

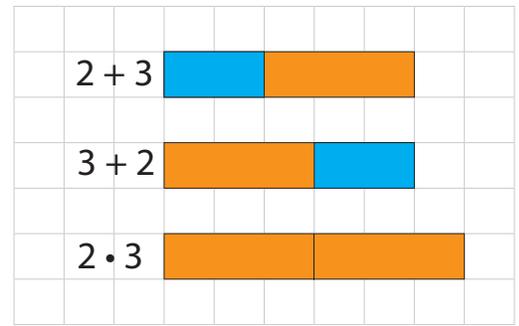
Tetrahedra



Expressions and Equations

Teacher Guide

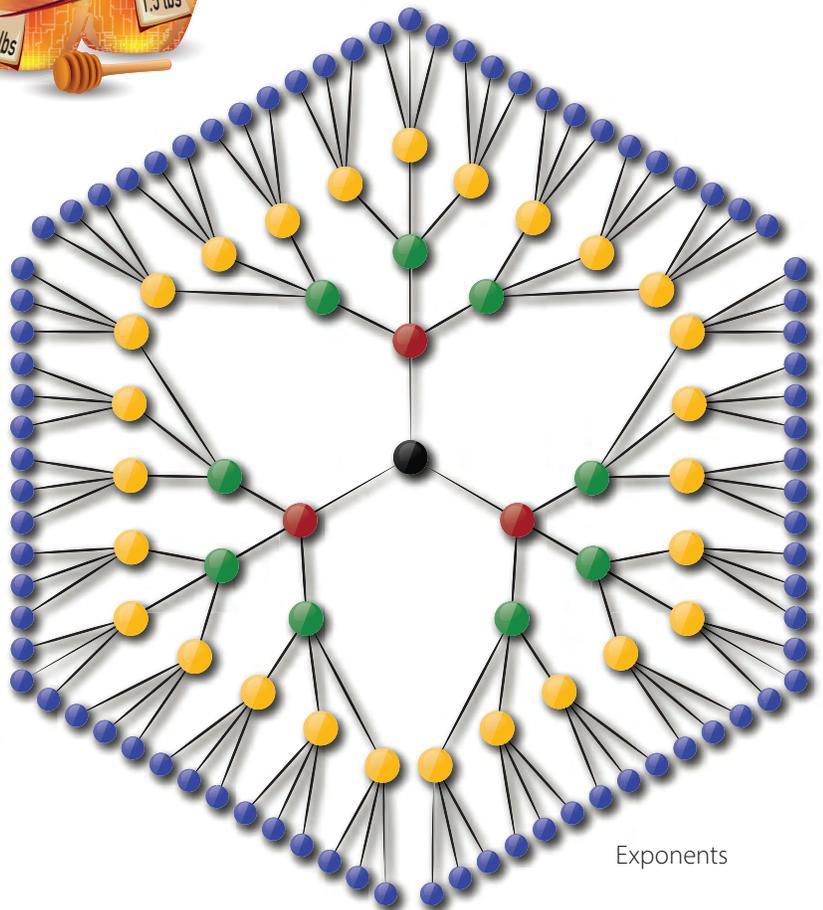
Two Related Quantities



Using Diagrams to Show That Expressions are Equivalent



Staying in Balance



Exponents

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Expressions and Equations

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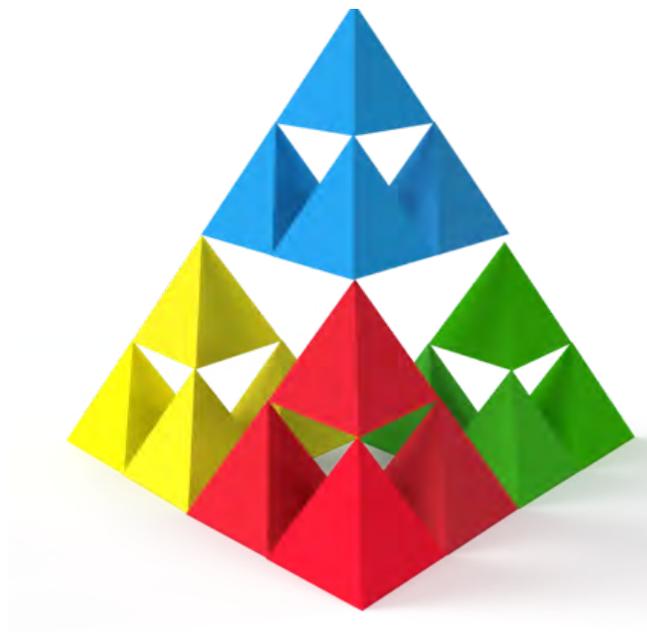
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Expressions and Equations

Teacher Guide

Core Knowledge Mathematics™

Expressions and Equations

Unit Narrative

Students begin the unit by working with linear equations that have single occurrences of one variable, e.g., $x + 1 = 5$ and $4x = 2$. They represent relationships with tape diagrams and with linear equations, explaining correspondences between these representations. They examine values that make a given linear equation true or false, and what it means for a number to be a solution to an equation. Solving equations of the form $px = q$ where p and q are rational numbers can produce complex fractions (i.e., quotients of fractions), so students extend their understanding of fractions to include those with numerators and denominators that are not whole numbers.

The second section introduces balanced and unbalanced “hanger diagrams” as a way to reason about solving the linear equations of the first section. Students write linear equations to represent situations, including situations with percentages, solve the equations, and interpret the solutions in the original contexts (MP2), specifying units of measurement when appropriate (MP6). They represent linear expressions with tape diagrams and use the diagrams to identify values of variables for which two linear expressions are equal. Students write linear expressions such as $6w + 24$ and $6(w + 4)$ and represent them with area diagrams, noting the connection with the distributive property (MP7). They use the distributive property to write equivalent expressions.

In the third section of the unit, students write expressions with whole-number exponents and whole-number, fraction, or variable bases. They evaluate such expressions, using properties of exponents strategically (MP5). They understand that a solution to an equation in one variable is a number that makes the equation true when the number is substituted for all instances of the variable. They represent algebraic expressions and equations in order to solve problems. They determine whether pairs of numerical exponential expressions are equivalent and explain their reasoning (MP3). By examining a list of values, they find solutions for simple exponential equations of the form $a = b^x$, e.g., $2^x = 32$, and simple quadratic and cubic equations, e.g., $64 = x^3$.

In the last section of the unit, students represent collections of equivalent ratios as equations. They use and make connections between tables, graphs, and linear equations that represent the same relationships (MP1).

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as interpreting, describing and explaining. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Interpret

- tape diagrams involving letters that stand for numbers (Lesson 1)
- the parts of an equation (Lesson 2)

- descriptions of situations (Lesson 6)
- numerical expressions involving exponents (Lesson 13)
- different representations of the same relationship between quantities (Lesson 17)

Describe

- how parts of an equation represent parts of a story (Lesson 2)
- solutions to equations (Lesson 2)
- stories represented by given equations (Lesson 5)
- patterns of growth that can be represented using exponents (Lesson 12)
- relationships between independent and dependent variables (Lesson 16)

Explain

- the meaning of a solution using hanger diagrams (Lesson 3)
- how to solve an equation (Lesson 4)
- how to use equations to solve percent problems (Lesson 7)
- how to determine whether two expressions are equivalent, including with reference to diagrams (Lesson 8)
- strategies for determining whether expressions are equivalent (Lesson 13)
- the process of evaluating variable exponential expressions (Lesson 15)

In addition, students are expected to compare equations with balanced hanger diagrams and with descriptions of situations, represent quantities with mathematical expressions, generalize about equivalent numerical expressions using rectangle diagrams and the distributive property, justify claims about equivalent variable expressions using rectangle diagrams and the distributive property, and justify reasoning when evaluating and comparing numerical expressions with exponents.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

Learning Targets

Expressions and Equations

Lesson 1: Tape Diagrams and Equations

- I can tell whether or not an equation could represent a tape diagram.
- I can use a tape diagram to represent a situation.

Lesson 2: Truth and Equations

- I can match equations to real life situations they could represent.
- I can replace a variable in an equation with a number that makes the equation true, and know that this number is called a solution to the equation.

Lesson 3: Staying in Balance

- I can compare doing the same thing to the weights on each side of a balanced hanger to solving equations by subtracting the same amount from each side or dividing each side by the same number.
- I can explain what a balanced hanger and a true equation have in common.
- I can write equations that could represent the weights on a balanced hanger.

Lesson 4: Practice Solving Equations and Representing Situations with Equations

- I can explain why different equations can describe the same situation.
- I can solve equations that have whole numbers, fractions, and decimals.

Lesson 5: A New Way to Interpret a over b

- I understand the meaning of a fraction made up of fractions or decimals, like $\frac{2.1}{0.07}$ or $\frac{\frac{4}{5}}{\frac{3}{2}}$.
- When I see an equation, I can make up a story that the equation might represent, explain what the variable represents in the story, and solve the equation.

Lesson 6: Write Expressions Where Letters Stand for Numbers

- I can use an expression that represents a situation to find an amount in a story.
- I can write an expression with a variable to represent a calculation where I do not know one of the numbers.

Lesson 7: Revisit Percentages

- I can solve percent problems by writing and solving an equation.

Lesson 8: Equal and Equivalent

- I can explain what it means for two expressions to be equivalent.
- I can use a tape diagram to figure out when two expressions are equal.
- I can use what I know about operations to decide whether two expressions are equivalent.

Lesson 9: The Distributive Property, Part 1

- I can use a diagram of a rectangle split into two smaller rectangles to write different expressions representing its area.
- I can use the distributive property to help do computations in my head.

Lesson 10: The Distributive Property, Part 2

- I can use a diagram of a split rectangle to write different expressions with variables representing its area.

Lesson 11: The Distributive Property, Part 3

- I can use the distributive property to write equivalent expressions with variables.

Lesson 12: Meaning of Exponents

- I can evaluate expressions with exponents and write expressions with exponents that are equal to a given number.
- I understand the meaning of an expression with an exponent like 3^5 .

Lesson 13: Expressions with Exponents

- I can decide if expressions with exponents are equal by evaluating the expressions or by understanding what exponents mean.

Lesson 14: Evaluating Expressions with Exponents

- I know how to evaluate expressions that have both an exponent and addition or subtraction.
- I know how to evaluate expressions that have both an exponent and multiplication or division.

Lesson 15: Equivalent Exponential Expressions

- I can find solutions to equations with exponents in a list of numbers.
- I can replace a variable with a number in an expression with exponents and operations and use the correct order to evaluate the expression.

Lesson 16: Two Related Quantities, Part 1

- I can create tables and graphs that show the relationship between two amounts in a given ratio.
- I can write an equation with variables that shows the relationship between two amounts in a given ratio.

Lesson 17: Two Related Quantities, Part 2

- I can create tables and graphs to represent the relationship between distance and time for something moving at a constant speed.
- I can write an equation with variables to represent the relationship between distance and time for something moving at a constant speed.

Lesson 18: More Relationships

- I can create tables and graphs that show different kinds of relationships between amounts.
- I can write equations that describe relationships with area and volume.

Lesson 19: Tables, Equations, and Graphs, Oh My!

- I can create a table and a graph that represent the relationship in a given equation.
- I can explain what an equation tells us about the situation.

lesson	new terminology	
	receptive	productive
6.6.1	value (of a variable)	operation
6.6.2	variable coefficient solution to an equation true equation / false equation	value (of a variable)
6.6.3	each side balanced hanger	
6.6.4	solve (an equation)	each side
6.6.6		equation
6.6.7		true equation / false equation
6.6.8	equivalent expressions	
6.6.9	term distributive property area as a product area as a sum	
6.6.12	to the power	
6.6.13	base (of an exponent)	to the power exponent
6.6.14		solution to an equation
6.6.16	independent variable dependent variable	variable relationship
6.6.17	coordinate plane coordinates	
6.6.18	horizontal axis vertical axis plot	

Required Materials

Colored pencils

Graph paper

Sticky notes

Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart

paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Section: Equations in One Variable

Lesson 1: Tape Diagrams and Equations

Goals

- Draw tape diagrams to represent equations of the forms $x + p = q$ and $px = q$.
- Interpret (orally and in writing) tape diagrams that represent equations of the form $p + x = q$ or $px = q$.
- Use tape diagrams to find unknown values in equations of the forms $x + p = q$ and $px = q$ and explain (orally) the solution method.

Learning Targets

- I can tell whether or not an equation could represent a tape diagram.
- I can use a tape diagram to represent a situation.

Lesson Narrative

The purpose of this lesson is to help students remember from earlier grades how tape diagrams can be used to represent operations. There are two roles that tape diagrams (or any diagrams) can play: helping to visualize a relationship, and helping to solve a problem. The focus here is the first of these, so that later students can use diagrams for the second of these. In this lesson, students both interpret tape diagrams and create their own.

Note that the terms “solution” and “variable” aren’t defined until the next lesson, nor should any solution methods be generalized yet. Students should engage with the activities and reason about unknown quantities in ways that make sense to them.

Alignments

Addressing

- 6.EE.B.6: Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

Building Towards

- 6.EE.B.5: Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

- 6.EE.B.6: Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- 6.EE.B.7: Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display
- MLR7: Compare and Connect
- Think Pair Share

Student Learning Goals

Let's see how tape diagrams and equations can show relationships between amounts.

1.1 Which Diagram is Which?

Warm Up: 5 minutes

Students recall tape diagram representations of addition and multiplication relationships.

For relationships involving multiplication, we follow the convention that the first factor is the number of groups and the second is the number in each group. But students do not have to follow that convention; they may use their understanding of the commutative property of multiplication to represent relationships in ways that make sense to them.

Building Towards

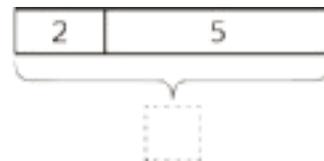
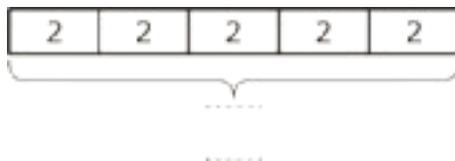
- 6.EE.B.6

Launch

Give students 2 minutes of quiet think time, followed by a whole-class discussion.

Student Task Statement

1. Here are two diagrams. One represents $2 + 5 = 7$. The other represents $5 \cdot 2 = 10$. Which is which? Label the length of each diagram.



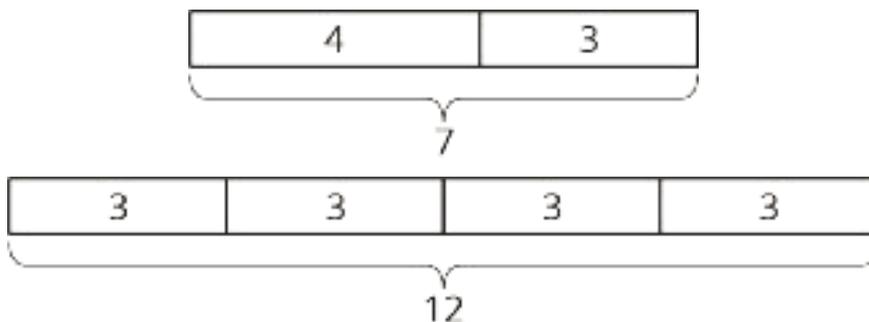
2. Draw a diagram that represents each equation.

$$4 + 3 = 7$$

$$4 \cdot 3 = 12$$

Student Response

1. $5 \cdot 2 = 10$ on the left and $2 + 5 = 7$ on the right.
- 2.



Activity Synthesis

Invite students to share their responses and rationale. The purpose of the discussion is to give students an opportunity to articulate how operations can be represented by tape diagrams. Some questions to guide the discussion:

- “Where do you see the 5 in the first diagram?” (There are 5 equal parts represented by 5 same-size boxes.)
- “How did you find the length of the first diagram?” (Either computed $2 + 2 + 2 + 2 + 2$ or $5 \cdot 2$.)
- “Explain how you knew what the diagrams for $4 + 3 = 7$ and $4 \cdot 3 = 12$ should look like. How are they alike? How are they different?”
- “How did you represent $4 \cdot 3$? How are they alike? How are they different?” (Some may represent $4 \cdot 3$ as 4 groups of size 3, while some may represent as 3 groups of size 4.)

1.2 Match Equations and Tape Diagrams

10 minutes

In this first activity on tape diagram representations of equations with variables, students use what they know about relationships between operations to identify multiple equations that match a given diagram. It is assumed that students have seen representations like these in prior grades. If this is not the case, return to the examples in the warm-up and ask students to write other equations for each of the diagrams. For example, the relationship between the quantities 2, 5, and 7 expressed by the equation $2 + 5 = 7$ can also be written as $2 = 7 - 5$, $5 = 7 - 2$, $7 = 2 + 5$, and $7 - 2 = 5$. Ask students to explain how these equations match the parts of the tape diagram.

Note that the word “variable” is not defined until the next lesson. It is not necessary to use that term with students yet. Also, we are sneaking in the equivalent expressions $x + x + x + x$ and $4 \cdot x$

because these equivalent ways of writing it should be familiar from earlier grades, but *equivalent expressions* are defined more carefully later in this unit. Even though this familiar example appears, the general idea of equivalent expressions is not a focus of this lesson.

Building Towards

- 6.EE.B.6
- 6.EE.B.7

Instructional Routines

- MLR2: Collect and Display
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 2 minutes of quiet work time. Then, ask them to share their responses with their partner and follow with whole-class discussion.

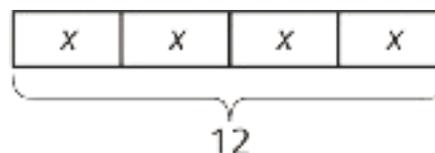
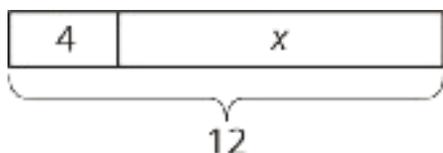
If necessary, explain that the x in each diagram is just standing in for a number.

Anticipated Misconceptions

Students may not have much experience with a letter standing in for a number. If students resist engaging, explain that the x is just standing in for a number. Students may prefer to figure out the value that x must take to make each diagram make sense (8 in the first diagram and 3 in the second diagram) before thinking out which equations can represent each diagram.

Student Task Statement

Here are two tape diagrams. Match each equation to one of the tape diagrams.



- $4 + x = 12$
- $12 = 4 + x$
- $12 \div 4 = x$
- $12 - x = 4$
- $4 \cdot x = 12$
- $12 = 4 \cdot x$
- $12 - 4 = x$
- $x = 12 - 4$
- $x + x + x + x = 12$

Student Response

Left diagram:

- $4 + x = 12$
- $12 = 4 + x$
- $12 - x = 4$
- $12 - 4 = x$

- $x = 12 - 4$

Right diagram:

- $12 \div 4 = x$
- $4 \cdot x = 12$
- $12 = 4 \cdot x$
- $x + x + x + x = 12$

Activity Synthesis

Focus the discussion on the reason that more than one equation can describe each tape diagram; namely, the same relationship can be expressed in more than one way. These ideas should be familiar to students from prior work. Ideas that may be noted:

- A multiplicative relationship can be expressed using division.
- An additive relationship can be expressed using subtraction.
- It does not matter how expressions are arranged around an equal sign. For example, $4 + x = 12$ and $12 = 4 + x$ mean the same thing.
- Repeated addition can be represented with multiplication. For example, $4x$ is another way to express $x + x + x + x$.

Students are likely to express these ideas using informal language, and that is okay. Encourage them to revise their statements using more precise language, but there is no reason to insist they use particular terms.

Some guiding questions:

- "How can you tell if a diagram represents addition or multiplication?"
- "Once you were sure about one equation, how did you find others that matched the same diagram?"
- Regarding any two equations that represent the same diagram: "What is the same about the equations? What is different?"

Access for English Language Learners

Writing, Representing, Conversing: MLR2 Collect and Display. While pairs are working, circulate and listen to student talk about the similarities and differences between the tape diagrams and the equations. Ask students to explain how these equations match the parts of the tape diagram. Write down common or important phrases you hear students say about each representation onto a visual display of both the tape diagrams and the equations. This will help the students use mathematical language during their paired and whole-group discussions.

Design Principle(s): Support sense-making; Cultivate conversation

1.3 Draw Diagrams for Equations

15 minutes

In this activity, students draw tape diagrams to match given equations. Then, they reason about the unknown value that makes the equation true, a process also known as solving the equation. Students should not be shown strategies to solve but rather should reason with equations or diagrams in ways that make sense to them. As they work, monitor for students who use the diagrams to find unknown quantities and for those who use the equations.

Building Towards

- 6.EE.B.5
- 6.EE.B.7

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect

Launch

Give students 5 minutes quiet work time followed by a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. For each equation, provide students with a blank template of a tape diagram for students to complete and find the unknown quantities.

Supports accessibility for: Visual-spatial processing; Organization

Anticipated Misconceptions

Students might draw a box with 3 for the equation $18 = 3 \cdot y$. Ask students about the meaning of multiplication and specifically what $3 \cdot y$ means. Ask how they could represent 3 equal groups with unknown size y .

Students might think they need to show an unknown number (y) of equal groups of 3. While this is possible, showing 3 equal groups with unknown size y is simpler to represent.

Student Task Statement

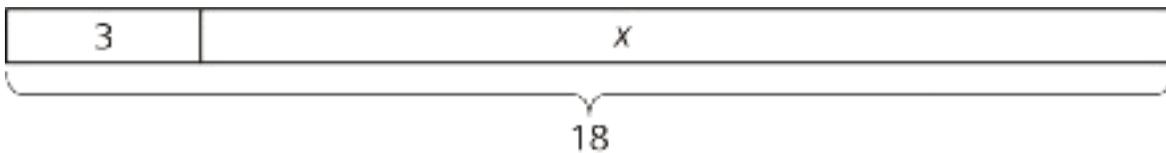
For each equation, draw a diagram and find the value of the unknown that makes the equation true.

1. $18 = 3 + x$

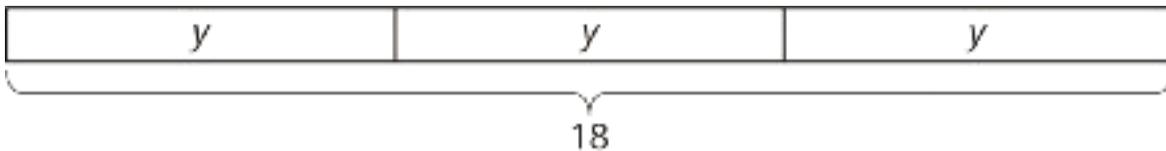
2. $18 = 3 \cdot y$

Student Response

1. For $18 = 3 + x$, the value is 15.



2. For $18 = 3 \cdot y$, the value is 6.



Are You Ready for More?

You are walking down a road, seeking treasure. The road branches off into three paths. A guard stands in each path. You know that only one of the guards is telling the truth, and the other two are lying. Here is what they say:

- Guard 1: The treasure lies down this path.
- Guard 2: No treasure lies down this path; seek elsewhere.
- Guard 3: The first guard is lying.

Which path leads to the treasure?

Student Response

Path 2 leads to the treasure.

Suppose Guard 1 is telling the truth. Then it would be true that Path 1 leads to the treasure. Then Guard 2's statement must be true as well. But only one of the guards is telling the truth. This means that Guard 1 must be lying.

Since Guard 1 is lying, Guard 3 is telling the truth about Guard 1 lying. That means that Guard 3 has the one true statement. Guard 2, then, is lying about his path being the wrong path, so the treasure lies down Path 2.

Activity Synthesis

Invite students to share their strategies for finding the values of x and y . Include at least one student who reasoned with the diagram and one who reasoned with the equation. Help students connect different methods by thinking about the relationships between the three quantities in each problem and how both the equations and the diagrams represent them.

Access for English Language Learners

Speaking, Listening, Representing: MLR7 Compare and Connect. Use this routine when students present their equations and tape diagrams. Draw students' attention to how the mathematical operations (addition, subtraction, multiplication, division) are represented in each relationship. For example, ask, "Where do you see multiplication in the diagram?" This will strengthen students' mathematical language use and reasoning about tape diagram representations of the equations.

Design Principle(s): Support sense-making; Maximize meta-awareness

Lesson Synthesis

To ensure that students understand the use and usefulness of tape diagrams in representing equations and finding unknown values, consider asking some of the following questions:

- "Why are tape diagrams useful to visualize a relationship?" (Answers vary. Sample response: You can see the way quantities are related.)
- "Where in the tape diagram do we see the equal sign that is in the equation it represents?" (The fact that the sum of the parts has the same value as the whole; the numbers and letters in the boxes add up to the total shown for the whole rectangle.)
- "Why can a diagram be represented by more than one equation?" (Because more than one operation can be used; for example, the same diagram can be represented by an addition or a subtraction equation. Because when two expressions are equal, it doesn't matter how they are arranged around the equal sign.)
- "Describe some ways to represent the relationship $23 + x = 132$ " (Tape diagram with two unequal parts, other equivalent equations like $x = 132 - 23$).

- “Describe some ways to represent the relationship $5x = 230$ ” (Tape diagram with 5 equal parts, other equivalent equations like $x = 230 \div 5$).

1.4 Finish the Diagrams

Cool Down: 5 minutes

Addressing

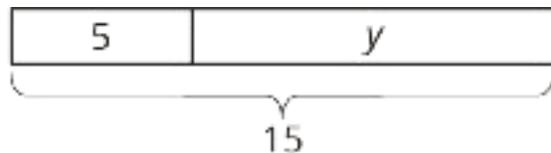
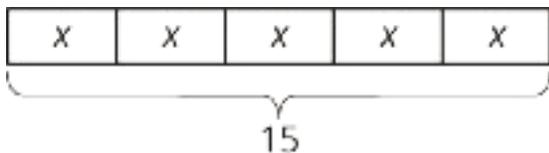
- 6.EE.B.6

Student Task Statement

Finish the first diagram so that it represents $5 \cdot x = 15$, and the second diagram so that it represents $5 + y = 15$.



Student Response



Student Lesson Summary

Tape diagrams can help us understand relationships between quantities and how operations describe those relationships.

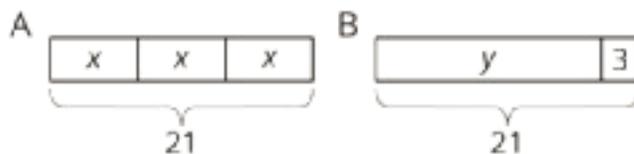


Diagram A has 3 parts that add to 21. Each part is labeled with the same letter, so we know the three parts are equal. Here are some equations that all represent diagram A:

$$x + x + x = 21$$

$$3 \cdot x = 21$$

$$x = 21 \div 3$$

$$x = \frac{1}{3} \cdot 21$$

Notice that the number 3 is not seen in the diagram; the 3 comes from counting 3 boxes representing 3 equal parts in 21.

We can use the diagram or any of the equations to reason that the value of x is 7.

Diagram B has 2 parts that add to 21. Here are some equations that all represent diagram B:


$$y + 3 = 21$$

$$y = 21 - 3$$

$$3 = 21 - y$$

We can use the diagram or any of the equations to reason that the value of y is 18.

Lesson 1 Practice Problems

Problem 1

Statement

Here is an equation: $x + 4 = 17$

- Draw a tape diagram to represent the equation.
- Which part of the diagram shows the quantity x ? What about 4? What about 17?
- How does the diagram show that $x + 4$ has the same value as 17?

Solution

- A tape diagram showing one part labeled x and another labeled 4 and a total of 17.
- The rectangle labeled x represents the quantity x , and the rectangle labeled 4 represents the quantity 4. The big rectangle (the combination of the two smaller ones) represents 17.
- The large rectangle is labeled 17, but it is also obtained by joining the two smaller rectangles labeled x and 4.

Problem 2

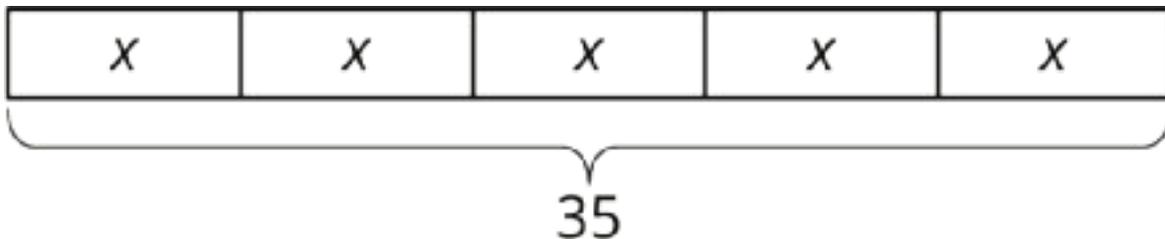
Statement

Diego is trying to find the value of x in $5 \cdot x = 35$. He draws this diagram but is not certain how to proceed.



- Complete the tape diagram so it represents the equation $5 \cdot x = 35$.
- Find the value of x .

Solution



-
- $x = 7$

Problem 3

Statement

Match each equation to one of the two tape diagrams.

a. $x + 3 = 9$

b. $3 \cdot x = 9$

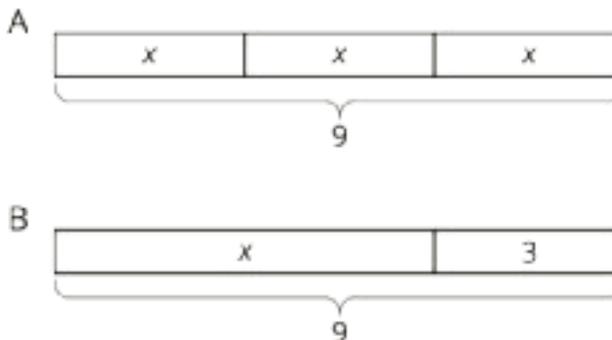
c. $9 = 3 \cdot x$

d. $3 + x = 9$

e. $x = 9 - 3$

f. $x = 9 \div 3$

g. $x + x + x = 9$



Solution

a. B

b. A

c. A

d. B

e. B

f. A

g. A

Problem 4

Statement

For each equation, draw a tape diagram and find the unknown value.

a. $x + 9 = 16$

b. $4 \cdot x = 28$

Solution

a. A tape diagram showing one part labeled 9 and another labeled x and a total of 16. The solution is 7.

b. A tape diagram showing 4 groups labeled x and a total of 28. The solution is 7.

Problem 5

Statement

A shopper paid \$2.52 for 4.5 pounds of potatoes, \$7.75 for 2.5 pounds of broccoli, and \$2.45 for 2.5 pounds of pears. What is the unit price of each item she bought? Show your reasoning.

Solution

Potatoes cost \$0.56 per pound, broccoli costs \$3.10 per pound, and pears costs \$0.98 per pound. Reasoning varies. Sample reasoning:

- $2.52 \div 4.5 = 252 \div 450$, which equals 0.56.
- $7.75 \div 2.5 = 775 \div 250$, which equals 3.1 or 3.10.
- $2.45 \div 2.5 = 245 \div 250$, which equals 0.98.

(From Unit 5, Lesson 13.)

Problem 6

Statement

A sports drink bottle contains 16.9 fluid ounces. Andre drank 80% of the bottle. How many fluid ounces did Andre drink? Show your reasoning.

Solution

13.52 fluid ounces ($0.8 \cdot 16.9 = 13.52$)

(From Unit 3, Lesson 14.)

Problem 7

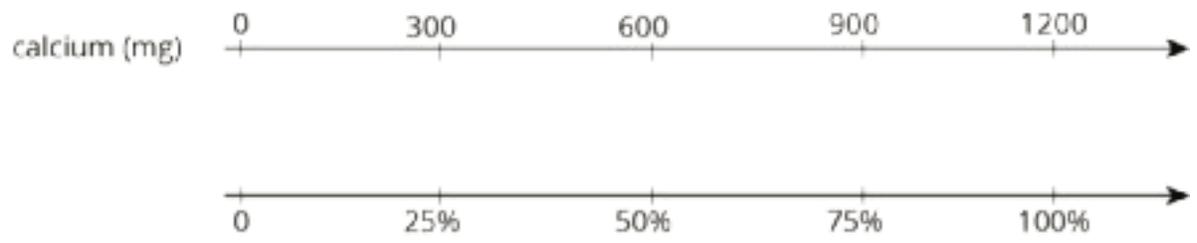
Statement

The daily recommended allowance of calcium for a sixth grader is 1,200 mg. One cup of milk has 25% of the recommended daily allowance of calcium. How many milligrams of calcium are in a cup of milk? If you get stuck, consider using the double number line.



Solution

300 mg. Sample reasoning using double number line:



(From Unit 3, Lesson 11.)

Lesson 2: Truth and Equations

Goals

- Comprehend the word “variable” to refer to a letter standing in for a number and recognize that a coefficient next to a variable indicates multiplication (in spoken and written language).
- Generate values that make an equation true or false and justify (orally and in writing) whether they are “solutions” to the equation.
- Use substitution to determine whether a given number makes an equation true.

Learning Targets

- I can match equations to real life situations they could represent.
- I can replace a variable in an equation with a number that makes the equation true, and know that this number is called a solution to the equation.

Lesson Narrative

Students begin the lesson by digging into what it means for an equation to be true or not true. They expand previously-held understandings of equations by thinking about the assumption that equations are always true. Students learn that a letter standing in for a number is called a variable. Students learn that, for an equation with a variable, a value of the variable that makes the equation true is called a solution of the equation. They find solutions to equations by using tape diagrams or reasoning about the meaning of "solution" once an equation is written.

This lesson is where "next to" notation is introduced (for example, $10m$ means $10 \cdot m$).

Alignments

Addressing

- 6.EE.B.5: Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Building Towards

- 6.EE.B.6: Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

Instructional Routines

- MLR2: Collect and Display
- MLR7: Compare and Connect
- Think Pair Share

Student Learning Goals

Let's use equations to represent stories and see what it means to solve equations.

2.1 Three Letters

Warm Up: 10 minutes

In this warm-up, students consider what it means for an equation to be true or false. They also practice substituting values for a letter and evaluating expressions with addition and multiplication. The term **variable** is introduced.

Addressing

- 6.EE.B.5

Launch

Allow students 2 minutes of quiet work time on the first part of the first question and ask them to pause their work. Ask a student to explain how they decided whether the equation was true or false given the values of a , b , and c . As the student explains, demonstrate this process by writing:

$$a + b = c$$

$$3 + 4 = 5$$

Since the sum of 3 and 4 is not 5, the equation is false for these values. Explain that a letter used to stand in for a number is called a **variable**. Throughout this unit, students will have many chances to understand and use this term.

Give students 2 minutes to complete the rest of the task.

Student Task Statement

1. The equation $a + b = c$ could be true or false.
 - a. If a is 3, b is 4, and c is 5, is the equation true or false?
 - b. Find new values of a , b , and c that make the equation true.
 - c. Find new values of a , b , and c that make the equation false.
2. The equation $x \cdot y = z$ could be true or false.
 - a. If x is 3, y is 4, and z is 12, is the equation true or false?
 - b. Find new values of x , y , and z that make the equation true.
 - c. Find new values of x , y , and z that make the equation false.

Student Response

Answers vary. Sample responses:

1. a. False
b. a is 1, b is 2, c is 3
c. a is 4, b is 5, c is 10
2. a. True
b. x is 3, y is 5, z is 15
c. x is 1, y is 2, z is 3

Activity Synthesis

The discussion should focus on the idea that an equation can be either true or false, and that the truth of an equation with variables depends on the values of the variables. This is an important idea to highlight, since throughout their work in previous grades, the assumption that equations were true was likely unstated. Additionally, the equals sign may have been used previously to signal that a calculation should be done rather than communicate that two expressions were equal to each other. Share that we can write $2 + 3 = 7$, which students and teachers likely would have previously said was “wrong” or “a mistake,” but can now be understood as a mathematical statement that is not true.

Invite students to share their values for true and false equations. If students struggle with expressing their ideas about equations, some guiding questions might help:

- “What does it mean when an equation contains a letter?” (The letter is called a variable; it is standing in for a number.)
- “What makes an equation true? What makes an equation false?” (If the expressions on each side have the same value, the equation is true. If the expressions on each side have different values, the equation is false.)
- “How can we determine whether an equation is true or false?” (Evaluate both sides and check whether the values on each side of the equation are equal. If they are, then the equation is true. If they are not, the equation is false.)

2.2 Storytime

15 minutes

In this activity, students see how an equation can represent a situation with an unknown amount. Students are presented with three stories. Each story involves the same three quantities: 5, 20, and an unknown amount x . Students think about the actions (running a number of miles, splitting up cups of cat food) and relationships (five times as many clubs) in the situations and consider the operations needed to describe them. They also make sense of what the unknown quantity represents in each story and how to show its relationship to the other two numbers and the quantities they represent. Monitor for one student for each situation who chooses a correct equation and has a reasonable way to explain their reasoning, either verbally or by using a diagram.

Building Towards

- 6.EE.B.6

Instructional Routines

- MLR7: Compare and Connect

Launch

It is likely that “next to” notation is new for students. Explain that $20x$ means the same thing as $20 \cdot x$, and we will frequently use this notation from now on. Explain that in the term $20x$, 20 is called the **coefficient**. Use this term throughout the lesson when the need naturally arises to name it.

Explain that for each situation, the task is to find *one* equation that represents it. Some of the equations will go unused.

Give students 5 minutes of quiet work time to answer the questions, followed by a whole-class discussion.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Activate or supply background knowledge. Invite students to create tape diagrams to represent each situation, before selecting the equations that match.

Supports accessibility for: Visual-spatial processing; Conceptual processing

Anticipated Misconceptions

Students who focus on key words might be misled in each situation. For the first situation, students might see the word “total” and decide they need to add 5 and 20. In the second situation, the words “five times as many” might prompt them to multiply 5 by 20. The third story poses some additional challenges: students see the word “divided” but there is no equation with division. Additionally, students might think that division always means divide the larger number by the smaller. Here are some ways to help students make sense of the situations and how equations can represent them:

- Suggest that students act out the situation or draw a picture. Focus on what quantity in the story each number or variable represents, and on the relationships among them.
- Use tape diagrams to represent quantities and think about where a situation describes a total and where it describes parts of the total.
- Ask students about the relationships between operations. For the third situation, ask what operation is related to dividing and might help describe the situation.

Student Task Statement

Here are three situations and six equations. Which equation best represents each situation? If you get stuck, consider drawing a diagram.

$$x + 5 = 20$$

$$x = 20 + 5$$

$$5x = 20$$

$$x + 20 = 5$$

$$5 \cdot 20 = x$$

$$20x = 5$$

1. After Elena ran 5 miles on Friday, she had run a total of 20 miles for the week. She ran x miles before Friday.
2. Andre's school has 20 clubs, which is five times as many as his cousin's school. His cousin's school has x clubs.
3. Jada volunteers at the animal shelter. She divided 5 cups of cat food equally to feed 20 cats. Each cat received x cups of food.

Student Response

1. $x + 5 = 20$
2. $5x = 20$
3. $20x = 5$

Activity Synthesis

For each situation, ask a student to share which equation they selected and the reason they chose it. If any students drew a diagram to help them reason about the situation, ask them to present their diagram and draw connections between the situation, equation, and diagram. Consider these questions for discussion:

- "All of the equations and situations had the same numbers. Describe how you decided which equations represented which situations."
- "Did any of the words in the stories confuse or mislead you? How did you move past the confusion?"
- "Did you rule out any equations right from the start? Which ones? Why?" (Students might notice that $x + 20 = 5$ has no solutions since they have not yet learned about negative numbers. They might also incorrectly claim that $20x = 5$ is impossible if they are focused only on whole numbers.)

Access for English Language Learners

Speaking, Representing, Writing: MLR7 Compare and Connect. When students share the equation they selected and the reason they chose it, ask the class questions to draw that attention to the connections between the situation, equation and diagram (e.g., “I see addition in the equation, where do you see addition in the situation?”). These exchanges help strengthen students’ mathematical language use and reasoning with multiple representations.

Design Principle(s): Maximize meta-awareness

2.3 Using Structure to Find Solutions

15 minutes

Having described situations with equations, students now solve equations by noticing and thinking about their structure and figuring out which value from a given set makes the equation true.

Addressing

- 6.EE.B.5

Instructional Routines

- MLR2: Collect and Display
- Think Pair Share

Launch

Remind students that a letter used to represent an unknown value is called a variable. An equation with a variable can be true or false. A value that can be used in place of the variable that makes the equation true is called a **solution** to the equation. In this task, students will look for solutions to equations. That means they will look for a value that can be used in place of a variable in an equation that makes the equation true.

Arrange students in groups of 2. Allow students 10 minutes to work quietly and share their responses with a partner, followed by a whole-class discussion.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Include the following terms and maintain the display for reference throughout the unit: variable, solution. Invite students to suggest language or diagrams to include on the display that will support their understanding of these terms.

Supports accessibility for: Memory; Language

Anticipated Misconceptions

Instead of solving, students might follow the operation symbol and combine the numbers in that way (for example, adding 12.6 and 4.1 to get 16.7 for the equation $12.6 = b + 4.1$). Encourage students to express the relationships of the equation in words and to draw diagrams that describe those statement. $12.6 = b + 4.1$ can be stated as "When a number is added to 4.1, the sum is 12.6." The tape diagram then shows the parts 4.1 and an unknown quantity b , and a total of 12.6.

Student Task Statement

Here are some equations that contain a **variable** and a list of values. Think about what each equation means and find a **solution** in the list of values. If you get stuck, consider drawing a diagram. Be prepared to explain why your solution is correct.

1. $1000 - a = 400$

2. $12.6 = b + 4.1$

3. $8c = 8$

4. $\frac{2}{3} \cdot d = \frac{10}{9}$

5. $10e = 1$

6. $10 = 0.5f$

7. $0.99 = 1 - g$

8. $h + \frac{3}{7} = 1$

List: $\frac{1}{8}$ $\frac{3}{7}$ $\frac{4}{7}$ $\frac{3}{5}$ $\frac{5}{3}$ $\frac{7}{3}$ 0.01 0.1 0.5
 1 2 8.5 9.5 16.7 20 400 600 1400

Student Response

1. 600

2. 8.5

3. 1

4. $\frac{5}{3}$

5. 0.1

6. 20

7. 0.01

8. $\frac{4}{7}$

Are You Ready for More?

One solution to the equation $a + b + c = 10$ is $a = 2, b = 5, c = 3$.

How many different whole-number solutions are there to the equation $a + b + c = 10$? Explain or show your reasoning.

Student Response

If $a, b,$ and c are *positive*, there are 36 solutions. If $a = 1$, the possibilities are that $b = 1$ and $c = 8$, $b = 2$ and $c = 7$, and so on, giving 8 solutions. If $a = 2$, then b and c could respectively be 1 and 7, 2 and 6, 3 and 5, etc. This gives 7 solutions for $a = 2$. If $a = 9$ and all numbers are positive, there are no possible numbers for both b and c . The total number of solutions (for a value of 1 through 8) is $8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$ or 36.

If $a, b,$ and c *non-negative* and includes 0, there are 66 solutions. If $a = 0$, there are 11 combinations of b and c . If $a = 1$, there are 10 combinations, and so on. The total number of solutions (for a value of 0 through 10) is $11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$, which is 66.

If $a, b,$ and c are *any* integers (including negative), then there is an unlimited number of solutions.

Activity Synthesis

The goal of the discussion is to ensure that students understand what it means to be a solution to an equation. To guide the discussion, consider the following:

- Solicit or display correct solutions.
- Choose a few equations and ask students to explain how they know a solution is correct.
- Highlight correct uses of the new terms variable, solution, and coefficient.

Access for English Language Learners

Conversing, Representing, Writing: MLR2 Collect and Display. Prepare students for the discussion by asking them to first share their responses to “Why are your solutions correct?” with a partner. Listen for, collect, and display vocabulary (e.g., coefficient, variable, true, false, solution) and include any diagrams that students use to represent each equation. Throughout the remainder of the lesson, continue to update collected student language and remind students to borrow language from the display as needed. This will help students use mathematical language during their paired and whole-group discussions.

Design Principles(s): Support sense-making; Cultivate conversation

Lesson Synthesis

The purpose of the discussion is to review appropriate use and understanding of new vocabulary and the concepts they represent. Consider asking some of the following questions:

- “What does it mean for an equation to be true? False? What does the equal sign have to do with whether the equation is true or false?” (A true equation has expressions with equal value on each side of the equal sign.)
- “Is an equation with a variable always true?” (It is only true for values of the variable that make the sides equal.)
- “What do we call a number written next to a variable? What does it mean?” (It is called the coefficient. Multiply the number by the value of the variable.)
- “Why might it be helpful to eliminate symbols that show multiplication?” (Answers vary. Sample response: It makes equations easier to read.)
- “What do we call a number that can be used in place of the variable that makes the equation true?” (It is called a solution of the equation.)

2.4 How Do You Know a Solution is a Solution?

Cool Down: 5 minutes

Addressing

- 6.EE.B.5

Student Task Statement

Explain how you know that 88 is a solution to the equation $\frac{1}{8}x = 11$ by completing the sentences:

The word “solution” means . . .

88 is a solution to $\frac{1}{8}x = 11$ because . . .

Student Response

The word “solution” means a value that makes the equation true.

88 is a solution to $\frac{1}{8}x = 11$, because if x is 88, the equation is $\frac{1}{8} \cdot 88 = 11$, which is true.

Student Lesson Summary

An equation can be true or false. An example of a true equation is $7 + 1 = 4 \cdot 2$. An example of a false equation is $7 + 1 = 9$.

An equation can have a letter in it, for example, $u + 1 = 8$. This equation is false if u is 3, because $3 + 1$ does not equal 8. This equation is true if u is 7, because $7 + 1 = 8$.

A letter in an equation is called a **variable**. In $u + 1 = 8$, the variable is u . A number that can be used in place of the variable that makes the equation true is called a **solution** to the equation. In $u + 1 = 8$, the solution is 7.

When a number is written next to a variable, the number and the variable are being multiplied. For example, $7x = 21$ means the same thing as $7 \cdot x = 21$. A number written next to a variable is called a **coefficient**. If no coefficient is written, the coefficient is 1. For example, in the equation $p + 3 = 5$, the coefficient of p is 1.

Glossary

- coefficient
- solution to an equation
- variable

Lesson 2 Practice Problems

Problem 1

Statement

Select **all** the true equations.

A. $5 + 0 = 0$

B. $15 \cdot 0 = 0$

C. $1.4 + 2.7 = 4.1$

D. $\frac{2}{3} \cdot \frac{5}{9} = \frac{7}{12}$

E. $4\frac{2}{3} = 5 - \frac{1}{3}$

Solution

["B", "C", "E"]

Problem 2

Statement

Mai's water bottle had 24 ounces in it. After she drank x ounces of water, there were 10 ounces left. Select **all** the equations that represent this situation.

A. $24 \div 10 = x$

B. $24 + 10 = x$

C. $24 - 10 = x$

D. $x + 10 = 24$

E. $10x = 24$

Solution

["C", "D"]

Problem 3

Statement

Priya has 5 pencils, each x inches in length. When she lines up the pencils end to end, they measure 34.5 inches. Select **all** the equations that represent this situation.

- A. $5 + x = 34.5$
- B. $5x = 34.5$
- C. $34.5 \div 5 = x$
- D. $34.5 - 5 = x$
- E. $x = (34.5) \cdot 5$

Solution

["B", "C"]

Problem 4

Statement

Match each equation with a solution from the list of values.

- | | |
|-----------------------------|-------------------|
| A. $2a = 4.6$ | 1. $\frac{8}{5}$ |
| B. $b + 2 = 4.6$ | 2. $1\frac{5}{8}$ |
| C. $c \div 2 = 4.6$ | 3. 2.3 |
| D. $d - 2 = 4.6$ | 4. 2.6 |
| E. $e + \frac{3}{8} = 2$ | 5. 6.6 |
| F. $\frac{1}{8}f = 3$ | 6. 9.2 |
| G. $g \div \frac{8}{5} = 1$ | 7. 24 |

Solution

- A: 3
- B: 4
- C: 6
- D: 5
- E: 2
- F: 7
- G: 1

Problem 5

Statement

The daily recommended allowance of vitamin C for a sixth grader is 45 mg. 1 orange has about 75% of the recommended daily allowance of vitamin C. How many milligrams are in 1 orange? If you get stuck, consider using the double number line.



Solution

33.75 mg. Sample reasoning using double number line diagram:



(From Unit 3, Lesson 11.)

Problem 6

Statement

There are 90 kids in the band. 20% of the kids own their own instruments, and the rest rent them.

- How many kids own their own instruments?
- How many kids rent instruments?
- What percentage of kids rent their instruments?

Solution

- 18 kids ($90 \cdot 0.2 = 18$)
- 72 kids ($90 - 18 = 72$)
- 80% ($100 - 20 = 80$)

(From Unit 3, Lesson 12.)

Problem 7

Statement

Find each product.

a. $(0.25) \cdot (1.4)$

b. $(0.061) \cdot (0.43)$

c. $(1.017) \cdot (0.072)$

d. $(5.226) \cdot (0.037)$

Solution

a. 0.35

b. 0.02623

c. 0.073224

d. 0.193362

(From Unit 5, Lesson 8.)

Lesson 3: Staying in Balance

Goals

- Interpret hanger diagrams (orally and in writing) and write equations that represent relationships between the weights on a balanced hanger diagram.
- Use balanced hangers to explain (orally and in writing) how to find solutions to equations of the form $x + p = q$ or $px = q$.

Learning Targets

- I can compare doing the same thing to the weights on each side of a balanced hanger to solving equations by subtracting the same amount from each side or dividing each side by the same number.
- I can explain what a balanced hanger and a true equation have in common.
- I can write equations that could represent the weights on a balanced hanger.

Lesson Narrative

The goal of this lesson is for students to understand that we can generally approach $p + x = q$ by subtracting the same thing from each side and that we can generally approach $px = q$ by dividing each side by the same thing. This is accomplished by considering what can be done to a hanger to keep it balanced.

Students are solving equations in this lesson in a different way than they did in the previous lessons. They are reasoning about things one could “do” to hangers while keeping them balanced alongside an equation that represents a hanger, so they are thinking about “doing” things to each side of an equation, rather than simply thinking “what value would make this equation true.”

Alignments

Addressing

- 6.EE.B.5: Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- 6.EE.B.6: Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- 6.EE.B.7: Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers.

Building Towards

- 6.EE.B.7: Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers.

Instructional Routines

- MLR2: Collect and Display
- MLR7: Compare and Connect
- Notice and Wonder
- Think Pair Share

Student Learning Goals

Let's use balanced hangers to help us solve equations.

3.1 Hanging Around

Warm Up: 10 minutes

Students encounter and reason about a concrete situation, hangers with equal and unequal weights on each side. They then see diagrams of balanced and unbalanced hangers and think about what must be true and false about the situations. In subsequent activities, students will use the hanger diagrams to develop general strategies for solving equations.

Building Towards

- 6.EE.B.7

Instructional Routines

- Notice and Wonder

Launch

Display the photo of socks and ask students, "What do you notice? What do you wonder?"



Give students 1 minute to think about the picture. Record their responses for all to see.

Things students may notice:

- There are two pink socks and two blue socks.
- The socks are clipped to either ends of two clothes hangers. The hangers are hanging from a rod.
- The hanger holding the pink socks is level; the hanger holding the blue socks is not level.

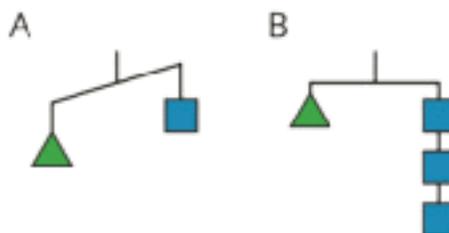
Things students may wonder:

- Why is the hanger holding the blue socks not level?
- Is something inside one of the blue socks to make it heavier than the other sock?
- What does this picture have to do with math?

Use the word “balanced” to describe the hanger on the left and “unbalanced” to describe the hanger on the right. Tell students that the hanger on the left is balanced because the two pink socks have an equal weight, and the hanger on the right is unbalanced because one blue sock is heavier than the other. Tell students that they will look at a diagram that is like the photo of socks, except with more abstract shapes, and they will reason about the weights of the shapes.

Give students 3 minutes of quiet work time followed by whole-class discussion.

Student Task Statement



For diagram A, find:

1. One thing that *must* be true
2. One thing that *could* be true or false
3. One thing that *cannot possibly* be true

For diagram B, find:

1. One thing that *must* be true
2. One thing that *could* be true or false
3. One thing that *cannot possibly* be true

Student Response

Answers vary. Sample responses:

Diagram A:

1. The triangle is heavier than the square.
2. The triangle could weigh 10 ounces and the square could weigh 6 ounces.
3. The square and the triangle weigh the same.

Diagram B:

1. One triangle weighs the same as three squares.
2. The triangle weighs three pounds and each square weighs one pound.
3. One square is heavier than the triangle.

Activity Synthesis

Ask students to share some things that must be true, could be true, and cannot possibly be true about the diagrams. Ask them to explain their reasoning. The purpose of this discussion is to understand how the hanger diagrams work. When the diagram is balanced, there is equal weight on each side. For example, since diagram B is balanced, we know that one triangle weighs the same as three squares. When the diagram is unbalanced, one side is heavier than the other. For example, since diagram A is unbalanced, we know that one triangle is heavier than one square.

3.2 Match Equations and Hangers

15 minutes

Students are presented with four hanger diagrams and are asked to match an equation to each hanger. They analyze relationships and find correspondences between the two representations.

Then students use the diagrams and equations to find the unknown value in each diagram. This value is a solution of the equation.

Notice that the hangers (and equations) for x and z are identical except that the variable appears on alternate sides of the equal sign. It may be obvious to some students that $3z = 6$ and $6 = 3x$ mean the same thing mathematically, but we know that in grade 6 many students do not have a well-developed understanding of what the equal sign means. So it is worth spending a little time to make explicit that these equations each have the solution 2. When we are writing an equation, it means the same thing if the two sides are swapped. Generally, $a = b$ means the same thing as $b = a$ where a and b represent any mathematical expression.

Building Towards

- 6.EE.B.7

Instructional Routines

- MLR2: Collect and Display
- Think Pair Share

Launch

Display the diagrams and explain that each square labeled with a 1 weighs 1 unit, and each shape labeled with a letter has an unknown weight.

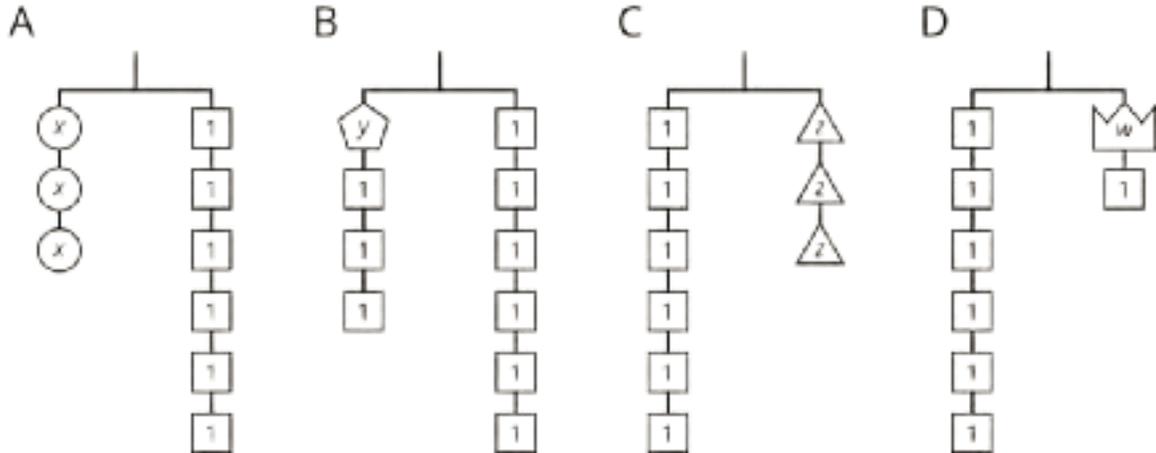
Arrange students in groups of 2. Give 5–10 minutes of quiet work time and time to share their responses with a partner, followed by a whole-class discussion.

Access for English Language Learners

Conversing, Representing, Writing: MLR2 Collect and Display. While pairs are working, circulate and listen to student talk about the hangers and the equations. Write down common phrases and terms you hear students say about each representation (e.g., level, equal, the same as, balanced, tilted, more than, less than, unbalanced, grouping). Write the students' words with the matched representation on a visual display and refer to it during the synthesis. This will help students connect everyday words and mathematical language for use during their paired and whole-group discussions.

Design Principle(s): Support sense-making

Student Task Statement



1. Match each hanger to an equation. Complete the equation by writing x , y , z , or w in the empty box.

$$\square + 3 = 6$$

$$3 \cdot \square = 6$$

$$6 = \square + 1$$

$$6 = 3 \cdot \square$$

2. Find a solution to each equation. Use the hangers to explain what each solution means.

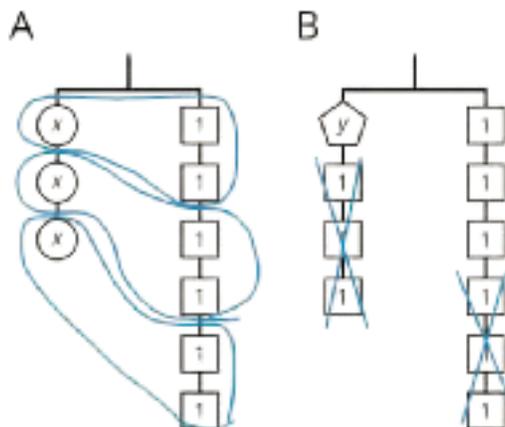
Student Response

1. A: $3 \cdot x = 6$, B: $y + 3 = 6$, C: $6 = 3 \cdot z$, D: $6 = w + 1$

2. A: x is 2, each circle weighs the same as 2 squares. B: y is 3, the pentagon weighs as much as 3 squares. C: z is 2, the z shape weighs the same as 2 squares. D: w is 5, the w shape weighs as much as 5 squares.

Activity Synthesis

Demonstrate two specific things for these specific examples: grouping the shapes on each side, and removing shapes from each side. In each case, the solution represents the weight of one shape. When you are done demonstrating, your diagrams might look like this:



Consider asking some of the following questions:

- “Explain how you know from looking at a hanger that it can be represented by an equation involving addition.”
- “Explain how you know from looking at a hanger that it can be represented by an equation involving multiplication.”
- “What are some moves that ensure that a balanced hanger stays balanced?”

3.3 Connecting Diagrams to Equations and Solutions

15 minutes

This activity continues the work of using a balanced hanger to develop strategies for solving equations. Students are presented with balanced hangers and are asked to write equations that represent them. They are then asked to explain how to use the diagrams, and then the equations, to reason about a solution. Students notice the structure of equations and diagrams and find correspondences between them and between solution strategies.

Building Towards

- 6.EE.B.7

Instructional Routines

- MLR7: Compare and Connect
- Think Pair Share

Launch

Draw students' attention to the diagrams in the task statement. Ensure they notice that the hangers are balanced and that each object is labeled with its weight. Some weights are labeled with numbers but some are unknown, so they are labeled with a variable.

Keep students in the same groups. Give 5–10 minutes of quiet work time and time to share their responses with a partner, followed by a whole-class discussion.

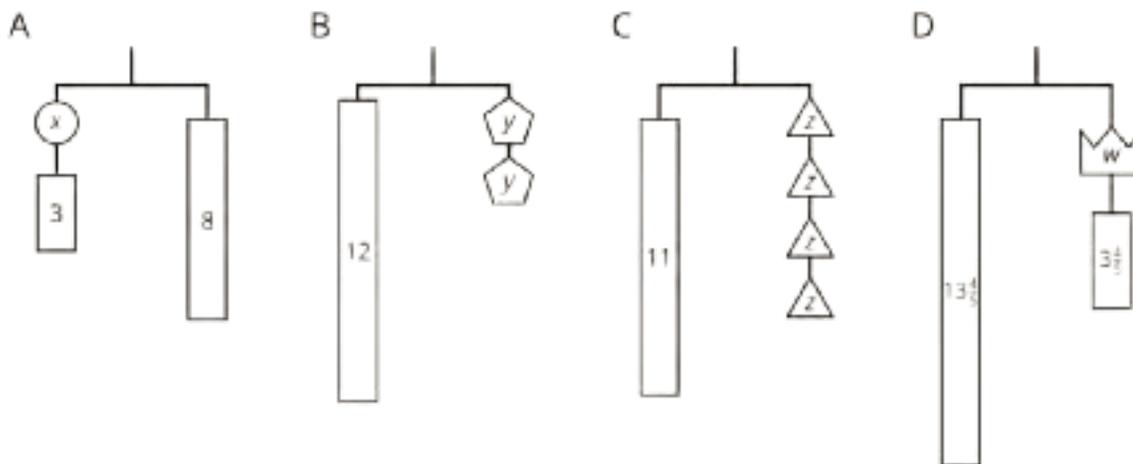
Access for Students with Disabilities

Representation: Internalize Comprehension. Demonstrate and encourage students to use color coding and annotations to highlight connections between representations. See activity synthesis from the previous activity for an example.

Supports accessibility for: Visual-spatial processing

Student Task Statement

Here are some balanced hangers. Each piece is labeled with its weight.



For each diagram:

1. Write an equation.
2. Explain how to reason with the *diagram* to find the weight of a piece with a letter.
3. Explain how to reason with the *equation* to find the weight of a piece with a letter.

Student Response

1. A: $x + 3 = 8$, B: $12 = 2y$, C: $11 = 4z$, D: $13\frac{4}{5} = w + 3\frac{4}{5}$
2.
 - a. $x = 5$. Together x and 3 weigh 8, so x weighs 5.
 - b. $y = 6$. 12 is twice the weight of y , so y weighs half of 12.
 - c. $z = \frac{11}{4}$. 11 is 4 times the weight of z , so z weighs a quarter of 11.
 - d. $w = 10$. Together w and $3\frac{4}{5}$ weight $13\frac{4}{5}$, so w weighs 10.
3.
 - a. Subtracting 3 from each side of the equation leaves $x = 5$.
 - b. The right side of the equation is equal to $2y$. After dividing each side of the equation by 2, the equation is $6 = y$.
 - c. Dividing each side of the equation by 4 leaves $\frac{11}{4} = z$.
 - d. Subtracting $3\frac{4}{5}$ from each side of the equation leaves $10 = w$.

Are You Ready for More?

When you have the time, visit the site <https://solve.edc.org/Mobiles.html> to solve some trickier puzzles that use hanger diagrams like the ones in this lesson. You can even build new ones. (If you want to do this during class, check with your teacher first!)

Activity Synthesis

Invite students to demonstrate, side by side, how they reasoned with both the diagram and the equation. For example, diagram A can be shown next to the equation $x + 3 = 8$. Cross out a piece representing 3 from each side, and write $x + 3 - 3 = 8 - 3$, followed by $x = 5$. Repeat for all four diagrams. For the diagrams represented by a multiplication equation, show dividing each side into equal-sized groups.

We want students to walk away with two things:

- An instant recognition of the structure of equations of the form $x + p = q$ and $px = q$ where p and q are specific, given numbers.
- A visual representation in their mind that can be used to support understanding of why for equations of the form $x + p = q$, you can subtract p from both sides, and for equations of the form $px = q$, you can divide both sides by p to find the solution.

Access for English Language Learners

Representing, Listening, Speaking: MLR7 Compare and Connect. Use this routine to help students consider audience when preparing to share their work. Ask students to prepare a visual display that shows their reasoning about the diagram and equation. Some students may wish to add notes or details to their displays to help communicate their thinking. Invite students to share their displays with a partner, and discuss “What is the same and what is different?” Listen for the ways students describe the correspondences between the structures of the equations and diagrams, and between their solution strategies. This will help students use mathematical language to connect equations with diagrams during the synthesis.

Design Principle(s): Optimize output; Maximize meta-awareness

Lesson Synthesis

Display the two equations $5x = 8$ and $5 + x = 8$. Ask students to draw a hanger to match each equation. Then have them work with a partner to solve the equation alongside finding the unknown value on the hanger. Ask students to compare the two strategies and discuss how they are alike and how they are different.

3.4 Weight of the Circle

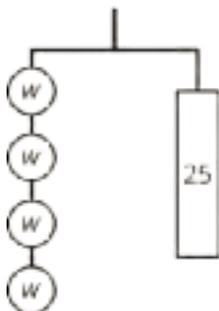
Cool Down: 5 minutes

Addressing

- 6.EE.B.5
- 6.EE.B.6
- 6.EE.B.7

Student Task Statement

Here is a balanced hanger.



1. Write an equation representing this hanger.
2. Find the weight of one circle. Show or explain how you found it.

Student Response

1. $4w = 25$
2. $\frac{25}{4}$ or $6\frac{1}{4}$

Student Lesson Summary

A hanger stays balanced when the weights on both sides are equal. We can change the weights and the hanger will stay balanced as long as both sides are changed in the same way. For example, adding 2 pounds to each side of a balanced hanger will keep it balanced. Removing half of the weight from each side will also keep it balanced.

An equation can be compared to a balanced hanger. We can change the equation, but for a true equation to remain true, the same thing must be done to both sides of the equal sign. If we add or subtract the same number on each side, or multiply or divide each side by the same number, the new equation will still be true.

This way of thinking can help us find solutions to equations. Instead of checking different values, we can think about subtracting the same amount from each side or dividing each side by the same number.

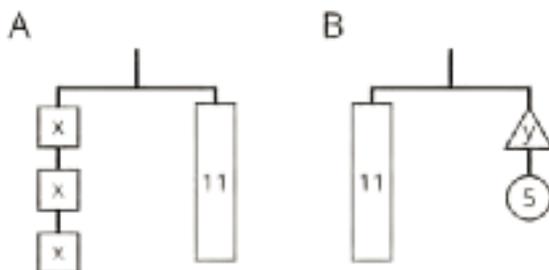


Diagram A can be represented by the equation $3x = 11$.

If we break the 11 into 3 equal parts, each part will have the same weight as a block with an x .

Splitting each side of the hanger into 3 equal parts is the same as dividing each side of the equation by 3.

- $3x$ divided by 3 is x .
- 11 divided by 3 is $\frac{11}{3}$.
- If $3x = 11$ is true, then $x = \frac{11}{3}$ is true.
- The solution to $3x = 11$ is $\frac{11}{3}$.

Diagram B can be represented with the equation $11 = y + 5$.

If we remove a weight of 5 from each side of the hanger, it will stay in balance.

Removing 5 from each side of the hanger is the same as subtracting 5 from each side of the equation.

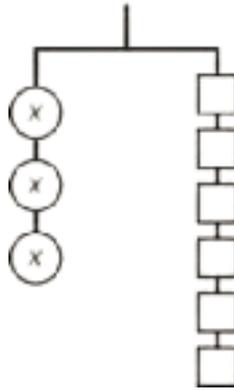
- $11 - 5$ is 6.
- $y + 5 - 5$ is y .
- If $11 = y + 5$ is true, then $6 = y$ is true.
- The solution to $11 = y + 5$ is 6.

Lesson 3 Practice Problems

Problem 1

Statement

Select all the equations that represent the hanger.



- A. $x + x + x = 1 + 1 + 1 + 1 + 1 + 1$
- B. $x \cdot x \cdot x = 6$
- C. $3x = 6$
- D. $x + 3 = 6$
- E. $x \cdot x \cdot x = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$

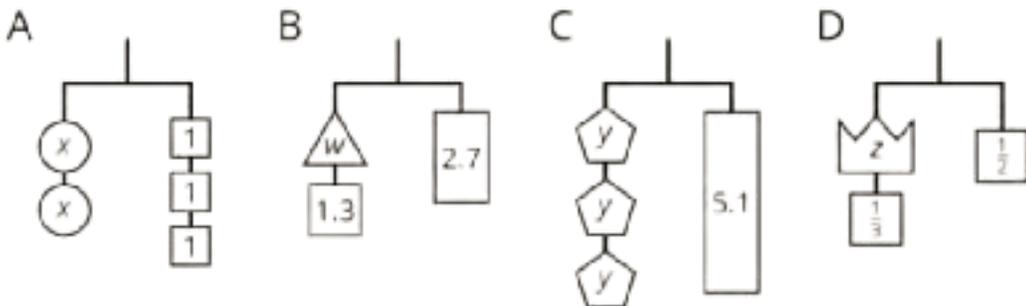
Solution

["A", "C"]

Problem 2

Statement

Write an equation to represent each hanger.



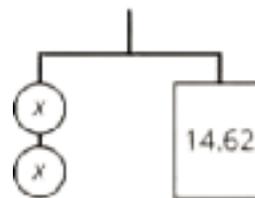
Solution

- a. $2x = 3$ (or equivalent)
- b. $w + 1.3 = 2.7$ (or equivalent)
- c. $3y = 5.1$ (or equivalent)
- d. $z + \frac{1}{3} = \frac{1}{2}$ (or equivalent)

Problem 3

Statement

- a. Write an equation to represent the hanger.
- b. Explain how to reason with the hanger to find the value of x .
- c. Explain how to reason with the equation to find the value of x .



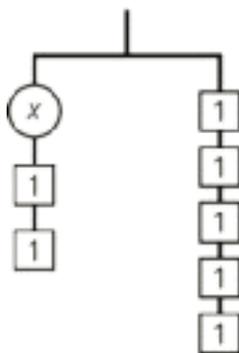
Solution

- a. $2x = 14.62$
- b. Each x can be grouped with half of the other side, so that means x is half of 14.62 or 7.31.
- c. 14.62 is twice x , so x must be 7.31, since $2 \cdot (7.31) = 14.62$.

Problem 4

Statement

Andre says that x is 7 because he can move the two 1s with the x to the other side.



Do you agree with Andre? Explain your reasoning.

Solution

Andre is not correct. Each 1 on the left balances with a 1 on the right. So taking away the two 1s on the left only leaves the hanger balanced if two 1s are removed on the right. This leaves x on the left and three 1s on the right, so $x = 3$.

Problem 5

Statement

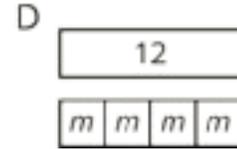
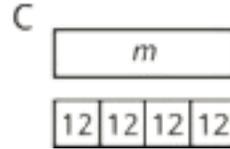
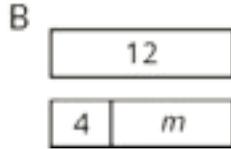
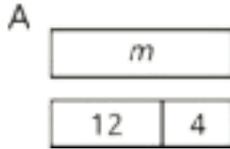
Match each equation to one of the diagrams.

a. $12 - m = 4$

b. $12 = 4 \cdot m$

c. $m - 4 = 12$

d. $\frac{m}{4} = 12$



Solution

- $12 - m = 4$ matches B
- $12 = 4 \cdot m$ matches D
- $m - 4 = 12$ matches A
- $\frac{m}{4} = 12$ matches C

(From Unit 6, Lesson 1.)

Problem 6

Statement

The area of a rectangle is 14 square units. It has side lengths x and y . Given each value for x , find y .

a. $x = 2\frac{1}{3}$

b. $x = 4\frac{1}{5}$

c. $x = \frac{7}{6}$

Solution

a. $y = 6$ ($14 \div 2\frac{1}{3} = 14 \div \frac{7}{3}$, and $14 \cdot \frac{3}{7} = 6$)

b. $y = 3\frac{1}{3}$ ($14 \div 4\frac{1}{5} = 14 \div \frac{21}{5}$, and $14 \cdot \frac{5}{21} = 3\frac{1}{3}$)

c. $y = 12$ ($14 \div \frac{7}{6} = 14 \cdot \frac{6}{7} = 12$)

(From Unit 4, Lesson 13.)

Problem 7

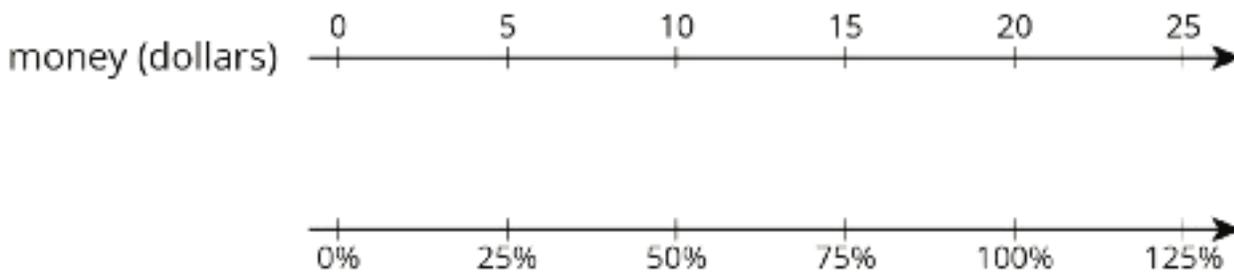
Statement

Lin needs to save up \$20 for a new game. How much money does she have if she has saved each percentage of her goal. Explain your reasoning.

- a. 25%
- b. 75%
- c. 125%

Solution

- a. \$5
- b. \$15
- c. \$25. Reasoning varies. Sample reasoning:



(From Unit 3, Lesson 11.)

Lesson 4: Practice Solving Equations and Representing Situations with Equations

Goals

- Interpret and coordinate sentences, equations, and diagrams that represent the same addition or multiplication situation.
- Solve equations of the form $x + p = q$ or $px = q$ and explain (in writing) the solution method.

Learning Targets

- I can explain why different equations can describe the same situation.
- I can solve equations that have whole numbers, fractions, and decimals.

Lesson Narrative

In this lesson, students consolidate their equation writing and solving skills. In the first activity they solve a variety of equations with different structures, and in the second they work to match equations to situations and solve them. Students may choose any strategy to solve equations, including drawing diagrams to reason about unknown quantities, looking at the structure of the equation, or doing the same thing to each side of the equation. They choose efficient tools and strategies for specific problems. This will help students develop flexibility and fluency in writing and solving equations.

Alignments

Addressing

- 6.EE.B: Reason about and solve one-variable equations and inequalities.
- 6.EE.B.5: Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- 6.EE.B.6: Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- 6.EE.B.7: Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers.
- 6.NS.B.3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Instructional Routines

- MLR2: Collect and Display

- MLR6: Three Reads
- MLR8: Discussion Supports
- Number Talk

Student Learning Goals

Let's solve equations by doing the same to each side.

4.1 Number Talk: Subtracting From Five

Warm Up: 5 minutes

The purpose of this number talk is to have students recall subtraction where regrouping needs to happen in preparation for the problems they will solve in the lesson.

Addressing

- 6.NS.B.3

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Student Task Statement

Find the value of each expression mentally.

$$5 - 2$$

$$5 - 2.1$$

$$5 - 2.17$$

$$5 - 2\frac{7}{8}$$

Student Response

- 3
- 2.9
- 2.83
- $2\frac{1}{8}$

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Did anyone have the same strategy but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to ___’s strategy?”
- “Do you agree or disagree? Why?”

Access for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, “First, I ___ because . . .” or “I noticed ___ so I . . .” Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Design Principle(s): Optimize output (for explanation)

4.2 Row Game: Solving Equations Practice

15 minutes

The purpose of this activity is for students to practice solving equations. Some students may use the “do the same to each side” strategy they developed in their work with balanced hangers. Others may use strategies like substituting values until they find a value that makes the equation true, or asking themselves questions like “2 times what is 18?” As students progress through the activity, the equations become more difficult to solve by strategies other than “do the same thing to each side.”

Addressing

- 6.EE.B

Instructional Routines

- MLR2: Collect and Display

Launch

Display an equation like $2x = 12$ or similar. Ask students to think about the balanced hangers of the last lesson and to recall how that helped us solve equations by doing the same to each side. Tell students that, after obtaining a solution via algebraic means, we end up with a variable on one side of the equal sign and a number on the other, e.g. $x = 6$. We can easily read the solution—in this case 6—from an equation with a letter on one side and a number on the other, and we often write solutions in this way. Tell students that the act of finding an equation's solution is sometimes called *solving* the equation.

Arrange students in groups of 2, and ensure that everyone understands how the row game works before students start working. Allow students 10 minutes of partner work followed by a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts. For example, after students have completed the first four rows of the table, check-in with either select groups of students or the whole class. Invite students to share the strategies they have used so far as well as any questions they have before continuing.

Supports accessibility for: Organization; Attention

Access for English Language Learners

Conversing, Representing, Writing: MLR2 Collect and Display. While pairs are working, circulate, collect and make a visual display of vocabulary, phrases and representations students use as they solve each situation. Make connections between how similar ideas might be communicated and represented in different ways. Look for and amplify phrases such as “I did the same thing to each side” or “I subtracted the same amount from both sides.” This helps students use mathematical language during paired and whole-group discussions.

Design Principle(s): Support sense-making

Student Task Statement

Solve the equations in one column. Your partner will work on the other column.

Check in with your partner after you finish each row. Your answers in each row should be the same. If your answers aren't the same, work together to find the error and correct it.

column A	column B
$18 = 2x$	$36 = 4x$
$17 = x + 9$	$13 = x + 5$
$8x = 56$	$3x = 21$
$21 = \frac{1}{4}x$	$28 = \frac{1}{3}x$
$6x = 45$	$8x = 60$
$x + 4\frac{5}{6} = 9$	$x + 3\frac{5}{6} = 8$
$\frac{5}{7}x = 55$	$\frac{3}{7}x = 33$
$\frac{1}{5} = 6x$	$\frac{1}{3} = 10x$
$2.17 + x = 5$	$6.17 + x = 9$
$\frac{20}{3} = \frac{10}{9}x$	$\frac{14}{5} = \frac{7}{15}x$
$14.88 + x = 17.05$	$3.91 + x = 6.08$
$3\frac{3}{4}x = 1\frac{1}{4}$	$\frac{7}{5}x = \frac{7}{15}$

Student Response

1. $x = 9$
2. $x = 8$
3. $x = 7$
4. $x = 84$
5. $x = 7\frac{1}{2}$
6. $x = 4\frac{1}{6}$
7. $x = 77$
8. $x = \frac{1}{30}$
9. $x = 2.83$

10. $x = 6$

11. $x = 2.17$

12. $x = \frac{1}{3}$

Activity Synthesis

Draw students' attention to $21 = \frac{1}{4}x$, and ask selected students to explain how they thought about solving this equation. Some may share strategies like "one-fourth of what number is 21?" Ideally, one student will say "divide each side by $\frac{1}{4}$ " and another will say "multiply each side by 4." From their studies in earlier units, students should understand that multiplying by 4 has the same result as dividing by $\frac{1}{4}$. Next, turn students attention to $\frac{5}{7}x = 55$ and ask them to describe the two ways to think about solving it. "Divide each side by $\frac{5}{7}$ " gets the same result as "Multiply each side by $\frac{7}{5}$."

4.3 Choosing Equations to Match Situations

15 minutes

The purpose of this activity is for students to practice matching equations to situations and then solving those equations using their new strategies. Monitor for students who draw diagrams (tape, hanger, or their own creations) that describe the relationships and those who solve the equations by doing the same to each side of one or more equations.

Addressing

- 6.EE.B.5
- 6.EE.B.6
- 6.EE.B.7

Instructional Routines

- MLR6: Three Reads

Launch

Allow students 10 minutes of quiet work time, followed by a whole-class discussion.

Access for English Language Learners

Reading: MLR6 Three Reads. Demonstrate this routine with the first situation to support reading comprehension. Use the first read to help students understand context. Ask, "What is this situation about?" (e.g., Clare and Mai each have a different number of books). After the second read, ask students "What are the quantities in the situation?" (e.g., the number of books Mai has, the number of books Clare has). After the third read, ask students to brainstorm possible strategies to connect the situation with the appropriate equation(s). Encourage students to repeat this routine themselves for each situation. This helps students connect the language in the word problem with the equation(s) while keeping the intended level of cognitive demand in the task.

Design Principle(s): Support sense-making

Anticipated Misconceptions

Students who continue to focus on key words misidentify the relationship in each situation. Encourage students to express the relationships in their own words and draw diagrams comparing the given quantities. For example, in the situation with Clare and Mai, they can draw a long rectangle representing Mai's books subdivided into two pieces. Filling in the information given in the story will help clear up the relationships; Clare's rectangle is labeled x , and she has 8 fewer books than Mai, so Mai's rectangle is labeled $x + 8$ and also 26. Alternatively, they can show that the piece labeled x must equal $26 - 8$.

Student Task Statement

Circle **all** of the equations that describe each situation. If you get stuck, consider drawing a diagram. Then find the solution for each situation.

1. Clare has 8 fewer books than Mai. If Mai has 26 books, how many books does Clare have?

- $26 - x = 8$
- $x = 26 + 8$
- $x + 8 = 26$
- $26 - 8 = x$
- $x = \underline{\hspace{2cm}}$

2. A coach formed teams of 8 from all the players in a soccer league. There are 14 teams. How many players are in the league?

- $y = 14 \div 8$
- $\frac{y}{8} = 14$
- $\frac{1}{8}y = 14$

○ $y = 14 \cdot 8$
 $y = \underline{\hspace{2cm}}$

3. Kiran scored 223 more points in a computer game than Tyler. If Kiran scored 409 points, how many points did Tyler score?

- $223 = 409 - z$
○ $409 - 223 = z$
○ $409 + 223 = z$
○ $409 = 223 + z$
 $z = \underline{\hspace{2cm}}$

4. Mai ran 27 miles last week, which was three times as far as Jada ran. How far did Jada run?

- $3w = 27$
○ $w = \frac{1}{3} \cdot 27$
○ $w = 27 \div 3$
○ $w = 3 \cdot 27$
 $w = \underline{\hspace{2cm}}$

Student Response

- $26 - x = 8, x + 8 = 26, 26 - 8 = x; x = 18$
- $\frac{y}{8} = 14, \frac{1}{8}y = 14, y = 14 \cdot 8; y = 112$
- $223 = 409 - z, 409 - 223 = z, 409 = 223 + z; z = 186$
- $3w = 27, w = \frac{1}{3} \cdot 27, w = 27 \div 3; w = 9$

Are You Ready for More?

Mai's mother was 28 when Mai was born. Mai is now 12 years old. In how many years will Mai's mother be twice Mai's age? How old will they be then?

Student Response

16 years; Mai will be 28 and her mother will be 56.

Activity Synthesis

Invite students to share their strategies for matching equations to the stories and for solving those equations. Include students who drew tape, hanger, or other types of diagrams to help them understand and reason about the relationships. Record the diagrams and strategies and have students compare them. Ask where they see information from the story in the parts of the diagrams and equations.

If no students bring it up, ask if any of the situations have a similar structure.

- The Clare/Mai and Kiran/Tyler situations share a similar structure where both the larger quantity and the difference between the smaller and larger quantities are known while the smaller quantity is unknown. Note that the first relationship is expressed with “fewer” and the second with “more.” This provides an opportunity for students to reason about the quantities, decontextualizing to see the similar structure and then contextualizing to understand the situations and answer questions.
- The soccer teams and Mai/Jada situations share similar structures in that equal parts add to a whole. The two problems differ in which quantities are known and unknown. In the soccer situation, the size of each group (8 players per team) and number of groups (14 teams) are known while the total is unknown. In the Mai/Jada multiplicative comparison situation, a total is known (27 miles) and the number of groups is known (3 times as many) but the size of each group is unknown.

Focusing on structure in this way helps students reason about the relationships between quantities in a situation, rather than focus on the words in the problem as hints to the operations needed in the equations.

For students who solved for the unknown by using the equations, ask which of the chosen equations they decided to solve and why.

Lesson Synthesis

The end of this lesson is a good place for students to take a moment and reflect on the learning of the past four lessons. Some questions to guide the discussion:

- “Describe some ways to understand how a situation can be represented mathematically.”
- “What have you learned about equations that surprised you?”
- “Share your thoughts about using diagrams to help understand relationships. Where have you seen diagrams used earlier this year? Where were they most helpful to you? Least helpful?”
- “Describe any connections you see between the types of diagrams used in the last four lessons.”

4.4 More Storytime

Cool Down: 5 minutes

Addressing

- 6.EE.B.5
- 6.EE.B.7

Student Task Statement

1. Write a story to match the equation $x + 2\frac{1}{2} = 10$.

2. Explain what x represents in your story.
3. Solve the equation. Explain or show your reasoning.

Student Response

Answers vary. Sample responses:

1. Lin likes to bake batches of muffins and share them with friends and family. She needs 10 cups of flour for her next batch, but only has $2\frac{1}{2}$ cups left. How much more flour does she need?
2. x represents the number of cups of flour Lin needs.
3. $x = 7\frac{1}{2}$. I subtracted $2\frac{1}{2}$ from each side of the equation.

Student Lesson Summary

Writing and solving equations can help us answer questions about situations.

Suppose a scientist has 13.68 liters of acid and needs 16.05 liters for an experiment. How many more liters of acid does she need for the experiment?

- We can represent this situation with the equation: $13.68 + x = 16.05$
- When working with hangers, we saw that the solution can be found by subtracting 13.68 from each side. This gives us some new equations that also represent the situation: $x = 16.05 - 13.68$
 $x = 2.37$
- Finding a solution in this way leads to a variable on one side of the equal sign and a number on the other. We can easily read the solution—in this case, 2.37—from an equation with a letter on one side and a number on the other. We often write solutions in this way.

Let's say a food pantry takes a 54-pound bag of rice and splits it into portions that each weigh $\frac{3}{4}$ of a pound. How many portions can they make from this bag?

- We can represent this situation with the equation: $\frac{3}{4}x = 54$
- We can find the value of x by dividing each side by $\frac{3}{4}$. This gives us some new equations that represent the same situation: $x = 54 \div \frac{3}{4}$
 $x = 72$
- The solution is 72 portions.

Lesson 4 Practice Problems

Problem 1

Statement

Select all the equations that describe each situation and then find the solution.

a. Kiran's backpack weighs 3 pounds less than Clare's backpack. Clare's backpack weighs 14 pounds. How much does Kiran's backpack weigh?

■ $x + 3 = 14$

■ $3x = 14$

■ $x = 14 - 3$

■ $x = 14 \div 3$

b. Each notebook contains 60 sheets of paper. Andre has 5 notebooks. How many sheets of paper do Andre's notebooks contain?

■ $y = 60 \div 5$

■ $y = 5 \cdot 60$

■ $\frac{y}{5} = 60$

■ $5y = 60$

Solution

a. $x + 3 = 14$, $x = 14 - 3$; $x = 11$, 11 pounds

b. $y = 5 \cdot 60$, $\frac{y}{5} = 60$; $y = 300$, 300 sheets

Problem 2

Statement

Solve each equation.

a. $2x = 5$

b. $y + 1.8 = 14.7$

c. $6 = \frac{1}{2}z$

d. $3\frac{1}{4} = \frac{1}{2} + w$

e. $2.5t = 10$

Solution

a. $x = \frac{5}{2}$ (or equivalent)

b. $y = 12.9$

c. $z = 12$

d. $w = 2\frac{3}{4}$ (or equivalent)

e. $t = 4$

Problem 3

Statement

For each equation, draw a tape diagram that represents the equation.

a. $3 \cdot x = 18$

b. $3 + x = 18$

c. $17 - 6 = x$

Solution

a. A tape diagram showing 3 groups labeled x and a total of 18.

b. A tape diagram showing one part labeled 3 and another labeled x and a total of 18.

c. A tape diagram showing one part labeled 6 and another labeled x and a total of 17.

(From Unit 6, Lesson 1.)

Problem 4

Statement

Find each product.

$(21.2) \cdot (0.02)$

$(2.05) \cdot (0.004)$

Solution

$(21.2) \cdot (0.02) = 0.424$

$(2.05) \cdot (0.004) = 0.0082$

(From Unit 5, Lesson 8.)

Problem 5

Statement

For a science experiment, students need to find 25% of 60 grams.

- Jada says, "I can find this by calculating $\frac{1}{4}$ of 60."
- Andre says, "25% of 60 means $\frac{25}{100} \cdot 60$."

Do you agree with either of them? Explain your reasoning.

Solution

Both are correct. Andre is right that 25% of a number means $\frac{25}{100}$ of that number. Jada is also right because $\frac{25}{100} = \frac{1}{4}$.

(From Unit 3, Lesson 13.)

Lesson 5: A New Way to Interpret a over b

Goals

- Comprehend that the notation $\frac{a}{b}$ can be used to represent division generally, and the numerator and denominator can include fractions, decimals, or variables.
- Describe (orally) a situation that could be represented by a given equation of the form $x + p = q$ or $px = q$.
- Express division as a fraction (in writing) when solving equations of the form $px = q$.

Learning Targets

- I understand the meaning of a fraction made up of fractions or decimals, like $\frac{2.1}{0.07}$ or $\frac{\frac{4}{5}}{\frac{3}{2}}$.
- When I see an equation, I can make up a story that the equation might represent, explain what the variable represents in the story, and solve the equation.

Lesson Narrative

In this lesson, students apply the general procedure they just learned for solving $px = q$ in order to define what $\frac{a}{b}$ means when a and b are not whole numbers. Up until now, students have likely only seen a fraction bar separating two whole numbers. This is because before grade 6, they couldn't divide arbitrary rational numbers. Now an expression like $\frac{2.5}{8.9}$ or $\frac{\frac{1}{2}}{\frac{3}{5}}$ can be well-defined. But the definition is not the same as what they learned for, for example, $\frac{2}{5}$ in grade 3, where they learned that $\frac{2}{5}$ is the number you get by partitioning the interval from 0 to 1 into 5 equal parts and then marking off 2 of the parts. That definition only works for whole numbers. However, in grade 5, students learned that $2 \div 3 = \frac{2}{3}$, so in grade 6 it makes sense to define $\frac{2.5}{8.9}$ as $2.5 \div 8.9$.

Alignments

Building On

- 6.NS.A.1: Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for $(\frac{2}{3}) \div (\frac{3}{4})$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(\frac{2}{3}) \div (\frac{3}{4}) = \frac{8}{9}$ because $\frac{3}{4}$ of $\frac{8}{9}$ is $\frac{2}{3}$. (In general, $(\frac{a}{b}) \div (\frac{c}{d}) = \frac{ad}{bc}$.) How much chocolate will each person get if 3 people share $\frac{1}{2}$ lb of chocolate equally? How many $\frac{3}{4}$ -cup servings are in $\frac{2}{3}$ of a cup of yogurt? How wide is a rectangular strip of land with length $\frac{3}{4}$ mi and area $\frac{1}{2}$ square mi?
- 6.NS.B.3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Addressing

- 6.EE.B: Reason about and solve one-variable equations and inequalities.
- 6.EE.B.5: Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
- 6.EE.B.6: Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- 6.EE.B.7: Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR2: Collect and Display
- Think Pair Share

Student Learning Goals

Let's investigate what a fraction means when the numerator and denominator are not whole numbers.

5.1 Recalling Ways of Solving

Warm Up: 5 minutes

The purpose of this warm-up is to apply what students have learned to some equations. Note that $0.07 \div 10$ and $10.1 - 7.2$ should be easy to evaluate given that work with fluently computing with decimals precedes this unit.

Addressing

- 6.EE.B.5
- 6.EE.B.7

Launch

Ask students to summarize what they learned in the previous lessons before setting them to work on this warm-up. Allow 1-2 minutes quiet think time, followed by a whole-class discussion.

Student Task Statement

Solve each equation. Be prepared to explain your reasoning.

$$0.07 = 10m$$

$$10.1 = t + 7.2$$

Student Response

1. $0.007 = m$
2. $2.9 = t$

Activity Synthesis

At the conclusion of the previous lesson, students should have seen that we can approach solving any equation of the form $px = q$ (where p and q are rational numbers and x is unknown) by dividing each side by p . Also, we can approach solving any equation of the form $x + p = q$ by subtracting p from each side. Discussion should focus on given $0.07 = 10m$, we can write $0.07 \div 10 = 10m \div 10$ and then $0.007 = m$.

5.2 Interpreting $\frac{a}{b}$

15 minutes

Students solve more equations of the form $px = q$ while interpreting the division as a fraction.

Building On

- 6.NS.A.1
- 6.NS.B.3

Addressing

- 6.EE.B

Instructional Routines

- MLR2: Collect and Display
- Think Pair Share

Launch

Arrange students in groups of 2. Give 5–10 minutes of quiet work time and time to share their responses with a partner, followed by a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge about division involving decimals and fractions. Allow students to use calculators to ensure inclusive participation in the activity.

Supports accessibility for: Memory; Conceptual processing

Access for English Language Learners

Conversing, Representing, Writing: MLR2 Collect and Display. As share their responses with a partner, circulate and listen to their conversations. Collect and display any vocabulary or representations students use (e.g., reciprocal, dividing, multiplying) to describe how to solve each equation. Continue to update collected student language once students move on to the activity. Remind students to borrow language from the display as needed. This will help student to use academic mathematical language during paired and group discussions to connect fractions with division.

Design Principle(s): Maximize meta-awareness; Support sense-making

Anticipated Misconceptions

Monitor for students who want to turn $\frac{35}{11}$ into a decimal, and reassure them that $\frac{35}{11}$ is a number.

Student Task Statement

Solve each equation.

1. $35 = 7x$

2. $35 = 11x$

3. $7x = 7.7$

4. $0.3x = 2.1$

5. $\frac{2}{5} = \frac{1}{2}x$

Student Response

1. 5

2. $\frac{35}{11}$

3. 1.1

4. 7

5. $\frac{4}{5}$

Are You Ready for More?

Solve the equation. Try to find some shortcuts.

$$\frac{1}{6} \cdot \frac{3}{20} \cdot \frac{5}{42} \cdot \frac{7}{72} \cdot x = \frac{1}{384}$$

Student Response

$x = 9$. Solution methods vary. One way is to factor each denominator and notice that there are many numbers that occur in both numerator and denominator.

Activity Synthesis

Define what $\frac{a}{b}$ means when a and b are not whole numbers. Tell students, "In third grade, when you saw something like $\frac{2}{5}$, you learned that that meant 'split up 1 into 5 equal pieces and take 2 of them.' But that definition only makes sense for whole numbers; it doesn't make sense for something like $\frac{2.1}{0.3}$ or $\frac{\frac{2}{5}}{\frac{1}{2}}$. From now on, when you see something like $\frac{2}{5}$, you'll know that that means the number $\frac{2}{5}$ that has a spot on the number line, but it also means '2 divided by 5.' The expression $\frac{2.1}{0.3}$ means 'the quotient of 2.1 and 0.3,' the expression $\frac{\frac{2}{5}}{\frac{1}{2}}$ means 'the quotient of two fifths and one half,' and generally, the expression $\frac{a}{b}$ means 'the quotient of a and b ' or ' a divided by b .'"

5.3 Storytime Again

15 minutes

This is a continuation of the activities Storytime and More Storytime from previous lessons. Over time in this unit, we are reminding students of work they should have done in previous grades with expressions that represent particular, concrete relationships. In grade 6, students are working toward producing such expressions themselves to represent a context.

Addressing

- 6.EE.B.6

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Remind students of work they did previously to match a situation with an equation. For example, they matched the equation $x + 5 = 20$ with the situation "After Elena ran 5 miles on Friday, she had run a total of 20 miles for the week. How many miles did she run before Friday?" In this activity, they come up with their own situations that can be represented by equations.

Keep students in the same groups. Clarify that for each equation, each partner will come up with a story, and one of those stories is chosen. Give students 5–10 minutes to work with their partner, followed by a whole-class discussion.

Anticipated Misconceptions

For students with limited fraction and decimal understanding, coming up with a reasonable story where the numbers are not whole can be daunting. You might suggest that students imagine

stories with similar structures that involve whole numbers, and then tweak the stories toward using the numbers given in the problems. Remind them that using fractions and decimals has to make sense in the situations, and encourage them to think about what kinds of situations those might be (measurement situations will usually work while those that involve counting discrete objects won't.)

Student Task Statement

Take turns with your partner telling a story that might be represented by each equation. Then, for each equation, choose one story, state what quantity x describes, and solve the equation. If you get stuck, consider drawing a diagram.

$$0.7 + x = 12$$

$$\frac{1}{4}x = \frac{3}{2}$$

Student Response

Answers vary. Sample responses:

1. $x = 11.3$. Diego went to the store to buy a box of crayons, which cost 70 cents. While there, he picked up some other items. The total amount he paid at checkout was \$12. Solving the equation gives 11.3. Diego spent \$11.30 on the other items he bought at the store.
2. $x = 6$. $\frac{1}{4}$ of a bottle of water contains $\frac{3}{2}$ cups of water. How many cups are in the whole bottle? The solution is $\frac{\frac{3}{2}}{\frac{1}{4}}$, which is 6. One bottle of water contains 6 cups.

Activity Synthesis

Invite students to share their stories. Ask each student to interpret the solution in terms of their situation.

Access for English Language Learners

Writing, Speaking: MLR1 Stronger and Clearer Each Time. Use this routine with successive pair shares to give students a structured opportunity to revise and refine their writing. For this activity, students should use the story for the equation they chose to solve. Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help teams strengthen their ideas and clarify their language (e.g., “How are the parts of the equation represented in your story?”, “Can you say more about how your solution fits in your story?”). Provide students with time to complete a final draft based on the feedback they receive about language and clarity.

Design Principle(s): Optimize output (for comparison)

Lesson Synthesis

Ask students to work with their partner. Each partner writes a number that is in fraction or decimal form. Have them choose one number to be the coefficient in an equation of the form $px = q$ and the second number the quantity on the other side of the equal sign. They then work together to write and evaluate the solution of the equation. Complete multiple rounds as time allows.

5.4 Choosing Solutions

Cool Down: 5 minutes

Addressing

- 6.EE.B.7

Student Task Statement

Select **all** the expressions that are solutions to $5 = \frac{2}{3}x$.

- $5 \cdot \frac{2}{3}$
- $\frac{5}{\frac{2}{3}}$
- $5 \div \frac{2}{3}$
- $\frac{15}{2}$
- $\frac{10}{3}$

Student Response

$\frac{5}{\frac{2}{3}}$, $5 \div \frac{2}{3}$, $\frac{15}{2}$

Student Lesson Summary

In the past, you learned that a fraction such as $\frac{4}{5}$ can be thought of in a few ways.

- $\frac{4}{5}$ is a number you can locate on the number line by dividing the section between 0 and 1 into 5 equal parts and then counting 4 of those parts to the right of 0.
- $\frac{4}{5}$ is the share that each person would have if 4 wholes were shared equally among 5 people. This means that $\frac{4}{5}$ is the result of *dividing* 4 by 5.

We can extend this meaning of *a fraction as a quotient* to fractions whose numerators and denominators are not whole numbers. For example, we can represent 4.5 pounds of rice divided into portions that each weigh 1.5 pounds as: $\frac{4.5}{1.5} = 4.5 \div 1.5 = 3$. In other words, $\frac{4.5}{1.5} = 3$ because the quotient of 4.5 and 1.5 is 3.

Fractions that involve non-whole numbers can also be used when we solve equations.

Suppose a road under construction is $\frac{3}{8}$ finished and the length of the completed part is $\frac{4}{3}$ miles. How long will the road be when completed?

We can write the equation $\frac{3}{8}x = \frac{4}{3}$ to represent the situation and solve the equation.

The completed road will be $3\frac{5}{9}$ or about 3.6 miles long.

$$\frac{3}{8}x = \frac{4}{3}$$

$$x = \frac{\frac{4}{3}}{\frac{3}{8}}$$

$$x = \frac{4}{3} \cdot \frac{8}{3}$$

$$x = \frac{32}{9} = 3\frac{5}{9}$$

Lesson 5 Practice Problems

Problem 1

Statement

Select all the expressions that equal $\frac{3.15}{0.45}$.

A. $(3.15) \cdot (0.45)$

B. $(3.15) \div (0.45)$

C. $(3.15) \cdot \frac{1}{0.45}$

D. $(3.15) \div \frac{45}{100}$

E. $(3.15) \cdot \frac{100}{45}$

F. $\frac{0.45}{3.15}$

Solution

["B", "C", "D", "E"]

Problem 2

Statement

Which expressions are solutions to the equation $\frac{3}{4}x = 15$? Select all that apply.

A. $\frac{15}{\frac{3}{4}}$

B. $\frac{15}{\frac{4}{3}}$

C. $\frac{4}{3} \cdot 15$

D. $\frac{3}{4} \cdot 15$

E. $15 \div \frac{3}{4}$

Solution

["A", "C", "E"]

Problem 3

Statement

Solve each equation.

$$4a = 32$$

$$4 = 32b$$

$$10c = 26$$

$$26 = 100d$$

Solution

a. $a = 8$

b. $b = \frac{1}{8}$

c. $c = 2.6$ (or equivalent)

d. $d = 0.26$ (or equivalent)

Problem 4

Statement

For each equation, write a story problem represented by the equation. For each equation, state what quantity x represents. If you get stuck, consider drawing a diagram.

a. $\frac{3}{4} + x = 2$

b. $1.5x = 6$

Solution

Answers vary. Sample response:

a. Jada ran for 2 miles. Elena ran for $\frac{3}{4}$ of a mile. How much further did Jada run than Elena? x represents the difference between the distance of Jada's run and Elena's run.

b. 1.5 times the amount a bucket holds makes 6 gallons. How many gallons does the bucket hold? x represents the volume in gallons that the bucket holds.

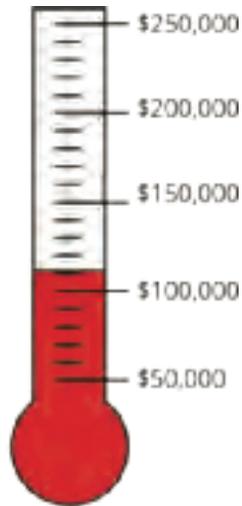
Problem 5

Statement

Write as many mathematical expressions or equations as you can about the image. Include a fraction, a decimal number, or a percentage in each.

FUNDRAISER

OUR GOAL:
\$250,000



Solution

Answers vary. Possible responses: $\frac{1}{5} \cdot 250,000 = 50,000$, 20% of 250,000 is 50,000, or $(0.2) \cdot 250,000 = 50,000$.

(From Unit 3, Lesson 13.)

Problem 6

Statement

In a lilac paint mixture, 40% of the mixture is white paint, 20% is blue, and the rest is red. There are 4 cups of blue paint used in a batch of lilac paint.

- How many cups of white paint are used?
- How many cups of red paint are used?
- How many cups of lilac paint will this batch yield?

If you get stuck, consider using a tape diagram.

Solution

- 8
- 8
- 20

(From Unit 3, Lesson 12.)

Problem 7

Statement

Triangle P has a base of 12 inches and a corresponding height of 8 inches. Triangle Q has a base of 15 inches and a corresponding height of 6.5 inches. Which triangle has a greater area? Show your reasoning.

Solution

Triangle Q has a larger area. The area of Triangle P is $\frac{1}{2} \cdot 12 \cdot 8$ or 48 square inches. The area of Triangle Q is $\frac{1}{2} \cdot 15 \cdot (6.5)$ or 48.75 square inches.

(From Unit 1, Lesson 9.)

Section: Equal and Equivalent

Lesson 6: Write Expressions Where Letters Stand for Numbers

Goals

- Explain (orally) how to create and solve an equation that represents a situation with an unknown amount.
- Write an expression with a variable to generalize the relationship between quantities in a situation.

Learning Targets

- I can use an expression that represents a situation to find an amount in a story.
- I can write an expression with a variable to represent a calculation where I do not know one of the numbers.

Lesson Narrative

This lesson is a shift from previous work in this unit. Up until now, we were focused on writing and solving equations. Starting in this lesson, we begin to focus on writing expressions to represent situations. Students write expressions that record operations with numbers and with letters standing in for numbers. Students can choose to represent expressions with tape diagrams if they wish (MP5).

Alignments

Addressing

- 6.EE.A.2.a: Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation “Subtract y from 5” as $5 - y$.
- 6.EE.A.2.c: Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.
- 6.EE.B.6: Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

Instructional Routines

- Algebra Talk

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display
- MLR8: Discussion Supports
- Think Pair Share

Student Learning Goals

Let's use expressions with variables to describe situations.

6.1 Algebra Talk: When x is 6

Warm Up: 5 minutes

The purpose of this algebra talk is to elicit strategies and understandings students have for evaluating an expression for a given value of its variable. These understandings help students develop fluency and will be helpful later in this lesson when students will need to be able to evaluate expressions.

Addressing

- 6.EE.A.2.c

Instructional Routines

- Algebra Talk
- MLR8: Discussion Supports

Launch

Give students a minute to see if they recall that x^2 means $x \cdot x$ (which they learned about in an earlier unit in this course), but if necessary, remind them what this notation means.

Display one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Student Task Statement

If x is 6, what is:

$$x + 4$$

$7 - x$

x^2

$\frac{1}{3}x$

Student Response

- 10
- 1
- 36
- 2

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone have the same strategy but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to ____'s strategy?"
- "Do you agree or disagree? Why?"

Access for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because . . ." or "I noticed ____ so I" Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Design Principle(s): Optimize output (for explanation)

6.2 Lemonade Sales and Heights

15 minutes

Throughout this unit students have been matching equations to tape diagrams, matching equations to situations, and solving equations. This lesson shifts the focus to writing the expressions that describe situations with an unknown quantity. Students use operations to calculate quantities and notice repeated patterns in those calculations. They replace a part of the calculation with a letter to represent any possible value and create an expression that represents the situation (MP8). Students learn that they can use these expressions to answer questions about specific values. Monitor for

students who use different strategies to answer the second part of each question. For each, select at least one student who uses a less-efficient method like trial-and-error and one student who writes and solves an equation. Note if any student represents a situation with a tape diagram—this can be presented to support reasoning about an unknown quantity using a variable.

Addressing

- 6.EE.A.2.a
- 6.EE.B.6

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display

Launch

Allow students 10 minutes quiet work time followed by a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Differentiate the degree of difficulty or complexity by beginning with more accessible values. Extend the given table, and begin by exploring values for money collected based on 1, 2, 3, 5, and 10 cups of lemonade sold. Draw students' attention to what changes and what stays the same each time they calculate the money collected.

Supports accessibility for: Conceptual processing

Access for English Language Learners

Conversing, Representing, Writing: MLR2 Collect and Display. During small-group discussion, listen for and collect the vocabulary and phrases students use to describe how to find the values of the table and how the expression represents the situation (e.g., "the number of cups is twice the number of dollars"). Make connections between how similar ideas are communicated and represented in different ways (e.g., "How do you see 'twice' in the tape diagrams and expressions?"). Remind students to borrow language from the display as needed. This will help students to use academic mathematical language during paired and group discussions when writing expressions representing situations with an unknown quantity.

Design Principle(s): Maximize meta-awareness

Student Task Statement

1. Lin set up a lemonade stand. She sells the lemonade for \$0.50 per cup.

- a. Complete the table to show how much money she would collect if she sold each number of cups.

lemonade sold (number of cups)	12	183	c
money collected (dollars)			

- b. How many cups did she sell if she collected \$127.50? Be prepared to explain your reasoning.

2. Elena is 59 inches tall. Some other people are taller than Elena.

- a. Complete the table to show the height of each person.

person	Andre	Lin	Noah
how much taller than Elena (inches)	4	$6\frac{1}{2}$	d
person's height (inches)			

- b. If Noah is $64\frac{3}{4}$ inches tall, how much taller is he than Elena?

Student Response

1. a.

lemonade sold (number of cups)	12	183	c
money collected (dollars)	6	91.50	$0.5c$

- b. 255 cups. (The number of cups is twice the the number of dollars.)

2. a.

person	Andre	Lin	Noah
how much taller they are than Elena (inches)	4	$6\frac{1}{2}$	d
person's height (inches)	63	$65\frac{1}{2}$	$59 + d$

- b. $5\frac{3}{4}$ inches. ($64\frac{3}{4}$ is $5\frac{3}{4}$ more than 59.)

Activity Synthesis

The goal of the discussion is to ensure students see that they can write a mathematical expression to represent a calculation, even if they do not know what one of the numbers is in the calculation. Select students to present who solved the second part of each problem with different strategies. If no student took an approach with equations, demonstrate that $0.5c = 127.50$ can represent the

situation and remind students they can find c with $\frac{127.5}{0.5}$. Similarly for the second problem, demonstrate that $59 + d = 64\frac{3}{4}$ can represent the situation and d can be found with $64\frac{3}{4} - 59$. Ask students to explain how the equations make use of the expressions they wrote in the tables and why they can write the equations in these ways.

6.3 Building Expressions

15 minutes

The purpose of this activity is to help students write expressions given a situation, then to solve equations involving the same situation. For the first three questions, the work is still scaffolded by providing numbers to use to calculate before prompting students to write an expression that uses a variable (MP8).

Addressing

- 6.EE.A.2.a
- 6.EE.B.6

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 5–10 minutes of quiet work time and time to share with a partner, followed by a whole-class discussion.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. For example, invite students to draw a picture or tape diagram to help as an intermediate step before writing an equation.

Supports accessibility for: Social-emotional skills; Conceptual processing

Anticipated Misconceptions

There are multiple quantities in each problem. Students may lose track of what the variable represents.

Students may not see the value in setting up an equation if they can solve it mentally. Later problems are less likely to be solved mentally, so encourage students to write and solve an equation each time.

Student Task Statement

1. Clare is 5 years older than her cousin.
 - a. How old would Clare be if her cousin is:
 - 10 years old?
 - 2 years old?
 - x years old?
 - b. Clare is 12 years old. How old is Clare's cousin?
2. Diego has 3 times as many comic books as Han.
 - a. How many comic books does Diego have if Han has:
 - 6 comic books?
 - n books?
 - b. Diego has 27 comic books. How many comic books does Han have?
3. Two fifths of the vegetables in Priya's garden are tomatoes.
 - a. How many tomatoes are there if Priya's garden has:
 - 20 vegetables?
 - x vegetables?
 - b. Priya's garden has 6 tomatoes. How many total vegetables are there?
4. A school paid \$31.25 for each calculator.
 - a. If the school bought x calculators, how much did they pay?
 - b. The school spent \$500 on calculators. How many did the school buy?

Student Response

1. a. 15 years old, 7 years old, $x + 5$ years old
b. 7 years old, since $x + 5 = 12$ is true when x is 7.
2. a. 18 books, $3n$ books
b. 9 books, since $3n = 27$ is true when n is 9.
3. a. 8 tomatoes, $\frac{2}{5}x$ tomatoes

b. 15 vegetables, since $\frac{2}{5}x = 6$ is true when x is 15.

4. a. $31.25x$

b. 16 calculators, since $31.25x = 500$ is true when x is 16.

Are You Ready for More?

Kiran, Mai, Jada, and Tyler went to their school carnival. They all won chips that they could exchange for prizes. Kiran won $\frac{2}{3}$ as many chips as Jada. Mai won 4 times as many chips as Kiran. Tyler won half as many chips as Mai.

1. Write an expression for the number of chips Tyler won. You should only use one variable: J , which stands for the number of chips Jada won.
2. If Jada won 42 chips, how many chips did Tyler, Kiran, and Mai each win?

Student Response

1. $\frac{4}{3}J$

2. Tyler has 56 chips. Kiran has 28 chips. Mai has 112 chips.

Activity Synthesis

We can often express one quantity in terms of another unknown quantity because we know a relationship between them. Sometimes we also know a value of this expression, and we can use that understanding to write an equation and solve for the unknown quantity. Consider asking some of the following questions to guide the discussion:

- “What facts describing each situation helped you to write the expression?”
- “How did you use the expression to write an equation? Why were you able to set the quantities on each side of the equation equal to each other? Can you give an example of this?” (They represent the same quantity in the story so they have to be equal; for example, the number of tomatoes in Priya’s garden is both 6 and $\frac{2}{5}$ of the number of vegetables, x , in her garden.)
- “What strategies did you use to solve each equation?”
- “How did you check that your solution was correct?” (The best way to check when there is a context is to go back to the original situation and see if the solution makes the statements true. Checking in the equation has the problem that you won’t catch if the equation you wrote does not correctly represent the situation.)

Access for English Language Learners

Listening, Conversing: MLR8 Discussion Supports. Support whole-class discussion by displaying and inviting students to use these sentence frames: "To solve each equation I ____ because ____." or "To check my work is correct, I can ____ because ____." As students share, encourage other students revoice or press for more explanation by asking, "So what I heard you say is ____" or "Can you tell me more about ____?"

Design Principle(s): Cultivate conversation; Optimize output

Lesson Synthesis

Keep students in the same groups. One partner makes up a story, similar to the situations they saw in the lesson, where they describe a relationship between two quantities. The second student assigns a value to one quantity and the other is unknown. As an example, tell students that you have half as many books as your friend, and you have 130 books. Your friend's number of books is the unknown quantity, let's call it b . You can then write the expression $\frac{1}{2}b$ to represent your number of books, and the equation $\frac{1}{2}b = 130$ to describe the situation. b can then be found with $\frac{130}{\frac{1}{2}}$.

Writing their own stories helps students reason about the meaning and structure of expressions and equations and the situations they represent.

6.4 Crazy Eights

Cool Down: 5 minutes

Addressing

- 6.EE.A.2.a
- 6.EE.B.6

Student Task Statement

A plant measured x inches tall last week and 8 inches tall this week.

1. Circle the expression that represents the number of inches the plant grew this week. Explain how you know.
 - $x - 8$
 - $8 - x$
2. For the expression *not* chosen, describe a situation that the expression might represent.

Student Response

1. $8 - x$. Sample explanation: Since the plant grew taller this week, 8 is greater than x . The difference of 8 and x is the amount that the plant grew.
2. Answers vary. Sample response: Elena has more roses than Lin. Lin has 8 roses and Elena has x roses. $x - 8$ represents how many more roses Elena has than Lin.

Student Lesson Summary

Suppose you share a birthday with a neighbor, but she is 3 years older than you. When you were 1, she was 4. When you were 9, she was 12. When you are 42, she will be 45.

If we let a represent your age at any time, your neighbor's age can be expressed $a + 3$.

your age	1	9	42	a
neighbor's age	4	12	45	$a + 3$

We often use a letter such as x or a as a placeholder for a number in expressions. These are called *variables* (just like the letters we used in equations, previously). Variables make it possible to write expressions that represent a calculation even when we don't know all the numbers in the calculation.

How old will you be when your neighbor is 32? Since your neighbor's age is calculated with the expression $a + 3$, we can write the equation $a + 3 = 32$. When your neighbor is 32 you will be 29, because $a + 3 = 32$ is true when a is 29.

Lesson 6 Practice Problems

Problem 1

Statement

Instructions for a craft project say that the length of a piece of red ribbon should be 7 inches less than the length of a piece of blue ribbon.

a. How long is the red ribbon if the length of the blue ribbon is:

10 inches?

27 inches?

x inches?

b. How long is the blue ribbon if the red ribbon is 12 inches?

Solution

a. 3 inches ($10 - 7 = 3$), 20 inches ($27 - 7 = 20$), $x - 7$ inches

b. 19 inches ($12 + 7 = 19$)

Problem 2

Statement

Tyler has 3 times as many books as Mai.

a. How many books does Mai have if Tyler has:

15 books?

21 books?

x books?

b. Tyler has 18 books. How many books does Mai have?

Solution

a. 5 books ($15 \div 3 = 5$), 7 books ($21 \div 3 = 7$), $\frac{x}{3}$ books

b. 6 books ($18 \div 3 = 6$)

Problem 3

Statement

A bottle holds 24 ounces of water. It has x ounces of water in it.

a. What does $24 - x$ represent in this situation?

b. Write a question about this situation that has $24 - x$ for the answer.

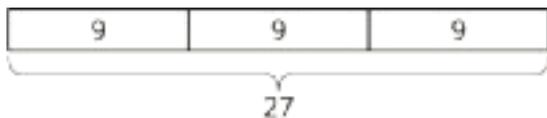
Solution

- The amount of water that has been removed from the bottle.
- Answers vary. Sample response: How many ounces of water did Jada drink from the full bottle if there are x ounces left?

Problem 4

Statement

Write an equation represented by this tape diagram using each of these operations.



- addition
- subtraction
- multiplication
- division

Solution

Answers vary. Sample responses:

- $9 + 9 + 9 = 27$
- $27 - 9 = 9 + 9$
- $3 \cdot 9 = 27$
- $27 \div 3 = 9$

(From Unit 6, Lesson 1.)

Problem 5

Statement

Select **all** the equations that describe each situation and then find the solution.

- Han's house is 450 meters from school. Lin's house is 135 meters closer to school. How far is Lin's house from school?
 - $z = 450 + 135$
 - $z = 450 - 135$
 - $z - 135 = 450$
 - $z + 135 = 450$
- Tyler's playlist has 36 songs. Noah's playlist has one quarter as many songs as Tyler's playlist. How many songs are on Noah's playlist?

■ $w = 4 \cdot 36$

■ $w = 36 \div 4$

■ $4w = 36$

■ $\frac{w}{4} = 36$

Solution

a. $z = 450 - 135, z + 135 = 450; z = 315$

b. $w = 36 \div 4, 4w = 36; w = 9$

(From Unit 6, Lesson 4.)

Problem 6

Statement

You had \$50. You spent 10% of the money on clothes, 20% on games, and the rest on books. How much money was spent on books?

Solution

\$35 Reasoning varies. Sample reasoning: \$5 was spent on books, because $50 \cdot 0.1 = 5$. \$10 was spend on games, because $50 \cdot 0.2 = 10$. \$15 is the combined amount spent on books and games. That leaves \$35, because $50 - 15 = 35$.

(From Unit 3, Lesson 12.)

Problem 7

Statement

A trash bin has a capacity of 50 gallons. What percentage of its capacity is each amount? Show your reasoning.

a. 5 gallons

b. 30 gallons

c. 45 gallons

d. 100 gallons

Solution

a. 5 gallons is 10% of 50 gallons, because $5 \div 50 = 0.1$.

b. 30 gallons is 60% of 50 gallons, because $30 \div 50 = 0.6$.

c. 45 gallons is 90% of 50 gallons, because $45 \div 50 = 0.9$.

d. 100 gallons is 200% of 50 gallons, because $100 \div 50 = 2$.

(From Unit 3, Lesson 14.)

Lesson 7: Revisit Percentages

Goals

- State explicitly what the chosen variable represents when creating an equation.
- Use equations to solve problems involving percentages and explain (orally) the solution method.
- Write equations of the form $px = q$ or equivalent to represent situations where the amount that corresponds to 100% is unknown.

Learning Targets

- I can solve percent problems by writing and solving an equation.

Lesson Narrative

Students learned about what percentages are and how to solve certain problems in an earlier unit. At the time, they did not learn an efficient procedure for finding B in “ $A\%$ of B is C ” given A and C , because they didn't have an efficient way to solve an equation of the form $px = q$. Now they do, so we briefly revisit this type of problem.

Alignments

Building On

- 6.RP.A.3.c: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

Addressing

- 6.EE.B.6: Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.
- 6.EE.B.7: Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers.
- 6.RP.A.3.c: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports
- Number Talk
- Think Pair Share

Student Learning Goals

Let's use equations to find percentages.

7.1 Number Talk: Percentages

Warm Up: 5 minutes

The purpose of this warm-up is to rekindle anything students remember about percentages and representations they use to reason about them.

Building On

- 6.RP.A.3.c

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Student Task Statement

Solve each problem mentally.

1. Bottle A contains 4 ounces of water, which is 25% of the amount of water in Bottle B. How much water is there in Bottle B?
2. Bottle C contains 150% of the water in Bottle B. How much water is there in Bottle C?
3. Bottle D contains 12 ounces of water. What percentage of the amount of water in Bottle B is this?

Student Response

1. 16 ounces
2. 24 ounces
3. 75%

Activity Synthesis

Invite students to share different representations and ways of reasoning. Record student strategies and nonchalantly write an equation for each in the process.

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Provide sentence frames to support students with explaining their strategies. For example, "I noticed that ____." or "First, I _____ because _____." When students share their answers with a partner, prompt them to rehearse what they will say when they share with the whole class. Rehearsing provides students with additional opportunities to clarify their thinking.

Design Principle(s): Optimize output (for explanation)

7.2 Representing a Percentage Problem with an Equation

20 minutes

Students perform repeated calculations and then generalize with an algebraic expression (MP8). The purpose of this activity is to help students see that any basic percentage problem like “ n percent of this is that” can be represented with an equation in the form $px = q$.

Addressing

- 6.EE.B.6
- 6.EE.B.7
- 6.RP.A.3.c

Instructional Routines

- MLR3: Clarify, Critique, Correct
- Think Pair Share

Launch

Using any insights from the warm-up as an example, remind students of any efficient method they know to compute a percentage. For example, 25% of 16 can be computed using $\frac{25}{100} \cdot 16$.

Arrange students in groups of 2. Give 5–10 minutes of quiet work time and time to share responses with a partner, followed by a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge about computing percentages. Allow students to use calculators to ensure inclusive participation in the activity.

Supports accessibility for: Memory; Conceptual processing

Anticipated Misconceptions

Students might not understand that we are trying to find the whole that we know an amount is a certain percent of. Encourage them to draw a tape diagram or a double number line to visualize the relationship between the three quantities.

Student Task Statement

1. Answer each question and show your reasoning.

a. Is 60% of 400 equal to 87?

b. Is 60% of 200 equal to 87?

c. Is 60% of 120 equal to 87?

2. 60% of x is equal to 87. Write an equation that expresses the relationship between 60%, x , and 87. Solve your equation.

3. Write an equation to help you find the value of each variable. Solve the equation.

60% of c is 43.2.

38% of e is 190.

Student Response

1. a. No, 60% of 400 equals $\frac{60}{100} \cdot 400$ or 240

b. No, 60% of 200 equals $\frac{60}{100} \cdot 200$ or 120

c. No, 60% of 120 equals $\frac{60}{100} \cdot 120$ or 72

2. $\frac{60}{100}x = 87$, $x = \frac{87}{\frac{60}{100}}$, $x = 145$

3. $\frac{60}{100}c = 43.2$, $c = 72$

$\frac{38}{100}e = 190$, $e = 500$

Activity Synthesis

The purpose of this discussion is for students to see how writing and solving an equation can be an efficient way to solve a problem about percentages. In the course of the discussion, they should see three equations written and solved. If any students used representations like tape diagrams or double number lines to reason about the problem, it can be advantageous to display these alongside the equations so that students can make connections between strategies they understand well and the more abstract strategy of writing and solving an equation.

Access for English Language Learners

Writing, Representing: MLR3 Clarify, Critique, Correct. Present an incorrect statement, “For 60% of c is 43.2, the value of c is 25.92 because 0.6 times 43.2 is 25.92.” Invite students to ask clarifying questions about the statement to identify the error. Invite students to work with a partner to write a correct statement using a representation such as a tape diagram or double number line. This will help students to visualize the relationship between the three quantities and use language to critique and create viable mathematical arguments.

Design Principle(s): Maximize meta-awareness

7.3 Puppies Grow Up, Revisited

10 minutes

In this activity, students are asked to write an equation but are not given a letter to use. This is an opportunity to explain to students that when they decide to use a letter to represent something, they need to state what the letter represents.

Addressing

- 6.EE.B.6
- 6.EE.B.7
- 6.RP.A.3.c

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Keep students in the same groups. Allow students 5 minutes of quiet work time and time to share responses with a partner, followed by a whole-class discussion.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. For example, invite students to draw a picture or tape diagram to help as an intermediate step before writing an equation.

Supports accessibility for: Social-emotional skills; Conceptual processing

Student Task Statement

1. Puppy A weighs 8 pounds, which is about 25% of its adult weight. What will be the adult weight of Puppy A?
2. Puppy B weighs 8 pounds, which is about 75% of its adult weight. What will be the adult weight of Puppy B?
3. If you haven't already, write an equation for each situation. Then, show how you could find the adult weight of each puppy by solving the equation.

Student Response

1. 32 pounds
2. $\frac{32}{3}$ pounds
3. Answers vary. Sample responses: Divide both sides of $8 = \frac{1}{4}x$ by $\frac{1}{4}$ to get $32 = x$. Divide both sides of $8 = \frac{3}{4}x$ by $\frac{3}{4}$ to get $y = \frac{32}{3}$ or about 10.7 pounds.

Are You Ready for More?

Diego wants to paint his room purple. He bought one gallon of purple paint that is 30% red paint and 70% blue paint. Diego wants to add more blue to the mix so that the paint mixture is 20% red, 80% blue.

1. How much blue paint should Diego add? Test the following possibilities: 0.2 gallons, 0.3 gallons, 0.4 gallons, 0.5 gallons.
2. Write an equation in which x represents the amount of paint Diego should add.
3. Check that the amount of paint Diego should add is a solution to your equation.

Student Response

1. 0.5 gallons is the correct amount of blue paint to add.
0.2 gallons: 75% blue paint, because the amount of blue paint divided by the total amount of paint is $\frac{0.9}{1.2} = 0.75$.
0.3 gallons: 77% blue paint, by similar reasoning

0.4 gallons: 78.6% blue paint

0.5 gallons: 80% blue paint

2. $0.7 + x = 0.8(1 + x)$

3. Yes.

Activity Synthesis

The focus of the discussion should be the selection of a variable to represent an unknown quantity. Invite students to share how they decided where to use a variable, what it represented in the story, what letter they used and why. Ask why it is important to state what the letter represents and where they made that statement in their solutions.

Access for English Language Learners

Speaking, Representing: MLR8 Discussion Supports. To support whole-class discussion about selecting a variable to represent an unknown quantity, provide sentence frames to help students explain their reasoning. For example, "I knew I needed to use a variable to represent ____ because ____." or "The variable I chose to represent ____ is ____, because ____."

Design Principle(s): Support sense-making

Lesson Synthesis

Students have been solving equations with fraction coefficients in the past few lessons so these percent problems are an application of their prior work. Consider asking some of the following questions to guide the discussion and help students recognize this connection:

- "How are the equations we wrote today related to the equations we have previously written with fractions? How do solution strategies compare?"
- "Can equations be used to solve other types of problems with percents? For example, where we know the part and the whole but not what percent the part is of the whole?" (Yes. For example, the equation $20p = 5$ and its solution $p = \frac{1}{4}$ or $\frac{25}{100}$ tells us that 5 is 25% of 20.)
- "Describe a situation where you know what percent a number is of another, but you don't know that second number. Explain to a partner how you would find the second number."

7.4 Fundraising for the Animal Shelter

Cool Down: 5 minutes

Addressing

- 6.EE.B.6
- 6.EE.B.7

- 6.RP.A.3.c

Student Task Statement

Noah raised \$54 to support the animal shelter, which is 60% of his fundraising goal.

1. Write an equation to represent the situation.
2. What is Noah's fundraising goal? Show or explain how you found it.

Student Response

1. $54 = \frac{60}{100}x$ (or equivalent)
2. \$90, divide both sides of the equation by $\frac{60}{100} = \frac{3}{5}$ to get $x = 54 \cdot \frac{5}{3} = 90$

Student Lesson Summary

If we know that 455 students are in school today and that number represents 70% attendance, we can write an equation to figure out how many students go to the school.

The number of students in school today is known in two different ways: as 70% of the students in the school, and also as 455. If s represents the total number of students who go to the school, then 70% of s , or $\frac{70}{100}s$, represents the number of students that are in school today, which is 455.

We can write and solve the equation:

$$\begin{aligned}\frac{70}{100}s &= 455 \\ s &= 455 \div \frac{70}{100} \\ s &= 455 \cdot \frac{100}{70} \\ s &= 650\end{aligned}$$

There are 650 students in the school.

In general, equations can help us solve problems in which one amount is a percentage of another amount.

Lesson 7 Practice Problems

Problem 1

Statement

A crew has paved $\frac{3}{4}$ of a mile of road. If they have completed 50% of the work, how long is the road they are paving?

Solution

$1\frac{1}{2}$ miles because $\frac{3}{4}$ is half (or 50%) of $\frac{6}{4}$ or $1\frac{1}{2}$.

Problem 2

Statement

40% of x is 35.

- Write an equation that shows the relationship of 40%, x , and 35.
- Use your equation to find x . Show your reasoning.

Solution

a. $0.4x = 35$

b. $x = 87.5$ ($35 \div 0.4 = 87.5$)

Problem 3

Statement

Priya has completed 9 exam questions. This is 60% of the questions on the exam.

- Write an equation representing this situation. Explain the meaning of any variables you use.
- How many questions are on the exam? Show your reasoning.

Solution

a. Answers vary. Sample responses: $9 = \frac{60}{100}q$ or $9 = 0.6q$ where q is the number of questions on the exam.

b. 15 because $9 \div (0.6) = 15$.

Problem 4

Statement

Answer each question. Show your reasoning.

20% of a is 11. What is a ?

75% of b is 12. What is b ?

80% of c is 20. What is c ?

200% of d is 18. What is d ?

Solution

- a. 55
- b. 16
- c. 25
- d. 9

Sample reasoning for "75% of b is 12":

- Using an equation: $\frac{75}{100}b = 12$, so $b = 12 \div \frac{75}{100}$, so $b = 12 \cdot \frac{100}{75}$, so $b = 16$.
- Using a table. To get from the first row to the second row, divide 75 and 12 each by 3. To get from the second to the third row, multiply the 25 and 4 each by 4.

percentage	number
75	12
25	4
100	16

Problem 5

Statement

For the equation $2n - 3 = 7$

- a. What is the variable?
- b. What is the coefficient of the variable?
- c. Which of these is the solution to the equation? 2, 3, 5, 7, n

Solution

- a. n

b. 2

c. 5

(From Unit 6, Lesson 2.)

Problem 6

Statement

Which of these is a solution to the equation $\frac{1}{8} = \frac{2}{5} \cdot x$?

A. $\frac{2}{40}$

B. $\frac{5}{16}$

C. $\frac{11}{40}$

D. $\frac{17}{40}$

Solution

B

(From Unit 6, Lesson 2.)

Problem 7

Statement

Find the quotients.

a. $0.009 \div 0.001$

b. $0.009 \div 0.002$

c. $0.0045 \div 0.001$

d. $0.0045 \div 0.002$

Solution

a. 9

b. 4.5

c. 4.5

d. 2.25

(From Unit 5, Lesson 13.)

Lesson 8: Equal and Equivalent

Goals

- Draw a diagram to represent the value of an expression for a given value of its variable.
- Explain (in writing) that some pairs of expressions are equal for one value of their variable but not for other values.
- Justify (orally, in writing, and through other representations) whether two expressions are “equivalent”, i.e., equal to each other for every value of their variable.

Learning Targets

- I can explain what it means for two expressions to be equivalent.
- I can use a tape diagram to figure out when two expressions are equal.
- I can use what I know about operations to decide whether two expressions are equivalent.

Lesson Narrative

In this lesson students are introduced to the idea of **equivalent expressions**. Two expressions are equivalent if they have the same value no matter what the value of the variable in them. Students use diagrams where the variable is represented by a generic length to decide if expressions are equivalent, and they show that expressions are not equivalent by giving values of the variable that make them unequal. They identify simple equivalent expressions using familiar facts about operations.

Alignments

Addressing

- 6.EE.A.4: Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.
- 6.EE.B.5: Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Instructional Routines

- Algebra Talk
- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports

Required Materials

Graph paper

Required Preparation

Graph paper in addition to grids printed with the tasks may or may not be necessary. It is recommended to have some on hand just in case.

Student Learning Goals

Let's use diagrams to figure out which expressions are equivalent and which are just sometimes equal.

8.1 Algebra Talk: Solving Equations by Seeing Structure

Warm Up: 5 minutes

In this algebra talk, students recall how to solve equations by considering what number can be substituted for the variable to make the equation true. (Note: $x^2 = 49$ of course has another solution if we allow solutions to be negative, but students haven't studied negative numbers yet, and don't study operations with negative numbers until grade 7, so it is unlikely to come up.)

Addressing

- 6.EE.B.5

Instructional Routines

- Algebra Talk
- MLR8: Discussion Supports

Launch

Display one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Student Task Statement

Find a solution to each equation mentally.

$$3 + x = 8$$

$$10 = 12 - x$$

$$x^2 = 49$$

$$\frac{1}{3}x = 6$$

Student Response

- 5
- 2
- 7
- 18

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone have the same strategy but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to ___'s strategy?"
- "Do you agree or disagree? Why?"

Access for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because . . ." or "I noticed ____ so I" Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Design Principle(s): Optimize output (for explanation)

8.2 Using Diagrams to Show That Expressions are Equivalent

20 minutes

Students use diagrams to show that expressions can be equivalent or expressions can be equal for only one value of their variable. Working through these tape diagrams with small whole numbers,

where students can count grids and use lengths to check their results, allows students to begin to generalize about equal and equivalent expressions.

Addressing

- 6.EE.A.4

Instructional Routines

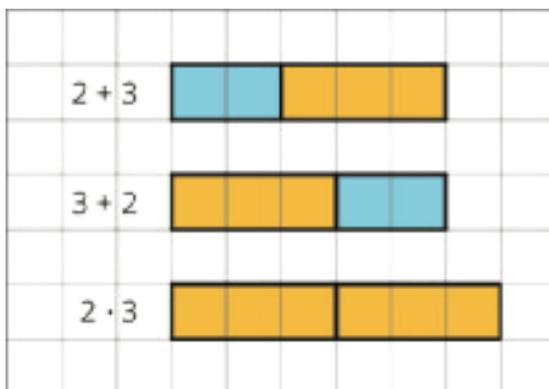
- MLR8: Discussion Supports

Launch

Provide access to graph paper. Ask students to draw diagrams that show:

$$2 + 3 = 3 + 2$$

$$2 + 3 \text{ does not equal } 2 \cdot 3$$



We can tell that $2 + 3$ and $3 + 2$ are equal because the length of the diagrams represent the value of each expression, and the diagrams are the same length. We can tell that $2 \cdot 3$ is not equal to these because this value is represented by the length of its diagram, and it's not the same length as the others. $2 + 3$ and $3 + 2$ are examples of expressions that are not identical, but are equal. Another example students have seen of this phenomenon are fractions like $\frac{1}{2}$ and $\frac{3}{6}$, which are not identical but equal.

When we start talking about expressions that have letters in them, the language gets more complicated, because expressions can be equal or not equal depending on the value the letter represents.

Arrange students in groups of 2. Ask students to work independently on each question and then check in with their partner, discussing and resolving any disagreements. Allow 15 minutes to work and share responses with a partner, followed by a whole-class discussion.

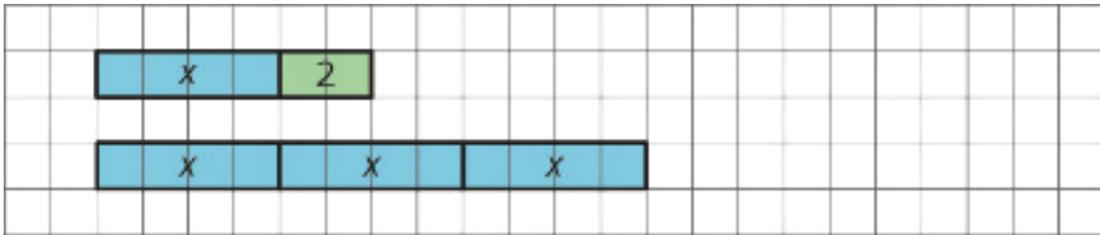
Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Begin with a small-group or whole-class demonstration and think aloud of the first question to remind students how to draw tape diagrams on grids. Keep the worked-out calculations on display for students to reference as they work.

Supports accessibility for: Memory; Conceptual processing

Student Task Statement

Here is a diagram of $x + 2$ and $3x$ when x is 4. Notice that the two diagrams are lined up on their left sides.



In each of your drawings below, line up the diagrams on one side.

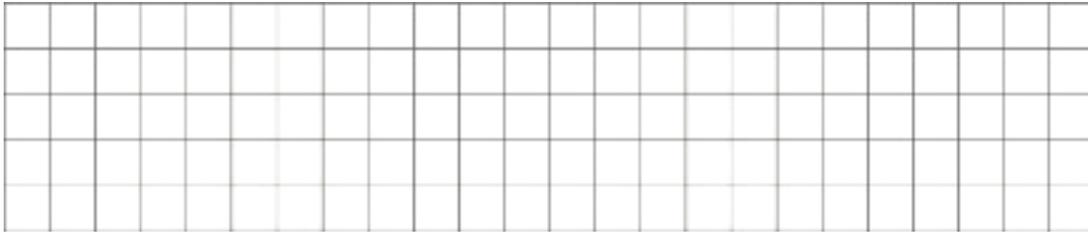
1. Draw a diagram of $x + 2$, and a separate diagram of $3x$, when x is 3.

A blank grid with 15 columns and 4 rows, intended for drawing the first set of diagrams.

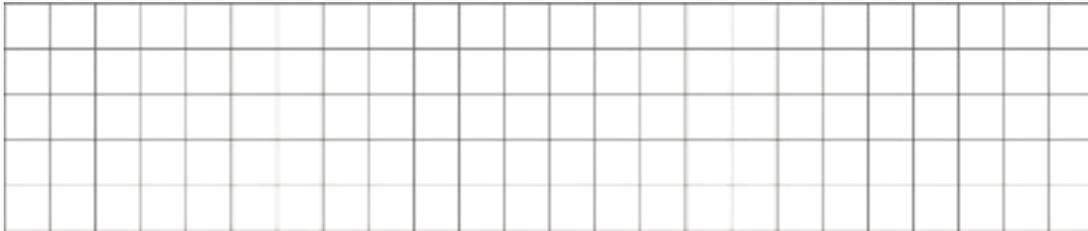
2. Draw a diagram of $x + 2$, and a separate diagram of $3x$, when x is 2.

A blank grid with 15 columns and 4 rows, intended for drawing the second set of diagrams.

3. Draw a diagram of $x + 2$, and a separate diagram of $3x$, when x is 1.



4. Draw a diagram of $x + 2$, and a separate diagram of $3x$, when x is 0.



5. When are $x + 2$ and $3x$ equal? When are they not equal? Use your diagrams to explain.

6. Draw a diagram of $x + 3$, and a separate diagram of $3 + x$.

7. When are $x + 3$ and $3 + x$ equal? When are they not equal? Use your diagrams to explain.

Student Response

1. Diagram shows a length of 5 for $x + 2$ and 9 for $3x$.
2. Diagram shows a length of 4 for $x + 2$ and 6 for $3x$.
3. Diagram shows a length of 3 for $x + 2$ and 3 for $3x$.
4. Diagram shows a length of 2 for $x + 2$ and 0 for $3x$.
5. They are equal when $x = 1$, not equal for other values. Explanations vary. Sample response: we can check the number of boxes or the lengths to see that the expressions have equal value when $x = 1$.
6. Answers vary. Diagrams should be the same length regardless of choice of x .
7. They are always equal. Answers vary. Sample response: The lengths will always be the same, even though one shows the 3 first and one shows x first.

Activity Synthesis

For the first sets of diagrams, if we consider $x + 2 = 3x$, we can see that this is true when x is 1, but not for the other values of x that we tried. For the second set of diagrams, if we consider $x + 3 = 3 + x$ we can see that this equation is always going to be true no matter what the value of x is. We call $x + 3$ and $3 + x$ *equivalent expressions*, because their values are equal no matter what the value of x is.

Access for English Language Learners

Representing, Conversing, Listening: MLR8 Discussion Supports. As students share their explanations for “When are $x + 2$ and $3x$ equal? When are they not equal?,” offer a sentence frame such as, “I know these expressions are equal (or not equal) when _____ because ...” Highlight diagrams that show the connection to the expressions. This will help students use mathematical language as they connect the representations of equal and not equal values of expressions.

Design Principle(s): Maximize meta-awareness

8.3 Identifying Equivalent Expressions

10 minutes

In this activity, students apply what they know about the meaning of operations and their properties to understand what is meant by “equivalent expressions.” The focus is more on building that understanding than it is about doing all the types they eventually need to be able to do.

It is expected that students will reason using what they know about operations on numbers and potentially use diagrams. For example they learned earlier this year that something $\div \frac{1}{3}$ is equivalent to that same thing $\cdot 3$. They can also reason that they know for example that $4 + 4 + 4 = 3 \cdot 4$, so $a + a + a = 3a$.

Addressing

- 6.EE.A.4

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Allow students 5 minutes of quiet work time, followed by a whole-class discussion.

Student Task Statement

Here is a list of expressions. Find any pairs of expressions that are equivalent. If you get stuck, try reasoning with diagrams.

$a + 3$	$a \div \frac{1}{3}$	$\frac{1}{3}a$	$\frac{a}{3}$	a
$a + a + a$	$a \cdot 3$	$3a$	$1a$	$3 + a$

Student Response

- $a + 3$ and $3 + a$
- $a \div \frac{1}{3}$ and $a \cdot 3$
- $a + a + a$ and $3a$ (these are also equivalent to $a \div \frac{1}{3}$ and $a \cdot 3$)
- $\frac{1}{3}a$ and $\frac{a}{3}$
- $1a$ and a

Are You Ready for More?

Below are four questions about equivalent expressions. For each one:

- Decide whether you think the expressions are equivalent.
- Test your guess by choosing numbers for x (and y , if needed).

1. Are $\frac{x \cdot x \cdot x \cdot x}{x}$ and $x \cdot x \cdot x$ equivalent expressions?

2. Are $\frac{x + x + x + x}{x}$ and $x + x + x$ equivalent expressions?

3. Are $2(x + y)$ and $2x + 2y$ equivalent expressions?

4. Are $2xy$ and $2x \cdot 2y$ equivalent expressions?

Student Response

1. Yes
2. No
3. Yes
4. No

Activity Synthesis

Invite students to share their pairs and reasoning. Include students who used diagrams.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Break the class into small discussion groups and then invite a representative from each group to report back to the whole class. This will provide students with additional opportunities to compare strategies and hear from others.

Supports accessibility for: Language; Social-emotional skills; Attention

Access for English Language Learners

Conversing: MLR3 Clarify, Critique, Correct. Use this routine to give students an opportunity to clarify a possible misunderstanding from the class. Display incorrect statement, “ $2xy$ and $2x \cdot 2y$ are equivalent expressions because there are two x 's and two y 's.” Ask students to clarify and critique this statement with a partner. Ask, “What error did this student make? Come up a counterexample to show that these expressions are not equivalent.” This will help students make sense of and define equivalent expressions.

Design Principle(s): Optimize output (for generalization); Cultivate conversation

Lesson Synthesis

The purpose of the discussion is to ensure students understand what is meant by equivalent expressions and how they are different from expressions that are just equal for a given value of their variable. Consider giving them some equivalent expressions, and ask if they can explain why they are equivalent without drawing diagrams. Examples:

- x and $x \cdot 1$
- $x + 1$ and $1 + x$
- $x \cdot 3$ and x
- x and $x + 0$
- $x + x + x$ and $3x$
- $x \div 4$ and $\frac{1}{4}x$

8.4 Decisions About Equivalence

Cool Down: 5 minutes

Addressing

- 6.EE.A.4

Student Task Statement

Decide if the expressions in each pair are equivalent. Explain how you know.

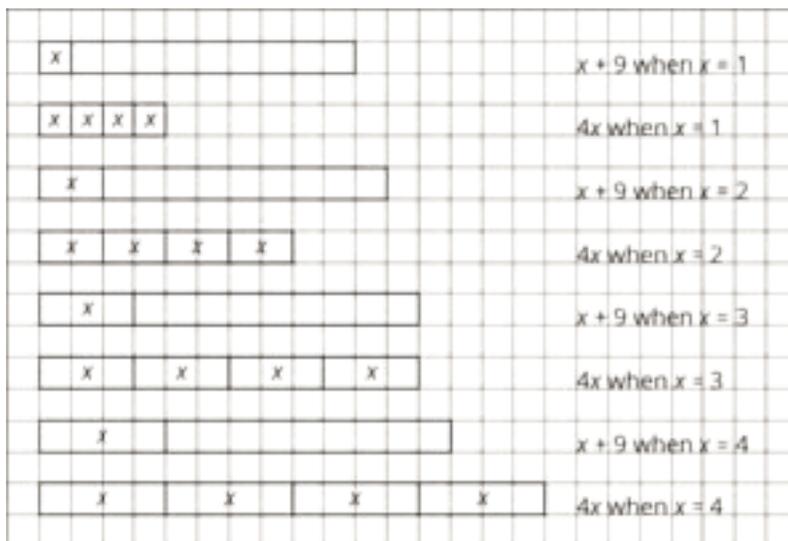
1. $x + x + x + x$ and $4x$
2. $5x$ and $x + 5$

Student Response

1. Equivalent, because the diagrams representing these expressions would have the same length for any value of x .
2. Not equivalent. For example, if $x = 1$, $5x = 5$ and $x + 5 = 6$, so they do not have the same value.

Student Lesson Summary

We can use diagrams showing lengths of rectangles to see when expressions are equal. For example, the expressions $x + 9$ and $4x$ are equal when x is 3, but are not equal for other values of x .



Sometimes two expressions are equal for only one particular value of their variable. Other times, they seem to be equal no matter what the value of the variable.

Expressions that are always equal for the same value of their variable are called **equivalent expressions**. However, it would be impossible to test every possible value of the variable. How can we know for sure that expressions are equivalent? We use the meaning of operations and properties of operations to know that expressions are equivalent. Here are some examples:

- $x + 3$ is equivalent to $3 + x$ because of the commutative property of addition.
- $4 \cdot y$ is equivalent to $y \cdot 4$ because of the commutative property of multiplication.
- $a + a + a + a + a$ is equivalent to $5 \cdot a$ because adding 5 copies of something is the same as multiplying it by 5.
- $b \div 3$ is equivalent to $b \cdot \frac{1}{3}$ because dividing by a number is the same as multiplying by its reciprocal.

In the coming lessons, we will see how another property, the distributive property, can show that expressions are equivalent.



Glossary

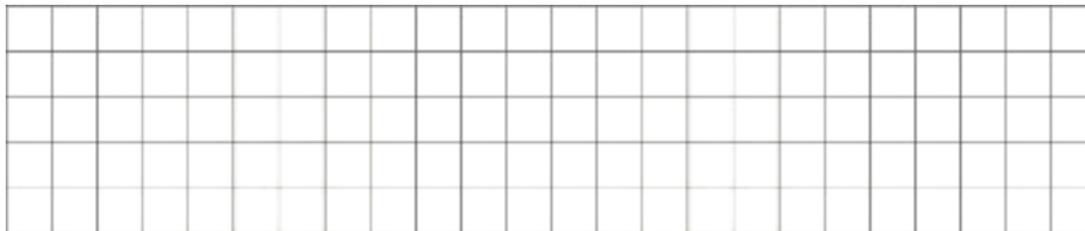
- equivalent expressions

Lesson 8 Practice Problems

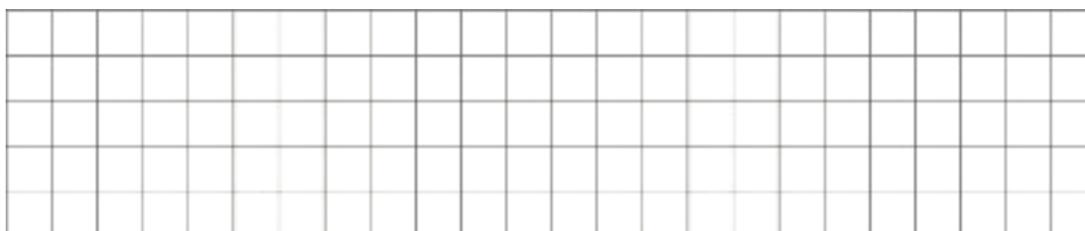
Problem 1

Statement

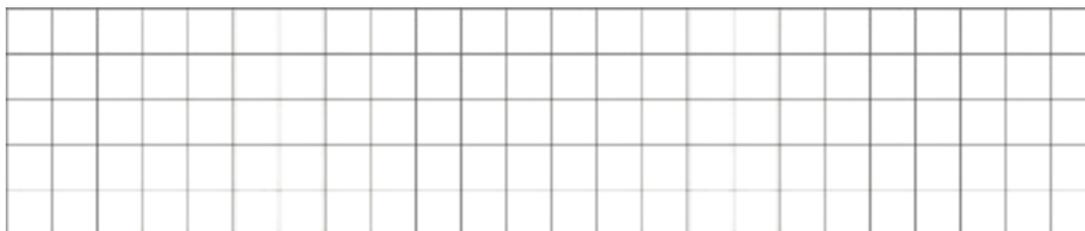
- a. Draw a diagram of $x + 3$ and a diagram of $2x$ when x is 1.



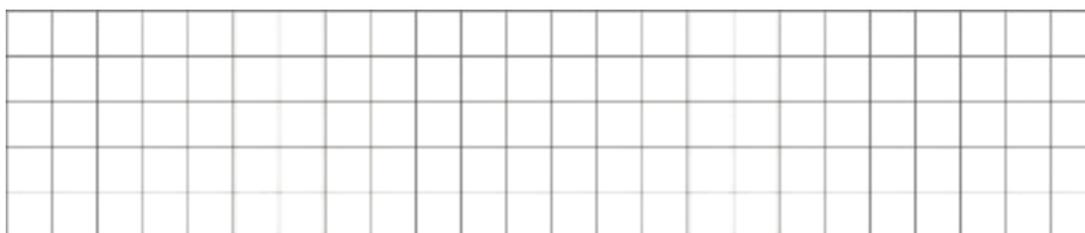
- b. Draw a diagram of $x + 3$ and of $2x$ when x is 2.



- c. Draw a diagram of $x + 3$ and of $2x$ when x is 3.



- d. Draw a diagram of $x + 3$ and of $2x$ when x is 4.



- e. When are $x + 3$ and $2x$ equal? When are they not equal? Use your diagrams to explain.

Solution

1 through 4

x		$x + 3$ when $x = 1$
x	x	$2x$ when $x = 1$
x		$x + 3$ when $x = 2$
x	x	$2x$ when $x = 2$
x		$x + 3$ when $x = 3$
x	x	$2x$ when $x = 3$
x		$x + 3$ when $x = 4$
x	x	$2x$ when $x = 4$

5. When $x = 3$ both expressions are 6. When x is less than 3, $3x$ is less than $x + 3$ and when x is larger than 3, $3x$ becomes larger than $x + 3$.

Problem 2

Statement

- Do $4x$ and $15 + x$ have the same value when x is 5?
- Are $4x$ and $15 + x$ equivalent expressions? Explain your reasoning.

Solution

- Yes, they both have the value of 20.
- No. Equivalent expressions have the same value no matter what number is used in place of the variable. Reasoning varies. Sample reasoning For example, when x is 1, $4x$ has the value 4 but $15 + x$ has the value 16.

Problem 3

Statement

- Check that $2b + b$ and $3b$ have the same value when b is 1, 2, and 3.
- Do $2b + b$ and $3b$ have the same value for all values of b ? Explain your reasoning.
- Are $2b + b$ and $3b$ equivalent expressions?

Solution

- When $b = 1$, they both take the value 3, when $b = 2$ they are both 6, and when $b = 3$ they both have the value 9.
- Yes, $2b + b$ is the same as $3b$. Both can be written as $b + b + b$.
- Yes, for any value of b , both $2b + b$ and $3b$ give 3 times the value of b .

Problem 4

Statement

80% of x is equal to 100.

- Write an equation that shows the relationship of 80%, x , and 100.
- Use your equation to find x .

Solution

- $0.8x = 100$
- $x = 125$

(From Unit 6, Lesson 7.)

Problem 5

Statement

For each story problem, write an equation to represent the problem and then solve the equation. Be sure to explain the meaning of any variables you use.

- Jada's dog was $5\frac{1}{2}$ inches tall when it was a puppy. Now her dog is $14\frac{1}{2}$ inches taller than that. How tall is Jada's dog now?
- Lin picked $9\frac{3}{4}$ pounds of apples, which was 3 times the weight of the apples Andre picked. How many pounds of apples did Andre pick?

Solution

- $t - 14\frac{1}{2} = 5\frac{1}{2}$ or equivalent, where t represents the height of Jada's dog now. Jada's dog is 20 inches tall.
- $9\frac{3}{4} = 3p$ or equivalent, where p represents the weight in pounds of the apples Andre picked. Andre picked $3\frac{1}{4}$ pounds of apples.

(From Unit 6, Lesson 5.)

Problem 6

Statement

Find these products.

- a. $(2.3) \cdot (1.4)$
- b. $(1.72) \cdot (2.6)$
- c. $(18.2) \cdot (0.2)$
- d. $15 \cdot (1.2)$

Solution

- a. 3.22
- b. 4.472
- c. 3.64
- d. 18

(From Unit 5, Lesson 8.)

Problem 7

Statement

Calculate $141.75 \div 2.5$ using a method of your choice. Show or explain your reasoning.

Solution

56.7. Sample reasonings:

- Multiply the dividend and divisor by 10 and calculate $1417.5 \div 25$.
- Multiply the dividend and divisor by 100 and calculate $14175 \div 250$. (Note that the fraction $\frac{14175}{250}$ can be simplified to $\frac{567}{10}$ as both the numerator and denominator are divisible by 25, but simplifying the fraction does not save time because finding $14175 \div 25$ is essentially equivalent to the problem of finding $141.75 \div 2.5$.)
- Write equivalent expression using fractions ($\frac{14175}{100} \div \frac{25}{10}$) and solve by finding $\frac{14175}{100} \cdot \frac{10}{25}$, which equals $\frac{14175}{250}$.

(From Unit 5, Lesson 13.)

Lesson 9: The Distributive Property, Part 1

Goals

- Generate equivalent numerical expressions that are related by the distributive property, and explain (orally or using other representations) the reasoning.
- Use an area diagram to make sense of equivalent numerical expressions that are related by the distributive property.

Learning Targets

- I can use a diagram of a rectangle split into two smaller rectangles to write different expressions representing its area.
- I can use the distributive property to help do computations in my head.

Lesson Narrative

This is the first of three lessons about the distributive property. In this lesson students recall the use of rectangle diagrams to represent the distributive property, and work with equations involving the distribute property with both addition and subtraction.

Alignments

Building On

- 3.MD.C.7.c: Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
- 3.OA.B.5: Apply properties of operations as strategies to multiply and divide. Students need not use formal terms for these properties. Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)

Building Towards

- 6.EE.A.3: Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.
- 6.EE.A.4: Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports
- Number Talk

Student Learning Goals

Let's use the distributive property to make calculating easier.

9.1 Number Talk: Ways to Multiply

Warm Up: 5 minutes

Students perform mental calculations by applying strategies involving the distributive property.

Building On

- 3.OA.B.5

Building Towards

- 6.EE.A.3

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Student Task Statement

Find each product mentally.

$$5 \cdot 102$$

$$5 \cdot 98$$

■ $5 \cdot 999$

Student Response

- 510, because $5 \cdot 102 = 5(100 + 2) = 500 + 10$
- 490, because $5 \cdot 98 = 5(100 - 2) = 500 - 10$
- 4,995, because $5 \cdot 999 = 5(1,000 - 1) = 5,000 - 5$

Activity Synthesis

Once students have had a chance to share a few different ways of reasoning about this product, focus on explanations using the distributive property like $5 \cdot 98 = 5 \cdot (90 + 8)$ or $5 \cdot (100 - 2)$, then writing out the two products from distributing. Remind students of the distributive property, and let them know they will spend the next few lessons working with it.

Access for English Language Learners

Speaking: MLR8 Discussion Supports: Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because . . ." or "I noticed ____ so I" Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Design Principle(s): Optimize output (for explanation)

9.2 Ways to Represent Area of a Rectangle

15 minutes

The purpose of this activity is to remind students of rectangle diagrams they worked with in a previous unit to represent multiplication. It is also to introduce the convention that for example the expression $6 \cdot 3 + 2$ equals 20. If we want the sum to be carried out before the product, we would need to use parentheses like $6 \cdot (3 + 2)$.

Building On

- 3.MD.C.7.c

Building Towards

- 6.EE.A.3
- 6.EE.A.4

Instructional Routines

- MLR8: Discussion Supports

Launch

Allow students 10 minutes of quiet work time, followed by a whole-class discussion.

Access for Students with Disabilities

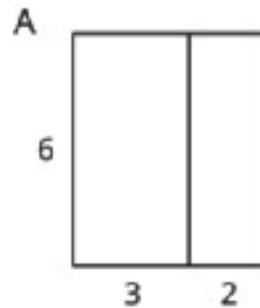
Representation: Internalize Comprehension. Activate or supply background knowledge about finding area. Some students may benefit from a review of the rectangle diagrams they used in a previous unit to represent multiplication.

Supports accessibility for: Memory; Conceptual processing

Student Task Statement

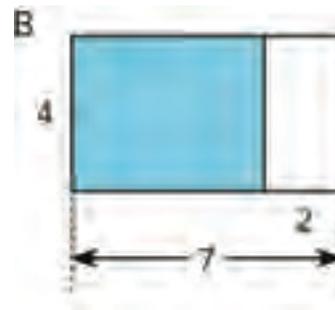
1. Select **all** the expressions that represent the area of the large, outer rectangle in figure A. Explain your reasoning.

- $6 + 3 + 2$
- $6 \cdot 3 + 6 \cdot 2$
- $6 \cdot 3 + 2$
- $6 \cdot 5$
- $6(3 + 2)$
- $6 \cdot 3 \cdot 2$



2. Select **all** the expressions that represent the area of the shaded rectangle on the left side of figure B. Explain your reasoning.

- $4 \cdot 7 + 4 \cdot 2$
- $4 \cdot 7 \cdot 2$
- $4 \cdot 5$
- $4 \cdot 7 - 4 \cdot 2$
- $4(7 - 2)$
- $4(7 + 2)$
- $4 \cdot 2 - 4 \cdot 7$



Student Response

1. $6 \cdot 3 + 6 \cdot 2$, $6 \cdot 5$, and $6(3 + 2)$. Explanations vary. Sample responses:
- These are all equal to 30.
 - $6 \cdot 3 + 6 \cdot 2$ is the sum of the areas of the two pieces, and the other two expressions are just the area of the whole rectangle.

2. $4 \cdot 5$, $4 \cdot 7 - 4 \cdot 2$, and $4(7 - 2)$. Explanations vary. Sample responses:
- These are all equal to 20.
 - $4 \cdot 7 - 4 \cdot 2$ is the area of the whole rectangle minus the unshaded part, and the other two expressions are just the area of the shaded part.

Activity Synthesis

Students may conclude that $6 \cdot 3 + 2$ represents the area of the rectangle. This is a good opportunity to introduce a convention. When we have multiplication and addition in the same expression, it is the convention that the multiplication is done first. So $6 \cdot 3 + 2$ equals $18 + 2$, or 20, so it doesn't represent the area of the rectangle, which we know to be 30 square units. If you want the addition to be done first, you need to use parentheses. Therefore, $6(3 + 2)$ does represent the area of the large rectangle. Remind students that "next to" implies multiplication.

By using the rectangle, we can tell that $6(3 + 2) = 6 \cdot 3 + 6 \cdot 2$. This is another example of two expressions that are equivalent because of the distributive property.

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Display sentence frames to support whole-class discussion. For example, "Expression ____ matches figure A because ____." or "Figure ____ cannot be represented by expression ____ because ____." Invite students to share their responses with a partner, and prompt them to rehearse what they will say when they share with the whole class. Rehearsing provides students with additional opportunities to clarify their thinking.

Design Principle(s): Optimize output (for explanation)

9.3 Distributive Practice

15 minutes

This is for practice going back and forth with the distributive property using numbers, but also to make the point that invoking the distributive property can help you do computations in your head. Some expressions that would be difficult to brute force become simpler after using the distributive property to write an equivalent expression.

Note that there is more than one way to rewrite the last row. For example, $24 - 16$ could also be written as $2(12 - 8)$ where $(12 - 8)$ is the difference of two terms. A *term* is a single number or variable, or variables and numbers multiplied together. This is fine, since there is no reason to insist that students use the greatest common factor at this time. Students should recognize that there is more than one factor that would work, and that the resulting expressions are equivalent. In a subsequent unit students will explicitly study the idea of a greatest common factor.

Building Towards

- 6.EE.A.3

Instructional Routines

- MLR2: Collect and Display

Launch

Give students the following setup: Suppose your business makes 15 items for \$17 each and sells them for \$20 each. How would you find your profit? One way is to write $15 \cdot 20 - 15 \cdot 17$. But that's a lot of calculations! An easier way to get the answer is to write $15(20 - 17)$, which is easier to figure out in your head.

Allow students 10 minutes of quiet work time, followed by a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts. For example, after students have completed the first two rows of the table, check-in with either select groups of students or the whole class. Invite students to share the strategies they have used so far as well as any questions they have before continuing.

Supports accessibility for: Organization; Attention

Anticipated Misconceptions

Students might understand how to expand an expression with parentheses but struggle with how to approach a sum. Encourage students to think about the rectangle diagrams they have seen and draw a diagram of a partitioned rectangle. Ask students what the sum represents and help them to see that it can represent the sum of the areas of the two smaller rectangles. Remind students that the rectangles have the same width, and ask what that width might have been to produce the two areas, what factor the two areas have in common. Then have them consider the other factors (the lengths) that would produce those products for the areas.

Student Task Statement

Complete the table. If you get stuck, skip an entry and come back to it, or consider drawing a diagram of two rectangles that share a side.

column 1	column 2	column 3	column 4	value
$5 \cdot 98$	$5(100 - 2)$	$5 \cdot 100 - 5 \cdot 2$	$500 - 10$	490
$33 \cdot 12$	$33(10 + 2)$			
		$3 \cdot 10 - 3 \cdot 4$	$30 - 12$	
	$100(0.04 + 0.06)$			
		$8 \cdot \frac{1}{2} + 8 \cdot \frac{1}{4}$		
			$9 + 12$	
			$24 - 16$	

Student Response

column 1	column 2	column 3	column 4	value
$5 \cdot 98$	$5(100 - 2)$	$5 \cdot 100 - 5 \cdot 2$	$500 - 10$	490
$33 \cdot 12$	$33(10 + 2)$	$33 \cdot 10 + 33 \cdot 2$	$330 + 66$	396
$3 \cdot 6$	$3(10 - 4)$	$3 \cdot 10 - 3 \cdot 4$	$30 - 12$	18
$100 \cdot 0.1$	$100(0.04 + 0.06)$	$100 \cdot 0.04 + 100 \cdot 0.06$	$4 + 6$	10
$8 \cdot \frac{3}{4}$	$8 \left(\frac{1}{2} + \frac{1}{4} \right)$	$8 \cdot \frac{1}{2} + 8 \cdot \frac{1}{4}$	$4 + 2$	6
$3 \cdot 7$	$3(3 + 4)$	$3 \cdot 3 + 3 \cdot 4$	$9 + 12$	21
$8 \cdot 1$	$8(3 - 2)$	$8 \cdot 3 - 8 \cdot 2$	$24 - 16$	8

Note that there is more than one correct response for the last row. For example, $24 - 16$ could also be rewritten as $2(12 - 8)$ or $4(6 - 4)$.

Are You Ready for More?

1. Use the distributive property to write two expressions that equal 360. (There are many correct ways to do this.)
2. Is it possible to write an expression like $a(b + c)$ that equals 360 where a is a fraction? Either write such an expression, or explain why it is impossible.

3. Is it possible to write an expression like $a(b - c)$ that equals 360? Either write such an expression, or explain why it is impossible.
4. How many ways do you think there are to make 360 using the distributive property?

Student Response

1. Answers vary. Possible expressions: $36(7 + 3)$, $10(20 + 16)$
2. Yes. For example, $\frac{1}{2}(700 + 20)$.
3. Yes. For example, $12(50 - 20)$.
4. There are infinite such expressions if you allow fractions or decimals, and quite a large number indeed even if you don't.

Activity Synthesis

Invite students to share their strategies and reasoning. Include students who used diagrams of partitioned rectangles. Ask if they noticed any interesting patterns, or if they want to share some examples of their own of calculations that can be made simpler by using the distributive property to write an equivalent expression.

Access for English Language Learners

Representing, Speaking, Listening: MLR2 Collect and Display. During whole-class discussion, create a visual display to record a list of the strategies students describe, such as diagrams of partitioned rectangles. Amplify use of mathematical words and phrases that students use, such as patterns, equivalent, sum, etc. that describe their process. Ask students which strategies worked best for them, and ask them to name what they have in common. Remind students that they can borrow language and strategies from the display as needed.

Design Principle(s): Support sense-making; Maximize meta-awareness

Lesson Synthesis

Arrange students in groups of 2. One partner writes a product of the form $a(b + c)$ or $a(b - c)$. The other partner writes an equivalent expression using the distributive property, then each student evaluates their expression. The partners compare which computation was simpler, took less time, etc. Then have one partner write a sum and the other see if they can write an equivalent expression. Evaluate, compare, repeat as time allows. (It may be necessary to give students three numbers to use for the first round, instead of asking them to think of 3 numbers to use.)

9.4 Complete the Equation

Cool Down: 5 minutes

Building Towards

- 6.EE.A.3

Student Task Statement

Write a number or expression in each empty box to create true equations.

$$1. 7(3 + 5) = \boxed{} + \boxed{}$$

$$2. 15 - 10 = \boxed{}(3 - 2)$$

Student Response

$$1. 7(3 + 5) = \boxed{21} + \boxed{35} \text{ or } 7(3 + 5) = \boxed{7 \cdot 3} + \boxed{7 \cdot 5} \text{ or equivalent}$$

$$2. 15 - 10 = \boxed{5}(3 - 2)$$

Student Lesson Summary

A **term** is a single number or variable, or variables and numbers multiplied together. Some examples of terms are 10, $8x$, ab , and $7yz$.

When we need to do mental calculations, we often come up with ways to make the calculation easier to do mentally.

Suppose we are grocery shopping and need to know how much it will cost to buy 5 cans of beans at 79 cents a can. We may calculate mentally in this way:

$$\begin{array}{r} 5 \cdot 79 \\ 5 \cdot 70 + 5 \cdot 9 \\ 350 + 45 \\ 395 \end{array}$$

In general, when we multiply two terms (or factors), we can break up one of the factors into parts, multiply each part by the other factor, and then add the products. The result will be the same as the product of the two original factors. When we break up one of the factors and multiply the parts we are using the distributive property.

The distributive property also works with subtraction. Here is another way to find $5 \cdot 79$:

$$\begin{array}{r} 5 \cdot 79 \\ 5 \cdot (80 - 1) \\ 400 - 5 \\ 395 \end{array}$$

Glossary

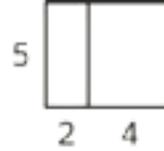
- term

Lesson 9 Practice Problems

Problem 1

Statement

Select all the expressions that represent the area of the large, outer rectangle.



- A. $5(2 + 4)$
- B. $5 \cdot 2 + 4$
- C. $5 \cdot 2 + 5 \cdot 4$
- D. $5 \cdot 2 \cdot 4$
- E. $5 + 2 + 4$
- F. $5 \cdot 6$

Solution

["A", "C", "F"]

Problem 2

Statement

Draw and label diagrams that show these two methods for calculating $19 \cdot 50$.

- First find $10 \cdot 50$ and then add $9 \cdot 50$.
- First find $20 \cdot 50$ and then take away 50.

Solution

- a. A 19-by-50 rectangle partitioned into two rectangles with dimensions 10 by 50 and 9 by 50.
- b. A 20-by-50 rectangle partitioned into a 1 by 50 and a 19 by 50. Shading or arrows indicate that the 19-by-50 rectangle is the one we want.

Problem 3

Statement

Complete each calculation using the distributive property.

$$\begin{array}{l} 98 \cdot 24 \\ (100 - 2) \cdot 24 \\ \dots \end{array}$$

$$\begin{array}{l} 21 \cdot 15 \\ (20 + 1) \cdot 15 \\ \dots \end{array}$$

$$\begin{array}{l} 0.51 \cdot 40 \\ (0.5 + 0.01) \cdot 40 \\ \dots \end{array}$$

Solution

- a. $(100 - 2) \cdot 24 = 2,400 - 48 = 2,352$
- b. $(20 + 1) \cdot 15 = 300 + 15 = 315$
- c. $(0.5 + 0.01) \cdot 40 = 20 + 0.4 = 20.4$

Problem 4

Statement

A group of 8 friends go to the movies. A bag of popcorn costs \$2.99. How much will it cost to get one bag of popcorn for each friend? Explain how you can calculate this amount mentally.

Solution

\$23.92. Reasoning varies. Sample reasoning: If the bags of popcorn were \$3 each, then this would be \$24 ($8 \cdot 3$). But $2.99 = 3 - 0.01$. So one cent has to be subtracted for each of the 8 bags of popcorn. That leaves \$23.92.

Problem 5

Statement

- a. On graph paper, draw diagrams of $a + a + a + a$ and $4a$ when a is 1, 2, and 3. What do you notice?
- b. Do $a + a + a + a$ and $4a$ have the same value for any value of a ? Explain how you know.

Solution

- a. See diagram

a. How old will Kiran's aunt be when Kiran is:

15 years old?

30 years old?

x years old?

b. How old will Kiran be when his aunt is 60 years old?

Solution

a. 32 years old ($15 + 17 = 32$), 47 years old ($30 + 17 = 47$), $x + 17$ years old.

b. 43 years old ($60 - 17 = 43$).

(From Unit 6, Lesson 6.)

Lesson 10: The Distributive Property, Part 2

Goals

- Generate algebraic expressions that represent the area of a rectangle with an unknown length.
- Justify (orally and using other representations) that algebraic expressions that are related by the distributive property are equivalent.

Learning Targets

- I can use a diagram of a split rectangle to write different expressions with variables representing its area.

Lesson Narrative

The purpose of this lesson is to extend the work with the distributive property in the previous lesson to situations where one of the quantities is represented by a variable, as in $2(x + 3) = 2x + 2 \cdot 3$. Students use the same rectangle diagrams as before to represent these situations, reinforcing the idea that the work they do with expressions is simply an extension of the work they previously did with numbers. They see that the distributive property can arise out of writing areas of rectangles in two different ways, which emphasizes the idea of equivalent expressions as being two different ways of writing the same quantity.

Alignments

Addressing

- 6.EE.A.2: Write, read, and evaluate expressions in which letters stand for numbers.
- 6.EE.A.3: Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.
- 6.EE.A.4: Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.

Instructional Routines

- MLR6: Three Reads
- MLR8: Discussion Supports

Student Learning Goals

Let's use rectangles to understand the distributive property with variables.

10.1 Possible Areas

Warm Up: 5 minutes

Students consider the area of a rectangle to reason about equivalent expressions and review symbolic notation for showing multiplication. This work supports the activities that follow, where students will explore and apply the distributive property by considering expressions for the areas of rectangles.

Addressing

- 6.EE.A.2

Launch

Allow students 2–3 minutes of quiet work time, followed by a whole-class discussion.

Anticipated Misconceptions

If students are struggling but they haven't drawn a diagram of a rectangle, suggest that they do so.

Student Task Statement

1. A rectangle has a width of 4 units and a length of m units. Write an expression for the area of this rectangle.
2. What is the area of the rectangle if m is:

3 units?	2.2 units?	$\frac{1}{5}$ unit?
----------	------------	---------------------
3. Could the area of this rectangle be 11 square units? Why or why not?

Student Response

1. $4m$ (or equivalent)
2. 12 square units, 8.8 square units, $\frac{4}{5}$ square units
3. Yes, the area could be 11 square units. m would have to be $\frac{11}{4}$ units, since $4 \cdot \frac{11}{4} = 11$.

Activity Synthesis

Select students to share their response to each question. Points to highlight:

- Rectangle areas can be found by multiplying length by width.
- Both $4m$ and $m \cdot 4$ are expressions for the area of this rectangle. These are equivalent expressions.
- Lengths don't have to be whole numbers. Neither do areas.

10.2 Partitioned Rectangles When Lengths are

Unknown

10 minutes

In this activity, students use expressions with variables to represent lengths of sides and areas of rectangles. These expressions are used to help students understand the distributive property and its use in creating equivalent expressions.

Addressing

- 6.EE.A.3
- 6.EE.A.4

Instructional Routines

- MLR6: Three Reads

Launch

Arrange students in groups of 2–3. Allow students 5 minutes to work with their groups, followed by a quick whole-class discussion.

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Maintain a display of important terms and vocabulary. During the launch take time to review the following terms from previous lessons that students will need to access for this activity: variable, area, length, width, expression.

Supports accessibility for: Memory; Language

Access for English Language Learners

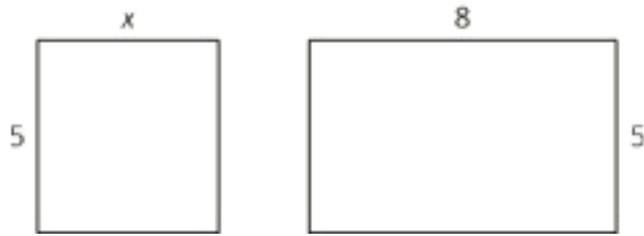
Reading: MLR6 Three Reads. Use this routine to support reading comprehension. In the first read, students read the situation with the goal of comprehending the text (e.g., the problem is about two rectangles with some dimensions given). In the second read, ask students to analyze the text to understand the mathematical structure (e.g., the width of both rectangles is 5. The length of one rectangle is 8 and the other rectangle's length is x). In the third read, ask students to brainstorm possible strategies to answer the follow-up questions. This routine helps students in reading comprehension and negotiating information in the text with a partner through mathematical language.

Design Principle(s): Support sense-making

Student Task Statement

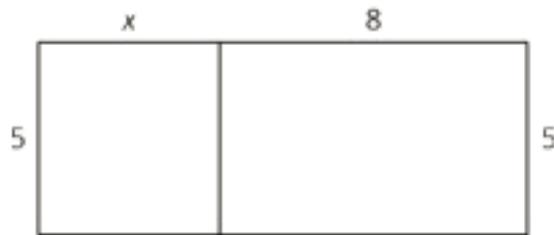
1. Here are two rectangles. The length and width of one rectangle are 8 and 5. The width of the other rectangle is 5, but its length is unknown so we labeled it x .

Write an expression for the sum of the areas of the two rectangles.



2. The two rectangles can be composed into one larger rectangle as shown.

What are the width and length of the new, large rectangle?



3. Write an expression for the total area of the large rectangle as the product of its width and its length.

Student Response

1. $5x + 40$ or $5x + 5 \cdot 8$
2. The length is $x + 8$ and width is 5 (or vice versa).
3. $5(x + 8)$ or $(x + 8) \cdot 5$

Activity Synthesis

Solicit students' responses to the first and third questions. Display two expressions for all to see. Expressions that are equivalent to these are fine; ensure everyone agrees that one is an acceptable response to the first question and the other is an acceptable response to the third question.

$$5 \cdot x + 5 \cdot 8$$

$$5(x + 8)$$

Ask students to look at the two expressions and think of something they notice and wonder. Here are some things that students might notice.

- The 5 appears twice in one expression and only once in the other.
- These expressions must be equivalent to each other, because they each represent the area of the same region.

- These look like an example of the distributive property, but with a letter.

10.3 Areas of Partitioned Rectangles

20 minutes

In this activity students are presented with several partitioned rectangles. They identify the length and width for each rectangle, and then write expressions for the area in two different ways: first as the product of the length times the width, where one of these measurements will be expressed as a sum, and then as the sum of the areas of the smaller rectangles that make up the large rectangle. Students reason that these two expressions must be equal since they both represent the total area of the partitioned rectangle. In this way they see several examples of the distributive property. Students may choose to assign values to the variable in each rectangle to check that their expressions for area are equal.

Addressing

- 6.EE.A.3
- 6.EE.A.4

Instructional Routines

- MLR8: Discussion Supports

Launch

Keep students in the same groups. Allow students 10 minutes to work with their groups, followed by a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Begin with a small-group or whole-class demonstration and think aloud of how to complete the first row of the table to remind students how to write expressions for the length, width and total area. Keep the worked-out calculations on display for students to reference as they work.

Supports accessibility for: Memory; Conceptual processing

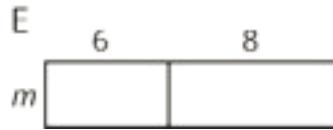
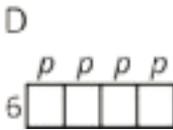
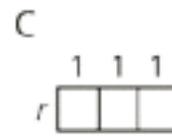
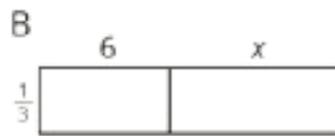
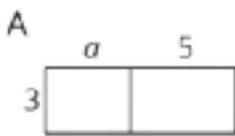
Access for English Language Learners

Listening, Representing: MLR8 Discussion Supports. To develop students' meta-awareness for writing equivalent expressions, demonstrate a think aloud about representing rectangle areas. As you talk, use mathematical language and highlight the connection between the written expressions and the chosen partitioned rectangle. Say, "For Rectangle A, if the width is 3, how can I write an expression for the length (e.g., $a + 5$)? If the length is represented as $a + 5$, then what is one way I can write an expression for the total area (e.g., a product of $3(a + 5)$)? What can be a second way (e.g., $3a + 3 \cdot 5$)?"

Design Principle(s): Maximize meta-awareness

Student Task Statement

For each rectangle, write expressions for the length and width and two expressions for the total area. Record them in the table. Check your expressions in each row with your group and discuss any disagreements.



rectangle	width	length	area as a product of width times length	area as a sum of the areas of the smaller rectangles
A				
B				
C				
D				
E				
F				

Student Response

Some answers vary. Width and length can be interchanged. Sample responses:

rectangle	width	length	area as a product of width times length	area as a sum of the areas of the smaller rectangles
A	3	$a + 5$	$3(a + 5)$	$3a + 15$
B	$\frac{1}{3}$	$6 + x$ or $x + 6$	$\frac{1}{3}(6 + x)$	$2 + \frac{1}{3}x$
C	r	3 or $1 + 1 + 1$	$r(1 + 1 + 1)$ or $3r$	$r + r + r$ or $1r + 1r + 1r$
D	6	$p + p + p + p$ or $4p$ as we saw above	$6(p + p + p + p)$ or $6(4p)$ or $24p$	$6p + 6p + 6p + 6p$
E	m	$6 + 8$ or 14	$m(6 + 8)$ or $14m$	$6m + 8m$
F	5	$3x + 8$	$5(3x + 8)$	$15x + 40$

Are You Ready for More?

Here is an area diagram of a rectangle.

	y	z
w	A	24
x	18	72

1. Find the lengths w , x , y , and z , and the area A . All values are whole numbers.
2. Can you find another set of lengths that will work? How many possibilities are there?

Student Response

There are four solutions to this problem. The value of A is always 6.

- $w = 1, x = 3, y = 6, z = 24, A = 6$
- $w = 2, x = 6, y = 3, z = 12, A = 6$
- $w = 3, x = 9, y = 2, z = 8, A = 6$
- $w = 6, x = 18, y = 1, z = 4, A = 6$

Activity Synthesis

The purpose of the discussion is to help students understand the distributive property and how it can be used to generate equivalent expressions.

Remind students about the term “coefficient” and the convention of writing the coefficient before the variable. Ask students why we consider one expression for area “a sum” and the other “a product” even though both expressions contain sums and products. (For example, we take $6x + 8$ to be a sum because we are adding two terms $6x$ and 8 , even though $6x$ is actually a product of 6 and x . Likewise, we take $2(3x + 4)$ to be a product because we note that 2 is multiplied by the quantity $3x + 4$, which contains a sum and a product.)

When appropriate, encourage students to use the word *term* to refer to things like $3a$, $6p$, and $15x$.

Finally, we want to make the point another way that each pair of expressions they wrote are equivalent to each other. For example, we can see that $5(3x + 8)$ and $15x + 40$ are equivalent because they both represent the area of figure G. However, we want to reinforce what equivalent means here. Ask each pair of students to choose any number, and evaluate both $5(3x + 8)$ and $15x + 40$ using the value they chose for x . No matter what value they choose, the expressions will yield the same value. For example, if they choose 2 as the value for x , $5(3 \cdot 2 + 8)$ is 70, and $15 \cdot 2 + 40$ is also 70.

Lesson Synthesis

Ask students to compare the expressions they saw today with the expressions they saw in the last lesson. How are they alike? How are they different? Students should see that their work with

expressions containing variables is an extension of the work they did in the last lesson with numbers.

Invite students to share any disagreements that arose within their group and how they were resolved.

10.4 Which Expressions Represent Area?

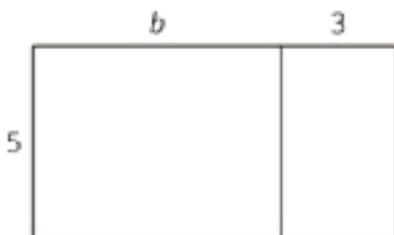
Cool Down: 5 minutes

Addressing

- 6.EE.A.3
- 6.EE.A.4

Student Task Statement

Select all the expressions that represent the large rectangle's total area.



- $3(5 + b)$
- $5(b + 3)$
- $5b + 15$
- $15 + 5b$
- $3 \cdot 5 + 3b$

Student Response

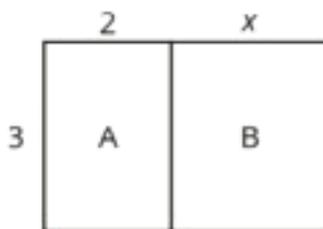
$$5(b + 3)$$

$$5b + 15$$

$$15 + 5b$$

Student Lesson Summary

Here is a rectangle composed of two smaller rectangles A and B.



Based on the drawing, we can make several observations about the area of the rectangle:

- One side length of the large rectangle is 3 and the other is $2 + x$, so its area is $3(2 + x)$.
- Since the large rectangle can be decomposed into two smaller rectangles, A and B, with no overlap, the area of the large rectangle is also the sum of the areas of rectangles A and B: $3(2) + 3(x)$ or $6 + 3x$.

- Since both expressions represent the area of the large rectangle, they are equivalent to each other. $3(2 + x)$ is equivalent to $6 + 3x$.

We can see that multiplying 3 by the sum $2 + x$ is equivalent to multiplying 3 by 2 and then 3 by x and adding the two products. This relationship is an example of the *distributive property*.

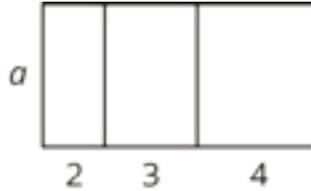
$$3(2 + x) = 3 \cdot 2 + 3 \cdot x$$

Lesson 10 Practice Problems

Problem 1

Statement

Here is a rectangle.



- Explain why the area of the large rectangle is $2a + 3a + 4a$.
- Explain why the area of the large rectangle is $(2 + 3 + 4)a$.

Solution

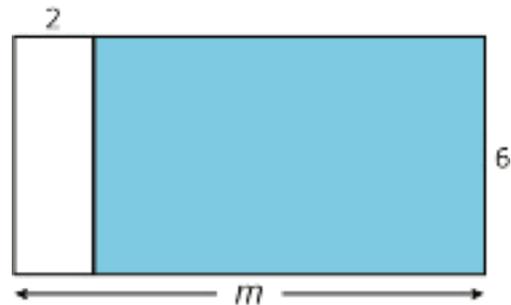
- The large rectangle is made up of three smaller rectangles whose areas are $2a$, $3a$, and $4a$.
- The large rectangle has height a and length $2 + 3 + 4$, so its area is $(2 + 3 + 4)a$.

Problem 2

Statement

Is the area of the shaded rectangle $6(2 - m)$ or $6(m - 2)$?

Explain how you know.



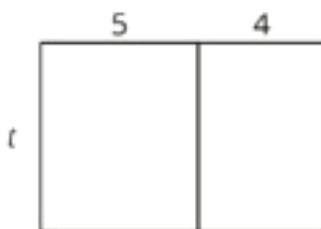
Solution

$6(m - 2)$. The width of the shaded rectangle is 6 . The length is what is left over if 2 is removed from m , so $m - 2$. So the area of the rectangle is $6(m - 2)$.

Problem 3

Statement

Choose the expressions that do *not* represent the total area of the rectangle. Select all that apply.



- A. $5t + 4t$
- B. $t + 5 + 4$
- C. $9t$
- D. $4 \cdot 5 \cdot t$
- E. $t(5 + 4)$

Solution

["B", "D"]

Problem 4

Statement

Evaluate each expression mentally.

- a. $35 \cdot 91 - 35 \cdot 89$
- b. $22 \cdot 87 + 22 \cdot 13$
- c. $\frac{9}{11} \cdot \frac{7}{10} - \frac{9}{11} \cdot \frac{3}{10}$

Solution

- a. 70, Sample reasoning: $35 \cdot 91 - 35 \cdot 89 = 35 \cdot (91 - 89) = 35 \cdot 2 = 70$
- b. 2,200, Sample reasoning: $22 \cdot 87 + 22 \cdot 13 = 22 \cdot (87 + 13) = 22 \cdot 100 = 2,200$
- c. $\frac{36}{110}$, Sample reasoning: $\frac{9}{11} \cdot \frac{7}{10} - \frac{9}{11} \cdot \frac{3}{10} = \frac{9}{11} \left(\frac{7}{10} - \frac{3}{10} \right) = \frac{9}{11} \cdot \frac{4}{10} = \frac{36}{110}$

(From Unit 6, Lesson 9.)

Problem 5

Statement

Select all the expressions that are equivalent to $4b$.

A. $b + b + b + b$

B. $b + 4$

C. $2b + 2b$

D. $b \cdot b \cdot b \cdot b$

E. $b \div \frac{1}{4}$

Solution

["A", "C", "E"]

(From Unit 6, Lesson 8.)

Problem 6

Statement

Solve each equation. Show your reasoning.

$$111 = 14a$$

$$13.65 = b + 4.88$$

$$c + \frac{1}{3} = 5\frac{1}{8}$$

$$\frac{2}{5}d = \frac{17}{4}$$

$$5.16 = 4e$$

Solution

a. $a = \frac{111}{14}$ (or equivalent)

b. $b = 8.77$

c. $c = 4\frac{19}{24}$ (or equivalent)

d. $d = \frac{85}{8}$ (or equivalent)

e. $e = 1.29$ (or equivalent)

(From Unit 6, Lesson 4.)

Problem 7

Statement

Andre ran $5\frac{1}{2}$ laps of a track in 8 minutes at a constant speed. It took Andre x minutes to run each lap. Select **all** the equations that represent this situation.



A. $(5\frac{1}{2})x = 8$

B. $5\frac{1}{2} + x = 8$

C. $5\frac{1}{2} - x = 8$

D. $5\frac{1}{2} \div x = 8$

E. $x = 8 \div (5\frac{1}{2})$

F. $x = (5\frac{1}{2}) \div 8$

Solution

["A", "E"]

(From Unit 6, Lesson 2.)

Lesson 11: The Distributive Property, Part 3

Goals

- Draw a diagram to justify that two expressions that are related by the distributive property are equivalent.
- Explain (orally) how to use the distributive property to identify or generate equivalent algebraic expressions.
- Use the distributive property to write equivalent algebraic expressions, including where the common factor is a variable.

Learning Targets

- I can use the distributive property to write equivalent expressions with variables.

Lesson Narrative

This is an optional lesson to practice identifying and writing equivalent expressions using the distributive property. If your students don't need additional practice at this point, this lesson can be skipped (or saved for a review days later) without missing any new material.

Alignments

Addressing

- 6.EE.A.2: Write, read, and evaluate expressions in which letters stand for numbers.
- 6.EE.A.3: Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.
- 6.EE.A.4: Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.

Instructional Routines

- MLR7: Compare and Connect
- MLR8: Discussion Supports

Student Learning Goals

Let's practice writing equivalent expressions by using the distributive property.

11.1 The Shaded Region

Warm Up: 5 minutes

Students reflect on equivalent expressions that represent the area of a shaded rectangle which is part of a larger rectangle of unknown width.

Addressing

- 6.EE.A.2
- 6.EE.A.3

Launch

Allow students 2 minutes of quiet work time, followed by a whole-class discussion.

Student Task Statement

A rectangle with dimensions 6 cm and w cm is partitioned into two smaller rectangles.

Explain why each of these expressions represents the area, in cm^2 , of the shaded region.

- $6w - 24$
- $6(w - 4)$



Student Response

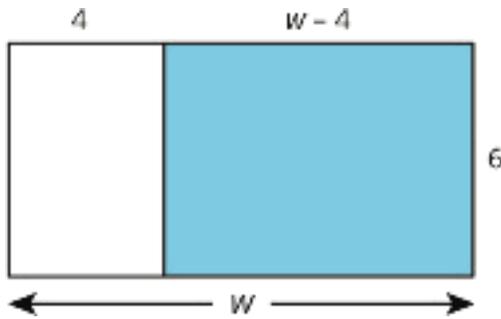
Answers vary. Sample responses:

- $6w - 24$: The area, in cm^2 , of the entire rectangle is $6w$. The area of the unshaded rectangle is $6 \cdot 4$ or 24 cm^2 . Subtracting the two, $6w - 24$, gives the area of the shaded rectangle.
- $6(w - 4)$: The length of the shaded rectangle is $w - 4$. Its width is 6 cm, so its area, in cm^2 , is $6(w - 4)$.

Activity Synthesis

Ask students to share their reasoning. $6w - 24$ should be straightforward: the area of the entire rectangle is $6w$, and the area of the unshaded portion is $6 \cdot 4$ or 24, so the area of the shaded portion is $6w - 24$.

Students may have more trouble with $6(w - 4)$. The key is to understand that the longer side of the shaded portion can be represented by $w - 4$.



11.2 Matching to Practice Distributive Property

Optional: 15 minutes

Students practice finding expressions that are equivalent because of the distributive property.

Addressing

- 6.EE.A.4

Instructional Routines

- MLR7: Compare and Connect

Launch

Allow students 10 minutes of quiet work time, followed by a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Remind students that they can use rectangle diagrams to help them match expressions.

Supports accessibility for: Social-emotional skills; Conceptual processing

Student Task Statement

Match each expression in column 1 to an equivalent expression in column 2. If you get stuck, consider drawing a diagram.

Column 1

1. $a(1 + 2 + 3)$

2. $2(12 - 4)$

3. $12a + 3b$

4. $\frac{2}{3}(15a - 18)$

5. $6a + 10b$

6. $0.4(5 - 2.5a)$

7. $2a + 3a$

Column 2

• $3(4a + b)$

• $12 \cdot 2 - 4 \cdot 2$

• $2(3a + 5b)$

• $(2 + 3)a$

• $a + 2a + 3a$

• $10a - 12$

• $2 - a$

Student Response

1. $a(1 + 2 + 3)$ and $a + 2a + 3a$

2. $2(12 - 4)$ and $12 \cdot 2 - 4 \cdot 2$

3. $12a + 3b$ and $3(4a + b)$

4. $\frac{2}{3}(15a - 18)$ and $10a - 12$

5. $6a + 10b$ and $2(3a + 5b)$

6. $0.4(5 - 2.5a)$ and $2 - a$

7. $2a + 3a$ and $(2 + 3)a$

Activity Synthesis

Invite students to share whether any of the matches were difficult to find and how they worked through the challenge.

Access for English Language Learners

Speaking: MLR7 Compare and Connect. Ask students to consider what is the same and what is different between the representations of expressions in each column. Highlight and demonstrate mathematical language used (e.g., distributive property, distribute, product, sum, difference, coefficient) to make connections among the matched expressions. These exchanges strengthen students' mathematical language use and reasoning about equivalent expressions in general and the distributive property specifically.

Design Principle(s): Maximize meta-awareness

11.3 Writing Equivalent Expressions Using the

Distributive Property

Optional: 15 minutes

Students practice working back and forth writing equivalent expressions with the distributive property.

Addressing

- 6.EE.A.3

Instructional Routines

- MLR8: Discussion Supports

Launch

Allow students 10 minutes of quiet work time, followed by a whole-class discussion.

Student Task Statement

The distributive property can be used to write equivalent expressions. In each row, use the distributive property to write an equivalent expression. If you get stuck, consider drawing a diagram.

product	sum or difference
$3(3 + x)$	
	$4x - 20$
$(9 - 5)x$	
	$4x + 7x$
$3(2x + 1)$	
	$10x - 5$
	$x + 2x + 3x$
$\frac{1}{2}(x - 6)$	
$y(3x + 4z)$	
	$2xyz - 3yz + 4xz$

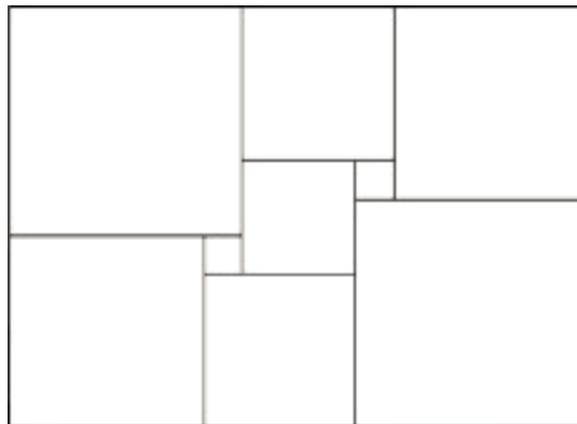
Student Response

product	sum or difference
$3(3 + x)$	$9 + 3x$
$4(x - 5)$	$4x - 20$
$(9 - 5)x$	$9x - 5x$
$(4 + 7)x$	$4x + 7x$
$3(2x + 1)$	$6x + 3$
$5(2x - 1)$	$10x - 5$
$x(1 + 2 + 3)$	$x + 2x + 3x$
$\frac{1}{2}(x - 6)$	$\frac{1}{2}x - 3$
$y(3x + 4z)$	$3xy + 4zy$
$z(2xy - 3y + 4x)$	$2xyz - 3yz + 4xz$

Note that in cases where factoring happens, expressions equivalent to these are also acceptable. For example, for $4x - 20$, equivalent expressions are $2(2x - 10)$ and $20(\frac{1}{5}x - 1)$ in addition to $4(x - 5)$.

Are You Ready for More?

This rectangle has been cut up into squares of varying sizes. Both small squares have side length 1 unit. The square in the middle has side length x units.



- Suppose that x is 3. Find the area of each square in the diagram. Then find the area of the large rectangle.

2. Find the side lengths of the large rectangle assuming that x is 3. Find the area of the large rectangle by multiplying the length times the width. Check that this is the same area you found before.
3. Now suppose that we do not know the value of x . Write an expression for the side lengths of the large rectangle that involves x .

Student Response

1. Answers are given in a sequence in which they can be derived:
Small squares: 1 square unit each
Center square: 9 square units
Top center: 16 square units
Top right: 25 square units
Bottom right: 36 square units
Bottom center: 16 square units
Bottom left: 25 square units
Top left: 36 square units
The area of the large rectangle is the sum of these numbers: 165 square units.
2. 11 units by 15 units. $11 \cdot 15 = 165$.
3. Answers vary, depending on how much they rewrite their expressions or on whether they combine like terms. Sample response: the length is $2x + (2x - 1)$, or $4x - 1$ units, and the width is $2x + (1 + x) + (2 + x)$, or $4x + 3$ units.

Activity Synthesis

Invite students to explain how they knew, when working backwards, what to put in front of the parentheses and what remained inside.

Access for English Language Learners

Listening, Representing: MLR8 Discussion Supports. To develop students' meta-awareness, think-aloud as you write an equivalent expression using the distributive property. As you talk, demonstrate mathematical language about the relationship between expressions on the same row of the table. For the second row, ask, "What number do I need that is a factor of both $4x$ and 20 (e.g., it's 4)? Because $4x$ equals 4 times x and since 20 equals 4 times 5, we can write $4x - 20$ as a product $4(x - 5)$."

Design Principle(s): Maximize meta-awareness

Lesson Synthesis

You might want to have students work on a creative visual display for the classroom that shows their understanding of the distributive property in both directions.

11.4 Writing Equivalent Expressions

Cool Down: 5 minutes

Addressing

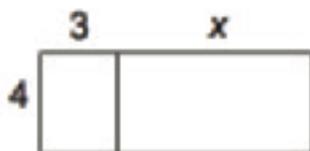
- 6.EE.A.3
- 6.EE.A.4

Student Task Statement

1. Use the distributive property to write an expression that is equivalent to $12 + 4x$.
2. Draw a diagram that shows the two expressions are equivalent.

Student Response

1. Answers vary. Sample response: $4(3 + x)$
2. Answers vary. Sample response:



Student Lesson Summary

The distributive property can be used to write a sum as a product, or write a product as a sum. You can always draw a partitioned rectangle to help reason about it, but with enough practice, you should be able to apply the distributive property without making a drawing.

Here are some examples of expressions that are equivalent due to the distributive property.

$$9 + 18 = 9(1 + 2)$$

$$2(3x + 4) = 6x + 8$$

$$2n + 3n + n = n(2 + 3 + 1)$$

$$11b - 99a = 11(b - 9a)$$

$$k(c + d - e) = kc + kd - ke$$

Lesson 11 Practice Problems

Problem 1

Statement

For each expression, use the distributive property to write an equivalent expression.

- a. $4(x + 2)$
- b. $(6 + 8) \cdot x$
- c. $4(2x + 3)$
- d. $6(x + y + z)$

Solution

- a. $4x + 4 \cdot 2$
- b. $6x + 8x$
- c. $8x + 4 \cdot 3$
- d. $6x + 6y + 6z$

Expressions that are equivalent to these are also acceptable, for example, $4x + 8$ for the first one.

Problem 2

Statement

Priya rewrites the expression $8y - 24$ as $8(y - 3)$. Han rewrites $8y - 24$ as $2(4y - 12)$. Are Priya's and Han's expressions each equivalent to $8y - 24$? Explain your reasoning.

Solution

Yes, the distributive property shows that each expression is equivalent to $8y - 24$.

Problem 3

Statement

Select all the expressions that are equivalent to $16x + 36$.

A. $16(x + 20)$

B. $x(16 + 36)$

C. $4(4x + 9)$

D. $2(8x + 18)$

E. $2(8x + 36)$

Solution

["C", "D"]

Problem 4

Statement

The area of a rectangle is $30 + 12x$. List at least 3 possibilities for the length and width of the rectangle.

Solution

Answers vary. Sample responses:

length	width
$10 + 4x$	3
$5 + 2x$	6
$15 + 6x$	2
$60 + 24x$	$\frac{1}{2}$
$3 + 1.2x$	10

Problem 5

Statement

Select **all** the expressions that are equivalent to $\frac{1}{2}z$.


$$10 = 4a$$

$$5b = 17.5$$

$$1.036 = 10c$$

$$0.6d = 1.8$$

$$15 = 0.1e$$

Solution

a. $a = 2.5$

b. $b = 3.5$

c. $c = 0.1036$

d. $d = 3$

e. $e = 150$

(From Unit 6, Lesson 5.)

Section: Expressions with Exponents

Lesson 12: Meaning of Exponents

Goals

- Describe (orally and in writing) a pattern that could be expressed using repeated multiplication.
- Generate and evaluate numerical expressions involving whole-number exponents.
- Interpret expressions with exponents larger than 3, and comprehend the phrase “to the power” or “to the” (in spoken language).

Learning Targets

- I can evaluate expressions with exponents and write expressions with exponents that are equal to a given number.
- I understand the meaning of an expression with an exponent like 3^5 .

Lesson Narrative

In their prior work with area and volume, students encountered expressions in which a number was squared or cubed. This lesson extends that work by considering exponent notation for any positive whole number exponent. In the warm-up, students first consider a dot pattern where the number of dots is repeatedly multiplied by 3 in each successive level. They next consider a situation involving money that repeatedly doubles, and engage in MP8 by connecting the repeated calculations to expressions involving exponents. Students then make use of the new shorthand notation to write expressions with exponents that evaluate to a given number.

Alignments

Addressing

- 6.EE.A.1: Write and evaluate numerical expressions involving whole-number exponents.

Building Towards

- 6.EE.A.1: Write and evaluate numerical expressions involving whole-number exponents.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR8: Discussion Supports
- Notice and Wonder

Student Learning Goals

Let’s see how exponents show repeated multiplication.

12.1 Notice and Wonder: Dots and Lines

Warm Up: 5 minutes

The purpose of this warm-up is to give students an opportunity to look for multiplication patterns in an image. While there are many things students may notice and wonder, the focus of the whole-group discussion should be the fact that each dot branches out to three more dots of a different color. These connections mean we have repeatedly growing groups of 3, so we can multiply by 3 to find the number of dots and lines at various stages. In the image, there are many other patterns students may see with the dots, lines and dot colors. Record or otherwise validate their observations, but don't dwell too long here.

Building Towards

- 6.EE.A.1

Instructional Routines

- Notice and Wonder

Launch

Arrange students in groups of 2. Tell students that they will look at a picture, and their job is to think of at least one thing they notice and at least one thing they wonder about the picture. Display the problem for all to see and give 1 minute of quiet think time. Ask students to give a signal when they have noticed or wondered about something. Tell them to share the things they noticed and wondered with a partner.

Access for Students with Disabilities

Representation: Internalize Comprehension. Guide information processing and visualization. To support working memory, show the image for a longer period of time. Students may also benefit from being explicitly told not to count the dots, but instead to look for helpful structure within the image.

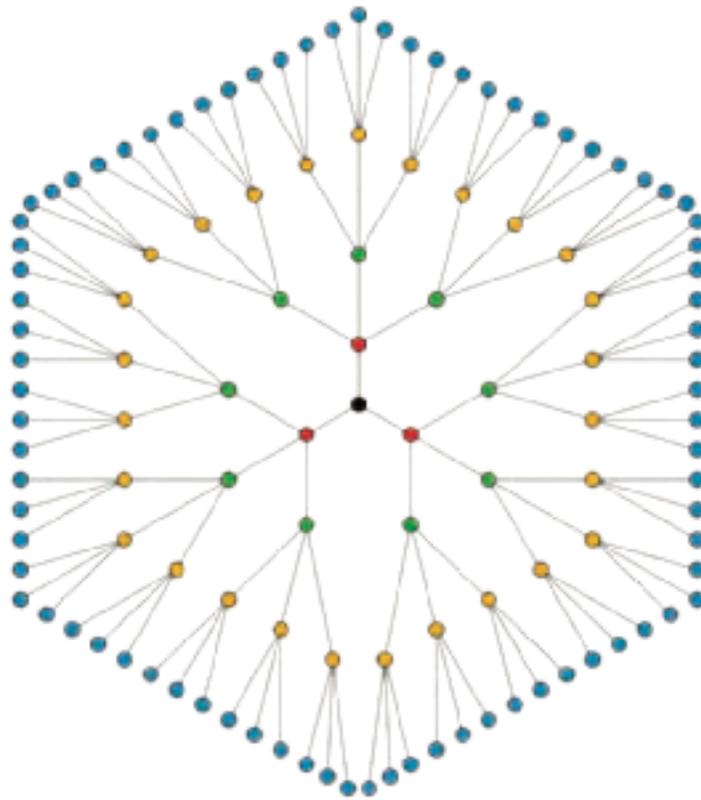
Supports accessibility for: Memory; Organization

Anticipated Misconceptions

Some students may try to count the dots in the two outer levels. To encourage students to use the patterns in the image, ask them if there is an easier way they could use their count from the level before to determine the next one.

Student Task Statement

What do you notice? What do you wonder?



Student Response

Answers vary. Possible responses:

Things students may notice:

- It looks like a hexagon.
- There are different colored dots.
- The black dot is the center.
- Each dot has three lines off of it.
- There is a vertical line of symmetry (and two other lines of symmetry).
- Each dot branches outward to three more dots of a different color.
- We can multiply the number of dots in one layer by 3 to find the number of dots in the next.

Things students may wonder:

- How many dots are in the outer layer?
- How many dots would there be in the next layer if we drew it?

- What would happen if there were only two dots connected to each one?

Activity Synthesis

After giving students a chance to share what they noticed and wondered with a partner, ask a few students to share with the whole group. Record and display their responses for all to see. After each response, ask the class if they agree or disagree.

12.2 The Genie's Offer

20 minutes (there is a digital version of this activity)

The purpose of this task is to show a simple context where exponent notation is naturally useful. The task lends itself to connecting repeated calculations with an expression involving exponents (MP8). This motivates creating a shorthand notation that can be used to answer the questions.

Addressing

- 6.EE.A.1

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

You find a brass bottle that looks really old. When you rub some dirt off the bottle, a genie appears! The genie offers you a reward. You must choose one:

- Take \$50,000; or
- Take a magical \$1 coin. The coin will turn into two coins on the first day. The two coins will turn into four coins on the second day. The four coins will double to 8 coins on the third day. The genie explains the doubling will continue for 28 days.

Ask students to close their books or devices. Display the scenario above for all to see, or explain it verbally. Ask students, "What do you notice? What do you wonder?" It is natural to wonder which is the better option. Poll the class and record the results. If possible, show the first few screens from the applet at ggbm.at/hvJbDbjg to help students see how the coins double each day, keeping the "Count" hidden. Use the Play and Pause buttons in the lower left corner of the screen. If it cannot be projected for all to see, ask students to describe what the first four days of the second offer would look like. Draw their descriptions for all to see.

Distribute scientific calculators to students or be ready to display [desmos.com/scientific](https://www.desmos.com/scientific). Follow with 5 minutes of quiet work time for students to complete the first two questions and then pause for discussion. Draw students' attention to the third question. Ask them, "How would you use the calculator to figure this out?" After a minute of quiet think time, solicit responses. Tell students how to calculate with exponents on the calculator and make the point that exponent notation is much more convenient for calculation and communication than writing out all the repeated factors. Give students 5 minutes to complete the last two questions, followed by whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Provide appropriate reading accommodations and supports to ensure students access to written directions, word problems and other text-based content. Display an image or video of a genie to activate prior knowledge of the context of the doubling problem.

Supports accessibility for: Language; Conceptual processing

Anticipated Misconceptions

Students might evaluate 2^5 as $2 \cdot 5$. If this happens, have them make a table showing the number of coins accumulated each day. It will soon be apparent that many more than 10 coins will be accumulated after 5 days.

Student Task Statement

You find a brass bottle that looks really old. When you rub some dirt off of the bottle, a genie appears! The genie offers you a reward. You must choose one:

\$50,000 or a magical \$1 coin.

The coin will turn into two coins on the first day. The two coins will turn into four coins on the second day. The four coins will double to 8 coins on the third day. The genie explains the doubling will continue for 28 days.

1. The number of coins on the third day will be $2 \cdot 2 \cdot 2$. Write an equivalent expression using exponents.
2. What do 2^5 and 2^6 represent in this situation? Evaluate 2^5 and 2^6 without a calculator. Pause for discussion.
3. How many days would it take for the number of magical coins to exceed \$50,000?
4. Will the value of the magical coins exceed a million dollars within the 28 days? Explain or show your reasoning.

Student Response

1. 2^3
2. The number of coins on the 5th day, the number of coins on the 6th day. $2^5 = 32$, $2^6 = 64$
3. 16 days
4. Answers vary. Sample response: On the 16th day, there are 65,536 coins. The next day there will be over 130,000, the next over 260,000, the next over half a million, so the next over a million. It will only take 4 more days to get to a million.

Are You Ready for More?

A scientist is growing a colony of bacteria in a petri dish. She knows that the bacteria are growing and that the number of bacteria doubles every hour.

When she leaves the lab at 5 p.m., there are 100 bacteria in the dish. When she comes back the next morning at 9 a.m., the dish is completely full of bacteria. At what time was the dish half full?

Student Response

8 a.m., because a half-full dish will take one hour to become a full dish.

Activity Synthesis

The goal of the discussion is for students to connect the idea of multiplying n factors of 2 to get the expression 2^n . This notation is convenient for communicating and computing when repeated multiplication is involved. Here are some questions for discussion:

- “How did you know what 2^5 represents? How did you evaluate it?”
- “How many times greater is 2^8 than 2^7 ? Could you answer this without evaluating both expressions?”
- “Why does writing expressions with exponents make them easier to work with and understand?”
- “How did you organize your work to answer questions 3 and 4?”
- “Use your calculator to find how much money you would get at the end of 28 days. Does it surprise you?”

Access for English Language Learners

Speaking and Listening: MLR1 Stronger and Clearer Each Time. Use this routine to provide students with a structured opportunity to refine their explanations about whether or not the value of the magical coins will exceed a million dollars within the 28 days. Give students time to meet with 2–3 partners, to share and get feedback on the first draft of their response. Provide students with prompts they can use to give each other feedback (e.g., “Can you explain how . . .”, “You should expand on . . .” etc.). This will give students an opportunity to strengthen their ideas and clarify their language.

Design Principle(s): Optimize output (for explanation)

12.3 Make 81

10 minutes

In this activity, students apply the meaning of exponents to practice writing and evaluating exponential expressions. Students gain experience experimenting with equivalent numerical expressions and engage in looking for structure (MP7) when they replace a portion of an expression with something equivalent to it.

Addressing

- 6.EE.A.1

Instructional Routines

- MLR8: Discussion Supports

Launch

An exponent is used to indicate multiplying a number by itself. For example, 2^4 means $2 \cdot 2 \cdot 2 \cdot 2$, so 2^4 equals 16.

There are different ways to say 2^4 . You can say “two raised to the power of four” or “two to the fourth power” or just “two to the fourth.”

Give students 5 minutes of quiet work time, followed by whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge about computing expressions with exponents. Allow students to use calculators to ensure inclusive participation in the activity.

Supports accessibility for: Memory; Conceptual processing

Access for English Language Learners

Speaking, Listening: MLR8 Discussion Supports. Before students work on the task, use choral repetition to help students develop the language used to state expressions with exponents. For example, display the expression $2^3 \cdot 2$ and the phrase “two to the third power times two” for all to see. Read the statement once, and invite students to repeat as you point to each part of the expression. This routine will develop students’ meta-awareness of the language of exponents.

Design Principle(s): Optimize output (for justification); Maximize meta-awareness

Anticipated Misconceptions

In expressions with multiplication and exponents, students might parse the expression incorrectly, or apply the order of operations incorrectly. For example, they might interpret $2^3 \cdot 2$ as 4^3 instead

of 2 multiplied by itself 4 times. Students might also multiply the base and exponent, $9^2 = 18$ instead of $9^2 = 81$.

To help students overcome these misconceptions, ask students to rewrite the expressions without exponents. For example, $2^3 \cdot 2 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$. Once students have demonstrated conceptual understanding of exponents and why exponents come before multiplication in the order of operations, students will be able to evaluate without rewriting expressions.

Student Task Statement

1. Here are some expressions. All but one of them equals 16. Find the one that is *not* equal to 16 and explain how you know.

$$2^3 \cdot 2$$

$$4^2$$

$$\frac{2^5}{2}$$

$$8^2$$

2. Write three expressions containing exponents so that each expression equals 81.

Student Response

1. 8^2 is not equal to 16. Explanations vary. Sample responses: $2^3 \cdot 2$ equals 16 because 2 is a factor 4 times. 4^2 is 16 because it is $4 \cdot 4$. $\frac{2^5}{2}$ is 16 because it is $32 \div 2$. 8^2 is not 16 because it is $8 \cdot 8$, not $8 \cdot 2$.

2. Answers vary. Sample responses: 9^2 , $\frac{9^3}{9}$, 3^4 , $\frac{3^5}{3}$, $3^3 \div \frac{1}{3}$, $3 \cdot 3^3$, $3^2 \cdot 3^2$, 81^1 , $\frac{81^2}{81}$

Activity Synthesis

The purpose of the discussion is to help students consider that, just as they did with addition, subtraction, multiplication, and division, they can express numbers in multiple ways using exponents. After reviewing the first question and addressing any misconceptions, ask students to share some of their expressions for the second question. Look for expressions that go beyond the more obvious choices and that involve other operations. If no students found more creative expressions, challenge them to come up with one more expression that involves an exponent and another operation. Ask students to share with the class what they came up with and have other students confirm that they evaluate to 81.

You might also ask students to write expressions that do *not* equal 81, but look like the error in question 1.

Lesson Synthesis

Ask students to think back to the dot pattern they saw at the beginning of the lesson and address some of the things they may have wondered:

- “Can we write an expression for the number of dots in the outer layer?” (3 with an exponent that is the number of the layer)

- “Based on that expression, how many dots were in the layer before? In the next layer? In the one after that? What part of the expression would change as we move from layer to layer? In what way would it change?” (The exponent changes, it increases by one for each higher layer and decreases by one for each lower layer)
- “What would happen if there were only 2 dots connected to each one? 4 dots? What part of the expression would change?” (The number that is being repeatedly multiplied would change.)
- “How can we use what we know about one layer to find out how many dots there are in another layer?” (Multiply or divide by 3 the same number of times as the layers moved. Notice incidentally that $a^n = a \cdot a^{n-1}$ but without using exponential notation.)

12.4 More 3's

Cool Down: 5 minutes

Addressing

- 6.EE.A.1

Student Task Statement

3^5 equals 243. Explain how to use that fact to quickly evaluate 3^6 .

Student Response

$$3^6 = 3^5 \cdot 3 = 729$$

Student Lesson Summary

When we write an expression like 2^n , we call n the exponent.

If n is a positive whole number, it tells how many factors of 2 we should multiply to find the value of the expression. For example, $2^1 = 2$, and $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$.

There are different ways to say 2^5 . We can say “two raised to the power of five” or “two to the fifth power” or just “two to the fifth.”

Lesson 12 Practice Problems

Problem 1

Statement

Select all the expressions that are equivalent to 64.

- A. 2^6
- B. 2^8
- C. 4^3
- D. 8^2
- E. 16^4
- F. 32^2

Solution

["A", "C", "D"]

Problem 2

Statement

Select all the expressions that equal 3^4 .

- A. 7
- B. 4^3
- C. 12
- D. 81
- E. 64
- F. 9^2

Solution

["D", "F"]

Problem 3

Statement

4^5 is equal to 1,024. Evaluate each expression.

a. 4^6

b. 4^4

c. $4^3 \cdot 4^2$

Solution

a. $4^6 = 4^5 \cdot 4 = 1,024 \cdot 4 = 4096$

b. $4^4 = 4^5 \div 4 = 1,024 \div 4 = 256$

c. $4^3 \cdot 4^2 = 4^5 = 1024$

Problem 4

Statement

$6^3 = 216$. Using exponents, write three more expressions whose value is 216.

Solution

Answers vary. Sample responses: $6^2 \cdot 6$, $\frac{6^4}{6}$, $2^3 \cdot 3^3$

Problem 5

Statement

Find two different ways to rewrite $3xy + 6yz$ using the distributive property.

Solution

Answers vary. Sample responses: $3(xy + 2yz)$, $3y(x + 2z)$, $y(3x + 6z)$.

(From Unit 6, Lesson 11.)

Problem 6

Statement

Solve each equation.

$$a - 2.01 = 5.5$$

$$b + 2.01 = 5.5$$

$$10c = 13.71$$

$$100d = 13.71$$

Solution

- a. $a = 7.51$
- b. $b = 3.49$
- c. $c = 1.371$
- d. $d = 0.1371$

(From Unit 6, Lesson 5.)

Problem 7

Statement

Which expressions represent the total area of the large rectangle? Select **all** that apply.



- A. $6(m + n)$
- B. $6n + m$
- C. $6n + 6m$
- D. $6mn$
- E. $(n + m)6$

Solution

["A", "C", "E"]

(From Unit 6, Lesson 10.)

Problem 8

Statement

Is each statement true or false? Explain your reasoning.

a. $\frac{45}{100} \cdot 72 = \frac{45}{72} \cdot 100$

b. 16% of 250 is equal to 250% of 16

Solution

a. False. Sample reasoning: The left side equals $45 \cdot \frac{72}{100}$ and the right side equals $45 \cdot \frac{100}{72}$. The left side is less than 45 and the right side is greater than 45.

b. True. Sample reasoning: 16% of 250 equals $\frac{16}{100} \cdot 250$. 250% of 16 is $\frac{250}{100} \cdot 16$. Each of these is equal to $\frac{16 \cdot 250}{100}$

(From Unit 3, Lesson 16.)

Lesson 13: Expressions with Exponents

Goals

- Critique (orally and in writing) arguments that claim two different numerical expressions are equal.
- Justify (orally and in writing) whether numerical expressions involving whole-number exponents are equal.

Learning Targets

- I can decide if expressions with exponents are equal by evaluating the expressions or by understanding what exponents mean.

Lesson Narrative

In this lesson, students analyze the structure of expressions (MP7) to apply their understanding of exponents. While they practice using the notation of expressions with exponents, students recall and apply their prior understanding of operations and connect those understandings to the meaning of exponents. They write, interpret, and evaluate expressions with exponent notation where the exponents are whole numbers and the bases may be whole numbers, fractions, or decimals. Students also apply their new understanding from earlier in the unit about determining whether equations are true or false.

Alignments

Addressing

- 6.EE.A.1: Write and evaluate numerical expressions involving whole-number exponents.

Instructional Routines

- MLR8: Discussion Supports
- Which One Doesn't Belong?

Student Learning Goals

Let's use the meaning of exponents to decide if equations are true.

13.1 Which One Doesn't Belong: Twos

Warm Up: 5 minutes

This warm-up prompts students to compare expressions. It encourages students to explain their reasoning, hold mathematical conversations, and gives you the opportunity to hear how they use terminology and talk about characteristics of the expressions in comparison to one another.

Addressing

- 6.EE.A.1

Instructional Routines

- Which One Doesn't Belong?

Launch

Arrange students in groups of 2–4. Display the questions for all to see. Ask students to indicate when they have noticed one expression that doesn't belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their small group. In their small groups, tell each student to share their reasoning why a particular question doesn't belong and together find at least one reason each question doesn't belong.

Student Task Statement

Which one doesn't belong?

$$2 \cdot 2 \cdot 2 \cdot 2$$

$$2^4$$

$$16$$

$$4 \cdot 2$$

Student Response

Answers vary. Sample responses:

$2 \cdot 2 \cdot 2 \cdot 2$ doesn't belong because it is the only expression that shows 4 repeated factors being multiplied.

16 doesn't belong because it is the only one that is just a number.

2^4 doesn't belong because it is the only expression that uses exponents.

$4 \cdot 2$ doesn't belong because it is the only expression that is not equal to 16.

Activity Synthesis

Ask each group to share one reason why a particular expression does not belong. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given are correct.

During the discussion, ask students to explain the meaning of any terminology they use, such as "exponents." Also, press students on unsubstantiated claims.

13.2 Is the Equation True?

15 minutes

The purpose of this task is to give students experience working with exponential expressions and to promote making use of structure (MP7) to compare exponential expressions. To this end, encourage students to rewrite expressions in a different form rather than evaluate them to a single number.

For students who are accustomed to viewing the equal sign as a directive that means “perform an operation,” tasks like these are essential to shifting their understanding of the meaning of the equal sign to one that supports work in algebra, namely, “The expressions on either side have the same value.”

Addressing

- 6.EE.A.1

Instructional Routines

- MLR8: Discussion Supports

Launch

Before students start working, it may be helpful to demonstrate how someone would figure out whether or not an equation is true without evaluating each expression. For example:

Is $4^2 = 2^3$ true? Well, let's see. We can rewrite each side like this:

$$4 \cdot 4 = 2 \cdot 2 \cdot 2$$

Then we can replace one of those $2 \cdot 2$'s with a 4, like this:

$$4 \cdot 4 = 4 \cdot 2$$

Now we can tell this equation is not true.

These problems can also be worked by directly evaluating expressions, which is fine, as it serves as practice evaluating exponential expressions.

Access for Students with Disabilities

Representation: Internalize Comprehension. Begin with the demonstration as described in the launch to support connections between new situations involving evaluating exponential expressions and prior understandings. Use color or annotations to highlight what changes and what stays the same at each step.

Supports accessibility for: Conceptual processing; Visual-spatial processing

Student Task Statement

Decide whether each equation is true or false, and explain how you know.

1. $2^4 = 2 \cdot 4$

2. $3 + 3 + 3 + 3 + 3 = 3^5$

3. $5^3 = 5 \cdot 5 \cdot 5$

4. $2^3 = 3^2$

5. $16^1 = 8^2$

6. $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = 4 \cdot \frac{1}{2}$

7. $\left(\frac{1}{2}\right)^4 = \frac{1}{8}$

8. $8^2 = 4^3$

Student Response

1. False, $2^4 = 2 \cdot 2 \cdot 2 \cdot 2$ and $2 \cdot 4 = 2 \cdot 2 \cdot 2$
2. False, $3 + 3 + 3 + 3 + 3 = 15$ and $3^5 = 243$
3. True, because $5 \cdot 5 \cdot 5$ is what 5^3 means
4. False, $2^3 = 8$ and $3^2 = 9$
5. False, $16^1 = 16$ and $8^2 = 64$
6. False, $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$ and $4 \cdot \frac{1}{2} = 2$
7. False, $\frac{1}{8}$ is $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$
8. True, both sides of the equation equal 64

Activity Synthesis

Invite students who evaluated the expressions and students who used structure or the meaning of exponents and operations to present their work. Compare and connect the strategies by noting where the use of structure might prove more efficient, where evaluating might be simpler, where thinking about the meaning of exponents and operations can make the true or false determination simpler.

Some guiding questions to highlight the meaning of exponents:

- “Can we switch the order with exponents like we can with addition and multiplication—specifically, are a^b and b^a equivalent? How do you know?” (No, you can try different values of a and b or use the meaning of exponents to see that a multiplied b times is not always the same as b multiplied a times.)
- “What change can we make to the equation $3 + 3 + 3 + 3 + 3 = 3^5$ to make it true?” (Change addition to multiplication on the left, or change the exponent to multiplication on the right.)
- “Your friend claims the equation $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = 4 \cdot \frac{1}{2}$ is true. What do you think they are misunderstanding? How can you convince them it is false?” (Possibly by saying that the left side shows multiplying $\frac{1}{2}$ 4 times, but the right side shows multiplying 4 by $\frac{1}{2}$, which means *adding* 4 copies of $\frac{1}{2}$, not multiplying them. You can show similar examples with other

numbers, but the best way to convince them is to talk about what exponents and multiplication mean.)

- “Can we show that $8^2 = 4^3$ is true without evaluating both sides? What understanding about the meaning of exponents and operations can help us?” (8^2 means $8 \cdot 8$ or $4 \cdot 2 \cdot 4 \cdot 2$, which also equals $4 \cdot 4 \cdot 2 \cdot 2$ or $4 \cdot 4 \cdot 4$. Another way to write $4 \cdot 4 \cdot 4$ is 4^3 . We are using the understanding that we can multiply in any order, or the commutative and associative properties of multiplication.)

Access for English Language Learners

Speaking, Writing: MLR8 Discussion Supports. Revoice language and push for clarity in reasoning when students discuss their strategies for determining whether the equations are true or false. Provide a sentence frame such as “The equation is true (or false) because _____.” This will strengthen students’ mathematical language use and reasoning when discussing the meaning of exponents and operations that can make the equivalence of expressions true or false.

Design Principle(s): Maximize meta-awareness

13.3 What’s Your Reason?

15 minutes

In this activity, students search for numerical expressions that are equivalent. Students construct arguments and critique the reasoning of others (MP3) as they explain to their partner why they think two expressions are equivalent and respond to their partner’s arguments about equivalence.

Addressing

- 6.EE.A.1

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. Partners work for 10 minutes, alternating every question which partner is explaining why a match is a match and listening to the explanation. If their partner disagrees, the partner explains why they don’t think the match is equivalent.

Encourage students to determine matches by looking for structure in the expressions and applying the meaning of exponents. It is not always necessary to evaluate the expressions in order to find equivalent expressions.

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. To help get students started, display sentence frames such as “____ and ____ are equivalent because . . .” and “I agree/ disagree because . . .”

Supports accessibility for: Language; Organization

Student Task Statement

In each list, find expressions that are equivalent to each other and explain to your partner why they are equivalent. Your partner listens to your explanation. If you disagree, explain your reasoning until you agree. Switch roles for each list. (There may be more than two equivalent expressions in each list.)

- $5 \cdot 5$
 - 2^5
 - 5^2
 - $2 \cdot 5$
- 4^3
 - 3^4
 - $4 \cdot 4 \cdot 4$
 - $4 + 4 + 4$
- $6 + 6 + 6$
 - 6^3
 - 3^6
 - $3 \cdot 6$
- 11^5
 - $11 \cdot 11 \cdot 11 \cdot 11 \cdot 11$
 - $11 \cdot 5$
 - 5^{11}

5. a. $\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$

b. $\left(\frac{1}{5}\right)^3$

c. $\frac{1}{15}$

d. $\frac{1}{125}$

6. a. $\left(\frac{5}{3}\right)^2$

b. $\left(\frac{3}{5}\right)^2$

c. $\frac{10}{6}$

d. $\frac{25}{9}$

Student Response

1. $5 \cdot 5$ and 5^2

2. 4^3 and $4 \cdot 4 \cdot 4$

3. $6 + 6 + 6$ and $3 \cdot 6$

4. 11^5 and $11 \cdot 11 \cdot 11 \cdot 11 \cdot 11$

5. $\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$ and $\left(\frac{1}{5}\right)^3$ and $\frac{1}{125}$

6. $\left(\frac{5}{3}\right)^2$ and $\frac{25}{9}$

Are You Ready for More?

What is the last digit of $3^{1,000}$? Show or explain your reasoning.

Student Response

The last digit is 1. Explanations vary. Sample response:

Some experimentation reveals a pattern—the first few powers of 3 are 3, 9, 27, 81, 243, 729, 2,187, 6,561, 19,683, etc. Specifically, the pattern of last digits goes 3, 9, 7, 1, 3, 9, 7, 1, 3, etc., repeating every four terms. So every exponent which is a multiple of 4, like $3^{1,000}$, evaluates to a number whose last digit is a 1.

Activity Synthesis

Invite students to describe their strategies for finding matches. As students respond, record the equivalent expressions using an equal sign.

Consider asking students:

- To share any expressions they had to work on to agree with their partner
- How they could find a match without evaluating every expression
- To describe some ways to recognize equivalence in expressions

Access for English Language Learners

Conversing: MLR8 Discussion Supports. To help students explain why two or more expressions in each list are equivalent to each other, provide sentence frames such as, “The expressions _____ and _____ are equivalent because _____.” To help students explain why they agree or disagree with their partner’s explanation, provide sentence frames such as, “I agree/ disagree with your reasoning because _____.” As students work on the task, listen for and amplify the language students use to explain the meaning of exponents. This routine will support rich and inclusive discussion about the meaning of exponents and the equivalence of numerical expressions.

Design Principle(s): Optimize output (for explanation); Cultivate conversation

Lesson Synthesis

Throughout the lesson, students saw many instances of typical errors and misconceptions when working with exponent notation. A possible activity for the end of the lesson is the creation of displays showing some of the errors that came up in the activity discussions. Students can work in groups to choose one error and create a visual display of why it is incorrect. For example, students might use a drawing similar to the dot picture in the warm-up of the last lesson to show the meaning of exponents while using an array to show the meaning of multiplication to illustrate that 3^5 is not the same number of dots as $3 \cdot 5$. Another display could show that the meaning of exponents is always the same, regardless of whether the number being repeatedly multiplied is a whole number, fraction, or decimal.

13.4 Coin Calculation

Cool Down: 5 minutes

Addressing

- 6.EE.A.1

Student Task Statement

Andre and Elena knew that after 28 days they would have 2^{28} coins, but they wanted to find out how many coins that actually is. Andre wrote:

$$2^{28} = 2 \cdot 28 = 56$$

Elena said, “No, exponents mean repeated multiplication. It should be $28 \cdot 28$, which works out to be 784.” Do you agree with either of them? Explain your reasoning.

Student Response

I disagree with both Andre and Elena. Andre thinks exponents are just a different way of writing multiplication of two numbers. Elena calculates 28^2 rather than 2^{28} .

Student Lesson Summary

When working with exponents, the bases don't have to always be whole numbers. They can also be other kinds of numbers, like fractions, decimals, and even variables. For example, we can use exponents in each of the following ways:

$$\left(\frac{2}{3}\right)^4 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}$$

$$(1.7)^3 = (1.7) \cdot (1.7) \cdot (1.7)$$

$$x^5 = x \cdot x \cdot x \cdot x \cdot x$$

Lesson 13 Practice Problems

Problem 1

Statement

Select all expressions that are equal to $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$.

- A. $3 \cdot 5$
- B. 3^5
- C. $3^4 \cdot 3$
- D. $5 \cdot 3$
- E. 5^3

Solution

["B", "C"]

Problem 2

Statement

Noah starts with 0 and then adds the number 5 four times. Diego starts with 1 and then multiplies by the number 5 four times. For each expression, decide whether it is equal to Noah's result, Diego's result, or neither.

- a. $4 \cdot 5$
- b. $4 + 5$
- c. 4^5
- d. 5^4

Solution

- a. Noah's
- b. Neither
- c. Neither
- d. Diego's

Problem 3

Statement

Decide whether each equation is true or false, and explain how you know.

a. $9 \cdot 9 \cdot 3 = 3^5$

b. $7 + 7 + 7 = 3 + 3 + 3 + 3 + 3 + 3 + 3$

c. $\frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} = \frac{3}{7}$

d. $4^1 = 4 \cdot 1$

e. $6 + 6 + 6 = 6^3$

Solution

a. True. Explanations vary. Sample explanation: The expression on the left is equivalent to $(3 \cdot 3) \cdot (3 \cdot 3) \cdot 3 = 3^5$.

b. True. Explanations vary. Sample explanation: Both sides of the equation are ways of writing $3 \cdot 7$.

c. False. $\frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} = \frac{1}{7^3}$ or $\frac{1}{343}$ which does not equal $\frac{3}{7}$.

d. True. Both sides equal 4.

e. False. $6^3 = 216$, but $6 + 6 + 6 = 18$.

Problem 4

Statement

a. What is the area of a square with side lengths of $\frac{3}{5}$ units?

b. What is the side length of a square with area $\frac{1}{16}$ square units?

c. What is the volume of a cube with edge lengths of $\frac{2}{3}$ units?

d. What is the edge length of a cube with volume $\frac{27}{64}$ cubic units?

Solution

a. $\frac{9}{25}$ square units ($\frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25}$)

b. $\frac{1}{4}$ units ($\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$)

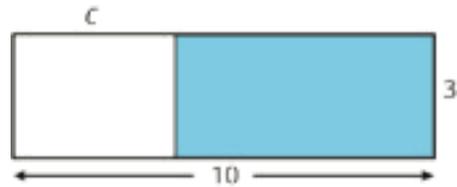
c. $\frac{8}{27}$ cubic units ($\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}$)

d. $\frac{3}{4}$ units ($\frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{27}{64}$)

Problem 5

Statement

Select all the expressions that represent the area of the shaded rectangle.



- A. $3(10 - c)$
- B. $3(c - 10)$
- C. $10(c - 3)$
- D. $10(3 - c)$
- E. $30 - 3c$
- F. $30 - 10c$

Solution

["A", "E"]

(From Unit 6, Lesson 10.)

Problem 6

Statement

A ticket at a movie theater costs \$8.50. One night, the theater had \$29,886 in ticket sales.

- a. Estimate about how many tickets the theater sold. Explain your reasoning.
- b. How many tickets did the theater sell? Explain your reasoning.

Solution

- a. About 3,000. Reasoning varies. Sample reasoning: If there were \$30,000 in sales and the tickets were \$10 each, then it would be 3000. The actual tickets are less than \$10 (while \$30,000 is very close to the total sales), so the actual answer should be more than 3000.
- b. 3,516. Reasoning varies. Sample reasoning: The number of tickets sold is $29,886 \div 8.5$, and this is 3,516.

(From Unit 5, Lesson 13.)

Problem 7

Statement

A fence is being built around a rectangular garden that is $8\frac{1}{2}$ feet by $6\frac{1}{3}$ feet. Fencing comes in panels. Each panel is $\frac{2}{3}$ of a foot wide. How many panels are needed? Explain or show your reasoning.

Solution

Answers vary. Possible solution (not reusing panel pieces): 46 panels. For the sides of length $8\frac{1}{2}$ feet, Jada needs $8\frac{1}{2} \div \frac{2}{3}$ panels. This is $\frac{51}{4} = 12\frac{3}{4}$ so these will use 13 panels of fencing. The other two sides each use $6\frac{1}{3} \div \frac{2}{3}$ panels of fencing, which is $9\frac{1}{2}$. This is 10 panels each. Possible solution (reusing panel pieces): 45 panels. The sides of length $8\frac{1}{2}$ feet each use $12\frac{3}{4}$ panels of fencing, for a total of $25\frac{1}{2}$. The other two sides each use $9\frac{1}{2}$ pieces of fencing for a total of 19 panels. Jada needs $44\frac{1}{2}$ panels, which means she needs 45 whole panels.

(From Unit 4, Lesson 12.)

Lesson 14: Evaluating Expressions with Exponents

Goals

- Evaluate numerical expressions that have an exponent and one other operation, and justify (orally) the process.
- Explain (orally and in writing) that the convention is to evaluate the exponent before the other operations in an expression with no grouping symbols.
- Interpret expressions with exponents that represent the surface area or volume of a cube.

Learning Targets

- I know how to evaluate expressions that have both an exponent and addition or subtraction.
- I know how to evaluate expressions that have both an exponent and multiplication or division.

Lesson Narrative

The focus of this lesson is evaluating expressions that have an exponent and one other operation by carrying out operations in the conventional order. This is accomplished through an example with surface area, where the context provides a clear reason for evaluating the exponential expression before performing the multiplication. Students practice evaluating numeric expressions that include exponents.

Alignments

Addressing

- 6.EE.A.1: Write and evaluate numerical expressions involving whole-number exponents.
- 6.EE.A.2.c: Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR6: Three Reads

Student Learning Goals

Let's find the values of expressions with exponents.

14.1 Revisiting the Cube

Warm Up: 10 minutes (there is a digital version of this activity)

The purpose of this warm-up is for students to recall previous understandings of area, volume, and surface area of cubes, and how to record these measurements as expressions using exponents. Students might respond with either verbal or numerical descriptions, saying, for example, "We can find the area of the square," or "The area of the square is 9 square units."

After students share their responses, display the following table for all to see and give students time to discuss the information with a partner. The table is used to encourage students to think about the expressions with exponents in addition to the numeric responses.

	side length of the square	area of the square	volume of the cube	surface area of the cube
as a number	3			
as an expression using an exponent	3			

Addressing

- 6.EE.A.1
- 6.EE.A.2.c

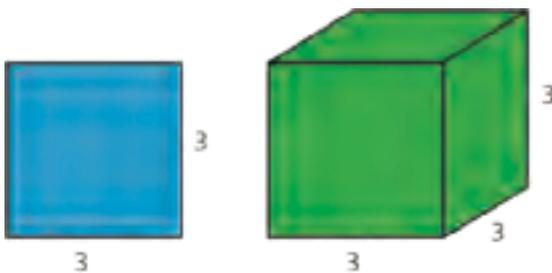
Launch

Arrange students in groups of 2. Display the table and give students 1 minute of quiet work time to complete as much of the table as they can. Then complete the table and discuss as a group.

If students have access to digital activities they can explore the applet and generate dimensions that can be determined. After sharing, they can complete the table.

Student Task Statement

Based on the given information, what other measurements of the square and cube could we find?



Student Response

Answers vary. Sample responses:

- We can find the area of the square.
- The area of the square is 9 square units.
- The perimeter is 12 units.
- We can find the volume of the cube.
- The volume of the cube is 27 cubic units.
- The surface area is 54 square units.

	side length of the square	area of the square	volume of the cube	surface area of the cube
as a number	3	9	27	54
as an expression using an exponent	3	3^2	3^3	$6(3^2)$

Activity Synthesis

Ask students to share their responses for the cells in the table. Poll the students on whether they agree or disagree with each response. Record and display the responses for all to see. As students share responses, ask the following questions to help clarify their answers:

- First row: What calculation did you do to arrive at that answer? Where are those measurements in the image?
- Second row: How did you decide on the exponent for your answer? Where are those measurements in the image?

	side length of the square	area of the square	volume of the cube	surface area of the cube
as a number	3	9	27	54
as an expression using an exponent	3	3^2	3^3	$6(3^2)$

In the next activity, students will analyze calculations of the surface area of a cube. Take time now to discuss why $6(3^2)$ expresses the surface area of this cube. Ask students to think about how they computed surface area, and then analyze this expression. Where did the 3^2 come from? (It's the area of one face of the cube.) Why are we multiplying by 6? (We want to add up 6 3^2 's, and that is the same as multiplying 3^2 by 6.)

14.2 Calculating Surface Area

10 minutes

In this activity, students use surface area as a context to extend the order of operations to expressions with exponents. The context provides a reason to evaluate the exponent before performing the multiplication.

Addressing

- 6.EE.A.1
- 6.EE.A.2.c

Instructional Routines

- MLR6: Three Reads

Launch

Give students 10 minutes of quiet work time, followed by a class discussion.

Access for English Language Learners

Reading: MLR6 Three Reads. Use this routine to support students' comprehension of the situation. In the first read, students read the text with the goal of comprehending the situation (e.g., Jada and Noah have different solutions for the surface area of the same cube). In the second read, ask students to identify important quantities that can be counted or measured (e.g., the side length of the cube; the number of faces of a cube; the area of each face of the cube). In the third read, reveal the question, "Do you agree with either of them? Explain your reasoning." Ask students to brainstorm possible strategies to answer the question (e.g., Find the area of each face of the cube. Multiply the area by 6 to find the surface area of the cube). This will help students concentrate on making sense of the situation before rushing to a solution or method.

Design Principle(s): Support sense-making

Student Task Statement

A cube has side length 10 inches. Jada says the surface area of the cube is 600 in^2 , and Noah says the surface area of the cube is $3,600 \text{ in}^2$. Here is how each of them reasoned:

Jada's Method:

$$\begin{aligned} &6 \cdot 10^2 \\ &6 \cdot 100 \\ &600 \end{aligned}$$

Noah's Method:

$$\begin{aligned} &6 \cdot 10^2 \\ &60^2 \\ &3,600 \end{aligned}$$

Do you agree with either of them? Explain your reasoning.

Student Response

Jada's solution is correct. Explanations vary. Sample response: The cube has 6 faces and each has an area of 10^2 or 100. The area calculation comes before multiplying by 6.

Activity Synthesis

In finding the surface area, there is a clear reason to find 10^2 and then multiply by 6. Tell students that sometimes it is not so clear in which order to evaluate operations. There is an order that we all generally agree on, and when we want something done in a different order, brackets are used to communicate what to do first. When an exponent occurs in the same expression as multiplication or division, we evaluate the exponent first, unless brackets say otherwise. Examples:

$(3 \cdot 4)^2 = 12^2 = 144$, since the brackets tell us to multiply $3 \cdot 4$ first. But $3 \cdot 4^2 = 3 \cdot 16 = 48$, because since there are no brackets, we evaluate the exponent before multiplying.

If students bring up PEMDAS or another mnemonic for remembering the order of operations, point out that PEMDAS can be misleading in indicating multiplication before division, and addition before subtraction. Discuss the convention that brackets or parentheses indicate that something should be evaluated first, followed by exponents, multiplication, or division (evaluated left to right), and last, addition or subtraction (evaluated left to right).

14.3 Row Game: Expression Explosion

15 minutes

In this activity, students use the order of operations to evaluate expressions with exponents. They engage in MP3 as they listen and critique their partner's reasoning when they do not agree on the answers.

Addressing

- 6.EE.A.1
- 6.EE.A.2.c

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Arrange students in groups of 2. Partners work individually on their expression in each row, then check their answers and discuss. Follow with a whole-class discussion.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they work together to find errors. For example, "How did you . . .?", "First, I ____ because...", "I agree/disagree because . . ."

Supports accessibility for: Language; Social-emotional skills

Student Task Statement

Evaluate the expressions in one of the columns. Your partner will work on the other column. Check with your partner after you finish each row. Your answers in each row should be the same. If your answers aren't the same, work together to find the error.

column A	column B
$5^2 + 4$	$2^2 + 25$
$2^4 \cdot 5$	$2^3 \cdot 10$
$3 \cdot 4^2$	$12 \cdot 2^2$
$20 + 2^3$	$1 + 3^3$
$9 \cdot 2^1$	$3 \cdot 6^1$
$\frac{1}{9} \cdot \left(\frac{1}{2}\right)^3$	$\frac{1}{8} \cdot \left(\frac{1}{3}\right)^2$

Student Response

1. 29
2. 80
3. 48
4. 28
5. 18
6. $\frac{1}{72}$

Are You Ready for More?

1. Consider this equation: $\square^2 + \square^2 = \square^2$. An example of 3 different whole numbers that could go in the boxes are 3, 4, and 5, since $3^2 + 4^2 = 5^2$. (That is, $9 + 16 = 25$.)

Can you find a different set of 3 whole numbers that make the equation true?

2. How many sets of 3 different whole numbers can you find?

3. Can you find a set of 3 different whole numbers that make this equation true?

$$\square^3 + \square^3 = \square^3$$

4. How about this one? $\square^4 + \square^4 = \square^4$

Once you have worked on this a little while, you can understand a problem that is famous in the history of math. (Alas, this space is too small to contain it.) If you are interested, consider doing some further research on *Fermat's Last Theorem*.

Student Response

1. Sample responses: {6, 8, 10}, {5, 12, 13}
2. Answers vary. There are an infinite number of these triples.
3. No. (No such triple exists.)
4. No. (No such triple exists.)

Activity Synthesis

The purpose of the discussion is to ensure that students understand and can apply the agreed on rules for order of operations when expressions contain exponents. Consider asking some of the following questions:

- "Were there any expressions that were difficult to evaluate? Why were they difficult?"
- "Did you disagree with your partner about any rows? How did you settle the disagreement?"
- "Did you learn anything new about evaluating expressions with exponents?"

Access for English Language Learners

Reading, Writing, Speaking: MLR3 Clarify, Critique, Correct. Present an incorrect solution based on a common misconception about evaluating expressions with exponents. For example, “The expression $1 + 3^3$ is equal to 64 because $1 + 3$ is 4 and 4^3 is 64.” Ask students to identify the error, critique the reasoning, and revise the original statement. As students discuss in partners, listen for students who state that exponents must be evaluated before addition, unless parenthesis say otherwise. Therefore, 3^3 must be evaluated first before adding 1. This routine will engage students in meta-awareness as they critique and correct the language used to evaluate expressions with exponents.

Design Principles(s): Cultivate conversation; Maximize meta-awareness

Lesson Synthesis

Ask students to write and evaluate a numerical expression with an exponent and one other operation. Then have students switch with a partner and evaluate the partner's expressions. Invite some students to share their expressions with the class.

14.4 Calculating Volumes

Cool Down: 5 minutes

Addressing

- 6.EE.A.1
- 6.EE.A.2.c

Student Task Statement

Jada and Noah wanted to find the total volume of a cube and a rectangular prism. They know the prism's volume is 20 cubic units, and they know the cube has side lengths of 10 units. Jada says the total volume is 27,000 cubic units. Noah says it is 1,020 cubic units. Here is how each of them reasoned:

Jada's Method:

$$\begin{array}{r} 20 + 10^3 \\ 30^3 \\ 27,000 \end{array}$$

Noah's Method:

$$\begin{array}{r} 20 + 10^3 \\ 20 + 1,000 \\ 1,020 \end{array}$$

Do you agree with either of them? Explain your reasoning.

Student Response

Noah's solution is correct. Reasoning varies. Sample reasoning: The cube has a volume of 1,000 cubic units and the additional 20 cubic units from the prism makes the total volume 1,020 cubic units. The exponent calculation comes before addition.

Student Lesson Summary

Exponents give us a new way to describe operations with numbers, so we need to understand how exponents get along with the other operations we know.

When we write $6 \cdot 4^2$, we want to make sure everyone agrees about how to evaluate this. Otherwise some people might multiply first and others compute the exponent first, and different people would get different values for the same expression!

Earlier we saw situations in which $6 \cdot 4^2$ represented the surface area of a cube with side lengths 4 units. When computing the surface area, we evaluate 4^2 first (or find the area of one face of the cube first) and then multiply the result by 6. In many other expressions that use exponents, the part with an exponent is intended to be evaluated first.

To make everyone agree about the value of expressions like $6 \cdot 4^2$, the convention is to *evaluate the part of the expression with the exponent first*. Here are a couple of examples:

$$6 \cdot 4^2$$

$$6 \cdot 16$$

$$96$$

$$45 + 5^2$$

$$45 + 25$$

$$70$$

If we want to communicate that 6 and 4 should be multiplied first and then squared, then we can use parentheses to group parts together:

$$(6 \cdot 4)^2$$

$$24^2$$

$$576$$

$$(45 + 5)^2$$

$$50^2$$

$$2,500$$

Lesson 14 Practice Problems

Problem 1

Statement

Lin says, "I took the number 8, and then multiplied it by the square of 3." Select **all** the expressions that equal Lin's answer.

A. $8 \cdot 3^2$

B. $(8 \cdot 3)^2$

C. $8 \cdot 2^3$

D. $3^2 \cdot 8$

E. 24^2

F. 72

Solution

["A", "D", "F"]

Problem 2

Statement

Evaluate each expression.

a. $7 + 2^3$

b. $9 \cdot 3^1$

c. $20 - 2^4$

d. $2 \cdot 6^2$

e. $8 \cdot \left(\frac{1}{2}\right)^2$

f. $\frac{1}{3} \cdot 3^3$

g. $\left(\frac{1}{5} \cdot 5\right)^5$

Solution

a. 15

b. 27

c. 4

d. 72

e. 2

f. 9

g. 1

Problem 3

Statement

Andre says, "I multiplied 4 by 5, then cubed the result." Select all the expressions that equal Andre's answer.

A. $4 \cdot 5^3$

B. $(4 \cdot 5)^3$

C. $(4 \cdot 5)^2$

D. $5^3 \cdot 4$

E. 20^3

F. 500

G. 8,000

Solution

["B", "E", "G"]

Problem 4

Statement

Han has 10 cubes, each 5 inches on a side.

a. Find the total volume of Han's cubes. Express your answer as an expression using an exponent.

b. Find the total surface area of Han's cubes. Express your answer as an expression using an exponent.

Solution

a. $10 \cdot 5^3 \text{ in}^3$

b. $10 \cdot 6 \cdot 5^2 \text{ in}^2$ or $60 \cdot 5^2 \text{ in}^2$

Problem 5

Statement

Priya says that $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{4}{3}$. Do you agree with Priya? Explain or show your reasoning.

Solution

Answers vary. Sample response: I disagree with Priya. $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3}$ is really $(\frac{1}{3})^4$, or $\frac{1}{81}$.

(From Unit 6, Lesson 13.)

Problem 6

Statement

Answer each question. Show your reasoning.

- 125% of e is 30. What is e ?
- 35% of f is 14. What is f ?

Solution

- $24. \frac{125}{100} \cdot e = 30$, so $e = 30 \div \frac{125}{100}$, so $e = 30 \cdot \frac{100}{125}$.
- $40. (0.35) \cdot f = 14$, so $f = 14 \div 0.35$.

(From Unit 6, Lesson 7.)

Problem 7

Statement

Which expressions are solutions to the equation $2.4y = 13.75$? Select **all** that apply.

- $13.75 - 1.4$
- $13.75 \cdot 2.4$
- $13.75 \div 2.4$
- $\frac{13.75}{2.4}$
- $2.4 \div 13.75$

Solution

["C", "D"]

(From Unit 6, Lesson 5.)

Problem 8

Statement

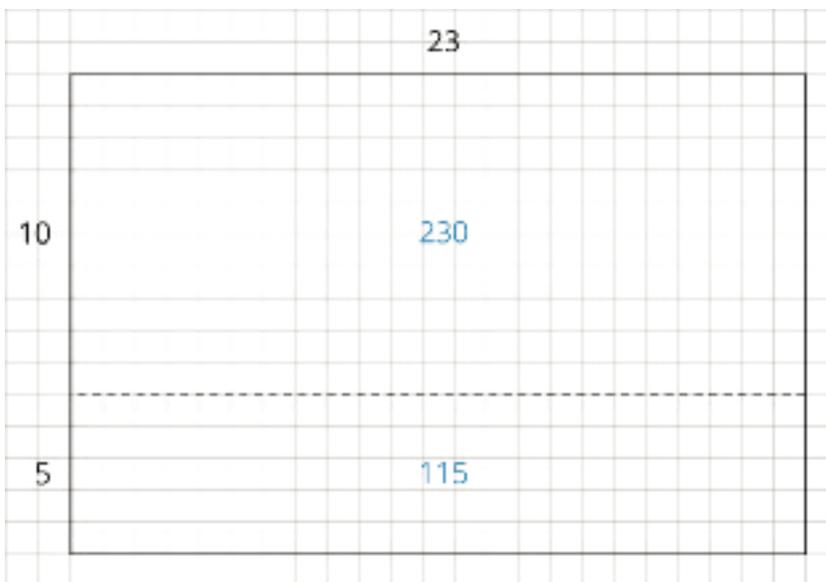
Jada explains how she finds $15 \cdot 23$:

"I know that ten 23s is 230, so five 23s will be half of 230, which is 115. 15 is 10 plus 5, so $15 \cdot 23$ is 230 plus 115, which is 345."

- Do you agree with Jada? Explain.
- Draw a 15 by 23 rectangle. Partition the rectangle into two rectangles and label them to show Jada's reasoning.

Solution

- Yes, Jada is calculating $15 \cdot 23$ by writing it as $(10 + 5) \cdot 23$ (using the distributive property). To find $5 \cdot 23$, she thinks of 5 as $\frac{10}{2}$. So Jada needs to multiply 23 by 10 (which gives her 230) and add half of this product (which is 115) to find the value of $(10 + 5) \cdot 23$.
-



(From Unit 5, Lesson 7.)

Lesson 15: Equivalent Exponential Expressions

Goals

- Describe (orally) the values that result from evaluating expressions in which a fraction is raised to a power.
- Determine whether a given value is a solution to an equation that includes an exponent.
- Evaluate expressions that have a variable, an exponent, and one other operation for a given value of the variable, carrying out the operations in the conventional order.

Learning Targets

- I can find solutions to equations with exponents in a list of numbers.
- I can replace a variable with a number in an expression with exponents and operations and use the correct order to evaluate the expression.

Lesson Narrative

In this lesson, students encounter expressions and equations with variables that also involve exponents. Students first evaluate expressions for given values of their variables. They learn that multiplication can be expressed without a dot or other symbol by placing a number, known as a coefficient, next to a variable or variable expression. In the next activity, students are presented with equations that contain a variable. They engage in MP7 by considering the structure of the equations and apply their understanding of exponents and operations to select a number from a list that, when replaced for the variable, makes the equation true. That number is a solution of the equation.

Alignments

Addressing

- 6.EE.A.1: Write and evaluate numerical expressions involving whole-number exponents.
- 6.EE.A.2.c: Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.
- 6.EE.B.5: Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports

Student Learning Goals

Let's investigate expressions with variables and exponents.

15.1 Up or Down?

Warm Up: 10 minutes

The purpose of this warm-up is for students to take two numbers to different powers and look for patterns. One number is a whole number and the other is a fraction that is the reciprocal of the whole number. Some students may notice they do not need to multiply $\frac{1}{3}$ after they complete the column for 3 because it is the reciprocal. This is a helpful pattern for students to notice, but also ask these students if their answer makes sense to ensure they understand the product is getting smaller as they multiply by further factors of $\frac{1}{3}$. Aside from the presence of exponents, these observations are largely a review of work from grade 5.

As students complete the table, monitor and select students who can describe some of the following patterns:

- The products in the 3 column increase in value as the exponent increases.
- The products in the $\frac{1}{3}$ column decrease in value as the exponent increases.
- The products in the $\frac{1}{3}$ column are reciprocals of the products in the corresponding 3 column.

Addressing

- 6.EE.A.1
- 6.EE.A.2.c

Launch

Give students 2 minutes of quiet work time, followed by a whole-group discussion.

Anticipated Misconceptions

Some students may need to sketch a diagram to help them find a fraction of another fraction.

Student Task Statement

Find the values of 3^x and $\left(\frac{1}{3}\right)^x$ for different values of x . What patterns do you notice?

x	3^x	$(\frac{1}{3})^x$
1		
2		
3		
4		

Student Response

x	3^x	$(\frac{1}{3})^x$
1	3	$\frac{1}{3}$
2	9	$\frac{1}{9}$
3	27	$\frac{1}{27}$
4	81	$\frac{1}{81}$

Many observations are possible. Possible responses:

- The products in the 3 column increase in value as the exponent increases.
- The products in the 3 column are multiplied by 3 each time you go down a row.
- The products in the $\frac{1}{3}$ column decrease in value as the exponent increases.
- The products in the $\frac{1}{3}$ column are multiplied by $\frac{1}{3}$ each time you go down a row.
- The products in the $\frac{1}{3}$ column are reciprocals of the products in the corresponding 3 column.

Activity Synthesis

Ask students to share responses to complete the table. Record and display the responses for all to see. Ask selected students to share the patterns they noticed in the table and ask others to explain why they think these patterns happen. If the following ideas do not arise from the students in the conversation, bring them to students' attention:

- The products in the 3 column increase in value as the exponent increases.
- The products in the 3 column are multiplied by 3 each time you go down a row.
- The products in the $\frac{1}{3}$ column decrease in value as the exponent increases.

- The products in the $\frac{1}{3}$ column are multiplied by $\frac{1}{3}$ each time you go down a row.
- The products in the $\frac{1}{3}$ column are reciprocals of the products in the corresponding 3 column.

15.2 What's the Value?

10 minutes

In this activity, students first encounter exponential expressions with variables.

Addressing

- 6.EE.A.1
- 6.EE.A.2.c

Instructional Routines

- MLR8: Discussion Supports

Launch

Recall with students that when we write $6x^2$, we mean to multiply 6 by the result of x^2 . Remind them that the number part of such a product is called the *coefficient* of the expression, so in this example, 6 is the coefficient of x^2 .

Give students 5 minutes of quiet work time to evaluate the expressions. Follow with whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts. After students have evaluated the first 2-3 expressions, check-in with either select groups of students or the whole class. Invite students to share the strategies they have used so far as well as any questions they have before continuing.

Supports accessibility for: Organization; Attention

Anticipated Misconceptions

Students may use the wrong order when evaluating expressions, such as in multiplying by 3 first in $3x^2$ and then squaring. Have them recall the last lesson where they practiced applying the agreed on order of operations in numeric expressions.

Students may interpret an expression like $3x$ as meaning 3 next to the digit x instead of as multiplication. For example, students may think that $3x$ means 35 instead of 15 when $x = 5$. Discuss how the shorthand notation of coefficients next to variables with no symbols between them tells us to multiply, and that it will simplify future work with expressions and equations.

Student Task Statement

Evaluate each expression for the given value of x .

1. $3x^2$ when x is 10
2. $3x^2$ when x is $\frac{1}{9}$
3. $\frac{x^3}{4}$ when x is 4
4. $\frac{x^3}{4}$ when x is $\frac{1}{2}$
5. $9 + x^7$ when x is 1
6. $9 + x^7$ when x is $\frac{1}{2}$

Student Response

1. 300
2. $\frac{1}{27}$
3. 16
4. $\frac{1}{32}$
5. 10
6. $9\frac{1}{128}$

Activity Synthesis

The purpose of the discussion is to ensure that students understand how to evaluate expressions with variables for a given value of the variable. It is also an opportunity for students to practice interpreting and using vocabulary like *coefficient*, *variable*, *power*, and *exponent*.

Some guiding questions:

- "In each expression, what is the coefficient?" (3, 3, $\frac{1}{4}$, $\frac{1}{4}$, 1, 1.)
- "Choose one of the expressions. Describe the steps that you carried out to evaluate the expression." (Sample response: I rewrote $9 + x^7$ as $9 + \left(\frac{1}{2}\right)^7$. I needed to first evaluate the exponent. I knew that $\left(\frac{1}{2}\right)^7$ means $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$, which is $\frac{1}{128}$. So then I wrote $9 + \frac{1}{128}$, which can also be written $9\frac{1}{128}$.)
- "How is evaluating the expressions when x is a fraction similar to when x is a whole number? How is it different?" (It's similar because you're still just multiplying x by itself a certain number of times. It's different because multiplying a fraction by a fraction is a bit more complicated than multiplying a whole number by itself.)

Access for English Language Learners

Speaking, Writing: MLR8 Discussion supports. Use this to amplify mathematical uses of language to communicate about how to evaluate exponential expressions. Revoice the term “coefficient” during student discussion and press for detail when identifying the coefficients and the order in which they are multiplied in each expression.

Design Principle(s): Support sense-making

15.3 Exponent Experimentation

15 minutes

In this activity, students continue their work with exponential expressions and recall what is meant by a *solution* to an equation as they look to replace a variable with a number that makes two expressions equivalent. Note that some of the equations also have solutions that are negative; however, since operations on negative numbers are not part of grade 6 standards, students are only expected to consider positive values in this task. This activity addresses student understanding of the meaning of the equal sign as one that supports work in algebra, namely, that the expressions on either side have the same value.

Addressing

- 6.EE.A.2.c
- 6.EE.B.5

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Ask students to close their books or devices. Display the equation $x^2 = 100$ and discuss what it would mean to find a *solution* to the equation. Remind students that a *solution* is a value for x that makes the equation true. Discuss why 10 is a solution, and why 50 is not a solution.

Give students 10 minutes of quiet work time, followed by a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge about exponential expressions and finding a solution to an equation. Allow students to use calculators to ensure inclusive participation in the activity.

Supports accessibility for: Memory; Conceptual processing

Student Task Statement

Find a solution to each equation in the list. (Numbers in the list may be a solution to more than one equation, and not all numbers in the list will be used.)

1. $64 = x^2$

2. $64 = x^3$

3. $2^x = 32$

4. $x = \left(\frac{2}{5}\right)^3$

5. $\frac{16}{9} = x^2$

6. $2 \cdot 2^5 = 2^x$

7. $2x = 2^4$

8. $4^3 = 8^x$

List:

$\frac{8}{125}$

$\frac{6}{15}$

$\frac{5}{8}$

$\frac{8}{9}$

1

$\frac{4}{3}$

2

3

4

5

6

8

Student Response

1. 8

2. 4

3. 5

4. $\frac{8}{125}$

5. $\frac{4}{3}$

6. 6

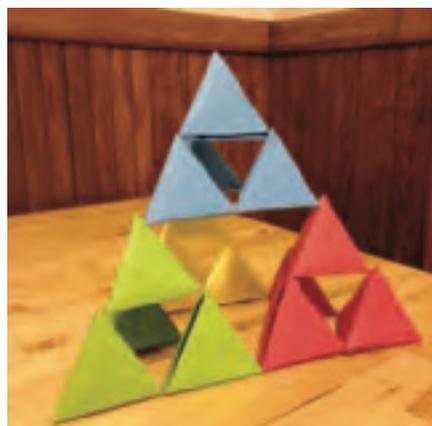
7. 8

8. 2

Are You Ready for More?

This fractal is called a Sierpinski Tetrahedron. A tetrahedron is a polyhedron that has four faces. (The plural of tetrahedron is tetrahedra.)

The small tetrahedra form four medium-sized tetrahedra: blue, red, yellow, and green. The medium-sized tetrahedra form one large tetrahedron.



1. How many small faces does this fractal have? Be sure to include faces you can't see. Try to find a way to figure this out so that you don't have to count every face.
2. How many small tetrahedra are in the bottom layer, touching the table?
3. To make an even bigger version of this fractal, you could take four fractals like the one pictured and put them together. Explain where you would attach the fractals to make a bigger tetrahedron.
4. How many small faces would this bigger fractal have? How many small tetrahedra would be in the bottom layer?
5. What other patterns can you find?

Student Response

1. 64 faces. Each small tetrahedron has 4 faces, and there are 16 small tetrahedra in the entire fractal. $4 \cdot 16 = 64$.
2. 9 tetrahedra
3. In the picture, we see four mid-sized tetrahedra. There is a mid-sized tetrahedron at the top. At each of the three bottom vertices the apex of a mid-sized tetrahedron is attached. We can follow this pattern with the large tetrahedra—attach the apex of a large tetrahedron to each of three bottom vertices of the kite in the picture.
4. 256 small faces and 27 tetrahedra in the bottom layer. Since we've quadrupled the number of small tetrahedra, the number of faces also quadruples. The new bottom layer will contain three copies of the bottom layer of the fractal pictured. Since the fractal we see has 9 tetrahedra in the bottom layer, the new kite should have 3 times more tetrahedra.
5. Answers vary.

Activity Synthesis

The discussion should focus on the meaning of a solution to an equation, the meaning of the equal sign, and how the meaning of exponents can help find solutions. Discussion:

- Explain what the equal sign in these equations tell us about the expressions on either side.
 - Describe your strategy for finding a number that makes each equation true.
 - Were there any equations that needed a different approach?
 - What was your strategy when x was the exponent?
 - Compare your strategy for the first question to your strategy for the last question.
-

Access for English Language Learners

Reading, Writing, Speaking: MLR3 Clarify, Critique, Correct. Present an incorrect solution based on a common misconception about solving equations with exponents. For example, “The solution to the equation $x = (\frac{2}{5})^3$ is $\frac{6}{15}$ because $(\frac{2}{5})^3$ is equivalent to $\frac{2}{5} \cdot 3$, which is $\frac{6}{15}$.” Ask students to identify the error, critique the reasoning, and revise the original statement. As students discuss in partners, listen for students who clarify the meaning of exponents and the equal sign. For example, $(\frac{2}{5})^3$ means $\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5}$, which is not equivalent to $\frac{2}{5} \cdot 3$. This routine will engage students in meta-awareness as they critique and correct the language used to find solutions to equations with exponents.

Design Principles(s): Cultivate conversation; Maximize meta-awareness

Lesson Synthesis

Ask students to reflect on their thinking in the activities they completed. Some questions to consider:

- “How did you make use of the meaning of operations and exponents in each activity?”
- “How did thinking about the meaning of operations and exponents help to find the value of x in the activity with the equations?”
- “Was there anything you learned in the first activity that helped you with the second activity?”
- “What have you noticed about a number less than 1 raised to a power? How does this compare to a number greater than 1 raised to a power?”

15.4 True Statements

Cool Down: 5 minutes

Addressing

- 6.EE.A.1
- 6.EE.B.5

Student Task Statement

Match each equation to a solution.

1. $2^x = 64$

2. $x = \left(\frac{2}{5}\right)^3$

3. $3 \cdot (3^4) = 3^x$

4. $\frac{16}{25} = x^2$

• $\frac{8}{125}$

• $\frac{4}{5}$

• 5

• 6

Student Response

1. 6

2. $\frac{8}{125}$

3. 5

4. $\frac{4}{5}$

Student Lesson Summary

In this lesson, we saw expressions that used the letter x as a variable. We evaluated these expressions for different values of x .

- To evaluate the expression $2x^3$ when x is 5, we replace the letter x with 5 to get $2 \cdot 5^3$. This is equal to $2 \cdot 125$ or just 250. So the value of $2x^3$ is 250 when x is 5.
- To evaluate $\frac{x^2}{8}$ when x is 4, we replace the letter x with 4 to get $\frac{4^2}{8} = \frac{16}{8}$, which equals 2. So $\frac{x^2}{8}$ has a value of 2 when x is 4.

We also saw equations with the variable x and had to decide what value of x would make the equation true.

- Suppose we have an equation $10 \cdot 3^x = 90$ and a list of possible solutions: 1, 2, 3, 9, 11. The only value of x that makes the equation true is 2 because $10 \cdot 3^2 = 10 \cdot 3 \cdot 3$, which equals 90. So 2 is the solution to the equation.

Lesson 15 Practice Problems

Problem 1

Statement

Evaluate each expression if $x = 3$.

- a. 2^x
- b. x^2
- c. 1^x
- d. x^1
- e. $(\frac{1}{2})^x$

Solution

- a. 8
- b. 9
- c. 1
- d. 3
- e. $\frac{1}{8}$

Problem 2

Statement

Evaluate each expression for the given value of each variable.

- a. $2 + x^3$, x is 3
- b. x^2 , x is $\frac{1}{2}$
- c. $3x^2 + y$, x is 5 y is 3
- d. $10y + x^2$, x is 6 y is 4

Solution

- a. 29
- b. $\frac{1}{4}$
- c. 78
- d. 76

Problem 3

Statement

Decide if the expressions have the same value. If not, determine which expression has the larger value.

- a. 2^3 and 3^2
- b. 1^{31} and 31^1
- c. 4^2 and 2^4
- d. $(\frac{1}{2})^3$ and $(\frac{1}{3})^2$

Solution

- a. Not equal. 3^2 has the larger value, because $2^3 = 8$ and $3^2 = 9$.
- b. Not equal. 31^1 has the larger value, because $1^{31} = 1$ and $31^1 = 31$.
- c. Equal. They both have 16 as their value.
- d. Not equal. $(\frac{1}{2})^3$, because $(\frac{1}{2})^3 = \frac{1}{8}$ and $(\frac{1}{3})^2 = \frac{1}{9}$ and $\frac{1}{8} > \frac{1}{9}$.

Problem 4

Statement

Match each equation to its solution.

- | | |
|---------------------------|------------|
| A. $7 + x^2 = 16$ | 1. $x = 1$ |
| B. $5 - x^2 = 1$ | 2. $x = 2$ |
| C. $2 \cdot 2^3 = 2^x$ | 3. $x = 3$ |
| D. $\frac{3^4}{3^x} = 27$ | 4. $x = 4$ |

Solution

- A: 3
- B: 2
- C: 4
- D: 1

Problem 5

Statement

An adult pass at the amusement park costs 1.6 times as much as a child's pass.

a. How many dollars does an adult pass cost if a child's pass costs:

\$5?

\$10?

w dollars?

b. A child's pass costs \$15. How many dollars does an adult pass cost?

Solution

a. 8 dollars ($1.6 \cdot 5 = 8$), 16 dollars, ($1.6 \cdot 10 = 16$), $1.6w$ dollars

b. 24 dollars ($1.6 \cdot 15 = 24$)

(From Unit 6, Lesson 6.)

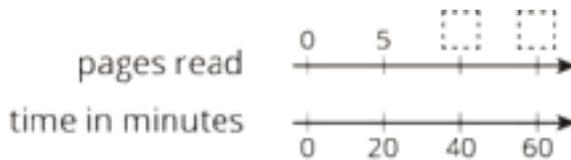
Problem 6

Statement

Jada reads 5 pages every 20 minutes. At this rate, how many pages can she read in 1 hour?

○ Use a double number line to find the answer.

○ Use a table to find the answer.



pages read	time in minutes
5	20

Which strategy do you think is better, and why?

Solution

○ 15 pages. The missing labels should be 10 and 15.

○ Answers vary. Sample responses:

pages read	time in minutes
5	20
0.25	1
15	60

pages read	time in minutes
5	20
10	40
15	60

Answers vary. Sample response: The table is more efficient, because I can skip values.

(From Unit 2, Lesson 14.)

Section: Relationships Between Quantities

Lesson 16: Two Related Quantities, Part 1

Goals

- Compare and contrast (orally) graphs and equations that represent a relationship between the same quantities but have the independent and dependent variables switched.
- Comprehend the terms “independent variable” and “dependent variable” (in spoken and written language).
- Create a table, graph, and equation to represent the relationship between quantities in a set of equivalent ratios.

Learning Targets

- I can create tables and graphs that show the relationship between two amounts in a given ratio.
- I can write an equation with variables that shows the relationship between two amounts in a given ratio.

Lesson Narrative

This lesson is the first of two that apply new understanding of algebraic expressions and equations to represent relationships between two quantities. Students use and make connections between tables, graphs, and equations that represent these relationships.

In this lesson, students revisit and extend their understanding of equivalent ratios. A familiar scenario of mixing paints in a given ratio provides the context for writing equations that represents the relationship between two quantities. Students then create a table of values that shows how changes in one quantity affect changes in the other, and graph the points from the table in the coordinate plane. They are invited to notice that these points lie on a line. Students will study proportional relationships in more depth in grade 7.

Students learn that relationships between two quantities can be described by two different but related equations with one quantity, the **dependent variable**, affected by changes in the other quantity, the **independent variable**. When people engage in mathematical modeling, which variable is considered independent and which is considered dependent is often the choice of the modeler (though sometimes the situation suggests choosing one way over the other). The context in this lesson was intentionally chosen because the context does not suggest a preference about which quantity is chosen as the independent variable.

Alignments

Addressing

- 6.EE.C.9: Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.
- 6.RP.A.1: Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2 : 1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”
- 6.RP.A.3.a: Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
- 6.RP.A.3.b: Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Student Learning Goals

Let's use equations and graphs to describe relationships with ratios.

16.1 Which One Would You Choose?

Warm Up: 5 minutes

The purpose of this warm-up is for students to remember that unit price can be used to figure out which price option is a better deal and also how to compute unit price. When students explain their reasoning, they may engage in constructing arguments and critiquing the reasoning of their classmates (MP3). The question, “Which one would you choose?” is purposefully asked because there is not one correct answer. While there is a choice that is a better deal, that is not the question. In defending their reasoning, students may have other reasons for their choice based on how they make sense of the context. For example, students might reason that a 5-gallon container is easier to store, or that 3 1-gallon containers are easier to share, or they might reject both options because they don't like honey.

As students work, listen for reasoning about the problem in different ways to share in the whole-class discussion. Choose at least one student who reasoned by finding the unit cost.

Addressing

- 6.RP.A.3.b

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Give 1 minute of quiet think time followed by 1 minute to discuss their responses with a partner. Follow with a whole-class discussion.

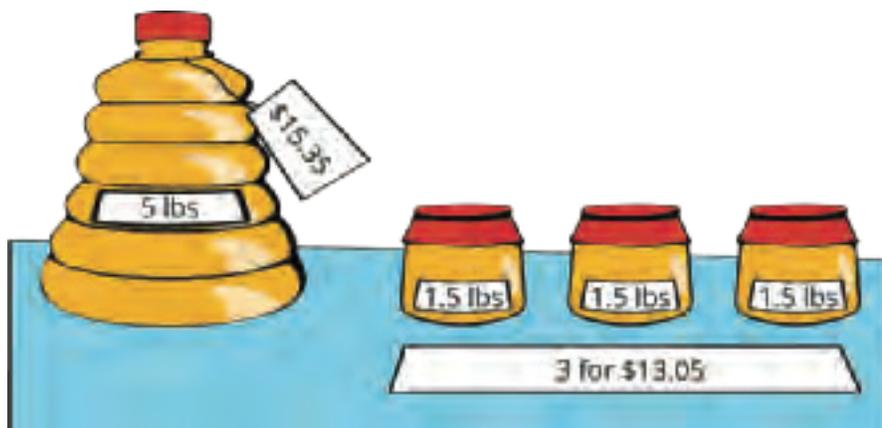
Anticipated Misconceptions

Students may choose the 5 pound jug because they assume that larger quantities are a better deal. Ask if they can be more precise in their reasoning and defend it mathematically.

Student Task Statement

Which one would you choose? Be prepared to explain your reasoning.

- A 5-pound jug of honey for \$15.35
- Three 1.5-pound jars of honey for \$13.05



Student Response

Answers vary. Unit costs are \$3.07 per pound for the large jug and \$2.90 per pound for the smaller jars.

Activity Synthesis

Poll the class to find who selected each of the options. Ask selected students to share their reasoning. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- "Who can restate ___'s reasoning in a different way?"

- “Does anyone want to add on to ____’s reasoning?”
- “Do you agree or disagree? Why?”

16.2 Painting the Set

25 minutes (there is a digital version of this activity)

This activity helps students recall what they know about solving problems in a ratio context, and then extends their thinking to consider writing two equations that relate the two quantities in the ratio and representing them with graphs.

Addressing

- 6.EE.C.9
- 6.RP.A.1
- 6.RP.A.3.a

Instructional Routines

- MLR8: Discussion Supports

Launch

Explain the meaning of independent variable and dependent variable. (This can be done before students start working, or you can have students pause after completing the table.) An example may help: Suppose Lin and her neighbor share the same birthday, but Lin is 3 years older. You can find the neighbor’s age by taking Lin’s age and subtracting 3. If Lin’s age is represented by L , and her neighbor’s age is represented by n , then the equation $n = L - 3$ describes the relationship. In this equation, the value of n depends on the value of L , so we call n the **dependent variable** and L the **independent variable**.

Give students 5-8 minutes of quiet work time, followed by a whole-class discussion.

Some students may not be familiar with the word “set” in this context. Explain that it means the scenery and other props and objects that are used on stage during a play or production.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Include the following terms and maintain the display for reference throughout the unit: dependent variable, independent variable. Invite students to suggest language or diagrams to include on the display that will support their understanding of these terms.

Supports accessibility for: Conceptual processing; Memory

Anticipated Misconceptions

For students who struggle to write an equation that relates the two quantities, help them to represent the situation in a concrete way, like with a tape diagram or a discrete diagram. You can also draw their attention to the completed table: "What can you do to each number in the r column to get the number in the y column?" Expressing the relationship in words can be a helpful step to expressing it with an equation.

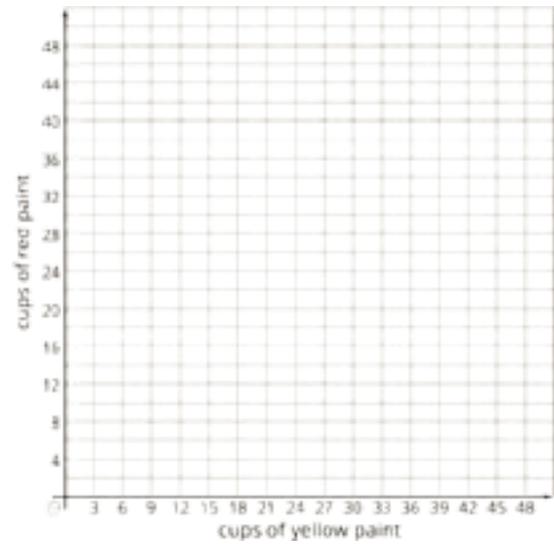
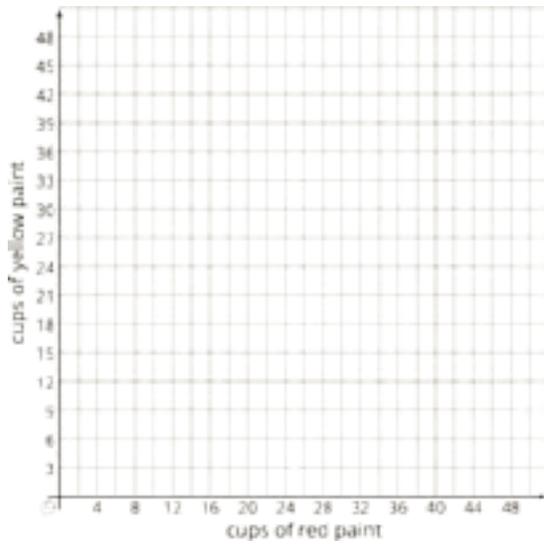
Student Task Statement

Lin needs to mix a specific shade of orange paint for the set of the school play. The color uses 3 parts yellow for every 2 parts red.

1. Complete the table to show different combinations of red and yellow paint that will make the shade of orange Lin needs.

cups of red paint (r)	cups of yellow paint (y)	total cups of paint (t)
2	3	
6		
		20
	18	
14		
16		
		50
	42	

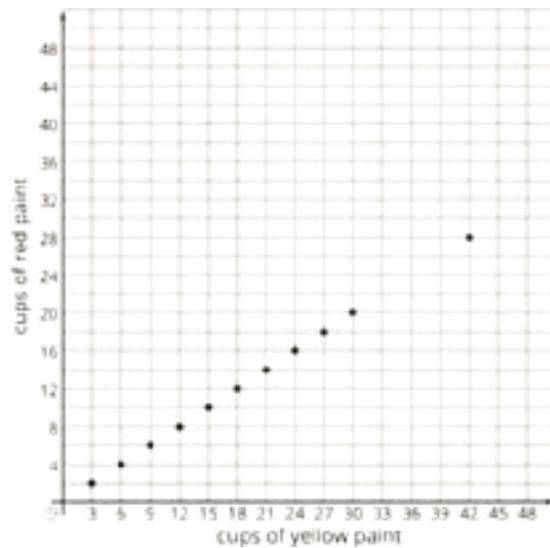
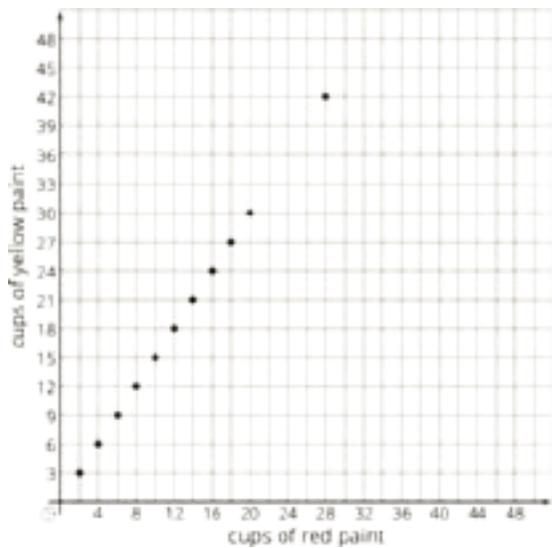
2. Lin notices that the number of cups of red paint is always $\frac{2}{5}$ of the total number of cups. She writes the equation $r = \frac{2}{5}t$ to describe the relationship. Which is the **independent variable**? Which is the **dependent variable**? Explain how you know.
3. Write an equation that describes the relationship between r and y where y is the independent variable.
4. Write an equation that describes the relationship between y and r where r is the independent variable.
5. Use the points in the table to create two graphs that show the relationship between r and y . Match each relationship to one of the equations you wrote.



Student Response

cups of red paint (r)	cups of yellow paint (y)	total cups of paint (t)
2	3	5
6	9	15
8	12	20
12	18	30
14	21	35
16	24	40
20	30	50
28	42	70

1. See table.
2. t is the independent variable and r is the dependent variable. The equation says that r is $\frac{2}{5}$ of t , so the value of r depends on the value of t .
3. $r = \frac{2}{3}y$
4. $y = \frac{3}{2}r$
5. First graph matches $y = \frac{3}{2}r$, second graph matches $r = \frac{2}{3}y$.



Are You Ready for More?

A fruit stand sells apples, peaches, and tomatoes. Today, they sold 4 apples for every 5 peaches. They sold 2 peaches for every 3 tomatoes. They sold 132 pieces of fruit in total. How many of each fruit did they sell?

Student Response

32 apples, 40 peaches, 60 tomatoes

Activity Synthesis

The discussion should focus on how all the representations (tables, equations, graphs) capture the same information but look at it in a different way.

Some guiding questions:

- “Why is it possible to write two different equations to describe the same situation?” (You can choose either quantity to be the dependent variable.)
- “What do you notice about the numbers that multiply by the independent variable in each equation?” (They are reciprocals, they represent the two unit rates for the situation.)
- “Are there other equations we could have written that describe this situation?” (Yes, for example, we can write equations to describe the relationship between the amount of yellow paint and the total number of cups: $y = \frac{3}{5}t$ and $t = \frac{5}{3}y$)
- “What do you notice about the similarities and differences between the graphs?” (Both have points that appear to lie on a line, both lines go up as you move to the right, the line for Lin’s equation slants up more, the coordinates of the points are reversed.)

Access for English Language Learners

Representing: MLR8 Discussion Supports. Ask students to identify where the independent and dependent variables are represented in the table, graph, and equation. Use sentence frames such as “For the equation $r = \frac{2}{5}t$, r is the ____ variable because _____. t is the ____ variable because _____.” This will help students make comparisons of the relationship between the variables and how they are represented.

Design Principle(s): Optimize output (for comparison)

Lesson Synthesis

In this lesson we revisited equivalent ratios by writing equations that represent sets of equivalent ratios and graphing them. Some guiding questions for discussion:

- “We wrote two equations to represent the relationship between cups of red paint and cups of yellow paint. How were these the same and how were they different?”
- “Explain the meaning of **dependent** and **independent** variable.”
- “How do you know which quantity to choose as the independent variable when you write an equation to describe a situation?” (It depends on what you know and what you need to calculate.)
- “When we first studied equivalent ratios, we used double number lines and tables. How do graphs and equations add to your understanding of equivalent ratios? Do they help in solving problems? If so, how?”

16.3 Baking Brownies

Cool Down: 5 minutes

Addressing

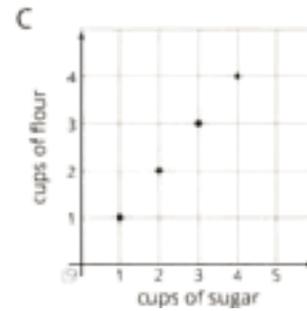
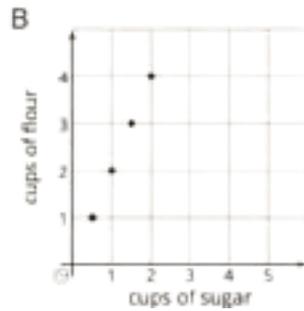
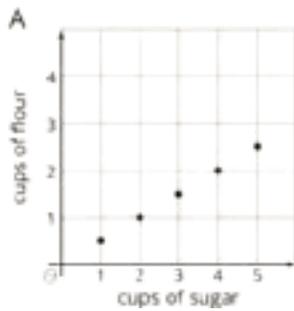
- 6.EE.C.9

Launch

For English Language Learners, clarify that the phrase “calls for” means something that is needed or required in the situation.

Student Task Statement

A brownie recipe calls for 1 cup of sugar and $\frac{1}{2}$ cup of flour to make one batch of brownies. To make multiple batches, the equation $f = \frac{1}{2}s$ where f is the number of cups of flour and s is the number of cups of sugar represents the relationship. Which graph also represents the relationship? Explain how you know.



Student Response

Graph A because for every cup of sugar, there is a half cup of flour. So, at 1 cup of sugar, there is $\frac{1}{2}$ cup of flour. At 2 cups of sugar, there is 1 cup of flour. For each batch, the sugar goes up by 1 cup and the flour goes up by a half cup.

Student Lesson Summary

Equations are very useful for describing sets of equivalent ratios. Here is an example.

A pie recipe calls for 3 green apples for every 5 red apples. We can create a table to show some equivalent ratios.

We can see from the table that r is always $\frac{5}{3}$ as large as g and that g is always $\frac{3}{5}$ as large as r .

green apples (g)	red apples (r)
3	5
6	10
9	15
12	20

We can write equations to describe the relationship between g and r .

- When we know the number of green apples and want to find the number of red apples, we can write:

$$r = \frac{5}{3}g$$

In this equation, if g changes, r is affected by the change, so we refer to g as the **independent variable** and r as the **dependent variable**.

We can use this equation with any value of g to find r . If 270 green apples are used, then $\frac{5}{3} \cdot (270)$ or 450 red apples are used.

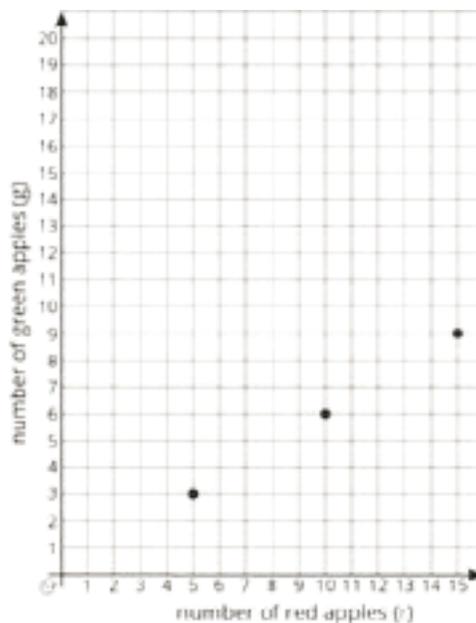
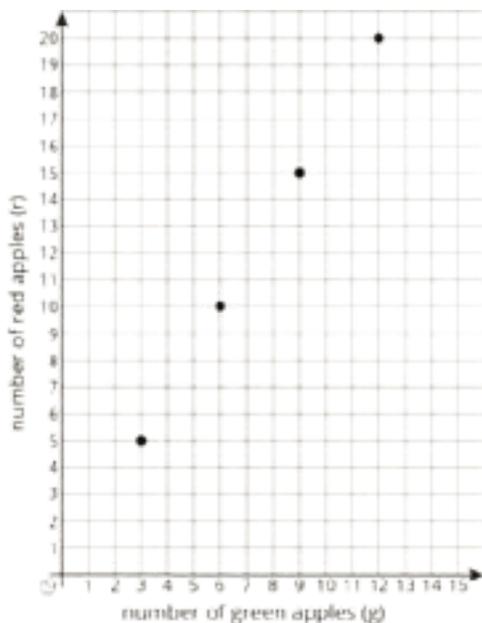
- When we know the number of red apples and want to find the number of green apples, we can write:

$$g = \frac{3}{5}r$$

In this equation, if r changes, g is affected by the change, so we refer to r as the independent variable and g as the dependent variable.

We can use this equation with any value of r to find g . If 275 red apples are used, then $\frac{3}{5} \cdot (275)$ or 165 green apples are used.

We can also graph the two equations we wrote to get a visual picture of the relationship between the two quantities.



Glossary

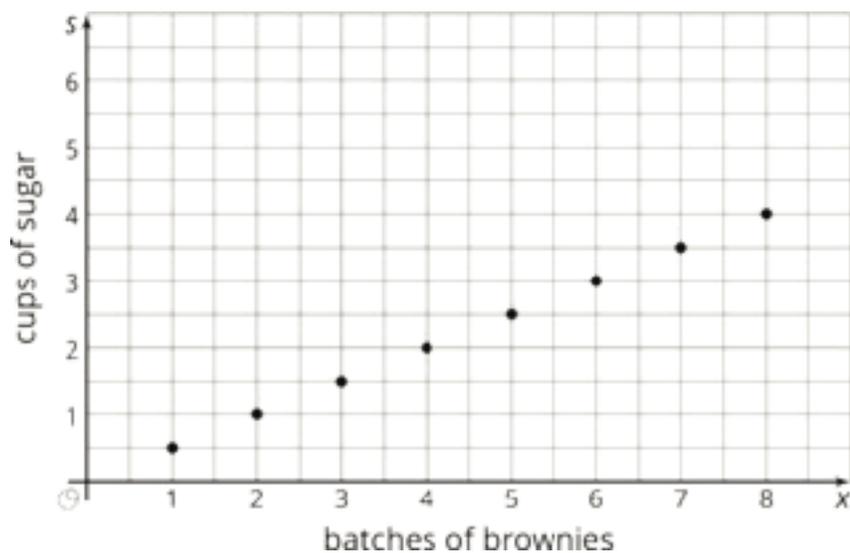
- dependent variable
- independent variable

Lesson 16 Practice Problems

Problem 1

Statement

Here is a graph that shows some values for the number of cups of sugar, s , required to make x batches of brownies.



- a. Complete the table so that the pair of numbers in each column represents the coordinates of a point on the graph.

x	1	2	3	4	5	6	7
s							

- b. What does the point $(8, 4)$ mean in terms of the amount of sugar and number of batches of brownies?
- c. Write an equation that shows the amount of sugar in terms of the number of batches.

Solution

a.

x	1	2	3	4	5	6	7
s	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$

- b. To make 8 batches of brownies, you need 4 cups of sugar.
- c. $s = \frac{1}{2}x$

Problem 2

Statement

Each serving of a certain fruit snack contains 90 calories.

- Han wants to know how many calories he gets from the fruit snacks. Write an equation that shows the number of calories, c , in terms of the number of servings, n .
- Tyler needs some extra calories each day during his sports season. He wants to know how many servings he can have each day if all the extra calories come from the fruit snack. Write an equation that shows the number of servings, n , in terms of the number of calories, c .

Solution

a. $c = 90n$

b. $n = \frac{c}{90}$ or $n = c \div 90$

Problem 3

Statement

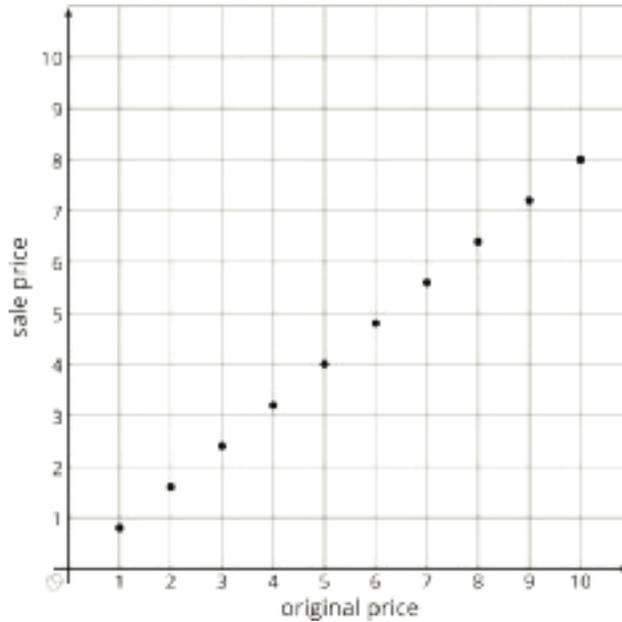
Kiran shops for books during a 20% off sale.

- What percent of the original price of a book does Kiran pay during the sale?
- Complete the table to show how much Kiran pays for books during the sale.
- Write an equation that relates the sale price, s , to the original price p .
- On graph paper, create a graph showing the relationship between the sale price and the original price by plotting the points from the table.

original price in dollars (p)	sale price in dollars (s)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Solution

- 80%
- Sale prices: 0.80, 1.60, 2.40, 3.20, 4.00, 4.80, 5.60, 6.40, 7.20, 8.00
- $s = 0.8p$
-



Problem 4

Statement

Evaluate each expression when x is 4 and y is 6.

- $(6 - x)^3 + y$
- $2 + x^3$
- $2^x - 2y$
- $\left(\frac{1}{2}\right)^x$
- $1^x + 2^x$
- $\frac{2^x}{x^2}$

Solution

- 14

- b. 66
- c. 4
- d. $\frac{1}{16}$
- e. 17
- f. 1

(From Unit 6, Lesson 15.)

Problem 5

Statement

Find $(12.34) \cdot (0.7)$. Show your reasoning.

Solution

8.638. Sample reasoning: $1,234 \cdot 7 = 8,638$. Because 12.34 is $\frac{1}{100}$ of 1,234 and 0.7 is $\frac{1}{10}$ of 7, the product 8,638 needs to be multiplied by $(\frac{1}{100} \cdot \frac{1}{10})$ or $\frac{1}{1,000}$.

(From Unit 5, Lesson 8.)

Problem 6

Statement

For each expression, write another division expression that has the same value and that can be used to help find the quotient. Then, find each quotient.

- a. $302.1 \div 0.5$
- b. $12.15 \div 0.02$
- c. $1.375 \div 0.11$

Solution

Answers vary. Sample response:

- a. $3,021 \div 5$. The quotient is 604.2.
- b. $1,215 \div 2$. The quotient is 607.5.
- c. $137.5 \div 11$. The quotient is 12.5

(From Unit 5, Lesson 13.)

Lesson 17: Two Related Quantities, Part 2

Goals

- Create a table, graph, and equation to represent the relationship between distance and time for an object moving at a constant speed.
- Identify (in writing) the independent and dependent variable in a equation.
- Interpret (orally and in writing) an equation that represents the relationship between distance and time for an object moving at a constant speed.

Learning Targets

- I can create tables and graphs to represent the relationship between distance and time for something moving at a constant speed.
- I can write an equation with variables to represent the relationship between distance and time for something moving at a constant speed.

Lesson Narrative

In this second lesson on representing relationships between two quantities, walking at a constant rate provides the context for writing an equation that represents the relationship. Students use and make connections between tables, graphs, and equations that represent the relationship between time and distance. They use their representations to compare rates and consider how each of the representations would change if the independent and dependent variables were switched.

Alignments

Addressing

- 6.EE.C.9: Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.
- 6.RP.A.3.a: Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
- 6.RP.A.3.b: Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Instructional Routines

- MLR7: Compare and Connect
- Notice and Wonder

Required Materials

Colored pencils

Student Learning Goals

Let's use equations and graphs to describe stories with constant speed.

17.1 Walking to the Library

Warm Up: 10 minutes

Students reason about the relationship between distance, rate, and time to solve a problem. The purpose is to reactivate what students know about constant speed contexts, where constant speed is represented by a set of equivalent ratios associating distance traveled and elapsed time. In the longer activity that follows, students represent a constant speed context using a table, equations, and graphs.

As students work, watch for different representations used (particularly tables) as well as for students who calculate each person's speed in miles per hour or each person's pace in hours per mile.

Addressing

- 6.EE.C.9
- 6.RP.A.3.a
- 6.RP.A.3.b

Instructional Routines

- Notice and Wonder

Launch

Consider starting off by having students close their books or devices, and display the following situation. Ask students, "What do you notice?" "What do you wonder?"

Lin and Jada each leave school at 3 p.m. to walk to the library. They each walk at a steady rate. When do they arrive?

Give them 1 minute to think of at least one thing they notice and one thing they wonder.

Students might notice that they leave at 3 p.m., that they walk at a steady rate (also called a constant speed), and that there is not enough information given to answer the question.

Students might wonder many things, but in order to answer the question, they would need to know:

- How fast do they each walk?
- How far away is the library?

Ask students to open their books or devices and use the additional information to solve the problem by any method they choose. If desired, remind students of tools that may be appropriate including double number lines or tables of equivalent ratios.

Student Task Statement

Lin and Jada each walk at a steady rate from school to the library. Lin can walk 13 miles in 5 hours, and Jada can walk 25 miles in 10 hours. They each leave school at 3:00 and walk $3\frac{1}{4}$ miles to the library. What time do they each arrive?

Student Response

Lin arrives at 4:15 and Jada arrives at 4:18. Explanations vary. Sample response:

Here is a table of equivalent ratios for Lin. Instead of $3\frac{1}{4}$ use $\frac{13}{4}$.

distance (miles)	time (hours)
13	5
1	$\frac{5}{13}$
$\frac{13}{4}$	$\frac{5}{4}$

Here is a table of equivalent ratios for Jada.

distance (miles)	time (hours)
25	10
1	$\frac{10}{25}$
$\frac{13}{4}$	$\frac{130}{100}$

To travel $3\frac{1}{4}$ miles it takes Lin $\frac{5}{4}$ or 1.25 hours, which is an hour and fifteen minutes. She arrives at 4:15.

To travel $3\frac{1}{4}$ miles it takes Jada $\frac{130}{100}$ or 1.3 hours, which is an hour and eighteen minutes. She arrives at 4:18.

Activity Synthesis

Invite students who used different representations and lines of reasoning to share. If no student mentions it, demonstrate how to represent on person's trip using a table of equivalent ratios with columns representing distance and time. Ask a student to explain how to use the table to reason about the distance traveled in 1 hour and the time it takes to travel 1 mile.

One way to reason is to notice that Lin can walk 26 miles in 10 hours, so she walks slightly faster than Jada (who can complete 25 miles in 10 hours) and should arrive a bit sooner. Both of these ways of reasoning are in preparation for the following activity.

17.2 The Walk-a-thon

25 minutes (there is a digital version of this activity)

This activity revisits the familiar context of traveling at a constant rate. Students calculate and compare the unit rates in miles per hour for three people and consider the graphs and equations that describe the distance–time relationship.

Addressing

- 6.EE.C.9
- 6.RP.A.3.a

Instructional Routines

- MLR7: Compare and Connect

Launch

Give students access to colored pencils and 5–8 minutes of quiet work time, followed by a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts. For example, after students have completed the table about the walk-a-thon, check-in with either select groups of students or the whole class. Invite students to share the strategies they have used so far as well as any questions they have before continuing.

Supports accessibility for: Organization; Attention

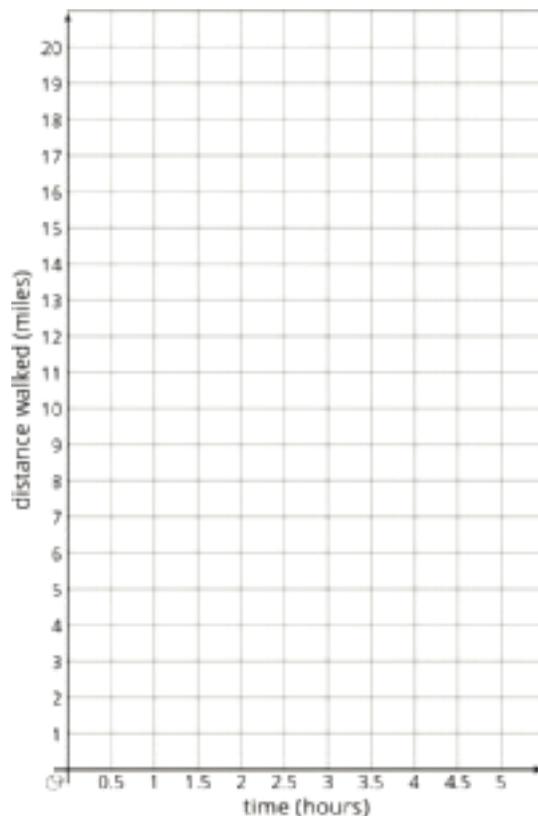
Student Task Statement

Diego, Elena, and Andre participated in a walk-a-thon to raise money for cancer research. They each walked at a constant rate, but their rates were different.

1. Complete the table to show how far each participant walked during the walk-a-thon.

time in hours	miles walked by Diego	miles walked by Elena	miles walked by Andre
1			
2	6		
	12	11	
5			17.5

- How fast was each participant walking in miles per hour?
- How long did it take each participant to walk one mile?
- Graph the progress of each person in the **coordinate plane**. Use a different color for each participant.



- Diego says that $d = 3t$ represents his walk, where d is the distance walked in miles and t is the time in hours.
 - Explain why $d = 3t$ relates the distance Diego walked to the time it took.

- b. Write two equations that relate distance and time: one for Elena and one for Andre.
6. Use the equations you wrote to predict how far each participant would walk, at their same rate, in 8 hours.
7. For Diego's equation and the equations you wrote, which is the dependent variable and which is the independent variable?

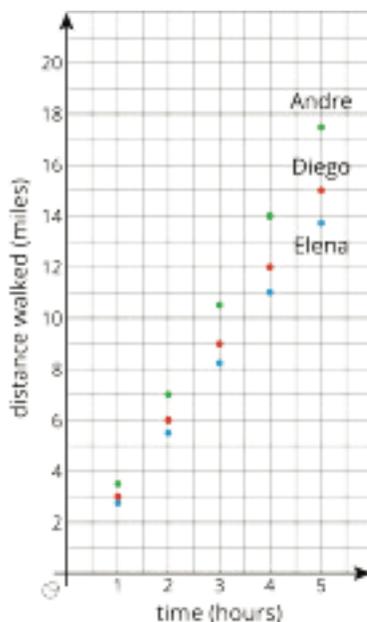
Student Response

1.

time in hours	miles walked by Diego	miles walked by Elena	miles walked by Andre
1	3	2.75 or $2\frac{3}{4}$	3.5
2	6	5.5 or $5\frac{1}{2}$	7
4	12	11	14
5	15	13.75 or $13\frac{3}{4}$	17.5

2. Diego: 3 miles per hour, Elena: 2.75 miles per hour, Andre: 3.5 miles per hour

3. Diego: $\frac{1}{3}$ hour, Elena: $\frac{4}{11}$ hour, Andre: $\frac{2}{7}$ hour



4.

5. a. Answers vary. Sample response: Diego walked 3 miles in 1 hour. So you can multiply the number of hours by 3 to find the distance.

b. Elena: $d = 2.75t$, Andre: $d = 3.5t$ or equivalent

6. Diego: 24 miles, Elena: 22 miles, Andre: 28 miles

7. Answers vary. Sample response: In Diego's equation, time is the independent variable and distance is the dependent variable. If the equations look instead like $t = \frac{1}{3}d$, the distance is the independent variable and time is the dependent variable.

Are You Ready for More?

- Two trains are traveling toward each other, on parallel tracks. Train A is moving at a constant speed of 70 miles per hour. Train B is moving at a constant speed of 50 miles per hour. The trains are initially 320 miles apart. How long will it take them to meet? One way to start thinking about this problem is to make a table. Add as many rows as you like.
- How long will it take a train traveling at 120 miles per hour to go 320 miles?
- Explain the connection between these two problems.

	train A	train B
starting position	0 miles	320 miles
after 1 hour	70 miles	270 miles
after 2 hours		

Student Response

- $2\frac{2}{3}$ hours, or 2 hours and 40 minutes.
- 2 hours and 40 minutes.
- Since trains A and B are moving toward each other, we can add their two speeds to find the rate at which their distance decreases. 70 miles per hour + 50 miles per hour = 120 miles per hour.

Activity Synthesis

The goal of the discussion is to ensure that students understand how each of the table, graph, and equations represent the situation and how they are connected to each other. Consider asking some of the following questions:

- "How can you determine from the table who walked the fastest and slowest?"
- "How can you determine from the graph who walked the fastest and slowest?"
- "How can you determine from the equations who walked the fastest and slowest?"

- “If distance was the independent variable, how would the equations and graphs be different?”

Access for English Language Learners

Representing, Speaking, Listening: MLR7 Compare and Connect. Invite students to prepare a visual display of their table, graph, and equations that relate distance and time for each participant. As students analyze each others' work, ask them to share what is especially clear in a particular representation. Listen for and amplify the language students use to describe how the distance traveled increases by a constant amount per hour and how this pattern can be seen on the table and graph. This will foster students' meta-awareness and support constructive conversations as they compare and connect the tables, graphs, and equations that represent the same situation.

Design Principles(s): Cultivate conversation; Maximize meta-awareness

Lesson Synthesis

Ask students to think about the different representations they used for a situation involving time, distance, and a constant rate. Invite their thoughts on which representations would be most helpful in finding unknown quantities in different situations. Ask what factors they would consider in deciding which quantity to set as the independent variable when writing an equation to describe a situation.

17.3 Interpret the Point

Cool Down: 5 minutes

Addressing

- 6.EE.C.9

Student Task Statement

During a walk-a-thon, Noah's time in hours, t , and distance in miles, d , are related by the equation $\frac{1}{3}d = t$. A graph of the equation includes the point (12, 4).

1. Identify the independent variable.
2. What does the point (12, 4) represent in this situation?
3. What point would represent the time it took to walk $7\frac{1}{2}$ miles?

Student Response

1. Distance, d , is the independent variable.
2. Answers vary. Sample responses: Noah can walk 12 miles in 4 hours. It takes Noah 4 hours to walk 12 miles.

3. $(7\frac{1}{2}, 2\frac{1}{2})$. $\frac{1}{3}(7\frac{1}{2}) = t, t = 2\frac{1}{2}$

Student Lesson Summary

Equations are very useful for solving problems with constant speeds. Here is an example.

A boat is traveling at a constant speed of 25 miles per hour.

1. How far can the boat travel in 3.25 hours?
2. How long does it take for the boat to travel 60 miles?

We can write equations to help us answer questions like these.

Let's use t to represent the time in hours and d to represent the distance in miles that the boat travels.

When we know the time and want to find the distance, we can write:

$$d = 25t$$

In this equation, if t changes, d is affected by the change, so we t is the independent variable and d is the dependent variable.

This equation can help us find d when we have any value of t . In 3.25 hours, the boat can travel $25(3.25)$ or 81.25 miles.

When we know the distance and want to find the time, we can write:

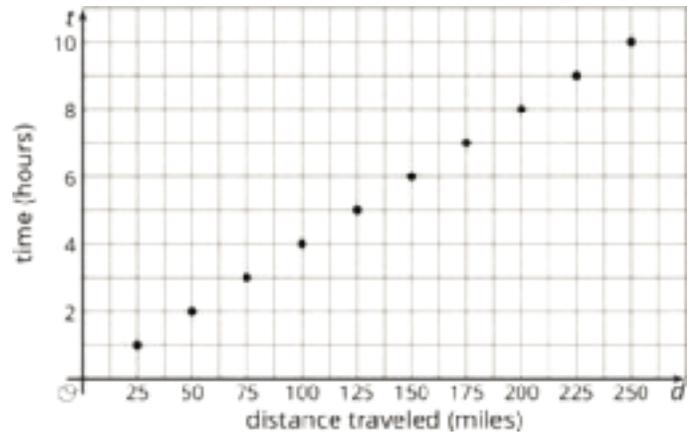
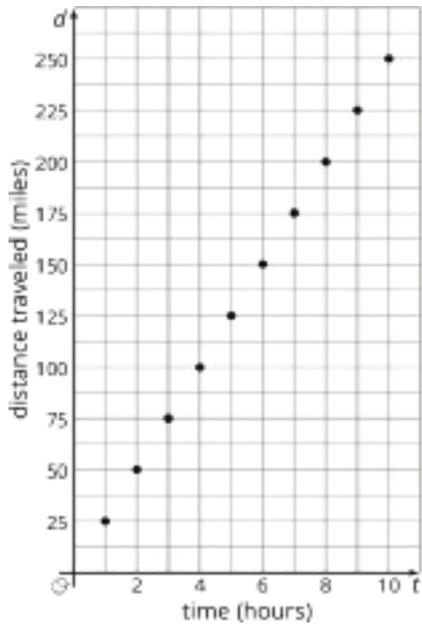
$$t = \frac{d}{25}$$

In this equation, if d changes, t is affected by the change, so we d is the independent variable and t is the dependent variable.

This equation can help us find t when for any value of d . To travel 60 miles, it will take $\frac{60}{25}$ or $2\frac{2}{5}$ hours.

These problems can also be solved using important ratio techniques such as a table of equivalent ratios. The equations are particularly valuable in this case because the answers are not round numbers or easy to quickly evaluate.

We can also graph the two equations we wrote to get a visual picture of the relationship between the two quantities:



Glossary

- coordinate plane

Lesson 17 Practice Problems

Problem 1

Statement

A car is traveling down a road at a constant speed of 50 miles per hour.

- Complete the table with the amounts of time it takes the car to travel certain distances, or the distances traveled for certain amounts of time.
- Write an equation that represents the distance traveled by the car, d , for an amount of time, t .
- In your equation, which is the dependent variable and which is the independent variable?

time (hours)	distance (miles)
2	
1.5	
t	
	50
	300
	d

Solution

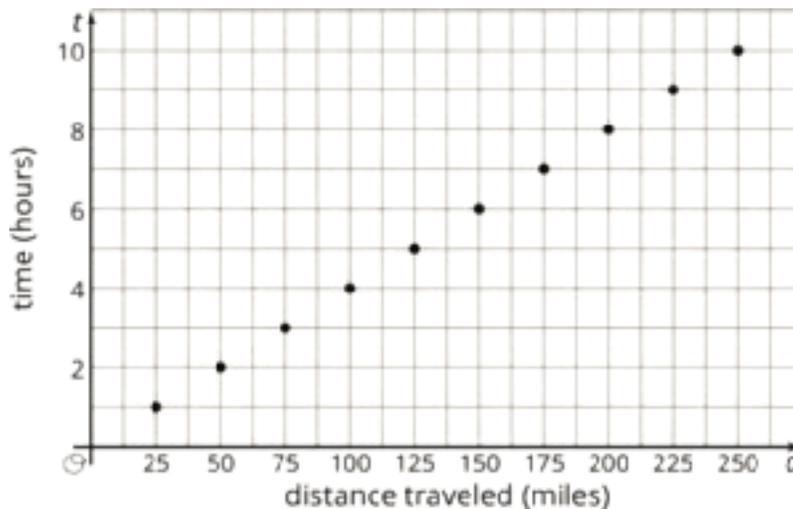
time (hours)	distance (miles)
2	100
1.5	75
t	$50t$
1	50
6	300
$\frac{1}{50}d$	d

- see table
- $d = 50t$
- t is the independent variable and d is the dependent variable.

Problem 2

Statement

The graph represents the amount of time in hours it takes a ship to travel various distances in miles.



- Write the coordinates of one point on the graph. What does the point represent?
- What is the speed of the ship in miles per hour?
- Write an equation that relates the time, t , it takes to travel a given distance, d .

Solution

- Answers vary. Sample response: $(75, 3)$. This point represents that the ship travels 75 miles in 3 hours.
- 25 miles per hour
- $d = 25t$ or $t = \frac{d}{25}$

Problem 3

Statement

Find a solution to each equation in the list that follows (not all numbers will be used):

- $2^x = 8$
- $2^x = 2$
- $x^2 = 100$
- $x^2 = \frac{1}{100}$
- $x^1 = 7$
- $2^x \cdot 2^3 = 2^7$

g. $\frac{2^x}{2^3} = 2^5$

List: $\frac{1}{10}$ $\frac{1}{3}$ 1 2 3 4 5 7 8 10 16

Solution

a. 3

b. 1

c. 10

d. $\frac{1}{10}$

e. 7

f. 4

g. 8

(From Unit 6, Lesson 15.)

Problem 4

Statement

Select all the expressions that are equivalent to $5x + 30x - 15x$.

A. $5(x + 6x - 3x)$

B. $(5 + 30 - 15) \cdot x$

C. $x(5 + 30x - 15x)$

D. $5x(1 + 6 - 3)$

E. $5(x + 30x - 15x)$

Solution

["A", "B", "D"]

(From Unit 6, Lesson 11.)

Problem 5

Statement

Evaluate each expression if x is 1, y is 2, and z is 3.

a. $7x^2 - z$



b. $(x + 4)^3 - y$

c. $y(x + 3^3)$

d. $(7 - y + z)^2$

e. $0.241x + x^3$

Solution

a. 4

b. 123

c. 56

d. 64

e. 1.241

(From Unit 6, Lesson 15.)

Lesson 18: More Relationships

Goals

- Coordinate (orally and in writing) graphs, tables, and equations that represent the same relationship.
- Create an equation and a graph to represent the relationship between two variables that are inversely proportional.
- Describe and interpret (orally and in writing) a graph that represents a nonlinear relationship between independent and dependent variables.

Learning Targets

- I can create tables and graphs that show different kinds of relationships between amounts.
- I can write equations that describe relationships with area and volume.

Lesson Narrative

This lesson is optional. It offers opportunities to look at multiple representations (equations, graphs, and tables) for some different contexts.

This final lesson on relationships between two quantities examines situations of constant area, constant volume, and a doubling relationship. Students have an opportunity to engage in MP7 as they notice the similar structures of the situations in the Making a Banner and Cereal Boxes activities, as well as connecting the Multiplying Mosquitoes activity to prior work with exponents and the Genie's coins situation from earlier in the unit. They may use those observations and knowledge to more easily solve the problems in the activities.

Consider offering students a choice about which one they work on. Then, in the lesson synthesis, invite students to share their work with the class and compare and contrast the representations of the different contexts.

Alignments

Addressing

- 6.EE.C.9: Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.

Instructional Routines

- MLR6: Three Reads

- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Which One Doesn't Belong?

Student Learning Goals

Let's use graphs and equations to show relationships involving area, volume, and exponents.

18.1 Which One Doesn't Belong: Graphs

Warm Up: 5 minutes

The purpose of this warm-up is to prompt students to reason about what a set of organized points in a coordinate plane might mean. This activity invites students to explain their reasoning and hold mathematical conversations, and allows you to hear how they use terminology and talk about points in a coordinate plane. To allow all students to access the activity, there is not one correct answer so students are able to choose any figure as long as they can support their reasoning. As students share their responses, listen for important ideas and terminology that will be helpful in upcoming work.

Addressing

- 6.EE.C.9

Instructional Routines

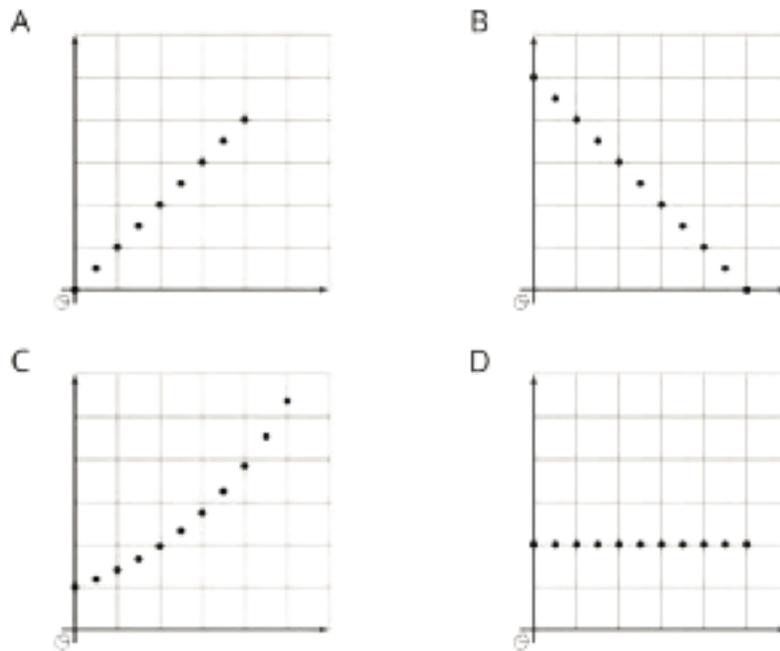
- Which One Doesn't Belong?

Launch

Arrange students in groups of 2-4. Display the image of the four figures for all to see. Ask students to indicate when they have noticed one figure that does not belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their group. After everyone has conferred in groups, ask the group to offer at least one reason *each* figure doesn't belong.

Student Task Statement

Which one doesn't belong?



Student Response

Answers vary. Possible responses:

A: The only one that starts at the origin; the only one with the points (1, 1), (2, 2), (3, 3) and (4, 4). Students might note that this graph looks like the graphs of ratio relationships.

B: The only one that decreases as you move to the right

C: The only one where the points are not on a line. Points are not equally spaced; the higher points are farther apart.

D: The only one that is flat, not increasing or decreasing as you move to the right

Activity Synthesis

After students have conferred in groups, invite each group to share one reason why a particular figure might not belong. Record and display the responses for all to see. After each response, poll the rest of the class if they agree or disagree. Since there is no single correct answer to the question of which shape does not belong, attend to students' explanations and ensure the reasons given are correct. During the discussion, prompt students with the following questions:

- "How would you describe the relationship between the two quantities represented by the two axes?"
- "Do you have any ideas about what quantities or relationships any of these graphs might represent?" (If students do have ideas, based on relationships they have explored or others they are thinking about, have them explain why the graph represents the relationship.)

18.2 Making a Banner

Optional: 15 minutes (there is a digital version of this activity)

In this activity, students consider the relationship between length and width for different rectangles with the same given area and are asked to compare strategies for finding various lengths and widths. They make sense of how the graph shows what happens to the width when the length changes and what the plotted points on the graph mean in the context of the problem.

Addressing

- 6.EE.C.9

Instructional Routines

- MLR6: Three Reads

Launch

Give students 10 minutes of quiet work time, followed by a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Remind students that they can draw rectangle diagrams to help them determine the missing values.
Supports accessibility for: Social-emotional skills; Conceptual processing

Access for English Language Learners

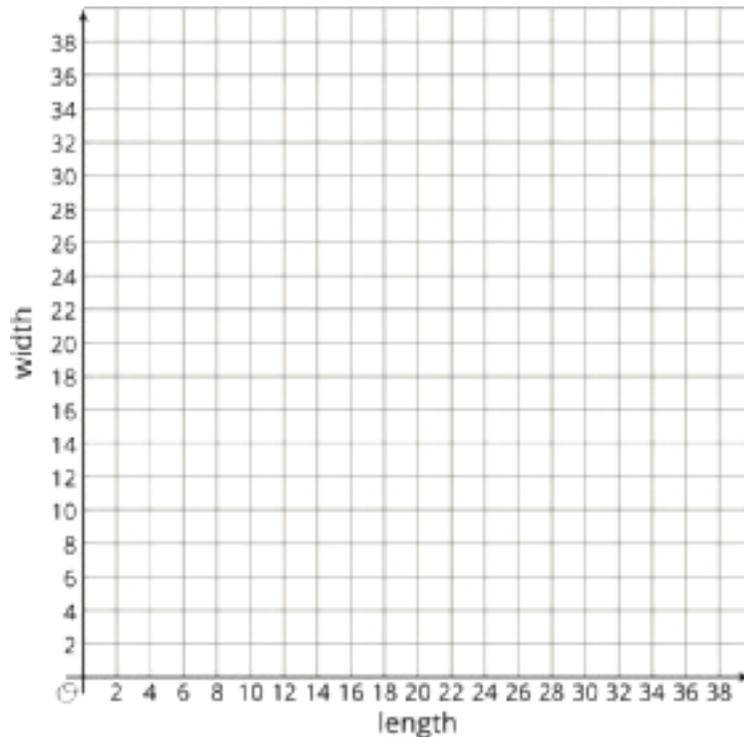
Reading: MLR6 Three Reads. Use this routine to support reading comprehension of this word problem, without solving it for students. In the first read, students read the problem with the goal of comprehending the situation (e.g., This problem is about creating a banner. Mai needs to buy material). Use the second read to identify the important quantities by asking students what can be counted or measured (e.g., The area of the banner needs to be 36 square units). In the third read, ask students to brainstorm possible strategies to answer the questions. This helps students connect the language in the word problem and the reasoning needed to solve the problem keeping the intended level of cognitive demand in the task.
Design Principle(s): Support sense-making; Maximize meta-awareness

Student Task Statement

Mai is creating a rectangular banner to advertise the school play. The material for the banner is sold by the square foot. Mai has enough money to buy 36 square feet of material. She is trying to decide on the length and width of the banner.

1. If the length is 6 feet, what is the width?

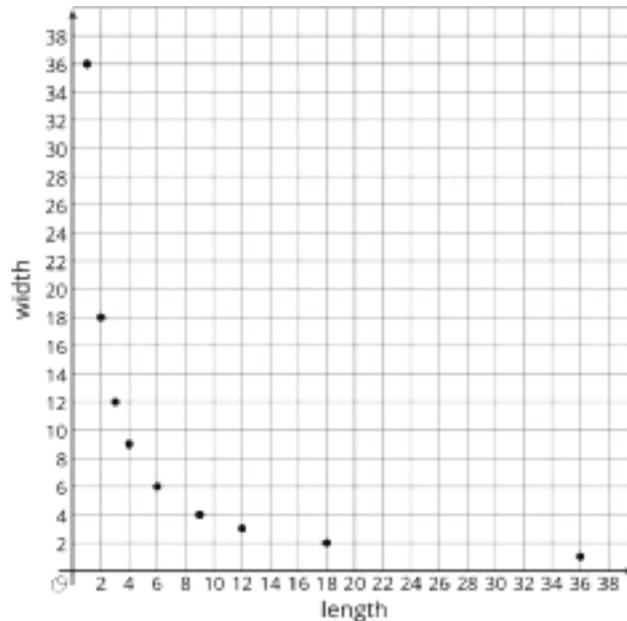
2. If the length is 4 feet, what is the width?
3. If the length is 9 feet, what is the width?
4. To find different combinations of length and width that give an area of 36 square feet, Mai uses the equation $w = \frac{36}{\ell}$, where w is the width and ℓ is the length. Compare your strategy and Mai's method for finding the width. How were they the same or different?
5. Use several combinations of length and width to create a graph that shows the relationship between the side lengths of various rectangles with area 36 square feet.



6. Explain how the graph describes the relationship between length and width for different rectangles with area 36.
7. Suppose Mai used the equation $\ell = \frac{36}{w}$ to find the length for different values of the width. Would the graph be different if she graphed length on the vertical axis and width on the horizontal axis? Explain how you know.

Student Response

1. 6 feet
2. 9 feet
3. 4 feet
4. Answers vary. Sample response: I looked for a number to multiply the length by to get 36.



5.

6. Answers vary. Sample response: As the length increases, the width decreases.

7. The graph would look the same. Explanation varies. Sample explanation: The values for length and width would just switch.

Activity Synthesis

The discussion should focus on the connection between the situation, the equation or strategy for finding combinations that make the area 36, and the graph that represents the relationship between length and width in the different rectangles.

Some guiding questions:

- “What does the point (2, 18) (for example) in the graph mean? In general, what does each point represent?” (each point represents a rectangle with area 36. (2, 18) represents a rectangle with length 2 and height 18)
- “Why does it make sense that the graph falls as you move to the right?” (The length and width are factors of a fixed product, so if one increases the other has to decrease)
- “Where do you see the area 36 in each of the situation, the equation or strategy for finding combinations, and the graph that shows those combinations?” (in the graph, the coordinates of each point multiply to 36)
- “Describe what the graph would look like if it were to extend more to the right. Name some points on the graph and describe their coordinates.” (Points could be $(\frac{9}{10}, 40)$, $(\frac{18}{25}, 50)$, $(\frac{1}{2}, 72)$. w would be a fraction less than 1 as ℓ gets larger than 36 because w and ℓ have to multiply to 36. The points will keep getting closer to the x -axis as w gets smaller.

18.3 Cereal Boxes

Optional: 15 minutes (there is a digital version of this activity)

This activity presents a situation with a similar structure to the area situation in the Making a Banner activity. Students consider different combinations of base areas and heights that keep the volume of a rectangular box at 225 cubic inches. They complete a table for given values of area and height, write an equation relating the area and height, and graph the relationship.

If students have completed the Making a Banner activity, prompt them to think about similarities and differences they noticed between the activities. Invite students to share these thoughts during the discussion. Students have an opportunity to engage in MP7 as they notice the similar structures of the two situations and use that knowledge to solve the problems in this activity.

Addressing

- 6.EE.C.9

Instructional Routines

- MLR7: Compare and Connect

Launch

Give students 10 minutes of quiet work time and follow with a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Provide Access for Physical Action. Provide access to tools and assistive technologies. Allow students to use the applet for this activity to facilitate plotting the ordered pairs from the table on a graph.

Supports accessibility for: Visual-spatial processing; Conceptual processing; Organization

Student Task Statement

A cereal manufacturer needs to design a cereal box that has a volume of 225 cubic inches and a height that is no more than 15 inches.

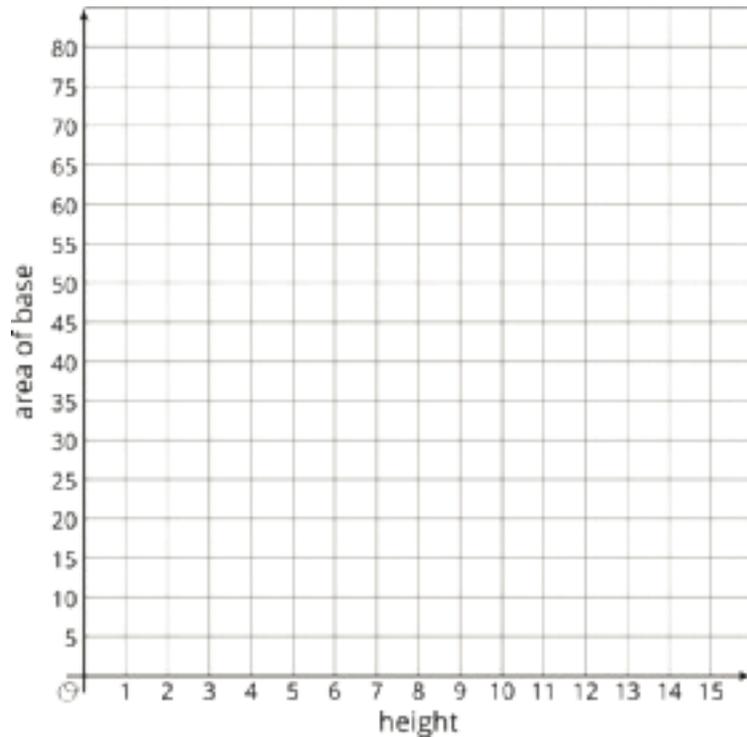
1. The designers know that the volume of a rectangular prism can be calculated by multiplying the area of its base and its height. Complete the table with pairs of values that will make the volume 225 in^3 .

height (in)		5	9	12		$7\frac{1}{2}$
area of base (in^2)	75				15	

2. Describe how you found the missing values for the table.

3. Write an equation that shows how the area of the base, A , is affected by changes in the height, h , for different rectangular prisms with volume 225 in^3 .

4. Plot the ordered pairs from the table on the graph to show the relationship between the area of the base and the height for different boxes with volume 225 in^3 .



Student Response

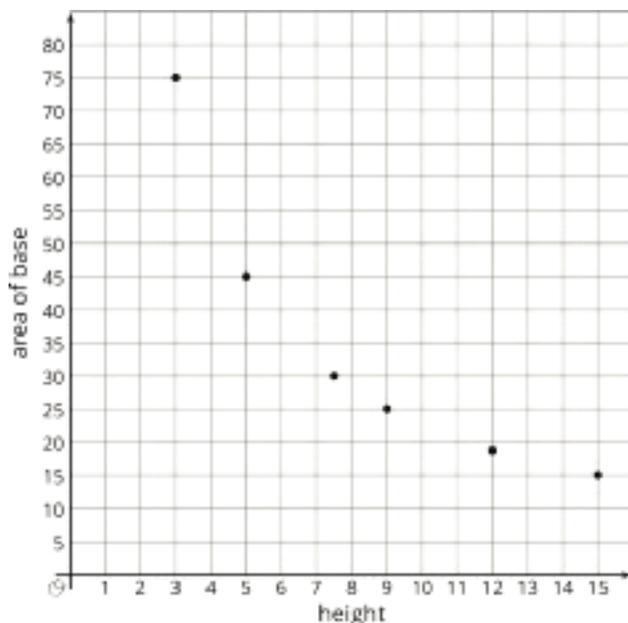
1.

height (in)	3	5	9	12	15	$7\frac{1}{2}$
area of base (in^2)	75	45	25	18.75	15	30

2. Answers vary. Sample response: Divide 225 by either the area or height.

3. Answers vary. Sample response: $A = \frac{225}{h}$ or equivalent

4.



Activity Synthesis

The purpose of the discussion is to connect this activity to the previous one and point out similarities and differences between them.

Consider asking the following questions:

- "How does this situation compare to the one in the Making a Banner activity?" (The structure of the equations was similar: the two quantities multiply to a constant.)
- "How did you decide on an equation that represents the relationship?"
- "How would the graph be different if height was on the vertical axis and area on the horizontal axis?" (It would look the same since the two numbers multiply to a constant.)

Access for English Language Learners

Representing, Speaking, Listening: MLR7 Compare and Connect. Invite students to prepare a visual display of their table, graph, and equation that represent the relationship between the area of the base and height of the cereal box. As students analyze each others' work, ask them to share what is especially clear in a particular representation. Listen for and amplify the language students use to describe how the area of the base decreases as the height of the cereal boxes increases. Invite students to identify where they see the volume of 225 cubic inches represented in the table, graph, and equation. Listen for and amplify the language students use to describe how the coordinates of each point on the graph multiply to 225. This will foster students' meta-awareness about language and support constructive conversations.

Design Principles(s): Cultivate conversation; Maximize meta-awareness

18.4 Multiplying Mosquitoes

Optional: 10 minutes (there is a digital version of this activity)

In this activity, students consider a doubling relationship where the exponent is a variable. Monitor for students who connect this activity to the lessons on exponents, or who recognize that the quantities in this relationship are changing with respect to each other in a different manner than previous examples they have seen. Have these students share during the discussion.

Addressing

- 6.EE.C.9

Instructional Routines

- MLR8: Discussion Supports

Launch

Give students 5-7 minutes of quiet work time, followed by a whole-class discussion.

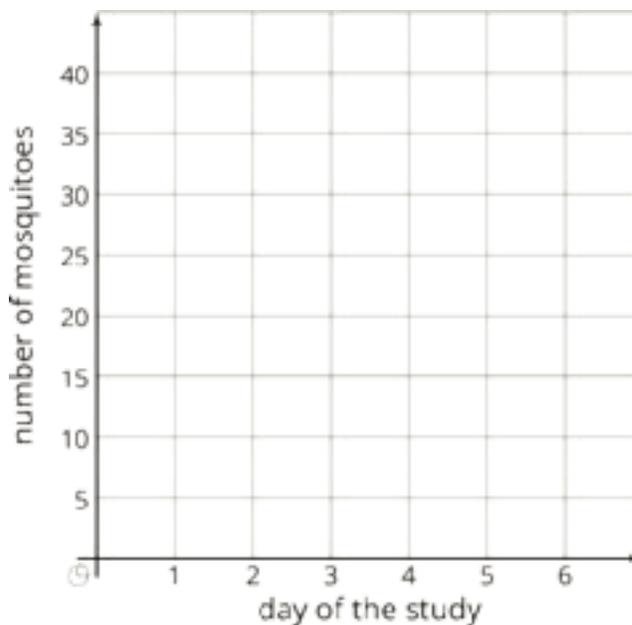
Student Task Statement

A researcher who is studying mosquito populations collects the following data:

day in the study (d)	number of mosquitoes (n)
1	2
2	4
3	8
4	16
5	32

1. The researcher said that, for these five days, the number of mosquitoes, n , can be found with the equation $n = 2^d$ where d is the day in the study. Explain why this equation matches the data.

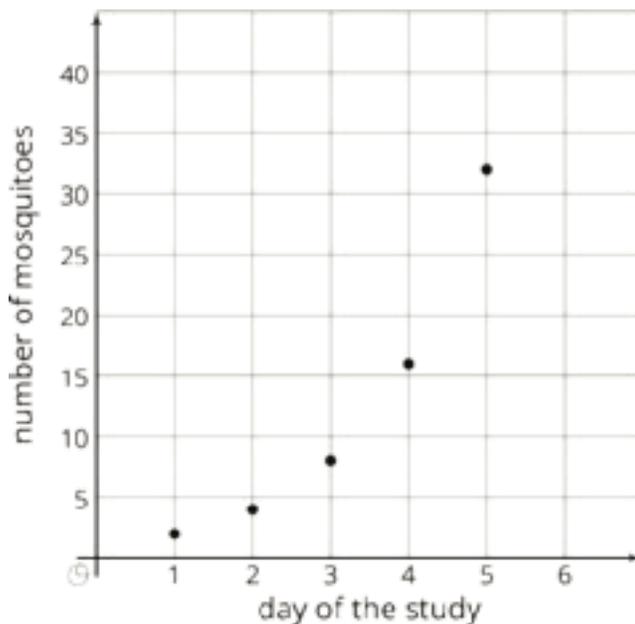
- Use the ordered pairs in the table to graph the relationship between number of mosquitoes and day in the study for these five days.



- Describe the graph. Compare how the data, equation, and graph illustrate the relationship between the day in the study and the number of mosquitoes.
- If the pattern continues, how many mosquitoes will there be on day 6?

Student Response

- Answers vary. Sample response: $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, and $2^5 = 32$, so the equation matches the data.



-

3. Answers vary. Sample response: The graph is increasing and rises very quickly after the first few days. We see bigger and bigger jumps from day to day in the data. In the equation, the exponent means that the numbers are doubling each day.

4. $2^6 = 64$, 64 mosquitoes

Activity Synthesis

The purpose of this discussion is to help students make connections between the table, graph, and equation that describe this situation. Ask students to share who connected this activity to the lessons on exponents, or who recognize that the quantities in this relationship are changing with respect to each other in a different manner than previous examples they have seen.

Consider asking some of these questions:

- “Which of the data, equation, or graph gives you more of a feel for how the mosquito population changes with the day in the study?”
- “Think back to the lesson on exponents with the dot pattern and the Genie activity. How do those situations compare with this mosquito situation? How would tables of data, graphs, and equations compare?”
- “What are some similarities and differences you noticed between all the relationships you've seen in the last few lessons?”

Access for English Language Learners

Speaking, Listening: MLR8 Discussion Supports. To support whole-class discussion, display sentence frames such as, “The dot pattern is similar to/ different than the mosquito situation because _____,” and “The genie situation is similar to/ different than the mosquito situation because _____.” This routine will support rich and inclusive discussion about the similarities and differences between all of the relationships students have seen in the last few lessons.

Design Principle(s): Optimize output (for comparison); Cultivate conversation

Lesson Synthesis

In this lesson we looked at three situations: rectangles with the same area, rectangular prisms with the same volume, and one quantity that doubles repeatedly each time another quantity is increased by 1. In each situation we examined the relationship between two quantities: length and width of the rectangle, area of the base and height, number of mosquitos and number of days.

Invite students to share and discuss their work with the class. To facilitate discussion, ask students to restate each other's explanations. Draw attention to the meaning of the components of each representation, and how the representations of the different situations are alike and different. For example, ask questions like,

- "What is the meaning of the coordinates of that specific point?"
- "How are the coordinates of a specific point connected to an equation that represents the situation?"
- "In the first activity, for a larger length, there is a *smaller* width. But in the last activity, for a larger number of days, there is a *larger* number of mosquitos. How come?"

18.5 Interpret the Point

Cool Down: 5 minutes

Addressing

- 6.EE.C.9

Student Task Statement

The equation $\frac{1}{4}P = s$ relates the perimeter P of any square and its side length s . A graph of the equation includes the point $(12, 3)$.

1. What does the point $(12, 3)$ represent in this situation?
2. What point would represent a square with perimeter $\frac{20}{21}$?

Student Response

1. It represents a square with side length 3 and perimeter 12.
2. $(\frac{20}{21}, \frac{5}{21})$

Student Lesson Summary

Equations can represent relationships between geometric quantities. For instance:

- If s is the side length of a square, then the area A is related to s by $A = s^2$.
- Sometimes the relationships are more specific. For example, the perimeter P of a rectangle with length l and width w is $P = 2l + 2w$. If we consider only rectangles with a length of 10, then the relationship between the perimeter and the width is $P = 20 + 2w$.

Here is another example of an equation with exponent expressing the relationship between quantities:

- A super ball is dropped from 10 feet. On each successive bounce, it only goes $\frac{1}{2}$ as high as on the previous bounce.

This means that on the first bounce, the ball will bounce 5 feet high, and then on the second bounce it will only go $2\frac{1}{2}$ feet high, and so on. We can represent this situation

with an equation to find how high the super ball will bounce after any number of bounces.

To find how high the super ball bounces on the n^{th} bounce, we have to multiply 10 feet (the initial height) by $\frac{1}{2}$ and multiply by $\frac{1}{2}$ again for each bounce thereafter; we need to do this n times. So the height, h , of the ball on the n^{th} bounce will be $h = 10\left(\frac{1}{2}\right)^n$. In this equation, the dependent variable, h , is affected by changes in the independent variable, n .

Equations and graphs can give us insight into different kinds of relationships between quantities and help us answer questions and solve problems.

Lesson 18 Practice Problems

Problem 1

Statement

Elena is designing a logo in the shape of a parallelogram. She wants the logo to have an area of 12 square inches. She draws bases of different lengths and tries to compute the height for each.

- Write an equation Elena can use to find the height, h , for each value of the base, b .
- Use your equation to find the height of a parallelogram with base 1.5 inches.

Solution

a. $h = \frac{12}{b}$

b. 8 inches

Problem 2

Statement

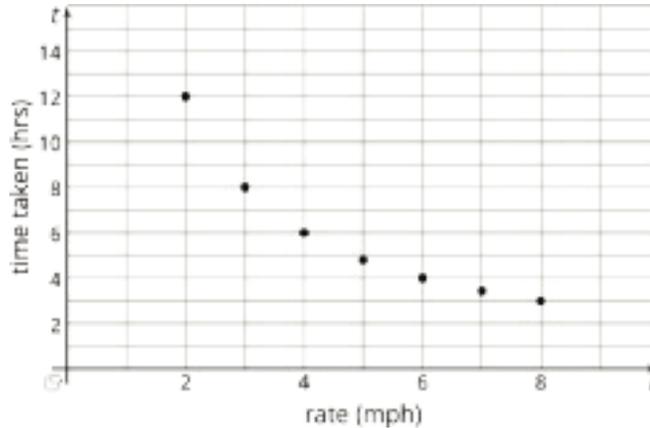
Han is planning to ride his bike 24 miles.

- How long will it take if he rides at a rate of:
3 miles per hour? 4 miles per hour? 6 miles per hour?
- Write an equation that Han can use to find t , the time it will take to ride 24 miles, if his rate in miles per hour is represented by r .
- On graph paper, draw a graph that shows t in terms of r for a 24-mile ride.

Solution

a. 8 hours, 6 hours, 4 hours

b. $t = 24 \div r$ or $t = \frac{24}{r}$.



c.

Problem 3

Statement

The graph of the equation $V = 10s^3$ contains the points $(2, 80)$ and $(4, 640)$.

- Create a story that is represented by this graph.
- What do the points mean in the context of your story?

Solution

Answers vary. Sample response: Lin and Jada each build a tower of 10 cubes. Lin's cubes have edge length 2 and Jada's have edge length 4. They use the equation $V = 10s^3$ to compute the volume of their towers.

Problem 4

Statement

You find a brass bottle that looks really old. When you rub some dirt off of the bottle, a genie appears! The genie offers you a reward. You must choose one:

\$50,000 or a magical \$1 coin.

The coin will turn into two coins on the first day. The two coins will turn into four coins on the second day. The four coins will double to 8 coins on the third day. The genie explains the doubling will continue for 28 days.

- a. Write an equation that shows the number of coins, n , in terms of the day, d .
- b. Create a table that shows the number of coins for each day for the first 15 days.
- c. Create a graph for days 7 through 12 that shows how the number of coins grows with each day.

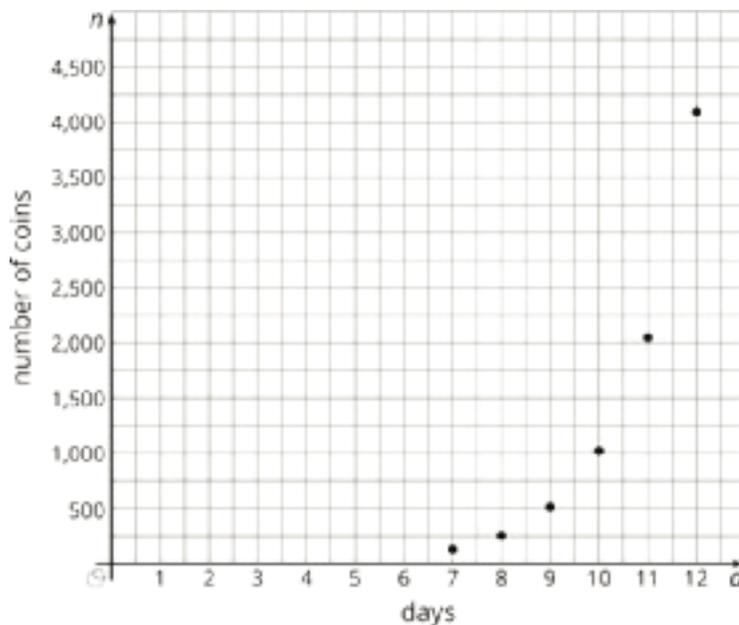
Solution

a. $n = 2^d$

b.

day (d)	number of coins (n)
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1,024
11	2,048
12	4,096
13	8,192
14	16,384
15	32,768

- c. The numbers are too large to show exact numbers on the vertical axis. It's best to mark increments of 500 or 1000 and approximate the positions.



Problem 5

Statement

At a market, 3.1 pounds of peaches cost \$7.72. How much did the peaches cost per pound? Explain or show your reasoning. Round your answer to the nearest cent.

Solution

\$2.49 per pound. Reasoning varies. Sample reasoning: Divide 7.72 by 3.1 to get the price of the peaches per pound in dollars. If dividend and divisor are both multiplied by 100, the value of the quotient does not change. Then calculate $772 \div 310$ by long division. The quotient is a little more than 2.490.

(From Unit 5, Lesson 13.)

Problem 6

Statement

Andre set up a lemonade stand last weekend. It cost him \$0.15 to make each cup of lemonade, and he sold each cup for \$0.35.

- If Andre collects \$9.80, how many cups did he sell?
- How much money did it cost Andre to make this amount of lemonade?
- How much money did Andre make in profit?

Solution

a. 28 ($9.80 \div 0.35 = 28$)

b. \$4.20 ($28 \cdot (0.15) = 4.20$)

c. \$5.60 ($9.80 - 4.20 = 5.60$)

(From Unit 5, Lesson 13.)

Section: Let's Put it to Work

Lesson 19: Tables, Equations, and Graphs, Oh My!

Goals

- Create a verbal description and a graph to represent the relationship shown in an equation and table.
- Identify tables and equations that represent the same relationship and justify (orally) the match.
- Interpret and critique (orally) different representations of the same relationship, i.e. table, equation, graph, and verbal description

Learning Targets

- I can create a table and a graph that represent the relationship in a given equation.
- I can explain what an equation tells us about the situation.

Lesson Narrative

In this culminating lesson, students look at several examples of equations that represent important relationships from real-world situations. In the first activity, students examine all 9 of the relationships briefly, matching an equation and a table that represent the same relationship. In the following activities, each student works with one of the relationships in more detail: interpreting the equation, continuing the table, and creating a graph. This gives students an opportunity to become an expert on one of these relationships and then use multiple representations to explain their understanding to others.

Alignments

Addressing

- 6.EE.A.2: Write, read, and evaluate expressions in which letters stand for numbers.
- 6.EE.B.7: Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p , q and x are all nonnegative rational numbers.
- 6.EE.C.9: Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.

Instructional Routines

- Group Presentations
- MLR7: Compare and Connect
- Take Turns

Required Materials

Graph paper

Sticky notes

Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart

paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Student Learning Goals

- Let's explore some equations from real-world situations.

19.1 Matching Equations and Tables

10 minutes

The purpose of this activity is to familiarize students with the 9 relationships they will continue to work with for the rest of this lesson. In this activity, students match equations and tables that represent the same relationship, without knowing what real-world situations the relationships represent. Students can make use of structure (MP7) as they narrow down which tables could possibly match each equation, such as recognizing whether the values for the dependent variable should be greater or less than the corresponding values for the independent variable, based on the operation in the equation.

Addressing

- 6.EE.C.9

Instructional Routines

- Take Turns

Launch

Arrange students in groups of 2. Ask students to take turns: one partner identifies a match and explains why they think it is a match. The other partner's job is to listen and make sure they agree. If they don't agree, the partners discuss until they come to an agreement. The students swap roles for the next equation. If necessary, demonstrate this protocol before students start working. Also, consider demonstrating productive ways to agree or disagree, for example, by explaining your mathematical thinking or asking clarifying questions.

Consider allowing students to use calculators to ensure inclusive participation in the activity.

Anticipated Misconceptions

Some students may struggle to relate the variables in the equation to the columns of the table. Remind them that when we have one variable expressed in terms of the other variable, we call the former the dependent variable and the latter the independent variable. For example, in the equation $a = b + 6$ we say that b is the independent variable and a is the dependent variable, because a is expressed in terms of b .

Student Task Statement

Match each equation with a table that represents the same relationship. Be prepared to explain your reasoning.

$$S - 2 = T$$

$$G = J + 13$$

$$P = I - 47.50$$

$$C + 273.15 = K$$

$$e = 6s$$

$$m = 8.96V$$

$$y = \frac{1}{12}x$$

$$t = \frac{d}{2.5}$$

$$g = 28.35z$$

Table 1:

independent variable	dependent variable
20	8
58.85	23.54
804	321.6

Table 2:

independent variable	dependent variable
5	18
36	49
75	88

Table 3:

independent variable	dependent variable
2.5	22.4
20	179.2
75	672

Table 4:

independent variable	dependent variable
20	$1\frac{2}{3}$
36	3
804	67

Table 5:

independent variable	dependent variable
58.85	11.35
175.5	128
804	756.5

Table 6:

independent variable	dependent variable
2.5	275.65
20	293.15
58.85	332

Table 7:

independent variable	dependent variable
5	3
20	18
36	34

Table 8:

independent variable	dependent variable
2.6	73.71
20	567
36	1,020.6

Table 9:

independent variable	dependent variable
2.6	15.6
36	216
58.85	353.1

Student Response

Table 1: $t = \frac{d}{2.5}$

Table 2: $G = J + 13$

Table 3: $m = 8.96V$

Table 4: $y = \frac{1}{12}x$

Table 5: $P = I - 47.50$

Table 6: $C + 273.15 = K$

Table 7: $S - 2 = T$

Table 8: $g = 28.35z$

Table 9: $e = 6s$

Activity Synthesis

Much of the discussion will take place between partners. Invite students to share how they identified tables and equations that match.

- What characteristics of the equation or table helped you narrow down the potential matches?
- Were there any matches you and your partner disagreed about? How did you work to reach an agreement?

19.2 Getting to Know an Equation

15 minutes

In the previous activity students looked at 9 different equations. In this activity, each student focuses on just one of those equations to learn about the real-world situation it represents. They interpret the variables and values in terms of the situation (MP7). Adding to the table they saw in the previous activity gives students an opportunity to practice both evaluating expressions and solving equations.

This activity also asks students to create a graph of their assigned relationship on graph paper. In previous activities, students were given a grid with the axes already labeled and numbered whenever they were asked to create a graph of a relationship. Here they must decide for themselves how to scale each axis.

Addressing

- 6.EE.A.2
- 6.EE.B.7
- 6.EE.C.9

Launch

Decide whether you want to assign the equations or allow students to select which equation they will work with. Make sure there is at least one student working with each equation.

Give students 5–10 minutes of quiet work time followed by partner and whole-class discussion. If there is more than one student working with the same equation, consider letting them share their answers and reasoning with each other, after they have had some time to work on their own.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts. Check in with students within the first 2-3 minutes of work time. Look for students who need additional support deciding how to scale each axis for table. Invite students to share strategies they have used so far, as well as any questions they have before continuing.

Supports accessibility for: Memory; Organization

Anticipated Misconceptions

Some students may struggle to understand what each step is asking them to do. Consider preparing an example to share with them, such as this for the equation $2.54i = c$, and point out which part of the example corresponds to the question they are on.

1. The number of centimeters is the product of the number of inches and 2.54.

2.

independent variable: length (inches)	dependent variable: length (centimeters)
5	12.7
36	91.44
75	190.5
60	
	300

3. If something is 36 inches long, this is equivalent to 91.44 when measured in centimeters.
4. In the table above, fill in 152.4 in the right column, next to the 60, because $2.54 \cdot 60 = 152.4$.
Fill in 118.11 in the left column, next to the 300, because $2.54 \cdot 118.11 \approx 300$
5. A graph with “length (inches)” on the horizontal axis, “length (centimeters)” on the vertical axis, and points at (5, 12.7), (36, 91.44), (75, 190.5), (60, 152.4), and (118.11, 300)

Some students may struggle with setting up their graph from scratch on graph paper. Prompt them to think about the maximum value they want to represent on each axis and what number they could count by to get to that maximum value in the amount of space they have.

Student Task Statement

The equations in the previous activity represent situations.

- $S - 2 = T$ where S is the number of sides on a polygon and T is the number of triangles you can draw inside it (from one vertex to the others, without overlapping)
- $G = J + 13$ where G is a day in the Gregorian calendar and J is the same day in the Julian calendar
- $P = I - 47.50$ where I is the amount of income and P is the profit after \$47.50 in expenses
- $C + 273.15 = K$ where C is a temperature in degrees Celsius and K is the same temperature in Kelvin
- $e = 6s$ where e is the total edge length of a regular tetrahedron and s is the length of one side
- $m = 8.96V$ where V is the volume of a piece of copper and m is its mass
- $y = \frac{1}{12}x$ where x is the number of eggs and y is how many dozens that makes

- $t = \frac{d}{2.5}$ where t is the amount of time it takes in seconds to jog a distance of d meters at a constant speed of 2.5 meters per second
- $g = 28.35z$ where g is the mass in grams and z is the same amount in ounces

Your teacher will assign you one of these equations to examine more closely.

1. Rewrite your equation using words. Use words like product, sum, difference, quotient, and term.
2. In the previous activity, you matched equations and tables. Copy the values from the table that matched your assigned equation into the first 3 rows of this table. Make sure to label what each column represents.

independent variable: _____	dependent variable: _____
60	
	300

3. Select one of the first 3 rows of the table and explain what those values mean in this situation.
4. Use your equation to find the values that complete the last 2 rows of the table. Explain your reasoning.
5. On graph paper, create a graph that represents this relationship. Make sure to label your axes.

Student Response

Answers vary. Sample response: For the equation $S - 2 = T$

1. The number of triangles in a polygon is the difference of the number of sides of the polygon and 2.
2. The first three rows of this table:

independent variable: number of sides of the polygon	dependent variable: number of triangles inside
5	3
20	18
36	34
60	58
302	300

- In a polygon with 5 sides, you can draw 3 triangles from one vertex that do not overlap.
- The last two rows of the table above, because $60 - 2 = 58$ and $300 + 2 = 302$.
- A graph with “number of sides of the polygon” on the horizontal axis, “number of triangles inside” on the vertical axis, and points at $(5, 3)$, $(20, 18)$, $(36, 34)$, $(60, 58)$, and $(302, 300)$.

Activity Synthesis

Continue to the next activity. The discussion of these relationships occurs after students have created their visual displays.

19.3 Sharing Your Equation with Others

15 minutes

The purpose of this activity is for students to use multiple representations to share with each other what they have learned about their assigned relationship.

Addressing

- 6.EE.C.9

Instructional Routines

- Group Presentations
- MLR7: Compare and Connect

Launch

Distribute tools for making a visual display. Give students 5–10 minutes of quiet work time, followed by a whole-class discussion or gallery walk. If time permits, consider having students research more information about their situation to add to their displays.

Access for English Language Learners

Representing, Conversing: MLR7 Compare and Connect. Use this routine to help students consider audience when preparing to display their work. Display the list of items that should be included each display. Ask students, “what kinds of details could you include on your display to help a reader understand what you have learned about your assigned relationship?” Record ideas and display for all to see. Examples of these types of details or annotations include: the order in which representations are organized on the display, attaching written notes or details to certain representations, using specific vocabulary or phrases, or using color or arrows to show connections between representations. If time allows, after the gallery walk, ask students to describe specific examples of additional details that other groups used that helped them to interpret and understand their displays.

Design Principle(s): Maximize meta-awareness; Optimize output

Student Task Statement

Create a visual display of your assigned relationships that includes:

- your equation along with an explanation of each variable
- a verbal description of the relationship
- your table
- your graph

If you have time, research more about your relationship and add more details or illustrations to help explain the situation.

Student Response

Answers vary.

Activity Synthesis

Conduct a gallery walk so students have a chance to read and discuss each other’s displays. Ask students to reflect on the following prompts:

- What is the same about their relationship and your relationship? What is different?
- What is the independent variable in their relationship? What is the dependent variable?
- What could they add to the display to make their explanation of the relationship even clearer?

Have students use sticky notes to leave questions or comments for the person who created the display. Give students a moment at the end to review any questions or comments left on their display.

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Family Support
Materials

Family Support Materials

Expressions and Equations

Here are the video lesson summaries for Grade 6, Unit 6: Expressions and Equations. Each video highlights key concepts and vocabulary that students learn across one or more lessons in the unit. The content of these video lesson summaries is based on the written Lesson Summaries found at the end of lessons in the curriculum. The goal of these videos is to support students in reviewing and checking their understanding of important concepts and vocabulary. Here are some possible ways families can use these videos:

- Keep informed on concepts and vocabulary students are learning about in class.
- Watch with their student and pause at key points to predict what comes next or think up other examples of vocabulary terms (the bolded words).
- Consider following the Connecting to Other Units links to review the math concepts that led up to this unit or to preview where the concepts in this unit lead to in future units.

Grade 6, Unit 6: Expressions and Equations	Vimeo	YouTube
Video 1: Understanding Equations (Lessons 1–3)	Link	Link
Video 2: Writing and Solving Equations (Lessons 4–7)	Link	Link
Video 3: Writing Equivalent Expressions (Lessons 8–11)	Link	Link
Video 4: Expressions with Exponents (Lessons 12–15)	Link	Link
Video 5: Relationships Between Quantities (Lessons 16–18)	Link	Link

Video 1

Video 'VLS G6U6V1 Understanding Equations (Lessons 1–3)' available here:
<https://player.vimeo.com/video/505730840>.

Video 2

Video 'VLS G6U6V2 Writing and Solving Equations (Lessons 4–7)' available here:
<https://player.vimeo.com/video/505735569>.

Video 3

Video 'VLS G6U6V3 Writing Equivalent Expressions (Lessons 8–11)' available here:
<https://player.vimeo.com/video/513058816>.

Video 4

Video 'VLS G6U6V4 Expressions with Exponents (Lessons 12–15)' available here:
<https://player.vimeo.com/video/521434518>.

Video 5

Video 'VLS G6U6V5 Relationships Between Quantities (Lessons 16–18)' available here:
<https://player.vimeo.com/video/530008085>.

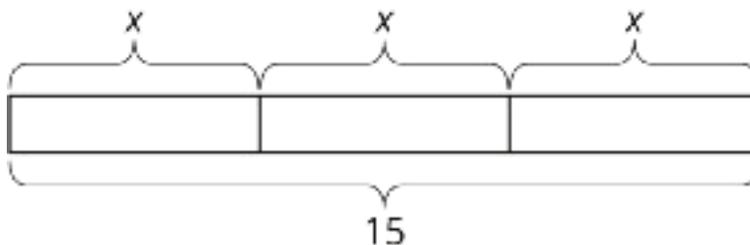
Connecting to Other Units

- *Coming soon*

Equations in One Variable

Family Support Materials 1

This week your student will be learning to visualize, write, and solve equations. They did this work in previous grades with numbers. In grade 6, we often use a letter called a **variable** to represent a number whose value is unknown. Diagrams can help us make sense of how quantities are related. Here is an example of such a diagram:



Since 3 pieces are labeled with the same variable x , we know that each of the three pieces represent the same number. Some equations that match this diagram are $x + x + x = 15$ and $15 = 3x$.

A **solution** to an equation is a number used in place of the variable that makes the equation true. In the previous example, the solution is 5. Think about substituting 5 for x in either equation: $5 + 5 + 5 = 15$ and $15 = 3 \cdot 5$ are both true. We can tell that, for example, 4 is *not* a solution, because $4 + 4 + 4$ does not equal 15.

Solving an equation is a process for finding a solution. Your student will learn that an equation like $15 = 3x$ can be solved by dividing each side by 3. Notice that if you divide each side by 3, $15 \div 3 = 3x \div 3$, you are left with $5 = x$, the solution to the equation.

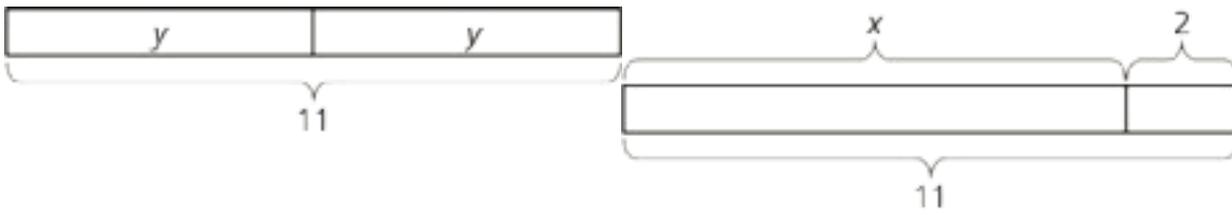
Here is a task to try with your student:

Draw a diagram to represent each equation. Then, solve each equation.

$$2y = 11$$

$$11 = x + 2$$

Solution:



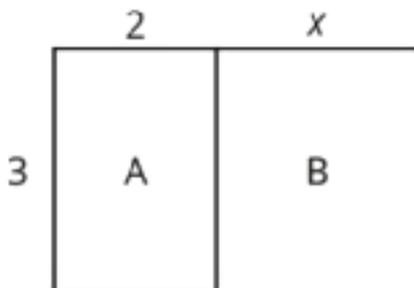
$$y = 5.5 \text{ or } y = \frac{11}{2}$$

$$x = 9$$

Equal and Equivalent

Family Support Materials 2

This week your student is writing mathematical expressions, especially expressions using the distributive property.



In this diagram, we can say one side length of the large rectangle is 3 units and the other is $x + 2$ units. So, the area of the large rectangle is $3(x + 2)$. The large rectangle can be partitioned into two smaller rectangles, A and B, with no overlap. The area of A is 6 and the area of B is $3x$. So, the area of the large rectangle can also be written as $3x + 6$. In other words,

$$3(x + 2) = 3x + 3 \cdot 2$$

This is an example of the distributive property.

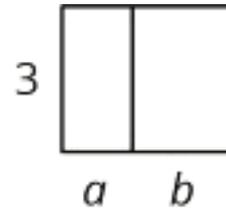
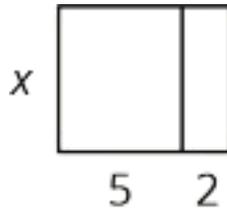
Here is a task to try with your student:

Draw and label a partitioned rectangle to show that each of these equations is always true, no matter the value of the letters.

- $5x + 2x = (5 + 2)x$
- $3(a + b) = 3a + 3b$

Solution:

Answers vary. Sample responses:



Expressions with Exponents

Family Support Materials 3

This week your student will be working with **exponents**. When we write an expression like 7^n , we call n the exponent. In this example, 7 is called the **base**. The exponent tells you how many factors of the base to multiply. For example, 7^4 is equal to $7 \cdot 7 \cdot 7 \cdot 7$. In grade 6, students write expressions with whole-number exponents and bases that are

- whole numbers like 7^4
- fractions like $\left(\frac{1}{7}\right)^4$
- decimals like 7.7^4
- variables like x^4

Here is a task to try with your student:

Remember that a solution to an equation is a number that makes the equation true. For example, a solution to $x^5 = 30 + x$ is 2, since $2^5 = 30 + 2$. On the other hand, 1 is not a solution, since 1^5 does not equal $30 + 1$. Find the solution to each equation from the list provided.

1. $n^2 = 49$ List: 0, 0.008, $\frac{1}{2}$, $\frac{9}{16}$, $\frac{6}{8}$, 0.8, 1, 2, 3, 4, 5, 6, 7

2. $4^n = 64$

3. $4^n = 4$

4. $\left(\frac{3}{4}\right)^2 = n$

5. $0.2^3 = n$

6. $n^4 = \frac{1}{16}$

7. $1^n = 1$

8. $3^n \div 3^2 = 3^3$

Solution:

1. 7, because $7^2 = 49$. (Note that -7 is also a solution, but in grade 6 students aren't expected to know about multiplying negative numbers.)
2. 3, because $4^3 = 64$
3. 1, because $4^1 = 4$
4. $\frac{9}{16}$, because $(\frac{3}{4})^2$ means $(\frac{3}{4}) \cdot (\frac{3}{4})$
5. 0.008, because 0.2^3 means $(0.2) \cdot (0.2) \cdot (0.2)$
6. $\frac{1}{2}$, because $(\frac{1}{2})^4 = \frac{1}{16}$
7. Any number! $1^n = 1$ is true no matter what number you use in place of n .
8. 5, because this can be rewritten $3^n \div 9 = 27$. What would we have to divide by 9 to get 27? 243, because $27 \cdot 9 = 243$. $3^5 = 243$.

Relationships Between Quantities

Family Support Materials 4

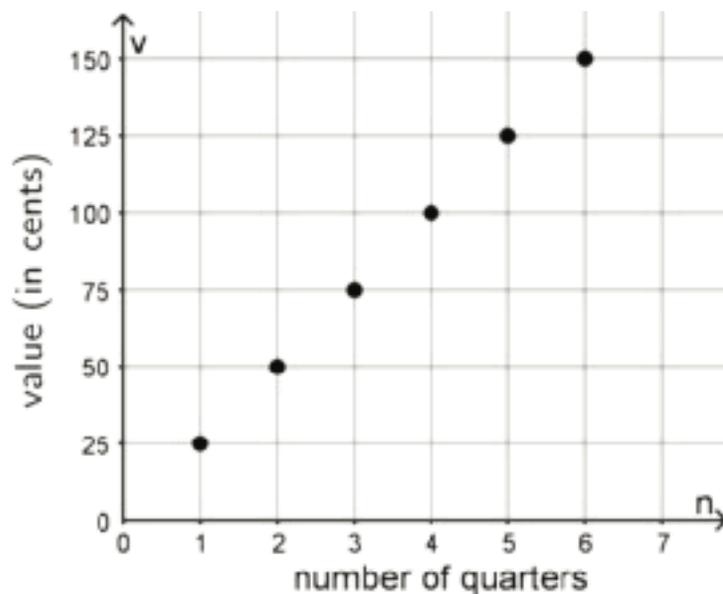
This week your student will study relationships between two quantities. For example, since a quarter is worth 25¢, we can represent the relationship between the number of quarters, n , and their value v in cents like this:

$$v = 25n$$

We can also use a table to represent the situation:

n	v
1	25
2	50
3	75

Or we can draw a graph to represent the relationship between the two quantities:



Here is a task to try with your student:

A shopper is buying granola bars. The cost of each granola bar is \$0.75.

1. Write an equation that shows the cost of the granola bars, c , in terms of the number of bars purchased, n .
2. Create a graph representing associated values of c and n .

3. What are the coordinates of some points on your graph? What do they represent?

Solution:

1. $c = 0.75n$. Every granola bar costs \$0.75 and the shopper is buying n of them, so the cost is $0.75n$.
2. Answers vary. One way to create a graph is to label the horizontal axis with "number of bars" with intervals, 0, 1, 2, 3, etc, and label the vertical axis with "total cost in dollars" with intervals 0, 0.25, 0.50, 0.75, etc.
3. If the graph is created as described in this solution, the first coordinate is the number of granola bars and the second is the cost in dollars for that number of granola bars. Some points on such a graph are (2, 1.50) and (10, 7.50)

Unit Assessments

Check Your Readiness A and B
Mid-Unit Assessment A and B
End-of-Unit Assessment A and B

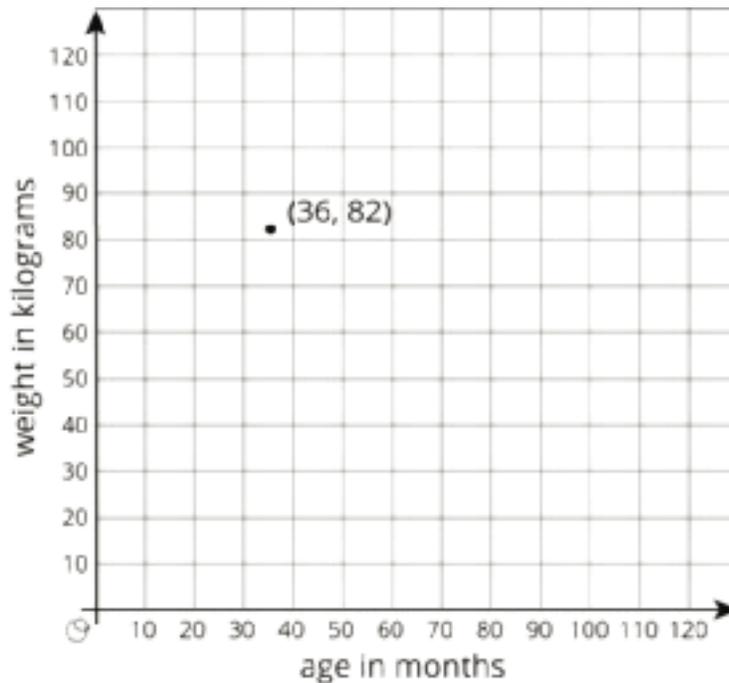
Expressions and Equations: Check Your Readiness (A)

1. a. Write 10^3 without an exponent.

b. What number is this? $5 \cdot 10^2 + 3$

2. A giant panda lives in the Washington, D.C. zoo.

a. What does the point shown tell you about the panda?



b. Another panda weighs 90 kilograms and is 110 months old. Plot the point that corresponds to this panda in the coordinate plane shown.

3. Describe what you would do to find the missing value in each equation. (You do not have to actually find the missing value.)

a. $? + 59\frac{5}{8} = 82\frac{2}{3}$

b. $87 \cdot ? = 9,526$

4. Without computing, select **all** of the expressions that have the same value as $81 \cdot (37 + 59)$.

A. $81 \cdot (59 + 37)$

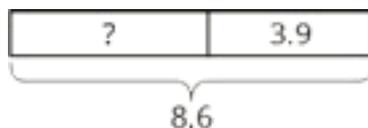
B. $(81 \cdot 37) + 59$

C. $(81 \cdot 37) + (81 \cdot 59)$

D. $81 + (37 \cdot 59)$

E. $(37 + 59) \cdot 81$

5. Select **all** the equations represented by this tape diagram.



A. $3.9 + ? = 8.6$

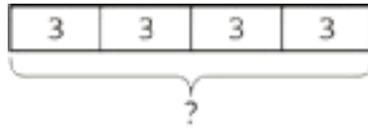
B. $8.6 + 3.9 = ?$

C. $8.6 = ? + 3.9$

D. $8.6 - 3.9 = ?$

E. $? - 3.9 = 8.6$

6. Select **all** the equations represented by this tape diagram.



- A. $4 + 3 = ?$
- B. $3 + 3 + 3 + 3 = ?$
- C. $? = 4 \cdot 3$
- D. $? = 3 \cdot 3 \cdot 3 \cdot 3$
- E. $4 \div 3 = ?$
- F. $4 = ? \div 3$

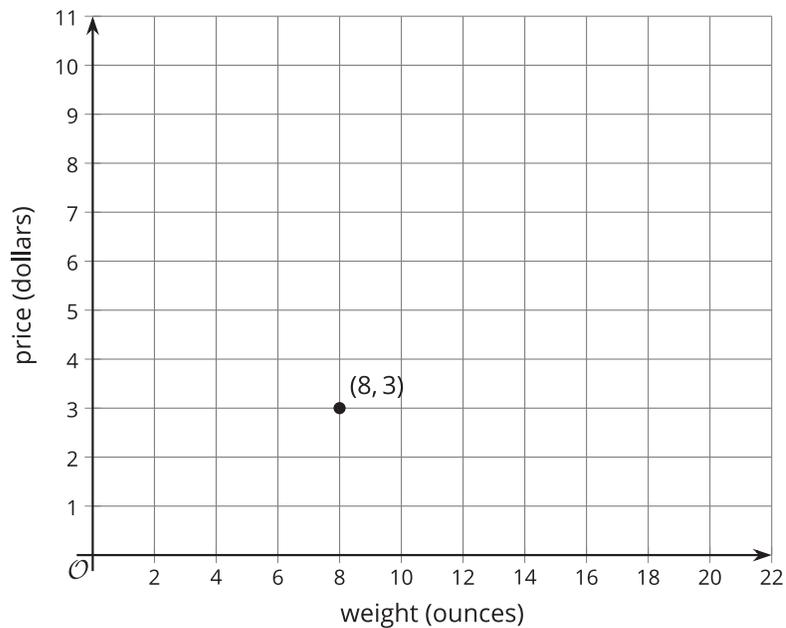
Expressions and Equations: Check Your Readiness (B)

1. a. Write 10^5 without an exponent.

b. What number is this? $7 \cdot 10^3 + 6$

2. A store sells blocks of cheese in several different sizes.

a. The point shown represents one block of cheese. What does the point's location tell you?



b. Another block of cheese costs \$7 and weighs 16 ounces. Plot and label a point to represent this block of cheese.

3. Describe what you would do to find the missing value in each equation. (You do not have to actually find the missing value.)

a. $? - 15\frac{2}{9} = 3\frac{1}{4}$

b. $? \cdot 35 = 2,380$

4. Without computing, select **all** of the expressions that have the same value as $74 \cdot (29 + 56)$.

A. $74 \cdot 29 + 56$

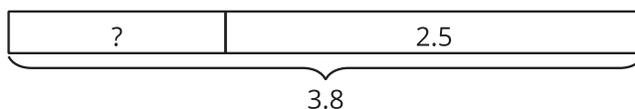
B. $74 + (29 + 56)$

C. $74 + (56 \cdot 29)$

D. $(74 \cdot 29) + (74 \cdot 56)$

E. $(56 + 29) \cdot 74$

5. Select **all** the equations represented by this tape diagram.



A. $2.5 + ? = 3.8$

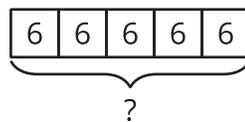
B. $3.8 = ? + 2.5$

C. $3.8 - 2.5 = ?$

D. $3.8 + 2.5 = ?$

E. $? - 2.5 = 3.8$

6. Select **all** the equations represented by this tape diagram.



A. $6 + 6 + 6 + 6 + 6 = ?$

B. $5 + 6 = ?$

C. $? = 6 \cdot 5$

D. $? = 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$

E. $? \div 5 = 6$

F. $5 = ? \div 6$

Expressions and Equations: Mid-Unit Assessment (A)

1. Select all the equations where $x = 3$ is a solution.

A. $x - 3 = 0$

B. $1 + x = 2$

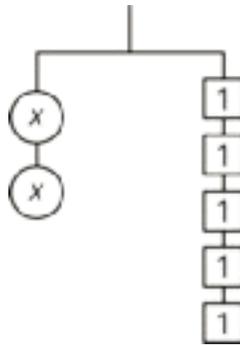
C. $9 - x = 3$

D. $6 = 2x$

E. $\frac{1}{5}x = 3$

F. $x^2 = 9$

2. Which equation matches the hanger diagram?



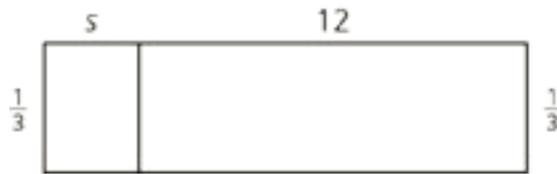
A. $x \cdot x = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$

B. $x = \frac{2}{5}$

C. $x + 3 = 5$

D. $2x = 5$

3. Select **all** the expressions that represent the total area of the rectangle.



A. $4s$

B. $\frac{1}{3}s + 12$

C. $\frac{1}{3} \cdot s + \frac{1}{3} \cdot 12$

D. $\frac{1}{3}s + 4$

E. $\frac{1}{3}(s + 12)$

4. $\frac{2}{9}$ of the students in a school are in sixth grade.

a. How many sixth graders are there if the school has 90 students?

b. How many sixth graders are there if the school has 27 students?

c. If the school has x students, write an expression for the number of sixth graders in terms of x .

d. How many students are in the school if 42 of them are sixth graders?

5. a. Write an equation that shows the relationship 30% of 140 is x .
- b. Write an equation that shows the relationship 64% of y is 40.
- c. 45% of z is 72. Find the value of z .

6. Diego is selling raffle tickets for \$1.75 per ticket.

- a. Complete the table to show how much money he would earn if he sold each number of tickets.

number of tickets sold	20	50	r
amount earned in dollars			

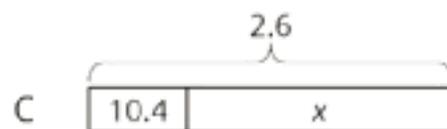
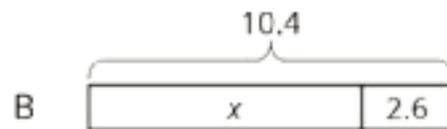
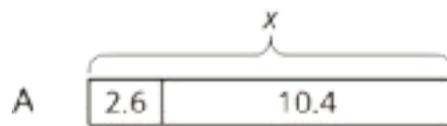
- b. How many tickets would Diego need to sell to earn \$140? Explain your reasoning.

7. Mai poured 2.6 liters of water into a partially filled pitcher. The pitcher then contained 10.4 liters.

a. Which diagram (A, B, or C) represents this situation?

b. Write an equation that represents this situation.

c. Solve the equation you wrote.



d. Explain what the solution to the equation means in this situation.

Expressions and Equations: Mid-Unit Assessment (B)

1. Select all the equations where $x = 5$ is a solution.

A. $x - 4 = 9$

B. $1 + x = 6$

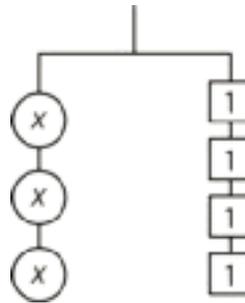
C. $2 = 7 - x$

D. $10x = 5$

E. $\frac{1}{10}x = \frac{1}{2}$

F. $x^2 = 5$

2. Which equation matches the hanger diagram?



A. $x \cdot x \cdot x = 1 \cdot 1 \cdot 1 \cdot 1$

B. $x = \frac{3}{4}$

C. $3x = 4$

D. $x + 1 = 4$

3. Select **all** the expressions that represent the total area of the rectangle.



A. $\frac{1}{5}c + 15$

B. $\frac{1}{5} \cdot c + \frac{1}{5} \cdot 15$

C. $\frac{1}{5}(c + 15)$

D. $3c$

E. $\frac{1}{5}c + 3$

4. $\frac{3}{7}$ of the students in a school are in sixth grade.

a. How many sixth graders are there if the school has 70 students?

b. How many sixth graders are there if the school has 28 students?

c. If the school has x students, write an expression for the number of sixth graders in terms of x .

d. How many students are in the school if 63 of them are sixth graders?

5. a. Write an equation that shows the relationship 40% of 120 is a .
- b. Write an equation that shows the relationship 55% of b is 30.
- c. 45% of c is 72. Find the value of c .

6. During a school event, Noah is selling pizza at the food stand for \$1.25 per slice.

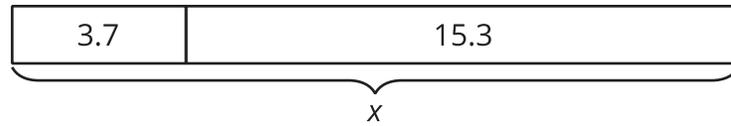
- a. Complete the table to show how much money he would earn if he sold each number of pizza slices.

number of pizza slices sold	20	50	p
amount of money earned in dollars			

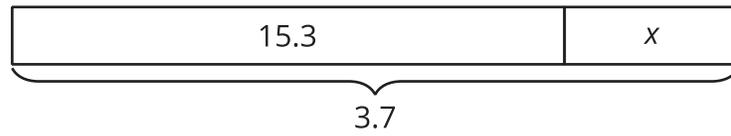
- b. How many slices of pizza would Noah need to sell to earn \$120? Explain your reasoning.

7. Jada poured 3.7 liters of water into a partially-filled bucket. The bucket then contained 15.3 liters.

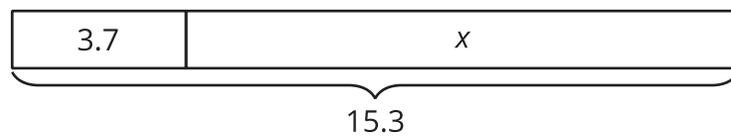
A



B



C



- Which diagram (A, B, or C) represents this situation?
- Write an equation that represents this situation.
- Solve the equation you wrote.
- Explain what the solution to the equation means in this situation.

Expressions and Equations: End-of-Unit Assessment (A)

Calculators should not be used.

1. Which expression is equal to 6^4 ?

- A. 10
- B. 24
- C. 4^6
- D. $6 \cdot 6 \cdot 6 \cdot 6$

2. Select **all** the expressions that have the same value.

- A. 2^4
- B. 2^6
- C. 2^8
- D. 4^3
- E. 8^2
- F. 8^8

3. Which expression is equivalent to $20c - 8d$?

A. $2(10c + 4d)$

B. $4(5c - 8d)$

C. $4(5c - 2d)$

D. $c(20 - 8d)$

4. Here is an expression: $3 \cdot 2^t$

a. Evaluate the expression when t is 1.

b. Evaluate the expression when t is 4.

5. a. Write an expression equivalent to $m + m + m + m$ that is a product of a coefficient and a variable.

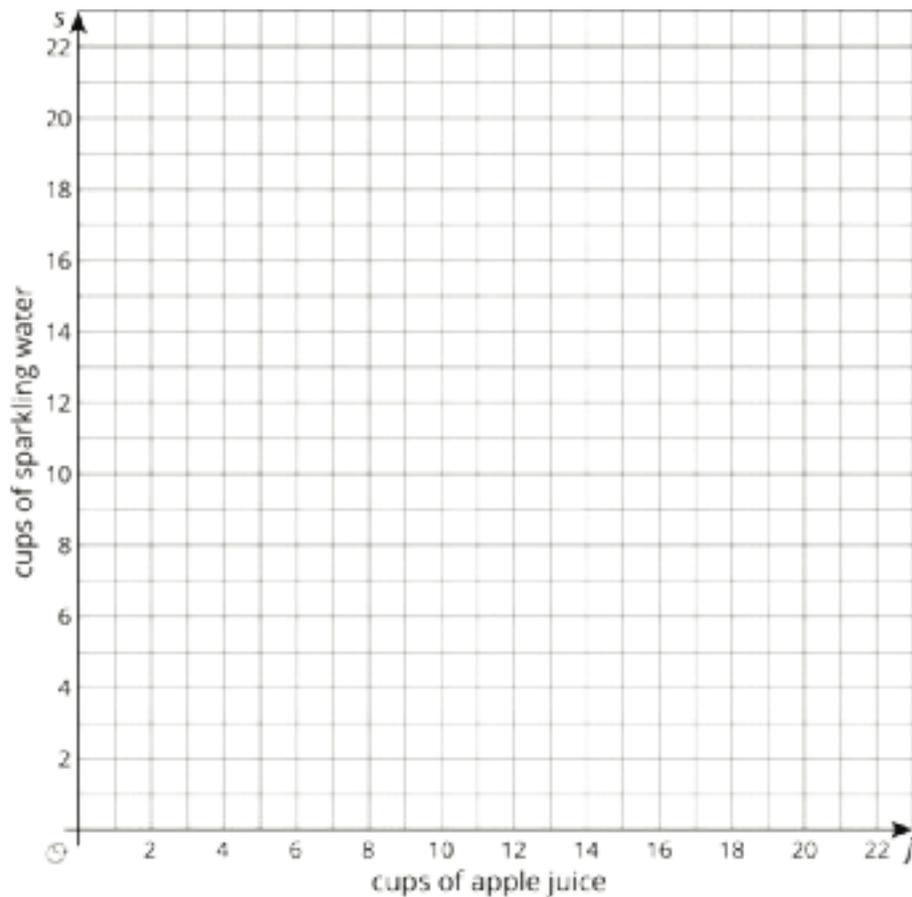
b. Write an expression equivalent to $m + m + m + m$ that is a sum of two terms.

6. Jada makes sparkling juice by mixing 2 cups of sparkling water with every 3 cups of apple juice.

a. How much sparkling water does Jada need if she uses 15 cups of apple juice?

b. How much apple juice does Jada need if she uses 6 cups of sparkling water?

c. Plot these pairs of measurements as points on the graph.



d. Let s represent the number of cups of sparkling water and j represent the number of cups of apple juice. Write an equation that shows how s and j are related.

7. This rectangle has a perimeter of 36 units.

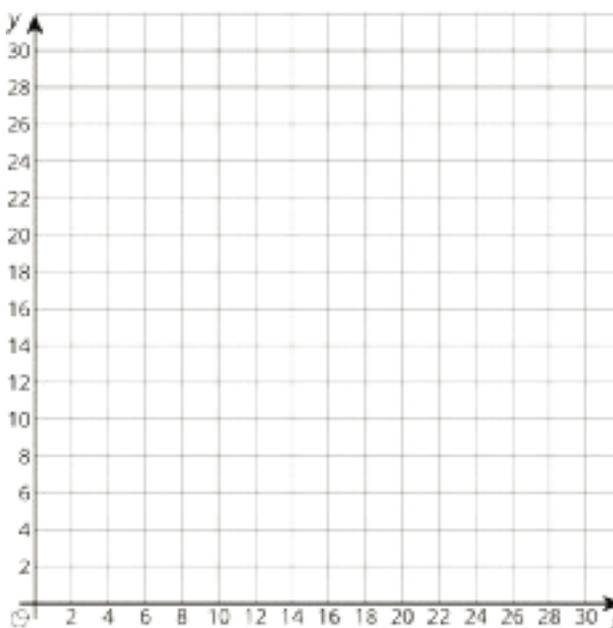


a. Create a table that shows the length and width of at least 3 different rectangles that also have a perimeter of 36 units.

b. Describe the relationship between the columns of your table.

c. Write an equation to represent the relationship. Identify the independent and dependent variables.

d. Plot the values in your table as points on the graph. Make sure to label the axes.



Expressions and Equations: End-of-Unit Assessment (B)

Calculators should not be used.

1. Which expression is equal to 7^3 ?

A. $7 \cdot 7 \cdot 7$

B. 21

C. 3^7

D. 10

2. Select **all** the expressions that have the same value.

A. 3^3

B. 3^4

C. 6^2

D. 6^3

E. 9^2

F. 9^3

3. Which expression is equivalent to $45a - 10b$?

A. $5(9a - 10b)$

B. $45(a - 10b)$

C. $5(9a + 2b)$

D. $5(9a - 2b)$

4. Here is an expression: $2 + 3^t$.

a. Evaluate the expression when t is 1.

b. Evaluate the expression when t is 4.

5. a. Write an expression equivalent to $b + b + b + b + b$ that is a product of a coefficient and a variable.

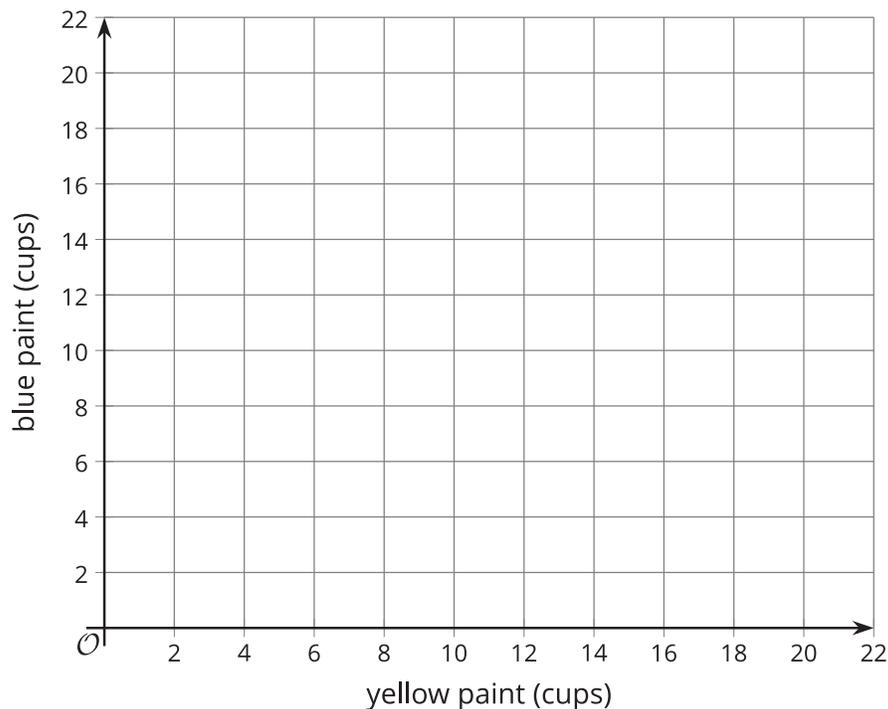
b. Write an expression equivalent to $b + b + b + b + b$ that is a sum of two terms.

6. Andre makes green paint by mixing 5 cups of yellow paint with 2 cups of blue paint.

a. To make the same shade of green, how much yellow paint does Andre need if he uses 8 cups of blue paint?

b. How much blue paint does Andre need if he uses 15 cups of yellow paint?

c. Plot these pairs of measurements as points on the graph.



d. Let y represent the cups of yellow paint and b represent the cups of blue paint. Write an equation that shows how y and b are related.

7. This rectangle has a perimeter of 28 units.



a. Create a table that shows the length and width of at least 3 different rectangles that also have a perimeter of 28 units.

b. Describe the relationship between the columns of your table.

c. Write an equation to represent the relationship. Identify the independent and dependent variables.

d. Plot the values in your table as points on the graph. Make sure to label the axes.



CKMath™
Core Knowledge **MATHEMATICS™**

Assessment Answer Keys

Check Your Readiness A and B
Mid-Unit Assessment A and B
End-of-Unit Assessment A and B

Assessment Answer Keys

Assessment : Check Your Readiness (A)

Teacher Instructions

This assesses students' incoming knowledge about concepts that support understanding of expressions and equations.

Problem 1

The content assessed in this problem is first encountered in Lesson 12: Meaning of Exponents.

This problem assesses basic knowledge of exponents. In Lesson 12 students will expand on their previous work with exponents.

If most students struggle with this item, plan to revisit the first part of this item after Lesson 12 Activity 1 and the second part before Lesson 14 Activity 3. Students may have only worked with exponential expressions in which 10 is the base and may only need a reminder about exponential notation.

Statement

1. Write 10^3 without an exponent.
2. What number is this? $5 \cdot 10^2 + 3$

Solution

1. 1,000 or $10 \cdot 10 \cdot 10$
2. 503

Aligned Standards

5.NBT.A.2

Problem 2

The content assessed in this problem is first encountered in Lesson 16: Two Related Quantities, Part 1.

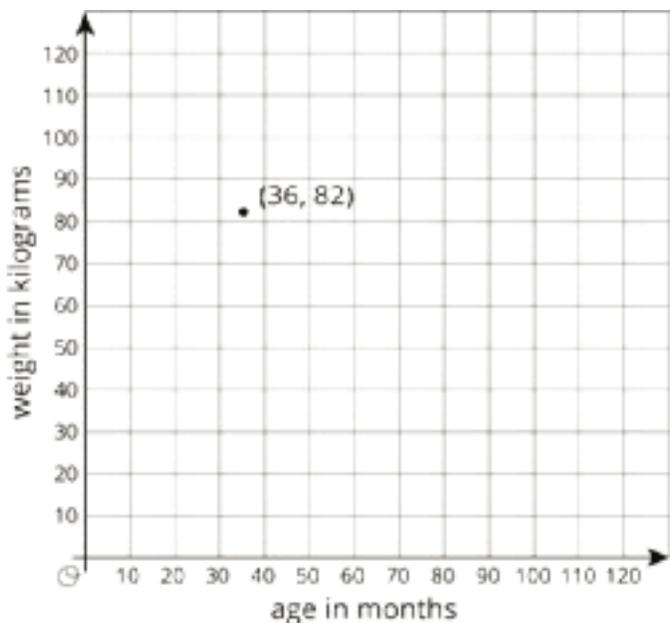
In this problem, students graph points in the first quadrant of a coordinate plane and interpret the points in a context. These skills will come up in the final section of this unit, in which students plot points from given relationships like distance vs. time and area vs. length.

If most students struggle with this item, plan to plot one pair of values together in the last question of Lesson 16 Activity 2. If students struggle with the given intervals in that activity, you might consider working through the first Practice Problem together.

Statement

A giant panda lives in the Washington, D.C. zoo.

1. What does the point shown tell you about the panda?



2. Another panda weighs 90 kilograms and is 110 months old. Plot the point that corresponds to this panda in the coordinate plane shown.

Solution

1. The panda is 36 months old and weighs 82 kilograms.
2. The point (110, 90) is plotted correctly.

Aligned Standards

5.G.A.2

Problem 3

The content assessed in this problem is first encountered in Lesson 3: Staying in Balance.

Students are not expected to know the standard ways to solve these equations, which will be codified in this unit, but some students may have been exposed to this material before. The purpose of this question is to assess where students are in their understanding of algebraic thinking.

If most students do well with this item, plan to study the "Are you ready for more?" in Lesson 3 as an example additional challenge that builds deep understanding of which moves are allowable and keep equations balanced.

Statement

Describe what you would do to find the missing value in each equation. (You do not have to actually find the missing value.)

1. $? + 59\frac{5}{8} = 82\frac{2}{3}$

2. $87 \cdot ? = 9,526$

Solution

1. Answers vary. Sample responses:

- I would subtract $59\frac{5}{8}$ from each side of the equation.
- I would find what number you have to add to $59\frac{5}{8}$ to get $82\frac{2}{3}$. You can keep adding tens, then ones, then fractions to get there.

2. Answers vary. Sample responses:

- I would divide each side of the equation by 87.
- I would find what number times 87 is 9,526, which is how many times 87 goes into 9,526.

Aligned Standards

1.OA.B.4, 3.OA.B.6

Problem 4

The content assessed in this problem is first encountered in Lesson 9: The Distributive Property, Part 1.

This problem assesses understanding of the distributive property of multiplication over addition. There is also a nested instance of recognizing the commutative property of addition and of multiplication within an expression involving parentheses. In this unit, students will explore the distributive property using both numbers and variables.

If most students struggle with this item, plan to replace Activity 1 with revisiting this question and asking students to articulate how someone could know, without computing, that A, C, and E are equal to the given expression.

Statement

Without computing, select all of the expressions that have the same value as $81 \cdot (37 + 59)$.

- A. $81 \cdot (59 + 37)$
- B. $(81 \cdot 37) + 59$
- C. $(81 \cdot 37) + (81 \cdot 59)$
- D. $81 + (37 \cdot 59)$
- E. $(37 + 59) \cdot 81$

Solution

["A", "C", "E"]

Aligned Standards

3.OA.B.5, 5.OA.A

Problem 5

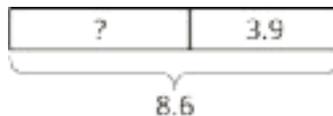
The content assessed in this problem is first encountered in Lesson 1: Tape Diagrams and Equations.

Students will use tape diagrams like this one to represent equations, specifically to solve equations in the form.

If most students struggle with this item, plan to provide additional support early on in the unit for students who weren't exposed to the tape diagram representation in earlier grades. This could be requiring particular students to draw a tape diagram when an activity prompt does not require one.

Statement

Select **all** the equations represented by this tape diagram.



- A. $3.9 + ? = 8.6$
- B. $8.6 + 3.9 = ?$
- C. $8.6 = ? + 3.9$
- D. $8.6 - 3.9 = ?$
- E. $? - 3.9 = 8.6$

Solution

["A", "C", "D"]

Aligned Standards

2.MD.B.5

Problem 6

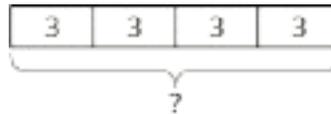
The content assessed in this problem is first encountered in Lesson 1: Tape Diagrams and Equations.

Students will use tape diagrams like this one to represent equations, specifically to solve equations in the form.

If most students struggle with this item, plan to provide additional support early on in the unit for students who weren't exposed to the tape diagram representation in earlier grades. This could be requiring particular students to draw a tape diagram when an activity prompt does not require one.

Statement

Select all the equations represented by this tape diagram.



- A. $4 + 3 = ?$
- B. $3 + 3 + 3 + 3 = ?$
- C. $? = 4 \cdot 3$
- D. $? = 3 \cdot 3 \cdot 3 \cdot 3$
- E. $4 \div 3 = ?$
- F. $4 = ? \div 3$

Solution

["B", "C", "F"]

Aligned Standards

3.OA.A

Assessment : Check Your Readiness (B)

Teacher Instructions

This assesses students' incoming knowledge about concepts that support understanding of expressions and equations.

Problem 1

The content assessed in this problem is first encountered in Lesson 12: Meaning of Exponents.

Beginning with lesson 12, students will expand on their previous work with exponents. This problem assesses prior knowledge of exponents.

If most students struggle with this item, plan to revisit the first part of this item after Lesson 12 Activity 1 and the second part before Lesson 14 Activity 3. Students may have only worked with exponential expressions in which 10 is the base and may only need a reminder about exponential notation.

Statement

1. Write 10^5 without an exponent.
2. What number is this? $7 \cdot 10^3 + 6$

Solution

1. 100,000 or $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10$
2. 7,006

Aligned Standards

5.NBT.A.2

Problem 2

The content assessed in this problem is first encountered in Lesson 16: Two Related Quantities, Part 1.

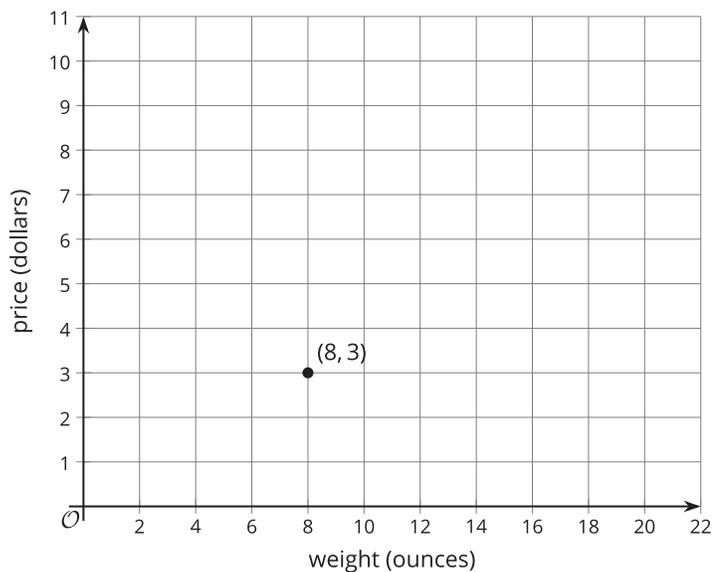
In this problem, students graph points in the first quadrant of a coordinate plane and interpret the points in a context. These skills will come up in the final section of this unit, in which students plot points from given relationships like distance vs. time and area vs. length.

If most students struggle with this item, plan to plot one pair of values together in the last question of Lesson 16 Activity 2. If students struggle with the given intervals in that activity, you might consider working through the first Practice Problem together.

Statement

A store sells blocks of cheese in several different sizes.

1. The point shown represents one block of cheese. What does the point's location tell you?



2. Another block of cheese costs \$7 and weighs 16 ounces. Plot and label a point to represent this block of cheese.

Solution

1. The block of cheese weighs 8 ounces and costs \$3.
2. The point (16, 7) is plotted correctly.

Aligned Standards

5.G.A.2

Problem 3

The content assessed in this problem is first encountered in Lesson 3: Staying in Balance.

Students are not expected to know the standard ways to solve these equations, which will be codified in this unit, but some students may have been exposed to this material before. The purpose of this question is to assess where students are in their understanding of algebraic thinking. Students with incorrect or blank answers do not need remediation. They will learn this content in the unit.

If most students do well with this item, plan to study the "Are you ready for more?" in Lesson 3 as an example additional challenge that builds deep understanding of which moves are allowable and keep equations balanced.

Statement

Describe what you would do to find the missing value in each equation. (You do not have to actually find the missing value.)

1. $? - 15\frac{2}{9} = 3\frac{1}{4}$

2. $? \cdot 35 = 2,380$

Solution

1. Answers vary. Sample responses:

◦ I would add $15\frac{2}{9}$ to $3\frac{1}{4}$.

◦ I would find what number is $3\frac{1}{4}$ away from $15\frac{2}{9}$. I would add the whole numbers, then add the fractions.

2. Answers vary. Sample responses:

◦ I would divide 2,380 by 35.

◦ I would find what number times 35 is 2,380, which is how many times 35 goes into 2,380.

Aligned Standards

1.OA.B.4, 3.OA.B.6

Problem 4

The content assessed in this problem is first encountered in Lesson 9: The Distributive Property, Part 1.

This problem assesses understanding of the distributive property of multiplication over addition. There is also a nested instance of recognizing the commutative property of addition and of multiplication within an expression involving parentheses. In this unit, students will explore the distributive property using both numbers and variables.

If most students struggle with this item, plan to replace Activity 1 with revisiting this question and asking students to articulate how someone could know, without computing, that A, C, and E are equal to the given expression.

Statement

Without computing, select all of the expressions that have the same value as $74 \cdot (29 + 56)$.

- A. $74 \cdot 29 + 56$
- B. $74 + (29 + 56)$
- C. $74 + (56 \cdot 29)$
- D. $(74 \cdot 29) + (74 \cdot 56)$
- E. $(56 + 29) \cdot 74$

Solution

["D", "E"]

Aligned Standards

3.OA.B.5, 5.OA.A

Problem 5

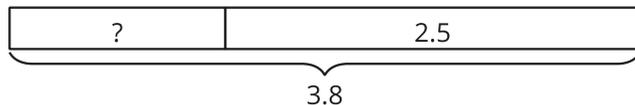
The content assessed in this problem is first encountered in Lesson 1: Tape Diagrams and Equations.

Students will use tape diagrams like this one to represent equations in the upcoming unit, specifically to solve equations in the form $x + p = q$.

If most students struggle with this item, plan to provide additional support early on in the unit for students who weren't exposed to the tape diagram representation in earlier grades. This could be requiring particular students to draw a tape diagram when an activity prompt does not require one.

Statement

Select **all** the equations represented by this tape diagram.



- A. $2.5 + ? = 3.8$
- B. $3.8 = ? + 2.5$
- C. $3.8 - 2.5 = ?$
- D. $3.8 + 2.5 = ?$
- E. $? - 2.5 = 3.8$

Solution

["A", "B", "C"]

Aligned Standards

2.MD.B.5

Problem 6

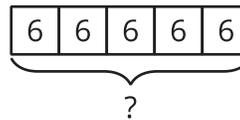
The content assessed in this problem is first encountered in Lesson 1: Tape Diagrams and Equations.

Students will use tape diagrams like this one to represent equations in the upcoming unit, specifically to solve equations in the form $px = q$.

If most students struggle with this item, plan to provide additional support early on in the unit for students who weren't exposed to the tape diagram representation in earlier grades. This could be requiring particular students to draw a tape diagram when an activity prompt does not require one.

Statement

Select **all** the equations represented by this tape diagram.



- A. $6 + 6 + 6 + 6 + 6 = ?$
- B. $5 + 6 = ?$
- C. $? = 6 \cdot 5$
- D. $? = 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$
- E. $? \div 5 = 6$
- F. $5 = ? \div 6$

Solution

["A", "C", "E", "F"]

Aligned Standards

3.OA.A

Assessment : Mid-Unit Assessment (A)

Teacher Instructions

Give this assessment after lesson 11.

Problem 1

Students selecting B may be subtracting instead of adding, since they are used to using inverse operations when solving equations. Students selecting C and E likely think that “solution to an equation” means “what comes after the = sign.” Students failing to select A may not understand how to substitute for a variable. Students failing to select D may think it cannot be true since the variable is on the right side. Students failing to select F may have a misconception about evaluating expressions that involve exponents.

Statement

Select all the equations where $x = 3$ is a solution.

A. $x - 3 = 0$

B. $1 + x = 2$

C. $9 - x = 3$

D. $6 = 2x$

E. $\frac{1}{5}x = 3$

F. $x^2 = 9$

Solution

["A", "D", "F"]

Aligned Standards

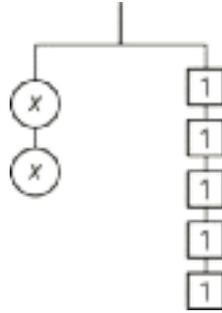
6.EE.B.5

Problem 2

Students selecting A did not notice the equation involves multiplication instead of addition. Students selecting B may have tried to solve the equation but made a mistake in the process. Students selecting C may not understand the difference between an equation of the form $p + x = q$ and $px = q$.

Statement

Which equation matches the hanger diagram?



A. $x \cdot x = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$

B. $x = \frac{2}{5}$

C. $x + 3 = 5$

D. $2x = 5$

Solution

D

Aligned Standards

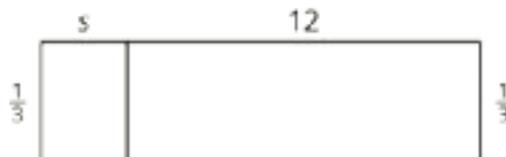
6.EE.B.7

Problem 3

Students not selecting C, D, and E may not understand both the s and 12 are multiplied by $\frac{1}{3}$.

Statement

Select **all** the expressions that represent the total area of the rectangle.



A. $4s$

B. $\frac{1}{3}s + 12$

C. $\frac{1}{3} \cdot s + \frac{1}{3} \cdot 12$

D. $\frac{1}{3}s + 4$

E. $\frac{1}{3}(s + 12)$

Solution

["C", "D", "E"]

Aligned Standards

6.EE.A.3, 6.EE.A.4

Problem 4

Students write an expression to represent a situation and solve a problem leading to writing and solving an equation in the form $px = q$.

Statement

$\frac{2}{9}$ of the students in a school are in sixth grade.

1. How many sixth graders are there if the school has 90 students?
2. How many sixth graders are there if the school has 27 students?
3. If the school has x students, write an expression for the number of sixth graders in terms of x .
4. How many students are in the school if 42 of them are sixth graders?

Solution

1. 20
2. 6
3. $\frac{2}{9}x$ or equivalent
4. 189

Aligned Standards

6.EE.B.6, 6.EE.B.7

Problem 5

Students write equations to represent relationships involving percentages. They may use a tape diagram or double number line to answer the last question. However, the first two questions may lead students to set up and solve an equation.

Statement

1. Write an equation that shows the relationship 30% of 140 is x .
2. Write an equation that shows the relationship 64% of y is 40.
3. 45% of z is 72. Find the value of z .

Solution

1. $(0.3) \cdot 140 = x$

2. $0.64y = 40$

3. 160

Aligned Standards

6.EE.B.6, 6.EE.B.7, 6.RP.A.3.c

Problem 6

First students complete the table to represent the relationship between the number of raffle tickets sold and the amount of money earned. Then they use the equation $1.75r = 140$ to determine the number of tickets it would take to earn \$140.

Statement

Diego is selling raffle tickets for \$1.75 per ticket.

1. Complete the table to show how much money he would earn if he sold each number of tickets.

number of tickets sold	20	50	r
amount earned in dollars			

2. How many tickets would Diego need to sell to earn \$140? Explain your reasoning.

Solution

1.

number of tickets sold	20	50	r
amount earned in dollars	35	87.50	$1.75r$

2. 80 tickets, because $\frac{140}{1.75} = 80$.

Minimal Tier 1 response:

- Work is complete and correct, with complete table and correct answer for part B.
- Sample:
- See table above
- $1.75r = 140, r = 140 \div 1.75, r = 80$

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.

- Acceptable errors: a reasonable response to part b is based on an incorrect expression in the last cell of the table.
- Sample errors: a substituted value for r is recorded in the last column of the table, but keeps the multiplicative relationship of 1.75.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: The table reflects a lack of understanding of the multiplicative relationship, which affects the equation in part b. Work involves a misinterpretation of the situation that affects all or most problem parts, but work does show understanding of writing equations to represent situations and interpreting solutions to equations.

Aligned Standards

6.EE.A.2.a, 6.EE.B.6

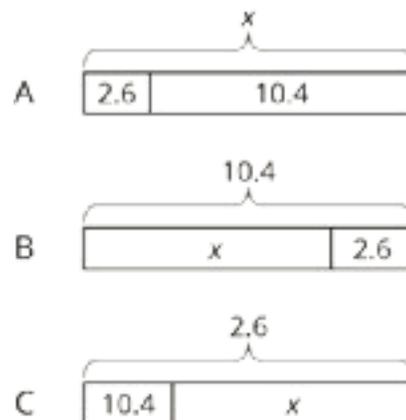
Problem 7

This problem asks students to demonstrate their skill in working in a context, writing and solving an equation of the form $x + p = q$. Students should determine the meaning of the variable from the context, and are asked to explain its meaning in the last part.

Statement

Mai poured 2.6 liters of water into a partially filled pitcher. The pitcher then contained 10.4 liters.

1. Which diagram (A, B, or C) represents this situation?
2. Write an equation that represents this situation.
3. Solve the equation you wrote.
4. Explain what the solution to the equation means in this situation.



Solution

1. B
2. $x + 2.6 = 10.4$ (or equivalent)
3. $x = 7.8$
4. The pitcher contained 7.8 liters of water before Mai added more water.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample:
 1. B
 2. $10.4 - 2.6 = x$
 3. $x = 7.8$
 4. 7.8 liters of water were in the pitcher before Mai added to it.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Acceptable errors: a reasonable response to part d is based on an incorrect solution to the equation.
- Sample errors: answer to part a is the only flawed response; an arithmetic error leads to an incorrect solution to the equation; response to part d is incomplete.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Acceptable errors: reasonable responses to parts c and d based on an incorrect equation.
- Sample errors: The equation in part b is incorrect; part c contains an incorrect interpretation of the solution to the equation; work involves a misinterpretation of the situation that affects all or most problem parts, but work does show understanding of writing equations to represent situations and interpreting solutions to equations.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: Multiple Tier 3 errors, major omissions.

Aligned Standards

6.EE.B.6, 6.EE.B.7

Assessment : Mid-Unit Assessment (B)

Problem 1

Students selecting A may be adding instead of subtracting, since they are used to using inverse operations when solving equations. Students selecting D and F likely think that “solution to an equation” means “what comes after the = sign.” Students failing to select B may not understand how to substitute for a variable. Students failing to select C may think it cannot be true since the variable is on the right side. Students failing to select E may have a misconception about evaluating expressions that involve fractions.

Statement

Select **all** the equations where $x = 5$ is a solution.

A. $x - 4 = 9$

B. $1 + x = 6$

C. $2 = 7 - x$

D. $10x = 5$

E. $\frac{1}{10}x = \frac{1}{2}$

F. $x^2 = 5$

Solution

["B", "C", "E"]

Aligned Standards

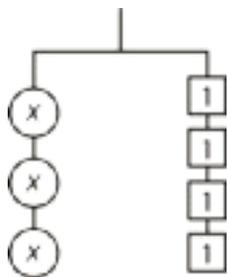
6.EE.B.5

Problem 2

Students selecting A did not notice the equation involves multiplication instead of addition. Students selecting B may have tried to solve the equation but made a mistake in the process. Students selecting D may not understand the difference between an equation of the form $p + x = q$ and $px = q$.

Statement

Which equation matches the hanger diagram?



A. $x \cdot x \cdot x = 1 \cdot 1 \cdot 1 \cdot 1$

B. $x = \frac{3}{4}$

C. $3x = 4$

D. $x + 1 = 4$

Solution

C

Aligned Standards

6.EE.B.7

Problem 3

Students not selecting B, C, and E may not understand both the c and 15 are multiplied by $\frac{1}{5}$.

Statement

Select all the expressions that represent the total area of the rectangle.



A. $\frac{1}{5}c + 15$

B. $\frac{1}{5} \cdot c + \frac{1}{5} \cdot 15$

C. $\frac{1}{5}(c + 15)$

D. $3c$

E. $\frac{1}{5}c + 3$

Solution

["B", "C", "E"]

Aligned Standards

6.EE.A.3, 6.EE.A.4

Problem 4

Students write an expression to represent a situation and solve a problem leading to writing and solving an equation in the form $px = q$.

Statement

$\frac{3}{7}$ of the students in a school are in sixth grade.

1. How many sixth graders are there if the school has 70 students?
2. How many sixth graders are there if the school has 28 students?
3. If the school has x students, write an expression for the number of sixth graders in terms of x .
4. How many students are in the school if 63 of them are sixth graders?

Solution

1. 30
2. 12
3. $\frac{3}{7}x$
4. 147

Aligned Standards

6.EE.B.6, 6.EE.B.7

Problem 5

Students write equations to represent relationships involving percentages. They may use a tape diagram or double number line to answer the last question. However, the first two questions may lead students to set up and solve an equation.

Statement

1. Write an equation that shows the relationship 40% of 120 is a .
2. Write an equation that shows the relationship 55% of b is 30.
3. 45% of c is 72. Find the value of c .

Solution

1. $(0.4) \cdot 120 = a$
2. $0.55b = 30$
3. 160

Aligned Standards

6.EE.B.6, 6.EE.B.7, 6.RP.A.3.c

Problem 6

First students complete the table to represent the relationship between the number of pizza slices sold and the amount of money earned. Then they use the equation $1.25p = 120$ to determine the number of pizza slices it would take to earn \$120.

Statement

During a school event, Noah is selling pizza at the food stand for \$1.25 per slice.

1. Complete the table to show how much money he would earn if he sold each number of pizza slices.

number of pizza slices sold	20	50	p
amount of money earned in dollars			

2. How many slices of pizza would Noah need to sell to earn \$120? Explain your reasoning.

Solution

1.

number of pizza slices sold	20	50	p
amount of money earned in dollars	25	62.50	$1.25p$

2. 96 slices of pizza, because $\frac{120}{1.25} = 96$

Minimal Tier 1 response:

- Work is complete and correct, with complete table and correct answer for part B.
- Sample: See table above
- $1.25p = 120, p = 120 \div 1.25, p = 96$

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Acceptable errors: a reasonable response to part b is based on an incorrect expression in the last cell of the table.
- Sample errors: a substituted value for p is recorded in the last column of the table, but keeps the multiplicative relationship of 1.25

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: The table reflects a lack of understanding of the multiplicative relationship, which affects the equation in part b. Work involves a misinterpretation of the situation that affects all or most problem parts, but work does show understanding of writing equations to represent situations and interpreting solutions to equations.

Aligned Standards

6.EE.A.2.a, 6.EE.B.6

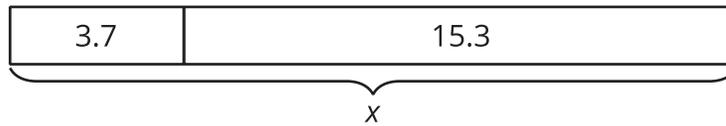
Problem 7

This problem asks students to demonstrate their skill in working in a context, writing and solving an equation of the form $x + p = q$. Students should determine the meaning of the variable from the context, and are asked to explain its meaning in the last part.

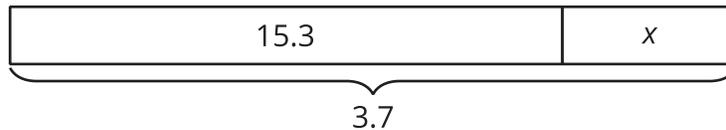
Statement

Jada poured 3.7 liters of water into a partially-filled bucket. The bucket then contained 15.3 liters.

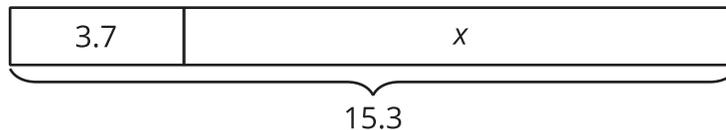
A



B



C



1. Which diagram (A, B, or C) represents this situation?
2. Write an equation that represents this situation.

3. Solve the equation you wrote.
4. Explain what the solution to the equation means in this situation.

Solution

1. C
2. $3.7 + x = 15.3$ (or equivalent)
3. $x = 11.6$
4. The bucket contained 11.6 liters of water before Jada added more water.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample:
 1. C
 2. $15.3 - 3.7 = x$
 3. $x = 11.6$
 4. 11.6 liters of water were in the bucket before Jada added to it.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Acceptable errors: a reasonable response to the last question is based on an incorrect solution to the equation.
- Sample errors: answer to the first question is the only flawed response; an arithmetic error leads to an incorrect solution to the equation; response to the last question is incomplete.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Acceptable errors: reasonable responses to the last two questions based on an incorrect equation.
- Sample errors: The equation is incorrect; there is an incorrect interpretation of the solution to the equation; work involves a misinterpretation of the situation that affects all or most problem parts, but work does show understanding of writing equations to represent situations and interpreting solutions to equations.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: Multiple Tier 3 errors, major omissions.

Aligned Standards

6.EE.B.6, 6.EE.B.7

Assessment : End-of-Unit Assessment (A)

Teacher Instructions

Calculators should not be used.

Student Instructions

Calculators should not be used.

Problem 1

Students selecting A are mistaking exponentiation for addition, while students selecting B are mistaking exponentiation for multiplication. Students selecting C switched the base and exponent, which almost never results in an equivalent expression.

Statement

Which expression is equal to 6^4 ?

- A. 10
- B. 24
- C. 4^6
- D. $6 \cdot 6 \cdot 6 \cdot 6$

Solution

D

Aligned Standards

6.EE.A.1

Problem 2

Students apply the meaning of bases and exponents to conclude that A, B and E all have equivalent values of 64.

Statement

Select all the expressions that have the same value.

A. 2^4

B. 2^6

C. 2^8

D. 4^3

E. 8^2

F. 8^8

Solution

["B", "D", "E"]

Aligned Standards

6.EE.A.1

Problem 3

Students selecting A have made a sign error or did not notice the sign change in the expression. Students selecting B have not distributed the 4 to the $8d$ term. Students selecting D have not distributed the c to the $8d$ term.

Statement

Which expression is equivalent to $20c - 8d$?

A. $2(10c + 4d)$

B. $4(5c - 8d)$

C. $4(5c - 2d)$

D. $c(20 - 8d)$

Solution

C

Aligned Standards

6.EE.A.4

Problem 4

Students evaluate an exponential expression for different values of the variable.

Statement

Here is an expression: $3 \cdot 2^t$

1. Evaluate the expression when t is 1.

2. Evaluate the expression when t is 4.

Solution

1. 6
2. 48

Aligned Standards

6.EE.A.1

Problem 5

Students demonstrate their understanding of the words sum, product, coefficient, and term by generating equivalent expressions with a specified structure.

Statement

1. Write an expression equivalent to $m + m + m + m$ that is a product of a coefficient and a variable.
2. Write an expression equivalent to $m + m + m + m$ that is a sum of two terms.

Solution

1. $4m$
2. Answers vary. Sample response: $2m + 2m$ or $3m + m$

Aligned Standards

6.EE.A.2.b, 6.EE.A.3

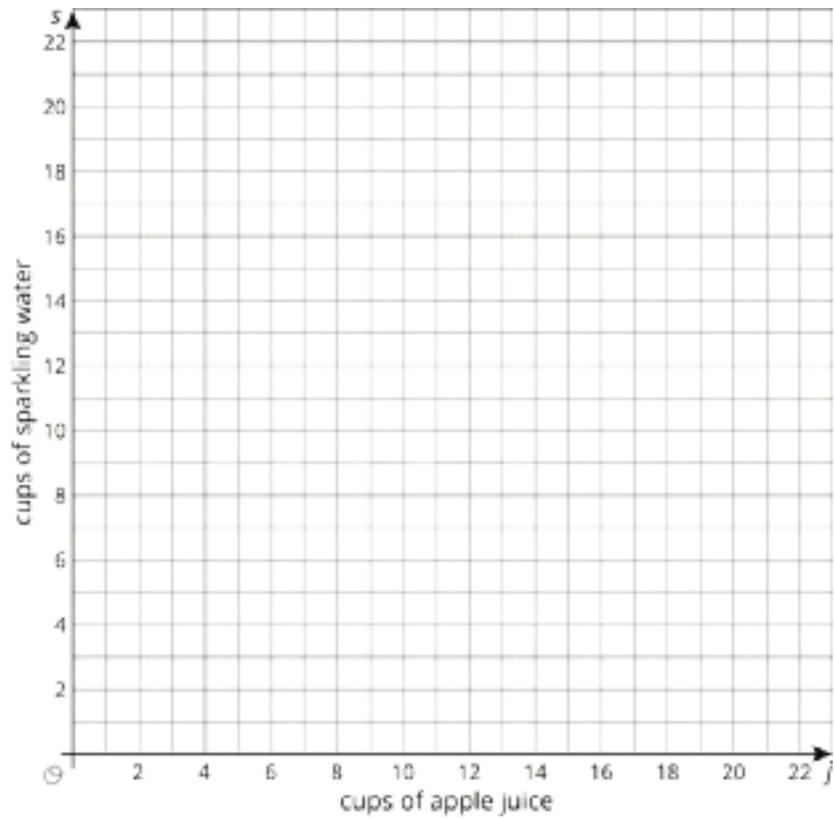
Problem 6

Students construct and evaluate a linear equation given a ratio. Points are then plotted to visualize the relationship.

Statement

Jada makes sparkling juice by mixing 2 cups of sparkling water with every 3 cups of apple juice.

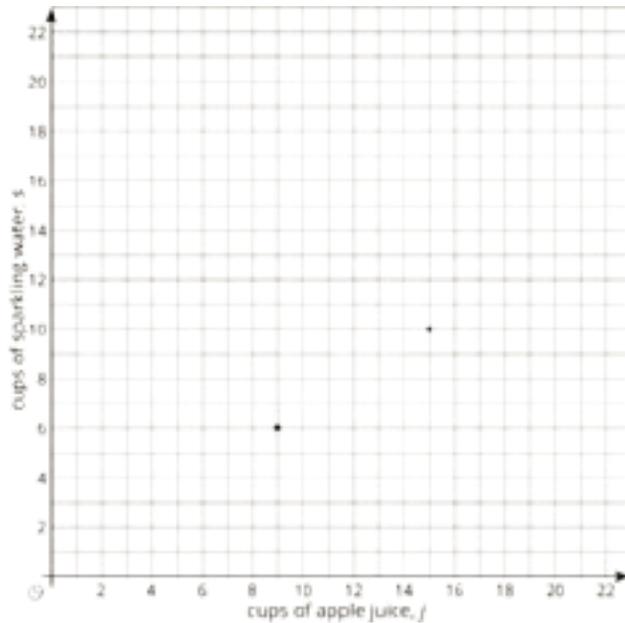
1. How much sparkling water does Jada need if she uses 15 cups of apple juice?
2. How much apple juice does Jada need if she uses 6 cups of sparkling water?
3. Plot these pairs of measurements as points on the graph.



4. Let s represent the number of cups of sparkling water and j represent the number of cups of apple juice. Write an equation that shows how s and j are related.

Solution

1. 10 cups
2. 9 cups



3.

4. $j = \frac{3}{2}s$ or equivalent

Minimal Tier 1 response:

- Work is complete and correct.
- Sample:
 1. 10
 2. 9
 3. (The points (9, 6) and (15, 10) appear on the graph)
 4. $j = 1.5s$

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: 3 out of 4 problem parts correct; consistently using $\frac{2}{3}$ as the unit rate, up to and including $j = \frac{2}{3}s$.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: 2 or more problem parts incorrect; using a constant of proportionality other than $\frac{3}{2}$ or $\frac{2}{3}$; assuming that there will always be one more cup of juice than cup of water.

Aligned Standards

6.EE.C.9, 6.RP.A.3.a

Problem 7

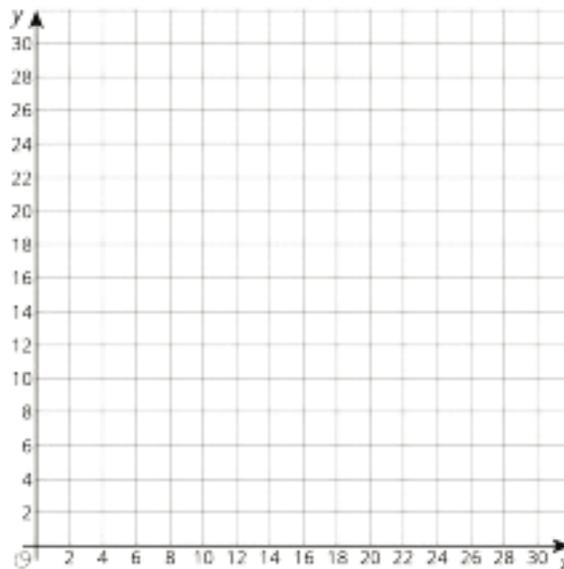
Students create a table, equation, and graph to represent a situation involving perimeter of a rectangle. Depending on how they view the relationship, the length and width could be either the independent or dependent variables.

Statement

This rectangle has a perimeter of 36 units.



1. Create a table that shows the length and width of at least 3 different rectangles that also have a perimeter of 36 units.
2. Describe the relationship between the columns of your table.
3. Write an equation to represent the relationship. Identify the independent and dependent variables.
4. Plot the values in your table as points on the graph. Make sure to label the axes.



Solution

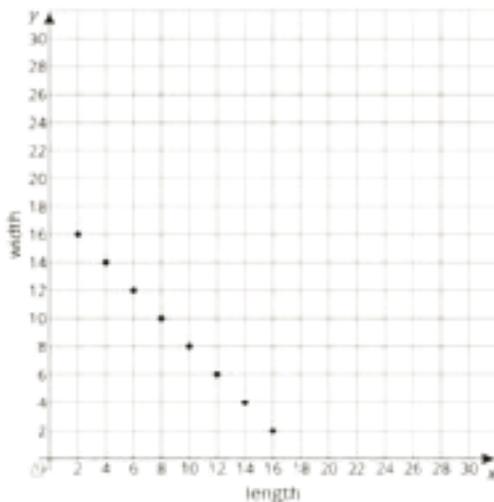
1. Answers vary. Sample response:

length	width
10	8
12	6
4	14

2. If we know the length of the rectangle, we can find its width by subtracting the length from 18.

3. Answers vary. Sample response:

- In the equation $w = 18 - \ell$ the rectangle's length is the independent variable and width is the dependent variable.
- In the equation $\ell = 18 - w$ the rectangle's width is the independent variable and length is the dependent variable.



4.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample:
 1. (A table, with at least 3 rows, where the sum of the two values in each row is 18.)
 2. $w + \ell = 18$
 3. $\ell = 18 - w$ The independent variable is the width and the dependent variable is the length.
 4. (A graph with at least 3 points that lie on the line $y = 18 - x$ and the axes labeled length and width.)

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Acceptable errors: the equation correctly represents the situation, but is not structured to express the dependent variable in terms of the independent variable.
- Sample errors: an arithmetic error leads to an incorrect solution; the axes on the graph are not labeled or are labeled with the variable identified as the independent variable on the vertical axis and the dependent variable on the horizontal axis.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Acceptable errors: values chosen for the length and width add up to 36 instead of 18; rectangles chosen have an area of 36 square units instead of a perimeter of 36 units.
- Sample errors: work involves a misinterpretation of the situation that affects all or most problem parts, but work does show understanding of creating equations, tables, and graphs to represent a situation.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: Multiple Tier 3 errors, major omissions.

Aligned Standards

6.EE.B.6, 6.EE.B.7

Assessment : End-of-Unit Assessment (B)

Teacher Instructions

Calculators should not be used.

Student Instructions

Calculators should not be used.

Problem 1

Students should be able to express expressions with exponents as repeated multiplication, as well as evaluate these expressions to equal a given number.

Statement

Which expression is equal to 7^3 ?

A. $7 \cdot 7 \cdot 7$

B. 21

C. 3^7

D. 10

Solution

A

Aligned Standards

6.EE.A.1

Problem 2

Students apply the meaning of bases and exponents to conclude that B and E both have equivalent values of 81.

Statement

Select **all** the expressions that have the same value.

- A. 3^3
- B. 3^4
- C. 6^2
- D. 6^3
- E. 9^2
- F. 9^3

Solution

["B", "E"]

Aligned Standards

6.EE.A.1

Problem 3

Students selecting A have not distributed the 5 to the $-10b$ term. Students selecting B have not distributed the 45 to the $-10b$ term. Students selecting C have made a sign error or did not notice the sign change in the expression.

Statement

Which expression is equivalent to $45a - 10b$?

- A. $5(9a - 10b)$
- B. $45(a - 10b)$
- C. $5(9a + 2b)$
- D. $5(9a - 2b)$

Solution

D

Aligned Standards

6.EE.A.3

Problem 4

Students evaluate exponential expressions when given a specific value for the unknown.

Statement

Here is an expression: $2 + 3^t$.

1. Evaluate the expression when t is 1.

2. Evaluate the expression when t is 4.

Solution

1. 5
2. 83

Aligned Standards

6.EE.A.1

Problem 5

Students demonstrate their understanding of the words sum, product, coefficient, and term by generating equivalent expressions with a specified structure.

Statement

1. Write an expression equivalent to $b + b + b + b + b$ that is a product of a coefficient and a variable.
2. Write an expression equivalent to $b + b + b + b + b$ that is a sum of two terms.

Solution

1. $5b$.
2. Answers vary. Sample response: $2b + 3b$ or $b + 4b$.

Aligned Standards

6.EE.A.2.b, 6.EE.A.3

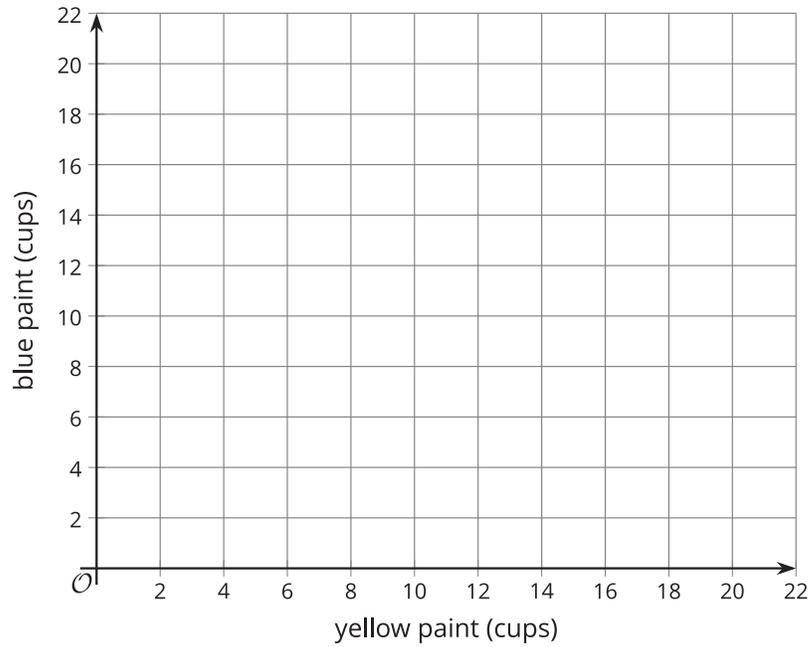
Problem 6

Students construct and evaluate a linear equation given a ratio. Points are then plotted to visualize the relationship.

Statement

Andre makes green paint by mixing 5 cups of yellow paint with 2 cups of blue paint.

1. To make the same shade of green, how much yellow paint does Andre need if he uses 8 cups of blue paint?
2. How much blue paint does Andre need if he uses 15 cups of yellow paint?
3. Plot these pairs of measurements as points on the graph.



4. Let y represent the cups of yellow paint and b represent the cups of blue paint. Write an equation that shows how y and b are related.

Solution

1. 20 cups
2. 6 cups
3. Points plotted at $(20, 8)$ and $(15, 6)$
4. $b = \frac{2}{5}y$ (or equivalent)

Minimal Tier 1 response:

- Work is complete and correct.
- Sample:
 1. 20
 2. 6
 3. The points $(20, 8)$ and $(15, 6)$ appear on the graph
 4. $b = \frac{2}{5}y$

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.

- Sample errors: 3 out of 4 problem parts correct; consistently using $\frac{2}{5}$ as the unit rate, up to and including $b = \frac{2}{5}y$.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: 2 or more problem parts incorrect; using a constant of proportionality other than $\frac{2}{5}$ or $\frac{5}{2}$; assuming that there will always be 3 more cups of yellow than blue.

Aligned Standards

6.EE.C.9

Problem 7

Students create a table, equation, and graph to represent a situation involving perimeter of a rectangle. Depending on how they view the relationship, the length and width could be either the independent or dependent variables.

Statement

This rectangle has a perimeter of 28 units.



1. Create a table that shows the length and width of at least 3 different rectangles that also have a perimeter of 28 units.
2. Describe the relationship between the columns of your table.
3. Write an equation to represent the relationship. Identify the independent and dependent variables.

4. Plot the values in your table as points on the graph. Make sure to label the axes.

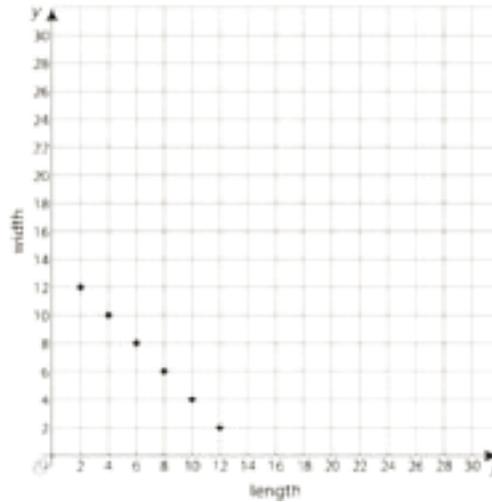


Solution

1. Answers vary. Sample response:

length	width
9	5
8	6
7	7

2. If we know the length of the rectangle, we can find its width by subtracting the length from 14.
3. Answers vary. Sample response:
- In the equation $w = 14 - l$ the rectangle's length is the independent variable and width is the dependent variable.
 - In the equation $l = 14 - w$ the rectangle's width is the independent variable and length is the dependent variable.



4.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample:
 1. (A table, with at least 3 rows, where the sum of the two values in each row is 14.)
 2. $w + l = 14$
 3. $l = 14 - w$ The independent variable is the width and the dependent variable is the length.
 4. (A graph with at least 3 points that lie on the line $y = 14 - x$ and the axes labeled length and width.)

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Acceptable errors: the equation correctly represents the situation, but is not structured to express the dependent variable in terms of the independent variable.
- Sample errors: an arithmetic error leads to an incorrect solution; the axes on the graph are not labeled or are labeled with the variable identified as the independent variable on the vertical axis and the dependent variable on the horizontal axis.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Acceptable errors: values chosen for the length and width add up to 28 instead of 14; rectangles chosen have an area of 28 square units instead of a perimeter of 28 units.

- Sample errors: work involves a misinterpretation of the situation that affects all or most problem parts, but work does show understanding of creating equations, tables, and graphs to represent a situation.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: Multiple Tier 3 errors, major omissions.

Aligned Standards

6.EE.B.6, 6.EE.B.7

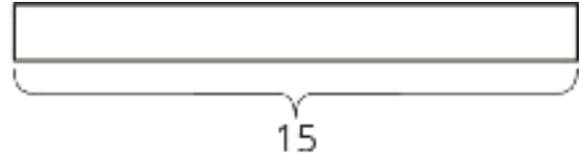
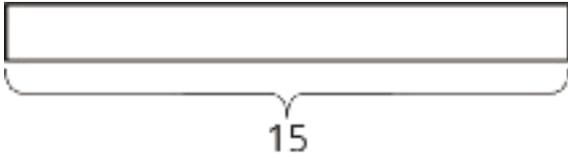
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Lesson
Cool Downs

Lesson 1: Tape Diagrams and Equations

Cool Down: Finish the Diagrams

Finish the first diagram so that it represents $5 \cdot x = 15$, and the second diagram so that it represents $5 + y = 15$.



Lesson 2: Truth and Equations

Cool Down: How Do You Know a Solution is a Solution?

Explain how you know that 88 is a solution to the equation $\frac{1}{8}x = 11$ by completing the sentences:

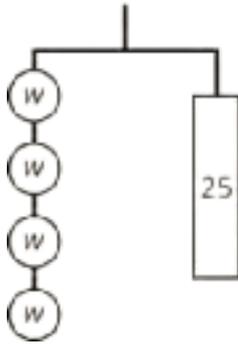
The word “solution” means . . .

88 is a solution to $\frac{1}{8}x = 11$ because . . .

Lesson 3: Staying in Balance

Cool Down: Weight of the Circle

Here is a balanced hanger.



1. Write an equation representing this hanger.

2. Find the weight of one circle. Show or explain how you found it.

Lesson 5: A New Way to Interpret a over b

Cool Down: Choosing Solutions

Select **all** the expressions that are solutions to $5 = \frac{2}{3}x$.

- $5 \cdot \frac{2}{3}$
- $\frac{5}{\frac{2}{3}}$
- $5 \div \frac{2}{3}$
- $\frac{15}{2}$
- $\frac{10}{3}$

Lesson 6: Write Expressions Where Letters Stand for Numbers

Cool Down: Crazy Eights

A plant measured x inches tall last week and 8 inches tall this week.

1. Circle the expression that represents the number of inches the plant grew this week. Explain how you know.

$x - 8$

$8 - x$

2. For the expression *not* chosen, describe a situation that the expression might represent.

Lesson 7: Revisit Percentages

Cool Down: Fundraising for the Animal Shelter

Noah raised \$54 to support the animal shelter, which is 60% of his fundraising goal.

1. Write an equation to represent the situation.

2. What is Noah's fundraising goal? Show or explain how you found it.

Lesson 8: Equal and Equivalent

Cool Down: Decisions About Equivalence

Decide if the expressions in each pair are equivalent. Explain how you know.

1. $x + x + x + x$ and $4x$

2. $5x$ and $x + 5$

Lesson 9: The Distributive Property, Part 1

Cool Down: Complete the Equation

Write a number or expression in each empty box to create true equations.

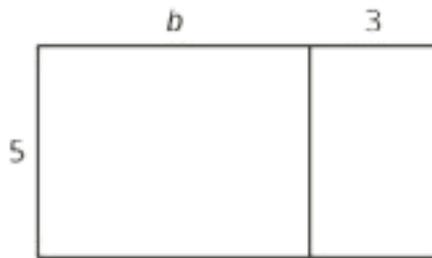
$$1. 7(3 + 5) = \boxed{} + \boxed{}$$

$$2. 15 - 10 = \boxed{}(3 - 2)$$

Lesson 10: The Distributive Property, Part 2

Cool Down: Which Expressions Represent Area?

Select **all** the expressions that represent the large rectangle's total area.



- $3(5 + b)$
- $5(b + 3)$
- $5b + 15$
- $15 + 5b$
- $3 \cdot 5 + 3b$

Lesson 11: The Distributive Property, Part 3

Cool Down: Writing Equivalent Expressions

1. Use the distributive property to write an expression that is equivalent to $12 + 4x$.

2. Draw a diagram that shows the two expressions are equivalent.

Lesson 12: Meaning of Exponents

Cool Down: More 3's

3^5 equals 243. Explain how to use that fact to quickly evaluate 3^6 .

Lesson 13: Expressions with Exponents

Cool Down: Coin Calculation

Andre and Elena knew that after 28 days they would have 2^{28} coins, but they wanted to find out how many coins that actually is. Andre wrote:

$$2^{28} = 2 \cdot 28 = 56$$

Elena said, "No, exponents mean repeated multiplication. It should be $28 \cdot 28$, which works out to be 784." Do you agree with either of them? Explain your reasoning.

Lesson 14: Evaluating Expressions with Exponents

Cool Down: Calculating Volumes

Jada and Noah wanted to find the total volume of a cube and a rectangular prism. They know the prism's volume is 20 cubic units, and they know the cube has side lengths of 10 units. Jada says the total volume is 27,000 cubic units. Noah says it is 1,020 cubic units. Here is how each of them reasoned:

Jada's Method:

$$\begin{array}{l} 20 + 10^3 \\ 30^3 \\ 27,000 \end{array}$$

Noah's Method:

$$\begin{array}{l} 20 + 10^3 \\ 20 + 1,000 \\ 1,020 \end{array}$$

Do you agree with either of them? Explain your reasoning.

Lesson 15: Equivalent Exponential Expressions

Cool Down: True Statements

Match each equation to a solution.

1. $2^x = 64$

• $\frac{8}{125}$

2. $x = \left(\frac{2}{5}\right)^3$

• $\frac{4}{5}$

3. $3 \cdot (3^4) = 3^x$

• 5

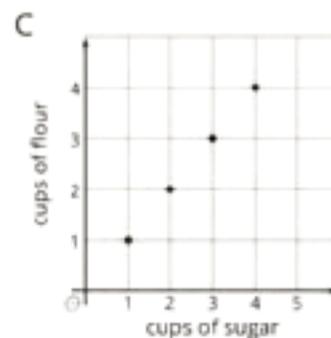
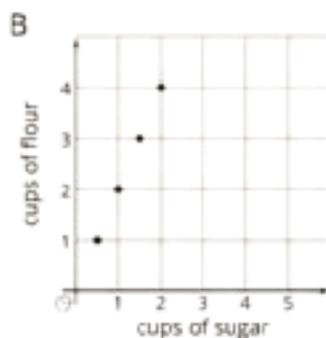
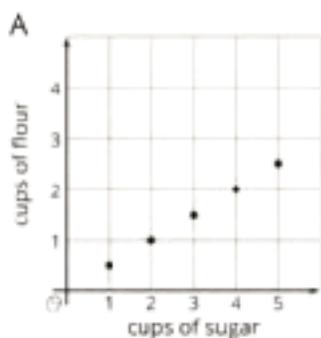
4. $\frac{16}{25} = x^2$

• 6

Lesson 16: Two Related Quantities, Part 1

Cool Down: Baking Brownies

A brownie recipe calls for 1 cup of sugar and $\frac{1}{2}$ cup of flour to make one batch of brownies. To make multiple batches, the equation $f = \frac{1}{2}s$ where f is the number of cups of flour and s is the number of cups of sugar represents the relationship. Which graph also represents the relationship? Explain how you know.



Lesson 17: Two Related Quantities, Part 2

Cool Down: Interpret the Point

During a walk-a-thon, Noah's time in hours, t , and distance in miles, d , are related by the equation $\frac{1}{3}d = t$. A graph of the equation includes the point $(12, 4)$.

1. Identify the independent variable.
2. What does the point $(12, 4)$ represent in this situation?
3. What point would represent the time it took to walk $7\frac{1}{2}$ miles?

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