Core Knowledge® MATHEMATICS

Expressions and Equations

Tetrahedra

Two Related Quantities

3 FOR $13.05

$15.35

5 lbs

1.5 lbs

1.5 lbs

1.5 lbs

2 + 3								
3 + 2								
2 · 3								

Using Diagrams to Show That Expressions are Equivalent

Staying in Balance

Exponents

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Expressions and Equations

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Expressions and Equations
Student Workbook

Core Knowledge Mathematics™

Lesson 1: Tape Diagrams and Equations

Let's see how tape diagrams and equations can show relationships between amounts.

1.1: Which Diagram is Which?

1. Here are two diagrams. One represents $2 + 5 = 7$. The other represents $5 \cdot 2 = 10$. Which is which? Label the length of each diagram.
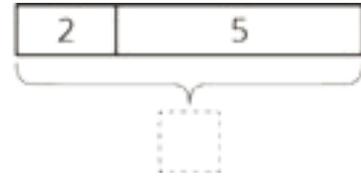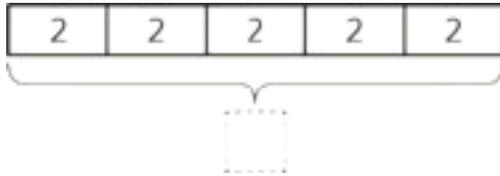
2. Draw a diagram that represents each equation.

$$4 + 3 = 7 \qquad\qquad 4 \cdot 3 = 12$$

1.2: Match Equations and Tape Diagrams

Here are two tape diagrams. Match each equation to one of the tape diagrams.
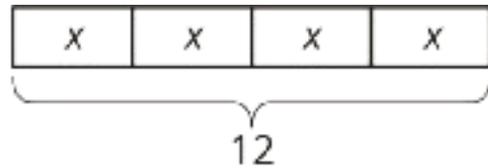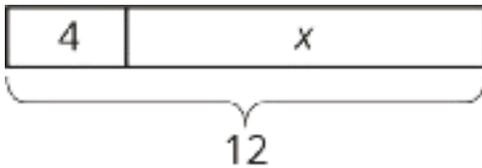
- $4 + x = 12$
- $12 \div 4 = x$
- $4 \cdot x = 12$
- $12 = 4 + x$
- $12 - x = 4$
- $12 = 4 \cdot x$
- $12 - 4 = x$
- $x = 12 - 4$
- $x + x + x + x = 12$

1.3: Draw Diagrams for Equations

For each equation, draw a diagram and find the value of the unknown that makes the equation true.

1. $18 = 3 + x$

2. $18 = 3 \cdot y$

Are you ready for more?

You are walking down a road, seeking treasure. The road branches off into three paths. A guard stands in each path. You know that only one of the guards is telling the truth, and the other two are lying. Here is what they say:

- Guard 1: The treasure lies down this path.
- Guard 2: No treasure lies down this path; seek elsewhere.
- Guard 3: The first guard is lying.

Which path leads to the treasure?

Lesson 1 Summary

Tape diagrams can help us understand relationships between quantities and how operations describe those relationships.
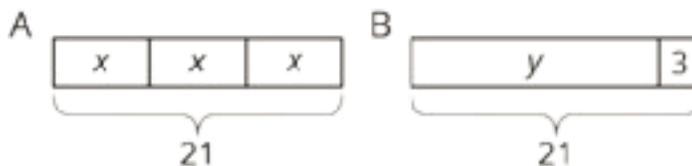
Diagram A has 3 parts that add to 21. Each part is labeled with the same letter, so we know the three parts are equal. Here are some equations that all represent diagram A:

$$x + x + x = 21$$

$$3 \cdot x = 21$$

$$x = 21 \div 3$$

$$x = \frac{1}{3} \cdot 21$$

Notice that the number 3 is not seen in the diagram; the 3 comes from counting 3 boxes representing 3 equal parts in 21.

We can use the diagram or any of the equations to reason that the value of x is 7.

Diagram B has 2 parts that add to 21. Here are some equations that all represent diagram B:

$$y + 3 = 21$$

$$y = 21 - 3$$

$$3 = 21 - y$$

We can use the diagram or any of the equations to reason that the value of y is 18.

Unit 6 Lesson 1 Cumulative Practice Problems

1. Here is an equation: $x + 4 = 17$

    a. Draw a tape diagram to represent the equation.

    b. Which part of the diagram shows the quantity x? What about 4? What about 17?

    c. How does the diagram show that $x + 4$ has the same value as 17?

2. Diego is trying to find the value of x in $5 \cdot x = 35$. He draws this diagram but is not certain how to proceed.
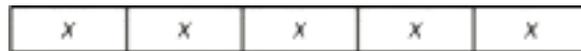
    a. Complete the tape diagram so it represents the equation $5 \cdot x = 35$.

    b. Find the value of x.

3. Match each equation to one of the two tape diagrams.

    a. $x + 3 = 9$

    b. $3 \cdot x = 9$

    c. $9 = 3 \cdot x$

    d. $3 + x = 9$

    e. $x = 9 - 3$

    f. $x = 9 \div 3$

    g. $x + x + x = 9$
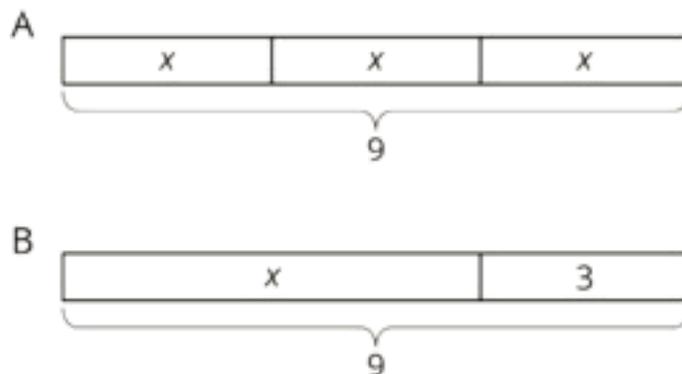
4. For each equation, draw a tape diagram and find the unknown value.

    a. $x + 9 = 16$

    b. $4 \cdot x = 28$

5. A shopper paid $2.52 for 4.5 pounds of potatoes, $7.75 for 2.5 pounds of broccoli, and $2.45 for 2.5 pounds of pears. What is the unit price of each item she bought? Show your reasoning.

(From Unit 5, Lesson 13.)

6. A sports drink bottle contains 16.9 fluid ounces. Andre drank 80% of the bottle. How many fluid ounces did Andre drink? Show your reasoning.

(From Unit 3, Lesson 14.)

7. The daily recommended allowance of calcium for a sixth grader is 1,200 mg. One cup of milk has 25% of the recommended daily allowance of calcium. How many milligrams of calcium are in a cup of milk? If you get stuck, consider using the double number line.

(From Unit 3, Lesson 11.)

Lesson 2: Truth and Equations

Let's use equations to represent stories and see what it means to solve equations.

2.1: Three Letters

1. The equation $a + b = c$ could be true or false.

   a. If a is 3, b is 4, and c is 5, is the equation true or false?

   b. Find new values of a, b, and c that make the equation true.

   c. Find new values of a, b, and c that make the equation false.

2. The equation $x \cdot y = z$ could be true or false.

   a. If x is 3, y is 4, and z is 12, is the equation true or false?

   b. Find new values of x, y, and z that make the equation true.

   c. Find new values of x, y, and z that make the equation false.

2.2: Storytime

Here are three situations and six equations. Which equation best represents each situation? If you get stuck, consider drawing a diagram.

$$x + 5 = 20 \qquad\qquad x = 20 + 5 \qquad\qquad 5x = 20$$

$$x + 20 = 5 \qquad\qquad 5 \cdot 20 = x \qquad\qquad 20x = 5$$

1. After Elena ran 5 miles on Friday, she had run a total of 20 miles for the week. She ran x miles before Friday.

2. Andre's school has 20 clubs, which is five times as many as his cousin's school. His cousin's school has x clubs.

3. Jada volunteers at the animal shelter. She divided 5 cups of cat food equally to feed 20 cats. Each cat received x cups of food.

2.3: Using Structure to Find Solutions

Here are some equations that contain a **variable** and a list of values. Think about what each equation means and find a **solution** in the list of values. If you get stuck, consider drawing a diagram. Be prepared to explain why your solution is correct.

1. $1000 - a = 400$

2. $12.6 = b + 4.1$

3. $8c = 8$

4. $\frac{2}{3} \cdot d = \frac{10}{9}$

5. $10e = 1$

6. $10 = 0.5f$

7. $0.99 = 1 - g$

8. $h + \frac{3}{7} = 1$

List: $\frac{1}{8}$ $\frac{3}{7}$ $\frac{4}{7}$ $\frac{3}{5}$ $\frac{5}{3}$ $\frac{7}{3}$ 0.01 0.1 0.5

1 2 8.5 9.5 16.7 20 400 600 1400

Are you ready for more?

One solution to the equation $a + b + c = 10$ is $a = 2$, $b = 5$, $c = 3$.

How many different whole-number solutions are there to the equation $a + b + c = 10$? Explain or show your reasoning.

Lesson 2 Summary

An equation can be true or false. An example of a true equation is $7 + 1 = 4 \cdot 2$. An example of a false equation is $7 + 1 = 9$.

An equation can have a letter in it, for example, $u + 1 = 8$. This equation is false if u is 3, because $3 + 1$ does not equal 8. This equation is true if u is 7, because $7 + 1 = 8$.

A letter in an equation is called a **variable**. In $u + 1 = 8$, the variable is u. A number that can be used in place of the variable that makes the equation true is called a **solution** to the equation. In $u + 1 = 8$, the solution is 7.

When a number is written next to a variable, the number and the variable are being multiplied. For example, $7x = 21$ means the same thing as $7 \cdot x = 21$. A number written next to a variable is called a **coefficient**. If no coefficient is written, the coefficient is 1. For example, in the equation $p + 3 = 5$, the coefficient of p is 1.

Unit 6 Lesson 2 Cumulative Practice Problems

1. Select **all** the true equations.

   A. $5 + 0 = 0$

   B. $15 \cdot 0 = 0$

   C. $1.4 + 2.7 = 4.1$

   D. $\frac{2}{3} \cdot \frac{5}{9} = \frac{7}{12}$

   E. $4\frac{2}{3} = 5 - \frac{1}{3}$

2. Mai's water bottle had 24 ounces in it. After she drank x ounces of water, there were 10 ounces left. Select **all** the equations that represent this situation.

   A. $24 \div 10 = x$

   B. $24 + 10 = x$

   C. $24 - 10 = x$

   D. $x + 10 = 24$

   E. $10x = 24$

3. Priya has 5 pencils, each x inches in length. When she lines up the pencils end to end, they measure 34.5 inches. Select **all** the equations that represent this situation.

   A. $5 + x = 34.5$

   B. $5x = 34.5$

   C. $34.5 \div 5 = x$

   D. $34.5 - 5 = x$

   E. $x = (34.5) \cdot 5$

4. Match each equation with a solution from the list of values.

A. $2a = 4.6$

B. $b + 2 = 4.6$

C. $c \div 2 = 4.6$

D. $d - 2 = 4.6$

E. $e + \frac{3}{8} = 2$

F. $\frac{1}{8}f = 3$

G. $g \div \frac{8}{5} = 1$

1. $\frac{8}{5}$

2. $1\frac{5}{8}$

3. 2.3

4. 2.6

5. 6.6

6. 9.2

7. 24

5. The daily recommended allowance of vitamin C for a sixth grader is 45 mg. 1 orange has about 75% of the recommended daily allowance of vitamin C. How many milligrams are in 1 orange? If you get stuck, consider using the double number line.
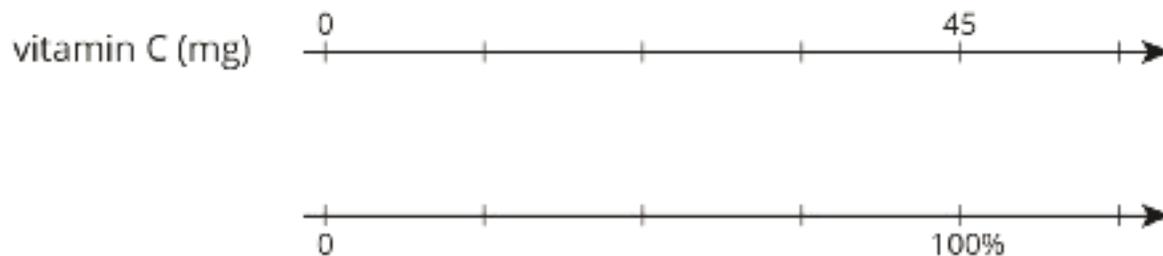
(From Unit 3, Lesson 11.)

6. There are 90 kids in the band. 20% of the kids own their own instruments, and the rest rent them.

   a. How many kids own their own instruments?

   b. How many kids rent instruments?

   c. What percentage of kids rent their instruments?

   (From Unit 3, Lesson 12.)

7. Find each product.

   a. $(0.25) \cdot (1.4)$

   b. $(0.061) \cdot (0.43)$

   c. $(1.017) \cdot (0.072)$

   d. $(5.226) \cdot (0.037)$

   (From Unit 5, Lesson 8.)

Lesson 3: Staying in Balance

Let's use balanced hangers to help us solve equations.

3.1: Hanging Around
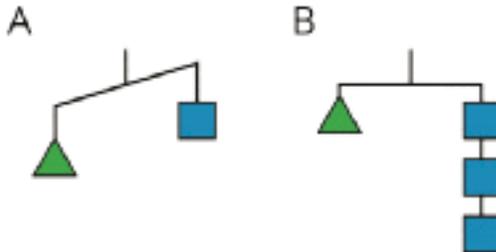
For diagram A, find:

    1. One thing that *must* be true

    2. One thing that *could* be true or false

    3. One thing that *cannot possibly* be true

For diagram B, find:

    1. One thing that *must* be true

    2. One thing that *could* be true or false

    3. One thing that *cannot possibly* be true

3.2: Match Equations and Hangers

A   B   C   D
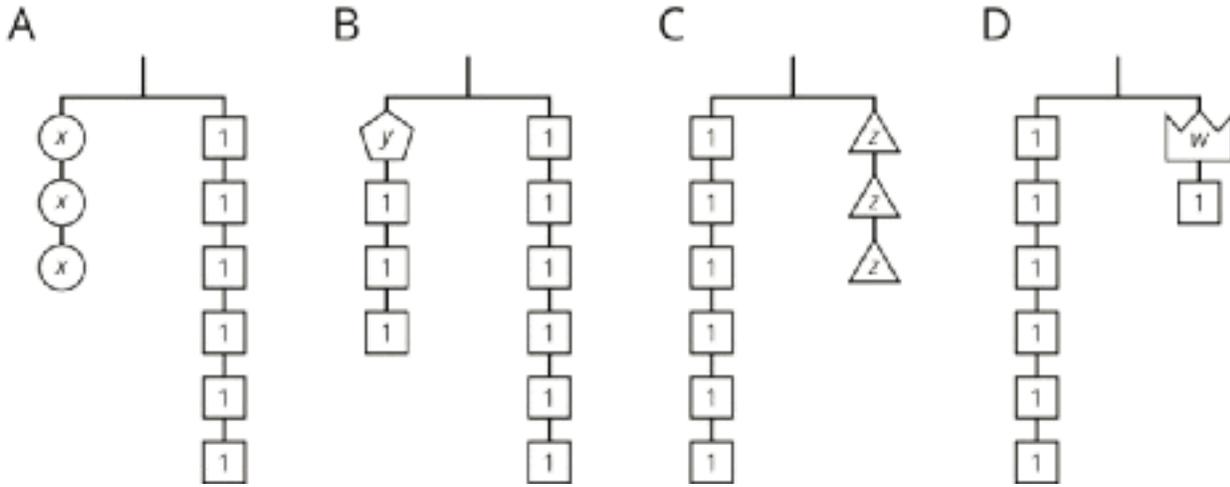
1. Match each hanger to an equation. Complete the equation by writing x, y, z, or w in the empty box.

$$\square + 3 = 6 \qquad 3 \cdot \square = 6 \qquad 6 = \square + 1 \qquad 6 = 3 \cdot \square$$

2. Find a solution to each equation. Use the hangers to explain what each solution means.

3.3: Connecting Diagrams to Equations and Solutions

Here are some balanced hangers. Each piece is labeled with its weight.
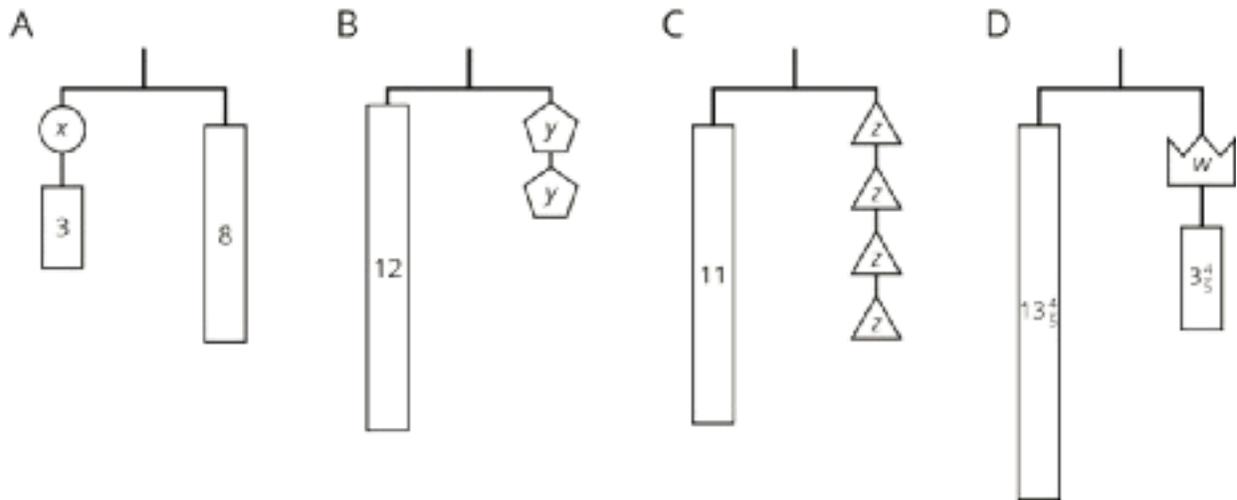
For each diagram:

1. Write an equation.

2. Explain how to reason with the *diagram* to find the weight of a piece with a letter.

3. Explain how to reason with the *equation* to find the weight of a piece with a letter.

Are you ready for more?

When you have the time, visit the site https://solveme.edc.org/Mobiles.html to solve some trickier puzzles that use hanger diagrams like the ones in this lesson. You can even build new ones. (If you want to do this during class, check with your teacher first!)

Lesson 3 Summary

A hanger stays balanced when the weights on both sides are equal. We can change the weights and the hanger will stay balanced as long as both sides are changed in the same way. For example, adding 2 pounds to each side of a balanced hanger will keep it balanced. Removing half of the weight from each side will also keep it balanced.

An equation can be compared to a balanced hanger. We can change the equation, but for a true equation to remain true, the same thing must be done to both sides of the equal sign. If we add or subtract the same number on each side, or multiply or divide each side by the same number, the new equation will still be true.

This way of thinking can help us find solutions to equations. Instead of checking different values, we can think about subtracting the same amount from each side or dividing each side by the same number.
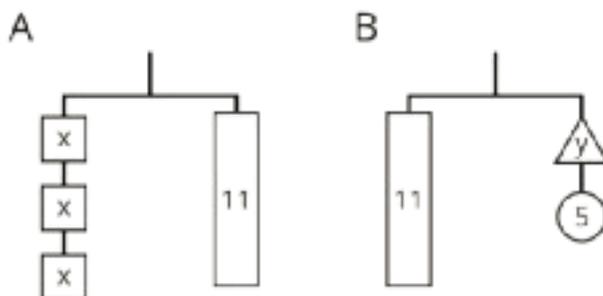
Diagram A can be represented by the equation $3x = 11$.

If we break the 11 into 3 equal parts, each part will have the same weight as a block with an x.

Splitting each side of the hanger into 3 equal parts is the same as dividing each side of the equation by 3.

- $3x$ divided by 3 is x.

- 11 divided by 3 is $\frac{11}{3}$.

- If $3x = 11$ is true, then $x = \frac{11}{3}$ is true.

- The solution to $3x = 11$ is $\frac{11}{3}$.

Diagram B can be represented with the equation $11 = y + 5$.

If we remove a weight of 5 from each side of the hanger, it will stay in balance.

Removing 5 from each side of the hanger is the same as subtracting 5 from each side of the equation.

- $11 - 5$ is 6.

- $y + 5 - 5$ is y.

- If $11 = y + 5$ is true, then $6 = y$ is true.

- The solution to $11 = y + 5$ is 6.

Unit 6 Lesson 3 Cumulative Practice Problems

1. Select **all** the equations that represent the hanger.
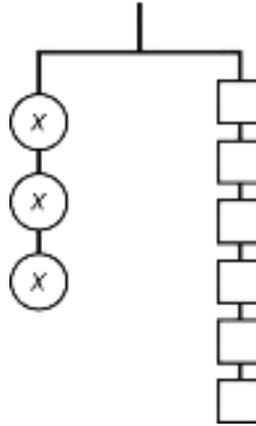
A. $x + x + x = 1 + 1 + 1 + 1 + 1 + 1$

B. $x \cdot x \cdot x = 6$

C. $3x = 6$

D. $x + 3 = 6$

E. $x \cdot x \cdot x = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$

2. Write an equation to represent each hanger.
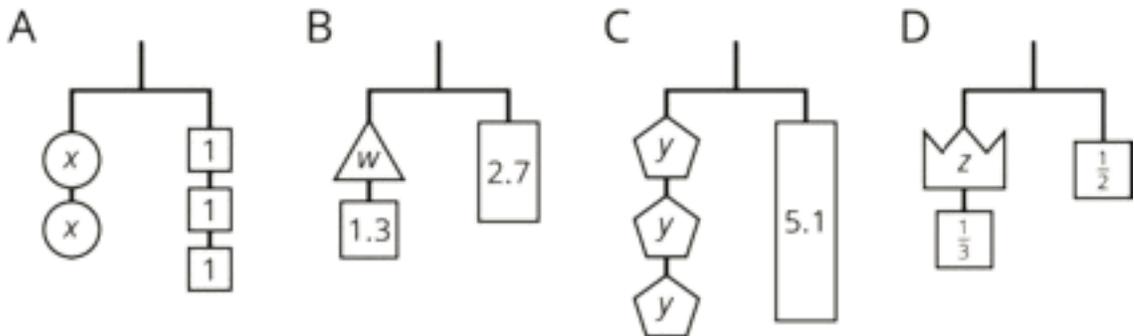
3.   a. Write an equation to represent the hanger.

       b. Explain how to reason with the hanger to find the value of x.
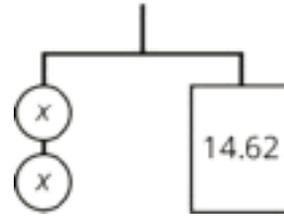
       c. Explain how to reason with the equation to find the value of x.

4. Andre says that x is 7 because he can move the two 1s with the x to the other side.
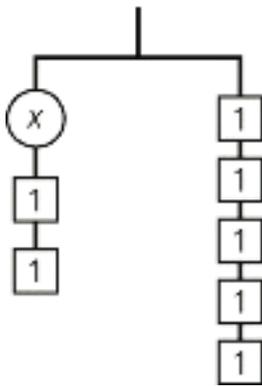
Do you agree with Andre? Explain your reasoning.

5. Match each equation to one of the diagrams.

       a. $12 - m = 4$

       b. $12 = 4 \cdot m$

       c. $m - 4 = 12$

       d. $\frac{m}{4} = 12$
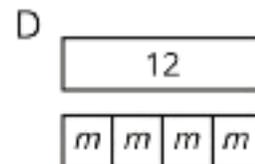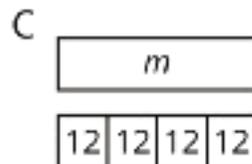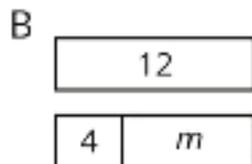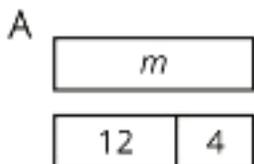
(From Unit 6, Lesson 1.)

6. The area of a rectangle is 14 square units. It has side lengths x and y. Given each value for x, find y.

    a. $x = 2\frac{1}{3}$

    b. $x = 4\frac{1}{5}$

    c. $x = \frac{7}{6}$

(From Unit 4, Lesson 13.)

7. Lin needs to save up $20 for a new game. How much money does she have if she has saved each percentage of her goal. Explain your reasoning.

    a. 25%

    b. 75%

    c. 125%

(From Unit 3, Lesson 11.)

Lesson 4: Practice Solving Equations and Representing Situations with Equations

Let's solve equations by doing the same to each side.

4.1: Number Talk: Subtracting From Five

Find the value of each expression mentally.

$5 - 2$

$5 - 2.1$

$5 - 2.17$

$5 - 2\frac{7}{8}$

4.2: Row Game: Solving Equations Practice

Solve the equations in one column. Your partner will work on the other column.

Check in with your partner after you finish each row. Your answers in each row should be the same. If your answers aren't the same, work together to find the error and correct it.

column A	column B
$18 = 2x$	$36 = 4x$
$17 = x + 9$	$13 = x + 5$
$8x = 56$	$3x = 21$
$21 = \frac{1}{4}x$	$28 = \frac{1}{3}x$
$6x = 45$	$8x = 60$
$x + 4\frac{5}{6} = 9$	$x + 3\frac{5}{6} = 8$
$\frac{5}{7}x = 55$	$\frac{3}{7}x = 33$
$\frac{1}{5} = 6x$	$\frac{1}{3} = 10x$
$2.17 + x = 5$	$6.17 + x = 9$
$\frac{20}{3} = \frac{10}{9}x$	$\frac{14}{5} = \frac{7}{15}x$
$14.88 + x = 17.05$	$3.91 + x = 6.08$
$3\frac{3}{4}x = 1\frac{1}{4}$	$\frac{7}{5}x = \frac{7}{15}$

4.3: Choosing Equations to Match Situations

Circle **all** of the equations that describe each situation. If you get stuck, consider drawing a diagram. Then find the solution for each situation.

1. Clare has 8 fewer books than Mai. If Mai has 26 books, how many books does Clare have?

   ○ $26 - x = 8$

   ○ $x = 26 + 8$

   ○ $x + 8 = 26$

   ○ $26 - 8 = x$

   $x = $ _____

2. A coach formed teams of 8 from all the players in a soccer league. There are 14 teams. How many players are in the league?

   ○ $y = 14 \div 8$

   ○ $\frac{y}{8} = 14$

   ○ $\frac{1}{8}y = 14$

   ○ $y = 14 \cdot 8$

   $y = $ _____

3. Kiran scored 223 more points in a computer game than Tyler. If Kiran scored 409 points, how many points did Tyler score?

   ○ $223 = 409 - z$

   ○ $409 - 223 = z$

   ○ $409 + 223 = z$

   ○ $409 = 223 + z$

   $z = $ _____

4. Mai ran 27 miles last week, which was three times as far as Jada ran. How far did Jada run?

   ○ $3w = 27$

   ○ $w = \frac{1}{3} \cdot 27$

   ○ $w = 27 \div 3$

   ○ $w = 3 \cdot 27$

   $w = $ _____

Are you ready for more?

Mai's mother was 28 when Mai was born. Mai is now 12 years old. In how many years will Mai's mother be twice Mai's age? How old will they be then?

Lesson 4 Summary

Writing and solving equations can help us answer questions about situations.

Suppose a scientist has 13.68 liters of acid and needs 16.05 liters for an experiment. How many more liters of acid does she need for the experiment?

- We can represent this situation with the equation: $$13.68 + x = 16.05$$

- When working with hangers, we saw that the solution can be found by subtracting 13.68 from each side. This gives us some new equations that also represent the situation:

$$x = 16.05 - 13.68$$
$$x = 2.37$$

- Finding a solution in this way leads to a variable on one side of the equal sign and a number on the other. We can easily read the solution—in this case, 2.37—from an equation with a letter on one side and a number on the other. We often write solutions in this way.

Let's say a food pantry takes a 54-pound bag of rice and splits it into portions that each weigh $\frac{3}{4}$ of a pound. How many portions can they make from this bag?

- We can represent this situation with the equation: $$\frac{3}{4}x = 54$$

- We can find the value of x by dividing each side by $\frac{3}{4}$. This gives us some new equations that represent the same situation:

$$x = 54 \div \frac{3}{4}$$
$$x = 72$$

- The solution is 72 portions.

Unit 6 Lesson 4 Cumulative Practice Problems

1. Select **all** the equations that describe each situation and then find the solution.

   a. Kiran's backpack weighs 3 pounds less than Clare's backpack. Clare's backpack weighs 14 pounds. How much does Kiran's backpack weigh?

   - ■ $x + 3 = 14$
   - ■ $3x = 14$
   - ■ $x = 14 - 3$
   - ■ $x = 14 \div 3$

   b. Each notebook contains 60 sheets of paper. Andre has 5 notebooks. How many sheets of paper do Andre's notebooks contain?

   - ■ $y = 60 \div 5$
   - ■ $y = 5 \cdot 60$
   - ■ $\frac{y}{5} = 60$
   - ■ $5y = 60$

2. Solve each equation.

   a. $2x = 5$

   b. $y + 1.8 = 14.7$

   c. $6 = \frac{1}{2}z$

   d. $3\frac{1}{4} = \frac{1}{2} + w$

   e. $2.5t = 10$

3. For each equation, draw a tape diagram that represents the equation.

    a. $3 \cdot x = 18$

    b. $3 + x = 18$

    c. $17 - 6 = x$

(From Unit 6, Lesson 1.)

4. Find each product.

    $(21.2) \cdot (0.02)$ $(2.05) \cdot (0.004)$

(From Unit 5, Lesson 8.)

5. For a science experiment, students need to find 25% of 60 grams.

    ○ Jada says, "I can find this by calculating $\frac{1}{4}$ of 60."

    ○ Andre says, "25% of 60 means $\frac{25}{100} \cdot 60$."

Do you agree with either of them? Explain your reasoning.

(From Unit 3, Lesson 13.)

Lesson 5: A New Way to Interpret a over b

Let's investigate what a fraction means when the numerator and denominator are not whole numbers.

5.1: Recalling Ways of Solving

Solve each equation. Be prepared to explain your reasoning.

$0.07 = 10m$ $10.1 = t + 7.2$

5.2: Interpreting $\frac{a}{b}$

Solve each equation.

1. $35 = 7x$

2. $35 = 11x$

3. $7x = 7.7$

4. $0.3x = 2.1$

5. $\frac{2}{5} = \frac{1}{2}x$

Are you ready for more?

Solve the equation. Try to find some shortcuts.

$$\frac{1}{6} \cdot \frac{3}{20} \cdot \frac{5}{42} \cdot \frac{7}{72} \cdot x = \frac{1}{384}$$

5.3: Storytime Again

Take turns with your partner telling a story that might be represented by each equation. Then, for each equation, choose one story, state what quantity x describes, and solve the equation. If you get stuck, consider drawing a diagram.

$$0.7 + x = 12 \qquad\qquad \frac{1}{4}x = \frac{3}{2}$$

Lesson 5 Summary

In the past, you learned that a fraction such as $\frac{4}{5}$ can be thought of in a few ways.

- $\frac{4}{5}$ is a number you can locate on the number line by dividing the section between 0 and 1 into 5 equal parts and then counting 4 of those parts to the right of 0.

- $\frac{4}{5}$ is the share that each person would have if 4 wholes were shared equally among 5 people. This means that $\frac{4}{5}$ is the result of *dividing* 4 by 5.

We can extend this meaning of *a fraction as a quotient* to fractions whose numerators and denominators are not whole numbers. For example, we can represent 4.5 pounds of rice divided into portions that each weigh 1.5 pounds as: $\frac{4.5}{1.5} = 4.5 \div 1.5 = 3$. In other words, $\frac{4.5}{1.5} = 3$ because the quotient of 4.5 and 1.5 is 3.

Fractions that involve non-whole numbers can also be used when we solve equations.

Suppose a road under construction is $\frac{3}{8}$ finished and the length of the completed part is $\frac{4}{3}$ miles. How long will the road be when completed?

We can write the equation $\frac{3}{8}x = \frac{4}{3}$ to represent the situation and solve the equation.

The completed road will be $3\frac{5}{9}$ or about 3.6 miles long.

$$\frac{3}{8}x = \frac{4}{3}$$

$$x = \frac{\frac{4}{3}}{\frac{3}{8}}$$

$$x = \frac{4}{3} \cdot \frac{8}{3}$$

$$x = \frac{32}{9} = 3\frac{5}{9}$$

Unit 6 Lesson 5 Cumulative Practice Problems

1. Select **all** the expressions that equal $\frac{3.15}{0.45}$.

    A. $(3.15) \cdot (0.45)$

    B. $(3.15) \div (0.45)$

    C. $(3.15) \cdot \frac{1}{0.45}$

    D. $(3.15) \div \frac{45}{100}$

    E. $(3.15) \cdot \frac{100}{45}$

    F. $\frac{0.45}{3.15}$

2. Which expressions are solutions to the equation $\frac{3}{4}x = 15$? Select **all** that apply.

    A. $\frac{15}{\frac{3}{4}}$

    B. $\frac{15}{\frac{4}{3}}$

    C. $\frac{4}{3} \cdot 15$

    D. $\frac{3}{4} \cdot 15$

    E. $15 \div \frac{3}{4}$

3. Solve each equation.

    $4a = 32$ $4 = 32b$ $10c = 26$ $26 = 100d$

4. For each equation, write a story problem represented by the equation. For each equation, state what quantity x represents. If you get stuck, consider drawing a diagram.

    a. $\frac{3}{4} + x = 2$

    b. $1.5x = 6$

5. Write as many mathematical expressions or equations as you can about the image. Include a fraction, a decimal number, or a percentage in each.

FUNDRAISER

OUR GOAL:

$250,000

$250,000

$200,000

$150,000

$100,000

$50,000

(From Unit 3, Lesson 13.)

6. In a lilac paint mixture, 40% of the mixture is white paint, 20% is blue, and the rest is red. There are 4 cups of blue paint used in a batch of lilac paint.

    a. How many cups of white paint are used?

    b. How many cups of red paint are used?

    c. How many cups of lilac paint will this batch yield?

    If you get stuck, consider using a tape diagram.

    (From Unit 3, Lesson 12.)

7. Triangle P has a base of 12 inches and a corresponding height of 8 inches. Triangle Q has a base of 15 inches and a corresponding height of 6.5 inches. Which triangle has a greater area? Show your reasoning.

    (From Unit 1, Lesson 9.)

Lesson 6: Write Expressions Where Letters Stand for Numbers

Let's use expressions with variables to describe situations.

6.1: Algebra Talk: When x is 6

If x is 6, what is:

$x + 4$

$7 - x$

x^2

$\frac{1}{3}x$

6.2: Lemonade Sales and Heights

1. Lin set up a lemonade stand. She sells the lemonade for $0.50 per cup.

    a. Complete the table to show how much money she would collect if she sold each number of cups.

lemonade sold (number of cups)	12	183	c
money collected (dollars)			

    b. How many cups did she sell if she collected $127.50? Be prepared to explain your reasoning.

2. Elena is 59 inches tall. Some other people are taller than Elena.

a. Complete the table to show the height of each person.

person	Andre	Lin	Noah
how much taller than Elena (inches)	4	$6\frac{1}{2}$	d
person's height (inches)			

b. If Noah is $64\frac{3}{4}$ inches tall, how much taller is he than Elena?

6.3: Building Expressions

1. Clare is 5 years older than her cousin.

a. How old would Clare be if her cousin is:

10 years old?

2 years old?

x years old?

b. Clare is 12 years old. How old is Clare's cousin?

2. Diego has 3 times as many comic books as Han.

    a. How many comic books does Diego have if Han has:

      6 comic books?

      n books?

    b. Diego has 27 comic books. How many comic books does Han have?

3. Two fifths of the vegetables in Priya's garden are tomatoes.

    a. How many tomatoes are there if Priya's garden has:

      20 vegetables?

      x vegetables?

    b. Priya's garden has 6 tomatoes. How many total vegetables are there?

4. A school paid $31.25 for each calculator.

    a. If the school bought x calculators, how much did they pay?

    b. The school spent $500 on calculators. How many did the school buy?

Are you ready for more?

Kiran, Mai, Jada, and Tyler went to their school carnival. They all won chips that they could exchange for prizes. Kiran won $\frac{2}{3}$ as many chips as Jada. Mai won 4 times as many chips as Kiran. Tyler won half as many chips as Mai.

1. Write an expression for the number of chips Tyler won. You should only use one variable: J, which stands for the number of chips Jada won.

2. If Jada won 42 chips, how many chips did Tyler, Kiran, and Mai each win?

Lesson 6 Summary

Suppose you share a birthday with a neighbor, but she is 3 years older than you. When you were 1, she was 4. When you were 9, she was 12. When you are 42, she will be 45.

If we let a represent your age at any time, your neighbor's age can be expressed $a + 3$.

your age	1	9	42	a
neighbor's age	4	12	45	$a + 3$

We often use a letter such as x or a as a placeholder for a number in expressions. These are called *variables* (just like the letters we used in equations, previously). Variables make it possible to write expressions that represent a calculation even when we don't know all the numbers in the calculation.

How old will you be when your neighbor is 32? Since your neighbor's age is calculated with the expression $a + 3$, we can write the equation $a + 3 = 32$. When your neighbor is 32 you will be 29, because $a + 3 = 32$ is true when a is 29.

Unit 6 Lesson 6 Cumulative Practice Problems

1. Instructions for a craft project say that the length of a piece of red ribbon should be 7 inches less than the length of a piece of blue ribbon.

    a. How long is the red ribbon if the length of the blue ribbon is:

    10 inches?   27 inches? x inches?

    b. How long is the blue ribbon if the red ribbon is 12 inches?

2. Tyler has 3 times as many books as Mai.

    a. How many books does Mai have if Tyler has:

    15 books?   21 books? x books?

    b. Tyler has 18 books. How many books does Mai have?

3. A bottle holds 24 ounces of water. It has x ounces of water in it.

    a. What does $24 - x$ represent in this situation?

    b. Write a question about this situation that has $24 - x$ for the answer.

4. Write an equation represented by this tape diagram using each of these operations.
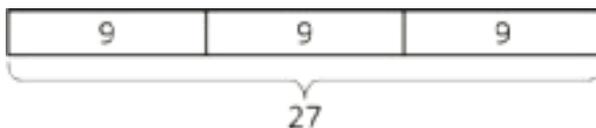
    a. addition

    b. subtraction

    c. multiplication

    d. division

(From Unit 6, Lesson 1.)

5. Select **all** the equations that describe each situation and then find the solution.

    a. Han's house is 450 meters from school. Lin's house is 135 meters closer to school. How far is Lin's house from school?

        ■ $z = 450 + 135$

        ■ $z = 450 - 135$

        ■ $z - 135 = 450$

        ■ $z + 135 = 450$

    b. Tyler's playlist has 36 songs. Noah's playlist has one quarter as many songs as Tyler's playlist. How many songs are on Noah's playlist?

        ■ $w = 4 \cdot 36$

        ■ $w = 36 \div 4$

        ■ $4w = 36$

        ■ $\frac{w}{4} = 36$

(From Unit 6, Lesson 4.)

6. You had $50. You spent 10% of the money on clothes, 20% on games, and the rest on books. How much money was spent on books?

(From Unit 3, Lesson 12.)

7. A trash bin has a capacity of 50 gallons. What percentage of its capacity is each amount? Show your reasoning.

    a. 5 gallons

    b. 30 gallons

    c. 45 gallons

    d. 100 gallons

(From Unit 3, Lesson 14.)

Lesson 7: Revisit Percentages

Let's use equations to find percentages.

7.1: Number Talk: Percentages

Solve each problem mentally.

1. Bottle A contains 4 ounces of water, which is 25% of the amount of water in Bottle B. How much water is there in Bottle B?

2. Bottle C contains 150% of the water in Bottle B. How much water is there in Bottle C?

3. Bottle D contains 12 ounces of water. What percentage of the amount of water in Bottle B is this?

7.2: Representing a Percentage Problem with an Equation

1. Answer each question and show your reasoning.

   a. Is 60% of 400 equal to 87?

   b. Is 60% of 200 equal to 87?

   c. Is 60% of 120 equal to 87?

2. 60% of x is equal to 87. Write an equation that expresses the relationship between 60%, x, and 87. Solve your equation.

3. Write an equation to help you find the value of each variable. Solve the equation.

60% of c is 43.2.   38% of e is 190.

7.3: Puppies Grow Up, Revisited

1. Puppy A weighs 8 pounds, which is about 25% of its adult weight. What will be the adult weight of Puppy A?

2. Puppy B weighs 8 pounds, which is about 75% of its adult weight. What will be the adult weight of Puppy B?

3. If you haven't already, write an equation for each situation. Then, show how you could find the adult weight of each puppy by solving the equation.

Are you ready for more?

Diego wants to paint his room purple. He bought one gallon of purple paint that is 30% red paint and 70% blue paint. Diego wants to add more blue to the mix so that the paint mixture is 20% red, 80% blue.

1. How much blue paint should Diego add? Test the following possibilities: 0.2 gallons, 0.3 gallons, 0.4 gallons, 0.5 gallons.

2. Write an equation in which x represents the amount of paint Diego should add.

3. Check that the amount of paint Diego should add is a solution to your equation.

Lesson 7 Summary

If we know that 455 students are in school today and that number represents 70% attendance, we can write an equation to figure out how many students go to the school.

The number of students in school today is known in two different ways: as 70% of the students in the school, and also as 455. If s represents the total number of students who go to the school, then 70% of s, or $\frac{70}{100}s$, represents the number of students that are in school today, which is 455.

We can write and solve the equation:

$$\frac{70}{100}s = 455$$
$$s = 455 \div \frac{70}{100}$$
$$s = 455 \cdot \frac{100}{70}$$
$$s = 650$$

There are 650 students in the school.

In general, equations can help us solve problems in which one amount is a percentage of another amount.

Unit 6 Lesson 7 Cumulative Practice Problems

1. A crew has paved $\frac{3}{4}$ of a mile of road. If they have completed 50% of the work, how long is the road they are paving?

2. 40% of x is 35.

   a. Write an equation that shows the relationship of 40%, x, and 35.

   b. Use your equation to find x. Show your reasoning.

3. Priya has completed 9 exam questions. This is 60% of the questions on the exam.

   a. Write an equation representing this situation. Explain the meaning of any variables you use.

   b. How many questions are on the exam? Show your reasoning.

4. Answer each question. Show your reasoning.

   20% of a is 11. What is a?   75% of b is 12. What is b?

   80% of c is 20. What is c?   200% of d is 18. What is d?

5. For the equation $2n - 3 = 7$

    a. What is the variable?

    b. What is the coefficient of the variable?

    c. Which of these is the solution to the equation? 2, 3, 5, 7, n

(From Unit 6, Lesson 2.)

6. Which of these is a solution to the equation $\frac{1}{8} = \frac{2}{5} \cdot x$?

    A. $\frac{2}{40}$

    B. $\frac{5}{16}$

    C. $\frac{11}{40}$

    D. $\frac{17}{40}$

(From Unit 6, Lesson 2.)

7. Find the quotients.

    a. $0.009 \div 0.001$

    b. $0.009 \div 0.002$

    c. $0.0045 \div 0.001$

    d. $0.0045 \div 0.002$

(From Unit 5, Lesson 13.)

Lesson 8: Equal and Equivalent

Let's use diagrams to figure out which expressions are equivalent and which are just sometimes equal.

8.1: Algebra Talk: Solving Equations by Seeing Structure

Find a solution to each equation mentally.

$3 + x = 8$

$10 = 12 - x$

$x^2 = 49$

$\frac{1}{3}x = 6$

8.2: Using Diagrams to Show That Expressions are Equivalent

Here is a diagram of $x + 2$ and $3x$ when x is 4. Notice that the two diagrams are lined up on their left sides.
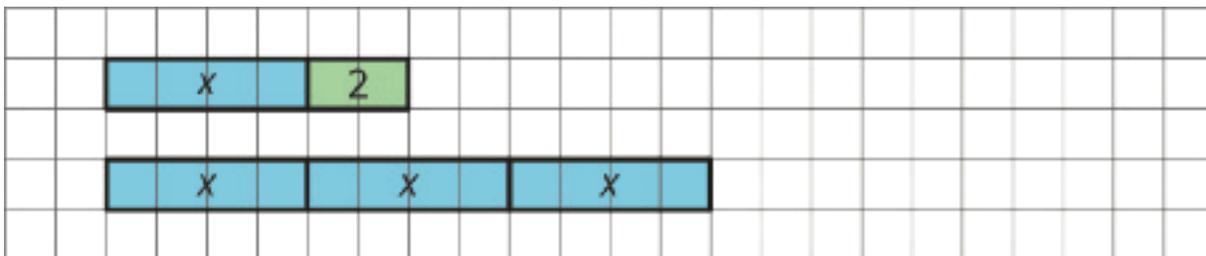
In each of your drawings below, line up the diagrams on one side.

1. Draw a diagram of $x + 2$, and a separate diagram of $3x$, when x is 3.
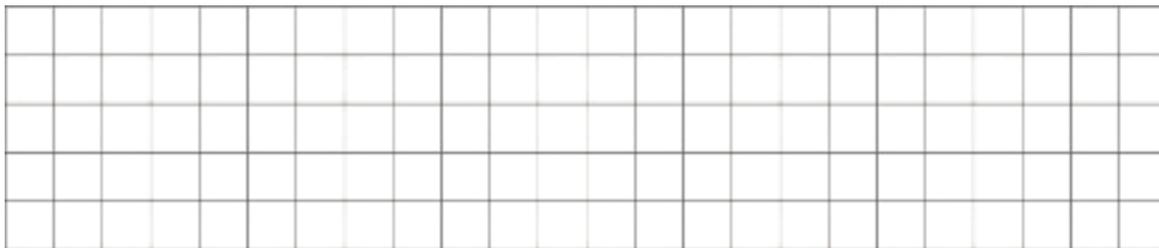
2. Draw a diagram of $x + 2$, and a separate diagram of $3x$, when x is 2.
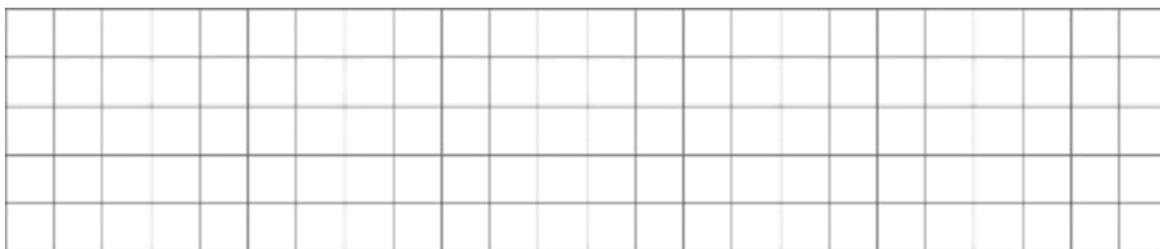
3. Draw a diagram of $x + 2$, and a separate diagram of $3x$, when x is 1.
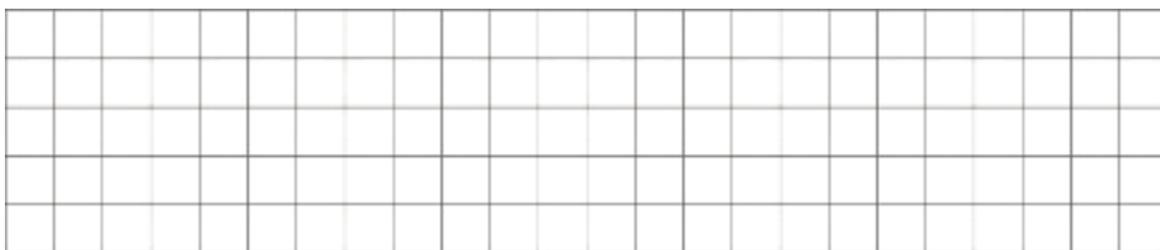
4. Draw a diagram of $x + 2$, and a separate diagram of $3x$, when x is 0.
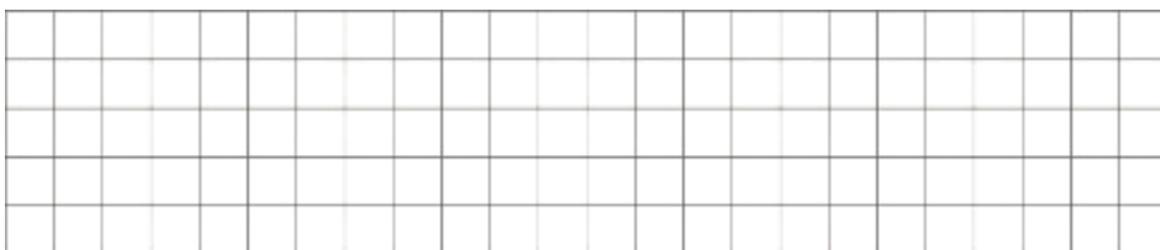
5. When are $x + 2$ and $3x$ equal? When are they not equal? Use your diagrams to explain.

6. Draw a diagram of $x + 3$, and a separate diagram of $3 + x$.

7. When are $x + 3$ and $3 + x$ equal? When are they not equal? Use your diagrams to explain.

8.3: Identifying Equivalent Expressions

Here is a list of expressions. Find any pairs of expressions that are equivalent. If you get stuck, try reasoning with diagrams.

$a + 3$ $\qquad\qquad$ $a \div \frac{1}{3}$ $\qquad\qquad$ $\frac{1}{3}a$ $\qquad\qquad$ $\frac{a}{3}$ $\qquad\qquad$ a

$a + a + a$ $\qquad\qquad$ $a \cdot 3$ $\qquad\qquad$ $3a$ $\qquad\qquad$ $1a$ $\qquad\qquad$ $3 + a$

Are you ready for more?

Below are four questions about equivalent expressions. For each one:

- Decide whether you think the expressions are equivalent.
- Test your guess by choosing numbers for x (and y, if needed).

1. Are $\dfrac{x \cdot x \cdot x \cdot x}{x}$ and $x \cdot x \cdot x$ equivalent expressions?

2. Are $\dfrac{x + x + x + x}{x}$ and $x + x + x$ equivalent expressions?

3. Are $2(x + y)$ and $2x + 2y$ equivalent expressions?

4. Are $2xy$ and $2x \cdot 2y$ equivalent expressions?

Lesson 8 Summary

We can use diagrams showing lengths of rectangles to see when expressions are equal. For example, the expressions $x + 9$ and $4x$ are equal when x is 3, but are not equal for other values of x.

Sometimes two expressions are equal for only one particular value of their variable. Other times, they seem to be equal no matter what the value of the variable.

Expressions that are always equal for the same value of their variable are called **equivalent expressions**. However, it would be impossible to test every possible value of the variable. How can we know for sure that expressions are equivalent? We use the meaning of operations and properties of operations to know that expressions are equivalent. Here are some examples:

- $x + 3$ is equivalent to $3 + x$ because of the commutative property of addition.
- $4 \cdot y$ is equivalent to $y \cdot 4$ because of the commutative property of multiplication.
- $a + a + a + a + a$ is equivalent to $5 \cdot a$ because adding 5 copies of something is the same as multiplying it by 5.
- $b \div 3$ is equivalent to $b \cdot \frac{1}{3}$ because dividing by a number is the same as multiplying by its reciprocal.

In the coming lessons, we will see how another property, the distributive property, can show that expressions are equivalent.

Unit 6 Lesson 8 Cumulative Practice Problems

1. a. Draw a diagram of $x + 3$ and a diagram of $2x$ when x is 1.
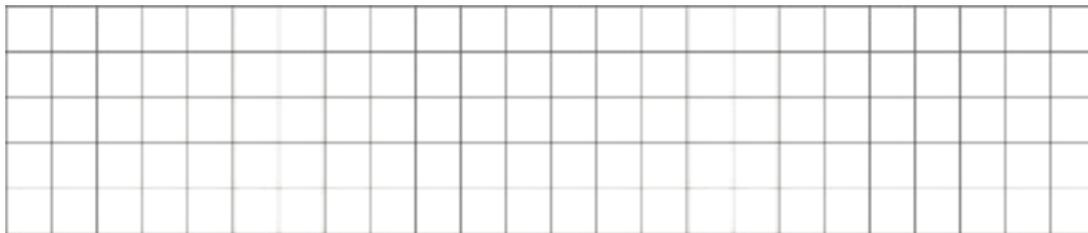
    b. Draw a diagram of $x + 3$ and of $2x$ when x is 2.
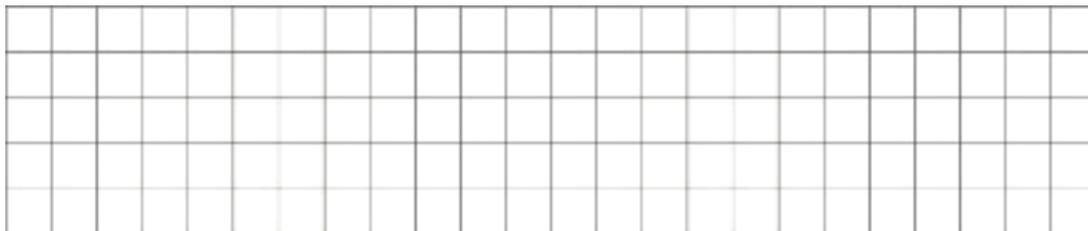
    c. Draw a diagram of $x + 3$ and of $2x$ when x is 3.
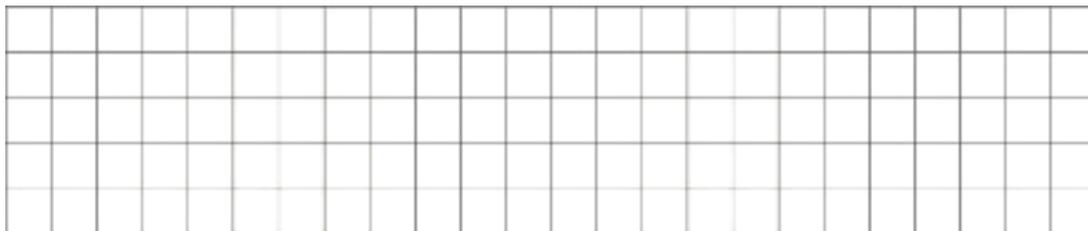
    d. Draw a diagram of $x + 3$ and of $2x$ when x is 4.
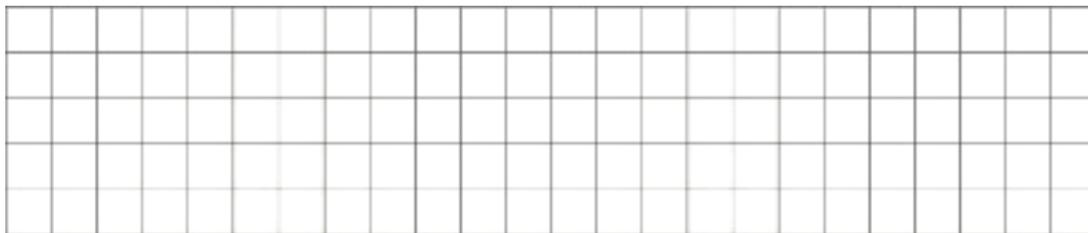
    e. When are $x + 3$ and $2x$ equal? When are they not equal? Use your diagrams to explain.

2.   a. Do $4x$ and $15 + x$ have the same value when x is 5?

      b. Are $4x$ and $15 + x$ equivalent expressions? Explain your reasoning.

3.   a. Check that $2b + b$ and $3b$ have the same value when b is 1, 2, and 3.

      b. Do $2b + b$ and $3b$ have the same value for all values of b? Explain your reasoning.

      c. Are $2b + b$ and $3b$ equivalent expressions?

4. 80% of x is equal to 100.

    a. Write an equation that shows the relationship of 80%, x, and 100.

    b. Use your equation to find x.

    (From Unit 6, Lesson 7.)

5. For each story problem, write an equation to represent the problem and then solve the equation. Be sure to explain the meaning of any variables you use.

  a. Jada's dog was $5\frac{1}{2}$ inches tall when it was a puppy. Now her dog is $14\frac{1}{2}$ inches taller than that. How tall is Jada's dog now?

  b. Lin picked $9\frac{3}{4}$ pounds of apples, which was 3 times the weight of the apples Andre picked. How many pounds of apples did Andre pick?

(From Unit 6, Lesson 5.)

6. Find these products.

  a. $(2.3) \cdot (1.4)$

  b. $(1.72) \cdot (2.6)$

  c. $(18.2) \cdot (0.2)$

  d. $15 \cdot (1.2)$

(From Unit 5, Lesson 8.)

7. Calculate $141.75 \div 2.5$ using a method of your choice. Show or explain your reasoning.

(From Unit 5, Lesson 13.)

Lesson 9: The Distributive Property, Part 1

Let's use the distributive property to make calculating easier.

9.1: Number Talk: Ways to Multiply

Find each product mentally.

$5 \cdot 102$

$5 \cdot 98$

$5 \cdot 999$

9.2: Ways to Represent Area of a Rectangle

1. Select **all** the expressions that represent the area of the large, outer rectangle in figure A. Explain your reasoning.

   - $6 + 3 + 2$
   - $6 \cdot 3 + 6 \cdot 2$
   - $6 \cdot 3 + 2$
   - $6 \cdot 5$
   - $6(3 + 2)$
   - $6 \cdot 3 \cdot 2$
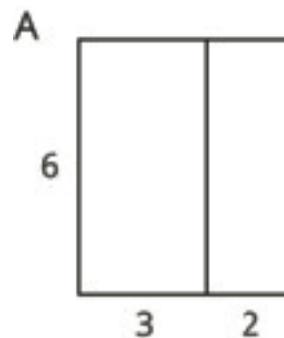
2. Select **all** the expressions that represent the area of the shaded rectangle on the left side of figure B. Explain your reasoning.

   - $4 \cdot 7 + 4 \cdot 2$
   - $4 \cdot 7 \cdot 2$
   - $4 \cdot 5$
   - $4 \cdot 7 - 4 \cdot 2$
   - $4(7 - 2)$
   - $4(7 + 2)$
   - $4 \cdot 2 - 4 \cdot 7$
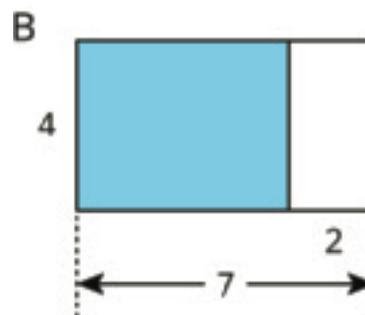
9.3: Distributive Practice

Complete the table. If you get stuck, skip an entry and come back to it, or consider drawing a diagram of two rectangles that share a side.

column 1	column 2	column 3	column 4	value
$5 \cdot 98$	$5(100 - 2)$	$5 \cdot 100 - 5 \cdot 2$	$500 - 10$	490
$33 \cdot 12$	$33(10 + 2)$			
		$3 \cdot 10 - 3 \cdot 4$	$30 - 12$	
	$100(0.04 + 0.06)$			
		$8 \cdot \frac{1}{2} + 8 \cdot \frac{1}{4}$		
			$9 + 12$	
			$24 - 16$	

Are you ready for more?

1. Use the distributive property to write two expressions that equal 360. (There are many correct ways to do this.)

2. Is it possible to write an expression like $a(b + c)$ that equals 360 where a is a fraction? Either write such an expression, or explain why it is impossible.

3. Is it possible to write an expression like $a(b - c)$ that equals 360? Either write such an expression, or explain why it is impossible.

4. How many ways do you think there are to make 360 using the distributive property?

Lesson 9 Summary

A **term** is a single number or variable, or variables and numbers multiplied together. Some examples of terms are 10, $8x$, ab, and $7yz$.

When we need to do mental calculations, we often come up with ways to make the calculation easier to do mentally.

Suppose we are grocery shopping and need to know how much it will cost to buy 5 cans of beans at 79 cents a can. We may calculate mentally in this way:

$$5 \cdot 79$$
$$5 \cdot 70 + 5 \cdot 9$$
$$350 + 45$$
$$395$$

In general, when we multiply two terms (or factors), we can break up one of the factors into parts, multiply each part by the other factor, and then add the products. The result will be the same as the product of the two original factors. When we break up one of the factors and multiply the parts we are using the distributive property.

The distributive property also works with subtraction. Here is another way to find $5 \cdot 79$:

$$5 \cdot 79$$
$$5 \cdot (80 - 1)$$
$$400 - 5$$
$$395$$

Unit 6 Lesson 9 Cumulative Practice Problems

1. Select **all** the expressions that represent the area of the large, outer rectangle.
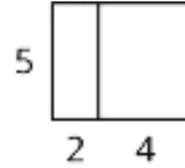
   A. $5(2 + 4)$

   B. $5 \cdot 2 + 4$

   C. $5 \cdot 2 + 5 \cdot 4$

   D. $5 \cdot 2 \cdot 4$

   E. $5 + 2 + 4$

   F. $5 \cdot 6$

2. Draw and label diagrams that show these two methods for calculating $19 \cdot 50$.

   ○ First find $10 \cdot 50$ and then add $9 \cdot 50$.

   ○ First find $20 \cdot 50$ and then take away 50.

3. Complete each calculation using the distributive property.

$$98 \cdot 24$$
$$(100 - 2) \cdot 24$$
$$\ldots$$

$$21 \cdot 15$$
$$(20 + 1) \cdot 15$$
$$\ldots$$

$$0.51 \cdot 40$$
$$(0.5 + 0.01) \cdot 40$$
$$\ldots$$

4. A group of 8 friends go to the movies. A bag of popcorn costs $2.99. How much will it cost to get one bag of popcorn for each friend? Explain how you can calculate this amount mentally.

5. a. On graph paper, draw diagrams of $a + a + a + a$ and $4a$ when a is 1, 2, and 3. What do you notice?

b. Do $a + a + a + a$ and $4a$ have the same value for any value of a? Explain how you know.

(From Unit 6, Lesson 8.)

6. 120% of x is equal to 78.

a. Write an equation that shows the relationship of 120%, x, and 78.

b. Use your equation to find x. Show your reasoning.

(From Unit 6, Lesson 7.)

7. Kiran's aunt is 17 years older than Kiran.

a. How old will Kiran's aunt be when Kiran is:

15 years old?   30 years old? x years old?

b. How old will Kiran be when his aunt is 60 years old?

(From Unit 6, Lesson 6.)

Lesson 10: The Distributive Property, Part 2

Let's use rectangles to understand the distributive property with variables.

10.1: Possible Areas

1. A rectangle has a width of 4 units and a length of m units. Write an expression for the area of this rectangle.

2. What is the area of the rectangle if m is:

   3 units?   2.2 units? $\frac{1}{5}$ unit?

3. Could the area of this rectangle be 11 square units? Why or why not?

10.2: Partitioned Rectangles When Lengths are Unknown

1. Here are two rectangles. The length and width of one rectangle are 8 and 5. The width of the other rectangle is 5, but its length is unknown so we labeled it x.

   Write an expression for the sum of the areas of the two rectangles.

2. The two rectangles can be composed into one larger rectangle as shown.

   What are the width and length of the new, large rectangle?

3. Write an expression for the total area of the large rectangle as the product of its width and its length.

10.3: Areas of Partitioned Rectangles

For each rectangle, write expressions for the length and width and two expressions for the total area. Record them in the table. Check your expressions in each row with your group and discuss any disagreements.

A

a 5

3

B

6 x

$\frac{1}{3}$

C

1 1 1

r

D

p p p p

6

E

6 8

m

F

$3x$ 8

5
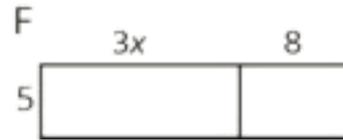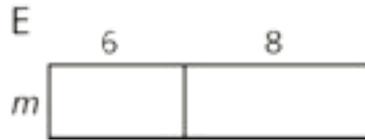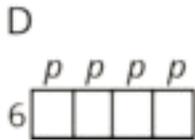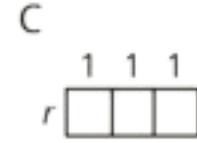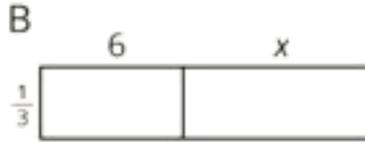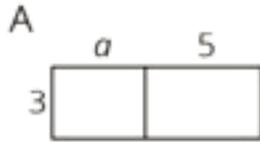
rectangle	width	length	area as a product of width times length	area as a sum of the areas of the smaller rectangles
A				
B				
C				
D				
E				
F				

Are you ready for more?

Here is an area diagram of a rectangle.

1. Find the lengths w, x, y, and z, and the area A. All values are whole numbers.

2. Can you find another set of lengths that will work? How many possibilities are there?

Lesson 10 Summary

Here is a rectangle composed of two smaller rectangles A and B.
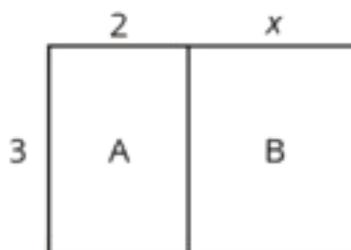
Based on the drawing, we can make several observations about the area of the rectangle:

- One side length of the large rectangle is 3 and the other is $2 + x$, so its area is $3(2 + x)$.

- Since the large rectangle can be decomposed into two smaller rectangles, A and B, with no overlap, the area of the large rectangle is also the sum of the areas of rectangles A and B: $3(2) + 3(x)$ or $6 + 3x$.

- Since both expressions represent the area of the large rectangle, they are equivalent to each other. $3(2 + x)$ is equivalent to $6 + 3x$.

We can see that multiplying 3 by the sum $2 + x$ is equivalent to multiplying 3 by 2 and then 3 by x and adding the two products. This relationship is an example of the *distributive property*.

$$3(2 + x) = 3 \cdot 2 + 3 \cdot x$$

Unit 6 Lesson 10 Cumulative Practice Problems

1. Here is a rectangle.
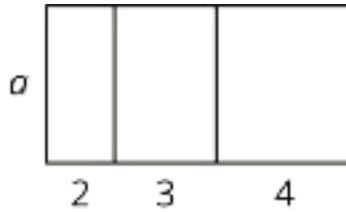
a. Explain why the area of the large rectangle is $2a + 3a + 4a$.

b. Explain why the area of the large rectangle is $(2 + 3 + 4)a$.

2. Is the area of the shaded rectangle $6(2 - m)$ or $6(m - 2)$?

Explain how you know.
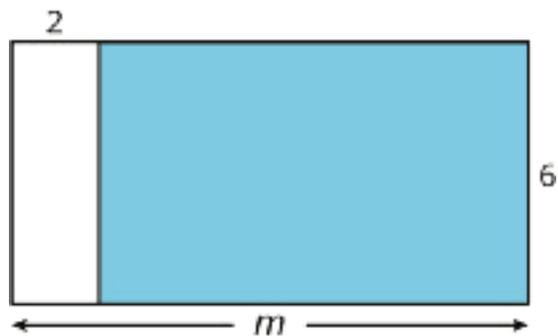
3. Choose the expressions that do *not* represent the total area of the rectangle. Select **all** that apply.
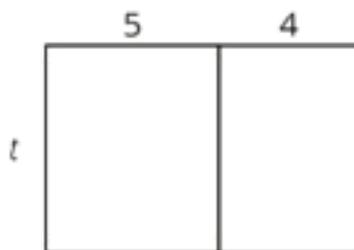
A. $5t + 4t$

B. $t + 5 + 4$

C. $9t$

D. $4 \cdot 5 \cdot t$

E. $t(5 + 4)$

4. Evaluate each expression mentally.

a. $35 \cdot 91 - 35 \cdot 89$

b. $22 \cdot 87 + 22 \cdot 13$

c. $\frac{9}{11} \cdot \frac{7}{10} - \frac{9}{11} \cdot \frac{3}{10}$

(From Unit 6, Lesson 9.)

5. Select **all** the expressions that are equivalent to $4b$.

A. $b + b + b + b$

B. $b + 4$

C. $2b + 2b$

D. $b \cdot b \cdot b \cdot b$

E. $b \div \frac{1}{4}$

(From Unit 6, Lesson 8.)

6. Solve each equation. Show your reasoning.

$$111 = 14a \qquad\qquad 13.65 = b + 4.88 \qquad\qquad c + \tfrac{1}{3} = 5\tfrac{1}{8}$$

$$\tfrac{2}{5}d = \tfrac{17}{4} \qquad\qquad 5.16 = 4e$$

(From Unit 6, Lesson 4.)

7. Andre ran $5\tfrac{1}{2}$ laps of a track in 8 minutes at a constant speed. It took Andre x minutes to run each lap. Select **all** the equations that represent this situation.

   A. $\left(5\tfrac{1}{2}\right) x = 8$

   B. $5\tfrac{1}{2} + x = 8$

   C. $5\tfrac{1}{2} - x = 8$

   D. $5\tfrac{1}{2} \div x = 8$

   E. $x = 8 \div \left(5\tfrac{1}{2}\right)$

   F. $x = \left(5\tfrac{1}{2}\right) \div 8$

(From Unit 6, Lesson 2.)

Lesson 11: The Distributive Property, Part 3

Let's practice writing equivalent expressions by using the distributive property.

11.1: The Shaded Region

A rectangle with dimensions 6 cm and w cm is partitioned into two smaller rectangles.

Explain why each of these expressions represents the area, in cm^2, of the shaded region.
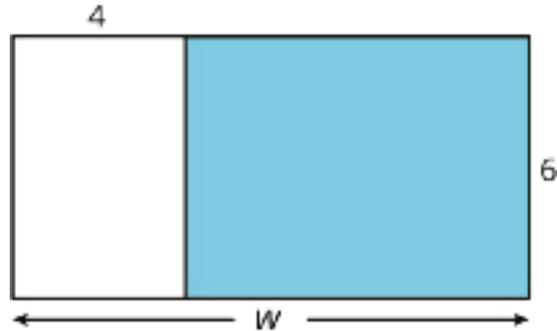
- $6w - 24$

- $6(w - 4)$

11.2: Matching to Practice Distributive Property

Match each expression in column 1 to an equivalent expression in column 2. If you get stuck, consider drawing a diagram.

Column 1

1. $a(1 + 2 + 3)$

2. $2(12 - 4)$

3. $12a + 3b$

4. $\frac{2}{3}(15a - 18)$

5. $6a + 10b$

6. $0.4(5 - 2.5a)$

7. $2a + 3a$

Column 2

- $3(4a + b)$

- $12 \cdot 2 - 4 \cdot 2$

- $2(3a + 5b)$

- $(2 + 3)a$

- $a + 2a + 3a$

- $10a - 12$

- $2 - a$

11.3: Writing Equivalent Expressions Using the Distributive Property

The distributive property can be used to write equivalent expressions. In each row, use the distributive property to write an equivalent expression. If you get stuck, consider drawing a diagram.

product	sum or difference
$3(3 + x)$	
	$4x - 20$
$(9 - 5)x$	
	$4x + 7x$
$3(2x + 1)$	
	$10x - 5$
	$x + 2x + 3x$
$\frac{1}{2}(x - 6)$	
$y(3x + 4z)$	
	$2xyz - 3yz + 4xz$

Are you ready for more?

This rectangle has been cut up into squares of varying sizes. Both small squares have side length 1 unit. The square in the middle has side length x units.
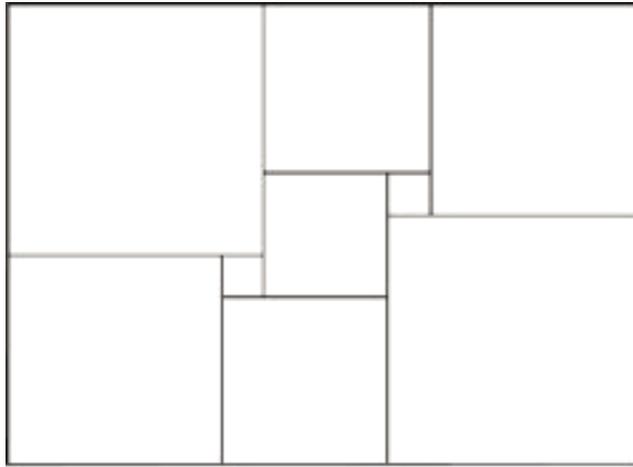
1. Suppose that x is 3. Find the area of each square in the diagram. Then find the area of the large rectangle.

2. Find the side lengths of the large rectangle assuming that x is 3. Find the area of the large rectangle by multiplying the length times the width. Check that this is the same area you found before.

3. Now suppose that we do not know the value of x. Write an expression for the side lengths of the large rectangle that involves x.

Lesson 11 Summary

The distributive property can be used to write a sum as a product, or write a product as a sum. You can always draw a partitioned rectangle to help reason about it, but with enough practice, you should be able to apply the distributive property without making a drawing.

Here are some examples of expressions that are equivalent due to the distributive property.

$$9 + 18 = 9(1 + 2)$$
$$2(3x + 4) = 6x + 8$$
$$2n + 3n + n = n(2 + 3 + 1)$$
$$11b - 99a = 11(b - 9a)$$
$$k(c + d - e) = kc + kd - ke$$

Unit 6 Lesson 11 Cumulative Practice Problems

1. For each expression, use the distributive property to write an equivalent expression.

    a. $4(x + 2)$

    b. $(6 + 8) \cdot x$

    c. $4(2x + 3)$

    d. $6(x + y + z)$

2. Priya rewrites the expression $8y - 24$ as $8(y - 3)$. Han rewrites $8y - 24$ as $2(4y - 12)$. Are Priya's and Han's expressions each equivalent to $8y - 24$? Explain your reasoning.

3. Select **all** the expressions that are equivalent to $16x + 36$.

    A. $16(x + 20)$

    B. $x(16 + 36)$

    C. $4(4x + 9)$

    D. $2(8x + 18)$

    E. $2(8x + 36)$

4. The area of a rectangle is $30 + 12x$. List at least 3 possibilities for the length and width of the rectangle.

5. Select **all** the expressions that are equivalent to $\frac{1}{2}z$.

   A. $z + z$

   B. $z \div 2$

   C. $z \cdot z$

   D. $\frac{1}{4}z + \frac{1}{4}z$

   E. $2z$

6.   a. What is the perimeter of a square with side length:

      3 cm?   7 cm? s cm?

      b. If the perimeter of a square is 360 cm, what is its side length?

      c. What is the area of a square with side length:

      3 cm?   7 cm? s cm?

      d. If the area of a square is 121 cm^2, what is its side length?

7. Solve each equation.

   $10 = 4a$ $5b = 17.5$ $1.036 = 10c$

   $0.6d = 1.8$ $15 = 0.1e$

Lesson 12: Meaning of Exponents

Let's see how exponents show repeated multiplication.

12.1: Notice and Wonder: Dots and Lines

What do you notice? What do you wonder?
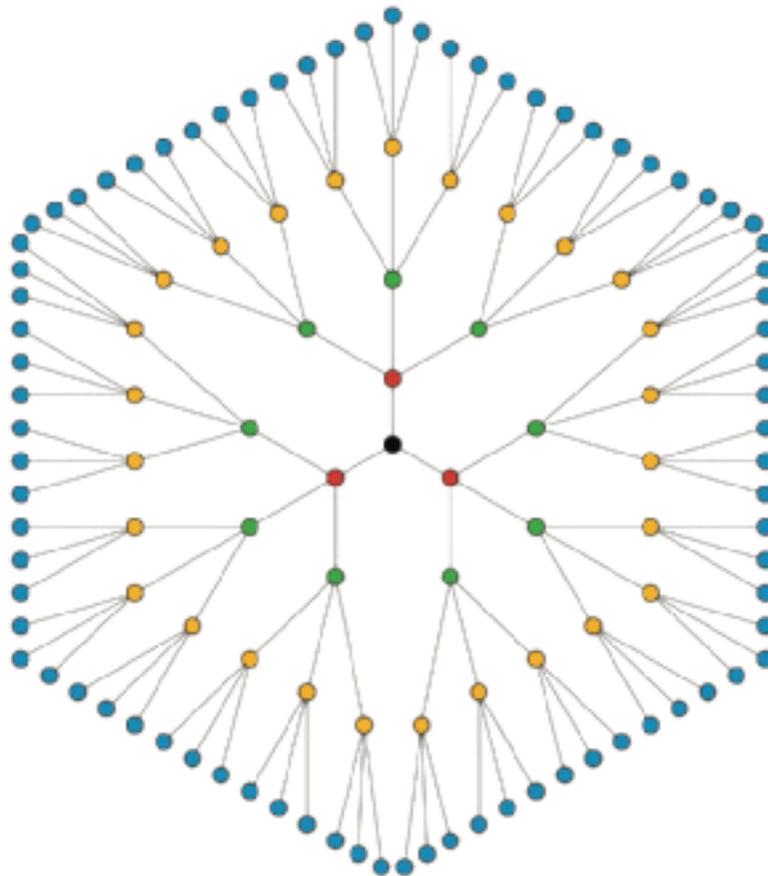
12.2: The Genie's Offer

You find a brass bottle that looks really old. When you rub some dirt off of the bottle, a genie appears! The genie offers you a reward. You must choose one:

$50,000 or a magical $1 coin.

The coin will turn into two coins on the first day. The two coins will turn into four coins on the second day. The four coins will double to 8 coins on the third day. The genie explains the doubling will continue for 28 days.

1. The number of coins on the third day will be $2 \cdot 2 \cdot 2$. Write an equivalent expression using exponents.

2. What do 2^5 and 2^6 represent in this situation? Evaluate 2^5 and 2^6 without a calculator. Pause for discussion.

3. How many days would it take for the number of magical coins to exceed $50,000?

4. Will the value of the magical coins exceed a million dollars within the 28 days? Explain or show your reasoning.

Are you ready for more?

A scientist is growing a colony of bacteria in a petri dish. She knows that the bacteria are growing and that the number of bacteria doubles every hour.

When she leaves the lab at 5 p.m., there are 100 bacteria in the dish. When she comes back the next morning at 9 a.m., the dish is completely full of bacteria. At what time was the dish half full?

12.3: Make 81

1. Here are some expressions. All but one of them equals 16. Find the one that is *not* equal to 16 and explain how you know.

$2^3 \cdot 2$ $\qquad\qquad$ 4^2 $\qquad\qquad$ $\frac{2^5}{2}$ $\qquad\qquad$ 8^2

2. Write three expressions containing exponents so that each expression equals 81.

Lesson 12 Summary

When we write an expression like 2^n, we call n the exponent.

If n is a positive whole number, it tells how many factors of 2 we should multiply to find the value of the expression. For example, $2^1 = 2$, and $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$.

There are different ways to say 2^5. We can say "two raised to the power of five" or "two to the fifth power" or just "two to the fifth."

Unit 6 Lesson 12 Cumulative Practice Problems

1. Select **all** the expressions that are equivalent to 64.

   A. 2^6

   B. 2^8

   C. 4^3

   D. 8^2

   E. 16^4

   F. 32^2

2. Select **all** the expressions that equal 3^4.

   A. 7

   B. 4^3

   C. 12

   D. 81

   E. 64

   F. 9^2

3. 4^5 is equal to 1,024. Evaluate each expression.

   a. 4^6

   b. 4^4

   c. $4^3 \cdot 4^2$

4. $6^3 = 216$. Using exponents, write three more expressions whose value is 216.

5. Find two different ways to rewrite $3xy + 6yz$ using the distributive property.

6. Solve each equation.

$a - 2.01 = 5.5$ $b + 2.01 = 5.5$

$10c = 13.71$ $100d = 13.71$

7. Which expressions represent the total area of the large rectangle? Select **all** that apply.

A. $6(m + n)$

B. $6n + m$

C. $6n + 6m$

D. $6mn$

E. $(n + m)6$

(From Unit 6, Lesson 10.)

8. Is each statement true or false? Explain your reasoning.

a. $\frac{45}{100} \cdot 72 = \frac{45}{72} \cdot 100$

b. 16% of 250 is equal to 250% of 16

(From Unit 3, Lesson 16.)

Lesson 13: Expressions with Exponents

Let's use the meaning of exponents to decide if equations are true.

13.1: Which One Doesn't Belong: Twos

Which one doesn't belong?

$2 \cdot 2 \cdot 2 \cdot 2$

2^4

16

$4 \cdot 2$

13.2: Is the Equation True?

Decide whether each equation is true or false, and explain how you know.

1. $2^4 = 2 \cdot 4$

2. $3 + 3 + 3 + 3 + 3 = 3^5$

3. $5^3 = 5 \cdot 5 \cdot 5$

4. $2^3 = 3^2$

5. $16^1 = 8^2$

6. $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = 4 \cdot \frac{1}{2}$

7. $\left(\frac{1}{2}\right)^4 = \frac{1}{8}$

8. $8^2 = 4^3$

13.3: What's Your Reason?

In each list, find expressions that are equivalent to each other and explain to your partner why they are equivalent. Your partner listens to your explanation. If you disagree, explain your reasoning until you agree. Switch roles for each list. (There may be more than two equivalent expressions in each list.)

1. a. $5 \cdot 5$

   b. 2^5

   c. 5^2

   d. $2 \cdot 5$

2. a. 4^3

   b. 3^4

   c. $4 \cdot 4 \cdot 4$

   d. $4 + 4 + 4$

3. a. $6 + 6 + 6$

   b. 6^3

   c. 3^6

   d. $3 \cdot 6$

4. a. 11^5

   b. $11 \cdot 11 \cdot 11 \cdot 11 \cdot 11$

   c. $11 \cdot 5$

   d. 5^{11}

5. a. $\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$

    b. $\left(\frac{1}{5}\right)^3$

    c. $\frac{1}{15}$

    d. $\frac{1}{125}$

6. a. $\left(\frac{5}{3}\right)^2$

    b. $\left(\frac{3}{5}\right)^2$

    c. $\frac{10}{6}$

    d. $\frac{25}{9}$

Are you ready for more?

What is the last digit of $3^{1,000}$? Show or explain your reasoning.

Lesson 13 Summary

When working with exponents, the bases don't have to always be whole numbers. They can also be other kinds of numbers, like fractions, decimals, and even variables. For example, we can use exponents in each of the following ways:

$$\left(\frac{2}{3}\right)^4 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}$$

$$(1.7)^3 = (1.7) \cdot (1.7) \cdot (1.7)$$

$$x^5 = x \cdot x \cdot x \cdot x \cdot x$$

Unit 6 Lesson 13 Cumulative Practice Problems

1. Select **all** expressions that are equal to $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$.

    A. $3 \cdot 5$

    B. 3^5

    C. $3^4 \cdot 3$

    D. $5 \cdot 3$

    E. 5^3

2. Noah starts with 0 and then adds the number 5 four times. Diego starts with 1 and then multiplies by the number 5 four times. For each expression, decide whether it is equal to Noah's result, Diego's result, or neither.

    a. $4 \cdot 5$

    b. $4 + 5$

    c. 4^5

    d. 5^4

3. Decide whether each equation is true or false, and explain how you know.

    a. $9 \cdot 9 \cdot 3 = 3^5$

    b. $7 + 7 + 7 = 3 + 3 + 3 + 3 + 3 + 3 + 3$

    c. $\frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} = \frac{3}{7}$

    d. $4^1 = 4 \cdot 1$

    e. $6 + 6 + 6 = 6^3$

4. a. What is the area of a square with side lengths of $\frac{3}{5}$ units?

b. What is the side length of a square with area $\frac{1}{16}$ square units?

c. What is the volume of a cube with edge lengths of $\frac{2}{3}$ units?

d. What is the edge length of a cube with volume $\frac{27}{64}$ cubic units?

5. Select **all** the expressions that represent the area of the shaded rectangle.
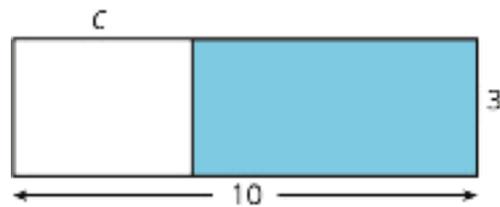
A. $3(10 - c)$

B. $3(c - 10)$

C. $10(c - 3)$

D. $10(3 - c)$

E. $30 - 3c$

F. $30 - 10c$

(From Unit 6, Lesson 10.)

6. A ticket at a movie theater costs $8.50. One night, the theater had $29,886 in ticket sales.

   a. Estimate about how many tickets the theater sold. Explain your reasoning.

   b. How many tickets did the theater sell? Explain your reasoning.

(From Unit 5, Lesson 13.)

7. A fence is being built around a rectangular garden that is $8\frac{1}{2}$ feet by $6\frac{1}{3}$ feet. Fencing comes in panels. Each panel is $\frac{2}{3}$ of a foot wide. How many panels are needed? Explain or show your reasoning.

(From Unit 4, Lesson 12.)

Lesson 14: Evaluating Expressions with Exponents

Let's find the values of expressions with exponents.

14.1: Revisiting the Cube

Based on the given information, what other measurements of the square and cube could we find?
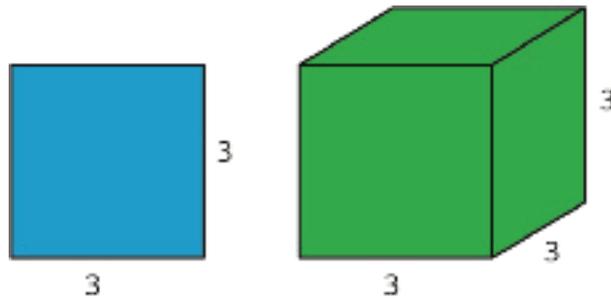
14.2: Calculating Surface Area

A cube has side length 10 inches. Jada says the surface area of the cube is 600 in^2, and Noah says the surface area of the cube is 3,600 in^2. Here is how each of them reasoned:

Jada's Method:

$6 \cdot 10^2$
$6 \cdot 100$
600

Noah's Method:

$6 \cdot 10^2$
60^2
$3,600$

Do you agree with either of them? Explain your reasoning.

14.3: Row Game: Expression Explosion

Evaluate the expressions in one of the columns. Your partner will work on the other column. Check with your partner after you finish each row. Your answers in each row should be the same. If your answers aren't the same, work together to find the error.

column A	column B
$5^2 + 4$	$2^2 + 25$
$2^4 \cdot 5$	$2^3 \cdot 10$
$3 \cdot 4^2$	$12 \cdot 2^2$
$20 + 2^3$	$1 + 3^3$
$9 \cdot 2^1$	$3 \cdot 6^1$
$\frac{1}{9} \cdot \left(\frac{1}{2}\right)^3$	$\frac{1}{8} \cdot \left(\frac{1}{3}\right)^2$

Are you ready for more?

1. Consider this equation: $\square^2 + \square^2 = \square^2$. An example of 3 different whole numbers that could go in the boxes are 3, 4, and 5, since $3^2 + 4^2 = 5^2$. (That is, $9 + 16 = 25$.)

   Can you find a different set of 3 whole numbers that make the equation true?

2. How many sets of 3 different whole numbers can you find?

3. Can you find a set of 3 different whole numbers that make this equation true?

$$\square^3 + \square^3 = \square^3$$

4. How about this one? $\square^4 + \square^4 = \square^4$

Once you have worked on this a little while, you can understand a problem that is famous in the history of math. (Alas, this space is too small to contain it.) If you are interested, consider doing some further research on *Fermat's Last Theorem*.

Lesson 14 Summary

Exponents give us a new way to describe operations with numbers, so we need to understand how exponents get along with the other operations we know.

When we write $6 \cdot 4^2$, we want to make sure everyone agrees about how to evaluate this. Otherwise some people might multiply first and others compute the exponent first, and different people would get different values for the same expression!

Earlier we saw situations in which $6 \cdot 4^2$ represented the surface area of a cube with side lengths 4 units. When computing the surface area, we evaluate 4^2 first (or find the area of one face of the cube first) and then multiply the result by 6. In many other expressions that use exponents, the part with an exponent is intended to be evaluated first.

To make everyone agree about the value of expressions like $6 \cdot 4^2$, the convention is to *evaluate the part of the expression with the exponent first*. Here are a couple of examples:

$$\begin{array}{cc}
6 \cdot 4^2 & 45 + 5^2 \\
6 \cdot 16 & 45 + 25 \\
96 & 70
\end{array}$$

If we want to communicate that 6 and 4 should be multiplied first and then squared, then we can use parentheses to group parts together:

$$\begin{array}{cc}
(6 \cdot 4)^2 & (45 + 5)^2 \\
24^2 & 50^2 \\
576 & 2{,}500
\end{array}$$

Unit 6 Lesson 14 Cumulative Practice Problems

1. Lin says, "I took the number 8, and then multiplied it by the square of 3." Select **all** the expressions that equal Lin's answer.

   A. $8 \cdot 3^2$

   B. $(8 \cdot 3)^2$

   C. $8 \cdot 2^3$

   D. $3^2 \cdot 8$

   E. 24^2

   F. 72

2. Evaluate each expression.

   a. $7 + 2^3$

   b. $9 \cdot 3^1$

   c. $20 - 2^4$

   d. $2 \cdot 6^2$

   e. $8 \cdot (\frac{1}{2})^2$

   f. $\frac{1}{3} \cdot 3^3$

   g. $(\frac{1}{5} \cdot 5)^5$

3. Andre says, "I multiplied 4 by 5, then cubed the result." Select **all** the expressions that equal Andre's answer.

   A. $4 \cdot 5^3$

   B. $(4 \cdot 5)^3$

   C. $(4 \cdot 5)^2$

   D. $5^3 \cdot 4$

   E. 20^3

   F. 500

   G. 8,000

4. Han has 10 cubes, each 5 inches on a side.

   a. Find the total volume of Han's cubes. Express your answer as an expression using an exponent.

   b. Find the total surface area of Han's cubes. Express your answer as an expression using an exponent.

5. Priya says that $\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{4}{3}$. Do you agree with Priya? Explain or show your reasoning.

(From Unit 6, Lesson 13.)

6. Answer each question. Show your reasoning.

   a. 125% of e is 30. What is e?

   b. 35% of f is 14. What is f?

   (From Unit 6, Lesson 7.)

7. Which expressions are solutions to the equation $2.4y = 13.75$? Select **all** that apply.

   A. $13.75 - 1.4$

   B. $13.75 \cdot 2.4$

   C. $13.75 \div 2.4$

   D. $\frac{13.75}{2.4}$

   E. $2.4 \div 13.75$

   (From Unit 6, Lesson 5.)

8. Jada explains how she finds $15 \cdot 23$:

   "I know that ten 23s is 230, so five 23s will be half of 230, which is 115. 15 is 10 plus 5, so $15 \cdot 23$ is 230 plus 115, which is 345."

   a. Do you agree with Jada? Explain.

   b. Draw a 15 by 23 rectangle. Partition the rectangle into two rectangles and label them to show Jada's reasoning.

   (From Unit 5, Lesson 7.)

Lesson 15: Equivalent Exponential Expressions

Let's investigate expressions with variables and exponents.

15.1: Up or Down?

Find the values of 3^x and $\left(\frac{1}{3}\right)^x$ for different values of x. What patterns do you notice?

x	3^x	$\left(\frac{1}{3}\right)^x$
1		
2		
3		
4		

15.2: What's the Value?

Evaluate each expression for the given value of x.

1. $3x^2$ when x is 10

2. $3x^2$ when x is $\frac{1}{9}$

3. $\frac{x^3}{4}$ when x is 4

4. $\frac{x^3}{4}$ when x is $\frac{1}{2}$

5. $9 + x^7$ when x is 1

6. $9 + x^7$ when x is $\frac{1}{2}$

15.3: Exponent Experimentation

Find a solution to each equation in the list. (Numbers in the list may be a solution to more than one equation, and not all numbers in the list will be used.)

1. $64 = x^2$

2. $64 = x^3$

3. $2^x = 32$

4. $x = \left(\frac{2}{5}\right)^3$

5. $\frac{16}{9} = x^2$

6. $2 \cdot 2^5 = 2^x$

7. $2x = 2^4$

8. $4^3 = 8^x$

List:

$\frac{8}{125}$ $\frac{6}{15}$ $\frac{5}{8}$ $\frac{8}{9}$ 1 $\frac{4}{3}$ 2   3   4   5   6   8

Are you ready for more?

This fractal is called a Sierpinski Tetrahedron. A tetrahedron is a polyhedron that has four faces. (The plural of tetrahedron is tetrahedra.)

The small tetrahedra form four medium-sized tetrahedra: blue, red, yellow, and green. The medium-sized tetrahedra form one large tetrahedron.
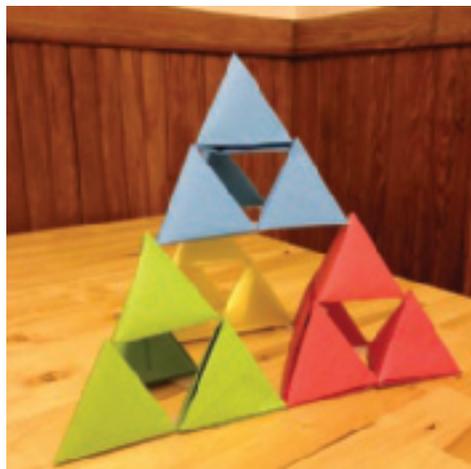
1. How many small faces does this fractal have? Be sure to include faces you can't see. Try to find a way to figure this out so that you don't have to count every face.

2. How many small tetrahedra are in the bottom layer, touching the table?

3. To make an even bigger version of this fractal, you could take four fractals like the one pictured and put them together. Explain where you would attach the fractals to make a bigger tetrahedron.

4. How many small faces would this bigger fractal have? How many small tetrahedra would be in the bottom layer?

5. What other patterns can you find?

Lesson 15 Summary

In this lesson, we saw expressions that used the letter x as a variable. We evaluated these expressions for different values of x.

- To evaluate the expression $2x^3$ when x is 5, we replace the letter x with 5 to get $2 \cdot 5^3$. This is equal to $2 \cdot 125$ or just 250. So the value of $2x^3$ is 250 when x is 5.

- To evaluate $\frac{x^2}{8}$ when x is 4, we replace the letter x with 4 to get $\frac{4^2}{8} = \frac{16}{8}$, which equals 2. So $\frac{x^2}{8}$ has a value of 2 when x is 4.

We also saw equations with the variable x and had to decide what value of x would make the equation true.

- Suppose we have an equation $10 \cdot 3^x = 90$ and a list of possible solutions: $1, 2, 3, 9, 11$. The only value of x that makes the equation true is 2 because $10 \cdot 3^2 = 10 \cdot 3 \cdot 3$, which equals 90. So 2 is the solution to the equation.

Unit 6 Lesson 15 Cumulative Practice Problems

1. Evaluate each expression if $x = 3$.

    a. 2^x

    b. x^2

    c. 1^x

    d. x^1

    e. $\left(\frac{1}{2}\right)^x$

2. Evaluate each expression for the given value of each variable.

    a. $2 + x^3$, x is 3

    b. x^2, x is $\frac{1}{2}$

    c. $3x^2 + y$, x is 5 y is 3

    d. $10y + x^2$, x is 6 y is 4

3. Decide if the expressions have the same value. If not, determine which expression has the larger value.

    a. 2^3 and 3^2

    b. 1^{31} and 31^1

    c. 4^2 and 2^4

    d. $\left(\frac{1}{2}\right)^3$ and $\left(\frac{1}{3}\right)^2$

4. Match each equation to its solution.

A. $7 + x^2 = 16$ 1. $x = 1$

B. $5 - x^2 = 1$ 2. $x = 2$

C. $2 \cdot 2^3 = 2^x$ 3. $x = 3$

D. $\frac{3^4}{3^x} = 27$ 4. $x = 4$

5. An adult pass at the amusement park costs 1.6 times as much as a child's pass.

   a. How many dollars does an adult pass cost if a child's pass costs:

   $5? $10? w dollars?

   b. A child's pass costs $15. How many dollars does an adult pass cost?

(From Unit 6, Lesson 6.)

6. Jada reads 5 pages every 20 minutes. At this rate, how many pages can she read in 1 hour?

   ○ Use a double number line to find the   ○ Use a table to find the
     answer.   answer.
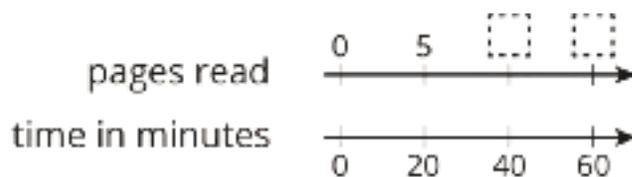
pages read	time in minutes
5	20

Which strategy do you think is better, and why?

(From Unit 2, Lesson 14.)

Lesson 16: Two Related Quantities, Part 1

Let's use equations and graphs to describe relationships with ratios.

16.1: Which One Would You Choose?

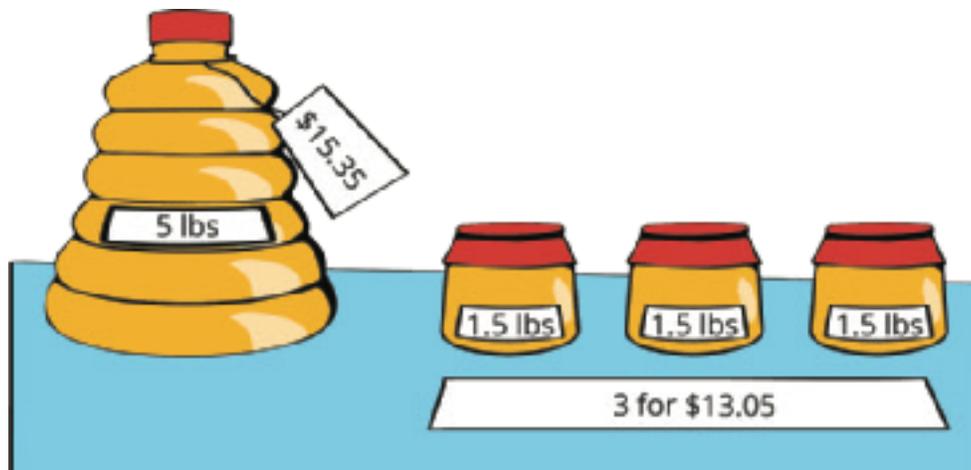Which one would you choose? Be prepared to explain your reasoning.

- A 5-pound jug of honey for $15.35

- Three 1.5-pound jars of honey for $13.05

16.2: Painting the Set

Lin needs to mix a specific shade of orange paint for the set of the school play. The color uses 3 parts yellow for every 2 parts red.

1. Complete the table to show different combinations of red and yellow paint that will make the shade of orange Lin needs.

cups of red paint (r)	cups of yellow paint (y)	total cups of paint (t)
2	3	
6		
		20
	18	
14		
16		
		50
	42	

2. Lin notices that the number of cups of red paint is always $\frac{2}{5}$ of the total number of cups. She writes the equation $r = \frac{2}{5}t$ to describe the relationship. Which is the **independent variable**? Which is the **dependent variable**? Explain how you know.

3. Write an equation that describes the relationship between r and y where y is the independent variable.

4. Write an equation that describes the relationship between y and r where r is the independent variable.

5. Use the points in the table to create two graphs that show the relationship between r and y. Match each relationship to one of the equations you wrote.

cups of red paint

cups of yellow paint
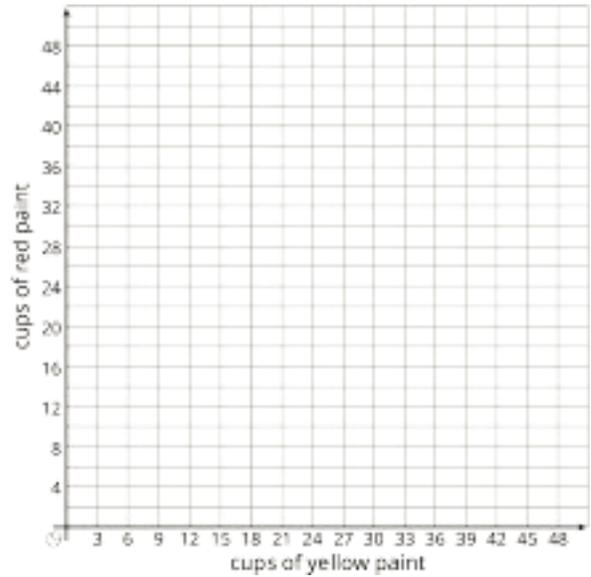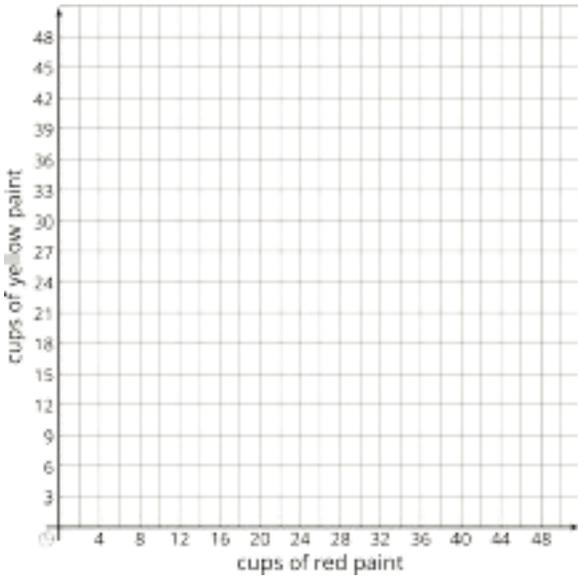
Are you ready for more?

A fruit stand sells apples, peaches, and tomatoes. Today, they sold 4 apples for every 5 peaches. They sold 2 peaches for every 3 tomatoes. They sold 132 pieces of fruit in total. How many of each fruit did they sell?

Lesson 16 Summary

Equations are very useful for describing sets of equivalent ratios. Here is an example.

A pie recipe calls for 3 green apples for every 5 red apples. We can create a table to show some equivalent ratios.

We can see from the table that r is always $\frac{5}{3}$ as large as g and that g is always $\frac{3}{5}$ as large as r.

green apples (g)	red apples (r)
3	5
6	10
9	15
12	20

We can write equations to describe the relationship between g and r.

- When we know the number of green apples and want to find the number of red apples, we can write:

$$r = \frac{5}{3}g$$

  In this equation, if g changes, r is affected by the change, so we refer to g as the **independent variable** and r as the **dependent variable**.

  We can use this equation with any value of g to find r. If 270 green apples are used, then $\frac{5}{3} \cdot (270)$ or 450 red apples are used.

- When we know the number of red apples and want to find the number of green apples, we can write:

$$g = \frac{3}{5}r$$

  In this equation, if r changes, g is affected by the change, so we refer to r as the independent variable and g as the dependent variable.

  We can use this equation with any value of r to find g. If 275 red apples are used, then $\frac{3}{5} \cdot (275)$ or 165 green apples are used.
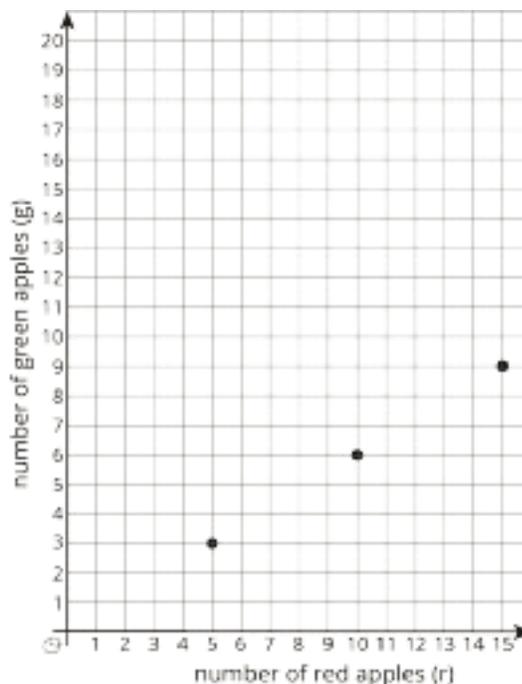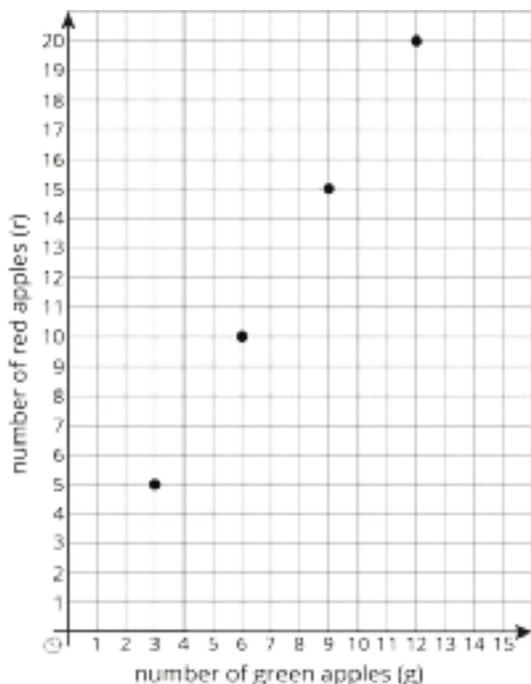
We can also graph the two equations we wrote to get a visual picture of the relationship between the two quantities.

Unit 6 Lesson 16 Cumulative Practice Problems

1. Here is a graph that shows some values for the number of cups of sugar, s, required to make x batches of brownies.

cups of sugar

batches of brownies
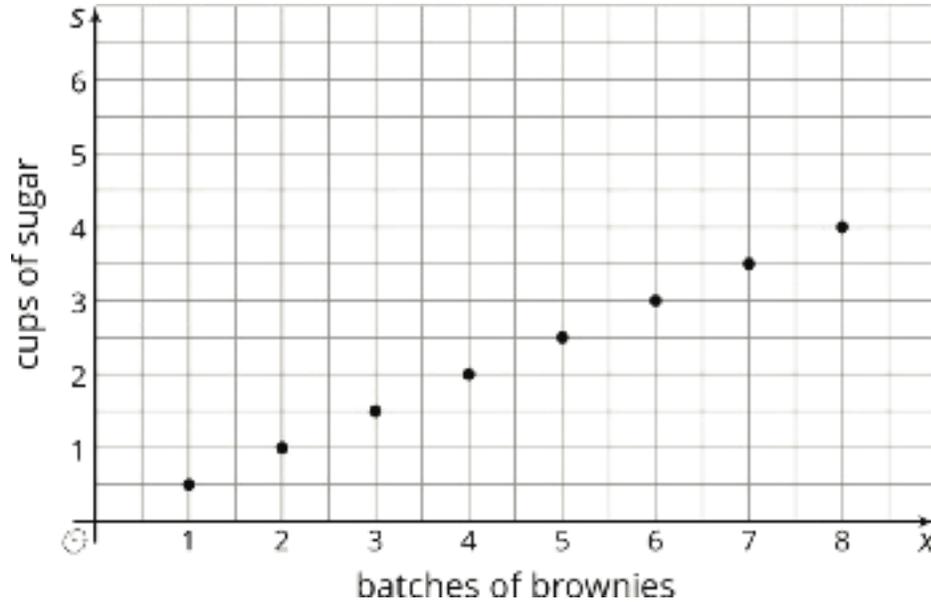
a. Complete the table so that the pair of numbers in each column represents the coordinates of a point on the graph.

x	1	2	3	4	5	6	7
s							

b. What does the point $(8, 4)$ mean in terms of the amount of sugar and number of batches of brownies?

c. Write an equation that shows the amount of sugar in terms of the number of batches.

2. Each serving of a certain fruit snack contains 90 calories.

    a. Han wants to know how many calories he gets from the fruit snacks. Write an equation that shows the number of calories, c, in terms of the number of servings, n.

    b. Tyler needs some extra calories each day during his sports season. He wants to know how many servings he can have each day if all the extra calories come from the fruit snack. Write an equation that shows the number of servings, n, in terms of the number of calories, c.

3. Kiran shops for books during a 20% off sale.

    a. What percent of the original price of a book does Kiran pay during the sale?

    b. Complete the table to show how much Kiran pays for books during the sale.

    c. Write an equation that relates the sale price, s, to the original price p.

    d. On graph paper, create a graph showing the relationship between the sale price and the original price by plotting the points from the table.

original price in dollars (p)	sale price in dollars (s)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

4. Evaluate each expression when x is 4 and y is 6.

    a. $(6 - x)^3 + y$

    b. $2 + x^3$

    c. $2^x - 2y$

    d. $\left(\frac{1}{2}\right)^x$

    e. $1^x + 2^x$

    f. $\frac{2^x}{x^2}$

(From Unit 6, Lesson 15.)

5. Find $(12.34) \cdot (0.7)$. Show your reasoning.

(From Unit 5, Lesson 8.)

6. For each expression, write another division expression that has the same value and that can be used to help find the quotient. Then, find each quotient.

    a. $302.1 \div 0.5$

    b. $12.15 \div 0.02$

    c. $1.375 \div 0.11$

(From Unit 5, Lesson 13.)

Lesson 17: Two Related Quantities, Part 2

Let's use equations and graphs to describe stories with constant speed.

17.1: Walking to the Library

Lin and Jada each walk at a steady rate from school to the library. Lin can walk 13 miles in 5 hours, and Jada can walk 25 miles in 10 hours. They each leave school at 3:00 and walk $3\frac{1}{4}$ miles to the library. What time do they each arrive?

17.2: The Walk-a-thon

Diego, Elena, and Andre participated in a walk-a-thon to raise money for cancer research. They each walked at a constant rate, but their rates were different.

1. Complete the table to show how far each participant walked during the walk-a-thon.

time in hours	miles walked by Diego	miles walked by Elena	miles walked by Andre
1			
2	6		
	12	11	
5			17.5

2. How fast was each participant walking in miles per hour?

3. How long did it take each participant to walk one mile?

4. Graph the progress of each person in the **coordinate plane**. Use a different color for each participant.
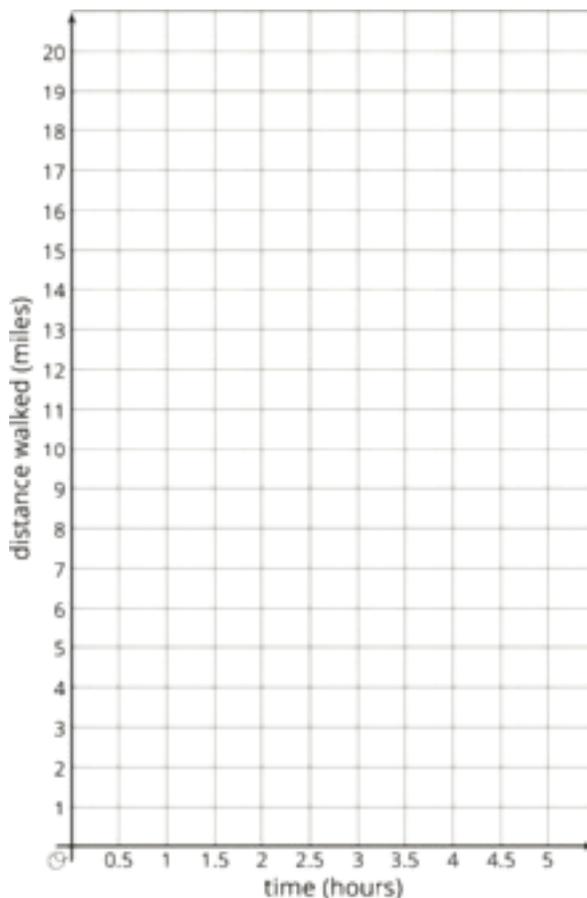
5. Diego says that $d = 3t$ represents his walk, where d is the distance walked in miles and t is the time in hours.

   a. Explain why $d = 3t$ relates the distance Diego walked to the time it took.

   b. Write two equations that relate distance and time: one for Elena and one for Andre.

6. Use the equations you wrote to predict how far each participant would walk, at their same rate, in 8 hours.

7. For Diego's equation and the equations you wrote, which is the dependent variable and which is the independent variable?

Are you ready for more?

1. Two trains are traveling toward each other, on parallel tracks. Train A is moving at a constant speed of 70 miles per hour. Train B is moving at a constant speed of 50 miles per hour. The trains are initially 320 miles apart. How long will it take them to meet? One way to start thinking about this problem is to make a table. Add as many rows as you like.

	train A	train B
starting position	0 miles	320 miles
after 1 hour	70 miles	270 miles
after 2 hours		

2. How long will it take a train traveling at 120 miles per hour to go 320 miles?

3. Explain the connection between these two problems.

Lesson 17 Summary

Equations are very useful for solving problems with constant speeds. Here is an example.

A boat is traveling at a constant speed of 25 miles per hour.

1. How far can the boat travel in 3.25 hours?

2. How long does it take for the boat to travel 60 miles?

We can write equations to help us answer questions like these.

Let's use t to represent the time in hours and d to represent the distance in miles that the boat travels.

When we know the time and want to find the distance, we can write:
$$d = 25t$$
In this equation, if t changes, d is affected by the change, so we t is the independent variable and d is the dependent variable.

This equation can help us find d when we have any value of t. In 3.25 hours, the boat can travel 25(3.25) or 81.25 miles.

When we know the distance and want to find the time, we can write:
$$t = \frac{d}{25}$$
In this equation, if d changes, t is affected by the change, so we d is the independent variable and t is the dependent variable.

This equation can help us find t when for any value of d. To travel 60 miles, it will take $\frac{60}{25}$ or $2\frac{2}{5}$ hours.

These problems can also be solved using important ratio techniques such as a table of equivalent ratios. The equations are particularly valuable in this case because the answers are not round numbers or easy to quickly evaluate.

We can also graph the two equations we wrote to get a visual picture of the relationship between the two quantities:
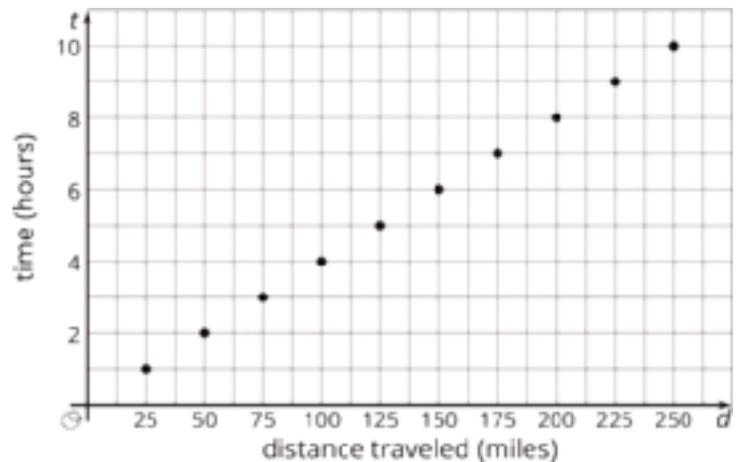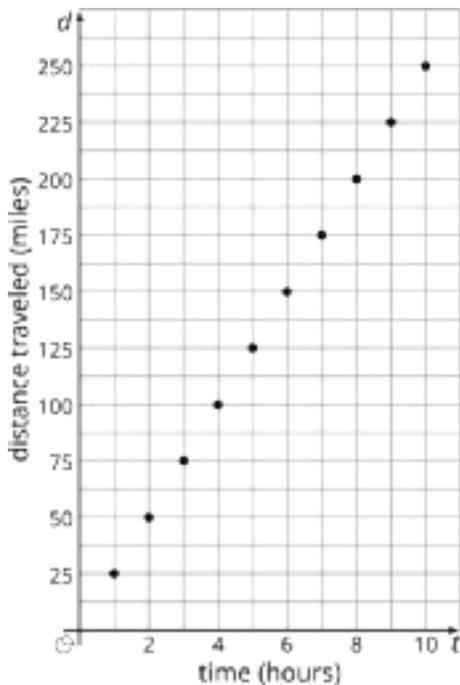
Unit 6 Lesson 17 Cumulative Practice Problems

1. A car is traveling down a road at a constant speed of 50 miles per hour.

   a. Complete the table with the amounts of time it takes the car to travel certain distances, or the distances traveled for certain amounts of time.

   b. Write an equation that represents the distance traveled by the car, d, for an amount of time, t.

   c. In your equation, which is the dependent variable and which is the independent variable?

time (hours)	distance (miles)
2	
1.5	
t	
	50
	300
	d

2. The graph represents the amount of time in hours it takes a ship to travel various distances in miles.
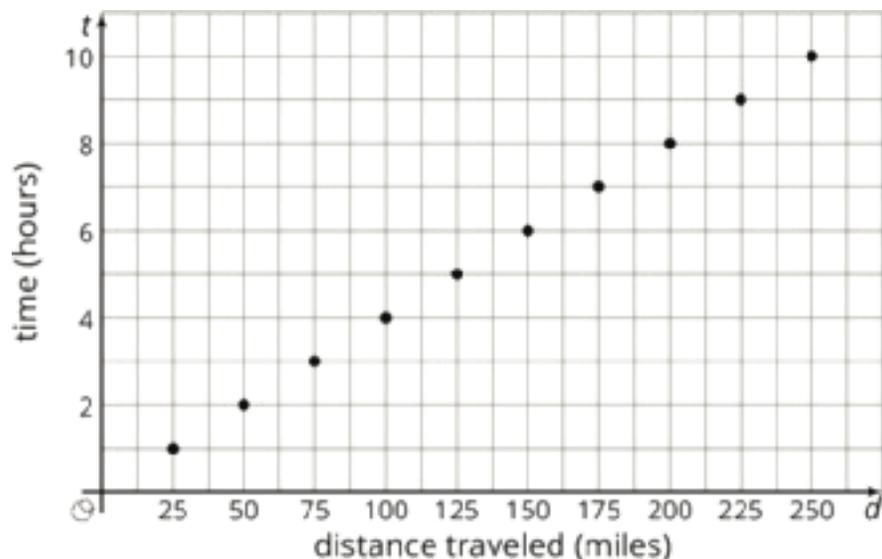
   a. Write the coordinates of one point on the graph. What does the point represent?

   b. What is the speed of the ship in miles per hour?

   c. Write an equation that relates the time, t, it takes to travel a given distance, d.

3. Find a solution to each equation in the list that follows (not all numbers will be used):

a. $2^x = 8$

b. $2^x = 2$

c. $x^2 = 100$

d. $x^2 = \frac{1}{100}$

e. $x^1 = 7$

f. $2^x \cdot 2^3 = 2^7$

g. $\frac{2^x}{2^3} = 2^5$

List: $\frac{1}{10}$ $\frac{1}{3}$ 1   2   3   4   5   7   8   10   16

(From Unit 6, Lesson 15.)

4. Select **all** the expressions that are equivalent to $5x + 30x - 15x$.

    A. $5(x + 6x - 3x)$

    B. $(5 + 30 - 15) \cdot x$

    C. $x(5 + 30x - 15x)$

    D. $5x(1 + 6 - 3)$

    E. $5(x + 30x - 15x)$

(From Unit 6, Lesson 11.)

5. Evaluate each expression if x is 1, y is 2, and z is 3.

    a. $7x^2 - z$

    b. $(x + 4)^3 - y$

    c. $y(x + 3^3)$

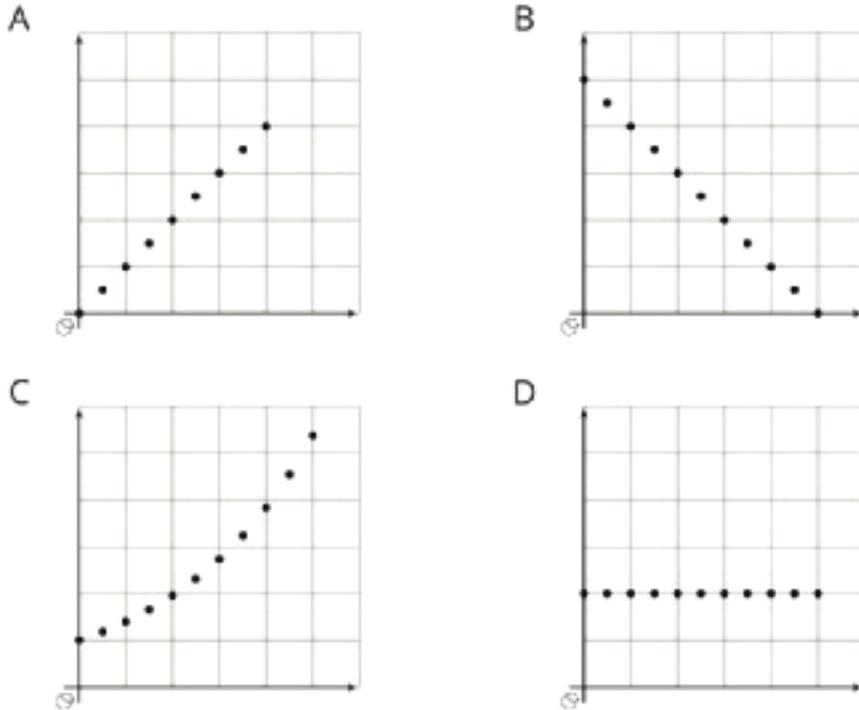   d. $(7 - y + z)^2$

    e. $0.241x + x^3$

(From Unit 6, Lesson 15.)

Lesson 18: More Relationships

Let's use graphs and equations to show relationships involving area, volume, and exponents.

18.1: Which One Doesn't Belong: Graphs

Which one doesn't belong?

A

B

C

D

18.2: Making a Banner

Mai is creating a rectangular banner to advertise the school play. The material for the banner is sold by the square foot. Mai has enough money to buy 36 square feet of material. She is trying to decide on the length and width of the banner.

    1. If the length is 6 feet, what is the width?

    2. If the length is 4 feet, what is the width?

    3. If the length is 9 feet, what is the width?

4. To find different combinations of length and width that give an area of 36 square feet, Mai uses the equation $w = \frac{36}{\ell}$, where w is the width and ℓ is the length.

   Compare your strategy and Mai's method for finding the width. How were they the same or different?

5. Use several combinations of length and width to create a graph that shows the relationship between the side lengths of various rectangles with area 36 square feet.
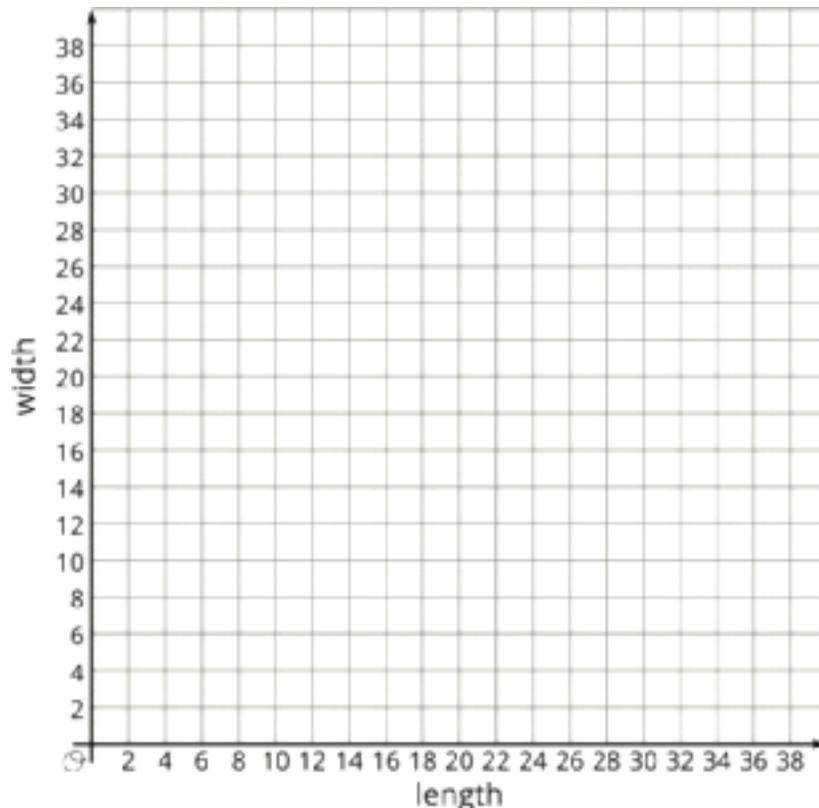
6. Explain how the graph describes the relationship between length and width for different rectangles with area 36.

7. Suppose Mai used the equation $\ell = \frac{36}{w}$ to find the length for different values of the width. Would the graph be different if she graphed length on the vertical axis and width on the horizontal axis? Explain how you know.

18.3: Cereal Boxes

A cereal manufacturer needs to design a cereal box that has a volume of 225 cubic inches and a height that is no more than 15 inches.

1. The designers know that the volume of a rectangular prism can be calculated by multiplying the area of its base and its height. Complete the table with pairs of values that will make the volume 225 in^3.

height (in)		5	9	12		$7\frac{1}{2}$
area of base (in^2)	75				15	

2. Describe how you found the missing values for the table.

3. Write an equation that shows how the area of the base, A, is affected by changes in the height, h, for different rectangular prisms with volume 225 in^3.

4. Plot the ordered pairs from the table on the graph to show the relationship between the area of the base and the height for different boxes box with volume 225 in^3.
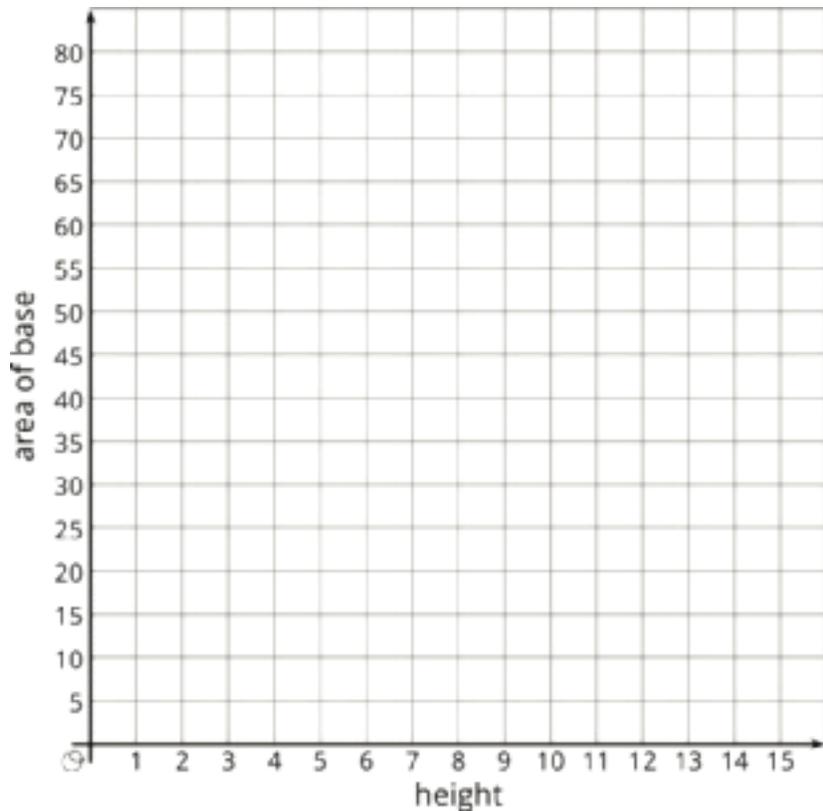
18.4: Multiplying Mosquitoes

A researcher who is studying mosquito populations collects the following data:

day in the study (d)	number of mosquitoes (n)
1	2
2	4
3	8
4	16
5	32

1. The researcher said that, for these five days, the number of mosquitoes, n, can be found with the equation $n = 2^d$ where d is the day in the study. Explain why this equation matches the data.

2. Use the ordered pairs in the table to graph the relationship between number of mosquitoes and day in the study for these five days.
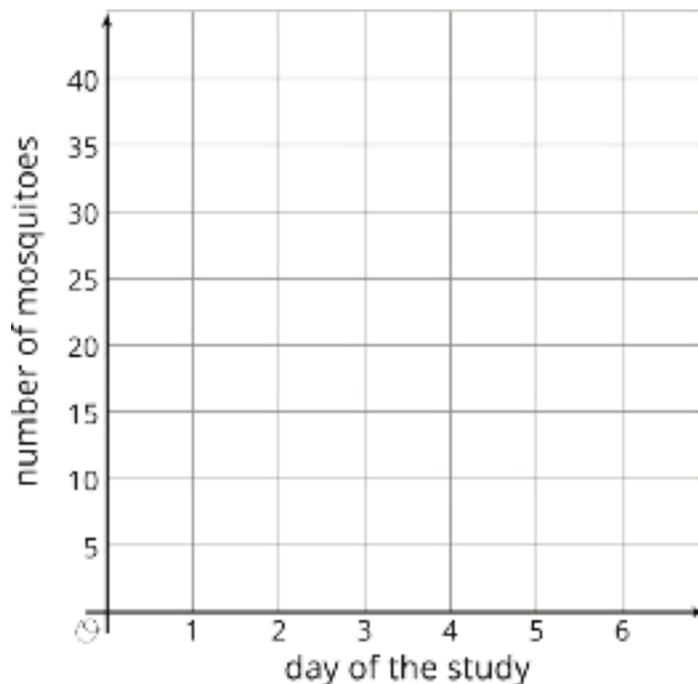
3. Describe the graph. Compare how the data, equation, and graph illustrate the relationship between the day in the study and the number of mosquitoes.

4. If the pattern continues, how many mosquitoes will there be on day 6?

Lesson 18 Summary

Equations can represent relationships between geometric quantities. For instance:

- If s is the side length of a square, then the area A is related to s by $A = s^2$.

- Sometimes the relationships are more specific. For example, the perimeter P of a rectangle with length l and width w is $P = 2l + 2w$. If we consider only rectangles with a length of 10, then the relationship between the perimeter and the width is $P = 20 + 2w$.

Here is another example of an equation with exponent expressing the relationship between quantities:

- A super ball is dropped from 10 feet. On each successive bounce, it only goes $\frac{1}{2}$ as high as on the previous bounce.

  This means that on the first bounce, the ball will bounce 5 feet high, and then on the second bounce it will only go $2\frac{1}{2}$ feet high, and so on. We can represent this situation with an equation to find how high the super ball will bounce after any number of bounces.

  To find how high the super ball bounces on the nth bounce, we have to multiply 10 feet (the initial height) by $\frac{1}{2}$ and multiply by $\frac{1}{2}$ again for each bounce thereafter; we need to do this n times. So the height, h, of the ball on the nth bounce will be $h = 10\left(\frac{1}{2}\right)^n$. In this equation, the dependent variable, h, is affected by changes in the independent variable, n.

Equations and graphs can give us insight into different kinds of relationships between quantities and help us answer questions and solve problems.

Unit 6 Lesson 18 Cumulative Practice Problems

1. Elena is designing a logo in the shape of a parallelogram. She wants the logo to have an area of 12 square inches. She draws bases of different lengths and tries to compute the height for each.

   a. Write an equation Elena can use to find the height, h, for each value of the base, b.

   b. Use your equation to find the height of a parallelogram with base 1.5 inches.

2. Han is planning to ride his bike 24 miles.

   a. How long will it take if he rides at a rate of:

      3 miles per hour?   4 miles per hour?   6 miles per hour?

   b. Write an equation that Han can use to find t, the time it will take to ride 24 miles, if his rate in miles per hour is represented by r.

   c. On graph paper, draw a graph that shows t in terms of r for a 24-mile ride.

3. The graph of the equation $V = 10s^3$ contains the points $(2, 80)$ and $(4, 640)$.

   a. Create a story that is represented by this graph.

   b. What do the points mean in the context of your story?

4. You find a brass bottle that looks really old. When you rub some dirt off of the bottle, a genie appears! The genie offers you a reward. You must choose one:

$50,000 or a magical $1 coin.

The coin will turn into two coins on the first day. The two coins will turn into four coins on the second day. The four coins will double to 8 coins on the third day. The genie explains the doubling will continue for 28 days.

    a. Write an equation that shows the number of coins, n, in terms of the day, d.

    b. Create a table that shows the number of coins for each day for the first 15 days.

    c. Create a graph for days 7 through 12 that shows how the number of coins grows with each day.

5. At a market, 3.1 pounds of peaches cost $7.72. How much did the peaches cost per pound? Explain or show your reasoning. Round your answer to the nearest cent.

(From Unit 5, Lesson 13.)

6. Andre set up a lemonade stand last weekend. It cost him $0.15 to make each cup of lemonade, and he sold each cup for $0.35.

    a. If Andre collects $9.80, how many cups did he sell?

    b. How much money did it cost Andre to make this amount of lemonade?

    c. How much money did Andre make in profit?

(From Unit 5, Lesson 13.)

Lesson 19: Tables, Equations, and Graphs, Oh My!

- Let's explore some equations from real-world situations.

19.1: Matching Equations and Tables

Match each equation with a table that represents the same relationship. Be prepared to explain your reasoning.

$$S - 2 = T \qquad\qquad G = J + 13 \qquad\qquad P = I - 47.50 \qquad\qquad C + 273.15 = K$$

$$e = 6s \qquad\qquad m = 8.96V \qquad\qquad y = \frac{1}{12}x \qquad\qquad t = \frac{d}{2.5}$$

$$g = 28.35z$$

Table 1:

independent variable	dependent variable
20	8
58.85	23.54
804	321.6

Table 2:

independent variable	dependent variable
5	18
36	49
75	88

Table 3:

independent variable	dependent variable
2.5	22.4
20	179.2
75	672

Table 4:

independent variable	dependent variable
20	$1\frac{2}{3}$
36	3
804	67

Table 5:

independent variable	dependent variable
58.85	11.35
175.5	128
804	756.5

Table 6:

independent variable	dependent variable
2.5	275.65
20	293.15
58.85	332

Table 7:			Table 8:			Table 9:	
independent variable	dependent variable		independent variable	dependent variable		independent variable	dependent variable
5	3		2.6	73.71		2.6	15.6
20	18		20	567		36	216
36	34		36	1,020.6		58.85	353.1

19.2: Getting to Know an Equation

The equations in the previous activity represent situations.

- $S - 2 = T$ where S is the number of sides on a polygon and T is the number of triangles you can draw inside it (from one vertex to the others, without overlapping)

- $G = J + 13$ where G is a day in the Gregorian calendar and J is the same day in the Julian calendar

- $P = I - 47.50$ where I is the amount of income and P is the profit after \$47.50 in expenses

- $C + 273.15 = K$ where C is a temperature in degrees Celsius and K is the same temperature in Kelvin

- $e = 6s$ where e is the total edge length of a regular tetrahedron and s is the length of one side

- $m = 8.96V$ where V is the volume of a piece of copper and m is its mass

- $y = \frac{1}{12}x$ where x is the number of eggs and y is how many dozens that makes

- $t = \frac{d}{2.5}$ where t is the amount of time it takes in seconds to jog a distance of d meters at a constant speed of 2.5 meters per second

- $g = 28.35z$ where g is the mass in grams and z is the same amount in ounces

Your teacher will assign you one of these equations to examine more closely.

1. Rewrite your equation using words. Use words like product, sum, difference, quotient, and term.

2. In the previous activity, you matched equations and tables. Copy the values from the table that matched your assigned equation into the first 3 rows of this table. Make sure to label what each column represents.

independent variable: _____	dependent variable: _____
60	
	300

3. Select one of the first 3 rows of the table and explain what those values mean in this situation.

4. Use your equation to find the values that complete the last 2 rows of the table. Explain your reasoning.

5. On graph paper, create a graph that represents this relationship. Make sure to label your axes.

19.3: Sharing Your Equation with Others

Create a visual display of your assigned relationships that includes:

- your equation along with an explanation of each variable
- a verbal description of the relationship
- your table
- your graph

If you have time, research more about your relationship and add more details or illustrations to help explain the situation.

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