

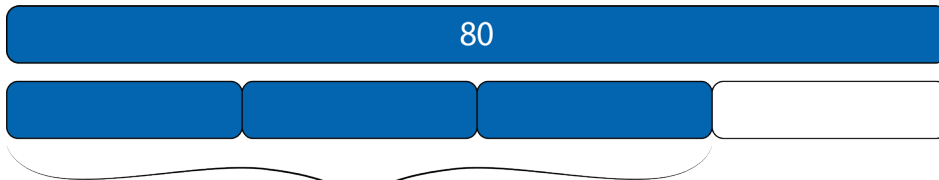
# Unit Rates and Percentages

## Teacher Guide

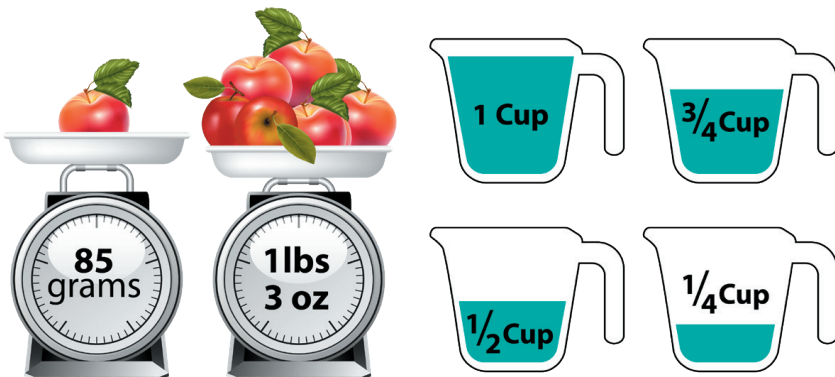
Comparing Prices



Estimation and Comparison



Percentages and Tape Diagrams



Standard Units of Measurement



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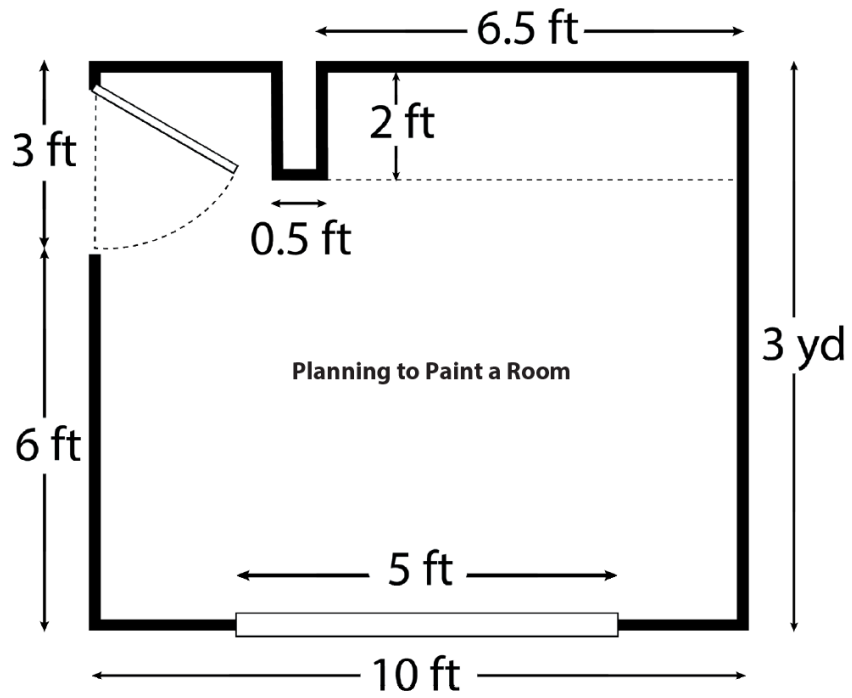
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# Unit Rates and Percentages

## Table of Contents

<b>Introduction: Unit Narrative</b> .....	1
<b>Student Learning Targets</b> .....	3
<b>Terminology</b> .....	6
<b>Required Materials</b> .....	7
<b>Lesson Plans and Student Task Statements:</b>	
Section 1: Lesson 1 <b>Units of Measurement</b> .....	8
Section 2: Lessons 2–4 <b>Unit Conversion</b> .....	23
Section 3: Lessons 5–9 <b>Rates</b> .....	70
Section 4: Lessons 10–16 <b>Percentages</b> .....	150
Let’s Put It to Work: Lesson 17 <b>Painting a Room</b> .....	246
<b>Teacher Resources</b> .....	255
Family Support Materials	
Unit Assessments	
Assessment Answer Keys	
Cool Downs (Lesson-level Assessments)	
Black Line Masters	





**Unit Rates and Percentages**  
**Teacher Guide**  
 Core Knowledge Mathematics™



# Unit Rates and Percentages

## Unit Narrative

In the previous unit, students began to develop an understanding of ratios and rates. They started to describe situations using terms such as “ratio,” “rate,” “equivalent ratios,” “per,” “constant speed,” and “constant rate” (MP6). They understood specific instances of the idea that  $a : b$  is equivalent to every other ratio of the form  $sa : sb$ , where  $s$  is a positive number. They learned that “at this rate” or “at the same rate” signals a situation that is characterized by equivalent ratios. Although the usefulness of ratios of the form  $\frac{a}{b} : 1$  and  $1 : \frac{b}{a}$  was highlighted, the term “unit rate” was not introduced.

In this unit, students find the two values  $\frac{a}{b}$  and  $\frac{b}{a}$  that are associated with the ratio  $a : b$ , and interpret them as rates per 1. For example, if a person walks 13 meters in 10 seconds at a constant rate, that means they walked at a speed of  $\frac{13}{10}$  meters per 1 second and a pace of  $\frac{10}{13}$  seconds per 1 meter.

Students learn that one of the two values ( $\frac{a}{b}$  or  $\frac{b}{a}$ ) may be more useful than the other in reasoning about a given situation. They find and use rates per 1 to solve problems set in contexts (MP2), attending to units and specifying units in their answers. For example, given item amounts and their costs, which is the better deal? Or given distances and times, which object is moving faster? Measurement conversions provide other opportunities to use rates.

Students observe that if two ratios  $a : b$  and  $c : d$  are equivalent, then  $\frac{a}{b} = \frac{c}{d}$ . The values  $\frac{a}{b}$  and  $\frac{c}{d}$  are called *unit rates* because they can be interpreted in the context from which they arose as rates per unit. Students note that in a table of equivalent ratios, the entries in one column are produced by multiplying a unit rate by the corresponding entries in the other column. Students learn that “percent” means “per 100” and indicates a rate. Just as a unit rate can be interpreted in context as a rate per 1, a percentage can be interpreted in the context from which it arose as a rate per 100. For example, suppose a beverage is made by mixing 1 cup of juice with 9 cups of water. The *percentage* of juice in 20 cups of the beverage is 2 cups and 10 *percent* of the beverage is juice. Interpreting the 10 as a rate: “there are 10 cups of juice per 100 cups of beverage” or, more generally, “there are 10 units of juice per 100 units of beverage.” The percentage—and the rate—indicate equivalent ratios of juice to beverage, e.g., 2 cups to 20 cups and 10 cups to 100 cups.

In this unit, tables and double number line diagrams are intended to help students connect percentages with equivalent ratios, and reinforce an understanding of percentages as rates per 100. Students should internalize the meaning of important benchmark percentages, for example, they should connect “75% of a number” with “ $\frac{3}{4}$  times a number” and “0.75 times a number.” Note that 75% (“seventy-five per hundred”) does not represent a fraction or decimal (which are numbers), but that “75% of a number” is calculated as a *fraction of* or a *decimal times* the number.

Work done in grades 4 and 5 supports learning about the concept of a percentage. In grade 5, students understand why multiplying a given number by a fraction less than 1 results in a product that is less than the original number, and why multiplying a given number by a fraction greater than 1 results in a product that is greater than the original number. This understanding of multiplication as scaling comes into play as students interpret, for example,

- 35% of 2 cups of juice as  $\frac{35}{100} \cdot 2$  cups of juice.
- 250% of 2 cups of juice as  $\frac{250}{100} \cdot 2$  cups of juice.

### **Progression of Disciplinary Language**

In this unit, teachers can anticipate students using language for mathematical purposes such as interpreting, explaining, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

#### **Interpret**

- a context in which identifying a unit rate is helpful (Lesson 1)
- unit rates in different contexts (Lesson 6)
- situations involving constant speed (Lesson 8)
- tape diagrams used to represent percentages (Lesson 12)
- situations involving measurement, rates, and cost (Lesson 17)

#### **Explain**

- reasoning for estimating and sorting measurements (Lesson 2)
- reasoning about relative sizes of units of measurement (Lesson 3)
- how to make decisions using rates (Lesson 9)
- reasoning about percentages (Lesson 11)
- strategies for finding missing information involving percentages (Lesson 14)

#### **Justify**

- reasoning about equivalent ratios and unit rates (Lesson 7)
- reasoning about finding percentages (Lessons 15 and 16)
- reasoning about costs and time (Lesson 17)

In addition, students have opportunities to generalize about unit ratios, unit rates, and percentages from multiple contexts and with reference to benchmark percentages, tape diagrams, and other mathematical representations. Students can also be expected to describe measurements and observations, describe and compare situations involving percentages, compare speeds, compare prices, and critique reasoning about costs and time.



# Learning Targets

## Unit Rates and Percentages

### Lesson 1: The Burj Khalifa

- I can see that thinking about “how much for 1” is useful for solving different types of problems.

### Lesson 2: Anchoring Units of Measurement

- I can name common objects that are about as long as 1 inch, foot, yard, mile, millimeter, centimeter, meter, or kilometer.
- I can name common objects that weigh about 1 ounce, pound, ton, gram, or kilogram, or that hold about 1 cup, quart, gallon, milliliter, or liter.
- When I read or hear a unit of measurement, I know whether it is used to measure length, weight, or volume.

### Lesson 3: Measuring with Different-Sized Units

- When I know a measurement in one unit, I can decide whether it takes more or less of a different unit to measure the same quantity.

### Lesson 4: Converting Units

- I can convert measurements from one unit to another, using double number lines, tables, or by thinking about “how much for 1.”
- I know that when we measure things in two different units, the pairs of measurements are equivalent ratios.

### Lesson 5: Comparing Speeds and Prices

- I understand that if two ratios have the same rate per 1, they are equivalent ratios.
- When measurements are expressed in different units, I can decide who is traveling faster or which item is the better deal by comparing “how much for 1” of the same unit.

### **Lesson 6: Interpreting Rates**

- I can choose which unit rate to use based on how I plan to solve the problem.
- When I have a ratio, I can calculate its two unit rates and explain what each of them means in the situation.

### **Lesson 7: Equivalent Ratios Have the Same Unit Rates**

- I can give an example of two equivalent ratios and show that they have the same unit rates.
- I can multiply or divide by the unit rate to calculate missing values in a table of equivalent ratios.

### **Lesson 8: More about Constant Speed**

- I can solve more complicated problems about constant speed situations.

### **Lesson 9: Solving Rate Problems**

- I can choose how to use unit rates to solve problems.

### **Lesson 10: What Are Percentages?**

- I can create a double number line with percentages on one line and dollar amounts on the other line.
- I can explain the meaning of percentages using dollars and cents as an example.

### **Lesson 11: Percentages and Double Number Lines**

- I can use double number line diagrams to solve different problems like “What is 40% of 60?” or “60 is 40% of what number?”

### **Lesson 12: Percentages and Tape Diagrams**

- I can use tape diagrams to solve different problems like “What is 40% of 60?” or “60 is 40% of what number?”

### **Lesson 13: Benchmark Percentages**

- When I read or hear that something is 10%, 25%, 50%, or 75% of an amount, I know what fraction of that amount they are referring to.

### **Lesson 14: Solving Percentage Problems**

- I can choose and create diagrams to help me solve problems about percentages.

**Lesson 15: Finding This Percent of That**

- I can solve different problems like “What is 40% of 60?” by dividing and multiplying.

**Lesson 16: Finding the Percentage**

- I can solve different problems like “60 is what percentage of 40?” by dividing and multiplying.

**Lesson 17: Painting a Room**

- I can apply what I have learned about unit rates and percentages to predict how long it will take and how much it will cost to paint all the walls in a room.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

lesson	new terminology	
	receptive	productive
6.3.1		at this rate
6.3.3	order	
6.3.5	(good / better / best) deal rate per 1	<b>unit price</b> same speed
6.3.6	<b>unit rate</b>	gallon
6.3.7	result	<b>unit rate</b>
6.3.8	<b>pace</b>	<b>speed</b>
6.3.9		<b>meters per second</b> (good / better / best) deal
6.3.10	<b>percentage</b> ___% of	
6.3.11		tick marks
6.3.12	___% as much	<b>tape diagram</b> ___% of
6.3.14	___% of	
6.3.15	regular price sale price	<b>percentage</b>

# Required Materials

**Base-ten blocks**

**Blank paper**

**Cuisenaire rods**

**Four-function calculators**

**Gallon-sized jug**

**Graduated cylinders**

**Household items**

**Inch cubes**

**Internet-enabled device**

**Liter-sized bottle**

**Materials assembled from the blackline master**

**Metal paper fasteners**

brass brads

**Meter sticks**

**Pre-assembled polyhedra**

**Pre-printed slips, cut from copies of the blackline master**

**Quart-sized bottle**

**Rulers**

**Salt**

**Scale**

a digital scale that can output in grams, kilograms, ounces, or pounds

**Scissors**

**Straightedges**

A rigid edge that can be used for drawing line segments. Sometimes a ruler is okay to use as a straightedge, but sometimes it is preferable to use an unruled straightedge, like a blank index card.

**String**

**Teaspoon**

**Tools for creating a visual display**

Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

**Tray**

**Yardsticks**

## Section: Units of Measurement

### Lesson 1: The Burj Khalifa

#### Goals

- Evaluate (orally) the usefulness of calculating a rate per 1 when solving problems involving unfamiliar rates.
- Explain (orally, in writing, and through other representations) how to solve a problem involving rates in a less familiar context, e.g., minutes per window.

#### Learning Targets

- I can see that thinking about “how much for 1” is useful for solving different types of problems.

#### Lesson Narrative

In the previous unit, students began to develop an understanding of ratios and familiarity with ratio and rate language. They represented equivalent ratios using discrete diagrams, double number lines, and tables. They learned that  $a : b$  is equivalent to every other ratio  $sa : sb$ , where  $s$  is a positive number. They learned that “at this rate” or “at the same rate” signals a situation that is characterized by equivalent ratios.

In this unit, students find the two values  $\frac{a}{b}$  and  $\frac{b}{a}$  that are associated with the ratio  $a : b$ , and interpret these values as rates per 1. For example, if a person walks 13 meters in 10 seconds, that means they walked  $\frac{13}{10}$  meters per 1 second and  $\frac{10}{13}$  seconds per 1 meter.

To kick off this work, in this lesson, students tackle a meaty problem that rewards finding and making sense of a rate per 1 (MP1). Note there is no need to use or define the term “rate per 1” with students in this lesson. All of the work and discussion takes place within a context, so students will be expected to understand and talk about, for example, the minutes per window or the meters climbed per minute, but they will not be expected to use or understand the more general term “rate per 1.”

#### Alignments

##### Building On

- 4.MD.A.1: Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...

- 5.MD.A.1: Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

### Addressing

- 6.RP.A.2: Understand the concept of a unit rate  $a/b$  associated with a ratio  $a : b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship. \$Expectations for unit rates in this grade are limited to non-complex fractions.

### Instructional Routines

- MLR6: Three Reads
- MLR7: Compare and Connect
- Think Pair Share

### Required Materials

#### Four-function calculators

### Required Preparation

All computations in this lesson can be done with methods students learned up through grade 5. However, you may wish to provide access to calculators to deemphasize computation and allow students to focus on reasoning about the context.

### Student Learning Goals

Let's investigate the Burj Khalifa building.

## 1.1 Estimating Height

### Warm Up: 10 minutes

This warm-up prompts students to reason about appropriate units of measurement in estimation and to review related work in grade 5 (converting across different-sized standard units within a given measurement system and using conversions to solve multi-step, real-world problems). It also allows them to form a reference for really tall things and convert measurements, which will be part of upcoming work.

As students discuss their estimates with a partner, monitor the discussions and identify students who use different strategies for estimating so they can share later.

### Building On

- 4.MD.A.1
- 5.MD.A.1

### Instructional Routines

- Think Pair Share

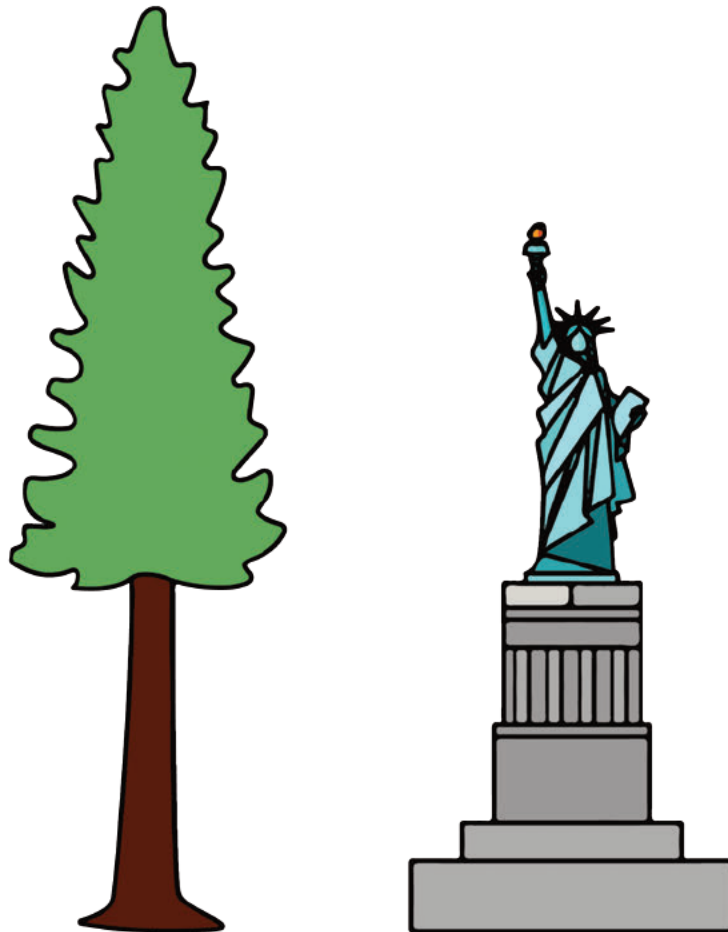
## Launch

Arrange students in groups of 2. Tell students they will be estimating the height of the tallest tree in the world, Hyperion. Ask students to give a signal when they have an estimate of the height of the tree. Give students 2 minutes of quiet think time followed by 3 minutes to discuss their estimates with a partner. Ask them to discuss the following questions, displayed for all to see:

- How close are your estimates to one another?
- How did you decide on the unit of measure?
- What was important to you in the image when making your estimate?
- Could you record your measurement using a different unit?

## Student Task Statement

Use the picture to estimate the height of Hyperion, the tallest known tree.





## Student Response

100 m or 400 ft or anything reasonably close to those values. Sample reasoning: Based on the height of [some building whose height is known in relation to the Statue of Liberty], I think the Statue of Liberty is about 300 feet tall. From the picture, it looks like Hyperion would be about 400 feet tall.

## Activity Synthesis

Invite selected students to share their estimates, how they chose their unit of measurement, and any information in the image that informs their estimates. After each explanation, solicit questions from the class that could help the student clarify his or her reasoning. Ask if there is another way to write each shared estimate in a different unit. Record the estimates and conversions and display them for all to see.

# 1.2 Window Washing

## 20 minutes

The purpose of this task is to give students a good reason to compute a rate “per 1.” Since the task statement doesn’t provide all the information needed to answer the question and does not suggest a solution pathway, it is an opportunity for students to make sense of the problem (MP1) and engage in some aspects of mathematical modeling (MP4).

Students learn about the Burj Khalifa, the tallest high-rise in the world, and try to determine how long it would take to wash all of its 24,348 windows given a specific rate of work by a window-washing crew. The fact that the total number of windows is not a multiple of the corresponding value in the given ratio (15 windows in 18 minutes) motivates students to identify the rate for 1 and then scale that number to answer the question.

This activity does not specify which “per 1” rate students should find. Some students are likely to first calculate  $\frac{5}{6}$  windows per minute instead of  $1\frac{1}{5}$  or 1.2 minutes per window. Students are not expected to name the above quantities as “rates per 1” or “unit rates” at this time. As students work, ask them to explain what the quantity they calculated (either windows per minute or minutes per window) means and how they plan to use it. If students are unsure where to start with the problem, ask, “At this rate, how long will it take the crew to finish 90 windows? 300 windows?”

## Addressing

- 6.RP.A.2

## Instructional Routines

- MLR7: Compare and Connect

## Launch

Ask students to close their workbooks or devices, and ask if they know what the Burj Khalifa is. Allow students who are familiar with the building to share what they know.

Then, display a picture of the building and a map of its location.



Once students see that the Burj Khalifa is the tallest artificial structure in the world and is located on the coast of the Arabian Gulf, surrounded by desert, ask what else they wonder about the building. Pause here to allow students time to think of a question. Keep the photo and map of the Burj Khalifa displayed.

Select students to share their questions, limiting it to one question per student. Record 5–10 questions for all to see. If no students wonder about the number of windows, say that you also have a question about the building and add to the list: “How long does it take to wash all the windows on the Burj Khalifa?”

Explain that your question came to mind after watching an online video of a window washing crew on the Burj Khalifa, using brushes and squeegees to clean the windows while harnessed to ropes.

Tell students they will now try to answer a question about the window washing. Give students 3–5 minutes of quiet think time to do so.

Answering the question requires knowing how many windows the Burj Khalifa has. The amount, *24,348 windows*, is readily available through online searches if students are allowed and able to go online. Otherwise, have the number ready to share, and consider announcing the value to the whole class only after several students have requested the information. Recognizing missing information and the steps needed to acquire it is part of the mathematical modeling process.

Some facts on the Burj Khalifa:

- Height: 2,722 ft (829.8 meters)
- Tallest artificial structure in the world (as of January 2016)

- 154 usable floors, 9 maintenance floors, 46 spire levels, 2 below-ground parking lots (Wikipedia)
  - 57 total elevators
  - It took 6 hours for an individual to climb to the top of the Burj Khalifa in 2011.
- 

### Support for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Chunk this task into more manageable parts to support students who benefit from support with organization and problem solving. For example, ask students to identify the rate for 1 window or 1 minute. Next, ask students to find how long it would take to wash a portion of the windows, such as 100 or 1000. After that, ask students how long it would take to wash all the windows of the Burj Khalifa.

*Supports accessibility for: Organization; Attention*

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### Support for English Language Learners

*Speaking: MLR7 Compare and Connect.* Use this routine when students present their strategy and representations to find how long it will take to wash all the windows. Ask students to consider what is the same and what is different about each approach. Draw students' attention to the different quantities they calculated (either windows per minute or minutes per window). These exchanges can strengthen students' mathematical language use and reasoning to make sense of strategies used to compute a rate "per 1."

*Design Principle(s): Maximize meta-awareness*

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### Anticipated Misconceptions

Students may get stuck when they try to find the minutes per window because 15 does not divide evenly into 18. Help them reason in terms of equivalent ratios or recall grade 5 methods for determining a quotient that is not a whole number.

### Student Task Statement



A window-washing crew can finish 15 windows in 18 minutes.

If this crew was assigned to wash all the windows on the outside of the Burj Khalifa, how long will the crew be washing at this rate?

### Student Response

To wash all 24,348 windows, it will take the crew 29,217.6 minutes. Representations vary. Sample response in a table:

number of windows	time in minutes
15	18
15,000	18,000
1	1.2
24,348	29,217.6

### Activity Synthesis

Invite a few students to explain their reasoning and solutions. Ask students who used both  $1\frac{1}{5}$  or 1.2 minutes per window and  $\frac{5}{6}$  windows per minute to share. Emphasize that both quantities are valid and have purpose, depending on the question you are answering.

Focus the discussion on strategies for identifying and correcting errors. As rate problems grow more complex, students become more likely to mix up numbers, calculate less-helpful rates per 1, or make arithmetic mistakes that are left unquestioned. If possible, have several students share approaches that did not work, and errors that were made, and ask them to explain how they knew they must have made an error.

If no students offer to share, have an example of a student error ready to display for the class. For example, a student may incorrectly calculate  $\frac{5}{6}$  minutes per window instead of  $1\frac{1}{5}$  minutes per window. Ask students to discuss with a partner what they think the error is and how they would help the student. Select 1–2 students to share their ideas with the class; while they share, make the corrections for all to see.

Lastly, students may point out that the answer, 29,217.6 minutes, is a total time and is not representative of how many normal work days it would take the crew to do the job. Time permitting, pursue this valuable line of thinking, as it shows that students are thinking about the math and the context (MP2).

## 1.3 Climbing the Burj Khalifa

### 15 minutes

The first activity of this lesson asked students how long a window-washing crew would be washing—i.e., time to complete an activity, which prompted students to calculate the amount of time per window instead of the number of windows per unit of time. Keeping in mind the context of the Burj Khalifa, this activity asks students to calculate how much of an activity would be finished—specifically, how much height would be scaled by a climber—in a given amount of time.

Students are likely to approach the first question by scaling down (dividing both the given height and the number of hours by 3), and the second question by calculating a rate per 1 hour (dividing the height and hours by 6), and then scaling up (multiplying by 5). The final question is meant to challenge students both in problem solving and in working with rational numbers. If students struggle to make sense of the problem, suggest they create a sketch of the situation to help determine what the third question is asking. As students work, identify several students who approach the task in different ways or use different representations to reason so they can share later.

### Addressing

- 6.RP.A.2

### Instructional Routines

- MLR6: Three Reads
- MLR7: Compare and Connect
- Think Pair Share

### Launch

Recap that in the previous activity, students tried to find the time it took to complete an activity—washing all the windows—given a certain rate of working. Tell students they are now to find the amount of an activity that's been completed, given a particular amount of time. Give students 5 minutes of quiet think time, and then a minute to share their responses with a partner. Ask students to be prepared to explain their thinking.

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### Support for Students with Disabilities

*Representation: Internalize Comprehension.* Activate or supply background knowledge. Provide students with access to a blank double number line to support information processing.  
*Supports accessibility for: Memory; Conceptual processing*

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### Support for English Language Learners

*Reading, Listening, Conversing: MLR6 Three Reads.* Use this routine to support reading comprehension of this word problem. In the first read, students read the problem with the goal of comprehending the situation (e.g., A climber is scaling outside of Burj Khalifa.). In the second read, ask students what can be counted or measured, without focusing on the values. Listen for, and amplify, the important quantities that vary in relation to each other in this situation: the height reached by the climber, in meters; the amount of time spent climbing, in hours. In the third read, ask students to brainstorm possible mathematical solution strategies to answer the questions. This helps students connect the language in the word problem and the reasoning needed to solve the problem.

*Design Principle(s): Support sense-making*

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### Anticipated Misconceptions

In the third question, students may struggle to divide 138 by 4. They may say the answer is 34 with a remainder of 2 or write 34.2, not knowing how to deal with the remainder. Ask those students what the remainder means and ask them to write the answer in terms of meters.

### Student Task Statement

In 2011, a professional climber scaled the outside of the Burj Khalifa, making it all the way to 828 meters (the highest point on which a person can stand) in 6 hours.

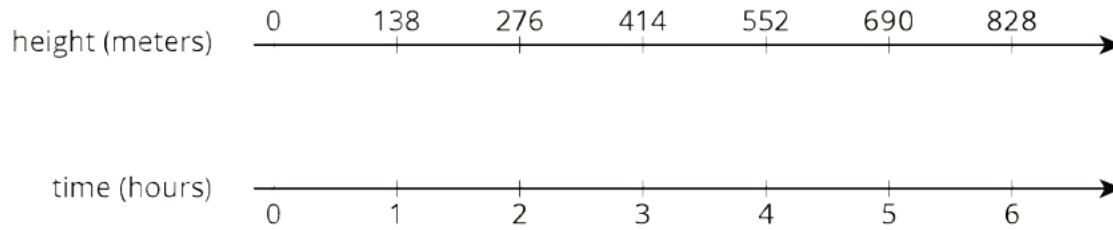
Assuming they climbed at the same rate the whole way:

1. How far did they climb in the first 2 hours?
2. How far did they climb in 5 hours?
3. How far did they climb in the final 15 minutes?

### Student Response

1. After two hours, the climber was at 276 meters.
2. After five hours, the climber was at 690 meters.

3. After  $5\frac{3}{4}$  hours, the climber was at 793.5 meters, so they climbed 34.5 meters in the last quarter hour. Representations vary. Sample responses:



time (hours)	height (meters)
6	828
1	138
$\frac{1}{4}$	34.5
$\frac{3}{4}$	103.5
$5\frac{3}{4}$	793.5

### Are You Ready for More?

Have you ever seen videos of astronauts on the Moon jumping really high? An object on the Moon weighs less than it does on Earth because the Moon has much less mass than Earth.

1. A person who weighs 100 pounds on Earth weighs 16.5 pounds on the Moon. If a boy weighs 60 pounds on Earth, how much does he weigh on the Moon?
2. Every 100 pounds on Earth are the equivalent to 38 pounds on Mars. If the same boy travels to Mars, how much would he weigh there?

### Student Response

1. The boy weighs 9.9 pounds on the Moon.
2. The boy would weigh 22.8 pounds on Mars.

### Activity Synthesis

For each question, invite 1–2 students with different solution paths to share their work, starting with those who used a more common approach. See MLR 7 (Compare and Connect) for more examples.

For example, in the second question, many students are likely to first calculate the number of meters per hour, and then multiply by 5 to find the number of meters in 5 hours. An alternative approach, but likely less common, is to calculate the number of meters per hour and then subtract that distance from the total height of the climb in 6 hours, since the distance climbed in the first hour and the last hour is assumed to be the same.

## Lesson Synthesis

In this lesson, we focused on finding and using the number of minutes per window and the number of meters per hour to solve problems in an efficient way.

Display the two main rates ( $1\frac{1}{5}$  minutes per window and 138 meters per hour) from this lesson.

Reinforce the usefulness of these quantities by questions such as:

- If the Burj Khalifa had 10,000 windows, how many minutes would it take the washing crew to clean all of them? 100,000 windows?
- How high is the climber after 2.5 hours? 2.25 hours? 2.2 hours?

## 1.4 Going Up?

Cool Down: 5 minutes

### Addressing

- 6.RP.A.2

### Student Task Statement

The fastest elevators in the Burj Khalifa can travel 330 feet in just 10 seconds. How far does the elevator travel in 11 seconds? Explain your reasoning.

### Student Response

363 feet. Possible strategies:

- If the elevator travels 330 feet in 10 seconds, then it is traveling 33 feet per second. Multiplying 33 by 11 gives 363 feet in 11 seconds.
- If the elevator travels 330 feet in 10 seconds, then it is traveling 33 feet per second. Adding 33 feet per second to 330 feet in 10 seconds gives 363 feet in 11 seconds.

### Student Lesson Summary

There are many real-world situations in which something keeps happening at the same rate. For example:

- a bus stop that is serviced by 4 buses per hour
- a washing machine that takes 45 minutes per load of laundry
- a school cafeteria that serves 15 students per minute



In situations like these, we can use equivalent ratios to predict how long it will take for something to happen some number of times, or how many times it will happen in a particular length of time.

For example, how long will it take the school cafeteria to serve 600 students?

The table shows that it will take the cafeteria 40 minutes to serve 600 students.

number of students	time in minutes
15	1
60	4
600	40

How many students can the cafeteria serve in 1 hour?



The double number line shows that the cafeteria can serve 900 students in 1 hour.

## Lesson 1 Practice Problems

### Problem 1

#### Statement

An elevator travels 310 feet in 10 seconds. At that speed, how far can this elevator travel in 12 seconds? Explain your reasoning.

#### Solution

372 feet.  $310 \div 10 = 31$ , so the elevator travels 31 feet per second. and  $31 \cdot 12 = 372$ .

### Problem 2

#### Statement

Han earns \$33.00 for babysitting 4 hours. At this rate, how much will he earn if he babysits for 7 hours? Explain your reasoning.

#### Solution

He will earn \$57.75 in 7 hours.  $33 \div 4 = 8.25$ , so the hourly rate is \$8.25. If he earns \$8.25 every hour, he will earn  $8.25 \cdot 7$  or \$57.75.

### Problem 3

#### Statement

The cost of 5 cans of dog food is \$4.35. At this price, how much do 11 cans of dog food cost? Explain your reasoning.

#### Solution

11 cans cost \$9.57.  $4.35 \div 5 = 0.87$ , so each can costs 87 cents, and  $0.87 \cdot 11 = 9.57$ .

### Problem 4

#### Statement

A restaurant has 26 tables in its dining room. It takes the waitstaff 10 minutes to clear and set 4 tables. At this rate, how long will it take the waitstaff to clear and set all the tables in the dining room? Explain or show your reasoning.

#### Solution

It will take 65 minutes, or 1 hour and 5 minutes. Sample strategy:

number of tables	time in minutes
4	10
1	2.5
26	65

### Problem 5

#### Statement

A sandwich shop serves 4 ounces of meat and 3 ounces of cheese on each sandwich. After making sandwiches for an hour, the shop owner has used 91 combined ounces of meat and cheese.

- How many combined ounces of meat and cheese are used on each sandwich?
- How many sandwiches were made in the hour?
- How many ounces of meat were used?
- How many ounces of cheese were used?

#### Solution

- 7 ounces

- b. 13 sandwiches
- c. 52 ounces of meat
- d. 39 ounces of cheese

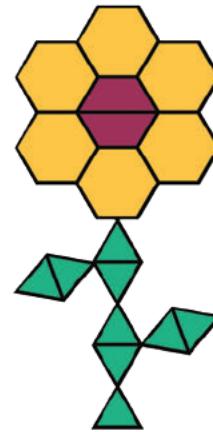
(From Unit 2, Lesson 16.)

## Problem 6

### Statement

Here is a flower made up of yellow hexagons, red trapezoids, and green triangles.

- a. How many copies of this flower pattern could you build if you had 30 yellow hexagons, 50 red trapezoids, and 60 green triangles?
- b. Of which shape would you have the most left over?



### Solution

I could build 5 copies of the flower pattern, because that would use all 30 of the yellow hexagons. I would have 40 red trapezoids left over.

(From Unit 2, Lesson 14.)

## Problem 7

### Statement

Match each quantity in the first list with an appropriate unit of measurement from the second list.

- A. the perimeter of a baseball field
- B. the area of a bed sheet
- C. the volume of a refrigerator
- D. the surface area of a tissue box
- E. the length of a spaghetti noodle
- F. the volume of a large lake
- G. the surface area of the the moon

1. centimeters (cm)
2. cubic feet (cu ft)
3. cubic kilometers (cu km)
4. meters (m)
5. square feet (sq ft)
6. square inches (sq in)
7. square kilometers (sq km)

### **Solution**

- A: 4
- B: 5
- C: 2
- D: 6
- E: 1
- F: 3
- G: 7

(From Unit 1, Lesson 16.)

## Section: Unit Conversion

### Lesson 2: Anchoring Units of Measurement

#### Goals

- Compare (orally) the relative size of different units of measure for one attribute, i.e., length, volume, weight or mass.
- Comprehend the approximate size of 1 “inch,” “foot,” “yard,” “mile,” “millimeter,” “centimeter,” “meter,” “kilometer,” “ounce,” “pound,” “ton,” “gram,” “kilogram,” “cup,” “quart,” “gallon,” “milliliter,” and “liter.”
- Identify which unit is closest to the length, volume, weight, or mass of a given object, and explain (orally) the reasoning.

#### Learning Targets

- I can name common objects that are about as long as 1 inch, foot, yard, mile, millimeter, centimeter, meter, or kilometer.
- I can name common objects that weigh about 1 ounce, pound, ton, gram, or kilogram, or that hold about 1 cup, quart, gallon, milliliter, or liter.
- When I read or hear a unit of measurement, I know whether it is used to measure length, weight, or volume.

#### Lesson Narrative

This lesson is optional. Students have worked with standard units of length since grade 2, and standard units of volume and mass since grade 3. This lesson is designed to anchor students’ perception of standard units of length, volume, weight, and mass with a collection of familiar objects that they can refer to in later lessons in preparation for using ratio reasoning to convert measurement units.

The main task of this lesson is a card-sorting activity in which students match common objects with their closest unit of length, volume, mass, or weight to establish anchor quantities for each unit of measurement. Since this lesson reinforces standards from previous grade levels instead of introducing grade 6 standards, if you believe that your students already have a firm grasp of these units of measurement, you may choose to skip this lesson.

#### Alignments

##### Building On

- 2.MD.A.3: Estimate lengths using units of inches, feet, centimeters, and meters.
- 4.MD.A.1: Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in

a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...

- 5.MD.A.1: Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

### **Building Towards**

- 6.RP.A.3.d: Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

### **Instructional Routines**

- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports
- Take Turns
- Think Pair Share

### **Required Materials**

**Household items**

**Meter sticks**

**Pre-printed slips, cut from copies of the blackline master**

**Quart-sized bottle**

**Rulers**

**Scissors**

**String**

**Yardsticks**

### **Required Preparation**

For the warm-up activity, each group of 2 students needs scissors and more string than necessary for their assigned unit of length. To distribute the string without wasting too much or giving away the actual lengths, consider dividing one ball of string ahead of time into equal spools, enough for every group to get one. The spools can then be reused class after class. Rotate the spools between groups assigned shorter and longer lengths, so that one spool does not run out long before the others. Only one of each of the rulers, meter sticks, and yardsticks is needed for demonstration purposes.

For the Measurements Card Sort activity, prepare 1 copy of the blackline master for each group of 4–6 students. These slips can be reused from one class to the next. If possible, copy each complete set on a different color of paper, so that a stray card can quickly be put back.

Also for the Measurements Card Sort activity, prepare several examples of real objects depicted on the cards, so the students can see them at actual size, especially any objects on the cards that may be unfamiliar to students. A real quart-sized bottle is an especially crucial example to have.

## Student Learning Goals

Let's see how big different things are.

# 2.1 Estimating Volume

### Warm Up: 10 minutes

This warm-up prompts students to reason about appropriate units of measurement in estimation and to review related work in grade 5 (converting across different-sized standard units within a given measurement system and using conversions to solve multi-step, real-world problems).

If there is time after sharing the estimates and reasoning, give the students the side lengths of the salt shaker (the length, width, and height are all 2.5 cm) and ask them to use that information to check the reasonableness of their answer. It may help to know that 1 cubic centimeter is the same volume as 1 milliliter.

### Building On

- 4.MD.A.1

### Instructional Routines

- Think Pair Share

### Launch

Arrange students in groups of 2. Tell students they will be estimating the volume of a tiny salt shaker. Ask students to give a signal when they have an estimate. Give students 2 minutes of quiet think time followed by 3 minutes to discuss their estimates with a partner. Ask them to discuss the following questions, displayed for all to see:

- How close are your estimates to one another?
- How did you decide on the unit of measure?
- What was important to you in the image when making your estimate?
- Could you record your measurement using a different unit?

### Student Task Statement

Estimate the volume of the tiny salt shaker.



### Student Response

Reasonable estimates would be close to  $\frac{1}{2}$  ounce, 3 teaspoons, 15 milliliters, or 15 cubic centimeters.

### Activity Synthesis

Invite a few students to share their estimates, how they chose their unit of measurement, and any information in the image that informs their estimates. After each explanation, solicit questions from the class that could help the student clarify his or her reasoning. Ask if there is another way to write each shared estimate in a different unit. Record the estimates and conversions and display them for all to see.

During the discussion, students may question if the volume indicates how much salt the shaker will realistically hold. This will depend on how high the salt is filled within the shaker. Welcome questions such as these and discuss how students' assumptions impacted their estimates.

## 2.2 Cutting String

### Optional: 10 minutes

This task is an opportunity to assess students' prior knowledge of standard units of length and find out the kinds of objects students already use as benchmarks for estimating length units.

Note that groups will likely produce their length of string pretty quickly. The majority of the time in this activity will be spent comparing and discussing with the whole group.

Expect some students who are assigned 1 meter to say that it is basically the same as 1 yard (which is acceptable during the group work). Be sure to address this in the class discussion.

### Building On

- 2.MD.A.3
- 4.MD.A.1



## Instructional Routines

- MLR8: Discussion Supports

### Launch

Hold up a pen, an envelope, or another object whose length is likely unfamiliar to students (unlike an index card or a letter-size paper, which are more likely to be familiar). Choose one length of the object and ask students to estimate how long it is in centimeters. (Consider taking a quick walk around the room with the object so students can get a closer look.) Ask them to share their estimate with a partner, and then reveal the actual length.

Tell students that people who work with certain units of length on a repeated basis can get very good at estimating lengths with those units. For example, someone who sews may be very good at estimating yards of fabric. Explain that they will cut a piece of string as close to their assigned length as possible without using a measurement tool.

Arrange students in groups of 2. Distribute scissors and string. Assign each group one of the following lengths: 1 centimeter, 1 foot, 1 inch, 1 meter, or 1 yard. Not all of these lengths have to be used, but each length to be used should be assigned to 2–3 different groups so their estimations can be compared at the end.

### Student Task Statement

Your teacher will assign you one of the following lengths:

1 centimeter, 1 foot, 1 inch, 1 meter, or 1 yard.

Estimate and cut a piece of string as close to your assigned length as you can without using a measurement tool.

### Student Response

Strings of varying lengths

### Activity Synthesis

Gather the strings into groups based on their assigned length. Display each group of strings for all to compare, starting with the groups assigned 1 foot. Then display a measuring tool next to the group of strings and show the actual assigned length. For the shorter lengths, it may be useful to project them using a document camera, or tape them to a colored piece of paper so they can be held up for all to see. Discuss the following:

- “How close are these estimates to each other?”
- “How close are these estimates to the actual length?”
- “What strategies or benchmarks were used to make estimates?”

Highlight any benchmark comparisons you heard students make when discussing with their partner (if the students themselves do not repeat these for the whole class). For example, a student might

mention that an inch is approximately the length of your thumb, or a yard is approximately the length of your arm. However, now is not the time to provide students with a list of benchmarks they did not mention themselves.

Lastly, hold up both the yardstick and the meter stick to compare their actual lengths. Ensure students notice that 1 meter is slightly longer than 1 yard. Then, hold up all the strings that were assigned to be 1 yard or 1 meter and have students tell how to regroup them based on which of the two units they were closer to.

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### Support for English Language Learners

*Speaking: MLR8 Discussion Supports.* Ask students to chorally repeat phrases that include measurements in context (e.g., “This piece of string measures 1 centimeter,” “The length of this piece of string is 1 foot,” etc.). Use this to amplify mathematical uses of language to communicate about units of measure.

*Design Principle(s): Support sense-making*

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## 2.3 Card Sort: Measurements

### Optional: 25 minutes

The previous activity included only units of length, which are the most familiar to students since they started working with length in second grade and because rulers, yardsticks, and meter sticks are common classroom tools. This activity expands the list to include units of volume, weight, and mass. First, students categorize the units by the attribute they measure: length, volume, and weight or mass. Then they go through each type of unit, matching each provided benchmark object with the closest unit of measurement.

Display several examples of real objects that are depicted on the cards, so the students can see them at actual size. The quart-sized bottle is an especially crucial real example to have, because many things that are packaged in quarts are also commonly available in other sizes.

After each group of students has sorted the units by attribute, review their categories and prompt them to fix any mistakes. It is important they have the units grouped correctly before they move on to matching the object pictures with the units. If students are very unfamiliar with any of the units of volume, weight, or mass, tell them one object that matches with that unit and have them decide by comparison how other objects should be matched.

Expect some students to sort the units into plausible categories but which are not aligned to the purposes of this activity. Clarify as needed.

- If they sort the units into customary and metric groups, say that all units of length should be grouped together, and if necessary, that there are two other categories.

- If students separated units of weight from units of mass, tell them that for the sake of this activity, weight and mass should be grouped together. If necessary, say that we are referring to the weight of objects on Earth’s surface.

Also expect students to equate units that are very close (e.g., to say 1 liter is basically the same as 1 quart). This is acceptable at this point and will be investigated further in the next lesson.

When students have completed the sorting and matching, they form new groups to analyze the matches made by one of the original groups. Those who are analyzing someone else’s work can voice their support or disagreement with the placements of the cards (MP3). One student—who now belongs to a new group but whose work with the original group is being analyzed—can defend the placement decisions to the others.

At the end of the discussion, students mix the cards up and put them back in the envelopes for the next class to use.

### **Building On**

- 4.MD.A.1

### **Building Towards**

- 6.RP.A.3.d

### **Instructional Routines**

- MLR3: Clarify, Critique, Correct
- Take Turns

### **Launch**

Arrange students in groups of 4–6 in two dimensions. (Assign each student to a group and then a label within it, so that new groups—consisting of one student from each the original groups—can be formed later).

Say to students that they have just looked at standard units of length, but as length is not the only measurable attribute, they will look at other attributes. Tell students this activity has two parts—a sorting-and-matching part and a discussion—and that they will complete each part in a different group.

Explain the sorting-and-matching activity:

“Your group will receive two sets of cards. One set contains units of measurements. Your job is to sort them based on the attribute they measure. For example, all units that measure length should be grouped together. The second set of cards contains pictures of objects. Your job is to match each one with an appropriate unit that can be used to measure the object.”

Distribute sets of cards to be sorted. Ask students to pause after their group has sorted the unit cards and have their work reviewed before moving on to match the object cards.

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### Support for Students with Disabilities

*Representation: Internalize Comprehension.* Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students cards a subset of the cards to start with and introduce the remaining cards once students have completed their initial sort or set of matches.

*Supports accessibility for: Conceptual processing; Organization*

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### Anticipated Misconceptions

Students may struggle to sort objects that weigh 1 pound versus 1 kilogram. Tell them one object that matches with each unit and have them decide the other objects by comparison.

#### Student Task Statement

Your teacher will give you some cards with the names of different units of measurement and other cards with pictures of objects.

1. Sort the units of measurement into groups based on the attribute they measure. Pause here so your teacher can review your groups.
2. Match each picture card that has “L” in the top right corner with the closest unit to the length of the object.
3. Match each picture card that has “V” in the top right corner with the closest unit to the volume of the object.
4. Match each picture card that has “WM” in the top right corner with the closest unit to the weight or mass of the object.

Your teacher will assign you a new group to discuss how you matched the objects. If you disagree, work to reach an agreement.

#### Student Response

1. Units grouped by attribute:

length	volume	weight or mass
1 inch 1 foot 1 yard 1 mile 1 millimeter 1 centimeter 1 meter 1 kilometer	1 cup 1 quart 1 gallon 1 milliliter 1 liter	1 ounce 1 pound 1 ton 1 gram 1 kilogram

2. Objects matched to units of length:

1 inch	1 foot	1 yard	1 mile
length of a thumb width of a quarter thickness of a hockey puck	length of a ruler length of a shoe length of a football length of a license plate	length from chest to fingers length of a baseball bat	distance run in 10 minutes

1 millimeter	1 centimeter	1 meter	1 kilometer
thickness of a dime	width of a pinky finger width of the head of a golf tee	length from fingers to opposite armpit length of a baseball bat and ball	distance walked in 10 minutes

3. Objects matched to units of volume:

1 cup	1 quart	1 gallon	1 milliliter	1 liter
measuring cup cup school milk carton	large sports drink bottle small paint can	large milk jug large paint can	raindrop 1s cube packet of artificial sweetener	reusable water bottle 1,000s cube half of a large soda bottle

4. Objects matched to units of weight or mass:

1 ounce	1 pound	1 ton	1 gram	1 kilogram
slice of bread birthday card mouse	loaf of bread jar of peanut butter box of 96 crayons hooded sweatshirt crow	small car draft horse	paperclip dollar bill 2 raisins	textbook bunch of bananas guinea pig

### Activity Synthesis

When most groups have finished matching the objects with the units, have them form new groups consisting of one person from each original group. Assign each new group a set of matched cards (matched by an original group) to analyze. Display and read aloud the following guiding questions:

- Did your original groups match the objects to the same units?
- Which objects did your groups match differently?
- Which objects or units were the easiest to match? Why?
- Which objects or units were the hardest to match? Why? Observe whether any object or unit was matched incorrectly by most of the class and tell what the correct match is.

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### Support for English Language Learners

*Representing, Listening, Conversing: MLR3 Clarify, Critique, Correct.* After the secondary groups finish analyzing the original matches, present a match showing a conceptual (or common) error to the whole class. For example, students may match objects that weigh 1 pound with objects that weigh 1 kilogram or objects that weigh 1 gallon with objects with 1 liter and may reason that either unit can be used to measure the object. Ask students to work in pairs to identify and analyze the mismatch, and write a justification of the revision that includes units of measure. If time allows, ask students to share their written justifications with the class. This will help students understand the difference between standards units of length, volume, weight, and mass.

*Design Principle(s): Cultivate conversation; Maximize meta-awareness*

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## Lesson Synthesis

After the preprinted slips for the sorting activity are put away, hold up real objects that match the objects pictured on some of the cards and ask the students to express the length, volume, weight, or mass of the object. For each unit of measure, consider having students record a benchmark object of their choice on a classroom display or in a notebook to serve as a reference for later.

## 2.4 So Much in Common

### Cool Down: 5 minutes

Students use their reinforced understanding of how big standard units are to compare a pair of lengths, a pair of volumes, and a pair of weight and mass. Both measurements in each pair have the same numerical value but different units.

### Building On

- 4.MD.A.1
- 5.MD.A.1

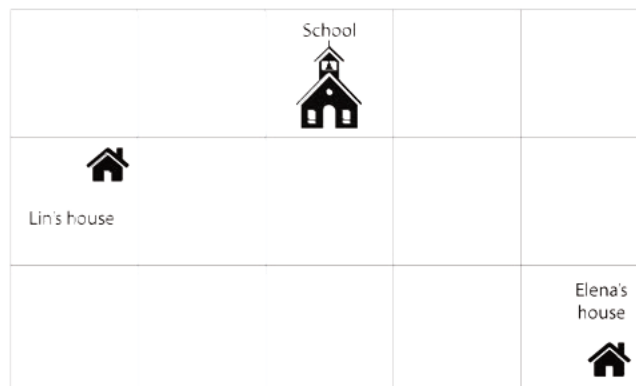
### Launch

Remind students to think about the benchmark objects for each unit of measurement while answering the questions.

### Student Task Statement

Lin and Elena have discovered they have so much in common.

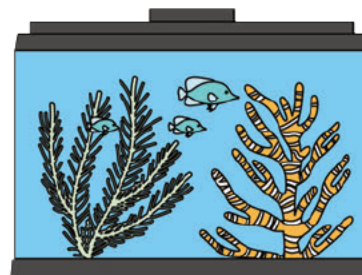
1. They each walk 500 units to school. Who walks 500 feet, and who walks 500 yards? Explain your reasoning.



2. They each have a fish tank holding 20 units of water. Whose tank holds 20 gallons, and whose holds 20 cups? Explain your reasoning.



Lin's fish bowl



Elena's fish tank

3. They each have a brother who weighs 40 units. Whose brother weighs 40 pounds, and whose weighs 40 kilograms? Explain your reasoning.



Elena's Brother



Lin's Brother

### Student Response

1. Lin walks 500 feet, and Elena walks 500 yards, because yards are longer than feet and Elena's house is farther away than Lin's. For example, it looks like 500 rulers could reach from Lin's house to the school, but it would take 500 yardsticks to reach from Elena's house to the school.
2. Elena's fish tank holds 20 gallons, and Lin's fish bowl holds 20 cups, because gallons are bigger than cups, and Elena's tank is larger. For example, it looks like Elena's fish tank could hold 20 large milk jugs of water while Lin's fish bowl could only hold 20 school milk cartons of water.
3. Lin's brother weighs 40 pounds, and Elena's brother weighs 40 kilograms, because kilograms are heavier than pounds and Elena's brother is bigger. For example, it looks like Lin's brother



would weigh as much as 40 boxes of crayons while Elena's brother would weigh as much as 40 textbooks.

## Student Lesson Summary

We can use everyday objects to estimate standard units of measurement.

For units of length:

- 1 millimeter is about the thickness of a dime.
- 1 centimeter is about the width of a pinky finger.
- 1 inch is about the length from the tip of your thumb to the first knuckle.
- 1 foot is the length of a football.
- 1 yard is about the length of a baseball bat.
- 1 meter is about the length of a baseball bat and ball.
- 1 kilometer is about the distance someone walks in ten minutes.
- 1 mile is about the distance someone runs in ten minutes.

For units of volume:

- 1 milliliter is about the volume of a raindrop.
- 1 cup is about the volume of a school milk carton.
- 1 quart is about the volume of a large sports drink bottle.
- 1 liter is about the volume of a reusable water bottle.
- 1 gallon is about the volume of a large milk jug.

For units of weight and mass:

- 1 gram is about the mass of a raisin.
- 1 ounce is about the weight of a slice of bread.
- 1 pound is about the weight of a loaf of bread.
- 1 kilogram is about the mass of a textbook.
- 1 ton is about the weight of a small car.

## Lesson 2 Practice Problems

### Problem 1

#### Statement

Select the unit from the list that you would use to measure each object.

- |  |                                   |
|--|-----------------------------------|
| a. The length of a pencil  | <input type="radio"/> centimeters |
| b. The weight or mass of a pencil                                | <input type="radio"/> cups        |
| c. The volume of a pencil  | <input type="radio"/> feet        |
| d. The weight or mass of a hippopotamus                          | <input type="radio"/> gallons     |
| e. The length of a hippopotamus                                  | <input type="radio"/> grams       |
| f. The length of a fingernail clipping                           | <input type="radio"/> inches      |
| g. The weight or mass of a fingernail clipping                   | <input type="radio"/> kilograms   |
| h. The volume of a sink  | <input type="radio"/> kilometers  |
| i. The volume of a bowl  | <input type="radio"/> liters      |
| j. The length of a chalkboard or whiteboard                      | <input type="radio"/> meters      |
| k. The weight or mass of a chalkboard or whiteboard              | <input type="radio"/> miles       |
| l. The length of the border between the United States and Canada | <input type="radio"/> milliliters |
|  | <input type="radio"/> millimeters |
|  | <input type="radio"/> ounces      |
|  | <input type="radio"/> pounds      |
|  | <input type="radio"/> quarts      |
|  | <input type="radio"/> tons        |
|  | <input type="radio"/> yards       |

#### Solution

Answers Vary. Possible responses:

- a. inches, centimeters
- b. grams, ounces
- c. milliliters
- d. pounds, kilograms, tons
- e. feet, yards, meters
- f. millimeters
- g. grams
- h. gallons, liters, quarts

- i. cups, liters, quarts
- j. feet, yards, meters
- k. kilograms, pounds
- l. kilometers, miles

## Problem 2

### Statement

When this pet hamster is placed on a digital scale, the scale reads 1.5.



What could be the units?

### Solution

Ounces. (Grams and milligrams are too small. Pounds and kilograms are too big.)

## Problem 3

### Statement

Circle the larger unit of measure. Then, determine if the unit measures distance, volume, or weight (or mass).

- a. meter or kilometer
- b. yard or foot
- c. cup or quart
- d. pound or ounce
- e. liter or milliliter
- f. gram or kilogram

### Solution

- a. Kilometer, distance
- b. Yard, distance

- c. Quart, volume
- d. Pound, weight (mass)
- e. Liter, volume
- f. Kilogram, weight (mass)

## Problem 4

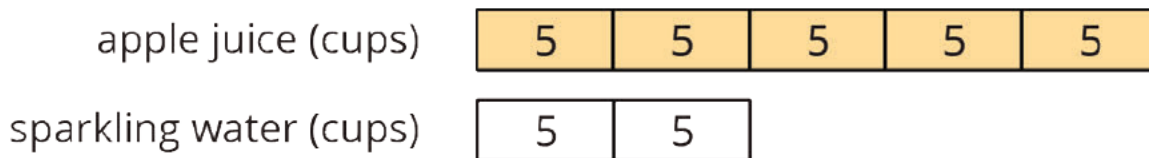
### Statement

Elena mixes 5 cups of apple juice with 2 cups of sparkling water to make sparkling apple juice. For a party, she wants to make 35 cups of sparkling apple juice. How much of each ingredient should Elena use? Explain or show your reasoning.

### Solution

25 cups of apple juice and 10 cups of sparkling water. Possible strategies:

- There are 7 cups of sparkling juice in each batch, since  $5 + 2 = 7$ . To make 35 cups Elena will need 5 batches since  $5 \cdot 7 = 35$ . 5 batches mean 25 cups of apple juice and 10 cups of sparkling water.
- Tape diagram:



(From Unit 2, Lesson 15.)

## Problem 5

### Statement

Lin bought 3 hats for \$22.50. At this rate, how many hats could she buy with \$60.00? If you get stuck, consider using the table.

number of hats	price in dollars

## Solution

8 hats. Sample reasoning:

number of hats	price in dollars
3	22.50
1	7.50
5	37.50
8	60

(From Unit 2, Lesson 12.)

## Problem 6

### Statement

Light travels about 180 million kilometers in 10 minutes. How far does it travel in 1 minute?  
How far does it travel in 1 second? Show your reasoning.

## Solution

Light travels about 18 million km in 1 minute.  $18,000,000 \div 60 = 300,000$ , so light travels about 300,000 km in one second.

(From Unit 2, Lesson 9.)

# Lesson 3: Measuring with Different-Sized Units

## Goals

- Generalize (orally and in writing) that it takes more of a smaller unit or fewer of a larger unit to measure the same quantity.
- Given a measurement in one unit, estimate what would be the same amount expressed in a different unit, and explain (orally) the reasoning.

## Learning Targets

- When I know a measurement in one unit, I can decide whether it takes more or less of a different unit to measure the same quantity.

## Lesson Narrative

This lesson develops students' familiarity with standard units of length, volume, weight, and mass through the tactile experiences of measuring objects. The main idea is that it takes more of a smaller unit and less of a larger unit to measure the same quantity. This idea is an important foundation for converting units of measurement using ratio reasoning in the next lesson (MP7).

## Alignments

### Building On

- 2.MD.A.2: Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

### Addressing

- 6.RP.A.3.d: Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

### Building Towards

- 6.RP.A.3.d: Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

### Instructional Routines

- MLR8: Discussion Supports

## Required Materials

**Base-ten blocks**

**Blank paper**

**Cuisenaire rods**

**Gallon-sized jug**

**Graduated cylinders**

**Household items**

**Inch cubes**

**Internet-enabled device**

**Liter-sized bottle**

**Materials assembled from the blackline master**

**Metal paper fasteners**

brass brads

**Meter sticks**

**Pre-assembled polyhedra**

**Quart-sized bottle**

**Rulers**

**Salt**

**Scale**

a digital scale that can output in grams, kilograms, ounces, or pounds

**Straightedges**

A rigid edge that can be used for drawing line segments. Sometimes a ruler is okay to use as a straightedge, but sometimes it is preferable to use an unruled straightedge, like a blank index card.

**Teaspoon**

**Tray**

## Required Preparation

For the first activity, prepare to display or distribute 6-cm and 9-cm Cuisenaire rods, which are often colored dark green and blue, respectively. If Cuisenaire rods are not available, small and large paper clips can be substituted.

For the second activity, identify where each station will be and set up the following materials:

For Station 1:

- From the first page of the blackline master, print the net for the 2-in by 2-in by 4-in box onto card stock, cut it out, and assemble it.
- Provide at least twenty inch cubes, one centimeter cube, and thirty 10-cm rods. The centimeter cube and 10-cm rods can come from a set of base-ten blocks or Cuisenaire rods. However, base-ten blocks are preferable so students can see how one rod is composed of ten centimeter cubes. Wooden inch cubes are available inexpensively at craft stores.

For Station 2:

- Identify something in the classroom that is about 20 feet long. Prepare a way to communicate to students that this is the object they are supposed to measure (but do not give away its length).
- Provide rulers and at least 2 meter sticks.

For Station 3:

- Prepare a way for students to be able to watch this video

Video 'Quarts and Liters' available here: <https://player.vimeo.com/video/304136597>.

- Provide an empty gallon-sized jug, quart-sized bottle, and liter-sized bottle for comparison.

For Station 4 (there are 3 different options):

1. *If students will weigh objects on a real scale:* Set up the scale and provide common household items for students to weigh. Note: The scale must be able to output in grams, kilograms, ounces, and pounds for this option to work.
2. *If students will use the digital scale simulation:* Prepare a way for students to access this widget <http://ggbm.at/eQQVYB7D>.
3. *If students will use the paper scale simulations:* Print pages 2–13 of the blackline master onto cardstock and cut out the scale images and output wheels. Make sure to cut out the two white windows on the base of each scale where the output wheels are supposed to show through. Assemble the paper scale simulations using metal fasteners so the output wheels can rotate behind the scale images.

For Station 5:

- On a tray for catching spills, provide a 100-ml graduated cylinder, a teaspoon, a straightedge for leveling off the teaspoon, and a small bowl with at least  $\frac{1}{2}$  cup of salt.

### Student Learning Goals

Let's measure things.

## 3.1 Width of a Paper

### Warm Up: 5 minutes

Students begin by thinking about length in terms of non-standard units—9-cm and 6-cm Cuisenaire rods—and consider how the size of units affects the number of units needed to express a length. If Cuisenaire rods are not available, modify the task to say: Does it take more large paper clips or small paper clips lined up end-to-end to measure the width of a piece of paper?

Some students may be able to reason that it takes more of the smaller unit than the larger unit to measure the same length; encourage them to articulate their reasoning. Others may need to visualize the situation by drawing or by measuring with actual rods (or paper clips).

### Building On

- 2.MD.A.2

### Building Towards

- 6.RP.A.3.d



## Launch

This activity is written to use 9-cm and 6-cm Cuisenaire rods, which are often blue and dark green, respectively. If your set of Cuisenaire rods has different colors, or if using small and large paper clips as substitutes, instruct students to modify the task accordingly.

Hold up the two sizes of rods or paper clips for the students to see. Give them quiet think time but not the manipulatives. Later, allow students to use the rods or paper clips to measure the paper if they need or wish to do so.

## Anticipated Misconceptions

Some students may assume that it will take more of the longer rods because they are used to associating the idea of “more” with “larger.” Encourage them to use the manipulatives to see that it actually takes fewer of the longer rods to reach across the paper.

### Student Task Statement

Your teacher will show you two rods. Does it take more green rods or blue rods lined up end to end to measure the width of a piece of printer paper?

## Student Response

It takes more green rods, because they are shorter than the blue rods.

## Activity Synthesis

Ask students to share their responses and reasoning. Highlight the fact that it takes more of a smaller unit and fewer of a larger unit to measure the same length.

# 3.2 Measurement Stations

**35 minutes (there is a digital version of this activity)**

In groups, students rotate through five different stations, where they measure one or more quantities using different units, and answer a series of summary questions afterward. Here are the quantities being measured and the units used at each station:

- **Station 1:** Volume of a box, in cubic inches and cubic centimeters.
- **Station 2:** Length, in meters and feet.
- **Station 3:** Volume of water, in gallons, quarts, and liters.  
(If desired, you can have students measure water with actual containers instead of watching the video <https://vimeo.com/illustrativemathematics/water>.)
- **Station 4:** Weights and masses of 2–3 objects, in ounces, pounds, grams, and kilograms.  
(You can have students weigh actual objects, use the digital simulation <http://ggbm.at/eQQVYB7D>, or use the paper simulations from the blackline master. If using one of the simulations instead of a real scale, prepare some real objects labeled with their weight or mass for students to hold and feel the weight of.)

- **Station 5:** Volume of salt, in milliliters and teaspoons.

You will need the blackline master for this activity. Page 1 is a net for the box needed for station 1. If you are using the paper scale simulation instead of a real scale or the applet, pages 2–13 are the parts needed to assemble Station 4.

### Addressing

- 6.RP.A.3.d

### Instructional Routines

- MLR8: Discussion Supports

### Launch

Tell students they will further investigate the idea of using different units to measure the same set of items. Introduce the five stations, what students are expected to do at each, the protocol for rotating through them, and the questions to answer at the end. Then, demonstrate how to use the straightedge to measure a level teaspoon of salt. If students do not use a level teaspoons of salt, they will not be able to answer the last set of questions about volume.

Arrange students into 5 groups and assign a starting station for each group.

If students have devices, Stations 3 and 4 can be digital.

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### Support for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, “I noticed \_\_\_\_ so I think...”

*Supports accessibility for: Language; Social-emotional skills*

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### Support for English Language Learners

*Speaking, Representing: MLR8 Discussion Supports.* Use this routine to support small-group discussion. As students rotate through stations, encourage students to solidify their own understanding by pressing for details and questioning their peers' explanations. Provide sentence frames for students to use, such as “I agree/disagree because . . .”, “How do you know . . .”, and “Can you give an example?” This will help students clarify their reasoning about comparing different measurements for the same quantity using different units.

*Design Principle(s): Support sense-making; Cultivate conversation*

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## Anticipated Misconceptions

At Station 1, students may count the number of base-10 centimeter rods rather than the number of centimeter cubes. Remind them that the question prompts for the number of cubes.

At Station 2, students may need reminders about measuring objects at the zero marking on the ruler and about keeping the ruler going straight, both of which will affect the answer. Show them they can measure along the edge of the object to make sure the ruler is not veering off in one direction or another.

At Station 4, students may be unclear about how to change the output unit on the scale for each object. Consider showing the class ahead of time. Students who are able to distinguish between weight and mass might say they cannot accurately compare their measurements. Clarify that we are talking only about the weight of the objects on Earth's surface.

At Station 5, some students may consistently use under-filled or rounded teaspoons of salt, so their data will not reveal the 5 : 1 ratio of milliliters to teaspoons. Repeat the demonstration of how to measure a level teaspoon for them.

Students may answer 3 milliliters for the question about 15 teaspoons because they divided by 5 instead of multiplying by 5. Encourage them to pay attention to which unit is bigger and ask what that tells them about which numerical value should be larger.

### Student Task Statement

#### Station 1

- Each large cube is 1 cubic inch. Count how many cubic inches completely pack the box without gaps.
- Each small cube is 1 cubic centimeter. Each rod is composed of 10 cubic centimeters. Count how many cubic centimeters completely fill the box.

	cubic inches	cubic centimeters
volume of the box		

#### Station 2

Your teacher showed you a length.

- Use the meter stick to measure the length to the nearest meter.
- Use a ruler to measure the length to the nearest foot.

	meters	feet
length of _____		

#### Station 3

If not using real water, open <https://vimeo.com/illustrativemathematics/water>.

- Count how many times you can fill the quart bottle from the gallon jug.
- Count how many times you can fill the liter bottle from the gallon jug.

	quarts	liters
1 gallon of water		

#### Station 4

If not using a real scale, open <http://ggbm.at/eQQVYB7D>.

- Select 2–3 different objects to measure on the scale.
- Record the weights in ounces, pounds, grams, and kilograms.

object	ounces	pounds	grams	kilograms

#### Station 5

- Count how many level teaspoons of salt fill the graduated cylinder to 20 milliliters, 40 milliliters, and 50 milliliters.
- Pour the salt back into the original container.

	milliliters	teaspoons
small amount of salt	20	
medium amount of salt	40	
large amount of salt	50	

After you finish all five stations, answer these questions with your group.

- a. Which is larger, a cubic inch or a cubic centimeter?
  - b. Did more cubic inches or cubic centimeters fit in the cardboard box? Why?
2. Did it take more feet or meters to measure the indicated length? Why?
3. Which is larger, a quart or a liter? Explain your reasoning.
4. Use the data from Station 4 to put the units of weight and mass in order from smallest to largest. Explain your reasoning.

5. a. About how many teaspoons of salt would it take to fill the graduated cylinder to 100 milliliters?
- b. If you poured 15 teaspoons of salt into an empty graduated cylinder, about how many milliliters would it fill?
- c. How many milliliters per teaspoon are there?
- d. How many teaspoons per milliliter are there?

### Student Response

	cubic inches	cubic centimeters
volume of the box	16	250

	meters	feet
length of _____	6	20

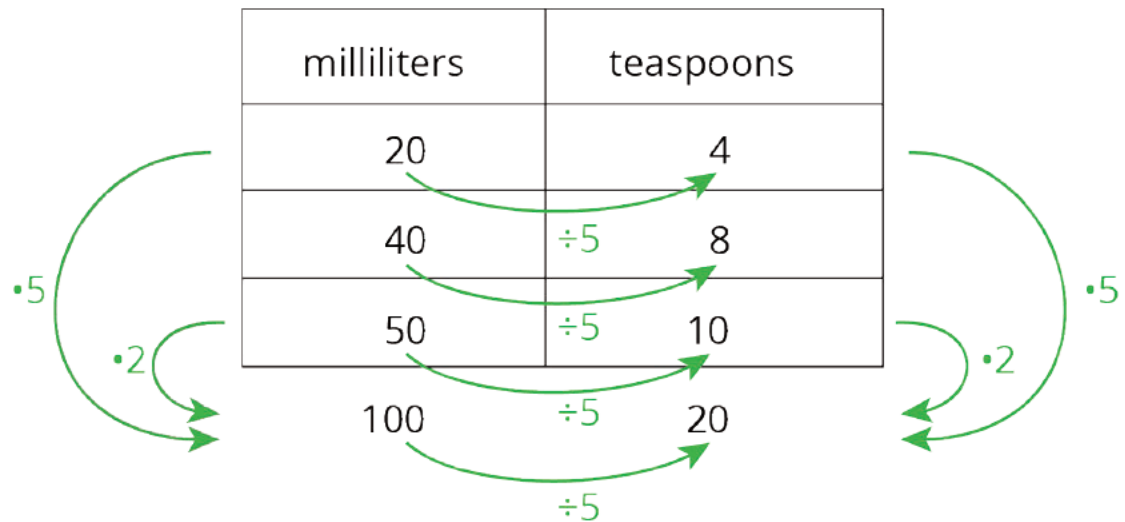
	quarts	liters
1 gallon of water	4	a little less than 4

object to weigh	ounces	pounds	grams	kilograms
Answers vary				

	milliliters	teaspoons
small amount of salt	20	4
medium amount of salt	40	8
large amount of salt	50	10

1. A cubic inch is larger. More cubic centimeters fit in the box because they are smaller.

2. It took more feet because feet are smaller than meters.
3. A liter is bigger than a quart because the gallon filled fewer of them.
4. From least to greatest, the units are gram, ounce, pound, and kilogram, because each object's weight was the largest number of grams, fewer ounces, even fewer pounds, and the smallest number of kilograms.
5. a. About 20 teaspoons for 100 milliliters of salt



- b. About 75 milliliters for 15 teaspoons of salt
- c. About 5 milliliters per teaspoon
- d. About  $\frac{1}{5}$  teaspoons per milliliter

### Are You Ready for More?

People in the medical field use metric measurements when working with medicine. For example, a doctor might prescribe medication in 10 mg tablets.

Brainstorm a list of reasons why healthcare workers would do this. Organize your thinking so it can be followed by others.

### Student Response

Answers vary and may include:

- Unit conversions are simpler
- Calculations are often simpler

### Activity Synthesis

Though much of the discussion will take place within groups, spend a few minutes ensuring that everyone understands the answers to the five questions. To conclude the activity, invite students to share anything that surprised them from the measuring work.

## Lesson Synthesis

If you measure the same quantity with different units, it will take more of the smaller unit and less of the larger one to express the measurement. For example, a jug that holds 2 gallons of liquid also holds 8 quarts of liquid. Quarts are four times smaller than gallons, so it takes four times as many quarts to measure the same volume of liquid.

To reinforce this idea, ask students questions such as:

- “What do quarts and gallons measure?” (Volume of a liquid)
- “Which is bigger: 1 quart, or 1 gallon?” (1 gallon. There are 4 quarts in 1 gallon.)
- “How many quarts are in 8 gallons?” (32. Since a quart is less than a gallon, you need more quarts to measure the same amount.)
- “How many gallons are in 8 quarts?” (2. Since a gallon is bigger than a quart, you need fewer gallons to measure the same amount.)

## 3.3 Which Measurement is Which?

Cool Down: 5 minutes

### Addressing

- 6.RP.A.3.d

### Anticipated Misconceptions

If students seem to be guessing on the first two questions, you can have them hold objects from the previous lesson that weigh close to 1 pound, 1 kilogram, 1 ounce, and 1 gram.

### Student Task Statement

1. Lin has a pet German Shepherd that weighs 38 when measured in one unit and 84 when measured in a different unit. Which measurement is in pounds, and which is in kilograms?  
38 \_\_\_\_\_                      84 \_\_\_\_\_
2. Elena has a pet parakeet that weighs 6 when measured in one unit and 170 when measured in a different unit. Which measurement is in ounces, and which is in grams?  
6 \_\_\_\_\_                      170 \_\_\_\_\_
3. Behind Lin’s house there is a kiddie pool that holds 180 or 680 units of water, depending on which unit you are using to measure. Which measurement is in gallons, and which is in liters?  
180 \_\_\_\_\_                      680 \_\_\_\_\_
4. Behind Elena’s house there is a portable storage container that holds 29 or 1024 units, depending on which unit you are using to measure. Which measurement is in cubic feet, and which is in cubic meters?  
29 \_\_\_\_\_                      1024 \_\_\_\_\_

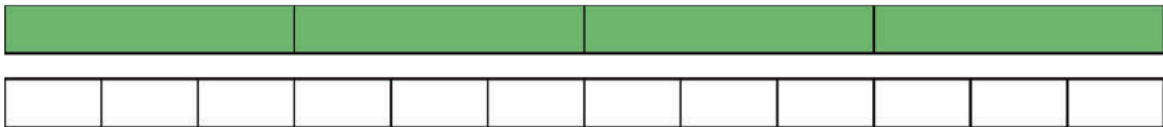
## Student Response

1. 38 is in kilograms and 84 is in pounds because a kilograms is heavier than a pound, so you need fewer kilograms to measure the same quantity.
2. 6 is in ounces and 170 is in grams because an ounce weighs more than a gram.
3. 180 is in gallons and 680 is in liters because gallons are a larger unit than liters.
4. 29 is in cubic meters and 1024 is in cubic feet because cubic meters are larger than cubic feet, so you need fewer of them to measure the same quantity.

## Student Lesson Summary

The size of the unit we use to measure something affects the measurement.

If we measure the same quantity with different units, it will take more of the smaller unit and fewer of the larger unit to express the measurement. For example, a room that measures 4 yards in length will measure 12 feet.



There are 3 feet in a yard, so one foot is  $\frac{1}{3}$  of a yard.

- It takes 3 times as many feet to measure the same length as it does with yards.
- It takes  $\frac{1}{3}$  as many yards to measure the same length as it does with feet.

## Lesson 3 Practice Problems

### Problem 1

#### Statement

Decide if each is a measurement of length, area, volume, or weight (or mass).

- a. How many centimeters across a handprint
- b. How many square inches of paper needed to wrap a box
- c. How many gallons of water in a fish tank
- d. How many pounds in a bag of potatoes
- e. How many feet across a swimming pool
- f. How many ounces in a bag of grapes
- g. How many liters in a punch bowl



h. How many square feet of grass in a lawn

## Solution

- a. Length
- b. Area
- c. Volume
- d. Weight (or mass)
- e. Length
- f. Weight (or mass)
- g. Volume
- h. Area

(From Unit 3, Lesson 2.)

## Problem 2

### Statement

Clare says, "This classroom is 11 meters long. A meter is longer than a yard, so if I measure the length of this classroom in yards, I will get less than 11 yards." Do you agree with Clare? Explain your reasoning.

## Solution

Clare is incorrect. Explanations vary. Sample explanation: Since yards are shorter than meters, more yards than meters are needed to measure the same length.

## Problem 3

### Statement

Tyler's height is 57 inches. What could be his height in centimeters?

- A. 22.4
- B. 57
- C. 144.8
- D. 3,551

## Solution

C

## Problem 4

### Statement

A large soup pot holds 20 quarts. What could be its volume in liters?

- A. 7.57
- B. 19
- C. 21
- D. 75.7

### Solution

B

## Problem 5

### Statement

Clare wants to mail a package that weighs  $4\frac{1}{2}$  pounds. What could this weight be in kilograms?

- A. 2.04
- B. 4.5
- C. 9.92
- D. 4,500

### Solution

A

## Problem 6

### Statement

Noah bought 15 baseball cards for \$9.00. Assuming each baseball card costs the same amount, answer the following questions.

- a. At this rate, how much will 30 baseball cards cost? Explain your reasoning.
- b. At this rate, how much will 12 baseball cards cost? Explain your reasoning.
- c. Do you think this information would be better represented using a table or a double number line? Explain your reasoning.

## Solution

- \$18.00, because 30 is twice as much as 15 and 18 is twice as much as 9.
- \$7.20, because each baseball card costs 60 cents, and 0.6 times 12 is 7.2.
- Answers vary. Sample response: A table would be more convenient, because the rows of the table can be listed in any order, and not all values between the ones needed have to be filled in.

(From Unit 2, Lesson 13.)

## Problem 7

### Statement

Jada traveled 135 miles in 3 hours. Andre traveled 228 miles in 6 hours. Both Jada and Andre traveled at a constant speed.

- How far did Jada travel in 1 hour?
- How far did Andre travel in 1 hour?
- Who traveled faster? Explain or show your reasoning.

## Solution

- Jada traveled 45 miles per hour because  $135 \div 3 = 45$ .
- Andre traveled 38 miles per hour because  $228 \div 6 = 38$ .
- Jada traveled faster because she covered a greater distance in the same amount of time.

(From Unit 2, Lesson 9.)

# Lesson 4: Converting Units

## Goals

- Choose and create a double number line diagram or table to solve problems involving unit conversion.
- Explain (orally) how to use a “rate per 1” to solve problems involving unit conversion.
- Recognize that when we measure things in two different units, the pairs of measurements are equivalent ratios.

## Learning Targets

- I can convert measurements from one unit to another, using double number lines, tables, or by thinking about “how much for 1.”
- I know that when we measure things in two different units, the pairs of measurements are equivalent ratios.

## Lesson Narrative

In grade 4, students began converting units of measurements by multiplying. The work in grade 5 expanded to include conversion by dividing, but was still restricted to units within the same measurement system. In this lesson, students progress to converting units that may be in different systems of measurement, using ratio reasoning and recently-learned strategies such as double number lines, tables, and multiplication or division of unit rates.

## Alignments

### Building On

- 5.NF.B.4.a: Interpret the product  $(a/b) \times q$  as  $a$  parts of a partition of  $q$  into  $b$  equal parts; equivalently, as the result of a sequence of operations  $a \times q \div b$ . For example, use a visual fraction model to show  $(2/3) \times 4 = 8/3$ , and create a story context for this equation. Do the same with  $(2/3) \times (4/5) = 8/15$ . (In general,  $(a/b) \times (c/d) = ac/bd$ .)

### Addressing

- 6.RP.A.3.d: Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

### Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display
- MLR3: Clarify, Critique, Correct
- MLR7: Compare and Connect

- MLR8: Discussion Supports
- Number Talk
- Think Pair Share

### Required Materials

Four-function calculators

### Student Learning Goals

Let's convert measurements to different units.

## 4.1 Number Talk: Fractions of a Number

### Warm Up: 10 minutes

This number talk encourages students to think about numbers and rely on what they know about structure, patterns, multiplication, division, and properties of operations to mentally solve a problem. Discussion of strategies is integral to the activity, but it may not be possible to share every possible strategy for each problem given limited time. Consider gathering only two or three different strategies per problem.

The factors in the problems are chosen such that their connections become increasingly more apparent as students progress. If such connections do not arise during discussions, make them explicit. Students should also be able to state that taking a fraction of a number involves multiplication and can be done with either multiplication or division.

### Building On

- 5.NF.B.4.a

### Instructional Routines

- MLR8: Discussion Supports
- Number Talk

### Launch

Display one problem at a time. Give students 1 minute of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Follow with a whole-class discussion.

---

### Support for Students with Disabilities

*Representation: Internalize Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.

*Supports accessibility for: Memory; Organization*

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### Student Task Statement

Find the values mentally.

$$\frac{1}{4} \text{ of } 32$$

$$\frac{3}{4} \text{ of } 32$$

$$\frac{3}{8} \text{ of } 32$$

$$\frac{3}{8} \text{ of } 64$$

### Student Response

- 8. Possible strategies:  $32 \div 4$  or  $4 \cdot 8$ .
- 24. Possible strategies:  $32 \div 4 \cdot 3$ , or since one of the factors in the first question tripled and the other remained constant, the product triples ( $8 \cdot 3$ ).
- 12. Possible strategies:  $32 \div 8 \cdot 3$ , or since  $\frac{3}{8}$  is half of  $\frac{3}{4}$ , the product is half of the product in the second question ( $24 \div 2$ ).
- 24. Possible strategies:  $64 \div 8 \cdot 3$ , or since one factor doubled from the previous problem and the other stayed the same, the product doubles ( $12 \cdot 2$ ).

### Activity Synthesis

Ask students to share their strategies for each problem. Record and display their explanations for all to see. If not mentioned by students as they discuss the last three problems, ask if or how the given factors impacted their strategy choice.

To involve more students in the conversation, consider asking:

- “Who can restate \_\_\_’s reasoning in a different way?”
- “Did anyone solve the problem the same way but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to \_\_\_’s strategy?”
- “Do you agree or disagree? Why?” If time permits, ask students if they notice any connections between the problems. Have them share any relationships they notice.

---

### Support for English Language Learners

*Speaking: MLR8 Discussion Supports.:* Display sentence frames to support students when they explain their strategy. For example, "First, I \_\_\_\_ because . . ." or "I noticed \_\_\_\_ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

*Design Principle(s): Optimize output (for explanation)*

---

## 4.2 Road Trip

### 15 minutes

The purpose of this activity is to help students understand that quantities measured using the same two units of measure form a set of equivalent ratios. All of the strategies and representations they have for reasoning about equivalent ratios can be used for reasoning about converting from one unit of measure to another. Any ratio  $a : b$  has two associated unit rates:  $\frac{a}{b}$  and  $\frac{b}{a}$ , with a particular meaning in the context. For example, since there are 8 kilometers in approximately 5 miles, there are  $\frac{8}{5}$  kilometers in 1 mile, and there are  $\frac{5}{8}$  of a mile in one kilometer. We want students to notice that finding "how much per 1" and reasoning with these unit rates is efficient, but to make sense of these efficient strategies by using familiar representations like double number lines and tables. In a constant speed context, students are explicitly asked to compute each unit rate, and then they are asked to solve a problem where either unit rate can be used. For the second problem, monitor for one student who uses each strategy to solve the problem:

- Creating a double number line or a table to represent the association between miles and the equivalent distance in kilometers as a set of equivalent ratios. (If both of these representations are used, it is fine to include both.)
- Converting 80 kilometers into 50 miles by evaluating  $80 \cdot \frac{5}{8}$  (in order to compare the resulting 50 miles per hour with 75 miles per hour)
- Converting 75 miles into 120 kilometers by evaluating  $75 \cdot \frac{8}{5}$  (in order to compare the resulting 120 kilometers per hour with 80 kilometers per hour)

Continuing to draw connections between representations of equivalent ratios and more efficient methods will help students make sense of the more efficient methods.

### Addressing

- 6.RP.A.3.d

### Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- MLR7: Compare and Connect

## Launch

Display the image from the task statement for all to see, tell students it is a traffic sign you might see while driving, and ask students to explain what it means. They will likely guess it is a speed limit sign and assume it means 80 miles per hour. If no one brings it up, tell students that this is a sign you might see while driving in Canada or another country that uses the metric system for more things than we use it for in the United States.

Give students 2 minutes of quiet work time and ask them to pause after the first question. Ensure that everyone has correct answers for the first question before proceeding with the second question. Follow with whole-class discussion.

---

### Support for Students with Disabilities

*Representation: Internalize Comprehension.* Provide appropriate reading accommodations and supports to ensure students access to written directions, word problems and other text-based content.

*Supports accessibility for: Language; Conceptual processing*

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### Anticipated Misconceptions

It is acceptable to express the answers to the first question in either fraction or decimal form. If students express uncertainty about carrying out the division of  $5 \div 8$  or  $8 \div 5$ , encourage them to express the quotient in fraction form.

#### Student Task Statement

Elena and her mom are on a road trip outside the United States. Elena sees this road sign.

Elena's mom is driving 75 miles per hour when she gets pulled over for speeding.



1. The police officer explains that 8 kilometers is approximately 5 miles.
  - a. How many kilometers are in 1 mile?
  - b. How many miles are in 1 kilometer?
2. If the speed limit is 80 kilometers per hour, and Elena's mom was driving 75 miles per hour, was she speeding? By how much?

#### Student Response

1. a. About  $\frac{8}{5}$  or 1.6 kilometers



b. About  $\frac{5}{8}$  or 0.625 miles

2. Yes, she was speeding by about 25 miles per hour or about 40 kilometers per hour. Possible strategies:

◦ Convert 80 kilometers per hour into miles per hour:  $80 \cdot \frac{5}{8} = 50$  and  $75 - 50 = 25$

◦ Convert 75 miles per hour into kilometers per hour:  $75 \cdot \frac{8}{5} = 120$  and  $120 - 80 = 40$

### Activity Synthesis

Focus discussion on different approaches to the second question. If any students with less-efficient methods were selected, have them go first in the sequence, or present one of these representations yourself. As students are presenting their work, encourage them to explain the meaning of any numbers used and the reason they decided to use particular operations. For example, if a student multiplies 80 by  $\frac{5}{8}$ , ask them to explain what  $\frac{5}{8}$  means in this context and why they decided to multiply 80 by it. It can be handy to have representations like double number lines or tables displayed to facilitate these explanations.

---

### Support for English Language Learners

*Representing, Listening, Speaking: MLR7 Compare and Connect.* Ask students to display their approaches to determine whether or not Elena’s mom was speeding. As students share their work, encourage them to explain the meaning of each quantity they use. For example, if they convert 80 miles per hour into kilometers per hour, where 80 is multiplied by  $\frac{5}{8}$ , ask what  $\frac{5}{8}$  means in this context and why they decide to multiply it by 80. If students used a table or a double number line, ask how these representations connect with other strategies. This will help students make sense of the various approaches to reason about equivalent ratios which can be used for reasoning about converting one unit of measure to another.

*Design Principle(s): Cultivate conversation; Maximize meta-awareness*

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## 4.3 Veterinary Weights

### 10 minutes

This activity is an opportunity to apply insights from the previous activity in a different context. In this activity, students convert between pounds and kilograms. The conversion factor is not given as a unit rate. As a result of the work in the previous activity, some students may compute and use unit rates, and some may still reason using various representations of equivalent ratios. The numbers are also purposely chosen such that the unit rate  $\frac{10}{22}$  does not have a convenient decimal equivalent, suggesting that fractions are sometimes much more convenient to work with than decimals. Students should have learned efficient methods for multiplying fractions in grade 5 (5.NF.B.4a), but may need support. Additionally, while all measurements within this activity are accurate with rounding to the nearest integer, you may choose to point out before or after the task

that  $\frac{10}{22}$  is a common approximation of the conversion factor from pounds to kilograms and not the true conversion factor.

As students work, identify those who computed and used the unit rates  $\frac{10}{22}$  and  $\frac{22}{10}$ . Highlight these strategies in the discussion, while continuing to refer to other representations to make sense of them as needed.

### Addressing

- 6.RP.A.3.d

### Instructional Routines

- MLR2: Collect and Display
- Think Pair Share

### Launch

Ask students to recall which is heavier: 1 pound or 1 kilogram? (1 kilogram is heavier.) Tell them that in this activity, they will be given weights in pounds and asked to express it in kilograms, and also the other way around. The pairs of measurements in pounds and kilograms for a set of objects are all equivalent ratios. Encourage students to consider how finding unit rates—how many kilograms in 1 pound and how many pounds in 1 kilogram—can make their work more efficient.

Give students quiet think time to complete the activity, and then time to share their explanation with a partner. Follow with whole-class discussion.

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### Support for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Chunk this task into more manageable parts to support students who benefit from support with organization and problem solving. For example, ask students to first draw a table or double number line to organize their thinking and then find a ratio for converting pounds to kilograms. Next, ask students to convert the weight of each dog separately. After that, ask students to complete the last question.

*Supports accessibility for: Organization; Attention*

---

### Anticipated Misconceptions

Students working with the unit rate  $\frac{10}{22}$  may want to convert it to a decimal and get bogged down. Encourage them to work with the fraction, reviewing strategies for multiplying by a fraction as necessary.

### Student Task Statement

A veterinarian uses weights in kilograms to figure out what dosages of medicines to prescribe for animals. For every 10 kilograms, there are 22 pounds.

1. Calculate each animal's weight in kilograms. Explain or show your reasoning. If you get stuck, consider drawing a double number line or table.
  - a. Fido the Labrador weighs 88 pounds.
  - b. Spot the Beagle weighs 33 pounds.
  - c. Bella the Chihuahua weighs  $5\frac{1}{2}$  pounds.
2. A certain medication says it can only be given to animals over 25 kilograms. How much is this in pounds?

### Student Response

1. Sample reasoning for each part: There are  $\frac{10}{22}$  kilograms per pound, so I can multiply by  $\frac{10}{22}$  to convert from pounds to kilograms.
  - a. 40, because  $88 \cdot \frac{10}{22} = 40$
  - b. 15, because  $33 \cdot \frac{10}{22} = 15$
  - c.  $2\frac{1}{2}$  (or equivalent).  $5\frac{1}{2} \cdot \frac{10}{22} = 2\frac{1}{2}$
2. 55 pounds. Sample reasoning: Since 10 kilograms is 22 pounds, 1 kilogram is  $\frac{22}{10}$  pounds. Therefore, 25 kilograms is 55 pounds because  $\frac{22}{10} \cdot 25 = 55$ .

Here is a table that may be used to organize the work:

weight (pounds)	weight (kilograms)
22	10
1	$\frac{10}{22}$
88	40
33	15
$5\frac{1}{2}$	$2\frac{1}{2}$
$\frac{22}{10}$	1
55	25

## Activity Synthesis

Invite one or more students to share who used the unit rates  $\frac{10}{22}$  and  $\frac{22}{10}$  as part of their work. Display a table of equivalent ratios as needed to help students make sense of this approach, including attending to the meaning of these numbers and the rationale for any operations used.

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### Support for English Language Learners

*Listening, Speaking: MLR2 Collect and Display.* As students share their explanations for the first and second questions with a partner followed with whole class discussion, listen for and scribe the words and phrases that students used when converting between pounds and kilograms. Some students may convert to decimals and find this method more challenging, while others may show reasoning using the unit rates  $\frac{10}{22}$  and  $\frac{22}{10}$ . Highlight key vocabulary that students use in the discussion, while continuing to refer to other representations to make sense of them as needed. This will help students increase sense-making while simultaneously support meta-awareness of language.

*Design Principle(s): Support sense-making; Maximize meta-awareness*

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## 4.4 Cooking with a Tablespoon

### Optional: 15 minutes

This optional activity is an opportunity to practice the methods in this lesson to convert between cups and tablespoons. The conversion factor is given in the form of a unit rate, so students only need to decide whether to multiply or divide. They might, however, choose to create a double number line diagram or a table to support their reasoning. Several of the measurements include fractions, giving students an opportunity to practice multiplying mixed numbers by whole numbers (5.NF.B.6) and dividing whole numbers that result in fractions (5.NF.B.3).

### Addressing

- 6.RP.A.3.d

### Instructional Routines

- MLR3: Clarify, Critique, Correct
- Think Pair Share

### Launch

Tell students they will now convert between tablespoons and cups. Just as with pairs of weights in pounds and kilograms, these pairs of tablespoons and cups can also be thought of as equivalent ratios. Welcome any strategies for reasoning about equivalent ratios, but encourage students to try to find efficient methods using multiplication and division.

Give students quiet think time to complete the activity and then time to share their explanation with a partner. Follow with whole-class discussion.

---

### Support for Students with Disabilities

*Representation: Internalize Comprehension.* Activate or supply background knowledge related to measuring using tablespoons and cups. Allow students to use calculators to ensure inclusive participation in the activity.

*Supports accessibility for: Memory; Conceptual processing*

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### Anticipated Misconceptions

Students may answer “zero cups” for the last question, because it is less than one. Ask them to consider what fraction of a cup would be equivalent to 6 tablespoons.

#### Student Task Statement

Diego is trying to follow a recipe, but he cannot find any measuring cups! He only has a tablespoon. In the cookbook, it says that 1 cup equals 16 tablespoons.

1. How could Diego use the tablespoon to measure out these ingredients?

$\frac{1}{2}$  cup almonds

$1\frac{1}{4}$  cup of oatmeal

$2\frac{3}{4}$  cup of flour

2. Diego also adds the following ingredients. How many cups of each did he use?

28 tablespoons of sugar

6 tablespoons of cocoa powder

#### Student Response

1. Cup to tablespoon conversions:

- a. 8 tablespoons of almonds, because  $\frac{1}{2} \cdot 16 = 8$ .

- b. 20 tablespoons of oatmeal, because  $\frac{1}{4} \cdot 16 = 4$  and  $16 + 4 = 20$ .

- c. 44 tablespoons of flour, because  $2\frac{3}{4} \cdot 16 = 44$ .

2. Tablespoon to cup conversions

- a.  $1\frac{3}{4}$  cups of sugar. Possible strategies:

- From earlier, 20 tablespoons is  $1\frac{1}{4}$  cups. For 28 tablespoons, you need an additional 8 tablespoons, or an additional  $\frac{1}{2}$  cup.  $1\frac{1}{4} + \frac{1}{2} = 1\frac{3}{4}$ .

- 1 tablespoon is  $\frac{1}{16}$  of a cup.  $\frac{1}{16} \cdot 28 = \frac{28}{16}$ , which is equivalent to  $1\frac{12}{16}$  and  $1\frac{3}{4}$ .

- b.  $\frac{3}{8}$  cup of cocoa powder. Possible strategies:

- Notice that to convert from tablespoons to cups, always divide by 16, so  $6 \div 16 = \frac{6}{16} = \frac{3}{8}$ .
- 1 tablespoon is  $\frac{1}{16}$  of a cup, so  $\frac{1}{16} \cdot 6 = \frac{6}{16} = \frac{3}{8}$ .

### Activity Synthesis

As in the previous task, select students to share based on their strategies, sequencing from less efficient to more efficient, being sure to highlight approaches using multiplication or division (by 16, or multiplication by  $\frac{1}{16}$ ). Record the representations or strategies students shared and display them for all to see.

When discussing the last strategy, ask students how they would know whether to multiply or to divide. Highlight that we multiply or divide depending on the information we have. Since 1 cup equals 16 tablespoons, if we know a quantity in cups, we can multiply it by 16 to find the number of tablespoons. On the other hand, if we know a quantity in tablespoons, we can divide it by 16 (or multiply by  $\frac{1}{16}$ ) to find the number of cups.

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### Support for English Language Learners

*Reading, Writing, Speaking: MLR3 Clarify, Critique, Correct.* Before students share their answers to the second question, present an incorrect solution and explanation or representation. For example, “Diego used zero cups of cocoa powder because 6 tablespoons is less than 1.” Ask students to identify the error(s), analyze the response in light of their own understanding of the problem, and work with a partner to propose an improved response. This will help students understand how a “rate per 1” could help convert between units.

*Design Principle(s): Optimize output (for explanation); Maximize meta-awareness*

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### Lesson Synthesis

In this lesson, students learned two important points:

- Two measurements of the same object in different units form equivalent ratios, and we can use all of our familiar tools to reason about equivalent ratios when we are thinking about converting units of measure.
- If we know a “rate per 1” that relates the two units, we can use it to convert one measurement to the other by multiplication or division.

To highlight the first point, choose and display a couple of tables of equivalent ratios from the lesson, each table showing two different units (e.g., pounds and kilograms, or cups and teaspoons). Ask students to explain how pairs of numbers in the table represent equivalent ratios and how to use equivalent ratios to convert between units of measurement.

Then, ask if and how a “rate per 1” could help us convert between units. Show examples from the lesson about how multiplying and dividing a rate per 1 helps us with conversion. For instance, we know that 1 kilogram is 2.2 pounds. With this information, we can convert 5 kilograms into 11 pounds, because  $5 \cdot (2.2) = 11$ . We can also convert 220 pounds into 100 kilograms, because  $220 \div 2.2 = 100$ .

## 4.5 Buckets

Cool Down: 5 minutes

### Addressing

- 6.RP.A.3.d

#### Student Task Statement

A large bucket holds 5 gallons of water, which is about the same as 19 liters of water.

A small bucket holds 2 gallons of water. About how many liters does it hold?

#### Student Response

$\frac{38}{5}$  (or 7.6 or equivalent). Possible strategy:

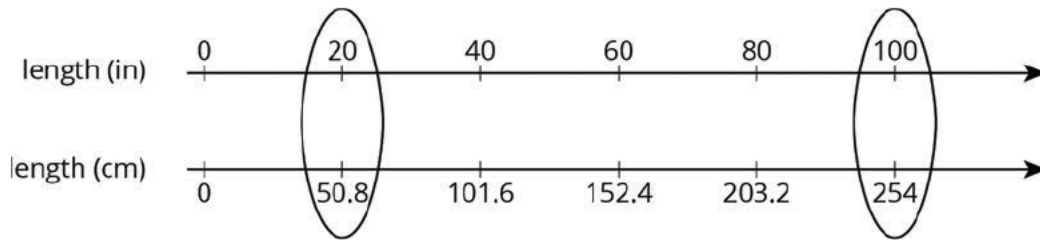
gallons	liters
5	19
1	$\frac{19}{5}$
2	$\frac{38}{5}$

#### Student Lesson Summary

When we measure something in two different units, the measurements form an equivalent ratio. We can reason with these equivalent ratios to convert measurements from one unit to another.

Suppose you cut off 20 inches of hair. Your Canadian friend asks how many centimeters of hair that was. Since 100 inches equal 254 centimeters, we can use equivalent ratios to find out how many centimeters equal 20 inches.

Using a double number line:



Using a table:

length (in)	length (cm)
100	254
1	2.54
20	50.8

One quick way to solve the problem is to start by finding out how many centimeters are in 1 inch. We can then multiply 2.54 and 20 to find that 20 inches equal 50.8 centimeters.

## Lesson 4 Practice Problems

### Problem 1

#### Statement

Priya's family exchanged 250 dollars for 4,250 pesos. Priya bought a sweater for 510 pesos. How many dollars did the sweater cost?

pesos	dollars
4,250	250
	25
	1
	3
510	

#### Solution

30 dollars



## Problem 2

### Statement

There are 3,785 milliliters in 1 gallon, and there are 4 quarts in 1 gallon. For each question, explain or show your reasoning.

- How many milliliters are in 3 gallons?
- How many milliliters are in 1 quart?

### Solution

- 11,355 milliliters, because  $3,785 \cdot 3 = 11,355$ .
- 946.25 milliliters, because  $3,785 \div 4 = 946.25$

## Problem 3

### Statement

Lin knows that there are 4 quarts in a gallon. She wants to convert 6 quarts to gallons, but cannot decide if she should multiply 6 by 4 or divide 6 by 4 to find her answer. What should she do? Explain or show your reasoning. If you get stuck, consider drawing a double number line or using a table.

### Solution

Lin should divide 6 by 4. Explanations vary. Sample explanations:

- A gallon is larger than a quart, so there are fewer than 6 gallons in 6 quarts.
- Table:

quarts	gallons
4	1
6	1.5

## Problem 4

### Statement

Tyler has a baseball bat that weighs 28 ounces. Find this weight in kilograms and in grams. (Note: 1 kilogram  $\approx$  35 ounces)

### Solution

0.8 kilograms ( $28 \div 35 = 0.8$ ) and 800 grams ( $0.8 \cdot 1,000 = 800$ )

## Problem 5

### Statement

Identify whether each unit measures length, volume, or weight (or mass).

- |                                  |                             |                                  |
|----------------------------------|-----------------------------|----------------------------------|
| <input type="radio"/> Mile       | <input type="radio"/> Liter | <input type="radio"/> Kilogram   |
| <input type="radio"/> Cup        | <input type="radio"/> Gram  | <input type="radio"/> Teaspoon   |
| <input type="radio"/> Pound      | <input type="radio"/> Pint  | <input type="radio"/> Milliliter |
| <input type="radio"/> Centimeter | <input type="radio"/> Yard  |                                  |

### Solution

- Length
- Volume
- Weight (or mass)
- Length
- Volume
- Weight (or mass)
- Volume
- Length
- Weight (or mass)
- Volume
- Volume

(From Unit 3, Lesson 1.)

## Problem 6

### Statement

A recipe for trail mix uses 7 ounces of almonds with 5 ounces of raisins. (Almonds and raisins are the only ingredients.) How many ounces of almonds would be in a one-pound bag of this trail mix? Explain or show your reasoning.

### Solution

$\frac{28}{3} = 9\frac{1}{3}$ , so there are  $9\frac{1}{3}$  ounces of almonds. There are multiple ways to find this, and one way is to know the original mix has 12 ounces and multiply by  $\frac{16}{12} = \frac{4}{3}$  to produce an equivalent ratio for a 16-ounce mix.

(From Unit 2, Lesson 11.)

## Problem 7

### Statement

An ant can travel at a constant speed of 980 inches every 5 minutes.

- a. How far does the ant travel in 1 minute?
- b. At this rate, how far can the ant travel in 7 minutes?

### Solution

- a. 196 inches per minute because  $980 \div 5 = 196$ .
- b. 1,372 inches because 196 times 7 is 1,372.

(From Unit 2, Lesson 9.)

## Section: Rates

### Lesson 5: Comparing Speeds and Prices

#### Goals

- Explain (orally and in writing) that if two ratios have the same rate per 1, they are equivalent ratios.
- Justify (orally and in writing) comparisons of speeds or prices.
- Recognize that calculating how much for 1 of the same unit is a useful strategy for comparing rates. Express these rates (in spoken and written language) using the word “per” and specifying the unit.

#### Learning Targets

- I understand that if two ratios have the same rate per 1, they are equivalent ratios.
- When measurements are expressed in different units, I can decide who is traveling faster or which item is the better deal by comparing “how much for 1” of the same unit.

#### Lesson Narrative

Previously, students found and used rates per 1 to solve problems in a context. This lesson is still about contexts, but it's more deliberately working toward the general understanding that when two ratios are associated with the same rate per 1, then they are equivalent ratios. Therefore, to determine whether two ratios are equivalent, it is useful to find and compare their associated rates per 1. In this lesson, we also want students to start to notice that dividing one of the quantities in a ratio by the other is an efficient way to find a rate per 1, while attending to the meaning of that number in the context (MP2).

Calculating rates per 1 is also a common way to compare rates in different situations. For example, suppose we find that one car is traveling 30 miles per hour and another car is traveling 40 miles per hour. The different rates tell us not only that the cars are traveling at different speeds, but which one is traveling faster. Similarly, knowing that one grocery store charges \$1.50 per item while another charges \$1.25 for the same item allows us to select the better deal even when the stores express the costs with rates such as “2 for \$3” or “4 for \$5.”

#### Alignments

##### Building On

- 5.NF.B.3: Interpret a fraction as division of the numerator by the denominator ( $a/b = a \div b$ ). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret  $3/4$  as the result of dividing 3 by 4, noting that  $3/4$  multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size  $3/4$ . If 9 people want to share a 50-pound sack of rice equally by weight, how

many pounds of rice should each person get? Between what two whole numbers does your answer lie?

### Addressing

- 6.RP.A.2: Understand the concept of a unit rate  $a/b$  associated with a ratio  $a : b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship. \$Expectations for unit rates in this grade are limited to non-complex fractions.
- 6.RP.A.3.b: Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

### Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR1: Stronger and Clearer Each Time
- MLR5: Co-Craft Questions
- MLR8: Discussion Supports

### Required Preparation

For the activity The Best Deal on Beans, consider gathering some examples of grocery store advertisements from newspapers or weekly fliers for deals like “3 for \$5.”

### Student Learning Goals

Let’s compare some speeds and some prices.

## 5.1 Closest Quotient

### Warm Up: 5 minutes

This warm-up prompts students to reason about the meaning of division by looking closely at the dividend and divisor. The expressions were purposely chosen to encourage more precise reasoning than roughly estimating. While some students may mentally solve each, encourage them to also think about the numbers in the problem without calculating. Ask them what would happen if the dividend or divisor increased or decreased. Expect students to think of fractions both as division and as numbers. Encourage connections between these two ideas.

### Building On

- 5.NF.B.3

### Instructional Routines

- MLR8: Discussion Supports

## Launch

Display one problem at a time. Give students 1 minute of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Follow with a whole-class discussion.

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### Support for Students with Disabilities

*Representation: Internalize Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.

*Supports accessibility for: Memory; Organization*

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### Support for English Language Learners

*Speaking, Reading Representing: MLR8 Discussion Supports.* Display a table that shows different representations and language used for  $\frac{1}{2}$ , 1, and  $1\frac{1}{2}$ . Highlight differences between similar-looking or similar-sounding language like “one”, “one half”, “a half”, “one and a half”, etc.

*Design Principle(s): Support sense-making; Maximize meta-awareness*

---

### Student Task Statement

Is the value of each expression closer to  $\frac{1}{2}$ , 1, or  $1\frac{1}{2}$ ?

1.  $20 \div 18$
2.  $9 \div 20$
3.  $7 \div 5$

### Student Response

1. Closer to 1. Possible strategy:  $20 \div 20 = 1$  and since 18 is less than 20, the quotient is more than 1 and since  $27 \div 18 = 1.5$  and 20 is less than 27, the quotient is less than 1.5. The distance from 1 and 1.5 could be reasoned about thinking about the size of the leftovers,  $\frac{2}{18}$  versus  $\frac{7}{18}$
2. Closer to  $\frac{1}{2}$ . Possible strategy: since  $\frac{10}{20} = \frac{1}{2}$  and since 9 is less than 10, the quotient is less than  $\frac{1}{2}$ , but only a small  $\frac{1}{20}$  away.
3. Closer to  $1\frac{1}{2}$ . Possible strategy:  $\frac{5}{5} = 1$ , the quotient is  $\frac{2}{5}$  over 1.  $\frac{2}{5} = 0.4$  which is closer to 1.5 than 1.

### Activity Synthesis

Discuss each problem one at a time with this structure:

- Ask students to indicate whether they think the expression is closer to  $\frac{1}{2}$ , 1, or  $1\frac{1}{2}$ .
- If everyone agrees on one answer, ask a few students to share their reasoning, recording it for all to see. If there is disagreement on an answer, ask students with opposing answers to explain their reasoning to come to an agreement on an answer.

## 5.2 More Treadmills

### 15 minutes

In this activity, students analyze the workouts of several people on a treadmill given time-distance ratios. The purpose of this activity is to remind students how speed contexts work and to start to nudge them toward more efficient ways to compare speeds. Students see that when such ratios can be expressed with the same number of meters per minute, the ratios are equivalent and the moving objects (people, cars, etc.) have the same speed.

Speed is typically expressed as a distance per 1 unit of time, so the task provides a familiar context for computing and using rates per 1. The numbers have been chosen such that any two workouts being compared has the same time, same distance, or same speed.

Encourage students to use “per 1” and “for each” language throughout, as this language supports the development of the concept of unit rate.

As students discuss the problems, listen closely for those who use these terms as well as descriptions of speed (e.g., “same speed,” “faster,” “slower”). Also notice students who make the connections between the rates per 1 they calculated in the first half of the task and use them to answer questions in the second half. Invite some of these students or groups to share later.

### Addressing

- 6.RP.A.2

### Instructional Routines

- MLR5: Co-Craft Questions

### Launch

Arrange students in groups of 3. Give students 2–3 minutes of quiet think time to complete the first three questions. Then, ask them to share their responses and complete the last three questions in their groups.

Specify that, when discussing the first three questions (comparisons of pairs of runners), each student in the group should take the lead on analyzing one sub-problem (i.e., sharing how the workouts of the two given runners are similar or different).

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### Support for Students with Disabilities

*Representation: Access for Perception.* Read the problem aloud. Students who both listen to and read the information will benefit from extra processing time. Check for understanding by asking 1-2 students to restate the problem in their own words.

*Supports accessibility for: Language*

---

### Support for English Language Learners

*Speaking, Writing: MLR5 Co-Craft Questions.* Display the constant speed of Tyler, Kiran, and Mai and ask pairs of students to write possible mathematical questions about the situation. They can also ask questions about information that might be missing, or even about assumptions that they think are important. Then, invite select pairs to share their questions with the class. Look for questions that require students to make comparisons about different speeds. Finally, reveal the actual questions students are expected to work on, and students are set to work. This routine creates space for students to produce the language of mathematical questions as well as develop the language used to talk about constant speed.

*Design Principle(s): Optimize output (for questioning); Cultivate conversation*

---

### Anticipated Misconceptions

If students are not sure how to begin, suggest that they try using a table or a double number line that associates meters and minutes.

#### Student Task Statement

Some students did treadmill workouts, each one running at a constant speed. Answer the questions about their workouts. Explain or show your reasoning.

- Tyler ran 4,200 meters in 30 minutes.
  - Kiran ran 6,300 meters in  $\frac{1}{2}$  hour.
  - Mai ran 6.3 kilometers in 45 minutes.
1. What is the same about the workouts done by:
    - a. Tyler and Kiran?
    - b. Kiran and Mai?
    - c. Mai and Tyler?
  2. At what rate did each of them run?



3. How far did Mai run in her first 30 minutes on the treadmill?

### Student Response

1.
  - a. Tyler and Kiran both ran for the same amount of time: 30 minutes. Kiran ran a greater distance in 30 minutes, so Kiran was running faster than Tyler.
  - b. Kiran and Mai ran the same distance, 6,300 meters, but Mai took more time than Kiran to run 6,300 meters, so Mai was running slower than Kiran.
  - c. Mai and Tyler both ran 140 meters per minute, so Mai and Tyler were running at the same speed. However they ran different distances and took different amounts of time to do so.
2. The tables show one possible strategy. Some students may reason with double number lines while others may simply calculate  $\frac{b}{a}$  for the given ratio  $a : b$ .  
Tyler ran 140 meters per minute.

distance (meters)	time (minutes)
4,200	30
1,400	10
140	1

Kiran ran 210 meters per minute.

distance (meters)	time (minutes)
6,300	30
2,100	10
1,050	5
210	1

Mai ran 140 meters per minute.

distance (meters)	time (minutes)
6,300	45
140	1

3. Mai ran 4,200 meters in 30 minutes, because she is going the same speed as Tyler and that is how far Tyler ran in 30 minutes.

### Are You Ready for More?

Tyler and Kiran each started running at a constant speed at the same time. Tyler ran 4,200 meters in 30 minutes and Kiran ran 6,300 meters in  $\frac{1}{2}$  hour. Eventually, Kiran ran 1 kilometer more than Tyler. How much time did it take for this to happen?

### Student Response

Just over 14 minutes. Kiran runs 2100 meters more than Tyler in 30 minutes. Each minute he runs 70 meters more, so it will take  $\frac{1,000}{70} = 14\frac{2}{7}$  minutes for him to run 1 kilometer more.

difference (meters)	time (minutes)
2,100	30
70	1
1,000	$\frac{1,000}{70}$

### Activity Synthesis

Focus the conversation on the questions in the second half of the activity, the idea of “same speed,” and the clues that two objects are moving equally fast or slow. To begin the conversation, ask: “How can you tell when things are going the same speed?” Give a moment of quiet think time before soliciting responses. Students may say: “They keep up with one another running on a track,” “same distance in the same time,” or “same miles per hour in a car,” etc.

Invite a few students to share their analyses of how the runners compare, starting with how Tyler's workout compares to Kiran's, and how Kiran's compares to Mai's. Descriptions such as “slower,” “faster,” or “higher or lower speed” should begin to emerge. After students share their analyses of Mai and Tyler's workouts, make sure to highlight that even though they ran different distances in different amounts of time, they each ran 140 meters per minute so we can say “they ran at the same speed.” This also means that Mai and Tyler's original ratios—4,200 : 30 and 6,300 : 45—are equivalent ratios.

In the last problem, students need to understand that since Mai and Tyler ran at the same speed they traveled the same distance for the first 30 minutes on the treadmill. This may be difficult for students to articulate with precision, so allowing multiple students to share their thinking may be beneficial.

## 5.3 The Best Deal on Beans

15 minutes

Students use and compare rates per 1 in a shopping context as they look for “the best deal.” The purpose of this activity is to remind students how unit price contexts work and to start to nudge them toward more efficient ways to compare unit prices.

While this task considers “the best deal” to mean having the lowest cost per unit, the phrase may have different meanings to students and should be discussed. For instance, students may bring up other considerations such as distance to store, store preference (e.g., some stores offer loyalty points), what else they need to purchase, and not wanting to buy in bulk when only a small quantity is needed. Discussing these real-life considerations, and choosing which to prioritize and which to disregard, is an important part of modeling with mathematics (MP4), but it is also appropriate to clarify that, for the purposes of this problem, we are looking for “the best deal” in the sense of the lowest cost per can.

As students work, monitor for students who use representations like double number lines or tables of equivalent ratios. These are useful for making sense of a strategy that divides the price by the number of cans to find the price per 1. Also monitor for students using more efficient strategies.

### **Addressing**

- 6.RP.A.2
- 6.RP.A.3.b

### **Instructional Routines**

- Anticipate, Monitor, Select, Sequence, Connect
- MLR1: Stronger and Clearer Each Time

### **Launch**

While some students may help with grocery shopping at home, it is likely many have not and will need extra information to understand what “the best deal” means.

Before students begin, ask if anyone is familiar with the weekly fliers that many stores send out to advertise special deals. Show students some advertisements from local stores, if available.

Ask students to share what “a good deal” and “the best deal” mean to them. Many students are likely to interpret these in terms of low prices (per item or otherwise) or “getting more for less money,” but some may have other practical or personal considerations. (Examples: it is not a good deal if you buy more than you can use before it goes bad. It is not a good deal if you have to travel a long distance to the store.) Acknowledge students’ perspectives and how “messy” such seemingly simple terms can be. Clarify that in this task, we are looking for “the best deal” in the sense of lowest cost per can.

5–10 minutes of quiet work time followed by whole-class discussion.

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### Support for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Provide students with a graphic organizer to organize their problem-solving strategy. The graphic organizer should ask students to identify what they need to find out, what information is provided, how they solved the problem, and why their answer is correct.

*Supports accessibility for: Language; Organization*

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### Support for English Language Learners

*Writing, Listening, Conversing: MLR1 Stronger and Clearer Each Time.* Display the ads about the special sales on 15-oz cans of baked beans. Ask students to write a brief explanation to answer the prompt, “Which store is offering the best deal? Explain your reasoning.” Ask each student to share their written explanation with 2–3 partners who will provide constructive feedback. Students can use ideas and language from each partner to refine and clarify the response through conversation, and then finally revise their original written response. Throughout this process, students should be pressed for details, and encouraged to press each other for details. This helps students to compare rates per 1 in a shopping context as they look for “the best deal.”

*Design Principle(s): Optimize output (for explanation); Cultivate conversation*

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### Anticipated Misconceptions

At first glance, students may look only at the number of cans in each offer or only at the price. Let students know that they need to consider the price per one can.

#### Student Task Statement

Four different stores posted ads about special sales on 15-oz cans of baked beans.

1. Which store is offering the best deal? Explain your reasoning.



2. The last store listed is also selling 28-oz cans of baked beans for \$1.40 each. How does that price compare to the other prices?

### Student Response

1. 8 for \$6 is the best deal at \$0.75 per can. 2 for \$3 is the worst deal at \$1.50 per can. Possible strategies:

- 8 for \$6

price (dollars)	cans
6	8
3	4
0.75	1

- \$10 for 10 cans means \$1 per can.

- 2 for \$3:

price (dollars)	cans
3	2
1.50	1

- 80 cents per can is the same as \$0.80 per can.

2. The last store's 28 oz can for \$1.40 is the same price per ounce as the first store's 15 oz can for \$0.75, because  $0.75 \div 15 = 0.05$  and  $1.40 \div 28 = 0.05$ .

### Activity Synthesis

If any students used a representation like a double number line or table to support their reasoning, select these students to share their strategy first. Keep these representations visible. Follow with explanations from students who used more efficient strategies, and use the representations to make connections to more efficient strategies. Highlight the use of division to compute the price per can and the use of “per 1” language. The purpose of this activity and this discussion is help students see that computing and comparing the price per 1 is an efficient way to compare rates in a unit price context.

### Lesson Synthesis

Previously, students compared rates of different ratios by showing that they are or are not equivalent, and by using diagrams and scale factors. In prior lessons students found rates per 1 as a way to determine equivalent ratios. Here rates per 1, in the form of speed and unit price, are deliberately calculated so that they can be compared.

To help students summarize their thinking, display a list of the stated ratios in each activity and how they would be written in “per 1” rate language, as shown here:

rate as given	rate per 1
4,200 meters in 30 minutes	140 meters per minute
6,300 meters in 30 minutes	210 meters per minute
6,300 meters in 45 minutes	140 meters per minute
-	-
8 cans for \$6	\$0.75 per can
10 cans for \$10	\$1.00 per can
2 cans for \$3	\$1.50 per can
80 cents per can	\$0.80 per can

Give students some quiet time to read through the list. Then, ask 2–3 students to share which rate they prefer for comparing and why (“I prefer the rates per 1 because I can just compare two numbers, since the 1 is the same.”).

## 5.4 A Sale on Sparkling Water

Cool Down: 5 minutes

### Addressing

- 6.RP.A.2
- 6.RP.A.3.b

### Student Task Statement

Bottles of sparkling water usually cost \$1.69 each. This week they are on sale for 4 bottles for \$5. You bought one last week and one this week. Did you pay more or less for the bottle this week? How much more or less?



### Student Response

I paid \$0.44 less this week. Possible strategy: Since 4 bottles cost \$5, each bottle costs  $\$5 \div 4$ , or \$1.25 this week. The difference is \$0.44, because  $1.69 - 1.25 = 0.44$ .

### Student Lesson Summary

Diego ran 3 kilometers in 20 minutes. Andre ran 2,550 meters in 17 minutes. Who ran faster? Since neither their distances nor their times are the same, we have two possible strategies:

- Find the time each person took to travel the *same distance*. The person who traveled that distance in less time is faster.
- Find the distance each person traveled in the *same time*. The person who traveled a longer distance in the same amount of time is faster.

It is often helpful to compare distances traveled in *1 unit* of time (1 minute, for example), which means finding the speed such as meters per minute.

Let's compare Diego and Andre's speeds in meters per minute.

distance (meters)	time (minutes)
3,000	20
1,500	10
150	1

distance (meters)	time (minutes)
2,550	17
150	1

Both Diego and Andre ran 150 meters per minute, so they ran at the same speed.

Finding ratios that tell us how much of quantity **A** per 1 unit of quantity **B** is an efficient way to compare rates in different situations. Here are some familiar examples:

- Car speeds in *miles per hour*.
- Fruit and vegetable prices in *dollars per pound*.

## Glossary

- unit price

## Lesson 5 Practice Problems

### Problem 1

#### Statement

Mai and Priya were on scooters. Mai traveled 15 meters in 6 seconds. Priya travels 22 meters in 10 seconds. Who was moving faster? Explain your reasoning.

#### Solution

Mai's scooter is faster.  $22 \div 10 = 2.2$ , so Priya's scooter travels at a rate of 2.2 meters per second.  $15 \div 6 = 2.5$ , so Mai's scooter travels at a rate of 2.5 meters per second.

### Problem 2

#### Statement

Here are the prices for cans of juice that are the same brand and the same size at different stores. Which store offers the best deal? Explain your reasoning.

Store X: 4 cans for \$2.48

Store Y: 5 cans for \$3.00

Store Z: 59 cents per can

#### Solution

Store Z has the best deal.  $2.48 \div 4 = 0.62$  or 62 cents per can.  $3 \div 5 = 0.6$  or 60 cents per can. 59 cents is the least expensive of the 3 options.



### Problem 3

#### Statement

Costs of homes can be very different in different parts of the United States.

- a. A 450-square-foot apartment in New York City costs \$540,000. What is the price per square foot? Explain or show your reasoning.
- b. A 2,100-square-foot home in Cheyenne, Wyoming, costs \$110 per square foot. How much does this home cost? Explain or show your reasoning.

#### Solution

- a. \$1,200 ( $540,000 \div 450 = 1,200$ )
- b. \$231,000 ( $2,100 \cdot 110 = 231,000$ )

### Problem 4

#### Statement

There are 33.8 fluid ounces in a liter. There are 128 fluid ounces in a gallon. About how many liters are in a gallon?

- a. 2
- b. 3
- c. 4
- d. 5

Is your estimate larger or smaller than the actual number of liters in a gallon? Explain how you know.

#### Solution

There are about 4 liters in a gallon. Sample explanation: This estimate is too big:  $4 \cdot 32 = 128$ , so  $4 \cdot (33.8)$  is larger than 128.

(From Unit 3, Lesson 4.)

### Problem 5

#### Statement

Diego is 165 cm tall. Andre is 1.7 m tall. Who is taller, Diego or Andre? Explain your reasoning.

#### Solution

Andre is taller. 1.7 m is 170 cm, and  $170 > 165$ .

(From Unit 3, Lesson 3.)

## Problem 6

### Statement

Name an object that could be about the same length as each measurement.

- |             |                  |
|-------------|------------------|
| a. 4 inches | a. 6 centimeters |
| b. 6 feet   | b. 2 millimeters |
| c. 1 meter  | c. 3 kilometers  |
| d. 5 yards  |                  |

### Solution

Answers vary. Sample response:

- a. Pencil
- b. Ladder
- c. Person's leg
- d. Tablecloth
- e. Insect
- f. Grain of rice
- g. Foot race

(From Unit 3, Lesson 2.)

## Lesson 6: Interpreting Rates

### Goals

- Calculate and interpret the two unit rates associated with a ratio, i.e.,  $\frac{a}{b}$  and  $\frac{b}{a}$  for the ratio  $a : b$ .
- Choose which unit rate to use to solve a given problem and explain the choice (orally and in writing).
- Comprehend the term “unit rate” (in spoken and written language) refers to a rate per 1.

### Learning Targets

- I can choose which unit rate to use based on how I plan to solve the problem.
- When I have a ratio, I can calculate its two unit rates and explain what each of them means in the situation.

### Lesson Narrative

In previous lessons students have calculated and worked with rates per 1. The purpose of this lesson is to introduce the two **unit rates**,  $\frac{a}{b}$  and  $\frac{b}{a}$ , associated with a ratio  $a : b$ . Each unit rate tells us how many of one quantity in the ratio there is per unit of the other quantity. An important goal is to give students the opportunity to see that both unit rates describe the same situation, but that one or the other might be preferable for answering a given question about the situation. Another goal is for students to recognize that they can just divide one number in a ratio by another to find a unit rate, rather than using a table or another representation as an intermediate step. The development of such fluency begins in this section and continues over time. In the Cooking Oatmeal activity, students have explicit opportunities to justify their reasoning and critique the reasoning of others (MP3).

### Alignments

#### Addressing

- 6.RP.A.2: Understand the concept of a unit rate  $a/b$  associated with a ratio  $a : b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship. \$Expectations for unit rates in this grade are limited to non-complex fractions.
- 6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- 6.RP.A.3.b: Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

## Building Towards

- 6.RP.A.2: Understand the concept of a unit rate  $a/b$  associated with a ratio  $a : b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship. \$Expectations for unit rates in this grade are limited to non-complex fractions.

## Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR6: Three Reads
- Think Pair Share

## Student Learning Goals

Let's explore unit rates.

# 6.1 Something per Something

## Warm Up: 5 minutes

This warm-up activates students' prior knowledge around "something per something" language. It gives them a chance to both recall and hear examples and contexts in which such language was used, either in past lessons or outside of the classroom, in preparation for the work ahead.

## Building Towards

- 6.RP.A.2

## Launch

Arrange students in groups of 3–4. Give students a minute of quiet think time to complete the first question, and then 2 minutes to share their ideas in groups and compile a list. Consider asking one or two volunteers to share an example or sharing one of your own. Challenge students to come up with something that is not an example of either unit price or speed, since these have already been studied.

If students are stuck, encourage them to think back to past lessons and see if they could remember any class activities in which the language of "per" was used or could be used.

## Student Task Statement

1. Think of two things you have heard described in terms of "something per something."
2. Share your ideas with your group, and listen to everyone else's idea. Make a group list of all unique ideas. Be prepared to share these with the class.

## Student Response

Answers vary. Sample responses:

- 40 miles per gallon

- \$2 per gallon
- 30 miles per hour
- \$1 per granola bar

Note that responses may lack quantities, such as “miles per hour” or “dollars per pound.”

### Activity Synthesis

Ask each group to share 1–2 of their examples and record unique responses for all to see.

After each group has shared, select one response (or more than one if time allows) that is familiar to students. For example, if one of the groups proposed 30 miles per hour, ask “What are some things we know for sure about an object moving 30 miles per hour?” (The object is traveling a distance of 30 miles every 1 hour.)

It is not necessary to emphasize “per 1” language at this point. The following activities in the lesson focus on the usefulness of “per 1” in the contexts of comparing multiple ratios.

## 6.2 Cooking Oatmeal

### 15 minutes

In this activity, students explore two unit rates associated with the ratio, think about their meanings, and use both to solve problems. The goals are to:

- Help students see that for every context that can be represented with a ratio  $a : b$  and an associated unit rate  $\frac{b}{a}$ , there is another unit rate  $\frac{a}{b}$  that also has meaning and purpose within the context.
- Encourage students to choose a unit rate flexibly depending on the question at hand. Students begin by reasoning whether the two rates per 1 (cups of oats per 1 cup of water, or cups of water per 1 cup of oats) accurately convey a given oatmeal recipe. As students work and discuss, notice those who use different representations (a table or a double number line diagram) or different arguments to make their case (MP3). Once students conclude that both Priya and Han’s rates are valid, they use the rates to determine unknown amounts of oats or water.

### Addressing

- 6.RP.A.3.b

### Instructional Routines

- MLR3: Clarify, Critique, Correct
- Think Pair Share

## Launch

Some students may not be familiar with oatmeal; others may only have experience making instant oatmeal, which comes in pre-measured packets. Explain that oatmeal is made by mixing a specific ratio of oats to boiling water.

Arrange students in groups of 2. Give students 2–3 minutes of quiet think time for the first question. Ask them to pause and share their response with their partner afterwards. Encourage partners to reach a consensus and to be prepared to justify their thinking. See MLR 3 (Clarify, Critique, Correct).

After partners have conferred, select several students to explain their reasoning and display their work for all to see. When the class is convinced that both Priya and Han are correct, ask students to complete the rest of the activity.

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### Support for Students with Disabilities

*Action and Expression: Develop Expression and Communication.* Invite students to talk about their ideas with their partner before writing them down. Display sentence frames to support students when they explain their ideas. For example, “I noticed \_\_\_\_ so I . . .”, “Both \_\_\_\_ and \_\_\_\_ are alike because . . .”, and “How do you know . . . ?”

*Supports accessibility for: Language; Organization*

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### Anticipated Misconceptions

Some students may think that Priya and Han cannot both be right because they came up with different numbers. Ask them to explain what each number means, so that they have a chance to notice that the numbers mean different things. Point out that the positioning of the number 1 appears in different columns within the table.

### Student Task Statement

Priya, Han, Lin, and Diego are all on a camping trip with their families. The first morning, Priya and Han make oatmeal for the group. The instructions for a large batch say, “Bring 15 cups of water to a boil, and then add 6 cups of oats.”

- Priya says, “The ratio of the cups of oats to the cups of water is 6 : 15. That’s 0.4 cups of oats per cup of water.”
- Han says, “The ratio of the cups of water to the cups of oats is 15 : 6. That’s 2.5 cups of water per cup of oats.”

1. Who is correct? Explain your reasoning. If you get stuck, consider using the table.

water (cups)	oats (cups)
15	6
1	
	1

2. The next weekend after the camping trip, Lin and Diego each decide to cook a large batch of oatmeal to have breakfasts ready for the whole week.
  - a. Lin decides to cook 5 cups of oats. How many cups of water should she boil?
  - b. Diego boils 10 cups of water. How many cups of oats should he add into the water?
3. Did you use Priya's rate (0.4 cups of oats per cup of water) or Han's rate (2.5 cups of water per cup of oats) to help you answer each of the previous two questions? Why?

### Student Response

The tables below include fractions and their decimal equivalents. These are included for the convenience of the teacher. The task statements do not require that students write both.

1. Priya and Han are both correct as shown by the table:

water (cups)	oats (cups)
15	6
1	$\frac{6}{15}$ or 0.4
$\frac{15}{6}$ or 2.5	1

2. The next weekend
  - a. 12.5. Multiply Han's rate 2.5 by 5 to find the amount of water needed for 5 cups of oats:

water (cups)	oats (cups)
2.5	1
12.5	5

- b. Multiply Priya's rate 0.4 by 10 to find the amount of oats needed for 10 cups of water:

water (cups)	oats (cups)
1	0.4
10	4

3. Answers vary. We can efficiently find how much water for 5 cups of oats using Han's rate and scaling up by 5. We can efficiently find how much oats for 10 cups of water using Priya's rate and scaling up by 10.

### Activity Synthesis

Focus the discussion on students' responses to the last question and how they knew which rate to use to solve for unknown amounts of oats and water. If not uncovered in students' explanations, highlight that when the amount of oats is known but the amount of water is not, it helps to use the "per 1 cup of oats" rate; a simple multiplication will tell us the missing quantity. Conversely, if the amount of water is known, it helps to use the "per 1 cup of water" rate. Since tables of equivalent ratios are familiar, use the completed table to support reasoning about how to use particular numbers to solve particular problems.

Consider connecting this idea to students' previous work. For example, when finding out how much time it would take to wash all the windows on the Burj Khalifa, it was simpler to use the "minutes per window" rate than the other way around, since the number of windows is known.

Leave the table for this activity displayed and to serve as a reference in the next activity.

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### Support for English Language Learners

*Writing, Listening, Conversing: MLR5 Clarify, Critique, and Correct.* Before discussing students' approaches to the final question. Present the following explanation: "I did not use either Priya's rate (0.4 cups of oats per cup of water) or Han's rate (2.5 cups of water per cup of oats) because they are not equal." Ask students to critique the reasoning and identify the error(s), as they work in pairs to propose an improved response that details how either rate can be used to answer the question. If students can't make the connections between these two rates, consider asking them what each number means. This will help students reflect on efficiently using a unit rate associated with the ratio, cups of water to the cups of oats.

*Design Principle(s): Optimize output (for explanation); Cultivate conversation; Maximize meta-awareness*

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## 6.3 Cheesecake, Milk, and Raffle Tickets

20 minutes



In this task, students calculate and interpret both  $\frac{a}{b}$  and  $\frac{b}{a}$  from a ratio  $a : b$  presented in a context. They work with less-familiar units. The term **unit rate** is introduced so that students have a general name for a “how many per 1” quantity.

In the first half of the task, students practice computing unit rates from ratios. In the second half they practice selecting the better unit rate to use ( $\frac{a}{b}$  or  $\frac{b}{a}$ ) based on the question posed.

As students work on the second half of the task, identify 1–2 students per question to share their choice of unit rate and how it was used to answer the question.

### Addressing

- 6.RP.A.2
- 6.RP.A.3.b

### Instructional Routines

- MLR6: Three Reads

### Launch

Recap that in the previous activity the ratio of 15 cups water for every 6 cups oats can be expressed as two rates “per 1”. These rates are 0.4 cups of oats per cup of water or 2.5 cups of water per cup of oats. Emphasize that, in a table, each of these rates reflects a value paired with a “1” in a row, and that both can be useful depending on the problem at hand. Tell students that we call 0.4 and 2.5 “unit rates” and that a unit rate means “the amount of one quantity for 1 of another quantity.”

Arrange students in groups of 2. Tell students that they will now solve some problems using unit rates. Give students 3–4 minutes to complete the first half of the task (the first three problems). Ask them to share their responses with their partner and come to an agreement before moving on to the second half. Clarify that “oz” is an abbreviation for “ounce.”

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### Support for Students with Disabilities

*Representation: Develop Language and Symbols.* Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of these terms. Include the following term and maintain the display for reference throughout the unit: unit rate.

*Supports accessibility for: Conceptual processing; Language*

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## Support for English Language Learners

*Writing, Listening: MLR6 Three Reads.* Use this routine to set students up to comprehend each situation, without solving the questions for them. In the first read, students read the situation with the goal of comprehending the situation (e.g., a cheesecake recipe contains cream cheese and sugar, Mai's family drinks milk regularly, Tyler has raffle tickets). In the second read, ask students to identify the important quantities by asking them what can be counted or measured (e.g., 12 oz of cream cheese with 15 oz of sugar, 10 gallons of milk every 6 weeks, \$16 for 4 raffle tickets). In the third read, ask students to brainstorm possible mathematical solution strategies to answer the question. This helps students connect the ratio language used in the problem to identify a strategy for finding a unit rate.

*Design Principle(s): Support sense-making*

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## Anticipated Misconceptions

If students are not sure how to use the unit rates they found for each situation to answer the second half of the task, remind them of how the oatmeal problem was solved. Suggest that this problem is similar because they can scale up from a unit rate to answer the questions.

### Student Task Statement

For each situation, find the **unit rates**.

1. A cheesecake recipe says, "Mix 12 oz of cream cheese with 15 oz of sugar."
  - How many ounces of cream cheese are there for every ounce of sugar?
  - How many ounces of sugar is that for every ounce of cream cheese?
2. Mai's family drinks a total of 10 gallons of milk every 6 weeks.
  - How many gallons of milk does the family drink per week?
  - How many weeks does it take the family to consume 1 gallon of milk?
3. Tyler paid \$16 for 4 raffle tickets.
  - What is the price per ticket?
  - How many tickets is that per dollar?
4. For each problem, decide which unit rate from the previous situations you prefer to use. Next, solve the problem, and show your thinking.
  - a. If Lin wants to make extra cheesecake filling, how much cream cheese will she need to mix with 35 ounces of sugar?
  - b. How many weeks will it take Mai's family to finish 3 gallons of milk?

c. How much would all 1,000 raffle tickets cost?

### Student Response

1. a. Lin's recipe calls for  $\frac{4}{5}$  or 0.8 ounce of cream cheese per ounce of sugar.
- b. The recipe calls for  $\frac{5}{4}$  or 1.25 ounces of sugar per ounce of cream cheese. Possible strategy:

cream cheese (oz)	sugar (oz)
12	15
4	5
1	1.25
0.8	1

2. a. Mai's family drinks  $\frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}$  gallons of milk per week.
- b. It takes the family 0.6 weeks (or a little more than half a week) to drink one gallon of milk.
3. a. Tyler paid \$4 per ticket, because  $16 \div 4 = 4$ .
- b. 0.25, or  $\frac{1}{4}$ , of a ticket costs a dollar.
4. a. Lin needs 28 ounces of cream cheese. You can multiply  $\frac{4}{5}$  ounces of cream cheese per ounce of sugar times 35 ounces of sugar.

cream cheese (oz)	sugar (oz)
12	15
4	5
1	1.25
0.8	1
25	35

$\cdot 35$  ↗
↖  $\cdot 35$

- b. It will take 1.8 weeks. You can multiply 0.6 weeks per gallon times 3 gallons.
- c. It would cost \$4,000. You can multiply \$4 per ticket times 1,000 tickets.

### Are You Ready for More?

Write a "deal" on tickets for Tyler's raffle that sounds good, but is actually a little worse than just buying tickets at the normal price.

## Student Response

Answers vary. One bad deal is 6 for \$25, when 6 tickets normally cost \$24.

## Activity Synthesis

Invite previously identified students to share their work on the second half (the last three questions) of the task.

Though the task prompts students to think in terms of unit rate, some students may still reason in ways that feel safer. For example, to find out how much cream cheese Lin would mix with 35 oz of sugar, they may double the 12 oz of cream cheese to 15 oz of sugar ratio to obtain 24 oz of cream cheese for 30 oz of sugar, and then add 4 oz more of cream cheese for the additional 5 oz of sugar. Such lines of reasoning show depth of understanding and should be celebrated. Guide students to also see, however, that some problems (such as the milk problem) can be more efficiently solved using unit rates.

For the ticket problem, students may comment that  $\frac{1}{4}$  of a ticket costing a dollar does not make sense, since it is not possible to purchase  $\frac{1}{4}$  of a ticket. Take this opportunity to applaud the student(s) for reasoning about the interpretation of the number in the context, which is an example of engaging in MP2. If students do not raise this concern, ask: “How can  $\frac{1}{4}$  of a ticket costing a dollar make sense?” Students may argue that the quantity, on its own, does not make sense. Challenge them to figure out how the rate could be used in the context of the problem. For example, ask, “If I had \$80, how many tickets could I buy? What if I had \$75? Can the ‘ $\frac{1}{4}$  of a ticket per dollar’ rate help answer these questions?”

## Lesson Synthesis

The important takeaways from this lesson are:

- Any ratio has two associated **unit rates**.
- Unit rates can often be calculated efficiently with a single operation (division or multiplication).
- Depending on the problem you want to solve, one unit rate might be more useful than the other.

Consider displaying this table from earlier in the lesson:

water (cups)	oats (cups)
15	6
1	
	1

Ask students and fill in the table as you go:

- What is a quick way to compute the number of cups of oats for 1 cup of water? ( $6 \div 15 = \frac{6}{15}$ )
- What is a quick way to compute the number of cups of water for 1 cup of oats? ( $15 \div 6 = \frac{15}{6}$ )
- For what types of problems is  $\frac{15}{6}$  easier to use? (Finding how many cups of water when we know the number of cups of oats.)
- For what types of problems is  $\frac{6}{15}$  easier to use? (Finding how many cups of oats when we know the number of cups of water.)

## 6.4 Buying Grapes by the Pound

Cool Down: 5 minutes

### Addressing

- 6.RP.A.3

#### Student Task Statement

Two pounds of grapes cost \$6.

1. Complete the table showing the price of different amounts of grapes at this rate.

grapes (pounds)	price (dollars)
2	6
	1
1	

2. Explain the meaning of each of the numbers you found.

## Student Response

1. Here is the completed table:

grapes (pounds)	price (dollars)
2	6
$\frac{1}{3}$	1
1	3

2. The price of  $\frac{1}{3}$  pound of grapes is \$1. 1 pound of grapes costs \$3.

## Student Lesson Summary

Suppose a farm lets us pick 2 pounds of blueberries for 5 dollars. We can say:

blueberries (pounds)	price (dollars)
2	5
1	$\frac{5}{2}$
$\frac{2}{5}$	1

- We get  $\frac{2}{5}$  pound of blueberries per dollar.
- The blueberries cost  $\frac{5}{2}$  dollars per pound.

The “cost per pound” and the “number of pounds per dollar” are the two *unit rates* for this situation.

A **unit rate** tells us how much of one quantity for 1 of the other quantity. Each of these numbers is useful in the right situation.

If we want to find out how much 8 pounds of blueberries will cost, it helps to know how much 1 pound of blueberries will cost.

blueberries (pounds)	price (dollars)
1	$\frac{5}{2}$
8	$8 \cdot \frac{5}{2}$

If we want to find out how many pounds we can buy for 10 dollars, it helps to know how many pounds we can buy for 1 dollar.

blueberries (pounds)	price (dollars)
$\frac{2}{5}$	1
$10 \cdot \frac{2}{5}$	10

Which unit rate is most useful depends on what question we want to answer, so be ready to find either one!

## Glossary

- unit rate

## Lesson 6 Practice Problems

### Problem 1

#### Statement

A pink paint mixture uses 4 cups of white paint for every 3 cups of red paint. The table shows different quantities of red and white paint for the same shade of pink. Complete the table.

white paint (cups)	red paint (cups)
4	3
	1
1	
	4
5	

#### Solution

Equivalent values are also acceptable.

white paint (cups)	red paint (cups)
4	3
$\frac{4}{3}$	1
1	$\frac{3}{4}$
$\frac{16}{3}$	4
5	$\frac{15}{4}$

## Problem 2

### Statement

A farm lets you pick 3 pints of raspberries for \$12.00.

- What is the cost per pint?
- How many pints do you get per dollar?
- At this rate, how many pints can you afford for \$20.00?
- At this rate, how much will 8 pints of raspberries cost?

### Solution

- Each pint costs  $\frac{12}{3}$  or \$4.
- You get  $\frac{3}{12}$  or  $\frac{1}{4}$  or 0.25 pints per dollar.
- You can afford 5 pints, because  $20 \div 4 = 5$  and  $(0.25) \cdot 20 = 5$ .
- 8 pints will cost \$32.00, because  $8 \cdot 4 = 32$ . Possible strategy:

pints of raspberries	cost in dollars
3	12
1	4
$\frac{1}{4}$	1
5	20
8	32



### Problem 3

#### Statement

Han and Tyler are following a polenta recipe that uses 5 cups of water for every 2 cups of cornmeal.

- Han says, "I am using 3 cups of water. I will need  $1\frac{1}{5}$  cups of cornmeal."
- Tyler says, "I am using 3 cups of cornmeal. I will need  $7\frac{1}{2}$  cups of water."

Do you agree with either of them? Explain your reasoning.

#### Solution

They are both correct. For every cup of water,  $\frac{2}{5}$  cup of cornmeal is used. For every cup of cornmeal,  $2\frac{1}{2}$  cups of water are used.

water (cups)	cornmeal (cups)
5	2
1	$\frac{2}{5}$
$2\frac{1}{2}$	1
3	$1\frac{1}{5}$
$7\frac{1}{2}$	3

### Problem 4

#### Statement

A large art project requires enough paint to cover 1,750 square feet. Each gallon of paint can cover 350 square feet. Each square foot requires  $\frac{1}{350}$  of a gallon of paint.

Andre thinks he should use the rate  $\frac{1}{350}$  gallons of paint per square foot to find how much paint they need. Do you agree with Andre? Explain or show your reasoning.

#### Solution

Answers vary. Sample responses:

- I agree with Andre. He needs enough paint for 1,750 square feet. Since each square foot requires  $\frac{1}{350}$  gallons of paint, Andre needs 5 gallons of paint because  $(1,750) \cdot \frac{1}{350} = 5$ .
- I disagree with Andre. It is easier to use the rate 350 square feet per gallon. This table shows that he needs 5 gallons of paint:

gallons of paint	area in square feet
1	350
5	1,750

## Problem 5

### Statement

Andre types 208 words in 4 minutes. Noah types 342 words in 6 minutes. Who types faster? Explain your reasoning.

### Solution

Noah types faster. He can type 5 more words per minute than Andre. Andre types at a rate of 52 words per minute, because  $208 \div 4 = 52$ . Noah types at a rate of 57 words per minute, because  $342 \div 6 = 57$ .

(From Unit 3, Lesson 5.)

## Problem 6

### Statement

A corn vendor at a farmer's market was selling a bag of 8 ears of corn for \$2.56. Another vendor was selling a bag of 12 for \$4.32. Which bag is the better deal? Explain or show your reasoning.

### Solution

The bag of 8 is better.  $2.56 \div 8 = 0.32$ , so each ear of corn is 32 cents. In the bag of 12, each ear of corn is 36 cents because  $4.32 \div 12 = 0.36$ .

(From Unit 3, Lesson 5.)

## Problem 7

### Statement

A soccer field is 100 meters long. What could be its length in yards?



- A. 33.3
- B. 91
- C. 100
- D. 109

**Solution**

D  
(From Unit 3, Lesson 3.)

# Lesson 7: Equivalent Ratios Have the Same Unit Rates

## Goals

- Apply reasoning about unit rates to complete a table of equivalent ratios, and explain (orally and in writing) the solution method.
- Explain (orally) that if two ratios are equivalent, they have the same rate per 1.
- Generalize that the unit rate is the factor that takes you from one column to the other column in a table of equivalent ratios.

## Learning Targets

- I can give an example of two equivalent ratios and show that they have the same unit rates.
- I can multiply or divide by the unit rate to calculate missing values in a table of equivalent ratios.

## Lesson Narrative

The purpose of this lesson is to make it explicit to students that equivalent ratios have the same unit rates. For instance, students can see that the ratios  $10 : 4$ ,  $15 : 6$ , and  $20 : 8$  all have unit rates of  $\frac{2}{5}$  and  $\frac{5}{2}$ . Interpreted in a context, this might mean, for example, that no matter how many ounces of raisins are purchased in bulk and how much is paid, the price per ounce will always match the \$0.40 per ounce rate marked on the price label.

$a$	$b$
10	4
15	6
20	8

This understanding gives new insights as students reason with tables. Up to this point, students have often been reasoning about the relationship from row to row, understanding that the rows contain equivalent ratios and the values in any row can be found by multiplying both quantities in another row by a scale factor. Here students see that they can also reason across columns, because the unit rate is the factor that relates the values in one column to those in the other (MP8). In grade 7, students will call the unit rate the *constant of proportionality* and write equations of the form  $y = kx$  to characterize these relationships.

Later in the lesson, students practice using unit rates and tables of equivalent ratios to find unknown quantities and compare rates in context.

## Alignments

### Addressing

- 6.RP.A.2: Understand the concept of a unit rate  $a/b$  associated with a ratio  $a : b$  with  $b \neq 0$ , and use rate language in the context of a ratio relationship. \$Expectations for unit rates in this grade are limited to non-complex fractions.
- 6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- 6.RP.A.3.b: Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

### Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Think Pair Share
- Which One Doesn't Belong?

### Student Learning Goals

Let's revisit equivalent ratios.

## 7.1 Which One Doesn't Belong: Comparing Speeds

### Warm Up: 10 minutes

This warm-up prompts students to compare four rates. It encourages students to explain their reasoning, hold mathematical conversations, and gives you the opportunity to hear how they use terminology and talk about rates in comparison to one another. To allow all students to access the activity, each rate has one obvious reason it does not belong. During the discussion, listen for the term "unit rate," speed, pace, and ways that students reason about whether two rates indicate the same speed.

### Addressing

- 6.RP.A.3.b

### Instructional Routines

- Which One Doesn't Belong?

## Launch

Arrange students in groups of 2–4. Display the rates for all to see. Ask students to indicate when they have noticed one that does not belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their small group. In their small groups, tell each student to share their reasoning why a particular question does not belong and together find at least one reason each question doesn't belong.

### Student Task Statement

Which one doesn't belong? Be prepared to explain your reasoning.

5 miles in 15 minutes

20 miles per hour

3 minutes per mile

32 kilometers per hour

### Student Response

- 5 miles in 15 minutes is the only rate not expressed as a unit rate.
- 3 minutes per mile is the only rate expressed as a pace instead of a speed.
- 20 miles per hour is the only one that sounds like we are used to talking about speeds or the only one that would be on a road sign in the United States.
- 32 kilometers per hour is the only one using metric units of length and is not *exactly* the same speed as the other 3 ( $32 \text{ km} = 19.8839 \text{ miles}$ ).

### Activity Synthesis

Ask each group to share one reason why a particular rate does not belong. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given are correct.

During the discussion, ask students to explain the meaning of any terminology they use, such as unit rate, speed, or pace. Also, press students on unsubstantiated claims.

## 7.2 Price of Burritos

### 10 minutes

In a previous lesson about treadmill workouts, students calculated unit rates and identified ratios that have matching unit rates as equivalent (i.e., if  $\frac{a}{b} = \frac{c}{d}$ , then  $a : b$  is equivalent to  $c : d$ ). Here they make sense of this relationship from the other direction; they see that when two ratios are equivalent, they have the same unit rate (i.e., if  $a : b$  is equivalent to  $c : d$ , then  $\frac{a}{b} = \frac{c}{d}$ ).

Students explore the above by noticing structures in a table of equivalent ratios—that in addition to the values in the rows being equivalent ratios, the values in the columns have a multiplicative relationship (MP7). They see, specifically, that dividing the values of the two quantities in the ratio

result in the same quotient—the associated unit rate—and that it can be used to reason about one quantity of the ratio when the other is known.

Students also begin to transition from numerical examples to encapsulating a relationship with variables as they generalize their observations above (MP8).

As students work and discuss, identify those who observed structures in the table and can describe them well. Also look for students who could explain how they know the per-item cost is the same given two ratios expressed in variables.

### **Addressing**

- 6.RP.A.2
- 6.RP.A.3

### **Instructional Routines**

- MLR1: Stronger and Clearer Each Time
- Think Pair Share

### **Launch**

Arrange students in groups of 2. Give students a few minutes of quiet think time to complete the first two questions, and then 1–2 minutes to discuss with a partner their observations about the values in the table. Ask them to complete the last two questions together afterwards.

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### **Support for Students with Disabilities**

*Representation: Internalize Comprehension.* Use color and annotations to illustrate student thinking. As students describe their calculations and the relationships they noticed in the tables, use color and annotation to scribe their thinking on a display of each problem so that it is visible for all students.

*Supports accessibility for: Visual-spatial processing; Conceptual processing*

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### Support for English Language Learners

*Writing, Speaking: MLR1 Stronger and Clearer Each Time.* Use this routine to help students refine their justifications for the final question, “Explain why, if you can buy  $b$  burritos for 4 dollars, or buy  $2 \cdot b$  burritos for  $2 \cdot c$  dollars, the cost per item is the same in either case.” Listeners should press for details and clarity as appropriate based on what each speaker produces. Provide students with prompts for feedback that will help individuals strengthen their ideas and clarify their language (e.g., “Why do you think that?”, “How could you use values to show your thinking?”, “Would your explanation work if you bought 4 burritos?”, etc.). Students can borrow ideas and language from each partner to strengthen their final product.

*Design Principle(s): Optimize output (for justification)*

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### Anticipated Misconceptions

Students may not realize that the third column asks for dollars per 1 burrito and instead write 14 dollars per 2 burritos or 28 dollars per 8 burritos. If this happens, remind students that “per burrito” means “per 1 burrito.”

### Student Task Statement

- Two burritos cost \$14. Complete the table to show the cost for 4, 5, and 10 burritos at that rate. Next, find the cost for a single burrito in each case.

number of burritos	cost in dollars	unit price (dollars per burrito)
2	14	
4		
5		
10		
$b$		

- What do you notice about the values in this table?
- Noah bought  $b$  burritos and paid  $c$  dollars. Lin bought twice as many burritos as Noah and paid twice the cost he did. How much did Lin pay per burrito?



	number of burritos	cost in dollars	unit price (dollars per burrito)
Noah	$b$	$c$	$\frac{c}{b}$
Lin	$2 \cdot b$	$2 \cdot c$	

4. Explain why, if you can buy  $b$  burritos for  $c$  dollars, or buy  $2 \cdot b$  burritos for  $2 \cdot c$  dollars, the cost per item is the same in either case.

### Student Response

1.

number of burritos	cost in dollars	unit price (dollars per burrito)
2	14	7
4	28	7
5	35	7
10	70	7
$b$	$b \cdot (7)$	7

2. Answers vary. Sample responses: The ratios are equivalent. No matter how many burritos you buy, it costs \$7 per burrito.

3.

	number of burritos	cost in dollars	unit price (dollars per burrito)
Noah	$b$	$c$	$\frac{c}{b}$
Lin	$2 \cdot b$	$2 \cdot c$	$\frac{c}{b}$

4. If you have  $b$  items for  $c$  dollars, then the unit rate is  $\frac{c}{b}$  dollars per burrito. If you have  $2b$  items for  $2c$  dollars, then the unit rate is  $\frac{2c}{2b}$  dollars per burrito. But  $\frac{c}{b}$  and  $\frac{2c}{2b}$  are equivalent fractions, so Noah and Lin each paid  $\frac{c}{b}$  per burrito.

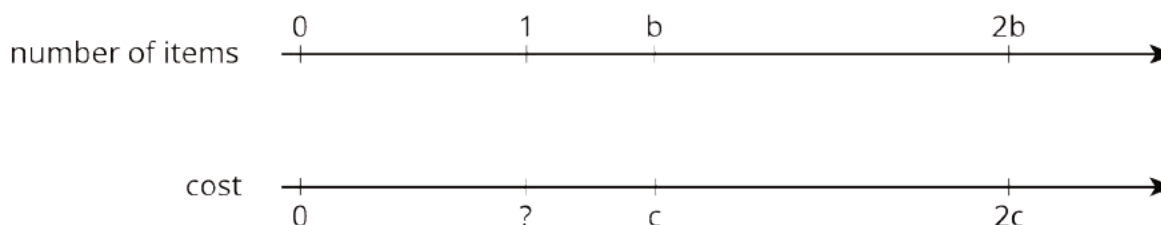
### Activity Synthesis

After students have conferred with a partner, debrief as a class. Focus the discussion on students' calculations and the relationships they noticed in the tables. Display a completed version of the first

table for all to see. Select previously identified students to share their observations. As they explain, illustrate their comments on the table. Students may bring up that:

- The ratios shown in the first two columns are equivalent across all rows.
- The ratios in all rows have the same dollar-per-item value (unit rate).
- The value in the last column can be found by dividing the dollar amount by the number of burritos. (If students use phrasing such as, “divide the second column by the first column,” encourage them to use more precise terms (MP6).) Highlight the first two observations above, or bring them up if students do not. The last observation above is welcome but do not need to be emphasized as it will be the focus of the next activity.

Invite selected students to share their reasoning on the last two questions. Attend to the last question in particular, as it may be challenging to digest given its abstract nature. Consider using a double number line to help students visualize how the unit rate is the same for  $b : c$  and  $2b : 2c$ , as shown.



Connect the location of 1 : ? with the unit rate, which students have by now recognized as having an unchanging value. Discuss how the unit rate is the same for any positive multiplier applied to  $b : c$ , since the multiplier would produce equivalent ratios.

## 7.3 Making Bracelets

### 10 minutes

At this point students understand that tables are a flexible tool for working with equivalent ratios. Up to this point, however, all actions performed on a table have started with both values of a known ratio  $a : b$  allowing students to move from row to row using multiplicative reasoning.

In this activity, however, students are asked to determine an unknown value of a ratio given only the other value of the ratio and a unit rate. Using their understanding of unit rate, equivalent ratios, and the relationship between the two (MP7), students learn how a unit rate is a factor that takes you from one column to another column in a table of equivalent ratios.

As students work, notice different approaches taken. For some students, the structure of the information in the table may not be apparent. Encourage them to refer to the tables in the preceding activity and think about the relationship between the ratios and unit rates there. Other

students may be inclined to create a different table—such as the ones below—as an intermediate step for completing the given table. If so, consider asking them to share first during whole-class discussion.

time in hours	number of bracelets	speed (bracelets per hour)
1	6	6
2	12	6

This intermediate strategy makes good use of the “1” rows in the table, but is less efficient than directly dividing or multiplying by the unit rate to move from one column value to another since students have to work out both unit rates instead of using only the given unit rate.

### Addressing

- 6.RP.A.2
- 6.RP.A.3

### Instructional Routines

- MLR7: Compare and Connect
- Think Pair Share

### Launch

Ask students to read the opening sentence and table. Explain that the first two columns show the ratio and the third column shows a unit rate associated with that ratio, similar to the structure of the tables in the previous activity.

Arrange students in groups of 2. Give students 3–4 minutes of quiet think time to complete the table and answer the questions, and then time to discuss their responses with a partner. Ask students to be mindful of how they go about completing the table and be prepared to explain their thinking.

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### Support for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Connect a new concept to one with which students have experienced success. For example, review the relationship between the number of burritos purchased, cost in dollars, and the unit price of a burrito from the previous task.

*Supports accessibility for: Social-emotional skills; Conceptual processing*

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### Student Task Statement

1. Complete the table. Then, explain the strategy you used to do so.

time in hours	number of bracelets	speed (bracelets per hour)
2		6
5		6
7		6
	66	6
	100	6



2. Here is a partially filled table from an earlier activity. Use the same strategy you used for the bracelet problem to complete this table.

number of burritos	cost in dollars	unit price (dollars per burrito)
	14	7
	28	7
5		7
10		7

3. Next, compare your results with those in the first table in the previous activity. Do they match? Explain why or why not.

### Student Response

1.

time in hours	number of bracelets	speed (bracelets per hour)
2	12	6
5	30	6
7	42	6
11	66	6
$16\frac{2}{3}$	100	6

Since the number of bracelets divided by the time in hours is 6 bracelets per hour, then the time in hours multiplied by 6 should give the number of bracelets. Using similar thinking, the number of bracelets divided by 6 gives the time in hours.

2.

number of burritos	cost in dollars	unit price (dollars per burrito)
2	14	7
4	28	7
5	35	7
10	70	7

3. Answers vary. Sample response: Yes, the strategy works and my table matches. To figure out cost in dollars from number of burritos and dollars per burrito, I can multiply 7 times the number of burritos. For example, 5 burritos cost \$35. To find values in the first column using the second two columns, I can divide cost in dollars by dollars per burrito. For example, 14 divided by 7 is 2, so I can buy 2 burritos for \$14.

### Activity Synthesis

After they have a chance to discuss with a partner, select a few students to share with the class their strategies for completing the table. Start with students using less efficient strategies, such as those that worked out the 1 rows. Progress toward using the given unit rate to navigate from column to column in efficient ways, such as multiplying the time in hours by 6 to find number of bracelets, and multiplying the number of bracelets by  $\frac{1}{6}$  to find time in hours.

### Support for English Language Learners

*Writing, Listening, Conversing: MLR7 Compare and Connect.* Assign students to prepare a visual display that shows how they completed the table for the first question, including a brief explanation of the strategy they used. Ask students to investigate each other's work by taking a tour of the visual displays and facilitate discussion about comparisons and connections of the different approaches or representations in their work. Prompt students with questions such as, "Did anyone solve the problem in a similar way, but would explain it differently?" "Why does this approach use \_\_\_\_, and this one does not? Is the outcome the same?." During the discussion, amplify language students use to communicate about unit rate, equivalent ratios, and the relationship between the two. This helps students to reflect on, and linguistically respond to, the comparisons and connections about the mathematical features that are key to this lesson.

*Design Principle(s): Cultivate conversation; Maximize meta-awareness*

## 7.4 How Much Applesauce?

**Optional: 10 minutes**

This optional activity is a chance to apply more sophisticated, newly-learned techniques to a familiar-looking problem about equivalent ratios. None of the given numbers are multiples of each other from row to row (for example, 7 isn't a multiple of 4), so this problem lends itself to reasoning about unit rates.

### Addressing

- 6.RP.A.3.b

### Instructional Routines

- MLR8: Discussion Supports

### Launch

If any student is familiar with making applesauce, ask them to explain how it is made. If not, explain: to make applesauce, you peel, core, and chop apples. Then, heat the apples gently in a saucepan for a while until they break down into a sauce. Finally, add flavors like lemon juice and cinnamon. If you know how many pounds of apples you start with, you can predict how many cups of applesauce after cooking.

### Student Task Statement

It takes 4 pounds of apples to make 6 cups of applesauce.

1. At this rate, how much applesauce can you make with:
  - a. 7 pounds of apples?
  - b. 10 pounds of apples?
2. How many pounds of apples would you need to make:
  - a. 9 cups of applesauce?
  - b. 20 cups of applesauce?

pounds of apples	cups of applesauce
4	6
7	
10	
	9
	20

### Student Response

To find the cups of applesauce, multiply the pounds of apples by 1.5.

To find the pounds of apples, divide the cups of applesauce by 1.5, or multiply by  $\frac{2}{3}$ .

pounds of apples	cups of applesauce
4	6
7	10.5
10	15
6	9
$13\frac{1}{3}$	20

### Are You Ready for More?

1. Jada eats 2 scoops of ice cream in 5 minutes. Noah eats 3 scoops of ice cream in 5 minutes. How long does it take them to eat 1 scoop of ice cream working together (if they continue eating ice cream at the same rate they do individually)?
2. The garden hose at Andre's house can fill a 5-gallon bucket in 2 minutes. The hose at his next-door neighbor's house can fill a 10-gallon bucket in 8 minutes. If they use both their garden hoses at the same time, and the hoses continue working at the same rate they did when filling a bucket, how long will it take to fill a 750-gallon pool?

### Student Response

1. 1 minute. Jada eats  $\frac{2}{5}$  scoop in 1 minute and Noah eats  $\frac{3}{5}$  scoop in 1 minute. So together, they eat 1 scoop in 1 minute.
2. 200 minutes or 3 hours and 20 minutes. The rate for Andre's hose is 2.5 gallons per minute, and the rate for his neighbor's hose is 1.25 gallons per minute. If they use the hoses at the same time, the pool will fill at a rate of 3.75 gallons per minute.  $750 \div 3.75 = 200$ , so it will take 200 minutes for the hoses to emit 750 gallons of water.

### Activity Synthesis

Highlight approaches where students compute how many pounds of apples per cup of applesauce and how many cups of applesauce per pound of apples and use multiplicative reasoning to move from column to column.

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### Support for English Language Learners

*Speaking, Listening, Conversing: MLR8 Discussion Supports.* As students are discussing how they make applesauce using the given information in the problem, press for details in students' explanations when finding how many pounds of apples are needed to make a recipe or how many cups of applesauce are made from some pounds of apples. Encourage think aloud by talking through their approaches when calculating the missing values in the table of equivalent ratios. This will help students make sense of the the problem and the approaches used to complete the table.

*Design Principle(s): Support sense-making; Cultivate conversation*

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## Lesson Synthesis

In this lesson, students learned about two new ideas around ratios and rates:

- Equivalent ratios have the same unit rate.
- Unit rates are the factors that takes you from one column to the other column in a table of equivalent ratios.

Summarize for students the two new ideas and, if possible, highlight how students used them in the final activity of the lesson.

## 7.5 Cheetah Speed

Cool Down: 5 minutes

### Addressing

- 6.RP.A.2
- 6.RP.A.3

### Student Task Statement

A cheetah can run at its top speed for about 25 seconds. Complete the table to represent a cheetah running at a constant speed. Explain or show your reasoning.

time (seconds)	distance (meters)	speed (meters per second)
4	120	
25		
	270	



## Student Response

time (seconds)	distance (meters)	speed (meters per second)
4	120	30
25	750	30
9	270	30

Sample reasoning: Since the cheetah runs 120 meters in 4 seconds, this is 30 meters per second because  $120 \div 4 = 30$ . Since it says the cheetah runs at a constant speed, the speed in each row is 30 meters per second. To find the distance run in 25 seconds, multiply 25 by 30. To find the time it takes to run 270 meters, divide 270 by 30.

## Student Lesson Summary

The table shows different amounts of apples selling at the same rate, which means all of the ratios in the table are equivalent. In each case, we can find the *unit price* in dollars per pound by dividing the price by the number of pounds.

apples (pounds)	price (dollars)	unit price (dollars per pound)
4	10	$10 \div 4 = 2.50$
8	20	$20 \div 8 = 2.50$
20	50	$50 \div 20 = 2.50$

The unit price is always the same. Whether we buy 4 pounds of apples for 10 dollars or 8 pounds of apples for 20 dollars, the apples cost 2.50 dollars per pound.

We can also find the number of pounds of apples we can buy per dollar by dividing the number of pounds by the price.

apples (pounds)	price (dollars)	pounds per dollar
4	10	$4 \div 10 = 0.4$
8	20	$8 \div 20 = 0.4$
20	50	$20 \div 50 = 0.4$

The number of pounds we can buy for a dollar is the same as well! Whether we buy 4 pounds of apples for 10 dollars or 8 pounds of apples for 20 dollars, we are getting 0.4 pounds per dollar.

This is true in all contexts: when two ratios are equivalent, their unit rates will be equal.

quantity $x$	quantity $y$	unit rate 1	unit rate 2
$a$	$b$	$\frac{a}{b}$	$\frac{b}{a}$
$s \cdot a$	$s \cdot b$	$\frac{s \cdot a}{s \cdot b} = \frac{a}{b}$	$\frac{s \cdot b}{s \cdot a} = \frac{b}{a}$

## Lesson 7 Practice Problems

### Problem 1

#### Statement

A car travels 55 miles per hour for 2 hours. Complete the table.

time (hours)	distance (miles)	miles per hour
1	55	55
$\frac{1}{2}$		
$1\frac{1}{2}$		
	110	

#### Solution

time (hours)	distance (miles)	miles per hour
1	55	55
$\frac{1}{2}$	27.5	55
$1\frac{1}{2}$	82.5	55
2	110	55

### Problem 2

#### Statement

The table shows the amounts of onions and tomatoes in different-sized batches of a salsa recipe.

Elena notices that if she takes the number in the tomatoes column and divides it by the corresponding number in the onions column, she always gets the same result.

What is the meaning of the number that Elena has calculated?

onions (ounces)	tomatoes (ounces)
2	16
4	32
6	48

## Solution

The recipe calls for 8 ounces of tomatoes per ounce of onions.

## Problem 3

### Statement

A restaurant is offering 2 specials: 10 burritos for \$12, or 6 burritos for \$7.50. Noah needs 60 burritos for his party. Should he buy 6 orders of the 10-burrito special or 10 orders of the 6-burrito special? Explain your reasoning.

## Solution

Answers vary. Possible reasoning: Noah should get 6 orders of the 10-burrito special. The 10-burrito special sells burritos at a rate of \$1.20 per burrito, because  $12 \div 10 = 1.20$ . The 6-burrito special sells at a rate of \$1.25 per burrito, because  $7.5 \div 6 = 1.25$ . The 10-burrito special is a better deal.

## Problem 4

### Statement

Complete the table so that the cost per banana remains the same.

number of bananas	cost in dollars	unit price (dollars per banana)
4		0.50
6		0.50
7		0.50
10		0.50
	10.00	0.50
	16.50	0.50

## Solution

number of bananas	cost in dollars	dollars per banana
4	2.00	0.50
6	3.00	0.50
7	3.50	0.50
10	5.00	0.50
20	10.00	0.50
33	16.50	0.50

## Problem 5

### Statement

Two planes travel at a constant speed. Plane A travels 2,800 miles in 5 hours. Plane B travels 3,885 miles in 7 hours. Which plane is faster? Explain your reasoning.

## Solution

Plane A is faster. Plane A travels  $2800 \div 5 = 560$  or 560 miles per hour. Plane B travels  $3,885 \div 7 = 555$ , or 555 miles per hour. Plane A travels a farther distance in one hour.

(From Unit 3, Lesson 5.)

## Problem 6

### Statement

A car has 15 gallons of gas in its tank. The car travels 35 miles per gallon of gas. It uses  $\frac{1}{35}$  of a gallon of gas to go 1 mile.

- How far can the car travel with 15 gallons? Show your reasoning.
- How much gas does the car use to go 100 miles? Show your reasoning.

## Solution

- a. 525 miles. Possible reasoning:

gallons of gas	miles car can travel
1	35
5	175
15	525

b.  $\frac{100}{35}$  (or  $\frac{20}{7}$  or  $2\frac{6}{7}$ ) gallons. Possible reasoning:

gallons of gas	miles car can travel
$\frac{1}{35}$	1
$\frac{10}{35}$	10
$\frac{100}{35}$	100

(From Unit 3, Lesson 6.)

## Problem 7

### Statement

A box of cereal weighs 600 grams. How much is this weight in pounds? Explain or show your reasoning. (Note: 1 kilogram = 2.2 pounds)

### Solution

1.32 pounds. Explanations vary. Possible explanation:

grams	pounds
1,000	2.2
100	0.22
500	1.1
600	1.32

(Note that for the first line of the table, 1 kilogram is written as 1,000 grams.)

(From Unit 3, Lesson 4.)

# Lesson 8: More about Constant Speed

## Goals

- Calculate unit rates that represent speed or pace, use them to determine unknown distances or elapsed times, and explain (orally) the solution method.
- Interpret a verbal (written) description of a situation involving two objects moving at constant speeds, and create a diagram or table to represent the situation.

## Learning Targets

- I can solve more complicated problems about constant speed situations.

## Lesson Narrative

This lesson allows students to practice working with equivalent ratios, tables that represent them, and associated unit rates in the familiar context of speed, time, and distance. Students use unit rates (speed or pace) and ratios (of time and distance) to find unknown quantities (e.g., given distances and times, find a constant speed or pace; and given a speed or pace, solve problems about distance and time).

## Alignments

### Addressing

- 6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- 6.RP.A.3.b: Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

## Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Group Presentations
- MLR1: Stronger and Clearer Each Time
- MLR6: Three Reads
- MLR8: Discussion Supports
- Notice and Wonder
- Think Pair Share

## Required Materials

### Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart

paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

## Required Preparation

Consider checking in advance whether there is a rail trail in your community, about which you could tell students during the Picnics on A Rail Trail activity.

### Student Learning Goals

Let's investigate constant speed some more.

## 8.1 Back on the Treadmill Again

### Warm Up: 10 minutes

Students have had experience determining speed given a ratio of time and distance. This task prompts students to use more than one strategy to solve speed-related problems (minutes passed and miles traveled) and practice reasoning in multiple ways, enabling them to see the connections across strategies.

There are several ways students can calculate how many miles Andre's dad could run in 30 minutes if traveling at a speed of 12 miles in 75 minutes. A few strategies:

- Using the speed. The ratio  $12 : 75$  has an associated unit rate of  $\frac{12}{75}$  or 0.16 miles per minute. To find the distance traveled in 30 minutes, multiply 0.16 miles per minute by 30.  $(0.16) \cdot 30 = 4.8$ , so he can run 4.8 miles in 30 minutes. Note that the rate per 1 associated with this unit rate is called **speed**.
- Using the pace.  $\frac{75}{12}$  or 6.25 minutes per mile is also a unit rate for the ratio. To find the distance traveled in 30 minutes, divide 30 by 6.25.  $30 \div 6.25 = 4.8$ . Note that the rate per 1 associated with this unit rate is called **pace**.
- Using a scale factor: Noticing that in the "time" column of a table, 75 multiplied by  $\frac{30}{75}$  is 30, and  $\frac{30}{75} = 0.4$ . The unknown number of miles is 4.8, because  $12 \cdot (0.4) = 4.8$ .

distance (miles)	time (minutes)
12	75
?	30

- Scaling up: Noticing that, going up in the "time" column, 75 is  $30 \cdot (2.5)$ . The unknown number of miles is then  $12 \div 2.5 = 4.8$ . As students work, notice different strategies being used so that they can be represented during discussion later.

## Addressing

- 6.RP.A.3

## Instructional Routines

- MLR1: Stronger and Clearer Each Time
- Think Pair Share

## Launch

Arrange students in groups of 2 and give them 3 minutes of quiet think time, followed by sharing with a partner and whole-class discussion. Ask students to be mindful of how they are thinking about the questions and be prepared to share their reasoning.

### Student Task Statement

While training for a race, Andre's dad ran 12 miles in 75 minutes on a treadmill. If he runs at that rate:

1. How long would it take him to run 8 miles?
2. How far could he run in 30 minutes?

### Student Response

1. It will take 50 minutes to run 8 miles. Possible strategies:

distance (miles)	time (minutes)
12	75
1	6.25
8	50

Annotations:  $\cdot \frac{1}{12}$  (from 12 to 1),  $\cdot 8$  (from 1 to 8) on the left;  $\cdot \frac{1}{12}$  (from 75 to 6.25),  $\cdot 8$  (from 6.25 to 50) on the right.

or

distance (miles)	time (minutes)
12	75
	$\cdot 6.25$
8	50
	$\cdot 6.25$

Annotations: Curved arrows from 12 to 8 and 75 to 50, with  $\cdot 6.25$  written below each arrow.

2. He can run 4.8 miles in 30 minutes. Possible strategies:



distance (miles)	time (minutes)
12	75
0.16	1
4.8	30

$\cdot \frac{1}{75}$  (from 12 to 0.16)  
 $\cdot 30$  (from 0.16 to 4.8)  
 $\cdot \frac{1}{75}$  (from 75 to 1)  
 $\cdot 30$  (from 1 to 30)

or

distance (miles)	time (minutes)
12	75
$\cdot 0.16$	
4.8	30

$\cdot 0.16$  (from 12 to 4.8)  
 $\cdot 0.16$  (from 75 to 30)

### Activity Synthesis

Invite students to share with their partner: their solution ideas and their explanations of how unit rates and scaling can be helpful. Then, give each student a new partner to repeat the process. Ask students to practice using mathematical language to be as clear as possible when sharing with the class, when and if they are called upon [see *MLR 1 (Stronger and Clearer Each Time)*].

Select students with different strategies to share with the class. Record their methods and display them for all to see. If any relevant strategies are missing, demonstrate them and add them to the display. Help students notice how unit rates and scaling can be helpful in solving similar problems.

Tell students when we find how the number of miles per minute or meters per second an object is moving, we are finding the **speed** of the object. When we find the number of minutes per mile or seconds per meter, we are finding the **pace** of the object.

## 8.2 Picnics on the Rail Trail

### 30 minutes

This task asks students to answer questions in the context of constant speed. If you wish to have students engage in more aspects of mathematical modeling (MP4), have students keep their books or devices closed and only display the stem that establishes the scenario. Ask students what they notice and wonder, and select questions to answer that are established through class discussion.

In this activity, students reason about the distances between the two friends, elapsed time, and speed. Students reason both quantitatively and abstractly (MP2); students can estimate some of the

solutions or check that they make sense in the given context (e.g., an earlier meeting time of the two friends would mean that one or both of them are traveling faster). As students work in groups, monitor the strategies they use to solve the last three problems, such as detailed diagrams of the path with marked-off distances, different ways of using tables, and so on.

### **Addressing**

- 6.RP.A.3

### **Instructional Routines**

- Group Presentations
- MLR6: Three Reads
- Notice and Wonder

### **Launch**

Share some information about the system of Rail Trails in the United States, as some students may be unfamiliar with non-motorized trails. Explain to students that, since they are built on old railway lines, these trails have very little gain or loss in elevation—making it reasonable to maintain a constant speed while walking, running, or cycling. Tell students about a Rail Trail near the school, if there is one.

If you (optionally) decide to take a less structured approach and compel students to engage in more aspects of mathematical modeling, have students keep their books or devices closed and only display the stem that establishes the scenario. Ask students, “What do you notice? What do you wonder?” Record the things they notice and wonder for all to see. Select questions that students posed for the class to explore that are similar to the questions in the task.

Arrange students in groups of 3–4. Give students a few minutes of quiet think time to complete the first two questions, and then time to discuss their responses with a partner. Encourage students to look at their partner’s approach and choices (e.g., how they work out the values in the table or sets up the table, how calculations are done, etc.). Ask students to pause for a brief whole-class discussion afterwards.

To make sure students are on the right track, display at least one student solution for each of the first two problems for all to see before they move on to complete the activity. Give students 8–10 minutes to work and discuss in groups, and tell them that each group will be assigned a problem to explain to the class. Provide each group with tools for creating a visual display.

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### Support for Students with Disabilities

*Representation: Internalize Comprehension.* Represent the same information through different modalities by using drawings or diagrams. If students are unsure where to begin, suggest that they draw a picture or diagram that represents the situation.

*Supports accessibility for: Conceptual processing; Visual-spatial processing*

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### Support for English Language Learners

*Reading: MLR6 Three Reads.* Use this routine to support reading comprehension of this word problem, without solving it for students. Use the first read to orient students to the situation by asking students to describe the situation without using numbers (e.g., Two friends live alongside different parts of a trail, one day they walk towards each other for a picnic). Use the second read to identify the important quantities by asking students what can be counted or measured (e.g., The distance between them is 24 miles to start, Kiran walks at a speed of 3 miles per hour, etc.). For the third read, reveal the first three questions and provide students with independent work time. Once students are ready, brainstorm possible solution strategies to respond to Kiran's suggestion: "If I walk 3 miles per hour toward you, and you walk 3.4 miles per hour toward me, it's the same as if you stay put and I jog 6.4 miles per hour." This will help students connect the language in the word problem about important quantities and rates with the reasoning needed to solve the problem.

*Design Principle(s): Support sense-making; Maximize meta-awareness*

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### Anticipated Misconceptions

Encourage students who are struggling to make sense of the mathematics to make a picture of the path and mark off distances after certain time periods.

Look for students misinterpreting expressions of time. For example, 2.5 hours after 8 a.m. is 10:30 a.m., not 10:50 a.m.

Students who are unsure about how to calculate distance apart in the table may benefit from creating a table with 4 columns: time in hours, how far Kiran has traveled, how far Clare has traveled, and the distance between them.

## Student Task Statement



Kiran and Clare live 24 miles away from each other along a rail trail. One Saturday, the two friends started walking toward each other along the trail at 8:00 a.m. with a plan to have a picnic when they meet.

Kiran walks at a **speed** of 3 miles per hour while Clare walks 3.4 miles per hour.

1. After one hour, how far apart will they be?
2. Make a table showing how far apart the two friends are after 0 hours, 1 hour, 2 hours, and 3 hours.
3. At what time will the two friends meet and have their picnic?
4. Kiran says "If I walk 3 miles per hour toward you, and you walk 3.4 miles per hour toward me, it's the same as if you stay put and I jog 6.4 miles per hour." What do you think Kiran means by this? Is he correct?
5. Several months later, they both set out at 8:00 a.m. again, this time with Kiran jogging and Clare still walking at 3.4 miles per hour. This time, they meet at 10:30 a.m. How fast was Kiran jogging?

## Student Response

1. After one hour Kiran has walked 3 miles and Clare has walked 3.4 miles, reducing their total distance apart to  $24 - 3 - 3.4 = 17.6$  miles.
2. Here is the table:

elapsed time (hours)	distance apart (miles)
0	24
1	17.6
2	11.2
3	4.8

3. Since the distance between them is decreasing by 6.4 miles each hour, they will meet after 3.75 hours, which will be at 11:45am.
4. What Kiran means is that each hour the total distance between them is decreasing by 6.4 miles, so the amount of time it will take them to meet is the same as if one person stays put

and the other travels at 6.4 miles per hour. However, the location of their meeting would change to Clare's house instead of somewhere in between the houses on the Rail Trail.

5. In 2.5 hours Clare traveled  $(3.4) \cdot (2.5) = 8.5$  miles.  $24 - 8.5 = 15.5$ . This means Kiran jogged the remaining 15.5 miles in 2.5 hours. 15.5 miles in 2.5 hours means Kiran jogged 6.2 miles per hour. Alternative solution: If they meet on the trail after 2.5 hours (8 am to 10:30 am), then their combined speed is 9.6 miles per hour, since  $\frac{24}{2.5} = 9.6$ . From the problem, Clare is walking 3.4 miles per hour, so Kiran must be jogging 6.2 miles per hour, since  $9.6 - 3.4 = 6.2$ .

### Are You Ready for More?

1. On his trip to meet Clare, Kiran brought his dog with him. At the same time Kiran and Clare started walking, the dog started running 6 miles per hour. When it got to Clare it turned around and ran back to Kiran. When it got to Kiran, it turned around and ran back to Clare, and continued running in this fashion until Kiran and Clare met. How far did the dog run?
2. The next Saturday, the two friends leave at the same time again, and Kiran jogs twice as fast as Clare walks. Where on the rail trail do Kiran and Clare meet?

### Student Response

1. We know that they meet after 3.75 hours. So the dog was running 6 miles per hour for 3.75 hours. Therefore, the dog ran 22.5 miles.
2. They meet 8 miles from Clare's starting point. It may appear this problem doesn't have enough information, but since Kiran will always travel twice as far as Clare, he must travel 16 miles and Clare must travel 8 miles.

### Activity Synthesis

Invite selected groups to present the solutions to their assigned problems. If possible, start from the most common strategies and move to the least common. Highlight effective uses of unit rates, equivalent ratios, scaling, and table representations in students' work.

## 8.3 Swimming and Biking

### Optional: 10 minutes

This optional activity is more opportunity to practice working with rates, in a new situation that involves constant speed of multiple people moving at the same time. This problem has less scaffolding than the previous activity. There are many different unit rates students may choose to calculate while solving this problem. Specifying the units and explaining the context for a rate gives students an opportunity to attend to precision (MP6).

Monitor for students that use different strategies to solve the problem:

- Creating a drawing or diagram that represents the situation

- Finding how far each person travels in the same amount of time (such as: In 24 minutes, Jada bikes 4 miles while her cousin swims 1 mile.)
- Finding how long it takes each person to travel the same distance (such as: To go 2 miles, it takes Jada 12 minutes and her cousin 48 minutes.)
- Calculating the pace of each individual
- Calculating the speed of each individual
- Calculating the combined speed of how fast they are moving away from each other

### Addressing

- 6.RP.A.3

### Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports

### Launch

Give students a few minutes of quiet think time to complete the two questions, then pause for a brief whole-class discussion afterwards.

### Support for English Language Learners

*Writing, Speaking: MLR8 Discussion Supports.* Provide sentence frames to students to support their explanations of their process for determining who was moving faster and by how much. For example, “First, I \_\_\_\_\_. Then, I \_\_\_\_\_.” or “I know that \_\_\_\_\_ is moving faster because \_\_\_\_\_.”  
*Design Principle(s): Support sense-making, Optimize output (for explanation)*

### Anticipated Misconceptions

Some students may confuse the meaning of the speed and pace, thinking that 24 minutes per mile is faster than 6 minutes per mile. Make sure they have labeled the units on their rates, and prompt them to consider what the words “per mile” tell us about the situation.

### Student Task Statement

Jada bikes 2 miles in 12 minutes. Jada’s cousin swims 1 mile in 24 minutes.

1. Who is moving faster? How much faster?
2. One day Jada and her cousin line up on the end of a swimming pier on the edge of a lake. At the same time, they start swimming and biking in opposite directions.
  - a. How far apart will they be after 15 minutes?

b. How long will it take them to be 5 miles apart?

### Student Response

1. Jada bikes faster than her cousin swims, by 7.5 miles per hour (or equivalent).
2. a.  $3\frac{1}{8}$  miles. Possible strategy: Jada bikes 1 mile in 6 minutes. There are  $2\frac{1}{2}$  groups of 6 minutes in 15 minutes. This means Jada bikes 2.5 miles in 15 minutes. Jada's cousin swims  $\frac{5}{8}$  mile in 15 minutes, because  $15 \div 24 = \frac{15}{24}$ , which is equivalent to  $\frac{5}{8}$ . The total distance between them is  $2\frac{4}{8} + \frac{5}{8}$ , or  $3\frac{1}{8}$  miles.  
  
b. 24 minutes. Possible strategy: Jada and her cousin are moving away from each other at a rate of 12.5 miles per hour.  $5 \div 12.5 = 0.4$ , so it will take them 0.4 hours to be 5 miles apart. This is equivalent to 24 minutes, because  $0.4 \cdot 60 = 24$ .

### Activity Synthesis

The key takeaway of this discussion is the idea that students can find and use different unit rates to solve the problem, so it is important to specify what a particular unit rate measures. Invite students who used different strategies to share how they solved the problem. Sequence the strategies from most common to least common. If any student used a drawing or diagram to represent the situation, consider having them share first. Some possible strategies to highlight include:

- Creating a double, triple, or quadruple number line diagram showing the elapsed time and distances traveled
- Finding how far they will travel in the same amount of time
- Calculating their individual paces in minutes per mile (6 minutes per mile for Jada, 24 minutes per mile for her cousin)
- Calculating their individual speeds in miles per minute ( $\frac{1}{6}$  mile per minute for Jada,  $\frac{1}{24}$  mile per minute for her cousin)
- Calculating their individual speeds in miles per hour (10 miles per hour for Jada, 2.5 miles per hour for her cousin)
- Calculating the combined speed of how fast they are moving away from each other ( $\frac{5}{24}$  mile per minute or 12.5 miles per hour)

Some unit rates can be more helpful than others, depending on the question we are trying to answer. Consider asking discussion questions like these:

- "Which unit rate was most helpful for answering how far apart they will be after 15 minutes?" (their speed, either in miles per minute or miles per hour)
- "Which unit rate was most helpful for answering how long it will take them to be 5 miles apart?" (their pace)

- “How did the fact that they were traveling away from each other affect the problem?” (The total distance between them at any point was the sum of the distance each person had traveled. The rate at which they were moving away from each other was the sum of their individual rates of travel.)

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### Support for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Break the class into small discussion groups and then invite a representative from each group to report back to the whole class.

*Supports accessibility for: Language; Social-emotional skills; Attention*

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## Lesson Synthesis

In this lesson we dealt with people traveling certain distances in a certain amount of time (at a constant speed). Let’s think about how Jada was traveling 2 miles in 12 minutes.

- “What are some ways to communicate her speed?” (A common way is to say she travels  $\frac{1}{6}$  of a mile per minute.)
- “How is it calculated?” (Divide 2 by 12.)
- “How would we calculate the other unit rate in this situation?” ( $12 \div 2$ )
- “What does it mean?” (It takes her 6 minutes to travel 1 mile. This is her pace.)
- “What are your favorite tools for making sense of and solving constant speed problems?” (Possible responses: double number lines, tables of equivalent ratios, dividing and multiplying.)

## 8.4 Penguin Speed

Cool Down: 5 minutes

### Addressing

- 6.RP.A.3.b

#### Student Task Statement

A penguin walks 10 feet in 6 seconds. At this speed:

1. How far does the penguin walk in 45 seconds?
2. How long does it take the penguin to walk 45 feet?

Explain or show your reasoning.



## Student Response

- 75 feet. Possible strategy: The penguin's speed is  $10 \div 6$ , or  $\frac{5}{3}$  feet per second. In 45 seconds, the penguin walks  $45 \cdot \frac{5}{3}$ , or 75 feet.
- 27 seconds. Possible strategy: The penguin's pace is  $6 \div 10$ , or 0.6 seconds per foot. To walk 45 feet, it takes the penguin  $45 \cdot 0.6$ , or 27 seconds.

## Student Lesson Summary

When two objects are each moving at a constant speed and their distance-to-time ratios are equivalent, we say that they are moving at the *same speed*. If their time-distance ratios are not equivalent, they are not moving at the same speed.

We describe **speed** in units of distance per unit of time, like *miles per hour* or *meters per second*.

- A snail that crawls 5 centimeters in 2 minutes is traveling at a rate of 2.5 centimeters per minute.
- A toddler that walks 9 feet in 6 seconds is traveling at a rate of 1.5 feet per second.
- A cyclist who bikes 20 kilometers in 2 hours is traveling at a rate of 10 kilometers per hour.

We can also use **pace** to describe distance and time. We measure pace in units such as *hours per mile* or *seconds per meter*.

- A snail that crawls 5 centimeters in 2 minutes has a pace of 0.4 minutes per centimeter.
- A toddler walking 9 feet in 6 seconds has a pace of  $\frac{2}{3}$  seconds per foot.
- A cyclist who bikes 20 kilometers in 2 hours has a pace of 0.1 hours per kilometer.

Speed and pace are reciprocals. Both can be used to compare whether one object is moving faster or slower than another object.

- An object with the higher speed is *faster* than one with a lower speed because the former travels a greater distance in the same amount of time.
- An object with the greater pace is *slower* than one with a smaller pace because the former takes more time to travel the same distance.

Because speed is a *rate per 1 unit of time* for ratios that relate distance and time, we can multiply the amount of time traveled by the speed to find the distance traveled.

time (minutes)	distance (centimeters)
2	5
1	2.5
4	$4 \cdot (2.5)$

## Glossary

- pace
- speed

## Lesson 8 Practice Problems

### Problem 1

#### Statement

A kangaroo hops 2 kilometers in 3 minutes. At this rate:

- a. How long does it take the kangaroo to travel 5 kilometers?      a. How far does the kangaroo travel in 2 minutes?

#### Solution

- a. 7.5 minutes (or equivalent)  
b.  $\frac{4}{3}$  kilometers (or equivalent)

### Problem 2

#### Statement

Mai runs around a 400-meter track at a constant speed of 250 meters per minute. How many minutes does it take Mai to complete 4 laps of the track? Explain or show your reasoning.

#### Solution

$\frac{32}{5}$  minutes (or equivalent). Possible responses:

○

distance (meters)	time (minutes)
250	1
500	2
400	1.6
1,600	6.4

- If each lap is 400 meters, then Mai runs 1,600 meters in 4 laps. Since every 250 meters takes her 1 minute to run, it would take her  $1,600 \div 250$  or 6.4 minutes to run 1,600 meters.

### Problem 3

#### Statement

At 10:00 a.m., Han and Tyler both started running toward each other from opposite ends of a 10-mile path along a river. Han runs at a pace of 12 minutes per mile. Tyler runs at a pace of 15 minutes per mile.

- How far does Han run after a half hour? After an hour?
- Do Han and Tyler meet on the path within 1 hour? Explain or show your reasoning.

#### Solution

- Han runs  $2\frac{1}{2}$  miles in a half hour and 5 miles in an hour. This table can be used to determine the distances.

time (minutes)	distance (miles)
12	1
1	$\frac{1}{12}$
30	$2\frac{1}{2}$
60	5

- No. Tyler travels 1 mile every 15 minutes, so he travels 4 miles in 60 minutes. Because Han travels 5 miles and Tyler travels 4 miles, and they are 10 miles apart, they are one mile apart after 1 hour.

### Problem 4

#### Statement

Two skateboarders start a race at the same time. Skateboarder A travels at a steady rate of 15 feet per second. Skateboarder B travels at a steady rate of 22 feet per second. After 4 minutes, how much farther will Skateboarder B have traveled? Explain your reasoning.

#### Solution

Skateboarder B will have traveled 1,680 feet farther. Possible reasoning: There are 240 seconds in 4 minutes, because  $4 \cdot 60 = 240$ . Skateboarder A travels 240 times 15, or 3,600 feet in 4 minutes. Skateboarder B travels 240 times 22, or 5,280 feet in 4 minutes, because  $5280 - 3600 = 1680$ .

(From Unit 2, Lesson 16.)

## Problem 5

### Statement

There are 4 tablespoons in  $\frac{1}{4}$  cup. There are 2 cups in 1 pint. How many tablespoons are there in 1 pint? If you get stuck, consider drawing a double number line or making a table.

### Solution

32 tablespoons

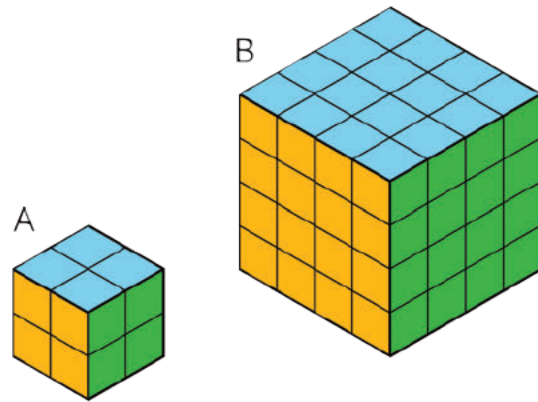
(From Unit 3, Lesson 4.)

## Problem 6

### Statement

Two larger cubes are made out of unit cubes. Cube A is 2 by 2 by 2. Cube B is 4 by 4 by 4. The side length of Cube B is twice that of Cube A.

- Is the surface area of Cube B also twice that of Cube A? Explain or show your reasoning.
- Is the volume of Cube B also twice that of Cube A? Explain or show your reasoning.



### Solution

- No. Sample reasoning: The surface area of Cube A is  $6 \cdot (2 \cdot 2)$  or 24 square units. The surface area of Cube B is  $6 \cdot (4 \cdot 4)$  or 96 square units. The surface area of B is 4 times that of A.
- No. Sample reasoning: The volume of Cube B is 64 cubic units because  $4^3 = 64$ . The volume of Cube A is 8 cubic units because  $2^3 = 8$ . 64 is not twice as much as 8.

(From Unit 1, Lesson 12.)

# Lesson 9: Solving Rate Problems

## Goals

- Apply reasoning about ratios and rates to convert and compare (in writing) distances expressed in different units.
- Apply reasoning about ratios and rates to justify (orally) whether a given price is a good deal.
- Practice grade 5 arithmetic with fractions and decimals.

## Learning Targets

- I can choose how to use unit rates to solve problems.

## Lesson Narrative

In previous lessons, students have used tables of equivalent ratios to reason about unit rates. In this lesson, students gain fluency working with unit rates without scaffolding (MP1). They choose what unit rate they want to use to solve a problem, divide to find the desired unit rate, and multiply or divide by the unit rate to answer questions. They may choose to create diagrams to represent the situations, but the problems do not prompt students to do so. The activity about which animal ran the farthest requires students to use multiple unit rates in a sequence to be able to convert all the measurements to the same unit.

## Alignments

### Addressing

- 6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- 6.RP.A.3.b: Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
- 6.RP.A.3.d: Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

### Building Towards

- 6.RP.A.3.c: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

### Instructional Routines

- MLR2: Collect and Display
- MLR7: Compare and Connect
- Poll the Class

- Take Turns

### Required Materials

Four-function calculators

blackline master

Pre-printed slips, cut from copies of the

### Required Preparation

Print and cut the blackline master so that each group of two students gets five cards A–E (or six cards A–F if you expect students to tackle the extension problem).

Optionally, purchase a four-pack of drinks for demonstration purposes in the Deal or No Deal activity.

Providing access to calculators is optional. All of the calculations in this lesson can be done using grade 5 techniques. If you would like students to practice arithmetic, don't offer calculators. If you think the calculations will present too much of a barrier to grade-level work, make them available.

### Student Learning Goals

Let's use unit rates like a pro.

## 9.1 Grid of 100

### Warm Up: 5 minutes

In this warm-up, students are asked to name the shaded portion of a 10-by-10 grid, which is equal to 1. To discourage students from counting every square, flash the image for a few seconds and then hide it. Flash it once more for students to check their thinking. Ask, "How did you see the shaded portion?" instead of "How did you solve for the shaded portion?" so students can focus on the structure of the fractional pieces and tenths in the image. Encourage students to name the shaded portion in fractions or decimals. Some students may also bring up percentages.

### Building Towards

- 6.RP.A.3.c

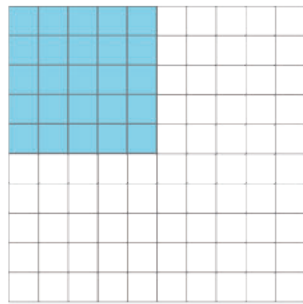
### Launch

Tell students you will show them a 10 x 10 grid for 3 seconds and that the entire grid represents 1. Their job is to find how much is shaded in the image and explain how they saw it.

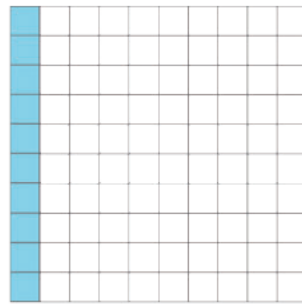
Display the image for 3 seconds and then hide it. Do this twice. Give students 15 seconds of quiet think time between each flash of the image. Encourage students who have one way of seeing the grid to consider another way to determine the size of the shaded portion while they wait.

### Student Task Statement

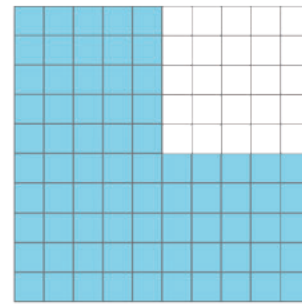
How much is shaded in each one?



A



B



C

### Student Response

A:  $\frac{25}{100}$ , 0.25,  $\frac{1}{4}$

B:  $\frac{1}{10}$ ,  $\frac{10}{100}$ , 0.1, 0.10

C:  $\frac{75}{100}$ , 0.75,  $\frac{3}{4}$

### Activity Synthesis

Invite students to share how they visualized the shaded portion of each image. Record and display their explanations for all to see. Solicit from the class alternative ways of quantifying the shaded portion and alternative ways of naming the size of the shaded portion (to elicit names in fractions, decimals, and percentages). To involve more students in the conversation, consider asking:

- “Who can restate \_\_\_’s reasoning in a different way?”
- “Did anyone solve the shaded portion the same way but would explain it differently?”
- “Did anyone solve the shaded portion in a different way?”
- “Does anyone want to add on to \_\_\_’s strategy?”
- “Do you agree or disagree? Why?”

## 9.2 Card Sort: Is it a Deal?

20 minutes

Students are given cards, each of which contains an original price and a new price, as shown.

B. Juice Boxes
Original: 10 for \$3.50
New Deal: 6 for \$2.40

Their job is to sort the cards into two piles: one pile for deals they would take, and another for those they would reject. There are many paths students could use to reason about whether or not to accept a deal. For example, if the original deal was \$3.50 for 10 juice boxes and the new deal is \$2.40 for 6 juice boxes, they could:

- Find and compare the unit rates for both the original pack and the new pack. If the unit rate is the same, the deal is fair. If the unit rate is lower, the clerk is offering a discount. If the unit rate is higher, the clerk is not being fair.

number of juice boxes	cost in dollars	dollars per box
10	3.50	0.35
6	2.40	0.40

- Find the unit rate in the original pack, apply it to the number of items in the new pack, and compare the costs for the same number of items in the original and new pricing schemes. This can be done in two ways, one focused more on column reasoning and the other on row reasoning, as shown.

number of juice boxes	cost in dollars
10	3.50
	$\cdot 0.35$
6	2.10

number of juice boxes	cost in dollars
10	3.50
1	0.35
6	2.10

- Use an abbreviated table and bypass calculating the unit rate. Find the multiplier to get from the original to the new number of items, and use the same multiplier to find what the price would be if the deal has not changed. Compare the actual new price to this projected price.

number of juice boxes	cost in dollars
10	3.50
6	?



As students work, attend to how they reason about the deals and make their decisions (deal or no deal).

### Addressing

- 6.RP.A.3

### Instructional Routines

- MLR2: Collect and Display
- Poll the Class
- Take Turns

### Launch

Show the picture on Card A (or use an actual 4-pack of a beverage with a missing bottle.) Present the following situation:

“You’ve entered a local shop to buy a 4-pack of drinks. You find one last pack of the drink you want on the shelf and, unfortunately, only 3 bottles remain in that pack. You decide to buy it anyway. You take the 3-pack to the check-out counter and ask the clerk to consider a fair price for the incomplete pack. If the cost of a 4-pack was \$3.16 and the clerk offers to sell the 3 pack for \$2.25, will you take the deal?”

Poll the class for their response and display how many students would and would not take the deal. Then, ask “How could you figure out if the deal is good or not?” Give students a moment of quiet think time to come up with strategies for solving such a problem and then invite a few students to share.

Arrange students in groups of 2. Give each group a set of five cards A–E (or six cards A–F if including the extension problem). Tell students their job is to sort the cards into a ‘Deal’ pile and a ‘No Deal’ pile. Instruct partners to collaborate in finding the answer for card A and divide up the remaining cards between them. Ask students to first work on their cards individually, then share their reasoning with their partner, and lastly, sort the cards together.

---

### Support for Students with Disabilities

*Representation: Internalize Comprehension.* Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students a subset of the cards to start with and introduce the remaining cards once students have identified which initial cards were good deals.

*Supports accessibility for: Conceptual processing; Organization*

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## Anticipated Misconceptions

Students who are not fluent in multiplication and division computation work from grade 5 may need some review in order to be successful in this activity.

### Student Task Statement

Your teacher will give you a set of cards showing different offers.

1. Find card A and work with your partner to decide whether the offer on card A is a good deal. Explain or show your reasoning.
2. Next, split cards B–E so you and your partner each have two.
  - a. Decide individually if your two cards are good deals. Explain your reasoning.
  - b. For each of your cards, explain to your partner if you think it is a good deal and why. Listen to your partner's explanations for their cards. If you disagree, explain your thinking.
  - c. Revise any decisions about your cards based on the feedback from your partner.
3. When you and your partner are in agreement about cards B–E, place all the cards you think are a good deal in one stack and all the cards you think are a bad deal in another stack. Be prepared to explain your reasoning.

### Student Response

Card A: Deal! The price per bottle is \$0.79 so a 3 pack should be \$2.37.

Card B: No Deal! The price per juice box is \$0.35 so 6 juice boxes should be \$2.10.

Card C: Deal! The price per bar is \$0.88 so 4 bars should be \$3.52.

Card D: Deal! The price per hummus container is \$0.90 so the cost for 10 should be \$9.00.

Card E: No Deal! The price per yogurt container is \$0.85 so containers should cost \$5.10.

### Are You Ready for More?

Time to make your own deal! Read the information on card F and then decide what you would charge if you were the clerk. When your teacher signals, trade cards with another group and decide whether or not you would take the other group's offer.

Keep in mind that you may offer a fair deal or an unfair deal, but the goal is to set a price close enough to the value it should be that the other group cannot immediately tell if the deal you offer is a good one.

### Student Response

Answers vary. A fair deal for F would be \$6.84 for 9 packs.

## Activity Synthesis

Select 2–3 students who used different but effective strategies to share their thinking with the class. Encourage students to listen to others' reasoning. Record the different strategies in one place and display them for all to see. Highlight any similarities and differences (e.g., whether a unit rate was used, whether students compare the original unit rate to the new quantity or the other way around, etc.)

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### Support for English Language Learners

*Writing, Listening, Conversing: MLR2 Collect and Display.* Listen and observe how students reason about the deals and make their decisions (deal or no deal), and make note of the different strategies students use to compare unit rates. Listen for language such as “the same,” “equal,” “unit rate,” “cost for the same number of items,” etc., and display these visually for the whole class to use as a reference. Continue to add to and refer to the display during the whole class discussion, making explicit connections between the language and the strategies used (i.e., whether students compare the original unit rate to the new quantity or the other way around). This will help students make sense of calculating and comparing unit rates while increasing meta-awareness of language.

*Design Principle(s): Support sense-making; Maximize meta-awareness*

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## 9.3 The Fastest of All

### 15 minutes

In this activity, students convert between customary and metric units in order to compare lengths. To make some measurements comparable to others, students need to perform multistep conversions and activate arithmetic skills from previous grades. Support students with computations as needed and provide access to calculators as appropriate. Share the following information with students when requested.

- 1 mile = 1,760 yards = 5,280 feet
- 1 yard = 3 feet
- 1 foot = 12 inches
- 1 kilometer = 1,000 meters
- 1 meter = 100 centimeters

Expect students to choose different units of measurements to make comparisons. As students work, identify those who opt for the same unit so that they can be partnered or grouped together for discussion.

## Addressing

- 6.RP.A.3.d

## Instructional Routines

- MLR7: Compare and Connect

## Launch

Give students 1–2 minutes to read the task, and then ask how they think they could compare these lengths. Students are likely to suggest converting all the lengths into the same unit of measurement. Ask students which units might be appropriate in this case and why. (Feet, yards, and meters are better choices than inches or miles.) After discussing some appropriate options, give students quiet think time to complete the activity, and then time to share their explanation with one or more students who have chosen to use the same unit of measurement.

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### Support for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* To support development of organizational skills, check in with students within the first 2-3 minutes of work time. Look for students who are converting all the distances to the same length.

*Supports accessibility for: Memory; Organization*

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## Anticipated Misconceptions

Some students may need to be prompted about the intermediate steps needed to compare units that require several conversions before they can be compared.

### Student Task Statement

Wild animals from around the world wanted to hold an athletic competition, but no one would let them on an airplane. They decided to just measure how far each animal could sprint in one minute and send the results to you to decide the winner.

You look up the following information about converting units of length:

$$1 \text{ inch} = 2.54 \text{ centimeters}$$

animal	sprint distance
cougar	1,408 yards
antelope	1 mile
hare	49,632 inches
kangaroo	1,073 meters
ostrich	1.15 kilometers
coyote	3,773 feet

1. Which animal sprinted the farthest?
2. What are the place rankings for all of the animals?

## Student Response

1. Antelope wins first place.

2. Antelope, Cougar, Hare, Coyote, Ostrich, Kangaroo. Possible strategies:

- Converting all measurements to feet:

Antelope sprinted 5,280 feet, because  $1 \cdot 1,760 \cdot 3 = 5,280$ .

Cougar sprinted 4,224 feet, because  $(1,408) \cdot 3 = 4,224$ .

Hare sprinted 4,136 feet, because  $49,632 \div 12 = 4,136$ .

Coyote sprinted 3,773 feet.

Ostrich sprinted almost 3,773 feet, because  $(1.15) \cdot (1,000) \cdot 100 \div 2.54 \div 12 = 3,772.97$ .

Kangaroo sprinted 3,520 feet, because  $(1,073) \cdot 100 \div 2.54 \div 12 = 3,520.34$ .

- Converting all measurements to yards:

Antelope sprinted 1,760 yards, because 1 mile = 1,760 yards.

Cougar sprinted 1,408 yards.

Hare sprinted  $1,378\frac{2}{3}$  yards, because  $49,632 \div 12 \div 3 = 1,378\frac{2}{3}$ .

Coyote sprinted  $1,257\frac{2}{3}$  yards, because  $3,773 \div 3 = 1,257\frac{2}{3}$ .

Ostrich sprinted almost  $1,257\frac{2}{3}$  yards, because  
 $(1.15) \cdot (1,000) \cdot 100 \div 2.54 \div 12 \div 3 = 1,257.66$ .

Kangaroo sprinted  $1,173\frac{1}{3}$  yards, because  $(1,073) \cdot 100 \div 2.54 \div 12 \div 3 \approx 1,173\frac{1}{3}$ .

- Converting all measurements to meters:

Antelope sprinted 1,609.34 meters, because  $1 \cdot (1,760) \cdot 3 \cdot 12 \cdot (2.54) \div 100 = 1,609.34$ .

Cougar sprinted 1,287.48 meters, because  $(1,408) \cdot 3 \cdot 12 \cdot 2.54 \div 100 = 1,287.48$ .

Hare sprinted 1,260.65 meters, because  $(49,632) \cdot (2.54) \div 100 = 1,260.65$ .

Coyote sprinted just over 1,150 meters, because  $3,773 \cdot 12 \cdot 2.54 \div 100 = 1,150.01$ .

Ostrich sprinted 1,150 meters, because  $(1.15) \cdot 1,000 = 1,150$ .

Kangaroo sprinted 1,073 meters.

## Activity Synthesis

Poll the class to see if they agree on who took first, second, third, and last place. If there is widespread agreement, invite two students to share: one student who converted all measurements to feet or yards, and another who converted everything to meters. If there are discrepancies, list the distances run by each animal in each unit of measurement and display them for all to analyze and double check. While the numerical values of the measurements in feet will all be greater than those in meters, the rank order will come out the same.

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### Support for English Language Learners

*Speaking, Representing: MLR7 Compare and Connect.* Use this routine when students present their strategy and representation for determining the place rankings for all of the animals. Ask students to consider what is the same and what is different about each approach. Draw students' attention to the different units of measurements used to make comparisons, while making connections to the strategies used to make conversions. These exchanges can strengthen students' mathematical language use and reasoning to make sense of strategies used to convert units to be able to make comparisons.

*Design Principle(s): Maximize meta-awareness*

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## Lesson Synthesis

Emphasize that when we want to compare rates, a straightforward way is to compare unit rates. For example, when we were comparing the best deal, an example was 10 juice boxes for \$3.50 or 6 juice boxes for \$2.40. It may be helpful to draw two tables or two double number lines to facilitate discussion. Questions to discuss:

- "What are two associated unit rates that we could compare?" (0.35 and 0.4)
- "How were they computed?" (Divide 3.5 by 10 and divide 2.4 by 6)
- "What do these numbers mean in this context?" (They are each the price per bottle for the different offers. For example, \$0.35 for 1 bottle.)

## 9.4 Tacos by the Pack

Cool Down: 5 minutes

### Addressing

- 6.RP.A.3.b

### Student Task Statement

A restaurant sells 10 tacos for \$8.49, or 6 of the same kind of taco for \$5.40.

Which is the better deal? Explain how you know.

## Student Response

The 10-taco offer is a better deal.

Based on the price per taco: The 10-taco offer is about \$0.85 per taco because  $8.49 \div 10 = 0.849$ . The 6-taco offer is \$0.90 per taco because  $5.40 \div 6 = 0.90$ .

Or, look at the cost of a common multiple number of tacos. The least common multiple is 30. Using the 10-taco offer, 30 tacos cost \$25.47. Using the 6-taco offer, 30 tacos cost \$27.00. The 10-taco offer is better.

Based on how much you get for a dollar (the calculations are more difficult with this approach): With the 10-taco offer, you get around 1.2 tacos per dollar because  $10 \div 8.49 \approx 1.2$ . With the 6-taco offer, you get around 1.1 tacos per dollar because  $6 \div 5.40 \approx 1.1$ .

## Student Lesson Summary

Sometimes we can find and use more than one unit rate to solve a problem.

Suppose a grocery store is having a sale on shredded cheese. A small bag that holds 8 ounces is sold for \$2. A large bag that holds 2 kilograms is sold for \$16. How do you know which is a better deal?

Here are two different ways to solve this problem:

Compare dollars per kilogram.

- The large bag costs \$8 per kilogram, because  $16 \div 2 = 8$ .
- The small bag holds  $\frac{1}{2}$  pound of cheese, because there are 16 ounces in 1 pound, and  $8 \div 16 = \frac{1}{2}$ .

The small bag costs \$4 per pound, because  $2 \div \frac{1}{2} = 4$ . This is about \$8.80 per kilogram, because there are about 2.2 pounds in 1 kilogram, and  $4.00 \cdot 2.2 = 8.80$ .

The large bag is a better deal, because it costs less money for the same amount of cheese.

Another way to solve the problem would be to compare the unit prices of each bag in dollars per ounce. Try it!

Compare ounces per dollar.

- With the small bag, we get 4 ounces per dollar, because  $8 \div 2 = 4$ .
- The large bag holds 2,000 grams of cheese. There are 1,000 grams in 1 kilogram, and  $2 \cdot 1,000 = 2,000$ . This means 125 grams per dollar, because  $2,000 \div 16 = 125$ .

There are about 28.35 grams in 1 ounce, and  $125 \div 28.35 \approx 4.4$ , so this is about 4.4 ounces per dollar.

The large bag is a better deal, because you get more cheese for the same amount of money.

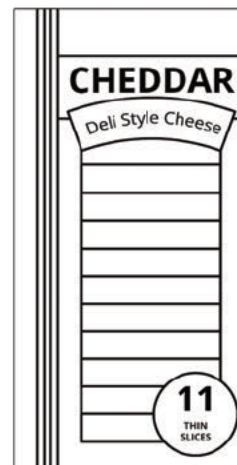
## Lesson 9 Practice Problems

### Problem 1

#### Statement

This package of sliced cheese costs \$2.97.

How much would a package with 18 slices cost at the same price per slice? Explain or show your reasoning.



#### Solution

\$4.86. Sample reasoning: The package of 11 slices costs \$2.97, so this is 27 cents per slice. A package of 18 slices at 27 cents per slice would cost \$4.86 because \$.

### Problem 2

#### Statement

A copy machine can print 480 copies every 4 minutes. For each question, explain or show your reasoning.

- How many copies can it print in 10 minutes?
- A teacher printed 720 copies. How long did it take to print?

#### Solution

- 1,200 copies, because the rate is 120 copies per minute, and  $120 \cdot 10 = 1,200$ .
- 6 minutes, because  $720 \div 120 = 6$

### Problem 3

#### Statement

Order these objects from heaviest to lightest.

(Note: 1 pound = 16 ounces, 1 kilogram  $\approx$  2.2 pounds, and 1 ton = 2,000 pounds)



item	weight
school bus	9 tons
horse	1,100 pounds
elephant	5,500 kilograms
grand piano	15,840 ounces

## Solution

school bus, elephant, horse, grand piano

item	weight	weight in pounds
school bus	9 tons	18,000
horse	1,100 pounds	1,100
elephant	5,500 kilograms	12,100
grand piano	15,840 ounces	990

## Problem 4

### Statement

Andre sometimes mows lawns on the weekend to make extra money. Two weeks ago, he mowed a neighbor's lawn for  $\frac{1}{2}$  hour and earned \$10. Last week, he mowed his uncle's lawn for  $\frac{3}{2}$  hours and earned \$30. This week, he mowed the lawn of a community center for 2 hours and earned \$30.

Which jobs paid better than others? Explain your reasoning.

### Solution

The first two jobs paid better. His neighbor and his uncle both paid \$20 per hour. For his neighbor, an hour of lawn mowing pays  $10 \cdot 2$  or \$20. His uncle paid \$30 per  $\frac{3}{2}$  hours, which means \$10 every  $\frac{1}{2}$  hour and \$20 every hour. The third job at the community center paid \$15 per hour, since  $30 \div 2 = 15$ .

(From Unit 3, Lesson 5.)

## Problem 5

### Statement

Calculate and express your answer in decimal form.

- a.  $\frac{1}{2} \cdot 17$
- b.  $\frac{3}{4} \cdot 200$
- c.  $(0.2) \cdot 40$
- d.  $(0.25) \cdot 60$

### Solution

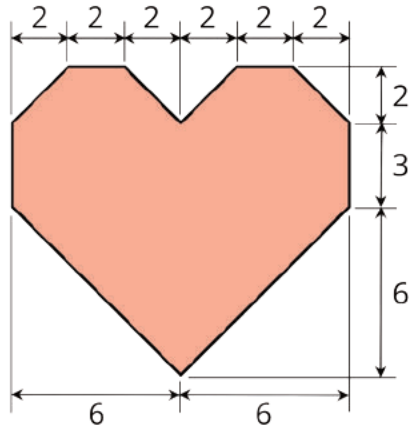
- a. 8.5
- b. 150
- c. 8
- d. 15

(From Unit 3, Lesson 1.)

## Problem 6

### Statement

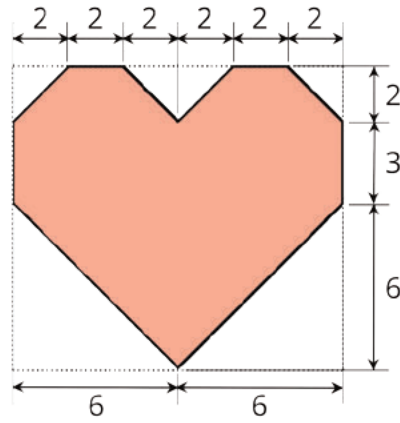
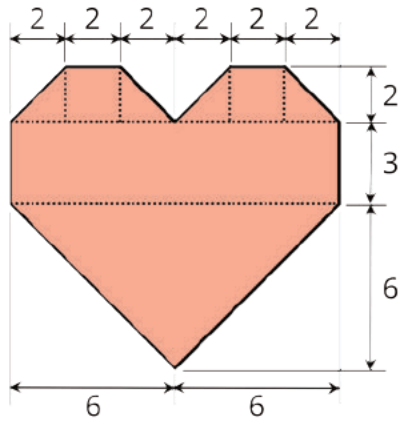
Here is a polygon.



- a. Decompose this polygon so that its area can be calculated. All measurements are in centimeters.
- b. Calculate its area. Organize your work so that it can be followed by others.

### Solution

- a. Answers vary. One strategy is to decompose the polygon into triangles and rectangles and adding up their areas. Another is to enclose it with a rectangle, find its area, and subtract the unshaded right triangles from it.



b. 88 square centimeters. Reasonings vary.

(From Unit 1, Lesson 11.)

## Section: Percentages

### Lesson 10: What Are Percentages?

#### Goals

- Comprehend the word “percentage” (in written and spoken language) and the symbol “%” (in written language) to mean a rate per 100.
- Draw and label a double number line diagram to represent percentages of a dollar and to find corresponding monetary values or percentages.

#### Learning Targets

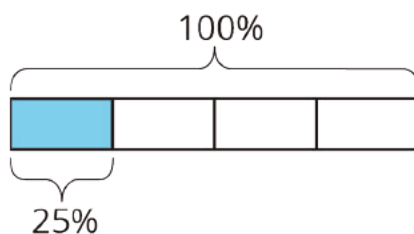
- I can create a double number line with percentages on one line and dollar amounts on the other line.
- I can explain the meaning of percentages using dollars and cents as an example.

#### Lesson Narrative

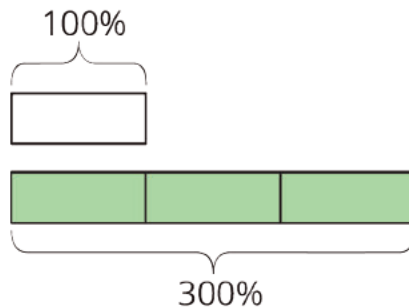
This lesson is the first of two that introduce students to **percentages** as a rate per 100 (MP6) and the ways they are used to describe different types of situations.

Percentages are commonly used in two ways:

1. To describe a part of a whole. For example, “Jada drank 25% of the bottle of water.” In this case, the percentage expressing the amount consumed is not bigger than 100% because it refers to a part of a whole, as shown in the diagram below.



2. To describe the size of one quantity as a percentage of another quantity. For example, “Jada drank 300% as much water as Diego did.” In this case, there is no restriction on the size of the percentage, because the percentage is describing a multiplicative comparison between two quantities, as shown below.



In the first usage there is a single quantity and we are describing a part of it; in the second usage we are comparing two quantities. Students may have prior exposure to percentages, but are likely to have only encountered the first usage and might not be able to make sense of percentages above 100% or those used in comparative contexts. This lesson exposes students to both applications of percentages.

Money is the main context for exploring percentages in this lesson and the warm up asks students to convert between dollars and cents providing an opportunity for the teacher to assess students' current abilities.

For the first several lessons exploring percentages, double number lines are the primary representation presented to students. This choice is intended to strongly communicate that we are working with percent *rates*, and that students can and should use all of the reasoning they have developed to deal with equivalent ratios and rates when dealing with rates per 100. That said, if students prefer to reason using tables or by multiplying or dividing by unit rates, they should not be discouraged from doing so.

## Alignments

### Building On

- 2.MD.C.8: Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using \$ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?
- 5.NBT.A.3: Read, write, and compare decimals to thousandths.
- 6.RP.A: Understand ratio concepts and use ratio reasoning to solve problems.

### Addressing

- 6.RP.A.3.c: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

### Building Towards

- 6.RP.A.3.c: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

### Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR7: Compare and Connect
- Think Pair Share

### Student Learning Goals

Let's learn about percentages.

## 10.1 Dollars and Cents

### Warm Up: 5 minutes

This warm-up prompts students to reason in monetary terms, preparing them for subsequent tasks in the lesson. It also provides insight into students' understanding of dollars and cents as well as their ability to reason mentally.

### Building On

- 2.MD.C.8
- 5.NBT.A.3

### Building Towards

- 6.RP.A.3.c

### Launch

Display questions for all to see. Ask students to solve them mentally.

### Anticipated Misconceptions

In response to "how many dollars are in one cent," students might say there are no dollars at all in one cent. Ask them what fraction of a dollar one cent represents.

### Student Task Statement

Find each answer mentally.

1. A sticker costs 25 cents. How many dollars is that?
2. A pen costs 1.50 dollars. How many cents is that?
3. How many cents are in one dollar?
4. How many dollars are in one cent?

### Student Response

1.  $\frac{1}{4}$  (or 0.25) of a dollar. There are four quarters in a dollar, and a quarter is 25 cents.

2. 150 cents. There are 100 cents in 1 dollar, so 1.50 dollars is multiplied by 100 to find the number of cents.
3. There are 100 cents in 1 dollar.
4.  $\frac{1}{100}$  (or 0.01) of a dollar. Since there are 100 cents in 1 dollar, 1 dollar is divided by 100.

### Activity Synthesis

After students solved all problems mentally, for each problem, ask 1–2 students to share their thinking. Pause between problems to give everyone time to reflect on the shared answers.

## 10.2 Coins

### 15 minutes

In this activity, students learn the definition of a **percentage** as a rate per 100 and apply this definition in the context of money. They label various coin amounts as percentages of 100 cents or 1 dollar.

Students are likely able to name the values of each coin and their individual percentages (in the first two questions) fairly quickly. Assigning a percentage to a group of coins (in the last two questions) adds complexity and should be the focus of the activity as students may use a variety of strategies. One possible strategy is to reason in terms of ratios. For example, a student may think that if a dime is 10% of a dollar, then 6 dimes is 60% of a dollar. This type of ratio thinking is a robust way for dealing with percent problems and should be encouraged early.

As students work, notice the strategies being used to solve the two problems and identify those with effective approaches so they can share later.

### Building Towards

- 6.RP.A.3.c

### Instructional Routines

- MLR3: Clarify, Critique, Correct

### Launch

Remind students that previously they have learned that a “rate per 1” tells us the amount of one quantity for 1 of another quantity. Explain that in this task, they will explore “rates per 100.”

Solicit a couple of ideas on what “rates per 100” might mean. Students are likely to suggest a description along the lines of “the amount of something for 100 of something else.” Tell students that a rate per 100 is called a percentage and that they will explore percentages in the context of money. Point out the half-dollar and dollar coins in the task, as some students may not be familiar with them.

Arrange students in groups of 2. Give students 3 minutes of quiet think time to begin work on the task. After that time, ask students to share their responses with a partner and complete the remaining questions together.

### Support for Students with Disabilities

*Action and Expression: Develop Expression and Communication.* Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their ideas. For example, “If \_\_\_\_ then \_\_\_\_ because...” or “How do you know...?”

*Supports accessibility for: Language; Organization*

### Anticipated Misconceptions

Students may notice a pattern particular to this activity—that the percent value is the same as that for cents—and carry that assumption forward and apply it incorrectly to situations in which 100% does not correspond to 100. This conversation is addressed in the Activity Synthesis.

### Student Task Statement

- Complete the table to show the values of these U.S. coins.



coin	penny	nickel	dime	quarter	half dollar	dollar
value (cents)						

The value of a quarter is 25% of the value of a dollar because there are 25 cents for every 100 cents.

1 Quarter

1 Dollar

- Write the name of the coin that matches each expression.



- 25% of a dollar
- 5% of a dollar
- 1% of a dollar
- 100% of a dollar
- 10% of a dollar
- 50% of a dollar

3. The value of 6 dimes is what **percent** of the value of a dollar?
4. The value of 6 quarters is what percent of the value of a dollar?

### Student Response

1. From left to right in the table: 1, 5, 10, 25, 50, 100
2.
  - a. Quarter
  - b. Nickel
  - c. Penny
  - d. Dollar
  - e. Dime
  - f. Half-dollar
3. The value of 6 dimes are 60 cents and a dollar is 100 cents, so 6 dimes are 60% of the value of 1 dollar.
4. The value of 6 quarters is 150 cents and a dollar is 100 cents, so 6 quarters is 150% of the value of 1 dollar.

### Are You Ready for More?

Find two different sets of coins that each make 120% of a dollar, where no type of coin is in both sets.

### Student Response

Answers vary. Sample response: A dollar and two dimes, four quarters and four nickels

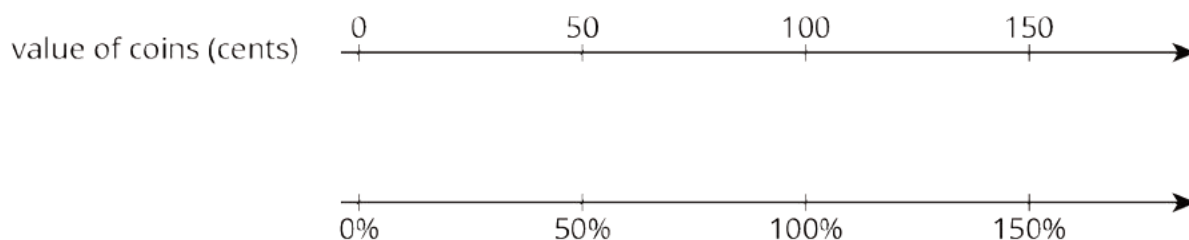
### Activity Synthesis

Focus the discussion on the ways students approached the last two questions and on precise use of language and notation (MP6). For example, in the first two problems students can write only a number or matched a coin to a pre-written phrase. In the last two problems, however, expressing a percentage with only a number and without the % symbol should be considered an incomplete answer.

Select students with successful strategies to share their thinking with the class. Display a concise version of their reasoning for all to see. Invite others to express support, disagreement, or questions (MP3).

If no one reasoned about percentages in terms of ratios (e.g., If a quarter is 25% of a dollar, 6 quarters are 150% of a dollar), illustrate it.

Many students may reason by noticing a pattern—that the number of cents in an amount matches its percentage of a dollar (e.g., 60 cents is 60%)—rather by thinking in terms of ratio or scaling. Since the pattern only holds up in the context of percentages of 100 of a quantity, students will need to be prompted to look more closely at the meaning of “rate per 100.” Conclude the discussion by displaying the following double number line with 100 at the 100%:



Point out that we were finding **percentages** of 100, so in the double number line, we line up 100% and 100 because 100% of 100 is 100.

---

### Support for English Language Learners

*Reading, Writing, Speaking: MLR3 Clarify, Critique, Correct.* Before students share their explanations for the final question, present an incorrect answer and explanation. For example, “The value of 6 quarters is 50% of the value of a dollar because the value of 6 quarters is 150 cents, which is 50 cents greater than 100 cents. This means that the value of 6 quarters is 50% of the value of a dollar.” Ask students to identify the error, critique the reasoning, and write a correct explanation. As students discuss in pairs, listen for students who identify and clarify the ambiguous language in the statement. For example, the author probably meant to say that 6 quarters is 50% greater than the value of a dollar, or that 6 quarters is 150% of the value of a dollar. This will help students understand how to use percentages to describe the size of one quantity as a percentage of another quantity.

*Design Principle(s): Optimize output (for explanation); Maximize meta-awareness*

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## 10.3 Coins on a Number Line

10 minutes

Previously, students found percentages of 100 cents. In this activity, they reason about percentages of 1 dollar.

One important question to think about here is how students know or decide how the numbers on the double number line diagram should be aligned. Students build on their extensive work on equivalent ratios and double number lines to make sense of percentages and “per 100” reasoning.

### Building On

- 6.RP.A

### Addressing

- 6.RP.A.3.c

### Instructional Routines

- MLR7: Compare and Connect
- Think Pair Share

### Launch

Recap that in the previous activity students found percentages of 100 cents. Tell students they will now find percentages of 1 dollar. Draw their attention to the fact that, on the double number line, the 1 dollar and 100% are lined up vertically to reflect this.

Keep students in the same groups. Give students 2–3 minutes of quiet think time, and then ask them to share their responses with their partner. Display and read aloud the following questions. Ask partners to use them to guide their discussion.

- How did each of you arrive at your answers for the first two questions?
- Where do your answers fall on the double number line diagram? How do you know?
- Are your answers the same for the third question? If they are not, can they both be correct? If they are, can you think of another answer that would also be correct?

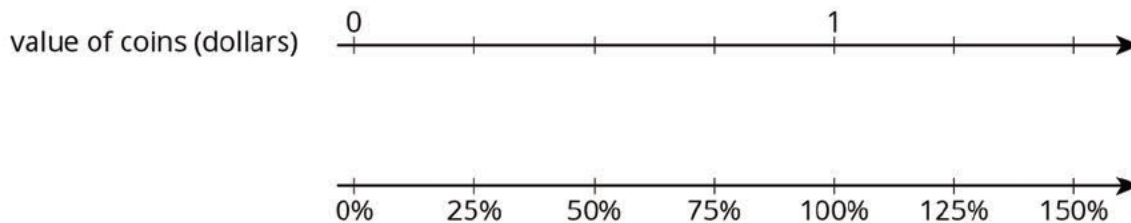
### Anticipated Misconceptions

Based on previous work with labeling number lines less than 1, students may label the tick marks with fractions instead of the decimal value of the coins. This may not be helpful for answering the first two questions, but provides an opportunity to discuss alternative ways to label the number line given the context of the problem. Consider prompting them to write fractional values as cents or to rewrite the cents as dollar values.

### Student Task Statement

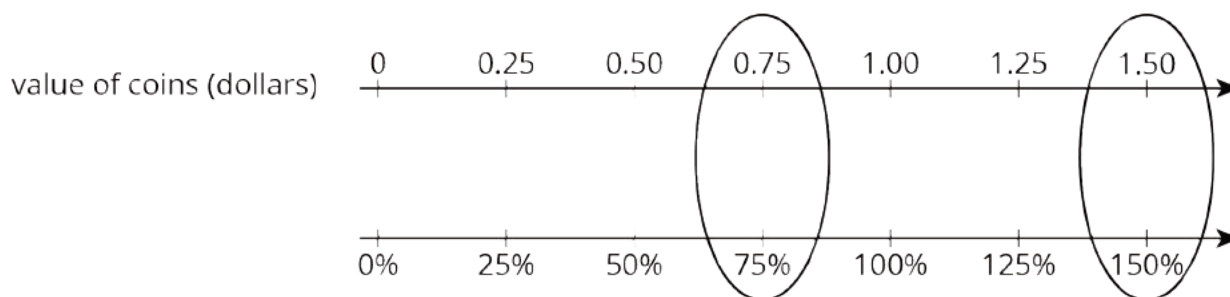


A \$1 coin is worth 100% of the value of a dollar. Here is a double number line that shows this.



1. The coins in Jada's pocket are worth 75% of a dollar. How much are they worth (in dollars)?
2. The coins in Diego's pocket are worth 150% of a dollar. How much are they worth (in dollars)?
3. Elena has 3 quarters and 5 dimes. What percentage of a dollar does she have?

### Student Response



1. \$0.75
2. \$1.50
3. 125%

### Activity Synthesis

Select students who used the provided double number line to share their reasoning. This is an opportunity to refresh students' number line reasoning. Some students may see the four equally spaced tick marks from 0 to 1 and conclude that each is worth 0.25, or  $\frac{1}{4}$ . Others may fill in the 0.50 first, as it is half of 1, then the 0.25 for half of 0.50, and then use additive thinking to fill in the other tick mark values along the top.

Some students may reason in terms of equivalent ratios and say, for example, that since 100 divided by 4 is 25, then  $1 \div 4 = 0.25$  must be 25% of 1. They would then assign the 0.25 value to the first tick mark and use additive thinking to conclude that 0.75 is 75% of a dollar. Ask students who used such an approach to present last to emphasize that the familiar ratio thinking applies to percentage problems as well, even though the % symbol may be unfamiliar. If no students took this approach, illustrate it to make this point.

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### Support for English Language Learners

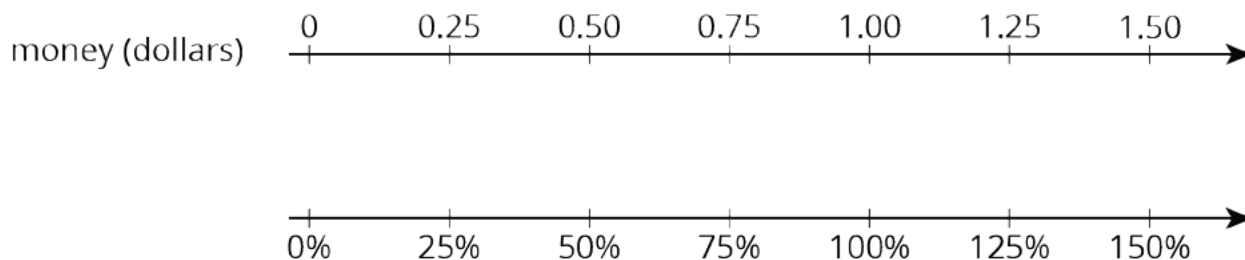
*Speaking, Listening: MLR7 Compare and Connect.* As students prepare a visual display of how they made sense of the problem, look for students who labeled the tick marks on the double number line with fractions or cents instead of dollar values. This may result in answers such as  $\frac{3}{4}$ , or 75 cents is 75% of a dollar rather than 0.75 is 75% of a dollar. Although 75 cents is 75% of a dollar, the number line should be labeled with the decimal value of the coins in dollars. As students investigate each other's work, ask students to share what worked or did not work well in the way they labeled the double number line. Is there a particular advantage to using decimals instead of fractions to label the double number line? Emphasize that although there are several ways to label the double number line given the context of the problem, certain methods are more helpful for answering the question. This will foster students' meta-awareness and support constructive conversations as they compare the various ways to label a double number line given a context.

*Design Principle(s): Cultivate conversation; Maximize meta-awareness*

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## Lesson Synthesis

Remind students that a **percentage** is a "rate per 100." We saw that the value of a quarter is 25% of the value of a dollar, because a quarter is worth 25 cents and a dollar is worth 100 cents. Reiterate that we found percentages of the value of a dollar using a double number line as shown here:



Here, 100% corresponds to 1 dollar, and this is reflected in the fact that the 1.00 and 100% are aligned in the double number line.

## 10.4 Eight Dimes

**Cool Down: 5 minutes**

The purpose of this activity is to see how students make sense of the percentage as a rate per 100.

### Addressing

- 6.RP.A.3.c

### Anticipated Misconceptions

In the first question, students may write that 8 dimes is 8% of the value of a dollar because they account for the number of coins but not account their value. In the second question, students may

put 130 cents as the answer, not differentiating between combination of coins and the value of the coins.

### Student Task Statement

1. Fill in the blank: The value of 8 dimes is \_\_\_\_\_% of the value of a dollar.
2. Name a combination of coins that is 130% of the value of a dollar.

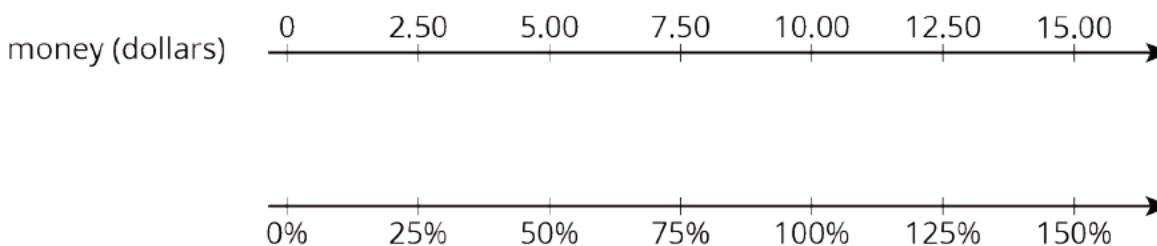
### Student Response

1. 80%. 8 dimes are 80 cents, and a dollar is 100 cents, so 8 dimes are 80% the value of 1 dollar.
2. Answers vary. Sample responses:
  - a. 1 dollar and 3 dimes
  - b. 5 quarters and 1 nickel
  - c. 13 dimes
  - d. 26 nickels

### Student Lesson Summary

A **percentage** is a *rate per 100*.

We can find percentages of \$10 using a double number line where 10 and 100% are aligned, as shown here:



Looking at the double number line, we can see that \$5.00 is 50% of \$10.00 and that \$12.50 is 125% of \$10.00.

### Glossary

- percent
- percentage

## Lesson 10 Practice Problems

### Problem 1

#### Statement

What percentage of a dollar is the value of each coin combination?

- a. 4 dimes
- b. 1 nickel and 3 pennies
- c. 5 quarters and 1 dime

## Solution

- a. 40%
- b. 8%
- c. 135%

## Problem 2

### Statement

- a. List three different combinations of coins, each with a value of 30% of a dollar.
- b. List two different combinations of coins, each with a value of 140% of a dollar.

## Solution

Answers vary. Sample response:

- a. 30 pennies, 6 nickels, or 3 dimes
- b. 140 pennies, 14 dimes, or 5 quarters and 3 nickels

## Problem 3

### Statement

The United States government used to make coins of many different values. For each coin, state its worth as a percentage of \$1.



$\frac{1}{2}$  cent



3 cents



20 cents



$\$2\frac{1}{2}$



\$5

## Solution

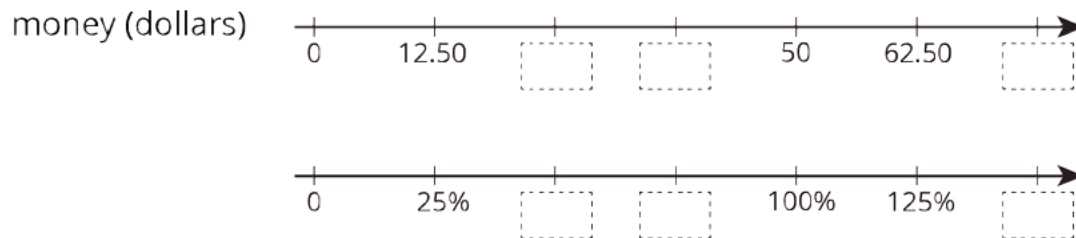
- a.  $\frac{1}{2}\%$
- b. 3%
- c. 20%

- d. 250%
- e. 500%

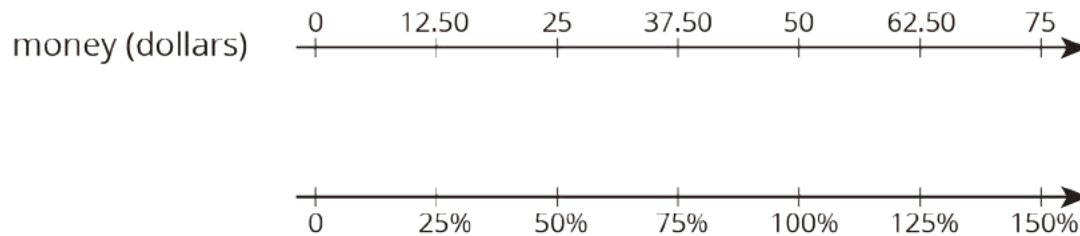
### Problem 4

#### Statement

Complete the double number to line show percentages of \$50.



#### Solution



### Problem 5

#### Statement

Elena bought 8 tokens for \$4.40. At this rate:

- a. How many tokens could she buy with \$6.05?
- b. How much do 19 tokens cost?

#### Solution

- a. 11 tokens
- b. \$10.45

(From Unit 3, Lesson 9.)

### Problem 6

#### Statement

A snail travels 10 cm in 4 minutes. At this rate:

- a. How long will it take the snail to travel 24 cm?



b. How far does the snail travel in 6 minutes?

## Solution

- a. 9.6 minutes (or equivalent)
- b. 15 cm

(From Unit 3, Lesson 8.)

## Problem 7

### Statement

a. 3 tacos cost \$18. Complete the table to show the cost of 4, 5, and 6 tacos at the same rate.

number of tacos	cost in dollars	rate in dollars per taco
3	18	
4		
5		
6		

b. If you buy  $t$  tacos for  $c$  dollars, what is the unit rate?

## Solution

a.

number of tacos	cost in dollars	rate in dollars per taco
3	18	6
4	24	6
5	30	6
6	36	6

b.  $\frac{c}{t}$  dollars per taco or  $\frac{t}{c}$  tacos per dollar.

(From Unit 3, Lesson 7.)

# Lesson 11: Percentages and Double Number Lines

## Goals

- Comprehend a phrase like “A% of B” (in written and spoken language) to refer to the value that makes a ratio with B that is equivalent to  $A : 100$ .
- Explain (orally) how to use a double number line diagram or table to solve problems such as “A% of B is ?” and “A% of ? is C.”
- State explicitly what one is finding the percentage of.

## Learning Targets

- I can use double number line diagrams to solve different problems like “What is 40% of 60?” or “60 is 40% of what number?”

## Lesson Narrative

In the previous lesson, students learned to find percentages of 100 and percentages of 1 in the context of money (100 cents and \$1). In this lesson, they explore percentages of quantities other than 100 and 1 in a variety of contexts. All of the tasks use comparison contexts—describing one quantity relative to another quantity—rather than part-whole contexts.

Students continue to have double number lines as a reasoning tool to use if they want. In several cases the double number line is provided. There are two reasons for this. First, the equal intervals on the provided double number line are useful for reasoning about percentages. Second, using the same representation that was used earlier for other ratio and rate reasoning reinforces the idea of a percentage as a rate per 100 (MP7). It is perfectly acceptable, however, for students to use strategies other than double number lines for solving percentage problems.

## Alignments

### Addressing

- 6.RP.A.3.c: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

### Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR1: Stronger and Clearer Each Time
- MLR7: Compare and Connect
- Think Pair Share

## Required Materials

### Tools for creating a visual display

Any way for students to create work that can be easily displayed to the class. Examples: chart

paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

### Student Learning Goals

Let's use double number lines to represent percentages.

## 11.1 Fundraising Goal

### Warm Up: 5 minutes

This warm-up is the first time students are asked to find  $A\%$  of  $B$  when  $B$  is not 100 or 1.

Students may approach the problem in a few different ways, with or without filling in values on the provided double number line diagram. For example, they may understand from the fundraising context of the problem that \$40 is 100% because it is the amount of the goal. From there, they may simply find half to \$40 for the 50% value and add that value to \$40 to find the 150% value. Other students may use equivalent ratio reasoning to calculate the value at 50% and 150%. As students work, notice the different strategies used and any misconceptions so they can be addressed during discussion.

### Addressing

- 6.RP.A.3.c

### Launch

Remind students that in the previous lesson, we found percentages of 100 and of 1 using double number lines. Explain that in this lesson we will find percentages of other numbers. Give students 2 minutes of quiet work time, and follow with a whole-class discussion. Encourage students to create a double number line to help them answer the questions if needed.

### Anticipated Misconceptions

Students may be surprised by a percentage greater than 100. If they are puzzled by this, explain that Andre raised more money than the goal.

### Student Task Statement

Each of three friends—Lin, Jada, and Andre—had the goal of raising \$40. How much money did each person raise? Be prepared to explain your reasoning.

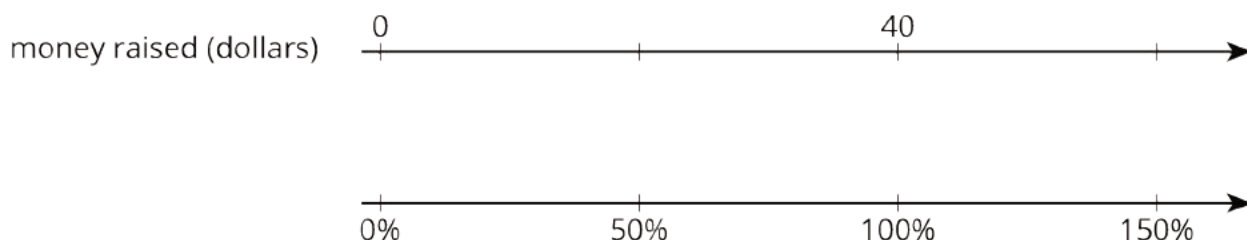
1. Lin raised 100% of her goal.
2. Jada raised 50% of her goal.
3. Andre raised 150% of his goal.

## Student Response

1. \$40. Lin raised \$40 since \$40 is 100% of the goal.
2. \$20. Half of 100 is 50, and half of \$40 is \$20, so Jada raised \$20, which is 50% of \$40.
3. \$60. Since the first tick mark is \$20, the third tick mark must be \$60. This means Andre raised \$60, which is 150% of the \$40 goal.

## Activity Synthesis

Consider displaying this double number line for all to see.



Invite a few students to share their solving strategies. One way to highlight the different techniques students use is to invite several students to explain how they calculated the amount of money raised by Andre. We can find 150% of 40 in several ways. For example, we can add the values of 50% of 40 and 100% of 40. We can also reason that since  $100 \cdot (1.5) = 150$  then  $40 \cdot (1.5) = 60$ , which means \$60 is 150% of \$40.

If not uncovered in students' explanations, explain that we are finding percentages of \$40, since this number—not 100 cents or 1 dollar—is the fundraising goal for the three friends. Since 100% of a goal of \$40 is \$40, the 100% and \$40 are lined up on the double number line.

Students who relied on the visual similarity between, for example, \$0.25 is 25% of 1 dollar in the previous lesson find this strategy unworkable here (as \$50 is not 50% of \$40). To encourage students to use their understanding of equivalent ratios to reason about percent problems, ask the class to explain—either when the above misconception arises or as a closing question—why \$50 is not 50% of \$40, but 50% of 100 cents is 50 cents.

## 11.2 Three-Day Biking Trip

### 15 minutes

In this activity, students find percentages of a value in a non-monetary context. They begin by assigning a value to 100% and reasoning about other percentages.

The double number line is provided to communicate that we can use all our skills for reasoning about equivalent ratios to reason about percentages. Providing this representation makes it more likely that students will use it, but it would be perfectly acceptable for them to use other strategies.

Monitor for students using these strategies:

- Use a double number line and reason that since 25% is  $\frac{1}{4}$  of 100%, 25% of 8 is  $8 \cdot \frac{1}{4} = 2$ . They may then skip count by the value for the first tick mark to find the values for other tick marks.
- Reason about 125% of the distance as 100% of the distance plus 25% of the distance, and add 8 and  $\frac{1}{4}$  of 8.
- Reason about 75% of 8 directly by multiplying 8 by  $\frac{3}{4}$  and 125% of 8 by multiply 8 by  $\frac{5}{4}$ .

### Addressing

- 6.RP.A.3.c

### Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect

### Launch

Arrange students in groups of 3–4. Provide tools for making a large visual display. Give students 2–3 minutes of quiet think time. Encourage students to use the double number line to help them answer the questions if needed. Afterwards, ask them to discuss their responses to the last two questions with their group and to create a visual display of one chosen strategy to be shared with the class.

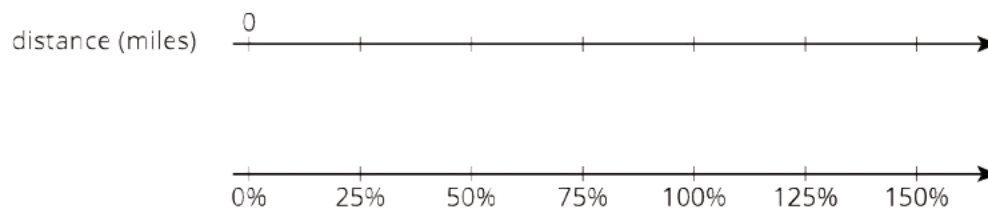
### Support for Students with Disabilities

*Representation: Internalize Comprehension.* Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.

*Supports accessibility for: Memory; Conceptual processing*

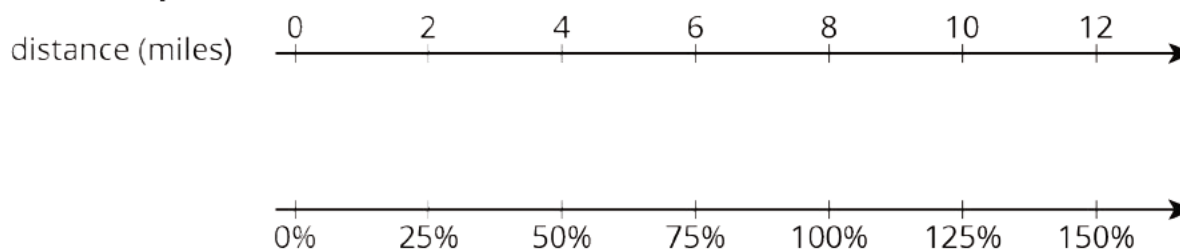
### Student Task Statement

Elena biked 8 miles on Saturday. Use the double number line to answer the questions. Be prepared to explain your reasoning.



1. What is 100% of her Saturday distance?
2. On Sunday, she biked 75% of her Saturday distance. How far was that?
3. On Monday, she biked 125% of her Saturday distance. How far was that?

## Student Response



1. 8 miles. She biked 8 miles total, so 100% of her distance is 8 miles.
2. 6 miles. 75% of 8 miles is 6 miles, so she biked 6 miles on Sunday.
3. 10 miles. Sample strategies:
  - Complete the double number line and observe that 125% aligns with 10 miles.
  - 25% of the distance is 2 miles, and 100% of the distance is 8 miles, and  $2 + 8 = 10$ .
  - Multiply 8 by 1.25 (or  $\frac{5}{4}$ ).

## Activity Synthesis

For each unique strategy, select one group to share their displays and explain their thinking. Sequence the presentations in the order they are presented in the Activity Narrative. If no students mention one or more of these strategies, bring them up. For example, if no one thought of 125% of the distance hiked as 100% plus 25%, present that approach and ask students to explain why it works.

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### Support for English Language Learners

*Speaking, Listening: MLR7 Compare and Connect.* As students prepare a visual display of how they made sense of the last question, look for groups with different methods for finding 125% of 8 miles. Some groups may reason that 25% of the distance is 2 miles and 100% of the distance is 8 miles, so 125% of the distance is the sum of 2 and 8. Others may reason that the product of 100 and 1.25 is 125. Since 125 is 125% of 100, then 125% of 8 miles is the product of 8 and 1.25. As students investigate each other's work, encourage students to compare other methods for finding 125% of 8 to their own. Which approach is easier to understand with the double number line? This will promote students' use of mathematical language as they make sense of the connections between the various methods for finding 125% of a quantity.

*Design Principle(s): Cultivate conversation; Maximize meta-awareness*

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## 11.3 Puppies Grow Up

15 minutes

Previously students were asked to find various percentages given 100% of a quantity. Here they are asked to find 100% of quantities given other percentages. The context does not explicitly state that

the values being sought (the adult weights of two puppies) are the values for 100%, so students will first need to make that connection.

Double number lines continue to be provided as a reasoning tool, but students may use a table of equivalent ratios or other methods. Those who use double number lines are likely to find them effective for the first question (find 100% of a quantity given 20%) but less straightforward for the second question (find 100% of a quantity given 30%). Since 100 is not a multiple of 30, students may use strategies such as subdividing the double number line into intervals of 10% and scaling up from there to find the value of 100%.

### Addressing

- 6.RP.A.3.c

### Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR1: Stronger and Clearer Each Time
- Think Pair Share

### Launch

Arrange students in groups of 2. Give students 3–4 minutes of quiet think time and then time to share their responses with their partner. Encourage students to refer to diagrams in previous activities if they are not sure how to get started. Students may need help interpreting the question to understand that 100% corresponds to the puppy's adult weight.

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### Support for English Language Learners

*Writing, Speaking, Listening: MLR1 Stronger and Clearer Each Time.* After students have had the opportunity to determine the adult weight of Jada's puppy, ask students to write a brief explanation of their process. Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language (e.g., "Can you explain how...?", "You should expand on...", etc.). Students can borrow ideas and language from each partner to refine and clarify their original explanation. This will help students refine their explanation and learn about other methods for finding the adult weight of a puppy.

*Design Principle(s): Optimize output (for explanation); Maximize meta-awareness*

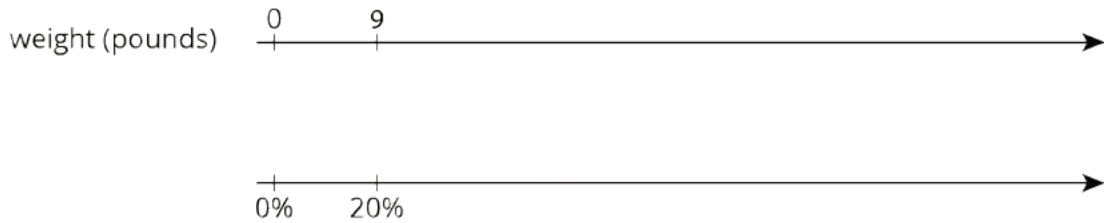
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### Anticipated Misconceptions

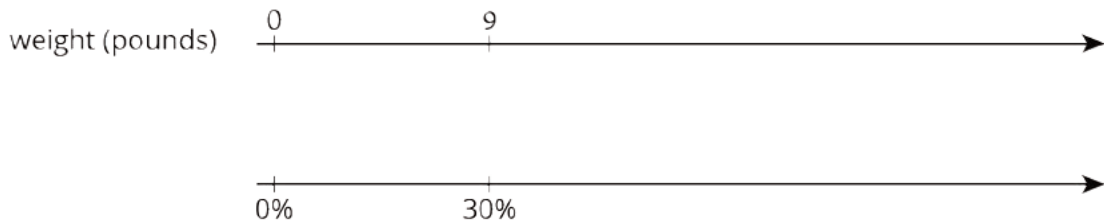
Students may stop before they reach 100% or go further than 100%. If this happens, explain that in this situation, the adult weight is at exactly 100%. Students may not use equal-sized increments between the tick marks they draw and label.

### Student Task Statement

1. Jada has a new puppy that weighs 9 pounds. The vet says that the puppy is now at about 20% of its adult weight. What will be the adult weight of the puppy?



2. Andre also has a puppy that weighs 9 pounds. The vet says that this puppy is now at about 30% of its adult weight. What will be the adult weight of Andre's puppy?

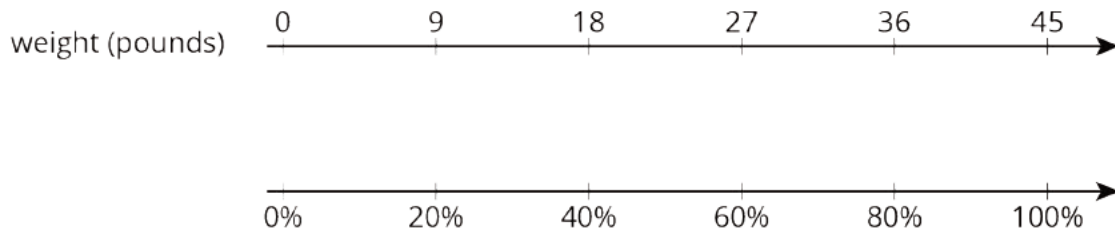


3. What is the same about Jada and Andre's puppies? What is different?

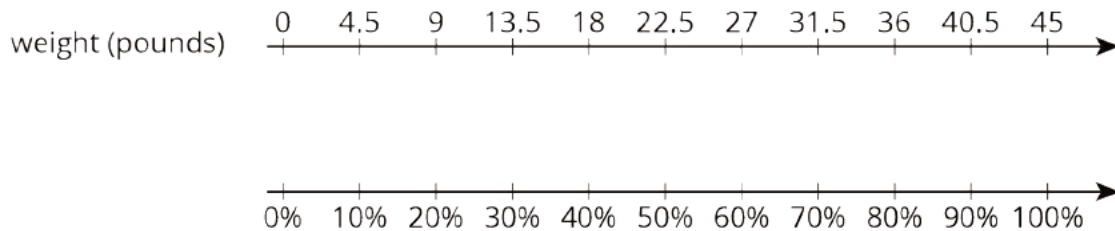
### Student Response

1. 45 pounds. Possible approaches:

- Complete the double number line with multiples of 9 and 20%:



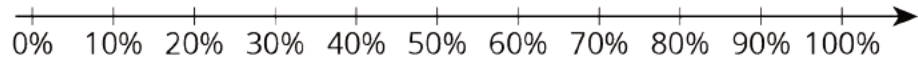
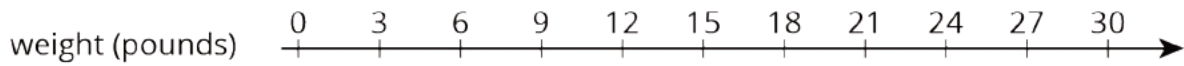
- Complete the double number line in 10% increments:



- 20% is  $\frac{1}{5}$  of 100%, so 100% is  $5 \cdot 9 = 45$ .
- 10% is half of 20%, so 10% is 4.5, and then 100% is 45.



2. 30 pounds. The strategies for this problem are similar to the previous ones, although they have to multiply 9 by  $\frac{10}{3}$  to go directly from 9 pounds to the puppy's adult weight, which might not occur to most students.



3. Both puppies weigh the same right now. The puppies will weigh different amounts when they are adults.

### Are You Ready for More?

A loaf of bread costs \$2.50 today. The same size loaf cost 20 cents in 1955.

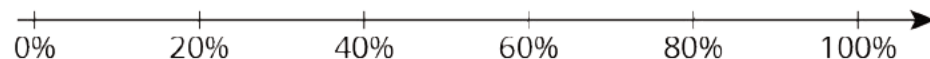
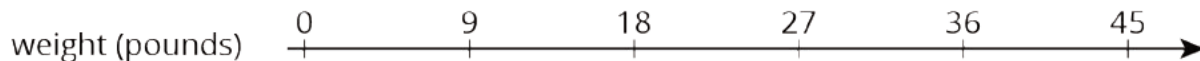
1. What percentage of today's price did someone in 1955 pay for bread?
2. A job pays \$10.00 an hour today. If the same percentage applies to income as well, how much would that job have paid in 1955?

### Student Response

1. 8%
2. \$0.80 an hour



### Activity Synthesis

Invite previously identified students to share their work. Start with someone who solved the first question using a double number line as follows, and follow with increasingly efficient strategies. Keep the number line displayed for all to see and to refer to throughout discussion.



If no students reasoned with a table, display this abbreviated table, or illustrate one student's approach and organize the steps in a table.

weight (pounds)	percentage
9	20
45	100

$\cdot 5$     $\cdot 5$

Follow a similar flow when discussing strategies for solving the second problem: start with a double number line and, if not mentioned by students, discuss how a table such as this one can be an efficient tool.

weight (pounds)	percentage
9	30
3	10
30	100

Annotations: On the left, a green arrow points from 30 to 3 with the label  $\cdot \frac{1}{3}$ , and another green arrow points from 3 to 9 with the label  $\cdot 10$ . On the right, a green arrow points from 100 to 30 with the label  $\cdot \frac{1}{3}$ , and another green arrow points from 30 to 10 with the label  $\cdot 10$ .

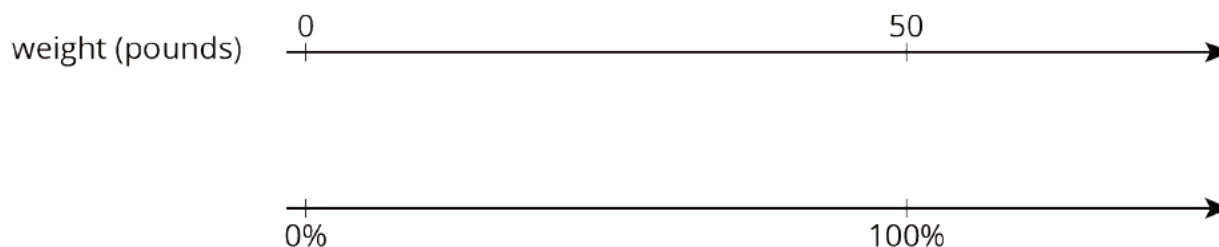
### Support for Students with Disabilities

*Representation: Internalize Comprehension.* Use color and annotations to illustrate connections between representations. For example, on a display, illustrate one student’s approach on both a double number line and a table. Support connections by highlighting how each step appears in each representation.

*Supports accessibility for: Visual-spatial processing; Conceptual processing*

## Lesson Synthesis

If you know the percentage and 100%, then you can find the percentage with a double number line by putting the value assigned to 100% opposite the tick mark labeled 100%. For example, if we want to find some percentage of 50 pounds, we can label 100% and 50 like this:



Display the table for all to see. Questions for discussion:

- What situation might this double number line represent?
- This says that 100% of 50 is 50. Where can we place some other percentages of 50?
- (If no one mentions a percentage greater than 100%) What about 110% of 50? Where would we place it? How would it be labeled?

## 11.4 A Medium Bottle of Juice

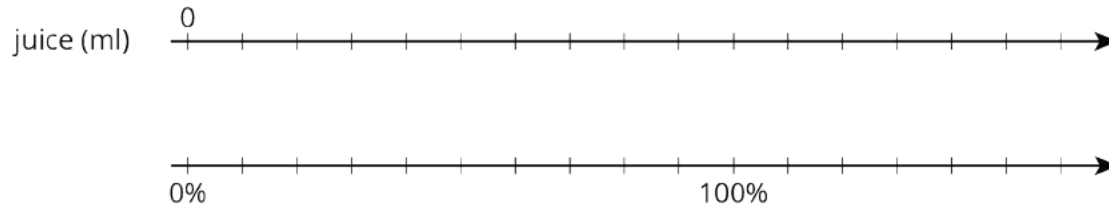
Cool Down: 5 minutes

### Addressing

- 6.RP.A.3.c

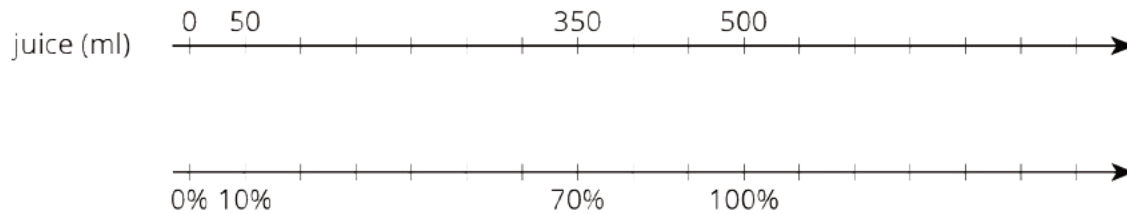
### Student Task Statement

A large bottle of juice contains 500 milliliters of juice. A medium bottle contains 70% as much juice as the large bottle. How many milliliters of juice are in the medium bottle?



### Student Response

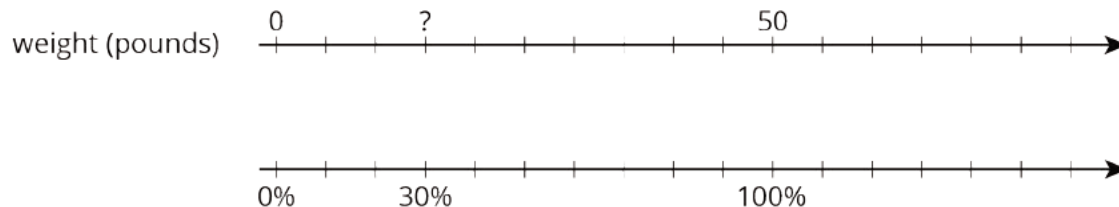
350 ml. Possible strategies: 10% of 500 is 50, so 70% of 500 is  $7 \cdot 50 = 350$ .



juice (ml)	percentage
500	100
50	10
350	70

### Student Lesson Summary

We can use a double number line to solve problems about percentages. For example, what is 30% of 50 pounds? We can draw a double number line like this:



We divide the distance between 0% and 100% and that between 0 and 50 pounds into ten equal parts. We label the tick marks on the top line by counting by 5s ( $50 \div 10 = 5$ ) and on the bottom line counting by 10% ( $100 \div 10 = 10$ ). We can then see that 30% of 50 pounds is 15 pounds.

We can also use a table to solve this problem.

weight (pounds)	percentage
50	100
5	10
15	30

$\cdot \frac{1}{10}$  and  $\cdot 3$  are shown as operations between rows on both sides of the table.

Suppose we know that 140% of an amount is \$28. What is 100% of that amount? Let's use a double number line to find out.



We divide the distance between 0% and 140% and that between \$0 and \$28 into fourteen equal intervals. We label the tick marks on the top line by counting by 2s and on the bottom line counting by 10%. We would then see that 100% is \$20.

Or we can use a table as shown.

money (dollars)	percentage
28	140
2	10
20	100

$\cdot \frac{1}{14}$  and  $\cdot 10$  are shown as operations between rows on both sides of the table.

## Glossary

- percent
- percentage

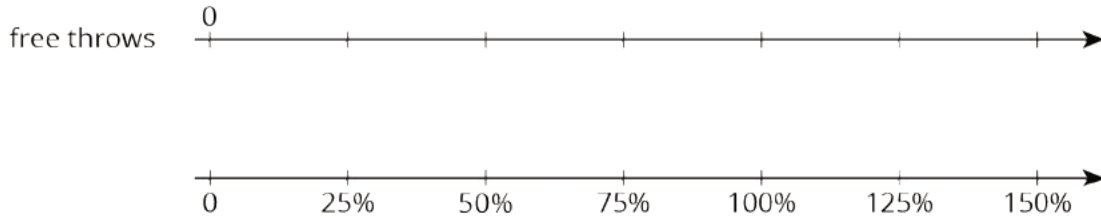
## Lesson 11 Practice Problems

### Problem 1

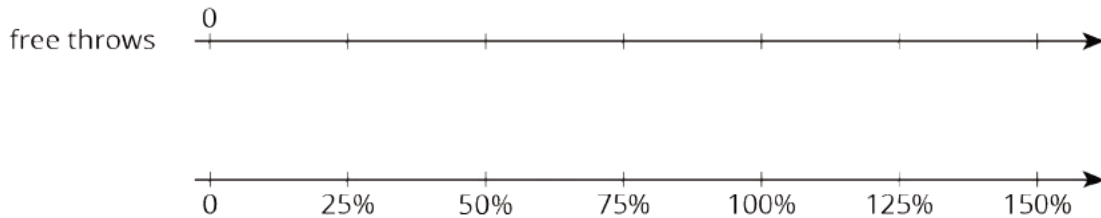
#### Statement

Solve each problem. If you get stuck, consider using the double number lines.

- a. During a basketball practice, Mai attempted 40 free throws and was successful on 25% of them. How many successful free throws did she make?



- b. Yesterday, Priya successfully made 12 free throws. Today, she made 150% as many. How many successful free throws did Priya make today?



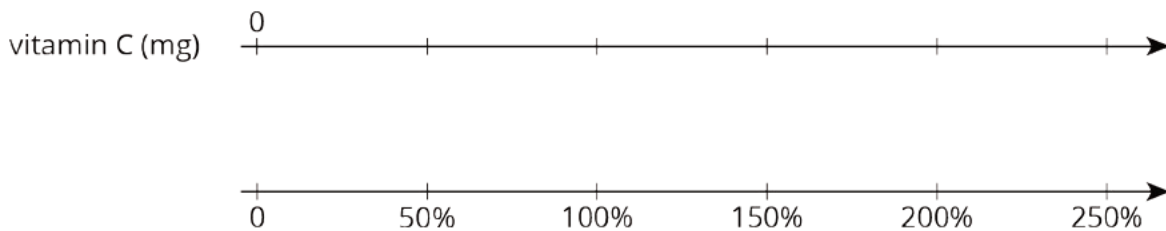
## Solution

- a. 10 free throws  
b. 18 free throws

## Problem 2

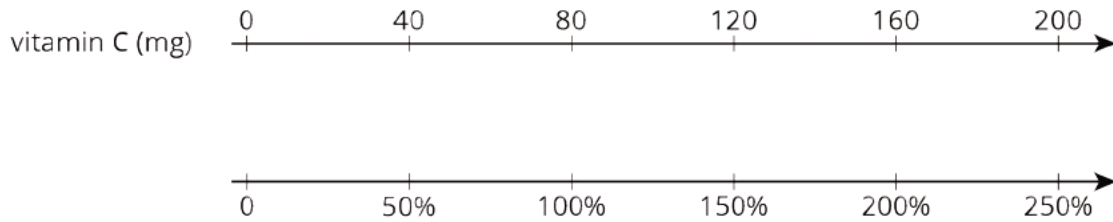
### Statement

A 16-ounce bottle of orange juice says it contains 200 milligrams of vitamin C, which is 250% of the daily recommended allowance of vitamin C for adults. What is 100% of the daily recommended allowance of vitamin C for adults?



## Solution

80 mg. Explanations vary. Sample explanation: On the double number line, place 200 above 250%. Dividing both of these by 5 gives 40 and 50%, so place 40 above 50%. Since 100% is double that, double 40 to get 80.



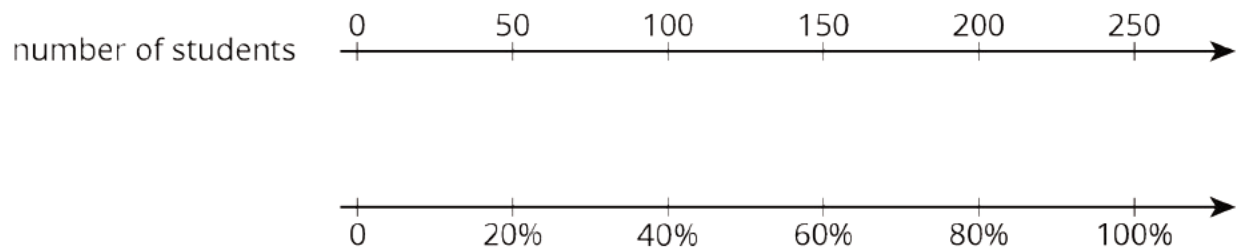
### Problem 3

#### Statement

At a school, 40% of the sixth-grade students said that hip-hop is their favorite kind of music. If 100 sixth-grade students prefer hip hop music, how many sixth-grade students are at the school? Explain or show your reasoning.

#### Solution

250. Explanations vary. Possible explanation:



### Problem 4

#### Statement

Diego has a skateboard, scooter, bike, and go-cart. He wants to know which vehicle is the fastest. A friend records how far Diego travels on each vehicle in 5 seconds. For each vehicle, Diego travels as fast as he can along a straight, level path.

vehicle	distance traveled
skateboard	90 feet
scooter	1,020 inches
bike	4,800 centimeters
go-cart	0.03 kilometers

- What is the distance each vehicle traveled in centimeters?
- Rank the vehicles in order from fastest to slowest.

## Solution

a. Skateboard: 2,743.2. Scooter: 2,590.8. Bike: 4,800. Go-cart: 3,000.

b. Bike, go-cart, skateboard, scooter

(From Unit 3, Lesson 9.)

## Problem 5

### Statement

It takes 10 pounds of potatoes to make 15 pounds of mashed potatoes. At this rate:

a. How many pounds of mashed potatoes can they make with 15 pounds of potatoes?

b. How many pounds of potatoes are needed to make 50 pounds of mashed potatoes?

## Solution

a. To find the amount of mashed potatoes, multiply the amount of potatoes by  $\frac{3}{2}$ ,  $22\frac{1}{2}$  pounds of mashed potatoes (or equivalent).

b. To find the potatoes, multiply the amount of mashed potatoes by  $\frac{2}{3}$ ,  $33\frac{1}{3}$  pounds of potatoes (or equivalent).

(From Unit 3, Lesson 7.)

# Lesson 12: Percentages and Tape Diagrams

## Goals

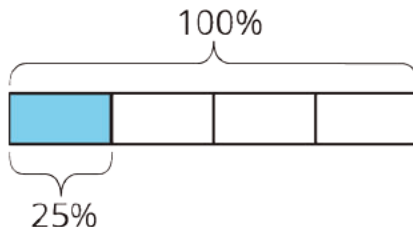
- Choose and create diagrams to solve problems such as  $A\%$  of  $B$  is  $?$  and  $A\%$  of  $?$  is  $C$ .
- Draw and label a tape diagram to represent a situation involving percentages.
- Interpret tape diagrams that represent multiplicative comparisons and express such comparisons using fractions and percentages.

## Learning Targets

- I can use tape diagrams to solve different problems like “What is 40% of 60?” or “60 is 40% of what number?”

## Lesson Narrative

In previous lessons students used double number lines to reason about percentages. Double number lines show different percentages when a given amount is identified as 100%, and emphasize that percentages are a rate per 100. In this lesson they use tape diagrams. Tape diagrams are useful for seeing the connection between percentages and fractions. For example, this tape diagram shows that 25% of a whole is the same as  $\frac{1}{4}$  of that whole by showing that 25% of the whole is one part when 100% of the whole is divided into four equal parts.



Tape diagrams are also useful in solving problems of the form  $A$  is  $B\%$  of  $C$  when you are given two of the numbers and must find the third. When reasoning about percentages, it is important to indicate the whole as 100%, just as it is important to indicate the whole when working with fractions (MP6).

## Alignments

### Addressing

- 6.RP.A.3.c: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

### Building Towards

- 6.RP.A.3.c: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.



## Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports
- Notice and Wonder
- Think Pair Share

## Student Learning Goals

Let's use tape diagrams to understand percentages.

# 12.1 Notice and Wonder: Tape Diagrams

### Warm Up: 5 minutes

The purpose of this warm-up is to elicit the idea that tape diagrams can be used to think about fractions of a whole as percentages of the whole, which will be useful when students interpret and draw tape diagrams in a later activity. While students may notice and wonder many things about these images, the important discussion points are that there are two rectangles of the same length, one of the rectangles is divided into four pieces of equal length, and a percentage is indicated.

### Building Towards

- 6.RP.A.3.c

## Instructional Routines

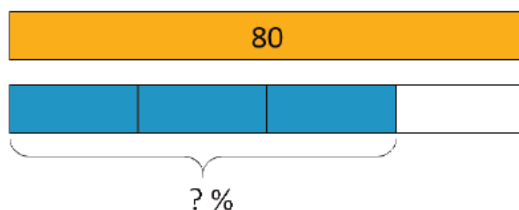
- Notice and Wonder

## Launch

Arrange students in groups of 2. Tell students that they will look at an image, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the image for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

## Student Task Statement

What do you notice? What do you wonder?



## Student Response

Things students may notice:

- It looks like a tape diagram.
- There are two rectangles that are the same length.
- One rectangle is yellow and the other is blue and white.
- One of the rectangles is divided into four pieces of equal length.
- One of the rectangles is labeled 80.

Things students may wonder:

- What do tape diagrams have to do with percentages?
- What do the different colors mean?
- What situation does this represent?
- What percentage should be used in place of the question mark?

## Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After each response, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to the images each time. If the four pieces of equal length do not come up during the conversation, ask students to discuss this idea. It is not necessary to decide what should be used in place of the question mark.

# 12.2 Revisiting Jada's Puppy

15 minutes

The purpose of this activity is for students to study and make sense of tape diagrams that can be used to see benchmark percentages in terms of fractions. The first question shows a percentage as a part of the whole, and the second shows a comparison between two quantities. Both situations can be described in terms of fractions or percentages. The second situation is important for making connections between percentages greater than 100 and fractions greater than 1.

## Building Towards

- 6.RP.A.3.c

## Instructional Routines

- MLR1: Stronger and Clearer Each Time
- Think Pair Share

## Launch

Give students 1 minute of quiet think time, and then have them turn to a partner to discuss the first question. Poll the class to be sure that everyone can see that the puppy is  $\frac{1}{5}$  of its adult weight. Ask the students what 100% is in this situation, and label the diagram with 100%. Give students 1 minute of quiet think time, and then have them discuss the second question with a partner.

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### Support for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Chunk this task into more manageable parts. Check in with students after the first 2-3 minutes of work time. Invite students to share the strategies they have used so far as well as any questions they have before continuing.

*Supports accessibility for: Organization; Attention*

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### Support for English Language Learners

*Writing, Speaking, Listening: MLR1 Stronger and Clearer Each Time.* After students have had the opportunity to think about the first question, ask students to write a brief explanation for how the puppy's current weight as a fraction of its adult weight is represented on the tape diagram. Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language (e.g., "Can you explain how...", "You should expand on...", etc.). Students can borrow ideas and language from each partner to refine and clarify their original explanation. This will help students refine their own explanation and learn about other ways to interpret the tape diagram.

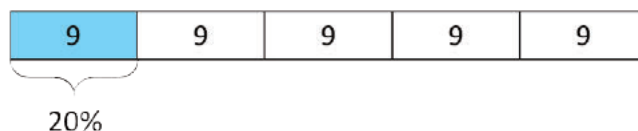
*Design Principle(s): Optimize output (for explanation); Maximize meta-awareness*

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### Student Task Statement

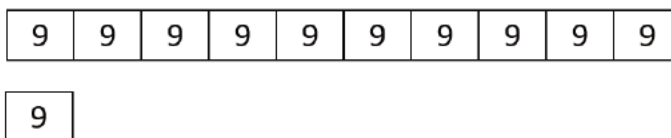
Jada has a new puppy that weighs 9 pounds. It is now at about 20% of its adult weight.

1. Here is a diagram that Jada drew about the weight of her puppy.



- a. The adult weight of the puppy will be 45 pounds. How can you see that in the diagram?
- b. What fraction of its adult weight is the puppy now? How can you see that in the diagram?

2. Jada's friend has a dog that weighs 90 pounds. Here is a diagram Jada drew that represents the weight of her friend's dog and the weight of her puppy.



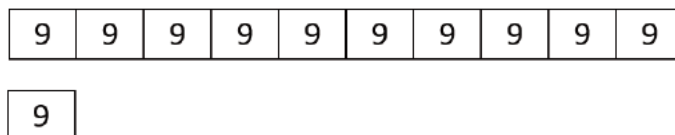
- How many times greater is the dog's weight than the puppy's?
- Compare the weight of the puppy and the dog using fractions.
- Compare the weight of the puppy and the dog using percentages.

### Student Response

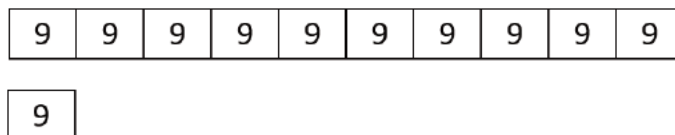
- For the diagram showing 20%.
  - The adult weight will be 45 pounds because there are 5 9's and  $5 \cdot 9 = 45$ .
  - The puppy is  $\frac{1}{5}$  of its adult weight because the whole is divided into 5 pieces of equal length.
- For the diagram showing ten 9's compared with one 9.
  - The dog's weight is 10 times greater than the weight of the puppy.
  - The puppy's weight is  $\frac{1}{10}$  the weight of the dog.
  - The puppy's weight is 10% the weight of the dog.

### Activity Synthesis

Display the second diagram for all to see.



Label the dog's weight with a 1, and ask the students how we should label the puppy's weight if we are comparing using fractions.



Display another copy of the second diagram. Ask students, "When we compare the puppy's weight to the dog's weight, what represents 100%? How should we label the diagram to show it? What should we label the puppy's weight?"

## 12.3 5 Dollars

15 minutes

The purpose of this activity is for students to describe multiplicative comparison problems given in terms of percentages using fractions.

### Addressing

- 6.RP.A.3.c

### Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

### Launch

Give students 3 minutes of quiet work time. Have them turn to a partner to discuss their answer to the first question. Then give them 3 minutes of quiet think time for the second question, followed by a whole-class discussion.

---

#### Support for Students with Disabilities

*Representation: Internalize Comprehension.* Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, ask students to use the same color to represent the amount of money Elena has compared to Noah in a tape diagram, written as a value, and represented as a fraction.

*Supports accessibility for: Visual-spatial processing*

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#### Support for English Language Learners

*Speaking: MLR8 Discussion Supports.* After students have had enough time to work on the first question, and to share their tape diagrams for the first question with a partner, bring the whole class back together. During the discussion, press for details in students' explanations by asking where they see Elena's \$2 and Noah's \$5 represented in the diagram. Use a visual display of the tape diagrams to annotate (or mark) student responses. Since Elena has 40% or  $\frac{2}{5}$  as much money as Noah, ask students where they see 40% or  $\frac{2}{5}$  represented in the tape diagram. As an additional challenge, since Noah has 250% or  $\frac{5}{2}$  as much money as Elena, ask students where they see 250% or  $\frac{5}{2}$  represented in the tape diagram. This will help students make sense of tape diagrams and see the relationship between percentages and fractions.

*Design Principle(s): Support sense-making*

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### Student Task Statement

Noah has \$5.

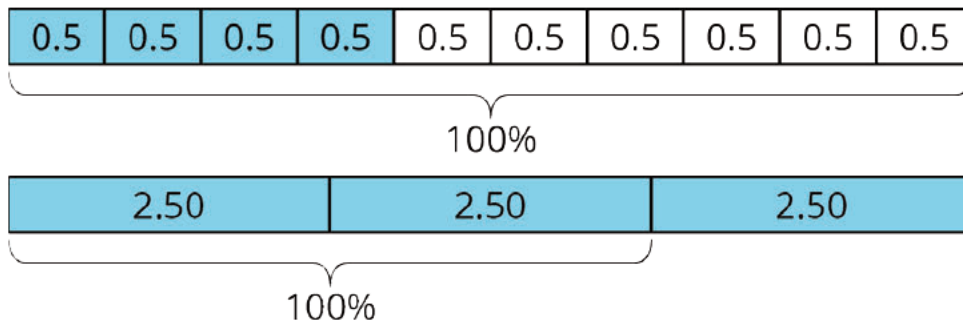
- Elena has 40% as much as Noah. How much does Elena have?
  - Compare Elena's and Noah's money using fractions. Draw a diagram to illustrate.
- Diego has 150% as much as Noah. How much does Diego have?
  - Compare Diego's and Noah's money using fractions. Draw a diagram to illustrate.

### Student Response

- Elena has \$2.
  - Answers vary. Sample responses: Elena has  $\frac{2}{5}$  as much money as Noah; Noah has  $\frac{5}{2}$  as much money as Elena.
- Diego has \$7.50.
  - Answers vary. Sample responses: Diego has  $1\frac{1}{2}$  as much money as Noah; Noah has  $\frac{2}{3}$  as much money as Diego.

### Activity Synthesis

Have students show and explain their diagrams. Then show these if no one has something equivalent:



## 12.4 Staying Hydrated

Optional: 10 minutes

In this activity, students explore percentages that describe parts of a whole. They find both  $B$  and  $C$ , where  $A\%$  of  $B$  is  $C$ , in the context of available and consumed water on a hike.

Students who use a double number line may notice that the value of  $B$  is the same in both questions, so the same double number line can be used to solve both parts of the problem. To solve the second question, however, the diagram needs to be partitioned with more tick marks.

As in the previous task, students may solve using other strategies, including by simply multiplying or dividing, i.e.,  $(1.5) \cdot 2 = 3.0$  and  $\frac{80}{100} \cdot (3.0) = 2.4$ . Encourage them to also explain their reasoning with a double number line, table, or tape diagram. Monitor for at least one student using each of these representations.

### Addressing

- 6.RP.A.3.c

### Instructional Routines

- MLR3: Clarify, Critique, Correct
- Think Pair Share

### Launch

Give students quiet think time to complete the activity and then time to share their explanation with a partner. Follow with a whole-class discussion.

### Anticipated Misconceptions

After students create a double number line diagram with tick marks at 50% and 100%, some may struggle to know how to fit 80% in between. Encourage them to draw and label tick marks at 10% increments or work with a table instead. Some students may think that the second question is asking for the amount of water Andre drank on the second part of the hike. Clarify that it is asking for his total water consumption on the entire hike.

### Student Task Statement

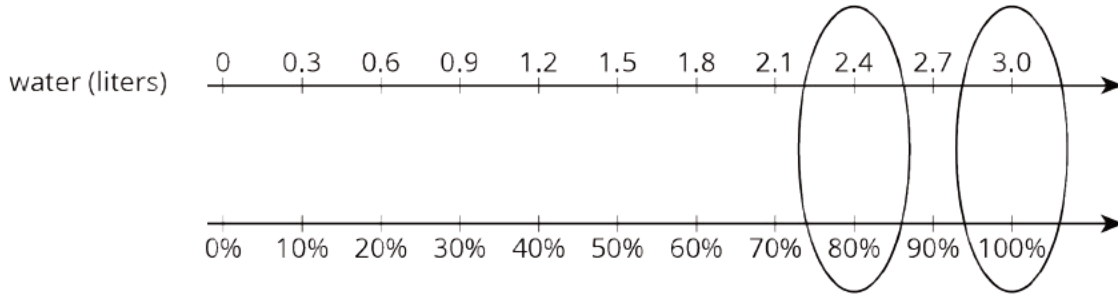
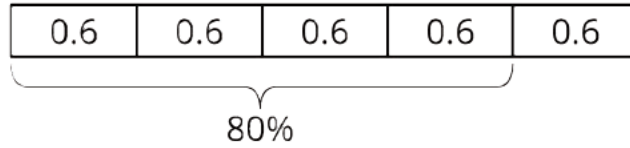
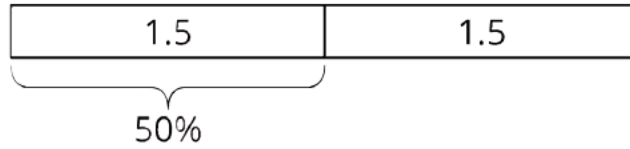
During the first part of a hike, Andre drank 1.5 liters of the water he brought.

1. If this is 50% of the water he brought, how much water did he bring?
2. If he drank 80% of his water on his entire hike, how much did he drink?

### Student Response

1. Andre brought 3 liters of water on the hike.
2. Andre drank 2.4 liters of water.

Possible strategies:



water (liters)	percentage
1.5	50
3.0	100
0.6	20
2.4	80

On the left side of the table, green arrows point from 1.5 to 3.0 (labeled  $\cdot 2$ ), from 3.0 to 0.6 (labeled  $\cdot \frac{1}{5}$ ), and from 0.6 to 2.4 (labeled  $\cdot 4$ ).  
 On the right side of the table, green arrows point from 50 to 100 (labeled  $\cdot 2$ ), from 100 to 20 (labeled  $\cdot \frac{1}{5}$ ), and from 20 to 80 (labeled  $\cdot 4$ ).

### Are You Ready for More?

Decide if each scenario is possible.

- Andre plans to bring his dog on his next hike, along with 150% as much water as he brought on this hike.
- Andre plans to drink 150% of the water he brought on his hike.

### Student Response

- This is possible because it means he will bring  $1\frac{1}{2}$  times as much water next time.
- This is not possible, because it means he will drink more water than he brought. He can only do so if he drinks someone else's water!



## Activity Synthesis

Select 1–2 students who used a double number line, a table of equivalent ratios, and a tape diagram to share their strategies. As students explain, illustrate and display those representations for all to see.

Ask students how they knew what 100% means in the context. At this point it is not necessary for students to formally conceptualize the two ways percentages are used (to describe parts of whole, and to describe comparative relationships). Drawing their attention to concrete and contextualized examples of both, however, serves to build this understanding intuitively.

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### Support for English Language Learners

*Reading, Writing, Speaking: MLR3 Clarify, Critique, Correct.* Before students share their answers for the second question, present an incorrect answer and explanation. For example, “If Andre drank 80% of his water on his entire hike, then he drank 1.2 liters of water because 0.8 times 1.5 liters is 1.2 liters.” Ask students to identify the error, critique the reasoning, and write a correct explanation. As students discuss in partners, listen for students who identify and clarify the assumption in the statement. For example, the author assumed that Andre brought 1.5 liters for his entire hike; however, the problem states that Andre drank 1.5 liters of the water he brought. This will remind students to carefully identify the quantity they are finding a percentage of.

*Design Principle(s): Optimize output (for explanation); Maximize meta-awareness*

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## Lesson Synthesis

If you are comparing two quantities using percentages, you can also compare them using fractions. Drawing a tape diagram can sometimes help us see how to do this more easily.

Questions for discussion:

- If you have 50% of the money needed to buy a book, what fraction is that?
- If you run 125% of your goal for the week, what fraction is that?

Seeing percentages in terms of fractions can help us solve percentage problems.

## 12.5 Small and Large

Cool Down: 5 minutes

### Addressing

- 6.RP.A.3.c

### Student Task Statement

Complete the statement with a situation and a unit of your choice. Then answer the question and draw a diagram.

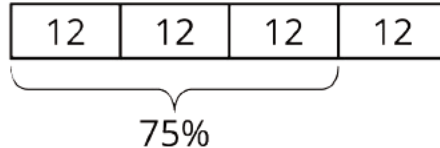
A small \_\_\_\_\_ holds 75% as much as a large \_\_\_\_\_.

1. If the small holds 36 units, how much does the large hold?
2. Draw a diagram to illustrate your answer.

### Student Response

Answers vary. Sample response. A small scoop holds 75% as much as a large scoop.

1. 48 units
2. Answers vary. Sample diagrams:

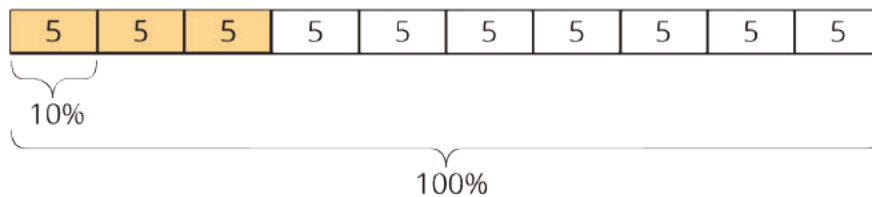


### Student Lesson Summary

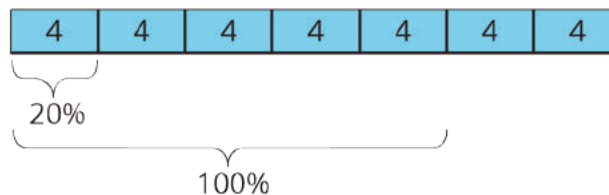
Tape diagrams can help us make sense of percentages.

Consider two problems that we solved earlier using double number lines and tables: “What is 30% of 50 pounds?” and “What is 100% of a number if 140% of it is 28?”

Here is a tape diagram that shows that 30% of 50 pounds is 15 pounds.



This diagram shows that if 140% of some number is 28, then that number must be 20.

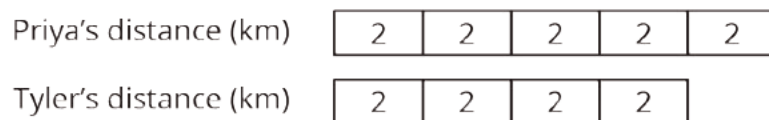


## Lesson 12 Practice Problems

### Problem 1

#### Statement

Here is a tape diagram that shows how far two students walked.



- What percentage of Priya's distance did Tyler walk?
- What percentage of Tyler's distance did Priya walk?

#### Solution

- 80%
- 125%

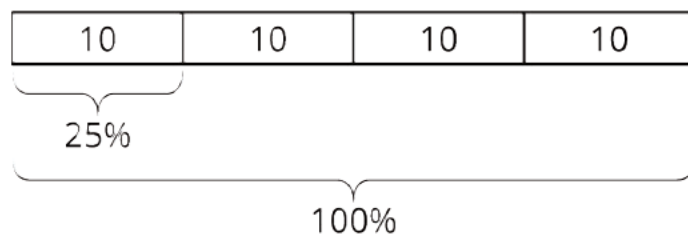
### Problem 2

#### Statement

A bakery makes 40 different flavors of muffins. 25% of the flavors have chocolate as one of the ingredients. Draw a tape diagram to show how many flavors have chocolate and how many don't.

#### Solution

Each unit in the tape diagram represents 25%, so 10 have chocolate and 30 do not.



### Problem 3

#### Statement

There are 70 students in the school band. 40% of them are sixth graders, 20% are seventh graders, and the rest are eighth graders.

- How many band members are sixth graders?
- How many band members are seventh graders?

c. What percentage of the band members are eighth graders? Explain your reasoning.

### Solution

a.  $28$  ( $70 \cdot 0.4 = 28$ )

b.  $14$  ( $70 \cdot 0.2 = 14$ )

c. 40% because the other percentages add up to 60% and that leaves 40%, because  $100 - 60 = 40$ .

### Problem 4

#### Statement

Jada has a monthly budget for her cell phone bill. Last month she spent 120% of her budget, and the bill was \$60. What is Jada's monthly budget? Explain or show your reasoning.

### Solution

\$50. Strategies vary. Sample reasoning: If 120% is 60, then 20% is 10, which I get by multiplying each by  $\frac{1}{6}$ . If 20% is 10, then 100% is 50, which I get by multiplying each by 5.

(From Unit 3, Lesson 11.)

### Problem 5

#### Statement

Which is a better deal, 5 tickets for \$12.50 or 8 tickets for \$20.16? Explain your reasoning.

### Solution

5 tickets for \$12.50 is a better deal. 5 tickets for \$12.50 equals a unit rate of \$2.50 per ticket, ( $12.50 \div 5 = 2.50$ ), and 8 tickets for \$20.16 equals a unit rate of \$2.52 per ticket, ( $20.16 \div 8 = 2.52$ ).

(From Unit 3, Lesson 9.)

### Problem 6

#### Statement

An athlete runs 8 miles in 50 minutes on a treadmill. At this rate:

a. How long will it take the athlete to run 9 miles?

b. How far can the athlete run in 1 hour?

### Solution

a. 56.25 minutes (or equivalent)

b. 9.6 miles (or equivalent)

(From Unit 3, Lesson 8.)

# Lesson 13: Benchmark Percentages

## Goals

- Explain (orally and in writing) how to solve problems involving the percentages 10%, 25%, 50%, and 75% by reasoning about the fractions  $\frac{1}{10}$ ,  $\frac{1}{4}$ ,  $\frac{1}{2}$ , and  $\frac{3}{4}$ .
- Generalize (orally) processes for calculating 10%, 25%, 50%, and 75% of a quantity.

## Learning Targets

- When I read or hear that something is 10%, 25%, 50%, or 75% of an amount, I know what fraction of that amount they are referring to.

## Lesson Narrative

The goal of this lesson is to help students understand the connection between benchmark percentages and common fractions (MP7). In these materials, we have identified 10%, 25%, 50%, and 75% as primary benchmark percentages and multiples of 10% as secondary benchmark percentages.

It is common to say that  $25\% = \frac{1}{4}$  or  $10\% = \frac{1}{10}$ . In these materials we avoid this usage and say rather that 25% of a quantity is  $\frac{1}{4}$  of that quantity, or that 10% of a quantity is  $\frac{1}{10}$  of that quantity.

This lesson builds on understanding of equivalent fractions, multiplying fractions, and dividing by unit fractions from grades 4 and 5.

## Alignments

### Building On

- 4.NF.B: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
- 5.NF.B: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

### Addressing

- 6.RP.A.3.c: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

### Building Towards

- 6.RP.A.3.c: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

### Instructional Routines

- MLR1: Stronger and Clearer Each Time

- MLR3: Clarify, Critique, Correct
- Think Pair Share

### Student Learning Goals

Let's contrast percentages and fractions.

## 13.1 What Percentage Is Shaded?

### Warm Up: 5 minutes

In this warm-up, students are presented with tape diagrams with a shaded portion, and they identify the percentage that is shaded.

### Building On

- 4.NF.B

### Building Towards

- 6.RP.A.3.c

### Instructional Routines

- Think Pair Share

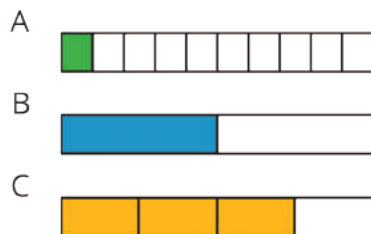
### Launch

Display the image in the task statement for all to see, and ask students to think of at least one thing they notice. Ask a few students to share something they notice. It is likely that students notice there are three tape diagrams of the same length, but the first is divided into 10 equal pieces, the second into 2 equal pieces, and the last into 4 equal pieces. Students may make claims about what fraction of each tape is shaded. Remind students that when we wonder, "What percent of something is shaded?" it is understood that the whole thing is 100%.

Arrange students in groups of 2. Give 1–2 minutes of quiet think time, followed by partner- and whole-class discussions.

### Student Task Statement

What percentage of each diagram is shaded?



### Student Response

A. 10%

B. 50%

C. 75%

### Activity Synthesis

Invite students to share how they reasoned about the percentage of each tape diagram that is shaded. Record and display their explanations for all to see. Highlight alternative ways of naming the size of the shaded portion, for example, " $\frac{3}{4}$  of the diagram" and "75% of the diagram."

## 13.2 Liters, Meters, and Hours

### 15 minutes

In this activity, students calculate three different benchmark percentages—50%, 10%, and 75%—given three different values that correspond to 100%. The repetition of the benchmark percentages allows students to notice regularity and engage in MP8. They generalize the patterns in their calculations to determine how to find those percentages when the 100% value is  $x$ .

### Building On

- 5.NF.B

### Addressing

- 6.RP.A.3.c

### Instructional Routines

- MLR1: Stronger and Clearer Each Time
- Think Pair Share

### Launch

Ask students to complete the first three sub-questions of each problem mentally. If necessary, clarify that "using mental math" means working out an answer without writing down their calculations and just recording the answer. For the last sub-question, ask them to write a sentence or two to explain their approach. Give students quiet think time to complete the activity and then time to share their explanation with a partner.

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### Support for Students with Disabilities

*Representation: Internalize Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.

*Supports accessibility for: Memory; Organization*

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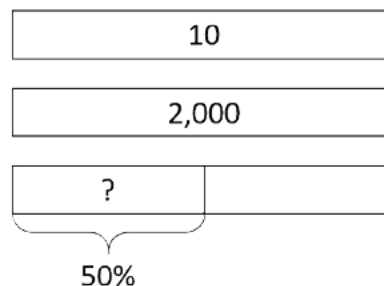


## Anticipated Misconceptions

If students struggle to get started with mental math, offer some scaffolding. For example, ask, “How much of 100% is 50%?” Suggest to students that if they know that 50% of a number is the same as  $\frac{1}{2}$  of that number, then they can think about what  $\frac{1}{2}$  of the number is.

### Student Task Statement

- How much is 50% of 10 liters of milk?
  - How far is 50% of a 2,000-kilometer trip?
  - How long is 50% of a 24-hour day?
  - How can you find 50% of any number?
- How far is 10% of a 2,000-kilometer trip?
  - How much is 10% of 10 liters of milk?
  - How long is 10% of a 24-hour day?
  - How can you find 10% of any number?
- How long is 75% of a 24-hour day?
  - How far is 75% of a 2,000-kilometer trip?
  - How much is 75% of 10 liters of milk?
  - How can you find 75% of any number?



### Student Response

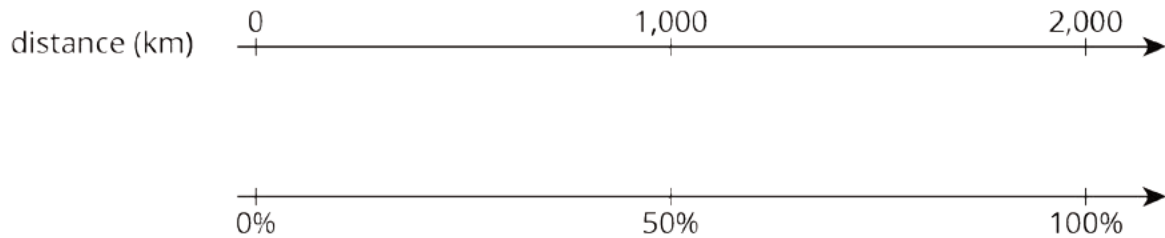
- 5 liters of milk
  - 1,000 kilometers
  - 12 hours
  - Divide by 2, or multiply by  $\frac{1}{2}$ .
- 200 kilometers
  - 1 liter of milk
  - 2.4 hours
  - Divide by 10, or multiply by  $\frac{1}{10}$ .
- 18 hours
  - 1,500 kilometers
  - 7.5 liters of milk

d. Divide by 4, and multiply by 3 (or multiply by  $\frac{3}{4}$ ).

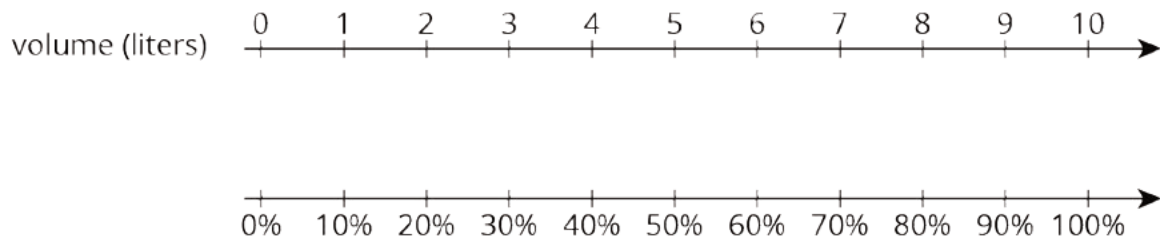
### Activity Synthesis

Highlight the following:

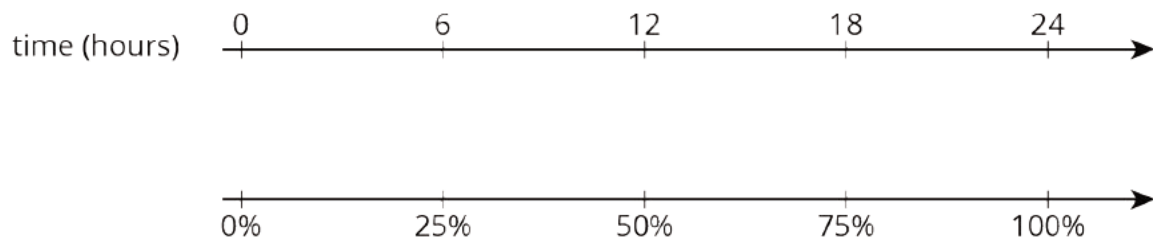
50% of a quantity is  $\frac{1}{2}$  of that quantity. We can calculate it by dividing the quantity by 2 or multiplying the quantity by  $\frac{1}{2}$ . If no students bring up the multiplication method, ask what fraction 50% reminds them of or what number they could multiply by to get the same answer; either  $\frac{1}{2}$  or 0.5 is fine.



10% of a quantity is  $\frac{1}{10}$  of that quantity. You can calculate it by dividing the quantity by 10, or multiplying the quantity by  $\frac{1}{10}$ .



75% of a quantity is  $\frac{3}{4}$  of that quantity. You can calculate it by dividing the quantity by four and then multiplying by 3, or by multiplying the quantity by  $\frac{3}{4}$ .



---

### Support for English Language Learners

*Writing, Speaking, Listening: MLR1 Stronger and Clearer Each Time.* After providing some independent think time, use this routine with successive pair shares to give students a structured opportunity to revise and refine their explanations for how to find 75% of any number. Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language (e.g., “Can you explain how...”, “You should expand on...”, etc.). Students can borrow ideas and language from each partner to refine and clarify their original explanation. This will help students refine their own explanation and learn about other strategies to find 75% of any number.

*Design Principles(s): Optimize output (for explanation); Maximize meta-awareness*

---

## 13.3 Nine is . . .

### 10 minutes

In this activity students find the values for 100% given different benchmark percentages. Students are likely to calculate the answers quickly. They are to spend the majority of the task time discussing how they reason about the questions.

### Building On

- 5.NF.B

### Addressing

- 6.RP.A.3.c

### Instructional Routines

- MLR3: Clarify, Critique, Correct
- Think Pair Share

### Launch

Give students quiet think time to complete the activity and then time to share their explanations with a partner.

---

### Support for Students with Disabilities

*Representation: Internalize Comprehension.* Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.

*Supports accessibility for: Memory; Conceptual processing*

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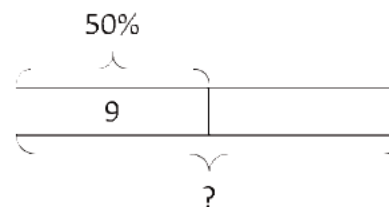
### Anticipated Misconceptions

Students may think 9 is 100% and try to find the percentage of the number given. You can use the diagram in the task statement to help them make sense of the first question and then encourage them to use mental math, or draw additional diagrams, to solve the other three questions.

### Student Task Statement

Explain how you can calculate each value mentally.

1. 9 is 50% of what number?
2. 9 is 25% of what number?
3. 9 is 10% of what number?
4. 9 is 75% of what number?
5. 9 is 150% of what number?



### Student Response

Answers vary. Sample responses:

1. Because 50% of a quantity is  $\frac{1}{2}$  of that quantity, I can multiply  $9 \cdot 2 = 18$ .
2. Because 25% of a quantity is  $\frac{1}{4}$  of that quantity, I can multiply  $9 \cdot 4 = 36$ .
3. Because 10% of a quantity is  $\frac{1}{10}$  of that quantity, I can multiply  $9 \cdot 10 = 90$ .
4. Because 75% of a quantity is  $\frac{3}{4}$  of that quantity, I can divide  $9 \div 3 = 3$  to find  $\frac{1}{4}$  of the quantity and then multiply  $3 \cdot 4 = 12$ .
5. Because 150% of a quantity is  $\frac{3}{2}$  of that quantity, I can divide  $9 \div 3 = 3$  to find  $\frac{1}{2}$  of the quantity and then multiply  $3 \cdot 2 = 6$ .

### Activity Synthesis

Invite students to share different strategies for answering the questions, with or without using fractions. For example, some may multiply the given 9 by 2, 4, and 10, respectively, to find values of 100% given the values of 50%, 25%, and 10% in the first three problems. Highlight any fractions students used to make sense of and solve the problems.

---

### Support for English Language Learners

*Reading, Writing, Speaking: MLR3 Clarify, Critique, Correct.* Before students share their answers for the first question, present an incorrect answer and explanation. For example, “9 is 50% of 4.5 because 9 times  $\frac{1}{2}$  is 4.5.” Ask students to identify the error, critique the reasoning, and write a correct explanation. Encourage students to use the tape diagram in the task statement to make sense of the question. As students discuss in partners, listen for students who clarify what it means for a quantity to be a percentage of another quantity. Also, listen for students who state that 50% of quantity is  $\frac{1}{2}$  of that quantity. Prompt students to share their critiques and corrected explanations with the class. This routine will engage students in meta-awareness as they clarify the meaning of the statement “9 is 50% of a number” as “9 is  $\frac{1}{2}$  of that number”.

*Design Principle(s): Optimize output (for explanation); Maximize meta-awareness*

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## 13.4 Matching the Percentage

**Optional: 10 minutes**

In this activity, students calculate benchmark percentages. Students are likely to calculate the answers quickly. They are to spend the majority of the task time discussing how they reason about the questions.

### Building On

- 5.NF.B

### Addressing

- 6.RP.A.3.c

### Instructional Routines

- Think Pair Share

### Launch

Give students quiet think time to complete the activity and then time to share their explanation with a partner.

---

### Support for Students with Disabilities

*Representation: Internalize Comprehension.* Provide appropriate reading accommodations and supports to ensure students access to written directions, word problems and other text-based content.

*Supports accessibility for: Language; Conceptual processing*

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### Anticipated Misconceptions

Because 5 goes into 20 four times, students may answer that 5 is 4% of 20. If this happens, explain that 5 is really  $\frac{1}{4}$  of 20 and ask them what percentage represents one quarter.

### Student Task Statement

Match the percentage that describes the relationship between each pair of numbers. One percentage will be left over. Be prepared to explain your reasoning.

- |                                |        |
|--------------------------------|--------|
| 1. 7 is what percentage of 14? | • 4%   |
| 2. 5 is what percentage of 20? | • 10%  |
| 3. 3 is what percentage of 30? | • 25%  |
| 4. 6 is what percentage of 8?  | • 50%  |
| 5. 20 is what percentage of 5? | • 75%  |
|                                | • 400% |

### Student Response

- 50%. 7 is half of 14, so 7 is 50% of 14.
- 25%. 5 times 4 is 20, so 5 is  $\frac{1}{4}$  (or 25%) of 20.
- 10%. 3 times 10 is 30, so 3 is  $\frac{1}{10}$  (or 10%) of 30.
- 75%. 2 is  $\frac{1}{4}$  of 8, so 6 is  $\frac{3}{4}$  (or 75%) of 8.
- 400%. 5 is 100% of 5, and 20 is 4 times that, so 20 is 400% of 5.

### Are You Ready for More?

- What percentage of the world's current population is under the age of 14?
- How many people is that?
- How many people are 14 or older?

### Student Response

Answers may vary depending on the year.

- 25% (as of 2017)
- 1.875 billion people are under 14 years old (as of 2017)
- 5.625 billion people are over 14 years old (as of 2017)

## Activity Synthesis

Invite students to share their mental math strategies. If needed, consider illustrating the relationship between  $B$  and  $C$  (where  $A\%$  of  $B$  is  $C$ ) using tape diagrams or double number line diagrams to help students visualize the association between the benchmark percentages and fractions.

## Lesson Synthesis

Certain percentages are easy to think about in terms of fractions. Ask students how they can think about each benchmark percentage by using a fraction. Demonstrate the correspondences using a double number line, tape diagram, or a table (as shown).

- 25% of a number is always  $\frac{1}{4}$  of that number.
- 50% of a number is always  $\frac{1}{2}$  of that number.
- 75% of a number is always  $\frac{3}{4}$  of that number
- 10% of a number is always  $\frac{1}{10}$  of that number.

value	percentage
$x$	100
$\frac{1}{4}x$	25
$\frac{1}{2}x$	50
$\frac{3}{4}x$	75

## 13.5 Around the Clock

### Cool Down: 5 minutes

In this activity, students find  $C$  (where  $A\%$  of  $B$  is  $C$ ), given benchmark percentages and a single value for  $B$  in the context of telling time.

### Addressing

- 6.RP.A.3.c

### Student Task Statement

Answer each question and explain your reasoning.

1. How long is 50% of 60 minutes?
2. How long is 10% of 60 minutes?

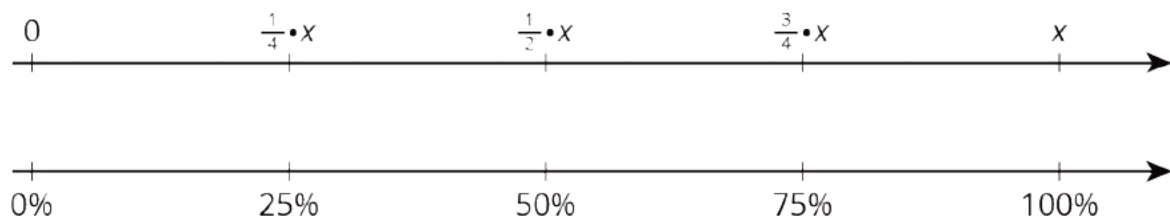
3. How long is 75% of 60 minutes?

### Student Response

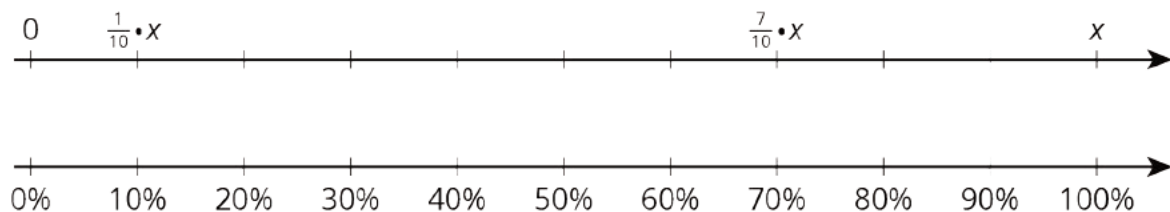
1. 30 minutes because it is  $\frac{1}{2}$  of an hour
2. 6 minutes because it is  $\frac{1}{10}$  of an hour
3. 45 minutes because it is  $\frac{3}{4}$  of an hour

### Student Lesson Summary

Certain percentages are easy to think about in terms of fractions.



- 25% of a number is always  $\frac{1}{4}$  of that number.  
For example, 25% of 40 liters is  $\frac{1}{4} \cdot 40$  or 10 liters.
- 50% of a number is always  $\frac{1}{2}$  of that number.  
For example, 50% of 82 kilometers  $\frac{1}{2} \cdot 82$  or 41 kilometers.
- 75% of a number is always  $\frac{3}{4}$  of that number.  
For example, 75% of 1 pound is  $\frac{3}{4}$  pound.
- 10% of a number is always  $\frac{1}{10}$  of that number.  
For example, 10% of 95 meters is 9.5 meters.
- We can also find multiples of 10% using tenths.  
For example, 70% of a number is always  $\frac{7}{10}$  of that number, so 70% of 30 days is  $\frac{7}{10} \cdot 30$  or 21 days.





## Lesson 13 Practice Problems

### Problem 1

#### Statement

- How can you find 50% of a number quickly in your head?
- Andre lives 1.6 km from school. What is 50% of 1.6 km?
- Diego lives  $\frac{1}{2}$  mile from school. What is 50% of  $\frac{1}{2}$  mile?

#### Solution

- Answers vary. Sample response: Divide the number by 2 (or multiply it by  $\frac{1}{2}$ ).
- 0.8 km (or equivalent)
- $\frac{1}{4}$  mile (or equivalent)

### Problem 2

#### Statement

There is a 10% off sale on laptop computers. If someone saves \$35 on a laptop, what was its original cost? If you get stuck, consider using the table.

savings (dollars)	percentage
35	10
?	100

#### Solution

\$350

### Problem 3

#### Statement

Explain how to calculate these mentally.

- 15 is what percentage of 30?
- 3 is what percentage of 12?
- 6 is what percentage of 10?

## Solution

Answers vary. Sample response:

- 50%. 15 is  $\frac{1}{2}$  of 30, so that is 50%.
- 25%. 3 is  $\frac{1}{4}$  of 12, so that is 25%.
- 60%.  $\frac{6}{10}$  is the same as  $\frac{3}{5}$ , and each  $\frac{1}{5}$  is 20%.

## Problem 4

### Statement

Noah says that to find 20% of a number he divides the number by 5. For example, 20% of 60 is 12, because  $60 \div 5 = 12$ . Does Noah's method always work? Explain why or why not.

### Solution

Yes. Answers vary. Sample response: 20% of a number is  $\frac{20}{100}$  times the number and  $\frac{20}{100} = \frac{1}{5}$ . Multiplying by  $\frac{1}{5}$  gives the same result as dividing by 5.

## Problem 5

### Statement

Diego has 75% of \$10. Noah has 25% of \$30. Diego thinks he has more money than Noah, but Noah thinks they have an equal amount of money. Who is right? Explain your reasoning.

### Solution

They each have \$7.50 ( $10 \cdot 0.75 = 7.50$  and  $30 \cdot 0.25 = 7.50$ ).

(From Unit 3, Lesson 10.)

## Problem 6

### Statement

Lin and Andre start walking toward each other at the same time from opposite ends of 22-mile walking trail. Lin walks at a speed of 2.5 miles per hour. Andre walks at a speed of 3 miles per hour.

Here is a table showing the distances traveled and how far apart Lin and Andre were over time. Use the table to find how much time passes before they meet.

elapsed time (hour)	Lin's distance (miles)	Andre's distance (miles)	distance apart (miles)
0	0	0	22
1	2.5	3	16.5
			0

## Solution

4 hours. Possible strategy:

elapsed time (hour)	Lin's distance (miles)	Andre's distance (miles)	distance apart (miles)
0	0	0	22
1	2.5	3	16.5
2	5	6	11
3	7.5	9	5.5
4	10	12	0

(From Unit 3, Lesson 8.)

# Lesson 14: Solving Percentage Problems

## Goals

- Choose and create a tape diagram, double number line diagram, or table to solve problems involving percentages and explain (orally) the solution method.
- Determine what information is needed to solve a problem involving percentages. Ask questions to elicit that information.

## Learning Targets

- I can choose and create diagrams to help me solve problems about percentages.

## Lesson Narrative

In previous lessons, students saw that a percentage is a rate per 100. They were provided with double number line diagrams to develop this understanding and to solve problems involving percentages. In this lesson, students solve similar problems but with less support. Because double number lines are not provided, students have opportunities to choose approaches that seem appropriate. Drawing a double number line is still a good strategy, but students may opt for tables or even more abbreviated reasoning methods.

## Alignments

### Building On

- 5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
- 6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

### Addressing

- 6.RP.A.3.c: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

### Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR4: Information Gap Cards
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Number Talk

## Required Materials

Pre-printed slips, cut from copies of the **blackline master**

## Required Preparation

You will need the Info Gap: Music Devices blackline master for this lesson. Make 1 copy for every 4 students, and cut them up ahead of time.

### Student Learning Goals

Let's solve more percentage problems.

# 14.1 Number Talk: Multiplication with Decimals

## Warm Up: 10 minutes

This number talk encourages students to rely on what they know about structure, patterns, decimal multiplication, and properties of operations to solve a problem mentally. Only two problems are given here so there is time to share many strategies and make connections between them.

## Building On

- 5.NBT.B.7

## Instructional Routines

- MLR8: Discussion Supports
- Number Talk

## Launch

Display one problem at a time. Give students 1 minute of quiet think time per problem, and ask students to give a signal when they have an answer and a strategy. Follow with a whole-class discussion.

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### Support for Students with Disabilities

*Representation: Internalize Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.

*Supports accessibility for: Memory; Organization*

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### Student Task Statement

Find the products mentally.

$$6 \cdot (0.8) \cdot 2$$

$$(4.5) \cdot (0.6) \cdot 4$$

### Student Response

- 9.6. Possible strategies:  $6 \cdot 2 = 12$  and  $\frac{8}{10} \cdot 12 = 9.6$  (or  $6 \cdot (0.8) = 4.8$  and  $(4.8) \cdot 2 = 9.6$ )
- 10.8. Possible strategies:  $(4.5) \cdot 4 = 18$  and  $\frac{6}{10} \cdot 18 = 10.8$  (or  $(0.6) \cdot 4 = 2.4$  and  $(2.4) \cdot (4.5) = 10.8$ )

### Activity Synthesis

Ask students to share their strategies for each problem. Record and display their explanations for all to see. If not mentioned by students, ask if or how the given factors impacted their strategy choice. To involve more students in the conversation, consider asking:

- Who can restate \_\_\_'s reasoning in a different way?
- Did anyone solve the problem the same way but would explain it differently?
- Did anyone solve the problem in a different way?
- Does anyone want to add on to \_\_\_\_'s strategy?
- Do you agree or disagree? Why?

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### Support for English Language Learners

*Speaking: MLR8 Discussion Supports.*: Display sentence frames to support students when they explain their strategy. For example, "First, I \_\_\_\_ because . . ." or "I noticed \_\_\_\_ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

*Design Principle(s): Optimize output (for explanation)*

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## 14.2 Coupons

### 10 minutes

In this activity, students solve percentage problems in the context of shopping. Students consider how discounts described as percentages translate into reduced prices and the other way around. (In other words, they find  $A$  and  $C$  where  $A\%$  of  $B$  is  $C$ ). In each problem, students need to first determine what value is associated with 100% and reason accordingly.

Students may choose to use double number lines or tables or simply reason without using a particular representation. For instance, to find 10% of \$15 they may first find 50% by dividing \$15 by 2, and then divide the resulting \$7.50 by 5 to obtain 10% of \$15. Those who have internalized the structure of percentages may be able to multiply or divide even more efficiently (e.g.,  $15 \cdot \frac{1}{10} = 1.5$ , or  $15 \div 10 = 1.5$ ). These strategies will be investigated more in the next lesson. For now, encourage students to also explain their reasoning with a double number line or table.

As students work, identify students who use different strategies so that they can share later.

### Building On

- 6.RP.A.3

### Addressing

- 6.RP.A.3.c

### Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect

### Launch

Display some coupons for all to see. Point out that some coupons specify amounts to be taken off in dollars (e.g., \$5 off) and some specify percentages (e.g., 10% off). Tell students that they will solve a couple of shopping problems that involve discounts.



Give students quiet think time to complete the activity and then time to share their explanation with a partner.

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### Support for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, "First, I \_\_\_ because...", "I noticed \_\_\_ so I...", and "How did you get...?"

*Supports accessibility for: Language; Social-emotional skills*

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### Anticipated Misconceptions

Since the first question asks students to find the dollar amount, some students may think that \$6 is the answer to the second question and not realize that it is asking them to find the percentage. Also, some students may try to find the sale price on the first question and the percentage of the sale price on the second question, instead of the discount and the percentage of the discount. Encourage them to revisit the questions or clarify what the questions ask.

### Student Task Statement

Han and Clare go shopping, and they each have a coupon. Answer each question and show your reasoning.

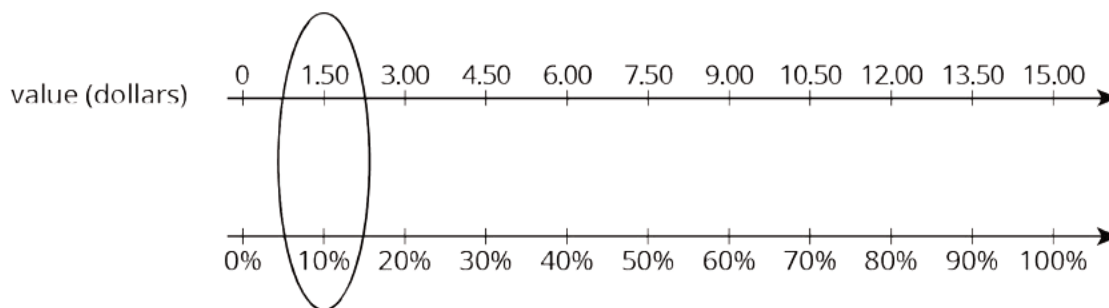
1. Han buys an item with a normal price of \$15, and uses a 10% off coupon. How much does he save by using the coupon?



2. Clare buys an item with a normal price of \$24, but saves \$6 by using a coupon. For what percentage off is this coupon?

### Student Response

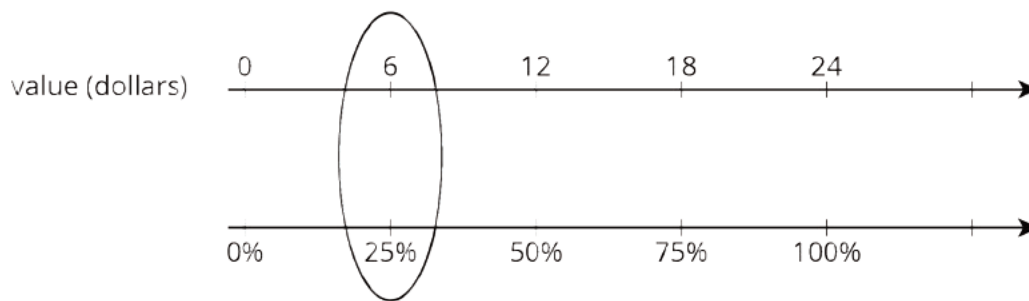
1. \$1.50. Possible strategies:  $\frac{10}{100} \cdot 15 = 1.50$  or  $(0.10) \cdot 15 = 1.50$



value (dollars)	percentage
15	100
1.5	10

2. 25%. Possible strategies:  $6 \div 24 = \frac{25}{100}$  or  $6 \div 24 = 0.25$





value (dollars)	percentage
24	100
6	25

### Are You Ready for More?

Clare paid full price for an item. Han bought the same item for 80% of the full price. Clare said, "I can't believe I paid 125% of what you paid, Han!" Is what she said true? Explain.

### Student Response

Yes. Han paid 80% or  $\frac{4}{5}$  what Clare paid. So Clare paid  $\frac{5}{4}$  or 125% of what Han paid.

### Activity Synthesis

Select students who used different representations: first a tape diagram, then a double number line or a table (or both, time permitting). As students explain, illustrate and display those representations for all to see. If no students mention using a double number line or a table, demonstrate at least one of these methods. When discussing double number lines (or tables), ask students if the same double number line (or table) could be used to solve both problems and discuss why not. Emphasize the fact that two separate double number lines (or tables) are necessary because the value for 100% is different in each case.

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### Support for English Language Learners

*Speaking, Listening: MLR7 Compare and Connect.* As students share how they made sense of the first question, make sure you hear from students with different strategies for finding the dollar amount Han saves by using the coupon. Some students may find 10% of \$15 by multiplying 15 by  $\frac{1}{10}$  or dividing 15 by 10. Others may draw a double number line where \$15 corresponds with 100% and figure out the dollar amount that corresponds with 10%. Others may use a table and reason that 10% is  $\frac{1}{10}$  of 100%, so the amount Han saves must be  $\frac{1}{10}$  of \$15.

Encourage students to make comparisons and connections between the various representations of the situation. Ask questions such as, “What is especially clear in this representation?” and “Where do you see the product of 15 and  $\frac{1}{10}$  represented in the diagram?” This will foster students’ meta-awareness and support constructive conversations as they compare and connect the various ways to find a percentage of a quantity.

*Design Principles(s): Cultivate conversation; Maximize meta-awareness*

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## 14.3 Info Gap: Music Devices

### 20 minutes

In this info gap activity, students find both  $A$  and  $C$  (where  $A\%$  of  $B$  is  $C$ ) in the context of buying a music device. The value of  $B$  is different in each of the two questions about the music device, so students who choose to draw diagrams or tables need to draw two. When answering the second question—expressing \$24 as a percentage of \$25—students may notice that drawing a complete double number line diagram with all 25 tick marks is rather time consuming. Encourage students to look for and make use of any noticeable structure (MP7) or pattern (MP8) to help them solve more efficiently.

Some students may, for example, see that if \$25 corresponds to 100%, then each dollar—and thus each tick mark—is  $100 \div 25$  or 4. They can then bypass drawing the rest of the tick marks and simply multiply  $4 \cdot 24$  to obtain 96. Some may notice that \$24 is \$1 away from \$25 and simply subtract the corresponding percentage from 100% ( $100 - 4 = 96$ ). A table provides another efficient structure for reasoning about this. Monitor for students who use different strategies.

The info gap structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need (MP1). It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need (MP6).

Here is the text of the cards for reference and planning:

Info Gap: Music Devices

### Problem Card 1

A store sells 3 different music devices: Device A, Device B, and Device C.

1. Which of the devices can Jada afford?
2. What percentage of the money needed for Device B does she have?

Info Gap: Music Devices

### Data Card 1

- Device A costs \$15.
- Device B costs \$25.
- Device C costs \$40.
- Jada has 60% of the money needed to buy Device C.

Info Gap: Music Devices

### Problem Card 2

The store starts selling another music device. Jada is interested in Device D, though she does not have enough money to buy it. How much does Device D cost?

Info Gap: Music Devices

### Data Card 2

- Jada has \$24.
- Jada has 40% of the money needed to buy Device D.

Note: If time is short, the second set of cards can be considered optional. It would be better for students to thoroughly understand one of these problems than to rush through both of them with less understanding.

## Addressing

- 6.RP.A.3.c

## Instructional Routines

- MLR4: Information Gap Cards

## Launch

Arrange students in groups of 2. In each group, distribute the first problem card to one student and a data card to the other student. After debriefing on the first problem, distribute the cards for the second problem, in which students switch roles.

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### Support for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Display or provide students with a physical copy of the written directions. Check for understanding by inviting students to rephrase directions in their own words. Keep the display of directions visible throughout the activity.

*Supports accessibility for: Memory; Organization*

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## Support for English Language Learners

*Conversing:* This activity uses MLR4 Information Gap to give students a purpose for discussing information necessary to solve problems involving percentages. Display questions or question starters for students who need a starting point such as: “Can you tell me . . . (specific piece of information)”, and “Why do you need to know . . . (that piece of information)?”

*Design Principle(s): Cultivate Conversation*

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### Student Task Statement

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

Continue to ask questions until you have enough information to solve the problem.

4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.
2. Ask your partner “*What specific information do you need?*” and wait for them to *ask* for information.

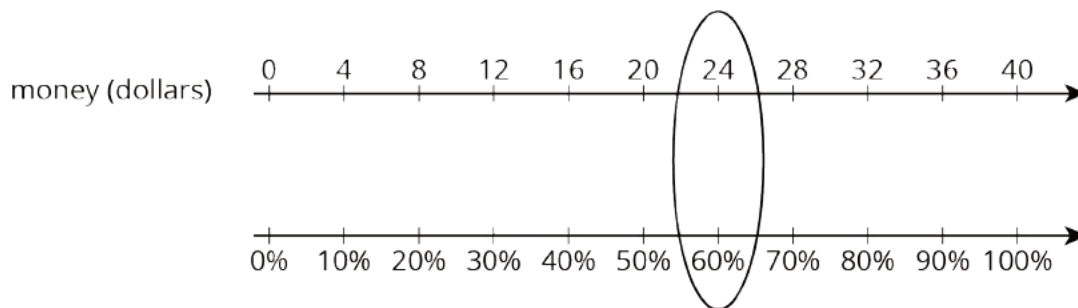
If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don’t have that information.

3. Before sharing the information, ask “*Why do you need that information?*” Listen to your partner’s reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

### Student Response

For Problem Card 1:

1. Jada can only afford Device A because 60% of \$40 is \$24. Possible strategy:



2. Jada has 96% of the money needed for Device B. Possible strategy: Set up a table, and reason that since 100 is  $25 \cdot 4$ , we also multiply 24 by 4.

money (dollars)	percentage
25	100
1	4
24	96

For Problem Card 2:

\$60. Sample reasoning: if \$24 is 40%, then \$6 is 10%. Therefore, \$60 is 100%.

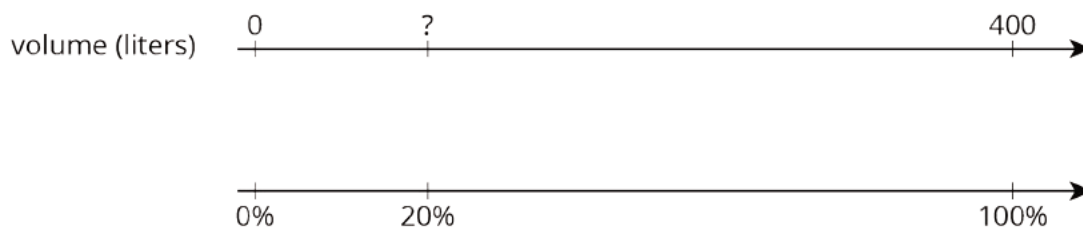
### Activity Synthesis

Select students with different strategies to share their approaches to the first question, starting with less efficient methods and ending with more efficient methods. Then, ask the class to predict how the same strategies might be used to solve the second question, and how the second problem could be solved more quickly.

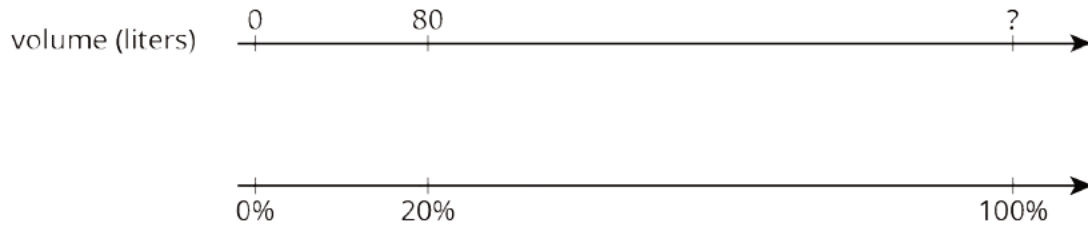
### Lesson Synthesis

We know that 20% of 400 liters is 80 liters. There are three different questions we can ask:

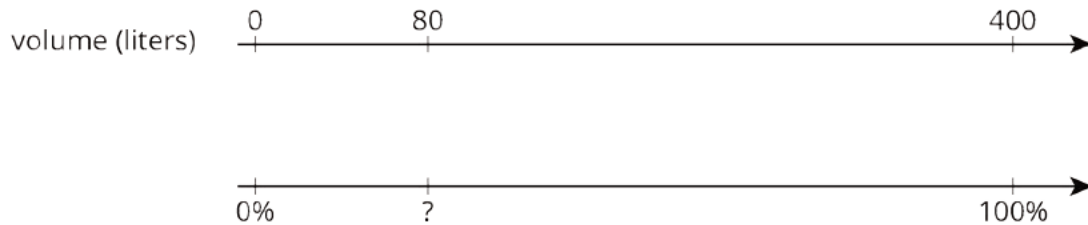
1. What is 20% of a 400 liter tank?



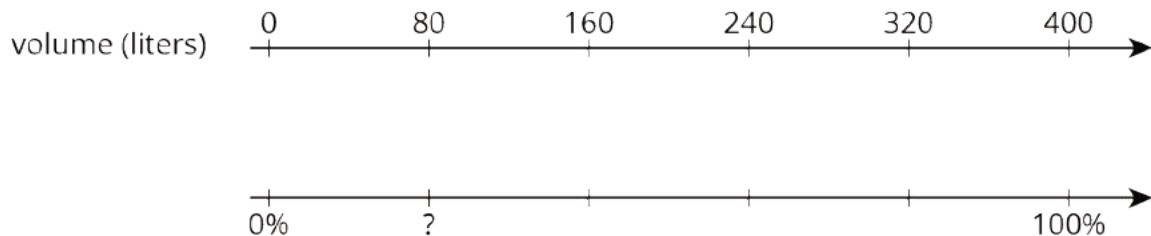
2. 20% of a full tank is 80 liters. How many liters are in a full tank?



3. 80 liters is what percentage of a 400 liter tank?



We can use a double number line to answer all three of these questions. In a previous lesson, we learned how to solve the first two kinds of problems. In this lesson, we also addressed the third kind of problem. We can begin solving by asking ourselves: what fraction of 400 is 80? The answer can tell us how many tick marks to place on the number line or how to divide the segment between 0 and 100%.  $80 \div 400 = \frac{1}{5}$ , so we can divide segment between 0 and 100% into five equal parts, as shown here:



Since  $\frac{1}{5}$  of 100 is 20, we know the percentage is 20. We can also use a table:

volume (liters)	percentage
400	100
80	20

## 14.4 Walking to School

Cool Down: 5 minutes

### Addressing

- 6.RP.A.3.c

### Student Task Statement

It takes Jada 20 minutes to walk to school. It takes Andre 80% as long to walk to school.

How long does it take Andre to walk to school?

### Student Response

16 minutes. Possible strategies:

- 10% of 20 minutes is 2 minutes.  $8 \cdot 2 = 16$ , so it takes 16 minutes for Andre to walk to school.
- Using a table:

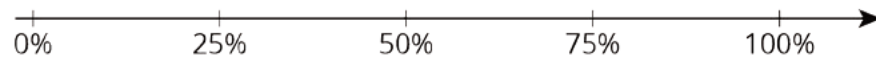
time (minutes)	percentage
20	100
2	10
16	80

### Student Lesson Summary

A pot can hold 36 liters of water. What percentage of the pot is filled when it contains 9 liters of water?

Here are two different ways to solve this problem:

- Using a double number line:



We can divide the distance between 0 and 36 into four equal intervals, so 9 is  $\frac{1}{4}$  of 36, or 9 is 25% of 36.

- Using a table:

volume (liters)	percentage
36	100
9	25

$\cdot \frac{1}{4}$  (left arrow)       $\cdot \frac{1}{4}$  (right arrow)

## Lesson 14 Practice Problems

### Problem 1

#### Statement

For each problem, explain or show your reasoning.

- 160 is what percentage of 40?
- 40 is 160% of what number?
- What number is 40% of 160?

#### Solution

Reasoning varies. Sample responses:

- 400%, because  $4 \cdot 40 = 160$ .
- 25, because  $40 \div 8 = 5$  is 20% of that number, and  $5 \cdot 5 = 25$  is 100% of that number.
- 64, because 10% of 160 is 16, and  $4 \cdot 16 = 64$ .

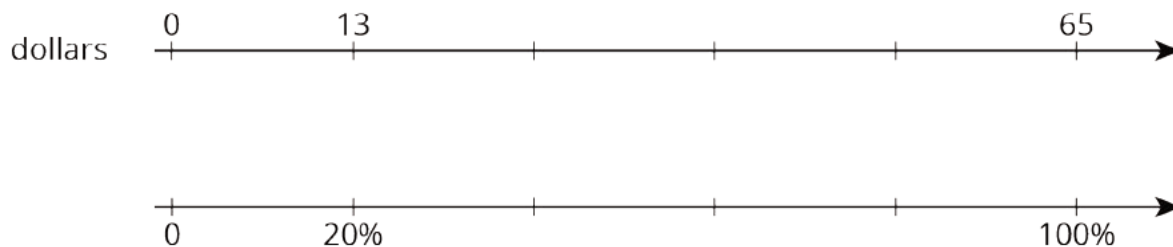
### Problem 2

#### Statement

A store is having a 20%-off sale on all merchandise. If Mai buys one item and saves \$13, what was the original price of her purchase? Explain or show your reasoning.

#### Solution

\$65. Possible reasoning:



Place \$13 at 20%. To get from 20% to 100%, multiply by 5. Therefore, also multiply 13 by 5.



### Problem 3

#### Statement

The original price of a scarf was \$16. During a store-closing sale, a shopper saved \$12 on the scarf. What percentage discount did she receive? Explain or show your reasoning.

#### Solution

75%. Possible explanations:

- $12 \div 16 = \frac{75}{100}$  (or  $12 \div 16 = 0.75$ )
- 

value (dollars)	percentage
16	100
12	75

### Problem 4

#### Statement

Select **all** the expressions whose value is larger than 100.

- A. 120% of 100
- B. 50% of 150
- C. 150% of 50
- D. 20% of 800
- E. 200% of 30
- F. 500% of 400
- G. 1% of 1,000

#### Solution

["A", "D", "F"]

### Problem 5

#### Statement

An ant travels at a constant rate of 30 cm every 2 minutes.

- a. At what pace does the ant travel per centimeter?
- b. At what speed does the ant travel per minute?

### Solution

- a. The pace is  $\frac{1}{15}$  of a minute per centimeter.
- b. The speed is 15 centimeters per minute.

(From Unit 3, Lesson 8.)

### Problem 6

#### Statement

Is  $3\frac{1}{2}$  cups more or less than 1 liter? Explain or show your reasoning. (Note: 1 cup  $\approx$  236.6 milliliters)

### Solution

Less. Explanations vary. Possible explanation:

cups	milliliters
1	236.6
0.5	118.3
3	709.8
3.5	828.1

(From Unit 3, Lesson 4.)

### Problem 7

#### Statement

Name a unit of measurement that is about the same size as each object.

- a. The distance of a doorknob from the floor is about 1 \_\_\_\_\_.
- b. The thickness of a fingernail is about 1 \_\_\_\_\_.
- c. The volume of a drop of honey is about 1 \_\_\_\_\_.
- d. The weight or mass of a pineapple is about 1 \_\_\_\_\_.
- e. The thickness of a picture book is about 1 \_\_\_\_\_.

- f. The weight or mass of a buffalo is about 1 \_\_\_\_\_.
- g. The volume of a flower vase is about 1 \_\_\_\_\_.
- h. The weight or mass of 20 staples is about 1 \_\_\_\_\_.
- i. The volume of a melon is about 1 \_\_\_\_\_.
- j. The length of a piece of printer paper is about 1 \_\_\_\_\_.

## Solution

- a. Yard or meter
- b. Millimeter
- c. Milliliter
- d. Kilogram or pound
- e. Centimeter or inch
- f. Ton
- g. Cup, quart, or liter
- h. Gram
- i. Gallon
- j. Foot

(From Unit 3, Lesson 2.)

# Lesson 15: Finding This Percent of That

## Goals

- Choose and create diagrams to calculate  $A\%$  of  $B$ , and explain (orally) the solution method.
- Generalize a process for finding  $A\%$  of  $B$  and justify (orally) why this can be abstracted as  $\frac{A}{100} \cdot B$ .
- Identify equivalent expressions that could be used to find  $A\%$  of  $B$  and justify (orally) that they are equivalent.

## Learning Targets

- I can solve different problems like “What is 40% of 60?” by dividing and multiplying.

## Lesson Narrative

Students have practiced solving three different types of percentage problems (corresponding to finding  $A$ ,  $B$ , or  $C$  respectively when  $A\%$  of  $B$  is  $C$ ). This lesson focuses on finding “ $A\%$  of  $B$ ” as efficiently as possible. While the previous lesson used numbers that students could calculate mentally, the numbers in this lesson are purposefully chosen to be difficult for students to calculate mentally or to represent on a double number line diagram, so as to motivate them to find the simplest way to do the calculation by hand.

The third activity hints at work students will do in grade 7, namely finding a constant of proportionality and writing an equation to represent a proportional relationship.

## Alignments

### Building On

- 5.NBT.B.6: Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

### Addressing

- 6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- 6.RP.A.3.c: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

### Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR7: Compare and Connect

- MLR8: Discussion Supports
- Number Talk

### Student Learning Goals

Let's solve percentage problems like a pro.

## 15.1 Number Talk: Decimals

### Warm Up: 5 minutes

The purpose of this number talk is to help students multiply and divide decimal numbers by 100 in preparation for their work with percentages later in the lesson.

### Building On

- 5.NBT.B.6

### Instructional Routines

- MLR8: Discussion Supports
- Number Talk

### Launch

Display one problem at a time. Give students 30 seconds of quiet think time for each problem, and ask them to give a signal when they have an answer and a strategy. Keep all previous problems displayed throughout the task.

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### Support for Students with Disabilities

*Representation: Internalize Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.

*Supports accessibility for: Memory; Organization*

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### Student Task Statement

Find the value of each expression mentally.

$$(0.23) \cdot 100$$

$$50 \div 100$$

$$145 \cdot \frac{1}{100}$$

$$7 \div 100$$

## Student Response

- 23. Possible reasoning: 23 hundredths times 100 is 23.
- 0.5. Possible reasoning: 50 is one half of 100, and one half can be written as 0.5.
- 1.45. Possible reasoning: 145 is bigger than 100 but close to 100, and the answer has to have the digits 1, 4, and 5, so it must be 1.45.
- 0.07. The answer is the same as 7 times  $\frac{1}{100}$ , which is  $\frac{7}{100}$ .

## Activity Synthesis

Select a couple of students to share their answer and strategies for each problem. Record and display their explanations for all to see. After evaluating all four expressions, ask students:

- How is multiplying by  $\frac{1}{100}$  related to division?
- What is important to remember about dividing a one digit number by 100?

To involve more students in the conversation, consider asking as the students share their ideas:

- Who can restate \_\_\_'s reasoning in a different way?
- Did anyone solve the problem the same way but would explain it differently?
- Did anyone solve the problem in a different way?
- Does anyone want to add on to \_\_\_'s strategy?
- Do you agree or disagree? Why?

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### Support for English Language Learners

*Speaking: MLR8 Discussion Supports.*: Display sentence frames to support students when they explain their strategy. For example, "First, I \_\_\_ because . . ." or "I noticed \_\_\_ so I . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

*Design Principle(s): Optimize output (for explanation)*

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## 15.2 Audience Size

### 15 minutes

In this activity, students encounter percentage problems that are inconvenient to solve by drawing double number lines, encouraging them to reason differently and begin noticing that  $P\%$  of a number is  $\frac{P}{100}$  times that number.

The first two questions, which ask students to find 30% and 140% of some values, can be solved using a familiar percentage, 10%, as a stepping stone. For example, to solve for 30% of 250, students may find 10% or  $\frac{1}{10}$  of 250 and then multiply the result by 3. This intermediate percentage does not work well for the last question, however, prompting them to find a workaround.

Students may resort to using a double number line, but may soon find it impractical. For example, they may decide to divide by 100% into 25 parts to find the value of 4%, which they can then multiply by 11. As time consuming drawing 25 tick marks is, it may encourage students to look for structure that may open a shorter path to the solution. For example, once they know that they need tickmarks every 4% and every 10 people, they can bypass the rest of the tick marks and multiply ( $4 \cdot 11 = 44$  and  $10 \cdot 11 = 110$ ) to find that 110 people attended literacy night.

Others may see that dividing the given value by 100 to find the value of 1% and then multiplying the result by the targeted percentage works well. Identify students who take this path so they can share later.

### Addressing

- 6.RP.A.3

### Instructional Routines

- MLR7: Compare and Connect

### Launch

Give students quiet think time to complete the activity and then time to share their explanation with a partner.

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### Support for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Chunk this task into more manageable parts. Check in with students after the first 2-3 minutes of work time. If students are finding that double number lines are not practical, invite others to share other strategies they have attempted so far.

*Supports accessibility for: Organization; Attention*

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### Student Task Statement

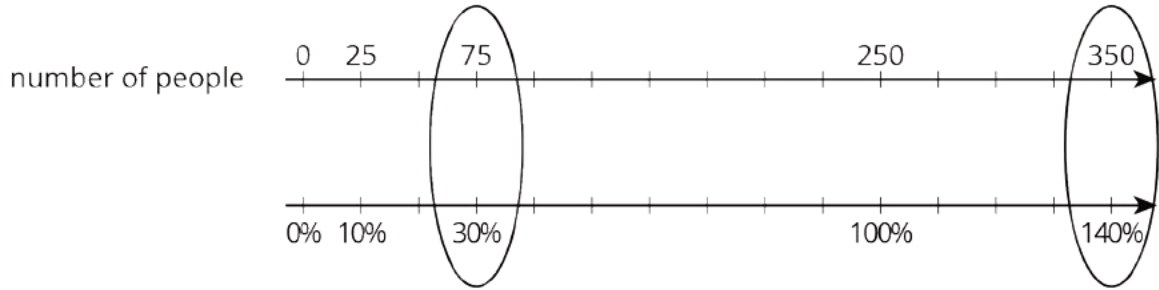
A school held several evening activities last month—a music concert, a basketball game, a drama play, and literacy night. The music concert was attended by 250 people. How many people came to each of the other activities?

1. Attendance at a basketball game was 30% of attendance at the concert.
2. Attendance at the drama play was 140% of attendance at the concert.

- 3. Attendance at literacy night was 44% of attendance at the concert.

**Student Response**

- 75 people attended the basketball game.
- 350 people attended the drama play. Possible strategies:



number of people	percentage
250	100
25	10
75	30
350	140

- 110 people attended literacy night. Possible strategies:
  - There is 4% for every 10 people, so multiply  $10 \cdot 11 = 110$ .
  - First find 1%, then multiply by 44 to find 44%.

number of people	percentage
250	100
2.5	1
110	44

**Are You Ready for More?**

50% of the people who attended the drama play also attended the music concert. What percentage of the people who attended the music concert also attended the drama play?

**Student Response**

70%. Half of 350 is 175, and 175 is 70% of 250.



## Activity Synthesis

Invite a couple of students to share their work on the first two questions, but focus the whole-class discussion on the last one. Select several students who effectively found 44% of 250 to share, saving the strategy involving 1% for last. If no one used this method, illustrate and explain it. Consider using a table in doing so, as shown below.

people	percentage
250	100
2.5	1
110	44

Guide students to see that this method of finding percentages can be generalized across all such problems, much like finding a unit rate is an effective way to solve any ratio problem.

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### Support for English Language Learners

*Representing, Speaking: MLR7 Compare and Connect.* As students share their strategies, make sure you hear from students with different strategies for calculating the number of people who attended literacy night. Encourage students to make comparisons and connections between when they are able to use familiar percentages, and when they cannot. Ask questions such as, “How did you decide which percentages to use?” and “How is using 1% as a strategy similar to or different from other percentages?” This will foster students’ meta-awareness and support constructive conversations as they compare and connect the various ways to find a percentage of a quantity.

*Design Principles(s): Cultivate conversation; Maximize meta-awareness*

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## 15.3 Everything is On Sale

15 minutes

In general,  $P\%$  of something is  $\frac{P}{100}$  times that thing. The purpose of this activity is to make this explicit. In this activity, students are asked to find 80% of several different values, generalize their process, and express their generalization in different ways. As they make repeated calculations, students look for and express regularity in their work (MP8) and see more explicitly that  $P\%$  of a number is  $\frac{P}{100}$  times that number. Once they arrive at one or more generalizations, students practice articulating why each generalization always works (MP3).

### Addressing

- 6.RP.A.3.c

## Instructional Routines

- MLR1: Stronger and Clearer Each Time

## Launch

Give students quiet think time to complete the activity and then time to share their explanation with a partner.

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### Support for Students with Disabilities

*Representation: Internalize Comprehension.* Activate or supply background knowledge about equivalent fractions. Incorporate explicit opportunities for review and practice if necessary. Allow students to use calculators to ensure inclusive participation in the activity.  
*Supports accessibility for: Memory; Conceptual processing*

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### Support for English Language Learners

*Writing, Speaking, Listening: MLR1 Stronger and Clearer Each Time.* Use this with successive pair shares to give students a structured opportunity to revise and refine their own strategies to determine 80% of any value to find a sale price. Make sure students include a table as well as an expression in their explanations. Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language (e.g., “Can you explain how...”, “Will this expression always work? How do you know?”, etc.). Students can borrow ideas and language from each partner to refine and clarify their original explanation. This will help students refine their own strategies and learn about other ways to find the sale price of an item.

*Design Principles(s): Optimize output (for explanation); Maximize meta-awareness*

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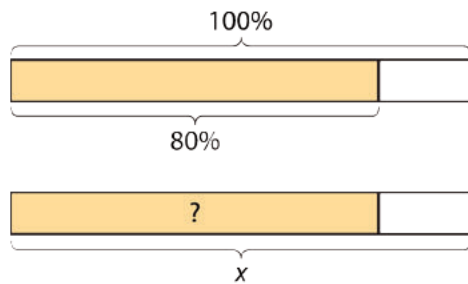
## Student Task Statement

During a sale, every item in a store is 80% of its regular price.

1. If the regular price of a T-shirt is \$10, what is its sale price?
2. The regular prices of five items are shown here. Find the sale price of each item.

	item 1	item 2	item 3	item 4	item 5
regular price	\$1	\$4	\$10	\$55	\$120
sale price					

3. You found 80% of many values. Was there a process you repeated over and over to find the sale prices? If so, describe it.



4. Select **all** of the expressions that could be used to find 80% of  $x$ . Be prepared to explain your reasoning.

$$\frac{8}{100} \cdot x$$

$$\frac{8}{10} \cdot x$$

$$\frac{8}{5} \cdot x$$

$$80 \cdot x$$

$$(0.8) \cdot x$$

$$\frac{80}{100} \cdot x$$

$$\frac{4}{10} \cdot x$$

$$\frac{4}{5} \cdot x$$

$$8 \cdot x$$

$$(0.08) \cdot x$$

### Student Response

- \$8
- Here is the table:

	item 1	item 2	item 3	item 4	item 5
normal price	\$1	\$4	\$10	\$55	\$120
sale price	\$0.80	\$3.20	\$8	\$44	\$96

- Answers vary. Sample response: Multiply each normal price by 0.8 (or by  $\frac{80}{100}$ ) to find the sale price.
- These expressions work:
  - $\frac{80}{100} \cdot x$  (See discussion.)
  - $\frac{8}{10} \cdot x$  because  $\frac{80}{100} = \frac{8}{10}$
  - $\frac{4}{5} \cdot x$  because  $\frac{8}{10} = \frac{4}{5}$
  - $(0.8) \cdot x$  because  $\frac{80}{100} = 0.8$

### Activity Synthesis

Make sure students have the correct values in the table for the second question. Then, discuss the expressions, starting with

$$\frac{80}{100} \cdot x.$$

Guide students to see that if we know the value of 100%, dividing that value by 100 (or equivalently, multiplying it by  $\frac{1}{100}$ ) tells us the corresponding value for 1%. We can then multiply that value by the desired percentage. Consider using a table to organize this argument, as shown below:

percentage	regular price
100	$x$
1	$\frac{1}{100} \cdot x$
80	$\frac{80}{100} \cdot x$

After everyone understands where this expression comes from, ask students to discuss the remaining expressions with a partner. This is a good opportunity for just-in-time review on equivalent fractions, if needed.

## Lesson Synthesis

The main idea in this lesson is that to find  $P\%$  of  $x$ , multiply:

$$\frac{P}{100} \cdot x$$

Ask students to describe a procedure for finding a percent of a number. If they struggle to describe a general method, ask about some specific examples, like “How could you find 32% of 500?” (You could multiply 500 by  $\frac{32}{100}$ .)

## 15.4 Ordering Percentages of Different Numbers

### Cool Down: 5 minutes

In addition to assessing students ability to find  $A\%$  of  $B$ , this cool-down allows students to practice arithmetic involving fractions or decimals (using grade 5 techniques). This is a good review for upcoming units on fractions and decimals.

### Addressing

- 6.RP.A.3.c

### Anticipated Misconceptions

Students may put these amounts in order based on just the percentages or just the amount that corresponds to 100%, not realizing that both parts of the expression affect the value. Demonstrate calculating 65% of 80 to get them on the right track.

### Student Task Statement

Order these three values from least to greatest. Explain or show your reasoning.

65% of 80

82% of 50

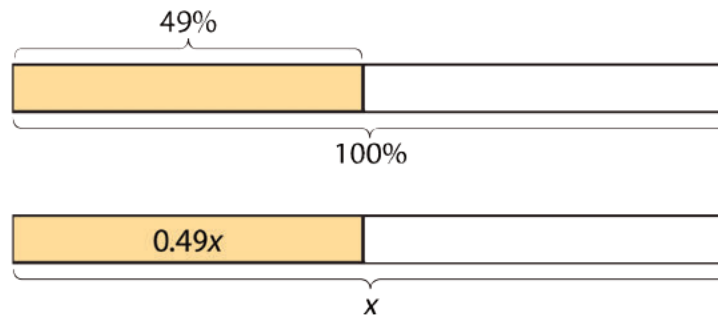
170% of 30

### Student Response

The least is 82% of 50, because  $(0.82) \cdot 50 = 41$ . Next is 170% of 30, because  $(1.7) \cdot 30 = 51$ . The greatest is 65% of 80,  $(0.65) \cdot 80 = 52$ .

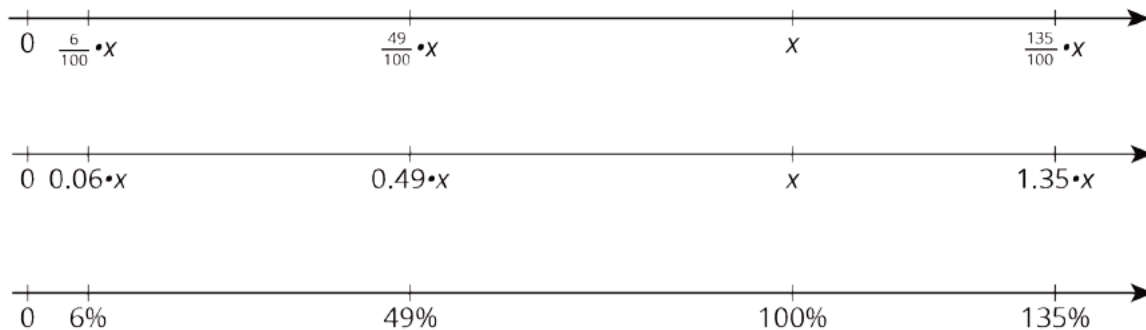
### Student Lesson Summary

To find 49% of a number, we can multiply the number by  $\frac{49}{100}$  or 0.49.



To find 135% of a number, we can multiply the number by  $\frac{135}{100}$  or 1.35.

To find 6% of a number, we can multiply the number by  $\frac{6}{100}$  or 0.06.



In general, to find  $P\%$  of  $x$ , we can multiply:

$$\frac{P}{100} \cdot x$$

## Lesson 15 Practice Problems

### Problem 1

#### Statement

- To find 40% of 75, Priya calculates  $\frac{2}{5} \cdot 75$ . Does her calculation give the correct value for 40% of 75? Explain or show how you know.
- If  $x$  represents a number, does  $\frac{2}{5} \cdot x$  always represent 40% of that number? Explain your reasoning.

#### Solution

- Yes. 40% is 0.4, and  $(0.4) \cdot 75 = 30$ . Using Priya's method:  $\frac{2}{5} \cdot 75 = 30$ .
- Yes. 40% of  $x$  is  $\frac{40}{100} \cdot x$ . This is the same as  $\frac{2}{5} \cdot x$ , since  $\frac{40}{100}$  and  $\frac{2}{5}$  are equivalent fractions.

### Problem 2

#### Statement

Han spent 75 minutes practicing the piano over the weekend. For each question, explain or show your reasoning.

- Priya practiced the violin for 152% as much time as Han practiced the piano. How long did she practice?
- Tyler practiced the clarinet for 64% as much time as Han practiced the piano. How long did he practice?

#### Solution

- 114 minutes. Sample reasoning: 152% of 75 minutes is  $\frac{152}{100} \cdot 75 = 114$ .
- 48 minutes. Sample reasoning: 64% of 75 minutes is  $\frac{64}{100} \cdot 75 = 48$ .

### Problem 3

#### Statement

Last Sunday 1,575 people visited the amusement park. 56% of the visitors were adults, 16% were teenagers, and 28% were children ages 12 and under. Find the number of adults, teenagers, and children that visited the park.

#### Solution

882 adults, 252 teenagers, and 441 children

## Problem 4

### Statement

Order from greatest to least:

- 55% of 180
- 300% of 26
- 12% of 700

### Solution

55% of 180, 12% of 700, 300% of 26.

## Problem 5

### Statement

Complete each statement.

- 20% of 60 is \_\_\_\_\_
- 25% of \_\_\_\_\_ is 6
- \_\_\_\_\_% of 100 is 14
- 50% of 90 is \_\_\_\_\_
- 10% of \_\_\_\_\_ is 7
- 30% of 70 is \_\_\_\_\_

### Solution

- a. 12
- b. 24
- c. 14
- d. 45
- e. 70
- f. 21

(From Unit 3, Lesson 14.)

## Problem 6

### Statement

A shopper needs 24 sandwich rolls. The store sells identical rolls in 2 differently sized packages. They sell a six-pack for \$5.28 and a four-pack for \$3.40. Should the shopper buy 4 six-packs or 6 four-packs? Explain your reasoning.

## Solution

6 four-packs is a better deal. The rolls in the six-pack are being sold at a rate of 88 cents each, because  $5.28 \div 6 = 0.88$ . The rolls in the four-pack are being sold at a rate of 85 cents each, because  $3.40 \div 4 = 0.85$ . The four-packs are a better deal, because the sandwich rolls have a cheaper unit rate.

(From Unit 3, Lesson 9.)

## Problem 7

### Statement

On a field trip, there are 3 chaperones for every 20 students. There are 92 people on the trip. Answer these questions. If you get stuck, consider using a tape diagram.

- a. How many chaperones are there?
- b. How many children are there?

## Solution

- a. 12
- b. 80

(From Unit 2, Lesson 15.)



# Lesson 16: Finding the Percentage

## Goals

- Critique or justify (orally) statements about percentages and equivalent numerical expressions.
- Generalize a process for finding the percentage that C is of B and justify (orally) why this can be abstracted as  $\frac{C}{B} \cdot 100$ .

## Learning Targets

- I can solve different problems like “60 is what percentage of 40?” by dividing and multiplying.

## Lesson Narrative

While students have found percentages with easy numbers before now, in this lesson they will develop a general structure (MP7) that will work for any numbers.

## Alignments

### Addressing

- 6.RP.A.3.c: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

### Instructional Routines

- MLR7: Compare and Connect
- MLR8: Discussion Supports
- True or False

### Student Learning Goals

Let’s find percentages in general.

## 16.1 True or False: Percentages

### Warm Up: 10 minutes

The purpose of this warm-up is to encourage students to reason about properties of operations in equivalent expressions. While students may evaluate each expression to determine if the statement is true or false, encourage students to think about the properties of arithmetic operations in their reasoning.

### Addressing

- 6.RP.A.3.c

## Instructional Routines

- True or False

## Launch

Display one problem at a time. Tell students to give a signal when they have decided if the equation is true or false. Give students 1 minute of quiet think time followed by a whole-class discussion.

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### Support for Students with Disabilities

*Representation: Internalize Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.

*Supports accessibility for: Memory; Organization*

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### Student Task Statement

Is each statement true or false? Be prepared to explain your reasoning.

1. 25% of 512 is equal to  $\frac{1}{4} \cdot 500$ .
2. 90% of 133 is equal to  $(0.9) \cdot 133$ .
3. 30% of 44 is equal to 3% of 440.
4. The percentage 21 is of 28 is equal to the percentage 30 is of 40.

### Student Response

Student explanations will vary.

1. False; 25% is equal to  $\frac{1}{4}$ , but 512 is not equal to 500.
2. True; These are equal, because 90% is equal to 0.9
3. True; These are equal because  $\frac{3}{10} \cdot 44 = \frac{3}{100} \cdot 440$
4. True; These are equal, because  $\frac{21}{28} = \frac{3}{4} = \frac{30}{40}$

### Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- Do you agree or disagree? Why?
- Who can restate \_\_\_'s reasoning in a different way?
- Does anyone want to add on to \_\_\_\_'s reasoning?

After each true equation, ask students if they could rely on the same reasoning to determine if other similar problems are equivalent. After each false equation, ask students how the problem could be changed to make the equation true.

## 16.2 Jumping Rope

15 minutes

The purpose of this activity is for students to see that to find what percentage one number is of another, divide them and then multiply by 100. First they find what percentage of 20 various numbers are, then they organize everything into a table. Using the table, students then describe the relationship they see between dividing the numbers by 20 and finding the percentage (MP8).

### Addressing

- 6.RP.A.3.c

### Instructional Routines

- MLR8: Discussion Supports

### Launch

Depending on students' strengths in grade 5 work, it may be beneficial to offer some strategies for writing a fraction in an equivalent decimal form before students start working. Display the fraction  $\frac{15}{25}$  and ask students how they might think about writing it in decimal form. Possible strategies are writing the equivalent fraction  $\frac{60}{100}$  and knowing that "60 hundredths" is written 0.60 or 0.6. Another possibility is to write the equivalent fraction  $\frac{3}{5}$ , which is equal to  $\frac{6}{10}$  and knowing that "6 tenths" can be written as 0.6.

Give students quiet think time to complete the activity and then time to share their explanations with a partner.

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### Support for Students with Disabilities

*Representation: Internalize Comprehension.* Provide appropriate reading accommodations and supports to ensure students access to written directions, word problems and other text-based content.

*Supports accessibility for: Language; Conceptual processing*

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### Support for English Language Learners

*Speaking: MLR8 Discussion Supports.* To amplify mathematical uses of language to communicate about equivalent fractions and percentages (e.g., 15 minutes is 75% of 20 minutes, 75% of 20 minutes is equivalent to  $\frac{15}{20}$ ), invite students to use these phrases when stating their ideas, revoicing and rephrasing as necessary.

*Design Principle(s): Support sense-making, Optimize output (for explanation)*

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### Student Task Statement

A school held a jump-rope contest. Diego jumped rope for 20 minutes.

1. Jada jumped rope for 15 minutes. What percentage of Diego's time is that?
2. Lin jumped rope for 24 minutes. What percentage of Diego's time is that?
3. Noah jumped rope for 9 minutes. What percentage of Diego's time is that?
4. Record your answers in this table. Write the quotients in the last column as decimals.

	time (minutes)	percentage	time $\div$ 20
Diego	20	100	$\frac{20}{20} = 1$
Jada	15		$\frac{15}{20} =$
Lin	24		$\frac{24}{20} =$
Noah	9		$\frac{9}{20} =$

5. What do you notice about the numbers in the last two columns of the table?

### Student Response

1. 15 minutes is 75% of 20 minutes.
2. 24 minutes is 120% of 20 minutes.
3. 9 minutes is 45% of 20 minutes.
4. Here is the table:

time (minutes)	percentage	time/20
20	100	$\frac{20}{20} = 1$
15	75	$\frac{15}{20} = 0.75$
24	120	$\frac{24}{20} = 1.2$
9	45	$\frac{9}{20} = 0.45$

5. The percentages in the second to last column are 100 times the decimals in the last column. To find what percentage a number is of 20, divide it by 20, and then multiply by 100.

### Activity Synthesis

Have students share their strategies for the first three problems. Then ask what they noticed about the last two columns. There are many ways to formulate the relationship. Make sure everyone sees that to find what percentage a number is of 20, divide it by 20 and then multiply by 100. A table showing the percentage for 1 minute might also help:

	time (minutes)	percentage
Diego	20	100
unit rate	1	5
Jada	15	75
Lin	24	120
Noah	9	45

Note that to find the percentage for 1 minute, we divide 100 by 20, and to find the percentage for any number of minutes, we multiply the result by that number of minutes. That is the same as dividing the number of minutes by 20 and multiplying by 100.

## 16.3 Restaurant Capacity

### 10 minutes

This activity gives students a chance to practice their new-found insight about how to find percentages. It also gives them an opportunity to practice dividing whole numbers in preparation for the next unit on base-ten numbers.

Note that it is expected and perfectly okay for students to revert to less-efficient methods that they trust. Monitor for students with more- and less-efficient methods. A less-efficient representation can be used to make sense of a more-efficient method, and these connections can be made in the discussion that follows the task.

### Addressing

- 6.RP.A.3.c

### Instructional Routines

- MLR7: Compare and Connect

### Launch

Give students quiet think time to complete the activity and then time to share their explanations with a partner.

---

### Support for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Provide students with a graphic organizer to organize their problem solving. The graphic organizer should ask students to identify what they need to find out, what information is provided, how they solved the problem, and why their answer is correct.

*Supports accessibility for: Language; Organization*

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### Student Task Statement

A restaurant has a sign by the front door that says, "Maximum occupancy: 75 people." Answer each question and explain or show your reasoning.

1. What percentage of its capacity is 9 people?
2. What percentage of its capacity is 51 people?
3. What percentage of its capacity is 84 people?

### Student Response

Reasonings vary.

1. 9 people is 12% of 75 people. Sample reasoning:
  - $\frac{9}{75}$  is equivalent to  $\frac{3}{25}$ , which is equivalent to  $\frac{12}{100}$ .
  - $\frac{9}{75} \cdot 100 = \frac{900}{75}$ , which is 12.
2. 51 people is 68% of 75 people. Sample reasonings:
  - $\frac{51}{75}$  is equivalent to  $\frac{17}{25}$ , which is equivalent to  $\frac{68}{100}$ .

- $\frac{51}{75} \cdot 100 = \frac{5,100}{75}$ , which is 68.
  - If 9 people is 12% of 75, then 45 people, which is  $5 \cdot 9$ , is  $5 \cdot 12$  or 60%. 6 more people is  $\frac{6}{9} \cdot 12$  or 8%. That means 51 people is  $(60 + 8)$  (or 68%).
3. 84 people is 112% of 75 people. Sample reasoning:
- 84 is 9 more than 75. We already know that 9 people is 12% and 75 people is 100%, so 84 people is  $(12 + 100)$  or 112%.
  - $\frac{84}{75} \cdot 100 = \frac{8,400}{75}$ , which is 112.

### Are You Ready for More?

Water makes up about 71% of Earth's surface, while the other 29% consists of continents and islands. 96% of all Earth's water is contained within the oceans as salt water, while the remaining 4% is fresh water located in lakes, rivers, glaciers, and the polar ice caps.

If the total volume of water on Earth is 1,386 million cubic kilometers, what is the volume of salt water? What is the volume of fresh water?

### Student Response

1,330,560,000 cubic kilometers of salt water

55,440,000 cubic kilometers of fresh water

### Activity Synthesis

Invite selected students to share various approaches to the three problems. Sequence less-efficient methods before more-efficient ones. Make sure all students have a chance to see the approach that is the focus of this lesson. For example, to find "9 is what percent of 75?" we can divide 9 by 75 and then multiply by 100.

### Support for English Language Learners

*Speaking, Listening: MLR7 Compare and Connect.* For the first question, "What percentage of its capacity is 9 people?", display a range of student approaches to share with the class. Some students may use a double number line or table and reason that 3 people is 4% of 75 people because 100% divided by 25 is 4%, and 75 people divided by 25 is 3 people. Then multiply 4% by 3 to find what percentage of 75 people is 9 people. Other students may use a more efficient method and reason that  $\frac{9}{75}$  is equivalent to  $\frac{3}{25}$ , which is equivalent to  $\frac{12}{100}$ . As students investigate each other's work, ask questions such as, "What is especially clear in this approach or representation?" and "Where do you see  $\frac{9}{75}$  or 12% represented in the diagram?" This will foster students' meta-awareness and support constructive conversations as they compare and connect the various ways to find what percentage one number is of another.

*Design Principles(s): Cultivate conversation; Maximize meta-awareness*

## Lesson Synthesis

The main idea in this lesson is that to find what percentage  $C$  is of  $B$ , multiply:

$$\frac{C}{B} \cdot 100$$

Ask students to describe a procedure for finding what percentage one number is of another number. If they struggle to describe a general method, ask about some specific examples, like “How could you find what percentage of 80 is 56?” (You could multiply 100 by  $\frac{56}{80}$ .)

## 16.4 Jet Fuel

Cool Down: 5 minutes

### Addressing

- 6.RP.A.3.c

### Student Task Statement

A jet plane can carry up to 200,000 liters of fuel. It used 130,000 liters of fuel during a flight. What percentage of the fuel capacity did it use on this flight?

### Student Response

$130,000 \div 200,000 = 0.65$ , so it burned 65% of its fuel capacity on the flight.

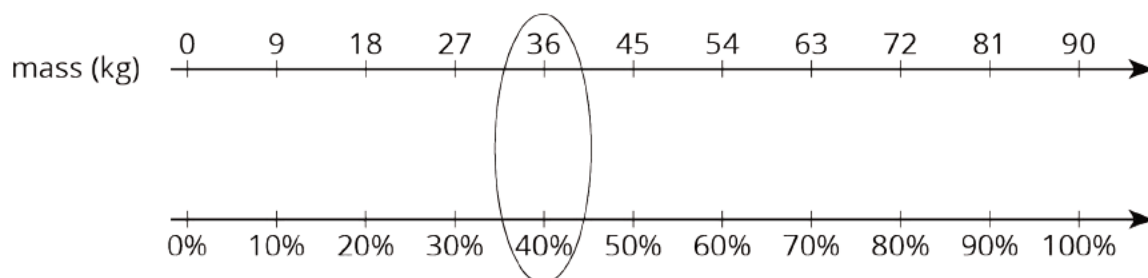
### Student Lesson Summary

What percentage of 90 kg is 36 kg? One way to solve this problem is to first find what percentage 1 kg is of 90, and then multiply by 36.

mass (kg)	percentage
90	100
1	$\frac{1}{90} \cdot 100$
36	$\frac{36}{90} \cdot 100$

Diagram illustrating the table with arrows and labels:  $\cdot \frac{1}{90}$  (left side, pointing to the 1 kg row),  $\cdot 36$  (left side, pointing to the 36 kg row),  $\cdot \frac{1}{90}$  (right side, pointing to the  $\frac{1}{90} \cdot 100$  row), and  $\cdot 36$  (right side, pointing to the  $\frac{36}{90} \cdot 100$  row).

From the table we can see that 1 kg is  $(\frac{1}{90} \cdot 100)\%$ , so 36 kg is  $(\frac{36}{90} \cdot 100)\%$  or 40% of 90. We can confirm this on a double number line:





In general, to find what percentage a number  $C$  is of another number  $B$  is to calculate  $\frac{C}{B}$  of 100%. We can find that by multiplying:

$$\frac{C}{B} \cdot 100$$

Suppose a school club has raised \$88 for a project but needs a total of \$160. What percentage of its goal has the club raised?

To find what percentage \$88 is of \$160, we find  $\frac{88}{160}$  of 100% or  $\frac{88}{160} \cdot 100$ , which equals  $\frac{11}{20} \cdot 100$  or 55. The club has raised 55% of its goal.

## Lesson 16 Practice Problems

### Problem 1

#### Statement

A sign in front of a roller coaster says "You must be 40 inches tall to ride." What percentage of this height is:

- a. 34 inches?
- b. 54 inches?

#### Solution

- a. 85%
- b. 135%

### Problem 2

#### Statement

At a hardware store, a tool set normally costs \$80. During a sale this week, the tool set costs \$12 less than usual. What percentage of the usual price is the savings? Explain or show your reasoning.

#### Solution

Reasoning varies. Sample response: 15%, because  $12 \div 80 = \frac{3}{20} = \frac{15}{100}$ .

### Problem 3

#### Statement

A bathtub can hold 80 gallons of water. The faucet flows at a rate of 4 gallons per minute. What percentage of the tub will be filled after 6 minutes?

## Solution

30%, because the tub will hold 24 gallons after 6 minutes, and 24 is 30% of 80.

## Problem 4

### Statement

The sale price of every item in a store is 85% of its usual price.

- The usual price of a backpack is \$30, what is its sale price?
- The usual price of a sweatshirt is \$18, what is its sale price?
- The usual price of a soccer ball is \$24.80, what is its sale price?

## Solution

- \$25.50
- \$15.30
- \$21.08

(From Unit 3, Lesson 15.)

## Problem 5

### Statement

A shopper needs 48 hot dogs. The store sells identical hot dogs in 2 differently sized packages. They sell a six-pack of hot dogs for \$2.10, and an eight-pack of hot dogs for \$3.12. Should the shopper buy 8 six-packs, or 6 eight-packs? Explain your reasoning.

## Solution

He should buy 8 six-packs. The hot dogs in the six-pack are being sold at a rate of 35 cents each, because  $2.10 \div 6 = 0.35$ . The hot dogs in the eight-pack are being sold at a rate of 39 cents each, because  $3.12 \div 8 = 0.39$ . The six-packs are a better deal, because the hot dogs have a cheaper unit rate.


(From Unit 3, Lesson 9.)

## Problem 6

### Statement

Elena is 56 inches tall.

- What is her height in centimeters? (Note: 100 inches = 254 centimeters)

-  b. What is her height in meters?

**Solution**

- a. 142.24 centimeters
- b. 1.42 meters

(From Unit 3, Lesson 4.)

## Section: Let's Put it to Work

### Lesson 17: Painting a Room

#### Goals

- Apply rates and percentages to calculate how long it will take and how much it will cost to complete a painting project, and explain (orally) the reasoning.
- Make simplifying assumptions and determine what information is needed to solve a problem about painting a room.

#### Learning Targets

- I can apply what I have learned about unit rates and percentages to predict how long it will take and how much it will cost to paint all the walls in a room.

#### Lesson Narrative

In this culminating lesson, students make material and cost estimates for a home improvement project, applying and integrating many concepts and skills from the past three units.

Students determine the area of the walls of a bedroom, estimate the amount of paint needed to paint them, and determine the cost associated with the project (MP4). Along the way, they reason about areas of two-dimensional figures, convert units of measurements, solve ratio and rate problems, and work with percentages. Though there is a single correct measure for the total area of the walls to be painted, the amount of paint needed will depend on some assumptions and decisions students make about the work involved. The problem requires students to make some decisions about how to approach the task and which tools to use (MP5).

Depending on instructional choices made, this lesson could take one or more class meetings. The time estimates for the two main activities are intentionally left blank because the time will vary based on instructional decisions made. Variables affecting the amount of time needed include how much guidance and autonomy students are given, how elaborate the presentation of their work is expected to be, and how much time is taken for sharing solutions at the end.

#### Alignments

##### Building On

- 4.MD.A: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

##### Addressing

- 6.G.A: Solve real-world and mathematical problems involving area, surface area, and volume.
- 6.RP.A: Understand ratio concepts and use ratio reasoning to solve problems.

## Instructional Routines

- MLR6: Three Reads

### Student Learning Goals

Let's see what it takes to paint a room.

## 17.1 Getting Ready to Paint

### Warm Up: 5 minutes

This warm-up prompts students to start thinking about painting a room, the central context of this culminating lesson. Since not all students would have had experience painting a room or seeing one painted, an introduction helps make sure that the context is understood.

Students brainstorm a list of necessary tools and consider what it might take to paint a room, including building an initial idea of associated costs.

### Building On

- 4.MD.A

### Launch

Tell students that their next project is about painting a room and their first task is to think about necessary supplies. Poll the class to find out who has experience painting a room. If possible, arrange students into small groups such that each group has at least one experienced person to help with the brainstorming. If no students (or only one or two) are familiar with room painting, consider finding and showing a video of someone doing detail painting. Ask students to notice the tools used in the work.

### Student Task Statement

What are some tools that are helpful when painting a room?

### Student Response

Possible responses: paint, paint brushes, paint rollers, drop cloths, painter's tape, mixing sticks, ladder, paint can opener

### Activity Synthesis

After students had a chance to brainstorm, invite groups to share their lists of necessary tools. Record and display the list for all to see and to refer to throughout the lesson. If possible, add the estimated cost of each item with the exception of the cost of the paint, since that will depend on the amount needed to paint the room. Alternatively, after the list is made, let students know that many hardware stores sell "painting kits" that include brushes, rollers, a tray, and a plastic drop sheet for around \$20.

## 17.2 How Much It Costs to Paint

### 35 minutes

This activity is comprised of two major parts: finding the total wall area to be painted and estimating needed supplies and associated costs. Students use a floor plan of a bedroom and a list of its features and measurements to calculate the room's total wall area. They then use the square footage to make purchasing decisions and estimate costs.

Because of the variations in the bedroom's walls and features, keeping track of the shapes to be included or excluded from the area calculation may be challenging to students. Look out for likely omissions or missed steps. For examples, students may neglect to:

- Include the 3 square feet of wall space above the door.
- Account for the south face of the small wall at one end of the closet. (At 0.5 feet wide and 8 feet tall, it accounts for 4 square feet of paintable surface area.)
- Account for the area above or below the window.
- Convert lengths to a common unit before determining area or before adding areas of different faces of the room. As students work, notice those who organize their work methodically and those who may need organizational support.

### Addressing

- 6.G.A
- 6.RP.A

### Instructional Routines

- MLR6: Three Reads

### Launch

Since students may not be familiar with floor plans, consider spending a few minutes making sense of one as a class before working on the task. Display a floor plan of the classroom and identify where and how different features of the classroom (doors, windows, closets, etc.) are represented on the the plan.

Give students 2–3 minutes to read the task statement individually and to note any questions that come to mind. Afterwards, ask a couple of students to restate the task in their own words, and spend a few minutes answering any clarifying questions students may have.

Arrange students in groups of 2. Give students approximately 15 minutes of quiet think time to complete the first question (determining the total wall area to be painted). Ask students to discuss their work with their partner afterwards, and to pause for a whole-class debrief before moving on to the rest of the task.

Select a few students to share their strategies and solutions for the total wall area. Discuss any disagreements or questions in the shared calculation or approach. Make sure that the class agrees on the total square feet before instructing students to complete the rest of the task individually.

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### Support for Students with Disabilities

*Engagement: Internalize Self Regulation.* Provide a project checklist that chunks the various steps of the project into a set of manageable tasks.

*Supports accessibility for: Organization; Attention*

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### Support for English Language Learners

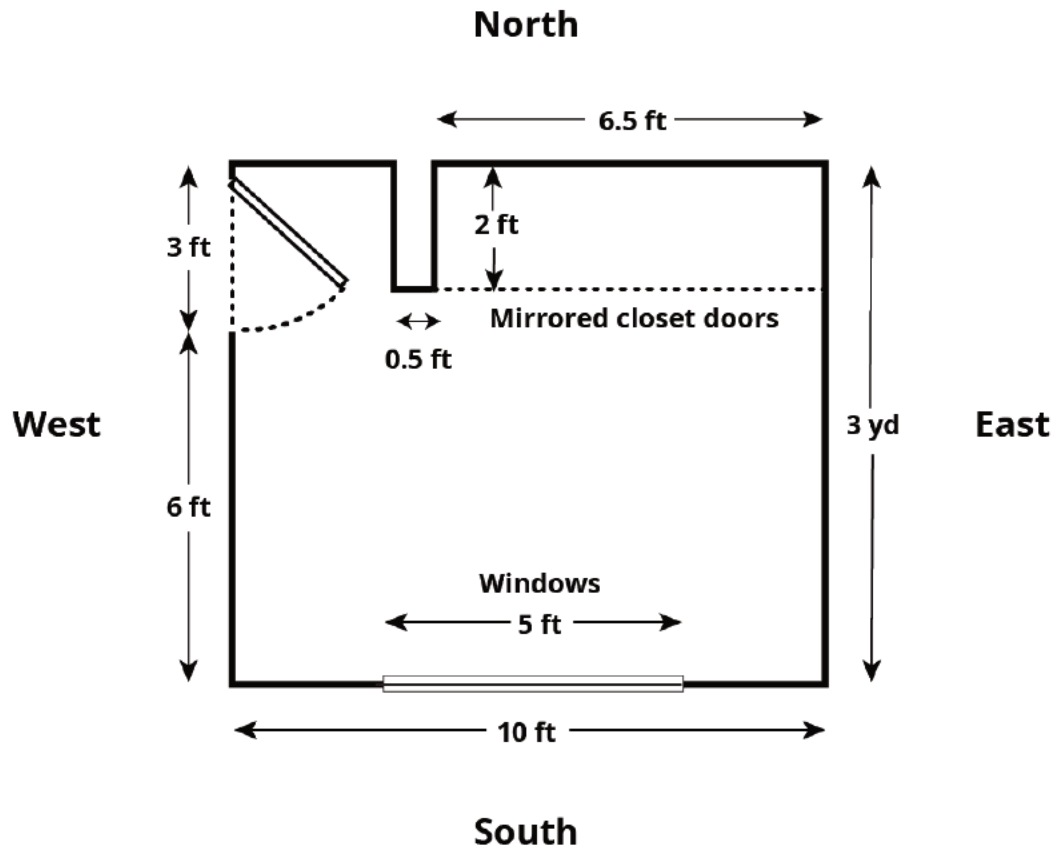
*Reading: MLR6 Three Reads.* Use this routine to help students make sense of the diagram and the accompanying text before unveiling the task questions. In the first read, students read the floor plan as well as the problem and with the goal of comprehending the situation (e.g., We need to repaint all the walls in a bedroom, except for a door and window.). In the second read, ask students to identify important quantities (e.g., The east wall is 3 yards long; the window is 5 feet long by 3 feet wide; the ceiling is 8 feet high). Use a visual display to help students make connections to the text by adding details to the floor plan. In the third read, reveal the questions and ask students to brainstorm possible solution strategies that connect to the important floor plan details and quantities they have listed. This routine supports students' reading comprehension, without solving the problem for them.

*Design Principle(s): Support sense-making; Maximize meta-awareness*

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### Student Task Statement

Here is the floor plan for a bedroom:



Imagine you are planning to repaint all the walls in this room, including inside the closet.

- The east wall is 3 yards long.
- The south wall is 10 feet long but has a window, 5 feet by 3 feet, that does not need to be painted.
- The west wall is 3 yards long but has a door, 7 feet tall and 3 feet wide, that does not need to be painted.
- The north wall includes a closet, 6.5 feet wide, with floor-to-ceiling mirrored doors that do not need to be painted. There is, however, a smaller wall between the west wall and the closet that does need to be painted on all sides. The wall is 0.5 feet wide and extends 2 feet into the room.
- The ceiling in this room is 8 feet high.
- All of the corners are right angles.

1. If you paint all the walls in the room, how many square feet do you need to cover?
2. An advertisement about the paint that you want to use reads: "Just 2 quarts covers 175 square feet!" If you need to apply two coats of paint on all the walls, how much paint do you need to buy?



- Paint can only be purchased in 1-quart, 1-gallon, and 5-gallon containers. How much will all supplies for the project cost if the cans of paint cost \$10.90 for a quart, \$34.90 for a gallon, and \$165.00 for 5 gallons?
- You have a coupon for 20% off all quart-sized paint cans. How does that affect the cost of the project?

### Student Response

- The total wall area that will be painted is 300 square feet. (North - 76, East - 72, South - 65, West - 51, small wall - 36)
- 6 quarts of paint should cover 525 square feet, which is too little for two coats, while 7 quarts should cover 612.5 sq feet, which is a bit more than two full coats. So, 7 quarts, or 1.75 gallons, of paint should be purchased. Purchasing 2 1-gallon containers or 1 1-gallon container and 3 1-quart containers instead of 7 1-quart containers would also work.
- Possible answers:

quarts	gallons	painting supplies	total cost
-	2 for \$69.80	\$20.00	\$89.80
7 for \$76.30	-	\$20.00	\$96.30
3 for \$32.70	1 for \$34.90	\$20.00	\$87.60

- Since the discount is only on the quart containers, option A above will stay the same price, option B is now \$81.04, and option C is now \$81.06.

### Activity Synthesis

Begin by asking 1–2 students to explain their calculations on how much paint is needed. Students should see that using the total wall area alone to find the volume of paint to purchase is not going to be adequate, since two full coats are needed, and paint can only be purchased in specific quantities. How students account for these factors in their decisions and calculations may vary. For example, some students may prefer buying fewer containers of a larger quantity while others may prefer buying more containers of a smaller quantity. Some may choose to have as little extra paint as possible, while other may choose otherwise. (To have enough paint, students may choose to purchase 7 quart-sized containers, 2 gallon-sized containers, or 3 quart-sized and 1 gallon-sized containers.)

Once students have a chance to discuss the various ways students chose to purchase paint, discuss their cost estimates. Consider setting up a few ranges of costs (e.g., \$80–\$84.99, \$85–\$89.99, \$90–\$94.99, \$95–\$99.99, \$100 or more, etc.) and polling students on where their cost estimates land. Their estimates vary depending on the containers chosen, supplies included, and other assumptions or decisions they made.

Ask, “Can all of these values be correct?” Give students a moment of quiet think time before selecting 2–3 to respond and to share how they calculated their estimates. Record and display students’ responses to help everyone make sense of how the different costs are possible. For example, students who decided that a minimum of 7 total quarts of paint are needed may have reasoned as follows: Cost of painting supplies + cost for 1 gallon container + cost of 3 quart containers = total cost. \$ For the final question of the activity, select a student with a successful strategy to share how they applied the coupon. Consider wrapping up the activity by asking how to purchase the paint and use the coupon to get the best deal.

## 17.3 How Long It Takes to Paint

### 15 minutes

In this activity, students use the area measures from the previous task to solve problems about the amount of painting time, using their understanding of ratio, rate, and percentage along the way. The problems can be approached in a number of ways, giving students additional opportunities to model with mathematics (MP4) and choose their tools (MP5).

### Addressing

- 6.RP.A

### Launch

Give students quiet think time to complete the activity and then time to share their work and solutions with a partner. Ask students to be ready to explain their partner’s strategy to the class (MP3).

### Anticipated Misconceptions

Some students may try to account for the amount of time that it takes the paint to dry between applying the first and second coat. Point out the the problem is only referring to the painting time, not the drying time.

### Student Task Statement

After buying the supplies, you start painting the east wall. It takes you 96 minutes to put two coats of paint on that wall (not including a lunch break between the two coats).

1. Your friend stops by to see how you are doing and comments that you are 25% finished with the painting. Are they correct?
2. Your friend offers to help you with the rest of the painting. It takes the two of you 150 more minutes of painting time to finish the entire room. How much time did your friend save you?

### Student Response

1. Your friend is incorrect, but not by much. The East wall is only  $\frac{72}{300} \cdot 100 = 24\%$  of the total area, not 25%.

2. Since it took you 96 minutes to finish 24% of the painting, it should take 304 minutes to finish the remaining 76% of the painting on your own. The friend saved you 154 minutes, because  $304 - 150 = 154$ .

### **Activity Synthesis**

After students have conferred with their partners and checked their solutions (which for this activity is important, since a correct answer to the first problem is needed to find a correct answer to the second problem), debrief as a class. Invite students who thought their partner used a particularly efficient strategy to share.

In the first problem, students may solve it by figuring out that 3 square feet is 1% and scaling it to 72 square feet to correspond to 24%. If this was the dominant strategy in the class, make sure to emphasize how students can solve this problem by calculating  $\frac{72}{300} \cdot 100$  directly.



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Family Support  
Materials

# Family Support Materials

## Unit Rates and Percentages

Here are the video lesson summaries for Grade 6, Unit 3 Unit Rates and Percentages. Each video highlights key concepts and vocabulary that students learn across one or more lessons in the unit. The content of these video lesson summaries is based on the written Lesson Summaries found at the end of lessons in the curriculum. The goal of these videos is to support students in reviewing and checking their understanding of important concepts and vocabulary. Here are some possible ways families can use these videos:

- Keep informed on concepts and vocabulary students are learning about in class.
- Watch with their student and pause at key points to predict what comes next or think up other examples of vocabulary terms (the bolded words).
- Consider following the Connecting to Other Units links to review the math concepts that led up to this unit or to preview where the concepts in this unit lead to in future units.

Grade 6, Unit 3: Unit Rates and Percentages	Vimeo	YouTube
Video 1: Converting Measurements (Lessons 2–4)	<a href="#">Link</a>	<a href="#">Link</a>
Video 2: Unit Rates (Lessons 5–8)	<a href="#">Link</a>	<a href="#">Link</a>
Video 3: Understanding Percentage (Lessons 10–13)	<a href="#">Link</a>	<a href="#">Link</a>
Video 4: Solving Percentage Problems (Lessons 14–16)	<a href="#">Link</a>	<a href="#">Link</a>

### Video 1

Video 'VLS G6U3V1 Converting Measurements (Lessons 2–4)' available here: <https://player.vimeo.com/video/469298365>.

### Video 2

Video 'VLS G6U3V2 Unit Rates (Lessons 5–8)' available here: <https://player.vimeo.com/video/470623725>.

### Video 3

Video 'VLS G6U3V3 Understanding Percentage (Lessons 10–13)' available here:  
<https://player.vimeo.com/video/469393213>.

### Video 4

Video 'VLS G6U3V4 Solving Percentage Problems (Lessons 14–16)' available here:  
<https://player.vimeo.com/video/471578428>.

# Units of Measurement

## Family Support Materials 1

If you weighed four objects in pounds, then weighed the same four objects in kilograms, you might come up with this table.

weight (pounds)	weight (kilograms)
22	10
88	40
33	15
40.7	18.5

Students are using what they know about ratios and rates to reason about measurements in different *units of measurement* such as pounds and kilograms. In earlier grades, students converted yards to feet using the fact that 1 yard is 3 feet, and kilometers to meters using the fact that 1 kilometer is 1,000 meters. Now in grade 6, students convert units that do not always use whole numbers.

Here is a task to try with your student:

Explain your strategy for each question.

1. Which is heavier, 1 pound or 1 kilogram?
2. A canoe weighs 99 pounds. How many kilograms does it weigh?
3. A watermelon weighs 12 kilograms. How many pounds does it weigh?

Solution:

Any correct strategy that your student understands and can explain is acceptable. Sample strategies:

1. 1 kilogram is heavier than 1 pound. When we weigh the same object in pounds and kilograms, the number of pounds is more than the number of kilograms. It takes fewer kilograms to express the weight of the same object, so each kilogram must be heavier than each pound. Another example of this idea: if we measure the length of a table in both meters and inches, the number of inches is more than the number of meters. Therefore, 1 inch must be shorter than 1 meter.



2. 45. Using the table, we can reason that 11 pounds is 5 kilograms. Multiplying each of these by 9 shows that 99 pounds is 45 kilograms.
3. 26.4. Using the table, we can find that each kilogram is equal to about 2.2 pounds. This means if we know an object's weight in kilograms, we can multiply by 2.2 to find its weight in pounds.  $12 \cdot (2.2) = 26.4$

## Rates

### Family Support Materials 2

Who biked faster: Andre, who biked 25 miles in 2 hours, or Lin, who biked 30 miles in 3 hours? One strategy would be to calculate a **unit rate** for each person. A unit rate is an equivalent ratio expressed as something “per 1.” For example, Andre’s rate could be written as “ $12\frac{1}{2}$  miles in 1 hour” or “ $12\frac{1}{2}$  miles *per 1 hour*.” Lin’s rate could be written “10 miles per 1 hour.” By finding the unit rates, we can compare the distance each person went in 1 hour to see that Andre biked faster.

Every ratio has *two* unit rates. In this example, we could also compute *hours per mile*: how many hours it took each person to cover 1 mile. Although not every rate has a special name, rates in “miles per hour” are commonly called **speed** and rates in “hours per mile” are commonly called **pace**.

Andre:

distance (miles)	time (hours)
25	2
1	0.08
12.5	1

Lin:

distance (miles)	time (hours)
30	3
10	1
1	0.1

Here is a task to try with your student:

Dry dog food is sold in bulk: 4 pounds for \$16.00.

1. At this rate, what is the cost *per pound* of dog food?
2. At this rate, what is the amount of dog food you can buy *per dollar*?

Solution:

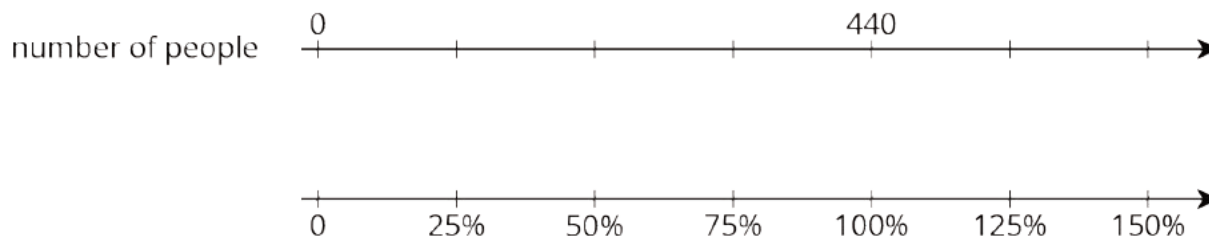
1. \$4.00 per pound because  $16 \div 4 = 4$ .
2. You get  $\frac{1}{4}$  or 0.25 of a pound per dollar because  $4 \div 16 = 0.25$ .

dog food (pounds)	cost (dollars)
4	16
1	4
0.25	1

## Percentages

### Family Support Materials 3

Let's say 440 people attended a school fundraiser last year. If 330 people were adults, what percentage of people were adults? If it's expected that the attendance this year will be 125% of last year, how many attendees are expected this year? A double number line can be used to reason about these questions.



Students use their understanding of “rates per 1” to find percentages, which we can think of as “rates per 100.” Double number lines and tables continue to support their thinking. The example about attendees of a fundraiser could also be organized in a table:

number of people	percentage
440	100%
110	25%
330	75%
550	125%

Toward the end of the unit, students develop more sophisticated strategies for finding percentages. For example, you can find 125% of 440 attendees by computing  $\frac{125}{100} \cdot 440$ . With practice, students will use these more efficient strategies and understand why they work.

Here is a task to try with your student:

For each question, explain your reasoning. If you get stuck, try creating a table or double number line for the situation.

1. A bottle of juice contains 16 ounces, and you drink 25% of the bottle. How many ounces did you drink?
2. You get 9 questions right in a trivia game, which is 75% of the questions. How many questions are in the game?
3. You planned to walk 8 miles, but you ended up walking 12 miles. What percentage of your planned distance did you walk?

Solution:

Any correct reasoning that a student understands and can explain is acceptable. Sample reasoning:

1. 4. 25% of the bottle is  $\frac{1}{4}$  of the bottle, and  $\frac{1}{4}$  of 16 is 4.
2. 12. If 9 questions is 75%, we can divide each by 3 to know that 3 questions is 25%. Multiplying each by 4 shows that 12 questions is 100%.
3. 150%. If 8 miles is 100%, then 4 miles is 50%, and 12 miles is 150%.

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# Unit Assessments

Check Your Readiness A and B  
End-of-Unit Assessment A and B

# Unit Rates and Percentages: Check Your Readiness (A)

1. The school orchestra sells snacks at their concerts to raise money. At the spring concert, they made  $\frac{9}{8}$  as much money as they made at their fall concert.

Did they make more money at the spring or the fall concert? Explain how you know.

2.
  - a. Kilometers and inches are some units used to measure length. What are some other units used to measure length?
  - b. Tablespoons and liters are some units used to measure volume. What are some other units used to measure volume?
3.
  - a. About how much does a textbook weigh?
  - b. About how much does a car weigh?
  - c. About how much does a mosquito weigh?
  - d. About how long is a textbook?
  - e. About how long is a car?
  - f. About how long is a mosquito?

4. Divide. Express each answer as a decimal.

$5 \div 4$

$30 \div 12$

$4 \div 5$

$12 \div 30$

5. Multiply. Express each answer as a decimal.

$\frac{1}{2} \cdot 15$

$(0.2) \cdot 60$

$\frac{3}{4} \cdot 200$

$(0.75) \cdot 20$



6. Write each fraction as a decimal.

a.  $\frac{65}{100}$

b.  $\frac{90}{100}$

c.  $\frac{140}{100}$

d.  $\frac{1}{2}$

e.  $\frac{1}{4}$

f.  $\frac{3}{4}$

g.  $\frac{5}{4}$

h.  $\frac{3}{2}$

## Unit Rates and Percentages: Check Your Readiness (B)

1. A baseball team sells snacks at their games to raise money. At their last game, they made  $\frac{8}{7}$  as much money as they made at their first game.

Did they make more money at their first or last game? Explain how you know.

2.
  - a. Cups and milliliters are some units used to measure volume. What are some other units used to measure volume?
  - b. Meters and yards are some units used to measure length. What are some other units used to measure length?
3.
  - a. About how much does a fly weigh?
  - b. About how much does a binder weigh?
  - c. About how much does a truck weigh?
  - d. About how long is a fly?
  - e. About how long is a binder?
  - f. About how long is a truck?

4. Divide. Express each answer as a decimal.

a.  $5 \div 2$

b.  $2 \div 5$

c.  $50 \div 4$

d.  $4 \div 50$

5. Multiply. Express each answer as a decimal.

a.  $\frac{1}{2} \cdot 13$

b.  $\frac{3}{4} \cdot 400$

c.  $(0.3) \cdot 60$

d.  $(0.75) \cdot 40$

6. Write each fraction as a decimal.

a.  $\frac{1}{4}$

b.  $\frac{1}{2}$

c.  $\frac{3}{4}$

d.  $\frac{9}{4}$

e.  $\frac{5}{2}$

f.  $\frac{55}{100}$

g.  $\frac{70}{100}$

h.  $\frac{130}{100}$

# Unit Rates and Percentages: End-of-Unit Assessment (A)

1. There are 15 pieces of fruit in a bowl and 6 of them are apples. What percentage of the pieces of fruit in the bowl are apples?
  - A. 0.06%
  - B. 0.4%
  - C. 6%
  - D. 40%
  
2. Select all of the trips that would take 2 hours.
  - A. Drive 60 miles per hour between Buffalo and Seneca Falls, which are 120 miles apart.
  - B. Walk 3 miles per hour to school, which is 1.5 miles away.
  - C. Take a train going 80 miles per hour from Albany to New York City, which are 160 miles apart.
  
3. Lin's family has completed 70% of a trip. They have traveled 35 miles. How far is the trip?
  - A. 24.5 miles
  - B. 50 miles
  - C. 59.5 miles
  - D. 200 miles

4. Lin runs 5 laps around a track in 6 minutes.

a. How many minutes per lap is that?

b. How many laps per minute is that?

c. If Lin runs 21 laps at the same rate, how long does it take her?

5. A ship captain is mapping a trip and wants to know the distance the ship will travel over certain time intervals.

time (hours)	distance (miles)
0.5	12.5
1	25
1.5	37.5

Assuming that the ship travels at a constant speed, what is its speed?

6. Which weighs more: a watermelon that weighs 7.5 kilograms or a baby that weighs 12 pounds? Explain your reasoning. Note: 1 pound is about 0.45 kilograms.

7. Elena and Jada are 12 miles apart on a path when they start moving toward each other. Elena runs at a constant speed of 5 miles per hour, and Jada walks at a constant speed of 3 miles per hour. How long does it take until Elena and Jada meet?

## Unit Rates and Percentages: End-of-Unit Assessment (B)

1. Lin's family has completed 60% of a trip. They have traveled 30 miles. How long is the whole trip?
  - A. 50 miles
  - B. 48 miles
  - C. 30 miles
  - D. 18 miles
  
2. It took 4 hours to drive 240 miles. At this rate, how long does it take to drive 90 miles?
  - A. 60 hours
  - B. 6 hours
  - C.  $2\frac{2}{3}$  hours
  - D.  $1\frac{1}{2}$  hours
  
3. There are 9 children in a class who take the bus to school, and there are 15 total children in the class. What percentage of the children take the bus to school?
  - A. 90%
  - B. 60%
  - C. 9%
  - D. 0.6%



4. It takes Andre 4 minutes to swim 5 laps.

a. How many laps per minute is that?

b. How many minutes per lap is that?

c. If Andre swims 22 laps at the same rate, how long does it take him?

5. A conductor is mapping a trip and records the distance the train travels over certain time intervals.

time (hours)	distance (miles)
0.5	22.5
1	45
1.5	67.5

The train travels at a constant speed. What is its speed?

6. Which weighs more: a pumpkin that weighs 3.2 kilograms, or a cat that weighs 9 pounds? Explain your reasoning. Note: 1 pound is about 0.45 kilograms.

7. Noah and Andre are 15 miles apart on a bike path when they start biking toward each other. Noah rides at a constant speed of 4 miles per hour, and Andre rides at a constant speed of 2 miles per hour. How long does it take until Noah and Andre meet?

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# Assessment Answer Keys

Check Your Readiness A and B  
End-of-Unit Assessment A and B

# Assessments

## Assessment : Check Your Readiness (A)

### Problem 1

The content assessed in this problem is first encountered in Lesson 1: The Burj Khalifa.

To do this problem, students need to identify that a fraction is greater than one, and realize that multiplying by a number greater than one results in a larger number. When finding and using rates, students need to think about whether the appropriate rate should be a fraction less than one or greater than one. In this unit, students see that there are two rates associated with quantities in a ratio, each the reciprocal of the other. They will reason about which of the two constants it makes sense to use when doing unit conversions, scaling up recipes and the like.

If most students struggle with this item, plan to use a student error in the Lesson 1 Activity 2 Synthesis that shows an incorrect calculation resulting in  $\frac{5}{6}$  instead of  $1\frac{1}{5}$  minutes per window. After discussing, show the table from the possible responses adding an extra row with a larger number of windows, like 30,000, and a blank for time in minutes. Ask students if the blank will be a number larger or smaller than 30,000. There will be several opportunities throughout this unit to interpret fractional rates.

### Statement

The school orchestra sells snacks at their concerts to raise money. At the spring concert, they made  $\frac{9}{8}$  as much money as they made at their fall concert.

Did they make more money at the spring or the fall concert? Explain how you know.

### Solution

They made more money at the spring concert.  $\frac{9}{8}$  is greater than 1 (since  $\frac{8}{8}$  is equal to 1), and when you multiply by a number greater than 1, you get a larger value.

### Aligned Standards

5.NF.B.5

### Problem 2

The content assessed in this problem is first encountered in Lesson 2: Anchoring Units of Measurement.

Basic units for measuring length are introduced in grade 2 and for volume in grade 3. In grades 4 and 5, students learn about more units and how to convert between them. This problem elicits some of the basic vocabulary students will use in this unit.

If most students struggle with this item, plan to do Lesson 2 which is an optional lesson. This lesson addresses vocabulary for length, volume, and weight. Depending on the level of support needed, you might do the entire lesson or focus on the card sort in Activity 3.

## Statement

1. Kilometers and inches are some units used to measure length. What are some other units used to measure length?
2. Tablespoons and liters are some units used to measure volume. What are some other units used to measure volume?

## Solution

Answers vary. Sample responses:

1. Feet, yards, miles, light years, millimeters, centimeters, meters
2. Cups, pints, quarts, gallons, barrels, milliliters, cubic feet

## Aligned Standards

2.MD.A.1, 3.MD.A.2

## Problem 3

The content assessed in this problem is first encountered in Lesson 2: Anchoring Units of Measurement.

This problem goes beyond 4.MD.A.1, which requires familiarity with relative size of different units of measure within a single system of measurement (for instance, 12 inches per 1 foot). In this problem, students select appropriate units and are also free to select their own system of measurement.

These items have variability in what their measurements might be. Look for whether students can come up with something reasonable in terms of order of magnitude and units of measure. Some students may be familiar with, say, pounds, but not know units of measure small enough to describe the weight of a mosquito. This problem is a good opportunity to pool students' knowledge of different units of measure.

If most students struggle with this item, plan to do Lesson 2 which is an optional lesson. Students can practice their estimation skills with length, volume, and weight during the Activity 2 card sort.

## Statement

1. About how much does a textbook weigh?
2. About how much does a car weigh?
3. About how much does a mosquito weigh?

4. About how long is a textbook?
5. About how long is a car?
6. About how long is a mosquito?

## Solution

Answers vary. Sample responses:

1. 1 pound or 1 kilogram
2. 1 ton
3. 2 mg
4. 1 foot
5. 4 to 5 yards
6. 1 cm

## Aligned Standards

4.MD.A.1

### Problem 4

The content assessed in this problem is first encountered in Lesson 5: Comparing Speeds and Prices.

Students will need to perform division when calculating unit rates and percentages. Keep an eye out for students who reverse the order of division or who misplace the decimal point.

If most students struggle with this item, plan to use items from the Practice Problems in Lessons 1, 2, and 3 to review division with decimal solutions before doing Lesson 5. In this Lesson, Activity 3 will offer more practice using a shopping context. Estimation warm-ups such as the warm-up in Lesson 5, Activity 1 can also be used to help students check the reasonableness of their answers.

## Statement

Divide. Express each answer as a decimal.

$$5 \div 4$$

$$30 \div 12$$

$$4 \div 5$$

$$12 \div 30$$

## Solution

1. 1.25
2. 0.8

3. 2.5

4. 0.4

## Aligned Standards

5.NBT.B.7

### Problem 5

The content assessed in this problem is first encountered in Lesson 4: Converting Units.

The first two parts of this problem involve multiplying a fraction by a whole number. The second two parts require keeping track of place value in decimal multiplication. Students will need to perform these types of calculations when using percent rates.

If most students struggle with this item, plan to do Lesson 4 Activity 1, attending to the relationship between multiplication and division with fractions as well as how to use the structure of previous problems to solve the current one. Students may use the skill of multiplying by a decimal rate in Lesson 6, although other strategies such as double number lines and tables of equivalent ratios might also be used. Help students make connections between multiplication and other strategies that rely on repeated addition.

### Statement

Multiply. Express each answer as a decimal.

$$\frac{1}{2} \cdot 15$$

$$(0.2) \cdot 60$$

$$\frac{3}{4} \cdot 200$$

$$(0.75) \cdot 20$$

### Solution

1. 7.5

2. 150

3. 12

4. 15

## Aligned Standards

5.NBT.B.5, 5.NBT.B.7

### Problem 6

The content assessed in this problem is first encountered in Lesson 8: More about Constant Speed.

The fractions in this problem are all multiples of the “benchmark” fractions  $\frac{1}{100}$ ,  $\frac{1}{2}$ , and  $\frac{1}{4}$ .

Check if students can recognize that some of the fractions are numbers that are greater than one. For incorrect answers, note whether students are still using multiples of  $\frac{1}{100}$ ,  $\frac{1}{2}$ , or  $\frac{1}{4}$ .

Some students may write the number 0.9 as 0.90. While this is technically correct, it is worth checking whether these students understand that 0.9 and 0.90 represent the same number.

If most students struggle with this item, plan to attend to the relationship between fractions and decimals in Lesson 8 Activity 1. For this task students will need to be fluent in their use of fractions and decimals to make sense of unit rates and scaling.

## Statement

Write each fraction as a decimal.

1.  $\frac{65}{100}$

2.  $\frac{90}{100}$

3.  $\frac{140}{100}$

4.  $\frac{1}{2}$

5.  $\frac{1}{4}$

6.  $\frac{3}{4}$

7.  $\frac{5}{4}$

8.  $\frac{3}{2}$

## Solution

1. 0.65

2. 0.9

3. 1.4

4. 0.5

5. 0.25

6. 0.75

7. 1.25

8. 1.5

## Aligned Standards

4.NF.C.6, 5.NBT.B.7

## Assessment : Check Your Readiness (B)

### Problem 1

The content assessed in this problem is first encountered in Lesson 1: The Burj Khalifa.

To do this problem, students need to identify that a fraction is greater than one, and realize that multiplying by a number greater than one results in a larger number. These concepts will be useful in this unit; when finding and using rates, students need to think about whether the appropriate rate should be a fraction less than one or greater than one. In the next unit, students will see that there are two rates associated with quantities in a ratio, with each being the reciprocal of the other. They will reason about which of the two constants it makes sense to use when doing unit conversions, scaling up recipes, and the like.

#### Statement

A baseball team sells snacks at their games to raise money. At their last game, they made  $\frac{8}{7}$  as much money as they made at their first game.

Did they make more money at their first or last game? Explain how you know.

#### Solution

They made more money at the last home game.  $\frac{8}{7}$  is greater than 1 (since  $\frac{7}{7}$  is equal to 1), and when you multiply by a number greater than 1, you get a larger value.

#### Aligned Standards

5.NF.B.5

### Problem 2

The content assessed in this problem is first encountered in Lesson 2: Anchoring Units of Measurement.

Basic units for measuring length are introduced in grade 2 and for volume in grade 3. In grades 4 and 5, students learn about more units and how to convert between them. This problem elicits some of the basic vocabulary students will use in this unit.

If most students struggle with this item, plan to do Lesson 2 which is an optional lesson. This lesson addresses vocabulary for length, volume, and weight. Depending on the level of support needed, you might do the entire lesson or focus on the card sort in Activity 3.

#### Statement

1. Cups and milliliters are some units used to measure volume. What are some other units used to measure volume?



2. Meters and yards are some units used to measure length. What are some other units used to measure length?

## Solution

Answers vary. Sample responses:

1. Tablespoons, pints, quarts, gallons, liters, cubic centimeters, barrels
2. Kilometers, feet, millimeters, centimeters, inches, light years, miles

## Aligned Standards

2.MD.A.1, 3.MD.A.2

### Problem 3

The content assessed in this problem is first encountered in Lesson 2: Anchoring Units of Measurement.

This problem goes beyond 4.MD.A.1, which requires familiarity with relative size of different units of measure within a single system of measurement (for instance, 12 inches per 1 foot). In this problem, students select appropriate units and are also free to select their own system of measurement. These items have variability in what their measurements might be. Look for whether students can come up with something reasonable in terms of order of magnitude and units of measure. Some students may be familiar with, say, pounds, but not know units of measure small enough to describe the weight of a fly. This problem is a good opportunity to pool students' knowledge of different units of measure.

If most students struggle with this item, plan to do Lesson 2 which is an optional lesson. Students can practice their estimation skills with length, volume, and weight during the Activity 2 card sort.

## Statement

1. About how much does a fly weigh?
2. About how much does a binder weigh?
3. About how much does a truck weigh?
4. About how long is a fly?
5. About how long is a binder?
6. About how long is a truck?

## Solution

Answers vary. Sample responses:

1. 1 mg

2. 3 pounds or 1 kg
3. 5,000 pounds or 2 tons
4. 1 cm
5. 12 inches or 1 foot
6. 15 feet or 5 yards

## Aligned Standards

4.MD.A.1

### Problem 4

The content assessed in this problem is first encountered in Lesson 5: Comparing Speeds and Prices.

Students will need to perform division when calculating unit rates and percentages. Keep an eye out for students who reverse the order of division or who misplace the decimal point.

If most students struggle with this item, plan to use items from the Practice Problems in Lessons 1, 2, and 3 to review division with decimal solutions before doing Lesson 5. In this Lesson, Activity 3 will offer more practice using a shopping context. Estimation warm-ups such as the warm-up in Lesson 5, Activity 1 can also be used to help students check the reasonableness of their answers.

### Statement

Divide. Express each answer as a decimal.

1.  $5 \div 2$

2.  $2 \div 5$

3.  $50 \div 4$

4.  $4 \div 50$

### Solution

1. 2.5

2. 0.4

3. 12.5

4. 0.08

## Aligned Standards

5.NBT.B.7

## Problem 5

The content assessed in this problem is first encountered in Lesson 4: Converting Units.

The first two parts of this problem involve multiplying a fraction by a whole number. The second two parts require keeping track of place value in decimal multiplication. Students will need to perform these types of calculations when using percent rates.

If most students struggle with this item, plan to do Lesson 4 Activity 1, attending to the relationship between multiplication and division with fractions as well as how to use the structure of previous problems to solve the current one. Students may use the skill of multiplying by a decimal rate in Lesson 6, although other strategies such as double number lines and tables of equivalent ratios might also be used. Help students make connections between multiplication and other strategies that rely on repeated addition.

### Statement

Multiply. Express each answer as a decimal.

1.  $\frac{1}{2} \cdot 13$

2.  $\frac{3}{4} \cdot 400$

3.  $(0.3) \cdot 60$

4.  $(0.75) \cdot 40$

### Solution

1. 6.5

2. 300

3. 18

4. 30

### Aligned Standards

5.NBT.B.5, 5.NBT.B.7

## Problem 6

The content assessed in this problem is first encountered in Lesson 8: More about Constant Speed.

The fractions in this problem are all multiples of the “benchmark” fractions  $\frac{1}{100}$ ,  $\frac{1}{2}$ , and  $\frac{1}{4}$ .

Check whether students can recognize the improper fractions as numbers that are greater than one. For incorrect answers, note whether students are still using multiples of 0.5, 0.25, or 0.01.

Some students may write the number 0.7 as 0.70. While this is technically correct, it is worth checking whether these students understand that 0.7 and 0.70 represent the same number.

If most students struggle with this item, plan to attend to the relationship between fractions and decimals in Lesson 8 Activity 1. For this task students will need to be fluent in their use of fractions and decimals to make sense of unit rates and scaling.

## Statement

Write each fraction as a decimal.

1.  $\frac{1}{4}$

2.  $\frac{1}{2}$

3.  $\frac{3}{4}$

4.  $\frac{9}{4}$

5.  $\frac{5}{2}$

6.  $\frac{55}{100}$

7.  $\frac{70}{100}$

8.  $\frac{130}{100}$

## Solution

1. 0.25

2. 0.5

3. 0.75

4. 2.25

5. 2.5

6. 0.55

7. 0.7

8. 1.3

## Aligned Standards

4.NF.C.6, 5.NBT.B.7

## Assessment : End-of-Unit Assessment (A)

### Problem 1

This problem is possible to solve without any calculations: 6 is a bit less than half of 15, and D is the only answer choice that is reasonably close. Students selecting A or C are using the given information of “6 apples” without calculating the percentage of the whole. Students selecting B have correctly divided 6 by 15, but have not multiplied by 100.

#### Statement

There are 15 pieces of fruit in a bowl and 6 of them are apples. What percentage of the pieces of fruit in the bowl are apples?

- A. 0.06%
- B. 0.4%
- C. 6%
- D. 40%

#### Solution

D

#### Aligned Standards

6.RP.A.3.c

### Problem 2

Students who incorrectly select B are reversing the ratio, dividing speed by distance or noticing that the speed is twice the distance. In choices A and C, the distance is twice the speed, which means that a person traveling at that rate would cover the distance in two hours.

#### Statement

Select **all** of the trips that would take 2 hours.

- A. Drive 60 miles per hour between Buffalo and Seneca Falls, which are 120 miles apart.
- B. Walk 3 miles per hour to school, which is 1.5 miles away.
- C. Take a train going 80 miles per hour from Albany to New York City, which are 160 miles apart.

#### Solution

["A", "C"]

## Aligned Standards

6.RP.A.3.b

### Problem 3

Students are not expected to take an algorithmic approach to find  $B$  in " $A\%$  of  $B$  is  $C$ ." They may use a double number line or table to keep track of values.

Students selecting A have computed 70% of 35, rather than using the information that 35 miles is 70% of the trip. Students selecting C have calculated 70% of 35, then added 35 more miles (since that distance is already traveled). Students selecting D may have calculated 200 as the solution to "35% of what is 70?", reversing the 35 and 70.

### Statement

Lin's family has completed 70% of a trip. They have traveled 35 miles. How far is the trip?

- A. 24.5 miles
- B. 50 miles
- C. 59.5 miles
- D. 200 miles

### Solution

B

## Aligned Standards

6.RP.A.3.c

### Problem 4

This problem has students calculate both unit rates that describe a situation. In the third part, students need to decide which of the two unit rates makes sense to use.

### Statement

Lin runs 5 laps around a track in 6 minutes.

1. How many minutes per lap is that?
2. How many laps per minute is that?
3. If Lin runs 21 laps at the same rate, how long does it take her?

### Solution

1. 1.2 minutes or 1 minute 12 seconds
2.  $\frac{5}{6}$  laps

$$3.21 \cdot (1.2) = 25.2 \text{ minutes}$$

## Aligned Standards

6.RP.A.2, 6.RP.A.3.b

### Problem 5

Students may think they may need to do extensive computations in this problem. In fact, looking at row 2 of the table is sufficient: the distance the ship travels in one hour is the speed (in miles per hour).

#### Statement

A ship captain is mapping a trip and wants to know the distance the ship will travel over certain time intervals.

time (hours)	distance (miles)
0.5	12.5
1	25
1.5	37.5

Assuming that the ship travels at a constant speed, what is its speed?

#### Solution

25 miles per hour

## Aligned Standards

6.RP.A.2

### Problem 6

This problem can be solved quickly using estimation, though students may do computations to convert pounds to kilograms or kilograms to pounds.

#### Statement

Which weighs more: a watermelon that weighs 7.5 kilograms or a baby that weighs 12 pounds? Explain your reasoning. Note: 1 pound is about 0.45 kilograms.

#### Solution

The watermelon weighs more. Possible strategies:

- 12 lbs is about 5.4 kg.

- Without computing anything, it can be reasoned that  $12 \cdot (0.45)$  is less than 6, so the baby must weigh less than the watermelon.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: A 12-pound baby weighs less than 6 kilograms, so the watermelon weighs more.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: correct unit conversion with incorrect interpretation; unit conversion multiplies kilograms by 0.45 or otherwise “goes the wrong way”; arithmetic errors in unit conversion.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: work does not involve unit conversion, either through estimation (as in the minimal Tier 1 response) or explicitly; unit conversion is attempted but with an incorrect conversion factor.

## Aligned Standards

6.RP.A.3.d

### Problem 7

This problem allows for a variety of strategies. Students who try to solve the problem in their head or by doing calculations may get stuck if they do not have an insight right away. Students who make a table or double number line will probably have their efforts pay off, since these strategies can help to figure out that Elena and Jada are 4 miles apart after 1 hour.

#### Statement

Elena and Jada are 12 miles apart on a path when they start moving toward each other. Elena runs at a constant speed of 5 miles per hour, and Jada walks at a constant speed of 3 miles per hour. How long does it take until Elena and Jada meet?

#### Solution

1.5 hours. Possible strategies:

- Realize that combined speeds are 8 miles per hour, so it takes 1.5 hours for them to cover 12 miles together.
- Construct a table or double number line to keep track of the distances covered over intervals of time.



Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Acceptable errors: a clear visual argument accompanied by minimal words.
- Sample: After 1 hour they are 4 miles apart. That was 8 miles, so it will take another half hour to get 4 miles closer. The total time is 1.5 hours.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: work has a helpful visual representation or table but contains one error that propagates; a written-only response is reasoned correctly but contains arithmetic errors.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: A table or visual representation is used to keep track of the information but contains flawed proportional reasoning; a written-only response involves some reference to location at a given time or to combined speed, with significant errors. Acceptable errors: deducing that the two students are 4 miles apart after 1 hour, with subsequent serious errors.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: no visual representation or explanation; explanation does not involve proportional reasoning.

## **Aligned Standards**

6.RP.A.3

## Assessment : End-of-Unit Assessment (B)

### Problem 1

Students are not expected to take an algorithmic approach to find  $B$  in  $A\%$  of  $B$  is  $C$ . They may use a double number line or table to keep track of values. Students selecting D have computed 60% of 30, rather than using the information that 30 miles is 60% of the trip. Students selecting C have subtracted  $60 - 30$ . Students selecting B have have calculated 60% of 30, then added 30 more miles (since that distance is already traveled).

#### Statement

Lin's family has completed 60% of a trip. They have traveled 30 miles. How long is the whole trip?

- A. 50 miles
- B. 48 miles
- C. 30 miles
- D. 18 miles

#### Solution

A

#### Aligned Standards

6.RP.A.3.c

### Problem 2

Students who incorrectly select A are dividing distance (240) by time (4), finding the speed of travel in miles per hour. Students selecting B may have divided 240 by 4, but realizing 60 hours was implausible, modified their answer by dividing by 10. Student choosing C divided 240 by 90, perhaps by performing operations on numbers in the problem until they arrived at a plausible answer.

#### Statement

It took 4 hours to drive 240 miles. At this rate, how long does it take to drive 90 miles?

- A. 60 hours
- B. 6 hours
- C.  $2\frac{2}{3}$  hours
- D.  $1\frac{1}{2}$  hours

## Solution

D

## Aligned Standards

6.RP.A.3.b

### Problem 3

This problem is possible to solve without any calculations; 9 is a bit more than half of 15, and B is the only answer choice that is reasonably close. Students selecting A or C are using the given information of “9 students” without calculating the percentage of the whole. Students selecting D have divided 9 by 15, but have not expressed the rate as a percentage.

### Statement

There are 9 children in a class who take the bus to school, and there are 15 total children in the class. What percentage of the children take the bus to school?

- A. 90%
- B. 60%
- C. 9%
- D. 0.6%

## Solution

B

## Aligned Standards

6.RP.A.3.c

### Problem 4

This problem has students calculate both unit rates that describe a situation. In the third part, students need to decide which of the two unit rates makes sense to use.

### Statement

It takes Andre 4 minutes to swim 5 laps.

1. How many laps per minute is that?
2. How many minutes per lap is that?
3. If Andre swims 22 laps at the same rate, how long does it take him?

## Solution

1.  $\frac{5}{4}$

2.  $\frac{4}{5}$  minutes, or 48 seconds per lap

3.  $22 \cdot (\frac{4}{5}) = 17.6$  minutes

## Aligned Standards

6.RP.A.2, 6.RP.A.3.b

### Problem 5

Students may think they may need to do extensive computations in this problem. In fact, looking at row 2 of the table is sufficient; the distance the ship travels in one hour is the speed (in miles per hour).

#### Statement

A conductor is mapping a trip and records the distance the train travels over certain time intervals.

time (hours)	distance (miles)
0.5	22.5
1	45
1.5	67.5

The train travels at a constant speed. What is its speed?

#### Solution

45 miles per hour

## Aligned Standards

6.RP.A.2

### Problem 6

This problem can be solved quickly using estimation, though students may do computations to convert pounds to kilograms or kilograms to pounds.

#### Statement

Which weighs more: a pumpkin that weighs 3.2 kilograms, or a cat that weighs 9 pounds? Explain your reasoning. Note: 1 pound is about 0.45 kilograms.

#### Solution

The cat weighs more. Possible strategies:

- 9 pounds is about 4.05 kg.

- Without computing, a student might reason that  $9 \cdot (0.45)$  is a little less than 4.5, so the cat must weigh more than the pumpkin.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: A 9-pound cat weighs a little less than 4.5 kilograms, so the cat weighs more.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: correct unit conversion with incorrect interpretation; unit conversion multiplies kilograms by 0.45 or otherwise “goes the wrong way”; arithmetic errors in unit conversion.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: work does not involve unit conversion, either through estimation (as in the minimal Tier 1 response) or explicitly; unit conversion is attempted but with an incorrect conversion factor.

## Aligned Standards

6.RP.A.3.d

### Problem 7

This problem allows for a variety of strategies. Students who try to solve the problem in their head or by doing calculations may get stuck if they do not have an insight right away. Students who make a table or double number line will probably have their efforts pay off, since these strategies can help to figure out that Noah and Andre are 9 miles apart after 1 hour.

#### Statement

Noah and Andre are 15 miles apart on a bike path when they start biking toward each other. Noah rides at a constant speed of 4 miles per hour, and Andre rides at a constant speed of 2 miles per hour. How long does it take until Noah and Andre meet?

#### Solution

2.5 hours. Possible strategies:

- Realize that combined speeds are 6 miles per hour, so it takes 2.5 hours for them to cover 15 miles together.
- Construct a table or double number line to keep track of the distances covered over intervals of time.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Acceptable errors: a clear visual argument accompanied by minimal words.
- Sample: After 1 hour, they are 9 miles apart, and after 2 hours, they are 3 miles apart. It will take another half hour to get 3 miles closer. The total time is 2.5 hours.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: work has a helpful visual representation or table but contains one error that propagates; a written-only response is reasoned correctly but contains arithmetic errors.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: A table or visual representation is used to keep track of the information but contains flawed proportional reasoning; a written-only response involves some reference to location at a given time or to combined speed, with significant errors.
- Acceptable errors: deducing that the two students are 9 miles apart after 1 hour, with subsequent serious errors.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: no visual representation or explanation; explanation does not involve proportional reasoning.

## **Aligned Standards**

6.RP.A.3

**CKMath**<sup>™</sup>  
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Lesson  
Cool Downs

# Lesson 1: The Burj Khalifa

## Cool Down: Going Up?

The fastest elevators in the Burj Khalifa can travel 330 feet in just 10 seconds. How far does the elevator travel in 11 seconds? Explain your reasoning.

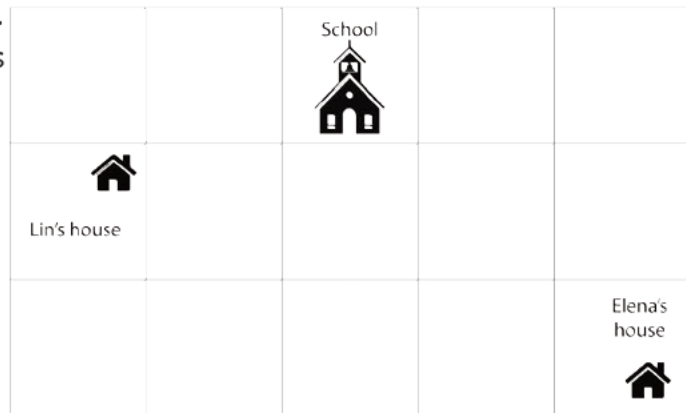


# Lesson 2: Anchoring Units of Measurement

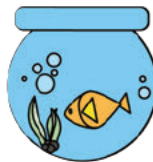
## Cool Down: So Much in Common

Lin and Elena have discovered they have so much in common.

1. They each walk 500 units to school. Who walks 500 feet, and who walks 500 yards? Explain your reasoning.



2. They each have a fish tank holding 20 units of water. Whose tank holds 20 gallons, and whose holds 20 cups? Explain your reasoning.



Lin's fish bowl



Elena's fish tank

3. They each have a brother who weighs 40 units. Whose brother weighs 40 pounds, and whose weighs 40 kilograms? Explain your reasoning.



Elena's Brother



Lin's Brother

## Lesson 3: Measuring with Different-Sized Units

### Cool Down: Which Measurement is Which?

1. Lin has a pet German Shepherd that weighs 38 when measured in one unit and 84 when measured in a different unit. Which measurement is in pounds, and which is in kilograms?

38 \_\_\_\_\_                      84 \_\_\_\_\_

2. Elena has a pet parakeet that weighs 6 when measured in one unit and 170 when measured in a different unit. Which measurement is in ounces, and which is in grams?

6 \_\_\_\_\_                      170 \_\_\_\_\_

3. Behind Lin's house there is a kiddie pool that holds 180 or 680 units of water, depending on which unit you are using to measure. Which measurement is in gallons, and which is in liters?

180 \_\_\_\_\_                      680 \_\_\_\_\_

4. Behind Elena's house there is a portable storage container that holds 29 or 1024 units, depending on which unit you are using to measure. Which measurement is in cubic feet, and which is in cubic meters?

29 \_\_\_\_\_                      1024 \_\_\_\_\_

## **Lesson 4: Converting Units**

### **Cool Down: Buckets**

A large bucket holds 5 gallons of water, which is about the same as 19 liters of water.

A small bucket holds 2 gallons of water. About how many liters does it hold?

# Lesson 5: Comparing Speeds and Prices

## Cool Down: A Sale on Sparkling Water

Bottles of sparkling water usually cost \$1.69 each. This week they are on sale for 4 bottles for \$5. You bought one last week and one this week. Did you pay more or less for the bottle this week? How much more or less?



## Lesson 6: Interpreting Rates

### Cool Down: Buying Grapes by the Pound

Two pounds of grapes cost \$6.

1. Complete the table showing the price of different amounts of grapes at this rate.

grapes (pounds)	price (dollars)
2	6
	1
1	

2. Explain the meaning of each of the numbers you found.

# Lesson 7: Equivalent Ratios Have the Same Unit Rates

## Cool Down: Cheetah Speed

A cheetah can run at its top speed for about 25 seconds. Complete the table to represent a cheetah running at a constant speed. Explain or show your reasoning.

time (seconds)	distance (meters)	speed (meters per second)
4	120	
25		
	270	



# Lesson 9: Solving Rate Problems

## Cool Down: Tacos by the Pack

A restaurant sells 10 tacos for \$8.49, or 6 of the same kind of taco for \$5.40.

Which is the better deal? Explain how you know.



# Lesson 10: What Are Percentages?

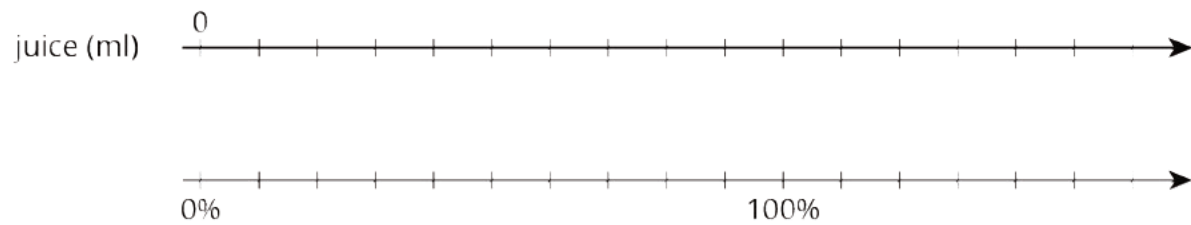
## Cool Down: Eight Dimes

1. Fill in the blank: The value of 8 dimes is \_\_\_\_\_% of the value of a dollar.
  
2. Name a combination of coins that is 130% of the value of a dollar.

# Lesson 11: Percentages and Double Number Lines

## Cool Down: A Medium Bottle of Juice

A large bottle of juice contains 500 milliliters of juice. A medium bottle contains 70% as much juice as the large bottle. How many milliliters of juice are in the medium bottle?



# Lesson 12: Percentages and Tape Diagrams

## Cool Down: Small and Large

Complete the statement with a situation and a unit of your choice. Then answer the question and draw a diagram.

A small \_\_\_\_\_ holds 75% as much as a large \_\_\_\_\_.

1. If the small holds 36 units, how much does the large hold?

2. Draw a diagram to illustrate your answer.

# Lesson 13: Benchmark Percentages

## Cool Down: Around the Clock

Answer each question and explain your reasoning.

1. How long is 50% of 60 minutes?

2. How long is 10% of 60 minutes?

3. How long is 75% of 60 minutes?

# Lesson 14: Solving Percentage Problems

## Cool Down: Walking to School

It takes Jada 20 minutes to walk to school. It takes Andre 80% as long to walk to school.

How long does it take Andre to walk to school?

# Lesson 15: Finding This Percent of That

## Cool Down: Ordering Percentages of Different Numbers

Order these three values from least to greatest. Explain or show your reasoning.

65% of 80

82% of 50

170% of 30

# Lesson 16: Finding the Percentage

## Cool Down: Jet Fuel

A jet plane can carry up to 200,000 liters of fuel. It used 130,000 liters of fuel during a flight. What percentage of the fuel capacity did it use on this flight?

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Blackline  
Masters



# Blackline Masters for Unit Rates and Percentages

address	title	students per copy	written on?	requires cutting?	card stock recommended?	color paper recommended?
Activity Grade6.3.2.3	Card Sort: Measurements	4	no	yes	no	yes
Activity Grade6.3.14.3	Info Gap: Music Devices	4	no	yes	no	no
Activity Grade6.3.3.2	Measurement Stations	30	yes	yes	yes	yes
Activity Grade6.3.9.2	Card Sort: Is it a Deal?	2	yes	yes	no	no

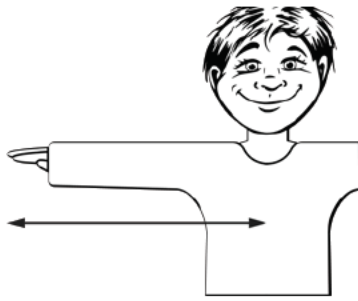
6.3.2.3 Card Sort: Measurements.

<b>1 centimeter</b>	<b>1 cup</b>	<b>1 foot</b>
<b>1 gallon</b>	<b>1 gram</b>	<b>1 inch</b>
<b>1 kilogram</b>	<b>1 kilometer</b>	<b>1 liter</b>
<b>1 meter</b>	<b>1 mile</b>	<b>1 milliliter</b>
<b>1 millimeter</b>	<b>1 ounce</b>	<b>1 pound</b>
<b>1 quart</b>	<b>1 ton</b>	<b>1 yard</b>

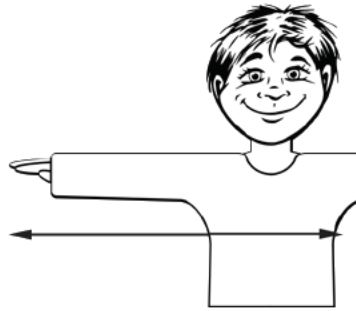
6.3.2.3 Card Sort: Measurements L



length of a thumb



length from chest to fingers



length from fingers to opposite armpit



width of a pinky finger



distance walked in 10 minutes



distance run in 10 minutes



length of a shoe



length of a ruler



width of a quarter



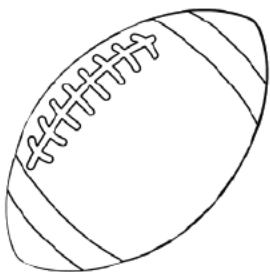
length of a license plate



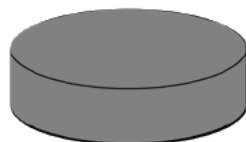
length of a baseball bat and ball



thickness of a dime



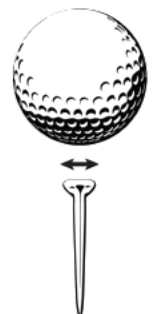
length of a football



thickness of a hockey puck

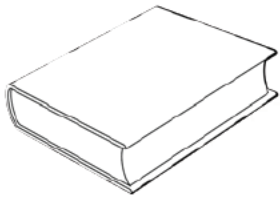


length of a baseball bat

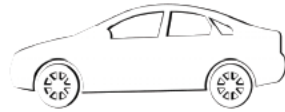


width of the head of a golf tee

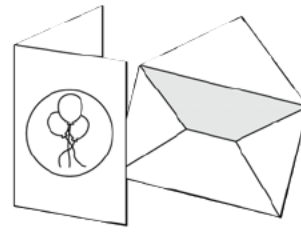
WM  
6.3.2.3 Card Sort: Measurements



**textbook**



**small car**



**birthday card**



**paperclip**

WM

WM

WM

WM

WM

WM



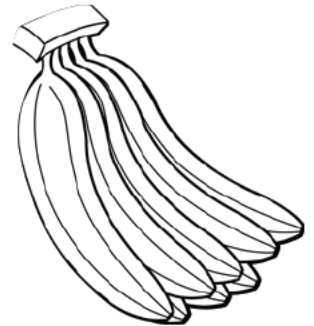
**hooded sweatshirt**



**dollar bill**



**box of 96 crayons**



**bunch of bananas**

WM

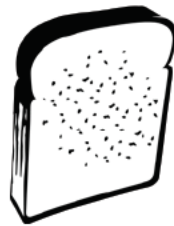
WM

WM

WM



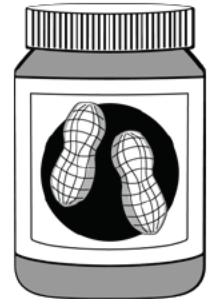
**2 raisins**



**slice of bread**



**loaf of bread**



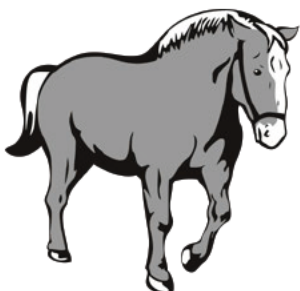
**jar of peanut butter**

WM

WM

WM

WM



**draft horse**



**crow**



**guinea pig**



**mouse**

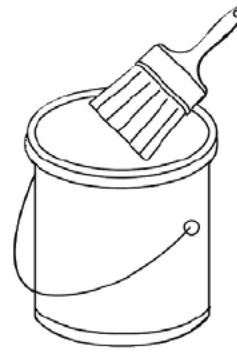
6.3.2.3 Card Sort: Measurements V



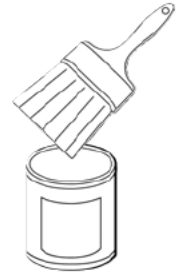
**school milk carton**



**packet of  
artificial sweetener**



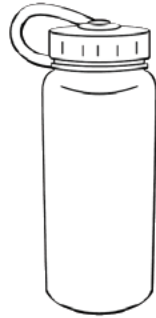
**large paint can**



**small paint can**



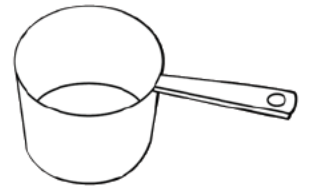
**large milk jug**



**reuseable water bottle**



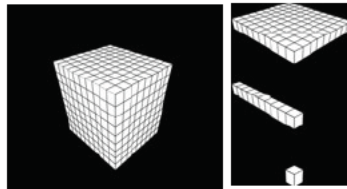
**large sports drink bottle**



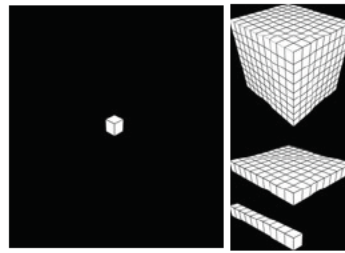
**measuring cup**



**half of a large  
soda bottle**



**1000s cube**



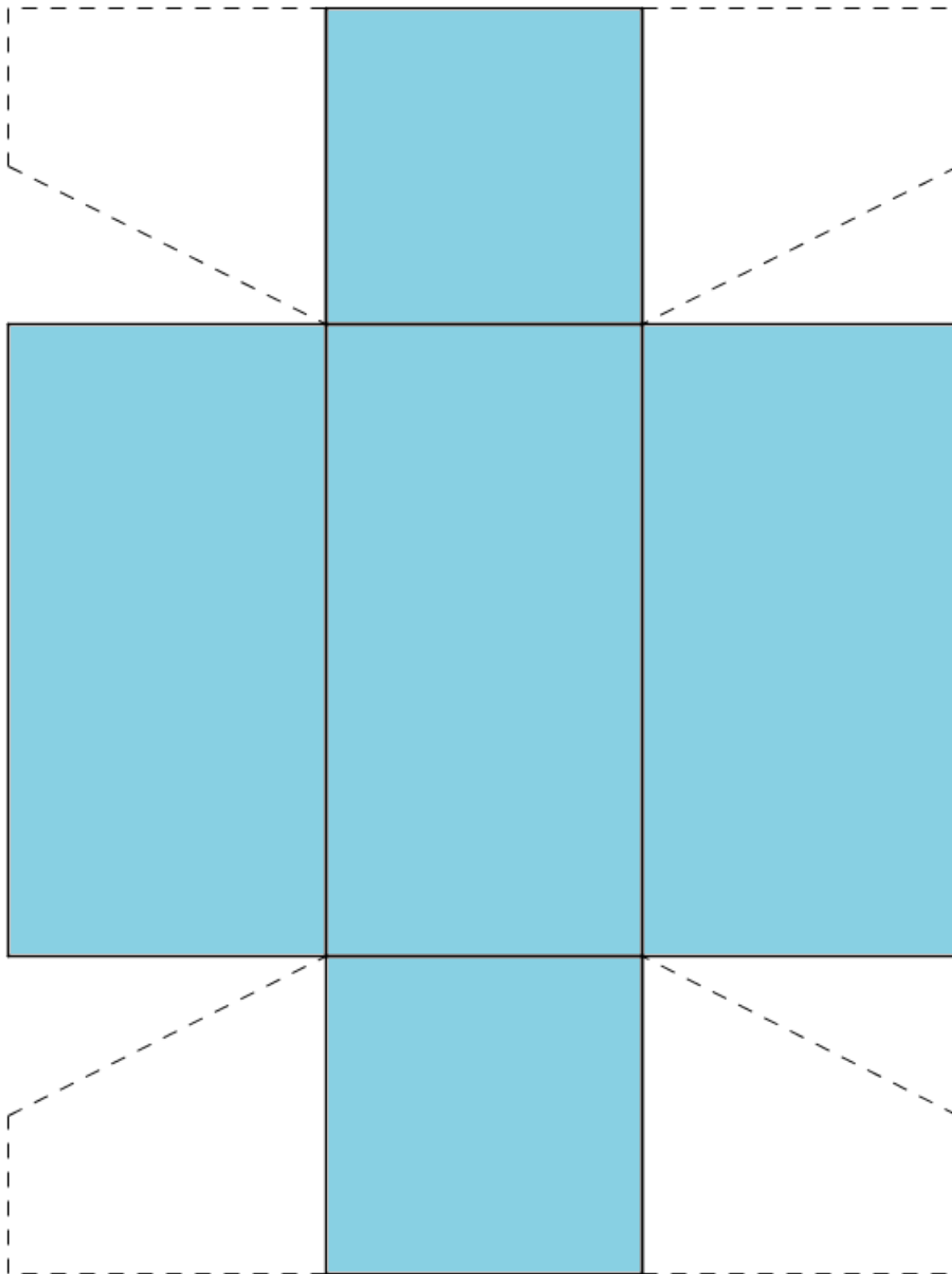
**1s cube**



**raindrop**

### 6.3.3.2 Measurement Stations.

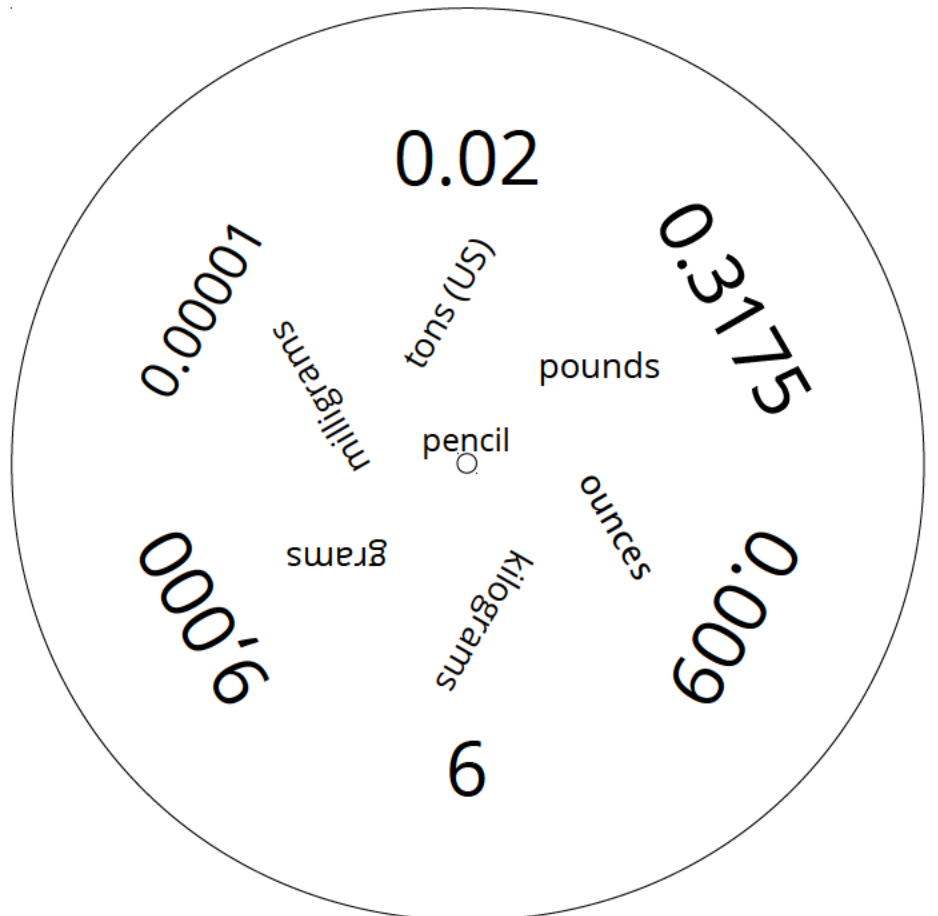
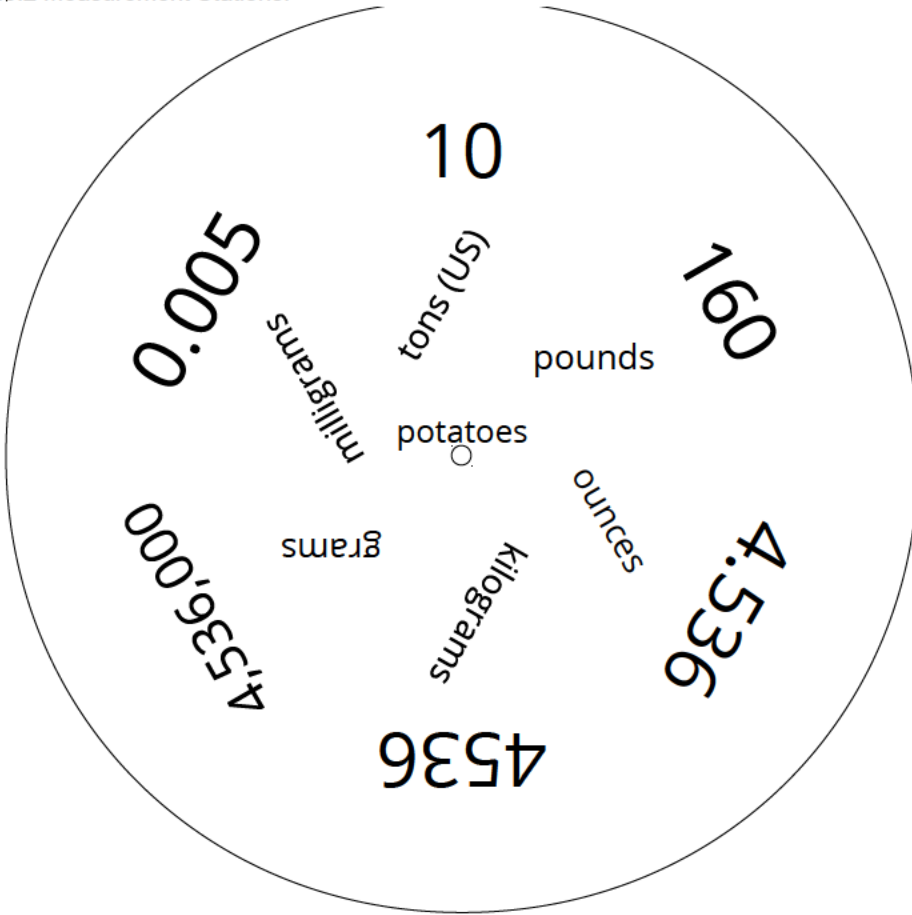
The entire net should be **slightly larger than** 6 inches by 8 inches when printed, before assembly. It is best to copy this net onto **card stock** before cutting it out **and gluing** it.



6.3.3.2 Measurement Stations.

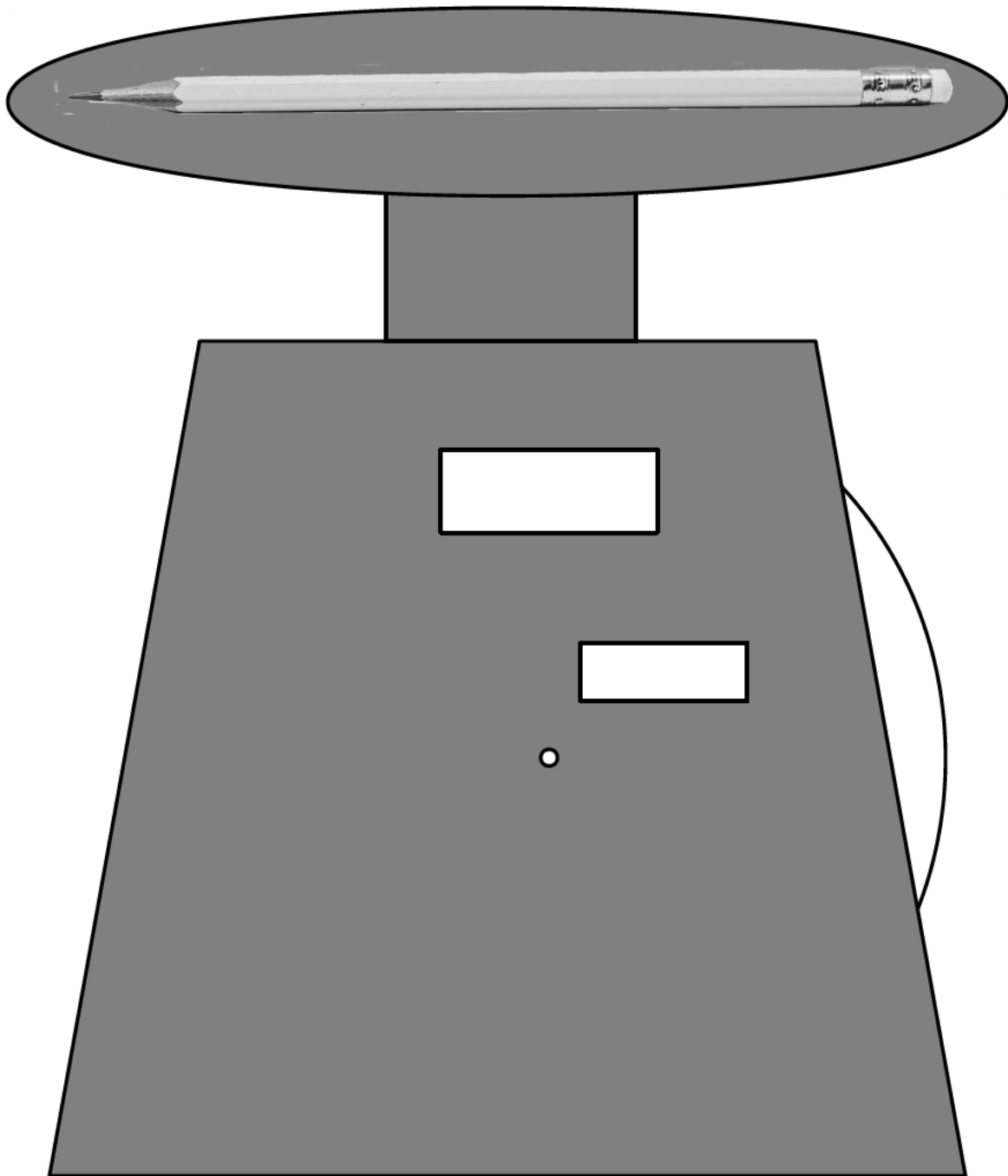


6.3.3.2 Measurement Stations.

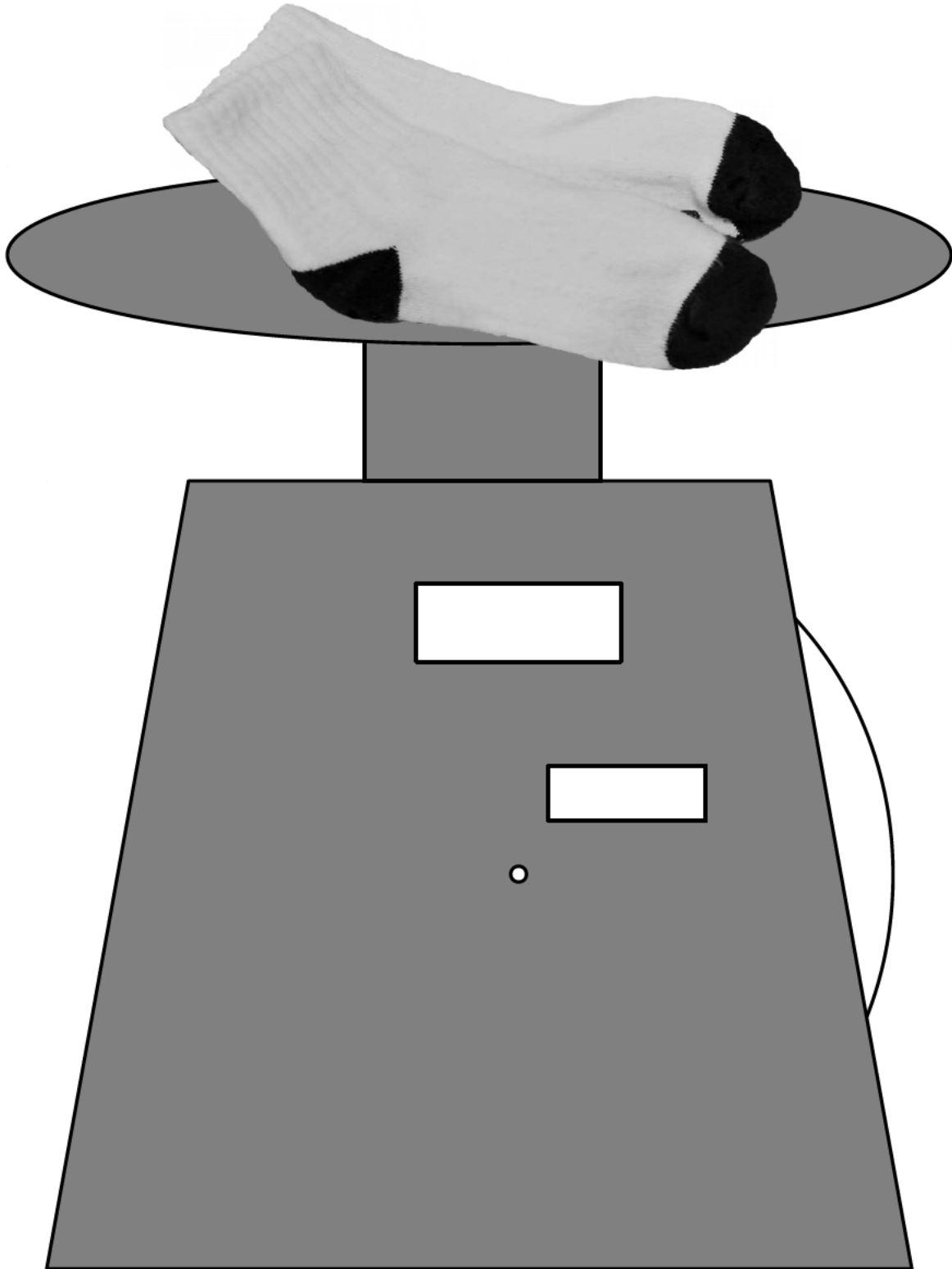




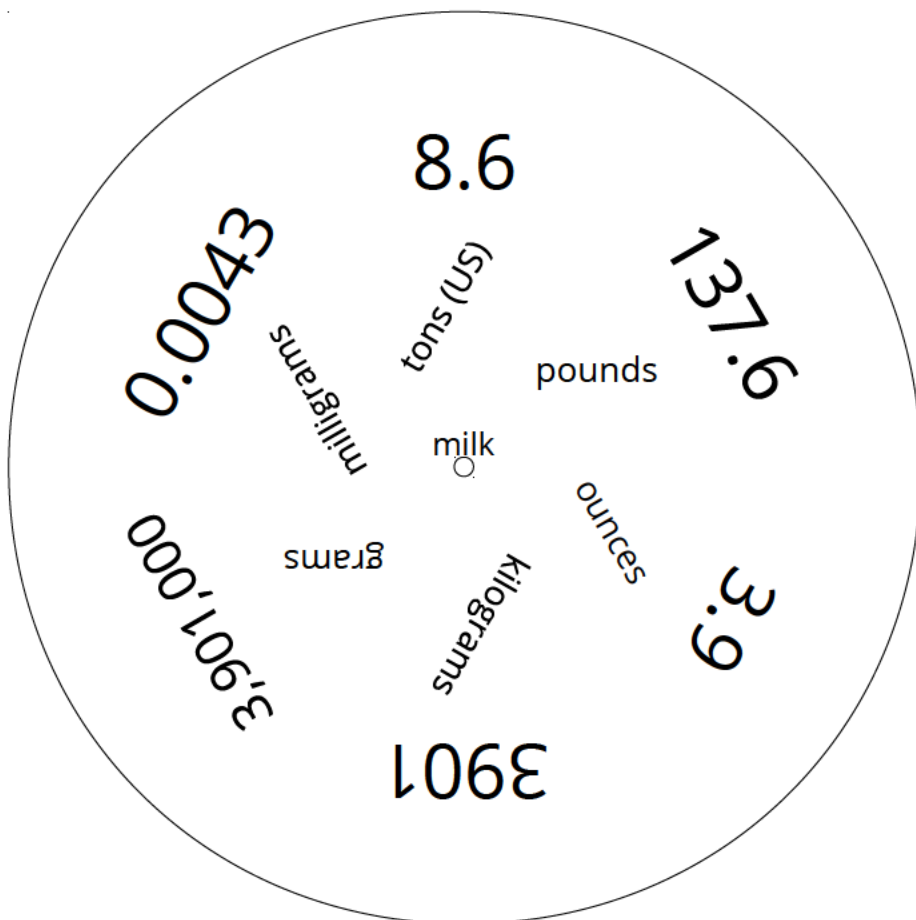
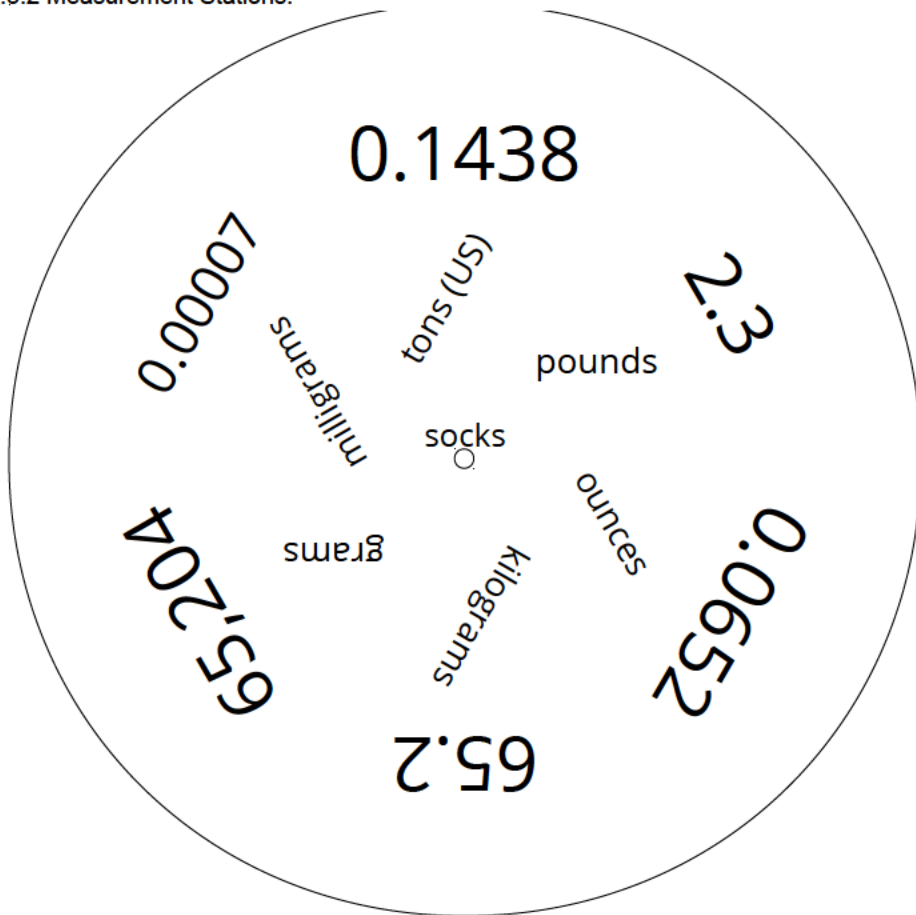
6.3.3.2 Measurement Stations.



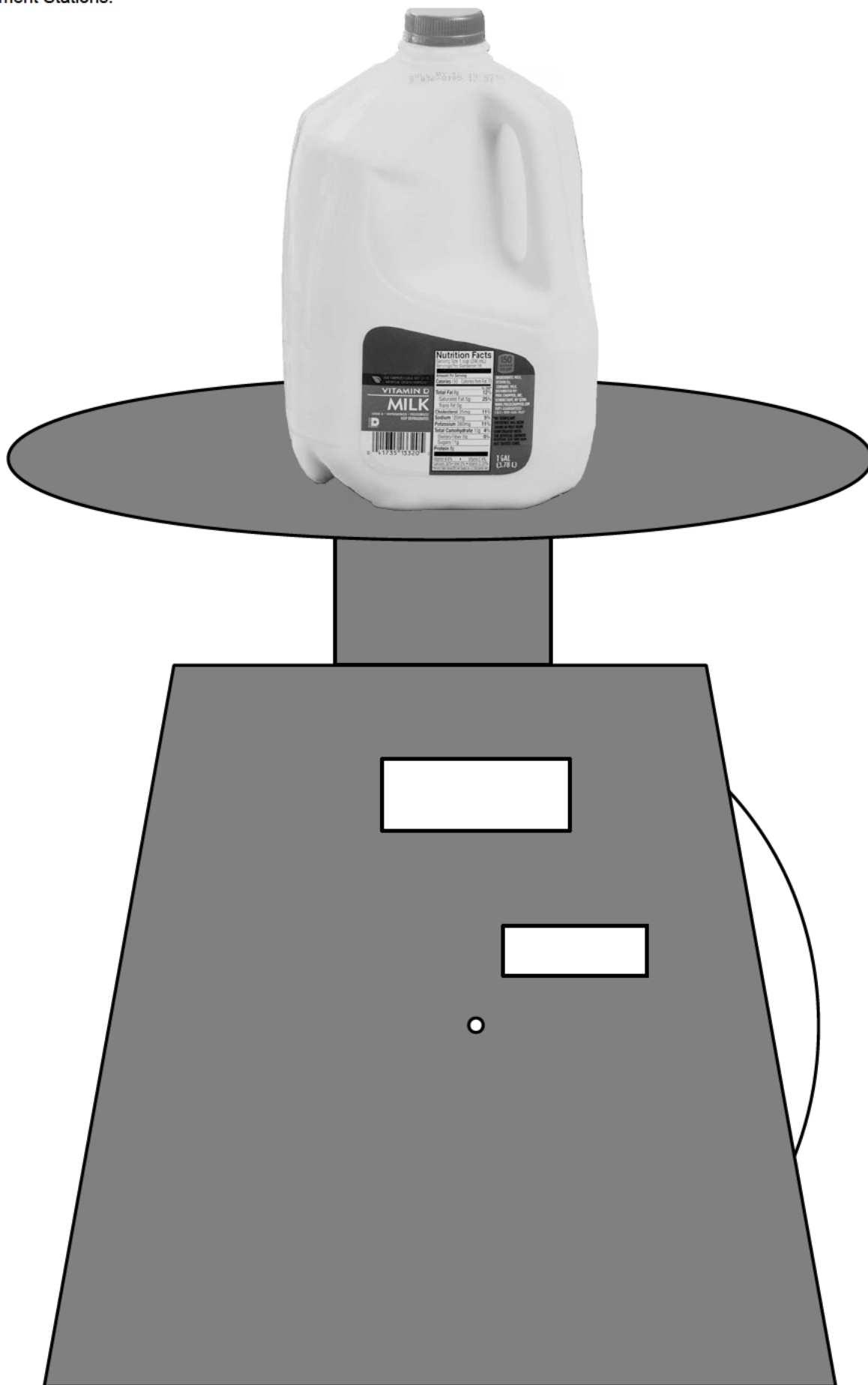
6.3.3.2 Measurement Stations.



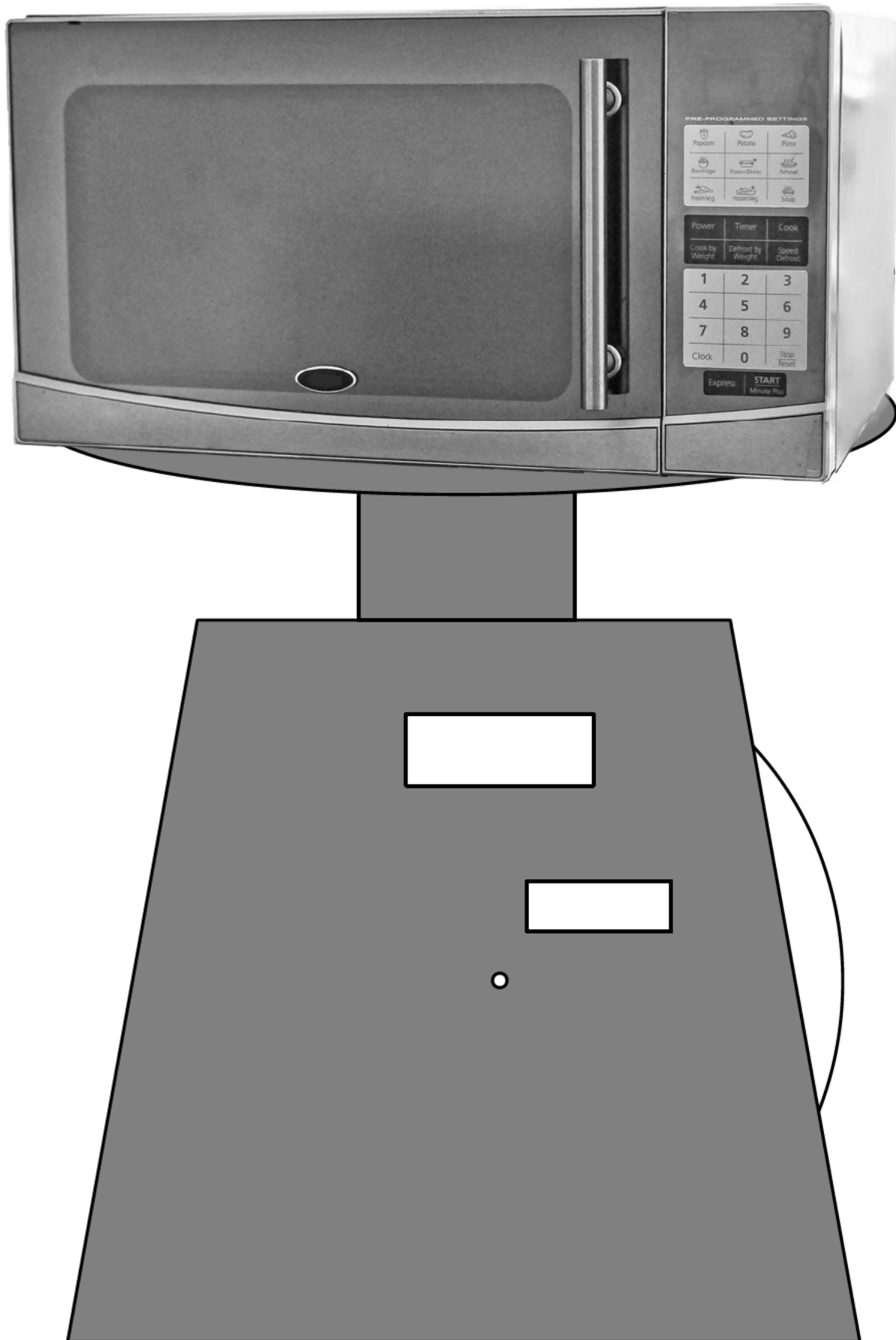
6.3.3.2 Measurement Stations.



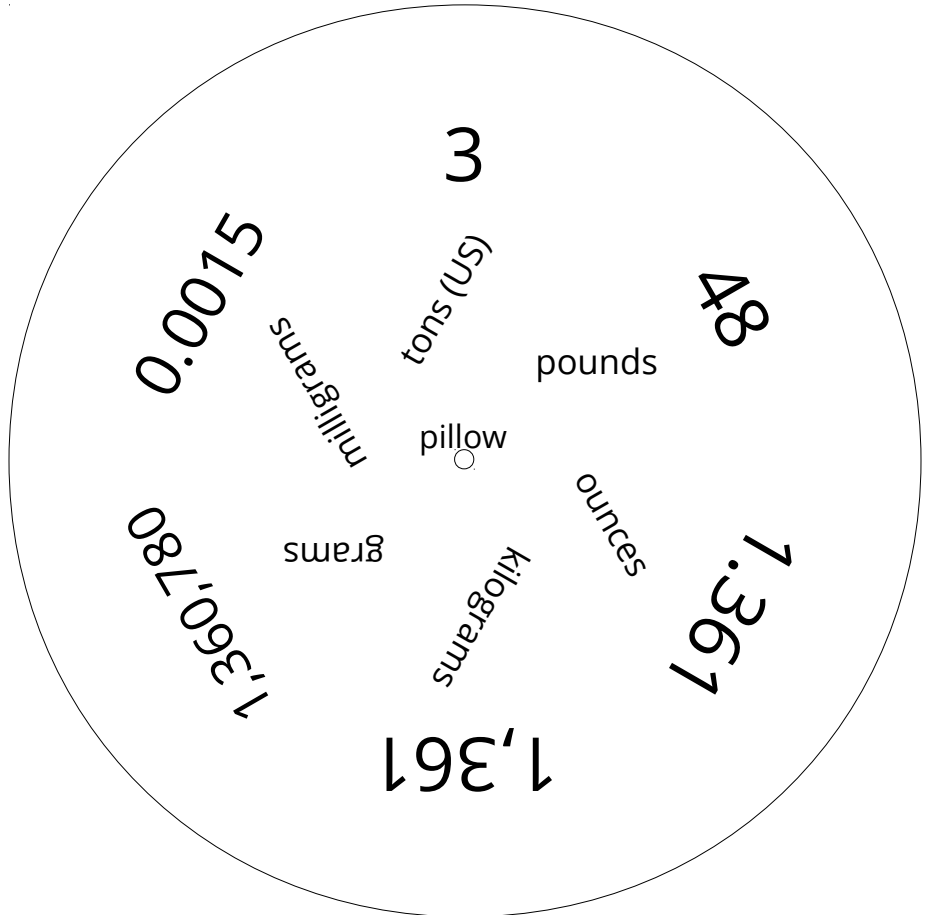
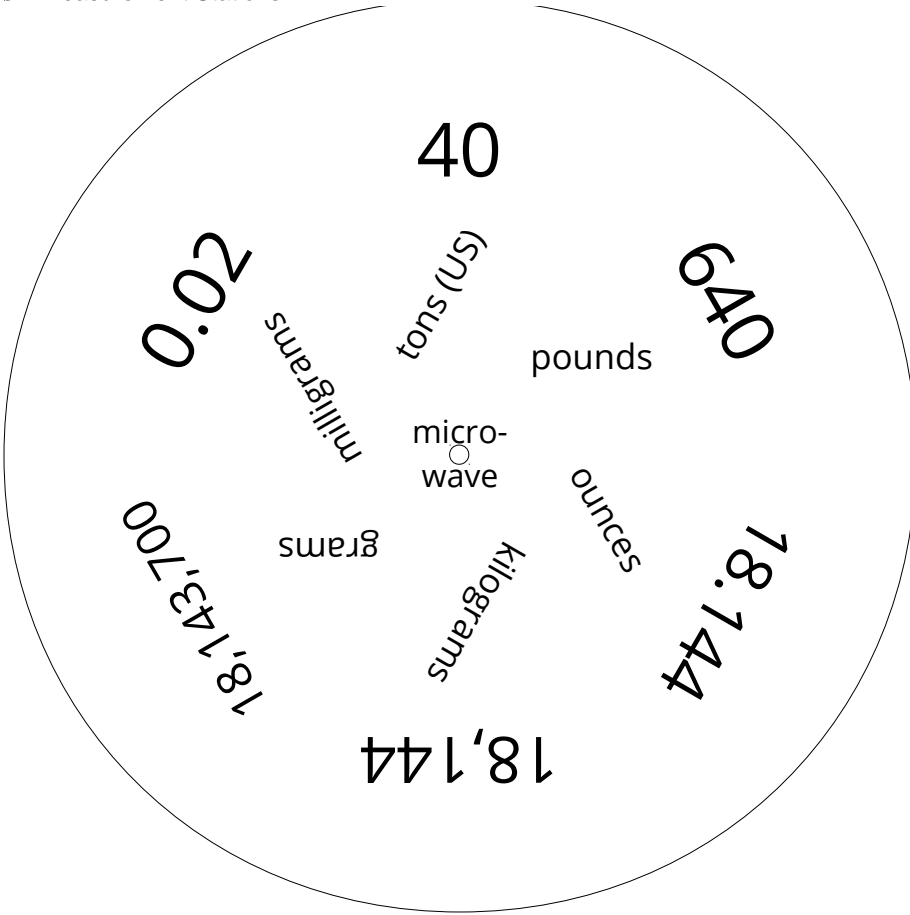
6.3.3.2 Measurement Stations.



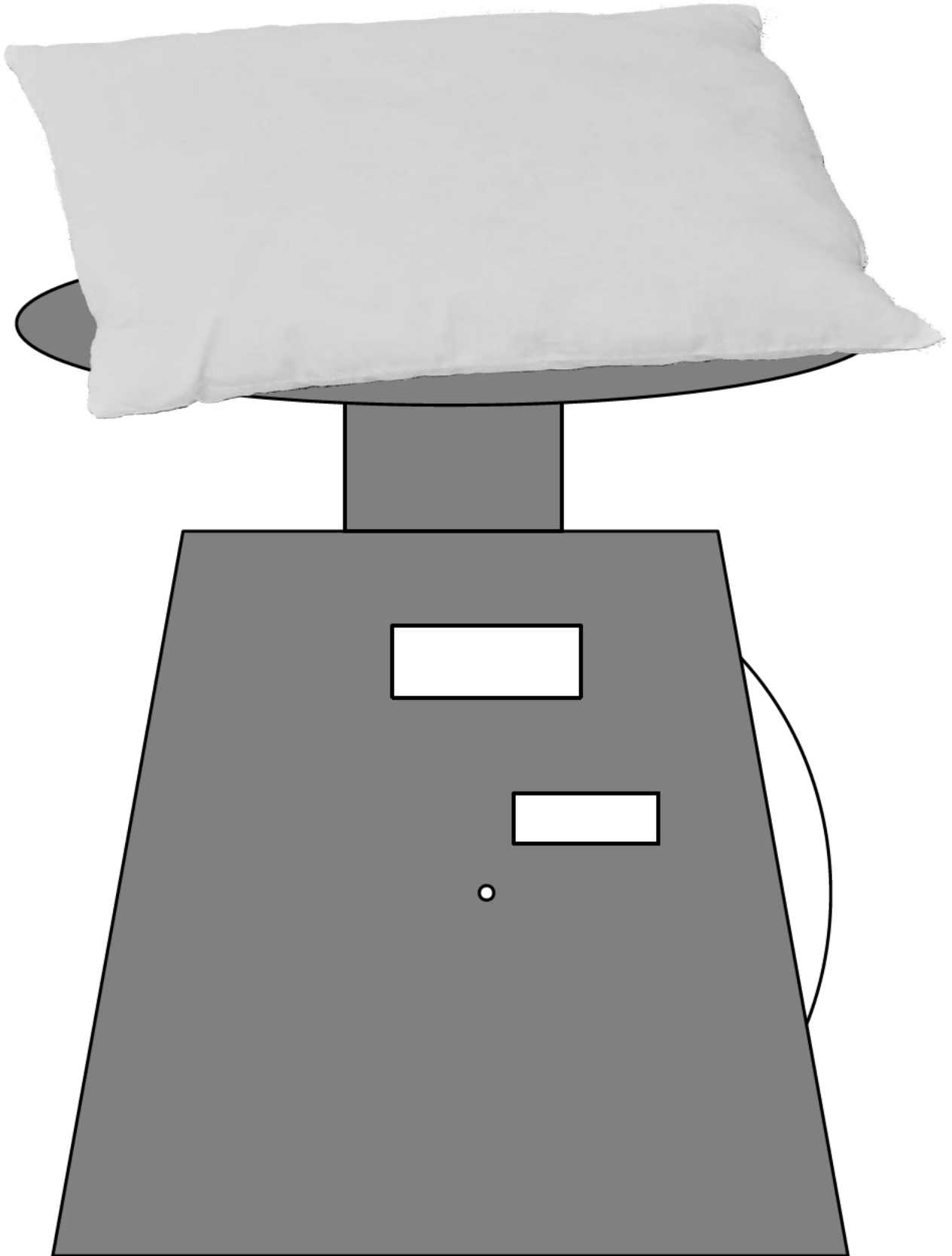
6.3.3.2 Measurement Stations.



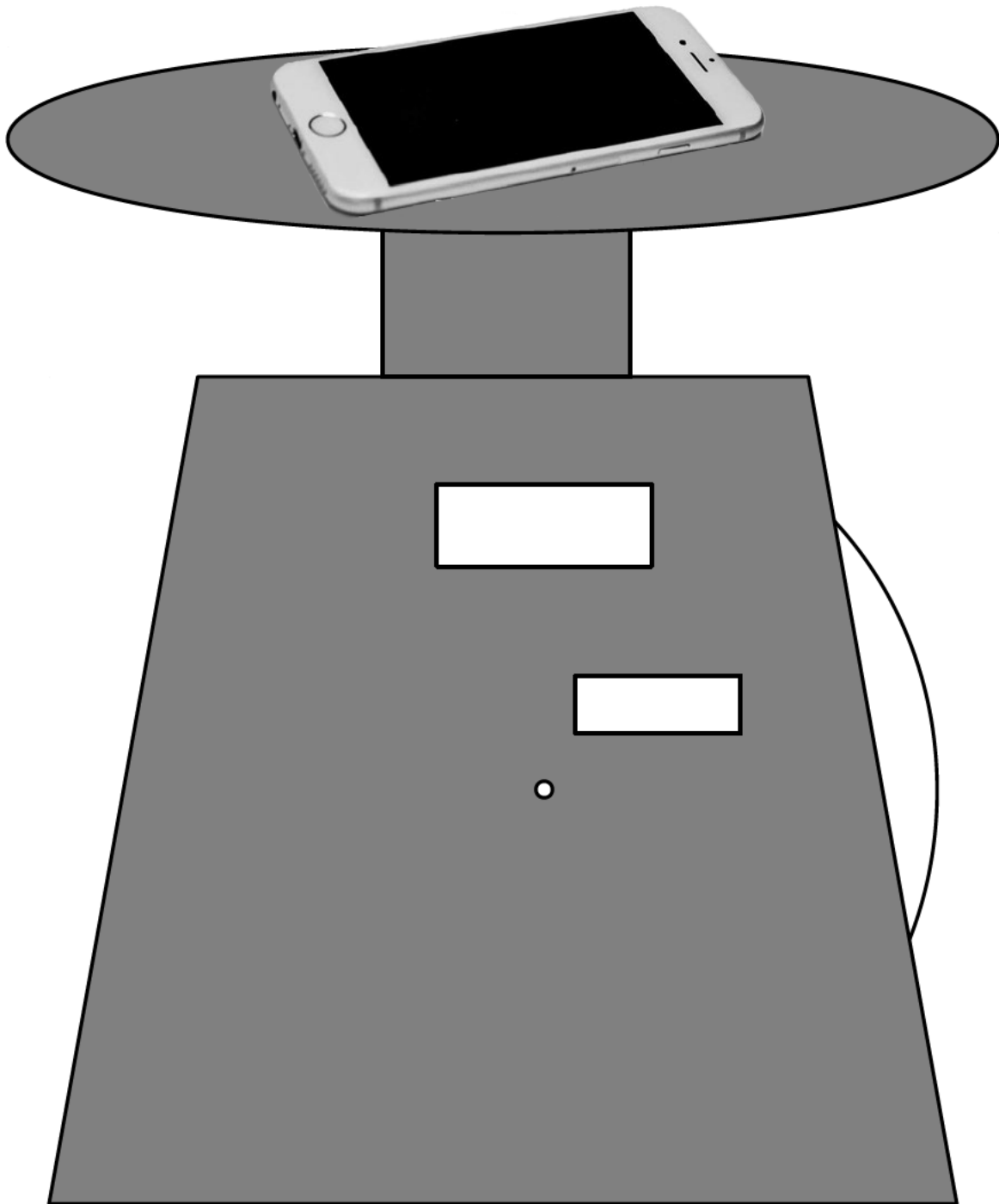
6.3.3.2 Measurement Stations.



6.3.3.2 Measurement Stations.

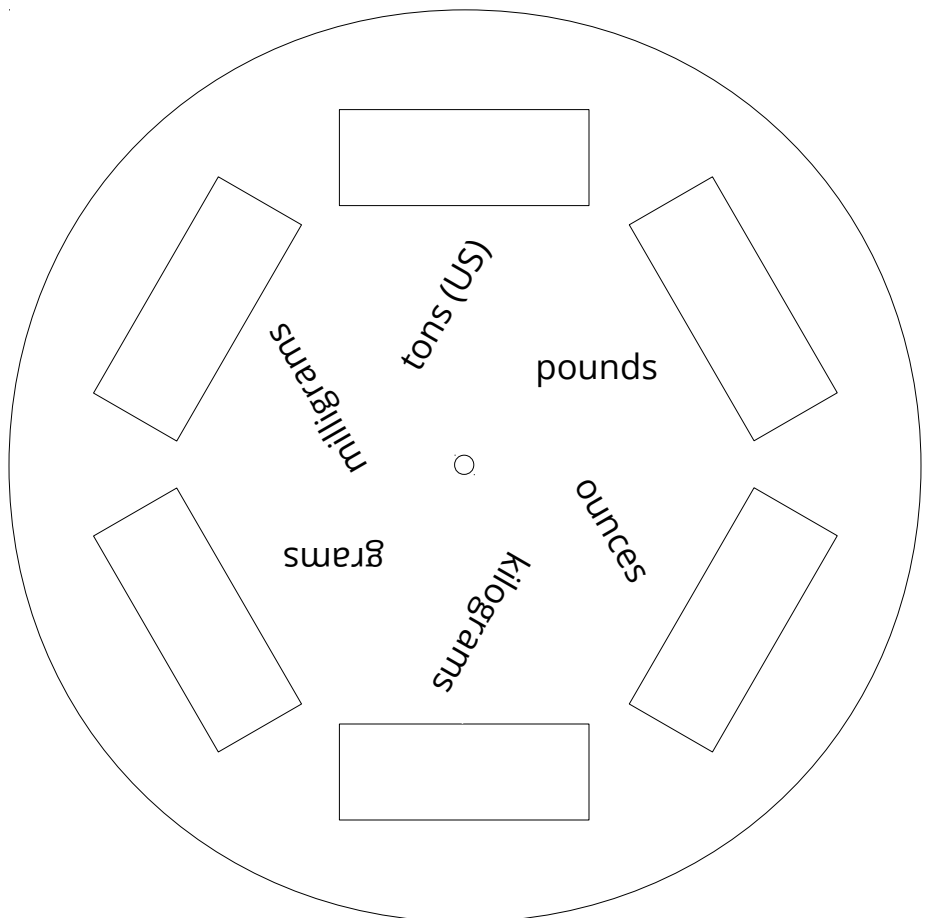
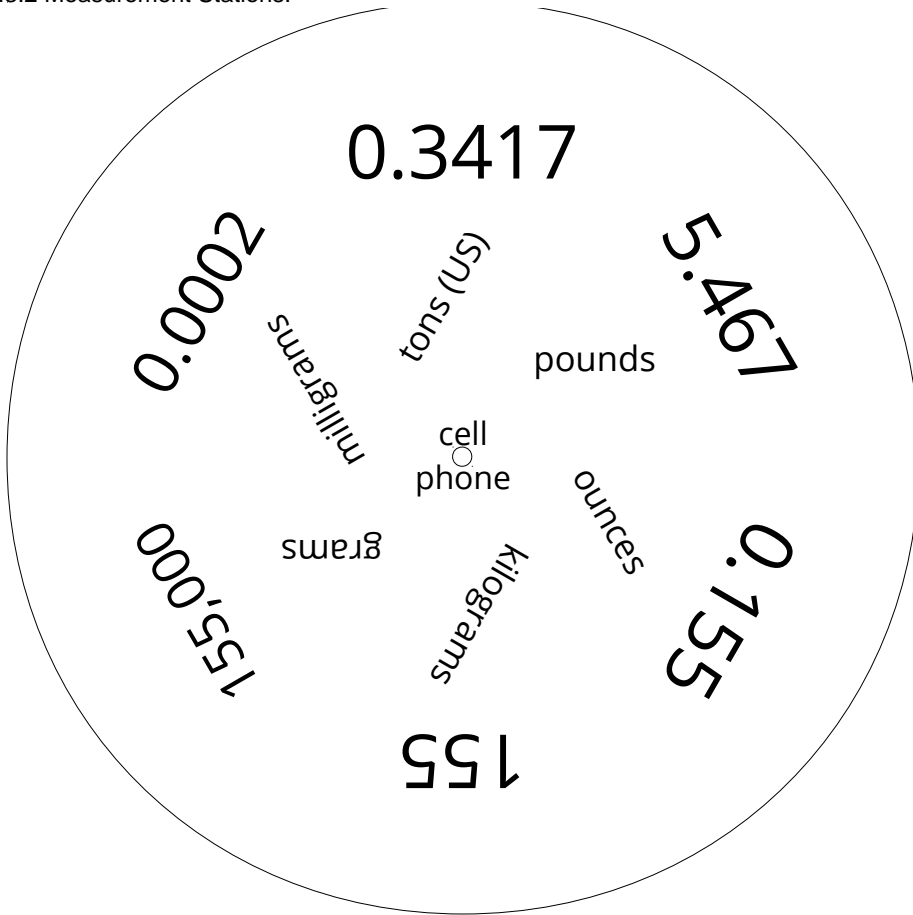


6.3.3.2 Measurement Stations.



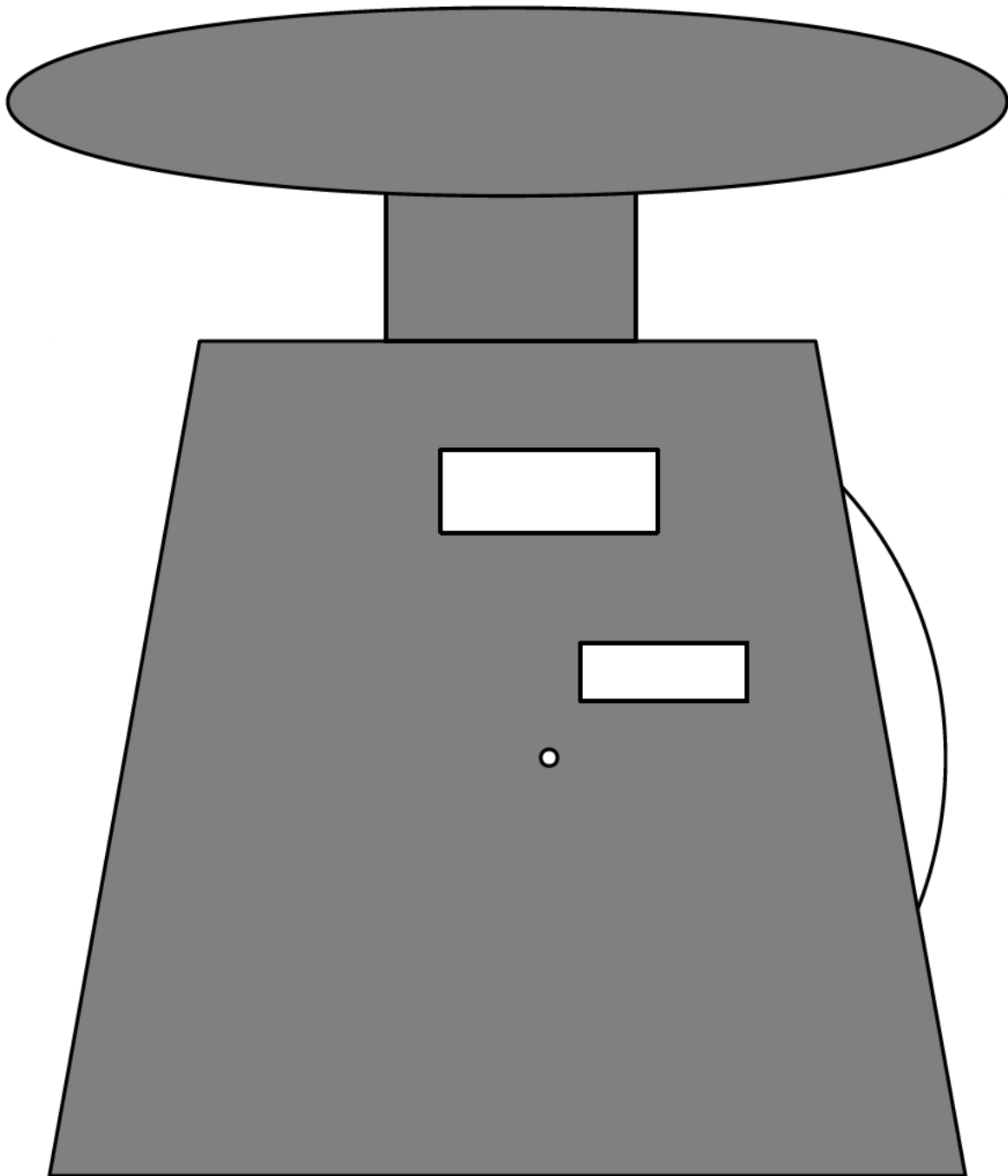


6.3.3.2 Measurement Stations.

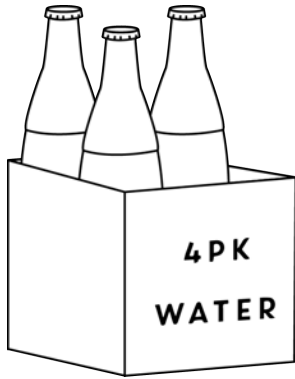


Invent your own:

6.3.3.2 Measurement Stations.

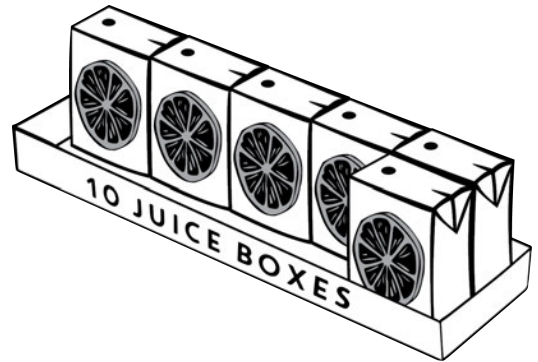


Card Sort: Is it a Deal?  
A. Drink Pack



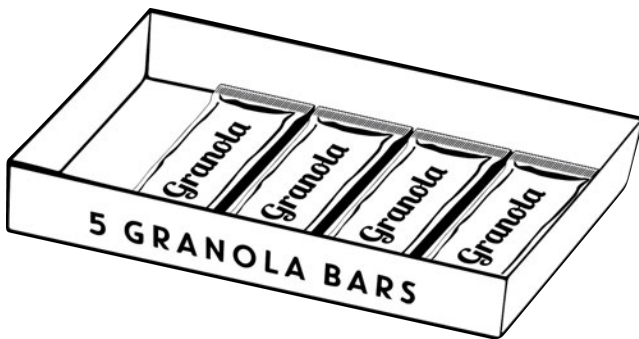
Original: 4 for \$3.16  
New Deal: 3 for \$2.25

Card Sort: Is it a Deal?  
B. Juice Boxes



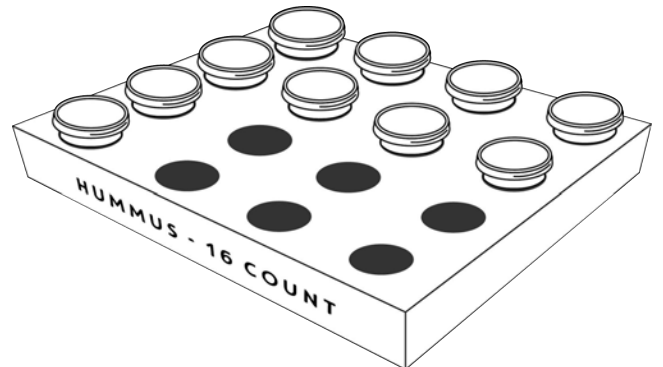
Original: 10 for \$3.50  
New Deal: 6 for \$2.40

Card Sort: Is it a Deal?  
C. Granola Bars



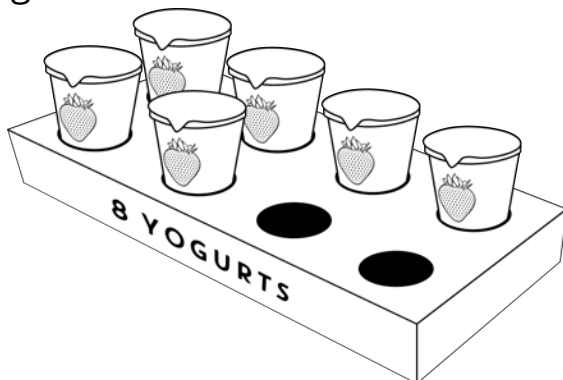
Original: 5 for \$4.40  
New Deal: 4 for \$3.12

Card Sort: Is it a Deal?  
D. Hummus



Original: 16 for \$14.40  
New Deal: 10 for \$9.00

Card Sort: Is it a Deal?  
E. Yogurt



Original: 8 for \$6.80  
New Deal: 6 for \$5.22

Card Sort: Is it a Deal?  
F. Fruit Snacks



Original: 12 for \$9.12  
New Deal: 9 for \_\_\_\_\_

Info Gap: Music Devices

### Problem Card 1

A store sells 3 different music devices:  
Device A, Device B, and Device C.

1. Which of the devices can Jada afford?
2. What percentage of the money needed for Device B does she have?

Info Gap: Music Devices

### Data Card 1

- Device A costs \$15.
- Device B costs \$25.
- Device C costs \$40.
- Jada has 60% of the money needed to buy Device C.

Info Gap: Music Devices

### Problem Card 2

The store starts selling another music device. Jada is interested in Device D, though she does not have enough money to buy it. How much does Device D cost?

Info Gap: Music Devices

### Data Card 2

- Jada has \$24.
- Jada has 40% of the money needed to buy Device D.

Info Gap: Music Devices

### Problem Card 1

A store sells 3 different music devices:  
Device A, Device B, and Device C.

1. Which of the devices can Jada afford?
2. What percentage of the money needed for Device B does she have?

Info Gap: Music Devices

### Data Card 1

- Device A costs \$15.
- Device B costs \$25.
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Info Gap: Music Devices

### Problem Card 2

The store starts selling another music device. Jada is interested in Device D, though she does not have enough money to buy it. How much does Device D cost?

Info Gap: Music Devices

### Data Card 2

- Jada has \$24.
- Jada has 40% of the money needed to buy Device D.

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