Equation

$$x + 8 = -5$$
$$(x + 8) + -8 = (-5) + -8$$
$$x = -13$$

Rational Number Arithmetic

Teacher Guide

Fractions of a Degree

10 meters

5 meters

-5 meters

-10 meters

vertical position of animals

Rational Number Arithmetic

Table of Contents

$$x + 8 = -5$$
$$(x + 8) + -8 = (-5) + -8$$
$$x = -13$$

Rational Number Arithmetic
Teacher Guide
Core Knowledge Mathematics™

Rational Number Arithmetic

Unit Narrative

In grade 6, students learned that the rational numbers comprise positive and negative fractions. They plotted rational numbers on the number line and plotted pairs of rational numbers in the coordinate plane. In this unit, students extend the operations of addition, subtraction, multiplication, and division from fractions to all rational numbers, written as decimals or in the form $\frac{a}{b}$.

The unit begins by revisiting ideas familiar from grade 6: how signed numbers are used to represent quantities such as measurements of temperature and elevation, opposites (pairs of numbers on the number line that are the same distance from zero), and absolute value.

In the second section of the unit, students extend addition and subtraction from fractions to all rational numbers. They begin by considering how changes in temperature and elevation can be $0.8 > -1.3$ represented—first with tables and number line diagrams, then with addition and subtraction expressions and equations. Initially, physical contexts provide meanings for sums and differences that include negative numbers. Students work with numerical addition and subtraction expressions and equations, becoming more fluent in computing sums and differences of signed numbers. Using the meanings that they have developed for addition and subtraction of signed numbers, they write equivalent numerical addition and subtraction expressions, e.g., $-8 + -3$ and $-8 - 3$; and they write different equations that express the same relationship.

The third section of the unit focuses on multiplication and division. It begins with problems about position, direction, constant speed, and constant velocity in which students represent quantities with number line diagrams and tables of numerical expressions with signed numbers. This allows products of signed numbers to be interpreted in terms of position and direction, using the understanding that numbers that are additive inverses are located at the same distance but opposite sides of the starting point. These examples illustrate how multiplication of how multiplication of fractions extends to rational numbers. The third lesson of this section focuses on computing products of signed numbers and is optional. In the fourth lesson, students use the relationship between multiplication and division to understand how division extends to rational numbers. In the process of solving problems set in contexts (MP4), they write and solve multiplication and division equations. $2 < x \quad 2 = x.$

In the fourth section of the unit, students work with expressions that use the four operations on rational numbers, making use of structure (MP7), e.g., to see without calculating that the product of two factors is positive because the values of the factors are both negative. They extend their use of the "next to" notation (which they used in expressions such as $5x$ and $6(3 + 2)$ in grade 6) to include negative numbers and products of numbers, e.g., writing $-5x$ and $(-5)(-10)$ rather than $(-5) \cdot (x)$ and $(-5) \cdot (-10)$. They extend their use of the fraction bar to include variables as well as numbers, writing $-8.5 \div x$ as well as $\frac{-8.5}{x}$. They solve problems that involve interpreting negative numbers in $x < 8$ context, for instance, when a negative number represents a rate at which water is flowing (MP2). $t > 10$

In the fifth section of the unit, students begin working with linear equations in one variable that have rational number coefficients. The focus of this section is representing situations with equations (MP4) and what it means for a number to be a solution for an equation, rather than methods for solving equations. Such methods are the focus of a later unit.

The last section of the unit is a lesson in which students use rational numbers in the context of stock-market situations, finding values of quantities such as the value of a portfolio or changes due to interest and depreciation (MP4).

Note. In these materials, an *expression* is built from numbers, variables, operation symbols (+, −, ·, ÷), parentheses, and exponents. (Exponents—in particular, negative exponents—are not a focus of this unit. Students work with integer exponents in grade 8 and non-integer exponents in high school.) An *equation* is a statement that two expressions are equal, thus always has an equal sign. *Signed numbers* include all rational numbers, written as decimals or in the form $\frac{a}{b}$.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as interpreting, representing, and generalizing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Interpret

- situations involving signed numbers (throughout Unit)
- tables with signed numbers (Lesson 3)
- bank statements with signed numbers (Lesson 4)

Represent

- addition of signed numbers on a number line (Lesson 2)
- situations involving signed numbers (Lessons 3 and 11)
- changes in elevation (Lesson 6)
- position, speed, and direction (Lesson 8)

Generalize

- about subtracting and adding signed numbers (Lesson 5)
- about differences and magnitude (Lesson 6)
- about multiplying negative numbers (Lesson 9)
- about additive and multiplicative inverses (Lesson 15)

In addition, students are expected to justify reasoning about distances on a number line and about negative numbers, account balances, and debt. Students are also expected to explain how to

determine changes in temperature, how to find information using inverses, and how to model situations involving signed numbers.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

Unit 5

Learning Targets

Rational Number Arithmetic

Lesson 1: Interpreting Negative Numbers

- I can compare rational numbers.

- I can use rational numbers to describe temperature and elevation.

Lesson 2: Changing Temperatures

- I can use a number line to add positive and negative numbers.

Lesson 3: Changing Elevation

- I understand how to add positive and negative numbers in general.

Lesson 4: Money and Debts

- I understand what positive and negative numbers mean in a situation involving money.

Lesson 5: Representing Subtraction

- I can explain the relationship between addition and subtraction of rational numbers.

- I can use a number line to subtract positive and negative numbers.

Lesson 6: Subtracting Rational Numbers

- I can find the difference between two rational numbers.

- I understand how to subtract positive and negative numbers in general.

Lesson 7: Adding and Subtracting to Solve Problems

- I can solve problems that involve adding and subtracting rational numbers.

Lesson 8: Position, Speed, and Direction

- I can multiply a positive number with a negative number.

- I can use rational numbers to represent speed and direction.

Lesson 9: Multiplying Rational Numbers

- I can explain what it means when time is represented with a negative number in a situation about speed and direction.

- I can multiply two negative numbers.

Lesson 10: Multiply!

- I can solve problems that involve multiplying rational numbers.

Lesson 11: Dividing Rational Numbers

- I can divide rational numbers.

Lesson 12: Negative Rates

- I can solve problems that involve multiplying and dividing rational numbers.

- I can solve problems that involve negative rates.

Lesson 13: Expressions with Rational Numbers

- I can add, subtract, multiply, and divide rational numbers.

- I can evaluate expressions that involve rational numbers.

Lesson 14: Solving Problems with Rational Numbers

- I can represent situations with expressions that include rational numbers.

- I can solve problems using the four operations with rational numbers.

Lesson 15: Solving Equations with Rational Numbers

- I can solve equations that include rational numbers and have rational solutions.

Lesson 16: Representing Contexts with Equations

- I can explain what the solution to an equation means for the situation.

- I can write and solve equations to represent situations that involve rational numbers.

Lesson 17: The Stock Market

- I can solve problems about the stock market using rational numbers and percentages.

lesson	new terminology	
	receptive	productive
7.5.1	**absolute value** degrees Celsius vertical elevation sea level	**positive number** **negative number**
7.5.2	signed numbers	temperature number line
7.5.3	sum opposite expression	
7.5.4	**deposit** **withdrawal** account balance debt	
7.5.6	difference	distance
7.5.7		**absolute value** x-coordinate y-coordinate
7.5.8	velocity	
7.5.11	**solution (to an equation)** factor	
7.5.13	**rational number**	sum difference
7.5.15	**variable** additive inverse multiplicative inverse	opposite **solution (to an equation)**
7.5.16	operation	
7.5.17		increase decrease

Required Materials

Copies of Instructional master
Four-function calculators
Pre-printed slips, cut from copies of the

Instructional
master Receipt
tape

Section: Interpreting Negative Numbers

Lesson 1: Interpreting Negative Numbers

Goals

- Interpret signed numbers in the contexts of temperature and elevation.

- Order rational numbers, and justify (orally) the comparisons.

- Plot points on a vertical or horizontal number line to represent rational numbers.

Learning Targets

- I can compare rational numbers.

- I can use rational numbers to describe temperature and elevation.

Lesson Narrative

In this lesson, students review what they learned about negative numbers in grade 6, including placing them on the number line, comparing and ordering them, and interpreting them in the contexts of temperature and elevation (MP2). The context of temperature helps build students' intuition about signed numbers because most students know what it means for a temperature to be negative and are familiar with representing temperatures on a number line (a thermometer). The context of elevation may be less familiar to students, but it provides a concrete (as well as cultural) example of one of the most fundamental uses of signed numbers: representing positions along a line relative to a reference point (sea level in this case). The number line is the primary representation for signed numbers in this unit, and the structure of the number line is used to make sense of the rules of signed number arithmetic in later lessons.

Alignments

Building On

- 6.NS.C: Apply and extend previous understandings of numbers to the system of rational numbers.

Addressing

- 7.NS.A.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

- 7.NS.A.1.b: Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

- 7.NS.A.1.c: Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

- 7.NS.A.2.d: Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Building Towards

- 7.NS.A.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- MLR2: Collect and Display

- MLR5: Co-Craft Questions

- MLR8: Discussion Supports

- Take Turns

Required Materials

Pre-printed slips, cut from copies of the **Instructional master**

Required Preparation

Print and cut up slips from the Rational Numbers Card Sort Instructional master. Prepare 1 copy for every 3 students. Students will need copies of both sets 1 and 2. Keep the slips from set 1 (Integers) separate from set 2 (Rational numbers that are not integers) for each group. Consider using different colors of paper so sets 1 and 2 are easier to separate.

> **Student Learning Goals**
>
> Let's review what we know about signed numbers.

1.1 Using the Thermometer

Warm Up: 5 minutes
The purpose of this warm-up is to remind students about negative numbers. The context of a weather thermometer works like a vertical number line. Students do not need to understand comparative temperatures in Celsius and Fahrenheit. The activity is written with temperatures in Celsius; however, the activity would work the same if the thermometer was labeled in Fahrenheit. These two different systems for measuring temperature is an opportunity to remind students that what counts as zero is abitrary and was chosen by someone as some point. If desired, explain to students that $0°$ Celsius is the freezing point of fresh water and $0°$ Fahrenheit is based on the freezing point of salt water.

Building On

- 6.NS.C

Building Towards

- 7.NS.A.1

Launch

Display the thermometer image for all to see. Explain that degrees Celsius is a way of measuring temperature, like degrees Fahrenheit—but it has a different zero point. Students may already know that $0°$ Celsius is based on the freezing point of water and $0°$ Fahrenheit on the freezing point of brine, but these were chosen by people; there's no reason they had to be this way. Give students 1 minute of quiet think time to examine the picture before they start writing.

Anticipated Misconceptions

Some students may think the missing number between 0 and -10 needs to have a magnitude larger than -10, such as -15, because on the positive side of the number line, numbers increase in magnitude as you go up.

> ### Student Task Statement
>
> Here is a weather thermometer. Three of the numbers have been left off.
>
>
>
> 1. What numbers go in the boxes?
>
> 2. What temperature does the thermometer show?
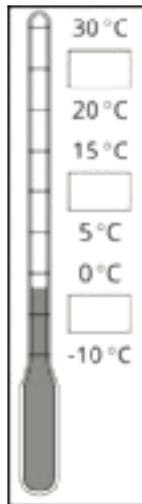
Student Response

1. 25, 10, -5

2. About -2$°C$

Activity Synthesis

Ask students to share their responses for the first question and explain their reasoning. After each response, ask students to indicate if they agree or disagree. If all students are in agreement, record and display the missing temperatures for all to see. If they disagree, have students explain their reasoning until they reach an agreement.

Ask students to share their responses to the second question. Because the thermometer is labeled in 5 degree increments, we have to estimate the temperature between $0°$ and $-5°$. Ask students to explain their reasoning and record and display possible responses for all to see. Highlight student responses that include the following ideas:

- The location of negative numbers below 0.

- The distance between numbers on the vertical number line.

1.2 Fractions of a Degree

5 minutes
In this activity, students return to the context of a thermometer to examine rational numbers that are not integers. Students compare and interpret the signed numbers to make sense of them in the context (MP2), including comparing a temperature that is not pictured to the temperatures that are pictured.

Building On
- 6.NS.C

Addressing
- 7.NS.A.1

Instructional Routines
- MLR5: Co-Craft Questions

Launch

Remind students of the warm-up problem about a weather thermometer. Instruct them to estimate when necessary.

Access for English Language Learners

Writing, Speaking: MLR5 Co-craft Questions. To help students make sense of the drawings in this problem and to increase their awareness of the language used when comparing signed numbers, show students just the images of the four thermometers. Ask pairs of students to write their own mathematical questions about the situation. Listen for how students use the idea of numbers being above or below zero. Ask pairs to share their questions with the whole class. Highlight specific questions that are related to comparing numbers above or below zero. This will help students develop meta-awareness of the language used when comparing signed numbers.
Design Principle(s): Maximize meta-awareness; Support sense-making<

Anticipated Misconceptions

Some students may struggle to estimate the temperature on the last thermometer, since it is between two markings. Ask them to tell what the temperature would be for the lines directly above and directly below the thermometer's level. Then ask what temperature would be halfway in between those two numbers.

Some students may struggle with comparing -4˚C to the temperatures shown on the thermometers. Prompt students to point out where -4˚C would be on the thermometer that is showing -3˚C.

Student Task Statement

1. What temperature is shown on each thermometer?

2. Which thermometer shows the highest temperature?

3. Which thermometer shows the lowest temperature?

4. Suppose the temperature outside is -4˚C. Is that colder or warmer than the coldest temperature shown? How do you know?
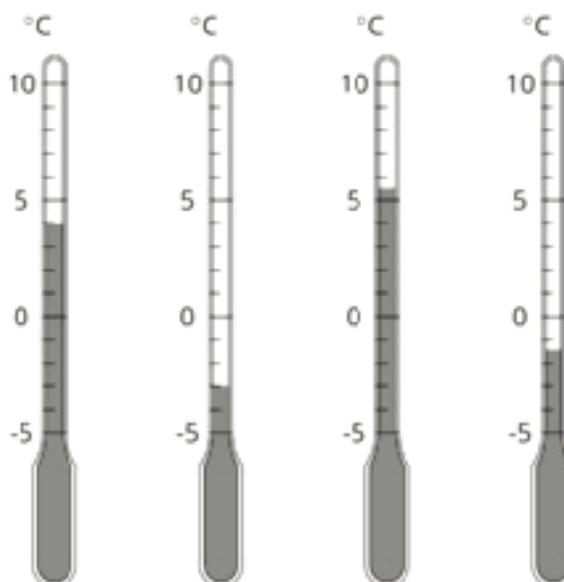
Student Response

1. 4˚C, -3˚C, 5.5˚C, -1.5˚C

2. The third thermometer

3. The second thermometer

4. It is colder because -4 < -3.

Activity Synthesis

Ask one or more students to share their response for the temperature for each thermometer.

When discussing the last question, first have students explain their reasoning until they come to an agreement that -4˚C is colder than -3˚C. Then, if not brought up in students' explanations, introduce the notation -4 < -3 and remind students that this is read, "Negative 4 is less than negative 3." Explain that -4 is farther away from zero than -3 is, and point to the location of -4 on a thermometer to show that is it below -3. On the negative side of the number line, that means -4 is less than -3. Familiarity with less than notation will be useful for describing their reasoning in the next activity.

Unit 5 Lesson 1

1.3 Seagulls Soar, Sharks Swim

10 minutes

The purpose of this activity is for students to continue interpreting signed numbers in context and to begin to compare their relative location. A vertical number line shows the heights above sea level or depths below sea level of various animals. The number line is labeled in 5 meter increments, so students have to interpolate the height or depth for some of the animals. Next, they are given the height or depth of other animals that are not pictured and asked to compare these to the animals shown.

As students work, monitor for whether they are expressing relative distances in words, for example "3 meters below," or if they are expressing the same idea with notation, as in -3 meters. Both are acceptable; these ideas are connected in the discussion that follows (MP2). Also monitor for students who notice that there are two possible answers for the last question.

Building On

- 6.NS.C

Addressing

- 7.NS.A.1

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- MLR2: Collect and Display

Launch

Display the image for all to see. Tell students to measure the height or depth of each animal's eyes, to the nearest meter. Remind students that we choose sea level to be our zero level, in the same way that we chose a zero level for temperature.

Access for Students with Disabilities

Representation: Internalize Comprehension. Represent the same information with additional structure. If students are unsure where to begin, suggest that they extend a straight horizontal line at each depth to determine the height or depth of each animal.
Supports accessibility for: Visual-spatial processing; Conceptual processing

---

Conversing, Representing: MLR2 Collect and Display. Use this routine to capture the language students use during discussion with a partner. Collect the language of opposites: "above" or "below." For example, "The albatross is 3 meters above the penguin or the penguin is 3 meters below the albatross." Then, identify students who use negative numbers to describe these differences to share their reasoning during the whole-class discussion. Ensure students connect this language to, "The difference in height is +3 (to represent above) or -2.5 (to represent below)."
Design Principle(s): Maximize meta-awareness

Anticipated Misconceptions

If students measure to the top or bottom of the animal, remind them that we are using the eyes of the animal to measure their height or depth.

Some students may struggle to visualize where the albatross, seagull, and clownfish are on the graph. Consider having them draw or place a marker where the new animal is located while comparing it to the other animals in the picture.

> **Student Task Statement**
>
> Here is a picture of some sea animals. The number line on the left shows the vertical position of each animal above or below sea level, in meters.
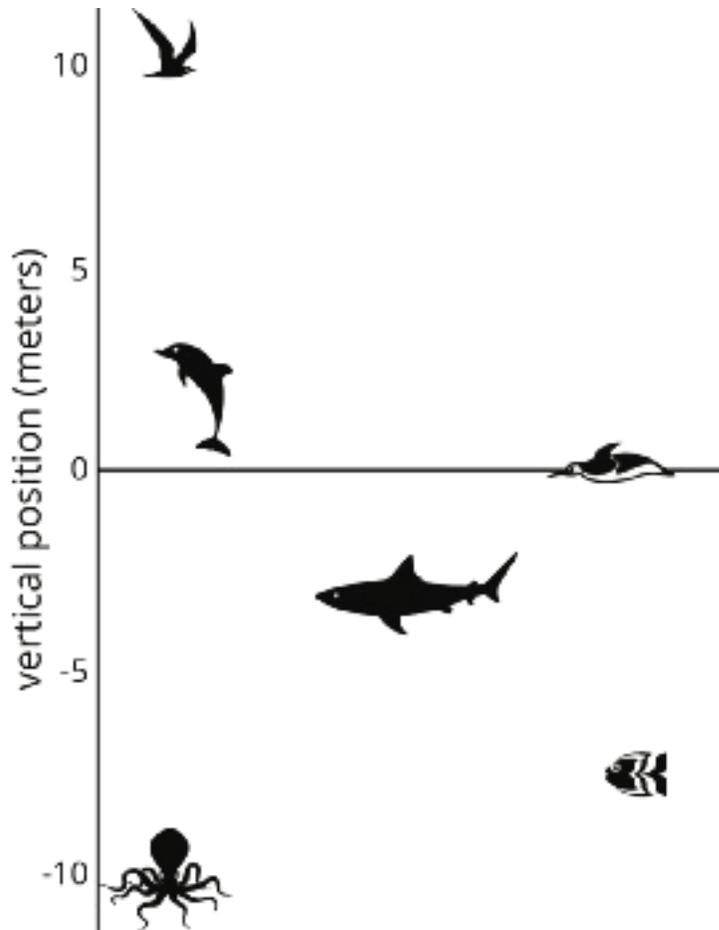
1. How far above or below sea level is each animal? Measure to their eye level.

2. A mobula ray is 3 meters above the surface of the ocean. How does its vertical position compare to the height or depth of:

   The jumping dolphin?   The flying seagull?   The octopus?

3. An albatross is 5 meters above the surface of the ocean. How does its vertical position compare to the height or depth of:

   The jumping dolphin?   The flying seagull?   The octopus?

4. A clownfish is 2 meters below the surface of the ocean. How does its vertical position compare to the height or depth of:

   The jumping dolphin?   The flying seagull?   The octopus?

5. The vertical distance of a new dolphin from the dolphin in the picture is 3 meters. What is its distance from the surface of the ocean?

Student Response

1. Seagull is at 10 m. Dolphin is at 3 m. Octopus is at -10 m. Shark is at -3 m. Fish is at -7 m. Penguin is at 0 m.

2. The mobula ray is
   a. 0 m above the dolphin

   b. 7 m below the seagull

   c. 13 m above the octopus

3. The albatross is
   a. 2 m above the dolphin

   b. 5 m below the seagull

   c. 15 m above the octopus

4. The clownfish is
   a. 5 m below the dolphin

   b. 12 m below the seagull

   c. 8 m above the octopus

5. Either 0 m or 6 m, depending on whether the new dolphin is 3 m above or below the dolphin in the picture.

Are You Ready for More?

The north pole is in the middle of the ocean. A person at sea level at the north pole would be 3,949 miles from the center of Earth. The sea floor below the north pole is at an elevation of approximately -2.7 miles. The elevation of the south pole is about 1.7 miles. How far is a person standing on the south pole from a submarine at the sea floor below the north pole?

Student Response

About 7,897 miles.

Activity Synthesis

The main point for students to get out of this activity is that we can represent distance above and below sea level using signed numbers. The depths of the shark, fish, and octopus can be expressed as approximately -3 m, -6 m, and -7.5 m respectively, because they are below sea level.

Signed numbers can also be used to represent the relative vertical position of different pairs of animals. Have selected students share their responses and reasoning for how the heights of the albatross, seabird, and clownfish compare to the dolphin, seagull, and octopus. Record and display their verbal descriptions using signed numbers. For example, if a student says the albatross is 7 meters below the seagull, write "-7".

Finally, ask whether students noticed the ambiguity in the last question (about the height of the new dolphin). Ask such a student to explain why there are two possible answers to the last question.

1.4 Card Sort: Rational Numbers

Optional: 15 minutes
This activity reviews ordering integers first, and then rational numbers second. Many of the numbers also have their additive inverse in the set, which can help students use the structure of the number line to order the numbers.

The previous activities in this lesson used vertical number lines to help students make sense of negative numbers being below 0. It is important that students also feel comfortable working with horizontal number lines. As students work on ordering these slips, it is likely they will automatically make the transition to using a horizontal orientation. Watch for any groups that continue to use a vertical orientation and prompt them to consider whether they have really ordered their numbers from least to greatest.

Monitor for students who specifically compare the magnitudes of numbers and translate that into the correct number order (such as $2.5 > 2$ so $-2.5 < -2$) are using the structure of the number line (MP7); ask them to share their reasoning in the whole-class discussion.

Building On
- 6.NS.C

Addressing
- 7.NS.A.1.b
- 7.NS.A.1.c
- 7.NS.A.2.d

Instructional Routines
- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports
- Take Turns

Launch

Arrange students in groups of 3. Distribute the first set of cards (integers) to each group. Instruct the students to put the cards in order from least to greatest. When a group has finished ordering the first set, give them the second set (rational numbers that are not integers) and have them add these in the correct locations.

Anticipated Misconceptions

Some students may struggle with ordering the negative numbers. For example, they may put -2.5 to the right of -2 since they are used to seeing 2.5 to the right of 2. Help students visualize a number line and figure out which number should be farther away from 0.

Student Task Statement

1. Your teacher will give your group a set of cards. Order the cards from least to greatest.

2. Pause here so your teacher can review your work. Then, your teacher will give you a second set of cards.

3. Add the new set of cards to the first set so that all of the cards are ordered from least to greatest.

Student Response

1. -23, -10, -9, -7, -6, -4, -3, -2, -1, 0, 1, 2, 3, 5, 8, 10, 11, 15, 22, 23

2. No answer needed.

3. -23, $-22\frac{3}{8}$, $-\frac{31}{3}$, -10, -9, -7.7, -7, -6, $-5\frac{5}{6}$, -4, -3, $-\frac{8}{3}$, -2.5, -2, $-\frac{9}{8}$, -1, $-\frac{1}{4}$, 0, $\frac{1}{4}$, 1, $\frac{9}{8}$, 2, 2.5, $\frac{8}{3}$, 3, 5, $5\frac{5}{6}$, 7.7, 8, 10, $\frac{31}{3}$, 11, 15, 22, $22\frac{3}{8}$, 23

Activity Synthesis

Select students to share their strategies when sorting. Highlight strategies that used the magnitudes of a number and its additive inverse.

Discuss:

- Which numbers were the hardest to order? Why?

- How did you decide where to put the fractions?

- How is, for example, $-\frac{9}{8}$ related to $\frac{9}{8}$?

Introduce the convention that number lines are usually drawn horizontally, with the negative numbers to the left of 0. If any groups put their slips in order vertically, considering having them

reposition their slips to match the orientation of a horizontal number line. Make sure students understand the meaning of the term "opposite" and absolute value notation.

Lesson Synthesis

Main learning points:

- Negative numbers can be used to represent quantities below a chosen zero point.

- Negative numbers can be ordered to the left side of zero on a horizontal number line.

- Absolute value, or magnitude, describes how far away from zero a value is.

Discussion questions:

- Which number is greater, -7 or -12?

- Which number has the greater magnitude, 7 or -12?

- How can we order negative numbers?

1.5 Signed Numbers

Cool Down: 5 minutes
For upcoming work in this unit, it is vital that students can correctly place positive and negative rational numbers on a number line, and that they can compare positive and negative rational numbers. If any students do poorly on this cool-down, they will have plenty of practice with placing positive and negative numbers on a number line in the next several lessons, but they may need more support in doing so.

Building On
- 6.NS.C

Addressing
- 7.NS.A.1

> **Student Task Statement**
>
> Here is a set of signed numbers: 7, -3, $\frac{1}{2}$, -0.8, 0.8, -$\frac{1}{10}$, -2
>
> 1. Order the numbers from least to greatest.
>
> 2. If these numbers represent temperatures in degrees Celsius, which is the coldest?
>
> 3. If these numbers represent elevations in meters, which is the farthest away from sea level?

1. -3, -2, -0.8, $-\frac{1}{10}$, $\frac{1}{2}$, 0.8, 7

2. -3

3. 7

Student Lesson Summary

We can use **positive numbers** and **negative numbers** to represent temperature and elevation.

When numbers represent temperatures, positive numbers indicate temperatures that are warmer than zero and negative numbers indicate temperatures that are colder than zero. This thermometer shows a temperature of -1 degree Celsius, which we write $-1°C$.
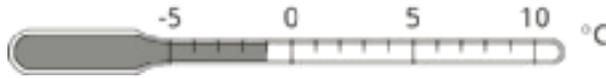
When numbers represent elevations, positive numbers indicate positions above sea level and negative numbers indicate positions below sea level.

We can see the order of signed numbers on a number line.

A number is always less than numbers to its right. So $-7 < -3$.

We use **absolute value** to describe how far a number is from 0. The numbers 15 and -15 are both 15 units from 0, so $|15| = 15$ and $|-15| = 15$. We call 15 and -15 *opposites*. They are on opposite sides of 0 on the number line, but the same distance from 0.

Glossary

- absolute value

- negative number

- positive number

Lesson 1 Practice Problems

Problem 1

Statement

It was -5°C in Copenhagen and -12°C in Oslo. Which city was colder?

Solution

It was colder in Oslo because -12 is less than -5.

Problem 2

Statement

a. A fish is 12 meters below the surface of the ocean. What is its elevation?

b. A sea bird is 28 meters above the surface of the ocean. What is its elevation?

c. If the bird is directly above the fish, how far apart are they?

Solution

a. -12 m

b. 28 m

c. 40 m

Problem 3

Statement

Compare using >, =, or <.

a. 3 _____ -3

b. 12 _____ 24

c. -12 _____ -24

d. 5 _____ -(-5)

e. 7.2 _____ 7

f. -7.2 _____ -7

g. -1.5 _____ $\frac{-3}{2}$

h. $\frac{-4}{5}$ _____ $\frac{-5}{4}$

i. $\frac{-3}{5}$ _____ $\frac{-6}{10}$

j. $\frac{-2}{3}$ _____ $\frac{1}{3}$

Solution

a. >

b. <

c. >

d. =

e. >

f. <

g. =

h. >

i. =

j. <

Problem 4

Statement

Han wants to buy a $30 ticket to a game, but the pre-order tickets are sold out. He knows there will be more tickets sold the day of the game, with a markup of 200%. How much should Han expect to pay for the ticket if he buys it the day of the game?

Solution

$90. A 100% increase of a $30 ticket is an additional $30, therefore a 200% increase of a $30 ticket would be an additional $60.

(From Unit 4, Lesson 7.)

Problem 5

Statement

A type of green paint is made by mixing 2 cups of yellow with 3.5 cups of blue.

a. Find a mixture that will make the same shade of green but a smaller amount.

b. Find a mixture that will make the same shade of green but a larger amount.

c. Find a mixture that will make a different shade of green that is bluer.

d. Find a mixture that will make a different shade of green that is more yellow.

Unit 5 Lesson 1

Solution

Answers vary. Sample response:

    a. 1 cup of yellow and 1.75 cups of blue

    b. 4 cups of yellow and 7 cups of blue

    c. 2 cups of yellow and 4 cups of blue.

    d. 2 cups of yellow and 2 cups of blue.

(From Unit 2, Lesson 1.)

Section: Adding and Subtracting Rational Numbers

Lesson 2: Changing Temperatures

Goals

- Determine the final temperature given the starting temperature and the change in temperature, and explain (orally and using other representations) the solution method.

- Explain (orally) how to create a number line diagram that represents adding signed numbers.

- Write an addition equation to represent a situation involving a temperature increase or decrease.

Learning Targets

- I can use a number line to add positive and negative numbers.

Lesson Narrative

In this lesson, students represent addition of signed numbers on a number line. There are different ways to do this; in this unit, the convention is that each addend is represented by an arrow and the sum is represented as a point on the number line. Positive addends are represented by arrows that point to the right, and negative addends by arrows that point to the left. The first arrow starts at zero; the next arrow starts where the first arrow ends. The sum is represented by a point on the number line where the arrow for the last addend ends.

This lesson uses the context of temperature to help students make sense of the addition equations. Students see that an increase in temperature can be represented as adding a positive value and a decrease in temperature can be represented as adding a negative value. When students use quantitative contexts like temperature to aid in abstract reasoning about numeric expressions with signed numbers, they engage in MP2.

Alignments

Addressing

- 7.NS.A.1.a: Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.

- 7.NS.A.1.b: Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

Building Towards

- 7.NS.A.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Instructional Routines

- MLR6: Three Reads
- MLR8: Discussion Supports
- Think Pair Share
- Which One Doesn't Belong?

Required Preparation

If desired, prepare to display a map showing the locations of:

- Houston, TX
- Orlando, FL
- Salt Lake City, UT
- Minneapolis, MN
- Fairbanks, AK

> **Student Learning Goals**
>
> Let's add signed numbers.

2.1 Which One Doesn't Belong: Arrows

Warm Up: 5 minutes
In this warm-up, students compare four number line diagrams with arrows. To give all students access the activity, each diagram has one obvious reason it does not belong. Students will use diagrams like these later in the lesson to represent sums of signed numbers, but for this activity, the goal is to just get them used to analyzing these types of diagrams carefully before they have to interpret them in terms of rational number arithmetic.

Building Towards

- 7.NS.A.1

Instructional Routines

- Which One Doesn't Belong?

Launch

Arrange students in groups of 2–4. Display the image of the four figures for all to see. Ask students to indicate when they have noticed one figure that does not belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their group. After everyone has conferred in groups, ask the group to offer at least one reason *each* figure doesn't belong.

Student Task Statement

Which pair of arrows doesn't belong?

1.

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

2.

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

3.

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10

4.

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10
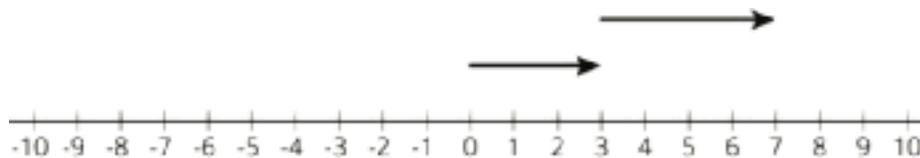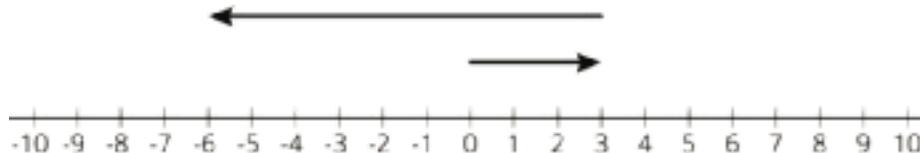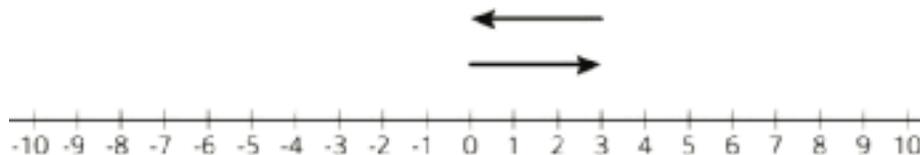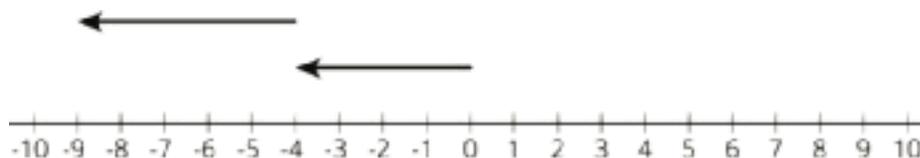
Student Response

1. The only one where both arrows point right.

2. The only one where the arrows point in opposite directions and are different lengths.

3. The only one where the arrows point in opposite directions and are the same length.

4. The only one where both arrows point left.

Activity Synthesis

After students have conferred in groups, invite each group to share one reason why a particular figure might not belong. Record and display the responses for all to see. After each response, poll the rest of the class if they agree or disagree. Since there is no single correct answer to the question of which diagram does not belong, attend to students' explanations and ensure the reasons given make sense.

Ask the students what they think the arrows might represent. After collecting responses, say we are going to represent positive and negative numbers and their sums using arrows on a number line.

Unit 5 Lesson 2

2.2 Warmer and Colder

15 minutes

In this activity, the context of temperature is used to help students make sense of adding signed numbers (MP2). First students reason about temperature increases and decreases. They represent these increases and decreases on a number line, and then connect these temperature changes with adding positive numbers for increases and adding negative numbers for decreases. Students repeatedly add numbers to 40 and then to -20 to see that adding a positive number is the same as moving to the right on the number line and adding a negative number is the same as moving to the left on the number line (MP8).

Addressing

- 7.NS.A.1.a

- 7.NS.A.1.b

Instructional Routines

- MLR8: Discussion Supports

- Think Pair Share

Launch

Arrange students in groups of 2. Ask them, "If the temperature starts at 40 degrees and increases 10 degrees, what will the final temperature be?" Show them this number line:

Explain how the diagram represents the situation, including the start temperature, the change, and the final temperature. Point out that in the table, this situation is represented by an equation where the initial temperature and change in temperature are added together to find the final temperature.

Next, ask students to think about the change in the second row of the table. Give students 1 minute of quiet work time to draw the diagram that shows a decrease of 5 degrees and to think about how they can represent this with an *addition* equation. Have them discuss with a partner for 1 minute. Ask a few students to share what they think the addition equation should be. Be sure students agree on the correct addition equation before moving on. Tell students they will be answering similar questions,

- first by reasoning through the temperature change using whatever method makes sense,

- then drawing a diagram to show the temperature change, and

- finally, by writing an equation to represent the situation.

Give students 4 minutes of quiet work time followed by partner and then whole group discussion.

Student Task Statement

1. Complete the table and draw a number line diagram for each situation.

	start (˚C)	change (˚C)	final (˚C)	addition equation
a	+40	10 degrees warmer	+50	$40 + 10 = 50$
b	+40	5 degrees colder		
c	+40	30 degrees colder		
d	+40	40 degrees colder		
e	+40	50 degrees colder		

a.

b.

c.

d.

e.
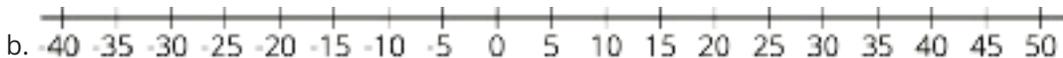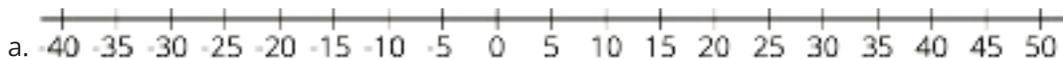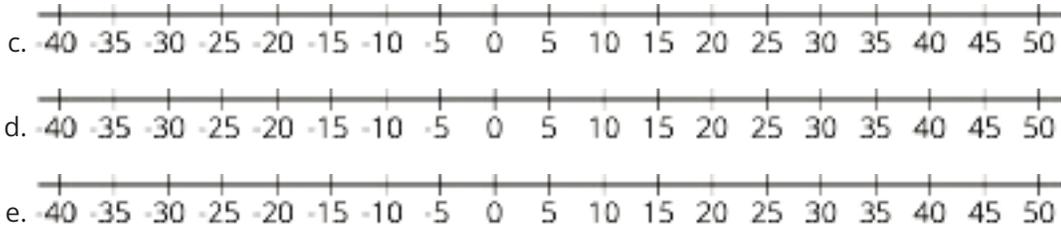
2. Complete the table and draw a number line diagram for each situation.

	start (°C)	change (°C)	final (°C)	addition equation
a	-20	30 degrees warmer		
b	-20	35 degrees warmer		
c	-20	15 degrees warmer		
d	-20	15 degrees colder		

a.

b.

c.

d.
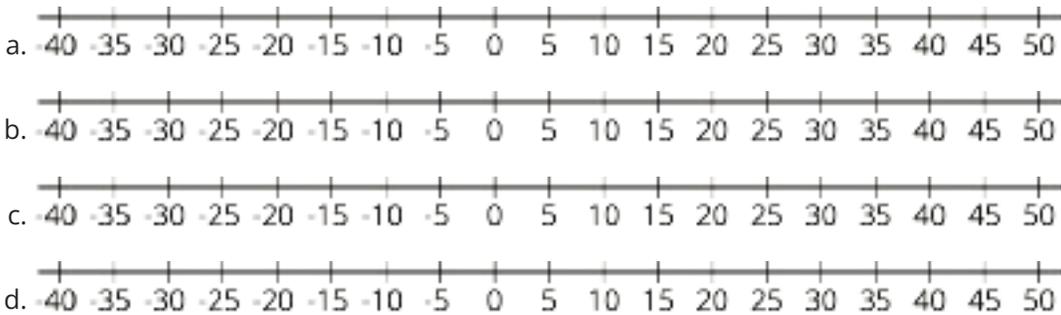
Student Response

1.

	start (°C)	change (°C)	final (°C)	addition equation
a	+40	10 degrees warmer	+50	$40 + 10 = 50$
b	+40	5 degrees colder	+35	$40 + \text{-}5 = 35$
c	+40	30 degrees colder	+10	$40 + \text{-}30 = 10$
d	+40	40 degrees colder	0	$40 + \text{-}40 = 0$
e	+40	50 degrees colder	-10	$40 + \text{-}50 = \text{-}10$

a. [shown in the launch]
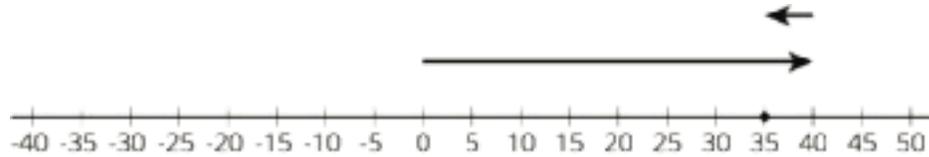
b.

c. A number line with an arrow pointing from 0 to 40, another arrow pointing from 40 to 10, and a dot at 10.

d. A number line with an arrow pointing from 0 to 40, another arrow pointing from 40 to 0, and a dot at 0.
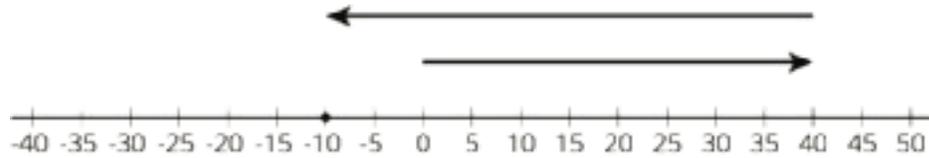
e.

2.

	start (°C)	change (°C)	final (°C)	addition equation
a	-20	30 degrees warmer	+10	-20 + 30 = 10
b	-20	35 degrees warmer	+15	-20 + 35 = 15
c	-20	15 degrees warmer	-5	-20 + 15 = -5
d	-20	15 degrees colder	-35	-20 + -15 = -35

a.
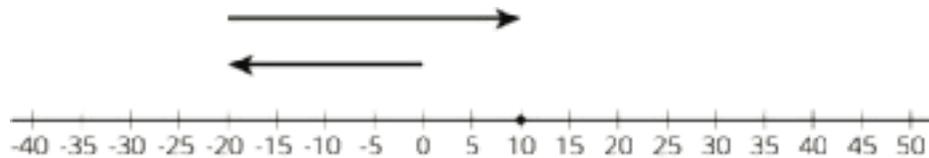
b.
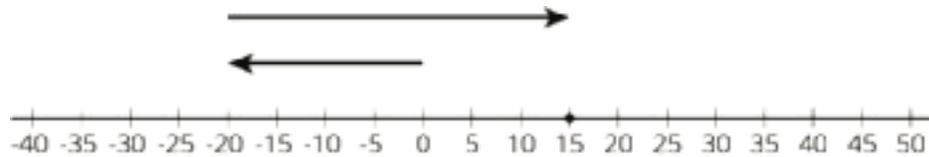
c.
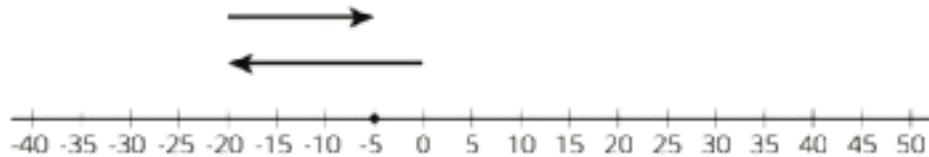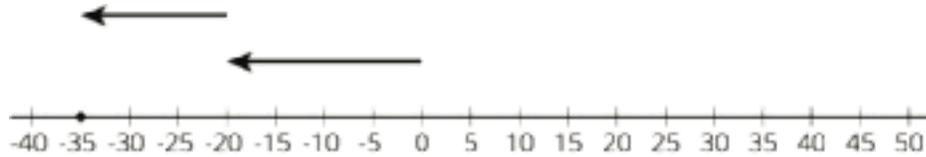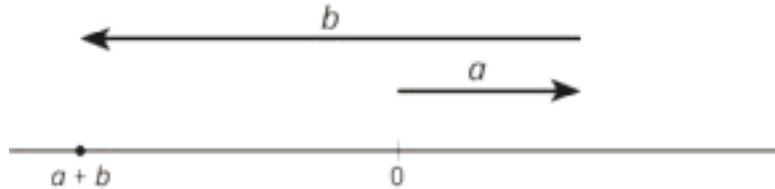
Unit 5 Lesson 2

d.

Student Response

$|a| - |b|$

Activity Synthesis

Ask students,

- "How can we represent an increase in temperature on a number line?" (An arrow pointing to the right.)

- "How can we represent a decrease in temperature on a number line?" (An arrow pointing to the left.)

- "How are positive numbers represented on a number line?" (Arrows pointing to the right.)

- "How are negative numbers represented on a number line?" (Arrows pointing to the left.)

- "How can we represent a sum of two numbers?" (But the arrows so the tail of the second is at the tip of the first.)

- "How can we determine the sum from the diagram?" (It is at the tip of the second arrow.)

- "What happens when we add a positive number to another number?" (We move to the right on the number line.)

- "What happens when we add a negative number to another number?" (We move to the left on the number line.)

2.3 Winter Temperatures

10 minutes (there is a digital version of this activity)

In this activity, students use what they learned in the previous activity to find temperature differences and connect them to addition equations. Students who use number line diagrams are using tools strategically (MP5). Students may draw number line diagrams in a variety of ways; what matters is that they can explain how their diagrams represent the situation. Students may think of these questions in terms of subtraction; that is completely correct, but the discussion should focus on how to think of these situations in terms of addition. Students will have an opportunity to connect addition and subtraction in a future lesson.

Addressing

- 7.NS.A.1.b

Instructional Routines

- MLR6: Three Reads

Launch

Before students start working, it may be helpful to display a map of the United States and point out the locations of the cities in the problem. Explain that in the northern hemisphere, it tends to be colder the farther north you are.

If students are using the digital version, ask them to think about and respond to the questions before testing out their conclusions with the embedded applet. If students jump right to using the applet, they might skip some deep thinking that results from figuring out how to represent the initial temperature, change in temperature, and final temperature, as well as miss out on practice representing these operations with a number line.

Access for English Language Learners

Reading: MLR6 Three Reads. Use this routine to support reading comprehension of this problem without solving it for students. Use the first read to orient students to the situation by asking students to describe it without using numbers (e.g., this problem is about temperatures in various cities; each city is warmer or colder than another). For the second read, identify the important quantities by asking students what can be counted or measured for each city (e.g., Houston is the only city with the actual temperature stated; the other temperatures given are relative values, such as "10 degrees warmer."). For the third read, brainstorm strategies that can be used to find the temperatures in the other cities. This helps students to comprehend the problem, identify mathematical relationships in the text, and develop some problem solving approaches.
Design Principle(s): Support sense-making

Anticipated Misconceptions

If students struggle to find the temperature in Minneapolis, thinking that $8 - 20$ doesn't have an answer, suggest they represent the decrease in temperature as $8 + (\text{-}20)$ and use a number line to reason about the resulting temperature.

Unit 5 Lesson 2

Student Task Statement

One winter day, the temperature in Houston is $8°$ Celsius. Find the temperatures in these other cities. Explain or show your reasoning.

1. In Orlando, it is $10°$ warmer than it is in Houston.

2. In Salt Lake City, it is $8°$ colder than it is in Houston.

3. In Minneapolis, it is $20°$ colder than it is in Houston.

4. In Fairbanks, it is $10°$ colder than it is in *Minneapolis*.

5. Write an addition equation that represents the relationship between the temperature in Houston and the temperature in Fairbanks.

Student Response

1. Orlando is $18°C$

2. Salt Lake City is $0°C$

3. Minneapolis is $-12°C$

4. Fairbanks is $-22°C$

5. $8 + (-20) + (-10) = -22$ or equivalent

Activity Synthesis

Ask students who used number lines to share their reasoning. If a students correctly describes the situation in terms of subtraction, acknowledge that perspective and then ask them if they can also think of it in terms of addition. If no students used a number line, demonstrate how to represent these situations a number line diagram. For example, we can draw this diagram for Minneapolis:

The initial temperature of 8 is represented by an arrow starting at 0 and going 8 units to the right. The decrease in temperature is represented by an arrow starting at 8 and going 20 units to the left. The final temperature is represented by a point at -12, where the second arrow ends. Remind them that we can represent this with an addition equation:

$$8 + (-20) = -12$$

Lesson Synthesis

Main learning points:

- We can represent a decrease as adding a negative number.

- We can represent addition on a number line with two arrows, the second arrow starting where the first arrow ends.

- We can represent a negative addend on a number line as an arrow pointing to the left.

Discussion questions:

- "How can you represent an increase or decrease in temperature using an addition equation?"

- "How can you represent an addition equation on a number line?"

2.4 Stories about Temperature

Cool Down: 10 minutes
Addressing

- 7.NS.A.1.b

Anticipated Misconceptions

For the second question, some students might not pay attention to the word *to*, and interpret the question as the temperature dropped *by* $16°$. Point out the word *to* in the question and tell them that their number line diagram should show the final temperature is $16°C$.

Student Task Statement

1. Write a story about temperatures that this expression could represent: $27 + (-11)$

2. Draw a number line diagram and write an expression to represent this situation: "On Tuesday at lunchtime, it was $29°C$. By sunset, the temperature had dropped to $16°C$."

Student Response

1. Answers vary. Sample response: It was 27 degrees at lunch time and by the evening the temperature has dropped 11 degrees.

2. Answers vary. Sample expression: $29 + (-13)$.
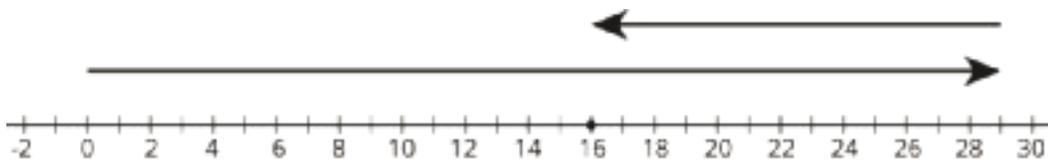
Student Lesson Summary

If it is $42°$ outside and the temperature increases by $7°$, then we can add the initial temperature and the change in temperature to find the final temperature.

$$42 + 7 = 49$$

If the temperature decreases by 7°, we can either subtract $42 - 7$ to find the final temperature, or we can think of the change as -7°. Again, we can add to find the final temperature.

$$42 + (-7) = 35$$

In general, we can represent a change in temperature with a positive number if it increases and a negative number if it decreases. Then we can find the final temperature by adding the initial temperature and the change. If it is 3° and the temperature decreases by 7°, then we can add to find the final temperature.

$$3 + (-7) = -4$$

We can represent signed numbers with arrows on a number line. We can represent positive numbers with arrows that start at 0 and point to the right. For example, this arrow represents +10 because it is 10 units long and it points to the right.

We can represent negative numbers with arrows that start at 0 and point to the left. For example, this arrow represents -4 because it is 4 units long and it points to the left.
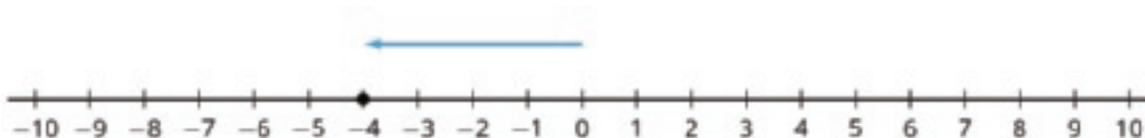
To represent addition, we put the arrows "tip to tail." So this diagram represents $3 + 5$:
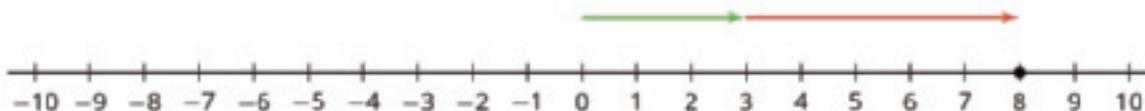
And this represents $3 + (-5)$:
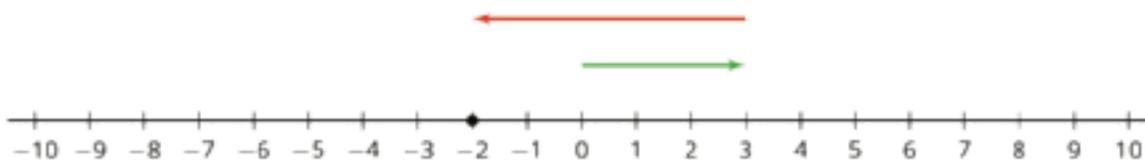
Lesson 2 Practice Problems
Problem 1
Statement

    a. The temperature is -2°C. If the temperature rises by 15°C, what is the new temperature?

    b. At midnight the temperature is -6°C. At midday the temperature is 9°C. By how much did the temperature rise?

Solution

    a. 13°C

    b. 15°C

Problem 2
Statement

Draw a diagram to represent each of these situations. Then write an addition expression that represents the final temperature.

    a. The temperature was $80°F$ and then fell $20°F$.

    b. The temperature was $-13°F$ and then rose $9°F$.

    c. The temperature was $-5°F$ and then fell $8°F$.

Solution

Answers vary. Sample responses:

    a. A number line diagram with an arrow that starts at 80, is 20 units long, and points to the left. $80 + (\text{-}20)$

    b. A number line diagram with an arrow that starts at -13, is 9 units long, and points to the right. $\text{-}13 + 9$

    c. A number line diagram with an arrow that starts at -5, is 8 units long, and points to the left. $\text{-}5 + (\text{-}8)$

Problem 3
Statement

Complete each statement with a number that makes the statement true.

    a. _____ < 7°C

Unit 5 Lesson 2

b. _____ < -3˚C

c. -0.8˚C < _____ < -0.1˚C

d. _____ > -2˚C

Solution

Answers vary. Sample responses:

a. 5˚C

b. -8˚C

c. -0.7˚C

d. 4˚C

(From Unit 5, Lesson 1.)

Problem 4

Statement

Decide whether each table could represent a proportional relationship. If the relationship could be proportional, what would be the constant of proportionality?

a. The number of wheels on a group of buses.

number of buses	number of wheels	wheels per bus
5	30	
8	48	
10	60	
15	90	

b. The number of wheels on a train.

number of train cars	number of wheels	wheels per train car
20	184	
30	264	
40	344	
50	424	

Solution

    a. It could be proportional. The constant of proportionality would be 6 wheels per bus.

    b. It is not proportional. (Every train car does not necessarily have the same number of wheels.)

(From Unit 2, Lesson 7.)

Problem 5

Statement

Noah was assigned to make 64 cookies for the bake sale. He made 125% of that number. 90% of the cookies he made were sold. How many of Noah's cookies were left after the bake sale?

Solution

8 cookies. He made 80 cookies, because $(1.25) \cdot 64 = 80$. 72 were sold, because $(0.9) \cdot 80 = 72$. Therefore 8 were left because $80 - 72 = 8$.

(From Unit 4, Lesson 7.)

Lesson 3: Changing Elevation

Goals

- Comprehend the term "opposite" (in spoken and written language) refers to numbers with the same magnitude but different signs.

- Create and interpret equations and diagrams that represent adding signed numbers in the context of elevation.

- Generalize (orally) a method for determining the sum of two signed numbers.

Learning Targets

- I understand how to add positive and negative numbers in general.

Lesson Narrative

In this lesson, students build towards fluency with adding signed numbers. They begin with the concrete context of elevations above and below sea level, but then move to more abstract work. They see that adding a number and its opposite gives a sum of 0. They contrast adding numbers with the same sign with adding numbers with different signs. Using the structure of opposites on the number line, they see that when adding two numbers with different signs, the sign of the sum will match the sign of the addend with the greater magnitude (MP7).

Alignments

Building On

- 6.NS.C: Apply and extend previous understandings of numbers to the system of rational numbers.

Addressing

- 7.NS.A.1.a: Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.

- 7.NS.A.1.b: Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.

- 7.NS.A.1.c: Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

- 7.NS.A.1.d: Apply properties of operations as strategies to add and subtract rational numbers.

Building Towards

- 7.NS.A.1.a: Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- MLR1: Stronger and Clearer Each Time

- MLR2: Collect and Display

- MLR7: Compare and Connect

- MLR8: Discussion Supports

- Think Pair Share

Required Materials

Receipt tape

Required Preparation

For optional activity, cut one strip of receipt tape, for every 2 students. Each strip of receipt tape should be at least 4 feet long. You may want to prepare some strips that are even longer, in case groups choose extra long objects.

> **Student Learning Goals**
>
> Let's solve problems about adding signed numbers.

3.1 That's the Opposite

Warm Up: 5 minutes
The purpose of this warm up is to think about how we might define opposites in different contexts.

Building On

- 6.NS.C

Building Towards

- 7.NS.A.1.a

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 2 minute of quiet work time followed by 1 minute of partner discussion, then follow with whole-class discussion.

Student Task Statement

1. Draw arrows on a number line to represents these situations:

    a. The temperature was -5 degrees. Then the temperature rose 5 degrees.

    b. A climber was 30 feet above sea level. Then she descended 30 feet.

2. What's the opposite?

    a. Running 150 feet east.

    b. Jumping down 10 steps.

    c. Pouring 8 gallons into a fish tank.

Student Response

1. Answers vary. Sample responses:
    a. A number line with an arrow pointing from 0 to -5 and another from -5 to 0. A point at 0.

    b. A number line with an arrow pointing from 0 to 30 and another from 30 to 0. A point at 0.

2. a. Running 150 feet west.

    b. Jumping up 10 steps.

    c. Pouring 8 gallons out of a fish tank.

Activity Synthesis

Ask a few students to share their responses. Tell students that there are many situations involving changes in quantities where we can represent the opposite action with the opposite value. In this lesson, we are going to investigate several situations like this.

3.2 Cliffs and Caves

15 minutes (there is a digital version of this activity)
The purpose of this activity is to see how to tell, from the equation, whether the sum will be positive, negative, or 0, without having to draw a number line diagram every time.

In this activity, students return to the context of height and depth to continue making sense of adding signed numbers. They use the structure of number line diagrams (MP7) to examine a variety of situations, which include starting in the positives, starting in the negatives, moving away from 0, moving towards 0, crossing over 0, and ending exactly on 0. From this variety of situations, students see that the lengths of the arrows in the number line diagrams give important information about the situation. This opens up the discussion of comparing the magnitude of the addends. When the

addends have opposite signs, the longer arrow (the number with the larger magnitude) determines the sign of the sum. $400 + (-150) = 250$ and $-200 + 150 = -50$.

This activity purposefully gives many problems to figure out, but only asks students to draw a diagram for 3 rows. Watch for students who think they need to draw a number line for every problem. Encourage them to look for and generalize from regularity in the problems (MP8) so that they don't need to always draw a diagram.

For students using the digital version of the materials, be sure they do not use the applet to represent and answer every question and bypass good thinking. Encourage students to figure out an answer for each question first, and then using the applet to check their work.

In preparation for dealing with subtraction of signed numbers, some of the problems involve determining how much the temperature changed, given the initial and final temperatures.

Addressing

- 7.NS.A.1.a

- 7.NS.A.1.b

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Explain that a *mountaineer* is someone who climbs mountains, and a *spelunker* is someone who explores caves and caverns. Arrange students in groups of 2. Give students 3 minute of quiet work time followed by partner discussion after the first question. Then have students work on the second question and follow with whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Represent the same information through different modalities by using physical objects to represent abstract concepts. Create an interactive display that allows students to experience physically moving an object (mountaineer) up and down a number line. Highlight connections between the act of physically moving the object up and down with arrows on a number line diagram.
Supports accessibility for: Conceptual processing; Visual-spatial processing

Anticipated Misconceptions

Some students may struggle with working backwards to fill in the change when given the final elevation. Ask them "What do you do to 400 to get to 50?" for example.

Student Task Statement

1. A mountaineer is climbing on a cliff. She is 400 feet above the ground. If she climbs up, this will be a positive change. If she climbs down, this will be a negative change.

   a. Complete the table.

	starting elevation (feet)	change (feet)	final elevation (feet)
A	+400	300 up	
B	+400	150 down	
C	+400	400 down	
D	+400		+50

   b. Write an addition equation and draw a number line diagram for B. Include the starting elevation, change, and final elevation in your diagram.

---

2. A spelunker is down in a cave next to the cliff. If she climbs down deeper into the cave, this will be a negative change. If she climbs up, whether inside the cave or out of the cave and up the cliff, this will be a positive change.

   a. Complete the table.

	starting elevation (feet)	change (feet)	final elevation (feet)
A	-200	150 down	
B	-200	100 up	
C	-200	200 up	
D	-200	250 up	
E	-200		-500

b. Write an addition equation and draw a number line diagram for C and D. Include the starting elevation, change, and final elevation in your diagram.

_____

_____

c. What does the expression $-75 + 100$ tell us about the spelunker? What does the value of the expression tell us?

Student Response

1.   a.

	starting elevation (feet)	change (feet)	final elevation (feet)
A	+400	300 up	+700
B	+400	150 down	+250
C	+400	400 down	0
D	+400	350 down	+50

b. $400 + (-150) = 250$.
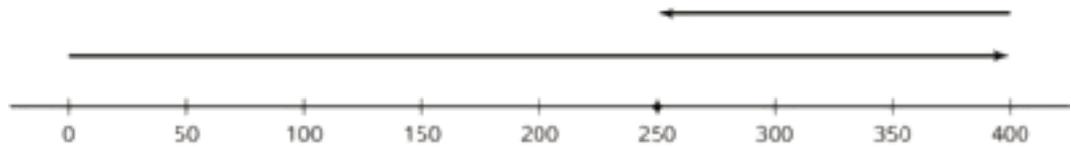
2.   a.

	starting elevation (feet)	change (feet)	final elevation (feet)
A	-200	150 down	-350
B	-200	100 up	-100
C	-200	200 up	0
D	-200	250 up	+50
E	-200	300 down	-500

b. C: $-200 + 200 = 0$.
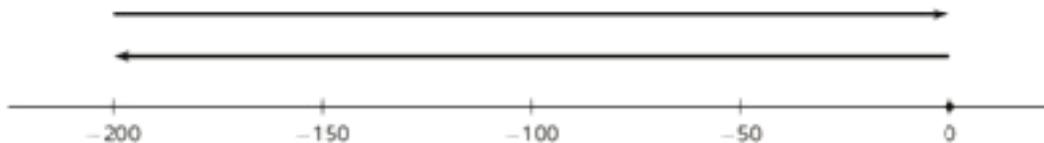
D: $-200 + 250 = 50$. A number line diagram with an arrow from 0 to -200, an arrow from -200 to 50, and a point at 50.

   c. The spelunker was at an elevation of -75 feet, then went 100 feet up. The spelunker is now at an elevation of 25 feet.

Activity Synthesis

The most important thing for students to get out of this activity is how to tell from the equation whether the sum will be positive, negative, or 0, without having to draw a number line diagram every time.

- Write all of the equations where the two addends have the same sign. The ask, "What happens when the two addends have the same sign?"

- Write all of the equations where the two addends have the opposite sign. The ask, "What happens when the two addends have opposite signs and . . .
  - the number with the larger magnitude is positive?"

  - the number with the larger magnitude is negative?"

- Ask, "How can you tell when the sum will be zero?"

Access for English Language Learners

Speaking, Listening, Writing: MLR1 Stronger and Clearer Each Time. To provide students with an opportunity to generalize about the sum of two addends, ask students to draft an initial response to the question "How can you tell from the equation whether the sum will be positive, negative, or 0?". Ask students to meet with 2–3 partners for feedback. Provide students with prompts for feedback that will help each other strengthen their ideas and clarify their language (e.g., "What if the signs of both numbers are the same/different?" or "How do you know if the sum will be positive/negative?", etc.). Students can borrow ideas and language from each partner to refine their explanation. This will help students to use mathematical language to generalize about the sums of rational numbers.
Design Principle(s): Optimize output (for generalization)

3.3 Adding Rational Numbers

10 minutes

In this activity, no scaffolding is given and students use any strategy to find the sums. Monitor for students who reason in different ways about the sums.

Addressing

- 7.NS.A.1.d

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- MLR8: Discussion Supports

- Think Pair Share

Launch

Arrange students in groups of 2. 2 minutes of quiet work time, followed by partner and whole-class discussion.

> **Student Task Statement**
>
> Find the sums.
>
> 1. $-35 + (30 + 5)$
>
> 2. $-0.15 + (-0.85) + 12.5$
>
> 3. $\frac{1}{2} + (-\frac{3}{4})$

Student Response

1. 0

2. 11.5

3. $-\frac{1}{4}$

> **Are You Ready for More?**
>
> Find the sum without a calculator.
>
> $$10 + 21 + 32 + 43 + 54 + (-54) + (-43) + (-32) + (-21) + (-10)$$

Student Response

0

Activity Synthesis

Select students to share their solutions. Sequence beginning with diagrams, then with more abstract reasoning. Help students connect the different reasoning strategies.

3.4 School Supply Number Line

Optional: 10 minutes
The purpose of this activity is to understand that even without knowing the actual numbers, knowing how the *signs* and *magnitudes* of two numbers compare is enough to determine whether their sum will be positive or negative.

In this activity, students use the relative position of numbers on the number line to compare them (MP7). Instead of working with specific given numbers, they start with two different, arbitrary lengths. They use these lengths to label various points on a number line. Students see that even though each group may have worked with different lengths, they should still get the same answers to the final set of questions, because everyone started with $a > b$. Students use their new insights to explain comparisons between signed numbers (MP3).

Identify a group that has a small difference between the lengths of their two objects, as well as a group that has a large difference between the lengths of their two objects, to contrast their number lines during the whole-class discussion.

Addressing

- 7.NS.A.1.b

- 7.NS.A.1.c

Instructional Routines

- MLR2: Collect and Display

- MLR7: Compare and Connect

Launch

Arrange students in groups of 2. Provide each group a blank strip of receipt tape. The length of tape has to be longer than four times the length of their longer object. Ensure that each group has access to two objects of appropriate length.

Anticipated Misconceptions

Some students might forget which symbol means greater than or less than.

Some students may be confused between comparing the value of the expression and the magnitude of the expression. Explain that the number to the left on a number line has the lesser value, even if it may have the greater magnitude (farther away from zero).

> ### Student Task Statement
>
> Your teacher will give you a long strip of paper.
>
> Follow these instructions to create a number line.
>
> 1. Fold the paper in half along its length and along its width.
>
> 2. Unfold the paper and draw a line along each crease.
>
> 3. Label the line in the middle of the paper 0. Label the right end of the paper + and the left end of the paper −.
>
> 4. Select two objects of different lengths, for example a pen and a gluestick. The length of the longer object is a and the length of the shorter object is b.
>
> 5. Use the objects to measure and label each of the following points on your number line.

a	$2b$	$-b$
b	$a+b$	$a+\text{-}b$
$2a$	$-a$	$b+\text{-}a$

6. Complete each statement using <, >, or =. Use your number line to explain your reasoning.

    a. a _____ b

    b. $-a$ _____ $-b$

    c. $a+\text{-}a$ _____ $b+\text{-}b$

    d. $a+\text{-}b$ _____ $b+\text{-}a$

    e. $a+\text{-}b$ _____ $-a+b$

Student Response

1–5. Answers vary. Sample response:
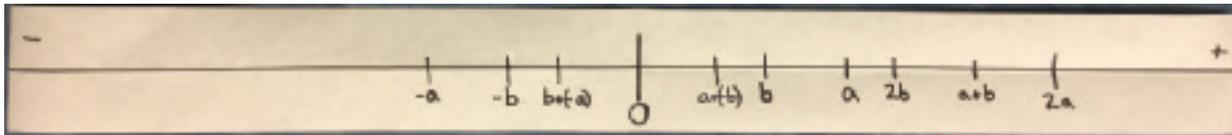
1.   a. $a > b$. Both values are positive and a is the value of the longer object.

    b. $-a < -b$. Both numbers are negative but the same distance away from zero as a and b.

    c. $a + (-a) = b + (-b)$. Both sides are zero.

    d. $a + (-b) > b + (-a)$. The expression on the left is greater than zero and the one on the right is less than zero.

    e. $a + (-b) > -a + b$. Since $b + (-a)$ is the same as $-a + b$, the same reason applies as before.

Activity Synthesis

The most important thing for students to understand is that even without knowing the actual numbers, knowing how the signs and magnitudes of two numbers compare is enough to determine whether their sum will be positive or negative.

Display the number lines from the selected groups (one with a small difference between a and b and one with a large difference).

Discuss:

- Which points are in the same relative position? (For example, $a + b$ is greater than $2b$ and less than $2a$ in both cases.)

- Which points are in different relative positions? (For example, $2b$ may be greater than a on one diagram but less than a on the other.)

For each part of the last question, ask students to indicate whether they used <, >, or =. Have students explain their reasoning until they come to an agreement. It may be helpful to point out the different positions on the number lines that students refer to during their explanations.

If desired, you can extend the discussion by highlighting the fact that all the comparisons in the task statement have the same answer for every group, but this didn't have to be the case. Ask students to invent another comparison that would have a different answer for some of the groups than others. For students who are struggling, ask them to connect positive and negative numbers and addition and subtraction to the previous contexts using MLR 7 (Compare and Connect).

Lesson Synthesis

Main learning points:

- To add two numbers with the same sign, we add their magnitudes (because the arrows point in the same direction) and keep the same sign for the sum.

- To add two numbers with different signs, we subtract their magnitudes (because the arrows point in the opposite direction) and use the sign of the number with the larger magnitude for the sum.

- When we add a number and its opposite, the sum is zero. These are called additive inverses.

Discussion questions:

- What is the opposite of 5? of -8? of $\frac{1}{3}$? of -0.6?

- What is the sum of a number and its opposite?

- Explain how to add two numbers with the same sign. With different signs.

3.5 Add 'Em Up

Cool Down: 5 minutes
Addressing

- 7.NS.A.1.b

> **Student Task Statement**
>
> Find each sum.
>
> 1. $56 + (-56)$
>
> 2. $-240 + 370$
>
> 3. $-5.7 + (-4.2)$

1. 0

2. 130

3. -9.9

Student Lesson Summary

The opposite of a number is the same distance from 0 but on the other side of 0.
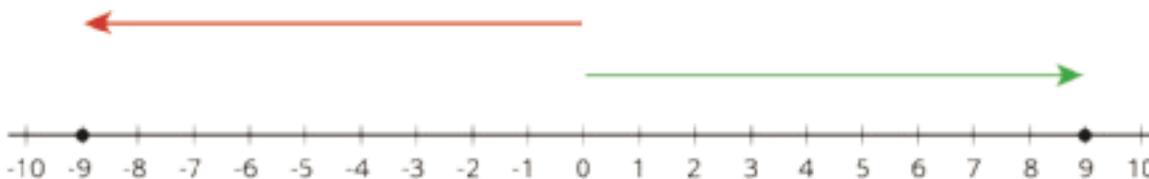
The opposite of -9 is 9. When we add opposites, we always get 0. This diagram shows that $9 + \text{-}9 = 0$.
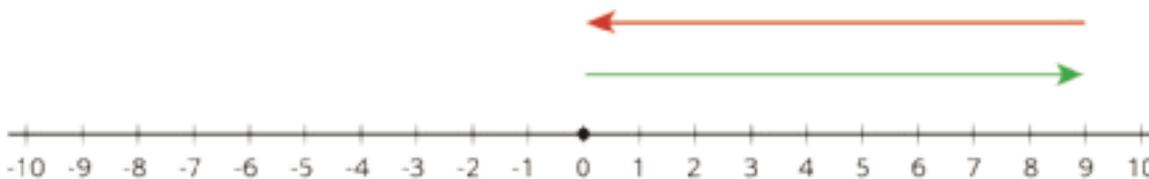
When we add two numbers with the same sign, the arrows that represent them point in the same direction. When we put the arrows tip to tail, we see the sum has the same sign.
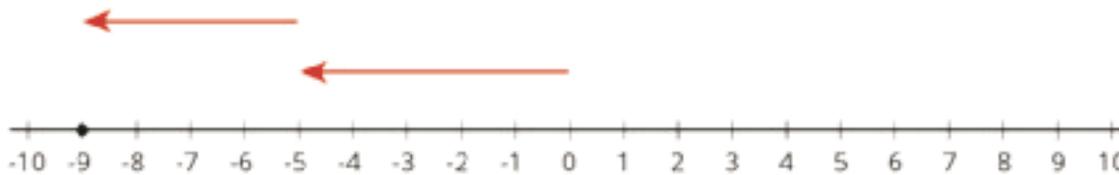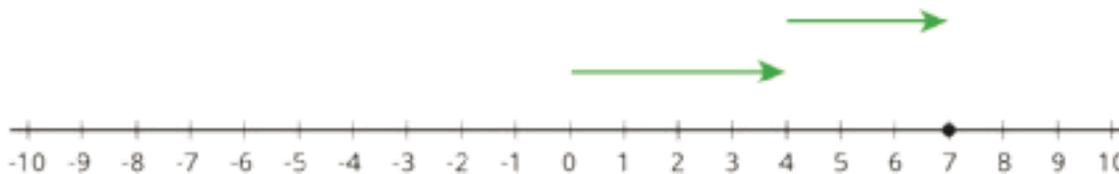
To find the sum, we add the magnitudes and give it the correct sign. For example, $(\text{-}5) + (\text{-}4) = \text{-}(5 + 4)$.

On the other hand, when we add two numbers with different signs, we subtract their magnitudes (because the arrows point in the opposite direction) and give it the sign of the number with the larger magnitude. For example, $(\text{-}5) + 12 = +(12 - 5)$.
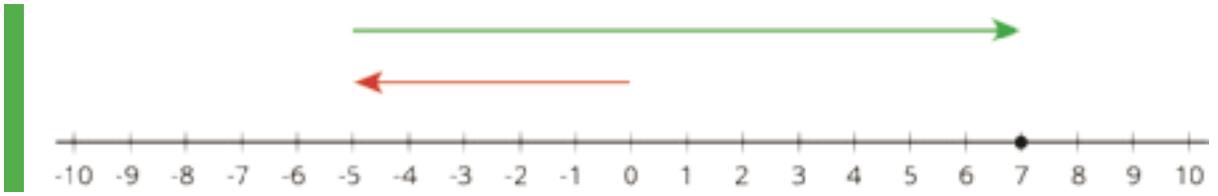
Lesson 3 Practice Problems
Problem 1
Statement

What is the final elevation if

    a. A bird starts at 20 m and changes 16 m?

    b. A butterfly starts at 20 m and changes -16 m?

    c. A diver starts at 5 m and changes -16 m?

    d. A whale starts at -9 m and changes 11 m?

    e. A fish starts at -9 meters and changes -11 meters?

Solution

a. 36 m because $20 + 16 = 36$

b. 4 m because $20 + (\text{-}16) = 4$

c. -11 m because $5 + (\text{-}16) = \text{-}11$

d. 2 m because $\text{-}9 + 11 = 2$

e. -20 m because $\text{-}9 + (\text{-}11) = \text{-}20$

Problem 2
Statement

One of the particles in an atom is called an electron. It has a charge of -1. Another particle in an atom is a proton. It has charge of +1. The charge of an atom is the sum of the charges of the electrons and the protons. A carbon atom has an overall charge of 0, because it has 6 electrons and 6 protons and $\text{-}6 + 6 = 0$. Find the overall charge for the rest of the elements on the list.

	charge from electrons	charge from protons	overall charge
carbon	-6	+6	0
neon	-10	+10	
oxide	-10	+8	
copper	-27	+29	
tin	-50	+50	

Solution

Carbon: $-6 + 6 = 0$

Neon: $-10 + 10 = 0$

Oxide: $-10 + 8 = -2$

Copper: $-27 + 29 = +2$

Tin: $-50 + 50 = 0$

Problem 3

Statement

Add.

$14.7 + 28.9$ $\qquad$ $-9.2 + 4.4$ $\qquad$ $-81.4 + (-12)$ $\qquad$ $51.8 + (-0.8)$

Solution

a. 43.6

b. -4.8

c. -93.4

d. 51

Problem 4

Statement

Last week, the price, in dollars, of a gallon of gasoline was g. This week, the price of gasoline per gallon increased by 5%. Which expressions represent this week's price, in dollars, of a gallon of gasoline? Select **all** that apply.

A. $g + 0.05$

B. $g + 0.05g$

C. $1.05g$

D. $0.05g$

E. $(1 + 0.05)g$

Solution

["B", "C", "E"]

(From Unit 4, Lesson 8.)

Problem 5

Statement

Decide whether each table could represent a proportional relationship. If the relationship could be proportional, what would be the constant of proportionality?

a. Annie's Attic is giving away $5 off coupons.

original price	sale price
$15	$10
$25	$20
$35	$30

b. Bettie's Boutique is having a 20% off sale.

original price	sale price
$15	$12
$25	$20
$35	$28

Solution

a. Not proportional.

b. Proportional. The constant of proportionality is 0.8 sale dollars per original dollar.

(From Unit 2, Lesson 7.)

Lesson 4: Money and Debts

Goals

- Apply addition of signed numbers to calculate an account balance after a deposit or withdrawal, and explain (orally and using other representations) the solution method.

- Explain (orally and in writing) how signed numbers can be used to represent situations involving money, including deposits or withdrawals, and assets or debts.

- Write an equation with an unknown addend to represent a situation where the amount of change is unknown.

Learning Targets

- I understand what positive and negative numbers mean in a situation involving money.

Lesson Narrative

In this lesson, students are introduced to using negative numbers in the context of money to represent debts or debits.

It is common to use money contexts to represent signed numbers. One point that often gets overlooked is that it is a *convention* that we do this, rather than a necessity. Any situation in which we use a negative number to represent a debt (for example), we could equally well just use a positive number and distinguish it by calling it a debt. The reason we use signed numbers in this context is that it allows us to represent a whole class of problems with the same expression. For example, if a person has $50 in the bank and writes a $20 check, we can represent the balance as $50 - 20$. If they had written an $80 check, we can still write the balance as $50 - 80$, as long as we have adopted the convention that negative numbers represent what the person owes the bank (and assuming the bank allows overdrafts). Using a mathematical structure (the signed numbers) to represent a context (a checking account balance) is an example of modeling with mathematics (MP4).

Alignments

Building On

- 6.NS.C: Apply and extend previous understandings of numbers to the system of rational numbers.

Addressing

- 7.NS.A.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Building Towards

- 7.NS.A.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- MLR5: Co-Craft Questions

- MLR8: Discussion Supports

- Think Pair Share

Required Preparation

This lesson presents opportunities to practice performing operations on signed values, but the emphasis is really on noticing that money can be represented with positive and negative values. If the computation requirements might get in the way of that understanding, consider providing access to calculators.

> **Student Learning Goals**
>
> Let's apply what we know about signed numbers to money.

4.1 Concert Tickets

Warm Up: 10 minutes
There are many ways to think about debt, and the way the lender views it differs the way from the borrower does. This warm-up introduces the idea that we can represent a debt with a signed number. From the perspective of the person who owes money, the debt is usually viewed as a negative number. From the perspective of the bank, it may be viewed as a positive number.

Building On

- 6.NS.C

Building Towards

- 7.NS.A.1

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Ask students,

"Priya is buying concert tickets for her and her friends with the money she earns at her part-time job. This month, she has earned $135. Can she buy three $50 tickets for a concert?"

Ask students to discuss with a partner for 1 minute.

Explain that sometimes we can borrow money from a bank to buy things we cannot afford at the time, and then pay the money back to the bank in the future. Give students time for partner discussion followed by whole-class discussion.

> **Student Task Statement**
>
> Priya wants to buy three tickets for a concert. She has earned $135 and each ticket costs $50. She borrows the rest of the money she needs from a bank and buys the tickets.
>
> 1. How can you represent the amount of money that Priya has after buying the tickets?
>
> 2. How much more money will Priya need to earn to pay back the money she borrowed from the bank?
>
> 3. How much money will she have after she pays back the money she borrowed from the bank?

Student Response

Answers vary. Sample response:

1. Since Priya owes the bank money, this could be represented by a negative number, -$15.

2. Priya needs to earn $15 more. Sample explanations:
   - Because $150 - 135 = 15$.

   - Because $-15 + 15 = 0$.

3. She will have $0 after she pays the money back.

Activity Synthesis

The most important thing for students to understand is that when representing a debt with a negative number, the additive inverse tells how much money is needed to pay off the debt. If Priya has -$15, then she needs $15 more to raise her balance back to $0.

If no students suggest that we can represent Priya's money using a negative number, introduce the idea.

4.2 Cafeteria Food Debt

10 minutes
In this activity, students solve problems about debts that can be represented with addition and subtraction equations. Some problems ask students to calculate the balance after the transaction and some questions ask students to calculate the amount of the transaction, given the starting and ending balances. Students draw number lines to represent each problem.

The series of questions involves a running balance after deposits and withdrawals are made. Students may represent this by drawing a new diagram for each question, or by adding on to the same diagram. Either approach will work.

The focus in this activity is on writing a new equation to represent each situation, and on creating a diagram to represent the situation. If students struggle, encourage them to think about what they have already learned about adding and subtracting integers, and assure them that they can use that understanding to reason about money. If students might struggle with the computations, consider providing access to a calculator to take the focus off of computation.

Addressing

- 7.NS.A.1

Instructional Routines

- MLR8: Discussion Supports

Launch

Remind students that a deposit is money paid into an account. If students do not read carefully, they may not realize that they are expected to write an equation and create a diagram for each question, and only record a numerical answer. Ensure they understand what they are expected to do before they begin working.

Give students quiet work time followed by whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts for students who benefit from support with organizational skills in problem solving. Give students time to work on the first 1-2 transactions before checking in. Invite 1-2 students to think aloud and share how the came up with an equation and how they represented the transaction on a number line. Record their thinking on a display chart and keep the work visible as students continue to work.
Supports accessibility for: Organization; Attention

Anticipated Misconceptions

Some students may struggle to write an equation for each problem. Prompt them to identify what amount is unknown in each situation.

> **Student Task Statement**
>
> At the beginning of the month Kiran had $24 in his school cafeteria account. Use a variable to represent the unknown quantity in each transaction below and write an equation to represent it. Then, represent each transaction on a number line. What is the unknown quantity in each case?

1. In the first week he spent $16 on lunches. How much was in his account then?

2. Then he deposited some more money and his account balance was $28. How much did he deposit?

3. Then he spent $34 on lunches the next week. How much was in his account then?

4. Then he deposited enough money to pay off his debt to the cafeteria. How much did he deposit?

5. Explain why it makes sense to use a negative number to represent Kiran's account balance when he owes money.

Student Response

Answers vary. Sample responses:

1. $24 + (\text{-}16) = n; n = 8$

2. $8 + m = 28; m = 20$

3. $28 + (\text{-}34) = p; p = \text{-}6$
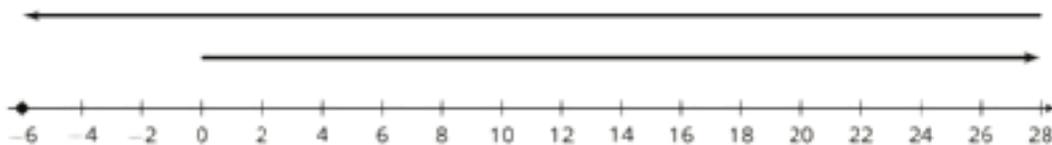
4. $(\text{-}6) + q = 0; q = 6$

5. It makes sense that his balance is negative because he must pay back a positive amount of money to get to zero and stop owing money.

Activity Synthesis

The most important thing for students to understand is that all the rules they have learned for adding and subtracting signed numbers still work when applied to the context of negative amounts of money.

Review each of the following types of computations and discuss how they apply to the school cafeteria situations:

- Adding numbers with the same sign

- Adding numbers with opposite signs

- Adding opposites makes 0

- Subtracting as addition with a missing addend

- Subtracting as adding the additive inverse

Access for English Language Learners

Speaking, Listening: MLR8 Discussion Supports. Demonstrate the use of mathematical language to describe thinking. For example, "If Kiran begins the month with $24, I know that he starts with a positive amount. I know he spends $16, and spending money is subtraction or negative. To create an equation, I need both values." To provide an opportunity for both listening and speaking, ask students, "Who can restate my reasoning in a different way?" This helps invite more student participation and meta-awareness of language and reasoning.
Design Principle(s): Optimize output (for explanation)

4.3 Bank Statement

10 minutes
In this activity, students see that withdrawals, in addition to debts, can also be represented using negative numbers. Students continue using addition and subtraction to solve problems about debt.

While solving the last problem, students may begin wondering about multiplying and dividing signed numbers, which will be addressed in the next several lessons.

You may wish to ask students to pause after the first question for discussion. The decision about which numbers to represent with positive versus negative values hinges on whether you are thinking from the perspective of the person or the perspective of the account. Point out that the final balance is represented with a negative number to show that the person owes the bank money (this should be brought out in the launch). Therefore, from the perspective of the account, deposits are positive values and withdrawals are negative values. It would be possible to proceed either way, but it will facilitate discussion later if everyone uses the same convention as a result of work on the first question.

As students work, monitor for students who are expressing their reasoning as addition and subtraction equations or expressions.

Addressing

- 7.NS.A.1

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- MLR5: Co-Craft Questions

Launch

Display the bank statement image for all to see, without the questions. Ask students to think of two things they notice and two things they wonder. Give students 1 minute of quiet think time. Select a few students to share. Make sure students understand the meaning of *deposit* and *withdrawal*.

Give students quiet work time followed by whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Reading, Writing: MLR5 Co-craft Questions. To support students in making sense of the bank statement, display the image of the bank statement without the remaining text of the problem. Clarify the meaning of the terms in the bank statement (e.g., deposit, withdrawal, balance) in this context. Note that the word "balance" has an everyday meaning ("I fell down when I lost my balance"). Then ask pairs of students to write down possible mathematical questions that might be asked about the situation. Invite pairs to share with the whole class. Highlight questions that relate to calculating a balance based on a given deposit or withdrawal. This will allow students to use mathematical language related to a bank statement as they reason about the accounting situation.
Design Principle(s): Cultivate conversation; Support sense-making

Student Task Statement

Here is a bank statement.

Responsible Bank
210 2nd Street
Anytown, MH 06930

Checking Account Statement
Page: 1 of 1

Andre Person
1729 Euclid Ave
Anytown, MH 06930

Statement Period	Account No.
2017-10-01 to 2017-11-01	1120635978

Date	Description	Withdrawals	Deposits	Balance
2017-10-03	Previous Balance			39.87
2017-10-05	Check Number 256	28.50		11.37
2017-10-06	ATM Deposit - Cash		45.00	56.37
2017-10-10	Wire Transfer	37.91		18.46
2017-10-17	Point of Sale - Grocery Store	16.43		2.03
2017-10-25	Funds Transfer from Savings		50.00	52.03
2017-10-28	Check Number 257	42.00		10.03
2017-10-29	Online Payment - Phone Services	72.50		-62.47

1. If we put withdrawals and deposits in the same column, how can they be represented?

2. Andre withdraws $40 to buy a music player. What is his new balance?

3. If Andre deposits $100 in this account, will he still be in debt? How do you know?

Student Response

1. We could use negative numbers to represent the withdrawals.

2. Andre's new balance will be -$102.47, because $-62.47 + (-40) = -102.47$.

3. Yes, he will still be in debt. Sample explanations:

- Because $-102.47 + 100 = -2.47$
- Because $|-102.47| > |100|$

> **Are You Ready for More?**
>
> The *national debt* of a country is the total amount of money the government of that country owes. Imagine everyone in the United States was asked to help pay off the national debt. How much would each person have to pay?

Student Response

Answers vary as the population and national debt of the United States changes. If the population of the United States is about 326 million and the national debt is about $19.9 trillion, each person would have to pay about $61,000.

Activity Synthesis

The most important thing for students to understand is that the rules for adding and subtracting signed numbers can help them solve problems about debts.

Select students to share their solutions. Ask students to indicate whether they agree, disagree, or have any clarifying questions.

Lesson Synthesis

Main learning points:

- We can use positive numbers to represent payments into a bank account (deposits) and negative numbers to represent money taken out of an account (withdrawals).
- We can also use a negative balance to represent debt (owing money).
- We can use the additive inverse to quickly find how much money is needed to reach a balance of zero.

Discussion questions:

- What words do we use to mean "money added into" or "money taken out of" an account?
- How can we represent owing money?
- Why does it make sense to use negative numbers to represent debt?
- How can we tell how much money is needed to pay off a debt?

4.4 Buying a Bike

Cool Down: 5 minutes
Addressing

- 7.NS.A.1

Student Task Statement

1. Clare has $150 in her bank account. She buys a bike for $200. What is Clare's account balance now?

2. If Clare earns $75 the next week from delivering newspapers and deposits it in her account, what will her account balance be then?

Student Response

1. Clare's balance is -$50.

2. Clare's new balance is $25.

Student Lesson Summary

Banks use positive numbers to represent money that gets put into an account and negative numbers to represent money that gets taken out of an account. When you put money into an account, it is called a **deposit**. When you take money out of an account, it is called a **withdrawal**.

People also use negative numbers to represent debt. If you take out more money from your account than you put in, then you owe the bank money, and your account balance will be a negative number to represent that debt. For example, if you have $200 in your bank account, and then you write a check for $300, you will owe the bank $100 and your account balance will be -$100.

starting balance	deposits and withdrawals	new balance
0	50	$0 + 50$
50	150	$50 + 150$
200	-300	$200 + (\text{-}300)$
-100		

In general, you can find a new account balance by adding the value of the deposit or withdrawal to it. You can also tell quickly how much money is needed to repay a debt using the fact that to get to zero from a negative value you need to add its opposite.

Glossary

- deposit
- withdrawal

Lesson 4 Practice Problems
Problem 1

Statement

The table shows five transactions and the resulting account balance in a bank account, except some numbers are missing. Fill in the missing numbers.

	transaction amount	account balance
transaction 1	200	200
transaction 2	-147	53
transaction 3	90	
transaction 4	-229	
transaction 5		0

Solution

	transaction amount	account balance
transaction 1	200	200
transaction 2	-147	53
transaction 3	90	143
transaction 4	-229	-86
transaction 5	86	0

Problem 2
Statement

a. Clare has $54 in her bank account. A store credits her account with a $10 refund. How much does she now have in the bank?

b. Mai's bank account is overdrawn by $60, which means her balance is -$60. She gets $85 for her birthday and deposits it into her account. How much does she now have in the bank?

c. Tyler is overdrawn at the bank by $180. He gets $70 for his birthday and deposits it. What is his account balance now?

d. Andre has $37 in his bank account and writes a check for $87. After the check has been cashed, what will the bank balance show?

Solution

a. $64 because $54 + 10 = 64$

b. $25 because $-60 + 85 = 25$

c. -$110 because $-180 + 70 = -110$

d. -$50 because $37 - 87 = -50$

Problem 3
Statement

Last week, it rained g inches. This week, the amount of rain decreased by 5%. Which expressions represent the amount of rain that fell this week? Select **all** that apply.

A. $g - 0.05$

B. $g - 0.05g$

C. $0.95g$

D. $0.05g$

E. $(1 - 0.05)g$

Solution

["B", "C", "E"]

(From Unit 4, Lesson 8.)

Problem 4
Statement

Decide whether or not each equation represents a proportional relationship.

a. Volume measured in cups (c) vs. the same volume measured in ounces (z): $c = \frac{1}{8}z$

b. Area of a square (A) vs. the side length of the square (s): $A = s^2$

c. Perimeter of an equilateral triangle (P) vs. the side length of the triangle (s): $3s = P$

d. Length (L) vs. width (w) for a rectangle whose area is 60 square units: $L = \frac{60}{w}$

Solution

a. yes

b. no

c. yes

d. no

(From Unit 2, Lesson 8.)

Problem 5

Statement

Add.

a. $5\frac{3}{4} + (-\frac{1}{4})$

b. $-\frac{2}{3} + \frac{1}{6}$

c. $-\frac{8}{5} + (-\frac{3}{4})$

Solution

a. $5\frac{1}{2}$

b. $\frac{-3}{6}$ or $\frac{-1}{2}$

c. $\frac{-47}{20}$

(From Unit 5, Lesson 3.)

Problem 6

Statement

In each diagram, x represents a different value.

A

B

C

D
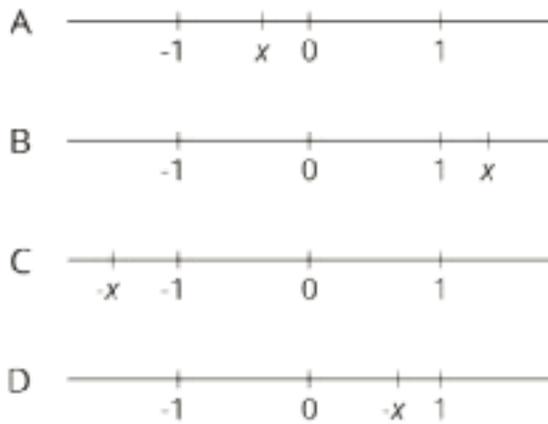
For each diagram,

    a. What is something that is *definitely* true about the value of x?

    b. What is something that *could be* true about the value of x?

Solution

Answers vary. Sample responses:

Diagram A:

    a. The value of x is definitely negative. The value of x is definitely greater than $\frac{-1}{2}$ and less than 0, since x is closer to 0 than it is to -1.

    b. The value of x could be $\frac{-1}{3}$ or -0.4; these values are negative and between 0 and $\frac{-1}{2}$.

Diagram B:

    a. The value of x is definitely positive. The value of x is definitely between 1 and 2, since x is a little greater than 1.

    b. The value of x could be 1.4 or $1\frac{1}{3}$; these values are positive and between 1 and 1.5.

Diagram C:

    a. The value of x is definitely positive, because -x is negative (for example, -(-1.5) = 1.5). The value of x is definitely between 1 and 2, since the distance of -x from 0 is a little greater than 1.

    b. The value of x could be $1\frac{1}{2}$ or 1.4; these values are halfway (or a little less than halfway) between 1 and 2.

Diagram D:

a. The value of x is definitely negative. The value of x is definitely less than $\frac{-1}{2}$ and greater than -1, since $-x$ is farther away from 0 than it is from 1.

b. The value of x could be -0.7 or $\frac{-2}{3}$; these values are between 0 and -1 but closer to -1.

(From Unit 5, Lesson 1.)

Lesson 5: Representing Subtraction

Goals

- Generalize (orally and in writing) that subtracting a number results in the same value as adding the additive inverse.

- Interpret a number line diagram that represents subtracting signed numbers as adding with an unknown addend.

- Use a number line diagram to find the difference of signed numbers, and explain (orally) the reasoning.

Learning Targets

- I can explain the relationship between addition and subtraction of rational numbers.

- I can use a number line to subtract positive and negative numbers.

Lesson Narrative

In this lesson, students represent a subtraction of signed numbers on a number line by relating it to an addition equation with a missing addend. The convention for representing subtraction on the number line fits with the convention for representing addition. When we represent $a + b = c$, we represent a with an arrow starting at zero, b with an arrow starting where the first arrow ends, and c with a point at the end of the second arrow. So when we want to represent $c - a = b$, we represent c with a point, a with an arrow starting at zero, and the difference b is the other arrow that is needed to reach from the end of the first arrow to the point.

At the beginning of the lesson, students see that a subtraction equation like $-8 - 3 = ?$ can be thought of as the related addition equation $3 + ? = $ -8. After repeatedly calculating differences this way (MP8), students recognize that the answer to each subtraction problem is the same number they would get by adding the opposite of the number. For example, by the end of the lesson, students see that $-8 - 3 = ?$ can also be thought of as $-8 + $ -3 $= ?$

Alignments

Building On

- 1.OA.B.4: Understand subtraction as an unknown-addend problem. For example, subtract $10 - 8$ by finding the number that makes 10 when added to 8.

Addressing

- 7.NS.A.1.c: Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

Building Towards

- 7.NS.A.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- MLR3: Clarify, Critique, Correct

- MLR8: Discussion Supports

> **Student Learning Goals**
>
> Let's subtract signed numbers.

5.1 Equivalent Equations

Warm Up: 5 minutes
The purpose of this warm-up is to refresh students' previous understanding about the relationship between addition and subtraction (MP7) so they can write related equations.

As students work, watch for any students who create a number line diagram to help them generate equations that express the same relationship a different way.

Building On

- 1.OA.B.4

Building Towards

- 7.NS.A.1

Launch

Give students 1 minute of quiet work time. Remind students that each new equation must include only the numbers in the original equation.

Anticipated Misconceptions

If students struggle to come up with other equations, encourage them to represent the relationship using a number line diagram, and then think about other operations they can use to show the same relationship with the same numbers.

> **Student Task Statement**
>
> Consider the equation $2 + 3 = 5$. Here are some more equations, using the same numbers, that express the same relationship in a different way:
>
> $$3 + 2 = 5 \qquad\qquad 5 - 3 = 2 \qquad\qquad 5 - 2 = 3$$

> For each equation, write two more equations, using the same numbers, that express the same relationship in a different way.
>
> 1. $9 + (-1) = 8$
>
> 2. $-11 + x = 7$

Student Response

1. For $9 + (-1) = 8$
   - $-1 + 9 = 8$
   - $8 - 9 = -1$
   - $8 - (-1) = 9$

2. For $-11 + x = 7$
   - $x + (-11) = 7$
   - $7 - x = -11$
   - $7 - (-11) = x$

Activity Synthesis

Ask selected students to share their equations that express the same relationship a different way. If any students created a number line diagram to explain their thinking, display this for all to see to facilitate connections between addition equations and related subtraction equations. Every addition equation has related subtraction equations and every subtraction equation has related addition equations; these are the most important takeaways from this activity.

5.2 Subtraction with Number Lines

10 minutes
The purpose of this activity is to apply the representation students have used while adding signed numbers, as well as the relationship between addition and subtraction, to begin subtracting signed numbers. Students are given number line diagrams showing one addend and the sum. They are asked to figure out what the other addend would be. Students examine how these addition equations with missing addends can be written using subtraction by analyzing and critiquing the reasoning of others (MP3).

Monitor for students who are using a consistent structure to analyze the diagrams to generalize and write related addition and subtraction equations (MP8). A template for this work might look something like:
$$a + \; ? = b$$
$$b - a = \; ?$$

Addressing

- 7.NS.A.1.c

Unit 5 Lesson 5

Instructional Routines

- MLR8: Discussion Supports

Launch

It may be useful to remind students how they represented addition on a number line in previous lessons. In particular, it is helpful to keep in mind that the two addends in an addition equation are drawn "tip-to-tail." You might use any number line diagrams created in the previous activity as an illustration of this idea.

Ask students to complete the questions for the first diagram and pause for discussion. Then, give students quiet work time to complete the remaining problems, followed by whole-class discussion.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. For example, reference previous activities where students used a representation while adding signed numbers to provide an entry point for this activity.
Supports accessibility for: Social-emotional skills; Conceptual processing

Access for English Language Learners

Speaking, Listening: MLR8 Discussion Support. To support students in producing statements about Mai's and Tyler's equations, provide sentence frames such as: "I agree/disagree with Mai/ Tyler because….".
Design Principle(s): Support sense-making; Optimize output (for critiques)

Anticipated Misconceptions

Some students may say they disagree with Tyler's equations for the number lines. Use fact families to help students see that subtraction equations are a valid way to represent problems involving finding a missing addend given a sum. It may help to remind them of the work they did in the warm-up.

> ### Student Task Statement
>
> 1. Here is an unfinished number line diagram that represents a sum of 8.
>
>
>
> a. How long should the other arrow be?
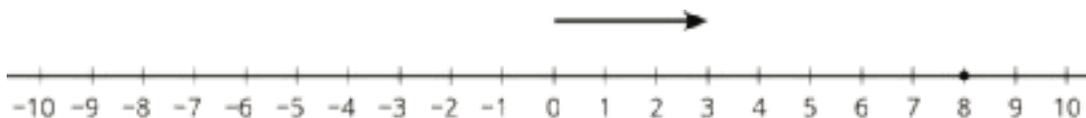
b. For an equation that goes with this diagram, Mai writes $3 + ? = 8$.
Tyler writes $8 - 3 = ?$. Do you agree with either of them?

c. What is the unknown number? How do you know?

2. Here are two more unfinished diagrams that represent sums.
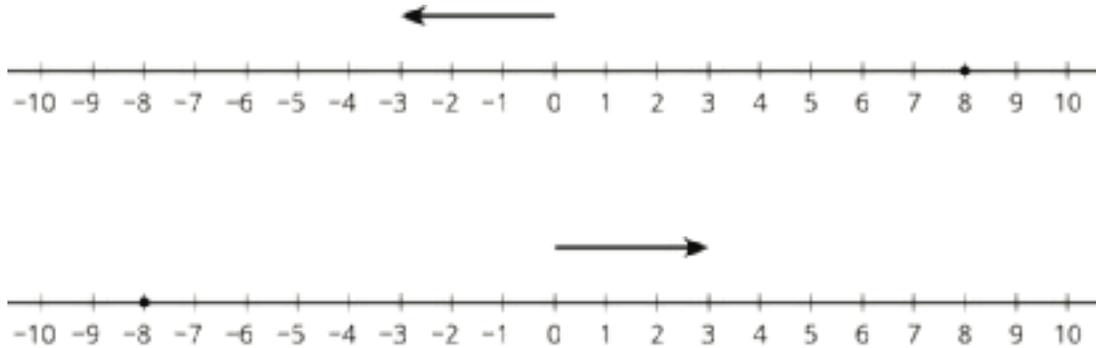
For each diagram:

a. What equation would Mai write if she used the same reasoning as before?

b. What equation would Tyler write if he used the same reasoning as before?

c. How long should the other arrow be?

d. What number would complete each equation? Be prepared to explain your reasoning.

3. Draw a number line diagram for $(\text{-}8) - (\text{-}3) = ?$ What is the unknown number? How do you know?

Student Response

1.
a. 5 units

b. Answers vary. Sample response: I agree with Mai because I want to know what to add to 3 to get 8 because 8 is the end point. I also agree with Tyler because the equations are equivalent.

c. +5. Sample explanation: The unknown number is 5 because to get from 3 to 8 you add on 5.

2. First Image:
a. Mai would write $(\text{-}3) + ? = 8$

b. Tyler would write $8 - (\text{-}3) = ?$

Unit 5 Lesson 5

c. The other arrow should be 11 units long.

d. The number is 11, because the other arrow is 11 units long and pointing to the right.

Second Image:
a. Mai would write $3 + ? = (-8)$

b. Tyler would write $(-8) - (3) = ?$

c. The other arrow should be 11 units long.

d. The number is -11, because the other arrow is 11 units long but pointing to the left.

3. Answers vary. Sample response:
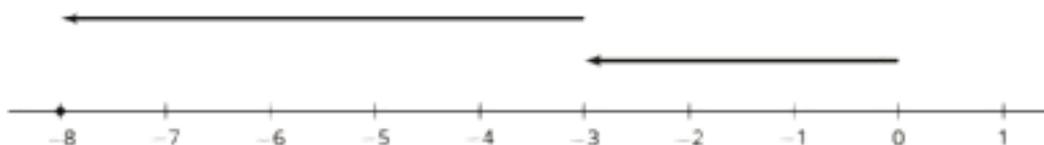
The unknown number is -5, because the arrow is 5 units long and pointing left.

Activity Synthesis

The most important things for students to understand is that subtraction equations can be written as addition equations with a missing addend and number line diagrams can help students figure out what the missing addend is. Students need to be comfortable with this way of representing subtraction for the next activity.

Ask at least one student to share their missing addend for each problem. Ask students to share their reasoning until they come to an agreement. Display two related equations for all to see and use as a reference in the following activity. They might look something like this, or you might choose to use numbers in a specific example rather than letters in a general example.

$$a + ? = b$$
$$b - a = ?$$

5.3 We Can Add Instead

15 minutes
In this activity, students begin to see that subtracting a signed number is equivalent to adding its opposite. First, students match expressions and number line diagrams. Then they add and subtract numbers to see that subtracting a number is the same as adding its opposite (MP8).

Monitor for students who see and can articulate the pattern that adding a number is the same as subtracting its opposite.

Addressing
- 7.NS.A.1.c

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR3: Clarify, Critique, Correct

Launch

Arrange students in groups of 2. Give students 3 minutes of quiet work time, then have them check in with a partner. Have them continue to complete the activity, and follow with a whole-group discussion.

Student Task Statement

1. Match each diagram to one of these expressions:

$3 + 7$ $\qquad$ $3 - 7$ $\qquad$ $3 + (\text{-}7)$ $\qquad$ $3 - (\text{-}7)$

a.

b.

c.

d.
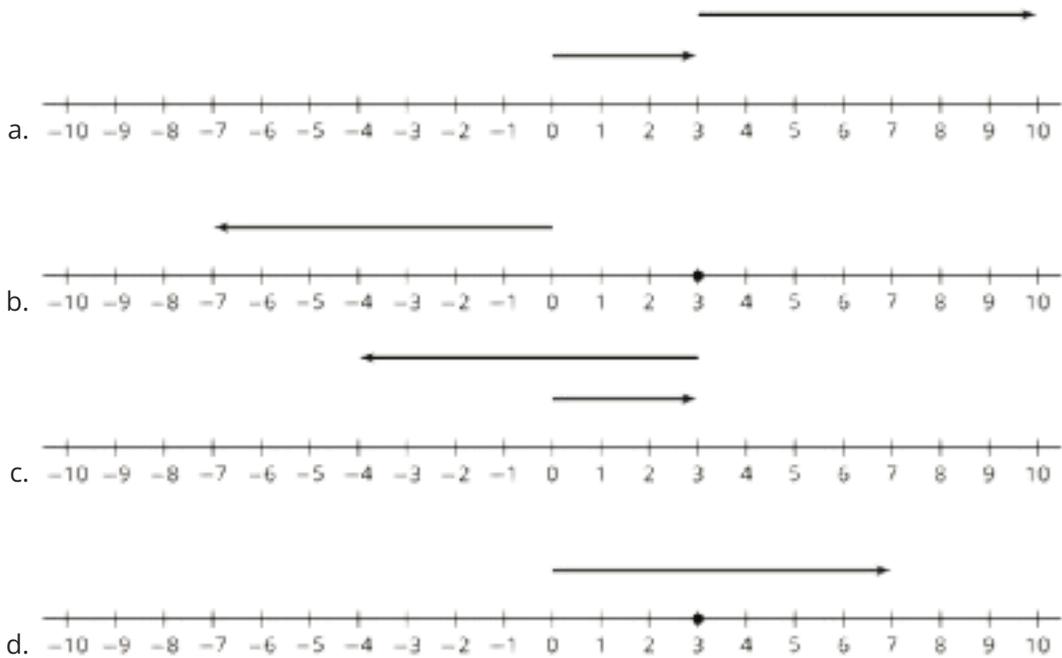
2. Which expressions in the first question have the same value? What do you notice?

3. Complete each of these tables. What do you notice?

expression	value
8 + (-8)	
8 − 8	
8 + (-5)	
8 − 5	
8 + (-12)	
8 − 12	

expression	value
-5 + 5	
-5 − (-5)	
-5 + 9	
-5 − (-9)	
-5 + 2	
-5 − (-2)	

Student Response

1.   a. $3 + 7$

     b. $3 − (-7)$

     c. $3 + (-7)$

     d. $3 − 7$

2. $3 + 7$ and $3 − (-7)$ have the same value. $3 + (-7)$ and $3 − 7$ have the same value. I notice that subtracting a number is the same as adding its opposite.

3. I notice that subtracting a number is the same as adding its opposite.

expression	value
8 + (-8)	0
8 − 8	0
8 + (-5)	3
8 − 5	3
8 + (-12)	-4
8 − 12	-4

expression	value
-5 + 5	0
-5 − (-5)	0
-5 + 9	4
-5 − (-9)	4
-5 + 2	-3
-5 − (-2)	-3

Are You Ready for More?

It is possible to make a new number system using *only* the numbers 0, 1, 2, and 3. We will write the symbols for adding and subtracting in this system like this: $2 \oplus 1 = 3$ and $2 \ominus 1 = 1$. The table shows some of the sums.

$\oplus$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3				

1. In this system, $1 \oplus 2 = 3$ and $2 \oplus 3 = 1$. How can you see that in the table?

2. What do you think $3 \oplus 1$ should be?

3. What about $3 \oplus 3$?

4. What do you think $3 \ominus 1$ should be?

5. What about $2 \ominus 3$?

6. Can you think of any uses for this number system?

Student Response

1. $1 \oplus 2 = 3$ is in the third row and fourth column and $2 \oplus 3 = 1$ is in the fourth row and fifth column.

2. Any responses are allowed. However, if we want $\oplus$ to satisfy the commutative and associative properties, then $3 \oplus 1 = 0$.

3. Assuming commutativity and associativity, $3 \oplus 3 = 2$.

4. Assuming $\ominus$ is the inverse of $\oplus$, $3 \ominus 1 = 2$.

5. Assuming $\ominus$ is the inverse of $\oplus$, $2 \ominus 3 = 3$.

6. Answers vary. For example, think about a dial which can only make 90° clockwise turns. The fact that making three such turns followed by two more turns is the same as making only one turn (since making four of those five 90° turns leaves you in the same position) could be represented by the statement $3 \oplus 2 = 1$. If we thought of the $\ominus$ operation as turning the dial counterclockwise, then the statement $2 \ominus 3 = 3$ reflects that two clockwise turns followed by three counterclockwise turns has the same effect as doing 3 clockwise turns.

Activity Synthesis

The most important takeaway is that subtracting a number gets the same answer as adding its opposite.

Select students to share what patterns they noticed. If no student mentions it, point out that subtracting a number is the same as adding its opposite. Ask students to help you list all of the pairs that show this.

Unit 5 Lesson 5

Then write this expression: $3 - 7$. Ask how it could be written as a sum? $3 + (\text{-}7)$. What numbers are both of these expressions equal to?

$3 - 7 = \text{-}4$

$3 + (\text{-}7) = \text{-}4$

For students who are ready to explore how knowing how to solve a one-step equation involving addition or subtraction (from grade 6) helps us show that subtracting a number is the same as adding its opposite, continue.

This is also true when solving equations. Write this equation:

$$x = 3 - 7$$

Ask how it can be written as a sum. Record students' responses. If no student writes

$$x + 7 = 3$$

then write that. Then point out that we can add -7 to each side:

$$x + 7 + \text{-}7 = 3 + \text{-}7$$
$$x = 3 + \text{-}7$$

There is nothing special about these numbers, because a number and its opposite always make a sum of 0. So subtracting a number is always the same as adding its opposite.

Access for English Language Learners

Writing, Conversing: MLR3 Clarify, Critique, Correct. Present an incorrect statement about adding and subtracting numbers that reflects a possible misunderstanding from the class. Display the following for all to see: "$(\text{-}5) - 3$ has the same value as $5 + (\text{-}3)$, because subtracting is the same as adding the opposite". Invite students to discuss this argument with a partner. Ask, "Do you agree with the statement? Why or why not?" Invite students to clarify and then critique the reasoning, and to write an improved response. This will help students use the language of justification to critique the reasoning related to subtraction of signed numbers.
Design Principle(s): Maximize meta-awareness; Cultivate conversation

Lesson Synthesis

Main takeaways:

- You can think of subtraction as addition with a missing addend: What number do I need to add to get from b to a?

- You can evaluate a subtraction expression by adding the opposite: $a - b = a + (-b)$. This works regardless of the sign for a or b.

Discussion questions

- How could we rewrite the expression $-5 - 3$ using addition? ($3 + ? = -5$, or more simply $-5 + (-3)$))

- Does this work for all numbers?

5.4 Same Value

Cool Down: 5 minutes
Addressing

- 7.NS.A.1.c

Student Task Statement

1. Which other expression has the same value as $(-14) - 8$? Explain your reasoning.

    a. $(-14) + 8$

    b. $14 - (-8)$

    c. $14 + (-8)$

    d. $(-14) + (-8)$

2. Which other expression has the same value as $(-14) - (-8)$? Explain your reasoning.

    a. $(-14) + 8$

    b. $14 - (-8)$

    c. $14 + (-8)$

    d. $(-14) + (-8)$

Student Response

1. $(-14) + (-8)$. Sample explanation: because adding -8 is the same as subtracting 8

2. $(-14) + 8$. Sample explanation: because subtracting -8 is the same as adding 8

Student Lesson Summary

The equation $7 - 5 = ?$ is equivalent to $? + 5 = 7$. The diagram illustrates the second equation.
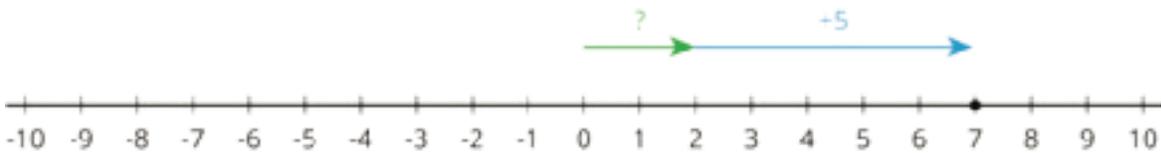
Notice that the value of $7 + (\text{-}5)$ is 2.
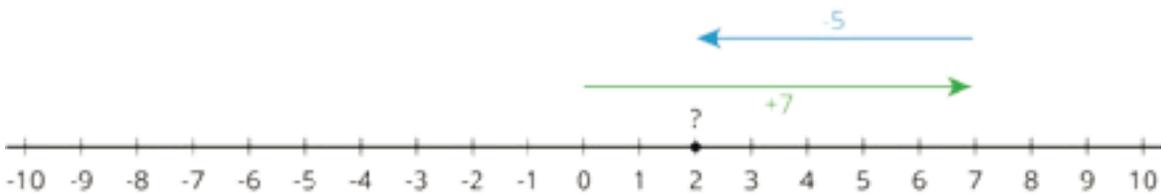
We can solve the equation $? + 5 = 7$ by adding -5 to both sides. This shows that
$7 - 5 = 7 + (\text{-}5)$

Likewise, $3 - 5 = ?$ is equivalent to $? + 5 = 3$.
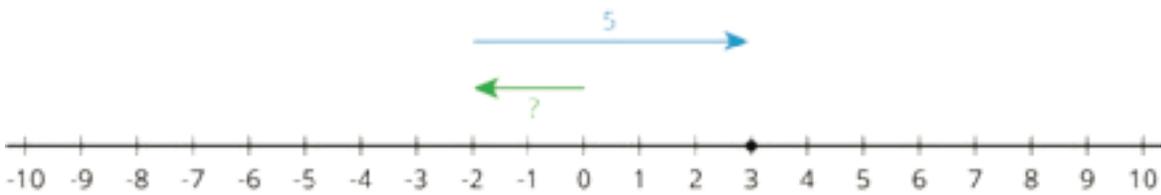
Notice that the value of $3 + (\text{-}5)$ is -2.
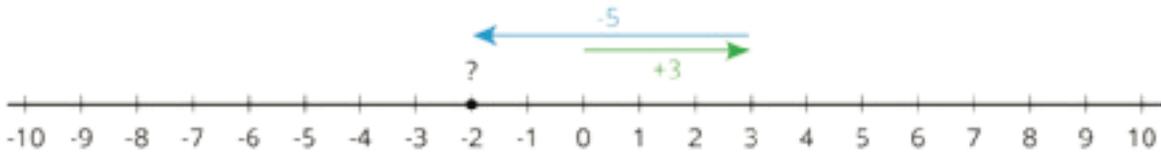
We can solve the equation $? + 5 = 3$ by adding -5 to both sides. This shows that
$3 - 5 = 3 + (\text{-}5)$

In general:

$$a - b = a + (\text{-}b)$$

If $a - b = x$, then $x + b = a$. We can add -b to both sides of this second equation to get that
$x = a + (\text{-}b)$

Lesson 5 Practice Problems
Problem 1
Statement
Write each subtraction equation as an addition equation.

    a. $a - 9 = 6$

    b. $p - 20 = \text{-}30$

    c. $z - (\text{-}12) = 15$

    d. $x - (\text{-}7) = \text{-}10$

Solution

    a. $a = 6 + 9$ (or $a = 9 + 6$)

    b. $p = \text{-}30 + 20$ (or $p = 20 + \text{-}30$)

    c. $z = 15 + (\text{-}12)$ (or $z = (\text{-}12) + 15$)

    d. $x = \text{-}10 + (\text{-}7)$ (or $x = \text{-}7 + (\text{-}10)$)

Problem 2
Statement
Find each difference. If you get stuck, consider drawing a number line diagram.

    a. $9 - 4$

    b. $4 - 9$

    c. $9 - (\text{-}4)$

    d. $\text{-}9 - (\text{-}4)$

    e. $\text{-}9 - 4$

    f. $4 - (\text{-}9)$

    g. $\text{-}4 - (\text{-}9)$

    h. $\text{-}4 - 9$

Solution

    a. 5

    b. -5

c. 13

d. -5

e. -13

f. 13

g. 5

h. -13

Problem 3

Statement

A restaurant bill is $59 and you pay $72. What percentage gratuity did you pay?

Solution

22%, because $13 \div 59 \approx 0.22$.

(From Unit 4, Lesson 10.)

Problem 4

Statement

Find the solution to each equation mentally.

a. $30 + a = 40$

b. $500 + b = 200$

c. $-1 + c = -2$

d. $d + 3{,}567 = 0$

Solution

a. $a = 10$

b. $b = -300$

c. $c = -1$

d. $d = -3{,}567$

Problem 5

Statement

One kilogram is 2.2 pounds. Complete the tables. What is the interpretation of the constant of proportionality in each case?

pounds	kilograms
2.2	1
11	
5.5	
1	

_____ kilogram per pound

kilograms	pounds
1	2.2
7	
30	
0.5	

_____ pounds per kilogram

Solution

Complete the tables. What is the interpretation of the constant of proportionality in each case?

a.

pounds	kilograms
2.2	1
11	5
5.5	2.5
1	0.45

0.45 kilogram per pound

Unit 5 Lesson 5

86

b.

kilograms	pounds
1	2.2
7	15.4
30	66
0.5	1.1

2.2 pounds per kilogram

(From Unit 2, Lesson 3.)

Lesson 6: Subtracting Rational Numbers

Goals

- Compare and contrast (orally) subtraction expressions that have the same numbers in the opposite order.

- Recognize that the "difference" of two numbers can be positive or negative, depending on the order they are listed, while the "distance" between two numbers is always positive.

- Subtract signed numbers, and explain (orally) the reasoning.

Learning Targets

- I can find the difference between two rational numbers.

- I understand how to subtract positive and negative numbers in general.

Lesson Narrative

In this lesson, students see that the *difference* between two numbers can be positive or negative, but the *distance* between two numbers is always positive. Using the geometry of the number line (MP7), they see that if you switch the order in which you subtract two numbers, the difference becomes its opposite.

For example, to find the difference in temperature between +70˚C and +32˚C we calculate $70 - 32 = 38$, so the difference is 38˚C. The distance between these two is also 38˚C. On the other hand, to find the difference in temperature between +32˚C and +70˚C we calculate $32 - 70 = -38$, so the difference is -38˚C. The distance is still 38˚C. In general, if $a - b = x$, then $b - a = -x$. By observing the outcome of several examples, students may conjecture that this is always true (MP8).

Alignments

Building On

- 6.EE.B: Reason about and solve one-variable equations and inequalities.

Addressing

- 7.NS.A.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

- 7.NS.A.1.c: Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

Building Towards

- 7.NS.A.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Instructional Routines

- MLR3: Clarify, Critique, Correct

- MLR8: Discussion Supports

- Number Talk

- Think Pair Share

Required Materials

Four-function calculators

Required Preparation

Use of calculators is optional. In this lesson, the important insights come from observing the outcome of evaluating expressions. Practice evaluating the expressions is of secondary importance.

> **Student Learning Goals**
>
> Let's bring addition and subtraction together.

6.1 Number Talk: Missing Addend

Warm Up: 5 minutes
The purpose of this number talk is to remind students about reasoning to find a missing addend and to rewrite each addition equation using subtraction. In this case, each problem is presented as an equation to solve. Previously in this unit, we have represented unknown values with question marks. Here, the unknown value is represented with a letter.

It may not be possible to share every possible strategy for the given limited time. Consider gathering only two distinctive strategies per problem.

Building On

- 6.EE.B

Building Towards

- 7.NS.A.1

Instructional Routines

- MLR8: Discussion Supports

- Number Talk

Launch

Display one problem at a time. Give students 30 seconds of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organization

> **Student Task Statement**
>
> Solve each equation mentally. Rewrite each addition equation as a subtraction equation.
>
> $247 + c = 458$
>
> $c + 43.87 = 58.92$
>
> $\frac{15}{8} + c = \frac{51}{8}$

Student Response

-
  - $c = 211$ Possible strategies:
    - Working backwards, I can subtract $458 - 247$ to find the missing addend 211.

    - I can make several guesses in a row until I find the right number:
      If $c = 200$, then $247 + 200 = 447$, which is too small.
      If $c = 210$, then $247 + 210 = 457$, which is still a little bit too small.
      If $c = 211$, then $247 + 211 = 458$, which is just right.

  - $c = 15.05$ Possible strategy:
    - $43.87 + 5 = 48.87$
      $48.87 + 10 = 58.87$
      $58.87 + 0.05 = 58.92$
      And then $5 + 10 + 0.05 = 15.05$

  - $c = \frac{36}{8}$ or equivalent. Possible strategy: $51 - 15 = 36$

-
  - $458 - 247 = 211$ (or $458 - 211 = 247$)

  - $58.92 - 43.87 = 15.05$ (or $58.92 - 15.05 = 43.87$)

  - $\frac{51}{8} - \frac{15}{8} = \frac{36}{8}$ (or $\frac{51}{8} - \frac{36}{8} = \frac{15}{8}$)

Unit 5 Lesson 6

Activity Synthesis

Ask students to share their reasoning. Record and display the responses for all to see. If students begin to talk about the distance between the given addend and sum when finding c, it may be helpful to draw a number line to represent their thinking. To involve more students in the conversation, use some of the following questions:

- "Who can restate ___'s reasoning in a different way?"

- "Did anyone find the value of n the same way, but would explain it differently?"

- "Did anyone find the value of n in a different way?"

- "Does anyone want to add on to _____'s strategy?"

- "Do you agree or disagree? Why?"

Access for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimize output (for explanation)

6.2 Expressions with Altitude

10 minutes
In this activity, students return to the familiar context of climbing up and down a cliff to apply what they have learned about subtracting signed numbers. They represent the change in elevation with an expression and then calculate the value of the expression. This activity does not provide a number line diagram or ask students to draw one, but some students may still choose to do so.

In this activity students are introduced to the idea that to find the difference between two values, we subtract one from the other. They use the context to make sure the order of the numbers in the subtraction expression correct. In the next activity, they attend to this explicitly in the abstract.

In this activity, no scaffolding is given, and students are free to use any strategy to find the differences.

Addressing
- 7.NS.A.1.c

Instructional Routines
- MLR3: Clarify, Critique, Correct

Launch

Arrange students in groups of 2. Give them 3 minutes of quiet work time, then have them check their progress with their partner. After students have come to agreement about the first few, they should finish the remainder. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Provide appropriate reading accommodations and supports to ensure students have access to written directions, word problems and other text-based content.
Supports accessibility for: Language; Conceptual processing

Anticipated Misconceptions

Some students may wonder why they are being asked to solve the same problems that they already have. Point out that they have learned new ways of representing these problems than what they did previously.

Student Task Statement

A mountaineer is changing elevations. Write an expression that represents the difference between the final elevation and beginning elevation. Then write the value of the change. The first one is done for you.

beginning elevation (feet)	final elevation (feet)	difference between final and beginning	change
+400	+900	$900 - 400$	+500
+400	+50		
+400	-120		
-200	+610		
-200	-50		
-200	-500		
-200	0		

beginning elevation (feet)	final elevation (feet)	difference between final and beginning	change
+400	+900	$900 - 400$	+500
+400	+50	$50 - 400$	-350
+400	-120	$-120 - 400$	-520
-200	+610	$610 - (-200)$	+810
-200	-50	$-50 - (-200)$	+150
-200	-500	$-500 - (-200)$	-300
-200	0	$0 - (-200)$	200

Are You Ready for More?

Fill in the table so that every row and every column sums to 0. Can you find another way to solve this puzzle?

	-12	0		5
0			-18	25
25		-18	5	-12
-12				-18
	-18	25	-12	

	-12	0		5
0			-18	25
25		-18	5	-12
-12				-18
	-18	25	-12	

Student Response

Answers vary. Sample responses:

-18	-12	0	25	5
0	5	-12	-18	25
25	0	-18	5	-12
-12	25	5	0	-18
5	-18	25	-12	0

-18	-12	0	25	5
0	-7	0	-18	25
25	0	-18	5	-12
-12	37	-7	0	-18
5	-18	25	-12	0

Activity Synthesis

Students should understand that to find the difference between two numbers, we subtract. Be sure they attend to the order in which the numbers appear in the subtraction expressions: the final elevation always comes first because the question asked for the difference between the final and the beginning elevations. Also reinforce the notion that to subtract a number, we can add its opposite. For familiar problems like $900 - 400$, this isn't necessary. But for problems like $610 - (\text{-}200)$ it is easier for some people than going through the process of reasoning about it as an addition problem like $? + \text{-}200 = 610$, although this is always an option, and it is good to reinforce that we get the same answer whenever students choose to solve it this way.

Draw attention to the final three lines in the table, which all involve subtracting a negative number. Make sure that students see that subtracting a negative results in the same answer as adding its opposite.

6.3 Does the Order Matter?

10 minutes
In this activity, students see that if you reverse the order of the two numbers in a subtraction expression, you get the same magnitude with the opposite sign (MP8).

For students who might overly struggle to evaluate the expressions, consider providing access to a calculator and showing them how to enter a negative value. The important insight here is the outcome of evaluating the expressions. Practice evaluating the expressions is of lesser importance.

In this activity, no supports are given or suggested and students are free to use any strategy to find the differences.

Addressing

- 7.NS.A.1

Instructional Routines

- MLR8: Discussion Supports

- Think Pair Share

Launch

Before working with the subtraction expressions in the task statement, consider telling students to close their books or devices and display these addition expressions for all to see. Discuss whether the order of the addends matters when adding signed numbers.

A	B
$3 + 2$	$2 + 3$
$5 + (\text{-}9)$	$(\text{-}9) + 5$
$(\text{-}11) + 2$	$2 + (\text{-}11)$
$(\text{-}6) + (\text{-}3)$	$(\text{-}3) + (\text{-}6)$
$(\text{-}1.2) + (\text{-}3.6)$	$(\text{-}3.6) + (\text{-}1.2)$
$(\text{-}2\frac{1}{2}) + (\text{-}3\frac{1}{2})$	$(\text{-}3\frac{1}{2}) + (\text{-}2\frac{1}{2})$

Arrange students in groups of 2. Give students quiet work time followed by partner and whole-class discussion.

Anticipated Misconceptions

Some students may try to interpret each subtraction expression as an addition equation with a missing addend and struggle to calculate the correct answer. Remind them that we saw another way to evaluate subtraction is by adding the additive inverse. Consider demonstrating how one of the subtraction expressions can be rewritten (e.g. $-11 - 2 = -11 + (-2)$).

Some students may struggle with deciding whether to add or subtract the magnitudes of the numbers in the problem. Prompt them to sketch a number line diagram and notice how the arrows compare.

Student Task Statement

1. Find the value of each subtraction expression.

A
$3 - 2$
$5 - (\text{-}9)$
$(\text{-}11) - 2$
$(\text{-}6) - (\text{-}3)$
$(\text{-}1.2) - (-3.6)$
$(\text{-}2\frac{1}{2}) - (\text{-}3\frac{1}{2})$

B
$2 - 3$
$(\text{-}9) - 5$
$2 - (\text{-}11)$
$(\text{-}3) - (\text{-}6)$
$(\text{-}3.6) - (\text{-}1.2)$
$(\text{-}3\frac{1}{2}) - (\text{-}2\frac{1}{2})$

2. What do you notice about the expressions in Column A compared to Column B?

3. What do you notice about their values?

Student Response

1.

A	B
1	-1
14	-14
-13	13
-3	3
2.4	-2.4
1	-1

2. Answers vary. Sample response: The numbers are the same, but they are subtracted in the opposite order.

3. Answers vary. Sample reponse: They are opposites (additive inverses).

Activity Synthesis

The most important thing for students to understand is that changing the order of the two numbers being subtracted will give the additive inverse of the original difference: $a - b = \text{-}(b - a)$. The two differences have the same magnitude but opposite signs. On a number line diagram, the arrows are the same length but pointing in opposite directions.

Consider displaying these unfinished number line diagrams as specific examples that students can refer to during the whole-class discussion:

-11 + 2
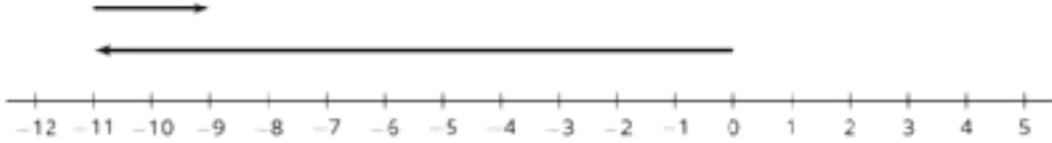
2 + (-11)
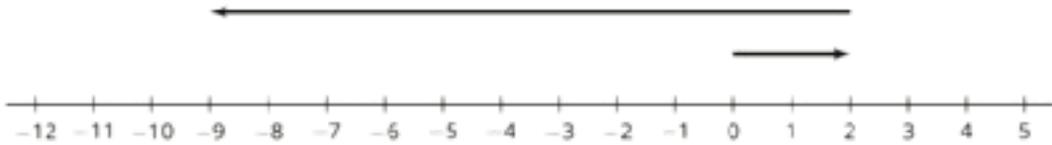
-11 − 2

2 − (-11)
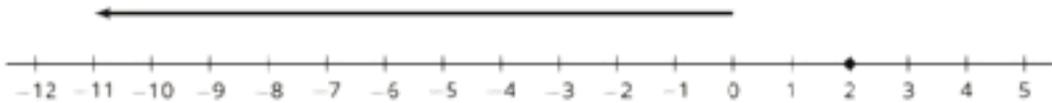
Discuss:

- Does changing the order of the numbers in an addition expression change the value? Why?

- Does changing the order of the numbers in a subtraction expression change the value? Why?

Unit 5 Lesson 6

Lesson Synthesis

Main takeaways:

- The difference between two numbers can be positive or negative, depending on their order: $(a - b) = \text{-}(b - a)$.

- The distance between two numbers is always positive. It does not depend on their order, because it is the magnitude of the difference: $|a - b| = |b - a|$.

Discussion questions:

- What is the difference between 12 and 10? ($12 - 10 = 2$)

- What is the difference between 10 and 12? ($10 - 12 = -2$)

- What is the distance between 12 and 10? ($|2| = 2$)

- What is the distance between 10 and 12? ($|\text{-}2| = 2$)

6.4 A Subtraction Expression

Cool Down: 5 minutes
Addressing

- 7.NS.A.1.c

Student Task Statement

Select all of the choices that are equal to $(\text{-}5) - (\text{-}12)$.

1. -7

2. 7

3. The difference between -5 and -12.

4. The difference between -12 and -5.

5. $(\text{-}5) + 12$

6. $(\text{-}5) + (\text{-}12)$

Student Response

7; The difference between -5 and -12; $\text{-}5 + 12$

Student Lesson Summary

When we talk about the difference of two numbers, we mean, "subtract them." Usually, we subtract them in the order they are named. For example, the difference of +8 and -6 is $8 - (\text{-}6)$.

The difference of two numbers tells you how far apart they are on the number line. 8 and -6 are 14 units apart, because $8 - (-6) = 14$:
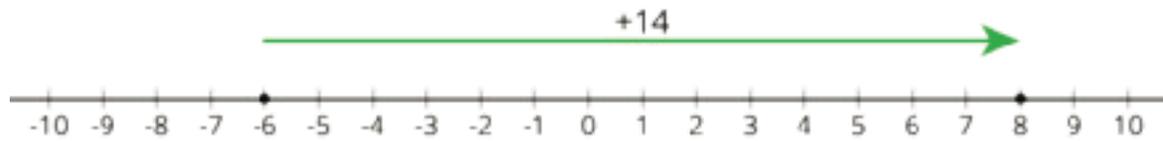
Notice that if you subtract them in the opposite order, you get the opposite number:

$$(-6) - 8 = -14$$

In general, the distance between two numbers a and b on the number line is $|a - b|$. Note that the *distance* between two numbers is always positive, no matter the order. But the *difference* can be positive or negative, depending on the order.

Lesson 6 Practice Problems
Problem 1
Statement
Write a sentence to answer each question:

    a. How much warmer is 82 than 40?

    b. How much warmer is 82 than -40?

Solution
Answers vary. Possible responses:

  a. 82 is 42 degrees warmer than 40.

  b. 82 is 122 degrees warmer than -40.

Problem 2
Statement
  a. What is the difference in height between 30 m up a cliff and 87 m up a cliff? What is the distance between these positions?

  b. What is the difference in height between an albatross flying at 100 m above the surface of the ocean and a shark swimming 30 m below the surface? What is the distance between them if the shark is right below the albatross?

Solution
  a. The difference in height is -57 m, because $30 - 87 =$ -57. The distance is 57 m, because $|\text{-}57| = 57$.

  b. The difference in height is 130 m, because $100 - (\text{-}30) = 130$. The distance is 130 m, because $|130| = 130$.

Problem 3
Statement
A company produces screens of different sizes. Based on the table, could there be a relationship between the number of pixels and the area of the screen? If so, write an equation representing the relationship. If not, explain your reasoning.

square inches of screen	number of pixels
6	31,104
72	373,248
105	544,320
300	1,555,200

Solution

It is a proportional relationship which can be represented as $p = 5{,}184 \cdot a$ where p represents the number of pixels and a is the area of the screen in square inches.

(From Unit 2, Lesson 8.)

Problem 4

Statement

Find each difference.

- $(-5) - 6$
- $35 - (-8)$

- $\frac{2}{5} - \frac{3}{5}$
- $-4\frac{3}{8} - (-1\frac{1}{4})$

Solution

a. -11

b. 43

c. $-\frac{1}{5}$

d. $-3\frac{1}{8}$

Problem 5

Statement

A family goes to a restaurant. When the bill comes, this is printed at the bottom of it:

Gratuity Guide For Your Convenience:
15% would be $4.89
18% would be $5.87
20% would be $6.52

How much was the price of the meal?
Explain your reasoning.

Solution

The bill was close to $32.60. We can't tell the exact amount because the suggested dollar amounts have been rounded to the hundredths place. $4.89 \div 0.15 = 32.60$, $5.87 \div 0.18 = 32.61$, and $6.52 \div 0.2 = 32.60$. The first and third quotients are exact while the middle quotient is rounded to the nearest hundredth.

(From Unit 4, Lesson 10.)

Problem 6

Statement

Which is a scaled copy of Polygon A? Identify a pair of corresponding sides and a pair of corresponding angles. Compare the areas of the scaled copies.
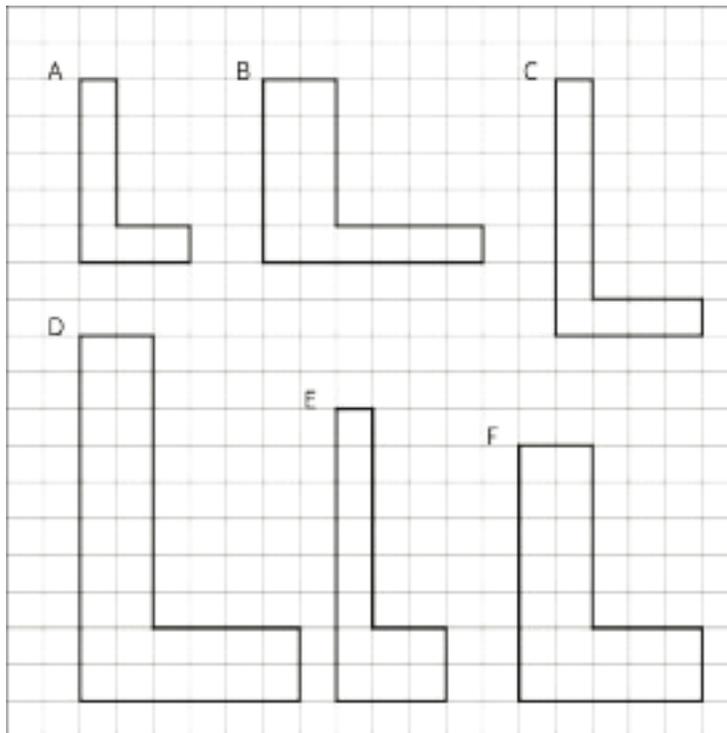
Solution

Polygon D is a scaled copy of Polygon A. This is true because all of the side lengths are doubled. The area of Polygon D is 4 times the area of Polygon A.

Answers vary. Sample markings:
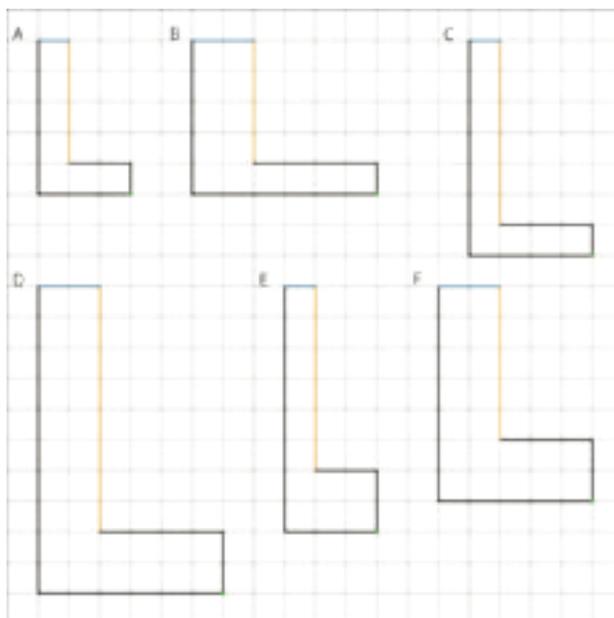
(From Unit 1, Lesson 2.)

Lesson 7: Adding and Subtracting to Solve Problems

Goals

- Apply addition and subtraction of signed numbers to solve problems in an unfamiliar context, and explain (orally and in writing) the solution method.

- Interpret signed numbers used to represent gains or losses in an unfamiliar context.

Learning Targets

- I can solve problems that involve adding and subtracting rational numbers.

Lesson Narrative

The purpose of this lesson is to put students' knowledge about addition and subtraction of signed numbers to use in real-life contexts. They work with tables that show the change, positive or negative, in quantities such as inventory or energy usage, and must make sense of these tables to answer questions about the context. An optional activity extends students' work with signed numbers on the number line to points in all four quadrants of the coordinate plane.

As students reason about quantities using signed numbers they engage in MP2.

Alignments

Building On

- 7.NS.A.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Addressing

- 7.NS.A.1.c: Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

- 7.NS.A.3: Solve real-world and mathematical problems involving the four operations with rational numbers.Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Building Towards

- 7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- MLR6: Three Reads

- MLR8: Discussion Supports
- Think Pair Share

> **Student Learning Goals**
>
> Let's apply what we know about signed numbers to different situations.

7.1 Positive or Negative?

Warm Up: 5 minutes
The purpose of this warm-up is to have students reason about an equation involving positive and negative rational numbers using what they have learned about operations with rational numbers.

Building On

- 7.NS.A.1

Building Towards

- 7.EE.B.4

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 30 seconds of quiet think time and ask them to give a signal when they have an answer and a strategy for the first question. Then have them discuss their reasoning with a partner. Ask for an explanation, and then ask if everyone agrees with that reasoning.

Then give students 30 seconds of quiet think time and ask them to give a signal when they have an answer for the second question. Then have them discuss their reasoning with a partner.

> **Student Task Statement**
>
> Without computing:
>
> 1. Is the solution to $-2.7 + x = -3.5$ positive or negative?
>
> 2. Select **all** the expressions that are solutions to $-2.7 + x = -3.5$.
>
>   a. $-3.5 + 2.7$
>
>   b. $3.5 - 2.7$
>
>   c. $-3.5 - (-2.7)$
>
>   d. $-3.5 - 2.7$

Student Response

1. The solution is negative.

2. $-3.5 + 2.7$ and $-3.5 - (-2.7)$

Activity Synthesis

Ask several student to share which expressions they chose for the second question. Discuss until everyone is in agreement about the answer to the second question.

7.2 Phone Inventory

10 minutes
Positive and negative numbers are often used to represent *changes* in a quantity. An increase in the quantity is positive, and a decrease in the quantity is negative. In this activity, students see an example of this convention and are asked to make sense of it in the given context.

Addressing

- 7.NS.A.3

Instructional Routines

- MLR8: Discussion Supports

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 30 seconds of quiet work time followed by 1 minute of partner discussion for the first two problems. Briefly, ensure everyone agrees on the interpretation of positive and negative numbers in this context, and then invite students to finish the rest of the questions individually. Follow with whole-class discussion.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Use virtual or concrete manipulatives to connect symbols to concrete objects or values. For example, demonstrate "change" by adding or taking away phones in a whole-class discussion.
Supports accessibility for: Visual-spatial processing; Conceptual processing

Student Task Statement

A store tracks the number of cell phones it has in stock and how many phones it sells.

The table shows the inventory for one phone model at the beginning of each day last week. The inventory changes when they sell phones or get shipments of phones into the store.

	inventory	change
Monday	18	-2
Tuesday	16	-5
Wednesday	11	-7
Thursday	4	-6
Friday	-2	20

1. What do you think it means when the change is positive? Negative?

2. What do you think it means when the inventory is positive? Negative?

3. Based on the information in the table, what do you think the inventory will be at on Saturday morning? Explain your reasoning.

4. What is the difference between the greatest inventory and the least inventory?

Student Response

Answers vary. Sample responses:

1. The inventory increases; the inventory decreases.

2. There are phones in the store that people can buy; someone ordered a phone but they are waiting for one to come into the store.

3. Sample response: I think the inventory will be 18, because the inventory one day is the sum of the inventory and the change on the previous day.

Unit 5 Lesson 7

108

4. The difference is 20: $18 - (\text{-}2) = 20$

Activity Synthesis

Tell students that we often use positive and negative to represent changes in a quantity. Typically, an increase in the quantity is positive, and a decrease in the quantity is negative.

Ask students what they answered for the second question and record their responses. Highlight one or two that describe the situation clearly.

Ask a few students to share their answer to the third question, and discuss any differences. Then discuss the answer to the last question.

7.3 Solar Power

15 minutes
It is common to use positive numbers to represent credit and negative numbers to represent debts on a bill. This task introduces students to this convention and asks them to solve addition and subtraction questions in that context. Note that whether a number should be positive or negative is often a choice, which means one must be very clear about explaining the interpretation of a signed number in a particular context (MP6).

For the second question, monitor for students who find the amount each week and sum those, and students who sum the value of the electricity used and the value of the electricity generated separately, and then find the sum of those.

Addressing

- 7.NS.A.3

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- MLR6: Three Reads

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 4 minutes of quiet work time followed by partner discussion. Follow with a whole-class discussion.

Student Task Statement

Han's family got a solar panel. Each month they get a credit to their account for the electricity that is generated by the solar panel. The credit they receive varies based on how sunny it is.

Here is their electricity bill from January.

Current charges: $83.56
Solar Credit: -$6.75
Amount due: $76.81

In January they used $83.56 worth of electricity and generated $6.75 worth of electricity.

1. In July they were traveling away from home and only used $19.24 worth of electricity. Their solar panel generated $22.75 worth of electricity. What was their amount due in July?

2. The table shows the value of the electricity they used and the value of the electricity they generated each week for a month. What amount is due for this month?

	used ($)	generated ($)
week 1	13.45	-6.33
week 2	21.78	-8.94
week 3	18.12	-7.70
week 4	24.05	-5.36

3. What is the difference between the value of the electricity generated in week 1 and week 2? Between week 2 and week 3?

Student Response

1. -$3.51

2. $49.07

3. Between week 1 and week 2 it is $2.61. Between week 2 and week 3 it is -$1.24.

Are You Ready for More?

While most rooms in any building are all at the same level of air pressure, hospitals make use of "positive pressure rooms" and "negative pressure rooms." What do you think it means to have negative pressure in this setting? What could be some uses of these rooms?

Student Response

Here the pressure of a room is being measured relative to the air pressure outside of the room, by taking the quantity

$$(\text{air pressure inside}) - (\text{air pressure outside})$$

So a positive pressure room is one where there air pressure inside the room is greater than the air pressure outside the room, and the reverse for negative pressure rooms. In positive pressure rooms, air does not naturally flow into the room, so it is a good place to keep patients who have a weakened immune system and are very susceptible to getting infected by airborne diseases. In negative pressure rooms, air does not naturally flow out of the room, so it is a good place to keep patients who are highly contagious.

Activity Synthesis

Ask one or more students to share their answer to the first question and resolve any discrepancies.

Ask selected students to share their reasoning for the second questions. Discuss the relative merits of different approaches to solving the problem.

Finish by going over the solution to the third question. Point out that the bill will reflect a negative number in the amount due section, but we can interpret this to mean that the family receives a credit, and it will be applied to their next bill.

7.4 Differences and Distances

Optional: 15 minutes (there is a digital version of this activity)
In grade 6, students practiced finding the horizontal or vertical distance between points on a coordinate plane. In this activity, students see that this can be done by subtracting the x or y-coordinates for the points (MP7). Students continue to work with the distinction between *distance* (which is unsigned) and *difference* (which is signed) (MP6). This prepares them finding the slope of a line and the diagonal distance between points in grade 8.

Addressing

- 7.NS.A.1.c

- 7.NS.A.3

Instructional Routines

- MLR8: Discussion Supports

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 3 minutes of quiet work time followed by partner discussion. Follow with a whole-class discussion.

> **Student Task Statement**
>
> Plot these points on the coordinate grid: $A = (5, 4), B = (5, \text{-}2), C = (\text{-}3, \text{-}2), D = (\text{-}3, 4)$
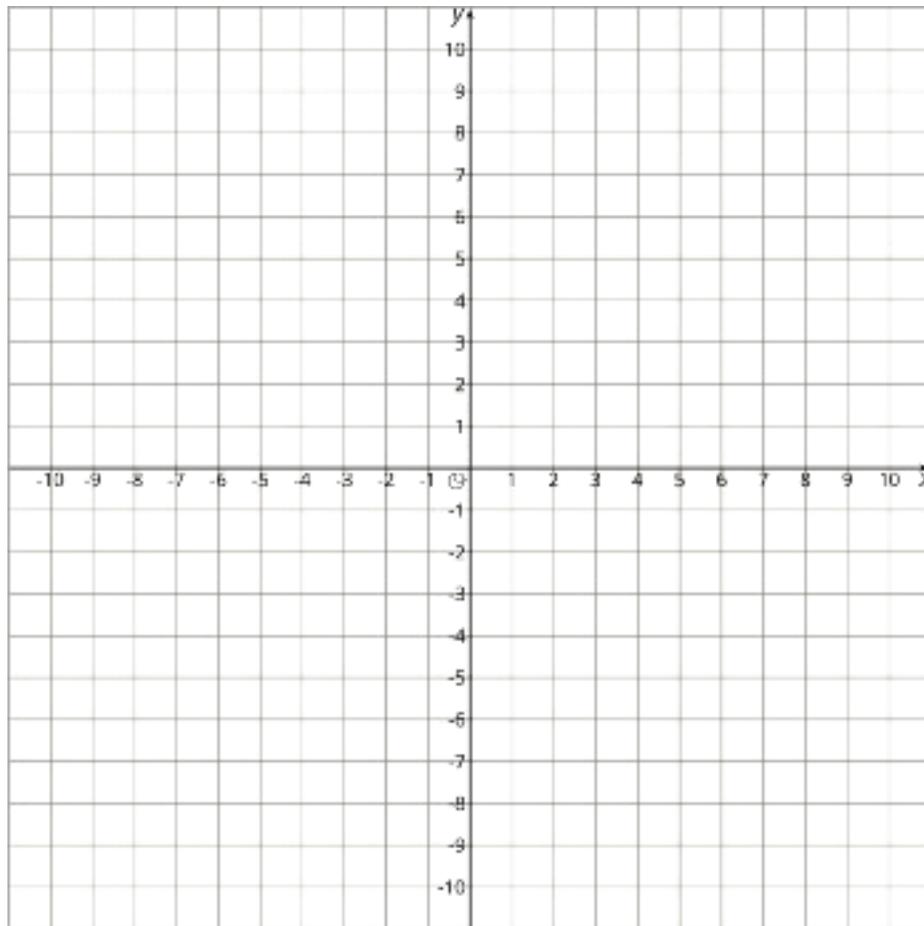
1. What shape is made if you connect the dots in order?

2. What are the side lengths of figure $ABCD$?

3. What is the difference between the x-coordinates of B and C?

4. What is the difference between the x-coordinates of C and B?

5. How do the differences of the coordinates relate to the distances between the points?

Student Response

1. a rectangle

2. 6 and 8

3. 8

4. -8

5. The absolute value of the difference is the distance.

Activity Synthesis

Main learning points:

- When two points in the coordinate plane lie on a horizontal line, you can find the distance between them by subtracting their x-coordinates.

- When two points in the coordinate plane lie on a vertical line, you can find the distance between them by subtracting their y-coordinates.

- The distance between two numbers is independent of the order, but the difference depends on the order.

Discussion questions:

- Explain what makes the distance between two points and the difference between two points distinct.

- Explain how you would find the vertical or horizontal distance between two points.

- Explain how you would find the vertical or horizontal difference between two points.

Access for English Language Learners

Representing, Speaking: *MLR8 Discussion Supports.* Before the whole-class discussion, invite students to discuss and prepare their responses to the discussion questions listed in the synthesis. Display the questions for all to see, and provide sentence frames that students can use to explain their reasoning. For example, "To find the vertical (or horizontal) distance between two points, first we ____, and then we ____.", and "To find the vertical (or horizontal) difference) between two points, first we ____, and then we ____." Listen for and amplify the ways students describe the distinction between distance (which is unsigned) and difference (which is signed). This opportunity to prepare in advance will provide students with additional opportunities to clarify their thinking, and to consider how they will communicate their reasoning.
Design Principle(s): *Support sense-making*

Lesson Synthesis

What are some situations where adding and subtracting rational numbers can help us solve problems?

7.5 Coffee Shop Cups

Cool Down: 5 minutes
Addressing

- 7.NS.A.3

Student Task Statement

Here is some record keeping from a coffee shop about their paper cups. Cups are delivered 2,000 at a time.

day	change
Monday	+2000
Tuesday	-125
Wednesday	-127
Thursday	+1719
Friday	-356
Saturday	-782
Sunday	0

1. Explain what a positive and negative number means in this situation

2. Assume the starting amount of coffee cups is 0. How many paper cups are left at the end of the week?

3. How many cups do you think were used on Thursday? Explain how you know.

Student Response

1. Interpretations vary. Positive might mean number of cups delivered or delivered minus used, negative might mean used up.

2. 2329

3. 281. Explanations vary. Sample explanation: It looks like some were delivered and some were used. Since they are delivered 2,000 at a time, then $2,000 - 1,719$ would be the number used.

Student Lesson Summary

Sometimes we use positive and negative numbers to represent quantities in context. Here are some contexts we have studied that can be represented with positive and negative numbers:

- temperature
- elevation
- inventory
- an account balance
- electricity flowing in and flowing out

In these situations, using positive and negative numbers, and operations on positive and negative numbers, helps us understand and analyze them. To solve problems in these situations, we just have to understand what it means when the quantity is positive, when it is negative, and what it means to add and subtract them.

When two points in the coordinate plane lie on a horizontal line, you can find the distance between them by subtracting their x-coordinates.

When two points in the coordinate plane lie on a vertical line, you can find the distance between them by subtracting their y-coordinates.
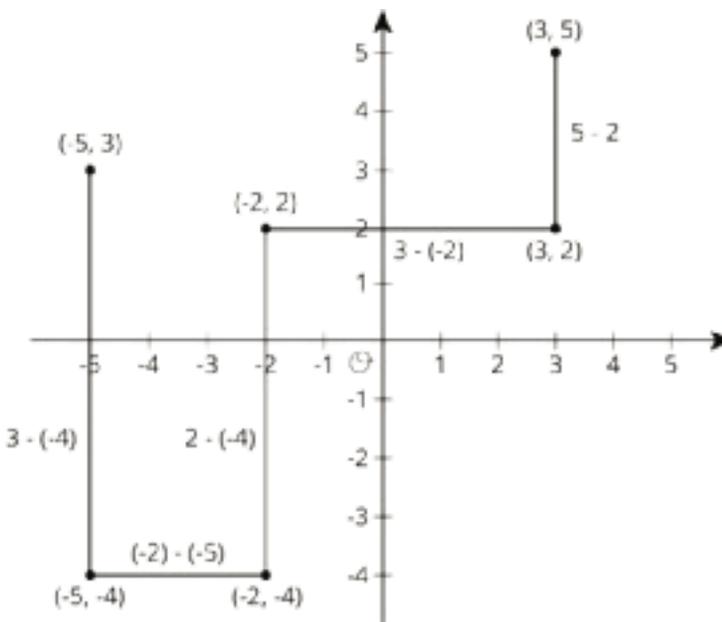
Remember: the *distance* between two numbers is independent of the order, but the *difference* depends on the order.

Lesson 7 Practice Problems
Problem 1

Statement

The table shows four transactions and the resulting account balance in a bank account, except some numbers are missing. Fill in the missing numbers.

	transaction amount	account balance
transaction 1	360	360
transaction 2	-22.50	337.50
transaction 3		182.35
transaction 4		-41.40

Solution

	transaction amount	account balance
transaction 1	360	360
transaction 2	-22.50	337.50
transaction 3	-155.15	182.35
transaction 4	-223.75	-41.40

Problem 2

Statement

The *departure from the average* is the difference between the actual amount of rain and the average amount of rain for a given month. The historical average for rainfall in Albuquerque, NM for June, July, and August is shown in the table.

June	July	August
0.67	1.5	1.57

a. Last June only 0.17 inches of rain fell all month. What is the difference between the average rainfall and the actual rainfall for last June?

> b. The departure from the average rainfall last July was -0.36 inches. How much rain fell last July?
>
> c. How much rain would have to fall in August so that the total amount of rain equals the average rainfall for these three months? What would the departure from the average be in August in that situation?

Solution

1. 0.5 inches. $0.67 - 0.17 = 0.50$
2. 1.14 inches. The departure from average was negative, the actual rainfall needed to be 0.36 inches less than the average rainfall.
3. 2.43 inches; 0.86 inches. The departure from the average in June was -0.5 and for July was -0.36, so for those two months it was $-0.5 + (-0.36) = -0.86$. So it will have to rain 0.86 more inches in August than usual to make that up and the departure from the average will be 0.86. $0.86 + 1.57 = 2.43$.

Problem 3

Statement

> a. How much higher is 500 than 400 m?
>
> b. How much higher is 500 than -400 m?
>
> c. What is the change in elevation from 8,500 m to 3,400 m?
>
> d. What is the change in elevation between 8,500 m and -300 m?
>
> e. How much higher is -200 m than 450 m?

Solution

a. 100 m, because $400 + 100 = 500$

b. 900 m, because $-400 + 900 = 500$

c. -5,100 m, because $8,500 + (-5,100) = 3,400$

d. -8,800 m, because $8,500 + (-8,800) = -300$

e. -650 m, because $450 + (-650) = -200$

(From Unit 5, Lesson 6.)

Problem 4

Statement

Tyler orders a meal that costs $15.

> a. If the tax rate is 6.6%, how much will the sales tax be on Tyler's meal?

b. Tyler also wants to leave a tip for the server. How much do you think he should pay in all? Explain your reasoning.

Solution

a. $0.99 because $15 \cdot 0.066 = 0.99$

b. Answers vary. Sample response: I think Tyler should pay $19 in all, because that would cover his meal, the sales tax, and a 20% tip for the server.

(From Unit 4, Lesson 10.)

Problem 5

Statement

In a video game, a character is healed at a constant rate as long as they are standing in a certain circle. Complete the table.

time in circle (seconds)	health gained (points)
4	100
10	
3	
	1,000

Solution

seconds in circle	health gained
4	100
10	250
3	75
40	1,000

(From Unit 2, Lesson 3.)

Section: Multiplying and Dividing Rational Numbers

Lesson 8: Position, Speed, and Direction

Goals

- Explain (orally and in writing) how signed numbers can be used to represent positions and speeds in opposite directions.

- Generalize (orally) that the product of a negative number and a positive number is negative.

- Write a multiplication equation to represent a situation involving constant speed with direction.

Learning Targets

- I can multiply a positive number with a negative number.

- I can use rational numbers to represent speed and direction.

Lesson Narrative

In this lesson, students are introduced to multiplying a negative number with a positive number, using the context of velocity, time, and position. In the next lesson, they multiply two negative numbers.

The context of elevation is an example of using signed numbers to represent the position of an object along a line relative to a reference position (sea level in the case of elevation). In the general case, zero represents the reference position, positive numbers represent positions on one side of the reference position, and negative numbers represent positions on the other side. In this lesson, students see that signed numbers can also be used to represent *speed with direction*. Scientists use the term *velocity* to describe the speed of an object in a specified direction. If one object is moving with a positive velocity, then any object moving in the opposite direction will have a negative velocity.

In previous units, students solved problems about moving objects, using the fact that the product of the (positive) speed and the (positive) travel time gives the (positive) distance traveled. In this lesson, students use several examples in the context of moving along a line to see that the product of a *negative* velocity and a positive travel time results in a *negative* position relative to the starting point.

Alignments

Addressing

- 7.NS.A.2.a: Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Instructional Routines

- MLR2: Collect and Display

- MLR8: Discussion Supports

Student Learning Goals

Let's use signed numbers to represent movement.

8.1 Distance, Rate, Time

Warm Up: 5 minutes
This activity reminds students of previous work they have done with constant speed situations, using $d = rt$ for the relationship between distance, rate, and time. This prepares students for representing movement in opposite directions using signed numbers in the rest of this lesson.

Students may fall back to earlier methods to make sense of these problems and come up with a solution, like creating a double number line or a table of equivalent ratios relating distance and time. These approaches are fine. In the discussion, though, ensure everyone understands using $d = rt$ to represent the relationship between distance traveled, elapsed time, and rate of travel for constant speed situations.

Addressing

- 7.RP.A

Launch

Ask students what they remember about problems involving distance, rate, and time. They might offer that distances traveled and elapsed time creates a set of equivalent ratios, or that the elapsed time can be multiplied by the speed to give the distance traveled. Give students 1 minute of quiet work time followed by whole-class discussion.

Anticipated Misconceptions

Some students may struggle knowing whether they should multiply or divide the numbers in each problem situation. Remind them of the equation $d = rt$.

Student Task Statement

1. An airplane moves at a constant speed of 120 miles per hour for 3 hours. How far does it go?

2. A train moves at constant speed and travels 6 miles in 4 minutes. What is its speed in miles per minute?

> 3. A car moves at a constant speed of 50 miles per hour. How long does it take the car to go 200 miles?

Student Response

1. 360 miles, because $120 \cdot 3 = 360$

2. $\frac{3}{2}$ or equivalent miles per minute, because $6 \div 4 = \frac{3}{2}$

3. 4 hours, because $200 \div 50 = 4$

Activity Synthesis

The most important thing for students to remember is that the equation $d = rt$ can be used to solve problems involving movement at a constant speed.

- To find the distance traveled, you can multiply the rate of travel (or speed) by the elapsed time.

- To find the rate of travel (or speed), you can divide the distance by the elapsed time.

- To find the elapsed time, you can divide the distance traveled by the rate of travel (or speed).

Consider drawing a table to facilitate the discussion of each problem and to remind students of the strategies they used while working with proportional relationships, such as using a scale factor or calculating the constant of proportionality. When relating distance and time in a constant speed situation, the speed is the constant of proportionality.

8.2 Going Left, Going Right

10 minutes
The purpose of this activity is to understand that a rate of travel at a constant speed (defined as velocity) can indicate the direction of travel, by using a negative or positive value to describe travel to the left or to the right of a location taken to be 0.

Students use their earlier understanding of a chosen zero point and describe their movement left (negative) or right (positive) along the number line, with different speeds. This will produce negative or positive end points depending on if they are moving to the left or the right. This will lead to students describing negative numbers multiplied by positive in the next activity.

Addressing

- 7.NS.A.2.a

Instructional Routines

- MLR2: Collect and Display

Launch

Remind students we have seen in earlier lessons that we can pick a location to represent zero, and then locations on either side are positive or negative.

Anticipated Misconceptions

Students may want to use the number line to help them with the position changes in the table.

Have students write in words how they calculated ending positions for the left and right if they get stuck trying to write an expression with variables.

Student Task Statement
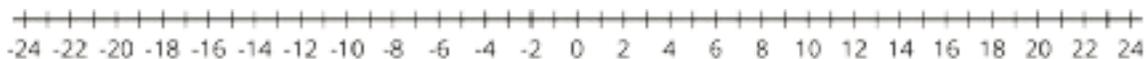
1. After each move, record your location in the table. Then write an expression to represent the ending position that uses the starting position, the speed, and the time. The first row is done for you.

starting position	direction	speed (units per second)	time (seconds)	ending position (units)	expression
0	right	5	3	+15	$0 + 5 \cdot 3$
0	left	4	6		
0	right	2	8		
0	right	6	2		
0	left	1.1	5		

2. How can you see the *direction* of movement in the expression?

3. Using a starting position p, a speed s, and a time t, write two expressions for an ending position. One expression should show the result of moving right, and one expression should show the result of moving left.

Student Response

1.

starting position	direction	speed (units per second)	time (seconds)	ending position	expression
0	right	5	3	+15	$0 + 5 \cdot 3$
0	left	4	6	-24	$0 - 4 \cdot 6$
0	right	2	8	+16	$0 + 2 \cdot 8$
0	right	6	2	+12	$0 + 6 \cdot 2$
0	left	1.1	5	-5.5	$0 - 1.1 \cdot 5$

2. The direction of movement is represented by whether you add to or subtract from 0.

3. Moving right $p + s \cdot t$. Moving left $p - s \cdot t$.

Activity Synthesis

Ask students to share how they could see the direction of travel in their expressions. It has to do with whether they added or subtracted the product st from zero. Ensure that everyone understands why $p + st$ represents final positions to the right of zero and $p - st$ represents positions to the left of zero.

Unit 5 Lesson 8

We saw, earlier in this unit, that subtracting a positive is the same as adding a negative. So let's write a *single* expression, $p + vt$ where instead of speed (which is always a positive number) we use a signed number for *speed plus direction* and we call this quantity *velocity*. Velocities when moving to the right will be represented by positive numbers, and velocities when moving to the left will be represented by negative numbers.

8.3 Velocity

15 minutes

The purpose of this activity is for students to encounter a concrete situation where multiplying two positive numbers results in a positive number, and multiplying a positive and a negative number results in a negative number.

Students use their earlier understanding of a chosen zero point, location relative to this as a positive or negative quantity and description of movement left (negative) or right (positive) along the number line, with different speeds. They extend their understanding to movement with positive and negative velocity and different times. This will produce negative or positive end points depending on if the velocity is negative or positive. Looking at a number of different examples will help students describe rules for identifying the sign of the product of a negative number with a positive number (MP8).

Addressing

- 7.NS.A.2.a

Instructional Routines

- MLR8: Discussion Supports

Launch
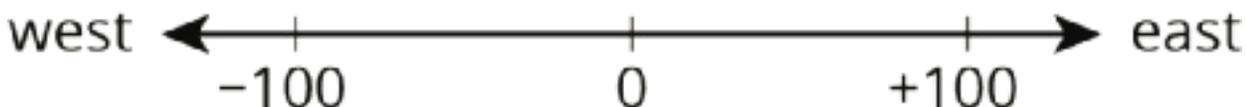
Display the image to remind students of west (left, negative) and east (right, positive) from the previous activity. Describe that we can talk about speed in a direction by calling it velocity and using a sign, so negative velocities represent movement west, and positive velocities represent movement east.

Access for Students with Disabilities

Representation: Internalize Comprehension. Begin with a physical demonstration of positive and negative velocity and different number times using the given number line to support connections between new situations and prior understandings. Consider using these prompts—"What does this demonstration have in common with moving left or right in the previous activity?" or "How is the direction of velocity related to the positive or negative sign?" *Supports accessibility for: Conceptual processing; Visual-spatial processing*

Anticipated Misconceptions

Encourage students who get stuck to use the provided number line to represent each situation.

Student Task Statement

A traffic safety engineer was studying travel patterns along a highway. She set up a camera and recorded the speed and direction of cars and trucks that passed by the camera. Positions to the east of the camera are positive, and to the west are negative.
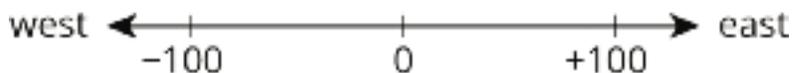
Vehicles that are traveling towards the east have a positive velocity, and vehicles that are traveling towards the west have a negative velocity.

1. Complete the table with the position of each vehicle if the vehicle is traveling at a constant speed for the indicated time period. Then write an equation.

velocity (meters per second)	time after passing the camera (seconds)	ending position (meters)	equation describing the position
+25	+10	+250	$25 \cdot 10 = 250$
-20	+30		
+32	+40		
-35	+20		
+28	0		

2. If a car is traveling east when it passes the camera, will its position be positive or negative 60 seconds after it passes the camera? If we multiply two positive numbers, is the result positive or negative?

Unit 5 Lesson 8

3. If a car is traveling west when it passes the camera, will its position be positive or negative 60 seconds after it passes the camera? If we multiply a negative and a positive number, is the result positive or negative?

Student Response

velocity (meters per second)	time after passing the camera (seconds)	ending position (meters)	equation describing position
+25	+10	+250	$25 \cdot 10 = 250$
-20	+30	-600	$(-20) \cdot 30 = -600$
+32	+40	+1280	$32 \cdot 40 = 1280$
-35	+20	-700	$-35 \cdot 20 = -700$
+28	0	0	$28 \cdot 0 = 0$

2. A car traveling east will be at a positive position 60 seconds after passing the camera—this means that if we multiply two positive numbers the result is positive.

3. A car traveling west will be at a negative position 60 seconds after passing the camera—this means that if we multiply a negative and a positive number the result is negative.

Are You Ready for More?

In many contexts we can interpret negative rates as "rates in the opposite direction." For example, a car that is traveling -35 miles per hour is traveling in the opposite direction of a car that is traveling 40 miles per hour.

1. What could it mean if we say that water is flowing at a rate of -5 gallons per minute?

2. Make up another situation with a negative rate, and explain what it could mean.

Student Response

1. Answers vary. Sample responses: A tank is draining out 5 gallons of water per minute. Due to tides, a creek is flowing upstream at a rate of 5 gallons per minute.

2. Answers vary.

Activity Synthesis

The most important thing for students to understand from this activity is that if we multiply two positive numbers the result is positive and that if we multiply a positive and a negative number the result is negative.

Ask a student to share their rationale about each problem. Display the number line from the launch, and place the negative answers in the context of the problem (to the west). Make sure the

distinction is made between the velocity (the direction of movement) and the position. Then, ensure students see that at least in this case, it appears that when we multiply two positive values, the product is positive. But when we multiply a positive by a negative value, the product is negative. We are going to take this to be true moving forward, even if the numbers represent other things.

Lesson Synthesis

Main takeaways:

- We can choose a zero point and then positive and negative numbers can represent positions to the right or left of this zero point.

- Signed numbers can also be used to represent speed in opposite directions. This is called velocity.

- A negative number multiplied by a positive gets a negative product.

Discussion questions:

- How can we represent a position to the left or right of a starting point without using direction words?

- How can we represent how fast something is moving to the left or right from a starting point? What word do we use to represent speed with a direction?

- What kind of number do you get when you multiply a negative number by a positive number?

8.4 Multiplication Expressions

Cool Down: 5 minutes
Addressing

- 7.NS.A.2.a

> **Student Task Statement**
>
> Four runners start at the same point; Lin, Elena, Diego, Andre. For each runner write a multiplication equation that describes their journey.

1. Lin runs for 25 seconds at 8.2 meters per second. What is her finish point?

2. Elena runs for 28 seconds and finishes at 250 meters. What is her velocity?

3. Diego runs for 32 seconds at -8.1 meters per second. What is his finish point?

4. Andre runs for 35 seconds and finishes at -285 meters. What is his velocity?

Student Response

1. $25 \cdot 8.2 = ?$

2. $28 \cdot ? = 250$

3. $32 \cdot (\text{-}8.1) = ?$

4. $35 \cdot ? = \text{-}285$

Student Lesson Summary

We can use signed numbers to represent the position of an object along a line. We pick a point to be the reference point, and call it zero. Positions to the right of zero are positive. Positions to the left of zero are negative.
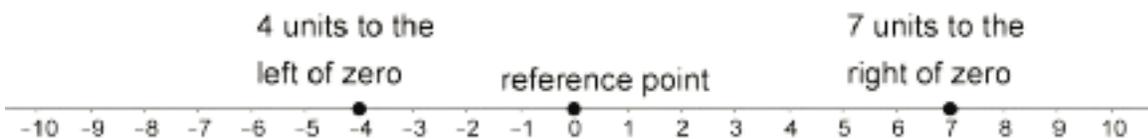
When we combine speed with direction indicated by the sign of the number, it is called *velocity*. For example, if you are moving 5 meters per second to the right, then your velocity is +5 meters per second. If you are moving 5 meters per second to the left, then your velocity is -5 meters per second.

If you start at zero and move 5 meters per second for 10 seconds, you will be $5 \cdot 10 = 50$ meters to the right of zero. In other words, $5 \cdot 10 = 50$.

If you start at zero and move -5 meters per second for 10 seconds, you will be $5 \cdot 10 = 50$ meters to the *left* of zero. In other words,

$$\text{-}5 \cdot 10 = \text{-}50$$

In general, a negative number times a positive number is a negative number.

Lesson 8 Practice Problems
Problem 1
Statement

A number line can represent positions that are north and south of a truck stop on a highway. Decide whether you want positive positions to be north or south of the truck stop. Then plot the following positions on a number line.

  a. The truck stop

  b. 5 miles north of the truck stop

  c. 3.5 miles south of the truck stop

Solution

Either choice is fine as long as students are consistent in the next part.

Problem 2
Statement

  a. How could you distinguish between traveling west at 5 miles per hour and traveling east at 5 miles per hour without using the words "east" and "west"?

  b. Four people are cycling. They each start at the same point. (0 represents their starting point.) Plot their finish points after five seconds of cycling on a number line

    ◦ Lin cycles at 5 meters per second

    ◦ Diego cycles at -4 meters per second

    ◦ Elena cycles at 3 meters per second

    ◦ Noah cycles at -6 meters per second

Solution

a. By giving the velocities opposite signs.

b.
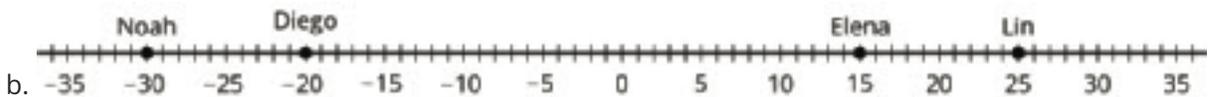
Problem 3
Statement

Find the value of each expression.

  a. $16.2 + \text{-}8.4$

b. $\frac{2}{5} - \frac{3}{5}$

c. -9.2 + -7

d. $-4\frac{3}{8} - (-1\frac{1}{4})$

Solution

a. 7.8

b. $\frac{-1}{5}$

c. -16.2

d. $-3\frac{1}{8}$

(From Unit 5, Lesson 6.)

Problem 4

Statement

For each equation, write two more equations using the same numbers that express the same relationship in a different way.

a. $3 + 2 = 5$

b. $7.1 + 3.4 = 10.5$

c. $15 - 8 = 7$

d. $\frac{3}{2} + \frac{9}{5} = \frac{33}{10}$

Solution

For each question, students should have 2 of the 3 equations listed.

a. $5 - 3 = 2; 5 - 2 = 3; 2 + 3 = 5$

b. $10.5 - 3.4 = 7.1; 10.5 - 7.1 = 3.4; 3.4 + 7.1 = 10.5$

c. $15 - 7 = 8; 8 + 7 = 15; 7 + 8 = 15$

d. $\frac{33}{10} - \frac{3}{2} = \frac{9}{5}; \frac{33}{10} - \frac{9}{5} = \frac{3}{2}; \frac{9}{5} + \frac{3}{2} = \frac{33}{10}$

(From Unit 5, Lesson 5.)

Problem 5

Statement

A shopper bought a watermelon, a pack of napkins, and some paper plates. In his state, there is no tax on food. The tax rate on non-food items is 5%. The total for the three items he bought was $8.25 before tax, and he paid $0.19 in tax. How much did the watermelon cost?

Solution

$4.45. The non-food items cost $0.19 \div 0.05 = 3.8$. The watermelon cost $8.25 - 3.8 = 4.45$.

(From Unit 4, Lesson 10.)

Problem 6

Statement

Which graphs could not represent a proportional relationship? Explain how you decided.
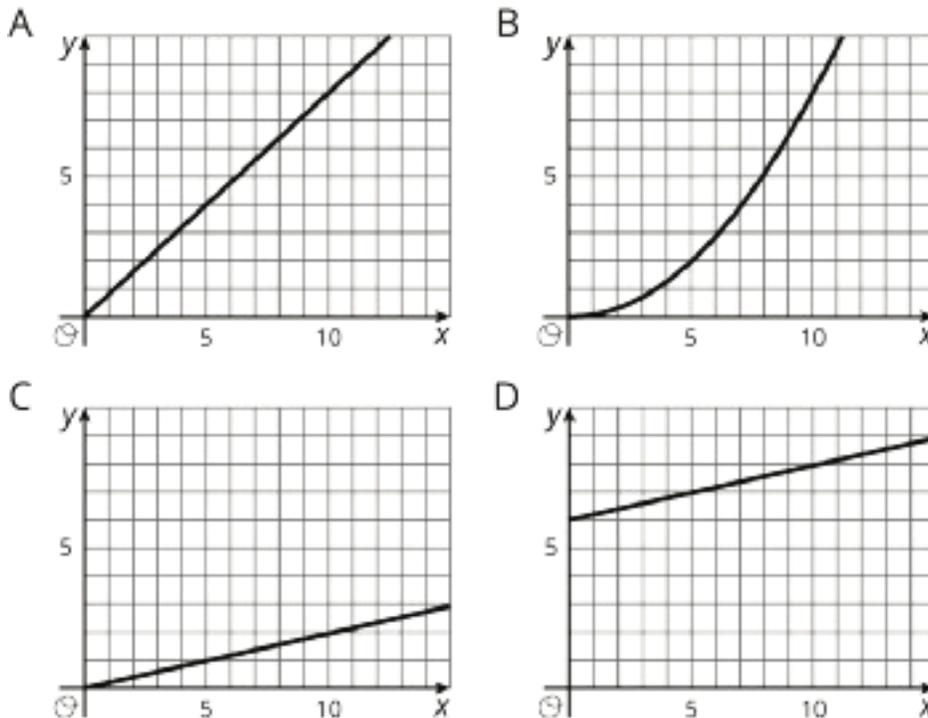
Solution

B and D are graphs that do not represent proportional relationships. B is not a straight line and D does not go through the origin.

(From Unit 2, Lesson 10.)

Lesson 9: Multiplying Rational Numbers

Goals

- Generalize (orally) that the product of two negative numbers is positive.

- Interpret signed numbers used to represent elapsed time before or after a chosen reference point.

- Use patterns to find the product of signed numbers, and explain (orally and using other representations) the reasoning.

Learning Targets

- I can explain what it means when time is represented with a negative number in a situation about speed and direction.

- I can multiply two negative numbers.

Lesson Narrative

The purpose of this lesson is to develop the rules for multiplying two negative numbers. Students use the familiar fact that

$$\text{distance} = \text{velocity} \times \text{time}$$

to make sense of this rule. They interpret negative time as time before a chosen starting time and then figure out what the position is of an object moving with a negative velocity at a negative time. An object moving with a negative velocity is moving from right to left along the number line. At a negative time it has not yet reached its starting point of zero, so it is to the right of zero, and therefore its position is positive. So a negative velocity times a negative time gives a positive position. When students connect reasoning about quantities with abstract properties of numbers, they engage in MP2.

There is also an optional activity where students can use another approach to understanding why the product of two negative numbers is positive, by examining patterns in an extended multiplication table that includes both positive and negative numbers (MP7).

Alignments

Building On

- 6.RP.A.3.b: Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Addressing

- 7.NS.A.2: Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

- 7.NS.A.2.a: Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

- 7.NS.A.2.c: Apply properties of operations as strategies to multiply and divide rational numbers.

- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

Instructional Routines

- MLR2: Collect and Display

- MLR5: Co-Craft Questions

- Think Pair Share

Required Materials

Copies of Instructional master

Required Preparation

It is optional to provide 1 copy of the Rational Numbers Multiplication Grid Instructional master to each student.

> **Student Learning Goals**
>
> Let's multiply signed numbers.

9.1 Before and After

Warm Up: 5 minutes
In this lesson, students will interpret negative time in context. The warm-up primes them for those interpretations.

Building On

- 6.RP.A.3.b

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 30 seconds of quiet think time, followed by partner discussion.

Student Task Statement

Where was the girl:

1. 5 seconds *after* this picture was taken? Mark her approximate location on the picture.

2. 5 seconds *before* this picture was taken? Mark her approximate location on the picture.

Student Response

1. Students should mark a position after her current position.

2. Students should mark a position before her current position, about equally distant from her current position as the previous mark.

Activity Synthesis

Ask students to come to agreement with their partners, and help them to productively resolve any discrepancies. Point out that if she is walking at a constant speed, then her positions before and after will be equally far from her position in the picture.

9.2 Backwards in Time

15 minutes
Students use their earlier understanding of a chosen zero point and description of positive and negative velocity, and extend this to include negative values for time to represent a time before the time assigned chosen as zero. This will produce different end points depending on if the velocity or time is negative or positive. Students use the context to help make sense of the arithmetic problems (MP2). Looking at a number of different examples will help students describe rules for identifying the sign of the product of two negative numbers (MP8). Students may choose to use a number line to help them in their reasoning; this is an example of using appropriate tools strategically (MP5).

Addressing

- 7.NS.A.2.a

Instructional Routines

- MLR5: Co-Craft Questions

Launch

Keep students in the same groups. Remind the students of movement east or west as positive or negative velocity.

This activity is the same context as one in the previous lesson, and the questions are related. So students should be able to get to work rather quickly. However, each question requires some careful thought, and one question builds on the other. Consider suggesting that students check in with their partner frequently and explain their thinking. Additionally, you might consider asking students to pause after each question for a quick whole-class discussion before continuing to the next question.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.
Supports accessibility for: Organization; Attention

Access for English Language Learners

Conversing, Writing: MLR5 Co-craft Questions. Display only the table of values from the task and ask students to write possible mathematical questions about the situation. Invite students to share their questions with a partner before selecting a few to share with the class. Highlight questions that connect the table to other representations, such as: "What equations can be written with the quantity's position and time?" or "How can each table entry be represented on a number line?" These questions help students look for and make use of structure to find the car's direction and velocity. This helps students produce the language of mathematical questions and talk about the relationships between the two quantities (e.g., position and time) prior to being asked to determine the car's direction and speed.
Design Principle(s): Cultivate conversation; Support sense-making

Anticipated Misconceptions

If students struggle to calculate the velocity, ask them how fast is the car going after 1 second.

> ### Student Task Statement
>
> A traffic safety engineer was studying travel patterns along a highway. She set up a camera and recorded the speed and direction of cars and trucks that passed by the camera. Positions to the east of the camera are positive, and to the west are negative.
>
> 1. Here are some positions and times for one car:

position (feet)	-180	-120	-60	0	60	120
time (seconds)	-3	-2	-1	0	1	2

a. In what direction is this car traveling?

b. What is its velocity?

2. a. What does it mean when the time is zero?

b. What could it mean to have a negative time?

3. Here are the positions and times for a different car whose velocity is -50 feet per second:

position (feet)				0	-50	-100
time (seconds)	-3	-2	-1	0	1	2

a. Complete the table with the rest of the positions.

b. In what direction is this car traveling? Explain how you know.

4. Complete the table for several different cars passing the camera.

	velocity (meters per second)	time after passing the camera (seconds)	ending position (meters)	equation
car C	+25	+10	+250	$25 \cdot 10 = 250$
car D	-20	+30		
car E	+32	-40		
car F	-35	-20		
car G	-15	-8		

5. a. If a car is traveling east when it passes the camera, will its position be positive or negative 60 seconds *before* it passes the camera?

b. If we multiply a positive number and a negative number, is the result positive or negative?

6. a. If a car is traveling west when it passes the camera, will its position be positive or negative 60 seconds *before* it passes the camera?

   b. If we multiply two negative numbers, is the result positive or negative?

Student Response

1. a. The car is traveling east.

   b. Its velocity is 60 feet per second.

2. a. It is the time the car passed the camera.

   b. It means time before the car passes the camera.

3. a.

position (feet)	-100	-50	0	50	100	150
time (seconds)	2	1	0	-1	-2	-3

   b. Travelling west because velocity is negative.

4. Fill out the table.

	velocity (meters per second)	time after passing camera (seconds)	ending position (meters)	expression
car C	+25	+10	+250	$25 \cdot 10 = 250$
car D	-20	+30	-600	$-20 \cdot 30 = -600$
car E	+32	-40	-1280	$32 \cdot (-40) = -1280$
car F	-35	-20	+700	$-35 \cdot (-20) = +700$
car G	-15	-8	+120	$-15 \cdot (-8) = +120$

5. a. A car traveling east will be be at a negative position before passing the camera.

   b. If we multiply a positive and a negative number the result is negative.

6. a. A car traveling west will be be at a positive position before passing the camera.

   b. If we multiply two negative numbers the result is positive.

Unit 5 Lesson 9

Activity Synthesis

The key thing for students to understand here is that a negative multiplied by another negative is a positive. The last two rows in the table and the final two questions are the keys to this so draw attention to the logical progression that movement in the negative direction will have a positive position when time is negative.

9.3 Cruising

Optional: 15 minutes (there is a digital version of this activity)
This is the first of two optional activities. The teacher may choose to implement either of the optional activities that would best reinforce the learning goals for their students.

In this optional activity, students find the position of a car traveling at a constant velocity at different positive and negative times, and plot the points in the coordinate plane. They see that just as with constant speed, the graph goes through $(0, 0)$, but because the velocity is negative it slants downward from left to right instead of passing through the first quadrant.

Addressing

- 7.NS.A.2

- 7.RP.A.2

Instructional Routines

- MLR2: Collect and Display

- Think Pair Share

Launch

Arrange students in groups of 2. Give them 4 minutes of quiet work time, followed by partner and then whole-class discussion.

Access for English Language Learners

Conversing, Reading: MLR2 Collect and Display. While partners talk about their work, circulate and listen to students describe where the car is at -3 seconds and at 6.5 seconds. Record common or important phrases you hear students say about each location on a visual display connecting the phrases to the table or coordinate grid. This will help students read and use mathematical language during their paired and whole-group discussions.
Design Principle(s): Support sense-making

Student Task Statement

Around noon, a car was traveling -32 meters per second down a highway. At exactly noon (when time was 0), the position of the car was 0 meters.

1. Complete the table.

time (s)	-10	-7	-4	-1	2	5	8	11
position (m)								

2. Graph the relationship between the time and the car's position.
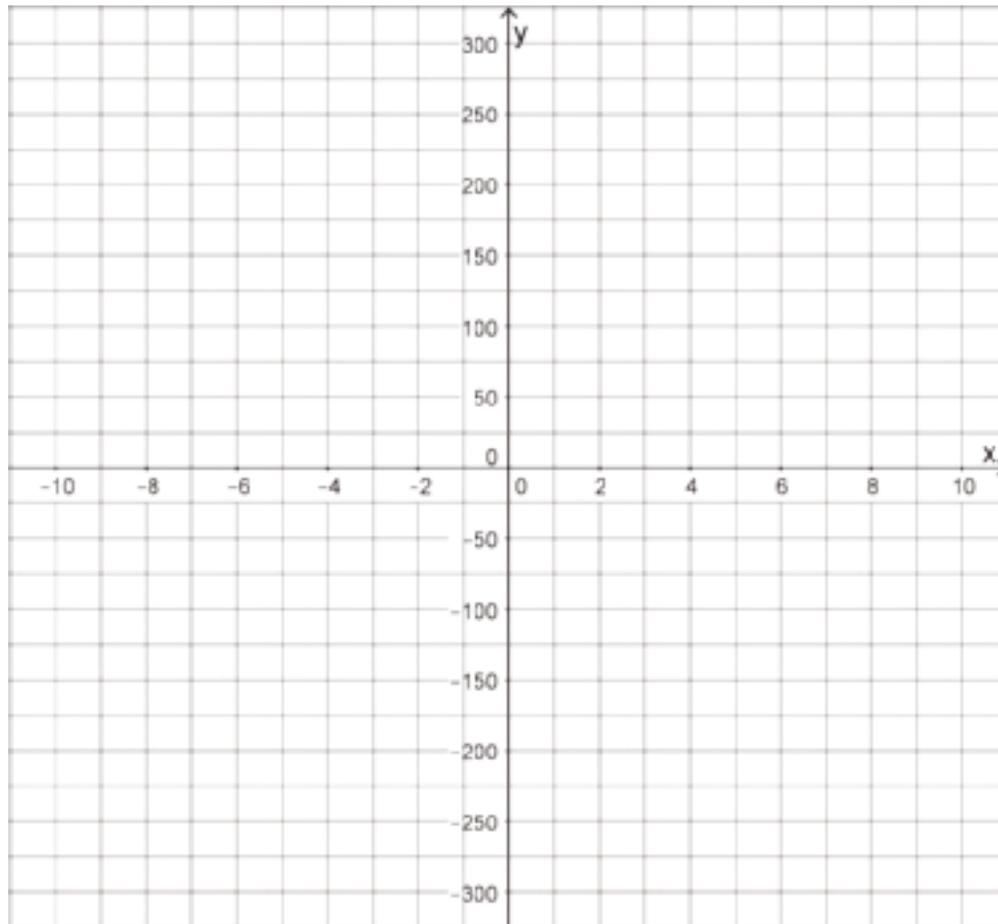
3. What was the position of the car at -3 seconds?

4. What was the position of the car at 6.5 seconds?

Student Response

1.

time (s)	-10	-7	-4	-1	2	5	8	11
position (m)	320	224	128	32	-64	-160	-256	-352

2. A line through $(0, 0)$ with a slope of -32

3. 96 meters

Unit 5 Lesson 9

4. -208 meters

> **Are You Ready for More?**
>
> Find the value of these expressions without using a calculator.
>
> $(-1)^2$ $(-1)^3$ $(-1)^4$ $(-1)^{99}$

Student Response

1, -1, 1, -1

Activity Synthesis

Ask students how the graph is similar and different from the graph of a proportional relationship. Poll the class for the last two questions. Ask students if they can see an equation that relates the time and the position of the car. Record their ideas and make sure everyone comes to agreement that if d is the position and t is the time, then $d = -32t$.

9.4 Rational Numbers Multiplication Grid

Optional: 10 minutes (there is a digital version of this activity)

In this optional activity, students revisit the representation of a multiplication chart, which may be familiar from previous grades; however, in this activity, the multiplication chart is extended to include negative numbers. Students identify and continue patterns (MP8) to complete the chart and see that it fits the patterns in the chart for the product of two negative numbers to be a positive number.

The Instructional master has a multiplication chart that also includes the factors 1.5, -1.5, 3.2, and -3.2, so that students can see how the patterns extend to rational numbers that are not integers. Encourage students to complete the rows and columns for the integers first and then come back to 1.5, -1.5, 3.2, and -3.2 later. Directions are included on the Instructional master for a way that students can fold their papers to hide the non-integers while they fill in the integers. If you want students to do this, it would be good to demonstrate and walk them through the process of folding their paper.

Addressing

- 7.NS.A.2.c

Launch

Arrange students in groups of 3. If desired, distribute 1 copy of the Instructional master to every student and instruct students to ignore the chart printed in their books or devices. (Also if desired, instruct students to fold their papers according to the directions on the top and right sides of the chart, so that the decimal rows and columns are temporarily hidden.) Give students 30 seconds of quiet think time. Have them share what patterns they notice about the numbers that are already filled in. Give the groups 5 minutes of work time followed by whole-class discussion.

If students have access to the digital materials, students can use the applet to complete the chart. The applet helps students focus on fewer of the numbers and patterns at a time, similar to the

purpose of folding the Instructional master. Also, the applet gives students immediate feedback on whether their answers are correct which helps them test their theories about ramifications of multiplying by a negative number.

Anticipated Misconceptions

Some students may need a reminder of how a mutliplication chart works: the factors are listed at the end of the rows and columns, and their products go in the boxes.

> **Student Task Statement**
>
> 1. Complete the *shaded* boxes in the multiplication square.

> 2. Look at the patterns along the rows and columns. Continue those patterns into the unshaded boxes.
>
> 3. Complete the whole table.
>
> 4. What does this tell you about multiplication with negative numbers?

Student Response

For the table printed in student books or devices:

5	-25	-20	-15	-10	-5	0	5	10	15	20	25
4	-20	-16	-12	-8	-4	0	4	8	12	16	20
3	-15	-12	-9	-6	-3	0	3	6	9	12	15
2	-10	-8	-6	-4	-2	0	2	4	6	8	10
1	-5	-4	-3	-2	-1	0	1	2	3	4	5
0	0	0	0	0	0	0	0	0	0	0	0
-1	5	4	3	2	1	0	-1	-2	-3	-4	-5
-2	10	8	6	4	2	0	-2	-4	-6	-8	-10
-3	15	12	9	6	3	0	-3	-6	-9	-12	-15
-4	20	16	12	8	4	0	-4	-8	-12	-16	-20
-5	25	20	15	10	5	0	-5	-10	-15	-20	-25
	-5	-4	-3	-2	-1	0	1	2	3	4	5

For the table on the Instructional master:

5	-25	-20	-16	-15	-10	-5	0	5	10	15	16	20	25
4	-20	-16	-12.8	-12	-8	-4	0	4	8	12	12.8	16	20
3	-15	-12	-9.6	-9	-6	-3	0	3	6	9	9.6	12	15
2	-10	-8	-6.4	-6	-4	-2	0	2	4	6	6.4	8	10
1.5	-7.5	-6	-4.8	-4.5	-3	-1.5	0	1.5	3	4.5	4.8	6	7.5
1	-5	-4	-3.2	-3	-2	-1	0	1	2	3	3.2	4	5
0	0	0	0	0	0	0	0	0	0	0	0	0	0
-1	5	4	3.2	3	2	1	0	-1	-2	-3	-3.2	-4	-5
-1.5	7.5	6	4.8	4.5	3	1.5	0	-1.5	-3	-4.5	-4.8	-6	-7.5
-2	10	8	6.4	6	4	2	0	-2	-4	-6	-6.4	-8	-10
-3	15	12	9.6	9	6	3	0	-3	-6	-9	-9.6	-12	-15
-4	20	16	12.8	12	8	4	0	-4	-8	-12	-12.8	-16	-20
-5	25	20	16	15	10	5	0	-5	-10	-15	-16	-20	-25
	-5	-4	-3.2	-3	-2	-1	0	1	2	3	3.2	4	5

Answer vary. Sample response: A positive number multiplied by a negative number is negative, but a negative number multiplied by a negative number is positive.

Activity Synthesis

The most important takeaway is that it makes sense for the product of two negative numbers to be a positive number, whether or not the numbers are integers. This fits in with the patterns in the extended multiplication chart. Those patterns depend on the distributive property. For example, the reason the numbers in the top row go up by 5s is that $5(n + 1) = 5n + 5$. So when students extend the pattern to negative numbers, they are extending the distributive property.

Display a complete chart for all to see, and ask students to explain the ways in which the chart shows that the product of a negative and a negative is a positive. The general argument involves assuming that a pattern observed in a row or column will continue on the other side of 0.

5	-25	-20	-16	-15	-10	-5	0	5	10	15	16	20	25
4	-20	-16	-12.8	-12	-8	-4	0	4	8	12	12.8	16	20
3	-15	-12	-9.6	-9	-6	-3	0	3	6	9	9.6	12	15
2	-10	-8	-6.4	-6	-4	-2	0	2	4	6	6.4	8	10
1.5	-7.5	-6	-4.8	-4.5	-3	-1.5	0	1.5	3	4.5	4.8	6	7.5
1	-5	-4	-3.2	-3	-2	-1	0	1	2	3	3.2	4	5
0	0	0	0	0	0	0	0	0	0	0	0	0	0
-1	5	4	3.2	3	2	1	0	-1	-2	-3	-3.2	-4	-5
-1.5	7.5	6	4.8	4.5	3	1.5	0	-1.5	-3	-4.5	-4.8	-6	-7.5
-2	10	8	6.4	6	4	2	0	-2	-4	-6	-6.4	-8	-10
-3	15	12	9.6	9	6	3	0	-3	-6	-9	-9.6	-12	-15
-4	20	16	12.8	12	8	4	0	-4	-8	-12	-12.8	-16	-20
-5	25	20	16	15	10	5	0	-5	-10	-15	-16	-20	-25
	-5	-4	-3.2	-3	-2	-1	0	1	2	3	3.2	4	5

Lesson Synthesis

Key takeaways:

- Positive times are after a chosen zero time, and negative times are times before the chosen zero time.

- A positive times a positive is always positive.

- A negative times a positive or a positive times a negative is always negative.

- A negative times a negative is always positive.

Discussion questions:

- How can we represent a time that came before a specific zero point?

Unit 5 Lesson 9

- What kind of number do you get when you multiply a negative number by a positive number? Use a context from the lesson to explain why this makes sense.

- What kind of number do you get when you multiply a negative number by a negative number? Use a context from the lesson to explain why this makes sense.

9.5 True Statements

Cool Down: 5 minutes
Addressing

- 7.NS.A.2.a

> **Student Task Statement**
>
> Decide if each equation is true or false.
>
> 1. $7 \cdot 8 = 56$
>
> 2. $-7 \cdot 8 = 56$
>
> 3. $-7 \cdot -8 = -56$
>
> 4. $-7 \cdot -8 = 56$
>
> 5. $3.5 \cdot 12 = 42$
>
> 6. $-3.5 \cdot -12 = -42$
>
> 7. $-3.5 \cdot -12 = 42$
>
> 8. $-12 \cdot \frac{7}{2} = 42$

Student Response

1. True

2. False

3. False

4. True

5. True

6. False

7. True

8. False

Student Lesson Summary

We can use signed numbers to represent time relative to a chosen point in time. We can think of this as starting a stopwatch. The positive times are after the watch starts, and negative times are times before the watch starts.
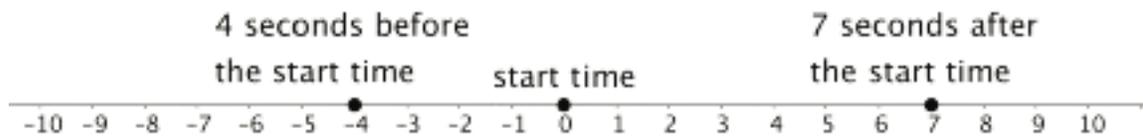
If a car is at position 0 and is moving in a positive direction, then for times after that (positive times), it will have a positive position. A positive times a positive is positive.
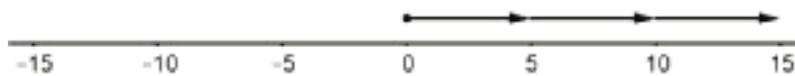
If a car is at position 0 and is moving in a negative direction, then for times after that (positive times), it will have a negative position. A negative times a positive is negative.
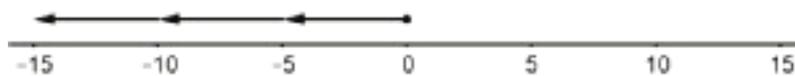
If a car is at position 0 and is moving in a positive direction, then for times *before* that (negative times), it must have had a negative position. A positive times a negative is negative.

If a car is at position 0 and is moving in a negative direction, then for times *before* that (negative times), it must have had a positive position. A negative times a negative is positive.
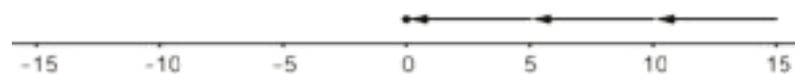
Here is another way of seeing this:

We can think of $3 \cdot 5$ as $5 + 5 + 5$, which has a value of 15.

We can think of $3 \cdot (-5)$ as -5 + -5 + -5, which has a value of -15.

We know we can multiply positive numbers in any order: $3 \cdot 5 = 5 \cdot 3$

If we can multiply signed numbers in any order, then $(-5) \cdot 3$ would also equal -15.

Now let's think about multiplying two negatives.

We can find $-5 \cdot (3 + -3)$ in two ways:

- Applying the distributive property:

$$-5 \cdot 3 + -5 \cdot (-3)$$

- Adding the numbers in parentheses:

$$-5 \cdot (0) = 0$$

This means that these expressions must be equal.

$$-5 \cdot 3 + -5 \cdot (-3) = 0$$

Multiplying the first two numbers gives

$$-15 + -5 \cdot (-3) = 0$$

Which means that

$$-5 \cdot (-3) = 15$$

There was nothing special about these particular numbers. This always works!

- A positive times a positive is always positive.
- A negative times a positive or a positive times a negative is always negative.
- A negative times a negative is always positive.

Lesson 9 Practice Problems

Problem 1

Statement

Fill in the missing numbers in these equations

    a. $-2 \cdot (-4.5) = ?$

    b. $(-8.7) \cdot (-10) = ?$

    c. $(-7) \cdot ? = 14$

    d. $? \cdot (-10) = 90$

Solution

    a. $-2 \cdot (-4.5) = 9$

    b. $(-8.7) \cdot (-10) = 87$

    c. $(-7) \cdot (-2) = 14$

    d. $(-9) \cdot (-10) = 90$

Problem 2

Statement

A weather station on the top of a mountain reports that the temperature is currently $0°C$ and has been falling at a constant rate of $3°C$ per hour. If it continues to fall at this rate, find each indicated temperature. Explain or show your reasoning.

    a. What will the temperature be in 2 hours?

    b. What will the temperature be in 5 hours?

    c. What will the temperature be in half an hour?

    d. What was the temperature 1 hour ago?

    e. What was the temperature 3 hours ago?

    f. What was the temperature 4.5 hours ago?

Solution

    a. The temperature will be $-6°C$, because $-3 \cdot 2 = -6$

    b. The temperature will be $-15°C$, because $-3 \cdot 5 = -15$

    c. The temperature will be $-1.5°C$, because $-3 \cdot 0.5 = -1.5$

d. The temperature was 3˚C, because -3 · (-1) = 3

e. The temperature was 9˚C, because -3 · (-3) = 9

f. The temperature was 13.5˚C, because -3 · (-4.5) = 13.5

Problem 3

Statement

Find the value of each expression.

    a. $\frac{1}{4} \cdot (-12)$

    b. $-\frac{1}{3} \cdot 39$

    c. $(-\frac{4}{5}) \cdot (-75)$

    d. $-\frac{2}{5} \cdot (-\frac{3}{4})$

    e. $\frac{8}{3} \cdot -42$

Solution

    a. -3

    b. -13

    c. 60

    d. $\frac{3}{10}$

    e. -112

Problem 4

Statement

To make a specific hair dye, a hair stylist uses a ratio of $1\frac{1}{8}$ oz of red tone, $\frac{3}{4}$ oz of gray tone, and $\frac{5}{8}$ oz of brown tone.

    a. If the stylist needs to make 20 oz of dye, how much of each dye color is needed?

    b. If the stylist needs to make 100 oz of dye, how much of each dye color is needed?

Solution

    a. 9 oz red, 6 oz gray, and 5 oz brown. The given amounts of dye make $2\frac{1}{2}$ oz so each quantity needs to be multiplied by 8 to get 20 oz.

b. 45 oz red, 30 oz gray, and 25 oz brown. 100 oz is 5 batches of 20 oz.

(From Unit 4, Lesson 2.)

Problem 5

Statement

a. Here are the vertices of rectangle $FROG$: $(-2, 5), (-2, 1), (6, 5), (6, 1)$.
Find the perimeter of this rectangle. If you get stuck, try plotting the points on a coordinate plane.

b. Find the area of the rectangle $FROG$.

c. Here are the coordinates of rectangle $PLAY$: $(-11, 20), (-11, -3), (-1, 20), (-1, -3)$. Find the perimeter and area of this rectangle. See if you can figure out its side lengths without plotting the points.

Solution

a. 24 units. The short side of the rectangle has length 4 units, because $5 - 1 = 4$. The long side has length 8 units, because $6 - (-2) = 8$. The perimeter is 24 units, because $4 + 4 + 8 + 8 = 24$.

b. 32 square units, because $8 \cdot 4 = 32$

c. The perimeter is 66 units and the area is 230 square units. The short side has length 10 units, because $-1 - (-11) = 10$. The long side has length 23 units, because $20 - (-3) = 23$. The perimeter is 66 units, because $2(10 + 23) = 66$.

(From Unit 5, Lesson 7.)

Lesson 10: Multiply!

Goals

- Identify multiplication expressions that are equal, and justify (orally) that they are equal.

- Multiply rational numbers, including multiplication expressions with three factors, and explain (orally and in writing) the reasoning.

Learning Targets

- I can solve problems that involve multiplying rational numbers.

Lesson Narrative

The purpose of this optional lesson is to provide students with practice multiplying rational numbers. Students see products of three numbers, and figure out what to do with a product of three negative numbers by grouping two of them together and multiplying those first. They see products involving unit fractions, reminding them that dividing by a whole number is the same as multiplying by its reciprocal. This will be useful in the next lesson when they start to divide rational numbers.

Alignments

Building On

- 6.EE.A.2.b: Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.

Addressing

- 7.NS.A.2.c: Apply properties of operations as strategies to multiply and divide rational numbers.

Building Towards

- 7.EE.B: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Instructional Routines

- MLR8: Discussion Supports

- Think Pair Share

- Which One Doesn't Belong?

Required Materials

Pre-printed slips, cut from copies of the   **Instructional master**

Required Preparation

For the Card Sort: Matching Expressions activity, prepare 1 copy of the Instructional master for each group of 2 students. If possible, copy each complete set on a different color of paper, so that a stray card can quickly be put back.

> **Student Learning Goals**
>
> Let's get more practice multiplying signed numbers.

10.1 Which One Doesn't Belong: Expressions

Warm Up: 5 minutes

This warm-up prompts students to compare four expressions. It encourages students to explain their reasoning, hold mathematical conversations, and gives you the opportunity to hear how they use terminology and talk about the expressions in comparison to one another. To allow all students to access the activity, each expression has one obvious reason it does not belong.

Building On

- 6.EE.A.2.b

Building Towards

- 7.EE.B

Instructional Routines

- Which One Doesn't Belong?

Launch

Arrange students in groups of 2–4. Display the expressions for all to see. Ask students to indicate when they have noticed one expression that does not belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their small group. In their small groups, tell each student to share their reasoning why a particular expression does not belong and together find at least one reason each expression doesn't belong.

> **Student Task Statement**
>
> Which expression doesn't belong?
>
> $7.9x$ $\qquad\qquad\qquad\qquad$ $7.9 + x$
>
> $7.9 \cdot (\text{-}10)$ $\qquad\qquad\qquad\qquad$ -79

Student Response

Answers vary. Sample responses: $7.9x$ is the only product of a decimal and a variable.

$7.9 \cdot (\text{-}10)$ is the only one with parentheses; is the only one that is a product of two known numbers.

$7.9 + x$ is the only sum.

-79 is the only plain number.

Activity Synthesis

Ask each group to share one reason why a particular expression does not belong. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given make sense.

10.2 Card Sort: Matching Expressions

Optional: 10 minutes
This activity reminds students of the links between positive fractions and multiplication and prepares them to think about division as multiplication by the reciprocal; this will be important for dividing negative numbers. Students will use earlier work from grade 6 and their work in previous lessons in this unit to extend what they know about division of positive rationals to all rational numbers (MP7).

Ask students as they are working if there is an easy way to tell if two expressions are not equivalent, making note of students who reason about how many negative numbers are multiplied, and what the outcome will be. For example, they may have first gone through and marked whether each product would be positive or negative before doing any arithmetic.

Addressing
- 7.NS.A.2.c

Instructional Routines
- MLR8: Discussion Supports

Launch
Ask students to recall the rules they have previously used about multiplication of signed numbers.

This is the first encounter with an expression where three integers are multiplied, so students might need to see an example of evaluating an expression like this one step at a time. Display an expression like this for all to see, and ask students how they might go about evaluating it:
$$(-2) \cdot (-3) \cdot (-4)$$
The key insight is that you can consider only one product, and replace a pair of numbers with the product. In this example, you can replace $(-2) \cdot (-3)$ with 6. Then, you are just looking at $(6) \cdot (-4)$, which we already know how to evaluate.

Arrange students in groups of 2. Distribute sets of cards.

Student Task Statement

Your teacher will give you cards with multiplication expressions on them. Match the expressions that are equal to each other. There will be 3 cards in each group.

Student Response

-12	$-1 \cdot 12$	$-1 \cdot (-3) \cdot (-4)$	$-2 \cdot 6$
-15	$-1 \cdot (-3) \cdot (-5)$	$-3 \cdot 5$	$1 \cdot -15$
+15	$1 \cdot (-3) \cdot (-5)$	$(-3) \cdot (-5)$	$1 \cdot 15$
+8	$-\frac{1}{2} \cdot (-16)$	$-\frac{1}{4} \cdot -32$	$2 \cdot 4$
-8	$-\frac{1}{2} \cdot 16$	$-64 \cdot \frac{1}{8}$	$2 \cdot (-4)$
+12	$-1 \cdot (-3) \cdot 4$	$-1 \cdot (-2) \cdot 6$	$-1 \cdot (-12)$

Activity Synthesis

Ask the previously identified students to share their rationale for identifying those that do not match.

Consider highlighting the link between multiplying by a fraction and dividing by a whole number. If desired, ask students to predict the values of some division expressions with signed numbers. For example, students could use the expression $-64 \cdot \frac{1}{8}$ to predict the value of $-64 \div 8$. However, it is not necessary for students to learn rules for dividing signed numbers at this point. That will be the focus of future lessons.

10.3 Row Game: Multiplying Rational Numbers

Optional: 10 minutes
This optional activity gives students an opportunity to practice multiplying signed numbers. The solutions to the problems in each row are the same, so students can check their work with a partner.

Addressing

- 7.NS.A.2.c

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Make sure students know how to play a row game. Give students 5–6 minutes of partner work time followed by whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts. For example, after students have completed the first 2-3 rows of the table, check-in with either select groups of students or the whole class. Invite students to share how they have applied generalizations about multiplying signed numbers from the previous activity so far.
Supports accessibility for: Conceptual processing; Organization; Memory

Student Task Statement

Evaluate the expressions in one of the columns. Your partner will work on the other column. Check in with your partner after you finish each row. Your answers in each row should be the same. If your answers aren't the same, work together to find the error.

column A	column B
$790 \div 10$	$(7.9) \cdot 10$
$-\frac{6}{7} \cdot 7$	$(0.1) \cdot -60$
$(2.1) \cdot -2$	$(-8.4) \cdot \frac{1}{2}$
$(2.5) \cdot (-3.25)$	$-\frac{5}{2} \cdot \frac{13}{4}$
$-10 \cdot (3.2) \cdot (-7.3)$	$5 \cdot (-1.6) \cdot (-29.2)$

Student Response

Row 1: 79

Row 2: -6

Row 3: -4.2

Row 4: -8.125

Row 5: 233.6

Are You Ready for More?

A sequence of rational numbers is made by starting with 1, and from then on, each term is one more than the reciprocal of the previous term. Evaluate the first few expressions in the sequence. Can you find any patterns? Find the 10th term in this sequence.

$$ 1 \qquad 1 + \frac{1}{1} \qquad 1 + \frac{1}{1 + 1} \qquad 1 + \frac{1}{1 + \frac{1}{1+1}} \qquad 1 + \frac{1}{1 + \frac{1}{1+\frac{1}{1+1}}} \qquad \dots $$

Student Response

The 10th term is $\frac{89}{55}$. Instead of calculating this from an extremely long fraction representation, we can look for patterns in applying the "one more than the reciprocal" rule. The first few terms are 1, 2, $\frac{3}{2}$, $\frac{5}{3}$, $\frac{8}{5}$. To find the next one, applying that rule gives $1 + \frac{8}{5} = 1 + \frac{5}{8} = \frac{8}{8} + \frac{5}{8} = \frac{13}{8}$. This leads to an interesting sequence in the numerators and denominators: Given a term $\frac{a}{b}$ in the sequence, the next term will be $\frac{a+b}{a}$. That is, the old numerator becomes the new denominator, and to get the new numerator, you add together the old numerator and old denominator. This observation makes it very easy to continue the sequence to the 10th term.

Unit 5 Lesson 10

$$1, 2, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \frac{21}{13}, \frac{34}{21}, \frac{55}{34}, \frac{89}{55}.$$

Activity Synthesis

Ask students, "Were there any rows that you and your partner did not get the same answer?" Invite students to share how they came to an agreement on the final answer for the problems in those rows.

Consider asking some of the following questions:

- "Did you and your partner use the same strategy for each row?"

- "What was the same and different about both of your strategies?"

- "Did you learn a new strategy from your partner?"

- "Did you try a new strategy while working on these questions?"

Lesson Synthesis

Display a number line with the numbers -1, 0, and 1 labeled. Ask students to give examples of multiplications problems with a product that is:

- greater than 1 (Sample responses: $5 \cdot 3$ or $-5 \cdot -3$)

- less than -1 (Sample responses: $5 \cdot -3$ or $-5 \cdot -3 \cdot -1$)

- between 0 and 1 (Sample responses: $\frac{1}{5} \cdot \frac{1}{3}$ or $-\frac{1}{5} \cdot -\frac{1}{3}$)

- between -1 and 0 (Sample responses: $\frac{1}{5} \cdot -\frac{1}{3}$ or $-\frac{1}{5} \cdot -\frac{1}{3} \cdot -1$)

10.4 Making Mistakes

Cool Down: 5 minutes
Addressing

- 7.NS.A.2.c

> **Student Task Statement**
>
> Noah was doing some homework and answered the following questions. Do you agree with his answers? If you disagree, explain your reasoning.
>
> 1. $2.7 \cdot -2.5 = -6.75$
>
> 2. $-\frac{3}{4} \cdot -\frac{5}{7} = -\frac{15}{28}$
>
> 3. $5.5 \cdot -\frac{3}{5} = 3.3$

Student Response

1. agree

2. disagree; a negative times a negative is positive

3. disagree; a positive times a negative is negative

Student Lesson Summary

- A positive times a positive is always positive.

  For example, $\frac{3}{5} \cdot \frac{7}{8} = \frac{21}{40}$.

- A negative times a negative is also positive.

  For example, $-\frac{3}{5} \cdot -\frac{7}{8} = \frac{21}{40}$.

- A negative times a positive or a positive times a negative is always negative.

  For example, $\frac{3}{5} \cdot -\frac{7}{8} = -\frac{3}{5} \cdot \frac{7}{8} = -\frac{21}{40}$.

- A negative times a negative times a negative is also negative.

  For example, $-3 \cdot -4 \cdot -5 = -60$.

Lesson 10 Practice Problems

Problem 1

Statement

Evaluate each expression:

a. $-12 \cdot \frac{1}{3}$

b. $-12 \cdot -\frac{1}{3}$

c. $12 \cdot \left(-\frac{5}{4}\right)$

d. $-12 \cdot \left(-\frac{5}{4}\right)$

Solution

a. -4

b. 4

c. -15

d. 15

Problem 2

Statement

Evaluate each expression:

a. $-1 \cdot 2 \cdot 3$

b. $-1 \cdot (-2) \cdot 3$

c. $-1 \cdot (-2) \cdot (-3)$

Solution

a. -6

b. 6

c. -6

Problem 3

Statement

Order each set of numbers from least to greatest.

a. 4, 8, -2, -6, 0

b. -5, -5.2, 5.5, $-5\frac{1}{2}$, $\frac{-5}{2}$

Solution

a. -6, -2, 0, 4, 8

b. $-5\frac{1}{2}$, -5.2, -5, $\frac{-5}{2}$, 5.5

(From Unit 5, Lesson 1.)

Problem 4

Statement

$30 + \text{-}30 = 0$.

a. Write another sum of two numbers that equals 0.

b. Write a sum of three numbers that equals 0.

c. Write a sum of four numbers that equals 0, none of which are opposites.

Solution

Answers vary. Sample response:

a. $\text{-}589 + 589$

b. $\text{-}589 + 500 + 89$

c. $\text{-}589 + 500 + 90 + (\text{-}1)$

(From Unit 5, Lesson 3.)

Problem 5

Statement

A submarine is searching for underwater features. It is accompanied by a small aircraft and an underwater robotic vehicle.

At one time the aircraft is 200 m above the surface, the submarine is 55 m below the surface, and the underwater robotic vehicle is 227 m below the surface.

a. What is the difference in height between the submarine and the aircraft?

b. What is the distance between the underwater robotic vehicle and the submarine?

Solution

We have to assume they are all directly above or below each other to answer the question.

 a. 255 m

 b. 172 m

Problem 6
Statement

 a. Clare is cycling at a speed of 12 miles per hour. If she starts at a position chosen as zero, what will her position be after 45 minutes?

 b. Han is cycling at a speed of -8 miles per hour; if he starts at the same zero point, what will his position be after 45 minutes?

 c. What will the distance between them be after 45 minutes?

Solution

 a. 9 miles ($12 \cdot 0.75 = 9$)

 b. -6 miles (-8 $\cdot 0.75 =$ -6)

 c. 15 miles

Problem 7
Statement

Fill in the missing numbers in these equations

 a. $(-7) \cdot ? = -14$

 b. $? \cdot 3 = -15$

 c. $? \cdot 4 = 32$

 d. $-49 \cdot 3 = ?$

Solution

 a. $(-7) \cdot 2 = -14$

 b. $(-5) \cdot 3 = -15$

c. $8 \cdot 4 = 32$

d. $(-49) \cdot 3 = -147$

Lesson 11: Dividing Rational Numbers

Goals

- Apply multiplication and division of signed numbers to solve problems involving constant speed with direction, and explain (orally) the reasoning.

- Generalize (orally) a method for determining the quotient of two signed numbers.

- Generate a division equation that represents the same relationship as a given multiplication equation with signed numbers.

Learning Targets

- I can divide rational numbers.

Lesson Narrative

In this lesson, students complete their work extending all four operations to signed numbers by studying division. They use the relationship between multiplication and division to develop rules for dividing signed numbers. In preparation for the next lesson on negative rates of change, students look at a context, drilling a well, that is modeled by an equation $y = kx$ where k is a negative number. This builds on their previous work with proportional relationships.

Alignments

Building On

- 7.NS.A: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Addressing

- 7.NS.A.2: Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

- 7.NS.A.2.b: Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

Building Towards

- 7.EE.B.4.a: Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Instructional Routines

- MLR6: Three Reads

- MLR8: Discussion Supports

- Think Pair Share

> **Student Learning Goals**
>
> Let's divide signed numbers.

11.1 Tell Me Your Sign

Warm Up: 5 minutes
For this warm-up students use what they have learned about multiplication and division with rational numbers to answer questions about the solution to an equation.

Building On

- 7.NS.A

Building Towards

- 7.EE.B.4.a

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 1 minute of quiet think time and ask them to discuss their reasoning with a partner. Follow with a whole-class discussion.

> **Student Task Statement**
>
> Consider the equation: $-27x = -35$
>
> Without computing:
>
> 1. Is the **solution** to this equation positive or negative?
>
> 2. Are either of these two numbers solutions to the equation?
> $$\frac{35}{27} \qquad\qquad -\frac{35}{27}$$

Student Response

1. Positive

2. The first one

Activity Synthesis

Ask students to share their reasoning.

11.2 Multiplication and Division

10 minutes

The purpose of this activity is to understand that the division facts for rational numbers are simply a consequence of the multiplication done previously. Students work several numerical examples relating multiplication to division, and then articulate a rule for the sign of a quotient based on the signs of the dividend and divisor (MP8).

Monitor for students who identify and describe the rule clearly.

Addressing

- 7.NS.A.2.b

Instructional Routines

- MLR6: Three Reads

- Think Pair Share

Launch

Remind students that we can rearrange division equations to be multiplication equations, and vice versa. It may be useful to demonstrate with positive numbers if students struggle to recall this. For example, ask how we could rewrite $10 \div 2 = 5$ as a multiplication equation. Students can also reference the multiplication table from a previous lesson, if needed.

Arrange students in groups of 2. Give students 4 minute of quiet work time followed by 2 minutes of partner discussion, then follow with whole-class discussion.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. For example, reference examples from the previous lesson on signed multiplication to provide an entry point into this activity.
Supports accessibility for: Social-emotional skills; Conceptual processing

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Anticipated Misconceptions

Have students refer to their previous work with signed multiplication to help them with the division statements.

Student Task Statement

1. Find the missing values in the equations

    a. $-3 \cdot 4 = ?$

    b. $-3 \cdot ? = 12$

    c. $3 \cdot ? = 12$

    d. $? \cdot -4 = 12$

    e. $? \cdot 4 = -12$

2. Rewrite the unknown factor problems as division problems.

3. Complete the sentences. Be prepared to explain your reasoning.

    a. The sign of a positive number divided by a positive number is always:

    b. The sign of a positive number divided by a negative number is always:

    c. The sign of a negative number divided by a positive number is always:

    d. The sign of a negative number divided by a negative number is always:

4. Han and Clare walk towards each other at a constant rate, meet up, and then continue past each other in opposite directions. We will call the position where they meet up 0 feet and the time when they meet up 0 seconds.

- ○ Han's velocity is 4 feet per second.
- ○ Clare's velocity is -5 feet per second.

a. Where is each person 10 seconds before they meet up?

b. When is each person at the position -10 feet from the meeting place?

Student Response

1.   a. -12

     b. -4

     c. 4

     d. -3

     e. -3

2. Answers vary. Possible responses:

     a. NA

     b. $12 \div (-3) = -4$

     c. $12 \div 4 = 3$

     d. $12 \div (-4) = -3$

     e. $-12 \div 4 = -3$

3.   a. positive

     b. negative

     c. negative

     d. positive

4.   a. Han was at -40 feet and Clare was at 50 feet.

     b. Han was there at -2.5 seconds and Clare was there at 2 seconds.

Are You Ready for More?

It is possible to make a new number system using *only* the numbers 0, 1, 2, and 3. We will write the symbols for multiplying in this system like this: $1 \otimes 2 = 2$. The table shows some of the products.

$\otimes$	0	1	2	3
0	0	0	0	0
1		1	2	3
2			0	2
3				

1. In this system, $1 \otimes 3 = 3$ and $2 \otimes 3 = 2$. How can you see that in the table?

2. What do you think $2 \otimes 1$ is?

3. What about $3 \otimes 3$?

4. What do you think the solution to $3 \otimes n = 2$ is?

5. What about $2 \otimes n = 3$?

Student Response

1. The table shows $1 \otimes 3 = 3$ in the cell in the "1" row and the "3" column. The table shows $2 \otimes 3 = 2$ in the cell in the "2" row and the "3" column.

2. It could be anything, but if multiplication is commutative, then $2 \otimes 1 = 2$.

3. It could be anything, but if we fill out the table assuming multiplication is commutative, we can get almost all of the entries:

$\otimes$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	

Looking at the table, setting $3 \otimes 3 = 1$ would make things symmetrical.

A more sophisticated argument requires us to know how to add these numbers, which we saw in an earlier extension. If the distributive property holds and $1 \oplus 2 = 3$, then $3 \otimes 3 = (1 \oplus 2) \otimes 3 = (1 \otimes 3) \oplus (2 \otimes 3) = 3 \oplus 2$. In the earlier extension we suggested that $3 \oplus 2 = 1$.

4. $2 \otimes 3 = 2$ so if multiplication is commutative, then $3 \otimes 2 = 2$, so $n = 2$.

5. Looking at all of the $2\otimes$, there is no value of n that makes this equation true.

Activity Synthesis

Select students who can articulate and explain what they have learned about the sign of a quotient based on the signs of the dividend and divisor (note that students do not need to use these words, they can just give examples). Highlight that the division facts are simply a consequence of the multiplication done previously. Moving forward, we are going to take these facts to be true for all rational numbers.

11.3 Drilling Down

10 minutes (there is a digital version of this activity)

The purpose of this activity is to use the new skills of multiplying and dividing rational numbers to represent and solve problems in a new context (MP4). In this activity students are using using multiplication and division of negatives and working with a proportional relationship with a negative constant of proportionality. Students should use what they know about proportional relationships to help them (for example; points lie on a straight line, line passes through zero, there is a point $(1, k)$ that lies on the line).

Addressing

- 7.NS.A.2

Instructional Routines

- MLR8: Discussion Supports

Launch

Remind students that we can model positions below the surface with negative values, so drilling 30 feet down is represented with -30 feet.

Ask students what they remember about proportional relationships. Examples:

- Often represented with an equation in the form $y = kx$.

- The constant of proportionality, often called k, is the change in y for a change by 1 in x.

- A graph representing a proportional relationship is a line through (0,0) and $(1, k)$

Arrange students in groups of 3 during the discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Provide appropriate reading accommodations and supports to ensure students have access to written directions, word problems and other text-based content.
Supports accessibility for: Language; Conceptual processing

Anticipated Misconceptions

Some students may struggle to get the length of time out of the phrase "one full day of continuous use," wondering whether the drill was running at night. Let them know the drill has been going for 24 hours.

> ### Student Task Statement
>
> A water well drilling rig has dug to a height of -60 feet after one full day of continuous use.

1. Assuming the rig drilled at a constant rate, what was the height of the drill after 15 hours?

2. If the rig has been running constantly and is currently at a height of -147.5 feet, for how long has the rig been running?

3. Use the coordinate grid to show the drill's progress.
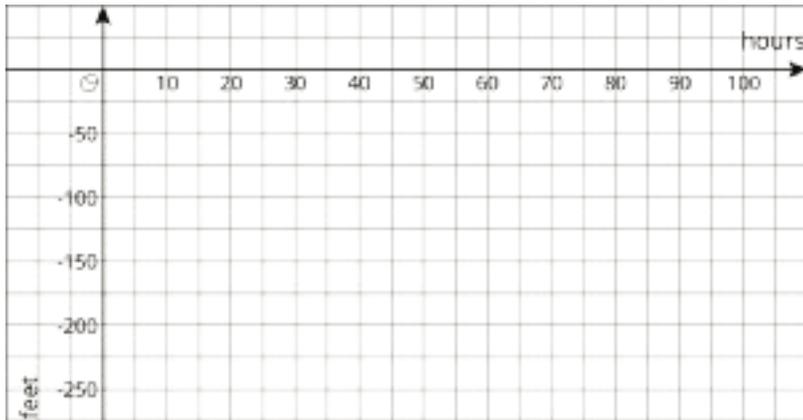
4. At this rate, how many hours will it take until the drill reaches -250 feet?

Student Response

1. The drill drills -60 ÷ 24 or -2.5 feet in one hour. After 15 hours it has drilled -37.5 feet, because $-2.5 \cdot 15 = -37.5$.

2. The drill has been running for 59 hours, because $-147.5 \div (-2.5)$ is 59.

3. A ray starting at $(0, 0)$ and passing through $(24, -60)$ and $(59, -147.5)$.

4. 100 hours because $-250 \div (-2.5) = 100$

Activity Synthesis

Have students to share their solutions with each other in groups of three and work to come to agreement.

To wrap up, emphasize that because we can now complete any calculation with any rational number, we can extend constants of proportionality k to include negative values. Because the operations are now well-defined, we can answer questions using such a constant.

Lesson Synthesis

Key takeaways:

- Recall that we can write a division problem as a multiplication problem.

- A positive divided by a negative is negative.

- A negative divided by a positive is negative.

- A negative divided by a negative is positive.

Discussion questions:

- What kind of number do you get when you divide a negative number by a positive number? Use a multiplication equation to explain why this makes sense.

- What kind of number do you get when you divide a negative number by a negative number? Use a multiplication equation to explain why this makes sense.

11.4 Matching Division Expressions

Cool Down: 5 minutes
Addressing
- 7.NS.A.2.b

> **Student Task Statement**
>
> Match each expression with its value.

1. $15 \div 12$
2. $12 \div (-15)$
3. $12 \div 15$
4. $15 \div (-12)$

- -0.8
- 0.8
- -1.25
- 1.25

Student Response

1. $15 \div 12 = 1.25$
2. $12 \div (-15) = -0.8$
3. $12 \div 15 = 0.8$
4. $15 \div (-12) = -1.25$

Student Lesson Summary

Any division problem is actually a multiplication problem:

- $6 \div 2 = 3$ because $2 \cdot 3 = 6$
- $6 \div -2 = -3$ because $-2 \cdot -3 = 6$
- $-6 \div 2 = -3$ because $2 \cdot -3 = -6$
- $-6 \div -2 = 3$ because $-2 \cdot 3 = -6$

Because we know how to multiply signed numbers, that means we know how to divide them.

- The sign of a positive number divided by a negative number is always negative.
- The sign of a negative number divided by a positive number is always negative.
- The sign of a negative number divided by a negative number is always positive.

A number that can be used in place of the variable that makes the equation true is called a **solution** to the equation. For example, for the equation $x \div -2 = 5$, the solution is -10, because it is true that $-10 \div -2 = 5$.

Glossary

- solution to an equation

Lesson 11 Practice Problems

Problem 1

Statement

Find the quotients:

    a. $24 \div -6$

    b. $-15 \div 0.3$

    c. $-4 \div -20$

Solution

-4, -50, 0.2

Problem 2

Statement

Find the quotients.

    a. $\frac{2}{5} \div \frac{3}{4}$

    b. $\frac{9}{4} \div \frac{-3}{4}$

    c. $\frac{-5}{7} \div \frac{-1}{3}$

    d. $\frac{-5}{3} \div \frac{1}{6}$

Solution

    a. $\frac{8}{15}$

    b. -3

    c. $\frac{15}{7}$

    d. -10

Problem 3

Statement

Is the solution positive or negative?

    a. $2 \cdot x = 6$

    b. $-2 \cdot x = 6.1$

c. $2.9 \cdot x = -6.04$

d. $-2.473 \cdot x = -6.859$

Solution

a. Positive

b. Negative

c. Negative

d. Positive

Problem 4

Statement

Find the solution mentally.

a. $3 \cdot -4 = a$

b. $b \cdot (-3) = -12$

c. $-12 \cdot c = 12$

d. $d \cdot 24 = -12$

Solution

a. $a = -12$

b. $b = 4$

c. $c = -1$

d. $d = \frac{-1}{2}$

Problem 5

Statement

In order to make a specific shade of green paint, a painter mixes $1\frac{1}{2}$ quarts of blue paint, 2 cups of green paint, and $\frac{1}{2}$ gallon of white paint. How much of each color is needed to make 100 cups of this shade of green paint?

Solution

Blue: $37\frac{1}{2}$ cups, green: $12\frac{1}{2}$ cups, white: 50 cups. There are 4 cups in a quart so $1\frac{1}{2}$ quarts is 6 cups. There are 16 cups in a gallon so $\frac{1}{2}$ gallons is 8 cups. So the ratio of cups of blue to cups of green to cups of white is 6 to 2 to 8 or, equivalently, 3 to 1 to 4. $3 + 1 + 4 = 8$. $100 \div 8 = 12.5$ So, the painter

needs 37.5 cups of blue because $3 \cdot (12.5) = 37.5$. The painter needs 12.5 cups of green because $1 \cdot (12.5) = 12.5$. And the painter needs 50 cups of white because $4 \cdot (12.5) = 50$.

(From Unit 4, Lesson 2.)

Problem 6

Statement

Here is a list of the highest and lowest elevation on each continent.

	highest point (m)	lowest point (m)
Europe	4,810	-28
Asia	8,848	-427
Africa	5,895	-155
Australia	4,884	-15
North America	6,198	-86
South America	6,960	-105
Antartica	4,892	-50

a. Which continent has the largest difference in elevation? The smallest?

b. Make a display (dot plot, box plot, or histogram) of the data set and explain why you chose that type of display to represent this data set.

Solution

a. Asia has the largest difference in elevation. $8,848 - (\text{-}427) = 9,275$. Europe has the smallest difference in elevation. $4,810 - (\text{-}28) = 4,838$

b. Answers vary. Possible solutions:
- Box plot:
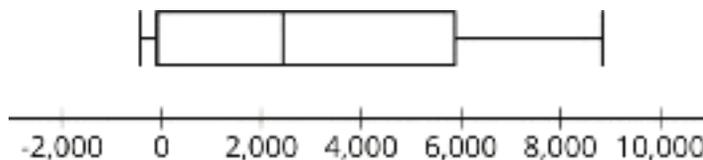
- Histogram:
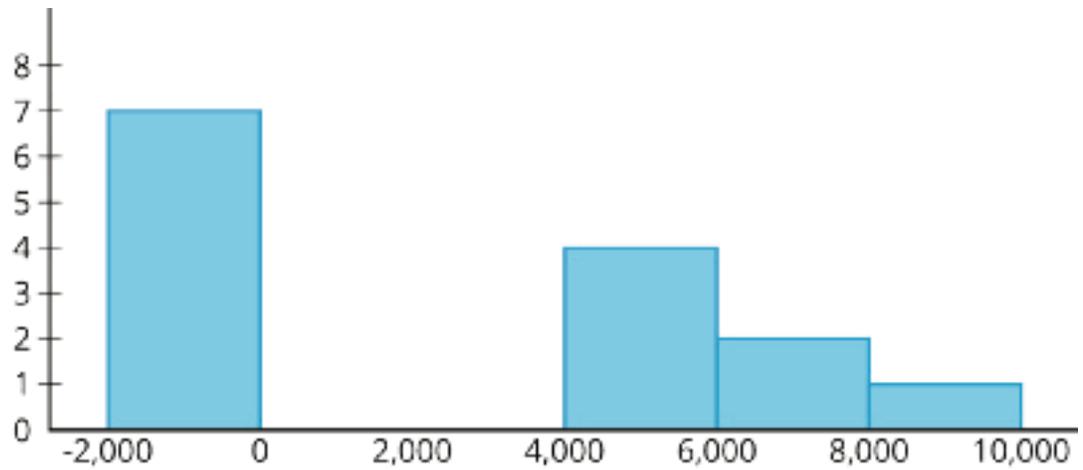
I chose to make a histogram because it can show the gap in the middle of the data better than a box plot can. Also, this data set would be hard to see on a dot plot because some of the elevations are very close to each other, but none of them are exactly the same.

(From Unit 5, Lesson 3.)

Unit 5 Lesson 11

Lesson 12: Negative Rates

Goals

- Apply operations with signed numbers to solve problems involving constant rates, and explain (orally) the solution method.

- Explain (orally and in writing) how signed numbers can be used to represent situations involving constant rates.

- Write an equation of the form $y = -kx$ to represent a situation that involves descending at a constant rate.

Learning Targets

- I can solve problems that involve multiplying and dividing rational numbers.

- I can solve problems that involve negative rates.

Lesson Narrative

The purpose of this lesson is to introduce students to negative rates of change, which will become important when they start learning about linear functions in later lessons. Students apply their understanding of operating with signed numbers to solve problems in context. The first problem involves a fish tank that is being filled and drained. The second problem deals with historic voyages in a bathyscaphe (deep-sea submarine) and a high-altitude hot air balloon. When students reason quantitatively about what it means for a rate to be negative, they engage in MP2.

The activities in this lesson involve more reading than most lessons. Be prepared to support students with unfamiliar words.

Alignments

Addressing

- 7.EE.B.3: Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. $

- 7.NS.A.3: Solve real-world and mathematical problems involving the four operations with rational numbers.Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

Instructional Routines

- MLR3: Clarify, Critique, Correct

- MLR6: Three Reads

- MLR8: Discussion Supports

- Think Pair Share

> **Student Learning Goals**
>
> Let's apply what we know about signed numbers.

12.1 Grapes per Minute

Warm Up: 5 minutes
The purpose of this warm-up is to review "per" language.

Addressing

- 7.RP.A.2

Launch

Arrange students in groups of 2. Give students 1 minute of quiet work time followed by 1 minute of partner discussion, then follow with whole-class discussion.

> **Student Task Statement**
>
> 1. If you eat 5 grapes per minute for 8 minutes, how many grapes will you eat?
>
> 2. If you hear 9 new songs per day for 3 days, how many new songs will you hear?
>
> 3. If you run 15 laps per practice, how many practices will it take you to run 30 laps?

Student Response

1. 40

2. 27

3. 2

Activity Synthesis

For each question, ask a student to share their response and reasoning. Resolve any disagreements that come up. Remind students that whenever we see the word "per", that means "for every 1".

12.2 Water Level in the Aquarium

10 minutes
This activity builds students understanding of how negative rates can be used to model directed change. Students use their knowledge of dividing and multiplying negative numbers to answer questions involving rates. They are not expected to express these as relationships of the form $y = kx$ in this activity, though some students might. In the second question students will also need

to convert between different rates. Pay close attention to students conversion between units in the last question as the rates are over different times. Identify those who convert to the same units (either minutes or hours) (MP6).

Addressing

- 7.NS.A.3

Instructional Routines

- MLR6: Three Reads

- MLR8: Discussion Supports

Launch

For each question, ask students to read the whole prompt individually before they start working on it. Remind students that they need to justify their answers. At the end of the activity put the students into groups of two. Use MLR 8 (Discussion Supports) to help students create context for this activity by providing or guiding students in creating visual diagrams that illustrate what is happening with the aquariums.

Access for Students with Disabilities

Representation: Access for Perception. Read the statements aloud. Students who both listen to and read the information will benefit from extra processing time. If students are unsure where to begin, suggest that they draw a diagram to help organize the information provided.
Supports accessibility for: Language

Access for English Language Learners

Reading: MLR6 Three Reads. Use this routine with the first problem to support students' reading comprehension. In the first read, students read the situation with the goal of understanding what the situation is about (e.g., an aquarium has a system to maintain the correct water level; too much water and it will overflow; too little water and the fish will get sick). If needed, discuss the meaning of unfamiliar terms at this time, or display a diagram to clarify the context. Use the second read to identify the important quantities by asking students what can be counted or measured, without focusing on specific values. Listen for, and amplify, the important quantities that vary in relation to each other: amount of water in the aquarium, in liters; amount of time spent filling or draining water, in minutes. After the third read, ask students to brainstorm possible strategies to answer the questions. This helps students connect the language in the word problems with the reasoning needed to solve the problems.
Design Principle(s): Support sense-making

Student Task Statement

1. A large aquarium should contain 10,000 liters of water when it is filled correctly. It will overflow if it gets up to 12,000 liters. The fish will get sick if it gets down to 4,000 liters. The aquarium has an automatic system to help keep the correct water level. If the water level is too low, a faucet fills it. If the water level is too high, a drain opens.

   One day, the system stops working correctly. The faucet starts to fill the aquarium at a rate of 30 liters per minute, and the drain opens at the same time, draining the water at a rate of 20 liters per minute.

   a. Is the water level rising or falling? How do you know?

   b. How long will it take until the tank starts overflowing or the fish get sick?

2. A different aquarium should contain 15,000 liters of water when filled correctly. It will overflow if it gets to 17,600 liters.

   One day there is an accident and the tank cracks in 4 places. Water flows out of each crack at a rate of $\frac{1}{2}$ liter per hour. An emergency pump can re-fill the tank at a rate of 2 liters per minute. How many minutes must the pump run to replace the water lost each hour?

Student Response

1. Represent the filling as 30 liters per minute and the drain as -20 liters per minute.
   a. This means that the water level is rising at $30 + (\text{-}20)$ or 10 liters per minute.

   b. The tank needs an extra 2,000 liters before it overflows. At 10 liters per minute this takes $2000 \div 10$ or 200 minutes.

2. The tank has 4 cracks, which leak at 0.5 liters per hour. This is a total of -2 liters per hour. The emergency pump is 2 liters per minute, so must run for one minute every hour to compensate.

Activity Synthesis

Ask students to compare their simplifying assumptions and solution methods. In a whole group discussion bring out the assumptions students have made. Compare the different rates in the answers to the last question.

12.3 Up and Down with the Piccards

15 minutes
This activity builds on students' previous work with proportional relationships, as well as their understanding of multiplying and dividing signed numbers, to model different historical scenarios involving ascent and descent, and students must explain their reasoning (MP3). While equations of

the form $y = kx$ are not technically proportional relationships if k is negative, students can still work with these equations. Identify students who convert between seconds and hours.

Addressing

- 7.EE.B.3
- 7.NS.A.3
- 7.RP.A.2

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2. Introduce the activity by asking students where they think the deepest part of the ocean is. It may be helpful to display a map showing the location of the Challenger Deep, but this is not required. Explain that Jacques Piccard had to design a specific type of submarine to make such a deep descent. Use MLR 8 (Discussion Supports) to help students create context for this activity. Provide pictures of Jacques Piccard with a submersible and Auguste Piccard with a hot air balloon. Guide students in creating visual diagrams that illustrate what is happening with the trench, ocean surface, and balloon. Remind students that they need to explain their reasoning. Provide for a quiet work time followed by partner and whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Provide appropriate reading accommodations and supports to ensure students have access to written directions, word problems and other text-based content.
Supports accessibility for: Language; Conceptual processing

Anticipated Misconceptions

Some students may be confused by the correct answer to the last question, thinking that $51{,}683 - 52{,}940 = -1{,}257$ means that Auguste landed his balloon below sea level. Explain that Auguste launched his balloon from a mountain, to help him reach as high of an altitude as possible. We have chosen to use zero to represent this starting point, instead of sea level, for this part of the activity. Therefore, a vertical position of -1,257 feet means that Auguste landed below his starting point, but not below sea level.

Student Task Statement

1. Challenger Deep is the deepest known point in the ocean, at 35,814 feet below sea level. In 1960, Jacques Piccard and Don Walsh rode down in the Trieste and became the first people to visit the Challenger Deep.

    a. If sea level is represented by 0 feet, explain how you can represent the depth of a submarine descending from sea level to the bottom of Challenger Deep.

    b. Trieste's descent was a change in depth of -3 feet per second. We can use the relationship $y = -3x$ to model this, where y is the depth (in feet) and x is the time (in seconds). Using this model, how much time would the Trieste take to reach the bottom?

    c. It took the Trieste 3 hours to ascend back to sea level. This can be modeled by a different relationship $y = kx$. What is the value of k in this situation?

2. The design of the Trieste was based on the design of a hot air balloon built by Auguste Piccard, Jacques's father. In 1932, Auguste rode in his hot-air balloon up to a record-breaking height.

    a. Auguste's ascent took 7 hours and went up 51,683 feet. Write a relationship $y = kx$ to represent his ascent from his starting location.

    b. Auguste's descent took 3 hours and went down 52,940 feet. Write another relationship to represent his descent.

    c. Did Auguste Piccard end up at a greater or lesser altitude than his starting point? How much higher or lower?

Student Response

1. a. We can use negative numbers to represent how many feet below sea level a submarine is.

    b. It would take 11,938 seconds to descend to the sea floor, or about 3 hours and 19 minutes. Substituting the depth of -35,814 in for y gives the equation $-35,814 = -3x$. Solving the equation gives 11,938 for x, because $-35,814 \div (-3) = 11,938$.

    c. $k = \frac{35814}{10800}$; It took 10,800 seconds to go up 35,814 feet, because $3 \cdot 60 \cdot 60 = 10,800$. (This is a vertical change of approximately 3.32 feet per second.)

2. a. The relationship is $y = \frac{51683}{25200}x$. It took 25,200 seconds to go up 51,683 feet, because $7 \cdot 60 \cdot 60 = 25,200$. (This is a vertical change of approximately 2.05 feet per second.)

b. The relationship is $y = \frac{-52940}{10800}x$, because it took 10,800 seconds to go down 52,940 feet. (This is a vertical change of approximately -4.9 feet per second.)

c. He ended up lower because $51{,}683 - 52{,}940 = $ -1,257, so 1,257 feet lower than his starting point.

Are You Ready for More?

During which part of either trip was a Piccard changing vertical position the fastest? Explain your reasoning.

- Jacques's descent
- Jacques's ascent
- Auguste's ascent
- Auguste's descent

Student Response

Auguste's descent was the fastest.

Activity Synthesis

First, have students compare their solutions with a partner and describe what is the same and what is different. This will help students be prepared to explain their reasoning to the whole class.

Next, select students to share with the class. Highlight solutions that correctly operarure with negatives, those convert between seconds and hours, and those that state their assumptions clearly. Help students make sense of each equation by asking questions such as:

- After 1 second, by how much have they changed vertical position? After 10 seconds? After 100 seconds?
- How can you tell from the equation whether they are going up or down?
- How can you tell from the equation the total distance or total time of their ascent or descent?
- What does 0 represent in this situation?

The most important thing for students to get out of this activity is how different operations with signed numbers were helpful for representing the situation and solving the problems.

Writing, Representing: MLR3 Clarify, Critique, Correct. To begin the whole-class discussion, display the following incorrect statements for all to see: "Since they are under water, submarines have negative ascent and descent, so the k will always be negative." and "A hot air balloon will always have a positive ascent and descent because it is in the air, so the k will always be negative." Give students 1–2 minutes of quiet think time to identify the errors, and to critique the reasoning. Invite students to share their thinking with a partner, and to work together to correct one of the statements. Listen for the ways students interpret each statement, paying attention to the connections they make between vertical distances, and how changes in vertical position are represented. For each situation, select 1–2 groups to share their revised statements with the class. This will help clarify language and understanding about how ascent and descent can be represented by an equation.
Design Principle(s): Optimize output (for explanation); Maximize meta-awareness

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Lesson Synthesis

Key takeaways:

- Recall we can represent speed with direction (velocity) using signed numbers. We can do this with vertical movement (in fact with any rate).

- The convention is that up is the positive direction and down is the negative direction.

Discussion questions:

- What other rates have you encountered where it makes sense to have positive and negative values?

12.4 Submarines

Cool Down: 5 minutes
Addressing

- 7.NS.A.3

- 7.RP.A.2

> **Student Task Statement**
>
> 1. A submarine is descending to examine the seafloor 2,100 feet below the surface. It takes the submarine 2 hours to make this descent. Write an equation to represent the relationship between the submarine's elevation and time.
>
> 2. Another submarine's descent can be represented as $y = -240x$, where y is the elevation and x is time in hours. How long will it take this submarine to make the descent?

Student Response

1. $y = -1050x$ where y is the elevation and x is the hour

2. 8.75 hours

Student Lesson Summary

We saw earlier that we can represent speed with direction using signed numbers. Speed with direction is called *velocity*. Positive velocities always represent movement in the opposite direction from negative velocities.

We can do this with vertical movement: moving up can be represented with positive numbers, and moving down with negative numbers. The magnitude tells you how fast, and the sign tells you which direction. (We could actually do it the other way around if we wanted to, but usually we make up positive and down negative.)

Lesson 12 Practice Problems
Problem 1
Statement

Describe a situation where each of the following quantities might be useful.

   a. -20 gallons per hour

   b. -10 feet per minute

   c. -0.1 kilograms per second

Solution

Answers vary. Sample responses:

   a. Water leaking out of a tank

   b. An airplane descending

   c. Gravel being emptied out of a truck

Problem 2
Statement

A submarine is only allowed to change its depth by rising toward the surface in 60-meter stages. It starts off at -340 meters.

   a. At what depth is it after:

      i. 1 stage

      ii. 2 stages

      iii. 4 stages

   b. How many stages will it take to return to the surface?

Solution

   a.   i. -280 m; $-340 + 60 = -280$

        ii. -220 m; $-340 + 120 = -220$

        iii. -100m; $-340 + 240 = -100$

   b. 6 stages of change. $340 \div 60 = 5.7$ so the submarine would need 6 stages

Problem 3

Statement

Some boats were traveling up and down a river. A satellite recorded the movements of several boats.

    a. A motor boat traveled -3.4 miles per hour for 0.75 hours. How far did it go?

    b. A tugboat traveled -1.5 miles in 0.3 hours. What was its velocity?

    c. What do you think that negative distances and velocities could mean in this situation?

Solution

a. -2.55 miles; $-3.4 \cdot 0.75 = -2.55$

b. -5 miles per hour; $-1.5 \div 0.3 = -5$

c. Someone had to choose one direction to be positive and the other to be negative. Positive distances could mean distances in the positive direction and negative distances mean distances in the other direction. Positive velocities could mean it was moving in the positive direction and negative velocities mean it was moving in the negative direction.

Problem 4

Statement

    a. A cookie recipe uses 3 cups of flour to make 15 cookies. How many cookies can you make with this recipe with 4 cups of flour? (Assume you have enough of the other ingredients.)

    b. A teacher uses 36 centimeters of tape to hang up 9 student projects. At that rate, how much tape would the teacher need to hang up 10 student projects?

Solution

a. 20 cookies; based on the information one can determine that 1 cup of flour makes 5 cookies, so 4 cups of flour will make 20 cookies.

b. 40 centimeters; based on the information one can determine that 4 centimeters of tape will hang up 1 student project, so the teacher would need 40 centimeters of tape to hang 10 student projects.

(From Unit 4, Lesson 3.)

Problem 5

Statement

Evaluate each expression. When the answer is not a whole number, write your answer as a fraction.

  a. $-4 \cdot -6$

  b. $-24 \cdot \frac{-7}{6}$

  c. $4 \div -6$

  d. $\frac{4}{3} \div -24$

Solution

  a. 24

  b. 28

  c. $\frac{-2}{3}$ (or equivalent)

  d. $\frac{-1}{18}$ (or equivalent)

(From Unit 5, Lesson 11.)

Section: Four Operations with Rational Numbers

Lesson 13: Expressions with Rational Numbers

Goals

- Evaluate an expression for given values of the variable, including negative values, and compare (orally) the resulting values of the expression.

- Generalize (orally) about the relationship between additive inverses and about the relationship between multiplicative inverses.

- Identify numerical expressions that are equal, and justify (orally) that they are equal.

Learning Targets

- I can add, subtract, multiply, and divide rational numbers.

- I can evaluate expressions that involve rational numbers.

Lesson Narrative

As students start to gain fluency with rational number arithmetic, they encounter complicated numerical expressions, and algebraic expressions with variables, and there is a danger that they might lose the connection between those expressions and numbers on the number line. The purpose of this lesson is to help students make sense of expressions, and reason about their position on the number line, for example whether the number is positive or negative, which of two numbers is larger, or whether two expressions represent the same number. They work through common misconceptions that can arise about expressions involving variables, for example the misconception that -x must always be a negative number. (It is positive if x is negative.) In the last activity they reason about expressions in a and b given the positions of a and b on a number line without a given scale, in order to develop the idea that you can always think of the letters in an algebraic expression as numbers and deduce, for example, that $\frac{1}{4}a$ is a quarter of the way from 0 to a on the number line, even if you don't know the value of a.

When students look at a numerical expression and see without calculation that it must be positive because it is a product of two negative numbers, they are making use of structure (MP7).

Alignments

Addressing

- 7.NS.A: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

- 7.NS.A.3: Solve real-world and mathematical problems involving the four operations with rational numbers.Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Instructional Routines

- MLR2: Collect and Display
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Notice and Wonder
- Take Turns
- True or False

Required Materials

Pre-printed slips, cut from copies of the   **Instructional master**

Required Preparation

Print and cut up slips from the Card Sort: The Same But Different Instructional master. Prepare 1 copy for every 2 students. If possible, copy each complete set on a different color of paper, so that a stray slip can quickly be put back.

> **Student Learning Goals**
>
> Let's develop our signed number sense.

13.1 True or False: Rational Numbers

Warm Up: 5 minutes

The purpose of this warm-up is for students to reason about numeric expressions using what they know about operations with negative and positive numbers without actually computing anything.

Addressing

- 7.NS.A

Instructional Routines

- True or False

Launch

Explain to students that we sometimes leave out the small dot that tells us that two numbers are being multiplied. So when a number, or a number and a variable, are next to each other without a symbol between them, that means we are finding their product. For example, $(-5) \cdot (-10)$ can be written $(-5)(-10)$ and $-5 \cdot x$ can be written $-5x$.

Display one problem at a time. Give students 1 minute of quiet think time per problem and ask them to give a signal when they have an answer. Follow with a whole-class discussion.

> **Student Task Statement**
>
> Decide if each statement is true or false. Be prepared to explain your reasoning.
>
> 1. (-38.76)(-15.6) is negative
>
> 2. $10,000 - 99,999 < 0$
>
> 3. $\left(\frac{3}{4}\right)\left(-\frac{4}{3}\right) = 0$
>
> 4. $(30)(-80) - 50 = 50 - (30)(-80)$

Student Response

1. false

2. true

3. false

4. false

Activity Synthesis

Ask students to share their reasoning.

13.2 Card Sort: The Same But Different

10 minutes
In this activity students continue to build fluency operating with signed numbers as they match different expressions that have the same value. Students look for and use the relationship between inverse operations (MP7).

As students work, identify groups that make connections between the operations, for example they notice that subtracting a number is the same as adding its opposite, or that dividing is the same as multiplying by the multiplicative inverse.

Addressing

- 7.NS.A.3

Instructional Routines

- MLR7: Compare and Connect

- Take Turns

Launch

Arrange students in groups of 2. Distribute pre-cut slips from the Instructional
master.

Anticipated Misconceptions

If students struggle to find matches, encourage them to think of the operations in different ways. You might ask:

- How else can you think of ____ (subtraction, addition, multiplication, division)?

> ### Student Task Statement
>
> Your teacher will give you a set of cards. Group them into pairs of expressions that have the same value.

Student Response

answer	one expression	other expression
-17	$-10 + (-7)$	$-10 - 7$
-4	$-1 \cdot 4$	$1 \div (-\frac{1}{4})$
-3	$-10 + 7$	$-10 - -7$
-2.5	$15 \div (-6)$	$-15 \cdot \frac{1}{6}$
-2	$8 \div (-4)$	$8 \cdot (-\frac{1}{4})$
-1	$1 - 2$	$1 + -2$
2	$8 \div 4$	$(8)(\frac{1}{4})$
2.5	$15 \cdot \frac{1}{6}$	$-15 \div -6$
3	$1 + 2$	$1 - (-2)$
4	$(1)(4)$	$1 \div \frac{1}{4}$

Activity Synthesis

Select students to share their strategies. Highlight stategies that compared the structure of the expressions instead of just the final answers.

Discuss:

- Subtracting a number is equivalent to adding the additive inverse.

- A number and its additive inverse have oppostie signs (but the same magnitude).

- Dividing by a number is equivalent to multiplying by the multiplicative inverse.

- A number and its multiplicative inverse have the same sign (but different magnitudes).

If desired, have students order their pairs of equivalent expressions from least to greatest to review ordering rational numbers.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Break the class into small discussion groups and then invite a representative from each group to report back to the whole class.
Supports accessibility for: Attention; Social-emotional skills

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Access for English Language Learners

Speaking, Representing: MLR7 Compare and Connect. Use this routine to support whole-class discussion. Ask students to consider what is the same and what is different about the structure of each expression. Draw students' attention to the association between quantities (e.g., adding the additive inverse is subtracting; multiplying by the multiplicative inverse is dividing). These exchanges strengthen students' mathematical language use and reasoning to describe when comparing inverse-operation relationships involving rational numbers.
Design Principle(s): Support sense-making; Maximize meta-awareness

---

13.3 Near and Far From Zero

15 minutes
In the previous activity, students interpreted the meaning of $-x$ when x represented a positive value and when x represented a negative value. The purpose of this activity is to understand that variables can have negative values, but if we compare two expressions containing the same variable, it is not possible to know which expression is larger or smaller (without knowing the values of the variables). For example, if we know that a is positive, then we know that $5a$ is greater than $4a$; however, if a can be any rational number, then it is possible for $4a$ to be greater than $5a$, or equal to $5a$.

Addressing
- 7.NS.A

Instructional Routines
- MLR8: Discussion Supports

Launch
$$a, b, \text{-}a, \text{-}4b, \text{-}a + b, a \div \text{-}b, a^2, b^3$$

Display the list of expressions for all to see. Ask students: Which expression do you think has the largest value? Which has the smallest? Which is closest to zero? It is reasonable to guess that $-4b$ is the smallest and b^3 is the largest. (This guess depends on supposing that b is positive. But this activity is designed to elicit the understanding that it is necessary to know the value of variables to be able to compare expressions. For the launch, you are just looking for students to register their initial suspicions.)

If you know from students' previous work that they struggle with basic operations on rational numbers, it might be helpful to demonstrate a few of the computations that will come up as they work on this activity. Tell students, "Say that a represents 10, and b represents -2. What would be the value of . . .

- $-b$

- b^3

- $a \cdot \frac{1}{b}$

- $\frac{a}{b} \div a$

- $a + \frac{1}{b}$

- $\left(\frac{1}{b}\right)^2$

Arrange students in groups of 2. Encourage them to check in with their partner as they evaluate each expression, and work together to resolve any discrepancies.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.
Supports accessibility for: Organization; Attention

Anticipated Misconceptions

If students struggle to find the largest value, smallest value, or value closest to zero in the set, encourage them to create a number line to help them reason about the positions of different candidates.

Student Task Statement

a	b	$-a$	$-4b$	$-a + b$	$a \div -b$	a^2	b^3
$-\frac{1}{2}$	6						
$\frac{1}{2}$	-6						
-6	$-\frac{1}{2}$						

1. For each set of values for a and b, evaluate the given expressions and record your answers in the table.

2. When $a = -\frac{1}{2}$ and $b = 6$, which expression:

   has the largest value?   has the smallest value?   is the closest to zero?

3. When $a = \frac{1}{2}$ and $b = -6$, which expression:

   has the largest value?   has the smallest value?   is the closest to zero?

4. When $a = -6$ and $b = -\frac{1}{2}$, which expression:

   has the largest value?   has the smallest value?   is the closest to zero?

Student Response

1.

a	b	$-a$	$-4b$	$-a + b$	$a \div -b$	a^2	b^3
$-\frac{1}{2}$	6	$\frac{1}{2}$	-24	$6\frac{1}{2}$	$\frac{1}{12}$	$\frac{1}{4}$	216
$\frac{1}{2}$	-6	$-\frac{1}{2}$	24	$-6\frac{1}{2}$	$\frac{1}{12}$	$\frac{1}{4}$	-216
-6	$-\frac{1}{2}$	6	2	$5\frac{1}{2}$	-12	36	$-\frac{1}{8}$

2. a. $b^3 = 216$

   b. $-4b = -24$

   c. $a \div -b = \frac{1}{12}$

3. a. $-4b = 24$

   b. $b^3 = -216$

c. $a \div \text{-}b = \frac{1}{12}$

4. a. $a^2 = 36$

   b. $a \div \text{-}b = \text{-}12$

   c. $b^3 = \text{-}\frac{1}{8}$

> **Are You Ready for More?**
>
> Are there any values could you use for a and b that would make all of these expressions have the same value? Explain your reasoning.

Student Response

No. If $a = 0$ and $b = 0$, then all of the expressions will also have a value of 0, except for $a \div (\text{-}b)$, because $0 \div 0$ is undefined.

Activity Synthesis

Display the completed table for all to see. Ask selected students to share their values that are largest, smallest, and closest to zero from each set and explain their reasoning.

Ask students if any of these results were surprising? What caused the surprising result? Some possible observations are:

- b^3 was both the largest and smallest value at different times. 6 and -6 are both relatively far from 0, so b^3 is a large number when b is positive and a small number when b is negative.

- $a \div \text{-}b$ was often the closest to zero, because it had the same absolute value no matter the sign of a and b.

Access for English Language Learners

Speaking, Representing: *MLR8 Discussion Supports.* Use this routine to support whole-class discussion. After each student shares, provide the class with the following sentence frames to help them respond: "I agree because ….” or "I disagree because ….” If necessary, revoice student ideas to demonstrate mathematical language use by restating a statement as a question in order to clarify, apply appropriate language, and involve more students. *Design Principle(s)*: *Support sense-making*

13.4 Seagulls and Sharks Again

Optional: 10 minutes (there is a digital version of this activity)
The purpose of this activity is for students to interpret an expression in terms of the position it represents on a number line and to interpret the meaning of an associated equation. Expressions are equal when they represent the same position on a number line. This activity uses a familiar

context that students have encountered before, so that they can more quickly engage with the meaning of the expressions. In this activity, students use the structure of the number line to reason about the relative values of expressions (MP7).

In grade 6 and earlier in this unit, students have seen fractions with a negative sign in front of the entire fraction. This activity is the first time students see a fraction with a negative sign in the numerator, in the equation $c = \frac{-a}{2}$. Students can apply what they know about dividing signed numbers to make sense of expressions like this.

Addressing

- 7.NS.A

Instructional Routines

- MLR2: Collect and Display

- Notice and Wonder

Launch

Display the diagram for all to see. Ask students to share anything they notice and wonder about the diagram. Some things to notice are that

- The seagull is above the water and the shark is below the water.

- a and b represent the vertical position of the seagull and shark, respectively.

- a represents a positive value and b represents a negative value.

- a is farther from 0 than b, so $|a| > |b|$.

To facilitate engagement with the task, consider demonstrating the placement of one animal before students start working. For example, ask students, "If there were a minnow with vertical position m, and $m = \frac{1}{3}b$, where is the minnow?" Help students interpret the equation. Explain that the value of m only gives the vertical position. The *horizontal* position of the minnow is unknown. Encourage students to plot all of the new points *on* the vertical axis (as specified in the task statement).

Keep students in the same groups as the previous activity. Encourage students to check in with their partner periodically, and work together to resolve any discrepancies.

Classes using the digital version have an applet to use. Students can choose their own vertical positions for a and b. Teachers may allow students to turn on a grid to facilitate the placement of the other animals.

Conversing, Representing, Writing: MLR2 Collect and Display. As students work, circulate and listen to students' conversations as they decide where on the vertical axis to show the position of each new animal. Write down common or important phrases you hear students say as they reason about the relative value of each expression (e.g., twice as far, opposite direction, half the distance, etc.). Write the students' words and expressions on a whole-class display of the task's diagram. This will help students interpret and use mathematical language during their paired and whole-group discussions.
Design Principle(s): Support sense-making

Anticipated Misconceptions

For students who are struggling to measure out a length of a or b or a sum, difference, or multiple of them, suggest that they measure and cut strips of paper for the lengths of a and b to help guide them. Ask how they could use the strips to find other distances such as $a - b$ and $\frac{a}{2}$.

Student Task Statement
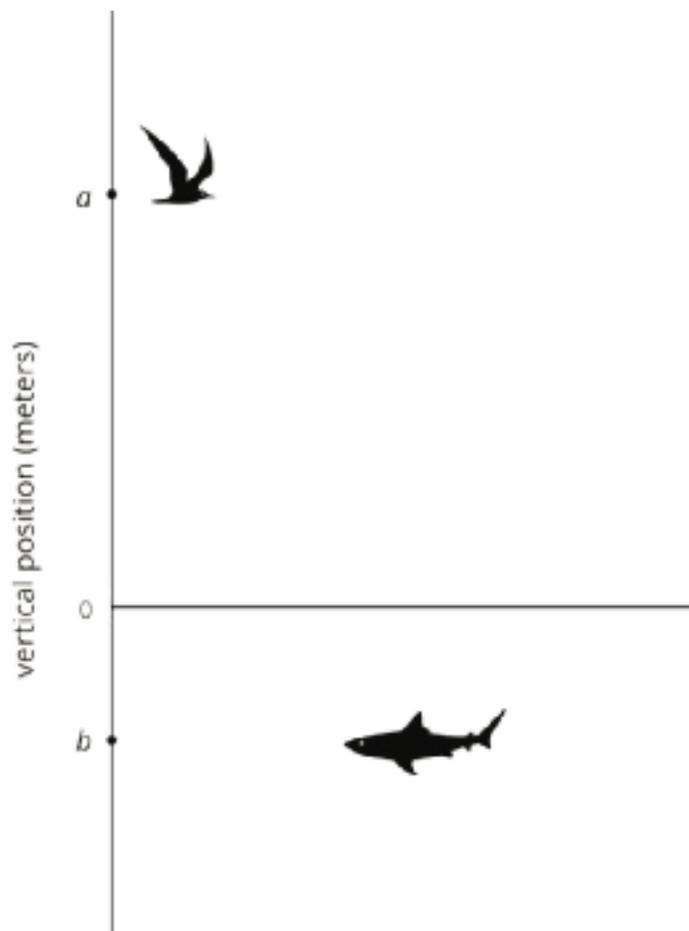
> A seagull has a vertical position a, and a shark has a vertical position b. Draw and label a point on the vertical axis to show the vertical position of each new animal.
>
> 1. A dragonfly at d, where $d = \text{-}b$
>
> 2. A jellyfish at j, where $j = 2b$
>
> 3. An eagle at e, where $e = \frac{1}{4}a$.
>
> 4. A clownfish at c, where $c = \frac{\text{-}a}{2}$
>
> 5. A vulture at v, where $v = a + b$
>
> 6. A goose at g, where $g = a - b$

Student Response
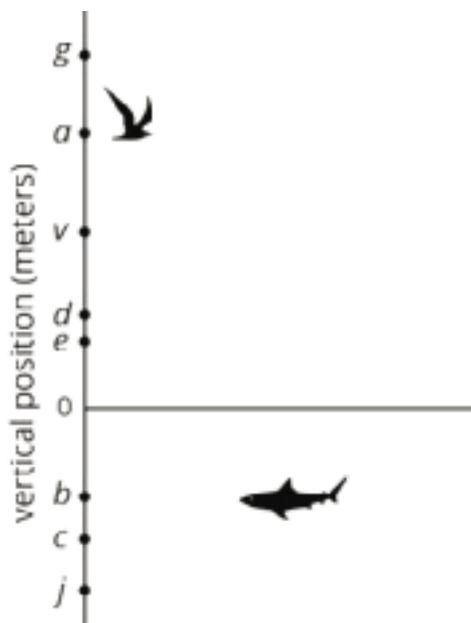
Activity Synthesis

Display the diagram from the task statement. Ask selected students to share their responses and the reasoning behind them, and record them on the diagram. As the discussion proceeds, illustrate the meaning of the equal sign by saying, for example, "We could either label *this point* with v, or we could label it with $a + b$, since we know $v = a + b$. Since these expressions are equal, they represent the same position on the number line."

If not brought up by students, consider drawing attention to the equation $c = \frac{\text{-}a}{2}$ and discuss how this relates to what they have learned about dividing signed numbers. It is important for students to understand that $\frac{\text{-}a}{2} = \text{-}\frac{a}{2}$ as well as $\frac{a}{\text{-}2}$, but these are not equal to $\frac{\text{-}a}{\text{-}2}$.

Lesson Synthesis

Key takeaways:

- Develop flexible thinking with all four operations across the rational numbers.

- Recognize multiplicative and additive inverses and the difference between them.

Discussion questions:

- Explain the rules for arithmetic with negative numbers.

- Can you give an example of a number whose additive inverse is the same as its multiplicative inverse? Why not?

13.5 Make Them True

Cool Down: 5 minutes
Addressing

- 7.NS.A.3

> **Student Task Statement**
>
> In each equation, select an operation to make the equation true.
>
> 1. $24 \underline{\quad} \frac{3}{4} = 18$
>
> 2. $24 \underline{\quad} \text{-}\frac{3}{4} = \text{-}32$
>
> 3. $12 \underline{\quad} 15 = \text{-}3$
>
> 4. $12 \underline{\quad} \text{-}15 = 27$
>
> 5. $\text{-}18 \underline{\quad} \text{-}\frac{3}{4} = 24$

Student Response

1. $24 \cdot \frac{3}{4} = 18$

2. $24 \div \text{-}\frac{3}{4} = \text{-}32$

3. $12 - 15 = \text{-}3$

4. $12 - \text{-}15 = 27$

5. $\text{-}18 \div \text{-}\frac{3}{4} = 24$

> # Student Lesson Summary
>
> We can represent sums, differences, products, and quotients of **rational numbers**, and combinations of these, with numerical and algebraic expressions.

Sums:	Differences:	Products:	Quotients:
$\frac{1}{2} + \text{-9}$	$\frac{1}{2} - \text{-9}$	$(\frac{1}{2})(\text{-9})$	$\frac{1}{2} \div \text{-9}$
$\text{-8.5} + x$	$\text{-8.5} - x$	$\text{-8.5}x$	$\frac{\text{-8.5}}{x}$

We can write the product of two numbers in different ways.

- By putting a little dot between the factors, like this: $\text{-8.5} \cdot x$.

- By putting the factors next to each other without any symbol between them at all, like this: $\text{-8.5}x$.

We can write the quotient of two numbers in different ways as well.

- By writing the division symbol between the numbers, like this: $\text{-8.5} \div x$.

- By writing a fraction bar between the numbers like this: $\frac{\text{-8.5}}{x}$.

When we have an algebraic expression like $\frac{\text{-8.5}}{x}$ and are given a value for the variable, we can find the value of the expression. For example, if x is 2, then the value of the expression is -4.25, because $\text{-8.5} \div 2 = \text{-4.25}$.

Glossary

- rational number

Lesson 13 Practice Problems

Problem 1

Statement

The value of x is $\frac{-1}{4}$. Order these expressions from least to greatest:

x $\qquad\qquad$ $1 - x$ $\qquad\qquad$ $x - 1$ $\qquad\qquad$ $-1 \div x$

Solution

$x - 1$, x, $1 - x$, $-1 \div x$.

The expressions' values are $\frac{-5}{4}$, $\frac{-1}{4}$, $\frac{5}{4}$, and 4.

Problem 2

Statement

Here are four expressions that have the value $\frac{-1}{2}$:

$\frac{-1}{4} + \left(\frac{-1}{4}\right)$ $\qquad\qquad$ $\frac{1}{2} - 1$ $\qquad\qquad$ $-2 \cdot \frac{1}{4}$ $\qquad\qquad$ $-1 \div 2$

Write five expressions: a sum, a difference, a product, a quotient, and one that involves at least two operations that have the value $\frac{-3}{4}$.

Solution

Answers vary. Sample response: $\frac{-1}{4} + \left(\frac{-1}{2}\right)$, $\frac{1}{4} - 1$, $-3 \cdot \frac{1}{4}$, $-3 \div 4$, $1 \div 4 - 1$

Problem 3

Statement

Find the value of each expression.

a. $-22 + 5$

b. $-22 - (-5)$

c. $(-22)(-5)$

d. $-22 \div 5$

Solution

a. -17

b. -17

c. 110

d. -4.4

Problem 4

Statement

The price of an ice cream cone is $3.25, but it costs $3.51 with tax. What is the sales tax rate?

Solution

a. 8% (Any answer between 7.85% and 8.15% is acceptable.)

(From Unit 4, Lesson 10.)

Problem 5

Statement

Two students are both working on the same problem: A box of laundry soap has 25% more soap in its new box. The new box holds 2 kg. How much soap did the old box hold?

○ Here is how Jada set up her double number line.
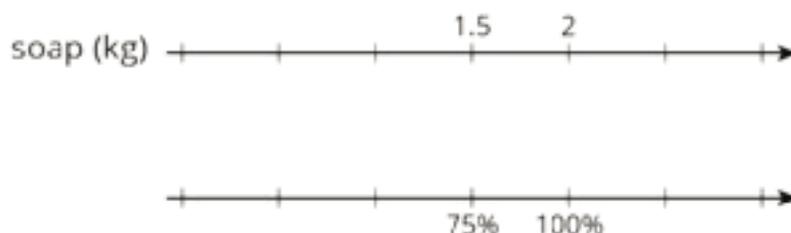
○ Here is how Lin set up her double number line.

Do you agree with either of them? Explain or show your reasoning.

Solution

Answers vary. Sample response: I agree with Lin. The soap in the old box represents 100% and the new box now holds 125% which is 2 kg.

(From Unit 4, Lesson 7.)

Problem 6

Statement

a. A coffee maker's directions say to use 2 tablespoons of ground coffee for every 6 ounces of water. How much coffee should you use for 33 ounces of water?

b. A runner is running a 10 km race. It takes her 17.5 minutes to reach the 2.5 km mark. At that rate, how long will it take her to run the whole race?

Solution

a. 11 tablespoons; from the information, one can determine that 1 tablespoon of coffee will be used for every 3 ounces of water, therefore for 33 ounces of water one would need 11 tablespoons of coffee.

b. 70 minutes; from the information, one can determine that it would take the runner 7 minutes to run 1 km, therefore it would take the runner 70 minutes to run 10 km.

(From Unit 4, Lesson 3.)

Lesson 14: Solving Problems with Rational Numbers

Goals

- Apply operations with rational numbers to solve problems involving repeated gains or losses, and present (orally, in writing, and using other representations) the solution method.

Learning Targets

- I can represent situations with expressions that include rational numbers.

- I can solve problems using the four operations with rational numbers.

Lesson Narrative

In this lesson students put together what they have learned about rational number arithmetic and the interpretation of negative quantities, such as negative time or negative rates of change. They solve problems with rational numbers in the context of a negative flow rate from a tank and negative charges on an electricity bill or a bank account. The problems in this section are designed so that it is natural to solve them by filling in tables or making numerical calculations. In the next lesson, students will move towards solving algebraic equations.

As students reason about the meaning of negative quantities, they engage in MP2.

Alignments

Building On

- 6.EE.B.7: Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

Addressing

- 7.NS.A.3: Solve real-world and mathematical problems involving the four operations with rational numbers.Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

Building Towards

- 7.EE.B.4.a: Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- MLR6: Three Reads

- MLR7: Compare and Connect

- Think Pair Share

- Which One Doesn't Belong?

> **Student Learning Goals**
>
> Let's use all four operations with signed numbers to solve problems.

14.1 Which One Doesn't Belong: Equations

Warm Up: 5 minutes

This warm-up prompts students to compare four equations. It encourages students to explain their reasoning, hold mathematical conversations, and gives you the opportunity to hear how they use terminology and talk about the equations in comparison to one another. To allow all students to access the activity, each equation has one obvious reason it does not belong.

Building On

- 6.EE.B.7

Building Towards

- 7.EE.B.4.a

Instructional Routines

- Which One Doesn't Belong?

Launch

Arrange students in groups of 2–4. Display the equations for all to see. Ask students to indicate when they have noticed one equation that does not belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their small group. In their small groups, tell each student to share their reasoning why a particular equation does not belong and together find at least one reason each equation doesn't belong.

> **Student Task Statement**
>
> Which equation doesn't belong?
>
> $\frac{1}{2}x = \text{-}50$ $x + 90 = \text{-}100$
>
> $\text{-}60t = 30$ $\text{-}0.01 = \text{-}0.001x$

Student Response

Answers vary. Sample responses:

$\frac{1}{2}x = \text{-}50$ is the only one with a fraction.

Unit 5 Lesson 14

$-60t = 30$ is the only one with a different variable; is the only one with a solution whose absolute value is less than 1.

$x + 90 = -100$ is the only one with addition instead of multiplication.

$-0.01 = -0.001x$ is the only one with a positive solution; is the only one with decimals; is the only one with the variable on the right-hand side of the equation.

Activity Synthesis

Ask each group to share one reason why a particular equation does not belong. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given make sense.

14.2 Draining and Filling a Tank

10 minutes
The purpose of this activity is for students to use the four operations on rational numbers solve a problem about water in a tank. The activity presents another example where negative time is used; this time to describe before a sensor starts working. Students examine the change as a separate column before using the starting point to model the draining of the tank.

Students who see that they are doing the same computations over and over and see that the structure of the expression is the same every time are expressing regularity in repeated reasoning (MP8). Monitor for different explanations for the last question.

Addressing

- 7.NS.A.3

- 7.RP.A.2

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- MLR7: Compare and Connect

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 4 minutes of quiet work time, followed by partner and whole-class discussion.

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts. For example, after students have completed the first 2-3 rows of the first table, check-in with either select groups of students or the whole class. Invite students to share how they have applied generalizations from previous lessons about using the four operations on rational numbers so far.
Supports accessibility for: Conceptual processing; Organization; Memory

Anticipated Misconceptions

The last question might be hard for students because they have had the table to support calculating the answers. Ask students how can they use the previous entries in the table to help them calcualte the answer? What is something they see changing in the expressions in the table that would change in this question?

Student Task Statement

A tank of water is being drained. Due to a problem, the sensor does not start working until some time into the draining process. The sensor starts its recording at time zero when there are 770 liters in the tank.

1. Given that the drain empties the tank at a constant rate of 14 liters per minute, complete the table:

time after sensor starts (minutes)	change in water (liters)	expression	water in the tank (liters)
0	0	$770 + (0)(\text{-}14)$	770
1	-14	$770 + (1)(\text{-}14)$	756
5	-70		
10			

2. Later, someone wants to use the data to find out how long the tank had been draining before the sensor started. Complete this table:

time after sensor starts (minutes)	change in water (liters)	expression	water in the tank (liters)
1	-14	$770 + (1)(-14)$	756
0	0	$770 + (0)(-14)$	770
-1	14	$770 + (-1)(-14)$	784
-2	28		
-3			
-4			
-5			

3. If the sensor started working 15 minutes into the tank draining, how much was in the tank to begin with?

Student Response

1.

time after sensor starts (minutes)	change in water (liters)	expression	water in the tank (liters)
0	0	$770 + (0)(-14)$	770
1	-14	$770 + (1)(-14)$	756
5	-70	$770 + (5)(-14)$	700
10		$770 + (10)(-14)$	630

2.

time after sensor starts (minutes)	change in water (liters)	expression	water in the tank (liters)
1	-14	$770 + (1)(-14)$	756
0	0	$770 + (0)(-14)$	770
-1	14	$770 + (-1)(-14)$	784
-2	28	$770 + (-2)(-14)$	798
-3		$770 + (-3)(-14)$	812
-4		$770 + (-4)(-14)$	826
-5		$770 + (-5)(-14)$	840

3. 980 liters

Activity Synthesis

Make sure students have filled out the tables appropriately. Select students to share their reasoning for the last one.

14.3 Buying and Selling Power

15 minutes
The purpose of this activity is for students to use the four operations on rational numbers solve real-world problems. Monitor for students who solved the problem using different representations and approaches.

Addressing

- 7.NS.A.3

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR6: Three Reads

Launch

Arrange students in groups of 2–4. Given them 4 minutes of quiet work time, followed by small group and then whole-class discussion.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because _____. Then, I...," "I noticed _____ so I..." and "I tried _____ and what happened was..."
Supports accessibility for: Language; Social-emotional skills

Access for English Language Learners

Reading: MLR6 Three Reads. Use this routine to support reading comprehension of this word problem, without solving it for students. In the first read, students read the problem with the goal of comprehending the situation (e.g., A utility company charges for energy use.). In the second read, ask students to identify the quantities and relationships, without focusing on specific values. Listen for, and amplify, the quantities that vary in relation to each other in this situation: electricity used, in kilowatt-hours; amount spent on electricity used, in dollars; electricity generated and not used, in kilowatt-hours; amount credited for electricity generated and not used, in dollars. Invite students to draw a diagram to represent the relationships among these quantities. After the third read, ask students to brainstorm possible strategies to answer the questions. This helps students connect the language in the word problem and the reasoning needed to solve the problem, maintaining the intended level of cognitive demand in the task.
Design Principle(s): Support sense-making

> ## Student Task Statement
>
> A utility company charges $0.12 per kilowatt-hour for energy a customer uses. They give a credit of $0.025 for every kilowatt-hour of electricity a customer with a solar panel generates that they don't use themselves.
>
> A customer has a charge of $82.04 and a credit of -$4.10 on this month's bill.
>
> 1. What is the amount due this month?

2. How many kilowatt-hours did they use?

3. How many kilowatt-hours did they generate that they didn't use themselves?

Student Response

1. $77.94

2. About 684 kilowatt-hours

3. 164 kilowatt-hours

Are You Ready for More?

1. Find the value of the expression without a calculator.

$$(2)(\text{-}30) + (\text{-}3)(\text{-}20) + (\text{-}6)(\text{-}10) - (2)(3)(10)$$

2. Write an expression that uses addition, subtraction, multiplication, and division and only negative numbers that has the same value.

Student Response

1. 0

2. Answers vary. Sample response: $-10 \div \text{-}2 + (\text{-}2)(\text{-}2)(\text{-}2) - \text{-}3$

Activity Synthesis

Select students to share their solution. Help them make connections between different solution approaches.

Lesson Synthesis

In this lesson we saw that signed numbers can be used to represent situations where amounts are changing different ways.

In the activity about the water tank,

- What did a positive amount represent? (water added to the tank)

- What did a negative amount represent? (water drained from the tank)

In the activity about the price of electricity,

- What did a positive amount represent? (money the customer owed the company)

- What did a negative amount represent? (money the company owed the customer)

14.4 Charges and Checks

Cool Down: 5 minutes

Addressing

- 7.NS.A.3

Student Task Statement

Lin's sister has a checking account. If the account balance ever falls below zero, the bank charges her a fee of $5.95 per day. Today, the balance in Lin's sister's account is -$2.67.

1. If she does not make any deposits or withdrawals, what will be the balance in her account after 2 days?

2. In 14 days, Lin's sister will be paid $430 and will deposit it into her checking account. If there are no other transactions besides this deposit and the daily fee, will Lin continue to be charged $5.95 each day after this deposit is made? Explain or show your reasoning.

Student Response

1. -$14.57

2. No. Reasoning varies. Sample explanation: even if the fee was $10 per day, that would total $140, which is much less than what she will deposit.

Student Lesson Summary

We can apply the rules for arithmetic with rational numbers to solve problems.

In general: $a - b = a + \text{-}b$

If $a - b = x$, then $x + b = a$. We can add $\text{-}b$ to both sides of this second equation to get that $x = a + \text{-}b$

Remember: the *distance* between two numbers is independent of the order, but the *difference* depends on the order.

And when multiplying or dividing:

- The sign of a positive number multiplied or divided by a negative number is always negative.
- The sign of a negative number multiplied or divided by a positive number is always negative.
- The sign of a negative number multiplied or divided by a negative number is always positive.

Lesson 14 Practice Problems

Problem 1

Statement

A bank charges a service fee of $7.50 per month for a checking account.

A bank account has $85.00. If no money is deposited or withdrawn except the service charge, how many months until the account balance is negative?

Solution

12, because $85 \div 7.50 = 11\frac{1}{3}$ which means it will take 12 months to have a negative balance in the account

Problem 2

Statement

The table shows transactions in a checking account.

January	February	March	April
-38.50	250.00	-14.00	-86.80
126.30	-135.20	99.90	-570.00
429.40	35.50	-82.70	100.00
-265.00	-62.30	-1.50	-280.10

a. Find the total of the transactions for each month.

b. Find the mean total for the four months.

Solution

a. January: 252.20; February: 88; March 1.70; April: -836.90

b. -123.75, because $[252.20 + 88 + 1.70 + (-836.9)] \div 4 = -123.75$

Problem 3

Statement

A large aquarium of water is being filled with a hose. Due to a problem, the sensor does not start working until some time into the filling process. The sensor starts its recording at the time zero minutes. The sensor initially detects the tank has 225 liters of water in it.

a. The hose fills the aquarium at a constant rate of 15 liters per minute. What will the sensor read at the time 5 minutes?

b. Later, someone wants to use the data to find the amount of water at times before the sensor started. What should the sensor have read at the time -7 minutes?

Solution

a. 300 liters, because $225 + 15 \cdot 5 = 300$

b. 120 liters, because $225 + 15 \cdot \text{-}7 = 120$

Problem 4

Statement

A furniture store pays a wholesale price for a mattress. Then, the store marks up the retail price to 150% of the wholesale price. Later, they put the mattress on sale for 50% off of the retail price. A customer just bought the mattress on sale and paid $1,200.

a. What was the retail price of the mattress, before the discount?

b. What was the wholesale price, before the markup?

Solution

a. $, because $1{,}200 \div 0.5 = 2{,}400$.

b. $, because $2{,}400 \div 1.5 = 1{,}600$.

(From Unit 4, Lesson 11.)

Problem 5

Statement

a. A restaurant bill is $21. You leave a 15% tip. How much do you pay including the tip?

b. Which of the following represents the amount a customer pays including the tip of 15% if the bill was b dollars? Select **all** that apply.

- $15b$
- $b + 0.15b$
- $1.15b$
- $1.015b$
- $b + \frac{15}{100}b$
- $b + 0.15$
- $0.15b$

Solution

a. $24.15

b. $b + 0.15b$, $1.15b$, $b + \frac{15}{100}b$

(From Unit 4, Lesson 10.)

Section: Solving Equations When There Are Negative Numbers

Lesson 15: Solving Equations with Rational Numbers

Goals

- Explain (orally and in writing) how to solve an equation of the form $x + p = q$ or $px = q$, where p, q, and x are rational numbers.

- Generalize (orally) the usefulness of additive inverses and multiplicative inverses for solving equations of the form $x + p = q$ or $px = q$.

- Generate an equation of the form $x + p = q$ or $px = q$ to represent a situation involving rational numbers.

Learning Targets

- I can solve equations that include rational numbers and have rational solutions.

Lesson Narrative

The purpose of this lesson is to get students thinking about how to solve equations involving rational numbers. In grade 6, students solved equations of the form $px = q$ and $x + p = q$ and saw that additive and multiplicative inverses (opposites and reciprocals) were useful for solving them. However, that work in grade 6 did not include equations with negative values of p or q or with negative solutions. This lesson builds on the ideas of the last lesson and brings together the work on equations in grade 6 with the work on operations on rational numbers from earlier in grade 7.

Alignments
Building On

- 6.EE.B.5: Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

- 7.NS.A: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Addressing

- 7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

- 7.EE.B.4.a: Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

- 7.NS.A.3: Solve real-world and mathematical problems involving the four operations with rational numbers.Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Building Towards

- 7.EE.B: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- MLR6: Three Reads

- MLR7: Compare and Connect

- MLR8: Discussion Supports

- Number Talk

- Take Turns

Required Preparation

Print and cut up cards from the Card Sort: Matching Inverses Instructional master. Prepare 1 set of cards for every 2 students.

> **Student Learning Goals**
>
> Let's solve equations that include negative values.

15.1 Number Talk: Opposites and Reciprocals

Warm Up: 5 minutes

The purpose of this number talk is to:

- Remind students that the sum of a number and a number of the same magnitude with the opposite sign is zero.

- Remind students that the product of a number and its reciprocal is one.

- Establish common vocabulary for referring to these numerical relationships.

There may not be time for students to share every possible strategy. Consider gathering only one strategy for the equations with one variable, and a few strategies for the equations with two

variables, since these have many possible answers, and they serve to generalize the relationship and provide an opportunity to introduce or reintroduce relevant vocabulary.

Building On

- 7.NS.A

Building Towards

- 7.EE.B

Instructional Routines

- MLR8: Discussion Supports

- Number Talk

Launch

Display one equation at a time. (When you get to $c \cdot d = 1$, ensure students understand that c and d represent different numbers.) Give students 30 seconds of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organization

Anticipated Misconceptions

Some students may use the strategy of guess and check for equations with two variables and use inverse operations to solve equations with one variable. Ask these students what they notice about the numbers in the equation and encourage them to find the value of the variable without using inverse operations.

> **Student Task Statement**
>
> The **variables** a through h all represent *different* numbers. Mentally find numbers that make each equation true.
>
> $\frac{3}{5} \cdot \frac{5}{3} = a$
>
> $7 \cdot b = 1$
>
> $c \cdot d = 1$
>
> $\text{-}6 + 6 = e$

$$11 + f = 0$$

$$g + h = 0$$

Student Response

- $a = 1$

- $b = \frac{1}{7}$

- Answers for c and d vary. Possible response: Possible response: $c = 3$ and $d = \frac{1}{3}$

- $e = 0$

- $f = -11$

- Answers for g and h vary. Possible response: $g = 5$ and $h = -5$

Activity Synthesis

Ask students to share their reasoning for each problem. Record and display the responses for all to see.

If the following ideas do not arise as students share their reasoning, make these ideas explicit:

- The sum of a number and its opposite is 0.

- The product of a number and its reciprocal is 1.

- If you want to find a number that you can add to something and get 0 as a sum, use its opposite.

- If you want to find a number that you can multiply something by and get 1 as a product, use its reciprocal.

To involve more students in the conversation, ask some of the following questions:

- "Who can restate ___'s reasoning in a different way?"

- "Does anyone want to add on to ____'s strategy?"

- "Do you agree or disagree? Why?"

Access for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because . . ." or "I noticed ____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimize output (for explanation)

Unit 5 Lesson 15

15.2 Match Solutions

10 minutes (there is a digital version of this activity)

Students solved equations of the form $x + p = q$ and $px = q$ in grade 6, but the equations only involved positive values. This activity bridges their understanding of a solution to an equation as a value that makes the equation true with their understanding of operations involving negative numbers from this unit. This activity builds on the work students have done in this lesson evaluating expressions at different values.

Monitor for students who:

- take an arithmetic approach by substituting in values and evaluating. (Does -2 · (-4.5) = -9?)

- take an algebraic approach by writing an equivalent equation using an inverse operation (If $-2 \cdot x = -9$, then $x = -9 \div -2$.)

Building On

- 6.EE.B.5

Addressing

- 7.EE.B.4.a

- 7.NS.A.3

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- MLR7: Compare and Connect

Launch

Allow students 5 minutes quiet work time followed by whole class discussion.

The digital version has an applet that allows students to see how many correct answers they have at any time.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support development of organizational skills, check in with students within the first 2-3 minutes of work time. Check to make sure students matched the correct solution to the first equation and can explain their reasoning with an arithmetic or algebraic approach.
Supports accessibility for: Memory; Organization

Student Task Statement

1. Match each equation to its solution.

   a. $\frac{1}{2}x = -5$

   b. $-2x = -9$

   c. $-\frac{1}{2}x = \frac{1}{4}$

   d. $-2x = 7$

   e. $x + -2 = -6.5$

   f. $-2 + x = \frac{1}{2}$

   1. $x = -4.5$

   2. $x = -\frac{1}{2}$

   3. $x = -10$

   4. $x = 4.5$

   5. $x = 2\frac{1}{2}$

   6. $x = -3.5$

Be prepared to explain your reasoning.

Student Response

1. a. 3

    b. 4

    c. 2

    d. 6

    e. 1

    f. 5

Activity Synthesis

Select an equation for which there is a student who took an arithmetic approach and a student who took an algebraic approach. Ask students to share their reasoning for why a solution is correct. Sequence arithmetic approaches before algebraic approaches. An example of an arithmetic approach: "I know that -3.5 is the solution to $-2x = 7$, because I know that $-2 \cdot (-3.5) = 7$. An example of an algebraic approach: "If $-2x = 7$, then I know that $x = 7 \div -2$.) Record their work, side by side, for all to see. If there are no equations for which students took both approaches, present both approaches anyway. Tell students that either approach is valid, but that in the next unit they will see some more complicated equations for which one approach might be simpler than the other.

15.3 Trip to the Mountains

20 minutes
In this activity, students interpret equations that represent situations (MP2). The purpose is for students to see that equations of the form $x + p = q$ can be solved by adding the opposite of p to the equation, regardless of whether p is positive or negative. Students also see that equations of the form $px = q$ can be solved by multiplying the equation by the reciprocal of p. Through this work, students see that the structure of equations can be used to reason about a path to a solution (MP7) even when negative values are included or when a variable can represent a negative number.

Addressing

- 7.EE.B.4

- 7.NS.A.3

Instructional Routines

- MLR6: Three Reads

Launch

Give students 5–6 minutes of quiet work time followed by whole-class discussion.

Reading: MLR6 Three Reads. Use this routine to help students interpret the representations of the situation in this activity. Use the first read to help students comprehend the different stages of situation (e.g., hiking a mountain, temperature falls, cost of the trip) without using numbers. Use the second read to identify the important quantities by asking students what can be counted or measured (e.g., increased elevation 290 feet, now at 450 feet; temperature fell 4 degrees, now is at 32 degrees; 3 times as many students this year, now 42 students; cost is $\frac{2}{3}$ of last year's, cost $32 this year). For the third read, ask students to brainstorm possible strategies to answer the given questions.
Design Principle(s): Support sense-making

Anticipated Misconceptions

Students may be misled by words to add or multiply (or subtract or divide) by the wrong numbers. For example, the word "increased" in question 1 may lead students to simply add the numbers they see, while the words "three times as many" in question 4 may lead students to multiply the numbers in the problem. Encourage students to make sense of the situations by acting them out or using visual diagrams, which will help them understand the actions and relationships in the stories.

Student Task Statement

The Hiking Club is on a trip to hike up a mountain.

1. The members increased their elevation 290 feet during their hike this morning. Now they are at an elevation of 450 feet.

    a. Explain how to find their elevation before the hike.

    b. Han says the equation $e + 290 = 450$ describes the situation. What does the variable e represent?

    c. Han says that he can rewrite his equation as $e = 450 + \text{-}290$ to solve for e. Compare Han's strategy to your strategy for finding the beginning elevation.

2. The temperature fell 4 degrees in the last hour. Now it is 21 degrees. Write and solve an equation to find the temperature it was 1 hour ago.

3. There are 3 times as many students participating in the hiking trip this year than last year. There are 42 students on the trip this year.

    a. Explain how to find the number of students that came on the hiking trip last year.

    b. Mai says the equation $3s = 42$ describes the situation. What does the variable s represent?

c. Mai says that she can rewrite her equation as $s = \frac{1}{3} \cdot 42$ to solve for s. Compare Mai's strategy to your strategy for finding the number of students on last year's trip.

4. The cost of the hiking trip this year is $\frac{2}{3}$ of the cost of last year's trip. This year's trip cost $32. Write and solve an equation to find the cost of last year's trip.

Student Response

1. Answers vary. Sample responses:
   a. Subtract the change of 290 from the current elevation of 450.

   b. e represents the starting elevation

   c. Han used a variable to represent the unknown quantity, wrote an equation to describe the situation, and then solved by adding the opposite.

2. 25 degrees. $t - 4 = 21, t - 4 + 4 = 21 + 4, t = 25$

3. Answers vary. Sample responses:
   a. Divide this year's number by 3, or multiply this year's number by $\frac{1}{3}$.

   b. s represents the number of students on the trip last year.

   c. Mai used a variable to represent the unknown quantity, wrote an equation to describe the situation, and then solved by multiplying by the reciprocal.

4. 48. $\frac{2}{3}c = 32, c = 32 \cdot \frac{3}{2}, c = 48$

Are You Ready for More?

A number line is shown below. The numbers 0 and 1 are marked on the line, as are two other rational numbers a and b.

Decide which of the following numbers are positive and which are negative.

$a - 1$ $\qquad$ $a - 2$ $\qquad$ $-b$ $\qquad$ $a + b$ $\qquad$ $a - b$ $\qquad$ $ab + 1$

Student Response

1. $a - 1$ is positive because a is bigger than 1

2. $a - 2$ is negative because the distance between 1 and a is less than the distance between 0 and 1. So a is less than 2.

3. $-b$ is positive because b is a negative number.

4. $a + b$ is negative because the distance from b to zero is greater than the distance from 0 to a.

5. $a - b$ is positive because b is a negative value.

6. $ab + 1$ is negative because both a and b have magnitudes greater than 1, so ab will have magnitude greater than 1. Also, ab will be negative as a is positive and b is negative. So $ab < -1$.

Activity Synthesis

Tell students: "We learned four things about the hiking trip in this activity: the students were climbing, the temperature was falling, there were more students this year than last, and the cost of the trip was less this year than last." Then ask them:

- "Think about how you knew what operation described the rise in elevation, fall in temperature, rise in number of students, and fall in the cost. How did you know whether the situation used adding or multiplying?" (This conversation can highlight the problem with relying on "key words" . For example, when students see "times as many" , they might want to multiply the numbers they see in the problem. Encourage students to make sense of the situations by acting them out or drawing diagrams.)

- "How did you decide how to solve for the unknown quantity?"

- "What are some ways to know that a situation involves negative values?"

15.4 Card Sort: Matching Inverses

Optional: 10 minutes
The Instructional master is a set of matching cards with fractions and integers. The students first recall the work from the previous section about additive inverses by matching them. They then match multiplicative inverses.

When matching multiplicative inverses, students should now use the fact that division follows the same structure as multiplication to identify that negative numbers require a negative inverse and positive numbers require a positive inverse. Monitor for students who use this step to make an initial sort.

Addressing

- 7.NS.A.3

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- MLR8: Discussion Supports

- Take Turns

Launch

Remind students of the meaning of additive inverse and multiplicative inverse. If $x + y = 0$, then x and y are additive inverses. If $x \cdot y = 1$, then x and y are multiplicative inverses. Display these definitions for all to see and use as a reference while they work through this activity. Ask students for some examples of additive inverse, and add these to the display (for example, 7 and -7, because $7 + \text{-}7 = 0$). Ask students for some examples of multiplicative inverses, and add these to the display (for example, $\frac{1}{7}$ and 7, because $\frac{1}{7} \cdot 7 = 1$).

If necessary, demonstrate productive ways for partners to communicate during a matching activity. For example, partners take turns identifying a match and explaining why they think it is a match. The other partner either accepts their explanation, or explains why they don't think it's a match. Then they change roles for the next match.

Arrange students in groups of 2. Distribute one set of paper slips per group. Instruct students to first match the numbers that are additive inverses. Then, they will re-sort the same cards into pairs of multiplicative inverses. Consider asking students to pause their work after matching the additive inverses for discussion before proceeding to match multiplicative inverses.

Access for English Language Learners

Conversing: MLR8 Discussion Supports. Display sentence frames to support students as they explain their reasoning for each match. For example, "___ matches ___ because . . .", and "I know that ___ and ___ are additive/multiplicative inverses because . . ." Encourage students to respond to the matches their partner makes using, "I agree/disagree, because . . ."
Design Principle(s): Support sense-making; Cultivate conversation

---

Anticipated Misconceptions

Students might need a reminder of the difference between the additive inverse and multiplicative inverse.

> ### Student Task Statement
>
> Your teacher will give you a set of cards with numbers on them.
>
> 1. Match numbers with their additive inverses.
> 2. Next, match numbers with their multiplicative inverses.
> 3. What do you notice about the numbers and their inverses?

Student Response

Additive Inverses:

$\frac{2}{1}$ and -2; $\frac{5}{10}$ and -0.5; 3 and -3; $\frac{1}{3}$ and $\text{-}\frac{1}{3}$; 4 and -4; 0.25 and $\text{-}\frac{1}{4}$; $\frac{10}{2}$ and -5; 0.2 and -0.2; 10 and $\text{-}\frac{10}{1}$; 0.1 and $\text{-}\frac{1}{10}$; 25 and -25; $\frac{4}{100}$ and $\text{-}\frac{4}{100}$

Multiplicative Inverses:

2 and $\frac{5}{10}$; -2 & -0.5; 3 and $\frac{1}{3}$; -3 and $-\frac{1}{3}$; 4 and 0.25; -4 and $-\frac{1}{4}$; $\frac{10}{2}$ and 0.2; -5 and -0.2; 10 and 0.1; $-\frac{10}{1}$ and $-\frac{1}{10}$; 25 and $\frac{4}{100}$; -25 and $-\frac{4}{100}$

Activity Synthesis

The most important thing to recognize is that multiplicative inverses require that the numbers have the same sign in order for the product to be positive, and so negative numbers require a negative multiplicative inverse and positive numbers require a positive inverse. Contrast this with additive inverses, which must have opposite signs in order for their sum to be 0. Select students that used that strategy, as well as some who used calculation, to share their thinking and draw out this conclusion.

Lesson Synthesis

In this lesson students represented situations with equations and used inverses as a strategy to solve them. Bring the activities together by asking students:

- How can we solve an equation like $x + (-9.2) = 7.5$? (We can add the opposite of -9.2 to 7.5.)

- How can we solve an equation like $x \cdot (-9.2) = 7.5$? (We can multiply 7.5 by the reciprocal of -9.2.)

- Suppose we know that 60 is $\frac{4}{5}$ of a number. What is the difference between writing the equation $\frac{4}{5}x = 60$ and writing the equation $x = 60 \cdot \frac{5}{4}$? (The first equation describes the situation while the second shows a way to rewrite the equation to solve for the unknown.)

15.5 Hiking Trip

Cool Down: 5 minutes
Addressing

- 7.EE.B.4.a

- 7.NS.A.3

> **Student Task Statement**
>
> The Hiking Club is taking another trip. The hike leader's watch shows that they gained 296 feet in altitude from their starting position.
>
> Their altitude is now 285 feet, but there is no record of their starting altitude.
>
> Write and solve an equation to represent this situation and find their starting altitude.

Student Response

$x + 296 = 285$; $x = -11$

Student Lesson Summary

To solve the equation $x + 8 = -5$, we can add the opposite of 8, or -8, to each side:

$$x + 8 = -5$$
$$(x + 8) + -8 = (-5) + -8$$
$$x = -13$$

Because adding the opposite of a number is the same as subtracting that number, we can also think of it as subtracting 8 from each side.

We can use the same approach for this equation:

$$-12 = t + -\frac{2}{9}$$
$$(-12) + \frac{2}{9} = \left(t + -\frac{2}{9}\right) + \frac{2}{9}$$
$$-11\frac{7}{9} = t$$

To solve the equation $8x = -5$, we can multiply each side by the reciprocal of 8, or $\frac{1}{8}$:

$$8x = -5$$
$$\frac{1}{8}(8x) = \frac{1}{8}(-5)$$
$$x = -\frac{5}{8}$$

Because multiplying by the reciprocal of a number is the same as dividing by that number, we can also think of it as dividing by 8.

We can use the same approach for this equation:

$$-12 = -\frac{2}{9}t$$
$$-\frac{9}{2}(-12) = -\frac{9}{2}\left(-\frac{2}{9}t\right)$$
$$54 = t$$

Glossary

- variable

Lesson 15 Practice Problems
Problem 1
Statement

Solve.

    a. $\frac{2}{5}t = 6$

    b. $-4.5 = a - 8$

    c. $\frac{1}{2} + p = -3$

    d. $12 = x \cdot 3$

    e. $-12 = -3y$

Solution

    a. $t = 15$

    b. $a = 3.5$

    c. $p = -3\frac{1}{2}$

    d. $x = 4$

    e. $y = 4$

Problem 2
Statement

Match each equation to a step that will help solve the equation.

    A. $5x = 0.4$ 1. Multiply each side by 5.

    B. $\frac{x}{5} = 8$ 2. Multiply each side by -5.

    C. $3 = \frac{-x}{5}$ 3. Multiply each side by $\frac{1}{5}$.

    D. $7 = -5x$ 4. Multiply each side by $\frac{-1}{5}$.

Solution

    ○ A: 3

    ○ B: 1

    ○ C: 2

    ○ D: 4

Problem 3

Statement

Evaluate each expression if x is $\frac{2}{5}$, y is -4, and z is -0.2.

    a. $x + y$

    b. $2x - z$

    c. $x + y + z$

    d. $y \cdot x$

Solution

  a. $-3\frac{3}{5}$ (or equivalent)

  b. 1

  c. -3.8 (or equivalent)

  d. $\frac{-8}{5}$ (or equivalent)

(From Unit 5, Lesson 13.)

Problem 4

Statement

    a. Write an equation where a number is added to a variable, and a solution is -8.

    b. Write an equation where a number is multiplied by a variable, and a solution is $\frac{-4}{5}$.

Solution

Answers vary. Sample responses:

  a. $x + 2 = -6$

  b. $-5x = 4$

Problem 5

Statement

The markings on the number line are evenly spaced. Label the other markings on the number line.

Solution

-2.5, -2, -1.5, -1, -0.5, 0, 0.5, 1, 1.5

(From Unit 5, Lesson 8.)

Problem 6

Statement

In 2012, James Cameron descended to the bottom of Challenger Deep in the Marianas Trench; the deepest point in the ocean. The vessel he rode in was called DeepSea Challenger.

Challenger Deep is 35,814 feet deep at its lowest point

a. DeepSea Challenger's descent was a change in depth of (-4) feet per second. We can use the equation $y = -4x$ to model this relationship, where y is the depth and x is the time in seconds that have passed. How many seconds does this model suggest it would take for DeepSea Challenger to reach the bottom?

b. To end the mission DeepSea Challenger made a one-hour ascent to the surface. How many seconds is this?

c. The ascent can be modeled by a different proportional relationship $y = kx$. What is the value of k in this case?

Solution

a. 8,953.5 seconds, because $-35,814 \div -4 = 8,953.5$

b. 3,600 seconds, because $60 \cdot 60 = 3,600$

c. It took 3,600 seconds to go 35,814 feet up. This means the proportional relationship is $y = \frac{35814}{3600}x.$

(From Unit 5, Lesson 12.)

Lesson 16: Representing Contexts with Equations

Goals

- Coordinate (orally and in writing) verbal descriptions, equations, and diagrams that represent the same situation involving an unknown amount in the context of temperature or elevation.

- Write equations of the form $x + p = q$ or $px = q$ to represent and solve a problem in an unfamiliar context, and present the solution method (using words and other representations).

Learning Targets

- I can explain what the solution to an equation means for the situation.

- I can write and solve equations to represent situations that involve rational numbers.

Lesson Narrative

In the previous lesson students looked at methods for solving equations with rational numbers. In this lesson students choose equations that represent a context, and write their own equations given a context. Students are also encouraged to look at the structure of an equation and decide if its solution is positive or negative, without solving it (MP7).

Alignments

Building On

- 7.NS.A: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

- 7.NS.A.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Addressing

- 7.EE.B.4.a: Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

- 7.NS.A.3: Solve real-world and mathematical problems involving the four operations with rational numbers.Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Building Towards

- 7.EE.B.4.a: Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the

operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- Group Presentations

- MLR2: Collect and Display

- MLR7: Compare and Connect

- MLR8: Discussion Supports

> **Student Learning Goals**
>
> Let's write equations that represent situations.

16.1 Don't Solve It

Warm Up: 5 minutes
For this warm-up students use what they have learned about arithmetic with negative and positive numbers to determine the sign of solutions to equations. Students who focus on the signs of the numbers and the relative magnitudes without actually computing are at an advantage.

Building On

- 7.NS.A

Building Towards

- 7.EE.B.4.a

Launch

Display one problem at a time. Give students 30 seconds of quiet think time per problem and ask them to give a signal when they have an answer. Follow with a whole-class discussion.

> **Student Task Statement**
>
> Is the solution positive or negative?
>
> $(-8.7)(1.4) = a$
>
> $-8.7b = 1.4$
>
> $-8.7 + c = -1.4$
>
> $-8.7 - d = -1.4$

Student Response

a is negative

b is negative

c is positive

d is negative

Activity Synthesis

For each question, ask at least one student to explain their reasoning. Make sure there is agreement for each question on whether the solution is positive or negative. Ask if anyone used the third question to help them answer the fourth.

16.2 Warmer or Colder than Before?

10 minutes (there is a digital version of this activity)
In this activity, students work with changing temperatures to build understanding of equations that represent situations with negative coefficients, variables, and solutions. Students choose from a bank of equations to find two equations, one that represents the situation using a variable and the other representing the path to solve for the variable. They interpret the meaning of the variable in the context of each situation, solve for the value of the variable that makes the equations true, and explain how the equations and their solutions describe the situation. Students engage in MP2 as they contextualize and decontextualize between the contexts of changing temperatures and the equations that represent them.

Note that the last question involves some ambiguity. In order to select the anticipated response, students need to use the assumption that the temperature is less at midnight than it is at 9. Since it's the last question, they could also use some process of elimination to help lead them to making the assumption.

Addressing

- 7.EE.B.4.a

- 7.NS.A.3

Instructional Routines

- MLR8: Discussion Supports

Launch

Give students 5 minutes of quiet work time followed by whole-class discussion.

Students using the digital activity have a thermometer applet to help visualize the changes in temperature.

Access for Students with Disabilities

Representation: Internalize Comprehension. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. For example, to get students started, provide a smaller bank of equations and only the first two situations. Once students have successfully completed the three steps for each, introduce the remaining equations and situations.
Supports accessibility for: Conceptual processing; Organization

Access for English Language Learners

Representing, Listening: MLR8 Discussion Supports. Demonstrate the first situation to clarify what students need to do by thinking aloud using mathematical language and reasoning. This will help invite more student participation, conversations, and meta-awareness of language representing situations with negative coefficients, defining variables, and determining values that make equations true.
Design Principle(s): Support sense-making; Maximize meta-awareness

Anticipated Misconceptions

For students who struggle to understand how equations represent these situations, suggest that they draw a number line in the form of a thermometer and show the changes along the number line while reasoning about the rising and falling temperatures.

> ### Student Task Statement
>
> For each situation,
>
> - Find *two* equations that could represent the situation from the bank of equations. (Some equations will not be used.)
> - Explain what the variable v represents in the situation.
> - Determine the value of the variable that makes the equation true, and explain your reasoning.
>
> Bank of equations:

$-3v = 9$	$v = -16 + 6$	$v = \frac{1}{3} \cdot (\text{-}6)$	$v + 12 = 4$
$-4 \cdot 3 = v$	$v = 4 + (\text{-}12)$	$v = -16 - (6)$	$v = 9 + 3$
$-4 \cdot \text{-}3 = v$	$-3v = \text{-}6$	$-6 + v = \text{-}16$	$-4 = \frac{1}{3}v$
$v = -\frac{1}{3} \cdot 9$	$v = -\frac{1}{3} \cdot (\text{-}6)$	$v = 4 + 12$	$4 = 3v$

1. Between 6 a.m. and noon, the temperature rose 12 degrees Fahrenheit to 4 degrees Fahrenheit.

2. At midnight the temperature was -6 degrees. By 4 a.m. the temperature had fallen to -16 degrees.

3. The temperature is 0 degrees at midnight and dropping 3 degrees per hour. The temperature is -6 degrees at a certain time.

4. The temperature is 0 degrees at midnight and dropping 3 degrees per hour. The temperature is 9 degrees at a certain time.

5. The temperature at 9 p.m. is one third the temperature at midnight.

Student Response

Explanations vary. Sample responses:

1. $v = 4 + (\text{-}12)$, $v + 12 = 4$, v represents the temperature at 6 a.m., $v = \text{-}8$. Adding 12 to -8 will bring the temperature to 4.

2. $-6 + v = \text{-}16$, $v = -16 + 6$, v represents the change in temperature between midnight and 4 a.m., $v = \text{-}10$. The temperature had to drop 10 degrees to go from -6 to -16.

3. $-3v = \text{-}6$, $v = -\frac{1}{3} \cdot (\text{-}6)$, v represents the number of hours after midnight that it took to reach -6 degrees, $v = 2$. Dropping at a rate of 3 degrees per hour it will take 2 hours to go from 0 degrees to -6 degrees, so the time is 2 a.m.

4. $-3v = 9$, $v = -\frac{1}{3} \cdot 9$, v represents the change in time from midnight when the temperature was 9 degrees, $v = \text{-}3$. If the temperature has been dropping at 3 degrees per hour, and is at 0 degrees at midnight, then the temperature was 9 degrees sometime before midnight, so the change in time is negative. It took 3 hours for the temperature to fall from 9 degrees to 0 degrees, so the time was 3 hours before midnight, or 9 p.m.

5. $-4 = \frac{1}{3}v$, $-4 \cdot 3 = v$, v represents the temperature at midnight, $v = \text{-}12$. We expect that the temperature at midnight will be lower than the temperature at 9 p.m., so $-4 = \frac{1}{3}v$ matches this situation. (Note that $v = \frac{1}{3} \cdot \text{-}6$ could also work, if v represents the temperature at 9 p.m., but there is no equivalent equation to partner it with.)

Activity Synthesis

This discussion has two goals: First, for students to connect the equation that represents a situation to the equation that represents the solution strategy, and second, to ensure that students understand how to represent negative quantities in an equation. Ask students to share examples of how they chose:

- the equation that represents the situation

- the equation that represents the solution strategy

and in what order they chose them. Also ask for examples of how they knew that a situation required an equation with negative values.

16.3 Animals Changing Altitudes

Optional: 15 minutes
In this optional activity, students use expressions and number line diagrams to represent situations involving the changing height and depth of sea animals. They discuss how there is more than one correct way to write an equation that represents each situation. As students work, identify students who are writing their equations differently.

Building On

- 7.NS.A.1

Building Towards

- 7.EE.B.4.a

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- MLR2: Collect and Display

Launch

Access for English Language Learners

Reading, Representing, Conversing: *MLR2 Collect and Display*. While students work, circulate and listen to students talk about the similarities and differences between the changing height and depth of sea animals. Write down common or important phrases you hear students say about there being more than one correct way to write an equation that represents each representation. Write the students' words on a visual display of the expressions and number line diagrams. This will help students read and use mathematical language during their paired and whole-group discussions.
Design Principle(s): Support sense-making

Anticipated Misconceptions

Some students may struggle to match the verbal descriptions with the number line diagrams. Prompt them to examine whether each situation is looking for the animal's final altitude or change in altitude. If it is asking for a change in altitude, the number line will have one arrow and one point, instead of two arrows.

Student Task Statement

1. Match each situation with a diagram.

    a. A penguin is standing 3 feet above sea level and then dives down 10 feet. What is its depth?

    b. A dolphin is swimming 3 feet below sea level and then jumps up 10 feet. What is its height at the top of the jump?

    c. A sea turtle is swimming 3 feet below sea level and then dives down 10 feet. What is its depth?

    d. An eagle is flying 10 feet above sea level and then dives down to 3 feet above sea level. What was its change in altitude?

    e. A pelican is flying 10 feet above sea level and then dives down reaching 3 feet below sea level. What was its change in altitude?

    f. A shark is swimming 10 feet below sea level and then swims up reaching 3 feet below sea level. What was its change in depth?

2. Next, write an equation to represent each animal's situation and answer the question. Be prepared to explain your reasoning.

Diagrams

A

B
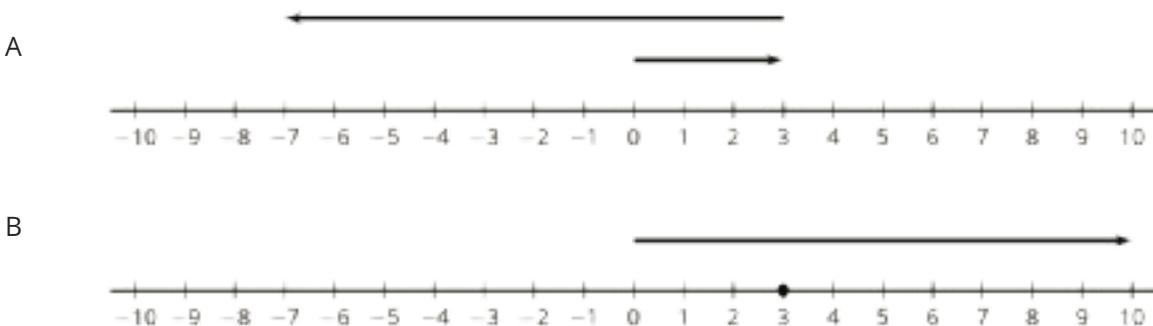
C
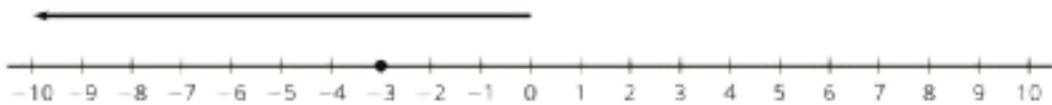
D
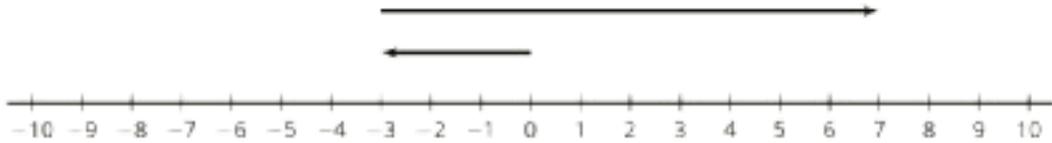
E
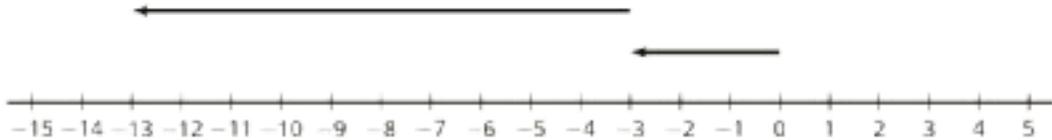
F

Student Response

1.   a. Diagram A

    b. Diagram D

    c. Diagram E

    d. Diagram B

    e. Diagram F

    f. Diagram C

2. Answers vary. Sample responses:

    a. $3 - 10 = a$; $a = $ -7

    b. -3 $+ 10 = b$; $b = 7$

    c. -3 $- 10 = c$; $c = $ -13

    d. $10 - d = 3$; $d = 7$

    e. $10 - e = $ -3; $e = 13$

    f. -10 $+ f = $ -3; $f = 7$

Activity Synthesis

Poll the class on which diagram matched which situation. The majority of the discussion should focus on how the students wrote the equations to represent each situation. Have previously identified students share their equations with the class and have the class discuss whether their equations correctly represent the situation. Try to get at least two different equations for every situation.

16.4 Equations Tell a Story

20 minutes
Unlike the previous activities, this activity gives students a chance to generate an equation by themselves, in preparation for the work in the upcoming lessons.

Addressing

- 7.EE.B.4.a

- 7.NS.A.3

Instructional Routines

- Group Presentations

- MLR7: Compare and Connect

Launch

Arrange students in groups of 2–3 and provide tools for making a visual display. Assign one situation to each group. Note that the level of difficulty increases for the situations, so this is an opportunity to differentiate by assigning more or less challenging situations to different groups.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Provide prompts, reminders, guides, rubrics, or checklists that focus on increasing the length of on-task orientation in the face of distractions. For example, create an exemplar display including all required components, highlighting the explanation for how inverses were used in problem solving.
Supports accessibility for: Attention; Social-emotional skills

Speaking, Listening, Representing: MLR7 Compare and Connect. As students prepare the visual display of their given situation, look for students with different strategies for writing and solving their equations. As students analyze each other's work, ask students to share what worked well in a particular approach when representing the situations with equations. Then encourage students to make connections between the use of multiplicative and arithmetic inverses when solving for the unknown quantities. During this discussion, listen for and amplify language students use to make sense of how the operations in the equation represent the relationships in the situation. This will foster students' meta-awareness and support constructive conversations as they compare strategies for writing and solving variable equations, and make connections between the situations and visual representations of equations and solutions on the visual display.
Design Principle(s): Cultivate conversation; Maximize meta-awareness

Student Task Statement

Your teacher will assign your group *one* of these situations. Create a visual display about your situation that includes:

- An equation that represents your situation
- What your variable and each term in the equation represent
- How the operations in the equation represent the relationships in the story
- How you use inverses to solve for the unknown quantity
- The solution to your equation

1. As a $7\frac{1}{4}$ inch candle burns down, its height decreases $\frac{3}{4}$ inch each hour. How many hours does it take for the candle to burn completely?

2. On Monday $\frac{1}{9}$ of the enrolled students in a school were absent. There were 4,512 students present. How many students are enrolled at the school?

3. A hiker begins at sea level and descends 25 feet every minute. How long will it take to get to an elevation of -750 feet?

4. Jada practices the violin for the same amount of time every day. On Tuesday she practices for 35 minutes. How much does Jada practice in a week?

5. The temperature has been dropping $2\frac{1}{2}$ degrees every hour and the current temperature is $-15°\text{F}$. How many hours ago was the temperature $0°\text{F}$?

6. The population of a school increased by 12%, and now the population is 476. What was the population before the increase?

7. During a 5% off sale, Diego pays $74.10 for a new hockey stick. What was the original price?

8. A store buys sweaters for $8 and sells them for $26. How many sweaters does the store need to sell to make a profit of $990?

Student Response

Answers vary. Sample responses:

1. $-\frac{3}{4}h = -7\frac{1}{4}$ where h represents how many hours the candle has been burning, $-\frac{3}{4}h$ represents how much the height of the candle has changed, and $-7\frac{1}{4}$ represents the entire candle burning down. The situation involves multiplication by a negative to show how much of the candle has burned away after each hour. The equation can be solved by multiplying by $-\frac{4}{3}$. The solution is $h = 9\frac{1}{2}$, which means that at this rate it will take $9\frac{1}{2}$ hours for the candle to burn down completely.

2. $(s - \frac{1}{9}s) = 4512$ where s represents the total number of students enrolled at the school, $\frac{1}{9}s$ represents the students that were absent, and 4512 represents the students that were at school on Monday. Students may not know how to solve this because they haven't practiced any equations with more than one variable yet. Students can reason that if $\frac{1}{9}$ of the students are absent, then $\frac{8}{9}$ of the students are present. They can write $\frac{8}{9}s = 4512$ and solve $s = \frac{9}{8} \cdot 4512 = 5076$. This means that there are 5076 students enrolled at the school.

3. $-25m = -750$ where m represents how many minutes they have been hiking, $-25m$ represents the elevation they have hiked to after m minutes, and -750 represents the goal elevation they are trying to get to. The situation involves multiplication by a negative to show how far the hiker has descended after the number of minutes. The equation can be solved by multiplying by $-\frac{1}{25}$. The solution is $m = 30$, which means that at this rate it will take the hiker 30 minutes to reach an elevation of -750 feet.

4. $35 \cdot 7 = p$ (or $\frac{1}{7}p = 35$) where p represents the total amount of time Jada practices in a week, $\frac{1}{7}p$ represents the amount of time Jada practices on one day, and 35 is the number of minutes that she practiced on Tuesday. The situation involves multiplication by a fraction to show that the time she practices on Tuesday is $\frac{1}{7}$ of the total time she practices during the week. The equation can be solved by multiplying by 7. The solution is $p = 245$, which means that Jada practices a total of 245 minutes, or 4 hours and 5 minutes, during the week.

5. $-2\frac{1}{2}h = -15$ where h represents the number of hours since the temperature was 0 degrees Fahrenheit, $-2\frac{1}{2}h$ represents how much the temperature has dropped after h hours, and -15 represents the current temperature. The situation involves multiplication by a negative to show how far the temperature has dropped after the number of hours. The equation can be solved by multiplying by $-\frac{2}{5}$ because that is the reciprocal of $-\frac{5}{2}$ which is equivalent to $-2\frac{1}{2}$. The solution is $h = 6$, which means that the temperature was 0 degrees Fahrenheit 6 hours ago.

6. $p + 0.12p = 476$ where p represents the school's population before the increase, $0.12p$ represents 12% of the original population that the school increased by, and 476 represents the new population of the school after the increase. The equation can be solved by reasoning that 112% of the original population is 476. Then $1.12p = 476$ and $p = 476 \div 1.12 = 425$. This means that the school population before the increase was 425 students.

7. $p - 0.05p = 74.10$, where p represents the original price of the hockey stick, $0.05p$ represents the discount of 5% of the original price, and 74.10 represents the price Diego pays for the stick, assuming there is no sales tax. The equation can be solved by reasoning that 95% of the original price is 74.10. Then $0.95p = 74.10$ and $p = 74.10 \div 0.95 = 78$. This means that the original price of the hockey stick was $78.

8. $(26 - 8)s = 990$, where s represents the number of sweaters sold, $(26 - 8)$ represents the profit the store makes on each sweater, and 990 represents the total profit they want to make. The equation can be solved by dividing by $26 - 8 = 18$. The solution is $s = 55$, which means the store has to sell 55 sweaters to make the desired profit.

Are You Ready for More?

Diego and Elena are 2 miles apart and begin walking towards each other. Diego walks at a rate of 3.7 miles per hour and Elena walks 4.3 miles per hour. While they are walking, Elena's dog runs back and forth between the two of them, at a rate of 6 miles per hour. Assuming the dog does not lose any time in turning around, how far has the dog run by the time Diego and Elena reach each other?

Student Response

Diego and Elena are approaching each other at a rate of $3.7 + 4.3$ or 8 miles per hour. We can write the equation $2 = 8t$ to find the time, t, it takes them to cover 2 miles together. Solving the equation, we find $t = \frac{1}{4}$. This means they walk, and the dog runs, for a quarter of an hour. In that time, the dog covers $(\frac{1}{4})(6)$, or 1.5 miles.

Activity Synthesis

Invite groups to present their solutions or to view all the solutions on display. The discussion should focus on

- how students decided what their variable would represent

- how to write the equation

- how to solve the equation

- how to interpret the solution in terms of the context.

Students should also address any equations with negative quantities and discuss how they represent the situations.

Lesson Synthesis

- When writing an equation to represent a situation, how do you decide what your variable represents?

- How do you solve the equation?

16.5 Floating Above a Sunken Canoe

Cool Down: 5 minutes
Addressing

- 7.EE.B.4.a

- 7.NS.A.3

Student Task Statement

A balloon is floating above a lake and a sunken canoe is below the surface of the lake. The balloon's vertical position is 12 meters and the canoe's is -4.8 meters. The equation $12 + d = \text{-}4.8$ represents this situation.

1. What does the variable d represent?

2. What value of d makes the equation true? Explain your reasoning.

Student Response

1. The difference in elevation. ("Change" in elevation should also be accepted.)

2. -16.8. The equation $12 + d = \text{-}4.8$ can be rewritten $d = \text{-}4.8 - 12$, and $\text{-}4.8 - 12 = \text{-}16.8$.

Student Lesson Summary

We can use variables and equations involving signed numbers to represent a story or answer questions about a situation.

For example, if the temperature is -3°C and then falls to -17°C, we can let x represent the temperature change and write the equation:

$$\text{-}3 + x = \text{-}17$$

We can solve the equation by adding 3 to each side. Since $\text{-}17 + 3 = \text{-}14$, the change is -14°C.

Here is another example: if a starfish is descending by $\frac{3}{2}$ feet every hour then we can solve

$$\text{-}\frac{3}{2}h = \text{-}6$$

to find out how many hours h it takes the starfish to go down 6 feet.

We can solve this equation by multiplying each side by $\text{-}\frac{2}{3}$. Since $\text{-}\frac{2}{3} \cdot \text{-}6 = 4$, we know it will take the starfish 4 hours to descend 6 feet.

Lesson 16 Practice Problems
Problem 1
Statement

Match each situation to one of the equations.

A. A whale was diving at a rate of 2 meters per second. How long will it take for the whale to get from the surface of the ocean to an elevation of -12 meters at that rate?

B. A swimmer dove below the surface of the ocean. After 2 minutes, she was 12 meters below the surface. At what rate was she diving?

C. The temperature was -12 degrees Celsius and rose to 2 degrees Celsius. What was the change in temperature?

D. The temperature was 2 degrees Celsius and fell to -12 degrees Celsius. What was the change in temperature?

1. $-12 + x = 2$

2. $2 + x = -12$

3. $-2x = -12$

4. $2x = -12$

Solution

- A: 3

- B: 4

- C: 1

- D: 2

Problem 2
Statement

Starting at noon, the temperature dropped steadily at a rate of 0.8 degrees Celsius every hour.

For each of these situations, write and solve an equation and describe what your variable represents.

a. How many hours did it take for the temperature to decrease by 4.4 degrees Celsius?

> b. If the temperature after the 4.4 degree drop was -2.5 degrees Celsius, what was the temperature at noon?

Solution

a. $-0.8h = -4.4$, where h is the number of hours it took for the temperature to decrease. Solve the equation by multiplying each side by $\frac{-1}{0.8}$. It took $5\frac{1}{2}$ hours for the temperature to drop 4.4 degrees Celsius.

b. $T - 4.4 = -2.5$, where T is the temperature at noon. The temperature at noon was 1.9 degrees Celsius.

Problem 3

Statement

Kiran mixes $\frac{3}{4}$ cups of raisins, 1 cup peanuts, and $\frac{1}{2}$ cups of chocolate chips to make trail mix. How much of each ingredient would he need to make 10 cups of trail mix? Explain your reasoning.

Solution

The ingredients listed will make $2\frac{1}{4}$ cups of trail mix. So to get 10 cups of trail mix, multiply each amount by $10 \div 2\frac{1}{4} = \frac{40}{9}$. Kiran will need $\frac{30}{9}$ cups of raisins, $\frac{40}{9}$ cups of peanuts, and $\frac{20}{9}$ cups of chocolate chips.

(From Unit 4, Lesson 3.)

Problem 4

Statement

Find the value of each expression.

a. $12 + -10$

b. $-5 - 6$

c. $-42 + 17$

d. $35 - -8$

e. $-4\frac{1}{2} + 3$

Solution

a. 2

b. -11

c. -25

d. 43

e. $-1\frac{1}{2}$

(From Unit 5, Lesson 6.)

Problem 5

Statement

The markings on the number line are evenly spaced. Label the other markings on the number line.

Solution

-75, -60, -45, -30, -15, 0, 15, 30, 45

(From Unit 5, Lesson 8.)

Problem 6

Statement

Kiran drinks 6.4 oz of milk each morning. How many days does it take him to finish a 32 oz container of milk?

a. Write and solve an equation for the situation.

b. What does the variable represent?

Solution

a. $6.4n = 32$. The solution is $n = 5$.

b. The variable n represents the number of days it takes Kiran to finish the container of milk.

Section: Let's Put It to Work

Lesson 17: The Stock Market

Goals

- Apply operations with rational numbers to calculate a stock's new value, change in value, or change expressed as a signed percentage of the previous value.

- Compare (orally and in writing) changes in the value of different stocks, including the dollar amount and the percentage of the previous value.

- Interpret (orally) tables that represent the values of different stocks in the stock market.

Learning Targets

- I can solve problems about the stock market using rational numbers and percentages.

Lesson Narrative

In the previous unit, students worked with percent increase and decrease. In this lesson, students see how signed numbers can be applied to representing changes in the stock market (MP4). This includes representing a decrease (in the price of an individual stock or the value of a stock portfolio) as a negative percentage of the original value.

The names given in the activities are generic, and the data is based on real stock market data from 2016. You may wish to replace them with up to date real companies and data.

Alignments

Addressing

- 7.EE.B.3: Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. $

- 7.NS.A.3: Solve real-world and mathematical problems involving the four operations with rational numbers.Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Instructional Routines

- MLR1: Stronger and Clearer Each Time

- MLR5: Co-Craft Questions

- MLR8: Discussion Supports

Required Materials

Copies of Instructional master

Required Preparation

Print the Your Own Stock Portfolio Instructional master. Each student will need 1 set of copies.

> **Student Learning Goals**
>
> Let's learn about the Stock Market.

17.1 Revisiting Interest and Depreciation

Warm Up: 5 minutes
The warm-up reminds students of their understanding of percentage increase and decrease in preparation for the shares questions later in the lesson. Some students will use different strategies; identify those for the discussion after the activity.

Addressing

- 7.EE.B.3

Launch

Remind students that they can use any strategies from sixth grade for answering the percentages questions.

Anticipated Misconceptions

If students forget about using an equation to calculate percent increase or decrease, remind them that they can use the equation $y = kx$ to figure out the answers to the questions.

> **Student Task Statement**
>
> 1. Lin deposited $300 in a savings account that has a 2% interest rate per year. How much is in her account after 1 year? After 2 years?
>
> 2. Diego wants to sell his bicycle. It cost $150 when he bought it but has depreciated by 15%. How much should he sell it for?

Student Response

1. After one year: $300 \cdot 1.02 = 306$, After two years: $306 \cdot 1.02 = 312.12$; so she has $312.12 in her account.

2. $150 \cdot 0.85 = 127.5$ so $127.50.

Activity Synthesis

The most important thing is to recall efficient methods of calculating percent increase and decrease, such as setting up an equation in the form A% of B is C. Select students to share their methods in preparation for the next activity.

17.2 Gains and Losses

10 minutes

Students are introduced to the idea of the stock market and how the value changes. They apply their understanding of negative numbers to percentage change as well as money (MP4).

Addressing

- 7.EE.B.3

- 7.NS.A.3

Instructional Routines

- MLR5: Co-Craft Questions

Launch

Introduce the concept of shares in a company. They have a specific value at a specific time that can change up or down. They represent a measure of the worth of the company. We sometimes express the change as a value in dollars, so investors can see how much money they will make or lose. We can also present these as a percentage to compare companies in terms of growth.

Before students begin working on the task consider asking how they would represent an increase or decrease in value so that it was easily apparent.

To familiarize students with the information in the table, consider asking which companies' stocks increased in value and which companies' stocks decreased in value from day 1 to day 2. Have students explain their reasoning and come to an agreement before anyone begins doing calculations.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Student Task Statement

1. Here is some information from the stock market in September 2016. Complete the table.

company	value on day 1 (dollars)	value on day 2 (dollars)	change in value (dollars)	change in value as a percentage of day 1 value
Mobile Tech Company	107.95	111.77	3.82	3.54%
Electrical Appliance Company		114.03	2.43	2.18%
Oil Corporation	26.14	25.14		-3.83%
Department Store Company	7.38	7.17		
Jewelry Company		70.30		2.27%

2. Which company's change in dollars had the largest magnitude?

3. Which company's change in percentage had the largest magnitude?

Student Response

1.

company	value on day 1 (dollars)	value on day 2 (dollars)	change in value (dollars)	change in value as a percentage of day 1 value
Mobile Tech Company	107.95	111.77	3.82	3.54
Electrical Appliance Company	111.6	114.03	2.43	2.18
Oil Corporation	26.14	25.14	-1.00	-3.83
Department Store Company	7.38	7.17	-0.21	-2.85
Jewelry Company	68.74	70.30	1.56	2.27

2. Mobile Tech Company's change has the largest magnitude.

3. Oil Corp's change as a percentage of its original value had the largest magnitude.

Activity Synthesis

The important part of this activity is that students use a directed quantity for percentage change to model decrease in value as negative and increase as positive. Ask students how they would prefer the information to be presented, and ask them to support their conclusions.

17.3 What is a Stock Portfolio?

15 minutes
This task introduces the concept of a stock portfolio being a selection of stocks an investor might own to try to make money—students examine the change in portfolio and evaluate the value. They must use both positive and negative change, and percentage change.

Addressing

- 7.EE.B.3

- 7.NS.A.3

Instructional Routines

- MLR8: Discussion Supports

Launch

Introduce the concept of a portfolio of shares in a company.

Student Task Statement

A person who wants to make money by investing in the stock market usually buys a portfolio, or a collection of different stocks. That way, if one of the stocks decreases in value, they won't lose all of their money at once.

1. Here is an example of someone's stock portfolio. Complete the table to show the total value of each investment.

name	price (dollars)	number of shares	total value (dollars)
Technology Company	107.75	98	
Airline Company	133.54	27	
Film Company	95.95	135	
Sports Clothing Company	58.96	100	

2. Here is the same portfolio the next year. Complete the table to show the new total value of each investment.

company	old price (dollars)	price change	new price (dollars)	number of shares	total value (dollars)
Technology Company	107.75	+2.43%		98	
Airline Company	133.54	-7.67%		27	
Film Company	95.95		87.58	135	
Sports Clothing Company	58.96	-5.56%		100	

3. Did the entire portfolio increase or decrease in value over the year?

Student Response

1.

name	price (dollars)	number of shares	total value (dollars)
Technology Company	107.75	98	10,559.50
Airline Company	133.54	27	3605.58
Film Company	95.95	135	12,953.25
Sports Clothing Company	58.96	100	5,896

	company	old price (dollars)	price change	new price (dollars)	number of shares	total value (dollars)
2.	Technology Company	107.75	+2.43%	110.37	98	10,816.26
	Airline Company	133.54	-7.67%	123.30	27	3,329.10
	Film Company	95.95	-8.72%	87.58	135	11,823.3
	Sports Clothing Company	58.96	-5.56%	55.68	100	5,568

3. The portfolio has decreased in total value from $33,014.33 to $31,5365.66.

Activity Synthesis

The important part of this activity is that it extends on the context previously examined, ahead of students building their own portfolio in a later activity.

17.4 Your Own Stock Portfolio

15 minutes
Distribute copies of the "Stocks Prices"

Students calculate their own portfolio

Distribute copies of the "Changes in Stock Prices after 3 Months" so they can calculate the change.

Addressing
- 7.EE.B.3
- 7.NS.A.3

Instructional Routines
- MLR1: Stronger and Clearer Each Time

Launch

Distribute copies of the Stock Prices Instructional Master. Have prepared the Changes in Stock Prices after 3 Months Instructional Master.

Student Task Statement

Your teacher will give you a list of stocks.

1. Select a combination of stocks with a total value close to, but no more than, $100.

2. Using the new list, how did the total value of your selected stocks change?

Student Response

1. Answers vary.

2. Answers vary.

Activity Synthesis

Have students trade with a partner and check each other's work. (Answers vary because each student worked with a different selection of stocks.)

Have students examine the "Changes in Stock Prices after 3 Months" page and discuss:

- How can you tell if the price of a stock increased or decreased from looking at this list?

- Which company's stock price increased or decreased by the most?

- Without doing any calculations, predict which company's stock price had the largest increase as a percentage of its starting value.

- Without doing any calculations, predict which company's stock price had the largest decrease as a percentage of its starting value.

Writing, Speaking: MLR1 Stronger and Clearer Each Time. Use this routine to give students a structured opportunity to revise and refine their initial response to: "Using the new list, how did the total value of your selected stock change?". Ask students to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help pairs strengthen their ideas and clarify their language (e.g., "What is a quantity that changed?", and "What other details are important?", etc.). Students can borrow ideas and language from each partner to strengthen their final response.
Design Principle(s): Optimize output (for description)

Lesson Synthesis

Key learning points:

- There are a number of contexts in which using negative numbers to represent directed change is important; the stock market is one of them.

- Percentage change down can be represented with a negative percentage

Discussion questions:

- Which way of showing the change do you prefer, percentage or monetary value?

- Can you think of other situations in life where using a negative to represent a change would be useful?

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Family Support
Materials

Family Support Materials

Rational Number Arithmetic

Here are the video lesson summaries for Grade 7, Unit 5: Rational Number Arithmetic. Each video highlights key concepts and vocabulary that students learn across one or more lessons in the unit. The content of these video lesson summaries is based on the written Lesson Summaries found at the end of lessons in the curriculum. The goal of these videos is to support students in reviewing and checking their understanding of important concepts and vocabulary. Here are some possible ways families can use these videos:

- Keep informed on concepts and vocabulary students are learning about in class.
- Watch with their student and pause at key points to predict what comes next or think up other examples of vocabulary terms (the bolded words).
- Consider following the Connecting to Other Units links to review the math concepts that led up to this unit or to preview where the concepts in this unit lead to in future units.

Grade 7, Unit 5: Rational Number Arithmetic	Vimeo	YouTube
Video 1: Adding Rational Numbers (Lessons 1–4)	Link	Link
Video 2: Subtracting Rational Numbers (Lessons 5–7)	Link	Link
Video 3: Multiplying and Dividing Rational Numbers (Lessons 8–11)	Link	Link
Video 4: Solving With Rational Numbers (Lessons 12–16)	Link	Link

Video 1

Video 'VLS G7U5V1 Adding Rational Numbers (Lessons 1–4)' available here: https://player.vimeo.com/video/494808053.

Video 2

Video 'VLS G7U5V2 Subtracting Rational Numbers (Lessons 5–7)' available here: https://player.vimeo.com/video/495520145.

Video 3

Video 'VLS G7U5V3 Multiplying and Dividing Rational Numbers (Lessons 8–11)' available here: https://player.vimeo.com/video/503252065.

Video 4

Video 'VLS G7U5V4 Solving With Rational Numbers (Lessons 12–16)' available here: https://player.vimeo.com/video/503606703.

Connecting to Other Units

- *Coming soon*

Adding and Subtracting Rational Numbers

Family Support Materials 1

This week your student will be adding and subtracting with negative numbers. We can represent this on a number line using arrows. The arrow for a positive number points right, and the arrow for a negative number points left. We add numbers by putting the arrows tail to tip.

For example, here is a number line that shows $-5 + 12 = 7$.
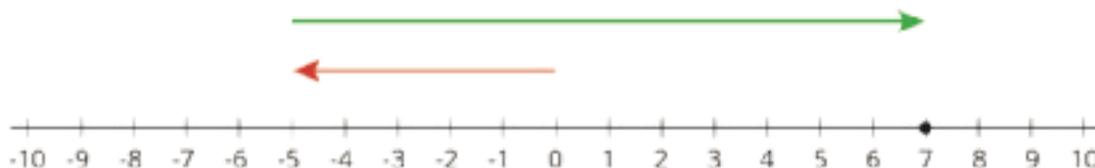
The first number is represented by an arrow that starts at 0 and points 5 units to the left. The next number is represented by an arrow that starts directly above the tip of the first arrow and points 12 units to the right. The answer is 7 because the tip of this arrow ends above the 7 on the number line.

In elementary school, students learned that every addition equation has two related subtraction equations. For example, if we know $3 + 5 = 8$, then we also know $8 - 5 = 3$ and $8 - 3 = 5$.

The same thing works when there are negative numbers in the equation. From the previous example, $-5 + 12 = 7$, we also know $7 - 12 = -5$ and $7 - -5 = 12$.

Here is a task to try with your student:

1. Use the number line to show $3 + -5$.

2. What does your answer tell you about the value of:
   a. $-2 - 3$?

   b. $-2 - -5$?

Solution:

1. The first arrow starts at 0 and points 3 units to the right. The next arrow starts at the tip of the first arrow and points 5 units to the left. This arrow ends above the -2, so $3 + -5 = -2$.
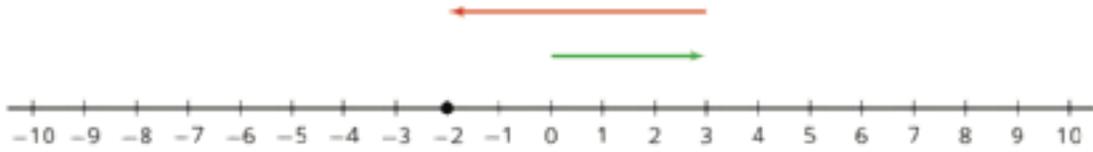
2. From the addition equation $3 + \text{-}5 = \text{-}2$, we get the related subtraction equations:

    a. $\text{-}2 - 3 = \text{-}5$

    b. $\text{-}2 - \text{-}5 = 3$

Multiplying and Dividing Rational Numbers

Family Support Materials 2

This week your student will be multiplying and dividing with negative numbers. The rules for multiplying positive and negative numbers are designed to make sure that addition and multiplication work the same way they always have.

For example, in elementary school students learned to think of "4 times 3" as 4 groups of 3, like $4 \cdot 3 = 3 + 3 + 3 = 12$. We can think of "4 times -3" the same way: $4 \cdot \text{-3} = (\text{-3}) + (\text{-3}) + (\text{-3}) + (\text{-3}) = \text{-12}$. Also, an important property of multiplication is that we can multiply numbers in either order. This means that $\text{-3} \cdot 4 = 4 \cdot \text{-3} = \text{-12}$.

What about -3 · -4? It may seem strange, but the answer is 12. To understand why this is, we can think of -4 as $(0 - 4)$.

$$(\text{-3}) \cdot (\text{-4})$$

$$(\text{-3}) \cdot (0 - 4)$$

$$(\text{-3} \cdot 0) - (\text{-3} \cdot 4)$$

$$0 - \text{-12}$$

$$12$$

After more practice, your student will be able to remember this without needing to think through examples:

- A positive times a negative is a negative.

- A negative times a positive is a negative.

- A negative times a negative is a positive.

Here is a task to try with your student:

1. Calculate $5 \cdot \text{-2}$.

2. Use your answer to the previous question to calculate:
   a. -2 · 5

   b. -2 · -5

   c. -5 · -2

Solution:

1. The answer is -10. We can think of $5 \cdot \text{-}2$ as 5 groups of -2, so

$5 \cdot \text{-}2 = (\text{-}2) + (\text{-}2) + (\text{-}2) + (\text{-}2) + (\text{-}2) = \text{-}10$

2.

    a. The answer is -10. We can multiply numbers in either order, so

       $\text{-}2 \cdot 5 = 5 \cdot \text{-}2 = \text{-}10$

    b. The answer is 10. We can think of -5 as $(0 - 5)$, and $\text{-}2 \cdot (0 - 5) = 0 - \text{-}10 = 10$.

    c. The answer is 10. Possible Strategies:

        ■ We can think of -2 as $(0 - 2)$, and $\text{-}5 \cdot (0 - 2) = 0 - \text{-}10 = 10$.

        ■ We can multiply numbers in either order, so $\text{-}5 \cdot \text{-}2 = \text{-}2 \cdot \text{-}5 = 10$.

Four Operations with Rational Numbers

Family Support Materials 3

This week your student will use what they know about negative numbers to solve equations.

- The *opposite* of 5 is -5, because $5 + -5 = 0$. This is also called the additive inverse.

- The *reciprocal* of 5 is $\frac{1}{5}$, because $5 \cdot \frac{1}{5} = 1$. This is also called the multiplicative inverse.

Thinking about opposites and reciprocals can help us solve equations. For example, what value of x makes the equation $x + 11 = $ -4 true?

$$x + 11 = \text{-}4$$
$$x + 11 + \text{-}11 = \text{-}4 + \text{-}11$$
$$x = \text{-}15$$

11 and -11 are opposites.

The solution is -15.

What value of y makes the equation $\frac{\text{-}1}{3}y = 6$ true?

$$\frac{\text{-}1}{3}y = 6$$
$$\text{-}3 \cdot \frac{\text{-}1}{3}y = \text{-}3 \cdot 6$$
$$y = \text{-}18$$

$\frac{\text{-}1}{3}$ and -3 are reciprocals.

The solution is -18.

Here is a task to try with your student:

Solve each equation:

$25 + a = 17$ $\text{-}4b = \text{-}30$ $\frac{\text{-}3}{4}c = 12$

Solution:

1. -8, because $17 + \text{-}25 = \text{-}8$.

2. 7.5 or equivalent, because $\frac{\text{-}1}{4} \cdot \text{-}30 = 7.5$.

3. -16, because $\frac{\text{-}4}{3} \cdot 12 = \text{-}16$.

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Unit Assessments

Check Your Readiness A and B
End-of-Unit Assessment A and B

Rational Number Arithmetic: Check Your Readiness (A)

Do not use a calculator.

1. a. A dolphin is at the surface of the ocean. What is the dolphin's elevation?

    b. The dolphin dives 20 feet below the surface of the ocean. What is its new elevation?

2. Place and label the following numbers on the number line.

$$-1 \qquad 1.75 \qquad -1.75 \qquad -2 \qquad -2\frac{1}{2} \qquad -\frac{5}{2} \qquad \frac{9}{4}$$
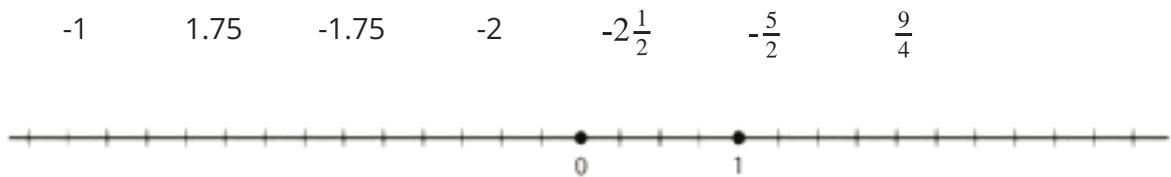
3. Kiran says that $-2 < -5$. Do you agree with Kiran? Explain or show your reasoning.

4. a. Put these numbers in order of their distance from 0 on the number line, starting with the smallest distance:

$$-26, \ -12.7, \ -\frac{9}{2}, \ \frac{2}{100}, \ 7, \ 11\frac{1}{4}, \ \frac{41}{2}, \ 35$$

    b. Find the values of $|-12.7|$ and $|11\frac{1}{4}|$.

5. a. Andre received and spent money in the following ways last month. For each example, write a positive or negative number to represent the change in money from his perspective.

      i. His uncle gave him $25 as a present.

      ii. He earned $18 dollars tutoring.

      iii. He spent $10 on a book.

   b. What does 0 mean in this situation?

6. a. Plot and label these four points in the coordinate plane.
$$A = (2, 3), \ B = (\text{-}2, 1), \ C = (4, \text{-}5), \ D = (\text{-}3, \text{-}5)$$
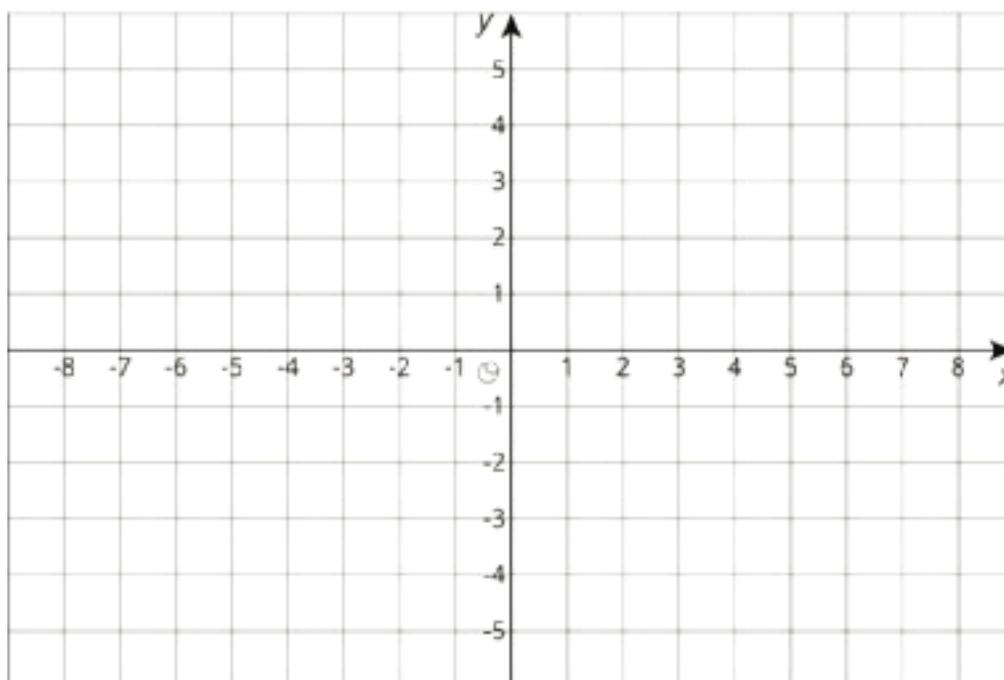
   b. Find the distance between C and D.

Rational Number Arithmetic: Check Your Readiness (B)

Do not use a calculator.

1. When a construction worker is on a platform 10 feet above the ground, we can describe her elevation as +10 feet.

    a. A construction worker digs down to 30 feet below the surface of the ground. What is her elevation?

    b. The construction worker returns to the surface of the ground. What is her elevation?

2. Plot and label each number on the number line.

    2   2.25   -2.25   -1 $\frac{5}{4}$ $-1\frac{3}{4}$ $-\frac{7}{4}$
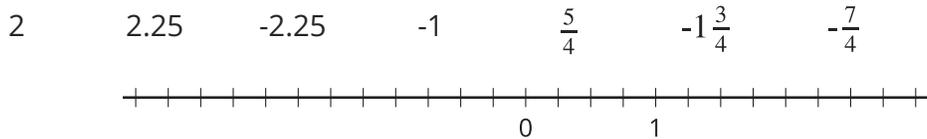
3. Mai says that -8 > -4. Do you agree with her? Explain or show your reasoning.

4.   a. Put these numbers in order of their distance from 0 on the number line, starting with the smallest distance:

$$44, \frac{50}{4}, 14\frac{1}{8}, 3, \frac{5}{50}, -\frac{7}{3}, -9.6, -59$$

   b. Find the values of |-9.6| and $|14\frac{1}{8}|$.

5. a. Diego received and spent money in the following ways last week. For each example, write a positive or negative number to represent the change in money from his perspective.

      i. He earned $30 dollars by cutting the neighbor's lawn.

      ii. He spent $14 going to the movies.

      iii. He spent $3.55 on an ice cream cone.

   b. What does 0 mean in this situation?

6. a. Plot and label these four points on the coordinate plane.

$$A = (-7, 5) , B = (-4, -5), C = (6, -2), D = (6, 3)$$
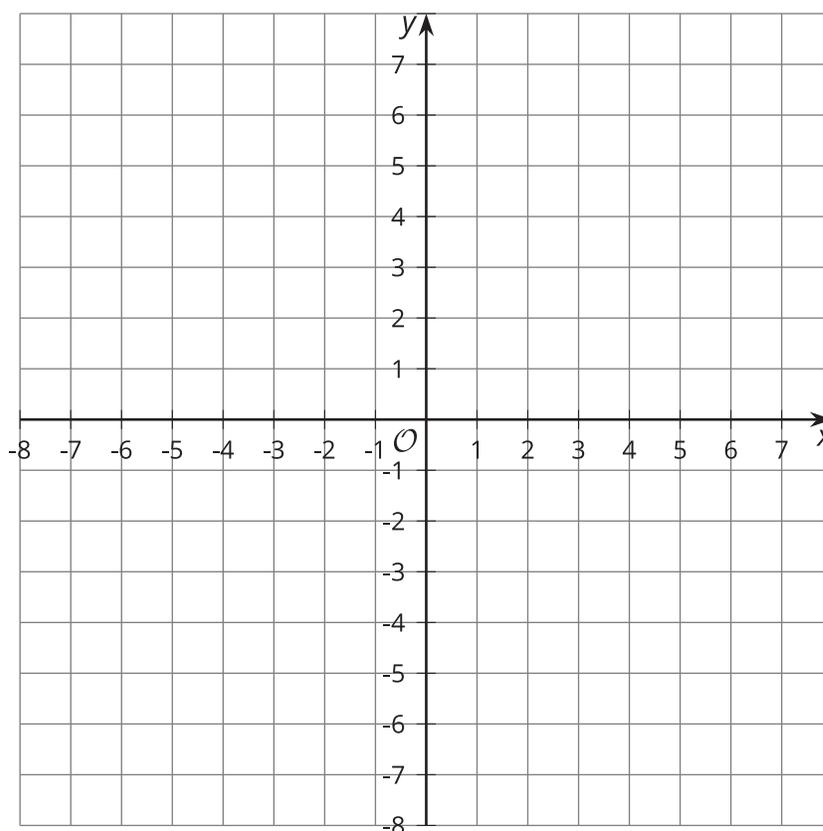
   b. Find the distance between C and D.

Rational Number Arithmetic: End-of-Unit Assessment (A)

Do not use a calculator.

1. Select **all** the true statements.

   A. $2.3 + (-2.3)$ is equal to zero.

   B. $(-3.7) + (-4.1)$ is positive.

   C. $-2.6 - (-\frac{12}{4})$ is positive.

   D. $\frac{5}{2} + (-2.5)$ is negative.

   E. $72 - (-100)$ is negative.

2. A heron is perched in a tree 50 feet above sea level. Directly below the heron, a pelican is flying 17 feet above sea level. Directly below the birds is a trout, swimming 23 feet below sea level.

   Select **all** the true statements.

   A. The difference in height between the pelican and the heron is -33 feet.

   B. The difference in height between the pelican and the heron is 33 feet.

   C. The distance between the heights of the pelican and heron is -33 feet.

   D. The difference in height between the pelican and the trout is -40 feet.

   E. The difference in height between the pelican and the trout is 40 feet.

   F. The distance between the heights of the pelican and the trout is 40 feet.

3. Let $x = -\frac{11}{8}$ and $y = -\frac{11}{4}$.
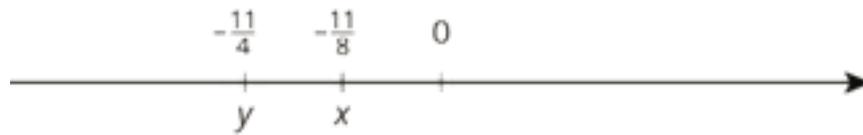
Which expression has a negative value?

    A. $x + y$

    B. $x - y$

    C. $x \cdot y$

    D. $\frac{x}{y}$

4. Calculate the value of each expression.

    a. $(-8) \cdot \frac{2}{3}$

    b. $(-8) \cdot (-\frac{2}{3})$

    c. $-\frac{18}{3}$

    d. $\frac{18}{(-3)}$

    e. $\frac{(-18)}{(-3)}$

5. Solve each equation.

    $\frac{1}{3}a = -5$ $12 - b = 12.5$ $-0.1 = -10c$

6. a. The table shows transactions from five different bank accounts. Fill in the missing numbers.

	old account balance	transaction amount	new account balance
account 1	352	-48	304
account 2	432		512
account 3	75	-100	
account 4		52	12
account 5	-10	-20	-30

   b. Explain what each of the numbers -10, -20, and -30 tells you about Account 5.

7. Jada walks up to a tank of water that can hold up to 10 gallons. When it is active, a drain empties water from the tank at a constant rate. When Jada first sees the tank, it contains 7 gallons of water. Three minutes later, the tank contains 5 gallons of water.

   a. At what rate is the amount of water in the tank changing? Use a signed number, and include the unit of measurement in your answer.

   b. How many more minutes will it take for the tank to drain completely? Explain or show your reasoning.

   c. How many minutes *before* Jada arrived was the water tank completely full? Explain or show your reasoning.

Rational Number Arithmetic: End-of-Unit Assessment (B)

Do not use a calculator.

1. Select **all** expressions whose value is negative.

   A. $\frac{-15}{4}$

   B. $\frac{15}{-4}$

   C. $\frac{-15}{-4}$

   D. $(-8) \cdot \frac{3}{4}$

   E. $(-8) \cdot (-\frac{3}{4})$

2. A sunken ship is resting at 3,000 feet below sea level. Directly above the ship, a whale is swimming 1,960 feet below sea level. Directly above the ship and the whale is a plane flying at 8,500 feet above sea level.

   Select the true statement.

   A. The distance between the heights of the whale and the plane is -10,460 feet.

   B. The difference in height between the whale and the plane is 10,460 feet.

   C. The difference in height between the whale and the ship is -1,040 feet.

   D. The distance between the heights of the whale and the ship is 1,040 feet.

3. Let $x = -\frac{5}{6}$ and $y = \frac{4}{3}$.

$$
\begin{array}{c}
\quad x \qquad\qquad\qquad\qquad\qquad\qquad y \\
\hline
-\frac{5}{6} \qquad\qquad 0 \qquad\qquad\qquad\qquad \frac{4}{3}
\end{array}
$$

Select **all** the expressions that have a positive value.

A. $x \cdot y$
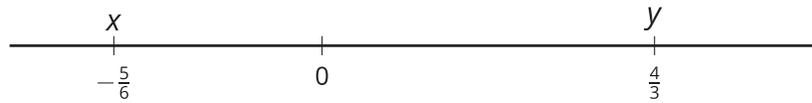
B. $\frac{y}{x}$

C. $\frac{x}{y}$

D. $x + y$

E. $x - y$

F. $y - x$

4. Calculate the value of each expression.

a. $60 - (\text{-}80)$

b. $2.3 + (\text{-}4.9)$

c. $4.5 + (\text{-}4.5)$

5. Solve each equation.

$\text{-}5a = \text{-}0.2$ $\qquad\qquad b + 14.5 = 10 \qquad\qquad 2 = \text{-}\frac{1}{5}c$

6.   a. Lin went hiking on five different weekends. The table shows the elevation changes in feet for each of her five hikes. Fill in the missing numbers.

	initial elevation	elevation change	final elevation
hike 1	768	-96	672
hike 2		82	20
hike 3	62	-100	
hike 4	354		129
hike 5	-20	-40	-60

    b. Explain what each of the numbers -20, -40, and -60 tells you about hike 5.

7. A scuba diver is diving at a constant rate when her team on the surface requests a status update. She looks at her watch which says her elevation is 18 feet below sea level. 8 seconds later, her elevation is 20 feet below sea level.

    a. At what rate is her elevation changing? Use a signed number, and include the unit of measurement in your answer.

    b. How many more seconds until she reaches her goal depth of 50 feet? Explain or show your reasoning.

    c. How many seconds *before* her team requested an update was the diver at the surface of the water? Explain or show your reasoning.

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Assessment
Answer Keys

Check Your Readiness A and B
End-of-Unit Assessment A and B

Assessment Answer Keys

Assessment : Check Your Readiness (A)

Teacher Instructions

Calculators should not be used.

Student Instructions

Do not use a calculator.

Problem 1

The content assessed in this problem is first encountered in Lesson 1: Interpreting Negative Numbers.

In grade 6, students studied elevation as a context for signed numbers. In this unit, they will return to this context.

If most students struggle with this item, plan to launch Activity 3 in Lesson 1 with a review of sea level and elevation, since this context will be used throughout the unit. You might look up your current elevation and have students discuss why that might be the case, especially if you are far from the ocean. You might also show pictures of videos highlighting the depth at which different animals hang out in the ocean to practice talk about feet below sea level.

> ### Statement
>
> 1. A dolphin is at the surface of the ocean. What is the dolphin's elevation?
>
> 2. The dolphin dives 20 feet below the surface of the ocean. What is its new elevation?

Solution

1. 0 feet

2. -20 feet

Aligned Standards

6.NS.C.5

Problem 2

The content assessed in this problem is first encountered in Lesson 2: Changing Temperatures.

This problem has students order and compare signed numbers, including fractions and decimals; this was work that they did in grade 6. In this unit, students will make sense of operations on signed numbers by using number lines to visualize these operations.

If most students struggle with this item, plan to do the optional activity in Lesson 1 Activity 4, Card Sort with Rational Numbers. The work with thermometers and vertical number lines in the warm-up and previous activities can help students recall what they learned about ordering rational numbers in 6th grade. Allow students to arrange the cards vertically or horizontally, and discuss how most number lines we see in this unit will be horizontal.

Statement

Place and label the following numbers on the number line.

$$-1 \qquad 1.75 \qquad -1.75 \qquad -2 \qquad -2\frac{1}{2} \qquad -\frac{5}{2} \qquad \frac{9}{4}$$

Solution

Aligned Standards

6.NS.C.6

Problem 3

The content assessed in this problem is first encountered in Lesson 1: Interpreting Negative Numbers.

This is another problem about comparing signed numbers. Student responses should provide insight into the ways in which students are conceptualizing negative numbers and their magnitudes.

If most students struggle with this item, plan to do the optional activity in Lesson 1 Activity 4, Card Sort with Rational Numbers. This will give students an opportunity to consider magnitude while ordering integers and rational numbers. Student will continue to work with this concept in future lessons. Students will have more opportunities to work on this skill in Lessons 2 and 3 as well.

Statement

Kiran says that -2 < -5. Do you agree with Kiran? Explain or show your reasoning.

Assessment: Check Your Readiness (A)

Solution

No. Explanations vary. Sample explanations include a number line showing the true ordering of -2 and -5, or a response describing how negative numbers with a small magnitude are greater than negative numbers with a large magnitude.

Aligned Standards

6.NS.C.7

Problem 4

The content assessed in this problem is first encountered in Lesson 1: Interpreting Negative Numbers.

Students are asked to rank numbers according to their magnitude. The concept of magnitude, or distance from zero, is important for students to understand as they make sense of operations on signed numbers. While the main purpose of this problem is to make sure that students recall that "distance from zero" is without regard to sign, the problem will also reveal if students need to practice comparing decimals, fractions, and whole numbers.

If most students struggle with this item, plan to do the optional activity in Lesson 1 Activity 4, Card Sort with Rational Numbers. This will give students an opportunity to consider magnitude while ordering integers and rational numbers. Student will continue to work with this concept in future lessons. Students will have more opportunities to work on this skill in Lessons 2 and 3 as well.

Statement

1. Put these numbers in order of their distance from 0 on the number line, starting with the smallest distance:

$$-26,\ -12.7,\ -\tfrac{9}{2},\ \tfrac{2}{100},\ 7,\ 11\tfrac{1}{4},\ \tfrac{41}{2},\ 35$$

2. Find the values of $|-12.7|$ and $|11\tfrac{1}{4}|$.

Solution

1. $\tfrac{2}{100},\ -\tfrac{9}{2},\ 7,\ 11\tfrac{1}{4},\ -12.7,\ \tfrac{41}{2},\ -26,\ 35$

2. $|-12.7| = 12.7$. $|11\tfrac{1}{4}| = 11\tfrac{1}{4}$.

Aligned Standards

6.NS.C.7

Problem 5

The content assessed in this problem is first encountered in Lesson 4: Money and Debts.

In grade 6, students learned that we often use positive and negative numbers to represent money contexts. Students will return to this context in this unit and develop it further.

If most students struggle with this item, plan to use this problem for some error analysis before doing Lesson 4. Activity 1, Concert Tickets, will provide some extra practice with this concept. Another option is to engage students in a shopping role play recording calculations from the perspective of the shopper and the seller.

Statement

1. Andre received and spent money in the following ways last month. For each example, write a positive or negative number to represent the change in money from his perspective.

    a. His uncle gave him $25 as a present.

    b. He earned $18 dollars tutoring.

    c. He spent $10 on a book.

2. What does 0 mean in this situation?

Solution

1.   a. 25 or +25

      b. 18 or +18

      c. -10

2. That Andre neither received nor spent money.

Aligned Standards

6.NS.C.5

Problem 6

The content assessed in this problem is first encountered in Lesson 7: Adding and Subtracting to Solve Problems.

Graphing in the coordinate plane requires a different kind of visual interpretation of signed numbers: right or left as well as up or down. Students learned to graph signed numbers on the coordinate plane in grade 6.

If most students struggle with this item, plan to do the optional Lesson 7 Activity 4, Differences and Distances. Students will continue thinking about distance (which is unsigned) and difference (which is signed).

Statement

1. Plot and label these four points in the coordinate plane.

Assessment: Check Your Readiness (A)

4

$$A = (2, 3), \quad B = (\text{-}2, 1), \quad C = (4, \text{-}5), \quad D = (\text{-}3, \text{-}5)$$

2. Find the distance between C and D.

Solution
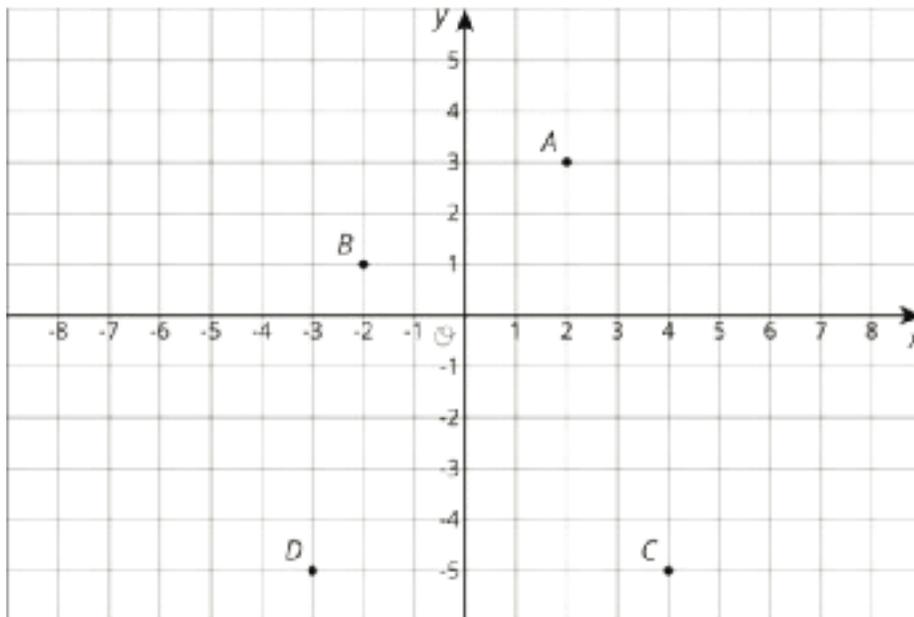
1.

2. 7 units

Aligned Standards

6.NS.C.6.b, 6.NS.C.8

5

Assessment : Check Your Readiness (B)

Teacher Instructions

Calculators should not be used.

Student Instructions

Do not use a calculator.

Problem 1

The content assessed in this problem is first encountered in Lesson 1: Interpreting Negative Numbers.

In grade 6, students studied elevation as a context for signed numbers. In this unit, they will return to this context.

If most students struggle with this item, plan to launch Activity 3 in Lesson 1 with a review of sea level and elevation, since this context will be used throughout the unit. You might look up your current elevation and have students discuss why that might be the case, especially if you are far from the ocean. You might also show pictures of videos highlighting the depth at which different animals hang out in the ocean to practice talk about feet below sea level.

Statement

When a construction worker is on a platform 10 feet above the ground, we can describe her elevation as +10 feet.

1. A construction worker digs down to 30 feet below the surface of the ground. What is her elevation?

2. The construction worker returns to the surface of the ground. What is her elevation?

Solution

1. -30 feet

2. 0 feet

Aligned Standards

6.NS.C.5

Problem 2

The content assessed in this problem is first encountered in Lesson 2: Changing Temperatures.
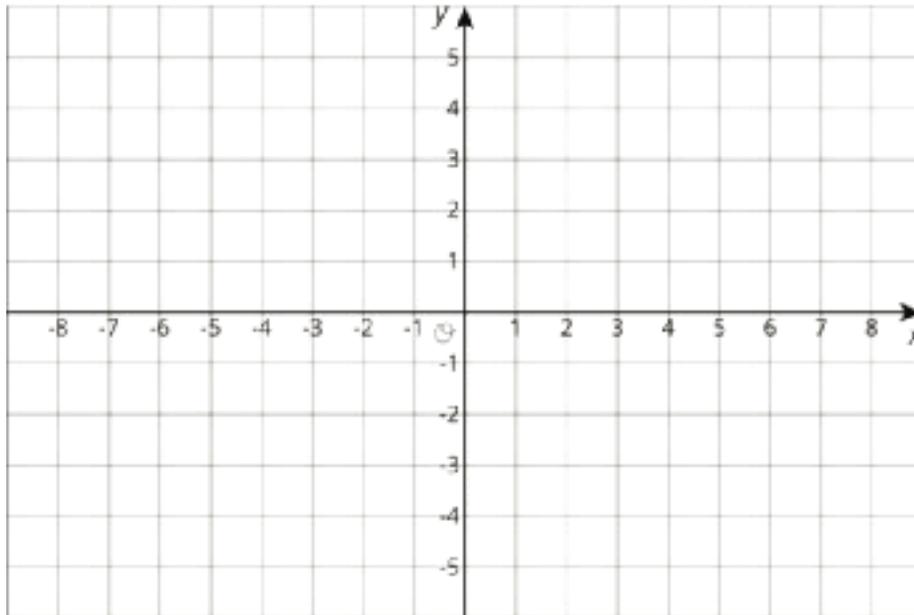
This problem has students order and compare signed numbers, including fractions and decimals; this was work that they did in grade 6. In this unit, students will make sense of operations on signed numbers by using number lines to visualize these operations.

If most students struggle with this item, plan to do the optional activity in Lesson 1 Activity 4, Card Sort with Rational Numbers. The work with thermometers and vertical number lines in the warm-up and previous activities can help students recall what they learned about ordering rational numbers in 6th grade. Allow students to arrange the cards vertically or horizontally, and discuss how most number lines we see in this unit will be horizontal.

Statement

Plot and label each number on the number line.

2 2.25 -2.25 -1 $\frac{5}{4}$ $-1\frac{3}{4}$ $-\frac{7}{4}$

Solution

Aligned Standards

6.NS.C.6

Problem 3

The content assessed in this problem is first encountered in Lesson 1: Interpreting Negative Numbers.

This is another problem about comparing signed numbers. Student responses should provide insight into the ways in which students are conceptualizing negative numbers and their magnitudes.

If most students struggle with this item, plan to do the optional activity in Lesson 1 Activity 4, Card Sort with Rational Numbers. This will give students an opportunity to consider magnitude while ordering integers and rational numbers. Student will continue to work with this concept in future lessons. Students will have more opportunities to work on this skill in Lessons 2 and 3 as well.

Statement

Mai says that $-8 > -4$. Do you agree with her? Explain or show your reasoning.

Solution

No. Explanations vary. Sample explanations include a number line showing that -8 lies to the left of -4, or a response describing how negative numbers with a small magnitude are greater than negative numbers with a large magnitude.

Aligned Standards

6.NS.C.7

Problem 4

The content assessed in this problem is first encountered in Lesson 1: Interpreting Negative Numbers.

Students are asked to rank numbers according to their magnitude. The concept of magnitude, or distance from zero, is important for students to understand as they make sense of operations on signed numbers. While the main purpose of this problem is to make sure that students recall that "distance from zero" is without regard to sign, the problem will also reveal if students need to practice comparing decimals, fractions, and whole numbers.

If most students struggle with this item, plan to do the optional activity in Lesson 1 Activity 4, Card Sort with Rational Numbers. This will give students an opportunity to consider magnitude while ordering integers and rational numbers. Student will continue to work with this concept in future lessons. Students will have more opportunities to work on this skill in Lessons 2 and 3 as well.

Statement

1. Put these numbers in order of their distance from 0 on the number line, starting with the smallest distance:

$$44, \frac{50}{4}, 14\frac{1}{8}, 3, \frac{5}{50}, -\frac{7}{3}, -9.6, -59$$

2. Find the values of $|-9.6|$ and $|14\frac{1}{8}|$.

Solution

1. $\frac{5}{50}, -\frac{7}{3}, 3, -9.6, \frac{50}{4}, 14\frac{1}{8}, 44, -59$

2. 9.6 and $14\frac{1}{8}$

Aligned Standards

6.NS.C.7

Problem 5

The content assessed in this problem is first encountered in Lesson 4: Money and Debts.

In grade 6, students learned that we often use positive and negative numbers to represent money contexts. Students will return to this context in this unit, and develop it further.

If most students struggle with this item, plan to use this problem for some error analysis before doing Lesson 4. Activity 1, Concert Tickets, will provide some extra practice with this concept. Another option is to engage students in a shopping role play recording calculations from the perspective of the shopper and the seller.

Statement

1. Diego received and spent money in the following ways last week. For each example, write a positive or negative number to represent the change in money from his perspective.

    a. He earned $30 dollars by cutting the neighbor's lawn.

    b. He spent $14 going to the movies.

    c. He spent $3.55 on an ice cream cone.

2. What does 0 mean in this situation?

Solution

1. Changes in money:
    a. 30 or +30

    b. -14

    c. -3.55

2. He neither spent nor received money.

Aligned Standards

6.NS.C.5

Problem 6

The content assessed in this problem is first encountered in Lesson 7: Adding and Subtracting to Solve Problems.

Graphing in the coordinate plane requires a different kind of visual interpretation of signed numbers: right or left as well as up or down. Students learned to graph signed numbers on the coordinate plane in grade 6.

If most students struggle with this item, plan to do the optional Lesson 7 Activity 4, Differences and Distances. Students will continue thinking about distance (which is unsigned) and difference (which is signed).

Statement

1. Plot and label these four points on the coordinate plane.

$$A = (\text{-}7, 5)\,,\ B = (\text{-}4, \text{-}5),\ C = (6, \text{-}2),\ D = (6, 3)$$

2. Find the distance between C and D.

Solution

1. See graph.

2. 5 units
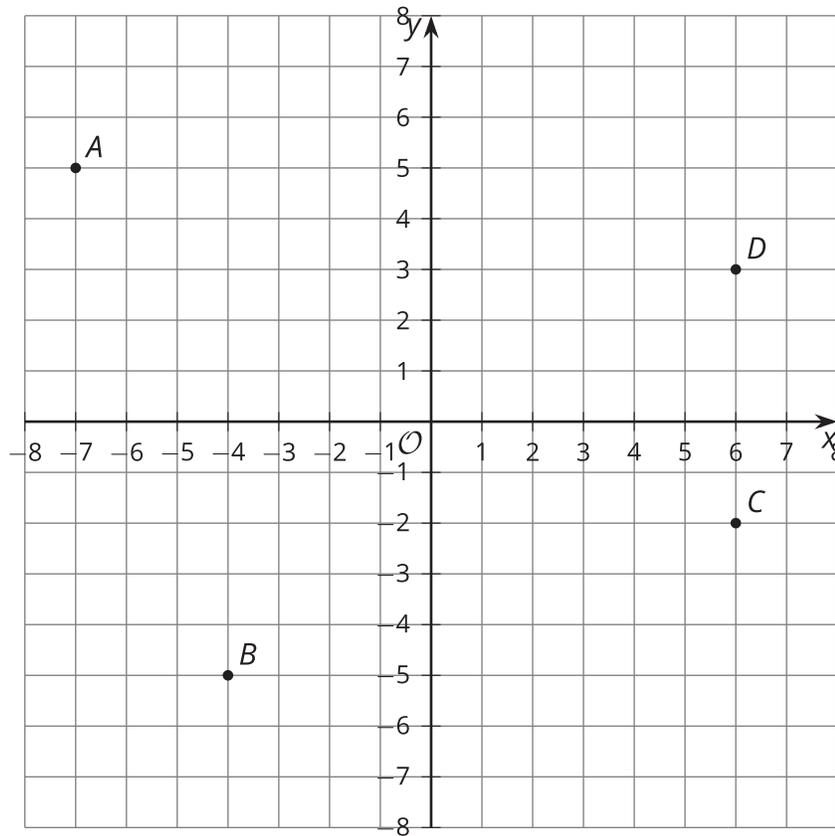
Aligned Standards

6.NS.C.6.b, 6.NS.C.8

Assessment : End-of-Unit Assessment (A)

Teacher Instructions

Calculators should not be used.

Student Instructions

Do not use a calculator.

Problem 1

This problem targets students' understanding of sums and differences of two rational numbers. Some students may wish to draw number lines to help.

Students failing to select A may have a major misconception about opposites. Students selecting B may have misread the problem as $(-3.7) - (-4.1)$. Students failing to select C may have misread the additional negative sign inside the parentheses, or made an error in the value of $-\frac{12}{4}$. Students selecting D may have failed to recognize that $\frac{5}{2}$ and -2.5 are opposites, or may not know that zero is neither positive nor negative. Students selecting E most likely have a serious misunderstanding about subtracting a negative number, using $72 - 100$ accidentally.

> ## Statement
>
> Select **all** the true statements.
>
> A. $2.3 + (-2.3)$ is equal to zero.
>
> B. $(-3.7) + (-4.1)$ is positive.
>
> C. $-2.6 - (-\frac{12}{4})$ is positive.
>
> D. $\frac{5}{2} + (-2.5)$ is negative.
>
> E. $72 - (-100)$ is negative.

Solution

["A", "C"]

Aligned Standards

7.NS.A.1

Problem 2

Students selecting B but not selecting A or selecting D but not selecting E may be remembering the convention that "the difference between a and b" is defined as $a - b$, not $b - a$ incorrectly. Students selecting other incorrect combinations of A, B, D, and E have deeper confusions. Students

selecting C are selecting a negative distance, and may need to be reminded about distance versus difference. Students failing to select F may also be confusing distance with difference.

Statement

A heron is perched in a tree 50 feet above sea level. Directly below the heron, a pelican is flying 17 feet above sea level. Directly below the birds is a trout, swimming 23 feet below sea level.

Select **all** the true statements.

A. The difference in height between the pelican and the heron is -33 feet.

B. The difference in height between the pelican and the heron is 33 feet.

C. The distance between the heights of the pelican and heron is -33 feet.

D. The difference in height between the pelican and the trout is -40 feet.

E. The difference in height between the pelican and the trout is 40 feet.

F. The distance between the heights of the pelican and the trout is 40 feet.

Solution

["A", "E", "F"]

Aligned Standards

7.NS.A.1.c

Problem 3

Students selecting an incorrect option may have a misunderstanding about the meaning of that operation, or may have misinterpreted addition as moving to the right on a number line.

The numbers are small enough that the answers can be computed, but it is simpler to use the sign and size of the numbers. Watch for students who compute here, and ask them what shortcuts they might take.

Statement

Let $x = -\frac{11}{8}$ and $y = -\frac{11}{4}$.
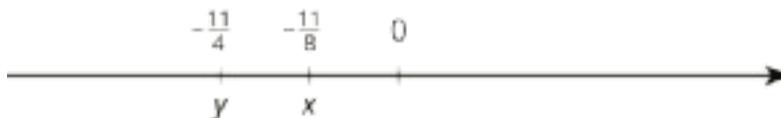
Which expression has a negative value?

A. $x + y$

B. $x - y$

C. $x \cdot y$

D. $\frac{x}{y}$

Solution

A

Aligned Standards

7.NS.A

Problem 4

Students should see the relationships between the first two expressions, and between the last three.

Statement

Calculate the value of each expression.

1. $(-8) \cdot \frac{2}{3}$

2. $(-8) \cdot (-\frac{2}{3})$

3. $-\frac{18}{3}$

4. $\frac{18}{(-3)}$

5. $\frac{(-18)}{(-3)}$

Solution

1. $-\frac{16}{3}$ or $-5\frac{1}{3}$

2. $\frac{16}{3}$ or $5\frac{1}{3}$

3. -6

4. -6

5. 6

Aligned Standards

7.NS.A.1.b, 7.NS.A.2.b, 7.NS.A.3

Problem 5

Students solve equations of the form $x + p = q$ and $px = q$ when the coefficients or solution is negative.

> ## Statement
>
> Solve each equation.
>
> $\frac{1}{3}a = \text{-}5$
> $\qquad\qquad 12 - b = 12.5$
> $\qquad\qquad \text{-}0.1 = \text{-}10c$

Solution

1. $a = \text{-}15$

2. $b = \text{-}0.5$ or equivalent

3. $c = 0.01$ or equivalent

Aligned Standards

7.EE.B.4.a

Problem 6

Students work with signed numbers in a money context and interpret the meaning of negative numbers in that context.

> ## Statement
>
> 1. The table shows transactions from five different bank accounts. Fill in the missing numbers.
>
> | | old account balance | transaction amount | new account balance |
> |---|---|---|---|
> | account 1 | 352 | -48 | 304 |
> | account 2 | 432 | | 512 |
> | account 3 | 75 | -100 | |
> | account 4 | | 52 | 12 |
> | account 5 | -10 | -20 | -30 |
>
> 2. Explain what each of the numbers -10, -20, and -30 tells you about Account 5.

Solution

1.

	old account balance	transaction amount	new account balance
account 1	352	-48	304
account 2	432	80	512
account 3	75	-100	-25
account 4	-40	52	12
account 5	-10	-20	-30

2. Answers vary. Sample response: The account balance was -10, which means they owe the bank $10. Then they wrote a check for $20, so the new balance is -30, so now they owe the bank $30.

Minimal Tier 1 response:

- Work is complete and correct.

- Sample:

1. See table.

2. The person was $10 in debt, then they had to pay somebody $20, and now they are $30 in debt.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.

- Sample errors: One incorrect entry in the table; response for part b acknowledges debt but is flawed in other ways.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.

- Sample errors: multiple incorrect entries in the table, including systematic errors like always adding the two known numbers or omitting negative signs; response for part b does not involve debt; multiple error types under Tier 2 response.

Aligned Standards

7.NS.A.1, 7.NS.A.3

Problem 7

As mentioned in the solution rubric, a student finding an incorrect rate should be given credit for correct calculations based on that rate.

If discussing this problem, look for multiple solution methods, especially those that involve working with signed numbers for both the rate and number of gallons. While it is not necessary to use signed numbers to solve these problems, it is an excellent way to work here. Do not penalize students who solve the problem without using signed number arithmetic, although they must use a signed number for the rate.

Statement

Jada walks up to a tank of water that can hold up to 10 gallons. When it is active, a drain empties water from the tank at a constant rate. When Jada first sees the tank, it contains 7 gallons of water. Three minutes later, the tank contains 5 gallons of water.

1. At what rate is the amount of water in the tank changing? Use a signed number, and include the unit of measurement in your answer.

2. How many more minutes will it take for the tank to drain completely? Explain or show your reasoning.

3. How many minutes *before* Jada arrived was the water tank completely full? Explain or show your reasoning.

Solution

1. $-\frac{2}{3}$ gallons per minute. 2 gallons leave in 3 minutes, which as a unit rate is $\frac{2}{3}$ gallons in 1 minute. The value is negative because the amount is decreasing.

2. $\frac{15}{2}$ minutes (or equivalent). 5 gallons must be drained at a rate of $-\frac{2}{3}$ gallons per minute. It will take $\frac{15}{2}$ minutes since $(-5) \div (-\frac{2}{3}) = \frac{15}{2}$.

3. $\frac{9}{2}$ minutes (or equivalent). 3 gallons have drained from the tank when Jada arrives. It took $\frac{9}{2}$ minutes since $3 \div (-\frac{2}{3}) = -\frac{9}{2}$.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.

- Sample:

1. $-\frac{2}{3}$ gallons per minute

2. 7.5, because 1 gallon drains every 1.5 minutes. $(1.5) \cdot 5 = 7.5$.

3. 4.5, because $(1.5) \cdot 3 = 4.5$.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.

- Sample errors: minor visible calculation errors in multiplying, dividing, or determining the rate; incorrect or omitted units used on rate; rate of $\frac{2}{3}$ instead of $-\frac{2}{3}$; correct answers without justification.

- Acceptable errors: an incorrect rate coming from a calculation error is used correctly through the rest of the problem.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.

- Sample errors: incorrect rate from conceptual misunderstanding, such as $\frac{3}{2}$ gallons per minute; omitted rate; invalid methods or omissions on either of the time questions.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.

- Sample errors: incorrect or omitted rate, and invalid methods or omissions on at least one of the time questions.

Aligned Standards

7.NS.A.2, 7.NS.A.3

Assessment : End-of-Unit Assessment (B)

Teacher Instructions

Calculators should not be used.

Student Instructions

Do not use a calculator.

Problem 1

Students selecting an incorrect option may have a misunderstanding about the meaning of that operation, or may have misinterpreted the sign of the number. The numbers are small enough that the answers can be computed, but it is simpler to use the sign and size of the numbers. Watch for students who compute here, and ask them what shortcuts they might take.

Statement

Select **all** expressions whose value is negative.

A. $\frac{-15}{4}$

B. $\frac{15}{-4}$

C. $\frac{-15}{-4}$

D. $(-8) \cdot \frac{3}{4}$

E. $(-8) \cdot (-\frac{3}{4})$

Solution

["A", "B", "D"]

Aligned Standards

7.NS.A.1.b, 7.NS.A.2.b, 7.NS.A.3

Problem 2

Students selecting B and C may be remembering the convention that "the difference between a and b" is defined as $a - b$, not $b - a$ incorrectly. Students selecting A are selecting a negative distance, and may need to be reminded about distance versus difference.

Statement

A sunken ship is resting at 3,000 feet below sea level. Directly above the ship, a whale is swimming 1,960 feet below sea level. Directly above the ship and the whale is a plane flying at 8,500 feet above sea level.

Select the true statement.

    A. The distance between the heights of the whale and the plane is -10,460 feet.

    B. The difference in height between the whale and the plane is 10,460 feet.

    C. The difference in height between the whale and the ship is -1,040 feet.

    D. The distance between the heights of the whale and the ship is 1,040 feet.

Solution

D

Aligned Standards

7.NS.A.1.c

Problem 3

Students selecting an incorrect option may have a misunderstanding about the meaning of that operation, or may have misinterpreted addition as moving to the right on a number line. The numbers are small enough that the answers can be computed, but it is simpler to use the sign and size of the numbers. Watch for students who compute here, and ask them what shortcuts they might take.

Statement

Let $x = -\frac{5}{6}$ and $y = \frac{4}{3}$.

Select **all** the expressions that have a positive value.

    A. $x \cdot y$
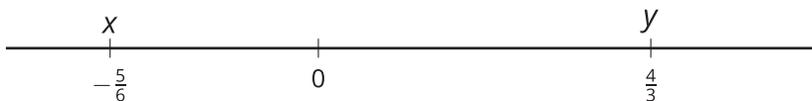
    B. $\frac{y}{x}$

    C. $\frac{x}{y}$

    D. $x + y$

    E. $x - y$

    F. $y - x$

Solution

["D", "F"]

Aligned Standards

7.NS.A

Problem 4

This problem targets students' understanding of sums and differences of two rational numbers. Some students may wish to draw number lines to help.

> ### Statement
>
> Calculate the value of each expression.
>
> 1. $60 - (-80)$
>
> 2. $2.3 + (-4.9)$
>
> 3. $4.5 + (-4.5)$

Solution

1. 140

2. -2.6

3. 0

Aligned Standards

7.NS.A.1

Problem 5

Students solve equations of the form $x + p = q$ and $px = q$ when the coefficients or solution is negative.

> ### Statement
>
> Solve each equation.
>
> $-5a = -0.2$ $b + 14.5 = 10$ $2 = -\frac{1}{5}c$

Solution

1. $a = 0.04$ or equivalent

2. $b = -4.5$

3. $c = -10$

Aligned Standards

7.EE.B.4.a

Problem 6

Students work with signed numbers in a hiking/elevation context and interpret the meaning of negative numbers in that context.

Statement

1. Lin went hiking on five different weekends. The table shows the elevation changes in feet for each of her five hikes. Fill in the missing numbers.

	initial elevation	elevation change	final elevation
hike 1	768	-96	672
hike 2		82	20
hike 3	62	-100	
hike 4	354		129
hike 5	-20	-40	-60

2. Explain what each of the numbers -20, -40, and -60 tells you about hike 5.

Solution

1.

	initial elevation	elevation change	final elevation
hike 1	768	-96	672
hike 2	**-62**	82	20
hike 3	62	-100	**-38**
hike 4	354	**-225**	129
hike 5	-20	-40	-60

2. Answers vary. Sample response: Lin's starting elevation was 20 feet below sea level. She descended 40 more feet. Her final elevation was 60 feet below sea level. (Note that Lin may also have descended 60 feet and then climbed 20 feet or equivalent: the important thing is that she wound up 40 feet below her starting point.)

Minimal Tier 1 response:

- Work is complete and correct.

Assessment: End-of-Unit Assessment (B)

22

- Sample:

1. See table.

2. Lin started hiking at 20 feet below sea level, descended 40 more feet, and now is at 60 feet below sea level.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.

- Sample errors: One incorrect entry in the table; response for part b acknowledges that a negative elevation change corresponds to downward motion but is flawed in other ways.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.

- Sample errors: multiple incorrect entries in the table, including systematic errors like always adding the two known numbers or omitting negative signs; response for part b does not involve hiking downward; multiple error types under Tier 2 response.

Aligned Standards

7.NS.A.1, 7.NS.A.3

Problem 7

As mentioned in the solution rubric, a student finding an incorrect rate should be given credit for correct calculations based on that rate. If discussing this problem, look for multiple solution methods, especially those that involve working with signed numbers for both the rate and elevation. While it is not necessary to use signed numbers to solve these problems, it is an excellent way to work here. Do not penalize students who solve the problem without using signed number arithmetic, although they must use a signed number for the rate.

Statement

A scuba diver is diving at a constant rate when her team on the surface requests a status update. She looks at her watch which says her elevation is 18 feet below sea level. 8 seconds later, her elevation is 20 feet below sea level.

1. At what rate is her elevation changing? Use a signed number, and include the unit of measurement in your answer.

2. How many more seconds until she reaches her goal depth of 50 feet? Explain or show your reasoning.

3. How many seconds *before* her team requested an update was the diver at the surface of the water? Explain or show your reasoning.

Solution

1. $-\frac{1}{4}$ feet per second or equivalent. Her elevation decreases 2 feet in 8 seconds, which as a unit rate is $\frac{2}{8}$ feet in 1 second. The value is negative because her elevation is decreasing.

2. 120 more seconds (or equivalent). 30 feet of elevation must be lost at a rate of $-\frac{1}{4}$ feet per second. It will take 120 seconds since $-30 \div (-\frac{1}{4}) = 120$.

3. 72 seconds (or equivalent). She had descended 18 feet at a rate of $-\frac{1}{4}$ feet per second, and $-18 \div (-\frac{1}{4}) = 72$.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.

- Sample:

1. $-\frac{2}{8}$ feet per second.

2. She needs to descend 30 more feet, which is 15 times the 2 feet we know about, so it takes 120 seconds because $15 \cdot 8 = 120$.

3. 72, because it takes 4 seconds to descend 1 foot, and $18 \cdot 4 = 72$.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.

- Sample errors: minor visible calculation errors in multiplying, dividing, or determining the rate; incorrect or omitted units used on rate; correct answers without justification.

- Acceptable errors: an incorrect rate coming from a calculation error is used correctly through the rest of the problem.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.

- Sample errors: incorrect rate from conceptual misunderstanding, such as $\frac{8}{20}$ feet every second; omitted rate; invalid methods or omissions on either of the time questions.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.

- Sample errors: incorrect or omitted rate, and invalid methods or omissions on at least one of the time questions.

Assessment: End-of-Unit Assessment (B)

Aligned Standards

7.NS.A.2, 7.NS.A.3

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Lesson
Cool Downs

Lesson 1: Interpreting Negative Numbers

Cool Down: Signed Numbers

Here is a set of signed numbers: 7, -3, $\frac{1}{2}$, -0.8, 0.8, -$\frac{1}{10}$, -2

    1. Order the numbers from least to greatest.

    2. If these numbers represent temperatures in degrees Celsius, which is the coldest?

    3. If these numbers represent elevations in meters, which is the farthest away from sea level?

Lesson 2: Changing Temperatures

Cool Down: Stories about Temperature

1. Write a story about temperatures that this expression could represent: $27 + (-11)$

2. Draw a number line diagram and write an expression to represent this situation: "On Tuesday at lunchtime, it was $29°C$. By sunset, the temperature had dropped to $16°C$."

Lesson 3: Changing Elevation

Cool Down: Add 'Em Up

Find each sum.

1. $56 + (-56)$

2. $-240 + 370$

3. $-5.7 + (-4.2)$

Lesson 4: Money and Debts

Cool Down: Buying a Bike

1. Clare has $150 in her bank account. She buys a bike for $200. What is Clare's account balance now?

2. If Clare earns $75 the next week from delivering newspapers and deposits it in her account, what will her account balance be then?

Lesson 5: Representing Subtraction

Cool Down: Same Value

1. Which other expression has the same value as $(-14) - 8$? Explain your reasoning.

   a. $(-14) + 8$

   b. $14 - (-8)$

   c. $14 + (-8)$

   d. $(-14) + (-8)$

2. Which other expression has the same value as $(-14) - (-8)$? Explain your reasoning.

   a. $(-14) + 8$

   b. $14 - (-8)$

   c. $14 + (-8)$

   d. $(-14) + (-8)$

Lesson 6: Subtracting Rational Numbers

Cool Down: A Subtraction Expression

Select all of the choices that are equal to $(-5) - (-12)$.

1. -7

2. 7

3. The difference between -5 and -12.

4. The difference between -12 and -5.

5. $(-5) + 12$

6. $(-5) + (-12)$

Lesson 7: Adding and Subtracting to Solve Problems

Cool Down: Coffee Shop Cups

Here is some record keeping from a coffee shop about their paper cups. Cups are delivered 2,000 at a time.

day	change
Monday	+2000
Tuesday	-125
Wednesday	-127
Thursday	+1719
Friday	-356
Saturday	-782
Sunday	0

1. Explain what a positive and negative number means in this situation

2. Assume the starting amount of coffee cups is 0. How many paper cups are left at the end of the week?

3. How many cups do you think were used on Thursday? Explain how you know.

Lesson 8: Position, Speed, and Direction

Cool Down: Multiplication Expressions

Four runners start at the same point; Lin, Elena, Diego, Andre. For each runner write a multiplication equation that describes their journey.

1. Lin runs for 25 seconds at 8.2 meters per second. What is her finish point?

2. Elena runs for 28 seconds and finishes at 250 meters. What is her velocity?

3. Diego runs for 32 seconds at -8.1 meters per second. What is his finish point?

4. Andre runs for 35 seconds and finishes at -285 meters. What is his velocity?

Lesson 9: Multiplying Rational Numbers

Cool Down: True Statements

Decide if each equation is true or false.

1. $7 \cdot 8 = 56$

2. $-7 \cdot 8 = 56$

3. $-7 \cdot -8 = -56$

4. $-7 \cdot -8 = 56$

5. $3.5 \cdot 12 = 42$

6. $-3.5 \cdot -12 = -42$

7. $-3.5 \cdot -12 = 42$

8. $-12 \cdot \frac{7}{2} = 42$

Lesson 10: Multiply!

Cool Down: Making Mistakes

Noah was doing some homework and answered the following questions. Do you agree with his answers? If you disagree, explain your reasoning.

1. $2.7 \cdot -2.5 = -6.75$

2. $-\frac{3}{4} \cdot -\frac{5}{7} = -\frac{15}{28}$

3. $5.5 \cdot -\frac{3}{5} = 3.3$

Lesson 11: Dividing Rational Numbers

Cool Down: Matching Division Expressions

Match each expression with its value.

1. $15 \div 12$

2. $12 \div (-15)$

3. $12 \div 15$

4. $15 \div (-12)$

- -0.8
- 0.8
- -1.25
- 1.25

Lesson 12: Negative Rates

Cool Down: Submarines

1. A submarine is descending to examine the seafloor 2,100 feet below the surface. It takes the submarine 2 hours to make this descent. Write an equation to represent the relationship between the submarine's elevation and time.

2. Another submarine's descent can be represented as $y = -240x$, where y is the elevation and x is time in hours. How long will it take this submarine to make the descent?

Lesson 13: Expressions with Rational Numbers

Cool Down: Make Them True

In each equation, select an operation to make the equation true.

1. $24 \underline{\quad} \frac{3}{4} = 18$

2. $24 \underline{\quad} \text{-}\frac{3}{4} = \text{-}32$

3. $12 \underline{\quad} 15 = \text{-}3$

4. $12 \underline{\quad} \text{-}15 = 27$

5. $\text{-}18 \underline{\quad} \text{-}\frac{3}{4} = 24$

Lesson 14: Solving Problems with Rational Numbers

Cool Down: Charges and Checks

Lin's sister has a checking account. If the account balance ever falls below zero, the bank charges her a fee of $5.95 per day. Today, the balance in Lin's sister's account is -$2.67.

1. If she does not make any deposits or withdrawals, what will be the balance in her account after 2 days?

2. In 14 days, Lin's sister will be paid $430 and will deposit it into her checking account. If there are no other transactions besides this deposit and the daily fee, will Lin continue to be charged $5.95 each day after this deposit is made? Explain or show your reasoning.

Lesson 15: Solving Equations with Rational Numbers

Cool Down: Hiking Trip

The Hiking Club is taking another trip. The hike leader's watch shows that they gained 296 feet in altitude from their starting position.

Their altitude is now 285 feet, but there is no record of their starting altitude.

Write and solve an equation to represent this situation and find their starting altitude.

Lesson 16: Representing Contexts with Equations

Cool Down: Floating Above a Sunken Canoe

A balloon is floating above a lake and a sunken canoe is below the surface of the lake. The balloon's vertical position is 12 meters and the canoe's is -4.8 meters. The equation $12 + d = \text{-}4.8$ represents this situation.

    1. What does the variable d represent?

    2. What value of d makes the equation true? Explain your reasoning.

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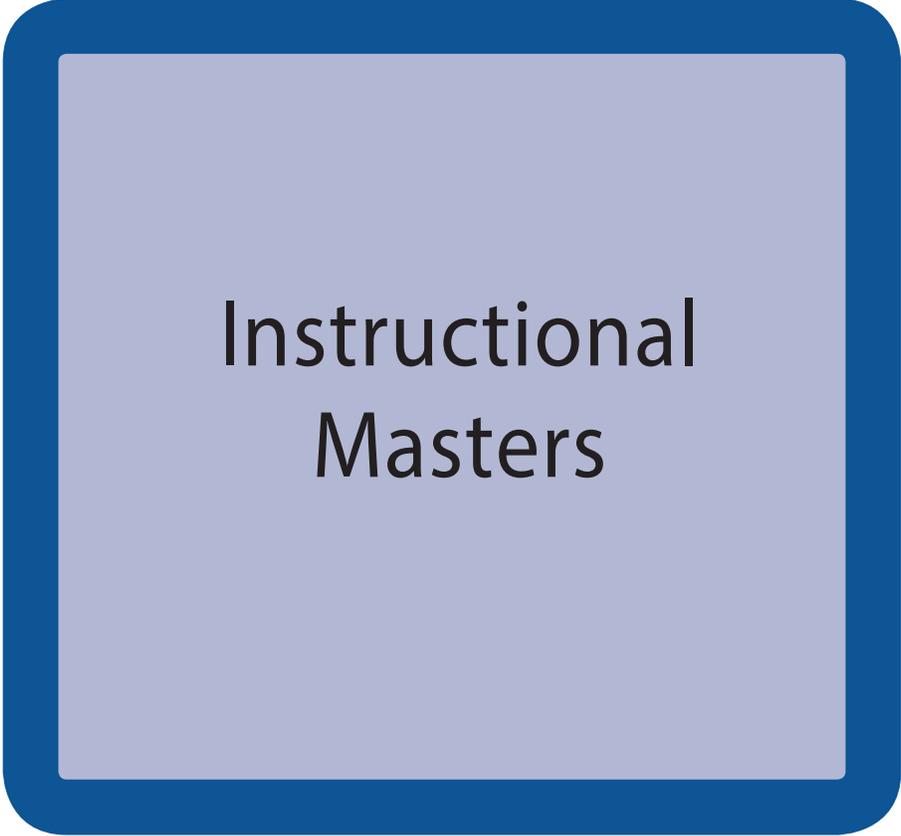
Instructional
Masters

Instructional Masters for Rational Number Arithmetic

address	title	students per copy	written on?	requires cutting?	card stock recommended?	color paper recommended?
Activity Grade7.5.17.4	Your Own Stock Portfolio	1	no	no	no	no
Activity Grade7.5.15.4	Card Sort: Matching Inverses	1	no	yes	no	yes
Activity Grade7.5.9.4	Rational Numbers Multiplication Grid	1	yes	no	no	no
Activity Grade7.5.10.2	Card Sort: Matching Equations	1	no	yes	no	yes
Activity Grade7.5.1.4	Card Sort: Rational Numbers	1	no	yes	no	yes
Activity Grade7.5.13.2	Card Sort: The Same But Different	1	no	yes	no	yes

Set 1 (Integers)

-23	-10	-9	-7
-6	-4	-3	-2
-1	0	1	2
3	5	8	10
11	15	22	23

Set 2 (Rational numbers that are not integers)

$\dfrac{1}{4}$	$\dfrac{9}{8}$	2.5	$\dfrac{8}{3}$
$-\dfrac{1}{4}$	$-\dfrac{9}{8}$	-2.5	$-\dfrac{8}{3}$
$5\dfrac{5}{6}$	7.7	$\dfrac{31}{3}$	$22\dfrac{3}{8}$
$-5\dfrac{5}{6}$	-7.7	$-\dfrac{31}{3}$	$-22\dfrac{3}{8}$

7.5.9.4 Rational Numbers Multiplication Grid.

×	5	4	3	2	1.5	1	0	-1	-1.5	-2	-3	-4	-5
-5													
-4													
-3.2													
-3													
-2													
-1				-2									
0	0	0	0	0	0	0	0	0					
1	5	4	3	2	1.5	1	0	0					
2	10	8	6	4	3	2	0	0					
3	15	12	9	6	4.5	3	0	0					
3.2	16	12.8	9.6	6.4	4.8	3.2	0	0					
4	20	16	12	8	6	4	0	0					
5													

(fold back) 1st ↓

2nd (fold in to here) ↓

(fold in to here) 4th ↓

3rd (fold back) ↓

(fold back) ← 5th

← 6th

(fold in to here)

(fold in to here)

← 7th (fold back)

← 8th

Card Sort: Matching Expressions	Card Sort: Matching Expressions	Card Sort: Matching Expressions
$-1 \cdot 12$	$-64 \cdot \frac{1}{8}$	$1 \cdot 15$
Card Sort: Matching Expressions	**Card Sort: Matching Expressions**	**Card Sort: Matching Expressions**
$-1 \cdot (-3) \cdot (-5)$	$-1 \cdot (-2) \cdot 6$	$-1 \cdot (-12)$
Card Sort: Matching Expressions	**Card Sort: Matching Expressions**	**Card Sort: Matching Expressions**
$1 \cdot (-3) \cdot (-5)$	$-\frac{1}{4} \cdot -32$	$-2 \cdot 6$
Card Sort: Matching Expressions	**Card Sort: Matching Expressions**	**Card Sort: Matching Expressions**
$-\frac{1}{2} \cdot (-16)$	$-3 \cdot 5$	$2 \cdot (-4)$
Card Sort: Matching Expressions	**Card Sort: Matching Expressions**	**Card Sort: Matching Expressions**
$-\frac{1}{2} \cdot 16$	$-1 \cdot (-3) \cdot (-4)$	$2 \cdot 4$
Card Sort: Matching Expressions	**Card Sort: Matching Expressions**	**Card Sort: Matching Expressions**
$-1 \cdot (-3) \cdot 4$	$(-3) \cdot (-5)$	$1 \cdot -15$

Card Sort: The Same But Different

$$1 + 2$$

Card Sort: The Same But Different

$$1 - 2$$

Card Sort: The Same But Different

$$-10 + 7$$

Card Sort: The Same But Different

$$-10 + (-7)$$

Card Sort: The Same But Different

$$-15 \div -6$$

Card Sort: The Same But Different

$$1 - (-2)$$

Card Sort: The Same But Different

$$1 + -2$$

Card Sort: The Same But Different

$$-10 - -7$$

Card Sort: The Same But Different

$$-10 - 7$$

Card Sort: The Same But Different

$$15 \cdot \frac{1}{6}$$

Card Sort: The Same But Different

$$(1)(4)$$

Card Sort: The Same But Different

$$-1 \cdot 4$$

Card Sort: The Same But Different

$$8 \div 4$$

Card Sort: The Same But Different

$$8 \div (-4)$$

Card Sort: The Same But Different

$$15 \div (-6)$$

Card Sort: The Same But Different

$$1 \div \frac{1}{4}$$

Card Sort: The Same But Different

$$1 \div \left(-\frac{1}{4}\right)$$

Card Sort: The Same But Different

$$(8)\left(\frac{1}{4}\right)$$

Card Sort: The Same But Different

$$(8) \cdot \left(-\frac{1}{4}\right)$$

Card Sort: The Same But Different

$$-15 \cdot \frac{1}{6}$$

Card Sort: Matching Inverses	Card Sort: Matching Inverses	Card Sort: Matching Inverses	Card Sort: Matching Inverses
$\dfrac{2}{1}$	-2	3	-3
Card Sort: Matching Inverses	Card Sort: Matching Inverses	Card Sort: Matching Inverses	Card Sort: Matching Inverses
$\dfrac{5}{10}$	-0.5	$\dfrac{1}{3}$	$-\dfrac{1}{3}$
Card Sort: Matching Inverses	Card Sort: Matching Inverses	Card Sort: Matching Inverses	Card Sort: Matching Inverses
4	-4	$\dfrac{10}{2}$	-5
Card Sort: Matching Inverses	Card Sort: Matching Inverses	Card Sort: Matching Inverses	Card Sort: Matching Inverses
0.25	$-\dfrac{1}{4}$	0.2	-0.2
Card Sort: Matching Inverses	Card Sort: Matching Inverses	Card Sort: Matching Inverses	Card Sort: Matching Inverses
10	$-\dfrac{10}{1}$	25	-25
Card Sort: Matching Inverses	Card Sort: Matching Inverses	Card Sort: Matching Inverses	Card Sort: Matching Inverses
0.1	$-\dfrac{1}{10}$	$\dfrac{4}{100}$	$-\dfrac{4}{100}$

Stock Prices

company name	price ($)
Marine Exploration Company	2.02
Drilling Products Company	2.18
Solar Power Company	3.07
Office Supply Company	3.31
Headphone Company	6.14
Home Appliance Company	6.74
Renewable Energy Company	8.83
Sporting Goods Company	9.27
Health Food Company	9.53
Fast Food Company	9.62
Shoe Company 1	11.28
Bookstore Company	11.35
Sports Technology Company	12.22
Teddy Bear Company	13.42
Microtechnology Company	13.76
Clothing Company 1	15.93
Social Media Company	16.91
Clothing Company 2	17.81
Clothing Company 3	21.22
Online Retail Company	23.41

company name	price ($)
Video Game Company	26.58
Clothing Company 4	27.50
Movie Theater Company	27.61
Shoe Company 2	29.72
Biotechnology Company	29.79
Genetics Company	30.60
Automotive Company	31.46
Grocery Store Company	36.79
Clothing Company 5	38.05
Sports Clothing Company	40.13
Household Goods Company	43.22
Restaurant Company	48.14
Computer Company	51.17
Amusement Park Company	57.95
Jewelry Company	60.64
Pizza Company	68.00
Cereal Company	71.32
Megastore Company	73.02
Clothing Company 6	89.62
Farm Supply Company	91.18

Changes in Stock Prices after 3 Months

company name	change ($)	company name	change ($)
Marine Exploration Company	+1.57	Video Game Company	+1.01
Drilling Products Company	-1.23	Clothing Company 4	+7.02
Solar Power Company	+0.09	Movie Theater Company	+3.48
Office Supply Company	+0.26	Shoe Company 2	-6.82
Headphone Company	+0.20	Biotechnology Company	+37.26
Home Appliance Company	-1.81	Genetics Company	-10.02
Renewable Energy Company	-0.36	Automotive Company	+16.72
Sporting Goods Company	+4.34	Grocery Store Company	-7.11
Health Food Company	+6.47	Clothing Company 5	+13.83
Fast Food Company	+1.18	Sports Clothing Company	-1.45
Shoe Company 1	-2.99	Household Goods Company	-0.11
Bookstore Company	-0.05	Restaurant Company	+1.92
Sports Technology Company	+2.62	Computer Company	+6.43
Teddy Bear Company	-3.06	Amusement Park Company	-4.34
Microtechnology Company	+4.02	Jewelry Company	+11.99
Clothing Company 1	+1.93	Pizza Company	+10.85
Social Media Company	+6.14	Cereal Company	-7.44
Clothing Company 2	-1.92	Megastore Company	-0.90
Clothing Company 3	+1.02	Clothing Company 6	+11.52
Online Retail Company	+9.49	Farm Supply Company	-23.83

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