



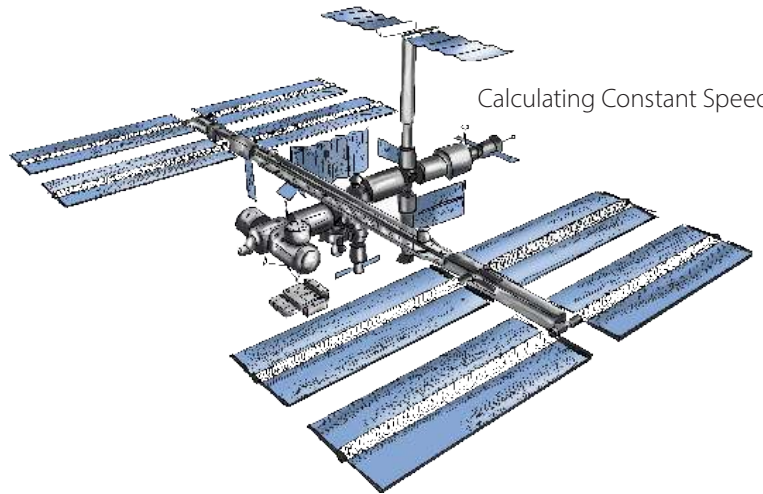
# Introducing Ratios

## Student Workbook

Using Recipes



Unit Prices



Calculating Constant Speed



Describing Ratios Using Illustrations

Solving Fermi Problems



The Recipe



How do we fix our mistake?



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# Introducing Ratios

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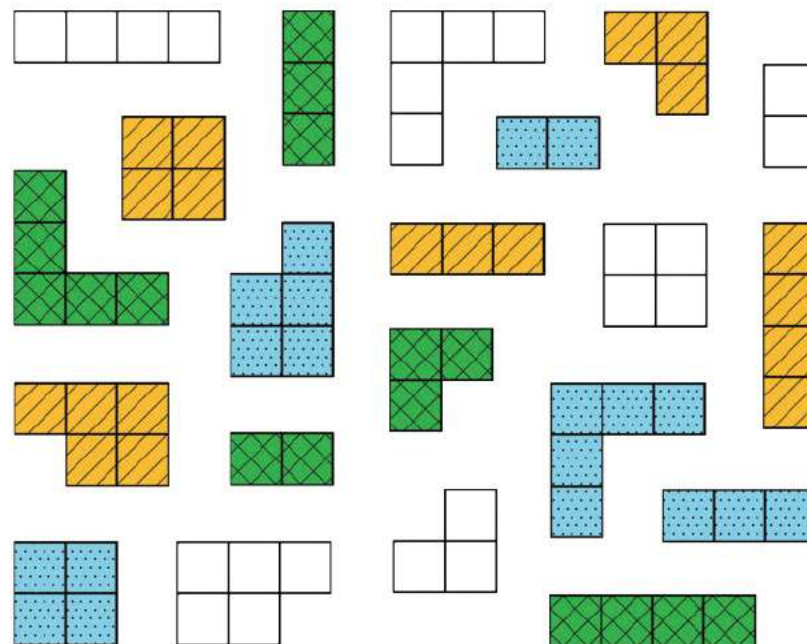


## **Introducing Ratios Student Workbook**

Core Knowledge Mathematics™

# Lesson 1: Introducing Ratios and Ratio Language

## 1.1: What Kind and How Many?



Think of different ways you could sort these figures. What categories could you use? How many groups would you have?

## 1.2: The Teacher's Collection

1. Think of a way to sort your teacher's collection into two or three categories. Count the items in each category, and record the information in the table.

|                 |  |  |  |
|-----------------|--|--|--|
| category name   |  |  |  |
| category amount |  |  |  |

Pause here so your teacher can review your work.

2. Write at least two sentences that describe **ratios** in the collection. Remember, there are many ways to write a ratio:

- The ratio of *one category* to *another category* is \_\_\_\_\_ to \_\_\_\_\_.
- The ratio of *one category* to *another category* is \_\_\_\_\_ : \_\_\_\_\_.
- There are \_\_\_\_\_ of *one category* for every \_\_\_\_\_ of *another category*.

## 1.3: The Student's Collection

1. Sort your collection into three categories. You can experiment with different ways of arranging these categories. Then, count the items in each category, and record the information in the table.

|                 |  |  |  |
|-----------------|--|--|--|
| category name   |  |  |  |
| category amount |  |  |  |

2. Write at least two sentences that describe **ratios** in the collection. Remember, there are many ways to write a ratio:

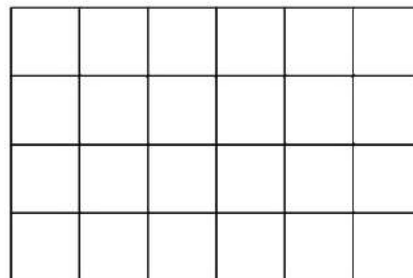
- The ratio of *one category* to *another category* is \_\_\_\_\_ to \_\_\_\_\_.
- The ratio of *one category* to *another category* is \_\_\_\_\_ : \_\_\_\_\_.
- There are \_\_\_\_\_ of *one category* for every \_\_\_\_\_ of *another category*.

Pause here so your teacher can review your sentences.

3. Make a visual display of your items that clearly shows one of your statements. Be prepared to share your display with the class.

### Are you ready for more?

1. Use two colors to shade the rectangle so there are 2 square units of one color for every 1 square unit of the other color.
2. The rectangle you just colored has an area of 24 square units. Draw a different shape that does *not* have an area of 24 square units, but that can also be shaded with two colors in a 2 : 1 ratio. Shade your new shape using two colors.



### Lesson 1 Summary

A **ratio** is an association between two or more quantities. There are many ways to describe a situation in terms of ratios. For example, look at this collection:

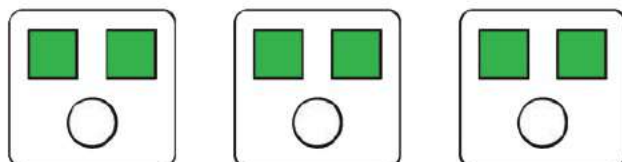


Here are some correct ways to describe the collection:



- The ratio of squares to circles is 6 : 3.
- The ratio of circles to squares is 3 to 6.

Notice that the shapes can be arranged in equal groups, which allow us to describe the shapes using other numbers.



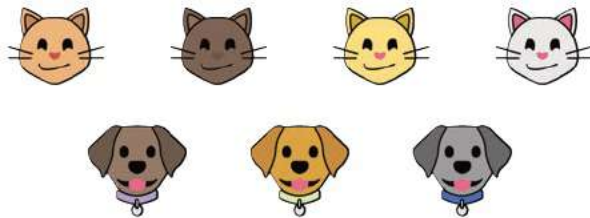
- There are 2 squares for every 1 circle.
- There is 1 circle for every 2 squares.

## Unit 2 Lesson 1 Cumulative Practice Problems

1. In a fruit basket there are 9 bananas, 4 apples, and 3 plums.

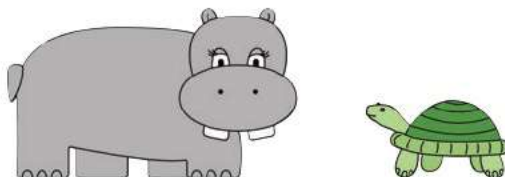
- a. The ratio of bananas to apples is \_\_\_\_\_ : \_\_\_\_\_.
- b. The ratio of plums to apples is \_\_\_\_\_ to \_\_\_\_\_.
- c. For every \_\_\_\_\_ apples, there are \_\_\_\_\_ plums.
- d. For every 3 bananas there is one \_\_\_\_\_.

2. Complete the sentences to describe this picture.



- a. The ratio of dogs to cats is \_\_\_\_\_.
- b. For every \_\_\_\_\_ dogs, there are \_\_\_\_\_ cats.

3. Write two different sentences that use ratios to describe the number of eyes and legs in this picture.



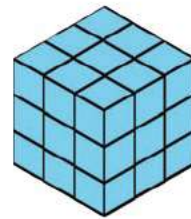
4. Choose an appropriate unit of measurement for each quantity.

- a. area of a rectangle ☐ cm
- b. volume of a prism ☐  $\text{cm}^3$
- c. side of a square ☐  $\text{cm}^2$
- d. area of a square
- e. volume of a cube

(From Unit 1, Lesson 17.)

5. Find the volume and surface area of each prism.

- a. Prism A: 3 cm by 3 cm by 3 cm



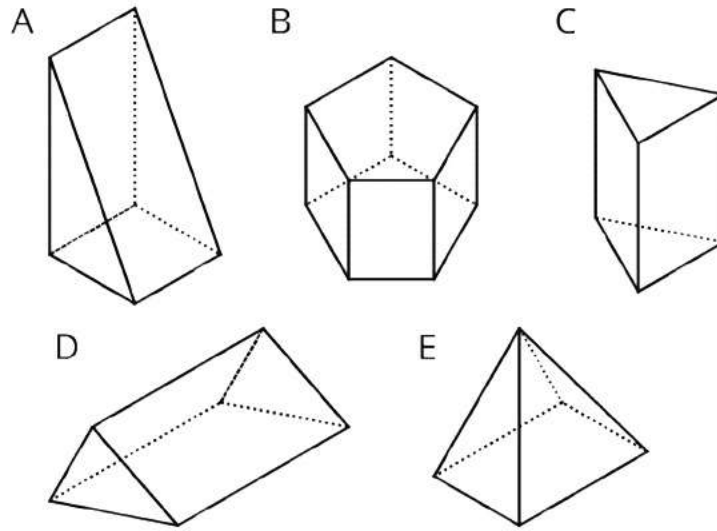
- b. Prism B: 5 cm by 5 cm by 1 cm



- c. Compare the volumes of the prisms and then their surface areas. Does the prism with the greater volume also have the greater surface area?

(From Unit 1, Lesson 16.)

6. Which figure is a triangular prism? Select all that apply.



A. A

B. B

C. C

D. D

E. E

(From Unit 1, Lesson 13.)

## Lesson 2: Representing Ratios with Diagrams

### 2.1: Number Talk: Dividing by 4 and Multiplying by $\frac{1}{4}$

Find the value of each expression mentally.

$$24 \div 4$$

$$\frac{1}{4} \cdot 24$$

$$24 \cdot \frac{1}{4}$$

$$5 \div 4$$

### 2.2: A Collection of Snap Cubes

Here is a collection of snap cubes.



1. Choose two of the colors in the image, and draw a diagram showing the number of snap cubes for these two colors.
2. Trade papers with a partner. On their paper, write a sentence to describe a ratio shown in their diagram. Your partner will do the same for your diagram.
3. Return your partner's paper. Read the sentence written on your paper. If you disagree, explain your thinking.

## 2.3: Blue Paint and Art Paste

Elena mixed 2 cups of white paint with 6 tablespoons of blue paint.

Here is a diagram that represents this situation.

white paint (cups)



blue paint (tablespoons)



1. Discuss each statement, and circle **all** those that correctly describe this situation. Make sure that both you and your partner agree with each circled answer.

- a. The ratio of cups of white paint to tablespoons of blue paint is 2 : 6.
- b. For every cup of white paint, there are 2 tablespoons of blue paint.
- c. There is 1 cup of white paint for every 3 tablespoons of blue paint.
- d. There are 3 tablespoons of blue paint for every cup of white paint.
- e. For each tablespoon of blue paint, there are 3 cups of white paint.
- f. For every 6 tablespoons of blue paint, there are 2 cups of white paint.
- g. The ratio of tablespoons of blue paint to cups of white paint is 6 to 2.

2. Jada mixed 8 cups of flour with 2 pints of water to make paste for an art project.

- a. Draw a diagram that represents the situation.

- b. Write at least two sentences describing the ratio of flour and water.

## 2.4: Card Sort: Spaghetti Sauce

Your teacher will give you cards describing different recipes for spaghetti sauce. In the diagrams:

- a circle represents a cup of tomato sauce
- a square represents a tablespoon of oil
- a triangle represents a teaspoon of oregano



1. Take turns with your partner to match a sentence with a diagram.
  - a. For each match that you find, explain to your partner how you know it's a match.
  - b. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.
2. After you and your partner have agreed on all of the matches, check your answers with the answer key. If there are any errors, discuss why and revise your matches.
3. There were two diagrams that each matched with two different sentences. Which were they?
  - Diagram \_\_\_\_\_ matched with both sentences \_\_\_\_\_ and \_\_\_\_\_.
  - Diagram \_\_\_\_\_ matched with both sentences \_\_\_\_\_ and \_\_\_\_\_.
4. Select one of the other diagrams and invent another sentence that could describe the ratio shown in the diagram.

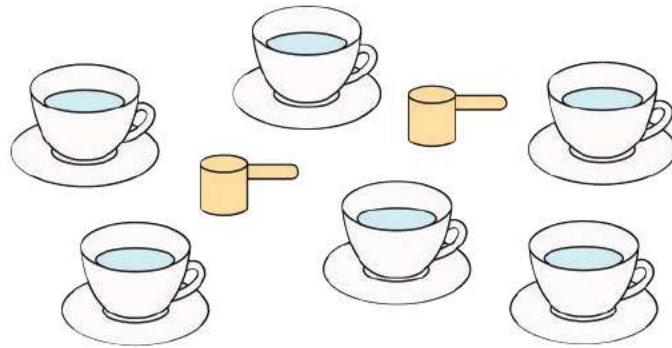
### Are you ready for more?

Create a diagram that represents any of the ratios in a recipe of your choice. Is it possible to include more than 2 ingredients in your diagram?

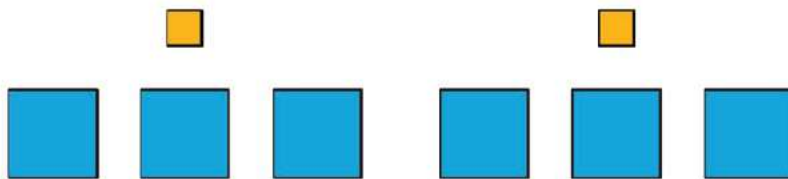
## Lesson 2 Summary

Ratios can be represented using diagrams. The diagrams do not need to include realistic details. For example, a recipe for lemonade says, "Mix 2 scoops of lemonade powder with 6 cups of water."

Instead of this:



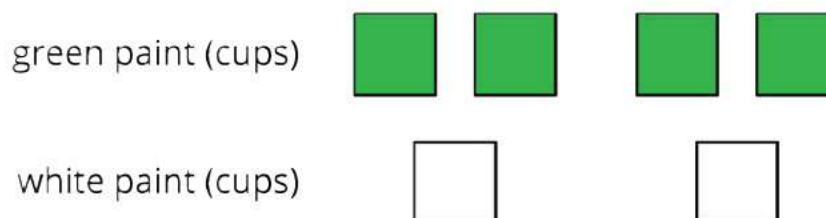
We can draw something like this:



This diagram shows that the ratio of cups of water to scoops of lemonade powder is 6 to 2. We can also see that for every scoop of lemonade powder, there are 3 cups of water.

## Unit 2 Lesson 2 Cumulative Practice Problems

1. Here is a diagram that describes the cups of green and white paint in a mixture.



Select all the statements that correctly describe this diagram

- A. The ratio of cups of white paint to cups of green paint is 2 to 4.
  - B. For every cup of green paint, there are two cups of white paint.
  - C. The ratio of cups of green paint to cups of white paint is 4 : 2.
  - D. For every cup of white paint, there are two cups of green paint.
  - E. The ratio of cups of green paint to cups of white paint is 2 : 4.
2. To make a snack mix, combine 2 cups of raisins with 4 cups of pretzels and 6 cups of almonds.

- a. Create a diagram to represent the quantities of each ingredient in this recipe.

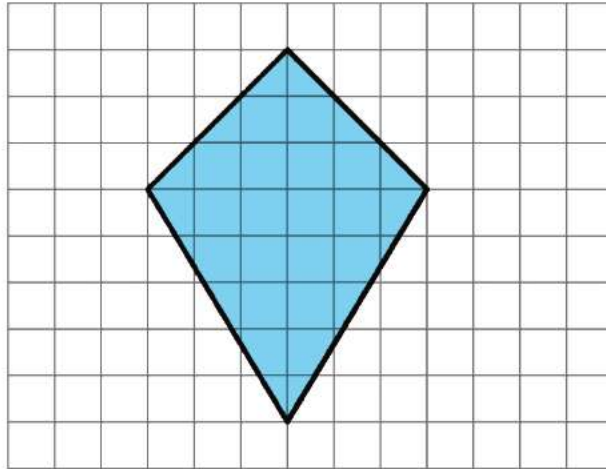
- b. Use your diagram to complete each sentence.

- The ratio of \_\_\_\_\_ to \_\_\_\_\_ to \_\_\_\_\_ is \_\_\_\_\_ : \_\_\_\_\_ : \_\_\_\_\_.
- There are \_\_\_\_\_ cups of pretzels for every cup of raisins.
- There are \_\_\_\_\_ cups of almonds for every cup of raisins.

3. a. A square is 3 inches by 3 inches. What is its area?
- b. A square has a side length of 5 feet. What is its area?
- c. The area of a square is 36 square centimeters. What is the length of each side of the square?

(From Unit 1, Lesson 17.)

4. Find the area of this quadrilateral. Explain or show your strategy.



(From Unit 1, Lesson 11.)

5. Complete each equation with a number that makes it true.

a.  $\frac{1}{8} \cdot 8 = \underline{\hspace{2cm}}$

a.  $\frac{1}{8} \cdot 7 = \underline{\hspace{2cm}}$

b.  $\frac{3}{8} \cdot 8 = \underline{\hspace{2cm}}$

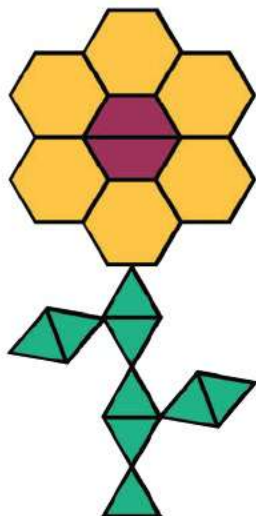
b.  $\frac{3}{8} \cdot 7 = \underline{\hspace{2cm}}$

(From Unit 2, Lesson 1.)

## Lesson 3: Recipes

### 3.1: Flower Pattern

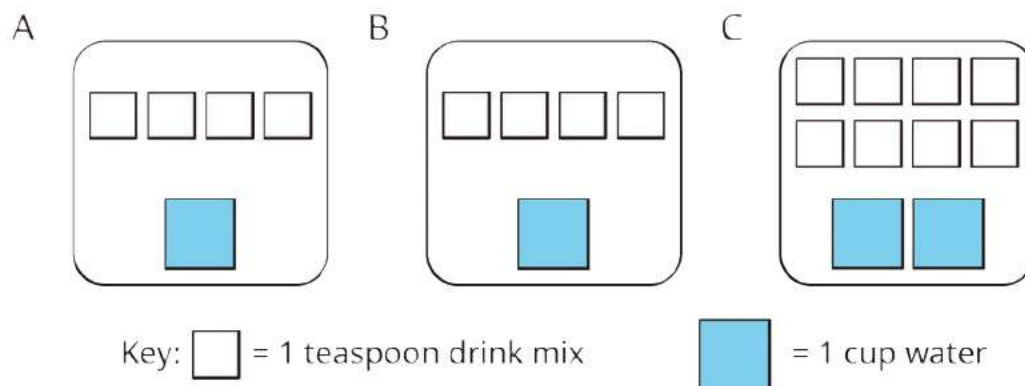
This flower is made up of yellow hexagons, red trapezoids, and green triangles.



1. Write sentences to describe the ratios of the shapes that make up this pattern.
2. How many of each shape would be in two copies of this flower pattern?

### 3.2: Powdered Drink Mix

Here are diagrams representing three mixtures of powdered drink mix and water:



1. How would the taste of Mixture A compare to the taste of Mixture B?
  
2. Use the diagrams to complete each statement:
  - a. Mixture B uses \_\_\_\_\_ cups of water and \_\_\_\_\_ teaspoons of drink mix. The ratio of cups of water to teaspoons of drink mix in Mixture B is \_\_\_\_\_.
  - b. Mixture C uses \_\_\_\_\_ cups of water and \_\_\_\_\_ teaspoons of drink mix. The ratio of cups of water to teaspoons of drink mix in Mixture C is \_\_\_\_\_.
3. How would the taste of Mixture B compare to the taste of Mixture C?

### Are you ready for more?

Sports drinks use sodium (better known as salt) to help people replenish electrolytes. Here are the nutrition labels of two sports drinks.

A

| Nutrition Facts                                  |             |                |
|--------------------------------------------------|-------------|----------------|
| Serving Size 8 fl oz (240 mL)                    |             |                |
| Serving Per Container 4                          |             |                |
| Amount Per Serving                               |             |                |
| Calories 50                                      |             |                |
|                                                  |             | % Daily Value* |
| <b>Total Fat</b>                                 | 0 g         | 0%             |
| <b>Sodium</b>                                    | 110 mg      | 5%             |
| <b>Potassium</b>                                 | 30 mg       | 1%             |
| <b>Total Carbohydrate</b>                        | 14 g        | 5%             |
|                                                  | Sugars 14 g |                |
| <b>Protein</b>                                   | 0 g         |                |
| % Daily Value are based on a 2,000 calorie diet. |             |                |

B

| Nutrition Facts                                  |             |                |
|--------------------------------------------------|-------------|----------------|
| Serving Size 12 fl oz (355 mL)                   |             |                |
| Serving Per Container about 2.5                  |             |                |
| Amount Per Serving                               |             |                |
| Calories 80                                      |             |                |
|                                                  |             | % Daily Value* |
| <b>Total Fat</b>                                 | 0 g         | 0%             |
| <b>Sodium</b>                                    | 150 mg      | 6%             |
| <b>Potassium</b>                                 | 35 mg       | 1%             |
| <b>Total Carbohydrate</b>                        | 21 g        | 7%             |
|                                                  | Sugars 20 g |                |
| <b>Protein</b>                                   | 0 g         |                |
| % Daily Value are based on a 2,000 calorie diet. |             |                |

1. Which of these drinks is saltier? Explain how you know.
2. If you wanted to make sure a sports drink was less salty than both of the ones given, what ratio of sodium to water would you use?

### 3.3: Batches of Cookies

A recipe for one batch of cookies calls for 5 cups of flour and 2 teaspoons of vanilla.

1. Draw a diagram that shows the amount of flour and vanilla needed for *two* batches of cookies.
  
  
  
  
  
  
  
  
  
  
2. How many batches can you make with 15 cups of flour and 6 teaspoons of vanilla?  
Show the additional batches by adding more ingredients to your diagram.
  
  
  
  
  
  
  
  
  
  
3. How much flour and vanilla would you need for 5 batches of cookies?
  
  
  
  
  
  
  
  
  
  
4. Whether the ratio of cups of flour to teaspoons of vanilla is  $5 : 2$ ,  $10 : 4$ , or  $15 : 6$ , the recipes would make cookies that taste the same. We call these **equivalent ratios**.
  - a. Find another ratio of cups of flour to teaspoons of vanilla that is equivalent to these ratios.
  
  
  
  
  
  
  
  
  
  
  - b. How many batches can you make using this new ratio of ingredients?

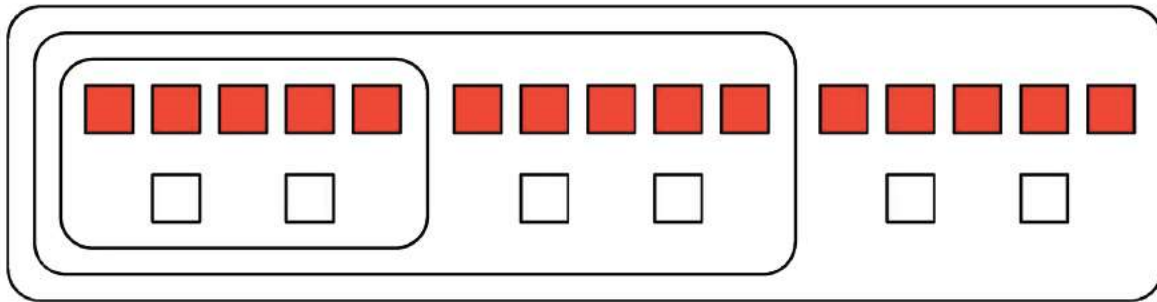
### Lesson 3 Summary

A recipe for fizzy juice says, "Mix 5 cups of cranberry juice with 2 cups of soda water."

To double this recipe, we would use 10 cups of cranberry juice with 4 cups of soda water.

To triple this recipe, we would use 15 cups of cranberry juice with 6 cups of soda water.

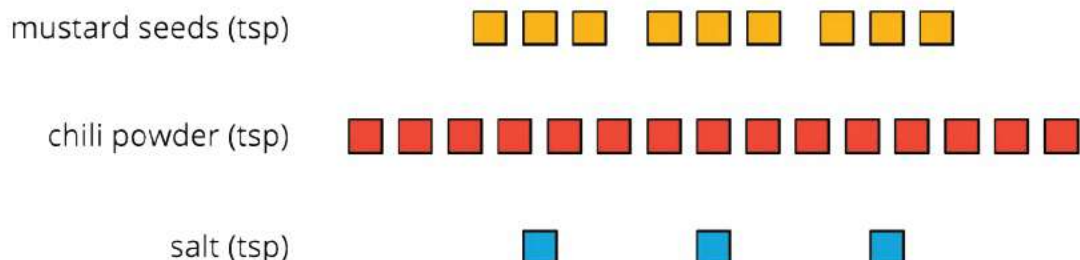
This diagram shows a single batch of the recipe, a double batch, and a triple batch:



We say that the ratios  $5 : 2$ ,  $10 : 4$ , and  $15 : 6$  are **equivalent**. Even though the amounts of each ingredient within a single, double, or triple batch are not the same, they would make fizzy juice that tastes the same.

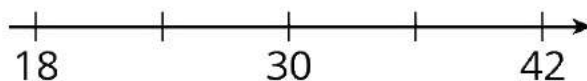
## Unit 2 Lesson 3 Cumulative Practice Problems

1. A recipe for 1 batch of spice mix says, "Combine 3 teaspoons of mustard seeds, 5 teaspoons of chili powder, and 1 teaspoon of salt." How many batches are represented by the diagram? Explain or show your reasoning.



2. Priya makes chocolate milk by mixing 2 cups of milk and 5 tablespoons of cocoa powder. Draw a diagram that clearly represents two batches of her chocolate milk.
3. In a recipe for fizzy grape juice, the ratio of cups of sparkling water to cups of grape juice concentrate is 3 to 1.
- Find two more ratios of cups of sparkling water to cups of juice concentrate that would make a mixture that tastes the same as this recipe.
  - Describe another mixture of sparkling water and grape juice that would taste different than this recipe.

4. Write the missing number under each tick mark on the number line.



(From Unit 2, Lesson 1.)

5. At the kennel, there are 6 dogs for every 5 cats.

a. The ratio of dogs to cats is \_\_\_\_\_ to \_\_\_\_\_.

b. The ratio of cats to dogs is \_\_\_\_\_ to \_\_\_\_\_.

c. For every \_\_\_\_\_ dogs there are \_\_\_\_\_ cats.

d. The ratio of cats to dogs is \_\_\_\_\_ : \_\_\_\_\_.

(From Unit 2, Lesson 1.)

6. Elena has 80 unit cubes. What is the volume of the largest cube she can build with them?

(From Unit 1, Lesson 17.)

7. Fill in the blanks to make each equation true.

a.  $3 \cdot \frac{1}{3} = \underline{\hspace{2cm}}$

a.  $5 \cdot \underline{\hspace{2cm}} = 1$

b.  $10 \cdot \frac{1}{10} = \underline{\hspace{2cm}}$

b.  $17 \cdot \underline{\hspace{2cm}} = 1$

c.  $19 \cdot \frac{1}{19} = \underline{\hspace{2cm}}$

c.  $b \cdot \underline{\hspace{2cm}} = 1$

d.  $a \cdot \frac{1}{a} = \underline{\hspace{2cm}}$

(As long as  $a$  does not equal 0.)

(From Unit 2, Lesson 1.)

## Lesson 4: Color Mixtures

### 4.1: Number Talk: Adjusting a Factor

Find the value of each product mentally.

$$6 \cdot 15$$

$$12 \cdot 15$$

$$6 \cdot 45$$

$$13 \cdot 45$$

### 4.2: Turning Green

Your teacher mixed milliliters of blue water and milliliters of yellow water in the ratio 5 : 15.

1. Doubling the original recipe:

a. Draw a diagram to represent the amount of each color that you will combine to double your teacher's recipe.

b. Use a marker to label an empty cup with the ratio of blue water to yellow water in this double batch.

c. Predict whether these amounts of blue and yellow will make the same shade of green as your teacher's mixture. Next, check your prediction by measuring those amounts and mixing them in the cup.

- d. Is the ratio in your mixture equivalent to the ratio in your teacher's mixture?  
Explain your reasoning.

2. Tripling the original recipe:

- a. Draw a diagram to represent triple your teacher's recipe.
- b. Label an empty cup with the ratio of blue water to yellow water.
- c. Predict whether these amounts will make the same shade of green. Next, check your prediction by mixing those amounts.
- d. Is the ratio in your new mixture equivalent to the ratio in your teacher's mixture? Explain your reasoning.

3. Next, invent your own recipe for a *bluer* shade of green water.

- a. Draw a diagram to represent the amount of each color you will combine.

- b. Label the final empty cup with the ratio of blue water to yellow water in this recipe.
- c. Test your recipe by mixing a batch in the cup. Does the mixture yield a bluer shade of green?
- d. Is the ratio you used in this recipe equivalent to the ratio in your teacher's mixture? Explain your reasoning.

### **Are you ready for more?**

Someone has made a shade of green by using 17 ml of blue and 13 ml of yellow. They are sure it cannot be turned into the original shade of green by adding more blue or yellow. Either explain how more can be added to create the original green shade, or explain why this is impossible.

## **4.3: Perfect Purple Water**

The recipe for Perfect Purple Water says, "Mix 8 ml of blue water with 3 ml of red water."

Jada mixes 24 ml of blue water with 9 ml of red water. Andre mixes 16 ml of blue water with 9 ml of red water.

1. Which person will get a color mixture that is the same shade as Perfect Purple Water? Explain or show your reasoning.
  
  
  
  
  
  
  
  
  
  
2. Find another combination of blue water and red water that will also result in the same shade as Perfect Purple Water. Explain or show your reasoning.

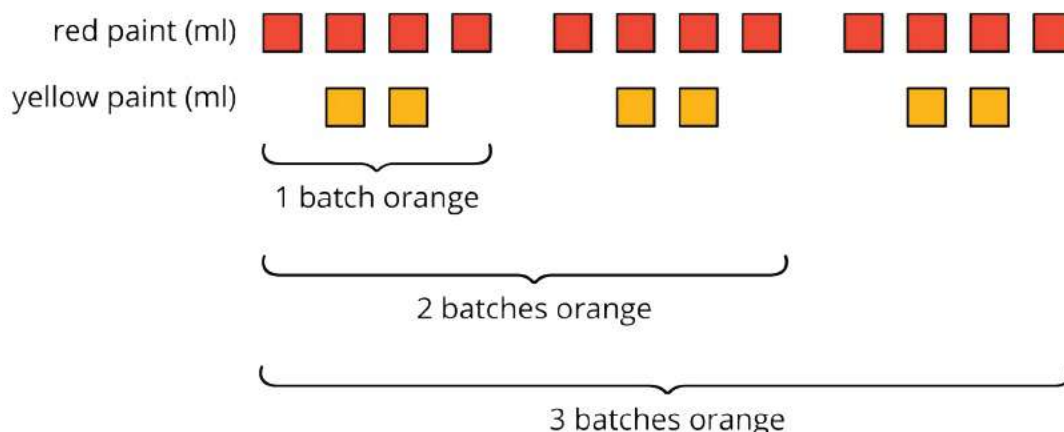
## Lesson 4 Summary

When mixing colors, doubling or tripling the amount of each color will create the same shade of the mixed color. In fact, you can always multiply the amount of *each* color by *the same number* to create a different amount of the same mixed color.

For example, a batch of dark orange paint uses 4 ml of red paint and 2 ml of yellow paint.

- To make two batches of dark orange paint, we can mix 8 ml of red paint with 4 ml of yellow paint.
- To make three batches of dark orange paint, we can mix 12 ml of red paint with 6 ml of yellow paint.

Here is a diagram that represents 1, 2, and 3 batches of this recipe.



We say that the ratios  $4 : 2$ ,  $8 : 4$ , and  $12 : 6$  are *equivalent* because they describe the same color mixture in different numbers of batches, and they make the same shade of orange.

## Unit 2 Lesson 4 Cumulative Practice Problems

1. Here is a diagram showing a mixture of red paint and green paint needed for 1 batch of a particular shade of brown.



Add to the diagram so that it shows 3 batches of the same shade of brown paint.

2. Diego makes green paint by mixing 10 tablespoons of yellow paint and 2 tablespoons of blue paint. Which of these mixtures produce the same shade of green paint as Diego's mixture? Select **all** that apply.
- A. For every 5 tablespoons of blue paint, mix in 1 tablespoon of yellow paint.
  - B. Mix tablespoons of blue paint and yellow paint in the ratio 1 : 5.
  - C. Mix tablespoons of yellow paint and blue paint in the ratio 15 to 3.
  - D. Mix 11 tablespoons of yellow paint and 3 tablespoons of blue paint.
  - E. For every tablespoon of blue paint, mix in 5 tablespoons of yellow paint.
3. To make 1 batch of sky blue paint, Clare mixes 2 cups of blue paint with 1 gallon of white paint.
- a. Explain how Clare can make 2 batches of sky blue paint.
  - b. Explain how to make a mixture that is a darker shade of blue than the sky blue.
  - c. Explain how to make a mixture that is a lighter shade of blue than the sky blue.

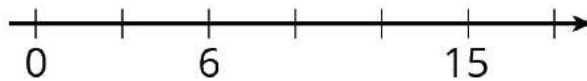
4. A smoothie recipe calls for 3 cups of milk, 2 frozen bananas and 1 tablespoon of chocolate syrup.

a. Create a diagram to represent the quantities of each ingredient in the recipe.

b. Write 3 different sentences that use a ratio to describe the recipe.

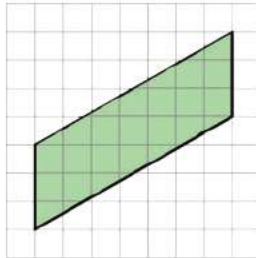
(From Unit 2, Lesson 2.)

5. Write the missing number under each tick mark on the number line.



(From Unit 2, Lesson 1.)

6. Find the area of the parallelogram. Show your reasoning.



(From Unit 1, Lesson 4.)

7. Complete each equation with a number that makes it true.

a.  $11 \cdot \frac{1}{4} = \underline{\hspace{2cm}}$

a.  $13 \cdot \frac{1}{99} = \underline{\hspace{2cm}}$

b.  $7 \cdot \frac{1}{4} = \underline{\hspace{2cm}}$

b.  $x \cdot \frac{1}{y} = \underline{\hspace{2cm}}$

(As long as  $y$  does not equal 0.)

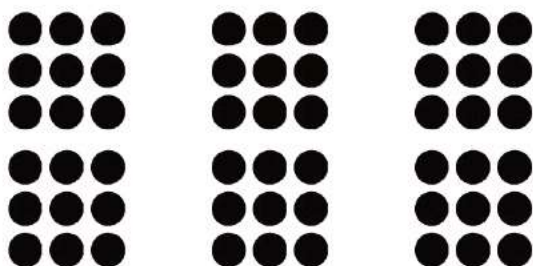
c.  $13 \cdot \frac{1}{27} = \underline{\hspace{2cm}}$

(From Unit 2, Lesson 1.)

# Lesson 5: Defining Equivalent Ratios

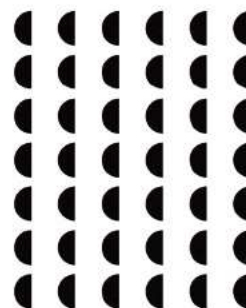
## 5.1: Dots and Half Dots

Dot Pattern 1:



Dot Pattern 2:

● = 1



## 5.2: Tuna Casserole

Here is a recipe for tuna casserole.

---

### Ingredients

- 3 cups cooked elbow-shaped pasta
- 6 ounce can tuna, drained
- 10 ounce can cream of chicken soup
- 1 cup shredded cheddar cheese
- $1\frac{1}{2}$  cups French fried onions



### Instructions

Combine the pasta, tuna, soup, and half of the cheese. Transfer into a 9 inch by 18 inch baking dish. Put the remaining cheese on top. Bake 30 minutes at 350 degrees. During the last 5 minutes, add the French fried onions. Let sit for 10 minutes before serving.

- 
1. What is the ratio of the ounces of soup to the cups of shredded cheese to the cups of pasta in one batch of casserole?
  2. How much of each of these 3 ingredients would be needed to make:
    - a. twice the amount of casserole?
    - b. half the amount of casserole?
    - c. five times the amount of casserole?
    - d. one-fifth the amount of casserole?
  3. What is the ratio of cups of pasta to ounces of tuna in one batch of casserole?
  4. How many batches of casserole would you make if you used the following amounts of ingredients?
    - a. 9 cups of pasta and 18 ounces of tuna?
    - b. 36 ounces of tuna and 18 cups of pasta?

- c. 1 cup of pasta and 2 ounces of tuna?

**Are you ready for more?**

The recipe says to use a 9 inch by 18 inch baking dish. Determine the length and width of a baking dish with the same height that could hold:

1. Twice the amount of casserole
2. Half the amount of casserole
3. Five times the amount of casserole
4. One-fifth the amount of casserole

**5.3: What Are Equivalent Ratios?**

The ratios 5 : 3 and 10 : 6 are equivalent ratios.

1. Is the ratio 15 : 12 equivalent to these? Explain your reasoning.
2. Is the ratio 30 : 18 equivalent to these? Explain your reasoning.
3. Give two more examples of ratios that are equivalent to 5 : 3.
4. How do you know when ratios are equivalent and when they are *not* equivalent?
5. Write a definition of *equivalent ratios*.

Pause here so your teacher can review your work and assign you a ratio to use for your visual display.

6. Create a visual display that includes:

- the title "Equivalent Ratios"
- your best definition of *equivalent ratios*
- the ratio your teacher assigned to you
- at least two examples of ratios that are equivalent to your assigned ratio
- an explanation of how you know these examples are equivalent
- at least one example of a ratio that is *not* equivalent to your assigned ratio
- an explanation of how you know this example is *not* equivalent

Be prepared to share your display with the class.

## Lesson 5 Summary

All ratios that are equivalent to  $a : b$  can be made by multiplying both  $a$  and  $b$  by the same number.

For example, the ratio  $18 : 12$  is equivalent to  $9 : 6$  because both 9 and 6 are multiplied by the same number: 2.

$$\begin{array}{ccc} & 9 : 6 & \\ \cdot 2 \downarrow & & \downarrow \cdot 2 \\ & 18 : 12 & \end{array}$$

$3 : 2$  is also equivalent to  $9 : 6$ , because both 9 and 6 are multiplied by the same number:  $\frac{1}{3}$ .

$$\begin{array}{ccc} & 9 : 6 & \\ \cdot \frac{1}{3} \downarrow & & \downarrow \cdot \frac{1}{3} \\ & 3 : 2 & \end{array}$$

Is  $18 : 15$  equivalent to  $9 : 6$ ?

No, because 18 is  $9 \cdot 2$ , but 15 is *not*  $6 \cdot 2$ .

$$\begin{array}{ccc} & 9 : 6 & \\ \cdot 2 \downarrow & & \downarrow \text{Nope.} \\ & 18 : 15 & \end{array}$$

## Unit 2 Lesson 5 Cumulative Practice Problems

1. Each of these is a pair of equivalent ratios. For each pair, explain why they are equivalent ratios or draw a diagram that shows why they are equivalent ratios.

a.  $4 : 5$  and  $8 : 10$

a.  $2 : 7$  and  $10,000 : 35,000$

b.  $18 : 3$  and  $6 : 1$

2. Explain why  $6 : 4$  and  $18 : 8$  are not equivalent ratios.

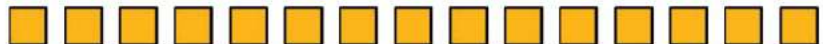
3. Are the ratios  $3 : 6$  and  $6 : 3$  equivalent? Why or why not?

4. This diagram represents 3 batches of light yellow paint. Draw a diagram that represents 1 batch of the same shade of light yellow paint.

white paint (cups)



yellow paint (cups)



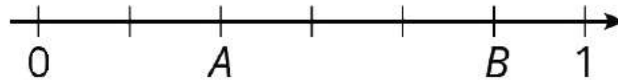
(From Unit 2, Lesson 4.)

5. In the fruit bowl there are 6 bananas, 4 apples, and 3 oranges.

- a. For every 4 \_\_\_\_\_, there are 3 \_\_\_\_\_.
- b. The ratio of \_\_\_\_\_ to \_\_\_\_\_ is 6 : 3.
- c. The ratio of \_\_\_\_\_ to \_\_\_\_\_ is 4 to 6.
- d. For every 1 orange, there are \_\_\_\_\_ bananas.

(From Unit 2, Lesson 1.)

6. Write fractions for points *A* and *B* on the number line.



(From Unit 2, Lesson 1.)

## Lesson 6: Introducing Double Number Line Diagrams

### 6.1: Number Talk: Adjusting Another Factor

Find the value of each product mentally.

$$(4.5) \cdot 4$$

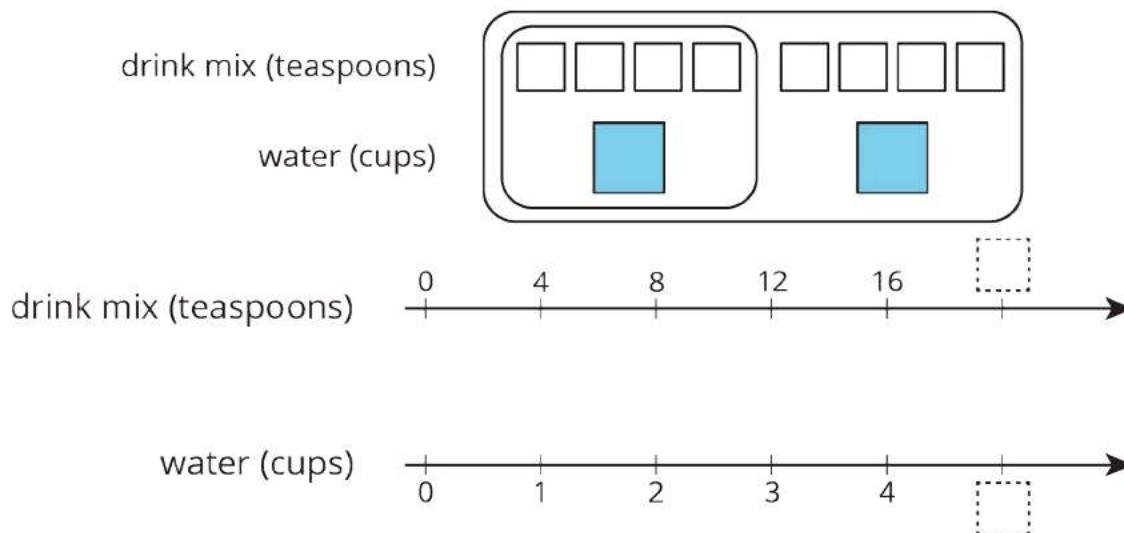
$$(4.5) \cdot 8$$

$$\frac{1}{10} \cdot 65$$

$$\frac{2}{10} \cdot 65$$

## 6.2: Drink Mix on a Double Number Line

The other day, we made drink mixtures by mixing 4 teaspoons of powdered drink mix for every cup of water. Here are two ways to represent multiple batches of this recipe:



1. How can we tell that  $4 : 1$  and  $12 : 3$  are equivalent ratios?
2. How are these representations the same? How are these representations different?
3. How many teaspoons of drink mix should be used with 3 cups of water?
4. How many cups of water should be used with 16 teaspoons of drink mix?
5. What numbers should go in the empty boxes on the double number line diagram?  
What do these numbers mean?

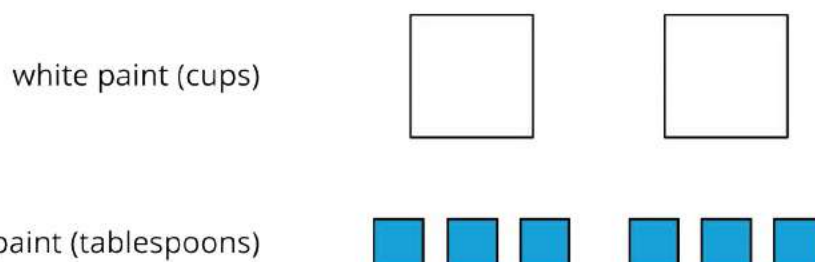
### **Are you ready for more?**

Recall that a *perfect square* is a number of objects that can be arranged into a square. For example, 9 is a perfect square because 9 objects can be arranged into 3 rows of 3. 16 is also a perfect square, because 16 objects can be arranged into 4 rows of 4. In contrast, 12 is not a perfect square because you can't arrange 12 objects into a square.

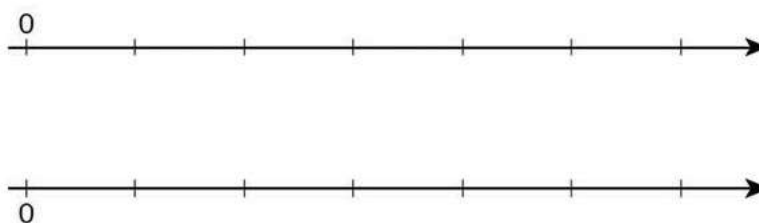
1. How many whole numbers starting with 1 and ending with 100 are perfect squares?
2. What about whole numbers starting with 1 and ending with 1,000?

### 6.3: Blue Paint on a Double Number Line

Here is a diagram showing Elena's recipe for light blue paint.



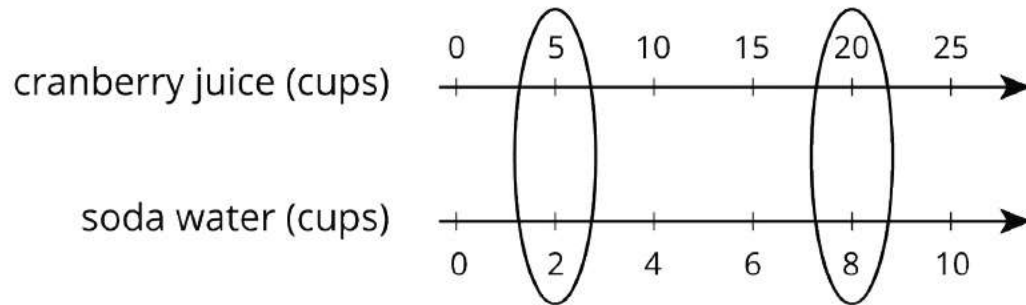
1. Complete the double number line diagram to show the amounts of white paint and blue paint in different-sized batches of light blue paint.



2. Compare your double number line diagram with your partner. Discuss your thinking. If needed, revise your diagram.
3. How many cups of white paint should Elena mix with 12 tablespoons of blue paint? How many batches would this make?
4. How many tablespoons of blue paint should Elena mix with 6 cups of white paint? How many batches would this make?
5. Use your double number line diagram to find another amount of white paint and blue paint that would make the same shade of light blue paint.
6. How do you know that these mixtures would make the same shade of light blue paint?

## Lesson 6 Summary

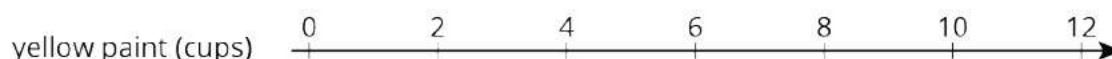
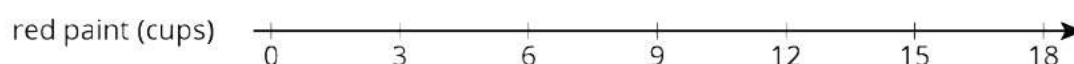
You can use a double number line diagram to find many equivalent ratios. For example, a recipe for fizzy juice says, "Mix 5 cups of cranberry juice with 2 cups of soda water." The ratio of cranberry juice to soda water is  $5 : 2$ . Multiplying both ingredients by the same number creates equivalent ratios.



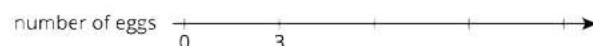
This double number line shows that the ratio  $20 : 8$  is equivalent to  $5 : 2$ . If you mix 20 cups of cranberry juice with 8 cups of soda water, it makes 4 times as much fizzy juice that tastes the same as the original recipe.

## Unit 2 Lesson 6 Cumulative Practice Problems

1. A particular shade of orange paint has 2 cups of yellow paint for every 3 cups of red paint. On the double number line, circle the numbers of cups of yellow and red paint needed for 3 batches of orange paint.



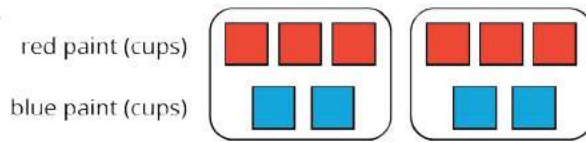
2. This double number line diagram shows the amount of flour and eggs needed for 1 batch of cookies.



- Complete the diagram to show the amount of flour and eggs needed for 2, 3, and 4 batches of cookies.
  - What is the ratio of cups of flour to eggs?
  - How much flour and how many eggs are used in 4 batches of cookies?
- How much flour is used with 6 eggs?
  - How many eggs are used with 15 cups of flour?

3. Here is a representation showing the amount of red and blue paint that make 2 batches of purple paint.

- a. On the double number line, label the tick marks to represent amounts of red and blue paint used to make batches of this shade of purple paint.



- a. How many batches are made with 12 cups of red paint?



- b. How many batches are made with 6 cups of blue paint?

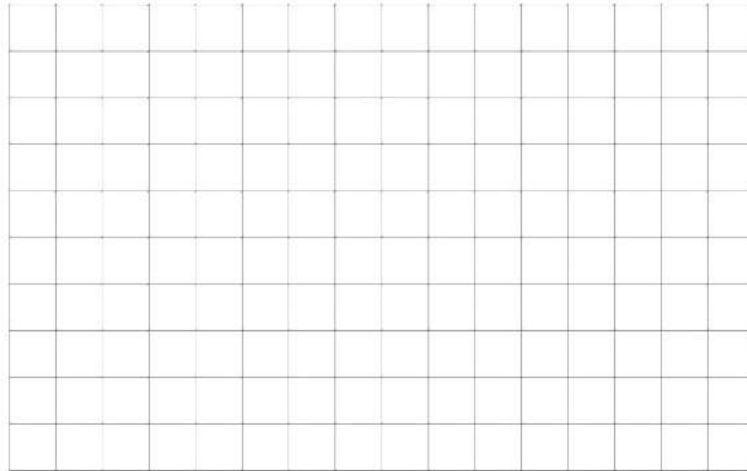


4. Diego estimates that there will need to be 3 pizzas for every 7 kids at his party. Select all the statements that express this ratio.

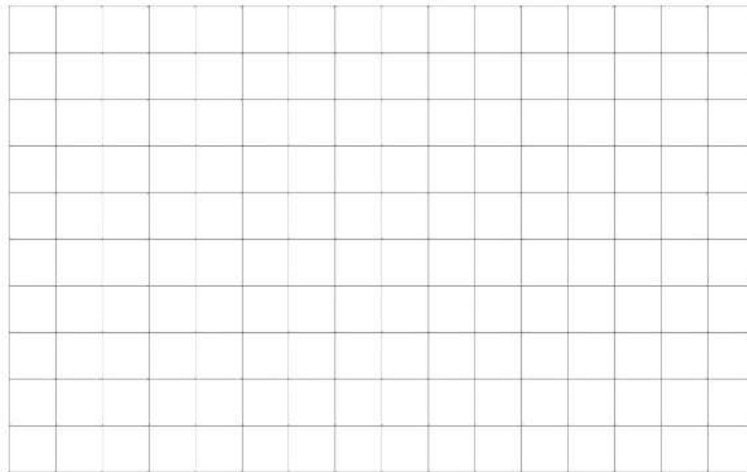
- A. The ratio of kids to pizzas is 7 : 3.
- B. The ratio of pizzas to kids is 3 to 7.
- C. The ratio of kids to pizzas is 3 : 7.
- D. The ratio of pizzas to kids is 7 to 3.
- E. For every 7 kids there need to be 3 pizzas.

(From Unit 2, Lesson 1.)

5. a. Draw a parallelogram that is not a rectangle that has an area of 24 square units. Explain or show how you know the area is 24 square units.



- b. Draw a triangle that has an area of 24 square units. Explain or show how you know the area is 24 square units.



(From Unit 1, Lesson 6.)

# Lesson 7: Creating Double Number Line Diagrams

## 7.1: Ordering on a Number Line

1. Locate and label the following numbers on the number line:

$\frac{1}{2}$

$\frac{1}{4}$

$1\frac{3}{4}$

$1.5$

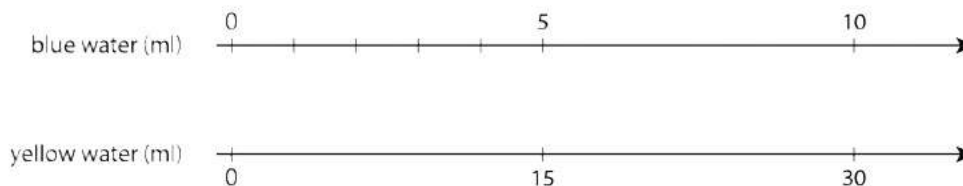
$1.75$



2. Based on where you placed the numbers, locate and label four more fractions or decimals on the number line.

## 7.2: Just a Little Green

The other day, we made green water by mixing 5 ml of blue water with 15 ml of yellow water. We want to make a very small batch of the same shade of green water. We need to know how much yellow water to mix with only 1 ml of blue water.



1. On the number line for blue water, label the four tick marks shown.
2. On the number line for yellow water, draw and label tick marks to show the amount of yellow water needed for each amount of blue water.
3. How much yellow water should be used for 1 ml of blue water? Circle where you can see this on the double number line.
4. How much yellow water should be used for 11 ml of blue water?
5. How much yellow water should be used for 8 ml of blue water?
6. Why is it useful to know how much yellow water should be used with 1 ml of blue water?

### 7.3: Art Paste on a Double Number Line

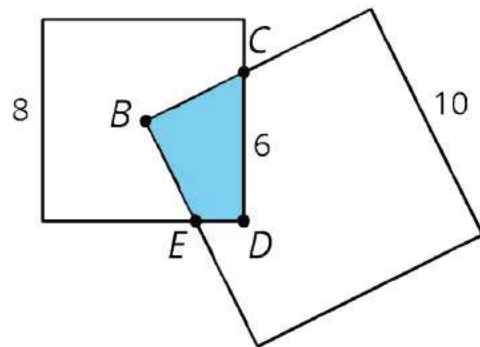
A recipe for art paste says “For every 2 pints of water, mix in 8 cups of flour.”

1. Follow the instructions to draw a double number line diagram representing the recipe for art paste.
  - a. Use a ruler to draw two parallel lines.
  - b. Label the first line “pints of water.” Label the second line “cups of flour.”
  - c. Draw at least 6 equally spaced tick marks that line up on both lines.
  - d. Along the water line, label the tick marks with the amount of water in 0, 1, 2, 3, 4, and 5 batches of art paste.
  - e. Along the flour line, label the tick marks with the amount of flour in 0, 1, 2, 3, 4, and 5 batches of art paste.
2. Compare your double number line diagram with your partner’s. Discuss your thinking. If needed, revise your diagram.
3. Next, use your double number line to answer these questions:
  - a. How much flour should be used with 10 pints of water?
  - b. How much water should be used with 24 cups of flour?
  - c. How much flour per pint of water does this recipe use?

#### Are you ready for more?

A square with side of 10 units overlaps a square with side of 8 units in such a way that its corner *B* is placed exactly at the center of the smaller square. As a result of the

overlapping, the two sides of the large square intersect the two sides of the small square exactly at points  $C$  and  $E$ , as shown. The length of  $CD$  is 6 units.



What is the area of the overlapping region  $CDEB$ ?

## 7.4: Revisiting Tuna Casserole

The other day, we looked at a recipe for tuna casserole that called for 10 ounces of cream of chicken soup for every 3 cups of elbow-shaped pasta.

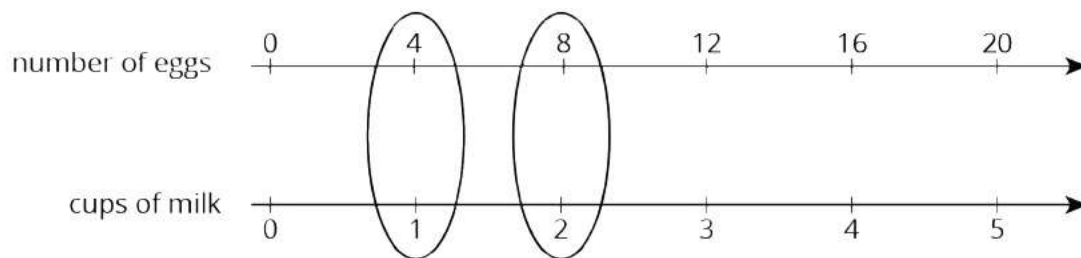
1. Draw a double number line diagram that represents the amounts of soup and pasta in different-sized batches of this recipe.
2. If you made a large amount of tuna casserole by mixing 40 ounces of soup with 15 cups of pasta, would it taste the same as the original recipe? Explain or show your reasoning.
3. The original recipe called for 6 ounces of tuna for every 3 cups of pasta. Add a line to your diagram to represent the amount of tuna in different batches of casserole.
4. How many ounces of soup should you mix with 30 ounces of tuna to make a casserole that tastes the same as the original recipe?

## Lesson 7 Summary

Here are some guidelines to keep in mind when drawing a double number line diagram:

- The two parallel lines should have labels that describe what the numbers represent.
- The tick marks and numbers should be spaced at equal intervals.
- Numbers that line up vertically make equivalent ratios.

For example, the ratio of the number of eggs to cups of milk in a recipe is 4 : 1. Here is a double number line that represents the situation:



We can also say that this recipe uses "4 eggs per cup of milk" because the word **per** means "for each."

## Unit 2 Lesson 7 Cumulative Practice Problems

1. A recipe for cinnamon rolls uses 2 tablespoons of sugar per teaspoon of cinnamon for the filling. Complete the double number line diagram to show the amount of cinnamon and sugar in 3, 4, and 5 batches.



2. One batch of meatloaf contains 2 pounds of beef and  $\frac{1}{2}$  cup of bread crumbs. Complete the double number line diagram to show the amounts of beef and bread crumbs needed for 1, 2, 3, and 4 batches of meatloaf.



3. A recipe for tropical fruit punch says, "Combine 4 cups of pineapple juice with 5 cups of orange juice."
- a. Create a double number showing the amount of each type of juice in 1, 2, 3, 4, and 5 batches of the recipe.
- b. If 12 cups of pineapple juice are used with 20 cups of orange juice, will the recipe taste the same? Explain your reasoning.

c. The recipe also calls for  $\frac{1}{3}$  cup of lime juice for every 5 cups of orange juice. Add a line to your diagram to represent the amount of lime juice in different batches of tropical fruit punch.

4. One batch of pink paint uses 2 cups of red paint and 7 cups of white paint. Mai made a large amount of pink paint using 14 cups of red paint.

a. How many batches of pink paint did she make?

b. How many cups of white paint did she use?

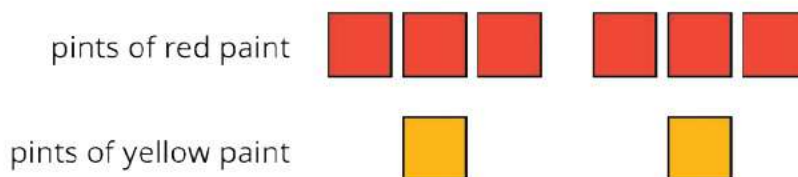
(From Unit 2, Lesson 4.)

5. a. Find three different ratios that are equivalent to the ratio 3 : 11.

b. Explain why your ratios are equivalent.

(From Unit 2, Lesson 5.)

6. Here is a diagram that represents the pints of red and yellow paint in a mixture.



Select **all** statements that accurately describe the diagram.

A. The ratio of yellow paint to red paint is 2 to 6.

B. For every 3 pints of red paint, there is 1 pint of yellow paint.

C. For every pint of yellow paint, there are 3 pints of red paint.

D. For every pint of yellow paint there are 6 pints of red paint.

E. The ratio of red paint to yellow paint is 6 : 2.

(From Unit 2, Lesson 2.)

## Lesson 8: How Much for One?

### 8.1: Number Talk: Remainders in Division

Find the quotient mentally.

$$246 \div 12$$

### 8.2: Grocery Shopping

Answer each question and explain or show your reasoning. If you get stuck, consider drawing a double number line diagram.

1. Eight avocados cost \$4.

- a. How much do 16 avocados cost?
- b. How much do 20 avocados cost?
- c. How much do 9 avocados cost?



2. Twelve large bottles of water cost \$9.

- a. How many bottles can you buy for \$3?
- b. What is the cost per bottle of water?
- c. How much would 7 bottles of water cost?



3. A 10-pound sack of flour costs \$8.

- a. How much does 40 pounds of flour cost?
- b. What is the cost per pound of flour?

### Are you ready for more?

It is commonly thought that buying larger packages or containers, sometimes called *buying in bulk*, is a great way to save money. For example, a 6-pack of soda might cost \$3 while a 12-pack of the same brand costs \$5.

Find 3 different cases where it is not true that buying in bulk saves money. You may use the internet or go to a local grocery store and take photographs of the cases you find. Make sure the products are the same brand. For each example that you find, give the quantity or size of each, and describe how you know that the larger size is not a better deal.

## 8.3: More Shopping

1. Four bags of chips cost \$6.

- a. What is the cost per bag?
- b. At this rate, how much will 7 bags of chips cost?

2. At a used book sale, 5 books cost \$15.

- a. What is the cost per book?
- b. At this rate, how many books can you buy for \$21?

3. Neon bracelets cost \$1 for 4.

- a. What is the cost per bracelet?
- b. At this rate, how much will 11 neon bracelets cost?

Pause here so your teacher can review your work.

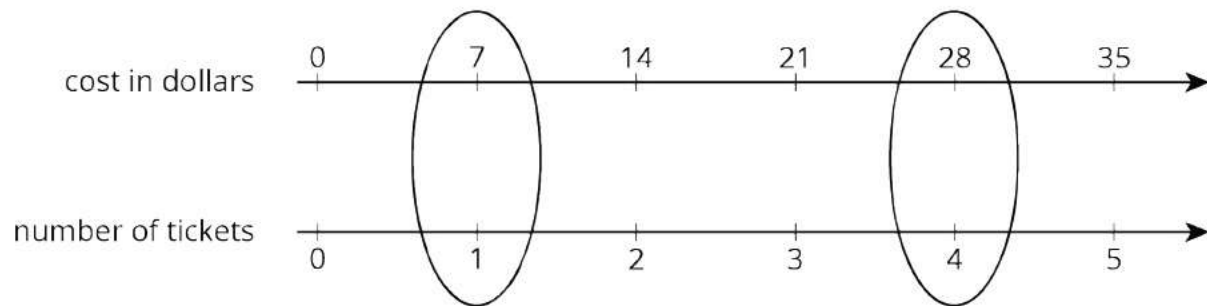


4. Your teacher will assign you one of the problems. Create a visual display that shows your solution to the problem. Be prepared to share your solution with the class.

## Lesson 8 Summary

The **unit price** is the price of 1 thing—for example, the price of 1 ticket, 1 slice of pizza, or 1 kilogram of peaches.

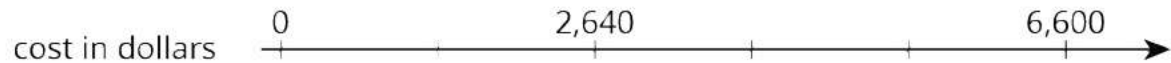
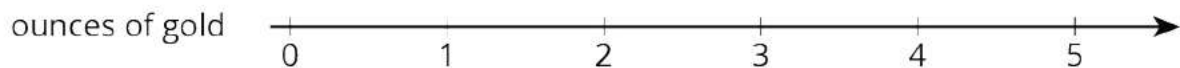
If 4 movie tickets cost \$28, then the unit price would be the cost *per* ticket. We can create a double number line to find the unit price.



This double number line shows that the cost for 1 ticket is \$7. We can also find the unit price by dividing,  $28 \div 4 = 7$ , or by multiplying,  $28 \cdot \frac{1}{4} = 7$ .

## Unit 2 Lesson 8 Cumulative Practice Problems

1. In 2016, the cost of 2 ounces of pure gold was \$2,640. Complete the double number line to show the cost for 1, 3, and 4 ounces of gold.



2. The double number line shows that 4 pounds of tomatoes cost \$14. Draw tick marks and write labels to show the prices of 1, 2, and 3 pounds of tomatoes.



3. 4 movie tickets cost \$48. At this rate, what is the cost of:

a. 5 movie tickets?

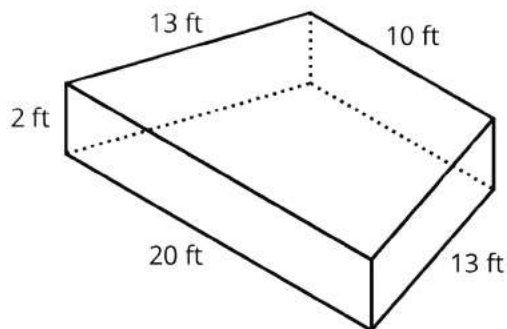
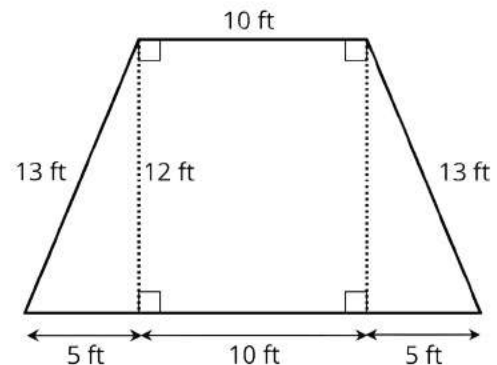
b. 11 movie tickets?

4. Priya bought these items at the grocery store. Find each unit price.
- a. 12 eggs for \$3. How much is the cost per egg?
  - b. 3 pounds of peanuts for \$7.50. How much is the cost per pound?
  - c. 4 rolls of toilet paper for \$2. How much is the cost per roll?
  - d. 10 apples for \$3.50. How much is the cost per apple?

5. Clare made a smoothie with 1 cup of yogurt, 3 tablespoons of peanut butter, 2 teaspoons of chocolate syrup, and 2 cups of crushed ice.
- Kiran tried to double this recipe. He used 2 cups of yogurt, 6 tablespoons of peanut butter, 5 teaspoons of chocolate syrup, and 4 cups of crushed ice. He didn't think it tasted right. Describe how the flavor of Kiran's recipe compares to Clare's recipe.
  - How should Kiran change the quantities that he used so that his smoothie tastes just like Clare's?

(From Unit 2, Lesson 3.)

6. A drama club is building a wooden stage in the shape of a trapezoidal prism. The height of the stage is 2 feet. Some measurements of the stage are shown here.



What is the area of all the faces of the stage, excluding the bottom? Show your reasoning. If you get stuck, consider drawing a net of the prism.

(From Unit 1, Lesson 15.)

## Lesson 9: Constant Speed

### 9.1: Number Talk: Dividing by Powers of 10

Find the quotient mentally.

$$30 \div 10$$

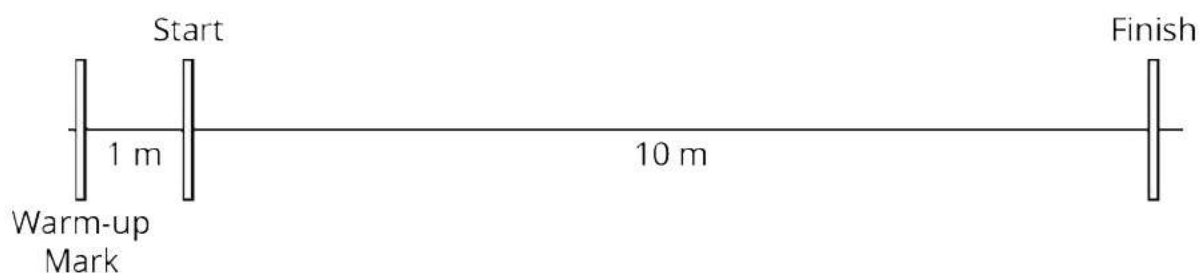
$$34 \div 10$$

$$3.4 \div 10$$

$$34 \div 100$$

### 9.2: Moving 10 Meters

Your teacher will set up a straight path with a 1-meter warm-up zone and a 10-meter measuring zone. Follow the following instructions to collect the data.



1.
  - a. The person with the stopwatch (the “timer”) stands at the finish line. The person being timed (the “mover”) stands at the warm-up line.
  - b. On the first round, the mover starts moving *at a slow, steady speed* along the path. When the mover reaches the start line, they say, “Start!” and the timer starts the stopwatch.
  - c. The mover keeps moving steadily along the path. When they reach the finish line, the timer stops the stopwatch and records the time, rounded to the nearest second, in the table.
  - d. On the second round, the mover follows the same instructions, but this time, moving *at a quick, steady speed*. The timer records the time the same way.
  - e. Repeat these steps until each person in the group has gone twice: once at a slow, steady speed, and once at a quick, steady speed.

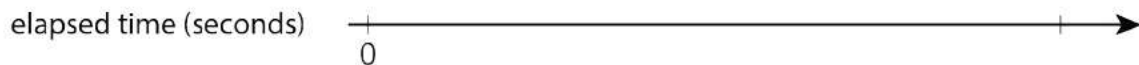
| your slow moving time (seconds) | your fast moving time (seconds) |
|---------------------------------|---------------------------------|
|                                 |                                 |

2. After you finish collecting the data, use the double number line diagrams to answer the questions. Use the times your partner collected while you were moving.

Moving slowly:



Moving quickly:



a. Estimate the distance in meters you traveled in 1 second when moving slowly.

b. Estimate the distance in meters you traveled in 1 second when moving quickly.

c. Trade diagrams with someone who is not your partner. How is the diagram representing someone moving slowly different from the diagram representing someone moving quickly?

## 9.3: Moving for 10 Seconds

Lin and Diego both ran for 10 seconds, each at their own constant speed. Lin ran 40 meters and Diego ran 55 meters.

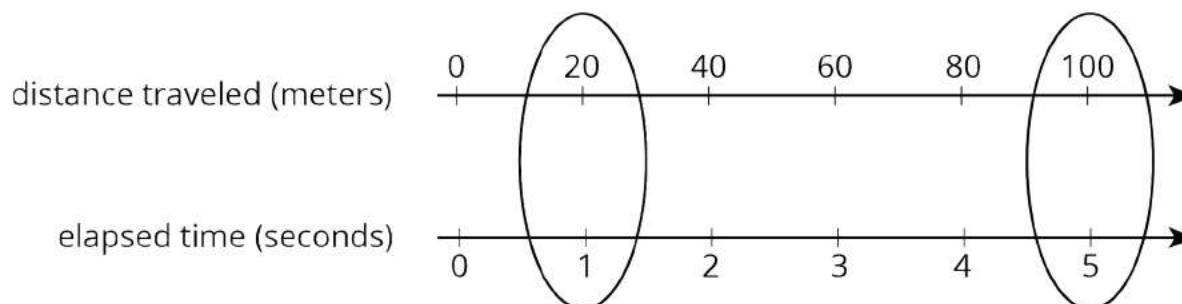
1. Who was moving faster? Explain your reasoning.
2. How far did each person move in 1 second? If you get stuck, consider drawing double number line diagrams to represent the situations.
3. Use your data from the previous activity to find how far *you* could travel in 10 seconds at your quicker speed.
4. Han ran 100 meters in 20 seconds at a constant speed. Is this speed faster, slower, or the same as Lin's? Diego's? Yours?

**Are you ready for more?**

Lin and Diego want to run a race in which they will both finish when the timer reads exactly 30 seconds. Who should get a head start, and how long should the head start be?

## Lesson 9 Summary

Suppose a train traveled 100 meters in 5 seconds at a constant speed. To find its speed in meters per second, we can create a double number line:



The double number line shows that the train's speed was 20 meters per second. We can also find the speed by dividing:  $100 \div 5 = 20$ .

Once we know the speed in meters per second, many questions about the situation become simpler to answer because we can multiply the amount of time an object travels by the speed to get the distance. For example, at this rate, how far would the train go in 30 seconds? Since  $20 \cdot 30 = 600$ , the train would go 600 meters in 30 seconds.

## Unit 2 Lesson 9 Cumulative Practice Problems

1. Han ran 10 meters in 2.7 seconds. Priya ran 10 meters in 2.4 seconds.

a. Who ran faster? Explain how you know.

b. At this rate, how long would it take each person to run 50 meters? Explain or show your reasoning.

2. A scooter travels 30 feet in 2 seconds at a constant speed.

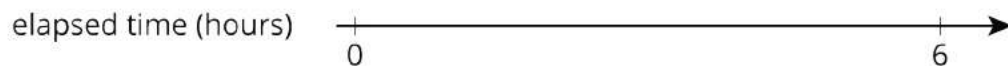


a. What is the speed of the scooter in feet per second?

b. Complete the double number line to show the distance the scooter travels after 1, 3, 4, and 5 seconds.

c. A skateboard travels 55 feet in 4 seconds. Is the skateboard going faster, slower, or the same speed as the scooter?

3. A cargo ship traveled 150 nautical miles in 6 hours at a constant speed. How far did the cargo ship travel in one hour?



4. A recipe for pasta dough says, "Use 150 grams of flour per large egg."

- a. How much flour is needed if 6 large eggs are used?
- b. How many eggs are needed if 450 grams of flour are used?

(From Unit 2, Lesson 3.)

5. The grocery store is having a sale on frozen vegetables. 4 bags are being sold for \$11.96. At this rate, what is the cost of:

- a. 1 bag
- b. 9 bags

(From Unit 2, Lesson 8.)

6. A pet owner has 5 cats. Each cat has 2 ears and 4 paws.

- a. Complete the double number line to show the numbers of ears and paws for 1, 2, 3, 4, and 5 cats.

number of ears  $\begin{array}{c} 0 \\ + \end{array} \longrightarrow$

- b. If there are 3 cats in the room, what is the ratio of ears to paws?

number of paws  $\begin{array}{c} + \\ 0 \end{array} \longrightarrow$

- a. If there are 4 cats in the room, what is the ratio of paws to ears?
- b. If all 5 cats are in the room, how many more paws are there than ears?

(From Unit 2, Lesson 7.)

7. Each of these is a pair of equivalent ratios. For each pair, explain why they are equivalent ratios or draw a representation that shows why they are equivalent ratios.

- a. 5 : 1 and 15 : 3
- b. 25 : 5 and 10 : 2
- c. 198 : 1,287 and 2 : 13

(From Unit 2, Lesson 5.)

# Lesson 10: Comparing Situations by Examining Ratios

## 10.1: Treadmills

Mai and Jada each ran on a treadmill. The treadmill display shows the distance, in miles, each person ran and the amount of time it took them, in minutes and seconds.

Here is Mai's treadmill display:



Here is Jada's treadmill display:



1. What is the same about their workouts? What is different about their workouts?
2. If each person ran at a constant speed the entire time, who was running faster? Explain your reasoning.

## 10.2: Concert Tickets

Diego paid \$47 for 3 tickets to a concert. Andre paid \$141 for 9 tickets to a concert. Did they pay at the same rate? Explain your reasoning.

## 10.3: Sparkling Orange Juice

Lin and Noah each have their own recipe for making sparkling orange juice.

- Lin mixes 3 liters of orange juice with 4 liters of soda water.
- Noah mixes 4 liters of orange juice with 5 liters of soda water.

How do the two mixtures compare in taste? Explain your reasoning.

### Are you ready for more?

1. How can Lin make her sparkling orange juice taste the same as Noah's just by adding more of one ingredient? How much will she need?
2. How can Noah make his sparkling orange juice taste the same as Lin's just by adding more of one ingredient? How much will he need?

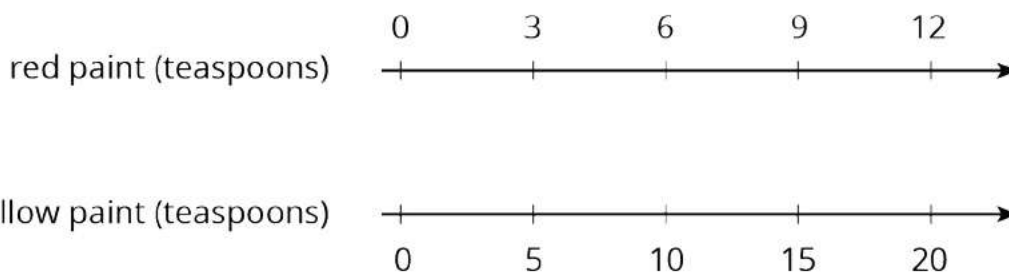
## Lesson 10 Summary

Sometimes we want to know whether two situations are described by the **same rate**. To do that, we can write an equivalent ratio for one or both situations so that one part of their ratios has the same value. Then we can compare the other part of the ratios.

For example, do these two paint mixtures make the same shade of orange?

- Kiran mixes 9 teaspoons of red paint with 15 teaspoons of yellow paint.
- Tyler mixes 7 teaspoons of red paint with 10 teaspoons of yellow paint.

Here is a double number line that represents Kiran's paint mixture. The ratio 9 : 15 is equivalent to the ratios 3 : 5 and 6 : 10.



For 10 teaspoons of yellow paint, Kiran would mix in 6 teaspoons of red paint. This is less red paint than Tyler mixes with 10 teaspoons of yellow paint. The ratios 6 : 10 and 7 : 10 are not equivalent, so these two paint mixtures would not be the same shade of orange.

When we talk about two things happening at the same rate, we mean that the ratios of the quantities in the two situations are equivalent. There is also something specific about the situation that is the same.

- If two ladybugs are moving at the same rate, then they are traveling at the *same constant speed*.
- If two bags of apples are selling for the same rate, then they have the *same unit price*.
- If we mix two kinds of juice at the same rate, then the mixtures have the *same taste*.
- If we mix two colors of paint at the same rate, then the mixtures have the *same shade*.

## Unit 2 Lesson 10 Cumulative Practice Problems

1. A slug travels 3 centimeters in 3 seconds. A snail travels 6 centimeters in 6 seconds. Both travel at constant speeds. Mai says, "The snail was traveling faster because it went a greater distance." Do you agree with Mai? Explain or show your reasoning.

2. If you blend 2 scoops of chocolate ice cream with 1 cup of milk, you get a milkshake with a stronger chocolate flavor than if you blended 3 scoops of chocolate ice cream with 2 cups of milk. Explain or show why.

3. There are 2 mixtures of light purple paint.

- Mixture A is made with 5 cups of purple paint and 2 cups of white paint.
- Mixture B is made with 15 cups of purple paint and 8 cups of white paint.

Which mixture is a lighter shade of purple? Explain your reasoning.

4. Tulip bulbs are on sale at store A, at 5 for \$11.00, and the regular price at store B is 6 for \$13. Is each store pricing tulip bulbs at the same rate? Explain how you know.

5. A plane travels at a constant speed. It takes 6 hours to travel 3,360 miles.

- a. What is the plane's speed in miles per hour?
- b. At this rate, how many miles can it travel in 10 hours?

(From Unit 2, Lesson 9.)

6. A pound of ground beef costs \$5. At this rate, what is the cost of:

- a. 3 pounds?
- b.  $\frac{1}{2}$  pound?
- c.  $\frac{1}{4}$  pound?
- d.  $\frac{3}{4}$  pound?
- e.  $3\frac{3}{4}$  pounds?

(From Unit 2, Lesson 8.)

7. In a triple batch of a spice mix, there are 6 teaspoons of garlic powder and 15 teaspoons of salt. Answer the following questions about the mix. If you get stuck, create a double number line.

- a. How much garlic powder is used with 5 teaspoons of salt?
- b. How much salt is used with 8 teaspoons of garlic powder?
- c. If there are 14 teaspoons of spice mix, how much salt is in it?
- d. How much more salt is there than garlic powder if 6 teaspoons of garlic powder are used?

(From Unit 2, Lesson 7.)

# Lesson 11: Representing Ratios with Tables

## 11.1: How Is It Growing?

Look for a pattern in the figures.

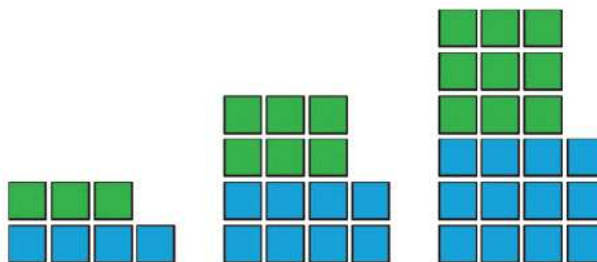
1. How many total tiles will be in:

a. the 4th figure?

b. the 5th figure?

c. the 10th figure?

2. How do you see it growing?



## 11.2: A Huge Amount of Sparkling Orange Juice

Noah's recipe for one batch of sparkling orange juice uses 4 liters of orange juice and 5 liters of soda water.

1. Use the double number line to show how many liters of each ingredient to use for different-sized batches of sparkling orange juice.

orange juice (liters) 

soda water (liters) 

2. If someone mixes 36 liters of orange juice and 45 liters of soda water, how many batches would they make?
3. If someone uses 400 liters of orange juice, how much soda water would they need?
4. If someone uses 455 liters of soda water, how much orange juice would they need?
5. Explain the trouble with using a double number line diagram to answer the last two questions.

### 11.3: Batches of Trail Mix

A recipe for trail mix says: “Mix 7 ounces of almonds with 5 ounces of raisins.” Here is a table that has been started to show how many ounces of almonds and raisins would be in different-sized batches of this trail mix.

| almonds (oz) | raisins (oz) |
|--------------|--------------|
| 7            | 5            |
| 28           |              |
|              | 10           |
| 3.5          |              |
|              | 250          |
| 56           |              |

1. Complete the table so that ratios represented by each row are equivalent.

2. What methods did you use to fill in the table?

3. How do you know that each row shows a ratio that is equivalent to 7 : 5? Explain your reasoning.

#### Are you ready for more?

You have created a best-selling recipe for chocolate chip cookies. The ratio of sugar to flour is 2 : 5.

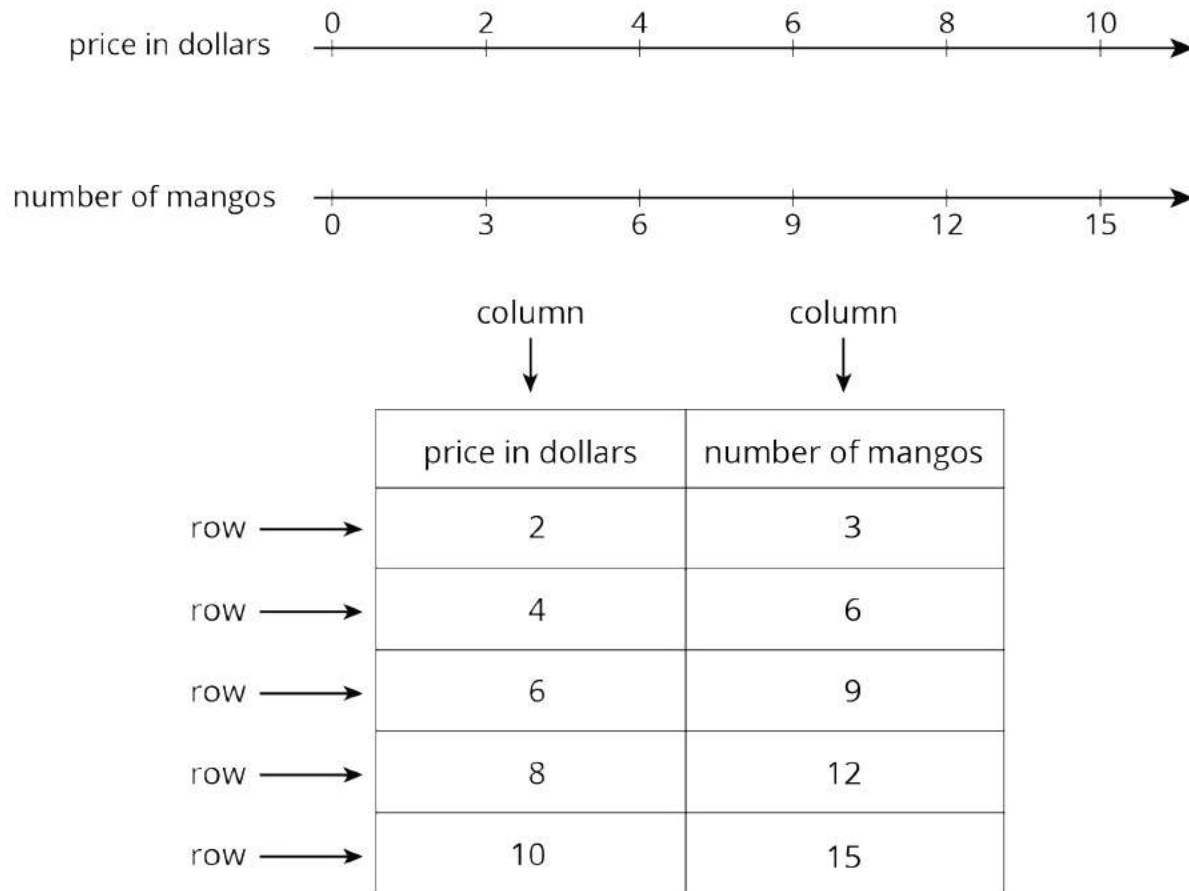
Create a table in which each entry represents amounts of sugar and flour that might be used at the same time in your recipe.

- One entry should have amounts where you have fewer than 25 cups of flour.
- One entry should have amounts where you have between 20–30 cups of sugar.
- One entry can have any amounts using more than 500 units of flour.

## Lesson 11 Summary

A **table** is a way to organize information. Each horizontal set of entries is called a *row*, and each vertical set of entries is called a *column*. (The table shown has 2 columns and 5 rows.) A table can be used to represent a collection of equivalent ratios.

Here is a double number line diagram and a table that both represent the situation: “The price is \$2 for every 3 mangos.”



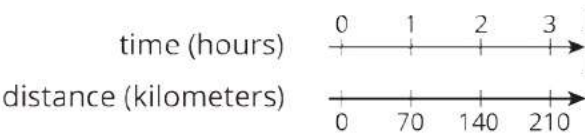
# Unit 2 Lesson 11 Cumulative Practice Problems

1. Complete the table to show the amounts of yellow and red paint needed for different-sized batches of the same shade of orange paint.

| yellow paint<br>(quarts) | red paint<br>(quarts) |
|--------------------------|-----------------------|
| 5                        | 6                     |
|                          |                       |
|                          |                       |
|                          |                       |

Explain how you know that these amounts of yellow paint and red paint will make the same shade of orange as the mixture in the first row of the table.

2. A car travels at a constant speed, as shown on the double number line.

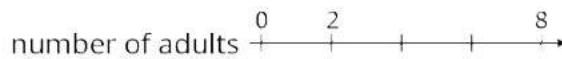


How far does the car travel in 14 hours?  
Explain or show your reasoning.

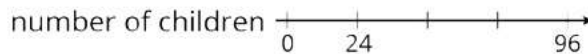
3. The olive trees in an orchard produce 3,000 pounds of olives a year. It takes 20 pounds of olives to make 3 liters of olive oil. How many liters of olive oil can this orchard produce in a year? If you get stuck, consider using the table.

| olives (pounds) | olive oil (liters) |
|-----------------|--------------------|
| 20              | 3                  |
| 100             |                    |
| 3,000           |                    |

4. At a school recess, there needs to be a ratio of 2 adults for every 24 children on the playground. The double number line represents the number of adults and children on the playground at recess.



- a. Label each remaining tick mark with its value.



- b. How many adults are needed if there are 72 children? Circle your answer on the double number line.

(From Unit 2, Lesson 6.)

5. While playing basketball, Jada's heart rate goes up to 160 beats per minute. While jogging, her heart beats 25 times in 10 seconds. Assuming her heart beats at a constant rate while jogging, which of these activities resulted in a higher heart rate? Explain your reasoning.

(From Unit 2, Lesson 10.)

6. A shopper bought the following items at the farmer's market:

a. 6 ears of corn for \$1.80. What was the cost per ear?

b. 12 apples for \$2.88. What was the cost per apple?

c. 5 tomatoes for \$3.10. What was the cost per tomato?

(From Unit 2, Lesson 8.)

# Lesson 12: Navigating a Table of Equivalent Ratios

## 12.1: Number Talk: Multiplying by a Unit Fraction

Find the product mentally.

$$\frac{1}{3} \cdot 21$$

$$\frac{1}{6} \cdot 21$$

$$(5.6) \cdot \frac{1}{8}$$

$$\frac{1}{4} \cdot (5.6)$$

## 12.2: Comparing Taco Prices

| number of<br>tacos | price in<br>dollars |
|--------------------|---------------------|
|                    |                     |
|                    |                     |
|                    |                     |
|                    |                     |
|                    |                     |

Use the table to help you solve these problems. Explain or show your reasoning.

1. Noah bought 4 tacos and paid \$6. At this rate, how many tacos could he buy for \$15?
2. Jada's family bought 50 tacos for a party and paid \$72. Were Jada's tacos the same price as Noah's tacos?

## 12.3: Hourly Wages

Lin is paid \$90 for 5 hours of work. She used the table to calculate how much she would be paid at this rate for 8 hours of work.



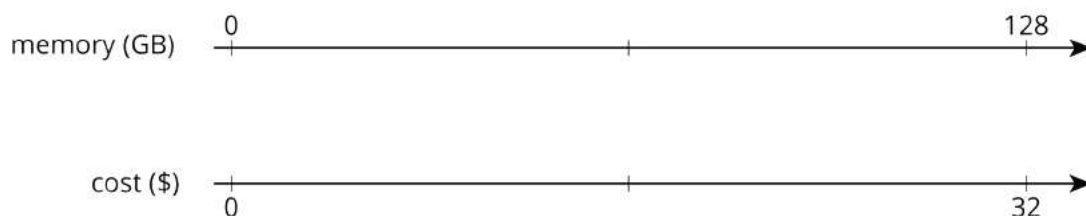
| amount earned (\$) | time worked (hours) |
|--------------------|---------------------|
| 90                 | 5                   |
| 18                 | 1                   |
| 144                | 8                   |
|                    |                     |
|                    |                     |

1. What is the meaning of the 18 that appears in the table?
2. Why was the number  $\frac{1}{5}$  used as a multiplier?
3. Explain how Lin used this table to solve the problem.
4. At this rate, how much would Lin be paid for 3 hours of work? For 2.1 hours of work?

## 12.4: Zeno's Memory Card

In 2016, 128 gigabytes (GB) of portable computer memory cost \$32.

1. Here is a double number line that represents the situation:



One set of tick marks has already been drawn to show the result of multiplying 128 and 32 each by  $\frac{1}{2}$ . Label the amount of memory and the cost for these tick marks.

Next, keep multiplying by  $\frac{1}{2}$  and drawing and labeling new tick marks, until you can no longer clearly label each new tick mark with a number.

2. Here is a table that represents the situation. Find the cost of 1 gigabyte. You can use as many rows as you need.

[illegible]

3. Did you prefer the double number line or the table for solving this problem? Why?

### **Are you ready for more?**

A kilometer is 1,000 meters because *kilo* is a prefix that means 1,000. The prefix *mega* means 1,000,000 and *giga* (as in gigabyte) means 1,000,000,000. One byte is the amount of memory needed to store one letter of the alphabet. About how many of each of the following would fit on a 1-gigabyte flash drive?

1. letters
2. pages
3. books
4. movies
5. songs

## Lesson 12 Summary

Finding a row containing a “1” is often a good way to work with tables of equivalent ratios. For example, the price for 4 lbs of granola is \$5. At that rate, what would be the price for 62 lbs of granola?

Here are tables showing two different approaches to solving this problem. Both of these approaches are correct. However, one approach is more efficient.

- Less efficient

| granola (lbs) | price (\$) |
|---------------|------------|
| 4             | 5          |
| 8             | 10         |
| 16            | 20         |
| 32            | 40         |
| 64            | 80         |
| 62            | 77.50      |

- More efficient

| granola (lbs) | price (\$) |
|---------------|------------|
| 4             | 5          |
| 1             | 1.25       |
| 62            | 77.50      |

Notice how the more efficient approach starts by finding the price for 1 lb of granola.

Remember that dividing by a whole number is the same as multiplying by a unit fraction. In this example, we can divide by 4 or multiply by  $\frac{1}{4}$  to find the unit price.

## Unit 2 Lesson 12 Cumulative Practice Problems

1. Priya collected 2,400 grams of pennies in a fundraiser. Each penny has a mass of 2.5 grams. How much money did Priya raise? If you get stuck, consider using the table.

| number of pennies | mass in grams |
|-------------------|---------------|
| 1                 | 2.5           |
|                   |               |
|                   |               |
|                   |               |
|                   |               |
|                   | 2,400         |

2. Kiran reads 5 pages in 20 minutes. He spends the same amount of time per page. How long will it take him to read 11 pages? If you get stuck, consider using the table.

| time in minutes | number of pages |
|-----------------|-----------------|
| 20              | 5               |
|                 | 1               |
|                 | 11              |

3. Mai is making personal pizzas. For 4 pizzas, she uses 10 ounces of cheese.

| number of pizzas | ounces of cheese |
|------------------|------------------|
| 4                | 10               |
|                  |                  |
|                  |                  |

a. How much cheese does Mai use per pizza?

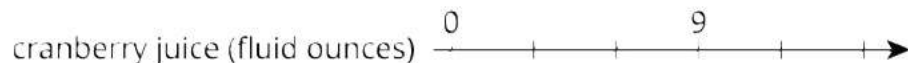
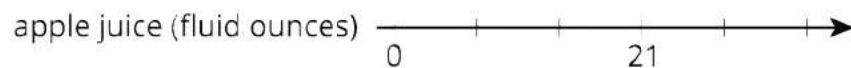
b. At this rate, how much cheese will she need to make 15 pizzas?

4. Clare is paid \$90 for 5 hours of work. At this rate, how many seconds does it take for her to earn 25 cents?

5. A car that travels 20 miles in  $\frac{1}{2}$  hour at constant speed is traveling at the same speed as a car that travels 30 miles in  $\frac{3}{4}$  hour at a constant speed. Explain or show why.

(From Unit 2, Lesson 10.)

6. Lin makes her favorite juice blend by mixing cranberry juice with apple juice in the ratio shown on the double number line. Complete the diagram to show smaller and larger batches that would taste the same as Lin's favorite blend.



(From Unit 2, Lesson 6.)

7. Each of these is a pair of equivalent ratios. For each pair, explain why they are equivalent ratios or draw a representation that shows why they are equivalent ratios.

a.  $600 : 450$  and  $60 : 45$

b.  $60 : 45$  and  $4 : 3$

c.  $600 : 450$  and  $4 : 3$

(From Unit 2, Lesson 5.)

# Lesson 13: Tables and Double Number Line Diagrams

## 13.1: Number Talk: Constant Dividend

Find the quotients mentally.

$$150 \div 2$$

$$150 \div 4$$

$$150 \div 8$$

Locate and label the quotients on the number line.



## 13.2: Moving 3,000 Meters

The other day, we saw that Han can run 100 meters in 20 seconds.

Han wonders how long it would take him to run 3,000 meters at this rate. He made a table of equivalent ratios.

1. Do you agree that this table represents the situation? Explain your reasoning.

|       |     |
|-------|-----|
| 20    | 100 |
| 10    | 50  |
| 1     | 5   |
| 3,000 |     |

2. Complete the last row with the missing number.

3. What question about the situation does this number answer?

4. What could Han do to improve his table?

5. Priya can bike 150 meters in 20 seconds. At this rate, how long would it take her to bike 3,000 meters?

|  |  |
|--|--|
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

6. Priya's neighbor has a dirt bike that can go 360 meters in 15 seconds. At this rate, how long would it take them to ride 3,000 meters?

### 13.3: The International Space Station

The International Space Station orbits around the Earth at a constant speed. Your teacher will give you either a double number line or a table that represents this situation. Your partner will get the other representation.



1. Complete the parts of your representation that you can figure out for sure.
2. Share information with your partner, and use the information that your partner shares to complete your representation.
3. What is the speed of the International Space Station?
4. Place the two completed representations side by side. Discuss with your partner some ways in which they are the same and some ways in which they are different.
5. Record at least one way that they are the same and one way they are different.

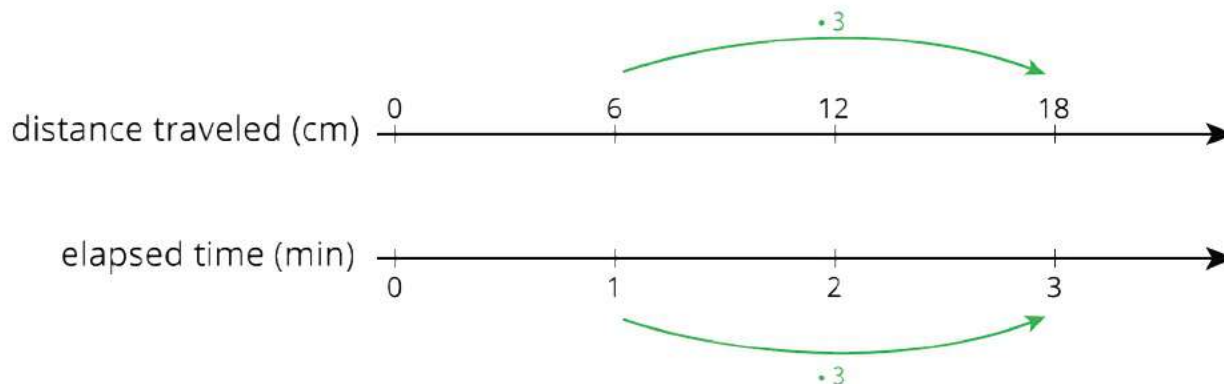
#### Are you ready for more?

Earth's circumference is about 40,000 kilometers and the orbit of the International Space Station is just a bit more than this. About how long does it take for the International Space Station to orbit Earth?

## Lesson 13 Summary

On a double number line diagram, we put labels in front of each line to tell what the numbers represent. On a table, we put labels at the top of each column to tell what the numbers represent.

Here are two different ways we can represent the situation: “A snail is moving at a constant speed down a sidewalk, traveling 6 centimeters per minute.”



| distance traveled (cm) | elapsed time (min) |
|------------------------|--------------------|
| 12                     | 2                  |
| 6                      | 1                  |
| 60                     | 10                 |
| 18                     | 3                  |

Green curved arrows with a small green dot and the number "3" are shown on the left and right sides of the table, pointing from the 6 and 18 rows to the 60 row.

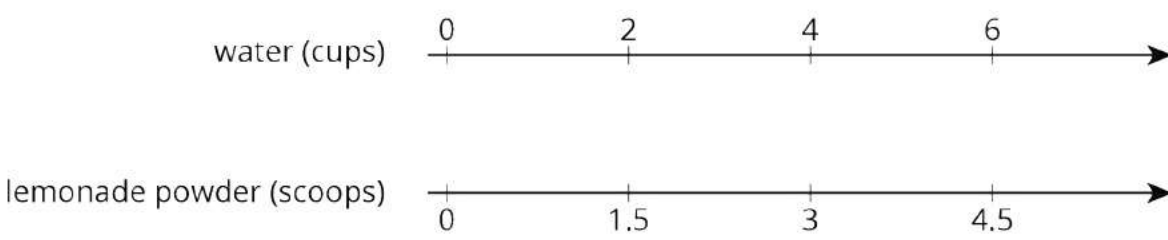
Both double number lines and tables can help us use multiplication to make equivalent ratios, but there is an important difference between the two representations.

On a double number line, the numbers on each line are listed in order. With a table, you can write the ratios in any order. For this reason, sometimes a table is easier to use to solve a problem.

For example, what if we wanted to know how far the snail travels in 10 minutes? Notice that 60 centimeters in 10 minutes is shown on the table, but there is not enough room for this information on the double number line.

# Unit 2 Lesson 13 Cumulative Practice Problems

1. The double number line shows how much water and how much lemonade powder to mix to make different amounts of lemonade.



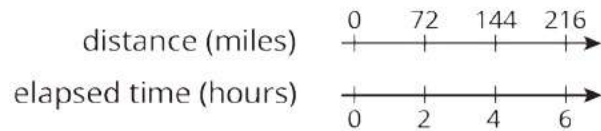
Make a table that represents the same situation.

2. A bread recipe uses 3 tablespoons of olive oil for every 2 cloves of crushed garlic.

- a. Complete the table to show different-sized batches of bread that taste the same as the recipe.
- b. Draw a double number line that represents the same situation.
- c. Which representation do you think works better in this situation? Explain why.

| olive oil<br>(tablespoons) | crushed garlic<br>(cloves) |
|----------------------------|----------------------------|
| 3                          | 2                          |
| 1                          |                            |
| 2                          |                            |
| 5                          |                            |
| 10                         |                            |

3. Clare travels at a constant speed, as shown on the double number line.



At this rate, how far does she travel in each of these intervals of time? Explain or show your reasoning. If you get stuck, consider using a table.

- a. 1 hour
- b. 3 hours
- c. 6.5 hours

4. Lin and Diego travel in cars on the highway at constant speeds. In each case, decide who was traveling faster and explain how you know.

- a. During the first half hour, Lin travels 23 miles while Diego travels 25 miles.
- b. After stopping for lunch, they travel at different speeds. To travel the next 60 miles, it takes Lin 65 minutes and it takes Diego 70 minutes.

(From Unit 2, Lesson 9.)

5. A sports drink recipe calls for  $\frac{5}{3}$  tablespoons of powdered drink mix for every 12 ounces of water. How many batches can you make with 5 tablespoons of drink mix and 36 ounces of water? Explain your reasoning.

(From Unit 2, Lesson 3.)

6. In this cube, each small square has side length 1 unit.
- What is the surface area of this cube?
  - What is the volume of this cube?

(From Unit 1, Lesson 18.)

## **Lesson 14: Solving Equivalent Ratio Problems**

### **14.1: What Do You Want to Know?**

Consider the problem: A red car and a blue car enter the highway at the same time and travel at a constant speed. How far apart are they after 4 hours?

What information would you need to be able to solve the problem?

## 14.2: Info Gap: Hot Chocolate and Potatoes

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

Continue to ask questions until you have enough information to solve the problem.

4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

1. Silently read your card.
2. Ask your partner “*What specific information do you need?*” and wait for them to *ask* for information.

If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don’t have that information.

3. Before sharing the information, ask “*Why do you need that information?*” Listen to your partner’s reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

### 14.3: Comparing Reading Rates

- Lin read the first 54 pages from a 270-page book in the last 3 days.
- Diego read the first 100 pages from a 320-page book in the last 4 days.
- Elena read the first 160 pages from a 480-page book in the last 5 days.

If they continue to read every day at these rates, who will finish first, second, and third? Explain or show your reasoning.

#### Are you ready for more?

The ratio of cats to dogs in a room is 2 : 3. Five more cats enter the room, and then the ratio of cats to dogs is 9 : 11. How many cats and dogs were in the room to begin with?

## Lesson 14 Summary

To solve problems about something happening at the same rate, we often need:

- Two pieces of information that allow us to write a ratio that describes the situation.
- A third piece of information that gives us one number of an equivalent ratio. Solving the problem often involves finding the other number in the equivalent ratio.

Suppose we are making a large batch of fizzy juice and the recipe says, “Mix 5 cups of cranberry juice with 2 cups of soda water.” We know that the ratio of cranberry juice to soda water is 5 : 2, and that we need 2.5 cups of cranberry juice per cup of soda water.

We still need to know something about the size of the large batch. If we use 16 cups of soda water, what number goes with 16 to make a ratio that is equivalent to 5 : 2?

To make this large batch taste the same as the original recipe, we would need to use 40 cups of cranberry juice.

| cranberry juice (cups) | soda water (cups) |
|------------------------|-------------------|
| 5                      | 2                 |
| 2.5                    | 1                 |
| 40                     | 16                |

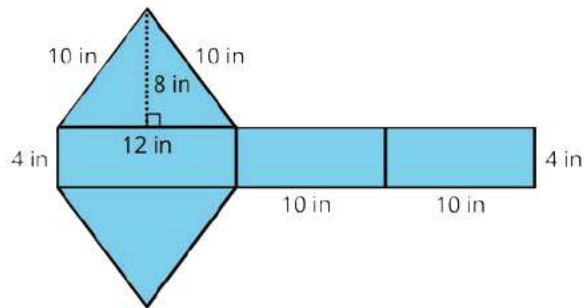
## Unit 2 Lesson 14 Cumulative Practice Problems

1. A chef is making pickles. He needs 15 gallons of vinegar. The store sells 2 gallons of vinegar for \$3.00 and allows customers to buy any amount of vinegar. Decide whether each of the following ratios correctly represents the price of vinegar.
  - a. 4 gallons to \$3.00
  - b. 1 gallon to \$1.50
  - c. 30 gallons to \$45.00
  - d. \$2.00 to 30 gallons
  - e. \$1.00 to  $\frac{2}{3}$  gallon
  
2. A caterer needs to buy 21 pounds of pasta to cater a wedding. At a local store, 8 pounds of pasta cost \$12. How much will the caterer pay for the pasta there?
  - a. Write a ratio for the given information about the cost of pasta.
  
  - b. Would it be more helpful to write an equivalent ratio with 1 pound of pasta as one of the numbers, or with \$1 as one of the numbers? Explain your reasoning, and then write that equivalent ratio.
  
  - c. Find the answer and explain or show your reasoning.

3. Lin is reading a 47-page book. She read the first 20 pages in 35 minutes.
- a. If she continues to read at the same rate, will she be able to complete this book in under 1 hour?
  - b. If so, how much time will she have left? If not, how much more time is needed? Explain or show your reasoning.
4. Diego can type 140 words in 4 minutes.
- a. At this rate, how long will it take him to type 385 words?
  - b. How many words can he type in 15 minutes?
- If you get stuck, consider creating a table.
5. A train that travels 30 miles in  $\frac{1}{3}$  hour at a constant speed is going faster than a train that travels 20 miles in  $\frac{1}{2}$  hour at a constant speed. Explain or show why.

(From Unit 2, Lesson 10.)

6. Find the surface area of the polyhedron that can be assembled from this net. Show your reasoning.



(From Unit 1, Lesson 14.)

## Lesson 15: Part-Part-Whole Ratios

### 15.1: True or False: Multiplying by a Unit Fraction

True or false?

$$\frac{1}{5} \cdot 45 = \frac{45}{5}$$

$$\frac{1}{5} \cdot 20 = \frac{1}{4} \cdot 24$$

$$42 \cdot \frac{1}{6} = \frac{1}{6} \cdot 42$$

$$486 \cdot \frac{1}{12} = \frac{480}{12} + \frac{6}{12}$$

### 15.2: Cubes of Paint

A recipe for maroon paint says, "Mix 5 ml of red paint with 3 ml of blue paint."

1. Use snap cubes to represent the amounts of red and blue paint in the recipe. Then, draw a sketch of your snap-cube representation of the maroon paint.
  - a. What amount does each cube represent?
  - b. How many milliliters of maroon paint will there be?
2.
  - a. Suppose each cube represents 2 ml. How much of each color paint is there?  
Red: \_\_\_\_\_ ml                      Blue: \_\_\_\_\_ ml                      Maroon: \_\_\_\_\_ ml
  - b. Suppose each cube represents 5 ml. How much of each color paint is there?  
Red: \_\_\_\_\_ ml                      Blue: \_\_\_\_\_ ml                      Maroon: \_\_\_\_\_ ml

3. a. Suppose you need 80 ml of maroon paint. How much red and blue paint would you mix? Be prepared to explain your reasoning.

Red: \_\_\_\_\_ ml

Blue: \_\_\_\_\_ ml

Maroon: 80 ml

- b. If the original recipe is for one batch of maroon paint, how many batches are in 80 ml of maroon paint?

### 15.3: Sneakers, Chicken, and Fruit Juice

Solve each of the following problems and show your thinking. If you get stuck, consider drawing a **tape diagram** to represent the situation.

1. The ratio of students wearing sneakers to those wearing boots is 5 to 6. If there are 33 students in the class, and all of them are wearing either sneakers or boots, how many of them are wearing sneakers?
2. A recipe for chicken marinade says, "Mix 3 parts oil with 2 parts soy sauce and 1 part orange juice." If you need 42 cups of marinade in all, how much of each ingredient should you use?

3. Elena makes fruit punch by mixing 4 parts cranberry juice to 3 parts apple juice to 2 parts grape juice. If one batch of fruit punch includes 30 cups of apple juice, how large is this batch of fruit punch?

### **Are you ready for more?**

Using the recipe from earlier, how much fruit punch can you make if you have 50 cups of cranberry juice, 40 cups of apple juice, and 30 cups of grape juice?

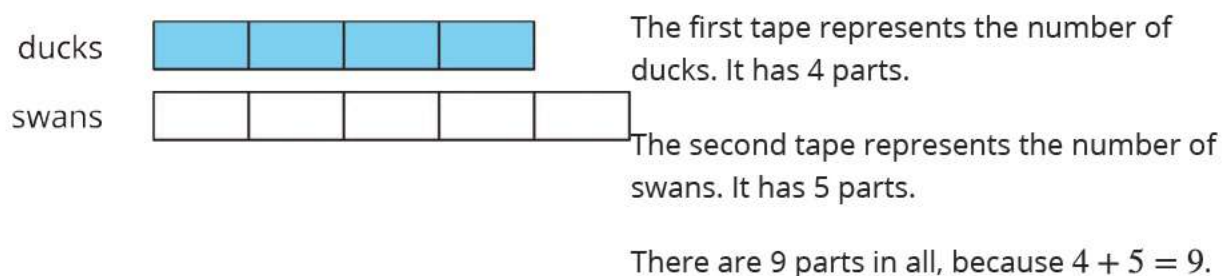
## **15.4: Invent Your Own Ratio Problem**

1. Invent another ratio problem that can be solved with a tape diagram and solve it. If you get stuck, consider looking back at the problems you solved in the earlier activity.
2. Create a visual display that includes:
  - The new problem that you wrote, without the solution.
  - Enough work space for someone to show a solution.
3. Trade your display with another group, and solve each other's problem. Include a tape diagram as part of your solution. Be prepared to share the solution with the class.
4. When the solution to the problem you invented is being shared by another group, check their answer for accuracy.

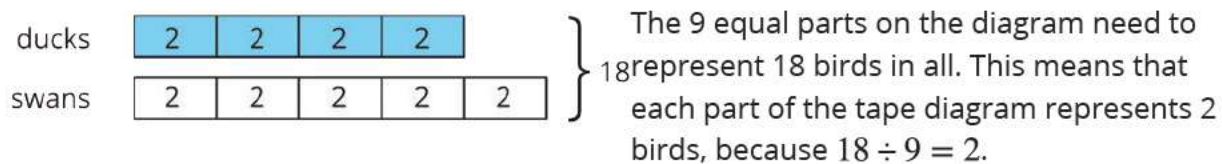
## Lesson 15 Summary

A **tape diagram** is another way to represent a ratio. All the parts of the diagram that are the same size have the same value.

For example, this tape diagram represents the ratio of ducks to swans in a pond, which is 4 : 5.



Suppose we know there are 18 of these birds in the pond, and we want to know how many are ducks.



There are 4 parts of the tape representing ducks, and  $4 \cdot 2 = 8$ , so there are 8 ducks in the pond.

## Unit 2 Lesson 15 Cumulative Practice Problems

1. Here is a tape diagram representing the ratio of red paint to yellow paint in a mixture of orange paint.

a. What is the ratio of yellow paint to red paint?

cups of red paint 

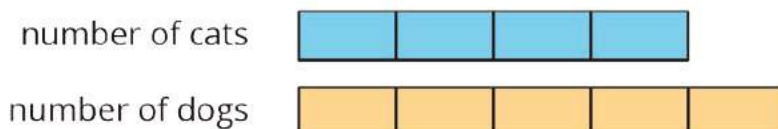
|   |   |   |
|---|---|---|
| 3 | 3 | 3 |
|---|---|---|

cups of yellow paint 

|   |   |
|---|---|
| 3 | 3 |
|---|---|

b. How many total cups of orange paint will this mixture yield?

2. At the kennel, the ratio of cats to dogs is 4 : 5. There are 27 animals in all. Here is a tape diagram representing this ratio.



- a. What is the value of each small rectangle?
- b. How many dogs are at the kennel?
- c. How many cats are at the kennel?
3. Last month, there were 4 sunny days for every rainy day. If there were 30 days in the month, how many days were rainy? Explain your reasoning. If you get stuck, consider using a tape diagram.

4. Noah entered a 100-mile bike race. He knows he can ride 32 miles in 160 minutes. At this rate, how long will it take him to finish the race? Use each table to find the answer. Next, explain which table you think works better in finding the answer.

Table A:

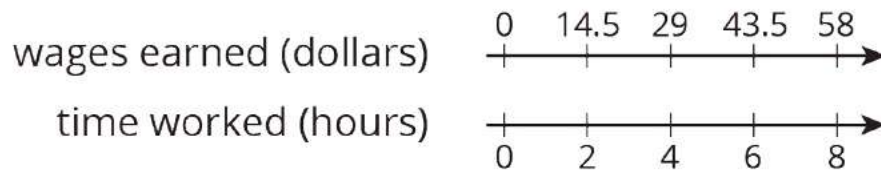
| distance<br>(miles) | elapsed time<br>(minutes) |
|---------------------|---------------------------|
| 32                  | 160                       |
| 1                   |                           |
| 100                 |                           |

Table B:

| distance<br>(miles) | elapsed time<br>(minutes) |
|---------------------|---------------------------|
| 32                  | 160                       |
| 96                  |                           |
| 4                   |                           |
| 100                 |                           |

(From Unit 2, Lesson 12.)

5. A cashier worked an 8-hour day, and earned \$58.00. The double number line shows the amount she earned for working different numbers of hours. For each question, explain your reasoning.



- a. How much does the cashier earn per hour?
- b. How much does the cashier earn if she works 3 hours?

(From Unit 2, Lesson 13.)

6. A grocery store sells bags of oranges in two different sizes.

- The 3-pound bags of oranges cost \$4.
- The 8-pound bags of oranges for \$9.

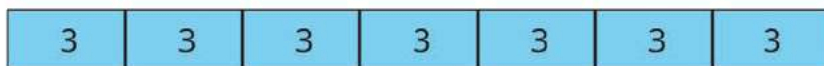
Which oranges cost less per pound? Explain or show your reasoning.

(From Unit 2, Lesson 10.)

# Lesson 16: Solving More Ratio Problems

## 16.1: You Tell the Story

Describe a situation with two quantities that this tape diagram could represent.

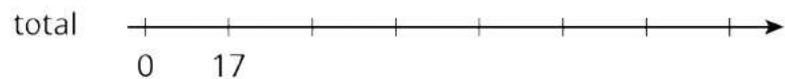


## 16.2: A Trip to the Aquarium

Consider the problem: A teacher is planning a class trip to the aquarium. The aquarium requires 2 chaperones for every 15 students. The teacher plans accordingly and orders a total of 85 tickets. How many tickets are for chaperones, and how many are for students?

1. Solve this problem in *one* of three ways:

Use a triple number line.

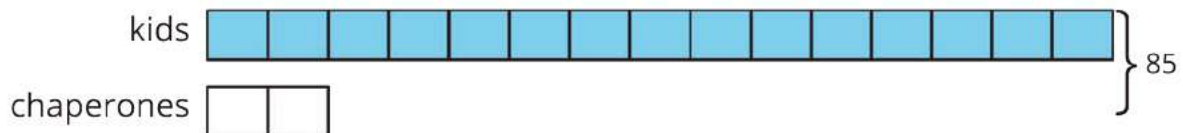


Use a table.

(Fill rows as needed.)

| kids | chaperones | total |
|------|------------|-------|
| 15   | 2          | 17    |
|      |            |       |
|      |            |       |
|      |            |       |
|      |            |       |

Use a tape diagram.



2. After your class discusses all three strategies, which do you prefer for this problem and why?

### Are you ready for more?

Use the digits 1 through 9 to create three equivalent ratios. Use each digit only one time.

$\square : \square$  is equivalent to  $\square\square : \square$  and  $\square\square : \square\square$

## 16.3: Salad Dressing and Moving Boxes

Solve each problem, and show your thinking. Organize it so it can be followed by others. If you get stuck, consider drawing a double number line, table, or tape diagram.

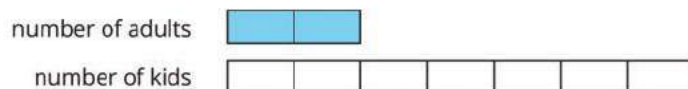
1. A recipe for salad dressing calls for 4 parts oil for every 3 parts vinegar. How much oil should you use to make a total of 28 teaspoons of dressing?
2. Andre and Han are moving boxes. Andre can move 4 boxes every half hour. Han can move 5 boxes every half hour. How long will it take Andre and Han to move all 72 boxes?

## Lesson 16 Summary

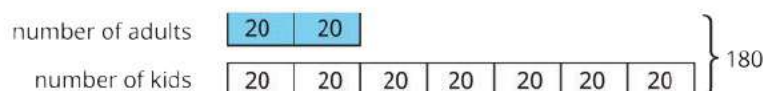
When solving a problem involving equivalent ratios, it is often helpful to use a diagram. Any diagram is fine as long as it correctly shows the mathematics and you can explain it.

Let's compare three different ways to solve the same problem: The ratio of adults to kids in a school is 2 : 7. If there is a total of 180 people, how many of them are adults?

- *Tape diagrams* are especially useful for this type of problem because both parts of the ratio have the same units ("number of people") and we can see the total number of parts.

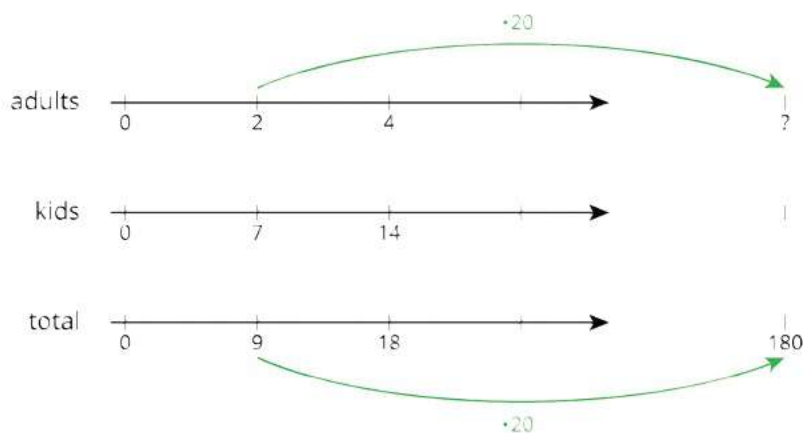


This tape diagram has 9 equal parts, and they need to represent 180 people total. That means each part represents  $180 \div 9$ , or 20 people.



Two parts of the tape diagram represent adults. There are 40 adults in the school because  $2 \cdot 20 = 40$ .

- *Double or triple number lines* are useful when we want to see how far apart the numbers are from one another. They are harder to use with very big or very small numbers, but they could support our reasoning.



- *Tables* are especially useful when the problem has very large or very small numbers.

| adults | kids | total |
|--------|------|-------|
| 2      | 7    | 9     |
| ?      |      | 180   |

•20 ↻      ↻ •20

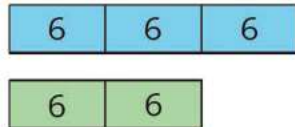
We ask ourselves, “9 times what is 180?” The answer is 20. Next, we multiply 2 by 20 to get the total number of adults in the school.

Another reason to make diagrams is to communicate our thinking to others. Here are some good habits when making diagrams:

- Label each part of the diagram with what it represents.
- Label important amounts.
- Make sure you read what the question is asking and answer it.
- Make sure you make the answer easy to find.
- Include units in your answer. For example, write “4 cups” instead of just “4.”
- Double check that your ratio language is correct and matches your diagram.

## Unit 2 Lesson 16 Cumulative Practice Problems

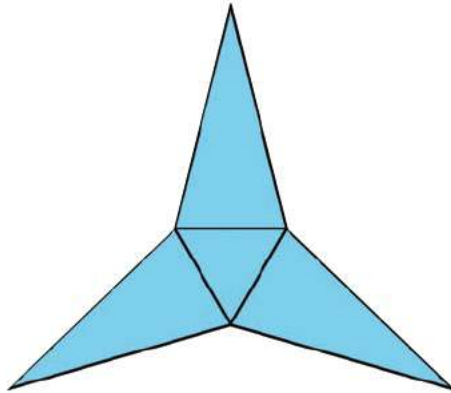
1. Describe a situation that could be represented with this tape diagram.



2. There are some nickels, dimes, and quarters in a large piggy bank. For every 2 nickels there are 3 dimes. For every 2 dimes there are 5 quarters. There are 500 coins total.
- a. How many nickels, dimes, and quarters are in the piggy bank? Explain your reasoning.
- b. How much are the coins in the piggy bank worth?
3. Two horses start a race at the same time. Horse A gallops at a steady rate of 32 feet per second and Horse B gallops at a steady rate of 28 feet per second. After 5 seconds, how much farther will Horse A have traveled? Explain or show your reasoning.
4. Andre paid \$13 for 3 books. Diego bought 12 books priced at the same rate. How much did Diego pay for the 12 books? Explain your reasoning.

(From Unit 2, Lesson 10.)

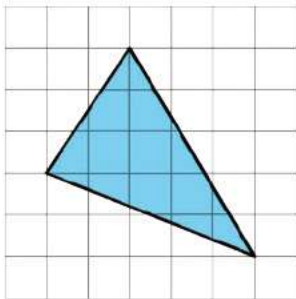
5. Which polyhedron can be assembled from this net?



- A. A triangular pyramid
- B. A trapezoidal prism
- C. A rectangular pyramid
- D. A triangular prism

(From Unit 1, Lesson 15.)

6. Find the area of the triangle. Show your reasoning. If you get stuck, consider drawing a rectangle around the triangle.

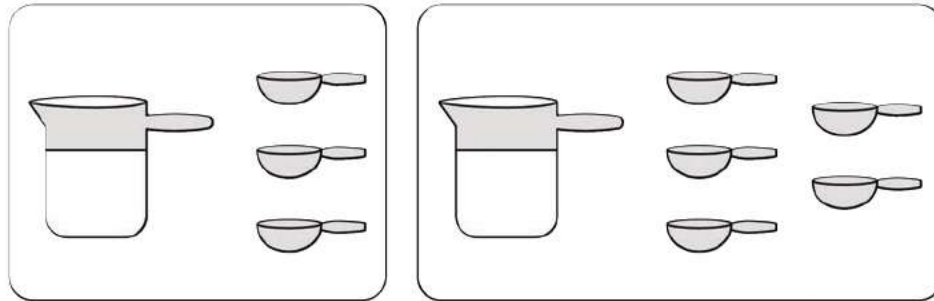


(From Unit 1, Lesson 10.)

## Lesson 17: A Fermi Problem

### 17.1: Fix It!

Andre likes a hot cocoa recipe with 1 cup of milk and 3 tablespoons of cocoa. He poured 1 cup of milk but accidentally added 5 tablespoons of cocoa.



1. How can you fix Andre's mistake and make his hot cocoa taste like the recipe?
2. Explain how you know your adjustment will make Andre's hot cocoa taste the same as the one in the recipe.

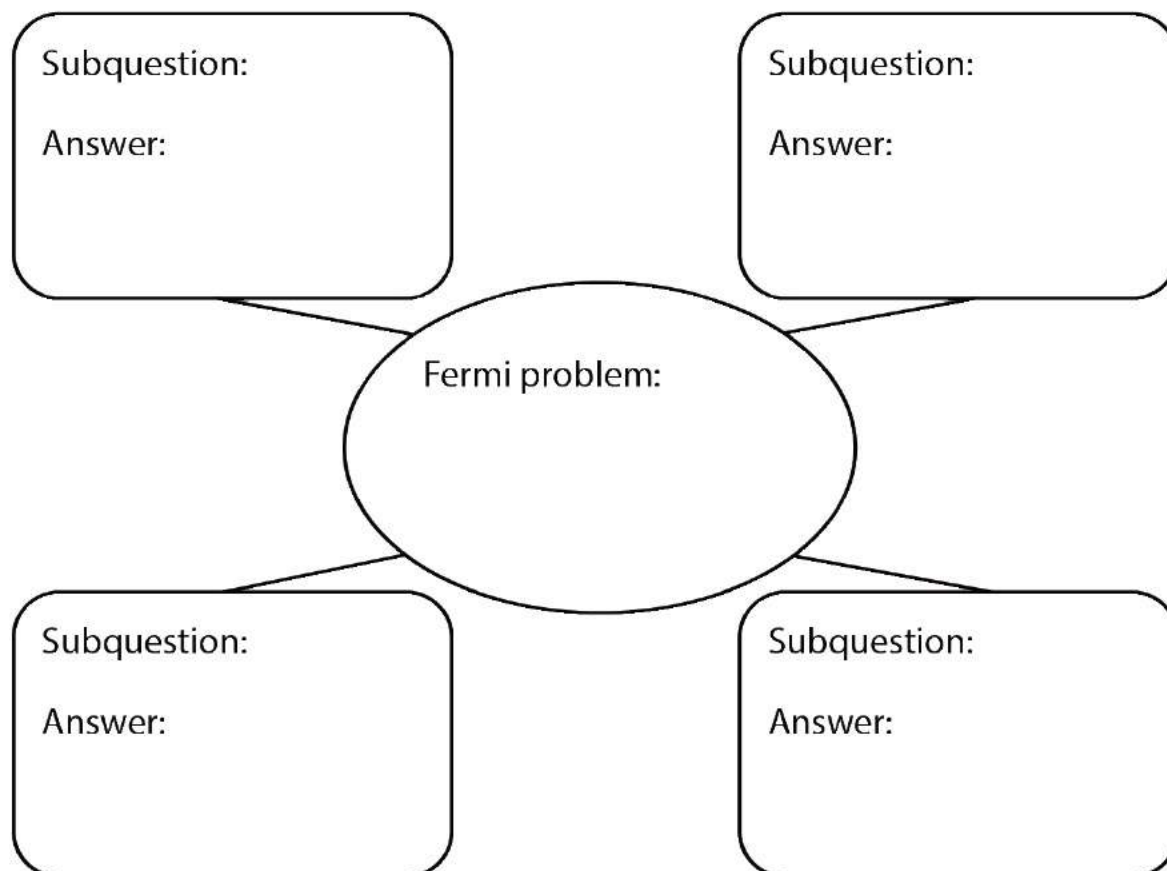
## 17.2: Who Was Fermi?

1. Record the Fermi question that your class will explore together.
2. Make an estimate of the answer. If making an estimate is too hard, consider writing down a number that would definitely be too low and another number that would definitely be too high.
3. What are some smaller sub-questions we would need to figure out to reasonably answer our bigger question?
4. Think about how the smaller questions above should be organized to answer the big question. Label each smaller question with a number to show the order in which they should be answered. If you notice a gap in the set of sub-questions (i.e., there is an unlisted question that would need to be answered before the next one could be tackled), write another question to fill the gap.

### 17.3: Researching Your Own Fermi Problem

1. Brainstorm at least five Fermi problems that you want to research and solve. If you get stuck, consider starting with "How much would it cost to . . .?" or "How long would it take to . . .?"
2. Pause here so your teacher can review your questions and approve one of them.

3. Use the graphic organizer to break your problem down into sub-questions.



4. Find the information you need to get closer to answering your question. Measure, make estimates, and perform any necessary calculations. If you get stuck, consider using tables or double number line diagrams.

5. Create a visual display that includes your Fermi problem and your solution. Organize your thinking so it can be followed by others.

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