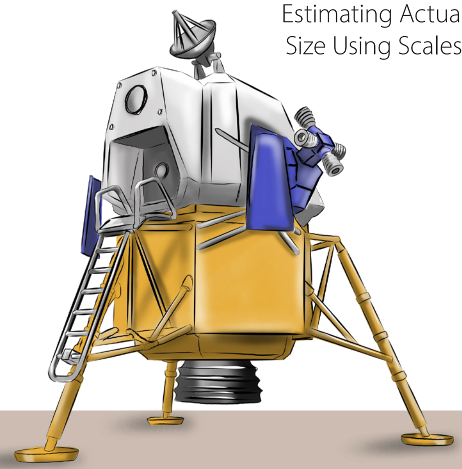
Core Knowledge® MATHEMATICS

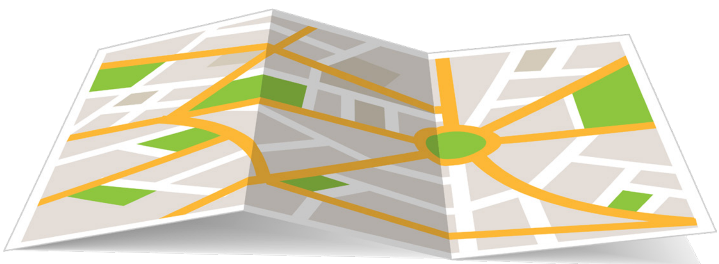
Scale Drawings

Teacher Guide

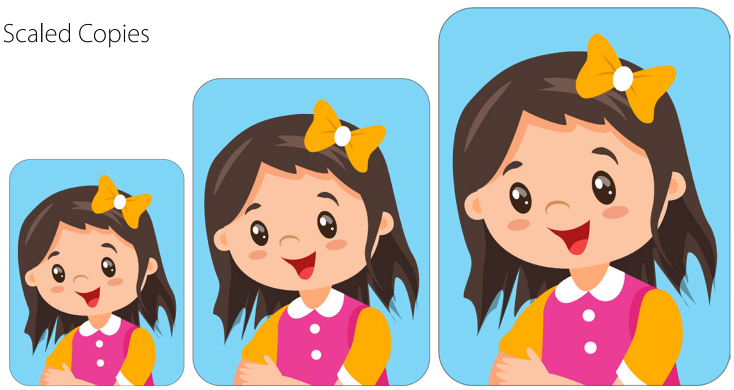
Estimating Actual Size Using Scales

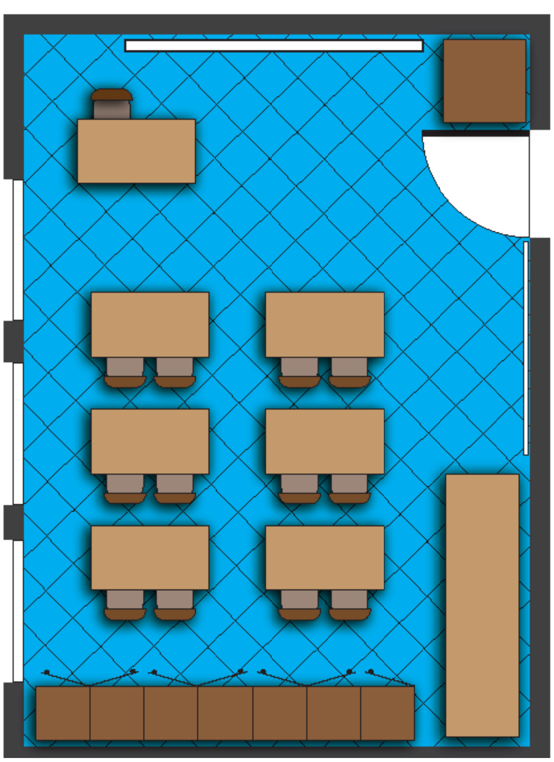


Using Maps

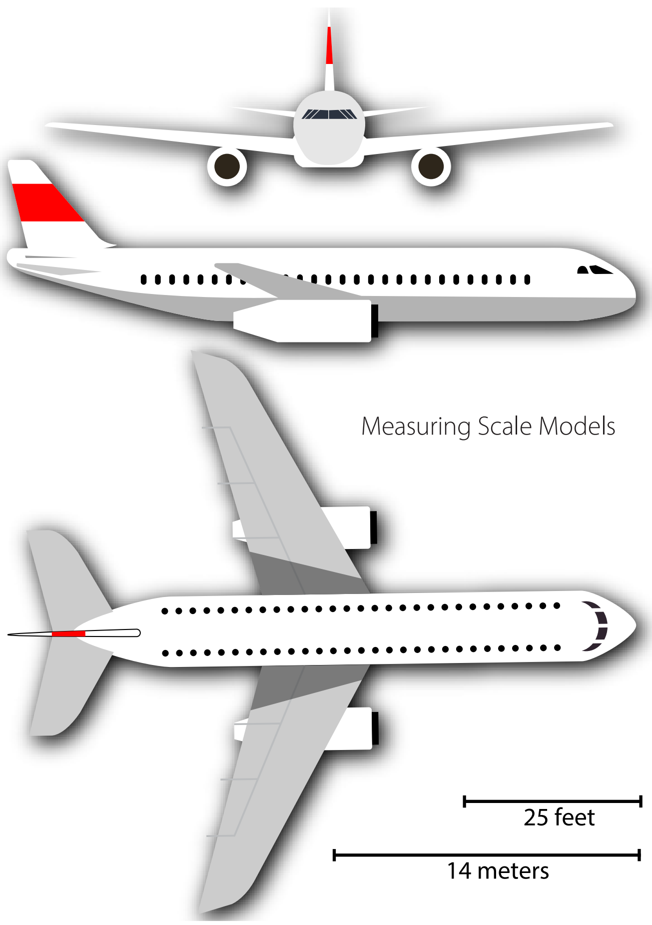


Scaled Copies





Classroom Floor Plans



Measuring Scale Models

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Scale Drawings

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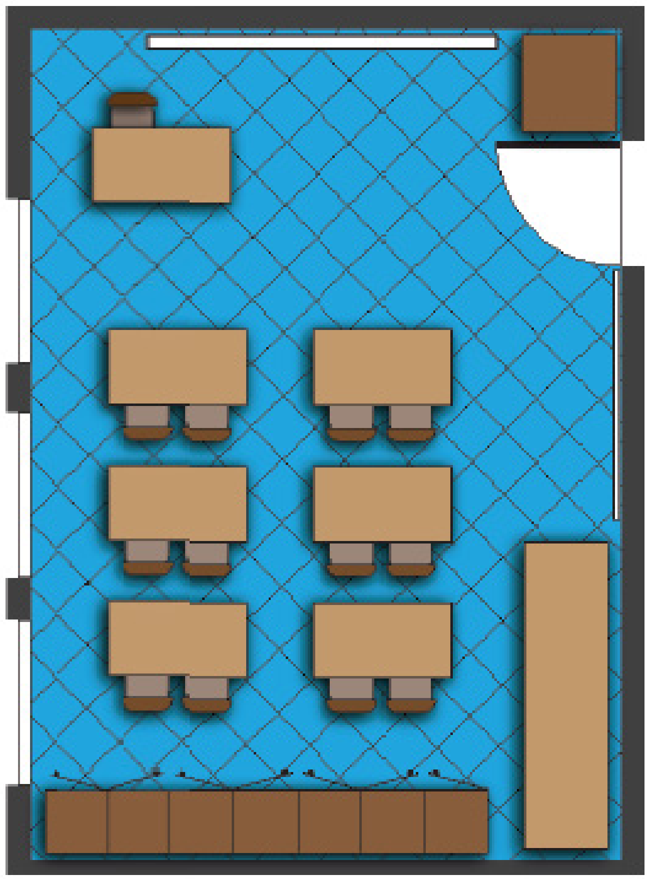


Scale Drawings

Teacher Guide

Core Knowledge Mathematics™

Scale Drawings

Unit Narrative

Work with scale drawings in grade 7 draws on earlier work with geometry and geometric measurement. Students began to learn about two- and three-dimensional shapes in kindergarten, and continued this work in grades 1 and 2, composing, decomposing, and identifying shapes. Students' work with geometric measurement began with length and continued with area. Students learned to "structure two-dimensional space," that is, to see a rectangle with whole-number side lengths as an array of unit squares, or rows or columns of unit squares. In grade 3, students distinguished between perimeter and area. They connected rectangle area with multiplication, understanding why (for whole-number side lengths) multiplying the side lengths of a rectangle yields the number of unit squares that tile the rectangle. They used area diagrams to represent instances of the distributive property. In grade 4, students applied area and perimeter formulas for rectangles to solve real-world and mathematical problems, and learned to use protractors. In grade 5, students extended the formula for the area of a rectangle to include rectangles with fractional side lengths. In grade 6, students built on their knowledge of geometry and geometric measurement to produce formulas for the areas of parallelograms and triangles, using these formulas to find surface areas of polyhedra.

In this unit, students study scaled copies of pictures and plane figures, then apply what they have learned to scale drawings, e.g., maps and floor plans. This provides geometric preparation for grade 7 work on proportional relationships as well as grade 8 work on dilations and similarity.

Students begin by looking at copies of a picture, some of which are to scale, and some of which are not. They use their own words to describe what differentiates scaled and non-scaled copies of a picture. As the unit progresses, students learn that all lengths in a scaled copy are multiplied by a scale factor and all angles stay the same. They draw scaled copies of figures. They learn that if the scale factor is greater than 1, the copy will be larger, and if the scale factor is less than 1, the copy will be smaller. They study how area changes in scaled copies of an image.

Next, students study scale drawings. They see that the principles and strategies that they used to reason about scaled copies of figures can be used with scale drawings. They interpret and draw maps and floor plans. They work with scales that involve units (e.g., "1 cm represents 10 km"), and scales that do not include units (e.g., "the scale is 1 to 100"). They learn to express scales with units as scales without units, and vice versa. They understand that actual lengths are products of a scale factor and corresponding lengths in the scale drawing, thus lengths in the drawing are the product of the actual lengths and the reciprocal of that scale factor. They study the relationship between regions and lengths in scale drawings. Throughout the unit, they discuss their mathematical ideas and respond to the ideas of others (MP3, MP6). In the culminating lesson of this unit, students make a floor plan of their classroom or some other room or space at their school. This is an opportunity for them to apply what they have learned in the unit to everyday life (MP4).

In the unit, several lesson plans suggest that each student have access to a *geometry toolkit*. Each toolkit contains tracing paper, graph paper, colored pencils, scissors, centimeter ruler, protractor (clear protractors with no holes that show radial lines are recommended), and an index card to use as a straightedge or to mark right angles. Providing students with these toolkits gives opportunities for students to develop abilities to select appropriate tools and use them strategically to solve problems (MP5). Note that even students in a digitally enhanced classroom should have access to such tools; apps and simulations should be considered additions to their toolkits, not replacements for physical tools.

Note that the study of scaled copies is limited to pairs of figures that have the same rotation and mirror orientation (i.e. that are not rotations or reflections of each other), because the unit focuses on scaling, scale factors, and scale drawings. In grade 8, students will extend their knowledge of scaled copies when they study translations, rotations, reflections, and dilations.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as representing, generalizing, and explaining. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Represent

- a scaled copy for a given scale factor (Lessons 3 and 5)
- distances using different scales (Lesson 11)
- relevant features of a classroom with a scale drawing (Lesson 13)

Generalize

- about corresponding distances and angles in scaled copies (Lesson 4)
- about scale factors greater than, less than, and equal to 1 (Lesson 5)
- about scale factors and area (Lessons 6 and 10)
- about scale factors with and without units (Lesson 12)

Explain

- how to use scale drawings to find actual distances (Lessons 7 and 11)
- how to use scale drawings to find actual distances, speed, and elapsed time (Lesson 8)
- how to use scale drawings to find actual areas (Lesson 12)

In addition, students are expected to describe features of scaled copies, justify and critique reasoning about scaled copies, and compare how different scales affect drawings. Over the course of the unit, teachers can support students' mathematical understandings by amplifying (not simplifying) language used for all of these purposes as students demonstrate and develop ideas.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students’ use of a new term in the lessons that follow where it was first introduced.

Learning Targets

Scale Drawings

Lesson 1: What are Scaled Copies?

- I can describe some characteristics of a scaled copy.
- I can tell whether or not a figure is a scaled copy of another figure.

Lesson 2: Corresponding Parts and Scale Factors

- I can describe what the scale factor has to do with a figure and its scaled copy.
- In a pair of figures, I can identify corresponding points, corresponding segments, and corresponding angles.

Lesson 3: Making Scaled Copies

- I can draw a scaled copy of a figure using a given scale factor.
- I know what operation to use on the side lengths of a figure to produce a scaled copy.

Lesson 4: Scaled Relationships

- I can use corresponding distances and corresponding angles to tell whether one figure is a scaled copy of another.
- When I see a figure and its scaled copy, I can explain what is true about corresponding angles.
- When I see a figure and its scaled copy, I can explain what is true about corresponding distances.

Lesson 5: The Size of the Scale Factor

- I can describe the effect on a scaled copy when I use a scale factor that is greater than 1, less than 1, or equal to 1.
- I can explain how the scale factor that takes Figure A to its copy Figure B is related to the scale factor that takes Figure B to Figure A.

Lesson 6: Scaling and Area

- I can describe how the area of a scaled copy is related to the area of the original figure and the scale factor that was used.

Lesson 7: Scale Drawings

- I can explain what a scale drawing is, and I can explain what its scale means.
- I can use actual distances and a scale to find scaled distances.
- I can use a scale drawing and its scale to find actual distances.

Lesson 8: Scale Drawings and Maps

- I can use a map and its scale to solve problems about traveling.

Lesson 9: Creating Scale Drawings

- I can determine the scale of a scale drawing when I know lengths on the drawing and corresponding actual lengths.
- I know how different scales affect the lengths in the scale drawing.
- When I know the actual measurements, I can create a scale drawing at a given scale.

Lesson 10: Changing Scales in Scale Drawings

- Given a scale drawing, I can create another scale drawing that shows the same thing at a different scale.
- I can use a scale drawing to find actual areas.

Lesson 11: Scales without Units

- I can explain the meaning of scales expressed without units.
- I can use scales without units to find scaled distances or actual distances.

Lesson 12: Units in Scale Drawings

- I can tell whether two scales are equivalent.
- I can write scales with units as scales without units.

Lesson 13: Draw It to Scale

- I can create a scale drawing of my classroom.
- When given requirements on drawing size, I can choose an appropriate scale to represent an actual object.

lesson	new terminology	
	receptive	**productive**
7.1.1	**scaled copy** original polygon	
7.1.2	**corresponding** **scale factor** figure segment	
7.1.4	quadrilateral measurement distance	**corresponding** **scale factor** original
7.1.5	**reciprocal**	
7.1.6	**area** one-dimensional two-dimensional	squared
7.1.7	**scale drawing** **scale** represent actual three-dimensional	**scaled copy**
7.1.8	estimate travel constant speed	**scale**
7.1.9	floorplan	
7.1.10	appropriate dimension	actual represent
7.1.11	scale without units ___ to ___	**scale drawing**
7.1.12	equivalent scales	___ to ___

Required Materials

Blank paper
Copies of blackline master
Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Graph paper
Measuring tools
Metric and customary unit conversion charts
Pattern blocks
Pre-printed slips, cut from copies of the blackline master
Rulers

Section: Scaled Copies

Lesson 1: What are Scaled Copies?

Goals

- Describe (orally) characteristics of scaled and non-scaled copies.
- Identify scaled copies of a figure and justify (orally and in writing) that the copy is a scaled copy.

Learning Targets

- I can describe some characteristics of a scaled copy.
- I can tell whether or not a figure is a scaled copy of another figure.

Lesson Narrative

This lesson introduces students to the idea of a **scaled copy** of a picture or a figure. Students learn to distinguish scaled copies from those that are not—first informally, and later, with increasing precision. They may start by saying that scaled copies have the same shape as the original figure, or that they do not appear to be distorted in any way, though they may have a different size. Next, they notice that the lengths of segments in a scaled copy vary from the lengths in the original figure in a uniform way. For instance, if a segment in a scaled copy is half the length of its counterpart in the original, then all other segments in the copy are also half the length of their original counterparts. Students work toward articulating the characteristics of scaled copies quantitatively (e.g., "all the segments are twice as long," "all the lengths have shrunk by one third," or "all the segments are one-fourth the size of the segments in the original"), articulating the relationships carefully (MP6) along the way.

The lesson is designed to be accessible to all students regardless of prior knowledge, and to encourage students to make sense of problems and persevere in solving them (MP1) from the very beginning of the course.

Alignments

Addressing

- 7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR8: Discussion Supports

- Take Turns
- Think Pair Share

Required Materials

Pre-printed slips, cut from copies of the blackline master

Required Preparation

You will need the Pairs of Scaled Polygons blackline master for this lesson. Print and cut slips A–J for the Pairs of Scaled Polygons activity. Prepare 1 copy for every 2 students. If possible, copy each complete set on a different color of paper, so that a stray slip can quickly be put back.

> **Student Learning Goals**
>
> Let's explore scaled copies.

1.1 Printing Portraits

Warm Up: 10 minutes (there is a digital version of this activity)
This opening task introduces the term **scaled copy**. It prompts students to observe several copies of a picture, visually distinguish scaled and unscaled copies, and articulate the differences in their own words. Besides allowing students to have a mathematical conversation about properties of figures, it provides an accessible entry into the concept and gives an opportunity to hear the language and ideas students associate with scaled figures.

Students are likely to have some intuition about the term "to scale," either from previous work in grade 6 (e.g., scaling a recipe, or scaling a quantity up or down on a double number line) or from outside the classroom. This intuition can help them identify scaled copies.

Expect them to use adjectives such as "stretched," "squished," "skewed," "reduced," etc., in imprecise ways. This is fine, as students' intuitive definition of scaled copies will be refined over the course of the lesson. As students discuss, note the range of descriptions used. Monitor for students whose descriptions are particularly supportive of the idea that lengths in a scaled copy are found by multiplying the original lengths by the same value. Invite them to share their responses later.

Addressing

- 7.G.A.1

Instructional Routines

- Think Pair Share

Launch

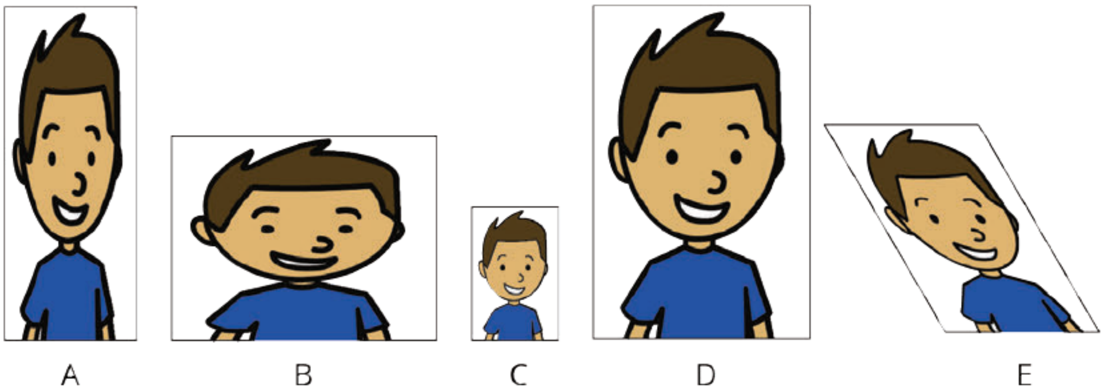
Arrange students in groups of 2. Give students 2–3 minutes of quiet think time and a minute to share their response with their partner.

If using the digital activity, have students work in groups of 2–3 to complete the activity. They should have quiet time in addition to share time, while solving the problem and developing language to describe scaling.

Student Task Statement

Here is a portrait of a student.

1. Look at Portraits A–E. How is each one the same as or different from the original portrait of the student?



2. Some of the Portraits A–E are **scaled copies** of the original portrait. Which ones do you think are scaled copies? Explain your reasoning.

3. What do you think "scaled copy" means?

Student Response

1. Answers vary. Sample response:

- Similarities: Pictures A–E are all based on the same original portrait. They all show the same boy wearing a blue shirt and brown hair. They all have the same white background.

- Differences: They all have different sizes; some have different shapes. Pictures A, B, and E have been stretched or somehow distorted. C and D are not stretched or distorted but are each of a different size than the original.

1. C and D are scaled copies. Sample explanation:

- A, B, and E are not scaled copies because they have changed in shape compared to the original portrait. Portrait A is stretched vertically, so the vertical side is now much longer than the horizontal side. B is stretched out sideways, so the horizontal sides are now longer than the vertical. E seems to have its upper left and lower right corners stretched out in opposite directions. The portrait is no longer a rectangle.

- C is a smaller copy and D is a larger copy of the original, but their shapes remain the same.

1. Answers vary. Sample definitions:

- A scaled copy is a copy of a picture that changes in size but does not change in shape.
- A scaled copy is a duplicate of a picture with no parts of it distorted, though it could be larger, smaller, or the same size.
- A scaled copy is a copy of a picture that has been enlarged or reduced in size but nothing else changes.

Activity Synthesis

Select a few students to share their observations. Record and display students' explanations for the second question. Consider organizing the observations in terms of how certain pictures are or are not distorted. For example, students may say that C and D are scaled copies because each is a larger or smaller version of the picture, but the face (or the sleeve, or the outline of the picture) has not changed in shape. They may say that A, B, and E are not scaled copies because something other than size has changed. If not already mentioned in the discussion, guide students in seeing features of C and D that distinguish them from A, B, and E.

Invite a couple of students to share their working definition of scaled copies. Some of the students' descriptions may not be completely accurate. That is appropriate for this lesson, as the goal is to build on and refine this language over the course of the next few lessons until students have a more precise notion of what it means for a picture or figure to be a scaled copy.

1.2 Scaling F

10 minutes (there is a digital version of this activity)

This task enables students to describe more precisely the characteristics of scaled copies and to refine the meaning of the term. Students observe copies of a line drawing on a grid and notice how the lengths of line segments and the angles formed by them compare to those in the original drawing.

Students engage in MP7 in multiple ways in this task. Identifying distinguishing features of the scaled copies means finding similarities and differences in the shapes. In addition, the fact that corresponding parts increase by the *same* scale factor is a vital structural property of scaled copies.

For the first question, expect students to explain their choices of scaled copies in intuitive, qualitative terms. For the second question, students should begin to distinguish scaled and unscaled copies in more specific and quantifiable ways. If it does not occur to students to look at lengths of segments, suggest they do so.

As students work, monitor for students who notice the following aspects of the figures. Students are not expected to use these mathematical terms at this point, however.

- The original drawing of the letter F and its scaled copies have equivalent width-to-height ratios.
- We can use a scale factor (or a multiplier) to compare the lengths of different figures and see if they are scaled copies of the original.
- The original figure and scaled copies have corresponding angles that have the same measure.

Addressing

- 7.G.A.1

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- Think Pair Share

Launch

Keep students in the same groups. Give them 3–4 minutes of quiet work time, and then 1–2 minutes to share their responses with their partner. Tell students that how they decide whether each of the seven drawings is a scaled copy may be very different than how their partner decides. Encourage students to listen carefully to each other's approach and to be prepared to share their strategies. Use gestures to elicit from students the words "horizontal" and "vertical" and ask groups to agree internally on common terms to refer to the parts of the F (e.g., "horizontal stems").

Support for Students with Disabilities

Engagement: Internalize Self Regulation. Display sentence frames to support small group discussion. For example, "That could/couldn't be true because...," "We can agree that...," and "Is there another way to say/do...?"
Supports accessibility for: Social-emotional skills; Organization; Language

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Support for English Language Learners

Speaking: Math Language Routine 1 Stronger and Clearer Each Time. This is the first time Math Language Routine 1 is suggested as a support in this course. In this routine, students are given a thought-provoking question or prompt and asked to create a first draft response in writing. Students meet with 2–3 partners to share and refine their response through conversation. While meeting, listeners ask questions such as, “What did you mean by . . .?” and “Can you say that another way?” Finally, students write a second draft of their response reflecting ideas from partners, and improvements on their initial ideas. The purpose of this routine is to provide a structured and interactive opportunity for students to revise and refine their ideas through verbal and written means.
Design Principle(s): Optimize output (for explanation)

How It Happens:

1. Use this routine to provide students a structured opportunity to refine their explanations for the first question: “Identify all the drawings that are scaled copies of the original letter F drawing. Explain how you know.” Allow students 2–3 minutes to individually create first draft responses in writing.

2. Invite students to meet with 2–3 other partners for feedback.

   Instruct the speaker to begin by sharing their ideas without looking at their written draft, if possible. Provide the listener with these prompts for feedback that will help their partner strengthen their ideas and clarify their language: “What do you mean when you say....?”, “Can you describe that another way?”, “How do you know that _ is a scaled copy?”, “Could you justify that differently?” Be sure to have the partners switch roles. Allow 1–2 minutes to discuss.

3. Signal for students to move on to their next partner and repeat this structured meeting.

4. Close the partner conversations and invite students to revise and refine their writing in a second draft.

   Provide these sentence frames to help students organize their thoughts in a clear, precise way: “Drawing _ is a scaled copy of the original, and I know this because....”, “When I look at the lengths, I notice that....”, and “When I look at the angles, I notice that....”

   Here is an example of a second draft:

   “Drawing 7 is a scaled copy of the original, and I know this because it is enlarged evenly in both the horizontal and vertical directions. It does not seem lopsided or stretched differently in one direction. When I look at the length of the top segment, it is 3 times as large as the original one, and the other segments do the same thing. Also, when I look at the angles, I notice that they are all right angles in both the original and scaled copy.”

5. If time allows, have students compare their first and second drafts. If not, have the students move on by working on the following problems.

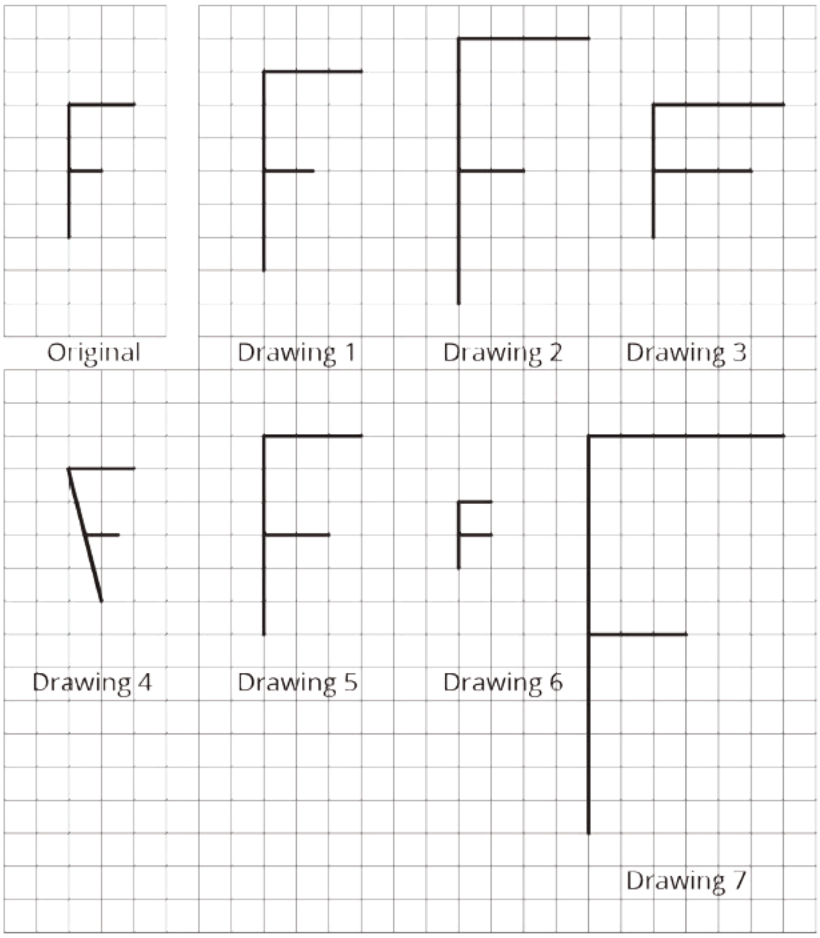
Anticipated Misconceptions

Students may make decisions by “eyeballing” rather than observing side lengths and angles. Encourage them to look for quantifiable evidence and notice lengths and angles.

Some may think vertices must land at intersections of grid lines (e.g., they may say Drawing 4 is not a scaled copy because the endpoints of the shorter horizontal segment are not on grid crossings). Address this during the whole-class discussion, after students have a chance to share their observations about segment lengths.

Student Task Statement

Here is an original drawing of the letter F and some other drawings.



1. Identify **all** the drawings that are scaled copies of the original letter F. Explain how you know.

2. Examine all the scaled copies more closely, specifically the lengths of each part of the letter F. How do they compare to the original? What do you notice?

3. On the grid, draw a different scaled copy of the original letter F.

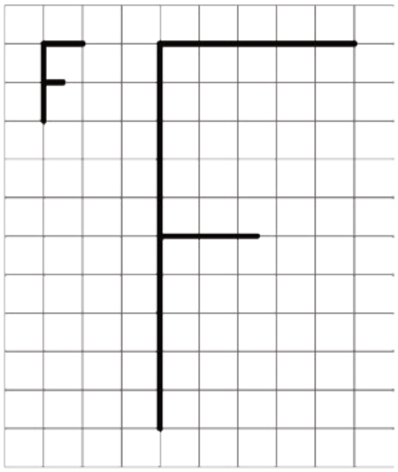
Student Response

1. Drawings 1, 2, and 7 are scaled copies of the original drawing. Explanations vary. Sample explanation: I know because they are not stretched differently in one direction. They are enlarged evenly in both vertical and horizontal directions.

2. Answers vary. Sample responses:

    - In the scaled copies, every segment is the same number of times as long as the matching segment in the original drawing.
    - In the scaled copies, all segments keep the same relationships as in the original. The original drawing of F is 4 units tall. Its top horizontal segment is 2 units wide and the shorter horizontal segment is 1 unit. In Drawing 1, the F is 6 units tall and 3 units wide; in Drawing 2, it is 8 units tall and 4 units wide, and in Drawing 7, it is 8 units tall and 4 units wide. In each scaled copy, the width is half of the height, just as in the original drawing of F, and the shorter horizontal segment is half of the longer one.

3. Drawings vary. Sample response:



Activity Synthesis

Display the seven copies of the letter F for all to see. For each copy, ask students to indicate whether they think each one is a scaled copy of the original F. Record and display the results for all to see. For contested drawings, ask 1–2 students to briefly say why they ruled these out.

Discuss the identified scaled and unscaled copies.

- What features do the scaled copies have in common? (Be sure to invite students who were thinking along the lines of scale factors and angle measures to share.)
- How do the other copies fail to show these features? (Sometimes lengths of sides in the copy use different multipliers for different sides. Sometimes the angles in the copy do not match the angles in the original.)

If there is a misconception that scaled copies must have vertices on intersections of grid lines, use Drawing 1 (or a relevant drawing by a student) to discuss how that is not the case.

Some students may not be familiar with words such as “twice,” “double,” or “triple.” Clarify the meanings by saying “two times as long” or “three times as long.”

1.3 Pairs of Scaled Polygons

15 minutes (there is a digital version of this activity)

In this activity, students hone their understanding of scaled copies by working with more complex figures. Students work with a partner to match pairs of polygons that are scaled copies. The polygons appear comparable to one another, so students need to look very closely at all side lengths of the polygons to tell if they are scaled copies.

As students confer with one another, notice how they go about looking for a match. Monitor for students who use precise language (MP6) to articulate their reasoning (e.g., “The top side of A is half the length of the top side of G, but the vertical sides of A are a third of the lengths of those in G.”).

You will need the Pairs of Scaled Polygons blackline master for this activity.

Addressing

- 7.G.A.1

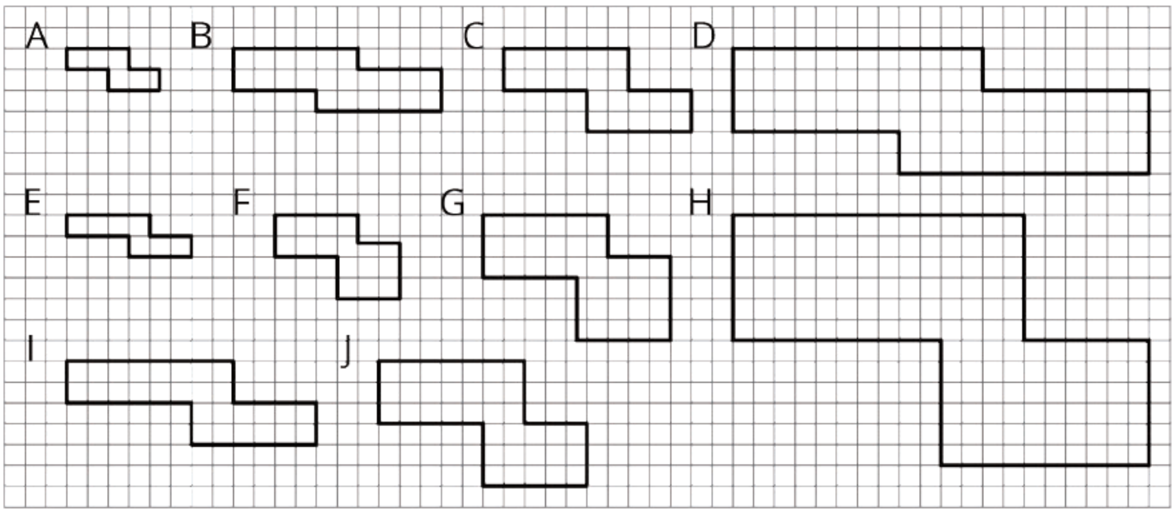
Instructional Routines

- MLR8: Discussion Supports
- Take Turns

Launch

Demonstrate how to set up and do the matching activity. Choose a student to be your partner. Mix up the cards and place them face-up. Tell them that each polygon has one and only one match (i.e., for each polygon, there is one and only one scaled copy of the polygon). Select two cards and then explain to your partner why you think the cards do or do not match. Demonstrate productive ways to agree or disagree (e.g., by explaining your mathematical thinking, asking clarifying questions, etc.).

Arrange students in groups of 2. Give each group a set of 10 slips cut from the blackline master. Encourage students to refer to a running list of statements and diagrams to refine their language and explanations of how they know one figure is a scaled copy of the other.



Support for Students with Disabilities

Representation: Internalize Comprehension. Provide a range of examples and counterexamples. During the demonstration of how to set up and do the matching activity, select two cards that do not match, and invite students to come up with a shared justification.
Supports accessibility for: Conceptual processing

Support for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support small-group discussion. As students take turns finding a match of two polygons that are scaled copies of one another and explaining their reasoning to their partner, display the following sentence frames for all to see: "____ matches ____ because . . ." and "I noticed ___ , so I matched . . . ." Encourage students to challenge each other when they disagree with the sentence frames "I agree because . . .", and "I disagree because . . . ." This will help students clarify their reasoning about scaled copies of polygons.
Design Principle(s): Support sense-making; Optimize output (for explanation)

Anticipated Misconceptions

Some students may think a figure has more than one match. Remind them that there is only one scaled copy for each polygon and ask them to recheck all the side lengths.

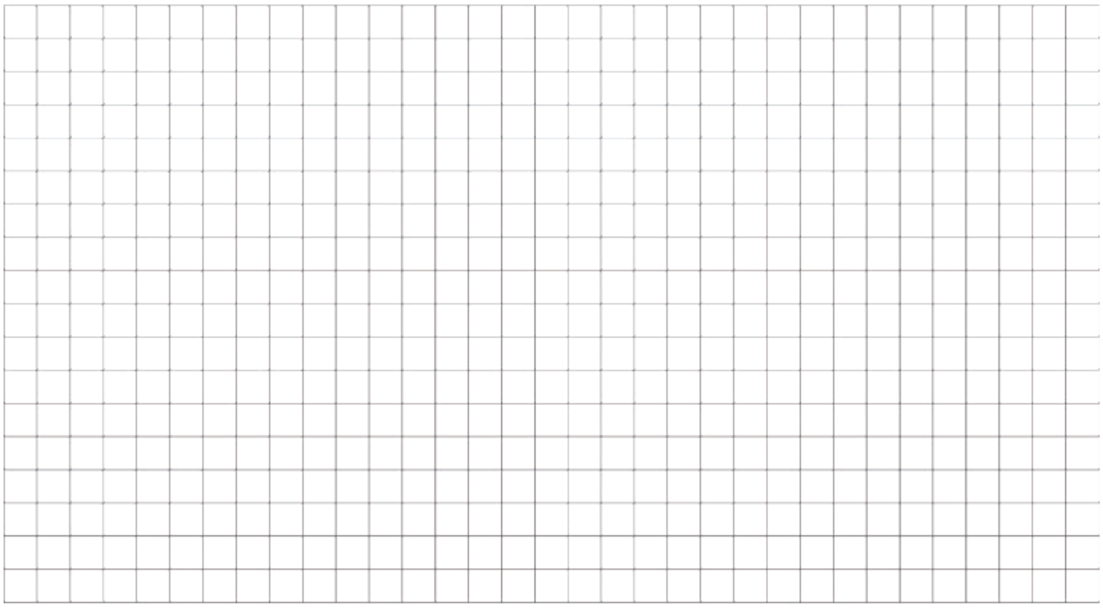
Some students may think that vertices must land at intersections of grid lines and conclude that, e.g., G cannot be a copy of F because not all vertices on F are on such intersections. Ask them to consider how a 1-unit-long segment would change if scaled to be half its original size. Where must one or both of its vertices land?

Student Task Statement

Your teacher will give you a set of cards that have polygons drawn on a grid. Mix up the cards and place them all face up.

1. Take turns with your partner to match a pair of polygons that are scaled copies of one another.
   a. For each match you find, explain to your partner how you know it's a match.
   b. For each match your partner finds, listen carefully to their explanation, and if you disagree, explain your thinking.
2. When you agree on all of the matches, check your answers with the answer key. If there are any errors, discuss why and revise your matches.

3. Select one pair of polygons to examine further. Draw both polygons on the grid. Explain or show how you know that one polygon is a scaled copy of the other.



Student Response

1. The following polygons are scaled versions of one another:
    - A and C
    - B and D
    - E and I
    - F and G
    - H and J
2. No answer needed.
3. Answers vary. Sample explanation for A and C: All the side lengths in C are twice as long as the lengths of the matching sides in A.

Are You Ready for More?

Is it possible to draw a polygon that is a scaled copy of both Polygon A and Polygon B? Either draw such a polygon, or explain how you know this is impossible.

Student Response

It's impossible to draw a polygon that is a scaled copy of both Polygon A and Polygon B. Sample explanations:

- If I draw a polygon that is a scaled copy of A, all the side lengths would be the same number of times larger or smaller than A, but they won't be the same number of times larger or smaller than B.

- A and B are not scaled copies of each other, so if I draw a scaled copy of one, it will not be a scaled copy of the other.

Activity Synthesis

The purpose of this discussion is to draw out concrete methods for deciding whether or not two polygons are scaled copies of one another, and in particular, to understand that just eyeballing to see whether they look roughly the same is not enough to determine that they are scaled copies.

Display the image of all the polygons. Ask students to share their pairings and guide a discussion about how students went about finding the scaled copies. Ask questions such as:

- When you look at another polygon, what exactly did you check or look for? (General shape, side lengths)
- How many sides did you compare before you decided that the polygon was or was not a scaled copy? (Two sides can be enough to tell that polygons are not scaled copies; all sides are needed to make sure a polygon is a scaled copy.)
- Did anyone check the angles of the polygons? Why or why not? (No; the sides of the polygons all follow grid lines.)

If students do not agree about some pairings after the discussion, ask the groups to explain their case and discuss which of the pairings is correct. Highlight the use of quantitative descriptors such as "half as long" or "three times as long" in the discussion. Ensure that students see that when a figure is a scaled copy of another, all of its segments are the same number of times as long as the corresponding segments in the other.

Lesson Synthesis

In this lesson, we encountered copies of a figure that are both scaled and not scaled. We saw different versions of a portrait of a student and of a letter F, as well as a variety of polygons that had some things in common.
In each case, we decided that some were scaled copies of one another and some were not. Consider asking students:

- What is a **scaled copy**?
- What are some characteristics of scaled copies? How are they different from figures that are not scaled copies?
- What specific information did you look for when determining if something was a scaled copy of an original?

While initial answers need not be particularly precise at this stage of the unit (for example, "scaled copies look the same but are a different size"), guide the discussion toward making careful statements that one could test. The lengths of segments in a scaled copy are related to the lengths in the original figure in a consistent way. For instance, if a segment in a scaled copy is half the length of its counterpart in the original, then all other segments in the copy are also half the length of their original counterparts. We might say, “All the segments are twice as long,” or “All the segments are one-third the size of the segments in the original.”

1.4 Scaling L

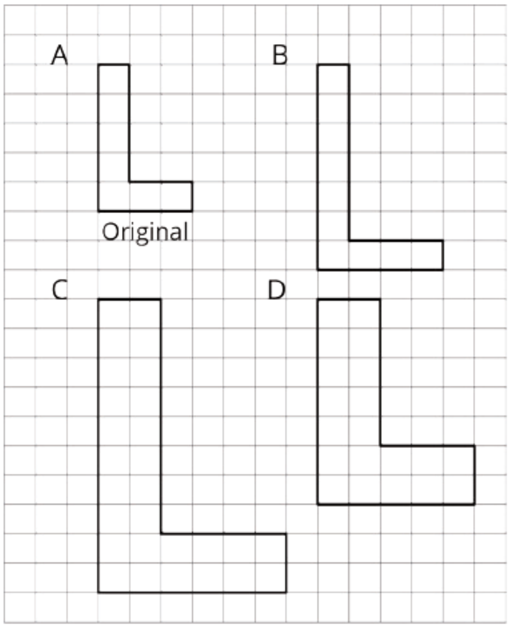
Cool Down: 5 minutes

Addressing

- 7.G.A.1

Student Task Statement

Are any of the figures B, C, or D scaled copies of figure A? Explain how you know.



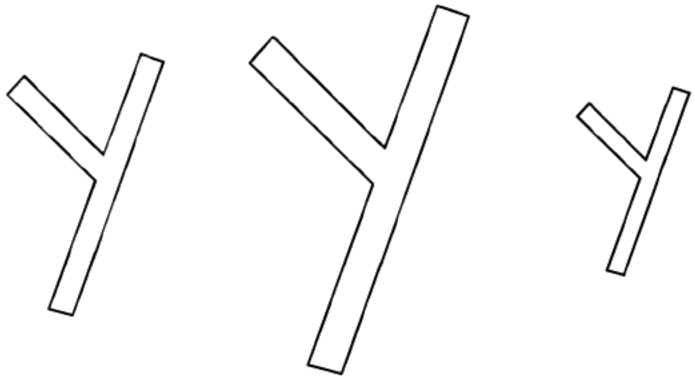
Student Response

Only figure C is a scaled copy of figure A. Sample explanation: In figure C, the length of each segment of the letter L is twice the length of the matching segment in A. In B, none of the segments are double the length. In figure D, some segments are double in length and some are not. So the block letters in B and D are not enlarged evenly.

Student Lesson Summary

What is a **scaled copy** of a figure? Let’s look at some examples.

The second and third drawings are both scaled copies of the original Y.



Original

However, here, the second and third drawings are *not* scaled copies of the original W.

Original

The second drawing is spread out (wider and shorter). The third drawing is squished in (narrower, but the same height).

We will learn more about what it means for one figure to be a scaled copy of another in upcoming lessons.

Glossary

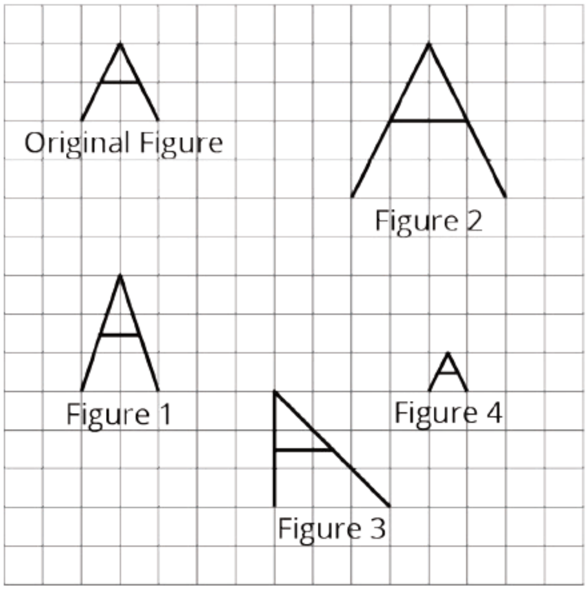
- scaled copy

Lesson 1 Practice Problems

Problem 1

Statement

Here is a figure that looks like the letter A, along with several other figures. Which figures are scaled copies of the original A? Explain how you know.



Solution

Figures 2 and 4 are scaled copies. Sample explanations:

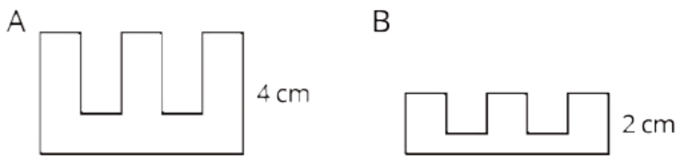
- The original A fits inside a square. The horizontal segment is halfway up the height of the square. The tip of the A is at the midpoint of the horizontal side of the square.
- Figure 1 inside a rectangle, not a square, so it is not a scaled copy. Figure 3 fits inside a square but the shape is different than the original letter A, since one of the legs of the A in Figure 3 is now vertical, so it also is not a scaled copy.
- Figure 2 is twice as high and twice as wide as the original A, and Figure 4 is half as tall and as wide, but in both figures the locations of the horizontal segment and the tip of the letter A still match the original.

Problem 2

Statement

Tyler says that Figure B is a scaled copy of Figure A because all of the peaks are half as tall.

Do you agree with Tyler? Explain your reasoning.



Solution

No. For the smaller figure to be a scaled copy, the figure would have to be half as wide as well.

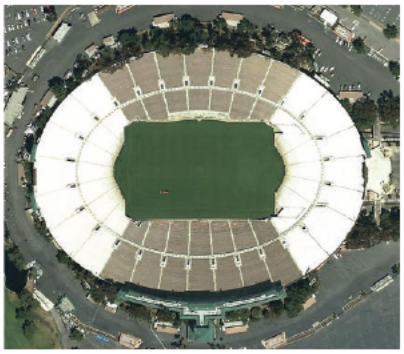
Problem 3

Statement

Here is a picture of the Rose Bowl Stadium in Pasadena, CA.

Here are some copies of the picture. Select **all** the pictures that are scaled copies of the original picture.

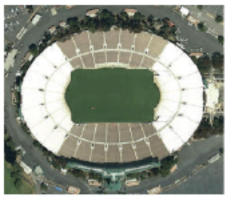
A



B

C

D



Solution

["A", "D"]

Problem 4

Statement

Complete each equation with a number that makes it true.

a. $5 \cdot$ _______ $= 15$

b. $4 \cdot$ _______ $= 32$

c. $6 \cdot$ ______ $= 9$

d. $12 \cdot$ ______ $= 3$

Solution

a. 3

b. 8

c. 1.5, $\frac{3}{2}$, or equivalent

d. 0.25, $\frac{1}{4}$, or equivalent

Lesson 2: Corresponding Parts and Scale Factors

Goals

- Comprehend the phrase “scale factor” and explain (orally) how it relates corresponding lengths of a figure and its scaled copy.
- Explain (orally) what it means to say one part in a figure “corresponds” to a part in another figure.
- Identify and describe (orally and in writing) corresponding points, corresponding segments, or corresponding angles in a pair of figures.

Learning Targets

- I can describe what the scale factor has to do with a figure and its scaled copy.
- In a pair of figures, I can identify corresponding points, corresponding segments, and corresponding angles.

Lesson Narrative

This lesson develops the vocabulary for talking about scaling and scaled copies more precisely (MP6), and identifying the structures in common between two figures (MP7).

Specifically, students learn to use the term **corresponding** to refer to a pair of points, segments, or angles in two figures that are scaled copies. Students also begin to describe the numerical relationship between the corresponding lengths in two figures using a **scale factor**. They see that when two figures are scaled copies of one another, the same scale factor relates their corresponding lengths. They practice identifying scale factors.

A look at the angles of scaled copies also begins here. Students use tracing paper to trace and compare angles in an original figure and its copies. They observe that in scaled copies the measures of corresponding angles are equal.

Alignments

Building On

- 5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
- 5.NF.B.4: Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

Addressing

- 7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Building Towards

- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

Instructional Routines

- MLR8: Discussion Supports
- Notice and Wonder
- Number Talk
- Think Pair Share

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Required Preparation

Prepare to display the images of the railroad crossing sign for the Corresponding Parts activity. Make sure students have access to their geometry toolkits, especially tracing paper and graph paper.

> **Student Learning Goals**
>
> Let’s describe features of scaled copies.

2.1 Number Talk: Multiplying by a Unit Fraction

Warm Up: 5 minutes
This number talk allows students to review multiplication strategies, refreshing the idea that multiplying by a unit fraction is the same as dividing by its whole number reciprocal. It encourages students to use the structure of base ten numbers and the properties of operations to find the product of two whole numbers (MP7). For example, a student might find $72 \cdot \frac{1}{9}$ (or $72 \div 9$) and then shift the decimal one place to the right in order to evaluate $(7.2) \cdot \frac{1}{9}$. Each problem was chosen to

elicit different approaches, so as students share theirs, ask how the factors in each problem impacted their strategies.

Before students begin, consider establishing a small, discreet hand signal (such as a thumbs-up) students can display to indicate they have an answer that they can support by reasoning. Discreet signaling is a quick way for teachers to gather feedback about timing. It also keeps students from being distracted or rushed by raised hands around the class.

Building On

- 5.NBT.B.7
- 5.NF.B.4

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students up to 1 minute of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Follow with a brief whole-class discussion.

Support for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organization

Student Task Statement

Find each product mentally.

$\frac{1}{4} \cdot 32$

$(7.2) \cdot \frac{1}{9}$

$\frac{1}{4} \cdot (5.6)$

Student Response

- $\frac{1}{4} \cdot 32 = 8$. Possible strategy: $32 \div 4 = 8$
- $(7.2) \cdot \frac{1}{9} = 0.8$. Possible strategy: $72 \div 9 = 8$ so $(7.2) \div 9 = 0.8$

- $\frac{1}{4} \cdot (5.6) = 1.4$. Possible strategy: $(5.6) \div 4 = 1.4$

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their explanations for all to see. If students express strategies in terms of division, ask if that strategy would work for any multiplication problem involving fractions. Highlight that these problems only involve unit fractions and division by the denominator is a strategy that works when multiplying by a unit fraction.

To involve more students in the conversation, consider asking:

- Who can restate ___'s reasoning in a different way?
- Did anyone solve the problem the same way but would explain it differently?
- Did anyone solve the problem in a different way?
- Does anyone want to add on to _____'s strategy?
- Do you agree or disagree? Why?

Support for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimize output (for explanation)

2.2 Corresponding Parts

15 minutes (there is a digital version of this activity)
This activity introduces important language students will apply to describe scaled copies. In particular, it introduces the important idea of corresponding parts. Students have previously analyzed corresponding sides in figures. Here they will begin to examine angles explicitly as well, understanding that corresponding angles in a figure and its scaled copy have the same measure.

Addressing

- 7.G.A.1

Instructional Routines

- MLR8: Discussion Supports
- Notice and Wonder

Launch

Tell students that in this lesson, they will look more closely at copies of figures and describe specific parts in them.

Display the designs (the three images in the activity statement) and the following descriptions for all to see. Ask students what they notice and what they wonder. After discussion, explain that the original design and its two copies have parts that correspond to one another. Point out some of their corresponding parts:

- The X-pattern going across each figure
- The curved outline of each figure
- The points K in the original sign, A in Copy 1, and U in Copy 2

Arrange students in groups of 2 and provide access to their geometry toolkits (especially tracing paper). Give students 2–3 minutes to complete the first two questions and another 2 minutes to discuss their responses with their partner. Ask students to pause their work for a quick class discussion afterwards.

Have a few students name a set of corresponding points, segments, and angles.

Then, ask students to indicate whether they think either copy is a scaled copy. Invite a couple of students to share their reasoning. When the class reaches an agreement that Copy 1 is a scaled copy and Copy 2 is not, ask students to complete the remaining questions individually and to use tracing paper as a tool.

Consider demonstrating to the class how to use tracing paper to compare angles. Tell or show students that the line segments forming an angle could be extended for easier tracing and comparison.

For classrooms using the digital version of the activity, the applet has a moveable angle tool to compare the angles in the copies with the angles in the original.

Support for Students with Disabilities

Action and Expression: Develop Expression and Communication. Maintain a display of important terms and vocabulary. During the launch take time to review the following terms from previous lessons that students will need to access for this activity: corresponding points, corresponding line segments, and corresponding angles.
Supports accessibility for: Memory; Language

Support for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to amplify mathematical uses of language to communicate about corresponding points, segments, and angles. As students share what they noticed between the three images, revoice their statements using the term "corresponding." Then, invite students to use the term "corresponding" when describing what they noticed. Some students may benefit from chorally repeating the phrases that include the word "corresponding" in context.
Design Principle(s): Optimize output (for explanation)

Student Task Statement

Here is a figure and two copies, each with some points labeled.

ORIGINAL COPY 1 COPY 2

1. Complete this table to show **corresponding parts** in the three figures.

original	copy 1	copy 2
point P		
segment LM		
	segment EF	
		point W
angle KLM		
		angle XYZ

2. Is either copy a scaled copy of the original figure? Explain your reasoning.

3. Use tracing paper to compare angle KLM with its corresponding angles in Copy 1 and Copy 2. What do you notice?

4. Use tracing paper to compare angle NOP with its corresponding angles in Copy 1 and Copy 2. What do you notice?

Student Response

1.

original	copy 1	copy 2
point P	point F	point Z
segment LM	segment BC	segment VW
segment OP	segment EF	segment YZ
point M	point C	point W
angle KLM	angle ABC	angle UVW
angle NOP	angle DEF	angle XYZ

2. Copy 1 is a scaled copy, but Copy 2 is not. Sample explanation: The original sign is a circle. Copy 1 is also a circle, only smaller. Copy 2 has been stretched sideways and shrunken vertically; its shape has changed into an oval, so it is not a scaled copy.
3. Angle ABC in Copy 1 corresponds to and has the same size as angle KLM. Angle UVW in Copy 2 also corresponds to angle KLM but is smaller in size than the original angle.
4. Angle DEF in Copy 1 corresponds to and has the same size as angle NOP. Angle XYZ in Copy 2 also corresponds to angle NOP but is larger in size than the original angle.

Activity Synthesis

Select a few students to share their observations about angles. Discuss the size of corresponding angles in figures that are scaled copies and those that are not. Ask questions such as:

- In the scaled copy, Copy 1, did the size of any angle change compared to its corresponding angle in the original sign? (No)
- In Copy 2, did the size of any angle change relative to its corresponding angle in the original sign? (Yes) Which ones? (Angle UVW has a different measure than angle KLM, for example.)
- What can you say about corresponding angles in two figures that are scaled copies of one another? (They have the same measure.)
- What can you say about corresponding angles in two figures that are *not* scaled copies? (They *might* not have the same measure.)

2.3 Scaled Triangles

15 minutes

In this activity, students continue to practice identifying corresponding parts of scaled copies. By organizing corresponding lengths in a table, students see that there is a single factor that relates each length in the original triangle to its corresponding length in a copy (MP8). They learn that this number is called a **scale factor**.

As students work on the first question, listen to how they reason about which triangles are scaled copies. Identify groups who use side lengths and angles as the basis for deciding. (Students are not expected to reason formally yet, but should begin to look to lengths and angles for clues.)

As students identify corresponding sides and their measures in the second and third questions, look out for confusion about corresponding parts. Notice how students decide which sides of the right triangles correspond.

If students still have access to tracing paper, monitor for students who use this tool strategically (MP5).

Addressing

- 7.G.A.1

Building Towards

- 7.RP.A.2

Instructional Routines

- Think Pair Share

Launch

Arrange students into groups of 4. Assign each student one of the following pairs of triangles in the first question.

- A and E
- B and F
- C and G
- D and H

Give students 2 minutes of quiet think time to determine if their assigned triangles are scaled copies of the original triangle. Give another 2–3 minutes to discuss their responses and complete the first question in groups.

Discuss briefly as a class which triangles are scaled copies and select a couple of groups who reasoned in terms of lengths and angles to explain their reasoning. Some guiding questions:

- What information did you use to tell scaled copies from those that are not?
- How were you able to tell right away that some figures are not scaled copies?

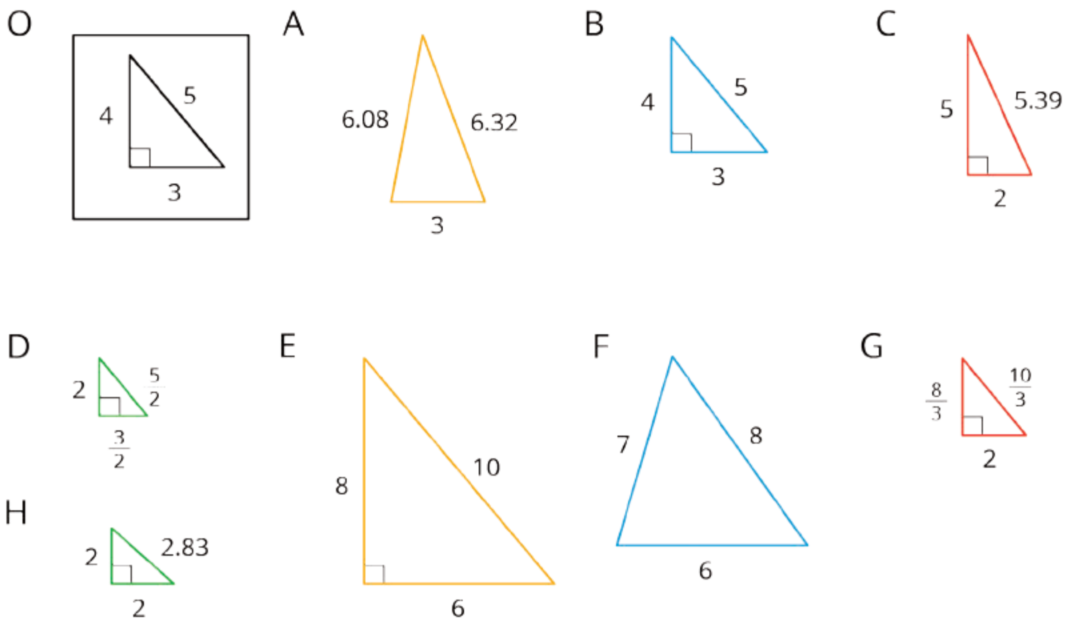
Give students quiet work time to complete the rest of the task after the class recognizes that A, C, F, and H are not scaled copies.

Anticipated Misconceptions

Students may think that Triangle F is a scaled copy because just like the 3-4-5 triangle, the sides are also three consecutive whole numbers. Point out that corresponding angles are not equal.

Student Task Statement

Here is Triangle O, followed by a number of other triangles.



Your teacher will assign you two of the triangles to look at.

1. For each of your assigned triangles, is it a scaled copy of Triangle O? Be prepared to explain your reasoning.
2. As a group, identify *all* the scaled copies of Triangle O in the collection. Discuss your thinking. If you disagree, work to reach an agreement.
3. List all the triangles that are scaled copies in the table. Record the side lengths that correspond to the side lengths of Triangle O listed in each column.

Triangle O	3	4	5

4. Explain or show how each copy has been scaled from the original (Triangle O).

Student Response

1. Answers vary depending on the pair of triangles students have. Triangles B, D, E, and G are scaled copies.
2. Triangles B, D, E, and G are scaled copies. Sample reasoning: B, D, E, and G have not changed in shape (they are still right triangles). Each of their sides are the same number of times as long as the corresponding sides in the original triangle. Triangles A and F do not have the same shape as Triangle O (their angles are all different), so they are not scaled copies. Triangles C, G, and H are right triangles but their sides are not the same number of times as long as the corresponding sides in the original triangle.
3.

Triangle O	3	4	5
Triangle B	3	4	5
Triangle D	$\frac{3}{2}$	2	$\frac{5}{2}$
Triangle E	6	8	10
Triangle G	2	$\frac{8}{3}$	$\frac{10}{3}$

4. Explanations vary. Sample explanations:
    - Triangle B is a same-size copy of the original. All the lengths stay the same.
    - In Triangle D, all the lengths are half of the original ones.
    - In Triangle E, all sides double in length.
    - In Triangle G, the lengths are $\frac{2}{3}$ times the corresponding lengths in the original triangle.

Are You Ready for More?

Choose one of the triangles that is not a scaled copy of Triangle O. Describe how you could change at least one side to make a scaled copy, while leaving at least one side unchanged.

Student Response

Answers vary. Sample response: on Triangle F, the side of length 7 could be extended to have length 10.

Activity Synthesis

Display the image of all triangles and invite a couple of students to share how they knew which sides of the triangles correspond. Then, display a completed table in the third question for all to see. Ask each group to present its observations about one triangle and how the triangle has been scaled from the original. Encourage the use of “corresponding” in their explanations. As students present, record or illustrate their reasoning on the table, e.g., by drawing arrows between rows and annotating with the operation students are describing, as shown here.

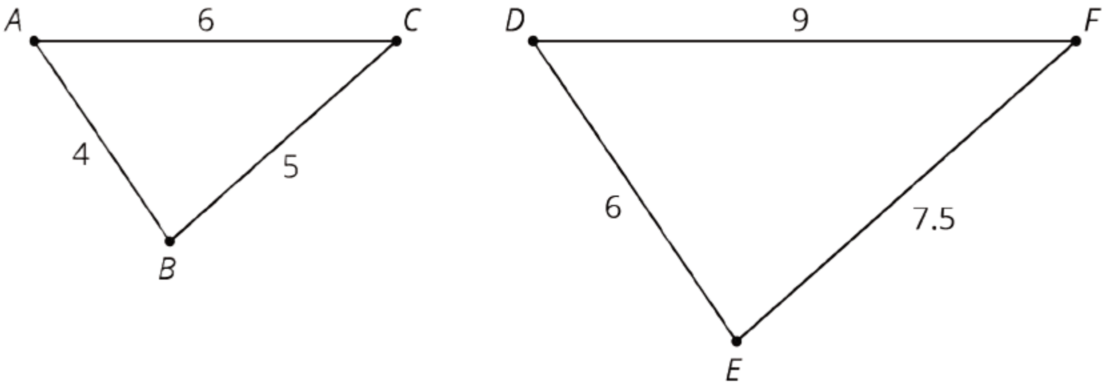
Triangle O	3	4	5
Triangle D $\cdot\frac{1}{2}$	$\frac{3}{2}$	2	$\frac{5}{2}$
Triangle E $\cdot 2$	6	8	10
Triangle B $\cdot 1$	3	4	5
Triangle G $\cdot\frac{2}{3}$	2	$\frac{8}{3}$	$\frac{10}{3}$

Use the language that students use to describe the side lengths and the numerical relationships in the table to guide students toward **scale factor**. For example: "You explained that the lengths in Triangle F are all twice those in the original triangle, so we can write those as "2 times" the original numbers. Lengths in Triangle A are half of those in the original; we can write "$\frac{1}{2}$ times" the original numbers. We call those multipliers—the 2 and the $\frac{1}{2}$-scale factors. We say that scaling Triangle O by a scale factor of 2 produces Triangle F, and that scaling Triangle O by $\frac{1}{2}$ produces Triangle A."

Lesson Synthesis

- What do we mean by **corresponding parts?**
- What is a **scale factor**? How does it work?

Students can use informal language to describe corresponding parts, and recognize a scale factor as a common ratio between the lengths of corresponding side lengths. In the figure, triangle DEF is a scaled copy of triangle ABC. We call parts that have the same position within each figure *corresponding parts*. For example, we refer to vertex E in triangle DEF and vertex B in triangle ABC as *corresponding points*; segment BC and segment EF as *corresponding segments*; and angle C (or angle BCA) and angle F (or angle EFD) as *corresponding angles*.



The segments in a scaled copy are always a certain number of times as long as the corresponding segments in the original figure. We call that number the *scale factor*. For example, the scale factor between ABC and its copy triangle DEF is $\frac{3}{2}$ or 1.5 because all lengths in triangle DEF are 1.5 times as long as the corresponding lengths in triangle ABC.

2.4 Comparing Polygons $ABCD$ and $PQRS$

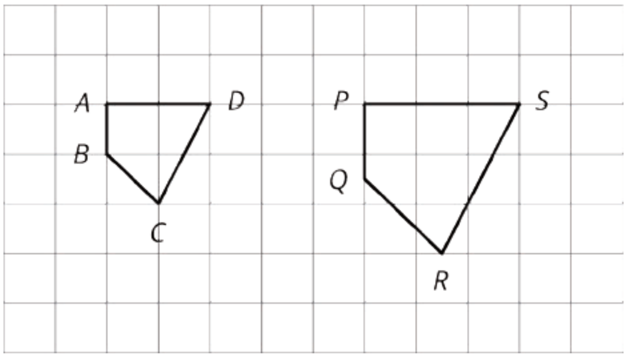
Cool Down: 5 minutes

Addressing

- 7.G.A.1

Student Task Statement

Polygon $PQRS$ is a scaled copy of polygon $ABCD$.



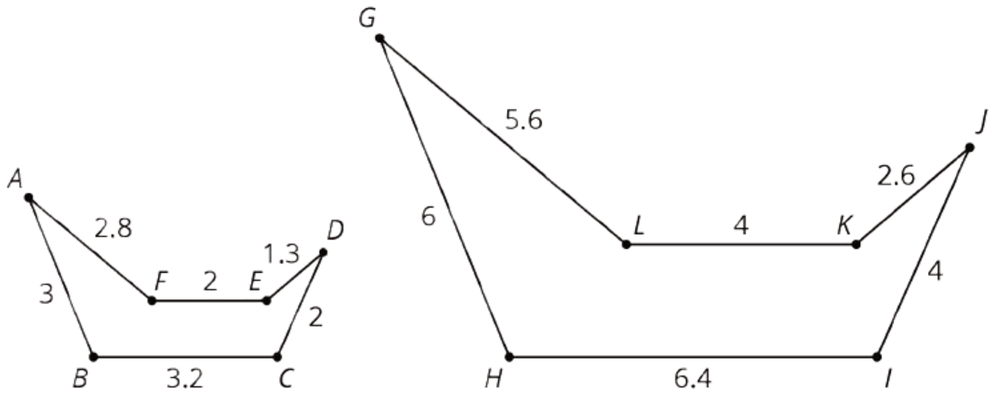
1. Name the angle in the scaled copy that corresponds to angle ABC.
2. Name the segment in the scaled copy that corresponds to segment AD.
3. What is the scale factor from polygon $ABCD$ to polygon $PQRS$?

Student Response

1. Angle PQR corresponds to angle ABC.
2. Segment PS corresponds to segment AD.
3. The scale factor is $\frac{3}{2}$ since $PS = 3$ and $AD = 2$.

Student Lesson Summary

A figure and its scaled copy have **corresponding parts**, or parts that are in the same position in relation to the rest of each figure. These parts could be points, segments, or angles. For example, Polygon 2 is a scaled copy of Polygon 1.



Polygon 1 Polygon 2

- Each point in Polygon 1 has a *corresponding point* in Polygon 2.
  For example, point B corresponds to point H and point C corresponds to point I.
- Each segment in Polygon 1 has a *corresponding segment* in Polygon 2.
  For example, segment AF corresponds to segment GL.
- Each angle in Polygon 1 also has a *corresponding angle* in Polygon 2.
  For example, angle DEF corresponds to angle JKL.

The **scale factor** between Polygon 1 and Polygon 2 is 2, because all of the lengths in Polygon 2 are 2 times the corresponding lengths in Polygon 1. The angle measures in Polygon 2 are the same as the corresponding angle measures in Polygon 1. For example, the measure of angle JKL is the same as the measure of angle DEF.

Glossary

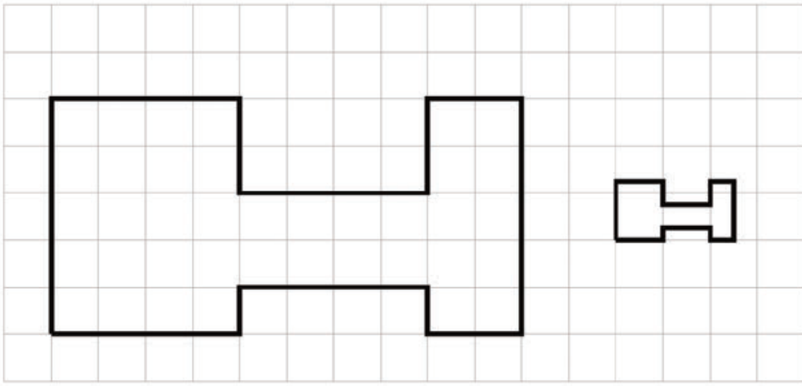
- corresponding
- scale factor

Lesson 2 Practice Problems

Problem 1

Statement

The second H-shaped polygon is a scaled copy of the first.

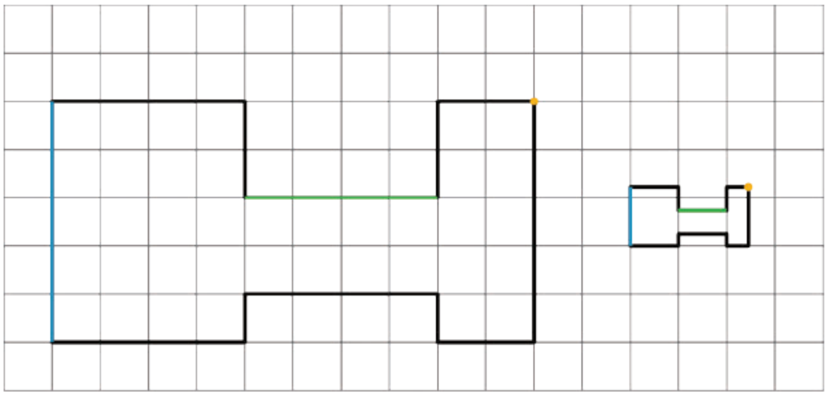


a. Show one pair of corresponding points and two pairs of corresponding sides in the original polygon and its copy. Consider using colored pencils to highlight corresponding parts or labeling some of the vertices.

b. What scale factor takes the original polygon to its smaller copy? Explain or show your reasoning.

Solution

a. Answers vary. Sample markings:



b. $\frac{1}{4}$ or 0.25. Sample explanation: The sides that are 4 units long in the original polygon are 1 unit long in the copy, which is one fourth of the original length.

Problem 2

Statement

Figure B is a scaled copy of Figure A. Select **all** of the statements that must be true:

A. Figure B is larger than Figure A.

B. Figure B has the same number of edges as Figure A.

C. Figure B has the same perimeter as Figure A.

D. Figure B has the same number of angles as Figure A.

E. Figure B has angles with the same measures as Figure A.

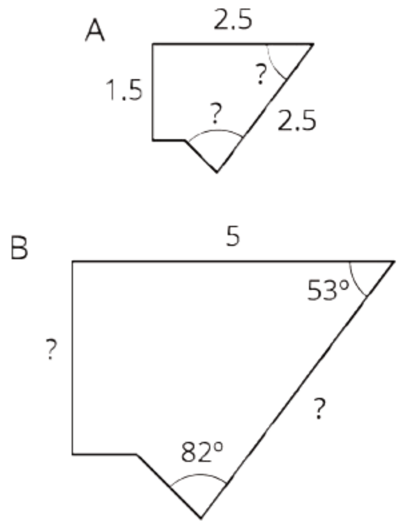
Solution

["B", "D", "E"]

Problem 3

Statement

Polygon B is a scaled copy of Polygon A.



a. What is the scale factor from Polygon A to Polygon B? Explain your reasoning.

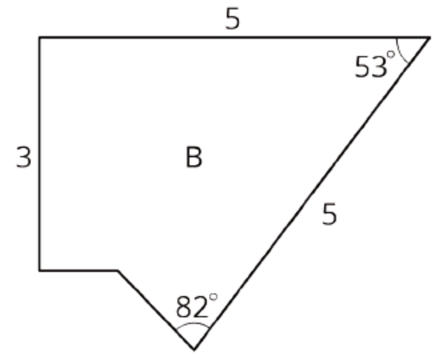
b. Find the missing length of each side marked with ? in Polygon B.

c. Determine the measure of each angle marked with ? in Polygon A.

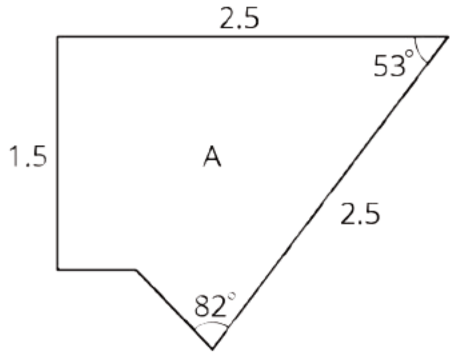
Solution

a. 2 because the top horizontal side has length 2.5 units in Polygon A and 5 units in Polygon B

b. All sides scale by the same factor of 2, so the side that is 2.5 units in Polygon A is 5 units in the copy, and the 1.5-unit-long one is 3 units in the copy



c. 53° and 82° because scaled copies have the same corresponding angles



Problem 4

Statement

Complete each equation with a number that makes it true.

a. $8 \cdot \_\_\_\_\_ = 40$

b. $8 + \_\_\_\_\_ = 40$

c. $21 \div \_\_\_\_\_ = 7$

d. $21 - \_\_\_\_\_ = 7$

e. $21 \cdot \_\_\_\_\_ = 7$

Solution

a. 5

b. 32

c. 3

d. 14

e. $\frac{1}{3}$

Lesson 3: Making Scaled Copies

Goals

- Critique (orally and in writing) different strategies (expressed in words and through other representations) for creating scaled copies of a figure.
- Draw a scaled copy of a given figure using a given scale factor.
- Generalize (orally and in writing) that the relationship between the side lengths of a figure and its scaled copy is multiplicative, not additive.

Learning Targets

- I can draw a scaled copy of a figure using a given scale factor.
- I know what operation to use on the side lengths of a figure to produce a scaled copy.

Lesson Narrative

In the previous lesson, students learned that we can use scale factors to describe the relationship between corresponding lengths in scaled figures. Here they apply this idea to draw scaled copies of simple shapes on and off a grid. They also strengthen their understanding that the relationship between scaled copies is multiplicative, not additive. Students make careful arguments about the scaling process (MP3), have opportunities to use tools like tracing paper or index cards strategically (MP5).

As students draw scaled copies and analyze scaled relationships more closely, encourage them to continue using the terms *scale factor* and *corresponding* in their reasoning.

Alignments

Building On

- 6.NS.B.3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Addressing

- 7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Building Towards

- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR1: Stronger and Clearer Each Time

- MLR3: Clarify, Critique, Correct
- MLR7: Compare and Connect
- Think Pair Share

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Required Preparation

Make sure students have access to their geometry toolkits, especially tracing paper and index cards.

> **Student Learning Goals**
>
> Let’s draw scaled copies.

3.1 More or Less?

Warm Up: 5 minutes
This warm-up prompts students to use what they know about numbers and multiplication to reason about decimal computations. The problems are designed to result in an answer very close to the given choices, so students must be more precise in their reasoning than simply rounding and calculating.

Whereas a number talk typically presents a numerical expression and asks students to explain strategies for evaluating it, this activity asks a slightly different question because students don't necessarily need to evaluate the expression. Rather, they are asked to judge whether the expression is greater than or less than a given value. Although this activity is not quite the same thing as a number talk, the discussion might sound quite similar.

Building On

- 6.NS.B.3

Launch

Display the problems for all to see. Give students 2 minutes of quiet think time. Tell students they may not have to calculate, but could instead reason using what they know about the numbers and operation in each problem. Ask students to give a signal when they have an answer and a strategy for every problem.

Anticipated Misconceptions

Students may attempt to solve each problem instead of reasoning about the numbers and operations. If a student is calculating an exact solution to each problem, ask them to look closely at the characteristics of the numbers and how an operation would affect those numbers.

Student Task Statement

For each problem, select the answer from the two choices.

1. The value of $25 \cdot (8.5)$ is:
    a. More than 205
    b. Less than 205
2. The value of $(9.93) \cdot (0.984)$ is:
    a. More than 10
    b. Less than 10
3. The value of $(0.24) \cdot (0.67)$ is:
    a. More than 0.2
    b. Less than 0.2

Student Response

1. More than 205. Since $8 \cdot 25 = 200$ and $0.5 \cdot 25 = 12.5$, then the product must be more than 205.
2. Less than 10. Since $9.93 \cdot 1 = 9.93$ and 0.9 is less than 1, then the product must be less than 10.
3. Less than 0.2. Since 0.24 is less than $\frac{1}{4}$ and 0.68 is less than 0.8, the product must be less than 0.2 which is $\frac{1}{4}$ of 0.8.

Activity Synthesis

Discuss each problem one at a time with this structure:

- Ask students to indicate which option they agree with.
- If everyone agrees on one answer, ask a few students to share their reasoning, recording it for all to see.
- If there is disagreement on an answer, ask students with differing answers to explain their reasoning and come to an agreement on an answer.

3.2 Drawing Scaled Copies

Optional: 10 minutes (there is a digital version of this activity)

Students continue to work with scaled copies of simple geometric figures, this time on a grid. When trying to scale non-horizontal and non-vertical segments, students may think of using tracing paper or a ruler to measure lengths and a protractor to measure angles. Make sure they have a chance to see how the structure of the grid can be useful for scaling the lengths of non-vertical and non-horizonal segments.

To create scaled copies, students need to attend to all parts of the original figure, or else the copy will not be scaled correctly. Use of the grid for scaling non-horizontal and non-vertical segments is a good example of using tools strategically (MP5).

As students work, monitor for students who find a way to scale segment lengths properly but neglect to consider the size of corresponding angles (especially in making a copy of Figure B and D).

Addressing

- 7.G.A.1

Building Towards

- 7.RP.A.2

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- Think Pair Share

Launch

Give students 3 minutes of quiet time to draw and another 3 minutes to share their drawings with a partner, check each other's work, and make revisions. Provide access to their geometry toolkits.

Representation: Internalize Comprehension. Check in with students after the first 2-3 minutes of work time. Check to make sure students have attended to all parts of the original figures.
Supports accessibility for: Conceptual processing; Organization

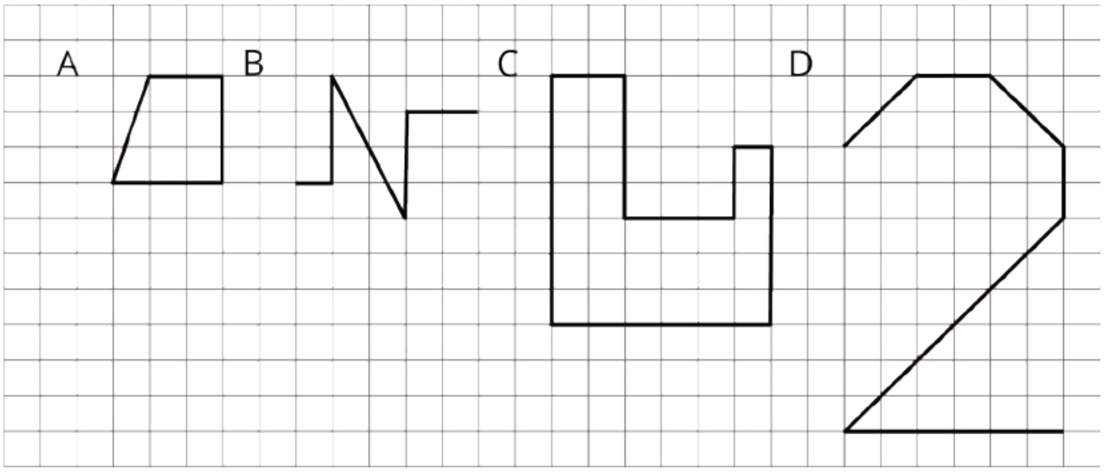
Support for English Language Learners

Speaking, Representing: MLR1 Stronger and Clearer Each Time. Use this routine to support productive discussion when students share their drawings with a partner. Give students time to meet with 2–3 partners, to share and get feedback on their scaled copies. Provide students with prompts for feedback that will help their partners strengthen their ideas and clarify their drawings (e.g., "How did you know how long to make each side length?", "How did you measure to make each angle"?", "How did you use the grid to create your scaled copy?"). Students can borrow ideas and language from each partner to strengthen their work. This provides students with an opportunity to produce verbal mathematical language in service of refining their ideas and their drawings.
Design Principle(s): Optimize output (for justification)

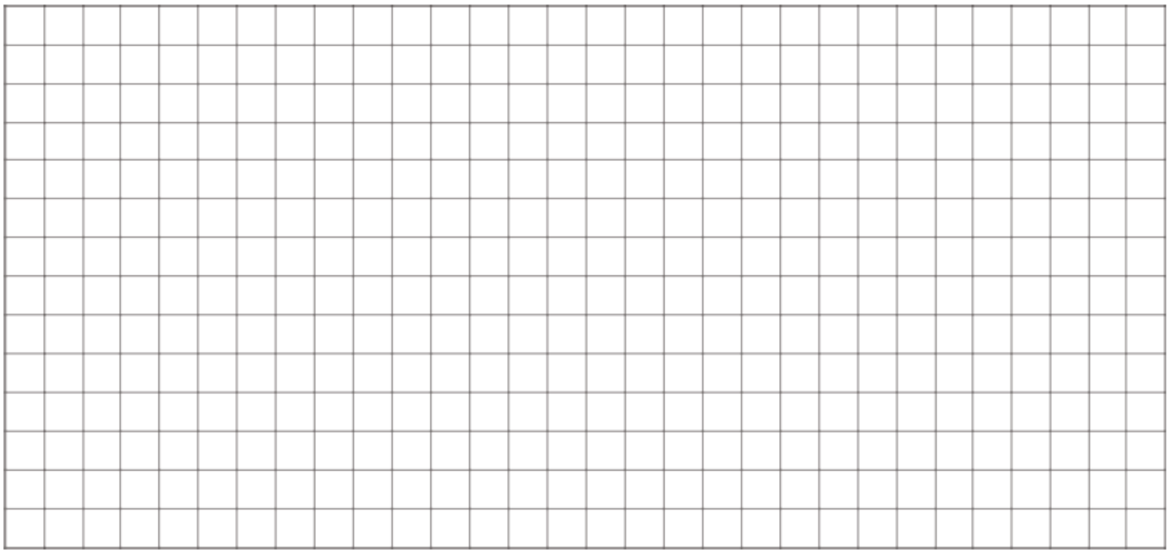
Anticipated Misconceptions

Some students may think that Figure C cannot be scaled by a factor of $\frac{1}{2}$ because some vertices will not land on intersections of grid lines. Clarify that the grid helps us see lengths in whole units but segments we draw on them are not limited to whole units in length.

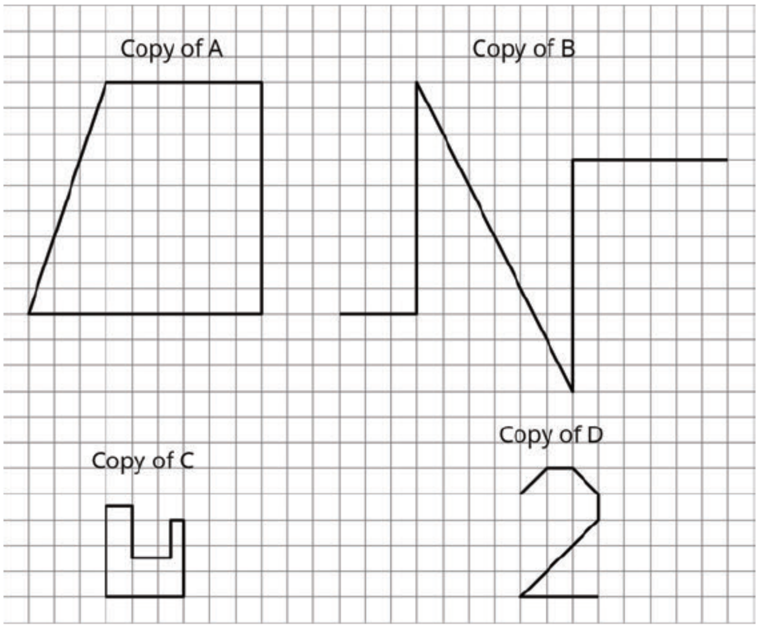
Student Task Statement



1. Draw a scaled copy of either Figure A or B using a scale factor of 3.
2. Draw a scaled copy of either Figure C or D using a scale factor of $\frac{1}{2}$.



Student Response



Activity Synthesis

Invite students to share their strategies of how they used the grid (or other tools) to make sure their drawings were scaled copies. Consider asking questions like:

- How did you know how long to make each side in your scaled copy?
- How did you know how big to make each angle in your scaled copy?
- If you made a mistake while drawing your scaled copy, how could you tell?

Model, prompt, and listen for the language students are using to distinguish between scaled and not scaled figures. Emphasize the usefulness of the grid in drawing and checking right angles, and for drawing and checking lengths of segments. All correct answers will be the same size and shape, but they could be drawn in different positions on the grid.

3.3 Which Operations? (Part 1)

10 minutes

The purpose of this activity is to contrast the effects of multiplying side lengths versus adding to side lengths when creating copies of a polygon. To find the corresponding side lengths on a scaled copy, the side lengths of a figure are all *multiplied* (or divided) by the same number.
However, students often mistakenly think that adding or subtracting the same number to all the side lengths will also create a scaled copy. When students recognize that there is a multiplicative relationship between the side lengths rather than an additive one, they are looking for and making use of structure (MP7).

Monitor for students who:

- notice that Diego's copy is no longer a polygon while Jada's still is
- notice that the relationships between side lengths in Diego's copy have changed (e.g., Side 1 is twice as long as Side 2 in the original but is not twice as long as Side 2 in the copy.) while in Jada's copy they have not
- notice that all the corresponding angles have equal measures (i.e., 90 or 270 degrees)
- describe Jada's copy as having all side lengths divided by 3
- describe Jada's copy as having all side lengths a third as long as their original lengths
- describe Jada's copy as having a scale factor of $\frac{1}{3}$

Addressing

- 7.G.A.1

Building Towards

- 7.RP.A.2

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR3: Clarify, Critique, Correct
- Think Pair Share

Launch

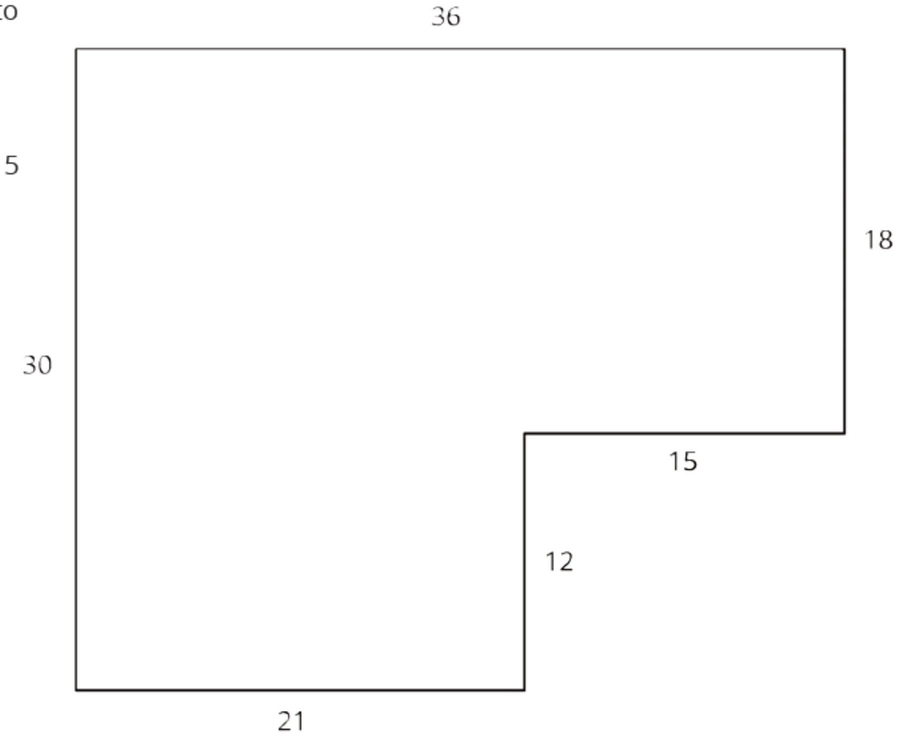
Give students 2–3 minutes of quiet think time, and then 2 minutes to share their thinking with a partner. See MLR 3 (Clarify, Critique, Correct) and use the strategy "Critique a Partial or Flawed Explanation".

Support for Students with Disabilities

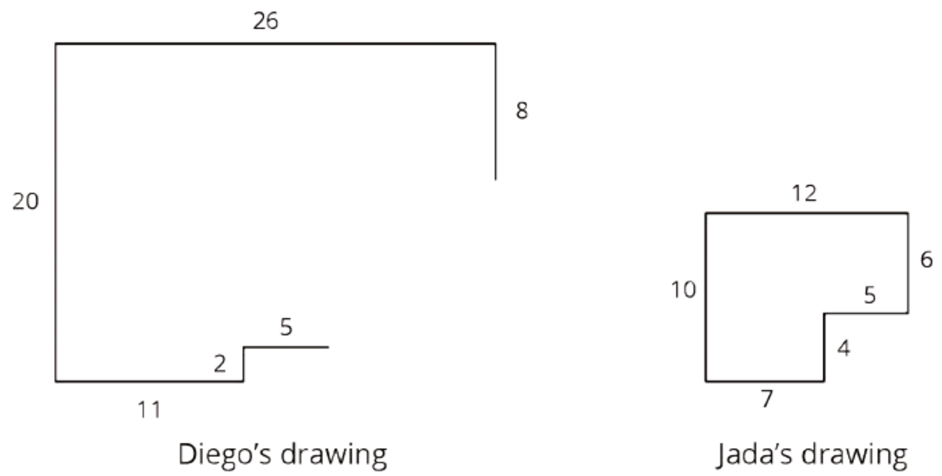
Engagement: Internalize Self Regulation. Demonstrate giving and receiving constructive feedback. Use a structured process and display sentence frames to support productive feedback. For example, “How did you get...?,” “How do you know...?,” and “That could/couldn’t be true because...”
Supports accessibility for: Social-emotional skills; Organization; Language

Student Task Statement

Diego and Jada want to scale this polygon so the side that corresponds to 15 units in the original is 5 units in the scaled copy.



Diego and Jada each use a different operation to find the new side lengths. Here are their finished drawings.



1. What operation do you think Diego used to calculate the lengths for his drawing?
2. What operation do you think Jada used to calculate the lengths for her drawing?
3. Did each method produce a scaled copy of the polygon? Explain your reasoning.

Student Response

1. Since we can get from 15 to 5 by subtracting 10, Diego may have subtracted 10 units from the length of every side. Subtracting 10 from each side length in the original gives Diego's picture.
2. Jada went from 15 to 5 by multiplying by $\frac{1}{3}$ or dividing by 3. Multiplying each side by $\frac{1}{3}$ in the original gives Jada's picture.
3. No, only Jada’s method produces a scaled copy. Sample explanation: Subtracting 10 from each length did not work because now the figure is no longer a polygon. There is a big gap between the two sides that should meet. To create a scaled copy, every length needs to be multiplied (or divided) by the same number.

Activity Synthesis

Invite previously-selected students to share their answers and reasoning. Sequence their explanations from most general to most technical.

Before moving to the next activity, consider asking questions like these:

- What is the scale factor used to create Jada’s drawing? What about for Diego’s drawing? ($\frac{1}{3}$ for Jada's; there isn't one for Diego's, because it is not a scaled copy.)
- What can you say about the corresponding angles in Jada and Diego’s drawings? (They are all equal, even though one is a scaled copy and one is not.)
- Subtraction of side lengths does not (usually) produce scaled copies. Do you think addition would work? (Answers vary.)

Note: There are rare cases when adding or subtracting the same length from each side of a polygon (and keeping the angles the same) *will* produce a scaled copy, namely if all side lengths are the same. If not mentioned by students, it is not important to discuss this at this point.

Support for English Language Learners

Representing, writing, and speaking: Math Language Routine 3 Clarify, Critique, Correct. This is the first time Math Language Routine 3 is suggested as a support in this course. In this routine, students are given an incorrect or incomplete piece of mathematical work. This may be in the form of a written statement, drawing, problem-solving steps, or another mathematical representation. Students analyze, reflect on, and improve the written work by correcting errors and clarifying meaning. Typical prompts are: "Is anything unclear?" and/or "Are there any reasoning errors?" The purpose of this routine is to engage students in analyzing mathematical thinking that is not their own, and to solidify their knowledge through communicating about conceptual errors and ambiguities in language.
Design Principle(s): Support sense-making; Optimize output (for reasoning)

How It Happens:

1. Play the role of Diego and present the following statement along with his flawed drawing to the class. "I used a scale factor of minus 10, and Jada used a scale factor of one third. So my drawing is a different kind of scaled copy from Jada's."

   Ask students, "What steps did Diego take to make the drawing?" and "Did he create a scaled copy? How do you know?"

2. Give students 1 minute of quiet think time to analyze the statement, and then 3 minutes to work on improving the statement with a partner.

   As pairs discuss, provide these sentence frames for scaffolding: "I believe Diego created the drawing by ___ because ___.", "Diego created/did not create a scaled copy. I know this because ___.", "You can't ___ because ___." Encourage the listener to ask clarifying questions by referring to the statement and the drawings. Allow each partner to take a turn as the speaker and listener.

   Listen for students identifying the type of operation used and justification for whether or not a scaled drawing was produced. Have the pairs reach a mutual understanding and agreement on a correct statement about Diego's drawing.

3. Invite 3 or 4 pairs to present their improved statement to the class, both orally and in writing. . Ask students to listen for order/time transition words (first, next, then, etc.), and any elements of justifications (e.g., First, ___ because ___.).

   Here are two sample improved statements:

   "I subtracted 10 from each side length and Jada used a scale factor of one third. So my drawing is not a scaled copy and Jada's is. Jada's is a scaled copy because I know that multiplying—not subtracting—creates a scaled copy. Her drawing created a polygon with

no gaps."
or
"I minused 10 from each side, but I should have realized that in order to scale 15 units in the original down to 5 units in the copy, you have to divide by 3. Jada used a scale factor of one third, which is the same as dividing by 3. My drawing is not a scaled copy and Jada's is because hers is not a polygon with no gaps, and minusing 10 is not a scale factor."

Call attention to statements that generalize that the method for finding the side lengths of a scaled copy is by multiplying or dividing, not adding or subtracting. Revoice student thoughts with an emphasis on knowing whether or not they created a scaled polygon.

4. Close the conversation on Diego's drawing, discuss the accuracy of Jada's scaled copy, and then move on to the next lesson activity.

3.4 Which Operations? (Part 2)

10 minutes

In the previous activity, students saw that subtracting the same value from all side lengths of a polygon did not produce a (smaller) scaled copy. This activity makes the case that adding the same value to all lengths also does not produce a (larger) scaled copy, reinforcing the idea that scaling involves multiplication.

This activity gives students a chance to draw a scaled copy without a grid and to use paper as a measuring tool. To create a copy using a scale factor of 2, students need to mark the length of each original segment and transfer it twice onto their drawing surface, reinforcing—in a tactile way—the meaning of scale factor. The angles in the polygon are right angles (and a 270 degree angle in one case) and can be made using the corner of an index card.

Some students may struggle to figure out how to use an index card or a sheet of paper to measure lengths. Before demonstrating, encourage them to think about how a length in the given polygon could be copied onto an index card and used as an increment for measuring. If needed, show how to mark the 4-unit length along the edge of a card and to use the mark to determine the needed lengths for the copy.

Addressing

- 7.G.A.1

Building Towards

- 7.RP.A.2

Instructional Routines

- MLR7: Compare and Connect
- Think Pair Share

Launch

Have students read the task statement and check that they understand which side of the polygon Andre would like to be 8 units long on his drawing. Provide access to index cards, so that students can use it as a measuring tool. Consider not explicitly directing students as to its use to give them a chance to use tools strategically (MP5). Give students 5–6 minutes of quiet work time, and then 2 minutes to share their work with a partner.

Anticipated Misconceptions

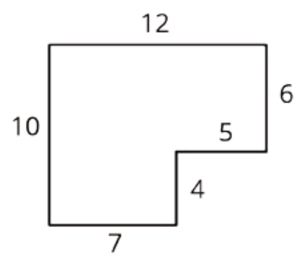
Some students might not be convinced that making each segment 4 units longer will not work. To show that adding 4 units would work, they might simply redraw the polygon and write side lengths that are 4 units longer, regardless of whether the numbers match the actual lengths. Urge them to check the side lengths by measuring. Tell them (or show, if needed) how the 4-unit length in Jada's drawing could be used as a measuring unit and added to all sides.

Other students might add 4 units to all sides and manage to make a polygon but changing the angles along the way. If students do so to make the case that the copy will not be scaled, consider sharing their illustrations with the class, as these can help to counter the idea that "scaling involves adding." If, however, students do this to show that adding 4 units all around does work, address the misconception. Ask them to recall the size of corresponding angles in scaled copies, or remind them that angles in a scaled copy are the same size as their counterparts in the original figure.

Student Task Statement

Andre wants to make a scaled copy of Jada's drawing so the side that corresponds to 4 units in Jada's polygon is 8 units in his scaled copy.

1. Andre says "I wonder if I should add 4 units to the lengths of all of the segments?" What would you say in response to Andre? Explain or show your reasoning.
2. Create the scaled copy that Andre wants. If you get stuck, consider using the edge of an index card or paper to measure the lengths needed to draw the copy.

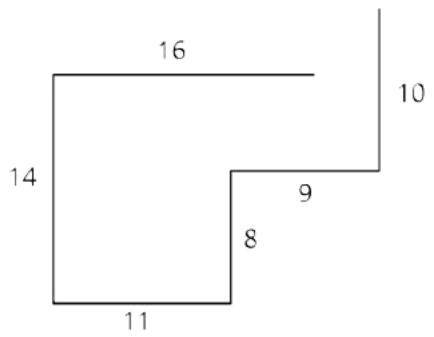


Jada's drawing

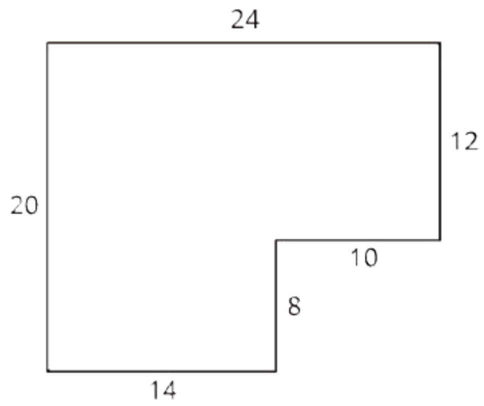
Student Response

1. Answers vary. Sample reasoning: Adding 4 units would not work because the shape of the copy would be different than the shape of the original. For example, in the original drawing, the top horizontal segment is 12 units and the two bottom horizontal segments (5 units and 7 units) also add up to 12 units. If we add 4 units to each segment, the top horizontal segment will be 16 units long, and the two bottom horizontal segments will be 9 units and 11 units, or a total of 20 units. There will be a gap where two segments should meet, or if we make the two ends meet, the angles will no longer be right angles. See the figure on the left.
2. See the figure on the right.

Adding 4 units to each side



A correctly drawn figure



Are You Ready for More?

The side lengths of Triangle B are all 5 more than the side lengths of Triangle A. Can Triangle B be a scaled copy of Triangle A? Explain your reasoning.

Student Response

Yes, if triangle A is equilateral then its side lengths are all the same. Adding 5 to each side, the lengths will still be the same and so triangle B will also be equilateral.

If triangle A is not equilateral then triangle B will not be a scaled copy of triangle A. To see why, notice that adding 5 to a side length of 5 doubles the side length. Adding 5 to a side length that greater than 5 changes the side by a scale factor less than 2 while adding 5 to a side length less than 5 changes the side length by a scale factor less than 2. So if one side length of triangle A is 5, all side lengths have to be 5 or else triangle B will not be a scaled copy of triangle A. This reasoning works for other side lengths than 5. In general, adding 5 to a *greater* side length uses a *smaller* scale factor.

Activity Synthesis

The purpose of the activity is to explicitly call out a potential misunderstanding of how scale factors work, emphasizing that scale factors work by multiplying existing side lengths by a common factor, rather than adding a common length to each.

Invite a couple of students to share their explanations or illustrations that adding 4 units to the length of each segment would not work (e.g. the copy is no longer a polygon, or the copy has angles that are different than in the original figure). Then, select a couple of other students to show their scaled copies and share how they created the copies. Consider asking:

- What scale factor did you use to create your copy? Why?
- How did you use an index card (or a sheet of paper) to measure the lengths for the copy?
- How did you measure the angles for the copy?

Support for English Language Learners

Speaking: Math Language Routine 7 Compare and Connect. This is the first time Math Language Routine 7 is suggested as a support in this course. In this routine, students are given a problem that can be approached using multiple strategies or representations, and are asked to prepare a visual display of their method. Students then engage in investigating the strategies (by means of a teacher-led gallery walk, partner exchange, group presentation, etc.), compare approaches, and identify correspondences between different representations. A typical discussion prompt is "What is the same and what is different?", comparing their own strategy to the others. The purpose of this routine is to allow students to make sense of mathematical strategies by identifying, comparing, contrasting, and connecting other approaches to their own, and to develop students' awareness of the language used through constructive conversations.
Design Principle(s): Maximize meta-awareness

How It Happens:

1. Use this routine to compare and contrast different methods for creating scaled copies of Jada's drawing. Before selecting students to share a display of their method with the whole class, first give students an opportunity to do this in a group of 3–4.

   Invite students to quietly investigate each other's work. Ask students to consider what is the same and what is different about each display. Invite students to give a step-by-step explanation of their method using this sentence frame: "In order to create the copy, first I.... Next,.... Then, ... Finally,....". Allow 1–2 minutes for each display and signal when it is time to switch.

2. Next, give each student the opportunity to add detail to their own display for 1-2 minutes. As students work on their displays, circulate the room to identify at least two different methods or two different ways of representing a method. Also look for methods that were only partially successful.

3. Consider selecting 1–2 students to share methods that were only partially successful in producing scaled copies. Then, select a couple of students to share displays of methods that did produce scaled copies.

   Draw students' attention to the approaches used in each drawing (e.g., adding the same value to each side length, not attending to the angles, multiplying by a common factor, not creating a polygon, etc.). Ask students, "Did this approach create a scaled copy? Why or why not?"

4. After the pre-selected students have finished sharing with the whole class, lead a discussion comparing, contrasting, and connecting the different approaches and representations.

In this discussion, demonstrate using the mathematical language "scale factor", "corresponding", and "multiplicative" to amplify student language.

Consider using these prompts:

- "How did the scale factor show up in each method?",
- "Why did the different approaches lead to the same outcome?",
- "What worked well in _____'s approach/representation? What did not work well?", and
- "What role does multiplication play in each approach?"

5. Close the discussion by inviting 3 students to revoice the incorrect method for creating a scaled drawing, and then invite 3 different students to revoice the correct method for creating a scaled drawing. Then, transition back to the Lesson Synthesis and Cool Down.

Lesson Synthesis

- How do we draw a scaled copy of a figure?
- Can we create scaled copies by adding or subtracting the same value from all lengths? Why or why not?

Scaling is a multiplicative process. To draw a scaled copy of a figure, we need to multiply all of the lengths by the scale factor. We saw in the lesson that adding or subtracting the same value to all lengths will not create scaled copies.

3.5 More Scaled Copies

Cool Down: 5 minutes

Addressing

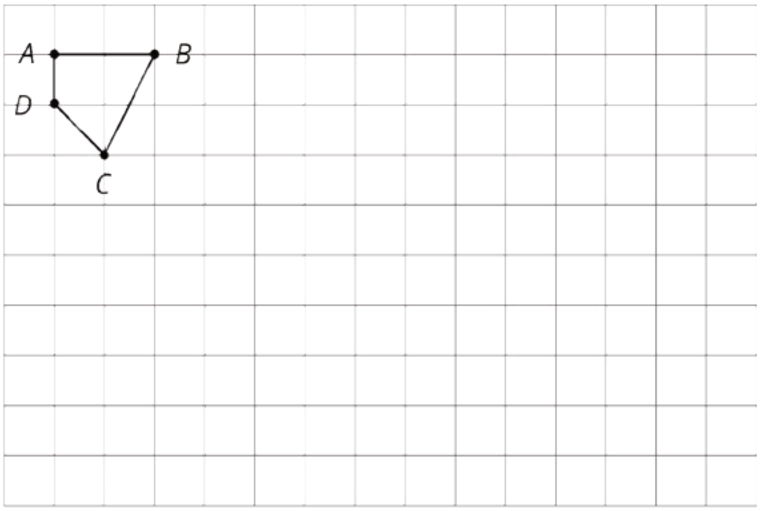
- 7.G.A.1

Building Towards

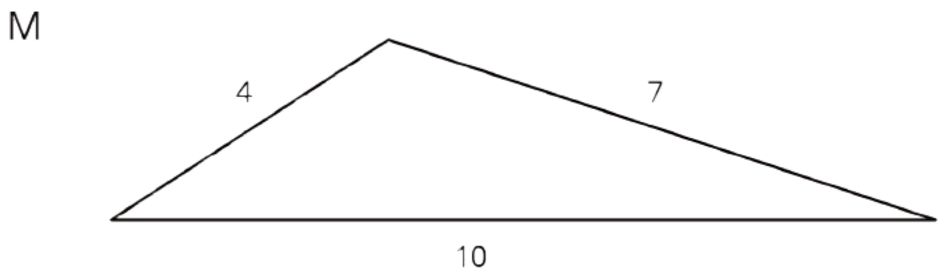
- 7.RP.A.2

Student Task Statement

1. Create a scaled copy of $ABCD$ using a scale factor of 4.



2. Triangle Z is a scaled copy of Triangle M.



Select all the sets of values that could be the side lengths of Triangle Z.

a. 8, 11, and 14.

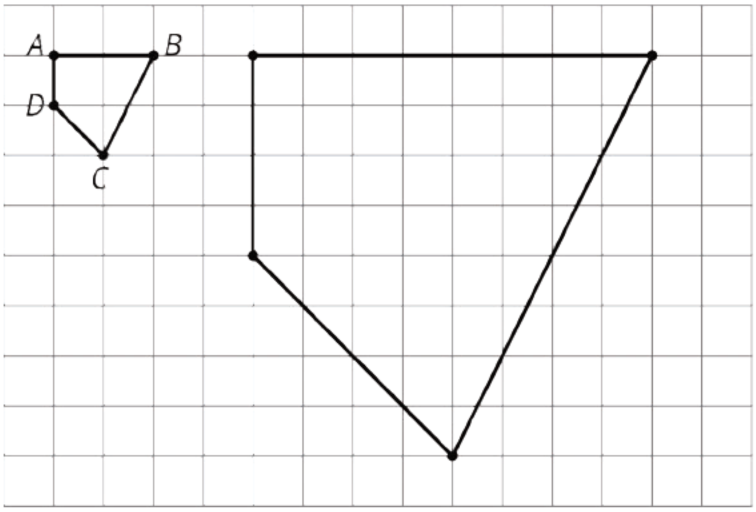
b. 10, 17.5, and 25.

c. 6, 9, and 11.

d. 6, 10.5, and 15.

e. 8, 14, and 20.

Student Response

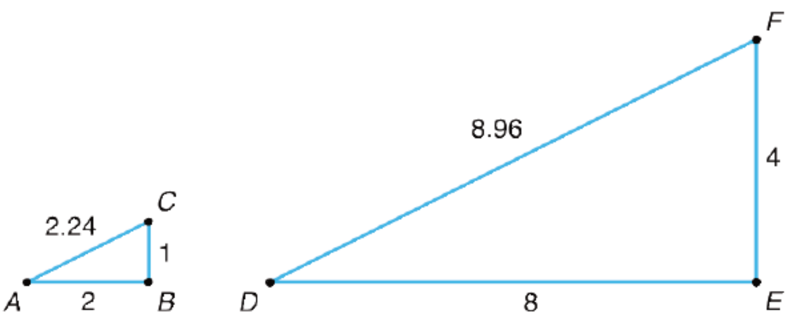
1. 

2. B, D, and E.

Student Lesson Summary

Creating a scaled copy involves *multiplying* the lengths in the original figure by a scale factor.

For example, to make a scaled copy of triangle ABC where the base is 8 units, we would use a scale factor of 4. This means multiplying all the side lengths by 4, so in triangle DEF, each side is 4 times as long as the corresponding side in triangle ABC.

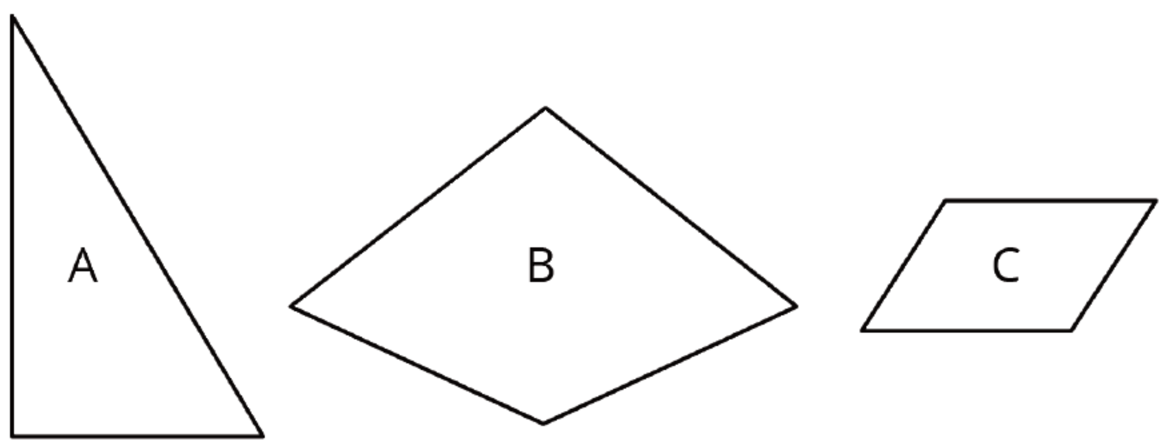


Lesson 3 Practice Problems

Problem 1

Statement

Here are 3 polygons.



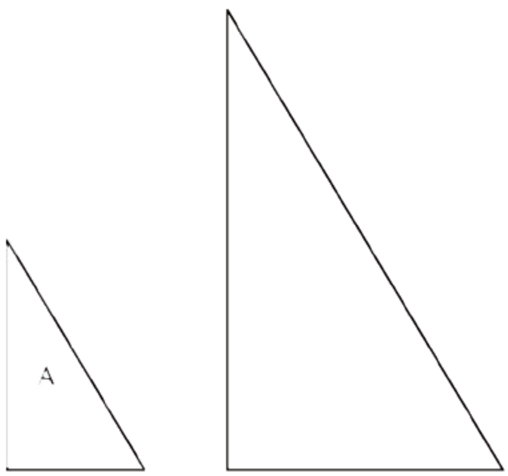
Draw a scaled copy of Polygon A using a scale factor of 2.

Draw a scaled copy of Polygon B using a scale factor of $\frac{1}{2}$.

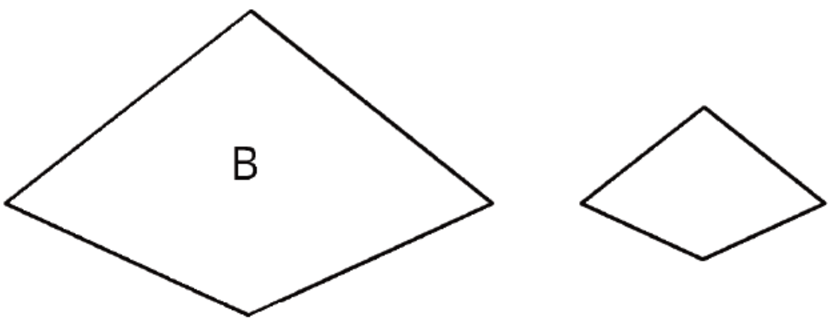
Draw a scaled copy of Polygon C using a scale factor of $\frac{3}{2}$.

Solution

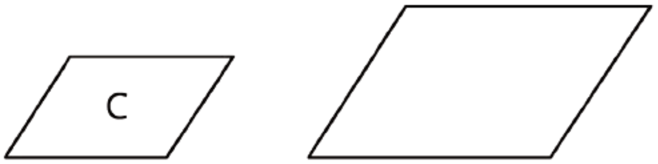
1.



2.



3.



Problem 2

Statement

Quadrilateral A has side lengths 6, 9, 9, and 12. Quadrilateral B is a scaled copy of Quadrilateral A, with its shortest side of length 2. What is the perimeter of Quadrilateral B?

Solution

The scale factor is $\frac{1}{3}$, so the side lengths of Quadrilateral B are 2, 3, 3, and 4. Summing these four numbers gives the perimeter of 12.

Problem 3

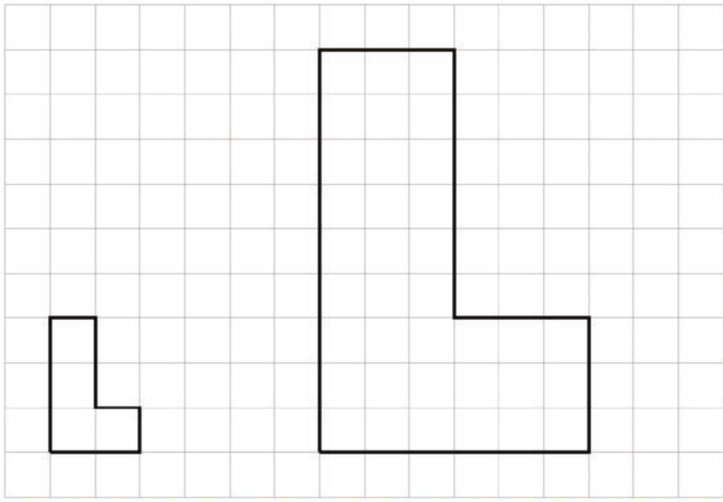
Statement

Here is a polygon on a grid.

Draw a scaled copy of this polygon that has a perimeter of 30 units. What is the scale factor? Explain how you know.

Solution

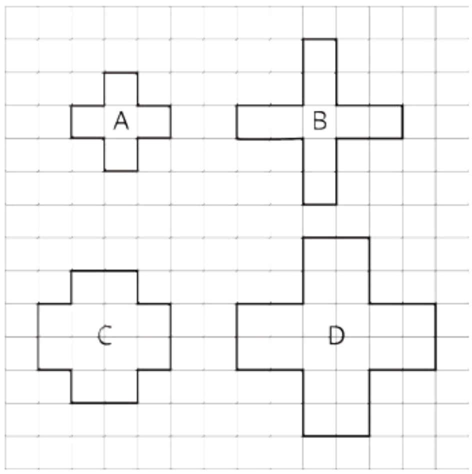
The perimeter of the original polygon is 10 units. Since the perimeter of a scaled copy is multiplied by the scale factor, a scale factor of 3 needs to be applied to get a copy with a perimeter of 30.



Problem 4

Statement

Priya and Tyler are discussing the figures shown below. Priya thinks that B, C, and D are scaled copies of A. Tyler says B and D are scaled copies of A. Do you agree with Priya, or do you agree with Tyler? Explain your reasoning.



Solution

Answers vary. Sample response: I agree with neither one. Only D is a scaled copy of A. In D, the length of each segment of the plus sign is twice the matching segments in A. In B and C, some segments are double the matching lengths in A but some are not.

(From Unit 1, Lesson 1.)

Section: Scale Factors

Lesson 4: Scaled Relationships

Goals

- Explain (orally and in writing) that corresponding angles in a figure and its scaled copies have the same measure.
- Identify (orally and in writing) corresponding distances or angles that can show that a figure is not a scaled copy of another.
- Recognize that corresponding distances in a figure and its scaled copy are related by the same scale factor as corresponding sides.

Learning Targets

- I can use corresponding distances and corresponding angles to tell whether one figure is a scaled copy of another.
- When I see a figure and its scaled copy, I can explain what is true about corresponding angles.
- When I see a figure and its scaled copy, I can explain what is true about corresponding distances.

Lesson Narrative

In previous lessons, students looked at the relationship between a figure and a scaled copy by finding the scale factor that relates the side lengths and by using tracing paper to compare the angles. This lesson takes both of these comparisons a step further.

- Students study *corresponding distances* between points that are not connected by segments, in both scaled and unscaled copies. They notice that when a figure is a scaled copy of another, corresponding distances that are not connected by a segment are also related by the same scale factor as corresponding sides.
- Students use protractors to test their observations about corresponding angles. They verify in several sets of examples that corresponding angles in a figure and its scaled copies are the same size.

Students use both insights—about angles and distances between points—to make a case for whether a figure is or is not a scaled copy of another (MP3). Practice with the use of protractors will help develop a sense for measurement accuracy, and how to draw conclusions from said measurements, when determining whether or not two angles are the same.

Alignments

Addressing

- 7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR1: Stronger and Clearer Each Time
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Notice and Wonder

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Required Preparation

Make sure students have access to their geometry toolkits, especially rulers and protractors.

> **Student Learning Goals**
>
> Let's find relationships between scaled copies.

4.1 Three Quadrilaterals (Part 1)

Warm Up: 5 minutes
This warm-up gives students a chance to practice identifying corresponding angles of scaled copies, measure angles using a protractor, and test their earlier conjecture that corresponding angles have the same measure.

Addressing

- 7.G.A.1

Instructional Routines

- Notice and Wonder

Launch

Have students look at the figures in the activity, and ask "What do you notice? What do you wonder?" Call out in particular questions about the angles in the figures (e.g., whether corresponding ones have the same measure). Tell students that they will test their previous observation about the angles of scaled figures, this time by using protractors instead of tracing paper.

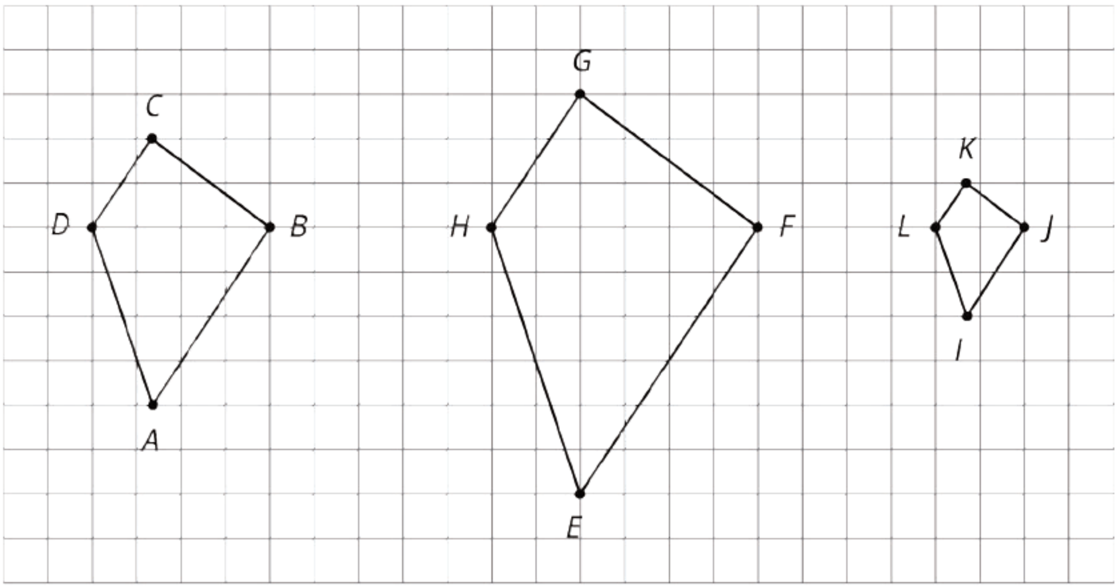
Provide access to protractors. Clear protractors with no holes and with radial lines printed on them are recommended here. Some angles may be challenging to measure because of the size of the polygons. If students find the sides of a polygon not long enough to accommodate angle measurements, suggest that they extend the lines, or demonstrate how to do so (especially if available protractors are opaque with holes in the middle).

Anticipated Misconceptions

Some students may read the wrong number on the protractor, moving down from the 180° mark instead of up from the 0° mark, or reading the measurement outside of one of the lines forming the angle instead of between the two lines. Clarify the angle being measured, how to line up the protractor, or how to read the markings correctly.

Student Task Statement

Each of these polygons is a scaled copy of the others.



1. Name two pairs of corresponding angles. What can you say about the sizes of these angles?
2. Check your prediction by measuring at least one pair of corresponding angles using a protractor. Record your measurements to the nearest 5°.

Student Response

1. Answers vary. Sample response:
    - Angles ABC and EFG (a.k.a. angles B and F)
    - Angles LIJ and HEF (a.k.a. angles I and E)

The corresponding angles of the polygons will be the same size.

2. At least two angles from one of these lists:

   - Angles A, E, and I each measure about 50°.
   - Angles B, F, and J each measure about 95°.
   - Angles C, G, and K each measure about 90°.
   - Angles D, H, and L each measure about 125°.

Activity Synthesis

Select a few students to share their angle measurements and poll the class briefly for agreement and disagreement. Discuss major discrepancies, if any. Students should be able to confirm that all corresponding angles in the scaled polygons are equal.

If desired, ask students whether recording the angles to the nearest 1 degree would be appropriate: in general, the thickness of the line segments and the markings on the protractor limit accuracy, so reporting to the nearest 5 degrees is appropriate (as long as none of the angles are too close to halfway between two increments).

4.2 Three Quadrilaterals (Part 2)

10 minutes
Students have seen that the lengths of corresponding segments in a figure and its scaled copy vary by the same scale factor. Here, they learn that in such a pair of figures, *any* corresponding distances—not limited to lengths of sides or segments—are related by the same scale factor. The side lengths of the polygons in this task cannot be easily determined, so students must look to other distances to compare.

Students must take care when they identify corresponding vertices and distances. As students work, urge them to attend to the order in which points or segments are listed.

If students are not sure what to make out of the values in the table (for the second question), encourage them to consider the corresponding distances of two figures at a time. For example, ask: What do you notice about the corresponding vertical distances in $IJKL$ and $EFGH$? What about the corresponding horizontal distances in those two figures?

Addressing

- 7.G.A.1

Instructional Routines

- MLR7: Compare and Connect

Launch

Arrange students in groups of 2. Ask if they can tell the lengths of segments GF or DC from the grid (without using rulers). Explain that they will explore another way to compare length measurements in scaled copies.

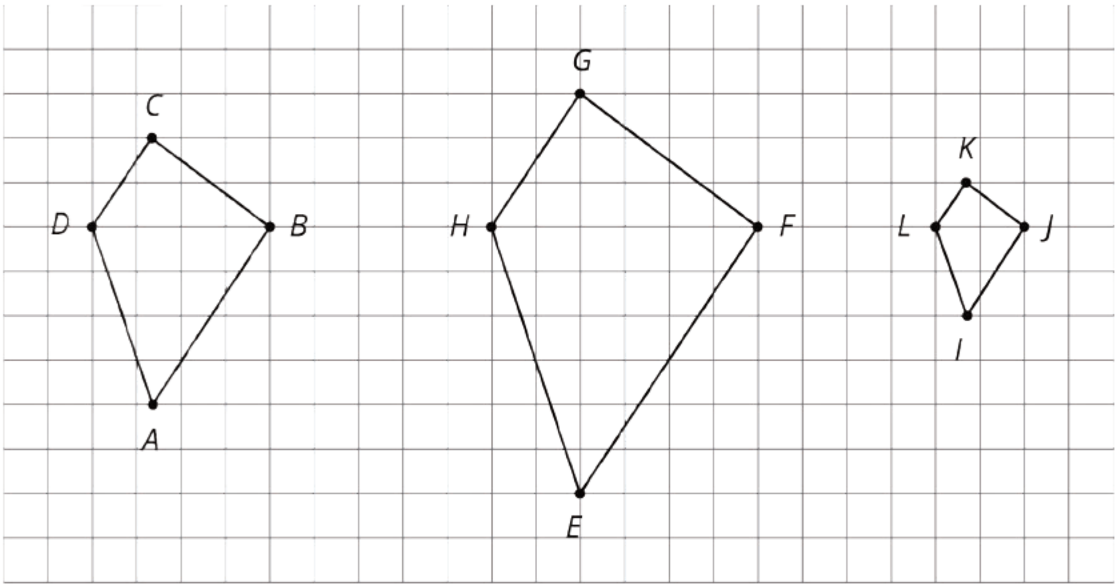
Give students 2–3 minutes of quiet work time for the first two questions, and 1 minute to discuss their responses with a partner before continuing on to the last question.

Anticipated Misconceptions

Students may list the corresponding vertices for distances in the wrong order. For example, instead of writing IK as the distance corresponding to DB, they may write KI. Remind students of the corresponding points by asking, "Which vertex in $IJKL$ corresponds to D? Which corresponds to B?" and have them match the order of the vertices accordingly.

Student Task Statement

Each of these polygons is a scaled copy of the others. You already checked their corresponding angles.



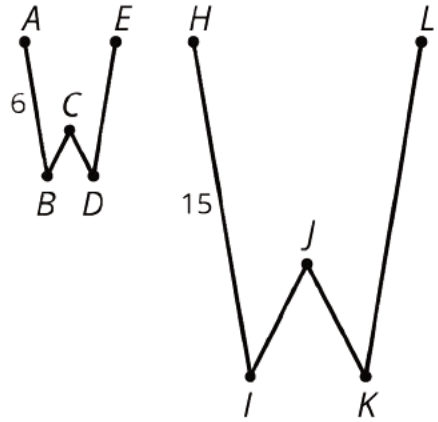
1. The side lengths of the polygons are hard to tell from the grid, but there are other *corresponding distances* that are easier to compare. Identify the distances in the other two polygons that correspond to DB and AC, and record them in the table.

quadrilateral	distance that corresponds to DB	distance that corresponds to AC
$ABCD$	$DB = 4$	$AC = 6$
$EFGH$		
$IJKL$		

2. Look at the values in the table. What do you notice?

   Pause here so your teacher can review your work.

3. The larger figure is a scaled copy of the smaller figure.



   a. If $AE = 4$, how long is the corresponding distance in the second figure? Explain or show your reasoning.

   b. If $IK = 5$, how long is the corresponding distance in the first figure? Explain or show your reasoning.

Student Response

1.

quadrilateral	distance that corresponds to DB	distance that corresponds to AC
$ABCD$	$DB = 4$	$AC = 6$
$EFGH$	$HF = 6$	$EG = 9$
$IJKL$	$LJ = 2$	$IK = 3$

2. These corresponding distances are related by the same scale factor even though they are not side lengths.

3. a. $HL = 10$. Sample explanation: $15 \div 6 = 2.5$, so the second figure is related to the first figure by a scale factor of 2.5. HL is the corresponding distance to AE and is also related by a factor of 2.5. $(2.5) \cdot 4 = 10$.

   b. $BD = 2$. Sample explanation: The scale factor from the small figure to the larger copy is 2.5, so dividing KI by 2.5 gives the corresponding distance in the original figure. $5 \div 2.5 = 2$.

Activity Synthesis

Display the completed tables for all to see. To highlight how all distances in a scaled copy (not just the side lengths of the figure) are related by the same scale factor, discuss:

- How does the vertical distance in $ABCD$ compare to that in $EFGH$? How do the horizontal distances in the two polygons compare? Do the pairs of vertical and horizontal distances share the same scale factor?
- How do the vertical distances in $EFGH$ and $IJKL$ compare? What about the horizontal distances? Is there a common scale factor? What is that scale factor?
- What scale factor relates the corresponding lengths and distances in the two drawings of the letter W?

Support for Students with Disabilities

Representation: Internalize Comprehension. Demonstrate and encourage students to use color coding and annotations to highlight connections between figures. For example, mark corresponding line segments using the same color.
Supports accessibility for: Visual-spatial processing

Support for English Language Learners

Speaking: MLR7 Compare and Connect. Use this routine to call attention to the different ways students may identify scale factors. Display the following statements: "The scale factor from $EFGH$ to $IJKL$ is 3," and "The scale factor from $EFGH$ to $IJKL$ is $\frac{1}{3}$." Give students 2 minutes of quiet think time to read and consider whether either or both of the statements are correct. Invite students to share their initial thinking with a partner before selecting 2–3 students to share with the class. In this discussion, listen for and amplify any comments that refer to the order of the original figure and its scaled copy, as well as those who identify corresponding vertices and distances. Draw students' attention to the different ways to describe the relationships between scaled copies and the original figure.
Design Principle(s): Maximize meta-awareness

4.3 Scaled or Not Scaled?

10 minutes

The purpose of this activity is for students to determine that figures are not scaled copies, even though they have either corresponding angles with equal measures or corresponding distances multiplied by the same scale factor. This shows that to determine one figure is a scaled copy of another, we have to check *both* the corresponding angles and the corresponding distances.

As students work, monitor for convincing arguments about why one polygon is or is not a scaled copy of the other. Ask them to present their cases during the whole-class discussion. As they present and analyze different arguments about the figures in the discussion, students will engage in MP3.

Addressing

- 7.G.A.1

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Keep students in the same groups. Provide access to geometry toolkits. Give students 6–7 minutes of quiet work time.

Support for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.
Supports accessibility for: Organization; Attention

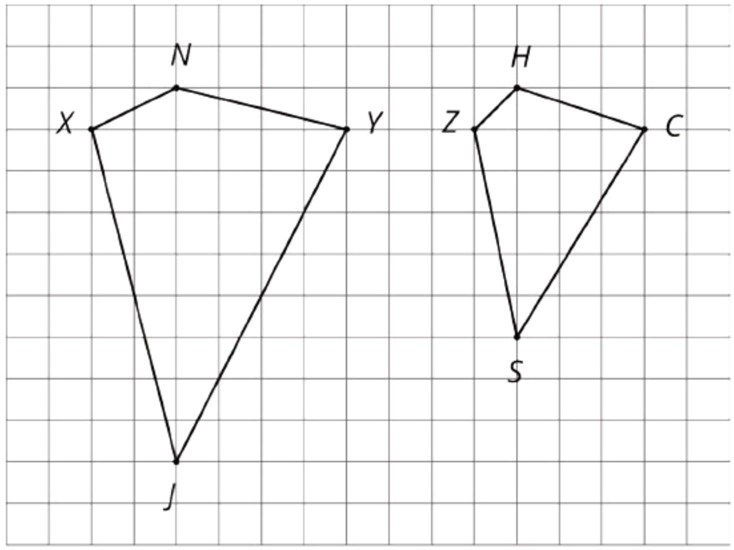
Anticipated Misconceptions

Students may rely on the appearance of the figures rather than analyze given information to draw conclusions about scaling. Urge them to look for information about distances and angles (and to think about which tools could help them find such information) to support their argument.

Some students may struggle with comparing the corresponding angles in the first pair of figures. Remind students of the tools that are at their disposal, and that they could extend the sides of the polygons, if needed, to make it easier to measure the angles.

Student Task Statement

Here are two quadrilaterals.



1. Mai says that Polygon $ZSCH$ is a scaled copy of Polygon $XJYN$, but Noah disagrees. Do you agree with either of them? Explain or show your reasoning.

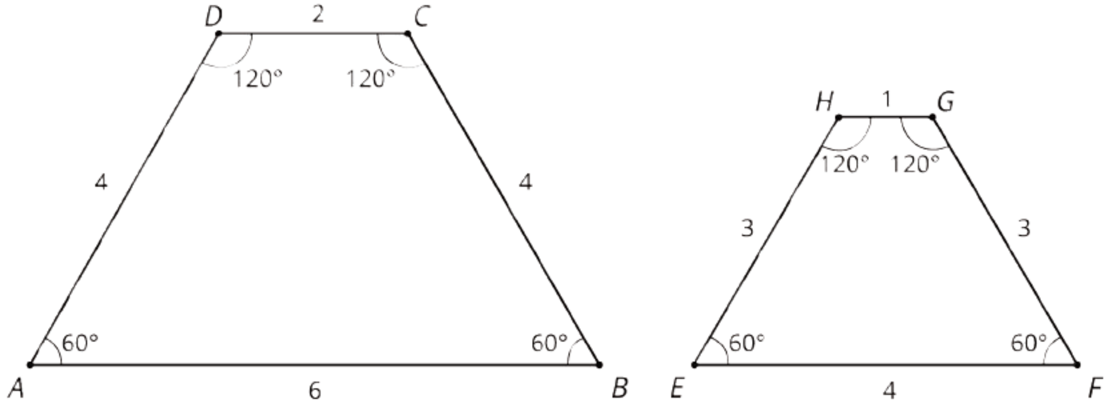
2. Record the corresponding distances in the table. What do you notice?

quadrilateral	horizontal distance	vertical distance
$XJYN$	$XY =$	$JN =$
$ZSCH$	$ZC =$	$SH =$

3. Measure at least three pairs of corresponding angles in $XJYN$ and $ZSCH$ using a protractor. Record your measurements to the nearest 5°. What do you notice?

4. Do these results change your answer to the first question? Explain.

5. Here are two more quadrilaterals.



Kiran says that Polygon $EFGH$ is a scaled copy of $ABCD$, but Lin disagrees. Do you agree with either of them? Explain or show your reasoning.

Student Response

1. Answers vary. Sample response: Noah is correct, because the corresponding angles are not equal. Mai may have noticed that the corresponding distances are multiplied by $\frac{3}{2}$ and thought this meant the polygons are similar.

2.

quadrilateral	horizontal distance	vertical distance
$XJYN$	$XY = 6$	$JN = 9$
$ZSCH$	$ZC = 4$	$SH = 6$

3. The corresponding angles are not all the same size. Rounded to the nearest 5°, the measures are:

$XJYN$	$ZSCH$
angle X measures 100°	angle Z measures 125°
angle J measures 40°	angle S measures 40°
angle Y measures 75°	angle C measures 75°
angle N measures 140°	angle H measures 115°

4. Since the corresponding angles are not equal, the polygons are definitely not scaled copies of one another.

5. Answers vary. Sample response: Lin is correct, because the corresponding distances are not multiplied by the same number (compared to $ABCD$, the top side in $EFGH$ is half as long, while the bottom side is two-thirds as long). Kiran may have noticed that the corresponding angles are equal and thought this meant the polygons are similar. I noticed that the scale factors for the corresponding sides are not the same. AB and EF are related by a scale factor of $\frac{2}{3}$, but DC and HG are related by a scale factor of $\frac{1}{2}$.

Are You Ready for More?

All side lengths of quadrilateral $MNOP$ are 2, and all side lengths of quadrilateral $QRST$ are 3. Does $MNOP$ have to be a scaled copy of $QRST$? Explain your reasoning.

Student Response

No. $MNOP$ could be a square and $QRST$ could be a rhombus that is not a square. Since the angles are different, $MNOP$ is not a scaled copy of $QRST$.

Activity Synthesis

The goal of this discussion is to make clear that angle measurements and distances are both important when deciding whether two polygons are scaled copies. To highlight the different arguments about whether one polygon is a scaled copy of another, consider debriefing with a role play. Ask four students to take on the roles of the four characters—Mai and Noah in the first question, and Kiran and Lin in the second—and make a brief argument about why they believe one figure is a scaled copy of the other in each case. Poll the class after each pair of cases are presented and find out with whom students agree.

Support for English Language Learners

Writing, speaking, and representing: MLR1 Stronger and Clearer Each Time. Use this routine to help students improve their writing, by providing them with multiple opportunities to clarify their explanations through conversation. At the appropriate time, give students time to meet with 2–3 partners to share and get feedback on their written justification for whether they agree with Mai or Noah. Display prompts for feedback that students can use to help their partner strengthen and clarify their ideas. For example, "Your explanation tells me . . .", "Can you say more about why you . . . ?", and "A detail (or word) you could add is _____, because . . . ." Give students with 3–4 minutes to revise their initial draft based on feedback from their peers.
Design Principle(s): Optimize output (for explanation)

4.4 Comparing Pictures of Birds

Optional: 10 minutes (there is a digital version of this activity)
In this activity, students use what they know about corresponding lengths and angles to show that one picture of a bird is *not* a scaled copy of the other. Unlike in previous tasks, minimal scaffolding is given here, so students need to decide what evidence is necessary to explain or show an absence of scaling. As students work, notice the different ways students use corresponding lengths and angles to think about scaling.

Identifying a specific measurement in one image and the corresponding measurements in the other (which show that one is not a scaled copy of the other) requires thinking about the structure of the pictures (MP7).

Monitor students for different approaches to show that one image is not a scaled copy of the other:

- Identifying corresponding angles in the two images which have different measures
- Identifying two pair of corresponding segments in the images with different scale factors

Select students who use these methods and ask them to present their approaches, in this sequence, during the discussion.

Addressing

- 7.G.A.1

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports

Launch

Arrange students in groups of 3–4. Provide access to geometry toolkits.

Give students 2 minutes of quiet work time followed by 1–2 minutes to discuss in groups how to find evidence that one picture is not a scaled copy of the other. Then, briefly discuss students' ideas as a class. Consider listing the ideas for all to see and discussing which ones are likely to be most effective.

If students' ideas deviate from drawing corresponding points and segments and comparing distances and angles, guide them with some prompts. For example:

- Pick a point that can be easily referenced (e.g., the tip of one wing) on one picture. Ask for the corresponding point on the other.
- Ask if that pair of corresponding points could help them determine if one picture is scaled from the other. If not, ask what else might be needed.
- Add another point and a segment connecting the two points. Ask if or how the segment could help, and so on.

Anticipated Misconceptions

Students may draw two segments that do not share a point, or choose non-corresponding points and segments on the two figures. Refer students to earlier work involving polygons and point out pairs of distances that could be used for comparison and those that could not be. Remind them that we can only compare the corresponding parts, not just any two parts.

Student Task Statement

Here are two pictures of a bird. Find evidence that one picture is not a scaled copy of the other. Be prepared to explain your reasoning.

Student Response

Answers vary. Sample reasoning:

- Corresponding angles do not match, so one picture cannot be a scaled copy of the other.
- The lengths of corresponding segments are not related by the same scale factor, so one picture cannot be a scaled copy of the other.

Activity Synthesis

Invite select students to share their reasoning. Ask how they knew that certain pairs of points in the images were corresponding points. Students could identify a unique feature such as the eyes or the tip of the wing. Make sure students understand that:

- Corresponding angles on an image and a scaled copy have the same measure

- Corresponding segments on an image and a scaled copy are related by the same scale factor

To conclude that an image is *not* a scaled copy of another, it is sufficient to find one pair of corresponding angle measures that are different or one pair of corresponding segments with different lengths.

Support for English Language Learners

Speaking: MLR8 Discussion Supports. Give students additional time to make sure that everyone in their group can explain or justify why the copy of the bird is not a scaled copy. Then, vary who is called on to represent the ideas of each group. This routine will prepare students for the role of group representative and to support each other to take on that role.
Design Principle(s): Optimize output (for explanation)

Lesson Synthesis

- Does a scale factor affect any other measurements other than segment lengths?
- How can we be sure that a figure is a scaled copy? What features do we check?

When a scaled copy is created from a figure, we know that:

- The distances between any two points in the original figure, even those not connected by segments, are scaled by the same scale factor.
- The corresponding angles in the original figure and scaled copies are congruent.

Polygons are a perfect context in which to apply these two ideas, being made up of line segments meeting at angles. So we can use these observations to check whether a polygon is actually a scaled copy of another. If all the corresponding angles are the same size and all corresponding distances are all scaled by the same factor, then we can conclude that it is a scaled copy of the other.

4.5 Corresponding Polygons

Cool Down: 5 minutes

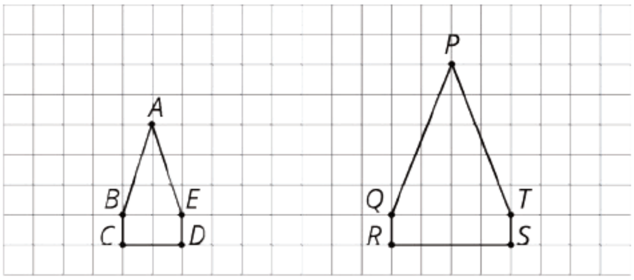
Addressing

- 7.G.A.1

Launch

Provide access to geometry toolkits.

> **Student Task Statement**
>
> Here are two polygons on a grid.



Is $PQRST$ a scaled copy of $ABCDE$? Explain your reasoning.

Student Response

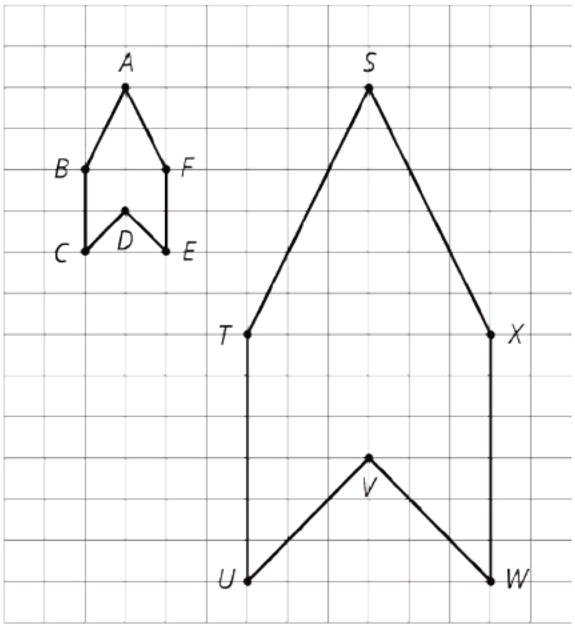
No. Sample explanation: $PQRST$ is not a scaled copy of $ABCDE$ because we need to use different scale factors when comparing corresponding lengths (1 for corresponding segments BC and QR and 2 for corresponding segments CD and RS). Also, not all of their corresponding angles are the same size. Angle A and angle P are not the same size.

Student Lesson Summary

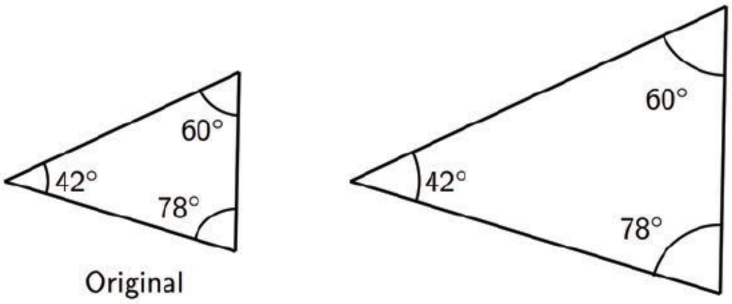
When a figure is a scaled copy of another figure, we know that:

- All distances in the copy can be found by multiplying the *corresponding distances* in the original figure by the same scale factor, whether or not the endpoints are connected by a segment.

  For example, Polygon $STUVWX$ is a scaled copy of Polygon $ABCDEF$. The scale factor is 3. The distance from T to X is 6, which is three times the distance from B to F.



- All angles in the copy have the same measure as the corresponding angles in the original figure, as in these triangles.



These observations can help explain why one figure is *not* a scaled copy of another.

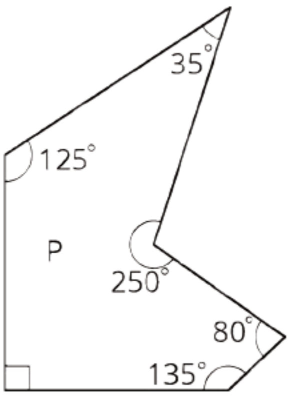
For example, even though their corresponding angles have the same measure, the second rectangle is not a scaled copy of the first rectangle, because different pairs of corresponding lengths have different scale factors, $2 \cdot \frac{1}{2} = 1$ but $3 \cdot \frac{2}{3} = 2$.

Lesson 4 Practice Problems

Problem 1

Statement

Select **all** the statements that must be true for *any* scaled copy Q of Polygon P.



A. The side lengths are all whole numbers.

B. The angle measures are all whole numbers.

C. Q has exactly 1 right angle.

D. If the scale factor between P and Q is $\frac{1}{5}$, then each side length of P is multiplied by $\frac{1}{5}$ to get the corresponding side length of Q.

E. If the scale factor is 2, each angle in P is multiplied by 2 to get the corresponding angle in Q.

F. Q has 2 acute angles and 3 obtuse angles.

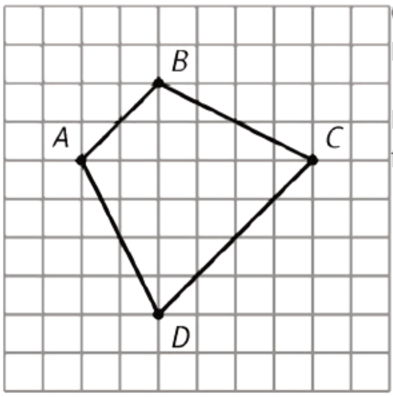
Solution

["B", "C", "D", "F"]

Problem 2

Statement

Here is Quadrilateral $ABCD$.



Quadrilateral $PQRS$ is a scaled copy of Quadrilateral $ABCD$. Point P corresponds to A, Q to B, R to C, and S to D.

If the distance from P to R is 3 units, what is the distance from Q to S? Explain your reasoning.

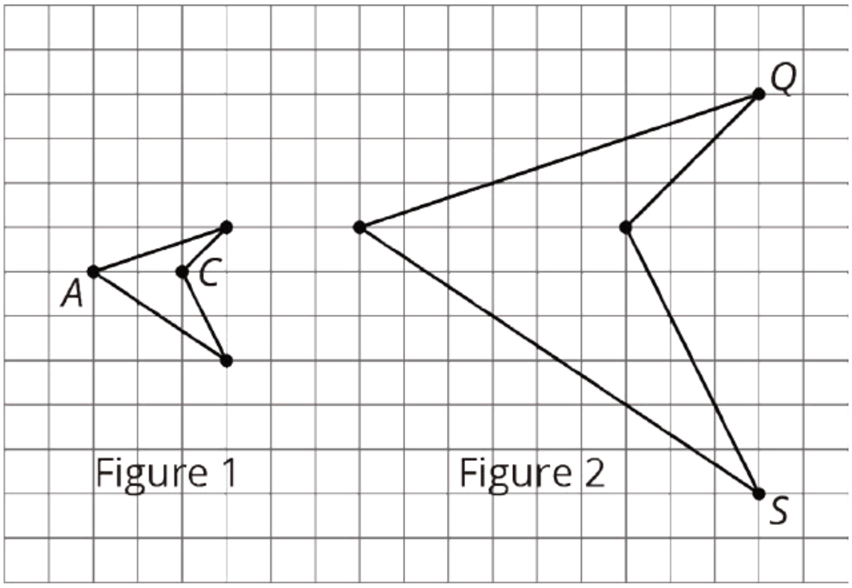
Solution

Since the lengths of AC and BD are 6, and AC corresponds to PR, the scale factor must be $\frac{1}{2}$. Since QS corresponds to BD, QS must also be 3 units long.

Problem 3

Statement

Figure 2 is a scaled copy of Figure 1.



a. Identify the points in Figure 2 that correspond to the points A and C in Figure 1. Label them P and R. What is the distance between P and R?

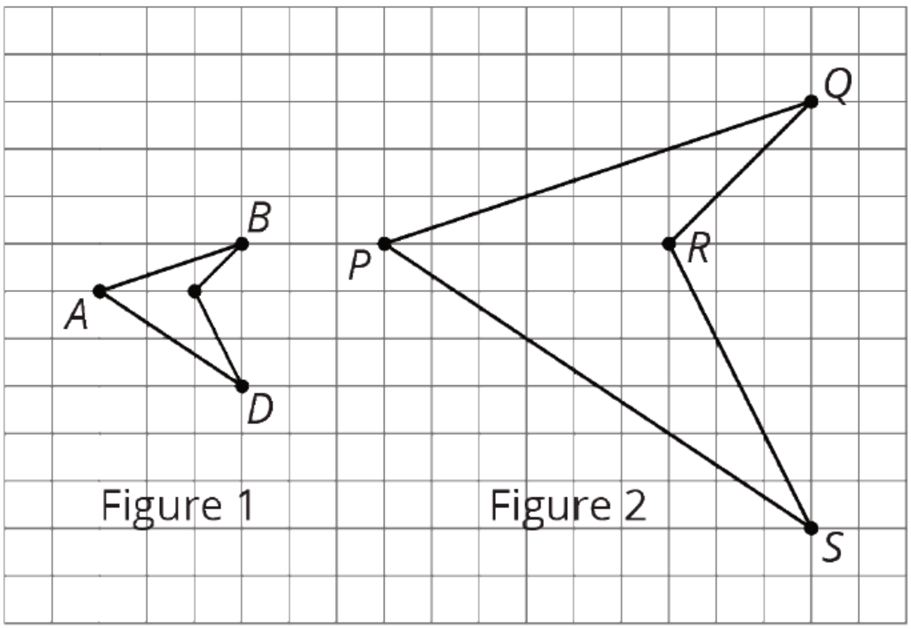
b. Identify the points in Figure 1 that correspond to the points Q and S in Figure 2. Label them B and D. What is the distance between B and D?

c. What is the scale factor that takes Figure 1 to Figure 2?

d. G and H are two points on Figure 1, but they are not shown. The distance between G and H is 1. What is the distance between the corresponding points on Figure 2?

Solution

a.



6 units

b. 3 units

c. 3 because distances between points in Figure 2 are three times the corresponding distances in Figure 1

d. 3 units because the scale factor is 3

Problem 4

Statement

To make 1 batch of lavender paint, the ratio of cups of pink paint to cups of blue paint is 6 to 5. Find two more ratios of cups of pink paint to cups of blue paint that are equivalent to this ratio.

Solution

Answers vary. Sample response: 12 cups of pink paint to 10 cups of blue paint and 18 cups of pink paint to 15 cups of blue paint. This is 2 batches and 3 batches, respectively, of this shade of lavender paint.

Lesson 5: The Size of the Scale Factor

Goals

- Describe (orally and in writing) how scale factors of 1, less than 1, and greater than 1 affect the size of scaled copies.
- Explain and show (orally and in writing) how to recreate the original figure given a scaled copy and its scale factor.
- Recognize (orally and in writing) the relationship between a scale factor of a scaled copy to its original figure is the "reciprocal" of the scale factor of the original figure to its scaled copy.

Learning Targets

- I can describe the effect on a scaled copy when I use a scale factor that is greater than 1, less than 1, or equal to 1.
- I can explain how the scale factor that takes Figure A to its copy Figure B is related to the scale factor that takes Figure B to Figure A.

Lesson Narrative

In this lesson, students deepen their understanding of scale factors in two ways:

1. They classify scale factors by size (less than 1, exactly 1, and greater than 1) and notice how each class of factors affects the scaled copies (MP8), and
2. They see that the scale factor that takes an original figure to its copy and the one that takes the copy to the original are reciprocals (MP7). This means that the scaling process is reversible, and that if Figure B is a scaled copy of Figure A, then Figure A is also a scaled copy of Figure B.

Students also continue to apply scale factors and what they learned about corresponding distances and angles to draw scaled copies without a grid.

Two of the activities, Scaling a Puzzle, and Missing Figure, Factor, or Copy, are optional. In Scaling a Puzzle, students scale the 6 pieces of a puzzle individually and then assemble them to make a scaled copy of the puzzle. The individual pieces are rectangular with line segments partitioning them into regions. Students need to think strategically about which measurements to take in order to scale the pieces accurately. In Missing Figure, Factor, or Copy, students gain fluency dealing with the different aspects of scaled copies, supplying the missing information in each case.

Alignments

Building On

- 5.NBT.B.6: Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

- 5.NF.B.4: Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
- 5.NF.B.5: Interpret multiplication as scaling (resizing), by:
- 6.NS.A.1: Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for $(2/3) \div (3/4)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9$ because $3/4$ of $8/9$ is $2/3$. (In general, $(a/b) \div (c/d) = ad/bc$.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?

Addressing

- 7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Building Towards

- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports
- Number Talk
- Think Pair Share

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Pre-printed slips, cut from copies of the blackline master

Required Preparation

Print and cut sets of slips for the sorting activity from the Scaled Copies Card Sort blackline master. Make enough copies so that each group of 3–4 students has a set. If possible, copy each complete set on a different color of paper, so that a stray slip can quickly be put back.

Print and cut puzzle pieces and blank squares for the Scaling a Puzzle activity from the Scaling a Puzzle blackline master. Make enough copies so that each group of 3 students has 1 original puzzle and 6 blank squares.

Make sure students have access to their geometry toolkits—especially rulers and protractors.

> **Student Learning Goals**
>
> Let's look at the effects of different scale factors.

5.1 Number Talk: Missing Factor

Warm Up: 10 minutes

This number talk encourages students to use structure and the relationship between multiplication and division to mentally solve problems involving fractions. It prompts students to think about how the size of factors impacts the size of the product. It reviews the idea of reciprocal factors in preparation for the work in the lesson.

Building On

- 5.NBT.B.6
- 5.NF.B.4
- 5.NF.B.5
- 6.NS.A.1

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Ask students what operation is meant when a number and a variable are placed right next to each other in an equation. (Multiplication)

Display one problem at a time. Give students 1 minute of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Follow with a whole-class discussion.

Support for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organization

Anticipated Misconceptions

Students might think that a product cannot be less than one of the factors, not realizing that one of the factors can be a fraction. Use examples involving smaller and familiar numbers to remind them that it is possible. Ask, for example, "What times 10 is 5?"

Student Task Statement

Solve each equation mentally.

$16x = 176$

$16x = 8$

$16x = 1$

$\frac{1}{5}x = 1$

$\frac{2}{5}x = 1$

Student Response

- 11. Possible strategy: $16 \cdot 10 = 160$ and $16 \cdot 1 = 16$, so $16 \cdot 11 = 176$
- $\frac{1}{2}$ or equivalent. Possible strategy: 16 divided by 2 is 8, and dividing by 2 is the same as multiplying by $\frac{1}{2}$.
- $\frac{1}{16}$. Possible strategy: It takes 16 of $\frac{1}{16}$ to make 1.
- 5. Possible strategy: There are 5 copies of $\frac{1}{5}$ in 1, so $5 \cdot \frac{1}{5} = 1$.
- $\frac{5}{2}$ or equivalent. Possible strategy: 5 groups of $\frac{2}{5}$ make 2, so $\frac{5}{2}$ groups of $\frac{2}{5}$ make 1.

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their explanations for all to see. To involve more students in the conversation, consider asking:

- Who can restate ___'s reasoning in a different way?
- Did anyone solve the problem the same way but would explain it differently?

- Did anyone solve the problem in a different way?
- Does anyone want to add on to _____'s strategy?
- Do you agree or disagree? Why?

Highlight that multiplying a factor by a fraction less than 1 results in a product that is less than one of the factors, and that two factors that multiply to be 1 are reciprocals.

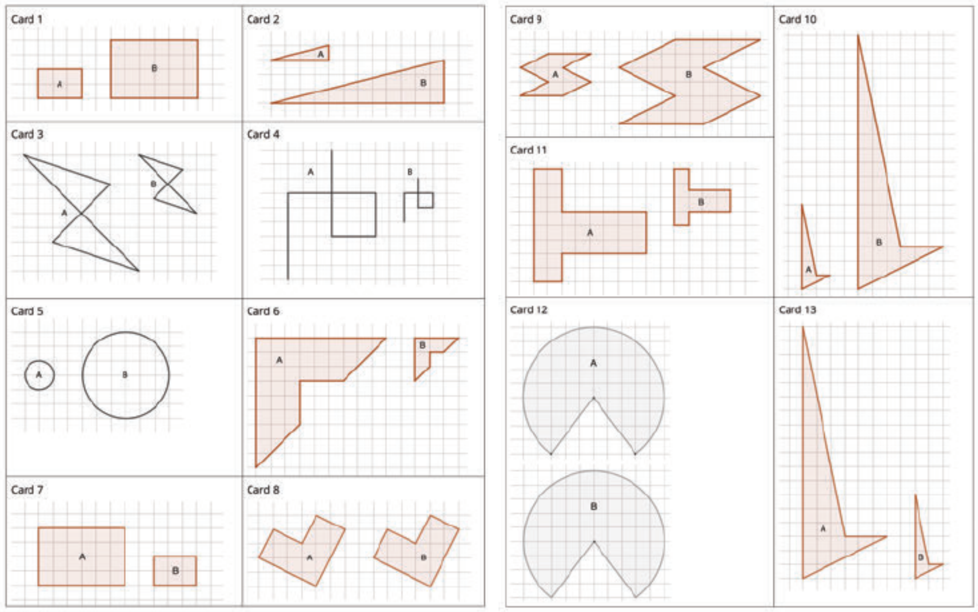
Support for English Language Learners

Speaking: *MLR8 Discussion Supports.*: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimize output (for explanation)

5.2 Card Sort: Scaled Copies

15 minutes
Students have studied many examples of scaled copies and know that corresponding lengths in a figure and its scaled copy are related by the same scale factor. The purpose of this activity is for students to examine how the *size* of the scale factor is related to the original figure and the scaled copy. The activity serves several purposes:

1. To reinforce students' awareness of scale factors
2. To draw attention to how scaled copies behave when the scale factor is 1, less than 1, and greater than 1; and
3. To help students notice that reciprocal scale factors reverse the scaling.



You will need the Scaled Copies Card Sort blackline master for this activity. Here is an image of the cards for your reference and planning.

Monitor for students who group the cards in terms of:

- Specific scale factors (e.g., 2, 3, $\frac{1}{2}$, etc.)
- Ranges of scale factors producing certain effects (e.g., factors producing larger, unchanged, or smaller copies)
- Reciprocal scale factors (e.g., one factor scales Figure A to B, and its reciprocal reverses the scaling)

Select groups who use each of these approaches (and any others) and ask them to share during the discussion.

Building On

- 5.NF.B.5

Addressing

- 7.G.A.1

Building Towards

- 7.RP.A.2

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR3: Clarify, Critique, Correct

Launch

Arrange students in groups of 3–4. Distribute one set of slips to each group. Give students 7–8 minutes of group work time, followed by whole-class discussion.

Support for Students with Disabilities

Action and Expression: Internalize Executive Functions. Begin with a small-group or whole-class demonstration and think aloud using Card 1 to remind students how to determine the scale factor between Figure A and Figure B. Keep the worked-out calculations on display for students to reference as they work.
Supports accessibility for: Memory; Conceptual processing

Anticipated Misconceptions

Students may sort by the types of figures rather than by how the second figure in each pair is scaled from the first. Remind students to sort based on how Figure A is scaled to create Figure B.

Students may think of the change in lengths between Figures A and B in terms of addition or subtraction, rather than multiplication or division. Remind students of an earlier lesson in which they explored the effect of subtracting the same length from each side of a polygon in order to scale it. What happened to the copy? (It did not end up being a polygon and was not a scaled copy of the original one.)

Students may be unclear as to how to describe how much larger or smaller a figure is, or may not recall the meaning of scale factor. Have them compare the lengths of each side of the figure. What is the common factor by which each side is multiplied?

Student Task Statement

Your teacher will give you a set of cards. On each card, Figure A is the original and Figure B is a scaled copy.

1. Sort the cards based on their scale factors. Be prepared to explain your reasoning.
2. Examine cards 10 and 13 more closely. What do you notice about the shapes and sizes of the figures? What do you notice about the scale factors?
3. Examine cards 8 and 12 more closely. What do you notice about the figures? What do you notice about the scale factors?

Student Response

1. Grouping categories vary. Sample categories:
    - Scale factor of 2: Cards 1 and 9.
    - Scale factor of $\frac{1}{2}$: Cards 3, 7, and 11.

- Scale factor of 3: Cards 2, 5, and 10.
- Scale factor of $\frac{1}{3}$: Cards 4, 6, and 13.
- Scale factor of 1: Cards 8 and 12.

2. Answers vary. Sample response: The shapes on both cards are the same, but on Card 10, the scaled copy is larger and the scale factor is 3. On Card 13, the scaled copy is smaller and the scale factor is $\frac{1}{3}$.

3. Answers vary. Sample response: The original and the copy are the same size on Cards 8 and 12. The copy is identical to the original. The scale factor is 1.

Are You Ready for More?

Triangle B is a scaled copy of Triangle A with scale factor $\frac{1}{2}$.

1. How many times bigger are the side lengths of Triangle B when compared with Triangle A?
2. Imagine you scale Triangle B by a scale factor of $\frac{1}{2}$ to get Triangle C. How many times bigger will the side lengths of Triangle C be when compared with Triangle A?
3. Triangle B has been scaled once. Triangle C has been scaled twice. Imagine you scale triangle A n times to get Triangle N, always using a scale factor of $\frac{1}{2}$. How many times bigger will the side lengths of Triangle N be when compared with Triangle A?

Student Response

1. $\frac{1}{2}$
2. $\frac{1}{4}$
3. $\left(\frac{1}{2}\right)^n$

Activity Synthesis

Select groups to explain their sorting decisions following the sequence listed in the Activity Narrative. If no groups sorted in terms of ranges of scale factors (less than 1, exactly 1, and greater than 1) or reciprocal scaling, ask:

- What can we say about the scale factors that produce larger copies? Smaller copies? Same-size copies?
- Some cards had the same pair of figures on them, just in a reversed order (i.e., pairs #1 and 7, #10 and 13). What do you notice about their scale factors?

Highlight the two main ideas of the lesson: 1) the effects of scale factors that are greater than 1, exactly 1, and less than 1; and 2) the reversibility of scaling. Point out that if Figure B is a scaled copy of Figure A, then A is also a scaled copy of B. In other words, A and B are scaled copies of one another, and their scale factors are reciprocals.

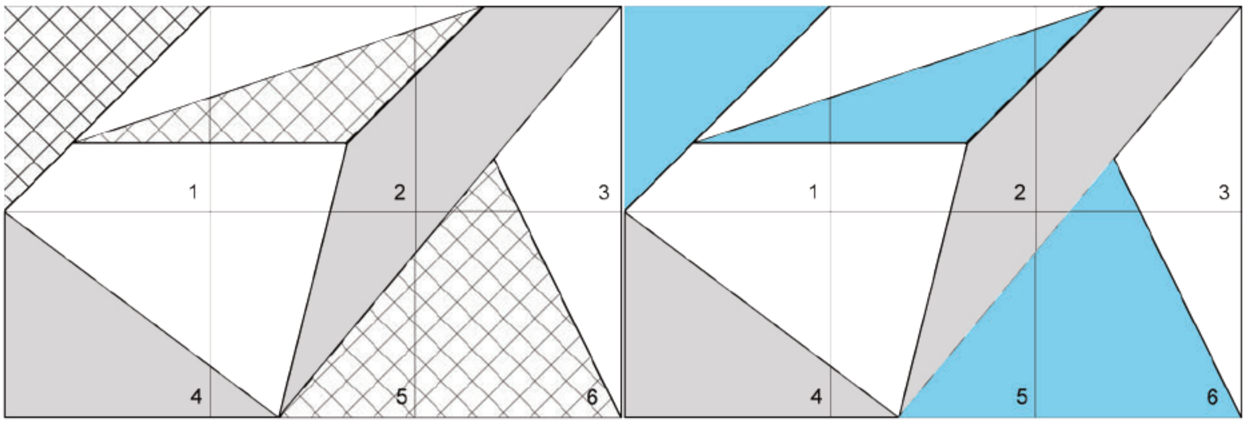
Suggest students add these observations to their answer for the last question.

Support for English Language Learners

Writing: MLR3 Clarify, Critique, Correct. Present an incorrect statement that reflects a possible misunderstanding from the class for the last prompt. For example, "The scale factor of cards 8 and 12 is 0 because the shapes are the same and there was no change." Prompt students to identify the error, and then write a correct version. In this discussion, highlight the use of disciplinary language by revoicing student ideas. This helps students evaluate, and improve on, the written mathematical arguments of others.
Design Principle(s): Maximize meta-awareness

5.3 Scaling A Puzzle

Optional: 15 minutes
This activity gives students a chance to apply what they know about scale factors, lengths, and angles and create scaled copies without the support of a grid. Students work in groups of 3 to complete a jigsaw puzzle, each group member scaling 2 non-adjacent pieces of a 6-piece puzzle with a scale factor of $\frac{1}{2}$. The group then assembles the scaled pieces and examines the accuracy of their scaled puzzle. Consider having students use a color in place of the cross hatching.



As students work, notice how they measure distances and whether they consider angles. Depending on how students determine scaled distances, they may not need to transfer angles. Look out for students who measure only the lengths of drawn segments rather than distances, e.g., between the corner of a square and where a segment begins. Suggest that they consider other measurements that might help them locate the beginning and end of a segment.

You will need the Scaling a Puzzle blackline master for this activity.

Addressing

- 7.G.A.1

Building Towards

- 7.RP.A.2

Launch

Arrange students in groups of 3. Give pre-cut puzzle squares 1 and 5 to one student in the group, squares 2 and 6 to a second student, and squares 3 and 4 to the third. After students have answered the first question, give each student 2 blank squares cut from the second section of the blackline master, whose sides are half of the side length of the puzzle squares. Provide access to geometry toolkits.

Support for Students with Disabilities

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. Before students begin, ask them to predict which tools, from their geometry toolkits, they anticipate they will need, and to describe how they might use them. During the activity, make sure that students are using their rulers and protractors correctly.
Supports accessibility for: Visual-spatial processing; Conceptual processing; Fine-motor skills

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Anticipated Misconceptions

Students may incorporate the scale factor when scaling line segments but neglect to do so when scaling distances between two points not connected by a segment. Remind them that all distances are scaled by the same factor.

Students may not remember to verify that the angles in their copies must remain the same as the original. Ask them to notice the angles and recall what happens to angles when a figure is a scaled copy.

Student Task Statement

Your teacher will give you 2 pieces of a 6-piece puzzle.

1. If you drew scaled copies of your puzzle pieces using a scale factor of $\frac{1}{2}$, would they be larger or smaller than the original pieces? How do you know?
2. Create a scaled copy of each puzzle piece on a blank square, with a scale factor of $\frac{1}{2}$.
3. When everyone in your group is finished, put all 6 of the original puzzle pieces together like this:

1	2	3
4	5	6

Next, put all 6 of your scaled copies together. Compare your scaled puzzle with the original puzzle. Which parts seem to be scaled correctly and which seem off? What might have caused those parts to be off?

4. Revise any of the scaled copies that may have been drawn incorrectly.

5. If you were to lose one of the pieces of the original puzzle, but still had the scaled copy, how could you recreate the lost piece?

Student Response

1. They would be smaller because $\frac{1}{2} < 1$.
2. Copies of puzzle pieces scaled by $\frac{1}{2}$.
3. Answers vary depending on how each student drew their pieces.
4. Revised copies of puzzle pieces.
5. Use the copy and a scale factor of 2 to recreate the original puzzle piece. Sample explanation: 2 is the reciprocal of $\frac{1}{2}$, so it would scale the copy back to the original.

Activity Synthesis

Much of the conversations about creating accurate scaled copies will have taken place among partners, but consider coming together as a class to reflect on the different ways students worked. Ask questions such as:

- How is this task more challenging than creating scaled copies of polygons on a grid?
- Besides distances or lengths, what helped you create an accurate copy?
- How did you know or decide which distances to measure?
- Before your drawings were assembled, how did you check if they were correct?

Student responses to these questions may differ: for example, for piece 6, the two lines can be drawn by measuring distances on the border of the puzzle piece; the angles work out correctly automatically. For piece 2, however, to get the three lines that meet in a point in the middle of the piece just right, students can either measure angles, or extend those line segments until they meet the border of the piece (and then measure distances).

5.4 Missing Figure, Factor, or Copy

Optional: 10 minutes

In this activity, students investigate different aspects of the figure, scaled copy, and scale factor trio. Given any two of these three, we can find the third. Students work with these different scenarios on a grid, dealing with scale factors greater than 1, less than 1, and equal to 1. In addition, they create a scaled copy of a non-polygonal figure off of a grid.

Both on and off of a grid, students need to decide what tools to use (MP5) in order to measure angles and side lengths in order to produce the scaled copies.

Addressing

- 7.G.A.1

Building Towards

- 7.RP.A.2

Instructional Routines

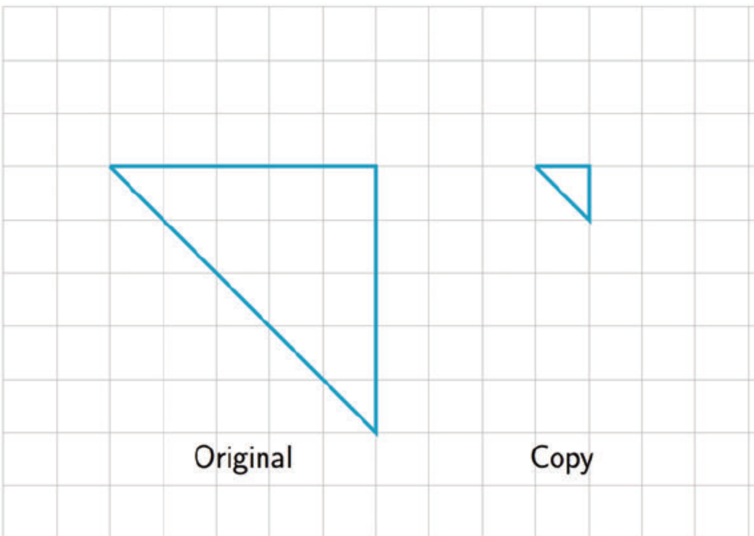
- MLR8: Discussion Supports
- Think Pair Share

Launch

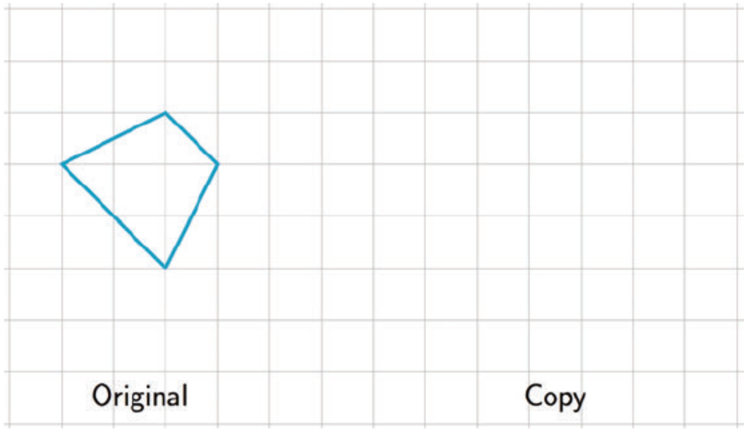
Arrange students in groups of 2. Provide access to geometry toolkits. Give students 3–4 minutes of quiet work time, followed by 2 minutes of group discussion and then whole-class discussion.

Student Task Statement

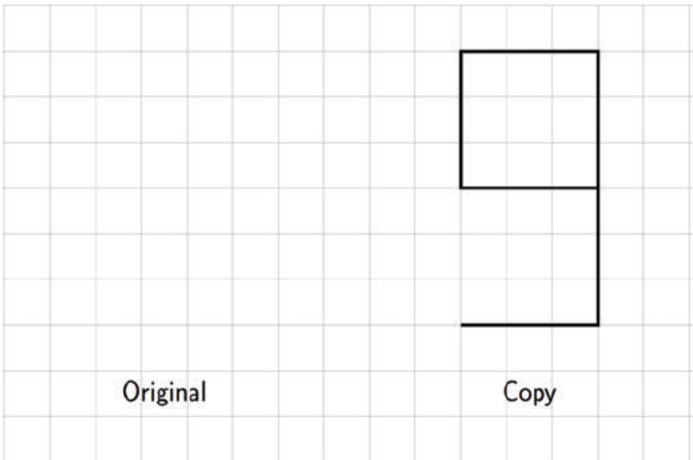
1. What is the scale factor from the original triangle to its copy? Explain or show your reasoning.



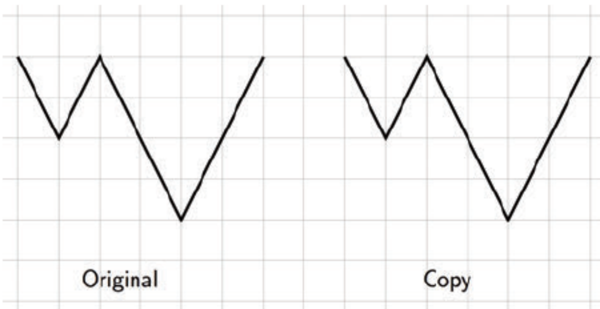
2. The scale factor from the original trapezoid to its copy is 2. Draw the scaled copy.



3. The scale factor from the original figure to its copy is $\frac{3}{2}$. Draw the original figure.



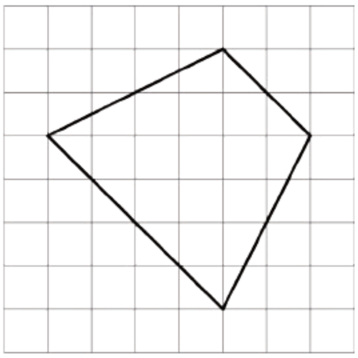
4. What is the scale factor from the original figure to the copy? Explain how you know.



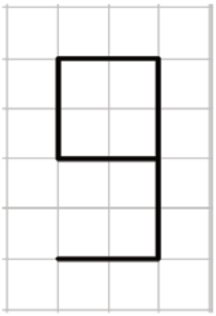
5. The scale factor from the original figure to its scaled copy is 3. Draw the scaled copy.

Student Response

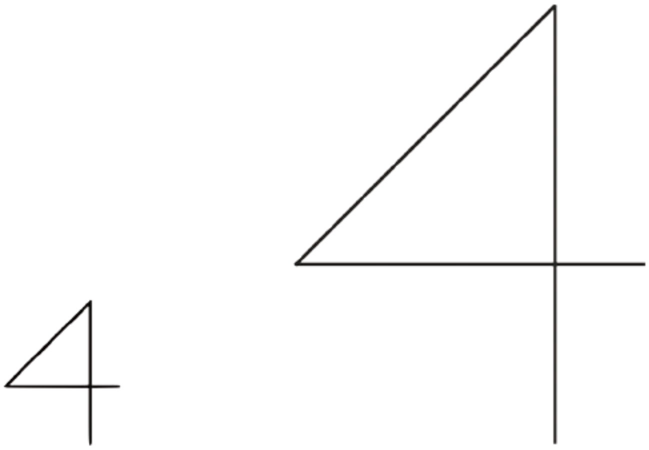
1. Scale factor: $\frac{1}{5}$ since the vertical and horizontal sides on the original are 5 grid units in length and the corresponding sides on the copy are 1 grid unit in length.
2. Scaled copy:



3. Original figure:



4. Scale factor: 1 since the figure and the copy are the same size.



5. Original Copy

Activity Synthesis

The purpose of this discussion is to ensure that students understand the connections between the original figure, scaled copy, and scale factor—given any two of the the three, they should understand a method for finding the third.

Ask students how they solved the first three questions. Which one was most challenging? Why? Individual responses will likely vary here, but the question with the missing original figure deserves special attention. This requires going backward since we have the scaled copy and the scale factor. This scenario also reinforces that the way to "undo" the scale factor of $\frac{3}{2}$ that has been applied to produce the copy is to apply a scale factor of $\frac{2}{3}$.

For the last problem, ask students how they used the segment lengths and angles of the original figure. They may have sketched the copy without checking angles. Ask them why measuring angles is important.

Support for English Language Learners

Speaking, Listening: MLR8 Discussion Supports. Use this routine to support whole-class discussion as students share their strategies and discuss which strategies they found to be easier or more challenging. Press for details in students' explanations by requesting that students challenge an idea, elaborate on an idea, or give an example. Revoice student ideas to demonstrate mathematical language use by restating a statement as a question in order to clarify, apply appropriate language, and involve more students.
Design Principle(s): Support sense-making

Lesson Synthesis

- What happens to the copy when it is created with a scale factor greater than 1? Less than 1? Exactly 1?
- How can we reverse the scaling to get back to the original figure when we have a scaled copy?

When the scale factor is greater than 1, the scaled copy is larger than the original. When it is less than 1, the copy is smaller than the original. A scale factor of exactly 1 produces a same-size copy.

Scaling can be reversed by using reciprocal factors. If we scale Figure A by a factor of 4 to obtain Figure B, we can scale B back to A using a factor of $\frac{1}{4}$. This means that if B is a scaled copy of A, A is also a scaled copy of B; they are scaled copies of each other.

5.5 Scaling a Rectangle

Cool Down: 5 minutes

Addressing

- 7.G.A.1

Building Towards

- 7.RP.A.2

Student Task Statement

A rectangle that is 2 inches by 3 inches has been scaled by a factor of 7.

1. What are the side lengths of the scaled copy?
2. Suppose you want to scale the copy back to its original size. What scale factor should you use?

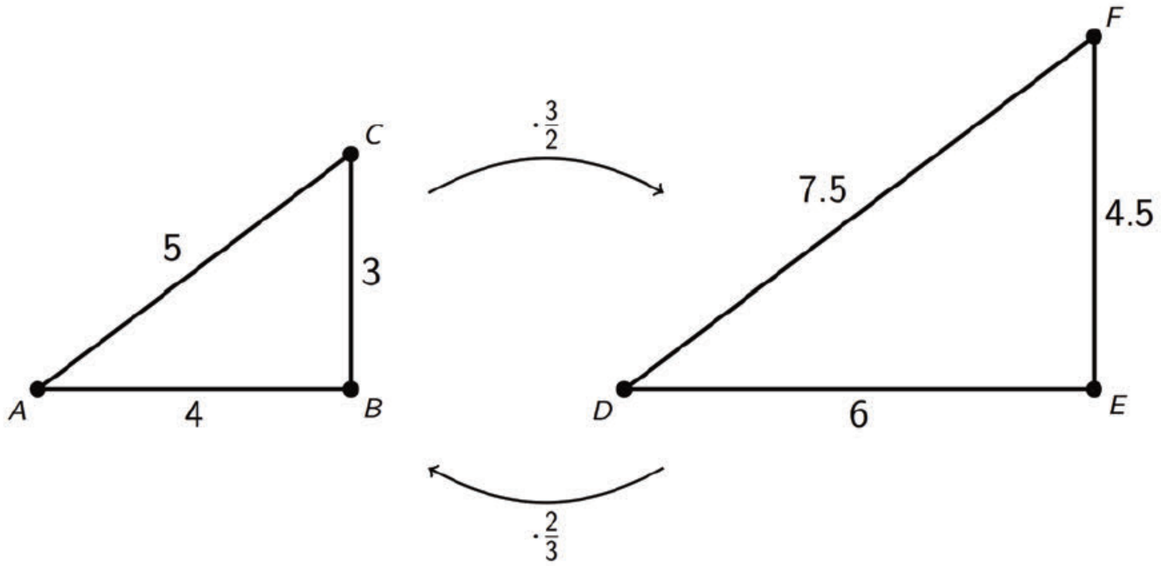
Student Response

1. 14 inches by 21 inches, because $2 \cdot 7 = 14$ and $3 \cdot 7 = 21$.
2. $\frac{1}{7}$, because it is the reciprocal of 7.

Student Lesson Summary

The size of the scale factor affects the size of the copy. When a figure is scaled by a scale factor greater than 1, the copy is larger than the original. When the scale factor is less than 1, the copy is smaller. When the scale factor is exactly 1, the copy is the same size as the original.

Triangle DEF is a larger scaled copy of triangle ABC, because the scale factor from ABC to DEF is $\frac{3}{2}$. Triangle ABC is a smaller scaled copy of triangle DEF, because the scale factor from DEF to ABC is $\frac{2}{3}$.



This means that triangles ABC and DEF are scaled copies of each other. It also shows that scaling can be reversed using **reciprocal** scale factors, such as $\frac{2}{3}$ and $\frac{3}{2}$.

In other words, if we scale Figure A using a scale factor of 4 to create Figure B, we can scale Figure B using the reciprocal scale factor, $\frac{1}{4}$, to create Figure A.

Glossary

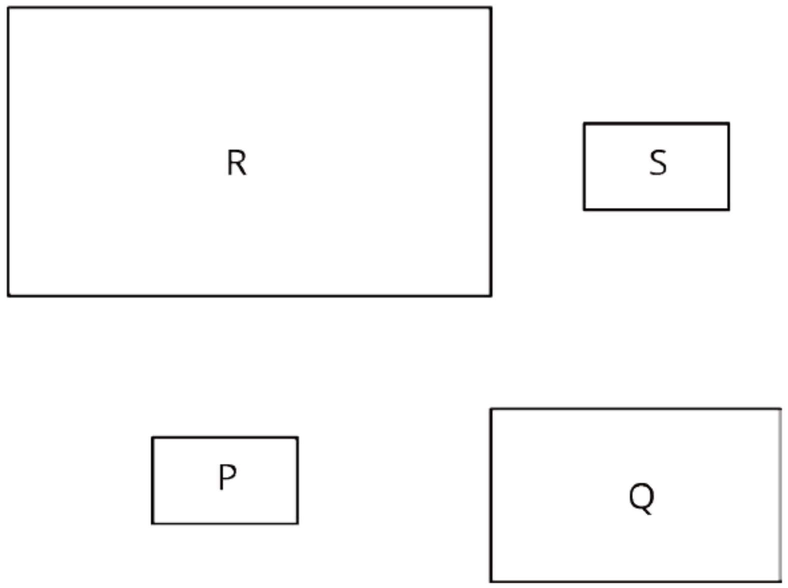
- reciprocal

Lesson 5 Practice Problems

Problem 1

Statement

Rectangles P, Q, R, and S are scaled copies of one another. For each pair, decide if the scale factor from one to the other is greater than 1, equal to 1, or less than 1.



a. from P to Q

b. from P to R

c. from Q to S

d. from Q to R

e. from S to P

f. from R to P

g. from P to S

Solution

a. Greater than 1

b. Greater than 1

c. Less than 1

d. Greater than 1

e. Equal to 1

f. Less than 1

g. Equal to 1

Problem 2

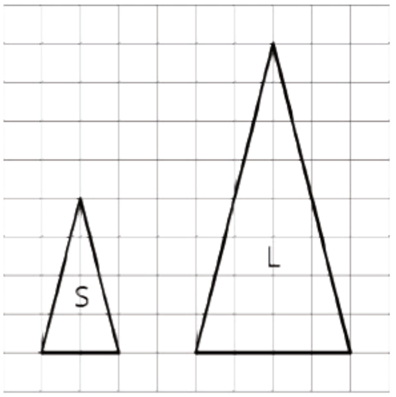
Statement

Triangle S and Triangle L are scaled copies of one another.

a. What is the scale factor from S to L?

b. What is the scale factor from L to S?

c. Triangle M is also a scaled copy of S. The scale factor from S to M is $\frac{3}{2}$. What is the scale factor from M to S?



Solution

a. 2

b. $\frac{1}{2}$

c. $\frac{2}{3}$. The two scale factors are reciprocals of each other.

Problem 3

Statement

Are two squares with the same side lengths scaled copies of one another? Explain your reasoning.

Solution

Yes. There is a scale factor of 1 between them.

Problem 4

Statement

Quadrilateral A has side lengths 2, 3, 5, and 6. Quadrilateral B has side lengths 4, 5, 8, and 10. Could one of the quadrilaterals be a scaled copy of the other? Explain.

Solution

No. For the shortest sides to match up, the scale factor from A to B would have to be 2. But scaling the side of A with length 3 by a factor of 2 would give a side of length 6, which doesn't match any of the side lengths of B.

(From Unit 1, Lesson 2.)

Problem 5

Statement

Select **all** the ratios that are equivalent to the ratio $12 : 3$.

A. 6 : 1

B. 1 : 4

C. 4 : 1

D. 24 : 6

E. 15 : 6

F. 1,200 : 300

G. 112 : 13

Solution

["C", "D", "F"]

Lesson 6: Scaling and Area

Goals

- Calculate and compare (orally and in writing) the areas of multiple scaled copies of the same shape.
- Generalize (orally) that the area of a scaled copy is the product of the area of the original figure and the “square” of the scale factor.
- Recognize that a two-dimensional attribute, like area, scales at a different rate than one-dimensional attributes, like length and distance.

Learning Targets

- I can describe how the area of a scaled copy is related to the area of the original figure and the scale factor that was used.

Lesson Narrative

This lesson is optional. In this lesson, students are introduced to how the area of a scaled copy relates to the area of the original shape. Students build on their grade 6 work with exponents to recognize that the area increases by the square of the scale factor by which the sides increased. Students will continue to work with the area of scaled shapes later in this unit and in later units in this course. Although the lesson is optional, it will be particularly helpful for students to have already had this introduction when they study the area of circles in a later unit.

In two of the activities in this lesson, students build scaled copies using pattern blocks as units of area. This work with manipulatives helps accustom students to a pattern that many find counterintuitive at first (MP8). (It is a common but false assumption that the area of scaled copies increases by the same scale factor as the sides.) After that, students calculate the area of scaled copies of parallelograms and triangles to apply the patterns they discovered in the hands-on activities (MP7).

Alignments

Building On

- 6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Addressing

- 7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

- 7.G.B.6: Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Building Towards

- 7.G.B.4: Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
- 7.G.B.6: Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
- 7.RP.A.2.a: Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Think Pair Share

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Pattern blocks
Pre-printed slips, cut from copies of the blackline master

Required Preparation

Prepare to distribute the pattern blocks, at least 16 blue rhombuses, 16 green triangles, 10 red trapezoids, and 7 yellow hexagons per group of 3–4 students.

Copy and cut up the Area of Scaled Parallelograms and Triangles blackline master so each group of 2 students can get 1 of the 2 shapes.

Student Learning Goals

Let's build scaled shapes and investigate their areas.

6.1 Scaling a Pattern Block

Warm Up: 10 minutes (there is a digital version of this activity)

By now, students understand that lengths in a scaled copy are related to the original lengths by the scale factor. Here they see that the area of a scaled copy is related to the original area by the *square* of the scale factor.

Students build scaled copies of a single pattern block, using blocks of the same shape to do so. They determine how many blocks are needed to create a copy at each specified scale factor. Each pattern block serves as an informal unit of area. Because each original shape has an area of 1 block, the $(\text{scale factor})^2$ pattern for the area of a scaled copy is easier to recognize.

Students use the same set of scale factors to build copies of three different shapes (a rhombus, a triangle, and a hexagon). They notice regularity in their repeated reasoning and use their observations to predict the number of blocks needed to build other scaled copies (MP8).

Addressing

- 7.G.A.1
- 7.G.B.6

Building Towards

- 7.RP.A.2.a

Launch

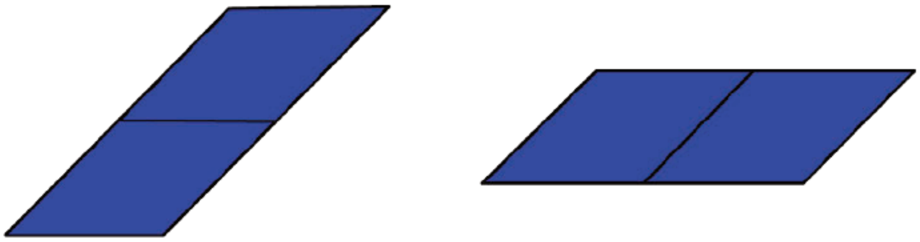
Arrange students in groups of 3–4. Distribute pattern blocks and ask students to use them to build scaled copies of each shape as described in the task. Each group would need at most 16 blocks each of the green triangle, the blue rhombus, and the red trapezoid. If there are not enough for each group to have a full set with 16 each of the green, blue, and red blocks, consider rotating the blocks of each color through the groups, or having students start with 10 blocks of each and ask for more as needed.

Give students 6–7 minutes to collaborate on the task and follow with a whole-class discussion. Make sure all students understand that "twice as long" means "2 times as long."

Using real pattern blocks is preferred, but the Digital Activity can replace the manipulatives if they are unavailable.

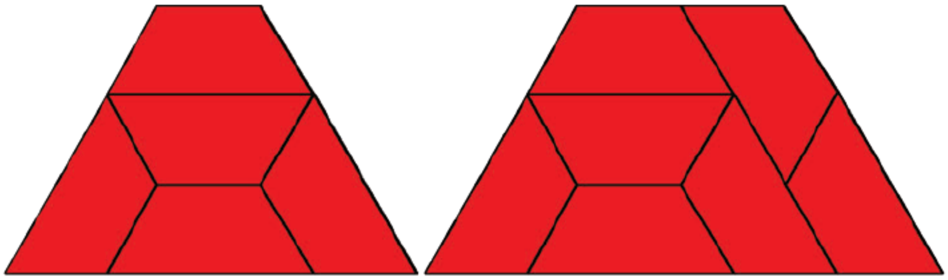
Anticipated Misconceptions

Some students may come up with one of these arrangements for the first question, because they assume the answer will take 2 blocks to build:



You could use one pattern block to demonstrate measuring the lengths of the sides of their shape, to show them which side they have not doubled.

Students may also come up with:



for tripling the trapezoid, because they triple the height of the scaled copy but they do not triple the length. You could use the process described above to show that not all side lengths have tripled.

Student Task Statement

Your teacher will give you some pattern blocks. Work with your group to build the scaled copies described in each question.

1. How many blue rhombus blocks does it take to build a scaled copy of Figure A:

    a. Where each side is twice as long?

    b. Where each side is 3 times as long?

    c. Where each side is 4 times as long?

2. How many green triangle blocks does it take to build a scaled copy of Figure B:

    a. Where each side is twice as long?

    b. Where each side is 3 times as long?

    c. Using a scale factor of 4?

3. How many red trapezoid blocks does it take to build a scaled copy of Figure C:

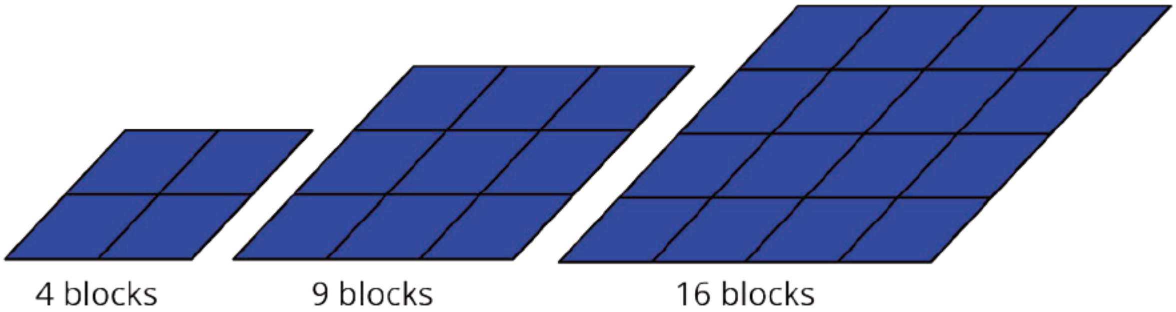
a. Using a scale factor of 2?

b. Using a scale factor of 3?

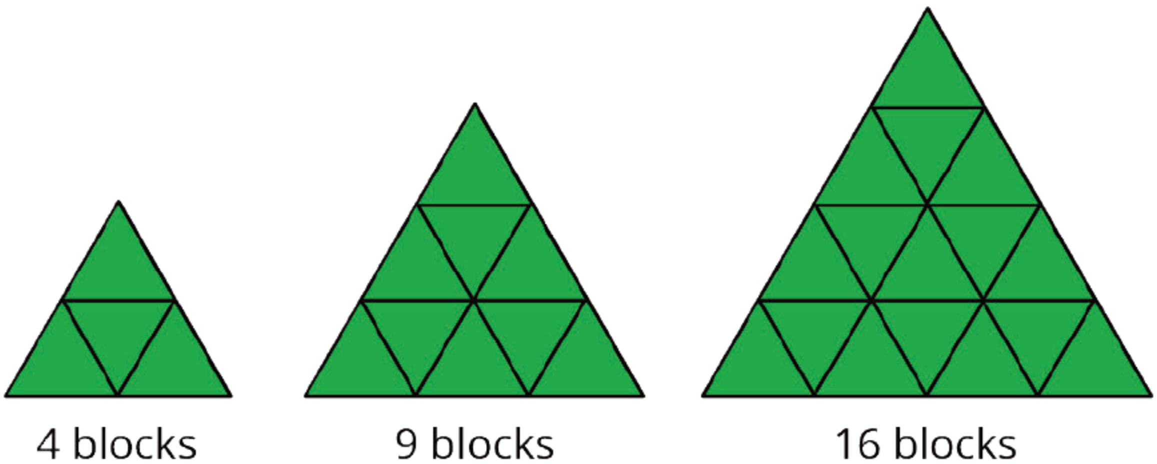
c. Using a scale factor of 4?

Student Response

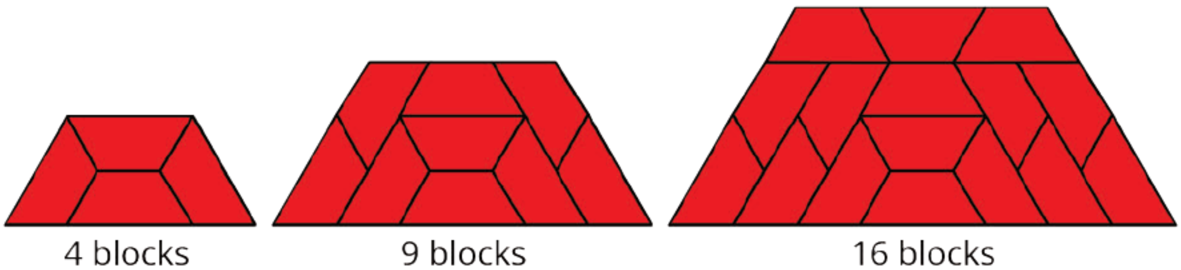
1.



2.



3.



Activity Synthesis

Display a table with only the column headings filled in. For the first four rows, ask different students to share how many blocks it took them to build each shape and record their answers in the table.

scale factor	number of blocks to build Figure A	number of blocks to build Figure B	number of blocks to build Figure C
1			
2			
3			
4			
5			
10			
s			
$\frac{1}{2}$			

To help student notice, extend, and generalize the pattern in the table, guide a discussion using questions such as these:

- In the table, how is the number of blocks related to the scale factor? Is there a pattern?
- How many blocks are needed to build scaled copies using scale factors of 5 or 10? How do you know?
- How many blocks are needed to build a scaled copy using any scale factor s?
- If we want a scaled copy where each side is half as long, how much of a block would it take? How do you know? Does the same rule still apply?

If not brought up by students, highlight the fact that the number of blocks it took to build each scaled shape equals the scale factor times itself, regardless of the shape (look at the table row for s). This rule applies to any factor, including those that are less than 1.

6.2 Scaling More Pattern Blocks

Optional: 10 minutes (there is a digital version of this activity)
This activity extends the conceptual work of the previous one by adding a layer of complexity. Here, the original shapes are comprised of more than 1 block, so the number of blocks needed to build their scaled copies is not simply $(\text{scale factor})^2$, but rather $n \times (\text{scale factor})^2$, where n is the number of blocks in the original shape. Students begin to think about how the scaled area relates to the original area, which is no longer 1 area unit. They notice that the pattern $(\text{scale factor})^2$ presents itself in the factor by which the original number of blocks has changed, rather than in the total number of blocks in the copy.

As in the previous task, students observe regularity in repeated reasoning (MP8), noticing that regardless of the shapes, starting with n pattern blocks and scaling by s uses ns^2 pattern blocks.

Also as in the previous task, the shape composed of trapezoids might be more challenging to scale than those composed of rhombuses and triangles. Prepare to support students scaling the red shape by offering some direction or additional time, if feasible.

As students work, monitor for groups who notice that the pattern of squared scale factors still occurs here, and that it is apparent if the original number of blocks is taken into account. Select them to share during class discussion.

Addressing

- 7.G.A.1
- 7.G.B.6

Building Towards

- 7.G.B.4

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports

Launch

Keep students in the same groups, or form combined groups if there are not enough blocks. Assign one shape for each group to build (or let groups choose a shape, as long as all 3 shapes are equally represented). To build a copy of each given shape using a scale factor of 2, groups will need 12 blue rhombuses, 8 red trapezoids, or 16 green triangles. To completely build a copy of each given shape with a scale factor of 3, they would need 27 blue rhombuses, 18 red trapezoids, and 36 green triangles; however, the task prompts them to stop building when they know what the answer will be.

Give students 6–7 minutes to build their shapes and complete the task. Remind them to use the same blocks as those in the original shape and to check the side lengths of each built shape to make sure they are properly scaled.

Using real pattern blocks is preferred, but the Digital Activity can replace the manipulatives if they are unavailable.

Support for Students with Disabilities

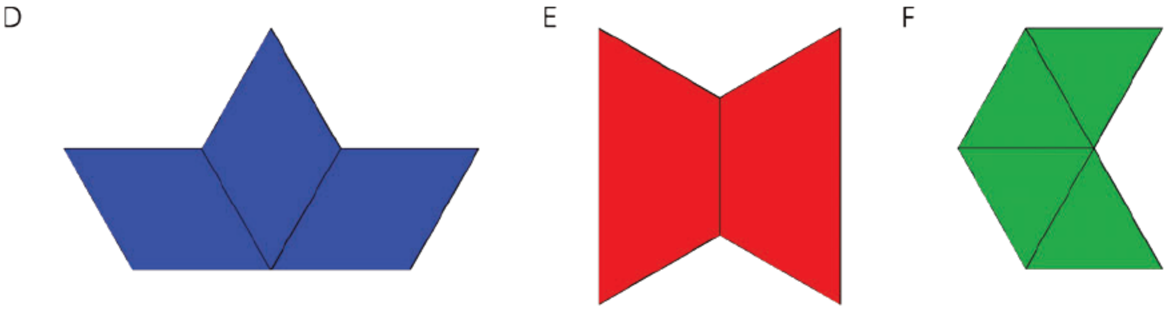
Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organization in problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.
Supports accessibility for: Organization; Attention

Anticipated Misconceptions

Students may forget to check that the lengths of all sides of their shape have been scaled and end with an inaccurate count of the pattern blocks. Remind them that all segments must be scaled by the same factor.

Student Task Statement

Your teacher will assign your group one of these figures.



1. Build a scaled copy of your assigned shape using a scale factor of 2. Use the same shape blocks as in the original figure. How many blocks did it take?
2. Your classmate thinks that the scaled copies in the previous problem will each take 4 blocks to build. Do you agree or disagree? Explain you reasoning.
3. Start building a scaled copy of your assigned figure using a scale factor of 3. Stop when you can tell for sure how many blocks it would take. Record your answer.
4. How many blocks would it take to build scaled copies of your figure using scale factors 4, 5, and 6? Explain or show your reasoning.
5. How is the pattern in this activity the same as the pattern you saw in the previous activity? How is it different?

Student Response

1. Answers vary based on the original figure: 12 blue rhombuses, 8 red trapezoids, or 16 green triangles.

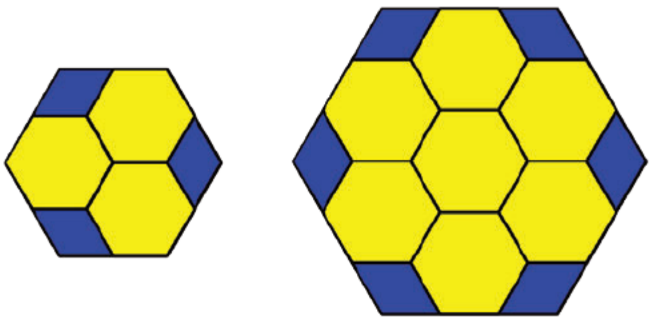
2. Each block in the pattern is replaced by 4 blocks in the scaled copy. There is more than one block in each pattern so the scaled copies of the patterns require more than 4 blocks.
3. Answers vary based on the original figure: 27 blue rhombuses, 18 red trapezoids, or 36 green triangles.
4. Blue rhombuses needed for scaled copies of Figure D: $3 \cdot 4^2 = 48, 3 \cdot 5^2 = 75, 3 \cdot 6^2 = 108.$
Red trapezoids needed for scaled copies of Figure E: $2 \cdot 4^2 = 32, 2 \cdot 5^2 = 50, 2 \cdot 6^2 = 72.$
Green triangles needed for scaled copies of Figure F: $4 \cdot 4^2 = 64, 4 \cdot 5^2 = 100, 4 \cdot 6^2 = 144.$
5. At first glance, the pattern does not seem the same because the answers are not 4 and 9. However, each individual block still scales by 4 and then 9, so you have to multiply that by the number of blocks in the original shape to get the number of blocks in the scaled copy.

Are You Ready for More?

1. How many blocks do you think it would take to build a scaled copy of one yellow hexagon where each side is twice as long? Three times as long?
2. Figure out a way to build these scaled copies.
3. Do you see a pattern for the number of blocks used to build these scaled copies? Explain your reasoning.

Student Response

1. It should take 4 blocks and 9 blocks following the pattern for the other shapes.
2. Answers vary. Sample response: here is a way to build the scaled copies using yellow hexagons and blue rhombuses:



3. The pattern does not work if you only count the number of blocks; however, it does work if you consider the size of each block being used. The first hexagon took 6 blocks to build: 3 yellow hexagons and 3 blue rhombuses, but 3 blue rhombuses cover the same area as 1 yellow hexagon, so the size of the scaled copy is equivalent to 4 yellow hexagons, because $3 + \frac{3}{3} = 4$. Similarly, the total size of the scaled copy with scale factor 3 is equivalent to 9 yellow hexagons.

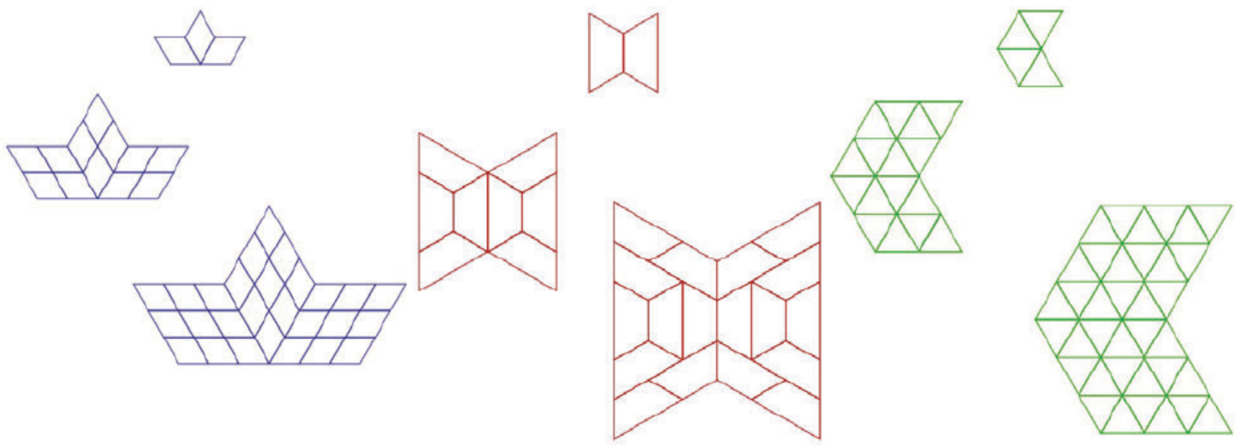
Activity Synthesis

The goal of this discussion is to ensure that students understand that the pattern for the number of blocks in the scaled copies depends *both* on the scale factor and on the number of blocks in the pattern.

Display a table with only the column headings filled in. Poll the class on how many blocks it took them to build each scaled copy using the factors of 2 and 3. Record their answers in the table.

scale factor	number of blocks to build Figure D	number of blocks to build Figure E	number of blocks to build Figure F
1	3	2	4
2			
3			
4			
5			
6			
s			

Consider displaying the built shapes or pictures of them for all to see.



Invite selected students to share the pattern that their groups noticed and used to predict the number of blocks needed for copies with scale factors 4, 5, and 6. Record their predictions in the table. Discuss:

- How does the pattern for the number of blocks in this activity compare to the pattern in the previous activity? Are they related? How?

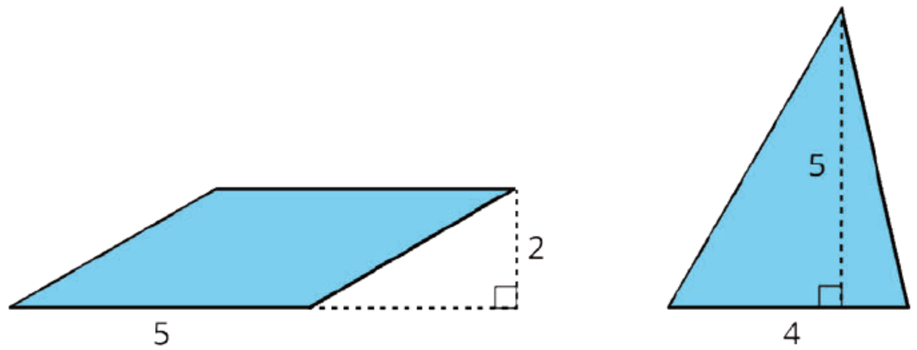
- For each figure, how many blocks does it take to build a copy using any scale factor s?

Support for English Language Learners

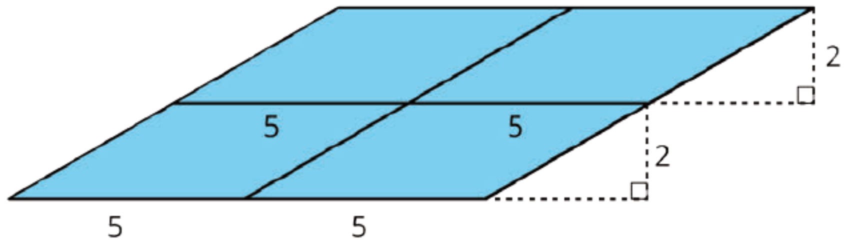
Speaking: MLR8 Discussion Supports. Give students additional time to make sure that everyone in the group can describe the patterns they noticed and the ways they predicted the number of blocks needed for copies with scale factors 4, 5, and 6. Vary who is called on to represent the ideas of each group. This routine will provide students additional opportunities to prepare for and share their thinking publicly.
Design Principle(s): Optimize output (for explanation)

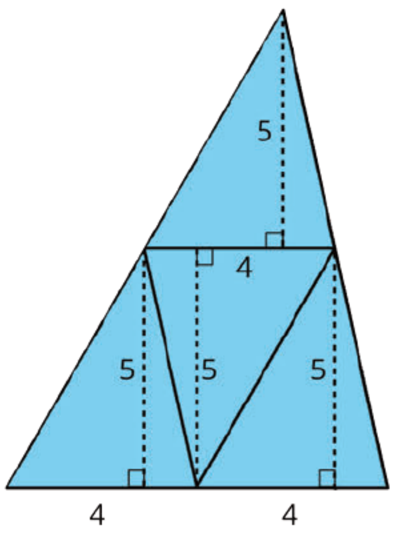
6.3 Area of Scaled Parallelograms and Triangles

Optional: 15 minutes
In this activity, students transfer what they learned with the pattern blocks to calculate the area of other scaled shapes (MP8). In groups of 2, students draw scaled copies of either a parallelogram or a triangle and calculate the areas. Then, each group compares their results with those of a group that worked on the other shape. They find that the scaled areas of two shapes are the same (even though the starting shapes are different and have different measurements) and attribute this to the fact that the two shapes had the same original area and were scaled using the same scale factors.



While students are not asked to reason about scaled areas by tiling (as they had done in the previous activities), each scaled copy can be tiled to illustrate how length measurements have scaled and how the original area has changed. Some students may choose to draw scaled copies and think about scaled areas this way.





As students find the areas of copies with scale factors 5 and $\frac{3}{5}$ without drawing (for the last question), monitor for these methods, depending on their understanding of or comfort with the $(\text{scale factor})^2$ pattern:

- Scaling the original base and height and then multiplying to find the area
- Multiplying the original area by the square of the scale factor

Select students using each approach. Invite them to share their reasoning, sequenced in this order, during the discussion.

You will need the Area of Scaled Parallelograms and Triangles blackline master for this activity.

Building On

- 6.G.A.1

Addressing

- 7.G.A.1

Building Towards

- 7.G.B.6

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect
- Think Pair Share

Launch

Arrange students in groups of 2. Provide access to geometry toolkits.

Distribute slips showing the parallelogram to half the groups and the triangle to the others. Give students 1 minute of quiet work time for the first question, and then time to complete the rest of the task with their partner.

Support for Students with Disabilities

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. For example, draw students attention to the warm-up and remind them how to build scaled copies of the rhombus with scale factors of 2, 3, and 4. Ask students how they can use this technique to draw scaled copies of the parallelogram or triangle with scale factors of 2, 3, and 5.
Supports accessibility for: Social-emotional skills; Conceptual processing

Anticipated Misconceptions

Students may not remember how to calculate the area of parallelograms and triangles. Make sure that they have the correct area of 10 square units for their original shape before they calculate the area of their scaled copies.

When drawing their scaled copies, some students might not focus on making corresponding angles equal. As long as they scale the base and height of their polygon correctly, this will not impact their area calculations. If time permits, however, prompt them to check their angles using tracing paper or a protractor.

Some students might focus unnecessarily on measuring other side lengths of their polygon, instead of attending only to base and height. If time is limited, encourage them to scale the base and height carefully and check or measure the angles instead.

Student Task Statement

1. Your teacher will give you a figure with measurements in centimeters. What is the **area** of your figure? How do you know?

2. Work with your partner to draw scaled copies of your figure, using each scale factor in the table. Complete the table with the measurements of your scaled copies.

scale factor	base (cm)	height (cm)	area (cm^2)
1			
2			
3			
$\frac{1}{2}$			
$\frac{1}{3}$			

3. Compare your results with a group that worked with a different figure. What is the same about your answers? What is different?

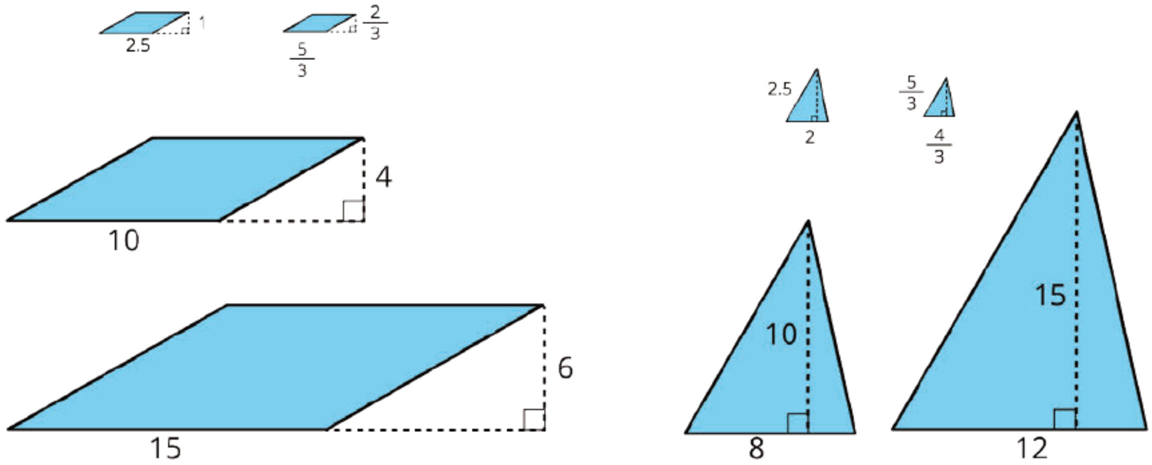
4. If you drew scaled copies of your figure with the following scale factors, what would their areas be? Discuss your thinking. If you disagree, work to reach an agreement. Be prepared to explain your reasoning.

scale factor	area (cm^2)
5	
$\frac{3}{5}$	

Student Response

1. The area of either shape is 10 square units, because $5 \cdot 2 = 10$ and $\frac{1}{2} \cdot 4 \cdot 5 = 10$.

2.



For the parallelogram:

scale factor	base	height	area
1	5	2	10
2	10	4	40
3	15	6	90
$\frac{1}{2}$	2.5	1	2.5
$\frac{1}{3}$	$\frac{5}{3}$	$\frac{2}{3}$	$\frac{10}{9}$

For the triangle:

scale factor	base	height	area
1	4	5	10
2	8	10	40
3	12	15	90
$\frac{1}{2}$	2	2.5	2.5
$\frac{1}{3}$	$\frac{4}{3}$	$\frac{5}{3}$	$\frac{10}{9}$

3. The areas are the same for each scale factor, even though the dimensions are different. Specifically, the bases of the parallelograms are equal to the heights of the triangles.

4.

scale factor	area
5	250
$\frac{3}{5}$	3.6

Activity Synthesis

Invite selected students to share their solutions. Then focus class discussion on two themes: how the values in the tables for the two shapes compare, and how students determined the scaled areas for the scale factors 5 and $\frac{3}{5}$. Ask questions such as:

- What did you notice when you compared your answerswithanother group that worked with the other figure? (When the scale factors are the same, the scaled areas are the same, though the bases and heights are different.)
- How did you find the scaled areas for scale factors of 5 and $\frac{3}{5}$? (By scaling the original base and height and multiplying the scaled measurements; by multiplying the original area by $(\text{scale factor})^2$.)
- How is the process for finding scaled area here the same as and different than that in the previous activities with pattern blocks? (The area units are different; the pattern of squaring the scale factor is the same.)

Highlight the connection between the two ways of finding scaled areas. Point out that when we multiply the base and height each by the scale factor and then multiply the results, we are essentially multiplying the original lengths by the scale factor two times. The effect of this process is the same as multiplying the original area by $(\text{scale factor})^2$.

Support for English Language Learners

Representing, Speaking, and Listening: MLR7 Compare and Connect. Invite students to prepare a display that shows their approach to finding the areas for scale factors of 5 and $\frac{3}{5}$. Ask students to research how other students approached the problem, in search of a method that is different from their own. Challenge students to describe why the different approaches result in the same answers. During the whole-class discussion, emphasize the language used to explain the different strategies, especially phrases related to “squaring” and “multiplying a number by itself.” This will strengthen students' mathematical language use and reasoning based on the relationship between scale factors and area.
Design Principle(s): Maximize meta-awareness

Lesson Synthesis

- If all the dimensions of a scaled copy are twice as long as in the original shape, will the area of the scaled copy be twice as large? (No)
    - Why not? (Both the length and the width get multiplied by 2, so the area gets multiplied by 4.)
- If the scale factor is 5, how many times larger will the scaled copy's area be? (25 times larger)

6.4 Enlarged Areas

Cool Down: 5 minutes

The first question gives students only the area of the original shape—but not the dimensions—to encourage them to find the area of the scaled copy by multiplying by the $(\text{scale factor})^2$; however, students can also choose a length and a width for the rectangle that would give the correct original area, and then scale those dimensions by the scale factor to calculate the area. The second question only asks students to find the $(\text{scale factor})^2$, but not to multiply by it.

Building On

- 6.G.A.1

Addressing

- 7.G.A.1

Building Towards

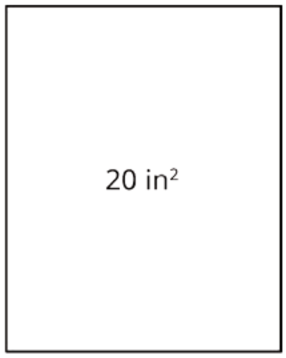
- 7.G.B.6

Anticipated Misconceptions

Some students may multiply the original shape's area by just the scale factor, instead of by the $(\text{scale factor})^2$, getting $80\ \text{in}^2$. Students who do not understand the generalized rule for how scaling affects area might still be able to answer the first question correctly. They could assume some dimensions for the original rectangle that would give it an area of $20\ \text{in}^2$, scale those dimensions by the given scale factor, and then multiply those scaled dimensions to find the new area.

Student Task Statement

1. Lin has a drawing with an area of 20 in^2. If she increases all the sides by a scale factor of 4, what will the new area be?



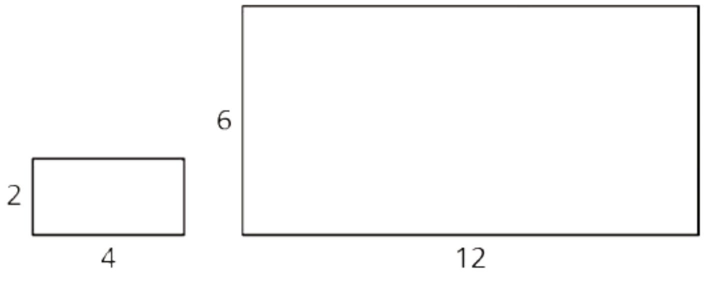
2. Noah enlarged a photograph by a scale factor of 6. The area of the enlarged photo is how many times as large as the area of the original?

Student Response

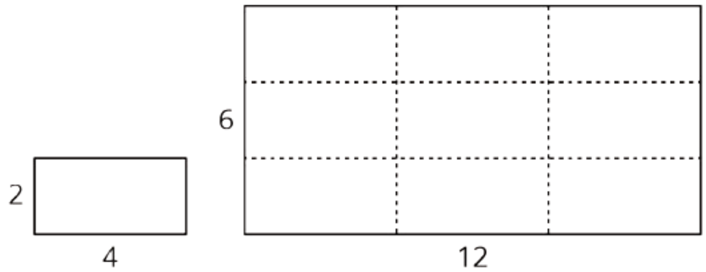
1. 320 in^2, Possible strategies:
   - $20 \cdot 4^2 = 320$
   - If the rectangle is 4 inches by 5 inches, the scaled copy will be $4 \cdot 4$ inches by $4 \cdot 5$ inches and $(4 \cdot 4) \cdot (4 \cdot 5) = 16 \cdot 20 = 320$.
   - If the rectangle is 2 inches by 10 inches, the scaled copy will be $4 \cdot 2$ inches by $4 \cdot 10$ inches and $(4 \cdot 2) \cdot (4 \cdot 10) = 8 \cdot 40 = 320$.
2. 36 times as large, because $6^2 = 36$.

Student Lesson Summary

Scaling affects lengths and areas differently. When we make a scaled copy, all original lengths are multiplied by the scale factor. If we make a copy of a rectangle with side lengths 2 units and 4 units using a scale factor of 3, the side lengths of the copy will be 6 units and 12 units, because $2 \cdot 3 = 6$ and $4 \cdot 3 = 12$.



The area of the copy, however, changes by a factor of (scale factor)^2. If each side length of the copy is 3 times longer than the original side length, then the area of the copy will be 9 times the area of the original, because $3 \cdot 3$, or 3^2, equals 9.



In this example, the area of the original rectangle is 8 units^2 and the area of the scaled copy is 72 units^2, because $9 \cdot 8 = 72$. We can see that the large rectangle is covered by 9 copies of the small rectangle, without gaps or overlaps. We can also verify this by multiplying the side lengths of the large rectangle: $6 \cdot 12 = 72$.

Lengths are one-dimensional, so in a scaled copy, they change by the scale factor. Area is two-dimensional, so it changes by the *square* of the scale factor. We can see this is true for a rectangle with length l and width w. If we scale the rectangle by a scale factor of s, we get a rectangle with length $s \cdot l$ and width $s \cdot w$. The area of the scaled rectangle is $A = (s \cdot l) \cdot (s \cdot w)$, so $A = (s^2) \cdot (l \cdot w)$. The fact that the area is multiplied by the square of the scale factor is true for scaled copies of other two-dimensional figures too, not just for rectangles.

Glossary

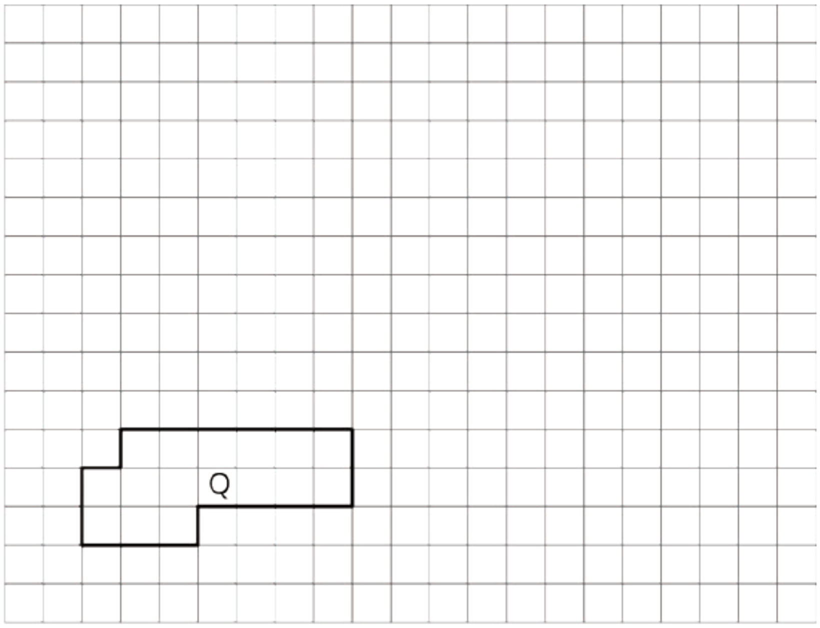
- area

Lesson 6 Practice Problems

Problem 1

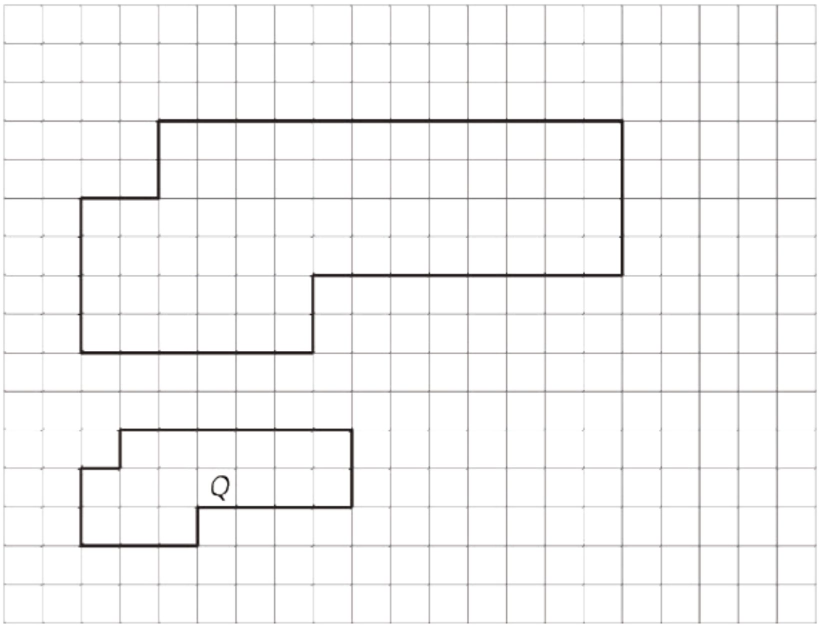
Statement

On the grid, draw a scaled copy of Polygon Q using a scale factor of 2. Compare the perimeter and area of the new polygon to those of Q.



Solution

The perimeter of Q is 20 units, and the area of Q is 16 square units. The perimeter of the scaled copy is 40 units, and its area is 64 square units. The perimeter is multiplied by the scale factor of 2, and the area is multiplied by the square of the scale factor, which is 4.



Problem 2

Statement

A right triangle has an area of 36 square units.

If you draw scaled copies of this triangle using the scale factors in the table, what will the areas of these scaled copies be? Explain or show your reasoning.

scale factor	area ($units^2$)
1	36
2	
3	
5	
$\frac{1}{2}$	
$\frac{2}{3}$	

Solution

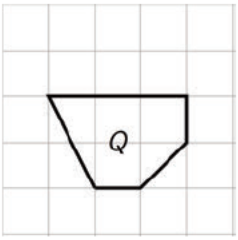
The area of each scaled triangle is the same as the area of the original triangle (36 square units) multiplied by the square of the scale factor:

scale factor	area ($units^2$)
1	36
2	144
3	324
5	900
$\frac{1}{2}$	9
$\frac{2}{3}$	16

Problem 3

Statement

Diego drew a scaled version of a Polygon P and labeled it Q.



If the area of Polygon P is 72 square units, what scale factor did Diego use to go from P to Q? Explain your reasoning.

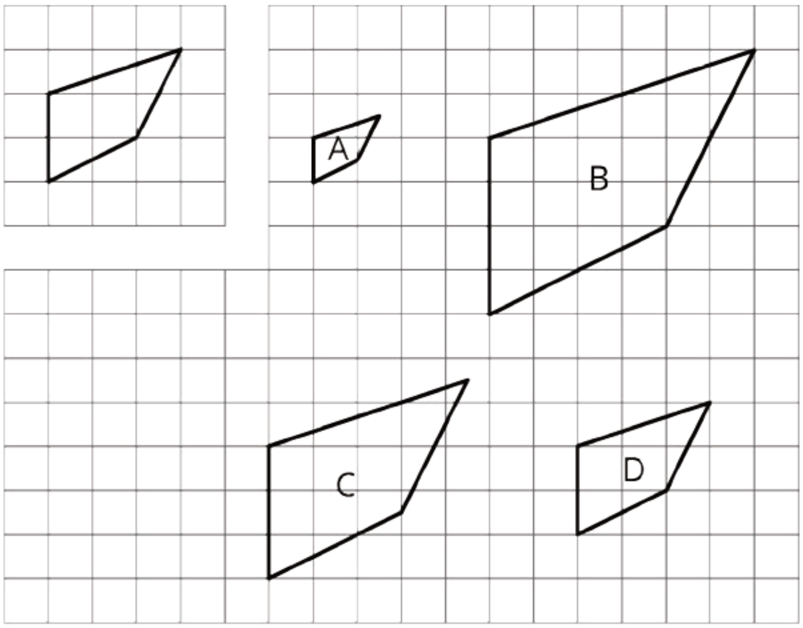
Solution

$\frac{1}{4}$. The area of Q is 4.5 square units (3 whole square units, one 2 unit by 1 unit right triangle, and one 1 unit by 1 unit right triangle). This area is $\frac{1}{16}$ of the area of P. This means the scale factor is $\frac{1}{4}$ because $\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$.

Problem 4

Statement

Here is an unlabeled polygon, along with its scaled copies Polygons A–D. For each copy, determine the scale factor. Explain how you know.



Solution

a. $\frac{1}{2}$ because the vertical side on the copy is $\frac{1}{2}$ the length of the vertical side on the original

b. 2 because the vertical side on the copy is twice the length of the vertical side on the original

c. $\frac{3}{2}$ because the vertical side on the copy is $\frac{3}{2}$ the length of the vertical side on the original

d. 1 because the original and the copy have the same size

(From Unit 1, Lesson 2.)

Problem 5

Statement

Solve each equation mentally.

a. $\frac{1}{7} \cdot x = 1$

b. $x \cdot \frac{1}{11} = 1$

c. $1 \div \frac{1}{5} = x$

Solution

a. $x = 7$

b. $x = 11$

c. $x = 5$

(From Unit 1, Lesson 5.)

Section: Scale Drawings

Lesson 7: Scale Drawings

Goals

- Describe (orally) what a “scale drawing” is.
- Explain (orally and in writing) how to use scales and scale drawings to calculate actual and scaled distances.
- Interpret the “scale” of a scale drawing.

Learning Targets

- I can explain what a scale drawing is, and I can explain what its scale means.
- I can use actual distances and a scale to find scaled distances.
- I can use a scale drawing and its scale to find actual distances.

Lesson Narrative

Up to this point, students have been exploring scaled copies, or two-dimensional images that have been recreated at certain scale factors. In this lesson, they begin to look at **scale drawings**, or scaled two-dimensional representations of actual objects or places. Students see that although scale drawings capture three-dimensional objects or places, they show scaled measurements in only two of the dimensions, and that all information is projected onto a plane.

In this and upcoming lessons, students see that the principles and strategies they used to reason about scaled copies are applicable to scale drawings (MP7). For example, previously they saw scale factor as a number that describes how lengths in a figure correspond to lengths in a copy of the figure (and vice versa). Now they see that **scale** serves a similar purpose: it describes how the lengths in an actual object are related to the lengths on a drawn representation of it. They learn that scale can be expressed in a number of ways, and use scale and scale drawings to find actual and scaled lengths.

Students begin by interpreting given scale drawings. In subsequent lessons, they will create or reproduce scale drawings at specified scales, as well as determine appropriate scales to use, given restrictions in the size of drawing.

Alignments

Addressing

- 7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR1: Stronger and Clearer Each Time
- MLR8: Discussion Supports

Required Materials

Copies of blackline master
Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Required Preparation

Prepare to display the examples and non-examples of scale drawings for all to see. Consider adding to the collection a local map showing the actual route of a train or bus line (example of scale drawing) and a diagrammatic transit map (non-example).

Ensure students have access to geometry toolkits, especially centimeter rulers and index cards or paper to use as a measuring tool.

You will need the Sizing Up a Basketball Court blackline master for this lesson. Prepare one copy per student.

Student Learning Goals

Let's explore scale drawings.

7.1 What is a Scale Drawing?

Warm Up: 5 minutes
This activity encourages students to notice characteristics of scale drawings by observing examples and counterexamples, and to articulate what a scale drawing is. Though students are not expected to come up with precise definitions, they are likely able to intuit that scale drawings are accurate two-dimensional depictions of what they represent, in the sense that all shapes, arrangements of parts, and relative sizes match those of the actual objects.

Expect student observations about scale drawings to be informal and not mathematical. For example, they might say that a scale drawing looks just like the object it is portraying, with the parts shown having the right size and being in the right places in the drawing. Or that in a scale drawing, a smaller part in the actual object does not end up being larger in the drawing.

Like any mathematical model of a real situation, a scale drawing captures some important aspects of the real object and ignores other aspects. It may not be apparent to students that scale drawings prioritize features of one plane of the object (and sometimes features of other planes parallel to it) and ignore other surfaces and dimensions. Notice students who show insights around this idea so they can share later.

Addressing

- 7.G.A.1

Launch

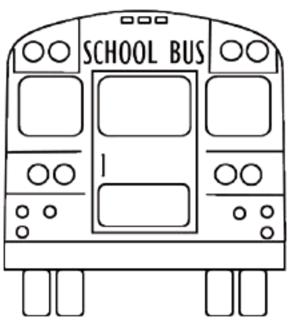
Arrange students in groups of 2. Before students look at the materials, poll the class to find out who has seen scale drawings. Ask a few students who are familiar with them to give a couple of examples of scale drawings they have seen. Then, give students 2 minutes to observe the examples and counterexamples of scale drawings and discuss in groups what they think a scale drawing is.

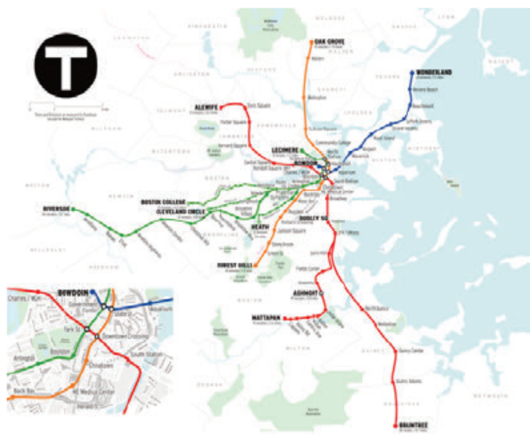
Anticipated Misconceptions

If students struggle to characterize scale drawings, offer prompts to encourage them to look closer. For example, ask: "How do the shapes and sizes of the objects in the drawings compare to those of the actual objects?" Students may say that sizes of the objects in the scale drawings are smaller than those of the actual objects. Ask them if any parts of the scale drawings are distorted, compared to the actual object—ask them to focus on the two images of the quarter, one of which is circular in shape while the other is not.

Student Task Statement

Here are some drawings of a school bus, a quarter, and the subway lines around Boston, Massachusetts. The first three drawings are **scale drawings** of these objects.





The next three drawings are *not* scale drawings of these objects.

Discuss with your partner what a scale drawing is.

Student Response

Answers vary. Sample descriptions:

- A scale drawing is a drawing that shows the object accurately and all parts in the drawing match the parts in the actual object.
- No parts in a scale drawing are distorted.
- A scale drawing is like a scaled copy of a real object, but it is a drawing that shows one flat surface of the object.

Activity Synthesis

Ask a few students to share what they noticed about characteristics of scale drawings and to compare and contrast scaled copies and scale drawings. Discuss questions such as the following. Record common themes and helpful descriptions.

- What do the examples have or show that the counterexamples do not?
- How are scale drawings like scaled copies you saw in earlier lessons? How are they different than scaled copies?
- What aspects of the bus, coin, and the city of Boston do the scale drawings show? What aspects of the actual objects do scale drawings *not* show?

Notice misconceptions, but it is not necessary to address them right away, as students' understanding will be shaped in this and upcoming lessons. Tell students that they will continue to analyze scale drawings and revise their definitions in upcoming activities.

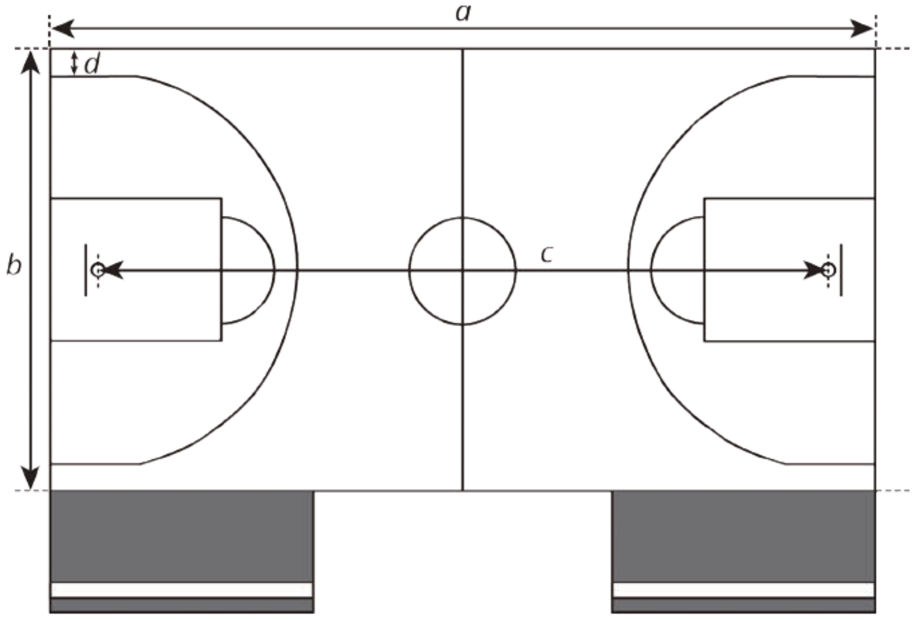
7.2 Sizing Up a Basketball Court

15 minutes

In this introductory activity, students explore the meaning of **scale.** They begin to see that a scale communicates the relationship between lengths on a drawing and corresponding lengths in the objects they represent, and they learn some ways to express this relationship:

- "a units on the drawing represent b units of of actual length"
- "at a scale of a units (on the drawing) to b units (actual)"
- "a units (on the drawing) for every b units (actual)"

Students measure lengths on a scale drawing and use a given scale to find corresponding lengths on a basketball court (MP2). Because students are measuring to the nearest tenth of a centimeter, some of the actual measurements they calculate will not have the precision of the official measurements. For example, the official measurement for d is 0.9 m.



You will need the Sizing Up a Basketball Court blackline master for this activity.

Addressing

- 7.G.A.1

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Ask students if they have ever played basketball or seen a basketball court. If so, where? If some students have played basketball or seen a basketball court, ask them if they could throw a basketball across the width of a basketball court. What about across the full length of the court?

Arrange students in groups of 2. Distribute a copy of the blackline master and a ruler to each student. Give students 6–7 minutes of quiet work time to complete the first three questions. Ask them to share their responses with their partner before completing the remaining questions.

Support for English Language Learners

Writing, Speaking, Listening: MLR1 Stronger and Clearer Each Time. After students have completed the first three questions, provide them an opportunity to refine their reasoning for the third question. Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language. For example, "How did you use the scale in your calculations?", "Why did you multiply each measurement from the drawing by 2?", and "A detail (or word) you could add is _____, because . . . ." Students can borrow ideas and language from each partner to refine and clarify their original explanation. This will help students refine their own explanation and learn about other ways to find actual measurements by using the measurements from a scale drawing.
Design Principles(s): Optimize output (for explanation); Maximize meta-awareness

Anticipated Misconceptions

Instead of using the scale to find actual measurements, students might try to convert distances in centimeters to meters (14 cm is 0.14 m). Explain that the distances they measured on paper could be converted to meters, but then the results are still lengths on paper, just expressed in meters, rather than the measurements of the actual basketball court. Draw students' attention to the statement "1 cm represents 2 m" on the scale drawing and ask them to think about how to use it to find actual measurements.

Student Task Statement

Your teacher will give you a scale drawing of a basketball court. The drawing does not have any measurements labeled, but it says that 1 centimeter represents 2 meters.

1. Measure the distances on the scale drawing that are labeled a–d to the nearest tenth of a centimeter. Record your results in the first row of the table.

2. The statement "1 cm represents 2 m" is the **scale** of the drawing. It can also be expressed as "1 cm to 2 m," or "1 cm for every 2 m." What do you think the scale tells us?

3. How long would each measurement from the first question be on an actual basketball court? Explain or show your reasoning.

measurement	(a) length of court	(b) width of court	(c) hoop to hoop	(d) 3 point line to sideline
scale drawing				
actual court				

4. On an actual basketball court, the bench area is typically 9 meters long.

    a. Without measuring, determine how long the bench area should be on the scale drawing.

    b. Check your answer by measuring the bench area on the scale drawing. Did your prediction match your measurement?

Student Response

1. 14 cm, 7.5 cm, 12.4 cm, 0.5 cm, although students may round measurements differently.
2. Answers vary. Sample responses:
    - The scale tells us how the lengths on the drawing compare to actual lengths.
    - The scale tells us how to use the measurements on the drawing to find actual measurements.
3. Sample reasoning: Since every centimeter represents 2 meters, I multiplied each measurement from the drawing by 2 to find the actual measurement in meters.

measurement	(a) length of court	(b) width of court	(c) hoop to hoop	(d) 3 point line to sideline
scale drawing	14 cm	7.5 cm	12.4 cm	0.5 cm
actual court	28 m	15 m	24.8 m	1 m

4. 4.5 cm. Answers vary.

Activity Synthesis

Before debriefing as a class, display the table showing only the scaled distances so students can do a quick check of their measurements (which they may round differently). Explain to students that the distances on a scale drawing are often referred to as “scaled distances.” The distances on the basketball court, in this case, are called *actual* distances.

Focus the discussion on the meaning of scale and how students used the given scale to find actual distances. Invite a few students to share their response to the second and third questions. To further students' understanding of scale, discuss:

- Does "1 cm for every 2 m" mean that the actual distance is twice that on the drawing? Why or why not?
- Which parts of the court should be drawn by using "1 cm for every 2 m" rule?
- Can we reverse the order in which we list the scaled and actual distances? For example, can we say "2 m of actual distance to 1 cm on the drawing" or "2 m to 1 cm"?

Note that the scaled distance is conventionally stated first, but the actual distance represented could also come first as long as it is clear from the context.

If needed, a short discussion about accuracy of measurements on the scale drawing might highlight some possible sources of measurement error including:

- Not measuring in a straight line
- The lines on the scale drawing have width, and this could contribute a small error depending on whether the measurement is from the inside or the outside of the lines

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Support for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Include the following terms and maintain the display for reference throughout the unit: scale, scaled distance, scale drawing. Invite students to suggest language or diagrams to include on the display that will support their understanding of these terms.
Supports accessibility for: Memory; Language

7.3 Tall Structures

15 minutes
This activity introduces students to graphic scales. Students interpret them, use them to find actual measurements (heights of tall buildings), and express them non-graphically. In earlier lessons, students used markings on an index card or sheet of paper to measure a drawing and create scaled copies. Here, they use non-standard measuring tools again to solve problems (MP5). Students make markings on an unmarked straightedge to measure scaled lengths on a drawing, and then use the given scale to determine actual lengths. This measuring strategy builds on students' work with measurement in grades 1 and 2 (i.e., placing multiple copies of a shorter object end to end and expressing lengths in terms of the number of objects).

As students work, encourage them to be as precise as possible in making their marks and in estimating lengths that are less than 1 scale-segment long. Most distances students are measuring here are not line segments but rather distances between a point (the tip of a building) and a line (the ground).

Monitor the different ways students reason about scaled and actual distances. Here are two likely approaches for finding the difference between the Burj Khalifa and the Eiffel Tower (the second question):

- Find the actual height of each tower and then find their difference.
- Find the difference in scaled heights, and then use the scale to find the difference in actual heights.

Select students who use these approaches to share during the whole-class discussion in this sequence.

Addressing

- 7.G.A.1

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports

Launch

Display the scale drawing of the structures. Before beginning the work of the task, students may be interested in or eager to share the locations of the structures. Consider taking a few minutes to elicit what they know, or display a world map showing the locations of the structures.

Ask students what the segment labeled with "0 m" and "100 m" might mean. Some students are likely to say that it also conveys a scale. Verify that a scale can indeed be communicated graphically; an actual distance is not represented by a numerical measurement, but rather, by the length of the segment.

Provide access to index cards or sheets of paper students can use to measure. Tell students to check their answers to the first question with a partner. Tell them to discuss as necessary until they reach an agreement before proceeding to work on the rest of the problems. Give students 4–5 minutes of quiet work time and partner discussion.

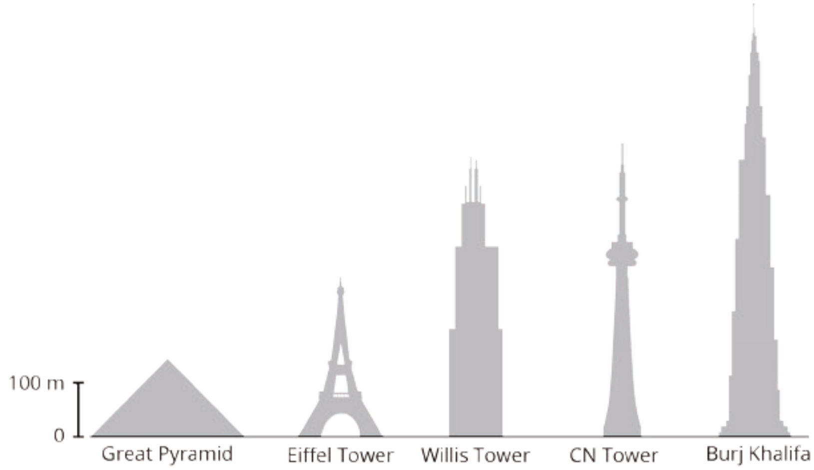
Anticipated Misconceptions

Students may not measure heights of the buildings at a right angle from the ground line. Remind students that heights are to be measured perpendicular to the ground or base line.

If needed, demonstrate how to use the edge of a sheet of paper or an index card to measure lengths on a scale drawing with a graphic scale.

Student Task Statement

Here is a scale drawing of some of the world's tallest structures.



1. About how tall is the actual Willis Tower? About how tall is the actual Great Pyramid? Be prepared to explain your reasoning.

2. About how much taller is the Burj Khalifa than the Eiffel Tower? Explain or show your reasoning.

3. Measure the line segment that shows the scale to the nearest tenth of a centimeter. Express the scale of the drawing using numbers and words.

Student Response

1. The Willis Tower is a bit more than 500 m tall. It takes about 5 of the segment lengths to measure the height. The Great Pyramid is approximately 150 m. Its height is about $1\frac{1}{2}$ segments long.

2. The Burj Khalifa is approximately 550 m taller than the Eiffel Tower. Sample explanations:

    - It takes about 3 of the 100-m segments to measure the Eiffel Tower, so it is about 300 m tall. It takes about $8\frac{1}{2}$ of the 100-m segments to measure the Burj Khalifa, so it is about 850 meters tall. $850 - 300 = 550$.

    - It takes about 3 of the 100-m segments to measure the Eiffel Tower and about $8\frac{1}{2}$ segments to measure the Burj Khalifa. This is a difference of $5\frac{1}{2}$ segments on the scale drawing, which would be an actual difference of about 550 m.

    - The Burj Khalifa looks almost 3 times as tall as the Eiffel Tower, which would be a difference of between 500 and 600 m.

3. Answers vary depending on the printed size of the scale. Sample responses:

    - 0.7 cm on the drawing represents 100 m.

- 0.7 cm to 100 m.
- 0.7 cm for every 100 m in actual height.

Are You Ready for More?

The tallest mountain in the United States, Mount Denali in Alaska, is about 6,190 m tall. If this mountain were shown on the scale drawing, how would its height compare to the heights of the structures? Explain or show your reasoning.

Student Response

Mount Denali will be far taller (about 7.5 times taller) than the Burj Khalifa. Sample explanations:

- I found out that the Burj Khalifa is about 830 m. Mount Denali is 6,190 m, and 6,190 divided by 830 is about 7.5, so the mountain will be about 7.5 times taller than the Burj Khalifa.
- It takes about 62 of the 100-m segments to measure the mountain ($6,190 \div 100 = 61.9$). It takes about $8\frac{1}{4}$ of the 100-m segments to measure the Burj Khalifa. This means the mountain is about 7.5 times taller than the Burj, since 62 divided by $8\frac{1}{4}$ is about 7.5.

Activity Synthesis

Poll the class for their answers to the first question. Ask what might be some sources of discrepancies. The main issue here is measurement error, but there are also different methods to make the measurements and estimate the heights of the buildings. Two methods to estimate the heights of the buildings (and their limitations) are:

- Estimate how many times "taller" each building is compared to the line segment giving the scale. Then multiply this number by 100 m. The accuracy here is not very good unless, for example, the height of the building is very close to being a whole number times the length of the scale.
- Measure the segment giving the scale (in centimeters, for example), and then express the scale using centimeters (for example, 0.7 cm represents 100 m). Then measure each building and use the scale to find the actual height. Estimating and rounding will be necessary when measuring the scale and when measuring the buildings.

Ask previously selected and sequenced students to highlight the different approaches for comparing the heights of the Burj Khalifa and the Eiffel Tower. The main difference between the two strategies is the order of arithmetic. Taking the difference of the actual building heights means multiplying scaled heights first and then subtracting. Taking the difference of the scaled heights and then applying the scale factor means subtracting scaled heights first and then multiplying by the scale factor.

If time permits, discuss the scale drawing of the towers more broadly. Consider asking:

- Besides height information, what other information about the towers does the drawing show? (Widths of the buildings and their overall shapes.)
- What information does it *not* show? (The depth of each building, any projections or protrusions, shapes of different parts of the buildings, etc.)
- How is this scale drawing the same as that of the basketball court? (They both show information on a single plane and are drawn using a scale.) How are they different? (The basketball court is a flat surface, like the drawing of the court. The drawing of the towers is a side view or front view; the actual objects represented are not actually flat objects.)

Support for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion. For each response or observation that is shared, ask students to restate and/or revoice what they heard using precise mathematical language. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. This will provide more students with an opportunity to produce language as they interpret the reasoning of others. *Design Principle(s): Support sense-making*

Lesson Synthesis

- What is a **scale drawing**?
- How can we describe the **scale** for a scale drawing?
- How do we find distances using a scale drawing?

A scale drawing is a scaled representation of an object. The scale tells us how lengths on the drawing relate to lengths on the actual object. For example, in the basketball court activity, we saw that 1 centimeter on the drawing represented 2 meters of actual distance on the actual court.

If we have a scale drawing, we can use the scale to find lengths on the actual object. For example, if a line segment in the scale drawing of the basketball court is 5 cm, then it represents a 10 m line segment on a real court, because $2 \cdot 5 = 10$. It is important to remember that a scale drawing shows scaled measurements in only two dimensions, i.e., measurements of a particular surface of an object and those that have been projected onto a particular plane. For example, the drawing of the basketball court did not show the height of the basketball hoops.

7.4 Length of a Bus and Width of a Lake

Cool Down: 5 minutes

Addressing

- 7.G.A.1

Launch

If desired, provide access to four-function calculators.

Student Task Statement

1. A scale drawing of a school bus has a scale of $\frac{1}{2}$ inch to 5 feet. If the length of the school bus is $4\frac{1}{2}$ inches on the scale drawing, what is the actual length of the bus? Explain or show your reasoning.

2. A scale drawing of a lake has a scale of 1 cm to 80 m. If the actual width of the lake is 1,000 m, what is the width of the lake on the scale drawing? Explain or show your reasoning.

Student Response

1. 45 ft. Sample explanation: There are 9 groups of $\frac{1}{2}$ in $4\frac{1}{2}$. If $\frac{1}{2}$ inch represents 5 feet, then $4\frac{1}{2}$ inches represents $9 \cdot 5$ or 45 feet.

2. 12.5 cm. Sample reasoning: Since every 80 m is represented by 1 cm, 1,000 m is represented by $1{,}000 \div 80$ or 12.5 cm.

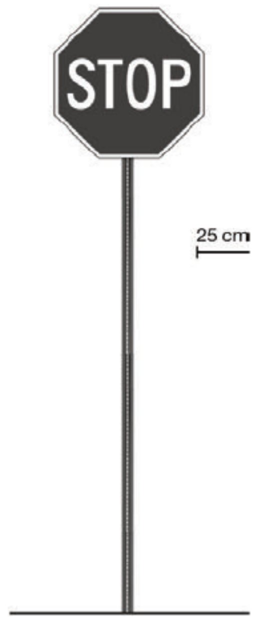
Student Lesson Summary

Scale drawings are two-dimensional representations of actual objects or places. Floor plans and maps are some examples of scale drawings. On a scale drawing:

- Every part corresponds to something in the actual object.
- Lengths on the drawing are enlarged or reduced by the same scale factor.
- A **scale** tells us how actual measurements are represented on the drawing. For example, if a map has a scale of "1 inch to 5 miles" then a $\frac{1}{2}$-inch line segment on that map would represent an actual distance of 2.5 miles

Sometimes the scale is shown as a segment on the drawing itself. For example, here is a scale drawing of a stop sign with a line segment that represents 25 cm of actual length.

The width of the octagon in the drawing is about three times the length of this segment, so the actual width of the sign is about $3 \cdot 25$, or 75 cm.



Because a scale drawing is two-dimensional, some aspects of the three-dimensional object are not represented. For example, this scale drawing does not show the thickness of the stop sign.

A scale drawing may not show every detail of the actual object; however, the features that are shown correspond to the actual object and follow the specified scale.

Glossary

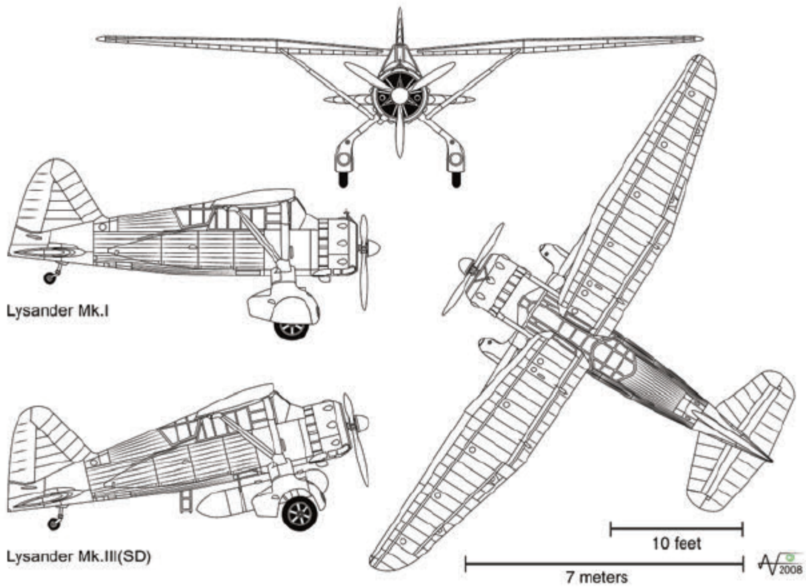
- scale
- scale drawing

Lesson 7 Practice Problems

Problem 1

Statement

The Westland Lysander was an aircraft used by the Royal Air Force in the 1930s. Here are some scale drawings that show the top, side, and front views of the Lysander.



Use the scales and scale drawings to approximate the actual lengths of:

a. the wingspan of the plane, to the nearest foot

b. the height of the plane, to the nearest foot

c. the length of the Lysander Mk. I, to the nearest meter

Solution

a. 46 feet

b. 12 feet

c. 9 meters

Problem 2

Statement

A blueprint for a building includes a rectangular room that measures 3 inches long and 5.5 inches wide. The scale for the blueprint says that 1 inch on the blueprint is equivalent to 10 feet in the actual building. What are the dimensions of this rectangular room in the actual building?

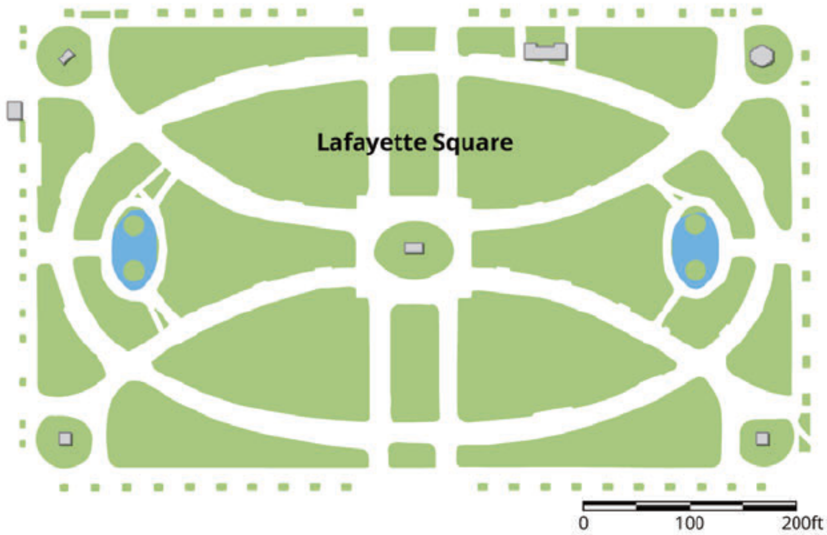
Solution

30 feet long and 55 feet wide

Problem 3

Statement

Here is a scale map of Lafayette Square, a rectangular garden north of the White House.



a. The scale is shown in the lower right corner. Find the actual side lengths of Lafayette Square in feet.

b. Use an inch ruler to measure the line segment of the graphic scale. About how many feet does one inch represent on this map?

Solution

a. About 800 ft by 500 ft

b. Answers vary depending on the size of printed scale. Sample response: 1 in represents 300 feet.

Problem 4

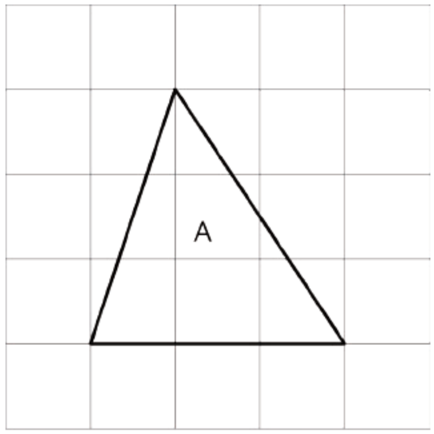
Statement

Here is Triangle A. Lin created a scaled copy of Triangle A with an area of 72 square units.

a. How many times larger is the area of of the scaled copy compared to that of Triangle A?

b. What scale factor did Lin apply to the Triangle A to create the copy?

c. What is the length of bottom side of the scaled copy?



Solution

a. 16 times larger ($72 \div 4.5 = 16$)

b. 4

c. 12 units

(From Unit 1, Lesson 6.)

Lesson 8: Scale Drawings and Maps

Goals

- Justify (orally and in writing) which of two objects was moving faster.
- Use a scale drawing to estimate the distance an object traveled, as well as its speed or elapsed time, and explain (orally and in writing) the solution method.

Learning Targets

- I can use a map and its scale to solve problems about traveling.

Lesson Narrative

This lesson is optional. In this lesson, students apply what they have learned about scale drawings to solve problems involving constant speed (MP1, MP2). Students are given a map with scale as well as a starting and ending point. In addition, they are either given the time the trip takes and are asked to estimate the speed or they are given the speed and asked to estimate how long the trip takes. In both cases, they need to make strategic use of the map and scale and they will need to estimate distances because the roads are not straight.

In the sixth grade, students have examined many contexts involving travel at constant speed. If a car travels at 30 mph, there is a ratio between the time of travel and the distance traveled. This can be represented in a ratio table, or on a graph, or with an equation. If d is the distance traveled in miles, and t is the amount of time in hours, then traveling at 30 mph can be represented by the equation $d = 30t$. Students may or may not use this representation as they work on the activities in this lesson. But they will gain further familiarity with this important context which they will examine in greater depth when they study ratios and proportional reasoning in grade 7, starting in the next unit.

Alignments

Building On

- 6.NS.B.2: Fluently divide multi-digit numbers using the standard algorithm.
- 6.RP.A.3.b: Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Addressing

- 7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Building Towards

- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.
- 7.RP.A.2.b: Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect
- Think Pair Share

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Required Preparation

Ensure students have access to geometry toolkits.

Student Learning Goals

Let's use scale drawings to solve problems.

8.1 A Train and a Car

Warm Up: 5 minutes
This warm-up serves two purposes. It refreshes the concept of distance, rate, and time of travel from grade 6, preparing students to use scale drawings to solve speed-related problems. It also allows students to estimate decimal calculations.

Students are likely to approach the question in a few different ways. As students work, notice students using each strategy.

- By finding or estimating the speed of the train in miles per hour and comparing this to the speed of the car
- By finding the distance the car travels in 4 hours and comparing it to the distance the train travels in 4 hours

Building On

- 6.NS.B.2
- 6.RP.A.3.b

Launch

Give students 3 minutes of quiet think time. Ask students to calculate the answer mentally and to give a signal when they have an answer and explanation. Follow with a whole-class discussion.

> ### Student Task Statement
>
> Two cities are 243 miles apart.
>
> - It takes a train 4 hours to travel between the two cities at a constant speed.
> - A car travels between the two cities at a constant speed of 65 miles per hour.
>
> Which is traveling faster, the car or the train? Be prepared to explain your reasoning.

Student Response

The car is traveling faster. Sample strategy: the speed of the train in miles per hour is $243 \div 4$. This is $(240 \div 4) + (3 \div 4) = 60\frac{3}{4}$, and that's slower than the car. Alternatively, in 4 hours, the car would travel $4 \cdot 65$ or 260 miles, and that's farther than the distance between the cities. So again, the conclusion is that the car is traveling faster.

Activity Synthesis

Invite students to share their strategies. Make sure to highlight different strategies, such as calculating the train's speed from the information and calculating how far the car would travel in 4 hours.

Record and display student explanations for all to see. To involve more students in the conversation, consider asking:

- Did anyone solve the problem in a different way?
- Does anyone want to add on to _____'s strategy?
- Do you agree or disagree? Why?

8.2 Driving on I-90

Optional: 15 minutes (there is a digital version of this activity)
Here, students use a scale and a scale drawing to answer a speed-related question. The task involves at least a couple of steps beyond finding the distance of travel and can be approached in several ways. Minimal scaffolding is given here, allowing students to model with mathematics more independently (MP4).

As students work, notice the different approaches they use to find the actual distance and to determine if the driver was speeding. Some likely variations:

- Comparing the *speed in miles per minute* (calculating the car's speed in miles per minute and converting the speed limit to miles per minute).
- Comparing the *speed in miles per hour* (finding the car's speed in miles per minute and converting it to miles per hour so it can be compared to the speed limit in miles per hour).
- Comparing the *time* it would take to travel the same distance at two different speeds (the car's and the limit).
- Comparing the *distance* traveled in the same amount of time at two different speeds (the car's and the limit).

Identify students using each method so they can share later.

For classrooms using the digital version of the activity, students will be measuring with the Distance or Length tool The tool will measure the shortest distance between two points or the length of a segment.

Addressing

- 7.G.A.1

Building Towards

- 7.RP.A.2.b

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

Launch

Tell students that they will now use a scale drawing (a map) to solve a problem about speed of travel. Survey the class on their familiarity with highway travel and speed limits. If some students are not familiar with speed limits, ask those who are to explain.

Arrange students in groups of 2 and provide access to geometry toolkits. Give students 5 minutes to work on the problem either individually or with their partner.

In the Digital Activity, students have choices about the number of points to plot along the route and whether or not they want to draw segments. Students need to pay attention to the map legend; turning on the grid helps them see that one unit on the grid is equivalent to 0.5 miles.

Support for Students with Disabilities

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. For example, draw students attention to the warm-up and remind them how they calculated the speed of the train in miles per hour. Ask students how they can use this method to calculate the speed of driver from Point A to Point B.
Supports accessibility for: Social-emotional skills; Conceptual processing

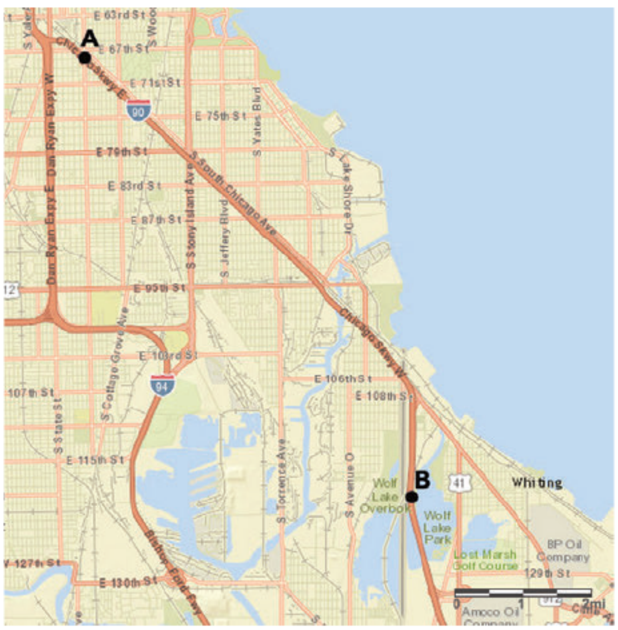
Anticipated Misconceptions

Students might not realize that they need to compare two quantities (either two speeds, two distances traveled in the same amount of time, or two durations of travel) in order to answer the question about speeding. Remind them that there are two potential scenarios here: the driver is obeying the speed limit or the driver is not obeying it.

Once students have found the distance between A and B to be about 8.5 miles, they might be inclined to divide 55 by 8.5 simply because 55 is a larger number. Using double number lines or a table to show the relationship between miles traveled and number of hours might be helpful, as might using friendlier examples of distances (e.g., "How long would it take to travel 110 miles? 11 miles?").

Student Task Statement

1. A driver is traveling at a constant speed on Interstate 90 outside Chicago. If she traveled from Point A to Point B in 8 minutes, did she obey the speed limit of 55 miles per hour? Explain your reasoning.



2. A traffic helicopter flew directly from Point A to Point B in 8 minutes. Did the helicopter travel faster or slower than the driver? Explain or show your reasoning.

Student Response

1. No, she did not. Sample explanations:
    - Using the scale and paper, I found the distance from Point A to Point B to be about 8.5 times the scale representing 1 mile, or 8.5 miles. If she traveled at 55 miles per hour, it would take about 0.15 hour or 9 minutes to travel 8.5 miles, since $8.5 \div 55 \approx 0.15$. Since she got from A to B in 8 minutes, she must have been going faster than the speed limit.
    - The distance between A and B is about 8.5 times the length of the segment representing 1 mile, so the distance is about 8.5 miles. She traveled 8.5 miles in 8 minutes, so her speed was about 1.06 miles per minute. The speed of 55 miles per hour is about 0.917 mile per minute, so the driver did not obey the speed limit.
2. The helicopter traveled slower, since the direct (straight-line) distance is shorter than the distance along the highway. Since it took the same amount of time to travel a shorter distance, the helicopter traveled more slowly.

Activity Synthesis

Ask students to indicate whether they believe the driver was speeding or not. Invite students who approached the task in different ways to share, highlighting methods that focus on:

- Calculating or estimating the speed the driver is traveling (in miles per minute or miles per hour)
- Finding how long it would take to make the trip at the speed limit
- Finding how far the driver would travel in 8 minutes going at the speed limit

Display their work or record or summarize it for all to see.

Ask students if they thought any method seems more efficient than others and why. Highlight that all the methods involved finding the distance traveled, and that the scale drawing and scale enabled us to find that distance.

One method for solving this problem which avoids the decimals that approximate fractional quantities is to observe that 55 miles per hour is the same as 55 miles in 60 minutes, so that is less than one mile per minute. So it will take more than 8.5 minutes to travel 8.5 miles at 55 miles per hour, and the driver must have been speeding.

8.3 Biking through Kansas

Optional: 10 minutes
In the previous activity, students were given a map with a scale and the amount of time it took to get from one place to another. They used this to estimate the speed of the trip. In this activity, students work with the speed and a map with scale to find the amount of time a trip will take.

The main strategy to expect is to measure the distance between the two locations on the map and use the scale to convert this to the distance between the actual cities. Then students can calculate how long it will take at 15 mph.

Addressing

- 7.G.A.1

Building Towards

- 7.RP.A

Instructional Routines

- MLR7: Compare and Connect
- Think Pair Share

Launch

Tell students that they will now use a scale drawing (a map) to solve a different problem about travel, this time focusing on how long it will take. Ask students what is the farthest they have ever biked. How long did it take? Do they know someone who has biked farther or for longer? If so, how far and how long?

Keep students in the same groups. Give students 4–5 minutes of quiet work time followed by partner and whole-class discussion.

Anticipated Misconceptions

The road from Garden City to Dodge City has many twists and bends. Students may not be sure how to treat these. Tell them to make their best estimate. Measuring many small segments of the road will have the advantage that those short segments are straight but it is time consuming. A good estimate will be sufficient here.

Student Task Statement

A cyclist rides at a constant speed of 15 miles per hour. At this speed, about how long would it take the cyclist to ride from Garden City to Dodge City, Kansas?

Student Response

Answers vary. Sample response 1: Using the scale, it appears to be about 50 miles from Garden City. In 3 hours, the cyclist would ride 45 miles, and the remaining 5 miles would take $\frac{1}{3}$ of an hour or 20 minutes. It would take the cyclist about 3 hours and 20 minutes.

Sample response 2: 15 mph is 15 miles in 60 minutes or 1 mile every 4 minutes. So 4 miles take 16 minutes. The (4 mile) scale fit a little more than 12 times, so that means the trip will take a little more than $12 \cdot 16$ minutes. That's 192 minutes or 3 hours and 12 minutes.

Are You Ready for More?

Jada finds a map that says, "Note: This map is not to scale." What do you think this means? Why is this information important?

Student Response

Answers vary. Sample response: it means that there is no one scale factor that relates distances on the map to distances in the place represented by the map. Some distances are distorted. If Jada were using her map to calculate how long it would take her to travel from one point to another on the map, her prediction may be inaccurate.

Activity Synthesis

First, have students compare answers with a partner and discuss their reasoning until they reach an agreement.

Next, invite students to share how they estimated the distance between the two cities (and how long it takes the cyclist to travel this distance). Ask students to consider the different distances students estimated the trip to be. What are some reasons for the differences? Possible explanations include:

- Measurement error

- The road is not straight and so needs to be approximated
- For students who lay out the scale over and over again to cover the distance, it is difficult to estimate the fraction of the scale at the last step

Because of these different sources of inaccuracy, reporting the distance as 50 miles is reasonable; reporting it as 52 miles would require a lot of time and measurements; and reporting it as 51.6 miles is not reasonable with the given scale and map.

Support for English Language Learners

Speaking, Listening: MLR7 Compare and Connect. As students work to determine the duration of the trip from Garden City to Dodge City, look for students with different strategies for estimating the distance between the two cities. As students investigate each other's work, ask students to share what worked well in a particular approach. During this discussion, listen for any comments that make the estimation of the distance more precise. Then encourage students to make connections between the various uses of constant speed to calculate the duration of the trip. Amplify language students use to make sense of the cyclist's constant speed and how it could be represented in the map. This will support constructive conversations as students compare strategies for calculating the duration of a trip and make connections between the quantity and visual representations of constant speed on the map.
Design Principle(s): Cultivate conversation; Maximize meta-awareness

Lesson Synthesis

A map with a scale helps estimate the distance between two places by measuring the distance on the map and using the scale to find the actual distance. Once the distance between two places is known:

- If we know how long the trip takes, we can calculate the speed by finding the quotient of the distance and the time.
- If we know the speed, we can calculate how long the trip takes by finding the quotient of the distance and the speed.

In both cases, care has to be taken regarding units. For example, if a 130-mile trip at a constant speed takes two hours, then the speed is 65 miles per hour, because $130 \div 2 = 65$. A 35-mile trip at 70 miles per hour takes $\frac{1}{2}$ hour, because $35 \div 70 = \frac{1}{2}$.

8.4 Walking Around the Botanical Garden

Cool Down: 5 minutes
Addressing

- 7.G.A.1

Building Towards

- 7.RP.A

Launch

Provide access to geometry toolkits. Make sure students know where the boundaries of the Botanical Garden are on the map.

Student Task Statement

Here is a map of the Missouri Botanical Garden. Clare walked all the way around the garden.

1. What is the actual distance around the garden? Show your reasoning.

2. It took Clare 30 minutes to walk around the garden at a constant speed. At what speed was she walking? Show your reasoning.

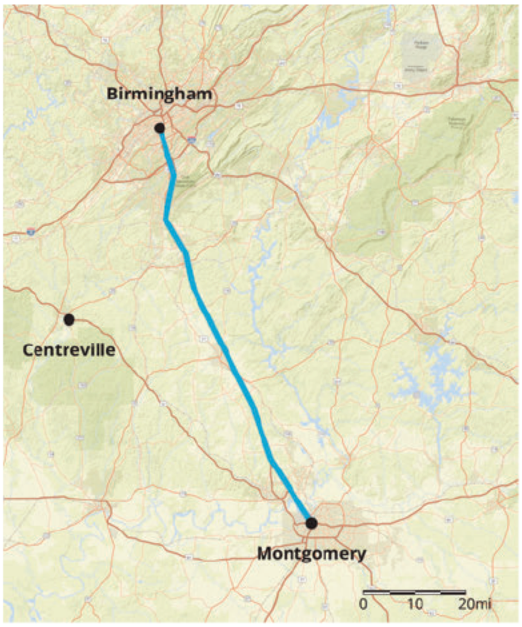
Student Response

1. It takes about 14 segments of the scale to measure the perimeter of the garden, and $14 \cdot 600 = 8{,}400$. So the distance around is about 8,400 feet.

2. If she walks for 30 minutes, that means she was traveling at about 280 feet per minute $(8{,}400 \div 30 = 280)$, or about 16,800 feet per hour $(280 \cdot 60 \approx 16{,}800)$.

Student Lesson Summary

Maps with scales are useful for making calculations involving speed, time, and distance. Here is a map of part of Alabama.



Suppose it takes a car 1 hour and 30 minutes to travel at constant speed from Birmingham to Montgomery. How fast is the car traveling?

To make an estimate, we need to know about how far it is from Birmingham to Montgomery. The scale of the map represents 20 miles, so we can estimate the distance between these cities is about 90 miles.

Since 90 miles in 1.5 hours is the same speed as 180 miles in 3 hours, the car is traveling about 60 miles per hour.

time (hours)	distance (miles)
1.5	90
3	180
1	60

($\cdot 2$ from 1.5 to 3 and from 90 to 180; $\cdot \frac{1}{3}$ from 3 to 1 and from 180 to 60)

Suppose a car is traveling at a constant speed of 60 miles per hour from Montgomery to Centreville. How long will it take the car to make the trip? Using the scale, we can estimate that it is about 70 miles. Since 60 miles per hour is the same as 1 mile per minute, it will take the car about 70 minutes (or 1 hour and 10 minutes) to make this trip.

Lesson 8 Practice Problems

Problem 1

Statement

Here is a map that shows parts of Texas and Oklahoma.

a. About how far is it from Amarillo to Oklahoma City? Explain your reasoning.

b. Driving at a constant speed of 70 miles per hour, will it be possible to make this trip in 3 hours? Explain how you know.

Solution

a. About 260 miles (but the road is not straight, so it is hard to tell the exact distance from the map)

b. No, a traveler can only go 210 miles in 3 hours, and the distance between the cities is definitely farther than that.

Problem 2

Statement

A local park is in the shape of a square. A map of the local park is made with the scale 1 inch to 200 feet.

a. If the park is shown as a square on the map, each side of which is one foot long, how long is each side of the square park?

b. If a straight path in the park is 900 feet long, how long would the path be when represented on the map?

Solution

a. 2,400 feet

b. 4.5 inches

Lesson 9: Creating Scale Drawings

Goals

- Compare and contrast (orally) different scale drawings of the same object, and describe (orally) how the scale affects the size of the drawing.
- Create a scale drawing, given the actual dimensions of the object and the scale.
- Determine the scale used to create a scale drawing and generate multiple ways to express it (in writing).

Learning Targets

- I can determine the scale of a scale drawing when I know lengths on the drawing and corresponding actual lengths.
- I know how different scales affect the lengths in the scale drawing.
- When I know the actual measurements, I can create a scale drawing at a given scale.

Lesson Narrative

In previous lessons, students have used scale drawings to calculate actual distances. This is the first lesson where students use the actual distance to calculate the scaled distance and create their own scale drawings. They see how different scale drawings can be created of the same actual thing, using different scales. They also see how the choice of scale influences the drawing. For example, a scale drawing with a scale of 1 cm to 5 m will be smaller than a scale drawing of the same object with a scale of 1 cm to 2 m (since each cm represents a larger distance, it takes fewer cm to represent the object). This prepares them for future lessons where they will recreate a given scale drawing at a different scale.

Noticing *how* scaled drawing change with the choice of scale develops important structural understanding of scale drawings (MP7).

Alignments

Building On

- 3.NF.A.3: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
- 5.NBT.A.3: Read, write, and compare decimals to thousandths.

Addressing

- 7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Notice and Wonder
- Number Talk
- Think Pair Share

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Required Preparation

Ensure students have access to geometry toolkits.

Student Learning Goals

Let's create our own scale drawings.

9.1 Number Talk: Which is Greater?

Warm Up: 5 minutes
In this number talk, students compare quantities involving division with whole numbers, decimals, and fractions. In each case, a strategy is available that does not require calculating the quantities. When the quantities are complex, there is motivation for using the structure of the expressions to compare (MP7) since the actual calculations would be more time-consuming.

Building On

- 3.NF.A.3
- 5.NBT.A.3

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 1 minute of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Follow with a whole-class discussion.

Support for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organization

Anticipated Misconceptions

Students may misinterpret the last question as $15 \cdot \frac{1}{3}$ or $15 \cdot \frac{1}{4}$. Point out that one way to interpret the first expression is "How many one-thirds are there in 15?"

Student Task Statement

Without calculating, decide which quotient is larger.

$11 \div 23$ or $7 \div 13$

$0.63 \div 2$ or $0.55 \div 3$

$15 \div \frac{1}{3}$ or $15 \div \frac{1}{4}$

Student Response

- $7 \div 13$ is greater, because it is greater than $\frac{1}{2}$ while $\frac{11}{23}$ is less than $\frac{1}{2}$.
- $0.63 \div 2$ is greater than $0.55 \div 3$ since $0.63 > 0.55$ and 0.63 is being divided by 2, whereas 0.55 is being divided into more equal parts (3).
- $15 \div \frac{1}{4}$ is greater than $15 \div \frac{1}{3}$ since $\frac{1}{4}$ is less than $\frac{1}{3}$, and dividing by a smaller (unit) fraction gives a larger quotient.

Activity Synthesis

Make sure to bring out different approaches for comparing the quantities, avoiding direct calculation where possible:

- $\frac{7}{13}$ and $\frac{11}{23}$ can both be compared with $\frac{1}{2}$, or students can find a common numerator or denominator, but this requires more calculations.
- 0.63 is greater than 0.55, and 2 is less than 3, or students might notice that $0.63 \div 2$ is greater than 0.3 while $0.55 \div 3$ is less than 0.2.

- Dividing by $\frac{1}{4}$ is equivalent to multiplying by 4 while dividing by $\frac{1}{3}$ is equivalent to multiplying by 3, so $15 \div \frac{1}{4}$ is greater than $15 \div \frac{1}{3}$.

Support for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimize output (for explanation)

9.2 Bedroom Floor Plan

10 minutes (there is a digital version of this activity)
In previous lessons in this unit, students have investigated the meaning of scale drawings and have used them to solve problems. The purpose of this activity is to prepare students for creating their own scale drawing. The discussion highlights that a scale can be expressed in different ways, or that different pairs of numbers may be used to show the same relationship. For example, a scale of 4 cm to 1 m is equivalent to a scale of 1 cm to 0.25 m.

As students work, notice those who use the language of scale appropriately, e.g., by saying that "every 4 cm on the drawing represents 1 m," or "every 0.25 m shows up as 1 cm on the drawing." Also notice those who do and do not attend to the relationship between actual and scaled lengths in finding missing measurements.

Addressing

- 7.G.A.1

Instructional Routines

- MLR7: Compare and Connect
- Think Pair Share

Launch

Tell students that a floor plan is a top-view drawing that shows a layout of a room or a building. Floor plans are usually scale drawings. Explain that sometimes the scale of a drawing is not specified, but we can still tell the scale if we know both the scaled and actual lengths.

Arrange students in groups of 2. Give students 4–5 minutes of quiet work time and partner discussion.

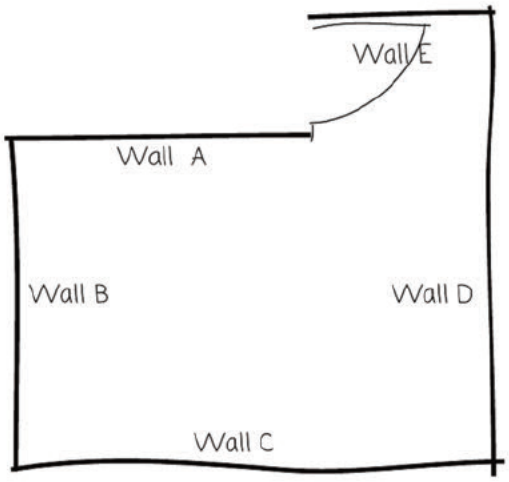
For students using the Digital Activity, teachers can allow exploration with the applet during think time.

Anticipated Misconceptions

Students may see that one value is 4 times the other and write the scale backwards, as "1 cm to 4 m." Prompt students to pay attention to the units and the meaning of each number.

Student Task Statement

Here is a rough sketch of Noah's bedroom (not a scale drawing).



Noah wants to create a floor plan that is a scale drawing.

1. The actual length of Wall C is 4 m. To represent Wall C, Noah draws a segment 16 cm long. What scale is he using? Explain or show your reasoning.

2. Find another way to express the scale.

3. Discuss your thinking with your partner. How do your scales compare?

4. The actual lengths of Wall A and Wall D are 2.5 m and 3.75 m. Determine how long these walls will be on Noah's scale floor plan. Explain or show your reasoning.

Student Response

1. 4 cm to 1 m; 1 cm to 0.25 m; or 16 cm to 4 m. Sample explanation: Since 16 cm represents an actual length of 4 m, then 1 cm must represent $\frac{1}{16}$ of 4 m, which is 0.25 m.

2. Answers vary depending on response to the first question.

3. Answers vary.

4. Wall A: 10 cm. Wall D: 15 cm. Sample explanations:
    - Since every 1 m is shown as 4 cm on the drawing, I multiplied the actual lengths, in meters, by 4 to find how many centimeters long the scale drawing should be.

    - Since every 1 cm represents 0.25 m, I divided the actual lengths, in meters, by 0.25 to find how many centimeters long the scale drawing should be.

Are You Ready for More?

If Noah wanted to draw another floor plan on which Wall C was 20 cm, would 1 cm to 5 m be the right scale to use? Explain your reasoning.

Student Response

No. Sample explanations:

- If he used 1 cm to 5 m, Wall C, which is 4 m long, will be less than 1 cm on the drawing—much smaller than what he wanted.
- If he used 1 cm to 5 m, a 20-cm segment would represent a 100-m long wall, which is not the length of Wall C.

Activity Synthesis

Focus the class discussion on two things:

1. Different ways to express the same scale
2. The relationship between scaled and actual lengths

Invite a couple of students to share how they determined the scale of the drawing. Students are likely to come up with several variations, e.g., 4 cm to 1 m, 1 cm to 0.25 m, 16 cm to 4 m, etc. Discuss how all of these express the same relationship and are therefore equivalent, especially how 4 cm to 1 m is equivalent to 1 cm to 0.25 m (or 1 cm to $\frac{1}{4}$ m).

Explain that although we can express a scale in multiple but equivalent ways, 1) scales are often simplified to show the actual distance for 1 scaled unit, and 2) it is common to express at least one distance (usually the scaled distance) as a whole number or a benchmark fraction (e.g., $\frac{1}{4}$, $\frac{1}{2}$) or a benchmark decimal (e.g., 0.25, 0.5, 0.75).

Given their work on scaled copies, students may be inclined to say that the scaled and actual lengths are related by a scale factor of 4. Ask: “Are the actual lengths four times the lengths on the drawing? Why or why not?” Point out that because the units for the two quantities are different, multiplying a scaled length in centimeters (e.g., 2.5 cm) by 4 will yield another length in centimeters (10 cm), which is not the actual length. It is not essential for students to know that the scale factor here is 250. That work will be explored in an upcoming lesson.

Support for English Language Learners

Speaking, Listening: MLR7 Compare and Connect. As students prepare a visual display of how they created the floor plan, look for students who expressed the scale in different ways. As students investigate each other's work, ask them to share what is especially clear about a particular approach. Then encourage students to explain why there are various yet equivalent ways to express the scale, such as 4 cm to 1 m and 1 cm to 0.25 m. Emphasize the language used to make sense of the different ways to express the same scale (e.g., Since 4 cm on the floor plan represents 1 m in the actual room, then 1 cm on the floor plan represents $\frac{1}{4}$ of 1 m, which is 0.25 m.) This will reinforce students' use of mathematical language related to equivalent scales.
Design Principles(s): Cultivate conversation; Maximize meta-awareness

9.3 Two Maps of Utah

15 minutes

In the previous activity, students calculated the scaled distances they would need to create a scale drawing, but did not actually create the scale drawing. In this activity, they create two different scale drawings of the state of Utah and notice how the scale impacts the drawing. One of the reasons choice of a scale is important is that we want to see the appropriate level of detail within a fixed space.

Addressing

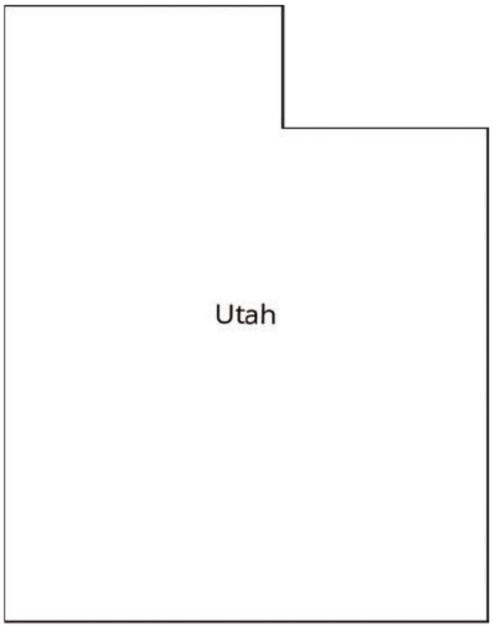
- 7.G.A.1

Instructional Routines

- MLR3: Clarify, Critique, Correct
- Notice and Wonder

Launch

Display the outline of Utah and ask students "What do you notice? What do you wonder?"



Ask students to describe the shape: a rectangle with a smaller rectangle removed in the upper right corner.

Give students 5–6 minutes of quiet work time followed by whole-class discussion.

Anticipated Misconceptions

Some students may get a shape that is not closed or does not have right angles if they did not measure carefully enough. Prompt them to double-check their measurement for a particular side of the state if you can easily tell which side is drawn incorrectly.

Students may think that a scale of 1 centimeter to 50 miles will produce a smaller scale drawing than a scale of 1 centimeter to 75 miles (because 50 is less than 75). Ask them how many centimeters it takes to represent 75 miles if 1 centimeter represents 50 miles (1.5) and how many centimeters it takes to represent 75 miles if 1 centimeter represents 75 miles (1).

Student Task Statement

A rectangle around Utah is about 270 miles wide and about 350 miles tall. The upper right corner that is missing is about 110 miles wide and about 70 miles tall.

1. Make a scale drawing of Utah where 1 centimeter represents 50 miles.

   Make a scale drawing of Utah where 1 centimeter represents 75 miles.

2. How do the two drawings compare? How does the choice of scale influence the drawing?

Student Response

1. A rectangle approximately 5.4 centimeters wide and 7 centimeters tall, missing an upper right corner which is approximately 2.2 centimeters wide and 1.4 centimeters tall and a rectangle

approximately 3.6 centimeters wide and 4.7 centimeters tall, missing an upper right corner which is approximately 1.5 centimeters wide and 1 centimeter tall.

2. The measurements in the 1 cm to 50 mile scale drawings are larger than the measurements in the 1 cm to 75 mile scale drawing. This makes sense because when 1 centimeter represents 50 miles, it takes 1.5 centimeters to represent 75 miles.

Activity Synthesis

Ask students what the two scale drawings share in common. Answers include: they both represent Utah, they both have the same shape, and they both can be used to measure distances in the actual state of Utah.

Ask students how the two scale drawings differ. The one at a scale of 1 centimeter to 50 miles is larger than the one at a scale of 1 centimeter to 75 miles.

Some students may notice that the scale drawing at a scale of 1 centimeter to 75 miles is actually a scaled copy of the other drawing, with a scale factor of 1.5. If so, ask them to share their observation linking scale drawings with scale copies.

Support for Students with Disabilities

Representation: Internalize Comprehension. Use color coding and annotations to highlight connections between representations in a problem. For example, annotate a display of the two scale drawings to make visible what the drawings share in common, and how they differ.
Supports accessibility for: Visual-spatial processing

Support for English Language Learners

Reading, Speaking: MLR3 Clarify, Critique, Correct. Before presenting the correct scale drawings of Utah, present an incorrect drawing and written explanation. For example, present a rectangle approximately 5.4 cm wide and 7 cm tall that is missing an upper right corner which is approximately 1.4 cm wide and 2.2 cm tall, and provide the statement: “Since 1 cm represents 50 miles, I divided 110 and 70 each by 50, and got 2.2 and 1.4. The small rectangle that is missing is 2.2 cm tall and 1.4 cm wide.” Ask students to identify the error, critique the reasoning, and revise the statement so that the drawing is a scale drawing of Utah. This will remind students about the characteristics of scale drawings discussed in previous lessons and how to determine whether a drawing qualifies as a scale drawing of an actual object.
Design Principle(s): Optimize output (for explanation); Maximize meta-awareness

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Lesson Synthesis

The size of the scale determines the size of the drawing. You can have different-sized scale drawings of the same actual object, but the size of the actual object doesn't change.

- "Suppose there are two scale drawings of the same house. One uses the scale of 1 cm to 2 m, and the other uses the scale 1 cm to 4 m. Which drawing is larger? Why?" (The one with the 1 cm to 2 m scale is larger, because it takes 2 cm on the drawing to represent 4 m of actual length.)
- "Another scale drawing of the house uses the scale of 5 cm to 10 m. How does its size compare to the other two?" (It is the same size as the drawing with the 1 cm to 2 m scale.)

Sometimes two different scales are actually equivalent, such as 5 cm to 10 m and 1 cm to 2 m. It is common to write a scale so that it tells you what one unit on the scale drawing represents (for example, 1 cm to 2 m).

9.4 Drawing a Pool

Cool Down: 5 minutes

Addressing

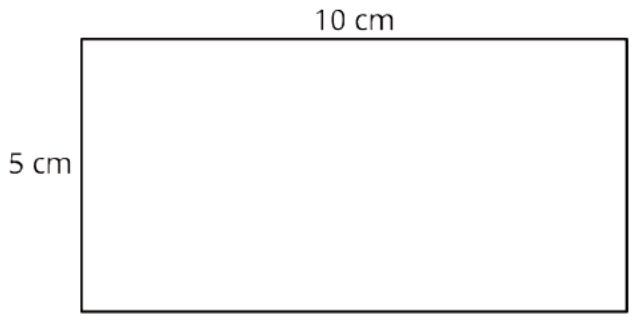
- 7.G.A.1

Student Task Statement

A rectangular swimming pool measures 50 meters in length and 25 meters in width.

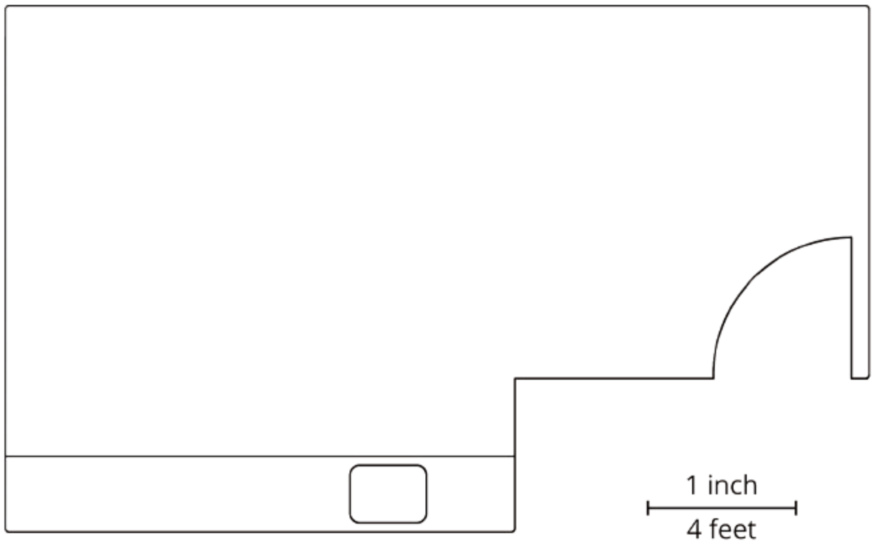
1. Make a scale drawing of the swimming pool where 1 centimeter represents 5 meters.
2. What are the length and width of your scale drawing?

Student Response



Student Lesson Summary

If we want to create a scale drawing of a room's floor plan that has the scale "1 inch to 4 feet," we can divide the actual lengths in the room (in feet) by 4 to find the corresponding lengths (in inches) for our drawing.



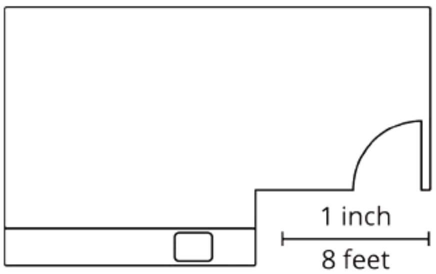
Suppose the longest wall is 15 feet long. We should draw a line 3.75 inches long to represent this wall, because $15 \div 4 = 3.75$.

There is more than one way to express this scale. These three scales are all equivalent, since they represent the same relationship between lengths on a drawing and actual lengths:

- 1 inch to 4 feet
- $\frac{1}{2}$ inch to 2 feet
- $\frac{1}{4}$ inch to 1 foot

Any of these scales can be used to find actual lengths and scaled lengths (lengths on a drawing). For instance, we can tell that, at this scale, an 8-foot long wall should be 2 inches long on the drawing because $\frac{1}{4} \cdot 8 = 2$.

The size of a scale drawing is influenced by the choice of scale. For example, here is another scale drawing of the same room using the scale 1 inch to 8 feet.



Notice this drawing is smaller than the previous one. Since one inch on this drawing represents twice as much actual distance, each side length only needs to be half as long as it was in the first scale drawing.

Lesson 9 Practice Problems

Problem 1

Statement

An image of a book shown on a website is 1.5 inches wide and 3 inches tall on a computer monitor. The actual book is 9 inches wide.

a. What scale is being used for the image?

b. How tall is the actual book?

Solution

a. 1 inch to 6 inches

b. 18 inches

Problem 2

Statement

The flag of Colombia is a rectangle that is 6 ft long with three horizontal strips.

- The top stripe is 2 ft tall and is yellow.
- The middle stripe is 1 ft tall and is blue.
- The bottom stripe is also 1 ft tall and is red.

a. Create a scale drawing of the Colombian flag with a scale of 1 cm to 2 ft.

b. Create a scale drawing of the Colombian flag with a scale of 2 cm to 1 ft.

Solution

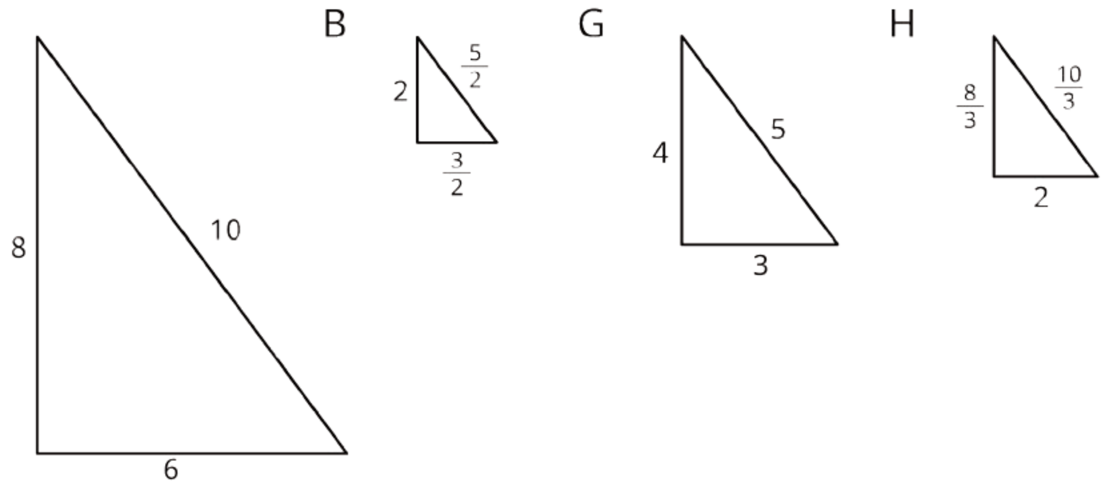
a. The flag will be 3 cm long and 2 cm tall. The yellow rectangle is 1 cm tall and the red and blue rectangles are each 0.5 cm tall.

b. The flag will be 12 cm long and 8 cm tall. The yellow rectangle is 4 cm tall and the red and blue rectangles are each 2 cm tall.

Problem 3

Statement

These triangles are scaled copies of each other.



For each pair of triangles listed, the area of the second triangle is how many times larger than the area of the first?

a. Triangle G and Triangle F

b. Triangle G and Triangle B

c. Triangle B and Triangle F

d. Triangle F and Triangle H

e. Triangle G and Triangle H

f. Triangle H and Triangle B

Solution

a. 4

b. $\frac{1}{4}$

c. 16

d. $\frac{1}{9}$

e. $\frac{4}{9}$

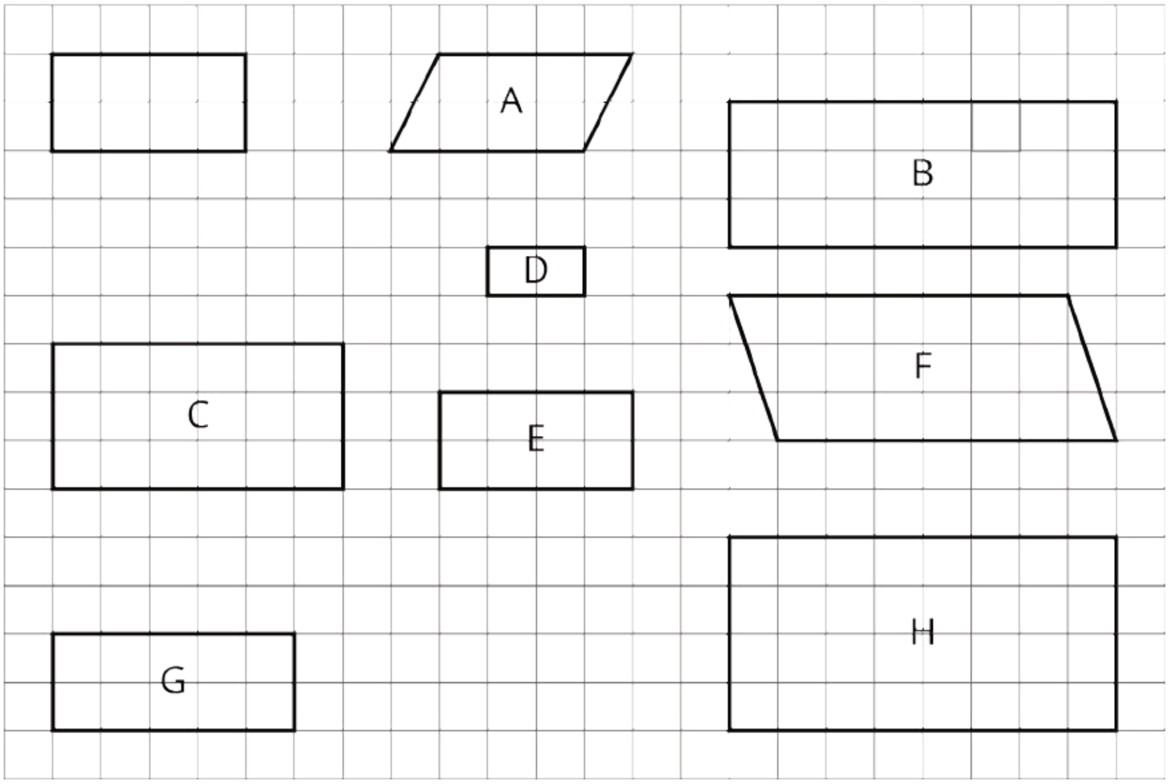
f. $\frac{9}{16}$

(From Unit 1, Lesson 6.)

Problem 4

Statement

Here is an unlabeled rectangle, followed by other quadrilaterals that are labeled.



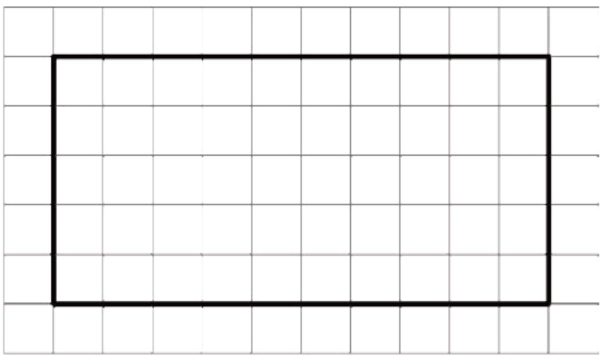
a. Select **all** quadrilaterals that are scaled copies of the unlabeled rectangle. Explain how you know.

b. On graph paper, draw a different scaled version of the original rectangle.

Solution

a. C, D, E, and H. Sample explanation: The length and width of each copy is related to the length and width of the original by the same factor and the corresponding angles are unchanged.

b. Drawings vary. Sample response:



(From Unit 1, Lesson 3.)

Lesson 10: Changing Scales in Scale Drawings

Goals

- Determine how much actual area is represented by one square unit in a scale drawing.
- Generalize (orally) that as the actual distance represented by one unit on the drawing increases, the size of the scale drawing decreases.
- Reproduce a scale drawing at a different scale and explain (orally) the solution method.

Learning Targets

- Given a scale drawing, I can create another scale drawing that shows the same thing at a different scale.
- I can use a scale drawing to find actual areas.

Lesson Narrative

In the previous lesson, students created multiple scale drawings using different scales. In this lesson, students are given a scale drawing and asked to recreate it at a different scale. Two possible strategies to produce these drawings are:

- Calculating the actual lengths and then using the new scale to find lengths on the new scale drawing.
- Relating the two scales and calculating the lengths for the new scale drawing using corresponding lengths on the given drawing.

In addition, students previously saw that the area of a scaled copy can be found by multiplying the area of the original figure by $(\text{scale factor})^2$. In this lesson, they extend this work in two ways:

- They compare areas of scale drawings of the same object with different scales.
- They examine how much area, on the actual object, is represented by 1 square centimeter on the scale drawing. For example, if the scale is 1 cm to 50 m, then 1 cm^2 represents $50 \boldsymbol{\cdot} 50$, or 2,500 m^2.

Throughout this lesson, students observe and explain structure (MP7), both when they reproduce a scale drawing at a different scale and when they study how the area of a scale drawing depends on the scale.

Alignments

Building On

- 2.MD.A: Measure and estimate lengths in standard units.

- 6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Addressing

- 7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Building Towards

- 7.G.B.6: Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.
- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display
- MLR8: Discussion Supports

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Pre-printed slips, cut from copies of the blackline master

Required Preparation

Print and cut the scales for the Same Plot, Different Drawings activity from the blackline master (1 set of scales per group of 5–6 students).

Ensure students have access to their geometry toolkits, especially centimeter rulers.

Student Learning Goals

Let's explore different scale drawings of the same actual thing.

10.1 Appropriate Measurements

Warm Up: 5 minutes

This warm-up prompts students to attend to precision in measurements, which will be important in upcoming work.

Building On

- 2.MD.A

Addressing

- 7.G.A.1

Building Towards

- 7.RP.A.3

Launch

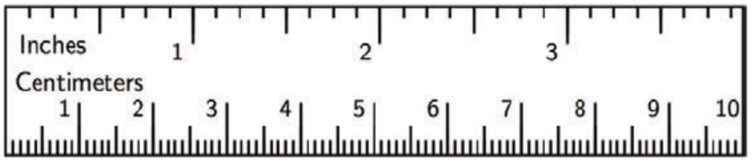
Arrange students in groups of 2. Give students 1 minute of quiet think time to estimate the size of their own foot in centimeters or inches, and a moment to share their estimate with a partner. Then, ask them to complete the task.

Anticipated Misconceptions

Some students may say the large foot is about $3\frac{1}{2}$ inches or about 9 centimeters long, because they assume the ruler shown in the first question is at the same scale as the feet shown in the second question. Explain that the images are drawn at different scales.

Student Task Statement

1. If a student uses a ruler like this to measure the length of their foot, which choices would be appropriate measurements? Select **all** that apply. Be prepared to explain your reasoning.



a. $9\frac{1}{4}$ inches

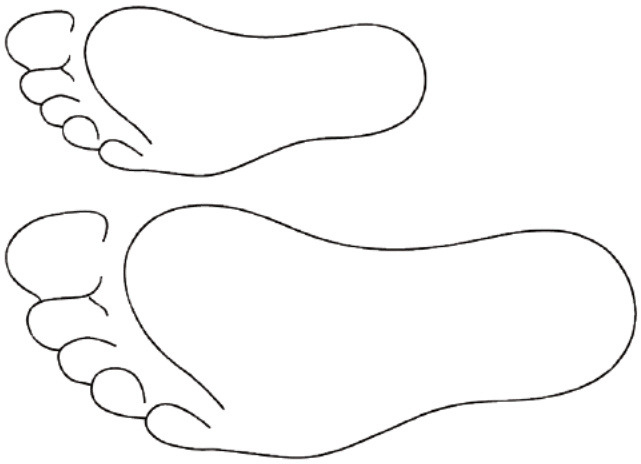
b. $9\frac{5}{64}$ inches

c. 23.47659 centimeters

d. 23.5 centimeters

e. 23.48 centimeters

2. Here is a scale drawing of an average seventh-grade student's foot next to a scale drawing of a foot belonging to the person with the largest feet in the world. Estimate the length of the larger foot.



Student Response

1. A and D would be the only appropriate measurements based on the markings on the given ruler. Since the ruler is only marked in $\frac{1}{8}$ inches and $\frac{1}{10}$ centimeter, we could not get measurements as precise as B, C, or E.

2. The largest foot in the world is about 1.5 times as long as the average seventh grader's foot. My foot is about 10 inches long, so the largest foot is about 15 inches or 1 foot and 3 inches long.

Activity Synthesis

Select a few students to share the measurements they think would be appropriate based on the given ruler. Consider displaying the picture of the ruler for all to see and recording students' responses on it. After each response, poll the class on whether they agree or disagree.

If students consider B, C, or E to be an appropriate measurement, ask them to share how to get such a level of precision on the ruler. Make sure students understand that reporting measurements to the nearest $\frac{1}{64}$ of an inch or to the hundred-thousandths of a centimeter would not be appropriate (i.e., show that the ruler does not allow for these levels of precision).

Choice E of 23.48 cm may merit specific attention. With the ruler, it is possible to *guess* that the hundredths place is an 8. This may even be correct. The problem with reporting the measurement in this way is that someone who sees this might misinterpret it and imagine that an extremely accurate measuring device was used to measure the foot, rather than this ruler. The way a measurement is reported reflects how the measurement was taken.

Next, invite students to share their estimates for the length of the large foot. Since it is difficult to measure the length of these feet very precisely, these measurements should not be reported with a high level of precision; the nearest centimeter would be appropriate.

10.2 Same Plot, Different Drawings

15 minutes

This activity serves several purposes: to allow students to practice creating scale drawings at given scales, to draw attention to the size of the scale drawing as one of the values in the scale changes, and to explore more fully the relationship between scaled area and actual area.

Each group member uses a different scale to calculate scaled lengths of the same plot of land, draw a scale drawing, and calculate its scaled area. The group then orders the different drawings and analyzes them. They think about how many square meters of actual area are represented by one square centimeter on each drawing. Students are likely to determine this value in two ways:

- By visualizing what a 1×1 centimeter square represents at a given scale (e.g., at a scale of 1 cm to 5 m, each 1 cm^2 represents $5 \cdot 5$, or 25 m^2).
- By dividing the actual area represented by the scale drawing by the area of their scale drawing

The relationships between scale, lengths in scale drawings, and area in scale drawings are all important examples of the mathematical structure (MP7) of scale drawings.

You will need the Same Plot, Different Drawings blackline master for this activity.

Building On

- 6.G.A.1

Addressing

- 7.G.A.1

Building Towards

- 7.G.B.6

Instructional Routines

- MLR2: Collect and Display

Launch

Display this map of a neighborhood in Philadelphia for all to see. Tell students that they are going to reproduce a map of the triangular piece of land at a different scale.

Tell students that the actual base of the triangle is 120 m and its actual height is 90 m. Ask, "What is the area of the plot of land?" (5400 square meters—one half the base times the height of the triangle.)

Arrange students in groups of 5–6 and provide access to centimeter graph paper. Assign each student in a group a different scale (from the blackline master) to use to create a scale drawing. Give students 3–4 minutes of quiet work time to answer the first 3 questions, and then 1–2 minutes to work on the last question in their groups.

Remind students to include the units in their measurements.

Support for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge about finding area of a triangle. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

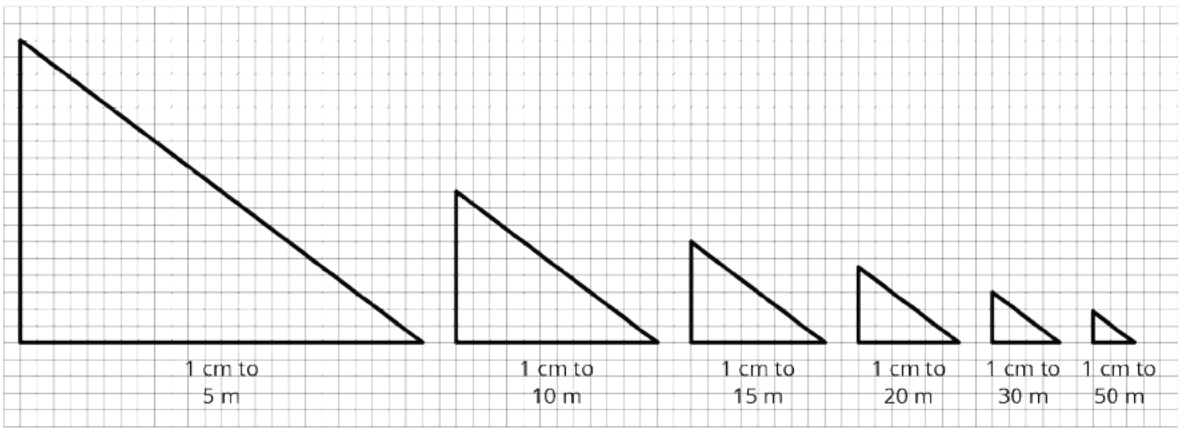
Student Task Statement

Here is a map showing a plot of land in the shape of a right triangle.

1. Your teacher will assign you a scale to use. On centimeter graph paper, make a scale drawing of the plot of land. Make sure to write your scale on your drawing.
2. What is the area of the triangle you drew? Explain or show your reasoning.
3. How many square meters are represented by 1 square centimeter in your drawing?
4. After everyone in your group is finished, order the scale drawings from largest to smallest. What do you notice about the scales when your drawings are placed in this order?

Student Response

1. Right triangles of various sizes:



2. Answers vary depending on the assigned scale. Possible solutions:
   - 216 cm^2, because $\frac{1}{2} \cdot 24 \cdot 18 = 216$.

- 54 cm^2, because $\frac{1}{2} \cdot 12 \cdot 9 = 54$.
- 24 cm^2, because $\frac{1}{2} \cdot 8 \cdot 6 = 24$.
- 13.5 cm^2, because $\frac{1}{2} \cdot 6 \cdot (4.5) = 13.5$.
- 6 cm^2, because $\frac{1}{2} \cdot 4 \cdot 3 = 6$.
- 2.16 cm^2, because $\frac{1}{2} \cdot (2.4) \cdot (1.8) = 2.16$.

3. Answers vary depending on the assigned scale. Possible solutions:

    - 25 m^2, because $5400 \div 216 = 25$.
    - 100 m^2, because $5400 \div 54 = 100$.
    - 225 m^2, because $5400 \div 24 = 225$.
    - 400 m^2, because $5400 \div 13.5 = 400$.
    - 900 m^2, because $5400 \div 6 = 900$.
    - 2,500 m^2, because $5400 \div 2.16 = 2500$.

4. The smaller the number of meters represented by one centimeter, the larger the scale drawing is.

Are You Ready for More?

Noah and Elena each make a scale drawing of the same triangular plot of land, using the following scales. Make a prediction about the size of each drawing. How would they compare to the scale drawings made by your group?

1. Noah uses the scale 1 cm to 200 m.
2. Elena uses the scale 2 cm to 25 m.

Student Response

1. Noah's drawing will be smaller than all the other drawings. The scale that created the smallest drawing so far was 1 cm to 50 m. Each length in a drawing done at 1 cm to 200 m will be 4 times as small as in the 1 cm-to-50 m drawing because every centimeter represents 4 times as much length.
2. The scale 2 cm to 25 m is equivalent to 1 cm to 12.5 m, so Elena's drawing will be larger than the 1 cm to 15 m drawing but smaller than the 1 cm to 10 m drawing.

Activity Synthesis

Focus the discussion on patterns or features students noticed in the different scale drawings. Ask questions such as:

- How does a change in the scale influence the size of the drawings?
  (As the length being represented by 1 cm gets larger, the size of the drawing decreases.)
- How do the lengths of the scale drawing where 1 cm represents 5 meters compare to the lengths of the drawing where 1 cm represents 15 meters? (They are three times as long.)
- How do the lengths of the scale drawing where 1 cm represents 5 meters compare to the lengths of the drawing where 1 cm represents 50 meters? (They are ten times as long.)
- How does the area of the scale drawing where 1 cm represents 5 meters compare to the area of the drawing where 1 cm represents 15 meters? (It is 9 times as great.)
- How does the area of the scale drawing where 1 cm represents 5 meters compare to the area of the drawing where 1 cm represents 50 meters? (It is 100 times as great.)

Help students to observe and formulate these patterns:

- As the number of meters represented by one centimeter increases, the lengths in the scale drawing decrease.
- As the number of meters represented by one centimeter increases, the area of the scale drawing also decreases, but it decreases by the square of the factor for the lengths (because finding the area means multiplying the length and width, both of which decrease by the same factor).

Support for English Language Learners

Conversing, Reading: Math Language Routine 2 Collect and Display. This is the first time Math Language Routine 2 is suggested as a support in this course. In this routine, the teacher circulates and listens to student talk while jotting down words, phrases, drawings, or writing students use. The language collected is displayed visually for the whole class to use throughout the lesson and unit. Generally, the display contains different examples of students using features of the disciplinary language functions, such as interpreting, justifying, or comparing. The purpose of this routine is to capture a variety of students' words and phrases in a display that students can refer to, build on, or make connections with during future discussions, and to increase students' awareness of language used in mathematics conversations.
Design Principle(s): Support sense-making; Maximize meta-awareness

How It Happens:

1. As students share their ideas in their groups about what they notice in the scale drawings, write down the language they use to describe how the scale affects the size of the scale drawing. Listen for the language students use to compare the lengths and areas of scale drawings with different scales.

   To support the discussion, provide these sentence frames: "As the value of the scale increases, the size of the drawing ___ because ___.", "When I compare the lengths of (choose two scale drawings), I notice that ___.", and "When I compare the areas of (choose two scale drawings), I notice that ___."

2. As groups close their conversation, display the language collected for all to reference.

3. Next, facilitate a whole-class discussion encouraging students to ask and respond to clarifying questions about the meaning of a word or phrase on the display.

   To prompt discussion, ask students, "What word or phrase is unclear? From the language I collected, what part does not make sense to you?"

   Here is an example:

   Student A: The phrase 'the triangle is 9 times greater' is unclear to me. I'm not sure what 9 times greater means.

   Teacher: [Point to the phrase on the display] Can someone clarify this phrase with specific details?

   Student B: The area of the triangle with a 1 cm to 5 m scale is 9 times greater than the area of the triangle with a 1 cm to 15 m scale.

Teacher: (pressing for more detail) Why is it 9 times greater? Can someone different explain or illustrate what 9 times greater means in this case?

Student C: [Student adds a sketch of both triangles to the display next to the phrase; teacher adds labels/arrows/calculations to the sketch while Student C explains] It's 9 times greater because 15 m divided by 5 m is 3, and since we're talking about area, you then calculate 3 times 3 to give you 9.

4. As time permits, continue this discussion until all questions have been addressed. If it did not arise through student-led questions and responses, help students generalize that as the number of meters represented by one centimeter increases, the lengths and areas of the scale drawing decreases.

   If students are having difficulty, consider using multiple examples from the activity to build up to the generalization. You may say, "Let's look at the case where we are comparing…." or "Can someone demonstrate the steps they took to compare…?"

5. Close this conversation by posting the display in the front of the classroom for students to reference for the remainder of the lesson, and then have students move on to the next activity.

10.3 A New Drawing of the Playground

15 minutes

Earlier, students created scale drawings given the actual dimensions and different scales. In this activity, instead of being given the actual dimensions, they are given a scale drawing to reproduce at a different scale.

There are two different types of reasoning students may apply. Monitor for students who:

- Use the scale drawing to find the dimensions of the actual school playground and then use those measurements to find the dimensions of the new scale drawing.
- Notice that in the given drawing, 1 centimeter represents 30 m, and in the new drawing, 1 centimeter represents 20 m. That means that each centimeter in the new drawing represents $\frac{2}{3}$ centimeters in the given one. So in the new drawing, the length of each side needs to be multiplied by a factor of $\frac{3}{2}$.

Select students using each strategy to share during the discussion, sequenced in this order.

Addressing

- 7.G.A.1

Building Towards

- 7.RP.A

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports

Launch

Tell students that they are going to reproduce a scale drawing using a different scale. The scale for the given drawing is 1 cm to 30 meters, and they are going to make a new scale drawing at a scale of 1 cm to 20 meters. Ask them if they think the new drawing will be larger or smaller than the given one.

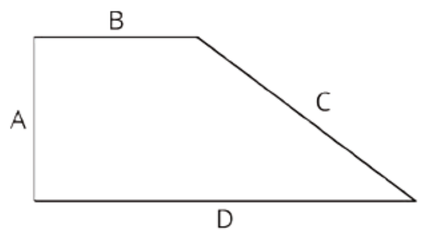
Arrange students in groups of 3. Make sure students have access to their geometry toolkits. Give students 5 minutes of quiet work time, followed by 3 minutes of group discussion.

Anticipated Misconceptions

Some students may not know how to begin the task. Prompt them to start by calculating the actual length of each side of the playground.

Student Task Statement

Here is a scale drawing of a playground.



The scale is 1 centimeter to 30 meters.

1. Make another scale drawing of the same playground at a scale of 1 centimeter to 20 meters.
2. How do the two scale drawings compare?

Student Response

1. Scaled copy of the drawing where each edge is 1.5 times as long as in the drawing.
2. The new drawing is larger. Sample explanation: When 1 cm represents 20 m, it takes 1.5 cm to represent 30 m. So the length measurements on the 1 cm to 20 m scale are 1.5 times as long as they are with the 1 cm to 30 m scale. The area measurements are 2.25 ($1.5 \cdot 1.5$) times as large.

Activity Synthesis

Invite selected students to share their work producing the new scale drawing. Ask students how the two scale drawings compare. Make sure that they recognize the shapes are the same (both represent the same playground) but the sizes are different.

Ask students if their prediction about which scale drawing would be larger was correct. Ask them to explain why the drawing at a scale of 1 cm to 20 m is larger than the drawing at a scale of 1 cm to 30 m. The important idea here is that when 1 cm on the scale drawing represents a *greater* distance, it takes fewer of those centimeters to describe the object. So the scale drawing at a scale of 1 cm to 30 m is smaller than the scale drawing at a scale of 1 cm to 20 m. Consider doing a demonstration in which you zoom in on a map with the scale showing.

To encourage students to think about the areas of the scale drawings like they did in the previous activity, consider asking questions like the following:

- "On the original map with the scale of 1 cm to 30 m, how much area does one square centimeter represent?" (900 cm^2)
- "On the new map with the scale of 1 cm to 20 m, how much area does one square centimeter represent?" (400 cm^2)
- "How many times as large as the original map is the new map?" (1.5 times for side lengths; $1.5 \cdot 1.5$, or 2.25, times for area)

Support for English Language Learners

Speaking, Listening: MLR8 Discussion Supports. Use this routine to support whole-class discussion. For each strategy that is shared, ask another student to restate what they heard using precise mathematical language. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. This provides more students with an opportunity to produce language as they interpret the reasoning of others. This will promote students' use of mathematical language as they make connections between the various ways to reproduce a scale drawing at a different scale.
Design Principle(s): Support sense-making

Lesson Synthesis

Sometimes we have a scale drawing and want to reproduce it at a different scale. Two common approaches are:

1. Using the original scale drawing to calculate the actual lengths and then using the actual lengths and the new scale to calculate the corresponding lengths on the new drawing.

2. Scaling lengths in the original scale drawing by a factor that relates the scales of the two drawings.

Suppose you have a map that uses the scale 1 cm to 200 m. You draw a new map of the same place using the scale 1 cm to 20 m.

- How does your new map compare to your original map? (The lengths are 10 times as long and the area is 100 times as large.)
- How much actual area does 1 cm^2 on your new map represent? (400 m^2)
- How much actual area did 1 cm^2 on your original map represent? (40,000 m^2)

10.4 Window Frame

Cool Down: 5 minutes

Addressing

- 7.G.A.1

Student Task Statement

Here is a scale drawing of a window frame that uses a scale of 1 cm to 6 inches.

Create another scale drawing of the window frame that uses a scale of 1 cm to 12 inches.

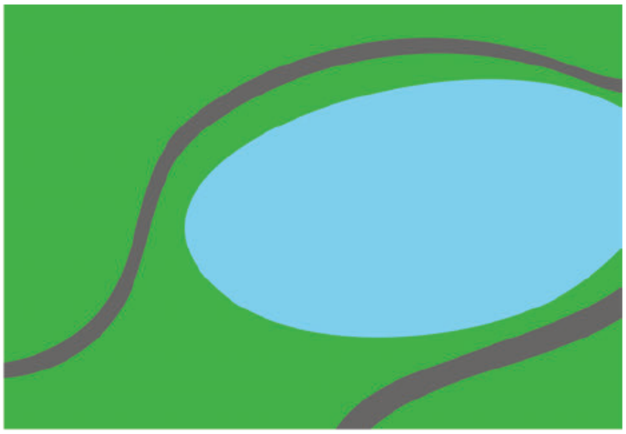
Student Response

Scaled copy of the drawing where each length is half as long as in the original.

Student Lesson Summary

Sometimes we have a scale drawing of something, and we want to create another scale drawing of it that uses a different scale. We can use the original scale drawing to find the size of the actual object. Then we can use the size of the actual object to figure out the size of our new scale drawing.

For example, here is a scale drawing of a park where the scale is 1 cm to 90 m.



The rectangle is 10 cm by 4 cm, so the actual dimensions of the park are 900 m by 360 m, because $10 \cdot 90 = 900$ and $4 \cdot 90 = 360$.

Suppose we want to make another scale drawing of the park where the scale is 1 cm to 30 meters. This new scale drawing should be 30 cm by 12 cm, because $900 \div 30 = 30$ and $360 \div 30 = 12$.

Another way to find this answer is to think about how the two different scales are related to each other. In the first scale drawing, 1 cm represented 90 m. In the new drawing, we would need 3 cm to represent 90 m. That means each length in the new scale drawing should be 3 times as long as it was in the original drawing. The new scale drawing should be 30 cm by 12 cm, because $3 \cdot 10 = 30$ and $3 \cdot 4 = 12$.

Since the length and width are 3 times as long, the area of the new scale drawing will be 9 times as large as the area of the original scale drawing, because $3^2 = 9$.

Lesson 10 Practice Problems

Problem 1

Statement

Here is a scale drawing of a swimming pool where 1 cm represents 1 m.

a. How long and how wide is the actual swimming pool?

b. Will a scale drawing where 1 cm represents 2 m be larger or smaller than this drawing?

c. Make a scale drawing of the swimming pool where 1 cm represents 2 m.

Solution

a. Answers vary. Sample response: The scale drawing is 10 cm long and 5 cm wide so the actual swimming pool is 10 m long and 5 m wide.

b. It will be smaller. Each centimeter will represent a larger distance so it will take fewer centimeters to represent the width and length of the swimming pool.

c. Answers vary. Sample response: The length and width will each be half as long as the given scale drawing. So the new scale drawing of the swimming pool will be 5 cm long and 2.5 cm wide.

Problem 2

Statement

A map of a park has a scale of 1 inch to 1,000 feet. Another map of the same park has a scale of 1 inch to 500 feet. Which map is larger? Explain or show your reasoning.

Solution

The map with a scale of 1 inch to 500 feet. It takes twice the number of units on this map to represent the same actual distance covered by the other map. For example, on the 1 inch to 1,000 feet map, it takes 1 inch to represent 1,000 feet in the actual park. On the 1 inch to 500 feet map, it takes 2 inches to represent the same 1,000 feet in the park.

Problem 3

Statement

On a map with a scale of 1 inch to 12 feet, the area of a restaurant is 60 in^2. Han says that the actual area of the restaurant is 720 ft^2. Do you agree or disagree? Explain your reasoning.

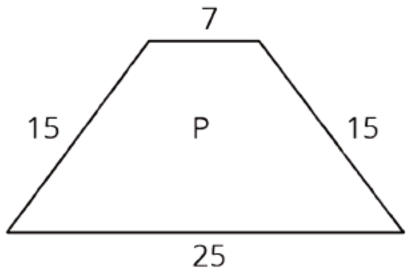
Solution

I disagree. Sample reasoning: At the scale of 1 inch to 12 feet, every 1 square inch represents 144 square feet, since $12 \cdot 12 = 144$. The actual area of the restaurant should be 8,640 square feet, because $60 \cdot 144 = 8,640$.

Problem 4

Statement

If Quadrilateral Q is a scaled copy of Quadrilateral P created with a scale factor of 3, what is the perimeter of Q?



Solution

186

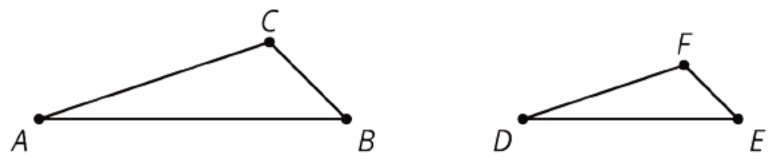
(From Unit 1, Lesson 3.)

Problem 5

Statement

Triangle DEF is a scaled copy of triangle ABC. For each of the following parts of triangle ABC, identify the corresponding part of triangle DEF.

- angle ABC
- angle BCA
- segment AC
- segment BA



Solution

- angle DEF
- angle EFD
- segment DF
- segment ED

(From Unit 1, Lesson 2.)

Section: Scales with and without Units

Lesson 11: Scales without Units

Goals

- Explain (orally and in writing) how to use scales without units to determine scaled or actual distances.
- Interpret scales expressed without units, e.g., “1 to 50,” (in spoken and written language).

Learning Targets

- I can explain the meaning of scales expressed without units.
- I can use scales without units to find scaled distances or actual distances.

Lesson Narrative

In previous lessons, students worked with scales that associated two distinct measurements—one for the distance on a drawing and one for actual distance. The units used in the two measurements are often different (centimeter and meter, inch and foot, etc.). In this lesson, students see that a scale can be expressed without units. For example, consider the scale 1 to 60. This means that every unit of length on the scale drawing represents an actual length that is 60 times its size, whatever the unit may be (inches, centimeters, etc.).

Expressing the scale as 1 to 60 highlights the scale factor relating the scale drawing to the actual object. Each measurement on the scale drawing is multiplied by 60 to find the corresponding measurement on the actual object. This relates closely to the scaled copies that were examined earlier in the unit in which each copy was related to the original by a scale factor. Students gain a better understanding of both scaled copies and scale drawings as they understanding the common underlying structure (MP7).

Alignments

Addressing

- 7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports
- Think Pair Share

Required Materials

Copies of blackline master

Rulers

Required Preparation

You will need the Apollo Lunar Module blackline master for this lesson. Prepare one copy per student.

Ensure students have access to geometry toolkits, especially rulers and graph paper.

> **Student Learning Goals**
>
> Let's explore a different way to express scales.

11.1 One to One Hundred

Warm Up: 5 minutes
This warm-up introduces students to a scale without units and invites them to interpret it using what they have learned about scales so far.

As students work and discuss, notice those who interpret the unitless scale as numbers having the same units, as well as those who see "1 to 100" as comparable to using a scale factor of 100. Invite them to share their thinking later.

Addressing

- 7.G.A.1

Instructional Routines

- Think Pair Share

Launch

Remind students that, until now, we have worked with scales that each specify two units—one for the drawing and one for the object it represents. Tell students that sometimes scales are given without units.

Arrange students in groups of 2. Give students 2 minutes of quiet think time and another minute to discuss their thinking with a partner.

Anticipated Misconceptions

Students might think that when no units are given, we can choose our own units, using different units for the 1 and the 100. This is a natural interpretation given students' work so far. Make note of this misconception, but address it only if it persists beyond the lesson.

> **Student Task Statement**
>
> A map of a park says its scale is 1 to 100.
>
> 1. What do you think that means?
>
> 2. Give an example of how this scale could tell us about measurements in the park.

Student Response

1. Answers vary. Sample responses:
    - Distances in the park are 100 times bigger than corresponding distances in the map.
    - One unit on the map represents 100 units of distance in the park.
2. Answers vary. Sample responses:
    - If a path is 6 inches long on the map, then we could tell that the actual path is 600 inches long.
    - We could use the scale to tell the size of the park. For example, if the park is 20 inches wide on the map, we can tell the actual park is 2,000 inches wide.

Activity Synthesis

Solicit students' ideas about what the scale means and ask for a few examples of how it could tell us about measurements in the park. If not already mentioned by students, point out that a scale written without units simply tells us how many times larger or smaller an actual measurement is compared to what is on the drawing. In this example, a distance in the park would be 100 times the corresponding distance on the map, so a distance of 12 cm on the map would mean 1,200 cm or 12 m in the park.

Explain that the distances could be in any unit, but because one is expressed as a number times the other, the unit is the same for both.

Tell students that we will explore this kind of scale in this lesson.

11.2 Apollo Lunar Module

15 minutes

In this activity, students use a scale drawing and a scale expressed without units to calculate actual lengths. Students will need to make a choice about which units to use, and some choices make the work easier than others.

Monitor for several paths students may take to determine actual heights of the objects in the drawing. Their choice of units could influence the number of conversions needed and the efficiency of their paths (as shown in the sample student responses). Select students with the following approaches, sequenced in this order, to share during the discussion.

- Measure in cm, find cm for actual spacecraft, then convert to m
- Measure in cm, convert to m for scale drawing, then find spacecraft measurement in m

One other approach students may use is to measure the scale drawing using an inch ruler. This leads to an extra conversion from inches to centimeters or meters. Ask them to consider the unit of interest. Discuss and highlight strategic choices of units during whole-class debriefing.

You will need the Apollo Lunar Module blackline master for this activity.

Addressing

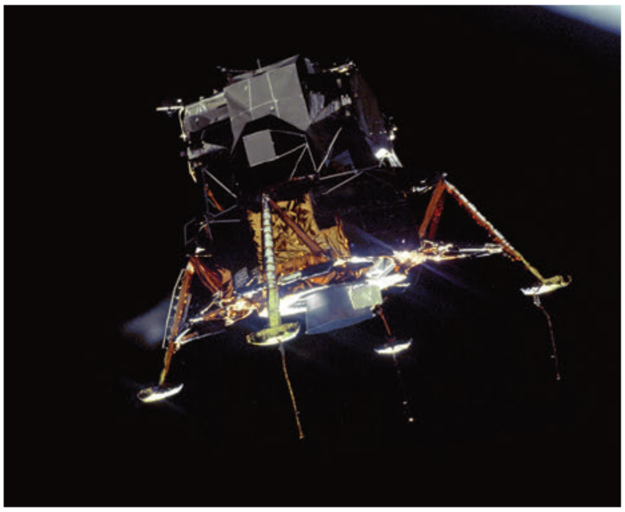
- 7.G.A.1

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports

Launch

Tell students that Neil Armstrong and Buzz Aldrin were the first people to walk on the surface of the Moon. The Apollo Lunar Module was the spacecraft used by the astronauts when they landed on the Moon in 1969. Consider displaying a picture of the landing module such as this one. Tell students that the landing module was one part of a larger spacecraft that was launched from Earth.



Solicit some guesses about the size of the spacecraft and about how the height of a person might compare to it. Explain to students that they will use a scale drawing of the Apollo Lunar Module to find out.

Arrange students in groups of 2. Give each student a scale drawing of the Apollo Lunar Module (from the blackline master). Provide access to centimeter and inch rulers. Give students 3–4 minutes to complete the first two questions. Ask them to pause briefly and discuss their responses with their partner before completing the rest of the questions.

Students are asked to find heights of people if they are drawn “to scale.” Explain that the phrase means “at the same scale” or “at the specified scale.”

Support for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts for students who benefit from support with organizational skills in problem solving. Check in with students within the first 2-3 minutes of work time to ensure that they have a place to start. If students are unsure how to begin finding the actual length of the landing gear or actual height of the spacecraft, suggest that they first find out the length on the drawing. *Supports accessibility for: Organization; Attention*

Anticipated Misconceptions

If students are unsure how to begin finding the actual length of the landing gear or actual height of the spacecraft, suggest that they first find out the length on the drawing.

Students may measure the height of the spacecraft in centimeters and then simply convert it to meters without using the scale. Ask students to consider the reasonableness of their answer (which is likely around 0.14 m) and remind them to take the scale into account.

Student Task Statement

Your teacher will give you a drawing of the Apollo Lunar Module. It is drawn at a scale of 1 to 50.

1. The "legs" of the spacecraft are its landing gear. Use the drawing to estimate the actual length of each leg on the sides. Write your answer to the nearest 10 centimeters. Explain or show your reasoning.
2. Use the drawing to estimate the actual height of the Apollo Lunar Module to the nearest 10 centimeters. Explain or show your reasoning.
3. Neil Armstrong was 71 inches tall when he went to the surface of the Moon in the Apollo Lunar Module. How tall would he be in the drawing if he were drawn with his height to scale? Show your reasoning.
4. Sketch a stick figure to represent yourself standing next to the Apollo Lunar Module. Make sure the height of your stick figure is to scale. Show how you determined your height on the drawing.

Student Response

1. The leg of the spacecraft is about 350 cm if you just include one straight segment. Sample reasoning:
    - The leg is about 7 cm on the drawing, so the actual length is $7 \cdot 50$ or 350 cm.

- The leg is about 2.75 inches on the drawing, so the actual length is 137.5 inches. $(2.75) \cdot 50 = 137.5$. Multiplying 137.5 by 2.54 gives the length in centimeters. $(137.5) \cdot (2.54) = 349.5$; this is 350 cm rounded to the nearest 10 cm.

2. The Lunar Module was about 7 meters tall. Sample explanations:
    - The spacecraft is about 14 cm tall on the drawing. The actual height is 50 times 14 cm, which is 700 cm. 700 cm is 7 m.
    - 14 cm is 0.14 m, because $14 \div 100 = 0.14$, and $(0.14) \cdot 50 = 7$, so the spacecraft is about 7 m tall.
    - The spacecraft is about 5.5 inches on the drawing. $(5.5) \cdot 50 = 275$. The actual height is about 275 inches, which is 698.5 cm. $275 \cdot (2.54) = 698.5$. 698.5 cm is 6.985 m, or about 7 m. (Do not highlight this solution in class discussion.)
3. Neil Armstrong would be about 1.4 inches tall in the scale drawing. Sample reasoning: $71 \div 50 \approx 1.4$.
4. Drawings vary depending on a student's height. Sample reasoning:
    - My height is 5 feet and 2 inches, which equals 62 inches. $(5 \cdot 12) + 2 = 62$. My height on the drawing is about $1\frac{1}{4}$ inches, since $62 \div 50 \approx 1.24$.
    - I am 155 cm tall. $155 \div 50 = 3.1$. My height is 3.1 cm on the drawing.

Are You Ready for More?

The table shows the distance between the Sun and 8 planets in our solar system.

1. If you wanted to create a scale model of the solar system that could fit somewhere in your school, what scale would you use?
2. The diameter of Earth is approximately 8,000 miles. What would the diameter of Earth be in your scale model?

planet	average distance (millions of miles)
Mercury	35
Venus	67
Earth	93
Mars	142
Jupiter	484
Saturn	887
Uranus	1,784
Neptune	2,795

Student Response

Answers vary. Sample response:

1. The gymnasium has a space of about 100 feet by 100 feet. The largest distance we need to represent is between the Sun and Neptune and this is about 2,800 million (or 3 billion) miles. So if 1 foot represents about 30 million miles, the solar system will fit.
2. 8,000 miles is one thousandth of 8 million miles and one millionth of 8,000 million miles. So the diameter of Earth will be about 3 millionths of the distance from Neptune to the Sun. This distance is represented by about 100 feet on the scale model, so the diameter of Earth will be about 3 millionths of 100 feet. This is about $\frac{1}{2,500}$ of an inch. This is smaller than a fine grain of sand!

Activity Synthesis

Invite selected students who measured using a centimeter ruler to share their strategies and solutions for the first two questions. Consider recording their reasoning for all to see. Highlight the multiplication of scaled measurements by 50 to find actual measurements. For example, the height of each leg is about 350 cm because $50 \cdot 7 = 350$.

Discuss whether or how units matter in problems involving unitless scales:

- Does it matter what unit we use to measure the drawing? Why or why not?
- Which unit is more efficient for measuring the height of the lunar module on the drawing—inches or centimeters? (Since the question asks for a height in meters, centimeters would be more efficient since it means fewer conversions. If the question asks for actual height in feet, inches would be a more strategic unit to use.)

Ask a few other students to share their responses to the last two questions. Select those who gave their heights in different units to share their solutions to the last problem. Highlight that, regardless of the starting unit, finding the length on the scale drawing involves dividing the actual measurement by 50. In other words, actual measurements can be translated to scaled measurements with a scale factor of $\frac{1}{50}$.

If time permits, consider displaying a photograph of one of the astronauts next to the Lunar Module, such as shown here, as a way to visually check the reasonableness of students' solutions.

Support for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion. For each response or observation that is shared, ask students to restate and/or revoice what they heard using precise mathematical language. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. This will provide more students with an opportunity to produce language as they interpret the reasoning of others.
Design Principle(s): Support sense-making

11.3 Same Drawing, Different Scales

15 minutes
In this activity, students explore the connection between a scale with units and one without units. Students are given two equivalent scales (one with units and the other without) and are asked to make sense of how the two could yield the same scaled measurements of an actual object. They also learn to rewrite a scale with units as a scale without units.

Students will need to attend to precision (MP6) as they work simultaneously with scales with units and without units. A scale of 1 inch to 16 feet is very different than a scale of 1 to 16, and students have multiple opportunities to address this subtlety in the activity.

As students work, identify groups that are able to reason clearly about why the two scales produce the same scale drawing. Two different types of reasoning to expect are:

- Using the two scales and the given dimensions of the parking lot to calculate and verify the student calculations.
- Thinking about the meaning of the scales, that is, in each case, the actual measurements are 180 times the measurements on the scale drawing.

Addressing

- 7.G.A.1

Instructional Routines

- MLR8: Discussion Supports

Launch

Ask students: "Is it possible to express the 1 to 50 scale of the Lunar Module as a scale with units? If so, what units would we use?" Solicit some ideas. Students are likely to say "1 inch to 50 inches," and "1 cm to 50 cm." Other units might also come up. Without resolving the questions, explain to students that their next task is to explore how a scale without units and one with units could express the same relationship between scaled lengths and actual lengths.

Keep students in the same groups. Provide access to rulers. Give partners 3–4 minutes to complete the first question and another 3–4 minutes of quiet work time for the last two questions.

Support for Students with Disabilities

Representation: Internalize Comprehension. Represent the same information through different modalities by using diagrams. If students are unsure where to begin, suggest that they draw a diagram to help organize the information provided.
Supports accessibility for: Conceptual processing; Visual-spatial processing

Anticipated Misconceptions

Some students may have trouble getting started. Suggest that they begin by treating each scale separately and find out, for instance, how a scale of 1 inch to 15 feet produces a drawing that is 8 inches by 5 inches.

Student Task Statement

A rectangular parking lot is 120 feet long and 75 feet wide.

- Lin made a scale drawing of the parking lot at a scale of 1 inch to 15 feet. The drawing she produced is 8 inches by 5 inches.
- Diego made another scale drawing of the parking lot at a scale of 1 to 180. The drawing he produced is also 8 inches by 5 inches.

1. Explain or show how each scale would produce an 8 inch by 5 inch drawing.
2. Make another scale drawing of the same parking lot at a scale of 1 inch to 20 feet. Be prepared to explain your reasoning.
3. Express the scale of 1 inch to 20 feet as a scale without units. Explain your reasoning.

Student Response

1. Answers vary. Sample explanations:
    - In Lin's case, 1 in represents 15 ft, so 120 ft is 8 in ($120 \div 15 = 8$) and 75 ft is 5 in ($75 \div 15 = 5$). In Diego's case, 1 unit on the drawing represents 180 of the same unit in the actual distance, so 1 in represents 180 in. 180 in is equal to 15 ft ($180 \div 12 = 15$). Since the scale here is also 1 in to 15 ft, the drawing will also be 8 in by 5 in.
    - 120 ft is 1,440 in ($120 \cdot 12 = 1{,}440$) and 75 ft is 900 in ($75 \cdot 12 = 900$). If the scale is 1 to 180, the sides of the parking lot will be $1{,}440 \div 180$ and $900 \div 180$, or 8 in and 5 in, respectively.
2. Drawing should show a 6 inch by $3\frac{3}{4}$ inch rectangle. Sample reasoning: $120 \div 20 = 6$. $75 \div 20 = 3\frac{3}{4}$.
3. 1 to 240. Sample explanation: 20 ft is 240 in, so 1 in on the drawing represents 240 in of actual distance.

Activity Synthesis

Select a couple of previously identified groups to share their responses to the first question and a couple of other groups for the other questions.

Highlight how scaled lengths and actual lengths are related by a factor of 180 in both scales, and that this factor is shown explicitly in one scale but not in the other.

- In the case of 1 to 180, we know that actual lengths are 180 times as long as scaled lengths (or scaled lengths are $\frac{1}{180}$ of actual lengths).
  If the scaled lengths are given in inches, we can use scaled lengths to find actual lengths in inches and, if desired, convert to feet afterward, and vice versa.
- In the case of 1 in to 15 ft, though we know that actual measurements are *not* 15 times longer than their corresponding measurements on a drawing (because 15 feet is not 15 times larger than 1 inch), it is not immediately apparent what factor relates the two measurements. Converting the units helps us see the scale factor. Since 1 foot equals 12 inches and $15 \cdot 12 = 180$, the scale of 1 in to 15 feet is equivalent to the scale of 1 in to 180 in, or 1 to 180.

Support for English Language Learners

Speaking, Representing: MLR8 Discussion Supports. Give students additional time to make sure that everyone in their group can explain their responses to the task statement questions. Invite groups to rehearse what they will say when they share with the whole class. Rehearsing provides students with additional opportunities to speak and clarify their thinking, and will improve the quality of explanations shared during the whole-class discussion. Vary who is selected to represent the work of the group, so that students get accustomed to preparing each other to fill that role.
Design Principle(s): Support sense-making; Cultivate conversation

Lesson Synthesis

- What does it mean when the scale on a scale drawing does not indicate any units?
- How is a scale without units the same as or different from a scale with units?
- How can a scale without units be used to calculate scaled or actual distances?

When a scale does not show units, the same unit is used for both the scaled distance and the actual distance. For instance, a scale of 1 to 500 means that 1 inch on the drawing represents 500 inches in actual distance, and 10 mm on a drawing represents 5,000 mm in actual distance. In other words, the actual distance is 500 times the distance on the drawing, and the scaled distance is $\frac{1}{500}$ of the actual distance. To calculate actual distances, we can multiply all distances on the drawing by the factor 500, regardless of the unit we choose or are given. Likewise, to find scaled distances, we multiply actual distances by $\frac{1}{500}$, regardless of the unit used. 500 and $\frac{1}{500}$ are scale factors that relate the two measurements (actual and scaled).

11.4 Scaled Courtyard Drawings

Cool Down: 5 minutes

Addressing

- 7.G.A.1

Student Task Statement

Andre drew a plan of a courtyard at a scale of 1 to 60. On his drawing, one side of the courtyard is 2.75 inches.

1. What is the actual measurement of that side of the courtyard? Express your answer in inches and then in feet.
2. If Andre made another courtyard scale drawing at a scale of 1 to 12, would this drawing be smaller or larger than the first drawing? Explain your reasoning.

Student Response

1. 165 in, which is 13.75 ft. Sample reasoning: $2.75 \cdot 60 = 165$. $165 \div 12 = 13.75$.
2. It would be larger. Sample explanation: A scale of 1 to 12 means the length on paper is $\frac{1}{12}$ of the original length (or 10 inches by 13.75 inches), so the drawing would be larger than one drawn at $\frac{1}{60}$ the original length.

Student Lesson Summary

In some scale drawings, the scale specifies one unit for the distances on the drawing and a different unit for the actual distances represented. For example, a drawing could have a scale of 1 cm to 10 km.

In other scale drawings, the scale does not specify any units at all. For example, a map may simply say the scale is 1 to 1,000. In this case, the units for the scaled measurements and actual measurements can be any unit, so long as the same unit is being used for both. So if a map of a park has a scale 1 to 1,000, then 1 inch on the map represents 1,000 inches in the park, and 12 centimeters on the map represent 12,000 centimeters in the park. In other words, 1,000 is the scale factor that relates distances on the drawing to actual distances, and $\frac{1}{1000}$ is the scale factor that relates an actual distance to its corresponding distance on the drawing.

A scale with units can be expressed as a scale without units by converting one measurement in the scale into the same unit as the other (usually the unit used in the drawing). For example, these scales are equivalent:

- 1 inch to 200 feet
- 1 inch to 2,400 inches (because there are 12 inches in 1 foot, and $200 \cdot 12 = 2{,}400$)
- 1 to 2,400

This scale tells us that all actual distances are 2,400 times their corresponding distances on the drawing, and distances on the drawing are $\frac{1}{2400}$ times the actual distances they represent.

Lesson 11 Practice Problems

Problem 1

Statement

A scale drawing of a car is presented in the following three scales. Order the scale drawings from smallest to largest. Explain your reasoning. (There are about 1.1 yards in a meter, and 2.54 cm in an inch.)

a. 1 in to 1 ft

b. 1 in to 1 m

c. 1 in to 1 yd

Solution

b, c, a. Explanations vary. Sample responses:

- Of the three units, 1 ft is the smallest unit, and 1 m is the largest. Therefore, a drawing with scale 1 in to 1 ft will require the most number units (the largest), and a drawing with scale 1 in to 1 m will require the least (the smallest).
- Each scale was converted into a scale without units. 1 in to 1 ft is equivalent to 1 to 12. 1 in to 1 m is equivalent to 2.54 cm to 100 cm, which is roughly 1 to 39. And 1 in to 1 yd is equivalent to 1 to 36.

Problem 2

Statement

Which scales are equivalent to 1 inch to 1 foot? Select **all** that apply.

A. 1 to 12

B. $\frac{1}{12}$ to 1

C. 100 to 0.12

D. 5 to 60

E. 36 to 3

F. 9 to 108

Solution

["A", "B", "D", "F"]

Problem 3

Statement

A model airplane is built at a scale of 1 to 72. If the model plane is 8 inches long, how many feet long is the actual airplane?

Solution

48 feet. The actual airplane is 72 times the length of the model. $8 \cdot 72 = 576$. 576 inches is 48 feet, as $576 \div 12 = 48$.

Problem 4

Statement

Quadrilateral A has side lengths 3, 6, 6, and 9. Quadrilateral B is a scaled copy of A with a shortest side length equal to 2. Jada says, "Since the side lengths go down by 1 in this scaling, the perimeter goes down by 4 in total." Do you agree with Jada? Explain your reasoning.

Solution

No. The side lengths of B are not each 1 less than those of A. The side lengths of B are $\frac{2}{3}$ of those of A, so they must be 2, 4, 4, and 6. The perimeter of A is 24 and the perimeter of B is 16, which is 8 less in total.

(From Unit 1, Lesson 3.)

Problem 5

Statement

Polygon B is a scaled copy of Polygon A using a scale factor of 5. Polygon A's area is what fraction of Polygon B's area?

Solution

$\frac{1}{25}$

(From Unit 1, Lesson 6.)

Problem 6

Statement

Figures R, S, and T are all scaled copies of one another. Figure S is a scaled copy of R using a scale factor of 3. Figure T is a scaled copy of S using a scale factor of 2. Find the scale factors for each of the following:

a. From T to S

b. From S to R

c. From R to T

d. From T to R

Solution

a. $\frac{1}{2}$

b. $\frac{1}{3}$

c. 6

d. $\frac{1}{6}$

(From Unit 1, Lesson 5.)

Lesson 12: Units in Scale Drawings

Goals

- Comprehend that the phrase "equivalent scales" refers to different scales that relate scaled and actual measurements by the same scale factor.
- Generate a scale without units that is equivalent to a given scale with units, or vice versa.
- Justify (orally and in writing) that scales are equivalent, including scales with and without units.

Learning Targets

- I can tell whether two scales are equivalent.
- I can write scales with units as scales without units.

Lesson Narrative

In previous lessons, students learned to express scales with or without units that can be the same or different. In this lesson, they analyze various scales and find that sometimes it is helpful to rewrite scales with units as scales without units in order to compare them. They see that equivalent scales relate scaled and actual measurements by the same scale factor, even though the scales may be expressed differently. For example, the scale 1 inch to 2.5 feet is equivalent to the scale 5 m to 150 m, because they are both at a scale of 1 to 30.

This lesson is also the culmination of students' work on scaling and area. Students have seen many examples of the relationship between scaled area and actual area, and now they must use this realization to find the area of an irregularly-shaped pool (MP7, MP8).

Here is some information about equal lengths that students may want to refer to during these activities.

Customary Units

1 foot (ft) = 12 inches (in)
1 yard (yd) = 36 inches
1 yard = 3 feet
1 mile = 5,280 feet

Metric Units

1 meter (m) = 1,000 millimeters (mm)
1 meter = 100 centimeters
1 kilometer (km) = 1,000 meters

Equal Lengths in Different Systems

1 inch = 2.54 centimeters
1 foot ≈ 0.30 meter
1 mile ≈ 1.61 kilometers

1 centimeter ≈ 0.39 inch
1 meter ≈ 39.37 inches
1 kilometer ≈ 0.62 mile

Alignments

Building On

- 6.RP.A.3.d: Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Addressing

- 7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Instructional Routines

- MLR5: Co-Craft Questions
- MLR8: Discussion Supports
- Notice and Wonder
- Take Turns
- Think Pair Share

Required Materials

Copies of blackline master
Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Metric and customary unit conversion charts
Pre-printed slips, cut from copies of the blackline master

Required Preparation

Note: This lesson contains optional activities. Decide which activities you will do before preparing the materials!

- For the Card Sort: Scales activity, print and cut the slips from the blackline master, so that each group of 3–4 students gets one complete set. If possible, copy each complete set on a different color of paper, so that a stray slip can quickly be put back.
- For the Pondering Pools activity, prepare one copy of the blackline master for every two students.

Ensure students have access to geometry toolkits. It is also recommended that a conversion chart for metric and customary units of length be provided while students are working on the activities in this lesson.

Student Learning Goals

Let's use different scales to describe the same drawing.

12.1 Centimeters in a Mile

Warm Up: 5 minutes

The goal of this warm-up is to review expressions in the context of conversions. This lesson will examine in depth equivalent scales, that is, scales that lead to the same size scale drawing. Checking whether or not two scales are equivalent often involves converting quantities to common units.

Building On

- 6.RP.A.3.d

Student Task Statement

There are 2.54 cm in an inch, 12 inches in a foot, and 5,280 feet in a mile. Which expression gives the number of centimeters in a mile? Explain your reasoning.

1. $\frac{2.54}{12 \cdot 5{,}280}$
2. $5{,}280 \cdot 12 \cdot (2.54)$
3. $\frac{1}{5{,}280 \cdot 12 \cdot (2.54)}$
4. $5{,}280 + 12 + 2.54$
5. $\frac{5{,}280 \cdot 12}{2.54}$

Student Response

B

Activity Synthesis

Ask one or more students to explain their reasoning for the correct choice $5{,}280 \cdot 12 \cdot (2.54)$. There are 2.54 centimeters in an inch and 12 inches in a foot, so that means there are $12 \cdot (2.54)$ centimeters in a foot. Then there are 5,280 feet in a mile, so that makes $5{,}280 \cdot 12 \cdot (2.54)$ centimeters in a mile. Students can also use common sense about measurements. A centimeter is a small unit of measure while a mile is quite large, so there have to be many centimeters in a mile.

Make sure to ask students what option C, $\frac{1}{2.54 \cdot 12 \cdot 5{,}280}$, represents in this setting. (The scale factor to convert from miles to centimeters.)

12.2 Card Sort: Scales

Optional: 15 minutes

The purpose of this activity is to give students more practice identifying equivalent scales, including some expressed without units and some with units. Students work with their group to sort slips into groups of equivalent scales and explain their reasoning. A key insight to uncover here is that when comparing scales, it can be helpful to convert them into equivalent scales in a particular format (e.g., without units, or using the same units).

You will need the Scales Card Sort blackline master for this activity.

Addressing

- 7.G.A.1

Instructional Routines

- MLR8: Discussion Supports
- Take Turns

Launch

Demonstrate how to set up and do the matching activity. Choose a student to be your partner. Mix up the rest of the cards and place them face up. Select two cards and then explain to your partner why you think the cards do or do not match. Demonstrate productive ways to agree or disagree (e.g., by explaining your mathematical thinking, asking clarifying questions, etc.).

Arrange students in groups of 4. If desired, arrange students in groups of 4–6 in two dimensions. (Assign each student into a group and then to a label within it, so that new groups—consisting one student from each of the original groups—can be formed later).

Give students 5–6 minutes to sort the slips, and another 2–3 minutes to check another group's work, followed by whole-class discussion.

Support for Students with Disabilities

Representation: Internalize Comprehension. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students cards a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.
Supports accessibility for: Conceptual processing; Organization

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Support for English Language Learners

Conversing: MLR8 Discussion Supports. Use this routine to help students describe the reasons for their card sorts. In groups of 4, students should take turns sorting 1–2 cards and explaining their reasoning to their group. Display the following sentence frames for all to see: “ ____ and ____ are equivalent because . . .”, and “I noticed ___ , so I matched . . .” Encourage students to challenge each other when they disagree. This will help students clarify their reasoning about equivalent scales.
Design Principle(s): Support sense-making; Maximize meta-awareness

Anticipated Misconceptions

If groups have trouble getting started, encourage them to think about different ways to express a scale, both with units and without units.

Students may sort the cards by the types (metric or customary; with units or without units) rather than by common scale factors. Remind students that scales that are equivalent have the same factor relating its scaled lengths to actual lengths.

Students may think that scales in metric units and those in customary units cannot be equivalent. For example, they may think that “1 inch to 1,000 inches” belongs in one group and “1 cm to 10 m” belongs in another. If this misconception arises and is not resolved in group discussions, address it during the activity synthesis.

Student Task Statement

Your teacher will give you some cards with a scale on each card.

1. Sort the cards into sets of equivalent scales. Be prepared to explain how you know that the scales in each set are equivalent. Each set should have at least two cards.
2. Trade places with another group and check each other’s work. If you disagree about how the scales should be sorted, work to reach an agreement.

   Pause here so your teacher can review your work.
3. Next, record one of the sets with three equivalent scales and explain why they are equivalent.

Student Response

1.
   - 1 centimeter to 1 meter, and 1 to 100
   - 1 centimeter to 1 kilometer, $\frac{1}{2}$ cm to 500 m, and 1 to 100,000
   - 1 inch to 8 feet, $\frac{1}{8}$ inch to 1 foot, and 1 to 96

    - 1 centimeter to 10 meters, 1 inch to 1,000 inches, and 1 millimeter to 1 meter
    - 1 foot to 1 mile, and 1 to 5,280
    - 1 inch to 1 mile, and 1 to 63,360
2. No answer necessary.
3. Answers vary. Sample response: There are 100,000 centimeters in one kilometer. Also, 100,000 groups of $\frac{1}{2}$ centimeter is 50,000 centimeters. This is the same length as 500 meters, because $50{,}000 \div 100 = 500$. That means the scales 1 cm to 1 km and $\frac{1}{2}$ cm to 500 m are both equivalent to the scale 1 to 100,000.

Activity Synthesis

Much of the discussion will happen in and between small groups, so a whole-class debrief may only be necessary to tie any loose ends. Invite a few students to share how their group reasoned about a couple of the scales (e.g., $\frac{1}{2}$ cm to 500 m, 1 mm to 1 m).

Address any questions that arose during sorting, common misconceptions, or unsettled disagreements between groups. For example, students may still be unclear about whether scales in customary and metric units can be equivalent. (i.e., Can “1 inch to 1,000 inches” and “1 centimeter to 10 meters” both go in the same group? Why or why not?) Help students see that as long as the two scales represent the same scale factor, they are equivalent and will produce the same scale drawing.

If time permits, consider asking students to order their groups of equivalent scales, starting with the ones that would produce the smallest drawing of the same actual thing to the ones that would produce the largest drawing. Invite students to explain their reasoning.

12.3 The World’s Largest Flag

15 minutes

In this activity, students use a scale without units to find actual and scaled distances that involve a wider range of numbers, from 0.02 to 2,000. They also return to thinking about how the area of a scale drawing relates to the area of the actual thing.

Students are likely to find scaled lengths in one of two ways: 1) by first converting the measurement in meters to centimeters and then dividing by 2,000; or 2) by dividing the measurement by 2,000 and then converting the result to centimeters. To find actual lengths, the same paths are likely, except that students will multiply by 2,000 and reverse the unit conversion. Identify students who use different approaches so they can share later.

Addressing

- 7.G.A.1

Instructional Routines

- Think Pair Share

Launch

Have students close their books or devices. Display an image of Tunisia's flag. Explain that Tunisia holds the world record for the largest version of a country flag. The record-breaking flag is nearly four soccer fields in length. Solicit from students a few guesses for a scale that would be appropriate to create a scale drawing of the flag on a sheet of paper. If asked, provide the length of the flag (396 m) and the size of the paper (letter size: $8\frac{1}{2}$ inches by 11 inches, or about 21.5 cm by 28 cm).

After hearing some guesses, explain to students that they will now solve problems about the scale and scale drawing of the giant Tunisian flag.

Arrange students in groups of 3–4. Provide access to a metric unit conversion chart. Give students 4–5 minutes of quiet work time, and then another 5 minutes to collaborate and discuss their work in groups.

During work time, assign one sub-problem from the second question for each group to present.

Support for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Anticipated Misconceptions

Students may be confused about whether to multiply or divide by 2,000 (or to multiply by 2,000 or by $\frac{1}{2,000}$) when finding the missing lengths. Encourage students to articulate what a scale of 1 to 2,000 means, or remind them that it is a shorthand for saying "1 unit on a scale drawing represents

2,000 of the same units in the object it represents." Ask them to now think about which of the two—actual or scaled lengths—is 2,000 times the other and which is $\frac{1}{2,000}$ of the other.

For the third question relating the area of the real flag to the scale model, if students are stuck, encourage them to work out the dimensions of each explicitly and to use this to calculate the scale factor between the areas.

Student Task Statement

As of 2016, Tunisia holds the world record for the largest version of a national flag. It was almost as long as four soccer fields. The flag has a circle in the center, a crescent moon inside the circle, and a star inside the crescent moon.

1. Complete the table. Explain or show your reasoning.

	flag length	flag height	height of crescent moon
actual	396 m		99 m
at 1 to 2,000 scale		13.2 cm	

2. Complete each scale with the value that makes it equivalent to the scale of 1 to 2,000. Explain or show your reasoning.

   a. 1 cm to __________ cm

   b. 1 cm to __________ m

   c. 1 cm to __________ km

   d. 2 m to __________ m

   e. 5 cm to __________ m

   f. __________ cm to 1,000 m

   g. __________ mm to 20 m

3. a. What is the area of the large flag?

   b. What is the area of the smaller flag?

   c. The area of the large flag is how many times the area of the smaller flag?

Student Response

1.

	flag length	flag height	height of crescent moon
actual	396 m	264 m	99 m
at 1 to 2,000 scale	19.8 cm	13.2 cm	4.95 cm

Sample reasoning:

- Length of flag: $396 \div 2,000 = 0.198$. 0.198 m is 19.8 cm.
- Height of flag: $(13.2) \cdot 2,000 = 26,400$. 26,400 cm is 264 m.
- Height of crescent moon on flag: $99 \div 2,000 = 0.0495$. 0.0495 m is 4.95 cm.

2. a. 1 cm to **2,000** cm. 2,000 times 1 cm is 2,000 cm.

   b. 1 cm to **20** m. I converted 2,000 cm to m. $2,000 \div 100 = 20$.

   c. 1 cm to **0.02** km. I converted 2000 cm to km. $2,000 \div 100,000 = 0.02$.

   d. 2 m to **4,000** m. 2,000 times 2 m is 4,000 m.

   e. 5 cm to **100** m. I know that 1 cm represents 20 m, so 5 cm represents $5 \cdot 20$.

   f. **50** cm to 1,000 m. I divided 1,000 m by 2,000, which is 0.5 m or 50 cm.

   g. **10** mm to 20 m. I know that 1 cm represents 20 m and 1 cm is 10 mm, so 10 mm represents 20 m.

3. a. The large flag is 396 m by 264 m, so its area is 104,544 m^2.

   b. The small flag is 19.8 cm by 13.2 cm, so its area is 261.36 cm^2.

   c. The scale factor for the height is 2,000 and the scale factor for the length is 2,000, so the area of the actual flag is $2,000 \cdot 2,000$, or 4,000,000 times the area of the scale drawing.

Activity Synthesis

Select a few students with differing solution paths to share their responses to the first question. Record and display their reasoning for all to see. Highlight two different ways for dealing with unit conversions. For example, in finding scaled lengths, one can either first convert the actual length in meters to centimeters and then multiply by $\frac{1}{2,000}$, or multiply by $\frac{1}{2,000}$ first, and then convert the quotient into centimeters.

Invite previously identified students to display and share their responses for the sub-problems in the second question. After each person shares, solicit questions or comments from the class.

Emphasize that all of the scales are equivalent because in each scale, a factor of 2,000 relates scaled distances to actual distances.

Reiterate the fact that a scale does not have to be expressed in terms of 1 scaled unit, as is shown in the last three sub-questions, but that 1 is often chosen because it makes the scale factor easier to see and can make calculations more efficient.

Make sure students understand why the scale factor for the area of the two flags is 4,000,000. (Both the length and the height of the large flag are 2,000 times the length and height of the small flag. So the area of the large flag is $2{,}000 \cdot 2{,}000$ times the area of the small flag. Alternatively, there are 10,000 square centimeters in a square meter, so in square centimeters, the area of the large flag is 1,053,360,000. Dividing this by the area of the small flag in square centimeters, 261.36, also gives 4,000,000.)

12.4 Pondering Pools

Optional: 10 minutes

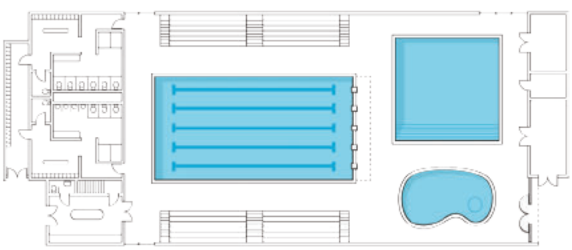
Previously, whenever students were asked to use a scale drawing to calculate the area of an actual region, they were able to find the dimensions of the actual region as an intermediate step. Each time, students were prompted to notice that the actual area was related to the scaled area by the $(\text{scale factor})^2$. Some students may have already become comfortable using this relationship to calculate the actual area directly from the scaled area, without needing to calculate the actual dimensions as an intermediate step.

The purpose of this activity is to help all students internalize this more efficient method. The question about the rectangular pool can be solved either way, but for the question about the kidney-shaped pool, students must rely on the relationship between scaled area and actual area.

As students work, monitor for those who express the scale of the drawing in different but equivalent ways (e.g., 3 cm to 15 m, 1 cm to 5 m, 1 to 500). Also monitor the different ways students find the area of the large rectangular pool:

- By first finding the actual side lengths of the pool in meters and then multiplying them
- By calculating the scaled area in square centimeters and multiplying it by 25 (or 5^2)

You will need the Pondering Pools blackline master for this activity.



Addressing

- 7.G.A.1

Instructional Routines

- MLR5: Co-Craft Questions
- Notice and Wonder

Launch

Give each student a copy of the blackline master. Invite students to share what they notice and what they wonder about the floor plan of the aquatic center.

Some things they might notice include:

- There are three different swimming pools on the floor plan.
- This floor plan has more details than others they have worked with previously, such as stairs and doors.

Some things they might wonder include:

- What is the scale of this drawing?
- How deep are these pools?
- Where is this aquatic center located?
- How much does it cost to get to use these pools?

Provide access to centimeter rulers. Give students 4–5 minutes of quiet work time, followed by whole-class discussion.

Support for English Language Learners

Representing, Conversing, Writing: Math Language Routine 5: Co-Craft Questions. This is the first time Math Language Routine 5 is suggested as a support in this course. In this routine, students are given a context or situation, often in the form of a problem stem with or without numerical values. Students develop mathematical questions that can be asked about the situation. A typical prompt is: "What mathematical questions could you ask about this situation?" The purpose of this routine is to allow students to make sense of a context before feeling pressure to produce answers, and to develop students' awareness of the language used in mathematics problems.
Design Principle(s): Cultivate conversation; Support sense-making

How It Happens:

1. Give each student a copy of the floor plan of the aquatic center. Do not allow students to see the follow-up questions for this situation.

   Ask students, "What mathematical questions could you ask about this situation?"

2. Give students 1 minute of individual time to jot some notes, and then 3 minutes to share ideas with a partner.

   As pairs discuss, support students in using conversation skills to generate and refine their questions collaboratively by seeking clarity, referring to students' written notes, and revoicing oral responses as necessary. Listen for how students refer to the scale of the drawing and talk about area in their discussion.

3. Ask each pair of students to contribute one written question to a poster, the whiteboard, or digital projection. Call on 2–3 pairs of students to present their question to the whole class, and invite the class to make comparisons among the questions shared and their own questions.

   Listen for questions intended to ask about the scale of the floor plan and the areas of the pools, and take note of those that use units and those that do not use units. Revoice student ideas with an emphasis on different but equivalent scales, as well as various methods for finding the areas of the pools, wherever it serves to clarify a question.

4. Reveal the three follow-up questions for this situation and give students a couple of minutes to compare them to their own and to those of their classmates. Identify similarities and differences.

   Consider providing these prompts: "Which of your questions is most similar to/different than the ones provided? Why?", "Is there a main mathematical concept that is present in both your questions and those provided? If so, describe it.", and "How do your questions relate to one of the lesson goals of comprehending equivalent scales?"

5. Invite students to choose one question to answer (from the class or from the curriculum), and then have students move on to the following problems.

Anticipated Misconceptions

Students may multiply the scaled area by 5 instead of by 5^2. Remind them to consider what 1 *square* centimeter represents, rather than what 1 centimeter represents.

Students may think that the last question cannot be answered because not enough information is given. Encourage them to revisit their previous work regarding how scaled area relates to actual area.

Student Task Statement

Your teacher will give you a floor plan of a recreation center.

1. What is the scale of the floor plan if the actual side length of the square pool is 15 m? Express your answer both as a scale with units and without units.
2. Find the actual area of the large rectangular pool. Show your reasoning.
3. The kidney-shaped pool has an area of 3.2 cm^2 on the drawing. What is its actual area? Explain or show your reasoning.

Student Response

1. The scale is 1 to 500, which could also be expressed as 1 cm to 5 m or 3 cm to 15 m. Sample reasoning: The side length of the square pool is 15 m. On the drawing, the side length measures 3 cm. The scale is 3 cm to 15 m, or 1 cm to 5 m. There are 500 cm in 5 m.
2. About 412.5 m^2. Sample explanations:
   - On the drawing, the pool is about 5.5 cm by 3 cm. The pool's actual measurements are 27.5 m by 15 m. Its area is 412.5 m^2, because $(27.5) \cdot 15 = 412.5$.
   - On the drawing, the pool is about 5.5 cm by 3 cm, so its area is about 16.5 cm^2. If 1 cm represents 5 m, then 1 cm^2 is 25 m^2 in actual area, so the area is 412.5 m^2, because $(16.5) \cdot 25 = 412.5$.
3. About 80 m^2. Sample explanation: I know that 1 cm^2 represents 25 m^2, so I multiplied 3.2 cm^2 by 25. $(3.2) \cdot 25 = 80$.

Are You Ready for More?

1. Square A is a scaled copy of Square B with scale factor 2. If the area of Square A is 10 units^2, what is the area of Square B?
2. Cube A is a scaled copy of Cube B with scale factor 2. If the volume of Cube A is 10 units^3, what is the volume of Cube B?
3. The four-dimensional Hypercube A is a scaled copy of Hypercube B with scale factor 2. If the "volume" of Hypercube A is 10 units^4, what do you think the "volume" of Hypercube B is?

Student Response

1. $\frac{10}{2^2}$
2. $\frac{10}{2^3}$
3. The answer to this depends on what it means to scale a hypercube in 4 dimensions! Assuming the pattern we see in 2 and 3 dimensions holds, we might suspect that the answer is $\frac{10}{2^4}$. That might even help us think about how to define scaling in four dimensions. If we use coordinates and think of scaling by a factor of 2 as multiplying all of the coordinates by a factor of 2, then it does, in fact, work the way we think it should based on the pattern in 2 and 3 dimensions.

Activity Synthesis

The goals of this discussion are to reinforce that there is more than one way to express the scale of a scale drawing and to see that, for a given problem, one way of expressing the scale may be more helpful than another.

First, invite students to share the scales they wrote for the first question. Record the answers for all to see. For each answer, poll the class on whether they agree that the scale is equivalent.

Next, ask selected students to share how they solved the questions about the area of the pools. Discuss:

- Were any of these scales easier to use when finding the actual area? Were any more difficult? Which ones?
- What might be some benefits of using one method over another for finding the actual area?

Lesson Synthesis

Scales can be expressed in many different ways, including using different units or not using any units.

- How can we express the scale 1 inch to 5 miles without units? (Since there are 12 inches in a foot and 5,280 feet in a mile, this is the same as 1 inch to 63,360 inches, or 1 to 63,360.)

A scale tells us how a distance on a scale drawing corresponds to an actual distance, and it can also tell us how an area on a drawing corresponds to an actual area.

If a map uses the scale 1 inch to 5 miles:

- How can we find the actual area of a region represented on the map? (Find the area on the map in square inches and multiply by 25, because 1 square inch represents 25 square miles.)
- How can we find a region's scaled area if we know its actual area? (Multiply the area of the actual region by $\frac{1}{25}$.)

12.5 Drawing the Backyard

Cool Down: 5 minutes

Addressing

- 7.G.A.1

Student Task Statement

Lin and her brother each created a scale drawing of their backyard, but at different scales. Lin used a scale of 1 inch to 1 foot. Her brother used a scale of 1 inch to 1 yard.

1. Express the scales for the drawings without units.
2. Whose drawing is larger? How many times as large is it? Explain or show your reasoning.

Student Response

1. Lin's scale of 1 inch to 1 foot can be written as 1 to 12. Her brother's scale of 1 inch to 1 yard can be written as 1 to 36.
2. Lin's drawing is larger. Sample explanations:
    - The lengths on Lin's plan are 3 times the corresponding lengths on her brother's drawing. The area of Lin's drawing is 9 times the area of her brother's drawing.
    - Since 1 yard equals 3 feet, the scale of Lin's brother's drawing is equivalent to 1 inch to 3 feet. Each inch on his drawing represents a longer distance than on Lin's drawing, so his drawing will require less space on paper.
    - At 1 inch to 1 foot, Lin's drawing will show $\frac{1}{12}$ of actual the distances. At 1 inch to 1 yard, or 1 inch to 3 feet, her brother's drawing will show $\frac{1}{36}$ of the actual distances. Since $\frac{1}{12}$ is larger than $\frac{1}{36}$, her drawing will be larger.

Student Lesson Summary

Sometimes scales come with units, and sometimes they don't. For example, a map of Nebraska may have a scale of 1 mm to 1 km. This means that each millimeter of distance on the map represents 1 kilometer of distance in Nebraska. Notice that there are 1,000 millimeters in 1 meter and 1,000 meters in 1 kilometer. This means there are $1{,}000 \cdot 1{,}000$ or 1,000,000 millimeters in 1 kilometer. So, the same scale without units is 1 to 1,000,000, which means that each unit of distance on the map represents 1,000,000 units of distance in Nebraska. This is true for *any* choice of unit to express the scale of this map.

Sometimes when a scale comes with units, it is useful to rewrite it without units. For example, let's say we have a different map of Rhode Island, and we want to use the two maps to compare the size of Nebraska and Rhode Island. It is important to know if the maps are at the same scale. The scale of the map of Rhode Island is 1 inch to 10 miles. There are 5,280 feet in 1 mile, and 12 inches in 1 foot, so there are 63,360 inches in 1 mile (because $5{,}280 \cdot 12 = 63{,}360$). Therefore, there are 633,600 inches in 10 miles. The scale of the map of Rhode Island without units is 1 to 633,600. The two maps are not at the same scale, so we should not use these maps to compare the size of Nebraska to the size of Rhode Island.

Here is some information about equal lengths that you may find useful.

Customary Units

1 foot (ft) = 12 inches (in)
1 yard (yd) = 36 inches
1 yard = 3 feet
1 mile = 5,280 feet

Metric Units

1 meter (m) = 1,000 millimeters (mm)
1 meter = 100 centimeters
1 kilometer (km) = 1,000 meters

Equal Lengths in Different Systems

1 inch = 2.54 centimeters
1 foot ≈ 0.30 meter
1 mile ≈ 1.61 kilometers
1 centimeter ≈ 0.39 inch
1 meter ≈ 39.37 inches
1 kilometer ≈ 0.62 mile

Lesson 12 Practice Problems

Problem 1

Statement

The Empire State Building in New York City is about 1,450 feet high (including the antenna at the top) and 400 feet wide. Andre wants to make a scale drawing of the front view of the Empire State Building on an $8\frac{1}{2}$-inch-by-11-inch piece of paper. Select a scale that you think is the most appropriate for the scale drawing. Explain your reasoning.

a. 1 inch to 1 foot

b. 1 inch to 100 feet

c. 1 inch to 1 mile

d. 1 centimeter to 1 meter

e. 1 centimeter to 50 meters

f. 1 centimeter to 1 kilometer

Solution

E, or 1 cm to 50 m, would be most appropriate. Explanations vary. Sample explanation: With A, B, and D, the scaled image will not fit on the page. For C and F, the image will be too small. Option E is just right because at 1 cm to 50 m, the height of the building is about 10 cm, and the width is about 3 cm.

Problem 2

Statement

Elena finds that the area of a house on a scale drawing is 25 square inches. The actual area of the house is 2,025 square feet. What is the scale of the drawing?

Solution

1 inch to 9 feet

Problem 3

Statement

Which of these scales are equivalent to 3 cm to 4 km? Select **all** that apply. Recall that 1 inch is 2.54 centimeters.

A. 0.75 cm to 1 km

B. 1 cm to 12 km

C. 6 mm to 2 km

D. 0.3 mm to 40 m

E. 1 inch to 7.62 km

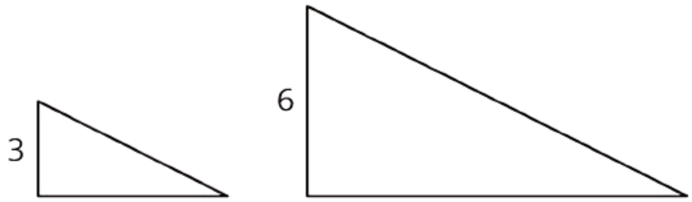
Solution

["A", "D"]

Problem 4

Statement

These two triangles are scaled copies of one another. The area of the smaller triangle is 9 square units. What is the area of the larger triangle? Explain or show how you know.



Solution

36 square units. When the lengths of a scaled copy are 2 times those of the original figure, the area of the copy is 4 times that of the original area: $4 \cdot 9 = 36$.

Problem 5

Statement

Water costs $1.25 per bottle. At this rate, what is the cost of:

a. 10 bottles?

b. 20 bottles?

c. 50 bottles?

Solution

a. $12.50 (because $10 \cdot 1.25 = 12.5$)

b. $25 (because $20 \cdot 1.25 = 25$)

c. $62.50 (because $50 \cdot 1.25 = 62.5$)

Problem 6

Statement

The first row of the table shows the amount of dish detergent and water needed to make a soap solution.

a. Complete the table for 2, 3, and 4 batches.

b. How much water and detergent is needed for 8 batches? Explain your reasoning.

number of batches	cups of water	cups of detergent
1	6	1
2		
3		
4		

Solution

a.

number of batches	cups of water	cups of detergent
1	6	1
2	12	2
3	18	3
4	24	4

b. 48 cups of water and 8 cups of dish detergent. Explanations vary. Sample response: 8 batches is 2 times 4 batches. Doubling 24 gives 48 and doubling 4 gives 8.

Section: Let's Put It to Work

Lesson 13: Draw It to Scale

Goals

- Compare, contrast, and critique (orally) scale drawings of the classroom.
- Generate an appropriate scale to represent an actual distance on a limited drawing size, and explain (orally) the reasoning.
- Make simplifying assumptions and determine what information is needed to create a scale drawing of the classroom.

Learning Targets

- I can create a scale drawing of my classroom.
- When given requirements on drawing size, I can choose an appropriate scale to represent an actual object.

Lesson Narrative

This culminating lesson is optional. Students use what they have learned in this unit to create a scale floor plan of their classroom.

The lesson is organized into three main parts:

- Part 1: Plan and measure. Each student sketches a rough floor plan of the classroom. In groups, they decide on necessary measurements to take, plan the steps and the tools for measuring, and carry out their plan (MP1).
- Part 2: Calculate and draw. Students select the paper to use for drawing, decide on a scale, and work individually to create their drawings. They choose their scale and method strategically, given their measurements and the constraints of their paper.
- Part 3: Reflect and discuss. In small groups, students explain their work, discuss and compare their floor plans (MP3), and evaluate the decisions they made in creating the scale drawing (MP4). As a class, they reflect on how the choice of scale, units, and paper affected the drawing process and the floor plans created.

Depending on the instructional choices made, this lesson could take one or more class meetings. The amount of time needed for each part might vary depending on factors such as:

- The size and complexity of the classroom, and whether measuring requires additional preparation or steps (e.g., moving furniture, taking turns, etc.).
- What the class or individual students decide to include in the floor plans.

- How much organizational support is given to students.
- How student work is ultimately shared with the class (not at all, informally, or with formal presentations).

Consider further defining the scope of work for students and setting a time limit for each part of the activity to focus students' work and optimize class time.

This activity can be modified so that students draw floor plans for different parts of the school—the cafeteria, the gym, the school grounds, and
so on—and their drawings could later be assembled as a scale floor plan of the school. If this version is chosen, coordinate the scale used by all students before they begin to draw.

Alignments

Addressing

- 7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Instructional Routines

- Group Presentations
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Think Pair Share

Required Materials

Blank paper
Graph paper
Measuring tools

Required Preparation

Make any available linear measuring tools available, which might include rulers, yardsticks, meter sticks, and tape measures, in centimeters and inches.

Prepare at least three different types of paper for each group, which could include:

- $8\frac{1}{2} \times 11$ printer paper
- 11×17 printer paper
- centimeter graph paper
- $\frac{1}{4}$-inch graph paper
- $\frac{1}{5}$-inch graph paper

Student Learning Goals

Let’s draw a floor plan.

13.1 Which Measurements Matter?

Warm Up: 5 minutes

This warm-up prepares students to create a scale floor plan of the classroom. Students brainstorm and make a list of the aspects of the classroom to include in a floor plan and the measurements to take.

Students are likely to note built-in fixtures, like walls, windows, and doors, as important components to measure. They may also include movable objects like furniture. As students work, identify those who list positions of objects (e.g., where a blackboard is on a wall, how far away the teacher’s desk is from the door, etc.). Invite them to share later.

Addressing

- 7.G.A.1

Launch

Tell students they will be creating a scale drawing of the classroom. Their first job is to think about what parts of the classroom to measure for the drawing. Give students 2 minutes of quiet think time to make a list, followed by 3 minutes of whole-class discussion. Ask students to be specific about the measurements they would include on the list.

Student Task Statement

Which measurements would you need in order to draw a scale floor plan of your classroom? List which parts of the classroom you would measure and include in the drawing. Be as specific as possible.

Student Response

Answers vary. Sample responses:

- The lengths of walls
- The size and location of windows and doors
- The size and location of fixed and movable furniture
- The measurements of different floor materials in the classroom

Activity Synthesis

Invite students to share their responses with the class, especially those who included measurements between objects in their lists. Record and display students’ responses for all to see and to serve as a reference during the main activity. Consider organizing students’ responses by type rather than by items (e.g., listing “furniture” instead of “chairs,” “desks,” etc.). Some guiding questions:

- Which parts of the classroom must be included in a scale floor plan? Which parts are less important?
- What measurements do we need?
- Besides lengths of walls and objects, what else would be helpful? (If no one mentioned the positions of objects, ask how we know where to place certain objects on the drawing.)
- Should we include vertical measurements? Why or why not?

13.2 Creating a Floor Plan (Part 1)

Optional: 15 minutes
The purpose of this activity is for students to make preparations to create their scale drawings. They sketch a rough floor plan of the classroom.
In groups, they plan the steps for making measurements and then carry out their plan.

Some things to notice as students work:

- As they draw their sketch, encourage them to focus on big-picture elements and not on details. It is not important that the sketch is neat or elaborate. What matters more is that it does not omit important features like the door.
- As they make plans for measuring and recording, encourage them to work systematically to minimize omissions and errors.
- Urge students to measure twice and record once. It is better to take a little more time to double check the measurements than to find out during drawing that they are off.

Addressing

- 7.G.A.1

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Give students 1–2 minutes to read the task statement individually and to ask any clarifying questions. Consider displaying a floor plan sketch of another room in the school. Emphasize that the sketch serves a similar purpose as an outline in writing. It does not need to be to scale, accurate, or elaborate, but it should show all the important pieces in the right places so it can be a reference in creating the scale drawing.

Arrange students in groups of 2–4. Smaller groups means that each individual student can be more involved in the measuring process, which is a benefit, but consider that it might also make the measuring process more time consuming (as it would mean more groups moving about in a confined space).

Distribute blank paper and give students 4–5 minutes to draw a sketch and to share it with a partner. Provide access to measuring tools. Give students another 4–5 minutes to plan in groups, and then time to measure (which may vary depending on size of classroom and other factors).

Support for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge about measurement. Check to see that students are able to use measuring tools and record measurements accurately.
Supports accessibility for: Memory; Conceptual processing

Student Task Statement

1. On a blank sheet of paper, make a *rough sketch* of a floor plan of the classroom. Include parts of the room that the class has decided to include or that you would like to include. Accuracy is not important for this rough sketch, but be careful not to omit important features like a door.
2. Trade sketches with a partner and check each other's work. Specifically, check if any parts are missing or incorrectly placed. Return their work and revise your sketch as needed.
3. Discuss with your group a plan for measuring. Work to reach an agreement on:
    - Which classroom features must be measured and which are optional.
    - The units to be used.
    - How to record and organize the measurements (on the sketch, in a list, in a table, etc.).
    - How to share the measuring and recording work (or the role each group member will play).
4. Gather your tools, take your measurements, and record them as planned. Be sure to double-check your measurements.
5. Make your own copy of all the measurements that your group has gathered, if you haven't already done so. You will need them for the next activity.

Student Response

Answers vary.

Activity Synthesis

After groups finish measuring, ask them to make sure that every group member has a copy of the measurements before moving on to the next part.

Consider briefly discussing what was challenging about doing the measuring. A few important issues which may come up include:

- Making sure that the measuring device stays in a straight line.
- It is hard to be accurate when the measuring device needs to be used *multiple* times in order to find the length of something long, such as a wall.
- Taking turns with other groups that are trying to measure the same thing.
- The measurements are not exact and need to be rounded.

Support for English Language Learners

Speaking: MLR8 Discussion Supports. As students describe their process for measuring features of the room, press for details in students' explanations by requesting that students challenge an idea, elaborate on an idea, or give an example of their measuring process. Provide a sentence frame such as: "It was challenging to measure _____ because . . . ." This will help students to produce and make sense of the language needed to communicate their own ideas. *Design Principle(s): Support sense-making; Optimize output (for explanation)*

13.3 Creating a Floor Plan (Part 2)

Optional: 15 minutes

In this activity, students use the measurements they just gathered to create their scale floor plans. Each student selects one of the paper options, decides on a scale to use, and works individually to create their drawing.

Support students as they reason about scale, scaled lengths, and how to go about creating the drawing. Encourage all to pay attention to units as they calculate scaled lengths. Ask students to think about the different ways that we can write a scale. If they struggle, remind students that a scale can be written in different units or written without units.

Support for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, invite students to draw one section of the room at a time and monitor students to ensure they are making progress throughout the activity.
Supports accessibility for: Organization; Attention

Addressing

- 7.G.A.1

Launch

Distribute at least three different types of paper for each group, which could include:

- $8\frac{1}{2} \times 11$ printer paper
- 11×17 printer paper
- Centimeter graph paper
- $\frac{1}{4}$-inch graph paper
- $\frac{1}{5}$-inch graph paper

Ask each group member to select a paper for their drawing. Encourage variation in paper selections. Explain that they should choose an appropriate scale based on the size of their paper, the size of the classroom, and their chosen units of measurement. This means that the floor plan must fit on the paper and not end up too small (e.g., if the paper is 11×17 inches, the floor plan should not be the size of a postcard).

Give students quiet time to create their floor plan. If the classroom layout is fairly complex, consider asking students to pause after they have completed a certain portion of the drawing (e.g., the main walls of the classroom) so their work may be checked. Alternatively, give them a minute to share their drawing-in-progress with a partner and discuss any issues.

Anticipated Misconceptions

Some students may pick a scale and start drawing without considering how large their completed floor plan will be. Encourage students to consider the size of their paper in order to determine an appropriate scale before they start drawing.

Student Task Statement

Your teacher will give you several paper options for your scale floor plan.

1. Determine an appropriate scale for your drawing based on your measurements and your paper choice. Your floor plan should fit on the paper and not end up too small.
2. Use the scale and the measurements your group has taken to draw a scale floor plan of the classroom. Make sure to:
    - Show the scale of your drawing.
    - Label the key parts of your drawing (the walls, main openings, etc.) with their actual measurements.
    - Show your thinking and organize it so it can be followed by others.

Student Response

Answers vary.

Are You Ready for More?

1. If the flooring material in your classroom is to be replaced with 10-inch by 10-inch tiles, how many tiles would it take to cover the entire room? Use your scale drawing to approximate the number of tiles needed.
2. How would using 20-inch by 20-inch tiles (instead of 10-inch by 10-inch tiles) change the number of tiles needed? Explain your reasoning.

Student Response

1. Answers vary.
2. It would reduce the number of tiles. Each 20-by-20 tile covers 4 times the area of each 10-by-10 tile, so it would take about $\frac{1}{4}$ as many tiles.

Activity Synthesis

Small-group and whole-class reflections will occur in the next activity.

13.4 Creating a Floor Plan (Part 3)

Optional: 15 minutes

In the final phase of the drawing project, students reflect on and revise their work. Students who chose the same paper option confer in small groups to analyze and compare their floor plans. They discuss their decisions, evaluate the accuracy of their drawings, and then revise them as needed.

After revision, students debrief as a class and discuss how the choice of scale, units, and paper affected the drawing process and the floor plans they created.

Addressing

- 7.G.A.1

Instructional Routines

- Group Presentations
- MLR7: Compare and Connect

Launch

Arrange students who use the same type and size of paper into small groups. Give them 8–10 minutes to share and explain their drawings. Display and read aloud questions such as the following. Ask students to use them to guide their discussion.

- What scale did you use? How did you decide on the scale?
- Do the scaled measurements in each drawing seem accurate? Do they represent actual measurements correctly?
- Did the scale seem appropriate for the chosen paper? Why or why not?
- What was the first thing you drew in your drawing? Why?
- How did you decide on the objects to show in your drawing?
- What aspects of your drawings are different?
- How could each floor plan be revised to better represent the classroom?

Student Task Statement

1. Trade floor plans with another student who used the same paper size as you. Discuss your observations and thinking.
2. Trade floor plans with another student who used a different paper size than you. Discuss your observations and thinking.
3. Based on your discussions, record ideas for how your floor plan could be improved.

Student Response

Answers vary.

Activity Synthesis

Before debriefing as a class, give students 4–5 minutes of quiet time to reflect. Ask them to write down ideas for revising their floor plan and strategies for creating accurate scale drawings based on their conversation.

Though much of the discussion will take place within the groups, debrief as a class so students can see floor plans created at a variety of scales and on different paper types or sizes. Display a range of scale drawings for all to see and discuss the following questions. (Alternatively, consider posting all students' work for a gallery walk and ask students to reflect on these questions.)

- What are the differences in these drawings?

- How did different scales impact the final drawing?
- How did the size of paper impact the choice of scale?
- What choices were really important when creating the scale drawing?
- Would these choices be the same if you were doing a different room in the school? Or some other building?

Support for English Language Learners

Representing, Conversing: *MLR7 Compare and Connect*. Use this routine to prepare students for the whole-class discussion. Invite students to quietly circulate and read at least 2 other posters or visual displays in the room prior to the whole-class discussion. Give students quiet think time to consider what is the same and what is different about the visual displays of the floor plans created at a variety of scales and on different paper types or sizes. Next, ask students to find a partner to discuss what they noticed. Listen for and amplify observations that include mathematical language and reasoning about how the choice of scale, units, and paper affect the process of creating scaled drawings.
Design Principle(s): Cultivate conversation

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Family Support Materials

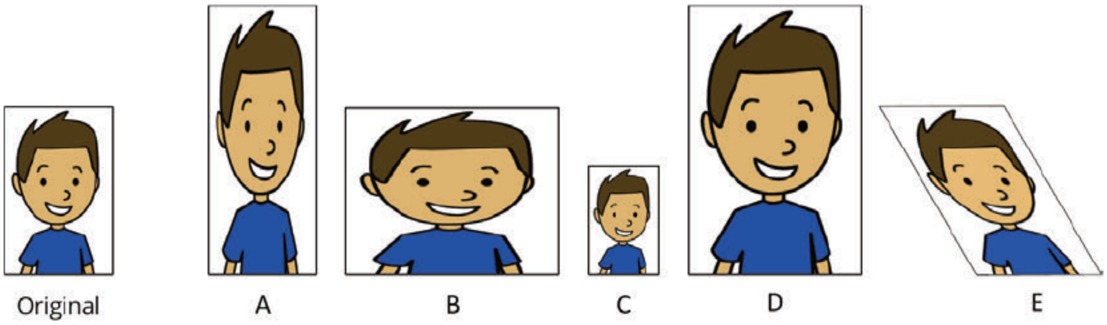
Family Support Materials

Scale Drawings

Scaled Copies

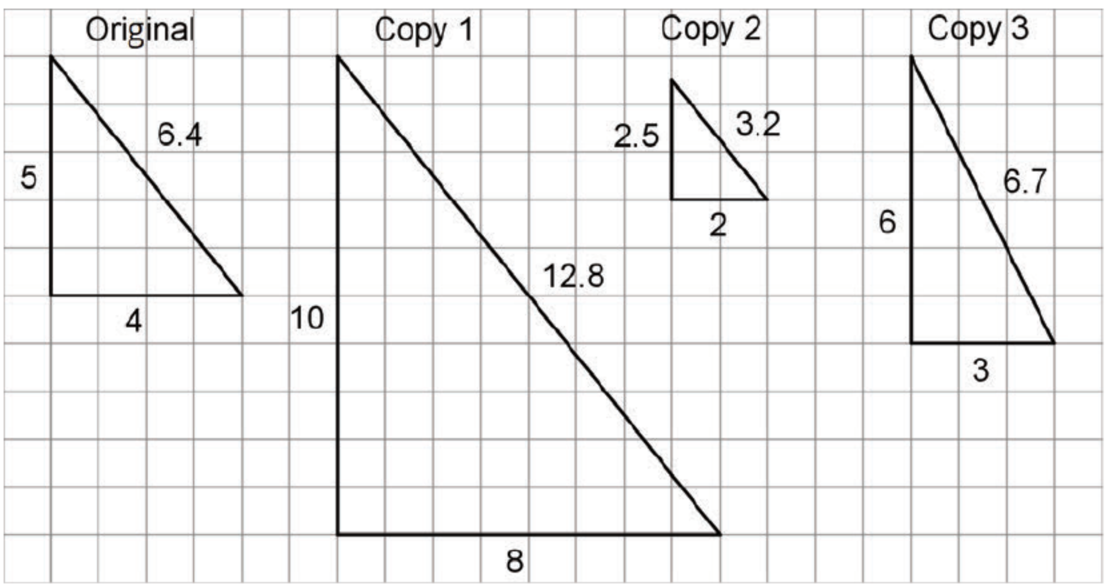
Family Support Materials 1

This week your student will learn about scaling shapes. An image is a **scaled copy** of the original if the shape is stretched in a way that does not distort it. For example, here is an original picture and five copies. Pictures C and D are scaled copies of the original, but pictures A, B, and E are not.



In each scaled copy, the sides are a certain number of times as long as the corresponding sides in the original. We call this number the **scale factor**. The size of the scale factor affects the size of the copy. A scale factor greater than 1 makes a copy that is larger than the original. A scale factor less than 1 makes a copy that is smaller.

Here is a task to try with your student:



1. For each copy, tell whether it is a scaled copy of the original triangle. If so, what is the scale factor?

2. Draw another scaled copy of the original triangle using a different scale factor.

Solution:

1.
    a. Copy 1 is a scaled copy of the original triangle. The scale factor is 2, because each side in Copy 1 is twice as long as the corresponding side in the original triangle. $5 \cdot 2 = 10$, $4 \cdot 2 = 8$, $(6.4) \cdot 2 = 12.8$

    b. Copy 2 is a scaled copy of the original triangle. The scale factor is $\frac{1}{2}$ or 0.5, because each side in Copy 2 is half as long as the corresponding side in the original triangle. $5 \cdot (0.5) = 2.5$, $4 \cdot (0.5) = 2$, $(6.4) \cdot (0.5) = 3.2$

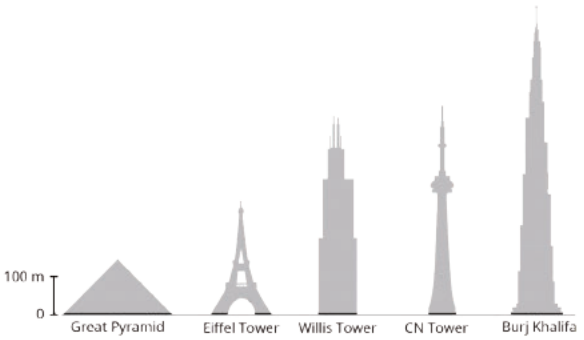
    c. Copy 3 is not a scaled copy of the original triangle. The shape has been distorted. The angles are different sizes and there is not one number we can multiply by each side length of the original triangle to get the corresponding side length in Copy 3.

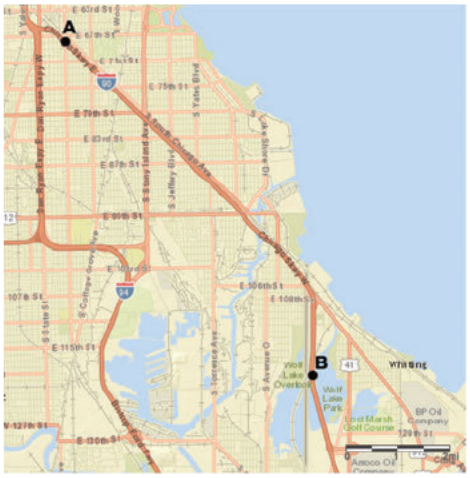
2. Answers vary. Sample response: A right triangle with side lengths of 12, 15, and 19.2 units would be a scaled copy of the original triangle using a scale factor of 3.

Scale Drawings

Family Support Materials 2

This week your student will be learning about scale drawings. A **scale drawing** is a two-dimensional representation of an actual object or place. Maps and floor plans are some examples of scale drawings.





The **scale** tells us what some length on the scale drawing represents in actual length. For example, a scale of “1 inch to 5 miles” means that 1 inch on the drawing represents 5 actual miles. If the drawing shows a road that is 2 inches long, we know the road is actually $2 \cdot 5$, or 10 miles long.

Scales can be written with units (e.g. 1 inch to 5 miles), or without units (e.g., 1 to 50, or 1 to 400). When a scale does not have units, the same unit is used for distances on the scale drawing and actual distances. For example, a scale of “1 to 50” means 1 centimeter on the drawing represents 50 actual centimeters,1 inch represents 50 inches, etc.

Here is a task to try with your student:

Kiran drew a floor plan of his classroom using the scale 1 inch to 6 feet.

1. Kiran's drawing is 4 inches wide and $5\frac{1}{2}$ inches long. What are the dimensions of the actual classroom?
2. A table in the classroom is 3 feet wide and 6 feet long. What size should it be on the scale drawing?

3. Kiran wants to make a larger scale drawing of the same classroom. Which of these scales could he use?
    a. 1 to 50

    b. 1 to 72

    c. 1 to 100

Solution:

1. 24 feet wide and 33 feet long. Since each inch on the drawing represents 6 feet, we can multiply by 6 to find the actual measurements. The actual classroom is 24 feet wide because $4 \cdot 6 = 24$. The classroom is 33 feet long because $5\frac{1}{2} \cdot 6 = 5 \cdot 6 + \frac{1}{2} \cdot 6 = 30 + 3 = 33$.

2. $\frac{1}{2}$ inch wide and 1 inch long. We can divide by 6 to find the measurements on the drawing. $6 \div 6 = 1$ and $3 \div 6 = \frac{1}{2}$.

3. A, 1 to 50. The scale "1 inch to 6 feet" is equivalent to the scale "1 to 72," because there are 72 inches in 6 feet. The scale "1 to 100" would make a scale drawing that is smaller than the scale "1 to 72," because each inch on the new drawing would represent more actual length. The scale "1 to 50" would make a scale drawing that is larger than the scale "1 to 72," because Kiran would need more inches on the drawing to represent the same actual length.

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Unit Assessments

Check Your Readiness A and B
End-of-Unit Assessment A and B

Scale Drawings: Check Your Readiness (A)

1. a. How many centimeters are there in one meter?

   b. How many meters are there in one kilometer?

   c. How many inches are there in one foot?

   d. How many feet are there in one yard?

2. There are 12 inches in 1 foot and 5,280 feet in 1 mile. Elena ran $2\frac{1}{2}$ miles.

   a. How many feet is that?

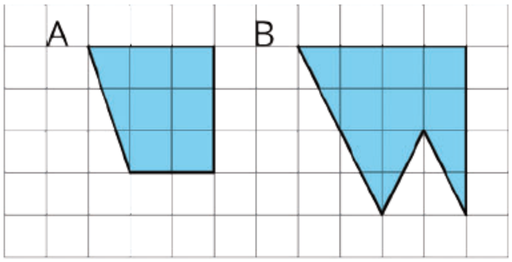
   b. How many inches is that?

3. Elena drank 3 liters of water yesterday. Jada drank $\frac{3}{4}$ times as much water as Elena. Lin drank twice as much water as Jada.

   a. Did Jada drink more or less water than Elena? Explain how you know.

   b. Did Lin drink more or less water than Elena? Explain how you know.

4. Each small square in the graph paper represents 1 square unit. Find the area of each figure. Explain your reasoning.



5. This rectangle has side lengths r and s.

For each expression, say whether it gives the *perimeter* of the rectangle, the *area* of the rectangle, or *neither*.

a. $r + s$

b. $r \cdot s$

c. $2r + 2s$

d. $r^2 + s^2$

6. A recipe for 1 loaf of bread calls for 2 cups of flour, 12 tablespoons of water, and 1 teaspoon of salt. The recipe can be scaled up to make multiple loaves of bread. Complete the table that shows the quantities to use for multiple loaves of bread.

number of loaves	cups of flour	tablespoons of water	teaspoons of salt
1	2	12	1
2	4		
4		48	
	6		

7. Here is a polygon.

Draw a scaled copy of the polygon with scale factor 3.

Scale Drawings: Check Your Readiness (B)

1. a. How many inches are there in one foot?

   b. How many centimeters are there in one meter?

   c. How many feet are there in one yard?

   d. How many meters are there in one kilometer?

2. There are 12 inches in 1 foot and 5,280 feet in 1 mile. Jada ran $3\frac{1}{4}$ miles.

   a. How many feet is that?

   b. How many inches is that?

3. Jada drank 2 liters of water yesterday. Andre drank $\frac{1}{4}$ times as much water as Jada. Lin drank three times as much water as Andre.

   a. Did Andre drink more or less water than Jada? Explain how you know.

   b. Did Lin drink more or less water than Jada? Explain how you know.

4. Each small square in the graph paper represents 1 square unit. Find the area of each figure. Explain your reasoning.

Figure A

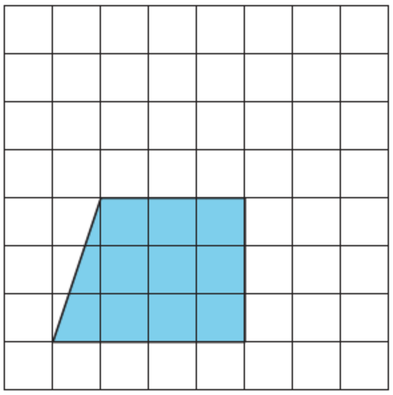
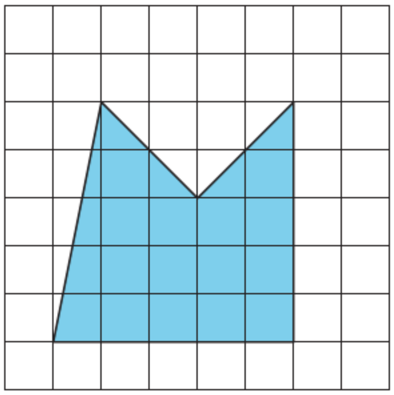
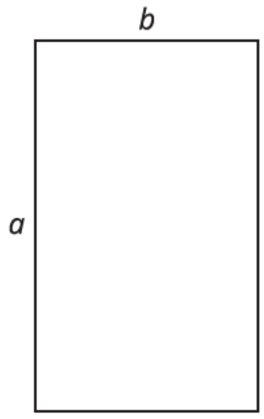


Figure B



5. This rectangle has side lengths a and b.



For each expression, say whether it gives the *perimeter* of the rectangle, the *area* of the rectangle, or *neither*.

a. $a \cdot b$

b. $a + b$

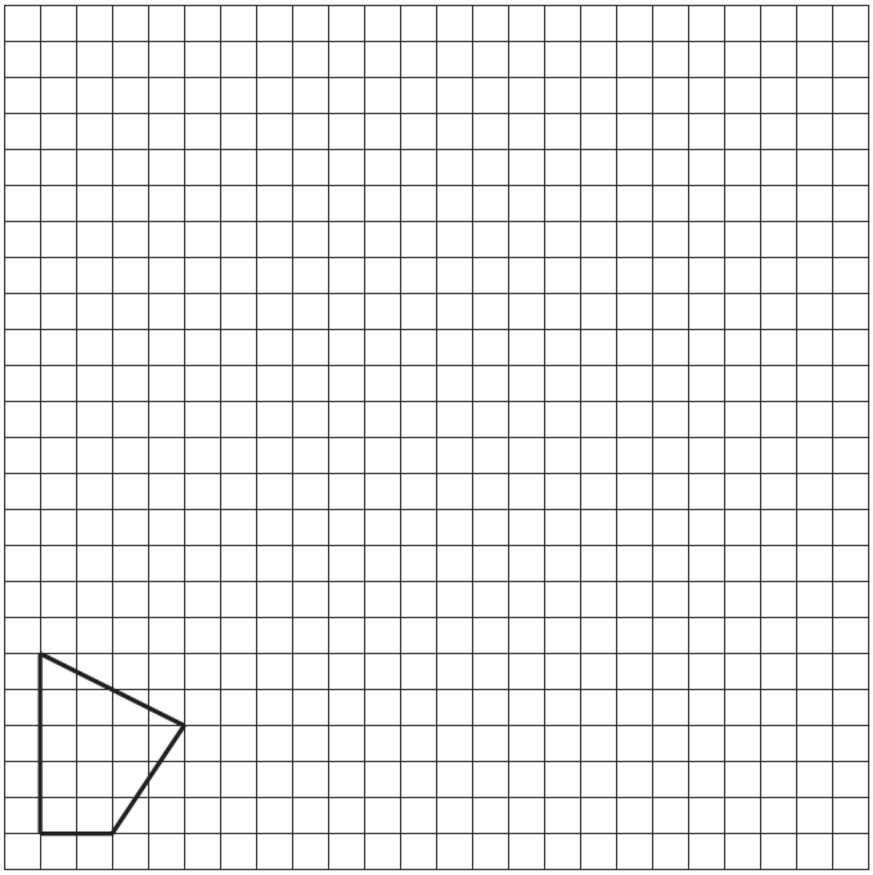
c. $a^2 + b^2$

d. $2a + 2b$

6. A recipe for 1 pizza crust calls for 2 cups of flour, 9 tablespoons of water, and 2 teaspoons of olive oil. The recipe can be scaled up to make multiple pizza crusts. Complete the table that shows the quantities to use for multiple pizza crusts.

number of pizza crusts	cups of flour	tablespoons of water	teaspoons of olive oil
1	2	9	2
2	4		
5		45	
	8		

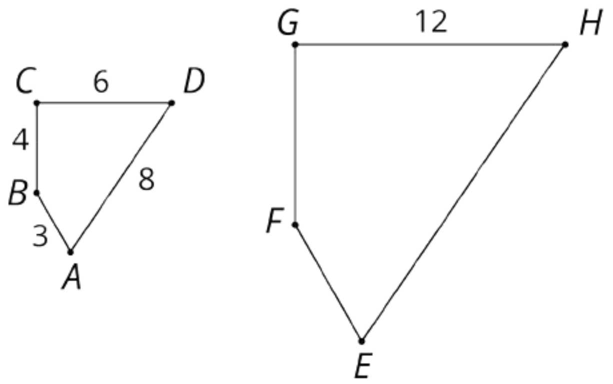
7. Here is a polygon. Draw a scaled copy of the polygon with scale factor 4.



Scale Drawings: End-of-Unit Assessment (A)

You will need a centimeter ruler for the problem in which you draw a bedroom floor plan.

1. Quadrilateral $EFGH$ is a scaled copy of quadrilateral $ABCD$. Select **all** of the true statements.



   A. Segment EF is twice as long as segment AB.

   B. Segment CD is twice as long as segment FG.

   C. The measure of angle HEF is twice the measure of angle DAB.

   D. The length of segment EH is 16 units.

   E. The area of $EFGH$ is twice the area of $ABCD$.

2. Rectangle A measures 9 inches by 3 inches. Rectangle B is a scaled copy of Rectangle A. Select **all** of the measurement pairs that could be the dimensions of Rectangle B.

   A. 4.5 inches by 1.5 inches

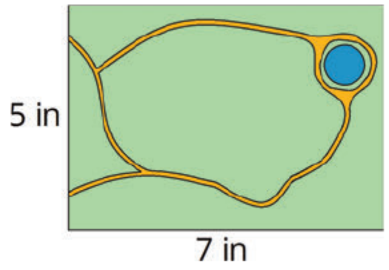
   B. 8 inches by 2 inches

   C. 10 inches by 4 inches

   D. 13.5 inches by 4.5 inches

   E. 90 inches by 30 inches

3. A scale drawing of a rectangular park is 5 inches wide and 7 inches long. The actual park is 280 yards long. What is the area of the actual park, in square yards?



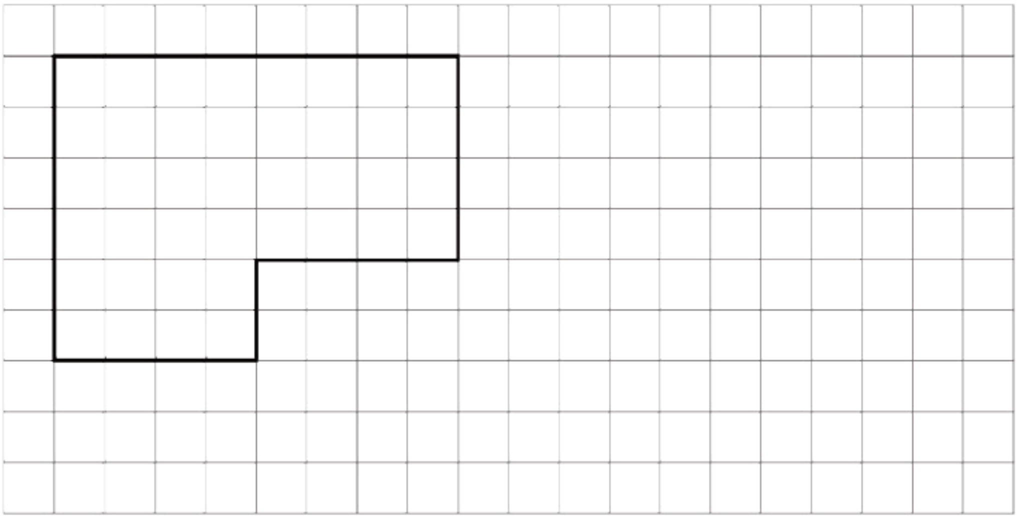
A. 35

B. 200

C. 1,400

D. 56,000

4. Here is a polygon. Draw a scaled copy of the polygon using a scale factor of $\frac{1}{2}$.



5. The scale of a map says that 4 cm represents 5 km.

a. What distance on the map (in centimeters) represents an actual distance of 4 kilometers?

b. What is the actual number of kilometers that is represented by 5 centimeters on the map?

6. Tyler has two different maps of Ohio.

    - The scale on the first map is 1 cm to 10 km. The distance from Cleveland to Cincinnati is 40 cm.
    - The scale on the second map is 1 cm to 50 km.

    What is the distance from Cleveland to Cincinnati on the second map? Explain your reasoning.

7. Elena wants to make a scale drawing of her bedroom. Her bedroom is a rectangle with length 5 m and width 3 m. She decides on a scale of 1 to 50.

    a. Draw and label the dimensions of a scale drawing of Elena’s bedroom, using a scale of 1 to 50.

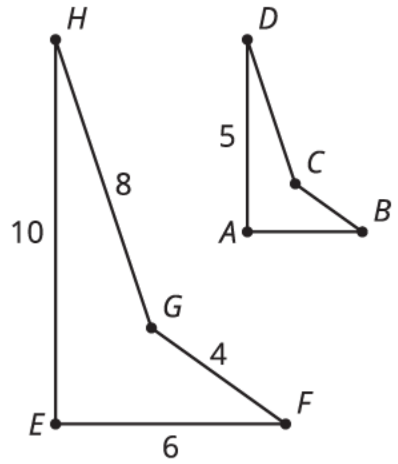
    b. Elena’s bedroom door is 0.8 m wide. How wide should the door be on the scale drawing? Explain how you know.

    c. Elena’s bed measures 4 cm by 3 cm on the scale drawing. What are the actual measurements of her bed?

Scale Drawings: End-of-Unit Assessment (B)

You will need a centimeter ruler for the problem in which you draw a kitchen floor plan.

1. Quadrilateral $ABCD$ is a scaled copy of quadrilateral $EFGH$. Select **all** of the true statements.



   A. Segment BC is half the length of segment EF.

   B. Segment CD is half the length of segment GH.

   C. The measure of angle ADC is equal to the measure of angle EHG.

   D. The length of segment AB is 3 units.

   E. The area of $ABCD$ is half the area of $EFGH$.

2. Rectangle A measures 8 inches by 2 inches. Rectangle B is a scaled copy of Rectangle A. Select **all** of the measurement pairs that could be the dimensions of Rectangle B.

   A. 40 inches by 10 inches

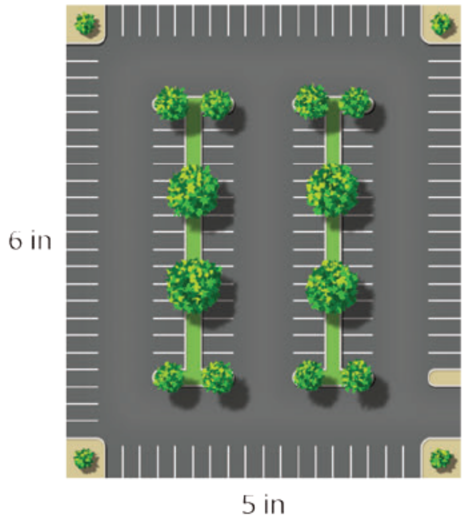
   B. 10 inches by 2.5 inches

   C. 9 inches by 3 inches

   D. 7 inches by 1 inch

   E. 6.4 inches by 1.6 inches

3. A scale drawing of a rectangular parking lot is 6 inches long and 5 inches wide. The actual parking is 240 feet long. What is the area of the actual parking lot, in square feet?



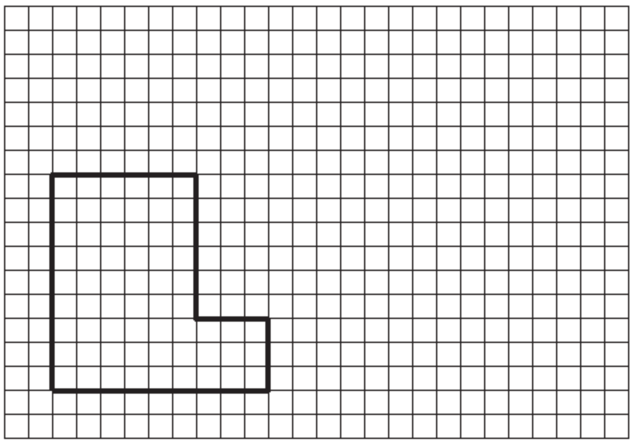
A. 48,000

B. 1,200

C. 200

D. 30

4. Here is a polygon. Draw a scaled copy of the polygon using a scale factor of $\frac{1}{3}$.



5. The scale of a map says that 8 cm represents 5 km.

a. What distance on the map (in centimeters) represents an actual distance of 8 kilometers?

b. What is the actual number of kilometers that is represented by 5 centimeters on the map?

6. There are two different maps of California.

    - The scale on the first map is 1 cm to 20 km. The distance from Fresno to San Francisco is 15 cm.
    - The scale on the second map is 1 cm to 100 km.

    What is the distance from Fresno to San Francisco on the second map? Explain your reasoning.

7. Mai wants to make a scale drawing of her kitchen. Her kitchen is a rectangle with length 6 m and width 2 m. She decides on a scale of 1 to 40.

    a. Draw and label the dimensions of a scale drawing of Mai's kitchen, using a scale of 1 to 40.

    b. Mai's kitchen door is 1.2 m wide. How wide should the door be on the scale drawing? Explain how you know.

    c. Mai's kitchen table measures 4 cm by 2.5 cm on the scale drawing. What are the actual measurements of her kitchen table?

CKMath™

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Assessment Answer Keys

Check Your Readiness A and B

End-of-Unit Assessment A and B

Assessments

Assessment : Check Your Readiness (A)

Problem 1

The content assessed in this problem is first encountered in Lesson 7: Scale Drawings.

Throughout unit 7.1, students work with length and area in a variety of contexts. Beginning in Lesson 7, students need to know how to convert units fluently and efficiently, and prior to Lesson 7 some students may use approaches that involve unit converstion.

If most students struggle with this item, plan to use this problem to create an anchor chart early in the unit for students to use as they think about appropriate scales to use throughout this unit.

Statement

1. How many centimeters are there in one meter?
2. How many meters are there in one kilometer?
3. How many inches are there in one foot?
4. How many feet are there in one yard?

Solution

1. 100
2. 1000
3. 12
4. 3

Aligned Standards

4.MD.A.1

Problem 2

The content assessed in this problem is first encountered in Lesson 11: Scales without Units.

Students perform multiple unit conversions, working with fractions and large numbers.

If most students struggle with this item, plan to practice unit conversions before doing Lesson 11. In grade 6 Unit 3 Lessons 3 and 4 you will find activities to support conversions with fractions and large numbers.

Statement

There are 12 inches in 1 foot and 5,280 feet in 1 mile. Elena ran $2\frac{1}{2}$ miles.

1. How many feet is that?
2. How many inches is that?

Solution

1. 13,200 ($2\frac{1}{2} \cdot 5{,}280 = 13{,}200$)
2. 158,400 ($12 \cdot 13{,}200 = 158{,}400$)

Aligned Standards

5.MD.A.1

Problem 3

The content assessed in this problem is first encountered in Lesson 2: Corresponding Parts and Scale Factors.

The language of scaling appears in grade 5. While the scaling of this unit is geometric in nature, the language and thought process associated with this grade 5 standard is very helpful for students.

If most students struggle with this item, plan to emphasize scaling language when launching and synthesizing Lesson 2 Activity 3, Scaled Triangles. As students are examining the triangles, they will encounter fractional multipliers 2, $\frac{1}{2}$, and $\frac{2}{3}$. MLR2: Collect and Display can be used to highlight student language such as "twice as big" and "half the size" and connect that language to scale factors.

Statement

Elena drank 3 liters of water yesterday. Jada drank $\frac{3}{4}$ times as much water as Elena. Lin drank twice as much water as Jada.

1. Did Jada drink more or less water than Elena? Explain how you know.
2. Did Lin drink more or less water than Elena? Explain how you know.

Solution

1. Less. $\frac{3}{4} < 1$, so multiplying by $\frac{3}{4}$ makes a positive number smaller.
2. More. Doubling $\frac{3}{4}$ of a quantity is the same as multiplying the quantity by $\frac{3}{2}$, and $\frac{3}{2} > 1$, so the quantity becomes larger.

Aligned Standards

5.NF.B.5.a

Problem 4

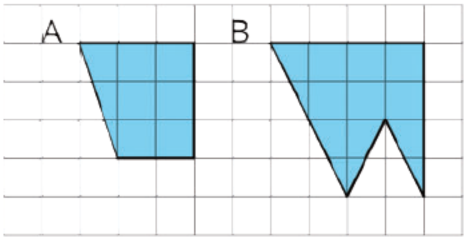
The content assessed in this problem is first encountered in Lesson 6: Scaling and Area.

One important use of scale models is to help students make calculations about the object represented. Part of these calculations use the scale while other parts involve finding the lengths or areas of the model. This task makes sure students can effectively calculate the areas of complex shapes.

If most students struggle with this item, plan to do Lesson 6 Activity 3, Area of Scaled Parallelograms and Triangles, focusing students on attending to the base and height of their polygon. During the activity launch include a discussion on how students can determine the area of figures if they aren't sure of a formula. During the launch, review the pre-unit diagnostic item featuring students who decomposed and rearranged as well as students who found parallelograms and triangles and can speak to how they used the base and height. If students need more support with calculating the area of complex shapes, refer to Grade 6 Unit 1.

Statement

Each small square in the graph paper represents 1 square unit. Find the area of each figure. Explain your reasoning.



Solution

Figure A has an area of $7\frac{1}{2}$ square units. It can be divided into a 2-unit-by-3-unit rectangle (with an area of 6 square units) and a 1-unit-by-3-unit triangle (with an area of $1\frac{1}{2}$ square units). Figure B has area of 10 square units. It can be surrounded by a 4-unit-by-4-unit square with two triangles removed. Those triangles have areas of 4 and 2. The area of Figure B is 10 square units since $16 - 4 - 2 = 10$.

Aligned Standards

6.G.A.1

Problem 5

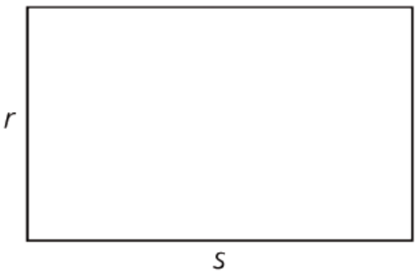
The content assessed in this problem is first encountered in Lesson 6: Scaling and Area.

Students have been calculating perimeter and area of figures since elementary school. In grade 6, they begin to use variables and exponential notation in these formulas. This exercise checks students' understanding of perimeter and area using variables.

If most students struggle with this item, plan to use this problem to review area and perimeter. Grade 6 Unit 1 Lessons 5 and 6 could also be used as review.

Statement

This rectangle has side lengths r and s.



For each expression, say whether it gives the *perimeter* of the rectangle, the *area* of the rectangle, or *neither*.

1. $r + s$
2. $r \cdot s$
3. $2r + 2s$
4. $r^2 + s^2$

Solution

1. Neither (It's half the perimeter.)
2. Area
3. Perimeter
4. Neither

Aligned Standards

6.EE.A.2.c

Problem 6

The content assessed in this problem is first encountered in Lesson 2: Corresponding Parts and Scale Factors.

Students scale figures up and down, which is similar to scaling recipes up and down. This item checks that students are comfortable representing scaling with a table.

If most students struggle with this item, plan to do Lesson 2 Activity 3, Scaled Triangles with extra attention to question 3 and the activity synthesis. The synthesis uses the table of scaled copies to begin thinking about and articulating scale factor.

Statement

A recipe for 1 loaf of bread calls for 2 cups of flour, 12 tablespoons of water, and 1 teaspoon of salt. The recipe can be scaled up to make multiple loaves of bread. Complete the table that shows the quantities to use for multiple loaves of bread.

number of loaves	cups of flour	tablespoons of water	teaspoons of salt
1	2	12	1
2	4		
4		48	
	6		

Solution

number of loaves	cups of flour	tablespoons of water	teaspoons of salt
1	2	12	1
2	4	24	2
4	8	48	4
3	6	36	3

Aligned Standards

6.RP.A.3.a

Problem 7

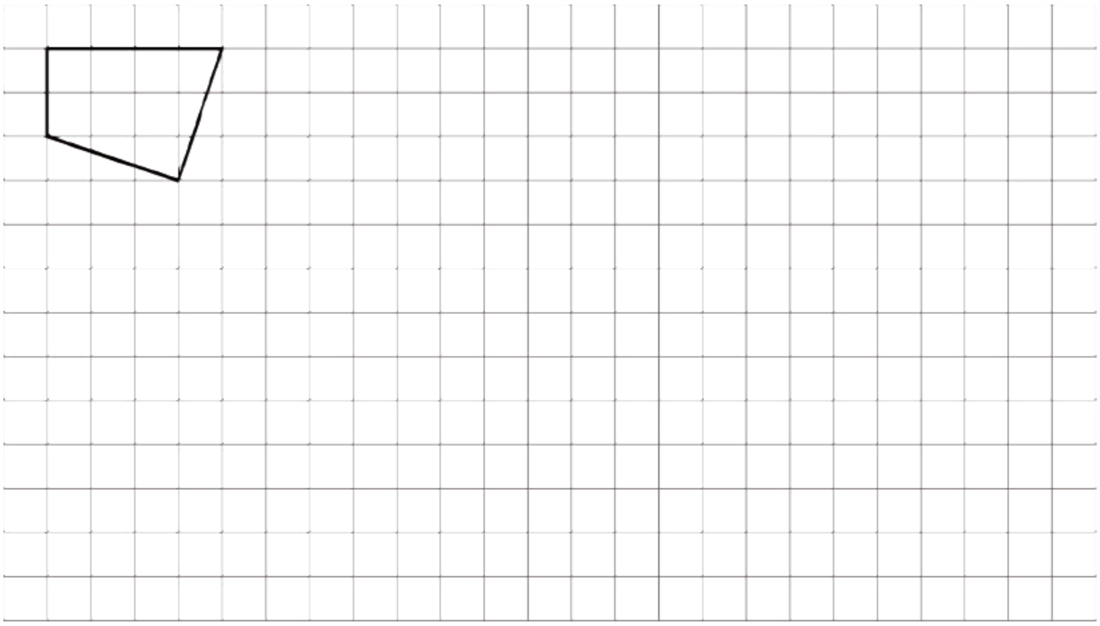
The content assessed in this problem is first encountered in Lesson 1: What are Scaled Copies?.

Some students may have seen or worked with scale drawings before. This item checks whether or not they can reproduce a drawing at given scale on a grid.

If most students do well with this item, it may be possible to skip Lesson 3 Optional Activity 2.

Statement

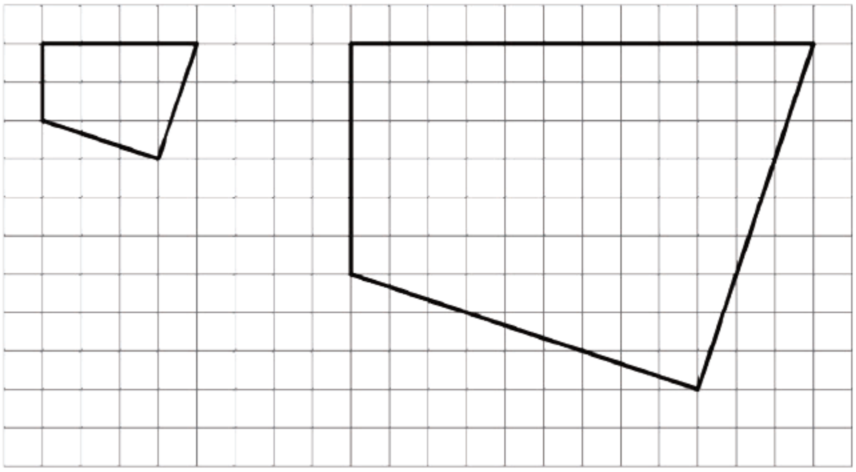
Here is a polygon.



Draw a scaled copy of the polygon with scale factor 3.

Solution

Answers vary. Sample response:



Aligned Standards

7.G.A.1

Assessment : Check Your Readiness (B)

Problem 1

The content assessed in this problem is first encountered in Lesson 7: Scale Drawings.

Throughout unit 7.1, students work with length and area in a variety of contexts. They need to know how to convert units fluently and efficiently.

If most students struggle with this item, plan to use this problem to create an anchor chart early in the unit for students to use as they think about appropriate scales to use throughout this unit.

Statement

1. How many inches are there in one foot?
2. How many centimeters are there in one meter?
3. How many feet are there in one yard?
4. How many meters are there in one kilometer?

Solution

1. 12
2. 100
3. 3
4. 1,000

Aligned Standards

4.MD.A.1

Problem 2

The content assessed in this problem is first encountered in Lesson 11: Scales without Units.

Students perform multiple unit conversions, working with fractions and large numbers.

If most students struggle with this item, plan to practice unit conversions before doing Lesson 11. In grade 6 Unit 3 Lessons 3 and 4 you will find activities to support conversions with fractions and large numbers.

Statement

There are 12 inches in 1 foot and 5,280 feet in 1 mile. Jada ran $3\frac{1}{4}$ miles.

1. How many feet is that?

2. How many inches is that?

Solution

1. 17,160 ($3\frac{1}{4} \cdot 5{,}280 = 17{,}160$)
2. 205,920 ($12 \cdot 17{,}160 = 205{,}920$)

Aligned Standards

5.MD.A.1

Problem 3

The content assessed in this problem is first encountered in Lesson 2: Corresponding Parts and Scale Factors.

The language of scaling appears in grade 5. While the scaling in this unit is geometric in nature, the language and thought process associated with this grade 5 standard is very helpful for students.

If most students struggle with this item, plan to emphasize scaling language when launching and synthesizing Lesson 2 Activity 3, Scaled Triangles. As students are examining the triangles, they will encounter fractional multipliers 2, $\frac{1}{2}$, and $\frac{2}{3}$. MLR2: Collect and Display can be used to highlight student language such as "twice as big" and "half the size" and connect that language to scale factors.

Statement

Jada drank 2 liters of water yesterday. Andre drank $\frac{1}{4}$ times as much water as Jada. Lin drank three times as much water as Andre.

1. Did Andre drink more or less water than Jada? Explain how you know.
2. Did Lin drink more or less water than Jada? Explain how you know.

Solution

1. Less. $\frac{1}{4} < 1$, so multiplying by $\frac{1}{4}$ makes a positive number smaller.
2. Less. Three times $\frac{1}{4}$ of a quantity is the same as multiplying the quantity by $\frac{3}{4}$, and $\frac{3}{4} < 1$, so the result is smaller.

Aligned Standards

5.NF.B.5.a

Problem 4

The content assessed in this problem is first encountered in Lesson 6: Scaling and Area.

One important use of scale models is to help students make calculations about the object represented. Part of these calculations use the scale while other parts involve finding the lengths or areas of the model. This task makes sure students can effectively calculate the areas of complex shapes.

If most students struggle with this item, plan to do Lesson 6 Activity 3, Area of Scaled Parallelograms and Triangles, focusing students on attending to the base and height of their polygon. During the activity launch include a discussion on how students can determine the area of figures if they aren't sure of a formula. During the launch, review the pre-unit diagnostic item featuring students who decomposed and rearranged as well as students who found parallelograms and triangles and can speak to how they used the base and height. If students need more support with calculating the area of complex shapes, refer to Grade 6 Unit 1.

Statement

Each small square in the graph paper represents 1 square unit. Find the area of each figure. Explain your reasoning.

Figure A

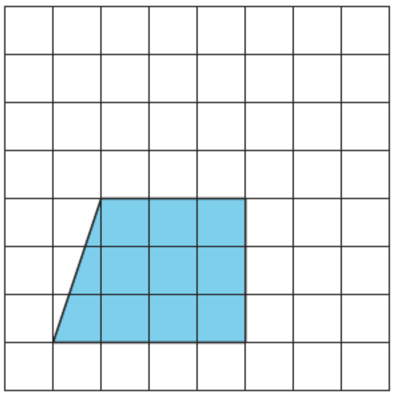
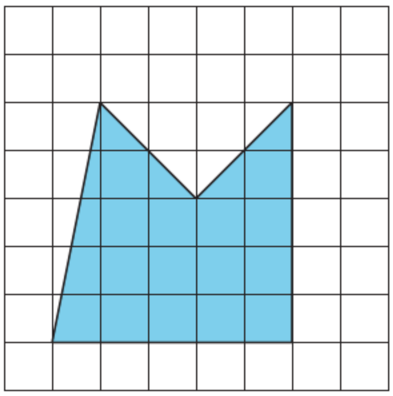


Figure B



Solution

Figure A has an area of $10\frac{1}{2}$ square units. It can be divided into a 3-unit-by-3-unit rectangle (with an area of 9 square units) and a 1-unit-by-3-unit triangle (with an area of $1\frac{1}{2}$ square units). Figure B has an area of $18\frac{1}{2}$ square units. It can be surrounded by a 5-unit-by-5-unit square with two triangles removed. Those triangles have areas of 4 and $2\frac{1}{2}$ square units. The area of Figure B is $18\frac{1}{2}$ square units since $25 - 4 - 2\frac{1}{2} = 18\frac{1}{2}$.

Aligned Standards

6.G.A.1

Problem 5

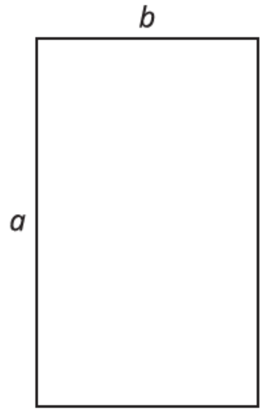
The content assessed in this problem is first encountered in Lesson 6: Scaling and Area.

Students have been calculating perimeter and area of figures since elementary school. In grade 6, they began to use variables and exponential notation in these formulas. This exercise checks students' understanding of perimeter and area using variables.

If most students struggle with this item, plan to use this problem to review area and perimeter. Grade 6 Unit 1 Lessons 5 and 6 could also be used as review.

Statement

This rectangle has side lengths a and b.



For each expression, say whether it gives the *perimeter* of the rectangle, the *area* of the rectangle, or *neither*.

1. $a \cdot b$
2. $a + b$
3. $a^2 + b^2$
4. $2a + 2b$

Solution

1. Area
2. Neither (It's half the perimeter)
3. Neither
4. Perimeter

Aligned Standards

6.EE.A.2.c

Problem 6

The content assessed in this problem is first encountered in Lesson 2: Corresponding Parts and Scale Factors.

Students scale figures up and down, which is similar to scaling recipes up and down. This item checks that students are comfortable representing scaling with a table.

If most students struggle with this item, plan to do Lesson 2 Activity 3, Scaled Triangles with extra attention to question 3 and the activity synthesis. The synthesis uses the table of scaled copies to begin thinking about and articulating scale factor.

Statement

A recipe for 1 pizza crust calls for 2 cups of flour, 9 tablespoons of water, and 2 teaspoons of olive oil. The recipe can be scaled up to make multiple pizza crusts. Complete the table that shows the quantities to use for multiple pizza crusts.

number of pizza crusts	cups of flour	tablespoons of water	teaspoons of olive oil
1	2	9	2
2	4		
5		45	
	8		

Solution

number of pizza crusts	cups of flour	tablespoons of water	teaspoons of olive oil
1	2	9	2
2	4	18	4
5	10	45	10
4	8	36	8

Aligned Standards

6.RP.A.3.a

Problem 7

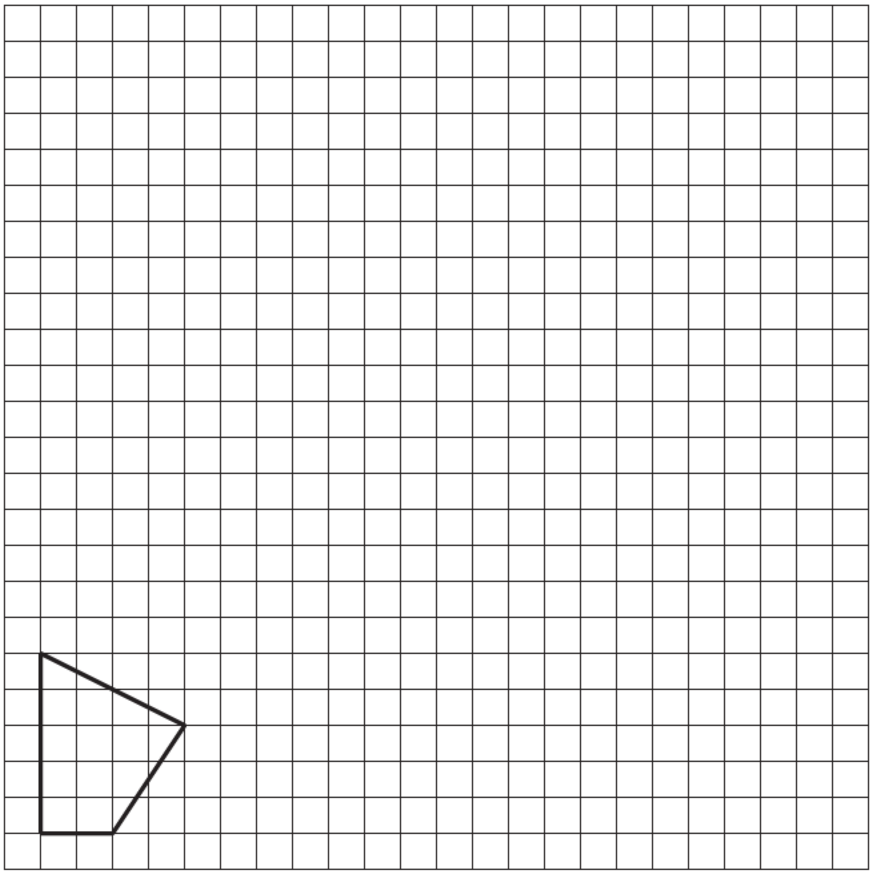
The content assessed in this problem is first encountered in Lesson 1: What are Scaled Copies?.

Some students may have seen or worked with scale drawings before. This item checks whether or not they can reproduce a drawing at a given scale on a grid.

If most students do well with this item, it may be possible to skip Lesson 3 Optional Activity 2.

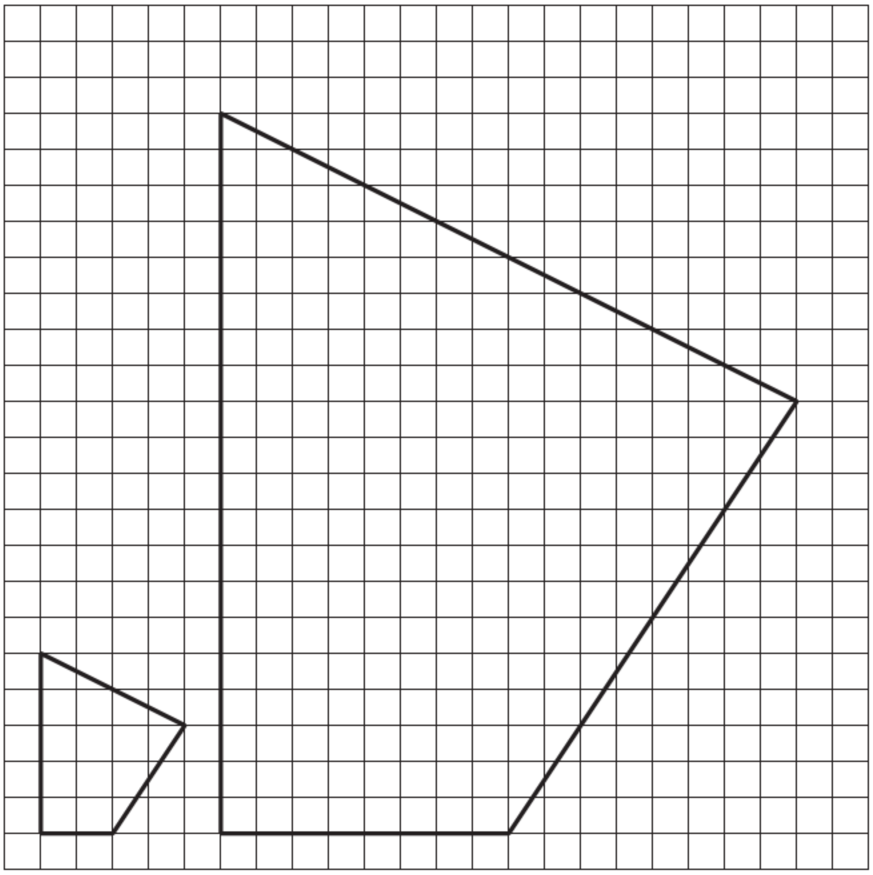
Statement

Here is a polygon. Draw a scaled copy of the polygon with scale factor 4.



Solution

Answers vary. Sample response:



Aligned Standards

7.G.A.1

Assessment : End-of-Unit Assessment (A)

Teacher Instructions

Provide access to a centimeter ruler for the problem in which students draw a bedroom floor plan.

Student Instructions

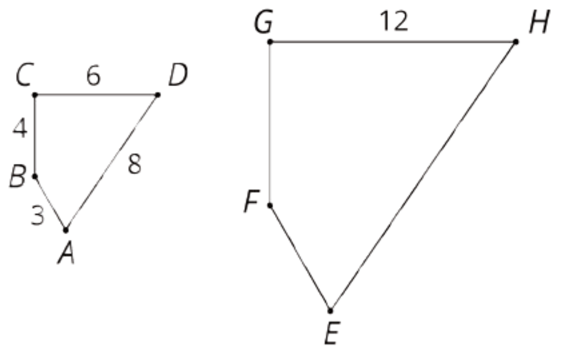
You will need a centimeter ruler for the problem in which you draw a bedroom floor plan.

Problem 1

Students selecting B are missing the fact that the two segments are not corresponding, possibly because they have not read the question carefully. Choice C may be tempting for some students because the side lengths of $ABCD$ are doubled, but the angle measures are not. Students selecting E are making a common mistake: when a figure is scaled up by a factor of two, its area quadruples rather than doubles.

Statement

Quadrilateral $EFGH$ is a scaled copy of quadrilateral $ABCD$. Select **all** of the true statements.



A. Segment EF is twice as long as segment AB.

B. Segment CD is twice as long as segment FG.

C. The measure of angle HEF is twice the measure of angle DAB.

D. The length of segment EH is 16 units.

E. The area of $EFGH$ is twice the area of $ABCD$.

Solution

["A", "D"]

Aligned Standards

7.G.A.1

Problem 2

The correct answer choices all involve sides lengths that are in a 9:3 ratio. Students selecting incorrect choices B or C may believe that adding or subtracting one from both lengths will maintain their ratio. The work in this problem helps students build towards developing proportional reasoning in the next unit.

Statement

Rectangle A measures 9 inches by 3 inches. Rectangle B is a scaled copy of Rectangle A. Select **all** of the measurement pairs that could be the dimensions of Rectangle B.

A. 4.5 inches by 1.5 inches

B. 8 inches by 2 inches

C. 10 inches by 4 inches

D. 13.5 inches by 4.5 inches

E. 90 inches by 30 inches

Solution

["A", "D", "E"]

Aligned Standards

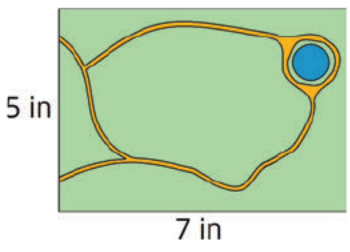
7.G.A.1

Problem 3

Students selecting A have calculated the area of the scale drawing rather than of the park. Students selecting B have found the correct width of the park, but not the area. Students selecting C have multiplied the area of the drawing by the scale factor, neglecting the fact that both the length and the width of the park are scaled by a factor of 40, so the area of the park will be 1,600 times greater than the area of the drawing.

Statement

A scale drawing of a rectangular park is 5 inches wide and 7 inches long. The actual park is 280 yards long. What is the area of the actual park, in square yards?



A. 35

B. 200

C. 1,400

D. 56,000

Solution

D

Aligned Standards

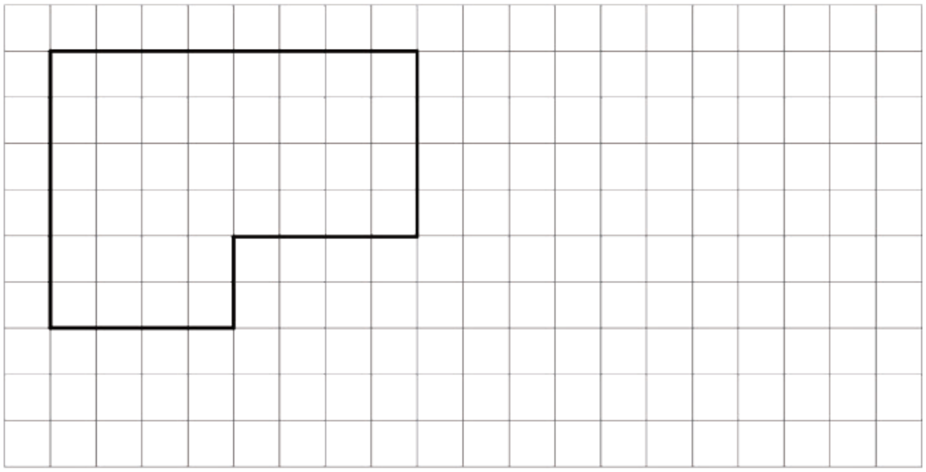
7.G.A.1

Problem 4

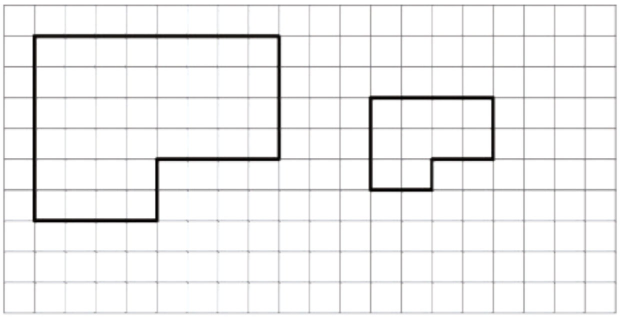
Students draw a scaled copy of a figure on a grid.

Statement

Here is a polygon. Draw a scaled copy of the polygon using a scale factor of $\frac{1}{2}$.



Solution



Minimal Tier 1 response:

- Work is complete and correct.

- Sample: see above. Acceptable errors: figure is somehow in a different orientation; figure overlaps original.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: correct drawing with scale factor 2 instead of scale factor $\frac{1}{2}$; minor error in determining dimensions of figure, such as a pair of segments 1 unit longer than they should be.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: drawing shows lack of use of any scale factor; multiple errors in determining dimensions; incorrect attempt at drawing with scale factor 2.

Aligned Standards

7.G.A.1

Problem 5

Students are given the scale of a map and asked to find lengths and distances using that scale. Common errors for this problem may include reversing the numbers (5 cm represents 4 km) and other errors typical of calculating and using unit rates such as multiplying rather than dividing.

Statement

The scale of a map says that 4 cm represents 5 km.

1. What distance on the map (in centimeters) represents an actual distance of 4 kilometers?
2. What is the actual number of kilometers that is represented by 5 centimeters on the map?

Solution

1. 3.2 cm or $\frac{16}{5}$ cm
2. 6.25 km or $\frac{25}{4}$ km

Aligned Standards

7.G.A.1

Problem 6

Students reason about maps drawn at two different scales. This problem can be solved using a wide variety of approaches, including ratio tables and double number lines.

Statement

Tyler has two different maps of Ohio.

- The scale on the first map is 1 cm to 10 km. The distance from Cleveland to Cincinnati is 40 cm.
- The scale on the second map is 1 cm to 50 km.

What is the distance from Cleveland to Cincinnati on the second map? Explain your reasoning.

Solution

The distance is 8 cm. Sample explanations:

- On the 1 cm : 50 km scale map, each centimeter represents 5 times as much actual distance as on the 1 cm: 10 km map. That means that on the 1 cm : 50 km map the distance from Cleveland to Cincinnati will be one fifth as much, 8 cm.
- The actual distance from Cleveland to Cincinnati is 400 km, because $40 \cdot 10 = 400$. The distance on the second map that represents 400 km is 8 cm, because $400 \div 50 = 8$.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: Lengths on the second map are five times smaller because 1 cm represents 50 km instead of 10 km. Divide 40 cm by 5 to get 8 cm.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: multiplication or division errors in otherwise correct work; work involves a correct substantive intermediate step (such as the actual distance from Cleveland to Cincinnati) but goes wrong after that; one mistake involving an "upside down" scale factor (or multiplying when division is called for); a correct answer without explanation.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: work does not involve proportional reasoning; an incorrect answer without explanation, even if close; multiple mistakes that involve inversion of scale factors.

Aligned Standards

7.G.A.1

Problem 7

Students make a scale drawing of a bedroom using a given scale. They then use that scale to calculate lengths of various objects.

Statement

Elena wants to make a scale drawing of her bedroom. Her bedroom is a rectangle with length 5 m and width 3 m. She decides on a scale of 1 to 50.

1. Draw and label the dimensions of a scale drawing of Elena's bedroom, using a scale of 1 to 50.
2. Elena's bedroom door is 0.8 m wide. How wide should the door be on the scale drawing? Explain how you know.
3. Elena's bed measures 4 cm by 3 cm on the scale drawing. What are the actual measurements of her bed?

Solution

1. The scale drawing is a rectangle, labeled with length 10 cm and width 6 cm (or equivalent units, such as 0.1 m and 0.06 m).
2. 1.6 cm (or equivalent units). Sample reasoning: 0.8 m is 80 cm. At 1 to 50, the width of the door is 1.6 cm, because $80 \div 50 = 1.6$.
3. 2 m by 1.5 m (or equivalent units). Actual measurements are 50 times as long as the corresponding measurements on the drawing.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample:

1. A rectangle, labeled 10 cm and 6 cm, with the larger side labeled 10 cm.
2. 1.6 cm. Because the scale is 1 to 50, the door's 80 cm width becomes $\frac{80}{50}$ cm in the scale drawing.
3. 2 m by 1.5 m

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.

- Sample errors: one calculation or conversion error; clear error in relative shape of rectangle; incomplete explanation of 1.6 cm calculation; describing instead of building and labeling the scale drawing.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: two or more error types from Tier 2 response; multiple calculation and conversion errors; scaling in wrong direction (multiplying or dividing when inappropriate); using incorrect scale factor.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: two or more error types from Tier 3 response; adding or subtracting when working with scale factor; misunderstanding of the meaning and use of "1 to 50."

Aligned Standards

7.G.A.1

Assessment : End-of-Unit Assessment (B)

Teacher Instructions

Provide access to a centimeter ruler for the problem in which students draw a kitchen floor plan.

Student Instructions

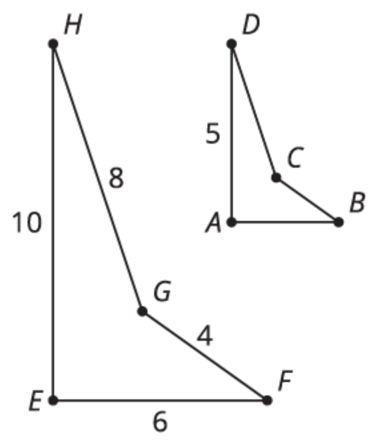
You will need a centimeter ruler for the problem in which you draw a kitchen floor plan.

Problem 1

Students selecting A are missing the fact that the two segments are not corresponding. Students selecting E are making a common mistake: when a figure is scaled by a factor of $\frac{1}{2}$, the scaled copy has $\frac{1}{4}$ the area (not $\frac{1}{2}$).

Statement

Quadrilateral $ABCD$ is a scaled copy of quadrilateral $EFGH$. Select **all** of the true statements.



A. Segment BC is half the length of segment EF.

B. Segment CD is half the length of segment GH.

C. The measure of angle ADC is equal to the measure of angle EHG.

D. The length of segment AB is 3 units.

E. The area of $ABCD$ is half the area of $EFGH$.

Solution

["B", "C", "D"]

Aligned Standards

7.G.A.1

Problem 2

The correct answer choices all involve side lengths that are in a $8 : 2$ ratio. Students selecting incorrect choices C or D may believe that adding or subtracting one from both lengths will maintain their ratio.

Statement

Rectangle A measures 8 inches by 2 inches. Rectangle B is a scaled copy of Rectangle A. Select **all** of the measurement pairs that could be the dimensions of Rectangle B.

A. 40 inches by 10 inches

B. 10 inches by 2.5 inches

C. 9 inches by 3 inches

D. 7 inches by 1 inch

E. 6.4 inches by 1.6 inches

Solution

["A", "B", "E"]

Aligned Standards

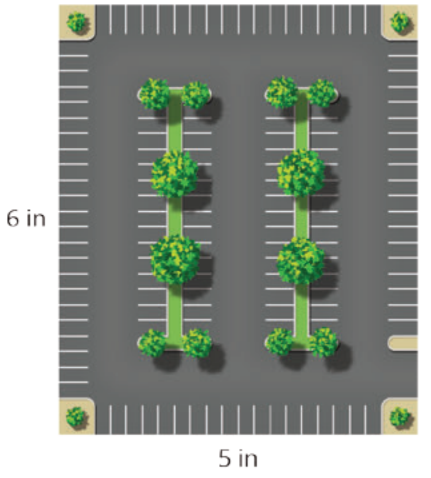
7.RP.A.2.a

Problem 3

Students selecting D have calculated the area of the scale drawing rather than of the park. Students selecting C have found the correct length of the park, but not the area. Students selecting B have multiplied the area of the drawing by the scale factor, neglecting the fact that both the length and the width of the park are scaled by a factor of 40, so the area of the park will be 1,600 times greater than the area of the drawing.

Statement

A scale drawing of a rectangular parking lot is 6 inches long and 5 inches wide. The actual parking is 240 feet long. What is the area of the actual parking lot, in square feet?



A. 48,000

B. 1,200

C. 200

D. 30

Solution

A

Aligned Standards

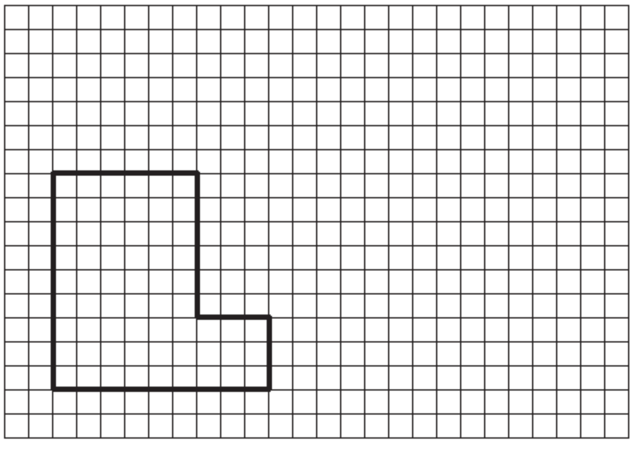
7.G.A.1

Problem 4

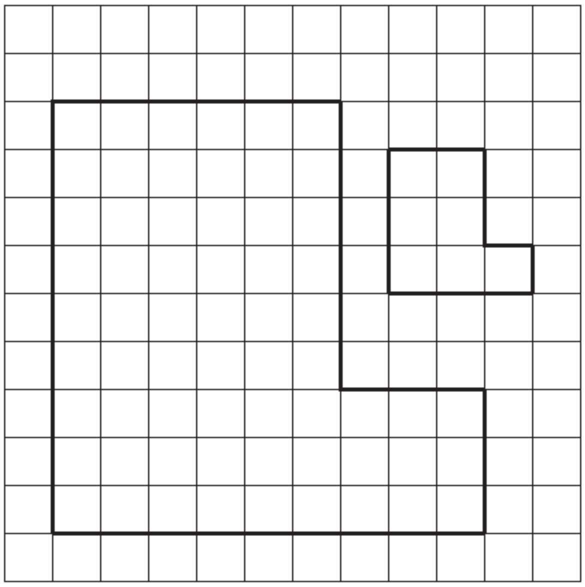
Students draw a scaled copy of a figure on a grid.

Statement

Here is a polygon. Draw a scaled copy of the polygon using a scale factor of $\frac{1}{3}$.



Solution



Minimal Tier 1 response:

- Work is complete and correct.
- Sample: see above. Acceptable differences from sample: figure is somehow in a different orientation; figure overlaps original.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: correct drawing with scale factor 3 instead of scale factor $\frac{1}{3}$; minor error in determining dimensions of figure, such as a pair of segments 1 unit longer than they should be.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: drawing shows lack of use of any scale factor; multiple errors in determining dimensions; incorrect attempt at drawing with scale factor 3.

Aligned Standards

7.G.A.1

Problem 5

Students are given the scale of a map and asked to find lengths and distances using that scale. Common errors for this problem may include reversing the numbers (4 cm represents 5 km) and other errors typical of calculating and using unit rates such as multiplying rather than dividing.

Statement

The scale of a map says that 8 cm represents 5 km.

1. What distance on the map (in centimeters) represents an actual distance of 8 kilometers?

2. What is the actual number of kilometers that is represented by 5 centimeters on the map?

Solution

1. 12.8 cm or $\frac{64}{5}$ cm

2. 3.125 km or $\frac{25}{8}$ km

Aligned Standards

7.G.A.1

Problem 6

Students reason about maps drawn at two different scales. This problem can be solved using a variety of approaches, including tables and double number lines.

Statement

There are two different maps of California.

- The scale on the first map is 1 cm to 20 km. The distance from Fresno to San Francisco is 15 cm.
- The scale on the second map is 1 cm to 100 km.

What is the distance from Fresno to San Francisco on the second map? Explain your reasoning.

Solution

The distance is 3 cm. Sample explanations:

- On the 1 cm : 20 km scale map, each centimeter represents 5 times as much actual distance as on the 1 cm: 100 km map. That means that on the 1 cm : 5 km map the distance from Fresno to San Francisco will be one fifth as much, 3 cm.

- The actual distance from Fresno to California is 300 km, because $15 \cdot 20 = 300$. The distance on the second map that represents 300 km is 3 cm, because $300 \div 100 = 3$.

Minimal Tier 1 response:

- Work is complete and correct.

- Sample: Lengths on the second map are five times smaller because 1 cm represents 20 km instead of 100 km. Divide 15 cm by 5 to get 3 cm.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: multiplication or division errors in otherwise correct work; work involves a correct substantive intermediate step (such as the actual distance from Fresno to San Francisco) but goes wrong after that; one mistake involving an “upside down” scale factor (or multiplying when division is called for); a correct answer without explanation.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: work does not involve proportional reasoning; an incorrect answer without explanation, even if close; multiple mistakes that involve inversion of scale factors.

Aligned Standards

7.G.A.1

Problem 7

Students make a scale drawing of a kitchen using a given scale. They then use that scale to calculate lengths of various objects.

Statement

Mai wants to make a scale drawing of her kitchen. Her kitchen is a rectangle with length 6 m and width 2 m. She decides on a scale of 1 to 40.

1. Draw and label the dimensions of a scale drawing of Mai’s kitchen, using a scale of 1 to 40.
2. Mai’s kitchen door is 1.2 m wide. How wide should the door be on the scale drawing? Explain how you know.
3. Mai’s kitchen table measures 4 cm by 2.5 cm on the scale drawing. What are the actual measurements of her kitchen table?

Solution

1. The scale drawing is a rectangle, labeled with length 15 cm and width 5 cm (or equivalent units, such as 0.15 m and 0.05 m).
2. 3 cm (or equivalent units). Sample reasoning: 1.2 m is 120 cm. At 1 to 40, the width of the door is 3 cm, because $120 \div 40 = 3$.

3. 1.6 m by 1 m (or equivalent units). Actual measurements are 40 times as long as the corresponding measurements on the drawing.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample:

1. A rectangle, labeled 15 cm and 5 cm, with the larger side labeled 15 cm.
2. 3 cm. Because the scale is 1 to 40, the door's 120 cm width becomes $120 \div 40$ cm in the scale drawing.
3. 1.6 m by 1 m

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: one calculation or conversion error; clear error in relative shape of rectangle; incomplete explanation of 3 cm calculation; describing instead of building and labeling the scale drawing.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: two or more error types from Tier 2 response; multiple calculation and conversion errors; scaling in wrong direction (multiplying or dividing when inappropriate); using incorrect scale factor.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: two or more error types from Tier 3 response; adding or subtracting when working with scale factor; misunderstanding of the meaning and use of "1 to 40."

Aligned Standards

7.G.A.1

CKMath™

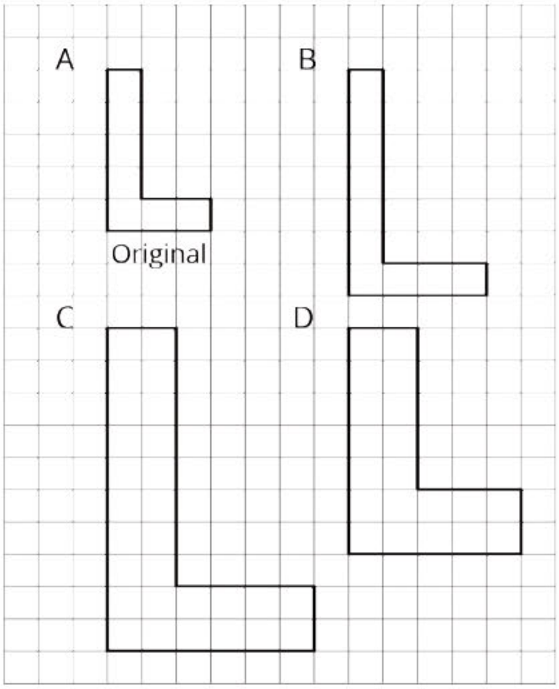
Core Knowledge MATHEMATICS™

Lesson Cool Downs

Lesson 1: What are Scaled Copies?

Cool Down: Scaling L

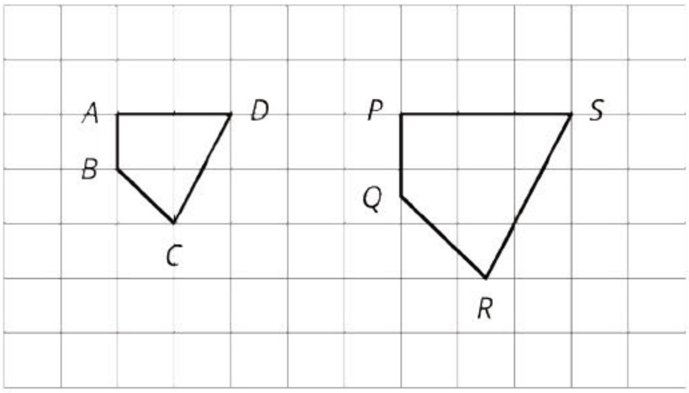
Are any of the figures B, C, or D scaled copies of figure A? Explain how you know.



Lesson 2: Corresponding Parts and Scale Factors

Cool Down: Comparing Polygons $ABCD$ and $PQRS$

Polygon $PQRS$ is a scaled copy of polygon $ABCD$.



1. Name the angle in the scaled copy that corresponds to angle ABC.

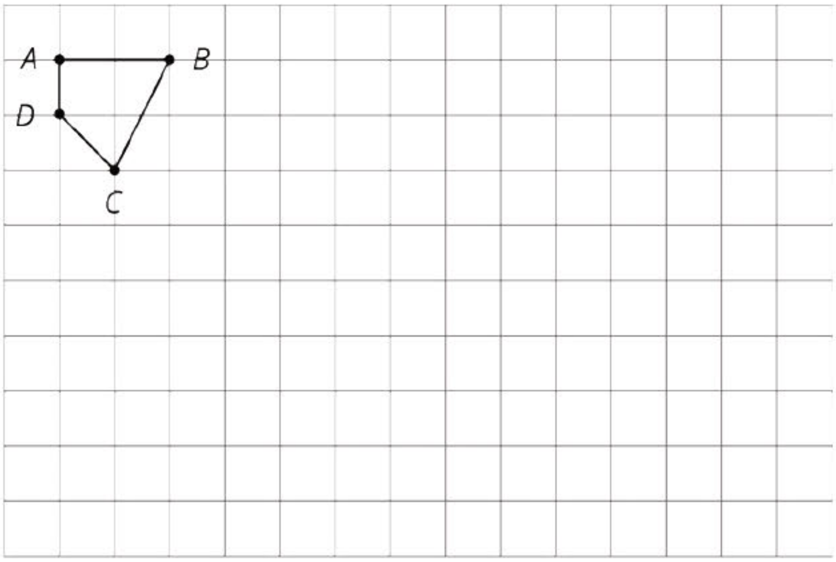
2. Name the segment in the scaled copy that corresponds to segment AD.

3. What is the scale factor from polygon $ABCD$ to polygon $PQRS$?

Lesson 3: Making Scaled Copies

Cool Down: More Scaled Copies

1. Create a scaled copy of $ABCD$ using a scale factor of 4.



2. Triangle Z is a scaled copy of Triangle M.

Select all the sets of values that could be the side lengths of Triangle Z.

a. 8, 11, and 14.

b. 10, 17.5, and 25.

c. 6, 9, and 11.

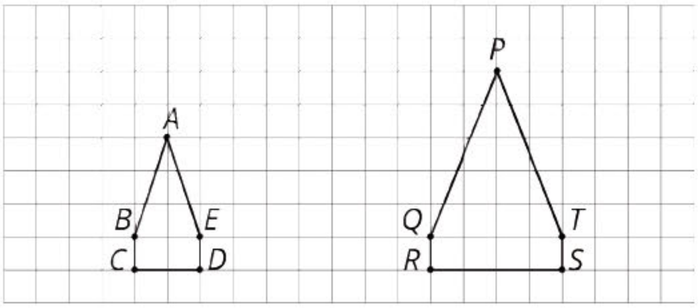
d. 6, 10.5, and 15.

e. 8, 14, and 20.

Lesson 4: Scaled Relationships

Cool Down: Corresponding Polygons

Here are two polygons on a grid.



Is $PQRST$ a scaled copy of $ABCDE$? Explain your reasoning.

Lesson 5: The Size of the Scale Factor

Cool Down: Scaling a Rectangle

A rectangle that is 2 inches by 3 inches has been scaled by a factor of 7.

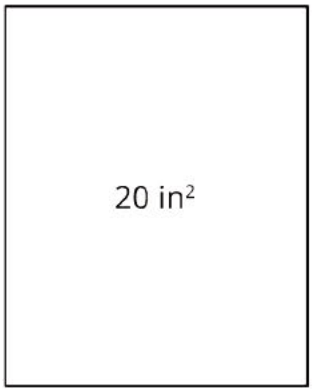
1. What are the side lengths of the scaled copy?

2. Suppose you want to scale the copy back to its original size. What scale factor should you use?

Lesson 6: Scaling and Area

Cool Down: Enlarged Areas

1. Lin has a drawing with an area of 20 in^2. If she increases all the sides by a scale factor of 4, what will the new area be?



2. Noah enlarged a photograph by a scale factor of 6. The area of the enlarged photo is how many times as large as the area of the original?

Lesson 7: Scale Drawings

Cool Down: Length of a Bus and Width of a Lake

1. A scale drawing of a school bus has a scale of $\frac{1}{2}$ inch to 5 feet. If the length of the school bus is $4\frac{1}{2}$ inches on the scale drawing, what is the actual length of the bus? Explain or show your reasoning.

2. A scale drawing of a lake has a scale of 1 cm to 80 m. If the actual width of the lake is 1,000 m, what is the width of the lake on the scale drawing? Explain or show your reasoning.

Lesson 8: Scale Drawings and Maps

Cool Down: Walking Around the Botanical Garden

Here is a map of the Missouri Botanical Garden. Clare walked all the way around the garden.

1. What is the actual distance around the garden? Show your reasoning.

2. It took Clare 30 minutes to walk around the garden at a constant speed. At what speed was she walking? Show your reasoning.

Lesson 9: Creating Scale Drawings

Cool Down: Drawing a Pool

A rectangular swimming pool measures 50 meters in length and 25 meters in width.

1. Make a scale drawing of the swimming pool where 1 centimeter represents 5 meters.

2. What are the length and width of your scale drawing?

Lesson 10: Changing Scales in Scale Drawings

Cool Down: Window Frame

Here is a scale drawing of a window frame that uses a scale of 1 cm to 6 inches.

Create another scale drawing of the window frame that uses a scale of 1 cm to 12 inches.

Lesson 11: Scales without Units

Cool Down: Scaled Courtyard Drawings

Andre drew a plan of a courtyard at a scale of 1 to 60. On his drawing, one side of the courtyard is 2.75 inches.

1. What is the actual measurement of that side of the courtyard? Express your answer in inches and then in feet.

2. If Andre made another courtyard scale drawing at a scale of 1 to 12, would this drawing be smaller or larger than the first drawing? Explain your reasoning.

Lesson 12: Units in Scale Drawings

Cool Down: Drawing the Backyard

Lin and her brother each created a scale drawing of their backyard, but at different scales. Lin used a scale of 1 inch to 1 foot. Her brother used a scale of 1 inch to 1 yard.

1. Express the scales for the drawings without units.

2. Whose drawing is larger? How many times as large is it? Explain or show your reasoning.

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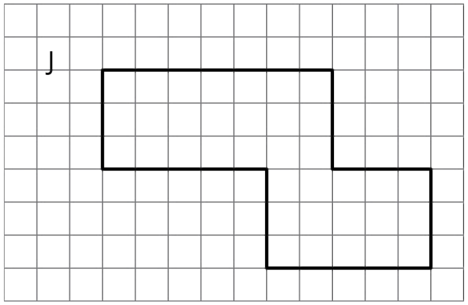
Instructional Masters

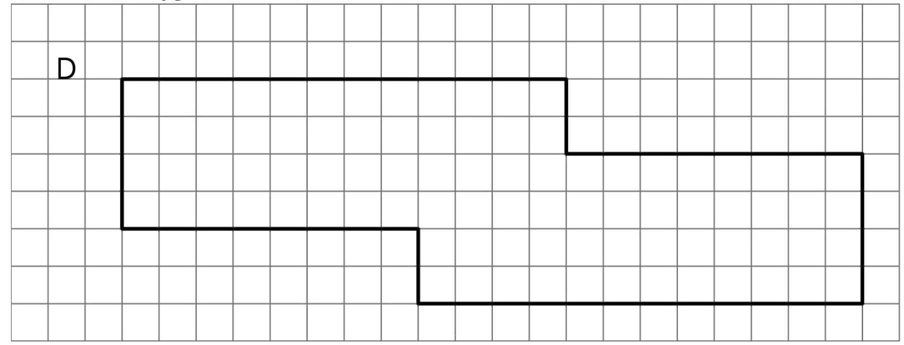
Instructional Masters for Scale Drawings

address	title	students per copy	written on?	requires cutting?	card stock recommended?	color paper recommended?
Activity Grade7.1.7.2	Sizing Up a Basketball Court	1	no	no	no	no
Activity Grade7.1.11.2	Apollo Lunar Module	1	yes	no	no	no
Activity Grade7.1.10.2	Same Plot, Different Drawings	24	no	yes	no	no
Activity Grade7.1.1.3	Pairs of Scaled Polygons	2	no	yes	no	yes
Activity Grade7.1.6.3	Area of Scaled Parallelograms and Triangles	6	no	yes	no	no
Activity Grade7.1.5.2	Scaled Copies Card Sort	3	no	yes	no	yes
Activity Grade7.1.12.4	Pondering Pools	2	no	no	no	no
Activity Grade7.1.5.3	Scaling A Puzzle	3	yes	yes	yes	no
Activity Grade7.1.12.2	Scales Card Sort	4	no	yes	no	yes

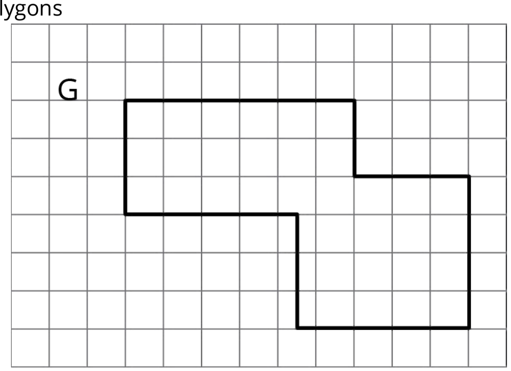
Pairs of Scaled Polygons



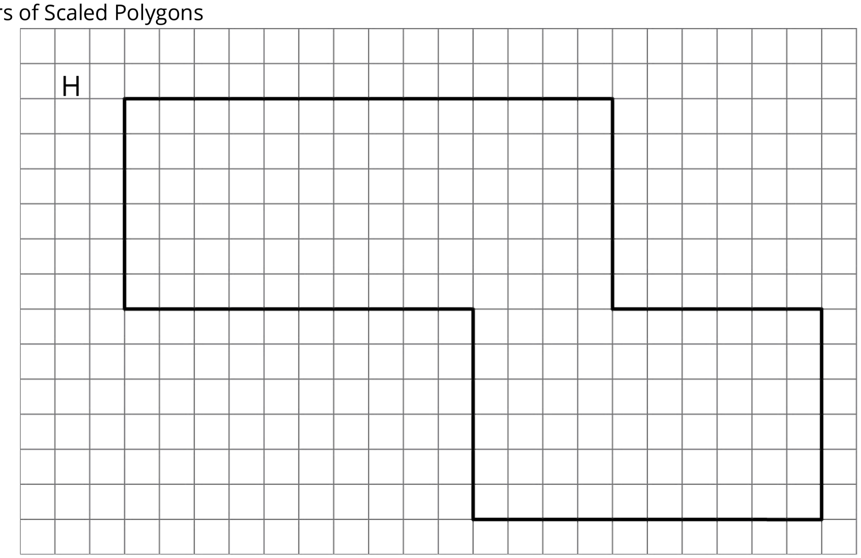
Pairs of Scaled Polygons



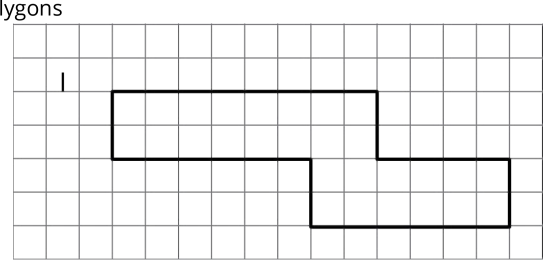
Pairs of Scaled Polygons



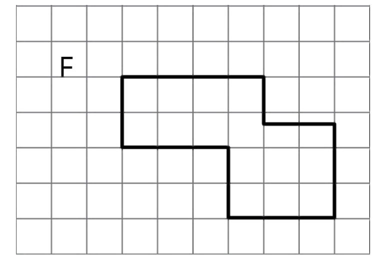
Pairs of Scaled Polygons



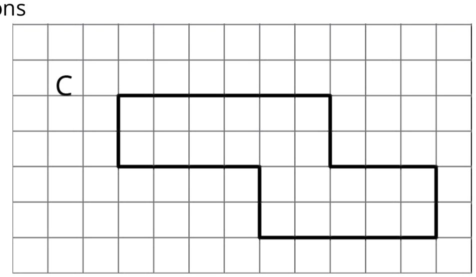
Pairs of Scaled Polygons



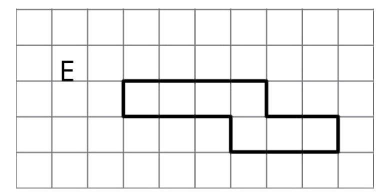
Pairs of Scaled Polygons



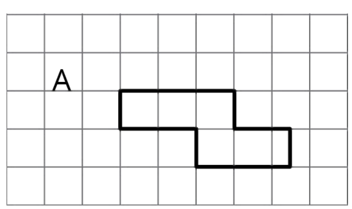
Pairs of Scaled Polygons



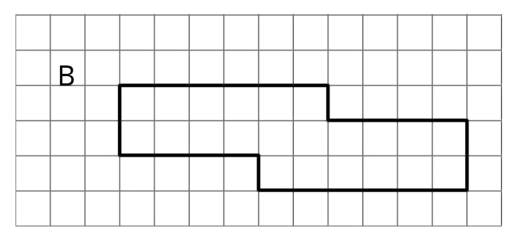
Pairs of Scaled Polygons



Pairs of Scaled Polygons

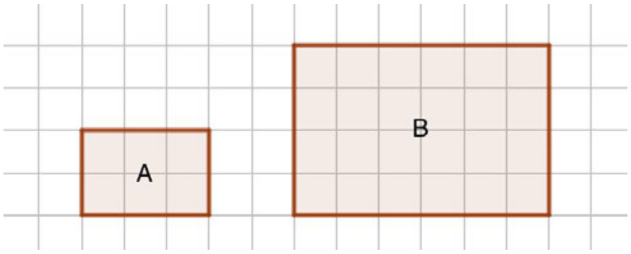


Pairs of Scaled Polygons

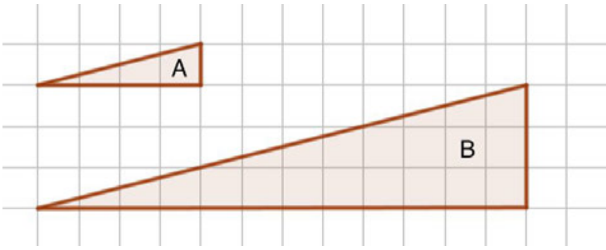


7.1.5.2 Card Sort: Scaled Copies .

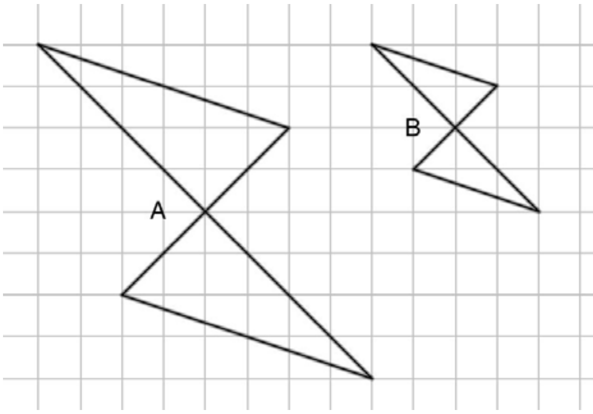
Scaled Copies Card Sort - Card 1



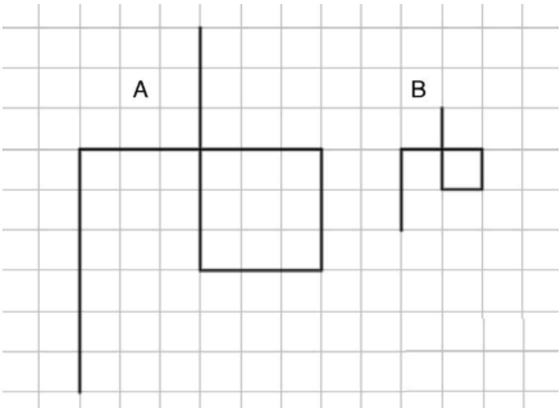
Scaled Copies Card Sort - Card 2



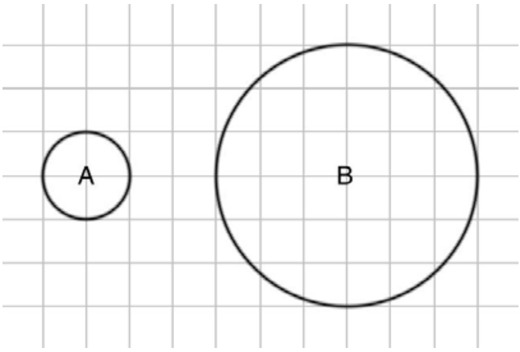
Scaled Copies Card Sort - Card 3



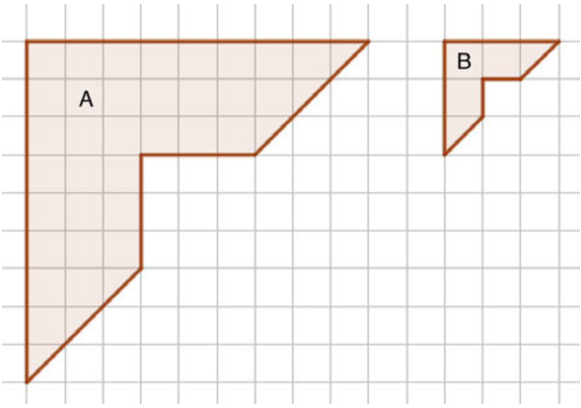
Scaled Copies Card Sort - Card 4



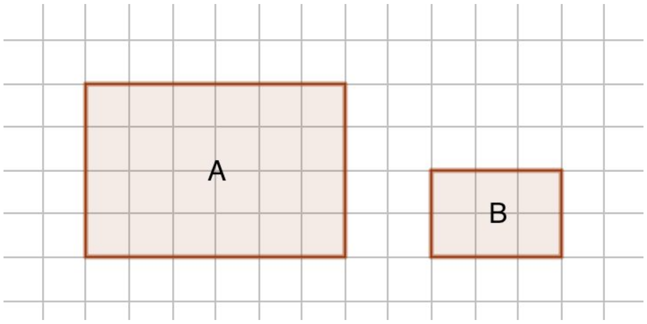
Scaled Copies Card Sort - Card 5



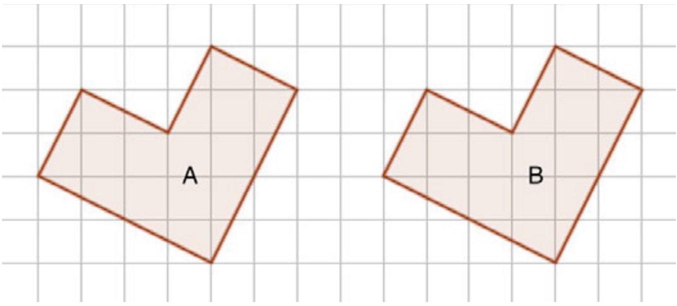
Scaled Copies Card Sort - Card 6



Scaled Copies Card Sort - Card 7

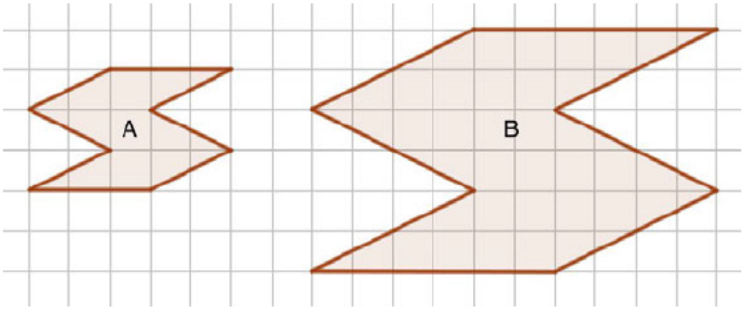


Scaled Copies Card Sort - Card 8

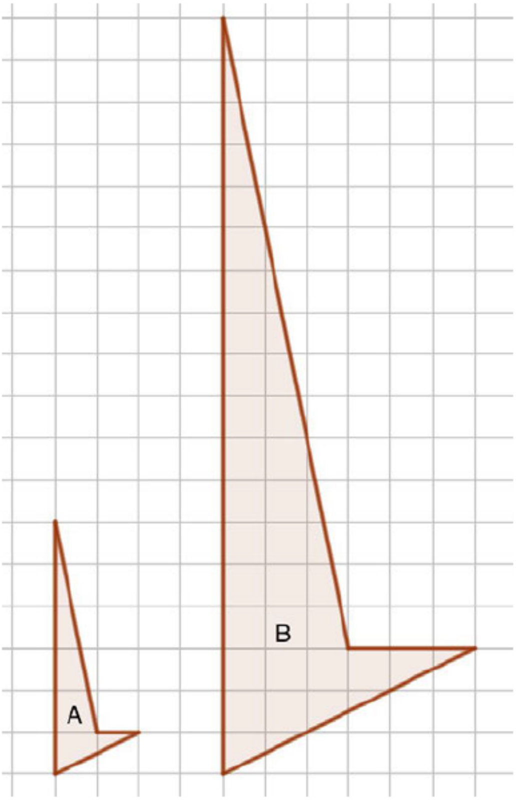


7.1.5.2 Card Sort: Scaled Copies .

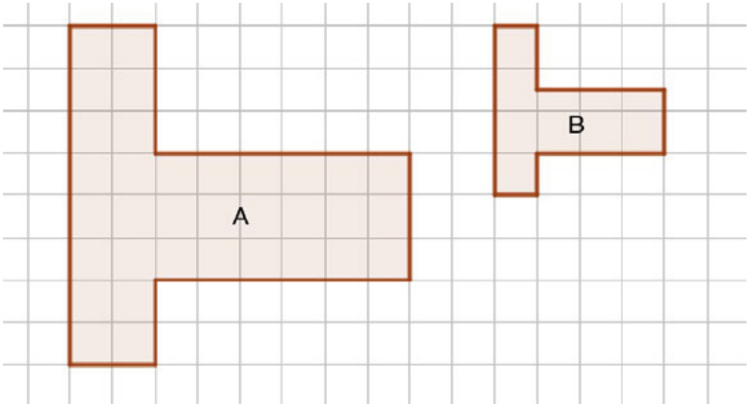
Scaled Copies Card Sort - Card 9



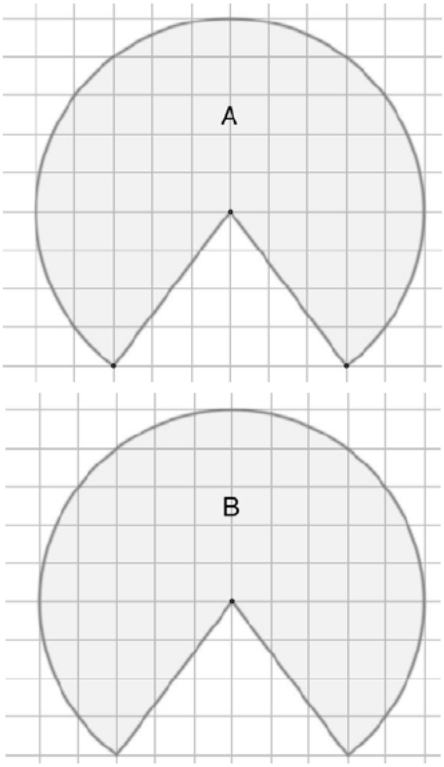
Scaled Copies Card Sort - Card 10



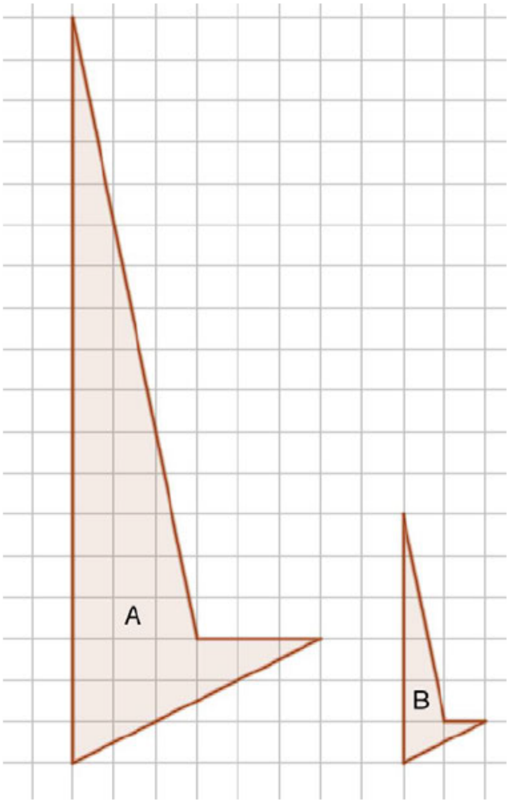
Scaled Copies Card Sort - Card 11

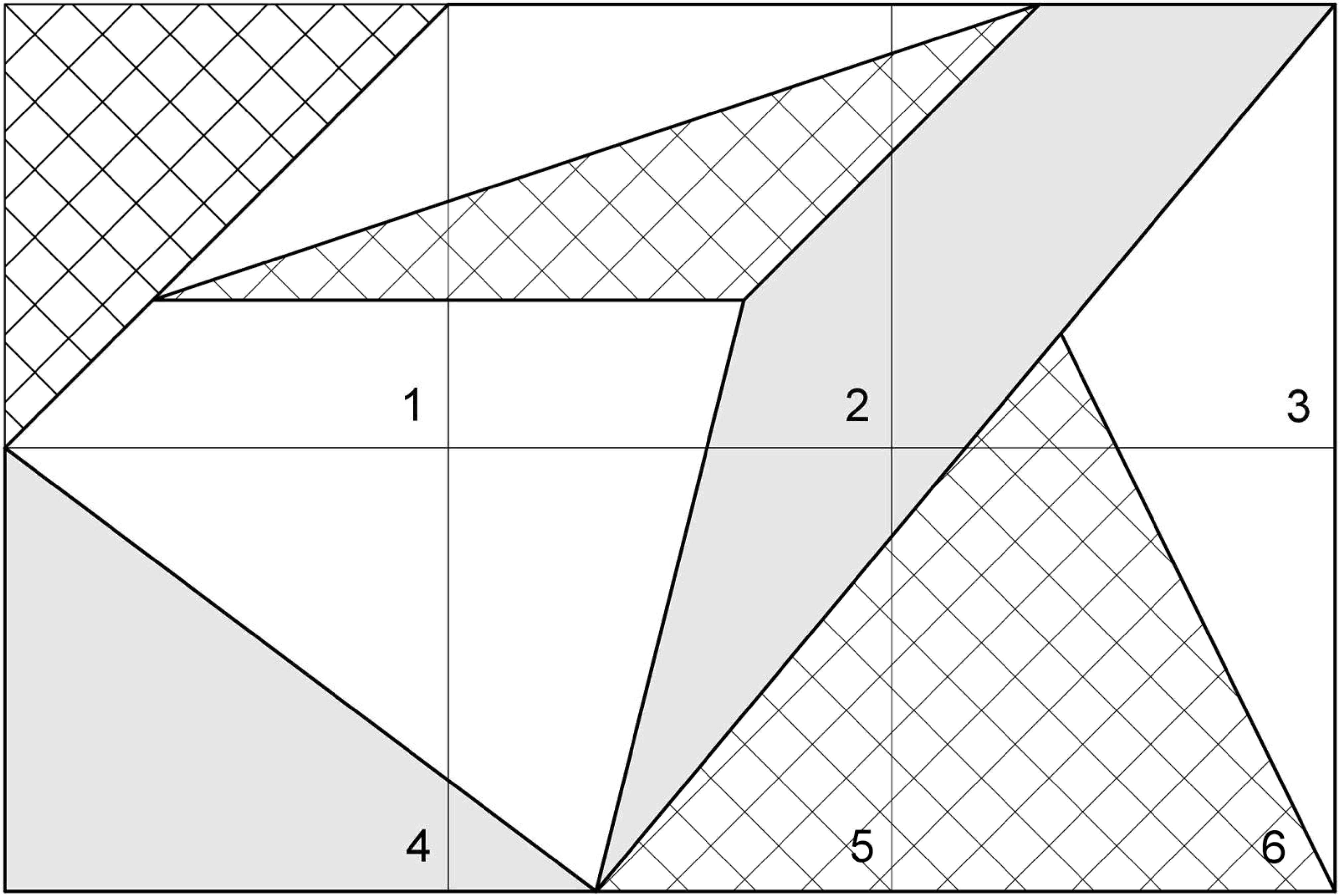


Scaled Copies Card Sort - Card 12



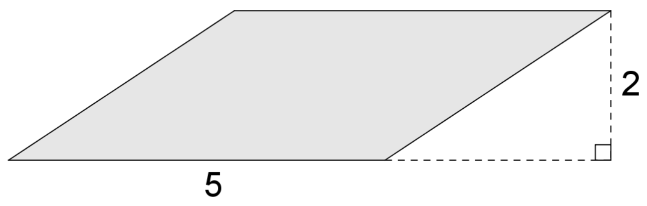
Scaled Copies Card Sort - Card 13



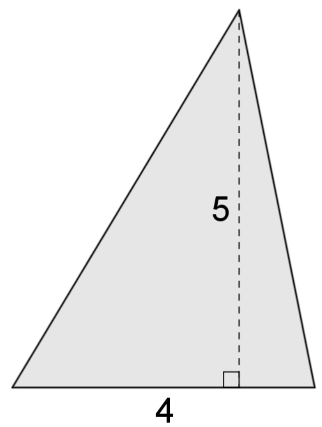

1
2
3
4
5
6

7.1.5.3 Scaling A Puzzle.

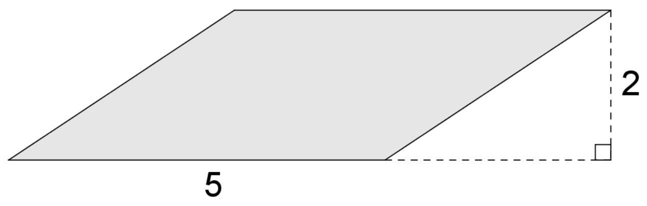
Area of Scaled Parallelograms and Triangles



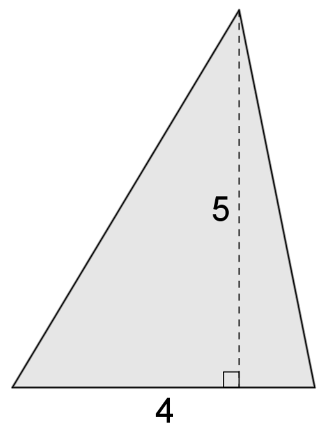
Area of Scaled Parallelograms and Triangles



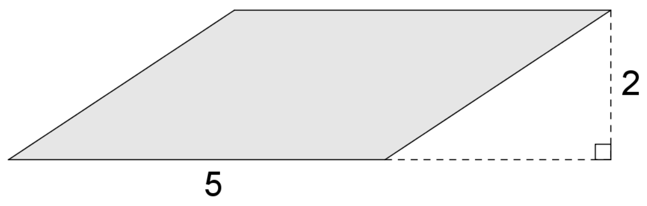
Area of Scaled Parallelograms and Triangles



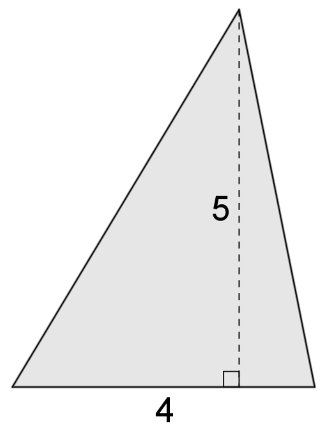
Area of Scaled Parallelograms and Triangles



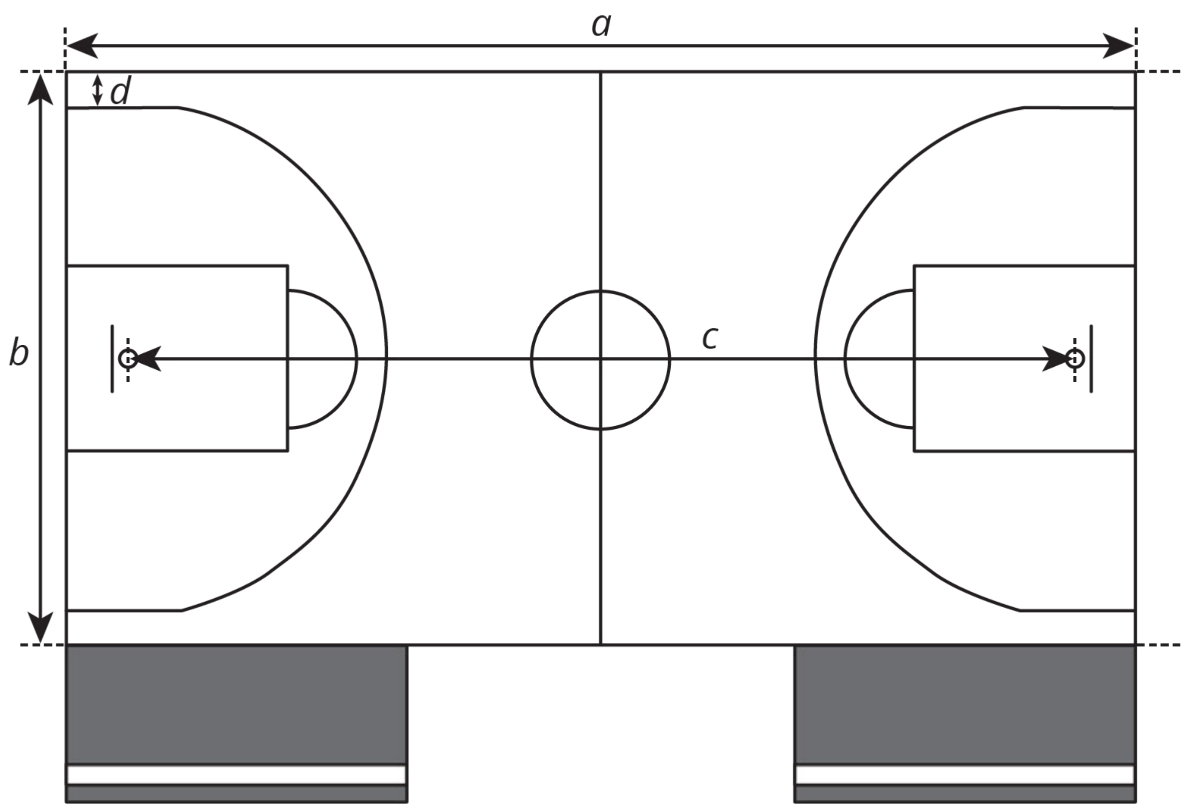
Area of Scaled Parallelograms and Triangles



Area of Scaled Parallelograms and Triangles



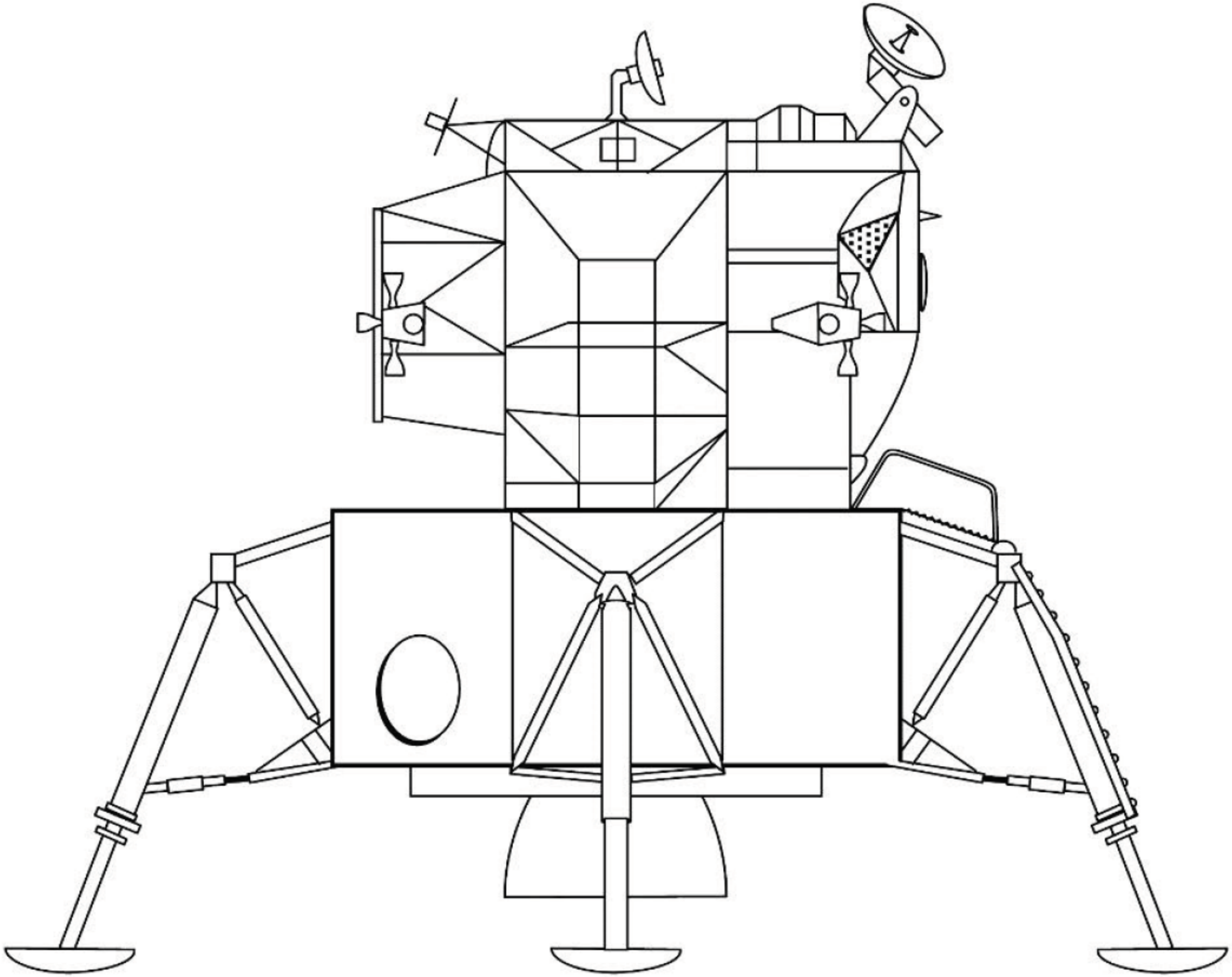
7.1.7.2 Sizing Up a Basketball Court.



1 cm represents 2 m

Same Plot, Different Drawings 1 cm to 5 m	Same Plot, Different Drawings 1 cm to 10 m	Same Plot, Different Drawings 1 cm to 15 m
Same Plot, Different Drawings 1 cm to 20 m	Same Plot, Different Drawings 1 cm to 30 m	Same Plot, Different Drawings 1 cm to 50 m
Same Plot, Different Drawings 1 cm to 5 m	Same Plot, Different Drawings 1 cm to 10 m	Same Plot, Different Drawings 1 cm to 15 m
Same Plot, Different Drawings 1 cm to 20 m	Same Plot, Different Drawings 1 cm to 30 m	Same Plot, Different Drawings 1 cm to 50 m
Same Plot, Different Drawings 1 cm to 5 m	Same Plot, Different Drawings 1 cm to 10 m	Same Plot, Different Drawings 1 cm to 15 m
Same Plot, Different Drawings 1 cm to 20 m	Same Plot, Different Drawings 1 cm to 30 m	Same Plot, Different Drawings 1 cm to 50 m
Same Plot, Different Drawings 1 cm to 5 m	Same Plot, Different Drawings 1 cm to 10 m	Same Plot, Different Drawings 1 cm to 15 m
Same Plot, Different Drawings 1 cm to 20 m	Same Plot, Different Drawings 1 cm to 30 m	Same Plot, Different Drawings 1 cm to 50 m

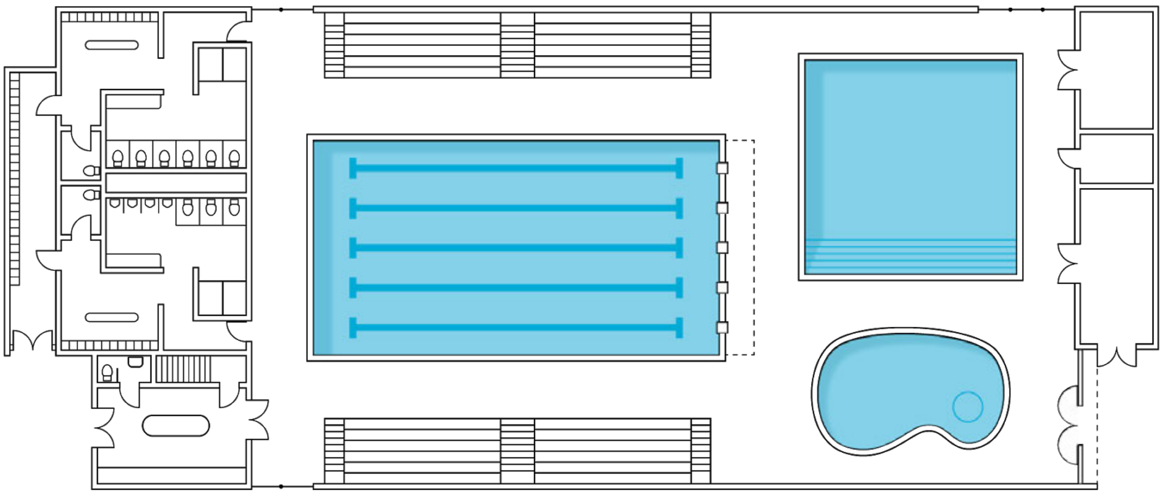
7.1.11 2 Apollo Lunar Module.

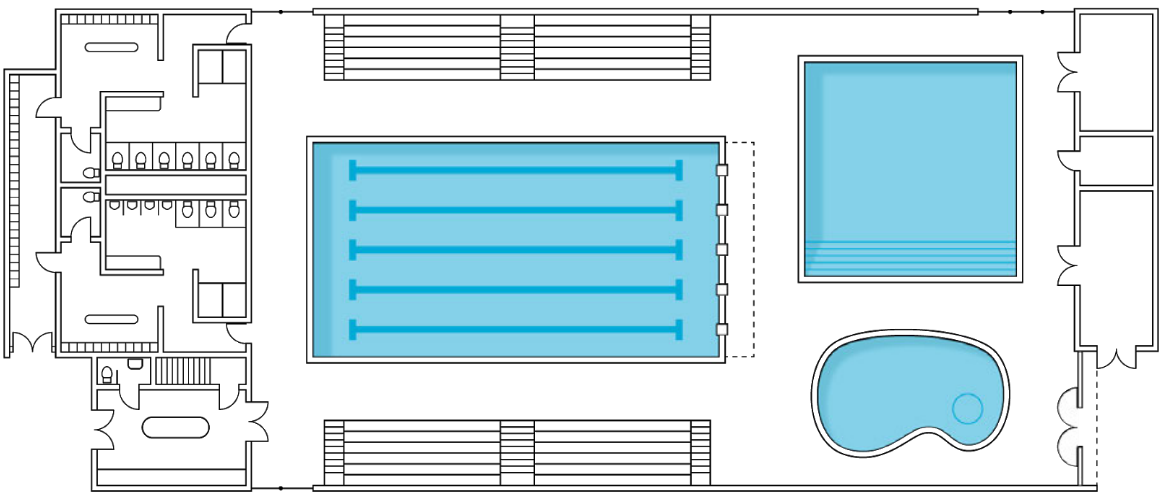


https://upload.wikimedia.org/wikipedia/commons/9/92/Manned_Moon_landers_LK_vs_LM_-_to_scale_drawing.png

Scales Card Sort 1 centimeter to 10 meters	Scales Card Sort $\frac{1}{2}$ centimeter to 500 meters
Scales Card Sort 1 centimeter to 1 meter	Scales Card Sort $\frac{1}{8}$ inch to 1 foot
Scales Card Sort 1 millimeter to 1 meter	Scales Card Sort 1 to 96
Scales Card Sort 1 centimeter to 1 kilometer	Scales Card Sort 1 to 100
Scales Card Sort 1 inch to 1,000 inches	Scales Card Sort 1 to 5,280
Scales Card Sort 1 foot to 1 mile	Scales Card Sort 1 to 63,360
Scales Card Sort 1 inch to 1 mile	Scales Card Sort 1 to 100,000
Scales Card Sort 1 inch to 8 feet	

7.1.12.4 Pondering Pools .





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