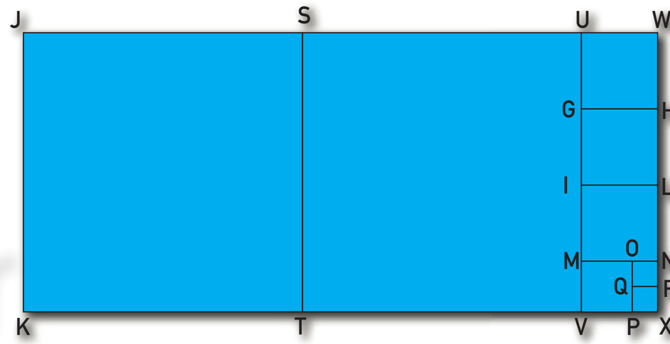
Core Knowledge® MATHEMATICS

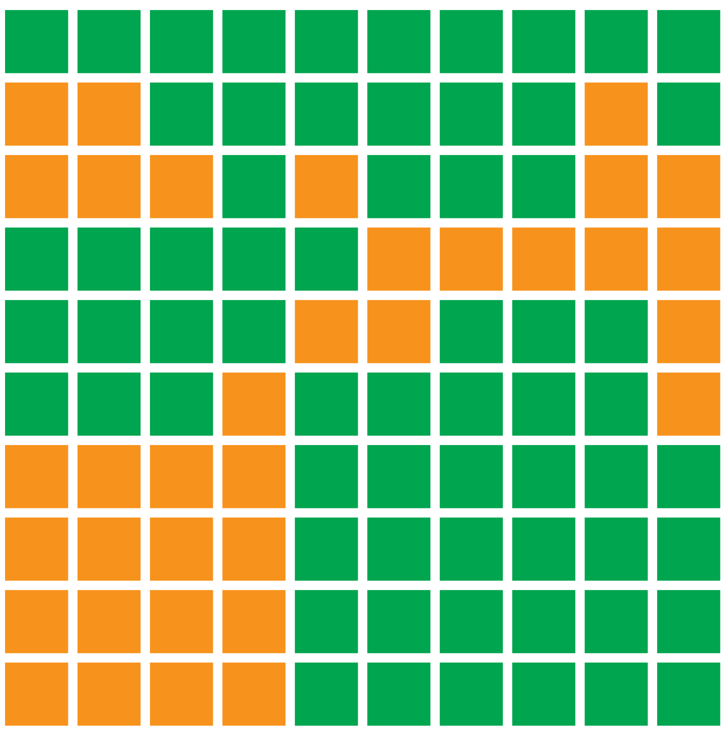
Putting it All Together

Student Workbook

Rectangle decomposed into squares.

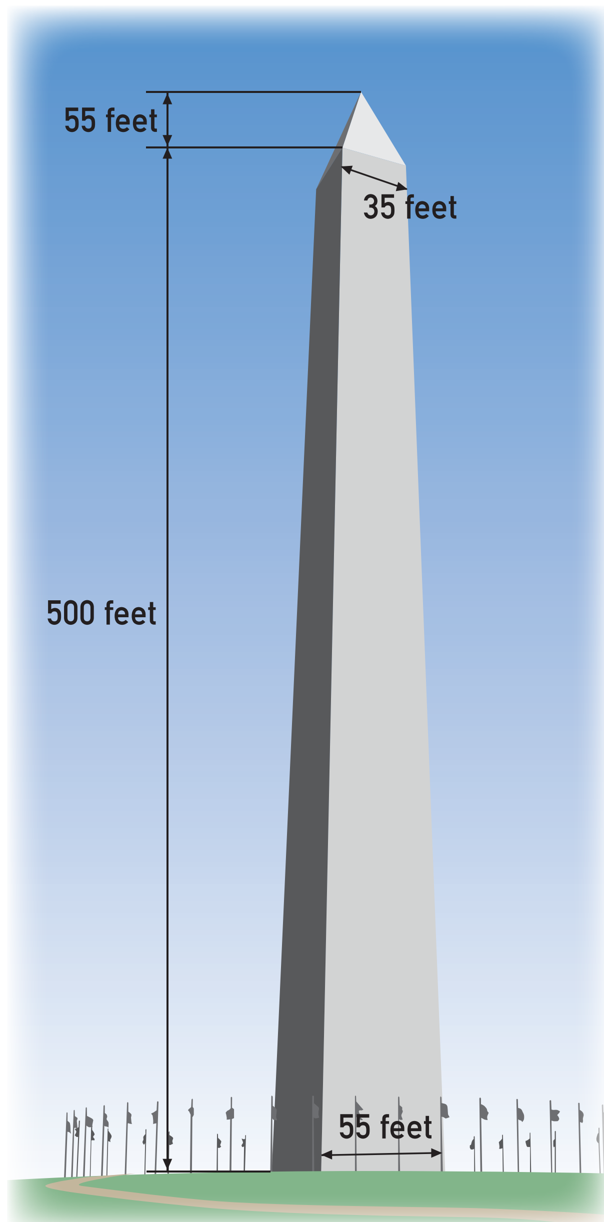


Squaretown's map



	✓	✗
class A	26	14
class B	31	19

Which Was "Yessier"?



Covering the Washington Monument

ISBN: 979-8-88970-888-9

Putting it All Together

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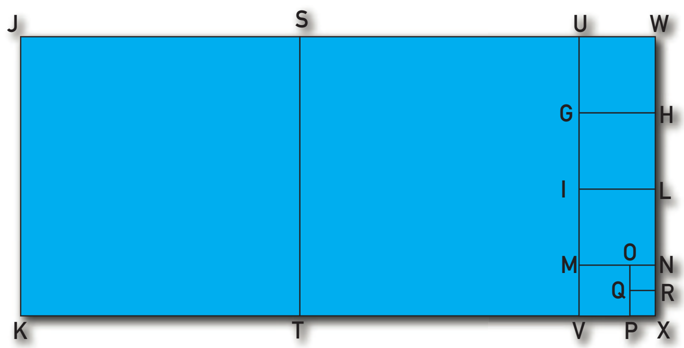


Putting it All Together

Student Workbook

Core Knowledge Mathematics™

Lesson 1: Fermi Problems

Let’s make some estimates.

1.1: Ant Trek

How long would it take an ant to run from Los Angeles to New York City?

1.2: Stacks and Stacks of Cereal Boxes

Imagine a warehouse that has a rectangular floor and that contains all of the boxes of breakfast cereal bought in the United States in one year.

If the warehouse is 10 feet tall, what could the side lengths of the floor be?

1.3: Covering the Washington Monument

How many tiles would it take to cover the Washington Monument?

Lesson 2: If Our Class Were the World

Let's use math to better understand our world.

2.1: All 7.4 Billion of Us

There are 7.4 billion people in the world. If the whole world were represented by a 30-person class:

- 14 people would eat rice as their main food.
- 12 people would be under the age of 20.
- 5 people would be from Africa.

1. How many people in the class would *not* eat rice as their main food?

2. What percentage of the people in the class would be under the age of 20?

3. Based on the number of people in the class representing people from Africa, how many people live in Africa?

2.2: About the People in the World

With the members of your group, write a list of questions about the people in the world. Your questions should begin with "How many people in the world. . ." Then, choose several questions on the list that you find most interesting.

2.3: If Our Class Were the World

Suppose your class represents all the people in the world.

Choose several characteristics about the world's population that you have investigated. Find the number of students in *your* class that would have the same characteristics.

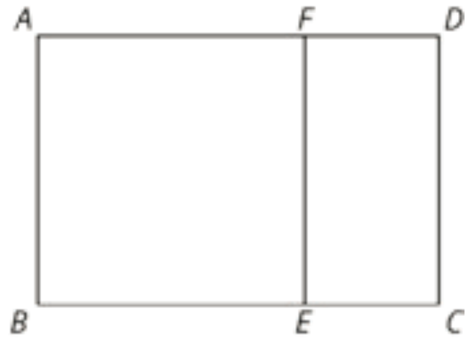
Create a visual display that includes a diagram that represents this information. Give your display the title "If Our Class Were the World."

Lesson 3: Rectangle Madness

Let's cut up rectangles.

3.1: Squares in Rectangles

1. Rectangle $ABCD$ is not a square. Rectangle $ABEF$ is a square.



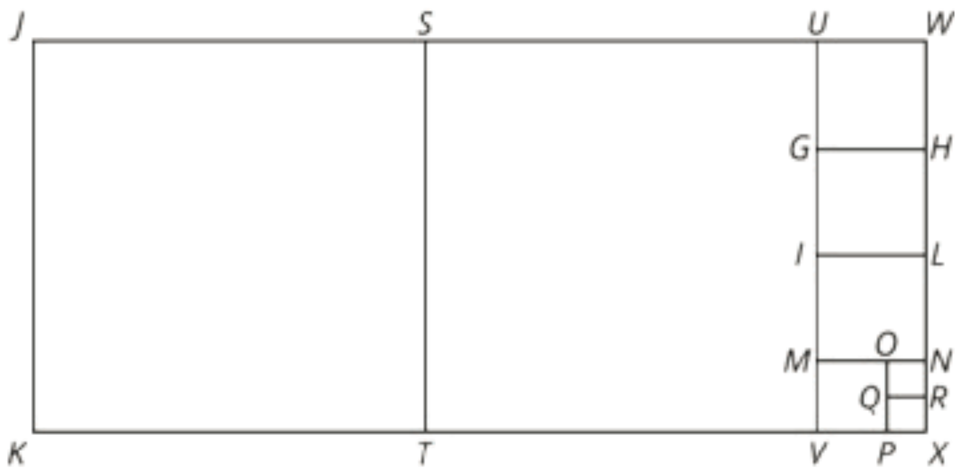
   a. Suppose segment AF were 5 units long and segment FD were 2 units long. How long would segment AD be?

   b. Suppose segment BC were 10 units long and segment BE were 6 units long. How long would segment EC be?

   c. Suppose segment AF were 12 units long and segment FD were 5 units long. How long would segment FE be?

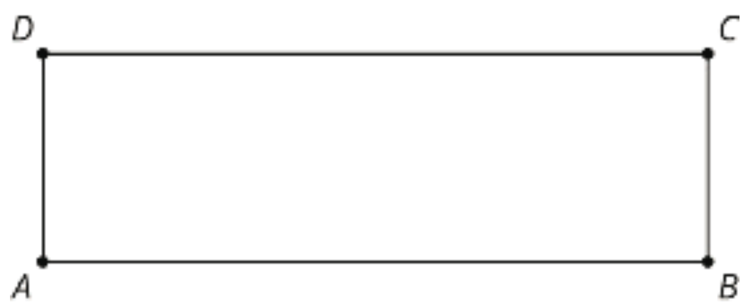
   d. Suppose segment AD were 9 units long and segment AB were 5 units long. How long would segment FD be?

2. Rectangle $JKXW$ has been decomposed into squares.



Segment JK is 33 units long and segment JW is 75 units long. Find the areas of all of the squares in the diagram.

3. Rectangle $ABCD$ is 16 units by 5 units.



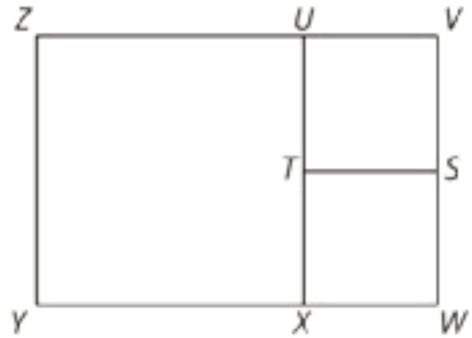
a. In the diagram, draw a line segment that decomposes $ABCD$ into two regions: a square that is the largest possible and a new rectangle.

b. Draw another line segment that decomposes the *new* rectangle into two regions: a square that is the largest possible and another new rectangle.

c. Keep going until rectangle $ABCD$ is entirely decomposed into squares.

d. List the side lengths of all the squares in your diagram.

Are you ready for more?



1. The diagram shows that rectangle $VWYZ$ has been decomposed into three squares. What could the side lengths of this rectangle be?

2. How many different side lengths can you find for rectangle $VWYZ$?

3. What are some rules for possible side lengths of rectangle $VWYZ$?

3.2: More Rectangles, More Squares

1. Draw a rectangle that is 21 units by 6 units.

   a. In your rectangle, draw a line segment that decomposes the rectangle into a new rectangle and a square that is as large as possible. Continue until the diagram shows that your original rectangle has been entirely decomposed into squares.

   b. How many squares of each size are in your diagram?

   c. What is the side length of the smallest square?

2. Draw a rectangle that is 28 units by 12 units.

   a. In your rectangle, draw a line segment that decomposes the rectangle into a new rectangle and a square that is as large as possible. Continue until the diagram shows that your original rectangle has been decomposed into squares.

   b. How many squares of each size are in your diagram?

   c. What is the side length of the smallest square?

3. Write each of these fractions as a mixed number with the smallest possible numerator and denominator:

   a. $\frac{16}{5}$

   b. $\frac{21}{6}$

   c. $\frac{28}{12}$

4. What do the fraction problems have to do with the previous rectangle decomposition problems?

3.3: Finding Equivalent Fractions

1. Accurately draw a rectangle that is 9 units by 4 units.

   a. In your rectangle, draw a line segment that decomposes the rectangle into a new rectangle and a square that is as large as possible. Continue until your original rectangle has been entirely decomposed into squares.

   b. How many squares of each size are there?

   c. What are the side lengths of the last square you drew?

   d. Write $\frac{9}{4}$ as a mixed number.

2. Accurately draw a rectangle that is 27 units by 12 units.

   a. In your rectangle, draw a line segment that decomposes the rectangle into a new rectangle and a square that is as large as possible. Continue until your original rectangle has been entirely decomposed into squares.

   b. How many squares of each size are there?

   c. What are the side lengths of the last square you drew?

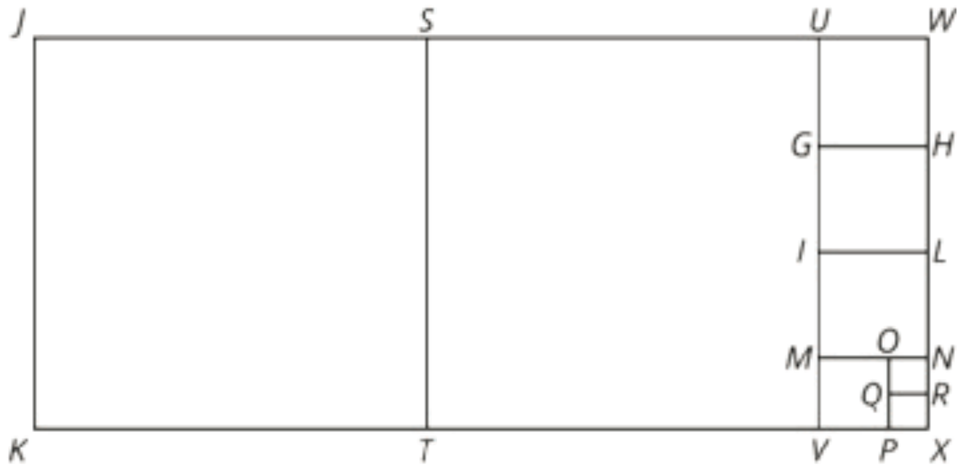
   d. Write $\frac{27}{12}$ as a mixed number.

   e. Compare the diagram you drew for this problem and the one for the previous problem. How are they the same? How are they different?

3. What is the greatest common factor of 9 and 4? What is the greatest common factor of 27 and 12? What does this have to do with your diagrams of decomposed rectangles?

Are you ready for more?

We have seen some examples of rectangle tilings. A *tiling* means a way to completely cover a shape with other shapes, without any gaps or overlaps. For example, here is a tiling of rectangle $KXWJ$ with 2 large squares, 3 medium squares, 1 small square, and 2 tiny squares.



Some of the squares used to tile this rectangle have the same size.

Might it be possible to tile a rectangle with squares where the squares are *all different sizes*?

If you think it is possible, find such a rectangle and such a tiling. If you think it is not possible, explain why it is not possible.

3.4: It’s All About Fractions

1. Accurately draw a 37-by-16 rectangle. (Use graph paper, if possible.)

   a. In your rectangle, draw a line segment that decomposes the rectangle into a new rectangle and a square that is as large as possible. Continue until your original rectangle has been entirely decomposed into squares.

   b. How many squares of each size are there?

   c. What are the dimensions of the last square you drew?

   d. What does this have to do with $2 + \frac{1}{3+\frac{1}{5}}$?

2. Consider a 52-by-15 rectangle.

a. In your rectangle, draw a line segment that decomposes the rectangle into a new rectangle and a square that is as large as possible. Continue until your original rectangle has been entirely decomposed into squares.

b. Write a fraction equal to this expression: $3 + \frac{1}{2+\frac{1}{7}}$.

c. Notice some connections between the rectangle and the fraction.

d. What is the greatest common factor of 52 and 15?

3. Consider a 98-by-21 rectangle.

a. In your rectangle, draw a line segment that decomposes the rectangle into a new rectangle and a square that is as large as possible. Continue until your original rectangle has been entirely decomposed into squares.

b. Write a fraction equal to this expression: $4 + \frac{1}{1+\frac{7}{14}}$.

c. Notice some connections between the rectangle and the fraction.

d. What is the greatest common factor of 98 and 21?

4. Consider a 121-by-38 rectangle.

a. Use the decomposition-into-squares process to write a continued fraction for $\frac{121}{38}$. Verify that it works.

b. What is the greatest common factor of 121 and 38?

Lesson 4: How Do We Choose?

Let's vote and choose a winner!

4.1: Which Was “Yessier”?

Two sixth-grade classes, A and B, voted on whether to give the answers to their math problems in poetry. The “yes” choice was more popular in both classes.

	yes	no
class A	24	16
class B	18	9

Was one class more in favor of math poetry, or were they equally in favor? Find three or more ways to answer the question.

4.2: Which Class Voted Purpler?

The school will be painted over the summer. Students get to vote on whether to change the color to purple (a “yes” vote), or keep it a beige color (a “no” vote).

The principal of the school decided to analyze voting results by class. The table shows some results.

In both classes, a majority voted for changing the paint color to purple. Which class was more in favor of changing?

	yes	no
class A	26	14
class B	31	19

4.3: Supermajorities

1. Another school is also voting on whether to change their school’s color to purple. Their rules require a $\frac{2}{3}$ supermajority to change the colors. A total of 240 people voted, and 153 voted to change to purple. Were there enough votes to make the change?

2. This school also is thinking of changing their mascot to an armadillo. To change mascots, a 55% supermajority is needed. How many of the 240 students need to vote “yes” for the mascot to change?

3. At this school, which requires more votes to pass: a change of mascot or a change of color?

4.4: Best Restaurant

A town’s newspaper held a contest to decide the best restaurant in town. Only people who subscribe to the newspaper can vote. 25% of the people in town subscribe to the newspaper. 20% of the subscribers voted. 80% of the people who voted liked Darnell’s BBQ Pit best.

Darnell put a big sign in his restaurant’s window that said, “80% say Darnell’s is the best!”

Do you think Darnell’s sign is making an accurate statement? Support your answer with:

- Some calculations
- An explanation in words
- A diagram that accurately represents the people in town, the newspaper subscribers, the voters, and the people who liked Darnell’s best

Lesson 5: More than Two Choices

Let's explore different ways to determine a winner.

5.1: Field Day

Students in a sixth-grade class were asked, “What activity would you most like to do for field day?” The results are shown in the table.

activity	number of votes
softball game	16
scavenger hunt	10
dancing talent show	8
marshmallow throw	4
no preference	2

1. What percentage of the class voted for softball?

2. What percentage did not vote for softball as their first choice?

5.2: School Lunches (Part 1)

Suppose students at our school are voting for the lunch menu over the course of one week. The following is a list of options provided by the caterer.

Menu 1: Meat Lovers

- Meat loaf
- Hot dogs
- Pork cutlets
- Beef stew
- Liver and onions

Menu 2: Vegetarian

- Vegetable soup and peanut butter sandwich
- Hummus, pita, and veggie sticks
- Veggie burgers and fries
- Chef's salad
- Cheese pizza every day
- Double desserts every day

Menu 3: Something for Everyone

- Chicken nuggets
- Burgers and fries
- Pizza
- Tacos
- Leftover day (all the week's leftovers made into a casserole)
- Bonus side dish: pea jello (green gelatin with canned peas)

Menu 4: Concession Stand

- Choice of hamburger or hot dog, with fries, every day

To vote, draw one of the following symbols next to each menu option to show your first, second, third, and last choices. If you use the slips of paper from your teacher, use only the column that says "symbol."

1. Meat Lovers __________
2. Vegetarian __________
3. Something for Everyone __________
4. Concession Stand __________

Here are two voting systems that can be used to determine the winner.

- Voting System #1. *Plurality*: The option with the most first-choice votes (stars) wins.
- Voting System #2. *Runoff*: If no choice received a majority of the votes, leave out the choice that received the fewest first-choice votes (stars). Then have another vote.

  If your first vote is still a choice, vote for that. If not, vote for your second choice that you wrote down.

  If there is still no majority, leave out the choice that got the fewest votes, and then vote again. Vote for your first choice if it's still in, and if not, vote for your second choice. If your second choice is also out, vote for your third choice.

1. How many people in our class are voting? How many votes does it take to win a majority?

2. How many votes did the top option receive? Was this a majority of the votes?

3. People tend to be more satisfied with election results if their top choices win. For how many, and what percentage, of people was the winning option:

   a. their first choice?

   b. their second choice?

   c. their third choice?

   d. their last choice?

4. After the second round of voting, did any choice get a majority? If so, is it the same choice that got a plurality in Voting System #1?

5. Which choice won?

6. How satisfied were the voters by the election results? For how many, and what percentage, of people was the winning option:

    a. their first choice?

    b. their second choice?

    c. their third choice?

    d. their last choice?

7. Compare the satisfaction results for the plurality voting rule and the runoff rule. Did one produce satisfactory results for more people than the other?

5.3: School Lunch (Part 2)

Let’s analyze a different election.

In another class, there are four clubs. Everyone in each club agrees to vote for the lunch menu exactly the same way, as shown in this table.

	Barbecue Club (21 members)	Garden Club (13 members)	Sports Boosters (7 members)	Film Club (9 members)
A. Meat Lovers	☆	✕	✕	✕
B. Vegetarian	☺	☆	😐	😐
C. Something for Everyone	😐	😐	☺	☆
D. Concession Stand	✕	☺	☆	☺

1. Figure out which option won the election by answering these questions.

    a. On the first vote, when everyone voted for their first choice, how many votes did each option get? Did any choice get a majority?

    b. Which option is removed from the next vote?

    c. On the second vote, how many votes did each of the remaining three menu options get? Did any option get a majority?

    d. Which menu option is removed from the next vote?

    e. On the third vote, how many votes did each of the remaining two options get? Which option won?

2. Estimate how satisfied all the voters were.
    a. For how many people was the winner their first choice?
    b. For how many people was the winner their second choice?
    c. For how many people was the winner their third choice?
    d. For how many people was the winner their last choice?
3. Compare the satisfaction results for the plurality voting rule and the runoff rule. Did one produce satisfactory results for more people than the other?

5.4: Just Vote Once

Your class just voted using the *instant runoff* system. Use the class data for following questions.

1. For our class, which choice received the most points?
2. Does this result agree with that from the runoff election in an earlier activity?
3. For the other class, which choice received the most points?
4. Does this result agree with that from the runoff election in an earlier activity?

5. The runoff method uses information about people's first, second, third, and last choices when it is not clear that there is a winner from everyone's first choices. How does the instant runoff method include the same information?

6. After comparing the results for the three voting rules (plurality, runoff, instant runoff) and the satisfaction surveys, which method do you think is fairest? Explain.

Are you ready for more?

Numbering your choices 0 through 3 might not really describe your opinions. For example, what if you really liked A and C a lot, and you really hated B and D? You might want to give A and C both a 3, and B and D both a 0.

1. Design a numbering system where the size of the number accurately shows how much you like a choice. Some ideas:
    - The same 0 to 3 scale, but you can choose more than one of each number, or even decimals between 0 and 3.
    - A scale of 1 to 10, with 10 for the best and 1 for the worst.

2. Try out your system with the people in your group, using the same school lunch options for the election.

3. Do you think your system gives a more fair way to make choices? Explain your reasoning.

5.5: Weekend Choices

Clare, Han, Mai, Tyler, and Noah are deciding what to do on the weekend. Their options are cooking, hiking, and bowling. Here are the points for their instant runoff vote. Each first choice gets 2 points, the second choice gets 1 point, and the last choice gets 0 points.

	cooking	hiking	bowling
Clare	2	1	0
Han	2	1	0
Mai	2	1	0
Tyler	0	2	1
Noah	0	2	1

1. Which activity won using the instant runoff method? Show your calculations and use expressions or equations.

2. Which activity would have won if there was just a vote for their top choice, with a majority or plurality winning?

3. Which activity would have won if there was a runoff election?

4. Explain why this happened.

Lesson 6: Picking Representatives

Let's think about fair representation.

6.1: Computers for Kids

A program gives computers to families with school-aged children. They have a certain number of computers to distribute fairly between several families. How many computers should each family get?

1. One month the program has 8 computers. The families have these numbers of school-aged children: 4, 2, 6, 2, 2.

   a. How many children are there in all?

   b. Counting all the children in all the families, how many children would use each computer? This is the number of children per computer. Call this number A.

   c. Fill in the third column of the table. Decide how many computers to give to each family if we use A as the basis for distributing the computers.

family	number of children	number of computers, using A
Baum	4	
Chu	2	
Davila	6	
Eno	2	
Farouz	2	

   d. Check that 8 computers have been given out in all.

2. The next month they again have 8 computers. There are different families with these numbers of children: 3, 1, 2, 5, 1, 8.

   a. How many children are there in all?

   b. Counting all the children in all the families, how many children would use each computer? This is the number of children per computer. Call this number B.

   c. Does it make sense that B is not a whole number? Why?

   d. Fill in the third column of the table. Decide how many computers to give to each family if we use B as the basis for distributing the computers.

family	number of children	number of computers, using B	number of computers, your way	children per computer, your way
Gray	3			
Hernandez	1			
Ito	2			
Jones	5			
Krantz	1			
Lo	8			

   e. Check that 8 computers have been given out in all.

   f. Does it make sense that the number of computers for one family is not a whole number? Explain your reasoning.

   g. Find and describe a way to distribute computers to the families so that each family gets a whole number of computers. Fill in the fourth column of the table.

h. Compute the number of children per computer in each family and fill in the last column of the table.

i. Do you think your way of distributing the computers is fair? Explain your reasoning.

6.2: School Mascot (Part 1)

A school is deciding on a school mascot. They have narrowed the choices down to the Banana Slugs or the Sea Lions.

The principal decided that each class gets one vote. Each class held an election, and the winning choice was the one vote for the whole class. The table shows how three classes voted.

	banana slugs	sea lions	class vote
class A	9	3	banana slug
class B	14	10	
class C	6	30	

1. Which mascot won, according to the principal's plan? What percentage of the votes did the winner get under this plan?

2. Which mascot received the most student votes in all? What percentage of the votes did this mascot receive?

3. The students thought this plan was not very fair. They suggested that bigger classes should have more votes to send to the principal. Make up a proposal for the principal where there are as few votes as possible, but the votes proportionally represent the number of students in each class.

4. Decide how to assign the votes for the results in the class. (Do they all go to the winner? Or should the loser still get some votes?)

5. In your system, which mascot is the winner?

6. In your system, how many representative votes are there? How many students does each vote represent?

6.3: Advising the School Board

1. In a very small school district, there are four schools, D, E, F, and G. The district wants a total of 10 advisors for the students. Each school should have at least one advisor.

school	number of students	number of advisors, using A
D	48	
E	12	
F	24	
G	36	

   a. How many students are in this district in all?

   b. If the advisors could represent students at different schools, how many students per advisor should there be? Call this number A.

   c. Using A students per advisor, how many advisors should each school have? Complete the table with this information for schools D, E, F, and G.

2. Another district has four schools; some are large, others are small. The district wants 10 advisors in all. Each school should have at least one advisor.

school	**number of students**	**number of advisors, using B**	**number of advisors, your way**	**students per advisor, your way**
Dr. King School	500			
O'Connor School	200			
Science Magnet School	140			
Trombone Academy	10			

a. How many students are in this district in all?

b. If the advisors didn't have to represent students at the same school, how many students per advisor should there be? Call this number B.

c. Using B students per advisor, how many advisors should each school have? Give your quotients to the tenths place. Fill in the first "number of advisors" column of the table. Does it make sense to have a tenth of an advisor?

d. Decide on a consistent way to assign advisors to schools so that there are only whole numbers of advisors for each school, and there is a total of 10 advisors among the schools. Fill in the "your way" column of the table.

e. How many students per advisor are there at each school? Fill in the last row of the table.

f. Do you think this is a fair way to assign advisors? Explain your reasoning.

6.4: School Mascot (Part 2)

The whole town gets interested in choosing a mascot. The mayor of the town decides to choose representatives to vote.

There are 50 blocks in the town, and the people on each block tend to have the same opinion about which mascot is best. Green blocks like sea lions, and gold blocks like banana slugs. The mayor decides to have 5 representatives, each representing a district of 10 blocks.

Here is a map of the town, with preferences shown.

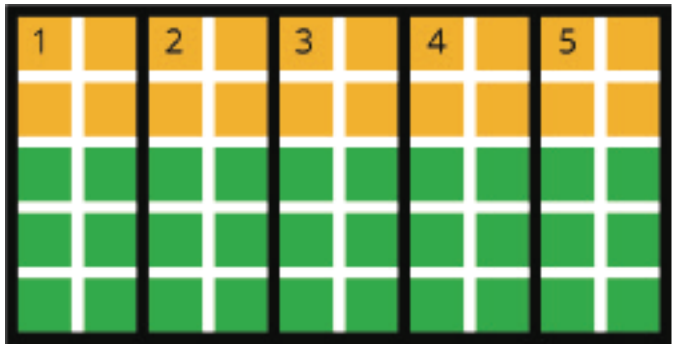
1. Suppose there were an election with each block getting one vote. How many votes would be for banana slugs? For sea lions? What percentage of the vote would be for banana slugs?

2. Suppose the districts are shown in the next map. What did the people in each district prefer? What did their representative vote? Which mascot would win the election?

Complete the table with this election's results.

district	number of blocks for banana slugs	number of blocks for sea lions	percentage of blocks for banana slugs	representative's vote
1	10	0		banana slugs
2				
3				
4				
5				

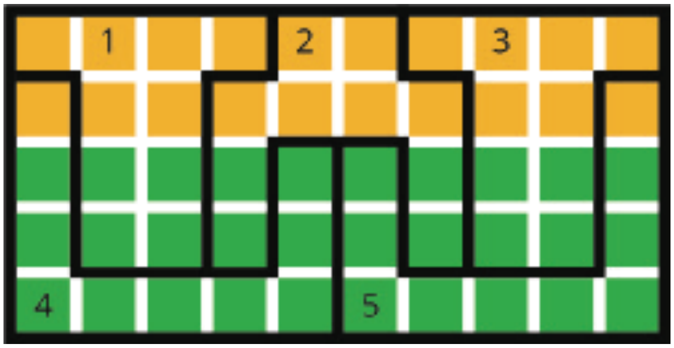
3. Suppose, instead, that the districts are shown in the new map below. What did the people in each district prefer? What did their representative vote? Which mascot would win the election?



Complete the table with this election's results.

district	number of blocks for banana slugs	number of blocks for sea lions	percentage of blocks for banana slugs	representative's vote
1				
2				
3				
4				
5				

4. Suppose the districts are designed in yet another way, as shown in the next map. What did the people in each district prefer? What did their representative vote? Which mascot would win the election?



Complete the table with this election's results.

district	number of blocks for banana slugs	number of blocks for sea lions	percentage of blocks for banana slugs	representative's vote
1				
2				
3				
4				
5				

5. Write a headline for the local newspaper for each of the ways of splitting the town into districts.

6. Which systems on the three maps of districts do you think are more fair? Are any totally unfair?

6.5: Fair and Unfair Districts

1. Smallville's map is shown, with opinions shown by block in green and gold. Decompose the map to create three connected, equal-area districts in two ways:

   a. Design three districts where *green* will win at least two of the three districts. Record results in Table 1.

Table 1:

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1				
2				
3				

   b. Design three districts where *gold* will win at least two of the three districts. Record results in Table 2.

Table 2:

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1				
2				
3				

2. Squaretown's map is shown, with opinions by block shown in green and gold. Decompose the map to create five connected, equal-area districts in two ways:

   a. Design five districts where *green* will win at least three of the five districts. Record the results in Table 3.

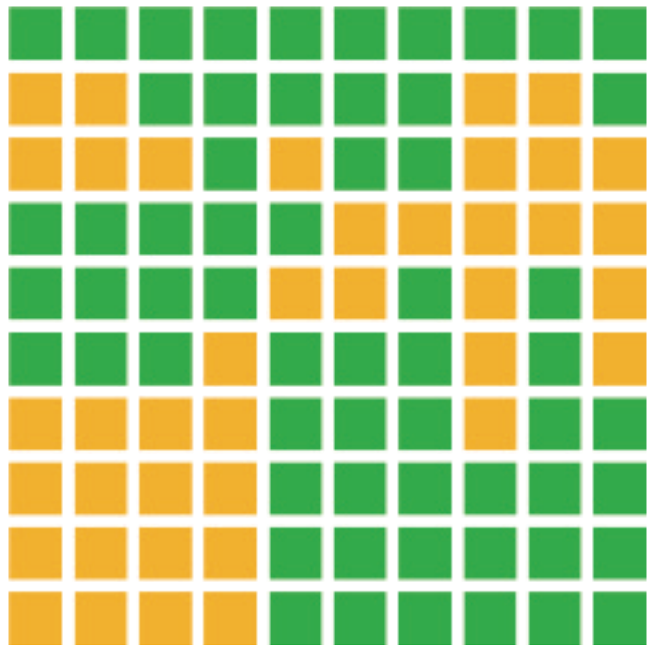


Table 3:

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1				
2				
3				
4				
5				

b. Design five districts where *gold* will win at least three of the five districts. Record the results in Table 4.

Table 4:

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1				
2				
3				
4				
5				

3. Mountain Valley's map is shown, with opinions by block shown in green and gold. (This is a town in a narrow valley in the mountains.) Can you decompose the map to create three connected, equal-area districts in the two ways described here?

   a. Design three districts where *green* will win at least 2 of the 3 districts. Record the results in Table 5.

Table 5:

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1				
2				
3				

   b. Design three districts where *gold* will win at least 2 of the 3 districts. Record the results in Table 6.

Table 6:

district	number of blocks for green	number of blocks for gold	percentage of blocks for green	representative's vote
1				
2				
3				

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