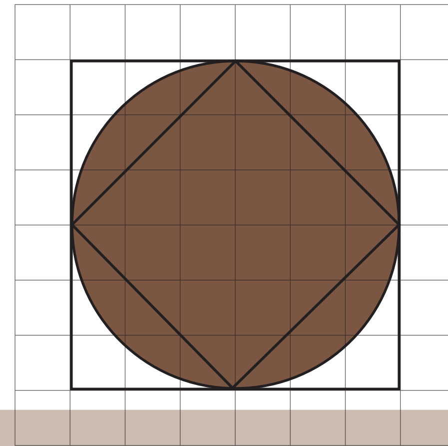
Core Knowledge® MATHEMATICS

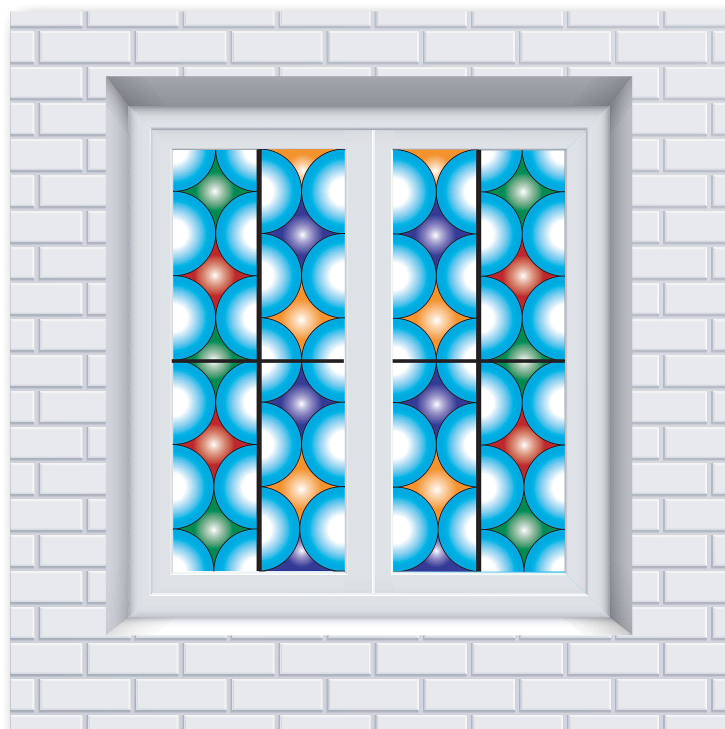
Measuring Circles

Student Workbook

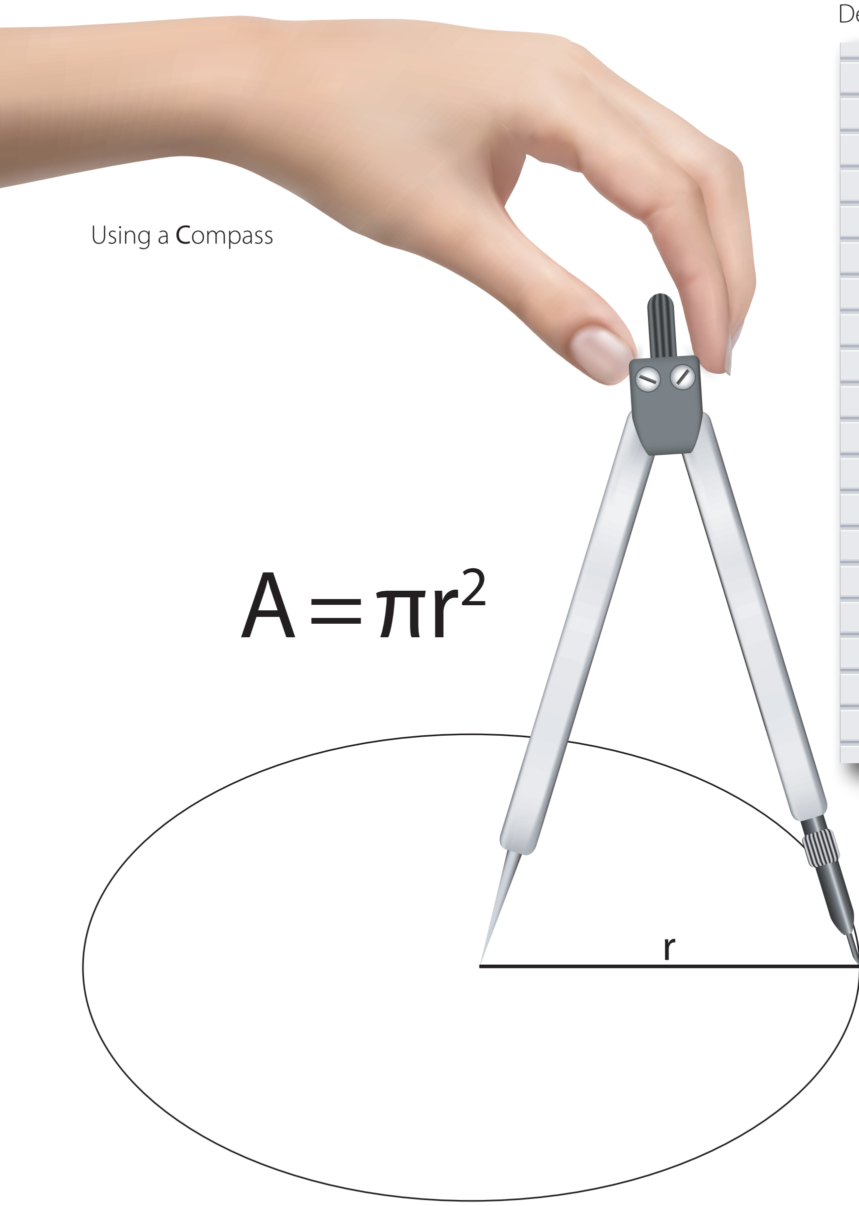
Comparing Areas of Different Shapes



Designing Windows Using Circles

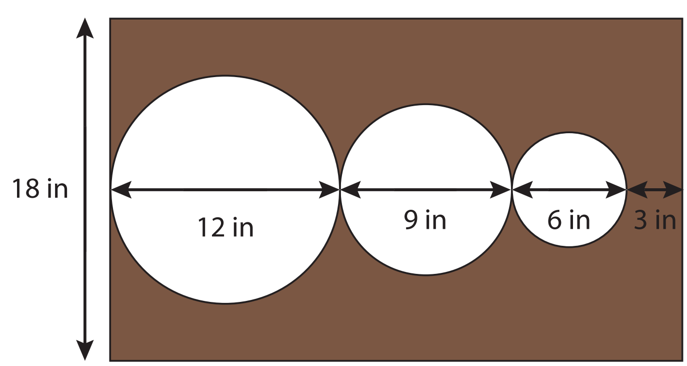


Using a Compass



$$A = \pi r^2$$

Calculating Area of Complex Shapes



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Measuring Circles

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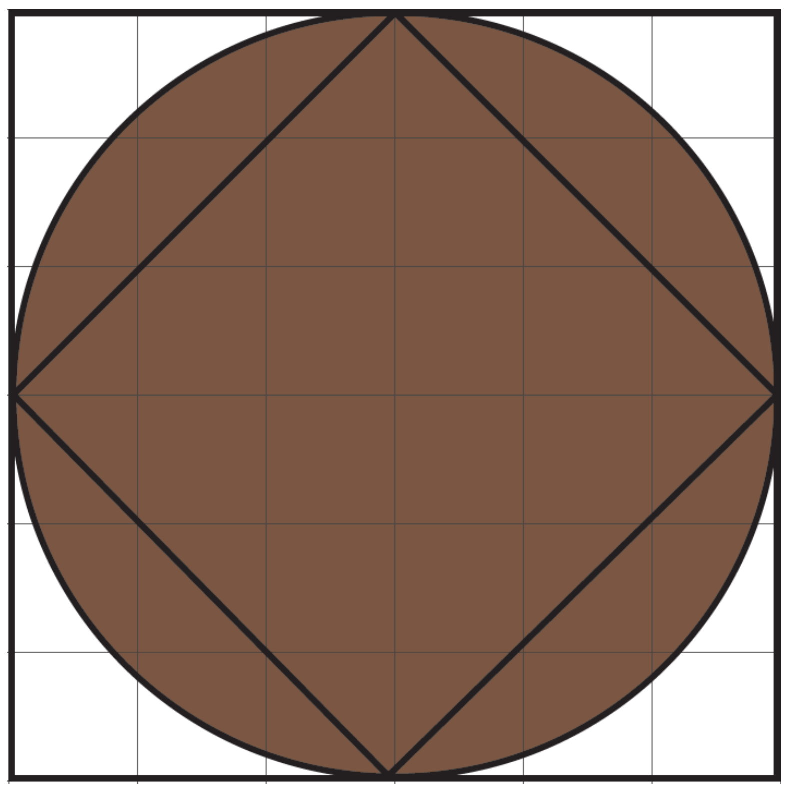


Measuring Circles

Student Workbook

Core Knowledge Mathematics™

Lesson 1: How Well Can You Measure?

1.1: Estimating a Percentage

A student got 16 out of 21 questions correct on a quiz. Use mental estimation to answer these questions.

1. Did the student answer less than or more than 80% of the questions correctly?

2. Did the student answer less than or more than 75% of the questions correctly?

1.2: Perimeter of a Square

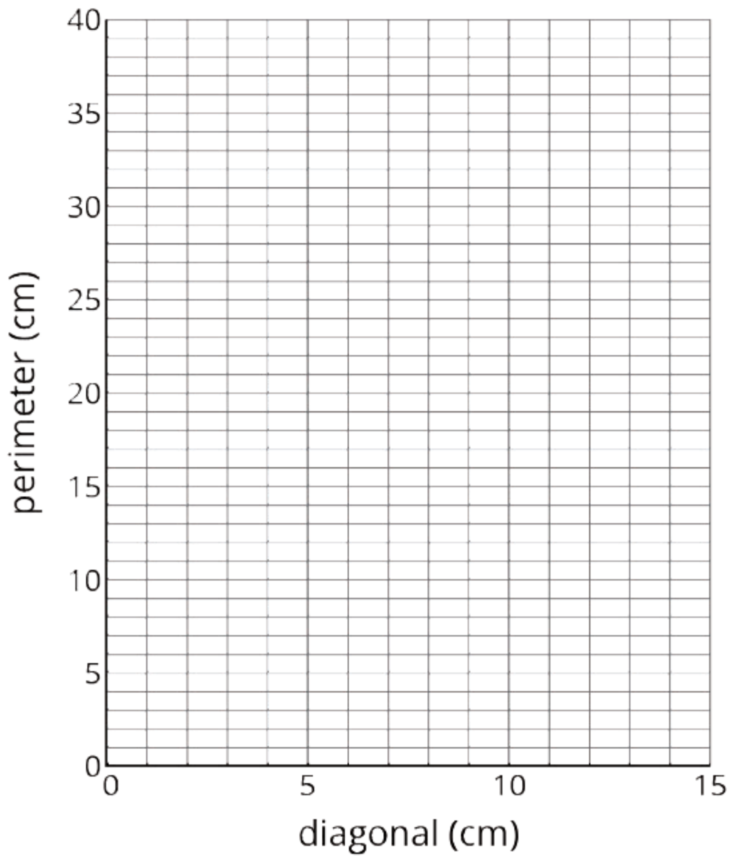
Your teacher will give you a picture of 9 different squares and will assign your group 3 of these squares to examine more closely.

1. For each of your assigned squares, measure the length of the diagonal and the perimeter of the square in centimeters.

   Check your measurements with your group. After you come to an agreement, record your measurements in the table.

	diagonal (cm)	perimeter (cm)
square A		
square B		
square C		
square D		
square E		
square F		
square G		
square H		
square I		

2. Plot the diagonal and perimeter values from the table on the coordinate plane.



3. What do you notice about the points on the graph?

Pause here so your teacher can review your work.

4. Record measurements of the other squares to complete your table.

1.3: Area of a Square

1. In the table, record the length of the diagonal for each of your assigned squares from the previous activity. Next, calculate the area of each of your squares.

	diagonal (cm)	area (cm^2)
square A		
square B		
square C		
square D		
square E		
square F		
square G		
square H		
square I		

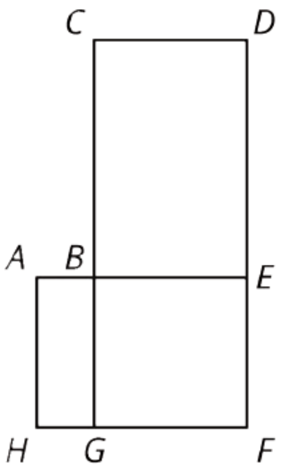
Pause here so your teacher can review your work. Be prepared to share your values with the class.

2. Examine the class graph of these values. What do you notice?

3. How is the relationship between the diagonal and area of a square the same as the relationship between the diagonal and perimeter of a square from the previous activity? How is it different?

Are you ready for more?

Here is a rough map of a neighborhood.



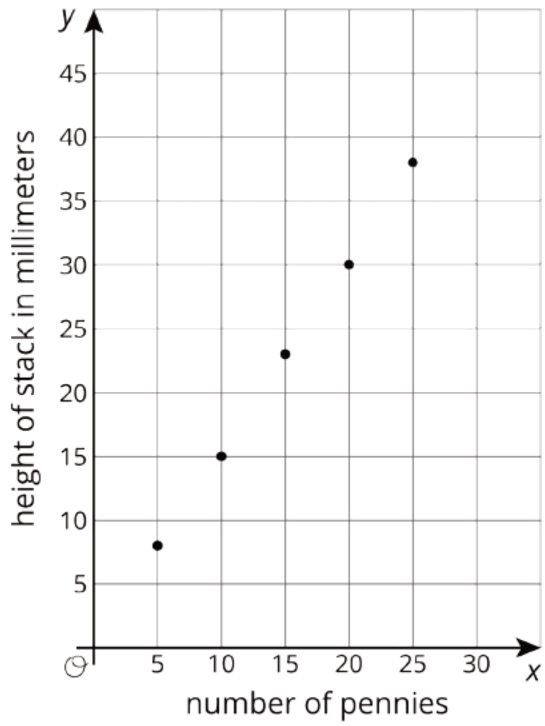
There are 4 mail routes during the week.

- On Monday, the mail truck follows the route A-B-E-F-G-H-A, which is 14 miles long.
- On Tuesday, the mail truck follows the route B-C-D-E-F-G-B, which is 22 miles long.
- On Wednesday, the truck follows the route A-B-C-D-E-F-G-H-A, which is 24 miles long.
- On Thursday, the mail truck follows the route B-E-F-G-B.

How long is the route on Thursdays?

Lesson 1 Summary

When we measure the values for two related quantities, plotting the measurements in the coordinate plane can help us decide if it makes sense to model them with a proportional relationship. If the points are close to a line through $(0, 0)$, then a proportional relationship is a good model. For example, here is a graph of the values for the height, measured in millimeters, of different numbers of pennies placed in a stack.



Because the points are close to a line through $(0, 0)$, the height of the stack of pennies appears to be proportional to the number of pennies in a stack. This makes sense because we can see that the heights of the pennies only vary a little bit.

An additional way to investigate whether or not a relationship is proportional is by making a table. Here is some data for the weight of different numbers of pennies in grams, along with the corresponding number of grams per penny.

number of pennies	grams	grams per penny
1	3.1	3.1
2	5.6	2.8
5	13.1	2.6
10	25.6	2.6

Though we might expect this relationship to be proportional, the quotients are not very close to one another. In fact, the metal in pennies changed in 1982, and older pennies are heavier. This explains why the weight per penny for different numbers of pennies are so different!

Unit 3 Lesson 1 Cumulative Practice Problems

1. Estimate the side length of a square that has a 9 cm long diagonal.

2. Select **all** quantities that are proportional to the diagonal length of a square.

    A. Area of the square

    B. Perimeter of the square

    C. Side length of the square

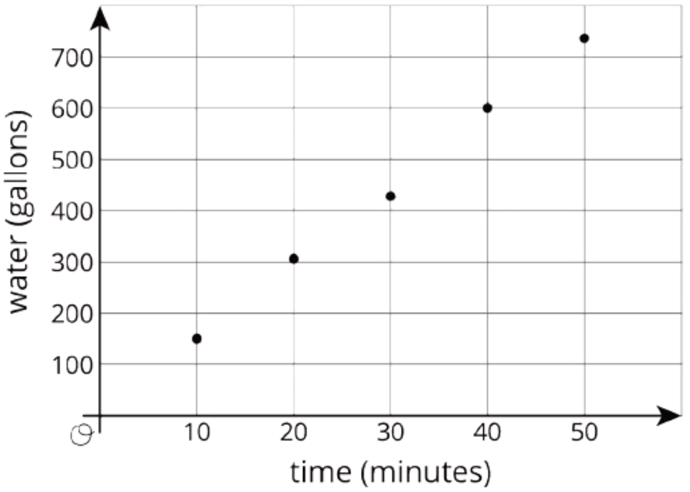
3. Diego made a graph of two quantities that he measured and said, “The points all lie on a line except one, which is a little bit above the line. This means that the quantities can’t be proportional.” Do you agree with Diego? Explain.

4. The graph shows that while it was being filled, the amount of water in gallons in a swimming pool was approximately proportional to the time that has passed in minutes.

   a. About how much water was in the pool after 25 minutes?

   b. Approximately when were there 500 gallons of water in the pool?

   c. Estimate the constant of proportionality for the gallons of water per minute going into the pool.



Lesson 2: Exploring Circles

2.1: How Do You Figure?

Here are two figures.

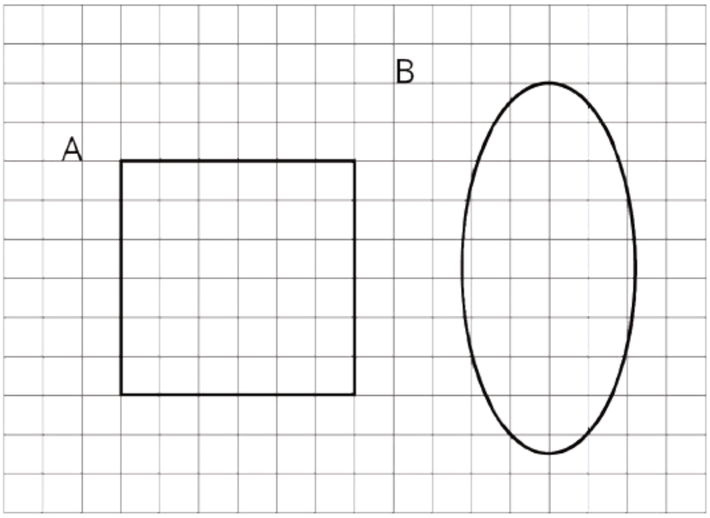


Figure C looks more like Figure A than like Figure B. Sketch what Figure C might look like. Explain your reasoning.

2.2: Sorting Round Objects

Your teacher will give you some pictures of different objects.

1. How could you sort these pictures into two groups? Be prepared to share your reasoning.

2. Work with your partner to sort the pictures into the categories that your class has agreed on. Pause here so your teacher can review your work.

3. What are some characteristics that all **circles** have in common?

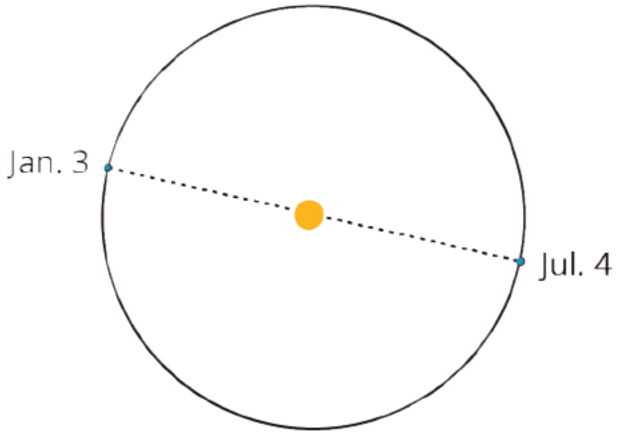
4. Put the circular objects in order from smallest to largest.

5. Select one of the pictures of a circular object. What are some ways you could measure the actual size of your circle?

Are you ready for more?

On January 3rd, Earth is 147,500,000 kilometers away from the Sun. On July 4th, Earth is 152,500,000 kilometers away from the Sun. The Sun has a radius of about 865,000 kilometers.

Could Earth's orbit be a circle with some point in the Sun as its center? Explain your reasoning.



2.3: Measuring Circles

Priya, Han, and Mai each measured one of the circular objects from earlier.

- Priya says that the bike wheel is 24 inches.
- Han says that the yo-yo trick is 24 inches.
- Mai says that the glow necklace is 24 inches.

1. Do you think that all these circles are the same size?

2. What part of the circle did each person measure? Explain your reasoning.

2.4: Drawing Circles

Draw and label each circle.

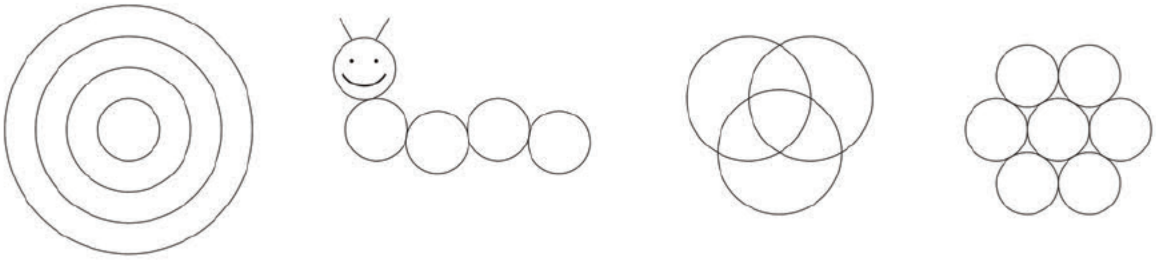
1. Circle A, with a **diameter** of 6 cm.

2. Circle B, with a **radius** of 5 cm. Pause here so your teacher can review your work.

3. Circle C, with a radius that is equal to Circle A’s diameter.

4. Circle D, with a diameter that is equal to Circle B's radius.

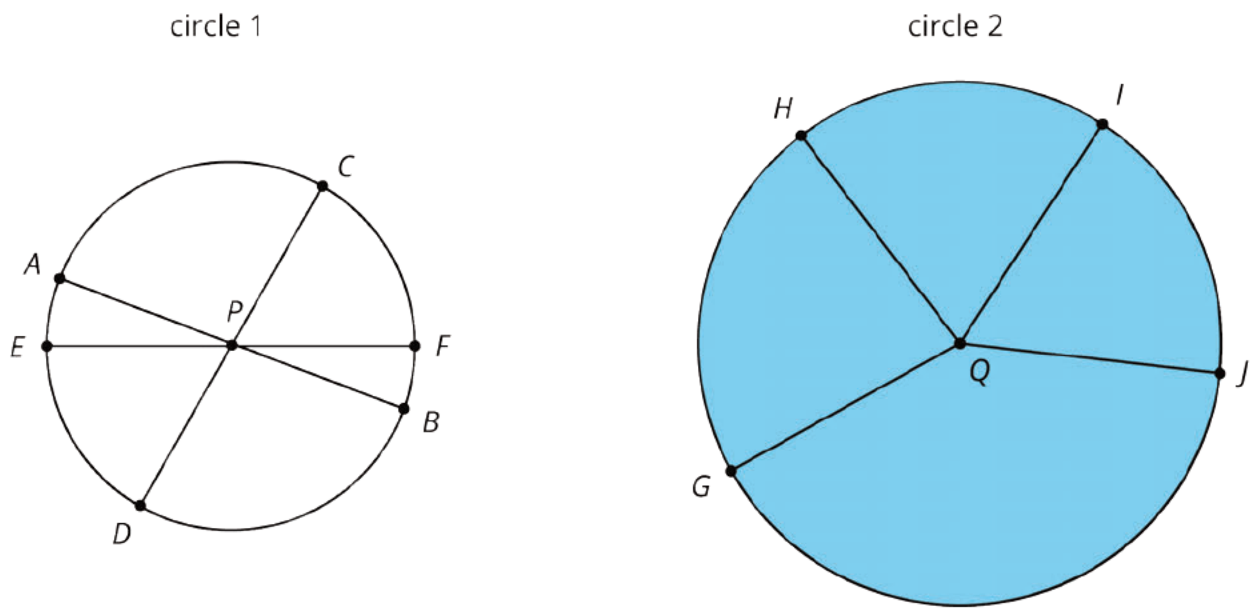
5. Use a compass to recreate one of these designs.



Lesson 2 Summary

A **circle** consists of all of the points that are the same distance away from a particular point called the *center* of the circle.

A segment that connects the center with any point on the circle is called a **radius**. For example, segments QG, QH, QI, and QJ are all radii of circle 2. (We say one radius and two radii.) The length of any radius is always the same for a given circle. For this reason, people also refer to this distance as the *radius* of the circle.



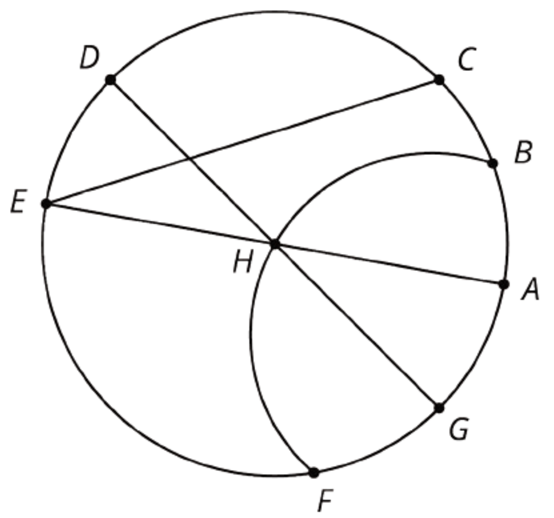
A segment that connects two opposite points on a circle (passing through the circle's center) is called a **diameter**. For example, segments AB, CD, and EF are all diameters of circle 1. All diameters in a given circle have the same length because they are composed of two radii. For this reason, people also refer to the length of such a segment as the *diameter* of the circle.

The **circumference** of a circle is the distance around it. If a circle was made of a piece of string and we cut it and straightened it out, the circumference would be the length of that string. A circle always encloses a circular region. The region enclosed by circle 2 is shaded, but the region enclosed by circle 1 is not. When we refer to the area of a circle, we mean the area of the enclosed circular region.

Unit 3 Lesson 2 Cumulative Practice Problems

1. Use a geometric tool to draw a circle. Draw and measure a radius and a diameter of the circle.

2. Here is a circle with center H and some line segments and curves joining points on the circle.



Identify examples of the following. Explain your reasoning.

a. Diameter

b. Radius

3. Lin measured the diameter of a circle in two different directions. Measuring vertically, she got 3.5 cm, and measuring horizontally, she got 3.6 cm. Explain some possible reasons why these measurements differ.

4. A small, test batch of lemonade used $\frac{1}{4}$ cup of sugar added to 1 cup of water and $\frac{1}{4}$ cup of lemon juice. After confirming it tasted good, a larger batch is going to be made with the same ratios using 10 cups of water. How much sugar should be added so that the large batch tastes the same as the test batch?

(From Unit 2, Lesson 1.)

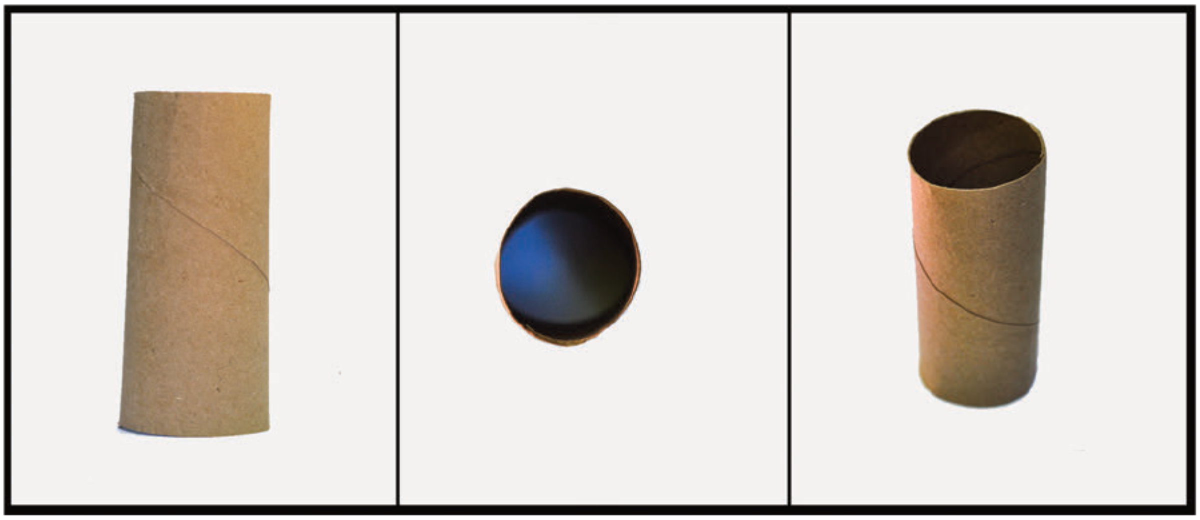
5. The graph of a proportional relationship contains the point with coordinates $(3, 12)$. What is the constant of proportionality of the relationship?

(From Unit 2, Lesson 13.)

Lesson 3: Exploring Circumference

3.1: Which Is Greater?

Clare wonders if the height of the toilet paper tube or the distance around the tube is greater. What information would she need in order to solve the problem? How could she find this out?



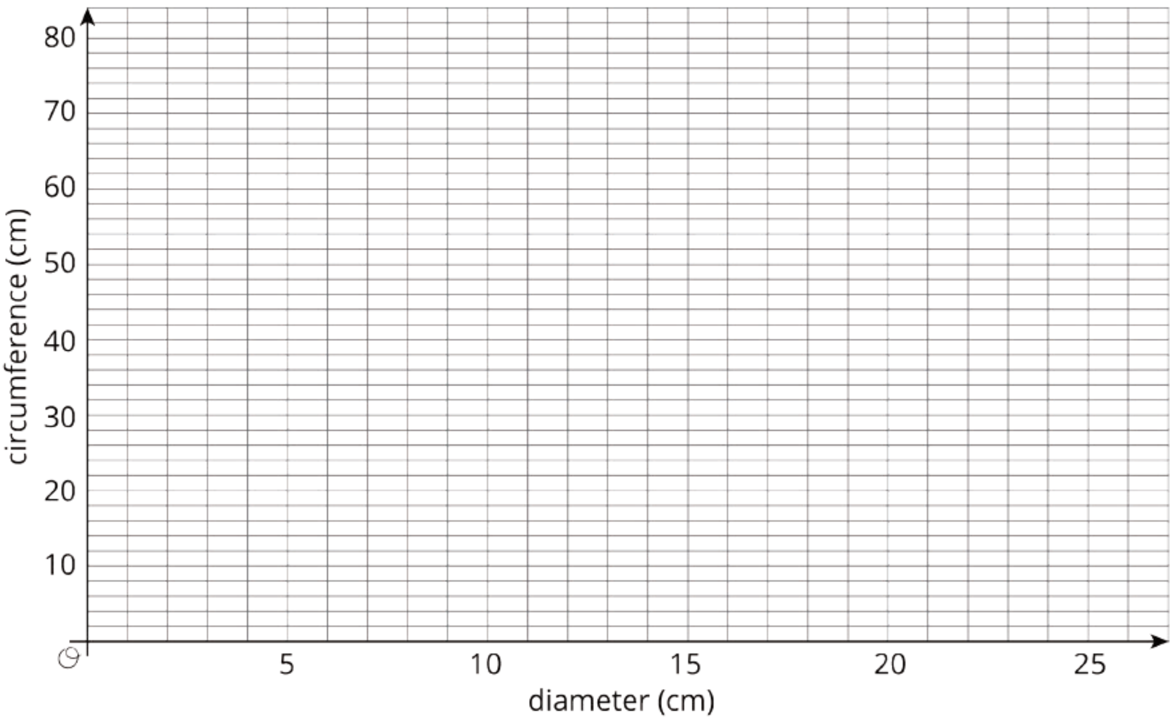
3.2: Measuring Circumference and Diameter

Your teacher will give you several circular objects.

1. Measure the diameter and the circumference of the circle in each object to the nearest tenth of a centimeter. Record your measurements in the table.

object	diameter (cm)	circumference (cm)

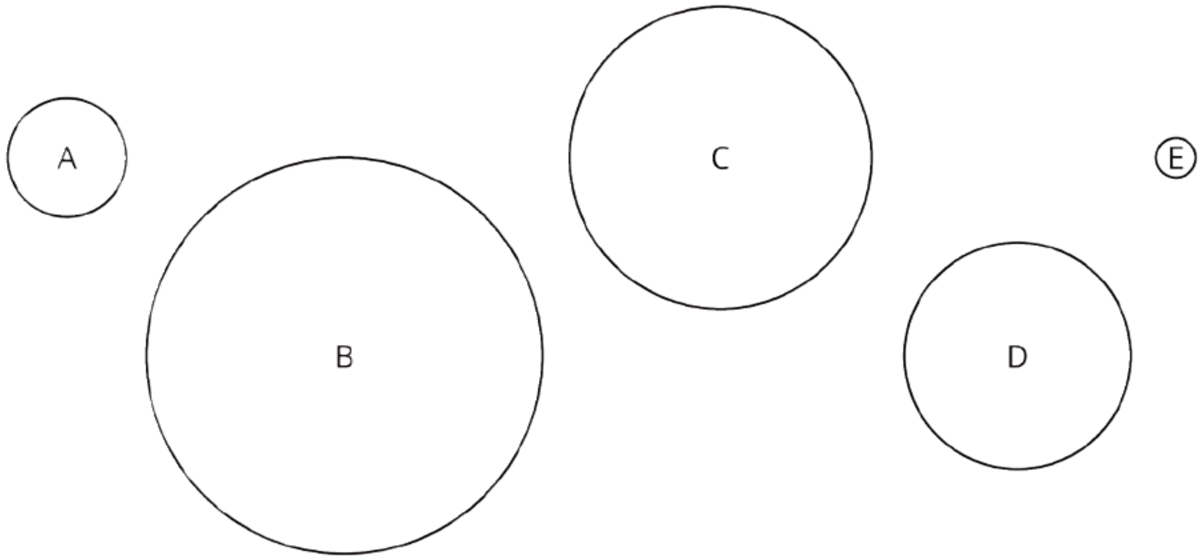
2. Plot the diameter and circumference values from the table on the coordinate plane. What do you notice?



3. Plot the points from two other groups on the same coordinate plane. Do you see the same pattern that you noticed earlier?

3.3: Calculating Circumference and Diameter

Here are five circles. One measurement for each circle is given in the table.



Use the constant of proportionality estimated in the previous activity to complete the table.

	diameter (cm)	circumference (cm)
circle A	3	
circle B	10	
circle C		24
circle D		18
circle E	1	

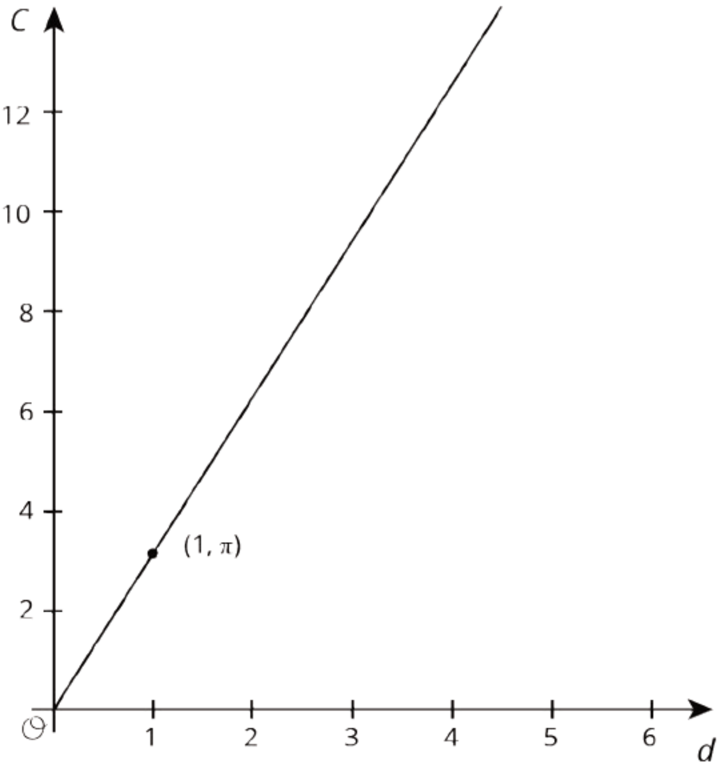
Are you ready for more?

The circumference of Earth is approximately 40,000 km. If you made a circle of wire around the globe, that is only 10 meters (0.01 km) longer than the circumference of the globe, could a flea, a mouse, or even a person creep under it?

Lesson 3 Summary

There is a proportional relationship between the diameter and circumference of any circle. That means that if we write C for circumference and d for diameter, we know that $C = kd$, where k is the constant of proportionality.

The exact value for the constant of proportionality is called π. Some frequently used approximations for π are $\frac{22}{7}$, 3.14, and 3.14159, but none of these is exactly π.



We can use this to estimate the circumference if we know the diameter, and vice versa. For example, using 3.1 as an approximation for π, if a circle has a diameter of 4 cm, then the circumference is about $(3.1) \cdot 4 = 12.4$ or 12.4 cm.

The relationship between the circumference and the diameter can be written as

$$C = \pi d$$

Unit 3 Lesson 3 Cumulative Practice Problems

1. Diego measured the diameter and circumference of several circular objects and recorded his measurements in the table.

object	diameter (cm)	circumference (cm)
half dollar coin	3	10
flying disc	23	28
jar lid	8	25
flower pot	15	48

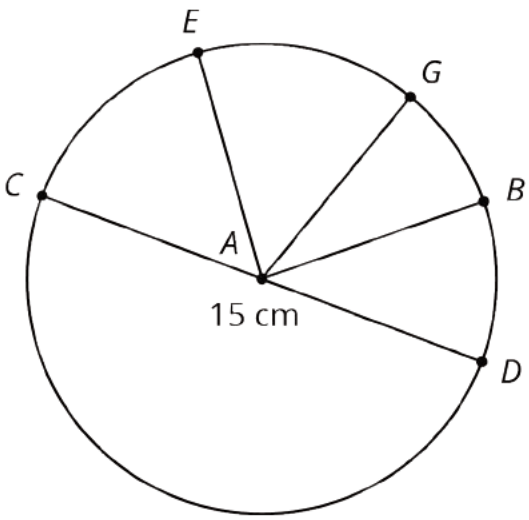
One of his measurements is inaccurate. Which measurement is it? Explain how you know.

2. Complete the table. Use one of the approximate values for π discussed in class (for example 3.14, $\frac{22}{7}$, 3.1416). Explain or show your reasoning.

object	diameter	circumference
hula hoop	35 in	
circular pond		556 ft
magnifying glass	5.2 cm	
car tire		71.6 in

3. A is the center of the circle, and the length of CD is 15 centimeters.

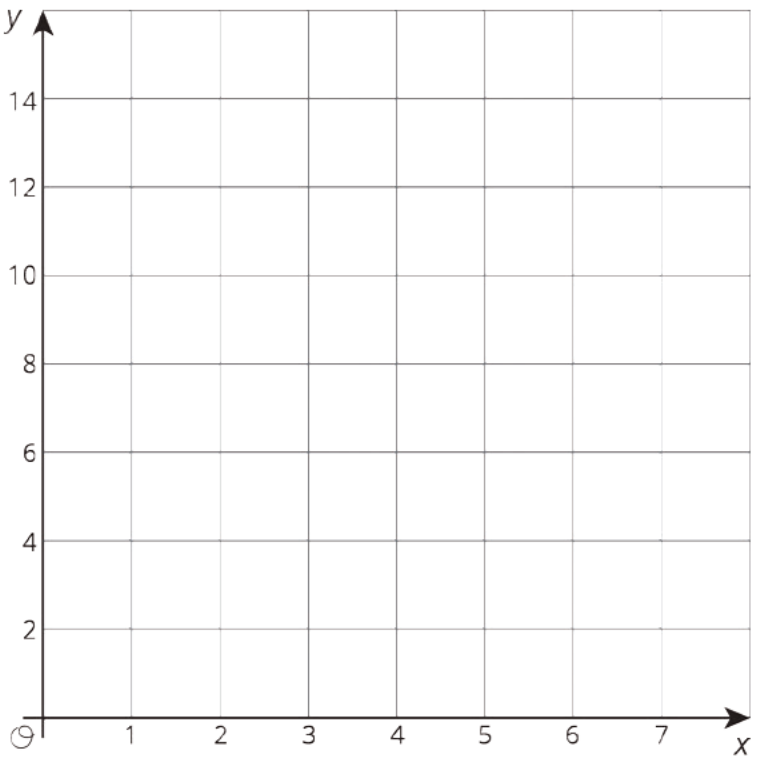
a. Name a segment that is a radius.
How long is it?



b. Name a segment that is a diameter.
How long is it?

(From Unit 3, Lesson 2.)

4. a. Consider the equation $y = 1.5x + 2$. Find four pairs of x and y values that make the equation true. Plot the points (x, y) on the coordinate plane.



b. Based on the graph, can this be a proportional relationship? Why or why not?

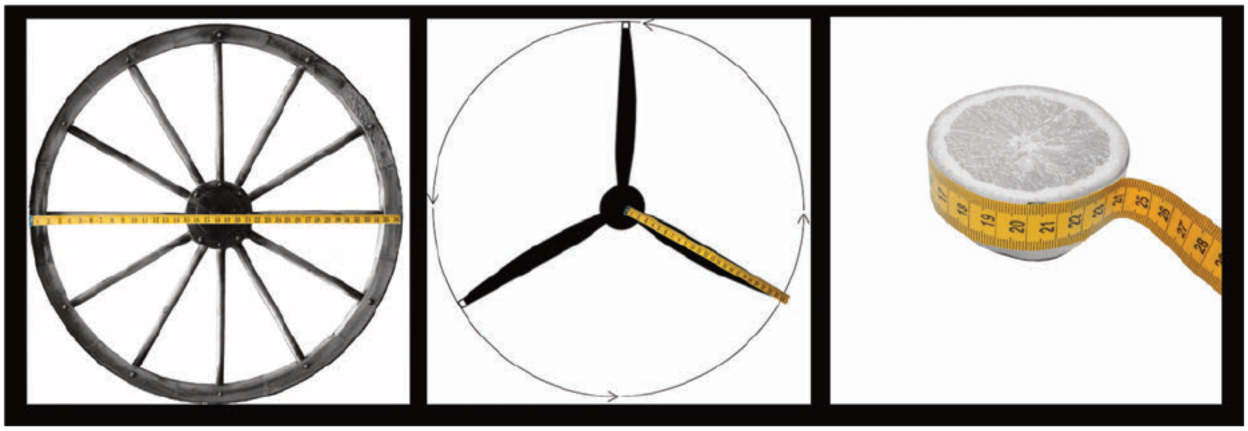
(From Unit 2, Lesson 10.)

Lesson 4: Applying Circumference

4.1: What Do We Know? What Can We Estimate?

Here are some pictures of circular objects, with measurement tools shown. The measurement tool on each picture reads as follows:

- Wagon wheel: 3 feet
- Plane propeller: 24 inches
- Sliced Orange: 20 centimeters



1. For each picture, which measurement is shown?

2. Based on this information, what measurement(s) could you estimate for each picture?

4.2: Using π

In the previous activity, we looked at pictures of circular objects. One measurement for each object is listed in the table.

Your teacher will assign you an approximation of π to use for this activity.

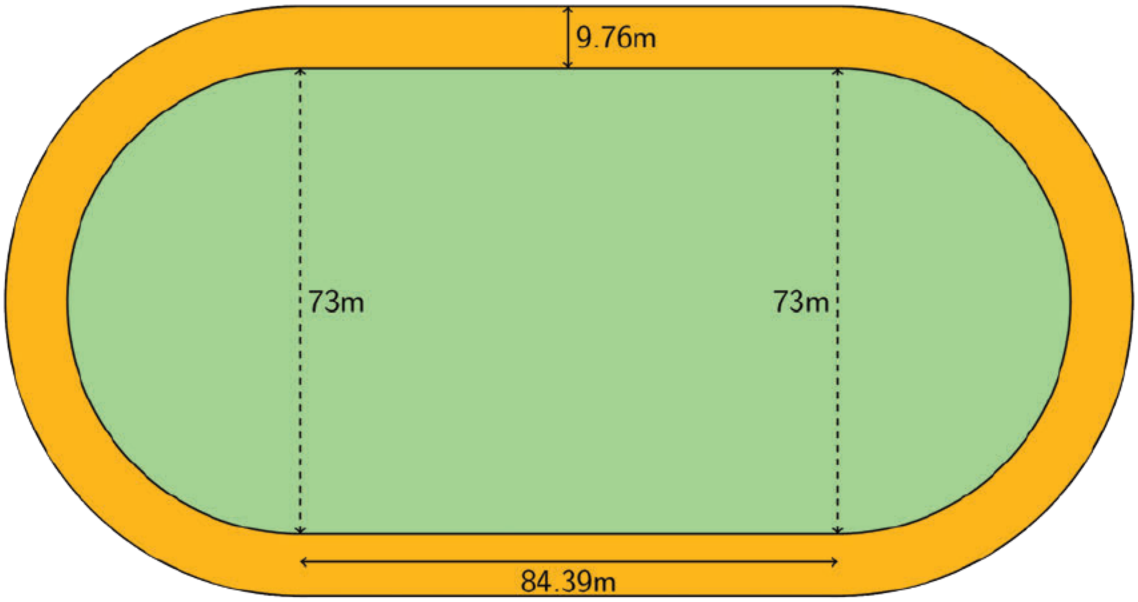
1. Complete the table.

object	radius	diameter	circumference
wagon wheel		3 ft	
airplane propeller	24 in		
orange slice			20 cm

2. A bug was sitting on the tip of the propeller blade when the propeller started to rotate. The bug held on for 5 rotations before flying away. How far did the bug travel before it flew off?

4.3: Around the Running Track

The field inside a running track is made up of a rectangle that is 84.39 m long and 73 m wide, together with a half-circle at each end.



1. What is the distance around the inside of the track? Explain or show your reasoning.

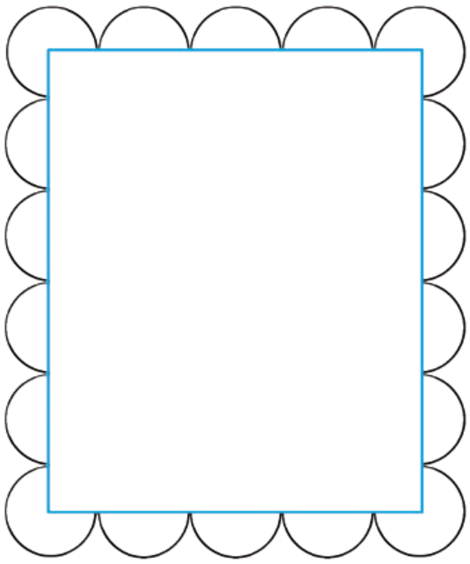
2. The track is 9.76 m wide all the way around. What is the distance around the outside of the track? Explain or show your reasoning.

Are you ready for more?

This size running track is usually called a 400-meter track. However, if a person ran as close to the "inside" as possible on the track, they would run less than 400 meters in one lap. How far away from the inside border would someone have to run to make one lap equal exactly 400 meters?

4.4: Measuring a Picture Frame

Kiran bent some wire around a rectangle to make a picture frame. The rectangle is 8 inches by 10 inches.



1. Find the perimeter of the wire picture frame. Explain or show your reasoning.

2. If the wire picture frame were stretched out to make one complete circle, what would its radius be?

Lesson 4 Summary

The circumference of a circle, C, is π times the diameter, d. The diameter is twice the radius, r. So if we know any one of these measurements for a particular circle, we can find the others. We can write the relationships between these different measures using equations:

$$d = 2r$$
$$C = \pi d$$
$$C = 2\pi r$$

If the diameter of a car tire is 60 cm, that means the radius is 30 cm and the circumference is $60 \cdot \pi$ or about 188 cm.

If the radius of a clock is 5 in, that means the diameter is 10 in, and the circumference is $10 \cdot \pi$ or about 31 in.

If a ring has a circumference of 44 mm, that means the diameter is $44 \div \pi$, which is about 14 mm, and the radius is about 7 mm.

Unit 3 Lesson 4 Cumulative Practice Problems

1. Here is a picture of a Ferris wheel. It has a diameter of 80 meters.

a. On the picture, draw and label a diameter.

b. How far does a rider travel in one complete rotation around the Ferris wheel?

2. Identify each measurement as the diameter, radius, or circumference of the circular object. Then, estimate the other two measurements for the circle.

a. The length of the minute hand on a clock is 5 in.

b. The distance across a sink drain is 3.8 cm.

c. The tires on a mining truck are 14 ft tall.

d. The fence around a circular pool is 75 ft long.

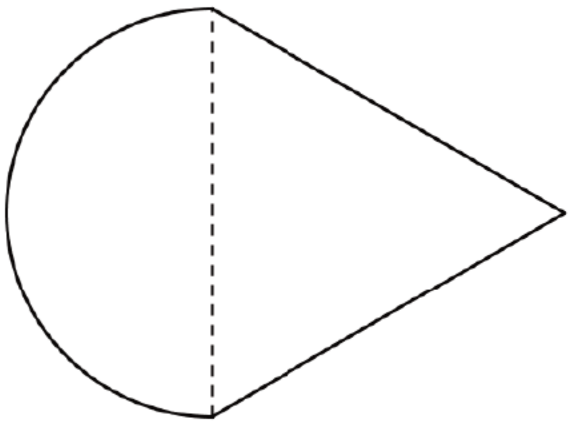
e. The distance from the tip of a slice of pizza to the crust is 7 in.

f. Breaking a cookie in half creates a straight side 10 cm long.

g. The length of the metal rim around a glass lens is 190 mm.

h. From the center to the edge of a DVD measures 60 mm.

3. A half circle is joined to an equilateral triangle with side lengths of 12 units. What is the perimeter of the resulting shape?



4. Circle A has a diameter of 1 foot. Circle B has a circumference of 1 meter. Which circle is bigger? Explain your reasoning. (1 inch = 2.54 centimeters)

5. The circumference of Tyler's bike tire is 72 inches. What is the diameter of the tire?

(From Unit 3, Lesson 3.)

Lesson 5: Circumference and Wheels

5.1: A Rope and a Wheel

Han says that you can wrap a 5-foot rope around a wheel with a 2-foot diameter because $\frac{5}{2}$ is less than pi. Do you agree with Han? Explain your reasoning.

5.2: Rolling, Rolling, Rolling

Your teacher will give you a circular object.

1. Follow these instructions to create the drawing:
    - On a separate piece of paper, use a ruler to draw a line all the way across the page.
    - Roll your object along the line and mark where it completes one rotation.
    - Use your object to draw tick marks along the line that are spaced as far apart as the diameter of your object.
2. What do you notice?

3. Use your ruler to measure each distance. Record these values in the first row of the table:

    a. the diameter of your object

    b. how far your object rolled in one complete rotation

    c. the quotient of how far your object rolled divided by the diameter of your object

object	diameter	distance traveled in one rotation	distance ÷ diameter

4. If you wanted to trace two complete rotations of your object, how long of a line would you need?

5. Share your results with your group and record their measurements in the table.

6. If each person in your group rolled their object along the entire length of the classroom, which object would complete the most rotations? Explain or show your reasoning.

5.3: Rotations and Distance

1. A car wheel has a diameter of 20.8 inches.

   a. About how far does the car wheel travel in 1 rotation? 5 rotations? 30 rotations?

   b. Write an equation relating the distance the car travels in inches, c, to the number of wheel rotations, x.

   c. About how many rotations does the car wheel make when the car travels 1 mile? Explain or show your reasoning.

2. A bike wheel has a radius of 13 inches.

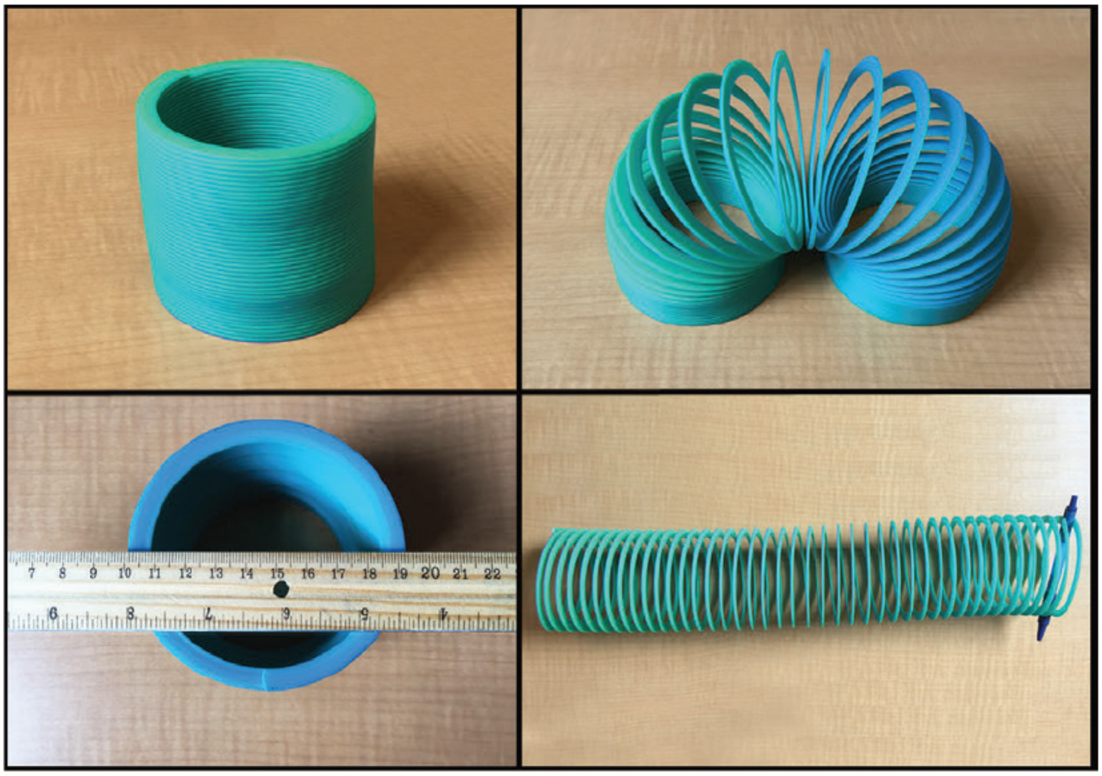
   a. About how far does the bike wheel travel in 1 rotation? 5 rotations? 30 rotations?

   b. Write an equation relating the distance the bike travels in inches, b, to the number of wheel rotations, x.

   c. About how many rotations does the bike wheel make when the bike travels 1 mile? Explain or show your reasoning.

Are you ready for more?

Here are some photos of a spring toy.



If you could stretch out the spring completely straight, how long would it be? Explain or show your reasoning.

5.4: Rotations and Speed

The circumference of a car wheel is about 65 inches.

1. If the car wheel rotates once per second, how far does the car travel in one minute?

2. If the car wheel rotates once per second, about how many miles does the car travel in one hour?

3. If the car wheel rotates 5 times per second, about how many miles does the car travel in one hour?

4. If the car is traveling 65 miles per hour, about how many times per second does the wheel rotate?

Lesson 5 Summary

The circumference of a circle is the distance around the circle. This is also how far the circle rolls on flat ground in one rotation. For example, a bicycle wheel with a diameter of 24 inches has a circumference of 24π inches and will roll 24π inches (or 2π feet) in one complete rotation. There is an equation relating the number of rotations of the wheel to the distance it has traveled. To see why, let's look at a table showing how far the bike travels when the wheel makes different numbers of rotations.

number of rotations	distance traveled (feet)
1	2π
2	4π
3	6π
10	20π
50	100π
x	?

In the table, we see that the relationship between the distance traveled and the number of wheel rotations is a proportional relationship. The constant of proportionality is 2π.

To find the missing value in the last row of the table, note that each rotation of the wheel contributes 2π feet of distance traveled. So after x rotations the bike will travel $2\pi x$ feet. If d is the distance, in feet, traveled when this wheel makes x rotations, we have the relationship:

$$d = 2\pi x$$

Unit 3 Lesson 5 Cumulative Practice Problems

1. The diameter of a bike wheel is 27 inches. If the wheel makes 15 complete rotations, how far does the bike travel?

2. The wheels on Kiran's bike are 64 inches in circumference. How many times do the wheels rotate if Kiran rides 300 yards?

3. The numbers are measurements of radius, diameter, and circumference of circles A and B. Circle A is smaller than circle B. Which number belongs to which quantity?

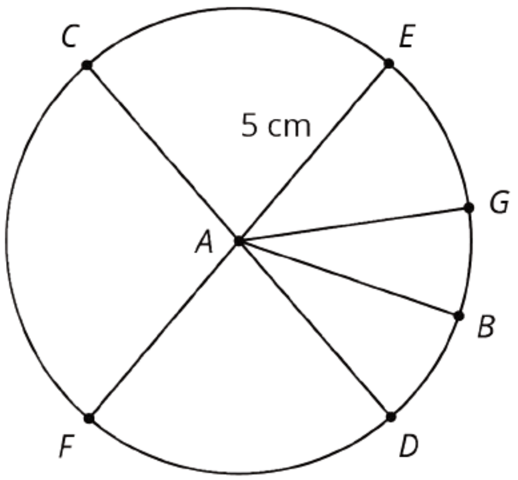
   2.5, 5, 7.6, 15.2, 15.7, 47.7

   (From Unit 3, Lesson 4.)

4. Circle A has circumference $2\frac{2}{3}$ m. Circle B has a diameter that is $1\frac{1}{2}$ times as long as Circle A's diameter. What is the circumference of Circle B?

(From Unit 3, Lesson 3.)

5. The length of segment AE is 5 centimeters.



a. What is the length of segment CD?

b. What is the length of segment AB?

c. Name a segment that has the same length as segment AB.

(From Unit 3, Lesson 2.)

Lesson 6: Estimating Areas

6.1: Mental Calculations

Find a strategy to make each calculation mentally:

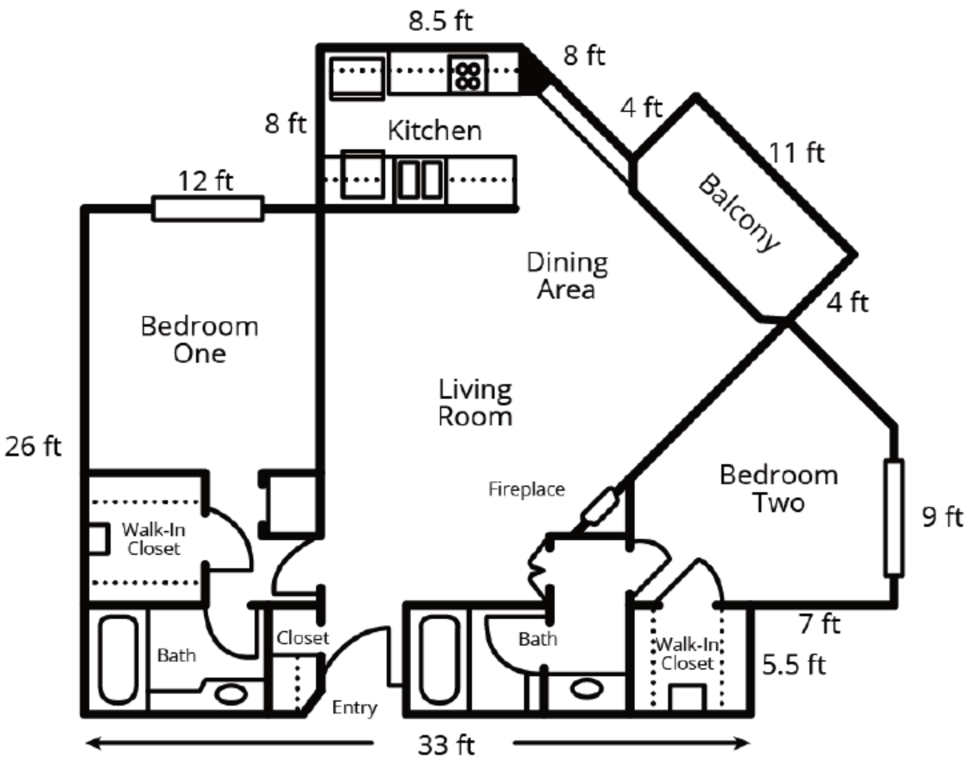
$599 + 87$

$254 - 88$

$99 \cdot 75$

6.2: House Floorplan

Here is a floor plan of a house. Approximate lengths of the walls are given. What is the approximate area of the home, including the balcony? Explain or show your reasoning.

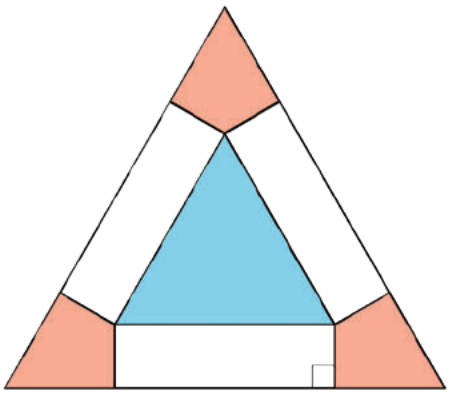


6.3: Area of Nevada

Estimate the area of Nevada in square miles. Explain or show your reasoning.

Are you ready for more?

The two triangles are equilateral, and the three pink regions are identical. The blue equilateral triangle has the same area as the three pink regions taken together. What is the ratio of the sides of the two equilateral triangles?



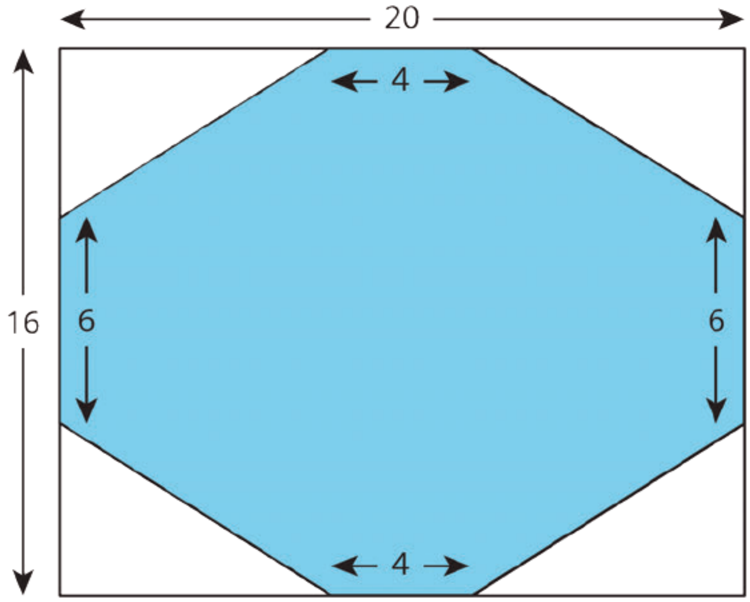
Lesson 6 Summary

We can find the area of some complex polygons by surrounding them with a simple polygon like a rectangle. For example, this octagon is contained in a rectangle.

The rectangle is 20 units long and 16 units wide, so its area is 320 square units. To get the area of the octagon, we need to subtract the areas of the four right triangles in the corners. These triangles are each 8 units long and 5 units wide, so they each have an area of 20 square units. The area of the octagon is

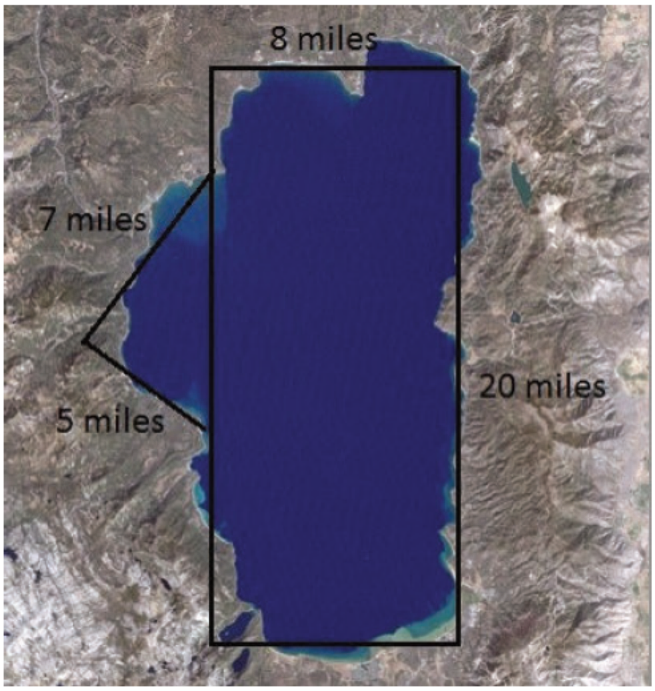
$$320 - (4 \cdot 20)$$

or 240 square units.



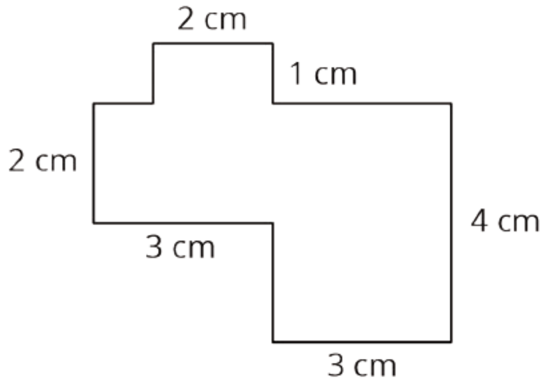
We can estimate the area of irregular shapes by approximating them with a polygon and finding the area of the polygon. For example, here is a satellite picture of Lake Tahoe with some one-dimensional measurements around the lake.

The area of the rectangle is 160 square miles, and the area of the triangle is 17.5 square miles for a total of 177.5 square miles. We recognize that this is an approximation, and not likely the exact area of the lake.



Unit 3 Lesson 6 Cumulative Practice Problems

1. Find the area of the polygon.



2. a. Draw polygons on the map that could be used to approximate the area of Virginia.

b. Which measurements would you need to know in order to calculate an approximation of the area of Virginia? Label the sides of the polygons whose measurements you would need. (Note: You aren't being asked to calculate anything.)

3. Jada's bike wheels have a diameter of 20 inches. How far does she travel if the wheels rotate 37 times?

(From Unit 3, Lesson 5.)

4. The radius of Earth is approximately 6,400 km. The equator is the circle around Earth dividing it into the northern and southern hemispheres. (The center of the earth is also the center of the equator.) What is the length of the equator?

(From Unit 3, Lesson 4.)

5. Here are several recipes for sparkling lemonade. For each recipe, describe how many tablespoons of lemonade mix it takes per cup of sparkling water.

    a. 4 tablespoons of lemonade mix and 12 cups of sparkling water

    b. 4 tablespoons of lemonade mix and 6 cups of sparkling water

    c. 3 tablespoons of lemonade mix and 5 cups of sparkling water

    d. $\frac{1}{2}$ tablespoon of lemonade mix and $\frac{3}{4}$ cups of sparkling water

(From Unit 2, Lesson 1.)

Lesson 7: Exploring the Area of a Circle

7.1: Estimating Areas

Your teacher will show you some figures. Decide which figure has the largest area. Be prepared to explain your reasoning.

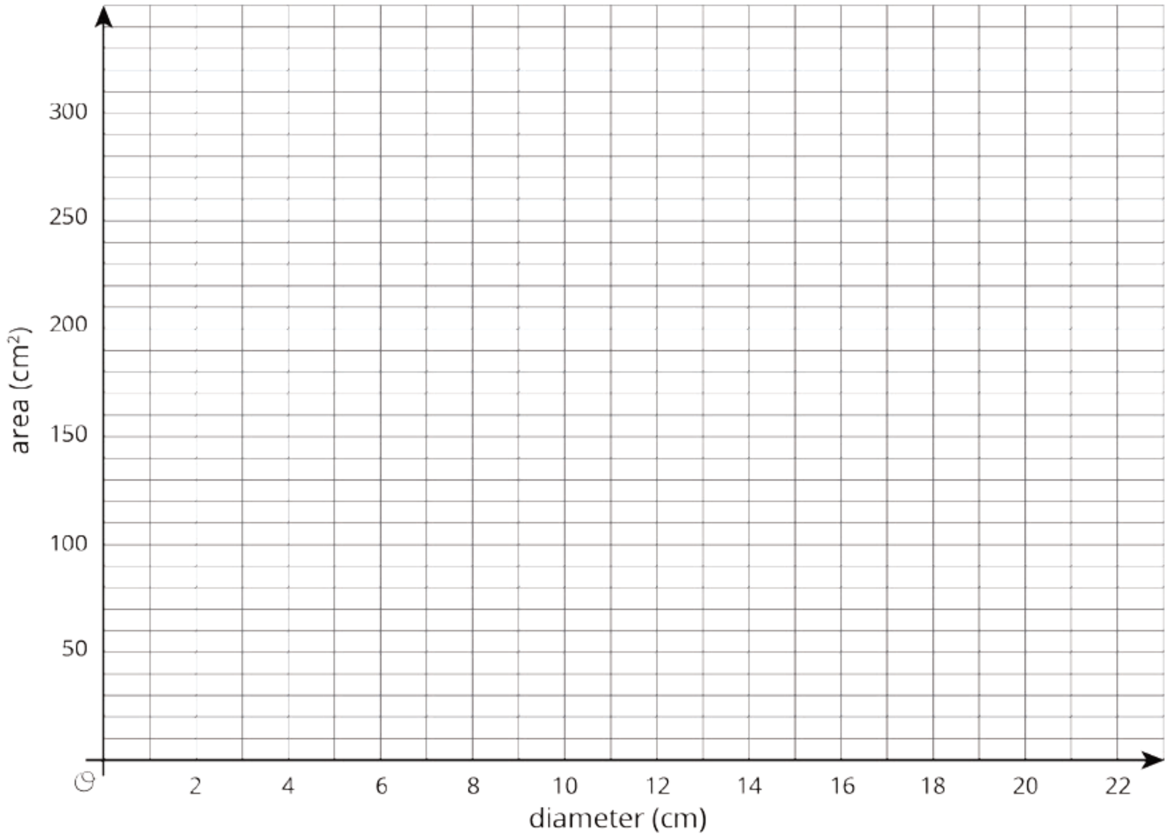
7.2: Estimating Areas of Circles

Your teacher will give your group two circles of different sizes.

1. For each circle, use the squares on the graph paper to measure the diameter and estimate the **area of the circle**. Record your measurements in the table.

diameter (cm)	estimated area (cm^2)

2. Plot the values from the table on the class coordinate plane. Then plot the class's data points on your coordinate plane.



3. In a previous lesson, you graphed the relationship between the diameter and circumference of a circle. How is this graph the same? How is it different?

Are you ready for more?

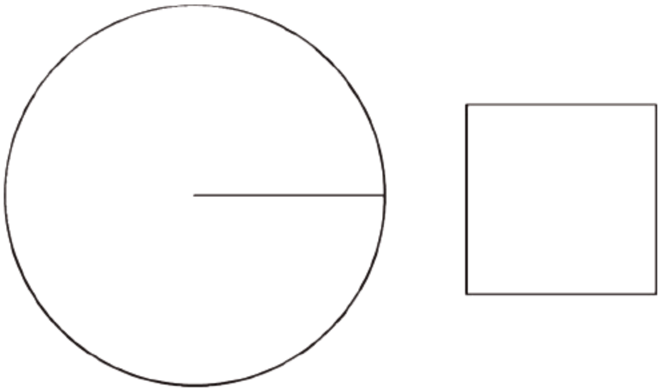
How many circles of radius 1 unit can you fit inside each of the following so that they do not overlap?

1. a circle of radius 2 units?
2. a circle of radius 3 units?
3. a circle of radius 4 units?

If you get stuck, consider using coins or other circular objects.

7.3: Covering a Circle

Here is a square whose side length is the same as the radius of the circle.



How many of these squares do you think it would take to cover the circle exactly?

Lesson 7 Summary

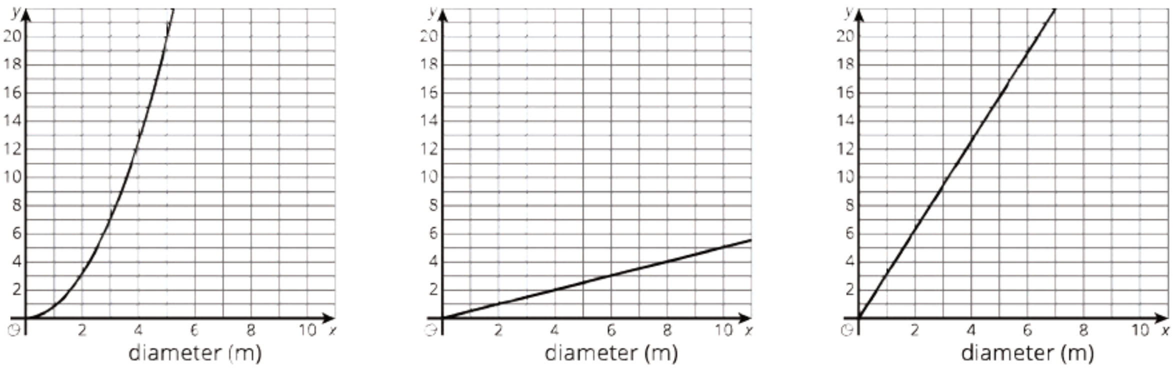
The circumference C of a circle is proportional to the diameter d, and we can write this relationship as $C = \pi d$. The circumference is also proportional to the radius of the circle, and the constant of proportionality is $2 \cdot \pi$ because the diameter is twice as long as the radius. However, the **area of a circle** is *not* proportional to the diameter (or the radius).

The area of a circle with radius r is a little more than 3 times the area of a square with side r so the area of a circle of radius r is approximately $3r^2$. We saw earlier that the circumference of a circle of radius r is $2\pi r$. If we write C for the circumference of a circle, this proportional relationship can be written $C = 2\pi r$.

The area A of a circle with radius r is approximately $3r^2$. Unlike the circumference, the area is not proportional to the radius because $3r^2$ cannot be written in the form kr for a number k. We will investigate and refine the relationship between the area and the radius of a circle in future lessons.

Unit 3 Lesson 7 Cumulative Practice Problems

1. The x-axis of each graph has the diameter of a circle in meters. Label the y-axis on each graph with the appropriate measurement of a circle: radius (m), circumference (m), or area (m^2).



2. Circle A has area 500 in^2. The diameter of circle B is three times the diameter of circle A. Estimate the area of circle B.

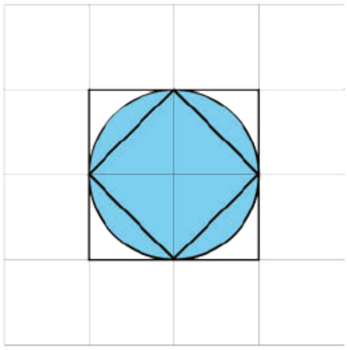
3. Lin's bike travels 100 meters when her wheels rotate 55 times. What is the circumference of her wheels?

(From Unit 3, Lesson 5.)

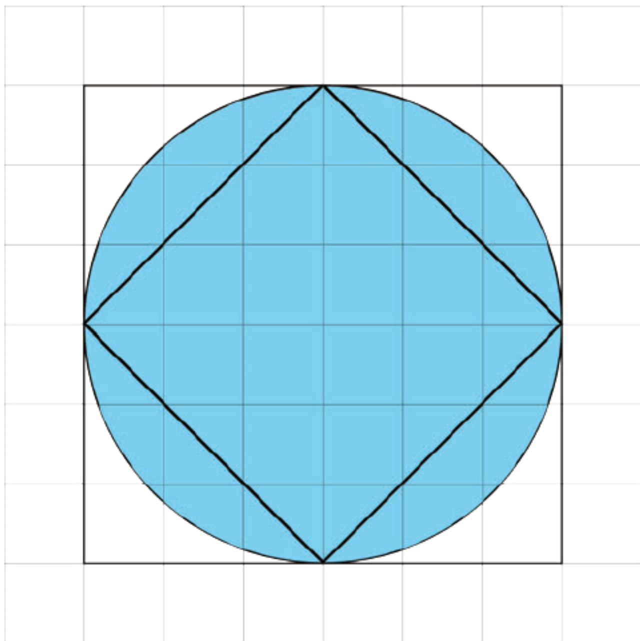
4. Priya drew a circle whose circumference is 25 cm. Clare drew a circle whose diameter is 3 times the diameter of Priya's circle. What is the circumference of Clare's circle?

(From Unit 3, Lesson 3.)

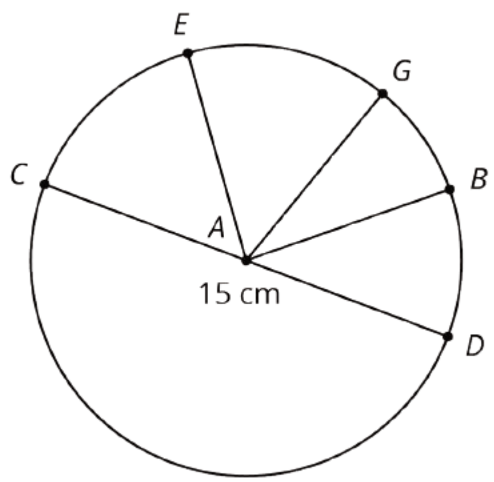
5. a. Here is a picture of two squares and a circle. Use the picture to explain why the area of this circle is more than 2 square units but less than 4 square units.



b. Here is another picture of two squares and a circle. Use the picture to explain why the area of this circle is more than 18 square units and less than 36 square units.



6. Point A is the center of the circle, and the length of CD is 15 centimeters. Find the circumference of this circle.

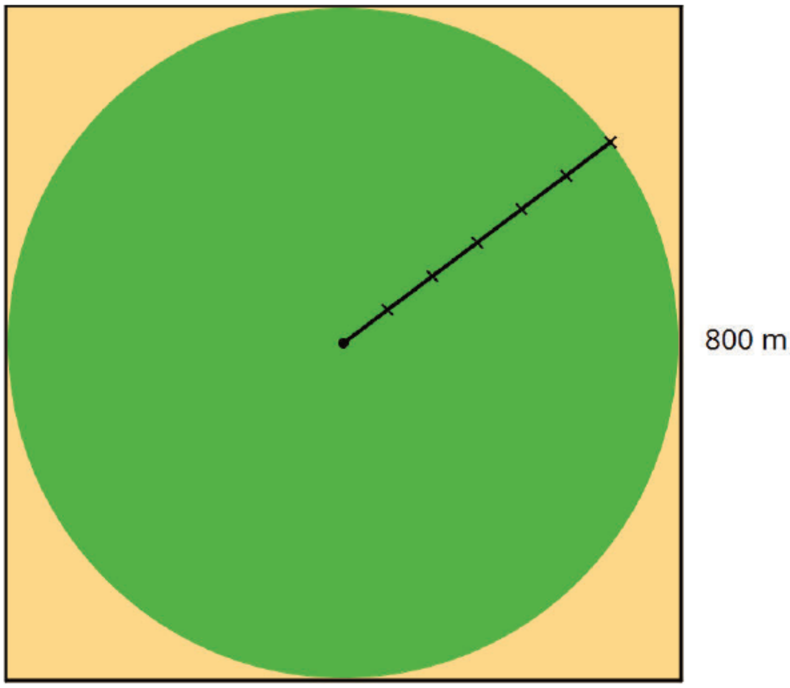


(From Unit 3, Lesson 3.)

Lesson 8: Relating Area to Circumference

8.1: Irrigating a Field

A circular field is set into a square with an 800 m side length. Estimate the field's area.



- About 5,000 m^2
- About 50,000 m^2
- About 500,000 m^2
- About 5,000,000 m^2
- About 50,000,000 m^2

8.2: Making a Polygon out of a Circle

Your teacher will give you a circular object, a marker, and two pieces of paper of different colors.

Follow these instructions to create a visual display:

1. Using a thick marker, trace your circle in two separate places on the same piece of paper.
2. Cut out both circles, cutting around the marker line.
3. Fold and cut one of the circles into fourths.
4. Arrange the fourths so that straight sides are next to each other, but the curved edges are alternately on top and on bottom. Pause here so your teacher can review your work.
5. Fold and cut the fourths in half to make eighths. Arrange the eighths next to each other, like you did with the fourths.
6. If your pieces are still large enough, repeat the previous step to make sixteenths.
7. Glue the remaining circle and the new shape onto a piece of paper that is a different color.

After you finish gluing your shapes, answer the following questions.

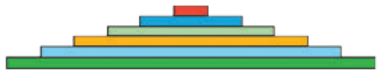
1. How do the areas of the two shapes compare?
2. What polygon does the shape made of the circle pieces most resemble?
3. How could you find the area of this polygon?

8.3: Making Another Polygon out of a Circle

Imagine a circle made of rings that can bend, but not stretch.

A circle is made of rings.

The rings are unrolled.



The circle has been made into a new shape.

1. What polygon does the new shape resemble?

2. How does the area of the polygon compare to the area of the circle?

3. How can you find the area of the polygon?

4. Show, in detailed steps, how you could find the polygon’s area in terms of the circle’s measurements. Show your thinking. Organize it so it can be followed by others.

5. After you finish, trade papers with a partner and check each other’s work. If you disagree, work to reach an agreement. Discuss:
    - Do you agree or disagree with each step?
    - Is there a way to make the explanation clearer?

6. Return your partner’s work, and revise your explanation based on the feedback you received.

8.4: Tiling a Table

Elena wants to tile the top of a circular table. The diameter of the table top is 28 inches. What is its area?

Are you ready for more?

A box contains 20 square tiles that are 2 inches on each side. How many boxes of tiles will Elena need to tile the table?

Lesson 8 Summary

If C is a circle's circumference and r is its radius, then $C = 2\pi r$. The area of a circle can be found by taking the product of half the circumference and the radius.

If A is the area of the circle, this gives the equation: $A = \frac{1}{2}(2\pi r) \cdot r$

This equation can be rewritten as: $A = \pi r^2$

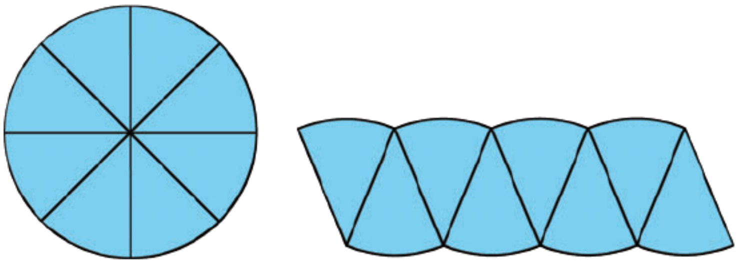
(Remember that when we have $r \cdot r$ we can write r^2 and we can say "r **squared**.")

This means that if we know the radius, we can find the area. For example, if a circle has radius 10 cm, then the area is about $(3.14) \cdot 100$ which is 314 cm^2.

If we know the diameter, we can figure out the radius, and then we can find the area. For example, if a circle has a diameter of 30 ft, then the radius is 15 ft, and the area is about $(3.14) \cdot 225$ which is approximately 707 ft^2.

Unit 3 Lesson 8 Cumulative Practice Problems

1. The picture shows a circle divided into 8 equal wedges which are rearranged.



   The radius of the circle is r and its circumference is $2\pi r$. How does the picture help to explain why the area of the circle is πr^2?

2. A circle's circumference is approximately 76 cm. Estimate the radius, diameter, and area of the circle.

3. Jada paints a circular table that has a diameter of 37 inches. What is the area of the table?

4. The Carousel on the National Mall has 4 rings of horses. Kiran is riding on the inner ring, which has a radius of 9 feet. Mai is riding on the outer ring, which is 8 feet farther out from the center than the inner ring is.

   a. In one rotation of the carousel, how much farther does Mai travel than Kiran?

   b. One rotation of the carousel takes 12 seconds. How much faster does Mai travel than Kiran?

(From Unit 3, Lesson 4.)

5. Here are the diameters of four coins:

coin	penny	nickel	dime	quarter
diameter	1.9 cm	2.1 cm	1.8 cm	2.4 cm

   a. A coin rolls a distance of 33 cm in 5 rotations. Which coin is it?

   b. A quarter makes 8 rotations. How far did it roll?

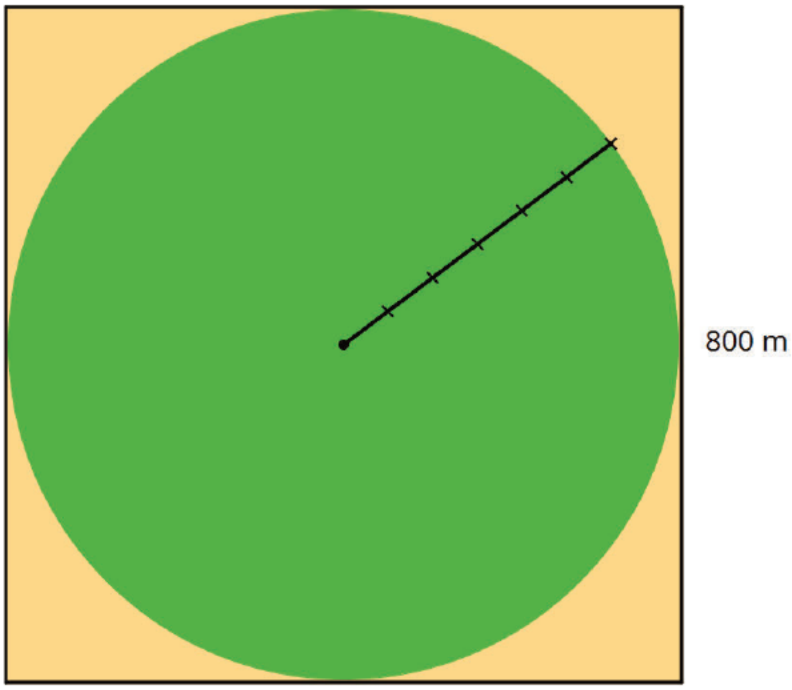
   c. A dime rolls 41.8 cm. How many rotations did it make?

(From Unit 3, Lesson 5.)

Lesson 9: Applying Area of Circles

9.1: Still Irrigating the Field

The area of this field is about 500,000 m^2. What is the field's area to the nearest square meter? Assume that the side lengths of the square are exactly 800 m.

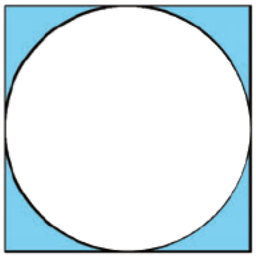


- 502,400 m^2
- 502,640 m^2
- 502,655 m^2
- 502,656 m^2
- 502,857 m^2

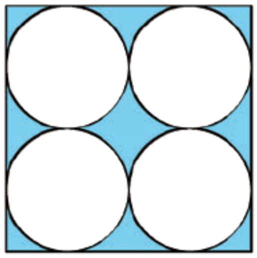
9.2: Comparing Areas Made of Circles

1. Each square has a side length of 12 units. Compare the areas of the shaded regions in the 3 figures. Which figure has the largest shaded region? Explain or show your reasoning.

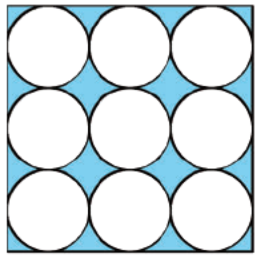
A



B

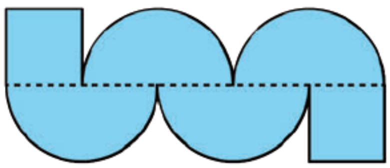


C

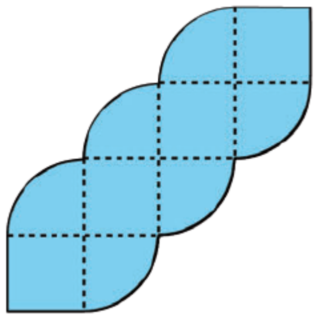


2. Each square in Figures D and E has a side length of 1 unit. Compare the area of the two figures. Which figure has more area? How much more? Explain or show your reasoning.

D



E

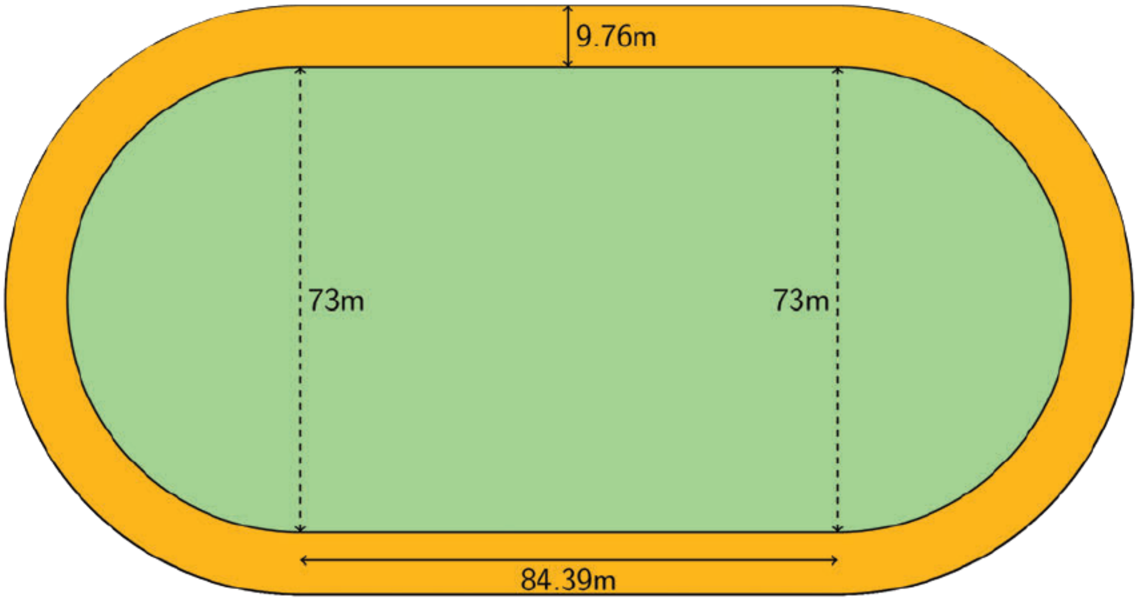


Are you ready for more?

Which figure has a longer perimeter, Figure D or Figure E? How much longer?

9.3: The Running Track Revisited

The field inside a running track is made up of a rectangle 84.39 m long and 73 m wide, together with a half-circle at each end. The running lanes are 9.76 m wide all the way around.



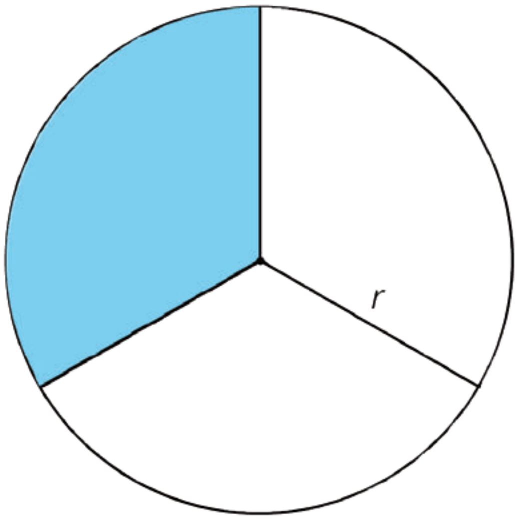
What is the area of the running track that goes around the field? Explain or show your reasoning.

Lesson 9 Summary

The relationship between A, the area of a circle, and r, its radius, is $A = \pi r^2$. We can use this to find the area of a circle if we know the radius. For example, if a circle has a radius of 10 cm, then the area is $\pi \cdot 10^2$ or 100π cm^2. We can also use the formula to find the radius of a circle if we know the area. For example, if a circle has an area of 49π m^2 then its radius is 7 m and its diameter is 14 m.

Sometimes instead of leaving π in expressions for the area, a numerical approximation can be helpful. For the examples above, a circle of radius 10 cm has area about 314 cm^2. In a similar way, a circle with area 154 m^2 has radius about 7 m.

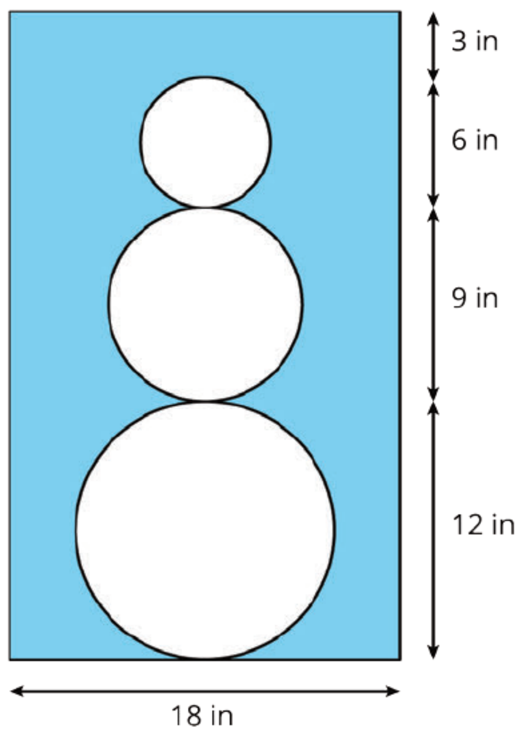
We can also figure out the area of a fraction of a circle. For example, the figure shows a circle divided into 3 pieces of equal area. The shaded part has an area of $\frac{1}{3}\pi r^2$.



Unit 3 Lesson 9 Cumulative Practice Problems

1. A circle with a 12-inch diameter is folded in half and then folded in half again. What is the area of the resulting shape?

2. Find the area of the shaded region. Express your answer in terms of π.



3. The face of a clock has a circumference of 63 in. What is the area of the face of the clock?

(From Unit 3, Lesson 8.)

4. Which of these pairs of quantities are proportional to each other? For the quantities that are proportional, what is the constant of proportionality?

   a. Radius and diameter of a circle

   b. Radius and circumference of a circle

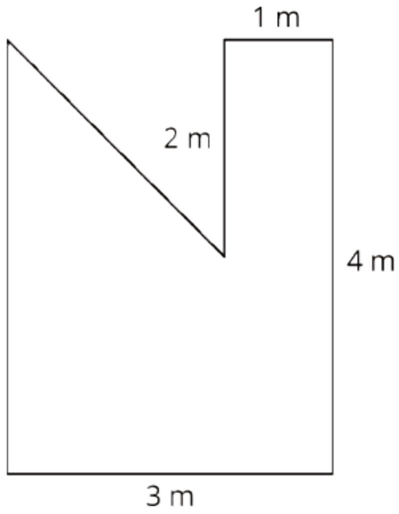
   c. Radius and area of a circle

   d. Diameter and circumference of a circle

   e. Diameter and area of a circle

(From Unit 3, Lesson 7.)

5. Find the area of this shape in two different ways.



(From Unit 3, Lesson 6.)

6. Elena and Jada both read at a constant rate, but Elena reads more slowly. For every 4 pages that Elena can read, Jada can read 5.

   a. Complete the table.

pages read by Elena	pages read by Jada
4	5
1	
9	
e	
	15
	j

   b. Here is an equation for the table: $j = 1.25e$. What does the 1.25 mean?

   c. Write an equation for this relationship that starts $e = \dots$

(From Unit 2, Lesson 5.)

Lesson 10: Distinguishing Circumference and Area

10.1: Filling the Plate

About how many cheese puffs can fit on the plate in a single layer? Be prepared to explain your reasoning.

10.2: Card Sort: Circle Problems

Your teacher will give you cards with questions about circles.

1. Sort the cards into two groups based on whether you would use the circumference or the area of the circle to answer the question. Pause here so your teacher can review your work.

2. Your teacher will assign you a card to examine more closely. What additional information would you need in order to answer the question on your card?

3. Estimate measurements for the circle on your card.

4. Use your estimates to calculate the answer to the question.

10.3: Visual Display of Circle Problem

In the previous activity you estimated the answer to a question about circles.

Create a visual display that includes:

- The question you were answering
- A diagram of a circle labeled with your estimated measurements
- Your thinking, organized so that others can follow it
- Your answer, expressed in terms of π and also expressed as a decimal approximation

10.4: Analyzing Circle Claims

Here are two students' answers for each question. Do you agree with either of them? Explain or show your reasoning.

1. How many feet are traveled by a person riding once around the merry-go-round?

   - Clare says, "The radius of the merry-go-round is about 4 feet, so the distance around the edge is about 8π feet."
   - Andre says, "The diameter of the merry-go-round is about 4 feet, so the distance around the edge is about 4π feet."

2. How much room is there to spread frosting on the cookie?

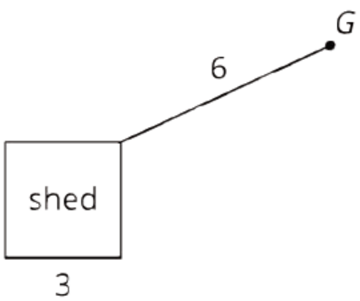
   - Clare says "The radius of the cookie is about 3 centimeters, so the space for frosting is about 6π cm^2."
   - Andre says "The diameter of the cookie is about 3 inches, so the space for frosting is about 2.25π in^2."

3. How far does the unicycle move when the wheel makes 5 full rotations?

- Clare says, "The diameter of the unicycle wheel is about 0.5 meters. In 5 complete rotations it will go about $\frac{5}{2}\pi$ m^2."
- Andre says, "I agree with Clare's estimate of the diameter, but that means the unicycle will go about $\frac{5}{4}\pi$ m."

Are you ready for more?

A goat (point G) is tied with a 6-foot rope to the corner of a shed. The floor of the shed is a square whose sides are each 3 feet long. The shed is closed and the goat can't go inside. The space all around the shed is flat, grassy, and the goat can't reach any other structures or objects. What is the area over which the goat can roam?



Lesson 10 Summary

Sometimes we need to find the circumference of a circle, and sometimes we need to find the area. Here are some examples of quantities related to the circumference of a circle:

- The length of a circular path.
- The distance a wheel will travel after one complete rotation.
- The length of a piece of rope coiled in a circle.

Here are some examples of quantities related to the area of a circle:

- The amount of land that is cultivated on a circular field.
- The amount of frosting needed to cover the top of a round cake.
- The number of tiles needed to cover a round table.

In both cases, the radius (or diameter) of the circle is all that is needed to make the calculation. The circumference of a circle with radius r is $2\pi r$ while its area is πr^2. The circumference is measured in linear units (such as cm, in, km) while the area is measured in square units (such as cm^2, in^2, km^2).

Unit 3 Lesson 10 Cumulative Practice Problems

1. For each problem, decide whether the circumference of the circle or the area of the circle is most useful for finding a solution. Explain your reasoning.

    a. A car's wheels spin at 1000 revolutions per minute. The diameter of the wheels is 23 inches. You want to know how fast the car is travelling.

    b. A circular kitchen table has a diameter of 60 inches. You want to know how much fabric is needed to cover the table top.

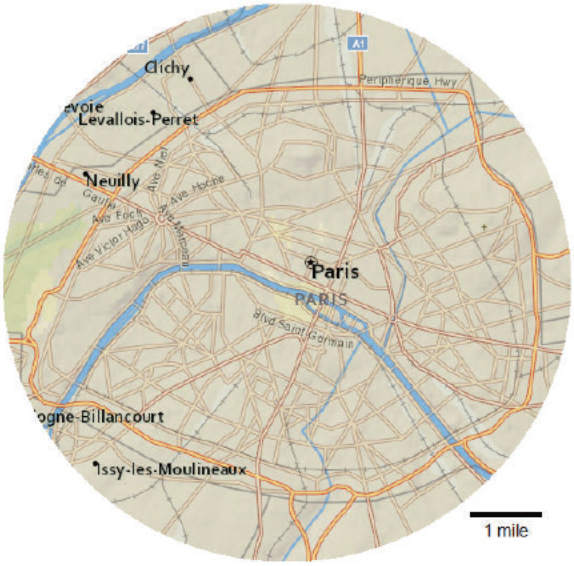
    c. A circular puzzle is 20 inches in diameter. All of the pieces are about the same size. You want to know about how many pieces there are in the puzzle.

    d. You want to know about how long it takes to walk around a circular pond.

2. The city of Paris, France is completely contained within an almost circular road that goes around the edge. Use the map with its scale to:

    a. Estimate the circumference of Paris.

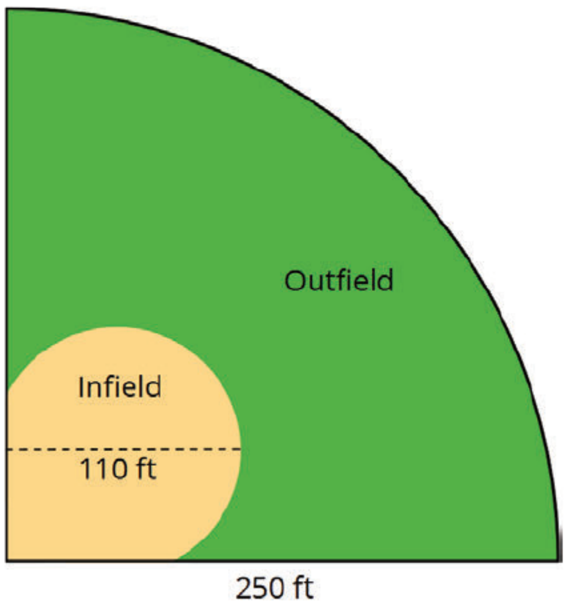
    b. Estimate the area of Paris.



3. Here is a diagram of a softball field:

    a. About how long is the fence around the field?

    b. About how big is the outfield?



4. While in math class, Priya and Kiran come up with two ways of thinking about the proportional relationship shown in the table.

x	y
2	?
5	1750

Both students agree that they can solve the equation $5k = 1750$ to find the constant of proportionality.

- Priya says, "I can solve this equation by dividing 1750 by 5."
- Kiran says, "I can solve this equation by multiplying 1750 by $\frac{1}{5}$."

a. What value of k would each student get using their own method?

b. How are Priya and Kiran's approaches related?

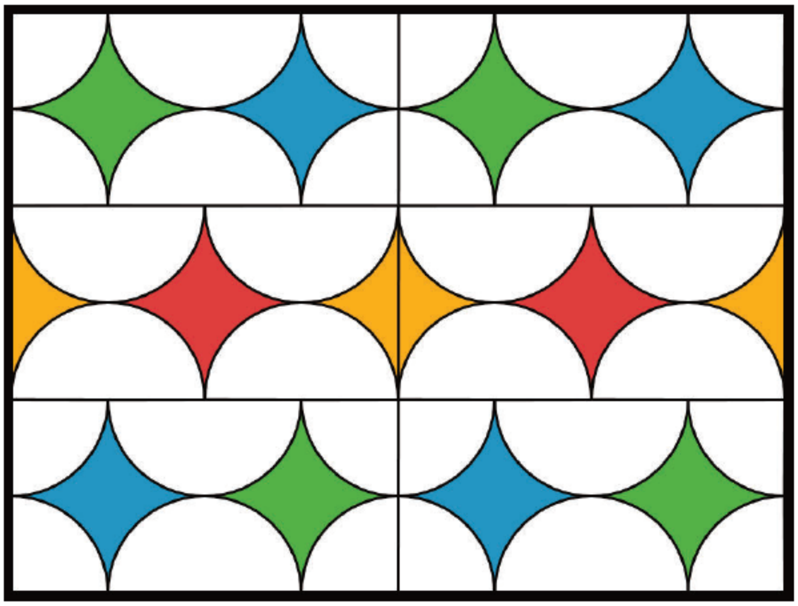
c. Explain how each student might approach solving the equation $\frac{2}{3}k = 50$.

(From Unit 2, Lesson 5.)

Lesson 11: Stained-Glass Windows

11.1: Cost of a Stained-Glass Window

The students in art class are designing a stained-glass window to hang in the school entryway. The window will be 3 feet tall and 4 feet wide. Here is their design.



They have raised $100 for the project. The colored glass costs $5 per square foot and the clear glass costs $2 per square foot. The material they need to join the pieces of glass together costs 10 cents per foot and the frame around the window costs $4 per foot.

Do they have enough money to cover the cost of making the window?

11.2: A Bigger Window

A local community member sees the school's stained-glass window and really likes the design. They ask the students to create a larger copy of the window using a scale factor of 3. Would $450 be enough to buy the materials for the larger window? Explain or show your reasoning.

11.3: Invent Your Own Design

Draw a stained-glass window design that could be made for less than $450. Show your thinking. Organize your work so it can be followed by others.

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