



Core Knowledge[®] MATHEMATICS

Introducing Proportional Relationships

Teacher Guide

Real World Proportions

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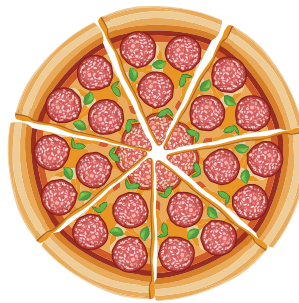
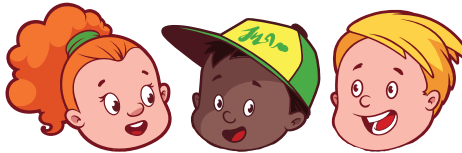
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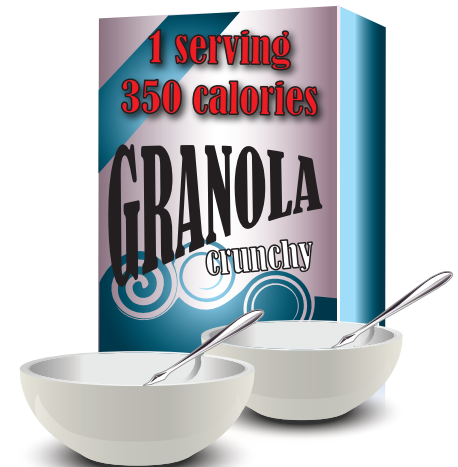
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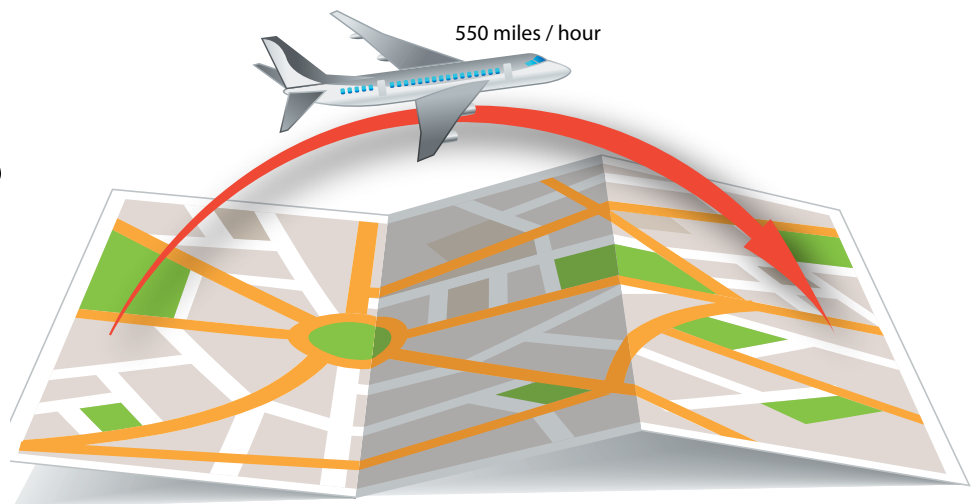
Saving Water:
A Bath or Shower?



Planning to Feed a Crowd



Calculating
Servings and Calories



Calculating Time and Distance Using Constant Speed

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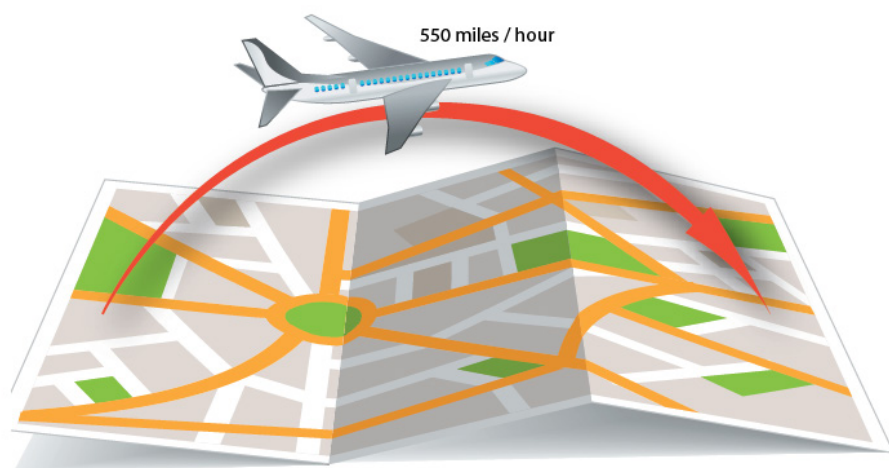
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Introducing Proportional Relationships

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**Introducing Proportional
Relationships
Teacher Guide**
Core Knowledge Mathematics™

Introducing Proportional Relationships

Unit Narrative

In this unit, students develop the idea of a proportional relationship out of the grade 6 idea of equivalent ratios. Proportional relationships prepare the way for the study of linear functions in grade 8.

In grade 6, students learned two ways of looking at equivalent ratios. First, if you multiply both values in a ratio $a : b$ by the same positive number s (called the scale factor) you get an equivalent ratio $sa : sb$. Second, two ratios are equivalent if they have the same *unit rate*. A unit rate is the “amount per 1” in a ratio; the ratio $a : b$ is equivalent to $\frac{a}{b} : 1$, and $\frac{a}{b}$ is a unit rate giving the amount of the first quantity per unit of the second quantity. You could also talk about the amount of the second quantity per unit of the first quantity, which is the unit rate $\frac{b}{a}$, coming from the equivalent ratio $1 : \frac{b}{a}$.

In a table of equivalent ratios, a multiplicative relationship between the pair of rows is given by a scale factor. By contrast, the multiplicative relationship between the columns is given by a unit rate. Every number in the second column is obtained by multiplying the corresponding number in the first column by one of the unit rates, and every number in the first column is obtained by multiplying the number in the second column by the other unit rate. The relationship between pairs of values in the two columns is called a *proportional relationship*, the unit rate that describes this relationship is called a *constant of proportionality*, and the quantity represented by the right column is said to be *proportional to* the quantity represented by the left. (Although a proportional relationship between two quantities represented by a and b is associated with *two* constants of proportionality, $\frac{a}{b}$ and $\frac{b}{a}$, throughout the unit, the convention is if a and b are, respectively, in the left and right columns of a table, then $\frac{b}{a}$ is the constant of proportionality for the relationship represented by the table.)

For example, if a person runs at a constant speed and travels 12 miles in 2 hours, then the distance traveled is proportional to the time elapsed, with constant of proportionality 6, because

$$\text{distance} = 6 \cdot \text{time}.$$

The time elapsed is proportional to distance traveled with constant of proportionality $\frac{1}{6}$, because

$$\text{time} = \frac{1}{6} \cdot \text{distance}.$$

Students learn that any proportional relationship can be represented by an equation of the form $y = kx$ where k is the constant of proportionality, that its graph lies on a line through the origin that passes through Quadrant I, and that the constant of proportionality indicates the steepness of the line. By the end of the unit, students should be able to easily work with common contexts associated with proportional relationships (such as constant speed, unit pricing, and measurement conversions) and be able to determine whether a relationship is proportional or not.

Because this unit focuses on understanding what a proportional relationship is, how it is represented, and what types of contexts give rise to proportional relationships, the contexts have been carefully chosen. The first tasks in the unit employ contexts such as servings of food, recipes, constant speed, and measurement conversion, that should be familiar to students from the grade 6 course. These contexts are revisited throughout the unit as new aspects of proportional relationships are introduced.

Associated with the contexts from the grade 6 course are derived units: miles per hour; meters per second; dollars per pound; or cents per minute. In this unit, students build on their grade 6 experiences in working with a wider variety of derived units, such as cups of flour per tablespoon of honey, hot dogs eaten per minute, and centimeters per millimeter. The tasks in this unit avoid discussion of measurement error and statistical variability, which will be addressed in later units.

On using the terms quantity, ratio, proportional relationship, unit rate, and fraction. In these materials, a *quantity* is a measurement that is or can be specified by a number and a unit, e.g., 4 oranges, 4 centimeters, “my height in feet,” or “my height” (with the understanding that a unit of measurement will need to be chosen, MP6). The term *ratio* is used to mean a type of association between two or more quantities. A *proportional relationship* is a collection of equivalent ratios.

A *unit rate* is the numerical part of a rate per 1 unit, e.g., the 6 in 6 miles per hour. The fractions $\frac{a}{b}$ and $\frac{b}{a}$ are never called ratios. The fractions $\frac{a}{b}$ and $\frac{b}{a}$ are identified as “unit rates” for the ratio $a : b$. In high school—after their study of ratios, rates, and proportional relationships—students discard the term “unit rate,” referring to a to b , $a : b$, and $\frac{a}{b}$ as “ratios.”

In grades 6–8, students write rates without abbreviated units, for example as “3 miles per hour” or “3 miles in every 1 hour.” Use of notation for derived units such as $\frac{\text{mi}}{\text{hr}}$ waits for high school—except for the special cases of area and volume. Students have worked with area since grade 3 and volume since grade 5. Before grade 6, they have learned the meanings of such things as sq cm and cu cm. After students learn exponent notation in grade 6, they also use cm^2 and cm^3 .

A *fraction* is a point on the number line that can be located by partitioning the segment between 0 and 1 into equal parts, then finding a point that is a whole number of those parts away from 0. A fraction can be written in the form $\frac{a}{b}$ or as a decimal.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as comparing, interpreting, and generalizing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Compare

- drink mixtures and figures (Lesson 1)
- approaches to solving problems involving proportional relationships (Lesson 6)
- proportional relationships with nonproportional relationships (Lesson 8)
- tables, descriptions, and graphs representing the same situations (Lesson 10)
- graphs of proportional relationships (Lesson 12)

Interpret

- representations showing equivalent ratios (Lesson 1)
- tables showing equivalent ratios (Lesson 2)
- situations involving proportional relationships (Lesson 6 and 9)
- how a graph represents features of a situation (Lesson 11)

Generalize

- about proportional relationships (Lesson 4)
- about equations that represent proportional relationships (Lesson 5)
- about how a constant of proportionality is represented by graphs and tables (Lesson 13)

In addition, students are expected to describe proportional relationships and constants of proportionality, explain how to determine whether or not a relationship is proportional and how to compare and represent situations with different constants of proportionality, justify whether or not a relationship is proportional, and represent proportional and nonproportional relationships in multiple ways.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

Learning Targets

Introducing Proportional Relationships

Lesson 1: One of These Things Is Not Like the Others

- I can use equivalent ratios to describe scaled copies of shapes.
- I know that two recipes will taste the same if the ingredients are in equivalent ratios.

Lesson 2: Introducing Proportional Relationships with Tables

- I can use a table to reason about two quantities that are in a proportional relationship.
- I understand the terms proportional relationship and constant of proportionality.

Lesson 3: More about Constant of Proportionality

- I can find missing information in a proportional relationship using a table.
- I can find the constant of proportionality from information given in a table.

Lesson 4: Proportional Relationships and Equations

- I can write an equation of the form $y = kx$ to represent a proportional relationship described by a table or a story.
- I can write the constant of proportionality as an entry in a table.

Lesson 5: Two Equations for Each Relationship

- I can find two constants of proportionality for a proportional relationship.
- I can write two equations representing a proportional relationship described by a table or story.

Lesson 6: Using Equations to Solve Problems

- I can find missing information in a proportional relationship using the constant of proportionality.
- I can relate all parts of an equation like $y = kx$ to the situation it represents.

Lesson 7: Comparing Relationships with Tables

- I can decide if a relationship represented by a table could be proportional and when it is definitely not proportional.

Lesson 8: Comparing Relationships with Equations

- I can decide if a relationship represented by an equation is proportional or not.

Lesson 9: Solving Problems about Proportional Relationships

- I can ask questions about a situation to determine whether two quantities are in a proportional relationship.
- I can solve all kinds of problem involving proportional relationships.

Lesson 10: Introducing Graphs of Proportional Relationships

- I know that the graph of a proportional relationship lies on a line through $(0, 0)$.

Lesson 11: Interpreting Graphs of Proportional Relationships

- I can draw the graph of a proportional relationship given a single point on the graph (other than the origin).
- I can find the constant of proportionality from a graph.
- I understand the information given by graphs of proportional relationships that are made of up of points or a line.

Lesson 12: Using Graphs to Compare Relationships

- I can compare two, related proportional relationships based on their graphs.
- I know that the steeper graph of two proportional relationships has a larger constant of proportionality.

Lesson 13: Two Graphs for Each Relationship

- I can interpret a graph of a proportional relationship using the situation.
- I can write an equation representing a proportional relationship from a graph.

Lesson 14: Four Representations

- I can make connections between the graphs, tables, and equations of a proportional relationship.
- I can use units to help me understand information about proportional relationships.

Lesson 15: Using Water Efficiently

- I can answer a question by representing a situation using proportional relationships.

lesson	new terminology	
	receptive	productive
7.2.1	equivalent ratios	
7.2.2	constant of proportionality proportional relationship value	equivalent ratios row column
7.2.3	___ is proportional to ___ relate constant	reciprocal per
7.2.4	equation quotient	___ is proportional to ___
7.2.5	steady situation	
7.2.6		equation quotient
7.2.7		constant of proportionality proportional relationship
7.2.8		constant
7.2.10	origin coordinate plane plot	
7.2.11	quantity axes coordinates	
7.2.13	x-coordinate y-coordinate	origin
7.2.14		axes
7.2.15	reasonable	

Required Materials

Colored pencils

Drink mix

A powder that is mixed with water to create a fruit-flavored or chocolate-flavored drink. Using a sugar-free drink mix is recommended, but *not* a mix that calls for adding a separate sweetener when mixing up the drink.

Four-function calculators

Graph paper

Internet-enabled device

Measuring cup

Measuring spoons

Mixing containers

Pre-printed slips, cut from copies of the blackline master

Rulers

Small disposable cups

Snap cubes

Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Water

Section: Representing Proportional Relationships with Tables

Lesson 1: One of These Things Is Not Like the Others

Goals

- Choose and create representations to compare ratios in the context of recipes or scaled copies.
- Coordinate (orally) different representations of a situation involving equivalent ratios, e.g., discrete diagrams, tables, or double number line diagrams.
- Determine which recipes or geometric figures involve equivalent ratios, and justify (orally, in writing, and through other representations) that they are equivalent.

Learning Targets

- I can use equivalent ratios to describe scaled copies of shapes.
- I know that two recipes will taste the same if the ingredients are in equivalent ratios.

Lesson Narrative

The activities in the lesson are intended to support initial, informal conversations about the key ideas in proportional relationships before the next lesson introduces the terms for those ideas. At the same time, there are opportunities to review work from grade 6 in representing ratios with tables and diagrams.

The tasks are intentionally not well-posed, that is, they do not have exact solutions. They are designed to give students an opportunity to think about how we can bring a mathematical lens to better understand common perceptual experiences (MP4), such as things that taste or look the same or different. Other possibilities include experiments with mixtures of paint, looking at videos of vehicles moving at different constant speeds, looking at faucets or other water sources that flow at different rates, and so on. The focus is on examination of a feature that can be represented as a *unit rate* (flavor, color intensity, speed, etc.) and beginning to analyze differences in that feature in terms of the two quantities involved (drink mix and water, two paint colors, time and distance, and so on). In the next lesson, this will be identified as the key idea motivating the concept of a

proportional relationship. Students may recognize, from their work in grade 6, associated quantities as **equivalent ratios** and reason in terms of scale factors and unit rates.

The second activity provides a bridge from students' work with scale drawings in the previous unit. In the first activity, students are given the relevant measurements; in the second, they are asked to think about how to quantify what they see, in particular, what measurements might help describe the picture.

The amount of time students spend on these activities can be adjusted based on the results of the diagnostic assessment. This lesson can be used to support just-in-time review of any ratio concepts from grade 6 that students struggled with in the diagnostic assessment.

Alignments

Building On

- 6.RP.A: Understand ratio concepts and use ratio reasoning to solve problems.

Addressing

- 7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Building Towards

- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Instructional Routines

- MLR2: Collect and Display
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Think Pair Share

Required Materials

Colored pencils

Drink mix

A powder that is mixed with water to create a fruit-flavored or chocolate-flavored drink. Using a sugar-free drink mix is recommended, but *not* a mix that calls for adding a separate sweetener when mixing up the drink.

Graph paper**Measuring cup****Measuring spoons****Mixing containers****Small disposable cups****Water****Required Preparation**

Make three mixtures:

- 1 cup of water with $1\frac{1}{2}$ teaspoons of powdered drink mix
- 2 cups of water with $\frac{1}{2}$ teaspoon of powdered drink mix
- 1 cup of water with $\frac{1}{4}$ teaspoon of powdered drink mix

Students will need three small cups each; they just need a few sips of the mixture in each cup.

Student Learning Goals

Let's remember what equivalent ratios are.

1.1 Remembering Double Number Lines

Warm Up: : 5 minutes

This activity prompts students to reason about equivalent ratios on a double number line and think of reasonable scenarios for these ratios as a review of their work in grade 6. As students discuss their answers with their partner, select students to share their answers during the whole-class discussion.

Building On

- 6.RP.A

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Display the double number line for all to see. Give students 2 minutes of quiet think time and ask them to give a signal when they have found the missing values. Ask students to compare their double number line with a partner and share the values they placed on the number line and their reasoning for each.

Anticipated Misconceptions

Students may struggle thinking of a scenario with a 1 : 2 ratio. For those students, ask them if they can draw a picture that would represent that ratio and label each line accordingly.

Student Task Statement

1. Complete the double number line diagram with the missing numbers.



2. What could each of the number lines represent? Invent a situation and label the diagram.
3. Make sure your labels include appropriate units of measure.

Student Response

1. The ratios are all equivalent to 1 : 2
2. Answers vary.
3. Answers vary.

Activity Synthesis

Invite selected students to explain how they reasoned about possible labels for each of the number lines and the units of each. After each student shares, invite others to agree, disagree, or question the reasonableness of the number line descriptions. If there is time, ask students to name other equivalent ratios that would appear if the double number line continued to the right.

1.2 Mystery Mixtures

: 15 minutes

The purpose of this activity is for students to articulate that the taste of the mixture depends both on the amount of water and the amount of drink mix used to make the mixture.

Ideally, students come into the class knowing how to draw and use diagrams or tables of equivalent ratios to analyze contexts like the one in the task. If the diagnostic assessment suggests that some students can and some students can't, make strategic pairings of students for this task.

Building On

- 6.RP.A

Building Towards

- 7.RP.A

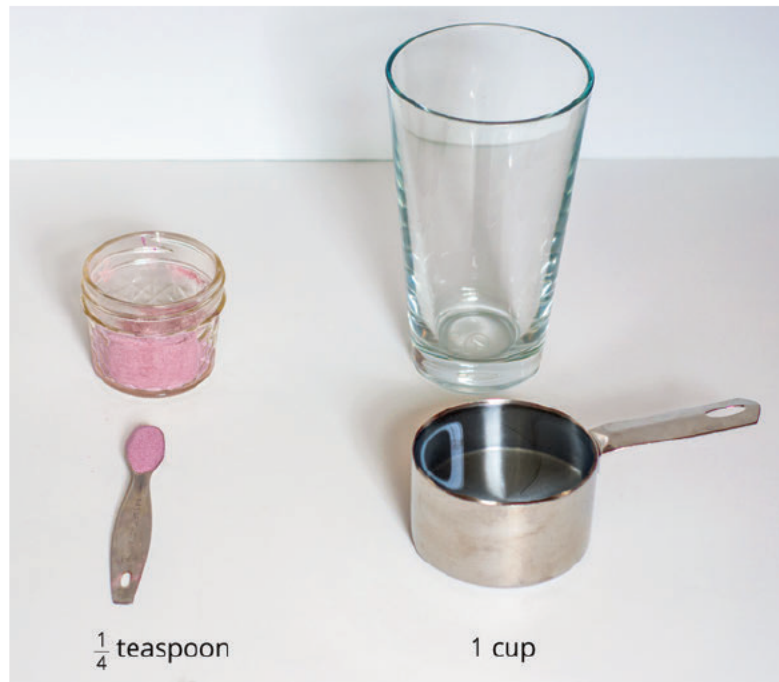
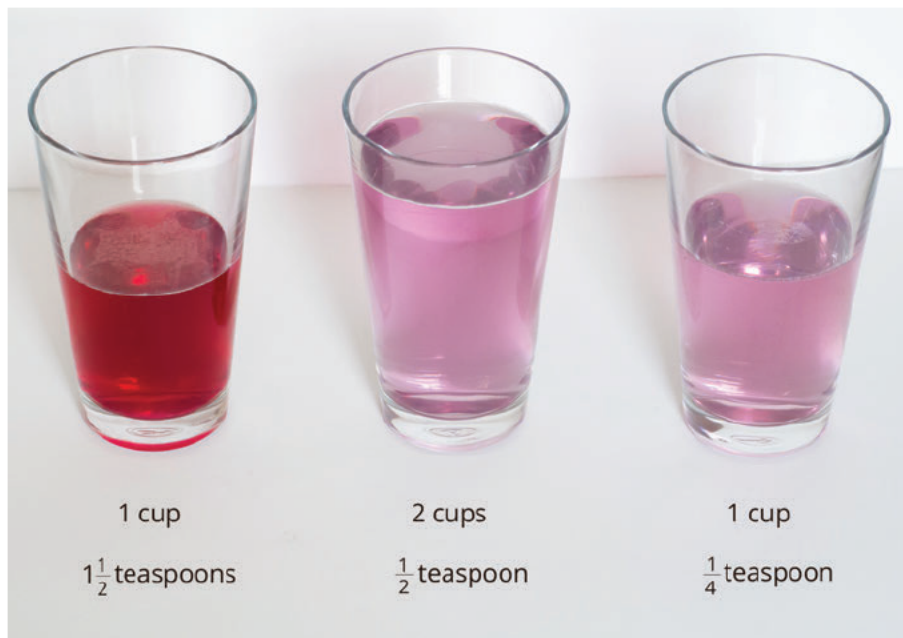
Instructional Routines

- MLR2: Collect and Display
- MLR7: Compare and Connect
- MLR8: Discussion Supports

Launch

Show students images of the drinks.





If possible, give each student three cups containing the drink mixtures.

Tell students to work through the first question and pause for a discussion. Ask questions like,

- “What does it mean to say that it has more drink mix in it?”

- “Imagine you take different amounts of the two that taste the same. There will be more drink mix in the larger amount, but it will not taste different. Why is that?”

The goal is to see that in the same quantity of each mixture (say a teaspoon), the more flavored drink mixture has more drink mix for the same amount of water. (Alternatively, we can say the more flavored drink mixture has less water for the same amount of drink mix.) Use MLR 8 (Discussion Supports) by making gestures or acting out facial expressions for “strength” of the mixture.

After the students have made some progress understanding this idea, the class should continue to the second question. If students finish quickly, press them to find the amount of drink mix per cup of water in each recipe, thus emphasizing the unit rate.

Support for English Language Learners

Conversing, Writing: MLR2 Collect and Display. Before students begin writing a response to the first question, invite them to discuss their thinking with a partner. Listen for vocabulary and phrases students use to describe how the amount of water and the amount of drink mix affects the taste of the mixture. Collect and display words and phrases such as “more drink mix,” “more water,” “tastes stronger/weaker,” etc., and then encourage students to use this language in their written responses, and during discussion.

Design Principle(s): Support sense-making

Student Task Statement

Your teacher will show you three mixtures. Two taste the same, and one is different.

1. Which mixture tastes different? Describe how it is different.

2. Here are the recipes that were used to make the three mixtures:

- 1 cup of water with $1\frac{1}{2}$ teaspoons of powdered drink mix
- 2 cups of water with $\frac{1}{2}$ teaspoon of powdered drink mix
- 1 cup of water with $\frac{1}{4}$ teaspoon of powdered drink mix

Which of these recipes is for the stronger tasting mixture? Explain how you know.

Student Response

1. The first mixture is different—it is stronger because it has more drink mix in it.
2. Answers vary. Possible responses: The first one has more drink mix, so it is the strongest one. There is more drink mix for every cup in the first one.

Are You Ready for More?

Salt and sugar give two distinctly different tastes, one salty and the other sweet. In a mixture of salt and sugar, it is possible for the mixture to be salty, sweet or both. Will any of these mixtures taste exactly the same?

- Mixture A: 2 cups water, 4 teaspoons salt, 0.25 cup sugar
- Mixture B: 1.5 cups water, 3 teaspoons salt, 0.2 cup sugar
- Mixture C: 1 cup water, 2 teaspoons salt, 0.125 cup sugar

Student Response

Mixture A and Mixture C will taste exactly the same. Mixture B will taste equally salty, but will be a little bit sweeter.

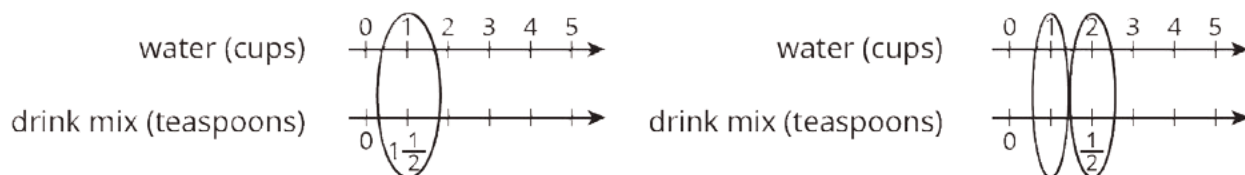
Activity Synthesis

The key takeaway from this activity is that the flavor depends on both how much drink mix *and* how much water there is in the mixture. For a given amount of water, the more drink mix you add, the stronger the mixture tastes. Likewise, for a given amount of drink mix, the more water you add, the weaker the mixture tastes. To compare the amount of flavor of two mixtures, when both the amounts of drink mix and the amounts of water are different in the two mixtures, we can write ratios equivalent to each situation so that we are comparing the amount of drink mix for the same amount of water or the amount of water for the same amount of drink mix. Computing a *unit rate* for each situation is a particular instance of this strategy. Make these ideas explicit if the students do not express them.

If students do not create them, draw discrete diagrams like this:



Or double number line diagrams like this:



For each mixture, identify correspondences between the discrete and number line diagrams, and between the diagrams and tables:

water (cups)	drink mix (teaspoons)
1	$1\frac{1}{2}$
2	3

water (cups)	drink mix (teaspoons)
2	$1\frac{1}{2}$
1	$\frac{1}{4}$

Ask questions like, “On the double number line diagram we see the 1 to $1\frac{1}{2}$ relationship at the first tick mark. Where do we see that relationship in the double tape diagram? In the table?”

Use MLR 7 (Compare and Connect) for students to compare methods of how they knew which recipe was strongest. Who used multiplication? Who used division? Who used a unit rate of water per drink mix teaspoon? Who used a unit rate of drink mix per water cup?

Support for Students with Disabilities

Representation: Internalize Comprehension. Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, use the same color to illustrate correspondences between the number line diagrams and ratio tables for each mixture.

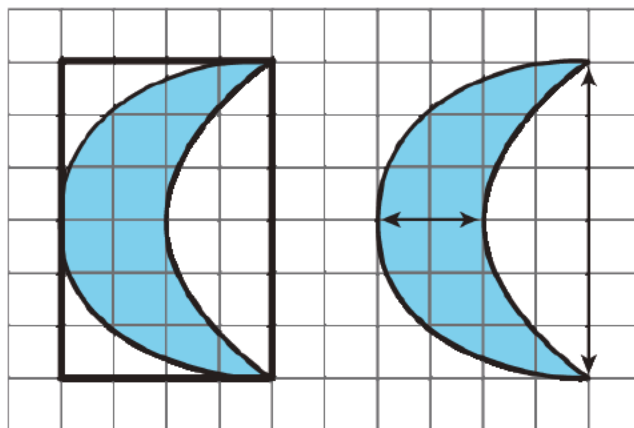
Supports accessibility for: Visual-spatial processing

1.3 Crescent Moons

: 15 minutes (there is a digital version of this activity)

In the previous unit, students studied scale drawings of real-world objects. In grade 8, they will study dilations and similarity. The purpose of this activity is to use students’ recent study of scale drawings as a transition to the study of proportional relationships. Initially, students may describe the difference between Moons A, B, C, and D in qualitative terms, e.g., “D is more squished than the others.” They may also use the term “scaled copies,” which appeared in the work of the previous unit, but struggle to identify measurements to use in these figures that consist only of curved sides. It is important to ask students to articulate what they mean by “squished” in quantitative terms, for example, by talking about the height relative to the width and helping students to define “height” and “width” of a moon in some appropriate way. Once students have that, they can note that the height of the enclosing rectangle is always one and a half times its width for Moons A, B,

and C, but not D, or they might note that the distance tip to tip is three times the width of the widest part of the moon for Moons A, B, and C, but not so for D.



As students explore these transformations, ask questions with the goal of having students articulate that for two images to look like scaled copies of each other, the ratios of the side lengths need to be the same.

In the third question, students are asked to represent the situation with tables and double number line diagrams, providing students with an opportunity to recall these representations from their work in grade 6.

Building On

- 6.RP.A

Addressing

- 7.G.A.1

Building Towards

- 7.RP.A

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports
- Think Pair Share

Launch

Give students 3 minutes of quiet think time and tell them to pause after the first two questions. After the quiet think time, ask students to discuss their answers with a partner to describe how Moon D is different. Use MLR 2 (Collect and Display) as students share.

Record the explanations that students are using to describe the moons. Ensure that students see some ways to measure lengths associated with the moons, then complete the last question.

If using the digital activity, give students 3 minutes of quiet think time and tell them to pause after the first two questions. After the quiet think time, students can discuss answers with a partner. Based on student conversations, you may want to have a whole-group discussion to ensure they see a way to measure the lengths associated with the moons before they attempt to answer the last question.

Support for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support development of organizational skills, check in with students within the first 2–3 minutes of work time. Check to make sure students have a method for comparing the measurements of each moon. For example, create a rectangle around each moon and compare the width-height ratios.

Supports accessibility for: Memory; Organization

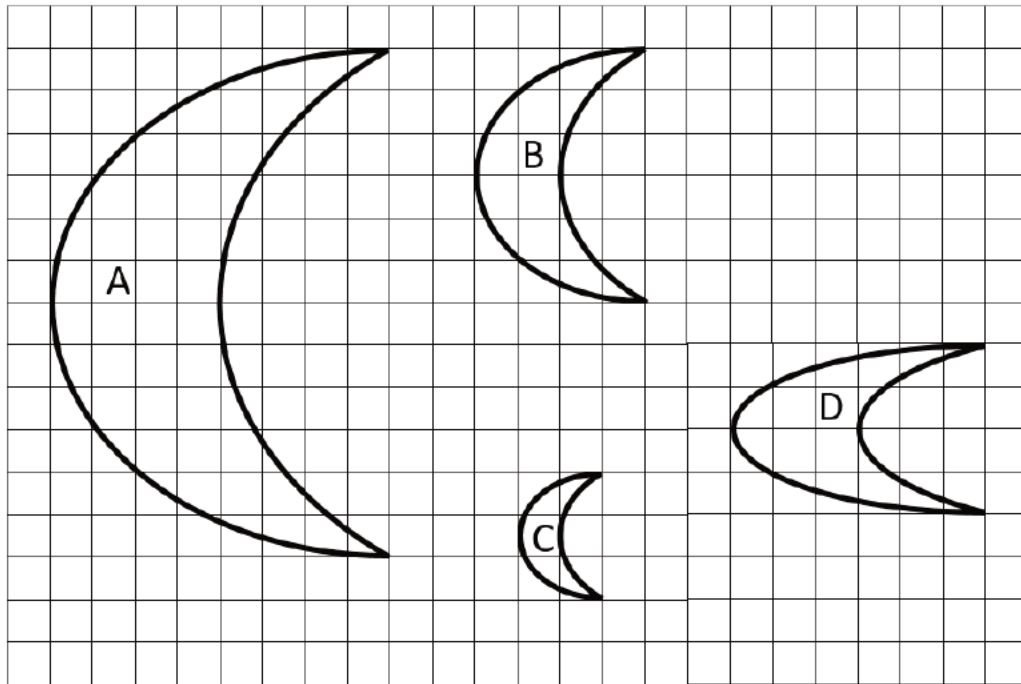
Anticipated Misconceptions

For question 2, students might attempt to find the area of each moon by counting individual square units. Suggest that they create a rectangle around each moon instead and compare the width-height ratios.

For question 3, if students are not sure how to set up these representations, providing a template may be helpful.

Student Task Statement

Here are four different crescent moon shapes.



1. What do Moons A, B, and C all have in common that Moon D doesn't?
2. Use numbers to describe how Moons A, B, and C are different from Moon D.
3. Use a table or a double number line to show how Moons A, B, and C are different from Moon D.

Student Response

1. Answers vary. Possible responses: Moon D is smashed down more than Moons A, B, and C. Moons A, B, and C are all taller than they are wide while Moon D is wider than it is tall.
2. Answers vary. Possible response: We could enclose the Moon with a rectangle and compare the ratio of width to height for each moon.
3. Answers vary. Possible responses: (Note that students might put the rows or columns in another order.)

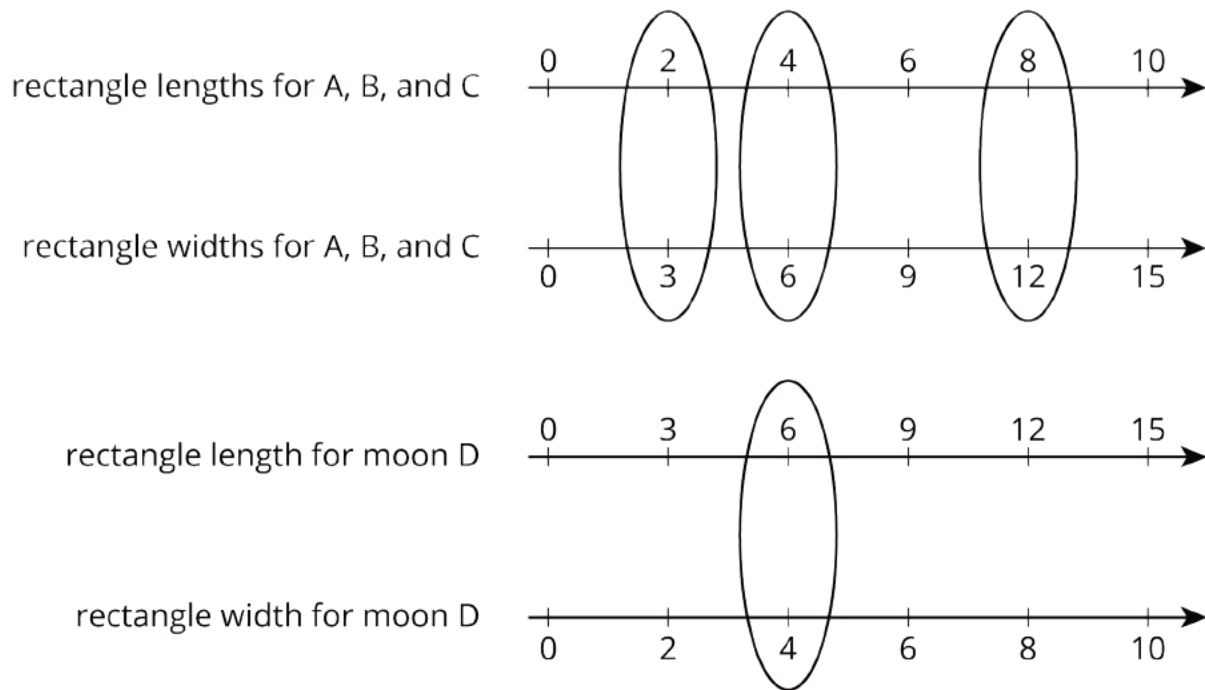
For students enclosing the moons with a rectangle:

moon	rectangle length (units)	rectangle width (units)	length ÷ width
C	2	3	$\frac{2}{3}$
B	4	6	$\frac{4}{6} = \frac{2}{3}$
A	8	12	$\frac{8}{12} = \frac{2}{3}$
D	6	4	$\frac{6}{4}$

For students measuring widest part and tip-to-tip:

moon	widest part (units)	tip-to-tip (units)	widest part ÷ tip-to-tip
C	1	3	$\frac{1}{3}$
B	2	6	$\frac{2}{6} = \frac{1}{3}$
A	4	12	$\frac{4}{12} = \frac{1}{3}$
D	3	4	$\frac{3}{4}$

Here is an example of double number line diagrams for enclosing rectangles:



Activity Synthesis

As students suggest ways to characterize the difference between Moons A, B, and C and Moon D, ask questions that help them clarify and make their statements more precise. For example,

- “What does it mean to be ‘smashed down?’ What measurements might you make to show that this is true?”
- “Is there anything else about what A, B, and C have in common that you can identify?”
- “What things might we measure about these moons to be able to talk about what makes them different in a more precise way?”
- “How can you represent the ratios of the measurements you are comparing using a table and a double number line diagram?”

If some students are still struggling with the tables or diagrams, ask students who were successful to share their representations with the class.

Support for English Language Learners

Speaking: MLR8 Discussion Supports. To aid students in producing statements about comparisons of proportional relationships. Provide sentence frames for students to use when they are comparing and contrasting such as: “All ____ have ____ except ____,” “What makes ____ different from the others is ____.” Ask students to further add to the statement to clarify the comparisons if they used descriptor words such as “wider, narrower, etc.” Improved statements should include mathematical language to describe the measurements (such as length, width, ratio, etc.). This will help students practice and develop language for comparisons.

Design Principle(s): Support sense-making

Lesson Synthesis

Revisit each activity (the drink mixture and the moons), and note that in each, there are two quantities. The ratios of those quantities are equivalent for all but one of the things, the one that is different in an important way. This unit is the study of situations where **equivalent ratios** characterize something important about a situation. As part of the discussion, use and emphasize ratio and rate language in contexts and review representations like double number line diagrams and tables of equivalent ratios.

- “In what important way were the drink mixtures the same and different?”
- “How could we tell using ratios that these were the same and different?”
- “In what important way were the moons the same and different?”
- “How could we tell using numbers that these were the same and different?”

1.4 Orangey-Pineapple Juice

Cool Down : 5 minutes

Building On

- 6.RP.A

Building Towards

- 7.RP.A

Student Task Statement

Here are three different recipes for Orange-Pineapple Juice. Two of these mixtures taste the same and one tastes different.

- Recipe 1: Mix 4 cups of orange juice with 6 cups of pineapple juice.
- Recipe 2: Mix 6 cups of orange juice with 9 cups of pineapple juice.
- Recipe 3: Mix 9 cups of orange juice with 12 cups of pineapple juice.

Which two recipes will taste the same, and which one will taste different? Explain or show your reasoning.

Student Response

Recipe 3, which requires $1\frac{1}{3}$ cups of pineapple juice for every 1 cup of orange juice, is different from Recipes 1 and 2, which both require $1\frac{1}{2}$ cups of pineapple juice for every 1 cup of orange juice.

recipe 1

recipe 2

recipe 3

orange juice (cups)	pineapple juice (cups)	orange juice (cups)	pineapple juice (cups)	orange juice (cups)	pineapple juice (cups)
4	6	6	9	9	12
2	3	2	3	3	4
1	$1\frac{1}{2}$	1	$1\frac{1}{2}$	1	$1\frac{1}{3}$

Double number line diagrams can be used to compare the recipes, for instance, by noting that for Recipes 1 and 2, you use 2 cups of orange juice for every 3 cups of pineapple juice, whereas with Recipe 3, you use $2\frac{1}{4}$ cups of orange juice for 3 cups of pineapple juice.

Student Lesson Summary

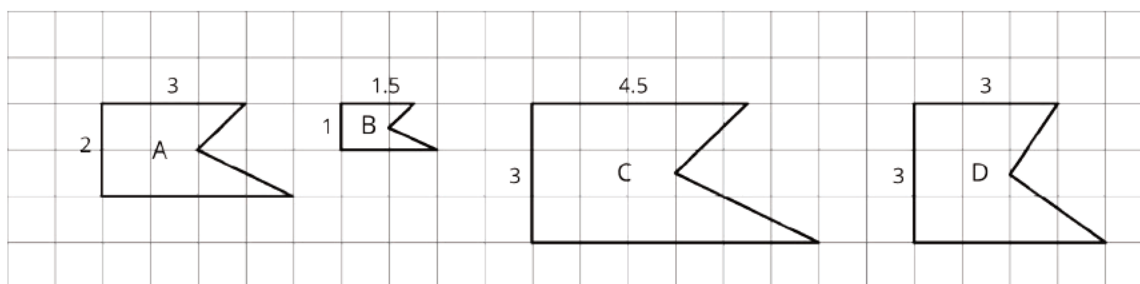
When two different situations can be described by **equivalent ratios**, that means they are alike in some important way.

An example is a recipe. If two people make something to eat or drink, the taste will only be the same as long as the ratios of the ingredients are equivalent. For example, all of the mixtures of water and drink mix in this table taste the same, because the ratios of cups of water to scoops of drink mix are all equivalent ratios.

water (cups)	drink mix (scoops)
3	1
12	4
1.5	0.5

If a mixture were not equivalent to these, for example, if the ratio of cups of water to scoops of drink mix were $6 : 4$, then the mixture would taste different.

Notice that the ratios of pairs of corresponding side lengths are equivalent in figures A, B, and C. For example, the ratios of the length of the top side to the length of the left side for figures A, B, and C are equivalent ratios. Figures A, B, and C are *scaled copies* of each other; this is the important way in which they are alike.



If a figure has corresponding sides that are not in a ratio equivalent to these, like figure D, then it's not a scaled copy. In this unit, you will study relationships like these that can be described by a set of equivalent ratios.

Glossary

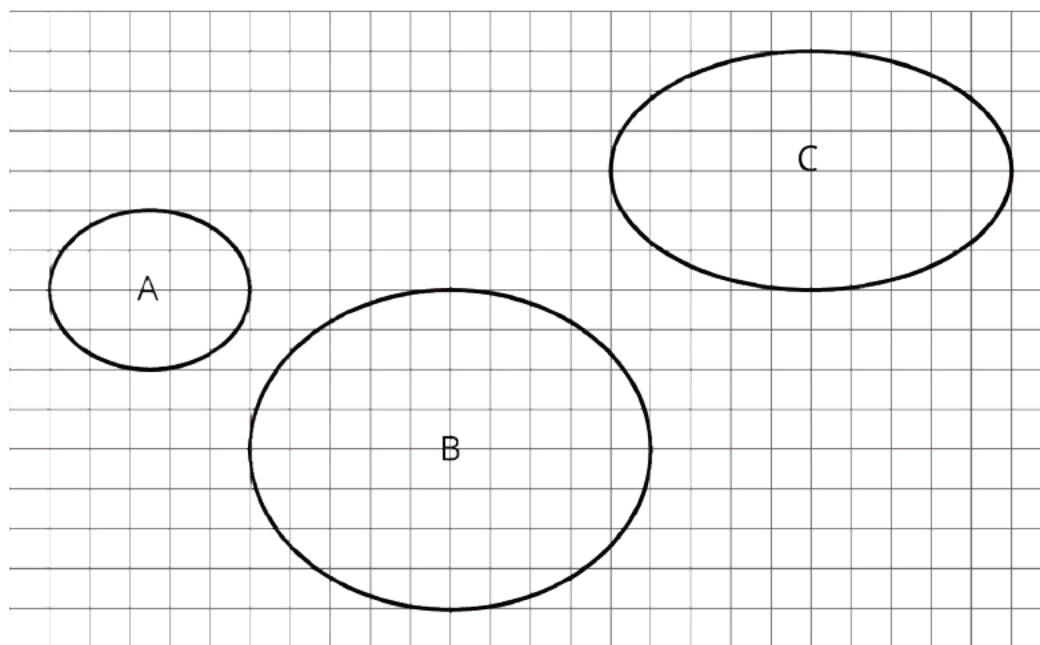
- equivalent ratios

Lesson 1 Practice Problems

1. Problem 1

Statement

Which one of these shapes is not like the others? Explain what makes it different by representing each width and height pair with a ratio.



Solution

C is different from A and B. For both A and B, the width:height ratio is 5:4. However, for C, the width is 10 units and the height is 6 units, so the width:height ratio is 5:3.

2. Problem 2

Statement

In one version of a trail mix, there are 3 cups of peanuts mixed with 2 cups of raisins. In another version of trail mix, there are 4.5 cups of peanuts mixed with 3 cups of raisins. Are the ratios equivalent for the two mixes? Explain your reasoning.

Solution

Yes, since 3 times 1.5 is 4.5 and 2 times 1.5 is 3.

3. Problem 3

Statement

For each object, choose an appropriate scale for a drawing that fits on a regular sheet of paper. Not all of the scales on the list will be used.

Objects

- a. A person
- b. A football field (120 yards by $53\frac{1}{3}$ yards)
- c. The state of Washington (about 240 miles by 360 miles)
- d. The floor plan of a house
- e. A rectangular farm (6 miles by 2 mile)

Scales

- 1 in : 1 ft
- 1 cm : 1 m
- 1: 1000
- 1 ft: 1 mile
- 1: 100,000
- 1 mm: 1 km
- 1: 10,000,000

Solution

Answers vary. Sample responses:

- a. 1 in :1 ft
- b. 1: 1000
- c. 1: 10,000,000
- d. 1cm: 1 m
- e. 1: 100,000

(From Unit 1, Lesson 12.)

4. Problem 4

Statement

Which scale is equivalent to 1 cm to 1 km?

- A. 1 to 1000
- B. 10,000 to 1
- C. 1 to 100,000
- D. 100,000 to 1
- E. 1 to 1,000,000

Solution

D

(From Unit 1, Lesson 11.)

5. Problem 5

Statement

- a. Find 3 different ratios that are equivalent to 7 : 3.
- b. Explain why these ratios are equivalent.

Solution

- a. Answers vary. Sample response: 14 : 6, 21 : 9, 28 : 12
- b. Answers vary. Sample response: 7 and 3 are each multiplied by 2, 3, and 4, respectively.

Lesson 2: Introducing Proportional Relationships with Tables

Goals

- Comprehend that the phrase “proportional relationship” (in spoken and written language) refers to when two quantities are related by multiplying by a “constant of proportionality.”
- Describe (orally and in writing) relationships between rows or between columns in a table that represents a proportional relationship.
- Explain (orally) how to calculate missing values in a table that represents a proportional relationship.

Learning Targets

- I can use a table to reason about two quantities that are in a proportional relationship.
- I understand the terms proportional relationship and constant of proportionality.

Lesson Narrative

The purpose of this lesson is to introduce the concept of a proportional relationship by looking at tables of equivalent ratios. Students learn that all entries in one column of the table can be obtained by multiplying entries in the other column by the *same* number. This number is called the constant of proportionality. The activities use contexts that make using the constant of proportionality the more convenient approach, rather than reasoning about equivalent ratios.

In any proportional relationship between two quantities x and y , there are two ways of viewing the relationship; y is proportional to x , or x is proportional to y . For example, the two tables below represent the same relationship between time elapsed and distance traveled for someone running at a constant rate. The first table shows that distance is proportional to time, with constant of proportionality 6, and the second table, representing the same information, shows that time is proportional to distance, with constant of proportionality $\frac{1}{6}$.

time (h)	distance (mi)	constant of proportionality	distance (mi)	time (h)	constant of proportionality
2	12	6	12	2	$\frac{1}{6}$
1	6	6	6	1	$\frac{1}{6}$
$\frac{1}{2}$	3	6	3	$\frac{1}{2}$	$\frac{1}{6}$

These tables illustrate the convention that when we say “ y is proportional to x ” we usually put x in the left hand column and y in the right hand column, so that multiplication by the constant of proportionality always goes from left to right. This is not a hard and fast rule, but it prepares students for later work on functions, where they will think of x as the independent variable and y as the dependent variable.

Alignments

Building On

- 6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Addressing

- 7.RP.A.2: Recognize and represent proportional relationships between quantities.
- 7.RP.A.2.a: Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- 7.RP.A.2.b: Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR6: Three Reads
- MLR8: Discussion Supports
- Notice and Wonder

Required Materials

Measuring cup

Measuring spoons

Required Preparation

A measuring cup and a tablespoon is optional—they may be handy for showing students who are unfamiliar with these kitchen tools.

Student Learning Goals

Let's solve problems involving proportional relationships using tables.

2.1 Notice and Wonder: Paper Towels by the Case

Warm Up: : 5 minutes

The purpose of this warm-up is to elicit the idea that you can use a table to see patterns between related quantities, which will be useful when students discuss how to use tables to learn about proportional relationships in a later activity. While students may notice and wonder many things about these images, the relationship between the rows (multiply both entries by the same number to get another row) and the relationship between the columns (multiply the number of cases by 12 to get the number of paper towels) are the important discussion points.

Addressing

- 7.RP.A.2.a
- 7.RP.A.2.b

Instructional Routines

- Notice and Wonder

Launch

Arrange students in groups of 2. Tell students that they will look at an image, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the image for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Student Task Statement

Here is a table that shows how many rolls of paper towels a store receives when they order different numbers of cases.

number of cases they order	number of rolls of paper towels
1	12
3	36
5	60
10	120

Note: Green arrows and the multiplier $\cdot 2$ are shown next to the table, indicating a doubling of the values from the third row to the fourth row.

What do you notice about the table? What do you wonder?

Student Response

Answers vary.

Some things to notice:

- To go from one row to another, multiply both columns by the same number.
- To find the number of rolls, multiply the number of cases by 12.
- There are 12 rolls in a case.

Some things to wonder:

- How much does a case cost?
- How many paper towels are on a roll?
- Why would you need 120 rolls of paper towels?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After each response, ask the class if they agree or disagree and to explain alternative ways

of thinking, referring back to the images each time. If the relationship between the number of cases and the number of paper towels does not come up during the conversation, ask students to discuss this idea.

2.2 Feeding a Crowd

: 15 minutes

The purpose of this task is to introduce students to the idea of a proportional relationship. From previous work, students should be familiar with the idea of equivalent ratios, and they may very well recognize the table as a set of equivalent ratios. Here, we are starting to expand this concept and the language associated with it to say that there is a proportional relationship between the cups of rice and number of people as well as the number of spring rolls and number of people. More generally, there is a proportional relationship between two quantities when the quantities are characterized by a set of equivalent ratios.

The context in this activity and the numbers used are intended to be accessible to all students so that they can focus on its mathematical structure (MP7) and the new terms introduced in the lesson without being distracted.

While students are working, monitor for groups using each of these approaches:

- Drawing that depicts 15 cups of rice and 45 people, organized into three people per cup. This representation of the problem can support all learners in moving forward. The teacher should highlight the organization of three people per cup, and identify correspondences between parts of the drawing and numbers in the table.
- Moving down the table, multiplying both numbers in a row by the same number. "Since I multiply 2 by 5 to get 10, I will multiply 6 by 5 and get 30. Since 9 times 5 is 45, I will also multiply 3 by 5 to get 15." If it comes up, the most appropriate name for the multiplier would be *scale factor*, but it is not necessary for students to call it this.
- Calculation and use of a unit rate. "I reasoned that 1 cup of rice must serve 3 people. So 10 cups of rice must serve 30, and 15 cups must serve 45."
- Moving across the table. "It looks like we always multiply number of cups of rice by 3 to get the number of people it serves, so I will multiply 10 cups by 3 to get 30, and divide the 45 by 3 to get 15."

Addressing

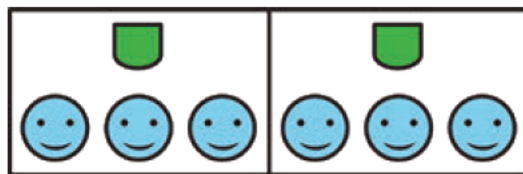
- 7.RP.A.2

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

Launch

Say, "Rice is a big part of the traditions and cultures of many families. Does your family cook rice, and if so, how?" Invite a student to describe the process (measure rice, measure water, simmer for a while). You use more rice for more people and less rice for fewer people. If students have trouble understanding or representing the context, show them the measuring cup so that they have a sense of its size, or draw a literal diagram that looks something like this:



Similarly, ask students if they have ever eaten a spring roll and invite them to describe what they are. While some spring rolls can be very large, the ones referred to in this activity are smaller.

Support for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because _____. Then, I....," "I noticed ____ so I....," and "I tried ____ and what happened was...."

Supports accessibility for: Language; Social-emotional skills

Student Task Statement

1. A recipe says that 2 cups of dry rice will serve 6 people. Complete the table as you answer the questions. Be prepared to explain your reasoning.

- 
- a. How many people will 10 cups of rice serve?
- b. How many cups of rice are needed to serve 45 people?

cups of rice	number of people
2	6
3	9
10	
	45

2. A recipe says that 6 spring rolls will serve 3 people. Complete the table.

number of spring rolls	number of people
6	3
30	
40	
	28

Student Response

1. a. 30
b. 15

cups of rice	number of people
2	6
3	9
10	30
15	45

2.

number of spring rolls	number of people
6	3
30	15
40	20
56	28

Activity Synthesis

Select students to share their approaches using the sequence listed in the Activity Narrative.

After students have shared their reasoning and connections have been drawn between the different approaches, introduce the term **proportional relationship**. For example, “Whenever we have a situation like this where two quantities are always in the same ratio, we say there is a proportional relationship between the quantities. So the relationship between the number of cups of rice and the number of people is a proportional relationship. The number of cups of rice is proportional to the number of people.” Write out some of these statements for all to see (in the next activity, students will need to write a statement like these about a different relationship). If the class has a word wall or students keep track of mathematical vocabulary in their notebooks, these new terms can be included there.

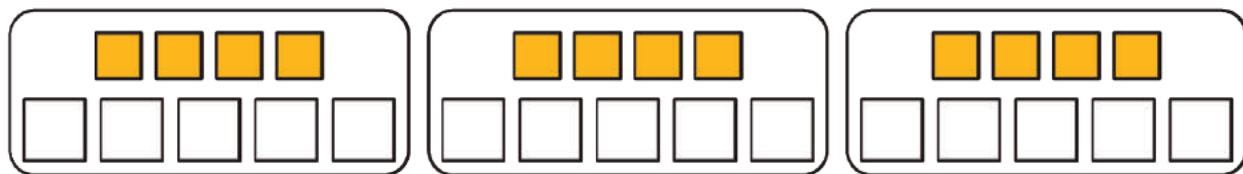
Notice that methods 2 and 3 involve identifying relationships of column entries, namely that the entry in the right-hand column is three times the corresponding entry in the left-hand column. This pattern can be explained in terms of the *unit rate* (people per cup of rice), which tells us how many people we can serve with a given amount of rice. The next activity is intended to build the understanding underlying this explanation.

After discussing the rice context thoroughly, ask students to share their solution approaches to the spring roll context. Ask students to identify and interpret the unit rate in this situation (0.5 people per spring roll may sound strange, but it means that 1 spring roll only halfway satisfies a person.)

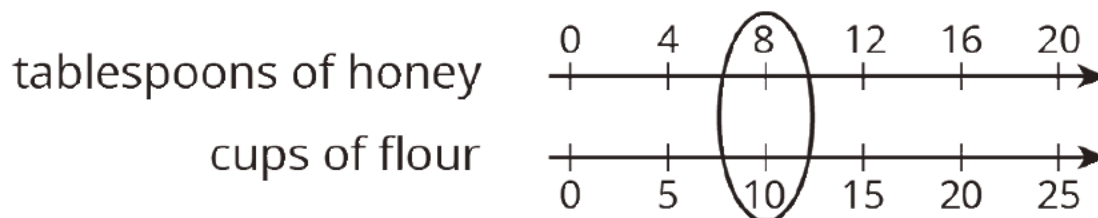
2.3 Making Bread Dough

: 10 minutes

In this activity, students grapple with finding missing values for ratios of whole numbers presented in a table where identifying a usable scale factor is not as easy as in the previous activity. This task is designed to encourage students to use a unit rate. Its context is intended to be familiar so that students can focus on mathematical structure (MP7) and the new terms (MP6) constant of proportionality and proportional relationship. If students are having difficulty understanding the scenario, consider drawing discrete diagrams like this:



This can be followed by a double number line diagram. Correspondences among the diagrams can be identified.



Anticipated approaches include use of scale factors, unit rate, and moving across columns. Because 13 is not a multiple of 4 or 8, students are more likely to use and see the value of using the unit rate or the relationship of the columns. Unit rates might be used: $\frac{5}{4}$ or 1.25 cups of flour per tablespoon of honey, or $\frac{4}{5}$ or 0.8 tablespoon of honey per cup of flour. Both approaches are correct, but the numbers are easier for the first approach, so this is the solution to have students share. While students are working, monitor the approaches used by each student or group.

Addressing

- 7.RP.A.2

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR6: Three Reads

Launch

Tell students that in this activity, they will think about a different proportional relationship. If necessary, show students the measuring cup and the tablespoon side by side to help make the context more concrete. You could even pantomime the first sentence in the activity: “measuring” 8 tablespoons of invisible honey and 10 cups of invisible flour.

Support for English Language Learners

Reading: MLR6 Three Reads. This is the first time Math Language Routine 6 is suggested as a support in this course. In this routine, students are supported in reading a mathematical text, situation, or word problem three times, each with a particular focus. During the first read, students focus on comprehending the situation; during the second read, students identify quantities; during the third read, students brainstorm possible strategies to answer the question. The question to be answered does not become a focus until the third read so that students can make sense of the whole context before rushing to a solution. The purpose of this routine is to support students' reading comprehension as they make sense of mathematical situations and information through conversation with a partner.

Design Principle(s): Support sense-making

How It Happens:

1. In the first read, students read the problem with the goal of comprehending the situation.

Invite a student to read the problem aloud while everyone else reads with them and then ask, "What is this situation about?"

Allow one minute to discuss with a partner, and then share with the whole class. A clear response would be: "A bakery uses a recipe to make bread. The recipe includes honey and flour."

2. In the second read, students analyze the mathematical structure of the story by naming quantities.

Invite students to read the problem aloud with their partner or select a different student to read to the class and then prompt students by asking, "What can be counted or measured in this situation? For now we don't need to focus on how many or how much of anything, but what can we count in this situation?" Give students one minute of quiet think time followed by another minute to share with their partner. Quantities may include: number of tablespoons of honey, number of cups of flour, size of the batches.

Listen for, and amplify, student language about the relationships among the amount of honey, the amount of flour, and the size of the the batches. Invite students to sketch a diagram to represent these relationships.

3. In the third read, students brainstorm possible strategies to complete the table.

Invite students to read the problem aloud with their partner or select a different student to read to the class. Instruct students to think of ways to approach the questions without actually completing the table. Consider using these questions to prompt students: “How would you approach this question?,” and “What strategy would you try first?”

Give students one minute of quiet think time followed by another minute to discuss with their partner. Provide these sentence frames as partners discuss: “To figure out how much flour is needed for a given amount of honey....”, “One way a constant of proportionality could help is....”

Sample responses include: “I would try to find the factor that gets us from 8 tablespoons to 20 tablespoons,” “I would draw a diagram to figure out how much flour is needed for 1 tablespoon of honey,” and, “I know that 10 to 8 is the same as 5 to 4 is the same as 1.25, and that is the constant of proportionality, so I know I need to multiply by 1.25.” This will help students concentrate on making sense of the situation before rushing to a solution or method.

4. As partners are discussing their strategies, select 1–2 students to share their ideas with the whole class.

As students are presenting ideas to the whole class, create a display that summarizes their ideas about how to complete the table. Listen for quantities that were mentioned during the second read, and include these on the display.

5. Post the summary where all students can use it as a reference.

Student Task Statement

A bakery uses 8 tablespoons of honey for every 10 cups of flour to make bread dough. Some days they bake bigger batches and some days they bake smaller

batches, but they always use the same ratio of honey to flour. Complete the table as you answer the questions. Be prepared to explain your reasoning.

1. How many cups of flour do they use with 20 tablespoons of honey?

honey (tbsp)	flour (c)
8	10
20	
13	
	20

2. How many cups of flour do they use with 13 tablespoons of honey?

3. How many tablespoons of honey do they use with 20 cups of flour?

4. What is the **proportional relationship** represented by this table?

Student Response

1. 25 cups of flour for 20 tablespoons of honey
2. $16\frac{1}{4}$ or 16.25 cups of flour for 13 tablespoons of honey
3. 16 tablespoons of honey for 20 cups of flour

honey (tbsp)	flour (c)
4	5
8	10
12	15
13	16.25
16	20
18	22.5
20	25

4. Answers vary. Possible responses:
 - The relationship between the number of tablespoons of honey and the number of cups of flour is proportional.
 - The relationship between the amount of honey and the amount of flour is a proportional relationship.
 - The table represents a proportional relationship between amount of honey and amount of flour.
 - The amount of honey is proportional to the amount of flour.

Activity Synthesis

Select students to share their solution approaches in this order:

1. Double each entry in the first row to get to row 4.
2. Divide each entry in the first row by 8, yielding the pair (1, 1.25). (1.25 can be called the unit rate, since 1.25 cups of flour are needed for 1 tablespoon of honey.) Multiply by 20 and 13 to get rows 2 and 3.

3. Notice any entry in the first column can be multiplied by $\frac{5}{4}$ or 1.25 to get the corresponding entry in the second column. Name this value the **constant of proportionality** for the proportional relationship.

Ensure that these are highlighted as part of the discussion and that students describe their methods using mathematical language (MP6). For example,

- Note that even though you multiply by a different scale factor to go from row to row in the table, the unit rate is always the same. Rename this “the constant of proportionality.” Note that It can always be found by finding how much of the second quantity per one of the first quantity.
- Ask students what the proportional relationship is in this situation.
- Ask students to interpret the constant of proportionality in the context: “What does the 1.25 tell us about?” (That there are 1.25 cups of flour per tablespoon of honey.) Because this can help them connect it to their work in grade 6 and prepare them for the next activity.

Support for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation such as: “First, I ____ because,” “I noticed ____ so I,” “Why did you...?,” “I agree/ disagree because....”

Supports accessibility for: Language; Social-emotional skills

2.4 Quarters and Dimes

Optional: : 10 minutes

The purpose of this optional activity is to practice using a proportional relationship in another context. The chosen numbers make the unit rate a useful tool to answer the questions.

Addressing

- 7.RP.A.2

Instructional Routines

- MLR8: Discussion Supports

Launch

Remind students of the value of a quarter and a dime.

Student Task Statement

4 quarters are equal in value to 10 dimes.

1. How many dimes equal the value of 6 quarters?
2. How many dimes equal the value of 14 quarters?
3. What value belongs next to the 1 in the table?
What does it mean in this context?

number of quarters	number of dimes
1	
4	10
6	
14	

Student Response

1. 15
2. 35
3. 2.5 means that two and a half dimes are worth the same amount as a single quarter.

number of quarters	number of dimes
1	2.5
4	10
6	15
14	35



Are You Ready for More?

Pennies made before 1982 are 95% copper and weigh about 3.11 grams each. (Pennies made after that date are primarily made of zinc). Some people claim that the value of the copper in one of these pennies is greater than the face value of the penny. Find out how much copper is worth right now, and decide if this claim is true.

Student Response

The cost of copper fluctuates, so the answer depends on the current value of copper. Assuming that copper costs about \$5.00 per kg, the copper in a penny would be worth about $0.95 \cdot 0.00311 \cdot 5 \approx 0.015$ or about 1.5 cents. As long as copper is worth at least \$3.38 per kilogram, then the copper in a pre-1982 penny will be worth more than 1 cent.

Activity Synthesis

Ask students for the value that belongs next to the 1 in the table. Invite several students to explain the significance of this number.

Support for English Language Learners

Speaking: MLR8 Discussion Supports. To provide a support students in precision of language when describing proportional relationships from a table, provide sentence frames for students to use, such as: “_____ is equal in value to _____, because”

Design Principle(s): Cultivate conversation

Lesson Synthesis

Briefly revisit the two activities, demonstrating the use of the new terms. It would be helpful to display a filled-in table for each to facilitate the conversation. For example,

- In the first activity, we looked at a **proportional relationship** between the amount of rice and the number of people served. Some people found missing values in the table by multiplying both values in one of the rows by a number, others used a *unit rate*—the number of people served per cup of rice. What was the **constant of proportionality** in that situation?
- In the second activity, we looked at a proportional relationship between the amount of honey and the amount of flour in a recipe. Some people found missing values in the table by multiplying both values in one of the rows by a number, others used a unit rate—the number of cups of flour per tablespoon of honey. What was the constant of proportionality in that situation?

If this has not already happened during the discussion, write a third column in each table that shows the constant of proportionality and label it.

2.5 Green Paint

Cool Down : 5 minutes

Building On

- 6.RP.A.3

Addressing

- 7.RP.A.2

Student Task Statement

When you mix two colors of paint in equivalent ratios, the resulting color is always the same. Complete the table as you answer the questions.

1. How many cups of yellow paint should you mix with 1 cup of blue paint to make the same shade of green? Explain or show your reasoning.

cups of blue paint	cups of yellow paint
2	10
1	

2. Make up a new pair of numbers that would make the same shade of green. Explain how you know they would make the same shade of green.

3. What is the proportional relationship represented by this table?

4. What is the *constant of proportionality*? What does it represent?

Student Response

1. You need 5 cups of yellow paint for 1 cup of blue paint. You can see this by multiplying the first row by a factor of $\frac{1}{2}$. Alternatively, you have to multiply 2 by $\frac{10}{2} = 5$ to get 10: multiplying 1 by 5 gives 5.
2. Answers vary. Sample response: 3 cups of blue paint mixed with 15 cups of yellow paint will also make the same shade of green. This can be obtained by multiplying the second row by a factor of 3 or choosing 3 for blue and then multiplying that by 5.
3. The relationship between the amount of blue paint and the amount of yellow paint is the proportional relationship represented by this table.

4. The constant of proportionality is 5. It represents the cups of yellow paint needed for 1 cup of blue paint.

Student Lesson Summary

If the ratios between two corresponding quantities are always equivalent, the relationship between the quantities is called a **proportional relationship**.

This table shows different amounts of milk and chocolate syrup. The ingredients in each row, when mixed together, would make a different total amount of chocolate milk, but these mixtures would all taste the same.

Notice that each row in the table shows a ratio of tablespoons of chocolate syrup to cups of milk that is equivalent to 4 : 1.

About the relationship between these quantities, we could say:

tablespoons of chocolate syrup	cups of milk
4	1
6	$1\frac{1}{2}$
8	2
$\frac{1}{2}$	$\frac{1}{8}$
12	3
1	$\frac{1}{4}$

- The relationship between amount of chocolate syrup and amount of milk is proportional.
- The relationship between the amount of chocolate syrup and the amount of milk is a proportional relationship.
- The table represents a proportional relationship between the amount of chocolate syrup and amount of milk.
- The amount of milk is proportional to the amount of chocolate syrup.

We could multiply any value in the chocolate syrup column by $\frac{1}{4}$ to get the value in the milk column. We might call $\frac{1}{4}$ a *unit rate*, because $\frac{1}{4}$ cups of milk are needed for 1 tablespoon of chocolate syrup. We also say that $\frac{1}{4}$ is the **constant of**

proportionality for this relationship. It tells us how many cups of milk we would need to mix with 1 tablespoon of chocolate syrup.

Glossary

- constant of proportionality
- proportional relationship

Lesson 2 Practice Problems

1. Problem 1

Statement

When Han makes chocolate milk, he mixes 2 cups of milk with 3 tablespoons of chocolate syrup. Here is a table that shows how to make batches of different sizes. Use the information in the table to complete the statements. Some terms are used more than once.

cups of milk	tablespoons of chocolate syrup
2	3
8	12
1	$\frac{3}{2}$
10	15

- The table shows a proportional relationship between _____ and _____.
- The scale factor shown is _____.
- The constant of proportionality for this relationship is _____.
- The units for the constant of proportionality are _____ per _____.

Bank of Terms: tablespoons of chocolate syrup, 4, cups of milk, cup of milk, $\frac{3}{2}$

Solution

- a. cups of milk, tablespoons of chocolate syrup
- b. 4
- c. $\frac{3}{2}$
- d. tablespoons of chocolate syrup, cup of milk

2. Problem 2

Statement

A certain shade of pink is created by adding 3 cups of red paint to 7 cups of white paint.

- a. How many cups of red paint should be added to 1 cup of white paint?

cups of white paint	cups of red paint
1	
7	3

- b. What is the constant of proportionality?

Solution

- a. $\frac{3}{7}$ cups of red paint
- b. $\frac{3}{7}$

3. Problem 3

Statement

A map of a rectangular park has a length of 4 inches and a width of 6 inches. It uses a scale of 1 inch for every 30 miles.

- a. What is the actual area of the park? Show how you know.
- b. The map needs to be reproduced at a different scale so that it has an area of 6 square inches and can fit in a brochure. At what scale should the map be reproduced so that it fits on the brochure? Show your reasoning.

Solution

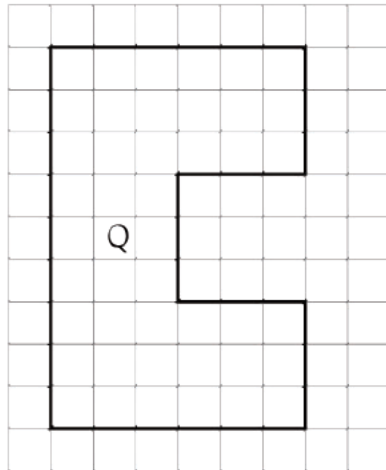
- a. 21,600 square miles. Sample reasoning: The area on the map is 24 square inches. 1 square inch represents 900 square miles, since $30 \cdot 30 = 90$. The actual area is $24 \cdot 900$, which equals 21,600 square miles.
- b. 1 inch to 60 miles. Sample explanations:
- If 21,600 square miles need to be represented by 6 square inches, each square inch needs to represent 3,600 square miles: $21,600 \div 6 = 3,600$. This means each 1-inch side of the square needs to be 60 miles.
 - The area of this new map is $\frac{1}{4}$ of the first map, since 6 is $\frac{1}{4}$ of 24. This means each 1 inch square has to represent 4 times as much area than the first. $900 \cdot 4 = 3,600$. If each 1 inch square represents 3,600 square miles, every 1 inch represents 60 miles.

(From Unit 1, Lesson 12.)

4. Problem 4

Statement

Noah drew a scaled copy of Polygon P and labeled it Polygon Q.



If the area of Polygon P is 5 square units, what scale factor did Noah apply to Polygon P to create Polygon Q? Explain or show how you know.

Solution

The area of polygon Q is 45 square units, so the area has scaled by a factor of 9, since $5 \cdot 9 = 45$. Since the area of a scaled copy varies from the original area by the square of the scale factor, the scale factor is 3.

(From Unit 1, Lesson 6.)

5. Problem 5

Statement

Select all the ratios that are equivalent to each other.

A. 4 : 7

B. 8 : 15

C. 16 : 28

D. 2 : 3

E. 20 : 35

Solution

["A", "C", "E"]

Lesson 3: More about Constant of Proportionality

Goals

- Compare, contrast, and critique (orally and in writing) different ways to express the constant of proportionality for a relationship.
- Explain (orally) how to determine the constant of proportionality for a proportional relationship represented in a table.
- Interpret the constant of proportionality for a relationship in the context of constant speed.

Learning Targets

- I can find missing information in a proportional relationship using a table.
- I can find the constant of proportionality from information given in a table.

Lesson Narrative

In this lesson, students continue to work with proportional relationships represented by tables using contexts familiar from previous grades: unit conversion and constant speed. They recognize the constant of proportionality as the conversion factor or the speed, and use it to answer questions about the context. Although students might continue to reason with equivalent ratios to solve problems, the contexts are designed so that it is more efficient to use the constant of proportionality. For example, when converting length measurements from feet to inches, it is more convenient to know the rule “multiply by 12” than to use an equivalent ratio with a different scale factor every time: “1 foot is 12 inches, so multiplying both quantities by 3 I see that 3 feet is 36 inches, and multiplying both quantities by 5 I see that 5 feet is 60 inches.”

Alignments

Building On

- 5.MD.A.1: Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

Addressing

- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

- 7.RP.A.2.a: Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- 7.RP.A.2.b: Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR8: Discussion Supports

Student Learning Goals

Let's solve more problems involving proportional relationships using tables.

3.1 Equal Measures

Warm Up: : 5 minutes

This warm-up prompts students to review work in grade 5 of converting across different-sized standard units within a given measurement system.

Building On

- 5.MD.A.1

Launch

Arrange students in groups of 2. Ask students to use the following numbers and units to record as many equivalent expressions as they can. Tell them they are allowed to reuse numbers and units. Give students 2 minutes of quiet think time followed by 1 minute to share their equations with their partner.

Student Task Statement

Use the numbers and units from the list to find as many equivalent measurements as you can. For example, you might write "30 minutes is $\frac{1}{2}$ hour."

You can use the numbers and units more than once.

1	$\frac{1}{2}$	0.3	centimeter
12	40	24	meter
0.4	0.01	$\frac{1}{10}$	hour
60	$3\frac{1}{3}$	6	feet
50	30	2	minute
		$\frac{2}{5}$	inch

Student Response

Answers vary. Possible responses:

- 1 cm is 0.01 m, 30 cm is 0.3 m, 40 cm is 0.4 m, $\frac{1}{2}$ m is 50 cm
- $\frac{1}{2}$ hr is 30 min, 1 hr is 60 min, $\frac{1}{10}$ hr is 6 min, $\frac{2}{5}$ hr is 0.4 hr or 24 min
- 24 in is 2 ft, 12 in is 1 ft, 12 in is 1 ft, 6 in is $\frac{1}{2}$ ft, 40 in is $3\frac{1}{3}$ ft

Activity Synthesis

Invite a few students to share equations that they had in common with their partner and ones that were different. Record these answers for all to see. After each equation is shared, ask students to give a signal if they had the same one recorded. Display the following questions for all to see and discuss:

- “What number(s) did you use the most? Why?”
- “If you could include two more cards to this selection, what would they be? Why?”

3.2 Centimeters and Millimeters

: 10 minutes

This activity has two purposes. First, it involves an important context that can be represented by proportional relationships, namely measurement conversion. Second, it introduces the idea that there are two constants of proportionality, and that they are reciprocals (also known as multiplicative inverses). Students understand why this is the case later when they use equations to represent proportional relationships. Students start to use “is proportional to” language to distinguish between the two constants of proportionality. During the discussion, students should be reminded that dividing by a number is equivalent to multiplying by its reciprocal. In principle, this is something students learn in grade 6, but they often need to be reminded. This will be needed in future lessons, so it is important to discuss it here.

Conversion of centimeters to millimeters has a constant of proportionality of 10 while conversion of millimeters to centimeters has a constant of proportionality of $\frac{1}{10}$. One way to explain why these two constants of proportionality are multiplicative inverses is to imagine starting with a measurement in centimeters, 15 cm for example. When we convert to millimeters we multiply by 10: $15 \text{ cm} = 150 \text{ mm}$. If we convert the measurement in millimeters back to centimeters, we know it will be 15, so the constant of proportionality we need to multiply by is $\frac{1}{10}$.

Addressing

- 7.RP.A.2.b

Instructional Routines

- MLR8: Discussion Supports

Launch

Tell students, “Let’s look at how centimeters and millimeters are related and how it is related to what we have been doing recently.”

Support for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Create and review a display that includes 1–2 examples to remind students that dividing by a number is equivalent to multiplying by its reciprocal. Keep the display visible throughout the activity.

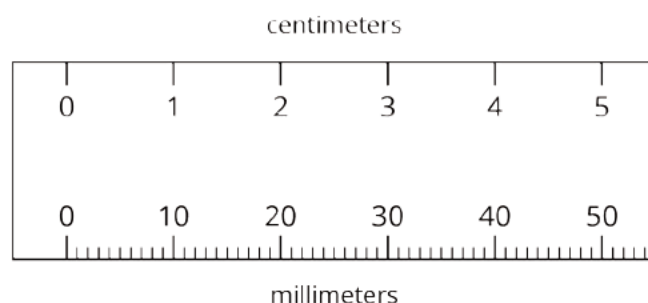
Supports accessibility for: Memory; Conceptual processing

Anticipated Misconceptions

Some students may say that the constants of proportionality are both 10 since you can divide by 10 in the second table. Tell students, “The constant of proportionality is what you *multiply* by. Can you find a way to *multiply* the numbers in the first column to get the numbers in second column?”

Student Task Statement

There is a proportional relationship between any length measured in centimeters and the same length measured in millimeters.



There are two ways of thinking about this proportional relationship.

1. If you know the length of something in centimeters, you can calculate its length in millimeters.

a. Complete the table.

b. What is the constant of proportionality?

length (cm)	length (mm)
9	
12.5	
50	
88.49	

2. If you know the length of something in millimeters, you can calculate its length in centimeters.

a. Complete the table.

b. What is the constant of proportionality?

length (mm)	length (cm)
70	
245	
4	
699.1	

3. How are these two constants of proportionality related to each other?

4. Complete each sentence:

a. To convert from centimeters to millimeters, you can multiply by _____.

b. To convert from millimeters to centimeters, you can divide by _____ *or* multiply by _____.

Student Response

1. a.

length (cm)	length (mm)
9	90
12.5	125
50	500
88.49	884.9

b. 10

2. a.

length (mm)	length (cm)
70	7
245	24.5
4	0.4
699.1	69.91

b. $\frac{1}{10}$ or 0.1.

3. 10 and $\frac{1}{10}$ are reciprocals.

4. a. To convert from centimeters to millimeters, you can multiply by 10.

b. To convert from millimeters to centimeters, you can divide by 10 *or* multiply by $\frac{1}{10}$.

Are You Ready for More?

1. How many square millimeters are there in a square centimeter?

2. How do you convert square centimeters to square millimeters? How do you convert the other way?

Student Response

1. 100
2. Multiply by 100. Multiply by $\frac{1}{100}$

Activity Synthesis

The goal of this discussion is to help students see the connection between this situation and the earlier tasks, so they can use the structure of the table (MP7) to find the constants of proportionality. Ask questions like, “Can you use any of the strategies we have been discussing in earlier problems to help you solve this problem?” as students work to help them make this connection.

If there is disagreement about the constants of proportionality, ask students to explain their thinking.

Look for students who have seen the connection between dividing by 10 and multiplying by $\frac{1}{10}$. Ask the students if there is a generalization to be made here. Help them articulate the idea that when you divide by a number, that is the same as multiplying by the reciprocal.

Support for English Language Learners

Speaking, Listening, Representing: MLR8 Discussion Supports. Use this routine to support whole-class discussion. As students share what they identified as the constant of proportionality and their strategies they used to identify the constant of proportionality, press for details in students’ explanations by requesting students to challenge an idea, elaborate on an idea, or give an example. In this discussion, demonstrate uses of disciplinary language functions such as detailing steps, describing and justifying reasoning, and questioning strategies. If necessary, revoice student ideas to demonstrate mathematical language, and invite students to chorally repeat phrases that include relevant vocabulary in context, such as “constant of proportionality”.

Design Principle(s): Support sense-making

3.3 Pittsburgh to Phoenix

: 15 minutes

This activity focuses on making connections between constant speed and proportional relationships, with special attention to the constant of proportionality. In grade 6, students solved problems involving constant speed, but they need opportunities to make the connection to proportional relationships; students who successfully make this connection are reasoning abstractly about contexts with constant speed (MP2). Students who need support in understanding the context can trace the segments in the map, labeling the distances they know and putting question marks for unknown distances. An empty double number line could also be a useful tool in helping students reason about the context.

The numbers are chosen so that students are more likely to use unit rate rather than scale factors. The goal of the discussion is for students to understand that when speed is constant, time elapsed and distance traveled are proportional. The constant of proportionality indicates the magnitude of the speed. For example, if distance is given in miles and time in hours, then the constant of proportionality indicates the speed in miles per hour.

Students might wonder about the route. If they ask about it, teachers can share some reasons why airplane routes are complex, e.g., the need to avoid congested areas and the fact that the shortest distances on the curved surface of Earth do not always correspond to lines on a map.

Addressing

- 7.RP.A.2

Instructional Routines

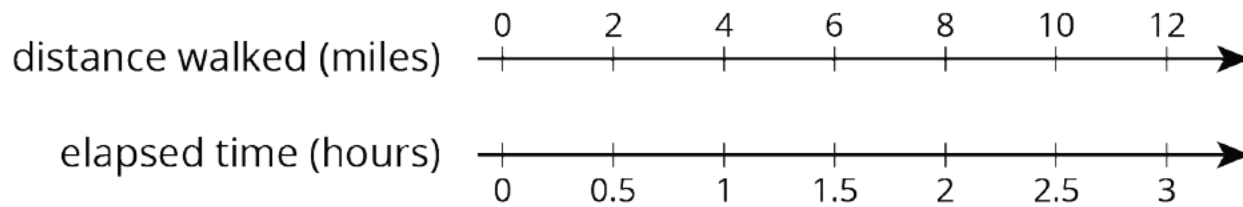
- MLR1: Stronger and Clearer Each Time

Launch

It may have been some time since students reasoned about constant speed, so before they start on this activity, consider posing some simple problems about constant speed. For example:

- Someone walks at a constant speed of 4 miles per hour. How much time does it take them to walk 4 miles? 8 miles? 20 miles? 2 miles? $\frac{1}{2}$ mile? How far do they get in 3 hours? In 10 minutes?
- Someone rides a bike at a constant speed. They go 30 miles in 2 hours. What was their speed? If students did some work with double number lines when learning about constant speed in grade 6, they may be inclined to create one to help them think through these launch questions and when they get into the activity. If no one suggests it, it might be worth showing a double number line when talking through

this launch, so that while working on the activity, students recall that these are useful tools. For example:



Support for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Allow students to use four-function calculators to ensure inclusive participation in the activity.

Supports accessibility for: Memory; Conceptual processing

Support for English Language Learners

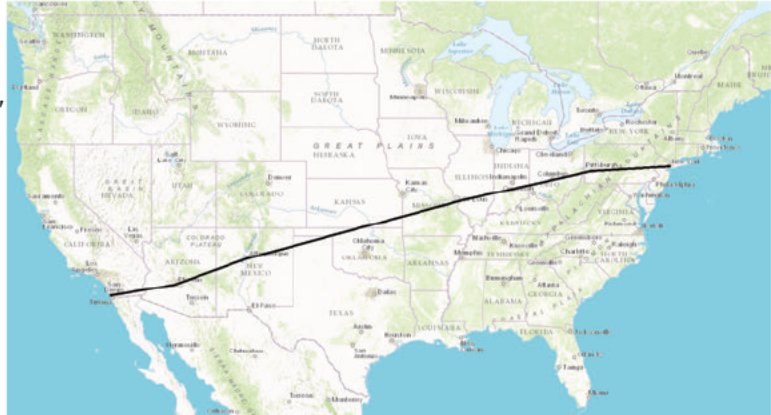
Speaking: MLR1 Stronger and Clearer Each Time. Use this routine with successive pair shares to give students a structured opportunity to revise and refine their response to the final question. Ask each student to meet with 2-3 other partners in a row for feedback. Provide students with prompts for feedback that will help teams strengthen their ideas, such as, "How do the units affect your answer?" or "Why do you think that value is a constant of proportionality?" Students can borrow ideas and language from each partner to strengthen their final opinion and reasons.

Design Principle(s): Optimize output (for explanation)

Student Task Statement

On its way from New York to San Diego, a plane flew over Pittsburgh, Saint Louis, Albuquerque, and Phoenix traveling at a constant speed.

Complete the table as you answer the questions. Be prepared to explain your reasoning.



segment	time	distance	speed
Pittsburgh to Saint Louis	1 hour	550 miles	
Saint Louis to Albuquerque	1 hour 42 minutes		
Albuquerque to Phoenix		330 miles	

1. What is the distance between Saint Louis and Albuquerque?
2. How many minutes did it take to fly between Albuquerque and Phoenix?
3. What is the proportional relationship represented by this table?
4. Diego says the constant of proportionality is 550. Andre says the constant of proportionality is $9\frac{1}{6}$. Do you agree with either of them? Explain your reasoning.

Student Response

segment	time	distance	speed
Pittsburgh to Saint Louis	1 hour	550 miles	550 miles per hour
Saint Louis to Albuquerque	1 hour 42 minutes	935 miles	550 miles per hour
Albuquerque to Phoenix	36 minutes	330 miles	550 miles per hour

- 935 miles. 42 minutes is $\frac{42}{60}$ hours or $\frac{7}{10}$ of an hour. $\frac{7}{10}$ of 550 is 385, and $385 + 550 = 935$.
- 36 minutes. 330 is $\frac{3}{5}$ of 550, and $\frac{3}{5}$ of 60 minutes is 36 minutes.
- The distance traveled is proportional to the elapsed time.
- Answers vary. Possible responses: Diego uses miles per hour, and Andre uses miles per minute.

segment	time	distance	speed
Pittsburgh to Saint Louis	1 hour	550 miles	550 miles per hour
Saint Louis to Albuquerque	1.7 hours	935 miles	550 miles per hour
Albuquerque to Phoenix	0.6 hours	330 miles	550 miles per hour

segment	time	distance	speed
Pittsburgh to Saint Louis	60 minutes	550 miles	$9\frac{1}{6}$ miles per minute
Saint Louis to Albuquerque	102 minutes	935 miles	$9\frac{1}{6}$ miles per minute
Albuquerque to Phoenix	36 minutes	330 miles	$9\frac{1}{6}$ miles per minute

Activity Synthesis

The goal of the discussion is for students to understand that when speed is constant, then distance traveled is proportional to elapsed time. The constant of proportionality is the speed. For example, if distance is given in miles and time in hours, then the constant of proportionality indicates the speed in miles per hour.

Begin by having a student share a table that is all in hours, miles, and miles per hour. Use this example to point out that every number in the first column can be multiplied by the speed to get the number in the second column. Ask:

- “Which quantities are in a proportional relationship? How do you know?”
- “What is the constant of proportionality in this case?”

If a student wrote their times in minutes and speed in miles per minute, you can have the same discussion if the class seems ready for that leap, but it is not required. In any case, summarize by making it explicit that when time and distance are in a proportional relationship, the constant of proportionality is the speed.

Lesson Synthesis

Briefly revisit the two contexts, demonstrating the use of new terms. For example,

- In the first activity, we examined the proportional relationship between millimeters and centimeters from two different perspectives and found two constants of proportionality. What were they? What is the relationship between the two constants of proportionality? (They are reciprocals.)

- In the second activity, we examined a proportional relationship between the time a plane flies and the distance it travels. What was the constant of proportionality in this task? What does the constant of proportionality represent in terms of the context? (Magnitude of speed.)

3.4 Fish Tank

Cool Down : 5 minutes

Addressing

- 7.RP.A.2.a
- 7.RP.A.2.b

Student Task Statement

Mai is filling her fish tank. Water flows into the tank at a constant rate. Complete the table as you answer the questions.

1. How many gallons of water will be in the fish tank after 3 minutes? Explain your reasoning.

time (minutes)	water (gallons)
0.5	0.8
1	
3	
	40

2. How long will it take to fill the tank with 40 gallons of water? Explain your reasoning.

3. What is the constant of proportionality?

Student Response

1. 4.8. If you double the first row (scale by 2), you get 1.6 gallons after 1 minute. If you triple the second row (scale by 3), you get 4.8 gallons after 3 minutes. Or you could scale the first row by 6 to get 4.8 gallons after 3 minutes.

2. 25 minutes. One way to find a scale factor to use is to divide 40 by 0.8: $\frac{40}{0.8} = 50$. That scale factor multiplied by the value in the first column of the first row is: $50 \cdot 0.5 = 25$.
3. 1.6 or equivalent. You can observe the amount of water that corresponds with 1 minute, or you can divide any value in the right column with its corresponding value in the left column.

Student Lesson Summary

When something is traveling at a constant speed, there is a proportional relationship between the time it takes and the distance traveled. The table shows the distance traveled and elapsed time for a bug crawling on a sidewalk.

distance traveled (cm)	elapsed time (sec)
$\frac{3}{2}$ →	1
1 →	$\frac{2}{3}$
3 →	2
10 →	$\frac{20}{3}$

$\cdot \frac{2}{3}$

We can multiply any number in the first column by $\frac{2}{3}$ to get the corresponding number in the second column. We can say that the elapsed time is proportional to the distance traveled, and the constant of proportionality is $\frac{2}{3}$. This means that the bug's *pace* is $\frac{2}{3}$ seconds per centimeter.

This table represents the same situation, except the columns are switched.

elapsed time (sec)	distance traveled (cm)
1 →	$\frac{3}{2}$
$\frac{2}{3}$ →	1
2 →	3
$\frac{20}{3}$ →	10

$\cdot \frac{3}{2}$

We can multiply any number in the first column by $\frac{3}{2}$ to get the corresponding number in the second column. We can say that the distance traveled is proportional to the elapsed time, and the constant of proportionality is $\frac{3}{2}$. This means that the bug's *speed* is $\frac{3}{2}$ centimeters per second.

Notice that $\frac{3}{2}$ is the reciprocal of $\frac{2}{3}$. When two quantities are in a proportional relationship, there are two constants of proportionality, and they are always reciprocals of each other. When we represent a proportional relationship with a table, we say the quantity in the second column is proportional to the quantity in the first column, and the corresponding constant of proportionality is the number we multiply values in the first column to get the values in the second.

Lesson 3 Practice Problems

1. Problem 1

Statement

Noah is running a portion of a marathon at a constant speed of 6 miles per hour.

Complete the table to predict how long it would take him to run different distances at that speed, and how far he would run in different time intervals.

time in hours	miles traveled at 6 miles per hour
1	
$\frac{1}{2}$	
$1\frac{1}{3}$	
	$1\frac{1}{2}$
	9
	$4\frac{1}{2}$

Solution

time in hours	miles traveled at 6 miles per hour
1	6
$\frac{1}{2}$	3
$1\frac{1}{3}$	8
$\frac{1}{4}$	$1\frac{1}{2}$
$1\frac{1}{2}$	9
$\frac{3}{4}$	$4\frac{1}{2}$

2. Problem 2

Statement

One kilometer is 1000 meters.

- a. Complete the tables. What is the interpretation of the constant of proportionality in each case?

meters	kilometers
1,000	1
250	
12	
1	

kilometers	meters
1	1,000
5	
20	
0.3	

The constant of proportionality tells us that: The constant of proportionality tells us that:

- b. What is the relationship between the two constants of proportionality?

Solution

a. i.

meters	kilometers
1,000	1
250	0.25
12	0.012
1	0.001

0.001 kilometers per meter

ii.

kilometers	meters
1	1,000
5	5,000
20	20,000
0.3	300

1000 meters per kilometer

b. 0.001 and 1000 are reciprocals of each other. This is easier to see if 0.001 is written as $\frac{1}{1000}$.

3. Problem 3

Statement

Jada and Lin are comparing inches and feet. Jada says that the constant of proportionality is 12. Lin says it is $\frac{1}{12}$. Do you agree with either of them? Explain your reasoning.

Solution

Jada is saying that there are 12 inches for every 1 foot. Lin is saying that there are $\frac{1}{12}$ foot for every 1 inch.

4. Problem 4

Statement

The area of the Mojave desert is 25,000 square miles. A scale drawing of the Mojave desert has an area of 10 square inches. What is the scale of the map?

Solution

1 inch to 50 miles

(From Unit 1, Lesson 12.)

5. Problem 5

Statement

Which of these scales is equivalent to the scale 1 cm to 5 km? Select **all** that apply.

- A. 3 cm to 15 km
- B. 1 mm to 150 km
- C. 5 cm to 1 km
- D. 5 mm to 2.5 km
- E. 1 mm to 500 m

Solution

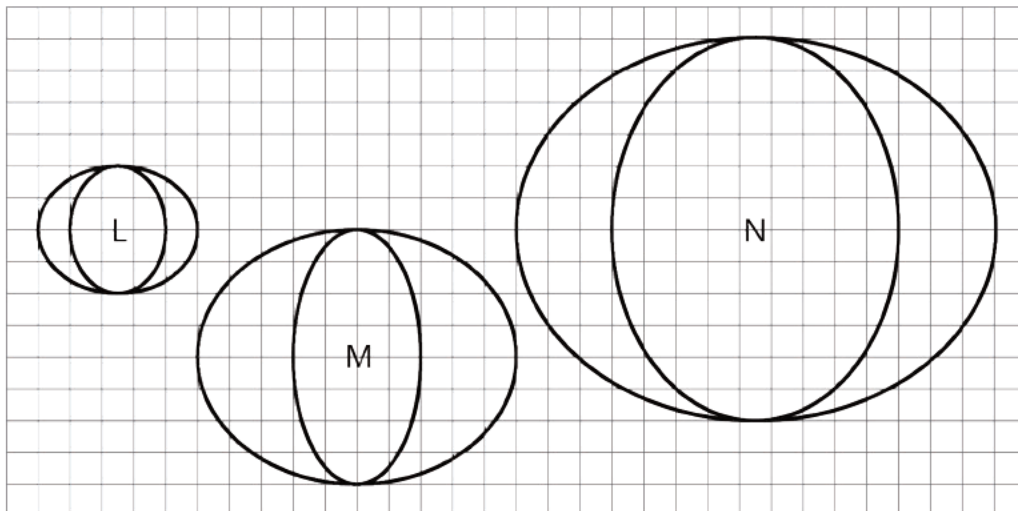
["A", "D", "E"]

(From Unit 1, Lesson 11.)

6. Problem 6

Statement

Which one of these pictures is not like the others? Explain what makes it different using ratios.



Solution

M is different from L and N. The width:height ratios for the outsides of the pictures are all equivalent to $5:4$. However, the width:height ratios of the insides of L and N both have a $3:4$ ratio of width:height, while the inside of M has a width of 4 units and a height of 8 units, making its ratio $1:2$.

Alternatively, the ratio of height to thickness at the widest part for L and N are both $4:1$. But M has a height of 8 units and a thickness of 3 units, making that ratio $8:3$.

(From Unit 2, Lesson 1.)

Section: Representing Proportional Relationships with Equations

Lesson 4: Proportional Relationships and Equations

Goals

- Generalize a process for finding missing values in a proportional relationship, and justify (orally) why this can be abstracted as $y = kx$, where k is the constant of proportionality.
- Generate an equation of the form $y = kx$ to represent a proportional relationship in a familiar context.
- Write the constant of proportionality to complete a row in the table of a proportional relationship where the value for the first quantity is 1.

Learning Targets

- I can write an equation of the form $y = kx$ to represent a proportional relationship described by a table or a story.
- I can write the the constant of proportionality as an entry in a table.

Lesson Narrative

In this lesson, students build on their work with tables and represent proportional relationships using equations of the form $y = kx$. The activities revisit contexts from the previous two lessons, presenting values in tables and focusing on the idea that for each table, there is a number k so that all values in the table satisfy the equation $y = kx$. By expressing the regularity of repeated calculations of values in the table with the equations, students are engaging in MP8.

Alignments

Building On

- 5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Addressing

- 7.RP.A.2: Recognize and represent proportional relationships between quantities.
- 7.RP.A.2.c: Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR5: Co-Craft Questions
- MLR6: Three Reads
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Number Talk
- Think Pair Share

Student Learning Goals

Let's write equations describing proportional relationships.

4.1 Number Talk: Division

Warm Up: : 5 minutes

This number talk encourages students to think about the numbers in division problems and how they can use the result of one division problem to find the answer to a similar problem with a different, but related, divisor. Four problems are given, however, given limited time it may not be possible to share every possible strategy. Consider gathering only two or three different strategies per problem. Each problem is chosen to elicit a slightly different reasoning, so, as students explain their strategies, ask how the factors impacted their product.

In the final question, ask students to choose a value for x for which they could easily find the quotient. If students do not use what they know based on the answer to the previous

question, ask them if they could use what they know about that equation to reason about the last expression.

Building On

- 5.NBT.B.7

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Display one problem at a time. Tell students to give a signal when they have an answer and a strategy. After each problem, give students 1 minute of quiet think time followed by a whole-class discussion. Pause after discussing the third question and tell students they will be using patterns they noticed in the previous problems to choose their own divisor for the last problem. Keep all problems displayed throughout the talk.

Support for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Student Task Statement

Find each quotient mentally.

$$645 \div 100$$

$$645 \div 50$$

$$48.6 \div 30$$

$$48.6 \div x$$

Student Response

- $645 \div 100 = 6.45$

- $645 \div 50 = 12.9$
- $48.6 \div 30 = 1.62$
- Answers vary. Students could double the divisor in the previous problem for a value of 60 for x and divide the previous quotient by 2 to arrive at a new quotient of 0.81. Students could also half the divisor in the previous problem for a value of 15 for x and multiply the previous quotient by 2 to arrive at a new quotient of 3.24.

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their explanations for all to see. To involve more students in the conversation, consider asking:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone solve the problem the same way but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to ____'s strategy?"
- "Do you agree or disagree? Why?"

For the fourth question, ask students to share their divisor choice and reasoning behind the choice. Record these divisors for all to see and ask the rest of the class to find the quotient based on the divisor choice. Ask students to refer back to patterns and regularity they noticed in the first three problems that influenced their decisions.

Support for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because . . ." or "I noticed ____ so I . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Design Principle(s): Optimize output (for explanation)

4.2 Feeding a Crowd, Revisited

: 10 minutes

This activity revisits a context seen previously. Students solved problems like this as early as grade 3 without formulating them in terms of ratios or rates (“If 1 cup of rice serves 3 people, how many people can you serve with 12 cups of rice?”). In this activity, they ultimately find an equation for the proportional relationship.

As students find missing values in the table, they should see that they can always multiply the number of food items by the constant of proportionality. When students see this pattern (MP7) and represent the number of people served by x cups of rice (or s spring rolls) as $3x$ (or $\frac{1}{2}s$), they are expressing regularity in repeated reasoning (MP8).

Only one row in each table is complete. Based on their experience in the previous lesson, students are more likely to multiply the entries in the left-hand column by 3 (or $\frac{1}{2}$) than to use scale factors, at least for the first and third rows. If they do, they are more likely to see how to complete the last row in each table.

Some students might use unit rates: If 1 cup of rice can serve 3 people, then x cups of rice can serve $3x$ people. So, students have different ways to generate $3x$ as the expression that represents the number of people served by x cups of rice: completing the table for numerical values and continuing the pattern to the last row; or finding the unit rate and using it in the case of x cups.

Monitor students for different approaches as they are working.

Addressing

- 7.RP.A.2.c

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect

Launch

Tell students that this activity revisits a context they worked on in an earlier lesson.

Anticipated Misconceptions

If students have trouble encapsulating the relationship with an expression, encourage them to draw diagrams or to verbalize the relationship in words.

Student Task Statement

1. A recipe says that 2 cups of dry rice will serve 6 people. Complete the table as you answer the questions. Be prepared to explain your reasoning.

a. How many people will 1 cup of rice serve?

b. How many people will 3 cups of rice serve? 12 cups? 43 cups?

c. How many people will x cups of rice serve?

cups of dry rice	number of people
1	
2	6
3	
12	
43	
x	

2. A recipe says that 6 spring rolls will serve 3 people. Complete the table as you answer the questions. Be prepared to explain your reasoning.

a. How many people will 1 spring roll serve?

b. How many people will 10 spring rolls serve? 16 spring rolls? 25 spring rolls?

c. How many people will n spring rolls serve?

number of spring rolls	number of people
1	
6	3
10	
16	
25	
n	

3. How was completing this table different from the previous table? How was it the same?

Student Response

1. a. 3 people since $1 \cdot 3 = 3$
b. 9 people. 36 people. 129 people.
c. $3x$ people.

cups of dry rice	number of people
1	3
2	6
3	9
12	36
43	129
x	$3x$

2. a. $\frac{1}{2}$. Since $1 \cdot \frac{1}{2} = \frac{1}{2}$
b. 5 people; 8 people; 12.5 people.
c. $\frac{1}{2}n$

number of spring rolls	number of people
1	$\frac{1}{2}$
6	3
10	5
16	8
25	12.5
n	$\frac{1}{2}n$

3. Answers vary.

Activity Synthesis

Select students who have used the following approaches in the given order:

- Recognize that to move from the first column to the second, you multiply by 3.
- Say that 3 is the constant of proportionality.
- Recognize that the equation $2 \cdot ? = 6$ can be used to find the constant of proportionality algebraically.
- Say that the 3 can be interpreted as the number of people per 1 cup of rice.

If no student sees that these insights are connected to prior work, explicitly connect them with the lessons of the previous two days. At the end of the discussion, suggest to students that we let y represent the number of people who can be served by x cups of rice. Ask students to write an equation that gives the relationship of x and y (this builds on work from grade 6, but it may be rusty). Be sure to write the equation where all students can see it and help students interpret its meaning in the context: “To find y , the number of people served, we can multiply the number of cups of rice, x , by 3.”

Support for Students with Disabilities

Representation: Internalize Comprehension. Use color and annotations to illustrate student thinking. As students describe their approaches, use color and annotations to scribe their thinking on a display.

Supports accessibility for: Visual-spatial processing; Conceptual processing

Support for English Language Learners

Conversing: MLR7 Compare and Connect. As students explain their different approaches to completing the tables, ask “What is similar, what is different?” about their methods. Draw students’ attention to the different ways the constant of proportionality was represented across the different strategies. For instance, when discussing first question, ask “Where do you see the 3 in each approach?” These exchanges strengthen students’ mathematical language use and reasoning based on constant of proportionality.

Design Principle(s): Cultivate conversation; Maximize meta-awareness

4.3 Denver to Chicago

: 10 minutes

This activity revisits a context seen previously. This time, students represent the proportional relationship between distance and time with an equation. Students once again make use of structure (MP7) and use repeated reasoning (MP8), but there is also a focus on moving back and forth between the abstract representation and the context (MP2).

As part of this activity, students calculate distance and speed. Students should know from grade 6 that speed is the quotient of distance traveled by amount of time elapsed, so they can divide 915 by 1.5 to get the speed. Students that do not begin the problem in that way can be directed back to the similar task in previous lessons to make connections and correct themselves. Once students have the speed, which is constant throughout this problem, they identify this as the constant of proportionality and use it to find the missing values.

Monitor for students who solve the problem in different ways.

Addressing

- 7.RP.A.2.c

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR5: Co-Craft Questions

Launch

Tell students that this activity revisits a context from an earlier lesson.

Support for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Provide students with access to calculators to ensure inclusive participation in the activity.

Supports accessibility for: Memory; Conceptual processing

Support for English Language Learners

Writing: MLR5 Co-Craft Questions. To help students consider the relationships between distance, time, and speed within this context, present to students the initial prompt and map image of the first question without the table. Ask students to write down possible mathematical questions that they can ask about this situation. Have pairs compare their questions and share out with the whole class. Listen for phrases such as “miles per hour,” “constant of proportionality,” “distance traveled,” “unit of time,” etc. This helps students produce the language of mathematical questions and talk about the relationships between the quantities in this task (e.g., distance, speed, and time) prior to being asked to analyze another’s reasoning.

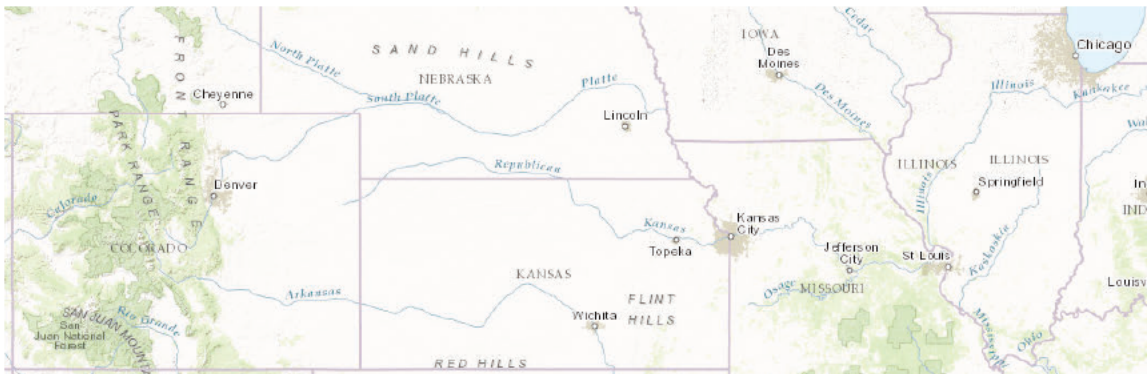
Design Principle(s): Maximize meta-awareness, Maximize output

Anticipated Misconceptions

Students who are having trouble understanding the task can draw a segment between Denver and Chicago and label it with the distance and the time. From there, they can draw a double number line diagram.

Student Task Statement

A plane flew at a constant speed between Denver and Chicago. It took the plane 1.5 hours to fly 915 miles.



1. Complete the table.

time (hours)	distance (miles)	speed (miles per hour)
1		
1.5	915	
2		
2.5		
t		

2. How far does the plane fly in one hour?
3. How far would the plane fly in t hours at this speed?
4. If d represents the distance that the plane flies at this speed for t hours, write an equation that relates t and d .
5. How far would the plane fly in 3 hours at this speed? in 3.5 hours? Explain or show your reasoning.

Student Response

1.

time (hours)	distance (miles)	speed (miles per hour)
1	610	610
1.5	915	610
2	1,220	610
2.5	1,525	610
t	$610t$	610

2. 610 miles since $915 \div 1.5 = 610$ for the speed and at a speed of 610 miles per hour, it would travel 610 miles after 1 hour.

3. $610t$ miles

4. Equation: $d = 610t$ or equivalent

5. 1,830 miles; 2,135 miles; I multiplied each number by 610.

Are You Ready for More?

A rocky planet orbits Proxima Centauri, a star that is about 1.3 parsecs from Earth. This planet is the closest planet outside of our solar system.

1. How long does it take light from Proxima Centauri to reach Earth? (A parsec is about 3.26 light years. A light year is the distance light travels in one year.)

- 
2. There are two twins. One twin leaves on a spaceship to explore the planet near Proxima Centauri traveling at 90% of the speed of light, while the other twin stays home on Earth. How much does the twin on Earth age while the other twin travels to Proxima Centauri? (Do you think the answer would be the same for the other twin? Consider researching “The Twin Paradox” to learn more.)

Student Response

1. $1.3 \cdot 3.26 \approx 4.24$ or about 4.24 years
2. $4.24 \div 0.9 \approx 4.7$ or about 4.7 years

Activity Synthesis

Select students to discuss and share their solutions. Ask them to identify difficulties which might include: getting started, noticing the pattern, dividing with decimals, completing the values in the table, creating the equation. This problem increases the level of difficulty by having so much missing information, and by using decimals in the table. It is important to identify if there are parts that are confusing for students to move them forward.

As part of the discussion, write the equation for all to see, and ask students to describe in words how to interpret its meaning in the context of the situation (To find d , the distance traveled by the plane in miles, multiply the hours of travel, t , by the plane’s speed in miles per hour, 610.).

4.4 Revisiting Bread Dough

Optional: : 10 minutes

This activity gives students more practice writing an equation that represents the proportional relationship examined in a previous lesson: the amount of flour and honey in a recipe. Students can then use their equation to answer additional questions about the situation.

Addressing

- 7.RP.A.2.c

Instructional Routines

- MLR6: Three Reads
- Think Pair Share

Launch

Tell students that this activity revisits a context from an earlier lesson. Give students quiet work time followed by partner discussion.

Support for English Language Learners

Reading: MLR6 Three Reads. Use this routine to support reading comprehension of the baking situation, without solving it for students. In the first read, students read the prompt with the goal of comprehending the situation (e.g., there is always the same ratio of honey to flour; different days have different numbers of batches). Delay asking students to complete the table. In the second read, ask students to analyze the prompts to understand the mathematical structure by naming quantities. Listen for, and amplify, the two important quantities that vary in relation to each other in this situation: f is the number of cups of flour; h is the number of tablespoons of honey. After the third read, ask students to brainstorm possible strategies to determine the amount of flour needed if you know how much honey will be used.

Design Principle(s): Support sense-making

Student Task Statement

A bakery uses 8 tablespoons of honey for every 10 cups of flour to make bread dough. Some days they bake bigger batches and some days they bake smaller batches, but they always use the same ratio of honey to flour.

1. Complete the table.

2. If f is the cups of flour needed for h tablespoons of honey, write an equation that relates f and h .

3. How much flour is needed for 15 tablespoons of honey? 17 tablespoons? Explain or show your reasoning.

honey (tbsp)	flour (c)
1	
8	10
16	
20	
h	

Student Response

1.

honey (tbsp)	flour (c)
1	$\frac{5}{4}$
8	10
16	20
20	25
h	$\frac{5}{4}h$

2. $f = \frac{5}{4}h$ or equivalent.

3. $18\frac{3}{4}$ cups. $21\frac{1}{4}$ cups. I multiplied each number by $\frac{5}{4}$.

Activity Synthesis

Ask students to compare answers with their partner and discuss their reasoning until they reach an agreement.

Then, invite students to share how they used their equation from question 2 to answer question 3 with the whole class.

Lesson Synthesis

Briefly revisit the three activities, demonstrating the use of new and old terms. For example:

- We examined a proportional relationship between cups of rice and people served. What was the *constant of proportionality* in this task? What did the constant of proportionality represent? What *equation* did we write for this situation?
- We examined a proportional relationship where we knew that a plane was flying at a constant speed. What was the constant of proportionality for this relationship? What does the constant of proportionality represent in terms of the context? What equation did we determine would represent this situation?

As a way to help students synthesize their learning, consider asking them to work with a partner and create a mind map of the features they have noticed these proportional relationships have in common. You might collect these to check for understanding. A class version can be created to be referenced, revised, or augmented during the unit.

4.5 It's Snowing in Syracuse

Cool Down : 5 minutes

Addressing

- 7.RP.A.2

Student Task Statement

Snow is falling steadily in Syracuse, New York. After 2 hours, 4 inches of snow has fallen.

1. If it continues to snow at the same rate, how many inches of snow would you expect after 6.5 hours? If you get stuck, you can use the table to help.

time (hours)	snow (inches)
	1
1	
2	4
6.5	
x	

2. Write an equation that gives the amount of snow that has fallen after x hours at this rate.

3. How many inches of snow will fall in 24 hours if it continues to snow at this rate?

Student Response

- 13 inches. Two inches fell in 1 hour, and 6.5 is $1 \cdot (6.5)$, and $2 \cdot (6.5) = 13$.
- $y = 2x$ where x is the number of hours that have passed and y is the depth of the accumulated snow.
- 48 inches. $24 \cdot 2 = 48$.

Student Lesson Summary

The table shows the amount of red paint and blue paint needed to make a certain shade of purple paint, called Venusian Sunset.

Note that “parts” can be *any* unit for volume. If we mix 3 cups of red with 12 cups of blue, you will get the same shade as if we mix 3 teaspoons of red with 12 teaspoons of blue.

red paint (parts)	blue paint (parts)
3	12
1	4
7	28
$\frac{1}{4}$	1
r	$4r$

The last row in the table says that if we know the amount of red paint needed, r , we can always multiply it by 4 to find the amount of blue paint needed, b , to mix with it to make Venusian Sunset. We can say this more succinctly with the equation $b = 4r$. So the amount of blue paint is proportional to the amount of red paint and the constant of proportionality is 4.

We can also look at this relationship the other way around.

If we know the amount of blue paint needed, b , we can always multiply it by $\frac{1}{4}$ to find the amount of red paint needed, r , to mix with it to make Venusian Sunset. So $r = \frac{1}{4}b$. The amount of blue paint is proportional to the amount of red paint and the constant of proportionality $\frac{1}{4}$.

blue paint (parts)	red paint (parts)
12	3
4	1
28	7
1	$\frac{1}{4}$
b	$\frac{1}{4}b$

In general, when y is proportional to x , we can always multiply x by the same number k —the constant of proportionality—to get y . We can write this much more succinctly with the equation $y = kx$.

Lesson 4 Practice Problems

1. Problem 1

Statement

A certain ceiling is made up of tiles. Every square meter of ceiling requires 10.75 tiles. Fill in the table with the missing values.

square meters of ceiling	number of tiles
1	
10	
	100
a	

Solution

square meters of ceiling	number of tiles
1	10.75
10	107.5
9.3	100
a	$10.75 \cdot a$

2. Problem 2

Statement

On a flight from New York to London, an airplane travels at a constant speed. An equation relating the distance traveled in miles, d , to the number of hours flying, t , is $t = \frac{1}{500}d$. How long will it take the airplane to travel 800 miles?

Solution

1.6 hours since $\frac{1}{500} \cdot 800 = 1.6$

3. Problem 3

Statement

Each table represents a proportional relationship. For each, find the constant of proportionality, and write an equation that represents the relationship.

s	P
2	8
3	12
5	20
10	40

Constant of proportionality:

Equation: $P =$

d	C
2	6.28
3	9.42
5	15.7
10	31.4

Constant of proportionality:

Equation: $C =$

Solution

a. Constant of proportionality: 4. Equation: $P = 4s$

b. Constant of proportionality: 3.14 Equation: $C = 3.14d$

4. Problem 4

Statement

A map of Colorado says that the scale is 1 inch to 20 miles or 1 to 1,267,200. Are these two ways of reporting the scale the same? Explain your reasoning.

Solution

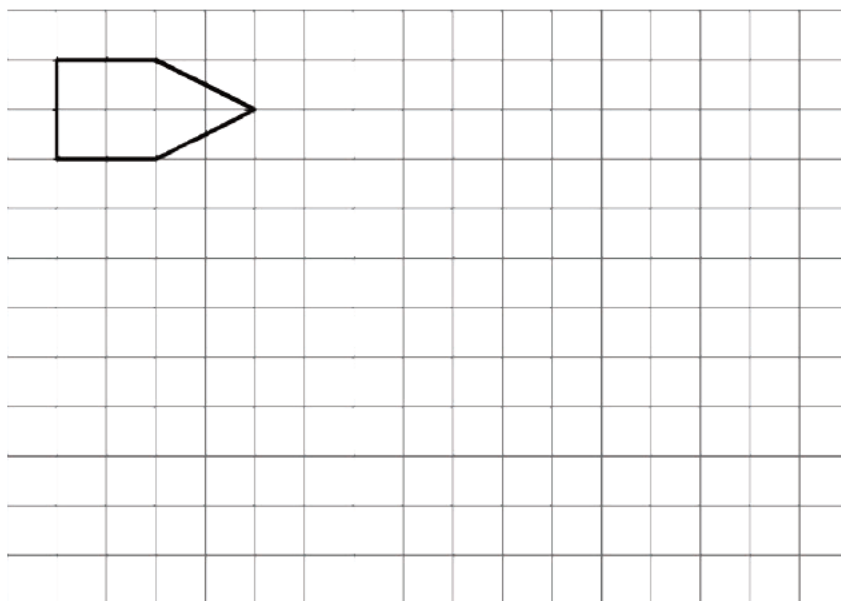
Yes. Sample response: There are 12 inches in a foot and 5280 feet in 1 mile, so that's 63,360 inches in a mile and 1,267,200 inches in 20 miles.

(From Unit 1, Lesson 11.)

5. Problem 5

Statement

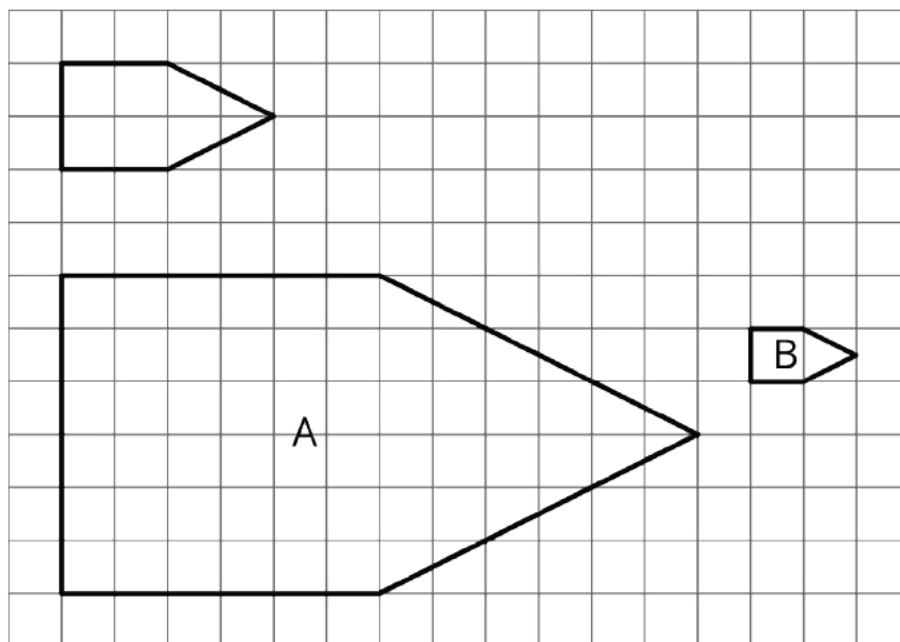
Here is a polygon on a grid.



- Draw a scaled copy of the polygon using a scale factor 3. Label the copy A.
- Draw a scaled copy of the polygon with a scale factor $\frac{1}{2}$. Label it B.
- Is Polygon A a scaled copy of Polygon B? If so, what is the scale factor that takes B to A?

Solution

a.



b. Yes, A is a scaled copy of B with a scale factor of 6.

(From Unit 1, Lesson 3.)

Lesson 5: Two Equations for Each Relationship

Goals

- Use the word “reciprocal” to explain (orally and in writing) that there are two related constants of proportionality for a proportional relationship.
- Write two equations that represent the same proportional relationship, i.e., $y = kx$ and $x = (\frac{1}{k})y$, and explain (orally) what each equation means.

Learning Targets

- I can find two constants of proportionality for a proportional relationship.
- I can write two equations representing a proportional relationship described by a table or story.

Lesson Narrative

In previous lessons students saw that a proportional relationships can be viewed in two ways, depending on which quantity you regard as being proportional to the other. In this lesson they write equations for these two ways, and they see why the two constants of proportionality associated with each way are reciprocals of each other.

The activities in this lesson use familiar contexts, but not identical situations from previous lessons: measurement conversions and water flowing at a constant rate. Students are expected to use methods developed earlier: organize data in a table, write and solve an equation to determine the constant of proportionality, and generalize from repeated calculations to arrive at an equation (MP8). After students write or use an equation, they interpret their answers in the context of the situation (MP2).

Alignments

Building On

- 5.OA.B: Analyze patterns and relationships.

Addressing

- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.
- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

- 7.RP.A.2.b: Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- 7.RP.A.2.c: Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR1: Stronger and Clearer Each Time
- MLR5: Co-Craft Questions
- MLR8: Discussion Supports
- Think Pair Share

Student Learning Goals

Let's investigate the equations that represent proportional relationships.

5.1 Missing Figures

Warm Up: : 5 minutes

This warm-up encourages students to look for regularity in how the tiles in the image are growing and describe this pattern (MP8) using ratios as a review of their work in grade 6. Students may use each color to reason about missing figures while others may reason about the way the tiles are arranged.

Building On

- 5.OA.B

Launch

Arrange students in groups of 2. Display the image for all to see and tell students that the collection of tiles is growing, but we can only see the second and fourth images. Ask students to look for a pattern in the sequence of figures and to give a signal when they have thought of one. Give students 1 minute of quiet think time, and then time to discuss their patterns and arrangements for Figures 1 and 3 with a partner. Tell students to use what they discussed in figuring out their answers to the next questions.

Student Task Statement

Here are the second and fourth figures in a pattern.

?



?



figure 1

figure 2

figure 3

figure 4

1. What do you think the first and third figures in the pattern look like?

2. Describe the 10th figure in the pattern.

Student Response

1. Figure 1 has 2 blue tiles and 3 yellow tiles and Figure 3 has 6 blue tiles and 9 yellow tiles.
2. Figure 10 would have 20 blue tiles and 30 yellow tiles.

Activity Synthesis

Invite students to share their responses and reasoning. Record and display the different ways of thinking for all to see. If possible, record the relevant reasoning on or near the images themselves. After each explanation, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to what is happening in the images each time.

Students may use each color to reason about missing figures while others may reason about the way the tiles are arranged. Emphasize both insights in addition to the usefulness of Figure 1 as students share their strategies.

5.2 Meters and Centimeters

: 10 minutes

This activity is intended to build confidence and facility in writing an equation for a proportional relationship. Students build on their understanding that measurement conversions can be represented by proportional relationships, which they studied in an earlier lesson and will revisit in future lessons. Students are expected to use methods developed earlier: organize data in a table, write and solve an equation to determine the constant of proportionality, and use repeated reasoning (MP8) to arrive at an equation. In addition, they are expected to identify the relationship between the constant of proportionality when going from centimeters to meters and vice versa (reciprocals). Since students have already explored a similar relationship (centimeters and millimeters) in a previous lesson, this activity may go very quickly.

As students work, look for students writing an equation like $100k = 1$ (for table 2) as a step to finding the constant of proportionality. Encourage them to say how they would solve the equation. Ask students to say why using this equation makes sense in the scenario.

Addressing

- 7.RP.A.2.b
- 7.RP.A.2.c

Instructional Routines

- MLR8: Discussion Supports

Launch

Introduce the task: “In a previous lesson, you examined the relationship between millimeters and centimeters. Today we will examine the relationship between centimeters and meters.”

Student Task Statement

There are 100 centimeters (cm) in every meter (m).

length (m)	length (cm)
1	100
0.94	
1.67	
57.24	
x	

length (cm)	length (m)
100	1
250	
78.2	
123.9	
y	

1. Complete each of the tables.
2. For each table, find the constant of proportionality.
3. What is the relationship between these constants of proportionality?
4. For each table, write an equation for the proportional relationship. Let x represent a length measured in meters and y represent the same length measured in centimeters.

Student Response

1. Tables:

length (m)	length (cm)
1	100
0.94	94
1.67	167
57.24	5,724
x	$100x$

length (cm)	length (m)
100	1
250	2.5
78.2	0.782
123.9	1.239
y	$0.01y$

2. The constant of proportionality for the first table is 100, and the second is 0.01 or $\frac{1}{100}$.

3. The constants of proportionality are reciprocals.

4. $y = 100x$ and $x = 0.01y$.

Are You Ready for More?

1. How many cubic centimeters are there in a cubic meter?
2. How do you convert cubic centimeters to cubic meters?



3. How do you convert the other way?

Student Response

1. 1,000,000
2. multiply by $\frac{1}{1,000,000}$
3. multiply by 1,000,000

Activity Synthesis

Invite students to share their answers. These questions can guide the discussion:

- "How can we find an equation for each table?"
- "Where does the constant of proportionality occur in the table and equation?"
- "What is the relationship between the two constants of proportionality? How can you use the equations to see why this should be true?"

The equations can help students see why the constants of proportionality are reciprocals:

$$\begin{aligned}y &= 100x \\ \left(\frac{1}{100}\right)y &= \frac{1}{100} \cdot (100x) \\ \left(\frac{1}{100}\right)y &= x \\ x &= \left(\frac{1}{100}\right)y\end{aligned}$$

This line of reasoning illustrated above should be accessible to students, because it builds on grade 6 work with expressions and equations.

Ask students to interpret the meaning of the equations in the context: "What do the equations tell us about the conversion from meters to centimeters and back?"

- Given the length in meters, to find the length in centimeters, multiply the number of centimeters by 100.
- Given the length in centimeters, to find the the length in meters, multiply the number of meters by $\frac{1}{100}$.

Support for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation such as: “First, I ____ because....,” “I noticed ____ so I....,” “Why did you . . .?,” and “I agree/disagree because....”

Supports accessibility for: Language; Social-emotional skills

Support for English Language Learners

Speaking: MLR8 Discussion supports. Provide students with sentence frames such as “I found the constant of proportionality by ____” to help explain their reasoning. Revoice student ideas to model mathematical language use by restating a response as a question in order to clarify, and apply appropriate language.

Design Principle(s): Maximize linguistic and cognitive awareness

5.3 Filling a Water Cooler

: 15 minutes

The theme continues by asking students to make sense of the two rates associated with a given proportional relationship. Here, students are asked to reason from an equation rather than a table, although they may find it helpful to create a table or graph (MP5). In this particular example, students work with both number of gallons per minute and number of minutes per gallon.

Monitor for students who are using different ways to decide if the cooler was filling faster before or after the flow rate was changed.

Addressing

- 7.RP.A.2.c

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR1: Stronger and Clearer Each Time

- Think Pair Share

Launch

Give students 5 minutes quiet work time followed by sharing with a partner and a whole-class discussion.

Support for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support development of organizational skills, check in with students within the first 2–3 minutes of work time. Check to make sure students have identified the correct equations that represent the relationship between w and t and can justify their reasoning.

Supports accessibility for: Memory; Organization

Support for English Language Learners

Writing, conversing: MLR1 Stronger and Clearer Each Time. Use this routine to help students improve their written mathematical argument for whether the cooler is filling faster before or after Priya changed the rate of water flow. Give students time to meet with 2–3 partners to share and get feedback on their initial drafts. Display feedback prompts that will help students strengthen their ideas and clarify their language. For example, “Can you explain how . . .?”, “Is there another way to say . . .?”, and “How do you know . . . ?”. Invite students to go back and revise or refine their written argument based on the feedback from peers. This will help students understand situations in which two different rates are associated with the same proportional relationship through communicating their reasoning with a partner.

Design Principle(s): Optimize output (for justification); Cultivate conversation

Anticipated Misconceptions

For the first question, if students struggle to identify the correct equations, encourage them to create two tables of values for the situation. Encourage them to create rows for both unit rates, in order to foster connections to prior learning.

Student Task Statement

It took Priya 5 minutes to fill a cooler with 8 gallons of water from a faucet that was flowing at a steady rate. Let w be the number of gallons of water in the cooler after t minutes.

1. Which of the following equations represent the relationship between w and t ? Select all that apply.
 - a. $w = 1.6t$
 - b. $w = 0.625t$
 - c. $t = 1.6w$
 - d. $t = 0.625w$
2. What does 1.6 tell you about the situation?
3. What does 0.625 tell you about the situation?
4. Priya changed the rate at which water flowed through the faucet. Write an equation that represents the relationship of w and t when it takes 3 minutes to fill the cooler with 1 gallon of water.
5. Was the cooler filling faster before or after Priya changed the rate of water flow? Explain how you know.

Student Response

1. Both $w = 1.6t$ and $t = 0.625w$ represent the relationship of w and t .
2. 1.6 tells you that water is flowing at 1.6 gallons per minute.
3. 0.625 tells you that it takes 0.625 minutes for a gallon of water to flow out of the faucet (or into the cooler).
4. $t = 3w$ or $w = \frac{1}{3}t$
5. The cooler filled faster before Priya changed the rate of water flow. Before the change, it took 0.625 minutes to get one gallon, but after, it took 3 minutes to get one gallon. (Alternatively, before the change she got 1.6 gallons per minute, but after the change she only got $\frac{1}{3}$ of a gallon per minute.)

Activity Synthesis

Select students to share their answers. Elicit both responses from the class, and be sure to identify connections between them.

Select students who used different explanations to share their answers to the last question.

5.4 Feeding Shrimp

Optional: : 10 minutes

This activity provides an additional opportunity for students to represent a proportional relationship with two related equations in a new context. This situation builds on the earlier work they did with feeding a crowd, but includes more complicated calculations. Students interpret the meaning of the constants of proportionality in the context of the situation and use the equations to answer questions.

The first few questions ask about 1 shrimp. The question about feeding 10 shrimp helps prepare students for work they will do in the next lesson with multiple quantities that are in proportional relationships to each other.

Addressing

- 7.RP.A.2

Instructional Routines

- MLR5: Co-Craft Questions

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 6 minutes of partner work time followed by whole-class discussion.

Support for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.

Supports accessibility for: Organization; Attention

Support for English Language Learners

Speaking, Reading: MLR5 Co-Craft Questions. Use this routine to help students interpret the language of proportional relationships, and to increase awareness of language used to talk about proportional relationships. Display only the first sentence of this problem ("At an aquarium, a shrimp is fed $\frac{1}{5}$ gram of food each feeding and is fed 3 times each day."), and ask students to write down possible mathematical questions that could be asked about the situation. Invite students to compare their questions before revealing the remainder of the question. Listen for and amplify any questions involving relationships between the two quantities in this task (amount of food and number of feedings).

Design Principle(s): Maximize meta-awareness; Support sense-making

Student Task Statement

At an aquarium, a shrimp is fed $\frac{1}{5}$ gram of food each feeding and is fed 3 times each day.

1. How much food does a shrimp get fed in one day?

2. Complete the table to show how many grams of food the shrimp is fed over different numbers of days.

number of days	food in grams
1	
7	
30	



3. What is the constant of proportionality? What does it tell us about the situation?
4. If we switched the columns in the table, what would be the constant of proportionality? Explain your reasoning.
5. Use d for number of days and f for amount of food in grams that a shrimp eats to write *two* equations that represent the relationship between d and f .
6. If a tank has 10 shrimp in it, how much food is added to the tank each day?
7. If the aquarium manager has 300 grams of shrimp food for this tank of 10 shrimp, how many days will it last? Explain or show your reasoning.

Student Response

1. $\frac{3}{5}$ grams

2.

number of days	food in grams
1	$\frac{3}{5}$
7	$4\frac{1}{5}$
30	18

3. $\frac{3}{5}$

4. $\frac{5}{3}$ since the reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$.

5. $f = \frac{3}{5}d$ and $d = \frac{5}{3}f$

6. 6 grams

7. 50 days

Activity Synthesis

Invite students to share their answers. Ask students which equation was most useful to answer each of the last two questions.

Lesson Synthesis

Briefly revisit the first two activities to summarize the important points of the lesson. For example:

- We examined the proportional relationship between meters and centimeters. Why were we able to write two equations for this situation? What were they? What were the constants of proportionality?
- We examined a proportional relationship where we knew how long it took to fill a water cooler with a certain amount of water. What were the constants of proportionality for this relationship? What equations did we determine would represent this situation?

- In each case, what was the relationship between the two constants of proportionality and between the two equations?

5.5 Flight of the Albatross

Cool Down : 5 minutes

Addressing

- 7.RP.A

Student Task Statement

An albatross is a large bird that can fly 400 kilometers in 8 hours at a constant speed. Using d for distance in kilometers and t for number of hours, an equation that represents this situation is $d = 50t$.

1. What are two constants of proportionality for the relationship between distance in kilometers and number of hours? What is the relationship between these two values?

2. Write another equation that relates d and t in this context.

Student Response

1. 50 and $\frac{1}{50}$. These numbers are reciprocals.
2. $t = \frac{1}{50}d$

Student Lesson Summary

If Kiran rode his bike at a constant 10 miles per hour, his distance in miles, d , is proportional to the number of hours, t , that he rode. We can write the equation

$$d = 10t$$

With this equation, it is easy to find the distance Kiran rode when we know how long it took because we can just multiply the time by 10.

We can rewrite the equation:

$$\begin{aligned} d &= 10t \\ \left(\frac{1}{10}\right)d &= t \\ t &= \left(\frac{1}{10}\right)d \end{aligned}$$

This version of the equation tells us that the amount of time he rode is proportional to the distance he traveled, and the constant of proportionality is $\frac{1}{10}$. That form is easier to use when we know his distance and want to find how long it took because we can just multiply the distance by $\frac{1}{10}$.

When two quantities x and y are in a proportional relationship, we can write the equation

$$y = kx$$

and say, “ y is proportional to x .” In this case, the number k is the corresponding constant of proportionality. We can also write the equation

$$x = \frac{1}{k}y$$

and say, “ x is proportional to y .” In this case, the number $\frac{1}{k}$ is the corresponding constant of proportionality. Each one can be useful depending on the information we have and the quantity we are trying to figure out.

Lesson 5 Practice Problems

1. Problem 1

Statement

The table represents the relationship between a length measured in meters and the same length measured in kilometers.

- Complete the table.
- Write an equation for converting the number of meters to kilometers. Use x for number of meters and y for number of kilometers.

meters	kilometers
1,000	1
3,500	
500	
75	
1	
x	

Solution

a.

meters	kilometers
1,000	1
3,500	3.5
500	0.5
75	0.075
1	0.001
x	$0.001x$

b. $y = 0.001x$ or equivalent

2. Problem 2

Statement

Concrete building blocks weigh 28 pounds each. Using b for the number of concrete blocks and w for the weight, write two equations that relate the two variables. One equation should begin with $w =$ and the other should begin with $b =$.

Solution

$$w = 28b \text{ and } b = \frac{1}{28}w$$

3. Problem 3

Statement

A store sells rope by the meter. The equation $p = 0.8L$ represents the price p (in dollars) of a piece of nylon rope that is L meters long.

- How much does the nylon rope cost per meter?
- How long is a piece of nylon rope that costs \$1.00?

Solution

a. \$0.80 or dollar or dollar.

b. 1.25 meters or $\frac{5}{4}$ meters or $1\frac{1}{4}$ meters.

4. Problem 4

Statement

The table represents a proportional relationship. Find the constant of proportionality and write an equation to represent the relationship.

a	y
2	$\frac{2}{3}$
3	1
10	$\frac{10}{3}$
12	4

Constant of proportionality: _____

Equation: $y =$

Solution

Constant of proportionality: $\frac{1}{3}$ Equation: $y = \frac{1}{3}a$

(From Unit 2, Lesson 4.)

5. Problem 5

Statement

On a map of Chicago, 1 cm represents 100 m. Select **all** statements that express the same scale.

- A. 5 cm on the map represents 50 m in Chicago.
- B. 1 mm on the map represents 10 m in Chicago.
- C. 1 km in Chicago is represented by 10 cm the map.
- D. 100 cm in Chicago is represented by 1 m on the map.

Solution

["B", "C"]

(From Unit 1, Lesson 8.)

Lesson 6: Using Equations to Solve Problems

Goals

- Generate an equation for a proportional relationship, given a description of the situation but no table.
- Interpret (orally) each part of an equation that represents a proportional relationship in an unfamiliar context.
- Use an equation to solve problems involving a proportional relationship, and explain (orally) the reasoning.

Learning Targets

- I can find missing information in a proportional relationship using the constant of proportionality.
- I can relate all parts of an equation like $y = kx$ to the situation it represents.

Lesson Narrative

In the previous two lessons students learned to represent proportional relationships with equations of the form $y = kx$. In this lesson they continue to write equations, and they begin to see situations where using the equation is a more efficient way of solving problems than other methods they have been using, such as tables and equivalent ratios.

The activities introduce new contexts and, for the first time, do not provide tables; students who still need tables should be given a chance to realize that and create tables for themselves. The activities are intended to motivate the usefulness of representing proportional relationships with equations, while at the same time providing some scaffolding for finding the equations.

As students use the abstract equation $y = kx$ to reason about quantitative situations, they engage in MP2.

Alignments

Building On

- 5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

- 6.RP.A.2: Understand the concept of a unit rate a/b associated with a ratio $a : b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $3/4$ cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.” Expectations for unit rates in this grade are limited to non-complex fractions.

Addressing

- 7.RP.A.2: Recognize and represent proportional relationships between quantities.
- 7.RP.A.2.c: Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR6: Three Reads
- MLR8: Discussion Supports
- Number Talk
- Think Pair Share

Required Materials

Four-function calculators

Student Learning Goals

Let’s use equations to solve problems involving proportional relationships.

6.1 Number Talk: Quotients with Decimal Points

Warm Up: : 5 minutes

The purpose of this Number Talk is to elicit strategies and understandings students have for determining how the size of a quotient changes when the decimal point in the divisor or dividend moves. These understandings help students develop fluency and will be

helpful later in this lesson when students will need to be able to check the reasonableness of their answers. While four problems are given in the first problem, it may not be possible to share answers for all of them.

Building On

- 5.NBT.B.7

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Arrange students in groups of 2. Display the first question. Give students 2 minutes of quiet think time and ask them to give a signal when they have an answer, and reasoning, to support their answer. Follow with a whole-class discussion. Display the second question and give students 1 minute of quiet think time.

Support for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Student Task Statement

Without calculating, order the quotients of these expressions from least to greatest.

$$42.6 \div 0.07$$

$$42.6 \div 70$$

$$42.6 \div 0.7$$

$$426 \div 70$$

Place the decimal point in the appropriate location in the quotient:

$$42.6 \div 7 = 608571$$

Use this answer to find the quotient of *one* of the previous expressions.

Student Response

- b, d, c, a
- - 6.08571
 - Answers vary. Possible responses in order a through d: 608.571; 0.608571; 60.8571; 6.08571

Activity Synthesis

Ask students to share where they placed the decimal point in the second question and their reasoning. After students share, ask the class if they agree or disagree. Ask selected students, who chose different problems to solve, to share quotients for the problems in the first question. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Did anyone have the same strategy but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to ____’s strategy?”
- “Do you agree or disagree? Why?”

Emphasize student reasoning based in place value that involve looking at the relationship between the dividend and divisor to determine the size of the quotient.

Support for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, “First, I ____ because . . .” or “I noticed ____ so I . . .” Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Design Principle(s): Optimize output (for explanation)

6.2 Concert Ticket Sales

: 15 minutes

This activity requires students to work with larger numbers, which is intended to encourage students to use an equation and notice the efficiencies of doing so. It also emphasizes the interpretation of the constant of proportionality in the context. In this case, the constant represents the cost of a single ticket, and makes it easy to identify which singer would make more money for similar ticket sales in a concert series. Note that asking students to give the revenues for different ticket sales encourages looking for and expressing regularity in repeated reasoning (MP8). The last set of questions ask students to interpret the constant of proportionality as represented in an equation in terms of the context (MP2).

Monitor for students who solve the problems using the following strategies and invite them to share during the whole-class discussion.

- writing many calculations, without any organization
- creating a table to organize their results
- writing an equation to encapsulate repeated reasoning

Addressing

- 7.RP.A.2

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports

Launch

Provide access to calculators. Consider using the names of actual performers to make the task more interesting to students.

Support for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Student Task Statement

A performer expects to sell 5,000 tickets for an upcoming concert. They want to make a total of \$311,000 in sales from these tickets.

1. Assuming that all tickets have the same price, what is the price for one ticket?
2. How much will they make if they sell 7,000 tickets?
3. How much will they make if they sell 10,000 tickets? 50,000? 120,000? a million? x tickets?
4. If they make \$404,300, how many tickets have they sold?
5. How many tickets will they have to sell to make \$5,000,000?

Student Response

1. \$62.20
2. \$435,400
3. \$622,000; \$3,110,000; \$7,464,000; \$62,200,000; $62.2x$
4. 6,500
5. 80,386

number of tickets sold	earnings in dollars
5,000	311,000
1	62.20
7,000	435,400
10,000	622,200
50,000	3,110,000
120,000	7,464,000
1,000,000	62,200,000
6,500	404,300
80,386	5,000,009.20
x	$62.20x$

Activity Synthesis

Select student responses to be shared with the whole class in discussion. Sequence their explanations from less efficient and organized to more efficient and organized. Discuss how the solutions are the same and different, and the advantages and disadvantages of each method. An important part of this discussion is correspondences and connections between different approaches.

Support for English Language Learners

Speaking, Listening: MLR8 Discussion Supports. Use this routine to support whole-class discussion. For each explanation that is shared, ask students to restate what they heard using precise mathematical language. Consider providing students time to restate what they heard to a partner before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. This provides more students with an opportunity to produce language as they interpret the reasoning of others.

Design Principle(s): Support sense-making

6.3 Recycling

: 15 minutes

This activity is intended to further develop students' ability to write equations to represent proportional relationships. It involves work with decimals and asks for equations that represent proportional relationships of different pairs of quantities, which increases the challenge of the task.

Students may solve the first two problems in different ways. Monitor for different solution approaches such as: using computations, using tables, finding the constant of proportionality, and writing equations.

Addressing

- 7.RP.A.2

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR6: Three Reads
- Think Pair Share

Launch

Arrange students in groups of 2. Provide access to calculators. Give 5 minutes quiet work time followed by sharing work with a partner.

Support for Students with Disabilities

Representation: Internalize Comprehension. Represent the same information through different modalities by using tables. If students are unsure where to begin, suggest that they draw a table to help organize the information provided.

Supports accessibility for: Conceptual processing; Visual-spatial processing

Support for English Language Learners

Reading: MLR6 Three Reads. Use this routine to support reading comprehension, without solving, for students. In the first read, students read the problem with the goal of comprehending the situation (e.g., The situation involves weight of cans and the amount of money made from recycling). In the second read, ask students to look for quantities without focusing on specific values. Listen for, and amplify, the quantities that vary in relation to each other in this situation: number of aluminum cans; total weight of aluminum cans, in kilograms; money earned, in dollars. In the third read, ask students to brainstorm possible strategies to calculate the weight of aluminum in one can and the amount of money earned from one can. This helps students connect the language in the word problem and the reasoning needed to solve the problem keeping the intended level of cognitive demand in the task.

Design Principle(s): Support sense-making

Anticipated Misconceptions

If students have trouble getting started, encourage them to create representations of the relationships, like a diagram or a table. If they are still stuck, suggest that they first find the weight and dollar value of 1 can.

Student Task Statement

Aluminum cans can be recycled instead of being thrown in the garbage. The weight of 10 aluminum cans is 0.16 kilograms. The aluminum in 10 cans that are recycled has a value of \$0.14.

1. If a family threw away 2.4 kg of aluminum in a month, how many cans did they throw away? Explain or show your reasoning.
2. What would be the recycled value of those same cans? Explain or show your reasoning.
3. Write an equation to represent the number of cans c given their weight w .
4. Write an equation to represent the recycled value r of c cans.
5. Write an equation to represent the recycled value r of w kilograms of aluminum.

Student Response

1. 150, because 2.4 is $(0.16) \cdot 15$, and $10 \cdot 15 = 150$.
2. \$2.10, because $(0.14) \cdot 15 = 2.1$.
3. $c = 62.5w$
4. $r = 0.014c$
5. $r = 0.875w$

Here is one way to organize the given information and solutions in a table:

number of cans (c)	weight in kilograms (w)	recycled value in dollars (r)
10	0.16	0.14
150	2.4	2.10
1	0.016	0.014
62.5	1	
$62.5w$	w	
c		$0.014c$
	1	0.875
	w	$0.875w$

Are You Ready for More?

The EPA estimated that in 2013, the average amount of garbage produced in the United States was 4.4 pounds per person per day. At that rate, how long would it take your family to produce a ton of garbage? (A ton is 2,000 pounds.)

Student Response

Answers vary. Sample responses: A family of two would take about 32 and a half weeks. A family of three would take about 21 and a half weeks. A family of four would take about 15 and a half weeks.

Activity Synthesis

Select students to share their methods: using computations, using tables, finding the constant of proportionality, writing equations. If students did not use equations to solve the first two problems, ask them how they can use the equations they found later in the activity to answer the first two questions.

If time permits, highlight connections between the equations generated, illustrated by the sequence of equations below.

$$\begin{aligned}r &= 0.014c \\r &= 0.014(62.5w) \\r &= 0.875w\end{aligned}$$

Lesson Synthesis

The activities in this lesson removed some scaffolds used in previous lessons (e.g., presenting a table) and included features (e.g., large numbers) intended to motivate use of equations. Remind students that throughout this lesson, they considered problem situations and created organized ways to get answers. Whether the numbers in the problem are whole numbers, large numbers, or decimals, if there is a proportional relationship between two quantities, their relationship can be represented by an equation of the form $y = kx$. The situations provided demonstrate the efficiency of equations for certain types of problems. Finding how many tickets should be sold in order to earn \$5 million and finding the relationship between number of cans and weight and recycled value are more elegantly and efficiently handled by equations than by calculations or tables.

- What were some helpful ways we organized information?
- What were some equations we found in this lesson?
- In each equation, what did the letters represent? What did the number mean?
 $y = 62.2x$, $c = 62.5w$, $r = 0.014c$, $r = 0.875w$

6.4 Granola

Cool Down : 5 minutes

Building On

- 6.RP.A.2

Addressing

- 7.RP.A.2.c

Student Task Statement

Based on her recipe, Elena knows that 5 servings of granola have 1,750 calories.

1. If she eats 2 servings of granola, how many calories does she eat?
2. If she wants to eat 175 calories of granola, how many servings should she eat?
3. Write an equation to represent the relationship between the number of calories and the number of servings of granola.

Student Response

1. 700 calories. $1,750 \div 5 = 350$, and $350 \cdot 2 = 700$.
2. $\frac{1}{2}$, because $175 = 350 \cdot \frac{1}{2}$.
3. If c represents the number of calories in s servings, then the equation could be either $c = 350s$ or $s = \frac{1}{350}c$.

Student Lesson Summary

Remember that if there is a proportional relationship between two quantities, their relationship can be represented by an equation of the form $y = kx$. Sometimes writing an equation is the easiest way to solve a problem.

For example, we know that Denali, the highest mountain peak in North America, is 20,300 feet above sea level. How many miles is that? There are 5,280 feet in 1 mile. This relationship can be represented by the equation

$$f = 5,280m$$

where f represents a distance measured in feet and m represents the same distance measured miles. Since we know Denali is 20,310 feet above sea level, we can write

$$20,310 = 5,280m$$

So $m = \frac{20,310}{5,280}$, which is approximately 3.85 miles.

Lesson 6 Practice Problems

1. Problem 1

Statement

A car is traveling down a highway at a constant speed, described by the equation $d = 65t$, where d represents the distance, in miles, that the car travels at this speed in t hours.

- What does the 65 tell us in this situation?
- How many miles does the car travel in 1.5 hours?
- How long does it take the car to travel 26 miles at this speed?

Solution

- The car travels 65 miles in 1 hour. Or, the car is traveling 65 miles per hour. Or, 65 miles per hour is the constant of proportionality.
- The car travels 97.5 miles in 1.5 hours.
- It takes the car $\frac{2}{5}$ of an hour, or 0.4 hours, or 24 minutes to travel 26 miles.

2. Problem 2

Statement

Elena has some bottles of water that each holds 17 fluid ounces.

- Write an equation that relates the number of bottles of water (b) to the total volume of water (w) in fluid ounces.
- How much water is in 51 bottles?
- How many bottles does it take to hold 51 fluid ounces of water?

Solution

- $w = 17b$ or $b = \frac{1}{17}w$
- 867 fluid ounces, because $17 \cdot 51 = 867$
- 3 bottles, because $51 \div 17 = 3$

3. Problem 3

Statement

There are about 1.61 kilometers in 1 mile. Let x represent a distance measured in kilometers and y represent the same distance measured in miles. Write two equations that relate a distance measured in kilometers and the same distance measured in miles.

Solution

$$x = 1.61y \text{ and } y = \frac{1}{1.61}x \text{ or } y = 0.62x$$

(From Unit 2, Lesson 5.)

4. Problem 4

Statement

In Canadian coins, 16 quarters is equal in value to 2 toonies.

number of quarters	number of toonies
1	
16	2
20	
24	

- Complete the table.
- What does the value next to 1 mean in this situation?

Solution

a.

number of quarters	number of toonies
1	$\frac{1}{8}$
16	2
20	2.5
24	3

- $\frac{1}{8}$ means that one-eighth of a toonie is worth the same as 1 quarter.

(From Unit 2, Lesson 2.)

5. Problem 5

Statement

Each table represents a proportional relationship. For each table:

- Fill in the missing parts of the table.
- Draw a circle around the constant of proportionality.

x	y
2	10
	15
7	
1	

a	b
12	3
20	
	10
1	

m	n
5	3
10	
	18
1	

Solution

x	y
2	10
3	15
7	35
1	5

$\cdot 2$

a	b
12	3
20	5
40	10
1	$\frac{1}{4}$

$\cdot 3$

m	n
5	3
10	6
30	18
1	$\frac{3}{5}$

$\cdot 3$

(From Unit 2, Lesson 2.)

6. Problem 6

Statement

Describe some things you could notice in two polygons that would help you decide that they were not scaled copies.

Solution

If they were not the same shape (for example, if one was a triangle and one was a square), they could not be scaled copies. I could find an angle measure in one that was not an angle measure of the other. I could find that a different scale factor would have to be used on one part of the pair than on another.

(From Unit 1, Lesson 4.)

Section: Comparing Proportional and Nonproportional Relationships

Lesson 7: Comparing Relationships with Tables

Goals

- Calculate and compare the quotients of the values in each row of a given table.
- Generate a different recipe for lemonade and describe (orally) how it would taste in comparison to a given recipe.
- Justify (orally) whether the values in a given table could or could not represent a proportional relationship.

Learning Targets

- I can decide if a relationship represented by a table could be proportional and when it is definitely not proportional.

Lesson Narrative

In the next two lessons students compare proportional and non-proportional relationships. In this lesson, students examine tables and explain whether the relationships represented are proportional, not proportional, or possibly proportional. At this point in the unit, students should be comfortable using the terms “proportional relationship,” “is proportional to,” and “constant of proportionality.” By the end of the next lesson, students should understand that equations of the form $y = kx$ with $k > 0$ characterize proportional relationships.

As students look at data from a context and reason about whether it makes sense quantitatively for the data to represent a proportional relationship, they are engaging in making viable arguments (MP3).

Alignments

Building On

- 6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Addressing

- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

Building Towards

- 7.RP.A.1: Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{\frac{1}{2}}{\frac{1}{4}}$ miles per hour, equivalently 2 miles per hour.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR5: Co-Craft Questions
- Think Pair Share

Required Materials

Four-function calculators

Required Preparation

Calculators can optionally be made available to take the focus off computation.

Student Learning Goals

Let's explore how proportional relationships are different from other relationships.

7.1 Adjusting a Recipe

Warm Up: : 5 minutes

This activity encourages students to reason about equivalent ratios in a context. During the discussion, emphasize the use of ratios and proportions in determining the effect on the taste of the lemonade.

Building On

- 6.RP.A.3

Launch

Arrange students in groups of 2. Give students 2 minutes of quiet think time.

Optionally, instead of the abstract recipe description, you could bring in a clear glass, measuring implements, and the lemonade ingredients. Pour in the ingredients and introduce the task that way.

Student Task Statement

A lemonade recipe calls for the juice of 5 lemons, 2 cups of water, and 2 tablespoons of honey.

Invent four new versions of this lemonade recipe:

1. One that would make more lemonade but taste the same as the original recipe.
2. One that would make less lemonade but taste the same as the original recipe.
3. One that would have a stronger lemon taste than the original recipe.
4. One that would have a weaker lemon taste than the original recipe.

Student Response

The base recipe has a ratio of number of lemons to cups of water to tablespoons of honey of 5:2:2. Answers vary. Sample responses:

1. 10 lemons, 4 cups of water, 4 tablespoons honey
2. $2\frac{1}{2}$ lemons, 1 cup water, 1 tablespoon honey
3. 8 lemons, 2 cups water, 2 tablespoons honey
4. 2 lemons, 2 cups water, 2 tablespoons honey

Activity Synthesis

Invite students to share their versions of the recipe with the class and record them for all to see. After each explanation, ask the class if they agree or disagree and how the new lemonade would taste. After recording at least 3 responses for each, ask students to describe any patterns they notice how the recipe was adjusted. If students do not mention ratios in their descriptions, be sure to ask them how the ratios changed in their new recipe.

7.2 Visiting the State Park

: 15 minutes

This activity provides the first example in this unit of a relationship that is not proportional. The second question focuses students' attention on the unit rates. If the relationship were proportional then regardless of the number of people in a vehicle, the cost per person would be the same. The question about the bus is to show students that they can't just scale up from 10. Students who write an equation also see that it is not of the form $y = kx$. In a later lesson students will learn that only equations of this form represent proportional relationships.

Monitor for students who approached this problem using different representations.

Addressing

- 7.RP.A.2

Building Towards

- 7.RP.A.1

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

Launch

Keep students in the same groups of 2. Give 5 minutes of quiet work time, followed by 5 minutes of students discussing responses with a partner, followed by a whole-class discussion.

Support for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support development of organizational skills, check in with students within the first 2–3 minutes of work time. Check to make sure students have accounted for the cost of the vehicle in their calculations of the total entrance cost for 4 people and 10 people.

Supports accessibility for: Memory; Organization

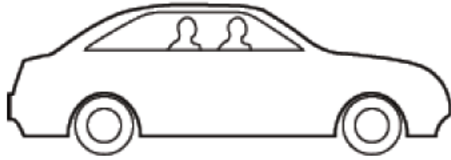
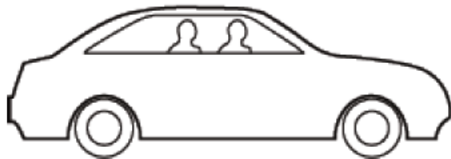
Anticipated Misconceptions

Some students may not account for the cost of the vehicle. They will get the following table with incorrect values and will need to be prompted to include the cost of the vehicle.

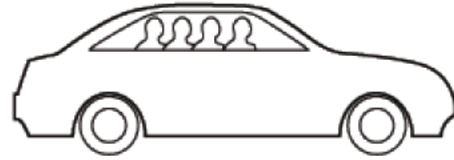
number of people in vehicle	total entrance cost in dollars
2	\$10
4	\$20
10	\$50

Teachers will want to circulate around the room keeping an eye out for this mistake and address it as soon as possible so that students spend most of their work time analyzing the non-proportional relationship. These diagrams may be helpful in illustrating to them that their resulting prices are including more than one vehicle. This gives them an opportunity to make sense of problems and persevere in solving them (MP1).

What you are paying for:



What you want to pay for:



Student Task Statement

Entrance to a state park costs \$6 per vehicle, plus \$2 per person in the vehicle.

1. How much would it cost for a car with 2 people to enter the park? 4 people? 10 people? Record your answers in the table.

number of people in vehicle	total entrance cost in dollars
2	
4	
10	

2. For each row in the table, if each person in the vehicle splits the entrance cost equally, how much will each person pay?

3. How might you determine the entrance cost for a bus with 50 people?

4. Is the relationship between the number of people and the total entrance cost a proportional relationship? Explain how you know.

Student Response

1.

number of people in vehicle	total entrance cost in dollars
2	\$10
4	\$14
10	\$26

2. With 2 people, \$5.00. With 4 people, \$3.50. With 10 people, \$2.60.
3. \$106. It still costs \$6 for the vehicle, plus \$2 for each of 50 people.
4. No. Explanations vary. Sample responses:
- Considering the ratio of people in the vehicle to total entrance cost, these are not equivalent ratios.
 - The cost per person is not the same for different numbers of people.
 - Each number of people and corresponding total entrance cost is not characterized by the same unit rate.

Are You Ready for More?

What equation could you use to find the total entrance cost for a vehicle with any number of people?

Student Response

Let p be number of people and c be total entrance cost in dollars. $c = 6 + 2p$.

Activity Synthesis

Select students to explain why they think the relationship is or is not proportional.

Reasons they may give:

- The cost per person is different for different number of people in a vehicle, i.e. the quotients of the entries in each row are not equal for all rows of the table.
- The ratio of people in the vehicle to total entrance cost are not equivalent ratios. You can't just multiply the entries in one row by the same constant to get the entries in another row.
- Each number of people and corresponding total entrance cost is not characterized by the same unit rate. You can't multiply the entries in the first column by the same number (constant of proportionality) to get the numbers in the second column.

Students who found an equation will also note that the equation is not of the same form as other equations, but they can't use this as a criterion until the class has established that only equations of this form represent proportional relationships. (This part of the discussion should come at the end of the next lesson, after students have analyzed lots of different equations.)

7.3 Running Laps

: 15 minutes

The purpose of this activity is to understand that discrete values in a table can be used as evidence that a relationship is proportional and can be used to know for sure that a relationship is not proportional, but can't be used to know for sure whether a relationship is definitely proportional. This activity builds on previous ones involving constant speed, but analyzes pace (minutes per lap) rather than speed (laps per minute). Explaining why the information given in the table is enough to conclude that Han didn't run at a constant pace but is not enough to know for sure whether Clare ran at a constant pace requires students to make a viable argument (MP3).

Addressing

- 7.RP.A.2

Building Towards

- 7.RP.A.1

Instructional Routines

- MLR5: Co-Craft Questions
- Think Pair Share

Launch

Keep students in the same groups of 2. Give 5 minutes of quiet work time, followed by 5 minutes of students discussing responses with a partner, followed by a whole-class discussion.

Support for English Language Learners

Conversing, Reading: MLR5 Co-Craft Questions. Use this routine to help students interpret the language of proportional relationships, and to increase awareness of language used to talk about proportional relationships. Display the first sentence of this problem ("Han and Clare were running laps around the track. The coach recorded their times at the end of laps 2, 4, 6, and 8.") and the tables of Han and Clare's run, and ask students to write down possible mathematical questions that could be asked about the situation. If needed, provide students with the question starter "Why is. . .?" Invite students to share their questions with the class before revealing the task questions. Listen for and amplify any questions involving proportional relationships or mathematical language, such as "constant pace," "not constant," "proportional," or "constant of proportionality." This will help students produce the language of mathematical questions and start noticing whether or not there is a proportional relationship in this task prior to starting the task.

Design Principle(s): Maximize meta-awareness; Support sense-making

Anticipated Misconceptions

Students are likely to answer that Clare is running at a constant pace because the minutes per lap shown in the table are the same for each lap. Because we only have four data points in a table, spaced at 5-minute intervals, Clare could still be speeding up and slowing down between the recorded times. However, given the data, it is reasonable to assume Clare is running at a constant pace for the purpose of estimating times or distances.

Student Task Statement

Han and Clare were running laps around the track. The coach recorded their times at the end of laps 2, 4, 6, and 8.

Han's run:

distance (laps)	time (minutes)	minutes per lap
2	4	
4	9	
6	15	
8	23	

Clare's run:

distance (laps)	time (minutes)	minutes per lap
2	5	
4	10	
6	15	
8	20	

1. Is Han running at a constant pace? Is Clare? How do you know?
2. Write an equation for the relationship between distance and time for anyone who is running at a constant pace.

Student Response

Here are the tables that have been completed correctly:

Han's run:

distance (laps)	time (minutes)	minutes per lap
2	4	2
4	9	2.25
6	15	2.5
8	23	2.875

Clare's run:

distance (laps)	time (minutes)	minutes per lap
2	5	2.5
4	10	2.5
6	15	2.5
8	20	2.5

1. Han is not running at a constant pace because the numbers in the third column are not the same; however, Clare may be. At the times recorded in the table, the minutes per lap are the same, but that does not guarantee that Clare's pace is constant between the recorded times. For example, she might stand still for half a minute, then complete a lap in 2 minutes.
2. The equation $t = 2.5d$ (where t represents time in minutes and d represents distance in laps) yields the entries in the table for Clare's times; this would be the equation if we knew that she were running at a constant pace.

Activity Synthesis

Invite students to explain why they think each person is or is not running at a constant pace. Point out to students that although the data points in the table for Clare are pairs in a proportional relationship, these four pairs of values do not guarantee that Clare ran at a

constant pace. She might have, but we don't know if she was running at a constant pace between the times that the coach recorded.

Ask the following questions:

- "Can you represent either relationship with an equation?" (The answer for Han is "no" and the answer for Clare is "yes, if she really ran at a constant pace between the points in time when the times were recorded." Write the equation for Clare together: $t = 2.5d$.)
- "Are the pairs of values in the table for Clare's run still values from a proportional relationship if we calculate laps per minute instead of minutes per lap? How does that change the equation?" (Yes, $d = 0.4t$.)

Lesson Synthesis

In this lesson, we learned some ways to tell whether a table could represent a proportional relationship. Revisit one or more of the activities in the lesson, highlighting the following points:

- If the quotient is the same for each row in the table, the table *could* represent a proportional relationship.
- It can be helpful to compute and write down this quotient for each row.
- The quotient is the constant of proportionality for the relationship (if the relationship is proportional).
- If all the quotients are not the same, the table definitely does not represent a proportional relationship.
- The relationship between the two quantities in a proportional relationship can be expressed using an equation of the form $y = kx$.

7.4 Apples and Pizza

Cool Down : 5 minutes

Addressing

- 7.RP.A.2

Building Towards

- 7.RP.A.1

Student Task Statement

1. Based on the information in the table, is the cost of the apples proportional to the weight of apples?

pounds of apples	cost of apples
2	\$3.76
3	\$5.64
4	\$7.52
5	\$9.40

2. Based on the information in the table, is the cost of the pizza proportional to the number of toppings?

number of toppings	cost of pizza
2	\$11.99
3	\$13.49
4	\$14.99
5	\$16.49

3. Write an equation for the proportional relationship.

Student Response

1. Possibly yes, the first table represents a proportional relationship because cost per pound of apples is the same in each row, 1.88 dollars per pound.

2. Definitely no, the second table does not represent a proportional relationship because cost per topping is not the same in each row. (An equation is $C = 1.50T + 8.99$ but students do not need to provide an equation.)
3. An equation relating the cost c to pounds of apples p is $c = 1.88p$.

Student Lesson Summary

Here are the prices for some smoothies at two different smoothie shops:

Smoothie Shop A

smoothie size (oz)	price (\$)	dollars per ounce
8	6	0.75
12	9	0.75
16	12	0.75
s	$0.75s$	0.75

Smoothie Shop B

smoothie size (oz)	price (\$)	dollars per ounce
8	6	0.75
12	8	0.67
16	10	0.625
s	???	???

For Smoothie Shop A, smoothies cost \$0.75 per ounce no matter which size we buy. There could be a proportional relationship between smoothie size and the price of the smoothie. An equation representing this relationship is

$$p = 0.75s$$

where s represents size in ounces and p represents price in dollars. (The relationship could still not be proportional, if there were a different size on the menu that did not have the same price per ounce.)

For Smoothie Shop B, the cost per ounce is different for each size. Here the relationship between smoothie size and price is definitely *not* proportional.

In general, two quantities in a proportional relationship will always have the same quotient. When we see some values for two related quantities in a table and we get the same quotient when we divide them, that means they might be in a proportional relationship—but if we can't see all of the possible pairs, we can't be



completely sure. However, if we know the relationship can be represented by an equation is of the form $y = kx$, then we are sure it is proportional.

Lesson 7 Practice Problems

1. Problem 1

Statement

Decide whether each table could represent a proportional relationship. If the relationship could be proportional, what would the constant of proportionality be?

- a. How loud a sound is depending on how far away you are.

distance to listener (ft)	sound level (dB)
5	85
10	79
20	73
40	67

- b. The cost of fountain drinks at Hot Dog Hut.

volume (fluid ounces)	cost (\$)
16	\$1.49
20	\$1.59
30	\$1.89

Solution

- a. Not proportional since the ratio of distance to listener to sound level is not always the same.
- b. Not proportional since the ratio of volume to cost is not always the same.

2. Problem 2

Statement

A taxi service charges \$1.00 for the first $\frac{1}{10}$ mile then \$0.10 for each additional $\frac{1}{10}$ mile after that.

Fill in the table with the missing information then determine if this relationship between distance traveled and price of the trip is a proportional relationship.

distance traveled (mi)	price (dollars)
$\frac{9}{10}$	
2	
$3\frac{1}{10}$	
10	

Solution

distance traveled (mi)	price (dollars)
$\frac{9}{10}$	1.80
2	2.90
$3\frac{1}{10}$	4.00
10	10.90

This is not a proportional relationship since the ratio of price to distance traveled is not always the same.

3. Problem 3

Statement

A rabbit and turtle are in a race. Is the relationship between distance traveled and time proportional for either one? If so, write an equation that represents the relationship.

Turtle's run:

distance (meters)	time (minutes)
108	2
405	7.5
540	10
1,768.5	32.75

Rabbit's run:

distance (meters)	time (minutes)
800	1
900	5
1,107.5	20
1,524	32.5

Solution

The distance might be proportional to the time for the turtle. The equation would be $d = 54 \cdot t$, where d represents the distance traveled in meters and t is the time in minutes.

4. Problem 4

Statement

For each table, answer: What is the constant of proportionality?

a	b
2	14
5	35
9	63
$\frac{1}{3}$	$\frac{7}{3}$

a	b
3	360
5	600
8	960
12	1440

a	b
75	3
200	8
1525	61
10	0.4

a	b
4	10
6	15
22	55
3	$7\frac{1}{2}$

Solution

- a. 7
- b. 120
- c. $\frac{1}{25}$ or equivalent
- d. $2\frac{1}{2}$ or equivalent

(From Unit 2, Lesson 2.)

5. Problem 5

Statement

Kiran and Mai are standing at one corner of a rectangular field of grass looking at the diagonally opposite corner. Kiran says that if the field were twice as long and twice as wide, then it would be twice the distance to the far corner. Mai says that it would be more than twice as far, since the diagonal is even longer than the side lengths. Do you agree with either of them?

Solution

Kiran is correct. If we scale the length and width of a rectangle by a factor of 2, then the diagonal will also scale by a factor of 2.

(From Unit 1, Lesson 4.)

Lesson 8: Comparing Relationships with Equations

Goals

- Compare and contrast (orally) equations that do and do not represent proportional relationships.
- Generalize that an equation equivalent to the form $y = kx$ can represent a proportional relationship.
- Use a table to determine whether a given equation represents a proportional relationship, and justify (in writing) the decision.

Learning Targets

- I can decide if a relationship represented by an equation is proportional or not.

Lesson Narrative

This lesson continues the work students did in the previous lesson on comparing proportional and nonproportional relationships. The focus is on students seeing the connection between the form of the equation and the kind of relationship it represents (MP7). Students should see by the end of this lesson that equations of the form $y = kx$ characterize proportional relationships.

Alignments

Building On

- 4.OA.C.5: Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.
- 6.EE.A.2: Write, read, and evaluate expressions in which letters stand for numbers.

Addressing

- 7.G.B.6: Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

- 7.RP.A.1: Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{\frac{1}{2}}{\frac{1}{4}}$ miles per hour, equivalently 2 miles per hour.
- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

Instructional Routines

- MLR8: Discussion Supports
- Notice and Wonder
- Think Pair Share

Required Materials

Four-function calculators

Snap cubes

Required Preparation

Calculators can optionally be made available to take the focus off computation.

Student Learning Goals

Let's develop methods for deciding if a relationship is proportional.

8.1 Notice and Wonder: Patterns with Rectangles

Warm Up: : 5 minutes

The purpose of this task is to elicit ideas that will be useful in the discussions in this lesson. While students may notice and wonder many things about these images, the relationship between the side lengths, perimeter, and area are the important discussion points.

Building On

- 4.OA.C.5

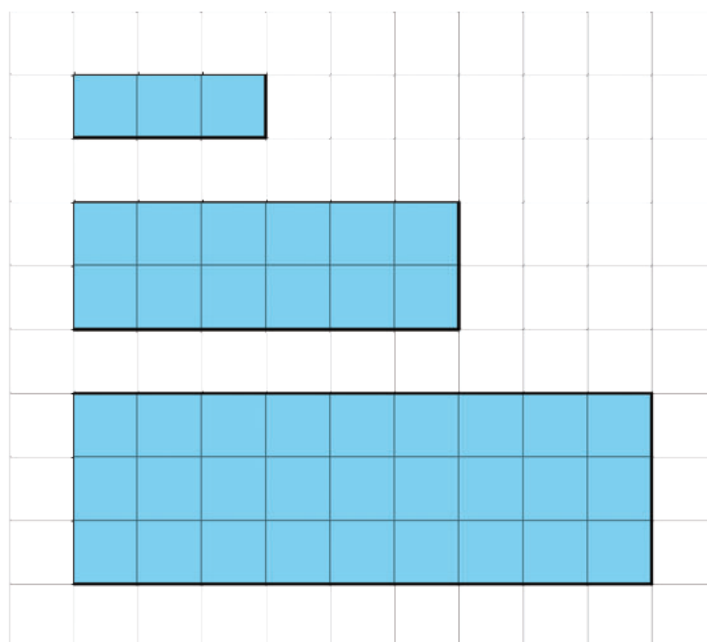
Instructional Routines

- Notice and Wonder

Launch

Arrange students in groups of 2. Tell students that they will look at a picture, and their job is to think of at least one thing they notice and at least one thing they wonder about the picture. Display the image for all to see. Ask students to give a signal when they have noticed or wondered about something and to think about the additional questions. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Student Task Statement



Do you see a pattern? What predictions can you make about future rectangles in the set if your pattern continues?

Student Response

Things (patterns) students may notice:

- The width increases by 3 each time.
- The height increases by 1 each time.

Things students may wonder:

- By how much does the area increase each time?
- By how much does the perimeter increase each time?
- Which rectangle will have a width of 10?

Activity Synthesis

Invite students to share the things they noticed and wondered. Record and display the different ways of thinking for all to see. If possible, record the relevant reasoning on or near the images themselves. If the two questions below the image do not come up during the conversation, ask students to discuss them. After each response, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to what is happening in the images each time.

8.2 More Conversions

: 15 minutes

Students have already looked at measurement conversions that can be represented by proportional relationships, both in grade 6 and in previous lessons in this unit. This task introduces a measurement conversion that is not associated with a proportional relationship. The discussion should start to move students from determining whether a relationship is proportional by examining a table, to making the determination from the equation.

Note that some students may think of the two scales on a thermometer like a double number line diagram, leading them to believe that the relationship between degrees Celsius and degrees Fahrenheit is proportional. If not mentioned by students, the teacher should point out that when a double number line is used to represent a set of equivalent ratios, the tick marks for 0 on each line need to be aligned.

Addressing

- 7.G.B.6

- 7.RP.A.2

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2. Provide access to calculators, if desired. Give students 5 minutes of quiet work time followed by students discussing responses with a partner, followed by whole-class discussion.

Support for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Anticipated Misconceptions

Some students may struggle with the fraction $\frac{9}{5}$ in the temperature conversion. Teachers can prompt them to convert the fraction to its decimal form, 1.8, before trying to evaluate the equation for the values in the table.

Student Task Statement

The other day you worked with converting meters, centimeters, and millimeters. Here are some more unit conversions.

1. Use the equation $F = \frac{9}{5}C + 32$, where F represents degrees Fahrenheit and C represents degrees Celsius, to complete the table.

temperature ($^{\circ}\text{C}$)	temperature ($^{\circ}\text{F}$)
20	
4	
175	

2. Use the equation $c = 2.54n$, where c represents the length in centimeters and n represents the length in inches, to complete the table.

length (in)	length (cm)
10	
8	
$3\frac{1}{2}$	

3. Are these proportional relationships? Explain why or why not.

Student Response

1.

temperature (°C)	temperature (°F)
20	68
4	39.2
175	347

2.

length (in)	length (cm)
10	25.4
8	20.32
$3\frac{1}{2}$	8.89

3. The temperature conversion does not determine a proportional relationship because the number of degrees Fahrenheit per degree Celsius is not the same. The length conversion does determine a proportional relationship because the number of centimeters per inch is the same.

Activity Synthesis

After discussing the work students did, ask, “What do you notice about the forms of the equations for each relationship?” After soliciting students’ observations, point out that the proportional relationship is of the form $y = kx$, while the nonproportional relationship is not.

Support for English Language Learners

Speaking: MLR8 Discussion Supports. To provide a support students in producing statements when they compare and contrast equations with proportional and nonproportional relationships. Provide sentence frames such as: “I noticed that _____,” “What makes _____ different from the others is _____,” or “The patterns show _____.” Invite students to use these sentence frames when students comprehend orally the patterns they notice with proportional equations.

Design Principle(s): Support sense-making

8.3 Total Edge Length, Surface Area, and Volume

: 15 minutes

This activity builds on the perimeter and area activity from the warm-up. Its goal is to use a context familiar from grade 6 to compare proportional and nonproportional relationships. The units for the quantities are purposely not given in the task statement to avoid giving away which relationships are not proportional. However, discussion should raise the possible units of measurement for edge length, surface area, and volume.

The focus of this task is whether or not relationships between the quantities are proportional, but there is also an opportunity for students to reinforce their understanding of geometry. Students should not spend too much time figuring out the surface area and volume. Watch carefully as students work and be ready to provide guidance or equations as needed, so students can get to the central purpose of the task, which is noticing the correspondences between the nature of relationships and the form of their equations. Strategic pairing of students and having snap cubes on hand can help struggling students complete the tables and determine equations.

Addressing

- 7.G.B.6
- 7.RP.A.2

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Display a cube (e.g. cardboard box) for all to see and ask:

- “How many edges are there?”
- “How long is one edge?”
- “How many faces are there?”
- “How large is one face?”

Give students 5 minutes of quiet work time followed by students discussing responses with a partner, followed by whole-class discussion.

Support for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.

Supports accessibility for: Organization; Attention

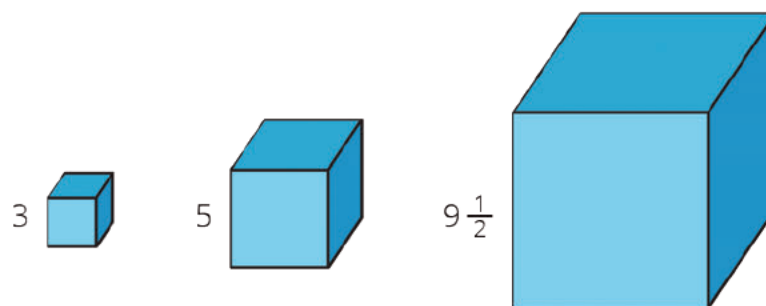
Anticipated Misconceptions

Some students may struggle to complete the tables. Teachers can use nets of cubes (flat or assembled) partitioned into square units to reinforce the process for finding total edge length, surface area, and volume of the cubes in the task. Snap cubes would also be appropriate supports.

If difficulties with the fractional side length $9\frac{1}{2}$ keep students from being able to find the surface area and volume or write the equations, the teacher can tell those students to replace $9\frac{1}{2}$ with 10 and retry their calculations. Their answers for surface area and volume will be different for that row in the table, but their equations and proportionality decisions will be the same. That way they can still learn the connection between the form of the equations and the nature of the relationships.

Student Task Statement

Here are some cubes with different side lengths. Complete each table. Be prepared to explain your reasoning.



1. How long is the total edge length of each cube?

side length	total edge length
3	
5	
$9\frac{1}{2}$	
s	

2. What is the surface area of each cube?

side length	surface area
3	
5	
$9\frac{1}{2}$	
s	

3. What is the volume of each cube?

side length	volume
3	
5	
$9\frac{1}{2}$	
s	

4. Which of these relationships is proportional? Explain how you know.

5. Write equations for the total edge length E , total surface area A , and volume V of a cube with side length s .

Student Response

1. A cube has 12 edges. Students may construct the third column below to reach this conclusion.

side length	total edge length	$\frac{\text{total edge length}}{\text{side length}}$
3	36	12
5	60	12
$9\frac{1}{2}$	114	12
s	$12s$	12

2. A cube has 6 faces each with an area of s^2 square units. Students may construct the third column below to reach this conclusion.

side length	surface area	$\frac{\text{surface area}}{\text{side length}}$
3	54	18
5	150	30
$9\frac{1}{2}$	$541\frac{1}{2}$	57
s	$6s^2$	$6s$

3. The bottom layer of a cube fits s^2 cubic units and s of these layers make up the cube. Students may construct the third column below to reach this conclusion.

side length	volume	$\frac{\text{volume}}{\text{side length}}$
3	27	9
5	125	25
$9\frac{1}{2}$	$857\frac{3}{8}$	$90\frac{1}{4}$
s	s^3	s^2

4. The relationship between side length and total edge length is proportional because the ratios in the third column is 12 for every side length. The relationships between side length and surface area, and between side length and volume are not proportional because the ratios in the third column of tables 2 and 3 are not the same for each side length.

5. $E = 12s$, $A = 6s^2$, $V = s^3$

Are You Ready for More?

1. A rectangular solid has a square base with side length ℓ , height 8, and volume V . Is the relationship between ℓ and V a proportional relationship?

- 
2. A different rectangular solid has length ℓ , width 10, height 5, and volume V . Is the relationship between ℓ and V a proportional relationship?
 3. Why is the relationship between the side length and the volume proportional in one situation and not the other?

Student Response

1. no
2. yes
3. In one situation, there are two unknown dimensions and in the other only one. So even though these situations look very similar, the relationship is different.

Activity Synthesis

Ask, "What do you notice about the equation for the relationship that is proportional?" Make sure students see that it is of the form $y = kx$, and the others are not. This realization is the central purpose of this task.

Ask students:

- "What could be possible units for the side lengths?" (linear measurements: centimeters, inches)
- "Then what would be the units for the surface area?" (square units: square centimeters, square inches)
- "What would be the units for the volume?" (cubic units: cubic centimeters, cubic inches)

Connect the units of measurements with the structure of the equation for each quantity: The side length and the units are raised to the same power.

8.4 All Kinds of Equations

Optional: : 10 minutes

This activity involves checking for a constant of proportionality in tables generated from simple equations that students should already be able to evaluate from their work with expressions and equations in grade 6. The purpose of this activity is to generalize about the forms of equations that do and do not represent proportional relationships. The relationships in this activity are presented without a context so that students can focus on the structure of the equations (MP7) without being distracted by what the variables represent.

Building On

- 6.EE.A.2

Addressing

- 7.RP.A.1
- 7.RP.A.2

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2. Provide access to calculators. Give students 5 minutes of quiet work time followed by students discussing responses with a partner, followed by whole-class discussion.

Support for Students with Disabilities

Engagement: Internalize Self Regulation. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. Invite students to choose and complete tables for four of the equations. Encourage students to complete tables for the equations $y = \frac{x}{4}$ and $y = 4x$.

Supports accessibility for: Organization; Attention

Anticipated Misconceptions

Students might struggle to see that the two proportional relationships have equations of the form $y = kx$ and to characterize the others as not having equations of that form. Students do not need to completely articulate this insight for themselves; this synthesis should emerge in the whole-class discussion.

Student Task Statement

Here are six different equations.

$$y = 4 + x$$

$$y = 4x$$

$$y = \frac{4}{x}$$

$$y = \frac{x}{4}$$

$$y = 4^x$$

$$y = x^4$$

$$y = 4 + x$$

x	y	$\frac{y}{x}$
2		
3		
4		
5		

$$y = 4x$$

x	y	$\frac{y}{x}$
2		
3		
4		
5		

$$y = \frac{4}{x}$$

x	y	$\frac{y}{x}$
2		
3		
4		
5		

$$y = \frac{x}{4}$$

x	y	$\frac{y}{x}$
2		
3		
4		
5		

$$y = 4^x$$

x	y	$\frac{y}{x}$
2		
3		
4		
5		

$$y = x^4$$

x	y	$\frac{y}{x}$
2		
3		
4		
5		

1. Predict which of these equations represent a proportional relationship.
2. Complete each table using the equation that represents the relationship.
3. Do these results change your answer to the first question? Explain your reasoning.
4. What do the equations of the proportional relationships have in common?

Student Response

1. Answers vary. Possible response: $y = 4x$ and $y = \frac{x}{4}$ represent proportional relationships, but the others do not.
- 2.

$$y = 4 + x$$

x	y	$\frac{y}{x}$
2	6	3
3	7	$2\frac{1}{3}$
4	8	2
5	9	$1\frac{4}{5}$

$$y = 4x$$

x	y	$\frac{y}{x}$
2	8	4
3	12	4
4	16	4
5	20	4

$$y = \frac{4}{x}$$

x	y	$\frac{y}{x}$
2	2	1
3	$\frac{4}{3}$	$\frac{4}{9}$
4	1	$\frac{1}{4}$
5	$\frac{4}{5}$	$\frac{4}{25}$

$$y = \frac{x}{4}$$

x	y	$\frac{y}{x}$
2	$\frac{1}{2}$	$\frac{1}{4}$
3	$\frac{3}{4}$	$\frac{1}{4}$
4	1	$\frac{1}{4}$
5	$\frac{5}{4}$	$\frac{1}{4}$

$$y = 4^x$$

x	y	$\frac{y}{x}$
2	16	8
3	64	$21\frac{1}{3}$
4	256	64
5	1,024	$204\frac{4}{5}$

$$y = x^4$$

x	y	$\frac{y}{x}$
2	16	8
3	81	27
4	256	64
5	625	125

3. Answers vary. Possible response: No, they just confirm that $y = 4x$ and $y = \frac{x}{4}$ represent proportional relationships, but the others do not.

4. Answers vary. Possible responses: The equations representing proportional relationships:

- can be written in the form $y = kx$
- do not contain any exponents or addition operations
- do not involve dividing by x

Activity Synthesis

Invite students to share what the equations for proportional relationships have in common, and by contrast, what is different about the other equations. At first glance, the

equation $y = \frac{x}{4}$ does not look like our standard equation for a proportional relationship, $y = kx$. Suggest to students that they rewrite the equation using the constant of proportionality they found after completing the table: $y = 0.25x$ which can also be expressed $y = \frac{1}{4}x$. If students do not express this idea themselves, remind them that they can think of dividing by 4 as multiplying by $\frac{1}{4}$.

Support for English Language Learners

Reading, Writing, Speaking: MLR3 Clarify, Critique, Correct. Before students share what they identified as similarities between the proportional and nonproportional relationships, present an incorrect explanation. For example, "There are no commonalities among the equations because they're all different." Ask students to identify the error, critique the reasoning, and write a correct description. As students discuss with a partner, listen for students who clarify that equations $y = 4x$ and $y = \frac{1}{4}x$ are of the form $y = kx$ and both represent proportional relationships. Invite students to share their critiques and corrected responses with the class. Listen for and amplify the language students use to describe the similarities and differences between equations that do and do not represent proportional relationships. This helps students evaluate, and improve upon, the written mathematical arguments of others, as they clarify how their understanding of proportional relationships and how they can be represented with equations.

Design Principle(s): Maximize meta-awareness

Lesson Synthesis

Review the findings from the activities and make explicit the fact that proportional relationships are characterized by equations of the form $y = kx$. Be sure to point out that this includes equations with other variables. For example:

$$y = 5.2x \quad d = 58t \quad a = 0.12B \quad W = 205n$$

This form characterizes proportional relationships due to a property we examined in previous lessons: if a table represents a proportional relationship between x and y , then the unit rates $\frac{y}{x}$ are always the same.

x	y	$\frac{y}{x}$
3	$3k$	k
5	$5k$	k
400	$400k$	k

If $\frac{y}{x} = k$, then $y = kx$ (as long as $x \neq 0$, but you don't need to mention this now unless a student brings it up.)

8.5 Tables and Chairs

Cool Down : 5 minutes

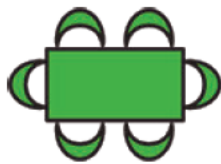
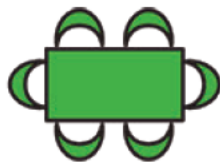
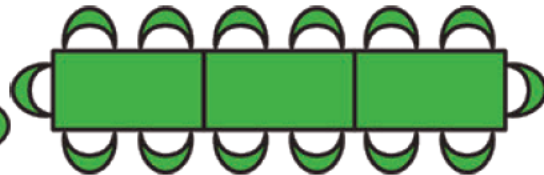
Addressing

- 7.RP.A.2

Student Task Statement

Andre is setting up rectangular tables for a party. He can fit 6 chairs around a single table. Andre lines up 10 tables end-to-end and tries to fit 60 chairs around them, but he is surprised when he cannot fit them all.

- Write an equation for the relationship between the number of chairs c and the number of tables t when:
 - the tables are apart from each other:
 - the tables are placed end-to-end:



2. Is the first relationship proportional? Explain how you know.

3. Is the second relationship proportional? Explain how you know.

Student Response

1. When the tables are apart: $c = 6t$ (or $t = \frac{1}{6}c$). When the tables are together: $c = 4t + 2$ (or $t = \frac{1}{4}c - \frac{1}{2}$).
2. This relationship is proportional. Possible reasons:
 - It can be represented with an equation of the form $c = kt$ (or $t = kc$).
 - There are 6 chairs per table no matter how many tables.
3. This relationship is not proportional. Possible reasons:
 - The number of chairs per table changes depending on how many tables there are.
 - The quotient of chairs and tables is not constant.
 - The relationship cannot be expressed with an equation of the form $c = kt$.

As shown below, the number of chairs per table is the same when the tables are apart, but it is not the same if the tables are pushed together.

With tables apart

tables	chairs	$\frac{\text{chairs}}{\text{tables}}$
1	6	6
2	12	6
3	18	6
4	24	6
10	60	6
t	$6t$	6

With tables end-to-end

tables	chairs	$\frac{\text{chairs}}{\text{tables}}$
1	6	6
2	10	5
3	14	4.667
4	18	4.5
10	42	4.2
t	$4t + 2$	

Student Lesson Summary

If two quantities are in a proportional relationship, then their quotient is always the same. This table represents different values of a and b , two quantities that are in a proportional relationship.

a	b	$\frac{b}{a}$
20	100	5
3	15	5
11	55	5
1	5	5

Notice that the quotient of b and a is always 5. To write this as an equation, we could say $\frac{b}{a} = 5$. If this is true, then $b = 5a$. (This doesn't work if $a = 0$, but it works otherwise.)

If quantity y is proportional to quantity x , we will always see this pattern: $\frac{y}{x}$ will always have the same value. This value is the constant of proportionality, which we often refer to as k . We can represent this relationship with the equation $\frac{y}{x} = k$ (as long as x is not 0) or $y = kx$.

Note that if an equation cannot be written in this form, then it does not represent a proportional relationship.

Lesson 8 Practice Problems

1. Problem 1

Statement

The relationship between a distance in yards (y) and the same distance in miles (m) is described by the equation $y = 1760m$.

- a. Find measurements in yards and miles for distances by completing the table.

distance measured in miles	distance measured in yards
1	
5	
	3,520
	17,600

- b. Is there a proportional relationship between a measurement in yards and a measurement in miles for the same distance? Explain why or why not.

Solution

a.

distance measured in miles	distance measured in yards
1	1,760
5	8,800
2	3,520
10	17,600

b. There is a proportional relationship. The constant of proportionality is 1760 yards per mile.

2. Problem 2

Statement

Decide whether or not each equation represents a proportional relationship.

- a. The remaining length (L) of 120-inch rope after x inches have been cut off: $120 - x = L$
- b. The total cost (t) after 8% sales tax is added to an item's price (p): $1.08p = t$
- c. The number of marbles each sister gets (x) when m marbles are shared equally among four sisters: $x = \frac{m}{4}$
- d. The volume (V) of a rectangular prism whose height is 12 cm and base is a square with side lengths s cm: $V = 12s^2$

Solution

- a. no
- b. yes
- c. yes
- d. no

3. Problem 3

Statement

- a. Use the equation $y = \frac{5}{2}x$ to complete the table.
Is y proportional to x and y ? Explain why or why not.

x	y
2	
3	
6	

- b. Use the equation $y = 3.2x + 5$ to complete the table.
Is y proportional to x and y ? Explain why or why not.

x	y
1	
2	
4	

Solution

a.

x	y
2	5
3	$\frac{15}{2}$
6	15

Yes, there is a proportional relationship between x and y since $\frac{y}{x} = \frac{5}{2}$ in each row.

b.

x	y
1	8.2
2	11.4
4	17.8

No, there is no proportional relationship between x and y . In the first row $\frac{y}{x} = 8.2$ but in the second row $\frac{y}{x} = 5.7$.

4. Problem 4

Statement

To transmit information on the internet, large files are broken into packets of smaller sizes. Each packet has 1,500 bytes of information. An equation relating packets to bytes of information is given by $b = 1,500p$ where p represents the number of packets and b represents the number of bytes of information.

- How many packets would be needed to transmit 30,000 bytes of information?
- How much information could be transmitted in 30,000 packets?
- Each byte contains 8 bits of information. Write an equation to represent the relationship between the number of packets and the number of bits.

Solution

- 20 packets
- 45,000,000 bytes
- $x = 12,000p$

(From Unit 2, Lesson 6.)

Lesson 9: Solving Problems about Proportional Relationships

Goals

- Decide whether it makes sense to represent a situation with a proportional relationship, and explain (orally) the reasoning.
- Determine what information is needed to solve a problem involving proportional relationships. Ask questions to elicit that information.
- Write an equation to represent a proportional relationship, and use the equation to solve problems about the situation.

Learning Targets

- I can ask questions about a situation to determine whether two quantities are in a proportional relationship.
- I can solve all kinds of problem involving proportional relationships.

Lesson Narrative

In the previous two lessons students have been comparing proportional and nonproportional relationships using tables and equations. In this lesson they learn to recognize proportional relationships from descriptions of the context. This lesson includes the first Info Gap activity students encounter in the grade 7 course. In the Info Gap activity, students keep asking questions until they get all the information needed to solve the problem (MP1, MP6). In this case, that means they must determine if there are quantities in a proportional relationship they can work with. They see the connection between the constant of proportionality and a constant rate in the situation, and they compare different proportional relationships using the constant of proportionality. They reason quantitatively about situations and connect their reasoning with the equation for a proportional relationship (MP3).

Alignments

Addressing

- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.
- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

Instructional Routines

- MLR4: Information Gap Cards
- MLR7: Compare and Connect

Required Materials

Pre-printed slips, cut from copies of the blackline master

Required Preparation

Make 1 copy of the Info Gap blackline master for every 2 students, and cut them up ahead of time.

Student Learning Goals

Let's solve problems about proportional relationships.

9.1 What Do You Want to Know?

Warm Up: : 10 minutes

This warm-up prepares students for the Info Gap activity that follows. First, students brainstorm what information they would need to know to solve a problem that involves constant speed. Then, the teacher demonstrates the process of asking a student why they need a specific piece of information before sharing it with them, in preparation for students following this procedure with their partner in the next activity.

Addressing

- 7.RP.A.2

Launch

Give students 1 minutes of quiet think time to brainstorm what information they would need to know to solve the problem, followed by a whole-class discussion demonstrating the procedure for the Info Gap activity.

Student Task Statement

Consider the problem: A person is running a distance race at a constant rate. What time will they finish the race?

What information would you need to be able to solve the problem?

Student Response

Answers vary. Sample responses:

- How long is the race?
- How fast is the person running?
- How far did the person run in 1 minute?
- What time did they start the race?

Activity Synthesis

Ask students:

- "What specific information do you need?"
- "Why do you need that information?"

Share each piece of information with the class after a student specifically asks for it (and explains why they need to know it).

- The race is 10,000 meters long.
- The race started at 9:15 a.m.
- In 1 minute, the person ran $156\frac{1}{4}$ meters.
- An equation relating distance and time is given by $d = 156\frac{1}{4} \cdot t$ where d represents distance in meters and t represents time in minutes.
- It takes 32 minutes for the person to run 5,000 meters.
- The person runs at a pace of 6.4 minutes per kilometer (or 1,000 meters).

After you share each piece of information, ask the class whether they have enough information to be able to solve the problem. When they think they do, give them 2 minutes to solve the problem and then have them share their strategies. (The person should finish the race at 10:19 a.m.)

Tell students that they will be working in groups of two in the next activity and that they will be using the same procedure that you just demonstrated to solve a problem.

9.2 Info Gap: Biking and Rain

: 30 minutes

In this info gap activity, students write equations for several proportional relationships given in the contexts of a bike ride and steady rainfall. They use the equations to make predictions.

The info gap structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need (MP1). It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need (MP6).

Here is the text of the cards for reference and planning:

Info Gap: Biking and Rain Problem Card 1

Mai and Noah each leave their houses at the same time and ride their bikes to the park.

1. For each person, write an equation that relates the distance they travel and the time.
2. Who will arrive at the park first?

Info Gap: Biking and Rain Data Card 1

- Noah lives 1 kilometer farther away from the park than Mai does.
- Mai lives 8,000 meters from the park.
- Noah lives 9,000 meters from the park.
- Mai and Noah each bike at a constant speed.
- Mai bikes 250 meters per minute.
- Noah bikes 300 meters per minute.

Info Gap: Biking and Rain Problem Card 2

A slow, steady rainstorm lasted all day. The rain was falling at a constant rate.

1. Write an equation that relates how much rain has fallen and how long it has been raining.
2. How long will it take for 5 cm of rain to fall?

Info Gap: Biking and Rain Data Card 2

- The rain storm lasted for 24 hours.
- 9.6 centimeters of rain fell during the storm.
- The rate of the rainfall was 2 millimeters of rain every 30 minutes.
- There are 10 millimeters in 1 centimeter.
- There are 60 minutes in 1 hour.

Addressing

- 7.RP.A.2

Instructional Routines

- MLR4: Information Gap Cards

Launch

Support for Students with Disabilities

Engagement: Develop Effort and Persistence. Display or provide students with a physical copy of the written directions. Check for understanding by inviting students to rephrase directions in their own words. Keep the display of directions visible throughout the activity.

Supports accessibility for: Memory; Organization

Support for English Language Learners

Conversing: This activity uses *MLR4 Information Gap* to give students a purpose for discussing information necessary to write equations for proportional relationships. Display questions or question starters for students who need a starting point such as: “Can you tell me . . . (specific piece of information)”, and “Why do you need to know . . . (that piece of information)?”

Design Principle(s): Cultivate Conversation

Student Task Statement

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*: If your teacher gives you the *data card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

Continue to ask questions until you have enough information to solve the problem.

4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

1. Silently read your card.
2. Ask your partner “*What specific information do you need?*” and wait for them to *ask* for information.

If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don’t have that information.

3. Before sharing the information, ask “*Why do you need that information?*” Listen to your partner’s reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Student Response

Card 1:

1. For Mai, $d = 250t$ where d represents distance in meters and t represents the amount of time in minutes. For Noah, $d = 300t$.
2. Noah will arrive first, since it will only take him 30 minutes ($9,000 \div 300 = 30$) while Mai takes 32 minutes ($8,000 \div 250 = 32$).

Card 2:

1. Answers vary. Possible responses:
 - $r = 0.4t$, where r represents the amount of rain that has fallen in centimeters and t represents the amount of time in hours.
 - $t = 2.5r$, where r represents the amount of rain that has fallen in centimeters and t represents the amount of time in hours.
 - $r = \frac{1}{150}t$, where r represents the amount of rain that has fallen in centimeters and t represents the amount of time in minutes.
 - $t = 150r$, where r represents the amount of rain that has fallen in centimeters and t represents the amount of time in minutes.
2. 12.5 hours

Activity Synthesis

Invite students to share their equations and predictions. Record their equations displayed for all to see. Ask them to explain how they determined the constant of proportionality as well as why it made sense to represent the situation with a proportional relationship.

9.3 Moderating Comments

Optional: : 10 minutes

In this activity students compute rates to decide which job applicant is working the fastest checking online comments. They compare rates and total number of comments checked, then see that using rates is the more useful information in this situation.

Addressing

- 7.RP.A.2

Instructional Routines

- MLR7: Compare and Connect

Launch

Keep students in the same groups of 2.

Support for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Student Task Statement

A company is hiring people to read through all the comments posted on their website to make sure they are appropriate. Four people applied for the job and were given one day to show how quickly they could check comments.

- Person 1 worked for 210 minutes and checked a total of 50,000 comments.
- Person 2 worked for 200 minutes and checked 1,325 comments every 5 minutes.
- Person 3 worked for 120 minutes, at a rate represented by $c = 331t$, where c is the number of comments checked and t is the time in minutes.
- Person 4 worked for 150 minutes, at a rate represented by $t = \left(\frac{3}{800}\right)c$.

1. Order the people from greatest to least in terms of total number of comments checked.

- 
2. Order the people from greatest to least in terms of how fast they checked the comments.

Student Response

1.
 - Person 2 checked a total of 53,000 comments, because $\frac{1,325}{5} \cdot 200 = 53,000$.
 - Person 1 checked a total of 50,000 comments.
 - Person 4 checked a total of 40,000 comments, because $\frac{800}{3} \cdot 150 = 40,000$.
 - Person 3 checked a total of 39,720 comments, because $331 \cdot 120 = 39,720$.
2.
 - Person 3 checked 331 comments per minute.
 - Person 4 checked about 267 comments per minute, because $800 \div 3 = 266.\overline{6}$.
 - Person 2 checked 265 comments per minute, because $1,325 \div 5 = 265$.
 - Person 1 checked about 238 comments per minute, because $50,000 \div 210 \approx 238$.



Are You Ready for More?

1. Write equations for each job applicant that allow you to easily decide who is working the fastest.
2. Make a table that allows you to easily compare how many comments the four job applicants can check.

Student Response

1. Person 1: $c = \frac{50,000}{210}t \approx 238.1t$. Person 2: $c = \frac{1,325}{5}t = 265t$. Person 3: $c = 331t$.
Person 4: $c = \frac{800}{3}t \approx 266.7t$

2.

person	time taken in minutes	comments per minute	total comments
1	210	$\frac{50,000}{210} \approx 238.1$	50,000
2	200	$\frac{1,325}{5} = 265$	53,000
3	120	331	39,720
4	150	$\frac{800}{3} \approx 266.7$	40,000

Activity Synthesis

Ask students which job applicant should get the job and why. If time permits, consider using MLR 1 (Stronger and Clearer Each Time).

Support for English Language Learners

Representing, Conversing: MLR7 Compare and Connect. Use this routine to prepare students for the whole-class discussion. At the appropriate time, invite students to create a visual display showing which job applicant should get the job and why. Allow students time to quietly circulate and analyze the selections and justifications in at least 2 other displays in the room. Give students quiet think time to consider what is the same and what is different and whether or not they agree. Next, ask students to find a partner to discuss what they noticed. Listen for and amplify observations that highlight advantages and disadvantages to each method of determining the top job applicant. This will help students identify situation when comparing rates is more effective than the sum of quantities.

Design Principle(s): Optimize output (for justification); Cultivate conversation

Lesson Synthesis

Whenever we have a situation involving constant rates, we are likely to have a proportional relationship between the quantities of interest.

- "What are some situations that we have seen where quantities were proportional to each other?"
- "When we are in a situation where we have a proportional relationship between two quantities, what information do we need to find an equation?"
- "How can we decide if a proportional relationship is a good representation of a particular situation?"
- "Equations are good tools to make predictions or decisions. When and how did we use an equation to make a prediction or a decision today?"

9.4 Steel Beams

Cool Down : 5 minutes

Addressing

- 7.RP.A

Student Task Statement

A steel beam can be cut to different lengths for a project. Assuming the weight of a steel beam is proportional to its length, what information would you need to know to write an equation that represents this relationship?

Student Response

Answers vary. Sample responses:

- Are the length of the steel beam and its weight in a proportional relationship?
- The weight of steel per 1 meter length of beam.
- The length of a steel beam that weighs 1 kilogram.
- The weight and length of a certain steel beam.

Student Lesson Summary

Whenever we have a situation involving constant rates, we are likely to have a proportional relationship between quantities of interest.

- When a bird is flying at a constant speed, then there is a proportional relationship between the flying time and distance flown.
- If water is filling a tub at a constant rate, then there is a proportional relationship between the amount of water in the tub and the time the tub has been filling up.
- If an aardvark is eating termites at a constant rate, then there is proportional relationship between the number of termites the aardvark has eaten and the time since it started eating.

Sometimes we are presented with a situation, and it is not so clear whether a proportional relationship is a good model. How can we decide if a proportional relationship is a good representation of a particular situation?

- If you aren't sure where to start, look at the quotients of corresponding values. If they are not always the same, then the relationship is definitely not a proportional relationship.
- If you can see that there is a single value that we always multiply one quantity by to get the other quantity, it is definitely a proportional relationship.

After establishing that it is a proportional relationship, setting up an equation is often the most efficient way to solve problems related to the situation.

Lesson 9 Practice Problems

1. Problem 1

Statement

For each situation, explain whether you think the relationship is proportional or not. Explain your reasoning.

- a. The weight of a stack of standard 8.5x11 copier paper vs. number of sheets of paper.



- b. The weight of a stack of different-sized books vs. the number of books in the stack.



Solution

- a. There is a proportional relationship between weight and number of sheets of paper. Each piece of paper has the same weight. To find the weight of a stack, multiply the number of sheets of paper by the weight of a single sheet of paper. (In case it comes up: We're assuming for this question that each piece of paper is the same weight. Manufacturing being what it is, though, we acknowledge that's not true.)
- b. The relationship between the number of books and the weight of the stack is not proportional. Each book has a different weight, the weight of the stack can't be determined by multiplying the number of books by the weight of one book.

2. Problem 2

Statement

Every package of a certain toy also includes 2 batteries.

- a. Are the number of toys and number of batteries in a proportional relationship? If so, what are the two constants of proportionality? If not, explain your reasoning.
- b. Use t for the number of toys and b for the number of batteries to write two equations relating the two variables.
 $b =$ $t =$

Solution

a. Yes. 2 and $\frac{1}{2}$ are the constants of proportionality

b. $b = 2t$ and $t = \frac{1}{2}b$

3. Problem 3

Statement

Lin and her brother were born on the same date in different years. Lin was 5 years old when her brother was 2.

a. Find their ages in different years by filling in the table.

Lin's age	Her brother's age
5	2
6	
15	
	25

b. Is there a proportional relationship between Lin's age and her brother's age? Explain your reasoning.

Solution

a.

Lin's age	Her brother's age
5	2
6	3
15	12
28	25

b. There is no proportional relationship. Every year, they each age by one year, so the ratio of their ages changes every year.

4. Problem 4

Statement

A student argues that $y = \frac{x}{9}$ does not represent a proportional relationship between x and y because we need to multiply one variable by the same constant to get the other one and not divide it by a constant. Do you agree or disagree with this student?

Solution

Disagree. Dividing by 9 is the same as multiplying by $\frac{1}{9}$. We can look at the equation $y = \frac{x}{9}$ as $y = \frac{1}{9}x$. Also, $\frac{y}{x} = \frac{1}{9}$ is constant for all corresponding values of x and y .

(From Unit 2, Lesson 8.)

5. Problem 5

Statement

Quadrilateral A has side lengths 3, 4, 5, and 6. Quadrilateral B is a scaled copy of Quadrilateral A with a scale factor of 2. Select all of the following that are side lengths of Quadrilateral B.

A. 5

B. 6

C. 7

D. 8

E. 9

Solution

["B", "D"]

(From Unit 1, Lesson 3.)

Section: Representing Proportional Relationships with Graphs

Lesson 10: Introducing Graphs of Proportional Relationships

Goals

- Compare and contrast (orally) graphs of relationships.
- Generalize (orally and in writing) that a proportional relationship can be represented in the coordinate plane by a line that includes the “origin” or by a collection of points that lie on such a line.
- Justify (orally) that a table and a graph represent the same relationship.

Learning Targets

- I know that the graph of a proportional relationship lies on a line through $(0, 0)$.

Lesson Narrative

This lesson introduces an important way of representing a proportional relationship: its graph. Students plot points on the graph from tables, and, by the end of the lesson, start to see that the graph of a proportional relationship always lies on a line that passes through $(0, 0)$. They match tables and graphs of given situations and articulate their reasons for each match (MP3).

Alignments

Building On

- 5.G.A: Graph points on the coordinate plane to solve real-world and mathematical problems.
- 6.NS.C.8: Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Addressing

- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

- 7.RP.A.2.a: Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Building Towards

- 7.RP.A.2.a: Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR8: Discussion Supports
- Think Pair Share

Required Materials

Pre-printed slips, cut from copies of the blackline master

Rulers

Required Preparation

Prepare Matching Tables and Graphs activity by printing one copy for each group of 2 students and cutting them up ahead of time. Prepare a few copies of an answer key and place them in envelopes for students to access to check their work when they finish.

Student Learning Goals

Let's see how graphs of proportional relationships differ from graphs of other relationships.

10.1 Notice These Points

Warm Up: : 5 minutes (there is a digital version of this activity)

This warm-up prepares students for graphing proportional relationships in the coordinate plane. They practice graphing coordinate points and notice that all points lie on a straight line.

Building On

- 5.G.A

Building Towards

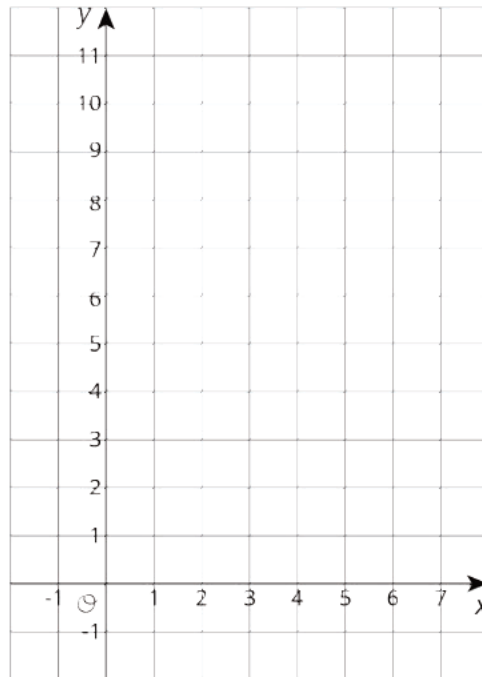
- 7.RP.A.2.a

Launch

Give students 3 minutes quiet work time followed by a whole-class discussion.

Student Task Statement

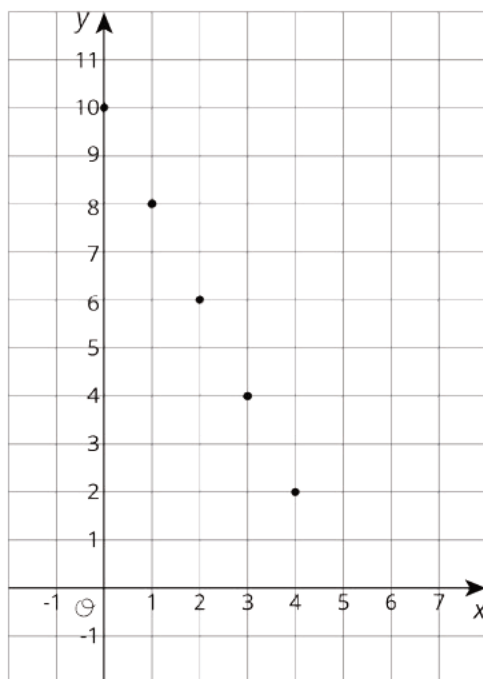
1. Plot the points $(0, 10)$, $(1, 8)$, $(2, 6)$, $(3, 4)$, $(4, 2)$.



2. What do you notice about the graph?

Student Response

1.



2. Answers vary. Sample responses:

- The points line up so that they could all be connected with a single line.
- The line goes down when reading left to right.
- Every time the x -coordinate goes up 1, the y -coordinate goes down 2.

Activity Synthesis

Invite students to share their observations. Ask if other students agree. If some students do not agree that the points lie on a straight line, ask which points break the pattern and give students a chance to self-correct their work.

10.2 T-shirts for Sale

: 10 minutes (there is a digital version of this activity)

This introductory activity asks students to plot points using tables of values that represent scenarios familiar from previous lessons. This activity is intended as a review of the coordinate plane, its axes, and plotting ordered pairs.

Building On

- 6.NS.C.8

Addressing

- 7.RP.A.2

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2. Provide access to rulers. Give students 5 minutes of quiet work time followed by students discussing responses with a partner, followed by whole-class discussion.

The second question reviews work from grades 5 and 6, but it is important that students understand how to plot points in the coordinate plane. Display the graph for all to see. Show students how to plot the pair from the first row in the table in the coordinate plane. Ask students to plot the remaining pairs and check with nearby students as they work. Be on the lookout for students plotting coordinates in the wrong order.

Student Task Statement

Some T-shirts cost \$8 each.

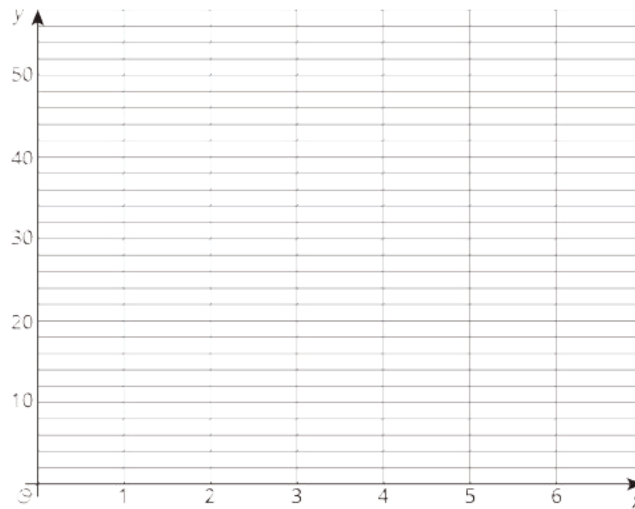
x	y
1	8
2	16
3	24
4	32
5	40
6	48

1. Use the table to answer these questions.
 - a. What does x represent?

b. What does y represent?

c. Is there a proportional relationship between x and y ?

2. Plot the pairs in the table on the coordinate plane.

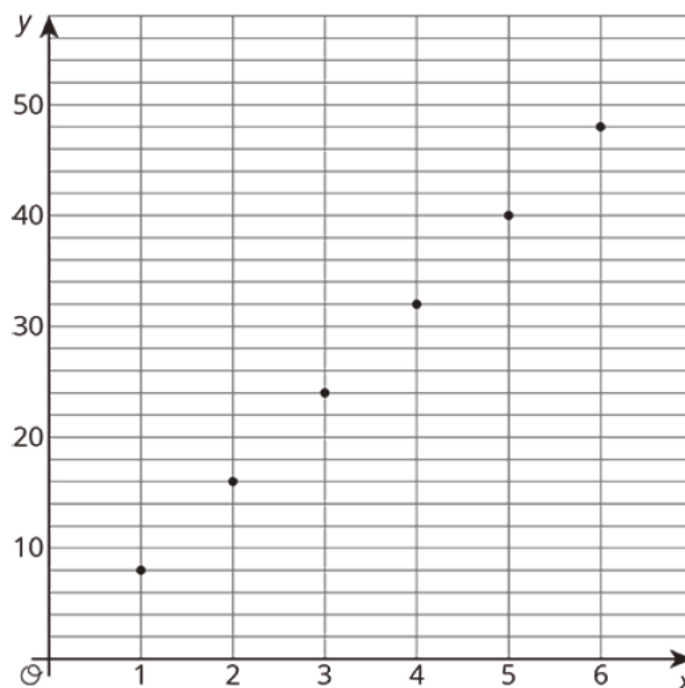


3. What do you notice about the graph?

Student Response

1.
 - a. x is the number of T-shirts
 - b. y is the total cost of those T-shirts
 - c. x is proportional to y . Students may identify 8 as the constant of proportionality.

2.



3. Students may notice that the points lie on a line.

Activity Synthesis

Discuss the first question if students had trouble with it.

Ask students to share their observations about the plotted points. Ask, “Could we buy 0 shirts? 7 T-shirts? 10 T-shirts? Can we buy half of a T-shirt?” Note that the graph consists of discrete points because only whole numbers of T-shirts make sense in this context; however, people often connect discrete points with a line to make the relationship more clear, even when the in-between values don’t make sense.

Ask the students, “Suppose instead of price per shirt, this graph displayed the cost of cherries that are \$8 per pound. Given that context, how should we change the graph?” Weights need not have integer values, so the graph is not restricted to discrete points. If you haven’t done so already, draw the ray starting at $(0, 0)$ that passes through the points.

Support for English Language Learners

Speaking: MLR8 Discussion Supports. As students describe their observations about the proportional relationship represented in the graph, revoice student ideas to demonstrate mathematical language use. Press for details in students' explanations by requesting that students challenge an idea, elaborate on an idea, or give an example. Show central concepts multi-modally by using different types of sensory inputs: acting out scenarios or inviting students to do so, using gestures, and talking about the context of selling T-shirts or cherries. This will help students to produce and make sense of the language needed to communicate their own ideas.

Design Principle(s): Support sense-making, Optimize output (for explanation)

10.3 Matching Tables and Graphs

: 25 minutes

Students work in pairs to match tables to graphs and to practice articulating their reasoning (MP3). This task is intended to foster understanding of correspondences between tables and graphs.

Students sort the graphs and justify their sorting schemes. Then, they compare the way they sorted their graphs with a different group. The purpose of this activity is to illustrate the idea that the graph of a proportional relationship is a line through the origin. Students will not have the tools for a formal explanation until grade 8.

Demonstrate how the matching activity works and how to have mathematical dialogue about the decisions being made (see the instructions in the task statement). When students finish the activity, they use an answer key to check their answers. If adjustments need to be made, students discuss any errors they made.

While students are working to match graphs to written descriptions and tables, circulate and ask them to justify their choices.

Addressing

- 7.RP.A.2

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Arrange students in groups of 2. Place copies of answer keys in envelopes.

Ask students to observe similarities and differences in the graphs and to create a rationale for sorting them. Provide access to sticky notes. If necessary, specify how the groups will trade places after they finish sorting their graphs into categories.

Demonstrate how to set up and conduct the matching activity. Choose a student to act as your partner. Mix up the cards and place them face-up. Point out that the cards contain either tables or graphs. Select one of each style of card and then explain to your partner why you think the cards do or do not match. Demonstrate productive ways to agree or disagree, e.g. by explaining your mathematical thinking, asking clarifying questions, etc.

Give each group cut-up slips for matching. Tell students to check their matches after they complete the activity using the answer keys.

Support for Students with Disabilities

Representation: Internalize Comprehension. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.

Supports accessibility for: Conceptual processing; Organization

Support for English Language Learners

Speaking and listening: MLR1 Stronger and Clearer Each Time. Use this with successive pair shares to give students a structured opportunity to revise and refine their response to “Which of the relationships are proportional?” Ask each student to meet with 2–3 other partners in a row for feedback. Display prompts for feedback that students can use to help their partner strengthen and clarify their ideas. For example, “Your explanation tells me . . .”, “Can you say more about why you . . .?”, and “A detail (or word) you could add is ____, because . . .” Give students with 3–4 minutes to revise their initial draft based on feedback from their peers.

Design Principle(s): Optimize output (for justification)

Anticipated Misconceptions

If students struggle to get starting making any matches, ask questions like “How would we expect this row in the table to look on the graph?” Or, “See this point on the graph? What corresponds to it in the table?”

A common misunderstanding is to assume that if the points lie on a line, then the graph represents a proportional relationship. Ask questions about the table to assist students in realizing the error.

Student Task Statement

Your teacher will give you papers showing tables and graphs.

1. Examine the graphs closely. What is the same and what is different about the graphs?
2. Sort the graphs into categories of your choosing. Label each category. Be prepared to explain why you sorted the graphs the way you did.
3. Take turns with a partner to match a table with a graph.
 - a. For each match you find, explain to your partner how you know it is a match.
 - b. For each match your partner finds, listen carefully to their explanation. If you disagree, work to reach an agreement.

Pause here so your teacher can review your work.

4. Trade places with another group. How are their categories the same as your group's categories? How are they different?

- 
5. Return to your original place. Discuss any changes you may wish to make to your categories based on what the other group did.
 6. Which of the relationships are proportional?
 7. What have you noticed about the graphs of proportional relationships? Do you think this will hold true for *all* graphs of proportional relationships?

Student Response

1. Answers vary. Sample response: All of the graphs have points that can be connected by a single, straight line. Some of the graphs will go through $(0, 0)$, but others will not.
2. Answers vary. Many students will sort the graphs into proportional and nonproportional. Some students may add more categories (i.e., not straight, straight but not proportional, proportional).
3. Here are the correct matches: 1H, 2B, 3G, 4D, 5A, 6E, 7F, 8I, 9C, 10J.
4. Answers vary.
5. No response required.
6. The proportional relationships are 2B, 4D, 7F, 8I, and 9C.
7. Answers vary. Possible responses: All points on a graph of a proportional relationship lie on a line. All such lines pass through $(0, 0)$. The constant of proportionality can be seen in the graphs as the y -coordinate when x is 1.

Are You Ready for More?

1. All the graphs in this activity show points where both coordinates are positive. Would it make sense for any of them to have one or more coordinates that are negative?
2. The equation of a proportional relationship is of the form $y = kx$, where k is a positive number, and the graph is a line through $(0, 0)$. What would the graph look like if k were a negative number?

Student Response

1. The temperature graph could have negative coordinates because temperatures can be negative.

2. The line would still be through the origin, but it would slant downward from left to right.

Activity Synthesis

Ask students how they determined which relationships were proportional. Invite volunteers to share the proportional relationships to ensure common understanding. Address any discrepancies through questioning: “How do you know the relationship is proportional? What have you learned about proportional relationships that applies here?”

Select students to share what they noticed about the characteristics of graphs of proportional relationships. Some observations might conclude:

- Points whose coordinates satisfy the relationship lie on a line.
- The line passes through the point $(0, 0)$.

This would be a good place to either introduce the term **origin** to refer to the point $(0, 0)$ (or to remind students of it, if they have encountered it before).

If time permits, discuss which written descriptions of proportional relationships would warrant “connecting the dots.” In other words, which proportional relationships are best represented with dots, and which are best represented with an unbroken line? Of the cards which describe a proportional relationship, it makes sense to draw an unbroken line for 7 and 8. The rest should use dots that are not connected. None include negative values without some assumptions (e.g., scenarios that involve owing money for card 2). Students should realize that even when the graph of a proportional relationship is represented by unconnected points, they lie on a line through the origin.

Lesson Synthesis

At the end of the lesson, make sure that students know that the graph of a proportional relationship lies on a line through the **origin**. (They will be able to explain why this is true in grade 8.) Students should understand that the context sometimes restricts which points on the line should be included in the graph.

10.4 Which Are Not Proportional

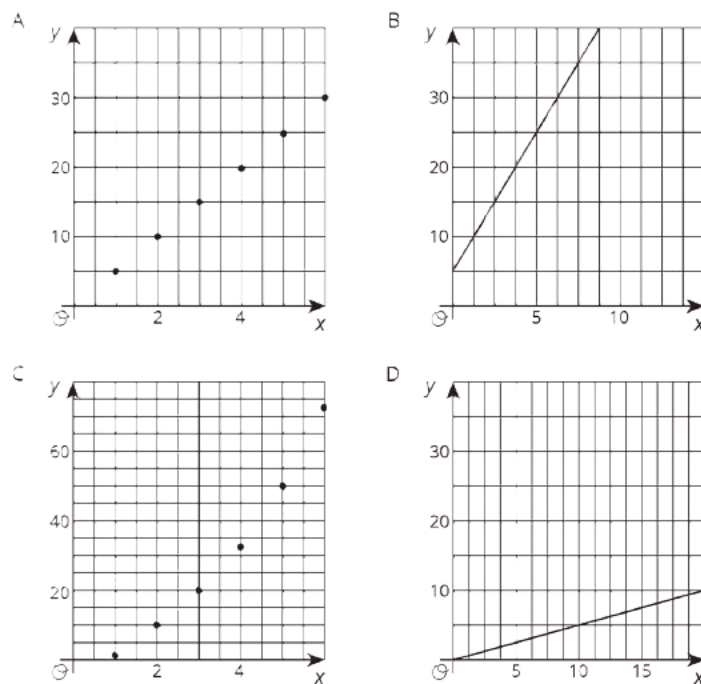
Cool Down : 5 minutes

Addressing

- 7.RP.A.2.a

Student Task Statement

Which graphs cannot represent a proportional relationship? Select **all** that apply. Explain how you know.

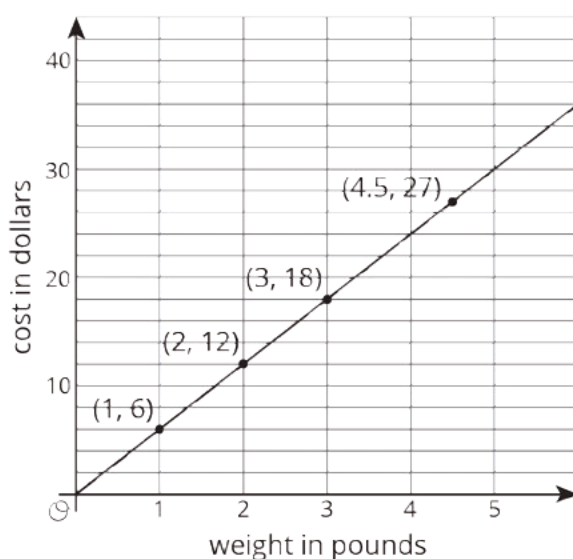


Student Response

B and C. Since graph B does not go through the origin, it cannot be a proportional relationship. Since the points in graph C cannot be connected by a single, straight line, it cannot be a proportional relationship.

Student Lesson Summary

One way to represent a proportional relationship is with a graph. Here is a graph that represents different amounts that fit the situation, "Blueberries cost \$6 per pound."



Different points on the graph tell us, for example, that 2 pounds of blueberries cost \$12, and 4.5 pounds of blueberries cost \$27.

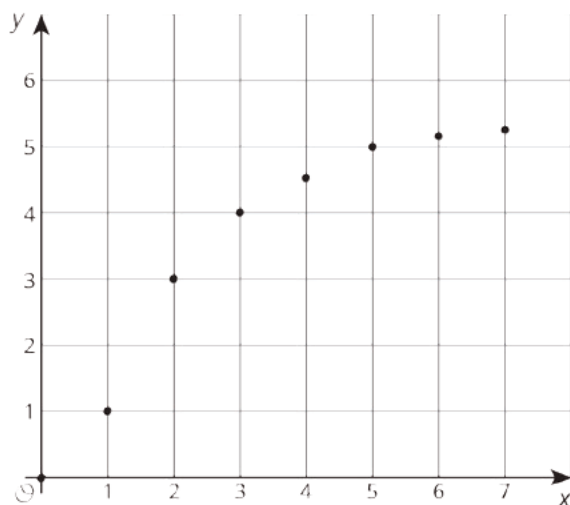
Sometimes it makes sense to connect the points with a line, and sometimes it doesn't. We could buy, for example, 4.5 pounds of blueberries or 1.875 pounds of blueberries, so all the points in between the whole numbers make sense in the situation, so any point on the line is meaningful.

If the graph represented the cost for different *numbers of sandwiches* (instead of pounds of blueberries), it might not make sense to connect the points with a line, because it is often not possible to buy 4.5 sandwiches or 1.875 sandwiches. Even if only points make sense in the situation, though, sometimes we connect them with a line anyway to make the relationship easier to see.

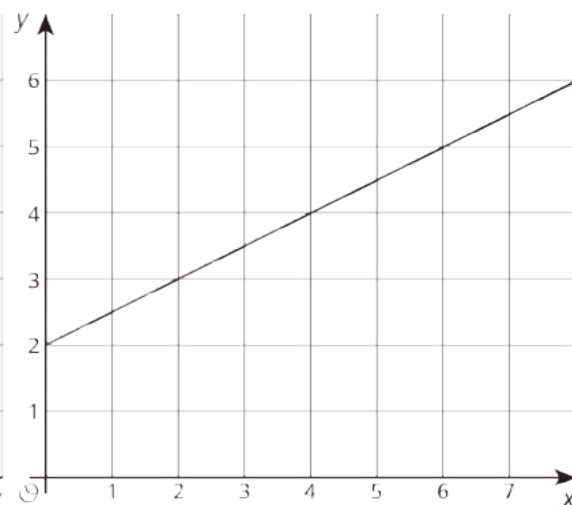
Graphs that represent proportional relationships all have a few things in common:

- Points that satisfy the relationship lie on a straight line.
- The line that they lie on passes through the **origin**, $(0, 0)$.

Here are some graphs that do *not* represent proportional relationships:



These points do not lie on a line.



This is a line, but it doesn't go through the origin.

Glossary

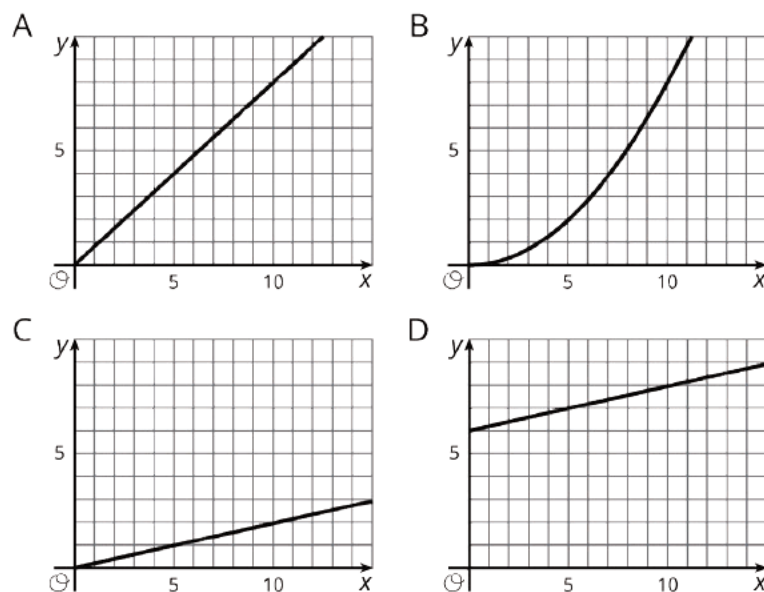
- coordinate plane
- origin

Lesson 10 Practice Problems

1. Problem 1

Statement

Which graphs could represent a proportional relationship?



- A. A
- B. B
- C. C
- D. D

Solution

["A", "C"]

2. Problem 2

Statement

A lemonade recipe calls for $\frac{1}{4}$ cup of lemon juice for every cup of water.

a. Use the table to answer these questions.

i. What does x represent?

ii. What does y represent?

iii. Is there a proportional relationship between x and y ?

b. Plot the pairs in the table in a coordinate plane.

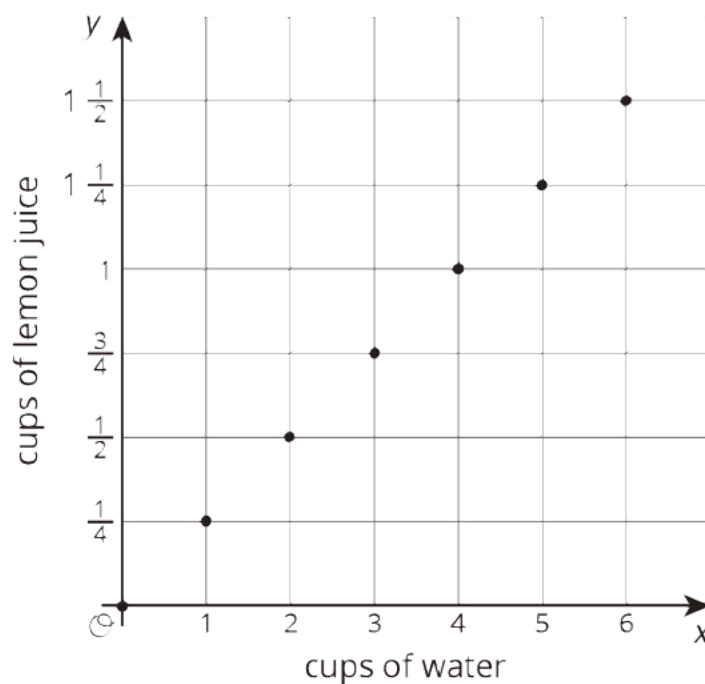
x	y
1	$\frac{1}{4}$
2	$\frac{1}{2}$
3	$\frac{3}{4}$
4	1

Solution

a. i. x represents the cups of water

ii. y represents the cups of lemon juice

iii. Yes



b.

3. Problem 3

Statement

Select all the pieces of information that would tell you x and y have a proportional relationship. Let y represent the distance in meters between a rock and a turtle's current position and x represent the time in minutes the turtle has been moving.

- A. $y = 3x$
- B. After 4 minutes, the turtle has walked 12 feet away from the rock.
- C. The turtle walks for a bit, then stops for a minute before walking again.
- D. The turtle walks away from the rock at a constant rate.

Solution

["A", "D"]

(From Unit 2, Lesson 9.)

4. Problem 4

Statement

Decide whether each table could represent a proportional relationship. If the relationship could be proportional, what would be the constant of proportionality?

- a. The sizes you can print a photo.

width of photo (inches)	height of photo (inches)
2	3
4	6
5	7
8	10

- b. The distance from which a lighthouse is visible.

height of a lighthouse (feet)	distance it can be seen (miles)
20	6
45	9
70	11
95	13

Solution

- a. Not proportional since the ratios of width to height are not all equivalent.
- b. Not proportional since the ratios of height to distance are not all equivalent.

(From Unit 2, Lesson 7.)

Lesson 11: Interpreting Graphs of Proportional Relationships

Goals

- Create the graph of a proportional relationship given only one pair of values, by drawing the line that connects the given point and $(0, 0)$.
- Identify the constant of proportionality from the graph of a proportional relationship.
- Interpret (orally and in writing) points on the graph of a proportional relationship.

Learning Targets

- I can draw the graph of a proportional relationship given a single point on the graph (other than the origin).
- I can find the constant of proportionality from a graph.
- I understand the information given by graphs of proportional relationships that are made of up of points or a line.

Lesson Narrative

In the previous lesson students learned that the graph of a proportional relationship lies on a line through the origin. (Students should come to use and understand “the origin” to mean $(0, 0)$.) In this lesson, they start to make connections between the graph and the context modeled by the proportional relationship, and between the graph and the equation for the proportional relationship. Given a graph, they think about what situation it might represent and learn the importance of being precise about saying which quantities are represented on each axis (MP6). They interpret the meaning of the point $(1, k)$ on the graph both in term of the constant of proportionality k in the equation $y = kx$ and in terms of a constant rate in the context.

Alignments

Addressing

- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.
- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

- 7.RP.A.2.d: Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR5: Co-Craft Questions
- Think Pair Share

Required Materials

Rulers

Student Learning Goals

Let's read stories from the graphs of proportional relationships.

11.1 What Could the Graph Represent?

Warm Up: : 5 minutes

This warm-up gives students an opportunity to think back to examples of proportional relationships they have encountered. Students are given a minute to think of some situations that could be represented by a graph. Several of their ideas should be shared with the class before students answer the remaining questions. During discussion, the characteristics of a graph of a proportional relationship should be reinforced.

Addressing

- 7.RP.A

Launch

Tell students that they will look at an unlabeled graph, and their job is to think of a situation that the graph could represent. Display the problem for all to see and give 1 minute of quiet think time. Ask students to give a signal when they have thought of a situation.

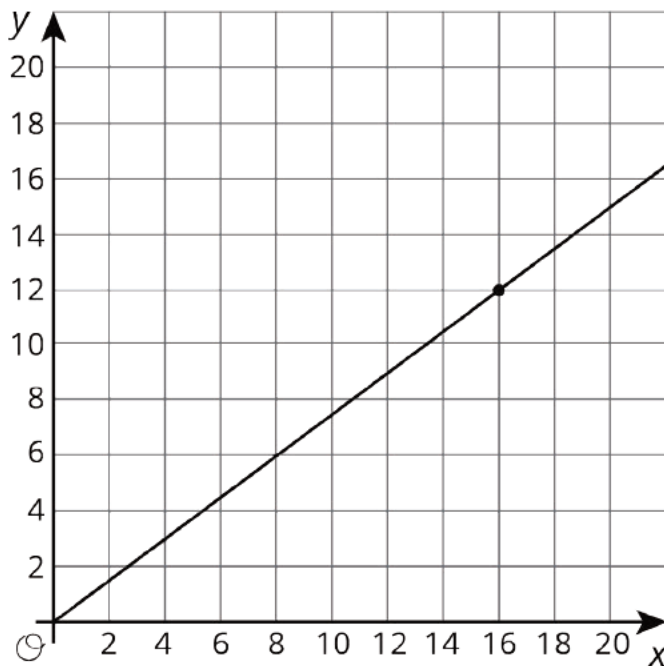
Invite some students to share their ideas and record the responses for all to see. (The purpose of this is to provide some inspiration to students who haven't come up with anything.) Ask students how they know all of the relationships are proportional. (Responses might include: when one value is 0 the other is 0, the situation

involves equivalent ratios, or that any pair of values in the relationship has the same unit rate.)

Ask students to complete the rest of the questions.

Student Task Statement

Here is a graph that represents a proportional relationship.



1. Invent a situation that could be represented by this graph.
2. Label the axes with the quantities in your situation.
3. Give the graph a title.
4. There is a point on the graph. What are its coordinates? What does it represent in your situation?

Student Response

Answers vary. Possible response:

1. A car is moving at a constant speed. We could say that its speed is $\frac{3}{4}$ miles per minute or its pace is $\frac{4}{3}$ minutes per mile.
2. The horizontal axis is labeled time (minutes) and the vertical axis is labeled distance (miles).
3. Distance Traveled by a Car and How Much Time It Takes
4. The coordinates of the point are (16, 12). In this situation, it means that the car travels 12 miles in 16 minutes.

Activity Synthesis

Ask a few students to share their situations and other responses. After each, ask the class if they need more information to understand the situation. After a few students have shared, ask the class to think about how all the situations were different and what they had in common. What sorts of things are always true about proportional relationships? Some possible responses might be:

- When one quantity is 0, the other is also 0.
- There is always the same amount of one quantity for every 1 of the other quantity.
- Context-specific considerations like constant speed, the same taste, or the same color.

Remind students that a coordinate point, (x, y) is made up of the “ x -coordinate” and the “ y -coordinate.”

11.2 Tyler's Walk

: 15 minutes

This activity is intended to further students' understanding of the graphs of proportional relationships in the following respects:

- points on the graph of a proportional relationship can be interpreted in the context represented (MP2)
- for these points, the quotient of the coordinates is—excepting $(0, 0)$ —the constant of proportionality

- if the first coordinate is 1, then the corresponding coordinate is k , the constant of proportionality.

Students explain correspondences between parts of the table and parts of the graph. The graph is simple so that students can focus on what a point means in the situation represented. Students need to realize, however, that the axes are marked in 10-unit intervals. The discussion questions are opportunities for students to construct viable arguments and critique the reasoning of others (MP3).

Addressing

- 7.RP.A.2

Instructional Routines

- MLR5: Co-Craft Questions
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 5 minutes of quiet work time followed by students discussing responses with a partner, followed by whole-class discussion.

Support for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.

Supports accessibility for: Organization; Attention

Support for English Language Learners

Speaking, Reading: MLR5 Co-Craft Questions. To help students make sense of graphs of proportional relationships, start by displaying only the first line of this task ("Tyler was at the amusement park. He walked at a steady pace from the ticket booth to the bumper cars.") and the graph. Ask students to write down possible mathematical questions that could be asked about the situation. Invite students to compare their questions before revealing the remainder of the questions. Listen for and amplify any questions involving correspondences between parts of the table and parts of the graph. This helps students produce the language of mathematical questions and talk about the relationship between distance and time.

Design Principle(s): Maximize meta-awareness; Support sense-making

Anticipated Misconceptions

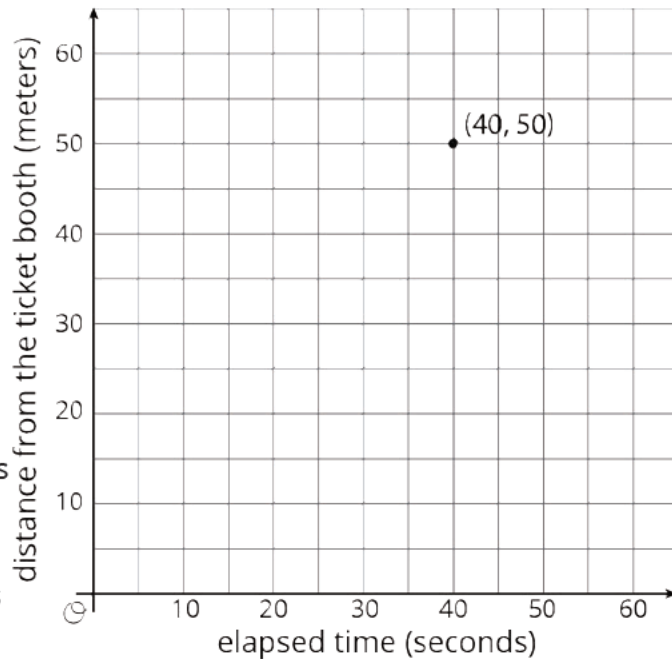
These questions can be used for discussion or for students who need scaffolding.

- "What quantities are shown in the graph?" (Distance in meters that Tyler is from the ticket booth and time elapsed in seconds since he started walking.)
- "How far is the ticket booth from the bumper cars?" (50 meters, assuming that Tyler walked in a straight line.) This is an opportunity for attention to precision (MP6) and making explicit assumptions about a situation (MP4).
- "Do the values in your table show a proportional relationship? How do you know?" (Based on prior lessons in this unit, students should identify the relationship as proportional because for every point the unit rate is the same.)
- "What do the coordinates of the points on the graph show?" (The first coordinate gives amount of time in seconds that elapsed since Tyler started walking. Its corresponding second coordinate shows how many meters away from the ticket booth Tyler was at the corresponding time, assuming that Tyler walked in a straight line.)

Student Task Statement

Tyler was at the amusement park. He walked at a steady pace from the ticket booth to the bumper cars.

1. The point on the graph shows his arrival at the bumper cars. What do the coordinates of the point tell us about the situation?



2. The table representing Tyler's walk shows other values of time and distance. Complete the table. Next, plot the pairs of values on the grid.

3. What does the point $(0, 0)$ mean in this situation?

time (seconds)	distance (meters)
0	0
20	25
30	37.5
40	50
1	

4. How far away from the ticket booth was Tyler after 1 second? Label the point on the graph that shows this information with its coordinates.

5. What is the constant of proportionality for the relationship between time and distance? What does it tell you about Tyler's walk? Where do you see it in the graph?

Student Response

1. 40 seconds after Tyler started walking, he was 50 meters from the ticket booth. The 40 represents the elapsed time in seconds since Tyler started walking away from the ticket booth; the 50 represents Tyler's distance in meters from the ticket booth at that time.
2. Students should write 1.25 in the empty cell of the table and plot $(0, 0)$, $(1, 1.25)$, $(20, 25)$, and $(30, 37.5)$.
3. Before any time passed, there was no distance between Tyler and the ticket booth.
4. Tyler was 1.25 meters from the ticket booth after 1 second. The corresponding point is $(1, 1.25)$.
5. The constant of proportionality is 1.25. It tells us that Tyler is walking at a speed of 1.25 meters per second. It appears as the second coordinate in $(1, 1.25)$.

Are You Ready for More?

If Tyler wanted to get to the bumper cars in half the time, how would the graph representing his walk change? How would the table change? What about the constant of proportionality?

Student Response

The graph would be steeper. For the same first coordinate, the second coordinate would be twice as big as in the original situation. The table would include $(0, 0)$, $(20, 40)$, and $(1, 2.5)$. Tyler would already arrive at the bumper cars after 20 seconds and his speed would be 2.5 meters per second.

Activity Synthesis

After students work on the task, it is important to discuss how the axis labels and the description in the task statement help us interpret points on the graph.

Consider asking these questions:

- "What quantities are shown in the graph?" (Distance in meters that Tyler is from the ticket booth and time elapsed in seconds since he started walking.)

- "How far is the ticket booth from the bumper cars?" (50 meters, assuming that Tyler walked in a straight line.) This is an opportunity for attention to precision (MP6) and making explicit assumptions about a situation (MP4).
- "Do the values in your table show a proportional relationship? How do you know?" (Based on prior lessons in this unit, students should identify the relationship as proportional because for every point the unit rate is the same.)
- "What do the coordinates of the points on the graph show?" (The first coordinate gives amount of time in seconds that elapsed since Tyler started walking. Its corresponding second coordinate shows how many meters away from the ticket booth Tyler was at the corresponding time, assuming that Tyler walked in a straight line.)

Ask students for the equation of this proportional relationship. Finally ask where students see k , the constant of proportionality, in each representation, the equation, the graph, the table, and the verbal description of the situation.

11.3 Seagulls Eat What?

: 15 minutes (there is a digital version of this activity)

In this activity, make sure students understand what it means when we draw a solid line instead of just points in a straight line to represent the proportional relationship.

Addressing

- 7.RP.A.2.d

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- Think Pair Share

Launch

Keep students in the same groups of 2. Give 5 minutes of quiet work time followed by partner and whole-class discussion.

Support for English Language Learners

Writing, Conversing: MLR1 Stronger and Clearer Each Time. Use this routine to help students improve their writing, by providing them with multiple opportunities to clarify their explanations through conversation. At the appropriate time, give students time to meet with 2–3 partners to share their response to the final question. Students should first check to see if they agree with each other about what the value of k means in the given context. Provide listeners with prompts for feedback that will help their partner add detail to strengthen and clarify their ideas. For example, students can ask their partner, “How did you calculate k ?”, “Why did you do...?”, or “What does k mean in this context?” Then provide students with 3–4 minutes to revise their initial draft based on feedback from their peers.

Design Principle(s): Optimize output (for justification); Maximize meta-awareness

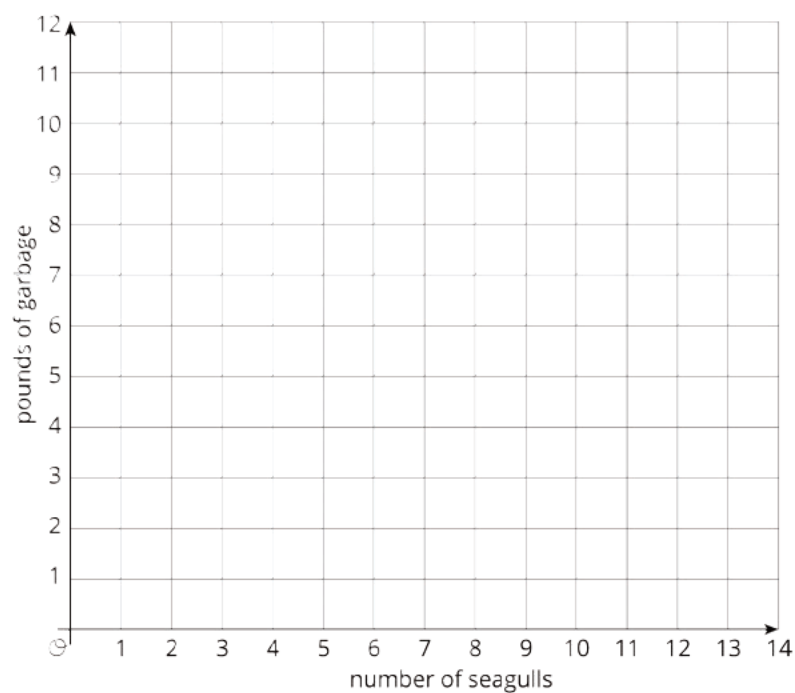
Anticipated Misconceptions

If students struggle to find k , encourage them to create a table with a few rows in it and ask them how they can use the table to find k .

Student Task Statement

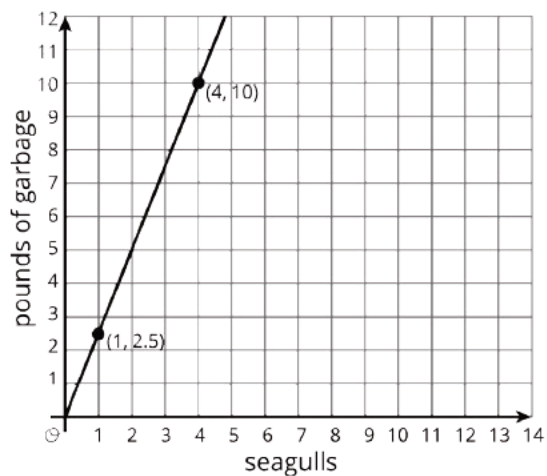
4 seagulls ate 10 pounds of garbage. Assume this information describes a proportional relationship.

1. Plot a point that shows the number of seagulls and the amount of garbage they ate.
2. Use a straight edge to draw a line through this point and $(0, 0)$.
3. Plot the point $(1, k)$ on the line. What is the value of k ? What does the value of k tell you about this context?



Student Response

1. Point $(4, 10)$ is plotted.
2. Line is drawn.
3. Point $(1, 2.5)$ is plotted. The value of k , 2.5, tells you the number of pounds of garbage consumed per seagull.



Activity Synthesis

Invite students to share their value and interpretation of k . Ask them for different ways to express this information. (Each seagull eats 2.5 pounds of garbage. Or: The rate of garbage consumption is 2.5 pounds per seagull.)

Ask students if it is possible to interpret the meaning of each point on the solid line. (No, only whole numbers of seagulls make sense.) Ask, why it is still useful to draw in the line. "How can it help us to learn more about the situation?" (It helps us to easily find out how much garbage different numbers of seagulls eat. It also helps us to estimate the value of k .)

Lesson Synthesis

Revisit the key insights from this lesson:

1. We can interpret points on a graph in terms of the context it represents.
2. The y -value that goes with the x -value of 1 is special because it shows us the value of the constant of proportionality. It can be seen using a table or a graph.

Display the completed graph from one of the activities. Choose a point on the graph and ask students to interpret its coordinates in the situation. Then choose the point with x -coordinate 1 and ask about the significance of its y -coordinate.

11.4 Filling a Bucket

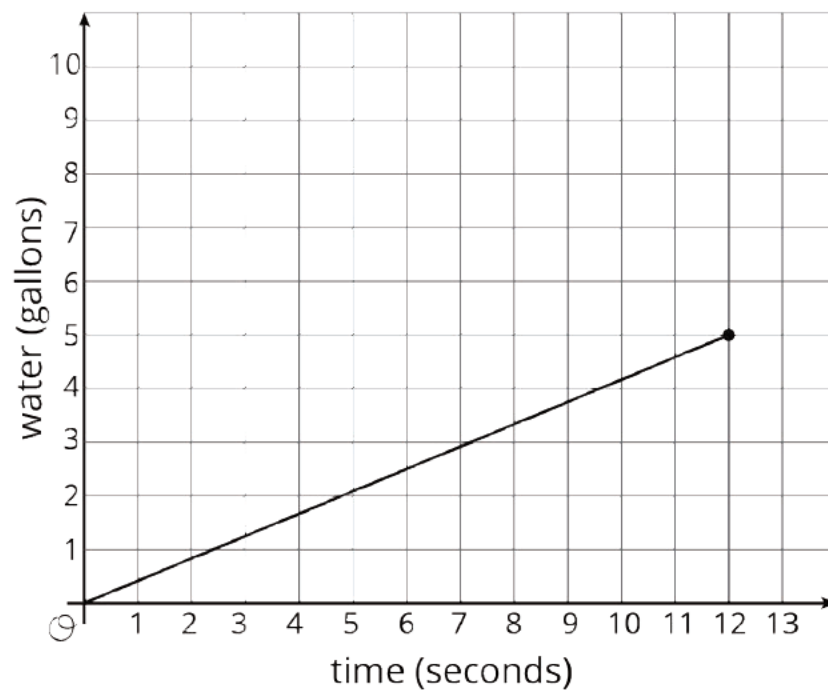
Cool Down : 5 minutes

Addressing

- 7.RP.A

Student Task Statement

Water runs from a hose into a bucket at a steady rate. The amount of water in the bucket for the time it is being filled is shown in the graph.



1. The point $(12, 5)$ is on the graph. What do the coordinates tell you about the water in the bucket?
2. How many gallons of water were in the bucket after 1 second? Label the point on the graph that shows this information.

Student Response

1. After 12 seconds, there were 5 gallons of water in the bucket.
2. $\frac{5}{12}$ or equivalent. The point $(1, \frac{5}{12})$ should be labeled.

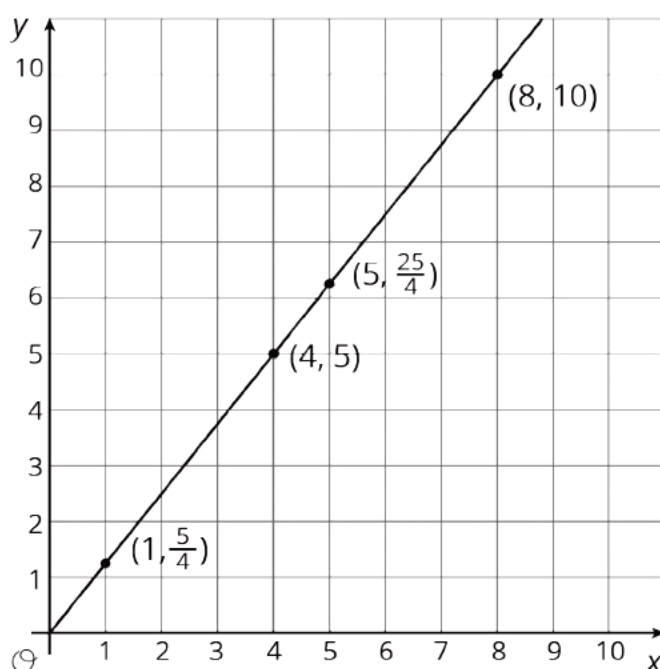
Student Lesson Summary

For the relationship represented in this table, y is proportional to x . We can see in the table that $\frac{5}{4}$ is the constant of proportionality because it's the y value when x is 1.

The equation $y = \frac{5}{4}x$ also represents this relationship.

x	y
4	5
5	$\frac{25}{4}$
8	10
1	$\frac{5}{4}$

Here is the graph of this relationship.



If y represents the distance in feet that a snail crawls in x minutes, then the point $(4, 5)$ tells us that the snail can crawl 5 feet in 4 minutes.

If y represents the cups of yogurt and x represents the teaspoons of cinnamon in a recipe for fruit dip, then the point $(4, 5)$ tells us that you can mix 4 teaspoons of cinnamon with 5 cups of yogurt to make this fruit dip.

We can find the constant of proportionality by looking at the graph, because $\frac{5}{4}$ is the y -coordinate of the point on the graph where the x -coordinate is 1. This could mean the snail is traveling $\frac{5}{4}$ feet per minute or that the recipe calls for $1\frac{1}{4}$ cups of yogurt for every teaspoon of cinnamon.

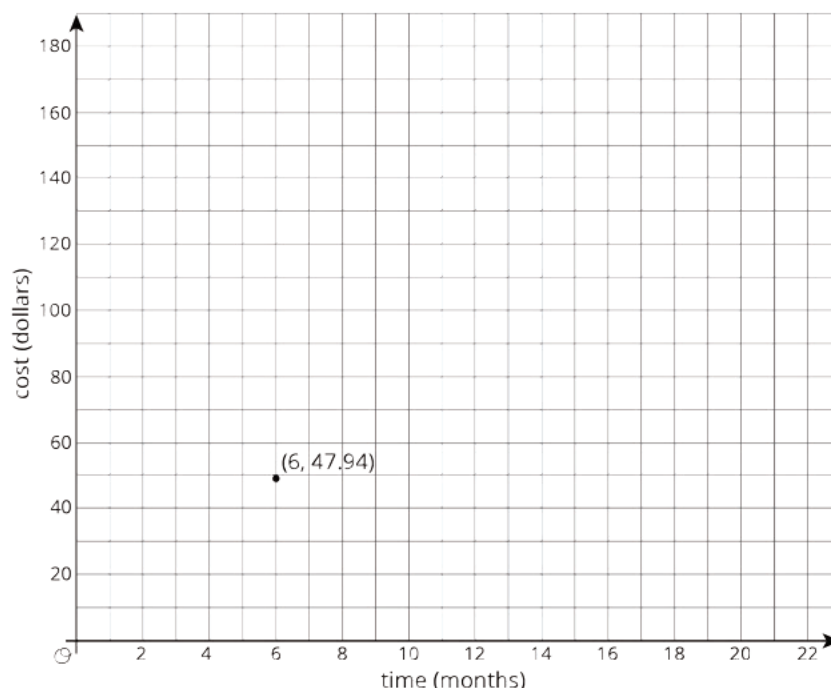
In general, when y is proportional to x , the corresponding constant of proportionality is the y -value when $x = 1$.

Lesson 11 Practice Problems

1. Problem 1

Statement

There is a proportional relationship between the number of months a person has had a streaming movie subscription and the total amount of money they have paid for the subscription. The cost for 6 months is \$47.94. The point $(6, 47.94)$ is shown on the graph below.

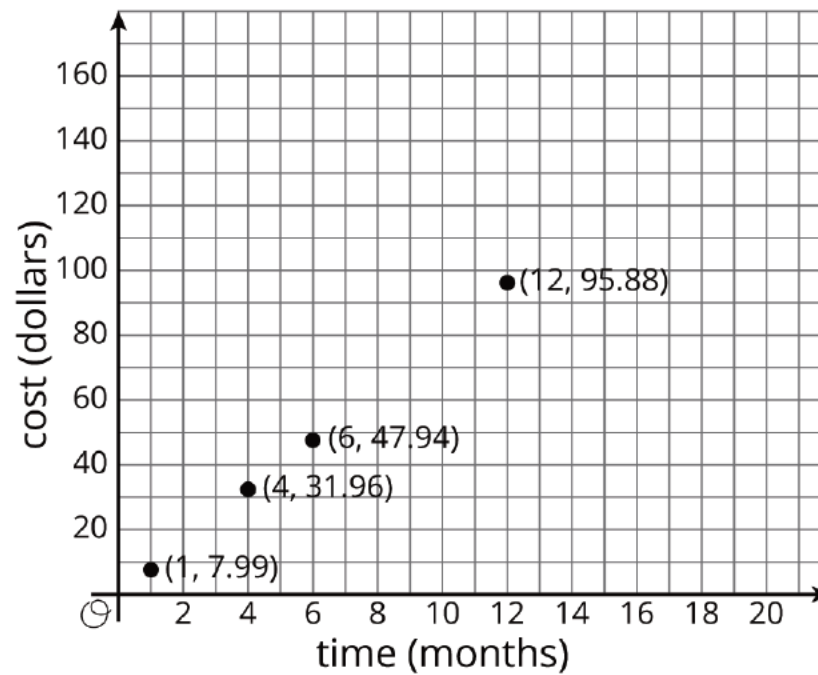


- What is the constant of proportionality in this relationship?
- What does the constant of proportionality tell us about the situation?
- Add at least three more points to the graph and label them with their coordinates.
- Write an equation that represents the relationship between C , the total cost of the subscription, and m , the number of months.

Solution

a. \$7.99

b. The movie streaming service costs \$7.99 for one month of service.



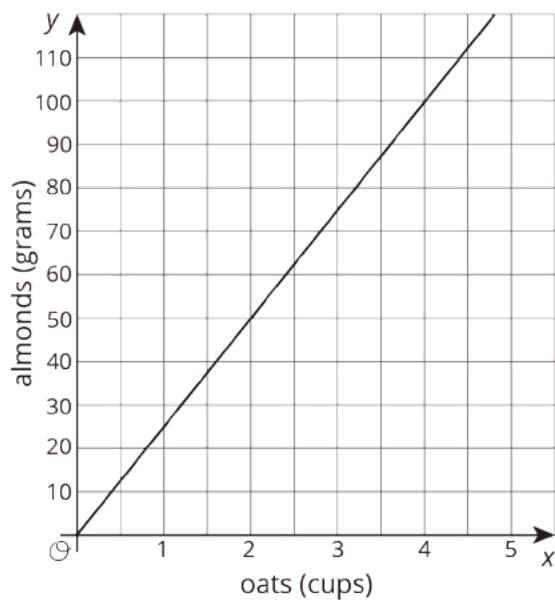
c.

d. $C = 7.99m$

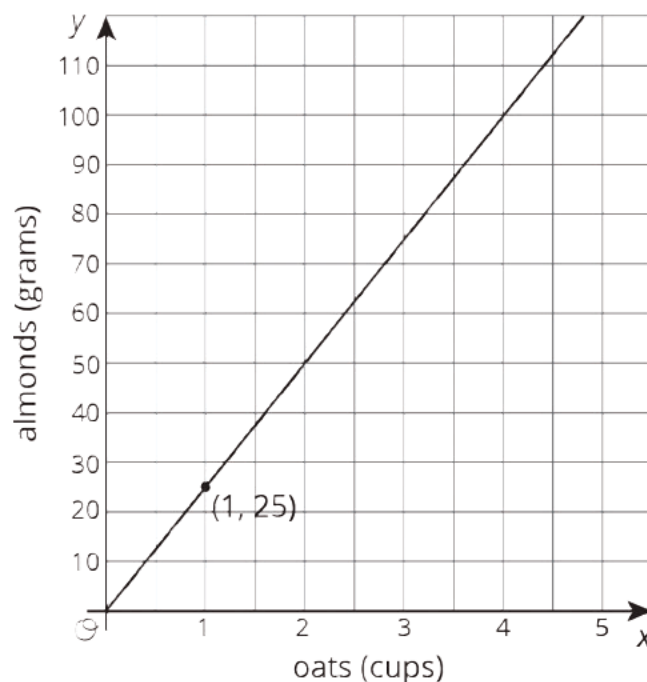
2. Problem 2

Statement

The graph shows the amounts of almonds, in grams, for different amounts of oats, in cups, in a granola mix. Label the point $(1, k)$ on the graph, find the value of k , and explain its meaning.



Solution



The point $(1, 25)$ is on the graph. It means that for each cup of oats there are 25 grams of almonds in the granola mix.

3. Problem 3

Statement

To make a friendship bracelet, some long strings are lined up then taking one string and tying it in a knot with each of the other strings to create a row of knots. A new string is chosen and knotted with the all the other strings to create a second row. This process is repeated until there are enough rows to make a bracelet to fit around your friend's wrist.

Are the number of knots proportional to the number of rows? Explain your reasoning.

Solution

Yes, since each row will have the same number of knots in it, the number of knots will always be a multiple of the number of rows.

(From Unit 2, Lesson 9.)

4. Problem 4



Statement

What information do you need to know to write an equation relating two quantities that have a proportional relationship?

Solution

A constant of proportionality and variables for the quantities.

(From Unit 2, Lesson 9.)

Lesson 12: Using Graphs to Compare Relationships

Goals

- Create and interpret graphs that show two different proportional relationships on the same axes.
- Generalize (orally and in writing) that when two different proportional relationships are graphed on the same axes, the steeper line has the greater constant of proportionality.

Learning Targets

- I can compare two, related proportional relationships based on their graphs.
- I know that the steeper graph of two proportional relationships has a larger constant of proportionality.

Lesson Narrative

In this lesson students continue their work with interpreting graphs of proportional relationships. An important goal of the lesson is for students to start to interpret the steepness of the graph in terms of the context. They use distance-versus-time graphs to decide which person from a group is going the fastest. They also work with graphs where the scale is not specified on each axis, and realize that they can still use graphs to compare rates.

Alignments

Building On

- 5.NF.B: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Addressing

- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

Building Towards

- 7.RP.A.1: Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{\frac{1}{2}}{\frac{1}{4}}$ miles per hour, equivalently 2 miles per hour.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports
- Number Talk

Required Materials

Colored pencils

Rulers

Required Preparation

Have available the information from the activity "Tyler's Walk" from the previous lesson.

Student Learning Goals

Let's graph more than one relationship on the same grid.

12.1 Number Talk: Fraction Multiplication and Division

Warm Up: : 5 minutes

The purpose of this Number Talk is to elicit strategies and understandings students have for multiplying and dividing fractions. These understandings help students develop fluency and will be helpful throughout this unit when students find constants of proportionality from graphs, tables, and equations. While four problems are given, it may not be possible to share every strategy. Consider gathering only two or three different strategies per problem, saving most of the time for the final question.

Building On

- 5.NF.B

Building Towards

- 7.RP.A.1

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

Support for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Student Task Statement

Find each product or quotient mentally.

$$\frac{2}{3} \cdot \frac{1}{2}$$

$$\frac{4}{3} \cdot \frac{1}{4}$$

$$4 \div \frac{1}{5}$$

$$\frac{9}{6} \div \frac{1}{2}$$

Student Response

- $\frac{1}{3}$. Explanations vary. Sample response: Half of two-thirds is one third.
- $\frac{1}{3}$. Explanations vary. Sample response: One fourth of four-thirds is one third.
- 20. Explanations vary. Sample response: There are 5 fifths in 1, so there are 20 fifths in 4.
- 3. Explanations vary. Sample response: $\frac{9}{6}$ is $\frac{3}{2}$. Dividing by $\frac{1}{2}$ is the same as multiplying by 2. Twice $\frac{3}{2}$ is 3.

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their answers and explanations for all to see.

To involve more students in the conversation, consider asking:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone solve the problem the same way but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to ____'s strategy?"
- "Do you agree or disagree? Why?"

If time permits, ask students if they notice any connections between the problems. Have them share any relationships they notice.

Support for English Language Learners

Speaking: MLR8 Discussion Supports: Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because . . ." or "I noticed ____ so I" Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Design Principle(s): Optimize output (for explanation)

12.2 Race to the Bumper Cars

: 15 minutes (there is a digital version of this activity)

In this activity, students graph three time-distance relationships along with the one from the previous lesson, “Tyler’s Walk.” One of these is not a proportional relationship, so students must pay close attention to the quantities represented. The purpose of this activity is to give students many opportunities to connect the different features of a graph with parts of the situation it represents. In particular, they attach meaning to any point that is on a graph, and they interpret the meaning of the distance when the time is 1 second as both the constant of proportionality of the relationship and the person’s speed in the context in meters per second. Comparing different but related situations and their graphs supports students as they make sense of the situation.

Addressing

- 7.RP.A.2

Launch

Arrange students in groups of 2–3. Provide access to colored pencils and rulers.

Tell students that this activity is tied to the activity titled “Tyler’s Walk” from the previous lesson. All references to Tyler going to the bumper cars come from the statements in that activity.

The digital version has an applet with options to change line colors and hide points. You may want to demonstrate the applet before students use it, perhaps graphing Tyler’s data from the previous activity together. Note: the applet can graph lines, rays, or segments. Your class can decide how to represent the data.

Support for Students with Disabilities

Representation: Internalize Comprehension. Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, use a different color for each person to highlight the connection between the table, graph, and constant of proportionality.

Supports accessibility for: Visual-spatial processing

Student Task Statement

Diego, Lin, and Mai went from the ticket booth to the bumper cars.

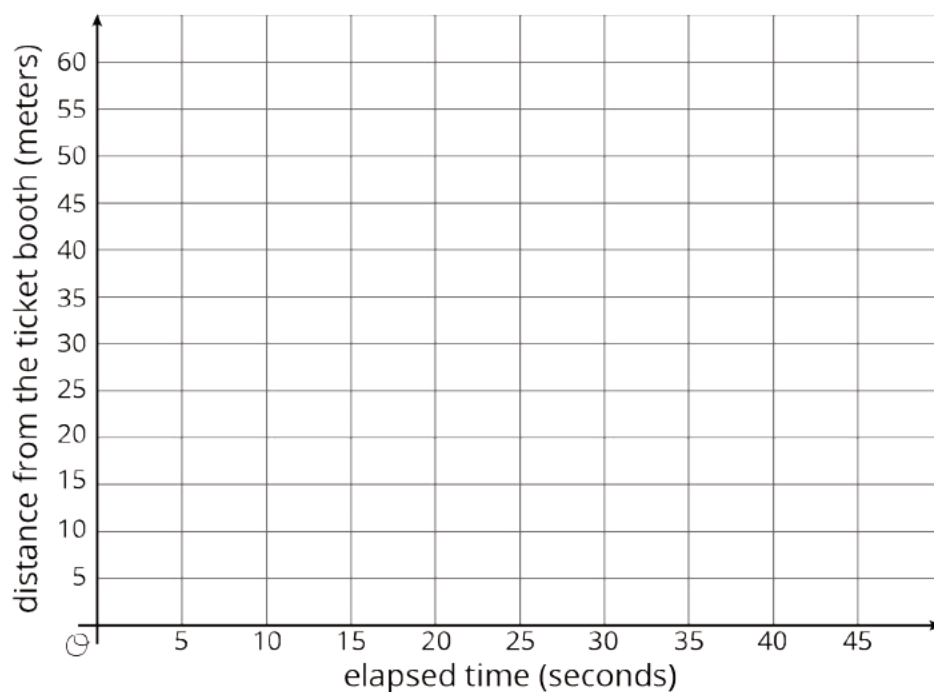
1. Use each description to complete the table representing that person's journey.
 - a. Diego left the ticket booth at the same time as Tyler. Diego jogged ahead at a steady pace and reached the bumper cars in 30 seconds.
 - b. Lin left the ticket booth at the same time as Tyler. She ran at a steady pace and arrived at the bumper cars in 20 seconds.
 - c. Mai left the booth 10 seconds later than Tyler. Her steady jog enabled her to catch up with Tyler just as he arrived at the bumper cars.

Diego's time (seconds)	Diego's distance (meters)
0	
15	
30	50
1	

Lin's time (seconds)	Lin's distance (meters)
	0
	25
20	50
1	

Mai's time (seconds)	Mai's distance (meters)
	0
	25
40	50
1	

2. Using a different color for each person, draw a graph of all four people's journeys (including Tyler's from the other day).



3. Which person is moving the most quickly? How is that reflected in the graph?

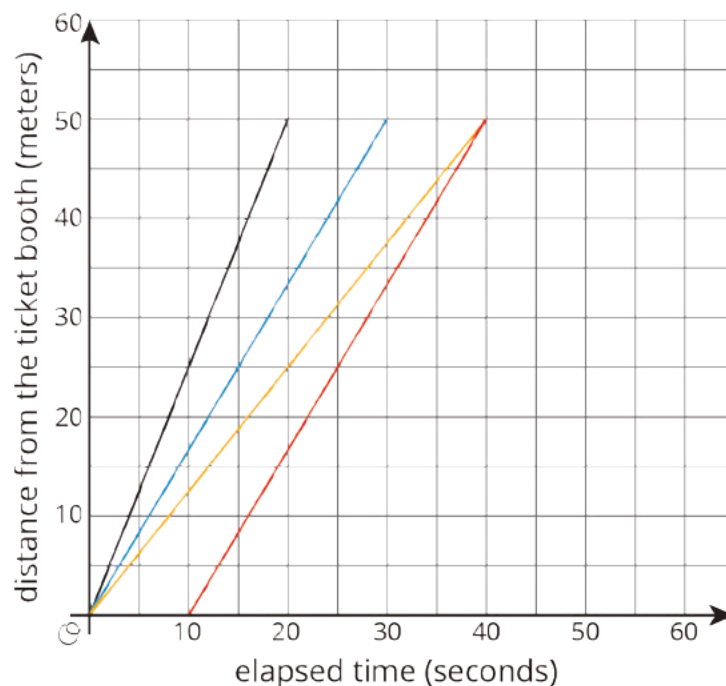
Student Response

1. Tables:

Diego's time (seconds)	Diego's distance (meters)
0	0
15	25
30	50
1	$\frac{5}{3}$

Lin's time (seconds)	Lin's distance (meters)
0	0
10	25
20	50
1	2.5

Mai's time (seconds)	Mai's distance (meters)
10	0
25	25
40	50
1	0



2.

3. Lin left the booth at the same time as Tyler but reached the bumper cars in 20 seconds; her time of arrival at the bumper cars is shown by the point (20, 50). Diego left at the same time as the first three, but it took him 30 seconds to reach the

bumper cars. His arrival at the bumper cars is shown by the point $(30, 50)$. Tyler's arrival at the bumper cars is shown by the point $(40, 50)$.

We don't know anything about Mai's distance from the ticket booth before she leaves 10 seconds after Tyler, so her graph does not include any points to the left of $x = 10$. The point $(10, 0)$ is on her graph. The pair $(40, 50)$ in her table shows that she and Tyler arrived at the bumper cars at the same time, 40 seconds after Tyler started walking, and that the point $(40, 50)$ is on her graph. Mai's graph does not represent a proportional relationship: The distance she travels is not proportional to time elapsed since Tyler left the ticket booth. (It is, however, proportional to time elapsed since *Mai* left the booth.)

- Lin is moving most quickly. Explanations vary for how you can see this on the graph. Sample responses:
- At any given time between 0 and 20 seconds, she has traveled the farthest.
- For any given distance between 0 and 50 meters, it takes her the least amount of time to get there.
- She is traveling at $2\frac{1}{2}$ meters per second, while Diego is traveling at $1\frac{2}{3}$ meters per second and Tyler at $1\frac{1}{4}$ meters per second. You can see this on the graph by looking at the points with x -coordinate 1.

Are You Ready for More?

Write equations to represent each person's relationship between time and distance.

Student Response

Let t represent elapsed time in seconds and d represent distance from the ticket booth in meters. Lin: $d = 2.5t$ or equivalent. Diego: $d = \frac{5}{3}t$ or equivalent. Tyler: $d = 1.25t$ or equivalent. Mai: $d = \frac{5}{3}(t - 10)$ or equivalent.

Activity Synthesis

Students may expect the graphs to intersect because everyone arrives at the same location. However, they did not arrive there at the same time (with the exception of Tyler and Mai). Because all characters traveled the same distance from the ticket booth and no further, the endpoints of their graphs lie on the same horizontal line $y = 50$, that is, they have the same y -coordinate. The points will vary in position from right to left depending on the number of seconds after Tyler left the ticket booth it took each person to arrive at the bumper cars. Note this features in a whole-class discussion.

The most important goals of the discussion are to attach meaning to any point that is on a graph, and to interpret the meaning of the distance when the time is 1 second as both the constant of proportionality of the relationship and the person's speed in the context in meters per second.

These questions may be used to facilitate the class discussion:

- "For each graph that shows a proportional relationship, what is the constant of proportionality?" (Tyler's was 1.25, Diego's was $1\frac{2}{3}$, Lin's was 2.5.)
- "How did you find them?" (Answers will vary, but students could have divided a y -coordinate by its associated x -coordinate.)
- "Where do constants of proportionality occur in the tables, and where do they occur on the graphs?" (They occur in the tables as the values in the second column that correspond to the value of 1 in the first column; on the graphs as the y -coordinate of points where the x -coordinate is 1.)
- "Which is the only graph that does not represent a proportional relationship?"
- "A classmate argues that Mai's graph must represent a proportional relationship, because she jogged at a steady rate. How do you answer?" (Mai's graph does not pass through the origin, so it does not represent a proportional relationship. That is, the distance she traveled is not proportional to the time elapsed. What does "time elapsed" mean? It is time elapsed since Tyler left the ticket booth. However, distance vs. time elapsed since *Mai* left the ticket booth until she arrived at the bumper cars is a proportional relationship.)

12.3 Space Rocks and the Price of Rope

: 10 minutes (there is a digital version of this activity)

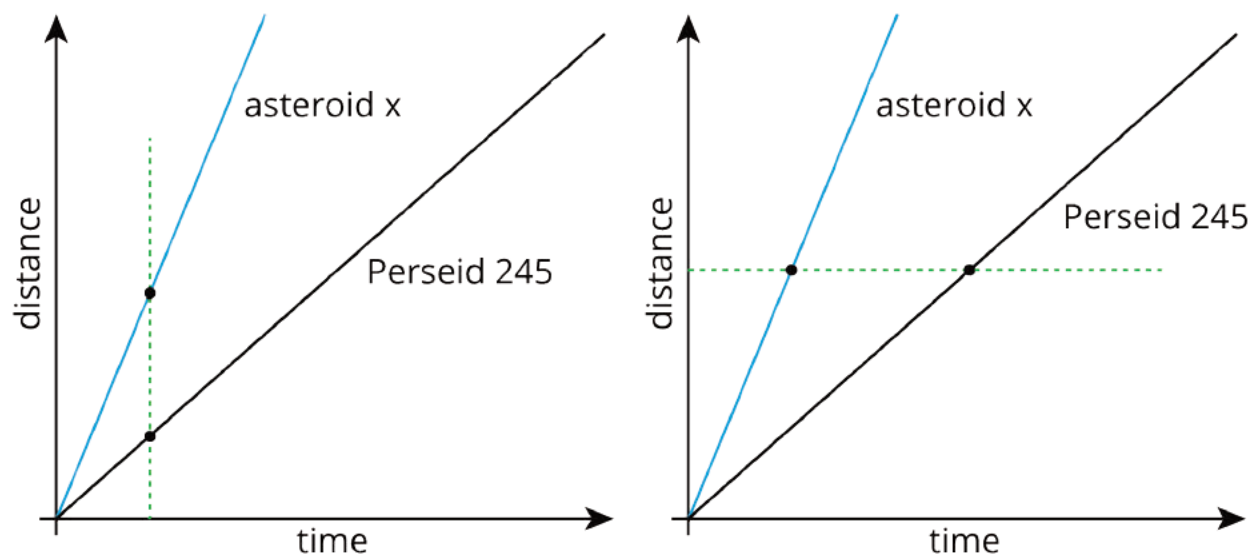
This activity is intended to help students interpret features of graphs in terms of the proportional relationships they represent. This task highlights two fundamental ideas:

- The steeper the graph, the greater the constant of proportionality.
- If the graph represents distance of an object vs. time, the constant of proportionality is the speed of the object.

Students can reason abstractly (MP2) by picking an arbitrary time and comparing the corresponding distances, or picking an arbitrary distance and comparing the corresponding times. Ideally, both of these ways of reasoning are shared in a whole-class discussion of the task because they will be needed in future work.

While students work, monitor for these approaches:

- At the same time, Asteroid x has traveled a greater distance (as in the graph with the vertical dashed line). We can't tell exactly where 1 unit of time is on the graph, but wherever it is, we can tell that Asteroid x has covered a greater distance.



- Asteroid x takes less time than Perseid 245 to cover the same distance. See the graph with the horizontal dashed line.
- When distance is proportional to time and distance is graphed against time, the constant of proportionality represents the magnitude of the speed (unit of distance traveled per unit of time). It was shown in the previous activity that a steeper line has

a greater constant of proportionality. Therefore, a line steeper than Perseid 245's line represents a greater speed.

Addressing

- 7.RP.A.2

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports

Launch

Keep students in the same groups of 2–3.

If using the digital activity, have students explore the applet to develop their reasoning around the following two concepts:

- The steeper the graph, the greater the constant of proportionality.
- If the graph represents distance vs. time, the constant of proportionality is the speed.

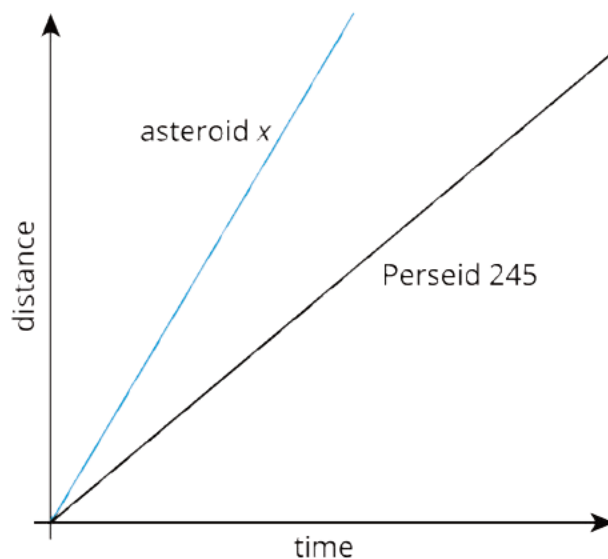
Support for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example: “First, I ____ because _____. Then, I....,” “I noticed ____ so I....,” and “I tried ____ and what happened was....”

Supports accessibility for: Language; Social-emotional skills

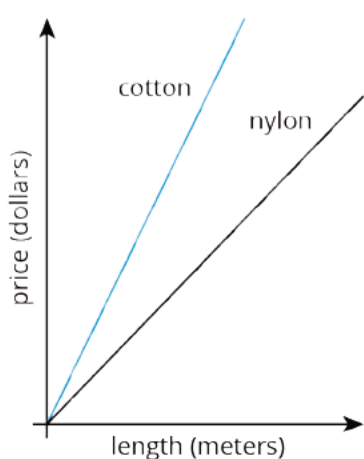
Student Task Statement

1. Meteoroid Perseid 245 and Asteroid x travel through the solar system. The graph shows the distance each traveled after a given point in time.



Is Asteroid x traveling faster or slower than Perseid 245? Explain how you know.

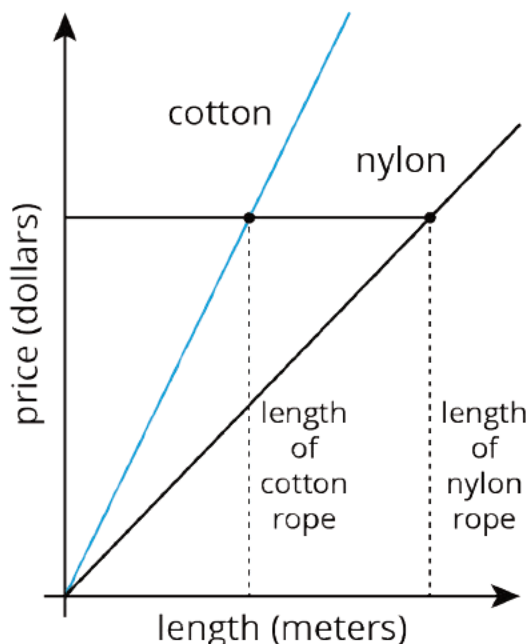
2. The graph shows the price of different lengths of two types of rope.



If you buy \$1.00 of each kind of rope, which one will be longer? Explain how you know.

Student Response

1. Asteroid x is traveling faster than Perseid 245. Explanations vary. Students might consider the same time on each graph and compare the distance traveled, they might consider the same distance traveled on each graph and compare the time it took, or they might reason about each object's speed in distance units per time unit.
2. The nylon rope would be longer. The graph shows that a greater length of nylon rope can be purchased for the same price as a shorter length of cotton rope.



Activity Synthesis

It is important that students not assume “steeper always means faster,” but that they understand why it is in this case by reasoning abstractly and attending to the referents for points on the graphs. If the same relationships were graphed with distance on the horizontal axis and time on the vertical axis, a steeper line would indicate a slower speed. If the same relationships were graphed on separate axes, their scales could be different. Because the graphs share the same axes, it is implicit that comparisons between them occur relative to the same units.

Try to find students who took each approach, and invite each to share reasoning with the class. Important points to highlight are:

- A steeper graph has a larger constant of proportionality and a larger constant of proportionality will have a steeper graph. (“When you look at two graphs of a proportional relationship, how can you tell which one has a greater constant of proportionality?”)

- In a distance vs. time graph, a steeper graph indicates a greater speed. (“When you look at two distance vs. time graphs, how can you tell which represents an object traveling at a greater speed?”)
- The two graphs in question 1 can be considered from two perspectives:
 - Same point in time, noting that one object has covered a greater distance. (“Which student focused on the two objects at the same moment in time? How did they know that Asteroid x was traveling faster?”)
 - Same distance, noting that one object took less time. (“The other student did not focus on the same moment in time. What did they focus on in her explanation?”)

Support for English Language Learners

Speaking: MLR8 Discussion Supports. As students describe their explanations for which asteroid moved faster or slower, revoice student ideas to demonstrate mathematical language use. Press for details in students’ explanations by requesting that students challenge an idea, elaborate on an idea, or give an example. Show central concepts multi-modally by using different types of sensory inputs: acting out scenarios or inviting students to do so, using gestures, and talking about the context of moving objects. This will help students to produce and make sense of the language needed to communicate their own ideas.

Design Principle(s): Support sense-making, Optimize output (for explanation)

Lesson Synthesis

It would be helpful to display the completed graph from the "Race to the Bumper Car" activity during discussion; the one that shows the distance time relationship for Tyler, Diego, Lin, and Mai’s trip from the ticket booth to the bumper cars. If the right technology is available, display this graph to facilitate discussion: <https://www.desmos.com/calculator/dpthqt3sld>. Lines and points can be shown and hidden by clicking the folder icons along the left side of the window. The graphed points, once turned on, can be dragged along the lines. Turn the coordinates on and off by clicking on a point. The most useful aspect of using this dynamic graph for this discussion is that the graph can be zoomed in to easily see a point when its x-coordinate is 1.

Revisit the connections made in the this activity.

- "How can we tell from the graph who had gone the farthest after 10 seconds?" (Find points of the graph with first coordinate 10 and compare second coordinate.)
- "How can we tell from the graph how long it took everybody to get to the bumper cars?" (Find the points on the graph when the second coordinate is 50.)
- "How can we tell from the graph who was moving the fastest?" (Find the constant of proportionality k by locating the point $(1, k)$, or in this case, see which graph is the steepest.)

12.4 Revisiting the Amusement Park

Cool Down : 5 minutes

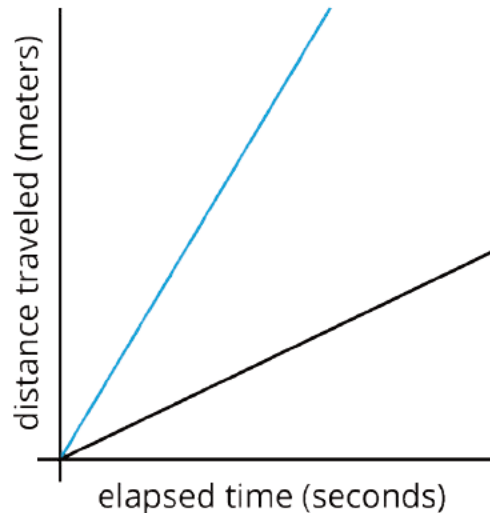
Addressing

- 7.RP.A.2

Student Task Statement

Noah and Diego left the amusement park's ticket booth at the same time. Each moved at a constant speed toward his favorite ride. After 8 seconds, Noah was 17 meters from the ticket booth, and Diego was 43 meters away from the ticket booth.

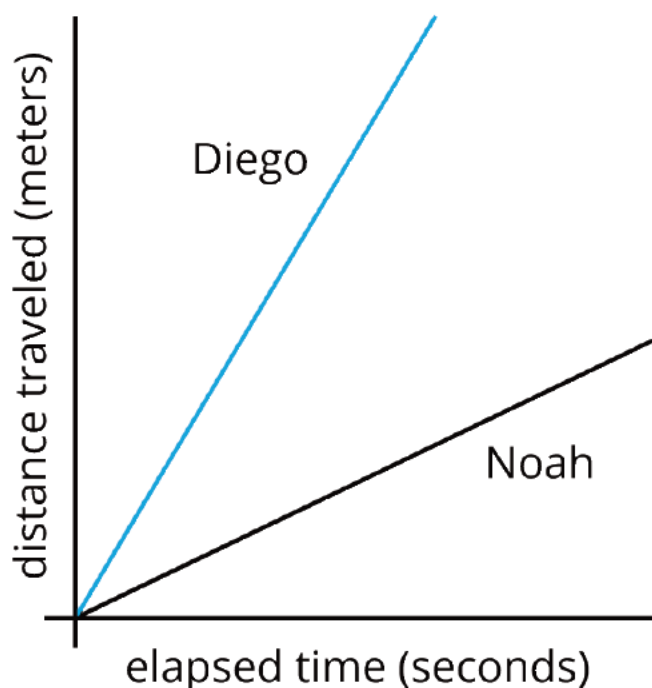
1. Which line represents the distance traveled by Noah, and which line represents the distance traveled by Diego? Label each line with one name.



2. Explain how you decided which line represents which person's travel.

Student Response

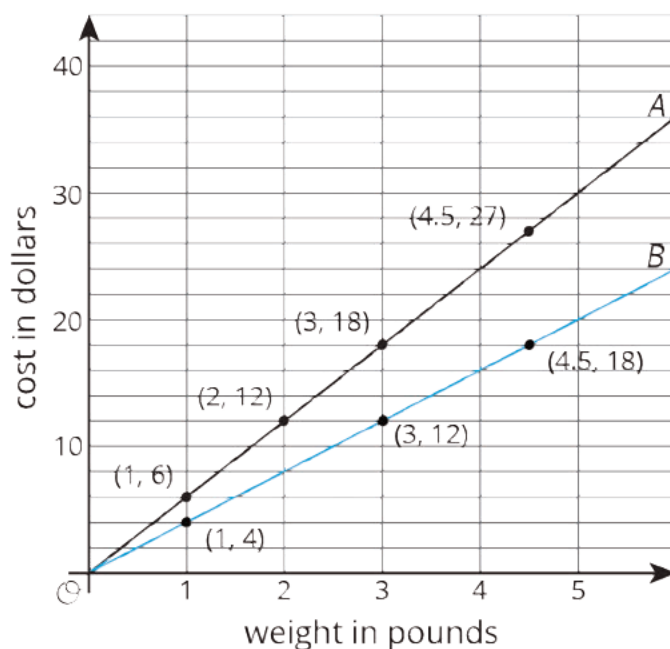
1. The steeper line represents the distance traveled by Diego.



2. Answers vary. Sample response: Diego had gone farther after 8 seconds. If you pick a time and look at which line represents a person who has gone farther, that is the steeper graph. So that must be Diego's line.

Student Lesson Summary

Here is a graph that shows the price of blueberries at two different stores. Which store has a better price?



We can compare points that have the same x value or the same y value. For example, the points $(2, 12)$ and $(3, 12)$ tell us that at store B you can get more pounds of blueberries for the same price.

The points $(3, 12)$ and $(3, 18)$ tell us that at store A you have to pay more for the same quantity of blueberries. This means store B has the better price.

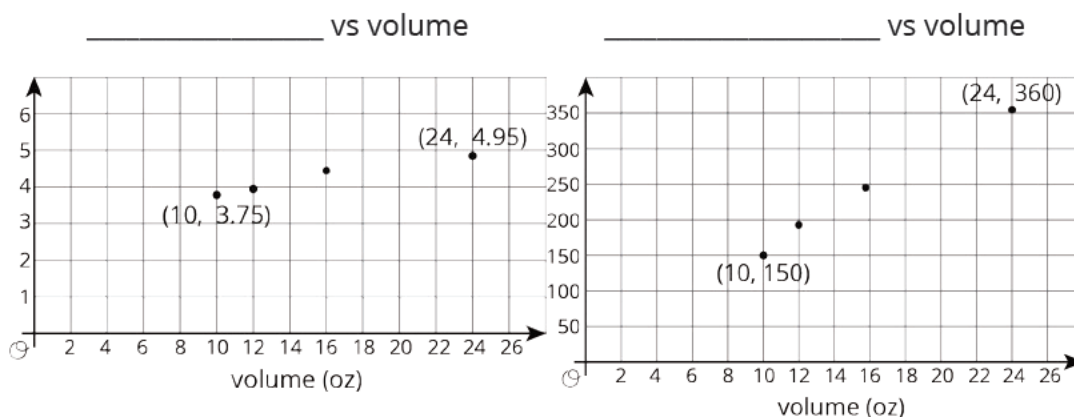
We can also use the graphs to compare the constants of proportionality. The line representing store B goes through the point $(1, 4)$, so the constant of proportionality is 4. This tells us that at store B the blueberries cost \$4 per pound. This is cheaper than the \$6 per pound unit price at store A.

Lesson 12 Practice Problems

1. Problem 1

Statement

The graphs below show some data from a coffee shop menu. One of the graphs shows cost (in dollars) vs. drink volume (in ounces), and one of the graphs shows calories vs. drink volume (in ounces).



- Which graph is which? Give them the correct titles.
- Which quantities appear to be in a proportional relationship? Explain how you know.
- For the proportional relationship, find the constant of proportionality. What does that number mean?

Solution

- a. The first graph is cost vs volume, and the second graph is calories vs volume. You can tell because the y-values are appropriate for cost in dollars on the first graph, and the y-values are appropriate for calories on the second.
- b. It appears there is a proportional relationship between calories and volume. The points appear to lie on a line that would pass through the origin. Also, it makes sense that every one ounce would contain the same number of calories. Regarding the cost relationship, the points do not appear to lie on a precise line, and the line definitely would not pass through the origin. This makes sense because there is more to the cost of a cup of coffee than the amount of coffee.
- c. The constant of proportionality is 15 calories per ounce, which can be found using $\frac{150}{10}$ or $\frac{360}{24}$. It means the coffee drink contains 15 calories in 1 ounce.

2. Problem 2

Statement

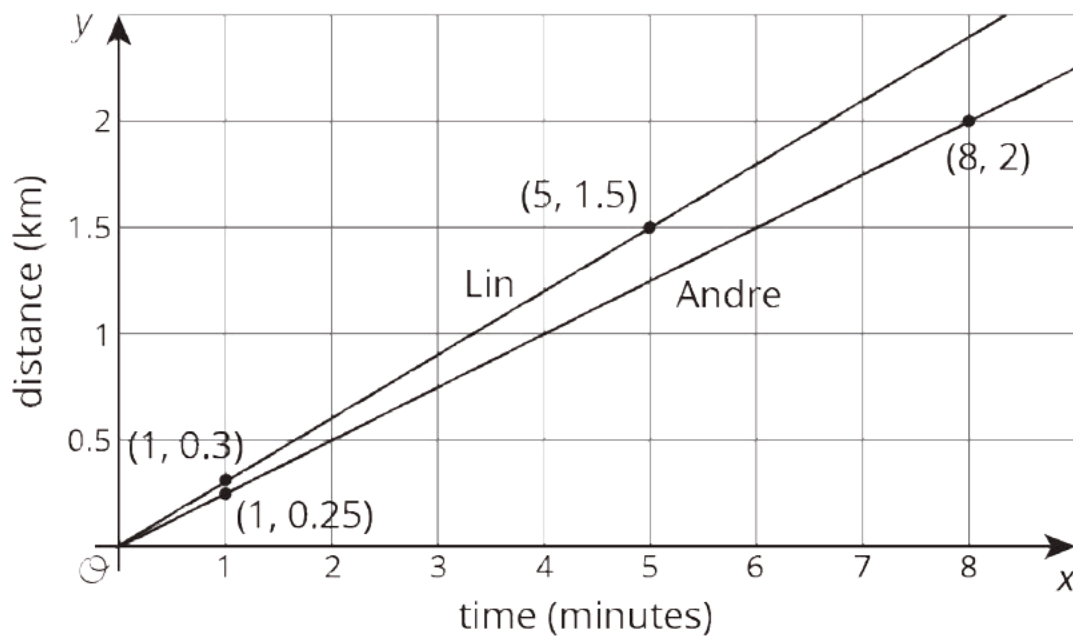
Lin and Andre biked home from school at a steady pace. Lin biked 1.5 km and it took her 5 minutes. Andre biked 2 km and it took him 8 minutes.

- a. Draw a graph with two lines that represent the bike rides of Lin and Andre.
- b. For each line, highlight the point with coordinates $(1, k)$ and find k .
- c. Who was biking faster?

Solution

Lin and Andre biked home from school at a steady pace. Lin biked 1.5 km and it took her 5 minutes. Andre biked 2 km and it took him 7 minutes.

a.



b. For Lin's graph, $k = 0.3$. For Andre's graph, $k = \frac{2}{8}$.

c. Lin is going slightly faster at 0.3 km per minutes. Andre is going $\frac{2}{8}$ or 0.25 km per minute.

3. **Problem 3**

Statement

Match each equation to its graph.

a. $y = 2x$

b. $y = \frac{4}{5}x$

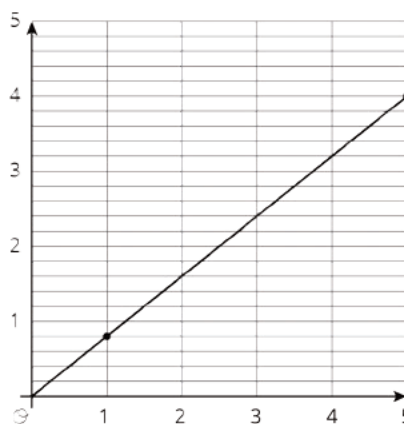
c. $y = \frac{1}{4}x$

d. $y = \frac{2}{3}x$

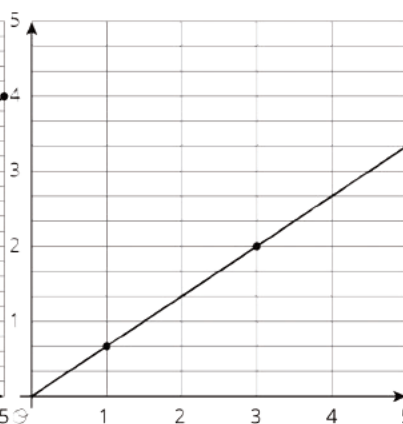
e. $y = \frac{4}{3}x$

f. $y = \frac{3}{2}x$

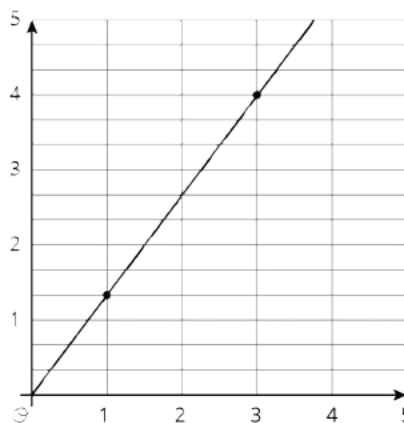
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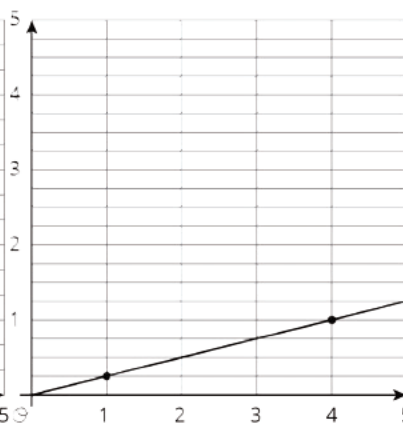
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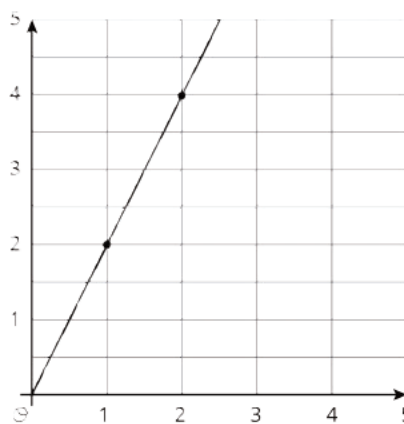
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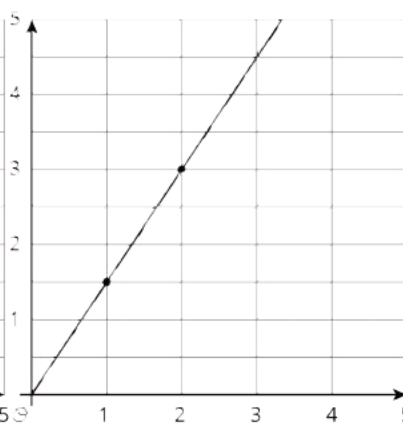
4



5



6



Solution

a. 5

b. 1

c. 4

d. 2

e. 3

f. 6

Lesson 13: Two Graphs for Each Relationship

Goals

- Coordinate (orally and in writing) tables, graphs, and equations that represent the same proportional relationship.
- Interpret two different graphs that represent the same proportional relationship, but have reversed which quantity is represented on each axis.
- Write an equation to represent a proportional relationship given only one pair of values or one point on the graph.

Learning Targets

- I can interpret a graph of a proportional relationship using the situation.
- I can write an equation representing a proportional relationship from a graph.

Lesson Narrative

In this lesson students focus on the relationship between the graph and the equation of a proportional relationship. They start with an activity designed to help them see all the different ways in which the graph and the equation are connected, for example the relation between a point (a, b) on the graph and the constant of proportionality $k = \frac{b}{a}$ in the equation and the fact that the point $(1, k)$ on the graph tells you the constant of proportionality. This prepares them for the next two activities where they see two ways to graph a proportional relationship, depending on which quantity goes on which axis. This connects with previous work with tables and equations, and gives students an opportunity to remember the fact that the constants of proportionality in the two ways are reciprocals.

As students connect the structure of an equation with features of the graph, they engage in MP7.

Alignments

Addressing

- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

Building Towards

- 7.EE.A: Use properties of operations to generate equivalent expressions.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display
- True or False

Required Materials

Rulers

Student Learning Goals

Let's use tables, equations, and graphs to answer questions about proportional relationships.

13.1 True or False: Fractions and Decimals

Warm Up: : 5 minutes

This warm-up encourages students to connect and reason algebraically about various computational relationships and patterns from previous exercises. While students may evaluate each side of the equation to determine if it is true or false, encourage students to think about the following ideas in each:

1. The multiplicative relationships between the factors. Multiplying one factor by 2 and dividing the other by 2 on the left side of the equation results in the two factors on the right hand side.
2. In this case, the factors on the left hand side of the equation are adjusted in the same manner as the first equation, however since the operation is division, this strategy results in one side being 4 times the value of the other.
3. This equation applies the same reasoning as the first equation except the factors are adjusted by multiplying and dividing by 4.

Building Towards

- 7.EE.A

Instructional Routines

- True or False

Launch

Display one problem at a time. Tell students to give a signal when they have an answer and a strategy. After each problem, give students 1 minute of quiet think time and follow with a whole-class discussion.

Anticipated Misconceptions

Students may think the same strategies that work for multiplication can be applied to division. For these students ask them for a context to demonstrate what happens when we double a dividend and halve the divisor.

Student Task Statement

Decide whether each equation is true or false. Be prepared to explain your reasoning.

1. $\frac{3}{2} \cdot 16 = 3 \cdot 8$

2. $\frac{3}{4} \div \frac{1}{2} = \frac{6}{4} \div \frac{1}{4}$

3. $(2.8) \cdot (13) = (0.7) \cdot (52)$

Student Response

1. True. $\frac{3}{2} \cdot 16 = 3 \cdot \frac{1}{12}$
2. False. $\frac{3}{4} \div \frac{1}{2} = \frac{6}{4}$ and $\frac{6}{4} \div \frac{1}{4} = 6$
3. True. $(2.8) \cdot 13 = (0.7 \cdot 4) \cdot 13$ and $0.7 \cdot (4 \cdot 13) = 0.7 \cdot (52)$

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their explanations for all to see. Ask students if or how the factors in the problem impacted the strategy choice. To involve more students in the conversation, consider asking:

- “Do you agree or disagree? Why?”
- “Who can restate ___’s reasoning in a different way?”
- “Does anyone want to add on to ____’s strategy?”
- “Will that strategy always work? How do you know?”

After each true equation, ask students if they could rely on the same reasoning to determine if other similar problems are equivalent. After each false equation, ask students how the problem could be changed to make the equation true.

13.2 Tables, Graphs, and Equations

: 20 minutes (there is a digital version of this activity)

Note: if it is possible in your local environment, we recommend using the digital version of this activity.

This activity is intended to help students identify correspondences between parts of a table, graph, and an equation of a line through the origin in the first quadrant. Students are guided to notice:

- Any pair of positive values (a, b) determine a proportional relationship.
- Given a point (a, b) other than the origin on the graph of a line through the origin, the constant of proportionality is always $\frac{b}{a}$.
- In an equation $y = \frac{b}{a}x$ that represents the relationship, the constant of proportionality appears as the coefficient of x .

- The constant of proportionality is the y -coordinate when x is 1, that is, $(1, \frac{b}{a})$ is a point on the graph.

In the print version, students plot one point and draw a ray that starts at the origin and passes through their point. They also create a table and an equation to represent the relationship. They respond to a series of questions about their representations. Then, they compare their representations of their relationship with the representations of other relationships created by members of their group.

In the digital version (recommended), students interact with a dynamic sketch while recording observations and responding to prompts. In the sketch, students are able to manipulate the graph of a proportional relationship, while changes to an associated table and equation automatically update.

Addressing

- 7.RP.A.2

Instructional Routines

- MLR2: Collect and Display

Launch

For the print version: Arrange students in groups of 3. Assign each student in each group a letter: A, B, or C. Provide access to rulers.

For the digital version: Arrange students in groups of 2–3 and have them complete the task.

Support for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organization and problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.

Supports accessibility for: Organization; Attention

Support for English Language Learners

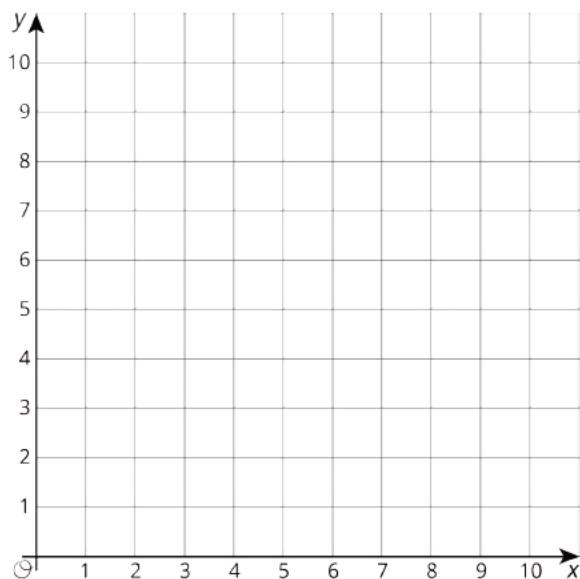
Speaking, Writing: MLR2 Collect and Display. While pairs or groups are working, circulate and listen to student talk about the connections they see between the tables, characteristics of the graphs, and the equations. Write down common or important phrases you hear students say about the connections onto a visual display. Throughout the remainder of the lesson, continue to update collected student language and remind students to borrow language from the display as needed. This will help students read and use mathematical language during paired and whole-class discussions.

Design Principle(s): Optimize output (for explanation); Maximize meta-awareness

Student Task Statement

Your teacher will assign you *one* of these three points:

$$A = (10, 4), B = (4, 5), C = (8, 5).$$



x	y	$\frac{y}{x}$
0		NA
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

1. On the graph, plot and label *only* your assigned point.
2. Use a ruler to line up your point with the origin, $(0, 0)$. Draw a line that starts at the origin, goes through your point, and continues to the edge of the graph.
3. Complete the table with the coordinates of points on your graph. Use a fraction to represent any value that is not a whole number.
4. Write an equation that represents the relationship between x and y defined by your point.
5. Compare your graph and table with the rest of your group. What is the same and what is different about:



a. your tables?

b. your equations?

c. your graphs?

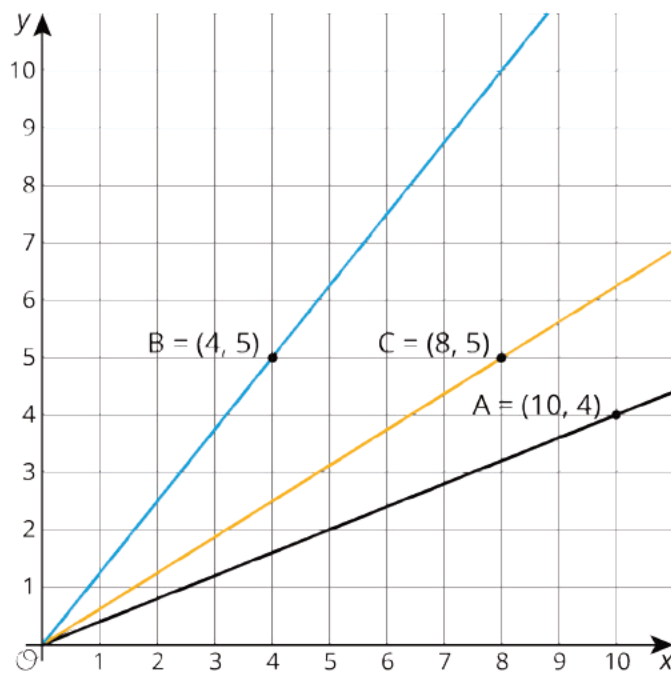
6. What is the y -coordinate of your graph when the x -coordinate is 1? Plot and label this point on your graph. Where do you see this value in the table? Where do you see this value in your equation?

7. Describe any connections you see between the table, characteristics of the graph, and the equation.

Student Response

1. See the graph

2. Students should create one of the following graphs.



3. Tables will differ depending on which point a student has been assigned. Any fraction equivalent to the one shown in the answer is acceptable.

Point A:

x	y	$\frac{y}{x}$
0	0	NA
1	$\frac{2}{5}$	$\frac{2}{5}$
2	$\frac{4}{5}$	$\frac{2}{5}$
3	$\frac{6}{5}$	$\frac{2}{5}$
4	$\frac{8}{5}$	$\frac{2}{5}$
5	2	$\frac{2}{5}$
6	$\frac{12}{5}$	$\frac{2}{5}$
7	$\frac{14}{5}$	$\frac{2}{5}$
8	$\frac{16}{5}$	$\frac{2}{5}$
9	$\frac{18}{5}$	$\frac{2}{5}$
10	4	$\frac{2}{5}$

Point B:

x	y	$\frac{y}{x}$
0	0	NA
1	$\frac{5}{4}$	$\frac{5}{4}$
2	$\frac{10}{4}$	$\frac{5}{4}$
3	$\frac{15}{4}$	$\frac{5}{4}$
4	5	$\frac{5}{4}$
5	$\frac{25}{4}$	$\frac{5}{4}$
6	$\frac{30}{4}$	$\frac{5}{4}$
7	$\frac{35}{4}$	$\frac{5}{4}$
8	10	$\frac{5}{4}$
9	$\frac{45}{4}$	$\frac{5}{4}$
10	$\frac{50}{4}$	$\frac{5}{4}$

Point C:

x	y	$\frac{y}{x}$
0	0	NA
1	$\frac{5}{8}$	$\frac{5}{8}$
2	$\frac{10}{8}$	$\frac{5}{8}$
3	$\frac{15}{8}$	$\frac{5}{8}$
4	$\frac{20}{8}$	$\frac{5}{8}$
5	$\frac{25}{8}$	$\frac{5}{8}$
6	$\frac{30}{8}$	$\frac{5}{8}$
7	$\frac{35}{8}$	$\frac{5}{8}$
8	5	$\frac{5}{8}$
9	$\frac{45}{8}$	$\frac{5}{8}$
10	$\frac{50}{8}$	$\frac{5}{8}$

4. *A*: $y = \frac{2}{5}x$ or equivalent. *B*: $y = \frac{5}{4}x$ or equivalent. *C*: $y = \frac{5}{8}x$ or equivalent.

5. Answers vary. Students may notice:

- When the y -coordinates are written as fractions, there are consistencies among them, like perhaps they all have the same denominator.
- In each table, all of the $\frac{y}{x}$ values are equal.
- All of the graphs are lines through the origin, but they have different steepnesses.
- The equations all include a y and an x . They all include a different number, but the number corresponds to a value in its table.

6. $A: \frac{2}{5}$ or equivalent. $B: \frac{5}{4}$ or equivalent. $C: \frac{5}{8}$ or equivalent.

7. Answers vary. Connections should be made between the point $(1, k)$ that appears on both the graph and in the table and the equation $y = kx$.

Are You Ready for More?

The graph of an equation of the form $y = kx$, where k is a positive number, is a line through $(0, 0)$ and the point $(1, k)$.

1. Name at least one line through $(0, 0)$ that cannot be represented by an equation like this.
2. If you could draw the graphs of *all* of the equations of this form in the same coordinate plane, what would it look like?

Student Response

1. The x and y -axes are both examples. Any line through $(0, 0)$ and $(1, k)$ where k is negative is also an example.
2. It would look like you completely shaded in the first and third quadrants of the coordinate plane.

Activity Synthesis

When all students have completed the activity, ask them to share their responses with a partner or small group. After a minute, go around the room asking each group to share one thing from the last question, and display these for all to see. When each group has shared, ask if there were any important observations that were missed.

Ensure that all the important connections are highlighted:

- A graph of a line through the origin and passing through the first quadrant represents a proportional relationship.
- The value of $\frac{b}{a}$ computed from any point (a, b) on that line (other than the origin) is the constant of proportionality.
- An equation of the relationship is given by $y = kx$ where k is $\frac{b}{a}$ for any point (a, b) on the graph other than the origin.

13.3 Hot Dog Eating Contest

: 10 minutes (there is a digital version of this activity)

The purpose of this activity is to help students understand derived units and rates. It is intended to help students see that a proportional relationship between two quantities is associated with two rates. The first rate indicates how many hot dogs someone eats in one minute (number of hot dogs per minute), and the second indicates how many minutes it takes to eat one hot dog (number of minutes per hot dog). The need for care with units (MP6) will become very clear should any students express rates in seconds per hot dog. If any students make this choice, it can be an opportunity to make connections between different ways of representing the same situation.

Monitor for students who take each approach.

Addressing

- 7.RP.A.2

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

Launch

Keep students in the same groups.

Support for Students with Disabilities

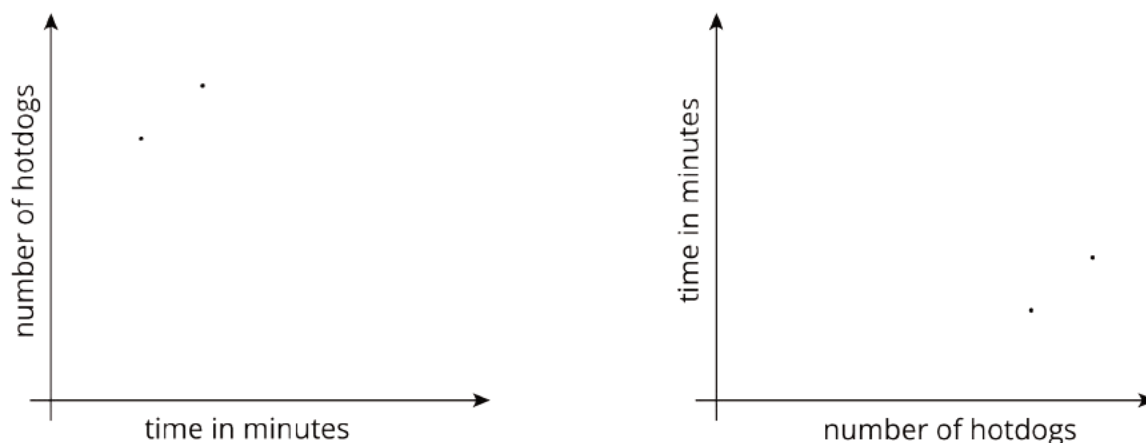
Representation: Internalize Comprehension. Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, use a different color for each person to highlight the connection between the graph, equation, and constant of proportionality.

Supports accessibility for: Visual-spatial processing

Student Task Statement

Andre and Jada were in a hot dog eating contest. Andre ate 10 hot dogs in 3 minutes. Jada ate 12 hot dogs in 5 minutes.

Here are two different graphs that both represent this situation.



1. On the first graph, which point shows Andre's consumption and which shows Jada's consumption? Label them.
2. Draw two lines: one through the origin and Andre's point, and one through the origin and Jada's point.

3. Write an equation for each line. Use t to represent time in minutes and h to represent number of hot dogs.

a. Andre:

b. Jada:

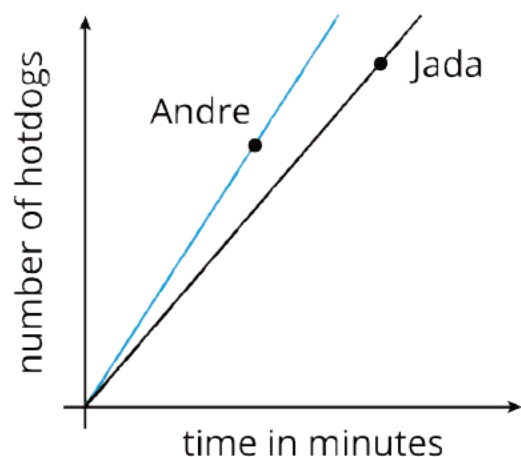
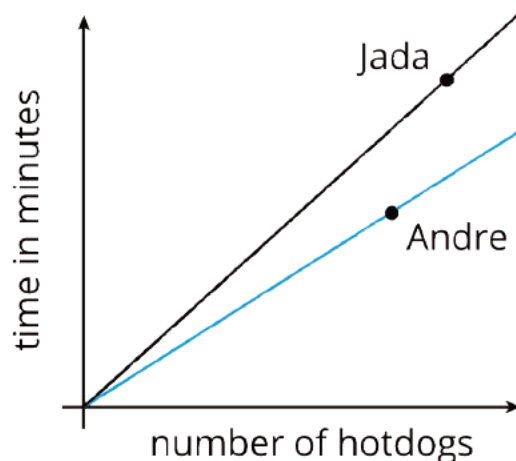
4. For each equation, what does the constant of proportionality tell you?

5. Repeat the previous steps for the second graph.

a. Andre:

b. Jada:

Student Response



1. Points are labeled.

2. See graph.

3. Andre: $h = \frac{10}{3}t$; Jada: $h = \frac{12}{5}t$ or $h = 2.4t$.

4. Andre eats $\frac{10}{3}$ (or $3\frac{1}{3}$ or approximately 3.33) hot dogs per minute. Jada eats $\frac{12}{5}$ (or $2\frac{2}{5}$ or 2.4) hot dogs per minute.

5. Points are labeled. See graph.

○ Andre: $t = \frac{3}{10}h$ or $t = 0.3h$. Jada: $t = \frac{5}{12}h$.

- Andre takes $\frac{3}{10}$ or 0.3 minutes per hot dog. Jada takes $\frac{5}{12}$ or approximately 0.42 minutes per hot dog. (Possibly: Andre takes 18 seconds per hot dog and Jada takes 25 seconds per hot dog.)

Activity Synthesis

Select students to share their reasoning. An important point to bring out in the discussion is that we can describe the rate of hot dog eating in two different ways as hot dogs per minute or minutes per hot dog.

Consider asking the following sequence of questions:

- "At what rate did Andre eat hot dogs?" (Some students might say $\frac{10}{3}$ or three and a third, while some say $\frac{3}{10}$ or 0.3.)
- "Well, you're both right, but we need more information to know what you're talking about. Can you be more precise?" (This will prompt students to modify their response and say " $\frac{10}{3}$ or three and a third hot dogs per minute" and " $\frac{3}{10}$ or 0.3 minutes per hot dog.")

Reassure students that either response is correct, as long as units are included. The important thing is that we communicate the meaning of the number clearly. Highlight the fact that $\frac{3}{10}$ and $\frac{10}{3}$ are reciprocals of each other.

Lesson Synthesis

Display the graphs and the corresponding equations from the "Hot Dog Eating Contest" activity. Ask student:

- "Do the graphs and equations tell the same story?"
- "How can you see the same information in both?"

When we have two quantities x and y in a proportional relationship, we have two choice for writing an equation, making a table, and drawing a graph to represent the relationship. Often the choice is arbitrary and if two people have made different choices, i.e. one views x as proportional to y and the other views y as proportional to x , the representations are related and still provide the same information.

Recall the new units used for the constants of proportionality in the activity: hot dogs per minute, minutes per hot dog. Note that if there is a proportional relationship between two

quantities with units A and B , then the associated rates are expressed in A s per B and B s per A .

13.4 Spicy Popcorn

Cool Down : 5 minutes

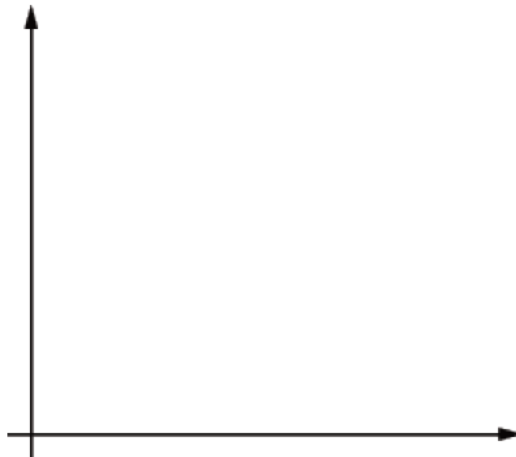
Addressing

- 7.RP.A.2

Student Task Statement

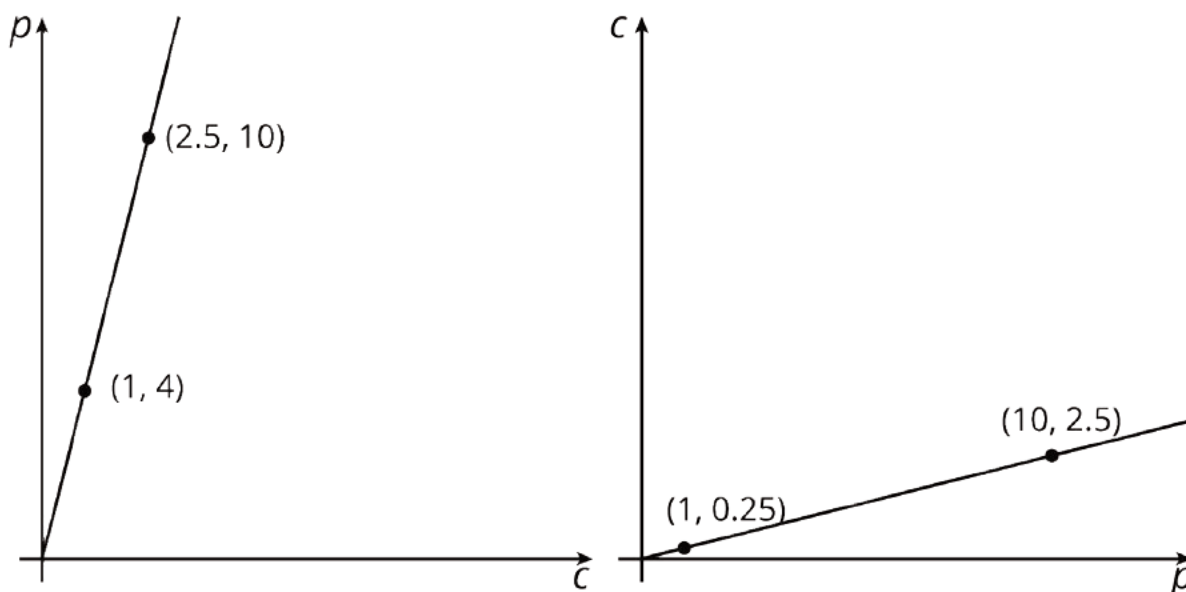
Elena went to a store where you can scoop your own popcorn and buy as much as you want. She bought 10 ounces of spicy popcorn for \$2.50.

1. How much does popcorn cost per ounce?
2. How much popcorn can you buy per dollar?
3. Write two different equations that represent this situation. Use p for ounces of popcorn and c for cost in dollars.
4. Choose one of your equations, and sketch its graph. Be sure to label the axes.



Student Response

1. \$0.25, because $2.50 \div 10 = 0.25$.
2. 4 ounces, because $10 \div 2.5 = 4$.
3. $p = 4c$ or $c = 0.25p$ or equivalent.
4. Students are only asked to create one of these graphs. It is not necessary that they plot and label any points, but it could be a helpful step in creating a reasonably accurate graph.

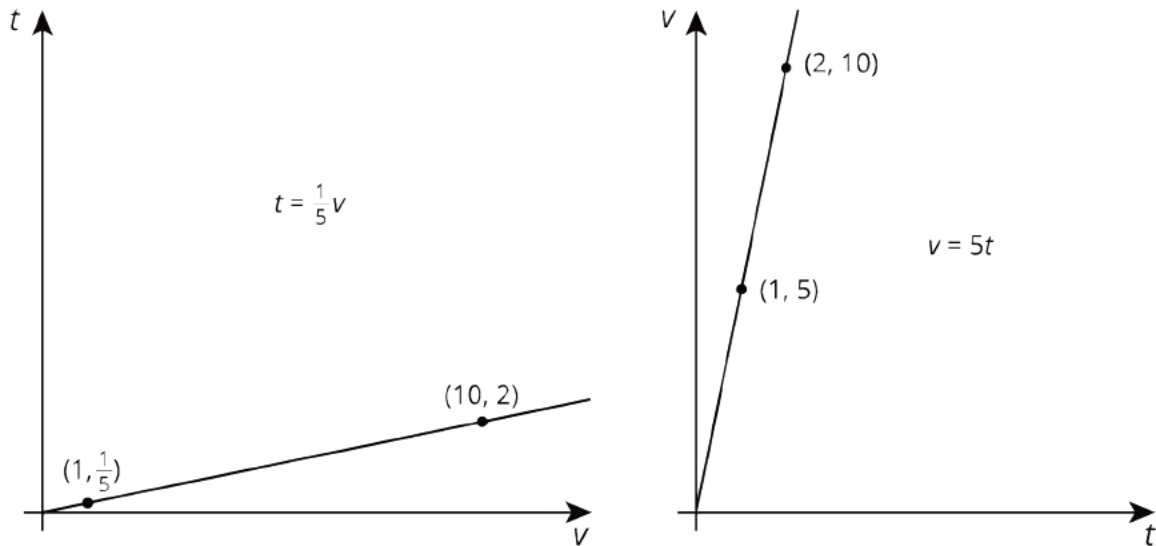


Student Lesson Summary

Imagine that a faucet is leaking at a constant rate and that every 2 minutes, 10 milliliters of water leaks from the faucet. There is a proportional relationship between the volume of water and elapsed time.

- We could say that the elapsed time is proportional to the volume of water. The corresponding constant of proportionality tells us that the faucet is leaking at a rate of $\frac{1}{5}$ of a minute per milliliter.
- We could say that the volume of water is proportional to the elapsed time. The corresponding constant of proportionality tells us that the faucet is leaking at a rate of 5 milliliters per minute.

Let's use v to represent volume in milliliters and t to represent time in minutes. Here are graphs and equations that represent both ways of thinking about this relationship:



Even though the relationship between time and volume is the same, we are making a different choice in each case about which variable to view as the independent variable. The graph on the left has v as the independent variable, and the graph on the right has t as the independent variable.

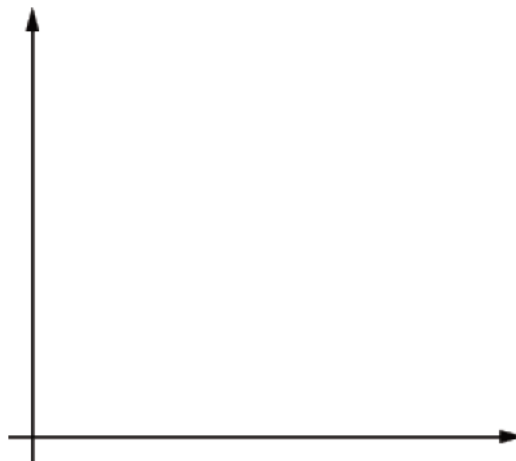
Lesson 13 Practice Problems

1. Problem 1

Statement

At the supermarket you can fill your own honey bear container. A customer buys 12 oz of honey for \$5.40.

- How much does honey cost per ounce?
- How much honey can you buy per dollar?
- Write two different equations that represent this situation. Use h for ounces of honey and c for cost in dollars.



- Choose one of your equations, and sketch its graph. Be sure to label the axes.

Solution

- \$0.45 per ounce
- About 2.2 ounces
- $c = 0.45h$; $h = 2.2c$
- Students should have one of two linear graphs going through the origin. Graph 1: $c = 0.45h$, horizontal axis label: h , honey (ounces); vertical axis label c , cost (\$); Graph 2: $h = 2.2c$, horizontal axis label: c , cost (\$) ; vertical axis label: h , honey (ounces)

2. Problem 2

Statement

The point $(3, \frac{6}{5})$ lies on the graph representing a proportional relationship.

Which of the following points also lie on the same graph? Select **all** that apply.

- A. $(1, 0.4)$
- B. $(1.5, \frac{6}{10})$
- C. $(\frac{6}{5}, 3)$
- D. $(4, \frac{11}{5})$
- E. $(15, 6)$

Solution

["A", "B", "E"]

Statement

A trail mix recipe asks for 4 cups of raisins for every 6 cups of peanuts. There is proportional relationship between the amount of raisins, r (cups), and the amount of peanuts, p (cups), in this recipe.

- a. Write the equation for the relationship that has constant of proportionality greater than 1. Graph the relationship.

Solution

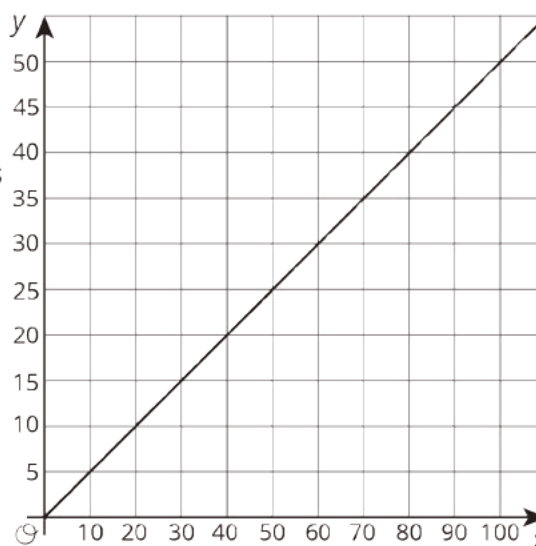
- a. $p = \frac{6}{4}r$. Students should have a graph of $p = \frac{6}{4}r$, label horizontal axis r (or "raisins (cups)") and vertical axis p (or "peanuts (cups)"). Since this is a proportional relationship, the graph should be linear and go through the origin.
- b. $r = \frac{4}{6}p$. Students should have a graph of $r = \frac{4}{6}p$, label horizontal axis p and vertical axis r . Since this is a proportional relationship, the graph should be linear and go through the origin. The slope of this graph should be less steep than the previous graph.

4. Problem 4

Statement

Here is a graph that represents a proportional relationship.

- a. Come up with a situation that could be represented by this graph.
- b. Label the axes with the quantities in your situation.
- c. Give the graph a title.
- d. Choose a point on the graph. What do the coordinates represent in your situation?



Solution

Answers vary. Sample response:

- a. For every 2 gallons of gray paint created, 1 gallon of black paint is used.
- b. Horizontal axis: gray paint (gallons). Vertical axis: black paint (gallons).
- c. Title: Amount of Black Paint Needed to Create Gray Paint
- d. The point (60, 30) means, in order to make 60 gallons of gray paint, 30 gallons of black paint is needed.

(From Unit 2, Lesson 11.)

Section: Let's Put it to Work

Lesson 14: Four Representations

Goals

- Calculate the constant of proportionality for a proportional relationship in an unfamiliar context, and express it (in writing) using the correct units.
- Critique (orally and in writing) presentations of proportional and nonproportional relationships.
- Invent and describe (in writing and using other representations) a proportional relationship and a nonproportional relationship.

Learning Targets

- I can make connections between the graphs, tables, and equations of a proportional relationship.
- I can use units to help me understand information about proportional relationships.

Lesson Narrative

In this lesson, students examine tables, equations, and graphs of proportional relationships, and use them to reason about relationships that are proportional as well as relationships that are not proportional.

This lesson requires students to use everything they have learned since the beginning of the unit. This unit is meant to help students understand:

- What is a proportional relationship?
- What kinds of situations can be represented by proportional relationships?
- What form does an equation of a proportional relationship have?
- What do graphs of proportional relationships look like?

The next lesson in this unit is intended as a bridge to Unit 7.4 which is about applications of proportional relationships. It gives an example of how proportional relationships, and the derived units they give rise to, help to solve problems about the real world.

Alignments

Building On

- 6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Addressing

- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.
- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

Instructional Routines

- Group Presentations
- MLR2: Collect and Display
- MLR7: Compare and Connect

Required Materials

Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Required Preparation

Students are asked to make displays of their work in groups of 2. Prepare materials for creating a visual display in this way such as markers, chart paper, board space, etc.

Student Learning Goals

Let's contrast relationships that are and are not proportional in four different ways.

14.1 Which is the Bluest?

Warm Up: : 5 minutes

In this warm-up, students are asked to reason which group of blocks is the bluest and explain how they arrived at that decision. The goal is to prompt students to visualize and articulate different ways they can use ratios, equivalent ratios and proportions to support their reasoning.

Building On

- 6.RP.A.3

Addressing

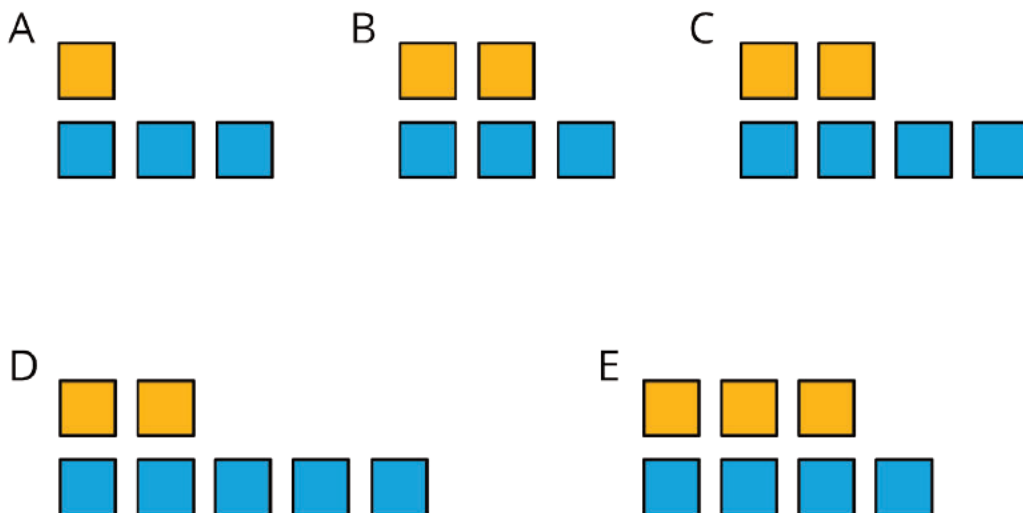
- 7.RP.A.2

Launch

Students in groups of 2. Tell students you will show them five groups of blocks. Their job is to determine which group of blocks is the bluest. Display the image for all to see. Give students 2 minutes of quiet think time. Encourage students who have one way of supporting their decision to think about another way while they wait.

Student Task Statement

1. Which group of blocks is the bluest?



2. Order the groups of blocks from least blue to bluest.

Student Response

1. A or D. A if looking at the amount of blue per yellow. D if looking at the total amount of blue or difference between blue and yellow.
2. Answers vary. Sample response: E, B, C, D, A when ordering by the amount of blue per total blocks.

Activity Synthesis

Ask students to share which group of blocks is the bluest and their reasoning. Record and display student explanations for all to see. To involve more students in the conversation, consider asking some of the following questions:

- Did anyone choose the same group of blocks but would explain it differently?
- Does anyone want to add an observation to the way ____ saw the blocks?
- Do you agree or disagree? Why? Ask students to order the groups of blocks from less blue to bluest after deciding on the bluest group of blocks.

14.2 One Scenario, Four Representations

: 20 minutes

In this activity, students choose from different lists of things to define their own proportional and nonproportional relationships. Some of the things on the list will be familiar and others will be unfamiliar. This is a significant change from previous activities where students were always given two quantities and they had to decide if they were proportional or not. This new step gives students the opportunity to think about what quantities are related to some of the items on the lists, which is an important step of modeling with mathematics (MP4).

This activity and the next go together. Students use the work from this activity to make a visual display of their work in the next activity.

Addressing

- 7.RP.A.2

Instructional Routines

- MLR2: Collect and Display

Launch

Arrange students in groups of 2.

The names of things in the task may be unfamiliar to both English Language Learners and fluent English speakers. Before students start, take some time to ensure they know the meaning of their chosen things or ask them to do some research on their meanings. You might ask each pair of students to choose a different unfamiliar word, spend five minutes to research it, and prepare a drawing or explanation to share with the class.

It is best to approve of students' choices before they work. For example, if students choose "legs" and "earthworms," that will not make for a very interesting relationship.

Support for Students with Disabilities

Action and Expression: Provide Access for Physical Action. Provide access to tools and assistive technologies such as a graphing calculator or graphing software. Some students may benefit from a checklist or list of steps to be able to use the calculator or software.

Supports accessibility for: Organization; Conceptual processing; Attention

Support for English Language Learners

Conversing, Writing: MLR2 Collect and Display. While pairs are working, circulate and listen to students talk about the relationships between quantities and justify whether the relationships are proportional. Write down common or important phrases you hear students say about the relationships. Throughout the remainder of the lesson, continue to update collected student language and remind students to borrow language from the display as needed. This will help students use mathematical language in their written work and in paired and whole group discussions.

Design Principle(s): Optimize output (for justification); Support sense-making

Anticipated Misconceptions

As students work, pay attention to the numbers they use in their tables. Students can be haphazard when choosing values, and their numbers may end up being unfriendly. You can ask them questions that encourage them to reason about what numbers would be friendlier. Also, depending on the things chosen, they may need to consider scales for their axes. Watch out for scales like $\{1, 2, 3, \dots\}$ for the number of legs on a centipede!

Creating a relationship that is not proportional may present too significant a challenge for struggling learners. An accommodation would be to change their task to creating only a

proportional relationship, or even assigning two quantities that are straightforward like “starfish legs vs. number of starfish.”

Student Task Statement

1. Select two things from different lists. Make up a situation where there is a *proportional relationship* between quantities that involve these things.

creatures	length	time	volume
◦ starfish	◦ centimeters	◦ nanoseconds	◦ milliliters
◦ centipedes	◦ cubits	◦ minutes	◦ gallons
◦ earthworms	◦ kilometers	◦ years	◦ bushels
◦ dinosaurs	◦ parsecs	◦ millennia	◦ cubic miles
body parts	area	weight	substance
◦ legs	◦ square microns	◦ nanograms	◦ helium
◦ eyes	◦ acres	◦ ounces	◦ oobleck
◦ neurons	◦ hides	◦ debens	◦ pitch
◦ digits	◦ square light-years	◦ metric tonnes	◦ glue

2. Select two other things from the lists, and make up a situation where there is a relationship between quantities that involve these things, but the relationship is *not* proportional.

- 
3. Your teacher will give you two copies of the “One Scenario, Four Representations” sheet. For each of your situations, describe the relationships in detail. If you get stuck, consider asking your teacher for a copy of the sample response.
- a. Write one or more sentences describing the relationship between the things you chose.
 - b. Make a table with titles in each column and at least 6 pairs of numbers relating the two things.
 - c. Graph the situation and label the axes.
 - d. Write an equation showing the relationship and explain in your own words what each number and letter in your equation means.
 - e. Explain how you know whether each relationship is proportional or not proportional. Give as many reasons as you can.

Student Response

Answers vary. Sample response:

One Scenario, Four Representations

The two quantities are: d yards or distance traveled during in the race in yards;
 t minutes or time in minutes that has elapsed in the race.

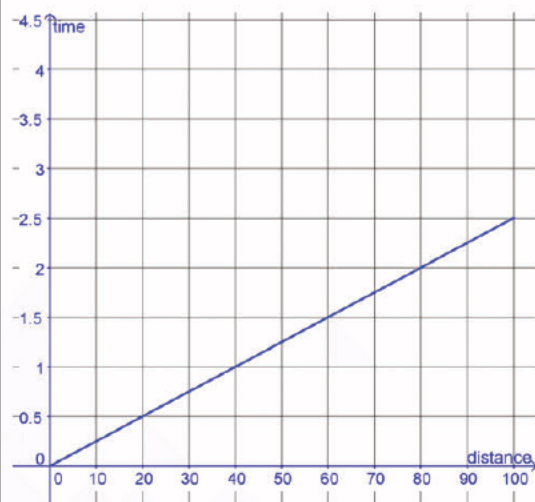
Verbal description: One or more complete sentences describing the relationship.

Adan and Mike are teammates in a 100-yd three-legged race. Their friend Ceril is timing them. Ceril notices that they pass the 20-yd marker at $\frac{1}{2}$ minute, the 40-yd marker at 1 minute, and the 60-yd marker at 1.5 minutes.

Table of Values

d	t
20	$\frac{1}{2}$
40	1
60	1.5
80	2
100	2.5
1	$\frac{1}{40}$

Graph
Label each axis!



Equation: $t = \frac{1}{40}d$

Explain in words what each letter and number in your equation means:

t represents the time in minutes that has elapsed in the race, d represents the distance in yards they have traveled, and $\frac{1}{40}$ is the constant of proportionality. It takes them $\frac{1}{40}$ of a minute to travel 1 yard.

Explain how you know the relationship is proportional. Find as many reasons as you can. This relationship is proportional because: each value of d in the table can be multiplied by $\frac{1}{40}$ to get the corresponding value of t . The graph is part of a line that goes through the origin and Quadrant I. The equation can be written in the form $d = kt$.

Activity Synthesis

Ask groups to trade their work with another group to give feedback about their analysis.

14.3 Make a Poster

Optional: : 15 minutes

In this activity, students make a visual display of their scenarios from the previous activity (after sharing their rough draft with another group and getting some feedback).

When the posters are complete and displayed around the room, students view each others' work and use sentence starters to give feedback and to critique the reasoning of others (MP3).

Addressing

- 7.RP.A.2

Instructional Routines

- Group Presentations
- MLR7: Compare and Connect

Launch

Keep students in the same groups. Tell them that they should incorporate the feedback they received when making their posters.

Support for Students with Disabilities

Engagement: Develop Effort and Persistence. Provide prompts, reminders, guides, rubrics, or checklists that focus on increasing the length of on-task orientation in the face of distractions. For example, provide a task checklist which makes all the required components of the poster explicit.

Supports accessibility for: Attention; Social-emotional skills

Student Task Statement

Create a visual display of your two situations that includes all the information from the previous activity.

Student Response

Answers vary.

Activity Synthesis

When the posters are complete, hang them around the room. Provide students with these sentence starters, and give them an opportunity to view their classmates' work and write their responses. This provides a structured way for students to critique the reasoning of others (MP3).

The most surprising combination of things was _____ because _____.

The group _____ should check their work where they _____.

I really liked when the group _____ did this _____ because _____.

Support for English Language Learners

Representing, Reading, Writing: MLR7 Compare and Connect. Use this routine when students share their visual displays. Direct attention to the different ways pairs explained quantities and their relationships and justified whether the relationship was proportional (e.g., distance traveled in yards and time elapsed in minutes, for every one minute they traveled 5 yards, the relationship is proportional because the graph of the line passes through the origin). Emphasize the language used to describe the proportional relationships and justify whether the relationship was proportional. These exchanges strengthen students' mathematical language use and reasoning of proportional relationships.

Design Principle(s): Maximize meta-awareness

Lesson Synthesis

Reflect on the following questions.

- "Describe any part of your work today that you would do differently, if you could start over."
- "Tell me about something new you learned in this class recently."

- "Tell me about any questions you still have, or anything that is confusing you."

14.4 Explain Their Work

Cool Down : 5 minutes

Addressing

- 7.RP.A

Student Task Statement

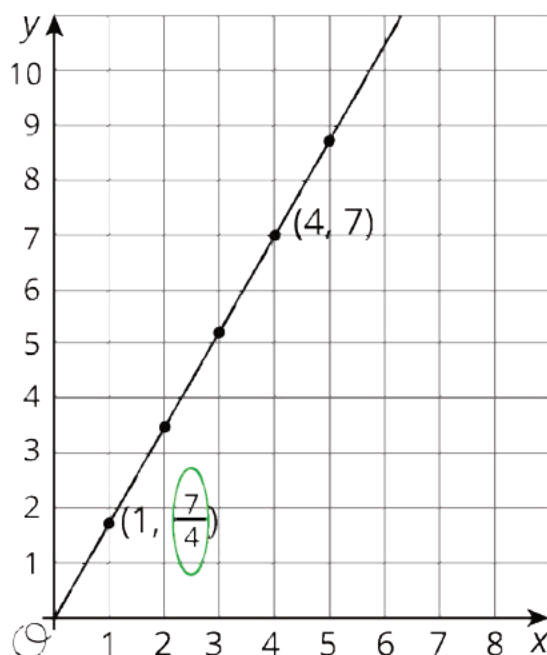
Choose a relationship that another group found and explain why it is a proportional relationship. Make sure to include the quantities they used and any important constants of proportionality.

Student Response

Answers vary. Sample response: In a 100 yard, Three Legged Race, distance in yards and time in minutes are proportional since each value of distance could be multiplied by $\frac{1}{40}$ to get the time. The constant of proportionality they used was $\frac{1}{40}$.

Student Lesson Summary

The constant of proportionality for a proportional relationship can often be easily identified in a graph, a table, and an equation that represents it. Here is an example of all three representations for the same relationship. The constant of proportionality is circled:



$$y = \left(\frac{7}{4}\right)x$$

x	y
0	0
1	$\frac{7}{4}$
2	$\frac{7}{2}$
3	$\frac{21}{4}$
4	7

On the other hand, some relationships are not proportional. If the graph of a relationship is not a straight line through the origin, if the equation cannot be expressed in the form $y = kx$, or if the table does not have a constant of proportionality that you can multiply by any number in the first column to get the associated number in the second column, then the relationship between the quantities is not a proportional relationship.

Lesson 14 Practice Problems

1. Problem 1

Statement

The equation $c = 2.95g$ shows how much it costs to buy gas at a gas station on a certain day. In the equation, c represents the cost in dollars, and g represents how many gallons of gas were purchased.

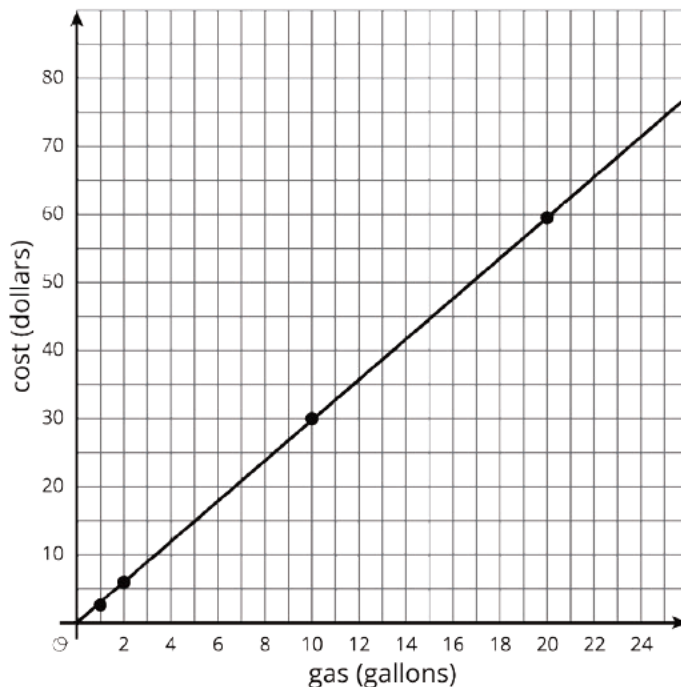
- a. Write down at least four (gallons of gas, cost) pairs that fit this relationship.
- b. Create a graph of the relationship.
- c. What does 2.95 represent in this situation?
- d. Jada's mom remarks, "You can get about a third of a gallon of gas for a dollar." Is she correct? How did she come up with that?

Solution

a. Answers vary. Sample response:

gallons of gas (g)	cost in dollars (c)
1	2.95
2	5.90
10	29.50
20	59.00

b. Answers vary. Sample response:



c. One gallon of gas costs \$2.95. Or, gas costs \$2.95 per gallon. Or, 2.95 dollars per gallon is the constant of proportionality.

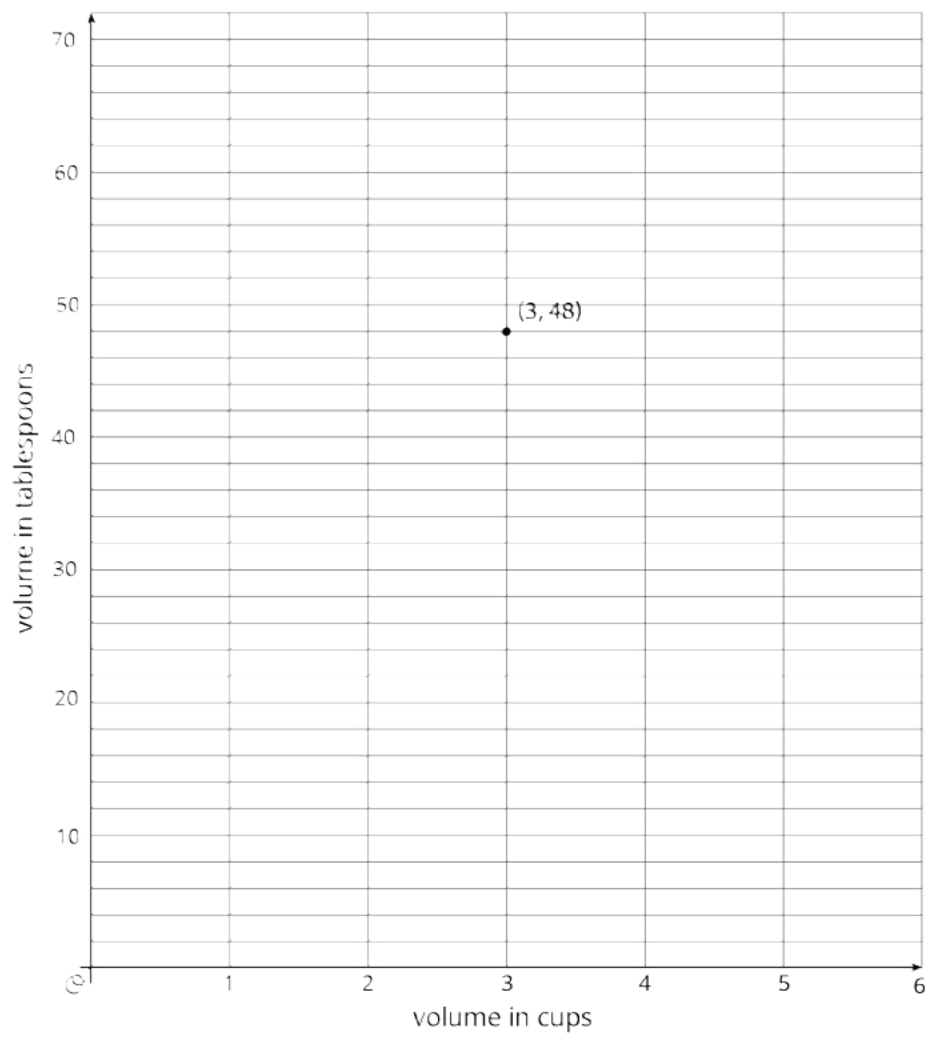
d. Since 2.95 is close to 3, Jada's mom reasoned that if it cost about 3 dollars per gallon, the reciprocal rate must be $\frac{1}{3}$ gallon per dollar. gallons of gas (g) cost (c).

2. Problem 2

Statement

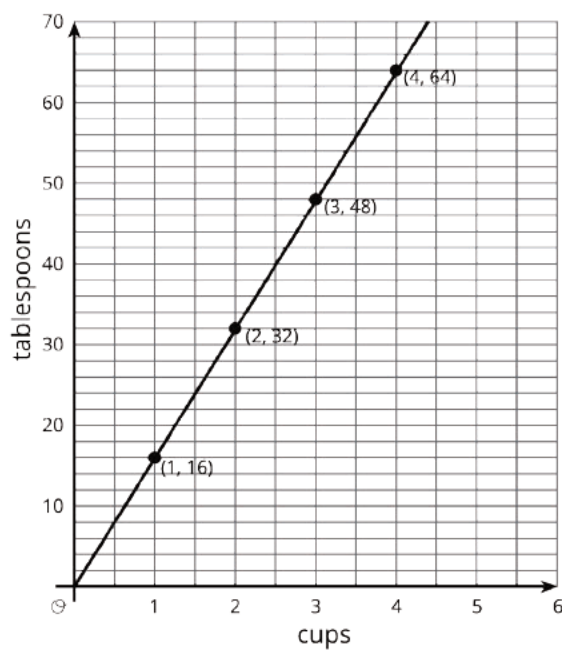
There is a proportional relationship between a volume measured in cups and the same volume measured in tablespoons. 3 cups is equivalent to 48 tablespoons, as shown in the graph.

- Plot and label at least two more points that represent the relationship.
- Use a straightedge to draw a line that represents this proportional relationship.
- For which value y is $(1, y)$ on the line you just drew?
- What is the constant of proportionality for this relationship?
- Write an equation representing this relationship. Use c for cups and t for tablespoons.



Solution

- a. See below
- b. See below
- c. 16
- d. 16 tablespoons per cup
- e. $t = 16c$



Lesson 15: Using Water Efficiently

Goals

- Apply reasoning developed in this unit to determine whether a proportional relationship models a situation about water usage.
- Make simplifying assumptions and determine what information is needed to solve a problem about water usage.
- Use proportional relationships to analyze (orally and in writing) a problem about water usage.

Learning Targets

- I can answer a question by representing a situation using proportional relationships.

Lesson Narrative

In this lesson, students use their understanding of proportional relationships to explore whether baths or showers use more water. The warm-up gives students a chance to think about what they would need to know in order to answer this open-ended question and to share their ideas with classmates. In the main activity, students seek out resources to help them answer the question and they create a display to report their findings.

If possible, allow students the chance to work on their own to find values to aid in their solutions. For example, sizes of typical bath tubs are usually listed on websites for hardware stores that carry baths for installation. If these resources are unavailable, some typical ranges are provided or reasonable estimates can be used.

Alignments

Addressing

- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

Instructional Routines

- Group Presentations
- MLR1: Stronger and Clearer Each Time
- MLR7: Compare and Connect

Required Materials

Internet-enabled device

Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Required Preparation

Internet enabled devices are only needed if students will conduct their own research. Tools for creating a visual display are only needed if students are making posters of their methods and results in the second activity.

Student Learning Goals

Let's investigate saving water.

15.1 Comparing Baths and Showers

Warm Up: : 5 minutes

This warm-up sets the stage for this lesson. Students are presented with the basic question of whether baths or showers use more water and they brainstorm information that might help them investigate the question.

Addressing

- 7.RP.A.2

Launch

"Some people take showers, some people take baths. There is disagreement over which one takes more water. What do you think?" Ask students to think for just a minute about whether they think a shower or a bath uses more water. (This is just to record their first instinct—they should not spend any time researching or calculating right now.) Poll the class and record the total for each category for all to see: "think a bath takes more water" and "think a shower takes more water."

Student Task Statement

Some people say that it uses more water to take a bath than a shower. Others disagree.

1. What information would you collect in order to answer the question?
2. Estimate some reasonable values for the things you suggest.

Student Response

Answers vary. Sample responses:

- Length of the shower: 10 minutes.
- Size of the bath tub: 40 gallons.
- How fast water comes out of the shower head: 2 gallons per minute
- How much the bath tub is filled with water: 50%

Activity Synthesis

Invite students to share their responses. Record and display their responses for all to see. If these quantities do not come up in conversation, ask students to discuss the ideas and provide reasonable estimates:

- Time spent in the shower
- Volume of the bath tub
- Rate of water coming out of the shower head

15.2 Saving Water: Bath or Shower?

: 20 minutes

When students are finding values to aid in their method, consider allowing them to research typical values online at hardware websites or search for values that would be useful. If these tools are not available, some values are provided here.

Values that may be useful for students:

- Typical (modern) shower heads have a flow rate of 1.9 to 2.5 gallons per minute. Older shower heads (pre-1992) could have flow rates up to 5.5 gallons per minute.
- Bath tubs hold approximately 120 to 180 gallons of water when completely filled to the top.
- The interior of a typical bath tub has an approximate width of 30 to 32 inches, length of 55 to 60 inches, and depth of 18 to 24 inches.
- There are approximately 230 cubic inches in 1 gallon of water.
- 1 liter of water is 1,000 cubic centimeters.
- 1 liter is approximately 0.26 gallons
- 1 inch is 2.54 centimeters.
- Typical showers last approximately 11 minutes although during a drought, it is recommended to reduce the time to about 5 minutes. During normal circumstances, some people appreciate much longer showers.

After students have made good progress in the activity, tell them to make a display (e.g. poster) to share their method and results.

This activity can take the rest of the class period, if desired.

Addressing

- 7.RP.A.2

Instructional Routines

- Group Presentations
- MLR7: Compare and Connect

Launch

Arrange students in groups of 2–4. Tell them either that they should research relative information and provide access to internet enabled devices or tell them that they can ask for information they need.

Support for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support development of organizational skills, check in with students within the first 2–3 minutes of work time. Check to make sure students have a valid method for comparing the water usage for a bath and a shower prior to allowing them to research typical values of measurements.

Supports accessibility for: Memory; Organization

Student Task Statement

1. Describe a method for comparing the water usage for a bath and a shower.
2. Find out values for the measurements needed to use the method you described. You may ask your teacher or research them yourself.
3. Under what conditions does a bath use more water? Under what conditions does a shower use more water?

Student Response

Answers vary. Sample responses:

- For a bath filled with 40 gallons of water and a shower head that uses 2 gallons per minute, any showers less than 20 minutes long will use less water than a bath. Longer showers will use more water than a bath.
- For a bath filled with 30 gallons of water and a shower head that uses 4 gallons per minute, any showers less than 7.5 minutes long will use less water than a bath. Longer showers will use more water than a bath.
- A 10 minute shower with a 3.5 gallon per minute shower head will use 35 gallons of water. Filling the bath with less water than that will use less water than the shower. A bath that is filled more than that 35 gallons will use more than the shower.

Activity Synthesis

Allow students to share their displays possibly through a gallery walk or ask them to present to the class. After students have had a chance to explore their work, ask them to share ideas they saw that were interesting and any methods they considered but did not use.

Support for English Language Learners

Writing, Representing: MLR7 Compare and Connect. Use this routine when students present their visual displays. Ask students to consider how comparisons were made between water usage of baths and showers. Draw students' attention to the relationships between quantities in each situation (e.g., How does the rate of water flow from the shower affect water usage? How does tub size affect the comparison?). Emphasize language used to make sense of strategies used to calculate and compare water usage.

Design Principle(s): Maximize meta-awareness; Support sense-making

15.3 Representing Water Usage

Optional: : 10 minutes

This optional activity gives additional review for the material from the unit in the context of this lesson. Students build on the work they did in the previous activity and see that water used in the shower is proportional to time spent in the shower, with constant of proportionality equal to the flow rate of the shower head.

Addressing

- 7.RP.A.2

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Keep students in the same groups.

Fine Motor Skills: Peer Tutors. Pair students with their previously identified peer tutors and allow students who struggle with fine motor skills to dictate graphing as needed.

Support for Students with Disabilities

Action and Expression: Provide Access for Physical Action. Provide access to tools and assistive technologies such as a graphing calculator or graphing software. Some students may benefit from a checklist or list of steps to be able to use the calculator or software.

Supports accessibility for: Organization; Conceptual processing; Attention

Support for English Language Learners

Writing, Conversing: MLR1 Stronger and Clearer Each Time. Use this routine to help students improve their writing, by providing them with multiple opportunities to clarify their explanations through conversation. Give students time to meet with 2–3 partners to share their response to the first question. Students should first check to see if they agree with each other on whether the two quantities they identified represent a proportional relationship. Provide listeners with prompts for feedback that will help their partner add detail to strengthen and clarify their ideas. For example, students can ask their partner, “How do you know they are proportional?” or “What do you mean . . . ?” Then provide students with 3–4 minutes to revise their initial draft based on feedback from their peers. This will help students understand quantities that do and do not represent proportional relationships through communicating their reasoning with a partner.

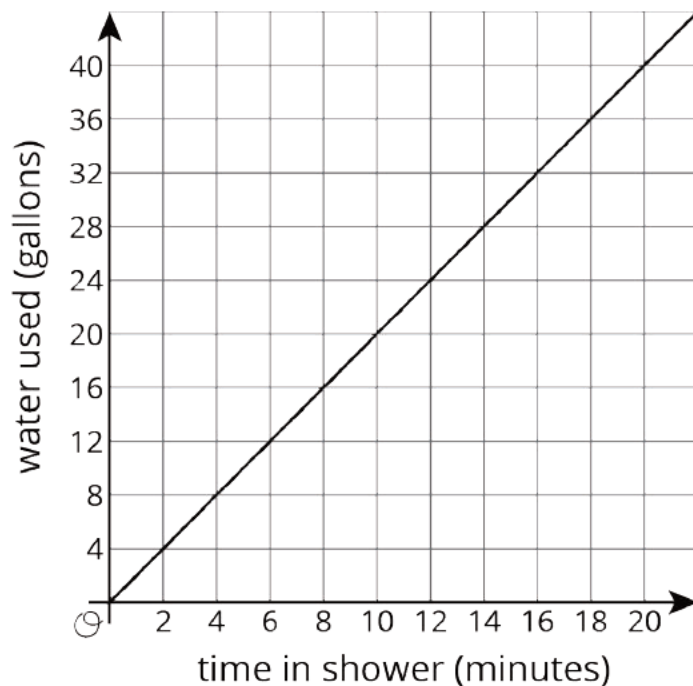
Design Principle(s): Optimize output (for explanation); Maximize meta-awareness

Student Task Statement

1. Continue considering the problem from the previous activity. Name two quantities that are in a proportional relationship. Explain how you know they are in a proportional relationship.
2. What are two constants of proportionality for the proportional relationship? What do they tell us about the situation?
3. On graph paper, create a graph that shows how the two quantities are related. Make sure to label the axes.
4. Write two equations that relate the quantities in your graph. Make sure to record what each variable represents.

Student Response

1. Amount of water used in the shower and time spent in the shower is a proportional relationship. Explanations vary. Sample explanation: The amount of water used divided by the amount of time spent in the shower is always the same constant.
2. Answers vary. Sample response: 2 gallons per minute and $\frac{1}{2}$ minute per gallon.
3. Answers vary. Sample response: Constant of proportionality: 2 gallons per minute.



4. Answers vary. Sample response: $w = 2t$ where w represents the amount of water used in the shower and t represents the amount of time in the shower.

Activity Synthesis

Instruct students to add this information to their displays from the previous activity. Invite students to share their reasoning.

Consider asking discussion questions like these:

- "Do you agree or disagree? Why?"
- "Did anyone solve the problem the same way but would explain it differently?"
- "Who can restate ___'s reasoning in a different way?"
- "Does anyone want to add on to ____'s strategy?"



Family Support Materials

Family Support Materials

Introducing Proportional Relationships

Representing Proportional Relationships with Tables

Family Support Materials 1

This week your student will learn about proportional relationships. This builds on the work they did with equivalent ratios in grade 6. For example, a recipe says “for every 5 cups of grape juice, mix in 2 cups of peach juice.” We can make different-sized batches of this recipe that will taste the same.

grape juice (cups)	peach juice (cups)
5	2
10	4
30	12
2.5	1

The amounts of grape juice and peach juice in each of these batches form equivalent ratios.

The relationship between the quantities of grape juice and peach juice is a **proportional relationship**. In a table of a proportional relationship, there is always some number that you can multiply by the number in the first column to get the number in the second column for any row. This number is called the **constant of proportionality**.

In the fruit juice example, the constant of proportionality is 0.4. There are 0.4 cups of peach juice per cup of grape juice.

grape juice (cups)	peach juice (cups)
5	2
10	4
30	12
2.5	1

Diagram illustrating the constant of proportionality (0.4) for the fruit juice example. The table shows the relationship between grape juice (cups) and peach juice (cups). The constant of proportionality is 0.4, indicated by green arrows and dots.

Here is a task you can try with your student:

Using the recipe “for every 5 cups of grape juice, mix in 2 cups of peach juice”

1. How much peach juice would you mix with 20 cups of grape juice?
2. How much grape juice would you mix with 20 cups of peach juice?

Solution:

1. 8 cups of peach juice. Sample reasoning: We can multiply any amount of grape juice by 0.4 to find the corresponding amount of peach juice, $20 \cdot (0.4) = 8$.
2. 50 cups of grape juice. Sample reasoning: We can *divide* any amount of peach juice by 0.4 to find the corresponding amount of grape juice, $20 \div 0.4 = 50$.

Representing Proportional Relationships with Equations

Family Support Materials 2

This week your student will learn to write equations that represent proportional relationships. For example, if each square foot of carpet costs \$1.50, then the cost of the carpet is proportional to the number of square feet.

The *constant of proportionality* in this situation is 1.5. We can multiply by the constant of proportionality to find the cost of a specific number of square feet of carpet.

carpet (square feet)	cost (dollars)
10	15.00
20	30.00
50	75.00

•1.5

•1.5

•1.5

We can represent this relationship with the equation $c = 1.5f$, where f represents the number of square feet, and c represents the cost in dollars. Remember that the cost of carpeting is always the number of square feet of carpeting times 1.5 dollars per square foot. This equation is just stating that relationship with symbols.

The equation for any proportional relationship looks like $y = kx$, where x and y represent the related quantities and k is the constant of proportionality. Some other examples are $y = 4x$ and $d = \frac{1}{3}t$. Examples of equations that do not represent proportional relationships are $y = 4 + x$, $A = 6s^2$, and $w = \frac{36}{L}$.

Here is a task to try with your student:

1. Write an equation that represents that relationship between the amounts of grape juice and peach juice in the recipe "for every 5 cups of grape juice, mix in 2 cups of peach juice."
2. Select **all** the equations that could represent a proportional relationship:
 - a. $K = C + 273$
 - b. $s = \frac{1}{4}p$
 - c. $V = s^3$
 - d. $h = 14 - x$

e. $c = 6.28r$

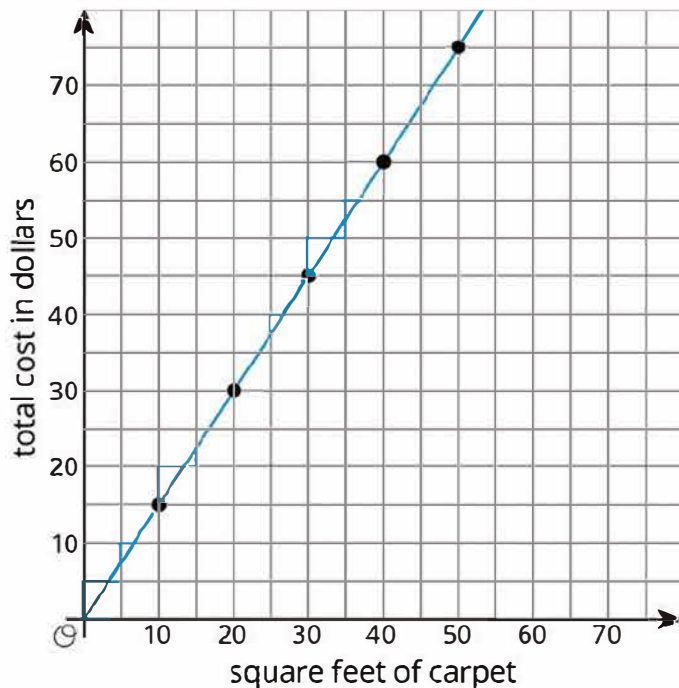
Solution:

1. Answers vary. Sample response: If p represents the number of cups of peach juice and g represents the number of cups of grape juice, the relationship could be written as $p = 0.4g$. Some other equivalent equations are $p = \frac{2}{5}g$, $g = \frac{5}{2}p$, or $g = 2.5p$.
2. B and E. For the equation $s = \frac{1}{4}p$, the constant of proportionality is $\frac{1}{4}$. For the equation $c = 6.28r$, the constant of proportionality is 6.28.

Representing Proportional Relationships with Graphs

Family Support Materials 3

This week your student will work with graphs that represent proportional relationships. For example, here is a graph that represents a relationship between the amount of square feet of carpet purchased and the cost in dollars.

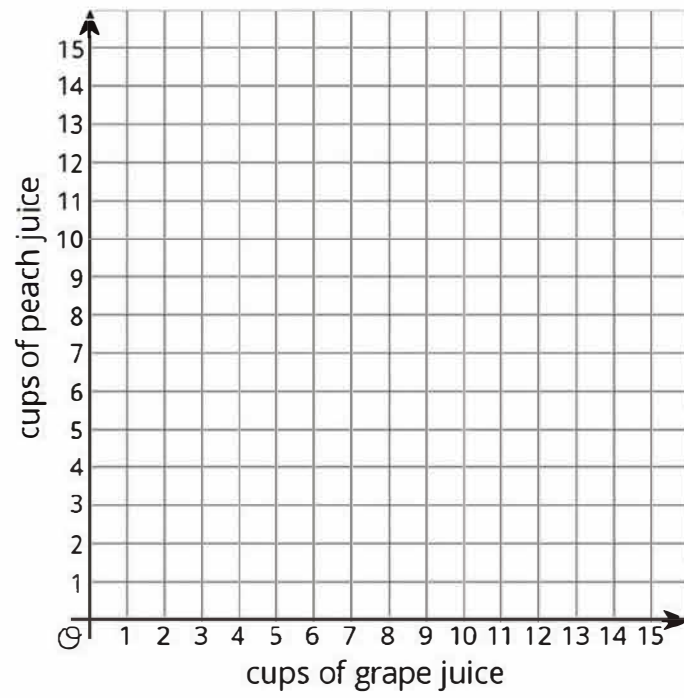


Each square foot of carpet costs \$1.50. The point (10, 15) on the graph tells us that 10 square feet of carpet cost \$15.

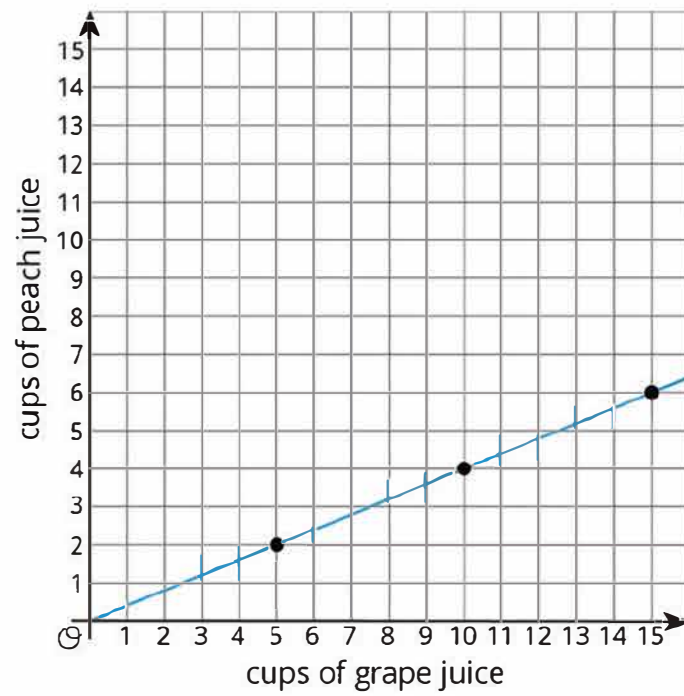
Notice that the points on the graph are arranged in a straight line. If you buy 0 square feet of carpet, it would cost \$0. Graphs of proportional relationships are always parts of straight lines including the point (0, 0).

Here is a task to try with your student:

Create a graph that represents the relationship between the amounts of grape juice and peach juice in different-sized batches of fruit juice using the recipe "for every 5 cups of grape juice, mix in 2 cups of peach juice."



Solution:



Unit Assessments

Check Your Readiness A and B
End-of-Unit Assessment A and B

Introducing Proportional Relationships: Check Your Readiness (A)

1. An airplane flew across the Pacific Ocean. The table shows the amount of time that had passed and the distance traveled when the airplane was traveling at a constant speed. Complete the table, and explain or show your reasoning.

elapsed time (hours)	distance traveled (miles)
2	
3	1,650
6	

2. Blueberries cost \$4.00 per pound. For each question, explain or show your reasoning.

a. How many pounds of blueberries can you buy for \$1.00?

b. How many pounds of blueberries can you buy for \$13.00?

3. Han made some hot chocolate by mixing 4 cups of milk with 6 tablespoons of cocoa.

a. How many tablespoons of cocoa per cup of milk is that?

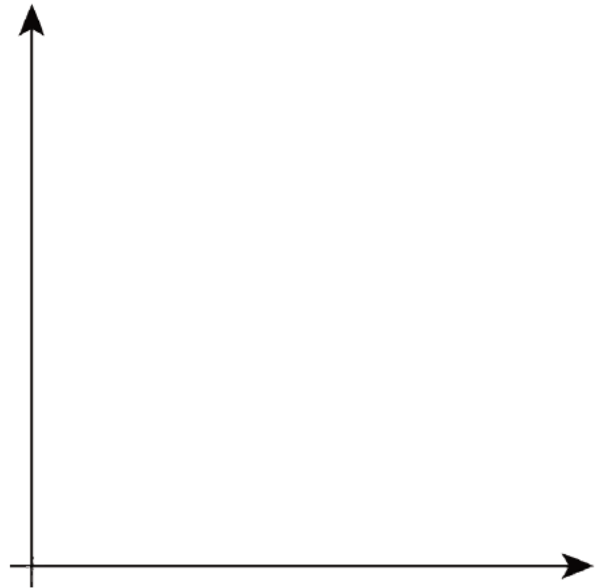
b. How many cups of milk per tablespoon of cocoa is that?

4. An area of 4 square yards is equal to 36 square feet. 10 square yards is equal to how many square feet? Explain or show your reasoning.

5. The ratio of the number of hippos to the number of crocodiles at a watering hole is 4 : 3. Draw a double number line diagram that would show the number of crocodiles if there were 20 hippos.

6. The table shows pairs of coordinates. Plot these in the coordinate plane. Be sure to label the axes.

x	y
4	3
2	6
5	0



7. If you mix red and white paint in different ratios, you will get different shades of pink paint. If the ratios are equivalent, the shades of pink will be the same.
- Mai mixed a batch of pink paint using 5 cups of red paint and 3 cups of white paint.
 - Priya mixed another batch of pink paint using 7 cups of red paint and 4 cups of white paint.

Are these two batches the same shade of pink? Explain.

Introducing Proportional Relationships: Check Your Readiness (B)

1. An airplane flew across the Atlantic Ocean. The table shows the amount of time and the distance traveled when the airplane was traveling at a constant speed. Complete the table with the missing values. Explain or show your reasoning.

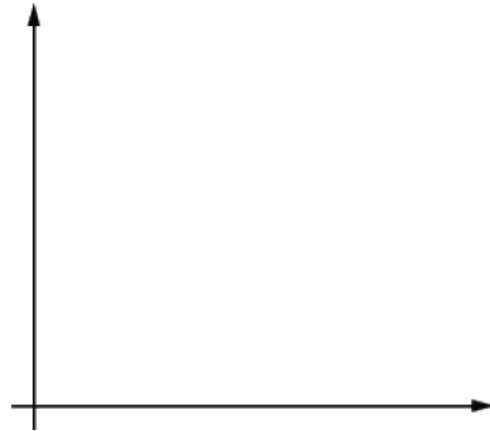
time (hours)	distance (miles)
2	
3	1,230
6	

2. Strawberries cost \$3.00 per pound. For each question, explain or show your reasoning.
- a. How many pounds of strawberries can you buy for \$1.00?
- b. How many pounds of strawberries can you buy for \$11.00?

3. Clare made some lemonade by mixing 6 cups of water with 8 tablespoons of lemonade powder.
- a. How many tablespoons of powder per cup of water is that?
 - b. How many cups of water per tablespoon of powder is that?
4. An area of 6 square yards is equal to 54 square feet. 9 square yards is equal to how many square feet? Explain or show your reasoning.
5. The ratio of the number of ducks to the number of turtles in a pond is 5 : 4. Draw a double number line diagram that would show the number of turtles if there were 25 ducks.

6. The table shows pairs of coordinates. Plot these in the coordinate plane. Be sure to label the axes.

x	y
3	2
4	5
0	6



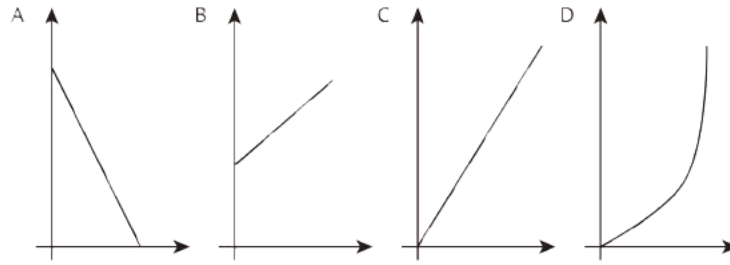
7. If you mix blue and yellow paint in different ratios, you will get different shades of green paint. If the ratios are equivalent, the shades of green will be the same.
- Clare mixed a batch of green paint using 8 cups of yellow paint and 5 cups of blue paint.
 - Jada mixed another batch of green paint using 6 cups of yellow paint and 4 cups of blue paint.

Are these two batches the same shade of green? Explain.

Introducing Proportional Relationships: End-of-Unit Assessment (A)

You will need a straightedge for this assessment.

1. Which graph represents a proportional relationship?



A. A

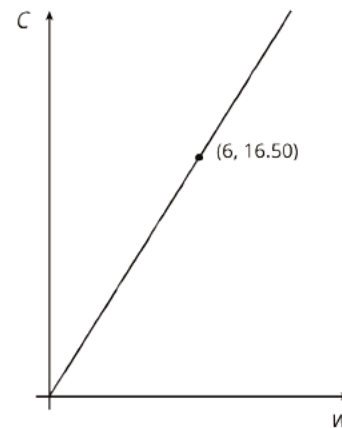
B. B

C. C

D. D

2. The graph shows the cost C in dollars of w pounds of blueberries, a proportional relationship.

Select all the true statements.



A. 1 pound of blueberries costs \$2.75.

B. 2.75 pounds of blueberries cost \$1.

C. 5 pounds of blueberries cost \$15.50.

D. 12 pounds of blueberries cost \$33.

E. The point $(3, 9)$ is on the graph of the proportional relationship.

3. Andre rode his bike at a constant speed. He rode 1 mile in 5 minutes.

Which of these equations represents the amount of time t (in minutes) that it takes him to ride a distance of d miles?

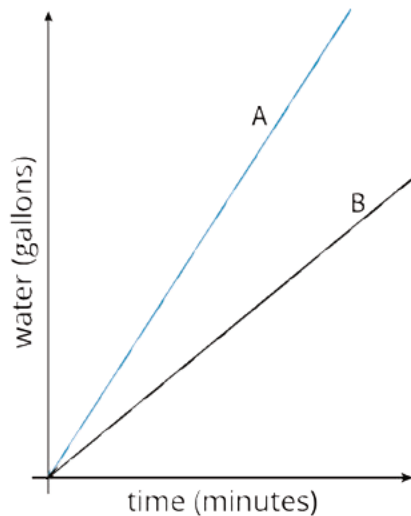
A. $t = 5d$

B. $t = \frac{1}{5}d$

C. $t = d + 4$

D. $t = d - 4$

4. The two lines represent the amount of water, over time, in two tanks that are the same size. Which container is filling more quickly? Explain how you know.



5. The table shows the weights of apples at a grocery store.

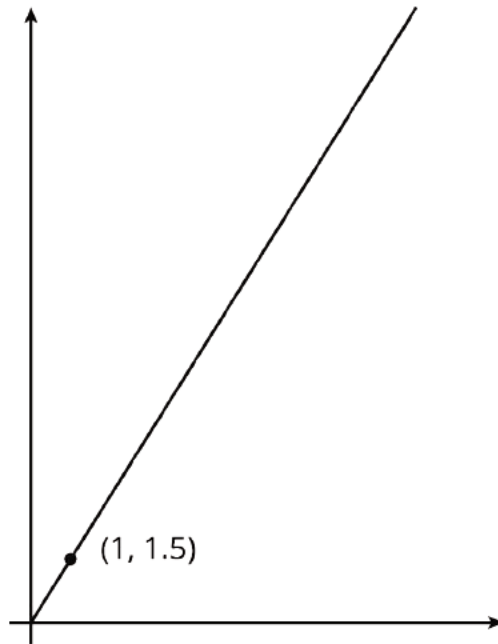
number of apples	weight in kilograms
2	
5	0.60
12	

Complete the table so that there is a proportional relationship between the number of apples and their weight.

6. The equation $F = \frac{9}{5}C + 32$ relates temperature measured in degrees Celsius, C , to degrees Fahrenheit, F .

Determine whether there is a proportional relationship between C and F . Explain or show your reasoning.

7. A recipe for salad dressing calls for 3 tablespoons of oil for every 2 tablespoons of vinegar. The line represents the relationship between the amount of oil and the amount of vinegar needed to make salad dressing according to this recipe. The point $(1, 1.5)$ is on the line.



- Label the axes appropriately.
- Write an equation that represents the proportional relationship between oil and vinegar. Indicate the meaning of each variable.
- Explain the meaning of the point $(1, 1.5)$ in terms of the situation.

Introducing Proportional Relationships: End-of-Unit Assessment (B)

You will need a straightedge for this assessment.

1. Which table represents a proportional relationship?

A.

0	0
2	4
3	9
4	16

B.

0	0
2	6
3	9
4	12

C.

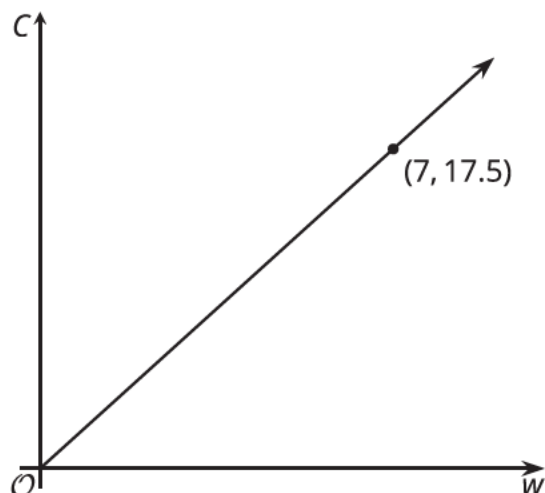
0	12
2	6
3	3
4	0

D.

0	6
2	12
3	15
4	18

2. The graph shows the cost C in dollars of w pounds of peanuts, a proportional relationship.

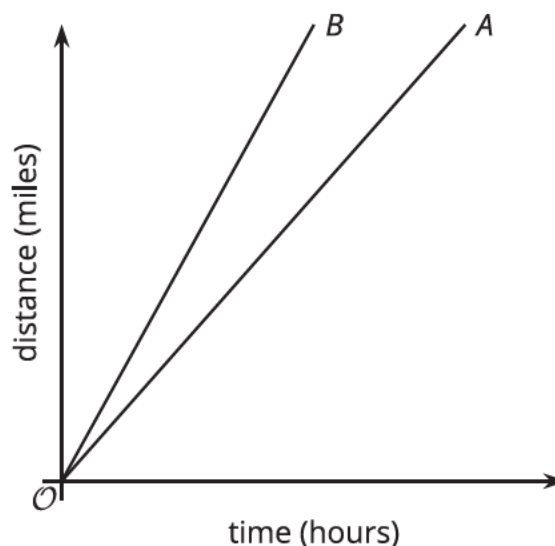
Select all the true statements.



- A. 2.5 pounds of peanuts costs \$1.
 - B. 1 pound of peanuts costs \$2.50.
 - C. 5 pounds of peanuts cost \$12.50.
 - D. 9 pounds of peanuts cost \$19.50.
 - E. The point $(4,10)$ is on the graph of the proportional relationship.
3. Kiran walked at a constant speed. He walked 1 mile in 15 minutes.
- Which of these equations represents the distance d (in miles) that Kiran walks in t minutes?

- A. $d = t + 14$
- B. $d = t - 14$
- C. $d = 15t$
- D. $d = \frac{1}{15}t$

4. The two lines represent the distance, over time, that two cars are traveling. Which car is traveling faster? Explain how you know.



5. The table shows the weights of bananas at a grocery store.

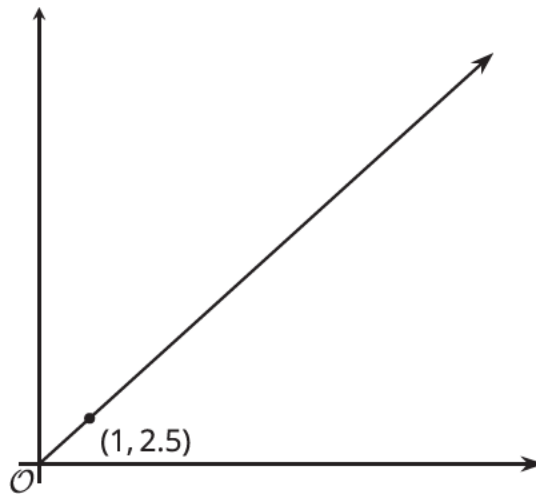
number of bananas	weight in pounds
3	
5	1.6
12	

Complete the table so there is a proportional relationship between the number of bananas and their weight.

6. The equation $S = 50 + 45w$ represents the savings, S , after w weeks working and depositing money into a savings account at the bank.

Determine whether there is a proportional relationship between S and w . Explain or show your reasoning.

7. A recipe for a smoothie calls for 5 cups of strawberries for every 2 cups of bananas. The line represents the relationship between the amount of strawberries and the amount of bananas needed to make a smoothie according to this recipe. The point $(1, 2.5)$ is on the line.



- Label the axes appropriately.
- Write an equation that represents the proportional relationship between strawberries and bananas. Indicate the meaning of each variable.
- Explain the meaning of the point $(1, 2.5)$ in terms of the situation.



Assessment Answer Keys

Check Your Readiness A and B
End-of-Unit Assessment A and B

Introducing Proportional Relationships

Assessment : Check Your Readiness (A)

Teacher Instructions

This diagnostic assessment focuses on the knowledge and skills from grade 6 needed for the unit. Together, the items assess understanding ratio concepts and use ratio reasoning to solve problems. The items also assess ratio and rate language in a variety of contexts. Some items can be completed using scale factors or using unit rates; different items are constructed so that one way is easier, and thus more likely, than another. Use of only one method suggests that the user understands that method.

Note that percents are not assessed here because they are not needed for this unit. Students will return to their work with percents in a later unit, and their knowledge related to percents will be assessed at the beginning of that unit.

Teachers can use the results of the diagnostic assessment to group students in future lessons and to think about where to focus in the first lesson, which provides opportunities to review most of these contexts while preparing students for the grade 7 work of the unit.

Problem 1

The content assessed in this problem is first encountered in Lesson 2: Introducing Proportional Relationships with Tables.

In this unit, students are expected to use tables to solve problems involving constant speed. This context should be a familiar one from grade 6.

The numbers are constructed so that it is simple to scale up and then down, making scaling a more strategic choice than computing a unit rate. Look carefully at student work to see what methods are used to find the missing values. Two different methods are presented in the solution.

If most students struggle with this item, plan to first do the activity Batches of Trail Mix in grade 6, unit 2, lesson 11, so that students can first see a simpler example of a table of equivalent ratios.

Statement

An airplane flew across the Pacific Ocean. The table shows the amount of time that had passed and the distance traveled when the airplane was traveling at a constant speed. Complete the table, and explain or show your reasoning.



elapsed time (hours)	distance traveled (miles)
2	
3	1,650
6	

Solution

Explanations vary. Sample explanations:

- Start with the last row. Find the missing value by doubling 1,650. Now find the distance corresponding to 2 hours by multiplying by $\frac{1}{3}$ or dividing by 3.

elapsed time (hours)	distance traveled (miles)
2	$\frac{1}{3} \cdot (3,300) = 1,100$
3	1650
6	$2 \cdot (1,650) = 3,300$

- Find the unit rate: that is, the number of miles the plane travels in 1 hour. Because 1 hour is $\frac{1}{3}$ of 3 hours, the plane will travel $\frac{1}{3}$ of 1,650 miles in 1 hour. This is 550 miles. This information can then be used to complete the table as shown.

elapsed time (hours)	distance traveled (miles)
2	$2 \cdot 550 = 1,100$
3	$3 \cdot 550 = 1,650$
6	$6 \cdot 550 = 3,300$

Aligned Standards

6.RP.A.1, 6.RP.A.3.a

Problem 2

The content assessed in this problem is first encountered in Lesson 2: Introducing Proportional Relationships with Tables.

The purpose of this assessment item is to see if students can find and use unit rates. Students are given one unit rate (number of dollars per pound), and then they need to use this to find the other unit rate (number of pounds per dollar). Teachers may wish to examine student work in the second question, which can be solved using either of the unit rates. If the unit rate for pounds per dollar is used, the second question is a multiplication problem. If the unit rate given in the task is used, the second question is a division problem. Either method works. Looking at student work also provides some insight into their thinking about ratios. For both questions, a double number line diagram or a table could be used strategically. If students use these representations correctly when not prompted, this foundational knowledge can be used as needed during the unit.

If most students struggle with this item, plan to emphasize techniques for finding both unit rates in the proportional relationship when solving problems in this unit. Show how both unit rates can be found when using representations like double number lines or tables of equivalent ratios.

Statement

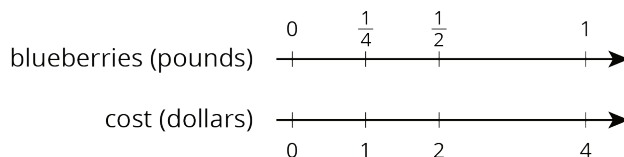
Blueberries cost \$4.00 per pound. For each question, explain or show your reasoning.

1. How many pounds of blueberries can you buy for \$1.00?
2. How many pounds of blueberries can you buy for \$13.00?

Solution

1. $\frac{1}{4}$ or 0.25.

Sample explanation:



2. $\frac{13}{4}$ or 3.25

Sample explanation:

blueberries (pounds)	cost (dollars)
$\frac{1}{4}$	1
1	4
$\frac{13}{4}$	13

Aligned Standards

6.RP.A.2, 6.RP.A.3.b

Problem 3

The content assessed in this problem is first encountered in Lesson 1: One of These Things Is Not Like the Others.

This item assesses student understanding of unit rates. Although students are not asked to use the unit rates to find other values, they need to find both unit rates for the context: tablespoons per cup and cups per tablespoon. Students might use discrete diagrams, double number line diagrams, or tables to arrive at their responses.

If most students struggle with this item, plan to first do the activity Cooking Oatmeal in grade 6, unit 3, lesson 6, to focus on ways to figure out both unit rates in a set of equivalent ratios.

Statement

Han made some hot chocolate by mixing 4 cups of milk with 6 tablespoons of cocoa.

1. How many tablespoons of cocoa per cup of milk is that?
2. How many cups of milk per tablespoon of cocoa is that?

Solution

1. $\frac{6}{4}$ (or equivalent)
2. $\frac{4}{6}$ (or equivalent)

Aligned Standards

6.RP.A.2, 6.RP.A.3.b

Problem 4

The content assessed in this problem is first encountered in Lesson 1: One of These Things Is Not Like the Others.

This task assesses students' ability to find equivalent ratios in a non-scaffolded but familiar context with friendly numbers. Because the context is familiar, students may already know that there are 9 square feet in a square yard and multiply 10 by the unit rate 9 to find out how many square feet there are in 10 square yards. Another method is to use the given information—36 square feet in 4 square yards—and multiply by a factor of $\frac{5}{2}$. This multiplication might occur in two steps (halving and then multiplying by 5). Yet another method is to note that 10, the desired number of square yards, is $4 + 4 + \frac{4}{2}$. In square feet, that corresponds to $36 + 36 + \frac{36}{2}$.

As with previous assessment items, double number line diagrams, double tape diagrams, or ratio tables could be used to complete the task.

If most students struggle with this item, plan to spend time ensuring students understand the structure of representations like double number lines and tables of equivalent ratios when opportunities arise throughout the unit.

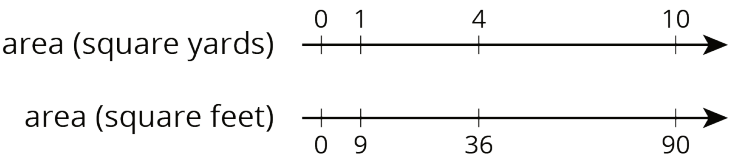
Statement

An area of 4 square yards is equal to 36 square feet. 10 square yards is equal to how many square feet? Explain or show your reasoning.

Solution

90 square feet. Explanations vary. Sample explanations:

- First find the unit rate (number of square feet per square yard) by dividing each number of square feet by the number of square yards: $36 \div 4 = 9$. Then multiply the unit rate by 10 to yield 90 square feet in 10 square yards.



-
-

area (square yards)	area (square feet)
4	36
40	360
10	90

Multiply the entries in the first row by 10. Then multiply the entries in the second row by $\frac{1}{4}$.

Aligned Standards

6.RP.A.1, 6.RP.A.3

Problem 5

The content assessed in this problem is first encountered in Lesson 1: One of These Things Is Not Like the Others.

Double number line diagrams provide a transition to graphing points in the coordinate plane, which becomes very important in grade 7 and later grades. These diagrams offer a little more flexibility than double tape diagrams, in that a wider range of values can be easily plotted on the line segments.

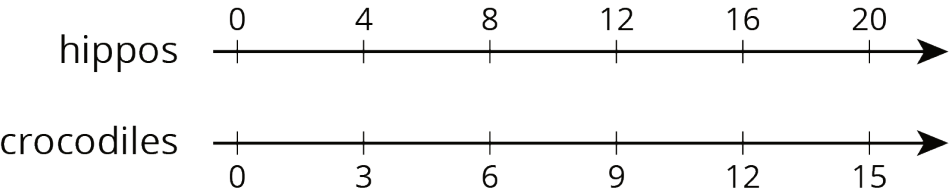
For many of the previous assessment items, double number line diagrams could be effectively used but are not necessary. This task provides a means of directly assessing whether students can use these important representations.

If most students do well with this item, it may be possible to skip the warm-up activity in Lesson 1 since students already have an understanding of double number lines.

Statement

The ratio of the number of hippos to the number of crocodiles at a watering hole is 4 : 3. Draw a double number line diagram that would show the number of crocodiles if there were 20 hippos.

Solution



Aligned Standards

6.RP.A.1, 6.RP.A.3

Problem 6

The content assessed in this problem is first encountered in Lesson 10: Introducing Graphs of Proportional Relationships.

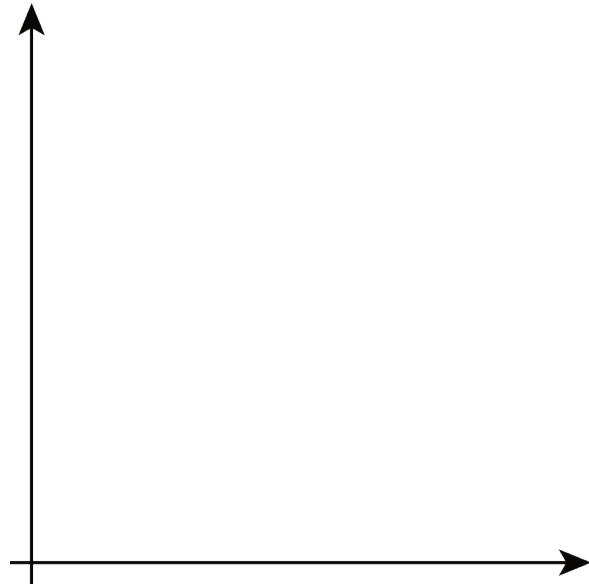
This task assesses whether students can graph points in the coordinate plane. They need to make a scale and label the axes.

If most students do well with this item, it may be possible to skip the warm-up activity in Lesson 10. Students who already have a solid understanding of plotting coordinates would be able to move right on to plotting and interpreting the graphs.

Statement

The table shows pairs of coordinates. Plot these in the coordinate plane. Be sure to label the axes.

x	y
4	3
2	6
5	0



Solution

The points (2, 6), (4, 3), and (5, 0) are plotted, a scale is indicated and the axes are labeled with the appropriate variable.

Aligned Standards

6.RP.A.3.a

Problem 7

The content assessed in this problem is first encountered in Lesson 7: Comparing Relationships with Tables.

This item has less scaffolding than earlier items and requires an explanation. Students may use a variety of methods, but given the numbers, there are two likely choices: One is to compute and compare the unit rates for each batch. The other is to find a batch of Mai's mixture and a batch of Priya's mixture that have the same amount of red paint (or same amount of white paint) to find out if the two batches also have the same amount of the other color of paint.

If most students struggle with this item, plan to spend time ensuring students understand representations of sets of equivalent ratios as opportunities arise. Success with this type of problem may be a good indication that students are ready for the grade 7 material on ratios and proportional relationships.

Statement

If you mix red and white paint in different ratios, you will get different shades of pink paint. If the ratios are equivalent, the shades of pink will be the same.

- Mai mixed a batch of pink paint using 5 cups of red paint and 3 cups of white paint.

- Priya mixed another batch of pink paint using 7 cups of red paint and 4 cups of white paint.

Are these two batches the same shade of pink? Explain.

Solution

No, they are different shades of pink. Priya's paint is redder than Mai's paint. Explanations vary. Sample explanations:

- Find unit rates for each. For Mai's pink paint, there are 5 cups of red paint for every 3 cups of white paint, so this means that in the mixture there are $\frac{5}{3}$ cups of red paint for every cup of white paint. For Priya's mixture, there are 7 cups of red paint for every 4 cups of white paint or $\frac{7}{4}$ cups of red paint for each cup of white paint. Since the two unit rates are different, the two shades of pink are different.
- Find a batch of each mixture with the same amounts of one type of paint. The tables below show two batches with the same amount of white paint. Here is a table for Mai's mixture.

cups of red paint	cups of white paint
5	3
20	12

Here is a table for Priya's mixture.

cups of red paint	cups of white paint
7	4
21	12

Looking at the second row of each table, we see that Priya's mixture has one extra cup of red paint for each 12 cups of white paint. So Priya's pink paint is a little bit redder than Mai's pink paint.

Aligned Standards

6.RP.A.2, 6.RP.A.3

Introducing Proportional Relationships

Assessment : Check Your Readiness (B)

Teacher Instructions

Assessment goals:

1. Understand and use ratio and rate language in a variety of contexts.
2. Find equivalent ratios using a scale factor.
3. Find unit rates in context.
4. Given one value of a ratio, use the unit rate to find the other.
5. Represent equivalent ratios in a table.
6. Represent a problem involving ratios with a double number line.
7. Graph points in the coordinate plane.

This diagnostic assessment focuses on the knowledge and skills from grade 6 needed for the unit. Together, the items assess understanding ratio concepts and use ratio reasoning to solve problems. The items also assess goal 1 because they cover a variety of contexts and a variety of ways of using ratio and rate language. Some items can be completed using scale factors or using unit rates; different items are constructed so that one way is easier, and thus more likely, than another. Use of only one method suggests that the user understands that method.

Note that percents are not assessed here because they are not needed for this unit. Students will return to their work with percents in a later unit, and their knowledge related to percents will be assessed at the beginning of that unit.

Teachers can use the results of the diagnostic assessment to group students in future lessons and to think about where to focus in the first lesson, which provides opportunities to review most of these contexts while preparing students for the grade 7 work of the unit.

Problem 1

The content assessed in this problem is first encountered in Lesson 2: Introducing Proportional Relationships with Tables.

In this unit, students are expected to use tables to solve problems involving constant speed. This context should be a familiar one from grade 6.

The numbers are constructed so that it is simple to scale up and then down, making scaling a more strategic choice than computing a unit rate. Look carefully at student work to see what methods are used to find the missing values. Two different methods are presented in the solution.

If most students struggle with this item, plan to first do the activity Batches of Trail Mix in grade 6, unit 2, lesson 11, so that students can first see a simpler example of a table of equivalent ratios.

Statement

An airplane flew across the Atlantic Ocean. The table shows the amount of time and the distance traveled when the airplane was traveling at a constant speed. Complete the table with the missing values. Explain or show your reasoning.

time (hours)	distance (miles)
2	
3	1,230
6	

Solution

Explanations vary. Sample explanations:

- Start with the last row. Find the missing value by doubling 1,230. Now find the distance corresponding to 2 hours by multiplying by $\frac{1}{3}$ or dividing by 3.

time (hours)	distance (miles)
2	$\frac{1}{3} \cdot (2,460) = 820$
3	1,230
6	$2 \cdot (1,230) = 2,460$

- Find the unit rate: that is, the number of miles the plane travels in 1 hour. Because 1 hour is $\frac{1}{3}$ of 3 hours, the plane will travel $\frac{1}{3}$ of 1,230 miles in 1 hour. This is 410 miles. This information can then be used to complete the table as shown.

time (hours)	distance (miles)
2	$2 \cdot 410 = 820$
3	$3 \cdot 410 = 1,230$
6	$6 \cdot 410 = 2,460$

Aligned Standards

6.RP.A.1, 6.RP.A.3.a

Problem 2

The content assessed in this problem is first encountered in Lesson 2: Introducing Proportional Relationships with Tables.

The purpose of this assessment item is to see if students can find and use unit rates. Students are given one unit rate (number of dollars per pound), and then they need to use this to find the other unit rate (number of pounds per dollar). Teachers may wish to examine student work in the second question, which can be solved using either of the unit rates. If the unit rate for pounds per dollar is used, the second question is a multiplication problem. If the unit rate given in the task is used, the second question is a division problem. Either method works. Looking at student work also provides some insight into their thinking about ratios. For both questions, a double number line diagram or a table could be used strategically. If students use these representations correctly when not prompted, this foundational knowledge can be used as needed during the unit.

If most students struggle with this item, plan to emphasize techniques for finding both unit rates in the proportional relationship when solving problems in this unit. Show how both unit rates can be found when using representations like double number lines or tables of equivalent ratios.

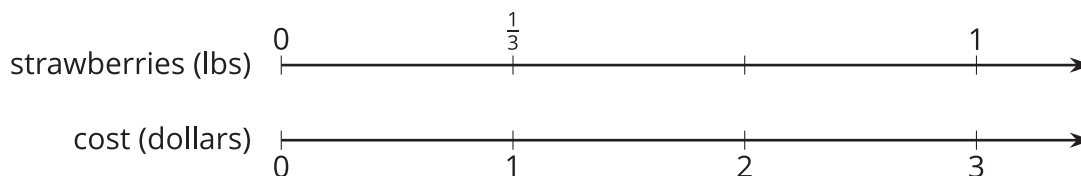
Statement

Strawberries cost \$3.00 per pound. For each question, explain or show your reasoning.

1. How many pounds of strawberries can you buy for \$1.00?
2. How many pounds of strawberries can you buy for \$11.00?

Solution

1. $\frac{1}{3}$ or 0.33



Sample explanation:

1. $\frac{11}{3}$ or 3.67 (if students multiply 0.33 by 11, then they could also have 3.63)

Sample explanation:

strawberries (pounds)	cost (dollars)
$\frac{1}{3}$	1
1	3
$\frac{11}{3}$	11

Aligned Standards

6.RP.A.2, 6.RP.A.3.b

Problem 3

The content assessed in this problem is first encountered in Lesson 1: One of These Things Is Not Like the Others.

This item assesses student understanding of unit rates. Although students are not asked to use the unit rates to find other values, they need to find both unit rates for the context: tablespoons per cup and cups per tablespoon. Students might use discrete diagrams, double number line diagrams, or tables to arrive at their responses.

If most students struggle with this item, plan to first do the activity Cooking Oatmeal in grade 6, unit 3, lesson 6, to focus on ways to figure out both unit rates in a set of equivalent ratios.

Statement

Clare made some lemonade by mixing 6 cups of water with 8 tablespoons of lemonade powder.

1. How many tablespoons of powder per cup of water is that?
2. How many cups of water per tablespoon of powder is that?

Solution

1. $\frac{8}{6}$ (or equivalent)
2. $\frac{6}{8}$ (or equivalent)

Aligned Standards

6.RP.A.2, 6.RP.A.3.b

Problem 4

The content assessed in this problem is first encountered in Lesson 1: One of These Things Is Not Like the Others.

This task assesses students’ ability to find equivalent ratios in a non-scaffolded but familiar context with friendly numbers. Because the context is familiar, students may already know that there are 9 square feet in a square yard and multiply 9 by the unit rate 9 to find out how many square feet there are in 9 square yards. Another method is to use the given information—54 square feet in 6 square yards—and multiply by a factor of $\frac{3}{2}$. This multiplication might occur in two steps (halving and then multiplying by 3). Yet another method is to note that 9, the desired number of square yards, is $6 + \frac{6}{2}$. In square feet, that corresponds to $54 + \frac{54}{2}$.

As with previous assessment items, double number line diagrams, double tape diagrams, or ratio tables could be used to complete the task.

If most students struggle with this item, plan to spend time ensuring students understand the structure of representations like double number lines and tables of equivalent ratios when opportunities arise throughout the unit.

Statement

An area of 6 square yards is equal to 54 square feet. 9 square yards is equal to how many square feet? Explain or show your reasoning.

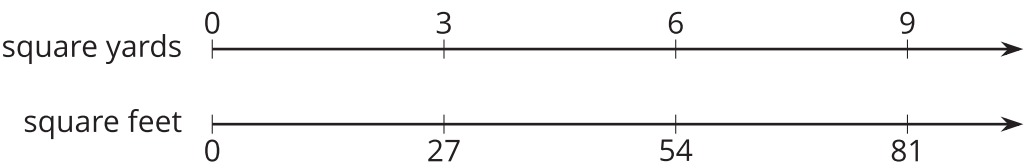
Solution

81 square feet. Explanations vary. Sample explanations:

- First find the unit rate (number of square feet per square yard) by dividing each number of square feet by the number of square yards: $54 \div 6 = 9$. Then multiply the unit rate by 9 to yield 81 square feet in 9 square yards.

area (square yards)	area (square feet)
3	27
6	54
9	81

Divide 6 and 54 by 2 to get the entries in the first row. Then multiply the entries in the first row by 3 to get the third row.



Aligned Standards

6.RP.A.1, 6.RP.A.3

Problem 5

The content assessed in this problem is first encountered in Lesson 1: One of These Things Is Not Like the Others.

Double number line diagrams provide a transition to graphing points in the coordinate plane, which becomes very important in grade 7 and later grades. These diagrams offer a little more flexibility than double tape diagrams, in that a wider range of values can be easily plotted on the line segments.

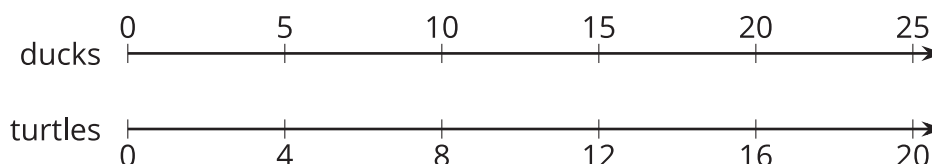
For many of the previous assessment items, double number line diagrams could be effectively used but are not necessary. This task provides a means of directly assessing whether students can use these important representations.

If most students do well with this item, it may be possible to skip the warm-up activity in Lesson 1 since students already have an understanding of double number lines.

Statement

The ratio of the number of ducks to the number of turtles in a pond is 5 : 4. Draw a double number line diagram that would show the number of turtles if there were 25 ducks.

Solution



Aligned Standards

6.RP.A.1, 6.RP.A.3

Problem 6

The content assessed in this problem is first encountered in Lesson 10: Introducing Graphs of Proportional Relationships.

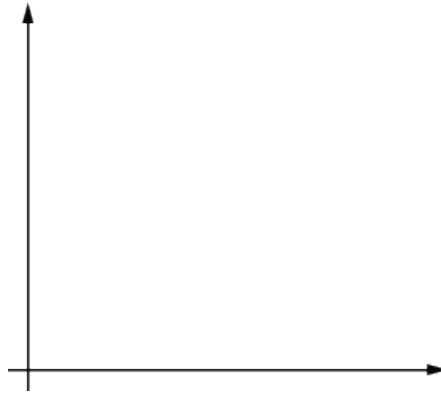
This task assesses whether students can graph points in the coordinate plane. They need to make a scale and label the axes.

If most students do well with this item, it may be possible to skip the warm-up activity in Lesson 10. Students who already have a solid understanding of plotting coordinates would be able to move right on to plotting and interpreting the graphs.

Statement

The table shows pairs of coordinates. Plot these in the coordinate plane. Be sure to label the axes.

x	y
3	2
4	5
0	6



Solution

The points (3,2), (4,5), and (0,6) are plotted, a scale is indicated and the axes are labeled with the appropriate variable.

Aligned Standards

6.RP.A.3.a

Problem 7

The content assessed in this problem is first encountered in Lesson 7: Comparing Relationships with Tables.

This item has less scaffolding than earlier items and requires an explanation. Students may use a variety of methods, but given the numbers, there are two likely choices. One is to compute and compare the unit rates for each batch. The other is to find a batch of Clare's mixture and a batch of Jada's mixture that have the same amount of blue paint (or same amount of yellow paint) to find out if the two batches also have the same amount of the other color of paint.

Success with this type of problem may be a good indication that students are ready for the grade 7 material on ratios and proportional relationships.

If most students struggle with this item, plan to spend time ensuring students understand representations of sets of equivalent ratios as opportunities arise. Success with this type of problem may be a good indication that students are ready for the grade 7 material on ratios and proportional relationships.

Statement

If you mix blue and yellow paint in different ratios, you will get different shades of green paint. If the ratios are equivalent, the shades of green will be the same.

- Clare mixed a batch of green paint using 8 cups of yellow paint and 5 cups of blue paint.
- Jada mixed another batch of green paint using 6 cups of yellow paint and 4 cups of blue paint.

■ Are these two batches the same shade of green? Explain.

Solution

No, they are different shades of green. Clare's paint is more yellow than Jada's paint. Explanations vary. Sample explanations:

- Find unit rates for each. For Clare's green paint, there are 8 cups of yellow paint for every 5 cups of blue paint, so this means that in the mixture there are $\frac{8}{5}$ cups of yellow for every cup of blue paint. For Jada's mixture, there are 6 cups of yellow for every 4 cups of blue or $\frac{6}{4}$ cups of yellow for each cup of blue. Since the two unit rates are different the two shades of green are different.
- Find a batch of each mixture with the same amounts of one type of paint. The tables below show two batches with the same amount of yellow paint. Here is a table for Clare's mixture.

cups of yellow paint	cups of blue paint
8	5
32	20

Here is a table for Jada's mixture.

cups of yellow paint	cups of blue paint
6	4
30	20

Looking at the second row of each table, we see that Clare's mixture has two extra cups of yellow paint for each 20 cups of blue paint. So Jada's green paint is a little bit more yellow than Clare's green paint.

Aligned Standards

6.RP.A.2, 6.RP.A.3

Introducing Proportional Relationships

Assessment : End-of-Unit Assessment (A)

Teacher Instructions

Provide access to a straightedge.

Student Instructions

You will need a straightedge for this assessment.

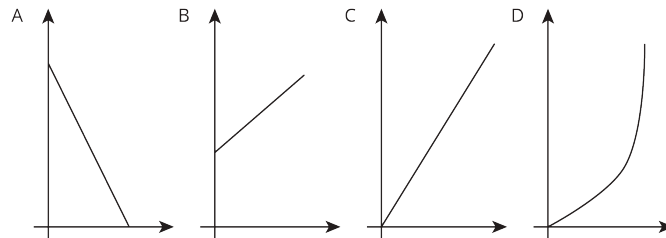
Problem 1

Students identify a proportional relationship from a graph. Students can correctly identify C and exclude A, B, and D without a complete understanding of proportional relationships by using the general rule that the graph of a proportional relationship lies on a ray in the first quadrant starting at $(0, 0)$.

Students selecting A or B may know the graph of a proportional relationship is linear, but may not recall that line must contain $(0, 0)$. Students selecting D may know the graph contains $(0, 0)$ but may not know that the graph must be linear.

Statement

Which graph represents a proportional relationship?



- A. A
- B. B
- C. C
- D. D

Solution

C

Aligned Standards

7.RP.A.2.a

Problem 2

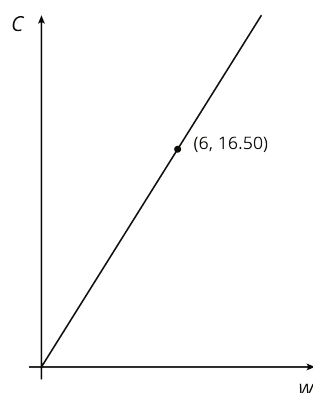
Students use the graph of a proportional relationship in order to find the unit rate, then to apply the unit rate to other questions about the situation. They are given a single point on the graph and work with these numbers to calculate the unit rate.

Students failing to select A may not have determined the correct unit rate of \$2.75. Students selecting B are using the unit rate in reverse, and may have the variables confused. Students selecting C have a misunderstanding about the nature of proportional relationships, since they subtracted 1 from each quantity. Students failing to select D may have made a calculation error, or failed to use the proportionality of doubling the values given in the table. Students selecting E might be eyeballing the graph, or estimating: the actual cost of 3 pounds is \$8.25, so (3, 9) is not on the graph.

Statement

The graph shows the cost C in dollars of w pounds of blueberries, a proportional relationship.

Select **all** the true statements.



- A. 1 pound of blueberries costs \$2.75.
- B. 2.75 pounds of blueberries cost \$1.
- C. 5 pounds of blueberries cost \$15.50.
- D. 12 pounds of blueberries cost \$33.
- E. The point (3, 9) is on the graph of the proportional relationship.

Solution

["A", "D"]

Aligned Standards

7.RP.A.2.d

Problem 3

Students need to recognize that when traveling at constant speed, the amount of time traveled is proportional to the distance traveled. No graphing or explanation is required and the context is a

familiar one, although the information is given in terms of pace in minutes per mile rather than speed in miles per minute.

Students selecting B have probably reversed the variables in context; they may have written $d = \frac{1}{5}t$ and ended there. They may also have misread the description as 5 miles in 1 minute, or may just be writing $\frac{1}{5}$ using their knowledge that division is a typical operation in rate contexts. Students selecting C have made an error involving the proportional relationship: even though 1 mile in 5 minutes makes this equation true, it is not a generally correct equation. Students selecting D have likely made two errors: the error from C, along with the error in reversing the variables' meaning.

Statement

Andre rode his bike at a constant speed. He rode 1 mile in 5 minutes.

Which of these equations represents the amount of time t (in minutes) that it takes him to ride a distance of d miles?

A. $t = 5d$

B. $t = \frac{1}{5}d$

C. $t = d + 4$

D. $t = d - 4$

Solution

A

Aligned Standards

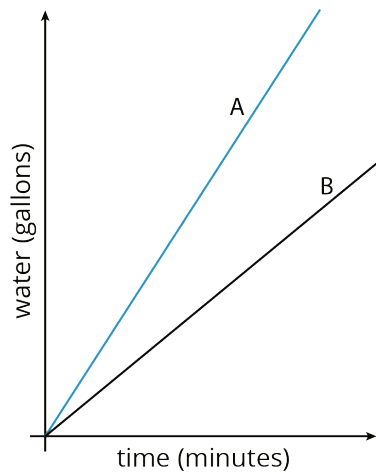
7.RP.A.2.c

Problem 4

This task requires students to interpret graphs of proportional relationships without numerical values. The rates at which the containers fill correspond to the constants of proportionality for the two relationships: the greater rate corresponds to the steeper graph.

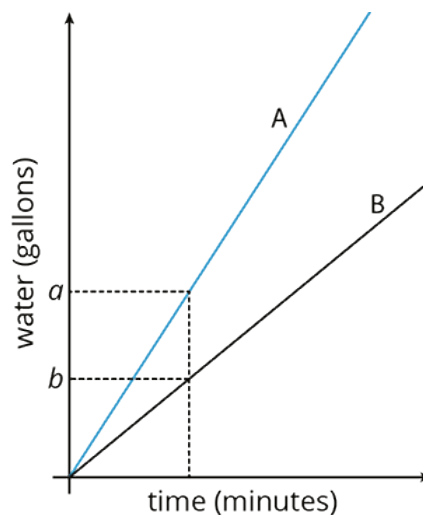
Statement

The two lines represent the amount of water, over time, in two tanks that are the same size. Which container is filling more quickly? Explain how you know.



Solution

Container A is filling more quickly. In each graph, the constant of proportionality represents the rate at which the water is flowing into the containers. The container that is filling more quickly is the one for which the graph is steeper: Container A is filling more quickly than Container B. Alternatively, choose a time and see how much water is in the two containers at that time. The graph below shows the amounts of water in the containers at a given time t .



Because $a > b$ at time t , more water has flowed into Container A than into Container B.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: Container A, because the graph for Container A is steeper.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.

- Sample errors: picking Container A with an incomplete or omitted explanation; picking Container B with a complete explanation of how a correct rate (minutes per gallon, for example) is larger.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: picking Container B with an incomplete or omitted explanation; claiming both containers are filling equally quickly.

Aligned Standards

7.RP.A.2.b

Problem 5

This item assesses students' ability to work with a proportional relationship defined by a table. While there is more than one possible way to complete the problem, the most likely method is to determine the unit rate of \$0.12 per apple, then multiply to determine the cost for 2 and for 12 apples.

Statement

The table shows the weights of apples at a grocery store.

number of apples	weight in kilograms
2	
5	0.60
12	

Complete the table so that there is a proportional relationship between the number of apples and their weight.

Solution

number of apples	weight in kilograms
2	0.24
5	0.60
12	1.44

Aligned Standards

7.RP.A.2.b

Problem 6

This item requires an explanation. Students' explanations involving graphs and tables should more clearly indicate their overall understanding of the concept of a proportional relationship. Students have done similar work in this unit, with a more directed table-building exercise, so this version is more open-ended.

Statement

The equation $F = \frac{9}{5}C + 32$ relates temperature measured in degrees Celsius, C , to degrees Fahrenheit, F .

Determine whether there is a proportional relationship between C and F . Explain or show your reasoning.

Solution

The relationship between degrees Celsius and degrees Fahrenheit is not proportional. Explanations vary. Sample explanations:

- Using a graph: Drawing the graph of the relationship shows it is not a line through the origin.
- Using a table: These three rows are generated from the equation.

temperature (degrees C)	temperature (degrees F)
0	32
10	50
20	68

In the second row, $50 = 10 \cdot 5$, but in the third row, $68 = 20 \cdot (3.4)$. Because 5 is different from 3.4, this is not a proportional relationship.

The first row of the table also shows that the relationship cannot be proportional because 32 is not a multiple of 0.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: No, because this graph is not a line through $(0, 0)$. (Response includes a correct graph.)
- Acceptable errors: axes of graph or headers of table are unlabeled.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: calculation errors while generating table or graph, including ones that lead to an incorrect conclusion based on the data (for example, accidentally using $F = \frac{9}{5}C \cdot 32$); incorrectly drawn graph.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: statement of a proportional relationship, unless based on incorrectly calculated data; answering either yes or no without explanation; answering based only on the equation, without building a table or graph.

Aligned Standards

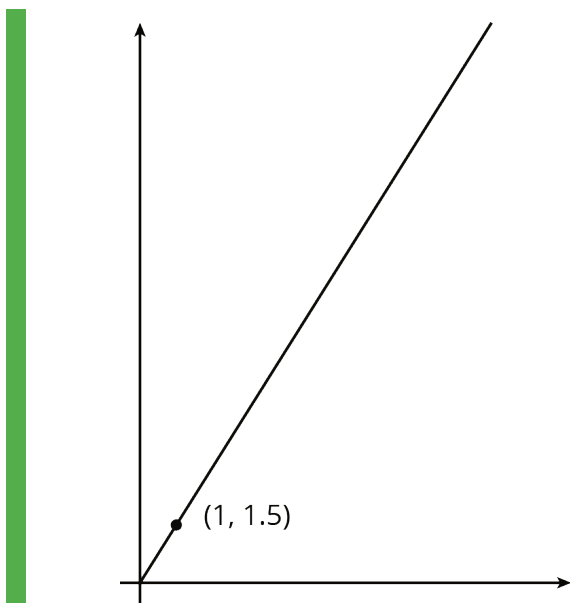
7.RP.A.2, 7.RP.A.2.a

Problem 7

This item is more challenging than earlier items. The axes are not labeled, and the coordinates of the point on the graph tell the amount of one quantity given one unit of the other quantity, rather than the values in the task statement. There are several ways students might reason about this problem, but however they do it, they need to explain their reasoning, requiring an understanding of the relationship between the constant of proportionality and the graph of the corresponding proportional relationship.

Statement

A recipe for salad dressing calls for 3 tablespoons of oil for every 2 tablespoons of vinegar. The line represents the relationship between the amount of oil and the amount of vinegar needed to make salad dressing according to this recipe. The point (1, 1.5) is on the line.



1. Label the axes appropriately.
2. Write an equation that represents the proportional relationship between oil and vinegar. Indicate the meaning of each variable.
3. Explain the meaning of the point $(1, 1.5)$ in terms of the situation.

Solution

1. The vertical axis should be labeled "oil (tablespoons)," and the horizontal axis should be labeled "vinegar (tablespoons)."
2. Answers vary. Sample response: If y is the number of tablespoons of oil, and x is the number of tablespoons of vinegar, then $y = 1.5x$.
3. The point $(1, 1.5)$ indicates that the recipe works with 1 tablespoon of vinegar and 1.5 tablespoons of oil. This point gives a unit rate.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample:
 1. The graph is labeled "oil (tbsp)" on the vertical and "vinegar (tbsp)" on the horizontal.
 2. $o = 1.5v$ where o is oil and v is vinegar.
 3. You could make the recipe with 1 tablespoon of vinegar and 1.5 tablespoons of oil.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: lack of units in either graph or description; one reversal of the quantities in the three parts.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: 2 or 3 reversals of the quantities (in total for the three parts); one part omitted or containing a more significant error than quantity reversal; two error types from Tier 2 response.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: more than one part omitted or containing a more significant error than quantity reversal; statements suggesting a non-proportional relationship.

Aligned Standards

7.RP.A.2.c, 7.RP.A.2.d

Introducing Proportional Relationships

Assessment : End-of-Unit Assessment (B)

Teacher Instructions

Provide access to a straightedge.

Student Instructions

You will need a straightedge for this assessment.

Problem 1

Students identify a proportional relationship from a table. Students can correctly exclude C and D without a complete understanding of proportional relationships by using the general rule that the graph of a proportional relationship lies on a ray in the first quadrant starting at $(0, 0)$. Students selecting C or D may know the graph of a proportional relationship is linear, but may not recall that line must contain $(0, 0)$. Students selecting A may know the graph contains $(0, 0)$ but may not know that the graph must be linear.

Statement

Which table represents a proportional relationship?

A.

0	0
2	4
3	9
4	16

B.

0	0
2	6
3	9
4	12

C.

0	12
2	6
3	3
4	0

D.

0	6
2	12
3	15
4	18

Solution

B

Aligned Standards

7.RP.A.2.a

Problem 2

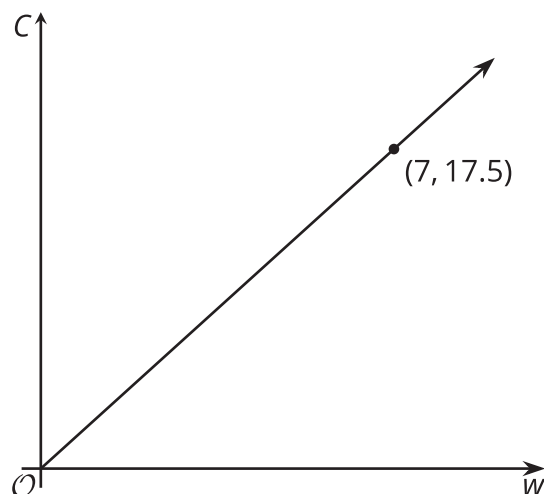
Students use the graph of a proportional relationship in order to find the unit rate, then to apply the unit rate to other questions about the situation. They are given a single point on the graph and work with these numbers to calculate the unit rate. Students selecting A are using the unit rate in reverse, and may have the variables confused. Students failing to select B may not have determined the correct unit rate of \$2.50. Students failing to select C may have made a calculation error.

Students selecting D have a misunderstanding about the nature of proportional relationships, since they added 2 to each quantity. Students failing to select E might be incorrectly eyeballing the graph or estimating: since the cost of 4 pounds is \$10, the point (4, 10) should be on the graph.

Statement

The graph shows the cost C in dollars of w pounds of peanuts, a proportional relationship.

Select **all** the true statements.



- A. 2.5 pounds of peanuts costs \$1.
- B. 1 pound of peanuts costs \$2.50.
- C. 5 pounds of peanuts cost \$12.50.
- D. 9 pounds of peanuts cost \$19.50.
- E. The point (4,10) is on the graph of the proportional relationship.

Solution

["B", "C", "E"]

Aligned Standards

7.RP.A.2.d

Problem 3

Students need to recognize that when traveling at constant speed, the amount of time traveled is proportional to the distance traveled. No graphing or explanation is required and the context is a familiar one.

Students selecting C have probably reversed the variables in context; they may have written $t = 15d$ and ended there. They may also have misread the description as 15 miles in 1 minute. Students selecting B have made an error involving the proportional relationship: even though 1 mile in 15 minutes makes this equation true, it is not a generally correct equation. Students selecting A have likely made two errors: the error from B, along with the error in reversing the variables' meaning.

Statement

Kiran walked at a constant speed. He walked 1 mile in 15 minutes.

Which of these equations represents the distance d (in miles) that Kiran walks in t minutes?

A. $d = t + 14$

B. $d = t - 14$

C. $d = 15t$

D. $d = \frac{1}{15}t$

Solution

D

Aligned Standards

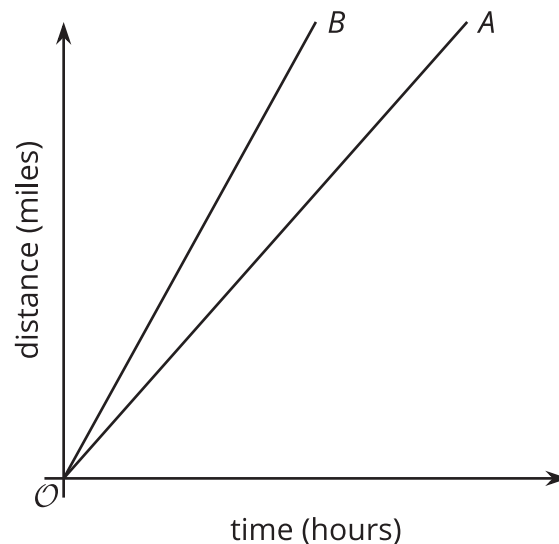
7.RP.A.2.c

Problem 4

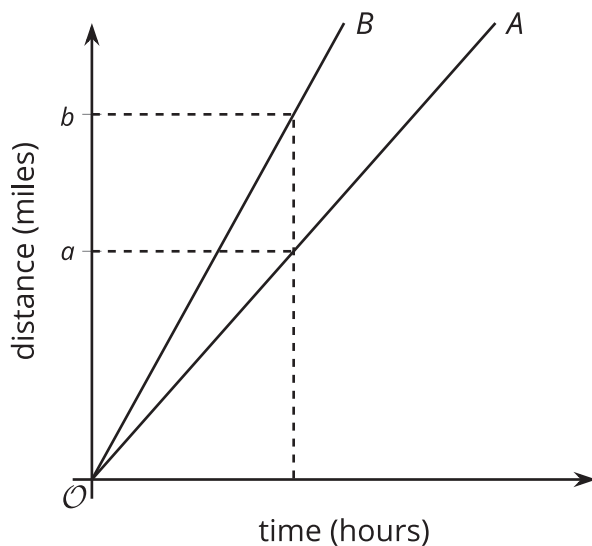
This task requires students to interpret graphs of proportional relationships without numerical values. The rates at which the containers fill correspond to the constants of proportionality for the two relationships: the greater rate corresponds to the steeper graph.

Statement

The two lines represent the distance, over time, that two cars are traveling. Which car is traveling faster? Explain how you know.



Solution



Car B is traveling faster. In each graph, the constant of proportionality represents the rate at which the cars are traveling. The car that is traveling faster is the one for which the graph is steeper: Car B is traveling faster than Car A. Alternatively, choose a time and see how many miles the two cars have traveled at that time. The graph below shows the distance traveled at the given time t .

Because $b > a$ at time t , Car B is moving faster than Car A.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: Car B, because the graph for Car B is steeper.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: picking Car B with an incomplete or omitted explanation; picking Car A with a complete explanation of how a correct rate (hours per mile, for example) is larger.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: picking Car A with an incomplete or omitted explanation; claiming both cars are traveling with equal speed.

Aligned Standards

7.RP.A.2.b

Problem 5

This item assesses students' ability to work with a proportional relationship defined by a table. While there is more than one possible way to complete the problem, the most likely method is to determine the unit rate of 0.32 pounds per banana, then multiply to determine the cost for 3 and for 12 bananas.

Statement

The table shows the weights of bananas at a grocery store.

number of bananas	weight in pounds
3	
5	1.6
12	

Complete the table so there is a proportional relationship between the number of bananas and their weight.

Solution

number of bananas	weight in pounds
3	0.96
5	1.6
12	3.84

Aligned Standards

7.RP.A.2.b

Problem 6

This item requires an explanation. Students' explanations involving graphs and tables should more clearly indicate their overall understanding of the concept of a proportional relationship. Students have done similar work in this unit, with a more directed table-building exercise, so this version is more open-ended.

Statement

The equation $S = 50 + 45w$ represents the savings, S , after w weeks working and depositing money into a savings account at the bank.

Determine whether there is a proportional relationship between S and w . Explain or show your reasoning.

Solution

- Using a graph: Drawing the graph of the relationship shows it is not a line through the origin.
- Using a table: These three rows are generated from the equation.

	number of weeks	savings in dollars
row 1	0	50
row 2	10	500
row 3	20	950

In the second row, $500 = 10 \cdot 50$, but in the third row, $950 = 20 \cdot (47.5)$. Because 50 is different from 47.5, this is not a proportional relationship.

The first row of the table also shows that the relationship cannot be proportional because 50 is not a multiple of 0.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: No, because this graph is not a line through (0,0). (Response includes a correct graph.)
- Acceptable errors: axes of graph or headers of table are unlabeled.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: calculation errors while generating table or graph, including ones that lead to an incorrect conclusion based on the data (for example, reversal of quantities $S = 45 + 50w$ or incorrectly drawn graph).

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: statement of a proportional relationship, unless based on incorrectly calculated data; answering either yes or no without explanation; answering based only on the equation, without building a table or graph.

Aligned Standards

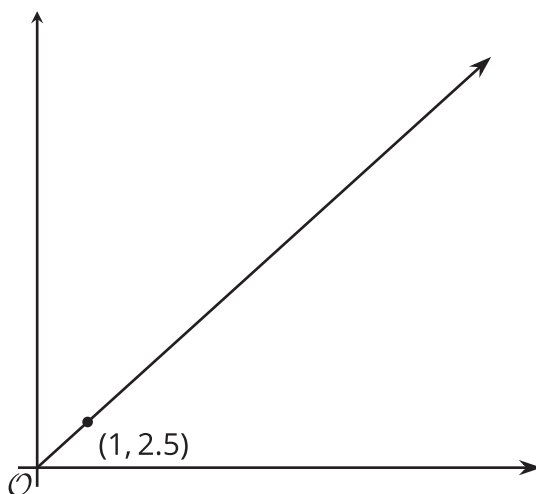
7.RP.A.2, 7.RP.A.2.a

Problem 7

This item is more challenging than earlier items. The axes are not labeled, and the coordinates of the point on the graph tell the amount of one quantity given one unit of the other quantity, rather than the values in the task statement. There are several ways students might reason about this problem, but however they do it, they need to explain their reasoning, requiring an understanding of the relationship between the constant of proportionality and the graph of the corresponding proportional relationship.

Statement

A recipe for a smoothie calls for 5 cups of strawberries for every 2 cups of bananas. The line represents the relationship between the amount of strawberries and the amount of bananas needed to make a smoothie according to this recipe. The point $(1, 2.5)$ is on the line.



1. Label the axes appropriately.
2. Write an equation that represents the proportional relationship between strawberries and bananas. Indicate the meaning of each variable.
3. Explain the meaning of the point $(1, 2.5)$ in terms of the situation.

Solution

1. The vertical axis should be labeled "strawberries (cups)," and the horizontal axis should be labeled "bananas (cups)."
2. Answers vary. Sample response: If y is the number of cups of strawberries, and x is the number of cups of bananas, then $y = 2.5x$.
3. The point $(1, 2.5)$ indicates that the recipe works with 1 cup of bananas and 2.5 cups of strawberries. This point gives a unit rate.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample:
- The graph is labeled “strawberries (cups)” on the vertical and bananas (cups)” on the horizontal.
- $s = 2.5b$ where s is cups of strawberries and b is cups of bananas.
- You could make the recipe with 1 cup of bananas and 2.5 of strawberries

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: lack of units in either graph or description; one reversal of the quantities in the three parts.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: 2 or 3 reversals of the quantities (in total for the three parts); one part omitted or containing a more significant error than quantity reversal; two error types from Tier 2 response.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: more than one part omitted or containing a more significant error than quantity reversal; statements suggesting a non-proportional relationship.

Aligned Standards

7.RP.A.2.c, 7.RP.A.2.d

Lesson Cool Downs

Lesson 1: One of These Things Is Not Like the Others

Cool Down: Orangey-Pineapple Juice

Here are three different recipes for Orangey-Pineapple Juice. Two of these mixtures taste the same and one tastes different.

- Recipe 1: Mix 4 cups of orange juice with 6 cups of pineapple juice.
- Recipe 2: Mix 6 cups of orange juice with 9 cups of pineapple juice.
- Recipe 3: Mix 9 cups of orange juice with 12 cups of pineapple juice.

Which two recipes will taste the same, and which one will taste different? Explain or show your reasoning.

Lesson 2: Introducing Proportional Relationships with Tables

Cool Down: Green Paint

When you mix two colors of paint in equivalent ratios, the resulting color is always the same. Complete the table as you answer the questions.

1. How many cups of yellow paint should you mix with 1 cup of blue paint to make the same shade of green? Explain or show your reasoning.

cups of blue paint	cups of yellow paint
2	10
1	

2. Make up a new pair of numbers that would make the same shade of green. Explain how you know they would make the same shade of green.

3. What is the proportional relationship represented by this table?

4. What is the *constant of proportionality*? What does it represent?

Lesson 3: More about Constant of Proportionality

Cool Down: Fish Tank

Mai is filling her fish tank. Water flows into the tank at a constant rate. Complete the table as you answer the questions.

1. How many gallons of water will be in the fish tank after 3 minutes? Explain your reasoning.

time (minutes)	water (gallons)
0.5	0.8
1	
3	
	40

2. How long will it take to fill the tank with 40 gallons of water? Explain your reasoning.

3. What is the constant of proportionality?

Lesson 4: Proportional Relationships and Equations

Cool Down: It's Snowing in Syracuse

Snow is falling steadily in Syracuse, New York. After 2 hours, 4 inches of snow has fallen.

1. If it continues to snow at the same rate, how many inches of snow would you expect after 6.5 hours? If you get stuck, you can use the table to help.

time (hours)	snow (inches)
	1
1	
2	4
6.5	
x	

2. Write an equation that gives the amount of snow that has fallen after x hours at this rate.

3. How many inches of snow will fall in 24 hours if it continues to snow at this rate?

Lesson 5: Two Equations for Each Relationship

Cool Down: Flight of the Albatross

An albatross is a large bird that can fly 400 kilometers in 8 hours at a constant speed. Using d for distance in kilometers and t for number of hours, an equation that represents this situation is $d = 50t$.

1. What are two constants of proportionality for the relationship between distance in kilometers and number of hours? What is the relationship between these two values?
2. Write another equation that relates d and t in this context.

Lesson 6: Using Equations to Solve Problems

Cool Down: Granola

Based on her recipe, Elena knows that 5 servings of granola have 1,750 calories.

1. If she eats 2 servings of granola, how many calories does she eat?
2. If she wants to eat 175 calories of granola, how many servings should she eat?
3. Write an equation to represent the relationship between the number of calories and the number of servings of granola.

Lesson 7: Comparing Relationships with Tables

Cool Down: Apples and Pizza

1. Based on the information in the table, is the cost of the apples proportional to the weight of apples?

pounds of apples	cost of apples
2	\$3.76
3	\$5.64
4	\$7.52
5	\$9.40

2. Based on the information in the table, is the cost of the pizza proportional to the number of toppings?

number of toppings	cost of pizza
2	\$11.99
3	\$13.49
4	\$14.99
5	\$16.49

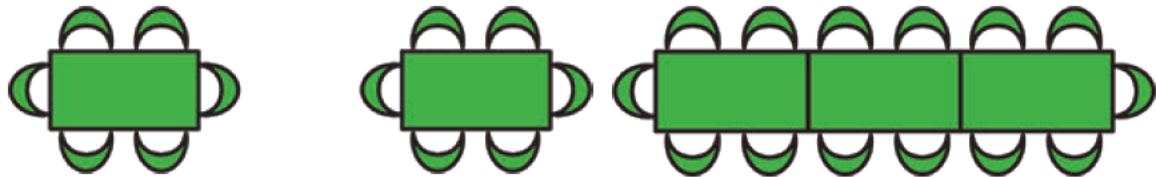
3. Write an equation for the proportional relationship.

Lesson 8: Comparing Relationships with Equations

Cool Down: Tables and Chairs

Andre is setting up rectangular tables for a party. He can fit 6 chairs around a single table. Andre lines up 10 tables end-to-end and tries to fit 60 chairs around them, but he is surprised when he cannot fit them all.

1. Write an equation for the relationship between the number of chairs c and the number of tables t when:
 - the tables are apart from each other:
 - the tables are placed end-to-end:



2. Is the first relationship proportional? Explain how you know.
3. Is the second relationship proportional? Explain how you know.

Lesson 9: Solving Problems about Proportional Relationships

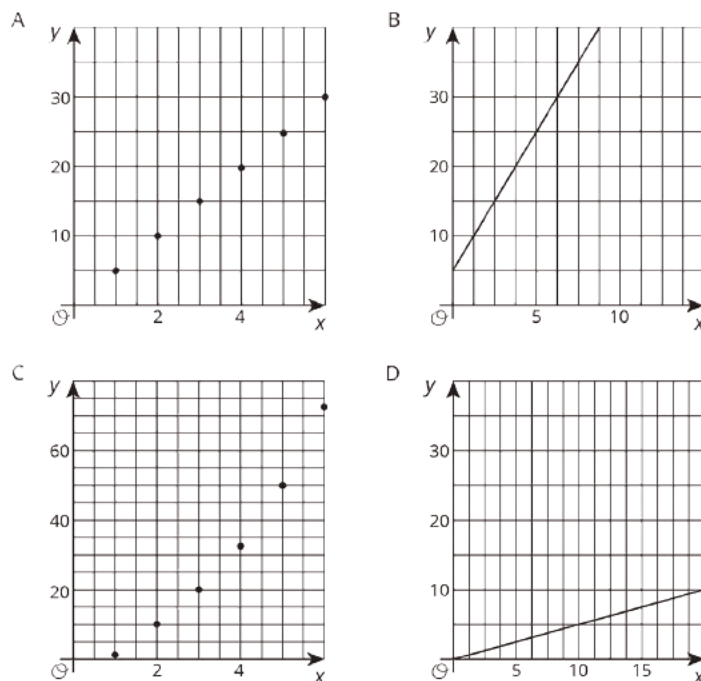
Cool Down: Steel Beams

A steel beam can be cut to different lengths for a project. Assuming the weight of a steel beam is proportional to its length, what information would you need to know to write an equation that represents this relationship?

Lesson 10: Introducing Graphs of Proportional Relationships

Cool Down: Which Are Not Proportional

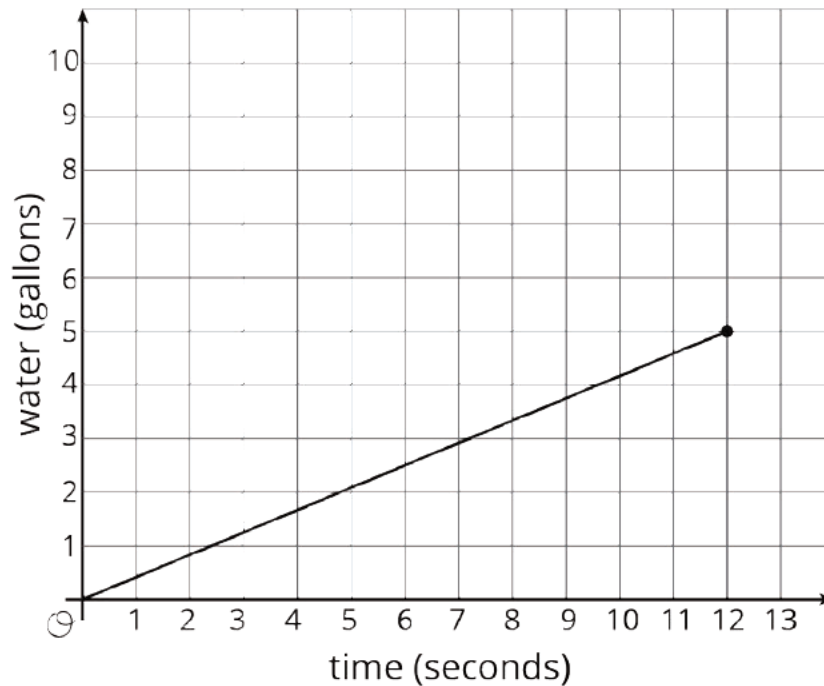
Which graphs cannot represent a proportional relationship? Select all that apply. Explain how you know.



Lesson 11: Interpreting Graphs of Proportional Relationships

Cool Down: Filling a Bucket

Water runs from a hose into a bucket at a steady rate. The amount of water in the bucket for the time it is being filled is shown in the graph.



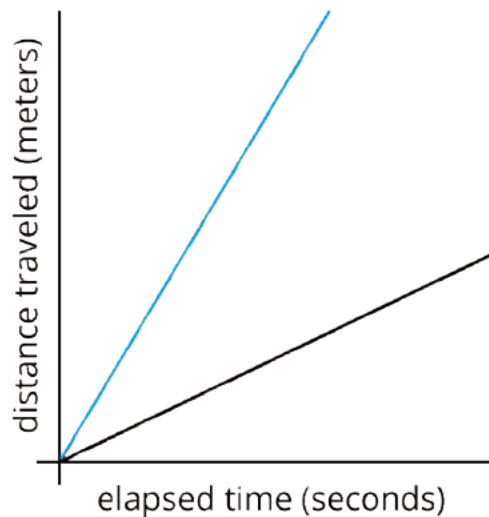
1. The point (12, 5) is on the graph. What do the coordinates tell you about the water in the bucket?
2. How many gallons of water were in the bucket after 1 second? Label the point on the graph that shows this information.

Lesson 12: Using Graphs to Compare Relationships

Cool Down: Revisiting the Amusement Park

Noah and Diego left the amusement park's ticket booth at the same time. Each moved at a constant speed toward his favorite ride. After 8 seconds, Noah was 17 meters from the ticket booth, and Diego was 43 meters away from the ticket booth.

1. Which line represents the distance traveled by Noah, and which line represents the distance traveled by Diego? Label each line with one name.



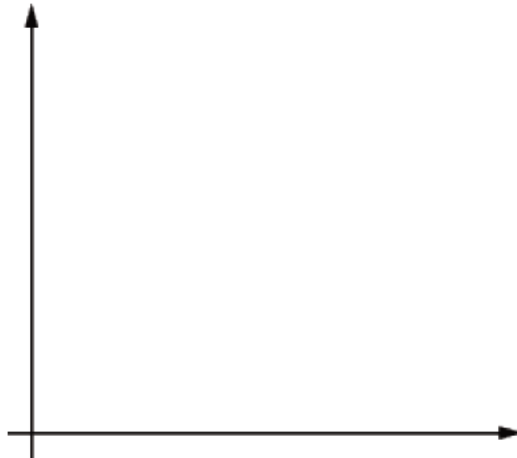
2. Explain how you decided which line represents which person's travel.

Lesson 13: Two Graphs for Each Relationship

Cool Down: Spicy Popcorn

Elena went to a store where you can scoop your own popcorn and buy as much as you want. She bought 10 ounces of spicy popcorn for \$2.50.

1. How much does popcorn cost per ounce?
2. How much popcorn can you buy per dollar?
3. Write two different equations that represent this situation. Use p for ounces of popcorn and c for cost in dollars.
4. Choose one of your equations, and sketch its graph. Be sure to label the axes.



Lesson 14: Four Representations

Cool Down: Explain Their Work

Choose a relationship that another group found and explain why it is a proportional relationship. Make sure to include the quantities they used and any important constants of proportionality.

Instructional Masters

Instructional Masters for Introducing Proportional Relationships

address	title	students per copy	written on?	requires cutting?	card stock recommended?	color paper recommended?
Activity Grade7.2.14.2	One Scenario, Four Representations	1	yes	no	no	no
Activity Grade7.2.9.2	Info Gap: Biking and Rain	4	no	yes	no	no
Activity Grade7.2.10.3	Matching Tables and Graphs	2	no	yes	no	yes

Info Gap: Biking and Rain
Problem Card 1

Mai and Noah each leave their houses at the same time and ride their bikes to the park.

- 1. For each person, write an equation that relates the distance they travel and the time.
- 2. Who will arrive at the park first?

Info Gap: Biking and Rain
Data Card 1

- Noah lives 1 kilometer farther away from the park than Mai does.
- Mai lives 8,000 meters from the park.
- Noah lives 9,000 meters from the park.
- Mai and Noah each bike at a constant speed.
- Mai bikes 250 meters per minute.
- Noah bikes 300 meters per minute.

Info Gap: Biking and Rain
Problem Card 2

A slow, steady rainstorm lasted all day. The rain was falling at a constant rate.

- 1. Write an equation that relates how much rain has fallen and how long it has been raining.
- 2. How long will it take for 5 cm of rain to fall?

Info Gap: Biking and Rain
Data Card 2

- The rain storm lasted for 24 hours.
- 9.6 centimeters of rain fell during the storm.
- The rate of the rainfall was 2 millimeters of rain every 30 minutes.
- There are 10 millimeters in 1 centimeter.
- There are 60 minutes in 1 hour.

Info Gap: Biking and Rain
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Matching Tables and Graphs

1. When you buy two shirts, you get the second one at half-price.

x	y
1	10
2	15
3	25
4	30
5	40
6	45

Matching Tables and Graphs

2. These t-shirts cost \$8 each.

x	y
1	8
2	16
4	32
5	40
7	56
8	64

Matching Tables and Graphs

3. In the science lab there is a chart to help students convert temperatures from Celsius to Fahrenheit.

x	y
0	32
10	50
20	68
30	86
40	104
50	122

Matching Tables and Graphs

4. She is planning on serving $\frac{1}{3}$ cup of rice per person.

x	y
1	$\frac{1}{3}$
2	$\frac{2}{3}$
3	1
4	$1\frac{1}{3}$
5	$1\frac{2}{3}$
6	2

Matching Tables and Graphs

5. Entrance to a state park costs \$6.00 per vehicle, plus \$2.00 per person in the vehicle. One vehicle can seat 6 people.

x	y
1	8
2	10
3	12
4	14
5	16
6	18

Matching Tables and Graphs

6. He measures the time that has elapsed after each lap he runs.

x	y
1	2
2	4
3	7
4	9
5	11
6	15

7. A recipe uses 2 tablespoons of honey for every 8 cups of flour.

x	y
2	8
3	12
6	24
7	28
10	40
12	48

8. She is filling her fish tank with water. The chart shows the gallons of water after so many minutes.

x	y
1	1.6
2	3.2
3	4.8
4	6.4
5	8.0
6	9.6

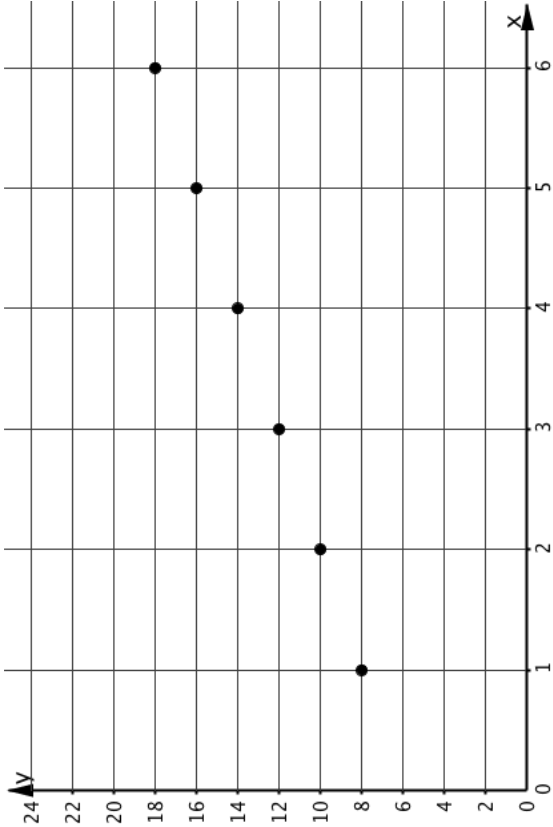
9. Ten empty aluminum cans weigh 0.15 kg.

x	y
10	0.15
20	0.30
25	0.375
40	0.60
50	0.75
60	0.90

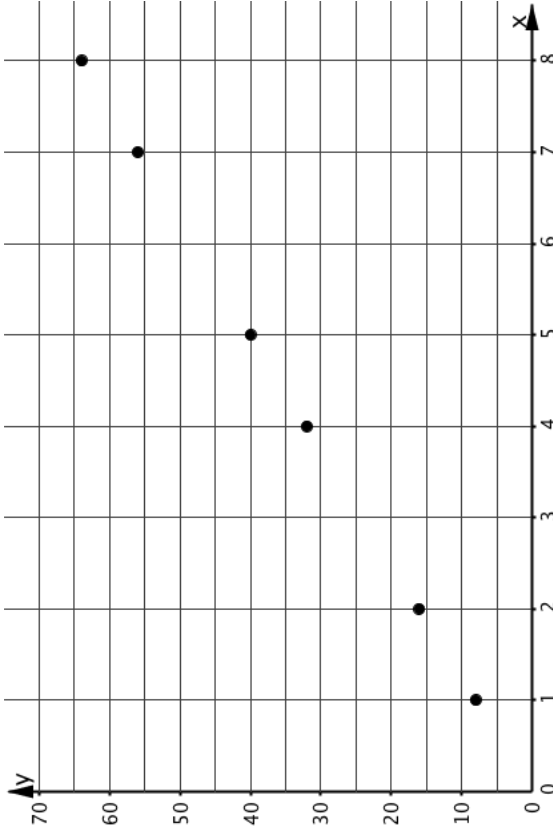
10. The area of a square is the square of the side length.

x	y
1	1
2	4
4	16
5	25
7	49
10	100

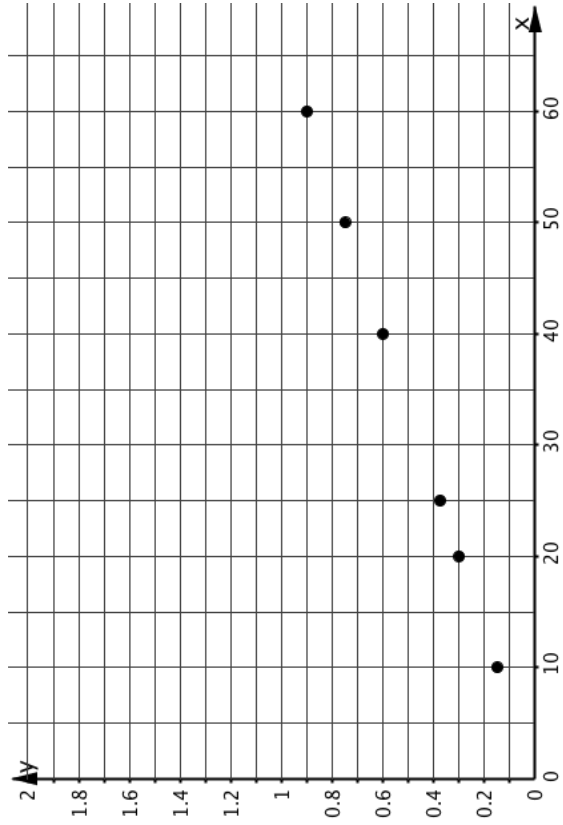
Matching Tables and Graphs
A.



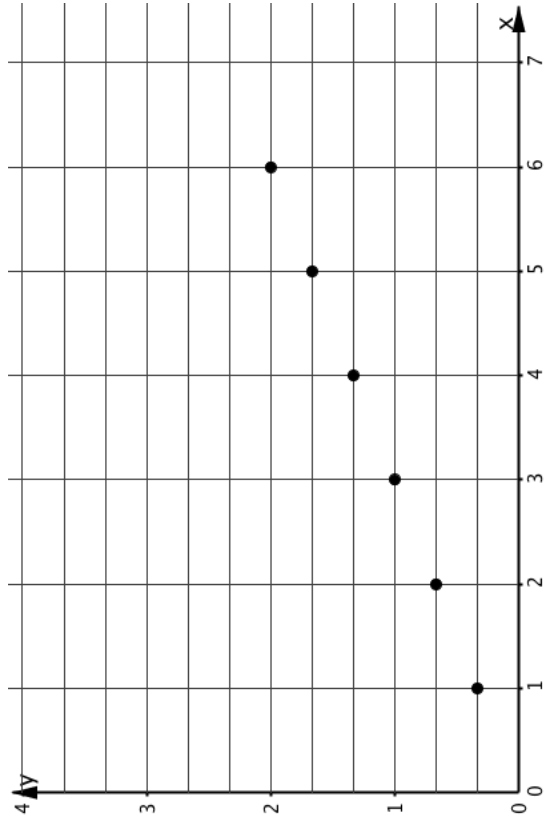
Matching Tables and Graphs
B.



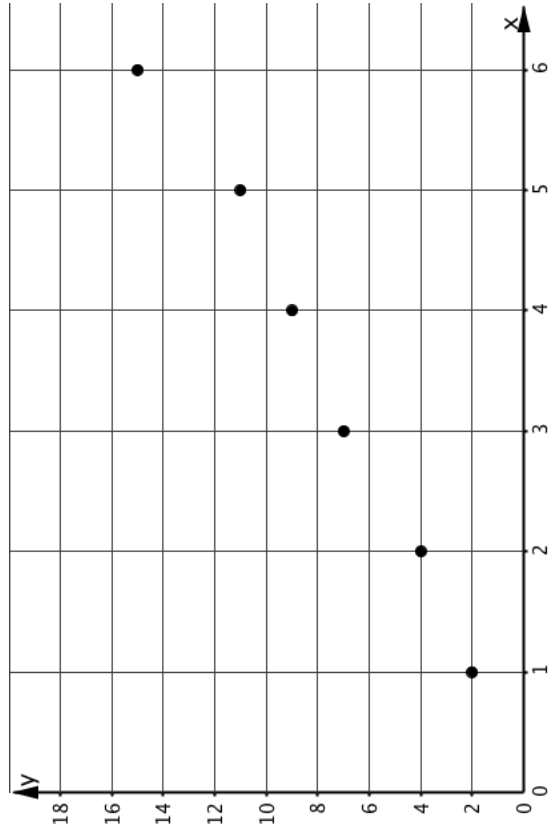
C.



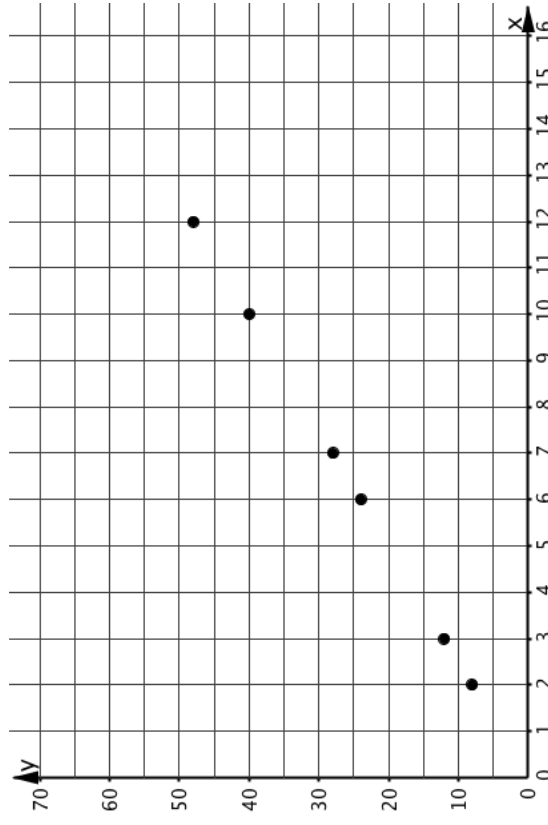
D.



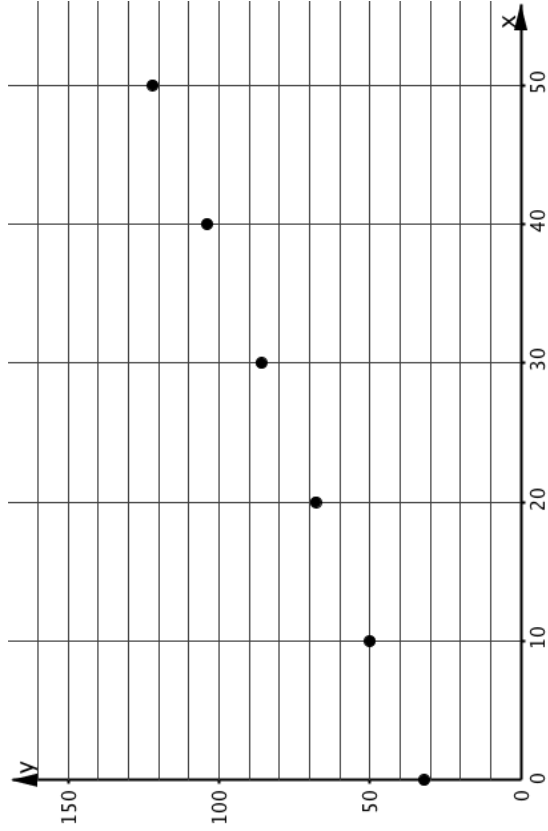
E.



F.

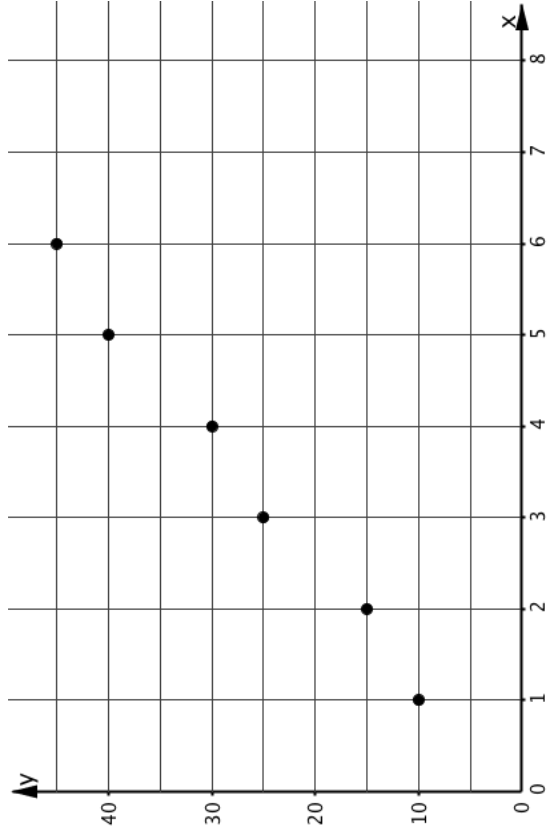


G.



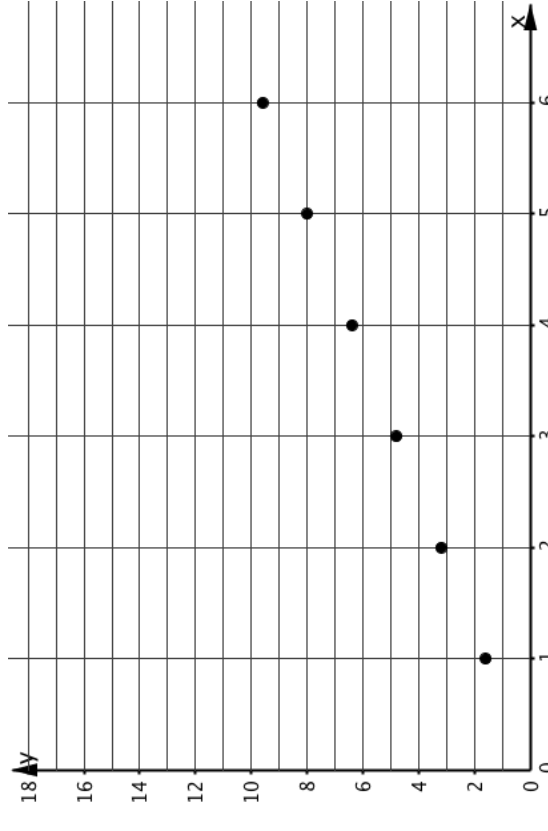
Matching Tables and Graphs

H.



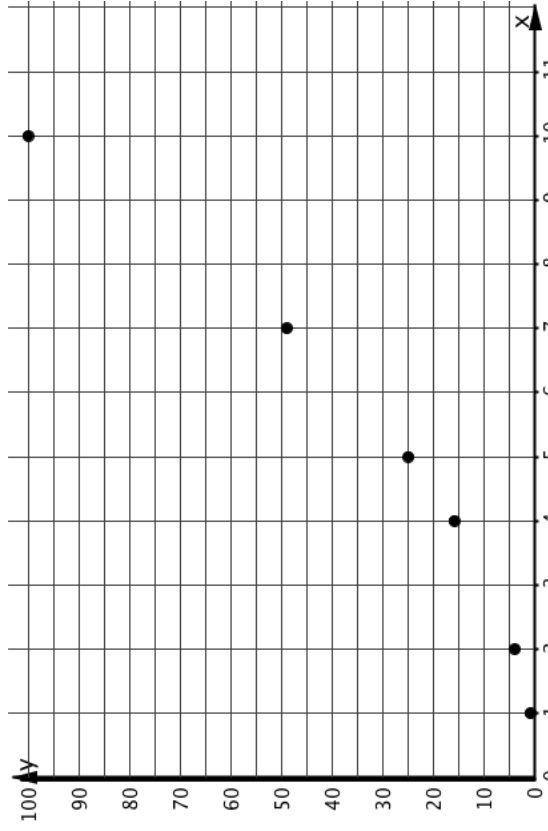
Matching Tables and Graphs

I.



Matching Tables and Graphs

J.

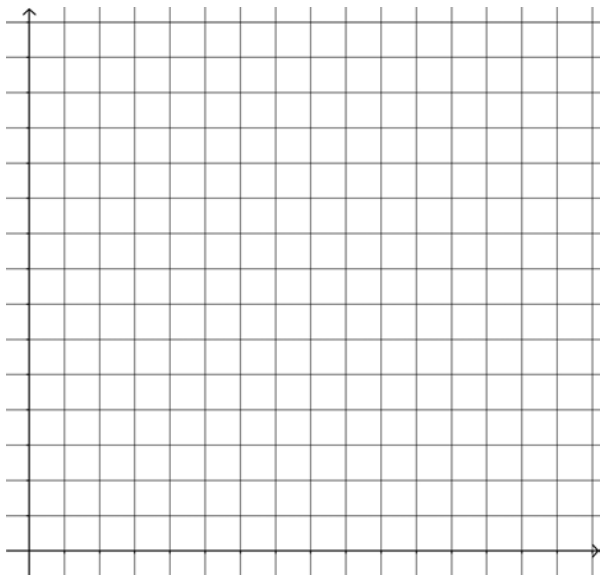


The two quantities are: _____ and _____.

Verbal Description: One or more complete sentences describing the relationship

Table of Values:

Graph:
Label each axis!



Equation:

Explain in words what each letter and number in your equation means:

Explain how you know the relationship is or is not proportional. Give as many reasons as you can:

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Editorial Director
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