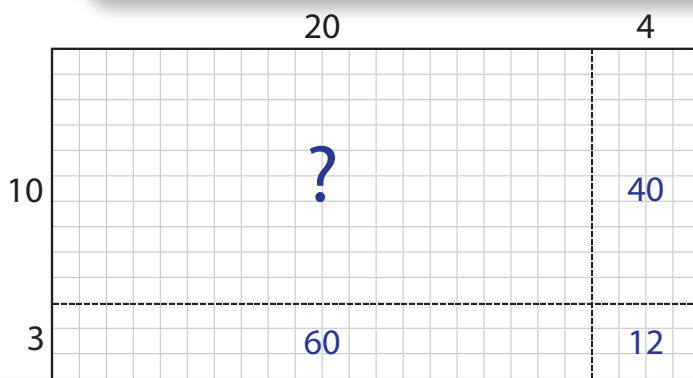
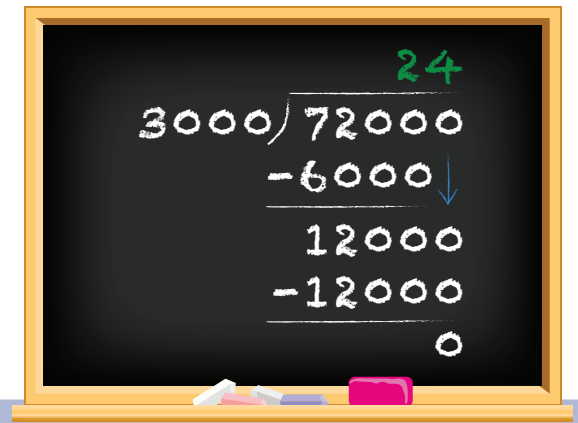




Arithmetic in Base Ten

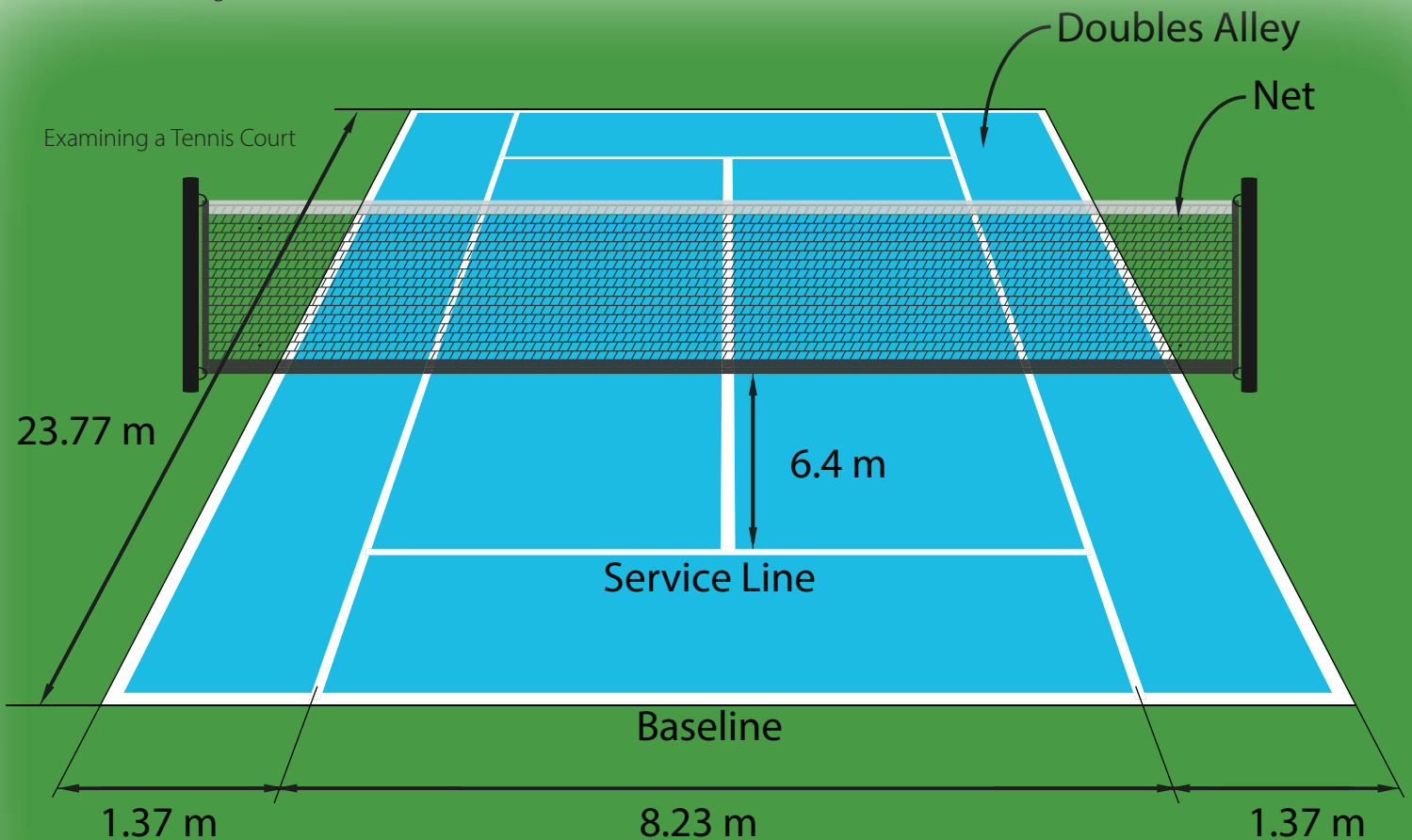
Teacher Guide



Finding the area of a rectangle

Vertical Calculations

$$\begin{array}{r} 10 \\ 40 \cancel{10} \\ 0.25103 \\ - 0.04671 \\ \hline 0.20432 \end{array}$$



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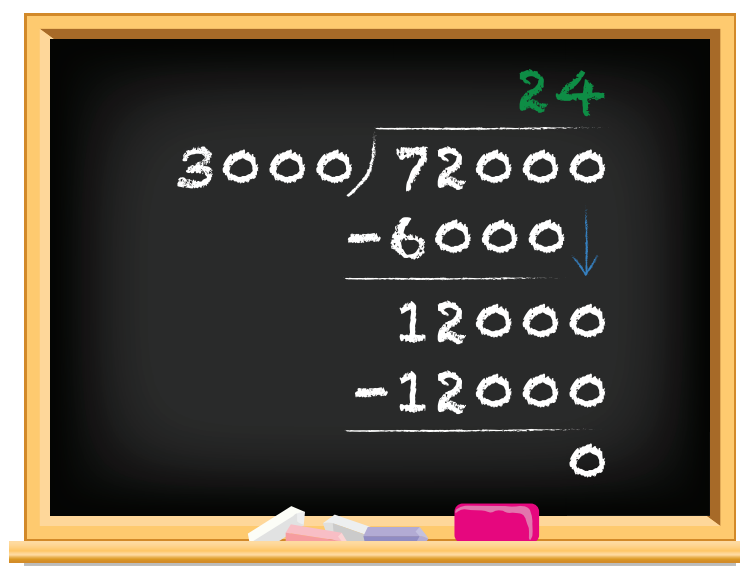
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Arithmetic in Base Ten

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Arithmetic in Base Ten
Teacher Guide
Core Knowledge Mathematics™

Dividing Fractions

Unit Narrative

By the end of grade 5, students learn to use efficient algorithms to fluently calculate sums, differences, and products of multi-digit whole numbers. They calculate quotients of multi-digit whole numbers with up to four-digit dividends and two-digit divisors. These calculations use strategies based on place value, the properties of operations, and the relationship between multiplication and division. Grade 5 students illustrate and explain these calculations with equations, rectangular arrays, and area diagrams.

In grade 5, students also calculate sums, differences, products, and quotients of decimals to hundredths, using concrete representations or drawings, and strategies based on place value, properties of operations, and the relationship between addition and subtraction. They connect their strategies to written methods and explain their reasoning.

In this unit, students learn an efficient algorithm for division and extend their use of other base-ten algorithms to decimals of arbitrary length. Because these algorithms rely on the structure of the base-ten system, students build on the understanding of place value and the properties of operations developed during earlier grades (MP7).

The unit begins with a lesson that revisits sums and differences of decimals to hundredths, and products of a decimal and whole number. The tasks are set in the context of shopping and budgeting, allowing students to be reminded of appropriate magnitudes for results of calculations with decimals.

The next section focuses on extending algorithms for addition, subtraction, and multiplication, which students used with whole numbers in earlier grades, to decimals of arbitrary length.

Students begin by using “base-ten diagrams,” diagrams analogous to base-ten blocks for ones, tens, and hundreds. These diagrams show, for example, ones as large squares, tenths as rectangles, hundredths as medium squares, thousandths as small rectangles, and ten-thousandths as small squares. These are designed so that the area of a figure that represents a base-ten unit is one tenth of the area of the figure that represents the base-ten unit of next highest value. Thus, a group of 10 figures that represent 10 like base-ten units can be replaced by a figure whose area is the sum of the areas of the 10 figures.

Students first calculate sums of two decimals by representing each number as a base-ten diagram, combining representations of like base-ten units and replacing representations of 10 like units by a representation of the unit of next highest value, e.g., 10 rectangles compose 1 large square. Next, they examine “vertical calculations,” representations of calculations with symbols that show one summand above the other, with the sum written below. They check each vertical calculation by representing it with base-ten diagrams. This is followed by a similar lesson on subtraction of decimals. The section concludes with a lesson designed to illustrate efficient algorithms and their advantages, and to promote their use.

The third section, multiplication of decimals, begins by asking students to estimate products of a whole number and a decimal, allowing students to be reminded of appropriate magnitudes for results of calculations with decimals. In this section, students extend their use of efficient algorithms for multiplication from whole numbers to decimals. They begin by writing products of decimals as products of fractions, calculating the product of the fractions, then writing the product as a decimal. They discuss the effect of multiplying by powers of 0.1, noting that multiplying by 0.1 has the same effect as dividing by 10. Students use area diagrams to represent products of decimals. The efficient multiplication algorithms are introduced and students use them, initially supported by area diagrams.

In the fourth section, students learn long division. They begin with quotients of whole numbers, first representing these quotients with base-ten diagrams, then proceeding to efficient algorithms, initially supporting their use with base-ten diagrams. Students then tackle quotients of whole numbers that result in decimals, quotients of decimals and whole numbers, and finally quotients of decimals.

The unit ends with two lessons in which students use calculations with decimals to solve problems set in real-world contexts. These require students to interpret diagrams, and to interpret results of calculations in the contexts from which they arose (MP2). The second lesson draws on work with geometry and ratios from previous units. Students fold papers of different sizes to make origami boxes of different dimensions, then compare the lengths, widths, heights, and surface areas of the boxes.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as explaining, interpreting and comparing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Explain

- processes of estimating and finding costs (Lesson 1)
- approaches to adding and subtracting decimals (Lesson 4)
- reasoning about products and quotients involving powers of 10 (Lesson 5)
- methods for multiplying decimals (Lesson 8)
- reasoning about relationships among measurements (Lesson 15)

Interpret

- representations of decimals (Lesson 2)
- base ten diagrams showing addition/subtraction of decimals (Lesson 3)
- area diagrams showing products of decimals (Lesson 7)
- base ten diagrams and long division when the quotient is a decimal value (Lesson 11)

Compare

- base ten diagrams with numerical calculations (Lesson 4)
- methods for multiplying decimals (Lesson 6)
- base ten diagrams showing quotients with partial quotient method (Lesson 9)
- previously studied methods for finding quotients with long division (Lesson 10)

In addition, students are expected to describe decimal values up to hundredths, generalize about multiplication by powers of 10 and about decimal measurements, critique approaches to operations on decimals, and justify strategies for finding quotients with reference to base-ten diagrams and more efficient algorithms.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

Learning Targets

Arithmetic in Base Ten

Lesson 1: Using Decimals in a Shopping Context

- I can use decimals to make estimates and calculations about money.

Lesson 2: Using Diagrams to Represent Addition and Subtraction

- I can use diagrams to represent and reason about addition and subtraction of decimals.
- I can use place value to explain addition and subtraction of decimals.
- I can use vertical calculations to represent and reason about addition and subtraction of decimals.

Lesson 3: Adding and Subtracting Decimals with Few Non-Zero Digits

- I can tell whether writing or removing a zero in a decimal will change its value.
- I know how to solve subtraction problems with decimals that require “unbundling” or “decomposing.”

Lesson 4: Adding and Subtracting Decimals with Many Non-Zero Digits

- I can solve problems that involve addition and subtraction of decimals.

Lesson 5: Decimal Points in Products

- I can use place value and fractions to reason about multiplication of decimals.

Lesson 6: Methods for Multiplying Decimals

- I can use area diagrams to represent and reason about multiplication of decimals.
- I know and can explain more than one way to multiply decimals using fractions and place value.

Lesson 7: Using Diagrams to Represent Multiplication

- I can use area diagrams and partial products to represent and find products of decimals.

Lesson 8: Calculating Products of Decimals

- I can describe and apply a method for multiplying decimals.
- I know how to use a product of whole numbers to find a product of decimals.

Lesson 9: Using the Partial Quotients Method

- I can use the partial quotients method to find a quotient of two whole numbers when the quotient is a whole number.

Lesson 10: Using Long Division

- I can use long division to find a quotient of two whole numbers when the quotient is a whole number.

Lesson 11: Dividing Numbers that Result in Decimals

- I can use long division to find the quotient of two whole numbers when the quotient is not a whole number.

Lesson 12: Dividing Decimals by Whole Numbers

- I can divide a decimal by a whole number.
- I can explain the division of a decimal by a whole number in terms of equal-sized groups.
- I know how multiplying both the dividend and the divisor by the same factor affects the quotient.

Lesson 13: Dividing Decimals by Decimals

- I can explain how multiplying dividend and divisor by the same power of 10 can help me find a quotient of two decimals.
- I can find the quotient of two decimals.

Lesson 14: Using Operations on Decimals to Solve Problems

- I can use addition, subtraction, multiplication, and division on decimals to solve problems.

Lesson 15: Making and Measuring Boxes

- I can use the four operations on decimals to find surface areas and reason about real-world problems.

lesson	new terminology	
	receptive	productive
6.5.1	digits budget at least	
6.5.2	base-ten diagram bundle vertical calculation	place value digits
6.5.3	unbundle	
6.5.4	method	
6.5.5	powers of 10	product decimal point
6.5.7	partial products	method
6.5.9	partial quotients	remainder
6.5.10	long division	divisor
6.5.13		long division
6.5.14	precision accuracy	
6.5.15	operation	

Required Materials

Copies of Instructional master

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider

cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Graph paper

Grocery store circulars

Grocery store advertisements from the newspaper or that are picked up at the store. If students have Internet access, you could substitute an online version of this.

Origami paper

Pre-printed slips, cut from copies of the Instructional master

Rulers

Lesson 1: Using Decimals in a Shopping Context

Goals

- Calculate sums and products of decimals in the context of money, and explain (orally and in writing) the calculation strategy
- Estimate sums, differences, products, and quotients of decimals in the context of money, and explain (orally) the estimation strategy.

Learning Targets

- I can use decimals to make estimates and calculations about money.

Lesson Narrative

In previous grades, students learned how to add, subtract, multiply, and divide whole numbers and decimals to the hundredths place. In this unit, they will extend this knowledge to include to *all* positive decimals.

This lesson activates students' previous experiences with the four operations, all in the context of planning for a party while staying within a budget (MP4). To do so, students make reasoned estimates and then compare them to actual calculated values. The lesson offers insights into students' understanding of operations and the structure of base-ten numbers before new concepts are introduced.

Alignments

Building On

- 5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Building Towards

- 6.NS.B.3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect

Required Materials

Grocery store circulars

Grocery store advertisements from the newspaper or that are picked up at the store. If

students have Internet access, you could substitute an online version of this.

Required Preparation

Pick up newspaper circulars from a local grocery store for students to use. Prepare enough for each group of 2 students to have a copy. Alternatively, prepare access to grocery advertisements online.

Student Learning Goals

Let's use what we know about decimals to make shopping decisions.

1.1 Snacks from the Concession Stand

Warm Up: 10 minutes

This activity allows students to review decimal work in a money context. This activity also offers insights into how they estimate and calculate sums, differences, and products of decimals. Both questions allow multiple paths of reasoning.

Monitor how students reason about situations involving adding, subtracting, and multiplying decimals. Also monitor for students using estimation to solve problems and how they go about doing so. Do they round the cents to the closest dollar, or do they look only at the dollar value to the left of the decimal point? (E.g. Some may round \$1.85 to \$2.00 because it is the closest whole dollar. Others may round to \$1.00 because "1" is the dollar amount in front of the decimal point.)

As students work, select those using different strategies so they can share during discussions. Note any misconceptions so that they can be addressed later.

Building On

- 5.NBT.B.7

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

Launch

Give students 2–3 minutes of quiet work time, and follow with a whole-class discussion.

Student Task Statement

Clare went to a concession stand that sells pretzels for \$3.25, drinks for \$1.85, and bags of popcorn for \$0.99 each. She bought at least one of each item and spent no more than \$10.



1. Could Clare have purchased 2 pretzels, 2 drinks, and 2 bags of popcorn? Explain your reasoning.
2. Could she have bought 1 pretzel, 1 drink, and 5 bags of popcorn? Explain your reasoning.

Student Response

1. No, one pretzel, one drink, and one bag of popcorn cost about \$6. So two of each would cost twice this much, about \$12.
2. No, one pretzel and one drink cost \$5.10. Five bags of popcorn cost \$4.95. Buying these items would cost 5 cents more than \$10.

Activity Synthesis

Ask selected students to share their responses. Record and display their strategies for adding, subtracting, and multiplying decimals for all to see. To involve more students in the conversation, consider asking some of the following questions:

- “Who can restate ___’s reasoning in a different way?”
- “Did anyone solve the problem the same way but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to ____’s strategy?”
- “Do you agree or disagree? Why?”

1.2 Planning a Dinner Party

30 minutes

In this activity, students perform decimal operations and estimate with money in a real-world context. They are asked to plan a dinner party for 8 guests with a \$50 budget. Students use an actual grocery store price list, select the foods they wish to serve, and determine an appropriate amount of each item. It is important to observe how students make choices in determining the amount of items and the cost of the items. Here are some ways they apply decimal skills along the way:

- Determine estimated costs: rounding
- Determine unit costs (per item or per guest): division
- Determine the subtotal and total costs: multiplication and addition
- Remove items if they go over budget: subtraction

Students are likely to check if their choices are within budget in two ways: by comparing their estimated total costs to \$50, or by comparing cost per guest to \$6.25 (which is $50 \div 8$). As students work, monitor for students who use each approach.

Building On

- 5.NBT.B.7

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect

Launch

Ask students if they have ever planned a party and what types of decisions are involved in the planning of a party. After hearing a few responses, arrange students in groups of 2. Provide each group with access to circulars from a local grocery store or to grocery advertisements online. Give students a minute to read the task statement. Give them another minute to preview a grocery store circular with a partner and briefly discuss which items they are interested in including at their party.

- Consider reviewing serving size and going over the second example in the table, in which the quantity sold is in bulk.
- Consider reviewing subtotal, which is among the values students are asked to find.
- Let students know that a good estimation for the amount of meat, poultry, or fish for each guest is 0.5 pound. Consider giving an example: "If you were going to serve turkey to 10 guests, how many pounds should you buy?" (At least 5 pounds, because $10 \cdot 0.5 = 5$.)
- For other items (such as pies or french fries), students will have to use their best judgment to decide how much is needed. Encourage them to discuss these decisions in their groups.
- Encourage students to focus on choosing items from the flier and keeping the choices relatively simple (e.g., if a student wants to make a salad, suggest choosing a prepared salad instead of individual ingredients).

Give students 15 minutes of quiet work time, but encourage them to make selections within the first 5–7 minutes so that they have ample time check their budget and to make revisions if necessary. Save at least 10 minutes for the sharing of menus and a whole-class discussion of the selection process. If time is a concern, consider removing an item from the budget worksheet (e.g., beverages) or pre-selecting some items.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support development of organizational skills, check in with students within the first 2–3 minutes of work time. Check to make sure groups have decided how much of each item on their menu is needed prior to estimating the subtotal and cost per person.

Supports accessibility for: Memory; Organization

Anticipated Misconceptions

When dividing prices to determine unit cost, students might not know what to make of a remainder in this context. For example, if lemons cost \$1 for 6, students may write "16 cents and a remainder of 2 cents" for the unit price. Prompt them to think about how the remainder could be divided as well.

Some students might write unit costs as fractions or mixed numbers, e.g., \$ or $33\frac{1}{3}$ cents. Prompt them to think about rounding these numbers to the nearest cent.

Student Task Statement

You are planning a dinner party with a budget of \$50 and a menu that consists of 1 main dish, 2 side dishes, and 1 dessert. There will be 8 guests at your party.

Choose your menu items and decide on the quantities to buy so you stay on budget. If you choose meat, fish, or poultry for your main dish, plan to buy at least 0.5 pound per person.

1. The budget is \$ _____ per guest.

2. Use the worksheet to record your choices and estimated costs. Then find the estimated total cost and cost per person. See examples in the first two rows.

item	quantity needed	advertised price	estimated subtotal (\$)	estimated cost per person (\$)
example main dish: fish	4 pounds	\$6.69 per pound	$4 \cdot 7 = 28$	$28 \div 8 = 3.50$
example dessert: cupcakes	8 cupcakes	\$2.99 per 6 cupcakes	$2 \cdot 3 = 6$	$6 \div 8 = 0.75$
main dish:				
side dish 1:				
side dish 2:				
dessert:				
estimated total				

3. Is your estimated total close to your budget? If so, continue to the next question. If not, revise your menu choices until your estimated total is close to the budget.
4. Calculate the actual costs of the two most expensive items and add them. Show your reasoning.
5. How will you know if your total cost for all menu items will or will not exceed your budget? Is there a way to predict this without adding all the exact costs? Explain your reasoning.

Student Response

1. The budget per person is $50 \div 8$, which is \$6.25.
2. Answers vary. Sample response:

item	quantity needed	advertised price	estimated subtotal (\$)	estimated cost per person (\$)
main dish: beef	4 pounds	\$5.89 per pound	$4 \cdot 6 = 24$	$24 \div 8 = 3$
side dish 1: corn on the cob	8 ears	\$2.00 for a bag of 6	$2 \cdot 2 = 4$	$4 \div 8 = 0.50$
side dish 2: French fries	2 bags	\$2.89 per bag	$2 \cdot 3 = 6$	$6 \div 8 = 0.75$
dessert: ice cream	1 tub	\$2.29 per gallon tub	$1 \cdot 2 = 2$	$2 \div 8 = 0.25$
estimate total			\$36	\$4.50

3. Answers vary.

4. Answers vary. Sample response: Most expensive: beef, which costs \$23.56. $4 \cdot (5.89) = 23.56$. Second most expensive: French fries, which cost \$5.78. $2 \cdot (2.89) = 5.78$. The combined cost of the two most expensive items is $23.56 + 5.78$, which is \$29.34.

5. Answers vary. Sample responses:

- The exact costs (not just the estimates) could be added up for one guest to check if \$50 is enough to buy all of the food.
- Without finding the exact costs, all estimates per guest could be made *too large*. If these numbers that are too large add up to less than \$6.25 then all of the items can be purchased without exceeding the budget. (even without knowing the exact cost). In the table for question 1, the first three estimates are all too high. The last two estimates are too low but only by a few cents. Since the total was less than \$5, this food can be bought with \$50.

Are You Ready for More?

How much would it cost to plant the grass on a football field? Explain or show your reasoning.

Student Response

Answers vary.

Activity Synthesis

The goal of this discussion is to highlight the decimal operations and estimations students did while planning the dinner party. Select a few students to share their menus with the entire class. Consider

displaying some of these questions for all to see and discuss. Choose questions that are relevant based on misconceptions you observed, if any, in the warm-up and in this activity.

- “How did you decide how much of each item to get?”
- “Were there any sale items that were sold in multiple quantities? If so, how did you decide how much to get?”
- “Were there any items that you did not choose because they were sold in an amount that was more than you needed?”
- “How did you determine if your menu choices are within budget? Did you look at total estimated cost, or estimated cost per person? Why?”
- “Were there items where it was difficult to estimate the cost per person? How so?”
- “Was your first planned menu in the right price range or did you need to revise?”
- “How did you decide which items to remove?”

Select previously identified students to share two ways of meeting the budget constraint (by comparing total cost or by comparing cost per guest). Briefly ask a few groups to share the approach they took and the merits of their approach. Invite other students to share how they divided the prices to find unit costs or cost per guest, and how they multiplied and added the prices to find sub-totals.

Access for English Language Learners

Speaking: MLR7 Compare and Connect. Use this routine when students present their menu and share strategies for meeting the budget constraint. Ask students to consider what is the same and what is different about each approach. In this discussion, emphasize language used to make sense of strategies for estimating and calculating with decimals. These exchanges can strengthen students' mathematical language use and reasoning of decimals.

Design Principle(s): Maximize meta-awareness; Support sense-making

Lesson Synthesis

In this lesson, we used what we know about decimals to make decisions about shopping and money. We noticed that sometimes it was helpful to round the dollars and cents and estimate, and other times it was necessary to be precise. Consider asking some of the following questions:

- “When was it appropriate to make an estimate, and when was it appropriate calculate the numbers precisely?”
- “How did you estimate sums and differences of decimals?”

- “How did you estimate products of decimals and whole numbers? What about quotients of decimals and whole numbers?”
- “How did you go about adding and subtracting decimals precisely?”
- “What strategies did you use to multiply and divide decimals precisely?”

1.3 How Did You Compute With Decimals?

Cool Down: 5 minutes

Building Towards

- 6.NS.B.3

Launch

As students review their calculation strategies, remind students that the sums and products of decimals should be exact. For example, if a student summed \$5.89 and \$1.45, they should not estimate the sum using \$6 and \$2.

Student Task Statement

Planning your menu involved many calculations with decimals. Reflect on how you made these calculations:

1. How did you compute sums of dollar amounts that were not whole numbers? For example, how did you compute the sum of \$5.89 and \$1.45? Use this example to explain your strategy.
2. How did you compute products of dollar amounts that were not whole numbers? For example, how did you compute the cost of 4 pounds of beef at \$5.89 per pound? Use this example to explain your strategy.

Student Response

1. Answers vary. Sample response: My strategy was to add the dollars and cents separately, and then combine the sums at the end. In this example, \$5 and \$1 would be added together to get \$6, and then add the 89 cents and 45 cents would be added together to get \$1.34. So the total would be \$7.34.
2. Answers vary. Sample response: My strategy was to first round the \$5.89 to \$5.90 to make it easier to multiply. Then, I found 4 times \$5, which is \$20, and 4 times \$0.90, which is \$3.60. These two products were added together to get \$23.60. The exact cost would be 4 cents less than \$23.60, because \$5.89 is 1 cent less than \$5.90, and 4 times 1 cent is 4 cents.

Student Lesson Summary

We often use decimals when dealing with money. In these situations, sometimes we round and make estimates, and other times we calculate the numbers more precisely.

There are many different ways we can add, subtract, multiply, and divide decimals. When we perform these calculations, it is helpful to understand the meanings of the digits in a number and the properties of operations. We will investigate how these understandings help us work with decimals in upcoming lessons.

Lesson 1 Practice Problems

Problem 1

Statement

Mai had \$14.50. She spent \$4.35 at the snack bar and \$5.25 at the arcade. What is the exact amount of money Mai has left?

- A. \$9.60
- B. \$10.60
- C. \$4.90
- D. \$5.90

Solution

C

Problem 2

Statement

A large cheese pizza costs \$7.50. Diego has \$40 to spend on pizzas. How many large cheese pizzas can he afford? Explain or show your reasoning.

Solution

5 pizzas. Sample reasoning: Each pizza costs about \$8, and $8 \cdot 5 = 40$.

Problem 3

Statement

Tickets to a show cost \$5.50 for adults and \$4.25 for students. A family is purchasing 2 adult tickets and 3 student tickets.

- a. Estimate the total cost.
- b. What is the exact cost?
- c. If the family pays \$25, what is the exact amount of change they should receive?

Solution

- a. \$24 ($6 + 6 + 4 + 4 + 4 = 24$)
- b. \$23.75 ($5.50 + 5.50 + 4.25 + 4.25 + 4.25 = 23.75$)
- c. \$1.25 ($25.00 - 23.75 = 1.25$)

Problem 4

Statement

Chicken costs \$3.20 per pound, and beef costs \$4.59 per pound. Answer each question and show your reasoning.

- a. What is the exact cost of 3 pounds of chicken?
- b. What is the exact cost of 3 pound of beef?
- c. How much more does 3 pounds of beef cost than 3 pounds of chicken?

Solution

- a. \$9.60 ($3.20 \cdot 3 = 9.60$)
- b. \$13.77 ($4.59 \cdot 3 = 13.77$)
- c. \$4.17 ($13.77 - 9.60 = 4.17$)

Problem 5

Statement

- a. How many $\frac{1}{5}$ -liter glasses can Lin fill with a $1\frac{1}{2}$ -liter bottle of water?
- b. How many $1\frac{1}{2}$ -liter bottles of water does it take to fill a 16-liter jug?

Solution

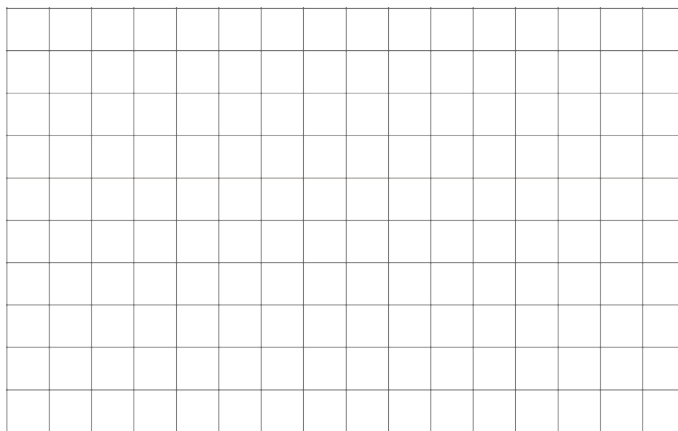
- a. $7\frac{1}{2}$ (or $\frac{15}{2}$). (She can fill 5 of the glasses with 1 liter and then another half of that or $2\frac{1}{2}$ with the other half liter, so that is $7\frac{1}{2}$ glasses. $1\frac{1}{2} \div \frac{1}{5} = \frac{15}{2}$ or $7\frac{1}{2}$.)
- b. $10\frac{2}{3}$ (or $\frac{32}{3}$). (This can be obtained by computing $16 \div 1\frac{1}{2}$, which is $16 \div \frac{3}{2}$ or $10\frac{2}{3}$. This is correct as 10 bottles give 15 liters, and then 1 more liter is $\frac{2}{3}$ of the bottle.)

(From Unit 4, Lesson 16.)

Problem 6

Statement

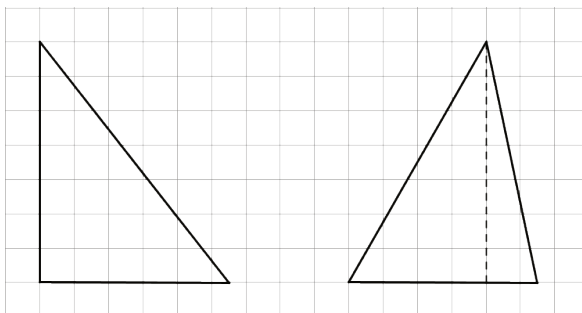
Let the side length of each small square on the grid represents 1 unit. Draw two different triangles, each with base $5\frac{1}{2}$ units and area $19\frac{1}{4}$ units².



Why does each of your triangles have area $19\frac{1}{4}$ units²? Explain or show your reasoning.

Solution

Drawings vary but should show a height of 7 units. Sample reasoning: The base times the height is 2 times the area of the triangle: $(\text{base}) \cdot (\text{height}) = 2 \cdot (19\frac{1}{4})$. Since $(19\frac{1}{4}) \div (5\frac{1}{2}) = 3\frac{1}{2}$, that means the height should be $2 \cdot (3\frac{1}{2}) = 7$.



(From Unit 4, Lesson 14.)


Problem 7

Statement

Find each quotient.

a. $\frac{5}{6} \div \frac{1}{6}$

b. $1\frac{1}{6} \div \frac{1}{12}$

 c. $\frac{10}{6} \div \frac{1}{24}$

Solution

a. 5

b. 14

c. 40

(From Unit 4, Lesson 10.)

Lesson 2: Using Diagrams to Represent Addition and Subtraction

Goals

- Compare and contrast (orally and in writing) vertical calculations and base-ten diagrams that represent adding and subtracting decimals.
- Explain (in words and through other representations) that adding and subtracting decimals requires combining digits that represent like base-ten units.
- Interpret and create diagrams that represent 10 like base-ten units being composed into 1 unit of higher place value, e.g., 10 tenths as 1 one, and comprehend the word “bundle” to refer to this concept.

Learning Targets

- I can use diagrams to represent and reason about addition and subtraction of decimals.
- I can use place value to explain addition and subtraction of decimals.
- I can use vertical calculations to represent and reason about addition and subtraction of decimals.

Lesson Narrative

This lesson is optional. Prior to grade 6, students have added and subtracted decimals to the hundredths using a variety of methods, all of which focus on understanding place value. This lesson reinforces their understanding of place-value relationships in preparation for computing sums and differences of any decimals algorithmically.

In this lesson, students use two methods—base-ten diagrams and vertical calculations—to find the sum and differences of decimals. Central to both methods is an understanding about the meaning of each digit in the numbers and how the different digits are related. Students recall that we only add the values of two digits if they represent the same base-ten units. They also recall that when the value of a base-ten unit is 10 or more we can express it with a different unit that is 10 times higher in value. For example, 10 tens can be expressed as 1 hundred, and 12 hundredths can be expressed as 1 tenth and 2 hundredths. This idea is made explicit both in the diagrams and in vertical calculations.

Alignments

Building On

- 5.NBT.A.1: Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.

- 5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Addressing

- 6.NS.B.3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Instructional Routines

- MLR7: Compare and Connect
- Think Pair Share

Required Materials

Graph paper

Instructional master

Pre-printed slips, cut from copies of the

Required Preparation

Students draw base-ten diagrams in this lesson. If drawing them is a challenge, consider giving students access to:

- Commercially produced base-ten blocks, if available.
- Print and cut up the Squares and Rectangles Instructional master. Prepare one copy for every student. These tools will be useful throughout the unit, so consider printing on card stock and organizing them for easy reuse.
- Digital applet of base-ten representations <https://www.geogebra.org/m/FXEZD466>.

Some students might find graph paper helpful for aligning the digits for vertical calculations. Consider having graph paper accessible for these activities: Finding Sums in Different Ways, Representing Subtraction, and Why or Why Not?.

Student Learning Goals

Let's represent addition and subtraction of decimals.

2.1 Changing Values

Warm Up: 5 minutes

The purpose of this warm-up is for students to review place value when working with decimals. There are many ways students might find the numbers represented by the large rectangle and large square. However, the focus as students work should be on understanding that each place represents a unit that is 10 times larger than the unit immediately to its right. This understanding can be represented by diagrams or by multiplication expressions (MP8).

Building On

- 5.NBT.A.1

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Tell students to look for patterns as they work. Give students 1–2 minute of quiet think time, followed by a brief partner discussion. Tell the partners to share their responses, come to an agreement on each answer, and discuss any patterns they noticed. Select students with correct responses to share during the whole-class discussion.

Anticipated Misconceptions

Some students may continually use skip counting (by 10, by 0.1, etc.) to find the value of the rectangle and the square, rather than making connections to place value. To help these students see a pattern connected to their skip counting, ask them to also write the multiplication expression that relates the value of each of the smaller units to that of the larger unit they compose.

Student Task Statement

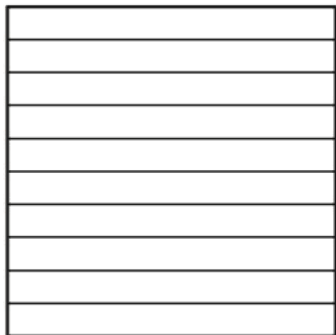
1. Here is a rectangle.



What number does the rectangle represent if each small square represents:

- a. 1
- b. 0.1
- c. 0.01
- d. 0.001

2. Here is a square.



What number does the square represent if each small rectangle represents:

- a. 10
- b. 0.1
- c. 0.00001

Student Response

1. a. 10

- b. 1
 - c. 0.1
 - d. 0.01
2. a. 100
- b. 1
 - c. 0.0001

Activity Synthesis

Ask selected students to share their values for the diagrams, how they found them, and what patterns they noticed. Record and display these values for all to see. Ask other students for the multiplication equation that represents each response to highlight that each decimal place value is 10 times the value of the unit to its right. For example, the recording for the first question should look like this:

$$10 \cdot 1 = 10$$

$$10 \cdot 0.1 = 1$$

$$10 \cdot 0.01 = 0.1$$

$$10 \cdot 0.001 = 0.01$$

If time permits, discuss some of these questions:

- “When you change what each small square represents, how did the change affect the value of the large rectangle?”
- “What might be some other numbers that the small square or long rectangle could represent?”
- “Why could these representations be called ‘base-ten diagrams?’”

2.2 Squares and Rectangles

Optional: 15 minutes (there is a digital version of this activity)

In this activity, students reinforce their understanding of the meaning of place value, i.e., that the value in ones place is 10 times the value of the place to its right (and $\frac{1}{10}$ the value of the place to its left). They draw base-ten diagrams to represent addition of decimals; they connect the process of regrouping numbers in the addition algorithm to the “bundling” of pieces in the diagram. For example, they see that 10 medium squares (0.01) can be composed or “bundled” to make 1 medium rectangle (0.1). Likewise, when the numbers in the ones place add up to be at least 10, we can group them and add 1 to the tens place.

As alternatives to drawing diagrams, consider having students use physical base-ten blocks (if available), a paper version of the base-ten figures (from the Instructional master), or this digital applet <https://ggbm.at/FXEZD466>.

Building On

- 5.NBT.A.1
- 5.NBT.B.7

Launch

Give students 1 minute of quiet time to study the diagrams and notice how they are structured. Then, have students identify the value of various units by pointing to them. For example, point to the medium square, and ask what it represents and how it relates to the large rectangle. Once students are familiar with the various pieces and their values, give students 10 minutes of quiet work time. Provide access to physical pattern blocks, cut-up paper copies of the base-ten figures, or the digital applet for representing base-ten numbers, if needed.



Classes using the digital activities have an interactive applet with virtual blocks. Note that the applet can only represent ones, tenths, and hundredths; this is so the pieces are large enough to manipulate. To use the bundling and unbundling features, the pieces must be aligned on the light blue grids. To bring a piece into the workspace, select one of the green tool icons and then click on the workspace. To move it, you must click on the Move tool

Access for Students with Disabilities

Representation: Develop Language and Symbols. Activate or supply background knowledge. Some students may benefit from continued access to physical base-ten blocks (if available), a paper version of the base-ten figures (from the Instructional master), or the digital applet.

Encourage students to begin with physical representations before drawing a diagram.

Supports accessibility for: Conceptual processing

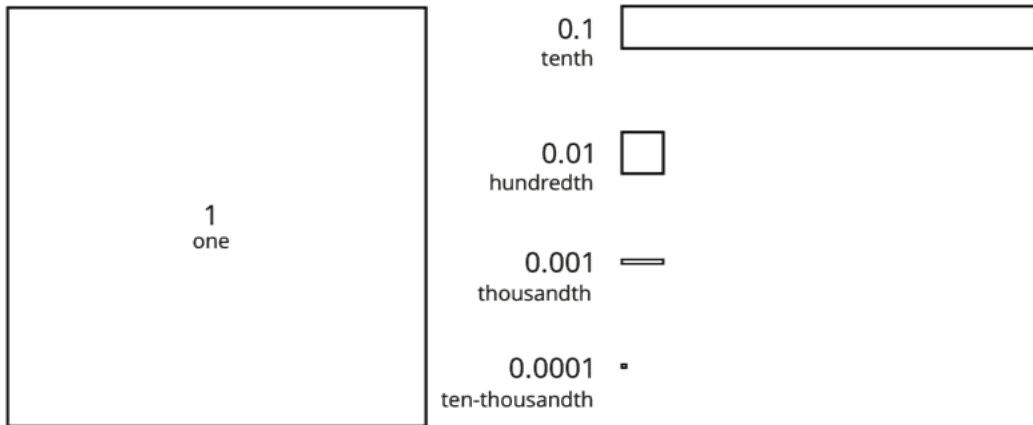
Anticipated Misconceptions

Students will need a solid understanding of place value to be successful with the activity. If they are unclear about the terms tenths, hundredths, thousandths, and ten thousandths, review place value before proceeding with the activity. Consider having students practice by counting by a certain base-ten unit (6 and 8 tenths, 6 and 9 tenths, 7, 7 and 1 tenth. . .), completing place value charts, reading decimals aloud, or by matching a decimal that is read aloud to its written numerical representation.

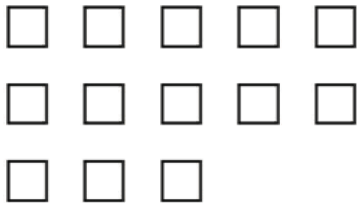
Student Task Statement

You may be familiar with base-ten blocks that represent ones, tens, and hundreds. Here are some diagrams that we will use to represent base-ten units.

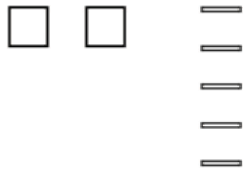
- A large square represents 1 one.
- A medium rectangle represents 1 tenth.
- A medium square represents 1 hundredth.
- A small rectangle represents 1 thousandth.
- A small square represents 1 ten-thousandth.



1. Here is the diagram that Priya drew to represent 0.13. Draw a different diagram that represents 0.13. Explain why both diagrams represent the same number.



2. Here is the diagram that Han drew to represent 0.025. Draw a different diagram that represents 0.025. Explain why both diagrams represent the same number.



3. For each number, draw or describe two different diagrams that represent it.

a. 0.1

b. 0.02

c. 0.004

4. Use diagrams of base-ten units to represent each sum. Think about how you could use as few units as possible to represent each number.

a. $0.03 + 0.05$

b. $0.006 + 0.007$

c. $0.4 + 0.7$

Student Response

Answers vary. Sample responses:

1. 1 tenth and 3 hundredths or 13 hundredths both represent 0.13. The tenth can be replaced with 10 of the hundredths to match what Priya drew.

2. 25 thousandths represent 0.025. 20 of the thousandths and can be replaced with the 2 hundredths to match what Han drew.

3. a. 1 tenth or 10 hundredths

b. 2 hundredths or 20 thousandths

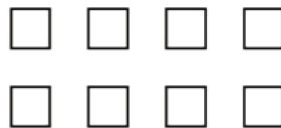
c. 4 thousandths or 40 ten-thousandths

4. a. 8 hundredths, 0.08

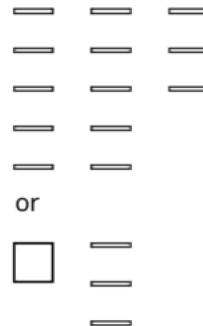
b. 13 thousandths or 1 hundredth and 3 thousandths, 0.013

c. 11 tenths or 1 one and 1 tenth, 1.1

a.



b.



c.

		or	<div></div>
			<div></div>

Activity Synthesis

Select a few students to share their responses to the questions, or display the correct answers for all to see. Discuss the following questions:

- “Is it always helpful or important to use as few base-ten figures as possible when representing addition of two numbers?” (Yes: fewer pieces mean less counting and smaller likelihood of making a counting mistake. No: once the pieces are bundled, it is harder to see the two numbers that make up the sum.)
- “When can you bundle a base-ten figure?” (When there are at least 10 of a unit.)
- “What would a figure that represents 10 look like?” (An extra-large rectangle, composed of 10 big squares.) “What about 100?” (A giant square made from 10 of the extra-large rectangles representing 10.)
- “Why might using base-ten diagrams for addition be cumbersome with larger multi-digit numbers?” (We would have to draw a lot of squares and rectangles. When there are more units, the diagrams become more complex.)

2.3 Finding Sums in Different Ways

Optional: 15 minutes (there is a digital version of this activity)

In this activity, students use symbols and diagrams to find a sum that requires regrouping of base-ten units. Use this activity to give students more explicit instruction on how to bundle smaller units into a larger one and additional practice on using addition algorithm to add decimals.

Again, consider having physical base-ten blocks, a paper version of the base-ten figures, or this digital applet ggbm.at/n9yaWPQj available as alternatives to diagram drawing, or to more concretely illustrate the idea of bundling and unbundling.

Building On

- 5.NBT.B.7

Addressing

- 6.NS.B.3

Instructional Routines

- MLR7: Compare and Connect

Launch

Arrange students in groups of 2. Give groups 4–5 minutes to discuss and answer the first question, and then 7–8 minutes of quiet time to complete the remaining questions. Provide access to base-ten representations, if needed.



Classes using the digital activities have an interactive applet with virtual blocks. Note that the applet can only represent ones, tenths, and hundredths, so the pieces are large enough to manipulate. To use the bundling and unbundling features, the pieces must be aligned on the light blue grids. To bring a piece into the workspace, select one of the green tool icons and then click on the workspace. To move it, you must click on the Move tool

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organization and problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.

Supports accessibility for: Organization; Attention

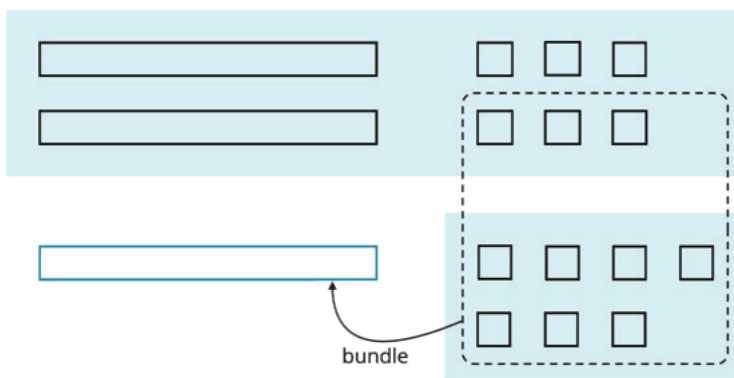
Anticipated Misconceptions

If students have difficulty drawing the diagrams to represent bundling, it might be helpful for them to work with actual base-ten blocks, paper printouts of base-ten blocks, or digital base-ten blocks so that they can physically trade 10 hundredths for 1 tenth or 10 tenths for 1 whole.

Student Task Statement

1. Here are two ways to calculate the value of $0.26 + 0.07$. In the diagram, each rectangle represents 0.1 and each square represents 0.01.

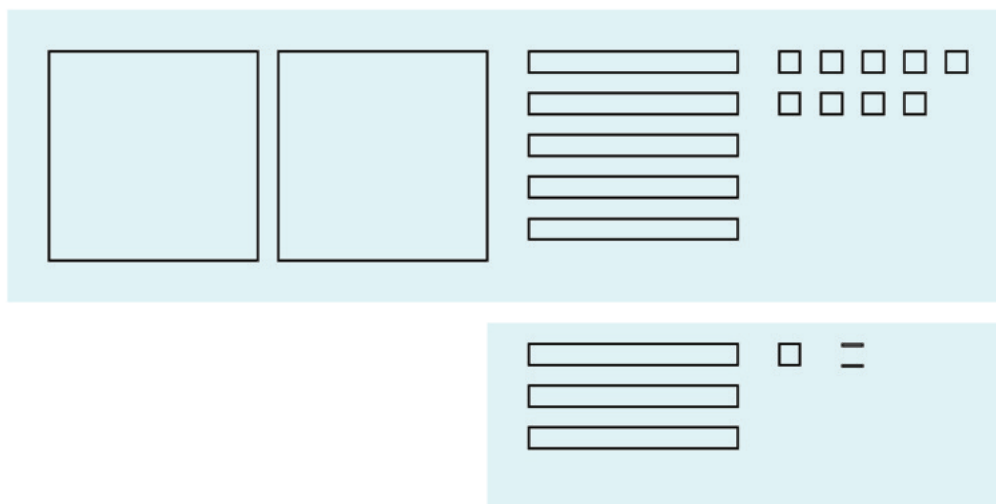
$$\begin{array}{r}
 1 \\
 0.26 \\
 + 0.07 \\
 \hline
 0.33
 \end{array}$$



Use what you know about base-ten units and addition to explain:

- Why ten squares can be “bundled” into a rectangle.
 - How this “bundling” is represented in the vertical calculation.
- Find the value of $0.38 + 0.69$ by drawing a diagram. Can you find the sum without bundling? Would it be useful to bundle some pieces? Explain your reasoning.
 - Calculate $0.38 + 0.69$. Check your calculation against your diagram in the previous question.
 - Find each sum. The larger square represents 1.

a.

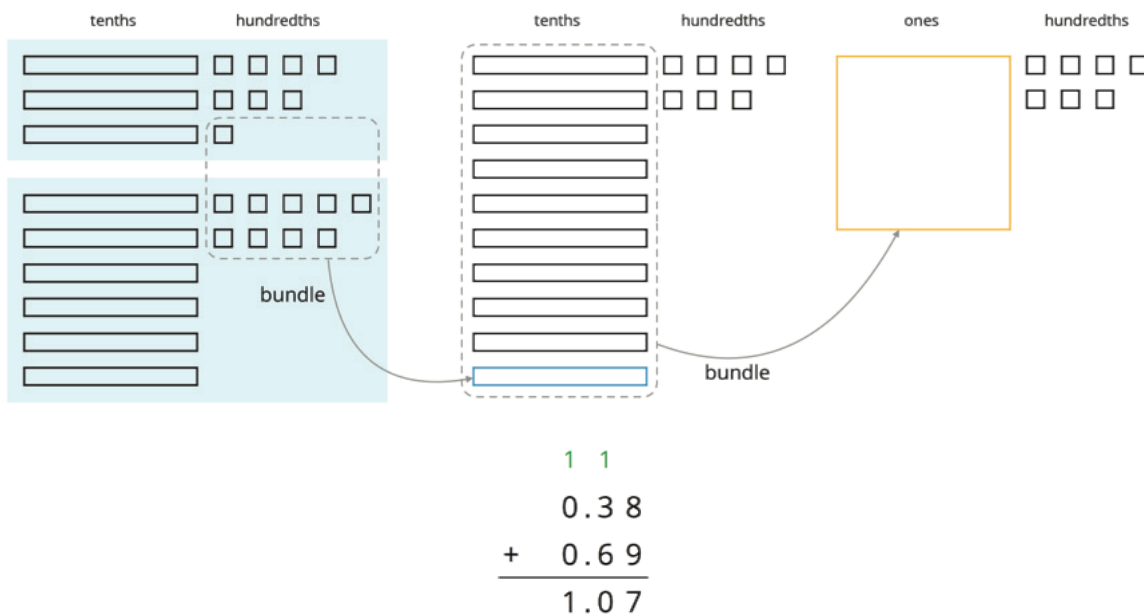


b.

$$\begin{array}{r}
 6.03 \\
 + 0.098 \\
 \hline
 \end{array}$$

Student Response

1. a. Ten squares can be bundled into a rectangle because the squares each represent $\frac{1}{100}$, and the rectangles represent $\frac{1}{10}$. There are ten hundredths in a tenth.
 b. In the computation, the 7 hundredths from 0.07 are combined with 3 of the hundredths from 0.26 to make a tenth.
2. $0.38 + 0.69 = 1.07$



- 3.
4. a. 2.902
 b. 6.128

Are You Ready for More?

A distant, magical land uses jewels for their bartering system. The jewels are valued and ranked in order of their rarity. Each jewel is worth 3 times the jewel immediately below it in the ranking. The ranking is red, orange, yellow, green, blue, indigo, and violet. So a red jewel is worth 3 orange jewels, a green jewel is worth 3 blue jewels, and so on.

1. If you had 500 violet jewels and wanted to trade so that you carried as few jewels as possible, which jewels would you have?
2. Suppose you have 1 orange jewel, 2 yellow jewels, and 1 indigo jewel. If you're given 2 green jewels and 1 yellow jewels, what is the fewest number of jewels that could represent the value of the jewels you have?

Student Response

1. 2 orange, 1 blue, 1 indigo, 2 violet

2. 2 orange, 2 green, 1 indigo

Activity Synthesis

Focus the whole-class debriefing on the idea of choosing appropriate tools to solve a problem, which is an important part of doing mathematics (MP5). Highlight how drawings can effectively help us understand what is happening when we add base-ten numbers before moving on to a more generalized method. Discuss:

- “In which place(s) did bundling happen when adding 0.38 and 0.69?” (In the hundredth and tenth places.) “Why?” (There is a total of 17 hundredths, and 10 hundredths can be bundled to make 1 tenth. This 1 tenth is added to the 3 tenths and 6 tenths, which makes 10 tenths. Ten tenths can be bundled into 1 one.)
- “How can the bundling process be represented in vertical calculations?” (We can show that the 8 hundredths and 9 hundredths make 1 tenth and 7 hundredths by recording 7 hundredths and writing a 1 above the 3 tenths in 0.38.)
- “Which method of calculating is more efficient?” (It depends on the complexity and size of the numbers. The drawings become hard when there are lots of digits or when the digits are large. The algorithm works well in all cases, but it is more abstract and requires that all bundling be recorded in the right places.)

Access for English Language Learners

Listening, Speaking, Representing: MLR7 Compare and Connect. Use this routine when students discuss different ways to calculate and represent decimal sums. Display three different representations for calculating the sum of 6.03 and 0.098 (using base-ten blocks, diagrams, and vertically). Ask students to identify where the bundling occurred and how it is shown in each representation. Draw students' attention to how the action of bundling is represented. For instance, in the base-ten blocks the pieces are physically joined or traded, in the diagram the hundredths are circled, and in the vertical calculation the “1” is notated. Emphasize the mathematical language used to make sense of the different ways to represent bundling. These exchanges strengthen students' mathematical language use and reasoning.

Design Principle(s): Support sense-making; Maximize meta-awareness

2.4 Representing Subtraction

Optional: 15 minutes

In this activity, students use base-ten diagrams and vertical calculations to perform subtraction. As with addition of decimals, students need to pay close attention to place value when calculating differences. They identify the need to pair the digits of like base-ten units when subtracting decimals and why it is helpful to line up the decimal points.

There is no decomposition or “unbundling” of a base-ten unit into smaller units in this activity, that will come up in the next activity.

As in previous activities, consider having students use physical base-ten blocks (if available), a paper version of the base-ten diagrams (from the Instructional master), or this digital applet <https://ggbm.at/n9yaWPQj>, as alternatives to drawing diagrams.

Building On

- 5.NBT.B.7

Addressing

- 6.NS.B.3

Instructional Routines

- Think Pair Share

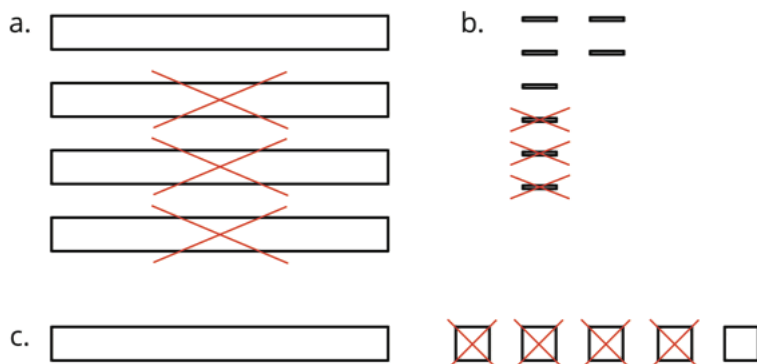
Launch

Arrange students in groups of 2. Review that the term “difference” means the result of a subtraction. Tell students that in this lesson they will use base-ten diagrams to determine differences and, in the diagrams, use X’s to indicate what is being taken away.

Give students 10 minutes of quiet work time. Ask them to pause after the third question, share their diagrams and calculations with a partner, and resolve any differences before finishing the activity.

Student Task Statement

1. Here are diagrams that represent differences. Removed pieces are marked with Xs. The larger rectangle represents 1 tenth. For each diagram, write a numerical subtraction expression and determine the value of the expression.



2. Express each subtraction in words.

- a. $0.05 - 0.02$
- b. $0.024 - 0.003$

c. $1.26 - 0.14$

3. Find each difference by drawing a diagram and by calculating with numbers. Make sure the answers from both methods match. If not, check your diagram and your numerical calculation.

a. $0.05 - 0.02$

b. $0.024 - 0.003$

c. $1.26 - 0.14$

Student Response

1. a. $0.4 - 0.3 = 0.1$

b. $0.008 - 0.003 = 0.005$

c. $0.15 - 0.04 = 0.11$

2. Answers vary. Possible responses:

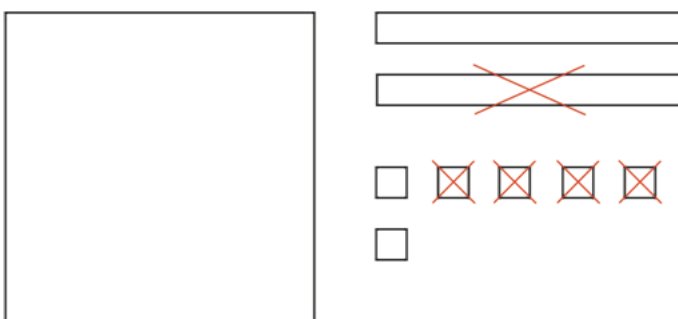
a. five hundredths minus two hundredths

b. the difference between twenty four thousandths and three thousandths

c. subtract fourteen hundredths from one and twenty six hundredths

a. 

b. 

c. 

3.

a.

$$\begin{array}{r} 0.05 \\ - 0.02 \\ \hline 0.03 \end{array}$$

b.

$$\begin{array}{r} 0.024 \\ - 0.003 \\ \hline 0.021 \end{array}$$

c.

$$\begin{array}{r} 1.26 \\ - 0.14 \\ \hline 1.12 \end{array}$$

Activity Synthesis

The goal of the whole-class discussion is to make sure students understand that when we perform subtraction without diagrams, it is essential to pay close attention to place value in the numbers. Select a few students to share their responses and reasonings for the last two questions. Highlight how the different sizes of the base-ten units in the diagram informs how we subtract one decimal from another. Then, discuss:

- How are addition and subtraction of decimal numbers similar? (It is important to attend to place value and to add or subtract numbers that represent the same base-ten units.)
- Did anyone find different results when using diagrams versus when calculating vertically? If so, where did the error happen and what might have caused it?
- Why is it helpful to line up the decimal points when calculating differences of decimals? (Aligning the points helps us align digits with the same place value.)
- Which is more efficient, using base-ten blocks or calculating the difference? (For some numbers, such as $1.26 - 0.14$, both methods are efficient. If the numbers contain more decimal places or larger digits, the diagrams would take a lot of time to draw.)

Lesson Synthesis

One main idea in this lesson is that addition of decimals beyond hundredths works the same way as addition of whole numbers and decimals up to hundredths: all of them rely on combining the values of like base-ten units. Another main idea is bundling: we can group 10 of any base-ten unit into 1 of a base-ten unit that is 10 times as large. The methods we used for adding in the lesson reflect both ideas.

- How do the pieces representing ones, tenths, hundredths, etc. of a base-ten diagram help us add two decimals? (We can combine the pieces that represent the same unit and see the value for each decimal place.)
- When might we want to bundle some of the base-ten pieces? (When we have at least 10 of the same unit.) Why? (It would make it simpler to show or tell the sum.)
- How is adding with vertical calculations similar to and different from using base-ten diagrams? (We still combine numbers based on their place values but without drawing figures to represent each number.)
- When using vertical calculations, how do we make sure that we add like base-ten units? (We line up digits that represent the same place value or line up the decimal point.)
- Which method of calculating is more efficient? (It depends on the size of the numbers, but vertical calculations tend to be quicker. Drawing becomes hard when the numbers have lots of digits, e.g., 2.315641, or when the digits are large, e.g., 9.999.)

2.5 Why or Why Not?

Cool Down: 5 minutes

Addressing

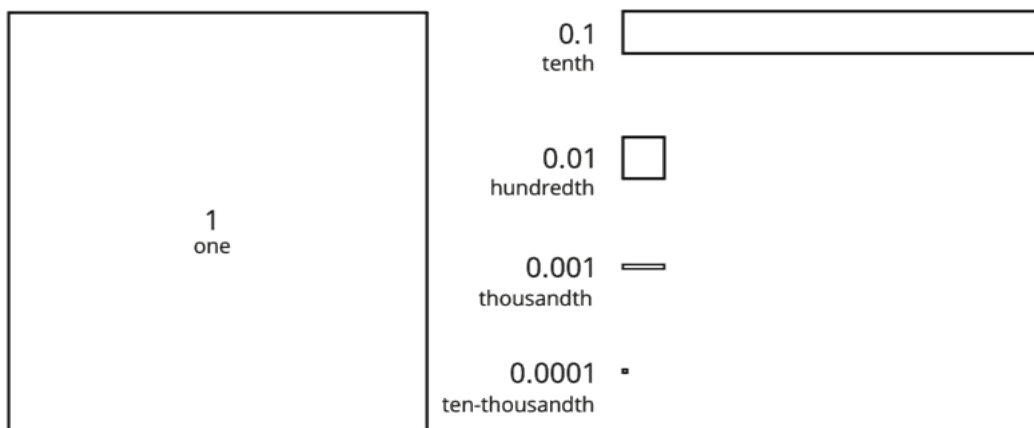
- 6.NS.B.3

Student Task Statement

Is this equation true?

$$0.025 + 0.17 = 0.042$$

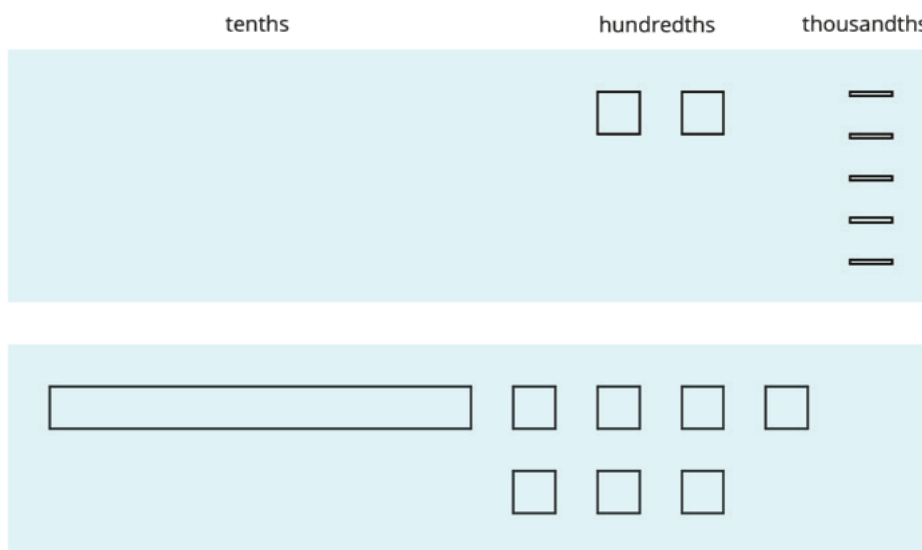
Use a diagram or numerical calculation to explain or show your reasoning. Here are diagrams that you could use to represent base-ten units.



Student Response

The equation is not true. Sample reasoning:

- First, 0.17 is larger than 0.042, so 0.042 cannot be the sum of 0.17 and another decimal.
- The diagram should show 1 medium rectangle (1 tenth), 9 medium squares (9 hundredths), and 5 small rectangles (5 thousandths).



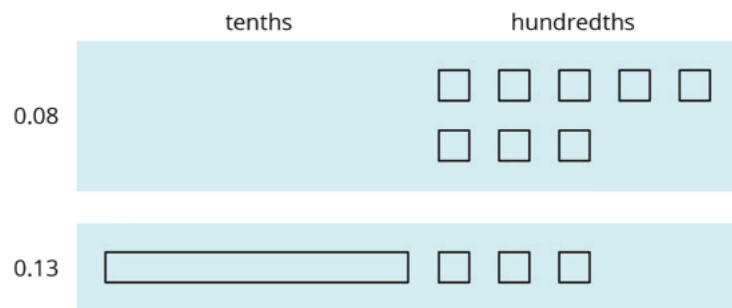
- $0.025 + 0.17 = 0.02 + 0.005 + 0.1 + 0.07 = 0.125 + 0.07 = 0.195$.
- Calculation with numbers should show the decimal points lining up and a sum of 0.195.

$$\begin{array}{r}
 0.025 \\
 + 0.170 \\
 \hline
 0.195
 \end{array}$$

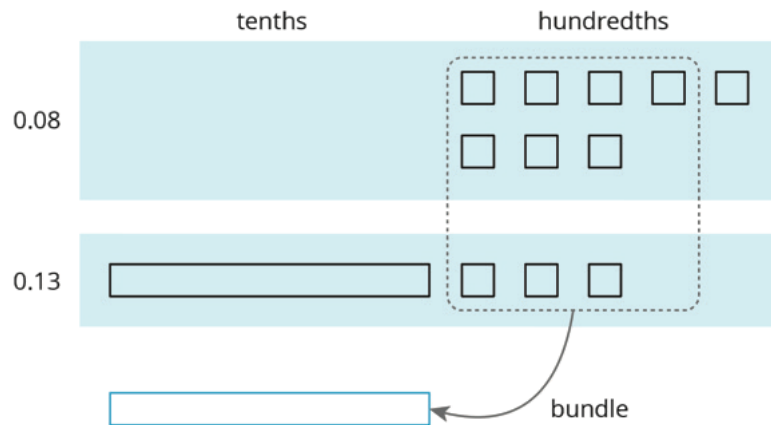
Student Lesson Summary

Base-ten diagrams represent collections of base-ten units—tens, ones, tenths, hundredths, etc. We can use them to help us understand sums of decimals.

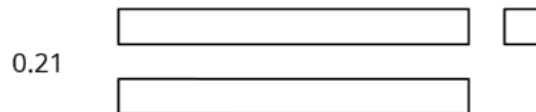
Suppose we are finding $0.08 + 0.13$. Here is a diagram where a square represents 0.01 and a rectangle (made up of ten squares) represents 0.1.



To find the sum, we can “bundle” (or compose) 10 hundredths as 1 tenth.



We now have 2 tenths and 1 hundredth, so $0.08 + 0.13 = 0.21$.

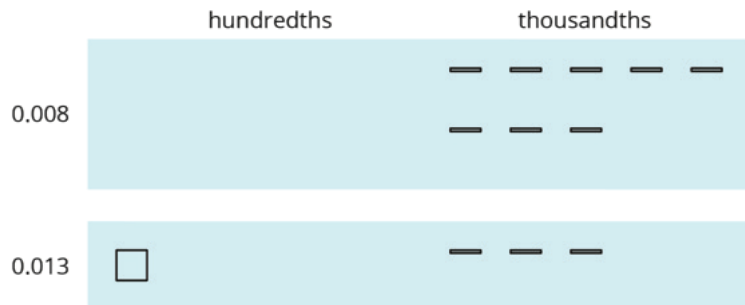


We can also use vertical calculation to find $0.08 + 0.13$.

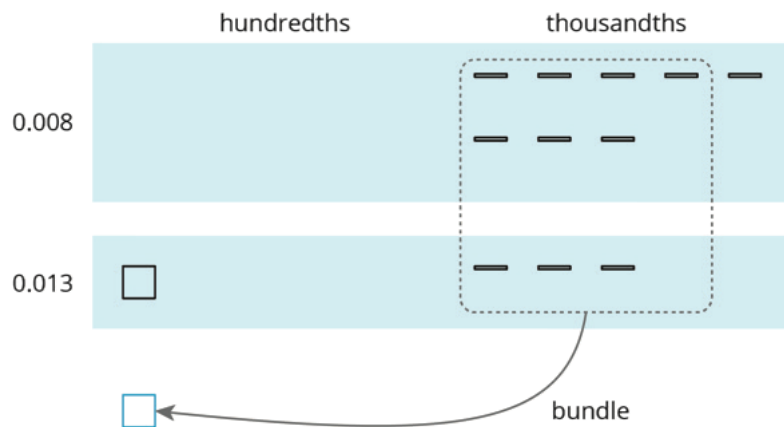
$$\begin{array}{r}
 \overset{1}{0.13} \\
 + 0.08 \\
 \hline
 0.21
 \end{array}$$

Notice how this representation also shows 10 hundredths are bundled (or composed) as 1 tenth.

This works for any decimal place. Suppose we are finding $0.008 + 0.013$. Here is a diagram where a small rectangle represents 0.001.



We can "bundle" (or compose) 10 thousandths as 1 hundredth.



The sum is 2 hundredths and 1 thousandth.

$$\begin{array}{r}
 0.021 \\
 \hline
 \end{array}$$

Here is a vertical calculation of $0.008 + 0.013$.

$$\begin{array}{r}
 \overset{1}{0.013} \\
 + 0.008 \\
 \hline
 0.021
 \end{array}$$


Lesson 2 Practice Problems

Problem 1


Statement

Use the given key to answer the questions.

0.1
tenth 

0.01
hundredth 

0.001
thousandth 

0.0001
ten-thousandth 

a. What number does this diagram represent?

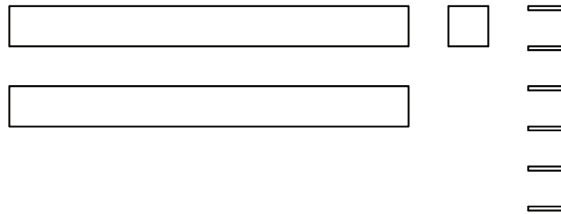


b. Draw a diagram that represents 0.216.

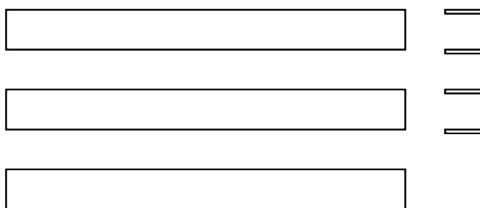
c. Draw a diagram that represents 0.304.

Solution

a. 0.025



b.

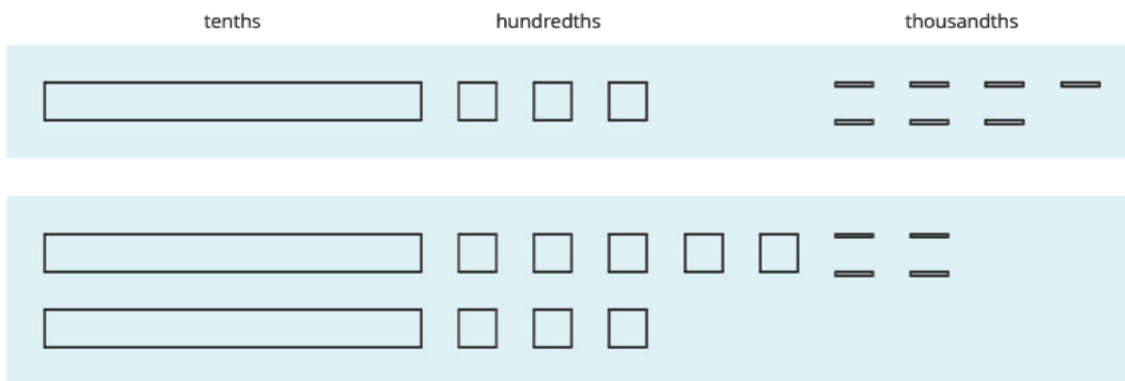


c.

Problem 2

Statement

Here are diagrams that represent 0.137 and 0.284.



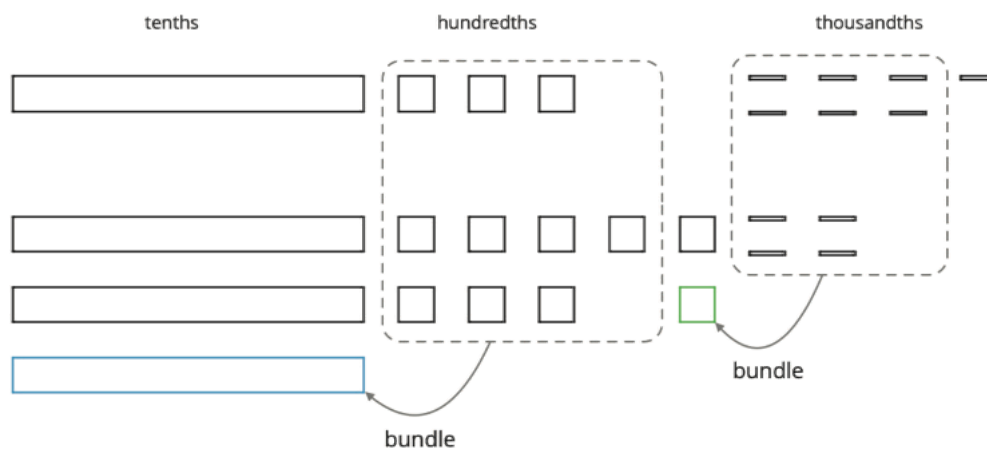
- Use the diagram to find the value of $0.137 + 0.284$. Explain your reasoning.
- Calculate the sum vertically.

$$\begin{array}{r}
 0.137 \\
 + 0.284 \\
 \hline
 \end{array}$$

- How was your reasoning about $0.137 + 0.284$ the same with the two methods? How was it different?

Solution

a.



b.

$$\begin{array}{r}
 0.137 \\
 + 0.284 \\
 \hline
 0.421
 \end{array}$$

- c. Responses vary. Sample response: Using the diagrams, 10 thousandths can be bundled to make 1 hundredth. Then 10 hundredths can be bundled to make 1 tenth. These values can then be combined. Without diagrams, 10 of the thousandths can be converted into 1 hundredth and 10 of the hundredths to 1 tenth. The methods are similar. The diagrams show the bundling, but the method without a diagram is faster.

Problem 3

Statement

For the first two problems, circle the vertical calculation where digits of the same kind are lined up. Then, finish the calculation and find the sum. For the last two problems, find the sum using vertical calculation.

a. $3.25 + 1$

$$\begin{array}{r} 3.25 \\ + 1.0 \\ \hline \end{array} \quad \begin{array}{r} 3.25 \\ + 1.0 \\ \hline \end{array} \quad \begin{array}{r} 3.25 \\ + 1 \\ \hline \end{array}$$

b. $0.5 + 1.15$

$$\begin{array}{r} 0.5 \\ + 1.15 \\ \hline \end{array} \quad \begin{array}{r} 0.5 \\ + 1.15 \\ \hline \end{array} \quad \begin{array}{r} 0.50 \\ + 1.150 \\ \hline \end{array}$$

c. $10.6 + 1.7$

d. $123 + 0.2$

Solution

- a. The second arrangement is correct. The sum is 4.25.
 b. The first arrangement is correct. The sum is 1.65.
 c. 12.3
 d. 123.2

Problem 4

Statement

Andre has been practicing his math facts. He can now complete 135 multiplication facts in 90 seconds.

- a. If Andre is answering questions at a constant rate, how many facts can he answer per second?
 b. Noah also works at a constant rate, and he can complete 75 facts in 1 minute. Who is working faster? Explain or show your reasoning.

Solution

a. 1.5 facts per second ($135 \div 9 = 1.5$)

b. Andre is faster, because Noah can only answer 1.25 facts per second. ($75 \div 60 = 1.25$)

(From Unit 2, Lesson 9.)

Lesson 3: Adding and Subtracting Decimals with Few Non-Zero Digits

Goals

- Add or subtract decimals, and explain the reasoning (using words and other representations).
- Comprehend the term “unbundle” means to decompose a larger base-ten unit into 10 units of lower place value (e.g., 1 tenth as 10 hundredths).
- Recognize and explain (orally) that writing additional zeros or removing zeros after the last non-zero digit in a decimal does not change its value.

Learning Targets

- I can tell whether writing or removing a zero in a decimal will change its value.
- I know how to solve subtraction problems with decimals that require “unbundling” or “decomposing.”

Lesson Narrative

As with addition, prior to grade 6 students have used various ways to subtract decimals to hundredths. Base-ten diagrams and vertical calculations are likewise used for subtracting decimals. “Unbundling,” which students have previously used to subtract whole numbers, is a key idea here. They recall that a base-ten unit can be expressed as another unit that is $\frac{1}{10}$ its size. For example, 1 tenth can be “unbundled” into 10 hundredths or into 100 thousandths. Students use this idea to subtract a larger digit from a smaller digit when both digits are in the same base-ten place, e.g., $0.012 - 0.007$. Rather than thinking of subtracting 7 thousandths from 1 hundredth and 2 thousandths, we can view the 1 hundredth as 10 thousandths and subtract 7 thousandths from 12 thousandths.

Unbundling also suggests that we can write a decimal in several equivalent ways. Because 0.4 can be viewed as 4 tenths, 40 hundredths, 400 thousandths, or 4,000 ten-thousandths, it can also be written as 0.40, 0.400, 0.4000, and so on; the additional zeros at the end of the decimal do not change its value. They use this idea to subtract a number with more decimal places from one with fewer decimal places (e.g., $2.5 - 1.028$). These calculations depend on making use of the structure of base-ten numbers (MP7).

The second activity is optional; it gives students additional opportunities to practice summing decimals.

Alignments

Building On

- 5.NBT.A: Understand the place value system.

Addressing

- 6.NS.B.3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR2: Collect and Display
- Think Pair Share

Required Preparation

Students draw base-ten diagrams in this lesson. If drawing them is a challenge, consider giving students access to:

- Commercially produced base-ten blocks, if available.
- Paper copies of squares and rectangles (to represent base-ten units), cut outs from copies of the Instructional master of the second lesson in the unit.
- Digital applet of base-ten representations <https://ggbm.at/zqxRkhMh>.

Some students might find it helpful to use graph paper to help them align the digits for vertical calculations. Consider having graph paper accessible for these activities: Representing Decimal Subtraction and Enough to Subtract?

Student Learning Goals

Let's add and subtract decimals.

3.1 Do the Zeros Matter?

Warm Up: 5 minutes

This warm-up prompts students to reason about regrouping and about when the zeros in a decimal affect the number that it represents. The mathematical work of interest is how students combine two decimals (e.g., in analyzing $1.009 + 0.391$, do they see that $0.009 + 0.001 = 0.010$?) and how they write the sum (e.g., do they know $1.4 = 1.40 = 1.400$?).

Building On

- 5.NBT.A

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 1 minute of quiet time to mentally add the decimals in the first problem and then another minute to discuss their answer and strategy with a partner.

Briefly discuss their strategies as a class, and then ask students to complete the true-or-false questions.

Anticipated Misconceptions

Some students may say, “You can take the zeros away after the decimal point and the number stays the same.” Although students could mean, for instance, that 12.9 is equal to 12.90, they might also mistakenly think 12.09 is equal 12.9. Ask these students to be more precise in their statement. Ask if the zero can be in any place after the decimal point or only in certain places.

Student Task Statement

1. Evaluate mentally: $1.009 + 0.391$
2. Decide if each equation is true or false. Be prepared to explain your reasoning.
 - a. $34.56000 = 34.56$
 - b. $25 = 25.0$
 - c. $2.405 = 2.45$

Student Response

1. 1.4 or 1.40 or 1.400. Strategies vary.
2.
 - a. true
 - b. true
 - c. false (405 thousandths does not have the same value as 45 hundredths)

Activity Synthesis

Ask students to indicate whether they think each equation is true or false and ask for an explanation for each. Students may simply say that we can or cannot just remove the zeros. Encourage them to use what they know about place values or comparison strategies to explain why one number is greater than, less than, or equal to the other. If students do not notice that the two numbers in the true-or-false questions have the same digits except for the missing zeros, point that out after each question.

If not mentioned by students in their explanations, ask:

- “Can zeros be written at the end of a decimal without changing the number that it represents?”
- “Can zeros be eliminated from the end of a decimal without changing the value?”
- “Can zeros be written or erased in the middle of a decimal without changing the value?”

3.2 Calculating Sums

Optional: 15 minutes (there is a digital version of this activity)

Here students continue to use diagrams to represent sums of decimals, but they also transition to writing addition calculations vertically. They think about the alignment of the digits in vertical calculations to help ensure that correct values are combined. This activity is optional, so students have the option to spend more time subtracting decimals in the next activity.

As in the previous activity, consider having students use physical base-ten blocks (if available), a paper version of the base-ten figures (from the Instructional master), or this digital applet ggbm.at/FXEZD466, as alternatives to drawing diagrams.

Addressing

- 6.NS.B.3

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- Think Pair Share

Launch

Remind students that the term “sum” means the result of an addition. Refer to the image from the previous activity that shows how several squares and rectangles were used to represent base-ten units. Tell students to use the same representations in this activity and to keep in mind that in the process of bundling they find more sums of decimals.

Arrange students in groups of 2. Give students 10 minutes of quiet work time, but encourage them to briefly discuss their responses with their partner after completing the second question and before continuing with the rest. Follow with a whole-class discussion.



Classes using the digital activities have an interactive applet with virtual blocks. In this activity, students must redefine the value of each block to represent the place values in each problem. To use the bundling and unbundling features, the pieces must be aligned on the light blue grids. To bring a piece into the workspace, select one of the green tool icons and then click on the workspace. To move it, you must click on the Move tool

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Encourage students to begin with physical representations before drawing a diagram. Provide access to physical or virtual base-ten blocks to support drawing diagrams.

Supports accessibility for: Conceptual processing

Access for English Language Learners

Conversing: MLR1 Stronger and Clearer Each Time. Use this routine to give students an opportunity to refine their explanation about which calculation is correct for $0.2 + 0.05$. At the appropriate time, give students time to meet with 2–3 partners to share their response. Display prompts students can use to provide feedback such as, “How did the alignment of the decimals change the sum?”, “Why is aligning decimal places important here?” and “A detail (or word) you could add is _____, because” As students discuss, listen to how they talk about combining units, aligning digits of the same place values, and whether it is necessary to add zeros. Give students 1–2 minutes to revise their writing based on their conversations.

Design Principle(s): Optimize output (for justification)

Student Task Statement

1. Andre and Jada drew base-ten diagrams to represent $0.007 + 0.004$. Andre drew 11 small rectangles. Jada drew only two figures: a square and a small rectangle.

Andre 

Jada 

- a. If both students represented the sum correctly, what value does each small rectangle represent? What value does each square represent?
- b. Draw or describe a diagram that could represent the sum $0.008 + 0.07$.
2. Here are two calculations of $0.2 + 0.05$. Which is correct? Explain why one is correct and the other is incorrect.

$$\begin{array}{r} 0.2 \\ + 0.05 \\ \hline 0.25 \end{array} \qquad \begin{array}{r} 0.2 \\ + 0.05 \\ \hline 0.07 \end{array}$$

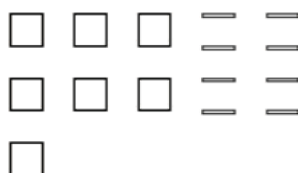
3. Compute each sum. If you get stuck, consider drawing base-ten diagrams to help you.
- a.

$$\begin{array}{r} 0.11 \\ + 0.005 \\ \hline \end{array}$$

- b. $0.209 + 0.01$
- c. $10.2 + 1.1456$

Student Response

1. a. A square represents 1 hundredth. A small rectangle represents 1 thousandth.
b. 7 squares (for 7 hundredths) and 8 small rectangles (for 8 thousandths)



2. The first response is correct, and the second response is incorrect. Sample reasoning:
Digits that represent unlike units were combined, so the sum would be off. Adding 2 tenths and 5 hundredths would not produce 7 hundredths.
3. a. 0.115
b. 0.219
c. 11.3456

Activity Synthesis

The goal of this discussion is to help students understand that vertical calculation is an efficient way to find the sums of decimals. Discuss:

- “When finding $0.008 + 0.07$, why do we not combine the 8 thousandths and 7 hundredths to make 15?” (Hundredths and thousandths are different units. If each hundredth is unbundled into 10 thousandths, we can add 70 thousandths and 8 thousandths to get 78 thousandths).
- “How do we use representations of base-ten numbers to add effectively and efficiently?” (Make sure to put together tenths with tenths, hundredths with hundredths, etc. Also, make sure that if a large square represents $\frac{1}{10}$ for one summand, it also represents $\frac{1}{10}$ for the other.)
- “When adding numbers without using base-ten diagrams or other representations, what can we do to help add them correctly?” (Pay close attention to place value so we combine only like units. It is helpful to line up the digits of the numbers so that numerals that represent the same place value are placed directly on top of one another.)

Additionally, consider using color coding to help students visualize the place-value structure, as shown here.

$$\begin{array}{r} 0.2 \\ + 0.05 \\ \hline 0.25 \end{array}$$

3.3 Subtracting Decimals of Different Lengths

25 minutes (there is a digital version of this activity)

In this activity, students encounter two variations of decimal subtraction in which regrouping is involved. They subtract a number with more decimal places from one with fewer decimal places (e.g., $0.1 - 0.035$), and subtract two digits that represent the same place value but where the value in the second number is greater than that in the starting number (e.g., in $1.12 - 0.47$, both the tenth and hundredth values in the second number is larger than those in the first).

Students represent these situations with base-ten diagrams and study how to perform them using vertical calculations. The big idea here is that of “unbundling” or of decomposing a unit with 10 of another unit that is $\frac{1}{10}$ its size to make it easier to subtract. In some cases, students would need to decompose twice before subtracting (e.g., a tenth into 10 hundredths, and then 1 hundredth into 10 thousandths).

Use the whole-class discussion to highlight the correspondences between the two methods and to illustrate how writing zeros at the end of a decimal helps us perform subtraction.

As in previous activities, consider having students use physical base-ten blocks (if available), a paper version of the base-ten figures (from the Instructional master), or this digital applet <https://ggbm.at/FXEZD466>, as alternatives to drawing diagrams.

Addressing

- 6.NS.B.3

Instructional Routines

- MLR2: Collect and Display

Launch

Keep students in the same groups of 2. Give partners 4–5 minutes to complete the first two questions. Then, give students 4–5 minutes of quiet time to complete the last question and follow with a whole-class discussion.



Classes using the digital activities have an interactive applet with virtual blocks. Note that students may need to reassign the values of the blocks to answer the questions. To use the bundling and unbundling features, the pieces must be aligned on the light blue grids. To bring a piece into the workspace, select one of the green tool icons and then click on the workspace. To move it, you must click on the Move tool Subtract by deleting with the delete tool

Use MLR2 (Collect and Display) to listen for and capture two or three different ways students refer to the idea of “unbundling” as they work on problems 1 and 2.

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Encourage students to begin with physical representations before drawing a diagram. Provide access to physical or virtual base-ten blocks to support drawing diagrams.

Supports accessibility for: Conceptual processing

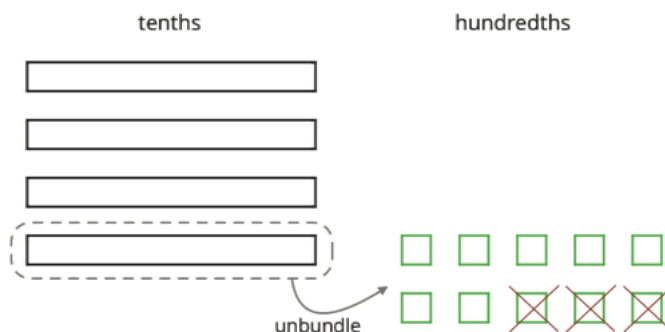
Anticipated Misconceptions

If students struggle to subtract two numbers that do not have the same number of decimal digits (such as in questions 3a and 3d), consider representing the subtraction with base-ten diagrams. For example, to illustrate $0.3 - 0.05$, start by drawing 3 large rectangles to represent 3 tenths. Replace 1 rectangle with 10 squares, each representing 1 hundredth. Cross out 5 squares to show subtraction of 5 hundredths. Alternatively, replace 3 rectangles (3 tenths) with 30 squares (30 hundredths), cross out 5 squares to show subtraction of 5 hundredths.

Student Task Statement

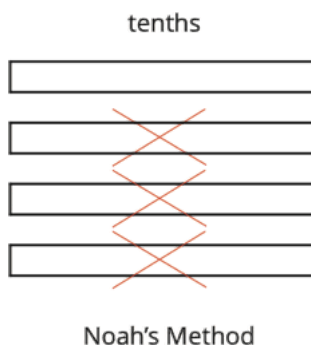
Diego and Noah drew different diagrams to represent $0.4 - 0.03$. Each rectangle represents 0.1. Each square represents 0.01.

- Diego started by drawing 4 rectangles to represent 0.4. He then replaced 1 rectangle with 10 squares and crossed out 3 squares to represent subtraction of 0.03, leaving 3 rectangles and 7 squares in his diagram.

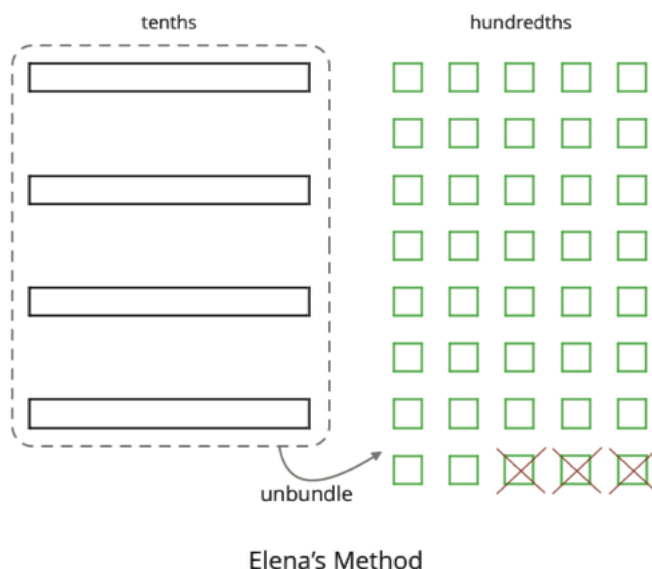


Diego's Method

- Noah started by drawing 4 rectangles to represent 0.4. He then crossed out 3 rectangles to represent the subtraction, leaving 1 rectangle in his diagram.



1. Do you agree that either diagram correctly represents $0.4 - 0.03$? Discuss your reasoning with a partner.
2. Elena also drew a diagram to represent $0.4 - 0.03$. She started by drawing 4 rectangles. She then replaced all 4 rectangles with 40 squares and crossed out 3 squares to represent subtraction of 0.03, leaving 37 squares in her diagram. Is her diagram correct? Discuss your reasoning with a partner.

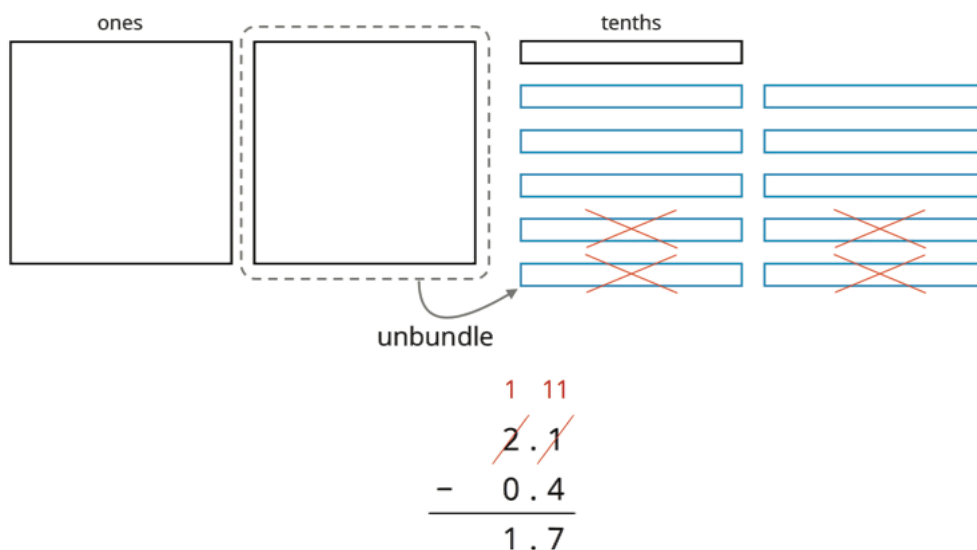


3. Find each difference. Explain or show your reasoning.
 - a. $0.3 - 0.05$
 - b. $2.1 - 0.4$
 - c. $1.03 - 0.06$
 - d. $0.02 - 0.007$

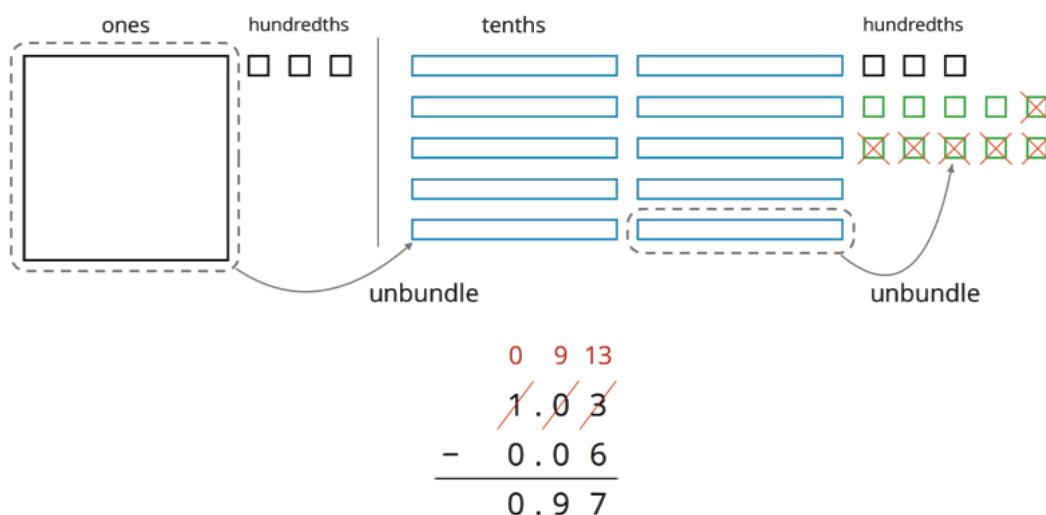
Student Response

1. Answers vary. Sample reasoning:

- I agree with Diego. Since 10 hundredths is 1 tenth, 1 rectangle can be replaced with 10 squares. Subtraction of 0.03 means taking away 3 hundredths or 3 small squares.
 - I disagree with Noah's representation. He removed 3 tenths not 3 hundredths.
2. Yes, her diagram is correct. Sample reasoning: Four rectangles is 4 tenths, which is equal to 40 hundredths. She correctly removed 3 hundredths from 40 hundredths.
3. a. 0.25. Sample reasoning: 0.3 is 3 tenths or 30 hundredths. Subtracting 0.05 or 5 hundredths from 30 hundredths leaves 25 hundredths, which is 0.25.
- b. 1.7. Sample reasoning:



c. 0.97. Sample reasoning:



- d. 0.013. Sample reasoning: 0.02 is 2 hundredths. One of the hundredths could be unbundled into 10 thousandths so that 7 thousandths could be subtracted. What remain are 1 hundredth and 3 thousandths, which is 0.013.

Are You Ready for More?

A distant, magical land uses jewels for their bartering system. The jewels are valued and ranked in order of their rarity. Each jewel is worth 3 times the jewel immediately below it in the ranking. The ranking is red, orange, yellow, green, blue, indigo, and violet. So a red jewel is worth 3 orange jewels, a green jewel is worth 3 blue jewels, and so on.

At the Auld Shoppe, a shopper buys items that are worth 2 yellow jewels, 2 green jewels, 2 blue jewels, and 1 indigo jewel. If they came into the store with 1 red jewel, 1 yellow jewel, 2 green jewels, 1 blue jewel, and 2 violet jewels, what jewels do they leave with? Assume the shopkeeper gives them their change using as few jewels as possible.

Student Response

2 orange jewels, 1 yellow jewel, 1 green jewel, and 2 indigo jewels

Activity Synthesis

The purpose of this discussion is for students to make connections between two different and correct ways to subtract decimals when unbundling is required. Ask:

- “What is the difference between Diego’s method and Elena’s method?” (Diego only breaks up 1 tenth into 10 hundredths, whereas Elena breaks up all 4 tenths into hundredths.)
- “What are some advantages to Diego’s method?” (Diego’s method is quicker to draw. It shows the 3 tenths and 7 hundredths. Elena would need to count how many hundredths she has.)
- “What are some advantages to Elena’s method?” (Elena’s diagram shows a difference of 37 hundredths, which matches how we say 0.37 in words.)

Consider connecting the diagrams for Diego’s and Elena’s work with numerical equations as shown here.

Diego’s	Elena’s
$\begin{aligned} &0.4 - 0.03 \\ &= 0.3 + 0.10 - 0.03 \\ &= 0.3 + 0.07 \\ &= 0.37 \end{aligned}$	$\begin{aligned} &0.4 - 0.03 \\ &= 0.40 - 0.03 \\ &= 0.37 \end{aligned}$

We can also show both calculations by arranging the numbers vertically. On the left, the 3 and 10 in red show Diego’s unbundling of the 4 hundredths. The calculation on the right illustrates Elena’s representation: the 0 written in blue helps us see 4 tenths as 40 hundredths, from which we can subtract 3 hundredths to get 37 hundredths. Use this example to reinforce that writing an additional zero at the end of a non-zero decimal does not change its value.

$$\begin{array}{r}
 \text{3 10} \\
 0.\cancel{4} \\
 - 0.03 \\
 \hline
 0.37
 \end{array}
 \qquad
 \begin{array}{r}
 0.40 \\
 - 0.03 \\
 \hline
 0.37
 \end{array}$$

To deepen students' understanding, consider asking:

- "In the problem $2.1 - 0.4$, how does unbundling in the diagram show up in the vertical calculation with numbers?" (In the diagram, we unbundle 1 whole to make 10 tenths. With the calculation, we rewrite 1 whole as 10 tenths (over the tenth place) and combine it to the given 1 tenth before subtracting 4 tenths.)
- "When might it be really cumbersome to subtract using base-ten diagrams? Can you give examples?" (When the numbers involve many decimal places, such as $113.004 - 6.056802$, or a problem with large digits, such as $7.758 - 0.869$.)

Emphasize that the algorithm (vertical calculation) for subtraction of decimals works like the algorithm for subtraction of whole numbers. The only difference is that the values involved in the subtraction problems can now include tenths, hundredths, thousandths, and so on. The key in both cases is to pay close attention to the place values of the digits in the two numbers.

Lesson Synthesis

In this lesson, we saw that decimal subtraction problems can be done with base-ten diagrams or with vertical calculations. In both cases, it is important to subtract the values that are in the same decimal place. We also saw that zeros can be written to or removed from the end of a decimal without changing the value of the number.

- When using base-ten blocks to represent subtraction of decimals, how do we remove a larger value from a smaller value that are in the same decimal place? For example, to find $4.5 - 2.7$, how do we remove 7 tenths from 5 tenths? (We unbundle a larger unit into 10 of a smaller unit; in this case, we exchange a 1 with 10 tenths, which allows us to subtract 7 tenths.)
- When calculating differences of decimals, why should we line up the decimal points or digits in the same decimal places? (The value of any digit in a base-ten number depends on its place. Lining up the decimal points and like units help us subtract correctly.)
- How do we subtract a number with more decimal places with one with fewer decimal places (e.g., $4.1 - 1.0935$)? (We can write zeros at the end of the shorter decimal to help us subtract.)
- Which are more efficient for finding differences, using base-ten diagrams or using vertical calculations? (It depends on the length of the number and the size of the digits. Base-ten diagrams may take a while to draw.)

3.4 Calculate the Difference

Cool Down: 5 minutes

Addressing

- 6.NS.B.3

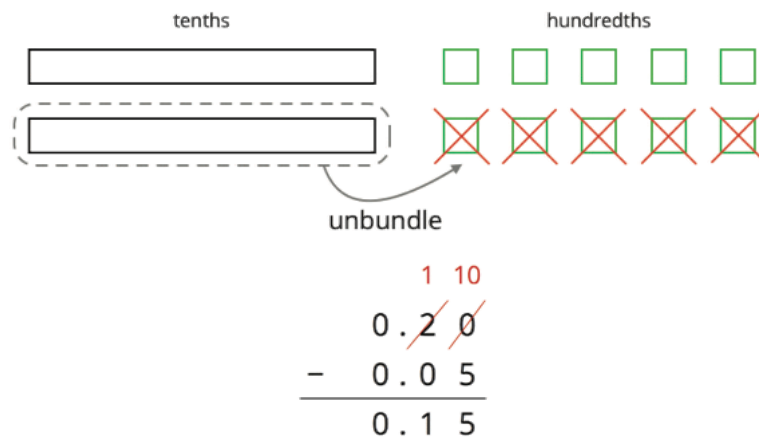
Student Task Statement

1. Find the sum $1.56 + 0.083$. Show your reasoning.
2. Find the difference $0.2 - 0.05$. Show your reasoning.
3. You need to be at least 39.37 inches tall (about a meter) to ride on a bumper car. Diego's cousin is 35.75 inches tall. How many more inches will he need to grow before Diego can take him on the bumper car ride? Explain or show your reasoning.

Student Response

1. $1.56 + 0.083 = 1.643$

2. $0.2 - 0.05 = 0.15$

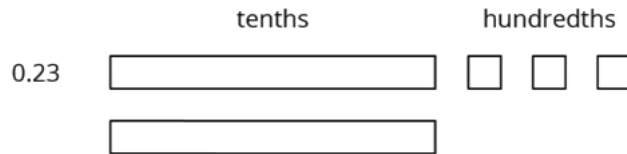


3. 3.62 inches taller

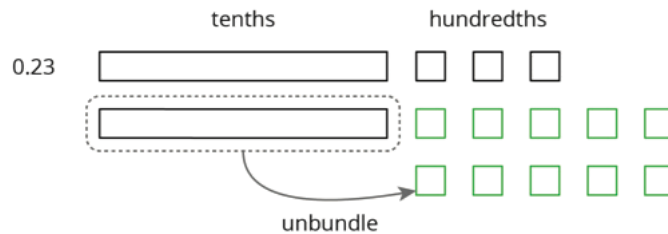
$$\begin{array}{r} 8 \ 13 \\ 3 \cancel{9}.\cancel{3} \ 7 \\ - 3 \ 5.\ 7 \ 5 \\ \hline 3.\ 6 \ 2 \end{array}$$

Student Lesson Summary

Base-ten diagrams can help us understand subtraction as well. Suppose we are finding $0.23 - 0.07$. Here is a diagram showing 0.23, or 2 tenths and 3 hundredths.



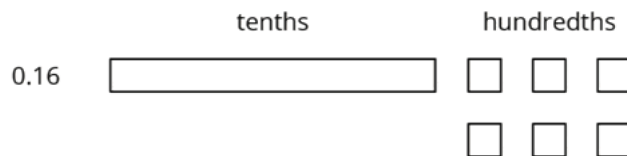
Subtracting 7 hundredths means removing 7 small squares, but we do not have enough to remove. Because 1 tenth is equal to 10 hundredths, we can “unbundle” (or decompose) one of the tenths (1 rectangle) into 10 hundredths (10 small squares).



We now have 1 tenth and 13 hundredths, from which we can remove 7 hundredths.



We have 1 tenth and 6 hundredths remaining, so $0.23 - 0.07 = 0.16$.

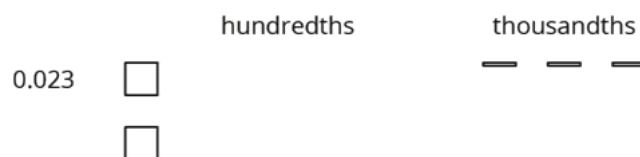


Here is a vertical calculation of $0.23 - 0.07$.

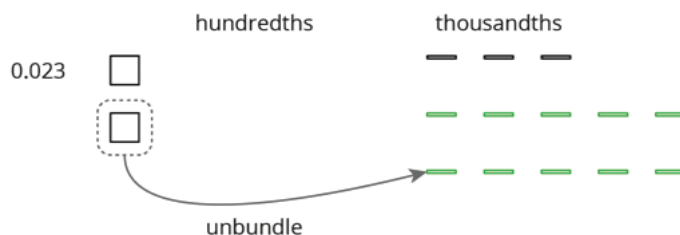
$$\begin{array}{r}
 1 \ 13 \\
 0.\cancel{2}\ \cancel{3} \\
 - \ 0.0\ 7 \\
 \hline
 0.1\ 6
 \end{array}$$

Notice how this representation also shows a tenth is unbundled (or decomposed) into 10 hundredths in order to subtract 7 hundredths.

This works for any decimal place. Suppose we are finding $0.023 - 0.007$. Here is a diagram showing 0.023.



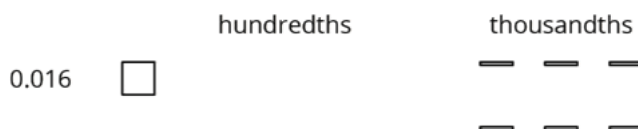
We want to remove 7 thousandths (7 small rectangles). We can “unbundle” (or decompose) one of the hundredths into 10 thousandths.



Now we can remove 7 thousandths.



We have 1 hundredth and 6 thousandths remaining, so $0.023 - 0.007 = 0.016$.



Here is a vertical calculation of $0.023 - 0.007$.

$$\begin{array}{r}
 02 \\
 - 00 \\
 \hline
 01
 \end{array}$$

The vertical calculation shows the subtraction of 0.007 from 0.023. The digits 2 and 3 are crossed out, and the result 0.016 is shown below the line. Above the 2 and 3, the numbers 1 and 13 are written in red, indicating the borrowing process.

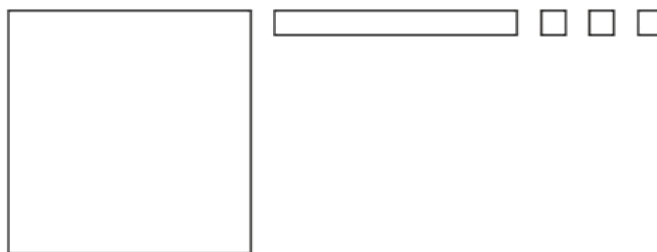
Lesson 3 Practice Problems

Problem 1

Statement

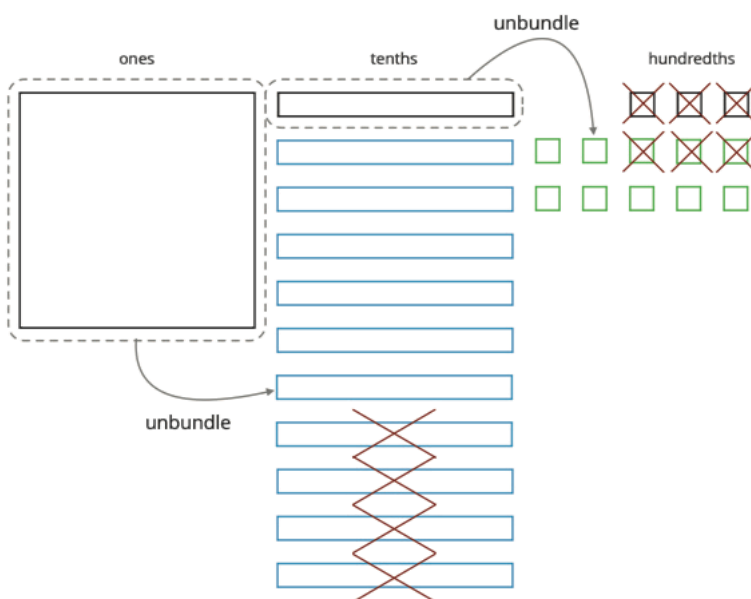
Here is a base-ten diagram that represents 1.13. Use the diagram to find $1.13 - 0.46$.

Explain or show your reasoning.



Solution

0.67. Sample response: First, unbundle 1 tenth into 10 hundredths and then take away 6 hundredths from the 13 hundredths, leaving 7 hundredths. Next, unbundle the 1 one as 10 tenths. After taking away 4 tenths, 6 tenths are left. So the answer is 0.67.



Problem 2

Statement

Compute the following sums. If you get stuck, consider drawing base-ten diagrams.

- $0.027 + 0.004$
- $0.203 + 0.01$
- $1.2 + 0.145$

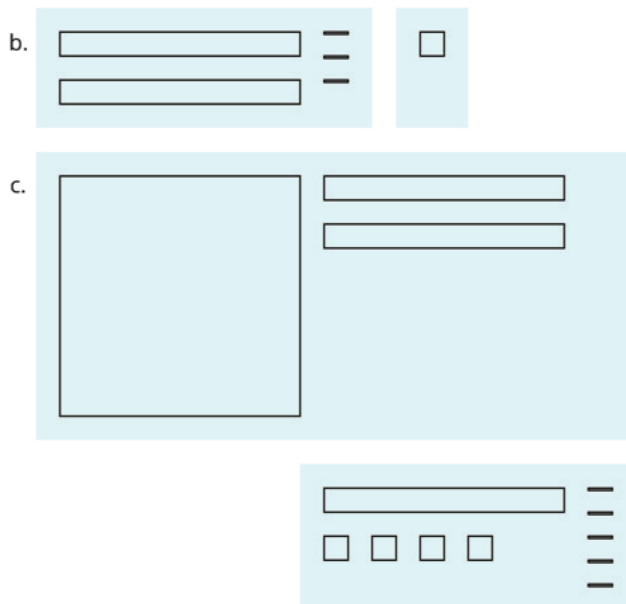
Solution

a. 0.031

b. 0.213

c. 1.345

(Diagrams for b and c shown here.)



Problem 3

Statement

A student said we cannot subtract 1.97 from 20 because 1.97 has two decimal digits and 20 has none. Do you agree with him? Explain or show your reasoning.

Solution

Disagree. Sample explanation: The number 1.97 is equal to 197 hundredths. 20 can be written as 20.00 or 2,000 hundredths. We can subtract 197 from 2,000 to get 1,803 hundredths, so $20 - 1.97 = 18.03$.

Problem 4

Statement

Decide which calculation shows the correct way to find $0.3 - 0.006$ and explain your reasoning.

A

$$\begin{array}{r} 0.3 \\ - 0.006 \\ \hline 0.306 \end{array}$$

B

$$\begin{array}{r} 0.3 \\ - 0.006 \\ \hline 0.097 \end{array}$$

C

$$\begin{array}{r} 0.30 \\ - 0.006 \\ \hline 0.024 \end{array}$$

D

$$\begin{array}{r} 0.300 \\ - 0.006 \\ \hline 0.294 \end{array}$$

Solution

D. Sample reasoning: It is the only one that shows the decimal points correctly lined up so that the same base-ten units are aligned vertically.

Problem 5

Statement

Complete the calculations so that each shows the correct difference.

a.

$$\begin{array}{r} 142.6 \\ - 1.4 \\ \hline \boxed{}\boxed{}\boxed{}.2 \end{array}$$

b.

$$\begin{array}{r} 38.60 \\ - 6.75 \\ \hline \boxed{}\boxed{}.\boxed{}5 \end{array}$$

c.

$$\begin{array}{r} 241.76 \\ - 2.18 \\ \hline \boxed{}\boxed{}\boxed{}.\boxed{}8 \end{array}$$

Solution

a.

$$\begin{array}{r} 142.6 \\ - 1.4 \\ \hline \boxed{1}\boxed{4}\boxed{1}.2 \end{array}$$

b.

$$\begin{array}{r} 38.60 \\ - 6.75 \\ \hline \boxed{3}\boxed{1}.\boxed{8}5 \end{array}$$

c.

$$\begin{array}{r} 241.76 \\ - 2.18 \\ \hline \boxed{2}\boxed{3}\boxed{9}.\boxed{5}8 \end{array}$$

Problem 6

Statement

The school store sells pencils for \$0.30 each, hats for \$14.50 each, and binders for \$3.20 each. Elena would like to buy 3 pencils, a hat, and 2 binders. She estimated that the cost will be less than \$20.

- Do you agree with her estimate? Explain your reasoning.
- Estimate the number of pencils could she buy with \$5. Explain or show your reasoning.

Solution

- a. Disagree. Sample reasoning: The hat costs more than \$14, and two binders cost more than \$6. Even without the pencils the cost is already more than \$20.
- b. Answers vary, but should be around 15 or 16. Sample reasoning: She could buy 3 pencils for every dollar, so for \$5, she could buy around 15 pencils.

(From Unit 5, Lesson 1.)

Problem 7

Statement

A rectangular prism measures $7\frac{1}{2}$ cm by 12 cm by $15\frac{1}{2}$ cm.

- a. Calculate the number of cubes with edge length $\frac{1}{2}$ cm that fit in this prism.
- b. What is the volume of the prism in cm^3 ? Show your reasoning. If you are stuck, think about how many cubes with $\frac{1}{2}$ -cm edge lengths fit into 1 cm^3 .

Solution

- a. 11,160 cubes
- b. 1395 cm^3 . Sample reasoning: Eight $\frac{1}{2}$ cm cubes fit in a 1 cm cube and 11,160 of these $\frac{1}{2}$ cm cubes fit in the prism. So, $11,160 \div 8$ of the 1 cm cubes fit in the prism. That means the volume of the prism in cm^3 is $11,160 \div 8 = 1395$.

(From Unit 4, Lesson 15.)

Problem 8

Statement

At a constant speed, a car travels 75 miles in 60 minutes. How far does the car travel in 18 minutes? If you get stuck, consider using the table.

minutes	distance in miles
60	75
6	
18	

Solution

22.5 miles (or equivalent). Possible strategy:

minutes	distance in miles
60	75
6	7.5
18	22.5

(From Unit 2, Lesson 12.)

Lesson 4: Adding and Subtracting Decimals with Many Non-Zero Digits

Goals

- Add or subtract decimals with multiple non-zero digits, and explain (orally) the solution method.
- Interpret a description (in written language) of a real-world situation involving decimals, and write an addition or subtraction problem to represent it.
- Recognize and explain (orally) that vertical calculation is an efficient strategy for adding and subtracting decimals, especially decimals with multiple non-zero digits.

Learning Targets

- I can solve problems that involve addition and subtraction of decimals.

Lesson Narrative

This lesson strengthens students' ability to add and subtract decimals, enabling them to work toward fluency. Students encounter longer decimals (beyond thousandths), find missing addends, and work with decimals in the context of situations. They decide which operation (addition or subtraction) to perform and which strategy to use when finding sums and differences. They also reinforce the idea that we can express a decimal in different but equivalent ways, and that writing additional zeros after the last non-zero digit in a decimal does not change its value. Students use this understanding to practice subtracting numbers with more decimal places from those with fewer decimal places (e.g., $1.9 - 0.4563$).

To solve these problems, students must lean heavily on their understanding of base-ten numbers (MP7). Given problems such as $7 - ? = 3.4567$ and $0.404 + ? = 1$, they need to think carefully about the meaning of each place value, the meaning of addition and subtraction, and potential paths toward the solution. Along the way, they also begin to see patterns in the calculations, which enables them to become increasingly fluent in finding sums and differences (MP8).

Alignments

Addressing

- 6.NS.B.3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR6: Three Reads

- MLR8: Discussion Supports
- Notice and Wonder
- Think Pair Share

Required Preparation

Some students might find it helpful to use graph paper to help them align the digits for vertical calculations. Consider having graph paper accessible for the Decimals All Around activity.

Student Learning Goals

Let's practice adding and subtracting decimals.

4.1 The Cost of a Photo Print

Warm Up: 5 minutes

This warm-up prompts students to review the placement of the digits when subtracting decimal numbers using an efficient algorithm. The first question gives students a chance to think about how different placements of the 5 (the first number) affects the subtraction and the difference. It also gives insight to how students interpret the 5 and its value. For examples:

- Would they place the decimal point directly after the 5?
- When placing 5 above 7 (in the tenths place), do they see it as 5 ones or as 5 tenths?
- Do they wonder if the 5 is missing a decimal point in the second and third case?

In the second and third questions, a context is introduced for the same subtraction. Notice how students interpret the problem now; the equivalence of 5 and 5.00 should become much more apparent in a money context. Identify a couple of students who have correct responses so they could share later.

Addressing

- 6.NS.B.3

Instructional Routines

- Notice and Wonder

Launch

Give students up to 1 minute to think about the first question and then ask them to share what they noticed and wondered. Then, give students 1 minute of quiet work time for the remaining questions and follow with a whole-class discussion.

Student Task Statement

1. Here are three ways to write a subtraction calculation. What do you notice? What do you wonder?

$$\begin{array}{r} 5 \\ - 0.17 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ - 0.17 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ - 0.17 \\ \hline \end{array}$$

2. Clare bought a photo for 17 cents and paid with a \$5 bill. Look at the previous question. Which way of writing the numbers could Clare use to find the change she should receive? Be prepared to explain how you know.
3. Find the amount of change that Clare should receive. Show your reasoning, and be prepared to explain how you calculate the difference of 0.17 and 5.

Student Response

1. Answers vary. Students may notice that no decimal point is shown in the 5 and that 5 is lined up with a different decimal place relative to the 0.17 each time. Students may wonder if the decimal point is missing in the 5 or if it is supposed to be lined up with the decimal point in 0.17.
2. The first setup (in which 5 and 0 line up vertically) is most conducive to correct subtraction. Sample reasoning: The 5 means \$5.00, and it helps line up the dollars (the ones) and the cents (the tenths and hundredths) when subtracting.
3. \$4.83

Activity Synthesis

Ask a few students to share which way they think Clare could write the calculation (in the second question) in order to find the amount of change. In this particular case, the first setup is the most conducive to correct computation, but there is not one correct answer. Students could find the answer with any one of the setups as long as they understand that the 5 represents \$5.00. If any students choose the second or third setup because they can mentally subtract the values without lining them up by place values, invite them to share their reasoning. Ask if they would use the same strategy for dealing with longer decimals (e.g., $5.23 - 0.4879$) and, if not, what approach might be more conducive to correct calculation in those cases.

Encourage more students to be involved in the conversation by asking questions such as:

- "Do you agree or disagree? Why?"
- "Can anyone explain ___'s reasoning in their own words?"
- "What is important for us to think about when subtracting this way?"
- "How could we have solved this problem mentally?"

4.2 Decimals All Around

15 minutes

This activity has two parts and serves two purposes. The first two problems aim to help students see the limits of using base-ten diagrams to add and subtract numbers and to think about choosing more efficient methods. The latter three questions prompt students to reason about addition and subtraction of numbers with more decimal places in the context of situations. Students need to determine the appropriate operations for the given situations, perform the calculations, and then relate their answers back to the contexts (MP2).

As students work, check that they remember to line up the decimal points, write zeros when needed, and label their answers with units of measurement (for the last three questions). Identify one student to present each question. For the last question, there are two likely methods for solving (as outlined in the solutions). If both methods arise, they should both be presented.

Addressing

- 6.NS.B.3

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR6: Three Reads

Launch

Ask students to read the first 2 problems and think about which method of reasoning might work best. Give students 5 minutes of quiet time to work on these problems. Observe which methods students choose and check their work for accuracy.

Afterwards, take a few minutes to discuss their answers and methods. Ask one or more of the following questions before moving onto the second half of the activity:

- “Would base-ten diagrams work well for $318.8 - 94.63$? Why or why not?” (No, it is time consuming to draw and keep track of so many pieces. Yes, I can draw 3 large squares for 300, 1 rectangle for 10, 8 small squares for 8, etc., remove pieces in the amount of 94.63, and count what is left.)
- “What challenges did you face in computing $0.02 - 0.0116$ using a diagram?” (I needed to take away 116 ten-thousandths from 2 hundredths, so I had to exchange hundredths with ten-thousandths, which meant drawing or showing many smaller pieces.) “What about using vertical calculations?” (The first number is two digits shorter than the second, which meant some extra steps.)
- “When might base-ten diagrams be a good choice for finding a difference (or sum)?” (When the numbers are simple or short so the diagrams do not take too long to draw.)

- “When might an algorithm be a good choice for finding a difference (or sum)?” (For a difference like $6.739258 - 0.8536672$, it will be time consuming to draw pictures and label each place value. The algorithm takes some time as well but is more efficient than drawing pictures.)

After this discussion, have students work independently on the remaining problems. During this time, monitor and select individuals to share their work during the whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support development of organizational skills, check in with students within the first 2–3 minutes of work time. Check to make sure students have proper decimal point and place value unit alignment prior to calculating the value of each expression. Some students may benefit from access to graph paper to aide in proper alignment of decimals.

Supports accessibility for: Organization; Visual-spatial processing

Access for English Language Learners

Reading: MLR6 Three Reads. Use this routine to support reading comprehension of the problem about Lin’s grandmother. After a shared reading of the question, ask students “what is this situation about?” (Lin’s grandmother ordered needles; the needles were different lengths). Use this read to clarify any unfamiliar language such as pharmacist or administer. After the second read, students list any quantities that can be counted or measured, without focusing on specific values (length of each type of needle, in inches). After the third read, invite students to discuss possible strategies to solve the problem. If needed, repeat this support with the remaining word problems. This helps students connect the language in the word problem and the reasoning needed to solve the problem.

Design Principle(s): Support sense-making

Anticipated Misconceptions

Students may benefit from using graph paper for calculations; they can place one digit in each box for proper decimal point placement and place value unit alignment. For the last question, students might confuse the amount of zeros in 1 millionth. It might be helpful to write 0.000001 on the board as reference.

Student Task Statement

1. Find the value of each expression. Show your reasoning.
 - a. $11.3 - 9.5$
 - b. $318.8 - 94.63$

c. $0.02 - 0.0116$

2. Discuss with a partner:

- Which method or methods did you use in the previous question? Why?
- In what ways were your methods effective? Was there an expression for which your methods did not work as well as expected?

3. Lin's grandmother ordered needles that were 0.3125 inches long to administer her medication, but the pharmacist sent her needles that were 0.6875 inches long. How much longer were these needles than the ones she ordered? Show your reasoning.

4. There is 0.162 liter of water in a 1-liter bottle. How much more water should be put in the bottle so it contains exactly 1 liter? Show your reasoning.

5. One micrometer is 1 millionth of a meter. A red blood cell is about 7.5 micrometers in diameter. A coarse grain of sand is about 70 micrometers in diameter. Find the difference between the two diameters in *meters*. Show your reasoning.

Student Response

1. a. 1.8

b. 224.17

c. 0.0084

2. Answers vary.

3. 0.375 inches. $0.6875 - 0.3125 = 0.375$

4. $1 - 0.162 = 0.838$, 0.838 liters

5. Answers vary. Sample responses:

- The blood cell is 7.5 micrometers, and the grain of sand is 70 micrometers. The difference is 62.5 micrometers. Since 1 micrometer is 0.000001 meters, this is 0.0000625 meters.
- The bacteria is 0.0000075 meters, and the grain of sand is 0.00007 meters. The difference is 0.0000625 meters.

Activity Synthesis

Discussions on the first 2 questions are outlined in the Launch section. Focus the discussion here on the last 3 questions, i.e., on how students interpreted the problems, determined which operation to perform, and found the sum or difference (including how they handled any regroupings). Select a few previously identified students to share their responses and reasoning.

If some students chose base-ten diagrams for the calculation, contrast its efficiency with that of numerical calculations. The numbers in these problems have enough decimal places that using

base-ten representations (physical blocks, drawings, or digital representations) would be cumbersome, making a numerical calculation appealing.

4.3 Missing Numbers

15 minutes

Students deepen their understanding of regrouping by tackling problems that are more challenging and that prompt them to notice and use structure (MP7). Students build on both their work with whole-number differences (such as $1,000 - 256$) to find differences such as $1 - 0.256$. To add and subtract digits, they may think in terms of bundling and unbundling base-ten units, but there are also other opportunities to use structure here. Let's take the example $1,000 - 256$. Since $1,000 = 999 + 1$, students could calculate $1,000 - 256$ by first finding $999 - 256 = 743$, and then adding 1 to get 744. They could use the same reasoning to find sums and differences of decimals.

As students work and discuss their responses, notice the different ways students reason about the addition and subtraction. Identify a few students with differing approaches to share later.

Addressing

- 6.NS.B.3

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 8–10 minutes of quiet work time and 2–3 minutes to discuss their answers with a partner. Follow with a whole-class discussion.

Anticipated Misconceptions

If students use a guess-and-check method without success, encourage them to try working backwards. Consider scaffolding with a simpler problem involving whole numbers, such as $10 - ? = 3$. We can find the answer by thinking: $3 + ? = 10$, or $10 - 3 = ?$.

Student Task Statement

Write the missing digits in each calculation so that the value of each sum or difference is correct. Be prepared to explain your reasoning.

1.

$$\begin{array}{r} 0.404 \\ + \\ \hline 1 \end{array}$$

2.

$$\begin{array}{r} 9.8765 \\ + \\ \hline 10 \end{array}$$

3.

$$\begin{array}{r} 0.7 \\ - \\ \hline 0.012 \end{array}$$

4.

$$\begin{array}{r} 7 \\ - \\ \hline 3.4567 \end{array}$$

5.

$$\begin{array}{r} 70 \\ - \\ \hline 0.0089 \end{array}$$

Student Response

1.

$$\begin{array}{r} 0.404 \\ + 0.596 \\ \hline 1.000 \end{array}$$

2.

$$\begin{array}{r} 9.8765 \\ + \\ \hline 10.0000 \end{array}$$

3.

$$\begin{array}{r} 0.700 \\ - 0.688 \\ \hline 0.012 \end{array}$$

4.

$$\begin{array}{r} 7.0000 \\ - 3.5433 \\ \hline 3.4567 \end{array}$$

5.

$$\begin{array}{r} 70.0000 \\ - 69.9911 \\ \hline 0.0089 \end{array}$$

Are You Ready for More?

In a cryptarithmic puzzle, the digits 0-9 are represented using the first 10 letters of the alphabet. Use your understanding of decimal addition to determine which digits go with the letters A, B, C, D, E, F, G, H, I, and J. How many possibilities can you find?

$$\begin{array}{r} I H F . I J \\ + J I I . F I \\ \hline E J I . I E \end{array}$$

Student Response

- E: 4, F: 0, H: 2, I: 1, J: 3
- E: 9, F: 0, H: 5, I: 2, J: 7

Activity Synthesis

Select previously identified students to share their responses. Ask students how they approached the addition and subtraction problems, whether they work backwards, write additional zeros, or use other strategies to find the missing numbers.

Conclude the discussion by asking students how their work would change in the fourth question ($7 - ? = 3.4567$) if they were to replace 7 with 6.9999. Give students a moment to think about the question, and then ask:

- “How might the number 6.9999 help us find the missing number?” (The 9s make it easier to find each missing digit, so we can find the difference between 6.9999 and 3.4567, and then add 0.0001 to the result because 6.9999 is 0.0001 less than 7.)
- “How would this method work for a problem such as the second question: $9.8765 + ? = 10$?” (We can replace 10 with 9.9999, determine the missing number, and then add 0.0001 to that number).

This strategy is effective because it eliminates the “extra zeros” and the need to compose or decompose.

Access for English Language Learners

Speaking, Listening, Representing: MLR8 Discussion Supports. Use this routine to support whole-class discussion. Display each of the five problems for all to see. For each response that is shared, ask students to restate what they heard using precise mathematical language. Annotate the display to illustrate the steps students describe, and label the strategy that was used next to each question (for example, “work backwards,” or “write additional zeros”). Invite students to suggest additional details to include on the display that will support their understanding of each approach.

Design Principle(s): Support sense-making

Lesson Synthesis

In this lesson, students practiced adding and subtracting numbers with many decimal places, both in and outside of the context of situations. They noticed the benefits of vertical calculations and used its structure to solve problems.

- Which problems did you solve with addition (or subtraction)? How did you know to use addition (or subtraction)?
- How did you find the sums or differences? Why did you choose that method?
- Did you use the same method for all problems? Why or why not?
- When subtracting a number with fewer decimal places by another number with more decimal places, such as $2.4 - 0.1587$, what strategies might be helpful? (We can think of 2.4 as 2.4000 and use unbundling to subtract 0.1587 from it. Or we can think of the 2.4 as $2.3999 + 0.0001$, line up and subtract 0.1587 from 2.3999, and then add 0.0001 back to the difference.)

4.4 Taller and Farther

Cool Down: 5 minutes

Addressing

- 6.NS.B.3

Student Task Statement

1. Diego is 59.5 inches tall. His brother is 40.125 inches tall. How much taller than his brother is Diego? Show your reasoning.
2. A runner has run 1.192 kilometers of a 10-kilometer race. How much farther does he need to run to finish the race? Show your reasoning.

Student Response

1. 19.375 inches because $59.5 - 40.125 = 19.375$
2. 8.808 km because $10 - 1.192 = 8.808$

Student Lesson Summary

Base-ten diagrams work best for representing subtraction of numbers with few non-zero digits, such as $0.16 - 0.09$. For numbers with many non-zero digits, such as $0.25103 - 0.04671$, it would take a long time to draw the base-ten diagram. With vertical calculations, we can find this difference efficiently.

Thinking about base-ten diagrams can help us make sense of this calculation.

$$\begin{array}{r} 0.25103 \\ - 0.04671 \\ \hline 0.20432 \end{array}$$

The diagram shows the vertical subtraction of $0.25103 - 0.04671$. Red lines indicate the unbundling process: a line from the 5 to the 4, a line from the 1 to the 6, and a line from the 0 to the 7. Above the 5, the number 4 is written, and above the 1, the number 0 is written. Above the 0, the number 10 is written, indicating that 1 ten-thousandth is unbundled to make 10 thousandths.

The thousandth in 0.25103 is unbundled (or decomposed) to make 10 ten-thousandths so that we can subtract 7 ten-thousandths. Similarly, one of the hundredths in 0.25103 is unbundled (or decomposed) to make 10 thousandths.

Lesson 4 Practice Problems

Problem 1

Statement

For each subtraction problem, circle the correct calculation.

a. $7.2 - 3.67$

b. $16 - 1.4$

a.

$$\begin{array}{r} 7.2 \\ - 3.67 \\ \hline 3.05 \end{array}$$

$$\begin{array}{r} 07.2 \\ - 3.67 \\ \hline 3.05 \end{array}$$

$$\begin{array}{r} 7.20 \\ - 3.67 \\ \hline 3.53 \end{array}$$

b.

$$\begin{array}{r} 16 \\ - 1.4 \\ \hline 0.2 \end{array}$$

$$\begin{array}{r} 16.0 \\ - 1.40 \\ \hline 0.20 \end{array}$$

$$\begin{array}{r} 16.0 \\ - 1.4 \\ \hline 14.6 \end{array}$$

Solution

a.

$$\begin{array}{r} 7.2 \\ - 3.67 \\ \hline 3.05 \end{array}$$

$$\begin{array}{r} 07.2 \\ - 3.67 \\ \hline 3.05 \end{array}$$

$$\begin{array}{r} 7.20 \\ - 3.67 \\ \hline 3.53 \end{array}$$

b.

$$\begin{array}{r} 16 \\ - 1.4 \\ \hline 0.2 \end{array}$$

$$\begin{array}{r} 16.0 \\ - 1.40 \\ \hline 0.20 \end{array}$$

$$\begin{array}{r} 16.0 \\ - 1.4 \\ \hline 14.6 \end{array}$$

Problem 2

Statement

Explain how you could find the difference of 1 and 0.1978.

Solution

Answers vary. Sample responses:

- 1 can be unbundled into 10,000 ten-thousandths. 0.1978 is 1,978 ten-thousandths. To find the difference, we subtract: $10,000 - 1,978 = 8,022$. The difference is 8,022 ten-thousandths or 0.8022.
- 1 can be written as 1.0000. In a vertical calculation, we can show the 1 being unbundled into 10 tenths, 1 of those tenths being unbundled into 10 hundredths, 1 of those hundredths being unbundled into 10 thousandths, and 1 of the thousandths being unbundled into 10 ten-thousandths. Subtracting 0.1978 from those digits gives us 0.8022.

$$\begin{array}{r} \overset{9}{\cancel{0}} \overset{9}{\cancel{0}} \overset{9}{\cancel{0}} \\ \overset{10}{\cancel{0}} \overset{10}{\cancel{0}} \overset{10}{\cancel{0}} \overset{10}{\cancel{0}} \\ 1 \overset{10}{\cancel{.0}} \overset{10}{\cancel{0}} \overset{10}{\cancel{0}} \overset{10}{\cancel{0}} \\ - 0.1978 \\ \hline 0.8022 \end{array}$$

Problem 3

Statement

A bag of chocolates is labeled to contain 0.384 pound of chocolates. The actual weight of the chocolates is 0.3798 pound.

- Are the chocolates heavier or lighter than the weight stated on the label? Explain how you know.
- How much heavier or lighter are the chocolates than stated on the label? Show your reasoning.

Solution

- Lighter. Reasoning varies. Sample reasoning: 0.3798 is 3,798 ten-thousandths. 0.384 is 384 thousandths, which is equal to 3,840 ten-thousandths, so 0.384 is greater than 0.3798.
- 0.0042 ounce lighter. Reasoning varies. Sample reasoning:
 - 3,840 ten-thousandths subtracted by 3,798 ten-thousandths is 22 ten-thousandths, because $3,840 - 3,798 = 42$.
 - 0.3798 is 0.0002 away from 0.3800, and 0.3800 is 0.004 away from 0.384, so 0.3798 is $(0.0002 + 0.004)$ or 0.0042 away from 0.384.
 - $0.384 - 0.3798 = 0.0042$.

$$\begin{array}{r}
 \overset{13}{\cancel{7} \cancel{3} \cancel{10}} \\
 0.3840 \\
 - 0.3798 \\
 \hline
 0.0042
 \end{array}$$

Problem 4

Statement

Complete the calculations so that each shows the correct sum.

a.

$$\begin{array}{r}
 1.036 \\
 + \boxed{} \boxed{} \boxed{} \boxed{} \\
 \hline
 4
 \end{array}$$

b.

$$\begin{array}{r}
 0.738 \\
 + \boxed{} \boxed{} \boxed{} \boxed{} \\
 \hline
 1
 \end{array}$$

c.

$$\begin{array}{r}
 0.5137 \\
 + \boxed{} \boxed{} \boxed{} \boxed{} \boxed{} \\
 \hline
 1
 \end{array}$$

Solution

- 2.964
- 0.262

c. 0.4863

a.

$$\begin{array}{r} 1.036 \\ + 2.964 \\ \hline 4.000 \end{array}$$

b.

$$\begin{array}{r} 0.738 \\ + 0.262 \\ \hline 1.000 \end{array}$$

c.

$$\begin{array}{r} 0.5137 \\ + 0.4863 \\ \hline 1.0000 \end{array}$$

Problem 5

Statement

A shipping company is loading cube-shaped crates into a larger cube-shaped container. The smaller cubes have side lengths of $2\frac{1}{2}$ feet, and the larger shipping container has side lengths of 10 feet. How many crates will fit in the large shipping container? Explain your reasoning.

Solution

64 crates. Reasoning varies. Sample reasoning:

- Four crates can fit in a length of 10 feet because $4 \cdot 2\frac{1}{2} = 10$. So the container can fit $4 \cdot 4 \cdot 4$ or 64 crates.
- The volume of the larger container is 1000 cubic feet because $10 \cdot 10 \cdot 10 = 1000$. The volume of a crate is $15\frac{5}{8}$, since $2\frac{1}{2} \cdot 2\frac{1}{2} \cdot 2\frac{1}{2} = 15\frac{5}{8}$. Then 64 crates fit inside the container because $1000 \div 15\frac{5}{8} = 64$.

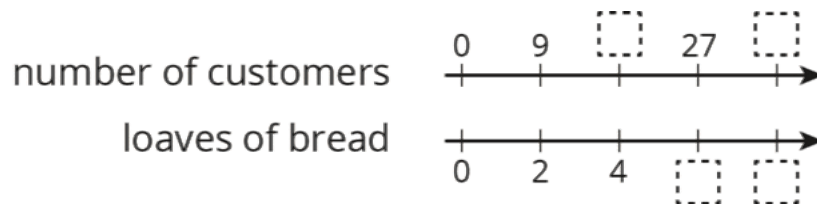
(From Unit 4, Lesson 14.)

Problem 6

Statement

For every 9 customers, the chef prepares 2 loaves of bread.

- a. Here is double number line showing varying numbers of customers and the loaves prepared. Complete the missing information.



- b. The same information is shown on a table. Complete the missing information.

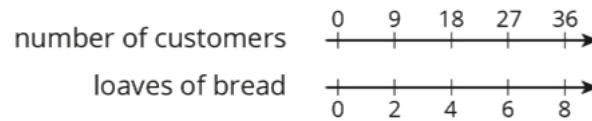
customers	loaves
9	2
	4
27	
	14
1	

c. Use either representation to answer these questions.

- How many loaves are needed for 63 customers?
- How many customers are there if the chef prepares 20 loaves?
- How much of a loaf is prepared for each customer?

Solution

a.



b.

customers	loaves
9	2
18	4
27	6
63	14
1	$\frac{2}{9}$ or equivalent

- c. ■ 14 loaves
- 90 customers
- $\frac{2}{9}$ of a loaf

(From Unit 2, Lesson 13.)

Lesson 5: Decimal Points in Products

Goals

- Generalize (orally and in writing) that the number of decimal places in a product is related to the number of decimal places in the factors.
- Justify (orally) the product of two decimals, which each have only one non-zero digit, by multiplying equivalent fractions that have a power of ten in the denominator.

Learning Targets

- I can use place value and fractions to reason about multiplication of decimals.

Lesson Narrative

In earlier grades, students have multiplied base-ten numbers up to hundredths (either by multiplying two decimals to tenths or by multiplying a whole number and a decimal to hundredths). Here, students use what they know about fractions and place value to calculate products of decimals beyond the hundredths. They express each decimal as a product of a whole number and a fraction, and then they use the commutative and associative properties to compute the product. For example, they see that $(0.6) \cdot (0.5)$ can be viewed as $6 \cdot (0.1) \cdot 5 \cdot (0.1)$ and thus as $(6 \cdot \frac{1}{10}) \cdot (5 \cdot \frac{1}{10})$. Multiplying the whole numbers and the fractions gives them $30 \cdot \frac{1}{100}$ and then 0.3.

Through repeated reasoning, students see how the number of decimal places in the factors can help them place the decimal point in the product (MP8).

Alignments

Building On

- 5.NBT.A.2: Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
- 5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Addressing

- 6.EE.A: Apply and extend previous understandings of arithmetic to algebraic expressions.
- 6.NS.B: Compute fluently with multi-digit numbers and find common factors and multiples.

Instructional Routines

- MLR2: Collect and Display

- MLR3: Clarify, Critique, Correct

Student Learning Goals

Let's look at products that are decimals.

5.1 Multiplying by 10

Warm Up: 5 minutes

In this warm-up, students use the structure (MP7) of a set of multiplication equations to see the relationship between two numbers that differ by a factor of a power of 10. Students evaluate the expression $x \cdot 10$ by considering the effect of multiplication by 10.

Building On

- 5.NBT.A.2

Addressing

- 6.EE.A

Launch

Ask students to answer the following questions without writing anything and to be prepared to explain their reasoning. Follow with whole-class discussion.

Student Task Statement

1. In which equation is the value of x the largest?

$$x \cdot 10 = 810$$

$$x \cdot 10 = 81$$

$$x \cdot 10 = 8.1$$

$$x \cdot 10 = 0.81$$

2. How many times the size of 0.81 is 810?

Student Response

1. The x has the largest value in the first equation. Sample explanations:
 - When multiplied by 10, the x in the first equation has the largest product.
 - Each x is one tenth of the product, and the largest product is 810.
2. 810 is 1,000 times the size of 0.81. Sample explanations:
 - Multiplying 0.81 by 10 moves the decimal place one digit to the right.
 - 810 is 10 times 81, and 81 is 100 times 0.81, so 810 must be 1,000 times 0.81.

Activity Synthesis

Ask students to share what they noticed about the first four equations. Record student explanations that connect multiplying a number by 10 with moving the decimal place.

Discuss how students could use their observations from the first question to multiply 0.81 by a number to get 810.

5.2 Fractionally Speaking: Powers of Ten

15 minutes

In grade 5, students recognize that multiplying a number by $\frac{1}{10}$ is the same as dividing the number by 10, and multiplying by $\frac{1}{100}$ is the same as dividing by 100. In this lesson, students will recognize and use the fact that multiplying by 0.1, 0.01, and 0.001 is equivalent to multiplying by $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1,000}$, respectively. In all cases, the essential point to understand is that in the base-ten system, the value of each place is $\frac{1}{10}$ the value of the place immediately to its left. Writing the decimals 0.1, 0.01, and 0.001 in fraction form will help students recognize how the position of the decimal point in the product is affected by the positions of the decimal points in the factors.

The structure of the base-ten system can serve as a guide for calculating products of decimals, and the goal of this lesson is to begin to uncover that structure (MP7).

Students might need to see decimal and fraction names. If so, display a place-value chart for reference.

Building On

- 5.NBT.B.7

Addressing

- 6.NS.B

Instructional Routines

- MLR2: Collect and Display

Launch

Arrange students in groups of 2. Ask one student in each group to complete the questions for Partner A, and have the other take the questions for Partner B. Then ask them to discuss their responses, answer the second question together, and pause for a brief class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge about multiplying a number by $\frac{1}{10}$ or $\frac{1}{100}$. Remind students that multiplying a number by $\frac{1}{10}$ is the same as dividing by 10 and multiplying a number by $\frac{1}{100}$ is the same as dividing by 100.

Supports accessibility for: Memory; Conceptual processing

Access for English Language Learners

Representing: MLR2 Collect and Display. Use this routine while students are working through the first two questions. As students work, circulate and listen for the connections students make between the problems. Write the students' words and phrases on a visual display and update it throughout the remainder of the lesson. Listen for language like "the same," reciprocal, and inverse operation. Remind students to borrow language from the display as needed.

Design Principle(s): Optimize output (for explanation); Maximize meta-awareness

Anticipated Misconceptions

Students may readily see that $36 \cdot (0.1)$ but be unsure about what to do when the factor being multiplied by the decimal 0.1 and 0.01 is also a decimal (e.g., $(24.5) \cdot (0.1)$). Encourage them to try writing both decimals (e.g., 24.5 and 0.1) as fractions, multiply, and convert the resulting fraction back to a decimal.

Student Task Statement

Work with a partner. One person solves the problems labeled "Partner A" and the other person solves those labeled "Partner B." Then compare your results.

1. Find each product or quotient. Be prepared to explain your reasoning.

Partner A

a. $250 \cdot \frac{1}{10}$

b. $250 \cdot \frac{1}{100}$

c. $48 \div 10$

d. $48 \div 100$

Partner B

a. $250 \div 10$

b. $250 \div 100$

c. $48 \cdot \frac{1}{10}$

d. $48 \cdot \frac{1}{100}$

2. Use your work in the previous problems to find $720 \cdot (0.1)$ and $720 \cdot (0.01)$. Explain your reasoning.

Pause here for a class discussion.

3. Find each product. Show your reasoning.

a. $36 \cdot (0.1)$

b. $(24.5) \cdot (0.1)$

c. $(1.8) \cdot (0.1)$

d. $54 \cdot (0.01)$

e. $(9.2) \cdot (0.01)$

4. Jada says: "If you multiply a number by 0.001, the decimal point of the number moves three places to the left." Do you agree with her? Explain your reasoning.

Student Response

1. For both Partner A and Partner B:
 - a. 25
 - b. 2.5
 - c. 4.8
 - d. 0.48
2. $720 \cdot (0.1) = 72$ and $720 \cdot (0.01) = 7.2$. Sample reasoning:
 - 0.1 is equal to $\frac{1}{10}$, so $720 \cdot (0.1) = 720 \cdot \frac{1}{10}$, which is equal to $720 \div 10$ or 72.
 - 0.01 is equal to $\frac{1}{100}$, so $720 \cdot (0.01) = 720 \cdot \frac{1}{100}$, which is equal to $720 \div 100$ or 7.2.
3.
 - a. 3.6. Sample reasoning: $36 \cdot (0.1)$ means 36 groups of 1 tenth or $36 \cdot \frac{1}{10}$, which equals 3.6.
 - b. 2.45. Sample reasoning: $(24.5) \cdot \frac{1}{10} = 24.5 \div 10$, which is 2.45.
 - c. 0.18. Sample reasoning: $(1.8) \cdot \frac{1}{10} = 1.8 \div 10$, which is 0.18.
 - d. 0.54. Sample reasoning: $54 \cdot (0.01)$ means 54 groups of 1 hundredth or 54 hundredths.
 - e. 0.092. Sample reasoning: $9.2 \div 100 = 0.092$.
4. I agree. Sample reasoning: Multiplying a number by a thousandth is the same as multiplying it by $\frac{1}{1,000}$, which is the same as dividing it by 1,000. Dividing by 1,000 moves the decimal point 3 places to the left.

Activity Synthesis

The purpose of this discussion is to highlight the placement of the decimal point in a product. Consider asking some of the following questions:

- "How does the size of a product compare to the size of the factor when the factor is multiplied by 0.1? How does the placement of the decimal point change?" (Multiplying by 0.1 makes the product ten times smaller than the factor. The decimal point moves to the left one place.)
- "How does the size of a product compare to the size of the factor when the factor is multiplied by 0.01?" (Multiplying by 0.01 makes the factor one hundred times smaller. The decimal point moves to the left two places.)

- “Can you predict the outcome of multiplying 750 by 0.1 or 0.01 without calculating? If so, how?” (Multiplying 750 by 0.1 would produce 75. Multiplying 750 by 0.01 would produce 7.5.)

5.3 Fractionally Speaking: Multiples of Powers of Ten

15 minutes

In this activity, students continue to think about products of decimals using fractions. They use what they know about $\frac{1}{10}$ and $\frac{1}{100}$, as well as the commutative and associative properties, to identify and write multiplication expressions that could help them find the product of two decimals.

While students may be able to start by calculating the value of each decimal product, the goal is for them to look for and use the structure of equivalent expressions (MP7), and later generalize the process to multiply any two decimals.

As students work, listen for the different ways students decide on which expressions are equivalent to $(0.6) \cdot (0.5)$. Identify a few students or groups with differing approaches so they can share later.

Building On

- 5.NBT.B.7

Addressing

- 6.NS.B

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Arrange students in groups of 2. Give groups 3–4 minutes to work on the first two questions, and then pause for a whole-class discussion. Discuss:

- Why is $(0.6) \cdot (0.5)$ equivalent to $6 \cdot \frac{1}{10} \cdot 5 \cdot \frac{1}{10}$? (0.6 is 6 tenths, which is the same as $6 \cdot \frac{1}{10}$, and 0.5 is 5 tenths, or $5 \cdot \frac{1}{10}$)
- Why is the expression $6 \cdot \frac{1}{10} \cdot 5 \cdot \frac{1}{10}$ equivalent to $6 \cdot 5 \cdot \frac{1}{10} \cdot \frac{1}{10}$? (We can ‘switch’ the places of 5 and $\frac{1}{10}$ in the multiplication and not change the product. This follows the commutative property of operations.)
- How did you find the value of $30 \cdot \frac{1}{100}$? (Multiplying by $\frac{1}{100}$ means dividing by 100, which moves the decimal point 2 places to the left, so the result is 0.30 or 0.3.)

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Maintain a display of important terms and vocabulary. During the launch take time to review terms that students will need to access for this activity. Invite students to suggest language or diagrams to include that will support their understanding of commutative and associative properties of operations.

Supports accessibility for: Memory; Language

Anticipated Misconceptions

If students try to use vertical calculation to find the products, ask them to instead do so by thinking of the decimals as fractions and about any patterns they observed.

Student Task Statement

1. Select **all** expressions that are equivalent to $(0.6) \cdot (0.5)$. Be prepared to explain your reasoning.
 - a. $6 \cdot (0.1) \cdot 5 \cdot (0.1)$
 - b. $6 \cdot (0.01) \cdot 5 \cdot (0.1)$
 - c. $6 \cdot \frac{1}{10} \cdot 5 \cdot \frac{1}{10}$
 - d. $6 \cdot \frac{1}{1,000} \cdot 5 \cdot \frac{1}{100}$
 - e. $6 \cdot (0.001) \cdot 5 \cdot (0.01)$
 - f. $6 \cdot 5 \cdot \frac{1}{10} \cdot \frac{1}{10}$
 - g. $\frac{6}{10} \cdot \frac{5}{10}$
2. Find the value of $(0.6) \cdot (0.5)$. Show your reasoning.
3. Find the value of each product by writing and reasoning with an equivalent expression with fractions.
 - a. $(0.3) \cdot (0.02)$
 - b. $(0.7) \cdot (0.05)$

Student Response

1. A, because $6 \cdot (0.1) \cdot 5 \cdot (0.1) = (0.6) \cdot (0.5)$
C, because $6 \cdot \frac{1}{10} \cdot 5 \cdot \frac{1}{10} = \frac{6}{10} \cdot \frac{5}{10} = 0.6 \cdot 0.5$

F, because $6 \cdot 5 \cdot \frac{1}{10} \cdot \frac{1}{10} = 6 \cdot \frac{1}{10} \cdot 5 \cdot \frac{1}{10}$

G, because $\frac{6}{10} \cdot \frac{5}{10} = (0.6) \cdot (0.5)$

2. 0.3. Sample reasoning: $(0.6) \cdot (0.5) = \frac{6}{10} \cdot \frac{5}{10}$, which is $\frac{30}{100}$ or 0.3.

3. Expressions vary.

a. 0.006. Sample reasoning: $\frac{3}{10} \cdot \frac{2}{100} = \frac{6}{1,000}$, which is 0.006.

b. 0.035. Sample reasoning: $(0.7) \cdot (0.05) = \frac{7}{10} \cdot \frac{5}{100}$, which is $\frac{35}{1,000}$ or 0.035.

Are You Ready for More?

Ancient Romans used the letter I for 1, V for 5, X for 10, L for 50, C for 100, D for 500, and M for 1,000. Write a problem involving merchants at an agora, an open-air market, that uses multiplication of numbers written with Roman numerals.

Student Response

Answers vary.

Activity Synthesis

Select several students to share their responses and reasonings for the last question.

To conclude, ask students to consider how writing $6 \cdot 5 \cdot \frac{1}{100}$ might be a favorable way to find $0.6 \cdot 0.5$. Students may respond that using whole numbers and fractions makes multiplication simpler; even if there is division, it is division by a power of 10. In future lessons, students will apply this reasoning to find products of more elaborate decimals, such as $(0.24) \cdot (0.011)$.

Access for English Language Learners

Writing, Speaking: MLR3 Clarify, Critique, Correct. Present an incorrect justification for one of the expressions from the first problem that was not equivalent (such as B or E). For example, “The expression $6 \cdot (0.01) \cdot 5 \cdot (0.1)$ is equivalent to $0.6 \cdot 0.5$ because the same whole numbers are used and where you put the zeros in a decimal doesn’t matter.” Ask students, “Is this reasoning correct? Why or why not?” Give students 1–2 minutes to write a brief explanation about why the expressions are not equivalent. Student responses should show attention to the placement of the decimal point and the place values of the digits. For students who need additional support, provide a sentence frame, such as “_____ is (equivalent/not equivalent) because _____.” This will help students produce clearer justifications that demonstrate their reasoning about what equivalence means.

Design Principle(s): Optimize output (for justification); Cultivate conversation

Lesson Synthesis

We can use our understanding of fractions and place value in calculating the product of two decimals. Writing decimals in fraction form can help us determine the number of decimal places the product will have and place the decimal point in the product.

- What are some ways to find $(0.4) \cdot (0.0007)$? (We can think of 0.4 as $\frac{4}{10}$ and 0.0007 as $\frac{7}{10,000}$, multiply the fractions to get $\frac{28}{100,000}$, and write the product as the decimal 0.00028. Or we can write 0.4 as $4 \cdot \frac{1}{10}$ and 0.0007 as $7 \cdot \frac{1}{10,000}$, multiply the whole numbers and the fractions, and again convert the fractional product into a decimal.)
- How might we tell which product will have a greater number of places to the right of the decimal point: $(0.03) \cdot (0.001)$ or $(0.3) \cdot (0.0001)$? (If we write the decimals as fractions and multiply them, we can see that both products equal $\frac{3}{100,000}$ or 0.00003, so they would have the same number of places to the right of the decimal point.)

5.4 Placing Decimal Points in Products

Cool Down: 5 minutes

Addressing

- 6.NS.B

Student Task Statement

1. Use what you know about decimals or fractions to explain why $(0.2) \cdot (0.002) = 0.0004$.
2. A rectangular plot of land is 0.4 kilometer long and 0.07 kilometer wide. What is its area in square kilometers? Show your reasoning.

Student Response

1. Answers vary. Sample response: 0.2 is $\frac{2}{10}$, and 0.002 is $\frac{2}{1,000}$. Multiplying the two we have:
 $\frac{2}{10} \cdot \frac{2}{1,000} = \frac{4}{10,000}$, which is 0.0004.
2. 0.028 square kilometers, because $(0.4) \cdot (0.07) = 0.028$

Student Lesson Summary

We can use fractions like $\frac{1}{10}$ and $\frac{1}{100}$ to reason about the location of the decimal point in a product of two decimals.

Let's take $24 \cdot (0.1)$ as an example. There are several ways to find the product:

- We can interpret it as 24 groups of 1 tenth (or 24 tenths), which is 2.4.
- We can think of it as $24 \cdot \frac{1}{10}$, which is equal to $\frac{24}{10}$ (and also equal to 2.4).

- Multiplying by $\frac{1}{10}$ has the same result as dividing by 10, so we can also think of the product as $24 \div 10$, which is equal to 2.4.

Similarly, we can think of $(0.7) \cdot (0.09)$ as 7 tenths times 9 hundredths, and write:

$$\left(7 \cdot \frac{1}{10}\right) \cdot \left(9 \cdot \frac{1}{100}\right)$$

We can rearrange whole numbers and fractions:

$$(7 \cdot 9) \cdot \left(\frac{1}{10} \cdot \frac{1}{100}\right)$$

This tells us that $(0.7) \cdot (0.09) = 0.063$.

$$63 \cdot \frac{1}{1,000} = \frac{63}{1,000}$$

Here is another example: To find $(1.5) \cdot (0.43)$, we can think of 1.5 as 15 tenths and 0.43 as 43 hundredths. We can write the tenths and hundredths as fractions and rearrange the factors.

$$\left(15 \cdot \frac{1}{10}\right) \cdot \left(43 \cdot \frac{1}{100}\right) = 15 \cdot 43 \cdot \frac{1}{1,000}$$

Multiplying 15 and 43 gives us 645, and multiplying $\frac{1}{10}$ and $\frac{1}{100}$ gives us $\frac{1}{1,000}$. So $(1.5) \cdot (0.43)$ is $645 \cdot \frac{1}{1,000}$, which is 0.645.

Lesson 5 Practice Problems

Problem 1

Statement

- a. Find the product of each number and $\frac{1}{100}$.

122.1

11.8

1350.1

1.704

- b. What happens to the decimal point of the original number when you multiply it by $\frac{1}{100}$? Why do you think that is? Explain your reasoning.

Solution

- a. 1.221
b. 0.118
c. 13.501
d. 0.01704

- e. Answers vary. Sample response: The decimal point moves 2 places to the left. Multiplying a decimal number by $\frac{1}{100}$ means dividing by 100, which moves the decimal point 2 places to the left.

Problem 2

Statement

Which expression has the same value as $(0.06) \cdot (0.154)$? Select **all** that apply.

- A. $6 \cdot \frac{1}{100} \cdot 154 \cdot \frac{1}{1,000}$
- B. $6 \cdot 154 \cdot \frac{1}{100,000}$
- C. $6 \cdot (0.1) \cdot 154 \cdot (0.01)$
- D. $6 \cdot 154 \cdot (0.00001)$
- E. 0.00924

Solution

["A", "B", "D", "E"]

Problem 3

Statement

Calculate the value of each expression by writing the decimal factors as fractions, then writing their product as a decimal. Show your reasoning.

- a. $(0.01) \cdot (0.02)$
- b. $(0.3) \cdot (0.2)$
- c. $(1.2) \cdot 5$
- d. $(0.9) \cdot (1.1)$
- e. $(1.5) \cdot 2$

Solution

- a. 0.0002 because $0.01 = \frac{1}{100}$ and $0.02 = \frac{2}{100}$, so the product is $\frac{2}{10,000}$
- b. 0.06 because $0.3 = \frac{3}{10}$ and $0.2 = \frac{2}{10}$, so the product is $\frac{6}{100}$
- c. 6 because $12 \cdot 5 = 60$ and 1.2 is one tenth of 12
- d. 0.99 because $0.9 = \frac{9}{10}$ and $1.1 = \frac{11}{10}$, so the product is $\frac{99}{100}$

e. 3 because $1.5 = \frac{3}{2}$ and twice this is 3

Problem 4

Statement

Write three numerical expressions that are equivalent to $(0.0004) \cdot (0.005)$.

Solution

Answers vary. Possible responses:

- $4 \cdot (0.0001) \cdot 5 \cdot (0.001)$
- $4 \cdot 5 \cdot (0.0001) \cdot (0.001)$
- $\frac{4}{10,000} \cdot \frac{5}{1,000}$
- $\frac{1}{10,000} \cdot 4 \cdot \frac{1}{1,000} \cdot 5$

Problem 5

Statement

Calculate each sum.

a. $33.1 + 1.95$

a. $1.075 + 27.105$

a. $0.401 + 9.28$

Solution

a. 35.05

b. 28.18

c. 9.681

(From Unit 5, Lesson 3.)

Problem 6

Statement

Calculate each difference. Show your reasoning.

a. $13.2 - 1.78$

a. $23.11 - 0.376$

a. $0.9 - 0.245$

Solution

a. 11.42

b. 22.734

c. 0.655 Sample reasoning:

$$\begin{array}{r}
 \text{a.} \quad \begin{array}{r}
 \overset{9}{\cancel{8}} \overset{10}{\cancel{0}} \overset{10}{\cancel{0}} \\
 0.\overset{8}{\cancel{9}} \overset{10}{\cancel{0}} \overset{10}{\cancel{0}} \\
 - 0.245 \\
 \hline
 0.655
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{b.} \quad \begin{array}{r}
 \overset{11}{\cancel{2}} \overset{1}{\cancel{1}} \\
 1.\overset{2}{\cancel{3}} \overset{1}{\cancel{2}} 0 \\
 - 1.78 \\
 \hline
 11.42
 \end{array}
 \end{array}$$

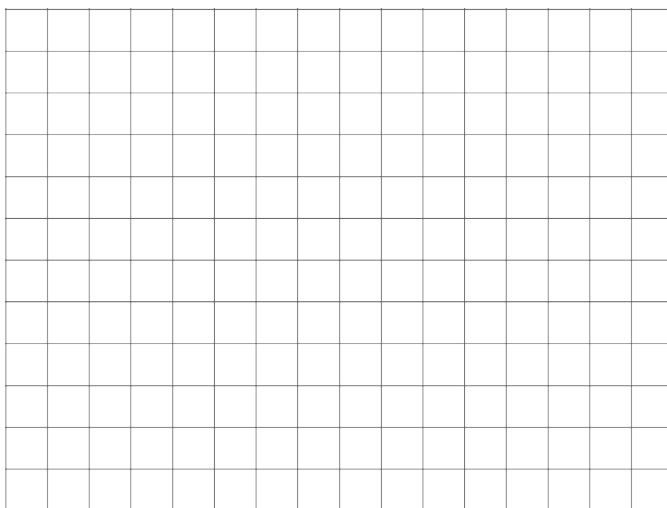
$$\begin{array}{r}
 \text{c.} \quad \begin{array}{r}
 \overset{2}{\cancel{3}} \overset{10}{\cancel{1}} \overset{10}{\cancel{1}} \overset{10}{\cancel{0}} \\
 2.\overset{2}{\cancel{3}} \overset{10}{\cancel{1}} \overset{10}{\cancel{1}} 0 \\
 - 0.376 \\
 \hline
 22.734
 \end{array}
 \end{array}$$

(From Unit 5, Lesson 4.)

Problem 7

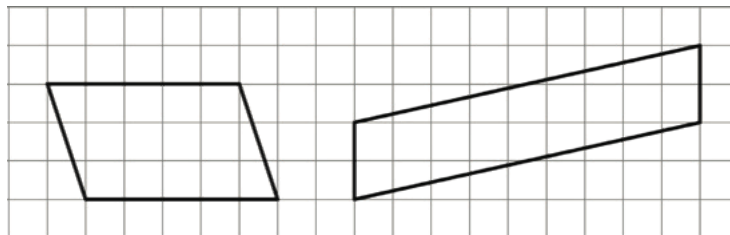
Statement

On the grid, draw a quadrilateral *that is not a rectangle* that has an area of 18 square units. Show how you know the area is 18 square units.



Solution

Answers vary. Sample responses:



(From Unit 1, Lesson 3.)

Lesson 6: Methods for Multiplying Decimals

Goals

- Interpret different methods for computing the product of decimals, and evaluate (orally) their usefulness.
- Justify (orally, in writing, and through other representations) where to place the decimal point in the product of two decimals with multiple non-zero digits.

Learning Targets

- I can use area diagrams to represent and reason about multiplication of decimals.
- I know and can explain more than one way to multiply decimals using fractions and place value.

Lesson Narrative

In this lesson, students continue to develop methods for computing products of decimals, including using area diagrams. They multiply decimals by expressing them as fractions, or by interpreting each decimal as a product of a whole number and a power of 10 and $\frac{1}{10}$. To multiply $(0.25) \cdot (1.6)$, for example, students may first multiply 0.25 by 100 and 1.6 by 10 to have whole numbers 25 and 16, multiply the whole numbers to get 400, and then multiply 400 by $\frac{1}{1,000}$ to invert the initial multiplication by 1,000. They may also think of 0.25 and 1.6 as $\frac{25}{100}$ and $\frac{16}{10}$, multiply the fractions, and then express the fractional product as a decimal.

In earlier grades, students used the area of rectangles to represent and find products of whole numbers and fractions. Here they do the same to represent and find products of decimals. They see that a rectangle that represents $4 \cdot 2$, for instance, can also be used to reason about $(0.4) \cdot (0.2)$, $(0.004) \cdot (0.002)$, or $40 \cdot 20$ because they all share a common structure. In this lesson, students extend their understanding of multiplication of fractions and multiplication using area diagrams by using previous methods to multiply any pair of decimals.

Alignments

Building On

- 5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Addressing

- 6.NS.B: Compute fluently with multi-digit numbers and find common factors and multiples.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR8: Discussion Supports
- Think Pair Share

Student Learning Goals

Let's look at some ways we can represent multiplication of decimals.

6.1 Equivalent Expressions

Warm Up: 5 minutes

The purpose of this warm-up is to help students recall that in an expression involving multiplication, the product is not affected by the order of the factors.

Building On

- 5.NBT.B.7

Launch

Set a timer for 15 seconds. Ask students to write as many expressions as they can think of that are equal to 0.6 without using addition or subtraction.

The purpose of the timer is to keep the number of unique expressions manageable, since you'll list them all during the activity synthesis. Adjust the duration of the timer if students need more or less time.

Student Task Statement

Write as many expressions as you can think of that are equal to 0.6. Do not use addition or subtraction.

Student Response

Answers vary. Sample responses:

- $6 \cdot 0.1$
- $0.1 \cdot 6$
- $\frac{6}{10}$
- $\frac{1}{10} \cdot 2 \cdot 3$
- $3 \cdot 0.2$

Activity Synthesis

Display for all to see all of the unique expressions that students create. Make sure everyone agrees that all of the expressions equal 0.6.

Make sure students see some expressions that illustrate the commutative property. For example, $0.1 \cdot 6$ and $6 \cdot 0.1$ are both equal to 0.6 because multiplication is commutative.

6.2 Using Properties of Numbers to Reason about Multiplication

20 minutes

This activity continues to develop the two methods for computing products of decimals introduced in the previous lesson. The first method uses the idea that multiplying a number by $\frac{1}{10}$ is the same as dividing the number by 10, multiplying by $\frac{1}{100}$ is the same as dividing by 100, and so on. The second method is to convert decimals to fractions, compute the product, then convert the product to a decimal. Students make sense of both methods and use one to solve a problem. As they continue to work through examples, students begin to notice a relationship between the location of decimal points in the factors and the product.

As they reason about the placement of the decimal and the relationship between decimals and fractions, students use the structure of the base-ten system (MP7). To reason correctly about the products of decimals, they also need to pay close attention to the digits and their place value.

Building On

- 5.NBT.B.7

Addressing

- 6.NS.B

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Arrange students in groups of 2. Give students 5–6 minutes to complete the first set of questions. Ask each student to study one of the two methods and then explain that method to their partner. After making sense of both methods together, each partner applies one method to solve a new problem. After the first question, have students pause for a brief discussion. Invite a student from each camp to share their reasoning. If not already mentioned in students' explanations, ask:

- Why might Elena have multiplied by 0.23 by 100 and 1.5 by 10? (Elena might have multiplied the factors by 100 and 10 to get them into whole numbers, which are easier to multiply.) What might be her reason for dividing 345 by 1,000? (Because she multiplied the original factors by

$(100 \cdot 10)$ or 1,000, so the product 345 is 1,000 times the original product and must therefore be divided by 1,000).

- How is Noah's method different than Elena's? (Noah converted each decimal into fractions and multiplied the fractions.)

Then give students another 7–8 minutes of quiet time to work on the second set of questions.

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. To help get students started, display sentence frames such as “____’s method makes more sense to me because . . .”

Supports accessibility for: Language; Organization

Student Task Statement

Elena and Noah used different methods to compute $(0.23) \cdot (1.5)$. Both calculations were correct.

$$(0.23) \cdot 100 = 23$$

$$(1.5) \cdot 10 = 15$$

$$23 \cdot 15 = 345$$

$$345 \div 1,000 = 0.345$$

Elena’s Method

$$0.23 = \frac{23}{100}$$

$$1.5 = \frac{15}{10}$$

$$\frac{23}{100} \cdot \frac{15}{10} = \frac{345}{1,000}$$

$$\frac{345}{1,000} = 0.345$$

Noah’s Method

1. Analyze the two methods, then discuss these questions with your partner.
 - Which method makes more sense to you? Why?
 - What might Elena do to compute $(0.16) \cdot (0.03)$? What might Noah do to compute $(0.16) \cdot (0.03)$? Will the two methods result in the same value?
2. Compute each product using the equation $21 \cdot 47 = 987$ and what you know about fractions, decimals, and place value. Explain or show your reasoning.
 - a. $(2.1) \cdot (4.7)$
 - b. $21 \cdot (0.047)$
 - c. $(0.021) \cdot (4.7)$

Student Response

1. Answers vary. Sample responses:
 - Elena might multiply 0.16 by 100 to get 16 and 0.03 by 100 to get 3, multiply 16 and 3 to get 48, and then divide 48 by 10,000 (because $100 \cdot 100 = 10,000$). Noah might write 0.16 as $\frac{16}{100}$ and 0.03 as $\frac{3}{100}$, and then multiply $\frac{16}{100} \cdot \frac{3}{100}$ to get $\frac{48}{10,000}$. Both methods would result in 0.0048.
2.
 - a. 9.87. Multiply each of the original factors by 10 (making a product of 100), and then divide 987 by 100. $987 \div 100 = 9.87$.
 - b. 0.987. Multiply the second original factor by 1,000, then divide 987 by 1,000. $987 \div 1,000 = 0.987$.
 - c. 0.0987. Multiply the original factors by 1,000 and 10 (making a product of 10,000), so then divide 987 by 10,000. $987 \div 10,000 = 0.0987$.

Activity Synthesis

Invite a couple of students to summarize the two different methods used in this activity. Poll the class to see which method they used in the second question. Ask if they preferred one method over the other and, if so, which one and why.

It is important students understand that the two methods presented here are mathematically equivalent. In Noah's method, the product of the numerators of his fractions (23 and 15) is the same as the product of Elena's whole numbers. Both of them then move the decimal point three places to the left because they need to divide by 1,000. For Noah, 1,000 is the denominator of his fraction. For Elena, the division by 1,000 reverts the initial multiplication by 1,000 she had performed so she could have whole-number factors.

Access for English Language Learners

Writing, Speaking, Listening: MLR1 Stronger and Clearer Each Time. Use this routine to help students improve their writing, by providing them with multiple opportunities to clarify their explanations through conversation. Give students time to meet with 2–3 partners, to share and get feedback on their responses. Display questions for students to ask their partners such as, "Can you say more about why you think Elena do that?" and "Why do you think Noah would do that?" Give students 1–2 minutes to revise their writing based on the feedback they received.

Design Principle(s): Optimize output (for explanation); Maximize meta-awareness

6.3 Using Area Diagrams to Reason about Multiplication

Optional: 15 minutes

Students have used area diagrams to reason about multiplication of whole numbers and fractions in previous grades. This task prepares them to use area diagrams to find products of decimals in upcoming lessons. In addition to using the structure of base-ten numbers in their reasoning, students also use the structure of the area diagram to help them find products of decimals (MP7). This activity illustrates how students' previous understandings of multiplication using area diagrams can be applied to the multiplication of any pair of decimals.

As students work, look and listen for different ways students might reason about the area of each unit square and of the large rectangle given particular side lengths. Identify a few students with correct reasoning so they can share later.

Building On

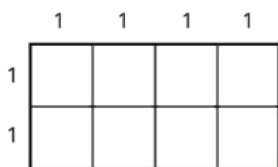
- 5.NBT.B.7

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Display this image for all to see.



Ask students to consider a rectangle composed of 8 squares. If the side length of each square is 1 unit, as shown:

- What is the area of each square? (The area of each square is 1 square unit, because $1 \cdot 1 = 1$.)
- What is the area of the rectangle? (The area of the rectangle is 8 square units, because it is made up of 8 squares and $8 \cdot 1 = 8$.)
- How can we express the area of the rectangle in terms of its length and width? (The area of the rectangle can also be expressed as $4 \cdot 2$.)

After this discussion, give students 5 minutes of quiet work time and 2–3 minutes to discuss their responses with a partner. Follow with a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organization and problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.

Supports accessibility for: Organization; Attention

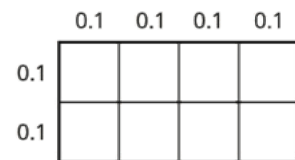
Anticipated Misconceptions

Some students might think that $(0.1) \cdot (0.1) = 0.1$ (just like $1 \cdot 1 = 1$). If this happens, have them write 0.1 in fraction form so they see that $\frac{1}{10} \cdot \frac{1}{10} = \frac{1}{100}$ or 0.01.

Student Task Statement

1. In the diagram, the side length of each square is 0.1 unit.

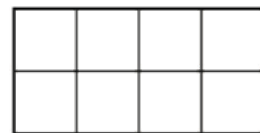
- a. Explain why the area of each square is *not* 0.1 square unit.



- b. How can you use the area of each square to find the area of the rectangle? Explain or show your reasoning.
- c. Explain how the diagram shows that the equation $(0.4) \cdot (0.2) = 0.08$ is true.

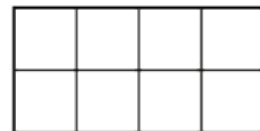
2. Label the squares with their side lengths so the area of this rectangle represents $40 \cdot 20$.

- a. What is the area of each square?
- b. Use the squares to help you find $40 \cdot 20$. Explain or show your reasoning.



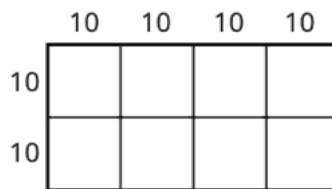
3. Label the squares with their side lengths so the area of this rectangle represents $(0.04) \cdot (0.02)$.

Next, use the diagram to help you find $(0.04) \cdot (0.02)$. Explain or show your reasoning.

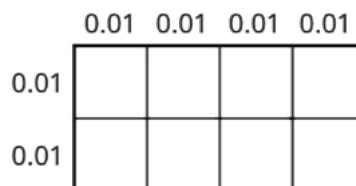


Student Response

1.
 - a. The area is not 0.1 square unit because $(0.1) \cdot (0.1)$ is 1 tenth times 1 tenth, which is 1 hundredth or 0.01 square units.
 - b. There are 8 squares in the rectangle, so the area of the rectangle is $8 \cdot (0.01) = 0.08$ square units.
 - c. The rectangle has side lengths of 0.4 and 0.2. The area of a rectangle is its length times its width, so the area of this rectangle is $(0.4) \cdot (0.2)$. We found out earlier that the area of the rectangle is 0.08 square units, so $(0.4) \cdot (0.2) = 0.08$.
2. The side length of each square is 10 units.



- a. The area of each square is $(10 \cdot 10)$ or 100 square units.
 - b. $40 \cdot 20 = 800$. Sample reasoning: There are 8 squares in the rectangle, so the area of the rectangle is $8 \cdot 100$ or 800 square units. Multiplying the side lengths of the rectangle, which are 40 units and 20 units, gives an area of 800 square units.
3. The side length of each square is 0.01 unit. $(0.04) \cdot (0.02) = 0.0008$. Sample reasoning: The area of each square is $(0.01) \cdot (0.01)$ or 0.0001 square units. There are 8 squares in the rectangles, so the area of the rectangle is $8 \cdot (0.0001)$ square units, which is 0.0008 square units.



Activity Synthesis

Select previously identified students to share their explanations and ask the class for agreement and disagreement.

Point out how the situation described in this task arises whenever we want to express an area given in one unit of measurement in terms of a different unit of measurement. Display for all to see a 2 by 4 rectangle with the side length of squares marked as 1 cm. After giving quiet think time, invite 1–2 students to explain their thinking for the following kinds of questions:

- If the sides of each small square are 1 cm by 1 cm, what is its area in cm^2 ? (1). What is the area of the rectangle in cm^2 ? (8)

- If the sides of each small square are 1 cm by 1 cm, what is the area of each small square in mm^2 ? (100). What is the area of the full rectangle in mm^2 ? (800)
- If the dimensions of each small square are 1 cm by 1 cm, what is the area of each small square in m^2 ? (There are 100 cm in a meter.)
 $(\frac{1}{100} \cdot \frac{1}{100} = \frac{1}{10,000})$.
- What is the area of the full rectangle in m^2 ? ($\frac{8}{10,000}$, or 0.0008)

Also point out that area diagrams are similar to the pieces in a base-ten diagram in that they can represent different values. Just as the same collection of base-ten figures can represent 103 or 0.103 (or many other numbers), so can the area of a rectangle composed of small-sized squares represent many different products. Students will continue to use area diagrams to multiply in the next lesson.

Access for English Language Learners

Speaking, Representing: MLR8 Discussion Supports. Use this routine to support whole-class discussion. After each student shares, provide the class with the following sentence frames to help them respond: "I agree because . . ." or "I disagree because . . ." If necessary, revoice student ideas to demonstrate mathematical language, and invite students to chorally repeat phrases that include relevant vocabulary in context.

Design Principle(s): Support sense-making

Lesson Synthesis

In this lesson, we saw additional ways to find the product of decimals: by converting the decimals to fractions and multiplying the fractions, and by using the area of a rectangle to represent multiplication.

- How can changing decimals to fractions help to find decimal products? (Writing the decimals as fractions allows us to use multiplication and division of whole numbers. It also tells us the size of the decimals relative to powers of $\frac{1}{10}$. Both kinds of information allow us to find the products.)
- How can an area diagram represent decimal products? (If the side lengths of a rectangle represent two factors, then the area of the rectangle represents the product of those factors. We can specify the unit of length to match that of the decimals, find the area of one unit square, and use the area of each unit square to find the area of the rectangle.)

6.4 Finding Products of Decimals

Cool Down: 5 minutes

Addressing

- 6.NS.B

Student Task Statement

1. Use the equation $135 \cdot 42 = 5,670$ and what you know about fractions, decimals, and place value to explain how to place the decimal point when you compute $(1.35) \cdot (4.2)$.
2. Which of the following is the correct value of $(0.22) \cdot (0.4)$? Show your reasoning.
 - a. 8.8
 - b. 0.88
 - c. 0.088
 - d. 0.0088

Student Response

1. $(1.35) \cdot (4.2) = 5.67$, because $135 \cdot 42 \cdot (0.01) \cdot (0.1) = (5,670) \cdot (0.001) = 5.67$.
2. C: 0.088. Sample explanations:
 - 0.22 is $\frac{22}{100}$ and 0.4 is $\frac{4}{10}$. The product is 0.088 because $\frac{22}{100} \cdot \frac{4}{10} = \frac{88}{1,000}$, and $\frac{88}{1,000}$ is 0.088.
 - 0.22 is $22 \cdot \frac{1}{100}$ and 0.4 is $4 \cdot \frac{1}{10}$, so $(0.22) \cdot (0.4) = 22 \cdot 4 \cdot \frac{1}{100} \cdot \frac{1}{10}$. Multiplying the whole numbers and the decimals gives: $88 \cdot \frac{1}{1,000}$, which equals 0.088.

Student Lesson Summary

Here are three other ways to calculate a product of two decimals such as $(0.04) \cdot (0.07)$.

- First, we can multiply each decimal by the same power of 10 to obtain whole-number factors.

$$(0.04) \cdot 100 = 4$$

$$(0.07) \cdot 100 = 7$$

Because we multiplied both 0.04 and 0.07 by 100 to get 4 and 7, the product 28 is $(100 \cdot 100)$ times the original product, so we need to divide 28 by 10,000.

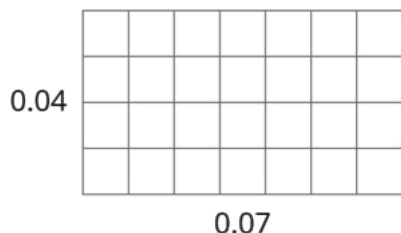
$$4 \cdot 7 = 28$$

$$28 \div 10,000 = 0.0028$$

- Second, we can write each decimal as a fraction, $0.04 = \frac{4}{100}$ and $0.07 = \frac{7}{100}$, and multiply them.

$$\frac{4}{100} \cdot \frac{7}{100} = \frac{28}{10,000} = 0.0028$$

- Third, we can use an area model. The product $(0.04) \cdot (0.07)$ can be thought of as the area of a rectangle with side lengths of 0.04 unit and 0.07 unit.



In this diagram, each small square is 0.01 unit by 0.01 unit. The area of each square, in square units, is therefore $(\frac{1}{100} \cdot \frac{1}{100})$, which is $\frac{1}{10,000}$.

Because the rectangle is composed of 28 small squares, the area of the rectangle, in square units, must be:

$$28 \cdot \frac{1}{10,000} = \frac{28}{10,000} = 0.0028$$

All three calculations show that $(0.04) \cdot (0.07) = 0.0028$.

Lesson 6 Practice Problems

Problem 1

Statement

Find each product. Show your reasoning.

- $(1.2) \cdot (0.11)$
- $(0.34) \cdot (0.02)$
- $120 \cdot (0.002)$

Solution

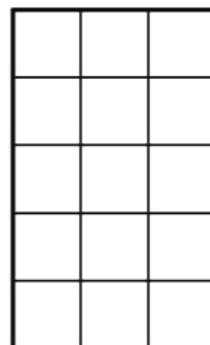
- 0.132. Sample reasoning: 1.2 is a tenth of 12 and 0.11 is a hundredth of 11, so the product of 1.2 and 0.11 is a thousandth of $12 \cdot 11$ or $\frac{1}{1,000} \cdot 132$, which is 0.132.
- 0.0068. Sample reasoning: $\frac{34}{100} \cdot \frac{2}{100} = \frac{68}{1,000}$ or 0.0068.
- 0.24. Sample reasoning: 0.002 is 2 thousandths or $2 \cdot \frac{1}{1,000}$, so the product of 120 and 0.002 is $120 \cdot 2 \cdot \frac{1}{1,000}$, which equals $\frac{240}{1,000}$ or 0.24.

Problem 2

Statement

You can use a rectangle to represent $(0.3) \cdot (0.5)$.

- What must the side length of each square represent for the rectangle to correctly represent $(0.3) \cdot (0.5)$?
- What area is represented by each square?
- What is $(0.3) \cdot (0.5)$? Show your reasoning.



Solution

- 0.1
- 0.01 square units
- The area is 0.15 because there are 15 squares, and $15 \cdot (0.01) = 0.15$.

Problem 3

Statement

One gallon of gasoline in Buffalo, New York costs \$2.29. In Toronto, Canada, one liter of gasoline costs \$0.91. There are 3.8 liters in one gallon.

- How much does one gallon of gas cost in Toronto? Round your answer to the nearest cent.
- Is the cost of gas greater in Buffalo or in Toronto? How much greater?

Solution

- \$. $(3.8) \cdot (0.91) = 3.458$, and this is closer to 3.46 than to 3.45.
- The cost of one gallon of gas is \$ more in Toronto.

Problem 4

Statement

Calculate each sum or difference.

$10.3 + 3.7$

$20.99 - 4.97$

$15.99 + 23.51$

$1.893 - 0.353$

Solution

- a. 14
- b. 16.02
- c. 39.5
- d. 1.54

(From Unit 5, Lesson 2.)

Problem 5

Statement

Find the value of $\frac{49}{50} \div \frac{7}{6}$ using any method.

Solution

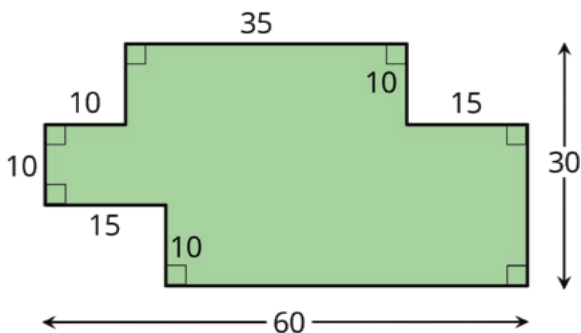
$\frac{21}{25}$ (or equivalent)

(From Unit 4, Lesson 11.)

Problem 6

Statement

Find the area of the shaded region. All angles are right angles. Show your reasoning.



Solution

1,400 square units. Reasoning varies. Sample reasoning: The region can be enclosed with a 60-by-30 rectangle, which has an area of 1,800 square units. Three of the corners of that rectangle have a rectangular region removed. The removed areas are 100 square units (upper left), 150 square units (lower left), and 150 square units (upper right). The area of the shaded region, in square units, is $1,800 - (100 + 150 + 150)$ or $1,800 - 400$, which is 1,400.

Problem 7

Statement

- Priya finds $(1.05) \cdot (2.8)$ by calculating $105 \cdot 28$, then moving the decimal point three places to the left. Why does Priya's method make sense?
- Use Priya's method to calculate $(1.05) \cdot (2.8)$. You can use the fact that $105 \cdot 28 = 2,940$.
- Use Priya's method to calculate $(0.0015) \cdot (0.024)$.

Solution

- $1.05 = \frac{1}{100} \cdot 105$ and $2.8 = \frac{1}{10} \cdot 28$, so $(1.05) \cdot (2.8) = \frac{1}{1,000} \cdot (105 \cdot 28)$. This is the same as finding $105 \cdot 28$ and then moving the decimal point three places to the left.
- Since $105 \cdot 28 = 2,940$, $(1.05) \cdot (2.8) = 2.940$ because the decimal point in 2,940 moved three places to the left.
- $15 \cdot 24 = 360$. The decimal needs to be moved 7 places to the left because the decimal point of 0.0015 was moved four places to the right to get 15, and the decimal point of 0.024 was moved three places to the right to get 24. So the answer is 0.0000360.

Lesson 7: Using Diagrams to Represent Multiplication

Goals

- Comprehend how the phrase “partial products” (in spoken and written language) refers to decomposing a multiplication problem.
- Coordinate area diagrams and vertical calculations that represent the same decimal multiplication problem.
- Use an area diagram to represent and justify (orally and in writing) how to find the product of two decimals.

Learning Targets

- I can use area diagrams and partial products to represent and find products of decimals.

Lesson Narrative

Students continue to use area diagrams to find products of decimals, while also beginning to generalize the process. They revisit two methods used to find products in earlier grades: decomposing a rectangle into sub-rectangles and finding the sum of their areas, and using the multiplication algorithm.

Students have previously seen that, in a rectangular area diagram, the side lengths can be decomposed by place value. For instance, in an 18 by 23 rectangle, the 18-unit side can be decomposed into 10 and 8 units (tens and ones), and the 23-unit side can be expressed as 20 and 3 (also tens and ones), creating four sub-rectangles whose areas constitute four partial products. The sum of these partial products is the product of 18 and 23. Students extend the same reasoning to represent and find products such as $(1.8) \cdot (2.3)$. Then, students explore how these partial products correspond to the numbers in the multiplication algorithm.

Students connect multiplication of decimals to that of whole numbers (MP7), look for correspondences between geometric diagrams and arithmetic calculations, and use these connections to calculate products of various decimals.

Alignments

Building On

- 4.NBT.B.5: Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
- 5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the

relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Addressing

- 6.NS.B.3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Instructional Routines

- MLR7: Compare and Connect

Required Materials

Graph paper

Rulers

Required Preparation

Some students might find it helpful to use graph paper to help them align the digits for vertical calculations. Consider having graph paper accessible for the last activity: Connecting Area Diagrams to Calculations with Decimals.

Student Learning Goals

Let's use area diagrams to find products.

7.1 Estimate the Product

Warm Up: 5 minutes

This warm-up prompts students to review multiplication of decimals and think about the size of a product given the decimals being multiplied. In grade 5, students multiplied decimals to hundredths using concrete representations or drawings, place-value strategies, and properties of operations. Though they may know how to find decimal products up to hundredths, they may not always consider the reasonableness of their answers. Students may simply perform the computation and “count the number of decimal places” to put the decimal point in the product. Discuss why each estimated product makes sense based on what they know about place value and multiplication.

Building On

- 5.NBT.B.7

Launch

Tell students you will display a multiplication expression along with three possible estimates of its value. Their job is to select the best estimate and be able to explain why it is the best. Display one problem at a time. Give students 30 seconds of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Follow with a whole-class discussion.

Anticipated Misconceptions

Some students might mix together an estimation strategy (e.g., rounding the factors) and the method of “counting the number of decimal places” (which is applied to precise calculations) and

put the decimal point in their estimate. For example, to estimate $74 \cdot (8.1)$, they might round the factors to 70 and 8 and find a product of 560. But because they see that there is a total of 1 place after the decimal point in the original factors, they move the decimal point in 560 one place to the left and choose 56 as their answer. Prompt students to think about the reasonableness of their answer relative to the factors (e.g., ask if 56 is a reasonable product of 70 and 8).

Student Task Statement

For each of the following products, choose the best estimate of its value. Be prepared to explain your reasoning.

1. $(6.8) \cdot (2.3)$

- ☐ 1.40
- ☐ 14
- ☐ 140

2. $74 \cdot (8.1)$

- ☐ 5.6
- ☐ 56
- ☐ 560

3. $166 \cdot (0.09)$

- ☐ 1.66
- ☐ 16.6
- ☐ 166

4. $(3.4) \cdot (1.9)$

- ☐ 6.5
- ☐ 65
- ☐ 650

Student Response

1. 14. Possible strategy: Round 6.8 to 7 and 2.3 to 2, and multiply. $7 \cdot 2 = 14$
2. 560. Possible strategy: Round 74 to 70 and 8.1 to 8, and multiply. $70 \cdot 8 = 560$
3. 16.6. Possible strategy: Round 0.09 to 0.1, and multiply. $166 \cdot (0.1) = 16.6$
4. 6.5. Possible strategy: Round 3.4 to 3 and 1.9 to 2, and multiply. $3 \cdot 2 = 6$, and 6.5 is closest to 6.

Activity Synthesis

Discuss one problem at a time. Ask students to share their response and reasoning. Record and display student explanations for all to see. If not mentioned by students in their explanations, ask if or how the given factors impacted how they estimated. To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Does anyone want to add on to ____’s strategy?”
- “Do you agree or disagree? Why?” If there is time, ask students to solve the final problem with a partner. Encourage them to solve it mentally first and then check with pencil and paper if necessary.

7.2 Connecting Area Diagrams to Calculations with Whole Numbers

Optional: 20 minutes (there is a digital version of this activity)

In this activity, students review two numerical methods for multiplication learned in previous grades and use them to calculate products of whole numbers. Students have had a chance to revisit area diagrams in the past couple of activities. Here they make explicit connections between numerical calculations and area diagrams. As in previous activities, place value plays a key role in how a rectangle can be partitioned to facilitate multiplication.

As students discuss and work, notice the ways they describe the correspondences between numerical calculations and the numbers in the area diagrams. Identify students who can clearly articulate their reasoning so they can share later.

Building On

- 4.NBT.B.5

Launch

Arrange students in groups of 2. Give groups 3–5 minutes to discuss and answer the first set of questions. Follow with a brief whole-class discussion. Select a few students to share their responses to the first set of questions. Ask students why factors are decomposed by place value (e.g., 20 and 4), as opposed to decomposing into any two numbers (e.g., 13 and 11)? (It is easier to multiply two numbers if there is only 1 non-zero digit in each.)

Give groups another 3–5 minutes to discuss and answer the second set of questions. Follow with a brief whole-class discussion. Highlight the relationship between the blue numbers in the calculations and the partial areas in the diagram. Discuss:

- The numbers in blue—in the calculations and area diagrams—are called “partial products.” Why might that be? (They represent products of parts of the factors.)

- How are the partial products in Calculation A different from those in Calculation B? (Calculation A lists the product of any two base-ten digits separately. Calculation B groups them.) How are they similar? (They both lead to the same final product.)

Next, give students a few minutes of quiet time to complete the remaining set of questions.

Students using the digital activity should complete the written work first, and then check their work using the applet. Encourage them to explore other multiplication expressions or find other products to deepen their understanding of the area diagram.

Access for Students with Disabilities

Representation: Internalize Comprehension. Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, use color coding to highlight connections between the partial products in the calculations and area diagrams.

Supports accessibility for: Visual-spatial processing

Anticipated Misconceptions

If students have trouble seeing the correspondence between the diagram and the numbers in Calculation B, consider asking them to shade the areas 60 and 12 in one color and the areas 200 and 40 in another color.

Student Task Statement

1. Here are three ways of finding the area of a rectangle that is 24 units by 13 units.

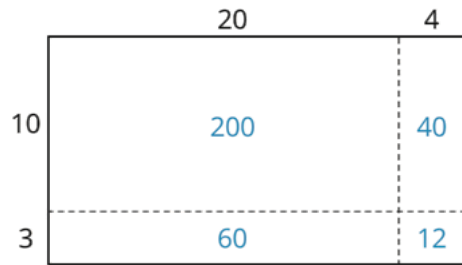


Diagram 1

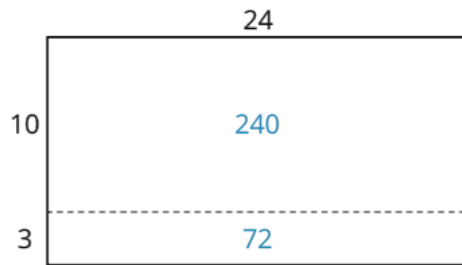


Diagram 2

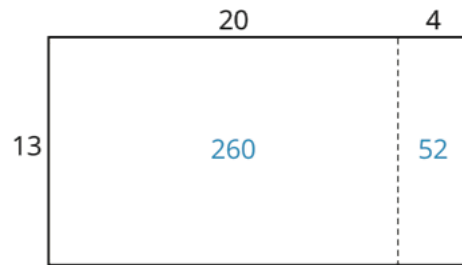


Diagram 3

- What do the diagrams have in common? How are they the same?
 - How are the diagrams different?
 - If you were to find the area of a rectangle that is 37 units by 19 units, which of the three ways of decomposing the rectangle would you use? Why?
2. You may be familiar with different ways to write multiplication calculations. Here are two ways to calculate 24 times 13.

$$\begin{array}{r}
 24 \\
 \times 13 \\
 \hline
 12 \\
 60 \\
 40 \\
 + 200 \\
 \hline
 312
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \text{partial products}$$

Calculation A

$$\begin{array}{r}
 24 \\
 \times 13 \\
 \hline
 72 \\
 + 240 \\
 \hline
 312
 \end{array}$$

Calculation B

- In Calculation A, how are each of the partial products obtained? For instance, where does the 12 come from?
- In Calculation B, how are the 72 and 240 obtained?
- Look at the diagrams in the first question. Which diagram corresponds to Calculation A? Which one corresponds to Calculation B?

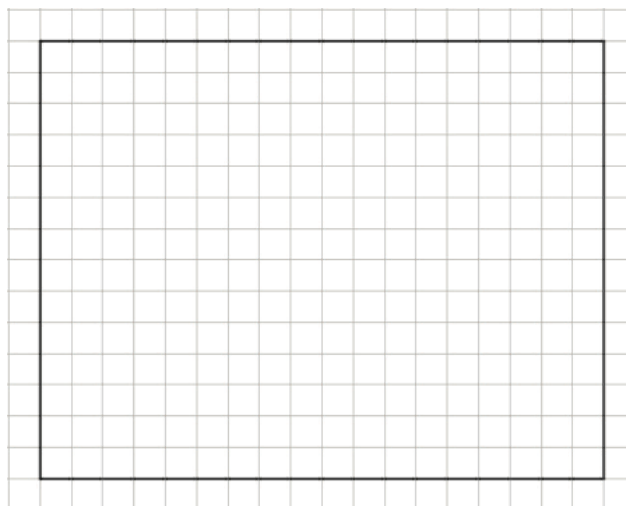
d. How are the partial products in Calculation A and the 72 and 240 in Calculation B related to the numbers in the diagrams?

3. Use the two following methods to find the product of 18 and 14.

○ Calculate numerically.

$$\begin{array}{r} 18 \\ \times 14 \\ \hline \end{array}$$

○ Here is a rectangle that is 18 units by 14 units. Find its area, in square units, by decomposing it. Show your reasoning.



4. Compare the values of $18 \cdot 14$ that you obtained using the two methods. If they are not the same, check your work.

Student Response

1. Answers vary. Sample responses:

- Similarities: They show the same rectangle with the same area. The sum of all the partial products is the same in all cases.
- Differences: Some side lengths are broken up by place value, others are not. Some have two sub-rectangles, others have four sub-rectangles.
- I would use the method as in Diagram 1. I would break both numbers into tens and ones because it is easier to multiply tens by tens and ones by ones. If I broke up only one number into tens and ones; the other number is still not friendly enough to multiply quickly.

2. Answers vary. Sample responses:

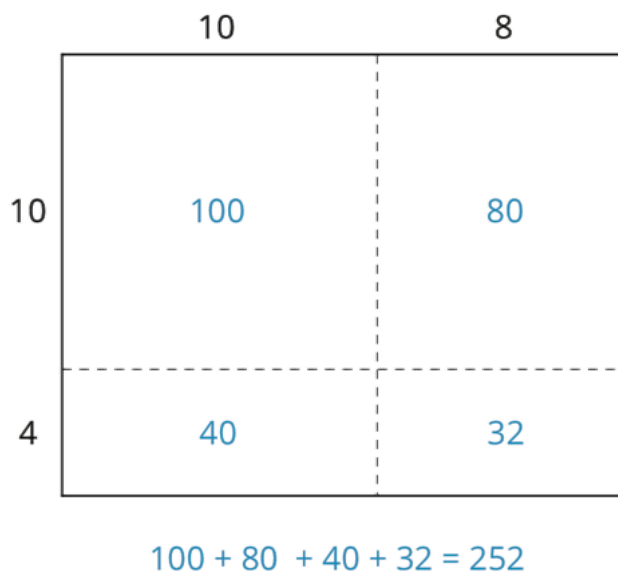
- In Calculation A, 12 is $3 \cdot 4$, 60 is $3 \cdot 20$, 40 is $10 \cdot 4$, and 200 is $10 \cdot 20$.
- In Calculation B, 72 is $12 + 60$, and 240 is $40 + 200$. We can also see 72 as $3 \cdot 24$ and 240 as $10 \cdot 24$.
- Calculation A corresponds to Diagram 1. Calculation B corresponds to Diagram 2.

d. Diagram 1 shows 4 sub-rectangles; their areas are the partial products in the calculation: 12, 60, 40, 200. This makes sense because the side lengths of those rectangles are the numbers multiplied to get the partial products. Diagram 2 shows 2 sub-rectangles; their areas are the partial products of 72 and 240. The area of the bottom rectangle is the sum of $12 + 60$, and the area of the top rectangle is the sum of $40 + 200$.

3. a.

$$\begin{array}{r} 8 \\ x 1 4 \\ \hline 7 2 \\ + 1 8 \\ \hline 2 5 2 \end{array} \qquad \begin{array}{r} 8 \\ x 1 4 \\ \hline 7 2 \\ + 1 8 0 \\ \hline 2 5 2 \end{array}$$

b.



4. No response required.

Activity Synthesis

Conclude with a whole-class discussion that focuses on connecting the numbers in Calculation B to the multiplication algorithm students learned in grade 5. Discuss:

- Calculation B shows the multiplication algorithm we can use to multiply 24 and 13 even without an area diagram. What calculation would give us the 72? What about the 240? (The two numbers in blue come from multiplying each digit of the second factor—starting from the ones place—by the first factor. The 72 is the product of 3 and 24. The 24 is the product of 10 and 24, or the 1 in the tens place and the 24.)

- There is a variation to Calculation B in which the zero in the ones place is not written (as shown), but the result is the same. Why might it be fine to write “24” without the zero at the end? (The 24 is understood to mean “24 tens” because it is written with 4 in the tens place.)

$$\begin{array}{r}
 24 \\
 \times 13 \\
 \hline
 72 \\
 + 24 \\
 \hline
 312
 \end{array}$$

Also consider exploring other multiplication scenarios with this digital applet: <https://ggbm.at/K9B6Eg4H>.

7.3 Connecting Area Diagrams to Calculations with Decimals

20 minutes (there is a digital version of this activity)

This activity extends the previous optional activity to products of decimals. It opens with a slight variant of the product $24 \cdot 13$, namely $(2.4) \cdot (1.3)$. The side lengths of the area diagrams are multiplied by 0.1, but the reasoning involved is unchanged. While students can find the product using any of the previously developed pathways—i.e., multiplying the factors by powers of 0.1 or $\frac{1}{10}$, or by using fractions—the focus here is on using partial products and connecting them to the multiplication algorithm. In the next lesson, students will generalize the process and use the algorithm to compute products of other decimals.

Recognition and use of structure (MP7) are once again important here. The use of the same non-zero digits in the problems also gives students a chance to see regularity in the reasoning and to focus on how the decimal point affects the calculation (MP8).

Addressing

- 6.NS.B.3

Instructional Routines

- MLR7: Compare and Connect

Launch

Arrange students in groups of 2. Give students 1–2 minutes of quiet think time for the first problem. Pause for a brief whole-class discussion, making sure all students label each region correctly. Explain that the diagram is not to scale, and that when drawing an area diagram, it is fine to estimate appropriate side lengths.

Have partners analyze the calculations in the second question. Monitor student discussions to check for understanding. If necessary, pause to have a whole-class discussion on the interpretation of these calculations. Then give students 7–8 minutes of quiet time to complete the remaining questions. Follow up with a whole-class discussion to emphasize how partial area products relate to the calculations.

Students with access to the digital materials can also work in groups of 2. When using the applet, students adjust the values by moving the dots on the ends of the segments to match the calculation. After giving students 3–4 minutes to log-in and complete the first problem, pause to discuss, making sure all students label each region correctly. Then they should also use the applet to check their calculations. Explain that diagrams may not be to scale, and that when drawing an area diagram, it is fine to estimate appropriate side lengths.

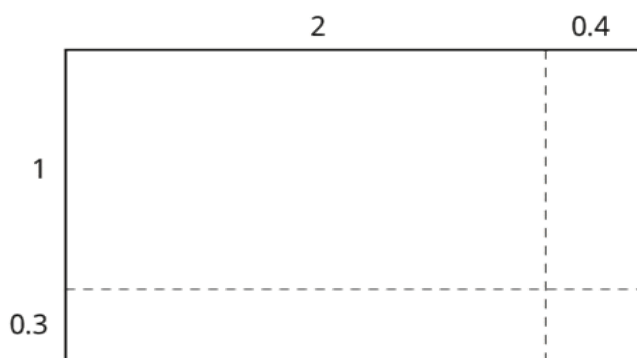
Access for Students with Disabilities

Representation: Internalize Comprehension. Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, use color coding to highlight connections between the partial products in the calculations and area diagrams.

Supports accessibility for: Visual-spatial processing

Student Task Statement

1. You can use area diagrams to represent products of decimals. Here is an area diagram that represents $(2.4) \cdot (1.3)$.



- Find the region that represents $(0.4) \cdot (0.3)$. Label it with its area of 0.12.
- Label the other regions with their areas.
- Find the value of $(2.4) \cdot (1.3)$. Show your reasoning.

2. Here are two ways of calculating $(2.4) \cdot (1.3)$.

$$\begin{array}{r}
 2.4 \\
 \times 1.3 \\
 \hline
 0.12 \\
 0.6 \\
 0.4 \\
 + 2 \\
 \hline
 3.12
 \end{array}
 \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \text{partial products}$$

Calculation A

$$\begin{array}{r}
 2.4 \\
 \times 1.3 \\
 \hline
 0.72 \\
 + 2.4 \\
 \hline
 3.12
 \end{array}$$

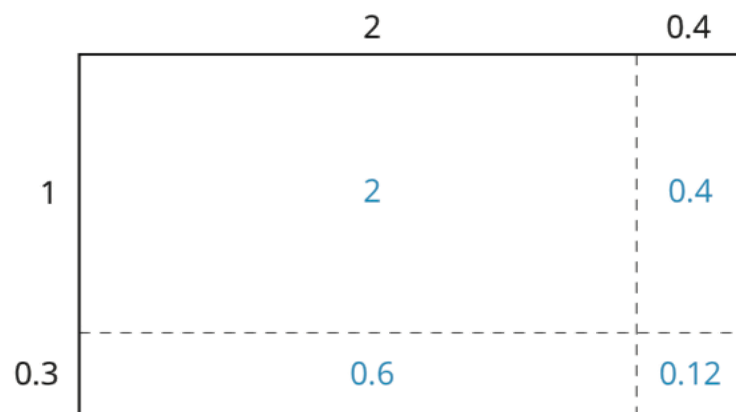
Calculation B

Analyze the calculations and discuss these questions with a partner:

- In Calculation A, where does the 0.12 and other partial products come from?
 - In Calculation B, where do the 0.72 and 2.4 come from?
 - In each calculation, why are the numbers below the horizontal line aligned vertically the way they are?
3. Find the product of $(3.1) \cdot (1.5)$ by drawing and labeling an area diagram. Show your reasoning.
 4. Show how to calculate $(3.1) \cdot (1.5)$ using numbers without a diagram. Be prepared to explain your reasoning. If you are stuck, use the examples in a previous question to help you.

Student Response

1. a. See next part for a completely labeled rectangle.
- b.



- c. The area of the rectangle is the sum of the sub-rectangles, which is 3.12:
 $2 + 0.4 + 0.6 + 0.12 = 3.12$.
2. a. In Calculation A:

■ $0.12 = (0.4) \cdot (0.3)$

■ $0.6 = 2 \cdot (0.3)$

■ $0.4 = (0.4) \cdot 1$

■ $2 = 2 \cdot 1$

In Calculation B: $0.72 = (0.3) \cdot (2.4)$ and $2.4 = 1 \cdot (2.4)$.

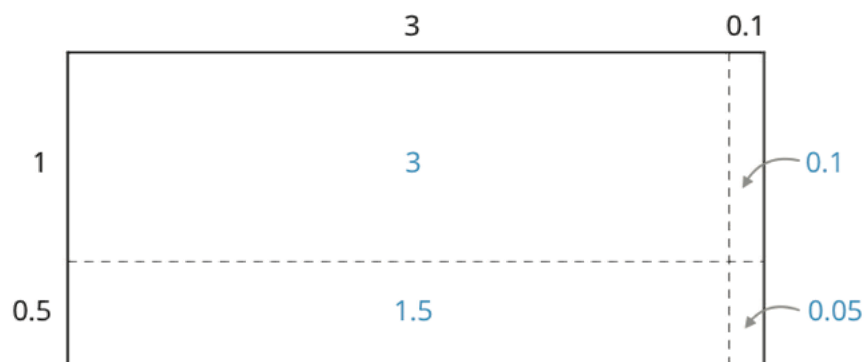
b. Sample reasonings:

■ The numbers are lined up so that digits in the same place value are aligned (for example, the 1 in 0.12, the 6 in 0.6, and the 4 in 0.4 are all in the tenths place).

■ The numbers are lined up this way so that like base-ten units can be easily added.

3. The sum of the sub-rectangles is 4.65, so that is the value of the product $(3.1) \cdot (1.5)$.

$$3 + 0.1 + 1.5 + 0.05 = 4.65.$$



4.

$$\begin{array}{r}
 3.1 \\
 \times 1.5 \\
 \hline
 0.05 \\
 1.5 \\
 0.1 \\
 + 3 \\
 \hline
 4.65
 \end{array}$$

Are You Ready for More?

How many hectares is the property of your school? How many morgens is that?

Student Response

Answers vary.

Activity Synthesis

To highlight the role of place value in multiplication, discuss the following questions:

- How does the diagram for the product $(2.4) \cdot (1.3)$ compare to that for $24 \cdot 13$? (They are the same; the only difference is that the decimal side lengths—the factors—are one-tenth of the whole-number ones.)
- How are the two calculations similar? How are they different? (There are no decimals in the partial products of $24 \cdot 13$. There are 0's at the end of the numbers in the calculation for $24 \cdot 13$ but not in the other calculation. The non-zero numbers are the same in the two calculations.)

	2.4		24
×	1.3	×	13
	0.12		12
	0.6		60
	0.4		40
+	2	+	200
	3.12		312

Prompt students to look at the close relationship between the partial-product calculations and the technique of using fractions to find products of decimals. Students previously saw that $(2.4) \cdot (1.3) = (24 \cdot \frac{1}{10}) \cdot (13 \cdot \frac{1}{10}) = (24 \cdot 13) \cdot (\frac{1}{10} \cdot \frac{1}{10}) = 312 \cdot \frac{1}{100} = 3.12$.

The two partial-product calculations, presented side-by-side, validate the previous reasoning. The calculation on the right shows the whole-number product of 24 and 13. The one on the left shows that when each decimal factor is one-tenth of the whole-number factor, the decimal product is one hundredth of the whole-number product, so the decimal point moves two places to the left.

Access for English Language Learners

Representing, Conversing: MLR7 Compare and Connect. Use this routine to help students make sense of each strategy. Invite students to create a visual display that clearly shows how they calculated $(3.1) \cdot (1.5)$, with and without a diagram. Students should consider how to display their calculations so that another student can interpret them. Some students may wish to add notes or details to their display to help communicate their thinking. Arrange students in groups of 2, and provide 2–3 minutes of quiet think time for students to read and interpret each other's work. Invite students to look for what is the same and what is different about their approaches, and to identify how 0.1 is represented in each method. This will support student understanding of decimal multiplication and strengthen their use of mathematical language.

Design Principle(s): Support sense-making; Maximize meta-awareness

7.4 Using the Partial Products Method

Optional: 10 minutes

This activity continues using partial products and area diagrams to compute products of decimals and connect area diagrams with vertical calculations. In these problems, the side lengths and areas of the rectangle are left blank. Students fill in these fields by decomposing the factors by place value and multiplying them. Like many of the activities in this unit, the work involves understanding the structure of the base-ten system used in multiplication calculations (MP7) and making sense of it from arithmetic and geometric perspectives.

Students also make another key connection about several previously developed ideas here. They see that we can write the non-zero digits of decimal factors as whole numbers, use vertical calculations to multiply them, and then attend to the decimal point in the product afterwards. In other words, they notice the algorithm for multiplication can streamline the reasoning processes they have used up to this point.

Addressing

- 6.NS.B.3

Launch

Arrange students in groups of 2. Give groups 1–2 minutes to make sense of the first set of questions and area diagram. Listen to their discussions to make sure they understand how to label the appropriate side lengths. If needed, pause the class and have a student or a group that correctly decomposed the side lengths by place value share their reasoning.

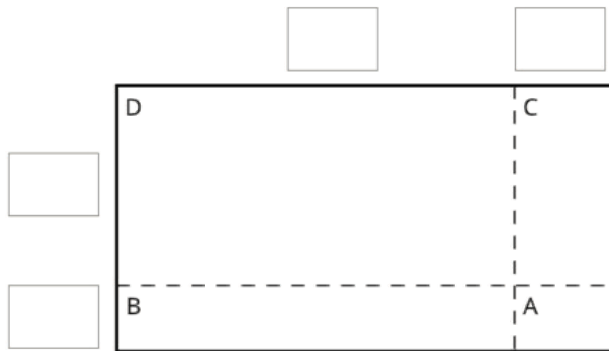
Give students 5 minutes of quiet work time for the rest of the task. Follow with a whole-class discussion.

Anticipated Misconceptions

If students label side lengths with numbers that are not single-digit multiples of powers of ten, remind them to work with base-ten units as practiced in the previous lesson. Provide them with one label for a length and see if they can then find the other sections.

Student Task Statement

1. Label the area diagram to represent $(2.5) \cdot (1.2)$ and to find that product.



- Decompose each number into its base-ten units (ones, tenths, etc.) and write them in the boxes on each side of the rectangle.
- Label Regions A, B, C, and D with their areas. Show your reasoning.
- Find the product that the area diagram represents. Show your reasoning.

2. Here are two ways to calculate $(2.5) \cdot (1.2)$. Each number with a box gives the area of one or more regions in the area diagram.

$ \begin{array}{r} 2.5 \\ \times 1.2 \\ \hline 0.1 \\ 0.4 \\ 0.5 \\ + 2.0 \\ \hline 3.00 \end{array} $	$ \begin{array}{r} 2.5 \\ \times 1.2 \\ \hline 0.5 \\ + 2.5 \\ \hline 3.00 \end{array} $
--	--

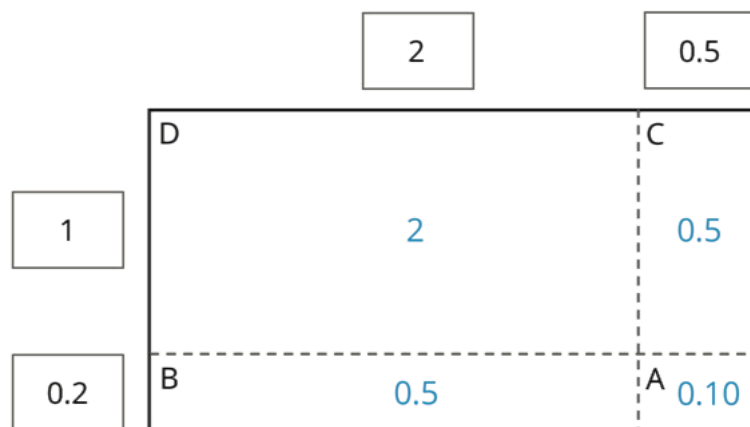
Calculation A

Calculation B

- In the boxes next to each number, write the letter(s) of the corresponding region(s).
- In Calculation B, which two numbers are being multiplied to obtain 0.5? Which numbers are being multiplied to obtain 2.5?

Student Response

1. a.



b. A: $(0.5) \cdot (0.2) = 0.1$, B: $(2) \cdot (0.2) = 0.4$, C: $(0.5) \cdot 1 = 0.5$,
D: $2 \cdot 1 = 2$

c. $0.1 + 0.4 + 0.5 + 2 = 3$, so $(2.5) \cdot (1.2) = 3$

2. a.

$\begin{array}{r} 2.5 \\ \times 1.2 \\ \hline 0.10 \\ 0.4 \\ 0.5 \\ + 2.0 \\ \hline 3.00 \end{array}$		<table><tr><td>A</td></tr><tr><td>B</td></tr><tr><td>C</td></tr><tr><td>D</td></tr></table>	A	B	C	D
A						
B						
C						
D						

$\begin{array}{r} 2.5 \\ \times 1.2 \\ \hline 0.5 \\ + 2.5 \\ \hline 3.00 \end{array}$		<table><tr><td>A + B</td></tr><tr><td>C + D</td></tr></table>	A + B	C + D
A + B				
C + D				

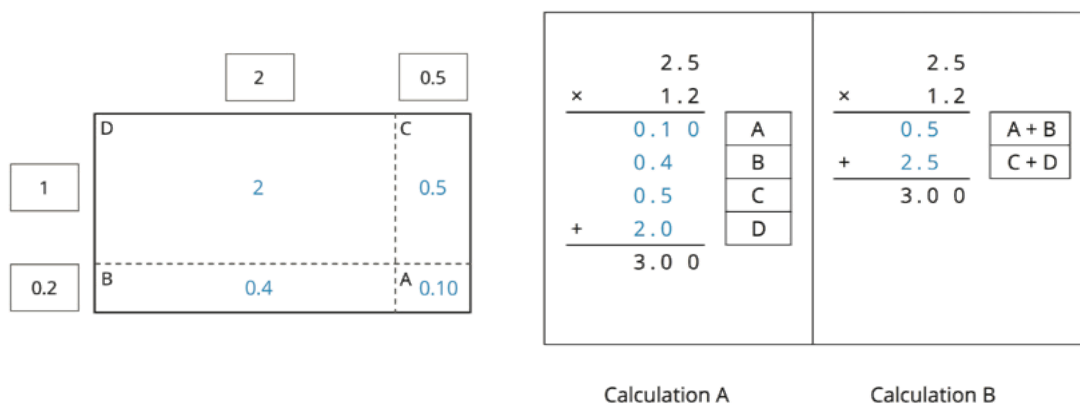
Calculation A

Calculation B

b. $0.5 = (0.2) \cdot (2.5)$ and $2.5 = (1.0) \cdot (2.5)$

Activity Synthesis

Display the solution shown here for all to see.



Select a few students to explain how they determined which factor belongs to which side of the rectangle and how they found each partial area. Then, ask how each blue number in the calculations, especially Calculation B, is obtained. If not mentioned in students' explanations, emphasize that the blue 0.5, represented by the area of A + B in the diagram, is the product of 0.2 and 2.5. To highlight this connection, consider shading or coloring the corresponding areas in the diagram and circling these values in the calculations. Likewise, the blue 2.5, represented by the area of C + D in the diagram, is the product of 1 and 2.5 in the calculation.

Lesson Synthesis

We can use the area of a rectangle to represent products of decimals, just as we have done so with whole numbers. The calculations of partial products and their sum can also be performed numerically and without a diagram. The numbers in a vertical calculation correspond closely to the partial areas in the diagram.

- How can a rectangle help us represent the product of two numbers? (The sides of a rectangle can represent the factors, and the area of the rectangle represents the product.)
- How do we use a rectangle to help us find the product of two numbers? (We can decompose each factor by place value—tens, ones, tenths, hundredths, etc., partition the rectangle into regions, and find the areas of these regions. The sum of these partial areas is the product of the two numbers. This method can be used with whole numbers and decimals.)
- How are the numbers in vertical calculations related to those in area diagrams? (The numbers in vertical calculations reflect the partial areas in diagram. These partial areas are added to find the product of the two factors.)

7.5 Find the Product

Cool Down: 5 minutes

Addressing

- 6.NS.B.3

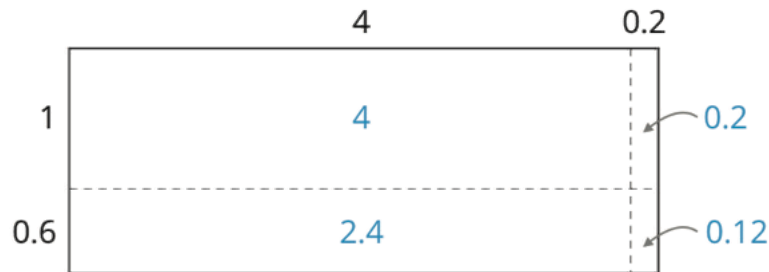
Student Task Statement

Find $(4.2) \cdot (1.6)$ by drawing an area diagram or using another method. Show your reasoning.

Student Response

The big rectangle represents 6.72 because

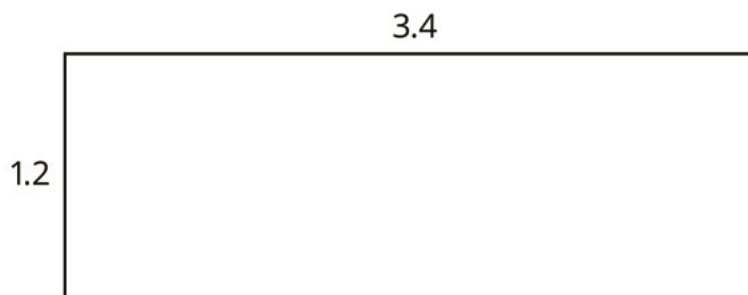
$$(4 \cdot 1) + (0.2 \cdot 1) + (4 \cdot 0.6) + (0.2 \cdot 0.6) = 4 + 0.2 + 2.4 + 0.12 = 6.72.$$



Student Lesson Summary

Suppose that we want to calculate the product of two numbers that are written in base ten. To explain how, we can use what we know about base-ten numbers and areas of rectangles.

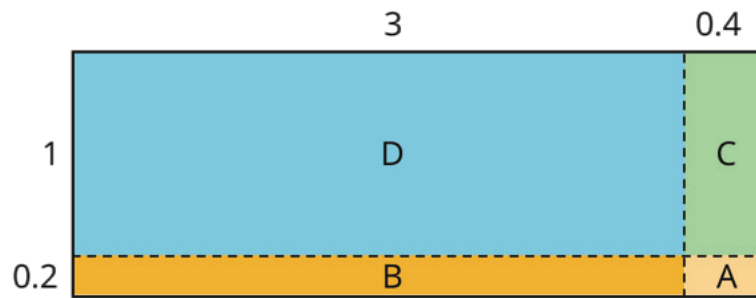
Here is a diagram of a rectangle with side lengths 3.4 units and 1.2 units.



Its area, in square units, is the product

$$(3.4) \cdot (1.2)$$

To calculate this product and find the area of the rectangle, we can decompose each side length into its base-ten units, $3.4 = 3 + 0.4$ and $1.2 = 1 + 0.2$, decomposing the rectangle into four smaller sub-rectangles.



We can rewrite the product and expand it twice:

$$\begin{aligned}
 (3.4) \cdot (1.2) &= (3 + 0.4) \cdot (1 + 0.2) \\
 &= (3 + 0.4) \cdot 1 + (3 + 0.4) \cdot 0.2 \\
 &= 3 \cdot 1 + 3 \cdot (0.2) + (0.4) \cdot 1 + (0.4) \cdot (0.2)
 \end{aligned}$$

In the last expression, each of the four terms is called a partial product. Each partial product gives the area of a sub-rectangle in the diagram. The sum of the four partial products gives the area of the entire rectangle.

We can show the horizontal calculations above as two vertical calculations.

$ \begin{array}{r} 3.4 \\ \times 1.2 \\ \hline 0.08 \\ 0.6 \\ 0.4 \\ + 3 \\ \hline 4.08 \end{array} $	$ \begin{array}{r} 3.4 \\ \times 1.2 \\ \hline 0.68 \\ + 3.4 \\ \hline 4.08 \end{array} $
<div style="display: inline-block; width: 30px; height: 20px; background-color: #f4a460; margin-bottom: 5px;"></div> A <div style="display: inline-block; width: 30px; height: 20px; background-color: #f4a460; margin-bottom: 5px;"></div> B <div style="display: inline-block; width: 30px; height: 20px; background-color: #a4d4a4; margin-bottom: 5px;"></div> C <div style="display: inline-block; width: 30px; height: 20px; background-color: #a4d4a4; margin-bottom: 5px;"></div> D	<div style="display: inline-block; width: 30px; height: 20px; background-color: #f4a460; margin-bottom: 5px;"></div> A + <div style="display: inline-block; width: 30px; height: 20px; background-color: #f4a460; margin-bottom: 5px;"></div> B <div style="display: inline-block; width: 30px; height: 20px; background-color: #a4d4a4; margin-bottom: 5px;"></div> C + <div style="display: inline-block; width: 30px; height: 20px; background-color: #a4d4a4; margin-bottom: 5px;"></div> D

The calculation on the left is an example of the partial products method. It shows the values of each partial product and the letter of the corresponding sub-rectangle. Each partial product gives an area:

- A is 0.2 unit by 0.4 unit, so its area is 0.08 square unit.
- B is 3 units by 0.2 unit, so its area is 0.6 square unit.
- C is 0.4 unit by 1 unit, so its area is 0.4 square unit.
- D is 3 units by 1 unit, so its area is 3 square units.
- The sum of the partial products is $0.08 + 0.6 + 0.4 + 3$, so the area of the rectangle is 4.08 square units.

The calculation on the right shows the values of two products. Each value gives a combined area of two sub-rectangles:

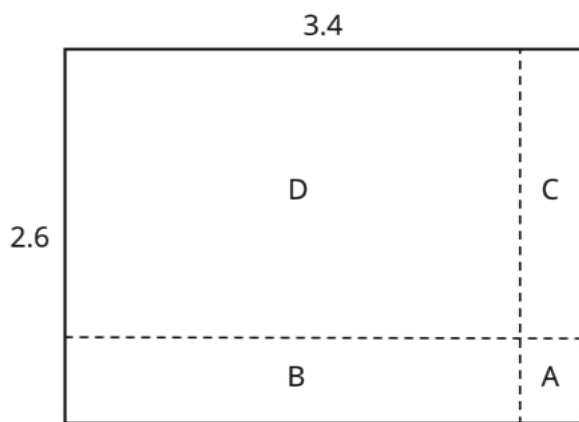
- The combined regions of A and B have an area of 0.68 square units; 0.68 is the value of $(3 + 0.4) \cdot 0.2$.
- The combined regions of C and D have an area of 3.4 square units; 3.4 is the value of $(3 + 0.4) \cdot 1$.
- The sum of the values of two products is $0.68 + 3.4$, so the area of the rectangle is 4.08 square units.

Lesson 7 Practice Problems

Problem 1

Statement

Here is a rectangle that has been partitioned into four smaller rectangles.



For each expression, choose the sub-rectangle whose area, in square units, matches the expression.

- $3 \cdot (0.6)$
- $(0.4) \cdot 2$
- $(0.4) \cdot (0.6)$
- $3 \cdot 2$

Solution

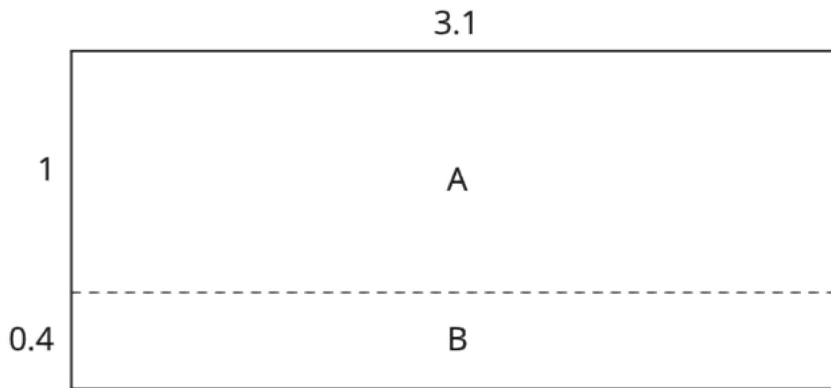
- B
- C
- A

d. D

Problem 2

Statement

Here is an area diagram that represents $(3.1) \cdot (1.4)$.



a. Find the areas of sub-rectangles A and B.

b. What is the area of the 3.1 by 1.4 rectangle?

Solution

a. Rectangle A: 3.1 square units, Rectangle B: 1.24 square units

b. 4.34 square units ($3.1 + 1.24 = 4.34$)

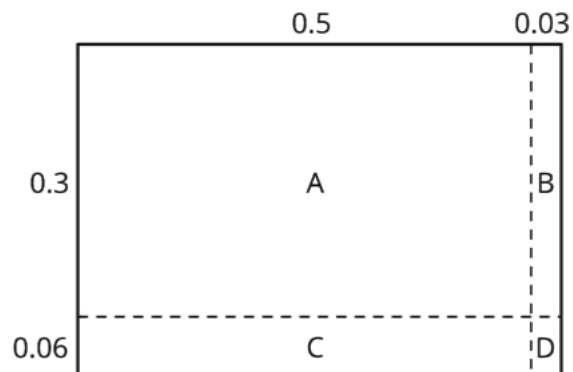
Problem 3

Statement

Draw an area diagram to find $(0.36) \cdot (0.53)$. Label and organize your work so that it can be followed by others.

Solution

0.1908. Sample diagram and reasoning:



◦ Area of A is $(0.5)(0.3) = 0.15$.

- Area of B is $(0.03)(0.3) = 0.009$.
- Area of C is $(0.5)(0.06) = 0.03$.
- Area of D is $(0.03)(0.06) = 0.0018$.
- The area of the rectangle, in square units, is $0.15 + 0.009 + 0.03 + 0.0018 = 0.1908$.

Problem 4

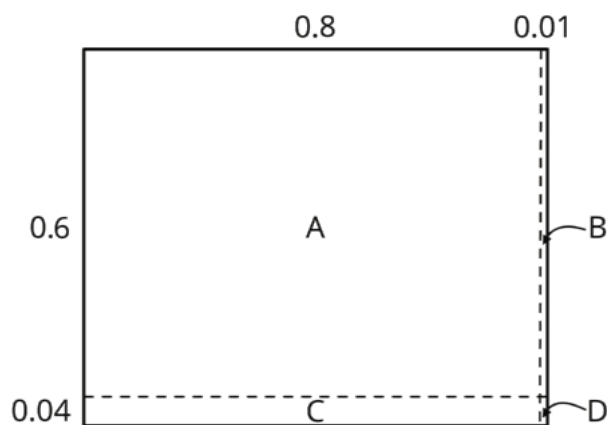
Statement

Find each product. Show your reasoning.

- $(2.5) \cdot (1.4)$
- $(0.64) \cdot (0.81)$

Solution

- 3.5. Sample reasoning: $2 \cdot (1.4) = 2.8$ and $(0.5) \cdot (1.4) = 0.7$. The product is $2.8 + 0.7$ or 3.5.
- 0.5184. Sample reasoning:



- Area of A is $(0.8)(0.6) = 0.48$.
- Area of B is $(0.01)(0.6) = 0.006$.
- Area of C is $(0.8)(0.04) = 0.032$.
- Area of D is $(0.01)(0.04) = 0.0004$.
- The area of the rectangle, in square units, is $0.48 + 0.006 + 0.032 + 0.0004 = 0.5184$.

Problem 5

Statement

Complete the calculations so that each shows the correct sum.

$$\begin{array}{r} 2.3\boxed{} \\ + \boxed{}.64 \\ \hline 9.\boxed{}5 \end{array}$$

$$\begin{array}{r} 2.3\boxed{} \\ + \boxed{}.64 \\ \hline 9.\boxed{}2 \end{array}$$

$$\begin{array}{r} 4.3\boxed{} \\ + \boxed{}.15 \\ \hline 6.\boxed{}2 \end{array}$$

$$\begin{array}{r} 1.5\boxed{} \\ + \boxed{}.38 \\ \hline 1.\boxed{}4 \end{array}$$

Solution

$$\begin{array}{r} 2.3\boxed{1} \\ + \boxed{7}.64 \\ \hline 9.\boxed{9}5 \end{array}$$

$$\begin{array}{r} 2.3\boxed{8} \\ + \boxed{6}.64 \\ \hline 9.\boxed{0}2 \end{array}$$

$$\begin{array}{r} 4.3\boxed{7} \\ + \boxed{2}.15 \\ \hline 6.\boxed{5}2 \end{array}$$

$$\begin{array}{r} 1.5\boxed{6} \\ + \boxed{0}.38 \\ \hline 1.\boxed{9}4 \end{array}$$

(From Unit 5, Lesson 3.)

Problem 6

Statement

Diego bought 12 mini muffins for \$4.20.

a. At this rate, how much would Diego pay for 4 mini muffins?

b. How many mini muffins could Diego buy with \$3.00? Explain or show your reasoning. If you get stuck, consider using the table.

number of mini muffins	price in dollars
12	4.20

Solution

a. \$1.40.

b. 8 mini muffins, which would cost \$2.80. He does not have enough money for 9 mini muffins, because that would cost \$3.15.

number of mini muffins	price in dollars
12	4.20
1	0.35
8	2.80
9	3.15

(From Unit 2, Lesson 12.)

Lesson 8: Calculating Products of Decimals

Goals

- Draw and label a diagram to check the answer to a decimal multiplication problem.
- Interpret a description (in written language) of a real-world situation involving multiplication of decimals, and write a multiplication problem to represent it.
- Use an algorithm to calculate the product of two decimals, and explain (orally) the solution method.

Learning Targets

- I can describe and apply a method for multiplying decimals.
- I know how to use a product of whole numbers to find a product of decimals.

Lesson Narrative

In this culminating lesson on multiplication, students continue to use the structure of base-ten numbers to make sense of calculations (MP7) and consolidate their understanding of the themes from the previous lessons. They see that multiplication of decimals can be accomplished by:

- thinking of the decimals as products of whole numbers and fractions;
- writing the non-zero digits of the factors as whole numbers, multiplying them, and moving the decimal point in the product;
- representing the multiplication with an area diagram and finding partial products; and
- using a multiplication algorithm, the steps of which can be explained with the reasonings above.

Alignments

Building On

- 5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Addressing

- 6.NS.B.3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Building Towards

- 6.EE.A: Apply and extend previous understandings of arithmetic to algebraic expressions.

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports
- Number Talk
- Think Pair Share

Required Preparation

Some students might find it helpful to use graph paper to help them align the digits for vertical calculations. Consider having graph paper accessible for the activity: Calculating Products of Decimals.

Student Learning Goals

Let's multiply decimals.

8.1 Number Talk: Twenty Times a Number

Warm Up: 5 minutes

The purpose of this number talk is to have students see structure related to the distributive property in preparation for the problems using area diagrams they will solve in the lesson.

Building On

- 5.NBT.B.7

Building Towards

- 6.EE.A

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Student Task Statement

Evaluate mentally.

$$20 \cdot 5$$

$$20 \cdot (0.8)$$

$$20 \cdot (0.04)$$

$$20 \cdot (5.84)$$

Student Response

- 100
- 16
- 0.8
- 116.8

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone have the same strategy but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to ____'s strategy?"
- "Do you agree or disagree? Why?"

Access for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because . . ." or "I noticed ____ so I" Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Design Principle(s): Optimize output (for explanation)

8.2 Calculating Products of Decimals

25 minutes

Students deepen and reinforce the ideas developed in previous activities: using area diagrams to find partial products, relating these partial products to the numbers in the algorithm, and using multiplication of whole numbers to find the product of decimals. By calculating products of decimals using vertical calculations of whole numbers, students extend their understanding of multiplication to include multiplication of any pair of decimals.

Finally, with a range of methods for multiplying decimals in hand, students choose a method to solve a contextual problem. The application invites students to use MP2, deciding what mathematical operations to perform based on the context and then using the context to understand how to deal with the result of the complex calculations.

Addressing

- 6.NS.B.3

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Arrange students in groups of 2. Ask students to discuss and agree on each step before moving on to the next step. Give partners 8–10 minutes to complete the first three questions and follow with a brief whole-class discussion.

Ask students to explain the first question using fractions. If not brought up by students, highlight the idea that $(2.5) \cdot (1.2)$ is equivalent to $25 \cdot (0.1) \cdot 12 \cdot (0.1)$, which is the same as $25 \cdot 12 \cdot (0.01)$ (and also $25 \cdot 12 \cdot \frac{1}{100}$). The example shows that we can treat the non-zero digits of the factors as whole numbers, use the algorithm to multiply them, and then multiply the product by some power of 0.1 or $\frac{1}{10}$ (or divide by some power of 10) and move the decimal point accordingly.

Give students 2–3 minutes of quiet work time on the last question. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Review the algorithm to multiply whole numbers.

Supports accessibility for: Memory; Conceptual processing

Access for English Language Learners

Speaking, Listening: MLR3 Clarify, Critique, Correct. Present an incorrect response that reflects a possible misunderstanding about decimal placement. Consider using this statement to open the discussion: "Since 46 times 9 is 414, then the product of 4.6 and 0.9 is 41.4." Ask pairs to clarify and critique by asking, "What questions do you have about this statement?" or "What errors do you notice?" Invite pairs to offer a correct response that includes a clear and correct mathematical explanation. This will help students explain how to find the product of numbers involving decimals.

Design Principle(s): Optimize output; Maximize meta-awareness

Anticipated Misconceptions

Students may not recall how to use the algorithm to multiply whole numbers. Consider reviewing the process prior to the activity.

Students may think that when calculating products, the decimal points need to line up. They may even write extra zeros at the end of a factor so there are the same amount of decimal places in each factor. Although this will not affect the answer, it is more efficient to align both factors to the right. If extra zeros are written at the end of a factor, there will be extra zeros accumulated in the calculation, and this can lead to careless errors.

Student Task Statement

1. A common way to find a product of decimals is to calculate a product of whole numbers, then place the decimal point in the product.

$$\begin{array}{r} 25 \\ \times 12 \\ \hline 50 \\ + 250 \\ \hline 300 \end{array}$$

Here is an example for $(2.5) \cdot (1.2)$.

Use what you know about decimals and place value to explain why the decimal point of the product is placed where it is.

$$25 \cdot 12 = 300$$

$$(2.5) \cdot (1.2) = 3.00$$

2. Use the method shown in the first question to calculate each product.
 - a. $(4.6) \cdot (0.9)$
 - b. $(16.5) \cdot (0.7)$
3. Use area diagrams to check your earlier calculations. For each problem:

- Decompose each number into its base-ten units and write them in the boxes on each side of the rectangle.
- Write the area of each lettered region in the diagram. Then find the area of the entire rectangle. Show your reasoning.

a. $(4.6) \cdot (0.9)$



b. $(16.5) \cdot (0.7)$



4. About how many centimeters are in 6.25 inches if 1 inch is about 2.5 centimeters? Show your reasoning.

Student Response

1. Answers vary. Sample responses: 2.5 is 0.1 times 25, and 1.2 is 0.1 times 12, so the product of 25 and 12 needs to be multiplied by $(0.1) \cdot (0.1)$. This is 0.01, which moves the decimal point two decimal places to the left. The number of decimal places in the product is the sum of the number of decimal places in the factors.

2. a. 4.14

b. 11.55

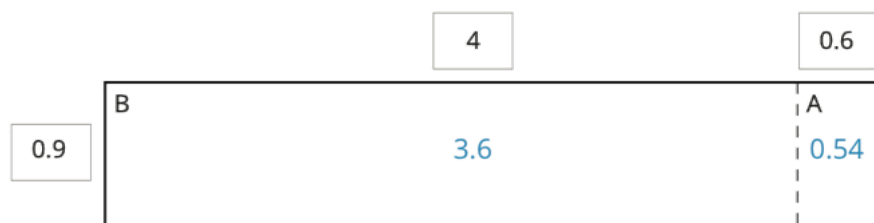
$$\begin{array}{r} 46 \\ \times 9 \\ \hline 414 \end{array}$$

$$\begin{aligned} 46 \cdot 9 &= 414 \\ (4.6) \cdot (0.9) &= 4.14 \end{aligned}$$

$$\begin{array}{r} 165 \\ \times 7 \\ \hline 1155 \end{array}$$

$$\begin{aligned} 165 \cdot 7 &= 1,155 \\ (16.5) \cdot (0.7) &= 11.55 \end{aligned}$$

3. a.



■ A: $(0.6) \cdot (0.9) = 0.54$, B: $4 \cdot (0.9) = 3.6$

■ $0.54 + 3.6 = 4.14$, so $(4.6) \cdot (0.9) = 4.14$

b.



■ A: $(0.5) \cdot (0.7) = 0.35$, B: $6 \cdot (0.7) = 4.2$, C: $10 \cdot (0.7) = 7$

■ $0.35 + 4.2 + 7 = 11.55$, so $(16.5) \cdot (0.7) = 11.55$

4. About 15.625 centimeters. $(6.25) \cdot (2.5) = 15.625$.

Activity Synthesis

Most of the discussions will have occurred in groups, but debrief as a class to tie a few ideas together. Ask a few students to share how they vertically calculated the products in the last several questions or to display the solutions for all to see. Discuss questions like:

- How did you know how to label the lengths of A, B, and C on the 16.5 by 0.7 rectangle? (The three digits in the number represent 10, 6, and 0.5, so the longest side is 10, the medium-length side is 6, and the shortest side is 0.5.)
- Which method—drawing an area diagram or using vertical calculations—do you prefer in finding products such as $(16.5) \cdot (0.7)$? Why? (Drawing an area diagram, because the visual representation helps us break up the calculation into smaller, more manageable pieces: $10 \cdot (0.7)$, $6 \cdot (0.7)$, and $(0.5) \cdot (0.7)$. Vertical calculation, because it is quicker to just multiply whole numbers and move the decimal point.)
- How did you know where to place the decimals in the last problem? (For part a, since 4.6 is 46 tenths and 0.9 is 9 tenths, we can compute $46 \cdot 9$ and then multiply the product by $(\frac{1}{10} \cdot \frac{1}{10})$ or by $\frac{1}{100}$ to find $(4.6) \cdot (0.9)$.)

8.3 Practicing Multiplication of Decimals

Optional: 15 minutes

This optional activity is an opportunity to practice the methods in this lesson to calculate products of decimals, and students have an opportunity to practice multiplying decimals in a real-world context. Students can choose to use area diagrams to help organize their work and support their

reasoning. However, the goal of the activity is to have students practice using the multiplication algorithm on decimals.

Addressing

- 6.NS.B.3

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Give students quiet think time to complete the activity and then time to share their explanation with a partner. Follow with whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Provide students with access to a blank area diagram to support information processing.

Supports accessibility for: Visual-spatial processing; Organization

Student Task Statement

1. Calculate each product. Show your reasoning. If you get stuck, consider drawing an area diagram to help.
 - a. $(5.6) \cdot (1.8)$
 - b. $(0.008) \cdot (7.2)$
2. A rectangular playground is 18.2 meters by 12.75 meters.
 - a. Find its area in square meters. Show your reasoning.
 - b. If 1 meter is approximately 3.28 feet, what are the approximate side lengths of the playground in feet? Show your reasoning.

Student Response

1. a. 10.08. Sample reasoning:
 - $56 \cdot 18 = 1,008$. 56 is 10 times 5.6 and 18 is 10 times 1.8, so 1,008 needs to be divided by 100. $1,008 \div 100 = 10.08$
 - $56 \cdot 18 = 1,008$. Each factor has 1 decimal place, so the product will have 2 decimal places. Moving the decimal point from the end of 1,008 two places to the left gives 10.08.

b. 0.0576. Sample reasoning: $\frac{8}{1000} \cdot \frac{72}{10} = \frac{576}{10000}$

2. a. 232.05 square meters. $(18.2) \cdot (12.75) = 232.05$

b. 59.696 feet and 41.82 feet. $(18.2) \cdot (3.28) = 59.696$ and $(12.75) \cdot (3.28) = 41.82$.

Are You Ready for More?

1. Write the following expressions as decimals.

a. $1 - 0.1$

b. $1 - 0.1 + 10 - 0.01$

c. $1 - 0.1 + 10 - 0.01 + 100 - 0.001$

2. Describe the decimal that results as this process continues.

3. What would happen to the decimal if all of the addition and subtraction symbols became multiplication symbols? Explain your reasoning.

Student Response

1. a. 0.9

b. 10.89

c. 110.889

2. The decimal would consist of 0.8 sandwiched between 1's on the left and 8's on the right, ending with a 9 in the smallest decimal place.

3. The decimal would be equal to just the last number in each expression.

Activity Synthesis

Select students to share their strategies, being sure to highlight approaches using area diagrams and vertical calculations. Record the representations or strategies students share and display them for all to see.

Access for English Language Learners

Representing, Conversing: MLR8 Discussion Supports. Use this routine help students prepare to share their strategies with the class. Give students time to discuss how they calculated each product for the first problem with a partner. Consider using this prompt to begin the conversation: “How did you use the multiplication algorithm to calculate the product of these two decimals? Provide a step-by-step explanation.” The listener should press for details by asking clarifying questions such as, “Why did you do that first?” and “Could you explain that a different way?” Allow each student an opportunity as the speaker and listener. This will help students build confidence in explaining the process of multiplying decimals before students are selected in the whole-class discussion.

Design Principle(s): Support sense-making; Cultivate conversation

Lesson Synthesis

We have learned several ways to calculate products of decimals—by using fractions, multiplying non-zero digits of the decimals, using area diagrams and finding partial products, and calculating vertically.

- How can working in fraction form help us find the product of two decimals?
- How can the product of two whole numbers (e.g., 48 and 19) help us find the product of two decimals with the same digits (e.g., 0.048 and 1.9)?
- How can we decompose decimal factors so they can be multiplied efficiently?

8.4 Calculate This!

Cool Down: 5 minutes

Addressing

- 6.NS.B.3

Student Task Statement

Calculate $(1.6) \cdot (0.215)$. Show your reasoning.

Student Response

$$\begin{array}{r} 215 \\ \times 16 \\ \hline 1290 \\ + 215 \\ \hline 3440 \end{array}$$

$$215 \cdot 16 = 3440$$

$$(0.215) \cdot (1.6) = 0.344$$

Student Lesson Summary

We can use $84 \cdot 43$ and what we know about place value to find $(8.4) \cdot (4.3)$.

Since 8.4 is 84 tenths and 4.3 is 43 tenths, then:

$$(8.4) \cdot (4.3) = \frac{84}{10} \cdot \frac{43}{10} = \frac{84 \cdot 43}{100}$$

That means we can compute $84 \cdot 43$ and then divide by 100 to find $(8.4) \cdot (4.3)$.

$$\begin{aligned} 84 \cdot 43 &= 3612 \\ (8.4) \cdot (4.3) &= 36.12 \end{aligned}$$

Using fractions such as $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1,000}$ allows us to find the product of two decimals using the following steps:

- Write each decimal factor as a product of a whole number and a fraction.
- Multiply the whole numbers.
- Multiply the fractions.
- Multiply the products of the whole numbers and fractions.

We know multiplying by fractions such as $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1,000}$ is the same as dividing by 10, 100, and 1,000, respectively. This means we can move the decimal point in the whole-number product to the left the appropriate number of spaces to correctly place the decimal point.

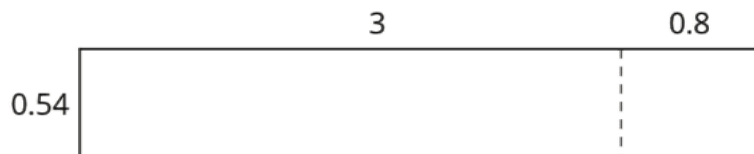
Lesson 8 Practice Problems

Problem 1

Statement

Here are an unfinished calculation of $(0.54) \cdot (3.8)$ and a 0.54-by-3.8 rectangle.

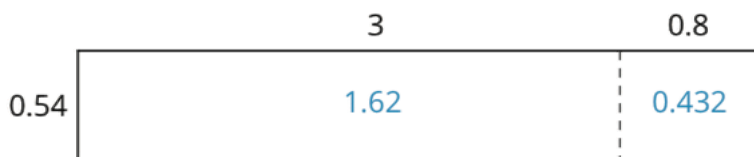
$$\begin{array}{r}
 43 \\
 0.54 \\
 \times 3.8 \\
 \hline
 0.432 \\
 1.62 \\
 \hline
 \end{array}$$



- a. Which part of the rectangle has an area of 0.432? Which part of the rectangle has an area of 1.62? Show your reasoning.
- b. What is $(0.54) \cdot (3.8)$?

Solution

- a. 0.432 is the area of the 0.8 by 0.54 rectangle because $(0.8) \cdot (0.54) = 0.432$. 1.62 is the area of the 3 by 0.54 rectangle because $3 \cdot (0.54) = 1.62$.



- b. 2.16 ($0.54 + 1.62 = 2.052$)

Problem 2

Statement

Explain how the product of 3 and 65 could be used to find $(0.03) \cdot (0.65)$.

Solution

Answers vary. Sample response: We can use vertical calculation to find 3 times 65, which equals 195. Because 0.03 is 3 hundredths and 0.65 is 65 hundredths, 195 will need to be multiplied by $(0.01) \cdot (0.01)$ or 0.0001. Multiplying by 0.0001 moves the decimal point 4 places to the left, so the product is 0.0195.

Problem 3

Statement

Use vertical calculation to find each product.

- a. $(5.4) \cdot (2.4)$
- b. $(1.67) \cdot (3.5)$

Solution

a. 12.96

b. 5.845

Problem 4

Statement

A pound of blueberries costs \$3.98 and a pound of clementines costs \$2.49. What is the combined cost of 0.6 pound of blueberries and 1.8 pounds of clementines? Round your answer to the nearest cent.

Solution

\$6.87. Sample reasoning: $(3.98) \cdot (0.6) = 2.388$, or about \$2.39. $(2.49) \cdot (1.8) = 4.482$, or about \$4.48. The combined cost is $2.39 + 4.48$ or 6.87.

Problem 5

Statement

Complete the calculations so that each shows the correct sum or difference.

a.

$$\begin{array}{r} 2.14\boxed{} \\ + 1.\boxed{}25 \\ \hline \boxed{}.865 \end{array}$$

b.

$$\begin{array}{r} 29.\boxed{}\boxed{} \\ + 1.42 \\ \hline \boxed{}\boxed{}.41 \end{array}$$

c.

$$\begin{array}{r} 6.1\boxed{}\boxed{} \\ - 1.009 \\ \hline \boxed{}.\boxed{}48 \end{array}$$

Solution

a.

$$\begin{array}{r} 2.14\boxed{0} \\ + 1.\boxed{7}25 \\ \hline \boxed{3}.865 \end{array}$$

b.

$$\begin{array}{r} 29.\boxed{9}\boxed{9} \\ + 1.42 \\ \hline \boxed{3}\boxed{1}.41 \end{array}$$

c.

$$\begin{array}{r} 6.1\boxed{5}\boxed{7} \\ - 1.009 \\ \hline \boxed{5}.\boxed{1}48 \end{array}$$

(From Unit 5, Lesson 3.)

Problem 6

Statement

Which has a greater value: $7.4 - 0.0022$ or $7.39 - 0.0012$? Show your reasoning.

Solution

$7.4 - 0.0022$ has a greater value. $7.4 - 0.0022 = 7.3978$ and $7.39 - 0.0012 = 7.3888$.

(From Unit 5, Lesson 4.)

Problem 7

Statement

Andre is planting saplings (baby trees). It takes him 30 minutes to plant 3 saplings. If each sapling takes the same amount of time to plant, how long will it take Andre to plant 14 saplings? If you get stuck, consider using the table.

number of saplings	time in minutes
3	30
1	
14	

Solution

140 minutes (or equivalent). Possible strategy:

number of saplings	time in minutes
3	30
1	10
14	140

(From Unit 2, Lesson 12.)

Lesson 9: Using the Partial Quotients Method

Goals

- Comprehend that the phrase “partial quotients” (in spoken and written language) refers to decomposing a division problem.
- Divide whole numbers that result in a whole-number quotient, and explain the reasoning (using words and other representations).
- Interpret different methods for computing the quotient of whole numbers, i.e., base-ten diagrams and partial quotients, and evaluate (orally) their usefulness.

Learning Targets

- I can use the partial quotients method to find a quotient of two whole numbers when the quotient is a whole number.

Lesson Narrative

Prior to grade 6, students reasoned about division of whole numbers and decimals to the hundredths in different ways. In this first lesson on division, they revisit two methods for finding quotients of whole numbers without remainder: using base-ten diagrams and using partial quotients. Reviewing these strategies reinforces students’ understanding of the underlying principles of base-ten division—which are based on the structure of place value, the properties of operations, and the relationship between multiplication and division—and paves the way for understanding the long division algorithm. Here, partial quotients are presented as vertical calculations, which also foreshadows long division.

In a previous unit, students revisited the two meanings of division—as finding the number of equal-size groups and finding the size of each group. Division is likewise interpreted in both ways here (MP2). When using base-ten diagrams or dividing by a small whole-number divisor, it is often natural to think about finding the size of each group. When using partial quotients, it may be more intuitive to think of division as finding the number of groups (e.g., $432 \div 16$ can be viewed as “how many 16s are in 432?”).

Alignments

Building On

- 5.NBT.B.6: Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Addressing

- 6.NS.B.2: Fluently divide multi-digit numbers using the standard algorithm.

Instructional Routines

- MLR5: Co-Craft Questions
- MLR7: Compare and Connect
- Think Pair Share

Required Materials

Graph paper

Required Preparation

Students draw base-ten diagrams in this lesson. If drawing them is a challenge, consider giving students access to:

- Commercially produced base-ten blocks, if available.
- Paper copies of squares and rectangles (to represent base-ten units), cut up from copies of the Instructional master of the second lesson in the unit.
- Digital applet of base-ten representations <https://ggbm.at/zqxRkhMh>

Some students might find it helpful to use graph paper to help them align the digits as they divide using the partial quotients method. Consider having graph paper accessible throughout the lesson.

Student Learning Goals

Let's divide whole numbers.

9.1 Using Base-Ten Diagrams to Calculate Quotients

Warm Up: 5 minutes

Prior to grade 6, students have solved division problems using their understanding of place value and the idea of creating equal-size groups. This warm-up relies on those concepts to prepare students for more-abstract work in later lessons. The divisor and dividend are chosen so that the hundreds in the dividend can be partitioned into equal groups but the tens cannot. The quotient, however, is a whole number. The key ideas that would enable students to ultimately divide a decimal by a decimal are present in this example:

- A number can be decomposed to make the division convenient, e.g., 372 can be viewed as $300 + 60 + 12$.
- Place value, expressed in the form of base-ten diagrams, plays a very important role in division.

Building On

- 5.NBT.B.6

Instructional Routines

- Think Pair Share

Launch

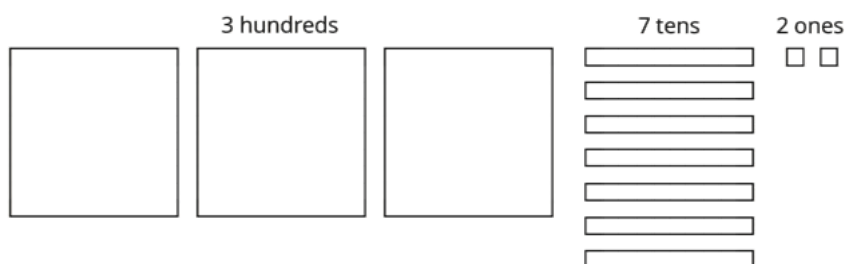
Arrange students in groups of 2. Display Elena's method for all to see and use as a reference. Give students 1 minute of quiet think time and 2 minutes to discuss with a partner. Follow with a whole-class discussion.

Anticipated Misconceptions

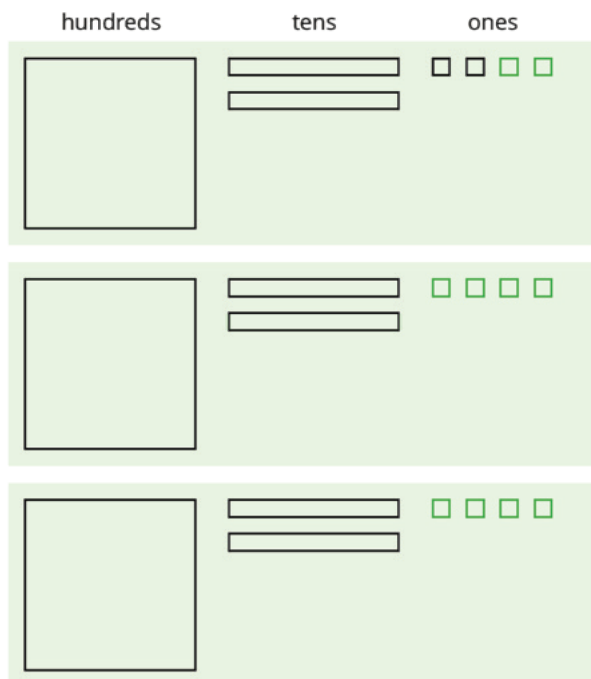
If students have difficulty making sense of Elena's method, consider demonstrating her process with actual base-ten blocks or paper cutouts.

Student Task Statement

Elena used base-ten diagrams to find $372 \div 3$. She started by representing 372.



She made 3 groups, each with 1 hundred. Then, she put the tens and ones in each of the 3 groups. Here is her diagram for $372 \div 3$.



Discuss with a partner:

- Elena's diagram for 372 has 7 tens. The one for $372 \div 3$ has only 6 tens. Why?
- Where did the extra ones (small squares) come from?

Student Response

Answers vary. Sample reasoning: Elena first put the 3 hundreds into 3 groups, placing 1 hundred in each group. Then she put 6 of the 7 tens into 3 groups, giving 2 tens to each group. She traded the remaining ten with 10 ones. Combining these 10 ones with the original 2 ones, she then has 12 ones. Elena put the 12 ones into 3 groups, putting 4 ones in each group. Each group then has 124, so $372 \div 3 = 124$.

Activity Synthesis

Highlight Elena's process of separating base-ten units into equal groups. Discuss:

- Which base-ten unit(s) did Elena unbundle or break up? (She unbundled a tens unit.)
- Why? What did unbundling accomplish? (She had only 1 ten left and there are 3 equal groups. Unbundling as smaller units made it possible to place the 1 ten in the 3 groups.)
- Is there another way that Elena could have made 3 equal groups out of the base-ten units? (She could have unbundled other larger units into smaller units—e.g., the 3 hundreds as 30 tens or all 7 tens as 70 ones—but it was not necessary.)
- How might one find $378 \div 3$ using Elena's method? (By thinking of 378 as 3 hundreds, 6 tens, and 18 ones and placing them into 3 equal groups.)

9.2 Using the Partial Quotients Method to Calculate Quotients

15 minutes

Here, students continue to find quotients of whole numbers by thinking about equal-size groups and place value. They learn that, in addition to using base-ten diagrams, they can also form equal-size groups using only numbers and by thinking in terms of partial quotients. Just as they had used diagrams to place base-ten units—first hundreds, then tens, and then ones—into equal groups until all units are placed, they can distribute base-ten units of a number into equal groups until all of the units are placed.

Building On

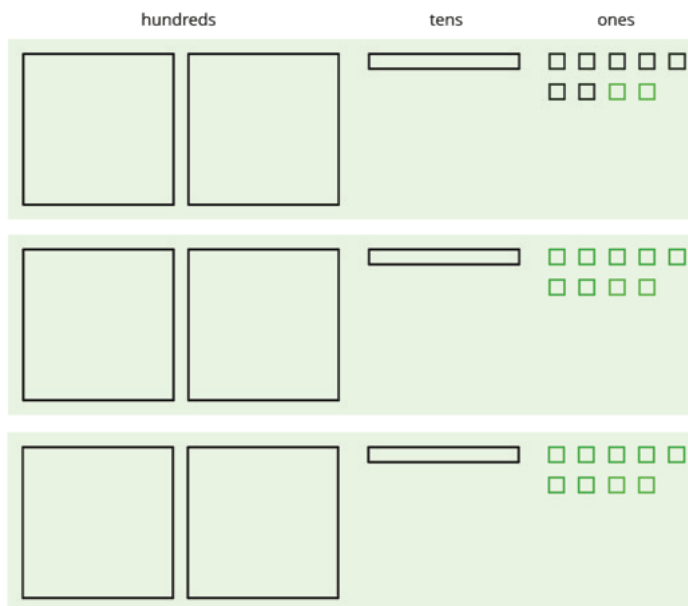
- 5.NBT.B.6

Instructional Routines

- MLR5: Co-Craft Questions

Launch

Keep students in groups of 2. Display the following diagram for all to see, and explain that it shows Elena's method of finding $657 \div 3$. Give students a minute to analyze the diagram, determine what the quotient is, and be prepared to explain what the diagram shows.



Give partners 1 minute to discuss their understanding of the diagram. Afterwards, consider asking a student to share with the class. Look for an explanation along the lines of the following:

- First, use the 6 hundreds to make 3 equal groups of 200.
- Then use 3 tens of the 5 tens to make 3 equal groups of 10.
- Unbundle the remaining 2 tens into 20 ones, combine them with the 7 ones, and split the 27 ones into 3 equal groups of 9.

If time permits, invite other students to elaborate on the presented explanations or share alternative analyses.

Keep Elena's method from the previous task displayed for students to reference. Give students 7–8 minutes to think about and discuss Andre's method in the first question and to complete the activity.

Provide access to graph paper. Tell students that they may find the grid helpful for aligning the digits when finding partial quotients.

Access for Students with Disabilities

Representation: Internalize Comprehension. Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, use color coding to highlight connections between the partial quotients in Andre's and Elena's method for calculating $657 \div 3$.

Supports accessibility for: Visual-spatial processing

Access for English Language Learners

Writing, Conversing: MLR5 Co-Craft Questions. Display Andre's division method without revealing the questions that follow. Ask students to write down questions they have for Andre. Invite students to compare their questions with a partner before sharing with the whole class. Highlight questions that ask about what specific numbers mean or represent in the given work. Finally, reveal the actual questions students are expected to work on. This will help students produce language related to representations of division.

Design Principle(s): Cultivate conversation; Maximize meta-awareness

Anticipated Misconceptions

When using the partial quotients method, students might make subtraction or multiplication errors because they did not line up the numbers appropriately. Prompt students to compare the structure of Andre's work with their own or to check if they have aligned like units in their vertical calculations.

Student Task Statement

1. Andre calculated $657 \div 3$ using a method that was different from Elena's.

He started by writing the dividend (657) and the divisor (3).

$$3 \overline{) 657}$$

He then subtracted 3 groups of different amounts from 657, starting with 3 groups of 200...

$$\begin{array}{r} 200 \\ 3 \overline{) 657} \\ - 600 \\ \hline 57 \end{array}$$

... then 3 groups of 10, and then 3 groups of 9.

$$\begin{array}{r} 9 \\ 10 \\ 200 \\ 3 \overline{) 657} \\ - 600 \\ \hline 57 \\ - 30 \\ \hline 27 \\ - 27 \\ \hline 0 \end{array}$$

Andre calculated $200 + 10 + 9$ and then wrote 219.

$$\begin{array}{r} \boxed{219} \\ 9 \\ 10 \\ 200 \\ 3 \overline{) 657} \\ - 600 \\ \hline 57 \\ - 30 \\ \hline 27 \\ - 27 \\ \hline 0 \end{array}$$

- Andre subtracted 600 from 657. What does the 600 represent?
- Andre wrote 10 above the 200, and then subtracted 30 from 57. How is the 30 related to the 10?
- What do the numbers 200, 10, and 9 represent?
- What is the meaning of the 0 at the bottom of Andre's work?

2. How might Andre calculate $896 \div 4$? Explain or show your reasoning.

Student Response

- The 600 represents 3 groups of 2 hundreds.
 - The 30 represents 3 tens that were distributed equally into 3 groups (1 ten in each).
 - The 200, 10, and 9 represents the contents of each group: 2 hundreds, 1 ten, and 9 ones.
 - The 0 means that after subtracting 3 times 219 from 657, there is no remainder.
- Andre could calculate $896 \div 4$ like this.

$$\begin{array}{r}
 \boxed{224} \\
 4 \\
 20 \\
 200 \\
 4 \overline{) 896} \\
 \underline{- 800} \\
 96 \\
 \underline{- 80} \\
 16 \\
 \underline{- 16} \\
 0
 \end{array}$$

Activity Synthesis

Make sure students understand how the steps in Elena's method and Andre's method correspond. Discuss:

- Elena's diagram shows 3 groups of 2 hundreds. Where in Andre's method do we see the same value? (In the 600 subtracted from 657.)
- Where in Elena's work do we see the 30 that Andre subtracts from 57? (In the 3 groups of 1 ten.)
- Do Andre and Elena both get the same answer? Why? (Yes, because they both distributed 657 into 3 equal groups.)

Tell students that Andre's method is an example of the partial quotients method, in which we divide a part of the dividend at a time, obtaining part of the quotient each time. In this case, the first partial quotient is 200, next is 210 (from $200 + 10$), and last is the quotient 219 (from $200 + 10 + 9$). We can still view a division expression such as $657 \div 3$ as a question asking "How many are in each group if I divide 657 into 3 equal groups?"

With the partial quotients method, we can take any amount we choose out of 657 and place it into the 3 equal groups. It is often helpful to take out the amount in each place value and distribute it into groups. The values placed in each group are partial quotients. Once we have distributed all of 657, we can add the partial quotients to find $657 \div 3$.

If time allows, discuss how to find the value of $655 \div 5$ using the partial quotients method. Ask students what value might be reasonable to take out first, second, etc. (e.g., taking out 100 or 120 is a reasonable first move).

9.3 What's the Quotient?

15 minutes

In this lesson, students choose how to perform division—by drawing diagrams or by using the partial quotients method. As they work with larger dividends and divisors, students observe the merits and potential drawbacks of each method. They see that base-ten diagrams are useful because they are concrete and help to visualize the meaning of division, but drawing all the pieces is cumbersome if the numbers are large. The partial quotients method relies on the same principles but will work for any numbers without the need for elaborate drawings. Students use these observations to decide on appropriate methods to use.

Earlier, when introducing Andre’s method and partial quotients, we had interpreted $657 \div 3$ as answering the question: “How much is in each group if 657 is divided into 3 equal groups?” For example, when Andre took out 600, we interpreted it as: “600 is 3 groups of 200.” This interpretation is helpful for making the connection to Elena’s base-ten diagrams, in which she divided base-ten units into 3 groups.

Note, however, that we can also interpret the division expression as asking: “How many groups of 3 are in 657?” This interpretation can likewise be represented using diagrams, though it would be highly impractical to draw 219 groups of 3. It is not impractical for students to think about partial quotients this way, however. For instance, they could take out 600, ask “how many groups of 3 are in 600?” and write down a partial quotient of 200 to represent 200 groups of 3 (instead of 3 groups of 200).

Building On

- 5.NBT.B.6

Instructional Routines

- MLR7: Compare and Connect

Launch

Remind students that Elena drew base-ten diagrams to represent equal groups, while Andre took out smaller amounts from the dividend, found those quotients, and then combined the partial quotients together. Tell students that in this activity they choose a method to perform division. Encourage them to refer to Elena and Andre’s methods from the previous activities, or display Elena and Andre’s methods for all to see.

Keep students in groups of 2. Give students 2–3 minutes to discuss the first question with a partner and 8–10 minutes of quiet work time on the remaining questions. Provide access to graph paper.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Use virtual or concrete manipulatives to connect symbols to concrete objects or values. For example, provide access to physical or virtual base-ten blocks as alternatives to drawing diagrams.

Supports accessibility for: Conceptual processing

Student Task Statement

- Find the quotient of $1,332 \div 9$ using one of the methods you have seen so far. Show your reasoning.
- Find each quotient and show your reasoning. Use the partial quotients method at least once.
 - $1,115 \div 5$
 - $665 \div 7$
 - $432 \div 16$

Student Response

148. Sample reasoning: First, I subtract 9 groups of 100, which leaves 432. Then, I subtract 9 groups of 40 and that leaves 72. I know that 72 is 9 groups of 8. So $1,332 \div 9$ is $100 + 40 + 8$, which equals 148.

$$\begin{array}{r}
 \boxed{148} \\
 8 \\
 40 \\
 100 \\
 9 \overline{) 1332} \\
 \underline{- 900} \\
 432 \\
 \underline{- 360} \\
 72 \\
 \underline{- 72} \\
 0
 \end{array}$$

2.

a.

$$\begin{array}{r}
 \boxed{223} \\
 3 \\
 20 \\
 200 \\
 5 \overline{) 1115} \\
 \underline{- 1000} \\
 115 \\
 \underline{- 100} \\
 15 \\
 \underline{- 15} \\
 0
 \end{array}$$

b.

$$\begin{array}{r}
 \boxed{95} \\
 5 \\
 90 \\
 7 \overline{) 665} \\
 \underline{- 630} \\
 35 \\
 \underline{- 35} \\
 0
 \end{array}$$

c.

$$\begin{array}{r}
 \boxed{27} \\
 2 \\
 5 \\
 20 \\
 16 \overline{) 432} \\
 \underline{- 320} \\
 112 \\
 \underline{- 80} \\
 32 \\
 \underline{- 32} \\
 0
 \end{array}$$

Activity Synthesis

Focus the discussion on showcasing the advantages and disadvantages of each method. Ask students to indicate which division method they preferred for the first two problems. Then, ask students who used base-ten diagrams to share the challenges of using this method to divide. Possible responses include:

- When the divisor is large, it takes a long time to draw all of the equal groups.
- When the dividend or divisor is large, it is difficult to tell how much I have used up or placed in groups and how much remains.
- When the dividend or divisor is large, it is difficult to check my work, i.e., to see that all of the equal groups add up to the value of the dividend.

Ask students who used the partial quotients to name the challenges of using this method for division? Possible responses include:

- I was not sure how to decide what amount to take out at each step.
- Sometimes I could not find familiar numbers to take out.
- I took out too little at a time, so it ended up taking a long time.
- I thought I had placed everything into equal groups but I ended up with a leftover.

Show that many of the challenges of using base-ten diagrams are eliminated by using the partial quotients method and explain that in future lessons the partial quotients method will be refined to help deal with its challenges.

Access for English Language Learners

Writing, Speaking, Listening: MLR7 Compare and Connect. As students discuss the first question with a partner, ask them to identify what is similar and what is different about the approaches used. Provide sentence frames to help students organize their thinking as they share their answer. For example, “In using _____, first, I _____, next, I _____, then, I _____, and finally, I _____.” Follow with a whole-class discussion centered around the advantages and disadvantages of each method. Consider charting the responses. Listen for comments noting the effects of a large dividend or divisor, leftover amounts, or decisions about what amount to take out. This will help students connect other students’ approaches to division to their own approach and decide which method is more efficient to use and why.

Design Principle(s): Optimize output (for comparison); Maximize meta-awareness

Lesson Synthesis

Recall that one way to think of division is as a process of splitting a value into equal-size groups and finding the size of *one* group. We can represent the groups and contents of each group using

base-ten diagrams. Let's take $456 \div 4$ as an example. We start by representing 456 with drawings of 4 hundreds, 5 tens, and 6 ones.

- How might we start dividing the pieces? (We can do it by place value. We draw 4 groups, put 1 hundred into each group, and then put 1 ten into each group.)
- What happens if there's a remainder, e.g., after putting 1 ten into each group, we still have 1 ten left? (We can unbundle the 1 ten into 10 ones.)
- What does unbundling accomplish? (It allows us to have smaller units to divide. Here we combine the 10 ones and 6 ones, and then divide 16 ones into 4 groups of 4.)
- How do we find the value of the quotient? (The value of all the pieces in each group is the quotient.)

We can also find quotients *without* drawing a diagram and by using the partial quotients method.

- How is the partial quotient method similar to drawing base-ten diagrams? (We still pay attention to place value and think in terms of equal-size groups.)
- How is it different than drawing diagrams? (With partial quotients, we use only numbers, and we don't have to divide the entirety of each base-ten unit at once. We can decide the amount to divide each round, e.g., we could first divide 200 into 4 groups, then another 200 into 4 groups, etc.)
- What might be the advantages of using base-ten diagrams? (It is concrete. It helps us visualize the number being divided.)
- What are the advantages of using partial quotients to divide numbers? (There is no need to draw all the pieces being divided or all the groups. There is flexibility in how we divide.)
- Do you have a preferred method for finding decimal quotients? Explain your reasoning.

9.4 Dividing by 11

Cool Down: 5 minutes

Addressing

- 6.NS.B.2

Student Task Statement

Calculate $4,235 \div 11$ using any method.

Student Response

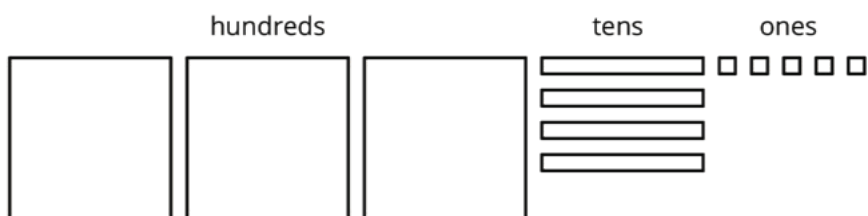
385. Sample reasoning:

$$\begin{array}{r}
 \boxed{385} \\
 5 \\
 80 \\
 300 \\
 11 \overline{) 4235} \\
 \underline{- 3300} \\
 935 \\
 \underline{- 880} \\
 55 \\
 \underline{- 55} \\
 0
 \end{array}$$

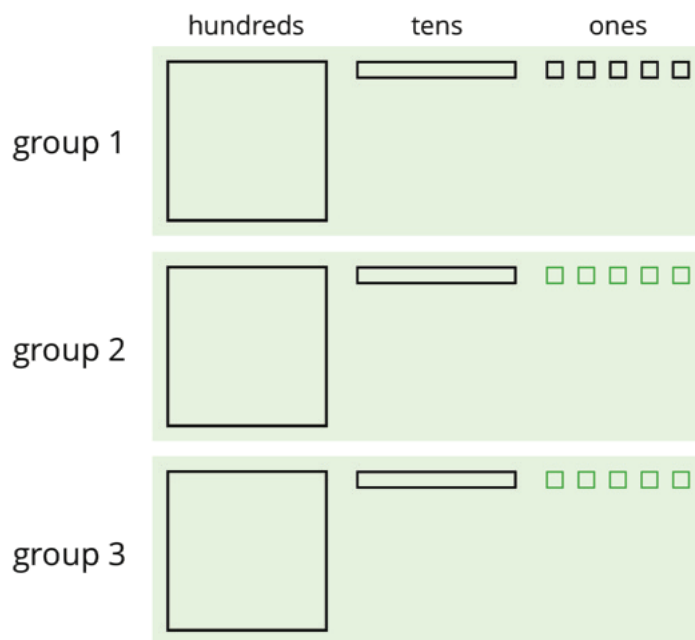
Student Lesson Summary

We can find the quotient $345 \div 3$ in different ways.

One way is to use a base-ten diagram to represent the hundreds, tens, and ones and to create equal-sized groups.



We can think of the division by 3 as splitting up 345 into 3 equal groups.



Each group has 1 hundred, 1 ten, and 5 ones, so $345 \div 3 = 115$. Notice that in order to split 345 into 3 equal groups, one of the tens had to be unbundled or decomposed into 10 ones.

Another way to divide 345 by 3 is by using the partial quotients method, in which we keep subtracting 3 groups of some amount from 345.

$$\begin{array}{r}
 \boxed{1 \ 1 \ 5} \\
 \begin{array}{r}
 5 \\
 10 \\
 100 \\
 \hline
 3 \overline{) 345} \\
 - 300 \quad \leftarrow 3 \text{ groups of } 100 \\
 \hline
 45 \\
 - 30 \quad \leftarrow 3 \text{ groups of } 10 \\
 \hline
 15 \\
 - 15 \quad \leftarrow 3 \text{ groups of } 5 \\
 \hline
 0
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \boxed{1 \ 1 \ 5} \\
 \begin{array}{r}
 50 \\
 50 \\
 15 \\
 \hline
 3 \overline{) 345} \\
 - 45 \quad \leftarrow 3 \text{ groups of } 15 \\
 \hline
 300 \\
 - 150 \quad \leftarrow 3 \text{ groups of } 50 \\
 \hline
 150 \\
 - 150 \quad \leftarrow 3 \text{ groups of } 50 \\
 \hline
 0
 \end{array}
 \end{array}$$

- In the calculation on the left, first we subtract 3 groups of 100, then 3 groups of 10, and then 3 groups of 5. Adding up the partial quotients ($100 + 10 + 5$) gives us 115.
- The calculation on the right shows a different amount per group subtracted each time (3 groups of 15, 3 groups of 50, and 3 more groups of 50), but the total amount in each of the 3 groups is still 115. There are other ways of calculating $345 \div 3$ using the partial quotients method.

Both the base-ten diagrams and partial quotients methods are effective. If, however, the dividend and divisor are large, as in $1,248 \div 26$, then the base-ten diagrams will be time-consuming.

Lesson 9 Practice Problems

Problem 1

Statement

Here is one way to find $2,105 \div 5$ using partial quotients. Show a different way of using partial quotients to divide 2,105 by 5.

$$\begin{array}{r}
 \boxed{421} \\
 1 \\
 20 \\
 400 \\
 5 \overline{) 2105} \\
 \underline{- 2000} \\
 105 \\
 \underline{- 100} \\
 5 \\
 \underline{- 5} \\
 0
 \end{array}$$

Solution

Responses vary. Sample response:

$$\begin{array}{r}
 \boxed{421} \\
 21 \\
 200 \\
 200 \\
 5 \overline{) 2105} \\
 \underline{- 1000} \\
 1105 \\
 \underline{- 1000} \\
 105 \\
 \underline{- 105} \\
 0
 \end{array}$$

Problem 2

Statement

Andre and Jada both found $657 \div 3$ using the partial quotients method, but they did the calculations differently, as shown here.

$$\begin{array}{r}
 \boxed{219} \\
 9 \\
 10 \\
 200 \\
 3 \overline{) 657} \\
 \underline{- 600} \\
 57 \\
 \underline{- 30} \\
 27 \\
 \underline{- 27} \\
 0
 \end{array}$$

Andre's Work

$$\begin{array}{r}
 \boxed{219} \\
 9 \\
 60 \\
 100 \\
 50 \\
 3 \overline{) 657} \\
 \underline{- 150} \\
 507 \\
 \underline{- 300} \\
 207 \\
 \underline{- 180} \\
 27 \\
 \underline{- 27} \\
 0
 \end{array}$$

Jada's Work

a. How is Jada's work the same as Andre's work? How is it different?

b. Explain why they have the same answer.

Solution

- a. Similarities: Andre and Jada both subtracted multiples of 3 several times. They both added the numbers being multiplied by 3 to find the quotient, and both ended up with 219. Differences: Jada subtracted multiples of 3 more times than Andre and the multiples of 3 that she subtracted were different.
- b. Andre and Jada have the same answer, since they both calculated $657 \div 3$ by subtracting multiples of 3 until there was no remainder and both gave the number of multiples of 3 subtracted as the answer.

Problem 3

Statement

Which might be a better way to evaluate $1,150 \div 46$: drawing base-ten diagrams or using the partial quotients method? Explain your reasoning.

Solution

Answers vary. Sample response: The partial quotient method works better. Dividing 1,150 into 46 equal groups by drawing will take too long. With the partial quotient method, the groups don't need to be drawn.

Problem 4

Statement

Here is an incomplete calculation of $534 \div 6$.

Write the missing numbers (marked with "?") that would make the calculation complete.

$$\begin{array}{r} \boxed{89} \\ 9 \\ 80 \\ 6 \overline{) 534} \\ - ? \\ \hline ? \\ - ? \\ \hline 0 \end{array}$$

Solution

$$\begin{array}{r} \boxed{89} \\ 9 \\ 80 \\ 6 \overline{) 534} \\ - 480 ? \\ \hline 54 ? \\ - 54 ? \\ \hline 0 \end{array}$$

Problem 5

Statement

Use the partial quotients method to find $1,032 \div 43$.

Solution

Responses vary. Sample response:

$$\begin{array}{r} \boxed{24} \\ 4 \\ 20 \\ 43 \overline{) 1032} \\ - 860 \\ \hline 172 \\ - 172 \\ \hline 0 \end{array}$$

Problem 6

Statement

Which of the polygons has the greatest area?

- A. A rectangle that is 3.25 inches wide and 6.1 inches long.
- B. A square with side length of 4.6 inches.
- C. A parallelogram with a base of 5.875 inches and a height of 3.5 inches.
- D. A triangle with a base of 7.18 inches and a height of 5.4 inches.

Solution

B

(From Unit 5, Lesson 8.)

Problem 7

Statement

One micrometer is a millionth of a meter. A certain spider web is 4 micrometers thick. A fiber in a shirt is 1 hundred-thousandth of a meter thick.

- a. Which is wider, the spider web or the fiber? Explain your reasoning.
- b. How many meters wider?

Solution

- a. The fiber is wider. 1 hundred-thousandth is 10 millionths, and 10 millionths is more than 4 millionths (it's 6 hundred thousandths more).
- b. 6 hundred-thousandths

(From Unit 5, Lesson 4.)

Lesson 10: Using Long Division

Goals

- Interpret the long division method, and compare and contrast it (orally) with other methods for computing the quotient of whole numbers.
- Recognize and explain (orally) that long division is an efficient strategy for dividing numbers, especially with multi-digit dividends.
- Use long division to divide whole numbers that result in a whole-number quotient, and multiply the quotient by the divisor to check the answer.

Learning Targets

- I can use long division to find a quotient of two whole numbers when the quotient is a whole number.

Lesson Narrative

This lesson introduces students to **long division**. Students see that in long division the meaning of each digit is intimately tied to its place value, and that it is an efficient way to find quotients. In the partial quotients method, all numbers and their meaning are fully and explicitly written out. For example, to find $657 \div 3$ we write that there are at least 3 groups of 200, record a subtraction of 600, and show a difference of 57. In long division, instead of writing out all the digits, we rely on the position of any digit—of the quotient, of the number being subtracted, or of a difference—to convey its meaning, which simplifies the calculation.

In addition to making sense of long division and using it to calculate quotients, students also analyze some place-value errors commonly made in long division (MP3).

Alignments

Building On

- 4.NBT.B.6: Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Addressing

- 6.NS.B.2: Fluently divide multi-digit numbers using the standard algorithm.

Building Towards

- 6.EE.A: Apply and extend previous understandings of arithmetic to algebraic expressions.

Instructional Routines

- MLR3: Clarify, Critique, Correct

- MLR8: Discussion Supports
- Number Talk
- Think Pair Share

Required Preparation

Some students might find it helpful to use graph paper to help them align the digits as they divide using long division and the partial quotients method. Consider having graph paper accessible throughout the lesson.

Student Learning Goals

Let's use long division.

10.1 Number Talk: Estimating Quotients

Warm Up: 5 minutes

This number talk prompts students to make reasonable estimates of quotients using their knowledge of numbers, division, and structures. Only two problems are given to allow more time for students to share their estimation strategies.

Making reasonable estimates helps to develop arithmetic fluency. Here, it relies on understanding the relationship between multiplication and division, and on the different properties of operations (commutative, associative, and distributive). For example, to find $500 \div 7$, students might think of the multiplication equation $7 \cdot ? = 500$. Since they know that $7 \cdot 100 = 700$ and $7 \cdot 30 = 210$, and that $500 = 700 - 200$, they could reason that 500 would be approximately $7 \cdot (100 - 30)$ or $7 \cdot 70$.

Building On

- 4.NBT.B.6

Building Towards

- 6.EE.A

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Display each problem one at a time for all to see. Give students 1 minute of quiet think time and ask them to give a signal when they have an answer and a strategy. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Anticipated Misconceptions

Students might try to calculate exact answers and take longer to produce an answer. Encourage them to approximate the actual answer by rounding the dividend or divisor or starting with friendlier numbers.

Student Task Statement

Estimate these quotients mentally.

$$500 \div 7$$

$$1,394 \div 9$$

Student Response

Answers vary. Sample responses:

- 70, because $7 \cdot 7 = 49$, so $70 \cdot 7 = 490$, which is almost 500.
- 150, because $100 \cdot 9 = 900$ and $50 \cdot 9 = 450$, so $150 \cdot 9 = 1,350$.

Activity Synthesis

Invite students to share their strategies. Record and display student explanations for all to see. Ask students to explain if or how the dividend or divisor impacted their choice of strategy. To involve more students in the conversation, consider asking:

- “Did anyone reason about the problem the same way but would explain it differently?”
- “Did anyone estimate in a different way?”
- “Does anyone want to add on to ____’s reasoning?”
- “Do you agree or disagree? Why?”

At the end of the discussion, if time permits, ask a few students to share a story problem or context that $1,394 \div 9$ could represent.

Access for English Language Learners

Speaking: MLR8 Discussion Supports: Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because . . ." or "I noticed ____ so I" Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Design Principle(s): Optimize output (for explanation)

10.2 Lin Uses Long Division

25 minutes

This activity introduces the use of **long division** to calculate a quotient of whole numbers. Students make sense of the process of long division by studying an annotated example and relating it to the use of partial quotients and base-ten diagrams. They begin to see that long division is a variant of the partial quotients method, but it is calculated and recorded differently.

In the partial quotients method, the division is done in installments, resulting in a series of partial quotients. The size of each installment can vary, but it is always a multiple of the divisor.

Each partial quotient is written above the dividend and stacked; the sum of all partial quotients is the quotient.

In long division, the division is performed digit by digit, from the largest place to the smallest, so the resulting quotient is also recorded one digit at a time. In each step, one more digit of the quotient is calculated. Students notice that although only one digit of the quotient is written down at a time, the value that it represents is communicated through its placement.

To become proficient in long division requires time, encounters with a variety of division problems, and considerable practice. Students will have opportunities to study the algorithm more closely and to use it to divide increasingly more challenging numbers over several upcoming lessons.

Addressing

- 6.NS.B.2

Instructional Routines

- MLR3: Clarify, Critique, Correct
- Think Pair Share

Launch

Tell students that they will now consider a third method—called long division—for solving the same division problem that Elena and Andre had calculated using base-ten diagrams and the partial quotients method. Encourage students to refer to their work on those activities, or display Elena

and Andre's methods for all to see. Ask a couple of students to briefly explain how Elena and Andre calculated $657 \div 3$.

Arrange students in groups of 2. Give students 2–3 minutes of quiet think time to make sense of the annotated example of long division, and then 3–4 minutes to discuss the first set of questions with a partner. Follow with a whole-class discussion before students use long division to answer the second set of questions.

Display Lin's method for all to see and ask a student to explain what Lin had done in his or her own words. Then, discuss students' responses to the first set of questions and these questions:

- In the first step, Lin divided 6 by 3 to get 2. Did it matter where Lin wrote the 2? Why did she put it over the 6? (Yes, it mattered. Because the 6 represents 600, she was really dividing 600 by 3, which is 200. The 2 needs to be written in the hundreds place to tell us its actual value.)
- After writing down the 2, Lin subtracted 6. Why? And why was the result of the subtraction not 651 (since $657 - 6 = 651$)? (Though she wrote a subtraction of 6, she actually subtracted 600. Because she had just divided 600 by 3, that portion of 657 is already accounted for.)
- Could Lin have written the full amounts being subtracted instead of just the non-zero digit (e.g., subtracting by 600, 50 and 7, instead of subtracting by 6, 5, and 7 after aligning them to certain places)? (Yes, it would involve more writing, but it works just as well.)
- How is this process similar to and different than the partial quotients method? (It is a similar idea of taking out a certain multiple of 3 at a time, but in long division we do it digit by digit and in the order of place value—from the largest unit to the smallest.)

Consider demonstrating the long-division process with another example such as $912 \div 4$ before asking students to complete the rest of the task. Provide access to graph paper. Tell students that the grid could help them line up the digits.

Access for Students with Disabilities

Representation: Access for Perception. Read Lin's method for calculating the quotient of $657 \div 3$ aloud. Students who both listen to and read the information will benefit from extra processing time.

Supports accessibility for: Language

Student Task Statement

Lin has a method of calculating quotients that is different from Elena's method and Andre's method. Here is how she found the quotient of $657 \div 3$:

Lin arranged the numbers for vertical calculations.

Her plan was to divide each digit of 657 into 3 groups, starting with the 6 hundreds.

$$3 \overline{) 657}$$

There are 3 groups of 2 in 6, so Lin wrote 2 at the top and subtracted 6 from the 6, leaving 0.

Then, she brought down the 5 tens of 657.

$$\begin{array}{r} 2 \\ 3 \overline{) 657} \\ - 6 \downarrow \\ \hline 05 \end{array}$$

There are 3 groups of 1 in 5, so she wrote 1 at the top and subtracted 3 from 5, which left a remainder of 2.

$$\begin{array}{r} 21 \\ 3 \overline{) 657} \\ - 6 \\ \hline 5 \\ - 3 \\ \hline 2 \end{array}$$

She brought down the 7 ones of 657 and wrote it next to the 2, which made 27.

There are 3 groups of 9 in 27, so she wrote 9 at the top and subtracted 27, leaving 0.

$$\begin{array}{r} 219 \\ 3 \overline{) 657} \\ - 6 \downarrow \\ \hline 5 \\ - 3 \downarrow \\ \hline 27 \\ - 27 \\ \hline 0 \end{array}$$

- Discuss with your partner how Lin's method is similar to and different from drawing base-ten diagrams or using the partial quotients method.
 - Lin subtracted $3 \cdot 2$, then $3 \cdot 1$, and lastly $3 \cdot 9$. Earlier, Andre subtracted $3 \cdot 200$, then $3 \cdot 10$, and lastly $3 \cdot 9$. Why did they have the same quotient?
 - In the third step, why do you think Lin wrote the 7 next to the remainder of 2 rather than adding 7 and 2 to get 9?
- Lin's method is called **long division**. Use this method to find the following quotients. Check your answer by multiplying it by the divisor.
 - $846 \div 3$
 - $1,816 \div 4$
 - $768 \div 12$

Student Response

- The 3 \cdot 2 Lin subtracted from 6 represents $3 \cdot 200$ subtracted from 600, since the 6 and the 2 are both in the hundreds place. Similarly, the 3 \cdot 1 Lin subtracted from 5 represents $3 \cdot 10$ subtracted from 5 tens (or 50). Lin's work shows the same steps Andre took without writing out as many digits in the calculations.
 - The 2 represents two tens or 20. The 7 represents 7 ones. So Lin is adding 2 tens to 7 ones, and the result is 27.

a.

$$\begin{array}{r}
 \textcolor{teal}{282} \\
 3 \overline{) 846} \\
 \underline{- 6} \\
 24 \\
 \underline{- 24} \\
 6 \\
 \underline{- 6} \\
 0
 \end{array}$$

b.

$$\begin{array}{r}
 \textcolor{teal}{454} \\
 4 \overline{) 1816} \\
 \underline{- 16} \\
 21 \\
 \underline{- 20} \\
 16 \\
 \underline{- 16} \\
 0
 \end{array}$$

c.

$$\begin{array}{r}
 \textcolor{teal}{64} \\
 12 \overline{) 768} \\
 \underline{- 72} \\
 48 \\
 \underline{- 48} \\
 0
 \end{array}$$

2.

Activity Synthesis

Display the worked-out long divisions for all to see. Select a student to explain the steps for at least one of the division problems. Highlight two ideas about long division: 1) we start by dividing the largest base-ten units and work toward smaller units, and 2) the placement of each digit of the quotient matters because it conveys the value of the digit.

a.

$$\begin{array}{r}
 \textcolor{teal}{282} \\
 3 \overline{) 846} \\
 \underline{- 6} \\
 24 \\
 \underline{- 24} \\
 6 \\
 \underline{- 6} \\
 0
 \end{array}$$

b.

$$\begin{array}{r}
 \textcolor{teal}{454} \\
 4 \overline{) 1816} \\
 \underline{- 16} \\
 21 \\
 \underline{- 20} \\
 16 \\
 \underline{- 16} \\
 0
 \end{array}$$

c.

$$\begin{array}{r}
 \textcolor{teal}{64} \\
 12 \overline{) 768} \\
 \underline{- 72} \\
 48 \\
 \underline{- 48} \\
 0
 \end{array}$$

Draw students' attention to the second problem ($1,816 \div 4$) or third problem ($768 \div 12$), in which the first digit of the dividend is smaller than the divisor. Select 1–2 students to share how they approached these situations. If not brought up in students' explanation, discuss how we could reason about these.

- "Let's take $1,816 \div 4$ as an example. If we were using base-ten diagrams, we would have 1 piece representing a thousand. How would we divide that piece into 4 groups?" (We would unbundle it into 10 hundreds, add them to the 8 pieces representing 8 hundreds, and then distribute the 18 hundreds into 4 groups.)
- "How can we apply the same idea to long division? If there is not enough thousands to divide into 4 groups, what can we do?" (We can think of the 1 thousand and 8 hundreds as 18 hundreds and divide that value instead.)

- “How many hundreds would go into each group if we divide 18 hundreds into 4 groups?” (4 hundreds, with a remainder of 2 hundreds.)
- “Where should we write the 4? Why?” (In the hundreds place, because it represents 4 hundreds.)
- “How do we deal with the 2 hundreds?” (Since there is not enough to distribute into 4 groups, we can unbundle them into 20 tens, combine them with the 1 ten, and divide 21 tens by 4.)

Access for English Language Learners

Writing, Speaking: MLR3 Clarify, Critique, and Correct. Use this routine to support whole-class discussion before students share their answer to $1,816 \div 4$. Display a calculation that shows incorrect placement of the 4 in the hundreds digit of the quotient above the 1 in the thousands digit of the dividend. Ask pairs to clarify and critique by asking, “What error was made? Why does the placement of each digit matter?” Give students 1–2 minutes to write a brief response. Look for students who address the placement of each digit by specifying the process they used, and invite these students to share with the class. This will help students clearly describe how to find quotients using long division.

Design Principle(s): Optimize output (for justification); Cultivate conversation

10.3 Dividing Whole Numbers

Optional: 10 minutes

In this activity, students continue to practice using long division to find quotients. Here, the presence of 0’s in the dividend and the quotient presents an added layer of complexity, prompting students to really make sense of the meaning of each digit in numbers they are dealing with (MP7).

Addressing

- 6.NS.B.2

Instructional Routines

- MLR8: Discussion Supports

Launch

Give students 6–7 minutes of quiet work time. Follow with a whole-class discussion. Provide access to graph paper.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation such as “First, I _____ because...,” “Why did you...?,” and “How did you get...?”

Supports accessibility for: Language; Social-emotional skills

Student Task Statement

1. Find each quotient.

a. $633 \div 3$

b. $1001 \div 7$

c. $2996 \div 14$

2. Here is Priya's calculation of $906 \div 3$.

$$\begin{array}{r} \\ 3 \overline{) 906} \\ \underline{- 9} \\ 6 \\ \underline{- 6} \\ 0 \end{array}$$

a. Priya wrote 320 for the value of $906 \div 3$. Check her answer by multiplying it by 3. What product do you get and what does it tell you about Priya's answer?

b. Describe Priya's mistake, then show the correct calculation and answer.

Student Response

a.

$$\begin{array}{r} \\ 3 \overline{) 633} \\ \underline{- 6} \\ 3 \\ \underline{- 3} \\ 3 \\ \underline{- 3} \\ 0 \end{array}$$

b.

$$\begin{array}{r} \\ 7 \overline{) 1001} \\ \underline{- 7} \\ 30 \\ \underline{- 28} \\ 21 \\ \underline{- 21} \\ 0 \end{array}$$

c.

$$\begin{array}{r} \\ 14 \overline{) 2996} \\ \underline{- 28} \\ 19 \\ \underline{- 14} \\ 56 \\ \underline{- 56} \\ 0 \end{array}$$

1.

2.

- a. $320 \cdot 3 = 960$. Priya made a mistake. When the result of $906 \div 3$ is multiplied by 3, it should equal 906.
- b. Priya made a mistake when she placed the 2 of the quotient in the tens place, above the 0 in 906. This would mean that she is taking away 3 times 20, or 60. Since there are only 6 ones remaining, she should have taken away 3 times 2, not 3 times 20. Instead of putting the 2 in the tens place of 906, she should have placed a 0 there and placed the 2 in the ones place (over the 6). The correct answer is 302.

Activity Synthesis

Focus class discussion on attending to the meaning of each digit in performing division. Discuss:

- How did you deal with the 0's in 1,001? Would they cause any difficulty when doing long division? (I brought down the first 0 and then performed division like I would have done with any other digit. After subtracting one 7 from 10, I was left with 3. Putting a 0 after the 3 changes the value to 30. So even though the 0 alone has no value, it changes the value of the numbers in front of it).
- How can you check your answer to a division problem such as $1,001 \div 147$? (We can check by multiplying the quotient by the divisor. If the division was done correctly, then $143 \cdot 7 = 1,001$, which is true).
- What happens if you check Priya's answer for $906 \div 3$? ($320 \cdot 3 = 960$, so this tells us that Priya's answer is incorrect).

Make sure students notice that although checking an answer can tell you that you have made a mistake, it will not necessarily identify *where* the mistake is. It only works if you perform the multiplication correctly.

Access for English Language Learners

Conversing: MLR8 Discussion Supports. Before the class discussion, give students the opportunity to meet with a partner to share their ideas using the three reflection questions. Ask students to focus their discussion on ways to check an answer. Provide these sentence frames: "To check my answer, I", "I know my answer is correct/incorrect because", "Using Priya's work as an example, I". The listener should press for details by asking clarifying questions such as, "How do you know that method works to check your answer?" and "Could you explain that using $1001 \div 7$?" Give each student an opportunity to be the speaker and the listener. This will help students communicate their thinking around verifying answers to long division problems.

Design Principle(s): Cultivate conversation; Maximize meta-awareness

Lesson Synthesis

Long division is another method for finding quotients. It follows similar lines of reasoning for dividing with base-ten diagrams or using the partial quotients method. All three methods rely on the structure of the base-ten number system.

- How is dividing using long division similar to dividing by drawing base-ten diagrams? (Even though one method involves drawing and the other involves using only numbers, they rely on the same principle of dividing base-ten units into equal-sized groups. In both methods, when there is not enough of a unit to divide equally into groups, we can unbundle the unit into the next smaller base-ten units.)
- How is long division similar to and different from the partial quotients method? (They are similar in that we divide in “installments,” but in the partial quotients method, we can decide on the size of each installment or each group being subtracted from the dividend. In long division, we follow a very specific order based on place value and we divide digit by digit—from left to right, and subtract as large a group as possible at any step. In long division, we also do not write out all the numbers in our calculations; we use one digit at a time and rely on its place in the base-ten system to convey its value.)
- Which method for finding quotients do you think is the most efficient? (It depends on the numbers involved. If the numbers are large or long, drawing would be laborious and prone to error, and using partial quotients might mean a whole lot of steps. Long division might be simpler because we are reasoning with one digit and one place-value unit at a time.)

10.4 Dividing by 15

Cool Down: 5 minutes

Addressing

- 6.NS.B.2

Student Task Statement

Use long division to find the value of $1,875 \div 15$.

Student Response

125

$$\begin{array}{r}
 15 \overline{) 1875} \\
 \underline{- 15} \\
 37 \\
 \underline{- 30} \\
 75 \\
 \underline{- 75} \\
 0
 \end{array}$$

Student Lesson Summary

Long division is another method for calculating quotients. It relies on place value to perform and record the division.

When we use long division, we work from left to right and with one digit at a time, starting with the leftmost digit of the dividend. We remove the largest group possible each time, using the placement of the digit to indicate the size of each group. Here is an example of how to find $948 \div 3$ using long division.

$$\begin{array}{r}
 316 \\
 3 \overline{) 948} \\
 \underline{- 9} \longleftarrow 3 \text{ groups of 3 (hundreds)} \\
 4 \\
 \underline{- 3} \longleftarrow 3 \text{ groups of 1 (ten)} \\
 18 \\
 \underline{- 18} \longleftarrow 3 \text{ groups of 6 (ones)} \\
 0
 \end{array}$$

- We start by dividing 9 hundreds into 3 groups, which means 3 hundreds in each group. Instead of writing 300, we simply write 3 in the hundreds place, knowing that it means 3 hundreds.
- There are no remaining hundreds, so we work with the tens. We can make 3 groups of 1 ten in 4 tens, so we write 1 in the tens place above the 4 of 948. Subtracting 3 tens from 4 tens, we have a remainder of 1 ten.
- We know that 1 ten is 10 ones. Combining these with the 8 ones from 948, we have 18 ones. We can make 3 groups of 6, so we write 6 in the ones place.

In total, there are 3 groups of 3 hundreds, 1 ten, and 6 ones in 948, so $948 \div 3 = 316$.

Glossary

- long division

Lesson 10 Practice Problems

Problem 1

Statement

Kiran is using long division to find $696 \div 12$.

$$12 \overline{) 696}$$

He starts by dividing 69 by 12. In which decimal place should Kiran place the first digit of the quotient (5)?

- A. Hundreds
- B. Tens
- C. Ones
- D. Tenths

Solution

B

Problem 2

Statement

Here is a long-division calculation of $917 \div 7$.

$$\begin{array}{r} 131 \\ 7 \overline{) 917} \\ \underline{- 7} \\ 21 \\ \underline{- 21} \\ 7 \\ \underline{- 7} \\ 0 \end{array}$$

- a. There is a 7 under the 9 of 917. What does this 7 represent?
- b. What does the subtraction of 7 from 9 mean?
- c. Why is a 1 written next to the 2 from $9 - 7$?

Solution

- a. Answers vary. Sample response: The 7 under the 9 represents 700 (because it is written directly under the hundreds place of 917).
- b. Answers vary. Sample response: It means a subtraction of 7 groups of 1 hundred from 9 hundreds.

- c. Answers vary. Sample response: To represent the 10 in 917. There is 2 hundreds left after 7 hundreds are subtracted from 9 hundreds. The 2 hundreds is combined with the 1 ten from 917, which makes 21 tens.

Problem 3

Statement

Han's calculation of $972 \div 9$ is shown here.

$$\begin{array}{r}
 180 \\
 9 \overline{) 972} \\
 \underline{- 9} \\
 72 \\
 \underline{- 72} \\
 0 \\
 \underline{- 0} \\
 0
 \end{array}$$

- Find $180 \cdot 9$.
- Use your calculation of $180 \cdot 9$ to explain how you know Han has made a mistake.
- Identify and correct Han's mistake.

Solution

- $180 \cdot 9 = 1620$
- If Han were correct, the product of 180 and 9 would be 972.
- Answers vary. Sample response: Han's mistake is that when he brought down the 7 from 972 and saw that 7 tens could not be divided into 9 groups (or 7 is not a multiple of 9), he did not write 0 above the 7 before bringing down the 2 ones. Here is the correct long division calculation:

$$\begin{array}{r}
 108 \\
 9 \overline{) 972} \\
 \underline{- 9} \\
 7 \\
 \underline{- 0} \\
 72 \\
 \underline{- 72} \\
 0
 \end{array}$$

Problem 4

Statement

Find each quotient.

a.

$$5 \overline{)465}$$

b.

$$12 \overline{)924}$$

c.

$$3 \overline{)1107}$$

Solution

a. 93

b. 77

c. 369

a.

$$\begin{array}{r} 93 \\ 5 \overline{)465} \\ \underline{-45} \\ 15 \\ \underline{-15} \\ 0 \end{array}$$

b.

$$\begin{array}{r} 77 \\ 12 \overline{)924} \\ \underline{-84} \\ 84 \\ \underline{-84} \\ 0 \end{array}$$

c.

$$\begin{array}{r} 369 \\ 3 \overline{)1107} \\ \underline{-9} \\ 20 \\ \underline{-18} \\ 27 \\ \underline{-27} \\ 0 \end{array}$$

Problem 5

Statement

One ounce of a yogurt contains of 1.2 grams of sugar. How many grams of sugar are in 14.25 ounces of yogurt?

A. 0.171 grams

B. 1.71 grams

C. 17.1 grams

D. 171 grams

Solution

C

(From Unit 5, Lesson 7.)

Problem 6

Statement

The mass of one coin is 16.718 grams. The mass of a second coin is 27.22 grams. How much greater is the mass of the second coin than the first? Show your reasoning.

Solution

10.502 grams, because $27.22 - 16.718 = 10.502$

(From Unit 5, Lesson 4.)

Lesson 11: Dividing Numbers that Result in Decimals

Goals

- Interpret different methods for computing a quotient that is not a whole number, and express it (orally and in writing) in terms of “unbundling.”
- Use long division to divide whole numbers that result in a quotient with a decimal, and explain (orally) the solution method.

Learning Targets

- I can use long division to find the quotient of two whole numbers when the quotient is not a whole number.

Lesson Narrative

So far, students have divided whole numbers that result in whole-number quotients. In the next three lessons, they work toward performing division in which the divisor, dividend, and quotient are decimals. In this lesson, they perform division of two whole numbers that result in a terminating decimal. Students divide using all three techniques introduced in this unit: base-ten diagrams, partial quotients, and long division. They apply this skill to calculate the (terminating) decimal expansion of some fractions.

Students analyze, explain, and critique various ways of reasoning about division (MP3).

Alignments

Building On

- 4.NBT.B.6: Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Addressing

- 6.NS.B.2: Fluently divide multi-digit numbers using the standard algorithm.

Building Towards

- 6.EE.A: Apply and extend previous understandings of arithmetic to algebraic expressions.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR7: Compare and Connect
- MLR8: Discussion Supports

- Number Talk
- Think Pair Share

Required Preparation

Students may choose to draw base-ten diagrams in this lesson. If drawing them is a challenge, consider giving students access to:

- Commercially produced base-ten blocks, if available.
- Paper copies of squares and rectangles (to represent base-ten units), cut up from copies of the Instructional master of the second lesson in the unit.
- Digital applet of base-ten representations <https://www.geogebra.org/m/FXEZD466>

Some students might find it helpful to use graph paper to help them align the digits as they divide using long division and the partial quotients method. Consider having graph paper accessible throughout the lesson.

Student Learning Goals

Let's find quotients that are not whole numbers.

11.1 Number Talk: Evaluating Quotients

Warm Up: 5 minutes

This number talk encourages students to think about the numbers in a computation problem and rely on what they know about structure, patterns, and division to mentally solve a problem. Four expressions are given. The first three expressions are partial quotients that could help students evaluate the last expression of $496 \div 8$. Be sure to allot more time to discuss the final expression and to draw out its connection to the other expressions.

Building On

- 4.NBT.B.6

Building Towards

- 6.EE.A

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Display one expression at a time, or ask students to work on one expression at a time and begin when cued. Give students 30 seconds of quiet think time per question and ask them to give a signal

when they have an answer and can explain their strategy. Select 1–2 students to briefly discuss how they reasoned about the quotient.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Anticipated Misconceptions

Some students may start from scratch when evaluating the final question. Ask them how they could use what they have already done to help them with the final question.

Student Task Statement

Find the quotients mentally.

$$400 \div 8$$

$$80 \div 8$$

$$16 \div 8$$

$$496 \div 8$$

Student Response

- 50. Strategies vary. Possible strategy: $(40 \div 8) \cdot 10$.
- 10. Strategies vary.
- 2. Strategies vary.
- 62. Strategies vary. Possible strategy: $496 \div 8 = (400 \div 8) + (80 \div 8) + (16 \div 8)$.

Activity Synthesis

Ask students to share their reasoning for each quotient. Record and display their explanations for all to see; this will help students see the connections between the first three expressions and the last one. To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Did anyone find the quotient the same way but would explain it differently?”
- “Did anyone find the quotient in a different way?”
- “Does anyone want to add on to ____’s reasoning?”
- “Do you agree or disagree? Why?”

If not mentioned by students, highlight how the quotients of $(400 \div 8) + (80 \div 8) + (16 \div 8)$ can be used to find $496 \div 8$.

Access for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because . . ." or "I noticed ____ so I" Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Design Principle(s): Optimize output (for explanation)

11.2 Keep Dividing

Optional: 20 minutes

This activity extends division techniques used to find whole-number quotients to divide whole numbers resulting in (terminating) decimal quotients. In these problems, students see remainders in the ones place. In order to continue the division process, the ones are broken into tenths. Conceptually, this is the same unbundling idea that is used when hundreds are broken into tens or when tens are broken into ones.

If students have trouble drawing the diagrams to represent unbundling, consider providing actual base-ten blocks or paper cut-outs of base-ten units (from the Instructional master used earlier in the unit) so that they can physically trade the pieces (e.g., 2 ones for 20 tenths).

Addressing

- 6.NS.B.2

Instructional Routines

- MLR7: Compare and Connect
- Think Pair Share

Launch

Arrange students in groups of 2. Provide access to graph paper. For the first question, give students 1 minute of quiet think time to analyze Mai's work and 2–3 minutes to discuss their observations with their partner. Pause for a whole-class discussion, making sure that all students understand how Mai dealt with the remainder.

Give students 5–7 minutes to complete the final two questions. Follow with a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Encourage students to begin with physical representations before drawing a diagram. Provide access to base-ten blocks or paper cut-outs of base-ten units to support drawing diagrams.

Supports accessibility for: Conceptual processing

Student Task Statement

Mai used base-ten diagrams to calculate $62 \div 5$. She started by representing 62.



She then made 5 groups, each with 1 ten. There was 1 ten left. She unbundled it into 10 ones and distributed the ones across the 5 groups.

Here is Mai's diagram for $62 \div 5$.

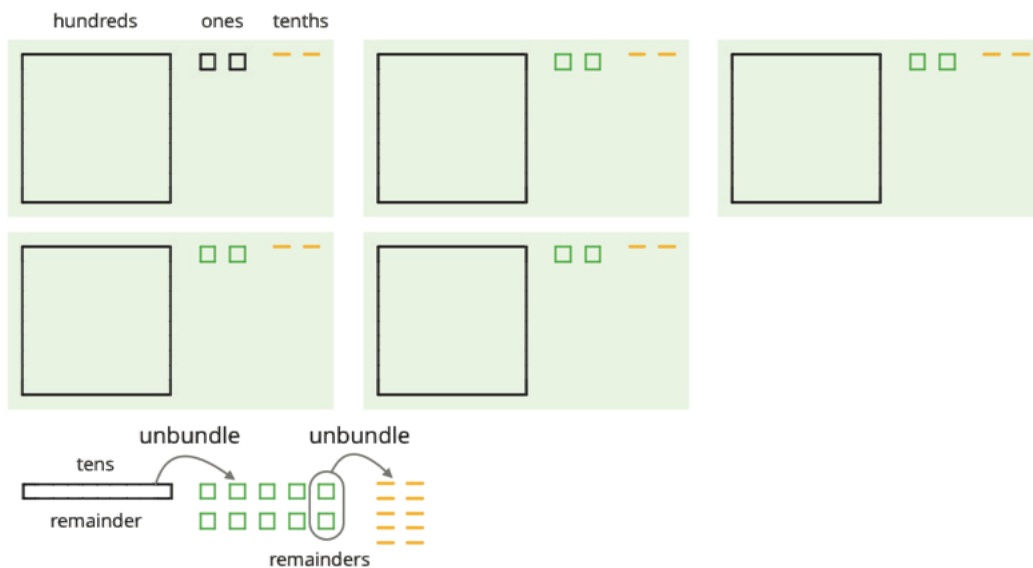


1. Discuss these questions with a partner and write down your answers:

- a. Mai should have a total of 12 ones, but her diagram shows only 10. Why?
 - b. She did not originally have tenths, but in her diagram each group has 4 tenths. Why?
 - c. What value has Mai found for $62 \div 5$? Explain your reasoning.
2. Find the quotient of $511 \div 5$ by drawing base-ten diagrams or by using the partial quotients method. Show your reasoning. If you get stuck, work with your partner to find a solution.
3. Four students share a \$271 prize from a science competition. How much does each student get if the prize is shared equally? Show your reasoning.

Student Response

1.
 - a. Mai unbundled two ones blocks to make 20 tenths. So instead of 12 ones, her diagram has 10 ones and 20 tenths.
 - b. In order to complete the division into 5 equal groups, Mai needed to unbundle 2 of her ones to make 20 tenths. She then placed 4 tenths in each of the equal groups.
 - c. Mai has divided 62 into 5 equal groups of 1 ten, 2 ones, and 4 tenths. So $62 \div 5 = 12.4$.
2. 102.2. Sample reasoning:



$$\begin{array}{r}
 \boxed{102.2} \\
 0.2 \\
 2 \\
 100 \\
 5 \overline{) 511} \\
 \underline{- 500} \\
 11 \\
 \underline{- 10} \\
 10 \\
 \underline{- 10} \\
 0
 \end{array}$$

3. \$67.75. Sample reasoning: There are 4 groups of \$67 in \$268, so each student gets \$67 dollars, and the group must split the remaining \$3 evenly. Since 300 pennies can be divided into 4 groups of 75 pennies, each student received \$0.75. This means that each student gets \$67.75.

Activity Synthesis

The goal of this discussion is for students to relate their earlier work on division, which resulted in whole-number quotients, to division involving decimal quotients. Below is an example of how the discussion may go, along with questions to ask students and some possible responses. Begin the discussion by reminding students of the work they have previously done to evaluate $657 \div 3$.

How is computation for $62 \div 5$ similar to that for $657 \div 3$?

- The method of division is the same: we divide a given number (62 or 657) into equal groups until everything is distributed.
- We divide by using place value, unbundling one unit into ten of a smaller unit as needed. For $62 \div 5$, the 2 ones can be broken into 20 tenths, while in $657 \div 3$, the 2 tens were unbundled into 20 ones.



How is computation for $62 \div 5$ different from that for $657 \div 3$?

- There is no remainder for $657 \div 3$, while there is a remainder of 2 for $62 \div 5$.
- We need to write a decimal point and work with tenths in $62 \div 5$.

It is important to stress that the methods and steps are the same in both computations. The big new idea here is that sometimes a division problem of whole numbers does not end when we get to the ones place. In these cases, we have to add a decimal because the number being divided involves tenths, hundredths, or smaller base-ten units.

Access for English Language Learners

Representing, Speaking: MLR7 Compare and Connect. Use this routine to give students an opportunity to compare approaches for finding the quotient of $511 \div 5$. Ask students to share their approach with a partner. Invite groups to discuss what is the same and what it different about finding a quotient either by drawing base-ten diagrams or by using the partial quotients method. Listen for opportunities to highlight language such as “remainder,” and “tenths.” This will help students connect division techniques and extend them to find decimal quotients.

Design Principle(s): Optimize output; Cultivate conversation

11.3 Using Long Division to Calculate Quotients

25 minutes

In this activity, students use long division to divide whole numbers whose quotient is not a whole number. Previously, students found the quotient of $62 \div 5$ using base-ten diagrams and the partial quotients method. Because the long division is a particular version of the partial quotients method,

and because students have been introduced to long division, they have the tools to divide whole numbers that result in a (terminating) decimal. In this activity, students evaluate and critique the reasoning of others (MP3).

Addressing

- 6.NS.B.2

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Arrange students in groups of 2. Provide access to graph paper. Give students 7–8 minutes to analyze and discuss Lin’s work with a partner and then complete the second set of questions.

Pause to discuss students’ analyses and at least one of the division problems. Students should understand that, up until reaching the decimal point, long division works the same for $62 \div 5 = 12.4$ as it does for $657 \div 3 = 219$. In $62 \div 5$, however, there is a remainder of 2 ones, and we need to convert to the next smaller place value (tenths), change the 2 ones into 20 tenths, and then divide these into 5 equal groups of 4 tenths.

Prepare students to do the last set of questions by setting up the long division of $5 \div 4$ for all to see. Discuss the placement of the decimal point. Reiterate that we can write extra zeros at the end of the dividend, following the decimal point, so that remainders can be worked with. Have students work through this problem and the others. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Access for Perception. Display and read Lin’s method for calculating the quotient of $62 \div 5$ aloud. Students who both listen to and read the information will benefit from extra processing time.

Supports accessibility for: Language; Conceptual processing

Anticipated Misconceptions

Students may be perplexed by the repeating decimals in the last question and think that they have made a mistake. Ask them to compare their work with a partner’s, and then clarify during discussion that some decimals do repeat. Because this work comes into focus in grade 7, the goal here is simply for students to observe that not all decimals terminate.

Student Task Statement

Here is how Lin calculated $62 \div 5$.

Lin set up the numbers for long division.

$$5 \overline{) 62}$$

She subtracted 5 times 1 from the 6, which leaves a remainder of 1.

She wrote the 2 from 62 next to the 1, which made 12, and subtracted 5 times 2 from 12.

$$\begin{array}{r} 1 \\ 5 \overline{) 62} \\ - 5 \\ \hline 12 \\ - 10 \\ \hline 2 \end{array}$$

Lin drew a vertical line and a decimal point, separating the ones and tenths place.

$12 - 10$ is 2. She wrote 0 to the right of the 2, which made 20.

$$\begin{array}{r} 12 \\ 5 \overline{) 62} \\ - 5 \\ \hline 12 \\ - 10 \\ \hline 20 \end{array}$$

Lastly, she subtracted 5 times 4 from 20, which left no remainder.

At the top, she wrote 4 next to the decimal point.

$$\begin{array}{r} 12.4 \\ 5 \overline{) 62} \\ - 5 \\ \hline 12 \\ - 10 \\ \hline 20 \\ - 20 \\ \hline 0 \end{array}$$

1. Discuss with your partner:

- Lin put a 0 after the remainder of 2. Why? Why does this 0 not change the value of the quotient?
- Lin subtracted 5 groups of 4 from 20. What value does the 4 in the quotient represent?
- What value did Lin find for $62 \div 5$?

2. Use long division to find the value of each expression. Then pause so your teacher can review your work.

a. $126 \div 8$

b. $90 \div 12$

3. Use long division to show that:

a. $5 \div 4$, or $\frac{5}{4}$, is 1.25.

b. $4 \div 5$, or $\frac{4}{5}$, is 0.8.

c. $1 \div 8$, or $\frac{1}{8}$, is 0.125.

d. $1 \div 25$, or $\frac{1}{25}$, is 0.04.

4. Noah said we cannot use long division to calculate $10 \div 3$ because there will always be a remainder.

- a. What do you think Noah meant by “there will always be a remainder”?

b. Do you agree with him? Explain your reasoning.

Student Response

1. Answers vary. Sample response:

- Lin put a 0 after the 2 because she could not take any more 5's from 2 but wanted to continue the calculation to the tenths place. The 0 represents 0 tenths and 20 tenths has the same value as 2 ones.
- The 4 in the quotient represents 4 tenths. 20 tenths is 5 equal groups of 4 tenths, and so 4 tenths is added to the quotient.
- The value of $62 \div 5$ is 12.4. There was no remainder after putting 1 ten, 2 ones, and 4 tenths into 5 equal groups.

2. a. $126 \div 8 = 15.75$

b. $90 \div 12 = 7.5$

a.

$$\begin{array}{r}
 15.75 \\
 8 \overline{) 126} \\
 \underline{- 8} \\
 46 \\
 \underline{- 40} \\
 60 \\
 \underline{- 56} \\
 40 \\
 \underline{- 40} \\
 0
 \end{array}$$

b.

$$\begin{array}{r}
 7.5 \\
 12 \overline{) 90} \\
 \underline{- 84} \\
 60 \\
 \underline{- 60} \\
 0
 \end{array}$$

3.

a.

$$\begin{array}{r}
 1.25 \\
 4 \overline{) 5} \\
 \underline{- 4} \\
 10 \\
 \underline{- 8} \\
 20 \\
 \underline{- 20} \\
 0
 \end{array}$$

b.

$$\begin{array}{r}
 0.8 \\
 5 \overline{) 4} \\
 \underline{- 0} \\
 40 \\
 \underline{- 40} \\
 0
 \end{array}$$

c.

$$\begin{array}{r}
 0.125 \\
 8 \overline{) 1} \\
 \underline{- 0} \\
 10 \\
 \underline{- 8} \\
 20 \\
 \underline{- 16} \\
 40 \\
 \underline{- 40} \\
 0
 \end{array}$$

d.

$$\begin{array}{r}
 0.04 \\
 25 \overline{) 1} \\
 \underline{- 0} \\
 10 \\
 \underline{- 0} \\
 100 \\
 \underline{- 100} \\
 0
 \end{array}$$

4. a. Write a decimal point and a 0 because 3 is larger than 1. There are 3 threes in 10 with a remainder of 1. Then write another 0 after the decimal point resulting in 10 again. 3 threes can continue to be taken out, but there is *always* 1 remaining.
- b. Yes, after writing another 0 after the decimal point, there is one remaining after taking out all of the threes.

Activity Synthesis

Focus the whole-class discussion on the third and fourth sets of questions. Ask a few students to show their long division for all to see and to explain their steps. Some ideas to bring to uncover:

- Problems like $1 \div 25$ are challenging because the first step is 0: there are *zero* groups of 25 in 1. This means that we need to introduce a decimal and put a 0 to the right of the decimal. But one 0 is not enough. It is not until we add the second 0 to the right of the decimal that we can find 4 groups of 25 in 100. Because we moved two places to the right of the decimal, these 4 groups are really 0.04, which is the quotient of 1 by 25.
- Problems like $1 \div 3$ are not fully treated until grade 7. At this point, we can observe that the long division process will go on and on because there is always a remainder of 1.

Some questions to ask students that highlight these points include:

- When you found $1 \div 8$, what was your first step? (I put a 0 above the dividend 1 and added a decimal because I cannot take any 8's from 1.)
- When you found $1 \div 25$, what were your first few steps? (I put a 0 above the dividend 1 and added a decimal because I cannot take any 25's from 1. I needed to add another 0 to the right of the decimal because I still cannot take any 25's from 10.)
- Why do we not write 0's in advance to the right of the decimal? (We do not know in advance if we will need these 0's or how many of them we might need, so we usually write them as needed.)

Another valuable link to make is to connect the values for the decimals in Problem 4 to percentages. For example, the fact that $\frac{4}{5} = 0.8$ means that $\frac{4}{5}$ of a quantity is 80% of that quantity. Similarly $\frac{1}{25}$ of a quantity is the same as 4% of that quantity.

Access for English Language Learners

Writing, Listening, Conversing: MLR1 Stronger and Clearer Each Time. Use this routine to help students improve their written responses to the question, “Noah said we cannot use long division to calculate $10 \div 3$ because there will always be a remainder. Do you agree or disagree with this statement?” Give students time to meet with 2–3 partners to share and get feedback on their responses. Provide listeners with prompts that will help their partners strengthen their ideas and clarify their language (e.g., “Can you explain why there is always a remainder?”, “What number always remains?”, and “Could you try to explain this using a different example?”). Give students 1–2 minutes to revise their writing based on the feedback they received. Communicating their reasoning with a partner will help students understand situations in which long division does not work

Design Principle(s): Optimize output (for explanation); Cultivate conversation

Lesson Synthesis

In this lesson, we saw that the quotient of two whole numbers can result in a decimal value. We can observe how this happens with base-ten blocks and by calculating with long division.

- When using base-ten blocks to divide, how can we work with a remainder of ones? (Unbundle the ones into tenths.)
- When calculating with long division, how can we keep dividing when there is a remainder? (Put a decimal point after the dividend and follow it with zeros. This allows you to bring down a zero and continue the calculation.)
- How do we divide a whole number that is smaller than the divisor, for instance $4 \div 5$? (Start by writing a 0 for the quotient and follow it with a decimal point. In the example of $4 \div 5$, the 0 means there is not enough ones to divide equally into 5 groups. Then, unbundle the number into ten of the next smaller unit so that it can be divided. In this example, 4 ones can be unbundled into 40 tenths, which can then be divided by 5.)

11.4 Calculating Quotients

Cool Down: 5 minutes

Addressing

- 6.NS.B.2

Student Task Statement

Use long division to find each quotient. Show your computation and write your answer as a decimal.

1. $22 \div 5$

$$2.7 \div 8$$

Student Response

a.

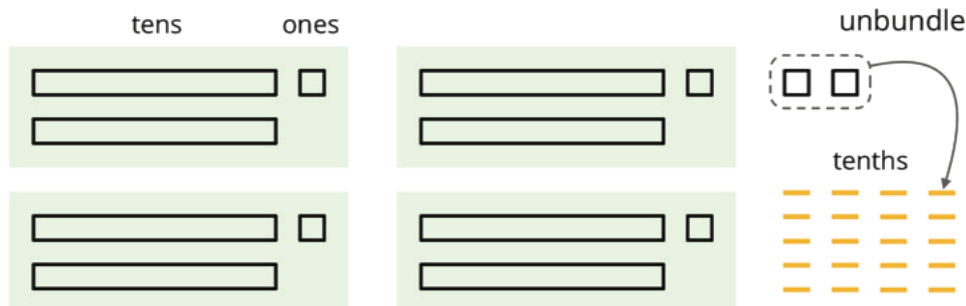
$$\begin{array}{r} 44 \\ 5 \overline{) 22} \\ \underline{- 20} \\ 20 \\ \underline{- 20} \\ 0 \end{array}$$

b.

$$\begin{array}{r} 0875 \\ 8 \overline{) 70} \\ \underline{- 0} \\ 70 \\ \underline{- 64} \\ 60 \\ \underline{- 56} \\ 40 \\ \underline{- 40} \\ 0 \end{array}$$

Student Lesson Summary

Dividing a whole number by another whole number does not always produce a whole-number quotient. Let's look at $86 \div 4$, which we can think of as dividing 86 into 4 equal groups.



We can see in the base-ten diagram that there are 4 groups of 21 in 86 with 2 ones left over. To find the quotient, we need to distribute the 2 ones into the 4 groups. To do this, we can unbundle or decompose the 2 ones into 20 tenths, which enables us to put 5 tenths in each group.

Once the 20 tenths are distributed, each group will have 2 tens, 1 one, and 5 tenths, so $86 \div 4 = 21.5$.

$$\begin{array}{r}
 21.5 \\
 4 \overline{) 86} \\
 \underline{- 8} \\
 6 \\
 \underline{- 4} \\
 20 \\
 \underline{- 20} \\
 0
 \end{array}$$

We can also calculate $86 \div 4$ using long division.

The calculation shows that, after removing 4 groups of 21, there are 2 ones remaining. We can continue dividing by writing a 0 to the right of the 2 and thinking of that remainder as 20 tenths, which can then be divided into 4 groups.

To show that the quotient we are working with now is in the tenth place, we put a decimal point to the right of the 1 (which is in the ones place) at the top. It may also be helpful to draw a vertical line to separate the ones and the tenths.

There are 4 groups of 5 tenths in 20 tenths, so we write 5 in the tenths place at the top. The calculation likewise shows $86 \div 4 = 21.5$.

Lesson 11 Practice Problems

Problem 1

Statement

Use long division to show that the fraction and decimal in each pair are equal.

$\frac{3}{4}$ and 0.75

$\frac{3}{50}$ and 0.06

$\frac{7}{25}$ and 0.28

Solution

a.

$$\begin{array}{r}
 0.75 \\
 4 \overline{) 3.00} \\
 \underline{- 0} \\
 30 \\
 \underline{- 28} \\
 20 \\
 \underline{- 20} \\
 0
 \end{array}$$

b.

$$\begin{array}{r}
 0.06 \\
 50 \overline{) 3.00} \\
 \underline{- 0} \\
 30 \\
 \underline{- 30} \\
 00 \\
 \underline{- 00} \\
 00 \\
 \underline{- 00} \\
 0
 \end{array}$$

c.

$$\begin{array}{r}
 0.28 \\
 25 \overline{) 7.00} \\
 \underline{- 0} \\
 70 \\
 \underline{- 50} \\
 20 \\
 \underline{- 20} \\
 00 \\
 \underline{- 00} \\
 0
 \end{array}$$

Problem 2

Statement

Mai walked $\frac{1}{8}$ of a 30-mile walking trail. How many miles did Mai walk? Explain or show your reasoning.

Solution

3.75 miles. Reasoning varies. Sample reasoning: $\frac{1}{8}$ of 30 is $30 \div 8 = 3.75$

Problem 3

Statement

Use long division to find each quotient. Write your answer as a decimal.

a. $99 \div 12$

b. $216 \div 5$

c. $1,988 \div 8$

Solution

a.

$$\begin{array}{r} 8 \overline{) 99.00} \\ \underline{-96} \\ 30 \\ \underline{-24} \\ 60 \\ \underline{-60} \\ 0 \end{array}$$

b.

$$\begin{array}{r} 43 \overline{) 216.0} \\ \underline{-20} \\ 16 \\ \underline{-15} \\ 10 \\ \underline{-10} \\ 0 \end{array}$$

c.

$$\begin{array}{r} 248 \overline{) 1988.0} \\ \underline{-16} \\ 38 \\ \underline{-32} \\ 68 \\ \underline{-64} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

Problem 4

Statement

Tyler reasoned: " $\frac{9}{25}$ is equivalent to $\frac{18}{50}$ and to $\frac{36}{100}$, so the decimal of $\frac{9}{25}$ is 0.36."

a. Use long division to show that Tyler is correct.

b. Is the decimal of $\frac{18}{50}$ also 0.36? Use long division to support your answer.

Solution

a.

$$\begin{array}{r} 0.36 \\ 25 \overline{) 9.00} \\ \underline{-0} \\ 90 \\ \underline{-75} \\ 150 \\ \underline{-150} \\ 0 \end{array}$$

b.

$$\begin{array}{r} 0.36 \\ 50 \overline{) 18.00} \\ \underline{-0} \\ 180 \\ \underline{-150} \\ 300 \\ \underline{-300} \\ 0 \end{array}$$

Yes, the decimal of $\frac{18}{50}$ is also 0.36.

Problem 5

Statement

Complete the calculations so that each shows the correct difference.

a.

$$\begin{array}{r} 5 \\ - \\ \hline 4.329 \end{array}$$

b.

$$\begin{array}{r} 1 \\ - \\ \hline 0.015 \end{array}$$

c.

$$\begin{array}{r} 1 \\ - \\ \hline 0.863 \end{array}$$

Solution

a. 0.671

b. 0.985

c. 0.137

$$\begin{array}{r} 99 \\ 4 \cancel{10} \cancel{10} \cancel{10} \\ 5. \cancel{0} \cancel{0} \cancel{0} \\ - \\ \hline 0.671 \\ 4.329 \end{array}$$

$$\begin{array}{r} 99 \\ 0 \cancel{10} \cancel{10} \cancel{10} \\ 1. \cancel{0} \cancel{0} \cancel{0} \\ - \\ \hline 0.985 \\ 0.015 \end{array}$$

$$\begin{array}{r} 99 \\ 0 \cancel{10} \cancel{10} \cancel{10} \\ 1. \cancel{0} \cancel{0} \cancel{0} \\ - \\ \hline 0.137 \\ 0.863 \end{array}$$

(From Unit 5, Lesson 4.)

Problem 6

Statement

Use the equation $124 \cdot 15 = 1,860$ and what you know about fractions, decimals, and place value to explain how to place the decimal point when you compute $(1.24) \cdot (0.15)$.

Solution

1.24 is $124 \cdot (0.01)$ and 0.15 is $15 \cdot (0.01)$. So $(1.24) \cdot (0.15)$ can be written as $124 \cdot 15 \cdot (0.01) \cdot (0.01)$, which is $(1,860) \cdot (0.0001)$ or 0.186.

(From Unit 5, Lesson 6.)

Lesson 12: Dividing Decimals by Whole Numbers

Goals

- Compare and contrast (orally and using other representations) division problems with whole-number and decimal dividends
- Divide decimals by whole numbers, and explain the reasoning (orally and using other representations).
- Generalize (orally and in writing) that multiplying both the dividend and the divisor by the same factor does not change the quotient.

Learning Targets

- I can divide a decimal by a whole number.
- I can explain the division of a decimal by a whole number in terms of equal-sized groups.
- I know how multiplying both the dividend and the divisor by the same factor affects the quotient.

Lesson Narrative

This lesson serves two purposes. The first is to show that we can divide a decimal by a whole number the same way we divide two whole numbers. Students first represent a decimal dividend with base-ten diagrams. They see that, just like the units representing powers of 10, those for powers of 0.1 can also be divided into groups. They then divide using another method—partial quotients or long division—and notice that the principle of placing base-ten units into equal-size groups is likewise applicable.

The second is to uncover the idea that the value of a quotient does not change if both the divisor and dividend are multiplied by the same factor. Students begin exploring this idea in problems where the factor is a multiple of 10 (e.g. $8 \div 1 = 80 \div 10$). This work prepares students to divide two decimals in the next lesson.

Alignments

Building On

- 5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Addressing

- 6.NS.B.3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Building Towards

- 6.EE.A: Apply and extend previous understandings of arithmetic to algebraic expressions.
- 6.NS.B.2: Fluently divide multi-digit numbers using the standard algorithm.

Instructional Routines

- MLR2: Collect and Display
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Number Talk

Required Preparation

Some students might find it helpful to use graph paper to help them align the digits as they divide using long division and the partial quotients method. Consider having graph paper accessible throughout the lesson.

Student Learning Goals

Let's divide decimals by whole numbers.

12.1 Number Talk: Dividing by 4

Warm Up: 5 minutes

The purpose of this number talk is to help students use the structure of base-ten numbers and the distributive property to solve a division problem involving decimals.

Building On

- 5.NBT.B.7

Building Towards

- 6.EE.A

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.

Supports accessibility for: Memory; Organization

Student Task Statement

Find each quotient mentally.

$$80 \div 4$$

$$12 \div 4$$

$$1.2 \div 4$$

$$81.2 \div 4$$

Student Response

- 20
- 3
- 0.3
- 20.3

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Did anyone have the same strategy but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to ____’s strategy?”
- “Do you agree or disagree? Why?”

Highlight the use of the distributive property in finding $81.2 \div 4$. Students should recognize that since $81.2 = 80 + 1.2$, we have $81.2 \div 4 = (80 \div 4) + (1.2 \div 4)$. To make this clear, consider explaining that the division could be equivalently represented by $81.2 \cdot \frac{1}{4} = (80 + 1.2) \cdot \frac{1}{4} = (80 \cdot \frac{1}{4}) + (1.2 \cdot \frac{1}{4})$.

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Provide sentence frames to support students with explaining their strategies. For example, “I noticed that _____, so I _____.” or “First, I _____ because _____.” When students share their answers with a partner, prompt them to rehearse what they will say when they share with the full group. Rehearsing provides opportunities to clarify their thinking.

Design Principle(s): Optimize output (for explanation)

12.2 Using Diagrams to Represent Division

15 minutes

Students have learned several effective methods to divide a whole number by a whole number, including cases when there is a remainder. The goal of this task is to introduce a method for dividing a decimal number by a whole number. Students notice that the steps in the division process are the same as when dividing a whole number by a whole number, whether the division is done with base-ten diagrams, as in this task, or using partial quotients or the division algorithm as in future tasks. Here, students need to think even more carefully about place value and where the decimal point goes in the quotient.

Throughout this activity, students rely on their understanding of equivalent expressions to interpret the unbundling in Elena’s process. For example, to unbundle a one into ten tenths means going between the expressions 1 and $0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1$.

Addressing

- 6.NS.B.3

Instructional Routines

- MLR2: Collect and Display

Launch

Give students 1–2 minutes of quiet think time for students to analyze Elena’s work. Pause and discuss with the whole class. Select a couple of students to share their analyses of what Elena had done to divide a decimal by a whole number. Then, give students 8–9 minutes to complete the questions and follow with a whole-class discussion.

Anticipated Misconceptions

Some students may stop dividing when they reach a remainder rather than unbundling the remainder into smaller units. Remind them that they can continue to divide the remainder by unbundling and to refer to Elena’s worked-out example or those from earlier lessons, if needed.

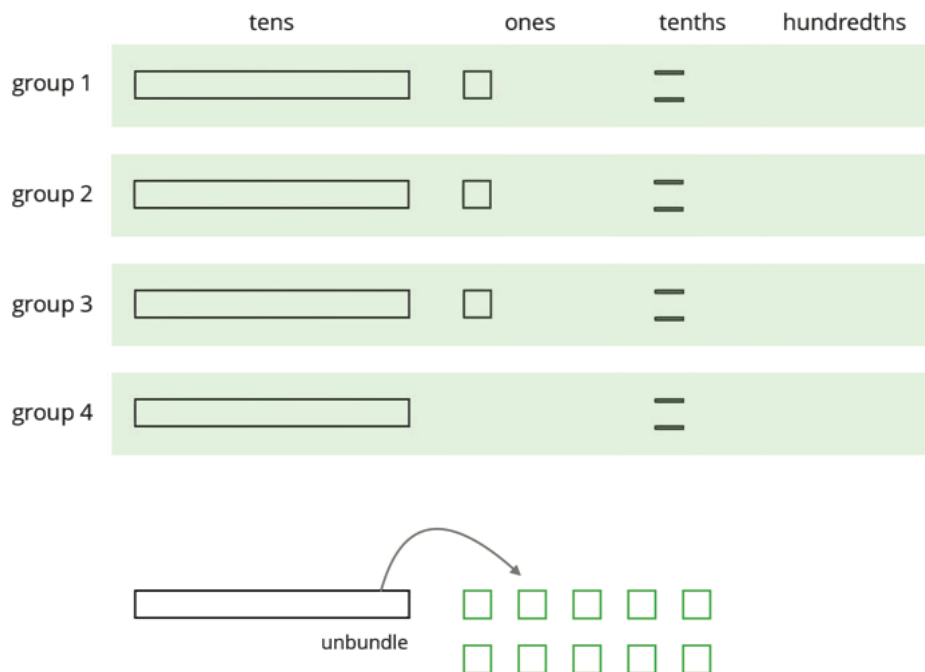
Student Task Statement

To find $53.8 \div 4$ using diagrams, Elena began by representing 53.8.



She placed 1 ten into each group, unbundled the remaining 1 ten into 10 ones, and went on distributing the units.

This diagram shows Elena's initial placement of the units and the unbundling of 1 ten.



1. Complete the diagram by continuing the division process. How would you use the available units to make 4 equal groups?

As the units get placed into groups, show them accordingly and cross out those pieces from the bottom. If you unbundle a unit, draw the resulting pieces.

2. What value did you find for $53.8 \div 4$? Be prepared to explain your reasoning.

3. Use long division to find $53.8 \div 4$. Check your answer by multiplying it by the divisor 4.
4. Use long division to find $77.4 \div 5$. If you get stuck, you can draw diagrams or use another method.

Student Response

1.

	tens	ones	tenths	hundredths
group 1				
group 2				
group 3				
group 4				

2. 13.45. Each group has 1 ten, 3 ones, 4 tenths, and 5 hundredths.

3. $53.8 \div 4 = 13.45$ and $(13.45) \cdot 4 = 53.8$

$$\begin{array}{r}
 13.45 \\
 4 \overline{) 53.8} \\
 \underline{- 4} \\
 13 \\
 \underline{- 12} \\
 18 \\
 \underline{- 16} \\
 20 \\
 \underline{- 20} \\
 0
 \end{array}$$

4. 15.48. Sample reasonings:

o

$$\begin{array}{r}
 15.48 \\
 5 \overline{) 77.4} \\
 \underline{- 5} \\
 27 \\
 \underline{- 25} \\
 24 \\
 \underline{- 20} \\
 40 \\
 \underline{- 40} \\
 0
 \end{array}$$

- There are five groups of 15 in 77 with 2 left over. There are 0.4 groups of 5 in 2.4 with 0.4 remaining. There are 0.08 groups of 5 in 0.40, so 77.4 can be divided into five equal groups of 15.48.

Are You Ready for More?

A distant, magical land uses jewels for their bartering system. The jewels are valued and ranked in order of their rarity. Each jewel is worth 3 times the jewel immediately below it in the ranking. The ranking is red, orange, yellow, green, blue, indigo, and violet. So a red jewel is worth 3 orange jewels, a green jewel is worth 3 blue jewels, and so on.

A group of 4 craftsmen are paid 1 of each jewel. If they split the jewels evenly amongst themselves, which jewels does each craftsman get?

Student Response

Each craftsman gets 1 orange, 1 green, and 1 indigo jewel. Together the four craftsmen must share 1 violet jewel.

Activity Synthesis

Ask a student or two to display and explain their work. Ask if others performed the division the same way and if there are disagreements.

Then, focus the discussion on the connections between a division problem with a whole-number dividend (such as $62 \div 5$) and that with a decimal dividend (such as $53.8 \div 4$). Discuss:

- How is the division problem $53.8 \div 4$ similar to $62 \div 5$ from a previous lesson?
 - In both problems, when we get to the final place value (tenths for $53.8 \div 4$ and one for $62 \div 5$), there is still a remainder.
 - In both problems, to complete the division and find the quotient we need to introduce a new place value (hundredths for $53.8 \div 4$, and tenths for $62 \div 5$).
 - We have to unbundle at every step in both division problems.
- How is the division problem $53.8 \div 4$ different to $62 \div 5$ from a previous lesson?

- There is already a decimal in 53.8 : we had to write the decimal point for $62 \div 5$.
- The quotient $53.8 \div 4$ goes to the hundredths place, so there is an extra step and an additional place value.

If we were to rewrite $62 \div 5$ as $62.0 \div 5$ (which is what is needed in order to complete the division), then the two division problems look similar. The biggest difference between $53.8 \div 4$ and $62 \div 5$ is that the former problem has an answer in the hundredths while the answer to the latter only has tenths.

Access for English Language Learners

Representing, Speaking, Listening: MLR2 Collect and Display. As students discuss how the division problem $53.8 \div 4$ is related to $62 \div 5$ from a previous lesson, create a two-column display with “similarities” and “differences” for the headers. Circulate through the groups and record student language in the appropriate column. Look for phrases such as “whole-number dividend,” “decimal dividend,” and “place value.” This will help students compare and contrast different types of division problems by recognizing the structure of the numbers.

Design Principle(s): Support sense-making; Maximize meta-awareness

12.3 Dividends and Divisors

15 minutes

In this activity, students study some carefully chosen quotients where the dividends are decimal numbers. The key goal here is to notice that there are other quotients of whole numbers that are equivalent to these quotients of decimals. In other words, when the dividend is a terminating decimal number, we can find an equivalent quotient whose dividend is a whole number. In combination with the previous task, this gives students the tools they need to divide a decimal number by a decimal number.

This activity strongly supports MP7. Students notice that, when working with a fraction, multiplying the numerator and denominator in a fraction by 10 does not change the value of the fraction. They use this insight to develop a way to divide decimal numbers in subsequent activities. This work develops students’ understanding of equivalent expressions by emphasizing that, for example, $8 \div 1 = (8 \cdot 10) \div (1 \cdot 10)$. Eventually, students will recognize the equivalence of $8 \div 1$ to statements such as $(8 \cdot y) \div (1 \cdot y)$. However, in this activity, students only examine situations where the dividend and divisor are multiplied by powers of 10.

Building Towards

- 6.NS.B.2

Instructional Routines

- MLR7: Compare and Connect

Launch

Display the following image of division calculations for all to see.

$$\begin{array}{r} 8 \\ 1 \overline{) 8} \\ - 8 \\ \hline 0 \end{array} \qquad \begin{array}{r} 8 \\ 100 \overline{) 800} \\ - 800 \\ \hline 0 \end{array} \qquad \begin{array}{r} 8 \\ 10000 \overline{) 80000} \\ - 80000 \\ \hline 0 \end{array}$$

Ask students what quotient each calculation shows. ($8 \div 1$, $800 \div 100$, and $80,000 \div 10,000$). Give students 1–2 minutes to notice and wonder about the dividends, divisors, and quotients in the three calculations. Ask them to give a signal when they have at least one observation and one question. If needed, remind students that the 8, 800, and 80,000 are the dividends and the 1, 100, and 10,000 are the divisors.

Invite a few students to share their observations and questions. They are likely to notice:

- Each calculation shows that the value of the corresponding quotient is 8; it is the same for all three calculations.
- All calculations have an 8 in the dividend and a 1 in the divisor.
- All calculations take one step to solve.
- Each divisor is 100 times the one to the left of it.
- Each dividend is 100 times the one to the left of it.
- Each dividend and each divisor have 2 more zeros than in the calculation immediately to their left.

They may wonder:

- Why are the quotients equal even though the divisors and dividends are different?
- Would $80 \div 10$ and $8,000 \div 10$ also produce a quotient of 8?
- Are there other division expressions with an 8 in the dividend and a 1 in the divisor and no other digits but zeros that would also produce a quotient of 8?

Without answering their questions, tell students that they'll analyze the sizes of dividends and divisors more closely to help them reason about quotients of numbers in base ten.

Give students 7–8 minutes of quiet work time to answer the four questions followed by a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organization and problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.

Supports accessibility for: Organization; Attention

Student Task Statement

Analyze the dividends, divisors, and quotients in the calculations, and then answer the questions.


$$\begin{array}{r} \overline{24} \\ 3 \overline{) 72} \\ \underline{- 6} \\ 12 \\ \underline{- 12} \\ 0 \end{array}$$

$$\begin{array}{r} \overline{24} \\ 30 \overline{) 720} \\ \underline{- 60} \\ 120 \\ \underline{- 120} \\ 0 \end{array}$$

$$\begin{array}{r} \overline{24} \\ 300 \overline{) 7200} \\ \underline{- 600} \\ 1200 \\ \underline{- 1200} \\ 0 \end{array}$$

$$\begin{array}{r} \overline{24} \\ 3000 \overline{) 72000} \\ \underline{- 6000} \\ 12000 \\ \underline{- 12000} \\ 0 \end{array}$$

- Complete each sentence. In the calculations shown:
 - Each dividend is _____ times the dividend to the left of it.
 - Each divisor is _____ times the divisor to the left of it.
 - Each quotient is _____ the quotient to the left of it.
- Suppose we are writing a calculation to the right of $72,000 \div 3,000$. Which expression has a quotient of 24? Be prepared to explain your reasoning.
 - $72,000 \div 30,000$
 - $720,000 \div 300,000$
 - $720,000 \div 30,000$
 - $720,000 \div 3,000$
- Suppose we are writing a calculation to the left of $72 \div 3$. Write an expression that would also give a quotient of 24. Be prepared to explain your reasoning.
- Decide which of the following expressions would have the same value as $250 \div 10$. Be prepared to share your reasoning.
 - $250 \div 0.1$

- 
- b. $25 \div 1$
 - c. $2.5 \div 1$
 - d. $2.5 \div 0.1$
 - e. $2,500 \div 100$
 - f. $0.25 \div 0.01$

Student Response

1.
 - 10 times
 - 10 times
 - “equal to” or “the same size as”
2. c. $720,000 \div 30,000$
3. $7.2 \div 0.3$
4. B, D, E, and F

Activity Synthesis

Ask students to write a reflection using the following prompt:

What happens to the value of the quotient when both the divisor and the dividend are multiplied by the same power of 10? Use examples to show your thinking.

The goal of this discussion is to make sure students understand that the value of a quotient does not change when both the divisor and the dividend are multiplied by the same power of ten. Ask students to explain why $\frac{25}{20} = \frac{250}{200}$. Possible responses include:

- Both the numerator and denominator of $\frac{250}{200}$ have a factor of 10, so the fraction can be written as $\frac{25}{20}$.
- Both fractions are equivalent to $\frac{5}{4}$.
- Dividing 250 by 200 and 25 by 20 both give a value of 1.25.

Tell students that their observations here will help them divide decimals in upcoming activities.

Access for English Language Learners

Speaking, Listening: MLR7 Compare and Connect. After students have answered the four questions in this activity, ask them to work in groups of 2–4 and identify what is similar and what is different about the approaches they used in analyzing the dividends, divisors, and quotients for the last question. Lead a whole-class discussion that draws students attention to what worked or did not work well when deciding which expressions have the same value as $250 \div 10$ (i.e., using powers of 10, long division, unbundling, etc). Look for opportunities to highlight mathematical language and reasoning involving multiplying or dividing by powers of 10. This will foster students' meta-awareness and support constructive conversations as they develop understanding of equivalent quotients.

Design Principles(s): Cultivate conversation; Maximize meta-awareness

Lesson Synthesis

In this lesson, we saw that we can divide a decimal by a whole number the same way we divide two whole numbers. We used base-ten diagrams to show how this is true.

- How do we use base-ten diagrams to show division of a decimal by a whole number, for example, $0.8 \div 5$? (We can draw eight of a type of figures to represent 8 tenths and distribute them into 5 groups.)
- What do we do with remainders? For example, in the case of $0.8 \div 5$, how do we deal with the remainder of 3 tenths? (We can unbundle each tenth into 10 hundredths and distribute the 30 hundredths into 5 equal groups. Each group would have 6 hundredths.)
- How do we know what the quotient of $0.8 \div 5$ is? (It is the value of each group: 1 tenth and 6 hundredths, or 0.16.)

We also thought about division of decimals a different way—by multiplying both the dividend and divisor by the same power of 10, which gives us an equivalent division expression.

- What are some division expressions that are equivalent to $30 \div 0.1$? ($300 \div 1$; $3,000 \div 10$)
- What happens to the value of a quotient when both the divisor and the dividend are multiplied by the same power of 10? (The value does not change.)

12.4 The Same Quotient

Cool Down: 5 minutes

Addressing

- 6.NS.B.3

Student Task Statement

1. Use long division to find the value of $43.5 \div 3$. If you get stuck, you can draw base-ten diagrams. Be sure to say what each type of figure represents in your diagrams.
2. Explain why all of these expressions have the same value.

$$100 \div 5$$

$$10 \div 0.5$$

$$1 \div 0.05$$

Student Response

1. 14.5

$$\begin{array}{r} 14.5 \\ 3 \overline{) 43.5} \\ \underline{- 3} \\ 13 \\ \underline{- 12} \\ 15 \\ \underline{- 15} \\ 0 \end{array}$$

2. Answers vary. Sample reasonings:
- Each dividend is 20 times the divisor.
 - Each dividend is 10 times the next dividend. Each divisor is 10 times the next divisor. The quotient of each pair of numbers is therefore the same.

Student Lesson Summary

We know that fractions such as $\frac{6}{4}$ and $\frac{60}{40}$ are equivalent because:

- The numerator and denominator of $\frac{60}{40}$ are each 10 times those of $\frac{6}{4}$.
- Both fractions can be simplified to $\frac{3}{2}$.
- 600 divided by 400 is 1.5, and 60 divided by 40 is also 1.5.

Just like fractions, division expressions can be equivalent. For example, the expressions $540 \div 90$ and $5,400 \div 900$ are both equivalent to $54 \div 9$ because:

- They all have a quotient of 6.

- The dividend and the divisor in $540 \div 90$ are each 10 times the dividend and divisor in $54 \div 9$. Those in $5,400 \div 900$ are each 100 times the dividend and divisor in $54 \div 9$. In both cases, the quotient does not change.

This means that an expression such as $5.4 \div 0.9$ also has the same value as $54 \div 9$. Both the dividend and divisor of $5.4 \div 0.9$ are $\frac{1}{10}$ of those in $54 \div 9$.

In general, multiplying a dividend and a divisor by the same number does not change the quotient. Multiplying by powers of 10 (e.g., 10, 100, 1,000, etc.) can be particularly useful for dividing decimals, as we will see in an upcoming lesson.

Lesson 12 Practice Problems

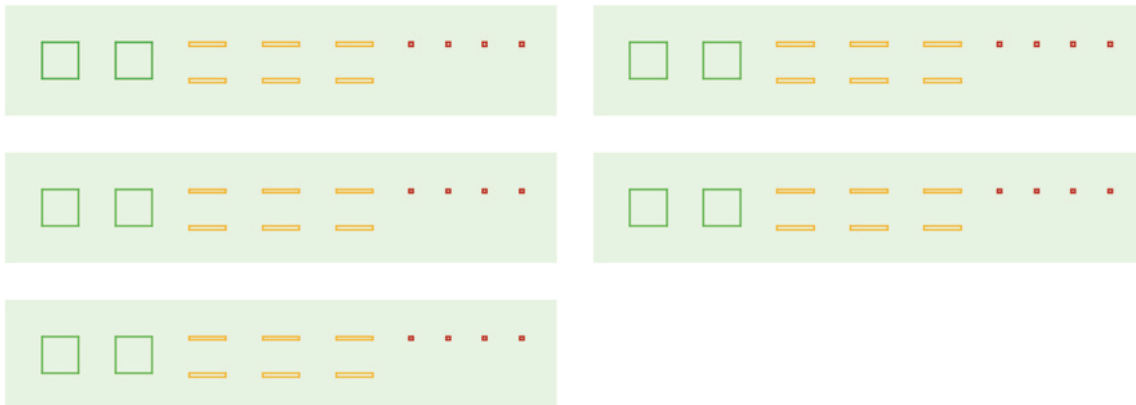
Problem 1

Statement

Here is a diagram representing a base-ten number. The large rectangle represents a unit that is 10 times the value of the square. The square represents a unit that is 10 times the value of the small rectangle.



Here is a diagram showing the number being divided into 5 equal groups.



- If a large rectangle represents 1,000, what division problem did the second diagram show? What is its answer?
- If a large rectangle represents 100, what division problem did the second diagram show? What is its answer?
- If a large rectangle represents 10, what division problem did the second diagram show? What is its answer?

Solution

- a. $1,320 \div 5$. The answer is 264.
- b. $132 \div 5$. The answer is 26.4.
- c. $13.2 \div 5$. The answer is 2.64.

Problem 2

Statement

- a. Explain why all of these expressions have the same value.

$$4.5 \div 0.09$$

$$45 \div 0.9$$

$$450 \div 9$$

$$4500 \div 90$$

- b. What is the common value?

Solution

- a. Answers vary. Sample response: The expressions all have the same value because the numerator and denominator are *both* being multiplied by 10 to get from one expression to the one above it. This does not affect the quotient (because dividing the 10 in the numerator by the 10 in the denominator results in 1).
- b. 50

Problem 3

Statement

Use long division to find each quotient.

$$a. 7.89 \div 2$$

$$a. 39.54 \div 3$$

$$a. 0.176 \div 5$$

Solution

- a. 3.945
- b. 13.18
- c. 0.0352

Possible calculations:

a.
$$\begin{array}{r} 3945 \\ 2 \overline{) 7890} \\ \underline{-6} \\ 18 \\ \underline{-18} \\ 09 \\ \underline{-8} \\ 10 \\ \underline{-10} \\ 0 \end{array}$$

b.
$$\begin{array}{r} 1486 \\ 3 \overline{) 3954} \\ \underline{-3} \\ 9 \\ \underline{-9} \\ 5 \\ \underline{-3} \\ 24 \\ \underline{-24} \\ 0 \end{array}$$

c.
$$\begin{array}{r} 00352 \\ 5 \overline{) 01760} \\ \underline{-015} \\ 26 \\ \underline{-25} \\ 10 \\ \underline{-10} \\ 0 \end{array}$$

Problem 4

Statement

Four students set up a lemonade stand. At the end of the day, their profit is \$17.52. How much money do they each have when the profit is split equally? Show or explain your reasoning.

Solution

\$4.38. Answers vary. Sample explanation: Four people are sharing \$17.52 equally, so each person gets $\$17.52 \div 4$. Each person can be given \$4, and then \$1.52 remains. Each person can be given \$0.30, and then \$0.32 remains. So they each get \$0.08 more. That means each person gets a total of $4 + 0.30 + 0.08$ or \$4.38.

Problem 5

Statement

- A standard sheet of paper in the United States is 11 inches long and 8.5 inches wide. Each inch is 2.54 centimeters. How long and wide is a standard sheet of paper in centimeters?
- A standard sheet of paper in Europe is 21.0 cm wide and 29.7 cm long. Which has the greater area, the standard sheet of paper in the United States or the standard sheet of paper in Europe? Explain your reasoning.

Solution

- 27.94 cm by 21.59 cm
- The European paper. Reasoning varies. Sample reasoning: The difference in the length is substantially larger than the difference in width, so the European paper probably has a larger

area. Calculating shows the standard European paper is 623.7 sq cm while the standard United States paper is 603.2246 sq cm.

(From Unit 5, Lesson 8.)

Lesson 13: Dividing Decimals by Decimals

Goals

- Compare and contrast (orally and using other representations) division problems with whole-number and decimal divisors.
- Divide whole numbers or decimals by decimals, and explain the reasoning (orally and using other representations), including choosing to divide a different expression that gets the same quotient.
- Generate another division expression that has the same value as a given expression, and justify (orally) that they are equal.

Learning Targets

- I can explain how multiplying dividend and divisor by the same power of 10 can help me find a quotient of two decimals.
- I can find the quotient of two decimals.

Lesson Narrative

In the previous lesson, students learned how to divide a decimal by a whole number. They also saw that multiplying both the dividend and the divisor by the same power of 10 does not change the quotient. In this lesson, students integrate these two understandings to find the quotient of two decimals. They see that to divide a number by a decimal, they can simply multiply both the dividend and divisor by a power of 10 so that both numbers are whole numbers. Doing so makes it simpler to use long division, or another method, to find the quotient. Students then practice using this principle to divide decimals in both abstract and contextual situations.

Alignments

Addressing

- 6.EE.A.4: Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.
- 6.NS.B.2: Fluently divide multi-digit numbers using the standard algorithm.
- 6.NS.B.3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Building Towards

- 6.NS.B.2: Fluently divide multi-digit numbers using the standard algorithm.

- 6.NS.B.3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Think Pair Share

Student Learning Goals

Let's divide decimals by decimals.

13.1 Same Values

Warm Up: 5 minutes

In this warm-up, students continue the decimal division work from the previous lesson and do so in the context of money. The work reinforces the idea that the value of a quotient does not change if the numerator and denominator are both multiplied by the same power of 10.

Addressing

- 6.EE.A.4

Launch

Give students a moment to read the first question and to estimate whether the quotient will be less than 1 or more than 1. Ask them to give a signal when they have an estimate and can explain it. Ask one student from the “more than 1” group to explain their reasoning and another from the “less than 1” group to do the same. Clarify that the quotient will be less than 1 and give students a few minutes to complete the questions. If time is limited, ask students to work only on the second question. Follow with a whole-class discussion.

Student Task Statement

1. Use long division to find the value of $5.04 \div 7$.

2. Select **all** of the quotients that have the same value as $5.04 \div 7$. Be prepared to explain how you know.

- a. $5.04 \div 70$
- b. $50.4 \div 70$
- c. $504,000 \div 700$
- d. $504,000 \div 700,000$

Student Response

1. 0.72. Student work should show long division.
2. B and D. Sample reasoning:
 - Long division can be used to divide 50.4 by 70 and still get 0.72.
 - $5.04 \div 7 = 50.4 \div 70$ because 5.04 and 7 can be multiplied by 10 to get 50.4 and 70. Multiplying both numbers by the same non-zero factor does not change the quotient.
 - 504,000 is 5.04 groups of 100,000, and 700,000 is 7 groups of 100,000. Since the unit of the groups is the same (100,000), examining how many times 7 goes into 5.04, which could simply be represented with $5.04 \div 7$, gives the answer.

Activity Synthesis

Focus the whole-class discussion on the second question. Select several students to explain why choices b and d are correct and why a and c are not. Students should see that $5.04 \div 70$ and $504,000 \div 700$ are not equivalent expressions to $5.04 \div 7$ because the dividend and divisors in each pair are *not* results of multiplying the 5.04 and 7 by the same factor or the same power of 10.

13.2 Placing Decimal Points in Quotients

15 minutes

The goal of this task is to show that we can calculate quotients of two decimals by “moving the decimal point” (multiplying both numbers by an appropriate power of 10) and, as a result, work only with whole numbers. Students can calculate the quotient of whole numbers using long division or another method of their choice. Students also have an opportunity to evaluate and critique another’s reasoning (MP3).

Students use the structure of base-ten numbers (MP7) to move the decimal point (through multiplication by an appropriate power of 10), and they use their understanding of equivalent expressions to know that multiplying both the numbers in a division by the same factor does not change the value of the quotient. Both pieces of knowledge allow students to replace a quotient of decimal numbers with a quotient of whole numbers.

Building Towards

- 6.NS.B.2
- 6.NS.B.3

Instructional Routines

- MLR7: Compare and Connect
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 3 minutes of quiet time to consider how to find the first quotient. Encourage them to think of more than one way to do so, if possible. Then, give partners 2–3 minutes to discuss their methods and another 2–3 minutes to find the second quotient together. Follow with a brief whole-class discussion, reviewing the first two questions. If not brought up by a student, discuss the equivalent expressions $300 \div 12$ and $1,800 \div 4$. Consider bringing up the first expression and asking students to find an analogous expression for the second problem.

Ask students to finish the last problem and follow with a whole-class discussion.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. For example, remind students about the quotients that have the same value as $5.04 \div 7$ in the previous activity. Explain to students that multiplying both the numbers in a division by the same factor does not change the value of the quotient. Ask students how they can use this idea to find a quotient that has the same value as $3 \div 0.12$.

Supports accessibility for: Social-emotional skills; Conceptual processing

Access for English Language Learners

Speaking, Listening: MLR7 Compare and Connect. After students have 3 minutes of quiet time to consider how to find the first quotient, ask them to create a visual display that shows their strategy and a brief explanation. Give students time to meet with 2–3 partners, to share discuss connections they notice between their different approaches. Follow up with a whole-class discussion to identify and highlight correspondences between different approaches or representations you observe in the room. Listen for and amplify key phrases such as “multiply both numbers by 10,” “move the decimal point,” “use whole numbers,” or “create an equivalent expression.” This will help students make sense of mathematical strategies by relating and connecting other approaches to their own.

Design Principles(s): Cultivate conversation; Maximize meta-awareness

Student Task Statement

1. Think of one or more ways to find $3 \div 0.12$. Show your reasoning.
2. Find $1.8 \div 0.004$. Show your reasoning. If you get stuck, think about what equivalent division expression you could write.
3. Diego said, "To divide decimals, we can start by moving the decimal point in both the dividend and divisor by the same number of places and in the same direction. Then we find the quotient of the resulting numbers."

Do you agree with Diego? Use the division expression $7.5 \div 1.25$ to support your answer.

Student Response

1. $3 \div 0.12 = 25$. Reasonings vary. Sample reasonings:
 - $10 \cdot (0.12) = 1.2$ and $20 \cdot (0.12) = 2.4$. These mean there are at least 20 groups of 0.12 in 3. The value 3 is 0.6 away from 2.4. There are 5 groups of 0.12 in 0.6 (i.e., $5 \cdot (0.12) = 0.6$). So in total, there are $(20 + 5)$ or 25 groups of 0.12 in 3.
 - $3 \div 12$ is $\frac{1}{4}$ or 0.25. Because the divisor 0.12 is a hundredth of 12, the quotient $3 \div 0.12$ must be 100 times 0.25, which is 25.
 - 0.12 can be written as $\frac{12}{100}$, so the division can be written as $3 \div \frac{12}{100}$, which equals $3 \cdot \frac{100}{12}$ or $\frac{300}{12}$. The quotient is 25 because $300 \div 12 = 25$.
 - We can multiply both 3 and 0.12 by 100 to get 300 and 12, and then simply find $300 \div 12$. Multiplying both the dividend and divisor by the same number does not change the quotient. $300 \div 12 = 25$.
2. $1.8 \div 0.004 = 450$. Reasonings vary. Sample reasonings:
 - 1.8 is $\frac{18}{10}$ and 0.004 is $\frac{4}{1,000}$. The quotient $\frac{18}{10} \div \frac{4}{1,000}$ can be found by multiplying $\frac{18}{10} \cdot \frac{1,000}{4}$, which equals $\frac{18,000}{40}$ or 450.
 - $1.8 \div 0.004$ is equivalent to $1,800 \div 4$, which is 450.
3. Agree. Sample explanation: We can multiply both 7.5 and 1.25 in $7.5 \div 1.25$ by 100 to have a whole-number dividend and divisor. Multiplying both by 100 moves the decimal point 2 places to the right, so 7.5 becomes 750 and 1.25 becomes 125. Then, we can divide 750 by 125.

Are You Ready for More?

Can we create an equivalent division expression by multiplying both the dividend and divisor by a number that is *not* a multiple of 10 (for example: 4, 20, or $\frac{1}{2}$)? Would doing so produce the same quotient? Explain or show your reasoning.

Student Response

Yes. Explanations vary. Sample response: A division expression such as $1.8 \div 0.004$ can be thought of as the fraction $\frac{1.8}{0.004}$. Equivalent fractions can be found by multiplying the numerator and denominator by the same number. This means that equivalent division expressions can be found by multiplying the dividend and divisor by the same number.

Activity Synthesis

The goal of this discussion is to help students recognize when division expressions are equivalent.

Ask students to write a division expression that looks like it might be equivalent to either $3 \div 0.12$ or $1.8 \div 0.004$ but has different decimal point locations. Select a few students to share their expression with the class.

13.3 Two Ways to Calculate Quotients of Decimals

Optional: 15 minutes

This lesson demonstrates how the division of two equivalent expressions (e.g., $48.78 \div 9$ and $4878 \div 900$) result in the same quotient. By looking at worked-out calculations, students reinforce their understanding about what each part of the calculations represent. The advantage of representing a quotient with a different equivalent expression is so students can simplify problems involving division of decimals by rewriting expressions using whole numbers.

They use the structure of base-ten numbers (MP7) to move the decimal point (through multiplication by an appropriate power of 10). They also use their understanding of equivalent expressions to know that multiplying both the numbers in a division by the same factor does not change the value of the quotient. Both pieces of knowledge allow students to replace a quotient of decimal numbers with a quotient of whole numbers.

Building Towards

- 6.NS.B.2
- 6.NS.B.3

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. Give partners 5 minutes to discuss the first problem and then quiet work time for the second problem. Follow with a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. To help get students started, display sentence frames such as “Both ____ and ____ are alike because...” and “____ and ____ are different because...”

Supports accessibility for: Language; Organization

Student Task Statement

1. Here are two calculations of $48.78 \div 9$. Work with your partner to answer the following questions.

$$\begin{array}{r} 5.42 \\ 9 \overline{) 48.78} \\ \underline{- 45} \\ 37 \\ \underline{- 36} \\ 18 \\ \underline{- 18} \\ 0 \end{array}$$

Calculation A

$$\begin{array}{r} 5.42 \\ 900 \overline{) 4878} \\ \underline{- 4500} \\ 3780 \\ \underline{- 3600} \\ 1800 \\ \underline{- 1800} \\ 0 \end{array}$$

Calculation B

- How are the two calculations the same? How are they different?
 - Look at Calculation A. Explain how you can tell that the 36 means “36 tenths” and the 18 means “18 hundredths.”
 - Look at Calculation B. What do the 3600 and 1800 mean?
 - We can think of $48.78 \div 9 = 5.42$ as saying, “There are 9 groups of 5.42 in 48.78.” We can think of $4878 \div 900 = 5.42$ as saying, “There are 900 groups of 5.42 in 4878.” How might we show that both statements are true?
- 2.
- Explain why $51.2 \div 6.4$ has the same value as $5.12 \div 0.64$.
 - Write a division expression that has the same value as $51.2 \div 6.4$ but is easier to use to find the value. Then, find the value using long division.

Student Response

- 1.
- Answers vary. Sample response: Both calculations have the same quotient. They both have the same non-zero digits in the dividend and divisor. In Calculation A, the decimal point in the dividend stays. In Calculation B, both the divisor and dividend have been

multiplied by 100 so that they are whole numbers. The decimal point in Calculation B is two places to the right of that in Calculation A.

b. The 3 is in the ones place and the 6 is in the tenths place, so 36 means 36 tenths. The 1 in 18 is in the tenths place, so it has the same value as 10 hundredths. The 8 is in the hundredth place. Together, they make 18 hundredths.

c. The 3600 represents 3600 tenths, and the 1800 represents 1800 hundredths.

d. Answers vary. Sample response:

■ We can multiply the number of groups and the total amount by 100.

$$9 \cdot (5.42) = 48.78 \text{ and } 900 \cdot (5.42) = 4878.$$

■ 900 is 100 times 9. So if 9 groups of 5.42 equal 48.78, then 900 groups of 5.42 equal 100 times 48.78, which is 4,878.

2. a. $51.2 = 10 \cdot (5.12)$ and $6.4 = 10 \cdot (0.64)$. Multiplying the dividend and the divisor by the same number (10) does not affect the value of the quotient.

b. Answers vary. Sample responses: $512 \div 64$ and $5,120 \div 640$. The long division should show a quotient of 8.

Activity Synthesis

The goal of this discussion is for students to contrast the two division methods used in the task. Discuss:

- Why is the value of $48.78 \div 9$ the same as the value of $4,878 \div 900$? (The numbers in the second expression are both multiplied by 100, and this does not change the value of the quotient.)
- What are some advantages of calculating $48.78 \div 9$ with the decimal intact (the method on the left)? (It is fast, and we don't need to deal with a bunch of 0's. Also, if the numbers are from a contextual problem, we could better make meaning of them in their original form.)
- What are some advantages of calculating $4,878 \div 900$ with long division? (These are whole numbers, and we are familiar with how to divide whole numbers. Also, we could express this as a fraction and write an equivalent fraction of $\frac{543}{100}$, which then tells us that its value is 5.43.)

End the discussion by telling students that they will next look at quotients where both the divisor and the dividend are decimals. The method used here of multiplying both numbers by a power of 10 will apply in that situation as well.

Access for English Language Learners

Speaking: MLR8 Discussion Supports. As students discuss contrasts between the two calculations of $48.78 \div 9$, press for details and mathematical language in their explanations. Encourage students to elaborate on the idea that multiplying the dividend and divisor by the same factor does not change the value of the quotient. Provide these sentence frames to support the discussion: “The values are the same because”, “I can use powers of 10 by”, “Some advantages of Calculation A/Calculation B are”. This will help students make sense of the structure of base-ten numbers to move the decimal point through multiplication by an appropriate power of 10.

Design Principles(s): Support sense-making

13.4 Practicing Division with Decimals

15 minutes

In this activity, students practice calculating quotients of decimals by using any method they prefer. Then, they extend their practice to calculate the division of decimals in a real-world context. While students could use ratio techniques (e.g., a ratio table) to answer the last question, encourage them to use the division of decimal numbers. The application of division to solve real-world problems illustrates MP4.

As students work on the first three problems, monitor for groups in which students have different strategies used on the same question.

Addressing

- 6.NS.B.2
- 6.NS.B.3

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Arrange students in groups of 3–5. Give groups 5–7 minutes to work through and discuss the first three questions. Ask them to consult with you if there is a disagreement about a correct answer in their group. (If this happens, let them know which student’s work is correct and have that student explain their thinking so all group members are in agreement.)

After all group members have answered the first three questions and have the same answer, have them complete the last question. Follow with a whole-class discussion.

Anticipated Misconceptions

Some students might have trouble calculating because their numbers are not aligned so the place-value associations are lost. Suggest that they use graph paper for their calculations. They can place one digit in each box for proper decimal point and place-value alignment.

Student Task Statement

Find each quotient. Discuss your quotients with your group and agree on the correct answers. Consult your teacher if the group can't agree.

- $106.5 \div 3$
- $58.8 \div 0.7$
- $257.4 \div 1.1$
- Mai is making friendship bracelets. Each bracelet is made from 24.3 cm of string. If she has 170.1 cm of string, how many bracelets can she make? Explain or show your reasoning.

Student Response

(Long division calculations for questions 1–3 are shown after question 3.)

- 35.5.
84. The dividend and divisor in $58.8 \div 0.7$ can each be multiplied by 10 to get 588 and 7. The quotient $58.8 \div 0.7$ has the same value as the quotient $588 \div 7$, which equals 84.
234. The dividend and divisor can be multiplied by 10 to get 2,574 and 11. The quotient $257.4 \div 1.1$ has the same value as the quotient $2,574 \div 11$, which equals 234.

1.

$$\begin{array}{r} 35.5 \\ 3 \overline{) 106.5} \\ \underline{- 9} \\ 16 \\ \underline{- 15} \\ 15 \\ \underline{- 15} \\ 0 \end{array}$$

2.

$$\begin{array}{r} 84 \\ 7 \overline{) 588} \\ \underline{- 56} \\ 28 \\ \underline{- 28} \\ 0 \end{array}$$

3.

$$\begin{array}{r} 234 \\ 11 \overline{) 2574} \\ \underline{- 22} \\ 37 \\ \underline{- 33} \\ 44 \\ \underline{- 44} \\ 0 \end{array}$$

- She can make 7 bracelets. $170.1 \div 24.3$ is equivalent to $1,701 \div 243$, which is 7.

Activity Synthesis

The purpose of this discussion is to highlight the different strategies used to answer the division questions. Select a previously identified group that used different strategies on one of the first three questions. Ask each student in the group to explain their strategy and why they chose it. For the fourth question, ask students:

- “Could the answer be found by calculating the quotient of this expression: $24.3 \div 170.1$?” (No, because the question is asking how many pieces of string of length 24.3 are in the long string of length 170.1. This is equivalent to asking how many groups of 24.3 are in 170.1 or $170.1 \div 24.3$.)
- “What would the quotient $24.3 \div 170.1$ represent in the context of the problem?” (This quotient would represent what fraction the length of bracelet string is of the full length of string.)

Access for English Language Learners

Writing, Speaking, Conversing: MLR3 Clarify, Critique, and Correct. Before students share their answers, present an incorrect solution that uses long division to find the quotient of $106.5 \div 3$. Consider using this statement to open the discussion: “Sam believes the quotient of $106.5 \div 3$ is 355 because he multiplied 106.5 by 10 and then got his answer.” Ask pairs to identify the ambiguity or error, and critique the reasoning presented in the statement. Guide discussion by asking, “What do you think Sam was thinking when performing this division problem?”, “What strategy did he use to find the quotient?” and/or “What is unclear?” Invite pairs to offer a response that includes a correct version of Sam’s long division (e.g., the long division work for $1,065 \div 30$). This will help students explain how to use multiplication by powers of 10 in both the divisor and dividend to create equivalent expressions.

Design Principle(s): Maximize meta-awareness; Cultivate conversation

Lesson Synthesis

In this lesson, we saw that we can divide decimals by decimals by first making the decimals whole numbers. As long as we multiply both numbers by the same number, the value of the quotient will not change.

- What equivalent expression can we write to help us find $18.4 \div 0.2$? (We can multiply both numbers by 10 to get $184 \div 2$, which is equivalent to the original expression.)
- How might we find the quotient of $184 \div 2$? (We can use any methods learned so far: base-ten diagrams, partial quotients, or long division.)
- Do we always multiply the dividend and divisor by 10? For example, what number should we multiply to enable us to find $1.25 \div 0.005$? (We can multiply by any power of 10. In this example, we should multiply both numbers by 1,000 to turn the 0.005 into 5, so that we can find $1,250 \div 5$.)
- Why is it helpful to multiply by a power of 10 instead of another number that is not a power of 10? (Because we are working with base-ten numbers, multiplying by a power of 10 allows us to easily “remove” the decimal point from a decimal so that we end up with a whole number.)

13.5 The Quotient of Two Decimals

Cool Down: 5 minutes

Building Towards

- 6.NS.B.2
- 6.NS.B.3

Student Task Statement

1. Write two division expressions that have the same value as $36.8 \div 2.3$.
2. Find the value of $36.8 \div 2.3$. Show your reasoning.

Student Response

1. Answers vary. Sample responses: $3.68 \div 0.23$ and $368 \div 23$.
2. 16. Sample reasoning:

$$\begin{array}{r} 16 \\ 23 \overline{) 368} \\ \underline{- 23} \\ 138 \\ \underline{- 138} \\ 8 \\ \underline{- 8} \\ 0 \end{array}$$

Student Lesson Summary

One way to find a quotient of two decimals is to multiply each decimal by a power of 10 so that both products are whole numbers.

If we multiply both decimals by the same power of 10, this does not change the value of the quotient. For example, the quotient $7.65 \div 1.2$ can be found by multiplying the two decimals by 10 (or by 100) and instead finding $76.5 \div 12$ or $765 \div 120$.

To calculate $765 \div 120$, which is equivalent to $76.5 \div 12$, we could use base-ten diagrams, partial quotients, or long division. Here is the calculation with long division:

$$\begin{array}{r}
 6.375 \\
 120 \overline{) 765} \\
 \underline{- 720} \\
 450 \\
 \underline{- 360} \\
 900 \\
 \underline{- 840} \\
 600 \\
 \underline{- 600} \\
 0
 \end{array}$$

Lesson 13 Practice Problems

Problem 1

Statement

A student said, "To find the value of $109.2 \div 6$, I can divide 1,092 by 60."

- Do you agree with her? Explain your reasoning.
- Calculate the quotient of $109.2 \div 6$ using any method of your choice.

Solution

- Yes. Reasoning varies. Sample reasoning: As long as both dividend and divisor are multiplied by the same power of 10 (or just the same non-zero number), the quotient has the same value.
- 18.2. Methods vary. Sample response: 109.2 (dividend) and 6 (divisor) can be multiplied by 10 to get $1,092 \div 60$. The value of this quotient is 18.2.

Problem 2

Statement

Here is how Han found $31.59 \div 13$:

$$\begin{array}{r}
 2.43 \\
 13 \overline{) 31.59} \\
 \underline{- 26} \\
 55 \\
 \underline{- 52} \\
 39 \\
 \underline{- 39} \\
 0
 \end{array}$$

- At the second step, Han subtracts 52 from 55. How do you know that these numbers represent tenths?
- At the third step, Han subtracts 39 from 39. How do you know that these numbers represent hundredths?
- Check that Han's answer is correct by calculating the product of 2.43 and 13.

Solution

- Explanations vary. Sample explanation: The second 5 of the 55 is written in the tenths column (directly under the tenths place of 31.59), so it represents 5 tenths. The first 5 of 55 is written in the ones column (directly under the ones place of 31.59), so it represents 5 ones, which is 50 tenths. So, the 55 represents 55 tenths. The 2 of 52 is written in the tenths column and the 5 is in the ones column, so the 52 represents 52 tenths.
- Explanations vary. Sample explanation: The 9 of 39 is written in the hundredths column (directly under the hundredths place of 31.59), so it represents 9 hundredths. The 3 of 39 is written in the tenths column (directly under the tenths place of 31.59), so it represents 3 tenths, which is 30 hundredths. So, the 39 represents 39 hundredths.
- $(2.43) \cdot 13 = 31.59$, so Han is correct. Calculations vary. Sample calculation: $(2.43) \cdot 13 = (2.43) \cdot (10 + 3) = 24.3 + 6 + 1.29 = 31.59$.

Problem 3

Statement

- Write two division expressions that have the same value as $61.12 \div 3.2$.
- Find the value of $61.12 \div 3.2$. Show your reasoning.

Solution

- Answers vary. Sample responses: $611.2 \div 32$ and $6.112 \div 0.32$.
- 19.1 Reasoning varies. Sample reasoning:

$$\begin{array}{r}
 19.1 \\
 32 \overline{) 611.2} \\
 \underline{- 320} \\
 291 \\
 \underline{- 288} \\
 32 \\
 \underline{- 32} \\
 0
 \end{array}$$

Problem 4

Statement

A bag of pennies weighs 5.1 kilograms. Each penny weighs 2.5 grams. About how many pennies are in the bag?

- A. 20
- B. 200
- C. 2,000
- D. 20,000

Solution

C

Problem 5

Statement

Find each difference. If you get stuck, consider drawing a diagram.

$2.5 - 1.6$

$0.72 - 0.4$

$11.3 - 1.75$

$73 - 1.3$

Solution

a. $2.5 - 1.6 = 0.9$

$$\begin{array}{r}
 1 \quad 15 \\
 \cancel{2}.\cancel{5} \\
 \underline{- 1.6} \\
 0.9
 \end{array}$$

b. $0.72 - 0.4 = 0.32$

$$\begin{array}{r} 0.72 \\ - 0.40 \\ \hline 0.32 \end{array}$$

c. $11.3 - 1.75 = 9.55$

$$\begin{array}{r} 1012 \\ 0210 \\ 1130 \\ - 175 \\ \hline 9.55 \end{array}$$

d. $73 - 1.3 = 71.7$

$$\begin{array}{r} 210 \\ 730 \\ - 13 \\ \hline 71.7 \end{array}$$

(From Unit 5, Lesson 3.)

Problem 6

Statement

Plant B is $6\frac{2}{3}$ inches tall. Plant C is $4\frac{4}{15}$ inches tall. Complete the sentences and show your reasoning.

- Plant C is _____ times as tall as Plant B.
- Plant C is _____ inches _____ (taller or shorter) than Plant B.

Solution

- Plant C is $\frac{16}{25}$ times as tall as Plant B. Plant B is $6\frac{2}{3}$ inches and Plant C is $4\frac{4}{15}$ inches tall.

Reasoning varies. Sample reasoning: $4\frac{4}{15} \div 6\frac{2}{3} = \frac{64}{15} \div \frac{20}{3} = \frac{64}{15} \cdot \frac{3}{20} = \frac{16}{25}$

- Plant C is $2\frac{2}{5}$ inches shorter than Plant B. $(6\frac{2}{3} - 4\frac{4}{15} = 6\frac{10}{15} - 4\frac{4}{15} = 2\frac{6}{15} = 2\frac{2}{5})$

(From Unit 4, Lesson 12.)

Problem 7

Statement

At a school, 460 of the students walk to school.

- a. The number of students who take public transit is 20% of the number of students who walk. How many students take public transit?
- b. The number of students who bike to school is 5% of the number of students who walk. How many students bike to school?
- c. The number of students who ride the school bus is 110% of the number of students who walk. How many students ride the school bus?

Solution

- a. 92 students ($460 \cdot 0.2 = 92$)
- b. 23 students ($460 \cdot 0.05 = 23$)
- c. 506 students ($460 \cdot 1.10 = 506$)

(From Unit 3, Lesson 15.)

Lesson 14: Using Operations on Decimals to Solve Problems

Goals

- Apply operations with decimals to solve problems about the dimensions of a sports field or court, and explain (orally, in writing, and using other representations) the solution method.
- Choose whether an exact answer or an estimate is appropriate for a given problem.
- Interpret a verbal description or diagram that represents the dimensions of a sports field or court.

Learning Targets

- I can use addition, subtraction, multiplication, and division on decimals to solve problems.

Lesson Narrative

In this lesson, students apply their knowledge of operations on decimals to two sporting contexts. They analyze the distance between hurdles in a 110-meter hurdle race. In this situation, students use the given context to determine which arithmetic operations are relevant and use them to solve the problems. Additionally, they draw or use a diagram to help them make sense of the measurements, as well as to communicate their reasoning about the measurements (MP3). The numbers used in the problems reflect measurements that can be accurately measured on site, so the decimals can all be calculated by hand.

Alignments

Building On

- 5.OA.A.2: Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7, then multiply by 2” as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.

Addressing

- 6.NS.B.2: Fluently divide multi-digit numbers using the standard algorithm.
- 6.NS.B.3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Instructional Routines

- Group Presentations
- MLR2: Collect and Display
- MLR5: Co-Craft Questions

- MLR6: Three Reads
- Think Pair Share

Required Materials

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Student Learning Goals

Let's solve some problems using decimals.

14.1 Close Estimates

Warm Up: 5 minutes

The purpose of this warm-up is to encourage students to think about the reasonableness of a quotient by looking closely at the values of the dividend and divisor. The digits in the answer choices reflect those in the actual answer. While some students may try to mentally evaluate each one precisely, encourage them to think about their answer in relation to the numbers in the expressions. For each expression, ask students if the actual answer would be greater than or less than their estimate and how they know.

Building On

- 5.OA.A.2

Launch

Display one expression at a time, or ask students to work on one expression at a time and begin when cued. Give students 1 minute of quiet think time per question and ask them to give a signal when they have an answer and can explain their strategy. Follow each question with a brief whole-class discussion.

Discuss each problem one at a time with this structure:

- Ask students to indicate which option they chose.
- If everyone agrees on one answer, ask a few students to share their reasoning. Record and display the explanations for all to see. If there is disagreement on an answer, ask students with opposing answers to explain their reasoning to come to an agreement on an answer.

Student Task Statement

For each expression, choose the best estimate of its value.

1. $76.2 \div 15$

- ☐ 0.5
- ☐ 5
- ☐ 50

2. $56.34 \div 48$

- ☐ 1
- ☐ 10
- ☐ 100

3. $124.3 \div 20$

- ☐ 6
- ☐ 60
- ☐ 600

Student Response

1. 5, because 76.2 is close to 75 and $15 \cdot 5 = 75$.
2. 1, because 48 and 56 are close to one another, so the answer must be close to 1.
3. 6, because 124.3 is close to 120, and 20 fits into 120 six times, so the quotient must be close to 6.

Activity Synthesis

Ask students to share their general strategies for these problems. To involve more students in the conversation, consider asking:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone solve the problem the same way but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to ____'s strategy?"
- "Do you agree or disagree? Why?"

14.2 Applying Division with Decimals

Optional: 15 minutes

In this activity, students practice calculating quotients of decimals by multiplying both the divisor and the dividend by an appropriate power of 10. Then, they extend their practice to calculate quotients of decimals in real-world contexts. Problem A reiterates earlier experiences with the “how much is in each group” and “how many groups” interpretations of division. Problem B recalls students’ prior work with ratios and determining rate of speed. While students can use ratio techniques (e.g. a ratio table) to answer these questions, encourage them to use division of decimals. The application of division to solve real world problems illustrates MP4.

Addressing

- 6.NS.B.2
- 6.NS.B.3

Instructional Routines

- Group Presentations
- MLR2: Collect and Display

Launch

Arrange students in groups of 3–5. Assign each group Problem A or B and have students circle the problem they are assigned. Give groups 5–7 minutes to work on their assigned problem. If time permits, consider giving students access to tools for creating a visual display. Have them create a simple visual display to showcase their solutions and prepare a short presentation in which they explain their reasoning and calculations.

Give students 2–3 minutes to review one another’s work followed by groups’ presentations of their displays.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Provide prompts, reminders, guides, rubrics, or checklists that focus on increasing the length of on-task orientation in the face of distractions. For example, provide a rubric or checklist to ensure the contents of the visual display are made explicit.

Supports accessibility for: Attention; Social-emotional skills

Access for English Language Learners

Speaking: MLR2 Collect and Display. Use this routine to capture the language students use as they calculate quotients of decimals in real-world contexts. Circulate and listen to students talk during small-group and whole-class discussion. Record the words, phrases, and writing students use to describe their strategies. Capture student language that reflects a variety of ways to show how they are making sense of each problem such as, “how many in each group”, “how many groups” or “multiply by the same factor.” Display for the whole class to see and to use as a reference throughout the lesson. This will help students develop meta-awareness of the language of division.

Design Principle(s): Support sense-making; Maximize meta-awareness

Student Task Statement

Your teacher will assign to you either Problem A or Problem B. Work together as a group to answer the questions. Be prepared to create a visual display to show your reasoning with the class.

Problem A:

A piece of rope is 5.75 meters in length.

1. If it is cut into 20 equal pieces, how long will each piece be?
2. If it is cut into 0.05-meter pieces, how many pieces will there be?

Problem B:

A tortoise travels 0.945 miles in 3.5 hours.

1. If it moves at a constant speed, how many miles per hour is it traveling?
2. At this rate, how long will it take the tortoise to travel 4.86 miles?



Student Response

Problem A:

1. 0.2875 meters. Each piece of rope is $5.75 \div 20$ meters long. This is a “how many in each group” division problem with the groups being the pieces of rope and the “how much in each group” being the length of the rope.

2. 115. There are $5.75 \div 0.05$ pieces of rope. This is a “how many groups” division problem with the groups being the pieces of rope and the “how much in each group” being the length of the rope.

Problem B:

1. 0.27 miles per hour. The tortoise traveled 0.945 miles in 3.5 hours. To find the speed, divide the distance by the number of hours, 3.5. The speed is given by $0.945 \div 3.5$, which is the same value as $9.45 \div 35$.
2. 18 hours, $4.86 \div 0.27 = 18$

Activity Synthesis

Have each group post their display and read aloud each problem to the class. Then give all students 2–3 minutes to circulate and look at the displays. Ask them to think about how the work on each display is similar to or different from their own group’s work. Afterwards, ask all groups to share their display. As each group presents, be sure they explain how they approached the problems and why these problems apply decimal division.

For Problem A ask:

- “Does the problem use the ‘how many groups’ or ‘how many in each group’ interpretation of division? How do you know?”

For Problem B ask:

- “What is the ratio relationship?” (Speed is the ratio to distance to time.) “If you know the rate per one hour, how can you determine the time it will take to travel any given distance?” (Divide the distance by the unit rate.)

After all groups present, ask:

- “Did groups that solved the same problem approach it in the same way? If not, how did strategies differ?”
- “What methods did students use for calculating quotients of decimals?”

14.3 Distance between Hurdles

20 minutes

Sports provide many good contexts for doing arithmetic. This is the first of two activities using a sporting context. Students use arithmetic with decimals to study the 110-meter hurdle race. The first question prompts students to draw a diagram to capture and make sense of all of the given information. The second prompts them to find the distance between hurdles.

As students work, find a variety of work samples (particularly ones that make use of a drawing) to share with the class during the discussion.

Addressing

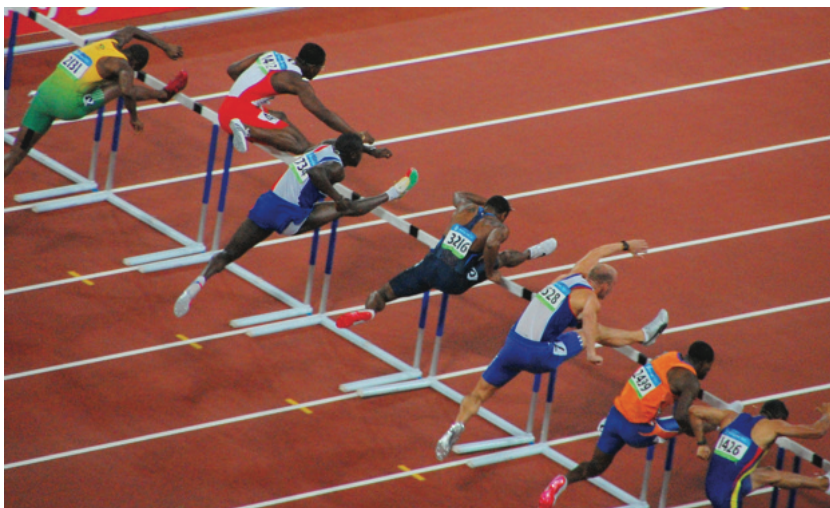
- 6.NS.B.3

Instructional Routines

- MLR6: Three Reads

Launch

Give a brief overview of hurdle races. Ask students if they have had the chance to watch track and field competitions. Have them describe what a hurdle race is. Display images such as the following or show a short video of a hurdle race.



Tell students that they will now use what they have learned about decimals to solve a couple of problems involving hurdles.

Arrange students in groups of 2. Give students about 5–6 minutes to draw a diagram for the first question and discuss their drawing with their partner. Then, give them another 5–6 minutes to complete the other two questions. Follow with a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support development of organizational skills, check in with students within the first 2–3 minutes of work time. Check to make sure students have calculated the correct distance between the first and last hurdles. Also, check to make sure students understand that there are only 9 spaces between 10 hurdles prior to calculating the distance between the hurdles.

Supports accessibility for: Memory; Organization

Access for English Language Learners

Reading, Listening, Conversing: MLR6 Three Reads. Use this routine to support reading comprehension of this problem, without solving it for students. In the first read, students read the problem with the goal of understanding the situation (e.g., equally-spaced hurdles on a race track). After the second read, ask students to identify the quantities that can be used or measured (e.g., number of equally-spaced hurdles, length of the first and final hurdles, length of the race track, etc.). After the third read, ask students to discuss possible strategies, referencing the relevant quantities identified in the second read. This will help students make sense of the problem context before being asked to solve it.

Design Principle(s): Support sense-making; Maximize meta-awareness

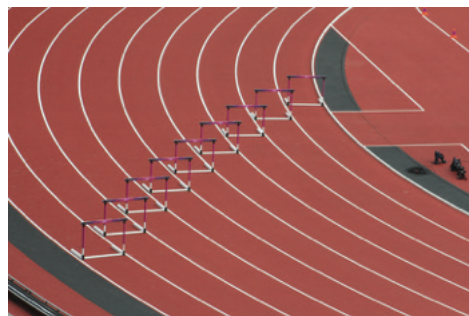
Anticipated Misconceptions

Students may not realize that there are only 9 spaces—not 10 spaces— between 10 hurdles, leading them to miscalculate the distance between hurdles. Have students study the number of spaces in their diagram, or ask them to think about how many spaces are between 2 hurdles, 3 hurdles, 4 hurdles, etc. and extend the pattern to 10 hurdles.

A calculation error in dividing may lead to a quotient with a non-terminating decimal. Look out for arithmetic errors when students calculate the distance between the first and last hurdles (82.26 meters) and when students perform division. If students end up with a non-terminating decimal for their answer, ask them to revisit each step and see where an error might have occurred.

Student Task Statement

There are 10 equally-spaced hurdles on a race track.
The first hurdle is 13.72 meters from the start line.
The final hurdle is 14.02 meters from the finish line.
The race track is 110 meters long.

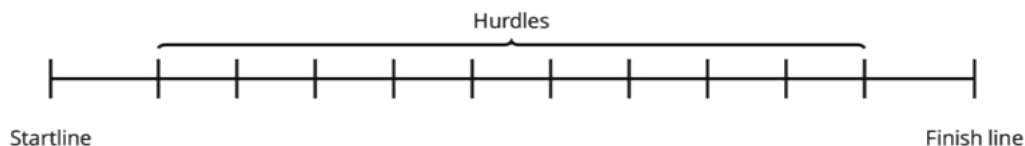


1. Draw a diagram that shows the hurdles on the race track. Label all known measurements.
2. How far are the hurdles from one another? Explain or show your reasoning.
3. A professional runner takes 3 strides between each pair of hurdles. The runner leaves the ground 2.2 meters *before* the hurdle and returns to the ground 1 meter *after* the hurdle.

About how long are each of the runner's strides between the hurdles? Show your reasoning.

Student Response

1. Diagrams vary. Sample diagram:



2. 9.14 meters. Sample reasoning: The distance between the first and last hurdles, in meters, is $110 - 13.72 - 14.02 = 82.26$. Since they are equally spaced, the ten equally spaced hurdles divide the track into 9 equal parts between the first and last hurdles. The distance between them is $82.26 \div 9 = 9.14$ meters.
3. 1.98 meters (or about 2 meters). Sample reasoning: There are 9.14 meters between the hurdles. The runner comes down 1 meter beyond the previous hurdle and takes off 2.2 meters before the following hurdle so that means the 3 strides cover a distance of $9.14 - 1 - 2.2 = 5.94$ meters. Each of the three strides covers $5.94 \div 3 = 1.98$ meters, just under 2 meters.

Activity Synthesis

Select one or more previously identified students to share their diagrams for the first question. Ask other students if theirs are comparable to these, and if not, where differences exist. If not mentioned by students, be sure to highlight proper labeling of the parts of the diagram. Then, discuss the second and third questions. For the second question, discuss:

- How many 'gaps' are there between the hurdles? (9)
- What is the distance from the first hurdle to the last hurdle? (82.26 meters)
- What arithmetic operation is applied to the two numbers, 9 and 82.26? Why? (Division, because the 10 equally spaced hurdles divide 82.26 meters into 9 equal groups.)

For the third question, discuss:

- How far does the runner go in three strides? How do you know?
- Is 1.98 meters (the exact answer) an appropriate answer for the question? Why or why not? (Most likely not, because the runner is not going to control their strides and jumps to the nearest centimeter. About 2 meters would be a more appropriate answer.)

Consider asking a general question about hurdle races: Is it important for the runner that the hurdles be placed as closely as possible to the correct location? The answer is yes, because runners train to take a precise number of strides and to hone their jumps to be as regular as possible. Moving a hurdle a few centimeters is unlikely to create a problem, but moving a hurdle by a meter would ruin the runners' regular rhythm in the race.

14.4 Examining a Tennis Court

Optional: 20 minutes

In this activity, students study the dimensions of a tennis court and apply their understanding of decimals to solve problems in another sporting context. This activity is optional, because it is an opportunity for students to further practice solving problems with decimals.

Visually, it appears as if each half of the tennis court (divided by the net) is a square. Similarly, it appears as if the service line is about halfway between the net and the baseline. Calculations show that in both cases, however, neither half of the tennis court is a square, and that the service line is not half way between the baseline and the net.

Just as with the distances between hurdles in the previous activity, the dimensions of a tennis court are very precisely determined. It is also very important for professional tennis players to regularly play on courts that have consistent dimensions, as the smallest differences could affect whether the ball is in or out. The third question gets at how small the lines are, compared to the full service box.

There are some subtleties in this task related to measurement in the real world and the idealized version in the task. On a tennis court, the lines have width. For the first two questions of the task, the strips can be taken as dimensionless lines. In the final questions, students deal explicitly with these strips.

Addressing

- 6.NS.B.3

Instructional Routines

- MLR5: Co-Craft Questions
- Think Pair Share

Launch

Give a brief introduction to tennis and tennis courts. Ask students if they play tennis or have ever watched a tennis match. Look at the picture of the tennis court and discuss the purpose of the boundary lines. Have students locate various sections of the court by pointing to rectangles, parallel segments, right angles, and the service box. (More detailed measurements for the parts of a tennis court can be found online.)

Keep students in groups of 2. Give students 2 minutes of quiet time to read and think about how they might approach the questions. Have them share their thinking with a partner for another 2 minutes. Then, give students 7–8 minutes to complete the questions. Follow with a whole-class discussion.

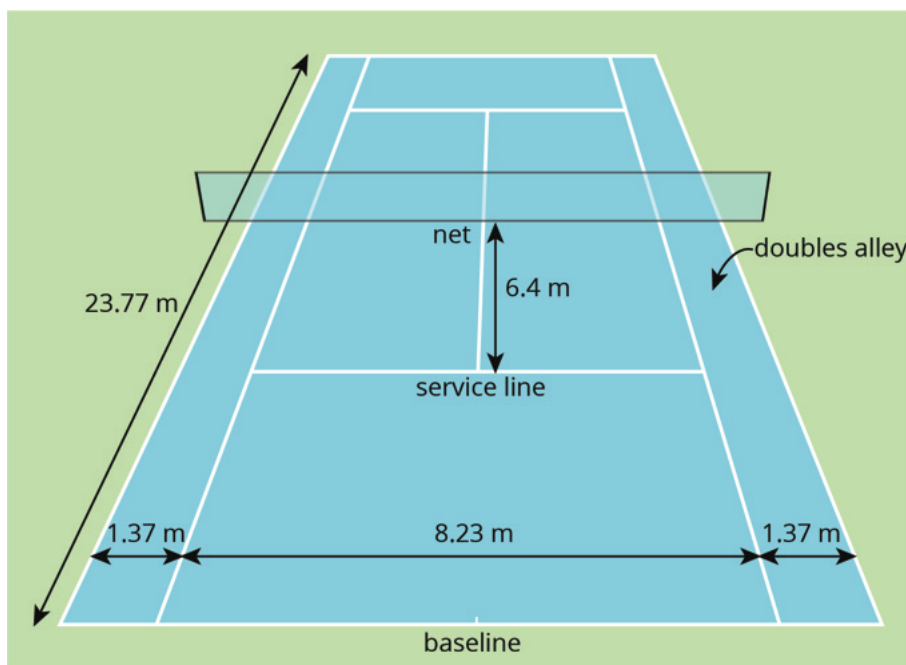
Access for English Language Learners

Writing, Speaking, Conversing: MLR5 Co-Craft Questions. Use this routine to provide students with an opportunity to develop questioning skills and understand the context of the situation. Begin by showing the diagram of the tennis court. Next, ask students to write down possible mathematical questions that might be asked about the diagram. These questions could include information that might be missing, or even assumptions that students think are important (e.g., each half looks like a square or distance between the service line and baseline). Next, invite students to compare the questions they generated with a partner before they share with the whole class. Finally, reveal the actual questions that students are expected to work on. This helps students develop their skills to generate questions and begin to comprehend the context of the problem before solving it.

Design Principle(s): Maximize meta-awareness; Cultivate conversation

Student Task Statement

Here is a diagram of a tennis court.



The full tennis court, used for doubles, is a rectangle. All of the angles made by the line segments in the diagram are right angles.

1. The net partitions the tennis court into two halves. Is each half a square? Explain your reasoning.
2. Is the service line halfway between the net and the baseline? Explain your reasoning.
3. Lines painted on a tennis court are 5 cm wide. A painter made markings to show the length and width of the court, then painted the lines to the outside of the markings.

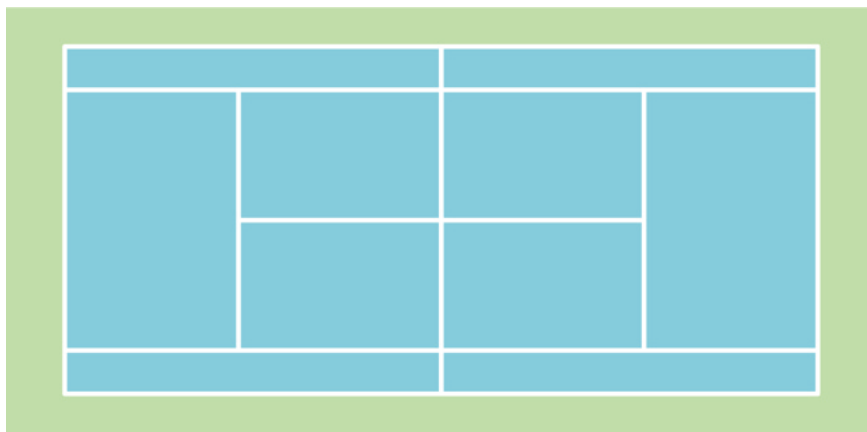
- a. Did the painter's mistake increase or decrease the overall size of the tennis court? Explain how you know.
- b. By how many square meters did the court's size change? Explain your reasoning.

Student Response

1. No, the tennis court is 23.77 meters long, and the net is in the middle. This means that each half of the court is $\frac{23.77}{2}$ meters long. This is 11.885 meters. Since the tennis court is only 10.97 meters wide, it is not a square.
2. No, the service line is 6.4 meters from the net. Since the length of the (half) court is 11.885 meters, this means that the service line is $11.885 - 6.4$ meters from the baseline. This is 5.485 meters. So the service line is almost a meter closer to the baseline than it is to the net.
3.
 - a. The painter's mistake made the court larger. The painter's outline of the court *begins* where the outline of the court is supposed to *end*.
 - b. The painter added two extra 23.77 m by 0.05 m strips and two 10.97 m by 0.05 m strips along the sides, and four 0.05 m by 0.05 m squares in the corners. All measurements are in meters. The two long strips make 2.377 square meters, while the two shorter strips add 1.097 square meters. The four small squares add another 0.01 square meters. This adds up to an extra 3.484 square meters.

Activity Synthesis

Here is a picture of the tennis court from directly above. In this picture we can see that the service box is not a square but it is difficult to determine, just by looking, whether or not each half of the court is a square:



On the other hand, it is possible to judge, from this picture, that the service line is closer to the baseline than it is to the net.

To reinforce these points, discuss:

- What do you look for to decide if you think a shape is a square? (4 equal sides, 4 right angles.)

- Can you tell by looking whether or not the service box is a square? (Yes, it looks significantly deeper than it is wide.)
- Can you tell by looking whether or not half of the full tennis court is a square? (Answers may vary, but it is close enough that it is not *clear* one way or another.)
- Would the painter's mistake change your answers to questions 1 or 2? (No, the sides of the quadrilaterals would all increase by 10 cm and they still would not be squares)
- Would you notice the painter's mistake if you were playing on the mis-painted court? (A professional player may notice, but an unseasoned player may not because 5 cm is very little compared to 11 meters and 24 meters.)

Lesson Synthesis

In this lesson, we used decimal operations to solve several problems involving measurements in sports. We noticed that diagrams can help us represent and communicate our mathematical thinking. We also saw that precision in our measurements and calculations matter.

- For which problems did you use a diagram? How did the diagram help you solve the problem?
- For which problems was it appropriate to round and estimate?
- For which problems was a precise answer necessary? For which ones was it not as critical to be precise?
- Find an example of how you applied each of the decimal operations to solve the problems in this lesson.

14.5 Middle School Hurdle Race

Cool Down: 5 minutes

Addressing

- 6.NS.B.3

Student Task Statement

Andre is running in an 80-meter hurdle race. There are 8 equally-spaced hurdles on the race track. The first hurdle is 12 meters from the start line and the last hurdle is 15.5 meters from the finish line.

1. Estimate how far the hurdles are from one another. Explain your reasoning.
2. Calculate how far the hurdles are from one another. Show your reasoning.

Student Response

1. Answers vary. Sample responses:
 - About 7 meters. The distance from the first hurdle to the last hurdle is about 50 meters, and there are 7 gaps between hurdles: $50 \div 7$ is about 7.

- Between 6 and 7 meters. This race is about $\frac{7}{10}$ as long as the 110-meter hurdle race, so the distance between the hurdles is about $\frac{7}{10}$ as much. In the 110-meter hurdle race, the hurdles are a little over 9 meters apart, so in the 80-meter race they are between 6 and 7 meters apart.
2. 7.5 meters. The distance, in meters, from the first to the last hurdle is $80 - 12 - 15.5 = 52.5$. There are 7 equal gaps between hurdles so each of these gaps, in meters, is $52.5 \div 7 = 7.5$.

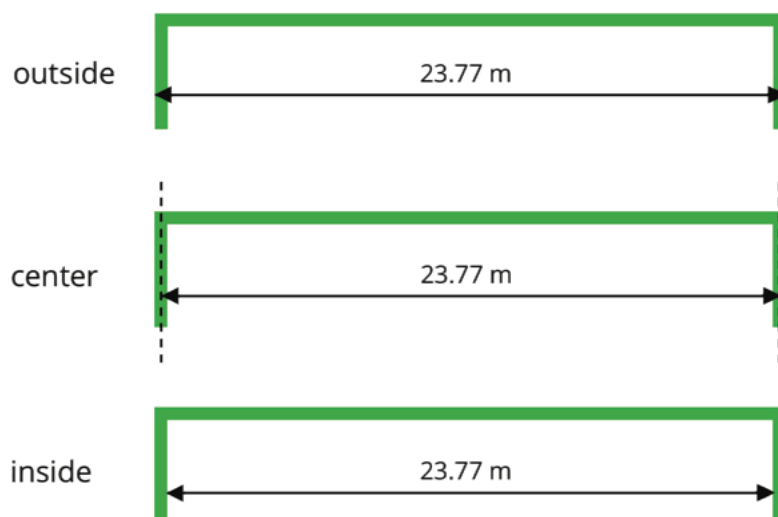
Student Lesson Summary

Diagrams can help us communicate and model mathematics. A clearly-labeled diagram helps us visualize what is happening in a problem and accurately communicate the information we need.

Sports offer great examples of how diagrams can help us solve problems. For example, to show the placement of the running hurdles in a diagram, we needed to know what the distances 13.72 and 14.02 meters tell us and the number of hurdles to draw. An accurate diagram not only helped us set up and solve the problem correctly, but also helped us see that there are only *nine* spaces between ten hurdles.

To communicate information clearly and solve problems correctly, it is also important to be precise in our measurements and calculations, especially when they involve decimals.

In tennis, for example, the length of the court is 23.77 meters. Because the boundary lines on a tennis court have a significant width, we would want to know whether this measurement is taken between the inside of the lines, the center of the lines, or the outside of the lines. Diagrams can help us attend to this detail, as shown here.



The accuracy of this measurement matters to the tennis players who use the court, so it matters to those who paint the boundaries as well. The tennis players practice their shots to be on or within certain lines. If the tennis court on which they play is not precisely measured,

their shots may not land as intended in relation to the boundaries. Court painters usually need to be sure their measurements are accurate to within $\frac{1}{100}$ of a meter or one centimeter.

Lesson 14 Practice Problems

Problem 1

Statement

A roll of ribbon was 12 meters long. Diego cut 9 pieces of ribbon that were 0.4 meter each to tie some presents. He then used the remaining ribbon to make some wreaths. Each wreath required 0.6 meter. For each question, explain your reasoning.

- How many meters of ribbon were available for making wreaths?
- How many wreaths could Diego make with the available ribbon?

Solution

- 8.4 meters. Reasoning varies. Sample reasoning: Diego used $9 \cdot (0.4)$ or 3.6 meters for the presents, which leaves 8.4 meters in the roll, because $12 - 3.6 = 8.4$.
- 14 wreaths. ($8.4 \div 0.6 = 14$)

Problem 2

Statement

The Amazon rainforest covered 6.42 million square kilometers in 1994. In 2014, it covered only $\frac{50}{59}$ as much. Which is closest to the area of the Amazon forest in 2014? Explain how you know without calculating the exact area.

- 6.4 million km^2
- 5.4 million km^2
- 4.4 million km^2
- 3.4 million km^2
- 2.4 million km^2

Solution

B

Problem 3

Statement

To get an A in her math class, Jada needs to have at least 90% of the total number of points possible. The table shows Jada's results before the final test in the class.

	Jada's points	total points possible
Homework	141	150
Test 1	87	100
Test 2	81	100
Test 3	91	100

- Does Jada have 90% of the total possible points *before* the final test? Explain how you know.
- Jada thinks that if she gets at least 92 out of 100 on the final test, she will get an A in the class. Do you agree? Explain.

Solution

- No, before the final exam, Jada has 400 points out of 450. But 90% of 450 is $(0.9) \cdot 450 = 405$. So she is 5 points short of 90%.
- Answers vary. Sample responses:
 - No, Jada is 5 points short of 90% before the last test, so her score on the last test has to be at least 5 points *more* than 90% to make up for this.
 - Maybe. This would give Jada 492 points out of 550. The value of $493 \div 550$ is a little more than 89.6. If the teacher rounds up, Jada will get an A.

Problem 4

Statement

Find the following differences. Show your reasoning.

◦ $0.151 - 0.028$

◦ $0.106 - 0.0315$

◦ $3.572 - 2.6014$

Solution

- 0.123. Sample reasoning: 0.151 is 151 thousandths and 0.028 is 28 thousandths. $151 - 28 = 123$, so the difference is 123 thousandths.
- 0.0745. Sample reasoning: 0.106 can be written as 0.1060 and the subtraction can be done using vertical calculation (as shown).

c. 0.9706. Sample reasoning: $3.572 - 2.6014$ can be thought of as $2.6014 + ? = 3.572$, and we can use vertical calculation to see what number, when added to 2.6014, makes 3.572 (the missing number is shown in boxes in the calculation).

b.
$$\begin{array}{r} 0.10510 \\ - 0.0315 \\ \hline 0.0745 \end{array}$$

c.
$$\begin{array}{r} 2.6014 \\ + \boxed{0}\boxed{9}\boxed{7}\boxed{0}\boxed{6} \\ \hline 3.5720 \end{array}$$

(From Unit 5, Lesson 4.)

Problem 5

Statement

Find these quotients. Show your reasoning.

◦ $24.2 \div 1.1$

◦ $13.25 \div 0.4$

◦ $170.28 \div 0.08$

Solution

a. 22. Reasoning varies. Sample reasoning (decomposing into sums of multiples of 11): $242 \div 11 = (220 + 22) \div 11 = 220 \div 11 + 22 \div 11 = 20 + 2 = 22$.

b. 33.125. Reasoning varies. Sample reasoning (decomposing into sums of multiples of 4 plus remainder): $132.5 \div 4 = (100 + 32 + .5) \div 4 = 25 + 8 + .125 = 33.125$.

c. 2,128.5. Reasoning varies. Sample reasoning (using long division):

$$\begin{array}{r} 2128.5 \\ 8 \overline{) 17028.0} \\ \underline{- 16} \\ 10 \\ \underline{- 8} \\ 22 \\ \underline{- 16} \\ 68 \\ \underline{- 64} \\ 40 \\ \underline{- 40} \\ 0 \end{array}$$

(From Unit 5, Lesson 13.)

Lesson 15: Making and Measuring Boxes

Goals

- Apply operations with decimals to calculate the surface area of paper boxes.
- Describe (orally) sources of measurement error, and justify an appropriate level of precision for reporting the answer.
- Measure and compare (orally and in writing) the dimensions of paper boxes.

Learning Targets

- I can use the four operations on decimals to find surface areas and reason about real-world problems.

Lesson Narrative

In this optional culminating lesson of the unit, students construct open-top origami boxes by folding different-size square paper. Before folding, they make conjectures about how the paper size affects the length and area measurements of the boxes. Later, they test their conjectures by finding and analyzing those measurements (MP8). While arithmetic operations on decimals are central to this work, students also build on their geometric work from earlier units. As they investigate the relationship between the side lengths of the origami paper and the edge lengths of the boxes, they also connect to their work on ratios.

This lesson is organized into two parts:

- Part 1: Measure, predict, and fold. Students carefully measure the sheets of square paper, predict the measurements of the boxes created from different-size sheets, and fold their paper into a box.
- Part 2: Measure, calculate, and compare. Students carefully measure the dimensions of the boxes, calculate their surface areas, and then compare the sizes of the boxes. They also reflect on the accuracy of their predictions from Part 1.

Depending on the instructional choices made, this lesson could take one or more class meetings. The time estimates are intentionally left blank because the amount of time needed might vary depending on factors such as:

- The size of the class.
- How familiar students are folding paper into shapes.
- How student work is ultimately shared with the class (not at all, informally, or with formal presentations).

Consider defining the scope of work further for students and setting a time limit for each part of the activity to focus students' work and optimize class time.

Alignments

Addressing

- 6.NS.B.3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports

Required Materials

**Copies of Instructional
master Origami paper**

Rulers

Required Preparation

Choose at least three different sizes of origami paper for students to use. Common length and width sizes of square origami paper include 6 inch, 7 inch, 8 inch, 9 inch, and 9.75 inch. If origami paper is not available, cut squares of paper from available paper (thinner is better). Pre-make sample boxes of different sizes.

In order to help students fold their own origami boxes, both an embedded video and printed instructions are provided. The printed instructions are in the Folding Paper Boxes Instructional master. If using the printed instructions, prepare 1 copy for every 2 students. These can be re-used with multiple classes.

Student Learning Goals

Let's use what we know about decimals to make and measure boxes.

15.1 Folding Paper Boxes

Optional:

In this activity, students work with decimals by building paper boxes, taking measurements of the paper and the boxes, and calculating surface areas. Although the units are specified in the problem, students need to measure very carefully in order to give an estimate to the nearest millimeter. Next, students compare the side lengths of several paper squares and estimate what the measurements of the boxes will be once the squares have been folded.

Closely monitor student progress on the first question to make sure that students are careful in their measurements. On the second question, make sure they think carefully about what level of precision to use in reporting the relationship between two measurements. For examples:

- If one sheet of square paper is very close to twice the length of another, reporting the answer as 2 is reasonable given the possible error in measurement.
- If the relationship is very close to a fraction (e.g. $\frac{3}{2}$ for the 6 inch and 9 inch squares) students might report the number as a fraction.
- If students report the quotient as a decimal, three digits (ones, tenths, hundredths) is appropriate because the measurements in the first problem have 3 digits.

Encourage students to make strong creases when folding their paper at the end of the activity. Suggest that they use the side of a thumbnail or a ruler to flatten the crease after making the initial fold. Students will use these boxes in the next activity.

Addressing

- 6.NS.B.3

Instructional Routines

- MLR2: Collect and Display

Launch

Choose at least three different sizes of square paper for students to use. To see the mathematical structure more clearly, the smallest should be 6 inches by 6 inches and the largest should be 12 inches by 12 inches. For the other size squares, common length and width sizes of origami paper include 7 inch, 8 inch, 9 inch, and 9.75 inch. Pre-make boxes of different sizes from square paper with lengths 6 inches, 8 inches, and 12 inches.

Arrange students in groups of 3–4. Provide each group with at least three different sizes of paper. Since the folding process is involved and students are asked to measure the squares they use to make the boxes, make some extra squares available for each group. Students will need metric rulers or tape measures, marked in millimeters.

Give students 10 minutes for the problems. When students are finished, demonstrate how to fold a paper square into a box and then have students fold their paper squares. This can be done with an accompanying video or with instructions such as those included in the Instructional master. In either case, consider going through the process with students step by step and practicing beforehand to make sure that it goes smoothly.

Note that these particular instructions make a box with a square base; the following activity, which prompts students to record the length and width of the box's base, is based on this premise. If a different origami construction is used, the instructions and possibly the task statements will need to be adjusted.

Video 'Origami Box' available here: <https://player.vimeo.com/video/304138309>.

Access for English Language Learners

Representing, Conversing: MLR2 Collect and Display. As students work on the problems, use this routine to capture the ways they communicate about the measurements, including the precision at which they were taken. Additionally, capture the ways students discuss the relationships between the dimensions of the sheets of paper and predicted dimensions of the boxes. Listen for terms such as quotient, divide, multiply, and rounding. Notate student language and relevant sketches/diagrams to reference while students are working and then discuss in the synthesis. This will help students communicate about the accuracy of the measurements and the size comparisons between boxes.

Design Principle(s): Support sense-making; Maximize meta-awareness

Anticipated Misconceptions

Check that students are obtaining accurate measurements to the nearest tenth of a centimeter. Are they placing the ruler at the edge of a side? Are they starting at zero?

Student Task Statement

Your teacher will demonstrate how to make an open-top box by folding a sheet of paper. Your group will receive 3 or more sheets of square paper. Each person in your group will make 1 box. Before you begin folding:

1. Record the side lengths of your papers, from the smallest to the largest.
 - Paper for Box 1: _____ cm
 - Paper for Box 2: _____ cm
 - Paper for Box 3: _____ cm
2. Compare the side lengths of the square sheets of paper. Be prepared to explain how you know.
 - a. The side length of the paper for Box 2 is _____ times the side length of the paper for Box 1.
 - b. The side length of the paper for Box 3 is _____ times the side length of the paper for Box 1.
3. Make some predictions about the measurements of the three boxes your group will make:
 - The surface area of Box 3 will be _____ as large as that of Box 1.
 - Box 2 will be _____ times as tall as Box 1.

- Box 3 will be _____ times as tall as Box 1.

Now you are ready to fold your paper into a box!

Student Response

Answers vary. Sample responses (based on paper sizes of 6 inches by 6 inches, 8 inches by 8 inches, and 12 inches by 12 inches):

1. Paper for Box 1: 15.2 cm, paper for Box 2: 20.3 cm, paper for Box 3: 30.5 cm.
2. The paper for Box 2 is about 1.3 times as long as the paper for Box 1, found by dividing 20.3 by 15.2. Alternatively, using the inch measurements, the paper for Box 2 is $\frac{4}{3}$ times as long as the paper for Box 1. The paper for Box 3 is about 2 times as long as the paper for Box 1, found by dividing 30.5 by 15.2 (or by dividing 12 by 6).
3. The surface area of Box 3 will be about four times the surface area of Box 1. Box 2 will be about 1.3 (or one and a third) times the height of Box 1. Box 3 will be 2 times as tall as Box 1.

Activity Synthesis

(Note: the discussion questions below assume square paper of side lengths 6 inches, 8 inches, and 12 inches were used).

The goal of this discussion is for students to think critically about the accuracy of their measurements and predictions. Consider asking some of these discussion questions. Sample responses are shown in parentheses, but expect students' answers to vary.

- What did you find for the length and width of the smallest square? (15.2 cm. Also expect some answers of 15.3 cm and possibly a wider range of values.)
- What was challenging about measuring the length of the squares? (The millimeters are so small that it was hard to tell which millimeter it was closest to. It was hard to measure *straight* across.)
- How confident are you about the accuracy of your measurements (For the first square, very confident about the 15 in 15.2 cm, but not confident about the 0.2.)
- How many times as long as Paper 1 was Paper 3? (About 2, very close to 2, or a decimal number that is close to 2.)
- What are some advantages and disadvantages of reporting the quotient of the side length of Paper 3 and that of Paper 1 as 2? (Advantage: it describes the general relationship clearly, and 2 is an easy number to grasp. Disadvantage: it was not exactly twice as long.)
- What are some advantages and disadvantages of reporting the quotient of the side length of Paper 3 and that of Paper 1 as 2.007 (or another value that is very close to 2 and is proposed by students)? (Advantage: the number is more accurate than 2. Disadvantage: it is too accurate. The measurements were done by hand and were not precise enough to judge the quotient to the nearest ten-thousandth.)

- How much taller do you think Box 3 will be compared to Box 1? (Twice, because the side length of the paper making Box 3 is twice as long and twice as wide as the that of paper making Box 1.)

15.2 Sizing Up Paper Boxes

Optional:

Using the boxes that they built in the previous activity, students now measure and compare the length, height, and surface area of the boxes. This work requires fluency in operations with decimal numbers and care in measurement. In measuring the dimensions of the box, there are multiple layers of imprecision that can be expected.

- The length, width, and height will not be an exact number of millimeters and so students round to the nearest millimeter. In some cases, this may essentially be a guess between two different values.
- The box is made by folding paper and this process is not exact. The box is therefore not exactly a rectangular prism with a square base, and measurements of the length, width, and height vary depending on which part of the box is measured.
- When finding the surface area of their box, students will add and multiply their measurements. In performing operations, any errors in the measurements propagate, making it challenging to trace where they originated.

Addressing

- 6.NS.B.3

Instructional Routines

- MLR8: Discussion Supports

Launch

Keep students in the same groups. Provide access to rulers. Give groups 8–10 minutes to complete the first table collaboratively. There are three sets of measurements and a surface area calculation for each box. Each student can fill out the row for the box that they made. If a group has more than three paper sizes, adjust the tables in the activity accordingly. Ask students to pause for a class discussion after they have completed the table.

Select a couple of groups to share their measurements and surface area calculations. Display their responses for all to see. Ask other groups if their responses are close, and if not, ask what their answers are. Come to a general agreement about the approximate measurements and areas.

Then, ask students to complete the remaining questions.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organization and problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.

Supports accessibility for: Organization; Attention

Anticipated Misconceptions

Students may neglect to attend to units of measurement when calculating area. Encourage them to attend to the units being used.

Student Task Statement

Now that you have made your boxes, you will measure them and check your predictions about how their heights and surface areas compare.

1. a. Measure the length and height of each box to the nearest tenth of a centimeter. Record the measurements in the table.

	side length of paper (cm)	length of box (cm)	height of box (cm)	surface area (sq cm)
Box 1				
Box 2				
Box 3				

- b. Calculate the surface area of each box. Show your reasoning and decide on an appropriate level of precision for describing the surface area (Is it the nearest 10 square centimeters, nearest square centimeter, or something else?). Record your answers in the table.
2. To see how many times as large one measurement is when compared to another, we can compute their quotient. Divide each measurement of Box 2 by the corresponding measurement for Box 1 to complete the following statements.
 - a. The length of Box 2 is _____ times the length of Box 1.
 - b. The height of Box 2 is _____ times the height of Box 1.
 - c. The surface area of Box 2 is _____ times the surface area of Box 1.

3. Find out how the dimensions of Box 3 compare to those of Box 1 by computing quotients of their lengths, heights, and surface areas. Show your reasoning.

- a. The length of Box 3 is _____ times the length of Box 1.
- b. The height of Box 3 is _____ times the height of Box 1.
- c. The surface area of Box 3 is _____ times the surface area of Box 1.

4. Record your results in the table.

	side length of paper	length of box	height of box	surface area
Box 2 compared to Box 1				
Box 3 compared to Box 1				

5. Earlier, in the first activity, you made predictions about how the heights and surface areas of the two larger boxes would compare to those of the smallest box. Discuss with your group:

- How accurate were your predictions? Were they close to the results you found by performing calculations?
- Let's say you had another piece of square paper to make Box 4. If the side length of this paper is 4 times the side length of the paper for Box 1, predict how the length, height, and surface area of Box 4 would compare to those of Box 1. How did you make your prediction?

Student Response

1. a. Answers vary. Sample response: The table below is filled out based on 6-inch, 8-inch, and 12-inch papers. Refer to the Activity Synthesis for examples of possible errors in these measurements.

	side length of paper (cm)	length of box (cm)	height of box (cm)	surface area (sq cm)
Box 1	15.2	5.4	2.7	87.5
Box 2	20.8	7.2	3.6	156
Box 3	30.5	10.8	5.4	350

b. Answers vary. See table for sample results.

2. Answers vary. See table in response to question 4.

3. Answers vary. See table in response to question 4.

4. Sample response: The table is filled out using three significant digits in each case.

	side length of paper	length of box	height of box	surface area
Box 2 compared to Box 1	1.34	1.3	1.3	1.78
Box 3 compared to Box 1	2.01	2	2	4

5. Answers vary. Sample response:

- My estimates were accurate. I thought the length and height of Box 3 would be twice those of Box 1 and that the length and height of Box 2 would be about one and a third times those of Box 1. I thought the surface area of Box 3 would be 4 times the surface area of Box 1. These answers are close to my results of my calculations. The discrepancies probably come from error in measurement.
- I predicted the length and height of this box to be 4 times as long as those of Box 1, and the surface area of Box 4 to be 16 times as large.

Activity Synthesis

(Note: the discussion questions below assume sheets of square paper of side lengths 6 inches, 8 inches, and 12 inches were used).

For each size of paper used, ask students to report their measurements for the volume of the box and record for all to see. Ask:

- “Is there variation in the measurements? Which ones?” (Yes, possibly for the dimensions of the squares; almost certainly for the dimensions of the boxes.)
- “Why do you think the measurements were not all the same?” (It was difficult to line up the edges of the paper with the ruler. It was also challenging to measure to the nearest millimeter.)
- “Were there extra difficulties measuring the boxes, as opposed to measuring the squares of paper?” (Yes, e.g. the sides of the box were not completely flat; the sides of the box were not identical; the height of the box was different at different places.)
- “How accurate do you think your measurements were?” (In each case, within 1 or 2 millimeters.)
- “How does this influence the way you record a result? For example, for the surface area, did anyone record their answer to hundredths of a square centimeter? Why or why not?” (Some students might have done this, but it is overly precise and does not take into account the imprecision in measurements mentioned above.)
- “In the table for the third question, did you record units for your measurements? Is it important to record units?” (It is important that the measurements of the differences between boxes must be in the *same* units.)

Also discuss with the class their predictions and measurements for how the heights and surface area of the boxes compare to one another. Ask:

- “How much larger did you think the surface area of Box 3 would be than the surface area of Box 1?” (4 times because it takes 4 of the smaller sheet of paper to make one of the larger ones. Some students may also say 2 times, because the height, length, and width are about 2 times as long as those of Box 1.)
- “Did your measurements match your prediction for the surface areas? Why might that be?” (Students who did not think the surface area of Box 3 would be 4 times as large as the surface area of Box 1 can discuss their old thinking and new. Because area is found by taking a product of length measurements and *all* measurements (length, width, height) of Box 3 are twice the corresponding measurements of Box 1, Box 3 will have 4 times as much surface area as Box 1.)

Access for English Language Learners

Listening, Speaking, Conversing: MLR8 Discussion Supports. Before the whole-class discussion, give students time to converse with their group about their predictions for how the heights and surface area of the boxes compare to one another. Provide this sentence frame for students to use: “My measurements matched/did not match my predictions because __.” Invite group members to press for details in their peers’ explanations and challenging ideas by asking, “Why did you make that prediction?”, “Could you explain that using a different example?”, or “What evidence do you have to support your reasoning?” This will help students solidify their reasoning, and improve their explanations before the whole-class discussion.

Design Principle(s): Cultivate conversation

Lesson Synthesis

The discussions of student work at the end of each activity provides opportunities to summarize takeaways from this lesson. Students can use this optional lesson to practice fluency with calculations involving decimals. Highlight instances where students have to make an estimate in order to proceed, figure out how to determine significant digits, or apply their understanding of units when measuring and recording information.



Family Support Materials

Family Support Materials

Arithmetic in Base Ten

Here are the video lesson summaries for Grade 6, Unit 5: Arithmetic in Base Ten. Each video highlights key concepts and vocabulary that students learn across one or more lessons in the unit. The content of these video lesson summaries is based on the written Lesson Summaries found at the end of lessons in the curriculum. The goal of these videos is to support students in reviewing and checking their understanding of important concepts and vocabulary. Here are some possible ways families can use these videos:

- Keep informed on concepts and vocabulary students are learning about in class.
- Watch with their student and pause at key points to predict what comes next or think up other examples of vocabulary terms (the bolded words).
- Consider following the Connecting to Other Units links to review the math concepts that led up to this unit or to preview where the concepts in this unit lead to in future units.

Grade 6, Unit 5: Arithmetic in Base Ten	Vimeo	YouTube
Video 1: Adding and Subtracting Decimals (Lessons 2–4)	Link	Link
Video 2: Multiplying Decimals (Lessons 5–8)	Link	Link
Video 3: Dividing Whole Numbers (Lessons 9–10)	Link	Link
Video 4: Dividing Decimals (Lessons 11–14)	Link	Link

Video 1

Video 'VLS G6U5V1 Adding and Subtracting Decimals (Lessons 2–4)' available here:
<https://player.vimeo.com/video/492582931>.

Video 2

Video 'VLS G6U5V2 Multiplying Decimals (Lessons 5–8)' available here:
<https://player.vimeo.com/video/500622718>.

Video 3

Video 'VLS G6U5V3 Dividing Whole Numbers (Lessons 9–10)' available here:
<https://player.vimeo.com/video/497426755>.

Video 4

Video 'VLS G6U5V4 Dividing Decimals (Lessons 11–14)' available here:
<https://player.vimeo.com/video/500082524>.

Connecting to Other Units

- *Coming soon*

Warming Up to Decimals

Family Support Materials 1

This week, your student will add and subtract numbers using what they know about the meaning of the digits. In earlier grades, your student learned that the 2 in 207.5 represents 2 *hundreds*, the 7 represents 7 *ones*, and the 5 represents 5 *tenths*. We add and subtract the digits that correspond to the same units like *hundreds* or *tenths*. For example, to find $10.5 + 84.3$, we add the tens, the ones, and the tenths separately, so $10.5 + 84.3 = 90 + 4 + 0.8 = 94.8$.

Any time we add digits and the sum is greater than 10, we can “bundle” 10 of them into the next higher unit. For example, $0.9 + 0.3 = 1.2$.

To add whole numbers and decimal numbers, we can arrange $0.921 + 4.37$ vertically, aligning the decimal points, and find the sum. This is a convenient way to be sure we are adding digits that correspond to the same units. This also makes it easy to keep track when we bundle 10 units into the next higher unit (some people call this “carrying”).

$$\begin{array}{r} 1 \\ 0.921 \\ + 4.37 \\ \hline 5.291 \end{array}$$

Here is a task to try with your student:

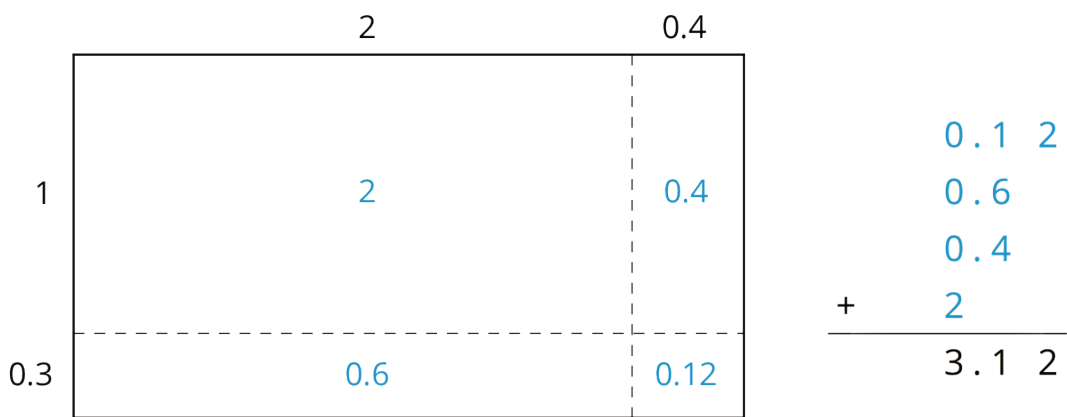
Find the value of $6.54 + 0.768$.

Solution: 7.308. Sample explanation: there are 8 thousandths from 0.768. Next, the 4 hundredths from 6.54 and 6 hundredths from 0.768 combined make 1 tenth. Together with the 5 tenths from 6.54 and the 7 tenths from 0.768 this is 13 tenths total or 1 and 3 tenths. In total, there are 7 ones, 3 tenths, no hundredths, and 8 thousandths.

Multiplying Decimals

Family Support Materials 2

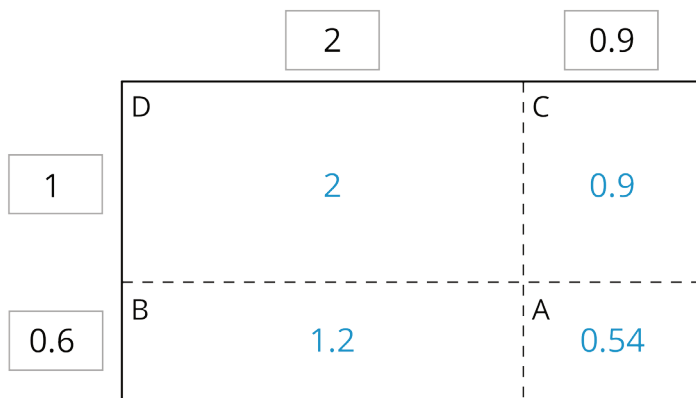
This week, your student will multiply decimals. There are a few ways we can multiply two decimals such as $(2.4) \cdot (1.3)$. We can represent the product as the area of a rectangle. If 2.4 and 1.3 are the side lengths of a rectangle, the product $(2.4) \cdot (1.3)$ is its area. To find the area, it helps to decompose the rectangle into smaller rectangles by breaking the side lengths apart by place value. The sum of the areas of all of the smaller rectangles, 3.12, is the total area.



Here is a task to try with your student:

Find $(2.9) \cdot (1.6)$ using an area model and partial products.

Solution: 4.64. The area of the rectangle (or the sum of the partial products) is:
 $2 + 0.9 + 1.2 + 0.54 = 4.64$



Dividing Decimals

Family Support Materials 3

This week, your student will divide whole numbers and decimals. We can think about division as breaking apart a number into equal-size groups.

For example, consider $65 \div 4$. We can image that we are sharing 65 grams of gold equally among 4 people. Here is one way to think about this:

- First give everyone 10 grams. Then 40 grams have been shared out, and 25 grams are left over. We can see this in the first example.
- If we give everyone 6 more grams, then 24 grams have been shared out, and 1 gram is left.
- If we give everyone 0.2 more grams, then 0.8 grams are shared out and 0.2 grams are left.
- If everyone gets 0.05 more grams next, then all of the gold has been shared equally.

Everyone gets $10 + 6 + 0.2 + 0.05 = 16.25$ grams of gold.

$$\begin{array}{r}
 \boxed{16.25} \\
 0.05 \\
 0.2 \\
 6 \\
 10 \\
 4 \overline{) 65} \\
 \underline{- 40} \quad \leftarrow 4 \text{ groups of } 10 \\
 25 \\
 \underline{- 24} \quad \leftarrow 4 \text{ groups of } 6 \\
 1.0 \\
 \underline{- .8} \quad \leftarrow 4 \text{ groups of } 0.2 \\
 .20 \\
 \underline{- .20} \quad \leftarrow 4 \text{ groups of } 0.05 \\
 0
 \end{array}$$

$$\begin{array}{r}
 \boxed{16.25} \\
 0.05 \\
 0.2 \\
 11 \\
 5 \\
 4 \overline{) 65} \\
 \underline{- 20} \\
 45 \\
 \underline{- 44} \\
 1.0 \\
 \underline{- .8} \\
 .20 \\
 \underline{- .20} \\
 0
 \end{array}$$

The calculation on the right shows different intermediate steps, but the quotient is the same. This approach is called the **partial quotients** method for dividing.

Here is a task to try with your student:

			1	1	2	
					2	
				1	0	
			1	0	0	
7	√	7	8	4		
		-	7	0	0	
				8	4	
		-	7	0		
				1	4	
		-	1	4		
				0		

Here is how Jada found $784 \div 7$ using the partial quotient method.

1. In the calculation, what does the subtraction of 700 represent?
2. Above the dividend 784, we see the numbers 100, 10, and 2. What do they represent?
3. How can we check if 112 is the correct quotient for $784 \div 7$?

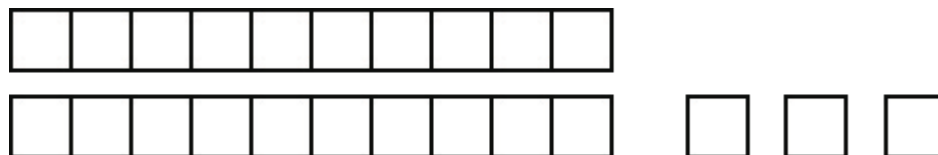
Solution

1. Subtraction of 7 groups of 100 from 784.
2. 100, 10, and 2 are the amounts distributed into each group over 3 rounds of dividing.
3. We can multiply $7 \cdot 112$ and see if it produces 784.

Unit Assessments

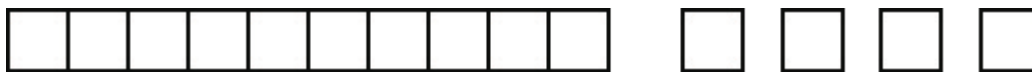
Arithmetic in Base Ten: Check Your Readiness (A)

1. This diagram shows three small squares and two rectangles composed of 10 small squares.



- a. Jada says this diagram can represent 230. What does a small square represent for Jada?
- b. Name a number greater than 230 that this diagram can also represent.
- c. Lin says this diagram can represent 2.3. What does a small square represent for Lin?
- d. Name a number less than 2.3 that this diagram can also represent.

2. This diagram shows four small squares and one rectangle composed of 10 small squares.



- a. Andre says that this diagram can represent 140. Do you agree with Andre? Explain why or why not.

- b. What is another value this diagram could represent? How do you know?

3. Here are some fractions:

$$\frac{490}{10}, \frac{49}{10}, \frac{490}{100}, \frac{49}{100}, \frac{490}{1000}, \frac{49}{1000}$$

- a. Select **all** the fractions that are equal to 4.9.
- b. Select **all** the fractions that are equal to 0.049.

4. a. Noah had \$5.25 and Lin had \$8.95. How much did they have altogether?

b. Diego had \$21.32 and gave \$9.50 to Elena. How much did he have left?

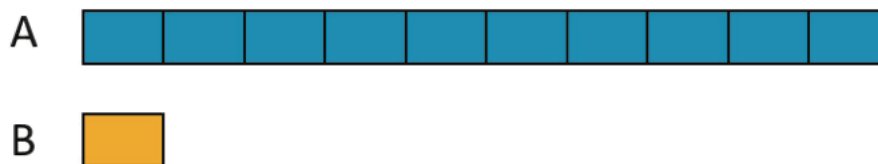
c. Andre deposited \$24.50 into his bank account each week for 5 weeks. What was the total amount Andre deposited?

d. Four friends did a job together that paid \$85.20. How much should each get if they all get an equal amount?

5. Compute $(4,803) \cdot 95$.

6. Noah has 4 bags of marbles with the same number in each bag. If there are 536 marbles altogether, how many marbles are in each bag? Explain how you know.

7. Rectangle A is 10 times as long as rectangle B.

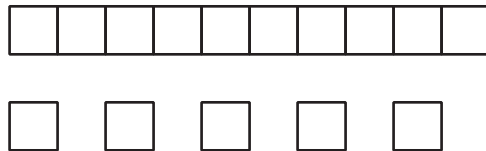


If rectangle A is 80 units long, select **all** the ways find the length of rectangle B.

- A. Multiply 80 by 10
- B. Multiply 80 by $\frac{1}{10}$
- C. Multiply 10 by 80
- D. Divide 80 by 10
- E. Divide 10 by 80
- F. Divide 80 by $\frac{1}{10}$

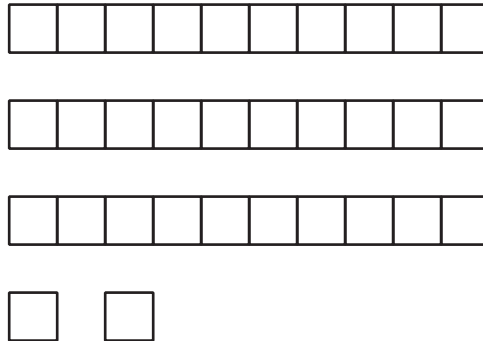
Arithmetic in Base Ten: Check Your Readiness (B)

1. This diagram shows five small squares and one rectangle composed of 10 small squares.



- a. Andre says this diagram can represent 1,500. What does a small square represent for Andre?
- b. Name a number greater than 1,500 that this diagram can also represent.
- c. Clare says this diagram can represent 1.5. What does a small square represent for Clare?
- d. Name a number less than 1.5 that this diagram can also represent.

2. This diagram shows two small squares and three rectangles each composed of 10 small squares.



- a. Priya says that this diagram can represent 320. Do you agree with Priya? Explain why or why not.
- b. What is another value this diagram could represent? How do you know?

3. Here are some fractions:

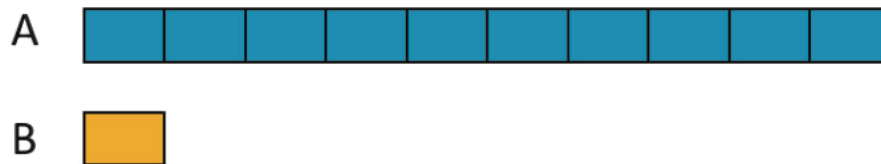
$$\frac{820}{100}, \frac{820}{1000}, \frac{82}{1000}, \frac{820}{10}, \frac{82}{10}, \frac{82}{100}$$

- a. Select **all** the fractions that are equal to 8.2.
- b. Select **all** the fractions that are equal to 0.082.

4. a. Lin had \$24.16 and spent \$8.50. How much did she have left?
- b. Elena had \$6.75 and Tyler had \$7.85. How much did they have altogether?
- c. Three friends did a job together that paid \$65.40. How much should each get if they all get an equal amount?
- d. Diego deposited \$36.50 into his bank account each week for 7 weeks. What was the total amount he deposited?
5. Compute $(3,902) \cdot 85$.

6. Kiran has 3 containers of blocks with the same number in each container. If there are 852 blocks altogether, how many blocks are in each container? Explain how you know.

7. Rectangle A is 10 times as long as Rectangle B.



If Rectangle A is 50 units long, select **all** the ways to find the length of Rectangle B.

- A. Multiply 50 by 10.
- B. Divide 50 by 10.
- C. Multiply 50 by $\frac{1}{10}$.
- D. Divide 50 by $\frac{1}{10}$.
- E. Multiply 10 by 50.
- F. Divide 10 by 50.

Arithmetic in Base Ten: Mid-Unit Assessment (A)

Calculators should not be used.

1. Which estimate is closest to the actual value of $(2.99548) \cdot (1.8342)$?
 - A. 4.8
 - B. 5.5
 - C. 6.2
 - D. 8.3

2. For part of a science experiment, Andre adds 0.25 milliliters of dye to 12.3 milliliters of water. How many milliliters does the mixture contain?
 - A. 12.05
 - B. 12.325
 - C. 12.55
 - D. 14.8

3. Select **all** the expressions that are greater than 3.
 - A. $1.67 + 1.4$
 - B. $2.97 + 0.004$
 - C. $3.017 - 0.05$
 - D. $4.5 - 1.47$
 - E. $5.503 - 2.52$

4. Four runners are training for long races. Noah ran 5.123 miles, Andre ran 6.34 miles, Jada ran 7.1 miles, and Diego ran 8 miles.

a. What is the total running distance of the four runners?

b. Jada ran how much farther than Noah?

5. One way to compute a 15% tip for a bill is to multiply it by 0.15.

A restaurant bill was \$42.40. Calculate the tip. Explain your reasoning in detail.

6. a. Find the product: $(0.061) \cdot (0.43)$

- b. Find the difference:

$$\begin{array}{r} 1 \\ - 0.4308 \\ \hline \end{array}$$

7. Two students have different ways to calculate $(0.25) \cdot (0.044)$ as a decimal. Show, in detail, how each student might compute this product.

- a. Elena says, "I find $25 \cdot 44$ and then move the decimal point."

- b. Diego says, "0.25 is the same as $\frac{1}{4}$, so I found $\frac{1}{4}$ of 0.044."

Arithmetic in Base Ten: Mid-Unit Assessment (B)

Calculators should not be used.

1. Which estimate is closest to the actual value of $(3.99437) \cdot (2.6851)$?

- A. 14.2
- B. 12.6
- C. 10.8
- D. 8.3

2. Select **all** the expressions that are greater than 5.

- A. $4.88 + 0.009$
- B. $3.76 + 1.3$
- C. $5.012 - 0.04$
- D. $7.702 - 2.71$
- E. $6.5 - 1.49$

3. Mai's backpack weighs 9 pounds. Han's backpack weighs 8.3 pounds. Priya's backpack weighs 7.75 pounds, and Kiran's backpack weighs 6.125 pounds. Select **all** the true statements.

- A. Han's backpack is 2.175 pounds heavier than Kiran's.
- B. Mai's backpack is 2.25 pounds heavier than Priya's.
- C. Han's backpack is 16.05 pounds heavier than Priya's.
- D. Kiran's backpack and Han's backpack together weigh 14.3125 pounds.
- E. The backpacks weigh 31.175 pounds together.

4. For part of a recipe, Jada adds 0.35 milliliters of vanilla extract to 16.4 milliliters of milk. How many milliliters does the mixture contain?

5. One way to compute a 20% discount is to multiply the price by 0.20. The price of a pair of sneakers is \$84.50. Calculate the discount. Explain or show your reasoning.

6. a. Find the difference:

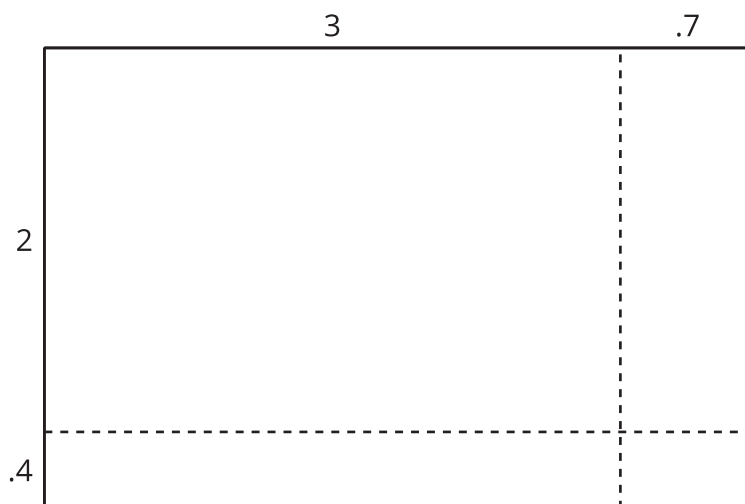
$$\begin{array}{r} 1 \\ - 0.5904 \\ \hline \end{array}$$

b. Find the product: $(0.072) \cdot (0.31)$

7. Two students have different ways to calculate $(2.4) \cdot (3.7)$. Show, in detail, how each student might compute this product.

a. Andre says, "I find $24 \cdot 37$ and then move the decimal point."

b. Lin says, "I drew this diagram and then found the area of each piece."



Arithmetic in Base Ten: End-of-Unit Assessment (A)

Calculators should not be used.

1. A woodworker wants to cut a board that is 8.225 feet long into 5 equal-length pieces. How long will each of the cut boards be?

A. 0.1645 feet
B. 1.645 feet
C. 4.1125 feet
D. 41.125 feet

2. Select **all** of the expressions that have the same value as $892 \div 8$.

A. $8920 \div 80$
B. $894 \div 10$
C. $89.2 \div 0.08$
D. $8.92 \div 0.8$
E. $0.892 \div 0.008$

3. Which is closest to the quotient $4,367 \div 0.004$?

A. 1,000
B. 10,000
C. 100,000
D. 1,000,000

4. One way to convert from inches to centimeters is to multiply the number of inches by 2.54. How many centimeters are there in $\frac{1}{4}$ inch?

5. Find each quotient using long division.

a. $2,247 \div 7$

b. $676 \div 13$

6. A sign in a bakery gives these options:

- 12 cupcakes for \$29
- 24 cupcakes for \$56
- 50 cupcakes for \$129

a. Find each unit price to the nearest cent, and show your reasoning.

b. Which option gives the lowest unit price?

7. A stack of 500 pieces of paper is 1.875 inches tall.

a. Diego guesses that each piece of paper is 0.015 inches thick. Explain how you know that Diego's answer is *not* correct.

b. Compute the thickness of each piece of paper. Show your reasoning.

Arithmetic in Base Ten: End-of-Unit Assessment (B)

Calculators should not be used.

1. A runner is preparing for a marathon that is cut into 5 equal sections. The marathon is 42.195 kilometers long. How long is each section of the marathon?

A. 210.975 km
B. 21.0975 km
C. 8.439 km
D. 0.8439 km

2. Select **all** of the expressions that are equal to 2.

A. $1,200 \div 60$
B. $1.2 \div 0.6$
C. $0.012 \div 0.06$
D. $0.5 \div 0.25$
E. $0.05 \div 0.025$

3. Which is closest to the quotient $5,278 \div 0.05$?

A. 1,000,000
B. 100,000
C. 10,000
D. 1,000

4. It takes 4.12 gallons of paint to paint a fence. How much paint is needed for $\frac{1}{5}$ of the fence?

5. Find each quotient using long division.

a. $3,328 \div 8$

b. $868 \div 14$

6. A sign in a bakery gives these options:

- 12 muffins for \$25
- 24 muffins for \$46
- 50 muffins for \$94

a. Find each unit price to the nearest cent, and show your reasoning.

b. Which option gives the lowest unit price?

7. A stack of 200 file folders is 5.125 inches tall.

a. Elena guesses that each file folder is 0.035 inches thick. Explain how you know that Elena's answer is *not* correct.

b. Compute the thickness of each file folder. Show your reasoning.



Assessment Answer Keys

Assessment Answer Keys

Arithmetic in Base Ten

Assessment : Check Your Readiness (A)

Teacher Instructions

This assesses students' incoming knowledge about concepts that support understanding of operations with base-ten numbers.

Problem 1

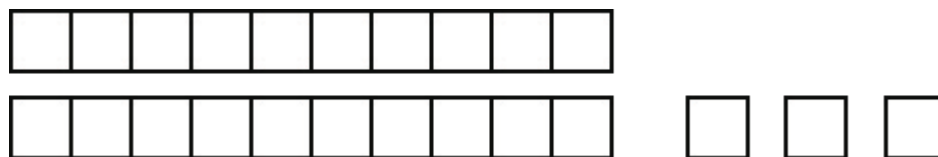
The content assessed in this problem is first encountered in Lesson 3: Adding and Subtracting Decimals with Few Non-Zero Digits.

This problem reintroduces students to base-ten diagrams, which are used throughout this unit. Students should understand that the unit square can have any value, but each strip of ten squares will have ten times this value. They should also understand that shifting digits to the left: for example, 230 vs 2300, means that each digit has ten times its previous value.

If most students struggle with this item, plan to do the optional Lesson 2, and the optional Lesson 3 Activity 2.

Statement

This diagram shows three small squares and two rectangles composed of 10 small squares.



1. Jada says this diagram can represent 230. What does a small square represent for Jada?
2. Name a number greater than 230 that this diagram can also represent.
3. Lin says this diagram can represent 2.3. What does a small square represent for Lin?
4. Name a number less than 2.3 that this diagram can also represent.

Solution

1. 10
2. Answers vary. Sample responses: 2,300; 23,000; 230,000; 460
3. 0.1
4. Answers vary. Sample responses: 0.23; 0.023; 0.0023; 1.15

Aligned Standards

5.NBT.A.1

Problem 2

The content assessed in this problem is first encountered in Lesson 3: Adding and Subtracting Decimals with Few Non-Zero Digits.

The main idea in this problem is that a digit in the hundreds place counts ten times more than a digit in the tens place. (And a digit in the thousands place counts ten times more than a digit in the hundreds place, etc.) Since there is one bundle of ten squares but four individual squares, that means that the 1 needs to go immediately to the left of the 4.

If most students struggle with this item, plan to do the optional Lesson 2 Activity 1, Changing Values.

Statement

This diagram shows four small squares and one rectangle composed of 10 small squares.



1. Andre says that this diagram can represent 140. Do you agree with Andre? Explain why or why not.
2. What is another value this diagram could represent? How do you know?

Solution

1. Yes. If a small square represents 10, then the rectangle represents $10 \cdot 10 = 100$, and the diagram represents 140.
2. Answers vary. The value selected should show understanding that the rectangle of 10 small squares is ten times larger than 1 small individual square. Possible responses:
 - 1,400 where the small square represents 100.
 - 14,000, where the small square represents 1,000.

Aligned Standards

5.NBT.A.1

Problem 3

The content assessed in this problem is first encountered in Lesson 5: Decimal Points in Products.

In this problem, students convert fractions to decimals, where each fraction has a denominator that is a power of 10. Students may use their understanding of place value to do this, or they may divide each numerator by the denominator.

If most students do well with this item, it may be possible to skip Lesson 5 Activity 2, Fractionally Speaking: Powers of 10.

Statement

Here are some fractions:

$$\frac{490}{10}, \frac{49}{10}, \frac{490}{100}, \frac{49}{100}, \frac{490}{1000}, \frac{49}{1000}$$

1. Select **all** the fractions that are equal to 4.9.
2. Select **all** the fractions that are equal to 0.049.

Solution

1. $\frac{49}{10}, \frac{490}{100}$
2. $\frac{49}{1000}$

Aligned Standards

4.NF.C.6

Problem 4

The content assessed in this problem is first encountered in Lesson 1: Using Decimals in a Shopping Context.

This problem assesses all four arithmetic operations with decimals: addition, subtraction, multiplication, and division. If most students do well with this item, plan to connect place value diagrams to pennies, dimes, and dollars in Lesson 3.

Statement

1. Noah had \$5.25 and Lin had \$8.95. How much did they have altogether?
2. Diego had \$21.32 and gave \$9.50 to Elena. How much did he have left?
3. Andre deposited \$24.50 into his bank account each week for 5 weeks. What was the total amount Andre deposited?
4. Four friends did a job together that paid \$85.20. How much should each get if they all get an equal amount?

Solution

1. \$14.20
2. \$11.82

3. \$122.50

4. \$21.30

Aligned Standards

5.NBT.B

Problem 5

The content assessed in this problem is first encountered in Lesson 7: Using Diagrams to Represent Multiplication.

Students will need to know how to perform multi-digit multiplication fluently before generalizing this work to decimal multiplication.

If most students struggle with this item, plan to do the optional Lesson 7 Activity 2, Connecting Area Diagrams to Calculations with Whole Numbers.

Statement

Compute $(4,803) \cdot 95$.

Solution

456,285

Aligned Standards

5.NBT.B.5

Problem 6

The content assessed in this problem is first encountered in Lesson 9: Using the Partial Quotients Method.

This problem assesses fluency in and understanding of whole-number division, another prerequisite skill for this unit. Students may explain the problem conceptually, as in the solution, or they may take a more formulaic approach.

If most students struggle with this item, plan to do some additional activities involving fair-sharing, including using manipulatives. Focus on dividing large numbers and ways to make the sharing easier, such as giving each person 10 at a time rather than 1 at a time.

Statement

Noah has 4 bags of marbles with the same number in each bag. If there are 536 marbles altogether, how many marbles are in each bag? Explain how you know.

Solution

There are 134 marbles in each bag.

There are 4 equal groups. Put 100 marbles into each group first, then you have 136 left. If you unbundle 100 into 10 tens, you will have 13 tens, and 3 tens can go in each group. You will have 1 ten left that can be unbundled into 10 ones. With 16 ones, 4 can go in each group.

Aligned Standards

4.NBT.B.6

Problem 7

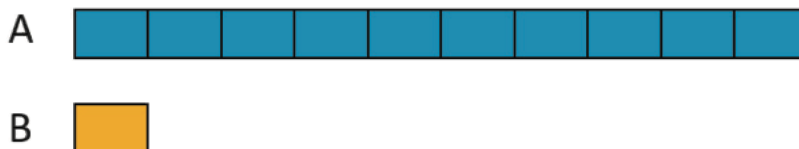
The content assessed in this problem is first encountered in Lesson 5: Decimal Points in Products.

Students who select D only may not understand that multiplying by $\frac{1}{10}$ is equivalent to dividing by 10. Students selecting F are making a common type of mistake: they are dividing by $\frac{1}{10}$ when they really mean to divide by 10.

If most students struggle with this item, plan to do Lesson 5 Activity 2, Fractionally Speaking: Powers of 10, launching it by doing some error analysis on this item from the Pre-Unit Diagnostic.

Statement

Rectangle A is 10 times as long as rectangle B.



If rectangle A is 80 units long, select **all** the ways find the length of rectangle B.

- A. Multiply 80 by 10
- B. Multiply 80 by $\frac{1}{10}$
- C. Multiply 10 by 80
- D. Divide 80 by 10
- E. Divide 10 by 80
- F. Divide 80 by $\frac{1}{10}$

Solution

["B", "D"]

Aligned Standards

5.NF.B.7

Arithmetic in Base Ten

Assessment : Check Your Readiness (B)

Teacher Instructions

This assesses students' incoming knowledge about concepts that support understanding of operations with base-ten numbers.

Problem 1

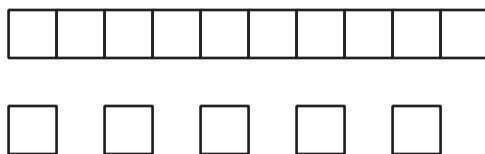
The content assessed in this problem is first encountered in Lesson 3: Adding and Subtracting Decimals with Few Non-Zero Digits.

This problem reintroduces students to base-ten diagrams, which are used throughout this unit. Students should understand that the unit square can have any value, but each strip of ten squares will have ten times this value. They should also understand that shifting digits to the left—for example, 150 vs. 1500—means that each digit has ten times its previous value.

If most students struggle with this item, plan to do the optional Lesson 2, and the optional Lesson 3 Activity 2.

Statement

This diagram shows five small squares and one rectangle composed of 10 small squares.



1. Andre says this diagram can represent 1,500. What does a small square represent for Andre?
2. Name a number greater than 1,500 that this diagram can also represent.
3. Clare says this diagram can represent 1.5. What does a small square represent for Clare?
4. Name a number less than 1.5 that this diagram can also represent.

Solution

1. 100

2. Answers vary. Sample responses: 15,000; 150,000; 3000

3. 0.1

4. Answers vary. Sample responses: 0.15; 0.015; 0.0015; 0.75

Aligned Standards

5.NBT.A.1

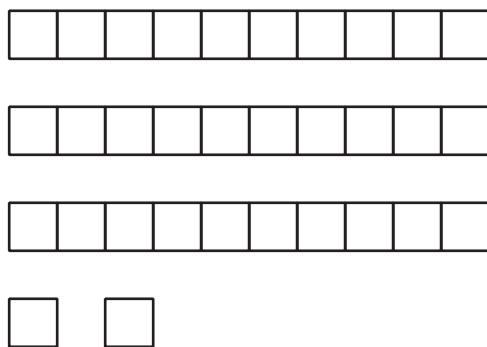
Problem 2

The main idea in this problem is that a digit in the hundreds place counts ten times more than a digit in the tens place. (And a digit in the thousands place counts ten times more than a digit in the hundreds place, etc.) Since there are three bundles of ten squares, but two individual squares, that means that the 3 needs to go immediately to the left of the 2.

If most students struggle with this item, plan to do the optional Lesson 2 Activity 1, Changing Values.

Statement

This diagram shows two small squares and three rectangles each composed of 10 small squares.



1. Priya says that this diagram can represent 320. Do you agree with Priya? Explain why or why not.
2. What is another value this diagram could represent? How do you know?

Solution

1. Yes. If a small square represents 10, then the rectangle represents $10 \cdot 10 = 100$. The three rectangles represent $3 \cdot 100 = 300$, and the whole diagram represents $300 + 10 + 10 = 320$.
2. Answers vary. The value selected should show understanding that the rectangles of 10 small squares are each ten times larger than 1 small individual square. Possible responses:
 - a. 3,200 where the small square represents 100.

b. 32,000, where the small square represents 1,000.

Aligned Standards

5.NBT.A.1

Problem 3

The content assessed in this problem is first encountered in Lesson 5: Decimal Points in Products.

In this problem, students convert fractions to decimals, where each fraction has a denominator that is a power of 10. Students may use their understanding of place value to do this, or they may divide each numerator by the denominator.

If most students do well with this item, it may be possible to skip Lesson 5 Activity 2, Fractionally Speaking: Powers of 10.

Statement

Here are some fractions:

$$\frac{820}{100}, \frac{820}{1000}, \frac{82}{1000}, \frac{820}{10}, \frac{82}{10}, \frac{82}{100}$$

1. Select **all** the fractions that are equal to 8.2.
2. Select **all** the fractions that are equal to 0.082.

Solution

1. $\frac{82}{10}, \frac{820}{100}$
2. $\frac{82}{1000}$

Aligned Standards

4.NF.C.6

Problem 4

The content assessed in this problem is first encountered in Lesson 1: Using Decimals in a Shopping Context.

This problem assesses all four arithmetic operations with decimals: addition, subtraction, multiplication, and division. If most students do well with this item, plan to connect place value diagrams to pennies, dimes, and dollars in Lesson 3.

Statement

1. Lin had \$24.16 and spent \$8.50. How much did she have left?
2. Elena had \$6.75 and Tyler had \$7.85. How much did they have altogether?

3. Three friends did a job together that paid \$65.40. How much should each get if they all get an equal amount?
4. Diego deposited \$36.50 into his bank account each week for 7 weeks. What was the total amount he deposited?

Solution

1. \$15.66
2. \$14.60
3. \$21.80
4. \$255.50

Aligned Standards

5.NBT.B

Problem 5

The content assessed in this problem is first encountered in Lesson 7: Using Diagrams to Represent Multiplication.

Students will need to know how to perform multi-digit multiplication fluently before generalizing this work to decimal multiplication.

If most students struggle with this item, plan to do the optional Lesson 7 Activity 2, Connecting Area Diagrams to Calculations with Whole Numbers.

Statement

Compute $(3,902) \cdot 85$.

Solution

331,670

Aligned Standards

5.NBT.B.5

Problem 6

The content assessed in this problem is first encountered in Lesson 9: Using the Partial Quotients Method.

This problem assesses fluency in and understanding of whole-number division, another prerequisite skill for this unit. Students may explain the problem conceptually, as in the solution, or they may take a more formulaic approach.

If most students struggle with this item, plan to do some additional activities involving fair-sharing, including using manipulatives. Focus on dividing large numbers and ways to make the sharing easier, such as giving each person 10 at a time rather than 1 at a time.

Statement

Kiran has 3 containers of blocks with the same number in each container. If there are 852 blocks altogether, how many blocks are in each container? Explain how you know.

Solution

There are 284 building blocks in each container. Sample reasoning: There are 3 equal groups. First, put 200 blocks into each group, then you have 252 left. If you unbundle 250 into tens, you will have 25 tens, and 8 tens can go in each group. You will have 1 ten left that can be unbundled into 10 ones. That gives a total of 12 ones, so 4 can go in each group.

Aligned Standards

4.NBT.B.6

Problem 7

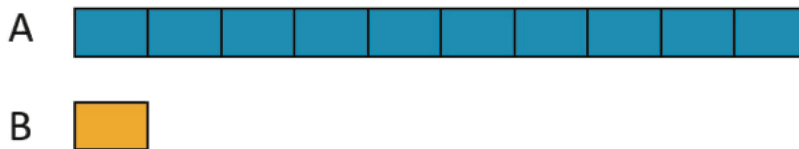
The content assessed in this problem is first encountered in Lesson 5: Decimal Points in Products.

Students who select B only may not understand that multiplying by $\frac{1}{10}$ is equivalent to dividing by 10. Students selecting D are making a common type of mistake—they are dividing by $\frac{1}{10}$ when they really mean to divide by 10.


If most students struggle with this item, plan to do Lesson 5 Activity 2, Fractionally Speaking: Powers of 10, launching it by doing some error analysis on this item from the Pre-Unit Diagnostic.

Statement

Rectangle A is 10 times as long as Rectangle B.



If Rectangle A is 50 units long, select **all** the ways to find the length of Rectangle B.

- 
- A. Multiply 50 by 10.
 - B. Divide 50 by 10.
 - C. Multiply 50 by $\frac{1}{10}$.
 - D. Divide 50 by $\frac{1}{10}$.
 - E. Multiply 10 by 50.
 - F. Divide 10 by 50.

Solution

["B", "C"]

Aligned Standards

5.NF.B.7

Arithmetic in Base Ten

Assessment : Mid-Unit Assessment (A)

Teacher Instructions

Give this assessment after lesson 8. Calculators should not be used.

Student Instructions

Calculators should not be used.

Problem 1

Students estimate a product of complicated decimals, and are not intended to calculate the product directly. The numbers are sufficiently complicated that finding the product will be time consuming. Students can reason that the product is less than $3 \cdot 2$, which eliminates all but choices A and B. The product is close to $3 \cdot 1.8 = 5.4$ which indicates that B is correct.

Statement

Which estimate is closest to the actual value of $(2.99548) \cdot (1.8342)$?

- A. 4.8
- B. 5.5
- C. 6.2
- D. 8.3

Solution

B

Aligned Standards

6.NS.B

Problem 2

This problem requires students to add decimals while paying careful attention to place value. Students selecting A have used subtraction instead of addition. Students selecting B or D have not lined up the decimal places correctly.

Statement

For part of a science experiment, Andre adds 0.25 milliliters of dye to 12.3 milliliters of water. How many milliliters does the mixture contain?

- A. 12.05
- B. 12.325
- C. 12.55
- D. 14.8

Solution

C

Aligned Standards

6.NS.B.3

Problem 3

Students estimate or perform addition and subtraction of decimals. Students selecting B may be thinking of $2.97 + 0.04$, which is 3.01. Likewise, students will reach the wrong conclusions in C and E if they make small mistakes in lining up the decimal places or if they accidentally add or omit the digit 0. Students failing to select A may have calculated 2.07, or failed to line up the decimal places correctly. Students failing to select D may have a misunderstanding about the values after a decimal point, thinking .47 is larger than .5.

Statement

Select **all** the expressions that are greater than 3.

- A. $1.67 + 1.4$
- B. $2.97 + 0.004$
- C. $3.017 - 0.05$
- D. $4.5 - 1.47$
- E. $5.503 - 2.52$

Solution

["A", "D"]

Aligned Standards

6.NS.B

Problem 4

Students use a context to recognize when to add and subtract decimals.

Statement

Four runners are training for long races. Noah ran 5.123 miles, Andre ran 6.34 miles, Jada ran 7.1 miles, and Diego ran 8 miles.

1. What is the total running distance of the four runners?
2. Jada ran how much farther than Noah?

Solution

1. 26.563 miles
2. 1.977 miles

Aligned Standards

6.NS.B

Problem 5

Students compute the tip on a bill by using a given method of multiplying the total by a decimal. Students then explain the process they used to complete the multiplication.

Statement

One way to compute a 15% tip for a bill is to multiply it by 0.15.

A restaurant bill was \$42.40. Calculate the tip. Explain your reasoning in detail.

Solution

\$6.36. Explanations vary. Sample explanation: I multiplied 42.40 by 0.1 to get 4.24 and then found half of that to get $0.05 \cdot 42.40 = 2.12$. Then I added the two answers to get \$6.36.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: 6.36, because $42.4 \cdot 0.15 = 6.36$ using the algorithm for multiplication.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: minor incorrect arithmetic on correct setup; incorrect decimal point of product.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: choice of operations invalid; incorrect decimal point of multiplicand ($42.4 \cdot 1.5$); multiple arithmetic errors.

Aligned Standards

6.NS.B

Problem 6

This problem asks students to compute a product and a difference. While students are not required to use an efficient algorithm for subtraction, it is likely they will do so, given the format provided.

Statement

1. Find the product: $(0.061) \cdot (0.43)$
2. Find the difference:

$$\begin{array}{r} 1 \\ - 0.4308 \\ \hline \end{array}$$

Solution

1. 0.02623
2. 0.5692

Aligned Standards

6.NS.B

Problem 7

Students explain two different methods for multiplying decimals. In the first method, students multiply the digits and then move the decimal point into position based on the place in the original values. In the second method, students convert the decimals to fractions, perform the multiplication there, and then convert back to a decimal.

Statement

Two students have different ways to calculate $(0.25) \cdot (0.044)$ as a decimal. Show, in detail, how each student might compute this product.

1. Elena says, "I find $25 \cdot 44$ and then move the decimal point."
2. Diego says, "0.25 is the same as $\frac{1}{4}$, so I found $\frac{1}{4}$ of 0.044."

Solution

1. $25 \cdot 44 = 1,100$. Since 0.25 was to the hundredths place and 0.044 was to the thousandths place, the last digit of the number 1,100 should be in the hundred thousandths place, meaning we should move the decimal point 5 places to the left. The answer is 0.011.
2. $0.044 = \frac{44}{1000}$, so $\frac{1}{4}$ of $\frac{44}{1000}$ is $\frac{11}{1000}$. This is 11 thousandths, which is 0.011.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample:
 1. $25 \cdot 44 = 1,100$ and the decimal point must be moved 5 places left.
 2. $\frac{1}{4}$ of 44 is 11, so $\frac{1}{4}$ of 0.044 is 0.011.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: incomplete explanation of how $25 \cdot 44$ relates; calculation of 5 decimal places off by 1; incomplete description of how to calculate $\frac{1}{4}$ of 0.044.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: two or more error types from Tier 2 response; one omitted or nonsensical response; relationship to $25 \cdot 44$ off by 2 or more decimal places or otherwise disconnected; incorrect operation used. If either part is completely correct, the response earns at least Tier 3.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: two or more error types from Tier 3 response.

Aligned Standards

6.NS.B

Arithmetic in Base Ten

Assessment : Mid-Unit Assessment (B)

Teacher Instructions

Give this assessment after lesson 8. Calculators should not be used.

Student Instructions

Calculators should not be used.

Problem 1

Students estimate a product of complicated decimals, and are not intended to calculate the product directly. The numbers are sufficiently complicated that finding the product will be time consuming. Students can reason that the product is less than $4 \cdot 3$, which eliminates A and B. The product is close to $4 \cdot 2.7 = 10.8$ which indicates that C is correct.

Statement

Which estimate is closest to the actual value of $(3.99437) \cdot (2.6851)$?

- A. 14.2
- B. 12.6
- C. 10.8
- D. 8.3

Solution

C

Aligned Standards

6.NS.B

Problem 2

Students estimate or perform addition and subtraction of decimals. Students selecting A may be thinking of $4.88 + 0.9$, which is 5.78. Likewise, students will reach the wrong conclusions in C and D if they make small mistakes in lining up the decimal places or if they accidentally add or omit the digit 0. Students failing to select B may have calculated 4.06, or failed to line up the decimal places correctly. Students failing to select E may have a misunderstanding about the values after a decimal point, thinking 0.49 is larger than 0.5.

Statement

Select **all** the expressions that are greater than 5.

- A. $4.88 + 0.009$
- B. $3.76 + 1.3$
- C. $5.012 - 0.04$
- D. $7.702 - 2.71$
- E. $6.5 - 1.49$

Solution

["B", "E"]

Aligned Standards

6.NS.B

Problem 3

Students use a context to recognize when to add and subtract decimals. Students selecting B have been misled by the fact that $9 - 7 = 2$. Students selecting C have added rather than subtracted. Students selecting D have concatenated the decimal parts of the two numbers to be added.

Statement

Mai's backpack weighs 9 pounds. Han's backpack weighs 8.3 pounds. Priya's backpack weighs 7.75 pounds, and Kiran's backpack weighs 6.125 pounds. Select **all** the true statements.

- A. Han's backpack is 2.175 pounds heavier than Kiran's.
- B. Mai's backpack is 2.25 pounds heavier than Priya's.
- C. Han's backpack is 16.05 pounds heavier than Priya's.
- D. Kiran's backpack and Han's backpack together weigh 14.3125 pounds.
- E. The backpacks weigh 31.175 pounds together.

Solution

["A", "E"]

Aligned Standards

6.NS.B

Problem 4

This problem requires students to add decimals while paying careful attention to place value.

Statement

For part of a recipe, Jada adds 0.35 milliliters of vanilla extract to 16.4 milliliters of milk. How many milliliters does the mixture contain?

Solution

16.75 milliliters

Aligned Standards

6.NS.B.3

Problem 5

Students compute the discount on a pair of sneakers by using a given method of multiplying the total by a decimal. Students explain the process they used to complete the multiplication.

Statement

One way to compute a 20% discount is to multiply the price by 0.20. The price of a pair of sneakers is \$84.50. Calculate the discount. Explain or show your reasoning.

Solution

\$16.90 discount. Explanations vary. Sample explanation: I multiplied \$84.50 by 0.1 to get \$8.45. Since the discount was double that amount, I multiplied \$8.45 by 2 to get \$16.90.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: \$16.90, because $84.50 \cdot 0.20 = 16.90$ using the algorithm for multiplication.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: minor incorrect arithmetic on correct setup; incorrect decimal point of product.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: choice of operations invalid; incorrect decimal point of multiplicand $84.5 \cdot 0.02$; multiple arithmetic errors.

Aligned Standards

6.NS.B

Problem 6

This problem asks students to compute a product and a difference. While students are not required to use an efficient algorithm for subtraction, it is likely they will do so, given the format provided.

Statement

1. Find the difference:

$$\begin{array}{r} 1 \\ - 0.5904 \\ \hline \end{array}$$

2. Find the product: $(0.072) \cdot (0.31)$

Solution

1. 0.4096
2. 0.02232

Aligned Standards

6.NS.B

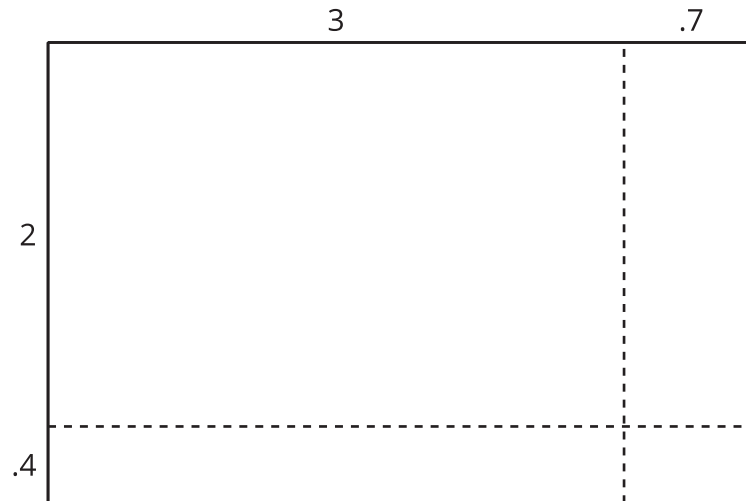
Problem 7

Students explain two different methods for multiplying decimals. In the first method, students multiply the digits and then move the decimal point into position based on the place in the original values. In the second method, students convert the decimals to fractions, perform the multiplication there, and then convert back to a decimal.

Statement

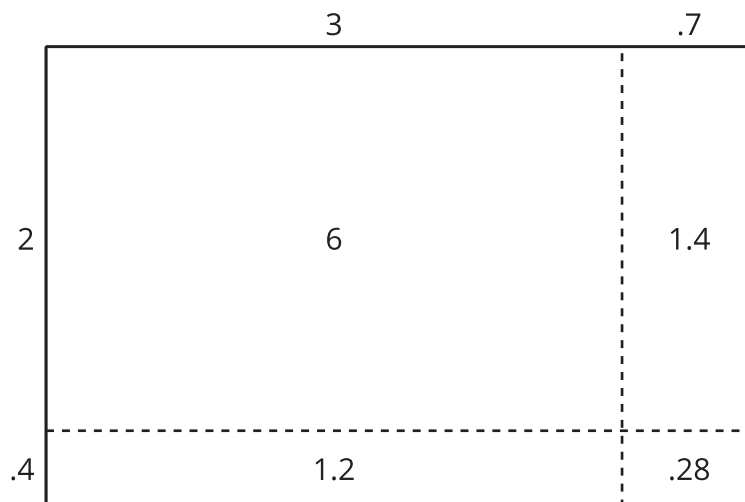
Two students have different ways to calculate $(2.4) \cdot (3.7)$. Show, in detail, how each student might compute this product.

1. Andre says, "I find $24 \cdot 37$ and then move the decimal point."
2. Lin says, "I drew this diagram and then found the area of each piece."



Solution

1. $24 \cdot 37 = 888$. Since 2.4 was to the tenths place and 3.7 was also to the tenths place, the last digit of the number 888 should be in the hundredths place, meaning that we should move the decimal two places to the left. The answer is 8.88.
2. The areas of the rectangles in the diagram are 6, 1.4, 1.2, and 0.28. Their sum is 8.88.



Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample:
- $24 \cdot 37 = 888$ and the decimal point must be moved 2 places left.

- $6 + 1.4 + 1.2 + 0.28 = 8.88$.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: incomplete explanation of how $24 \cdot 37$ relates; calculation of 2 decimal places off by 1; one or two errors in calculating the areas of the pieces of the rectangle.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: two or more error types from Tier 2 response; one omitted or nonsensical response; relationship to $24 \cdot 37$ off by 2 or more decimal places or otherwise disconnected; incorrect operation used. If either part is completely correct, the response earns at least Tier 3.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: two or more error types from Tier 3 response.

Aligned Standards

6.NS.B

Arithmetic in Base Ten

Assessment : End-of-Unit Assessment (A)

Teacher Instructions

Calculators should not be used.

Student Instructions

Calculators should not be used.

Problem 1

Students divide a decimal by a whole number in context. The method is not specified. Students selecting A placed the decimal point incorrectly in the quotient. Students selecting C or D multiplied instead of dividing.

Statement

A woodworker wants to cut a board that is 8.225 feet long into 5 equal-length pieces. How long will each of the cut boards be?

- A. 0.1645 feet
- B. 1.645 feet
- C. 4.1125 feet
- D. 41.125 feet

Solution

B

Aligned Standards

6.NS.B.2, 6.NS.B.3

Problem 2

Students examine different quotients of decimals and identify those that have the same value. The key here is thinking about multiplying both numbers of the quotient by the same power of 10: this does not change the quotient. Students selecting C or D have used incorrect powers of 10. Students selecting B may have a deeper misunderstanding of division, since this choice adds the same amount (2) to both numbers. Students failing to select A or E have made an error in counting digits or (in the case of E) using the decimal point correctly.

Statement

Select **all** of the expressions that have the same value as $892 \div 8$.

- A. $8920 \div 80$
- B. $894 \div 10$
- C. $89.2 \div 0.08$
- D. $8.92 \div 0.8$
- E. $0.892 \div 0.008$

Solution

["A", "E"]

Aligned Standards

6.NS.B.3

Problem 3

Students estimate a quotient of decimals. While they could find the exact answer, the numbers are not friendly and the expectation is that students will estimate and use their knowledge of place value. The quotient $4,367 \div 0.004$ has the same value as $4,367,000 \div 4$, which they can estimate more readily.

Students selecting A may just be dividing by 4, without paying attention to decimal places. Students selecting B or C are making mistakes related to place value. Some students may look at the two zeros after the decimal point in 0.004 and think that the decimal point should move two places to the right instead of three: these students would select C.

Statement

Which is closest to the quotient $4,367 \div 0.004$?

- A. 1,000
- B. 10,000
- C. 100,000
- D. 1,000,000

Solution

D

Aligned Standards

6.NS.B.2

Problem 4

Students multiply a decimal by a fraction in a unit conversion context. They have multiple options available to solve this problem:

- Convert 2.54 to a fraction and use multiplication of fractions (which they may or may not convert back to a decimal).
- Convert $\frac{1}{4}$ to a decimal and multiply two decimals.
- Calculate $2.54 \div 4$.

All three methods are equally effective.

Statement

One way to convert from inches to centimeters is to multiply the number of inches by 2.54.
How many centimeters are there in $\frac{1}{4}$ inch?

Solution

0.635 centimeters or equivalent

Aligned Standards

6.NS.B.2, 6.NS.B.3

Problem 5

This problem assesses students' recent work with the long division algorithm. Since a method is specified, students' work using other methods should not be given full credit.

Statement

Find each quotient using long division.

1. $2,247 \div 7$

2. $676 \div 13$

Solution

1. 321

2. 52

$$\begin{array}{r}
 \overline{) 2247} \\
 \underline{- 21} \\
 14 \\
 \underline{- 14} \\
 07 \\
 \underline{- 7} \\
 0
 \end{array}$$

$$\begin{array}{r}
 \overline{) 676} \\
 \underline{- 65} \\
 26 \\
 \underline{- 26} \\
 0
 \end{array}$$

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: see diagram.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Acceptable errors: a small mistake early on results in different calculations (with no other errors).
- Sample errors: One or two arithmetic mistakes, such as subtracting incorrectly; one or two mistakes in the algorithm such as bringing down the wrong number, provided there is evidence elsewhere in the work that the student knows how to do this; use of the method of partial quotients rather than long division.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: correct answers found by a different method, such as guess-and-check multiplication; consistent mistakes with the algorithm; work is only partially completed.

Aligned Standards

6.NS.B.2

Problem 6

Students calculate quotients of whole numbers resulting in decimals in context, then compare the resulting decimals to look for the best deal.

In addition to using long division, a student might strategically notice that equivalent fractions are very effective for the third calculation: $\frac{129}{50} = \frac{258}{100}$ gives the unit price of \$2.58 for 50 cupcakes.

Statement

A sign in a bakery gives these options:

- 12 cupcakes for \$29
 - 24 cupcakes for \$56
 - 50 cupcakes for \$129
1. Find each unit price to the nearest cent, and show your reasoning.
 2. Which option gives the lowest unit price?

Solution

1. The unit prices are \$2.42, \$2.33, and \$2.58. Reasoning varies, but most students will use long division.
2. 24 cupcakes for \$56 has the lowest unit price.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: (Accompanied by work showing long division or other calculation methods.) The unit prices are \$2.42, \$2.33, and \$2.58. 24 cupcakes for \$56 is the best deal.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Acceptable errors: incorrect selection of lowest unit rate comes from errors in calculation of unit rates.
- Sample errors: one or two errors in long division; incorrect selection of the best deal despite having calculated all three unit rates correctly.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: calculation of unit rates involves a conceptual error, such as dividing the number of cupcakes by the price; three or more errors in long division; correct selection of the best deal without calculation of each unit rate.

Aligned Standards

6.RP.A.3.b

Problem 7

In the second part of this problem, students will need to divide a decimal by a whole number. The numbers are chosen so that partial quotients or long division are the most likely effective choices. One challenge with this problem will be placing the decimal correctly in the quotient.

Statement

A stack of 500 pieces of paper is 1.875 inches tall.

1. Diego guesses that each piece of paper is 0.015 inches thick. Explain how you know that Diego's answer is *not* correct.
2. Compute the thickness of each piece of paper. Show your reasoning.

Solution

1. If each piece of paper were 0.015 inches thick, the stack would be 7.5 inches high:
 $500 \cdot (0.015) = 7.5$.
2. 0.00375 inches, because $1.875 \div 500 = 0.00375$.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample:
 1. This can't be right, because $500 \cdot (0.015)$ is 7.5.
 2. $1.875 \div 500$ is $(1.875) \cdot 2 \div 1000$, which is $3.75 \div 1000$. The decimal point moves 3 places to the left, so each piece is 0.00375 inches thick.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: arithmetic error in multiplication or division; result off by one decimal place but otherwise correct.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: invalid or omitted work on one of the two problem parts; major error in division; division result off by more than one decimal place; incorrect choice of operation.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: two or more error types from Tier 3 response.

Aligned Standards

6.NS.B.2

Arithmetic in Base Ten

Assessment : End-of-Unit Assessment (B)

Teacher Instructions

Calculators should not be used.

Student Instructions

Calculators should not be used.

Problem 1

Students divide a decimal by a whole number in context. The method is not specified. Students selecting A multiplied instead of dividing. Students selecting B multiplied instead of dividing, and placed the decimal incorrectly in the quotient. Students selecting D placed the decimal point incorrectly in the quotient.

Statement

A runner is preparing for a marathon that is cut into 5 equal sections. The marathon is 42.195 kilometers long. How long is each section of the marathon?

- A. 210.975 km
- B. 21.0975 km
- C. 8.439 km
- D. 0.8439 km

Solution

C

Aligned Standards

6.NS.B.2, 6.NS.B.3

Problem 2

Students examine different quotients of decimals and identify those that have the same value. One strategy is to think about multiplying both numbers of the quotient by the same power of 10 to result in a more familiar division problem. Students selecting A may be working too fast or failing to use number sense to check their answers. Students failing to select B, D, or E or selecting C may have made errors shifting the decimal. Students failing to select D or E may have rejected it out of hand since $25 \div 5 = 5$.

Statement

Select **all** of the expressions that are equal to 2.

- A. $1,200 \div 60$
- B. $1.2 \div 0.6$
- C. $0.012 \div 0.06$
- D. $0.5 \div 0.25$
- E. $0.05 \div 0.025$

Solution

["B", "D", "E"]

Aligned Standards

6.NS.B.3

Problem 3

Students estimate a quotient of decimals. While they could find the exact answer, the numbers are not friendly and the expectation is that students will estimate and use their knowledge of place value. The quotient $5,278 \div 0.05$ has the same value as $527,800 \div 5$, which they can estimate more readily. Students selecting A or C are making mistakes related to place value. Students selecting D may just be dividing by 5, without paying attention to place value. Some students may look at the one zero after the decimal point in 0.05 and think that the decimal point should move one place to the right instead of two: these students would select C.

Statement

Which is closest to the quotient $5,278 \div 0.05$?

- A. 1,000,000
- B. 100,000
- C. 10,000
- D. 1,000

Solution

B

Aligned Standards

6.NS.B.2

Problem 4

Students multiply a decimal by a fraction. They have multiple options available to solve this problem. All three methods are equally effective.

- Convert 4.12 to a fraction and use multiplication of fractions (which they may or may not convert back to a decimal).
- Convert $\frac{1}{5}$ to a decimal and multiply two decimals.
- Calculate $4.12 \div 5$.

Statement

It takes 4.12 gallons of paint to paint a fence. How much paint is needed for $\frac{1}{5}$ of the fence?

Solution

0.824 gallons or equivalent

Aligned Standards

6.NS.B.2, 6.NS.B.3

Problem 5

This problem assesses students' work with the long division algorithm. Since a method is specified, solutions using other methods should not be given full credit.

Statement

Find each quotient using long division.

1. $3,328 \div 8$

2. $868 \div 14$

Solution

1. 416

2. 62

$$\begin{array}{r}
 416 \\
 8 \overline{)3328} \\
 \underline{32} \\
 12 \\
 \underline{8} \\
 48 \\
 \underline{48} \\
 0
 \end{array}$$

$$\begin{array}{r}
 62 \\
 14 \overline{)868} \\
 \underline{84} \\
 28 \\
 \underline{28} \\
 0
 \end{array}$$

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: see diagram.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Acceptable errors: a small mistake early on results in different calculations (with no other errors).
- Sample errors: One or two arithmetic mistakes, such as subtracting incorrectly; one or two mistakes in the algorithm such as bringing down the wrong number, provided there is evidence elsewhere in the work that the student knows how to do this; use of the method of partial quotients rather than long division.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: correct answers found by a different method, such as guess-and-check multiplication; consistent mistakes with the algorithm; work is only partially completed.

Aligned Standards

6.NS.B.2

Problem 6

Students calculate quotients of whole numbers resulting in decimals in context, then compare the resulting decimals to look for the best deal. In addition to using long division, a student might strategically notice that equivalent fractions are very effective for the third calculation: $\frac{94}{50} = \frac{188}{100}$ gives the unit price of \$1.88 for 50 muffins.

Statement

A sign in a bakery gives these options:

- 12 muffins for \$25
- 24 muffins for \$46
- 50 muffins for \$94

1. Find each unit price to the nearest cent, and show your reasoning.
2. Which option gives the lowest unit price?

Solution

1. The unit prices are \$2.08, \$1.92, and \$1.88. Reasoning varies, but long division is a reasonable approach.
2. 50 muffins for \$94 has the lowest unit price.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: (Accompanied by work showing long division or other calculation methods.) The unit prices are \$2.08, \$1.92, and \$1.88. 50 muffins for \$94 is the best deal.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Acceptable errors: incorrect selection of lowest unit rate comes from errors in calculation of unit rates.
- Sample errors: one or two errors in long division; incorrect selection of the best deal despite having calculated all three unit rates correctly.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: calculation of unit rates involves a conceptual error, such as dividing the number of muffins by the price; three or more errors in long division; correct selection of the best deal without calculation of each unit rate.

Aligned Standards

6.RP.A.3.b

Problem 7

In the second part of this problem, students will need to divide a decimal by a whole number. The numbers are chosen so that partial quotients or long division are the most likely effective choices. One challenge with this problem will be placing the decimal correctly in the quotient.

Statement

A stack of 200 file folders is 5.125 inches tall.

1. Elena guesses that each file folder is 0.035 inches thick. Explain how you know that Elena's answer is *not* correct.
2. Compute the thickness of each file folder. Show your reasoning.

Solution

1. If each folder were 0.035 inches thick, the stack would be 7 inches high: $200 \cdot (0.035) = 7$.
2. 0.025625 inches, because $5.125 \div 200 = 0.025625$.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample:
 - This can't be right, because $200 \cdot (0.035)$ is 7.
 - $5.125 \div 200$ is $(5.125 \cdot 5) \div 1,000$, which is $25.625 \div 1,000$. The decimal point moves 3 places to the left, so each folder is 0.025625 inches thick.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: arithmetic error in multiplication or division; result off by one decimal place but otherwise correct.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: invalid or omitted work on one of the two problem parts; major error in division; division result off by more than one decimal place; incorrect choice of operation.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: two or more error types from Tier 3 response.

Aligned Standards

6.NS.B.2

Lesson Cool Downs

Lesson 1: Using Decimals in a Shopping Context

Cool Down: How Did You Compute With Decimals?

Planning your menu involved many calculations with decimals. Reflect on how you made these calculations:

1. How did you compute sums of dollar amounts that were not whole numbers? For example, how did you compute the sum of \$5.89 and \$1.45? Use this example to explain your strategy.
2. How did you compute products of dollar amounts that were not whole numbers? For example, how did you compute the cost of 4 pounds of beef at \$5.89 per pound? Use this example to explain your strategy.

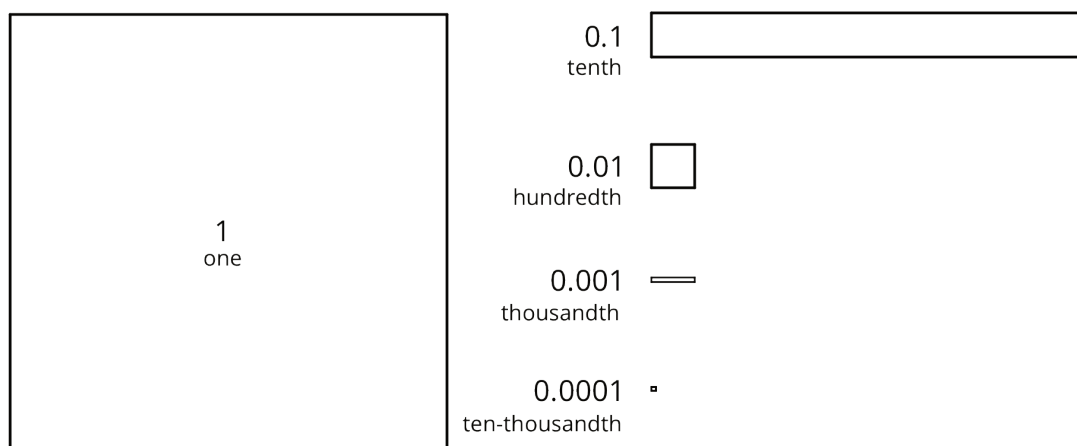
Lesson 2: Using Diagrams to Represent Addition and Subtraction

Cool Down: Why or Why Not?

Is this equation true?

$$0.025 + 0.17 = 0.042$$

Use a diagram or numerical calculation to explain or show your reasoning. Here are diagrams that you could use to represent base-ten units.



Lesson 3: Adding and Subtracting Decimals with Few Non-Zero Digits

Cool Down: Calculate the Difference

1. Find the sum $1.56 + 0.083$. Show your reasoning.
2. Find the difference $0.2 - 0.05$. Show your reasoning.
3. You need to be at least 39.37 inches tall (about a meter) to ride on a bumper car. Diego's cousin is 35.75 inches tall. How many more inches will he need to grow before Diego can take him on the bumper car ride? Explain or show your reasoning.

Lesson 4: Adding and Subtracting Decimals with Many Non-Zero Digits

Cool Down: Taller and Farther

1. Diego is 59.5 inches tall. His brother is 40.125 inches tall. How much taller than his brother is Diego? Show your reasoning.

2. A runner has run 1.192 kilometers of a 10-kilometer race. How much farther does he need to run to finish the race? Show your reasoning.

Lesson 5: Decimal Points in Products

Cool Down: Placing Decimal Points in Products

1. Use what you know about decimals or fractions to explain why $(0.2) \cdot (0.002) = 0.0004$.
2. A rectangular plot of land is 0.4 kilometer long and 0.07 kilometer wide. What is its area in square kilometers? Show your reasoning.

Lesson 6: Methods for Multiplying Decimals

Cool Down: Finding Products of Decimals

1. Use the equation $135 \cdot 42 = 5,670$ and what you know about fractions, decimals, and place value to explain how to place the decimal point when you compute $(1.35) \cdot (4.2)$.
2. Which of the following is the correct value of $(0.22) \cdot (0.4)$? Show your reasoning.
 - a. 8.8
 - b. 0.88
 - c. 0.088
 - d. 0.0088

Lesson 7: Using Diagrams to Represent Multiplication

Cool Down: Find the Product

Find $(4.2) \cdot (1.6)$ by drawing an area diagram or using another method. Show your reasoning.

Lesson 8: Calculating Products of Decimals

Cool Down: Calculate This!

Calculate $(1.6) \cdot (0.215)$. Show your reasoning.

Lesson 9: Using the Partial Quotients Method

Cool Down: Dividing by 11

Calculate $4,235 \div 11$ using any method.

Lesson 10: Using Long Division

Cool Down: Dividing by 15

Use long division to find the value of $1,875 \div 15$.

Lesson 11: Dividing Numbers that Result in Decimals

Cool Down: Calculating Quotients

Use long division to find each quotient. Show your computation and write your answer as a decimal.

1. $22 \div 5$

2. $7 \div 8$

Lesson 12: Dividing Decimals by Whole Numbers

Cool Down: The Same Quotient

1. Use long division to find the value of $43.5 \div 3$. If you get stuck, you can draw base-ten diagrams. Be sure to say what each type of figure represents in your diagrams.

2. Explain why all of these expressions have the same value.

$$100 \div 5$$

$$10 \div 0.5$$

$$1 \div 0.05$$

Lesson 13: Dividing Decimals by Decimals

Cool Down: The Quotient of Two Decimals

1. Write two division expressions that have the same value as $36.8 \div 2.3$.
2. Find the value of $36.8 \div 2.3$. Show your reasoning.

Lesson 14: Using Operations on Decimals to Solve Problems

Cool Down: Middle School Hurdle Race

Andre is running in an 80-meter hurdle race. There are 8 equally-spaced hurdles on the race track. The first hurdle is 12 meters from the start line and the last hurdle is 15.5 meters from the finish line.

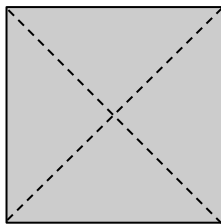
1. Estimate how far the hurdles are from one another. Explain your reasoning.
2. Calculate how far the hurdles are from one another. Show your reasoning.

Instructional Masters

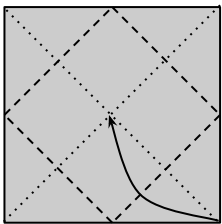
Instructional Masters for Arithmetic in Base Ten

address	title	students per copy	written on?	requires cutting?	card stock recommended?	color paper recommended?
Activity Grade6.5.2.2	Squares and Rectangles	1	no	yes	yes	no
Activity Grade6.5.15.1	Folding Paper Boxes	1	no	no	no	no

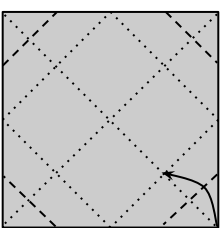
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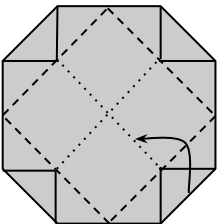
1. Start with a square sheet of paper. Fold it in half along each diagonal (the dashed lines), make a crease, and then unfold.



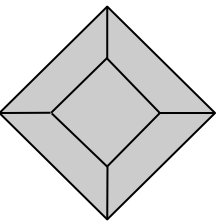
2. Fold one corner into the center, make a crease, and unfold. Repeat with the remaining three corners.



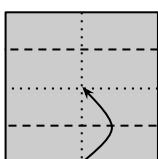
3. Fold the four corners to the crease line you made in the last step. Do not unfold!



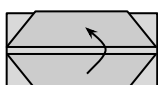
4. Re-fold along the creases you made in step 2.



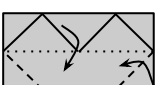
5. It should look like this now. Flip the whole thing over, and rotate it so that it looks like a square instead of a diamond.



6. Fold the left edge and the right edge to the center crease.



7. Fold over just the top layer from right to left.



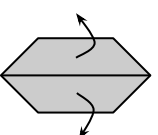
8. Fold in the top right corner and the bottom right corner. Then, flip the two top flaps from left to right.



9. Repeat the process on the left side. Fold in the top left corner and the bottom left corner.



10. Fold the top flap of paper back from right to left.



11. Pull the top layers away from each other, and allow your box to become three dimensional. Crease the edges to make it look like a box!

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