



Core Knowledge[®] MATHEMATICS

Introducing Proportional Relationships

Student Workbook

Real World Proportions

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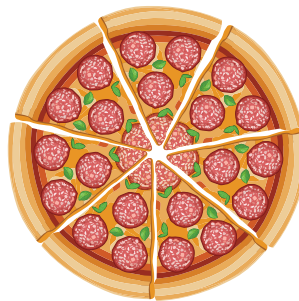
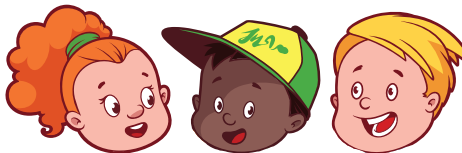
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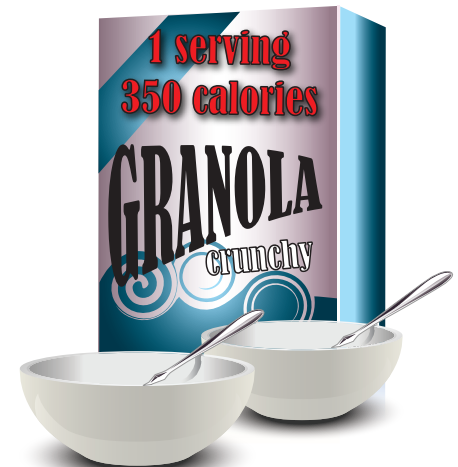
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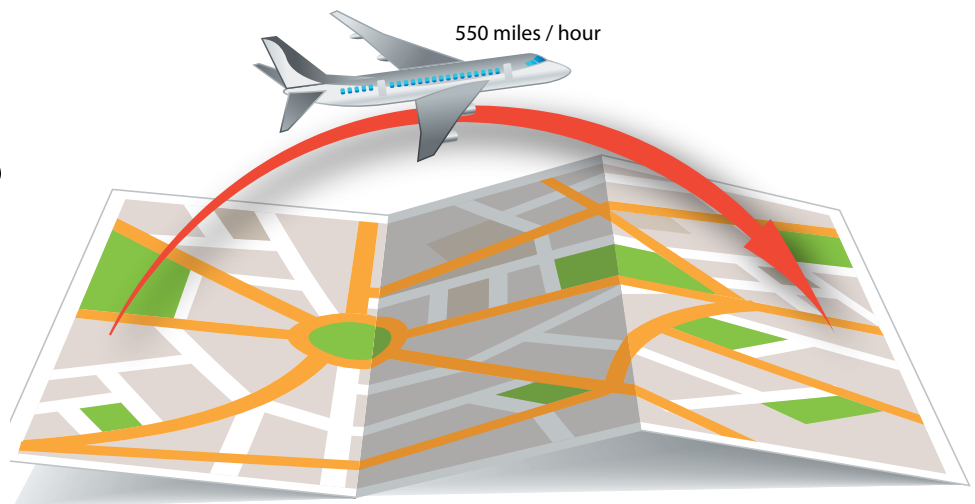
Saving Water:
A Bath or Shower?



Planning to Feed a Crowd



Calculating
Servings and Calories



Calculating Time and Distance Using Constant Speed

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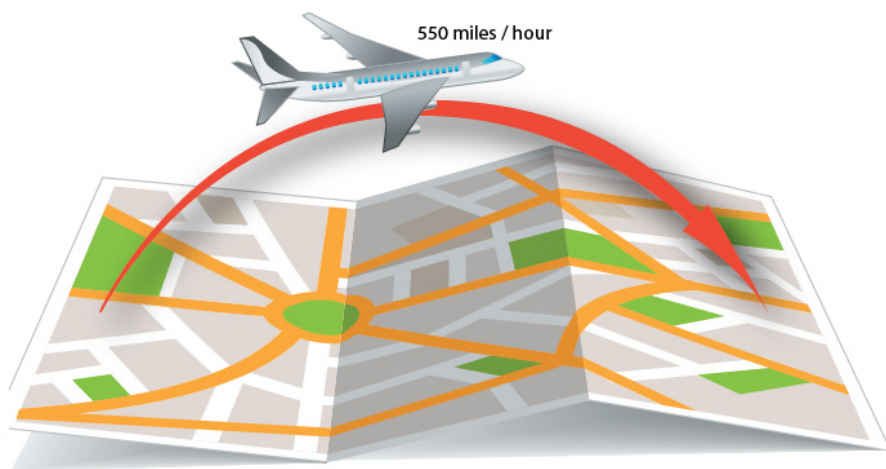
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Introducing Proportional Relationships

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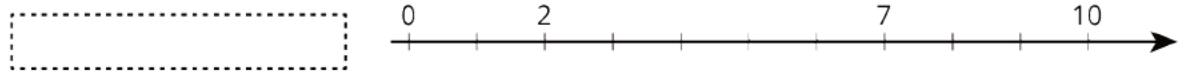
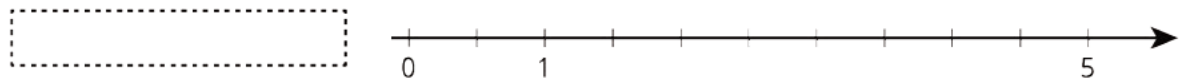


**Introducing Proportional
Relationships**
Student Workbook
Core Knowledge Mathematics™

Lesson 1: One of These Things Is Not Like the Others

1.1: Remembering Double Number Lines

1. Complete the double number line diagram with the missing numbers.



2. What could each of the number lines represent? Invent a situation and label the diagram.

3. Make sure your labels include appropriate units of measure.

1.2: Mystery Mixtures

Your teacher will show you three mixtures. Two taste the same, and one is different.

1. Which mixture tastes different? Describe how it is different.

2. Here are the recipes that were used to make the three mixtures:
 - 1 cup of water with $1\frac{1}{2}$ teaspoons of powdered drink mix
 - 2 cups of water with $\frac{1}{2}$ teaspoon of powdered drink mix
 - 1 cup of water with $\frac{1}{4}$ teaspoon of powdered drink mix

Which of these recipes is for the stronger tasting mixture? Explain how you know.

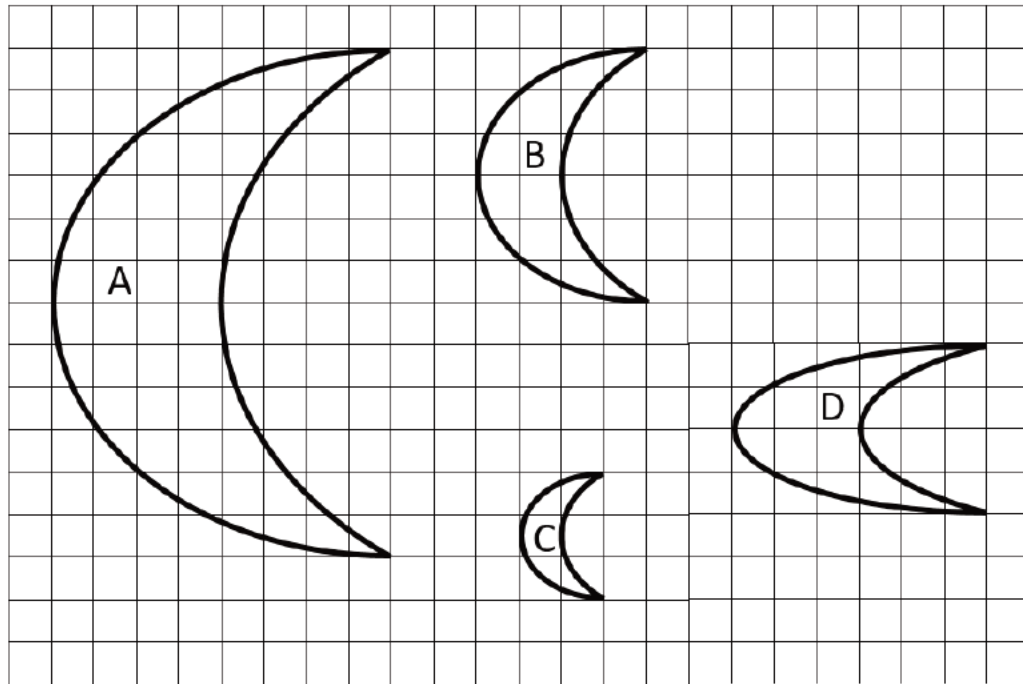
Are you ready for more?

Salt and sugar give two distinctly different tastes, one salty and the other sweet. In a mixture of salt and sugar, it is possible for the mixture to be salty, sweet or both. Will any of these mixtures taste exactly the same?

- Mixture A: 2 cups water, 4 teaspoons salt, 0.25 cup sugar
- Mixture B: 1.5 cups water, 3 teaspoons salt, 0.2 cup sugar
- Mixture C: 1 cup water, 2 teaspoons salt, 0.125 cup sugar

1.3: Crescent Moons

Here are four different crescent moon shapes.



1. What do Moons A, B, and C all have in common that Moon D doesn't?
2. Use numbers to describe how Moons A, B, and C are different from Moon D.
3. Use a table or a double number line to show how Moons A, B, and C are different from Moon D.

Lesson 1 Summary

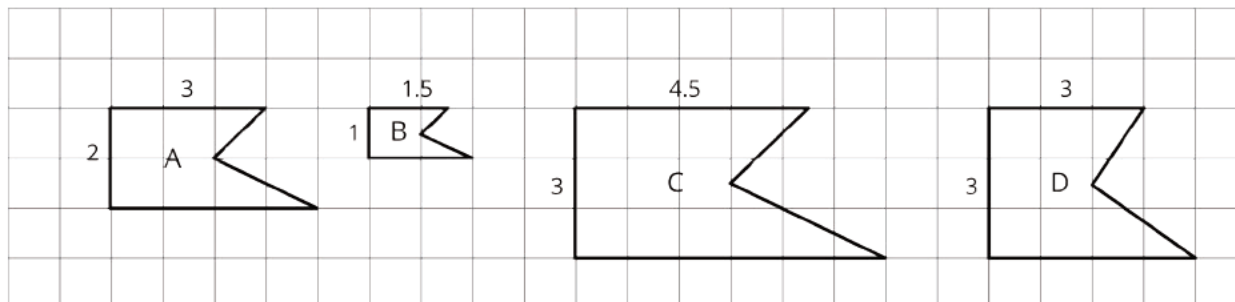
When two different situations can be described by **equivalent ratios**, that means they are alike in some important way.

An example is a recipe. If two people make something to eat or drink, the taste will only be the same as long as the ratios of the ingredients are equivalent. For example, all of the mixtures of water and drink mix in this table taste the same, because the ratios of cups of water to scoops of drink mix are all equivalent ratios.

water (cups)	drink mix (scoops)
3	1
12	4
1.5	0.5

If a mixture were not equivalent to these, for example, if the ratio of cups of water to scoops of drink mix were 6 : 4, then the mixture would taste different.

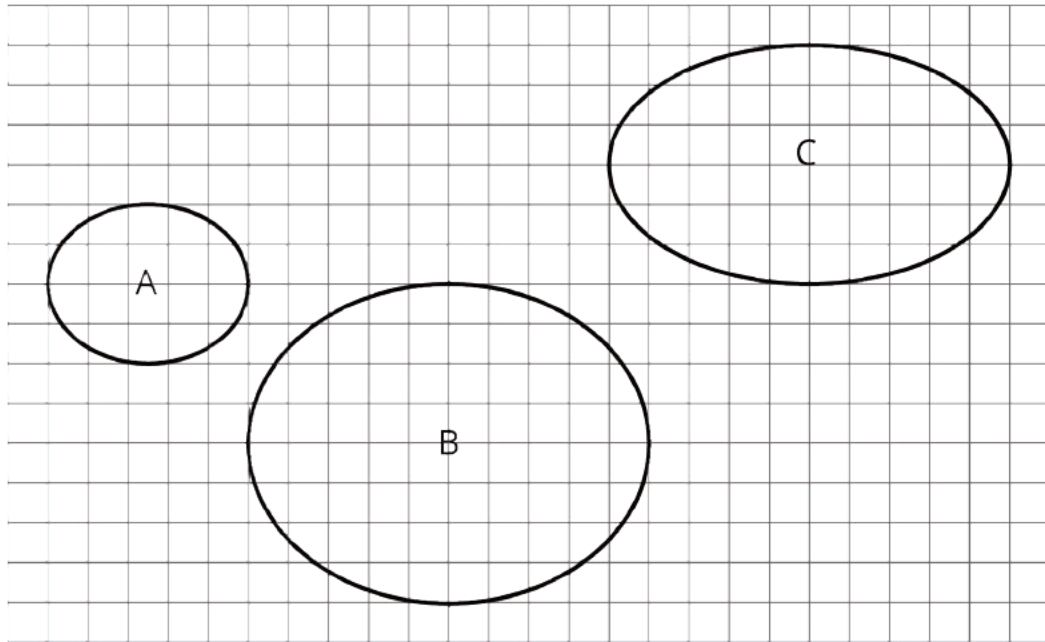
Notice that the ratios of pairs of corresponding side lengths are equivalent in figures A, B, and C. For example, the ratios of the length of the top side to the length of the left side for figures A, B, and C are equivalent ratios. Figures A, B, and C are *scaled copies* of each other; this is the important way in which they are alike.



If a figure has corresponding sides that are not in a ratio equivalent to these, like figure D, then it's not a scaled copy. In this unit, you will study relationships like these that can be described by a set of equivalent ratios.

Unit 2 Lesson 1 Cumulative Practice Problems

1. Which one of these shapes is not like the others? Explain what makes it different by representing each width and height pair with a ratio.



2. In one version of a trail mix, there are 3 cups of peanuts mixed with 2 cups of raisins. In another version of trail mix, there are 4.5 cups of peanuts mixed with 3 cups of raisins. Are the ratios equivalent for the two mixes? Explain your reasoning.

3. For each object, choose an appropriate scale for a drawing that fits on a regular sheet of paper. Not all of the scales on the list will be used.

Objects	Scales
a. A person	☐ 1 in : 1 ft
b. A football field (120 yards by $53\frac{1}{3}$ yards)	☐ 1 cm : 1 m
c. The state of Washington (about 240 miles by 360 miles)	☐ 1: 1000
	☐ 1 ft: 1 mile
d. The floor plan of a house	☐ 1: 100,000
e. A rectangular farm (6 miles by 2 mile)	☐ 1 mm: 1 km
	☐ 1: 10,000,000

(From Unit 1, Lesson 12.)

4. Which scale is equivalent to 1 cm to 1 km?

- A. 1 to 1000
- B. 10,000 to 1
- C. 1 to 100,000
- D. 100,000 to 1
- E. 1 to 1,000,000

(From Unit 1, Lesson 11.)

5. a. Find 3 different ratios that are equivalent to 7 : 3.
- b. Explain why these ratios are equivalent.

Lesson 2: Introducing Proportional Relationships with Tables

2.1: Notice and Wonder: Paper Towels by the Case

Here is a table that shows how many rolls of paper towels a store receives when they order different numbers of cases.

number of cases they order	number of rolls of paper towels
1	12
3	36
5	60
10	120

$\cdot 2$ $\cdot 2$

What do you notice about the table? What do you wonder?

2.2: Feeding a Crowd

1. A recipe says that 2 cups of dry rice will serve 6 people. Complete the table as you answer the questions. Be prepared to explain your reasoning.

a. How many people will 10 cups of rice serve?

b. How many cups of rice are needed to serve 45 people?

cups of rice	number of people
2	6
3	9
10	
	45

2. A recipe says that 6 spring rolls will serve 3 people. Complete the table.

number of spring rolls	number of people
6	3
30	
40	
	28

2.3: Making Bread Dough

A bakery uses 8 tablespoons of honey for every 10 cups of flour to make bread dough. Some days they bake bigger batches and some days they bake smaller batches, but they always use the same ratio of honey to flour. Complete the table as you answer the questions. Be prepared to explain your reasoning.

1. How many cups of flour do they use with 20 tablespoons of honey?

2. How many cups of flour do they use with 13 tablespoons of honey?

3. How many tablespoons of honey do they use with 20 cups of flour?

4. What is the **proportional relationship** represented by this table?

honey (tbsp)	flour (c)
8	10
20	
13	
	20

2.4: Quarters and Dimes

4 quarters are equal in value to 10 dimes.

1. How many dimes equal the value of 6 quarters?
2. How many dimes equal the value of 14 quarters?
3. What value belongs next to the 1 in the table? What does it mean in this context?

number of quarters	number of dimes
1	
4	10
6	
14	

Are you ready for more?

Pennies made before 1982 are 95% copper and weigh about 3.11 grams each. (Pennies made after that date are primarily made of zinc). Some people claim that the value of the copper in one of these pennies is greater than the face value of the penny. Find out how much copper is worth right now, and decide if this claim is true.

Lesson 2 Summary

If the ratios between two corresponding quantities are always equivalent, the relationship between the quantities is called a **proportional relationship**.

This table shows different amounts of milk and chocolate syrup. The ingredients in each row, when mixed together, would make a different total amount of chocolate milk, but these mixtures would all taste the same.

Notice that each row in the table shows a ratio of tablespoons of chocolate syrup to cups of milk that is equivalent to 4 : 1.

About the relationship between these quantities, we could say:

tablespoons of chocolate syrup	cups of milk
4	1
6	$1\frac{1}{2}$
8	2
$\frac{1}{2}$	$\frac{1}{8}$
12	3
1	$\frac{1}{4}$

- The relationship between amount of chocolate syrup and amount of milk is proportional.
- The relationship between the amount of chocolate syrup and the amount of milk is a proportional relationship.
- The table represents a proportional relationship between the amount of chocolate syrup and amount of milk.
- The amount of milk is proportional to the amount of chocolate syrup.

We could multiply any value in the chocolate syrup column by $\frac{1}{4}$ to get the value in the milk column. We might call $\frac{1}{4}$ a *unit rate*, because $\frac{1}{4}$ cups of milk are needed for 1 tablespoon of chocolate syrup. We also say that $\frac{1}{4}$ is the **constant of proportionality** for this relationship. It tells us how many cups of milk we would need to mix with 1 tablespoon of chocolate syrup.

Unit 2 Lesson 2 Cumulative Practice Problems

1. When Han makes chocolate milk, he mixes 2 cups of milk with 3 tablespoons of chocolate syrup. Here is a table that shows how to make batches of different sizes. Use the information in the table to complete the statements. Some terms are used more than once.

cups of milk	tablespoons of chocolate syrup
2	3
8	12
1	$\frac{3}{2}$
10	15

•4

•4

- The table shows a proportional relationship between _____ and _____.
- The scale factor shown is _____.
- The constant of proportionality for this relationship is _____.
- The units for the constant of proportionality are _____ per _____.

Bank of Terms: tablespoons of chocolate syrup, 4, cups of milk, cup of milk, $\frac{3}{2}$

2. A certain shade of pink is created by adding 3 cups of red paint to 7 cups of white paint.

- How many cups of red paint should be added to 1 cup of white paint?

cups of white paint	cups of red paint
1	
7	3

- What is the constant of proportionality?

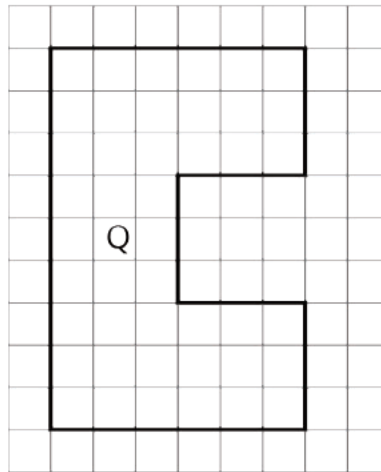
3. A map of a rectangular park has a length of 4 inches and a width of 6 inches. It uses a scale of 1 inch for every 30 miles.

a. What is the actual area of the park? Show how you know.

b. The map needs to be reproduced at a different scale so that it has an area of 6 square inches and can fit in a brochure. At what scale should the map be reproduced so that it fits on the brochure? Show your reasoning.

(From Unit 1, Lesson 12.)

4. Noah drew a scaled copy of Polygon P and labeled it Polygon Q.



If the area of Polygon P is 5 square units, what scale factor did Noah apply to Polygon P to create Polygon Q? Explain or show how you know.

(From Unit 1, Lesson 6.)

5. Select **all** the ratios that are equivalent to each other.

- A. 4 : 7
- B. 8 : 15
- C. 16 : 28
- D. 2 : 3
- E. 20 : 35

Lesson 3: More about Constant of Proportionality

3.1: Equal Measures

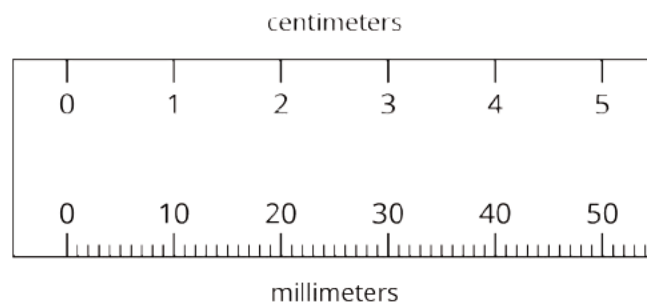
Use the numbers and units from the list to find as many equivalent measurements as you can. For example, you might write “30 minutes is $\frac{1}{2}$ hour.”

You can use the numbers and units more than once.

1	$\frac{1}{2}$	0.3	centimeter
12	40	24	meter
0.4	0.01	$\frac{1}{10}$	hour
60	$3\frac{1}{3}$	6	feet
50	30	2	minute
		$\frac{2}{5}$	inch

3.2: Centimeters and Millimeters

There is a proportional relationship between any length measured in centimeters and the same length measured in millimeters.



There are two ways of thinking about this proportional relationship.

1. If you know the length of something in centimeters, you can calculate its length in millimeters.
 - a. Complete the table.
 - b. What is the constant of proportionality?

length (cm)	length (mm)
9	
12.5	
50	
88.49	

2. If you know the length of something in millimeters, you can calculate its length in centimeters.

a. Complete the table.

b. What is the constant of proportionality?

length (mm)	length (cm)
70	
245	
4	
699.1	

3. How are these two constants of proportionality related to each other?

4. Complete each sentence:

a. To convert from centimeters to millimeters, you can multiply by _____.

b. To convert from millimeters to centimeters, you can divide by _____ *or* multiply by _____.

Are you ready for more?

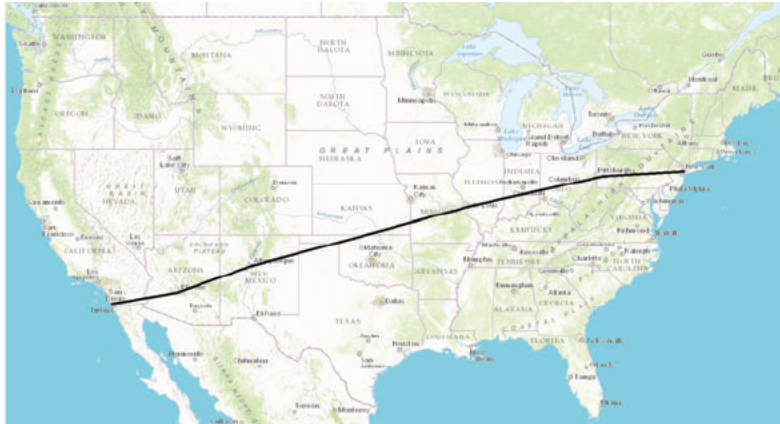
1. How many square millimeters are there in a square centimeter?

2. How do you convert square centimeters to square millimeters? How do you convert the other way?

3.3: Pittsburgh to Phoenix

On its way from New York to San Diego, a plane flew over Pittsburgh, Saint Louis, Albuquerque, and Phoenix traveling at a constant speed.

Complete the table as you answer the questions. Be prepared to explain your reasoning.



segment	time	distance	speed
Pittsburgh to Saint Louis	1 hour	550 miles	
Saint Louis to Albuquerque	1 hour 42 minutes		
Albuquerque to Phoenix		330 miles	

1. What is the distance between Saint Louis and Albuquerque?
2. How many minutes did it take to fly between Albuquerque and Phoenix?
3. What is the proportional relationship represented by this table?
4. Diego says the constant of proportionality is 550. Andre says the constant of proportionality is $9\frac{1}{6}$. Do you agree with either of them? Explain your reasoning.

Lesson 3 Summary

When something is traveling at a constant speed, there is a proportional relationship between the time it takes and the distance traveled. The table shows the distance traveled and elapsed time for a bug crawling on a sidewalk.

distance traveled (cm)	elapsed time (sec)
$\frac{3}{2}$	1
1	$\frac{2}{3}$
3	2
10	$\frac{20}{3}$

$$\cdot \frac{2}{3}$$

We can multiply any number in the first column by $\frac{2}{3}$ to get the corresponding number in the second column. We can say that the elapsed time is proportional to the distance traveled, and the constant of proportionality is $\frac{2}{3}$. This means that the bug's *pace* is $\frac{2}{3}$ seconds per centimeter.

This table represents the same situation, except the columns are switched.

elapsed time (sec)	distance traveled (cm)
1	$\frac{3}{2}$
$\frac{2}{3}$	1
2	3
$\frac{20}{3}$	10

$$\cdot \frac{3}{2}$$

We can multiply any number in the first column by $\frac{3}{2}$ to get the corresponding number in the second column. We can say that the distance traveled is proportional to the elapsed time, and the constant of proportionality is $\frac{3}{2}$. This means that the bug's *speed* is $\frac{3}{2}$ centimeters per second.

Notice that $\frac{3}{2}$ is the reciprocal of $\frac{2}{3}$. When two quantities are in a proportional relationship, there are two constants of proportionality, and they are always reciprocals of each other. When we represent a proportional relationship with a table, we say the quantity in the second column is proportional to the quantity in the first column, and the corresponding constant of proportionality is the number we multiply values in the first column to get the values in the second.

Unit 2 Lesson 3 Cumulative Practice Problems

1. Noah is running a portion of a marathon at a constant speed of 6 miles per hour.

Complete the table to predict how long it would take him to run different distances at that speed, and how far he would run in different time intervals.

time in hours	miles traveled at 6 miles per hour
1	
$\frac{1}{2}$	
$1\frac{1}{3}$	
	$1\frac{1}{2}$
	9
	$4\frac{1}{2}$

2. One kilometer is 1000 meters.

- a. Complete the tables. What is the interpretation of the constant of proportionality in each case?

meters	kilometers
1,000	1
250	
12	
1	

kilometers	meters
1	1,000
5	
20	
0.3	

The constant of proportionality tells us that:

The constant of proportionality tells us that:

- b. What is the relationship between the two constants of proportionality?

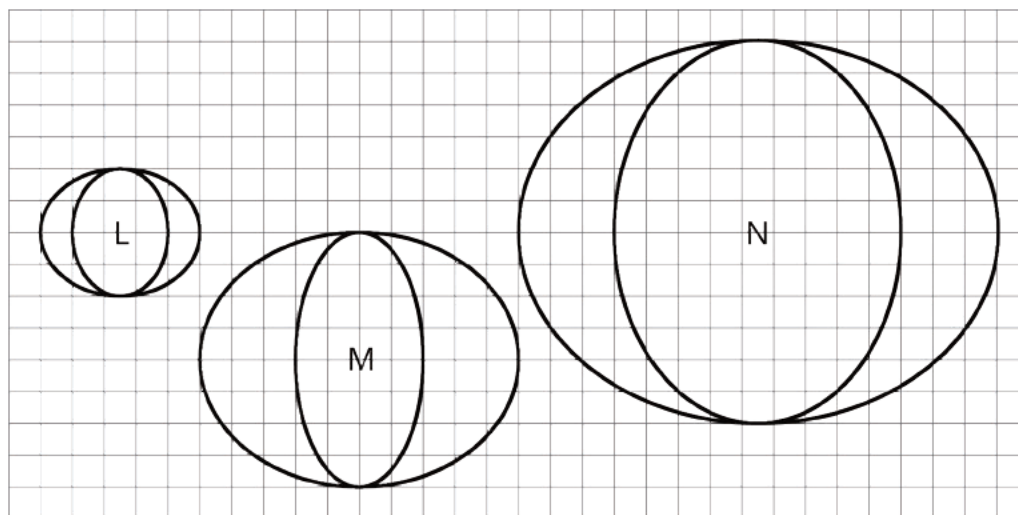
3. Jada and Lin are comparing inches and feet. Jada says that the constant of proportionality is 12. Lin says it is $\frac{1}{12}$. Do you agree with either of them? Explain your reasoning.
4. The area of the Mojave desert is 25,000 square miles. A scale drawing of the Mojave desert has an area of 10 square inches. What is the scale of the map?

(From Unit 1, Lesson 12.)

5. Which of these scales is equivalent to the scale 1 cm to 5 km? Select all that apply.
- A. 3 cm to 15 km
 - B. 1 mm to 150 km
 - C. 5 cm to 1 km
 - D. 5 mm to 2.5 km
 - E. 1 mm to 500 m

(From Unit 1, Lesson 11.)

6. Which one of these pictures is not like the others? Explain what makes it different using ratios.



(From Unit 2, Lesson 1.)

Lesson 4: Proportional Relationships and Equations

4.1: Number Talk: Division

Find each quotient mentally.

$$645 \div 100$$

$$645 \div 50$$

$$48.6 \div 30$$

$$48.6 \div x$$

4.2: Feeding a Crowd, Revisited

1. A recipe says that 2 cups of dry rice will serve 6 people. Complete the table as you answer the questions. Be prepared to explain your reasoning.

a. How many people will
1 cup of rice serve?

b. How many people will
3 cups of rice serve?
12 cups? 43 cups?

c. How many people will
 x cups of rice serve?

cups of dry rice	number of people
1	
2	6
3	
12	
43	
x	

2. A recipe says that 6 spring rolls will serve 3 people. Complete the table as you answer the questions. Be prepared to explain your reasoning.

a. How many people will 1 spring roll serve?

b. How many people will 10 spring rolls serve? 16 spring rolls? 25 spring rolls?

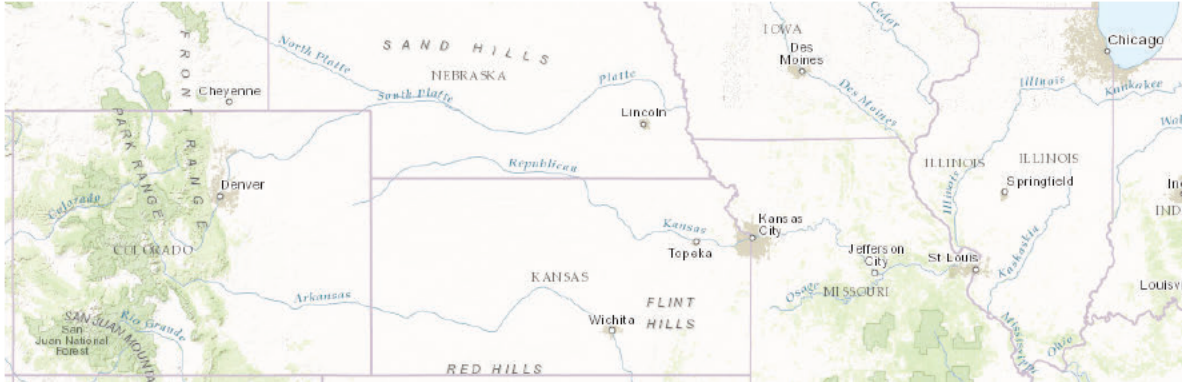
c. How many people will n spring rolls serve?

number of spring rolls	number of people
1	
6	3
10	
16	
25	
n	

3. How was completing this table different from the previous table? How was it the same?

4.3: Denver to Chicago

A plane flew at a constant speed between Denver and Chicago. It took the plane 1.5 hours to fly 915 miles.



1. Complete the table.

time (hours)	distance (miles)	speed (miles per hour)
1		
1.5	915	
2		
2.5		
t		

- How far does the plane fly in one hour?
- How far would the plane fly in t hours at this speed?
- If d represents the distance that the plane flies at this speed for t hours, write an equation that relates t and d .
- How far would the plane fly in 3 hours at this speed? in 3.5 hours? Explain or show your reasoning.

Are you ready for more?

A rocky planet orbits Proxima Centauri, a star that is about 1.3 parsecs from Earth. This planet is the closest planet outside of our solar system.

1. How long does it take light from Proxima Centauri to reach Earth? (A parsec is about 3.26 light years. A light year is the distance light travels in one year.)
2. There are two twins. One twin leaves on a spaceship to explore the planet near Proxima Centauri traveling at 90% of the speed of light, while the other twin stays home on Earth. How much does the twin on Earth age while the other twin travels to Proxima Centauri? (Do you think the answer would be the same for the other twin? Consider researching “The Twin Paradox” to learn more.)

4.4: Revisiting Bread Dough

A bakery uses 8 tablespoons of honey for every 10 cups of flour to make bread dough. Some days they bake bigger batches and some days they bake smaller batches, but they always use the same ratio of honey to flour.

1. Complete the table.
2. If f is the cups of flour needed for h tablespoons of honey, write an equation that relates f and h .
3. How much flour is needed for 15 tablespoons of honey? 17 tablespoons? Explain or show your reasoning.

honey (tbsp)	flour (c)
1	
8	10
16	
20	
h	

Lesson 4 Summary

The table shows the amount of red paint and blue paint needed to make a certain shade of purple paint, called Venusian Sunset.

Note that “parts” can be *any* unit for volume. If we mix 3 cups of red with 12 cups of blue, you will get the same shade as if we mix 3 teaspoons of red with 12 teaspoons of blue.

red paint (parts)	blue paint (parts)
3	12
1	4
7	28
$\frac{1}{4}$	1
r	$4r$

The last row in the table says that if we know the amount of red paint needed, r , we can always multiply it by 4 to find the amount of blue paint needed, b , to mix with it to make Venusian Sunset. We can say this more succinctly with the equation $b = 4r$. So the amount of blue paint is proportional to the amount of red paint and the constant of proportionality is 4.

We can also look at this relationship the other way around.

If we know the amount of blue paint needed, b , we can always multiply it by $\frac{1}{4}$ to find the amount of red paint needed, r , to mix with it to make Venusian Sunset. So $r = \frac{1}{4}b$. The amount of blue paint is proportional to the amount of red paint and the constant of proportionality $\frac{1}{4}$.

blue paint (parts)	red paint (parts)
12	3
4	1
28	7
1	$\frac{1}{4}$
b	$\frac{1}{4}b$

In general, when y is proportional to x , we can always multiply x by the same number k —the constant of proportionality—to get y . We can write this much more succinctly with the equation $y = kx$.

Unit 2 Lesson 4 Cumulative Practice Problems

1. A certain ceiling is made up of tiles. Every square meter of ceiling requires 10.75 tiles. Fill in the table with the missing values.

square meters of ceiling	number of tiles
1	
10	
	100
a	

2. On a flight from New York to London, an airplane travels at a constant speed. An equation relating the distance traveled in miles, d , to the number of hours flying, t , is $t = \frac{1}{500}d$. How long will it take the airplane to travel 800 miles?

3. Each table represents a proportional relationship. For each, find the constant of proportionality, and write an equation that represents the relationship.

s	P
2	8
3	12
5	20
10	40

Constant of proportionality:

Equation: $P =$

d	C
2	6.28
3	9.42
5	15.7
10	31.4

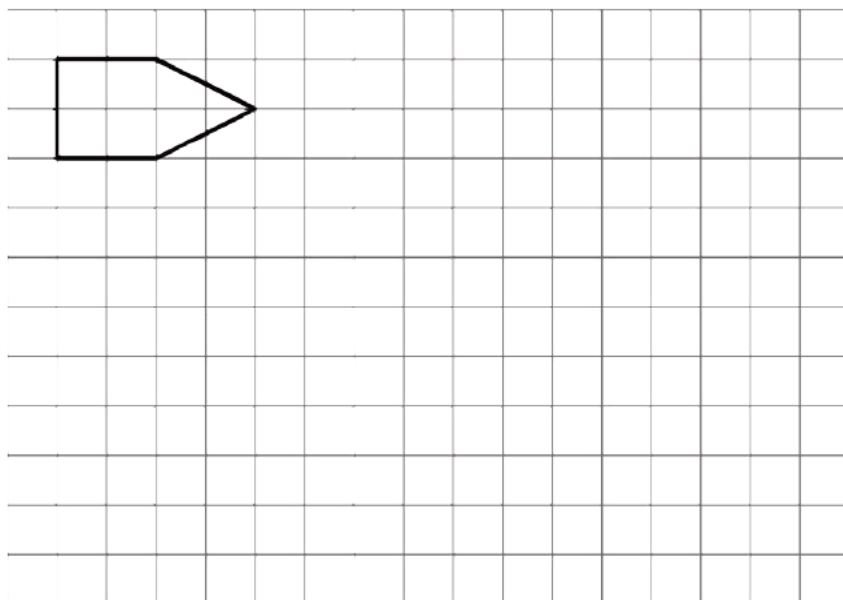
Constant of proportionality:

Equation: $C =$

4. A map of Colorado says that the scale is 1 inch to 20 miles or 1 to 1,267,200. Are these two ways of reporting the scale the same? Explain your reasoning.

(From Unit 1, Lesson 11.)

5. Here is a polygon on a grid.



- Draw a scaled copy of the polygon using a scale factor 3. Label the copy A.
- Draw a scaled copy of the polygon with a scale factor $\frac{1}{2}$. Label it B.
- Is Polygon A a scaled copy of Polygon B? If so, what is the scale factor that takes B to A?

(From Unit 1, Lesson 3.)

Lesson 5: Two Equations for Each Relationship

5.1: Missing Figures

Here are the second and fourth figures in a pattern.

?



?



figure 1

figure 2

figure 3

figure 4

1. What do you think the first and third figures in the pattern look like?

2. Describe the 10th figure in the pattern.

5.2: Meters and Centimeters

There are 100 centimeters (cm) in every meter (m).

length (m)	length (cm)
1	100
0.94	
1.67	
57.24	
x	

length (cm)	length (m)
100	1
250	
78.2	
123.9	
y	

1. Complete each of the tables.
2. For each table, find the constant of proportionality.
3. What is the relationship between these constants of proportionality?
4. For each table, write an equation for the proportional relationship. Let x represent a length measured in meters and y represent the same length measured in centimeters.

Are you ready for more?

1. How many cubic centimeters are there in a cubic meter?
2. How do you convert cubic centimeters to cubic meters?
3. How do you convert the other way?

5.3: Filling a Water Cooler

It took Priya 5 minutes to fill a cooler with 8 gallons of water from a faucet that was flowing at a steady rate. Let w be the number of gallons of water in the cooler after t minutes.

1. Which of the following equations represent the relationship between w and t ?
Select all that apply.
 - a. $w = 1.6t$
 - b. $w = 0.625t$
 - c. $t = 1.6w$
 - d. $t = 0.625w$
2. What does 1.6 tell you about the situation?
3. What does 0.625 tell you about the situation?
4. Priya changed the rate at which water flowed through the faucet. Write an equation that represents the relationship of w and t when it takes 3 minutes to fill the cooler with 1 gallon of water.
5. Was the cooler filling faster before or after Priya changed the rate of water flow?
Explain how you know.

5.4: Feeding Shrimp

At an aquarium, a shrimp is fed $\frac{1}{5}$ gram of food each feeding and is fed 3 times each day.

1. How much food does a shrimp get fed in one day?
2. Complete the table to show how many grams of food the shrimp is fed over different numbers of days.

number of days	food in grams
1	
7	
30	



3. What is the constant of proportionality? What does it tell us about the situation?
4. If we switched the columns in the table, what would be the constant of proportionality? Explain your reasoning.
5. Use d for number of days and f for amount of food in grams that a shrimp eats to write *two* equations that represent the relationship between d and f .
6. If a tank has 10 shrimp in it, how much food is added to the tank each day?
7. If the aquarium manager has 300 grams of shrimp food for this tank of 10 shrimp, how many days will it last? Explain or show your reasoning.

Lesson 5 Summary

If Kiran rode his bike at a constant 10 miles per hour, his distance in miles, d , is proportional to the number of hours, t , that he rode. We can write the equation

$$d = 10t$$

With this equation, it is easy to find the distance Kiran rode when we know how long it took because we can just multiply the time by 10.

We can rewrite the equation:

$$\begin{aligned}d &= 10t \\ \left(\frac{1}{10}\right)d &= t \\ t &= \left(\frac{1}{10}\right)d\end{aligned}$$

This version of the equation tells us that the amount of time he rode is proportional to the distance he traveled, and the constant of proportionality is $\frac{1}{10}$. That form is easier to use when we know his distance and want to find how long it took because we can just multiply the distance by $\frac{1}{10}$.

When two quantities x and y are in a proportional relationship, we can write the equation

$$y = kx$$

and say, “ y is proportional to x .” In this case, the number k is the corresponding constant of proportionality. We can also write the equation

$$x = \frac{1}{k}y$$

and say, “ x is proportional to y .” In this case, the number $\frac{1}{k}$ is the corresponding constant of proportionality. Each one can be useful depending on the information we have and the quantity we are trying to figure out.

Unit 2 Lesson 5 Cumulative Practice Problems

1. The table represents the relationship between a length measured in meters and the same length measured in kilometers.

a. Complete the table.

b. Write an equation for converting the number of meters to kilometers. Use x for number of meters and y for number of kilometers.

meters	kilometers
1,000	1
3,500	
500	
75	
1	
x	

2. Concrete building blocks weigh 28 pounds each. Using b for the number of concrete blocks and w for the weight, write two equations that relate the two variables. One equation should begin with $w =$ and the other should begin with $b =$.

3. A store sells rope by the meter. The equation $p = 0.8L$ represents the price p (in dollars) of a piece of nylon rope that is L meters long.

a. How much does the nylon rope cost per meter?

b. How long is a piece of nylon rope that costs \$1.00?

4. The table represents a proportional relationship. Find the constant of proportionality and write an equation to represent the relationship.

a	y
2	$\frac{2}{3}$
3	1
10	$\frac{10}{3}$
12	4

Constant of proportionality: _____

Equation: $y =$

(From Unit 2, Lesson 4.)

5. On a map of Chicago, 1 cm represents 100 m. Select **all** statements that express the same scale.

- A. 5 cm on the map represents 50 m in Chicago.
- B. 1 mm on the map represents 10 m in Chicago.
- C. 1 km in Chicago is represented by 10 cm the map.
- D. 100 cm in Chicago is represented by 1 m on the map.

(From Unit 1, Lesson 8.)

Lesson 6: Using Equations to Solve Problems

6.1: Number Talk: Quotients with Decimal Points

Without calculating, order the quotients of these expressions from least to greatest.

$$42.6 \div 0.07$$

$$42.6 \div 70$$

$$42.6 \div 0.7$$

$$426 \div 70$$

Place the decimal point in the appropriate location in the quotient: $42.6 \div 7 = 608571$

Use this answer to find the quotient of *one* of the previous expressions.

6.2: Concert Ticket Sales

A performer expects to sell 5,000 tickets for an upcoming concert. They want to make a total of \$311,000 in sales from these tickets.

1. Assuming that all tickets have the same price, what is the price for one ticket?
2. How much will they make if they sell 7,000 tickets?
3. How much will they make if they sell 10,000 tickets? 50,000? 120,000? a million?
 x tickets?
4. If they make \$404,300, how many tickets have they sold?
5. How many tickets will they have to sell to make \$5,000,000?

6.3: Recycling

Aluminum cans can be recycled instead of being thrown in the garbage. The weight of 10 aluminum cans is 0.16 kilograms. The aluminum in 10 cans that are recycled has a value of \$0.14.

1. If a family threw away 2.4 kg of aluminum in a month, how many cans did they throw away? Explain or show your reasoning.
2. What would be the recycled value of those same cans? Explain or show your reasoning.
3. Write an equation to represent the number of cans c given their weight w .
4. Write an equation to represent the recycled value r of c cans.
5. Write an equation to represent the recycled value r of w kilograms of aluminum.

Are you ready for more?

The EPA estimated that in 2013, the average amount of garbage produced in the United States was 4.4 pounds per person per day. At that rate, how long would it take your family to produce a ton of garbage? (A ton is 2,000 pounds.)

Lesson 6 Summary

Remember that if there is a proportional relationship between two quantities, their relationship can be represented by an equation of the form $y = kx$. Sometimes writing an equation is the easiest way to solve a problem.

For example, we know that Denali, the highest mountain peak in North America, is 20,310 feet above sea level. How many miles is that? There are 5,280 feet in 1 mile. This relationship can be represented by the equation

$$f = 5,280m$$

where f represents a distance measured in feet and m represents the same distance measured miles. Since we know Denali is 20,310 feet above sea level, we can write

$$20,310 = 5,280m$$

So $m = \frac{20,310}{5,280}$, which is approximately 3.85 miles.

Unit 2 Lesson 6 Cumulative Practice Problems

1. A car is traveling down a highway at a constant speed, described by the equation $d = 65t$, where d represents the distance, in miles, that the car travels at this speed in t hours.
 - a. What does the 65 tell us in this situation?
 - b. How many miles does the car travel in 1.5 hours?
 - c. How long does it take the car to travel 26 miles at this speed?
2. Elena has some bottles of water that each holds 17 fluid ounces.
 - a. Write an equation that relates the number of bottles of water (b) to the total volume of water (w) in fluid ounces.
 - b. How much water is in 51 bottles?
 - c. How many bottles does it take to hold 51 fluid ounces of water?
3. There are about 1.61 kilometers in 1 mile. Let x represent a distance measured in kilometers and y represent the same distance measured in miles. Write two equations that relate a distance measured in kilometers and the same distance measured in miles.

(From Unit 2, Lesson 5.)

4. In Canadian coins, 16 quarters is equal in value to 2 toonies.

number of quarters	number of toonies
1	
16	2
20	
24	

a. Complete the table.

b. What does the value next to 1 mean in this situation?

(From Unit 2, Lesson 2.)

5. Each table represents a proportional relationship. For each table:

a. Fill in the missing parts of the table.

b. Draw a circle around the constant of proportionality.

x	y
2	10
	15
7	
1	

a	b
12	3
20	
	10
1	

m	n
5	3
10	
	18
1	

(From Unit 2, Lesson 2.)

6. Describe some things you could notice in two polygons that would help you decide that they were not scaled copies.

(From Unit 1, Lesson 4.)

Lesson 7: Comparing Relationships with Tables

7.1: Adjusting a Recipe

A lemonade recipe calls for the juice of 5 lemons, 2 cups of water, and 2 tablespoons of honey.

Invent four new versions of this lemonade recipe:

1. One that would make more lemonade but taste the same as the original recipe.
2. One that would make less lemonade but taste the same as the original recipe.
3. One that would have a stronger lemon taste than the original recipe.
4. One that would have a weaker lemon taste than the original recipe.

7.2: Visiting the State Park

Entrance to a state park costs \$6 per vehicle, plus \$2 per person in the vehicle.

1. How much would it cost for a car with 2 people to enter the park? 4 people? 10 people? Record your answers in the table.

number of people in vehicle	total entrance cost in dollars
2	
4	
10	

2. For each row in the table, if each person in the vehicle splits the entrance cost equally, how much will each person pay?
3. How might you determine the entrance cost for a bus with 50 people?
4. Is the relationship between the number of people and the total entrance cost a proportional relationship? Explain how you know.

Are you ready for more?

What equation could you use to find the total entrance cost for a vehicle with any number of people?

7.3: Running Laps

Han and Clare were running laps around the track. The coach recorded their times at the end of laps 2, 4, 6, and 8.

Han's run:

distance (laps)	time (minutes)	minutes per lap
2	4	
4	9	
6	15	
8	23	

Clare's run:

distance (laps)	time (minutes)	minutes per lap
2	5	
4	10	
6	15	
8	20	

1. Is Han running at a constant pace? Is Clare? How do you know?
2. Write an equation for the relationship between distance and time for anyone who is running at a constant pace.

Lesson 7 Summary

Here are the prices for some smoothies at two different smoothie shops:

Smoothie Shop A

smoothie size (oz)	price (\$)	dollars per ounce
8	6	0.75
12	9	0.75
16	12	0.75
s	$0.75s$	0.75

Smoothie Shop B

smoothie size (oz)	price (\$)	dollars per ounce
8	6	0.75
12	8	0.67
16	10	0.625
s	???	???

For Smoothie Shop A, smoothies cost \$0.75 per ounce no matter which size we buy. There could be a proportional relationship between smoothie size and the price of the smoothie. An equation representing this relationship is

$$p = 0.75s$$

where s represents size in ounces and p represents price in dollars. (The relationship could still not be proportional, if there were a different size on the menu that did not have the same price per ounce.)

For Smoothie Shop B, the cost per ounce is different for each size. Here the relationship between smoothie size and price is definitely *not* proportional.

In general, two quantities in a proportional relationship will always have the same quotient. When we see some values for two related quantities in a table and we get the same quotient when we divide them, that means they might be in a proportional relationship—but if we can't see all of the possible pairs, we can't be completely sure. However, if we know the relationship can be represented by an equation is of the form $y = kx$, then we are sure it is proportional.

Unit 2 Lesson 7 Cumulative Practice Problems

1. Decide whether each table could represent a proportional relationship. If the relationship could be proportional, what would the constant of proportionality be?

a. How loud a sound is depending on how far away you are.

distance to listener (ft)	sound level (dB)
5	85
10	79
20	73
40	67

b. The cost of fountain drinks at Hot Dog Hut.

volume (fluid ounces)	cost (\$)
16	\$1.49
20	\$1.59
30	\$1.89

2. A taxi service charges \$1.00 for the first $\frac{1}{10}$ mile then \$0.10 for each additional $\frac{1}{10}$ mile after that.

Fill in the table with the missing information then determine if this relationship between distance traveled and price of the trip is a proportional relationship.

distance traveled (mi)	price (dollars)
$\frac{9}{10}$	
2	
$3\frac{1}{10}$	
10	

3. A rabbit and turtle are in a race. Is the relationship between distance traveled and time proportional for either one? If so, write an equation that represents the relationship.

Turtle's run:

distance (meters)	time (minutes)
108	2
405	7.5
540	10
1,768.5	32.75

Rabbit's run:

distance (meters)	time (minutes)
800	1
900	5
1,107.5	20
1,524	32.5

4. For each table, answer: What is the constant of proportionality?

a	b
2	14
5	35
9	63
$\frac{1}{3}$	$\frac{7}{3}$

a	b
3	360
5	600
8	960
12	1440

a	b
75	3
200	8
1525	61
10	0.4

a	b
4	10
6	15
22	55
3	$7\frac{1}{2}$

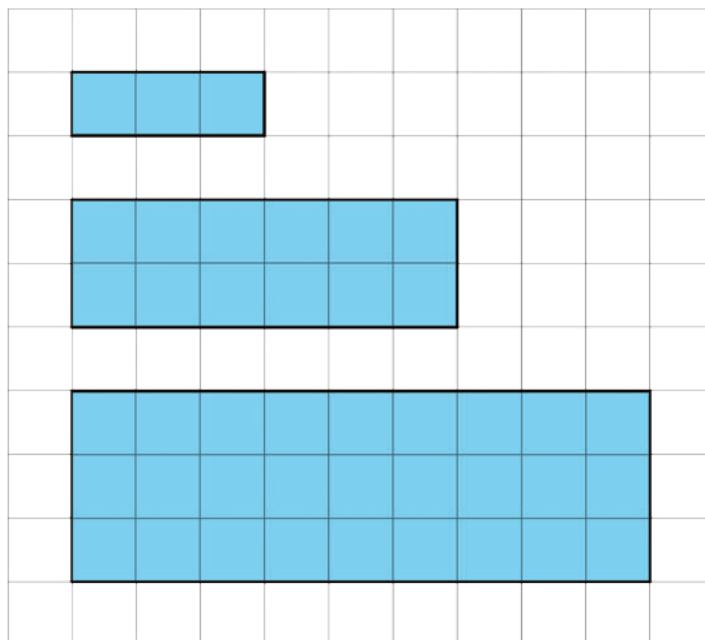
(From Unit 2, Lesson 2.)

5. Kiran and Mai are standing at one corner of a rectangular field of grass looking at the diagonally opposite corner. Kiran says that if the field were twice as long and twice as wide, then it would be twice the distance to the far corner. Mai says that it would be more than twice as far, since the diagonal is even longer than the side lengths. Do you agree with either of them?

(From Unit 1, Lesson 4.)

Lesson 8: Comparing Relationships with Equations

8.1: Notice and Wonder: Patterns with Rectangles



Do you see a pattern? What predictions can you make about future rectangles in the set if your pattern continues?

8.2: More Conversions

The other day you worked with converting meters, centimeters, and millimeters. Here are some more unit conversions.

1. Use the equation $F = \frac{9}{5}C + 32$, where F represents degrees Fahrenheit and C represents degrees Celsius, to complete the table.

temperature (°C)	temperature (°F)
20	
4	
175	

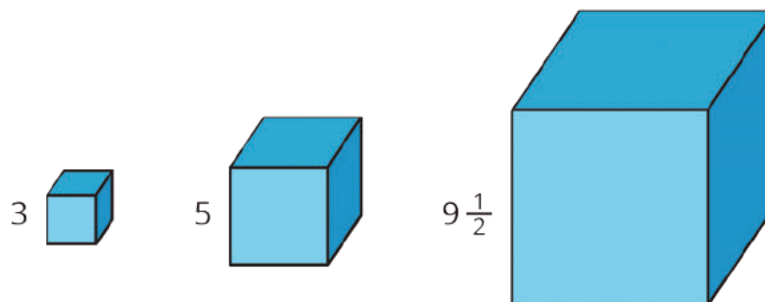
2. Use the equation $c = 2.54n$, where c represents the length in centimeters and n represents the length in inches, to complete the table.

length (in)	length (cm)
10	
8	
$3\frac{1}{2}$	

3. Are these proportional relationships? Explain why or why not.

8.3: Total Edge Length, Surface Area, and Volume

Here are some cubes with different side lengths. Complete each table. Be prepared to explain your reasoning.



1. How long is the total edge length of each cube?

side length	total edge length
3	
5	
$9\frac{1}{2}$	
s	

2. What is the surface area of each cube?

side length	surface area
3	
5	
$9\frac{1}{2}$	
s	

3. What is the volume of each cube?

side length	volume
3	
5	
$9\frac{1}{2}$	
s	

4. Which of these relationships is proportional? Explain how you know.

5. Write equations for the total edge length E , total surface area A , and volume V of a cube with side length s .

Are you ready for more?

1. A rectangular solid has a square base with side length ℓ , height 8, and volume V . Is the relationship between ℓ and V a proportional relationship?
2. A different rectangular solid has length ℓ , width 10, height 5, and volume V . Is the relationship between ℓ and V a proportional relationship?
3. Why is the relationship between the side length and the volume proportional in one situation and not the other?

8.4: All Kinds of Equations

Here are six different equations.

$$y = 4 + x$$

$$y = 4x$$

$$y = \frac{4}{x}$$

$$y = \frac{x}{4}$$

$$y = 4^x$$

$$y = x^4$$

$$y = 4 + x$$

x	y	$\frac{y}{x}$
2		
3		
4		
5		

$$y = 4x$$

x	y	$\frac{y}{x}$
2		
3		
4		
5		

$$y = \frac{4}{x}$$

x	y	$\frac{y}{x}$
2		
3		
4		
5		

$$y = \frac{x}{4}$$

x	y	$\frac{y}{x}$
2		
3		
4		
5		

$$y = 4^x$$

x	y	$\frac{y}{x}$
2		
3		
4		
5		

$$y = x^4$$

x	y	$\frac{y}{x}$
2		
3		
4		
5		

1. Predict which of these equations represent a proportional relationship.
2. Complete each table using the equation that represents the relationship.
3. Do these results change your answer to the first question? Explain your reasoning.
4. What do the equations of the proportional relationships have in common?

Lesson 8 Summary

If two quantities are in a proportional relationship, then their quotient is always the same. This table represents different values of a and b , two quantities that are in a proportional relationship.

a	b	$\frac{b}{a}$
20	100	5
3	15	5
11	55	5
1	5	5

Notice that the quotient of b and a is always 5. To write this as an equation, we could say $\frac{b}{a} = 5$. If this is true, then $b = 5a$. (This doesn't work if $a = 0$, but it works otherwise.)

If quantity y is proportional to quantity x , we will always see this pattern: $\frac{y}{x}$ will always have the same value. This value is the constant of proportionality, which we often refer to as k . We can represent this relationship with the equation $\frac{y}{x} = k$ (as long as x is not 0) or $y = kx$.

Note that if an equation cannot be written in this form, then it does not represent a proportional relationship.

Unit 2 Lesson 8 Cumulative Practice Problems

1. The relationship between a distance in yards (y) and the same distance in miles (m) is described by the equation $y = 1760m$.

a. Find measurements in yards and miles for distances by completing the table.

distance measured in miles	distance measured in yards
1	
5	
	3,520
	17,600

- b. Is there a proportional relationship between a measurement in yards and a measurement in miles for the same distance? Explain why or why not.

2. Decide whether or not each equation represents a proportional relationship.

a. The remaining length (L) of 120-inch rope after x inches have been cut off: $120 - x = L$

b. The total cost (t) after 8% sales tax is added to an item's price (p): $1.08p = t$

c. The number of marbles each sister gets (x) when m marbles are shared equally among four sisters: $x = \frac{m}{4}$

d. The volume (V) of a rectangular prism whose height is 12 cm and base is a square with side lengths s cm: $V = 12s^2$

3. a. Use the equation $y = \frac{5}{2}x$ to complete the table.
Is y proportional to x and y ? Explain why or why not.

x	y
2	
3	
6	

- b. Use the equation $y = 3.2x + 5$ to complete the table.
Is y proportional to x and y ? Explain why or why not.

x	y
1	
2	
4	

4. To transmit information on the internet, large files are broken into packets of smaller sizes. Each packet has 1,500 bytes of information. An equation relating packets to bytes of information is given by $b = 1,500p$ where p represents the number of packets and b represents the number of bytes of information.

- a. How many packets would be needed to transmit 30,000 bytes of information?
- b. How much information could be transmitted in 30,000 packets?
- c. Each byte contains 8 bits of information. Write an equation to represent the relationship between the number of packets and the number of bits.

(From Unit 2, Lesson 6.)

Lesson 9: Solving Problems about Proportional Relationships

9.1: What Do You Want to Know?

Consider the problem: A person is running a distance race at a constant rate. What time will they finish the race?

What information would you need to be able to solve the problem?

9.2: Info Gap: Biking and Rain

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

Continue to ask questions until you have enough information to solve the problem.

4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.
2. Ask your partner “*What specific information do you need?*” and wait for them to *ask* for information.

If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don’t have that information.

3. Before sharing the information, ask “*Why do you need that information?*” Listen to your partner’s reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

9.3: Moderating Comments

A company is hiring people to read through all the comments posted on their website to make sure they are appropriate. Four people applied for the job and were given one day to show how quickly they could check comments.

- Person 1 worked for 210 minutes and checked a total of 50,000 comments.
- Person 2 worked for 200 minutes and checked 1,325 comments every 5 minutes.
- Person 3 worked for 120 minutes, at a rate represented by $c = 331t$, where c is the number of comments checked and t is the time in minutes.
- Person 4 worked for 150 minutes, at a rate represented by $t = \left(\frac{3}{800}\right)c$.

1. Order the people from greatest to least in terms of total number of comments checked.
2. Order the people from greatest to least in terms of how fast they checked the comments.

Are you ready for more?

1. Write equations for each job applicant that allow you to easily decide who is working the fastest.
2. Make a table that allows you to easily compare how many comments the four job applicants can check.

Lesson 9 Summary

Whenever we have a situation involving constant rates, we are likely to have a proportional relationship between quantities of interest.

- When a bird is flying at a constant speed, then there is a proportional relationship between the flying time and distance flown.
- If water is filling a tub at a constant rate, then there is a proportional relationship between the amount of water in the tub and the time the tub has been filling up.
- If an aardvark is eating termites at a constant rate, then there is proportional relationship between the number of termites the aardvark has eaten and the time since it started eating.

Sometimes we are presented with a situation, and it is not so clear whether a proportional relationship is a good model. How can we decide if a proportional relationship is a good representation of a particular situation?

- If you aren't sure where to start, look at the quotients of corresponding values. If they are not always the same, then the relationship is definitely not a proportional relationship.
- If you can see that there is a single value that we always multiply one quantity by to get the other quantity, it is definitely a proportional relationship.

After establishing that it is a proportional relationship, setting up an equation is often the most efficient way to solve problems related to the situation.

Unit 2 Lesson 9 Cumulative Practice Problems

1. For each situation, explain whether you think the relationship is proportional or not. Explain your reasoning.

- a. The weight of a stack of standard 8.5x11 copier paper vs. number of sheets of paper.



- b. The weight of a stack of different-sized books vs. the number of books in the stack.



2. Every package of a certain toy also includes 2 batteries.

- a. Are the number of toys and number of batteries in a proportional relationship? If so, what are the two constants of proportionality? If not, explain your reasoning.

- b. Use t for the number of toys and b for the number of batteries to write two equations relating the two variables.

$b =$

$t =$

3. Lin and her brother were born on the same date in different years. Lin was 5 years old when her brother was 2.

a. Find their ages in different years by filling in the table.

Lin's age	Her brother's age
5	2
6	
15	
	25

b. Is there a proportional relationship between Lin's age and her brother's age? Explain your reasoning.

4. A student argues that $y = \frac{x}{9}$ does not represent a proportional relationship between x and y because we need to multiply one variable by the same constant to get the other one and not divide it by a constant. Do you agree or disagree with this student?

(From Unit 2, Lesson 8.)

5. Quadrilateral A has side lengths 3, 4, 5, and 6. Quadrilateral B is a scaled copy of Quadrilateral A with a scale factor of 2. Select all of the following that are side lengths of Quadrilateral B.

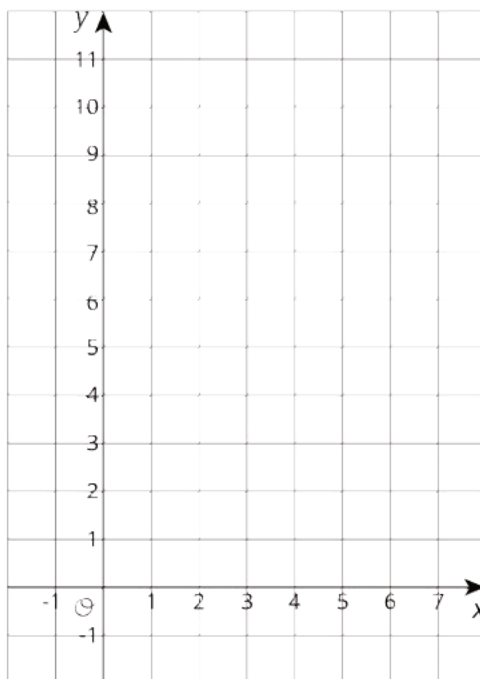
- A. 5
- B. 6
- C. 7
- D. 8
- E. 9

(From Unit 1, Lesson 3.)

Lesson 10: Introducing Graphs of Proportional Relationships

10.1: Notice These Points

1. Plot the points $(0, 10)$, $(1, 8)$, $(2, 6)$, $(3, 4)$, $(4, 2)$.



2. What do you notice about the graph?

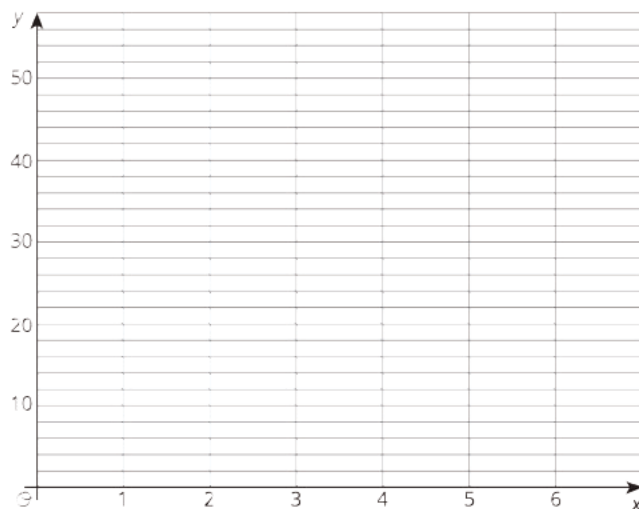
10.2: T-shirts for Sale

Some T-shirts cost \$8 each.

x	y
1	8
2	16
3	24
4	32
5	40
6	48

1. Use the table to answer these questions.
 - a. What does x represent?
 - b. What does y represent?
 - c. Is there a proportional relationship between x and y ?

2. Plot the pairs in the table on the coordinate plane.



3. What do you notice about the graph?

10.3: Matching Tables and Graphs

Your teacher will give you papers showing tables and graphs.

1. Examine the graphs closely. What is the same and what is different about the graphs?
2. Sort the graphs into categories of your choosing. Label each category. Be prepared to explain why you sorted the graphs the way you did.
3. Take turns with a partner to match a table with a graph.
 - a. For each match you find, explain to your partner how you know it is a match.
 - b. For each match your partner finds, listen carefully to their explanation. If you disagree, work to reach an agreement.

Pause here so your teacher can review your work.

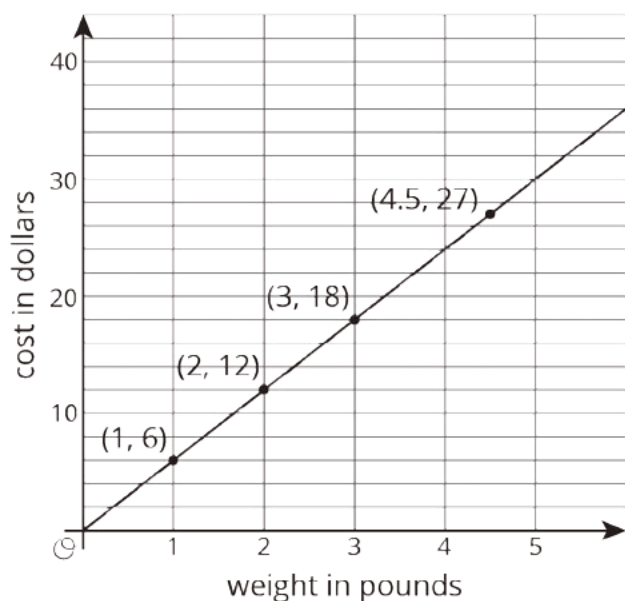
4. Trade places with another group. How are their categories the same as your group's categories? How are they different?
5. Return to your original place. Discuss any changes you may wish to make to your categories based on what the other group did.
6. Which of the relationships are proportional?
7. What have you noticed about the graphs of proportional relationships? Do you think this will hold true for *all* graphs of proportional relationships?

Are you ready for more?

1. All the graphs in this activity show points where both coordinates are positive. Would it make sense for any of them to have one or more coordinates that are negative?
2. The equation of a proportional relationship is of the form $y = kx$, where k is a positive number, and the graph is a line through $(0, 0)$. What would the graph look like if k were a negative number?

Lesson 10 Summary

One way to represent a proportional relationship is with a graph. Here is a graph that represents different amounts that fit the situation, "Blueberries cost \$6 per pound."



Different points on the graph tell us, for example, that 2 pounds of blueberries cost \$12, and 4.5 pounds of blueberries cost \$27.

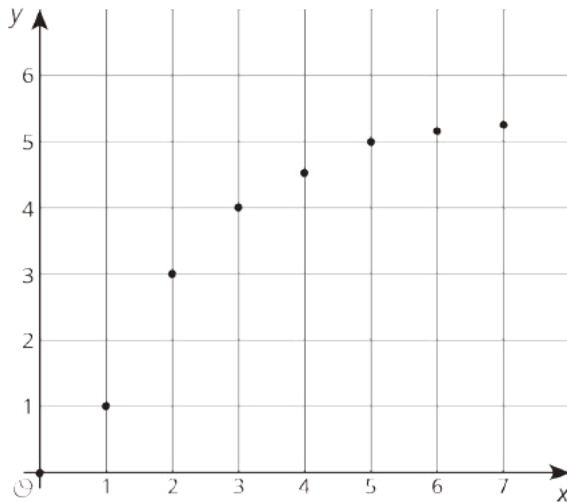
Sometimes it makes sense to connect the points with a line, and sometimes it doesn't. We could buy, for example, 4.5 pounds of blueberries or 1.875 pounds of blueberries, so all the points in between the whole numbers make sense in the situation, so any point on the line is meaningful.

If the graph represented the cost for different *numbers of sandwiches* (instead of pounds of blueberries), it might not make sense to connect the points with a line, because it is often not possible to buy 4.5 sandwiches or 1.875 sandwiches. Even if only points make sense in the situation, though, sometimes we connect them with a line anyway to make the relationship easier to see.

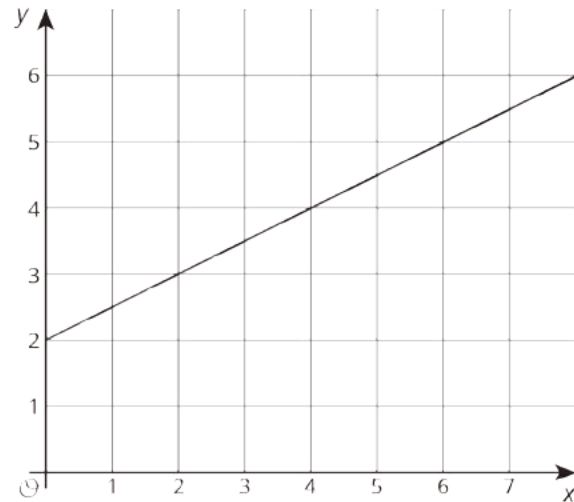
Graphs that represent proportional relationships all have a few things in common:

- Points that satisfy the relationship lie on a straight line.
- The line that they lie on passes through the origin, $(0, 0)$.

Here are some graphs that do *not* represent proportional relationships:



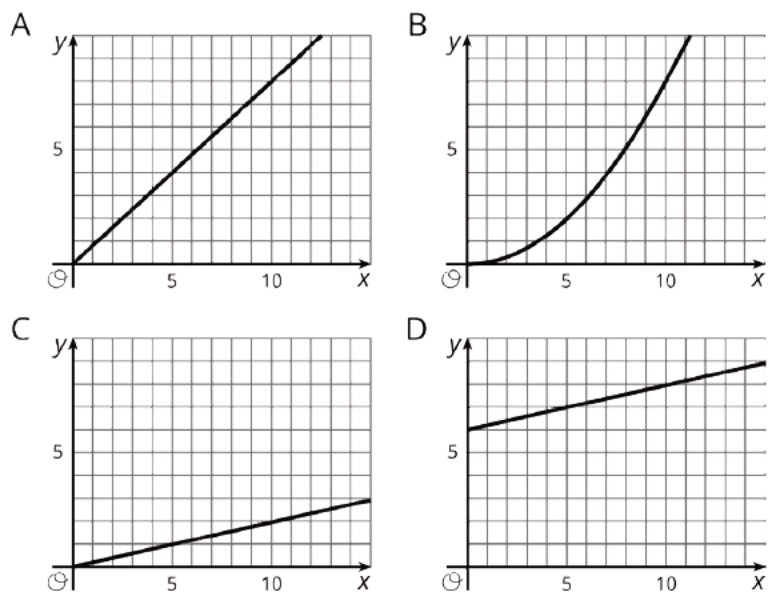
These points do not lie on a line.



This is a line, but it doesn't go through the origin.

Unit 2 Lesson 10 Cumulative Practice Problems

1. Which graphs could represent a proportional relationship?



- A. A
- B. B
- C. C
- D. D

2. A lemonade recipe calls for $\frac{1}{4}$ cup of lemon juice for every cup of water.

a. Use the table to answer these questions.

i. What does x represent?

ii. What does y represent?

iii. Is there a proportional relationship between x and y ?

b. Plot the pairs in the table in a coordinate plane.

x	y
1	$\frac{1}{4}$
2	$\frac{1}{2}$
3	$\frac{3}{4}$
4	1

3. Select all the pieces of information that would tell you x and y have a proportional relationship. Let y represent the distance in meters between a rock and a turtle's current position and x represent the time in minutes the turtle has been moving.

A. $y = 3x$

B. After 4 minutes, the turtle has walked 12 feet away from the rock.

C. The turtle walks for a bit, then stops for a minute before walking again.

D. The turtle walks away from the rock at a constant rate.

(From Unit 2, Lesson 9.)

4. Decide whether each table could represent a proportional relationship. If the relationship could be proportional, what would be the constant of proportionality?

a. The sizes you can print a photo.

width of photo (inches)	height of photo (inches)
2	3
4	6
5	7
8	10

b. The distance from which a lighthouse is visible.

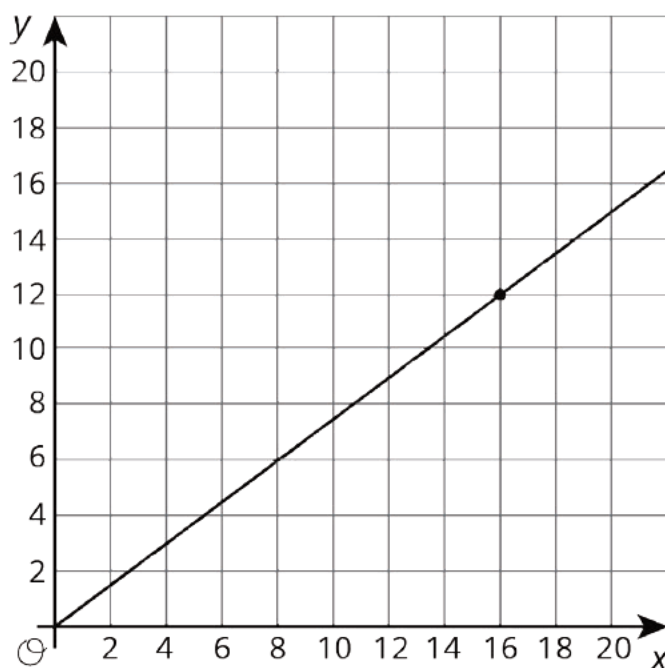
height of a lighthouse (feet)	distance it can be seen (miles)
20	6
45	9
70	11
95	13

(From Unit 2, Lesson 7.)

Lesson 11: Interpreting Graphs of Proportional Relationships

11.1: What Could the Graph Represent?

Here is a graph that represents a proportional relationship.

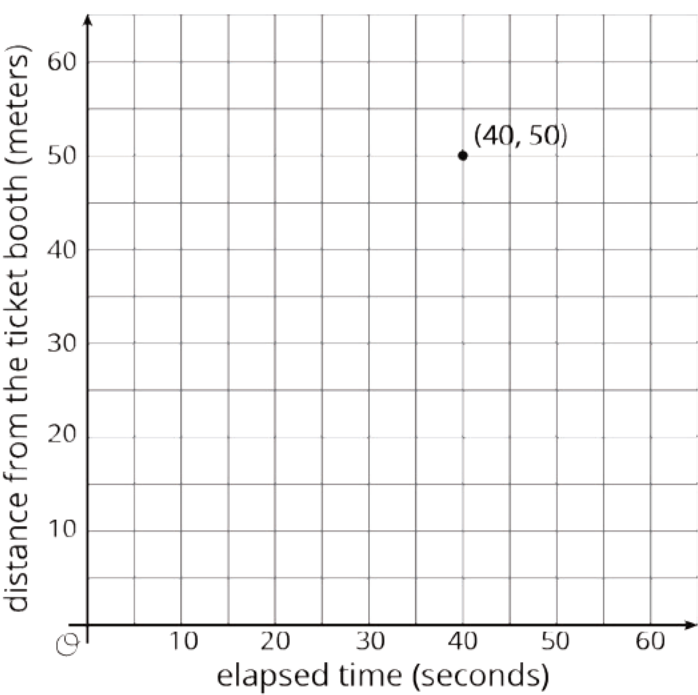


1. Invent a situation that could be represented by this graph.
2. Label the axes with the quantities in your situation.
3. Give the graph a title.
4. There is a point on the graph. What are its coordinates? What does it represent in your situation?

11.2: Tyler's Walk

Tyler was at the amusement park. He walked at a steady pace from the ticket booth to the bumper cars.

- 1. The point on the graph shows his arrival at the bumper cars. What do the coordinates of the point tell us about the situation?



- 2. The table representing Tyler's walk shows other values of time and distance. Complete the table. Next, plot the pairs of values on the grid.

- 3. What does the point (0, 0) mean in this situation?

time (seconds)	distance (meters)
0	0
20	25
30	37.5
40	50
1	

- 4. How far away from the ticket booth was Tyler after 1 second? Label the point on the graph that shows this information with its coordinates.
- 5. What is the constant of proportionality for the relationship between time and distance? What does it tell you about Tyler's walk? Where do you see it in the graph?

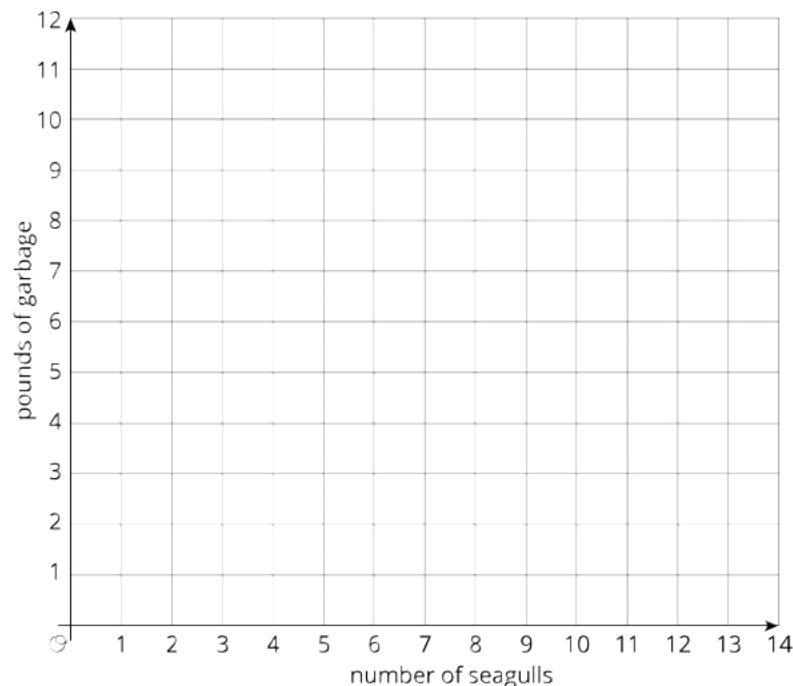
Are you ready for more?

If Tyler wanted to get to the bumper cars in half the time, how would the graph representing his walk change? How would the table change? What about the constant of proportionality?

11.3: Seagulls Eat What?

4 seagulls ate 10 pounds of garbage. Assume this information describes a proportional relationship.

1. Plot a point that shows the number of seagulls and the amount of garbage they ate.
2. Use a straight edge to draw a line through this point and $(0, 0)$.
3. Plot the point $(1, k)$ on the line. What is the value of k ? What does the value of k tell you about this context?



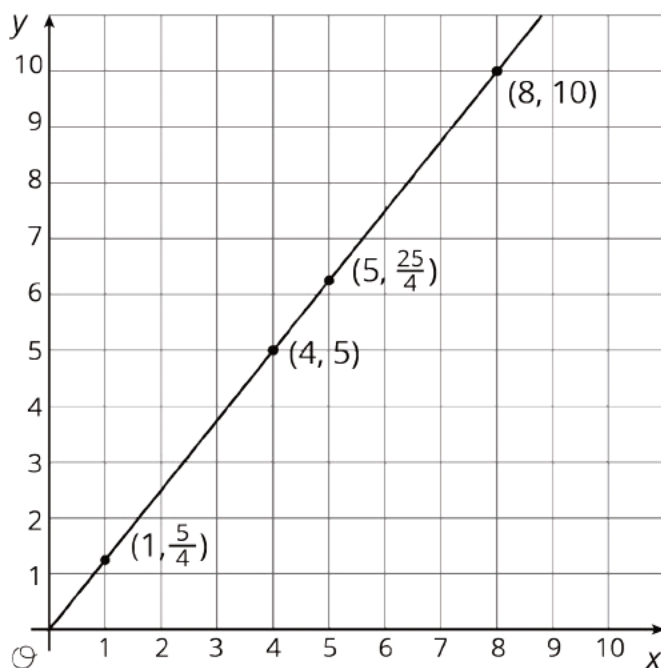
Lesson 11 Summary

For the relationship represented in this table, y is proportional to x . We can see in the table that $\frac{5}{4}$ is the constant of proportionality because it's the y value when x is 1.

The equation $y = \frac{5}{4}x$ also represents this relationship.

x	y
4	5
5	$\frac{25}{4}$
8	10
1	$\frac{5}{4}$

Here is the graph of this relationship.



If y represents the distance in feet that a snail crawls in x minutes, then the point (4, 5) tells us that the snail can crawl 5 feet in 4 minutes.

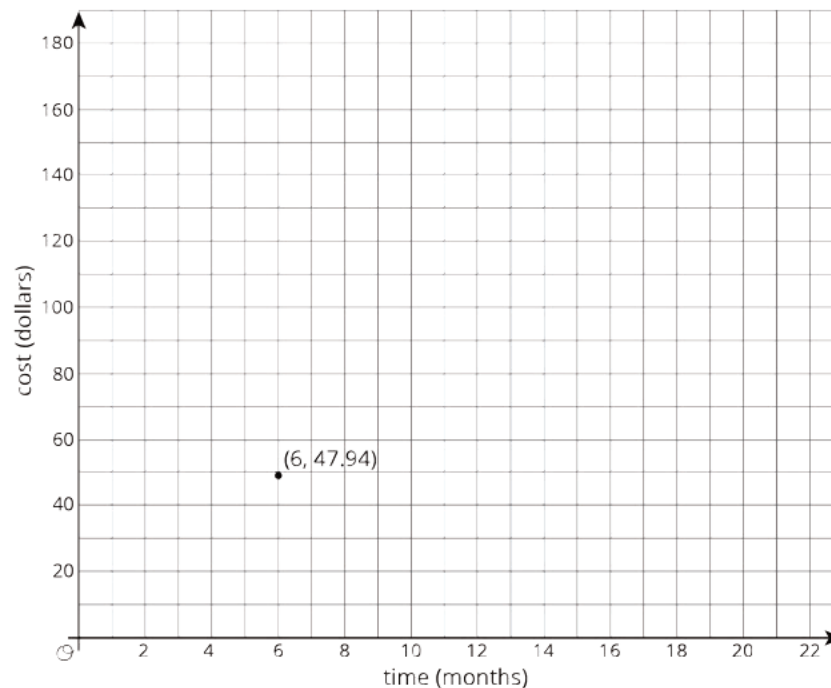
If y represents the cups of yogurt and x represents the teaspoons of cinnamon in a recipe for fruit dip, then the point (4, 5) tells us that you can mix 4 teaspoons of cinnamon with 5 cups of yogurt to make this fruit dip.

We can find the constant of proportionality by looking at the graph, because $\frac{5}{4}$ is the y -coordinate of the point on the graph where the x -coordinate is 1. This could mean the snail is traveling $\frac{5}{4}$ feet per minute or that the recipe calls for $1\frac{1}{4}$ cups of yogurt for every teaspoon of cinnamon.

In general, when y is proportional to x , the corresponding constant of proportionality is the y -value when $x = 1$.

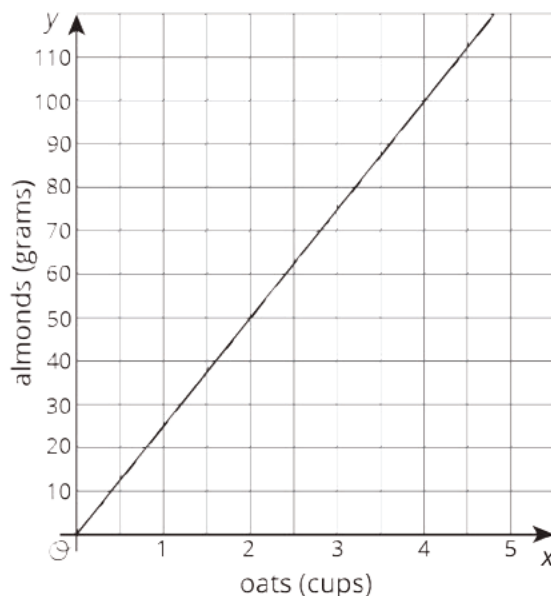
Unit 2 Lesson 11 Cumulative Practice Problems

1. There is a proportional relationship between the number of months a person has had a streaming movie subscription and the total amount of money they have paid for the subscription. The cost for 6 months is \$47.94. The point $(6, 47.94)$ is shown on the graph below.



- What is the constant of proportionality in this relationship?
- What does the constant of proportionality tell us about the situation?
- Add at least three more points to the graph and label them with their coordinates.
- Write an equation that represents the relationship between C , the total cost of the subscription, and m , the number of months.

2. The graph shows the amounts of almonds, in grams, for different amounts of oats, in cups, in a granola mix. Label the point $(1, k)$ on the graph, find the value of k , and explain its meaning.



3. To make a friendship bracelet, some long strings are lined up then taking one string and tying it in a knot with each of the other strings to create a row of knots. A new string is chosen and knotted with all the other strings to create a second row. This process is repeated until there are enough rows to make a bracelet to fit around your friend's wrist.

Are the number of knots proportional to the number of rows? Explain your reasoning.

(From Unit 2, Lesson 9.)

4. What information do you need to know to write an equation relating two quantities that have a proportional relationship?

(From Unit 2, Lesson 9.)

Lesson 12: Using Graphs to Compare Relationships

12.1: Number Talk: Fraction Multiplication and Division

Find each product or quotient mentally.

$$\frac{2}{3} \cdot \frac{1}{2}$$

$$\frac{4}{3} \cdot \frac{1}{4}$$

$$4 \div \frac{1}{5}$$

$$\frac{9}{6} \div \frac{1}{2}$$

12.2: Race to the Bumper Cars

Diego, Lin, and Mai went from the ticket booth to the bumper cars.

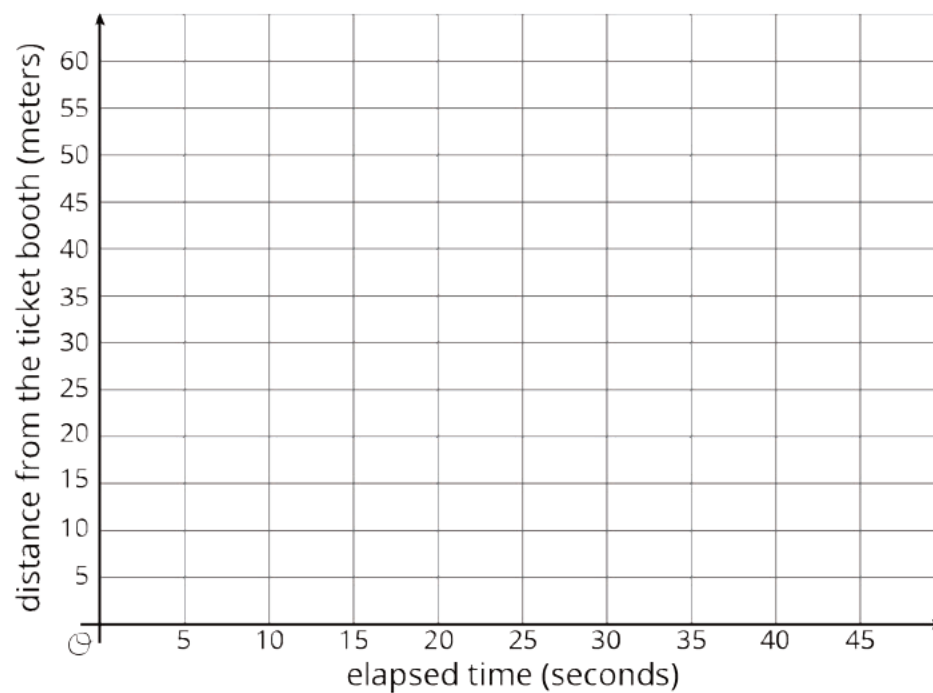
- 1. Use each description to complete the table representing that person’s journey.
 - a. Diego left the ticket booth at the same time as Tyler. Diego jogged ahead at a steady pace and reached the bumper cars in 30 seconds.
 - b. Lin left the ticket booth at the same time as Tyler. She ran at a steady pace and arrived at the bumper cars in 20 seconds.
 - c. Mai left the booth 10 seconds later than Tyler. Her steady jog enabled her to catch up with Tyler just as he arrived at the bumper cars.

Diego’s time (seconds)	Diego’s distance (meters)
0	
15	
30	50
1	

Lin’s time (seconds)	Lin’s distance (meters)
	0
	25
20	50
1	

Mai’s time (seconds)	Mai’s distance (meters)
	0
	25
40	50
1	

2. Using a different color for each person, draw a graph of all four people's journeys (including Tyler's from the other day).



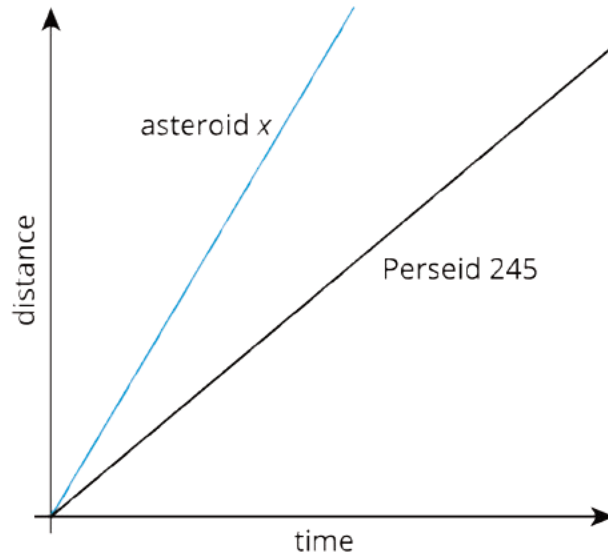
3. Which person is moving the most quickly? How is that reflected in the graph?

Are you ready for more?

Write equations to represent each person's relationship between time and distance.

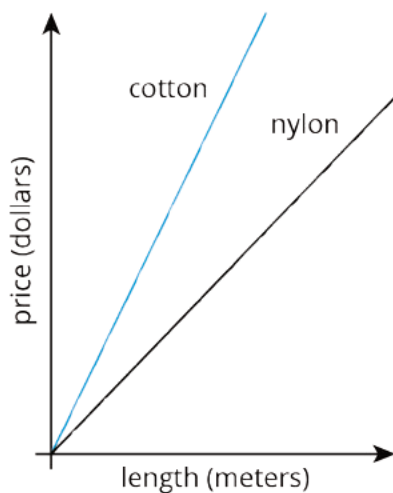
12.3: Space Rocks and the Price of Rope

1. Meteoroid Perseid 245 and Asteroid x travel through the solar system. The graph shows the distance each traveled after a given point in time.



Is Asteroid x traveling faster or slower than Perseid 245? Explain how you know.

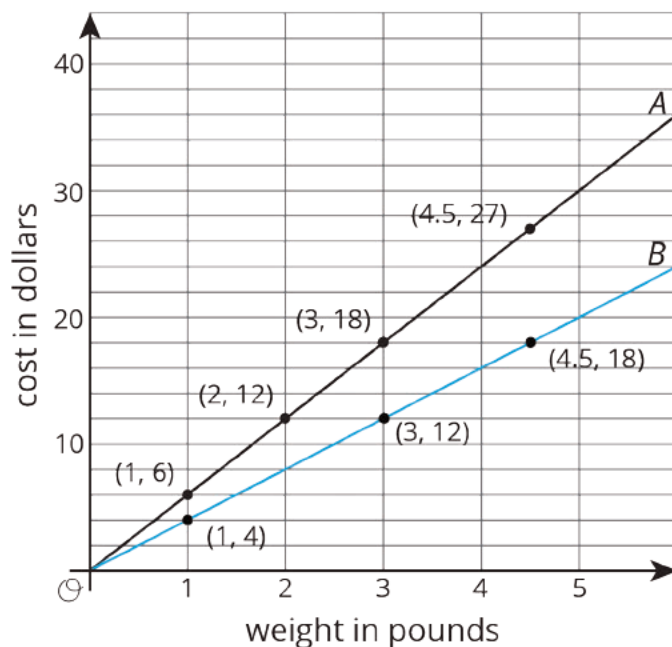
2. The graph shows the price of different lengths of two types of rope.



If you buy \$1.00 of each kind of rope, which one will be longer? Explain how you know.

Lesson 12 Summary

Here is a graph that shows the price of blueberries at two different stores. Which store has a better price?



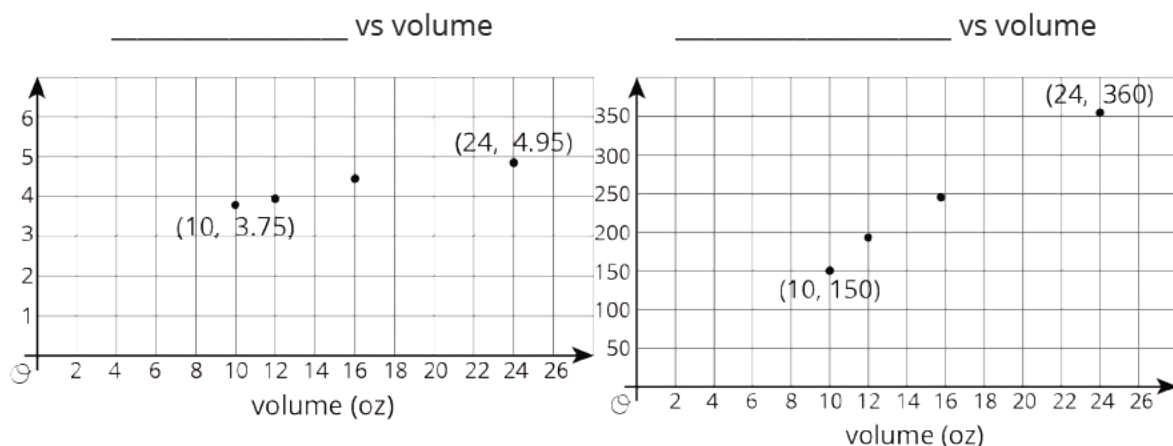
We can compare points that have the same x value or the same y value. For example, the points $(2, 12)$ and $(3, 12)$ tell us that at store B you can get more pounds of blueberries for the same price.

The points $(3, 12)$ and $(3, 18)$ tell us that at store A you have to pay more for the same quantity of blueberries. This means store B has the better price.

We can also use the graphs to compare the constants of proportionality. The line representing store B goes through the point $(1, 4)$, so the constant of proportionality is 4. This tells us that at store B the blueberries cost \$4 per pound. This is cheaper than the \$6 per pound unit price at store A.

Unit 2 Lesson 12 Cumulative Practice Problems

1. The graphs below show some data from a coffee shop menu. One of the graphs shows cost (in dollars) vs. drink volume (in ounces), and one of the graphs shows calories vs. drink volume (in ounces).



- Which graph is which? Give them the correct titles.
 - Which quantities appear to be in a proportional relationship? Explain how you know.
 - For the proportional relationship, find the constant of proportionality. What does that number mean?
2. Lin and Andre biked home from school at a steady pace. Lin biked 1.5 km and it took her 5 minutes. Andre biked 2 km and it took him 8 minutes.
- Draw a graph with two lines that represent the bike rides of Lin and Andre.
 - For each line, highlight the point with coordinates $(1, k)$ and find k .
 - Who was biking faster?

3. Match each equation to its graph.

a. $y = 2x$

b. $y = \frac{4}{5}x$

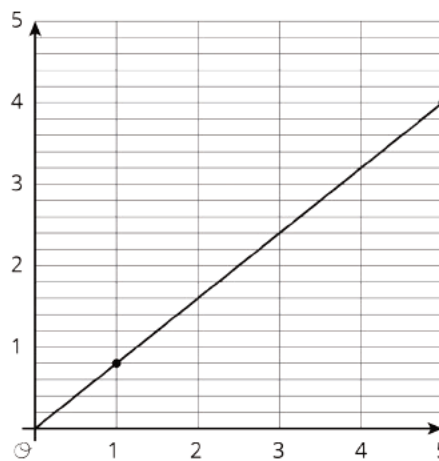
c. $y = \frac{1}{4}x$

d. $y = \frac{2}{3}x$

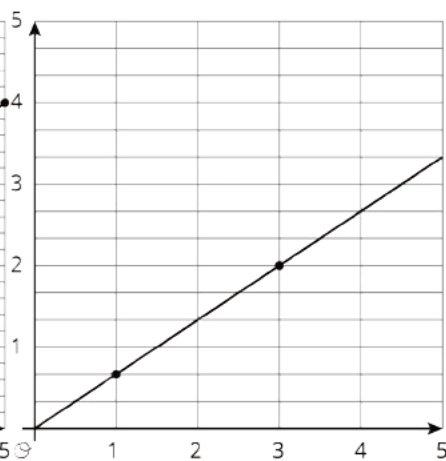
e. $y = \frac{4}{3}x$

f. $y = \frac{3}{2}x$

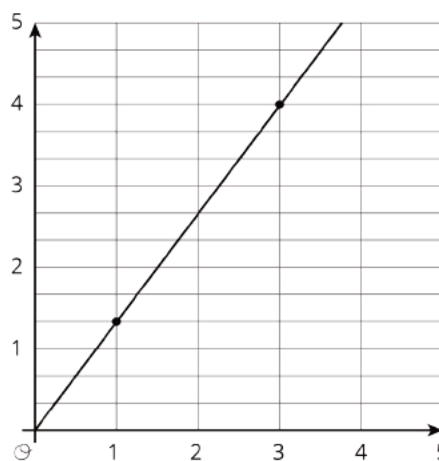
1



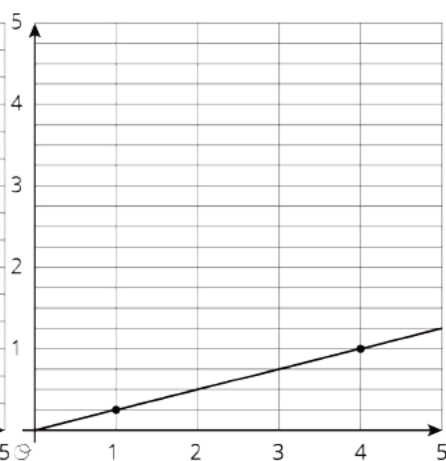
2



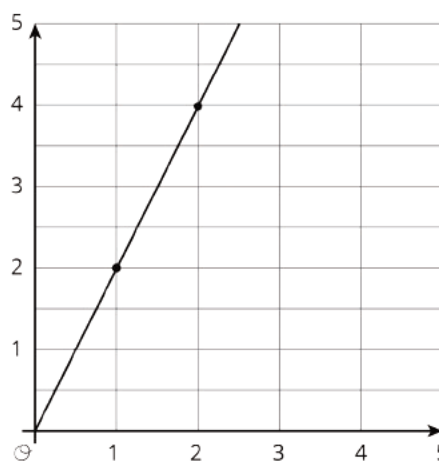
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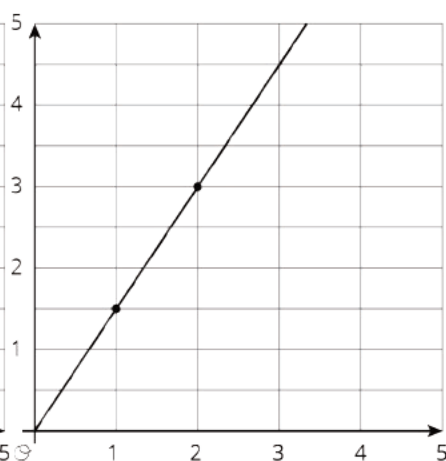
4



5



6



Lesson 13: Two Graphs for Each Relationship

13.1: True or False: Fractions and Decimals

Decide whether each equation is true or false. Be prepared to explain your reasoning.

1. $\frac{3}{2} \cdot 16 = 3 \cdot 8$

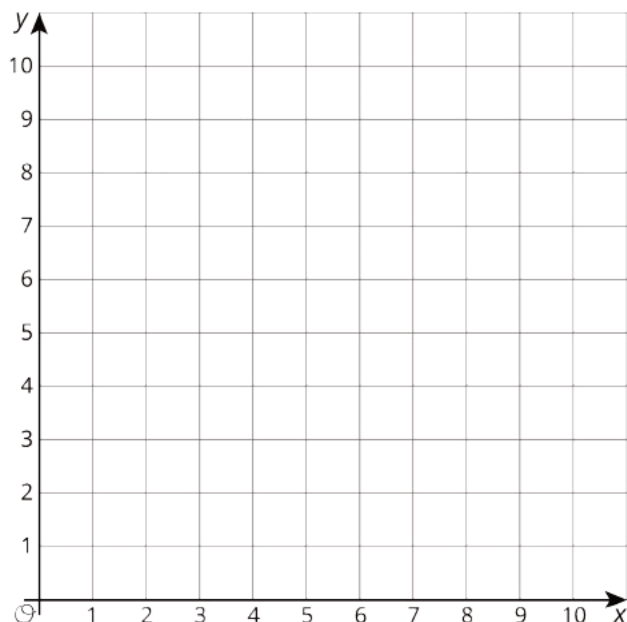
2. $\frac{3}{4} \div \frac{1}{2} = \frac{6}{4} \div \frac{1}{4}$

3. $(2.8) \cdot (13) = (0.7) \cdot (52)$

13.2: Tables, Graphs, and Equations

Your teacher will assign you *one* of these three points:

$$A = (10, 4), B = (4, 5), C = (8, 5).$$



x	y	$\frac{y}{x}$
0		NA
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

1. On the graph, plot and label *only* your assigned point.
2. Use a ruler to line up your point with the origin, $(0, 0)$. Draw a line that starts at the origin, goes through your point, and continues to the edge of the graph.
3. Complete the table with the coordinates of points on your graph. Use a fraction to represent any value that is not a whole number.
4. Write an equation that represents the relationship between x and y defined by your point.

5. Compare your graph and table with the rest of your group. What is the same and what is different about:
- a. your tables?
 - b. your equations?
 - c. your graphs?
6. What is the y -coordinate of your graph when the x -coordinate is 1? Plot and label this point on your graph. Where do you see this value in the table? Where do you see this value in your equation?
7. Describe any connections you see between the table, characteristics of the graph, and the equation.

Are you ready for more?

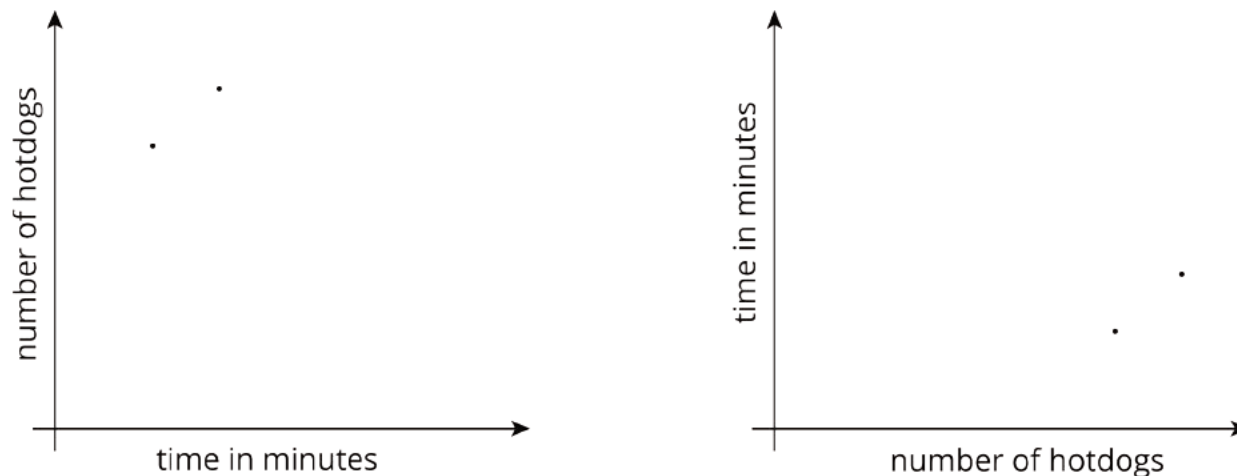
The graph of an equation of the form $y = kx$, where k is a positive number, is a line through $(0, 0)$ and the point $(1, k)$.

1. Name at least one line through $(0, 0)$ that cannot be represented by an equation like this.
2. If you could draw the graphs of *all* of the equations of this form in the same coordinate plane, what would it look like?

13.3: Hot Dog Eating Contest

Andre and Jada were in a hot dog eating contest. Andre ate 10 hot dogs in 3 minutes. Jada ate 12 hot dogs in 5 minutes.

Here are two different graphs that both represent this situation.



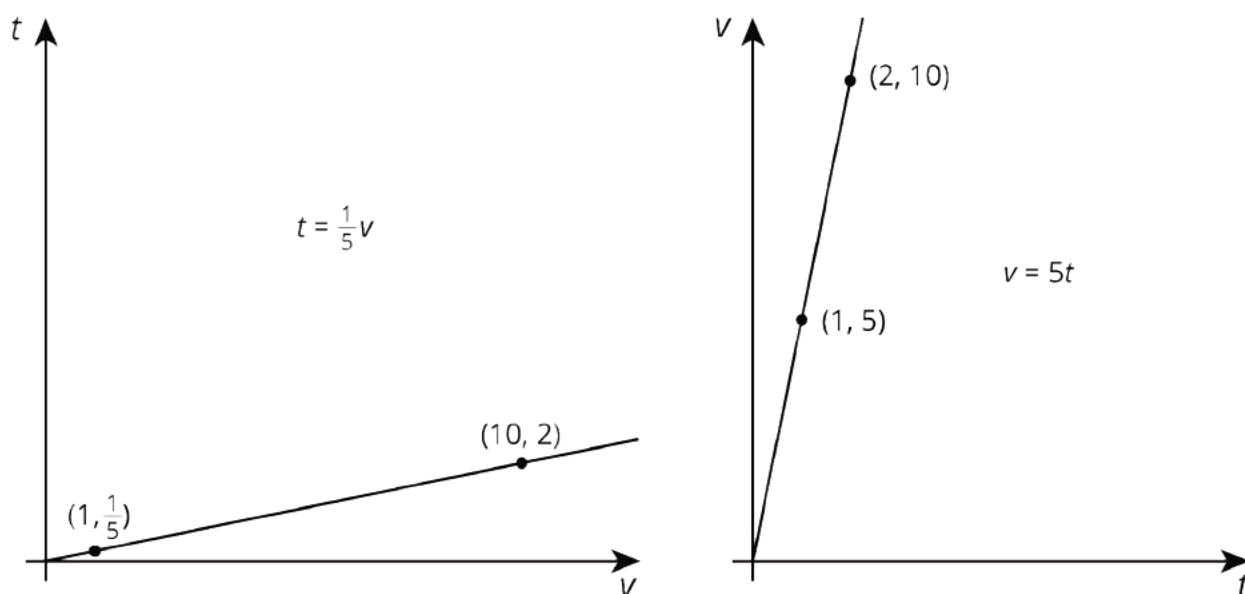
1. On the first graph, which point shows Andre's consumption and which shows Jada's consumption? Label them.
2. Draw two lines: one through the origin and Andre's point, and one through the origin and Jada's point.
3. Write an equation for each line. Use t to represent time in minutes and h to represent number of hot dogs.
 - a. Andre:
 - b. Jada:
4. For each equation, what does the constant of proportionality tell you?
5. Repeat the previous steps for the second graph.
 - a. Andre:
 - b. Jada:

Lesson 13 Summary

Imagine that a faucet is leaking at a constant rate and that every 2 minutes, 10 milliliters of water leaks from the faucet. There is a proportional relationship between the volume of water and elapsed time.

- We could say that the elapsed time is proportional to the volume of water. The corresponding constant of proportionality tells us that the faucet is leaking at a rate of $\frac{1}{5}$ of a minute per milliliter.
- We could say that the volume of water is proportional to the elapsed time. The corresponding constant of proportionality tells us that the faucet is leaking at a rate of 5 milliliters per minute.

Let's use v to represent volume in milliliters and t to represent time in minutes. Here are graphs and equations that represent both ways of thinking about this relationship:

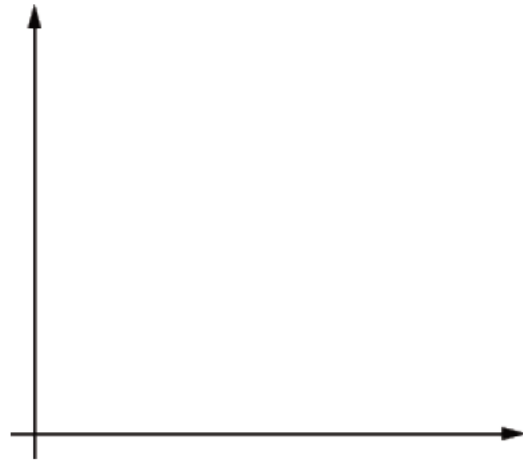


Even though the relationship between time and volume is the same, we are making a different choice in each case about which variable to view as the independent variable. The graph on the left has v as the independent variable, and the graph on the right has t as the independent variable.

Unit 2 Lesson 13 Cumulative Practice Problems

1. At the supermarket you can fill your own honey bear container. A customer buys 12 oz of honey for \$5.40.

- a. How much does honey cost per ounce?
- b. How much honey can you buy per dollar?
- c. Write two different equations that represent this situation. Use h for ounces of honey and c for cost in dollars.



- Choose one of your equations, and sketch its graph. Be sure to label the axes.
2. The point $(3, \frac{6}{5})$ lies on the graph representing a proportional relationship. Which of the following points also lie on the same graph? Select all that apply.
- A. $(1, 0.4)$
 - B. $(1.5, \frac{6}{10})$
 - C. $(\frac{6}{5}, 3)$
 - D. $(4, \frac{11}{5})$
 - E. $(15, 6)$

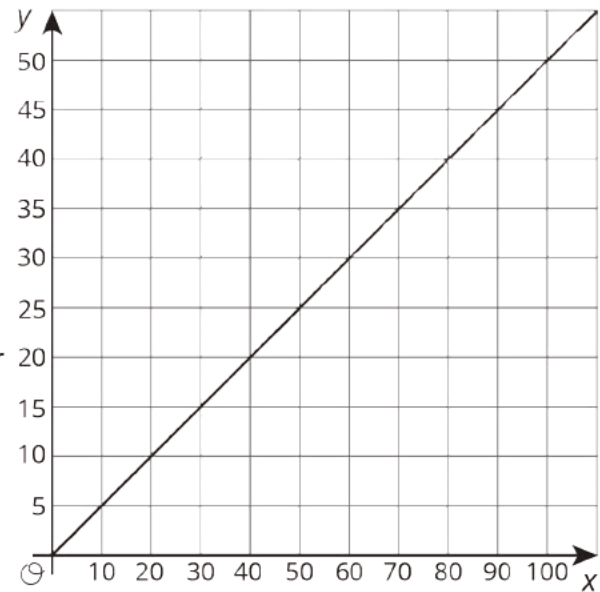
3. A trail mix recipe asks for 4 cups of raisins for every 6 cups of peanuts. There is proportional relationship between the amount of raisins, r (cups), and the amount of peanuts, p (cups), in this recipe.

a. Write the equation for the relationship that has constant of proportionality greater than 1. Graph the relationship.

b. Write the equation for the relationship that has constant of proportionality less than 1. Graph the relationship.

4. Here is a graph that represents a proportional relationship.

- Come up with a situation that could be represented by this graph.
- Label the axes with the quantities in your situation.
- Give the graph a title.
- Choose a point on the graph. What do the coordinates represent in your situation?

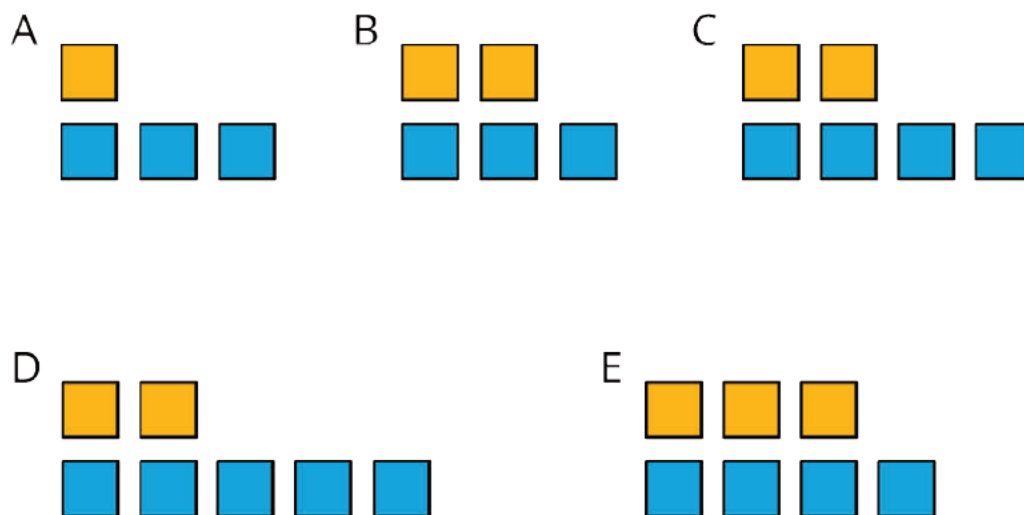


(From Unit 2, Lesson 11.)

Lesson 14: Four Representations

14.1: Which is the Bluest?

1. Which group of blocks is the bluest?



2. Order the groups of blocks from least blue to bluest.

14.2: One Scenario, Four Representations

1. Select two things from different lists. Make up a situation where there is a *proportional relationship* between quantities that involve these things.

creatures	length	time	volume
◦ starfish	◦ centimeters	◦ nanoseconds	◦ milliliters
◦ centipedes	◦ cubits	◦ minutes	◦ gallons
◦ earthworms	◦ kilometers	◦ years	◦ bushels
◦ dinosaurs	◦ parsecs	◦ millennia	◦ cubic miles
body parts	area	weight	substance
◦ legs	◦ square microns	◦ nanograms	◦ helium
◦ eyes	◦ acres	◦ ounces	◦ oobleck
◦ neurons	◦ hides	◦ debens	◦ pitch
◦ digits	◦ square light-years	◦ metric tonnes	◦ glue

2. Select two other things from the lists, and make up a situation where there is a relationship between quantities that involve these things, but the relationship is *not* proportional.

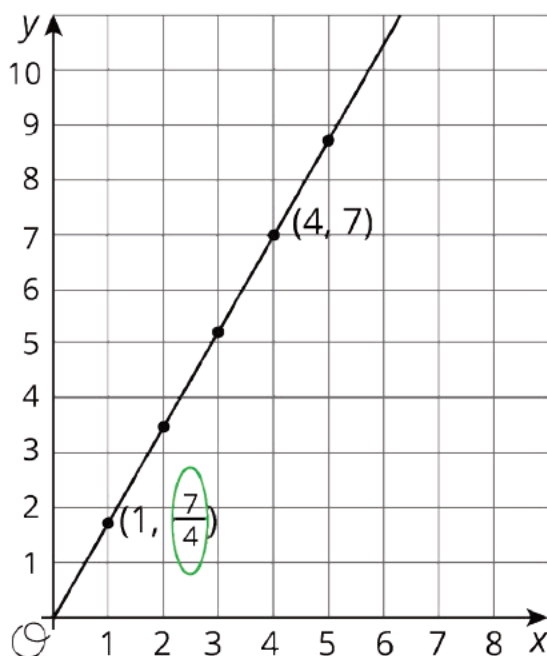
3. Your teacher will give you two copies of the “One Scenario, Four Representations” sheet. For each of your situations, describe the relationships in detail. If you get stuck, consider asking your teacher for a copy of the sample response.
- a. Write one or more sentences describing the relationship between the things you chose.
 - b. Make a table with titles in each column and at least 6 pairs of numbers relating the two things.
 - c. Graph the situation and label the axes.
 - d. Write an equation showing the relationship and explain in your own words what each number and letter in your equation means.
 - e. Explain how you know whether each relationship is proportional or not proportional. Give as many reasons as you can.

14.3: Make a Poster

Create a visual display of your two situations that includes all the information from the previous activity.

Lesson 14 Summary

The constant of proportionality for a proportional relationship can often be easily identified in a graph, a table, and an equation that represents it. Here is an example of all three representations for the same relationship. The constant of proportionality is circled:



$$y = \left(\frac{7}{4}\right)x$$

x	y
0	0
1	$\frac{7}{4}$
2	$\frac{7}{2}$
3	$\frac{21}{4}$
4	7

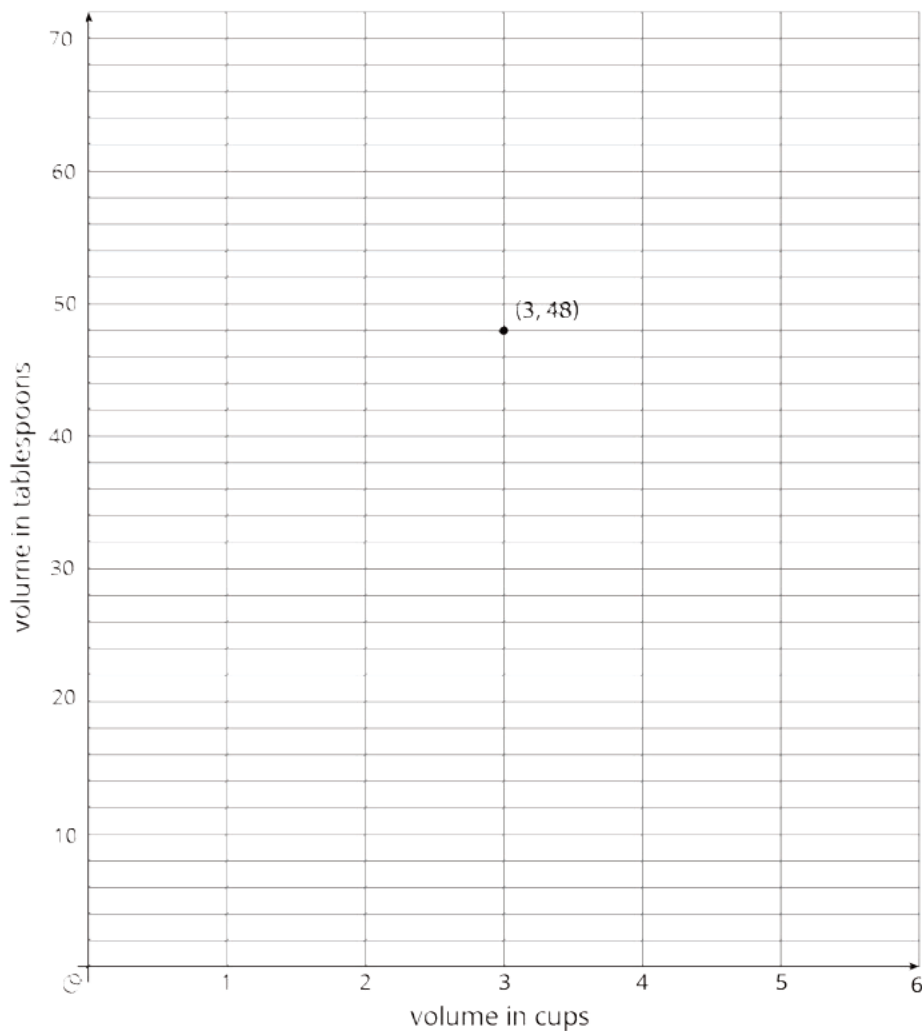
On the other hand, some relationships are not proportional. If the graph of a relationship is not a straight line through the origin, if the equation cannot be expressed in the form $y = kx$, or if the table does not have a constant of proportionality that you can multiply by any number in the first column to get the associated number in the second column, then the relationship between the quantities is not a proportional relationship.

Unit 2 Lesson 14 Cumulative Practice Problems

1. The equation $c = 2.95g$ shows how much it costs to buy gas at a gas station on a certain day. In the equation, c represents the cost in dollars, and g represents how many gallons of gas were purchased.
 - a. Write down at least four (gallons of gas, cost) pairs that fit this relationship.
 - b. Create a graph of the relationship.
 - c. What does 2.95 represent in this situation?
 - d. Jada's mom remarks, "You can get about a third of a gallon of gas for a dollar." Is she correct? How did she come up with that?

2. There is a proportional relationship between a volume measured in cups and the same volume measured in tablespoons. 3 cups is equivalent to 48 tablespoons, as shown in the graph.

- Plot and label at least two more points that represent the relationship.
- Use a straightedge to draw a line that represents this proportional relationship.
- For which value y is $(1, y)$ on the line you just drew?
- What is the constant of proportionality for this relationship?
- Write an equation representing this relationship. Use c for cups and t for tablespoons.



Lesson 15: Using Water Efficiently

15.1: Comparing Baths and Showers

Some people say that it uses more water to take a bath than a shower. Others disagree.

1. What information would you collect in order to answer the question?
2. Estimate some reasonable values for the things you suggest.

15.2: Saving Water: Bath or Shower?

1. Describe a method for comparing the water usage for a bath and a shower.
2. Find out values for the measurements needed to use the method you described. You may ask your teacher or research them yourself.
3. Under what conditions does a bath use more water? Under what conditions does a shower use more water?

15.3: Representing Water Usage

1. Continue considering the problem from the previous activity. Name two quantities that are in a proportional relationship. Explain how you know they are in a proportional relationship.
2. What are two constants of proportionality for the proportional relationship? What do they tell us about the situation?
3. On graph paper, create a graph that shows how the two quantities are related. Make sure to label the axes.
4. Write two equations that relate the quantities in your graph. Make sure to record what each variable represents.

Credits

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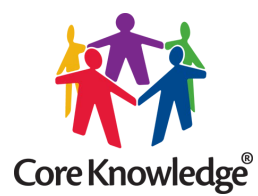
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