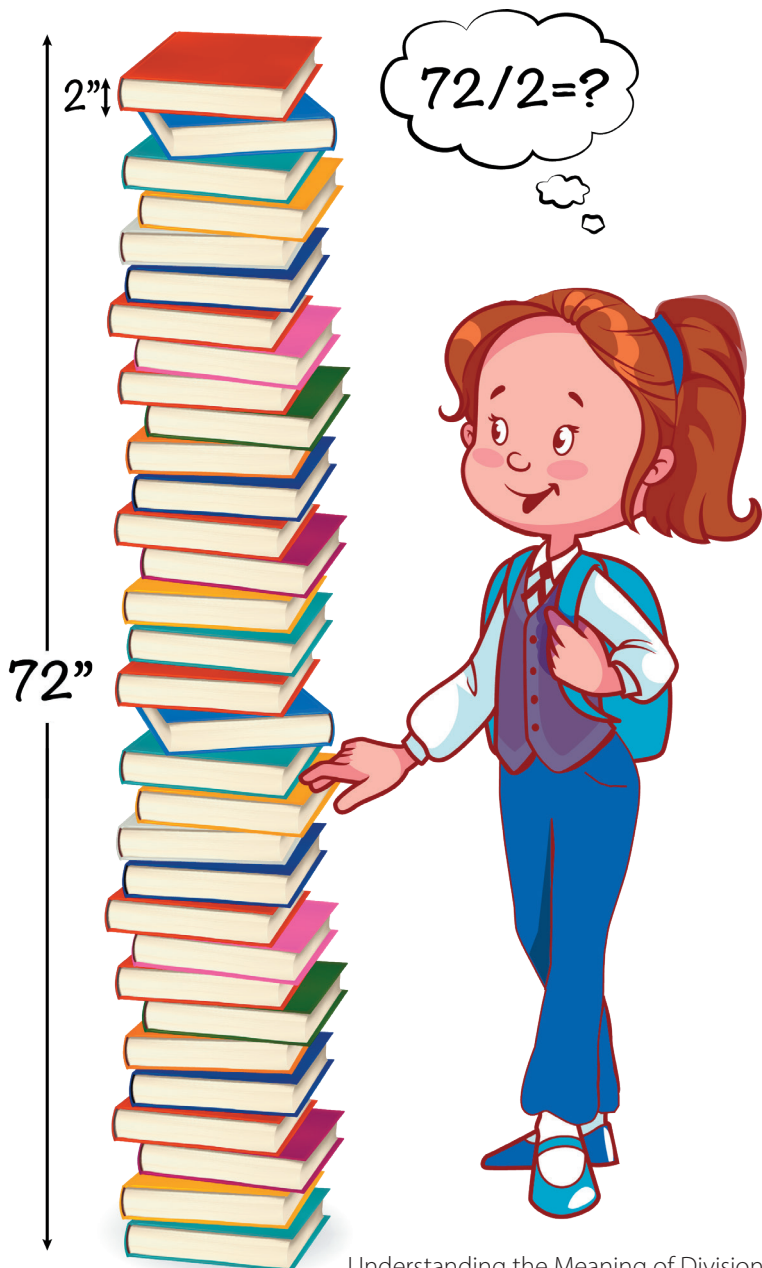
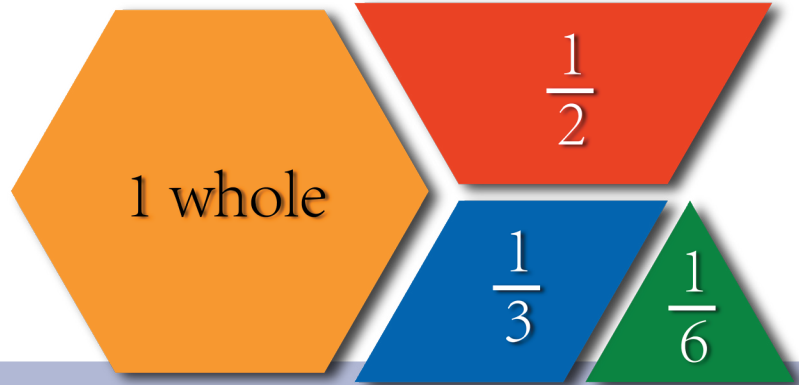
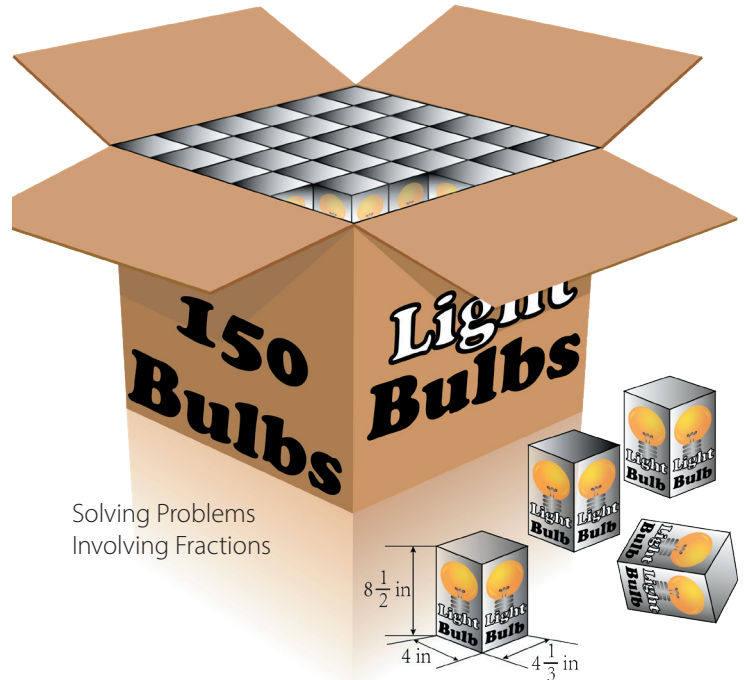


Dividing Fractions

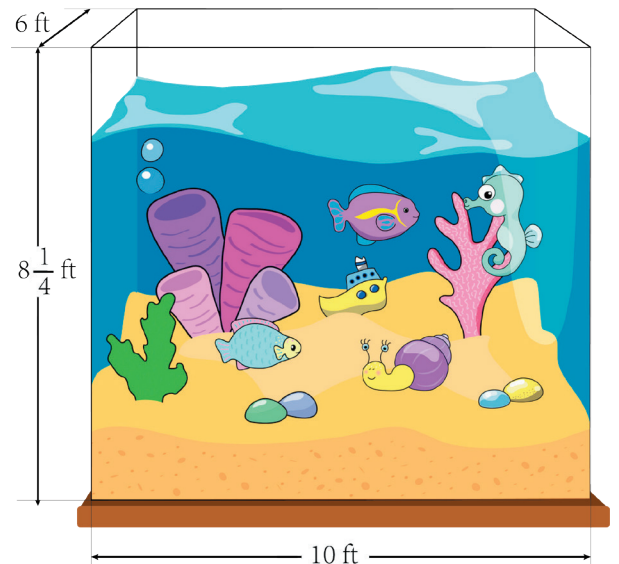
Student Workbook



Understanding the Meaning of Division



Solving Problems Involving Fractions



Calculating the Volume of Prisms

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Dividing Fractions

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Dividing Fractions Student Workbook

Core Knowledge Mathematics™

Lesson 1: Size of Divisor and Size of Quotient

1.1: Number Talk: Size of Dividend and Divisor

Find the value of each expression mentally.

$$5,000 \div 5$$

$$5,000 \div 2,500$$

$$5,000 \div 10,000$$

$$5,000 \div 500,000$$

1.2: All Stacked Up

- Here are several types of objects. For each type of object, estimate how many are in a stack that is 5 feet high. Be prepared to explain your reasoning.

Cardboard boxes



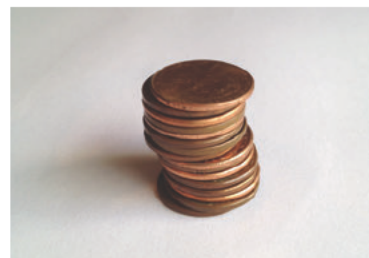
Notebooks



Bricks



Coins



2. A stack of books is 72 inches tall. Each book is 2 inches thick. Which expression tells us how many books are in the stack? Be prepared to explain your reasoning.

$72 \cdot 2$

$72 - 2$

$2 \div 72$

$72 \div 2$

3. Another stack of books is 43 inches tall. Each book is $\frac{1}{2}$ -inch thick. Write an expression that represents the number of books in the stack.

1.3: All in Order

Your teacher will give you two sets of papers with division expressions.

1. Without computing, estimate the quotients in each set and order them from greatest to least. Be prepared to explain your reasoning.

Pause here for a class discussion.

Record the expressions in each set in order from the greatest value to the least.

a. Set 1

b. Set 2

2. Without computing, estimate the quotients and sort them into the following three groups. Be prepared to explain your reasoning.

$30 \div \frac{1}{2}$

$9 \div 10$

$18 \div 19$

$15,000 \div 1,500,000$

$30 \div 0.45$

$9 \div 10,000$

$18 \div 0.18$

$15,000 \div 14,500$

Close to 0

Close to 1

Much larger than 1

Are you ready for more?

Write 10 expressions of the form $12 \div ?$ in a list ordered from least to greatest. Can you list expressions that have value near 1 without equaling 1? How close can you get to the value 1?

Lesson 1 Summary

Here is a division expression: $60 \div 4$. In this division, we call 60 the *dividend* and 4 the *divisor*. The result of the division is the quotient. In this example, the quotient is 15, because $60 \div 4 = 15$.

We don't always have to make calculations to have a sense of what a quotient will be. We can reason about it by looking at the size of the dividend and the divisor. Let's look at some examples.

- In $100 \div 11$ and in $18 \div 2.9$ the dividend is larger than the divisor. $100 \div 11$ is very close to $99 \div 11$, which is 9. The quotient $18 \div 2.9$ is close to $18 \div 3$ or 6.

In general, when a larger number is divided by a smaller number, the quotient is greater than 1.

- In $99 \div 101$ and in $7.5 \div 7.4$ the dividend and divisor are very close to each other. $99 \div 101$ is very close to $99 \div 100$, which is $\frac{99}{100}$ or 0.99. The quotient $7.5 \div 7.4$ is close to $7.5 \div 7.5$, which is 1.

In general, when we divide two numbers that are nearly equal to each other, the quotient is close to 1.

- In $10 \div 101$ and in $50 \div 198$ the dividend is smaller than the divisor. $10 \div 101$ is very close to $10 \div 100$, which is $\frac{10}{100}$ or 0.1. The division $50 \div 198$ is close to $50 \div 200$, which is $\frac{1}{4}$ or 0.25.

In general, when a smaller number is divided by a larger number, the quotient is less than 1.

Unit 4 Lesson 1 Cumulative Practice Problems

1. Order from smallest to largest:

- Number of pennies in a stack that is 1 ft high
- Number of books in a stack that is 1 ft high
- Number of dollar bills in a stack that is 1 ft high
- Number of slices of bread in a stack that is 1 ft high

2. Use each of the numbers 4, 40, and 4000 once to complete the sentences.

- a. The value of _____ \div 40.01 is close to 1.
- b. The value of _____ \div 40.01 is much less than 1.
- c. The value of _____ \div 40.01 is much greater than 1.

3. Without computing, decide whether the value of each expression is much smaller than 1, close to 1, or much greater than 1.

- a. $100 \div \frac{1}{1000}$
- b. $50\frac{1}{3} \div 50\frac{1}{4}$
- c. $4.7 \div 5.2$
- d. $2 \div 7335$
- e. $2,000,001 \div 9$
- f. $0.002 \div 2,000$

4. A rocking horse has a weight limit of 60 pounds.

a. What percentage of the weight limit is 33 pounds?

b. What percentage of the weight limit is 114 pounds?

c. What weight is 95% of the limit?

(From Unit 3, Lesson 16.)

5. Compare using $>$, $=$, or $<$.

a. 0.7 _____ 0.70

b. $0.03 + \frac{6}{10}$ _____ $0.30 + \frac{6}{100}$

c. 0.9 _____ 0.12

(From Unit 3, Lesson 15.)

6. Diego has 90 songs on his playlist. How many songs are there for each genre?

a. 40% rock

b. 10% country

c. 30% hip-hop

d. The rest is electronica

(From Unit 3, Lesson 14.)

7. A garden hose emits 9 quarts of water in 6 seconds. At this rate:

a. How long will it take the hose to emit 12 quarts?

b. How much water does the hose emit in 10 seconds?

(From Unit 3, Lesson 8.)

Lesson 2: Meanings of Division

2.1: A Division Expression

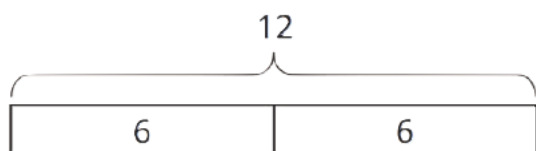
Here is an expression: $20 \div 4$.

What are some ways to think about this expression? Describe at least two meanings you think it could have.

2.2: Bags of Almonds

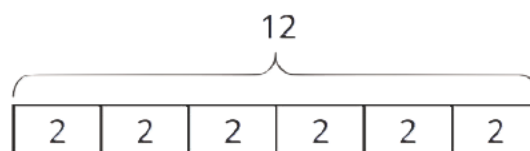
A baker has 12 pounds of almonds. She puts them in bags, so that each bag has the same weight.

Clare and Tyler drew diagrams and wrote equations to show how they were thinking about $12 \div 6$.



$$\underline{\quad} \cdot 6 = 12$$

Clare's diagram and equation



$$6 \cdot \underline{\quad} = 12$$

Tyler's diagram and equation

1. How do you think Clare and Tyler thought about $12 \div 6$? Explain what each diagram and the parts of each equation could mean about the situation with the bags of almonds. Make sure to include the meaning of the missing number.

Pause here for a class discussion.

2. Explain what each division expression could mean about the situation with the bags of almonds. Then draw a diagram and write a multiplication equation to show how you are thinking about the expression.

a. $12 \div 4$

b. $12 \div 2$

c. $12 \div \frac{1}{2}$

Are you ready for more?

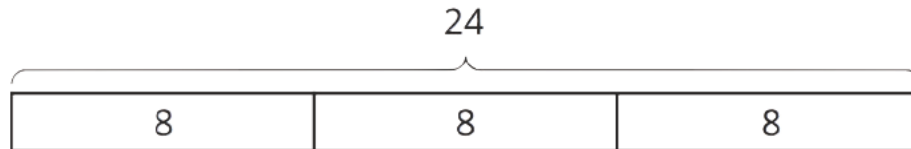
A loaf of bread is cut into slices.

1. If each slice is $\frac{1}{2}$ of a loaf, how many slices are there?
2. If each slice is $\frac{1}{5}$ of a loaf, how many slices are there?
3. What happens to the number of slices as each slice gets smaller?
4. What would dividing by 0 mean in this situation about slicing bread?

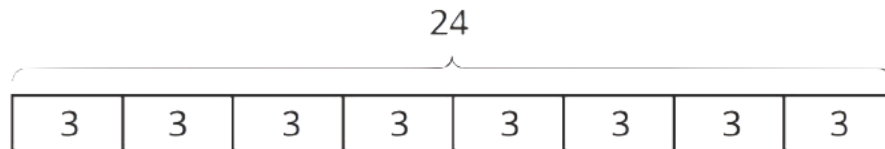
Lesson 2 Summary

Suppose 24 bagels are being distributed into boxes. The expression $24 \div 3$ could be understood in two ways:

- 24 bagels are distributed equally into 3 boxes, as represented by this diagram:



- 24 bagels are distributed into boxes, 3 bagels in each box, as represented by this diagram:



In both interpretations, the quotient is the same ($24 \div 3 = 8$), but it has different meanings in each case. In the first case, the 8 represents the number of bagels in each of the 3 boxes. In the second, it represents the number of boxes that were formed with 3 bagels in each box.

These two ways of seeing division are related to how 3, 8, and 24 are related in a multiplication. Both $3 \cdot 8$ and $8 \cdot 3$ equal 24.

- $3 \cdot 8 = 24$ can be read as “3 groups of 8 make 24.”
- $8 \cdot 3 = 24$ can be read as “8 groups of 3 make 24.”

If 3 and 24 are the only numbers given, the multiplication equations would be:

$$3 \cdot ? = 24$$

$$? \cdot 3 = 24$$

In both cases, the division $24 \div 3$ can be used to find the value of the “?” But now we see that it can be interpreted in more than one way, because the “?” can refer to *the size of a group* (as in “3 groups of what number make 24?”), or to *the number of groups* (as in “How many groups of 3 make 24?”).

Unit 4 Lesson 2 Cumulative Practice Problems

1. Twenty pounds of strawberries are being shared equally by a group of friends. The equation $20 \div 5 = 4$ represents the division of strawberries.

a. If the 5 represents the number of people, what does the 4 represent?

b. If the 5 represents the pounds of strawberries per person, what does the 4 represent?

2. A sixth-grade science club needs \$180 to pay for the tickets to a science museum. All tickets cost the same amount.

What could $180 \div 15$ mean in this situation? Describe two different possible meanings of this expression. Then, find the quotient and explain what it means in each case.

3. Write a multiplication equation that corresponds to each division equation.

a. $10 \div 5 = ?$

b. $4.5 \div 3 = ?$

c. $\frac{1}{2} \div 4 = ?$

4. Write a division or multiplication equation that represents each situation. Use a “?” for the unknown quantity.

a. 2.5 gallons of water are poured into 5 equally sized bottles. How much water is in each bottle?

b. A large bucket of 200 golf balls is divided into 4 smaller buckets. How many golf balls are in each small bucket?

c. Sixteen socks are put into pairs. How many pairs are there?

5. Find a value for a that makes each statement true.

- a. $a \div 6$ is greater than 1
- b. $a \div 6$ is equal to 1
- c. $a \div 6$ is less than 1
- d. $a \div 6$ is equal to a whole number

(From Unit 4, Lesson 1.)

6. Complete the table. Write each percentage as a percent of 1.

fraction	decimal	percentage
$\frac{1}{4}$	0.25	25% of 1
	0.1	
		75% of 1
$\frac{1}{5}$		
	1.5	
		140% of 1

(From Unit 3, Lesson 14.)

7. Jada walks at a speed of 3 miles per hour. Elena walks at a speed of 2.8 miles per hour. If they both begin walking along a walking trail at the same time, how much farther will Jada walk after 3 hours? Explain your reasoning.

(From Unit 3, Lesson 8.)

Lesson 3: Interpreting Division Situations

3.1: Dot Image: Properties of Multiplication



3.2: Homemade Jams

Draw a diagram, and write a multiplication equation to represent each situation. Then answer the question.

1. Mai had 4 jars. In each jar, she put $2\frac{1}{4}$ cups of homemade blueberry jam. Altogether, how many cups of jam are in the jars?
2. Priya filled 5 jars, using a total of $7\frac{1}{2}$ cups of strawberry jam. How many cups of jam are in each jar?
3. Han had some jars. He put $\frac{3}{4}$ cup of grape jam in each jar, using a total of $6\frac{3}{4}$ cups. How many jars did he fill?

3.3: Making Granola

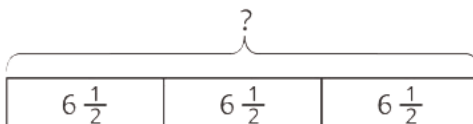
1. Consider the problem: To make 1 batch of granola, Kiran needs 26 ounces of oats. The only measuring tool he has is a 4-ounce scoop. How many scoops will it take to measure 26 ounces of oats?
 - a. Will the answer be more than 1 or less than 1?
 - b. Write a multiplication equation and a division equation that represent this situation. Use “?” to represent the unknown quantity.
 - c. Find the unknown quantity. If you get stuck, consider drawing a diagram.

2. The recipe calls for 14 ounces of mixed nuts. To get that amount, Kiran uses 4 bags of mixed nuts.
 - a. Write a mathematical question that might be asked about this situation.
 - b. What might the equation $14 \div 4 = ?$ represent in Kiran’s situation?
 - c. Find the quotient. Show your reasoning. If you get stuck, consider drawing a diagram.

Lesson 3 Summary

If a situation involves equal-sized groups, it is helpful to make sense of it in terms of the number of groups, the size of each group, and the total amount. Here are three examples to help us better understand such situations.

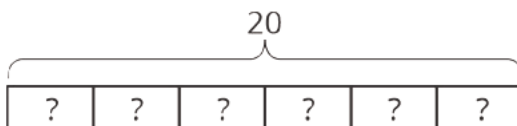
- Suppose we have 3 bottles with $6\frac{1}{2}$ ounces of water in each, and the total amount of water is not given. Here we have 3 groups, $6\frac{1}{2}$ ounces in each group, and an unknown total, as shown in this diagram:



We can express this situation as a multiplication problem. The unknown is the product, so we can simply multiply the 2 known numbers to find it.

$$3 \cdot 6\frac{1}{2} = ?$$

- Next, suppose we have 20 ounces of water to fill 6 equal-sized bottles, and the amount in each bottle is not given. Here we have 6 groups, an unknown amount in each, and a total of 20. We can represent it like this:



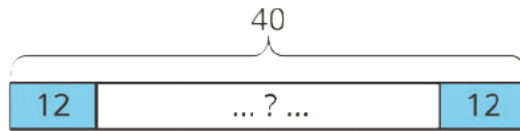
This situation can also be expressed using multiplication, but the unknown is a factor, rather than the product:

$$6 \cdot ? = 20$$

To find the unknown, we cannot simply multiply, but we can think of it as a division problem:

$$20 \div 6 = ?$$

- Now, suppose we have 40 ounces of water to pour into bottles, 12 ounces in each bottle, but the number of bottles is not given. Here we have an unknown number of groups, 12 in each group, and a total of 40.



Again, we can think of this in terms of multiplication, with a different factor being the unknown:

$$? \cdot 12 = 40$$

Likewise, we can use division to find the unknown:

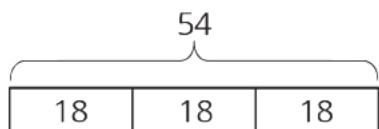
$$40 \div 12 = ?$$

Whenever we have a multiplication situation, one factor tells us *how many groups* there are, and the other factor tells us *how much is in each group*.

Sometimes we want to find the total. Sometimes we want to find how many groups there are. Sometimes we want to find how much is in each group. Anytime we want to find out how many groups there are or how much is in each group, we can represent the situation using division.

Unit 4 Lesson 3 Cumulative Practice Problems

1. Write a multiplication equation and a division equation that this diagram could represent.



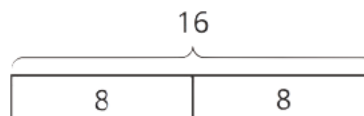
2. Consider the problem: Mai has \$36 to spend on movie tickets. Each movie ticket costs \$4.50. How many tickets can she buy?

- a. Write a multiplication equation and a division equation to represent this situation.
- b. Find the answer. Draw a diagram, if needed.

- c. Use the multiplication equation to check your answer.

3. Kiran said that this diagram can show the solution to $16 \div 8 = ?$ or $16 \div 2 = ?$, depending on how we think about the equations and the “?”.

Explain or show how Kiran is correct.



4. Write a sentence describing a situation that could be represented by the equation $4 \div 1\frac{1}{3} = ?$.

(From Unit 4, Lesson 2.)

5. Noah said, "When you divide a number by a second number, the result will always be smaller than the first number."

Jada said, "I think the result could be larger or smaller, depending on the numbers."

Do you agree with either of them? Explain or show your reasoning.

(From Unit 4, Lesson 1.)

6. Mini muffins cost \$3.00 per dozen.

- Andre says, "I have \$2.00, so I can afford 8 muffins."
- Elena says, "I want to get 16 muffins, so I'll need to pay \$4.00."

Do you agree with either of them? Explain your reasoning.

(From Unit 3, Lesson 7.)

7. A family has a monthly budget of \$2,400. How much money is spent on each category?

- a. 44% is spent on housing.
- b. 23% is spent on food.
- c. 6% is spent on clothing.
- d. 17% is spent on transportation.
- e. The rest is put into savings.

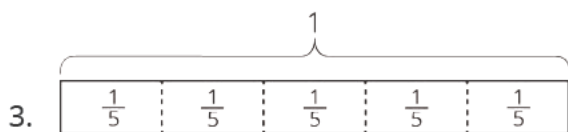
(From Unit 3, Lesson 15.)

Lesson 4: How Many Groups? (Part 1)

4.1: Equal-sized Groups

Write a multiplication equation and a division equation for each sentence or diagram.

1. Eight \$5 bills are worth \$40.
2. There are 9 thirds in 3 ones.



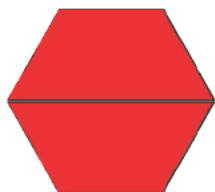
4.2: Reasoning with Pattern Blocks

Your teacher will give you pattern blocks as shown here. Use them to answer the questions.

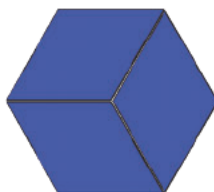


1. If a hexagon represents 1 whole, what fraction does each of the following shapes represent? Be prepared to show or explain your reasoning.
 - 1 triangle
 - 4 triangles
 - 1 hexagon and 1 trapezoid
 - 1 rhombus
 - 3 rhombuses
 - 1 trapezoid
 - 2 hexagons

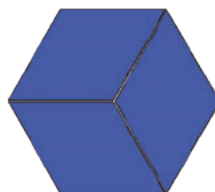
2. Here are Elena's diagrams for $2 \cdot \frac{1}{2} = 1$ and $6 \cdot \frac{1}{3} = 2$. Do you think these diagrams represent the equations? Explain or show your reasoning.



$$2 \cdot \frac{1}{2} = 1$$



$$6 \cdot \frac{1}{3} = 2$$



3. Use pattern blocks to represent each multiplication equation. Remember that a hexagon represents 1 whole.

a. $3 \cdot \frac{1}{6} = \frac{1}{2}$

b. $2 \cdot \frac{3}{2} = 3$

4. Answer the questions. If you get stuck, consider using pattern blocks.

a. How many $\frac{1}{2}$ s are in 4?

b. How many $\frac{2}{3}$ s are in 2?

c. How many $\frac{1}{6}$ s are in $1\frac{1}{2}$?

Lesson 4 Summary

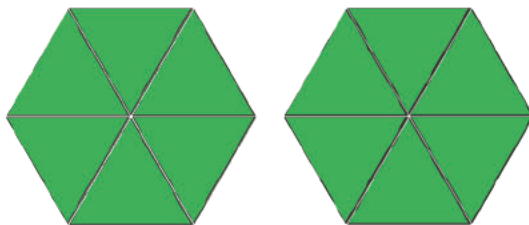
Some problems that involve equal-sized groups also involve fractions. Here is an example: "How many $\frac{1}{6}$ are in 2?" We can express this question with multiplication and division equations.

$$\begin{aligned} ? \cdot \frac{1}{6} &= 2 \\ 2 \div \frac{1}{6} &= ? \end{aligned}$$

Pattern-block diagrams can help us make sense of such problems. Here is a set of pattern blocks.



If the hexagon represents 1 whole, then a triangle must represent $\frac{1}{6}$, because 6 triangles make 1 hexagon. We can use the triangle to represent the $\frac{1}{6}$ in the problem.



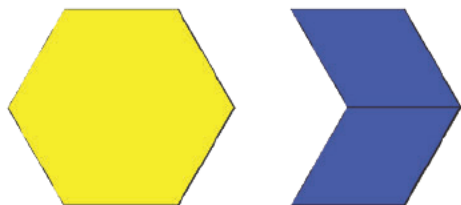
Twelve triangles make 2 hexagons, which means there are 12 groups of $\frac{1}{6}$ in 2.

If we write the 12 in the place of the "?" in the original equations, we have:

$$\begin{aligned} 12 \cdot \frac{1}{6} &= 2 \\ 2 \div \frac{1}{6} &= 12 \end{aligned}$$

Unit 4 Lesson 4 Cumulative Practice Problems

1. Consider the problem: A shopper buys cat food in bags of 3 lbs. Her cat eats $\frac{3}{4}$ lb each week. How many weeks does one bag last?
 - a. Draw a diagram to represent the situation and label your diagram so it can be followed by others. Answer the question.
 - b. Write a multiplication or division equation to represent the situation.
 - c. Multiply your answer in the first question (the number of weeks) by $\frac{3}{4}$. Did you get 3 as a result? If not, revise your previous work.
2. Use the diagram to answer the question: How many $\frac{1}{3}$ s are in $1\frac{2}{3}$? The hexagon represents 1 whole. Explain or show your reasoning.



3. Which question can be represented by the equation $? \cdot \frac{1}{8} = 3$?
 - A. How many 3s are in $\frac{1}{8}$?
 - B. What is 3 groups of $\frac{1}{8}$?
 - C. How many $\frac{1}{8}$ s are in 3?
 - D. What is $\frac{1}{8}$ of 3?

4. Write two division equations for each multiplication equation.

a. $15 \cdot \frac{2}{5} = 6$

b. $6 \cdot \frac{4}{3} = 8$

c. $16 \cdot \frac{7}{8} = 14$

5. Noah and his friends are going to an amusement park. The total cost of admission for 8 students is \$100, and all students share the cost equally. Noah brought \$13 for his ticket. Did he bring enough money to get into the park? Explain your reasoning.

(From Unit 4, Lesson 2.)

6. Write a division expression with a quotient that is:

a. greater than $8 \div 0.001$

b. less than $8 \div 0.001$

c. between $8 \div 0.001$ and $8 \div \frac{1}{10}$

(From Unit 4, Lesson 1.)

7. Find each unknown number.

a. 12 is 150% of what number?

b. 5 is 50% of what number?

c. 10% of what number is 300?

d. 5% of what number is 72?

e. 20 is 80% of what number?

(From Unit 3, Lesson 14.)

Lesson 5: How Many Groups? (Part 2)

5.1: Reasoning with Fraction Strips

Write a fraction or whole number as an answer for each question. If you get stuck, use the fraction strips. Be prepared to share your reasoning.

1. How many $\frac{1}{2}$ s are in 2?
2. How many $\frac{1}{5}$ s are in 3?
3. How many $\frac{1}{8}$ s are in $1\frac{1}{4}$?
4. $1 \div \frac{2}{6} = ?$
5. $2 \div \frac{2}{9} = ?$
6. $4 \div \frac{2}{10} = ?$

1										1									
$\frac{1}{2}$					$\frac{1}{2}$					$\frac{1}{2}$					$\frac{1}{2}$				
$\frac{1}{3}$			$\frac{1}{3}$			$\frac{1}{3}$			$\frac{1}{3}$			$\frac{1}{3}$			$\frac{1}{3}$				
$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$		$\frac{1}{4}$			
$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$		$\frac{1}{5}$			
$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$			
$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$		$\frac{1}{8}$			
$\frac{1}{9}$		$\frac{1}{9}$		$\frac{1}{9}$		$\frac{1}{9}$		$\frac{1}{9}$		$\frac{1}{9}$		$\frac{1}{9}$		$\frac{1}{9}$		$\frac{1}{9}$			

5.2: More Reasoning with Pattern Blocks

Your teacher will give you pattern blocks. Use them to answer the questions.

1. If the trapezoid represents 1 whole, what do each of the other shapes represent? Be prepared to show or explain your reasoning.



2. Use pattern blocks to represent each multiplication equation. Use the trapezoid to represent 1 whole.

a. $3 \cdot \frac{1}{3} = 1$

b. $3 \cdot \frac{2}{3} = 2$

3. Diego and Jada were asked "How many rhombuses are in a trapezoid?"
 - Diego says, " $1\frac{1}{3}$. If I put 1 rhombus on a trapezoid, the leftover shape is a triangle, which is $\frac{1}{3}$ of the trapezoid."
 - Jada says, "I think it's $1\frac{1}{2}$. Since we want to find out 'how many rhombuses,' we should compare the leftover triangle to a rhombus. A triangle is $\frac{1}{2}$ of a rhombus."

Do you agree with either of them? Explain or show your reasoning.

4. Select all the equations that can be used to answer the question: "How many rhombuses are in a trapezoid?"

$\frac{2}{3} \div ? = 1$

$1 \div \frac{2}{3} = ?$

$? \div \frac{2}{3} = 1$

$? \cdot \frac{2}{3} = 1$

$1 \cdot \frac{2}{3} = ?$

5.3: Drawing Diagrams to Show Equal-sized Groups

For each situation, draw a diagram for the relationship of the quantities to help you answer the question. Then write a multiplication equation or a division equation for the relationship. Be prepared to share your reasoning.

1. The distance around a park is $\frac{3}{2}$ miles. Noah rode his bicycle around the park for a total of 3 miles. How many times around the park did he ride?

2. You need $\frac{3}{4}$ yard of ribbon for one gift box. You have 3 yards of ribbon. How many gift boxes do you have ribbon for?

3. The water hose fills a bucket at $\frac{1}{3}$ gallon per minute. How many minutes does it take to fill a 2-gallon bucket?

Are you ready for more?

How many heaping teaspoons are in a heaping tablespoon? How would the answer depend on the shape of the spoons?

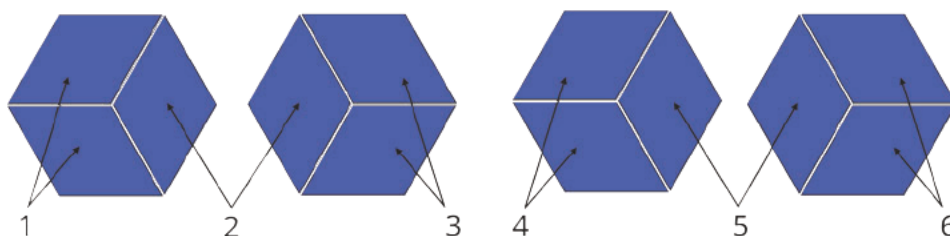
Lesson 5 Summary

Suppose one batch of cookies requires $\frac{2}{3}$ cup flour. How many batches can be made with 4 cups of flour?

We can think of the question as being: “How many $\frac{2}{3}$ are in 4?” and represent it using multiplication and division equations.

$$\begin{aligned} ? \cdot \frac{2}{3} &= 4 \\ 4 \div \frac{2}{3} &= ? \end{aligned}$$

Let’s use pattern blocks to visualize the situation and say that a hexagon is 1 whole.



Since 3 rhombuses make a hexagon, 1 rhombus represents $\frac{1}{3}$ and 2 rhombuses represent $\frac{2}{3}$. We can see that 6 pairs of rhombuses make 4 hexagons, so there are 6 groups of $\frac{2}{3}$ in 4.

Other kinds of diagrams can also help us reason about equal-sized groups involving fractions. This example shows how we might reason about the same question from above: “How many $\frac{2}{3}$ -cups are in 4 cups?”

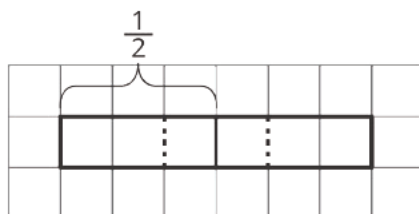


We can see each “cup” partitioned into thirds, and that there are 6 groups of $\frac{2}{3}$ -cup in 4 cups. In both diagrams, we see that the unknown value (or the “?” in the equations) is 6. So we can now write:

$$\begin{aligned} 6 \cdot \frac{2}{3} &= 4 \\ 4 \div \frac{2}{3} &= 6 \end{aligned}$$

Unit 4 Lesson 5 Cumulative Practice Problems

1. Use the tape diagram to find the value of $\frac{1}{2} \div \frac{1}{3}$. Show your reasoning.



2. What is the value of $\frac{1}{2} \div \frac{1}{3}$? Use pattern blocks to represent and find this value. The yellow hexagon represents 1 whole. Explain or show your reasoning.

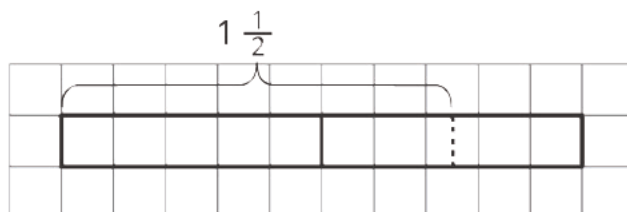


3. Use a standard inch ruler to answer each question. Then, write a multiplication equation and a division equation that answer the question.

- How many $\frac{1}{2}$ s are in 7?
- How many $\frac{3}{8}$ s are in 6?
- How many $\frac{5}{16}$ s are in $1\frac{7}{8}$?

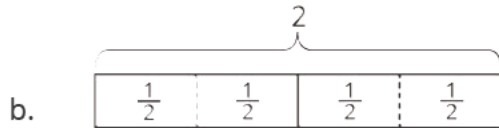


4. Use the tape diagram to answer the question: How many $\frac{2}{5}$ s are in $1\frac{1}{2}$? Show your reasoning.

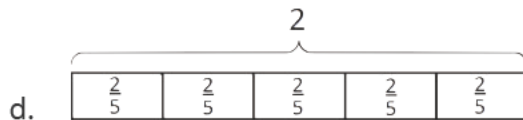


5. Write a multiplication equation and a division equation to represent each sentence or diagram.

a. There are 12 fourths in 3.



c. How many $\frac{2}{3}$ s are in 6?



(From Unit 4, Lesson 4.)

6. At a farmer's market, two vendors sell fresh milk. One vendor sells 2 liters for \$3.80, and another vendor sells 1.5 liters for \$2.70. Which is the better deal? Explain your reasoning.

(From Unit 3, Lesson 5.)

7. A recipe uses 5 cups of flour for every 2 cups of sugar.

a. How much sugar is used for 1 cup of flour?

b. How much flour is used for 1 cup of sugar?

c. How much flour is used with 7 cups of sugar?

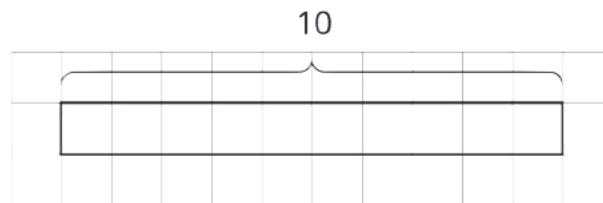
d. How much sugar is used with 6 cups of flour?

(From Unit 3, Lesson 6.)

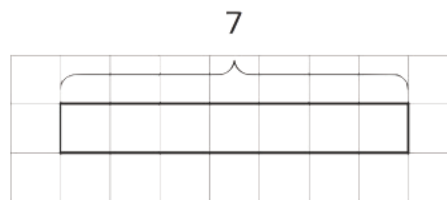
Lesson 6: Using Diagrams to Find the Number of Groups

6.1: How Many of These in That?

1. We can think of the division expression $10 \div 2\frac{1}{2}$ as the question: "How many groups of $2\frac{1}{2}$ are in 10?" Complete the tape diagram to represent this question. Then find the answer.



2. Complete the tape diagram to represent the question: "How many groups of 2 are in 7?" Then find the answer.

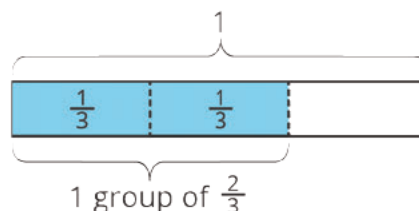


6.2: Representing Groups of Fractions with Tape Diagrams

To make sense of the question “How many $\frac{2}{3}$ s are in 1?,” Andre wrote equations and drew a tape diagram.

$$? \cdot \frac{2}{3} = 1$$

$$1 \div \frac{2}{3} = ?$$



- In an earlier task, we used pattern blocks to help us solve the equation $1 \div \frac{2}{3} = ?$. Explain how Andre’s tape diagram can also help us solve the equation.

- Write a multiplication equation and a division equation for each question. Then, draw a tape diagram and find the answer.

- How many $\frac{3}{4}$ s are in 1?

- How many $\frac{2}{3}$ s are in 3?

c. How many $\frac{3}{2}$ s are in 5?

6.3: Finding Number of Groups

1. Write a multiplication equation or a division equation for each question. Then, find the answer and explain or show your reasoning.

a. How many $\frac{3}{8}$ -inch thick books make a stack that is 6 inches tall?

b. How many groups of $\frac{1}{2}$ pound are in $2\frac{3}{4}$ pounds?

2. Write a question that can be represented by the division equation $5 \div 1\frac{1}{2} = ?$. Then, find the answer and explain or show your reasoning.

Lesson 6 Summary

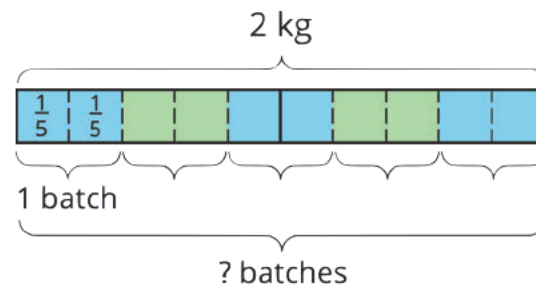
A baker used 2 kilograms of flour to make several batches of a pastry recipe. The recipe called for $\frac{2}{5}$ kilogram of flour per batch. How many batches did she make?

We can think of the question as: “How many groups of $\frac{2}{5}$ kilogram make 2 kilograms?” and represent that question with the equations:

$$? \cdot \frac{2}{5} = 2$$

$$2 \div \frac{2}{5} = ?$$

To help us make sense of the question, we can draw a tape diagram. This diagram shows 2 whole kilograms, with each kilogram partitioned into fifths.



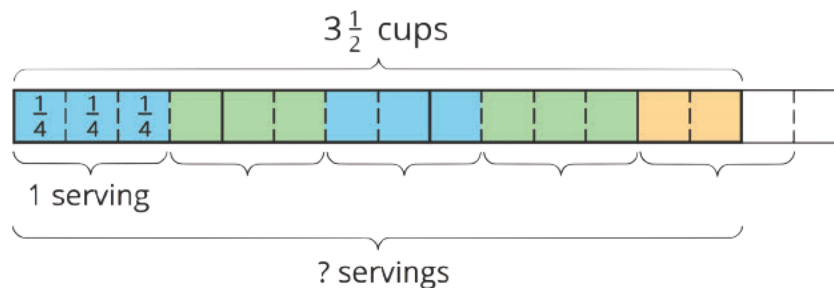
We can see there are 5 groups of $\frac{2}{5}$ in 2. Multiplying 5 and $\frac{2}{5}$ allows us to check this answer: $5 \cdot \frac{2}{5} = \frac{10}{5}$ and $\frac{10}{5} = 2$, so the answer is correct.

Notice the number of groups that result from $2 \div \frac{2}{5}$ is a whole number. Sometimes the number of groups we find from dividing may not be a whole number. Here is an example:

Suppose one serving of rice is $\frac{3}{4}$ cup. How many servings are there in $3\frac{1}{2}$ cups?

$$? \cdot \frac{3}{4} = 3\frac{1}{2}$$

$$3\frac{1}{2} \div \frac{3}{4} = ?$$



Looking at the diagram, we can see there are 4 full groups of $\frac{3}{4}$, plus 2 fourths. If 3 fourths make a whole group, then 2 fourths make $\frac{2}{3}$ of a group. So the number of servings (the “?” in each equation) is $4\frac{2}{3}$. We can check this by multiplying $4\frac{2}{3}$ and $\frac{3}{4}$.

$$4\frac{2}{3} \cdot \frac{3}{4} = \frac{14}{3} \cdot \frac{3}{4}, \text{ and } \frac{14}{3} \cdot \frac{3}{4} = \frac{14}{4}, \text{ which is indeed equivalent to } 3\frac{1}{2}.$$

Unit 4 Lesson 6 Cumulative Practice Problems

1. We can think of $3 \div \frac{1}{4}$ as the question "How many groups of $\frac{1}{4}$ are in 3?" Draw a tape diagram to represent this question. Then find the answer.

2. Describe how to draw a tape diagram to represent and answer $3 \div \frac{3}{5} = ?$ for a friend who was absent.

3. How many groups of $\frac{1}{2}$ day are in 1 week?
 - a. Write a multiplication equation or a division equation to represent the question.

 - b. Draw a tape diagram to show the relationship between the quantities and to answer the question. Use graph paper, if needed.

4. Diego said that the answer to the question "How many groups of $\frac{5}{6}$ are in 1?" is $\frac{6}{5}$ or $1\frac{1}{5}$. Do you agree with him? Explain or show your reasoning.

5. Select all the equations that can represent the question: "How many groups of $\frac{4}{5}$ are in 1?"

A. $? \cdot 1 = \frac{4}{5}$

B. $1 \cdot \frac{4}{5} = ?$

C. $\frac{4}{5} \div 1 = ?$

D. $? \cdot \frac{4}{5} = 1$

E. $1 \div \frac{4}{5} = ?$

(From Unit 4, Lesson 5.)

6. Calculate each percentage mentally.

a. What is 10% of 70?

b. What is 10% of 110?

c. What is 25% of 160?

d. What is 25% of 48?

e. What is 50% of 90?

f. What is 50% of 350?

g. What is 75% of 300?

h. What is 75% of 48?

(From Unit 3, Lesson 14.)

Lesson 7: What Fraction of a Group?

7.1: Estimating a Fraction of a Number

1. Estimate the quantities:

a. What is $\frac{1}{3}$ of 7?

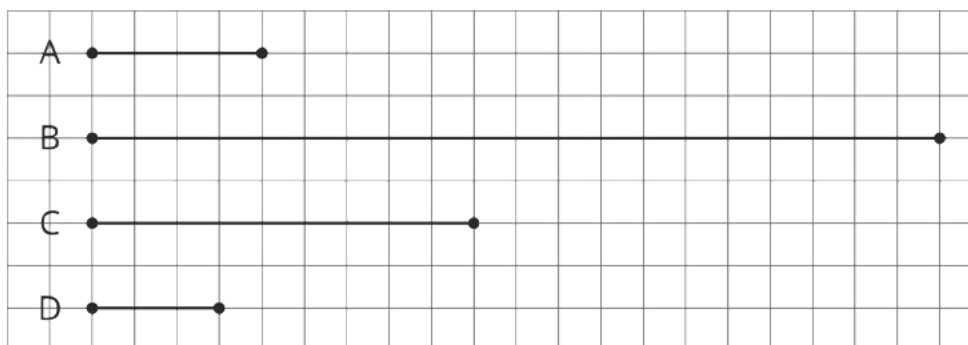
b. What is $\frac{4}{5}$ of $9\frac{2}{3}$?

c. What is $2\frac{4}{7}$ of $10\frac{1}{9}$?

2. Write a multiplication expression for each of the previous questions.

7.2: Fractions of Ropes

Here is a diagram that shows four ropes of different lengths.



1. Complete each sentence comparing the lengths of the ropes. Then, use the measurements shown on the grid to write a multiplication equation and a division equation for each comparison.

a. Rope B is _____ times as long as Rope A.

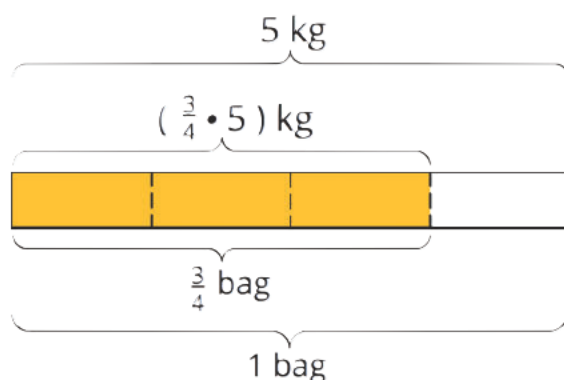
b. Rope C is _____ times as long as Rope A.

c. Rope D is _____ times as long as Rope A.

Lesson 7 Summary

It is natural to think about groups when we have more than one group, but we can also have a *fraction of a group*.

To find the amount in a fraction of a group, we can multiply the fraction by the amount in the whole group. If a bag of rice weighs 5 kg, $\frac{3}{4}$ of a bag would weigh $(\frac{3}{4} \cdot 5)$ kg.

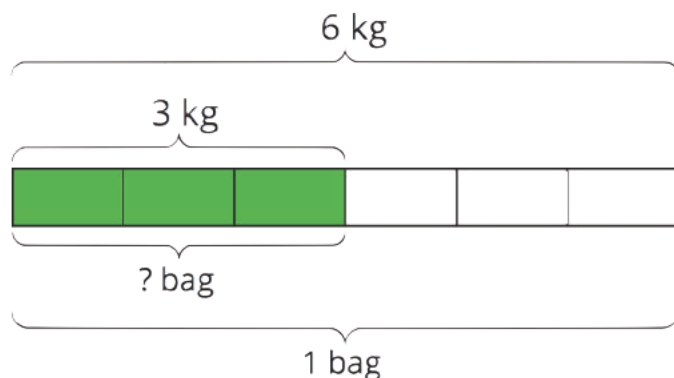


Sometimes we need to find what fraction of a group an amount is. Suppose a full bag of flour weighs 6 kg. A chef used 3 kg of flour. What fraction of a full bag was used? In other words, what fraction of 6 kg is 3 kg?

This question can be represented by a multiplication equation and a division equation, as well as by a diagram.

$$? \cdot 6 = 3$$

$$3 \div 6 = ?$$

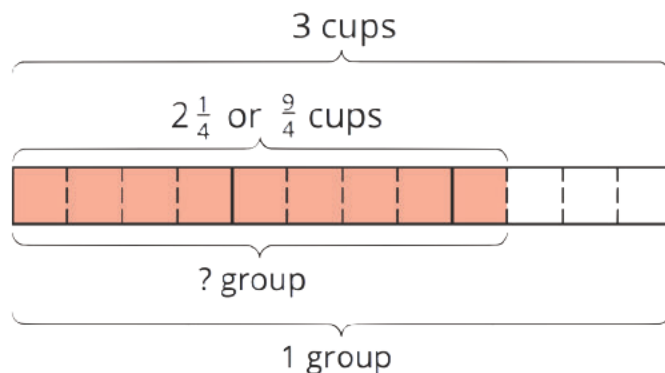


We can see from the diagram that 3 is $\frac{1}{2}$ of 6, and we can check this answer by multiplying:
 $\frac{1}{2} \cdot 6 = 3$.

In *any* situation where we want to know what fraction one number is of another number, we can write a division equation to help us find the answer.

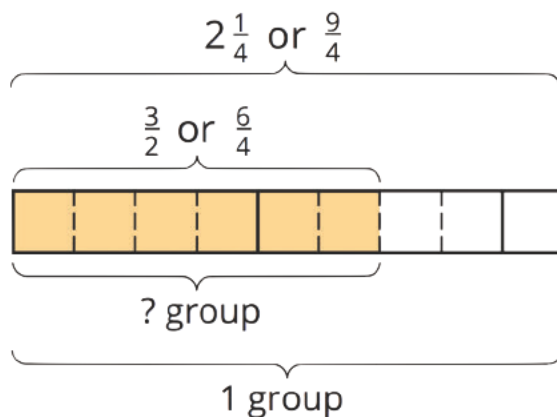
For example, “What fraction of 3 is $2\frac{1}{4}$?” can be expressed as $? \cdot 3 = 2\frac{1}{4}$, which can also be written as $2\frac{1}{4} \div 3 = ?$.

The answer to “What is $2\frac{1}{4} \div 3$?” is also the answer to the original question.



The diagram shows that 3 wholes contain 12 fourths, and $2\frac{1}{4}$ contains 9 fourths, so the answer to this question is $\frac{9}{12}$, which is equivalent to $\frac{3}{4}$.

We can use diagrams to help us solve other division problems that require finding a fraction of a group. For example, here is a diagram to help us answer the question: “What fraction of $\frac{9}{4}$ is $\frac{3}{2}$?” which can be written as $\frac{3}{2} \div \frac{9}{4} = ?$.



We can see that the quotient is $\frac{6}{9}$, which is equivalent to $\frac{2}{3}$. To check this, let’s multiply. $\frac{2}{3} \cdot \frac{9}{4} = \frac{18}{12}$, and $\frac{18}{12}$ is, indeed, equal to $\frac{3}{2}$.

4. Draw a tape diagram to represent the question: What fraction of $2\frac{1}{2}$ is $\frac{4}{5}$?
Then find the answer.

5. How many groups of $\frac{3}{4}$ are in each of these quantities?

a. $\frac{11}{4}$

b. $6\frac{1}{2}$

(From Unit 4, Lesson 6.)

6. Which question can be represented by the equation $4 \div \frac{2}{7} = ?$

A. What is 4 groups of $\frac{2}{7}$?

B. How many $\frac{2}{7}$ s are in 4?

C. What is $\frac{2}{7}$ of 4?

D. How many 4s are in $\frac{2}{7}$?

(From Unit 4, Lesson 4.)

Lesson 8: How Much in Each Group? (Part 1)

8.1: Inventing a Situation

1. Think of a situation with a question that can be represented by the equation $12 \div \frac{2}{3} = ?$. Describe the situation and the question.

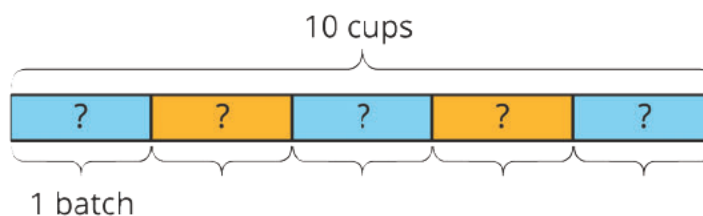
2. Trade descriptions with your partner, and answer your partner's question.

8.2: How Much in One Batch?

To make 5 batches of cookies, 10 cups of flour are required. Consider the question: How many cups of flour does each batch require?

We can write equations and draw a diagram to represent this situation.

$$5 \cdot ? = 10$$
$$10 \div 5 = ?$$



This helps us see that each batch requires 2 cups of flour.

For each question, write a multiplication equation and a division equation, draw a diagram, and find the answer.

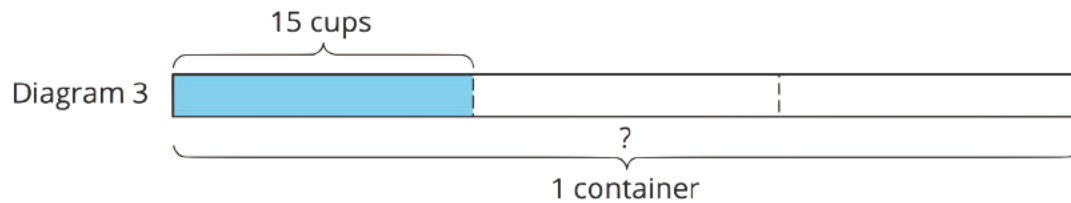
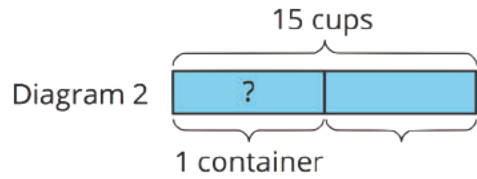
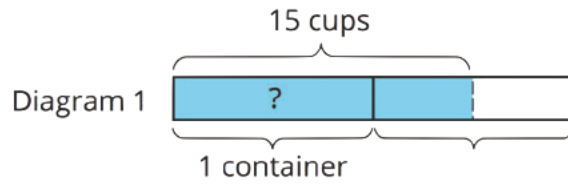
1. To make 4 batches of cupcakes, it takes 6 cups of flour. How many cups of flour are needed for 1 batch?

2. To make $\frac{1}{2}$ batch of rolls, it takes $\frac{5}{4}$ cups of flour. How many cups of flour are needed for 1 batch?

3. Two cups of flour make $\frac{2}{3}$ batch of bread. How many cups of flour make 1 batch?

8.3: One Container and One Section of Highway

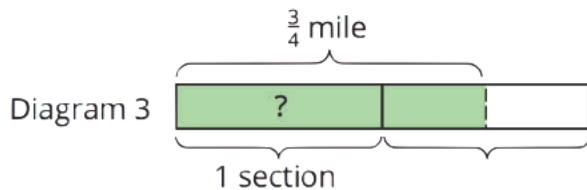
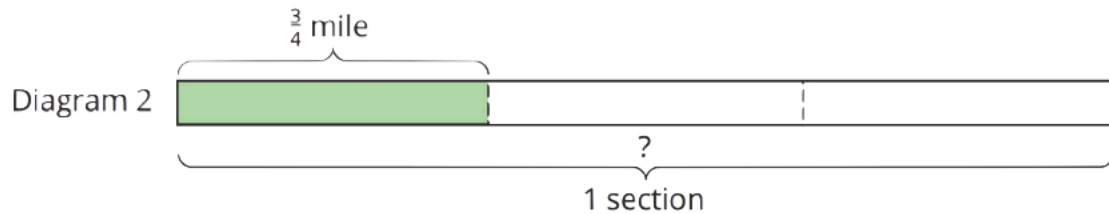
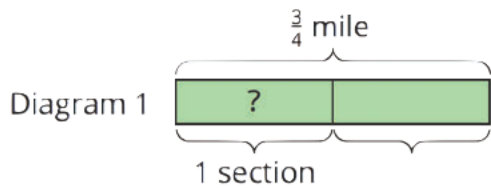
Here are three tape diagrams that represent situations about filling containers of water.



Match each situation to a diagram and use the diagram to help you answer the question. Then, write a multiplication equation and a division equation to represent the situation.

1. Tyler poured a total of 15 cups of water into 2 equal-sized bottles and filled each bottle. How much water was in each bottle?
2. Kiran poured a total of 15 cups of water into equal-sized pitchers and filled $1\frac{1}{2}$ pitchers. How much water was in the full pitcher?
3. It takes 15 cups of water to fill $\frac{1}{3}$ pail. How much water is needed to fill 1 pail?

Here are tape diagrams that represent situations about cleaning sections of highway.



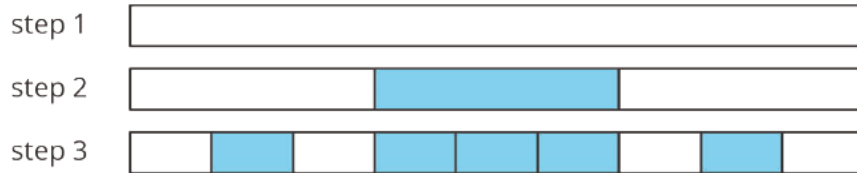
Match each situation to a diagram and use the diagram to help you answer the question. Then, write a multiplication equation and a division equation to represent the situation.

- Priya's class has adopted two equal sections of a highway to keep clean. The combined length is $\frac{3}{4}$ of a mile. How long is each section?
- Lin's class has also adopted some sections of highway to keep clean. If $1\frac{1}{2}$ sections are $\frac{3}{4}$ mile long, how long is each section?
- A school has adopted a section of highway to keep clean. If $\frac{1}{3}$ of the section is $\frac{3}{4}$ mile long, how long is the section?

Are you ready for more?

To make a Cantor ternary set:

- Start with a tape diagram of length 1 unit. This is step 1.
- Color in the middle third of the tape diagram. This is step 2.
- Do the same to each remaining segment that is not colored in. This is step 3.
- Keep repeating this process.

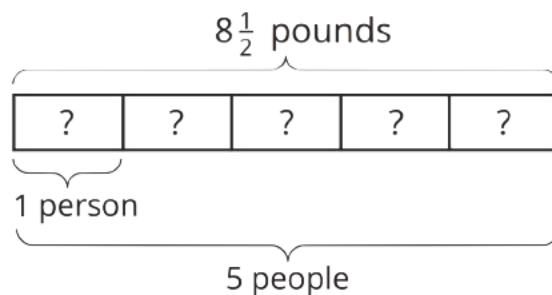


1. How much of the diagram is colored in after step 2? Step 3? Step 10?
2. If you continue this process, how much of the tape diagram will you color?
3. Can you think of a different process that will give you a similar result? For example, color the first fifth instead of the middle third of each strip.

Lesson 8 Summary

Sometimes we know the amount for *multiple* groups, but we don't know how much is in one group. We can use division to find out.

For example, if 5 people share $8\frac{1}{2}$ pounds of cherries equally, how many pounds of cherries does each person get?



We can represent this situation with a multiplication equation and a division equation:

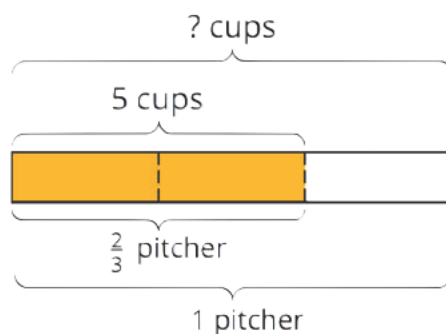
$$5 \cdot ? = 8\frac{1}{2}$$

$$8\frac{1}{2} \div 5 = ?$$

$8\frac{1}{2} \div 5$ can be written as $\frac{17}{2} \div 5$. Dividing by 5 is equivalent to multiplying by $\frac{1}{5}$, and $\frac{17}{2} \cdot \frac{1}{5} = \frac{17}{10}$. This means each person gets $1\frac{7}{10}$ pounds.

Other times, we know the amount for *a fraction* of a group, but we don't know the size of one whole group. We can also use division to find out.

For example, Jada poured 5 cups of iced tea in a pitcher and filled $\frac{2}{3}$ of the pitcher. How many cups of iced tea fill the entire pitcher?



We can represent this situation with a multiplication equation and a division equation:

$$\frac{2}{3} \cdot ? = 5$$

$$5 \div \frac{2}{3} = ?$$

The diagram can help us reason about the answer. If $\frac{2}{3}$ of a pitcher is 5 cups, then $\frac{1}{3}$ of a pitcher is half of 5, which is $\frac{5}{2}$. Because there are 3 thirds in 1 whole, there would be $(3 \cdot \frac{5}{2})$ or $\frac{15}{2}$ cups in one whole pitcher. We can check our answer by multiplying: $\frac{2}{3} \cdot \frac{15}{2} = \frac{30}{6}$, and $\frac{30}{6} = 5$.

Notice that in the first example, the number of groups is greater than 1 (5 people) and in the second, the number of groups is less than 1 ($\frac{2}{3}$ of a pitcher), but the division and multiplication equations for both situations have the same structures.

Unit 4 Lesson 8 Cumulative Practice Problems

1. For each situation, complete the tape diagram to represent and answer the question.

- a. Mai has picked 1 cup of strawberries for a cake, which is enough for $\frac{3}{4}$ of the cake. How many cups does she need for the whole cake?



- b. Priya has picked $1\frac{1}{2}$ cups of raspberries, which is enough for $\frac{3}{4}$ of a cake. How many cups does she need for the whole cake?



2. Consider the problem: Tyler painted $\frac{9}{2}$ square yards of wall area with 3 gallons of paint. How many gallons of paint does it take to paint each square yard of wall?

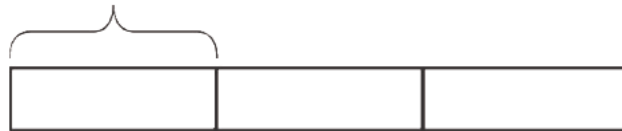
- a. Write multiplication and division equations to represent the situation.

- b. Draw a diagram to represent and answer the question.

3. Consider the problem: After walking $\frac{1}{4}$ mile from home, Han is $\frac{1}{3}$ of his way to school. What is the distance between his home and school?

a. Write multiplication and division equations to represent this situation.

b. Complete the diagram to represent and answer the question.



4. Here is a division equation: $\frac{4}{5} \div \frac{2}{3} = ?$

a. Write a multiplication equation that corresponds to the division equation.

b. Draw a diagram to represent and answer the question.

(From Unit 4, Lesson 7.)

5. Consider the problem: A set of books that are each 1.5 inches wide are being organized on a bookshelf that is 36 inches wide. How many books can fit on the shelf?

a. Write multiplication and division equations to represent the situation.

b. Find the answer. Draw a diagram, if needed.

c. Use the multiplication equation to check your answer.

(From Unit 4, Lesson 3.)

6. a. Without calculating, order the quotients from smallest to largest.

$$56 \div 8$$

$$56 \div 8,000,000$$

$$56 \div 0.000008$$

b. Explain how you decided the order of the three expressions.

c. Find a number n so that $56 \div n$ is greater than 1 but less than 7.

(From Unit 4, Lesson 1.)

Lesson 9: How Much in Each Group? (Part 2)

9.1: Number Talk: Greater Than 1 or Less Than 1?

Decide whether each quotient is greater than 1 or less than 1.

$$\frac{1}{2} \div \frac{1}{4}$$

$$1 \div \frac{3}{4}$$

$$\frac{2}{3} \div \frac{7}{8}$$

$$2\frac{7}{8} \div 2\frac{3}{5}$$

9.2: Two Water Containers



1. After looking at these pictures, Lin says, "I see the fraction $\frac{2}{5}$." Jada says, "I see the fraction $\frac{3}{4}$." What quantities are Lin and Jada referring to?

2. Consider the problem: How many liters of water fit in the water dispenser?
 - a. Write a multiplication equation and a division equation for the question.

 - b. Find the answer and explain your reasoning. If you get stuck, consider drawing a diagram.

 - c. Check your answer using the multiplication equation.

9.3: Amount in One Group

Write a multiplication equation and a division equation and draw a diagram to represent each situation. Then, find the answer and explain your reasoning.

1. Jada bought $3\frac{1}{2}$ yards of fabric for \$21. How much did each yard cost?

2. $\frac{4}{9}$ kilogram of baking soda costs \$2. How much does 1 kilogram of baking soda cost?

3. Diego can fill $1\frac{1}{5}$ bottles with 3 liters of water. How many liters of water fill 1 bottle?

4. $\frac{5}{4}$ gallons of water fill $\frac{5}{6}$ of a bucket. How many gallons of water fill the entire bucket?

Are you ready for more?

The largest sandwich ever made weighed 5,440 pounds. If everyone on Earth shares the sandwich equally, how much would you get? What fraction of a regular sandwich does this represent?

9.4: Inventing Another Situation

1. Think of a situation with a question that can be represented by $\frac{1}{3} \div \frac{1}{4} = ?$. Describe the situation and the question.
2. Trade descriptions with a partner.
 - Review each other's description and discuss whether each question matches the equation.
 - Revise your description based on the feedback from your partner.
3. Find the answer to your question. Explain or show your reasoning. If you get stuck, consider drawing a diagram.

Lesson 9 Summary

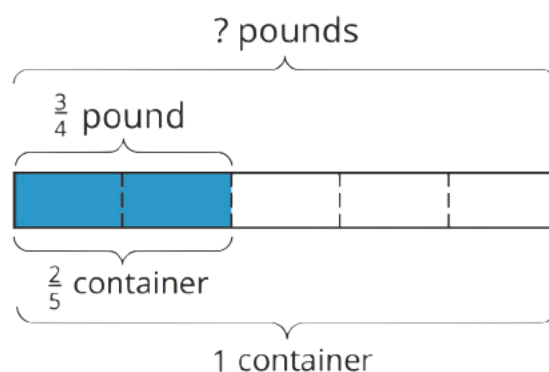
Sometimes we have to think carefully about how to solve a problem that involves multiplication and division. Diagrams and equations can help us.

For example, $\frac{3}{4}$ of a pound of rice fills $\frac{2}{5}$ of a container. There are two whole amounts to keep track of here: 1 whole pound and 1 whole container. The equations we write and the diagram we draw depend on what question we are trying to answer.

- How many pounds fill 1 container?

$$\frac{2}{5} \cdot ? = \frac{3}{4}$$

$$\frac{3}{4} \div \frac{2}{5} = ?$$

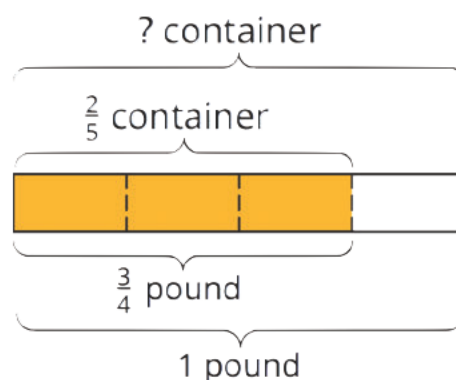


If $\frac{2}{5}$ of a container is filled with $\frac{3}{4}$ pound, then $\frac{1}{5}$ of a container is filled with half of $\frac{3}{4}$, or $\frac{3}{8}$, pound. One whole container then has $5 \cdot \frac{3}{8}$ (or $\frac{15}{8}$) pounds.

- What fraction of a container does 1 pound fill?

$$\frac{3}{4} \cdot ? = \frac{2}{5}$$

$$\frac{2}{5} \div \frac{3}{4} = ?$$



If $\frac{3}{4}$ pound fills $\frac{2}{5}$ of a container, then $\frac{1}{4}$ pound fills a third of $\frac{2}{5}$, or $\frac{2}{15}$, of a container. One whole pound then fills $4 \cdot \frac{2}{15}$ (or $\frac{8}{15}$) of a container.

Unit 4 Lesson 9 Cumulative Practice Problems

1. A group of friends is sharing $2\frac{1}{2}$ pounds of berries.
 - a. If each friend received $\frac{5}{4}$ of a pound of berries, how many friends are sharing the berries?

 - b. If 5 friends are sharing the berries, how many pounds of berries does each friend receive?

2. $\frac{2}{5}$ kilogram of soil fills $\frac{1}{3}$ of a container. Can 1 kilogram of soil fit in the container? Explain or show your reasoning.

3. After raining for $\frac{3}{4}$ of an hour, a rain gauge is $\frac{2}{5}$ filled. If it continues to rain at that rate for 15 more minutes, what fraction of the rain gauge will be filled?

a. To help answer this question, Diego wrote the equation $\frac{3}{4} \div \frac{2}{5} = ?$. Explain why this equation does *not* represent the situation.



b. Write a multiplication equation and a division equation that do represent the situation.

4. 3 tickets to the museum cost \$12.75. At this rate, what is the cost of:

a. 1 ticket?

b. 5 tickets?

(From Unit 2, Lesson 8.)

5. Elena went 60 meters in 15 seconds. Noah went 50 meters in 10 seconds. Elena and Noah both moved at a constant speed.

a. How far did Elena go in 1 second?

b. How far did Noah go in 1 second?

c. Who went faster? Explain or show your reasoning.

(From Unit 2, Lesson 9.)

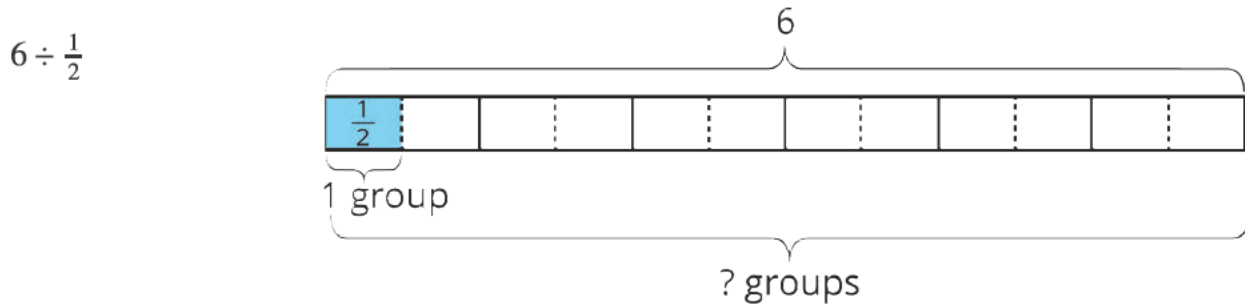
6. The first row in the table shows a recipe for 1 batch of trail mix. Complete the table to show recipes for 2, 3, and 4 batches of the same type of trail mix.

number of batches	cups of cereal	cups of almonds	cups of raisins
1	2	$\frac{1}{3}$	$\frac{1}{4}$
2			
3			
4			

(From Unit 2, Lesson 11.)

10.2: Dividing by Unit Fractions

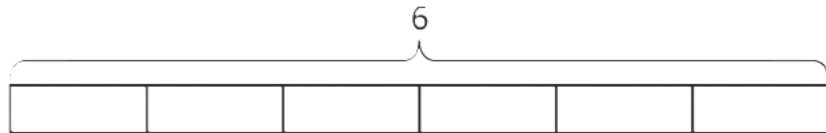
To find the value of $6 \div \frac{1}{2}$, Elena thought, "How many $\frac{1}{2}$ s are in 6?" and then she drew this tape diagram. It shows 6 ones, with each one partitioned into 2 equal pieces.



1. For each division expression, complete the diagram using the same method as Elena. Then, find the value of the expression.

a.

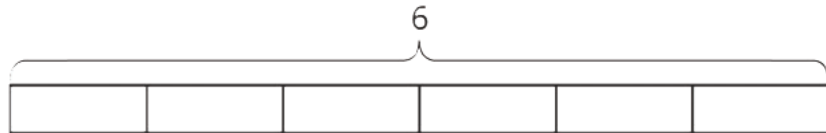
$$6 \div \frac{1}{3}$$



Value of the expression: _____

b.

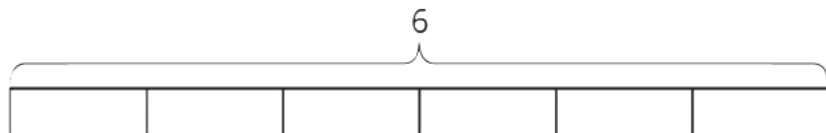
$$6 \div \frac{1}{4}$$



Value of the expression: _____

c.

$$6 \div \frac{1}{6}$$



Value of the expression: _____

2. Examine the expressions and answers more closely. Look for a pattern. How could you find how many halves, thirds, fourths, or sixths were in 6 without counting all of them? Explain your reasoning.

3. Use the pattern you noticed to find the values of these expressions. If you get stuck, consider drawing a diagram.

a. $6 \div \frac{1}{8}$

b. $6 \div \frac{1}{10}$

c. $6 \div \frac{1}{25}$

d. $6 \div \frac{1}{b}$

4. Find the value of each expression.

a. $8 \div \frac{1}{4}$

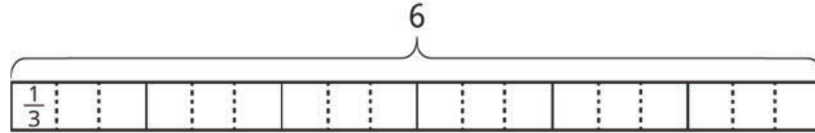
b. $12 \div \frac{1}{5}$

c. $a \div \frac{1}{2}$

d. $a \div \frac{1}{b}$

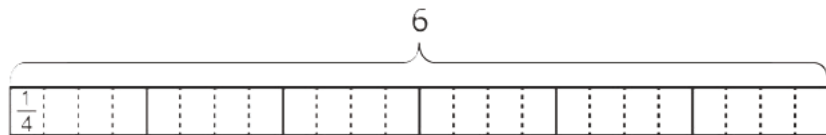
10.3: Dividing by Non-unit Fractions

1. To find the value of $6 \div \frac{2}{3}$, Elena started by drawing a diagram the same way she did for $6 \div \frac{1}{3}$.



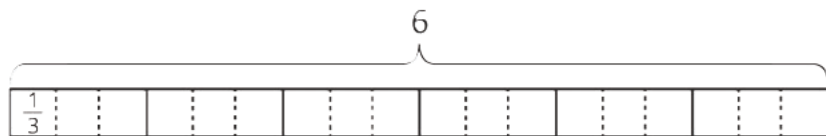
- a. Complete the diagram to show how many $\frac{2}{3}$ s are in 6.
- b. Elena says, "To find $6 \div \frac{2}{3}$, I can just take the value of $6 \div \frac{1}{3}$ and then either multiply it by $\frac{1}{2}$ or divide it by 2." Do you agree with her? Explain your reasoning.
2. For each division expression, complete the diagram using the same method as Elena. Then, find the value of the expression. Think about how you could find that value without counting all the pieces in your diagram.

a. $6 \div \frac{3}{4}$



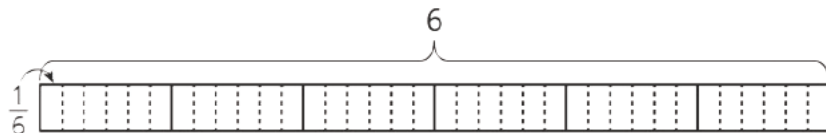
Value of the expression: _____

b. $6 \div \frac{4}{3}$



Value of the expression: _____

c. $6 \div \frac{4}{6}$



Value of the expression: _____

3. Elena examined her diagrams and noticed that she always took the same two steps to show division by a fraction on a tape diagram. She said:

“My first step was to divide each 1 whole into as many parts as the number in the denominator. So if the expression is $6 \div \frac{3}{4}$, I would break each 1 whole into 4 parts. Now I have 4 times as many parts.

My second step was to put a certain number of those parts into one group, and that number is the numerator of the divisor. So if the fraction is $\frac{3}{4}$, I would put 3 of the $\frac{1}{4}$ s into one group. Then I could tell how many $\frac{3}{4}$ s are in 6.”

Which expression represents how many $\frac{3}{4}$ s Elena would have after these two steps? Be prepared to explain your reasoning.

$6 \div 4 \cdot 3$

$6 \cdot 4 \div 3$

$6 \div 4 \div 3$

$6 \cdot 4 \cdot 3$

4. Use the pattern Elena noticed to find the values of these expressions. If you get stuck, consider drawing a diagram.

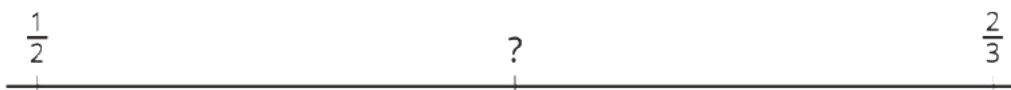
a. $6 \div \frac{2}{7}$

b. $6 \div \frac{3}{10}$

c. $6 \div \frac{6}{25}$

Are you ready for more?

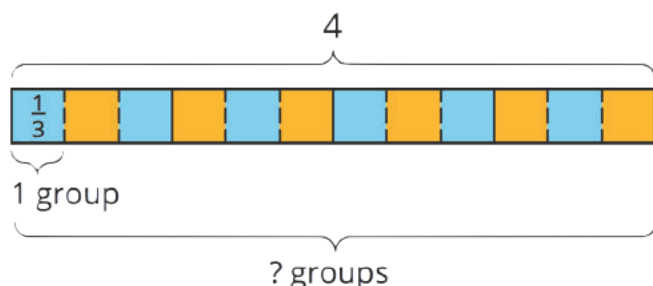
Find the missing value.



Lesson 10 Summary

To answer the question “How many $\frac{1}{3}$ s are in 4?” or “What is $4 \div \frac{1}{3}$?”, we can reason that there are 3 thirds in 1, so there are $(4 \cdot 3)$ thirds in 4.

In other words, dividing 4 by $\frac{1}{3}$ has the same result as multiplying 4 by 3.

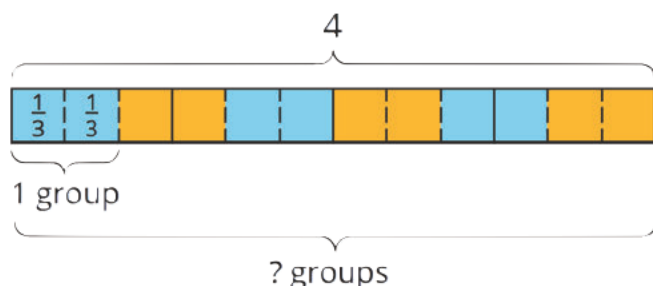


$$4 \div \frac{1}{3} = 4 \cdot 3$$

In general, dividing a number by a unit fraction $\frac{1}{b}$ is the same as multiplying the number by b , which is the reciprocal of $\frac{1}{b}$.

How can we reason about $4 \div \frac{2}{3}$?

We already know that there are $(4 \cdot 3)$ or 12 groups of $\frac{1}{3}$ s in 4. To find how many $\frac{2}{3}$ s are in 4, we need to put together every 2 of the $\frac{1}{3}$ s into a group. Doing this results in half as many groups, which is 6 groups. In other words:



$$4 \div \frac{2}{3} = (4 \cdot 3) \div 2$$

or

$$4 \div \frac{2}{3} = (4 \cdot 3) \cdot \frac{1}{2}$$

In general, dividing a number by $\frac{a}{b}$, is the same as multiplying the number by b and then dividing by a , or multiplying the number by b and then by $\frac{1}{a}$.

Unit 4 Lesson 10 Cumulative Practice Problems

1. Priya is sharing 24 apples equally with some friends. She uses division to determine how many people can have a share if each person gets a particular number of apples. For example, $24 \div 4 = 6$ means that if each person gets 4 apples, then 6 people can have apples. Here are some other calculations:

$$24 \div 4 = 6$$

$$24 \div 2 = 12$$

$$24 \div 1 = 24$$

$$24 \div \frac{1}{2} = ?$$

- a. Priya thinks the “?” represents a number less than 24. Do you agree? Explain or show your reasoning.
- b. In the case of $24 \div \frac{1}{2} = ?$, how many people can have apples?

2. Here is a centimeter ruler.

- a. Use the ruler to find $1 \div \frac{1}{10}$ and $4 \div \frac{1}{10}$.



- b. What calculation did you do each time?

- c. Use this pattern to find $18 \div \frac{1}{10}$.

- d. Explain how you could find $4 \div \frac{2}{10}$ and $4 \div \frac{8}{10}$.

3. Find each quotient.

a. $5 \div \frac{1}{10}$

b. $5 \div \frac{3}{10}$

c. $5 \div \frac{9}{10}$

4. Use the fact that $2\frac{1}{2} \div \frac{1}{8} = 20$ to find $2\frac{1}{2} \div \frac{5}{8}$. Explain or show your reasoning.

5. Consider the problem: It takes one week for a crew of workers to pave $\frac{3}{5}$ kilometer of a road. At that rate, how long will it take to pave 1 kilometer?

Write a multiplication equation and a division equation to represent the question. Then find the answer and show your reasoning.

(From Unit 4, Lesson 9.)

6. A box contains $1\frac{3}{4}$ pounds of pancake mix. Jada used $\frac{7}{8}$ pound for a recipe. What fraction of the pancake mix in the box did she use? Explain or show your reasoning. Draw a diagram, if needed.

(From Unit 4, Lesson 7.)

7. Calculate each percentage mentally.

a. 25% of 400

a. 75% of 200

a. 5% of 20

b. 50% of 90

b. 10% of 8,000

(From Unit 3, Lesson 14.)

Lesson 11: Using an Algorithm to Divide Fractions

11.1: Multiplying Fractions

Evaluate each expression.

1. $\frac{2}{3} \cdot 27$

2. $\frac{1}{2} \cdot \frac{2}{3}$

3. $\frac{2}{9} \cdot \frac{3}{5}$

4. $\frac{27}{100} \cdot \frac{200}{9}$

5. $(1\frac{3}{4}) \cdot \frac{5}{7}$

11.2: Dividing a Fraction by a Fraction

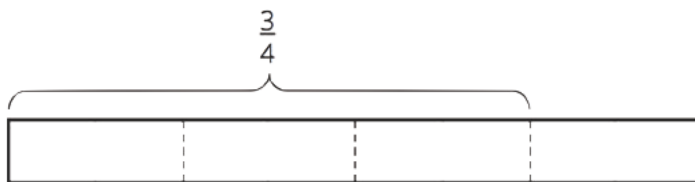
Work with a partner. One person works on the questions labeled “Partner A” and the other person works on those labeled “Partner B.”

1. Partner A: Find the value of each expression by completing the diagram.

a.

$$\frac{3}{4} \div \frac{1}{8}$$

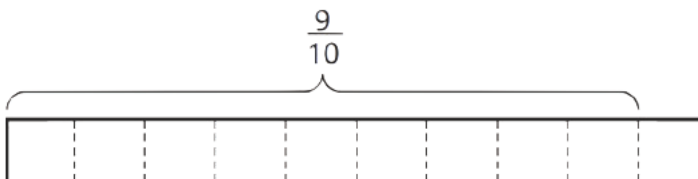
How many $\frac{1}{8}$ s in $\frac{3}{4}$?



b.

$$\frac{9}{10} \div \frac{3}{5}$$

How many $\frac{3}{5}$ s in $\frac{9}{10}$?



Partner B:

Elena said, "If I want to divide 4 by $\frac{2}{5}$, I can multiply 4 by 5 and then divide it by 2 or multiply it by $\frac{1}{2}$."

Find the value of each expression using the strategy Elena described.

a. $\frac{3}{4} \div \frac{1}{8}$

b. $\frac{9}{10} \div \frac{3}{5}$

2. What do you notice about the diagrams and expressions? Discuss with your partner.

3. Complete this sentence based on what you noticed:

To divide a number n by a fraction $\frac{a}{b}$, we can multiply n by _____ and then divide the product by _____.

4. Select all the equations that represent the sentence you completed.

$n \div \frac{a}{b} = n \cdot b \div a$

$n \div \frac{a}{b} = n \cdot a \div b$

$n \div \frac{a}{b} = n \cdot \frac{a}{b}$

$n \div \frac{a}{b} = n \cdot \frac{b}{a}$

11.3: Using an Algorithm to Divide Fractions

Calculate each quotient. Show your thinking and be prepared to explain your reasoning.

1. $\frac{8}{9} \div 4$

2. $\frac{3}{4} \div \frac{1}{2}$

3. $3\frac{1}{3} \div \frac{2}{9}$

4. $\frac{9}{2} \div \frac{3}{8}$

5. $6\frac{2}{5} \div 3$

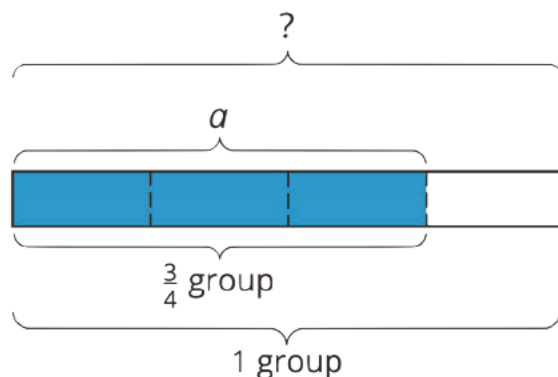
6. After biking $5\frac{1}{2}$ miles, Jada has traveled $\frac{2}{3}$ of the length of her trip. How long (in miles) is the entire length of her trip? Write an equation to represent the situation, and then find the answer.

Are you ready for more?

Suppose you have a pint of grape juice and a pint of milk. You pour 1 tablespoon of the grape juice into the milk and mix it up. Then you pour 1 tablespoon of this mixture back into the grape juice. Which liquid is more contaminated?

Lesson 11 Summary

The division $a \div \frac{3}{4} = ?$ is equivalent to $\frac{3}{4} \cdot ? = a$, so we can think of it as meaning “ $\frac{3}{4}$ of what number is a ?” and represent it with a diagram as shown. The length of the entire diagram represents the unknown number.



If $\frac{3}{4}$ of a number is a , then to find the number, we can first divide a by 3 to find $\frac{1}{4}$ of the number. Then we multiply the result by 4 to find the number.

The steps above can be written as: $a \div 3 \cdot 4$. Dividing by 3 is the same as multiplying by $\frac{1}{3}$, so we can also write the steps as: $a \cdot \frac{1}{3} \cdot 4$.

In other words: $a \div 3 \cdot 4 = a \cdot \frac{1}{3} \cdot 4$. And $a \cdot \frac{1}{3} \cdot 4 = a \cdot \frac{4}{3}$, so we can say that:

$$a \div \frac{3}{4} = a \cdot \frac{4}{3}$$

In general, dividing a number by a fraction $\frac{c}{d}$ is the same as multiplying the number by $\frac{d}{c}$, which is the reciprocal of the fraction.

Unit 4 Lesson 11 Cumulative Practice Problems

1. Select all the statements that show correct reasoning for finding $\frac{14}{15} \div \frac{7}{5}$.
 - A. Multiplying $\frac{14}{15}$ by 5 and then by $\frac{1}{7}$.
 - B. Dividing $\frac{14}{15}$ by 5, and then multiplying by $\frac{1}{7}$.
 - C. Multiplying $\frac{14}{15}$ by 7, and then multiplying by $\frac{1}{5}$.
 - D. Multiplying $\frac{14}{15}$ by 5 and then dividing by 7.
 - E. Multiplying $\frac{15}{14}$ by 7 and then dividing by 5.
2. Clare said that $\frac{4}{3} \div \frac{5}{2}$ is $\frac{10}{3}$. She reasoned: $\frac{4}{3} \cdot 5 = \frac{20}{3}$ and $\frac{20}{3} \div 2 = \frac{10}{3}$.

Explain why Clare's answer and reasoning are incorrect. Find the correct quotient.

3. Find the value of $\frac{15}{4} \div \frac{5}{8}$. Show your reasoning.

4. Consider the problem: Kiran has $2\frac{3}{4}$ pounds of flour. When he divides the flour into equal-sized bags, he fills $4\frac{1}{8}$ bags. How many pounds fit in each bag?

Write a multiplication equation and a division equation to represent the question. Then, find the answer and show your reasoning.

5. Divide $4\frac{1}{2}$ by each of these unit fractions.

a. $\frac{1}{8}$

b. $\frac{1}{4}$

c. $\frac{1}{6}$

(From Unit 4, Lesson 10.)

6. Consider the problem: After charging for $\frac{1}{3}$ of an hour, a phone is at $\frac{2}{5}$ of its full power. How long will it take the phone to charge completely?

Decide whether each equation can represent the situation.

a. $\frac{1}{3} \cdot ? = \frac{2}{5}$

b. $\frac{1}{3} \div \frac{2}{5} = ?$

c. $\frac{2}{5} \div \frac{1}{3} = ?$

d. $\frac{2}{5} \cdot ? = \frac{1}{3}$

(From Unit 4, Lesson 9.)

7. Elena and Noah are each filling a bucket with water. Noah's bucket is $\frac{2}{5}$ full and the water weighs $2\frac{1}{2}$ pounds. How much does Elena's water weigh if her bucket is full and her bucket is identical to Noah's?

a. Write multiplication and division equations to represent the question.

b. Draw a diagram to show the relationship between the quantities and to find the answer.

(From Unit 4, Lesson 8.)

Lesson 12: Fractional Lengths

12.1: Number Talk: Multiplication Strategies

Find the product mentally.

$$19 \cdot 14$$

12.2: Info Gap: How Many Would It Take?

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

Continue to ask questions until you have enough information to solve the problem.

4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.
2. Ask your partner “*What specific information do you need?*” and wait for them to *ask* for information.

If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don’t have that information.

3. Before sharing the information, ask “*Why do you need that information?*” Listen to your partner’s reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

Are you ready for more?

Lin has a work of art that is 14 inches by 20 inches. She wants to frame it with large paper clips laid end to end.

1. If each paper clip is $1\frac{3}{4}$ inch long, how many paper clips would she need? Show your reasoning and be sure to think about potential gaps and overlaps. Consider making a sketch that shows how the paper clips could be arranged.
2. How many paper clips are needed if the paper clips are spaced $\frac{1}{4}$ inch apart? Describe the arrangement of the paper clips at the corners of the frame.

12.3: How Many Times as Tall or as Far?

1. A second-grade student is 4 feet tall. Her teacher is $5\frac{2}{3}$ feet tall.
 - a. How many times as tall as the student is the teacher?
 - b. What fraction of the teacher's height is the student's height?
2. Find each quotient. Show your reasoning and check your answer.
 - a. $9 \div \frac{3}{5}$
 - b. $1\frac{7}{8} \div \frac{3}{4}$

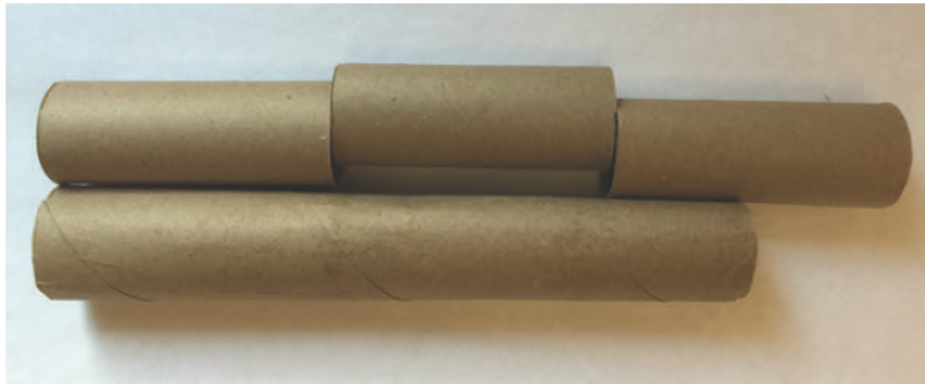
3. Write a division equation that can help answer each of these questions. Then find the answer. If you get stuck, consider drawing a diagram.

a. A runner ran $1\frac{4}{5}$ miles on Monday and $6\frac{3}{10}$ miles on Tuesday. How many times her Monday's distance was her Tuesday's distance?

b. A cyclist planned to ride $9\frac{1}{2}$ miles but only managed to travel $3\frac{7}{8}$ miles. What fraction of his planned trip did he travel?

12.4: Comparing Paper Rolls

The photo shows a situation that involves fractions.



1. Complete the sentences. Be prepared to explain your reasoning.

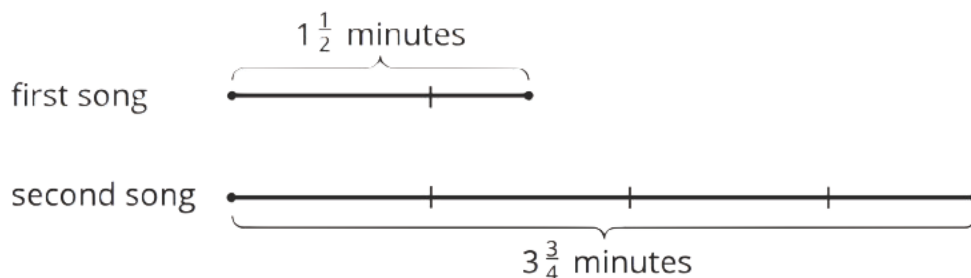
a. The length of the long tube is about _____ times the length of a short tube.

b. The length of a short tube is about _____ times the length of the long tube.

2. If the length of the long paper roll is $11\frac{1}{4}$ inches, what is the length of each short paper roll?

Lesson 12 Summary

Division can help us solve comparison problems in which we find out how many times as large or as small one number is compared to another. For example, a student is playing two songs for a music recital. The first song is $1\frac{1}{2}$ minutes long. The second song is $3\frac{3}{4}$ minutes long.



We can ask two different comparison questions and write different multiplication and division equations to represent each question.

- How many times as long as the first song is the second song?
- What fraction of the second song is the first song?

$$? \cdot 1\frac{1}{2} = 3\frac{3}{4}$$

$$3\frac{3}{4} \div 1\frac{1}{2} = ?$$

$$? \cdot 3\frac{3}{4} = 1\frac{1}{2}$$

$$1\frac{1}{2} \div 3\frac{3}{4} = ?$$

We can use the algorithm we learned to calculate the quotients.

$$= \frac{15}{4} \div \frac{3}{2}$$

$$= \frac{15}{4} \cdot \frac{2}{3}$$

$$= \frac{30}{12}$$

$$= \frac{5}{2}$$

$$= \frac{3}{2} \div \frac{15}{4}$$

$$= \frac{3}{2} \cdot \frac{4}{15}$$

$$= \frac{12}{30}$$

$$= \frac{2}{5}$$

This means the second song is $2\frac{1}{2}$ times as long as the first song.

This means the first song is $\frac{2}{5}$ as long as the second song.

Unit 4 Lesson 12 Cumulative Practice Problems

1. One inch is around $2\frac{11}{20}$ centimeters.



- a. How many centimeters long is 3 inches? Show your reasoning.
- b. What fraction of an inch is 1 centimeter? Show your reasoning.
- c. What question can be answered by finding $10 \div 2\frac{11}{20}$ in this situation?
2. A zookeeper is $6\frac{1}{4}$ feet tall. A young giraffe in his care is $9\frac{3}{8}$ feet tall.
- a. How many times as tall as the zookeeper is the giraffe?
- b. What fraction of the giraffe's height is the zookeeper's height?
3. A rectangular bathroom floor is covered with square tiles that are $1\frac{1}{2}$ feet by $1\frac{1}{2}$ feet. The length of the bathroom floor is $10\frac{1}{2}$ feet and the width is $6\frac{1}{2}$ feet.
- a. How many tiles does it take to cover the length of the floor?
- b. How many tiles does it take to cover the width of the floor?

4. The Food and Drug Administration (FDA) recommends a certain amount of nutrient intake per day called the “daily value.” Food labels usually show percentages of the daily values for several different nutrients—calcium, iron, vitamins, etc.

Consider the problem: In $\frac{3}{4}$ cup of oatmeal, there is $\frac{1}{10}$ of the recommended daily value of iron. What fraction of the daily recommended value of iron is in 1 cup of oatmeal?

Write a multiplication equation and a division equation to represent the question. Then find the answer and show your reasoning.

(From Unit 4, Lesson 11.)

5. What fraction of $\frac{1}{2}$ is $\frac{1}{3}$? Draw a tape diagram to represent and answer the question. Use graph paper if needed.

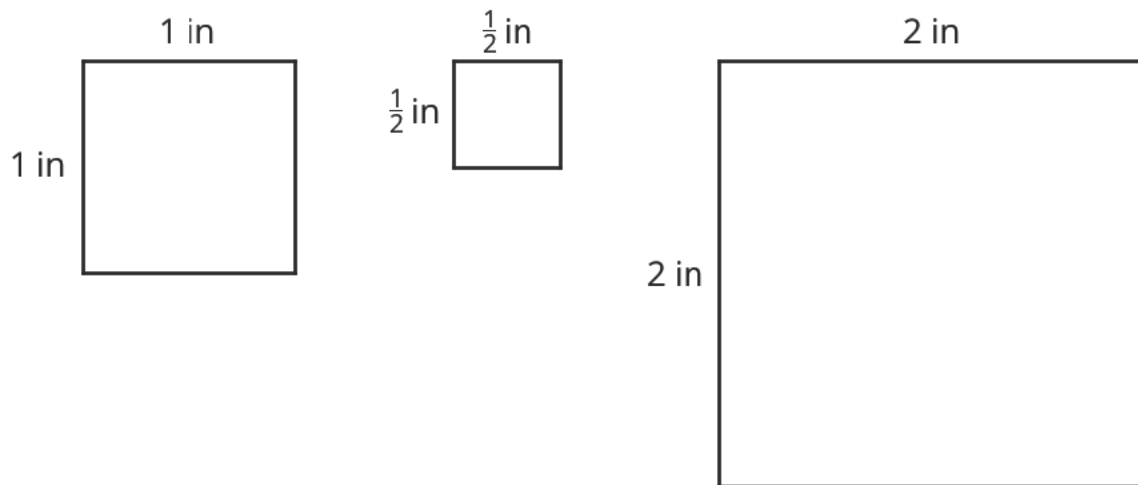
(From Unit 4, Lesson 7.)

6. Noah says, “There are $2\frac{1}{2}$ groups of $\frac{4}{5}$ in 2.” Do you agree with him? Draw a tape diagram to show your reasoning. Use graph paper, if needed.

(From Unit 4, Lesson 6.)

Lesson 13: Rectangles with Fractional Side Lengths

13.1: Areas of Squares



1. What do you notice about the areas of the squares?
2. Kiran says "A square with side lengths of $\frac{1}{3}$ inch has an area of $\frac{1}{3}$ square inches." Do you agree? Explain or show your reasoning.

13.2: Areas of Squares and Rectangles

Your teacher will give you graph paper and a ruler.

1. On the graph paper, draw a square with side lengths of 1 inch. Inside this square, draw another square with side lengths of $\frac{1}{4}$ inch.

Use your drawing to answer the questions.

- a. How many squares with side lengths of $\frac{1}{4}$ inch can fit in a square with side lengths of 1 inch?
- b. What is the area of a square with side lengths of $\frac{1}{4}$ inch? Explain or show your reasoning.

2. On the graph paper, draw a rectangle that is $3\frac{1}{2}$ inches by $2\frac{1}{4}$ inches.

For each question, write a division expression and then find the answer.

- a. How many $\frac{1}{4}$ -inch segments are in a length of $3\frac{1}{2}$ inches?

- b. How many $\frac{1}{4}$ -inch segments are in a length of $2\frac{1}{4}$ inches?

3. Use your drawing to show that a rectangle that is $3\frac{1}{2}$ inches by $2\frac{1}{4}$ inches has an area of $7\frac{7}{8}$ square inches.

13.3: Areas of Rectangles

Each of these multiplication expressions represents the area of a rectangle.

$2 \cdot 4$

$2\frac{1}{2} \cdot 4$

$2 \cdot 4\frac{3}{4}$

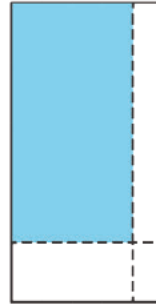
$2\frac{1}{2} \cdot 4\frac{3}{4}$

1. All regions shaded in light blue have the same area. Match each diagram to the expression that you think represents its area. Be prepared to explain your reasoning.

A



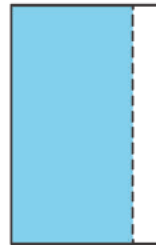
B



C



D



2. Use the diagram that matches $2\frac{1}{2} \cdot 4\frac{3}{4}$ to show that the value of $2\frac{1}{2} \cdot 4\frac{3}{4}$ is $11\frac{7}{8}$.

Are you ready for more?

The following rectangles are composed of squares, and each rectangle is constructed using the previous rectangle. The side length of the first square is 1 unit.



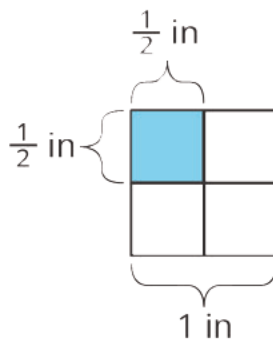
1. Draw the next four rectangles that are constructed in the same way. Then complete the table with the side lengths of the rectangle and the fraction of the longer side over the shorter side.

short side	long side	$\frac{\text{long side}}{\text{short side}}$
1		
1		
2		
3		

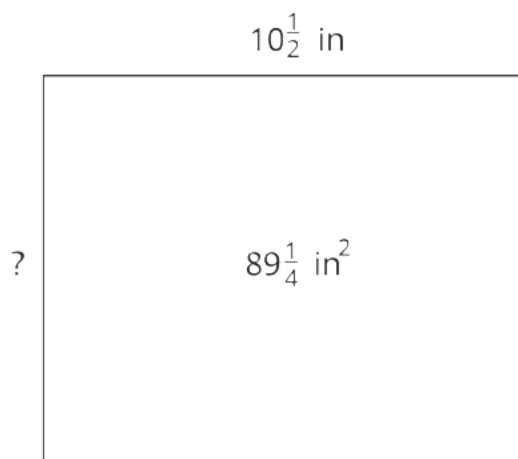
2. Describe the values of the fraction of the longer side over the shorter side. What happens to the fraction as the pattern continues?

Lesson 13 Summary

If a rectangle has side lengths a units and b units, the area is $a \cdot b$ square units. For example, if we have a rectangle with $\frac{1}{2}$ -inch side lengths, its area is $\frac{1}{2} \cdot \frac{1}{2}$ or $\frac{1}{4}$ square inches.



This means that if we know the *area* and *one side length* of a rectangle, we can divide to find the *other side length*.



If one side length of a rectangle is $10\frac{1}{2}$ in and its area is $89\frac{1}{4}$ in², we can write this equation to show their relationship:

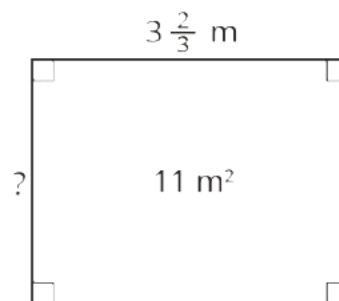
$$10\frac{1}{2} \cdot ? = 89\frac{1}{4}$$

Then, we can find the other side length, in inches, using division:

$$89\frac{1}{4} \div 10\frac{1}{2} = ?$$

Unit 4 Lesson 13 Cumulative Practice Problems

1. a. Find the unknown side length of the rectangle if its area is 11 m^2 . Show your reasoning.



- b. Check your answer by multiplying it by the given side length ($3\frac{2}{3}$). Is the resulting product 11? If not, revise your previous work.
2. A worker is tiling the floor of a rectangular room that is 12 feet by 15 feet. The tiles are square with side lengths $1\frac{1}{3}$ feet. How many tiles are needed to cover the entire floor? Show your reasoning.
3. A television screen has length $16\frac{1}{2}$ inches, width w inches, and area 462 square inches. Select all the equations that represent the relationship of the side lengths and area of the television.

A. $w \cdot 462 = 16\frac{1}{2}$

B. $16\frac{1}{2} \cdot w = 462$

C. $462 \div 16\frac{1}{2} = w$

D. $462 \div w = 16\frac{1}{2}$

E. $16\frac{1}{2} \cdot 462 = w$

4. The area of a rectangle is $17\frac{1}{2} \text{ in}^2$ and its shorter side is $3\frac{1}{2}$ in. Draw a diagram that shows this information. What is the length of the longer side?

5. A bookshelf is 42 inches long.

a. How many books of length $1\frac{1}{2}$ inches will fit on the bookshelf? Explain your reasoning.

b. A bookcase has 5 of these bookshelves. How many feet of shelf space is there? Explain your reasoning.

(From Unit 4, Lesson 12.)

6. Find the value of $\frac{5}{32} \div \frac{25}{4}$. Show your reasoning.

(From Unit 4, Lesson 11.)

7. How many groups of $1\frac{2}{3}$ are in each of these quantities?

a. $1\frac{5}{6}$

b. $4\frac{1}{3}$

c. $\frac{5}{6}$

(From Unit 4, Lesson 6.)

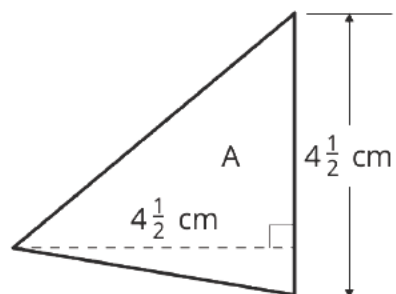
8. It takes $1\frac{1}{4}$ minutes to fill a 3-gallon bucket of water with a hose. At this rate, how long does it take to fill a 50-gallon tub? If you get stuck, consider using a table.

(From Unit 2, Lesson 14.)

Lesson 14: Fractional Lengths in Triangles and Prisms

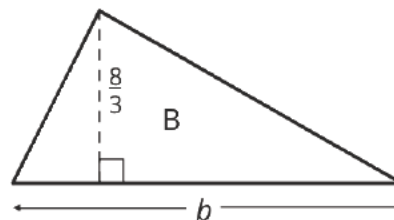
14.1: Area of Triangle

Find the area of Triangle A in square centimeters. Show your reasoning.

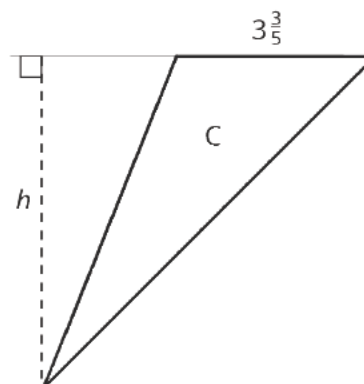


14.2: Bases and Heights of Triangles

1. The area of Triangle B is 8 square units. Find the length of b . Show your reasoning.



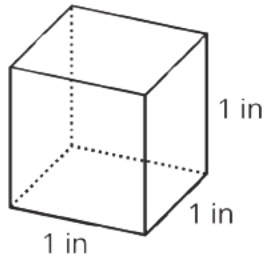
2. The area of Triangle C is $\frac{54}{5}$ square units. What is the length of h ? Show your reasoning.



14.3: Volumes of Cubes and Prisms

Your teacher will give you cubes that have edge lengths of $\frac{1}{2}$ inch.

1. Here is a drawing of a cube with edge lengths of 1 inch.



- a. How many cubes with edge lengths of $\frac{1}{2}$ inch are needed to fill this cube?
- b. What is the volume, in cubic inches, of a cube with edge lengths of $\frac{1}{2}$ inch?
Explain or show your reasoning.

2. Four cubes are piled in a single stack to make a prism. Each cube has an edge length of $\frac{1}{2}$ inch. Sketch the prism, and find its volume in cubic inches.

3. Use cubes with an edge length of $\frac{1}{2}$ inch to build prisms with the lengths, widths, and heights shown in the table.

a. For each prism, record in the table how many $\frac{1}{2}$ -inch cubes can be packed into the prism and the volume of the prism.

prism length (in)	prism width (in)	prism height (in)	number of $\frac{1}{2}$ -inch cubes in prism	volume of prism (in^3)
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$		
1	1	$\frac{1}{2}$		
2	1	$\frac{1}{2}$		
2	2	1		
4	2	$\frac{3}{2}$		
5	4	2		
5	4	$2\frac{1}{2}$		

b. Examine the values in the table. What do you notice about the relationship between the edge lengths of each prism and its volume?

4. What is the volume of a rectangular prism that is $1\frac{1}{2}$ inches by $2\frac{1}{4}$ inches by 4 inches? Show your reasoning.

Are you ready for more?

A unit fraction has a 1 in the numerator.

- These are unit fractions: $\frac{1}{3}$, $\frac{1}{100}$, $\frac{1}{1}$.
- These are *not* unit fractions: $\frac{2}{9}$, $\frac{8}{1}$, $2\frac{1}{5}$.

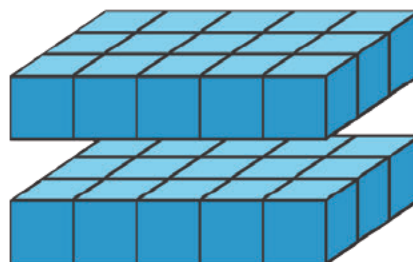
1. Find three unit fractions whose sum is $\frac{1}{2}$. An example is: $\frac{1}{8} + \frac{1}{8} + \frac{1}{4} = \frac{1}{2}$ How many examples like this can you find?

2. Find a box whose surface area in square units equals its volume in cubic units. How many like this can you find?

Lesson 14 Summary

If a rectangular prism has edge lengths of 2 units, 3 units, and 5 units, we can think of it as 2 layers of unit cubes, with each layer having $(3 \cdot 5)$ unit cubes in it. So the volume, in cubic units, is:

$$2 \cdot 3 \cdot 5$$



To find the volume of a rectangular prism with fractional edge lengths, we can think of it as being built of cubes that have a unit fraction for their edge length. For instance, if we build a prism that is $\frac{1}{2}$ -inch tall, $\frac{3}{2}$ -inch wide, and 4 inches long using cubes with a $\frac{1}{2}$ -inch edge length, we would have:

- A height of 1 cube, because $1 \cdot \frac{1}{2} = \frac{1}{2}$.
- A width of 3 cubes, because $3 \cdot \frac{1}{2} = \frac{3}{2}$.
- A length of 8 cubes, because $8 \cdot \frac{1}{2} = 4$.

The volume of the prism would be $1 \cdot 3 \cdot 8$, or 24 cubic units. How do we find its volume in cubic inches? We know that each cube with a $\frac{1}{2}$ -inch edge length has a volume of $\frac{1}{8}$ cubic inch, because $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$. Since the prism is built using 24 of these cubes, its volume, in cubic inches, would then be $24 \cdot \frac{1}{8}$, or 3 cubic inches.

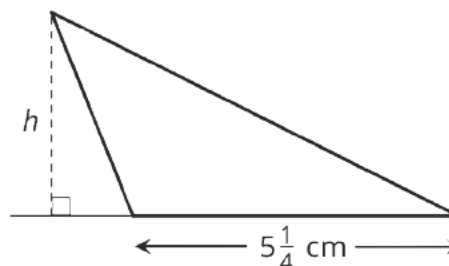
The volume of the prism, in cubic inches, can also be found by multiplying the fractional edge lengths in inches: $\frac{1}{2} \cdot \frac{3}{2} \cdot 4 = 3$

Unit 4 Lesson 14 Cumulative Practice Problems

1. Clare is using little wooden cubes with edge length $\frac{1}{2}$ inch to build a larger cube that has edge length 4 inches. How many little cubes does she need? Explain your reasoning.

2. The triangle has an area of $7\frac{7}{8}$ cm^2 and a base of $5\frac{1}{4}$ cm.

What is the length of h ? Explain your reasoning.



3. a. Which expression can be used to find how many cubes with edge length of $\frac{1}{3}$ unit fit in a prism that is 5 units by 5 units by 8 units? Explain or show your reasoning.
- $(5 \cdot \frac{1}{3}) \cdot (5 \cdot \frac{1}{3}) \cdot (8 \cdot \frac{1}{3})$
 - $5 \cdot 5 \cdot 8$
 - $(5 \cdot 3) \cdot (5 \cdot 3) \cdot (8 \cdot 3)$
 - $(5 \cdot 5 \cdot 8) \cdot (\frac{1}{3})$
- b. Mai says that we can also find the answer by multiplying the edge lengths of the prism and then multiplying the result by 27. Do you agree with her? Explain your reasoning.

4. A builder is building a fence with $6\frac{1}{4}$ -inch-wide wooden boards, arranged side-by-side with no gaps or overlaps. How many boards are needed to build a fence that is 150 inches long? Show your reasoning.

(From Unit 4, Lesson 12.)

5. Find the value of each expression. Show your reasoning and check your answer.

a. $2\frac{1}{7} \div \frac{2}{7}$

b. $\frac{17}{20} \div \frac{1}{4}$

(From Unit 4, Lesson 12.)

6. Consider the problem: A bucket contains $11\frac{2}{3}$ gallons of water and is $\frac{5}{6}$ full. How many gallons of water would be in a full bucket?

Write a multiplication and a division equation to represent the situation. Then, find the answer and show your reasoning.

(From Unit 4, Lesson 11.)

7. There are 80 kids in a gym. 75% are wearing socks. How many are *not* wearing socks?
If you get stuck, consider using a tape diagram.

(From Unit 3, Lesson 12.)

8. a. Lin wants to save \$75 for a trip to the city. If she has saved \$37.50 so far, what percentage of her goal has she saved? What percentage remains?

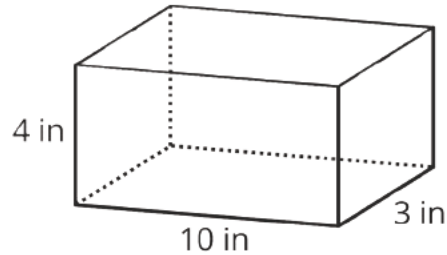
- b. Noah wants to save \$60 so that he can purchase a concert ticket. If he has saved \$45 so far, what percentage of his goal has he saved? What percentage remains?

(From Unit 3, Lesson 11.)

Lesson 15: Volume of Prisms

15.1: A Box of Cubes

1. How many cubes with an edge length of 1 inch fill this box?



2. If the cubes had an edge length of 2 inches, would you need more or fewer cubes to fill the box? Explain your reasoning.

3. If the cubes had an edge length of $\frac{1}{2}$ inch, would you need more or fewer cubes to fill the box? Explain your reasoning.

15.2: Cubes with Fractional Edge Lengths

1. Diego says that 108 cubes with an edge length of $\frac{1}{3}$ inch are needed to fill a rectangular prism that is 3 inches by 1 inch by $1\frac{1}{3}$ inch.
 - a. Explain or show how this is true. If you get stuck, consider drawing a diagram.

 - b. What is the volume, in cubic inches, of the rectangular prism? Explain or show your reasoning.

2. Lin and Noah are packing small cubes into a larger cube with an edge length of $1\frac{1}{2}$ inches. Lin is using cubes with an edge length of $\frac{1}{2}$ inch, and Noah is using cubes with an edge length of $\frac{1}{4}$ inch.
 - a. Who would need more cubes to fill the $1\frac{1}{2}$ -inch cube? Be prepared to explain your reasoning.

 - b. If Lin and Noah each use their small cubes to find the volume of the larger $1\frac{1}{2}$ -inch cube, will they get the same answer? Explain or show your reasoning.

15.3: Fish Tank and Baking Pan

1. A nature center has a fish tank in the shape of a rectangular prism. The tank is 10 feet long, $8\frac{1}{4}$ feet wide, and 6 feet tall.

- a. What is the volume of the tank in cubic feet?
Explain or show your reasoning.



- b. The nature center's caretaker filled $\frac{4}{5}$ of the tank with water. What was the volume of the water in the tank, in cubic feet? What was the height of the water in the tank? Explain or show your reasoning.

- c. Another day, the tank was filled with 330 cubic feet of water. The height of the water was what fraction of the height of the tank? Show your reasoning.

2. Clare's recipe for banana bread won't fit in her favorite pan. The pan is $8\frac{1}{2}$ inches by 11 inches by 2 inches. The batter fills the pan to the very top, and when baking, the batter spills over the sides. To avoid spills, there should be about an inch between the top of the batter and the rim of the pan.

Clare has another pan that is 9 inches by 9 inches by $2\frac{1}{2}$ inches. If she uses this pan, will the batter spill over during baking?

Are you ready for more?

1. Find the area of a rectangle with side lengths $\frac{1}{2}$ and $\frac{2}{3}$.
2. Find the volume of a rectangular prism with side lengths $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$.
3. What do you think happens if we keep multiplying fractions $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} \cdot \frac{5}{6} \dots$?

4. Find the area of a rectangle with side lengths $\frac{1}{1}$ and $\frac{2}{1}$.
5. Find the volume of a rectangular prism with side lengths $\frac{1}{1}$, $\frac{2}{1}$, and $\frac{1}{3}$.
6. What do you think happens if we keep multiplying fractions $\frac{1}{1} \cdot \frac{2}{1} \cdot \frac{1}{3} \cdot \frac{4}{1} \cdot \frac{1}{5} \dots$?

Lesson 15 Summary

If a rectangular prism has edge lengths a units, b units, and c units, the volume is the product of a , b , and c .

$$V = a \cdot b \cdot c$$

This means that if we know the *volume* and *two edge lengths*, we can divide to find the *third edge length*.

Suppose the volume of a rectangular prism is $400\frac{1}{2} \text{ cm}^3$, one edge length is $\frac{11}{2}$ cm, another is 6 cm, and the third edge length is unknown. We can write a multiplication equation to represent the situation:

$$\frac{11}{2} \cdot 6 \cdot ? = 400\frac{1}{2}$$

We can find the third edge length by dividing:

$$400\frac{1}{2} \div \left(\frac{11}{2} \cdot 6 \right) = ?$$

Unit 4 Lesson 15 Cumulative Practice Problems

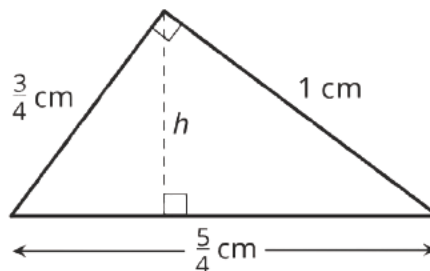
1. A pool in the shape of a rectangular prism is being filled with water. The length and width of the pool is 24 feet and 15 feet. If the height of the water in the pool is $1\frac{1}{3}$ feet, what is the volume of the water in cubic feet?

2. A rectangular prism measures $2\frac{2}{5}$ inches by $3\frac{1}{5}$ inches by 2 inch.
 - a. Priya said, "It takes more cubes with edge length $\frac{2}{5}$ inch than cubes with edge length $\frac{1}{5}$ inch to pack the prism." Do you agree with Priya? Explain or show your reasoning.

 - b. How many cubes with edge length $\frac{1}{5}$ inch fit in the prism? Show your reasoning.

 - c. Explain how you can use your answer in the previous question to find the volume of the prism in cubic inches.

3. a. Here is a right triangle. What is its area?



- b. What is the height h for the base that is $\frac{5}{4}$ units long? Show your reasoning.

(From Unit 4, Lesson 14.)

4. To give their animals essential minerals and nutrients, farmers and ranchers often have a block of salt—called “salt lick”—available for their animals to lick.

- a. A rancher is ordering a box of cube-shaped salt licks. The edge lengths of each salt lick are $\frac{5}{12}$ foot. Is the volume of one salt lick greater or less than 1 cubic foot? Explain your reasoning.



- b. The box that contains the salt lick is $1\frac{1}{4}$ feet by $1\frac{2}{3}$ feet by $\frac{5}{6}$ feet. How many cubes of salt lick fit in the box? Explain or show your reasoning.

5. a. How many groups of $\frac{1}{3}$ inch are in $\frac{3}{4}$ inch?

- b. How many inches are in $1\frac{2}{5}$ groups of $1\frac{2}{3}$ inches?

(From Unit 4, Lesson 12.)

6. Here is a table that shows the ratio of flour to water in an art paste. Complete the table with values in equivalent ratios.

cups of flour	cups of water
1	$\frac{1}{2}$
4	
	3
$\frac{1}{2}$	

(From Unit 2, Lesson 12.)

Lesson 16: Solving Problems Involving Fractions

16.1: Operations with Fractions

Without calculating, order the expressions according to their values from least to greatest. Be prepared to explain your reasoning.

$$\frac{3}{4} + \frac{2}{3}$$

$$\frac{3}{4} - \frac{2}{3}$$

$$\frac{3}{4} \cdot \frac{2}{3}$$

$$\frac{3}{4} \div \frac{2}{3}$$

16.2: Situations with $\frac{3}{4}$ and $\frac{1}{2}$

Here are four situations that involve $\frac{3}{4}$ and $\frac{1}{2}$.

- Before calculating, decide if each answer is greater than 1 or less than 1.
 - Write a multiplication equation or division equation for the situation.
 - Answer the question. Show your reasoning. Draw a tape diagram, if needed.
1. There was $\frac{3}{4}$ liter of water in Andre's water bottle. Andre drank $\frac{1}{2}$ of the water. How many liters of water did he drink?

 2. The distance from Han's house to his school is $\frac{3}{4}$ kilometers. Han walked $\frac{1}{2}$ kilometers. What fraction of the distance from his house to the school did Han walk?

 3. Priya's goal was to collect $\frac{1}{2}$ kilograms of trash. She collected $\frac{3}{4}$ kilograms of trash. How many times her goal was the amount of trash she collected?

4. Mai's class volunteered to clean a park with an area of $\frac{1}{2}$ square mile. Before they took a lunch break, the class had cleaned $\frac{3}{4}$ of the park. How many square miles had they cleaned before lunch?

16.3: Pairs of Problems

1. Work with a partner to write equations for the following questions. One person works on the questions labeled A1, B1, . . . , E1 and the other person works on those labeled A2, B2, . . . , E2.

A1. Lin's bottle holds $3\frac{1}{4}$ cups of water. She drank 1 cup of water. What fraction of the water in the bottle did she drink?

A2. Lin's bottle holds $3\frac{1}{4}$ cups of water. After she drank some, there were $1\frac{1}{2}$ cups of water in the bottle. How many cups did she drink?

B1. Plant A is $\frac{16}{3}$ feet tall. This is $\frac{4}{5}$ as tall as Plant B. How tall is Plant B?

B2. Plant A is $\frac{16}{3}$ feet tall. Plant C is $\frac{4}{5}$ as tall as Plant A. How tall is Plant C?

C1. $\frac{8}{9}$ kilogram of berries is put into a container that already has $\frac{7}{3}$ kilogram of berries. How many kilograms are in the container?

C2. A container with $\frac{8}{9}$ kilogram of berries is $\frac{2}{3}$ full. How many kilograms can the container hold?

D1. The area of a rectangle is $14\frac{1}{2}$ sq cm and one side is $4\frac{1}{2}$ cm. How long is the other side?

D2. The side lengths of a rectangle are $4\frac{1}{2}$ cm and $2\frac{2}{5}$ cm. What is the area of the rectangle?

E1. A stack of magazines is $4\frac{2}{5}$ inches high. The stack needs to fit into a box that is $2\frac{1}{8}$ inches high. How many inches too high is the stack?

E2. A stack of magazines is $4\frac{2}{5}$ inches high. Each magazine is $\frac{2}{5}$ -inch thick. How many magazines are in the stack?

2. Trade papers with your partner, and check your partner's equations. If you disagree, work to reach an agreement.
3. Your teacher will assign 2 or 3 questions for you to answer. For each question:
 - a. Estimate the answer before calculating it.
 - b. Find the answer, and show your reasoning.

16.4: Baking Cookies

Mai, Kiran, and Clare are baking cookies together. They need $\frac{3}{4}$ cup of flour and $\frac{1}{3}$ cup of butter to make a batch of cookies. They each brought the ingredients they had at home.

- Mai brought 2 cups of flour and $\frac{1}{4}$ cup of butter.
- Kiran brought 1 cup of flour and $\frac{1}{2}$ cup of butter.
- Clare brought $1\frac{1}{4}$ cups of flour and $\frac{3}{4}$ cup of butter.

If the students have plenty of the other ingredients they need (sugar, salt, baking soda, etc.), how many whole batches of cookies can they make? Explain your reasoning.

Lesson 16 Summary

We can add, subtract, multiply, and divide both whole numbers and fractions. Here is a summary of how we add, subtract, multiply, and divide fractions.

- To add or subtract fractions, we often look for a common denominator so the pieces involved are the same size. This makes it easy to add or subtract the pieces.

$$\frac{3}{2} - \frac{4}{5} = \frac{15}{10} - \frac{8}{10}$$

- To multiply fractions, we often multiply the numerators and the denominators.

$$\frac{3}{8} \cdot \frac{5}{9} = \frac{3 \cdot 5}{8 \cdot 9}$$

- To divide a number by a fraction $\frac{a}{b}$, we can multiply the number by $\frac{b}{a}$, which is the reciprocal of $\frac{a}{b}$.

$$\frac{4}{7} \div \frac{5}{3} = \frac{4}{7} \cdot \frac{3}{5}$$

Unit 4 Lesson 16 Cumulative Practice Problems

1. An orange has about $\frac{1}{4}$ cup of juice. How many oranges are needed to make $2\frac{1}{2}$ cups of juice? Select all the equations that represent this question.

A. $? \cdot \frac{1}{4} = 2\frac{1}{2}$

B. $\frac{1}{4} \div 2\frac{1}{2} = ?$

C. $? \cdot 2\frac{1}{2} = \frac{1}{4}$

D. $2\frac{1}{2} \div \frac{1}{4} = ?$

2. Mai, Clare, and Tyler are hiking from a parking lot to the summit of a mountain. They pass a sign that gives distances.

Parking lot: $\frac{3}{4}$ mile Summit: $1\frac{1}{2}$ miles

- Mai says: "We are one third of the way there."
- Clare says: "We have to go twice as far as we have already gone."
- Tyler says: "The total hike is three times as long as what we have already gone."

Do you agree with any of them? Explain your reasoning.

3. Priya's cat weighs $5\frac{1}{2}$ pounds and her dog weighs $8\frac{1}{4}$ pounds. First, estimate the number that would complete each sentence. Then, calculate the answer. If any of your estimates were not close to the answer, explain why that may be.

a. The cat is _____ as heavy as the dog.

b. Their combined weight is _____ pounds.

c. The dog is _____ pounds heavier than the cat.

4. Before refrigerators existed, some people had blocks of ice delivered to their homes. A delivery wagon had a storage box in the shape of a rectangular prism that was $7\frac{1}{2}$ feet by 6 feet by 6 feet. The cubic ice blocks stored in the box had side lengths $1\frac{1}{2}$ feet. How many ice blocks fit in the storage box?

A. 270

B. $3\frac{3}{8}$

C. 80

D. 180

(From Unit 4, Lesson 15.)

5. Fill in the blanks with 0.001, 0.1, 10, or 1000 so that the value of each quotient is in the correct column.

Close to $\frac{1}{100}$

○ _____ \div 9

○ $12 \div$ _____

Close to 1

○ _____ \div 0.12

○ $\frac{1}{8} \div$ _____

Greater than 100

○ _____ \div $\frac{1}{3}$

○ $700.7 \div$ _____

(From Unit 4, Lesson 1.)

6. A school club sold 300 shirts. 31% were sold to fifth graders, 52% were sold to sixth graders, and the rest were sold to teachers. How many shirts were sold to each group—fifth graders, sixth graders, and teachers? Explain or show your reasoning.

(From Unit 3, Lesson 15.)

7. Jada has some pennies and dimes. The ratio of Jada's pennies to dimes is 2 to 3.

a. From the information given, can you determine how many coins Jada has?

b. If Jada has 55 coins, how many of each kind of coin does she have?

c. How much are her coins worth?

(From Unit 2, Lesson 15.)

Lesson 17: Fitting Boxes into Boxes

17.1: Determining Shipping Costs (Part 1)

An artist makes necklaces. She packs each necklace in a small jewelry box that is $1\frac{3}{4}$ inches by $2\frac{1}{4}$ inches by $\frac{3}{4}$ inch.

A department store ordered 270 necklaces. The artist plans to ship the necklaces to the department store using flat-rate shipping boxes from the post office.

1. Consider the problem: Which of the flat-rate boxes should she use to minimize her shipping cost?

What other information would you need to be able to solve the problem?

2. Discuss this information with your group. Make a plan for using this information to find the most inexpensive way to ship the jewelry boxes. Once you have agreed on a plan, write down the main steps.

17.2: Determining Shipping Costs (Part 2)

Work with your group to find the best plan for shipping the boxes of necklaces. Each member of your group should select a different type of flat-rate shipping box and answer the following questions. Recall that each jewelry box is $1\frac{3}{4}$ inches by $2\frac{1}{4}$ inches by $\frac{3}{4}$ inch, and that there are 270 jewelry boxes to be shipped.

For each type of flat-rate shipping box:

1. Find how many jewelry boxes can fit into the box. Explain or show how the jewelry boxes can be packed in the shipping box. Draw a sketch to show your thinking, if needed.

2. Calculate the total cost of shipping all 270 jewelry boxes in shipping boxes of that type. Show your reasoning and organize your work so it can be followed by others.

17.3: Determining Shipping Costs (Part 3)

1. Share and discuss your work with the other members of your group. Your teacher will display questions to guide your discussion. Note the feedback from your group so you can use it to revise your work.

2. Using the feedback from your group, revise your work to improve its correctness, clarity, and accuracy. Correct any errors. You may also want to add notes or diagrams, or remove unnecessary information.

3. Which shipping boxes should the artist use? As a group, decide which boxes you recommend for shipping 270 jewelry boxes. Be prepared to share your reasoning.

Unit 4 Lesson 17 Cumulative Practice Problems

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