters in length.
- She only has 65 centimeters of lace available to use for the border.

Giving Advice

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