Core Knowledge® MATHEMATICS

Equation

Rational Number Arithmetic

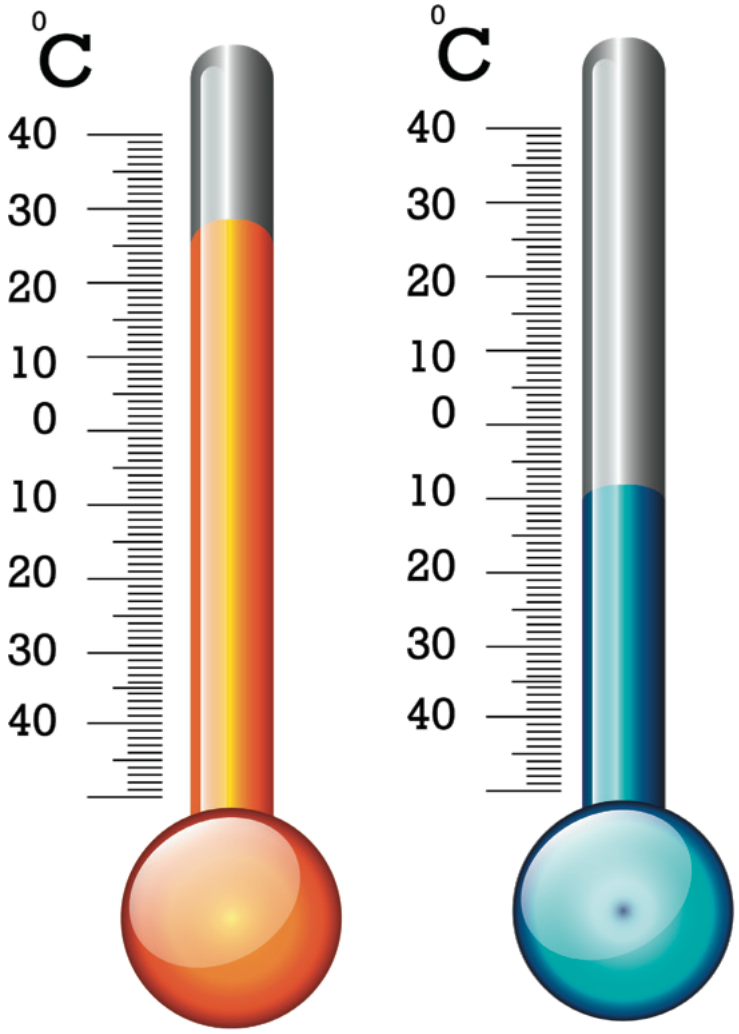
$x+8=-5$

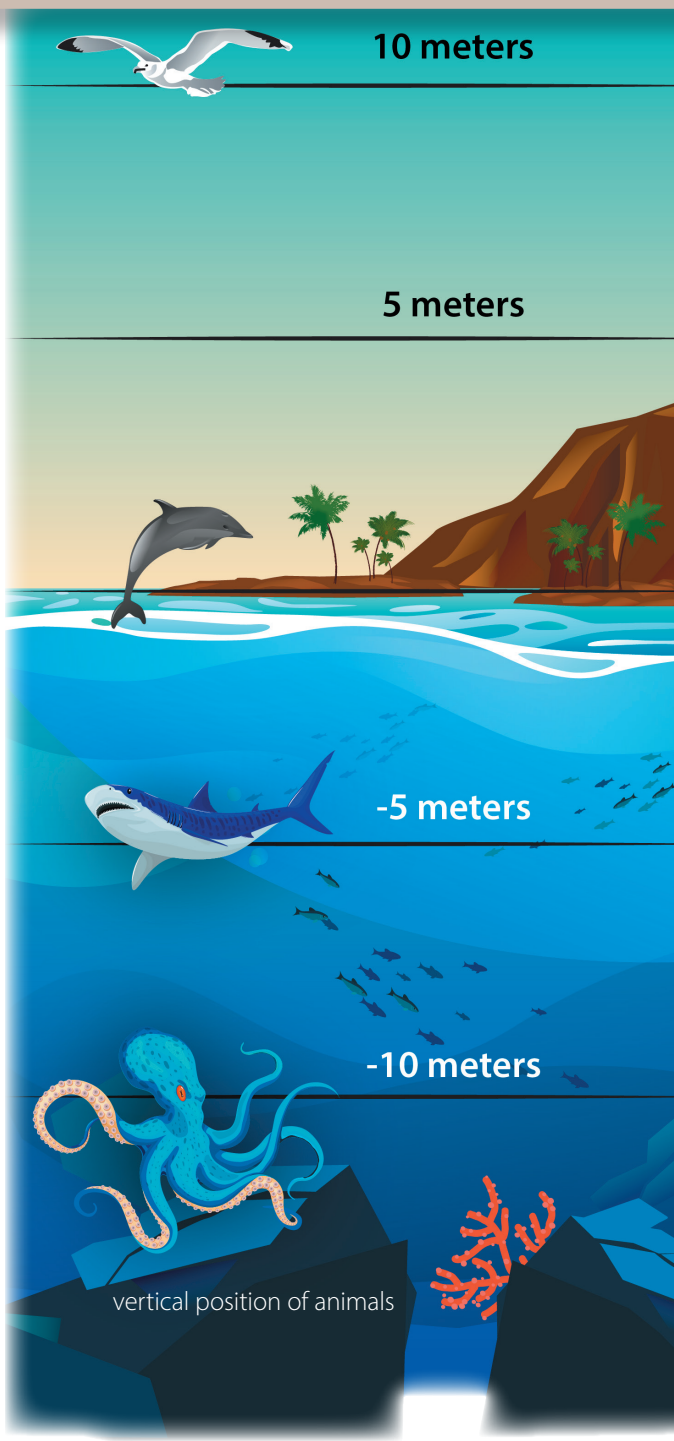
$(x+8)+-8=(-5)+-8$

$x=-13$

Student Workbook

Fractions of a Degree





vertical position of animals

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Rational Number Arithmetic

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$$x+8=-5$$
$$(x+8)+-8=(-5)+-8$$
$$x=-13$$

Rational Number Arithmetic

Student Workbook

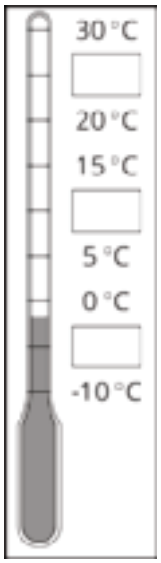
Core Knowledge Mathematics™

Lesson 1: Interpreting Negative Numbers

Let's review what we know about signed numbers.

1.1: Using the Thermometer

Here is a weather thermometer. Three of the numbers have been left off.

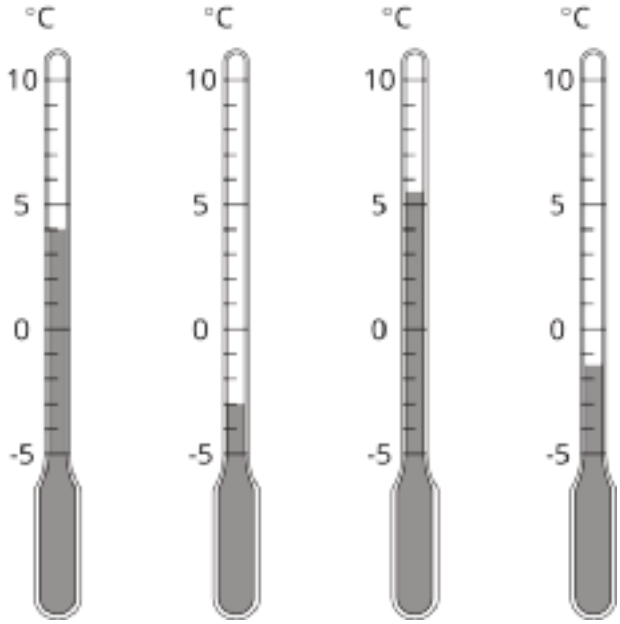


1. What numbers go in the boxes?

2. What temperature does the thermometer show?

1.2: Fractions of a Degree

1. What temperature is shown on each thermometer?



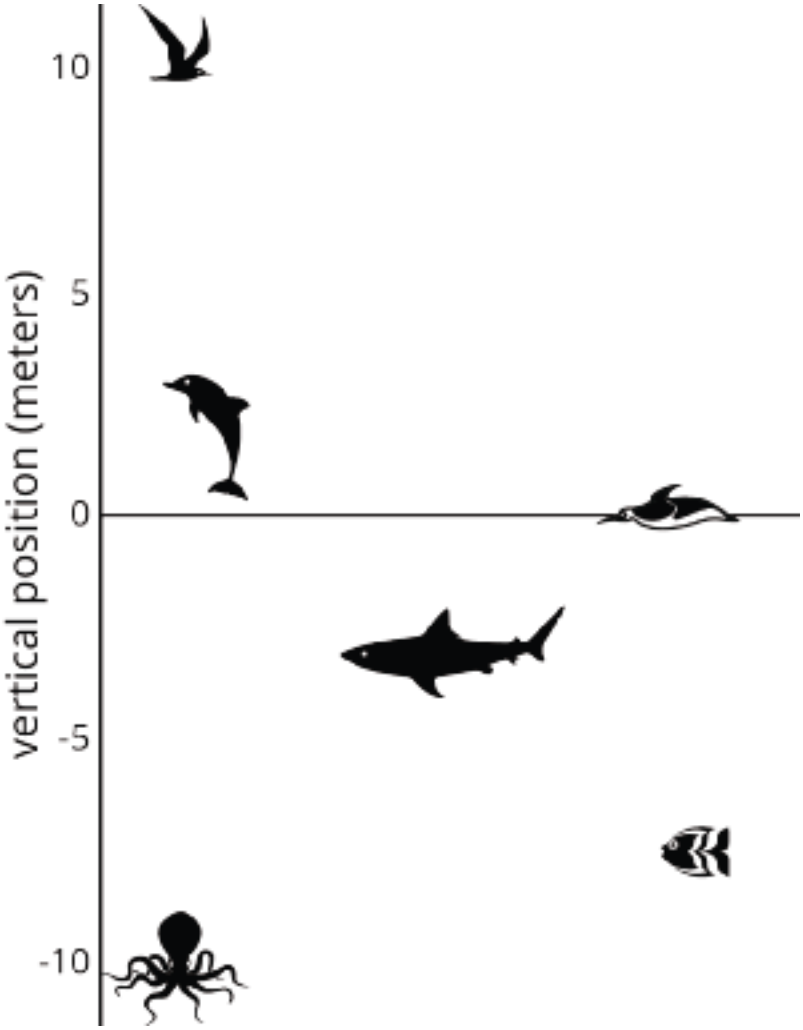
2. Which thermometer shows the highest temperature?

3. Which thermometer shows the lowest temperature?

4. Suppose the temperature outside is $-4^\circ C$. Is that colder or warmer than the coldest temperature shown? How do you know?

1.3: Seagulls Soar, Sharks Swim

Here is a picture of some sea animals. The number line on the left shows the vertical position of each animal above or below sea level, in meters.



1. How far above or below sea level is each animal? Measure to their eye level.

2. A mobula ray is 3 meters above the surface of the ocean. How does its vertical position compare to the height or depth of:

   The jumping dolphin? The flying seagull? The octopus?

3. An albatross is 5 meters above the surface of the ocean. How does its vertical position compare to the height or depth of:

   The jumping dolphin? The flying seagull? The octopus?

4. A clownfish is 2 meters below the surface of the ocean. How does its vertical position compare to the height or depth of:

   The jumping dolphin? The flying seagull? The octopus?

5. The vertical distance of a new dolphin from the dolphin in the picture is 3 meters. What is its distance from the surface of the ocean?

Are you ready for more?

The north pole is in the middle of the ocean. A person at sea level at the north pole would be 3,949 miles from the center of Earth. The sea floor below the north pole is at an elevation of approximately -2.7 miles. The elevation of the south pole is about 1.7 miles. How far is a person standing on the south pole from a submarine at the sea floor below the north pole?

1.4: Card Sort: Rational Numbers

1. Your teacher will give your group a set of cards. Order the cards from least to greatest.

2. Pause here so your teacher can review your work. Then, your teacher will give you a second set of cards.

3. Add the new set of cards to the first set so that all of the cards are ordered from least to greatest.

Lesson 1 Summary

We can use **positive numbers** and **negative numbers** to represent temperature and elevation.

When numbers represent temperatures, positive numbers indicate temperatures that are warmer than zero and negative numbers indicate temperatures that are colder than zero. This thermometer shows a temperature of -1 degree Celsius, which we write -1°C.

When numbers represent elevations, positive numbers indicate positions above sea level and negative numbers indicate positions below sea level.

We can see the order of signed numbers on a number line.

A number is always less than numbers to its right. So $-7 < -3$.

We use **absolute value** to describe how far a number is from 0. The numbers 15 and -15 are both 15 units from 0, so $|15| = 15$ and $|-15| = 15$. We call 15 and -15 *opposites*. They are on opposite sides of 0 on the number line, but the same distance from 0.

Unit 5 Lesson 1 Cumulative Practice Problems

1. It was $-5^\circ C$ in Copenhagen and $-12^\circ C$ in Oslo. Which city was colder?

2. a. A fish is 12 meters below the surface of the ocean. What is its elevation?

   b. A sea bird is 28 meters above the surface of the ocean. What is its elevation?

   c. If the bird is directly above the fish, how far apart are they?

3. Compare using >, =, or <.

   a. 3 _____ -3

   b. 12 _____ 24

   c. -12 _____ -24

   d. 5 _____ -(-5)

   e. 7.2 _____ 7

   f. -7.2 _____ -7

   g. -1.5 _____ $\frac{-3}{2}$

   h. $\frac{-4}{5}$ _____ $\frac{-5}{4}$

   i. $\frac{-3}{5}$ _____ $\frac{-6}{10}$

   j. $\frac{-2}{3}$ _____ $\frac{1}{3}$

4. Han wants to buy a $30 ticket to a game, but the pre-order tickets are sold out. He knows there will be more tickets sold the day of the game, with a markup of 200%. How much should Han expect to pay for the ticket if he buys it the day of the game?

(From Unit 4, Lesson 7.)

5. A type of green paint is made by mixing 2 cups of yellow with 3.5 cups of blue.

   a. Find a mixture that will make the same shade of green but a smaller amount.

   b. Find a mixture that will make the same shade of green but a larger amount.

   c. Find a mixture that will make a different shade of green that is bluer.

   d. Find a mixture that will make a different shade of green that is more yellow.

(From Unit 2, Lesson 1.)

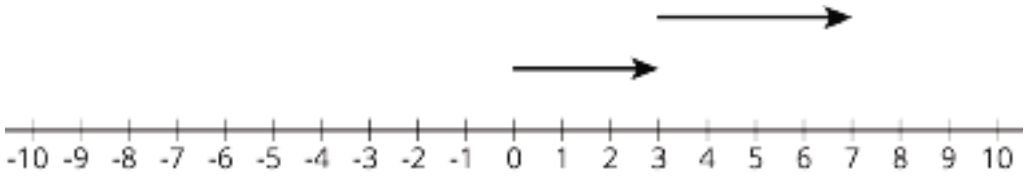
Lesson 2: Changing Temperatures

Let's add signed numbers.

2.1: Which One Doesn’t Belong: Arrows

Which pair of arrows doesn't belong?

1.



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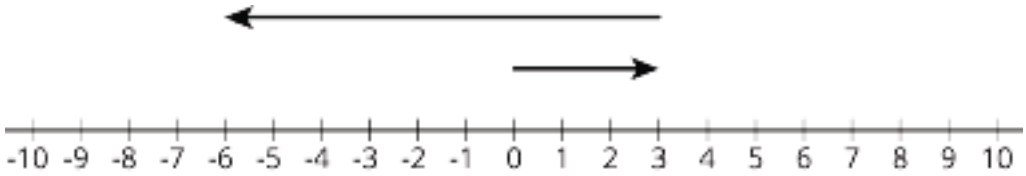

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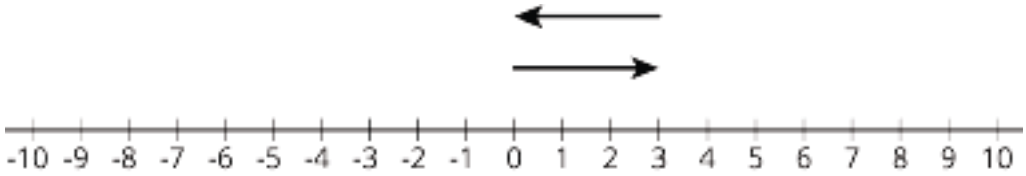

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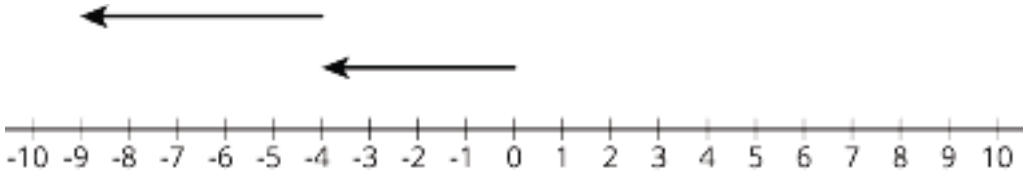
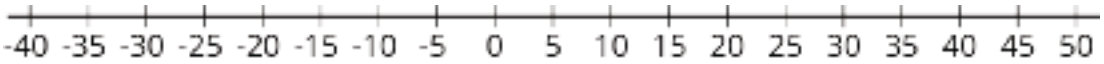
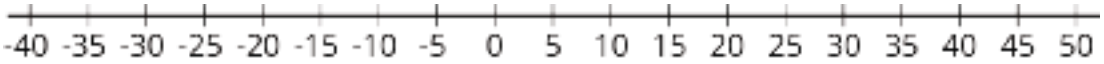


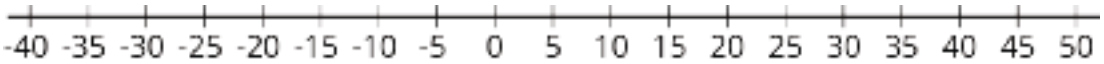
2.2: Warmer and Colder

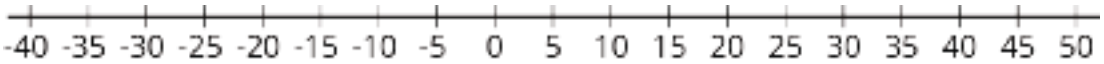
1. Complete the table and draw a number line diagram for each situation.

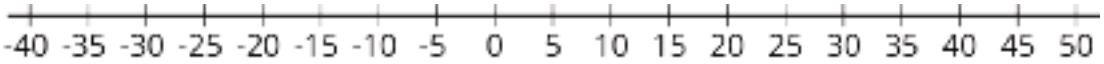
	start (°C)	change (°C)	final (°C)	addition equation
a	+40	10 degrees warmer	+50	$40 + 10 = 50$
b	+40	5 degrees colder		
c	+40	30 degrees colder		
d	+40	40 degrees colder		
e	+40	50 degrees colder		

a. 

b. 

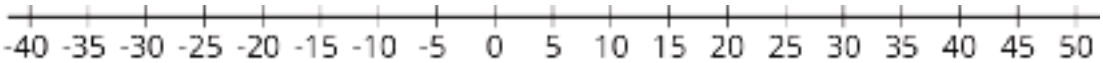
c. 

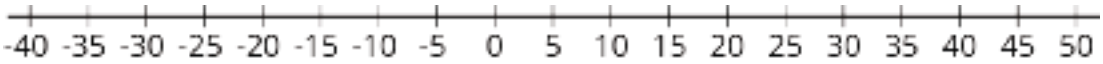
d. 

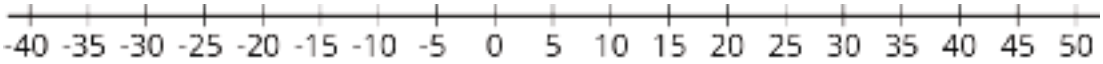
e. 

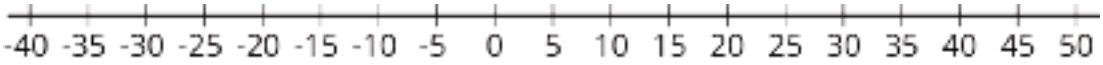
2. Complete the table and draw a number line diagram for each situation.

	start (°C)	change (°C)	final (°C)	addition equation
a	-20	30 degrees warmer		
b	-20	35 degrees warmer		
c	-20	15 degrees warmer		
d	-20	15 degrees colder		

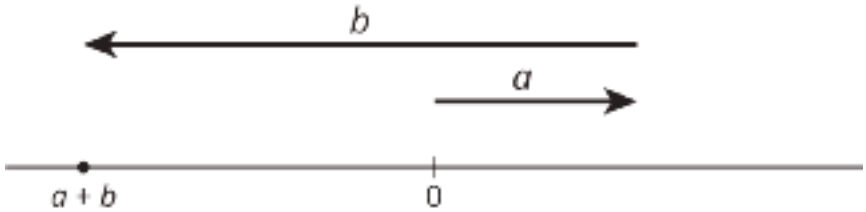
a. 

b. 

c. 

d. 

Are you ready for more?



For the numbers a and b represented in the figure, which expression is equal to $|a+b|$?

$|a|+|b|$ $\qquad$ $|a|-|b|$ $\qquad$ $|b|-|a|$

2.3: Winter Temperatures

One winter day, the temperature in Houston is 8° Celsius. Find the temperatures in these other cities. Explain or show your reasoning.

1. In Orlando, it is 10° warmer than it is in Houston.

2. In Salt Lake City, it is 8° colder than it is in Houston.

3. In Minneapolis, it is 20° colder than it is in Houston.

4. In Fairbanks, it is 10° colder than it is in *Minneapolis*.

5. Write an addition equation that represents the relationship between the temperature in Houston and the temperature in Fairbanks.

Lesson 2 Summary

If it is 42° outside and the temperature increases by 7°, then we can add the initial temperature and the change in temperature to find the final temperature.

$$42 + 7 = 49$$

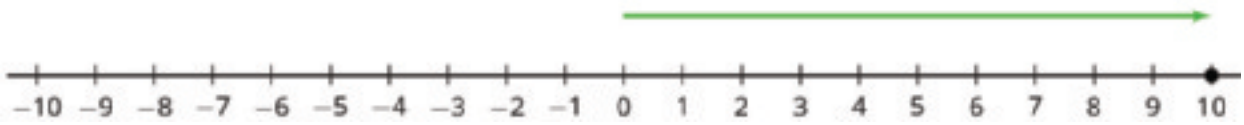
If the temperature decreases by 7°, we can either subtract $42 - 7$ to find the final temperature, or we can think of the change as -7°. Again, we can add to find the final temperature.

$$42 + (-7) = 35$$

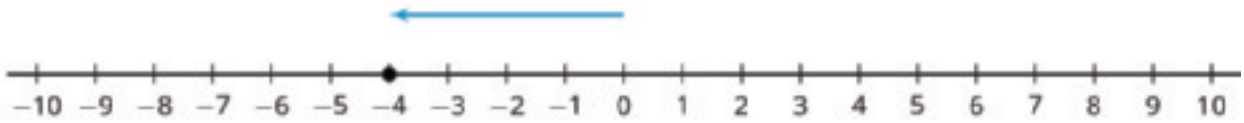
In general, we can represent a change in temperature with a positive number if it increases and a negative number if it decreases. Then we can find the final temperature by adding the initial temperature and the change. If it is 3° and the temperature decreases by 7°, then we can add to find the final temperature.

$$3 + (-7) = -4$$

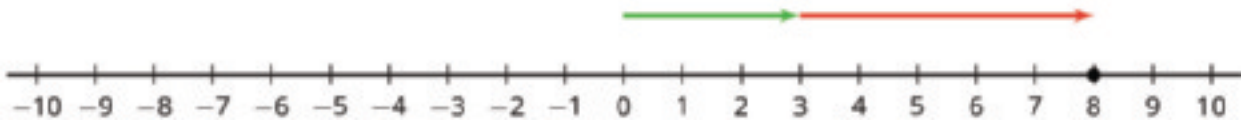
We can represent signed numbers with arrows on a number line. We can represent positive numbers with arrows that start at 0 and point to the right. For example, this arrow represents +10 because it is 10 units long and it points to the right.



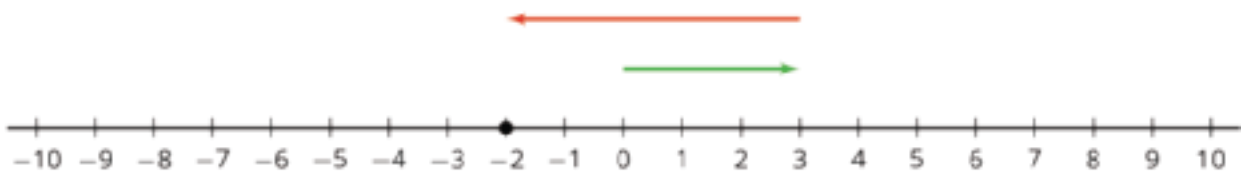
We can represent negative numbers with arrows that start at 0 and point to the left. For example, this arrow represents -4 because it is 4 units long and it points to the left.



To represent addition, we put the arrows "tip to tail." So this diagram represents $3 + 5$:



And this represents $3 + (-5)$:



Unit 5 Lesson 2 Cumulative Practice Problems

1. a. The temperature is -2°C. If the temperature rises by 15°C, what is the new temperature?

   b. At midnight the temperature is -6°C. At midday the temperature is 9°C. By how much did the temperature rise?

2. Draw a diagram to represent each of these situations. Then write an addition expression that represents the final temperature.

   a. The temperature was 80°F and then fell 20°F.

   b. The temperature was -13°F and then rose 9°F.

   c. The temperature was -5°F and then fell 8°F.

3. Complete each statement with a number that makes the statement true.

   a. _____ < 7°C

   b. _____ < -3°C

   c. -0.8°C < _____ < -0.1°C

   d. _____ > -2°C

(From Unit 5, Lesson 1.)

4. Decide whether each table could represent a proportional relationship. If the relationship could be proportional, what would be the constant of proportionality?

   a. The number of wheels on a group of buses.

number of buses	number of wheels	wheels per bus
5	30	
8	48	
10	60	
15	90	

   b. The number of wheels on a train.

number of train cars	number of wheels	wheels per train car
20	184	
30	264	
40	344	
50	424	

(From Unit 2, Lesson 7.)

5. Noah was assigned to make 64 cookies for the bake sale. He made 125% of that number. 90% of the cookies he made were sold. How many of Noah's cookies were left after the bake sale?

(From Unit 4, Lesson 7.)

Lesson 3: Changing Elevation

Let's solve problems about adding signed numbers.

3.1: That's the Opposite

1. Draw arrows on a number line to represents these situations:

   a. The temperature was -5 degrees. Then the temperature rose 5 degrees.

   b. A climber was 30 feet above sea level. Then she descended 30 feet.

2. What’s the opposite?

   a. Running 150 feet east.

   b. Jumping down 10 steps.

   c. Pouring 8 gallons into a fish tank.

3.2: Cliffs and Caves

1. A mountaineer is climbing on a cliff. She is 400 feet above the ground. If she climbs up, this will be a positive change. If she climbs down, this will be a negative change.

   a. Complete the table.

	starting elevation (feet)	change (feet)	final elevation (feet)
A	+400	300 up	
B	+400	150 down	
C	+400	400 down	
D	+400		+50

   b. Write an addition equation and draw a number line diagram for B. Include the starting elevation, change, and final elevation in your diagram.

2. A spelunker is down in a cave next to the cliff. If she climbs down deeper into the cave, this will be a negative change. If she climbs up, whether inside the cave or out of the cave and up the cliff, this will be a positive change.

   a. Complete the table.

	starting elevation (feet)	change (feet)	final elevation (feet)
A	-200	150 down	
B	-200	100 up	
C	-200	200 up	
D	-200	250 up	
E	-200		-500

   b. Write an addition equation and draw a number line diagram for C and D. Include the starting elevation, change, and final elevation in your diagram.

   c. What does the expression $-75 + 100$ tell us about the spelunker? What does the value of the expression tell us?

3.3: Adding Rational Numbers

Find the sums.

1. $-35 + (30 + 5)$

2. $-0.15 + (-0.85) + 12.5$

3. $\frac{1}{2} + (-\frac{3}{4})$

Are you ready for more?

Find the sum without a calculator.

$$10 + 21 + 32 + 43 + 54 + (-54) + (-43) + (-32) + (-21) + (-10)$$

3.4: School Supply Number Line

Your teacher will give you a long strip of paper.

Follow these instructions to create a number line.

1. Fold the paper in half along its length and along its width.
2. Unfold the paper and draw a line along each crease.
3. Label the line in the middle of the paper 0. Label the right end of the paper $+$ and the left end of the paper $-$.
4. Select two objects of different lengths, for example a pen and a gluestick. The length of the longer object is a and the length of the shorter object is b.
5. Use the objects to measure and label each of the following points on your number line.

a	$2b$	$-b$
b	$a + b$	$a + -b$
$2a$	$-a$	$b + -a$

6. Complete each statement using <, >, or =. Use your number line to explain your reasoning.

   a. a _____ b

   b. $-a$ _____ $-b$

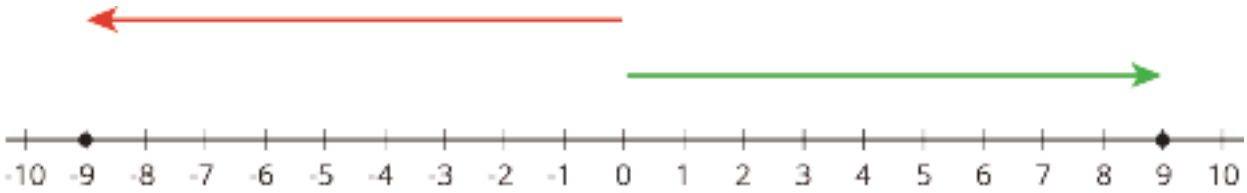
   c. $a + -a$ _____ $b + -b$

   d. $a + -b$ _____ $b + -a$

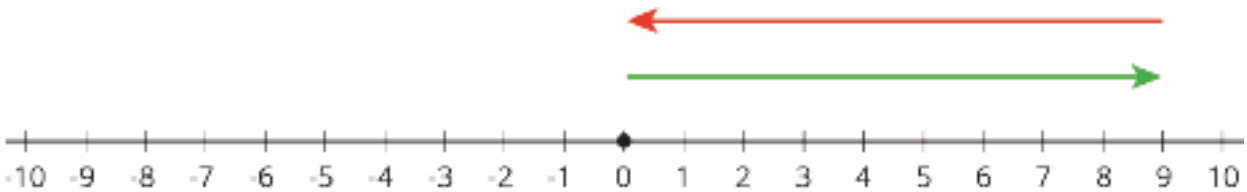
   e. $a + -b$ _____ $-a + b$

Lesson 3 Summary

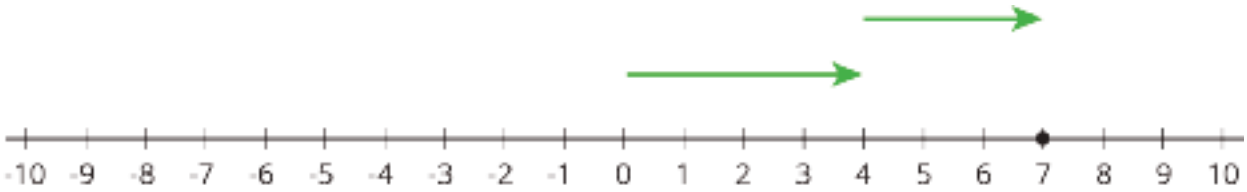
The opposite of a number is the same distance from 0 but on the other side of 0.

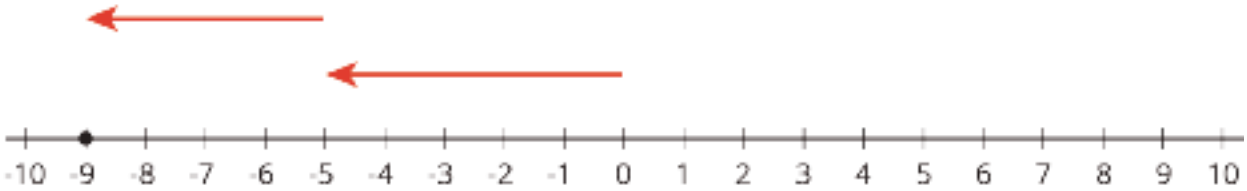


The opposite of -9 is 9. When we add opposites, we always get 0. This diagram shows that $9 + -9 = 0$.



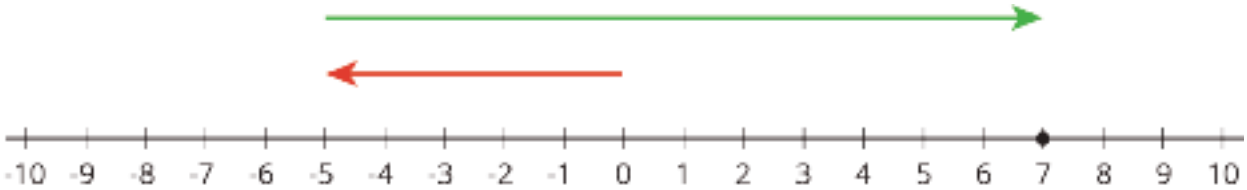
When we add two numbers with the same sign, the arrows that represent them point in the same direction. When we put the arrows tip to tail, we see the sum has the same sign.





To find the sum, we add the magnitudes and give it the correct sign. For example, $(-5) + (-4) = -(5 + 4)$.

On the other hand, when we add two numbers with different signs, we subtract their magnitudes (because the arrows point in the opposite direction) and give it the sign of the number with the larger magnitude. For example, $(-5) + 12 = +(12 - 5)$.



Unit 5 Lesson 3 Cumulative Practice Problems

1. What is the final elevation if

   a. A bird starts at 20 m and changes 16 m?

   b. A butterfly starts at 20 m and changes -16 m?

   c. A diver starts at 5 m and changes -16 m?

   d. A whale starts at -9 m and changes 11 m?

   e. A fish starts at -9 meters and changes -11 meters?

2. One of the particles in an atom is called an electron. It has a charge of -1. Another particle in an atom is a proton. It has charge of +1. The charge of an atom is the sum of the charges of the electrons and the protons. A carbon atom has an overall charge of 0, because it has 6 electrons and 6 protons and $-6 + 6 = 0$. Find the overall charge for the rest of the elements on the list.

	charge from electrons	charge from protons	overall charge
carbon	-6	+6	0
neon	-10	+10	
oxide	-10	+8	
copper	-27	+29	
tin	-50	+50	

3. Add.

$14.7 + 28.9$ $\quad$ $-9.2 + 4.4$ $\quad$ $-81.4 + (-12)$ $\quad$ $51.8 + (-0.8)$

4. Last week, the price, in dollars, of a gallon of gasoline was g. This week, the price of gasoline per gallon increased by 5%. Which expressions represent this week's price, in dollars, of a gallon of gasoline? Select **all** that apply.

   A. $g + 0.05$

   B. $g + 0.05g$

   C. $1.05g$

   D. $0.05g$

   E. $(1 + 0.05)g$

   (From Unit 4, Lesson 8.)

5. Decide whether each table could represent a proportional relationship. If the relationship could be proportional, what would be the constant of proportionality?

   a. Annie's Attic is giving away $5 off coupons.

original price	sale price
$15	$10
$25	$20
$35	$30

   b. Bettie's Boutique is having a 20% off sale.

original price	sale price
$15	$12
$25	$20
$35	$28

   (From Unit 2, Lesson 7.)

Lesson 4: Money and Debts

Let's apply what we know about signed numbers to money.

4.1: Concert Tickets

Priya wants to buy three tickets for a concert. She has earned $135 and each ticket costs $50. She borrows the rest of the money she needs from a bank and buys the tickets.

1. How can you represent the amount of money that Priya has after buying the tickets?

2. How much more money will Priya need to earn to pay back the money she borrowed from the bank?

3. How much money will she have after she pays back the money she borrowed from the bank?

4.2: Cafeteria Food Debt

At the beginning of the month Kiran had $24 in his school cafeteria account. Use a variable to represent the unknown quantity in each transaction below and write an equation to represent it. Then, represent each transaction on a number line. What is the unknown quantity in each case?

1. In the first week he spent $16 on lunches. How much was in his account then?

2. Then he deposited some more money and his account balance was $28. How much did he deposit?

3. Then he spent $34 on lunches the next week. How much was in his account then?

4. Then he deposited enough money to pay off his debt to the cafeteria. How much did he deposit?

5. Explain why it makes sense to use a negative number to represent Kiran's account balance when he owes money.

4.3: Bank Statement

Here is a bank statement.

Responsible Bank
210 2nd Street
Anytown, MH 06930

Checking Account Statement
Page: 1 of 1

Andre Person
1729 Euclid Ave
Anytown, MH 06930

Statement Period	Account No.
2017-10-01 to 2017-11-01	1120635978

Date	Description	Withdrawals	Deposits	Balance
2017-10-03	Previous Balance			39.87
2017-10-05	Check Number 256	28.50		11.37
2017-10-06	ATM Deposit - Cash		45.00	56.37
2017-10-10	Wire Transfer	37.91		18.46
2017-10-17	Point of Sale - Grocery Store	16.43		2.03
2017-10-25	Funds Transfer from Savings		50.00	52.03
2017-10-28	Check Number 257	42.00		10.03
2017-10-29	Online Payment - Phone Services	72.50		-62.47

1. If we put withdrawals and deposits in the same column, how can they be represented?

2. Andre withdraws $40 to buy a music player. What is his new balance?

3. If Andre deposits $100 in this account, will he still be in debt? How do you know?

Are you ready for more?

The *national debt* of a country is the total amount of money the government of that country owes. Imagine everyone in the United States was asked to help pay off the national debt. How much would each person have to pay?

Lesson 4 Summary

Banks use positive numbers to represent money that gets put into an account and negative numbers to represent money that gets taken out of an account. When you put money into an account, it is called a **deposit**. When you take money out of an account, it is called a **withdrawal**.

People also use negative numbers to represent debt. If you take out more money from your account than you put in, then you owe the bank money, and your account balance will be a negative number to represent that debt. For example, if you have $200 in your bank account, and then you write a check for $300, you will owe the bank $100 and your account balance will be -$100.

starting balance	deposits and withdrawals	new balance
0	50	$0 + 50$
50	150	$50 + 150$
200	-300	$200 + (-300)$
-100		

In general, you can find a new account balance by adding the value of the deposit or withdrawal to it. You can also tell quickly how much money is needed to repay a debt using the fact that to get to zero from a negative value you need to add its opposite.

Unit 5 Lesson 4 Cumulative Practice Problems

1. The table shows five transactions and the resulting account balance in a bank account, except some numbers are missing. Fill in the missing numbers.

	transaction amount	account balance
transaction 1	200	200
transaction 2	-147	53
transaction 3	90	
transaction 4	-229	
transaction 5		0

2. a. Clare has $54 in her bank account. A store credits her account with a $10 refund. How much does she now have in the bank?

   b. Mai's bank account is overdrawn by $60, which means her balance is -$60. She gets $85 for her birthday and deposits it into her account. How much does she now have in the bank?

   c. Tyler is overdrawn at the bank by $180. He gets $70 for his birthday and deposits it. What is his account balance now?

   d. Andre has $37 in his bank account and writes a check for $87. After the check has been cashed, what will the bank balance show?

3. Last week, it rained g inches. This week, the amount of rain decreased by 5%. Which expressions represent the amount of rain that fell this week? Select **all** that apply.

   A. $g - 0.05$

   B. $g - 0.05g$

   C. $0.95g$

   D. $0.05g$

   E. $(1 - 0.05)g$

   (From Unit 4, Lesson 8.)

4. Decide whether or not each equation represents a proportional relationship.

   a. Volume measured in cups (c) vs. the same volume measured in ounces (z): $c = \frac{1}{8}z$

   b. Area of a square (A) vs. the side length of the square (s): $A = s^2$

   c. Perimeter of an equilateral triangle (P) vs. the side length of the triangle (s): $3s = P$

   d. Length (L) vs. width (w) for a rectangle whose area is 60 square units: $L = \frac{60}{w}$

   (From Unit 2, Lesson 8.)

5. Add.

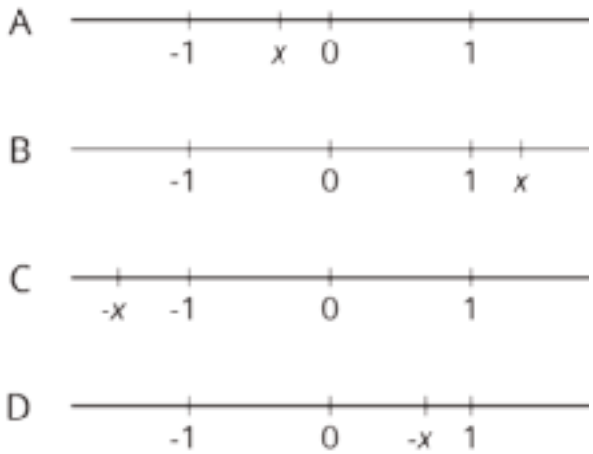
a. $5\frac{3}{4} + (-\frac{1}{4})$

b. $-\frac{2}{3} + \frac{1}{6}$

c. $-\frac{8}{5} + (-\frac{3}{4})$

(From Unit 5, Lesson 3.)

6. In each diagram, x represents a different value.



For each diagram,

a. What is something that is *definitely* true about the value of x?

b. What is something that *could be* true about the value of x?

(From Unit 5, Lesson 1.)

Lesson 5: Representing Subtraction

Let's subtract signed numbers.

5.1: Equivalent Equations

Consider the equation $2 + 3 = 5$. Here are some more equations, using the same numbers, that express the same relationship in a different way:

$$3 + 2 = 5 \qquad 5 - 3 = 2 \qquad 5 - 2 = 3$$

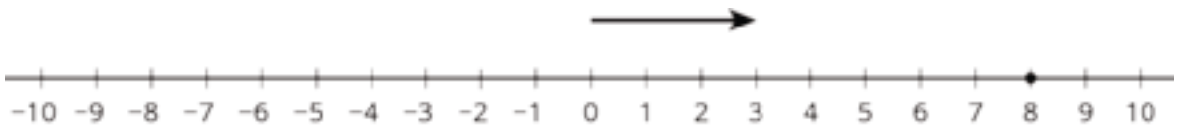
For each equation, write two more equations, using the same numbers, that express the same relationship in a different way.

1. $9 + (-1) = 8$

2. $-11 + x = 7$

5.2: Subtraction with Number Lines

1. Here is an unfinished number line diagram that represents a sum of 8.

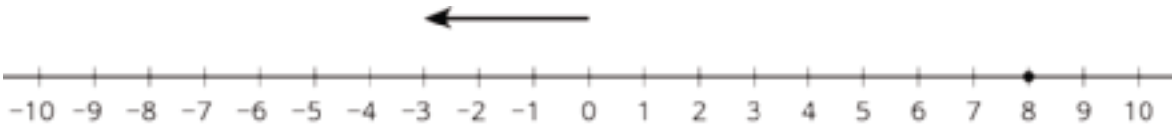


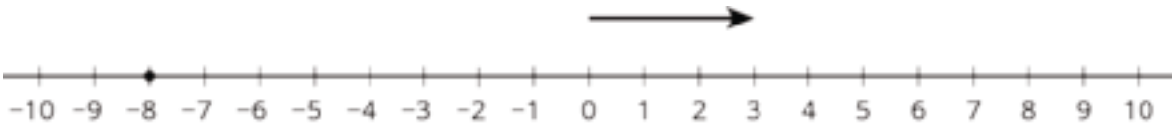
   a. How long should the other arrow be?

   b. For an equation that goes with this diagram, Mai writes $3 + ? = 8$. Tyler writes $8 - 3 = ?$. Do you agree with either of them?

   c. What is the unknown number? How do you know?

2. Here are two more unfinished diagrams that represent sums.





For each diagram:

a. What equation would Mai write if she used the same reasoning as before?

b. What equation would Tyler write if he used the same reasoning as before?

c. How long should the other arrow be?

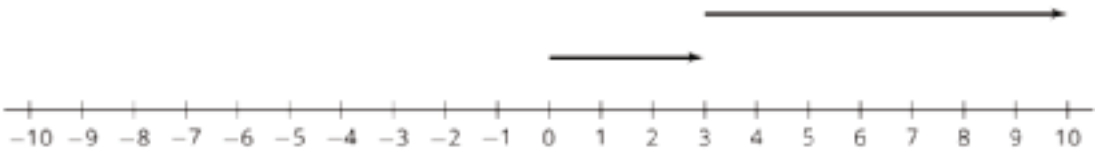
d. What number would complete each equation? Be prepared to explain your reasoning.

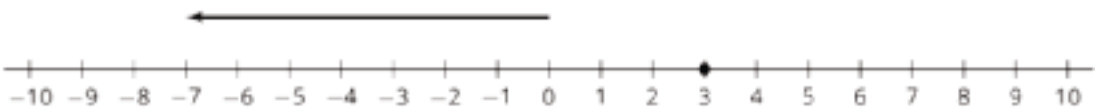
3. Draw a number line diagram for $(-8) - (-3) = ?$ What is the unknown number? How do you know?

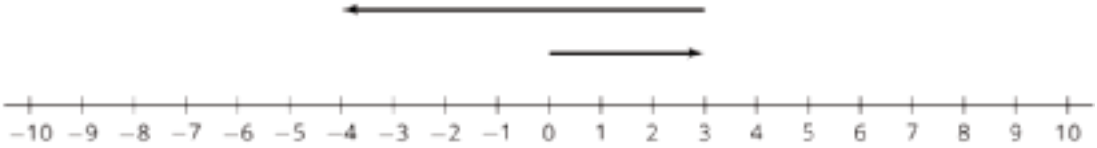
5.3: We Can Add Instead

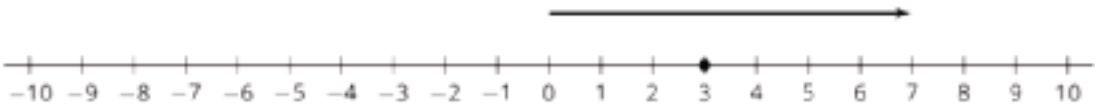
1. Match each diagram to one of these expressions:

$3 + 7$ $\quad$ $3 - 7$ $\quad$ $3 + (-7)$ $\quad$ $3 - (-7)$

a.


b.


c.


d.


2. Which expressions in the first question have the same value? What do you notice?

3. Complete each of these tables. What do you notice?

expression	value
$8 + (-8)$	
$8 - 8$	
$8 + (-5)$	
$8 - 5$	
$8 + (-12)$	
$8 - 12$	

expression	value
$-5 + 5$	
$-5 - (-5)$	
$-5 + 9$	
$-5 - (-9)$	
$-5 + 2$	
$-5 - (-2)$	

Are you ready for more?

It is possible to make a new number system using *only* the numbers 0, 1, 2, and 3. We will write the symbols for adding and subtracting in this system like this: $2 \oplus 1 = 3$ and $2 \ominus 1 = 1$. The table shows some of the sums.

$\oplus$	**0**	**1**	**2**	**3**
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3				

1. In this system, $1 \oplus 2 = 3$ and $2 \oplus 3 = 1$. How can you see that in the table?

2. What do you think $3 \oplus 1$ should be?

3. What about $3 \oplus 3$?

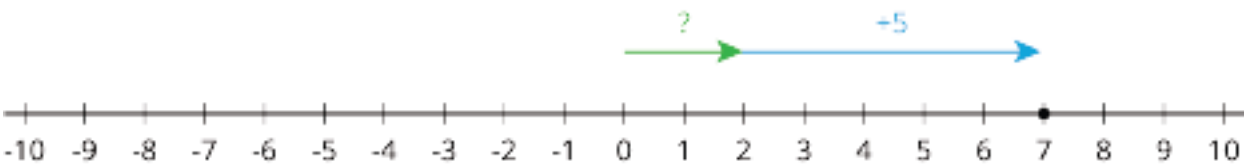
4. What do you think $3 \ominus 1$ should be?

5. What about $2 \ominus 3$?

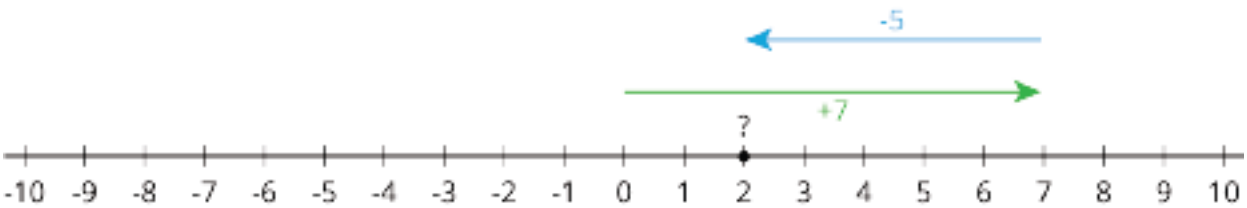
6. Can you think of any uses for this number system?

Lesson 5 Summary

The equation $7 - 5 = ?$ is equivalent to $? + 5 = 7$. The diagram illustrates the second equation.

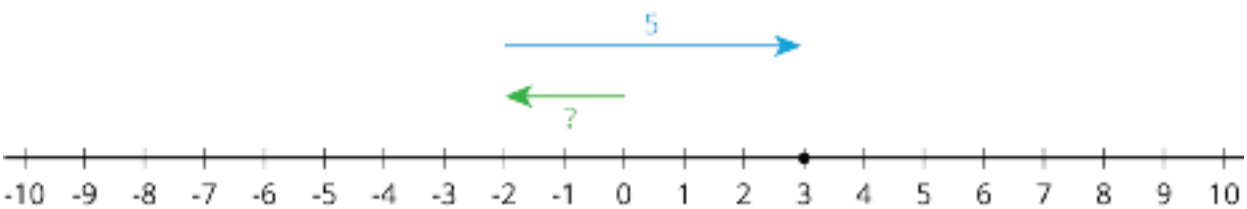


Notice that the value of $7 + (-5)$ is 2.

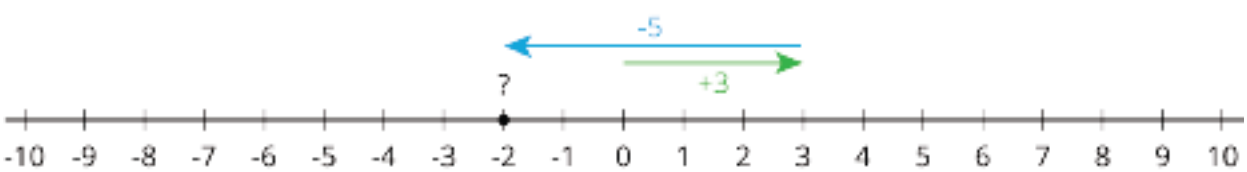


We can solve the equation $? + 5 = 7$ by adding -5 to both sides. This shows that $7 - 5 = 7 + (-5)$

Likewise, $3 - 5 = ?$ is equivalent to $? + 5 = 3$.



Notice that the value of $3 + (-5)$ is -2.



We can solve the equation $? + 5 = 3$ by adding -5 to both sides. This shows that $3 - 5 = 3 + (-5)$

In general:

$$a - b = a + (-b)$$

If $a - b = x$, then $x + b = a$. We can add $-b$ to both sides of this second equation to get that $x = a + (-b)$

Unit 5 Lesson 5 Cumulative Practice Problems

1. Write each subtraction equation as an addition equation.

   a. $a - 9 = 6$

   b. $p - 20 = -30$

   c. $z - (-12) = 15$

   d. $x - (-7) = -10$

2. Find each difference. If you get stuck, consider drawing a number line diagram.

   a. $9 - 4$

   b. $4 - 9$

   c. $9 - (-4)$

   d. $-9 - (-4)$

   e. $-9 - 4$

   f. $4 - (-9)$

   g. $-4 - (-9)$

   h. $-4 - 9$

3. A restaurant bill is $59 and you pay $72. What percentage gratuity did you pay?

(From Unit 4, Lesson 10.)

4. Find the solution to each equation mentally.

   a. $30 + a = 40$

   b. $500 + b = 200$

   c. $-1 + c = -2$

   d. $d + 3{,}567 = 0$

5. One kilogram is 2.2 pounds. Complete the tables. What is the interpretation of the constant of proportionality in each case?

_____ kilogram per pound

pounds	kilograms
2.2	1
11	
5.5	
1	

_____ pounds per kilogram

kilograms	pounds
1	2.2
7	
30	
0.5	

(From Unit 2, Lesson 3.)

Lesson 6: Subtracting Rational Numbers

Let's bring addition and subtraction together.

6.1: Number Talk: Missing Addend

Solve each equation mentally. Rewrite each addition equation as a subtraction equation.

$247 + c = 458$

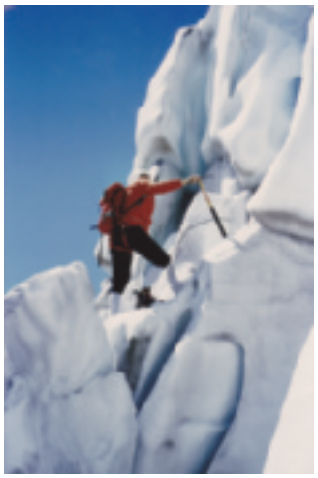
$c + 43.87 = 58.92$

$\frac{15}{8} + c = \frac{51}{8}$

6.2: Expressions with Altitude

A mountaineer is changing elevations. Write an expression that represents the difference between the final elevation and beginning elevation. Then write the value of the change. The first one is done for you.

beginning elevation (feet)	final elevation (feet)	difference between final and beginning	change
+400	+900	$900 - 400$	+500
+400	+50		
+400	-120		
-200	+610		
-200	-50		
-200	-500		
-200	0		



Are you ready for more?

Fill in the table so that every row and every column sums to 0. Can you find another way to solve this puzzle?

	-12	0		5
0			-18	25
25		-18	5	-12
-12				-18
	-18	25	-12	

	-12	0		5
0			-18	25
25		-18	5	-12
-12				-18
	-18	25	-12	

6.3: Does the Order Matter?

1. Find the value of each subtraction expression.

A
$3 - 2$
$5 - (-9)$
$(-11) - 2$
$(-6) - (-3)$
$(-1.2) - (-3.6)$
$(-2\frac{1}{2}) - (-3\frac{1}{2})$

B
$2 - 3$
$(-9) - 5$
$2 - (-11)$
$(-3) - (-6)$
$(-3.6) - (-1.2)$
$(-3\frac{1}{2}) - (-2\frac{1}{2})$

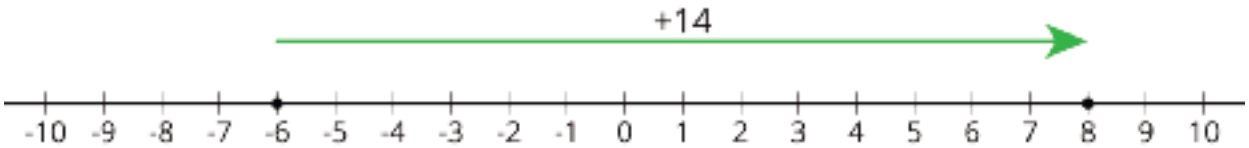
2. What do you notice about the expressions in Column A compared to Column B?

3. What do you notice about their values?

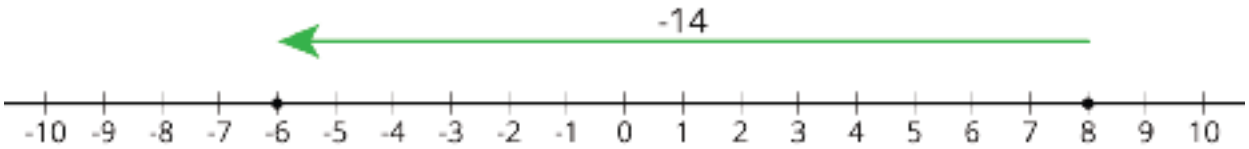
Lesson 6 Summary

When we talk about the difference of two numbers, we mean, “subtract them.” Usually, we subtract them in the order they are named. For example, the difference of +8 and -6 is $8 - (-6)$.

The difference of two numbers tells you how far apart they are on the number line. 8 and -6 are 14 units apart, because $8 - (-6) = 14$:



Notice that if you subtract them in the opposite order, you get the opposite number:



$$(-6) - 8 = -14$$

In general, the distance between two numbers a and b on the number line is $|a - b|$. Note that the *distance* between two numbers is always positive, no matter the order. But the *difference* can be positive or negative, depending on the order.

Unit 5 Lesson 6 Cumulative Practice Problems

1. Write a sentence to answer each question:

   a. How much warmer is 82 than 40?

   b. How much warmer is 82 than -40?

2. a. What is the difference in height between 30 m up a cliff and 87 m up a cliff? What is the distance between these positions?

   b. What is the difference in height between an albatross flying at 100 m above the surface of the ocean and a shark swimming 30 m below the surface? What is the distance between them if the shark is right below the albatross?

3. A company produces screens of different sizes. Based on the table, could there be a relationship between the number of pixels and the area of the screen? If so, write an equation representing the relationship. If not, explain your reasoning.

square inches of screen	number of pixels
6	31,104
72	373,248
105	544,320
300	1,555,200

(From Unit 2, Lesson 8.)

4. Find each difference.

- $(-5) - 6$
- $35 - (-8)$
- $\frac{2}{5} - \frac{3}{5}$
- $-4\frac{3}{8} - (-1\frac{1}{4})$

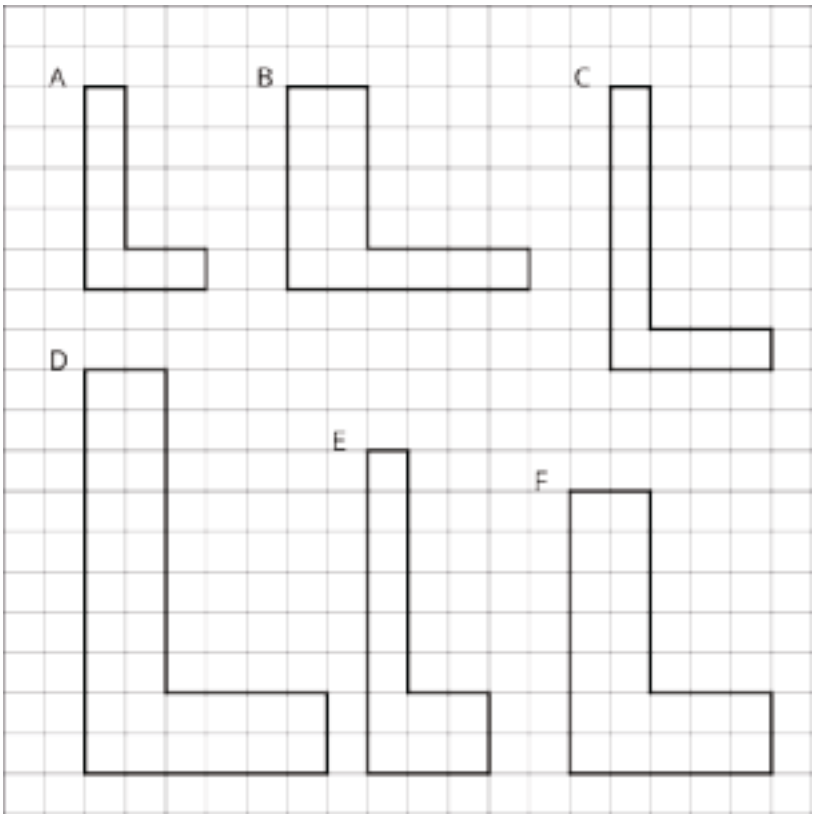
5. A family goes to a restaurant. When the bill comes, this is printed at the bottom of it:

Gratuity Guide For Your Convenience:
15% would be $4.89
18% would be $5.87
20% would be $6.52

How much was the price of the meal? Explain your reasoning.

(From Unit 4, Lesson 10.)

6. Which is a scaled copy of Polygon A? Identify a pair of corresponding sides and a pair of corresponding angles. Compare the areas of the scaled copies.



(From Unit 1, Lesson 2.)

Lesson 7: Adding and Subtracting to Solve Problems

Let's apply what we know about signed numbers to different situations.

7.1: Positive or Negative?

Without computing:

1. Is the solution to $-2.7 + x = -3.5$ positive or negative?

2. Select **all** the expressions that are solutions to $-2.7 + x = -3.5$.

   a. $-3.5 + 2.7$

   b. $3.5 - 2.7$

   c. $-3.5 - (-2.7)$

   d. $-3.5 - 2.7$

7.2: Phone Inventory

A store tracks the number of cell phones it has in stock and how many phones it sells.

The table shows the inventory for one phone model at the beginning of each day last week. The inventory changes when they sell phones or get shipments of phones into the store.

	inventory	change
Monday	18	-2
Tuesday	16	-5
Wednesday	11	-7
Thursday	4	-6
Friday	-2	20

1. What do you think it means when the change is positive? Negative?

2. What do you think it means when the inventory is positive? Negative?

3. Based on the information in the table, what do you think the inventory will be at on Saturday morning? Explain your reasoning.

4. What is the difference between the greatest inventory and the least inventory?

7.3: Solar Power

Han's family got a solar panel. Each month they get a credit to their account for the electricity that is generated by the solar panel. The credit they receive varies based on how sunny it is.

Here is their electricity bill from January.

Current charges: $83.56
Solar Credit: -$6.75
Amount due: $76.81

In January they used $83.56 worth of electricity and generated $6.75 worth of electricity.

1. In July they were traveling away from home and only used $19.24 worth of electricity. Their solar panel generated $22.75 worth of electricity. What was their amount due in July?

2. The table shows the value of the electricity they used and the value of the electricity they generated each week for a month. What amount is due for this month?

	used ($)	generated ($)
week 1	13.45	-6.33
week 2	21.78	-8.94
week 3	18.12	-7.70
week 4	24.05	-5.36

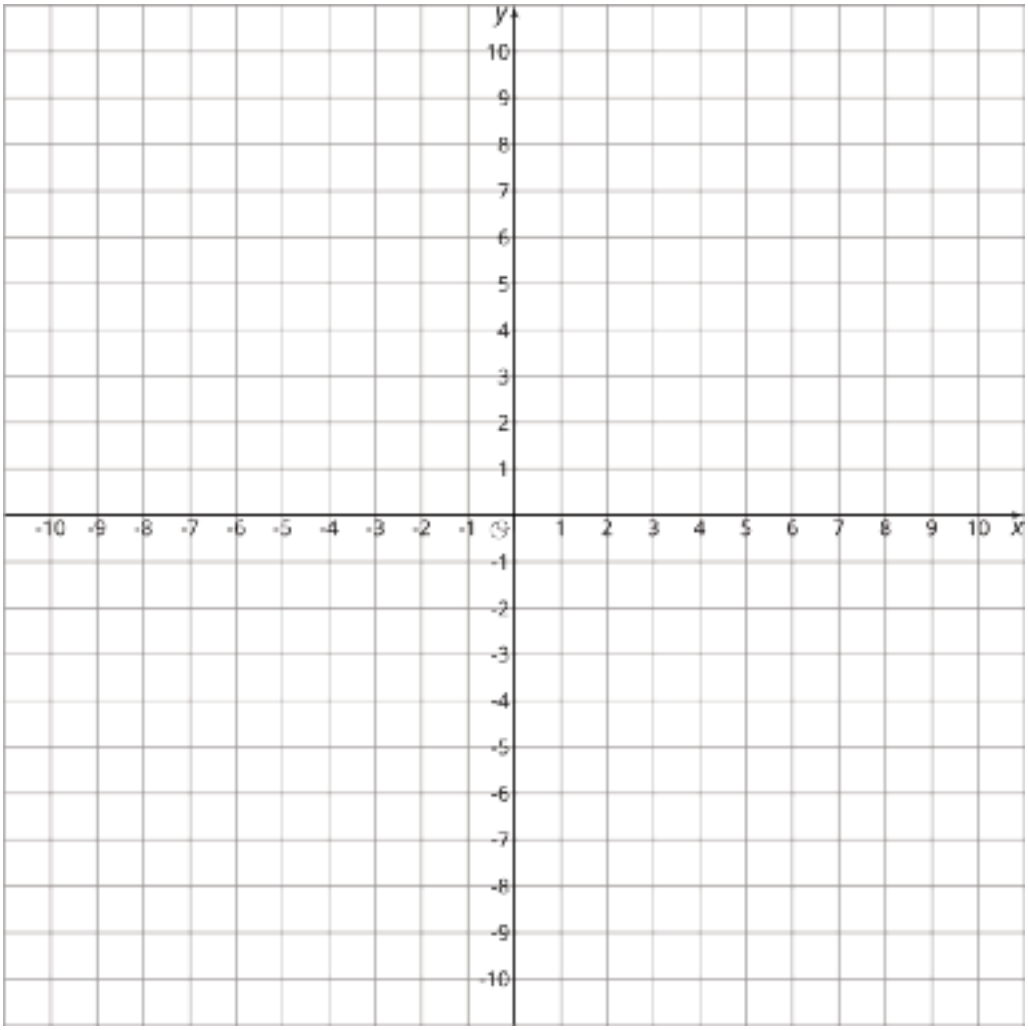
3. What is the difference between the value of the electricity generated in week 1 and week 2? Between week 2 and week 3?

Are you ready for more?

While most rooms in any building are all at the same level of air pressure, hospitals make use of "positive pressure rooms" and "negative pressure rooms." What do you think it means to have negative pressure in this setting? What could be some uses of these rooms?

7.4: Differences and Distances

Plot these points on the coordinate grid: $A = (5, 4)$, $B = (5, -2)$, $C = (-3, -2)$, $D = (-3, 4)$



1. What shape is made if you connect the dots in order?

2. What are the side lengths of figure $ABCD$?

3. What is the difference between the x-coordinates of B and C?

4. What is the difference between the x-coordinates of C and B?

5. How do the differences of the coordinates relate to the distances between the points?

Lesson 7 Summary

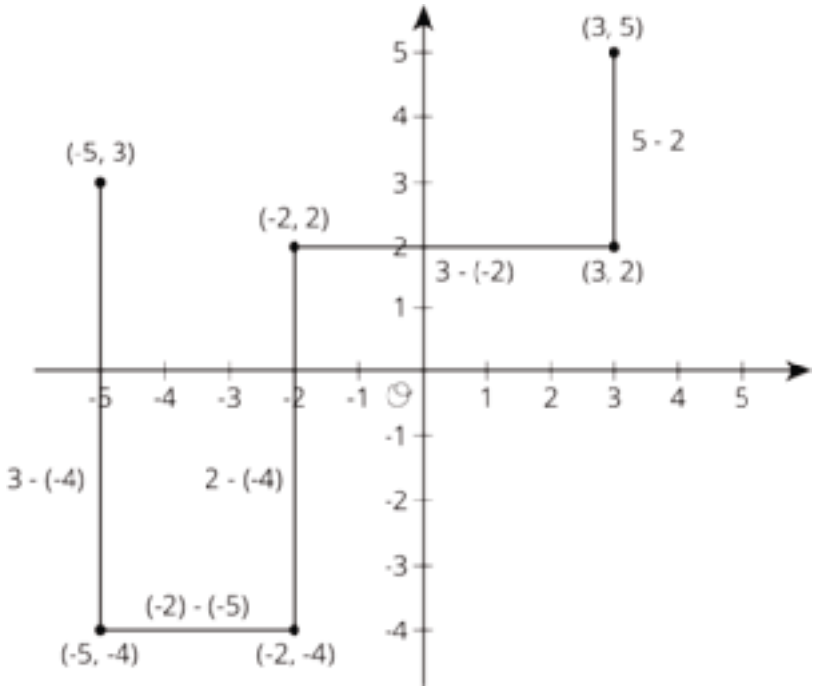
Sometimes we use positive and negative numbers to represent quantities in context. Here are some contexts we have studied that can be represented with positive and negative numbers:

- temperature
- elevation
- inventory
- an account balance
- electricity flowing in and flowing out

In these situations, using positive and negative numbers, and operations on positive and negative numbers, helps us understand and analyze them. To solve problems in these situations, we just have to understand what it means when the quantity is positive, when it is negative, and what it means to add and subtract them.

When two points in the coordinate plane lie on a horizontal line, you can find the distance between them by subtracting their x-coordinates.

When two points in the coordinate plane lie on a vertical line, you can find the distance between them by subtracting their y-coordinates.



Remember: the *distance* between two numbers is independent of the order, but the *difference* depends on the order.

Unit 5 Lesson 7 Cumulative Practice Problems

1. The table shows four transactions and the resulting account balance in a bank account, except some numbers are missing. Fill in the missing numbers.

	transaction amount	account balance
transaction 1	360	360
transaction 2	-22.50	337.50
transaction 3		182.35
transaction 4		-41.40

2. The *departure from the average* is the difference between the actual amount of rain and the average amount of rain for a given month. The historical average for rainfall in Albuquerque, NM for June, July, and August is shown in the table.

June	July	August
0.67	1.5	1.57

   a. Last June only 0.17 inches of rain fell all month. What is the difference between the average rainfall and the actual rainfall for last June?

   b. The departure from the average rainfall last July was -0.36 inches. How much rain fell last July?

   c. How much rain would have to fall in August so that the total amount of rain equals the average rainfall for these three months? What would the departure from the average be in August in that situation?

3. a. How much higher is 500 than 400 m?

   b. How much higher is 500 than -400 m?

   c. What is the change in elevation from 8,500 m to 3,400 m?

   d. What is the change in elevation between 8,500 m and -300 m?

   e. How much higher is -200 m than 450 m?

(From Unit 5, Lesson 6.)

4. Tyler orders a meal that costs $15.

   a. If the tax rate is 6.6%, how much will the sales tax be on Tyler's meal?

   b. Tyler also wants to leave a tip for the server. How much do you think he should pay in all? Explain your reasoning.

(From Unit 4, Lesson 10.)

5. In a video game, a character is healed at a constant rate as long as they are standing in a certain circle. Complete the table.

time in circle (seconds)	health gained (points)
4	100
10	
3	
	1,000

(From Unit 2, Lesson 3.)

Lesson 8: Position, Speed, and Direction

Let's use signed numbers to represent movement.

8.1: Distance, Rate, Time

1. An airplane moves at a constant speed of 120 miles per hour for 3 hours. How far does it go?

2. A train moves at constant speed and travels 6 miles in 4 minutes. What is its speed in miles per minute?

3. A car moves at a constant speed of 50 miles per hour. How long does it take the car to go 200 miles?

8.2: Going Left, Going Right

1. After each move, record your location in the table. Then write an expression to represent the ending position that uses the starting position, the speed, and the time. The first row is done for you.

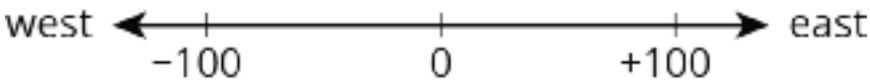
starting position	direction	speed (units per second)	time (seconds)	ending position (units)	expression
0	right	5	3	+15	$0 + 5 \cdot 3$
0	left	4	6		
0	right	2	8		
0	right	6	2		
0	left	1.1	5		

2. How can you see the *direction* of movement in the expression?

3. Using a starting position p, a speed s, and a time t, write two expressions for an ending position. One expression should show the result of moving right, and one expression should show the result of moving left.

8.3: Velocity

A traffic safety engineer was studying travel patterns along a highway. She set up a camera and recorded the speed and direction of cars and trucks that passed by the camera. Positions to the east of the camera are positive, and to the west are negative.



Vehicles that are traveling towards the east have a positive velocity, and vehicles that are traveling towards the west have a negative velocity.

1. Complete the table with the position of each vehicle if the vehicle is traveling at a constant speed for the indicated time period. Then write an equation.

velocity (meters per second)	time after passing the camera (seconds)	ending position (meters)	equation describing the position
+25	+10	+250	$25 \cdot 10 = 250$
-20	+30		
+32	+40		
-35	+20		
+28	0		

2. If a car is traveling east when it passes the camera, will its position be positive or negative 60 seconds after it passes the camera? If we multiply two positive numbers, is the result positive or negative?

3. If a car is traveling west when it passes the camera, will its position be positive or negative 60 seconds after it passes the camera? If we multiply a negative and a positive number, is the result positive or negative?

Are you ready for more?

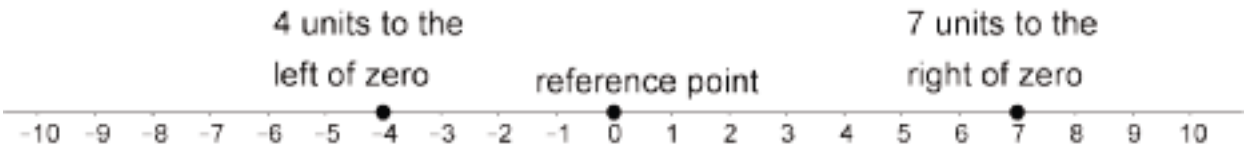
In many contexts we can interpret negative rates as "rates in the opposite direction." For example, a car that is traveling -35 miles per hour is traveling in the opposite direction of a car that is traveling 40 miles per hour.

1. What could it mean if we say that water is flowing at a rate of -5 gallons per minute?

2. Make up another situation with a negative rate, and explain what it could mean.

Lesson 8 Summary

We can use signed numbers to represent the position of an object along a line. We pick a point to be the reference point, and call it zero. Positions to the right of zero are positive. Positions to the left of zero are negative.



When we combine speed with direction indicated by the sign of the number, it is called *velocity*. For example, if you are moving 5 meters per second to the right, then your velocity is +5 meters per second. If you are moving 5 meters per second to the left, then your velocity is -5 meters per second.

If you start at zero and move 5 meters per second for 10 seconds, you will be $5 \cdot 10 = 50$ meters to the right of zero. In other words, $5 \cdot 10 = 50$.

If you start at zero and move -5 meters per second for 10 seconds, you will be $5 \cdot 10 = 50$ meters to the *left* of zero. In other words,

$$-5 \cdot 10 = -50$$

In general, a negative number times a positive number is a negative number.

Unit 5 Lesson 8 Cumulative Practice Problems

1. A number line can represent positions that are north and south of a truck stop on a highway. Decide whether you want positive positions to be north or south of the truck stop. Then plot the following positions on a number line.

   a. The truck stop

   b. 5 miles north of the truck stop

   c. 3.5 miles south of the truck stop

2. a. How could you distinguish between traveling west at 5 miles per hour and traveling east at 5 miles per hour without using the words “east” and “west”?

   b. Four people are cycling. They each start at the same point. (0 represents their starting point.) Plot their finish points after five seconds of cycling on a number line

      - Lin cycles at 5 meters per second
      - Diego cycles at -4 meters per second
      - Elena cycles at 3 meters per second
      - Noah cycles at -6 meters per second

3. Find the value of each expression.

a. $16.2 + -8.4$

b. $\frac{2}{5} - \frac{3}{5}$

c. $-9.2 + -7$

d. $-4\frac{3}{8} - (-1\frac{1}{4})$

(From Unit 5, Lesson 6.)

4. For each equation, write two more equations using the same numbers that express the same relationship in a different way.

a. $3 + 2 = 5$

b. $7.1 + 3.4 = 10.5$

c. $15 - 8 = 7$

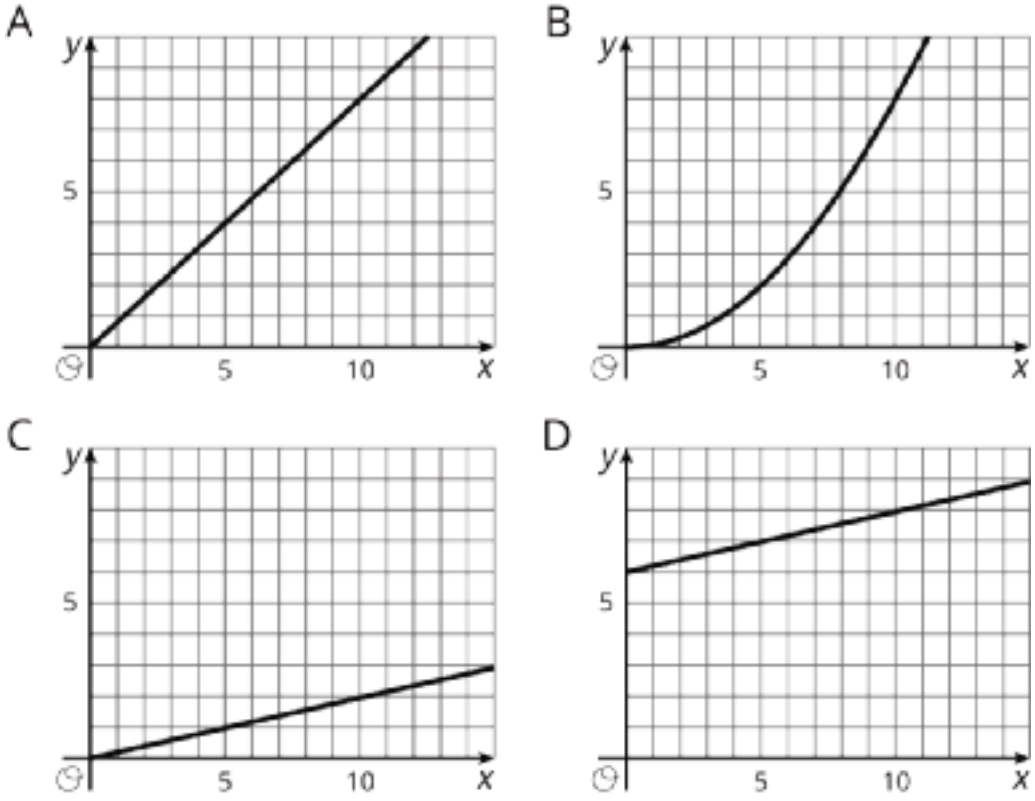
d. $\frac{3}{2} + \frac{9}{5} = \frac{33}{10}$

(From Unit 5, Lesson 5.)

5. A shopper bought a watermelon, a pack of napkins, and some paper plates. In his state, there is no tax on food. The tax rate on non-food items is 5%. The total for the three items he bought was $8.25 before tax, and he paid $0.19 in tax. How much did the watermelon cost?

(From Unit 4, Lesson 10.)

6. Which graphs could not represent a proportional relationship? Explain how you decided.



(From Unit 2, Lesson 10.)

Lesson 9: Multiplying Rational Numbers

Let's multiply signed numbers.

9.1: Before and After

Where was the girl:

1. 5 seconds *after* this picture was taken? Mark her approximate location on the picture.
2. 5 seconds *before* this picture was taken? Mark her approximate location on the picture.

9.2: Backwards in Time

A traffic safety engineer was studying travel patterns along a highway. She set up a camera and recorded the speed and direction of cars and trucks that passed by the camera. Positions to the east of the camera are positive, and to the west are negative.

1. Here are some positions and times for one car:

position (feet)	-180	-120	-60	0	60	120
time (seconds)	-3	-2	-1	0	1	2

   a. In what direction is this car traveling?

   b. What is its velocity?

2. a. What does it mean when the time is zero?

b. What could it mean to have a negative time?

3. Here are the positions and times for a different car whose velocity is -50 feet per second:

position (feet)				0	-50	-100
time (seconds)	-3	-2	-1	0	1	2

a. Complete the table with the rest of the positions.

b. In what direction is this car traveling? Explain how you know.

4. Complete the table for several different cars passing the camera.

	velocity (meters per second)	**time after passing the camera (seconds)**	**ending position (meters)**	**equation**
car C	+25	+10	+250	$25 \cdot 10 = 250$
car D	-20	+30		
car E	+32	-40		
car F	-35	-20		
car G	-15	-8		

5. a. If a car is traveling east when it passes the camera, will its position be positive or negative 60 seconds *before* it passes the camera?

b. If we multiply a positive number and a negative number, is the result positive or negative?

6. a. If a car is traveling west when it passes the camera, will its position be positive or negative 60 seconds *before* it passes the camera?

b. If we multiply two negative numbers, is the result positive or negative?

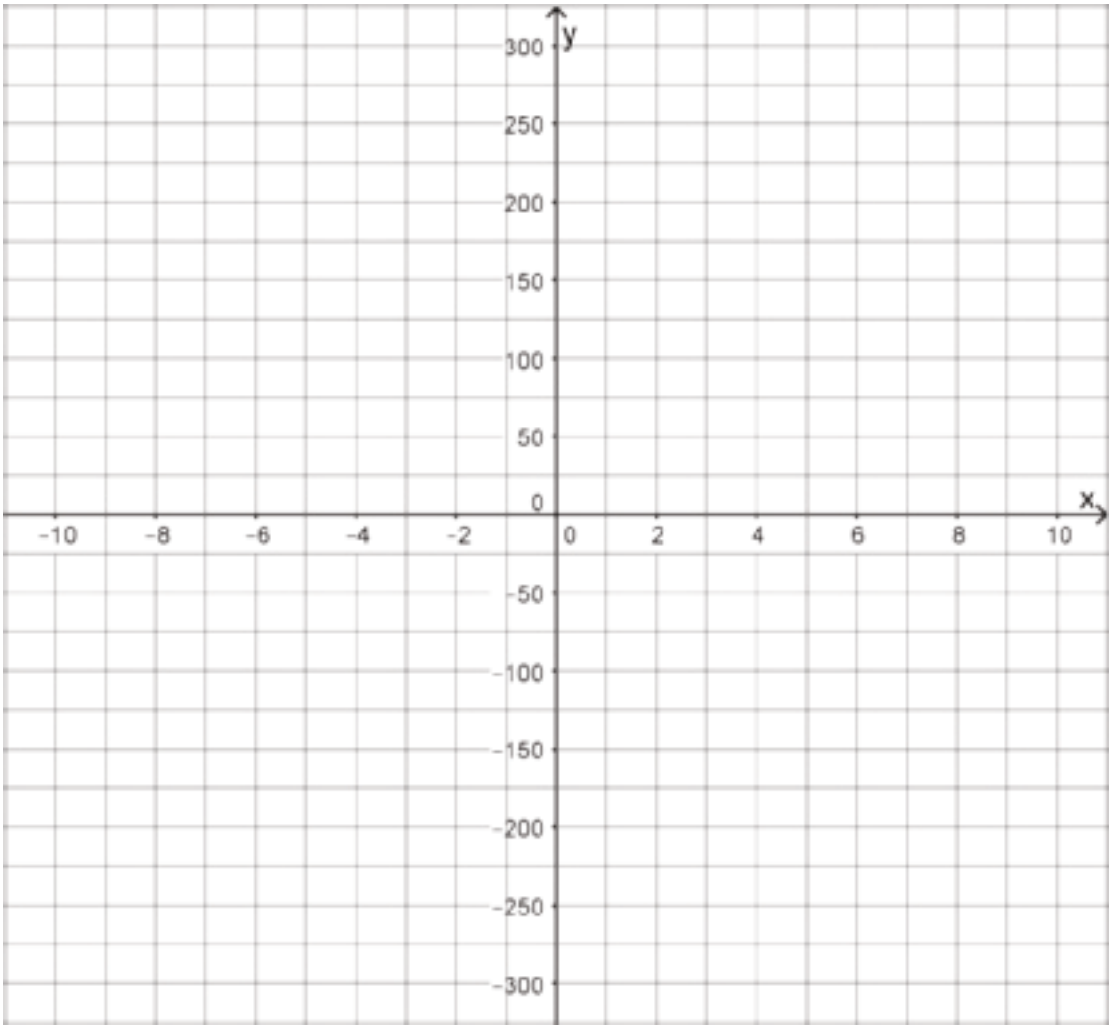
9.3: Cruising

Around noon, a car was traveling -32 meters per second down a highway. At exactly noon (when time was 0), the position of the car was 0 meters.

1. Complete the table.

time (s)	-10	-7	-4	-1	2	5	8	11
position (m)								

2. Graph the relationship between the time and the car's position.



3. What was the position of the car at -3 seconds?

4. What was the position of the car at 6.5 seconds?

Are you ready for more?

Find the value of these expressions without using a calculator.

$(-1)^2$ $(-1)^3$ $(-1)^4$ $(-1)^{99}$

9.4: Rational Numbers Multiplication Grid

1. Complete the *shaded* boxes in the multiplication square.

5						0	5	10	15	20	
4						0	4	8	12	16	
3						0	3	6	9	12	
2					-2	0	2	4	6	8	
1						0	1	2	3	4	
0						0	0	0	0	0	
-1											
-2											
-3											
-4											
-5											
	-5	-4	-3	-2	-1	0	1	2	3	4	5

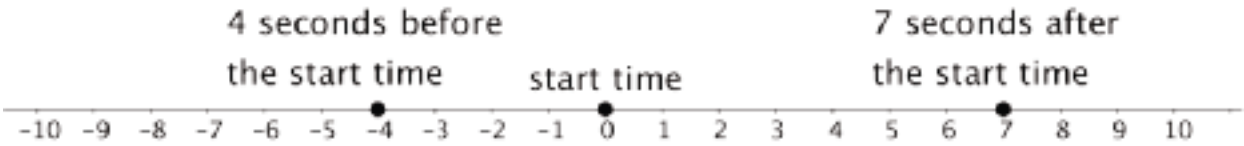
2. Look at the patterns along the rows and columns. Continue those patterns into the unshaded boxes.

3. Complete the whole table.

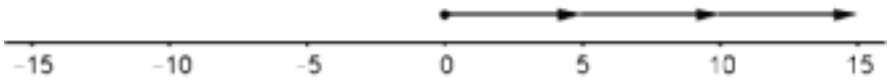
4. What does this tell you about multiplication with negative numbers?

Lesson 9 Summary

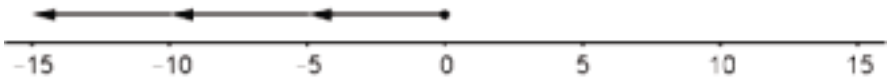
We can use signed numbers to represent time relative to a chosen point in time. We can think of this as starting a stopwatch. The positive times are after the watch starts, and negative times are times before the watch starts.



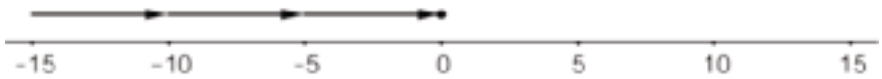
If a car is at position 0 and is moving in a positive direction, then for times after that (positive times), it will have a positive position. A positive times a positive is positive.



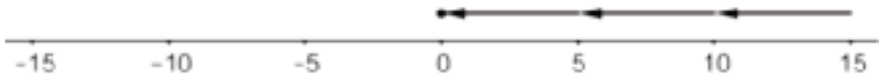
If a car is at position 0 and is moving in a negative direction, then for times after that (positive times), it will have a negative position. A negative times a positive is negative.



If a car is at position 0 and is moving in a positive direction, then for times *before* that (negative times), it must have had a negative position. A positive times a negative is negative.



If a car is at position 0 and is moving in a negative direction, then for times *before* that (negative times), it must have had a positive position. A negative times a negative is positive.



Here is another way of seeing this:

We can think of $3 \cdot 5$ as $5 + 5 + 5$, which has a value of 15.

We can think of $3 \cdot (-5)$ as $-5 + -5 + -5$, which has a value of -15.

We know we can multiply positive numbers in any order: $3 \cdot 5 = 5 \cdot 3$

If we can multiply signed numbers in any order, then $(-5) \cdot 3$ would also equal -15.

Now let's think about multiplying two negatives.

We can find $-5 \cdot (3 + -3)$ in two ways:

- Applying the distributive property:

$$-5 \cdot 3 + -5 \cdot (-3)$$

- Adding the numbers in parentheses:

$$-5 \cdot (0) = 0$$

This means that these expressions must be equal.

$$-5 \cdot 3 + -5 \cdot (-3) = 0$$

Multiplying the first two numbers gives

$$-15 + -5 \cdot (-3) = 0$$

Which means that

$$-5 \cdot (-3) = 15$$

There was nothing special about these particular numbers. This always works!

- A positive times a positive is always positive.
- A negative times a positive or a positive times a negative is always negative.
- A negative times a negative is always positive.

Unit 5 Lesson 9 Cumulative Practice Problems

1. Fill in the missing numbers in these equations

   a. $-2 \cdot (-4.5) = ?$

   b. $(-8.7) \cdot (-10) = ?$

   c. $(-7) \cdot ? = 14$

   d. $? \cdot (-10) = 90$

2. A weather station on the top of a mountain reports that the temperature is currently 0°C and has been falling at a constant rate of 3°C per hour. If it continues to fall at this rate, find each indicated temperature. Explain or show your reasoning.

   a. What will the temperature be in 2 hours?

   b. What will the temperature be in 5 hours?

   c. What will the temperature be in half an hour?

d. What was the temperature 1 hour ago?

e. What was the temperature 3 hours ago?

f. What was the temperature 4.5 hours ago?

3. Find the value of each expression.

a. $\frac{1}{4} \cdot (-12)$

b. $-\frac{1}{3} \cdot 39$

c. $(-\frac{4}{5}) \cdot (-75)$

d. $-\frac{2}{5} \cdot (-\frac{3}{4})$

e. $\frac{8}{3} \cdot -42$

4. To make a specific hair dye, a hair stylist uses a ratio of $1\frac{1}{8}$ oz of red tone, $\frac{3}{4}$ oz of gray tone, and $\frac{5}{8}$ oz of brown tone.

   a. If the stylist needs to make 20 oz of dye, how much of each dye color is needed?

   b. If the stylist needs to make 100 oz of dye, how much of each dye color is needed?

(From Unit 4, Lesson 2.)

5. a. Here are the vertices of rectangle $FROG$: $(-2, 5), (-2, 1), (6, 5), (6, 1)$. Find the perimeter of this rectangle. If you get stuck, try plotting the points on a coordinate plane.

   b. Find the area of the rectangle $FROG$.

   c. Here are the coordinates of rectangle $PLAY$: $(-11, 20), (-11, -3), (-1, 20), (-1, -3)$. Find the perimeter and area of this rectangle. See if you can figure out its side lengths without plotting the points.

(From Unit 5, Lesson 7.)

Lesson 10: Multiply!

Let's get more practice multiplying signed numbers.

10.1: Which One Doesn't Belong: Expressions

Which expression doesn't belong?

$7.9x$

$7.9 + x$

$7.9 \cdot (-10)$

-79

10.2: Card Sort: Matching Expressions

Your teacher will give you cards with multiplication expressions on them. Match the expressions that are equal to each other. There will be 3 cards in each group.

10.3: Row Game: Multiplying Rational Numbers

Evaluate the expressions in one of the columns. Your partner will work on the other column. Check in with your partner after you finish each row. Your answers in each row should be the same. If your answers aren't the same, work together to find the error.

column A	column B
$790 \div 10$	$(7.9) \cdot 10$
$-\frac{6}{7} \cdot 7$	$(0.1) \cdot -60$
$(2.1) \cdot -2$	$(-8.4) \cdot \frac{1}{2}$
$(2.5) \cdot (-3.25)$	$-\frac{5}{2} \cdot \frac{13}{4}$
$-10 \cdot (3.2) \cdot (-7.3)$	$5 \cdot (-1.6) \cdot (-29.2)$

Are you ready for more?

A sequence of rational numbers is made by starting with 1, and from then on, each term is one more than the reciprocal of the previous term. Evaluate the first few expressions in the sequence. Can you find any patterns? Find the 10th term in this sequence.

$$1 \qquad 1+\frac{1}{1} \qquad 1+\frac{1}{1+1} \qquad 1+\frac{1}{1+\frac{1}{1+1}} \qquad 1+\frac{1}{1+\frac{1}{1+\frac{1}{1+1}}} \qquad \ldots$$

Lesson 10 Summary

- A positive times a positive is always positive.

  For example, $\frac{3}{5} \cdot \frac{7}{8} = \frac{21}{40}$.

- A negative times a negative is also positive.

  For example, $-\frac{3}{5} \cdot -\frac{7}{8} = \frac{21}{40}$.

- A negative times a positive or a positive times a negative is always negative.

  For example, $\frac{3}{5} \cdot -\frac{7}{8} = -\frac{3}{5} \cdot \frac{7}{8} = -\frac{21}{40}$.

- A negative times a negative times a negative is also negative.

  For example, $-3 \cdot -4 \cdot -5 = -60$.

Unit 5 Lesson 10 Cumulative Practice Problems

1. Evaluate each expression:

   a. $-12 \cdot \frac{1}{3}$

   b. $-12 \cdot -\frac{1}{3}$

   c. $12 \cdot \left(-\frac{5}{4}\right)$

   d. $-12 \cdot \left(-\frac{5}{4}\right)$

2. Evaluate each expression:

   a. $-1 \cdot 2 \cdot 3$

   b. $-1 \cdot (-2) \cdot 3$

   c. $-1 \cdot (-2) \cdot (-3)$

3. Order each set of numbers from least to greatest.

   a. 4, 8, -2, -6, 0

   b. -5, -5.2, 5.5, $-5\frac{1}{2}$, $\frac{-5}{2}$

(From Unit 5, Lesson 1.)

4. $30 + -30 = 0$.

   a. Write another sum of two numbers that equals 0.

   b. Write a sum of three numbers that equals 0.

   c. Write a sum of four numbers that equals 0, none of which are opposites.

(From Unit 5, Lesson 3.)

5. A submarine is searching for underwater features. It is accompanied by a small aircraft and an underwater robotic vehicle.

   At one time the aircraft is 200 m above the surface, the submarine is 55 m below the surface, and the underwater robotic vehicle is 227 m below the surface.

   a. What is the difference in height between the submarine and the aircraft?

   b. What is the distance between the underwater robotic vehicle and the submarine?

(From Unit 5, Lesson 6.)

6. a. Clare is cycling at a speed of 12 miles per hour. If she starts at a position chosen as zero, what will her position be after 45 minutes?

b. Han is cycling at a speed of -8 miles per hour; if he starts at the same zero point, what will his position be after 45 minutes?

c. What will the distance between them be after 45 minutes?

(From Unit 5, Lesson 8.)

7. Fill in the missing numbers in these equations

a. $(-7) \cdot ? = -14$

b. $? \cdot 3 = -15$

c. $? \cdot 4 = 32$

d. $-49 \cdot 3 = ?$

(From Unit 5, Lesson 9.)

Lesson 11: Dividing Rational Numbers

Let's divide signed numbers.

11.1: Tell Me Your Sign

Consider the equation: $-27x = -35$

Without computing:

1. Is the **solution** to this equation positive or negative?

2. Are either of these two numbers solutions to the equation?

$$\frac{35}{27} \qquad -\frac{35}{27}$$

11.2: Multiplication and Division

1. Find the missing values in the equations

   a. $-3 \cdot 4 = ?$

   b. $-3 \cdot ? = 12$

   c. $3 \cdot ? = 12$

   d. $? \cdot -4 = 12$

   e. $? \cdot 4 = -12$

2. Rewrite the unknown factor problems as division problems.

3. Complete the sentences. Be prepared to explain your reasoning.

   a. The sign of a positive number divided by a positive number is always:

   b. The sign of a positive number divided by a negative number is always:

   c. The sign of a negative number divided by a positive number is always:

   d. The sign of a negative number divided by a negative number is always:

4. Han and Clare walk towards each other at a constant rate, meet up, and then continue past each other in opposite directions. We will call the position where they meet up 0 feet and the time when they meet up 0 seconds.

   - Han's velocity is 4 feet per second.
   - Clare's velocity is -5 feet per second.

   a. Where is each person 10 seconds before they meet up?

   b. When is each person at the position -10 feet from the meeting place?

Are you ready for more?

It is possible to make a new number system using *only* the numbers 0, 1, 2, and 3. We will write the symbols for multiplying in this system like this: $1 \otimes 2 = 2$. The table shows some of the products.

$\otimes$	**0**	**1**	**2**	**3**
0	0	0	0	0
1		1	2	3
2			0	2
3				

1. In this system, $1 \otimes 3 = 3$ and $2 \otimes 3 = 2$. How can you see that in the table?

2. What do you think $2 \otimes 1$ is?

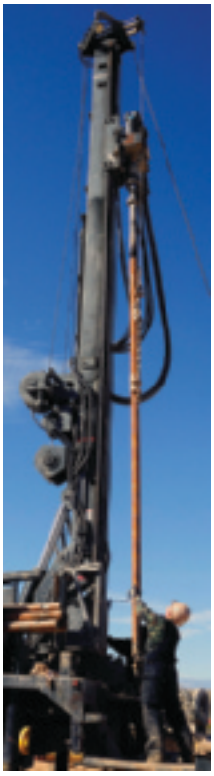
3. What about $3 \otimes 3$?

4. What do you think the solution to $3 \otimes n = 2$ is?

5. What about $2 \otimes n = 3$?

11.3: Drilling Down

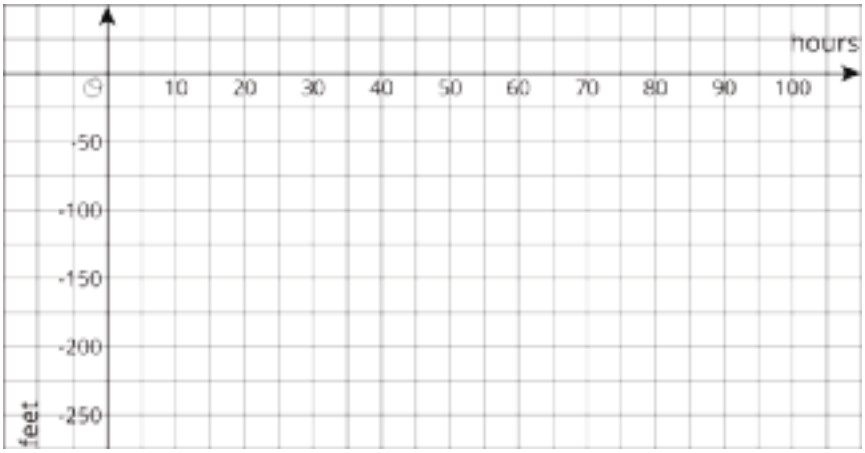
A water well drilling rig has dug to a height of -60 feet after one full day of continuous use.



1. Assuming the rig drilled at a constant rate, what was the height of the drill after 15 hours?

2. If the rig has been running constantly and is currently at a height of -147.5 feet, for how long has the rig been running?

3. Use the coordinate grid to show the drill's progress.



4. At this rate, how many hours will it take until the drill reaches -250 feet?

Lesson 11 Summary

Any division problem is actually a multiplication problem:

- $6 \div 2 = 3$ because $2 \cdot 3 = 6$
- $6 \div -2 = -3$ because $-2 \cdot -3 = 6$
- $-6 \div 2 = -3$ because $2 \cdot -3 = -6$
- $-6 \div -2 = 3$ because $-2 \cdot 3 = -6$

Because we know how to multiply signed numbers, that means we know how to divide them.

- The sign of a positive number divided by a negative number is always negative.
- The sign of a negative number divided by a positive number is always negative.
- The sign of a negative number divided by a negative number is always positive.

A number that can be used in place of the variable that makes the equation true is called a **solution** to the equation. For example, for the equation $x \div -2 = 5$, the solution is -10, because it is true that $-10 \div -2 = 5$.

Unit 5 Lesson 11 Cumulative Practice Problems

1. Find the quotients:

   a. $24 \div -6$

   b. $-15 \div 0.3$

   c. $-4 \div -20$

2. Find the quotients.

   a. $\frac{2}{5} \div \frac{3}{4}$

   b. $\frac{9}{4} \div \frac{-3}{4}$

   c. $\frac{-5}{7} \div \frac{-1}{3}$

   d. $\frac{-5}{3} \div \frac{1}{6}$

3. Is the solution positive or negative?

   a. $2 \cdot x = 6$

   b. $-2 \cdot x = 6.1$

   c. $2.9 \cdot x = -6.04$

   d. $-2.473 \cdot x = -6.859$

4. Find the solution mentally.

   a. $3 \cdot -4 = a$

   b. $b \cdot (-3) = -12$

   c. $-12 \cdot c = 12$

   d. $d \cdot 24 = -12$

5. In order to make a specific shade of green paint, a painter mixes $1\frac{1}{2}$ quarts of blue paint, 2 cups of green paint, and $\frac{1}{2}$ gallon of white paint. How much of each color is needed to make 100 cups of this shade of green paint?

(From Unit 4, Lesson 2.)

6. Here is a list of the highest and lowest elevation on each continent.

	highest point (m)	lowest point (m)
Europe	4,810	-28
Asia	8,848	-427
Africa	5,895	-155
Australia	4,884	-15
North America	6,198	-86
South America	6,960	-105
Antartica	4,892	-50

a. Which continent has the largest difference in elevation? The smallest?

b. Make a display (dot plot, box plot, or histogram) of the data set and explain why you chose that type of display to represent this data set.

(From Unit 5, Lesson 3.)

Lesson 12: Negative Rates

Let's apply what we know about signed numbers.

12.1: Grapes per Minute

1. If you eat 5 grapes per minute for 8 minutes, how many grapes will you eat?

2. If you hear 9 new songs per day for 3 days, how many new songs will you hear?

3. If you run 15 laps per practice, how many practices will it take you to run 30 laps?

12.2: Water Level in the Aquarium

1. A large aquarium should contain 10,000 liters of water when it is filled correctly. It will overflow if it gets up to 12,000 liters. The fish will get sick if it gets down to 4,000 liters. The aquarium has an automatic system to help keep the correct water level. If the water level is too low, a faucet fills it. If the water level is too high, a drain opens.

   One day, the system stops working correctly. The faucet starts to fill the aquarium at a rate of 30 liters per minute, and the drain opens at the same time, draining the water at a rate of 20 liters per minute.

   a. Is the water level rising or falling? How do you know?

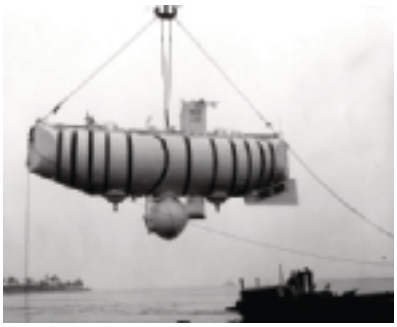
   b. How long will it take until the tank starts overflowing or the fish get sick?

2. A different aquarium should contain 15,000 liters of water when filled correctly. It will overflow if it gets to 17,600 liters.

   One day there is an accident and the tank cracks in 4 places. Water flows out of each crack at a rate of $\frac{1}{2}$ liter per hour. An emergency pump can re-fill the tank at a rate of 2 liters per minute. How many minutes must the pump run to replace the water lost each hour?

12.3: Up and Down with the Piccards

1. Challenger Deep is the deepest known point in the ocean, at 35,814 feet below sea level. In 1960, Jacques Piccard and Don Walsh rode down in the Trieste and became the first people to visit the Challenger Deep.



   a. If sea level is represented by 0 feet, explain how you can represent the depth of a submarine descending from sea level to the bottom of Challenger Deep.

   b. Trieste's descent was a change in depth of -3 feet per second. We can use the relationship $y = -3x$ to model this, where y is the depth (in feet) and x is the time (in seconds). Using this model, how much time would the Trieste take to reach the bottom?

   c. It took the Trieste 3 hours to ascend back to sea level. This can be modeled by a different relationship $y = kx$. What is the value of k in this situation?

2. The design of the Trieste was based on the design of a hot air balloon built by Auguste Piccard, Jacques's father. In 1932, Auguste rode in his hot-air balloon up to a record-breaking height.

   a. Auguste's ascent took 7 hours and went up 51,683 feet. Write a relationship $y = kx$ to represent his ascent from his starting location.

   b. Auguste's descent took 3 hours and went down 52,940 feet. Write another relationship to represent his descent.

   c. Did Auguste Piccard end up at a greater or lesser altitude than his starting point? How much higher or lower?

Are you ready for more?

During which part of either trip was a Piccard changing vertical position the fastest? Explain your reasoning.

- Jacques's descent
- Jacques's ascent
- Auguste's ascent
- Auguste's descent

Lesson 12 Summary

We saw earlier that we can represent speed with direction using signed numbers. Speed with direction is called *velocity*. Positive velocities always represent movement in the opposite direction from negative velocities.

We can do this with vertical movement: moving up can be represented with positive numbers, and moving down with negative numbers. The magnitude tells you how fast, and the sign tells you which direction. (We could actually do it the other way around if we wanted to, but usually we make up positive and down negative.)

Unit 5 Lesson 12 Cumulative Practice Problems

1. Describe a situation where each of the following quantities might be useful.

   a. -20 gallons per hour

   b. -10 feet per minute

   c. -0.1 kilograms per second

2. A submarine is only allowed to change its depth by rising toward the surface in 60-meter stages. It starts off at -340 meters.

   a. At what depth is it after:

      i. 1 stage

      ii. 2 stages

      iii. 4 stages

   b. How many stages will it take to return to the surface?

3. Some boats were traveling up and down a river. A satellite recorded the movements of several boats.

   a. A motor boat traveled -3.4 miles per hour for 0.75 hours. How far did it go?

   b. A tugboat traveled -1.5 miles in 0.3 hours. What was its velocity?

   c. What do you think that negative distances and velocities could mean in this situation?

4. a. A cookie recipe uses 3 cups of flour to make 15 cookies. How many cookies can you make with this recipe with 4 cups of flour? (Assume you have enough of the other ingredients.)

   b. A teacher uses 36 centimeters of tape to hang up 9 student projects. At that rate, how much tape would the teacher need to hang up 10 student projects?

(From Unit 4, Lesson 3.)

5. Evaluate each expression. When the answer is not a whole number, write your answer as a fraction.

   a. $-4 \cdot -6$

   b. $-24 \cdot \frac{-7}{6}$

   c. $4 \div -6$

   d. $\frac{4}{3} \div -24$

(From Unit 5, Lesson 11.)

Lesson 13: Expressions with Rational Numbers

Let's develop our signed number sense.

13.1: True or False: Rational Numbers

Decide if each statement is true or false. Be prepared to explain your reasoning.

1. $(-38.76)(-15.6)$ is negative

2. $10,000 - 99,999 < 0$

3. $\left(\frac{3}{4}\right)\left(-\frac{4}{3}\right) = 0$

4. $(30)(-80) - 50 = 50 - (30)(-80)$

13.2: Card Sort: The Same But Different

Your teacher will give you a set of cards. Group them into pairs of expressions that have the same value.

13.3: Near and Far From Zero

a	b	$-a$	$-4b$	$-a+b$	$a \div -b$	a^2	b^3
$-\frac{1}{2}$	6						
$\frac{1}{2}$	-6						
-6	$-\frac{1}{2}$						

1. For each set of values for a and b, evaluate the given expressions and record your answers in the table.

2. When $a = -\frac{1}{2}$ and $b = 6$, which expression:

   has the largest value? has the smallest value? is the closest to zero?

3. When $a = \frac{1}{2}$ and $b = -6$, which expression:

   has the largest value? has the smallest value? is the closest to zero?

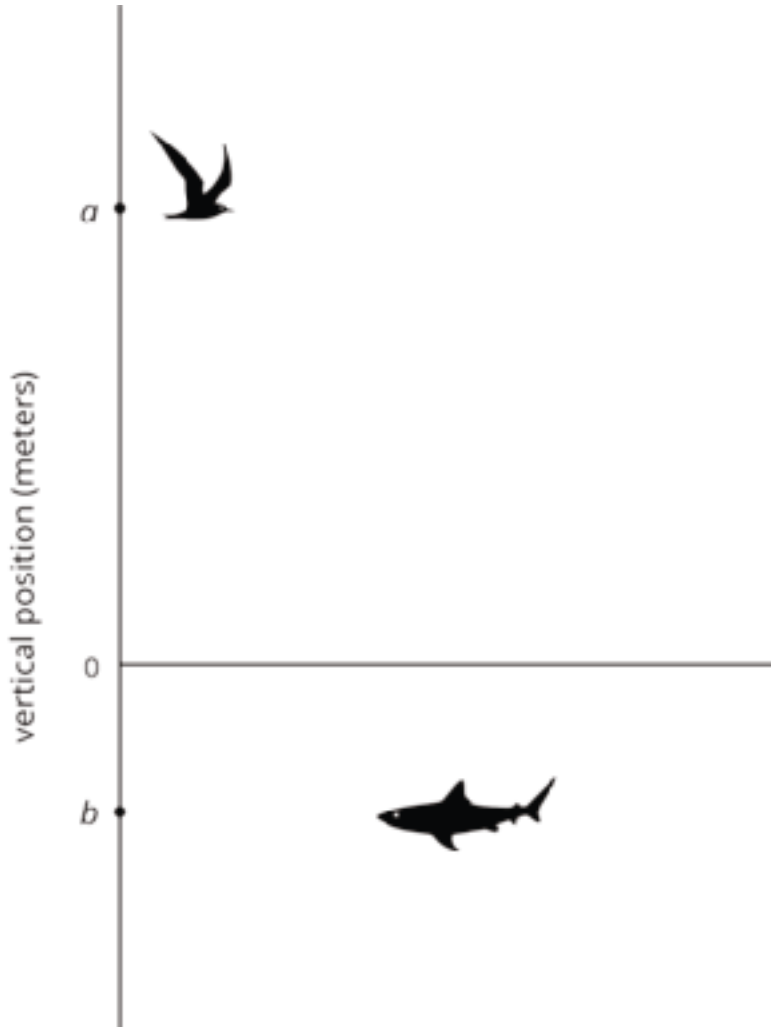
4. When $a = -6$ and $b = -\frac{1}{2}$, which expression:

   has the largest value? has the smallest value? is the closest to zero?

Are you ready for more?

Are there any values could you use for a and b that would make all of these expressions have the same value? Explain your reasoning.

13.4: Seagulls and Sharks Again



A seagull has a vertical position a, and a shark has a vertical position b. Draw and label a point on the vertical axis to show the vertical position of each new animal.

1. A dragonfly at d, where $d = -b$
2. A jellyfish at j, where $j = 2b$
3. An eagle at e, where $e = \frac{1}{4}a$.
4. A clownfish at c, where $c = \frac{-a}{2}$
5. A vulture at v, where $v = a + b$
6. A goose at g, where $g = a - b$

Lesson 13 Summary

We can represent sums, differences, products, and quotients of **rational numbers**, and combinations of these, with numerical and algebraic expressions.

Sums:	Differences:	Products:	Quotients:
$\frac{1}{2} + -9$	$\frac{1}{2} - -9$	$(\frac{1}{2})(-9)$	$\frac{1}{2} \div -9$
$-8.5 + x$	$-8.5 - x$	$-8.5x$	$\frac{-8.5}{x}$

We can write the product of two numbers in different ways.

- By putting a little dot between the factors, like this: $-8.5 \cdot x$.
- By putting the factors next to each other without any symbol between them at all, like this: $-8.5x$.

We can write the quotient of two numbers in different ways as well.

- By writing the division symbol between the numbers, like this: $-8.5 \div x$.
- By writing a fraction bar between the numbers like this: $\frac{-8.5}{x}$.

When we have an algebraic expression like $\frac{-8.5}{x}$ and are given a value for the variable, we can find the value of the expression. For example, if x is 2, then the value of the expression is -4.25, because $-8.5 \div 2 = -4.25$.

Unit 5 Lesson 13 Cumulative Practice Problems

1. The value of x is $\frac{-1}{4}$. Order these expressions from least to greatest:

   x $\quad$ $1 - x$ $\quad$ $x - 1$ $\quad$ $-1 \div x$

2. Here are four expressions that have the value $\frac{-1}{2}$:

   $\frac{-1}{4} + \left(\frac{-1}{4}\right)$ $\quad$ $\frac{1}{2} - 1$ $\quad$ $-2 \cdot \frac{1}{4}$ $\quad$ $-1 \div 2$

   Write five expressions: a sum, a difference, a product, a quotient, and one that involves at least two operations that have the value $\frac{-3}{4}$.

3. Find the value of each expression.

   a. $-22 + 5$

   b. $-22 - (-5)$

   c. $(-22)(-5)$

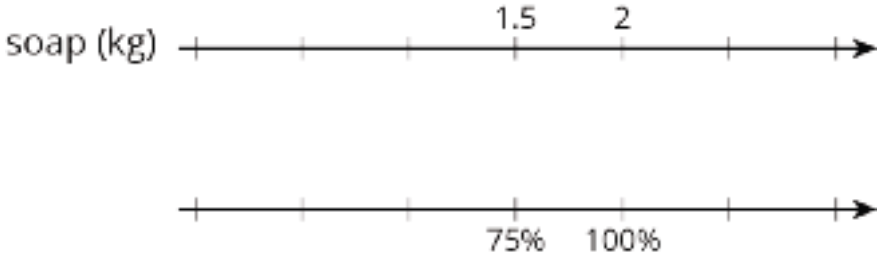
   d. $-22 \div 5$

4. The price of an ice cream cone is \$3.25, but it costs \$3.51 with tax. What is the sales tax rate?

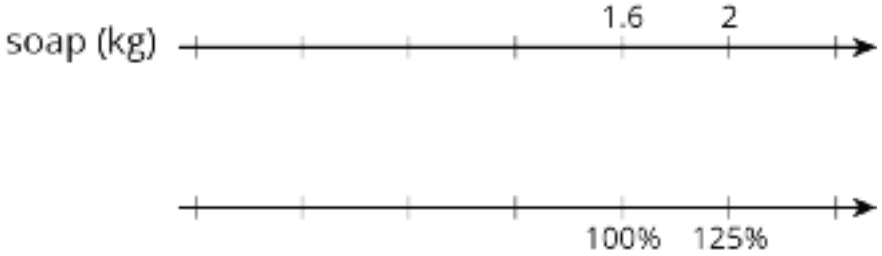
   (From Unit 4, Lesson 10.)

5. Two students are both working on the same problem: A box of laundry soap has 25% more soap in its new box. The new box holds 2 kg. How much soap did the old box hold?

   - Here is how Jada set up her double number line.



   - Here is how Lin set up her double number line.



   Do you agree with either of them? Explain or show your reasoning.

   (From Unit 4, Lesson 7.)

6. a. A coffee maker's directions say to use 2 tablespoons of ground coffee for every 6 ounces of water. How much coffee should you use for 33 ounces of water?

   b. A runner is running a 10 km race. It takes her 17.5 minutes to reach the 2.5 km mark. At that rate, how long will it take her to run the whole race?

   (From Unit 4, Lesson 3.)

Lesson 14: Solving Problems with Rational Numbers

Let's use all four operations with signed numbers to solve problems.

14.1: Which One Doesn't Belong: Equations

Which equation doesn't belong?

$\frac{1}{2}x = -50$

$x + 90 = -100$

$-60t = 30$

$-0.01 = -0.001x$

14.2: Draining and Filling a Tank

A tank of water is being drained. Due to a problem, the sensor does not start working until some time into the draining process. The sensor starts its recording at time zero when there are 770 liters in the tank.

1. Given that the drain empties the tank at a constant rate of 14 liters per minute, complete the table:

time after sensor starts (minutes)	change in water (liters)	expression	water in the tank (liters)
0	0	$770 + (0)(-14)$	770
1	-14	$770 + (1)(-14)$	756
5	-70		
10			

2. Later, someone wants to use the data to find out how long the tank had been draining before the sensor started. Complete this table:

time after sensor starts (minutes)	change in water (liters)	expression	water in the tank (liters)
1	-14	$770 + (1)(-14)$	756
0	0	$770 + (0)(-14)$	770
-1	14	$770 + (-1)(-14)$	784
-2	28		
-3			
-4			
-5			

3. If the sensor started working 15 minutes into the tank draining, how much was in the tank to begin with?

14.3: Buying and Selling Power

A utility company charges $0.12 per kilowatt-hour for energy a customer uses. They give a credit of $0.025 for every kilowatt-hour of electricity a customer with a solar panel generates that they don't use themselves.

A customer has a charge of $82.04 and a credit of -$4.10 on this month's bill.

1. What is the amount due this month?

2. How many kilowatt-hours did they use?

3. How many kilowatt-hours did they generate that they didn't use themselves?

Are you ready for more?

1. Find the value of the expression without a calculator.

$$(2)(-30) + (-3)(-20) + (-6)(-10) - (2)(3)(10)$$

2. Write an expression that uses addition, subtraction, multiplication, and division and only negative numbers that has the same value.

Lesson 14 Summary

We can apply the rules for arithmetic with rational numbers to solve problems.

In general: $a - b = a + -b$

If $a - b = x$, then $x + b = a$. We can add $-b$ to both sides of this second equation to get that $x = a + -b$

Remember: the *distance* between two numbers is independent of the order, but the *difference* depends on the order.

And when multiplying or dividing:

- The sign of a positive number multiplied or divided by a negative number is always negative.
- The sign of a negative number multiplied or divided by a positive number is always negative.
- The sign of a negative number multiplied or divided by a negative number is always positive.

Unit 5 Lesson 14 Cumulative Practice Problems

1. A bank charges a service fee of $7.50 per month for a checking account.

   A bank account has $85.00. If no money is deposited or withdrawn except the service charge, how many months until the account balance is negative?

2. The table shows transactions in a checking account.

January	February	March	April
-38.50	250.00	-14.00	-86.80
126.30	-135.20	99.90	-570.00
429.40	35.50	-82.70	100.00
-265.00	-62.30	-1.50	-280.10

   a. Find the total of the transactions for each month.

   b. Find the mean total for the four months.

3. A large aquarium of water is being filled with a hose. Due to a problem, the sensor does not start working until some time into the filling process. The sensor starts its recording at the time zero minutes. The sensor initially detects the tank has 225 liters of water in it.

   a. The hose fills the aquarium at a constant rate of 15 liters per minute. What will the sensor read at the time 5 minutes?

   b. Later, someone wants to use the data to find the amount of water at times before the sensor started. What should the sensor have read at the time -7 minutes?

4. A furniture store pays a wholesale price for a mattress. Then, the store marks up the retail price to 150% of the wholesale price. Later, they put the mattress on sale for 50% off of the retail price. A customer just bought the mattress on sale and paid $1,200.

   a. What was the retail price of the mattress, before the discount?

   b. What was the wholesale price, before the markup?

(From Unit 4, Lesson 11.)

5. a. A restaurant bill is $21. You leave a 15% tip. How much do you pay including the tip?

b. Which of the following represents the amount a customer pays including the tip of 15% if the bill was b dollars? Select **all** that apply.

- $15b$
- $b + 0.15b$
- $1.15b$
- $1.015b$
- $b + \frac{15}{100}b$
- $b + 0.15$
- $0.15b$

(From Unit 4, Lesson 10.)

Lesson 15: Solving Equations with Rational Numbers

Let's solve equations that include negative values.

15.1: Number Talk: Opposites and Reciprocals

The **variables** a through h all represent *different* numbers. Mentally find numbers that make each equation true.

$\frac{3}{5} \cdot \frac{5}{3} = a$

$7 \cdot b = 1$

$c \cdot d = 1$

$-6 + 6 = e$

$11 + f = 0$

$g + h = 0$

15.2: Match Solutions

1. Match each equation to its solution.

a. $\frac{1}{2}x = -5$	1. $x = -4.5$
b. $-2x = -9$	2. $x = -\frac{1}{2}$
c. $-\frac{1}{2}x = \frac{1}{4}$	3. $x = -10$
d. $-2x = 7$	4. $x = 4.5$
e. $x + -2 = -6.5$	5. $x = 2\frac{1}{2}$
f. $-2 + x = \frac{1}{2}$	6. $x = -3.5$

Be prepared to explain your reasoning.

15.3: Trip to the Mountains

The Hiking Club is on a trip to hike up a mountain.

1. The members increased their elevation 290 feet during their hike this morning. Now they are at an elevation of 450 feet.

    a. Explain how to find their elevation before the hike.

    b. Han says the equation $e + 290 = 450$ describes the situation. What does the variable e represent?

    c. Han says that he can rewrite his equation as $e = 450 + \text{-}290$ to solve for e. Compare Han's strategy to your strategy for finding the beginning elevation.

2. The temperature fell 4 degrees in the last hour. Now it is 21 degrees. Write and solve an equation to find the temperature it was 1 hour ago.

3. There are 3 times as many students participating in the hiking trip this year than last year. There are 42 students on the trip this year.

    a. Explain how to find the number of students that came on the hiking trip last year.

b. Mai says the equation $3s = 42$ describes the situation. What does the variable s represent?

c. Mai says that she can rewrite her equation as $s = \frac{1}{3} \cdot 42$ to solve for s. Compare Mai's strategy to your strategy for finding the number of students on last year's trip.

4. The cost of the hiking trip this year is $\frac{2}{3}$ of the cost of last year's trip. This year's trip cost $32. Write and solve an equation to find the cost of last year's trip.

Are you ready for more?

A number line is shown below. The numbers 0 and 1 are marked on the line, as are two other rational numbers a and b .

Decide which of the following numbers are positive and which are negative.

$a - 1$ $a - 2$ $-b$ $a + b$ $a - b$ $ab + 1$

15.4: Card Sort: Matching Inverses

Your teacher will give you a set of cards with numbers on them.

1. Match numbers with their additive inverses.
2. Next, match numbers with their multiplicative inverses.
3. What do you notice about the numbers and their inverses?

Lesson 15 Summary

To solve the equation $x + 8 = -5$, we can add the opposite of 8, or -8, to each side:

$$\begin{aligned} x + 8 &= -5 \\ (x + 8) + -8 &= (-5) + -8 \\ x &= -13 \end{aligned}$$

Because adding the opposite of a number is the same as subtracting that number, we can also think of it as subtracting 8 from each side.

We can use the same approach for this equation:

$$\begin{aligned} -12 &= t + -\frac{2}{9} \\ (-12) + \frac{2}{9} &= \left(t + -\frac{2}{9}\right) + \frac{2}{9} \\ -11\frac{7}{9} &= t \end{aligned}$$

To solve the equation $8x = -5$, we can multiply each side by the reciprocal of 8, or $\frac{1}{8}$:

$$\begin{aligned} 8x &= -5 \\ \frac{1}{8}(8x) &= \frac{1}{8}(-5) \\ x &= -\frac{5}{8} \end{aligned}$$

Because multiplying by the reciprocal of a number is the same as dividing by that number, we can also think of it as dividing by 8.

We can use the same approach for this equation:

$$\begin{aligned} -12 &= -\frac{2}{9}t \\ -\frac{9}{2}(-12) &= -\frac{9}{2}\left(-\frac{2}{9}t\right) \\ 54 &= t \end{aligned}$$

Unit 5 Lesson 15 Cumulative Practice Problems

1. Solve.

   a. $\frac{2}{5}t = 6$

   b. $-4.5 = a - 8$

   c. $\frac{1}{2} + p = -3$

   d. $12 = x \cdot 3$

   e. $-12 = -3y$

2. Match each equation to a step that will help solve the equation.

A. $5x = 0.4$	1. Multiply each side by 5.
B. $\frac{x}{5} = 8$	2. Multiply each side by -5.
C. $3 = \frac{-x}{5}$	3. Multiply each side by $\frac{1}{5}$.
D. $7 = -5x$	4. Multiply each side by $\frac{-1}{5}$.

3. Evaluate each expression if x is $\frac{2}{5}$, y is -4, and z is -0.2.

   a. $x + y$

   b. $2x - z$

   c. $x + y + z$

   d. $y \cdot x$

(From Unit 5, Lesson 13.)

4. a. Write an equation where a number is added to a variable, and a solution is -8.

   b. Write an equation where a number is multiplied by a variable, and a solution is $\frac{-4}{5}$.

5. The markings on the number line are evenly spaced. Label the other markings on the number line.

(From Unit 5, Lesson 8.)

6. In 2012, James Cameron descended to the bottom of Challenger Deep in the Marianas Trench; the deepest point in the ocean. The vessel he rode in was called DeepSea Challenger.

   Challenger Deep is 35,814 feet deep at its lowest point

   a. DeepSea Challenger's descent was a change in depth of (-4) feet per second. We can use the equation $y = -4x$ to model this relationship, where y is the depth and x is the time in seconds that have passed. How many seconds does this model suggest it would take for DeepSea Challenger to reach the bottom?

   b. To end the mission DeepSea Challenger made a one-hour ascent to the surface. How many seconds is this?

   c. The ascent can be modeled by a different proportional relationship $y = kx$. What is the value of k in this case?

(From Unit 5, Lesson 12.)

Lesson 16: Representing Contexts with Equations

Let’s write equations that represent situations.

16.1: Don't Solve It

Is the solution positive or negative?

$(-8.7)(1.4) = a$

$-8.7b = 1.4$

$-8.7 + c = -1.4$

$-8.7 - d = -1.4$

16.2: Warmer or Colder than Before?

For each situation,

- Find *two* equations that could represent the situation from the bank of equations. (Some equations will not be used.)
- Explain what the variable v represents in the situation.
- Determine the value of the variable that makes the equation true, and explain your reasoning.

Bank of equations:

$-3v = 9$	$v = -16 + 6$	$v = \frac{1}{3} \cdot (-6)$	$v + 12 = 4$
$-4 \cdot 3 = v$	$v = 4 + (-12)$	$v = -16 - (6)$	$v = 9 + 3$
$-4 \cdot -3 = v$	$-3v = -6$	$-6 + v = -16$	$-4 = \frac{1}{3}v$
$v = -\frac{1}{3} \cdot 9$	$v = -\frac{1}{3} \cdot (-6)$	$v = 4 + 12$	$4 = 3v$

1. Between 6 a.m. and noon, the temperature rose 12 degrees Fahrenheit to 4 degrees Fahrenheit.

2. At midnight the temperature was -6 degrees. By 4 a.m. the temperature had fallen to -16 degrees.

3. The temperature is 0 degrees at midnight and dropping 3 degrees per hour. The temperature is -6 degrees at a certain time.

4. The temperature is 0 degrees at midnight and dropping 3 degrees per hour. The temperature is 9 degrees at a certain time.

5. The temperature at 9 p.m. is one third the temperature at midnight.

16.3: Animals Changing Altitudes

1. Match each situation with a diagram.

    a. A penguin is standing 3 feet above sea level and then dives down 10 feet. What is its depth?

    b. A dolphin is swimming 3 feet below sea level and then jumps up 10 feet. What is its height at the top of the jump?

    c. A sea turtle is swimming 3 feet below sea level and then dives down 10 feet. What is its depth?

    d. An eagle is flying 10 feet above sea level and then dives down to 3 feet above sea level. What was its change in altitude?

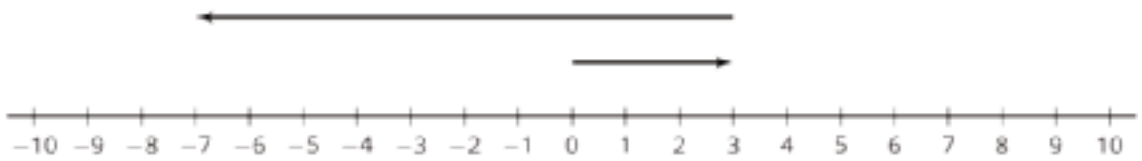
    e. A pelican is flying 10 feet above sea level and then dives down reaching 3 feet below sea level. What was its change in altitude?

    f. A shark is swimming 10 feet below sea level and then swims up reaching 3 feet below sea level. What was its change in depth?

2. Next, write an equation to represent each animal's situation and answer the question. Be prepared to explain your reasoning.

Diagrams

A



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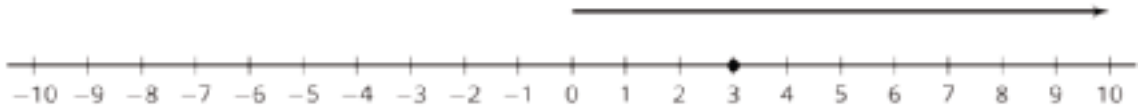

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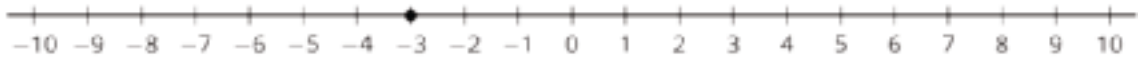

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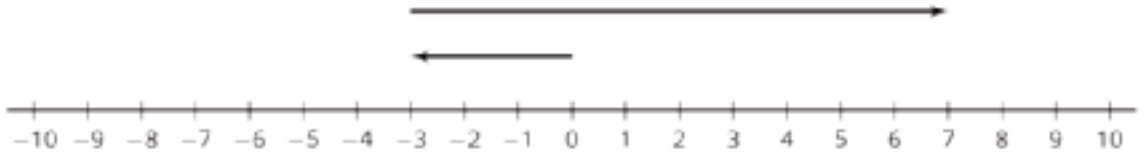

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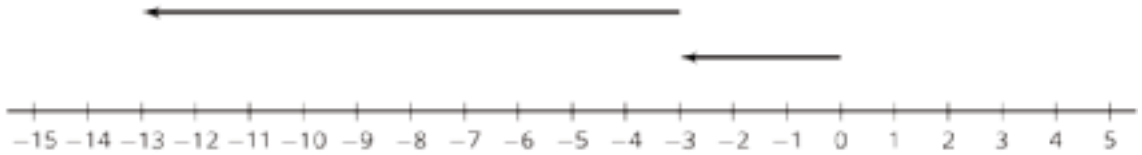

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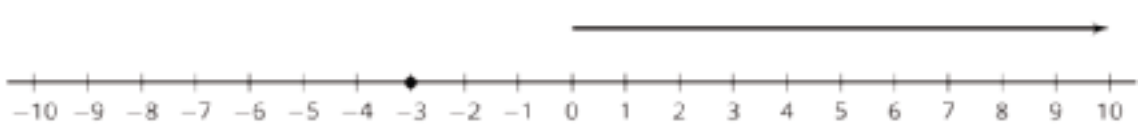


16.4: Equations Tell a Story

Your teacher will assign your group *one* of these situations. Create a visual display about your situation that includes:

- An equation that represents your situation
- What your variable and each term in the equation represent
- How the operations in the equation represent the relationships in the story
- How you use inverses to solve for the unknown quantity
- The solution to your equation

1. As a $7\frac{1}{4}$ inch candle burns down, its height decreases $\frac{3}{4}$ inch each hour. How many hours does it take for the candle to burn completely?

2. On Monday $\frac{1}{9}$ of the enrolled students in a school were absent. There were 4,512 students present. How many students are enrolled at the school?

3. A hiker begins at sea level and descends 25 feet every minute. How long will it take to get to an elevation of -750 feet?

4. Jada practices the violin for the same amount of time every day. On Tuesday she practices for 35 minutes. How much does Jada practice in a week?

5. The temperature has been dropping $2\frac{1}{2}$ degrees every hour and the current temperature is -15°F. How many hours ago was the temperature 0°F?

6. The population of a school increased by 12%, and now the population is 476. What was the population before the increase?

7. During a 5% off sale, Diego pays $74.10 for a new hockey stick. What was the original price?

8. A store buys sweaters for $8 and sells them for $26. How many sweaters does the store need to sell to make a profit of $990?

Are you ready for more?

Diego and Elena are 2 miles apart and begin walking towards each other. Diego walks at a rate of 3.7 miles per hour and Elena walks 4.3 miles per hour. While they are walking, Elena's dog runs back and forth between the two of them, at a rate of 6 miles per hour. Assuming the dog does not lose any time in turning around, how far has the dog run by the time Diego and Elena reach each other?

Lesson 16 Summary

We can use variables and equations involving signed numbers to represent a story or answer questions about a situation.

For example, if the temperature is -3°C and then falls to -17°C, we can let x represent the temperature change and write the equation:

$$-3 + x = -17$$

We can solve the equation by adding 3 to each side. Since $-17 + 3 = -14$, the change is -14°C.

Here is another example: if a starfish is descending by $\frac{3}{2}$ feet every hour then we can solve

$$-\frac{3}{2}h = -6$$

to find out how many hours h it takes the starfish to go down 6 feet.

We can solve this equation by multiplying each side by $-\frac{2}{3}$. Since $-\frac{2}{3} \cdot -6 = 4$, we know it will take the starfish 4 hours to descend 6 feet.

Unit 5 Lesson 16 Cumulative Practice Problems

1. Match each situation to one of the equations.

   A. A whale was diving at a rate of 2 meters per second. How long will it take for the whale to get from the surface of the ocean to an elevation of -12 meters at that rate?

   B. A swimmer dove below the surface of the ocean. After 2 minutes, she was 12 meters below the surface. At what rate was she diving?

   C. The temperature was -12 degrees Celsius and rose to 2 degrees Celsius. What was the change in temperature?

   D. The temperature was 2 degrees Celsius and fell to -12 degrees Celsius. What was the change in temperature?

   1. $-12 + x = 2$
   2. $2 + x = -12$
   3. $-2x = -12$
   4. $2x = -12$

2. Starting at noon, the temperature dropped steadily at a rate of 0.8 degrees Celsius every hour.

   For each of these situations, write and solve an equation and describe what your variable represents.

   a. How many hours did it take for the temperature to decrease by 4.4 degrees Celsius?

   b. If the temperature after the 4.4 degree drop was -2.5 degrees Celsius, what was the temperature at noon?

3. Kiran mixes $\frac{3}{4}$ cups of raisins, 1 cup peanuts, and $\frac{1}{2}$ cups of chocolate chips to make trail mix. How much of each ingredient would he need to make 10 cups of trail mix? Explain your reasoning.

(From Unit 4, Lesson 3.)

4. Find the value of each expression.

   a. $12 + -10$

   b. $-5 - 6$

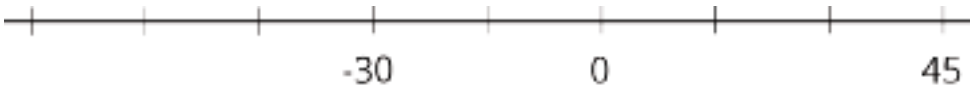
   c. $-42 + 17$

   d. $35 - -8$

   e. $-4\frac{1}{2} + 3$

(From Unit 5, Lesson 6.)

5. The markings on the number line are evenly spaced. Label the other markings on the number line.



(From Unit 5, Lesson 8.)

6. Kiran drinks 6.4 oz of milk each morning. How many days does it take him to finish a 32 oz container of milk?

   a. Write and solve an equation for the situation.

   b. What does the variable represent?

Lesson 17: The Stock Market

Let's learn about the Stock Market.

17.1: Revisiting Interest and Depreciation

1. Lin deposited $300 in a savings account that has a 2% interest rate per year. How much is in her account after 1 year? After 2 years?

2. Diego wants to sell his bicycle. It cost $150 when he bought it but has depreciated by 15%. How much should he sell it for?

17.2: Gains and Losses

1. Here is some information from the stock market in September 2016. Complete the table.

company	value on day 1 (dollars)	value on day 2 (dollars)	change in value (dollars)	change in value as a percentage of day 1 value
Mobile Tech Company	107.95	111.77	3.82	3.54%
Electrical Appliance Company		114.03	2.43	2.18%
Oil Corporation	26.14	25.14		-3.83%
Department Store Company	7.38	7.17		
Jewelry Company		70.30		2.27%

2. Which company's change in dollars had the largest magnitude?

3. Which company's change in percentage had the largest magnitude?

17.3: What is a Stock Portfolio?

A person who wants to make money by investing in the stock market usually buys a portfolio, or a collection of different stocks. That way, if one of the stocks decreases in value, they won't lose all of their money at once.

1. Here is an example of someone's stock portfolio. Complete the table to show the total value of each investment.

name	price (dollars)	number of shares	total value (dollars)
Technology Company	107.75	98	
Airline Company	133.54	27	
Film Company	95.95	135	
Sports Clothing Company	58.96	100	

2. Here is the same portfolio the next year. Complete the table to show the new total value of each investment.

company	old price (dollars)	price change	new price (dollars)	number of shares	total value (dollars)
Technology Company	107.75	+2.43%		98	
Airline Company	133.54	-7.67%		27	
Film Company	95.95		87.58	135	
Sports Clothing Company	58.96	-5.56%		100	

3. Did the entire portfolio increase or decrease in value over the year?

17.4: Your Own Stock Portfolio

Your teacher will give you a list of stocks.

1. Select a combination of stocks with a total value close to, but no more than, $100.

2. Using the new list, how did the total value of your selected stocks change?

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