

Proportional Relationships and Percentages

Percent increase and decrease

Student Workbook



Tax and Tip



A train is traveling at...



Measuring a soccer field

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Proportional Relationships and Percentages

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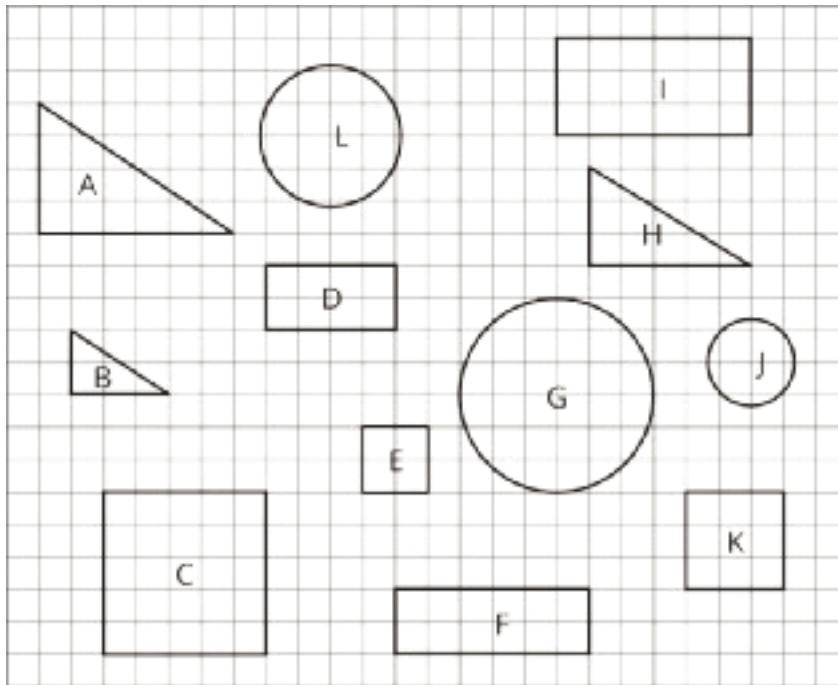
Proportional Relationships and Percentages
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Core Knowledge Mathematics™

Lesson 1: Lots of Flags

Let's explore the U.S. flag.

1.1: Scaled or Not?

1. Which of the geometric objects are scaled versions of each other?



2. Pick two of the objects that are scaled copies and find the scale factor.

1.2: Flags Are Many Sizes

One standard size for the United States flag is 19 feet by 10 feet. On a flag of this size, the union (the blue rectangle in the top-left corner) is $7\frac{5}{8}$ feet by $5\frac{3}{8}$ feet.

There are many places that display flags of different sizes.

- Many classrooms display a U.S. flag.
- Flags are often displayed on stamps.
- There was a flag on the space shuttle.
- Astronauts on the Apollo missions had a flag on a shoulder patch.



1. Choose one of the four options and decide on a size that would be appropriate for this flag. Find the size of the union.

2. Share your answer with another group that used a different option. What do your dimensions have in common?

1.3: What Percentage Is the Union?

On a U.S. flag that is 19 feet by 10 feet, the union is $7\frac{5}{8}$ feet by $5\frac{3}{8}$ feet. For each question, first estimate the answer and then compute the actual percentage.

1. What **percentage** of the flag is taken up by the union?

2. What percentage of the flag is red? Be prepared to share your reasoning.

Are you ready for more?

The largest U.S. flag in the world is 225 feet by 505 feet.

1. Is the ratio of the length to the width equivalent to 1 : 1.9, the ratio for official government flags?

2. If a square yard of the flag weighs about 3.8 ounces, how much does the entire flag weigh in pounds?

Lesson 1 Summary

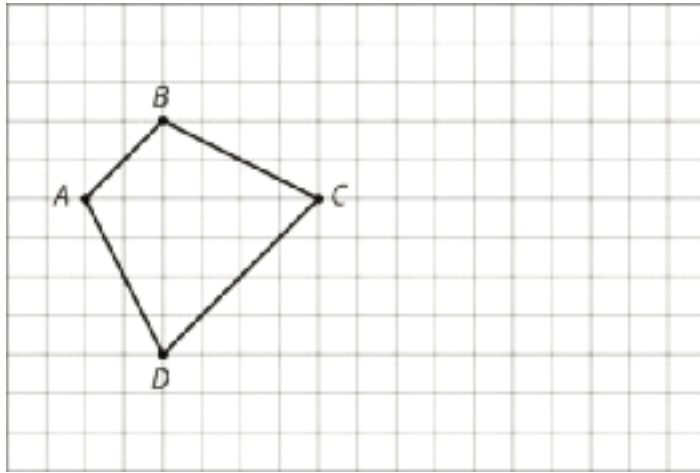
Imagine you have a painting that is 15 feet wide and 5 feet high. To sketch a scaled copy of the painting, the ratio of the width and height of a scaled copy must be equivalent to 15 : 5. What is the height of a scaled copy that is 2 feet across?

width	height
15	5
2	h

We know that the height is $\frac{1}{3}$ the width, so $h = \frac{1}{3} \cdot 2$ or $\frac{2}{3}$.

Sometimes ratios include fractions and decimals. We will be working with these kinds of ratios in the next few lessons.

4. On the grid, draw a scaled copy of quadrilateral ABCD with a scale factor $\frac{2}{3}$.



(From Unit 1, Lesson 4.)

5. Solve each equation mentally.

a. $\frac{5}{2} \cdot x = 1$

b. $x \cdot \frac{7}{3} = 1$

c. $1 \div \frac{11}{2} = x$

(From Unit 1, Lesson 5.)

6. Lin has a scale model of a modern train. The model is created at a scale of 1 to 48.

a. The height of the model train is 102 millimeters. What is the actual height of the train in meters? Explain your reasoning.

b. On the scale model, the distance between the wheels on the left and the wheels on the right is $1\frac{1}{4}$ inches. The state of Wyoming has old railroad tracks that are 4.5 feet apart. Can the modern train travel on those tracks? Explain your reasoning.

(From Unit 1, Lesson 11.)

Lesson 2: Ratios and Rates With Fractions

Let's calculate some rates with fractions.

2.1: Number Talk: Division

Find each quotient mentally.

$$5 \div \frac{1}{3}$$

$$2 \div \frac{1}{3}$$

$$\frac{1}{2} \div \frac{1}{3}$$

$$2\frac{1}{2} \div \frac{1}{3}$$

2.2: A Train is Traveling at . . .

A train is traveling at a constant speed and goes 7.5 kilometers in 6 minutes. At that rate:

1. How far does the train go in 1 minute?



2. How far does the train go in 100 minutes?

2.3: Comparing Running Speeds

Lin ran $2\frac{3}{4}$ miles in $\frac{2}{5}$ of an hour. Noah ran $8\frac{2}{3}$ miles in $\frac{4}{3}$ of an hour.

1. Pick one of the questions that was displayed, but don't tell anyone which question you picked. Find the answer to the question.

2. When you and your partner are both done, share the answer you got (do not share the question) and ask your partner to guess which question you answered. If your partner can't guess, explain the process you used to answer the question.

3. Switch with your partner and take a turn guessing the question that your partner answered.

Are you ready for more?

Nothing can go faster than the speed of light, which is 299,792,458 meters per second. Which of these are possible?

1. Traveling a billion meters in 5 seconds.

2. Traveling a meter in 2.5 nanoseconds. (A nanosecond is a billionth of a second.)

3. Traveling a parsec in a year. (A parsec is about 3.26 light years and a light year is the distance light can travel in a year.)

2.4: Scaling the Mona Lisa

In real life, the Mona Lisa measures $2\frac{1}{2}$ feet by $1\frac{3}{4}$ feet. A company that makes office supplies wants to print a scaled copy of the Mona Lisa on the cover of a notebook that measures 11 inches by 9 inches.

1. What size should they use for the scaled copy of the Mona Lisa on the notebook cover?
2. What is the scale factor from the real painting to its copy on the notebook cover?
3. Discuss your thinking with your partner. Did you use the same scale factor? If not, is one more reasonable than the other?

Lesson 2 Summary

There are 12 inches in a foot, so we can say that for every 1 foot, there are 12 inches, or the ratio of feet to inches is 1 : 12. We can find the **unit rates** by dividing the numbers in the ratio:

$1 \div 12 = \frac{1}{12}$
so there is $\frac{1}{12}$ foot per inch.

$12 \div 1 = 12$
so there are 12 inches per foot.

The numbers in a ratio can be fractions, and we calculate the unit rates the same way: by dividing the numbers in the ratio. For example, if someone runs $\frac{3}{4}$ mile in $\frac{11}{2}$ minutes, the ratio of minutes to miles is $\frac{11}{2} : \frac{3}{4}$.

$\frac{11}{2} \div \frac{3}{4} = \frac{22}{3}$, so the person's pace is $\frac{22}{3}$ minutes per mile.

$\frac{3}{4} \div \frac{11}{2} = \frac{3}{22}$, so the person's speed is $\frac{3}{22}$ miles per minute.

Unit 4 Lesson 2 Cumulative Practice Problems

1. A cyclist rode 3.75 miles in 0.3 hours.

- a. How fast was she going in miles per hour?

- b. At that rate, how long will it take her to go 4.5 miles?

2. A recipe for sparkling grape juice calls for $1\frac{1}{2}$ quarts of sparkling water and $\frac{3}{4}$ quart of grape juice.

- a. How much sparkling water would you need to mix with 9 quarts of grape juice?

- b. How much grape juice would you need to mix with $\frac{15}{4}$ quarts of sparkling water?

- c. How much of each ingredient would you need to make 100 quarts of punch?

3. At a deli counter,

- Someone bought $1\frac{3}{4}$ pounds of ham for \$14.50.
- Someone bought $2\frac{1}{2}$ pounds of turkey for \$26.25.
- Someone bought $\frac{3}{8}$ pounds of roast beef for \$5.50.

Which meat is the least expensive per pound? Which meat is the most expensive per pound? Explain how you know.

4. a. Draw a scaled copy of the circle using a scale factor of 2.



b. How does the circumference of the scaled copy compare to the circumference of the original circle?

c. How does the area of the scaled copy compare to the area of the original circle?

(From Unit 3, Lesson 10.)

5. Jada has a scale map of Kansas that fits on a page in her book. The page is 5 inches by 8 inches. Kansas is about 210 miles by 410 miles. Select **all** scales that could be a scale of the map. (There are 2.54 centimeters in an inch.)

- A. 1 in to 1 mi
- B. 1 cm to 1 km
- C. 1 in to 10 mi
- D. 1 ft to 100 mi
- E. 1 cm to 200 km
- F. 1 in to 100 mi
- G. 1 cm to 1000 km

(From Unit 1, Lesson 11.)

Lesson 3: Revisiting Proportional Relationships

Let's use constants of proportionality to solve more problems.

3.1: Recipe Ratios

A recipe calls for $\frac{1}{2}$ cup sugar and 1 cup flour. Complete the table to show how much sugar and flour to use in different numbers of batches of the recipe.

sugar (cups)	flour (cups)
$\frac{1}{2}$	1
$\frac{3}{4}$	
	$1\frac{3}{4}$
1	
	$2\frac{1}{2}$

3.2: The Price of Rope

Two students are solving the same problem: At a hardware store, they can cut a length of rope off of a big roll, so you can buy any length you like. The cost for 6 feet of rope is \$7.50. How much would you pay for 50 feet of rope, at this rate?

1. Kiran knows he can solve the problem this way.

length of rope (feet)	price of rope (dollars)
6	7.50
1	1.25
50	

What would be Kiran's answer?

2. Kiran wants to know if there is a more efficient way of solving the problem. Priya says she can solve the problem with only 2 rows in the table.

length of rope (feet)	price of rope (dollars)
6	7.50
50	

What do you think Priya's method is?

3.3: Swimming, Manufacturing, and Painting

1. Tyler swims at a constant speed, 5 meters every 4 seconds. How long does it take him to swim 114 meters?

distance (meters)	time (seconds)
5	4
114	

2. A factory produces 3 bottles of sparkling water for every 8 bottles of plain water. How many bottles of sparkling water does the company produce when it produces 600 bottles of plain water?

number of bottles of sparkling water	number of bottles of plain water

3. A certain shade of light blue paint is made by mixing $1\frac{1}{2}$ quarts of blue paint with 5 quarts of white paint. How much white paint would you need to mix with 4 quarts of blue paint?

4. For each of the previous three situations, write an equation to represent the proportional relationship.

Are you ready for more?

Different nerve signals travel at different speeds.

- Pressure and touch signals travel about 250 feet per second.
 - Dull pain signals travel about 2 feet per second.
1. How long does it take you to feel an ant crawling on your foot?
 2. How much longer does it take to feel a dull ache in your foot?

3.4: Finishing the Race and More Orange Juice

1. Lin runs $2\frac{3}{4}$ miles in $\frac{2}{5}$ of an hour. Tyler runs $8\frac{2}{3}$ miles in $\frac{4}{3}$ of an hour. How long does it take each of them to run 10 miles at that rate?
2. Priya mixes $2\frac{1}{2}$ cups of water with $\frac{1}{3}$ cup of orange juice concentrate. Diego mixes $1\frac{2}{3}$ cups of water with $\frac{1}{4}$ cup orange juice concentrate. How much concentrate should each of them mix with 100 cups of water to make juice that tastes the same as their original recipe? Explain or show your reasoning.

Lesson 3 Summary

If we identify two quantities in a problem and one is proportional to the other, then we can calculate the constant of proportionality and use it to answer other questions about the situation. For example, Andre runs at a constant speed, 5 meters every 2 seconds. How long does it take him to run 91 meters at this rate?

In this problem there are two quantities, time (in seconds) and distance (in meters). Since Andre is running at a constant speed, time is proportional to distance. We can make a table with distance and time as column headers and fill in the given information.

distance (meters)	time (seconds)
5	2
91	

To find the value in the right column, we multiply the value in the left column by $\frac{2}{5}$ because $\frac{2}{5} \cdot 5 = 2$. This means that it takes Andre $\frac{2}{5}$ seconds to run one meter.

At this rate, it would take Andre $\frac{2}{5} \cdot 91 = \frac{182}{5}$, or 36.4 seconds to walk 91 meters. In general, if t is the time it takes to walk d meters at that pace, then $t = \frac{2}{5}d$.

Unit 4 Lesson 3 Cumulative Practice Problems

1. It takes an ant farm 3 days to consume $\frac{1}{2}$ of an apple. At that rate, in how many days will the ant farm consume 3 apples?

2. To make a shade of paint called jasper green, mix 4 quarts of green paint with $\frac{2}{3}$ cups of black paint. How much green paint should be mixed with 4 cups of black paint to make jasper green?

3. An airplane is flying from New York City to Los Angeles. The distance it travels in miles, d , is related to the time in seconds, t , by the equation $d = 0.15t$.

a. How fast is it flying? Be sure to include the units.

b. How far will it travel in 30 seconds?

c. How long will it take to go 12.75 miles?

4. A grocer can buy strawberries for \$1.38 per pound.
- Write an equation relating c , the cost, and p , the pounds of strawberries.
 - A strawberry order cost \$241.50. How many pounds did the grocer order?
5. Crater Lake in Oregon is shaped like a circle with a diameter of about 5.5 miles.
- How far is it around the perimeter of Crater Lake?
 - What is the area of the surface of Crater Lake?

(From Unit 3, Lesson 10.)

6. A 50-centimeter piece of wire is bent into a circle. What is the area of this circle?

(From Unit 3, Lesson 8.)

7. Suppose Quadrilaterals A and B are both squares. Are A and B necessarily scaled copies of one another? Explain.

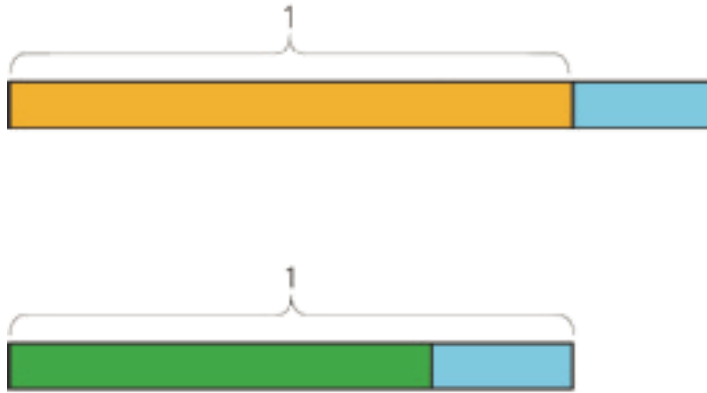
(From Unit 1, Lesson 2.)

Lesson 4: Half as Much Again

Let's use fractions to describe increases and decreases.

4.1: Notice and Wonder: Tape Diagrams

What do you notice? What do you wonder?



4.2: Walking Half as Much Again

1. Complete the table to show the total distance walked in each case.

- a. Jada's pet turtle walked 10 feet, and then half that length again.
- b. Jada's baby brother walked 3 feet, and then half that length again.
- c. Jada's hamster walked 4.5 feet, and then half that length again.
- d. Jada's robot walked 1 foot, and then half that length again.
- e. A person walked x feet and then half that length again.

initial distance	total distance
10	
3	
4.5	
1	
x	

2. Explain how you computed the total distance in each case.

3. Two students each wrote an equation to represent the relationship between the initial distance walked (x) and the total distance walked (y).

◦ Mai wrote $y = x + \frac{1}{2}x$.

◦ Kiran wrote $y = \frac{3}{2}x$.

Do you agree with either of them? Explain your reasoning.

Are you ready for more?

Zeno jumped 8 meters. Then he jumped half as far again (4 meters). Then he jumped half as far again (2 meters). So after 3 jumps, he was $8 + 4 + 2 = 14$ meters from his starting place.

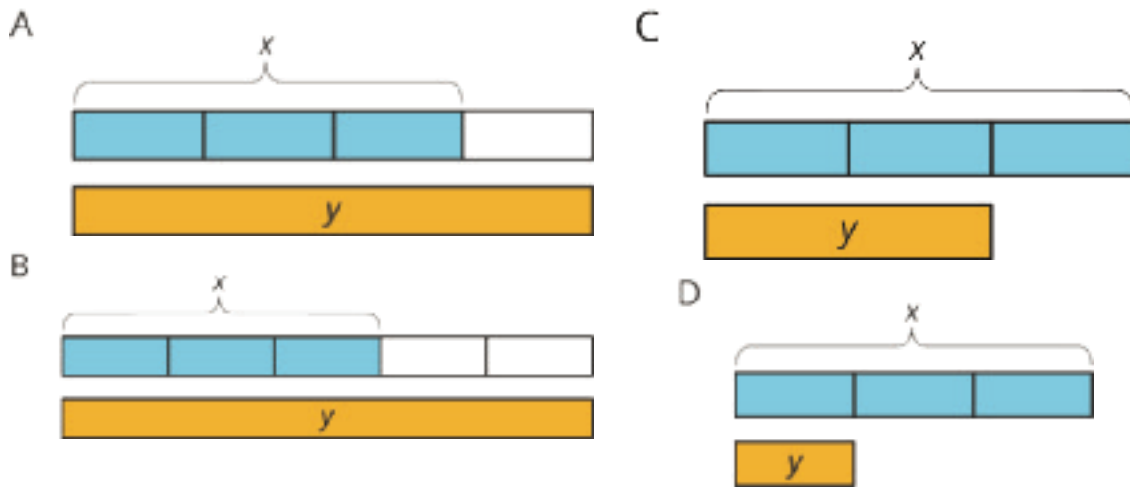
1. Zeno kept jumping half as far again. How far would he be after 4 jumps? 5 jumps? 6 jumps?

2. Before he started jumping, Zeno put a mark on the floor that was exactly 16 meters from his starting place. How close can Zeno get to the mark if he keeps jumping half as far again?

3. If you enjoyed thinking about this problem, consider researching Zeno's Paradox.

4.3: More and Less

1. Match each situation with a diagram. A diagram may not have a match.



- Han ate x ounces of blueberries. Mai ate $\frac{1}{3}$ less than that.
- Mai biked x miles. Han biked $\frac{2}{3}$ more than that.
- Han bought x pounds of apples. Mai bought $\frac{2}{3}$ of that.

2. For each diagram, write an equation that represents the relationship between x and y .

a. Diagram A:

b. Diagram B:

c. Diagram C:

d. Diagram D:

3. Write a story for one of the diagrams that doesn't have a match.

4.4: Card Sort: Representations of Proportional Relationships

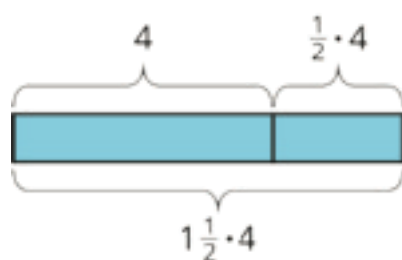
Your teacher will give you a set of cards that have proportional relationships represented three different ways: as descriptions, equations, and tables. Mix up the cards and place them all face-up.

1. Take turns with a partner to match a description with an equation and a table.
 - a. For each match you find, explain to your partner how you know it's a match.
 - b. For each match your partner finds, listen carefully to their explanation, and if you disagree, explain your thinking.
2. When you agree on all of the matches, check your answers with the answer key. If there are any errors, discuss why and revise your matches.

Lesson 4 Summary

Using the distributive property provides a shortcut for calculating the final amount in situations that involve adding or subtracting a fraction of the original amount.

For example, one day Clare runs 4 miles. The next day, she plans to run that same distance plus half as much again. How far does she plan to run the next day?



Tomorrow she will run 4 miles plus $\frac{1}{2}$ of 4 miles. We can use the distributive property to find this in one

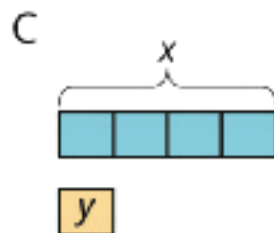
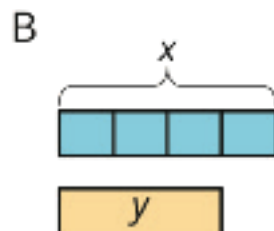
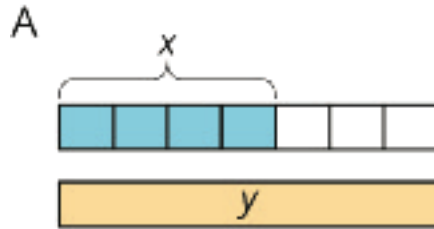
$$\text{step: } 1 \cdot 4 + \frac{1}{2} \cdot 4 = \left(1 + \frac{1}{2}\right) \cdot 4$$

Clare plans to run $1\frac{1}{2} \cdot 4$, or 6 miles.

This works when we decrease by a fraction, too. If Tyler spent x dollars on a new shirt, and Noah spent $\frac{1}{3}$ less than Tyler, then Noah spent $\frac{2}{3}x$ dollars since $x - \frac{1}{3}x = \frac{2}{3}x$.

Unit 4 Lesson 4 Cumulative Practice Problems

1. Match each situation with a diagram.



A. Diagram A

B. Diagram B

C. Diagram C

1. Diego drank x ounces of juice. Lin drank $\frac{1}{4}$ less than that.

2. Lin ran x miles. Diego ran $\frac{3}{4}$ more than that.

3. Diego bought x pounds of almonds. Lin bought $\frac{1}{4}$ of that.

2. Elena walked 12 miles. Then she walked $\frac{1}{4}$ that distance. How far did she walk all together? Select **all** that apply.

A. $12 + \frac{1}{4}$

B. $12 \cdot \frac{1}{4}$

C. $12 + \frac{1}{4} \cdot 12$

D. $12 \left(1 + \frac{1}{4}\right)$

E. $12 \cdot \frac{3}{4}$

F. $12 \cdot \frac{5}{4}$

3. Write a story that can be represented by the equation $y = x + \frac{1}{4}x$.

4. Select **all** ratios that are equivalent to 4 : 5.

A. 2 : 2.5

B. 2 : 3

C. 3 : 3.75

D. 7 : 8

E. 8 : 10

F. 14 : 27.5

(From Unit 4, Lesson 1.)

5. Jada is making circular birthday invitations for her friends. The diameter of the circle is 12 cm. She bought 180 cm of ribbon to glue around the edge of each invitation. How many invitations can she make?

(From Unit 3, Lesson 10.)

Lesson 5: Say It with Decimals

Let's use decimals to describe increases and decreases.

5.1: Notice and Wonder: Fractions to Decimals

A calculator gives the following decimal representations for some unit fractions:

$$\frac{1}{2} = 0.5$$

$$\frac{1}{7} = 0.142857143$$

$$\frac{1}{3} = 0.3333333$$

$$\frac{1}{8} = 0.125$$

$$\frac{1}{4} = 0.25$$

$$\frac{1}{9} = 0.1111111$$

$$\frac{1}{5} = 0.2$$

$$\frac{1}{10} = 0.1$$

$$\frac{1}{6} = 0.1666667$$

$$\frac{1}{11} = 0.0909091$$

What do you notice? What do you wonder?

5.2: Repeating Decimals

1. Use **long division** to express each fraction as a decimal.

$$\frac{9}{25}$$

$$\frac{11}{30}$$

$$\frac{4}{11}$$

2. What is similar about your answers to the previous question? What is different?

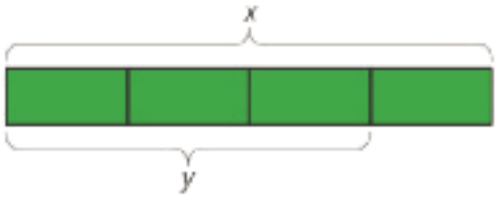
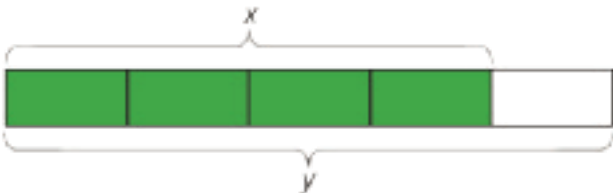
3. Use the decimal representations to decide which of these fractions has the greatest value. Explain your reasoning.

Are you ready for more?

One common approximation for π is $\frac{22}{7}$. Express this fraction as a decimal. How does this approximation compare to 3.14?

5.3: More and Less with Decimals

1. Match each diagram with a description and an equation.

	Diagrams:	Descriptions:	Equations:
A		An increase by $\frac{1}{4}$	$y = 1.\overline{6}x$
		An increase by $\frac{1}{3}$	$y = 1.\overline{3}x$
		An increase by $\frac{2}{3}$	$y = 0.75x$
B		A decrease by $\frac{1}{5}$	$y = 0.4x$
		A decrease by $\frac{1}{4}$	$y = 1.25x$

2. Draw a diagram for one of the unmatched equations.

5.4: Card Sort: More Representations

Your teacher will give you a set of cards that have proportional relationships represented 2 different ways: as descriptions and equations. Mix up the cards and place them all face-up.

Take turns with a partner to match a description with an equation.

1. For each match you find, explain to your partner how you know it's a match.
2. For each match your partner finds, listen carefully to their explanation, and if you disagree, explain your thinking.
3. When you have agreed on all of the matches, check your answers with the answer key. If there are any errors, discuss why and revise your matches.

Lesson 5 Summary

Long division gives us a way of finding decimal representations for fractions.

For example, to find a decimal representation for $\frac{9}{8}$, we can divide 9 by 8.

$$\begin{array}{r} 1.125 \\ 8 \overline{)9.000} \\ \underline{8} \\ 10 \\ \underline{8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

So $\frac{9}{8} = 1.125$.

Sometimes it is easier to work with the decimal representation of a number, and sometimes it is easier to work with its fraction representation. It is important to be able to work with both. For example, consider the following pair of problems:

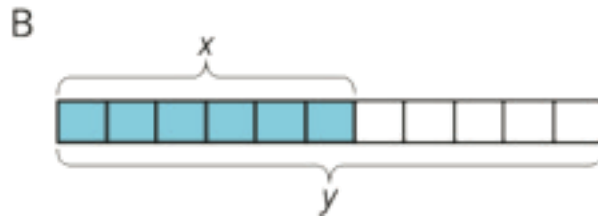
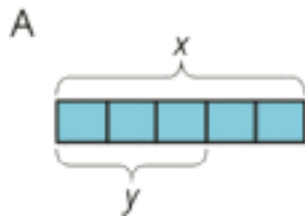
- Priya earned x dollars doing chores, and Kiran earned $\frac{6}{5}$ as much as Priya. How much did Kiran earn?
- Priya earned x dollars doing chores, and Kiran earned 1.2 times as much as Priya. How much did Kiran earn?

Since $\frac{6}{5} = 1.2$, these are both exactly the same problem, and the answer is $\frac{6}{5}x$ or $1.2x$.

When we work with percentages in later lessons, the decimal representation will come in especially handy.

Unit 4 Lesson 5 Cumulative Practice Problems

1. a. Match each diagram with a description and an equation.



Descriptions:

An increase by $\frac{2}{3}$

An increase by $\frac{5}{6}$

A decrease by $\frac{2}{5}$

A decrease by $\frac{5}{11}$

Equations:

$$y = 1.8\bar{3}x$$

$$y = 1.\bar{6}x$$

$$y = 0.6x$$

$$y = 0.4x$$

- b. Draw a diagram for one of the unmatched equations.

2. At the beginning of the month, there were 80 ounces of peanut butter in the pantry. Since then, the family ate 0.3 of the peanut butter. How many ounces of peanut butter are in the pantry now?

A. $0.7 \cdot 80$

B. $0.3 \cdot 80$

C. $80 - 0.3$

D. $(1 + 0.3) \cdot 80$

3. a. On a hot day, a football team drank an entire 50-gallon cooler of water and half as much again. How much water did they drink?
- b. Jada has 12 library books checked out and Han has $\frac{1}{3}$ less than that. How many books does Han have checked out?

(From Unit 4, Lesson 4.)

4. If x represents a positive number, select **all** expressions whose value is greater than x .

A. $(1 - \frac{1}{4})x$

B. $(1 + \frac{1}{4})x$

C. $\frac{7}{8}x$

D. $\frac{9}{8}x$

(From Unit 4, Lesson 4.)

5. A person's resting heart rate is typically between 60 and 100 beats per minute. Noah looks at his watch, and counts 8 heartbeats in 10 seconds.

a. Is his heart rate typical? Explain how you know.

b. Write an equation for h , the number of times Noah's heart beats (at this rate) in m minutes.

(From Unit 2, Lesson 6.)

Lesson 6: Increasing and Decreasing

Let's use percentages to describe increases and decreases.

6.1: Improving Their Game

Here are the scores from 3 different sports teams from their last 2 games.

sports team	total points in game 1	total points in game 2
football team	22	30
basketball team	100	108
baseball team	4	12

1. What do you notice about the teams' scores? What do you wonder?

2. Which team improved the most? Explain your reasoning.

6.2: More Cereal and a Discounted Shirt

1. A cereal box says that now it contains 20% more. Originally, it came with 18.5 ounces of cereal. How much cereal does the box come with now?



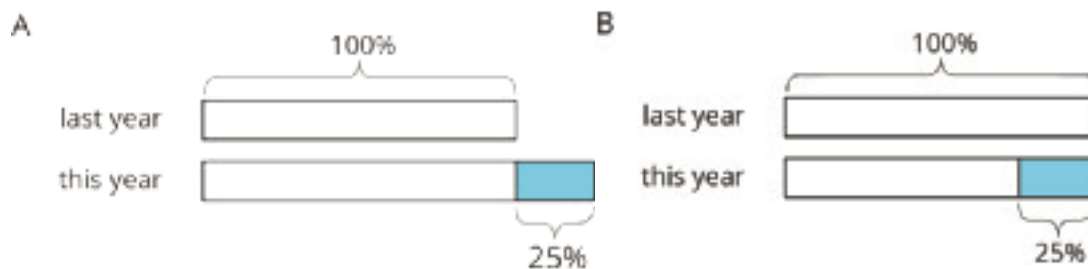
2. The price of a shirt is \$18.50, but you have a coupon that lowers the price by 20%. What is the price of the shirt after using the coupon?



6.3: Using Tape Diagrams

1. Match each situation to a diagram. Be prepared to explain your reasoning.

- a. Compared with last year's strawberry harvest, this year's strawberry harvest is a 25% increase.
- b. This year's blueberry harvest is 75% of last year's.
- c. Compared with last year, this year's peach harvest decreased 25%.
- d. This year's plum harvest is 125% of last year's plum harvest.



2. Draw a diagram to represent these situations.

- a. The number of ducks living at the pond increased by 40%.

- b. The number of mosquitoes decreased by 80%.

Are you ready for more?

What could it mean to say there is a 100% decrease in a quantity? Give an example of a quantity where this makes sense.

6.4: Agree or Disagree: Percentages

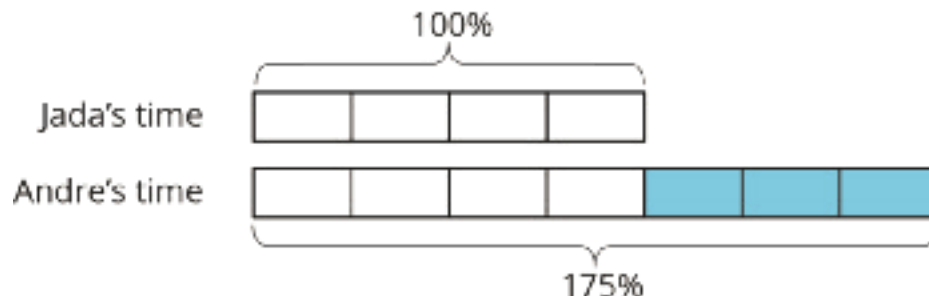
Do you agree or disagree with each statement? Explain your reasoning.

1. Employee A gets a pay raise of 50%. Employee B gets a pay raise of 45%. So Employee A gets the bigger pay raise.

2. Shirts are on sale for 20% off. You buy two of them. As you pay, the cashier says, "20% off of each shirt means 40% off of the total price."

Lesson 6 Summary

Imagine that it takes Andre $\frac{3}{4}$ more than the time it takes Jada to get to school. Then we know that Andre's time is $1\frac{3}{4}$ or 1.75 times Jada's time. We can also describe this in terms of percentages:

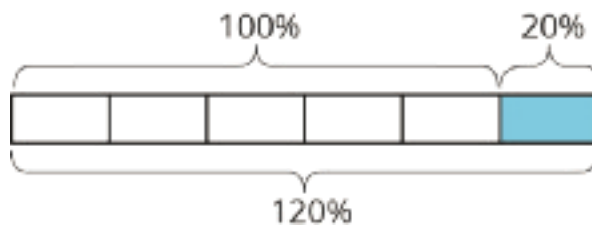


We say that Andre's time is 75% more than Jada's time. We can also see that Andre's time is 175% of Jada's time. In general, the terms **percent increase** and **percent decrease** describe an increase or decrease in a quantity as a percentage of the starting amount.

For example, if there were 500 grams of cereal in the original package, then "20% more" means that 20% of 500 grams has been added to the initial amount, $500 + (0.2) \cdot 500 = 600$, so there are 600 grams of cereal in the new package.

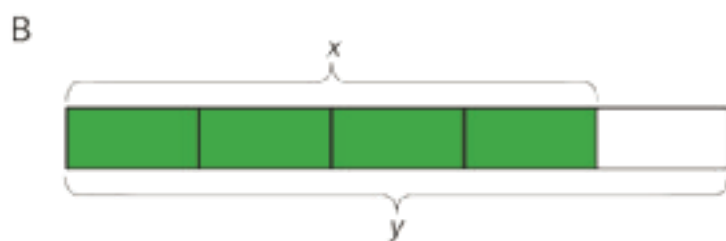
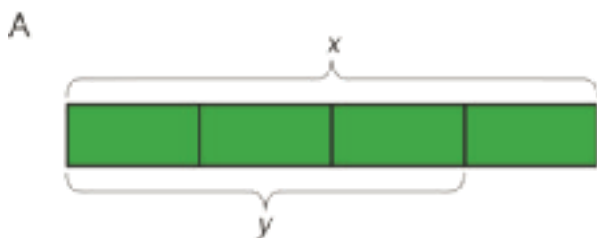
We can see that the new amount is 120% of the initial amount because

$$500 + (0.2) \cdot 500 = (1 + 0.2)500$$



Unit 4 Lesson 6 Cumulative Practice Problems

1. For each diagram, decide if y is an increase or a decrease relative to x . Then determine the percent increase or decrease.



2. Draw diagrams to represent the following situations.

a. The amount of flour that the bakery used this month was a 50% increase relative to last month.

b. The amount of milk that the bakery used this month was a 75% decrease relative to last month.

3. Write each percent increase or decrease as a percentage of the initial amount. The first one is done for you.

a. This year, there was 40% more snow than last year.

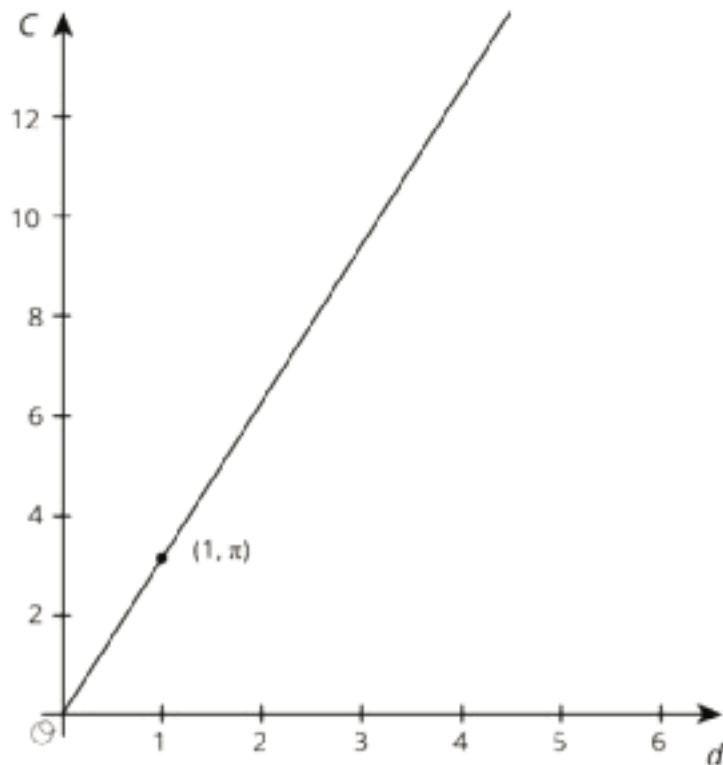
The amount of snow this year is 140% of the amount of snow last year.

b. This year, there were 25% fewer sunny days than last year.

c. Compared to last month, there was a 50% increase in the number of houses sold this month.

d. The runner's time to complete the marathon was a 10% less than the time to complete the last marathon.

4. The graph shows the relationship between the diameter and the circumference of a circle with the point $(1, \pi)$ shown. Find 3 more points that are on the line.



(From Unit 3, Lesson 3.)

5. Priya bought x grams of flour. Clare bought $\frac{3}{8}$ more than that. Select **all** equations that represent the relationship between the amount of flour that Priya bought, x , and the amount of flour that Clare bought, y .

A. $y = \frac{3}{8}x$

B. $y = \frac{5}{8}x$

C. $y = x + \frac{3}{8}x$

D. $y = x - \frac{3}{8}x$

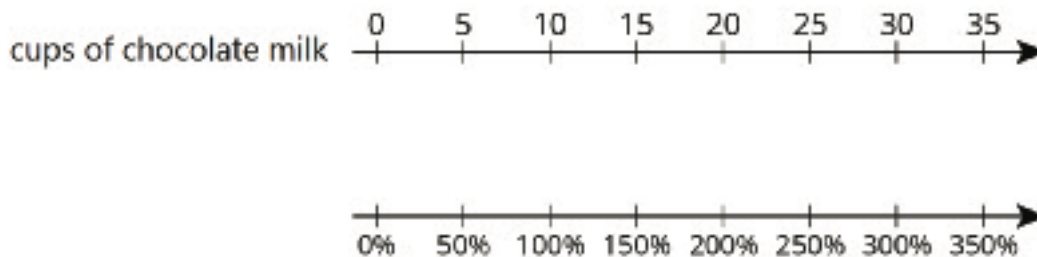
E. $y = \frac{11}{8}x$

(From Unit 4, Lesson 4.)

Lesson 7: One Hundred Percent

Let's solve more problems about percent increase and percent decrease.

7.1: Notice and Wonder: Double Number Line

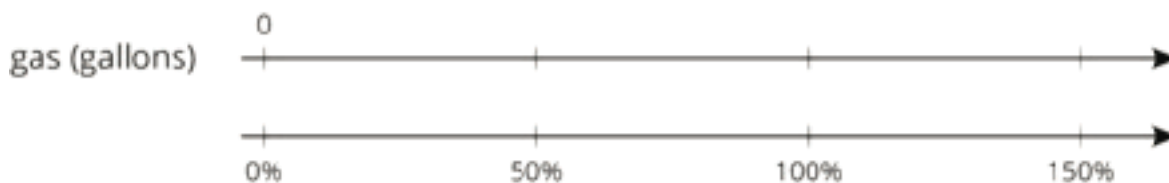


What do you notice? What do you wonder?

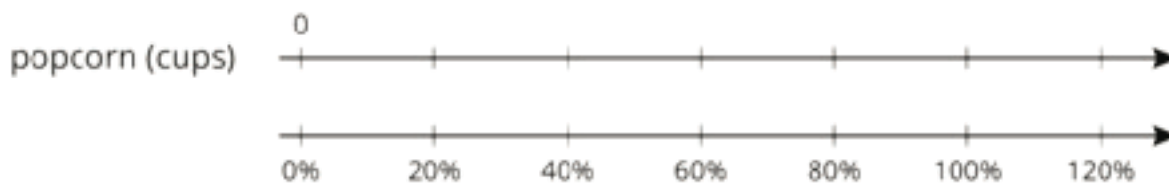
7.2: Double Number Lines

For each problem, complete the double number line diagram to show the percentages that correspond to the original amount and to the new amount.

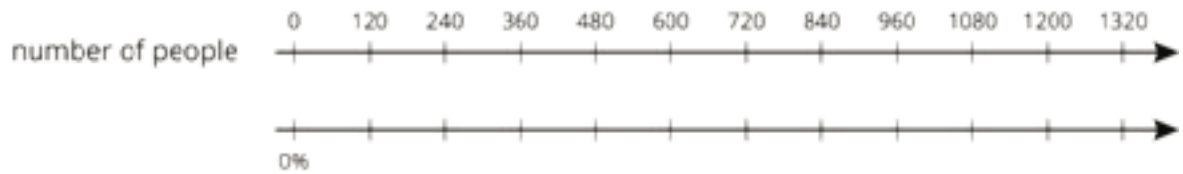
1. The gas tank in dad's car holds 12 gallons. The gas tank in mom's truck holds 50% more than that. How much gas does the truck's tank hold?



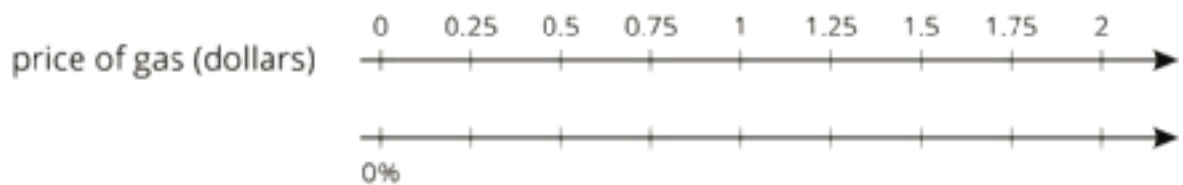
2. At a movie theater, the size of popcorn bags decreased 20%. If the old bags held 15 cups of popcorn, how much do the new bags hold?



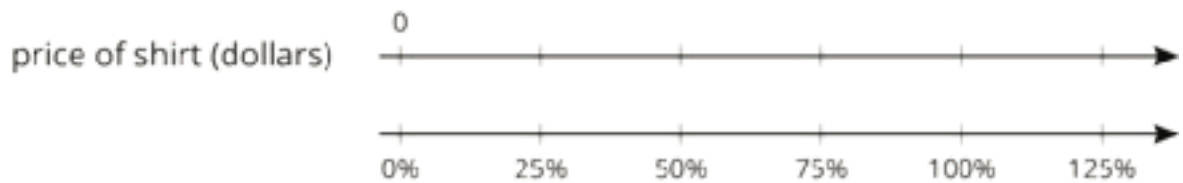
3. A school had 1,200 students last year and only 1,080 students this year. What was the percentage decrease in the number of students?



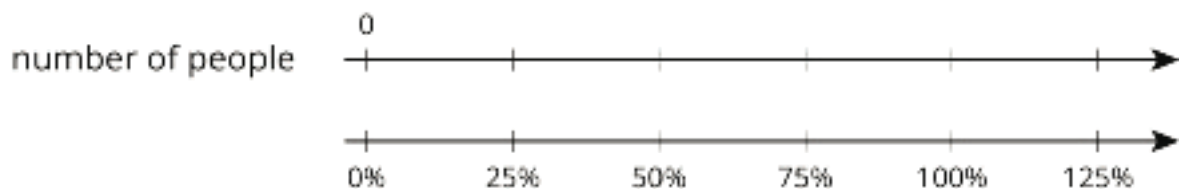
4. One week gas was \$1.25 per gallon. The next week gas was \$1.50 per gallon. By what percentage did the price increase?



5. After a 25% discount, the price of a T-shirt was \$12. What was the price before the discount?



6. Compared to last year, the population of Boom Town has increased 25%. The population is now 6,600. What was the population last year?

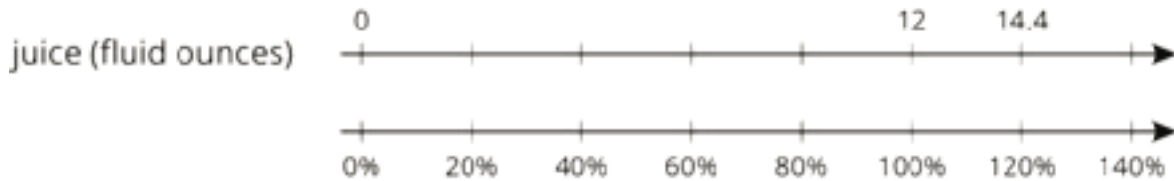


7.3: Representing More Juice

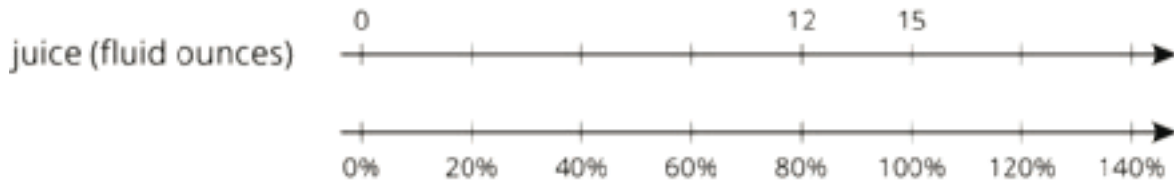
Two students are working on the same problem:

A juice box has 20% more juice in its new packaging. The original packaging held 12 fluid ounces. How much juice does the new packaging hold?

- Here is how Priya set up her double number line.



- Here is how Clare set up her double number line.



Do you agree with either of them? Explain or show your reasoning.

Are you ready for more?

Clare's diagram could represent a percent decrease. Describe a situation that could be represented with Clare's diagram.

7.4: Protecting the Green Sea Turtle

Green sea turtles live most of their lives in the ocean, but come ashore to lay their eggs. Some beaches where turtles often come ashore have been made into protected sanctuaries so the eggs will not be disturbed.

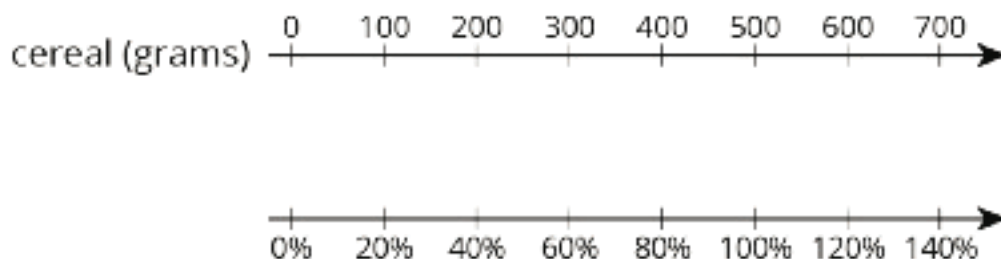


1. One sanctuary had 180 green sea turtles come ashore to lay eggs last year. This year, the number of turtles increased by 10%. How many turtles came ashore to lay eggs in the sanctuary this year?

2. At another sanctuary, the number of nesting turtles decreased by 10%. This year there were 234 nesting turtles. How many nesting turtles were at this sanctuary last year?

Lesson 7 Summary

We can use a double number line diagram to show information about percent increase and percent decrease:



The initial amount of cereal is 500 grams, which is lined up with 100% in the diagram. We can find a 20% *increase* to 500 by adding 20% of 500:

$$500 + (0.2) \cdot 500 = (1.20) \cdot 500 \\ = 600$$

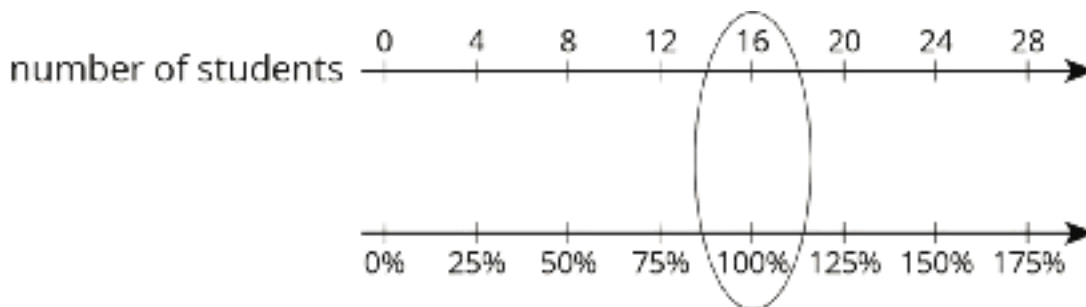
In the diagram, we can see that 600 corresponds to 120%.

If the initial amount of 500 grams is *decreased* by 40%, we can find how much cereal there is by subtracting 40% of the 500 grams:

$$500 - (0.4) \cdot 500 = (0.6) \cdot 500 \\ = 300$$

So a 40% decrease is the same as 60% of the initial amount. In the diagram, we can see that 300 is lined up with 60%.

To solve percentage problems, we need to be clear about what corresponds to 100%. For example, suppose there are 20 students in a class, and we know this is an increase of 25% from last year. In this case, the number of students in the class *last year* corresponds to 100%. So the initial amount (100%) is unknown and the final amount (125%) is 20 students.



Looking at the double number line, if 20 students is a 25% increase from the previous year, then there were 16 students in the class last year.

4. Elena walked 12 miles. Then she walked 0.25 that distance. How far did she walk all together? Select **all** that apply.

A. $12 + 0.25 \cdot 12$

B. $12(1 + 0.25)$

C. $12 \cdot 1.25$

D. $12 \cdot 0.25$

E. $12 + 0.25$

(From Unit 4, Lesson 5.)

5. A circle's circumference is 600 m. What is a good approximation of the circle's area?

A. 300 m^2

B. $3,000 \text{ m}^2$

C. $30,000 \text{ m}^2$

D. $300,000 \text{ m}^2$

(From Unit 3, Lesson 8.)

6. The equation $d = 3t$ represents the relationship between the distance (d) in inches that a snail is from a certain rock and the time (t) in minutes.

a. What does the number 3 represent?

b. How many minutes does it take the snail to get 9 inches from the rock?

c. How far will the snail be from the rock after 9 minutes?

(From Unit 2, Lesson 6.)

Lesson 8: Percent Increase and Decrease with Equations

Let's use equations to represent increases and decreases.

8.1: Number Talk: From 100 to 106

How do you get from one number to the next using multiplication or division?

From 100 to 106

From 100 to 90

From 90 to 100

From 106 to 100

8.2: Interest and Depreciation

1. Money in a particular savings account increases by about 6% after a year. How much money will be in the account after one year if the initial amount is \$100? \$50? \$200? \$125? x dollars? If you get stuck, consider using diagrams or a table to organize your work.

2. The value of a new car decreases by about 15% in the first year. How much will a car be worth after one year if its initial value was \$1,000? \$5,000? \$5,020? x dollars? If you get stuck, consider using diagrams or a table to organize your work.



8.3: Matching Equations

Match an equation to each of these situations. Be prepared to share your reasoning.

1. The water level in a reservoir is now 52 meters. If this was a 23% increase, what was the initial depth?

$$0.23x = 52$$

$$0.77x = 52$$

2. The snow is now 52 inches deep. If this was a 77% decrease, what was the initial depth?

$$1.23x = 52$$

$$1.77x = 52$$

Are you ready for more?

An astronaut was exploring the moon of a distant planet, and found some glowing goo at the bottom of a very deep crater. She brought a 10-gram sample of the goo to her laboratory. She found that when the goo was exposed to light, the total amount of goo increased by 100% every hour.

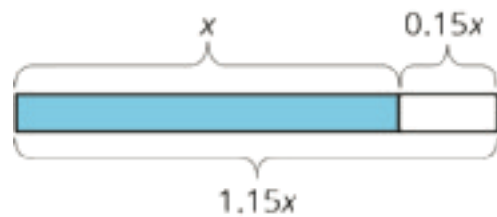
1. How much goo will she have after 1 hour? After 2 hours? After 3 hours? After n hours?
2. When she put the goo in the dark, it shrank by 75% every hour. How many hours will it take for the goo that was exposed to light for n hours to return to the original size?

8.4: Representing Percent Increase and Decrease: Equations

1. The gas tank in dad's car holds 12 gallons. The gas tank in mom's truck holds 50% more than that. How much gas does the truck's tank hold? Explain why this situation can be represented by the equation $(1.5) \cdot 12 = t$. Make sure that you explain what t represents.
2. Write an equation to represent each of the following situations.
 - a. A movie theater decreased the size of its popcorn bags by 20%. If the old bags held 15 cups of popcorn, how much do the new bags hold?
 - b. After a 25% discount, the price of a T-shirt was \$12. What was the price before the discount?
 - c. Compared to last year, the population of Boom Town has increased by 25%. The population is now 6,600. What was the population last year?

Lesson 8 Summary

We can use equations to express percent increase and percent decrease. For example, if y is 15% more than x ,



we can represent this using any of these equations:

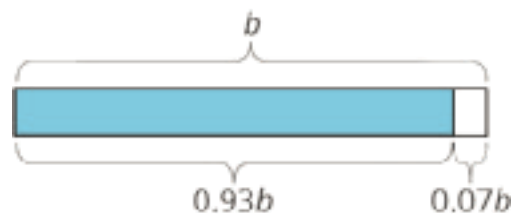
$$y = x + 0.15x$$

$$y = (1 + 0.15)x$$

$$y = 1.15x$$

So if someone makes an investment of x dollars, and its value increases by 15% to \$1250, then we can write and solve the equation $1.15x = 1250$ to find the value of the initial investment.

Here is another example: if a is 7% less than b ,



we can represent this using any of these equations:

$$a = b - 0.07b$$

$$a = (1 - 0.07)b$$

$$a = 0.93b$$

So if the amount of water in a tank decreased 7% from its starting value of b to its ending value of 348 gallons, then you can write $0.93b = 348$.

Often, an equation is the most efficient way to solve a problem involving percent increase or percent decrease.

4. Draw a diagram to represent each situation:

a. The number of miles driven this month was a 30% decrease of the number of miles driven last month.

b. The amount of paper that the copy shop used this month was a 25% increase of the amount of paper they used last month.

(From Unit 4, Lesson 6.)

5. Which decimal is the best estimate of the fraction $\frac{29}{40}$?

A. 0.5

B. 0.6

C. 0.7

D. 0.8

(From Unit 4, Lesson 5.)

6. Could 7.2 inches and 28 inches be the diameter and circumference of the same circle? Explain why or why not.

(From Unit 3, Lesson 3.)

Lesson 9: More and Less than 1%

Let's explore percentages smaller than 1%.

9.1: Number Talk: What Percentage?

Determine the percentage mentally.

10 is what percentage of 50?

5 is what percentage of 50?

1 is what percentage of 50?

17 is what percentage of 50?

9.2: Waiting Tables

During one waiter's shift, he delivered 13 appetizers, 17 entrées, and 10 desserts.

1. What percentage of the dishes he delivered were:

a. desserts?

b. appetizers?

c. entrées?

2. What do your percentages add up to?

9.3: Fractions of a Percent

1. Find each percentage of 60. What do you notice about your answers?

30% of 60

3% of 60

0.3% of 60

0.03% of 60

2. 20% of 5,000 is 1,000 and 21% of 5,000 is 1,050. Find each percentage of 5,000 and be prepared to explain your reasoning. If you get stuck, consider using the double number line diagram.

a. 1% of 5,000

b. 0.1% of 5,000

c. 20.1% of 5,000

d. 20.4% of 5,000



3. 15% of 80 is 12 and 16% of 80 is 12.8. Find each percentage of 80 and be prepared to explain your reasoning.

a. 15.1% of 80

b. 15.7% of 80

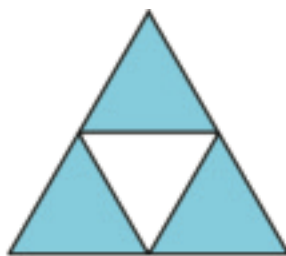
Are you ready for more?

To make Sierpinski's triangle,

- Start with an equilateral triangle. This is step 1.
- Connect the midpoints of every side, and remove the middle triangle, leaving three smaller triangles. This is step 2.
- Do the same to each of the remaining triangles. This is step 3.
- Keep repeating this process.



step 1



step 2



step 3

1. What percentage of the area of the original triangle is left after step 2? Step 3? Step 10?

2. At which step does the percentage first fall below 1%?

9.4: Population Growth

1. The population of City A was approximately 243,000 people, and it increased by 8% in one year. What was the new population?

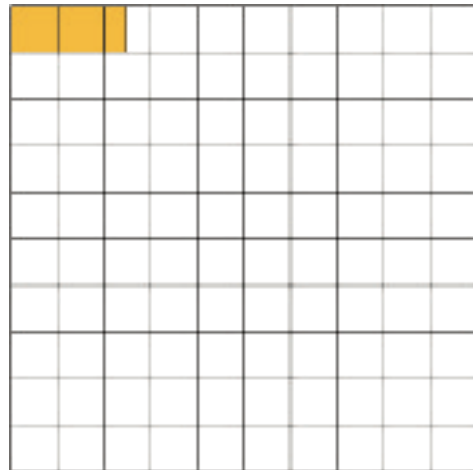
2. The population of city B was approximately 7,150,000, and it increased by 0.8% in one year. What was the new population?

Lesson 9 Summary

A percentage, such as 30%, is a rate per 100. To find 30% of a quantity, we multiply it by $30 \div 100$, or 0.3.

The same method works for percentages that are not whole numbers, like 7.8% or 2.5%. In the square, 2.5% of the area is shaded.

To find 2.5% of a quantity, we multiply it by $2.5 \div 100$, or 0.025. For example, to calculate 2.5% interest on a bank balance of \$80, we multiply $(0.025) \cdot 80 = 2$, so the interest is \$2.



We can sometimes find percentages like 2.5% mentally by using convenient whole number percents. For example, 25% of 80 is one fourth of 80, which is 20. Since 2.5 is one tenth of 25, we know that 2.5% of 80 is one tenth of 20, which is 2.

Unit 4 Lesson 9 Cumulative Practice Problems

1. The student government snack shop sold 32 items this week. For each snack type, what percentage of all snacks sold were of that type?

snack type	number of items sold
fruit cup	8
veggie sticks	6
chips	14
water	4

2. Select **all** the options that have the same value as $3\frac{1}{2}\%$ of 20.

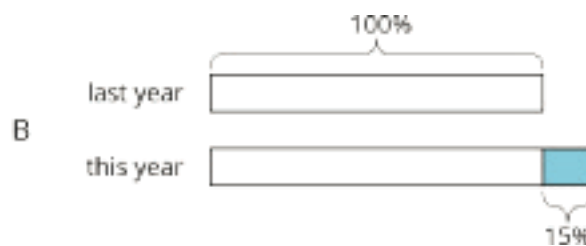
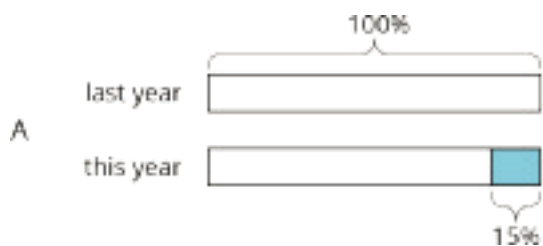
- A. 3.5% of 20
- B. $3\frac{1}{2} \cdot 20$
- C. $(0.35) \cdot 20$
- D. $(0.035) \cdot 20$
- E. 7% of 10

3. 22% of 65 is 14.3. What is 22.6% of 65? Explain your reasoning.

4. A bakery used 30% more sugar this month than last month. If the bakery used 560 kilograms of sugar last month, how much did it use this month?

(From Unit 4, Lesson 7.)

5. Match each situation to a diagram. The diagrams can be used more than once.



A. The amount of apples this year decreased by 15% compared with last year's amount.

1. Diagram A

B. The amount of pears this year is 85% of last year's amount.

2. Diagram B

C. The amount of cherries this year increased by 15% compared with last year's amount.

D. The amount of oranges this year is 115% of last year's amount.

(From Unit 4, Lesson 6.)

6. A certain type of car has room for 4 passengers.

a. Write an equation relating the number of cars (n) to the number of passengers (p).

b. How many passengers could fit in 78 cars?

c. How many cars would be needed to fit 78 passengers?

(From Unit 2, Lesson 6.)

Lesson 10: Tax and Tip

Let's learn about sales tax and tips.

10.1: Notice and Wonder: The Price of Sunglasses

You are on vacation and want to buy a pair of sunglasses for \$10 or less. You find a pair with a price tag of \$10. The cashier says the total cost will be \$10.45.



What do you notice? What do you wonder?

10.2: Shopping in Two Different Cities

Different cities have different sales tax rates. Here are the sales tax charges on the same items in two different cities. Complete the tables.

City 1

item	price (dollars)	sales tax (dollars)	total cost (dollars)
paper towels	8.00	0.48	8.48
lamp	25.00	1.50	
pack of gum	1.00		
laundry soap	12.00		
	x		

City 2

item	price (dollars)	sales tax (dollars)	total cost (dollars)
paper towels	8.00	0.64	8.64
lamp	25.00	2.00	
pack of gum	1.00		
laundry soap	12.00		
	x		

10.3: Shopping in a Third City

Here is the sales tax on the same items in City 3.

item	price (dollars)	sales tax (dollars)
paper towels	8.00	0.58
lamp	25.00	1.83
pack of gum	1.00	0.07
laundry soap	12.00	

1. What is the tax rate in this city?
2. For the sales tax on the laundry soap, Kiran says it should be \$0.84. Lin says it should be \$0.87. Do you agree with either of them? Explain your reasoning.

10.4: Dining at a Restaurant

1. Jada has a meal in a restaurant. She adds up the prices listed on the menu for everything they ordered and gets a subtotal of \$42.00.

Date: Sep. 12th	
Time: 6:55 PM	
Server: # 27	
Bread Stix	9.50
Chicken Parm	15.50
Chef Salad	12.00
Lemon Soda	2.00
Tea	3.00
Subtotal	42.00
Sales Tax	3.99
Total	45.99

- a. When the check comes, it says they also need to pay \$3.99 in sales tax. What percentage of the subtotal is the sales tax?
- b. After tax, the total is \$45.99. What percentage of the subtotal is the total?
- c. They actually pay \$52.99. The additional \$7 is a tip for the server. What percentage of the subtotal is the tip?

2. The tax rate at this restaurant is 9.5%.

Date: Sep 12th	
Time: 6:04 PM	
Server: # 27	
Bread Stix	9.50
Ravioli Bites	10.50
Cheesecake	4.95
Subtotal	24.95
Sales Tax	_____
Total	_____

Another person's subtotal is \$24.95.
How much will their sales tax be?

Date: Sep 12th	
Time: 7:12 PM	
Server: # 27	
Garden Salad	_____
Broccoli Bites	_____
Subtotal	_____
Sales Tax	1.61
Total	_____

Some other person's sales tax is \$1.61.
How much was their subtotal?

Are you ready for more?

Elena's cousins went to a restaurant. The part of the entire cost of the meal that was tax and tip together was 25% of the cost of the food alone. What could the tax rate and tip rate be?

Lesson 10 Summary

Many places have *sales tax*. A sales tax is an amount of money that a government agency collects on the sale of certain items. For example, a state might charge a tax on all cars purchased in the state. Often the tax rate is given as a percentage of the cost. For example, a state's tax rate on car sales might be 2%, which means that for every car sold in that state, the buyer has to pay a tax that is 2% of the sales price of the car.

Fractional percentages often arise when a state or city charges a sales tax on a purchase. For example, the sales tax in Arizona is 7.5%. This means that when someone buys something, they have to add 0.075 times the amount on the price tag to determine the total cost of the item.

For example, if the price tag on a T-shirt in Arizona says \$11.50, then the sales tax is $(0.075) \cdot 11.5 = 0.8625$, which rounds to 86 cents. The customer pays $11.50 + 0.86$, or \$12.36 for the shirt.

The total cost to the customer is the item price plus the sales tax. We can think of this as a percent increase. For example, in Arizona, the total cost to a customer is 107.5% of the price listed on the tag.

A *tip* is an amount of money that a person gives someone who provides a service. It is customary in many restaurants to give a tip to the server that is between 10% and 20% of the cost of the meal. If a person plans to leave a 15% tip on a meal, then the total cost will be 115% of the cost of the meal.

Unit 4 Lesson 10 Cumulative Practice Problems

1. In a city in Ohio, the sales tax rate is 7.25%. Complete the table to show the sales tax and the total price including tax for each item.

item	price before tax (\$)	sales tax (\$)	price including tax (\$)
pillow	8.00		
blanket	22.00		
trash can	14.50		

2. The sales tax rate in New Mexico is 5.125%. Select **all** the equations that represent the sales tax, t , you would pay in New Mexico for an item of cost c ?

A. $t = 5.125c$

B. $t = 0.5125c$

C. $t = 0.05125c$

D. $t = c \div 0.05125$

E. $t = \frac{5.125}{100}c$

3. Here are the prices of some items and the amount of sales tax charged on each in Nevada.

cost of item (\$)	sales tax (\$)
10	0.46
50	2.30
5	0.23

- a. What is the sales tax rate in Nevada?

- b. Write an expression for the amount of sales tax charged, in dollars, on an item that costs c dollars.

4. Find each amount:

a. 3.8% of 25

b. 0.2% of 50

c. 180.5% of 99

(From Unit 4, Lesson 9.)

5. On Monday, the high was 60 degrees Fahrenheit. On Tuesday, the high was 18% more. How much did the high increase from Monday to Tuesday?

(From Unit 4, Lesson 8.)

6. Complete the table. Explain or show your reasoning.

object	radius	circumference
ceiling fan	2.8 ft	
water bottle cap	13 mm	
bowl		56.5 cm
drum		75.4 in

(From Unit 3, Lesson 4.)

Lesson 11: Percentage Contexts

Let's learn about more situations that involve percentages.

11.1: Leaving a Tip

Which of these expressions represent a 15% tip on a \$20 meal? Which represent the total bill?

$$15 \cdot 20$$

$$20 + 0.15 \cdot 20$$

$$1.15 \cdot 20$$

$$\frac{15}{100} \cdot 20$$

11.2: A Car Dealership

A car dealership pays a wholesale price of \$12,000 to purchase a vehicle.

1. The car dealership wants to make a 32% profit.
 - a. By how much will they mark up the price of the vehicle?



- b. After the markup, what is the retail price of the vehicle?
2. During a special sales event, the dealership offers a 10% discount off of the retail price. After the discount, how much will a customer pay for this vehicle?

Are you ready for more?

This car dealership pays the salesperson a bonus for selling the car equal to 6.5% of the sale price. How much commission did the salesperson lose when they decided to offer a 10% discount on the price of the car?

11.3: Commission at a Gym

1. For each gym membership sold, the gym keeps \$42 and the employee who sold it gets \$8. What is the commission the employee earned as a percentage of the total cost of the gym membership?

2. If an employee sells a family pass for \$135, what is the amount of the commission they get to keep?

11.4: Card Sort: Percentage Situations

Your teacher will give you a set of cards. Take turns with your partner matching a situation with a descriptor. For each match, explain your reasoning to your partner. If you disagree, work to reach an agreement.

Lesson 11 Summary

There are many everyday situations where a percentage of an amount of money is added to or subtracted from that amount, in order to be paid to some other person or organization:

	goes to	how it works
sales tax	the government	added to the price of the item
gratuity (tip)	the server	added to the cost of the meal
interest	the lender (or account holder)	added to the balance of the loan, credit card, or bank account
markup	the seller	added to the price of an item so the seller can make a profit
markdown (discount)	the customer	subtracted from the price of an item to encourage the customer to buy it
commission	the salesperson	subtracted from the payment that is collected

For example,

- If a restaurant bill is \$34 and the customer pays \$40, they left \$6 dollars as a tip for the server. That is 18% of \$34, so they left an 18% tip. From the customer's perspective, we can think of this as an 18% increase of the restaurant bill.
- If a realtor helps a family sell their home for \$200,000 and earns a 3% commission, then the realtor makes \$6,000, because $(0.03) \cdot 200,000 = 6,000$, and the family gets \$194,000, because $200,000 - 6,000 = 194,000$. From the family's perspective, we can think of this as a 3% decrease on the sale price of the home.

Unit 4 Lesson 11 Cumulative Practice Problems

1. A car dealership pays \$8,350 for a car. They mark up the price by 17.4% to get the retail price. What is the retail price of the car at this dealership?
2. A store has a 20% off sale on pants. With this discount, the price of one pair of pants before tax is \$15.20. What was the original price of the pants?
 - A. \$3.04
 - B. \$12.16
 - C. \$18.24
 - D. \$19.00
3. Lin is shopping for a couch with her dad and hears him ask the salesperson, "How much is your commission?" The salesperson says that her commission is $5\frac{1}{2}\%$ of the selling price.
 - a. How much commission will the salesperson earn by selling a couch for \$495?
 - b. How much money will the store get from the sale of the couch?

4. A college student takes out a \$7,500 loan from a bank. What will the balance of the loan be after one year (assuming the student has not made any payments yet):

a. if the bank charges 3.8% interest each year?

b. if the bank charges 5.3% interest each year?

(From Unit 4, Lesson 9.)

5. Match the situations with the equations.

a. Mai slept for x hours, and Kiran slept for $\frac{1}{10}$ less than that.

$$y = 2.33x$$

$$y = 1.375x$$

b. Kiran practiced the piano for x hours, and Mai practiced for $\frac{2}{5}$ less than that.

$$y = 0.6x$$

$$y = 0.9x$$

c. Mai drank x oz of juice and Kiran drank $\frac{4}{3}$ more than that.

$$y = 0.75x$$

$$y = 1.6x$$

d. Kiran spent x dollars and Mai spent $\frac{1}{4}$ less than that.

$$y = 0.7x$$

$$y = 2.5x$$

e. Mai ate x grams of almonds and Kiran ate 1.5 times more than that.

f. Kiran collected x pounds of recycling and Mai collected $\frac{3}{10}$ less than that.

g. Mai walked x kilometers and Kiran walked $\frac{3}{8}$ more than that.

h. Kiran completed x puzzles and Mai completed $\frac{3}{5}$ more than that.

(From Unit 4, Lesson 5.)

Lesson 12: Finding the Percentage

Let's find unknown percentages.

12.1: Tax, Tip, and Discount

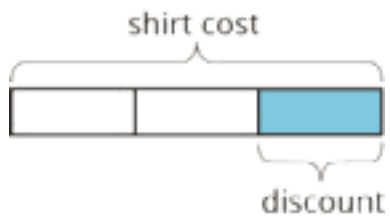
What percentage of the car price is the tax?



What percentage of the food cost is the tip?



What percentage of the shirt cost is the discount?



12.2: What Is the Percentage?

1. A salesperson sold a car for \$18,250 and their commission is \$693.50. What percentage of the sale price is their commission?

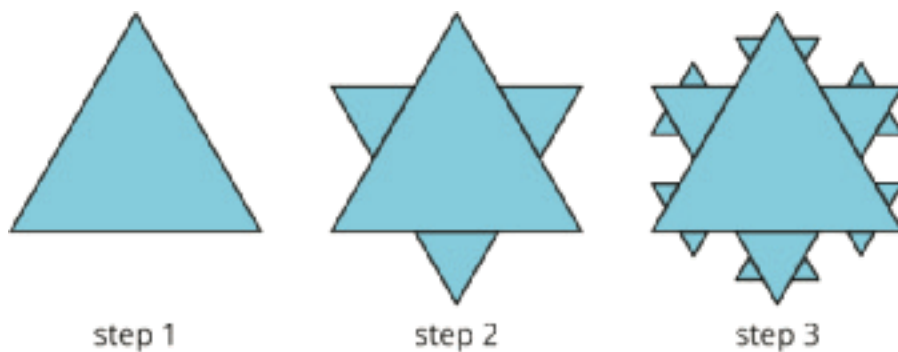
2. The bill for a meal was \$33.75. The customer left \$40.00. What percentage of the bill was the tip?

3. The original price of a bicycle was \$375. Now it is on sale for \$295. What percentage of the original price was the markdown?

Are you ready for more?

To make a Koch snowflake,

- Start with an equilateral triangle. This is step 1.
- Divide each side into 3 equal pieces. Construct a smaller equilateral triangle on the middle third. This is step 2.
- Do the same to each of the newly created sides. This is step 3.
- Keep repeating this process.



By what percentage does the perimeter increase at step 2? Step 3? Step 10?

12.3: Info Gap: Sporting Goods

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

Continue to ask questions until you have enough information to solve the problem.

4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.
2. Ask your partner “*What specific information do you need?*” and wait for them to *ask* for information.

If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don’t have that information.

3. Before sharing the information, ask “*Why do you need that information?*” Listen to your partner’s reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Lesson 12 Summary

To find a 30% increase over 50, we can find 130% of 50.

$$1.3 \cdot 50 = 65$$

To find a 30% decrease from 50, we can find 70% of 50.

$$0.7 \cdot 50 = 35$$

If we know the initial amount and the final amount, we can also find the percent increase or percent decrease. For example, a plant was 12 inches tall and grew to be 15 inches tall. What percent increase is this? Here are two ways to solve this problem:

The plant grew 3 inches, because $15 - 12 = 3$. We can divide this growth by the original height, $3 \div 12 = 0.25$. So the height of the plant increased by 25%.

The plant's new height is 125% of the original height, because $15 \div 12 = 1.25$. This means the height increased by 25%, because $125 - 100 = 25$.

Here are two ways to solve the problem: A rope was 2.4 meters long. Someone cut it down to 1.9 meters. What percent decrease is this?

The rope is now $2.4 - 1.9$, or 0.5 meters shorter. We can divide this decrease by the original length, $0.5 \div 2.4 = 0.208\overline{3}$. So the length of the rope decreased by approximately 20.8%.

The rope's new length is about 79.2% of the original length, because $1.9 \div 2.4 = 0.791\overline{6}$. The length decreased by approximately 20.8%, because $100 - 79.2 = 20.8$.

Unit 4 Lesson 12 Cumulative Practice Problems

1. A music store marks up the instruments it sells by 30%.

a. If the store bought a guitar for \$45, what will be its store price?

b. If the price tag on a trumpet says \$104, how much did the store pay for it?

c. If the store paid \$75 for a clarinet and sold it for \$100, did the store mark up the price by 30%?

2. A family eats at a restaurant. The bill is \$42. The family leaves a tip and spends \$49.77.

a. How much was the tip in dollars?

b. How much was the tip as a percentage of the bill?

3. The price of gold is often reported per ounce. At the end of 2005, this price was \$513. At the end of 2015, it was \$1060. By what percentage did the price per ounce of gold increase?

4. A phone keeps track of the number of steps taken and the distance traveled. Based on the information in the table, is there a proportional relationship between the two quantities? Explain your reasoning.

number of steps	distance in kilometers
950	1
2,852	3
4,845	5.1

(From Unit 2, Lesson 7.)

5. Noah picked 3 kg of cherries. Mai picked half as many cherries as Noah. How many total kg of cherries did Mai and Noah pick?

A. $3 + \frac{1}{2}$

B. $3 - \frac{1}{2}$

C. $(1 + \frac{1}{2}) \cdot 3$

D. $1 + \frac{1}{2} \cdot 3$

(From Unit 4, Lesson 4.)

Lesson 13: Measurement Error

Let's use percentages to describe how accurately we can measure.

13.1: Measuring to the Nearest

Your teacher will give you two rulers and three line segments labeled A, B, and C.

1. Use the centimeter ruler to measure each line segment to the nearest centimeter. Record these lengths in the first column of the table.
2. Use the millimeter ruler to measure each line segment to the nearest tenth of a centimeter. Record these lengths in the second column of the table.

line segment	length (cm) as measured with the first ruler	length (cm) as measured with the second ruler
A		
B		
C		

13.2: Measuring a Soccer Field

A soccer field is 120 yards long. Han measures the length of the field using a 30-foot-long tape measure and gets a measurement of 358 feet, 10 inches.

1. What is the amount of the error?

2. Express the error as a percentage of the actual length of the field.

13.3: Measuring Your Classroom

Your teacher will tell you which three items to measure. Keep using the paper rulers from the earlier activity.

1. Between you and your partner, decide who will use which ruler.
2. Measure the three items assigned by your teacher and record your measurements in the first column of the appropriate table.

Using the cm ruler:

item	measured length (cm)	actual length (cm)	difference	percentage

Using the mm ruler:

item	measured length (cm)	actual length (cm)	difference	percentage

3. After you finish measuring the items, share your data with your partner. Next, ask your teacher for the actual lengths.
4. Calculate the difference between your measurements and the actual lengths in both tables.
5. For each difference, what percentage of the actual length is this amount? Record your answers in the last column of the tables.

Are you ready for more?

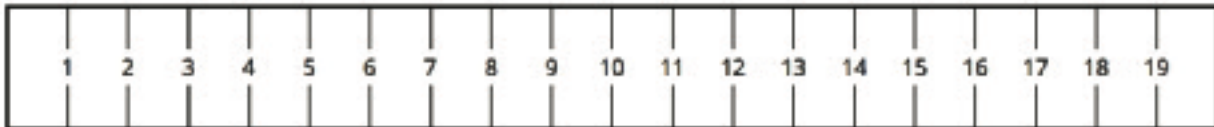
Before there were standard units of measurement, people often measured things using their hands or feet.

1. Measure the length of your foot to the nearest centimeter with your shoe on.
2. How many foot-lengths long is your classroom? Try to determine this as precisely as possible by carefully placing your heel next to your toe as you pace off the room.
3. Use this information to estimate the length of your classroom in centimeters.
4. Use a tape measure to measure the length of your classroom. What is the difference between the two measurements? Which one do you think is more accurate?

Lesson 13 Summary

When we are measuring a length using a ruler or measuring tape, we can get a measurement that is different from the actual length. This could be because we positioned the ruler incorrectly, or it could be because the ruler is not very precise. There is always at least a small difference between the actual length and a measured length, even if it is a microscopic difference!

Here are two rulers with different markings.



The second ruler is marked in millimeters, so it is easier to get a measurement to the nearest tenth of a centimeter with this ruler than with the first. For example, a line that is actually 6.2 cm long might be measured to be 6 cm long by the first ruler, because we measure to the nearest centimeter.

The **measurement error** is the positive difference between the measurement and the actual value. Measurement error is often expressed as a percentage of the actual value. We always use a positive number to express measurement error and, when appropriate, use words to describe whether the measurement is greater than or less than the actual value.

For example, if we get 6 cm when we measure a line that is actually 6.2 cm long, then the measurement error is 0.2 cm, or about 3.2%, because $0.2 \div 6.2 \approx 0.032$.

Unit 4 Lesson 13 Cumulative Practice Problems

1. The depth of a lake is 15.8 m.
 - a. Jada accurately measured the depth of the lake to the nearest meter. What measurement did Jada get?
 - b. By how many meters does the measured depth differ from the actual depth?
 - c. Express the measurement error as a percentage of the actual depth.

2. A watermelon weighs 8,475 grams. A scale measured the weight with an error of 12% under the actual weight. What was the measured weight?

3. Noah's oven thermometer gives a reading that is 2% greater than the actual temperature.
 - a. If the actual temperature is 325°F , what will the thermometer reading be?

 - b. If the thermometer reading is 76°F , what is the actual temperature?

4. At the beginning of the month, there were 80 ounces of peanut butter in the pantry. Now, there is $\frac{1}{3}$ less than that. How many ounces of peanut butter are in the pantry now?

A. $\frac{2}{3} \cdot 80$

B. $\frac{1}{3} \cdot 80$

C. $80 - \frac{1}{3}$

D. $(1 + \frac{1}{3}) \cdot 80$

(From Unit 4, Lesson 4.)

5. a. Fill in the table for side length and area of different squares.

side length (cm)	area (cm ²)
3	
100	
25	
s	

b. Is the relationship between the side length of a square and the area of a square proportional?

(From Unit 3, Lesson 7.)

Lesson 14: Percent Error

Let's use percentages to describe other situations that involve error.

14.1: Number Talk: Estimating a Percentage of a Number

Estimate.

25% of 15.8

9% of 38

1.2% of 127

0.53% of 6

0.06% of 202

14.2: Plants, Bicycles, and Crowds

1. Instructions to care for a plant say to water it with $\frac{3}{4}$ cup of water every day. The plant has been getting 25% too much water. How much water has the plant been getting?

2. The pressure on a bicycle tire is 63 psi. This is 5% higher than what the manual says is the correct pressure. What is the correct pressure?

3. The crowd at a sporting event is estimated to be 3,000 people. The exact attendance is 2,486 people. What is the **percent error**?

Are you ready for more?

A micrometer is an instrument that can measure lengths to the nearest micron (a micron is a millionth of a meter). Would this instrument be useful for measuring any of the following things? If so, what would the largest percent error be?

1. The thickness of an eyelash, which is typically about 0.1 millimeters.

2. The diameter of a red blood cell, which is typically about 8 microns.

3. The diameter of a hydrogen atom, which is about 100 picometers (a picometer is a trillionth of a meter).

14.3: Measuring in the Heat

A metal measuring tape expands when the temperature goes above 50°F . For every degree Fahrenheit above 50, its length increases by 0.00064%.

1. The temperature is 100 degrees Fahrenheit. How much longer is a 30-foot measuring tape than its correct length?

2. What is the percent error?

Lesson 14 Summary

Percent error can be used to describe any situation where there is a correct value and an incorrect value, and we want to describe the relative difference between them. For example, if a milk carton is supposed to contain 16 fluid ounces and it only contains 15 fluid ounces:

- the measurement error is 1 oz, and
- the percent error is 6.25% because $1 \div 16 = 0.0625$.

We can also use percent error when talking about estimates. For example, a teacher estimates there are about 600 students at their school. If there are actually 625 students, then the percent error for this estimate was 4%, because $625 - 600 = 25$ and $25 \div 625 = 0.04$.

5. For each story, write an equation that describes the relationship between the two quantities.

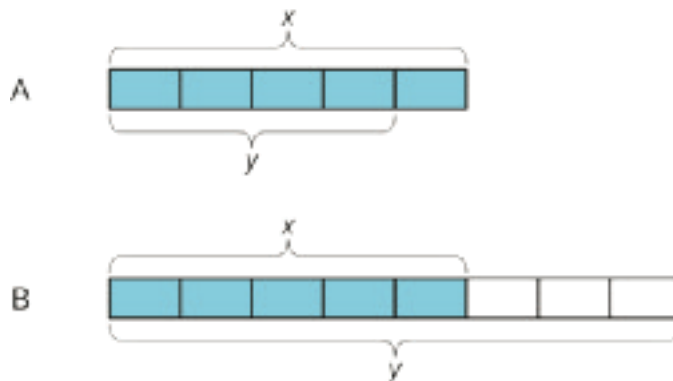
a. Diego collected x kg of recycling. Lin collected $\frac{2}{5}$ more than that.

b. Lin biked x km. Diego biked $\frac{3}{10}$ less than that.

c. Diego read for x minutes. Lin read $\frac{4}{7}$ of that.

(From Unit 4, Lesson 4.)

6. For each diagram, decide if y is an increase or a decrease of x . Then determine the percentage.



(From Unit 4, Lesson 12.)

7. Lin is making a window covering for a window that has the shape of a half circle on top of a square of side length 3 feet. How much fabric does she need?

(From Unit 3, Lesson 10.)

Lesson 15: Error Intervals

Let's solve more problems about percent error.

15.1: A Lot of Iron Ore

An industrial scale is guaranteed by the manufacturer to have a percent error of no more than 1%. What is a possible reading on the scale if you put 500 kilograms of iron ore on it?

15.2: Saw Mill

1. A saw mill cuts boards that are 16 ft long. After they are cut, the boards are inspected and rejected if the length has a percent error of 1.5% or more.
 - a. List some board lengths that should be accepted.

 - b. List some board lengths that should be rejected.

2. The saw mill also cuts boards that are 10, 12, and 14 feet long. An inspector rejects a board that was 2.3 inches too long. What was the intended length of the board?

15.3: Info Gap: Quality Control

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

Continue to ask questions until you have enough information to solve the problem.

4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.
2. Ask your partner “*What specific information do you need?*” and wait for them to *ask* for information.

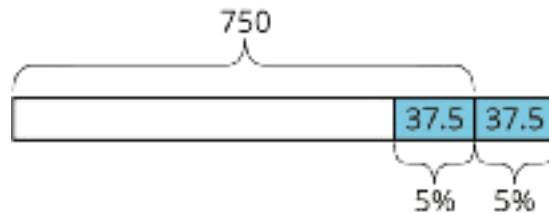
If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don’t have that information.

3. Before sharing the information, ask “*Why do you need that information?*” Listen to your partner’s reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Lesson 15 Summary

Percent error is often used to express a range of possible values. For example, if a box of cereal is guaranteed to have 750 grams of cereal, with a margin of error of less than 5%, what are possible values for the actual number of grams of cereal in the box? The error could be as large as $(0.05) \cdot 750 = 37.5$ and could be either above or below than the correct amount.



Therefore, the box can have anywhere between 712.5 and 787.5 grams of cereal in it, but it should not have 700 grams or 800 grams, because both of those are more than 37.5 grams away from 750 grams.

Unit 4 Lesson 15 Cumulative Practice Problems

1. Jada measured the height of a plant in a science experiment and finds that, to the nearest $\frac{1}{4}$ of an inch, it is $4\frac{3}{4}$ inches.
 - a. What is the largest the actual height of the plant could be?
 - b. What is the smallest the actual height of the plant could be?
 - c. How large could the percent error in Jada's measurement be?
2. The reading on a car's speedometer has 1.6% maximum error. The speed limit on a road is 65 miles per hour.
 - a. The speedometer reads 64 miles per hour. Is it possible that the car is going over the speed limit?
 - b. The speedometer reads 66 miles per hour. Is the car definitely going over the speed limit?
3. Water is running into a bathtub at a constant rate. After 2 minutes, the tub is filled with 2.5 gallons of water. Write two equations for this proportional relationship. Use w for the amount of water (gallons) and t for time (minutes). In each case, what does the constant of proportionality tell you about the situation?

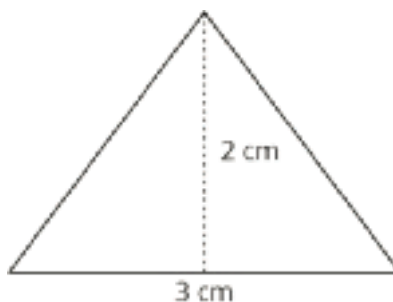
(From Unit 2, Lesson 5.)

4. Noah picked 3 kg of cherries. Jada picked half as many cherries as Noah. How many total kg of cherries did Jada and Noah pick?

- A. $3 + 0.5$
- B. $3 - 0.5$
- C. $(1 + 0.5) \cdot 3$
- D. $1 + 0.5 \cdot 3$

(From Unit 4, Lesson 5.)

5. Here is a shape with some measurements in cm.



a. Complete the table showing the area of different scaled copies of the triangle.

scale factor	area (cm^2)
1	
2	
5	
s	

b. Is the relationship between the scale factor and the area of the scaled copy proportional?

(From Unit 3, Lesson 7.)

Lesson 16: Posing Percentage Problems

Let's explore how percentages are used in the news.

16.1: Sorting the News

Your teacher will give you a variety of news clippings that include percentages.

1. Sort the clippings into two piles: those that are about increases and those that are about decreases.
2. Were there any clippings that you had trouble deciding which pile they should go in?

16.2: Investigating

In the previous activity, you sorted news clippings into two piles.

1. For each pile, choose one example. Draw a diagram that shows how percentages are being used to describe the situation.

a. Increase Example:

b. Decrease Example:

2. For each example, write *two* questions that you can answer with the given information. Next, find the answers. Explain or show your reasoning.

16.3: Displaying the News

1. Choose the example that you find the most interesting. Create a visual display that includes:
 - a title that describes the situation
 - the news clipping
 - your diagram of the situation
 - the two questions you asked about the situation
 - the answers to each of your questions
 - an explanation of how you calculated each answer

Pause here so your teacher can review your work.

2. Examine each display. Write one comment and one question for the group.
3. Next, read the comments and questions your classmates wrote for your group. Revise your display using the feedback from your classmates.

Lesson 16 Summary

Statements about percentage increase or decrease need to specify what the whole is to be mathematically meaningful. Sometimes advertisements, media, etc. leave the whole ambiguous in order to make somewhat misleading claims. We should be careful to think critically about what mathematical claim is being made.

For example, if a disinfectant claims to "kill 99% of all bacteria," does it mean that

- It kills 99% of the number of bacteria on a surface?
- Or is it 99% of the types of bacteria commonly found inside the house?
- Or 99% of the total mass or volume of bacteria?
- Does it even matter if the remaining 1% are the most harmful bacteria?

Resolving questions of this type is an important step in making informed decisions.

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