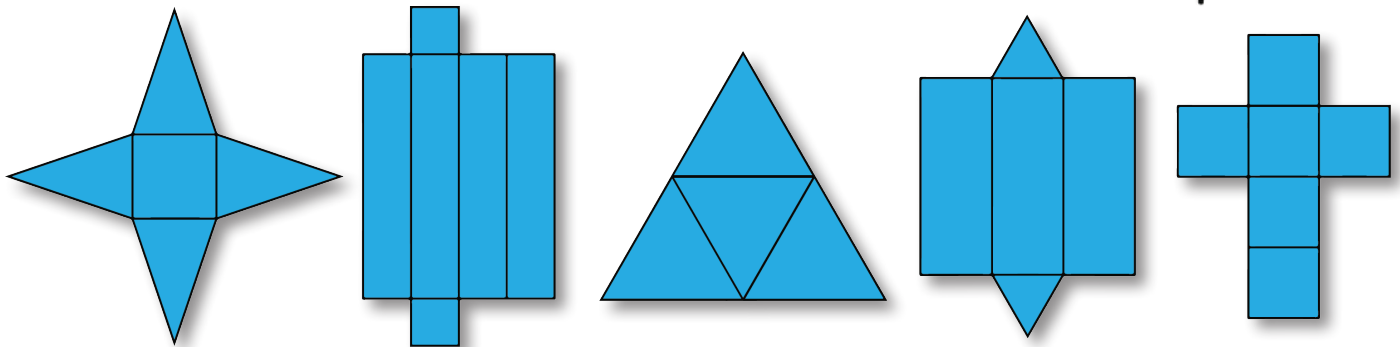


Designing a Tent

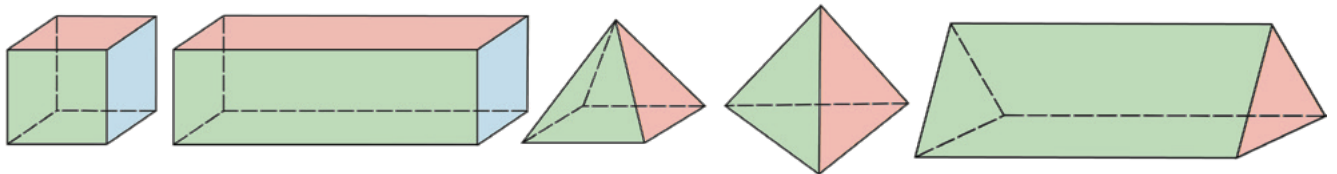
Area and Surface Area



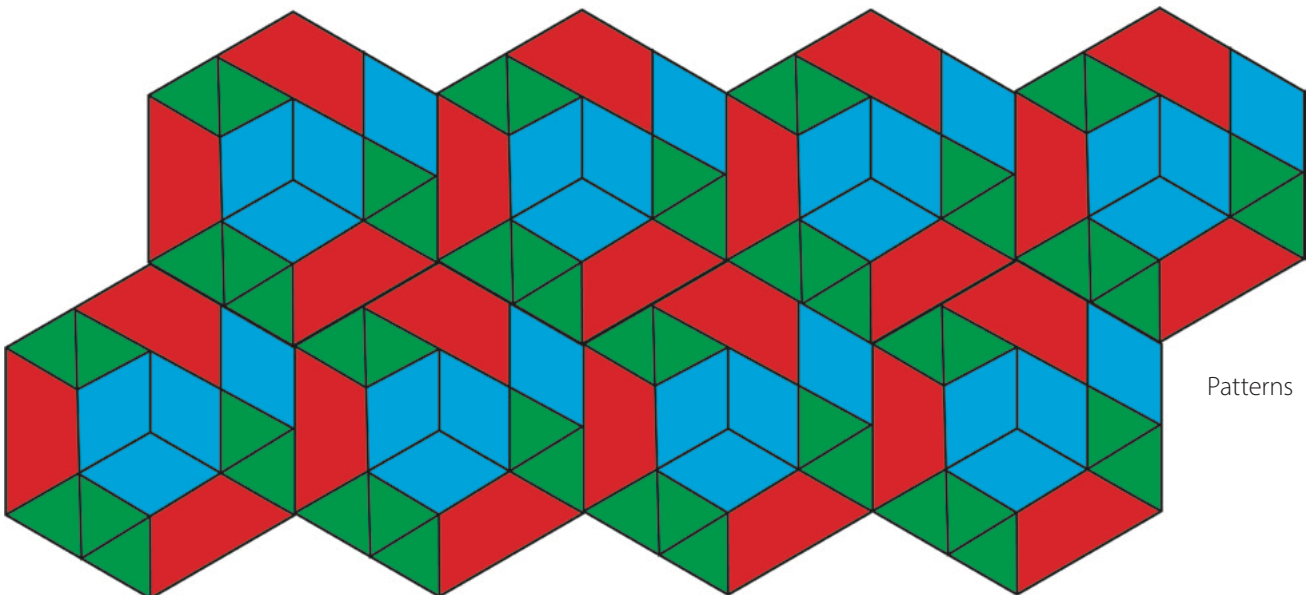
Teacher Guide



Nets and Surface Area



Prisms



Patterns

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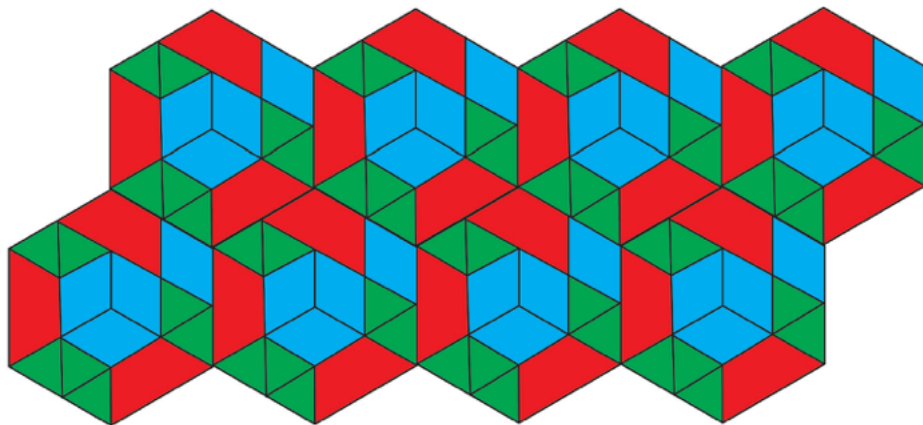
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Area and Surface Area

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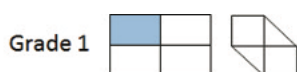


Area and Surface Area
Teacher Guide
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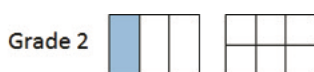
Area and Surface Area

Unit Narrative

Work with area in grade 6 draws on earlier work with geometry and geometric measurement. Students began to learn about two- and three-dimensional shapes in kindergarten, and continued this work in grades 1 and 2, composing, decomposing, and identifying shapes. Students' work with geometric measurement began with length and continued with area. Students learned to "structure two-dimensional space," that is, to see a rectangle with whole-number side lengths as composed of an array of unit squares or composed of iterated rows or iterated columns of unit squares. In grade 3, students distinguished between perimeter and area. They connected rectangle area with multiplication, understanding why (for whole-number side lengths) multiplying the side lengths of a rectangle yields the number of unit squares that tile the rectangle. They used area diagrams to represent instances of the distributive property. In grade 4, students applied area and perimeter formulas for rectangles to solve real-world and mathematical problems, and learned to use protractors. In grade 5, students extended the formula for the area of rectangles to rectangles with fractional side lengths.



Partition rectangles and circles into halves and quarters. Compose figures in the plane.



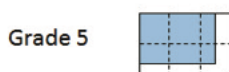
Partition rectangles and circles into thirds. Partition rectangles into squares and count them.



Find whole-number areas. Multiply to find areas of rectangles. Distinguish between perimeter and area.



Apply area and perimeter formulas for rectangles in real-world contexts.



Tile to find areas of rectangles with fractional side-lengths. Multiply to find these areas.



Understand the area of a triangle is half of the product of one of its side-lengths and its corresponding height. Find areas of polygons.

In grade 6, students extend their reasoning about area to include shapes that are not composed of rectangles. Doing this draws on abilities developed in earlier grades to compose and decompose shapes, for example, to see a rectangle as composed of two congruent right triangles. Through activities designed and sequenced to allow students to make sense of problems and persevere in solving them (MP1), students build on these abilities and their knowledge of areas of rectangles to find the areas of polygons by decomposing and rearranging them to make figures whose areas they can determine (MP7). They learn strategies for finding areas of parallelograms and triangles, and use regularity in repeated reasoning (MP8) to develop formulas for these areas, using geometric properties to justify the correctness of these formulas. They use these formulas to solve problems. They understand that any polygon can be decomposed into triangles, and use this knowledge to find areas of polygons. Students find the surface areas of polyhedra with triangular and rectangular surfaces. They study, assemble, and draw nets for polyhedra and use nets to determine surface

areas. Throughout, they discuss their mathematical ideas and respond to the ideas of others (MP3, MP6).

Because grade 6 students will be writing algebraic expressions and equations involving the letter x and x is easily confused with \times , these materials use the “dot” notation, e.g., $2 \cdot 3$, for multiplication instead of the “cross” notation, e.g., 2×3 . The dot notation will be new for many students, and they will need explicit guidance in using it.

Many of the lessons in this unit ask students to work on geometric figures that are not set in a real-world context. This design choice respects the significant intellectual work of reasoning about area. Tasks set in real-world contexts that involve areas of polygons are often contrived and hinder rather than help understanding. Moreover, mathematical contexts are legitimate contexts that are worthy of study. Students do have an opportunity at the end of the unit to tackle a real-world application (MP2, MP4).

In grade 6, students are likely to need physical tools in order to check that one figure is an identical copy of another where “identical copy” is defined as follows:

One figure is an *identical copy* of another if one can be placed on top of the other so that they match up exactly.

In grade 8, students will understand “identical copy of” as “congruent to” and understand congruence in terms of rigid motions, that is, motions such as reflection, rotation, and translation. In grade 6, students do not have any way to check for congruence except by inspection, but it is not practical to cut out and stack every pair of figures one sees. Tracing paper is an excellent tool for verifying that figures “match up exactly,” and students should have access to this and other tools at all times in this unit. Thus, each lesson plan suggests that each student should have access to a *geometry toolkit*, which contains tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles. Providing students with these toolkits gives opportunities for students to develop abilities to select appropriate tools and use them strategically to solve problems (MP5). Note that even students in a digitally enhanced classroom should have access to such tools; apps and simulations should be considered additions to their toolkits, not replacements for physical tools. In this grade, all figures are drawn and labeled so that figures that look congruent actually are congruent; in later grades when students have the tools to reason about geometric figures more precisely, they will need to learn that visual inspection is not sufficient for determining congruence. Also note that all arguments laid out in this unit can (and should) be made more precise in later grades, as students’ geometric understanding deepens.

Progression of Disciplinary Language

In this unit, teachers can anticipate students using language for mathematical purposes such as comparing, explaining, and describing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

Compare

- geometric patterns and shapes (Lesson 1)
- strategies for finding areas of shapes (Lesson 3) and polygons (Lesson 11)
- the characteristics of prisms and pyramids (Lesson 13)
- the measures and units of 1-, 2-, and 3-dimensional attributes (Lesson 16)
- representations of area and volume (Lesson 17)

Explain

- how to find areas by composing (Lesson 3)
- strategies used to find areas of parallelograms (Lesson 4) and triangles (Lesson 8)
- how to determine the area of a triangle using its base and height (Lesson 9)
- strategies to find surface areas of polyhedra (Lesson 14)

Describe

- observations about decomposition of parallelograms (Lesson 7)
- information needed to find the surface area of rectangular prisms (Lesson 12)
- the features of polyhedra and their nets (Lesson 13)
- the features of polyhedra (Lesson 15)
- relationships among features of a tent and the amount of fabric needed for the tent (Lesson 19)

In addition, students are expected to justify claims about the base, height, or area of shapes, generalize about the features of parallelograms and polygons, interpret relevant information for finding the surface area of rectangular prisms, and represent the measures and units of 2- and 3-dimensional figures. Over the course of the unit, teachers can support students' mathematical understandings by amplifying (not simplifying) language used for all of these purposes as students demonstrate and develop ideas.

The table shows lessons where new terminology is first introduced, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms from the glossary appear bolded. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

Unit Learning Targets

Area and Surface Area Lesson

1: Tiling the Plane

- I can explain the meaning of area.

Lesson 2: Finding Area by Decomposing and Rearranging

- I can explain how to find the area of a figure that is composed of other shapes.
- I know how to find the area of a figure by decomposing it and rearranging the parts.
- I know what it means for two figures to have the same area.

Lesson 3: Reasoning to Find Area

- I can use different reasoning strategies to find the area of shapes.

Lesson 4: Parallelograms

- I can use reasoning strategies and what I know about the area of a rectangle to find the area of a parallelogram.
- I know how to describe the features of a parallelogram using mathematical vocabulary.

Lesson 5: Bases and Heights of Parallelograms

- I can identify pairs of base and height of a parallelogram.
- I can write and explain the formula for the area of a parallelogram.
- I know what the terms "base" and "height" refer to in a parallelogram.

Lesson 6: Area of Parallelograms

- I can use the area formula to find the area of any parallelogram.

Lesson 7: From Parallelograms to Triangles

- I can explain the special relationship between a pair of identical triangles and a parallelogram.

Lesson 8: Area of Triangles

- I can use what I know about parallelograms to reason about the area of triangles.

Lesson 9: Formula for the Area of a Triangle

- I can use the area formula to find the area of any triangle.
- I can write and explain the formula for the area of a triangle.
- I know what the terms “base” and “height” refer to in a triangle.

Lesson 10: Bases and Heights of Triangles

- I can identify pairs of base and corresponding height of any triangle.
- When given information about a base of a triangle, I can identify and draw a corresponding height.

Lesson 11: Polygons

- I can describe the characteristics of a polygon using mathematical vocabulary.
- I can reason about the area of any polygon by decomposing and rearranging it, and by using what I know about rectangles and triangles.

Lesson 12: What is Surface Area?

- I know what the surface area of a three-dimensional object means.

Lesson 13: Polyhedra

- I can describe the features of a polyhedron using mathematical vocabulary.
- I can explain the difference between prisms and pyramids.
- I understand the relationship between a polyhedron and its net.

Lesson 14: Nets and Surface Area

- I can match polyhedra to their nets and explain how I know.
- When given a net of a prism or a pyramid, I can calculate its surface area.

Lesson 15: More Nets, More Surface Area

- I can calculate the surface area of prisms and pyramids.
- I can draw the nets of prisms and pyramids.

Lesson 16: Distinguishing Between Surface Area and Volume

- I can explain how it is possible for two polyhedra to have the same surface area but different volumes, or to have different surface areas but the same volume.
- I know how one-, two-, and three-dimensional measurements and units are different.

Lesson 17: Squares and Cubes

- I can write and explain the formula for the volume of a cube, including the meaning of the exponent.
- When I know the edge length of a cube, I can find the volume and express it using appropriate units.

Lesson 18: Surface Area of a Cube

- I can write and explain the formula for the surface area of a cube.
- When I know the edge length of a cube, I can find its surface area and express it using appropriate units.

Lesson 19: Designing a Tent

- I can apply what I know about the area of polygons to find the surface area of three-dimensional objects.
- I can use surface area to reason about real-world objects.

lesson	new terminology	
	receptive	productive
6.1.1	<p>area region plane gap</p>	
6.1.2	<p>compose decompose rearrange two-dimensional</p>	
6.1.3	<p>shaded strategy</p>	
6.1.4	<p>parallelogram opposite (sides or angles)</p>	<p>quadrilateral</p>
6.1.5	<p>base (of a parallelogram or triangle) height corresponding expression represent</p>	
6.1.6	<p>horizontal vertical</p>	
6.1.7	<p>identical</p>	<p>parallelogram</p>
6.1.8		<p>base (of a parallelogram or triangle) height compose decompose rearrange</p>
6.1.9	<p>opposite vertex</p>	
6.1.10	<p>vertex edge</p>	
6.1.11	<p>polygon</p>	<p>horizontal vertical</p>

lesson	new terminology	
	receptive	productive
6.1.12	face surface area	area region
6.1.13	polyhedron net prism pyramid base (of a prism or pyramid) three-dimensional	polygon vertex edge face
6.1.15		prism pyramid
6.1.16	volume appropriate quantity	two-dimensional three-dimensional
6.1.17	squared cubed exponent edge length	
6.1.18	value (of an expression)	squared cubed net
6.1.19	estimate description	surface area volume

Required Materials

Copies of Instructional master

Demonstration nets with and without flaps

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When

compasses are required in grades 6-8 they are listed as a separate Required Material.

Glue or glue sticks

Nets of polyhedra

Pre-assembled or commercially produced polyhedra

Pre-assembled or commercially produced tangrams

Pre-printed slips, cut from copies of the Instructional master

Rulers

Scissors

Snap cubes

Sticky notes

Tape

Section: Reasoning to Find Area

Lesson 1: Tiling the Plane

Goals

- Compare (orally) areas of the shapes that make up a geometric pattern.
- Comprehend that the word “area” (orally and in writing) refers to how much of the plane a shape covers.

Learning Targets

- I can explain the meaning of area.

Lesson Narrative

Students start the first lesson of the school year by recalling what they know about **area** (note that students studied the areas of rectangles with whole-number side lengths in grade 3 and with fractional side lengths in grade 5). The mathematics they explore is not complicated, so it offers a low threshold for entry. The lesson does, however, uncover two important ideas:

- If two figures can be placed one on top of the other so that they match up exactly, then they have the same area.
- The area of a **region** does not change when the region is decomposed and rearranged.

At the end of this lesson, students are asked to write their best definition of area. It is important to let them formulate their definition in their own words. For English learners, it is especially important that they be encouraged to use their own words and also to use words of their peers. In the next lesson, students will revisit the definition of area as the number of square units that cover a region without gaps or overlaps.

As the first set of problems in a problem-based curriculum, students will also begin their year-long work on making sense of problems and persevering in solving them (MP1). This opening lesson leaves space for teachers to begin setting classroom routines and their expectations for mathematical discourse (MP3).

In all of the lessons in this unit, students should have access to their geometry toolkits, which should contain tracing paper, graph paper, colored pencils, scissors, and an index card. Students may not need all (or even any) of these tools to solve a particular problem. However, to make strategic choices about when to use which tools (MP5), students need to have opportunities to make those choices. Apps and simulations should supplement rather than replace physical tools.

Notes on terminology. In these materials, when we talk about a figure such as a rectangle, triangle, or circle, we usually mean the boundary of the figure (e.g., the sides of the rectangle), not including the region inside. However, we also use shorthand language such as “the area of a rectangle” to mean

the “the area of the region inside the rectangle.” The term shape could refer to a figure with or without its interior. Although the terms figure, region, and shape are used without being defined precisely for students, help students understand that sometimes our focus is on the boundary (which in this unit will always be composed of black line segments), and sometimes it is on the region inside (which in this unit will always be shown in color and referred to as “the shaded region”).

Alignments

Building On

- 3.G.A: Reason with shapes and their attributes.

Building Towards

- 6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display
- Think Pair Share
- Which One Doesn't Belong?

Required Materials

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Required Preparation

Assemble geometry toolkits. It would be best if students have access to these toolkits at all times throughout the unit. Toolkits include tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

Student Learning Goals

Let's look at tiling patterns and think about area.

1.1 Which One Doesn't Belong: Tilings

Warm Up: 10 minutes

This warm-up prompts students to compare four geometric patterns, explain their reasoning, and hold mathematical conversations. It allows you to hear how students use terminology in describing geometric characteristics.

Observing patterns gives every student an entry point. Each figure has at least one reason it does not belong. The patterns also urge students to think about shapes that cover the plane without gaps and overlaps, which supports future conversations about the meaning of area.

Before students begin, consider establishing a small, discreet hand signal that students can display to indicate they have an answer they can support with reasoning. This signal could be a thumbs-up, a certain number of fingers that tells the number of responses they have, or another subtle signal. This is a quick way to see if students have had enough time to think about the problem. It also keeps students from being distracted or rushed by hands being raised around the class.

Anticipate students to describe the patterns in terms of:

- Colors (blue, green, yellow, white, or no color)
- Size of shapes or other measurements
- Geometric shapes (polygons, squares, pentagons, hexagons, etc.)
- Relationships of shapes (whether each side of the polygons meets the side of another polygon, what polygon is attached to each side, whether there is a gap between polygons, etc.)

Building On

- 3.G.A

Instructional Routines

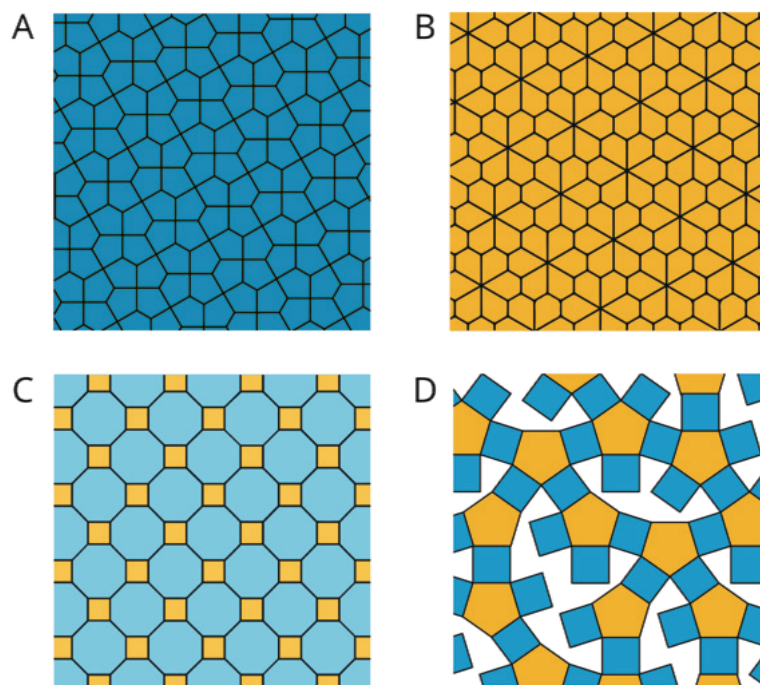
- Which One Doesn't Belong?

Launch

Arrange students in groups of 2–4. Display the four patterns for all to see. Give students 1 minute of quiet think time and ask them to indicate when they have noticed one pattern that does not belong and can explain why. Encourage them to think of more than one possibility. When the minute is up, give students 2 minutes to share their response with their group, and then together find at least one reason, if possible, that each pattern doesn't belong.

Student Task Statement

Which pattern doesn't belong?



Student Response

Answers vary. Sample responses:

- A: It doesn't have any yellow. Groups of four pentagons make hexagonal shapes that interlock without gaps.
- B: It doesn't have any blue. Groups of six pentagons make flower-like shapes that interlock without gaps.
- C: It doesn't have any pentagons. It has octagons and squares. The polygons that make up the patterns are very different in size.
- D: It has gaps between the shapes. Not all of the colored polygons meet another colored polygon on all sides. It has white (or non-filled) shapes that are more complex than other colored shapes. It is the only one where all the polygons have the same side length.

Activity Synthesis

After students shared their observations in groups, invite each group to share one reason why a particular figure might not belong. Record and display the responses for all to see. After each response, poll the rest of the class to see if others made the same observation.

Since there is no single correct answer to the question of which pattern does not belong, attend to students' explanations, and make sure the reasons given are correct. Prompt students to explain the meaning of any terminology they use (names of polygons or angles, parts of polygons, area, etc.) and to substantiate their claims. For example, a student may claim that Pattern D does not

belong because its polygons all have the same side length. Ask how they know that is the case, and whether that is true for the white (or non-filled) polygon.

Explain to students that covering a two-dimensional region with copies of the same shape or shapes such that there are no gaps or overlaps is called "tiling" the plane. Patterns A, B, and C are examples of tiling. Tell students that we explore more tilings in upcoming activities.

1.2 More Red, Green, or Blue?

25 minutes (there is a digital version of this activity)

This activity asks students to compare the amounts of the plane covered by two tiling patterns, with the aim of supporting two big ideas of the unit:

- If two figures can be placed one on top of the other so that they match up exactly, then they have the same area.
- A region can be decomposed and rearranged without changing its area.

Students are likely to notice that in each pattern:

- The same three polygons (triangles, rhombuses, and trapezoids) are used as tiles.
- The entire tiling pattern is composed of these hexagons.
- The shapes are arranged without gaps and overlaps, but their arrangements are different.
- A certain set of smaller tiles form a larger hexagon. Each hexagon has 3 trapezoids, 4 rhombuses, and 7 triangles.

Expect some students to begin their comparison by counting each shape, either within a hexagon or the entire pattern. To effectively compare how much of the plane is covered by each shape, however, they need to be aware of the relationships *between* the shapes. For example, two green triangles can be placed on top of a blue rhombus so that they match up exactly, which tells us that two green triangles cover the same amount of the plane as one blue rhombus. Monitor for such an awareness. (It is not necessary for students to use the word "area" in their explanations. At this point, phrases such as "they match up" or "two triangles make one rhombus" suffice.)

If students are not sure how to approach the questions, encourage them to think about whether any tools in their geometry toolkits could help. (For example, they could use tracing paper to trace entire patterns or certain shapes to make comparison, or use a straightedge to extend lines within the pattern. Some students may be inclined to cut out and compare the shapes.) Pattern tiles, if available, can be offered as well.

During the partner discussion, monitor for groups who discuss the following ideas so that they can share later, in this sequence:

- Relationships between two shapes: E.g., 2 triangles make a rhombus, and 3 triangles make a trapezoid.

- Relative overall quantities: E.g., there are 64 green triangles, 32 blue rhombuses (which have the same area as 64 triangles), and 24 red trapezoids (which have the same area as 72 triangles), so there is more red.
- Relative quantities in a hexagon: E.g., in each hexagon there are 7 green triangles, 4 rhombuses (which have the same area as 8 triangles), and 3 trapezoids (which have the same area as 9 triangles).

Classrooms using the digital activity have the option for students to use an applet that allows for the pattern to be isolated and also framed. This might assist students in focusing on how many of each shape comprise the pattern.

Building Towards

- 6.G.A.1

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display
- Think Pair Share

Launch

Arrange students in groups of 2. Ask one partner to analyze Pattern A and the other to analyze Pattern B. Tell students that their job is to compare the amount of the plane covered by each shape in the pattern.

Before students begin, introduce students to the geometry toolkits, and explain that they can use the toolkits for help, if needed. Give students 7–8 minutes of quiet think time. Then, ask students to share their responses with their partners and follow with a whole-class discussion.

Support for Students with Disabilities

Representation: Access for Perception. Provide physical objects such as pattern blocks for students to view or manipulate. Before students begin, introduce students to the geometry toolkits and pattern blocks, and ask students to brainstorm how they might use these tools to help them analyze each pattern. Monitor for students who make comparisons based on the number of shapes instead of accounting for the area covered by each shape. Use pattern blocks to clarify.
Supports accessibility for: Visual-spatial processing; Conceptual processing

Support for English Language Learners

Conversing, Speaking, Listening: Math Language Routine 2 Collect and Display.

This is the first time Math Language Routine 2 is suggested as a support in this course. In this routine, the teacher circulates and listens to students talk while writing down the words, phrases, or drawings students produce. The language collected is displayed visually for the whole class to use throughout the lesson and unit. Generally, the display contains different examples of students using features of the disciplinary language functions, such as interpreting, justifying, or comparing. The purpose of this routine is to capture a variety of students' words and phrases in a display that students can refer to, build on, or make connections with during future discussions, and to increase students' awareness of language used in mathematics conversations.

Design Principle(s): Support sense-making

How It Happens:

1. After assigning students to work on Pattern A or B, circulate around the room and collect examples of language students are using to compare areas of polygons. Focus on capturing a variety of language describing the relationship between the size of two shapes, comparing overall quantities of shapes to equivalent areas of other shapes, and comparing relevant quantities in a hexagon. Aim to capture a range of student language that includes formal, precise, complete ideas and informal, ambiguous, and partial ideas. Plan to publicly update and revise this display throughout the lesson and unit. If pairs are stuck, consider using these questions to elicit conversation: "How many green triangles, blue rhombuses, and red trapezoids are in each pattern?", "Three triangles is equivalent to how many trapezoids?", and "Which shapes make up a hexagon?"

If using the applet, have pairs use the applet together. Check that students focus on how many of each shape comprise the pattern by hiding, moving, and turning the shapes.

2. Create a display that includes visual representations of the words and phrases collected. Group language about Pattern A on one side of the display and language about Pattern B on the other side.
3. Close this conversation by posting the display in the front of the classroom for students to reference for the remainder of the lesson, and then have students move on to discussing other aspects of the activity. Continue to publicly update and revise the display throughout the lesson and unit.

Anticipated Misconceptions

Students may say more of the area is covered by the color they see the most in each image, saying, for example, "It just looks like there is more red." Ask these students if there is a way to prove their observations.

Students may only count the number of green triangles, red trapezoids, and blue rhombuses but not account for the area covered by each shape. If they suggest that the shape with the largest number of pieces covers the most amount of the plane, ask them to test their hypothesis. For example, ask, "Do 2 triangles cover more of the plane than 1 trapezoid?"

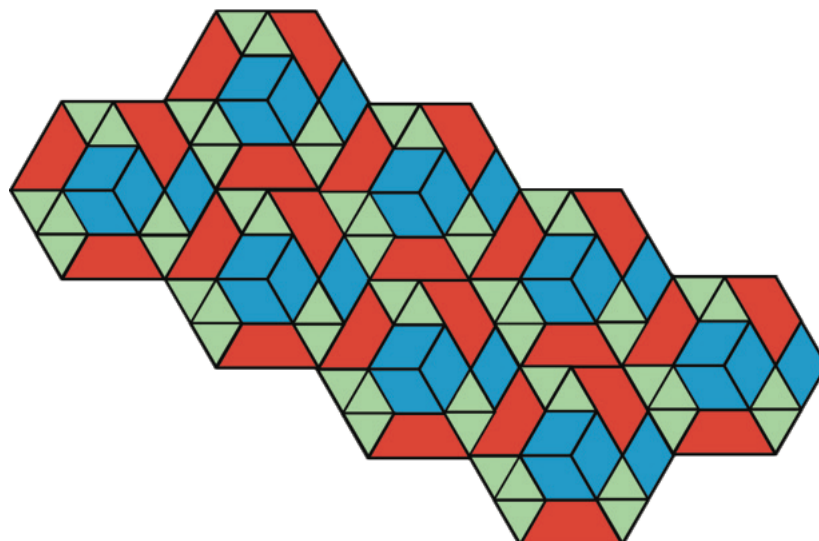
Students may not recall the terms trapezoid, rhombus, and triangle. Consider reviewing the terms, although they do not need to know the formal definitions to work on the task.

Student Task Statement

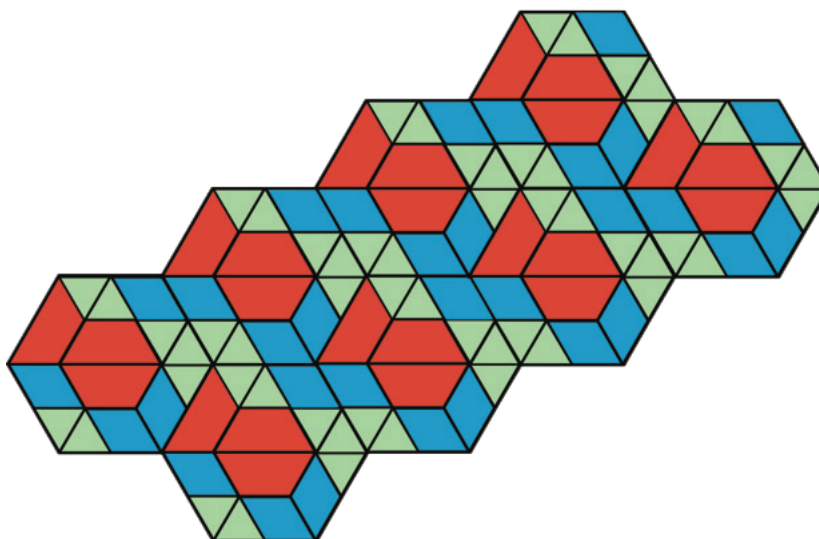
Your teacher will assign you to look at Pattern A or Pattern B.

In your pattern, which shape covers more of the plane: blue rhombuses, red trapezoids, or green triangles? Explain how you know.

Pattern A



Pattern B





Student Response

In both Patterns A and B, more of the plane is covered by red trapezoids than green triangles or blue rhombuses. Possible explanations:

- Patterns A and B are each made of 56 green triangles, 32 blue rhombuses, 24 red trapezoids.
 - One red trapezoid covers the same amount of the plane as 3 green triangles, so 24 red trapezoids cover the same amount of the plane as 72 green triangles, which are more than the 56 green triangles.
 - Each blue rhombus covers the same amount of the plane as 2 green triangles, so the 32 rhombuses cover the same amount of the plane as 64 green triangles, which are also more than the 56 green triangles.
- Each pattern is composed up of 8 hexagons. In each hexagon there are 3 red trapezoids, 4 blue rhombuses, and 7 green triangles.
 - Two red trapezoids can be arranged into a small hexagon. Three rhombuses can also be arranged into the same small hexagon. This means 2 trapezoids cover the same amount of the plane as 3 rhombuses.
 - Each large hexagon has 3 red trapezoids and 4 blue rhombuses. Since 2 trapezoids are equal to 3 rhombuses, we can just compare 1 trapezoid and 1 rhombus. We can see that 1 red trapezoid covers more of the plane than 1 rhombus.
 - Each large hexagon has 3 red trapezoids and 7 green triangles. One trapezoid covers the same amount of the plane as 3 triangles, so 3 trapezoids cover the same amount of the plane as 9 triangles, which are more than 7 green triangles.



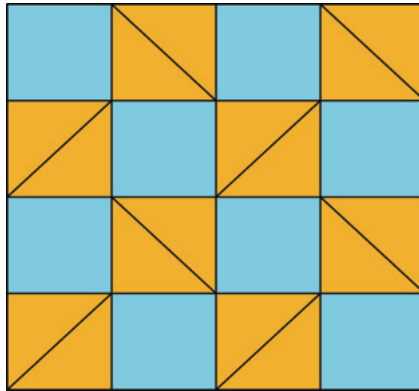
Are You Ready for More?

On graph paper, create a tiling pattern so that:

- The pattern has at least two different shapes.
- The same amount of the plane is covered by each type of shape.

Student Response

There are an infinite number of possibilities. Here is one:



Activity Synthesis

Select previously identified students or groups to share their answers and explanations. Sequence the explanations in the order listed in the Activity Narrative. To clarify the idea of comparing shapes by placing them on top of one another and seeing if or how they match, consider demonstrating using the digital applet.

Then, make it explicit that when we ask, “Which type of shape covers more of the plane?” we are asking them to compare the **areas** covered by the different types of shapes. To recast the comparisons of the shapes in terms of area, ask questions such as:

- “How does the area of the trapezoid compare to the area of the triangle?” (The area of the trapezoid is three times the area of the triangle.)
- “How does the area of the rhombus compare to the area of the triangle?” (The area of the rhombus is twice the area of the triangle.)
- “Is it possible to compare the area of the rhombuses in Pattern A and the area of the triangles in Pattern B? How?” (Yes, we can count the number of rhombuses in A and the number of triangles in B. Because 2 triangles have the same area as 1 rhombus, we divide the number of triangles by 2 to compare them.)

Lesson Synthesis

In this lesson, we have started to reason about what it means for two shapes to have the same **area**. We started doing mathematics and thinking about tools that can help us. Ask students:

- “What are some of the tools in the geometry toolkit and what are they used for?”
- “Draw two shapes that you know do not have the same area. How can you tell?”

Tell students that we will continue to think about area, to do and talk about mathematics, and to learn to use tools strategically.

1.3 What is Area?

Cool Down: 5 minutes

The purpose of this cool-down is to check how students are thinking about area after engaging in the activities. While the task prompts students to reflect on the work in this lesson, ideas about area from students' prior work in grades 3–5 may also emerge. Knowing the range of student thinking will help to inform the next day's lesson.

Student Task Statement

Think about your work today, and write your best definition of area.

Student Response

Answers vary. Sample responses:

- The amount of space inside a two-dimensional shape
- The measurement of the inside of a polygon
- The number of square units inside a shape

Student Lesson Summary

In this lesson, we learned about *tiling* the plane, which means covering a two-dimensional region with copies of the same shape or shapes such that there are no gaps or overlaps.

Then, we compared tiling patterns and the shapes in them. In thinking about which patterns and shapes cover more of the plane, we have started to reason about **area**.

We will continue this work, and to learn how to use mathematical tools strategically to help us do mathematics.

Glossary

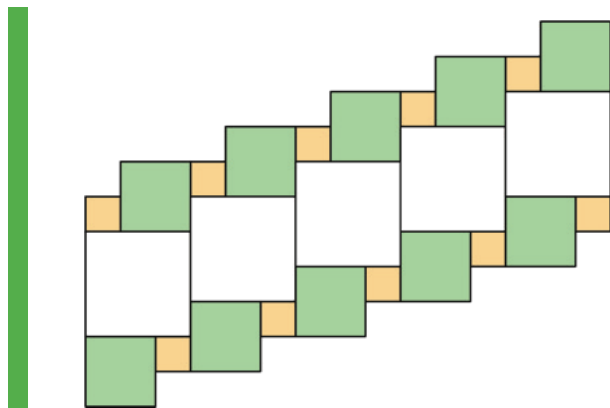
- area
- region

Lesson 1 Practice Problems

Problem 1

Statement

Which square—large, medium, or small—covers more of the plane? Explain your reasoning.



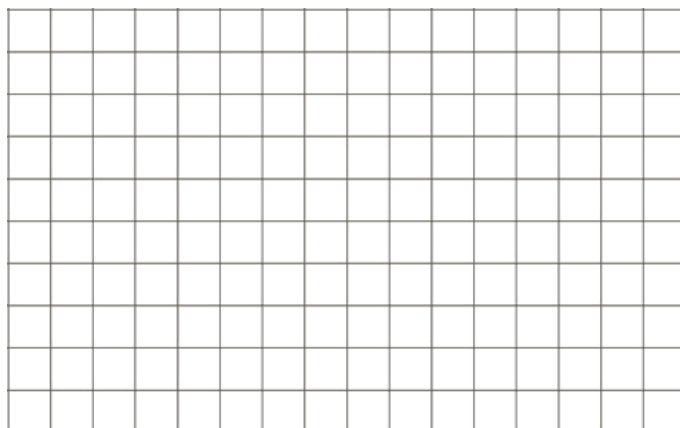
Solution

The large square covers more of the plane. Reasoning varies. Sample reasoning: A large square can fit exactly 9 small squares. A medium square can fit exactly 4 small squares. There are 5 large squares, which cover the same amount of the plane as 45 small squares. There are 10 medium squares, which cover the same amount of the plane as 40 small squares. There are only 10 small squares.

Problem 2

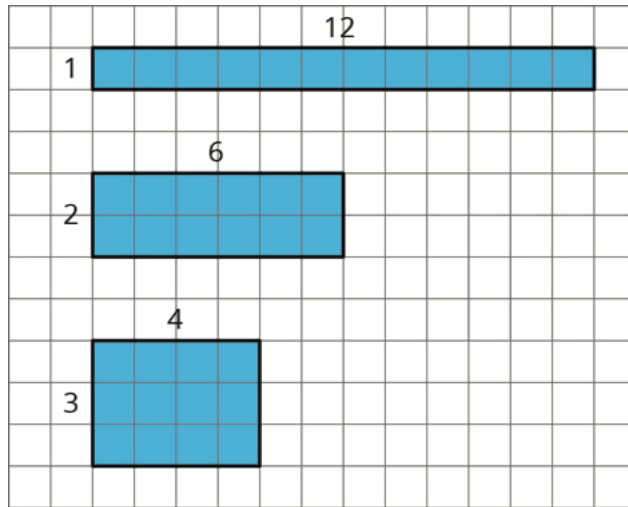
Statement

Draw three different quadrilaterals, each with an area of 12 square units.



Solution

Answers vary. Sample response:



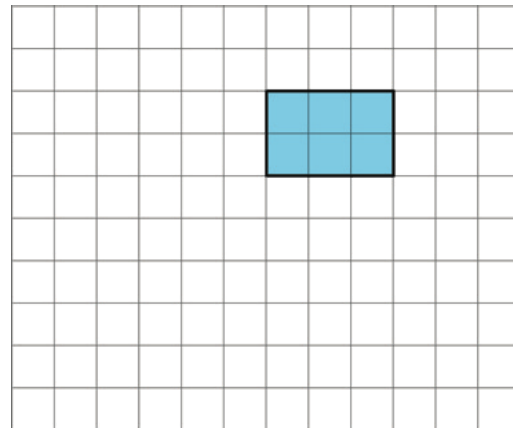
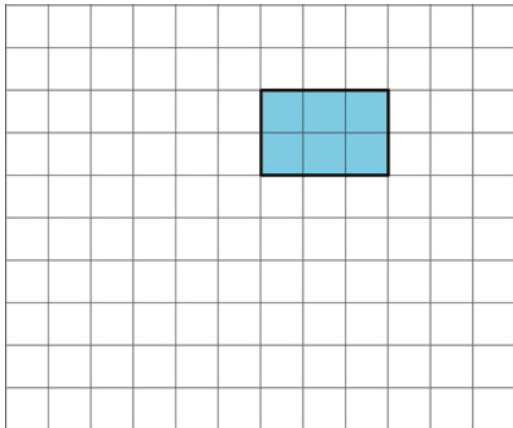
Problem 3

Statement

Use copies of the rectangle to show how a rectangle could:

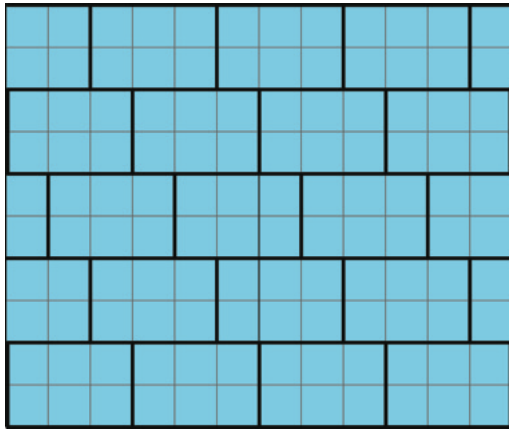
a. tile the plane.

b. *not* tile the plane.

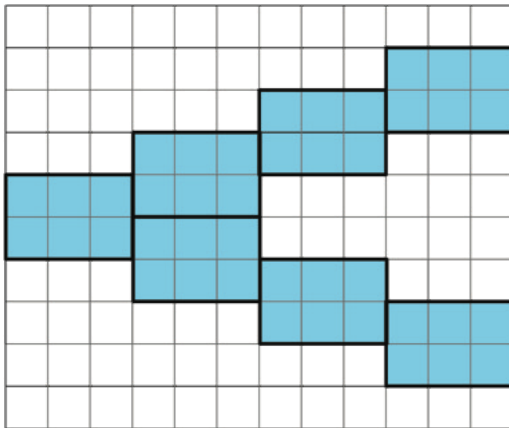


Solution

a. Answers vary. Sample response:



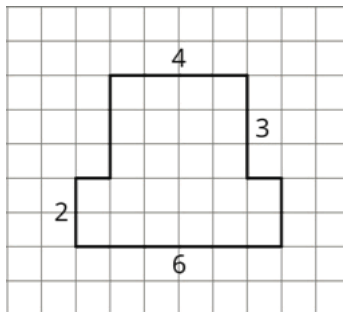
b. Answers vary. Sample response:



Problem 4

Statement

The area of this shape is 24 square units. Which of these statements is true about the area?
Select all that apply.



- A. The area can be found by counting the number of squares that touch the edge of the shape.
- B. It takes 24 grid squares to cover the shape without gaps and overlaps.
- C. The area can be found by multiplying the sides lengths that are 6 units and 4 units.
- D. The area can be found by counting the grid squares inside the shape.
- E. The area can be found by adding 4×3 and 6×2 .

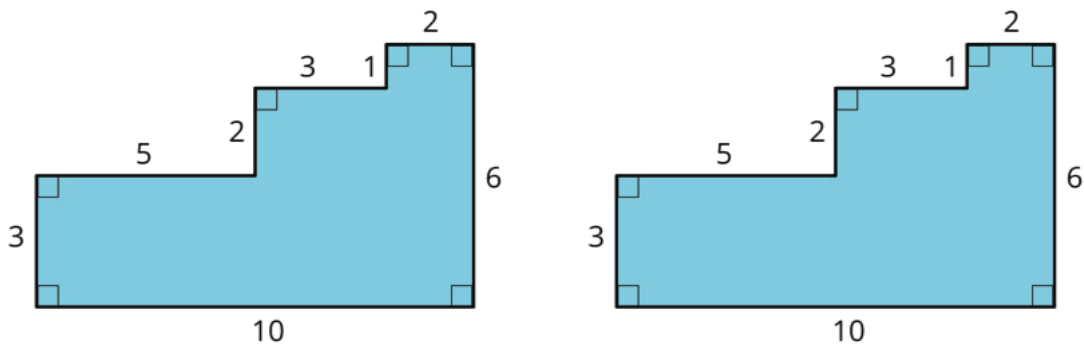
Solution

["B", "D", "E"]

Problem 5

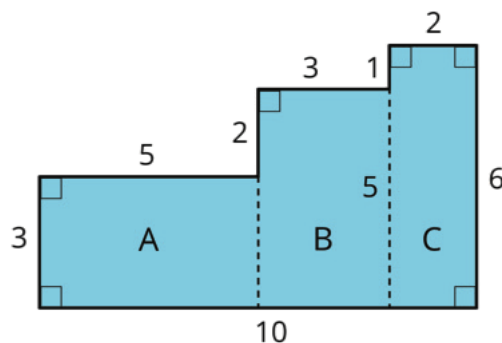
Statement

Here are two copies of the same figure. Show two different ways for finding the area of the shaded region. All angles are right angles.

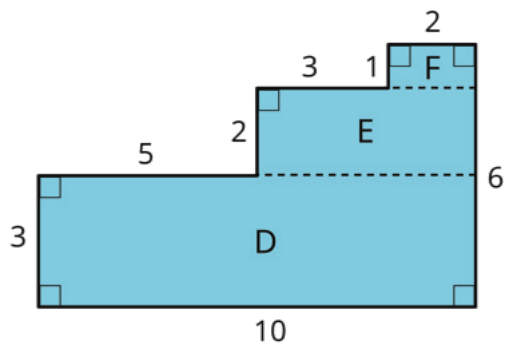


Solution

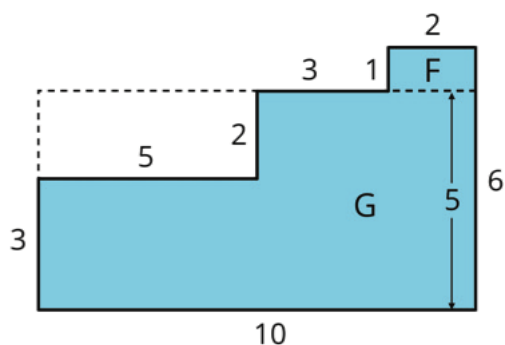
Answers vary. Sample strategies:



Area of A is 15 square units. Area of B is 15 square units. Area of C is 12 square units. The area of the entire region is $15 + 15 + 12$ or 42 square units.



Area of D is 30 square units. Area of E is 10 square units. Area of F is 2 square units. The area of the entire region is $30 + 10 + 2$ or 42 square units.



Area of F is 2 square units. Area of G is the area of the 10-by-5 rectangle subtracted by the area of a 5-by-2 rectangle in the upper left. $(10 \times 5) - (5 \times 2) = 50 - 10 = 40$, so the area of G is 40 square units. The total area is $40 + 2$ or 42 square units.

Problem 6

Statement

Which shape has a larger area: a rectangle that is 7 inches by $\frac{3}{4}$ inch, or a square with side length of $2\frac{1}{2}$ inches? Show your reasoning.

Solution

The square is larger. Its area is $2\frac{1}{2} \times 2\frac{1}{2} = \frac{5}{2} \times \frac{5}{2}$, which is $\frac{25}{4}$ or $6\frac{1}{4}$ square inches. The rectangle has an area of $5\frac{1}{4}$ square inches because $7 \times \frac{3}{4} = \frac{21}{4}$.

Lesson 2: Finding Area by Decomposing and Rearranging

Goals

- Calculate the area of a region by decomposing it and rearranging the pieces, and explain (orally and in writing) the solution method.
- Recognize and explain (orally) that if two figures can be placed one on top one other so that they match up exactly, they must have the same area.
- Show that area is additive by composing polygons with a given area.

Learning Targets

- I can explain how to find the area of a figure that is composed of other shapes.
- I know how to find the area of a figure by decomposing it and rearranging the parts.
- I know what it means for two figures to have the same area.

Lesson Narrative

This lesson begins by revisiting the definitions for **area** that students learned in earlier grades. The goal here is to refine their definitions (MP6) and come up with one that can be used by the class for the rest of the unit. They also learn to reason flexibly about two-dimensional figures to find their areas, and to communicate their reasoning clearly (MP3).

The area of two-dimensional figures can be determined in multiple ways. We can **compose** that figure using smaller pieces with known areas. We can **decompose** a figure into shapes whose areas we can determine and add the areas of those shapes. We can also decompose it and **rearrange** the pieces into a different but familiar shape so that its area can be found. The two key principles in this lesson are:

- Figures that match up exactly have equal areas. If two figures can be placed one on top of the other such that they match up exactly, then they have the same area.
- A figure can be decomposed and its pieces rearranged without changing its area. The sum of the areas of the pieces is equal to the area of the original figure. Likewise, if a figure is composed of non-overlapping pieces, its area is equal to the sum of the areas of the pieces. In other words, area is additive.

Students have used these principles since grade 3, but mainly to decompose squares, rectangles, and their composites (e.g., an L-shape) and rearrange them to form other such figures. In this lesson, they decompose triangles and rearrange them to form figures whose areas they know how to calculate.

A note about “two figures that match up exactly”: In grade 8, students will learn to refer to such figures as *congruent* and to describe congruence in terms of rigid motions (reflections, rotations, and translations). In these materials, the word congruent is not used in grade 6. A possibility is to use an informal term such as “identical,” so that students can talk about one figure being an “identical copy” of another. What “identical” means, however, might also require clarification (e.g., that it is independent of color and orientation).

Alignments

Building On

- 3.MD.C.5.b: A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

Addressing

- 6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Building Towards

- 6.G.A: Solve real-world and mathematical problems involving area, surface area, and volume.

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports
- Think Pair Share

Required Materials

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Pre-assembled or commercially produced tangrams

Required Preparation

Prepare 1 set of tangrams that contains 4 small, 1 medium, and 2 large right triangles for every 2 students. Print and cut out the Instructional master (printing on card stock is recommended), or use commercially-available tangrams. Note that the tangram pieces used here differs from a standard set in that two additional small triangles are used instead of a parallelogram.

A tangram applet is included for classrooms using the digital materials, but students can also be given the option of using physical tangrams instead of the digital tool.

Make sure students have access to their geometry toolkits, which should include tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

Student Learning Goals

Let's create shapes and find their areas.

2.1 What is Area?

Warm Up: 10 minutes

This warm-up activates and refines students' prior knowledge of area. It prompts students to articulate a definition of **area** that can be used for the rest of the unit. This definition of area is not new, but rather reiterates what students learned in grades 3–5.

Before this lesson, students explored tiling and tile patterns. Here, they analyze four ways a region is being tiled or otherwise fitted with squares. They decide which arrangements of squares can be used to find the area of the region and why, and use their analysis to write a definition of area. In identifying the most important aspects that should be included in the definition, students attend to precision (MP6).

Students' initial definitions may be incomplete. During partner discussions, note students who mention these components so they can share later:

- Plane or two-dimensional region
- Square units
- Covering a region completely without gaps or overlaps

Limit the whole-class discussion to 5–7 minutes to leave enough time for the work that follows.

Building On

- 3.MD.C.5.b

Building Towards

- 6.G.A

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 2 minutes of quiet think time for the first question, and ask them to be ready to explain their decision. Then, give partners 3–4 minutes to share their responses and to complete the second question together.

Anticipated Misconceptions

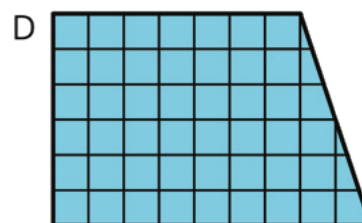
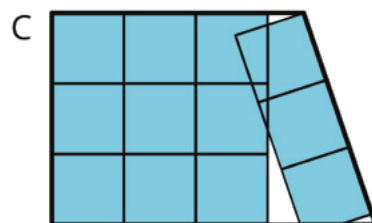
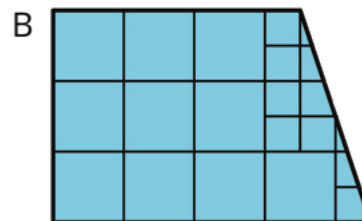
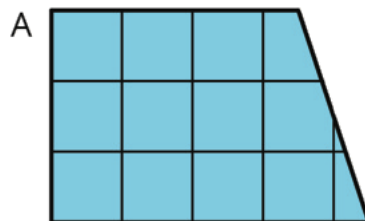
Students may focus on how they have typically found the area of a rectangle—by multiplying its side lengths—instead of thinking about what “the area of any region” means. Ask them to consider what the product of the side lengths of a rectangle actually tells us. (For example, if they say that the area of a 5-by-3 rectangle is 15, ask what the 15 means.)

Some students may think that none of the options, including A and D, could be used to find area because they involve partial squares, or because the partial squares do not appear to be familiar fractional parts. Use of benchmark fractions may help students see that the area of a region could be a non-whole number. For example, ask students if the area of a rectangle could be, say, $8\frac{1}{2}$ or $2\frac{1}{4}$ square units.

Student Task Statement

You may recall that the term **area** tells us something about the number of squares inside a two-dimensional shape.

1. Here are four drawings that each show squares inside a shape. Select **all** drawings whose squares could be used to find the area of the shape. Be prepared to explain your reasoning.



2. Write a definition of area that includes all the information that you think is important.

Student Response

1. A and D. B could be considered if the larger squares and the smaller ones are distinguished when determining area.
2. Answers vary, but the working definition should contain all of these components: "The area of a two-dimensional region (in square units) is the number of unit squares that cover the region without gaps or overlaps."

Activity Synthesis

For each drawing in the first question, ask students to indicate whether they think the squares could or couldn't be used to find the area. From their work in earlier grades students are likely to see that the number of squares in A and D can each tell us about the area. Given the recent work on tiling, students may decide that C is unhelpful. Discuss students' decisions and ask:

- "What is it about A and D that can help us find the area?" (The squares are all the same size. They are unit squares.)
- "What is it about C that might make it unhelpful for finding area?" (The squares overlap and do not cover the entire region, so counting the squares won't give us the area.)
- "If you think B cannot be used to find area, why not?" (We can't just count the number of squares and say that the number is the area because the squares are not all the same size.)
- "If you think we can use B to find area, how?" (Four small squares make a large square. If we count the number of large squares and the number of small squares separately, we can convert one to the other and find the area in terms of either one of them.)

If time permits, discuss:

- "How are A and D different?" (A uses larger unit squares and D uses smaller ones. Each size represent a different unit.)
- "Will they give us different areas?" (They will give us areas in different units, such as square inches and square centimeters.)

Select a few groups to share their definitions of area or what they think should be included in the class definition of area. The discussion should lead to a definition that conveys key aspects of area: The area of a two-dimensional region (in square units) is the number of unit squares that cover the region without gaps or overlaps.

Display the class definition and revisit as needed throughout this unit. Tell students this will be a working definition that can be revised as they continue their work in the unit.

2.2 Composing Shapes

25 minutes (there is a digital version of this activity)

In grade 3, students recognized that area is additive. They learned to find the area of a rectilinear figure by decomposing it into non-overlapping rectangles and adding their areas. Here students extend that understanding to non-rectangular shapes. They compose tangram pieces—consisting of triangles and a square—into shapes with certain areas. The square serves as a unit square. Because students have only one square, they need to use these principles in their reasoning:

- If two figures can be placed one on top of the other so that they match up exactly, then they have the same area.
- If a figure is decomposed and rearranged to compose another figure, then its area is the same as the area of the original figure.

Each question in the task aims to elicit discussions about these two principles. Though they may seem obvious, these principles still need to be stated explicitly (at the end of the lesson), as more-advanced understanding of the area of complex figures depends on them.

The terms **compose**, **decompose**, and **rearrange** will be formalized in an upcoming lesson, but throughout this lesson, look for opportunities to demonstrate their use as students describe their work with the tangram pieces. When students use “make” or “build,” “break,” and “move around,” recast their everyday terms using the more formal ones.

As students work, notice how they compose the pieces to create shapes with certain areas. Look for students whose reasoning illustrates the ideas outlined in the Activity Synthesis.

Demonstrate the use of the word “compose” by repeating students’ everyday language use and then recast using the formal terms here.

Addressing

- 6.G.A.1

Instructional Routines

- MLR2: Collect and Display
- Think Pair Share

Launch

Give each group of 2 students the following set of tangram pieces from the Instructional master or from commercially available sets. Note that the tangram pieces used here differ from a standard set in that two additional small triangles are used instead of a parallelogram.

- Square: 1
- Small triangles: 4
- Medium triangle: 1
- Large triangles: 2

It is important not to give them more than these pieces.

Give students 2–3 minutes of quiet think time for the first three questions. Ask them to pause afterwards and compare their solutions to their partner's. If they created the same shape for each question, ask them to create a different shape that has the same given area before moving on. Then, ask them to work together to answer the remaining questions.

Classrooms using the digital activities can use physical tangram pieces or an applet with the same shapes to determine the relationships between the areas. Applet is adapted from the work of [Harry Drew](#) in GeoGebra.

Support for English Language Learners

Speaking, Conversing: MLR2 Collect and Display. Circulate and listen to the ways students describe composing, decomposing, and rearranging the shapes. On a display, write down common phrases you hear students say about each, such as “building,” “breaking apart,” “moving.” Include relevant pictures or drawings. Update the display as needed throughout the remainder of the lesson. Remind students to borrow language from the display as needed. This will help students use mathematical language during their paired and whole-group discussions.

Design Principle(s): Maximize meta-awareness; Support sense-making

Anticipated Misconceptions

Students may consider the area to be the number of pieces in the compositions, instead of the number of square units. Remind them of the meaning of area, or prompt them to review the definition of area discussed in the warm-up activity.

Because the 2 large triangles in the tangram set can be arranged to form a square, students may consider that square to be the square unit rather than the smaller square composed of 2 small triangles. Ask students to review the task statement and verify the size of the unit square.

Student Task Statement

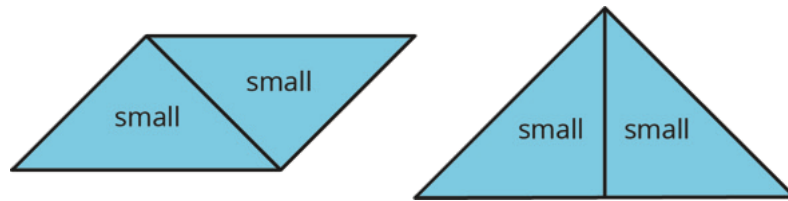
Your teacher will give you one square and some small, medium, and large right triangles. The area of the square is 1 square unit.

1. Notice that you can put together two small triangles to make a square. What is the area of the square composed of two small triangles? Be prepared to explain your reasoning.
2. Use your shapes to create a new shape with an area of 1 square unit that is not a square. Trace your shape.
3. Use your shapes to create a new shape with an area of 2 square units. Trace your shape.

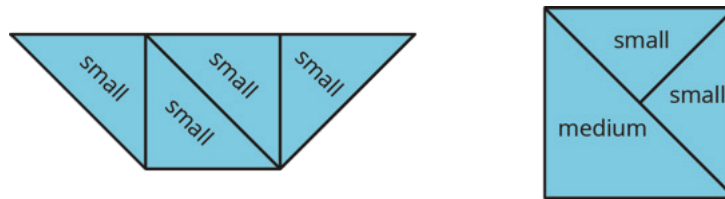
- Use your shapes to create a *different* shape with an area of 2 square units. Trace your shape.
- Use your shapes to create a new shape with an area of 4 square units. Trace your shape.

Student Response

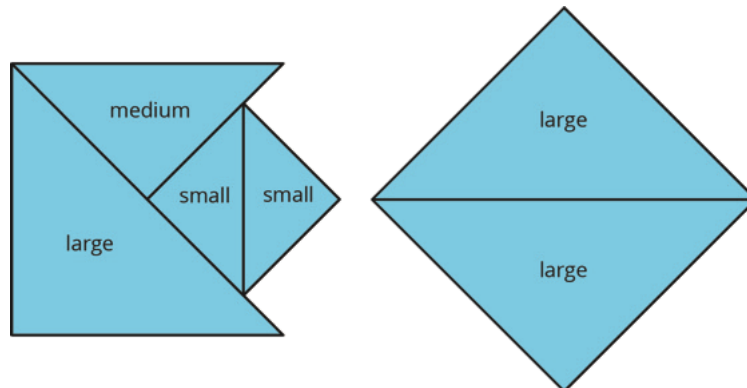
- The area of the square made from two small triangles is 1 square unit because it is identical to the given square with area 1 square unit. "Identical" means you can put one on top of the other and they match up exactly.
- Any composite of two small triangles.



- Any composite of four small triangles or two small triangles and one medium triangle. Sample responses:



- Any composite of four small triangles or two small triangles and one medium triangle.
- Any composite with an area of 4 square units. Some possibilities:

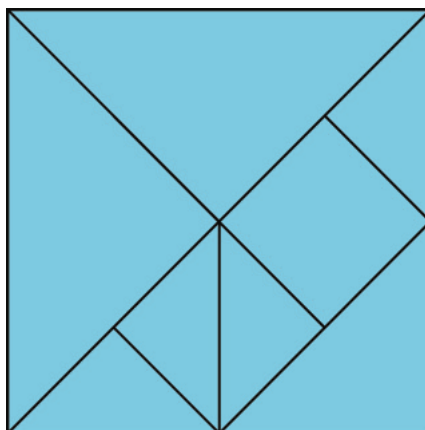


Are You Ready for More?

Find a way to use all of your pieces to compose a single large square. What is the area of this large square?

Student Response

The area is 8 square units. Sample response:



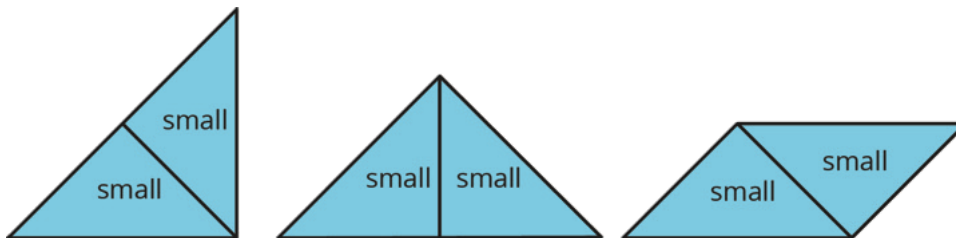
Activity Synthesis

Invite previously identified students (whose work illustrates the ideas shown here) to share. Name these moves explicitly as they come up: **compose**, **decompose**, and **rearrange**.

- First question: Two small triangles can be composed into a square that matches up exactly with the given square piece. This means that the two squares—the composite and the unit square—have the same area.

Tell students, “We say that if a region can be placed on top of another region so that they match up exactly, then they have the same **area**.”

- Second question: Two small triangles can be rearranged to compose a different figure but the area of that composite is still 1 square unit. These three shapes—each composed of two triangles—have the same area. If we rotate the first figure, it can be placed on top of the second so that they match up exactly. The third one has a different shape than the other two, but because it is made up of the same two triangles, it has the same area.



Emphasize: “If a figure is decomposed and rearranged as a new figure, the **area** of the new figure is the same as the area of the original figure.”

- Third and fourth questions: The composite figures could be formed in several ways: with only small triangles, with two triangles and a medium triangle, or with two small triangles and a square.

- Last question: A large triangle is needed here. To find its area, we need to either compose 4 smaller triangles into a large triangle, or to see that the large triangle could be decomposed into 4 smaller triangles, which can then be composed into 2 unit squares.

Support for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of these terms. Include the following terms and maintain the display for reference throughout the unit: area, compose, decompose, and rearrange.

Supports accessibility for: Conceptual processing; Language

2.3 Tangram Triangles

Optional: 15 minutes (there is a digital version of this activity)

In this activity, students use the areas of composite shapes from the previous activity to reason about the area of each tangram shape. Students may have recognized previously that the area of one small triangle is $\frac{1}{2}$ square unit, the area of one medium triangle is 1 square unit, and the area of one large triangle is 2 square units. Here they practice articulating how they know that these observations are true (MP3). The explanations could be written in words, or as clearly-labeled illustrations that support their answers.

As partners discuss, look for two ways of thinking about the area of each assigned triangle: by *composing* copies of the triangle into a square or a larger triangle, or by *decomposing* the triangle or the unit square into smaller pieces and *rearranging* the pieces. Identify at least one student who uses each approach.

Addressing

- 6.G.A.1

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2. Assign the first and second questions to one partner and the second and third questions to the other partner. Give each group access to the geometry toolkits and the same set of tangram pieces as used in the earlier activity.

Give students 3–4 minutes of quiet time to find the areas of their assigned triangles and to construct their explanations, followed by a few minutes to share their responses with their partner. Tell students that as one partner explains, the other should listen carefully and either agree or

disagree with the explanation. They should then come to an agreement about the answers and explanations.

Classrooms using the digital activities can use an applet to assist in determining the areas of the triangles.

Support for English Language Learners

Speaking, Writing: MLR8 Discussion Supports. Use this routine when students compare areas of triangles and squares to support the use of mathematical language. As students share their responses with their partner, circulate and encourage listeners to push speakers to use the language “compose,” “decompose,” or “rearrange” in their explanations. Look for students who name the square or larger triangles “composite figures” and amplify this language. Encourage students to borrow words and phrases from each other and to use this language in their written responses.

Design Principle(s): Cultivate conversation; Optimize output (for explanation)

Anticipated Misconceptions

If students initially have trouble determining the areas of the shapes, ask how they reasoned about areas in the previous activity. Have samples of composed and decomposed shapes that form one square unit available for students to reference.

Student Task Statement

Recall that the area of the square you saw earlier is 1 square unit. Complete each statement and explain your reasoning.

1. The area of the small triangle is _____ square units. I know this because . . .
2. The area of the medium triangle is _____ square units. I know this because . . .
3. The area of the large triangle is _____ square units. I know this because . . .

Student Response

1. $\frac{1}{2}$ square unit. Sample explanations:
 - Two small triangles can be put together to make a square, which has an area of one square unit. Because this composite shape matches the unit square exactly, their areas must be equal. This means that the area of each small triangle is half the area of the unit square.
 - A square can be decomposed into exactly two small triangles, so the area of each small triangle must be half of that of the square.
2. 1 square unit. Sample explanations:

- Two small triangles can be put together to make one medium triangle. Two triangles can also be put together to make a square with an area of 1 square unit. Because two small triangles make both a medium triangle and a square, the area of the medium triangle must be 1 square unit.
- One medium triangle can be decomposed into two small triangles. These can be rearranged into a square whose area is 1 square unit, so the area of the medium triangle is also 1 square unit.

3. 2 square units. Sample explanations:

- Two medium triangles can be arranged into one large triangle. Because the area of the medium triangle is 1 square unit, a figure that is composed of two of them has area 2 square units.
- A large triangle can be decomposed into 4 small triangles, which can in turn be rearranged into two squares. The combined area of the two squares is 2 square units.

Activity Synthesis

After partners shared and agreed on the correct areas and explanations, discuss with the class:

- “Did you and your partner use the same strategy to find the area of each triangle?”
- “How were your explanations similar? How were they different?”

Select two previously identified students to share their explanations: one who reasoned in terms of *composing* copies of their assigned triangle into another shape, and one who reasoned in terms of *decomposing* their triangle or the unit square into smaller pieces and *rearranging* them. If these approaches are not brought up by students, be sure to make them explicit at the end of the lesson.

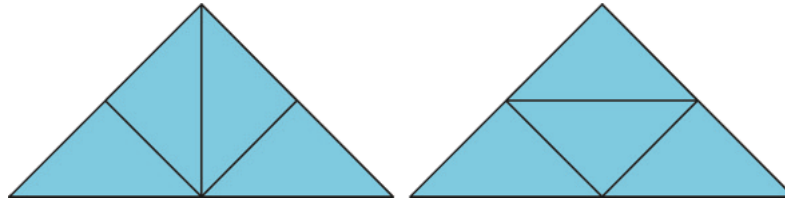
Lesson Synthesis

There are two principles that can help us reason about area:

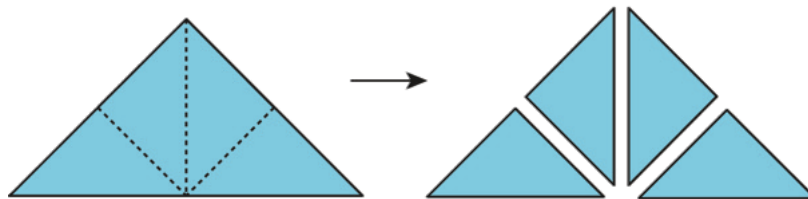
1. If two figures can be placed one on top of the other so that they match up exactly, then they have the same area.
2. The area of a figure can be found by adding the areas of its parts. If we **compose** (put together) a new figure from smaller pieces without overlapping them, then the sum of the areas of the pieces is the area of the new figure. Likewise, if we **decompose** (cut or break apart) a given figure into pieces, then the area of the given figure is the sum of the areas of the pieces. Even if we **rearrange** the pieces, the overall area does not change.

Here is an example. Suppose we know the area of a small triangle and wish to find the area of a large triangle. Demonstrate the following (using the tangram pieces, if possible):

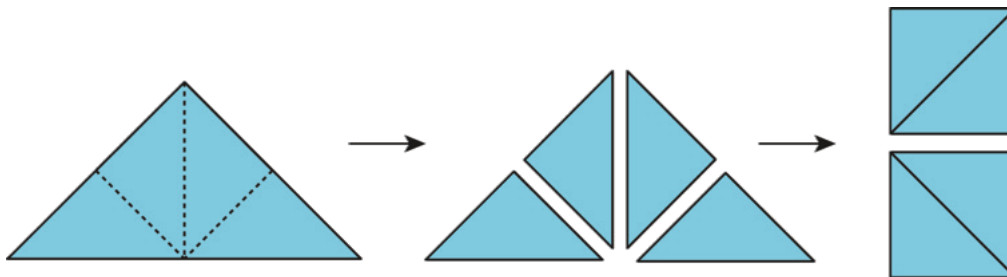
- We can use 4 small triangles to *compose* a large triangle. Here are two ways to do so. If we place a large triangle on top of a composition of 4 small triangles and they match up exactly, we know that the area of the large triangle is equal to the combined area of 4 small triangles.



- We can *decompose* the large triangle into 4 small triangles. Again, we can reason that the area of one large triangle is equal to the combined area of 4 small triangles.



- Suppose we don't know the area of a small triangle, but we do know the area of a square that is composed of 2 small triangles. We can *decompose* the large triangle into 4 small triangles and then *rearrange* them into 2 squares. We can reason that the area of the large triangle is equal to the combined area of 2 squares. This is because when the 4 rearranged small triangles are placed on top of two squares, they match up exactly.



We will look more deeply into these strategies in the next lesson.

2.4 Tangram Rectangle

Cool Down: 5 minutes

Addressing

- 6.G.A.1

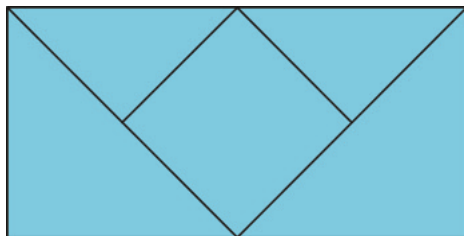
Launch

Give students access to the tangram shapes and geometry toolkits. Tell students that this figure is composed of two small right triangles, two medium right triangles, and a square, just like the ones they used earlier.

Note that students might not, at first, see the "square in the middle" as a square, or they might think of it a diamond (with unequal angles). Make sure that everyone understands that square-ness does not depend on how we turn the paper: A square is a rectangle (with all four angles being right angles) that has 4 equal sides.

Student Task Statement

The square in the middle has an area of 1 square unit. What is the area of the entire rectangle in square units? Explain your reasoning.



Student Response

The area is 4 square units. Possible strategies:

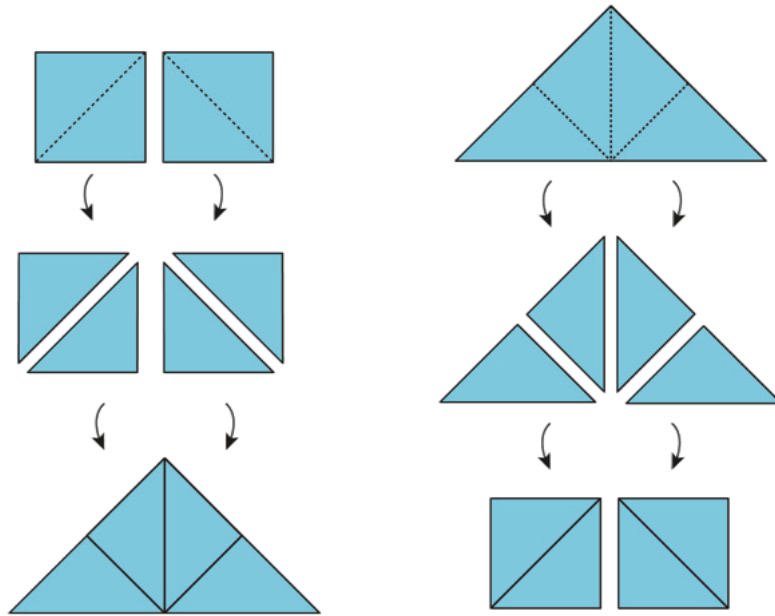
- Put together the two small triangles to make a square. Its area is 1 square unit. Decompose each medium triangle into two small triangles that can be arranged as a square. Each of these squares has area 1 square unit. Together with the square in the middle, the sum of the areas of these pieces is 4 square units.
- A small triangle has an area of $\frac{1}{2}$ square unit, and a medium triangle has an area of 1 square unit. $1 + 1 + 1 + \frac{1}{2} + \frac{1}{2} = 4$

Student Lesson Summary

Here are two important principles for finding **area**:

1. If two figures can be placed one on top of the other so that they match up exactly, then they have the *same area*.
2. We can **decompose** a figure (break a figure into pieces) and **rearrange** the pieces (move the pieces around) to find its area.

Here are illustrations of the two principles.



- Each square on the left can be decomposed into 2 triangles. These triangles can be rearranged into a large triangle. So the large triangle has the *same area* as the 2 squares.
- Similarly, the large triangle on the right can be decomposed into 4 equal triangles. The triangles can be rearranged to form 2 squares. If each square has an area of 1 square unit, then the area of the large triangle is 2 square units. We also can say that each small triangle has an area of $\frac{1}{2}$ square unit.

Glossary

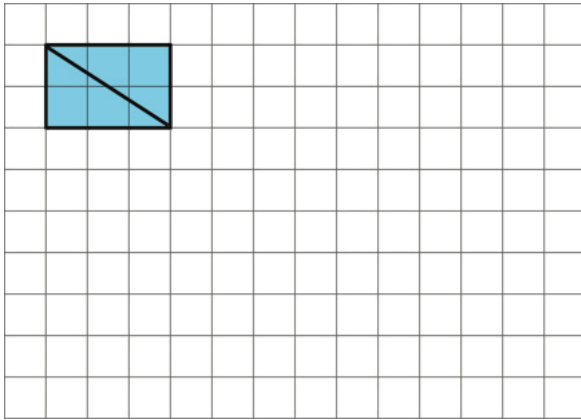
- compose
- decompose

Lesson 2 Practice Problems

Problem 1

Statement

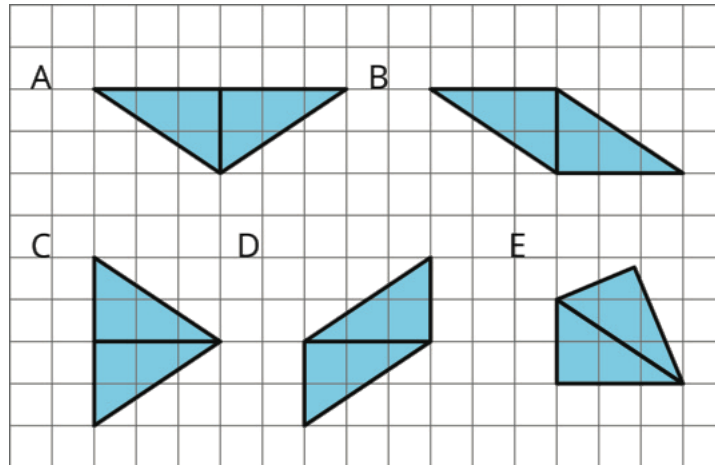
The diagonal of a rectangle is shown.



- Decompose the rectangle along the diagonal, and recompose the two pieces to make a *different* shape.
- How does the area of this new shape compare to the area of the original rectangle? Explain how you know.

Solution

a. Answers vary. Five different ways are shown.



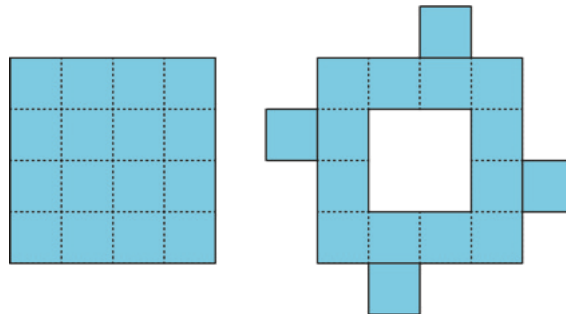
b. The areas are the same as all of the shapes are composed of two copies of the same triangle.

Problem 2

Statement

Priya decomposed a square into 16 smaller, equal-size squares and then cut out 4 of the small squares and attached them around the outside of original square to make a new figure.

How does the area of her new figure compare with that of the original square?



- A. The area of the new figure is greater.
- B. The two figures have the same area.
- C. The area of the original square is greater.
- D. We don't know because neither the side length nor the area of the original square is known.

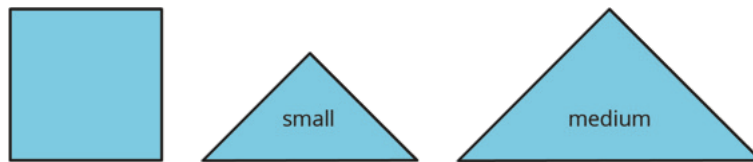
Solution

B

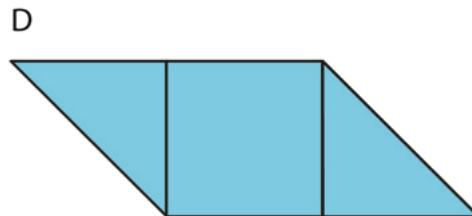
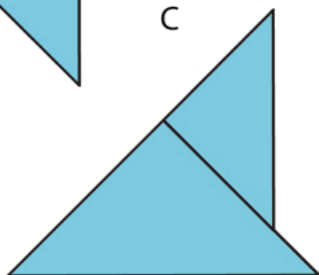
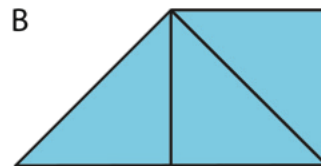
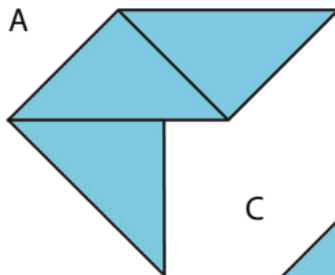
Problem 3

Statement

The area of the square is 1 square unit. Two small triangles can be put together to make a square or to make a medium triangle.



Which figure also has an area of $1\frac{1}{2}$ square units? Select **all** that apply.



- A. Figure A
- B. Figure B
- C. Figure C
- D. Figure D

Solution

["A", "B", "C"]

Problem 4

Statement

The area of a rectangular playground is 78 square meters. If the length of the playground is 13 meters, what is its width?

Solution

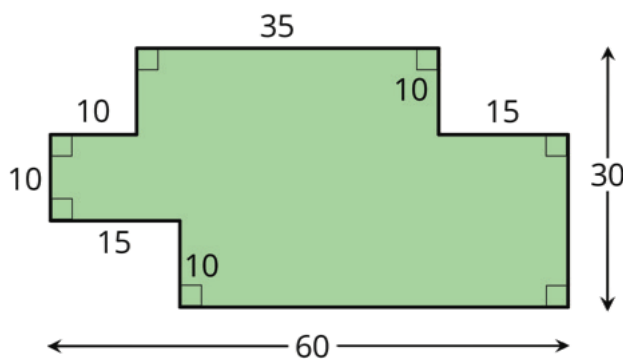
6 meters

(From Unit 1, Lesson 1.)

Problem 5

Statement

A student said, "We can't find the area of the shaded region because the shape has many different measurements, instead of just a length and a width that we could multiply."



Explain why the student's statement about area is incorrect.

Solution

Answers vary. Sample explanation: Area measures how many unit squares cover a region without gaps or overlaps. We multiply a length and a width when finding the area of a rectangle because that product tells us the number of unit squares in it. We can still find the area of a shape as shown, but first we will need to break it apart into rectangles whose areas we can find and then find the total area. We can also enclose the 30-by-60 region with a rectangle, find its area, and subtract the areas of the unshaded portions.

(From Unit 1, Lesson 1.)

Lesson 3: Reasoning to Find Area

Goals

- Compare and contrast (orally) different strategies for calculating the area of a polygon.
- Find the area of a polygon by decomposing, rearranging, subtracting or enclosing shapes, and explain (orally and in writing) the solution method.
- Include appropriate units (in spoken and written language) when stating the area of a polygon.

Learning Targets

- I can use different reasoning strategies to find the area of shapes.

Lesson Narrative

This lesson is the third of three lessons that use the following principles for reasoning about figures to find area:

- If two figures can be placed one on top of the other so that they match up exactly, then they have the same area.
- If a figure is composed from pieces that don't overlap, the sum of the areas of the pieces is the area of the figure. If a given figure is decomposed into pieces, then the area of the given figure is the sum of the areas of the pieces.

Following these principles, students can use several strategies to find the area of a figure. They can:

- Decompose it into shapes whose areas they can calculate.
- Decompose and rearrange it into shapes whose areas they can calculate.
- Consider it as a shape with one or more missing pieces, calculate the area of the shape, then subtract the areas of the missing pieces.
- Enclose it with a figure whose area they can calculate, consider the result as a region with missing pieces, and find its area using the previous strategy.

Use of these strategies involves looking for and making use of structure (MP7); explaining them involves constructing logical arguments (MP3). For now, rectangles are the only shapes whose areas students know how to calculate, but the strategies will become more powerful as students' repertoires grow. This lesson includes one figure for which the "enclosing" strategy is appropriate, however, that strategy is not the main focus of the lesson and is not included in the list of strategies at the end.

Note that these materials use the "dot" notation (for example $2 \cdot 3$) to represent multiplication instead of the "cross" notation (for example 2×3). This is because students will be writing many algebraic expressions and equations in this course, sometimes involving the letter x used as a

variable. This notation will be new for many students, and they will need explicit guidance in using it.

Alignments

Building On

- 3.MD.C.7.d: Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Addressing

- 6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR3: Clarify, Critique, Correct
- MLR7: Compare and Connect
- Think Pair Share

Required Materials

Copies of Instructional master

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Required Preparation

Make sure students have access to items in their geometry toolkits: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For the warm-up activity, prepare several copies of the pair of figures on the Instructional master, in case students propose cutting them out to compare the areas.

Student Learning Goals

Let's decompose and rearrange shapes to find their areas.

3.1 Comparing Regions

Warm Up: 5 minutes

This activity prompts students to use reasoning strategies from earlier lessons to compare the areas of two figures. It is also an opportunity to use (or introduce) tracing paper as a way to illustrate *decomposing* and *rearranging* a figure.

As students work, look for students who are able to explain or show how they know that the areas are equal. Some students may simply look at the figures and say, with no justification, that they have the same area. Urge them to think of a way to show that their conclusion is true.

Building On

- 3.MD.C.7.d

Launch

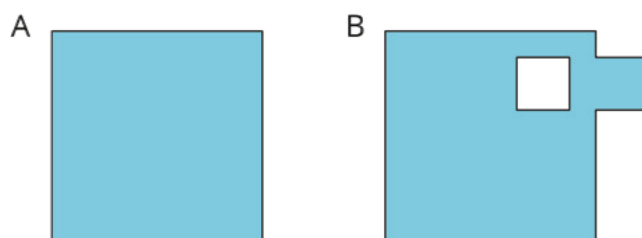
Give students access to their geometry toolkits, and allow for 2 minutes of quiet think time. Ask them to be ready to support their answer, and remind them to use the tools at their disposal. Have copies of the Instructional master ready for students who propose cutting the figures out for comparison or as a way to differentiate the activity.

Anticipated Misconceptions

Students may interpret the area of Figure B as the entire region inside the outer boundary including the unfilled square. Clarify that we want to compare the areas of only the shaded parts of Figure B and Figure A.

Student Task Statement

Is the area of Figure A greater than, less than, or equal to the area of the shaded region in Figure B? Be prepared to explain your reasoning.



Student Response

The areas are equal. Possible strategies:

- Measuring: Measure the side lengths of the small, unfilled rectangle and the small rectangle that are on the side of Figure B. Both rectangles have the same side lengths, so their areas are equal. This means the rectangle on the side fills the whole in the middle. Measure the side lengths of the large shaded square in Figure A and then in Figure B. Both have the same side lengths, so their areas are equal.

- Using scissors: Cut off the little square on the side of Figure B, and use it to fill the hole in the middle of Figure B. Then you get a square that matches up exactly with Figure A.
- Using tracing paper: Trace the boundary of the little square on the side of Figure B, move the tracing paper over the unshaded hole, and you can see that the little shaded square is exactly the same size as the hole. If you moved that little shaded square to fill the unshaded hole, you would get a big shaded square. If you trace the boundary of that big shaded square and put the tracing paper over Figure A, you can show that the boundary of that square matches up exactly with Figure A.

Activity Synthesis

Start the discussion by asking the students to indicate which of the three possible responses—area of Figure A is greater, area of Figure B is greater, or the areas are equal—they choose.

Select previously identified students to share their explanations. If no student mentioned using tracing paper, demonstrate the following.

- *Decomposing and rearranging Figure B:* Place a piece of tracing paper over Figure B. Draw the boundary of the small square, making a dotted auxiliary line to show its separation from the large square. Move the tracing paper so that the boundary of the small square matches up exactly with the boundary of the square-shaped hole in Figure B. Draw the boundary of the large square. Explain that the small square matches up exactly with the hole, so we know the small, shaded square and the hole have equal area.
- *Matching the two figures:* Move the tracing paper over Figure A so that the boundary of the rearranged Figure B matches up exactly with that of Figure A. Say, “When two figures that are overlaid one on top of another match up exactly, their areas are equal.”

Highlight the strategies and principles that are central to this unit. Tell students, “We just decomposed and rearranged Figure B so that it matches up exactly with Figure A. When two figures that are overlaid one on top of another match up exactly, we can say that their areas are equal.”

3.2 On the Grid

20 minutes

This activity gives students opportunities to find areas of regions using a variety of strategies. When working with a grid, students may start by counting squares, as they had done in earlier grades. However, the figures have been chosen to elicit the strategies listed in the Lesson Narrative.

- Figure A can be easily decomposed into rectangles.
- Figure B can be decomposed into rectangles. Or, more efficiently, it can be seen as a square with a missing piece, and the area of the inner unshaded square can be subtracted from the area of the larger square.

- Figure C can also be seen as having a missing piece, but subtracting the area of the unshaded shape does not work because the side lengths of the inner square are unknown. Instead, the shaded triangles can be decomposed and rearranged into rectangles.
- Figure D can be decomposed and rearranged into rectangles. It can also be viewed as the inner square of Figure C.

As students work, identify students who use these strategies and can illustrate or explain them clearly. Ask them to share later. Look for at least two strategies being used for each figure (one strategy as shown in the Student Response and at least one other).

Addressing

- 6.G.A.1

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR3: Clarify, Critique, Correct
- Think Pair Share

Launch

Tell students that they will find the areas of various figures on a grid. To encourage students to use a more grade-appropriate strategy for finding areas, show them a strategy from earlier grades. As a class, find the area of Figure A by counting the squares one by one aloud. Confirm that there are 24 square units, and then ask students to think about other ways to find the area of Figure A and other figures besides counting each square.

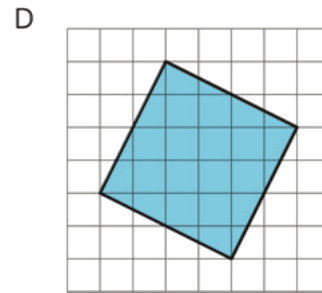
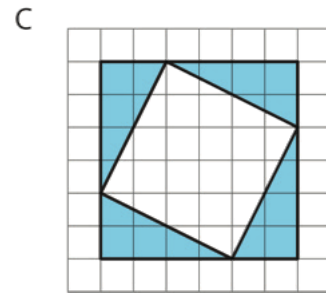
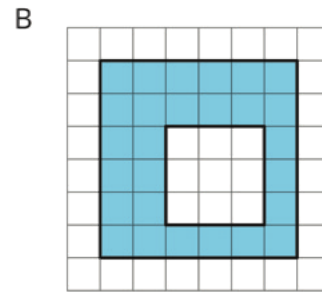
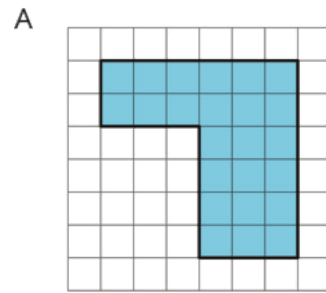
Arrange students in groups of 2. Ask one partner to start with Figures A and C, and the other with B and D. Give students 4–5 minutes of quiet think time, and provide access to their geometry toolkits. Then, give them a few minutes to share their responses with their partner. Emphasize that as one partner explains, the other should listen carefully and see if they agree or disagree with the answer and explanations.

Anticipated Misconceptions

Some students may count both complete and partial grid squares instead of looking for ways to decompose and rearrange larger shapes. Ask them if they can find a way to find the area by decomposing and rearranging larger pieces. The discussion at the end, during which everyone sees a variety of strategies, is especially important for these students.

Student Task Statement

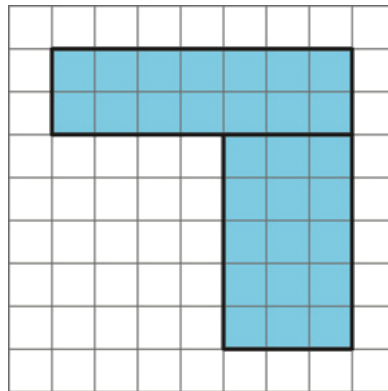
Each grid square is 1 square unit. Find the area, in square units, of each shaded region without counting every square. Be prepared to explain your reasoning.



Student Response

Strategies vary.

A: 24 square units. Sample strategy: Decompose the figure into rectangles. One way is shown here.
 $(2 \cdot 6) + (4 \cdot 3) = 24$

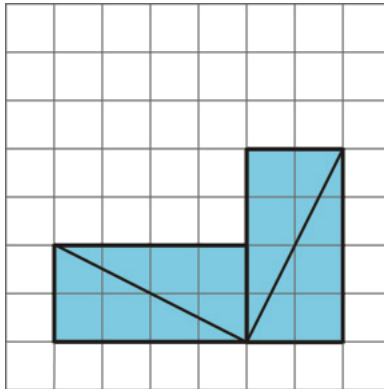


B: 27 square units. Sample strategies:

- Decompose the figure into four rectangles.
- Subtract the area of the inner square from the larger square. $(6 \cdot 6) - (3 \cdot 3) = 27$

C: 16 square units. Sample strategies:

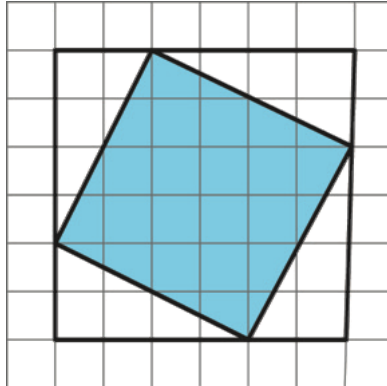
- Decompose into right triangles and rearrange into rectangles. $(2 \cdot 4) + (4 \cdot 2) = 16$



- Find the area of Figure D first, and then subtract it from the 6-by-6 square.

D: 20 square units. Sample strategies:

- Decompose the shaded square into four right triangles and a 2-by-2 square. Rearrange the right triangles into two rectangles that are each 2 units by 4 units, with a combined area of 16 square units. Adding the area of the small square (4 square units) gives a total of 20 square units.
- Notice that the shaded square is the inner square of Figure C, enclose it in a square as in Figure C, and subtract the areas of the four right triangles (i.e., the area of Figure C) from the area of the enclosing square. $(6 \cdot 6) - 16 = 20$

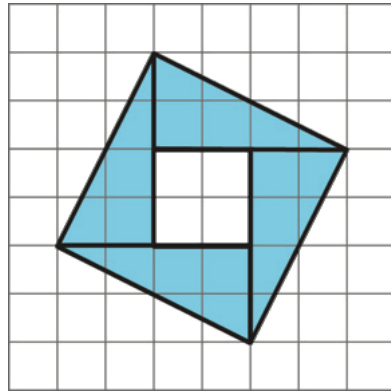


Are You Ready for More?

Rearrange the triangles from Figure C so they fit inside Figure D. Draw and color a diagram of your work.

Student Response

The triangles fit inside the square, with a smaller 2-by-2 square in the center.



This arrangement shows that the area of Figure D is 4 square units more than the area of Figure C.

Activity Synthesis

Discussion should center around how different strategies—decomposing, rearranging, subtracting, and enclosing—are used to find area. For each figure, select two students with different strategies to share their work, if possible. Sequence students' presentations so that, for each figure, a subtracting strategy comes last, as that is typically the most challenging.

Before sharing begins, explain to students that they should notice similarities and differences in the strategies shared and be ready to explain them. As students share their strategies, consider recording the moves on each figure for all to see. After each person shares, name the strategy, and poll the class to see if anyone else reasoned the same way. If one of these strategies does not appear in students' work, illustrate it for the class.

- Decomposing (A and B)
- Decomposing and rearranging (C and D)
- Subtracting (B)
- Enclosing, then subtracting (D)

If time permits, give partners a minute to talk to their partner about the similarities and differences they saw in the strategies used to find the areas of the four figures. Consider displaying sentence starters such as:

- The strategies used to find the areas of figures ___ and ___ are alike in that. . .
- The strategies used to find the areas of figures ___ and ___ are different in that. . .

Support for Students with Disabilities

Representation: Internalize Comprehension. Use color and annotations to highlight distinctions between strategies. Create a display that includes multiple copies of each figure. As students describe their strategies, use color and annotation to scribe their thinking so that it is visible for all students. Be sure to label each figure with the strategy described (decomposing, rearranging, subtracting, or enclosing).

Supports accessibility for: Visual-spatial processing; Conceptual processing

Support for English Language Learners

Speaking: Math Language Routine 3 Clarify, Critique, Correct. This is the first time Math Language Routine 3 is suggested as a support in this course. In this routine, students are given an incorrect or incomplete piece of mathematical work. This may be in the form of a written statement, drawing, problem-solving steps, or another mathematical representation. Pairs of students analyze, reflect on, and improve the written work by correcting errors and clarifying meaning. Typical prompts are “Is anything unclear?” or “Are there any reasoning errors?” The purpose of this routine is to engage students in analyzing mathematical thinking that is not their own, and to solidify their knowledge through communicating about conceptual errors and ambiguities in language.

Design Principle(s): Maximize meta-awareness

How It Happens:

1. In the class discussion for Figure C of this activity, provide the following “draft” explanation:
“I cut a square in the middle, then I saw that it was a bunch of triangles, so then I figured those out to get my answer.”

Prompt students to identify the ambiguity of this response. Ask students, “What do you think this person is trying to say? What is unclear? Did the author use any of the strategies we’ve been using to find area?”

2. Give students 1 minute of individual time to respond to the questions in writing, and then 3 minutes to discuss with a partner.

As pairs discuss, provide these sentence frames for scaffolding: “I think the author is trying to use the strategy _ because...”, “I think what the author meant by ‘figured it out’ was...”, and “The part that is most unclear to me is ... because...”. Encourage the listener to press for detail by asking follow-up questions to clarify the intended meaning of the statement. Allow each partner to take a turn as the speaker and listener.

3. Invite students to improve the “draft” response using the target vocabulary and structures.

The targeted vocabulary includes the names of the strategies (decomposing, rearranging, subtracting, enclosing, or any combination of them), and other terms from this lesson such as “alike,” “different,” “area,” and “polygon.”

The targeted structures of the response should include an explanation of each step, order/time transition words (first, next, then, etc.), and/or reasons for decisions made during steps.

Here is one example of an improved response:

“First, I cut out the square in the middle of the shaded regions. Next, I noticed that all the shaded regions were a bunch of right triangles. Then, I rearranged the triangles into rectangles. Lastly, I figured out the area of each rectangle and added them together to get my answer.”

4. Ask each pair of students to contribute their improved response to a poster, the whiteboard, or digital projection. Call on 2–3 pairs of students to present their response to the whole class, and invite the class to make comparisons among the responses shared and their own responses.
Listen for responses that include the strategy of decomposing and then rearranging the triangles. Emphasize that this is one way of figuring out the area of the shaded region.
5. Share one more improved response, discuss to reach a general understanding, and then move on to Figure D.

3.3 Off the Grid

15 minutes

In this activity, students apply the strategies they learned to find the areas of figures that are not on a grid.

- Figure A can easily be decomposed and rearranged.
- Figure B can be decomposed and rearranged into rectangles as for Figure C of the previous task. Students cannot use the strategy of subtracting the area of the inner square from that of the outer square because the side lengths of the inner square are unknown.
- For Figure C, students must subtract the area of the inner square from that of the outer square because there is not enough information to decompose and rearrange the shaded regions into rectangles.

As students discuss their approaches in groups, support them in naming the strategies and by asking clarifying questions. Notice any groups that may be stuck in a disagreement on the area of a particular figure. Identify students who observed that the same area-reasoning strategies can be applied both on and off the grid.

Students may not remember from earlier grades that if the measurements of side lengths of a rectangle are given in a particular unit, then the area is given in *square units*. Look for students who have trouble giving the appropriate area units (square centimeters) for these figures.

Addressing

- 6.G.A.1

Instructional Routines

- MLR7: Compare and Connect

Launch

Tell students that they will now find areas of figures that are not on a grid. Give students access to their geometry toolkits. Allow for 6–7 minutes of quiet time to find all three areas.

Then, arrange students into groups of 4, and ask each group to discuss their answers and strategies, using these guiding questions:

1. What units did you use for each area?
2. Compare your answers and strategies for finding the area of each figure.
 - How are your strategies the same? How are they different?
 - Which strategies are similar to the ones you used in the previous activity?

Support for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts. After students have found the area of Figure A, check in with select groups of students or the whole class. Invite students to share and name the strategy they used, and to predict a strategy they may find useful for Figure B.

Supports accessibility for: Organization; Attention

Anticipated Misconceptions

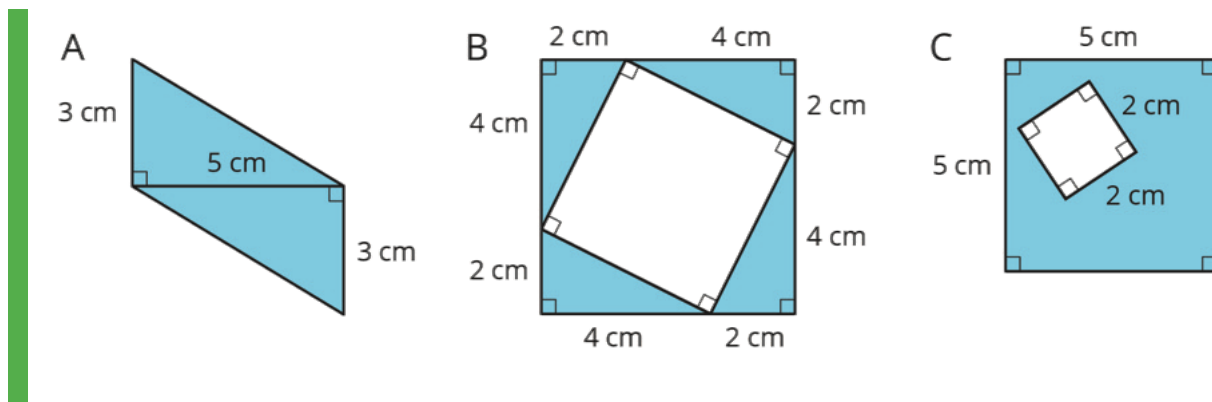
In Figure B, students may estimate the side lengths of the inner square so that its area could be subtracted from that of the outer square. They may struggle to see how the triangles could be rearranged. Suggest that they use tracing paper to help them in their thinking.

In Figures B and C, students may confuse finding area with finding perimeter. Remind them that area refers to the number of square units it takes to cover a region without gaps or overlaps.

Students might not be familiar with the symbols that indicate right angles and might think these symbols indicate square units. Remind them that those symbols indicate 90 degree angles.

Student Task Statement

Find the area of the shaded region(s) of each figure. Explain or show your reasoning.



Student Response

Reasoning varies.

Figure A: The area is 15 square centimeters. Sample reasoning: Decompose the two triangles and rearrange them to form a rectangle with side lengths of 5 centimeters and 3 centimeters.

Figure B: The area is 16 square centimeters. Sample reasoning: Decompose the triangles and rearrange them to form two rectangles with side lengths of 4 centimeters and 2 centimeters.

Figure C: The area is 21 square centimeters. Sample reasoning: Subtract the area of the inner square from the area of the outer square. $25 - 4 = 21$

Activity Synthesis

Reconvene briefly as a class to discuss the question, “Which strategies are similar to the ones you used in the previous activity?”

Select 1–2 previously identified students to share (those who noticed that they decomposed, rearranged, enclosed, and subtracted in both activities). Emphasize that the same strategies for finding area can be used whether we use the measurements indicated by a grid, or whether the measurements are given directly (without a grid).

Support for English Language Learners

Speaking: Math Language Routine 7 Compare and Connect. This is the first time Math Language Routine 7 is suggested as a support in this course. In this routine, students are given a problem that can be approached using multiple strategies or representations, and are asked to prepare a visual display of their method. Students then engage in investigating the strategies (by means of a teacher-led gallery walk, partner exchange, group presentation, etc.), compare approaches, and identify correspondences between different representations. A typical discussion prompt is, “What is the same and what is different?” regarding their own strategy and that of the others. The purpose of this routine is to allow students to make sense of mathematical strategies by identifying, comparing, contrasting, and connecting other approaches to their own, and to develop students’ awareness of the language used through constructive conversations.

Design Principle(s): Maximize meta-awareness

How It Happens:

1. Identify which Figure (A, B, or C) generated the most variety among students’ strategies. Invite students to create a visual display showing how they made sense of this figure. Students should include these features on their display:

Students should include these features on their display:

- the drawing of the figure
 - appropriate units
 - a representation of how the area was calculated
 - the name of the strategy used to find the area (decomposing, rearranging, subtracting, enclosing, or any combination of them)
2. Arrange students in groups of 4, and invite them to investigate each other’s work. Allow 1 minute for each display and signal when it is time to switch. Next, give each student the opportunity to add detail to their own display for 1-2 minutes.
 3. Circulate around the room and invite 2–3 students to present their display to the whole class. Be sure to select a variety of strategies.
 4. After the pre-selected students have finished presenting their displays, lead a discussion comparing, contrasting, and connecting the different approaches.

Consider using these prompts to amplify student language while comparing and contrasting the different approaches:

- “Why did different approaches for Figure _ lead to the same outcome?”

-
- “What worked well in this approach for Figure _? What did not work well?”
 - “What would make this strategy for Figure _ more complete or easy to understand?”

Consider using these prompts to amplify student language while connecting the different approaches:

- “Can you find any connections between the representations?”
- “Where are units used in each strategy?”
- “Is it possible to use the strategy of decomposing for this figure?”
- “What mathematical features do you see present in all of the representations?”

5. Close the discussion by inviting 3 students to revoice the strategies used in the presentations, and then transition back to the Lesson Synthesis and Cool-Down.

Lesson Synthesis

This lesson was all about identifying strategies for finding area and applying them to various figures. We reasoned about the area of a figure on and off a grid by:

- decomposing it into familiar shapes;
- decomposing it and rearranging the pieces into familiar shapes; or
- considering it as a shape with missing pieces, then subtracting the areas of the missing pieces from the area of the shape.

Ask students to go back through the activities and find problems in which these strategies were used—one strategy at a time. Tell students we will have lots of opportunity to use these strategies in upcoming lessons.

3.4 Maritime Flag

Cool Down: 5 minutes

This task does not explicitly ask students to state area units because one purpose of the task is to assess if students understand what units are appropriate given the information presented.

Addressing

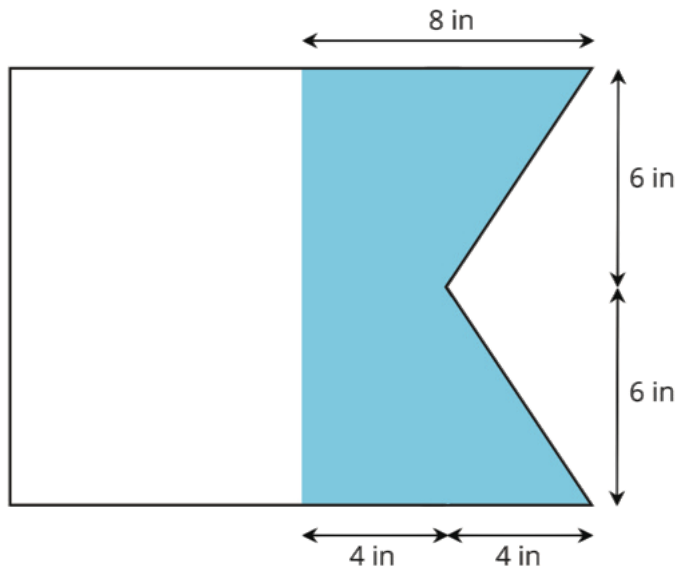
- 6.G.A.1

Launch

Give students access to their geometry toolkits.

Student Task Statement

A maritime flag is shown. What is the area of the shaded part of the flag? Explain or show your reasoning.



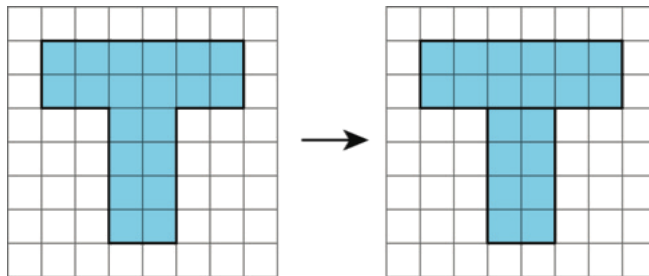
Student Response

72 square inches. Reasoning varies. Sample reasoning: If we draw a line down the middle of the shaded area, we would have a 4 inch-by-12 inch rectangle on the left and two right triangles. The 4-by-12 rectangle has an area of 48 square inches. The two triangles on the right can be composed into a 4 inch-by-6 inch rectangle, so their combined area is 24 square inches. $48 + 24 = 72$.

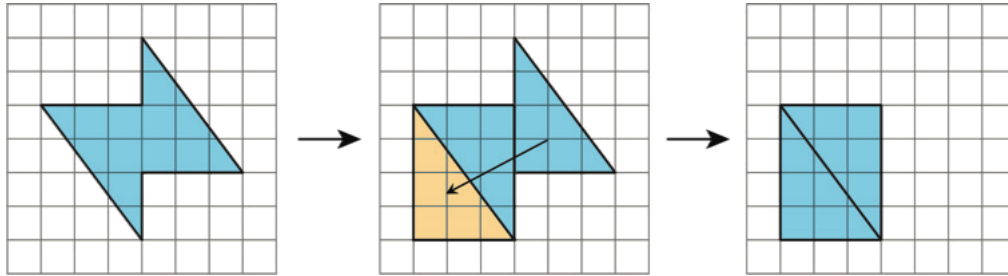
Student Lesson Summary

There are different strategies we can use to find the area of a region. We can:

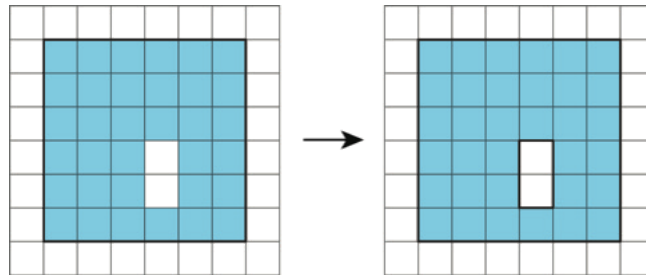
- Decompose it into shapes whose areas you know how to calculate; find the area of each of those shapes, and then add the areas.



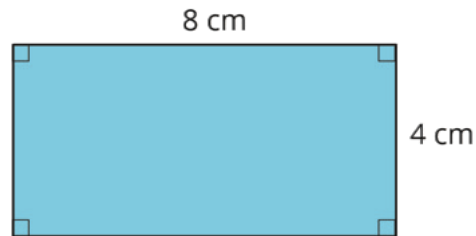
- Decompose it and rearrange the pieces into shapes whose areas you know how to calculate; find the area of each of those shapes, and then add the areas.



- Consider it as a shape with a missing piece; calculate the area of the shape and the missing piece, and then subtract the area of the piece from the area of the shape.



The area of a figure is always measured in square units. When both side lengths of a rectangle are given in centimeters, then the area is given in square centimeters. For example, the area of this rectangle is 32 square centimeters.

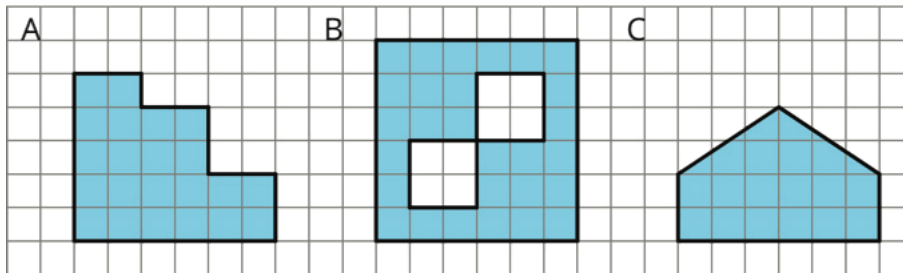


Lesson 3 Practice Problems

Problem 1

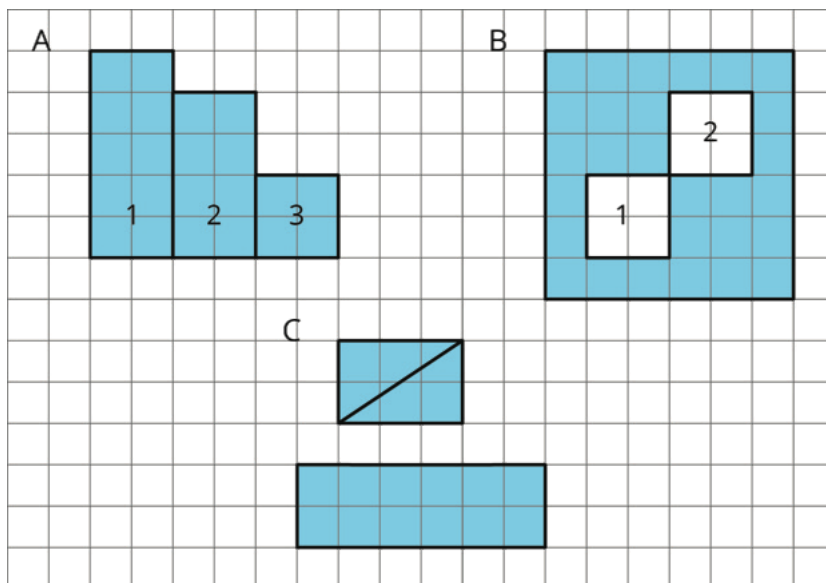
Statement

Find the area of each shaded region. Show your reasoning.



Solution

Answers vary. Sample response:



Shape A: 22 square units. The shaded region can be partitioned into rectangles. One way to do this is shown above. Rectangle 1 is 2 units by 5 units, so its area is 10 square units. Rectangle 2 is 2 units by 4 units, so its area is 8 square units. The area of Rectangle 3 is 4 square units. The total shaded area is 22 square units, since $10 + 8 + 4 = 22$.

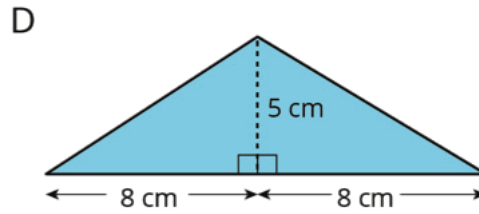
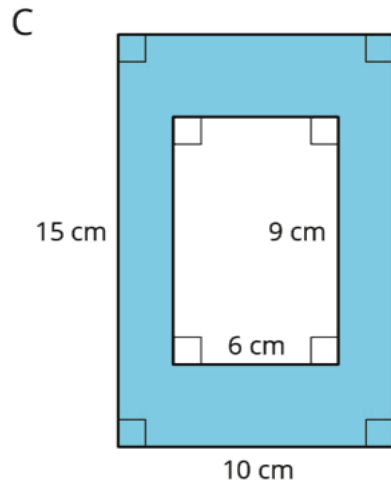
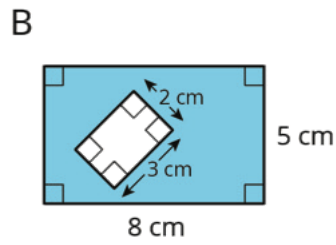
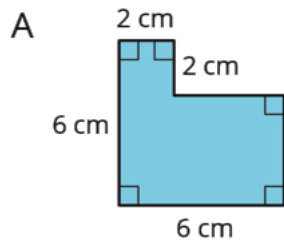
Shape B: 28 square units. The outer square is 6 units by 6 units, so its area is 36 square units. There are two smaller squares inside. Square 1 and Square 2 have been removed. Each small square has an area of 4 square units. To get the shaded area, compute $36 - 4 - 4$, which equals 28.

Shape C: 18 square units. The region can be recomposed to form a 2-by-6 rectangle and a 2-by-3 rectangle. $(2 \cdot 6) + (2 \cdot 3) = 18$.

Problem 2

Statement

Find the area of each shaded region. Show or explain your reasoning.



Solution

Reasoning varies. Sample responses:

A: 28 sq cm. A horizontal cut partitions this into a 2 cm-by-2 cm square (4 sq cm) and a 4 cm-by-6 cm rectangle (24 sq cm).

B: 34 sq cm. The outer rectangle has an area of 40 sq cm while the inner rectangle has an area of 6 sq cm. $40 - 6 = 34$.

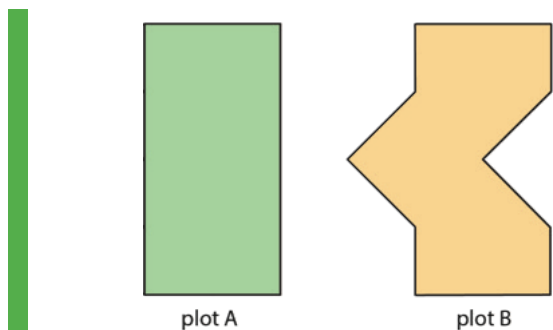
C: 96 sq cm. The outer rectangle has an area of 150 sq cm while the inner rectangle has an area of 54 sq cm. $150 - 54 = 96$.

D: 40 sq cm. The two right triangles can be put together to make a 5 cm-by-8 cm rectangle.

Problem 3

Statement

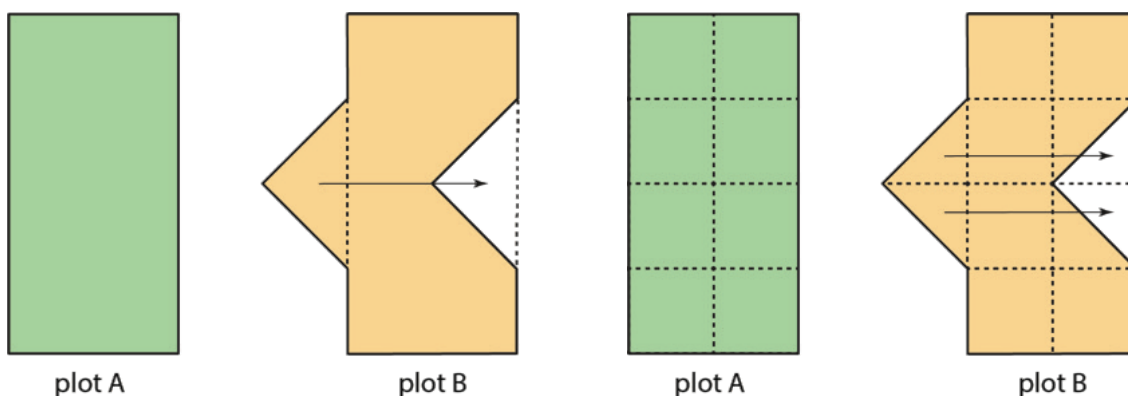
Two plots of land have very different shapes. Noah said that both plots of land have the same area.



Do you agree with Noah? Explain your reasoning.

Solution

Agree. Answers vary. Sample reasoning: The triangular shape that juts out from the left side of plot B can be cut off and moved to the right side of plot B. The resulting shape is a rectangle that matches exactly with the shape of plot A. We can use tracing paper to verify. Sample diagrams:



Problem 4

Statement

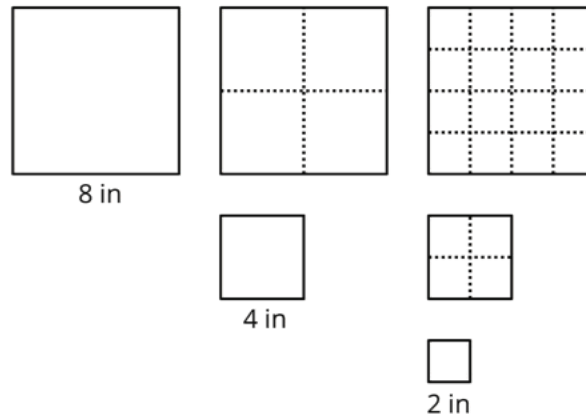
A homeowner is deciding on the size of tiles to use to fully tile a rectangular wall in her bathroom that is 80 inches by 40 inches. The tiles are squares and come in three side lengths: 8 inches, 4 inches, and 2 inches. State if you agree with each statement about the tiles. Explain your reasoning.

- Regardless of the size she chooses, she will need the same number of tiles.
- Regardless of the size she chooses, the area of the wall that is being tiled is the same.
- She will need two 2-inch tiles to cover the same area as one 4-inch tile.
- She will need four 4-inch tiles to cover the same area as one 8-inch tile.
- If she chooses the 8-inch tiles, she will need a quarter as many tiles as she would with 2-inch tiles.

Solution

Explanations vary. Sample explanations and diagram:

- a. Disagree. She will need fewer of the larger tiles and more of the smaller tiles.
- b. Agree. The region being covered does not change regardless of what tiles she chooses.
- c. Disagree. She will need four 2-inch tiles to cover the same area as one 4-inch tile.



- d. Agree. Two rows of two 4-inch tiles cover the same area as one 8-inch tile.
- e. Disagree. Because one 8-inch tile covers the same area as four 4-inch tiles, she will need $\frac{1}{16}$ as many 8-inch tiles as she would with 2-inch tiles.

(From Unit 1, Lesson 2.)

Section: Parallelograms

Lesson 4: Parallelograms

Goals

- Compare and contrast (orally) different strategies for determining the area of a parallelogram.
- Describe (orally and in writing) observations about the opposite sides and opposite angles of parallelograms.
- Explain (orally and in writing) how to find the area of a parallelogram by rearranging or enclosing it in a rectangle.

Learning Targets

- I can use reasoning strategies and what I know about the area of a rectangle to find the area of a parallelogram.
- I know how to describe the features of a parallelogram using mathematical vocabulary.

Lesson Narrative

Students were introduced to parallel lines in grade 4. While the standards do not explicitly state that students must work with parallelograms in grades 3–5, the geometry standards in those grades invite students to learn about and explore quadrilaterals of all kinds. The K–6 Geometry Progression gives examples of the kinds of work that students can do in this domain, including work with parallelograms.

In this lesson, students analyze the defining attributes of **parallelograms**, observe other properties that follow from that definition, and use reasoning strategies from previous lessons to find the areas of parallelograms.

By decomposing and rearranging parallelograms into rectangles, and by enclosing a parallelogram in a rectangle and then subtracting the area of the extra regions, students begin to see that parallelograms have related rectangles that can be used to find the area.

Throughout the lesson, students encounter various parallelograms that, because of their shape, encourage the use of certain strategies. For example, some can be easily decomposed and rearranged into a rectangle. Others—such as ones that are narrow and stretched out—may encourage students to enclose them in rectangles and subtract the areas of the extra pieces (two right triangles).

After working with a series of parallelograms, students attempt to generalize (informally) the process of finding the area of any parallelogram (MP8).

Note that these materials use the “dot” notation (for example $2 \cdot 3$) to represent multiplication instead of the “cross” notation (for example 2×3). This is because students will be writing many

algebraic expressions and equations in this course, sometimes involving the letter x used as a variable. This notation will be new for many students, and they will need explicit guidance in using it.

Alignments

Building On

- 4.G.A.2: Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
- 5.G.B: Classify two-dimensional figures into categories based on their properties.

Addressing

- 6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display
- MLR3: Clarify, Critique, Correct
- Notice and Wonder

Required Materials

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Student Learning Goals

Let's investigate the features and area of parallelograms.

4.1 Features of a Parallelogram

Warm Up: 10 minutes

Prior to grade 6, students learned that lines are parallel if they never intersect. In this activity, students look at the defining attributes of **parallelograms**—a quadrilateral whose opposite sides

are parallel. They observe other properties that follow from that definition—that opposite sides of a parallelogram have the same length and opposite angles have the same measure.

Students' initial investigation of parallelograms should involve lots of examples and non-examples, giving them opportunities to look for and express regularity in repeated reasoning (MP8) and seek and make use of structure (MP7). This activity assumes that students have had some exposure to parallelograms, but is also accessible to students who have not.

Building On

- 4.G.A.2
- 5.G.B

Instructional Routines

- Notice and Wonder

Launch

Display the image of figures A–F for all to see. Give students a minute to observe it and to prepare to share at least one thing they notice and one thing they wonder. When the minute is up, invite students to share their responses with the class.

Students may notice that:

- all except one figure (E) are quadrilaterals.
- figures C and E have sides that are all equal in length.
- figures A and B have two pairs of equal sides.
- figures B and C are rectangles.
- none of the sides in D are parallel.
- two of the sides in F are parallel.

They may wonder:

- why a hexagon is in the set.
- if the sides of figure A are all equal.
- if figure C is a parallelogram.

One or more students are likely to mention “parallelogram” in their observations or questions. Tell students that they will look closely at parallelograms in this lesson. Read aloud the opening sentences in the task statement. Clarify that A, B, and C are examples of **parallelograms**, and that D, E, and F are non-examples (i.e., they are not parallelograms).

Arrange students into groups of 2 and give them geometry toolkits. Give students 3 minutes of quiet think time to complete the task. Afterwards, give them a minute to discuss their answers and observations with their partner.

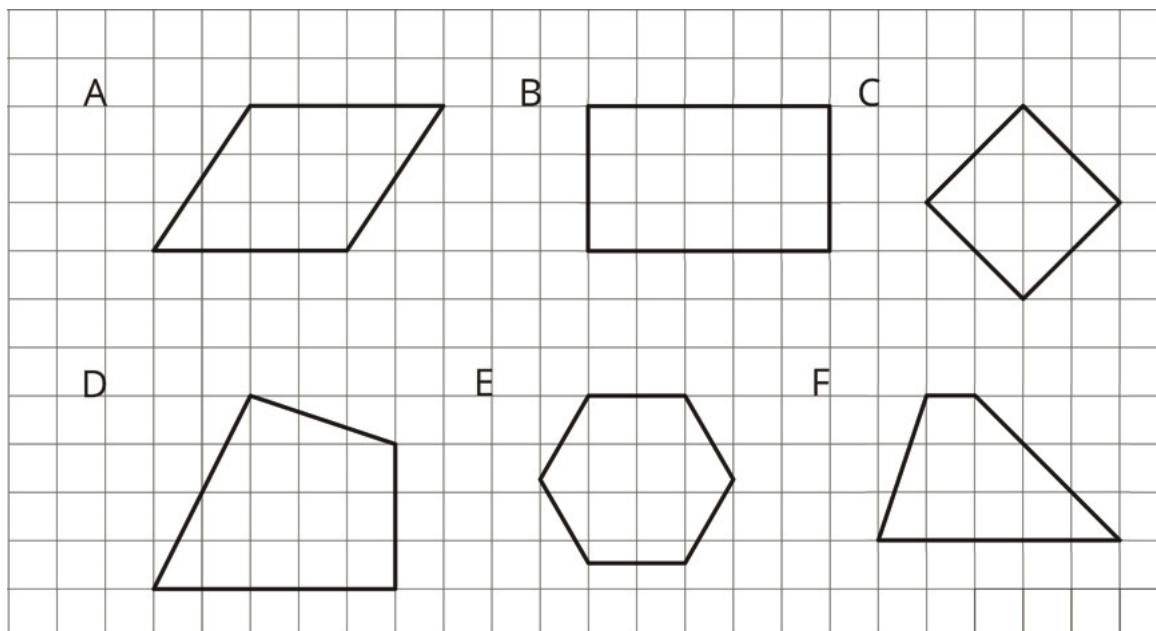
Anticipated Misconceptions

Students may have trouble seeing C as a square because of its orientation. They may also think that squares and rectangles are not parallelograms. Explain that the definition we are using for parallelograms is: a quadrilateral where both pairs of opposite sides are parallel. By that definition, rectangles and squares are special kinds of parallelograms.

If students wonder how they would know if two sides are parallel, explain that a consequence of “never intersecting” is that the length of a perpendicular line segment between them always has the same length. Students can use an index card to check this in figures A and C.

Student Task Statement

Figures A, B, and C are **parallelograms**. Figures D, E, and F are *not* parallelograms.



Study the examples and non-examples. What do you notice about:

1. the number of sides that a parallelogram has?
2. opposite sides of a parallelogram?
3. opposite angles of a parallelogram?

Student Response

1. Parallelograms have four sides.
2. Opposite sides of parallelograms are parallel and have equal length.

3. Opposite angles of parallelograms have equal size.

Activity Synthesis

Ask a few students to share their responses to the questions. After each response, ask students to indicate whether they agree. If a student disagrees, discuss the disagreement. Record the agreed-upon responses for all to see and highlight that:

- A parallelogram is a polygon with four sides, and both pairs of opposite sides are parallel.
- Opposite sides have equal length.
- Opposite angles have equal measure.

Tell students that for now we will just take properties about parallelograms as facts, and that later on in their schooling they will learn some ways to prove that they are always true.

If time permits, revisit figures D, E, and F. Ask students to explain why these are non-examples and see if students connect their explanations to the properties of parallelogram.

- “Why is figure D not a parallelogram?”
- “Why is figure E not a parallelogram? What about F?”

4.2 Area of a Parallelogram

15 minutes (there is a digital version of this activity)

In this activity, students explore different methods of decomposing a parallelogram and rearranging the pieces to find its area. Presenting the parallelograms on a grid makes it easier for students to see that the area does not change as they decompose and rearrange the pieces.

This investigation lays a foundation for upcoming work with area of triangles and other polygons. Here are the two key approaches for finding the area of parallelograms:

- Decompose the parallelogram, rearrange the parts into a rectangle, and multiply the side lengths of the rectangle to find the area.
- Enclose the parallelogram in a rectangle and subtract the combined area of the extra regions.

As students work and discuss, monitor for these approaches. Identify students whose reasoning can highlight the usefulness of a related rectangle for finding the area of a parallelogram.

Some students may begin by counting squares because it is a strategy used in earlier grades. This strategy is not reinforced here. Instead, encourage students to listen for and try more sophisticated, grade-appropriate methods shared during the class discussion.

In the digital version of the activity, students are given rectangles and right triangles as tools to visualize their reasoning (decomposition, rearrangement, and enclosure).

Addressing

- 6.G.A.1

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display

Launch

Arrange students in groups of 2–4. Ask students to find the area of the parallelogram using recently learned strategies and tools. Prepare extra copies of the task in case students wish to cut the parallelogram out.

Give students 5 minutes of quiet think time and access to their toolkits. Ask them to share their strategies with their group afterwards.

To encourage students to be mindful of their strategies and to plant the seed for the whole-class discussion, display and read aloud the following reflection questions before students begin working.

- Why did you decompose the parallelogram the way you did?
- Why did you rearrange the pieces the way you did?

For digital classrooms, project the applet to introduce it. Ask students to experiment with the given polygons to find the area of the parallelograms. For the second question, students are given the same starting parallelogram as in the first question. They will need to move the vertices to change it into a different parallelogram before finding its area.

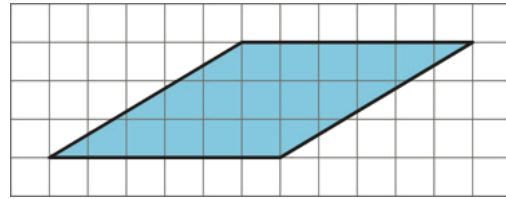
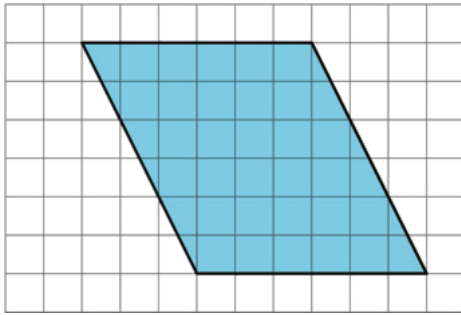
Support for English Language Learners

Speaking, Listening: MLR2 Collect and Display. Circulate and listen to students talk about the similarities and differences between the two approaches to finding the area. Write down common or important phrases you hear students say about each approach on a visual display with the labels “Decompose, Rearrange, Enclose.” As the lesson continues, update the visual display and remind students to borrow language from the display as needed. This will help students use mathematical language during their paired and whole-group discussions.

Design Principle(s): Support sense-making; Maximize meta-awareness

Student Task Statement

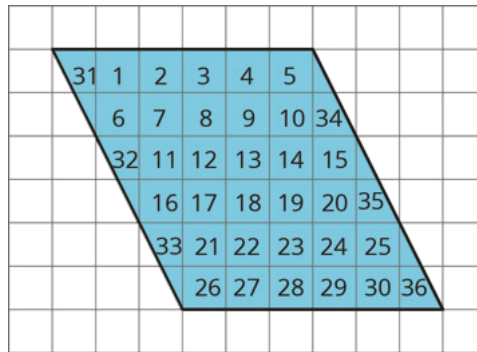
Find the area of each parallelogram. Show your reasoning.



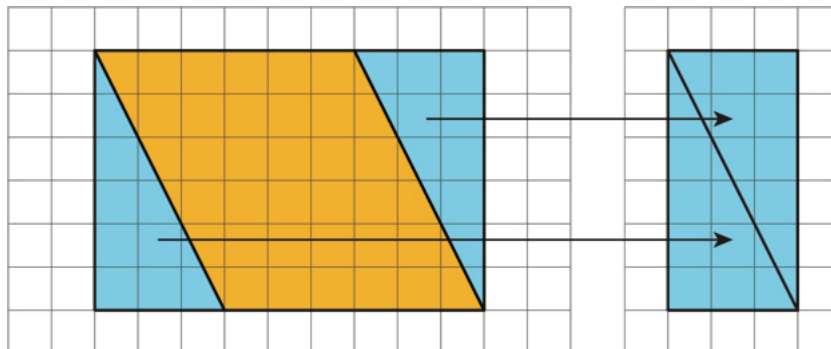
Student Response

1. 36 square units. Possible strategies:

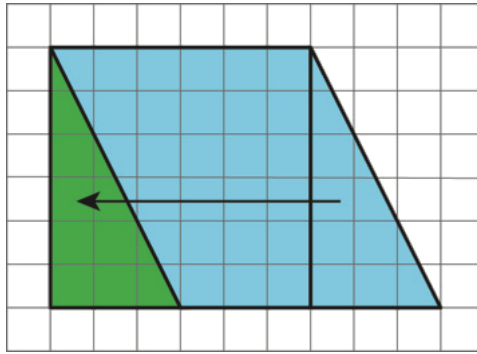
- Count the squares. This strategy is not encouraged or to be shared, but it may arise.



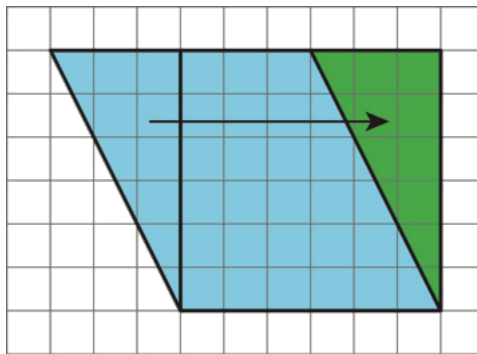
- Enclose the parallelogram within a rectangle and subtract the extra pieces. To subtract the area of the two right triangles, students may count the squares, or put them together to form a rectangle. $(6 \cdot 9) - (6 \cdot 3) = 36$.



- Decompose the parallelogram as shown and move the right triangle to form a rectangle. $6 \cdot 6 = 36$.



- Decompose the parallelogram as shown and move the right triangle to form a rectangle.
 $6 \cdot 6 = 36$.



2. 18 square units. Reasonings vary, but should involve decomposition and rearrangement, or other area reasoning strategies.

Activity Synthesis

Invite selected students to share how they found the area of the first parallelogram. Begin with students who decomposed the parallelogram in different ways. Follow with students who enclosed the parallelogram and rearranged the extra right triangles.

As students share, display and list the strategies for all to see. (Restate them in terms of decomposing, rearranging, and enclosing, as needed.) The list will serve as a reference for upcoming work. If one of the key strategies is not mentioned, illustrate it and add it to the list.

Use the reflection questions in the Launch to help highlight the usefulness of rectangles in finding the area of parallelograms. Consider using the applet to illustrate this point, <https://ggbm.at/kj5DcRvn>.

Support for Students with Disabilities

Representation: Internalize Comprehension. Use color and annotations to highlight distinctions between strategies. Create a display that includes multiple copies of each parallelogram. As students describe their strategies, use color and annotation to scribe their thinking so that it is visible for all students. Be sure to label each figure with the strategy described (decomposing, rearranging, or enclosing).

Supports accessibility for: Visual-spatial processing; Conceptual processing

4.3 Lots of Parallelograms

15 minutes

In this activity, students continue to reason about areas of parallelograms, both on and off the grid.

While there is not one correct way to find the area of a parallelogram, each parallelogram here is designed to elicit a particular strategy. Parallelograms A and C encourage decomposing and rearranging the parallelograms into a rectangle. Parallelogram B is not as easy to decompose and rearrange (though some students are likely to first try that approach), and may prompt students to enclose the parallelograms and subtract the areas of the extra pieces. The grid and its absence allow students to reason concretely and abstractly, respectively, about the measurements that they need to find the area.

As students work, monitor for uses of the two main strategies—decompose-and-rearrange and enclose-and-subtract. The three measurements in Parallelogram C may give students pause. Notice students who are able to reason about why the side that is 4.5 inches long is not needed for finding the area of the parallelogram.

With repeated reasoning, students may begin to see the segments and measurements that tend to be useful in finding area (MP8). There are no references to bases or heights in this lesson, but highlighting the segments and measurements that helped students reason about area here can help to support students' future work. In the next lesson, awareness of the measurements that affect the area of a parallelogram will be formalized.

Addressing

- 6.G.A.1

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR3: Clarify, Critique, Correct

Launch

Keep students in groups of 2–4 and ask them to find the areas of several more parallelograms. Give them 7–8 minutes of quiet think time, followed by a couple of minutes to share their strategies with their groups. Ask students to attempt at least the first two questions individually before discussing with their group. Provide access to their geometry toolkits.

To prepare for the whole-class discussion, consider asking students to think, as they work through the problems, about what measurements of the parallelogram seem to be helpful for finding its area.

Support for Students with Disabilities

Representation: Internalize Comprehension. Represent the same information through different modalities by using physical objects to represent abstract concepts. Some students may benefit by starting with an enlarged printed copy of the task to draw on or highlight for calculating areas of parallelograms.

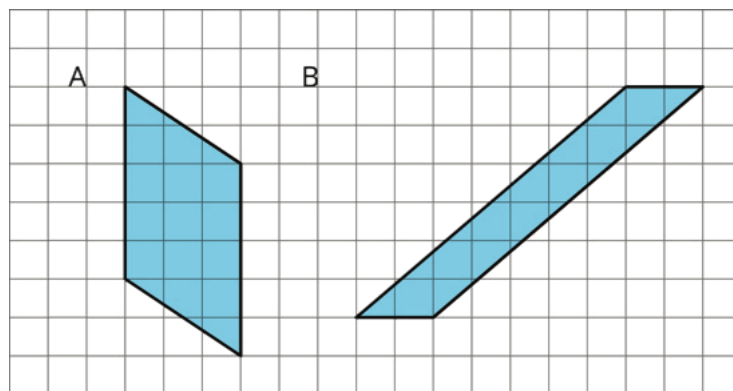
Supports accessibility for: Conceptual processing; Visual-spatial processing

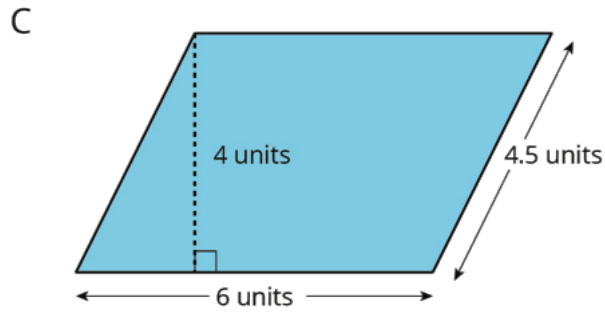
Anticipated Misconceptions

Some students may think that it is not possible to decompose and rearrange Parallelogram A because it has a pair of vertical sides instead of a pair of horizontal sides. Rotating their paper 90 degrees and back might help them see that they could still use the same reasoning strategy. Or they may find it helpful to first reason about area with the parallelogram rotated 90 degrees and then rotating it back to its original orientation.

Student Task Statement

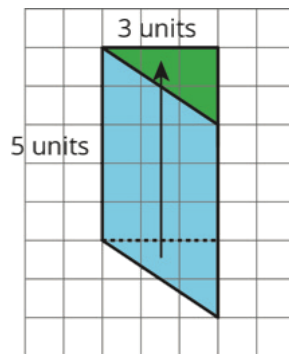
Find the area of each parallelogram. Show your reasoning.





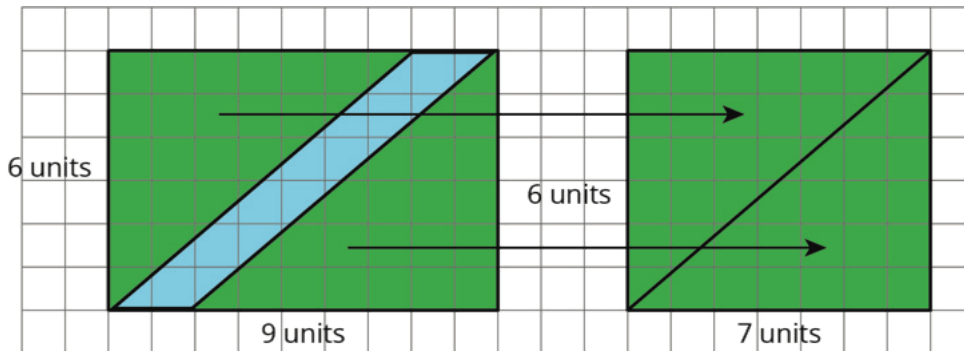
Student Response

Parallelogram A: 15 square units. Possible strategy: Decompose and rearrange the pieces to form a rectangle and multiply the side lengths of the rectangle to find the area. $5 \cdot 3 = 15$.



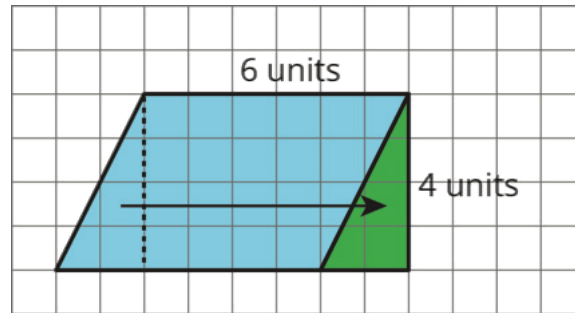
Parallelogram B: 12 square units. Possible strategy:

Enclose the parallelogram and subtract the area of the extra pieces. The area of the extra pieces is found by rearranging the triangles to form a rectangle. $6 \cdot 9 - 6 \cdot 7 = 12$.



Parallelogram C: 24 square units. Possible strategy:

Decompose the parallelogram and rearrange into a rectangle. Multiply the side lengths of the rectangle. $6 \cdot 4 = 24$.



Activity Synthesis

For each parallelogram, select 1–2 students with differing strategies to share their work with the class, starting with the less-efficient strategy. If an important strategy is not mentioned, bring it up and illustrate it. Briefly poll the class to see how others approached the problem. After hearing from students on each problem, consider asking questions such as the following. Focus the discussion on parallelograms B and C.

- “Is Parallelogram A different than others you have seen so far? How so?” (Students may answer yes or no, but some may see it as different because it has a pair of vertical sides.) “Can it still be decomposed and rearranged into a shape whose area we know how to calculate?” (Yes.)
- “Which strategy—decomposing and rearranging, or enclosing and subtracting—seems more practical for finding a parallelogram such as B? Why?” (Enclosing and subtracting, because it can be done in fewer steps. Decomposing the figure into small pieces could get confusing and lead to errors.)
- “If you decomposed C into a right triangle and another shape, how do you know that the cut-out piece actually fits on the other side, given that there’s no grid to verify?” (The two opposite sides of a parallelogram are parallel, so the longest (slanted) side of the right triangle that is rearranged would match up perfectly with the segment on the other side. Make available tracing paper or a copy of the drawing for cutting and rearranging physically so students can verify.)
- “Three measurements are shown for Parallelogram C. Which ones did you use? Which ones did you not use? Why and why not?” (The 4 units and 6 units are side lengths of a rectangle that has the same area of the parallelogram. If we decompose the parallelogram with a vertical cut and move the piece on the left to the right to make a rectangle, the 4.5-unit length is no longer relevant.)
- “Why did your strategy make the most sense to you for this parallelogram?”

To help students make connections and generalize their observations, ask questions such as:

- “When you decomposed and rearranged the parallelogram into another shape, did the area change?” (No.)

- “Why use a rectangle?”(We know how to find the area of a rectangle; we can multiply the two side lengths.)
- “For those of you who enclosed the parallelogram with a rectangle, how did the two right triangles help you?” (They can be combined into a rectangle, whose area we can find and subtract from the area of a large rectangle.)
- “Which measurements or lengths were useful for finding the area of the parallelogram?” (One side length of the parallelogram and the length of a perpendicular segment between that side and the opposite side.) Which lengths did you not use?” (The other side length.)

Support for English Language Learners

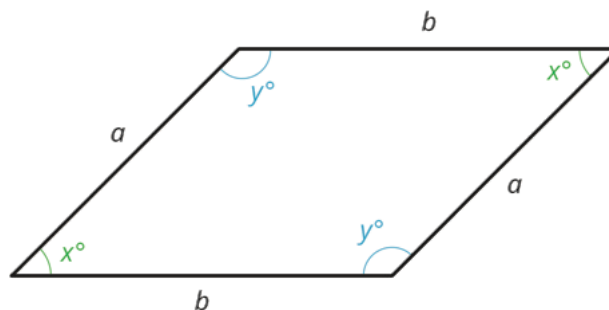
Writing, Speaking: MLR3 Clarify, Critique, Correct. Before students share their work, present the following incorrect statement to the class: “To calculate the area of parallelogram A, I counted 3 units on the diagonal side and 5 units on the vertical side. Then, I multiplied $5 \cdot 3$ to get 15 square units.” Ask students to identify the error, critique the reasoning, and write a correct explanation. As students discuss with a partner, listen for students who identify and can explain why the length of a diagonal segment cannot be easily used to find the area of a parallelogram. Invite students to share their critiques and corrected explanations with the class. Amplify the language students use to describe measurements that can be helpful for finding the area. This will support students’ use of mathematical language when the class discusses the strategies of decompose-and-rearrange and enclose-and-subtract.

Design Principle(s): Optimize output (for explanation); Maximize meta-awareness

Lesson Synthesis

Revisit the definition of a **parallelogram** with students: A parallelogram has four sides. The opposite sides of a parallelogram are parallel. Remind them that, as a result of the sides being parallel, it is also true that:

- The opposite sides of a parallelogram have equal length.
- The opposite angles of a parallelogram have equal measure.



Tell students that, while we are just taking these properties as facts for now, in later grades they will be able to prove these for themselves.

Briefly revisit the last task, displaying for all to see the multiple area strategies students used. Point out that in some cases, students chose to decompose and rearrange parts, and in others they chose to enclose the parallelogram with a rectangle and subtract the area of the extra pieces from the area of the rectangle. Ask about a couple of the parallelograms: "What was it about that parallelogram that prompted that particular choice?"

4.4 How Would You Find the Area?

Cool Down: 5 minutes

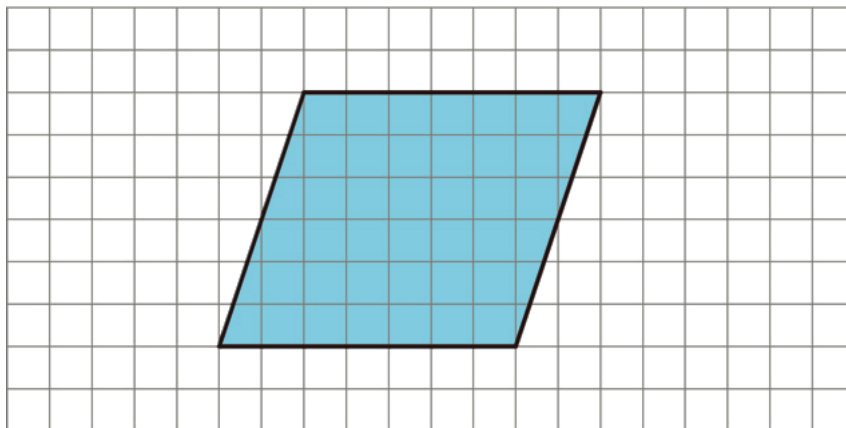
This activity sets the stage for the next lesson, which formalizes how to find the area of any parallelogram. Notice the strategies students are currently using to help make connections to the algebraic expression $b \cdot h$ that they will see in the next lesson.

Addressing

- 6.G.A.1

Student Task Statement

How would you find the area of this parallelogram? Describe your strategy.



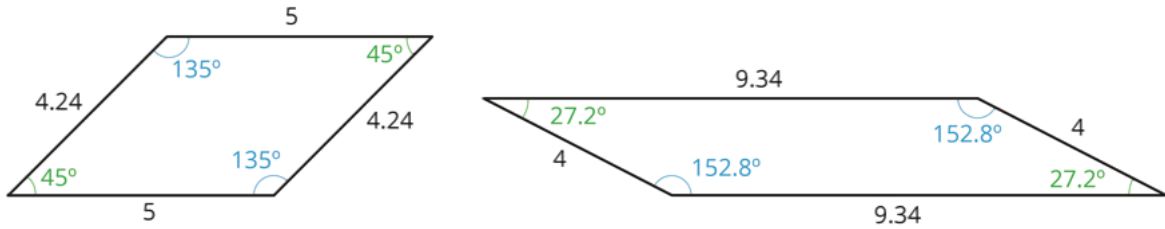
Student Response

Answers vary. Sample responses:

- Decompose a triangle from one side of the parallelogram and move it to the other side to make a rectangle. Multiply the side lengths of the rectangle.
- Draw a rectangle around the parallelogram, multiply the side lengths of the rectangle to find the area of the rectangle and subtract the combined area of the triangles that do not belong to the parallelogram.
- Count how many squares are across the bottom of the parallelogram and how many squares tall it is and multiply them.

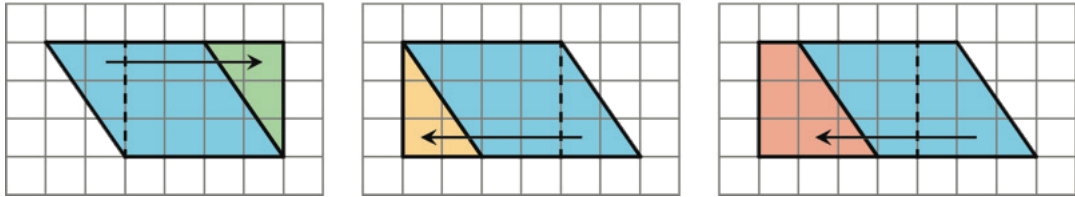
Student Lesson Summary

A **parallelogram** is a quadrilateral (it has four sides). The opposite sides of a parallelogram are parallel. It is also true that the opposite sides of a parallelogram have equal length, and the opposite angles of a parallelogram have equal measure.

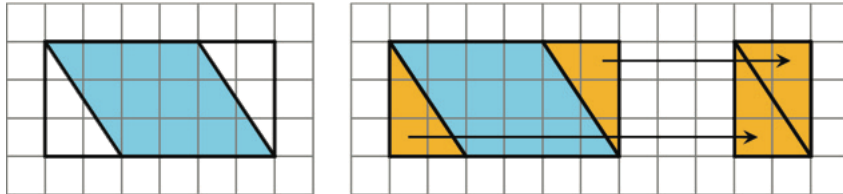


There are several strategies for finding the area of a **parallelogram**.

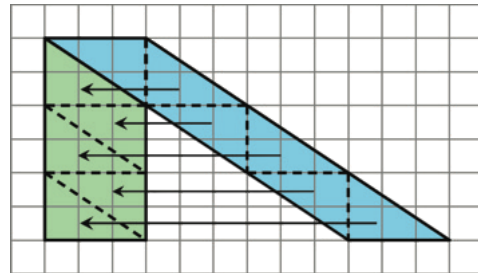
- We can decompose and rearrange a parallelogram to form a rectangle. Here are three ways:



- We can enclose the parallelogram and then subtract the area of the two triangles in the corner.



Both of these ways will work for any parallelogram. However, for some parallelograms the process of decomposing and rearranging requires a lot more steps than if we enclose the parallelogram with a rectangle and subtract the combined area of the two triangles in the corners.



Glossary

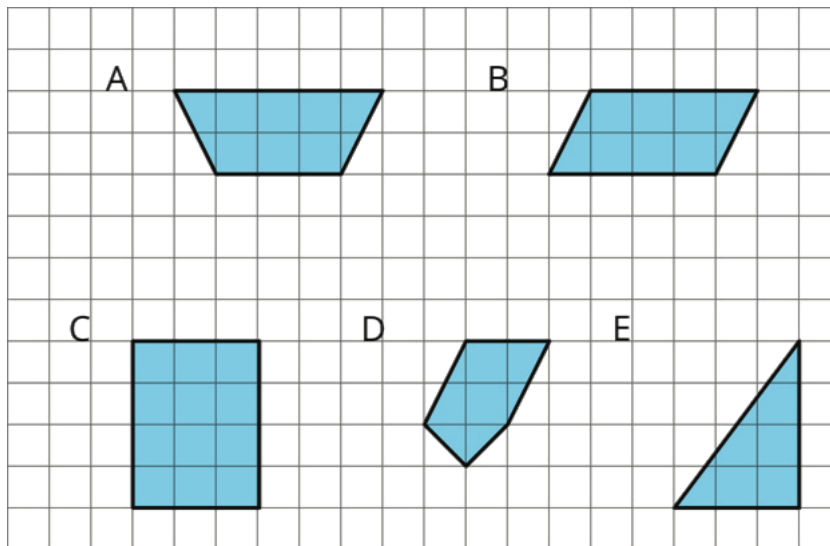
- parallelogram
- quadrilateral

Lesson 4 Practice Problems

Problem 1

Statement

Select **all** of the parallelograms. For each figure that is *not* selected, explain how you know it is not a parallelogram.



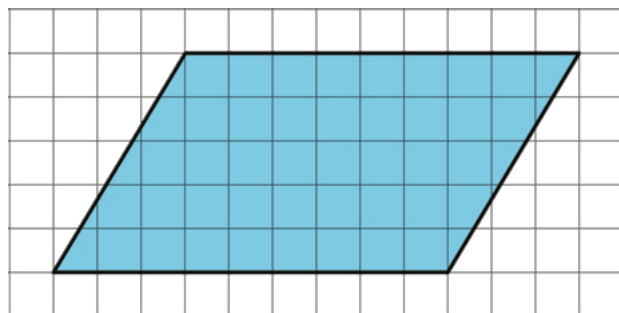
Solution

B and C are parallelograms (C is also a rectangle). A is a trapezoid (two opposite sides are not parallel and two are not the same length), D is a pentagon, and E is a (right) triangle.

Problem 2

Statement

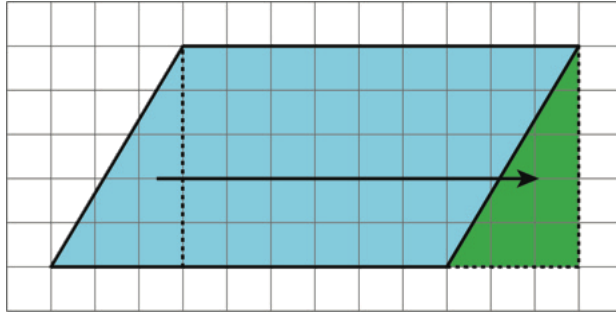
- a. Decompose and rearrange this parallelogram to make a rectangle.



- b. What is the area of the parallelogram? Explain your reasoning.

Solution

- a. Answers vary. Sample response: The diagram shows that we get a rectangle that is 5 units by 9 units by decomposing and rearranging.

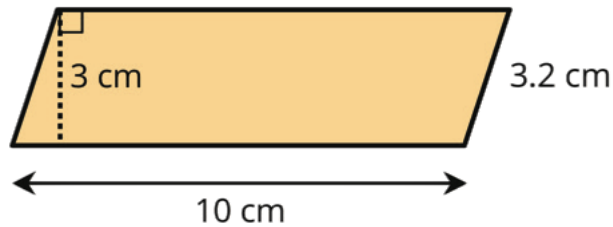


- b. The area of the parallelogram is the same as the area of the rectangle, which is 45 square units.

Problem 3

Statement

Find the area of the parallelogram.



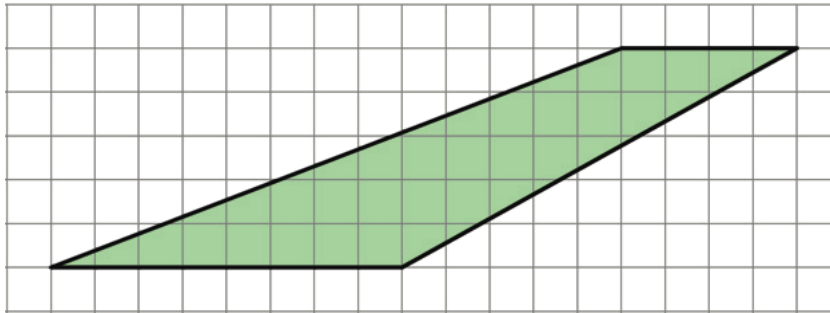
Solution

30 sq cm

Problem 4

Statement

Explain why this quadrilateral is *not* a parallelogram.



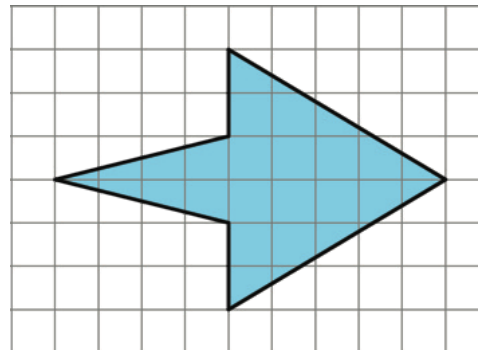
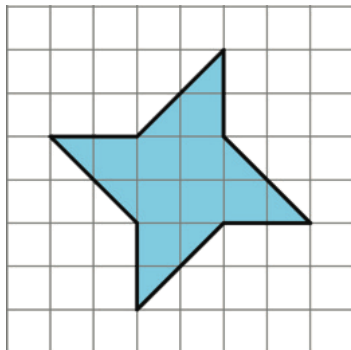
Solution

Explanations vary. Sample explanation: Opposite sides are not parallel and not the same length. Opposite angles are not equal.

Problem 5

Statement

Find the area of each shape. Show your reasoning.



Solution

12 square units, 19 square units. Reasoning varies.

(From Unit 1, Lesson 3.)

Problem 6

Statement

Find the area of the rectangle with each set of side lengths.

a. 5 in and $\frac{1}{3}$ in

b. 5 in and $\frac{4}{3}$ in

c. $\frac{5}{2}$ in and $\frac{4}{3}$ in

d. $\frac{7}{6}$ in and $\frac{6}{7}$ in

Solution

a. $\frac{5}{3}$ square inches

b. $\frac{20}{3}$ square inches

c. $\frac{10}{3}$ square inches

d. 1 square inch

(From Unit 1, Lesson 1.)

Lesson 5: Bases and Heights of Parallelograms

Goals

- Comprehend the terms “base” and “height” to refer to one side of a parallelogram and the perpendicular distance between that side and the opposite side.
- Generalize (orally) a process for finding the area of a parallelogram, using the length of a base and the corresponding height.
- Identify a base and the corresponding height for a parallelogram, and understand that there are two different base-height pairs for any parallelogram.

Learning Targets

- I can identify pairs of base and height of a parallelogram.
- I can write and explain the formula for the area of a parallelogram.
- I know what the terms “base” and “height” refer to in a parallelogram.

Lesson Narrative

Students begin this lesson by comparing two strategies for finding the area of a parallelogram. This comparison sets the stage both for formally defining the terms **base** and **height** and for writing a general formula for the area of a parallelogram. Being able to correctly identify a base-height pair for a parallelogram requires looking for and making use of structure (MP7).

The terms **base** and **height** are potentially confusing because they are sometimes used to refer to particular line segments, and sometimes to the length of a line segment or the distance between parallel lines. Furthermore, there are always two base-height pairs for any parallelogram, so asking for *the* base and *the* height is not, technically, a well-posed question. Instead, asking for *a* base and its corresponding height is more appropriate. As students clarify their intended meaning when using these terms, they are attending to precision of language (MP6). In these materials, the words “base” and “height” mean the numbers unless it is clear from the context that it means a segment and that there is no potential confusion.

By the end of the lesson, students both look for a pattern they can generalize to the formula for the area of a rectangle (MP8) and make arguments that explain why this works for all parallelograms (MP3).

Alignments

Addressing

- 6.EE.A.2.a: Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation “Subtract y from 5” as $5 - y$.
- 6.EE.A.2.c: Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including

those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.

- 6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Instructional Routines

- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Notice and Wonder
- Think Pair Share

Required Materials

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Student Learning Goals

Let's investigate the area of parallelograms some more.

5.1 A Parallelogram and Its Rectangles

Warm Up: 10 minutes (there is a digital version of this activity)

In this warm-up, students compare and contrast two ways of decomposing and rearranging a parallelogram on a grid such that its area can be found. It serves a few purposes: to reinforce the work done in the previous lesson; to allow students to practice communicating their observations; and to shed light on the features of a parallelogram that are useful for finding area—a base and a corresponding height.

The flow of key ideas—to be uncovered during discussion and gradually throughout the lesson—is as follows:

- There are multiple ways to decompose a parallelogram (with one cut) and rearrange it into a rectangle whose area we can determine.

- The cut can be made in different places, but to compose a rectangle, the cut has to be at a *right angle* to two opposite sides of the parallelogram.
- The length of one side of this newly composed rectangle is the same as the length of one side of the parallelogram. We use the term **base** to refer to this side.
- The length of the other side of the rectangle is the length of the cut we made to the parallelogram. We call this segment a **height** that corresponds to the chosen base.
- We use these two lengths to determine the area of the rectangle, and thus also the area of the parallelogram.

As students work and discuss, identify those who make comparisons in terms of the first two points so they could share later. Be sure to leave enough time to discuss the first four points as a class.

Addressing

- 6.G.A.1

Instructional Routines

- Think Pair Share

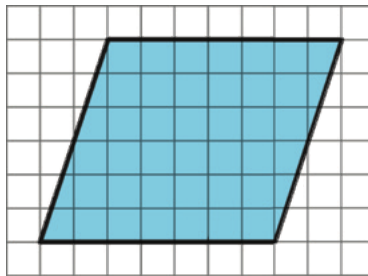
Launch

Arrange students in groups of 2. Give students 2 minutes of quiet think time and access to geometry toolkits. Ask them to share their responses with a partner afterwards.

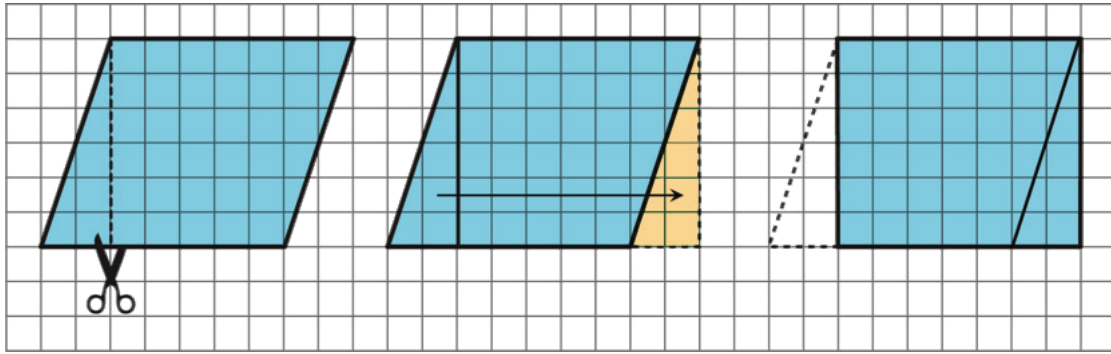
If using the digital activity, have students explore the applet for 3 minutes individually, and then discuss with a partner. Students will be able to see the cuts by dragging the point on the segment under the parallelogram.

Student Task Statement

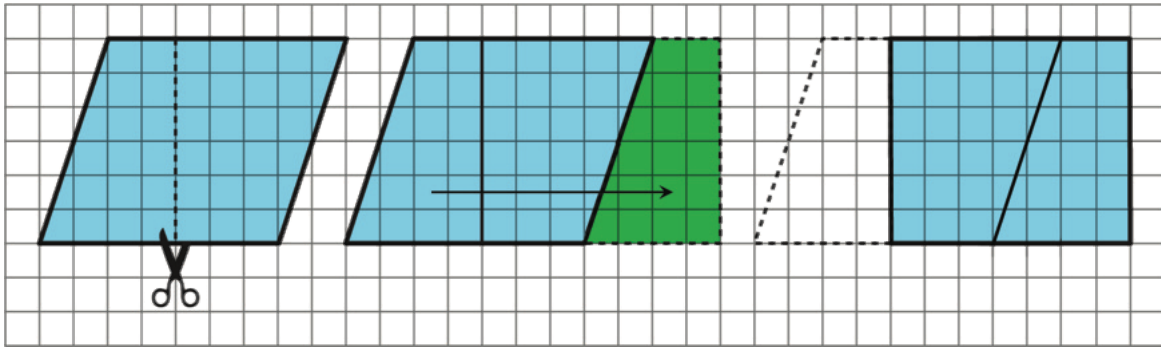
Elena and Tyler were finding the area of this parallelogram:



Here is how Elena did it:



Here is how Tyler did it:



How are the two strategies for finding the area of a parallelogram the same? How they are different?

Student Response

Answers vary. Sample responses:

- Similar: They both cut off a piece from the left of the parallelogram and moved it over to the right to make a rectangle. The rectangles they made are identical.
- Different: They cut the parallelogram at different places. Elena cut a right triangle from the left side and Tyler cut off a trapezoid. The rectangles they made are not in the same place.

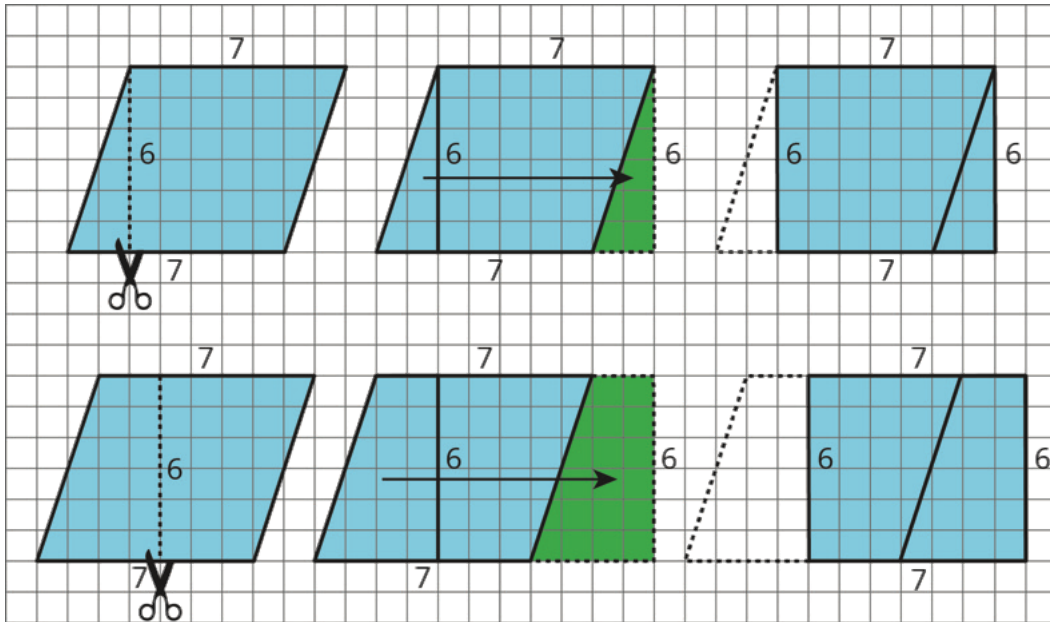
Activity Synthesis

Ask a few students to share what was the same and what was different about the methods they observed.

Highlight the following points on how Elena and Tyler's approaches are the same, though do not expect students to use the language. Instead, rely on pointing and gesturing to make clear what is meant. If any of these are not mentioned by the students, share them.

- The rectangles are identical; they have the same side lengths. (Label the side lengths of the rectangles.)
- The cuts were made in different places, but the length of the cuts was the same. (Label the lengths along the vertical cuts.)

- The horizontal sides of the parallelogram have the same length as the horizontal sides of the rectangle. (Point out how both segments have the same length.)
- The length of each cut is also the distance between the two horizontal sides of the parallelogram. It is also the vertical side length of the rectangle. (Point out how that distance stays the same across the horizontal length of the parallelogram.)



Begin to connect the observations to the terms **base** and **height**. For example, explain:

- “The two measurements that we see here have special names. The length of one side of the parallelogram—which is also the length of one side of the rectangle—is called a *base*. The length of the vertical cut segment—which is also the length of the vertical side of the rectangle—is called a *height* that corresponds to that base.”
- “Here, the side of the parallelogram that is 7 units long is also called a base. In other words, the word *base* is used for both the segment and the measurement.”

Tell students that we will explore bases and heights of a parallelogram in this lesson.

5.2 The Right Height?

20 minutes (there is a digital version of this activity)

Previously, students saw numerical examples of a base and a height of a parallelogram. This activity further develops the idea of base and height through examples and non-examples and error analysis.

Some important ideas to be uncovered here:

- In a parallelogram, the term “base” refers to the length of one side and “height” to the length of a perpendicular segment between that side and the opposite side.

- Any side of a parallelogram can be a base.
- There are always two base-height pairs for a given parallelogram.

Addressing

- 6.G.A.1

Instructional Routines

- MLR8: Discussion Supports
- Notice and Wonder
- Think Pair Share

Launch

Display the image of examples and non-examples of bases and heights for all to see. Read aloud the description for examples and non-examples. Give students a minute to observe it and to prepare to share at least one thing they notice and one thing they wonder about. When the minute is up, invite students to share their responses with the class, and record these for all to see. It isn't necessary to address their questions at this time.

Students may notice:

- Both sets of diagrams show the same 2 pairs of parallelograms and the same sides labeled "base."
- All the examples show a right-angle mark, a dashed segment, and a side labeled "base."
- Only one of the non-examples show a right-angle mark, but all of them show a dashed segment.
- In both examples and non-examples, there is one parallelogram with a dashed segment and a right angle shown outside of it.
- If the dashed segments are used to cut the first three parallelograms in the examples, the cut-out pieces could be rearranged to form a rectangle. The same cannot be done for the dashed segments in the non-examples.

They may wonder:

- why some dashed segments are inside the parallelogram and some are outside.
- what the rule might be for a dashed segment to be considered a height.
- what the bases and heights have to do with area.

Arrange students in groups of 2. Give students 4–5 minutes to complete the first question with their partner. Ask them to pause for a class discussion after the first question. Select a student or a group to make a case for whether each statement is true or false. If one or more students disagree,

ask them to explain their reasoning and discuss to reach a consensus. Before moving on to the next question, be sure students record the verified true statements so that they can be used as a reference later.

Give students 3–4 minutes of quiet time to answer the second question and another 2–3 minutes to share their responses with a partner. Ask them to focus partner conversations on the following questions, displayed for all to see:

- How do you know the parallelogram is labeled correctly or incorrectly?
- Is there another way a base and height could be labeled on this parallelogram?

After answering the questions, students using the digital activity can explore the applet ggbm.at/UnfbrN96 and use it to verify their responses and further their understanding of bases and heights. The applet is a dynamic parallelogram with a height displayed. It allows students to see placements of height in a variety of parallelograms and when any side is chosen as a base.

Support for English Language Learners

Listening, Speaking, Conversing: MLR8 Discussion Supports. As students share their responses with a partner, invite them to restate each others' reasoning using the terms base, height, and perpendicular. If needed, demonstrate the meaning of perpendicular multi-modally using manipulatives, drawings, and gestures. Encourage students to challenge each other when they disagree, using prompts such as "I agree because . . ." or "I disagree because . . ."

Design Principle(s): Support sense-making

Anticipated Misconceptions

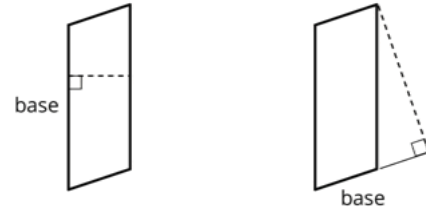
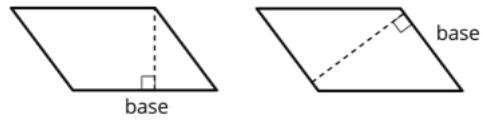
Students may not yet internalize that any side of parallelogram can be a base (they may think that a base must be the bottom, horizontal side), or that the height needs to be perpendicular to the base. Point out where the right angle symbols are located and how they relate to the height. Students may think a segment showing the height cannot be drawn outside of the parallelogram (as in Parallelogram C).

Students may relate how they think about the side lengths of a rectangle and inaccurately apply it to Parallelogram E.

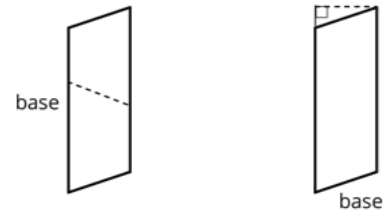
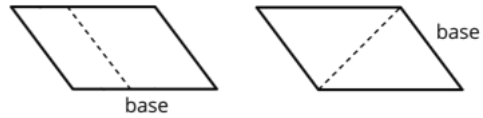
Student Task Statement

Study the examples and non-examples of **bases** and **heights** of parallelograms.

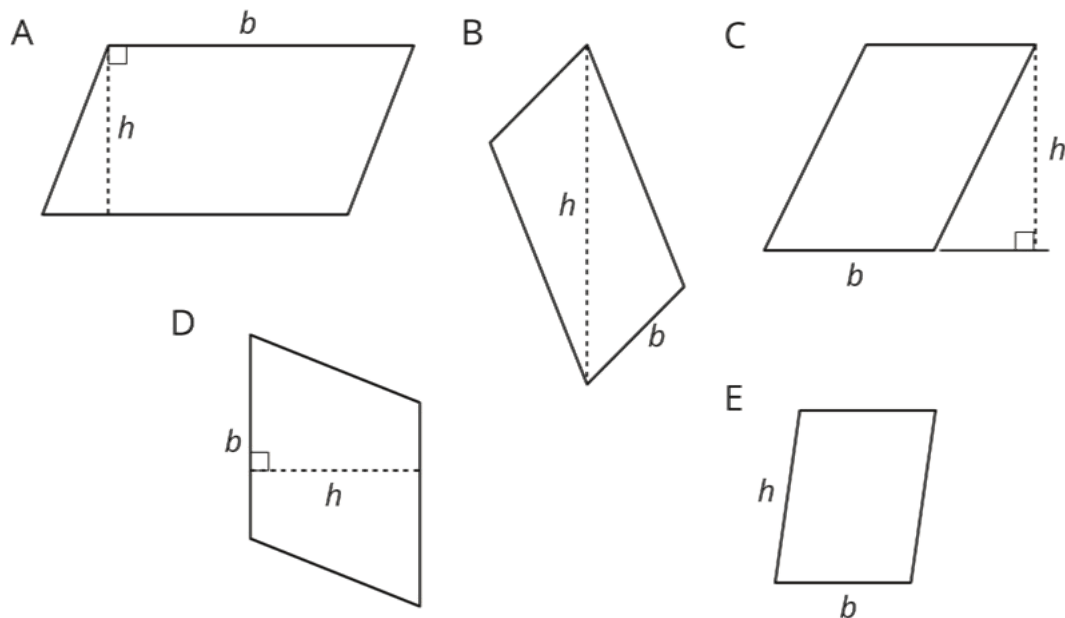
- Examples: The dashed segments in these drawings represent the corresponding height for the given base.



- Non-examples: The dashed segments in these drawings do *not* represent the corresponding height for the given base.



- Select **all** the statements that are true about bases and heights in a parallelogram.
 - Only a horizontal side of a parallelogram can be a base.
 - Any side of a parallelogram can be a base.
 - A height can be drawn at any angle to the side chosen as the base.
 - A base and its corresponding height must be perpendicular to each other.
 - A height can only be drawn inside a parallelogram.
 - A height can be drawn outside of the parallelogram, as long as it is drawn at a 90-degree angle to the base.
 - A base cannot be extended to meet a height.
- Five students labeled a base b and a corresponding height h for each of these parallelograms. Are all drawings correctly labeled? Explain how you know.



Student Response

1. Statements B, D, and F are true.
2. A, C, and D are correct. B and E are not correct because in each, the segment labeled with an h is not perpendicular to the side labeled with a b .

Activity Synthesis

Poll the class—with a quick agree-or-disagree signal—on whether each figure in the last question is labeled correctly with b and h . After each polling, ask a student to explain how they know it is correct or incorrect.

If a parallelogram is incorrectly labeled, ask where a correct height could be. If it is correctly labeled, ask students if there is another base and height that could be labeled on this parallelogram. Be sure students understand which parallelograms are labeled correctly before moving forward in this lesson.

An important point to emphasize: “We can choose any side of a parallelogram as a base. To find the height that corresponds to that base, draw a segment that joins the base and its opposite side; that segment has to be perpendicular to both.”

Consider using the applet ggbm.at/UnfbrN96 to further illustrate possible base-height pairs and reinforce students' understanding of them.

5.3 Finding the Formula for Area of Parallelograms

15 minutes

In previous lessons, students reasoned about the area of parallelograms by decomposing, rearranging, and enclosing them and by using what they know about the area of rectangles. They

also identified base-height pairs in parallelograms. Here, they use what they learned to find the area of new parallelograms, generalize the process, and write an expression for finding the area of any parallelogram.

As students discuss their work, monitor conversations for any disagreements between partners. Support them by asking clarifying questions:

- “How did you choose a base? How can you be sure that is the height?”
- “How did you find the area? Why did you choose that strategy for this parallelogram?”
- “Is there another way to find the area and to check our answer?”

Addressing

- 6.EE.A.2.a
- 6.G.A.1

Instructional Routines

- MLR7: Compare and Connect
- Think Pair Share

Launch

Keep students in groups of 2. Give students access to their geometry toolkits and 5–7 minutes of quiet think time to complete the first four rows of the table. Ask them to be prepared to share their reasoning. If time is limited, consider splitting up the work: have one partner work independently on parallelograms A and C, and the other partner on B and D. Encourage students to use their work from earlier activities (on bases and heights) as a reference.

Ask students to pause after completing the first four rows and to share their responses with their partner. Then, they should discuss how to write the expression for the area of any parallelogram. Students should notice that the area of every parallelogram is the product of a base and its corresponding height.

Support for Students with Disabilities

Representation: Internalize Comprehension. Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. As students describe their thinking, highlight the base-height pairs on each parallelogram and record the responses in the table.

Supports accessibility for: Visual-spatial processing; Conceptual processing

Anticipated Misconceptions

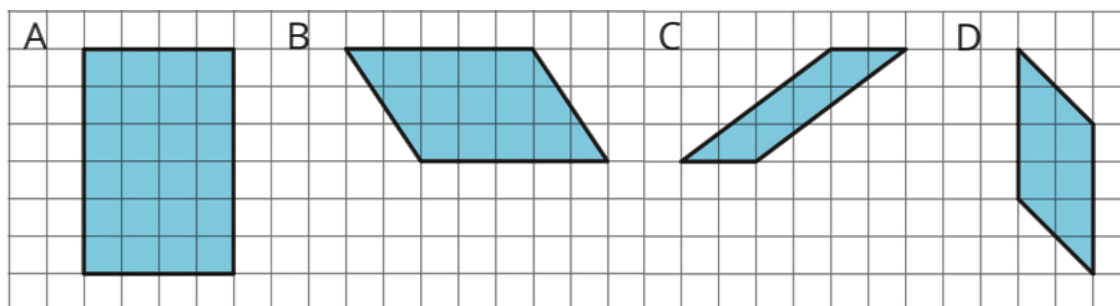
Finding a height segment outside of the parallelogram may still be a rather unfamiliar idea to students. Have examples from the “The Right Height?” section visible so they can serve as a reference in finding heights.

Students may say that the base of Parallelogram D cannot be determined because, as displayed, it does not have a horizontal side. Remind students that in an earlier activity we learned that any side of a parallelogram could be a base. Ask students to see if there is a side whose length can be determined.

Student Task Statement

For each parallelogram:

- Identify a base and a corresponding height, and record their lengths in the table.
- Find the area of the parallelogram and record it in the last column of the table.



parallelogram	base (units)	height (units)	area (sq units)
A			
B			
C			
D			
any parallelogram	b	h	

In the last row, write an expression for the area of any parallelogram, using b and h .

Student Response

While there are two possible base-height pairs, these are the easiest ones for students to use given the orientation of each parallelogram on the grid.

parallelogram	base (units)	height (units)	area (square units)
A	6 (or 4)	4 (or 6)	24
B	5	3	15
C	2	3	6
D	4	2	8
any parallelogram	b	h	$b \cdot h$

Are You Ready for More?

1. What happens to the area of a parallelogram if the height doubles but the base is unchanged? If the height triples? If the height is 100 times the original?
2. What happens to the area if both the base and the height double? Both triple? Both are 100 times their original lengths?

Student Response

1. The area doubles, triples, is multiplied by 100.
2. The area quadruples, is 9 times the original area, is 10,000 times the original area.

Activity Synthesis

Display the parallelograms and the table for all to see. Select a few students to share the correct answers for each parallelogram. As students share, highlight the base-height pairs on each parallelogram and record the responses in the table. Although only one base-height pair is named for each parallelogram, reiterate that there is another pair. Show the second pair on the diagram or ask students to point it out.

After all answers for the first four rows are shared, discuss the following questions, displayed for all to see:

- “How did you determine the expression for the area for any parallelogram?” (The areas of parallelograms A–D are each the product of base and height.)
- “Suppose you decompose a parallelogram with a cut and rearrange it into a rectangle. Does this expression for finding area still work? Why or why not?” (Yes. One side of the rectangle will be the same as the base of the parallelogram. The height of the parallelogram is also the height of the rectangle—both are perpendicular to the base.)
- “Do you think this expression will always work?”

Be sure everyone has the correct expression for finding the area of a parallelogram by the end of the discussion. The second discussion question is meant to elicit connections to the parallelogram’s

related rectangle as they decomposed and rearranged to find the area. The third question (about whether the expression will always work) is not meant to be proven here, so speculation on students' part is expected at this point. It is intended to prompt students to think of other differently-shaped parallelograms beyond the four shown here.

Support for English Language Learners

Representing, Conversing: MLR7 Compare and Connect. When students share their responses for the first four rows of the table, ask students to identify what is the same and what is different about how they determined the base and height for each parallelogram. Listen for and amplify student discussions that attempt to explain why their different approaches led to the same area. Then ask students if any side of a parallelogram can be used as the base. This will help students understand how the area of every parallelogram is product of the base and its corresponding height.

Design Principle(s): Support sense-making

Lesson Synthesis

In this lesson, we identified a **base** and a corresponding **height** in a parallelogram, and then wrote an algebraic expression for finding the area of any parallelogram.

- “How do you decide the base of a parallelogram?” (Any side can be a base. Sometimes one side is preferable over another because its length is known or easy to know.)
- “Once we have chosen a base, how can we identify a height that corresponds to it?” (Identify a perpendicular segment that connects that base and the opposite side; find the length of that segment.)
- “In how many ways can we identify a base and a height for a given parallelogram?” (There are two possible bases. For each base, many possible segments can represent the corresponding height.)
- “What is the relationship between the base and height of a parallelogram and its area?” (The area is the product of base and height.)

5.4 Parallelograms S and T

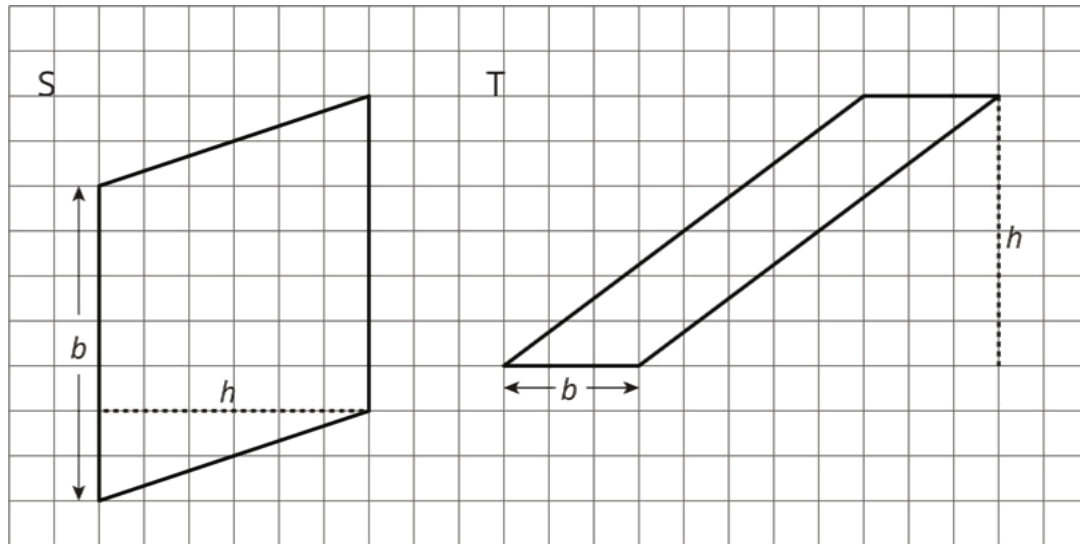
Cool Down: 5 minutes

Addressing

- 6.EE.A.2.c
- 6.G.A.1

Student Task Statement

Parallelograms S and T are each labeled with a base and a corresponding height.



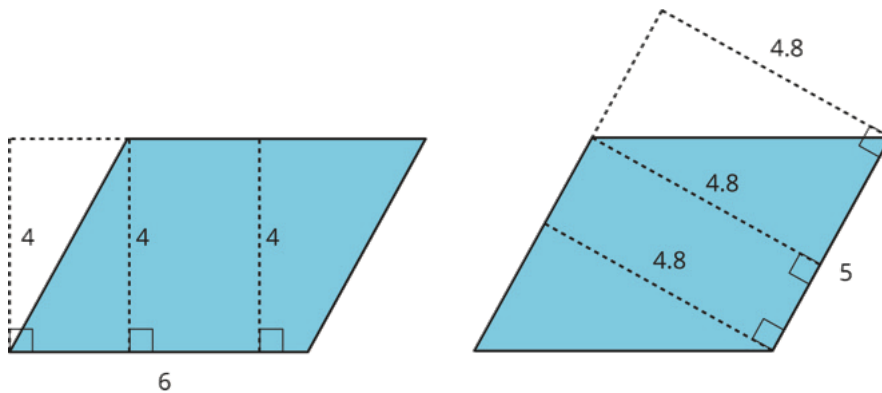
- What are the values of b and h for each parallelogram?
 - Parallelogram S: $b = \underline{\hspace{2cm}}$, $h = \underline{\hspace{2cm}}$
 - Parallelogram T: $b = \underline{\hspace{2cm}}$, $h = \underline{\hspace{2cm}}$
- Use the values of b and h to find the area of each parallelogram.
 - Area of Parallelogram S:
 - Area of Parallelogram T:

Student Response

- Parallelogram S: $b = 7, h = 6$
 - Parallelogram T: $b = 3, h = 6$
- Area of Parallelogram S: 42 square units. $7 \cdot 6 = 42$
 - Area of Parallelogram T: 18 square units. $3 \cdot 6 = 18$

Student Lesson Summary

- We can choose any of the four sides of a parallelogram as the **base**. Both the side (the segment) and its length (the measurement) are called the base.
- If we draw any perpendicular segment from a point on the base to the opposite side of the parallelogram, that segment will always have the same length. We call that value the **height**. There are infinitely many segments that can represent the height!



Here are two copies of the same parallelogram. On the left, the side that is the base is 6 units long. Its corresponding height is 4 units. On the right, the side that is the base is 5 units long. Its corresponding height is 4.8 units. For both, three different segments are shown to represent the height. We could draw in many more!

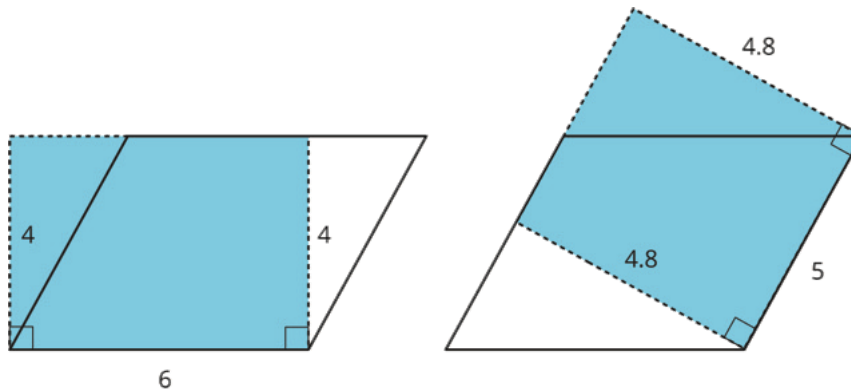
No matter which side is chosen as the base, the area of the parallelogram is the product of that base and its corresponding height. We can check this:

$$4 \times 6 = 24$$

and

$$4.8 \times 5 = 24$$

We can see why this is true by decomposing and rearranging the parallelograms into rectangles.



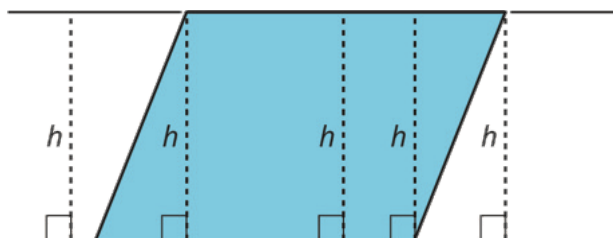
Notice that the side lengths of each rectangle are the base and height of the parallelogram. Even though the two rectangles have different side lengths, the products of the side lengths are equal, so they have the same area! And both rectangles have the same area as the parallelogram.

We often use letters to stand for numbers. If b is base of a parallelogram (in units), and h is the corresponding height (in units), then the area of the parallelogram (in square units) is the product of these two numbers.

$$b \cdot h$$

Notice that we write the multiplication symbol with a small dot instead of a \times symbol. This is so that we don't get confused about whether \times means multiply, or whether the letter x is standing in for a number.

In high school, you will be able to prove that a perpendicular segment from a point on one side of a parallelogram to the opposite side will always have the same length.



You can see this most easily when you draw a parallelogram on graph paper. For now, we will just use this as a fact.

Glossary

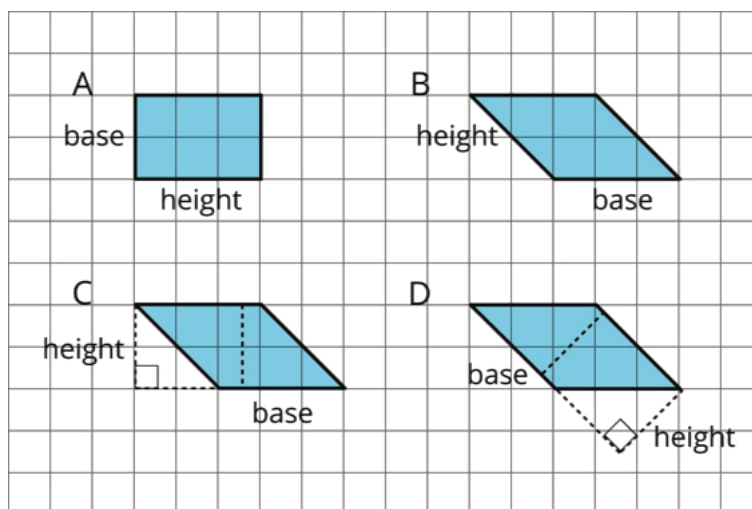
- base (of a parallelogram or triangle)
- height (of a parallelogram or triangle)

Lesson 5 Practice Problems

Problem 1

Statement

Select all parallelograms that have a correct height labeled for the given base.



- A. A
- B. B
- C. C
- D. D

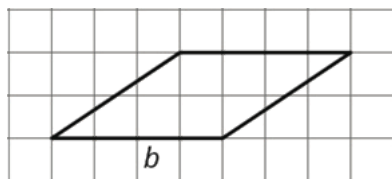
Solution

["A", "C", "D"]

Problem 2

Statement

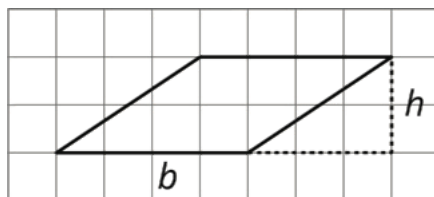
The side labeled b has been chosen as the base for this parallelogram.



Draw a segment showing the height corresponding to that base.

Solution

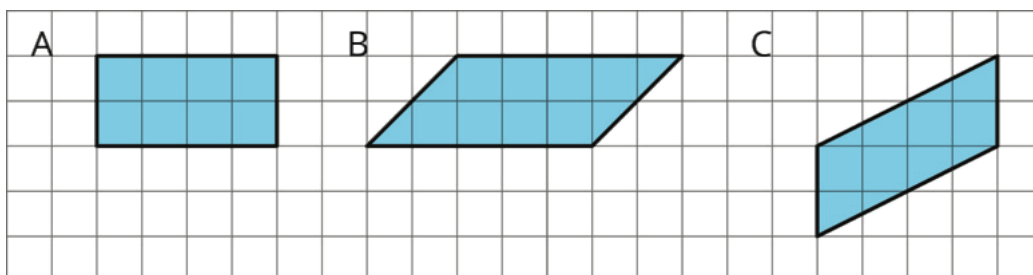
Answers vary, (The height can be any segment perpendicular to the base that joins the line containing the base to the line containing the side opposite the base). Sample response:



Problem 3

Statement

Find the area of each parallelogram.



Solution

A: 8 square units. (This is a 2-by-4 rectangle.)

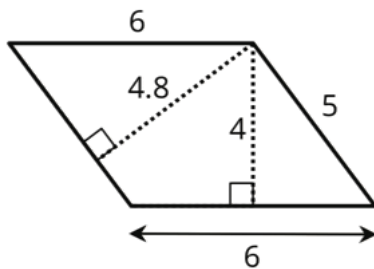
B: 10 square units. (The horizontal side is 5 units long and can be the base. The height for this base is 2 units.)

C: 8 square units. (The vertical side can be used as the base. The base is 2 units, and the height for this base is 4 units.)

Problem 4

Statement

If the side that is 6 units long is the base of this parallelogram, what is its corresponding height?



- A. 6 units
- B. 4.8 units
- C. 4 units
- D. 5 units

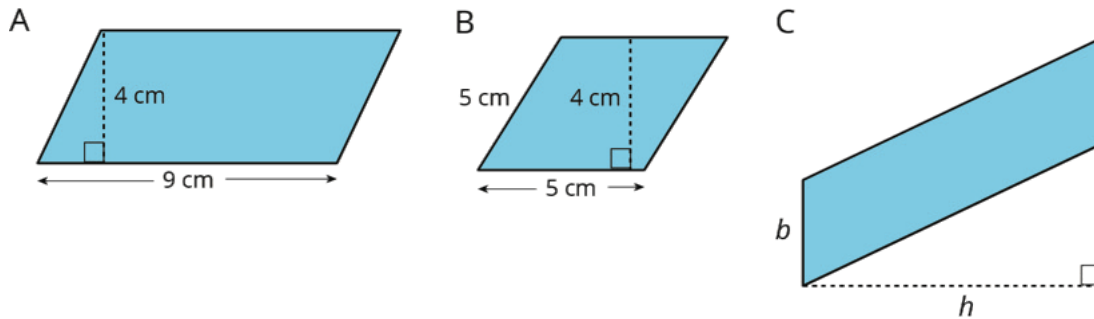
Solution

C

Problem 5

Statement

Find the area of each parallelogram.



Solution

A: 36 sq cm. (The base is 9 cm, and the height for that base is 4 cm.)

B: 20 sq cm. (The base is 5 cm, and the height for this base is 4 cm.)

C: bh . (The base is b , and the corresponding height is h .)

Problem 6

Statement

Do you agree with each of these statements? Explain your reasoning.

- A parallelogram has six sides.
- Opposite sides of a parallelogram are parallel.
- A parallelogram can have one pair or two pairs of parallel sides.
- All sides of a parallelogram have the same length.
- All angles of a parallelogram have the same measure.

Solution

- Disagree. A parallelogram is a quadrilateral.
- Agree. By definition, opposite sides of a parallelogram are parallel.
- Disagree. By definition, a parallelogram has two pairs of parallel sides.
- Disagree. Sometimes all sides of a parallelogram have the same length, but not always. Opposite sides of a parallelogram always have the same length.
- Disagree. Sometimes all angles of a parallelogram have the same measure (when the parallelogram is a rectangle), but not always. Opposite angles of a parallelogram have the same measure.

(From Unit 1, Lesson 4.)

Problem 7

Statement

A square with an area of 1 square meter is decomposed into 9 identical small squares. Each small square is decomposed into two identical triangles.

- a. What is the area, in square meters, of 6 triangles? If you get stuck, consider drawing a diagram.
- b. How many triangles are needed to compose a region that is $1\frac{1}{2}$ square meters?

Solution

- a. $\frac{6}{18}$ or $\frac{1}{3}$ square meter.
- b. 27 triangles. It takes 18 triangles to make an area of 1 square meter and 9 triangles to make an area of $\frac{1}{2}$ square meter. $18 + 9 = 27$.

(From Unit 1, Lesson 2.)

Lesson 6: Area of Parallelograms

Goals

- Apply the formula for area of a parallelogram to find the area, the length of the base, or the height, and explain (orally and in writing) the solution method.
- Choose which measurements to use for calculating the area of a parallelogram when more than one base or height measurement is given, and explain (orally and in writing) the choice.

Learning Targets

- I can use the area formula to find the area of any parallelogram.

Lesson Narrative

This lesson allows students to practice using the formula for the area of parallelograms, and to choose the measurements to use as a base and a corresponding height. Through repeated reasoning, they see that some measurements are more helpful than others. For example, if a parallelogram on a grid has a vertical side or horizontal side, both the base and height can be more easily determined if the vertical or horizontal side is used as a base.

Along the way, students see that parallelograms with the same base and the same height have the same area because the products of those two numbers are equal, even if the parallelograms look very different. This gives us a way to use given dimensions to find others.

Alignments

Building On

- 3.OA.A: Represent and solve problems involving multiplication and division.

Addressing

- 6.EE.A.2.c: Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.
- 6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Instructional Routines

- MLR8: Discussion Supports

Required Materials

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Student Learning Goals

Let's practice finding the area of parallelograms.

6.1 Missing Dots

Warm Up: 5 minutes

In this warm-up, students determine the number of dots in an image without counting and explain how they arrive at that answer. The activity also gives students a chance use decomposition and structure to quantify something, in a setting that is slightly different than what they have seen in this unit. To arrive at the total number of dots, students need to visualize and articulate how the dots can be decomposed, and use what they know about arrays, multiplication, and area to arrive at the number of interest. To encourage students to refer to the image in their explanation, ask students how they *saw* the dots instead of how they *found* the number of dots.

As in an earlier warm-up, consider establishing a small, discreet hand signal that students can display to indicate that they have an answer they can support with reasoning. This signal could be a thumbs-up, a certain number of fingers that tells the number of responses they have, or another subtle signal. This is a quick way to see if the students have had enough time to think about the problem. It also keeps students from being distracted or rushed by hands being raised around the class.

Building On

- 3.OA.A

Launch

Give students 1–2 minutes of quiet think time and ask them to give a signal showing how many solutions they have. Encourage students who have found one way of seeing the dots to think about another way while they wait.

Student Task Statement



How many dots are in the image?

How do you see them?

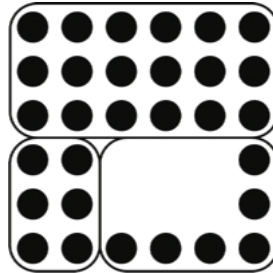
Student Response

30 dots. Strategies vary. Sample strategies:

- Decomposing the image into parts, then multiplying and adding.
 - $(2 \cdot 6) + (3 \cdot 4) + 6 = 30$



- $(3 \cdot 6) + (3 \cdot 2) + 6 = 30$



- Multiply to find the dots in the entire array and subtract the missing array of dots.
 - $(6 \cdot 6) - (2 \cdot 3) = 30$



Activity Synthesis

Ask students to share how many dots they saw and how they saw them. Record and display student explanations for all to see. To involve more students in the conversation, consider asking some of the following questions:

- “Who can restate the way ___ saw the dots in different words?”
- “Did anyone see the dots the same way but would explain it differently?”
- “Does anyone want to add an observation to the way ___ saw the dots?”
- “Do you agree or disagree? Why?”

6.2 More Areas of Parallelograms

25 minutes (there is a digital version of this activity)

This activity allows students to practice finding and reasoning about the area of various parallelograms—on and off a grid. Students need to make sense of the measurements and relationships in the given figures, identify an appropriate pair of base-height measurements to use, and recognize that two parallelograms with the same base-height measurements (or with different base-height measurements but the same product) have the same area.

As they work individually, notice how students determine base-height pairs to use. As they work in groups, listen to their discussions and identify those who can clearly explain how they found the area of each of the parallelograms.

Addressing

- 6.EE.A.2.c
- 6.G.A.1

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 4. Give each student access to their geometry toolkits and 5 minutes of quiet time to find the areas of the parallelograms in the first question. Then, assign each student one parallelogram (A, B, C or D). Ask each student to explain to the group, one at a time, how they found the area of the assigned parallelogram. After each student shares, check for agreement or disagreement from the rest of the group. Discuss any disagreement and come to a consensus on the correct answer before moving to the next parallelogram.

Afterwards, give students another 5–7 minutes of quiet work time to complete the rest of the activity.

For classrooms using the digital activity, arrange students in groups of 2. Ask each student to explain to their partner how they found the area of each parallelogram. When using the second applet, each student should each find one pair of quadrilaterals with equal area.

Support for Students with Disabilities

Representation: Internalize Comprehension. Use color and annotations to illustrate student thinking. As students describe how they calculated the area of each parallelogram, use color and annotations to scribe their thinking on a display. Ask students how they knew which measurements to use, and label each base and height accordingly.

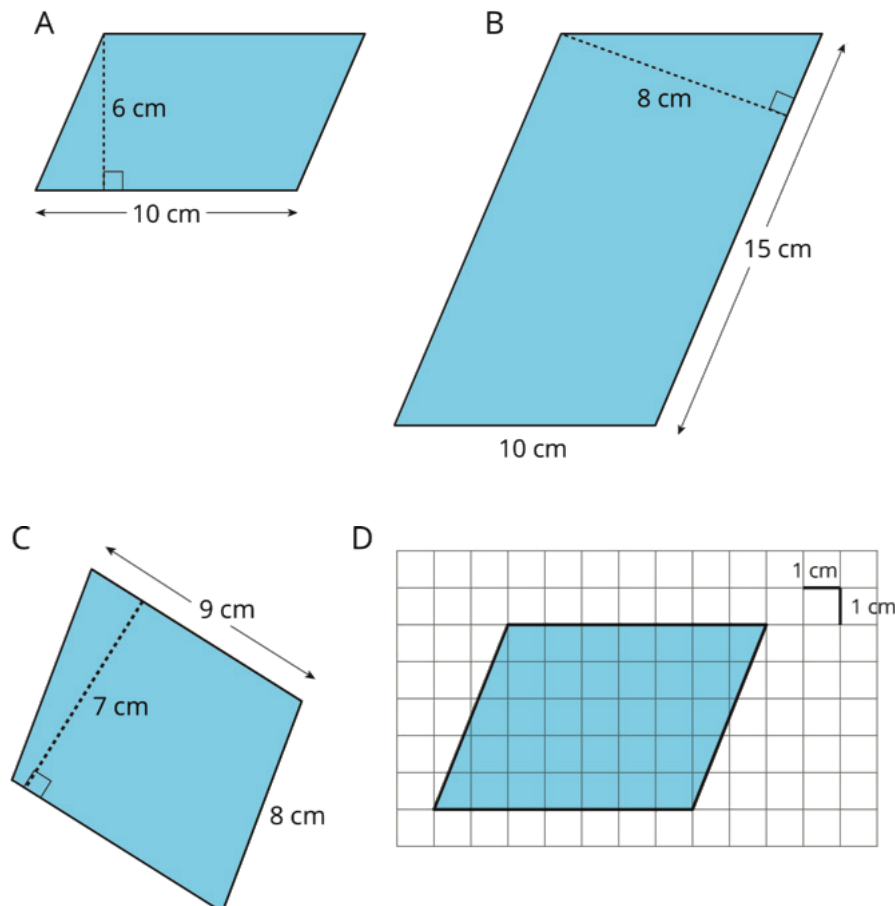
Supports accessibility for: Visual-spatial processing; Conceptual processing

Anticipated Misconceptions

Some students may continue to use visual reasoning strategies (decomposition, rearranging, enclosing, and subtracting) to find the area of parallelograms. This is fine at this stage, but to help them gradually transition toward abstract reasoning, encourage them to try solving one problem both ways—using visual reasoning and their generalization about bases and heights from an earlier lesson. They can start with one method and use the other to check their work.

Student Task Statement

1. Find the area of each parallelogram. Show your reasoning.



- In Parallelogram B, what is the corresponding height for the base that is 10 cm long? Explain or show your reasoning.
- Two different parallelograms P and Q both have an area of 20 square units. Neither of the parallelograms are rectangles.

On the grid, draw two parallelograms that could be P and Q.

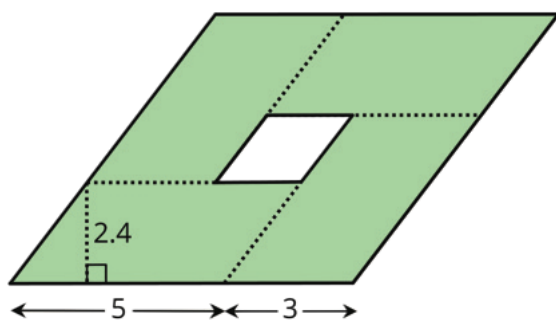


Student Response

- A: $10 \cdot 6 = 60$ square centimeters
B: $15 \cdot 8 = 120$ square centimeters
C: $9 \cdot 7 = 63$ square centimeters
D: $7 \cdot 5 = 35$ square centimeters
- 12 centimeters. Sample reasoning: We found the area of the parallelogram to be 120 square centimeters. If the side that is 10 centimeters is the base, then $10 \cdot h$ must equal 120, so the height must be $120 \div 10$ or 12 centimeters.
- Answers vary. Sample responses:
 - One parallelogram has a base of 10 units and a height of 2 units; another one has a base that is 4 units and a height that is 5 units.
 - One parallelogram has a base of 5 units and a height of 4 units; another one has a base that is 4 units and a height that is 5 units.
 - Two parallelograms with equal base and equal height but with different orientations, or with the pair of parallel bases positioned differently.

Are You Ready for More?

Here is a parallelogram composed of smaller parallelograms. The shaded region is composed of four identical parallelograms. All lengths are in inches.



What is the area of the unshaded parallelogram in the middle? Explain or show your reasoning.

Student Response

3.2 square inches. Reasonings vary. Sample responses:

- The area of one shaded parallelogram is 12 square inches, because one base is 5 inches and its corresponding height is 2.4 inches ($5 \cdot (2.4) = 12$). This means the corresponding height for the side that is 3 inches is 4 inches ($3 \cdot 4 = 12$). The height of the small parallelogram is the difference between 4 inches and 2.4 inches, which is 1.6 inches. The horizontal side of the the unshaded parallelogram, which can be a base, is 2 inches ($5 - 3 = 2$). The area of the unshaded parallelogram is therefore $2 \cdot (1.6)$ or 3.2 square inches.
- The base of the overall parallelogram is 8 inches ($5 + 3 = 8$). Its height is 6.4 inches ($4 + 2.4 = 6.4$). Its area is therefore $8 \cdot (6.4)$ or 51.2 square inches. The area of the four shaded parallelograms is $4 \cdot 12$ or 48 square inches. The area of the unshaded region is therefore $51.2 - 48$ or 3.2 square inches.

Activity Synthesis

Use whole-class discussion to draw out three important points:

1. We need base and height information to help us calculate the area of a parallelogram, so we generally look for the length of one side and the length of a perpendicular segment that connects the base to the opposite side. Other measurements may not be as useful.
2. A parallelogram generally has two pairs of base and height. Both pairs produce the same area (it's the same parallelogram), so the product of pair of numbers should equal the product of the other pair.
3. Two parallelograms with different pairs of base and height can have the same area, as long as their products are equal. So a 3-by-6 rectangle and a parallelogram with base 1 and height 18 will have the same area because $3 \cdot 6 = 1 \cdot 18$.

To highlight the first point, ask how students decided which measurements to use when calculating area.

- "When multiple measurements are shown, how did you know which of the measurements would help you find area?"
- "Which pieces of information in parallelograms B and C were not needed? Why not?"

To highlight the second point, select 1–2 previously identified students to share how they went about finding the missing height in the second question. Emphasize that the product $8 \cdot 15$ and that of 10 and the unknown h must be equal because both give us the area of the same parallelogram.

To highlight the last point, invite a few students to share their pair of parallelograms with equal area and an explanation of how they know the areas are equal. If not made explicit in students' explanations, stress that the base-height pairs must have the same product.

Support for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion when students explain how they created two parallelograms with equal area. For each explanation that is shared, ask students to restate what they heard using precise mathematical language. Consider providing students time to restate what they hear to a partner before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. If students are not able to restate, they should ask for clarification. Call students' attention to any words or phrases that helped to clarify the original statement, such as area, product, base, or height. This provides more students with an opportunity to produce language as they interpret the reasoning of others.

Design Principle(s): Support sense-making; Maximize meta-awareness

Lesson Synthesis

We used the formula for area to practice finding the area of various parallelograms.

- “When a parallelogram is on a grid, how do we know which side to choose for a base? Can we use any side?” (It is helpful to use a horizontal or a vertical side as a base; it would be easier to tell the length of that side and of its corresponding height.)
- “Off a grid, how do we know which measurements can help us find the area of a parallelogram?” (We need the length of one side of the parallelogram and of a perpendicular segment that connects that side to the opposite side.)
- “Do parallelograms that have the same area always look the same?” (No.) “Can you show an example?”
- “Do parallelograms that have the same base and height always look the same?” (No.) “Can you show an example?”
- “How can we draw two different parallelograms with the same area?” (We can find any two pairs of base-height lengths that have the same product. We can also use the same pair of numbers by draw the parallelograms differently.)

6.3 One More Parallelogram

Cool Down: 5 minutes

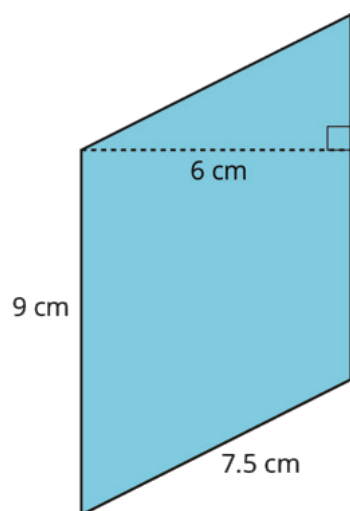
Addressing

- 6.G.A.1

Launch

Access to geometry toolkits.

Student Task Statement



1. Find the area of the parallelogram. Explain or show your reasoning.
2. Was there a length measurement you did not use to find the area? If so, explain why it was not used.

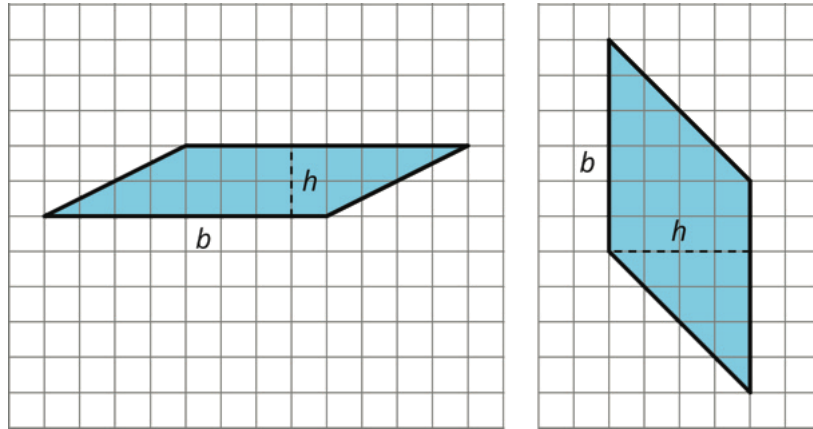
Student Response

1. 54 sq cm. A base is 9 cm and its corresponding height is 6 cm. $9 \cdot 6 = 54$.
2. The 7.5 cm length was not used. Explanations vary. Sample explanations:
 - If the side that is 7.5 cm was used to find area, we would need the length of a perpendicular segment between that side and the opposite side as its corresponding height. We don't have that information.
 - The parallelogram can be decomposed and rearranged into a rectangle by cutting it along the horizontal line and moving the right triangle to the bottom side. Doing this means the side that is 7.5 cm is no longer relevant. The rectangle is 6 cm by 9 cm; we can use those side lengths to find area.

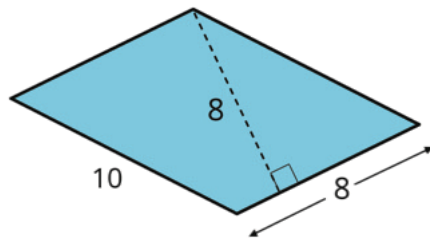
Student Lesson Summary

Any pair of base and corresponding height can help us find the area of a parallelogram, but some base-height pairs are more easily identified than others.

When a parallelogram is drawn on a grid and has *horizontal* sides, we can use a horizontal side as the base. When it has *vertical* sides, we can use a vertical side as the base. The grid can help us find (or estimate) the lengths of the base and of the corresponding height.

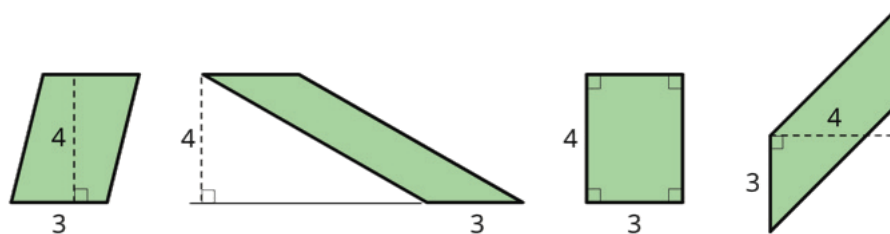


When a parallelogram is *not* drawn on a grid, we can still find its area if a base and a corresponding height are known.



In this parallelogram, the corresponding height for the side that is 10 units long is not given, but the height for the side that is 8 units long is given. This base-height pair can help us find the area.

Regardless of their shape, parallelograms that have the same base and the same height will have the same area; the product of the base and height will be equal. Here are some parallelograms with the same pair of base-height measurements.

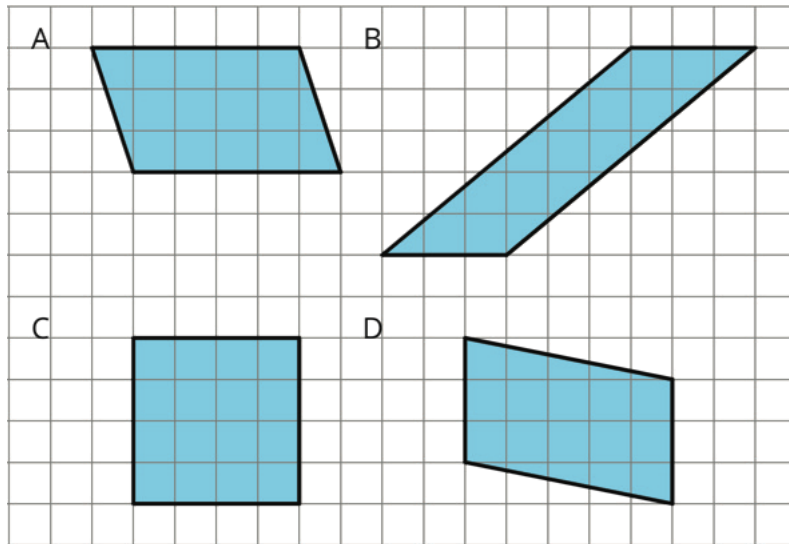


Lesson 6 Practice Problems

Problem 1

Statement

Which three of these parallelograms have the same area as each other?



- A. A
- B. B
- C. C
- D. D

Solution

["A", "B", "D"]

Problem 2

Statement

Which pair of base and height produces the greatest area? All measurements are in centimeters.

- A. $b = 4, h = 3.5$
- B. $b = 0.8, h = 20$
- C. $b = 6, h = 2.25$
- D. $b = 10, h = 1.4$

Solution

B

Problem 3

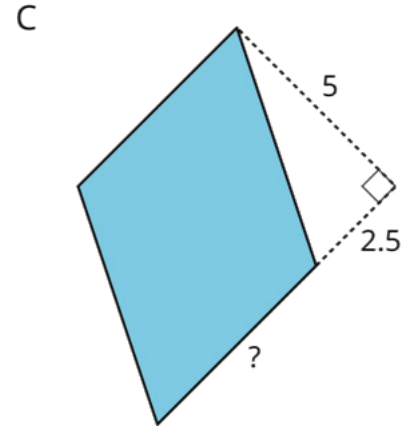
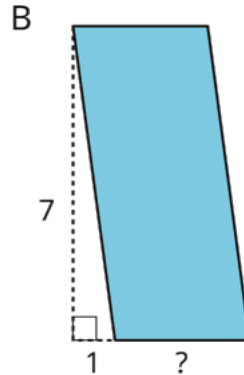
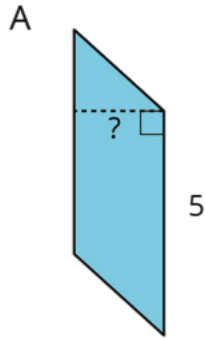
Statement

Here are the areas of three parallelograms. Use them to find the missing length (labeled with a "?") on each parallelogram.

A: 10 square units

B: 21 square units

C: 25 square units



Solution

A: 2 units

B: 3 units

C: 5 units

Problem 4

Statement

The Dockland Building in Hamburg, Germany is shaped like a parallelogram.



If the length of the building is 86 meters and its height is 55 meters, what is the area of this face of the building?

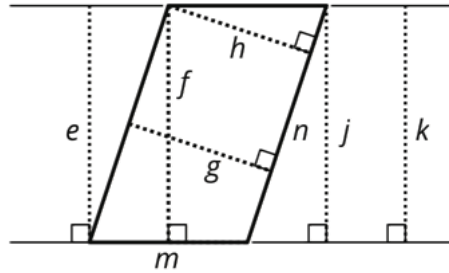
Solution

4,730 square meters ($86 \cdot 55 = 4,730$).

Problem 5

Statement

Select **all** segments that could represent a corresponding height if the side m is the base.



- A. e
- B. f
- C. g
- D. h
- E. j
- F. k
- G. n

Solution

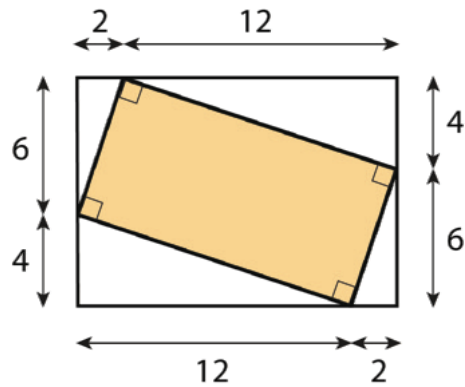
["A", "B", "E", "F"]

(From Unit 1, Lesson 5.)

Problem 6

Statement

Find the area of the shaded region. All measurements are in centimeters. Show your reasoning.



Solution

80 square centimeters. Sample reasoning: The area of the large rectangle is 140 square centimeters, because $14 \cdot 10 = 140$. The areas of the small, unshaded right triangles are each 6 square centimeters, because $6 \cdot 2 \div 2 = 6$. The areas of the larger, unshaded right triangles are each 24 square centimeters, because $4 \cdot 12 \div 2 = 24$. Subtracting the areas of the four unshaded right triangles from the area of the large rectangle yields 80: $140 - 6 - 6 - 24 - 24 = 80$.

(From Unit 1, Lesson 3.)

Section: Triangles

Lesson 7: From Parallelograms to Triangles

Goals

- Describe (orally and in writing) ways in which two identical triangles can be composed, i.e., into a parallelogram or into a rectangle.
- Show how any parallelogram can be decomposed into two identical triangles by drawing a diagonal, and generalize (in writing) that this property applies to all parallelograms, but not all quadrilaterals.

Learning Targets

- I can explain the special relationship between a pair of identical triangles and a parallelogram.

Lesson Narrative

This lesson prepares students to apply what they know about the area of parallelograms to reason about the area of triangles.

Highlighting the relationship between triangles and parallelograms is a key goal of this lesson. The activities make use of both the idea of *decomposition* (of a quadrilateral into triangles) and *composition* (of two triangles into a quadrilateral). The two-way study is deliberate, designed to help students view and reason about the area of a triangle differently. Students see that a parallelogram can always be decomposed into two identical triangles, and that any two identical triangles can always be composed into a parallelogram (MP7).

Because a lot happens in this lesson and timing might be tight, it is important to both prepare all the materials and consider grouping arrangements in advance.

Alignments

Addressing

- 6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Building Towards

- 6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Instructional Routines

- MLR2: Collect and Display

- MLR8: Discussion Supports

Required Materials

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty

paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Pre-printed slips, cut from copies of the Instructional master

Rulers

Required Preparation

Print pairs of triangles from the Instructional master for A Tale of Two Triangles (Part 2). If students are cutting out the triangles, use the first page only. If the triangles are to be pre-cut by the teacher, print the second and third pages. Prepare enough sets so that each group of 3-4 students has a complete set (2 copies each of triangles P-U).

For classes using the digital version of the activity, an applet is provided that can be used in place of, or in addition to, the cut out triangles.

Student Learning Goals

Let's compare parallelograms and triangles.

7.1 Same Parallelograms, Different Bases

Warm Up: 5 minutes

This warm-up reinforces students' understanding of bases and heights in a parallelogram. In previous lessons, students calculated areas of parallelograms using bases and heights. They have also determined possible bases and heights of a parallelogram given a whole-number area. They saw, for instance, that finding possible bases and heights of a parallelogram with an area of 20 square units means finding two numbers with a product of 20. Students extend that work here by working with decimal side lengths and area.

As students work, notice students who understand that the two identical parallelograms have equal area and who use that understanding to find the unknown base. Ask them to share later.

Addressing

- 6.G.A.1

Launch

Give students 2 minutes of quiet work time and access to their geometry toolkits.

Students should be adequately familiar with bases and heights to begin the warm-up. If needed, however, briefly review the relationship between a pair of base and height in a parallelogram, using questions such as:

- “Can we use any side of a parallelogram as a base?” (Yes.)
- “Is the height always the length of one of the sides of the parallelogram?” (No.)
- “Once we have identified a base, how do we identify a height?” (It can be any segment that is perpendicular to the base and goes from the base to the opposite side.)
- “Can a height segment be drawn outside of a parallelogram?” (Yes.)

Anticipated Misconceptions

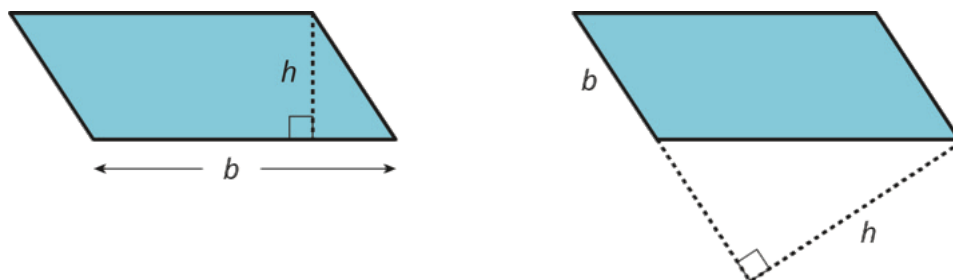
Some students may not know how to begin answering the questions because measurements are not shown on the diagrams. Ask students to label the parallelograms based on the information in the task statement.

Students may say that there is not enough information to answer the second question because only one piece of information is known (the height). Ask them what additional information might be needed. Prompt them to revisit the task statement and see what it says about the two parallelograms. Ask what they know about the areas of two figures that are identical.

Students may struggle to find the unknown base in the second question because the area of the parallelogram is a decimal and they are unsure how to divide a decimal. Ask them to explain how they would reason about it if the area was a whole number. If they understand that they need to divide the area by 2 (since the height is 2 cm), see if they could reason in terms of multiplication (i.e., 2 times what number is 2.4?) or if they could reason about the division using fractions (i.e., 2.4 can be seen as $2\frac{4}{10}$ or $\frac{24}{10}$; what is 24 tenths divided by 2?).

Student Task Statement

Here are two copies of a parallelogram. Each copy has one side labeled as the base b and a segment drawn for its corresponding height and labeled h .



1. The base of the parallelogram on the left is 2.4 centimeters; its corresponding height is 1 centimeter. Find its area in square centimeters.

2. The height of the parallelogram on the right is 2 centimeters. How long is the base of that parallelogram? Explain your reasoning.

Student Response

1. The area of the first parallelogram is 2.4 square centimeters. $(2.4) \cdot 1 = 2.4$
2. The area of the second parallelogram is also 2.4 square centimeters. Since the base and height must multiply to the same area of 2.4, the base must be 1.2 centimeters because $(1.2) \cdot 2 = 2.4$.

Activity Synthesis

Select 1–2 previously identified students to share their responses. If not already explained by students, emphasize that we know the parallelograms have the same area because they are identical, which means that when one is placed on top of the other they would match up exactly.

Before moving on, ask students: “How can we verify that the height we found is correct, or that the two pairs of bases and heights produce the same area?” (We can multiply the values of each pair and see if they both produce 2.4.)

7.2 A Tale of Two Triangles (Part 1)

15 minutes (there is a digital version of this activity)

In earlier lessons, students saw that a square can be decomposed into two identical isosceles right triangles. They concluded that the area of each of those triangles is half of the area of the square. They used this observation to determine the area of composite regions.

This activity helps students see that parallelograms other than squares can also be decomposed into two identical triangles by drawing a diagonal. They check this by tracing a triangle on tracing paper and then rotating it to match the other copy. The process prepares students to see any triangle as occupying half of a parallelogram, and consequently, as having one half of its area. To generalize about quadrilaterals that can be decomposed into identical triangles, students need to analyze the features of the given shapes and look for structure (MP7).

There are a number of geometric observations in this unit that must be taken for granted at this point in students' mathematical study. This is one of those instances. Students have only seen examples of a parallelogram being decomposable into two copies of the same triangle, or have only verified this conjecture through physical experimentation, but for the time being it can be considered a fact. Starting in grade 8, they will begin to prove some of the observations they have previously taken to be true.

Building Towards

- 6.G.A.1

Instructional Routines

- MLR2: Collect and Display

Launch

Arrange students in groups of 3–4. Give students access to geometry toolkits and allow for 2 minutes of quiet think time for the first two questions. Then, ask them to share their drawings with their group and discuss how they drew their lines. If group members disagree on whether a quadrilateral can be decomposed into two identical triangles, they should note the disagreement, but it is not necessary to come to an agreement. They will soon have a chance to verify their responses.

Next, ask students to use tracing paper to check that the pairs of triangles that they believe to be identical are indeed so (i.e., they would match up exactly if placed on top of one another). Tell students to divide the checking work among the members of their group to optimize time.

Though students have worked with tracing paper earlier in the unit, some may not recall how to use it to check the congruence of two shapes; some explicit guidance might be needed. Encourage students to work carefully and precisely. A straightedge can be used in tracing but is not essential and may get in the way. Once students finish checking the triangles in their list and verify that they are identical (or correct their initial response), ask them to answer the last question.

Students using the digital activity can decompose the shapes using an applet. Encourage students to use the segment tool rather than free-drawing a segment to divide the shapes.

Support for Students with Disabilities

Representation: Internalize Comprehension. Represent the same information through different modalities by providing access to a hands-on alternative. To determine which quadrilaterals can be decomposed into two identical triangles, some students may benefit from enlarged cut-outs of the quadrilaterals that they can manipulate, fold, or cut.

Supports accessibility for: Conceptual processing; Visual-spatial processing

Support for English Language Learners

Conversing, Representing: MLR2 Collect and Display. To help students reason about and use the mathematical language of decompose, diagonal, and identical, listen to students talk about how they are making their drawings. Record and display common or important phrases you hear students say as well as examples of their drawings. Continue to update collected student language throughout the lesson, and remind students to borrow language from the display as needed.

Design Principle(s): Support sense-making; Maximize meta-awareness

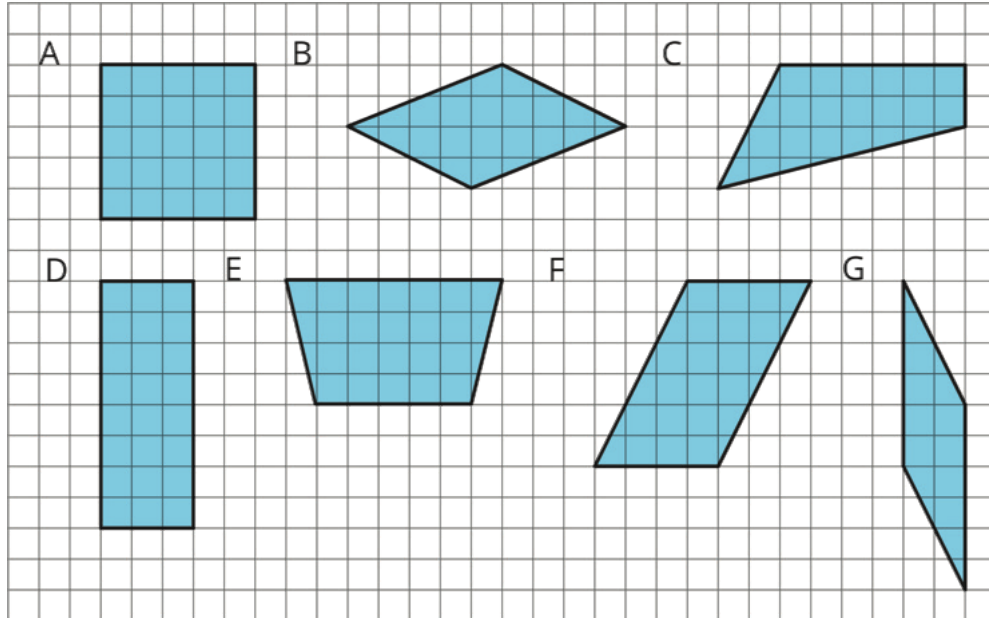
Anticipated Misconceptions

It may not occur to students to rotate triangles to check congruence. If so, tell students that we still consider two triangles identical even when one needs to be rotated to match the other.

Student Task Statement

Two polygons are identical if they match up exactly when placed one on top of the other.

1. Draw *one* line to decompose each polygon into two identical triangles, if possible. Use a straightedge to draw your line.



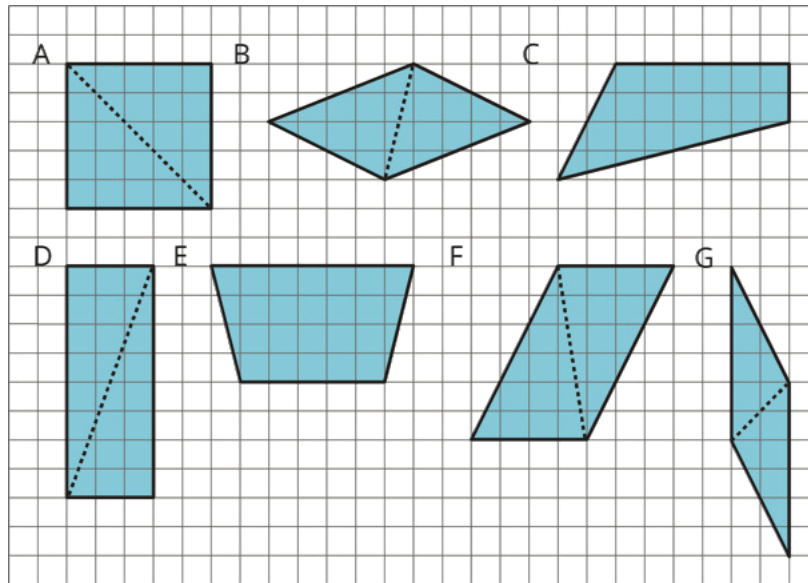
2. Which quadrilaterals can be decomposed into two identical triangles?

Pause here for a small-group discussion.

3. Study the quadrilaterals that can, in fact, be decomposed into two identical triangles. What do you notice about them? Write a couple of observations about what these quadrilaterals have in common.

Student Response

1. Answers vary. Sample response:



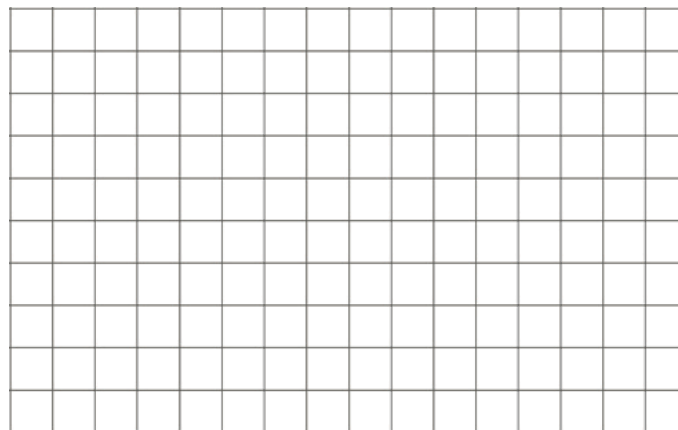
2. Answers vary. Quadrilaterals A, B, D, F, and G can be decomposed into two identical triangles.

3. Answers vary. Sample responses:

- They have two pairs of parallel sides and each pair has equal length.
- They are all parallelograms.
- The triangles are formed by drawing a diagonal connecting opposite vertices.
- Some triangles are right triangles, some are acute, and some are obtuse.
- For some quadrilaterals, there is more than one way to decompose it into two identical triangles.

Are You Ready for More?

On the grid, draw some other types of quadrilaterals that are not already shown. Try to decompose them into two identical triangles. Can you do it?



Come up with a rule about what must be true about a quadrilateral for it to be decomposed into two identical triangles.

Student Response

Answers vary.

Activity Synthesis

The discussion should serve two goals: to highlight how quadrilaterals can be decomposed into triangles and to help students make generalizations about the types of quadrilaterals that can be decomposed into two identical triangles. Consider these questions:

- How did you decompose the quadrilaterals into two triangles? (Connect opposite vertices, i.e. draw a diagonal.)
- Did the strategy of drawing a diagonal work for decomposing all quadrilaterals into two triangles? (Yes.) Are all of the resulting triangles identical? (No.)
- What is it about C and E that they cannot be decomposed into two identical triangles? (They don't have equal sides or equal angles. Their opposite sides are not parallel.)
- What do A, B, and D have that C and E do not? (A, B, and D have two pairs of parallel sides that are of equal lengths. They are parallelograms.)

Ask students to complete this sentence starter: For a quadrilateral to be decomposable into two identical triangles it must be (or have) . . .

If time permits, discuss how students verified the congruence of the two triangles.

- How did you check if the triangles are identical? Did you simply stack the traced triangle or did you do something more specific? (They may notice that it is necessary to rotate one triangle—or to reflect one triangle it twice—before the triangles could be matched up.)
- Did anyone use another way to check for congruence? (Students may also think in terms of the parts or composition of each triangle. E.g. “Both triangles have all the same side lengths; they both have a right angle”).

7.3 A Tale of Two Triangles (Part 2)

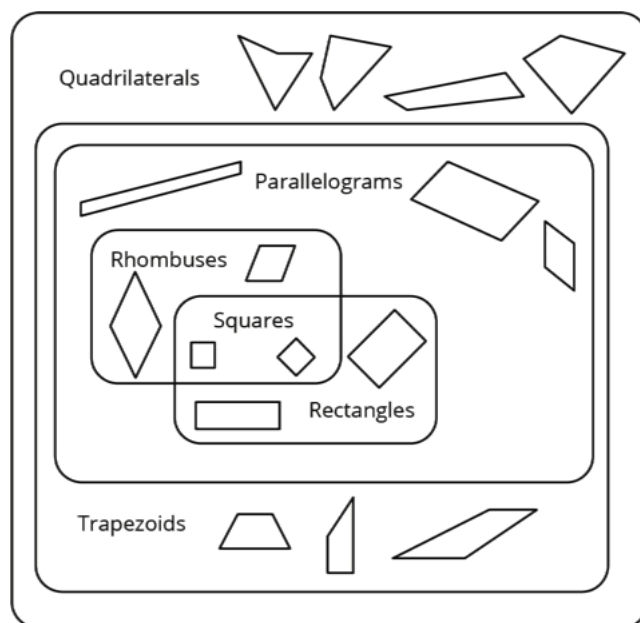
15 minutes (there is a digital version of this activity)

Previously, students decomposed quadrilaterals into two identical triangles. The work warmed them to the idea of a triangle as a *half* of a familiar quadrilateral. This activity prompts them to think the other way—to *compose* quadrilaterals using two identical triangles. It helps students see that two identical triangles of any kind can always be joined to produce a parallelogram. Both explorations prepare students to make connections between the area of a triangle and that of a parallelogram in the next lesson.

A key understanding to uncover here is that two identical copies of a triangle can be joined along any corresponding side to produce a parallelogram, and that more than one parallelogram can be formed.

As students work, look for different compositions of the same pair of triangles. Select students using different approaches to share later.

When manipulating the cutouts students are likely to notice that right triangles can be composed into rectangles (and sometimes squares) and that non-right triangles produce parallelograms that are not rectangles. Students may not immediately recall that squares and rectangles are also parallelograms. Consider preparing a reference for students to consult. Here is an example:



As before, students make generalizations here that they don't yet have the tools to justify them. This is appropriate at this stage. Later in their mathematical study they will learn to verify what they now take as facts.

For students using the digital activity, an applet can be used to compose triangles into other shapes.

Building Towards

- 6.G.A.1

Instructional Routines

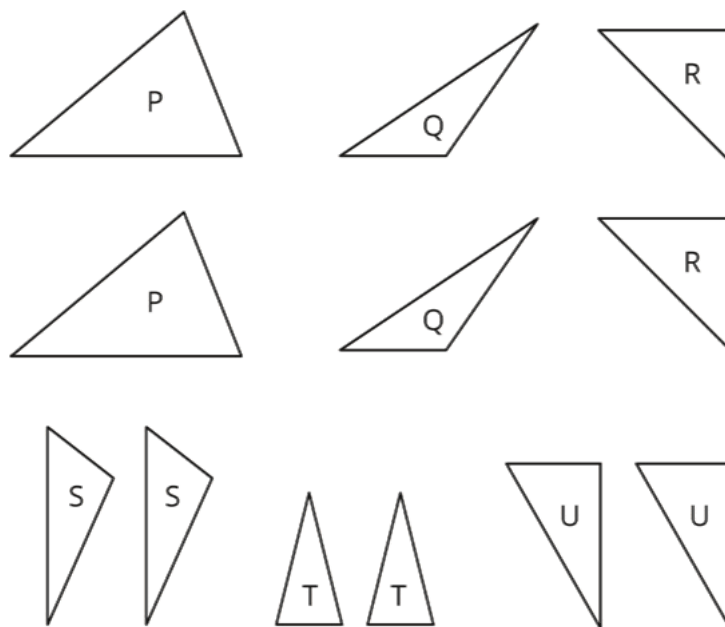
- MLR8: Discussion Supports

Launch

Keep students in the same groups. Give each group one set of triangles labeled P-U (two copies of each triangle) from the Instructional master and access to scissors if the triangles are not pre-cut. The set includes different types of triangles (isosceles right, scalene right, obtuse, acute, and equilateral). Ask each group member to take 1–2 pairs of triangles.

Reiterate that students learned that certain types of quadrilaterals can be decomposed into two identical triangles. Explain that they will now see if it is possible to *compose* quadrilaterals out of two identical triangles, and, if so, to find out what types of quadrilaterals would result.

Give students 1–2 minutes of quiet work time, and then 5 minutes to discuss their responses and answer the second question with their group.



Support for English Language Learners

Conversing: MLR8 Discussion Supports. To reinforce use of the language that students have previously learned about quadrilaterals, create and display the reference chart, as described, for students to consult. Use this display to help students visualize the different types of quadrilaterals. Ask students to discuss with a partner, “What is the same and different about the different types of quadrilaterals?” Tell students to take turns sharing what they notice or remember from previous lessons, then call on different groups to share what they notice with the whole class. When recording and displaying students’ observations, listen for opportunities to re-voice the mathematical terms that students used.

Design Principle(s): Support sense-making; Cultivate conversation

Anticipated Misconceptions

Students may draw incorrect conclusions if certain pieces of their triangles are turned over (to face down), or if it did not occur to them that the pieces could be moved. Ask them to try manipulating the pieces in different ways.

Seeing that two copies of a triangle can always be composed into a parallelogram, students might mistakenly conclude that any two copies of a triangle can *only* be composed into a parallelogram

(i.e., no other quadrilaterals can be formed from joining two identical triangles). Showing a counterexample may be a simple way to help students see that this is not the case.

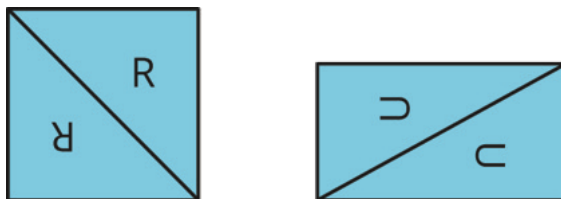
Student Task Statement

Your teacher will give your group several pairs of triangles. Each group member should take 1 or 2 pairs.

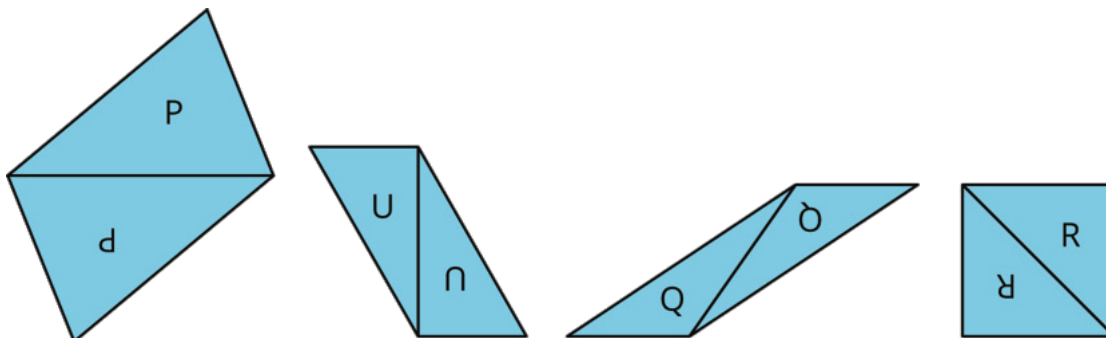
1.
 - a. Which pair(s) of triangles do you have?
 - b. Can each pair be composed into a rectangle? A parallelogram?
2. Discuss with your group your responses to the first question. Then, complete each statement with *All*, *Some*, or *None*. Sketch 1 or 2 examples to illustrate each completed statement.
 - a. _____ of these pairs of identical triangles can be composed into a *rectangle*.
 - b. _____ of these pairs of identical triangles can be composed into a *parallelogram*.

Student Response

1.
 - a. Answers vary. Yes for triangles R and U, no for the rest
 - b. Yes for all triangles
2.
 - a. *Some* of these pairs of triangles can be composed into a rectangle.



- b. *All* of these pairs of triangles can be composed into a *parallelogram*. Examples:



Activity Synthesis

The focus of this discussion would be to clarify whether or not two copies of each triangle can be composed into a rectangle or a parallelogram, and to highlight the different ways two triangles could be composed into a parallelogram.

Ask a few students who composed different parallelograms from the same pair of triangles to share. Invite the class to notice how these students ended up with different parallelograms. To help them see that a triangle can be joined along any side of its copy to produce a parallelogram, ask questions such as:

- Here is one way of composing triangles S into a parallelogram. Did anyone else do it this way? Did anyone obtain a parallelogram a different way?
- How many different parallelograms can be created with any two copies of a triangle? Why? (3 ways, because there are 3 sides along which the triangles could be joined.)
- What kinds of triangles can be used to compose a rectangle? How? (Right triangles, by joining two copies along the side opposite of the right angle.)
- What kinds of triangles can be used to compose a parallelogram? How? (Any triangle, by joining two copies along any side with the same length.)

Lesson Synthesis

Display and revisit representative works from the two main activities. Draw out key observations about the special connections between triangles and parallelograms.

First, we tried to decompose or break apart quadrilaterals into two identical triangles.

- “What strategy allowed us to do that?” (Drawing a segment connecting opposite vertices.)
- “Which types of quadrilaterals could always be decomposed into two identical triangles?” (Parallelograms.)
- “Can quadrilaterals that are not parallelograms be decomposed into triangles?” (Yes, but the resulting triangles may not be identical.)

Then, we explored the relationship between triangles and quadrilaterals the other way around. We tried to compose or create quadrilaterals from pairs of identical triangles.

- “What types of quadrilaterals were you able to compose with a pair of identical triangles?” (Parallelograms—some of them are rectangles.)
- “Does it matter what type of triangles was used?” (No. Any two copies of a triangle could be composed into a parallelogram.)
- “Was there a particular side along which the two triangles must be joined to form a parallelogram?” (No. Any of the three sides could be used.)

We saw how two identical copies of a triangle can be combined to make a parallelogram. This is true for any triangle. The reverse is also true: any parallelogram can be split into two identical triangles. In grade 8 we will acquire some tools to prove these observations. For now, we will take the special relationships between triangles and parallelograms as a fact. We will use them to find the area of any triangle in upcoming lessons.

7.4 A Tale of Two Triangles (Part 3)

Cool Down: 5 minutes

Building Towards

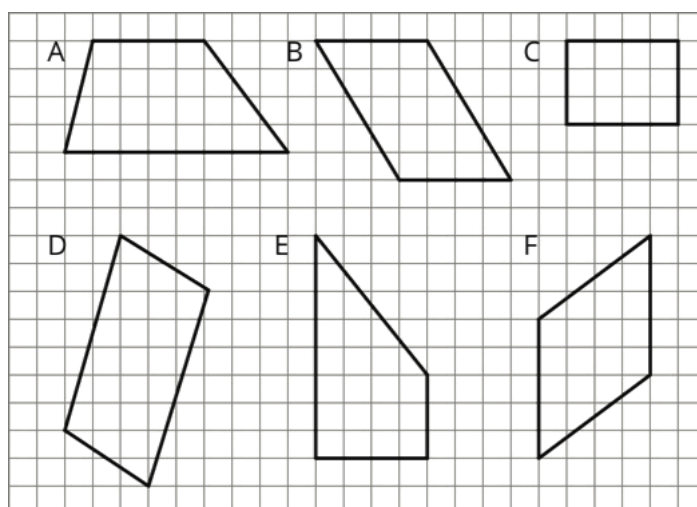
- 6.G.A.1

Launch

Give students access to their geometry toolkits if needed.

Student Task Statement

1. Here are some quadrilaterals.



- a. Circle all quadrilaterals that you think can be decomposed into two identical triangles using only one line.
- b. What characteristics do the quadrilaterals that you circled have in common?

2. Here is a right triangle. Show or briefly describe how two copies of it can be composed into a parallelogram.

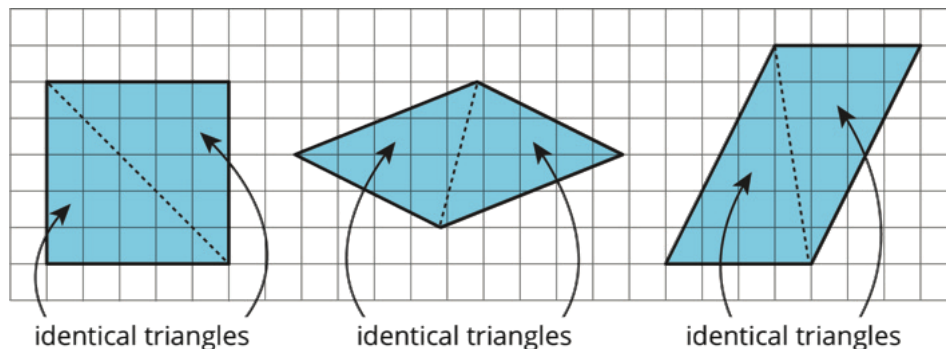


Student Response

1.
 - a. Quadrilaterals B, C, D, and F should be circled.
 - b. They all have two pairs of parallel sides. They are all parallelograms.
2. Answers vary. Sample response: Joining two copies of the triangle along a side that is the same length (e.g., the shortest side of one and the shortest side of the other) would make a parallelogram. (Three parallelograms are possible, since there are three sides at which the triangles could be joined. One of the parallelograms is a rectangle.)

Student Lesson Summary

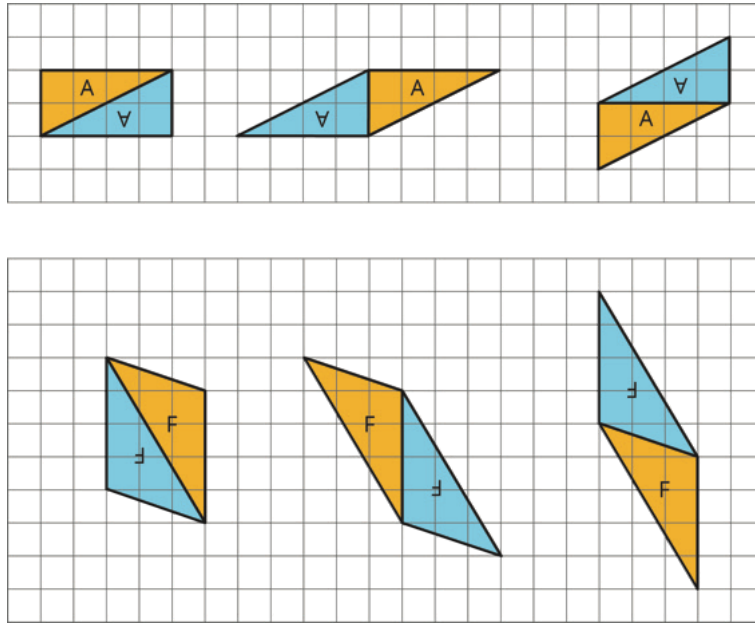
A parallelogram can always be decomposed into two identical triangles by a segment that connects opposite vertices.



Going the other way around, two identical copies of a triangle can always be arranged to form a parallelogram, regardless of the type of triangle being used.

To produce a parallelogram, we can join a triangle and its copy along any of the three sides, so the same pair of triangles can make different parallelograms.

Here are examples of how two copies of both Triangle A and Triangle F can be composed into three different parallelograms.



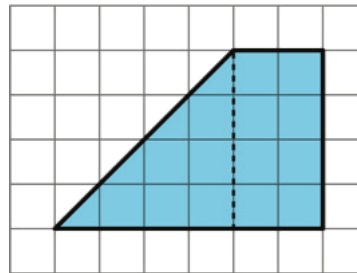
This special relationship between triangles and parallelograms can help us reason about the area of any triangle.

Lesson 7 Practice Problems

Problem 1

Statement

To decompose a quadrilateral into two identical shapes, Clare drew a dashed line as shown in the diagram.



- She said that the two resulting shapes have the same area. Do you agree? Explain your reasoning.
- Did Clare partition the figure into two identical shapes? Explain your reasoning.

Solution

- Yes, the rectangle is 2 units by 4 units, so it has an area of 8 square units. The triangle is half of a 4-by-4 square, so its area is also 8 square units.
- No, although the shapes have the same area, they are not identical shapes—one is a rectangle and the other a triangle.

Problem 2

Statement

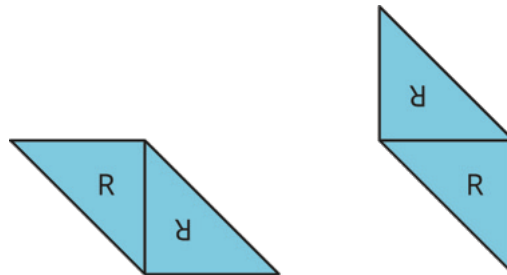
Triangle R is a right triangle. Can we use two copies of Triangle R to compose a parallelogram that is not a square?



If so, explain how or sketch a solution. If not, explain why not.

Solution

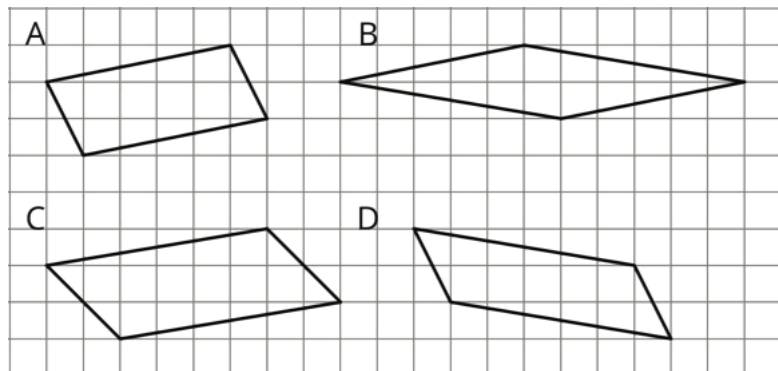
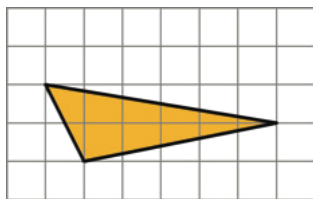
Yes, we can use two right triangles R to compose a parallelogram that is not a square by joining them along one of the shorter sides (the sides that make the right angle).



Problem 3

Statement

Two copies of this triangle are used to compose a parallelogram. Which parallelogram *cannot* be a result of the composition? If you get stuck, consider using tracing paper.



- A. A
- B. B
- C. C
- D. D

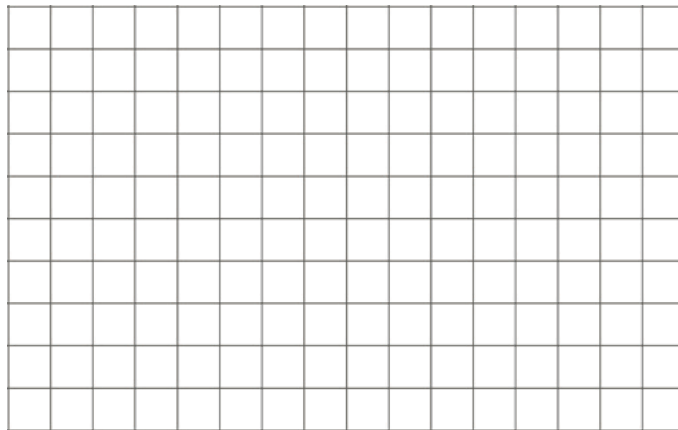
Solution

C

Problem 4

Statement

- a. On the grid, draw at least three different quadrilaterals that can each be decomposed into two identical triangles with a single cut (show the cut line). One or more of the quadrilaterals should have non-right angles.

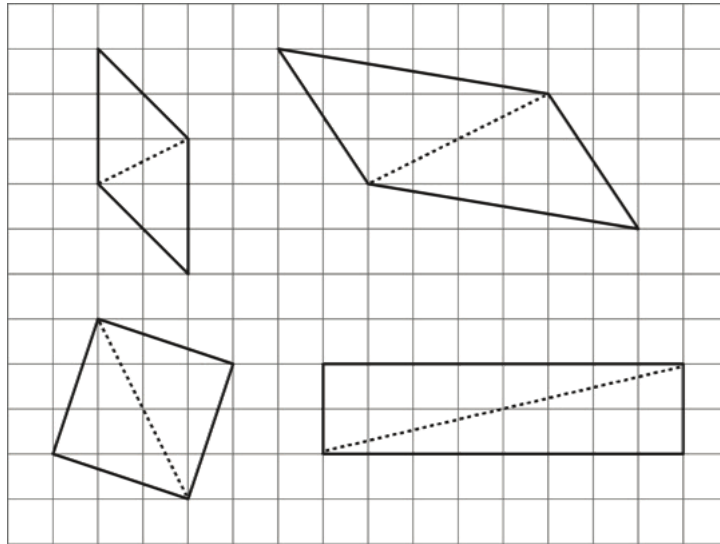


- b. Identify the type of each quadrilateral.

Solution

Answers vary. Sample responses:

- a.



- b. The top two are parallelograms. The bottom left one is a square. The bottom right one is a rectangle. (All of them are parallelograms.)

Problem 5

Statement

- A parallelogram has a base of 9 units and a corresponding height of $\frac{2}{3}$ units. What is its area?
- A parallelogram has a base of 9 units and an area of 12 square units. What is the corresponding height for that base?
- A parallelogram has an area of 7 square units. If the height that corresponds to a base is $\frac{1}{4}$ unit, what is the base?

Solution

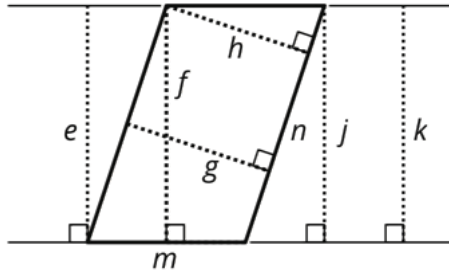
- $\frac{18}{3}$ square units (or equivalent)
- $\frac{12}{9}$ units (or equivalent)
- 28 units

(From Unit 1, Lesson 6.)

Problem 6

Statement

Select **all** the segments that could represent the height if side n is the base.



- A. e
- B. f
- C. g
- D. h
- E. m
- F. n
- G. j
- H. k

Solution

["C", "D"]

(From Unit 1, Lesson 5.)

Lesson 8: Area of Triangles

Goals

- Draw a diagram to show that the area of a triangle is half the area of an associated parallelogram.
- Explain (orally and in writing) strategies for using the base and height of an associated parallelogram to determine the area of a triangle.

Learning Targets

- I can use what I know about parallelograms to reason about the area of triangles.

Lesson Narrative

This lesson builds on students' earlier work decomposing and rearranging regions to find area. It leads students to see that, in addition to using area-reasoning methods from previous lessons, they can use what they know to be true about parallelograms (i.e. that the area of a parallelogram is $b \cdot h$) to reason about the area of triangles.

Students begin to see that the area of a triangle is half of the area of the parallelogram of the same height, or that it is the same as the area of a parallelogram that is half its height. They build this intuition in several ways:

- by recalling that two copies of a triangle can be composed into a parallelogram;
- by recognizing that a triangle can be recomposed into a parallelogram that is half the triangle's height; or
- by reasoning indirectly, using one or more rectangles with the same height as the triangle.

They apply this insight to find the area of triangles both on and off the grid.

Alignments

Addressing

- 6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display
- MLR7: Compare and Connect
- Notice and Wonder

- Think Pair Share

Required Materials

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty

paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Glue or glue sticks

Pre-printed slips, cut from copies of the Instructional master

Tape

Required Preparation

Students need access to tape *or* glue; it is not necessary to have both.

Each copy of the Instructional master contains two copies of each of parallelograms A, B, C, and D. Prepare enough copies so that each student receives two copies of a parallelogram.

Student Learning Goals

Let's use what we know about parallelograms to find the area of triangles.

8.1 Composing Parallelograms

Warm Up: 10 minutes

This warm-up has two aims: to solidify what students learned about the relationship between triangles and parallelograms and to connect their new insights back to the concept of area.

Students are given a right triangle and the three parallelograms that can be composed from two copies of the triangle. Though students are not asked to find the area of the triangle, they may make some important observations along the way. They are likely to see that:

- The triangle covers half of the region of each parallelogram.
- The base-height measurements for each parallelogram involve the numbers 6 and 4, which are the lengths of two sides of the triangle.
- All parallelograms have the same area of 24 square units.

These observations enable them to reason that the area of the triangle is half of the area of a parallelogram (in this case, any of the three parallelograms can be used to find the area of the triangle). In upcoming work, students will test and extend this awareness, generalizing it to help them find the area of any triangle.

Addressing

- 6.G.A.1

Instructional Routines

- Notice and Wonder
- Think Pair Share

Launch

Display the images of the triangle and the three parallelograms for all to see. Give students a minute to observe them. Ask them to be ready to share at least one thing they notice and one thing they wonder. Give students a minute to share their observations and questions with a partner.

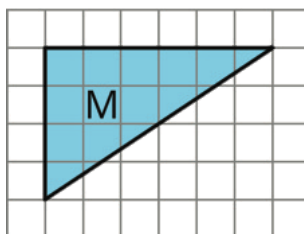
Give students 2–3 minutes of quiet time to complete the activity, and provide access to their geometry toolkits. Follow with a whole-class discussion.

Anticipated Misconceptions

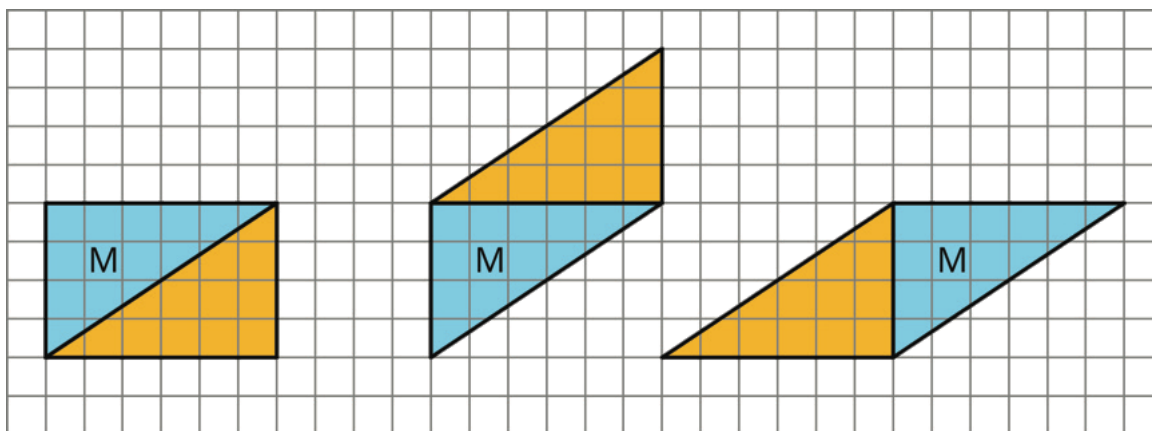
When identifying bases and heights of the parallelograms, some students may choose a non-horizontal or non-vertical side as a base and struggle to find its length and the length of its corresponding height. Ask them to see if there's another side that could serve as a base and has a length that can be more easily determined.

Student Task Statement

Here is Triangle M.



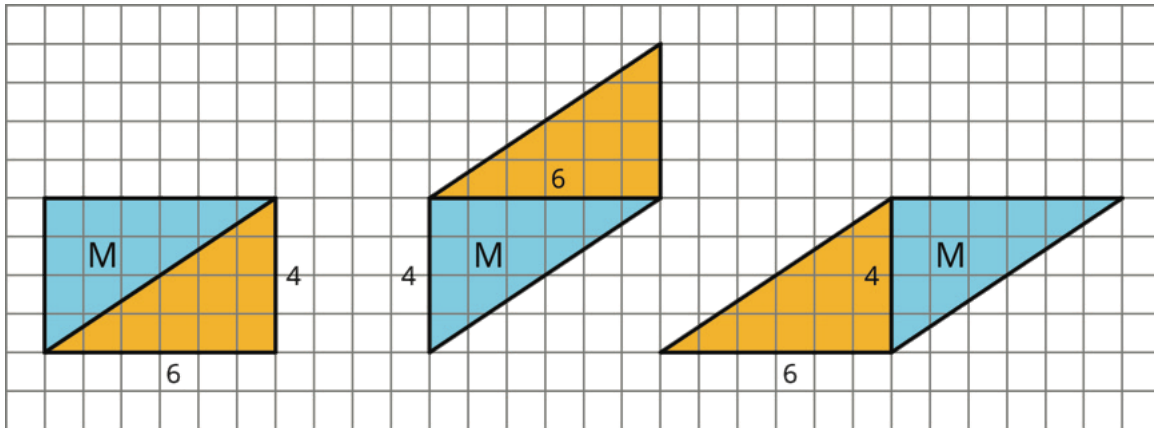
Han made a copy of Triangle M and composed three different parallelograms using the original M and the copy, as shown here.



1. For each parallelogram Han composed, identify a base and a corresponding height, and write the measurements on the drawing.
2. Find the area of each parallelogram Han composed. Show your reasoning.

Student Response

1. First parallelogram: $b = 6$ and $h = 4$, second parallelogram: $b = 4$ and $h = 6$, third parallelogram: $b = 6$ and $h = 4$



2. The area of all parallelograms is 24 square units. The base and height measurements for the parallelograms are 4 units and 6 units, or 6 units and 4 units. $4 \cdot 6 = 24$ and $6 \cdot 4 = 24$.

Activity Synthesis

Ask one student to identify the base, height, and area of each parallelogram, as well as how they reasoned about the area. If not already brought up by students in their explanations, discuss the following questions:

- “Why do all parallelograms have the same area even though they all have different shapes?” (They are composed of the same parts—two copies of the same right triangles. They have the same pair of numbers for their base and height. They all can be decomposed and rearranged into a 6-by-4 rectangle.)
- “What do you notice about the bases and heights of the parallelograms?” (They are the same pair of numbers.)
- “How are the base-height measurements related to the right triangle?” (They are the lengths of two sides of the right triangles.)
- “Can we find the area of the triangle? How?” (Yes, the area of the triangle is 12 square units because it is half of the area of every parallelogram, which is 24 square units.)

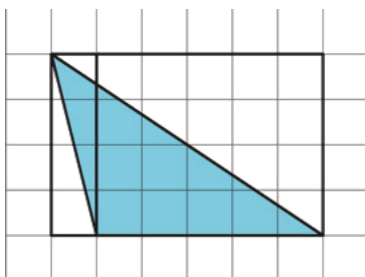
8.2 More Triangles

25 minutes

In this activity, students apply what they have learned to find the area of various triangles. They use reasoning strategies and tools that make sense to them. Students are not expected to use a formal procedure or to make a general argument. They will think about general arguments in an upcoming lesson.

Here are some anticipated paths students may take, from more elaborate to more direct. Also monitor for other approaches.

- Draw two smaller rectangles that decompose the given triangle into two right triangles. Find the area of each rectangle and take half of its area. Add the areas of the two right triangles. (This is likely used for B and D.)
For Triangle C, some students may choose to draw two rectangles around and on the triangle (as shown here), find half of the area of each rectangle, and *subtract* one area from the other.



- Enclose the triangle with one rectangle, find the area of the rectangle, and take half of that area. (This is likely used for right triangle A.)
- Duplicate the triangle to form a parallelogram, find the area of the parallelogram, and take half of its area. (Likely used with any triangle.)

Monitor the different strategies students use. Consider asking each student that uses a unique strategy to create a visual display of their work and to share it with the class later.

Addressing

- 6.G.A.1

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display

Launch

Tell students that they will now apply their observations from the past few activities to find the area of several triangles. Arrange students in groups of 2–3. Give students 6–8 minutes of quiet work time and a few more minutes to discuss their work with a partner. Ask them to confer with their group only after each person has attempted to find the area of at least two triangles. Provide access to their geometry toolkits (especially tracing paper).

Support for Students with Disabilities

Representation: Internalize Comprehension. Use color and annotations to illustrate student strategies. As students describe how they calculated the area of each triangle, use color and annotations to scribe their thinking on a display. Ask students how they knew which measurements to use, and label each base and height accordingly.

Supports accessibility for: Visual-spatial processing; Conceptual processing

Support for English Language Learners

Representing, Conversing: MLR2 Collect and Display. Use this routine to collect the initial language and representations students produce when finding the area of a triangle prior to formalizing a formula. As students work through the questions, circulate and observe the various strategies they use to find area. Take pictures of different strategies or sketch them onto a display. Look for students who decompose triangles, or who enclose triangles in a rectangle. While students confer with a partner, continue to add examples of student language to the display. During the whole-class discussion, invite students to borrow language from this display to help them explain their thinking.

Design Principle(s): Support sense-making; Maximize meta-awareness

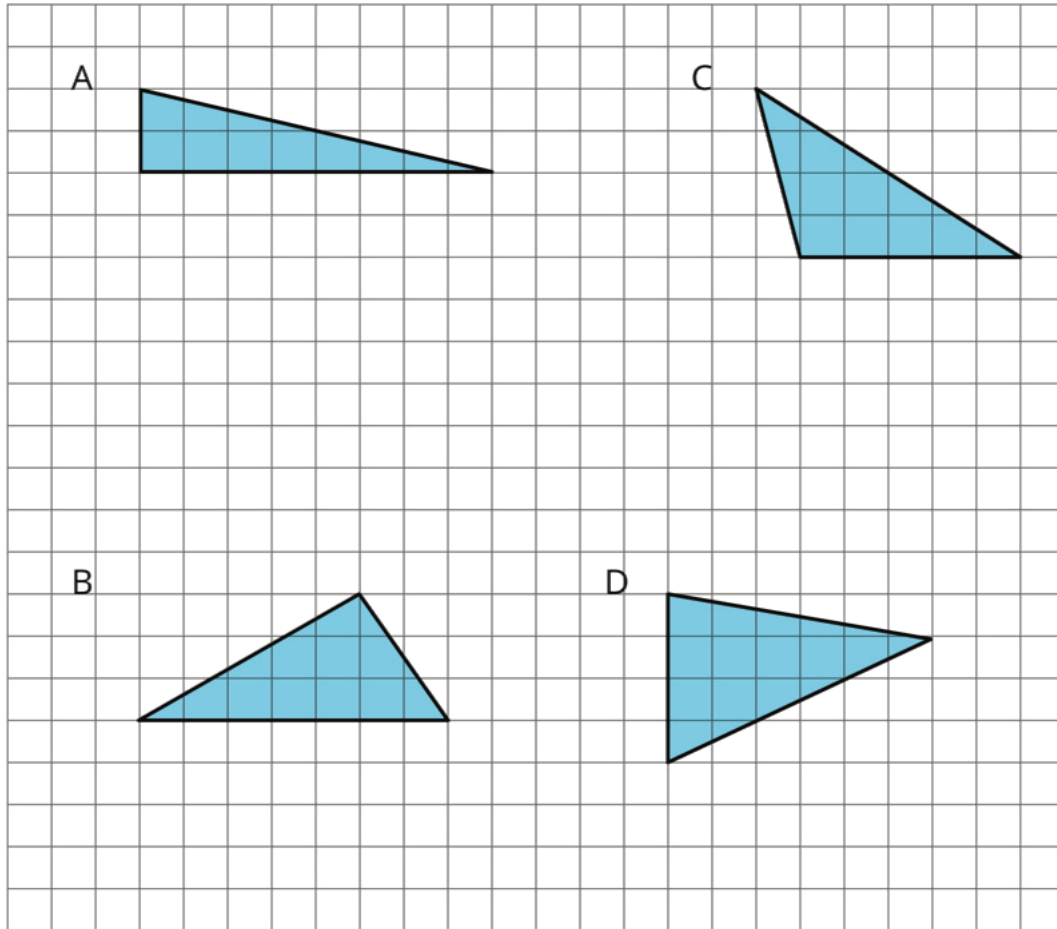
Anticipated Misconceptions

At this point students should not be counting squares to determine area. If students are still using this approach, steer them in the direction of recently learned strategies (decomposing, rearranging, enclosing, or duplicating).

Students may not recognize that the vertical side of Triangle D could be the base and try to measure the lengths the other sides. If so, remind them that any side of a triangle can be the base.

Student Task Statement

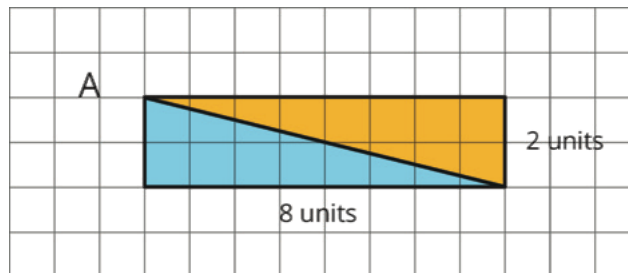
Find the areas of at least two of these triangles. Show your reasoning.



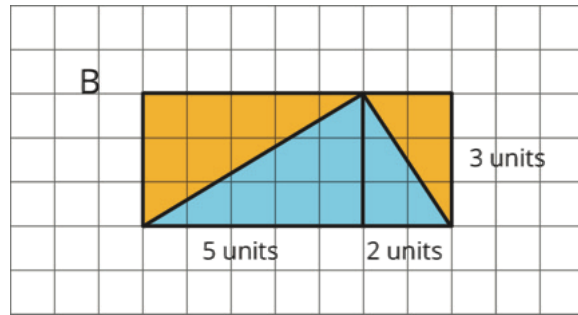
Student Response

Diagrams and explanations vary. Sample responses:

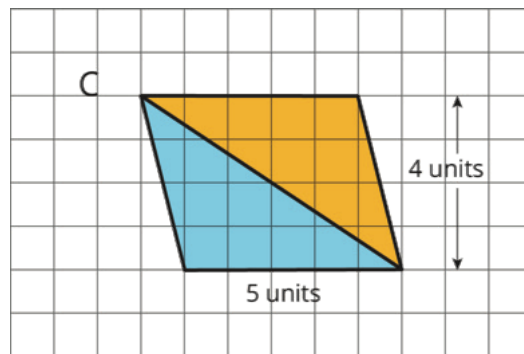
A: 8 square units. $8 \cdot 2 = 16$, so the area of the rectangle is 16 square units. The area of the triangle is half of that of the rectangle, so it is 8 square units.



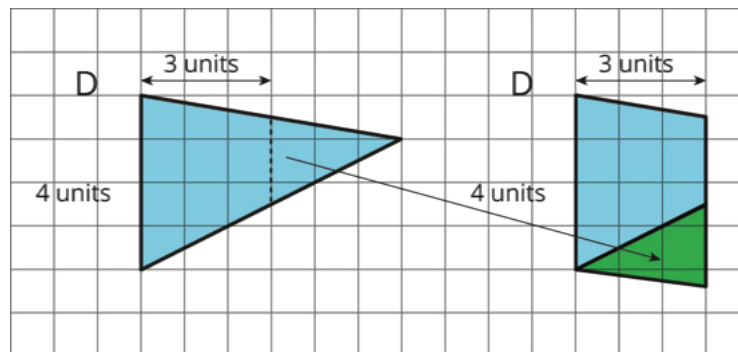
B: 10.5 square units. $5 \cdot 3 = 15$, so the area of the left rectangle is 15 square units. The area of the left triangle is then 7.5 square units. $2 \cdot 3 = 6$, so the area of the right rectangle is 6 square units, so area of the right triangle is 3 square units. The sum of the areas of the small triangles which make up the large triangle is $7.5 + 3 = 10.5$, so the large triangle has area 10.5 square units.



C: 10 square units. If we make a copy of the triangle, rotate it, and join them along the longest side we would get a parallelogram. The base length is 5 units and the height is 4 units, so the area of the parallelogram is 20 square units. The area of the triangle is half of that area, so it is 10 square units.



D: 12 square units. Decompose the triangle into a trapezoid and a small triangle by drawing a vertical line 3 units from the left side. Rotate the small triangle to line up with the bottom side of the trapezoid to create a parallelogram. To get the area of that parallelogram: $4 \cdot 3 = 12$.



Activity Synthesis

Though students may have conferred with one or more partners during the task, take a few minutes to come together as a class so that everyone has a chance to see a wider range of approaches.

Select previously identified students to explain their approach and display their reasoning for all to see. Start with the most-elaborate strategy (most likely a strategy that involves enclosing a triangle), and move toward the most direct (most likely duplicating the triangle to compose a parallelogram). After each student presents, ask the class:

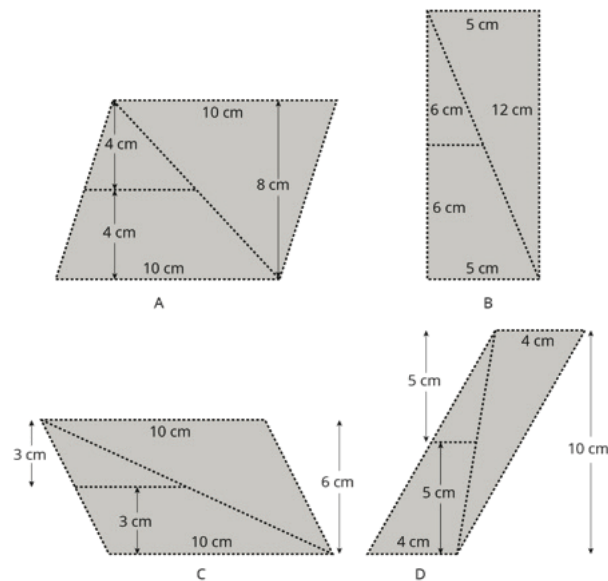
- “Did anyone else reason the same way?”
- “Did anyone else draw the same diagram but think about the problem differently?”
- “Can this strategy be used on another triangle in this set? Which one?”
- “Is there a triangle for which this strategy would *not* be helpful? Which one, and why not?”

8.3 Decomposing a Parallelogram

Optional: 25 minutes

By now students have more than one path for finding the area of a triangle. This optional activity offers one more lens for thinking about the relationship between triangles and parallelograms. Previously, students duplicated triangles to compose parallelograms. Here they see that a different set of parallelograms can be created from a triangle, not by duplicating it, but by *decomposing* it.

Students are assigned a parallelogram to be cut into two congruent triangles. They take one triangle and decompose it into smaller pieces by cutting along a line that goes through the midpoints of two sides. They then use these pieces to compose a new parallelogram (two parallelograms are possible) and find its area.



Students notice that the height of this new parallelogram is half of the original parallelogram, and the area is also half of that of the original parallelogram. Because the new parallelogram is composed of the same parts as a large triangle, the area of triangle is also half of that of the original parallelogram. This reasoning paves another way to understand the formula for the area of triangles.

Of the four given parallelograms, Parallelogram B is likely the most manageable for students. When decomposed, its pieces (each with a right angle) resemble those seen in earlier work on parallelograms. Consider this as you assign parallelograms to students.

Addressing

- 6.G.A.1

Instructional Routines

- MLR7: Compare and Connect

Launch

Tell students that they will investigate another way in which triangles and parallelograms are related. Arrange students in groups of 2–4. Assign a different parallelogram from the Instructional master to each student in the group. Give each student two copies of the parallelogram and access to a pair of scissors and some tape or glue.

Each parallelogram shows some measurements and dotted lines for cutting. For the first question, students who have parallelograms C and D should *not* cut off the measurements shown outside of the figures.

Give students 10 minutes to complete the activity, followed by a few minutes to discuss their work (especially the last three questions). Ask students who finish early to find someone with the same original parallelogram and compare their work.

Support for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills. For example, reveal only one question at a time, pausing to check for understanding before moving on.
Supports accessibility for: Organization; Attention

Anticipated Misconceptions

Students may struggle to form a new parallelogram because the two composing pieces are not both facing up (i.e. either the triangle or the trapezoid is facing down). Tell them that the shaded side of the cut-outs should face up.

Students may struggle to use the appropriate measurements needed to find the area of the parallelogram in the first question. They may multiply more numbers than necessary because the measurements are given. If this happens, remind them that only two measurements (base and height) are needed to determine the area of a parallelogram.

Student Task Statement

1. Your teacher will give you two copies of a parallelogram. Glue or tape *one* copy of your parallelogram here and find its area. Show your reasoning.

- Decompose the second copy of your parallelogram by cutting along the dotted lines. Take *only* the small triangle and the trapezoid, and rearrange these two pieces into a different parallelogram. Glue or tape the newly composed parallelogram on your paper.
- Find the area of the new parallelogram you composed. Show your reasoning.
- What do you notice about the relationship between the area of this new parallelogram and the original one?
- How do you think the area of the large triangle compares to that of the new parallelogram: Is it larger, the same, or smaller? Why is that?
- Glue or tape the remaining large triangle to your paper. Use any part of your work to help you find its area. Show your reasoning.

Student Response

Parallelogram A:

1. 80 sq cm. $10 \cdot 8 = 80$.

2.



3. 40 sq cm. $10 \cdot 4 = 40$.

Parallelogram B:

1. $5 \cdot 12 = 60$.

2.

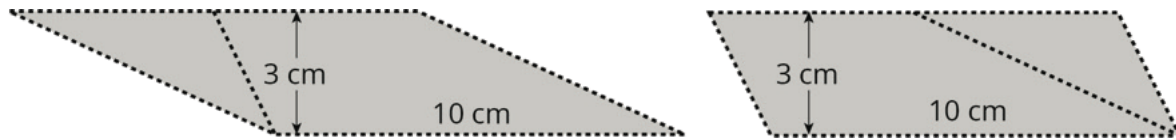


3. 30 sq cm. $5 \cdot 6 = 30$.

Parallelogram C:

1. 60 sq cm. $10 \cdot 6 = 60$.

2.

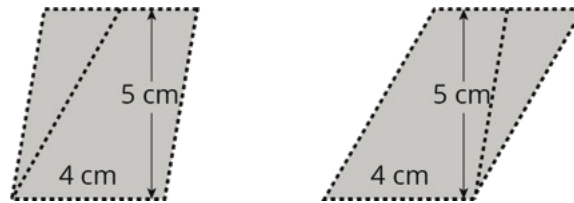


3. 30 sq cm. $10 \cdot 3 = 30$.

Parallelogram D:

1. 40 sq cm. $4 \cdot 10 = 40$.

2.



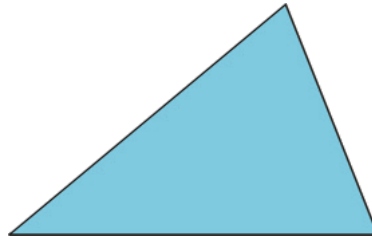
3. 20 sq cm. $4 \cdot 5 = 20$.

All Parallelograms:

1. The area of the new parallelogram is half the area of the original one.
2. Answers vary. Sample responses:
 - The area of the large triangle is the same as that of the new parallelogram. I know that because the trapezoid and little triangle together can be arranged into a triangle that is identical to the large triangle.
 - The new parallelogram and the large triangle have the same area since they are two halves of the original parallelogram.
3. Answers vary. Sample responses:
 - The large triangle in Parallelogram A has an area of 40 sq cm since that is the area of the new parallelogram.
 - The large triangle in Parallelogram D has an area of 20 sq cm since it is half of the original parallelogram, which has an area of 40 sq cm.

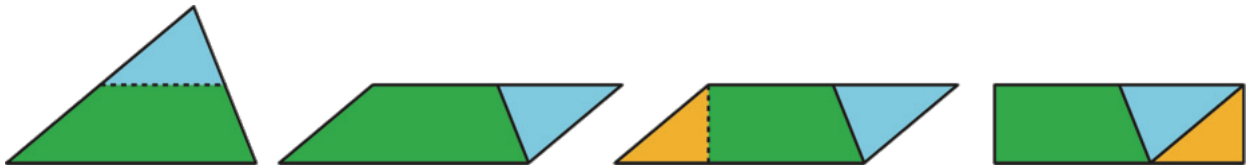
Are You Ready for More?

Can you decompose this triangle and rearrange its parts to form a rectangle? Describe how it could be done.



Student Response

Answers vary. Cutting the triangle at half its height, parallel to the base, creates a parallelogram, then another cut helps create a rectangle.



Activity Synthesis

For each parallelogram, invite a student to share with the class their new parallelogram (their answer to the second question). Then, ask if anyone who started with the same original figure created a different, new parallelogram. If a student created a different one, ask them to share it to the class. After all four parallelograms have been presented, discuss:

- “How many possible parallelograms can be created from each set of trapezoid and triangle?”
- “Do they all yield the same area? Why or why not?”
- “How does the area of the new parallelogram relate to the area of the original parallelogram?” (It is half the area of the original.) Why do you think that is?” (The new parallelogram is decomposed and rearranged from a triangle that is half of the original parallelogram. The original and new parallelograms have one side in common (they have the same length), but the height of the new parallelogram is half of that of the original.)
- “Can the area of the large triangle be determined? How?” (Yes. It has the same area as the new parallelogram because it is composed of the same pieces.)

If not already observed by students, point out that, just as in earlier investigations, we see that:

- The area of a triangle is half of that of a related parallelogram that share the same base.
- The triangle and the related parallelogram have at least one side in common.

Support for English Language Learners

Speaking, Listening: MLR7 Compare and Connect. Select students who use varying approaches to the last question to share their reasoning with the class and display their work for all to see. Ask “What is the same and what is different among the different approaches?” Listen for and amplify students’ use of mathematical language (e.g., triangle, parallelogram, trapezoid) and relationships (e.g., “half of”, “same as”). These exchanges strengthen students’ mathematical language use and reasoning.

Design Principle(s): Optimize meta-awareness; Support sense-making

Lesson Synthesis

In this lesson, we practiced using what we know about parallelograms to reason about areas of triangles. We duplicated a triangle to make a parallelogram, decomposed and rearranged a triangle into a parallelogram, or enclosed a triangle with one or more rectangles.

- “What can we say about the area of a triangle and that of a parallelogram with the same height?” (The area of the triangle is half of the area of the related parallelogram.)
- “In the second activity, we cut a triangle along a line that goes through the midpoints of two sides and rearranged the pieces into a parallelogram. What did we notice about the area and the height of the resulting parallelogram?” (It has the same area as the original triangle but half its height.)
- “How might we start finding the area of any triangle, in general?” (Start by finding the area of a related parallelogram whose base is also a side of the triangle.)

8.4 An Area of 14

Cool Down: 5 minutes

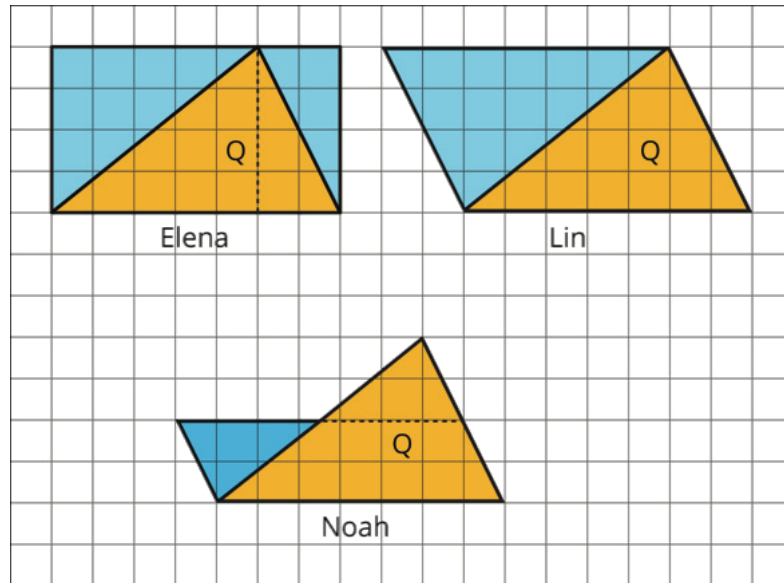
Students have explored several ways to reason about the area of a triangle. This cool-down prompts them to articulate at least one way to do so. Not all methods will be equally intuitive or clear to them. In writing a commentary about at least one approach, students can show what makes sense to them at this point.

Addressing

- 6.G.A.1

Student Task Statement

Elena, Lin, and Noah all found the area of Triangle Q to be 14 square units but reasoned about it differently, as shown in the diagrams. Explain *at least one* student’s way of thinking and why his or her answer is correct.



Student Response

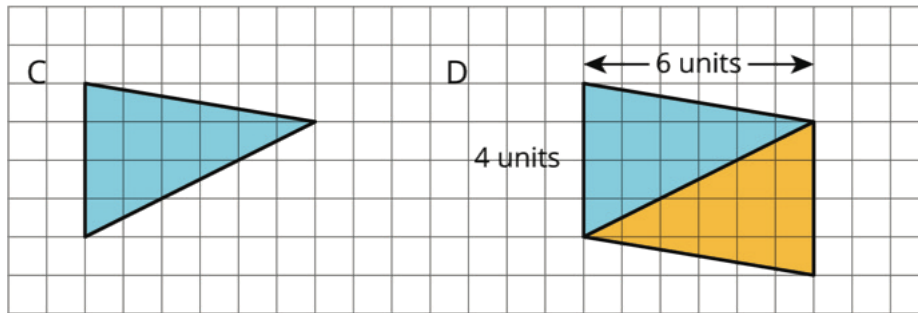
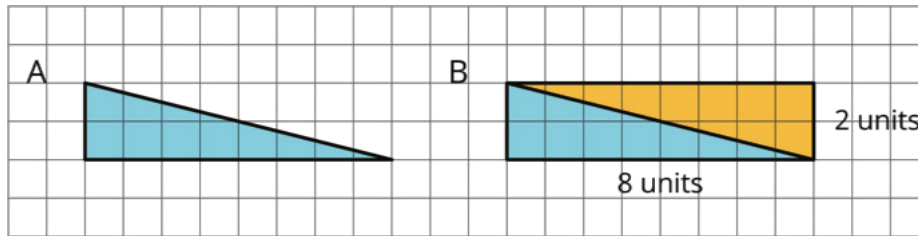
Explanations vary. Sample responses:

- Elena drew two rectangles that decomposed the triangle into two right triangles. She found the area of each right triangle to be half of the area of its enclosing rectangle. This means that the area of the original triangle is the sum of half of the area of the rectangle on the left and half of the rectangle on the right. Half of $(4 \cdot 5)$ plus half of $(4 \cdot 2)$ is $10 + 4$, so the area is 14 square units.
- Lin saw it as half of a parallelogram with the base of 7 units and height of 4 units (and thus an area of 28 square units). Half of 28 is 14.
- Noah decomposed the triangle by cutting it at half of the triangle's height, turning the top triangle around, and joining it with the bottom trapezoid to make a parallelogram. He then calculated the area of that parallelogram, which has the same base length but half the height of the triangle. $7 \cdot 2 = 14$, so the area is 14 square units.

Student Lesson Summary

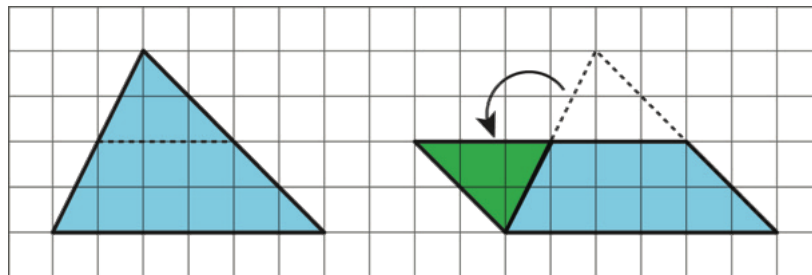
We can reason about the area of a triangle by using what we know about parallelograms. Here are three general ways to do this:

- Make a copy of the triangle and join the original and the copy along an edge to create a parallelogram. Because the two triangles have the same area, one copy of the triangle has half the area of that parallelogram.



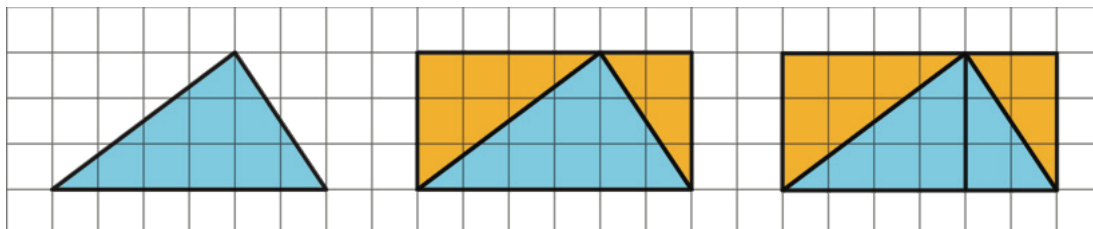
The area of Parallelogram B is 16 square units because the base is 8 units and the height 2 units. The area of Triangle A is half of that, which is 8 square units. The area of Parallelogram D is 24 square units because the base is 4 units and the height 6 units. The area of Triangle C is half of that, which is 12 square units.

- Decompose the triangle into smaller pieces and compose them into a parallelogram.



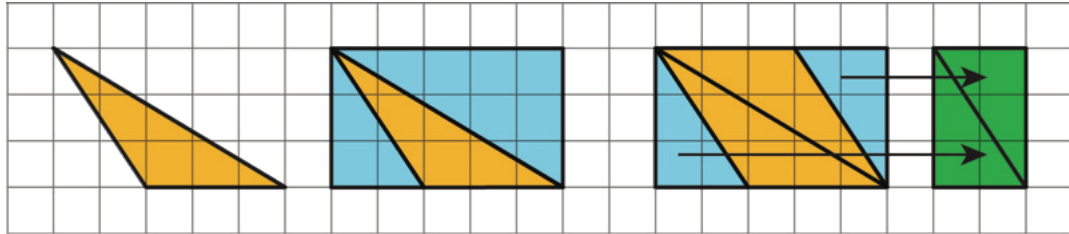
In the new parallelogram, $b = 6$, $h = 2$, and $6 \cdot 2 = 12$, so its area is 12 square units. Because the original triangle and the parallelogram are composed of the same parts, the area of the original triangle is also 12 square units.

- Draw a rectangle around the triangle. Sometimes the triangle has half of the area of the rectangle.



The large rectangle can be decomposed into smaller rectangles. The one on the left has area $4 \cdot 3$ or 12 square units; the one on the right has area $2 \cdot 3$ or 6 square units. The large triangle is also decomposed into two right triangles. Each of the right triangles is half of a smaller rectangle, so their areas are 6 square units and 3 square units. The large triangle has area 9 square units.

Sometimes, the triangle is half of what is left of the rectangle after removing two copies of the smaller right triangles.



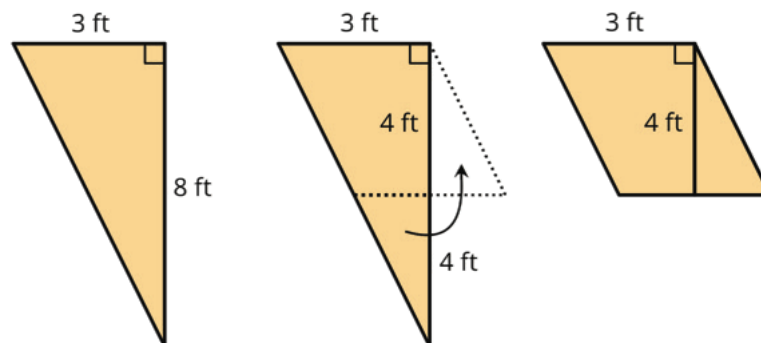
The right triangles being removed can be composed into a small rectangle with area $(2 \cdot 3)$ square units. What is left is a parallelogram with area $5 \cdot 3 - 2 \cdot 3$, which equals $15 - 6$ or 9 square units. Notice that we can compose the same parallelogram with two copies of the original triangle! The original triangle is half of the parallelogram, so its area is $\frac{1}{2} \cdot 9$ or 4.5 square units.

Lesson 8 Practice Problems

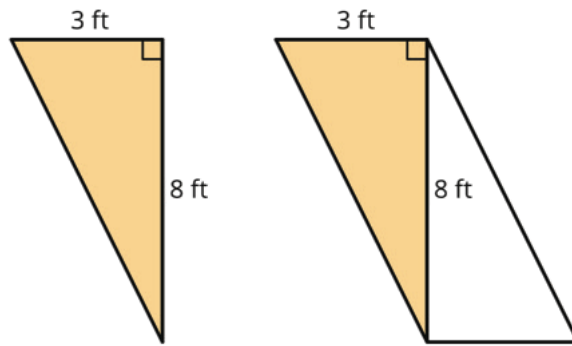
Problem 1

Statement

To find the area of this right triangle, Diego and Jada used different strategies. Diego drew a line through the midpoints of the two longer sides, which decomposes the triangle into a trapezoid and a smaller triangle. He then rearranged the two shapes into a parallelogram.



Jada made a copy of the triangle, rotated it, and lined it up against one side of the original triangle so that the two triangles make a parallelogram.



- Explain how Diego might use his parallelogram to find the area of the triangle.
- Explain how Jada might use her parallelogram to find the area of the triangle.

Solution

Answers vary. Sample explanations:

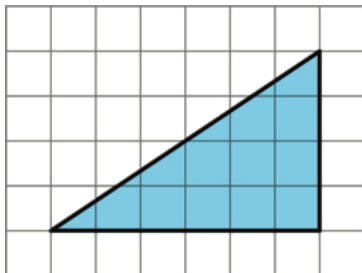
- Diego's parallelogram has a base of 3 feet and a height of 4 feet, so its area is 12 square feet. Because the original right triangle and the parallelogram are composed of the same parts, they have the same area. The area of the triangle is also 12 square feet.
- Jada's parallelogram has a base of 3 feet and a height of 8 feet, so its area is 24 square feet. Because it is composed of two copies of the right triangle, she could divide 24 by 2 to find the area of the triangle. $24 \div 2 = 12$ or 12 square feet.

Problem 2

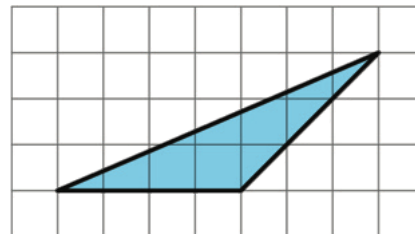
Statement

Find the area of the triangle. Explain or show your reasoning.

a.

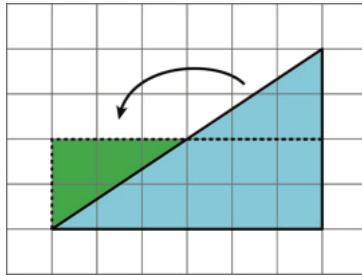


b.



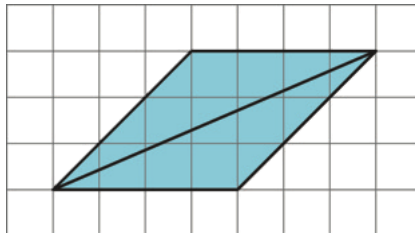
Solution

- 12 square units. Reasoning varies. Sample reasoning: Make a horizontal cut, and rearrange the pieces to make a rectangle. The rectangle is 2 units by 6 units, so its area is 12 square units.

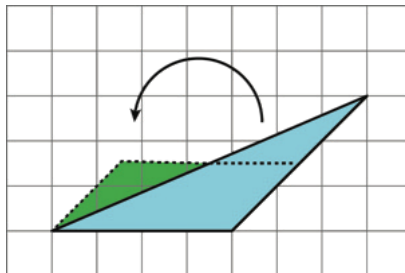


b. 6 square units. reasoning varies. Sample reasoning:

- Duplicate the triangle, and rearrange the pieces to make a parallelogram. The parallelogram has a base of 4 units and a height of 3 units, so its area is 12 square units. Since the parallelogram's area is double the triangle's, the triangle's area is 6 square units.



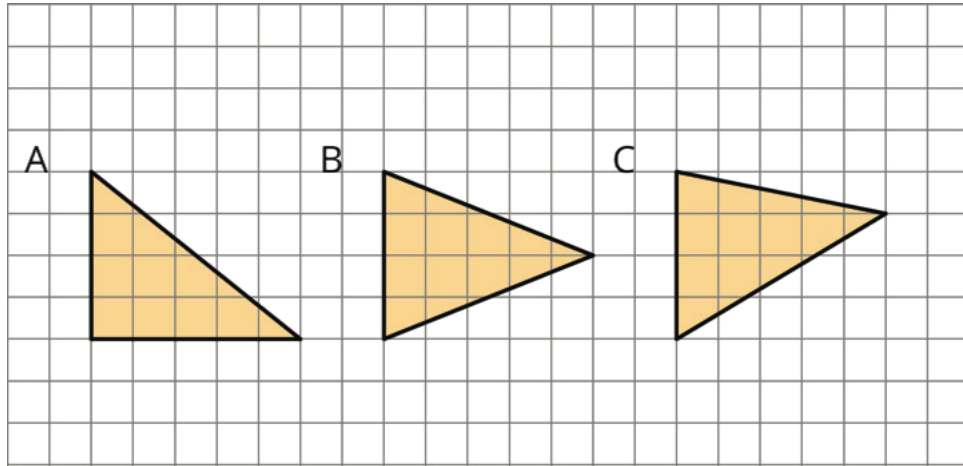
- Decompose the triangle with a cut line half-way between the base and the opposite vertex. Rearrange the smaller triangle to form a parallelogram. This parallelogram has a horizontal base of length 4 units and a height of 1.5 units, so its area is 6 square units. That means the area of the original triangle is 6 square units.



Problem 3

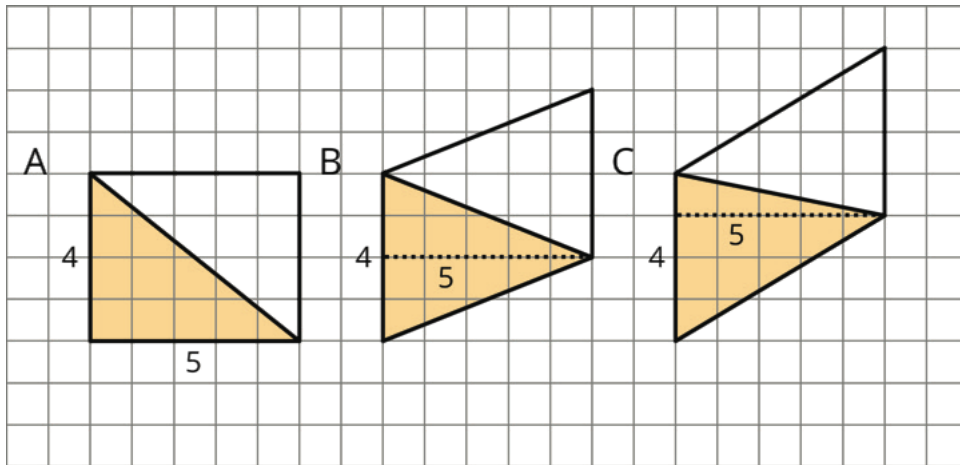
Statement

Which of the three triangles has the greatest area? Show your reasoning. If you get stuck, try using what you know about the area of parallelograms.



Solution

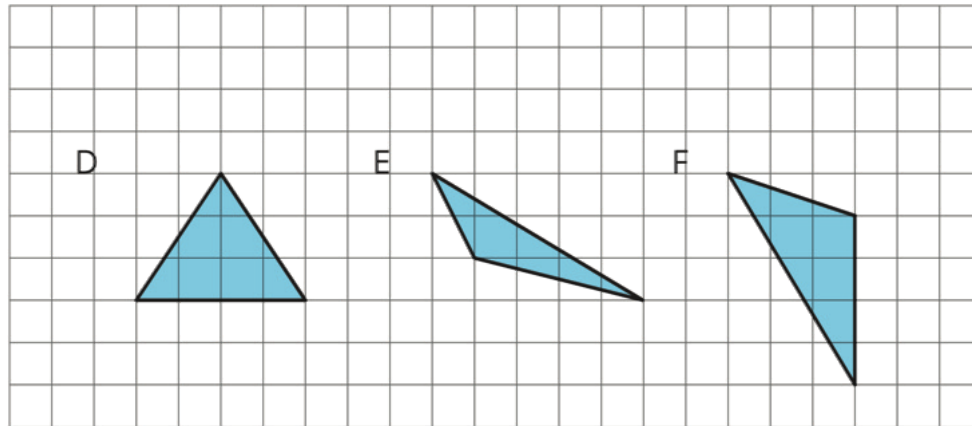
All three triangles have the same area of 10 square units. Reasoning varies. Sample reasoning: Two identical copies of each triangle can be composed into a parallelogram with a base of 5 units and a corresponding height of 4 units, which means an area of 20 square units. The area of each triangle is half of that of the parallelogram. $\frac{1}{2} \cdot 20 = 10$.



Problem 4

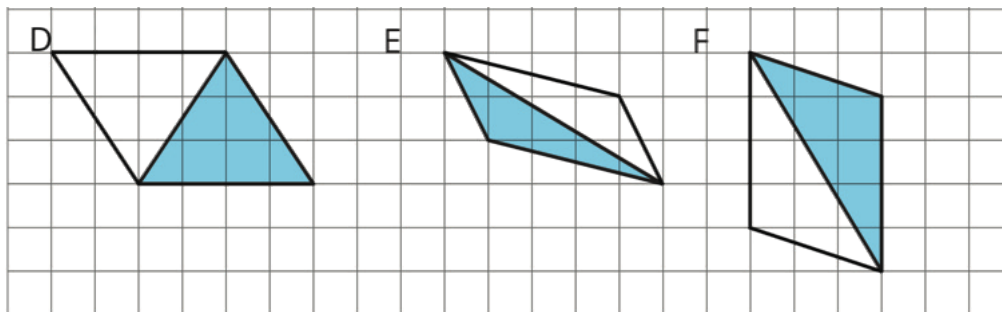
Statement

Draw an identical copy of each triangle such that the two copies together form a parallelogram. If you get stuck, consider using tracing paper.



Solution

Answers vary. Sample response:



(From Unit 1, Lesson 7.)

Problem 5

Statement

- A parallelogram has a base of 3.5 units and a corresponding height of 2 units. What is its area?
- A parallelogram has a base of 3 units and an area of 1.8 square units. What is the corresponding height for that base?
- A parallelogram has an area of 20.4 square units. If the height that corresponds to a base is 4 units, what is the base?

Solution

- 7 square units
- 0.6 units
- 5.1 units

(From Unit 1, Lesson 6.)

Lesson 9: Formula for the Area of a Triangle

Goals

- Compare, contrast, and critique (orally) different strategies for determining the area of a triangle.
- Generalize a process for finding the area of a triangle, and justify (orally and in writing) why this can be abstracted as $\frac{1}{2} \cdot b \cdot h$.
- Recognize that any side of a triangle can be considered its base, choose a side to use as the base when calculating the area of a triangle, and identify the corresponding height.

Learning Targets

- I can use the area formula to find the area of any triangle.
- I can write and explain the formula for the area of a triangle.
- I know what the terms “base” and “height” refer to in a triangle.

Lesson Narrative

In this lesson students begin to reason about area of triangles more methodically: by generalizing their observations up to this point and expressing the area of a triangle in terms of its **base** and **height**.

Students first learn about bases and heights in a triangle by studying examples and counterexamples. They then identify base-height measurements of triangles, use them to determine area, and look for a pattern in their reasoning to help them write a general formula for finding area (MP8). Students also have a chance to build an informal argument about why the formula works for any triangle (MP3).

Alignments

Addressing

- 6.EE.A.2.a: Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation “Subtract y from 5” as $5 - y$.
- 6.EE.A.2.c: Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.
- 6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Building Towards

- 6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports
- Notice and Wonder

Required Materials

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Student Learning Goals

Let's write and use a formula to find the area of a triangle.

9.1 Bases and Heights of a Triangle

Warm Up: 10 minutes

In this activity, students think about the meaning of **base** and **height** in a triangle by studying examples and non-examples. The goal is for them to see that in a triangle:

- Any side can be a base.
- A segment that represents a height must be drawn at a right angle to the base, but can be drawn in more than one place. The length of this perpendicular segment is the distance between the base and the vertex opposite it.
- A triangle can have three possible bases and three corresponding heights.

Students may draw on their experience with bases and heights in a parallelogram and observe similarities. Encourage this, as it would help them conceptualize base-height pairs in triangles.

As students discuss with their partners, listen for how they justify their decisions or how they know which statements are true (MP3).

Building Towards

- 6.G.A.1

Instructional Routines

- Notice and Wonder

Launch

Display the examples and non-examples of bases and heights for all to see. Give students a minute to observe them. Ask them to be ready to share at least one thing they notice and one thing they wonder. Give the class a minute to share some of their observations and questions.

Tell students they will now use the examples and non-examples to determine what is true about bases and heights in a triangle. Arrange students in groups of 2. Give them 2–3 minutes of quiet think time and then a minute to share their response with a partner.

Anticipated Misconceptions

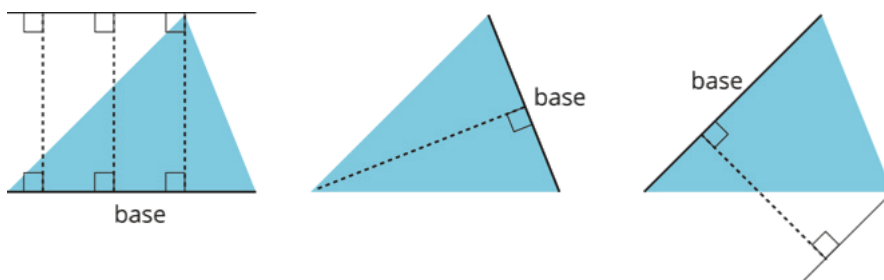
Some students may struggle to interpret the diagrams. Ask them to point out parts of the diagrams that might be unclear and clarify as needed.

Students may not remember from their experience with parallelograms that a height needs to be perpendicular to a base. Consider posting a diagram of a parallelogram—with its base and height labeled—in a visible place in the room so that it can serve as a reference.

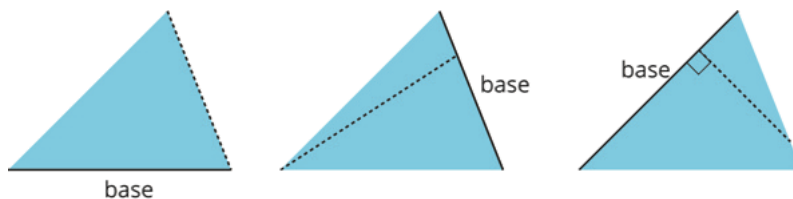
Student Task Statement

Study the examples and non-examples of **bases** and **heights** in a triangle.

- Examples: These dashed segments represent heights of the triangle.



- Non-examples: These dashed segments do *not* represent heights of the triangle.



Select **all** the statements that are true about bases and heights in a triangle.

1. Any side of a triangle can be a base.

2. There is only one possible height.
3. A height is always one of the sides of a triangle.
4. A height that corresponds to a base must be drawn at an acute angle to the base.
5. A height that corresponds to a base must be drawn at a right angle to the base.
6. Once we choose a base, there is only one segment that represents the corresponding height.
7. A segment representing a height must go through a vertex.

Student Response

Only statements 1 and 5 are true.

Activity Synthesis

For each statement, ask students to indicate whether they think it is true. For each “true” vote, ask one or two students to explain how they know (MP3). Do the same for each “false” vote. Encourage students to use the examples and counterexamples to support their argument (e.g., “The last statement is not true because the examples show dashed segments or heights that do not go through a vertex.”) which means more than one height.” Agree on the truth value of each statement before moving on. Record and display the true statements for all to see.

Students should see that only statements 1 and 5 are true—that any side of a triangle can be a base, and a segment for the corresponding height must be drawn at a right angle to the base. What is missing—an important gap to fill during discussion—is the length of any segment representing a height.

Ask students, “How long should a segment that shows a height be? If we draw a perpendicular line from the base, where do we stop?” Solicit some ideas from students.

Explain that the length of each perpendicular segment is the distance between the base and the vertex opposite of it. The **opposite vertex** is the vertex that is not an endpoint of the base. Point out the opposite vertex for each base. Clarify that the segment does not have to be drawn through the vertex (although that would be a natural place to draw it), as long as it maintains that distance between the base and the opposite vertex.

It is helpful to connect this idea to that of heights in a parallelogram. Consider duplicating the triangle and use the original and the copy to compose a parallelogram. The height for a chosen base in the triangle is also the height of the parallelogram with the same base.

Students will have many opportunities to make sense of bases and heights in this lesson and an upcoming one, so they do not need to know how to draw a height correctly at this point.

9.2 Finding a Formula for Area of a Triangle

20 minutes

This task culminates in writing a formula for the area of triangles. By now students are likely to have developed the intuition that the area of a triangle is half of that of a parallelogram with the same base and height. This activity encapsulates that work in an algebraic expression.

Students first find the areas of several triangles given base and height measurements. They then generalize the numerical work to arrive at an expression for finding the area of any triangle (MP8).

If needed, remind students how they reasoned about the area of triangles in the previous lesson (i.e. by composing a parallelogram, enclosing with one or more rectangles, etc.). Encourage them to refer to their previous work and use tracing paper as needed. Students might write $b \cdot h \div 2$ or $b \cdot h \cdot \frac{1}{2}$ as the expression for the area of any triangle. Any equivalent expression should be celebrated.

At the end of the activity, consider giving students a chance to reason more abstractly and deductively, i.e., to think about why the expression $b \cdot h \div 2$ would hold true for all triangles. See Activity Synthesis for prompts and diagrams that support such reasoning.

Addressing

- 6.EE.A.2.a
- 6.G.A.1

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2–3. Explain that they will now find the area of some triangles using what they know about base-height pairs in triangles and the relationships between triangles and parallelograms.

Give students 5–6 minutes to complete the activity and access to geometry toolkits, especially tracing paper. Ask them to find the area at least a couple of triangles independently before discussing with their partner(s).

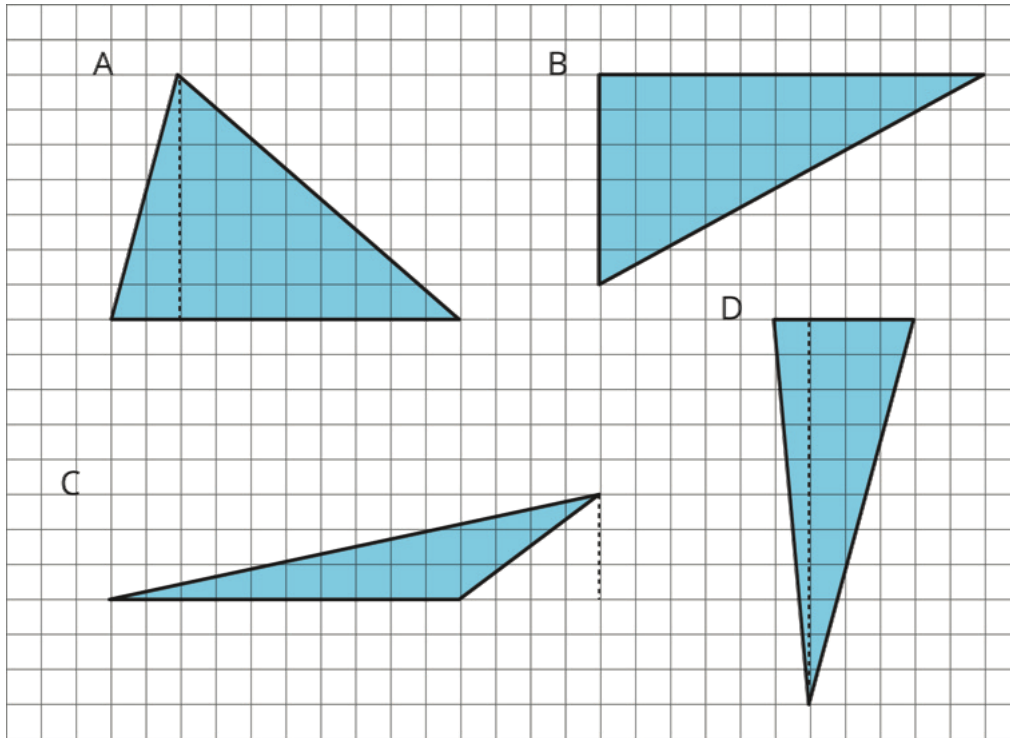
Anticipated Misconceptions

Students may not be inclined to write an expression using the variables b and h and instead replace the variables with numbers of their choice. Ask them to reflect on what they did with the numbers for the first four triangles. Then, encourage them to write the same operations but using the letters b and h rather than numbers.

Student Task Statement

For each triangle:

- Identify a base and a corresponding height, and record their lengths in the table.
- Find the area of the triangle and record it in the last column of the table.



triangle	base (units)	height (units)	area (square units)
A			
B			
C			
D			
any triangle	b	h	

In the last row, write an expression for the area of any triangle, using b and h .

Student Response

1.

triangle	base (units)	height (units)	area (square units)
A	10	7	35
B	11 (or 6)	6 (or 11)	33
C	10	3	15
D	4	11	22
any triangle	b	h	$b \cdot h \div 2$ (or equivalent)

2. Explanations vary. Sample responses:

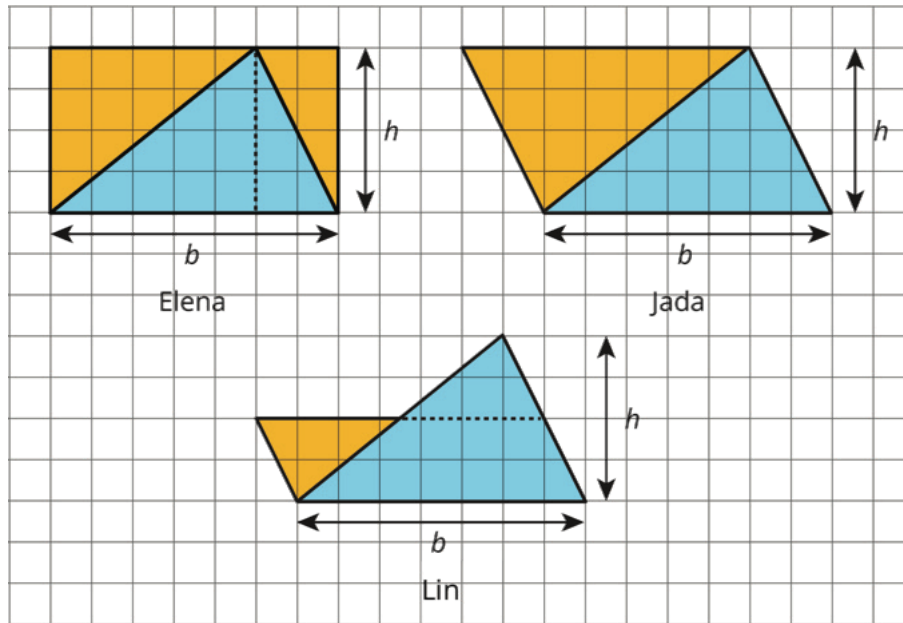
- We can make a parallelogram from any triangle using the same base and height. The triangle will be half of the parallelogram. The area of a parallelogram is the length of the base times the length of the height, so the area of the triangle will be $b \cdot h \div 2$.
- I can cut off the top half of a triangle and rotate it to make a parallelogram. That parallelogram has a base of b and a height that is half of the original triangle, which is $\frac{1}{2} \cdot h$, so its area is $b \cdot \frac{1}{2} \cdot h$. Since the parallelogram is just the triangle rearranged, the area of the triangle is also $\frac{1}{2}b \cdot h$.

Activity Synthesis

Select a few students to share their expression for finding the area of any triangle. Record each expression for all to see.

To give students a chance to reason logically and deductively about their expression, ask, "Can you explain why this expression is true for *any* triangle?"

Display the following diagrams for all to see. Give students a minute to observe the diagrams. Ask them to choose one that makes sense to them and use that diagram to explain or show in writing that the expression $b \cdot h \div 2$ works for finding the area of any triangle. (Consider giving each student an index card or a sheet of paper on which to write their reasoning so that their responses could be collected, if desired.)



When dealing only with the variables b and h and no numbers, students are likely to find Jada's and Lin's diagrams more intuitive to explain. Those choosing to use Elena's diagram are likely to suggest moving one of the extra triangles and joining it with the other to form a non-rectangular parallelogram with an area of $b \cdot h$. Expect students to be less comfortable reasoning in abstract terms than in concrete terms. Prepare to support them in piecing together a logical argument using only variables.

If time permits, select students who used different diagrams to share their explanation, starting with the most commonly used diagram (most likely Jada's). Ask other students to support, refine, or disagree with their arguments. If time is limited, consider collecting students' written responses now and discussing them in an upcoming lesson.

Support for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and concepts. Invite students to suggest language or diagrams to include that will support their understanding. Include the following terms and maintain the display for reference throughout the unit: opposite vertex, base (of a triangle), height (of a triangle), and formula for finding area.

Supports accessibility for: Memory; Language

Support for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion when students share their expressions for finding the area of any triangle. Provide students with 1–2 minutes of quiet think time to begin to consider why their expression is true for any triangle, before they continue work with a partner to complete their response. Select 1 or 2 pairs of students to share with the class, then call on students to restate their peers' reasoning. This will give more students the chance to use language to interpret and describe expressions for the area of triangles.

Design Principle(s): Support sense-making; Maximize meta-awareness

9.3 Applying the Formula for Area of Triangles

10 minutes

In this activity, students apply the expression they previously generated to find the areas of various triangles. Each diagram is labeled with two or three measurements. Before calculating, students think about which lengths can be used to find the area of each triangle.

As students work, notice students who choose different bases for Triangles B and D. Invite them to contribute to the discussion about finding the areas of right triangles later.

Addressing

- 6.EE.A.2.c
- 6.G.A.1

Instructional Routines

- MLR2: Collect and Display

Launch

Explain to students that they will now practice using their expression to find the area of triangles without a grid. For each triangle, ask students to be prepared to explain which measurement they choose for the base and which one for the corresponding height and why.

Keep students in groups of 2–4. Give students 5 minutes of quiet think time, followed by 1–2 minutes for discussing their responses in their group.

Support for English Language Learners

Representing: MLR2 Collect and Display. While pairs are working, circulate and listen to student talk about identifying the bases and corresponding heights for each of triangles. Write down common or important phrases you hear students say about each triangle, specifically focusing on how students make sense of the base and height of each triangle. Record the words students use to refer to each triangle and display them for all to see during the whole-class discussion.

Design Principle(s): Support sense-making; Maximize meta-awareness

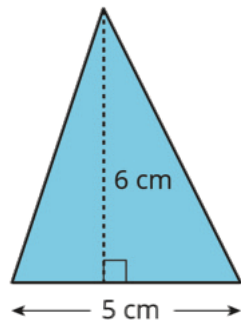
Anticipated Misconceptions

The extra measurement in Triangles C, D, and E may confuse some students. If they are unsure how to decide the measurement to use, ask what they learned must be true about a base and a corresponding height in a triangle. Urge them to review the work from the warm-up activity.

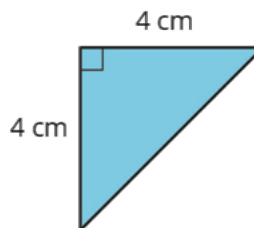
Student Task Statement

For each triangle, circle a base measurement that you can use to find the area of the triangle. Then, find the area of any *three* triangles. Show your reasoning.

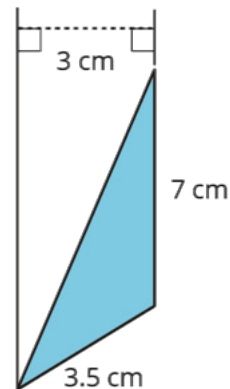
A



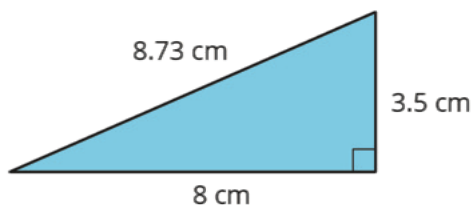
B



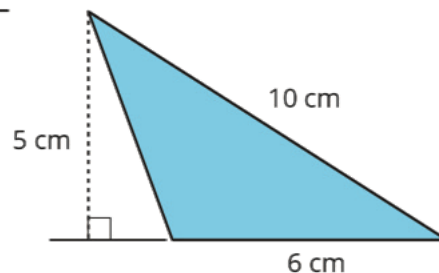
C



D

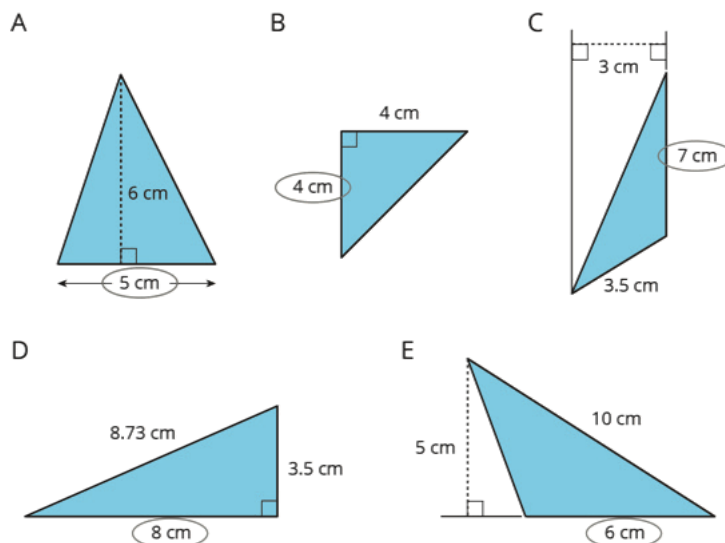


E



Student Response

In B and D either of the given pair of measurements can be the base.



Triangle A: 15 square cm, $b = 5$, $h = 6$, $A = 5 \cdot 6 \div 2 = 15$

Triangle B: 8 square cm, $b = 4$, $h = 4$, $A = 4 \cdot 4 \div 2 = 8$

Triangle C: 10.5 square cm, $b = 7$, $h = 3$, $A = 7 \cdot 3 \div 2 = 10.5$

Triangle D: 14 square cm, $b = 8$, $h = 3.5$, $A = 8 \cdot (3.5) \div 2 = 14$

Triangle E: 15 square cm, $b = 6$, $h = 5$, $A = 6 \cdot 5 \div 2 = 15$

Activity Synthesis

The aim of this whole-class discussion is to deepen students' awareness of the base and height of triangles. Discuss questions such as:

- For Triangle A, can we say that the 6-cm segment is the base and the 5-cm segment is the height? Why or why not? (No, the base of a triangle is one of its sides.)
- What about for Triangle C: Can the 3-cm segment serve as the base? Why or why not? (No, that segment is not a side of the triangle.)
- Can the 3.5-cm side in Triangle D serve as the base? Why or why not? (Yes, it is a side of the triangle, but because we don't have the height that corresponds to it, it is not helpful for finding the area here.)
- More than two measurements are given for Triangles C, D, and E. Which ones are helpful for finding area? (We need a base and a corresponding height, which means the length of one side of the triangle and the length of a perpendicular segment between that side and the opposite vertex.)

- When it comes to finding area, how are right triangles—like B and D—unique? (Either of the two sides that form the right angle could be the base or the height. In non-right triangles—like A, C, and E—the height segment is not a side of the triangle; a different line segment has to be drawn.)

Lesson Synthesis

The area of a parallelogram can be determined using base and height measurements. In this lesson you learned that we can do the same with triangles.

- “How do we locate the **base** of a triangle? How many possible bases are there?” (Any side of a triangle can be a base. There are 3 possible bases.)
- “How do we locate the **height** once we know the base?” (Find the length of a perpendicular segment that connects the base and its opposite vertex.)

We can use the base-height pair of measurements to find the area of a triangle quite simply.

- “What expression works for finding the area of a triangle?” ($\frac{1}{2} \cdot b \cdot h$ or $\frac{b \cdot h}{2}$)
- “Can you explain briefly why this expression or formula works?” (The area of a triangle is always half of the area of a related parallelogram that shares the same base and height.)

You learned that any side of the triangle can be the base, but not all sides can be the height.

- “Are there cases in which both the base and the height are sides of the triangle? When does that happen?” (Yes. In a right triangle, both the base and height can be the sides of the triangle.)

9.4 Two More Triangles

Cool Down: 5 minutes

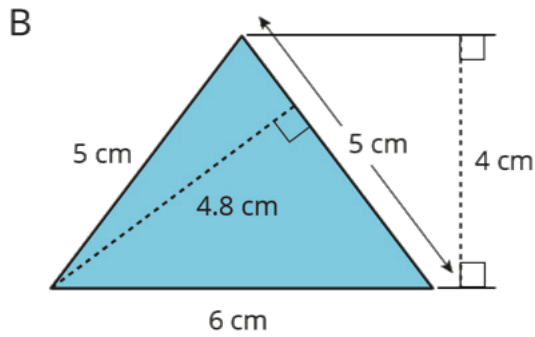
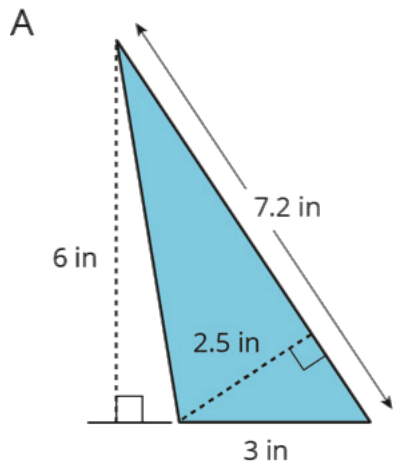
Students apply what they learned about the area formula and about the base and height of a triangle in this cool-down. Multiple measurements are given, so students need to be attentive in choosing the right pair of measurements that would allow them to calculate the area.

Addressing

- 6.EE.A.2.c
- 6.G.A.1

Student Task Statement

For each triangle, identify a base and a corresponding height. Use them to find the area. Show your reasoning.



Student Response

Answers vary. Possible responses:

Triangle A:

- $b = 3, h = 6$, area: $9 \text{ sq in}, \frac{1}{2} \cdot 3 \cdot 6 = 9$
- $b = 7.2, h = 2.5$, area: $9 \text{ sq in}, \frac{1}{2} \cdot (7.2) \cdot (2.5) = 9$

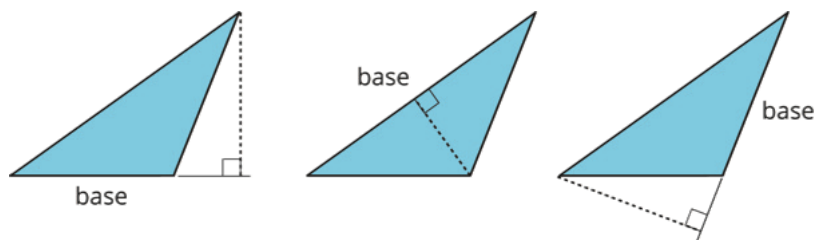
Triangle B:

- $b = 6, h = 4$, area: $12 \text{ sq in}, \frac{1}{2} \cdot 6 \cdot 4 = 12$
- $b = 5, h = 4.8$, area: $12 \text{ sq in}, \frac{1}{2} \cdot 5 \cdot (4.8) = 12$

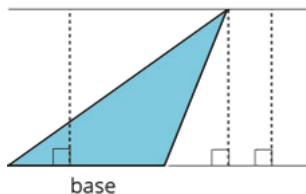
Student Lesson Summary

- We can choose any of the three sides of a triangle to call the **base**. The term “base” refers to both the side and its length (the measurement).
- The corresponding **height** is the length of a perpendicular segment from the base to the vertex opposite of it. The **opposite vertex** is the vertex that is *not* an endpoint of the base.

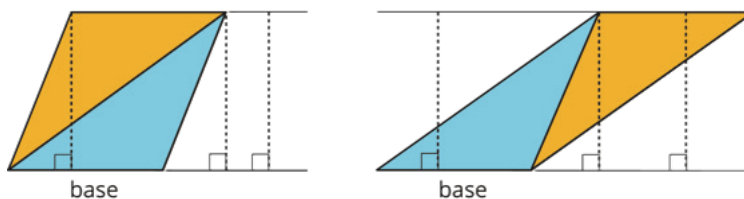
Here are three pairs of bases and heights for the same triangle. The dashed segments in the diagrams represent heights.



A segment showing a height must be drawn at a right angle to the base, but it can be drawn in more than one place. It does not have to go through the opposite vertex, as long as it connects the base and a line that is parallel to the base and goes through the opposite vertex, as shown here.



The base-height pairs in a triangle are closely related to those in a parallelogram. Recall that two copies of a triangle can be composed into one or more parallelograms. Each parallelogram shares at least one base with the triangle.

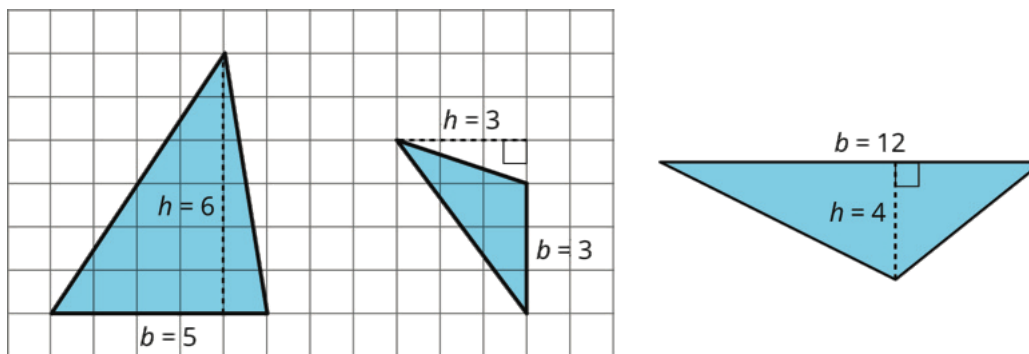


For any base that they share, the corresponding height is also shared, as shown by the dashed segments.

We can use the base-height measurements and our knowledge of parallelograms to find the area of any triangle.

- The formula for the area of a parallelogram with base b and height h is $b \cdot h$.
- A triangle takes up half of the area of a parallelogram with the same base and height. We can therefore express the area A of a triangle as:

$$A = \frac{1}{2} \cdot b \cdot h$$



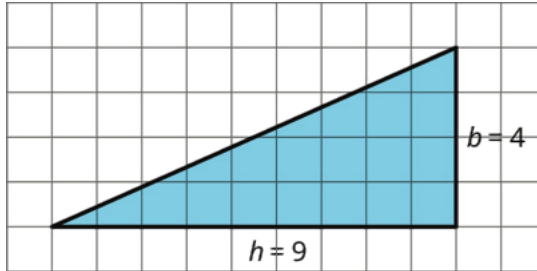
- The area of Triangle A is 15 square units because $\frac{1}{2} \cdot 5 \cdot 6 = 15$.
- The area of Triangle B is 4.5 square units because $\frac{1}{2} \cdot 3 \cdot 3 = 4.5$.

- The area of Triangle C is 24 square units because $\frac{1}{2} \cdot 12 \cdot 4 = 24$.

In each case, one side of the triangle is the base but neither of the other sides is the height. This is because the angle between them is not a right angle.

In right triangles, however, the two sides that are perpendicular can be a base and a height.

The area of this triangle is 18 square units whether we use 4 units or 9 units for the base.



Glossary

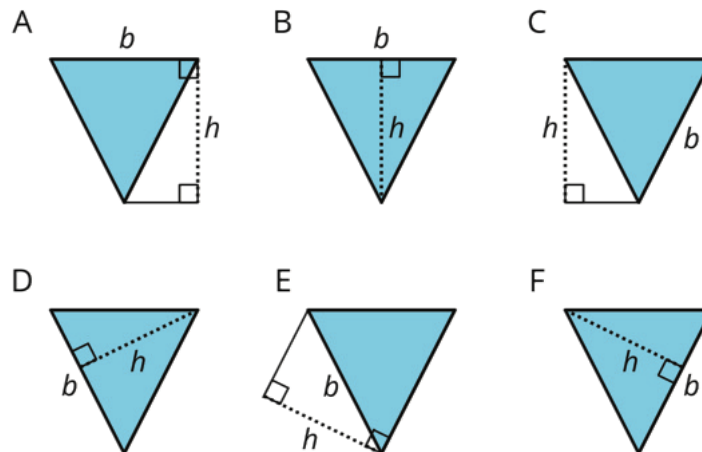
- opposite vertex

Lesson 9 Practice Problems

Problem 1

Statement

Select all drawings in which a corresponding height h for a given base b is correctly identified.



- A. A
- B. B
- C. C
- D. D
- E. E
- F. F

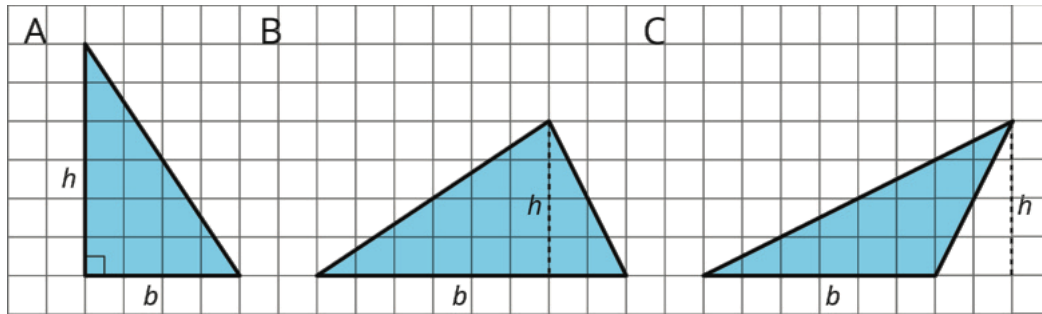
Solution

["A", "B", "D", "F"]

Problem 2

Statement

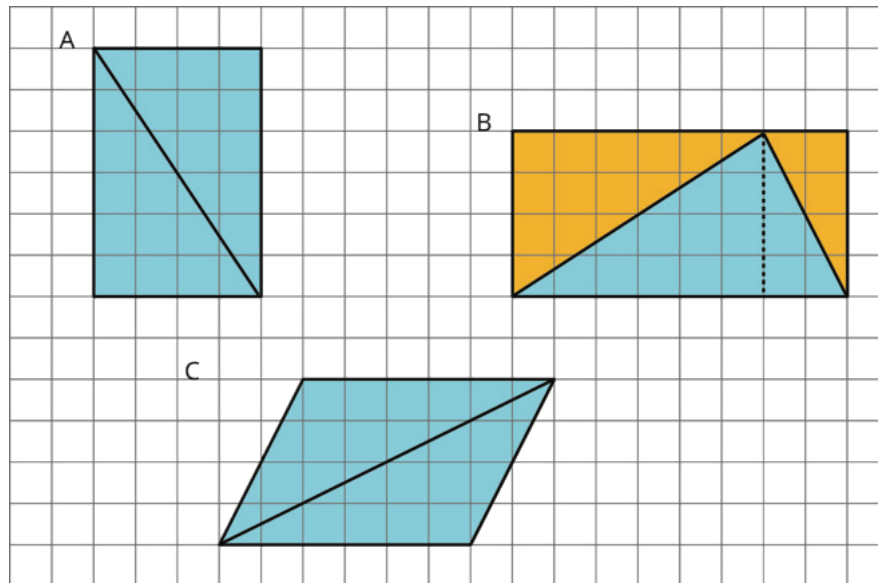
For each triangle, a base and its corresponding height are labeled.



- Find the area of each triangle.
- How is the area related to the base and its corresponding height?

Solution

- Triangle A: 12 square units, Triangle B: 16 square units, Triangle C: 12 square units

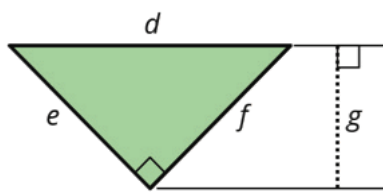


- In each case, the area of the triangle, in square units, is half of the base times its corresponding height, $\frac{b \cdot h}{2}$.

Problem 3

Statement

Here is a right triangle. Name a corresponding height for each base.



- a. Side d
- b. Side e
- c. Side f

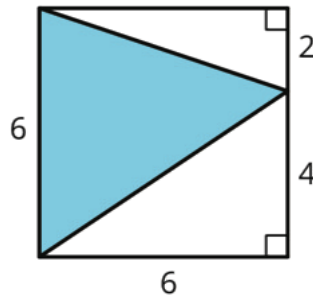
Solution

- a. Segment g
- b. Side f
- c. Side e

Problem 4

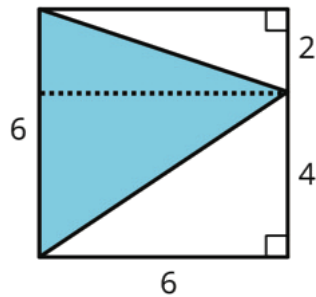
Statement

Find the area of the shaded triangle. Show your reasoning.



Solution

18 square units. Reasoning varies. One likely approach is by decomposing the triangle with a horizontal line to form two rectangles and to split the triangle into two smaller triangles. The top triangle is half of the top rectangle, so its area is $\frac{1}{2} \cdot 6 \cdot 2 = 6$. The bottom triangle is half of the bottom rectangle, so its area is $\frac{1}{2} \cdot 6 \cdot 4 = 12$. The area of the original triangle is $6 + 12$ or 18 square units.

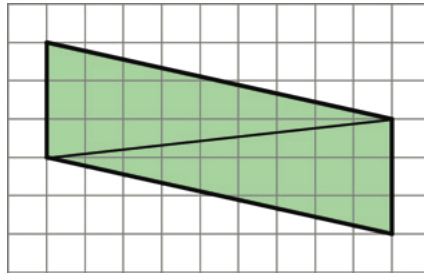


(From Unit 1, Lesson 8.)

Problem 5

Statement

Andre drew a line connecting two opposite corners of a parallelogram. Select **all** true statements about the triangles created by the line Andre drew.



- A. Each triangle has two sides that are 3 units long.
- B. Each triangle has a side that is the same length as the diagonal line.
- C. Each triangle has one side that is 3 units long.
- D. When one triangle is placed on top of the other and their sides are aligned, we will see that one triangle is larger than the other.
- E. The two triangles have the same area as each other.

Solution

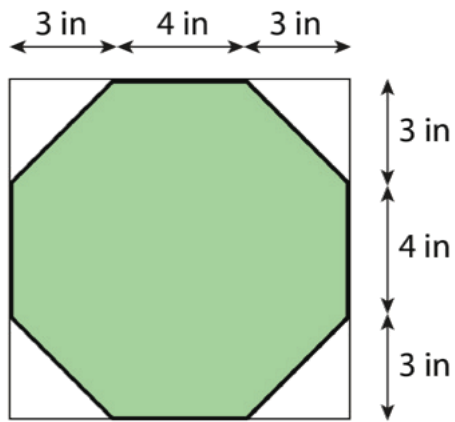
["B", "C", "E"]

(From Unit 1, Lesson 7.)

Problem 6

Statement

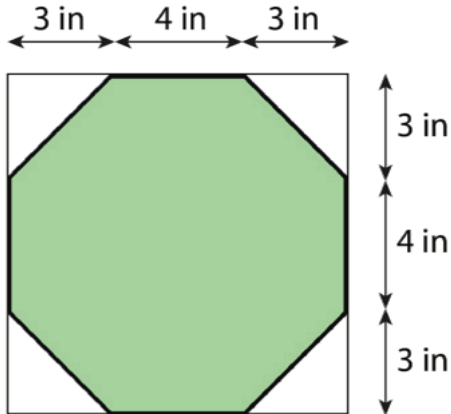
Here is an octagon. (Note: The diagonal sides of the octagon are *not* 4 inches long.)



- a. While estimating the area of the octagon, Lin reasoned that it must be less than 100 square inches. Do you agree? Explain your reasoning.
- b. Find the exact area of the octagon. Show your reasoning.

Solution

- a. Yes. Explanations vary. Sample explanation: The octagon fits in a square that is 10 inches by 10 inches, but with four corners of the square removed. The square has an area of 100 square inches, so the area of the octagon must be less than that.
- b. 82 square inches. Reasoning varies. Sample reasoning: A 10-inch-by-10-inch square that encloses the octagon has an area of 100 square inches. Two corner triangles compose a 3 inch-by-3 inch square, so their combined area is 9 square inches.
 $100 - 2(3 \cdot 3) = 100 - 18 = 82$.



(From Unit 1, Lesson 3.)

Lesson 10: Bases and Heights of Triangles

Goals

- Draw and label the height that corresponds to a given base of a triangle, making sure it is perpendicular to the base and the correct length.
- Evaluate (orally) the usefulness of different base-height pairs for finding the area of a given triangle.

Learning Targets

- I can identify pairs of base and corresponding height of any triangle.
- When given information about a base of a triangle, I can identify and draw a corresponding height.

Lesson Narrative

This lesson furthers students' ability to identify and work with a base and height in a triangle in two ways:

1. By learning to draw (not just to recognize) a segment to show the corresponding height for any given base, and
2. By learning to choose appropriate base-height pairs to enable area calculations.

Students have seen that the area of a triangle can be determined in multiple ways. Using the base and height measurements and the formula is a handy approach, but because there are three possible pairs of bases and heights, some care is needed in identifying the right combination of measurements. Some base-height pairs may be more practical or efficient to use than others, so it helps to be strategic in choosing a side to use as a base.

Alignments

Addressing

- 6.EE.A.2.c: Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.
- 6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports
- Think Pair Share

Required Materials

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Required Preparation

From the geometry toolkit, each student especially needs an index card for the Hunting for Heights activity.

Student Learning Goals

Let's use different base-height pairs to find the area of a triangle.

10.1 An Area of 12

Warm Up: 10 minutes (there is a digital version of this activity)

So far, students have determined area given a triangle and some measurements. In this warm-up, students are invited to reverse the process. They are given an area measure and are asked to create several triangles with that area.

Expect students to gravitate toward right triangles first (or to halve rectangles that have factors of 12 as their side lengths). This is a natural and productive starting point. Prompting students to create non-right triangles encourages them to apply insights from their experiences with non-right parallelograms.

As students work alone and discuss with partners, notice the strategies they use to draw their triangles and to verify their areas. Identify a few students with different strategies to share later.

Addressing

- 6.G.A.1

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 2–3 minutes of quiet think time and 2 minutes to share their drawings with their partner afterwards. Encourage students to refer to previous work as needed. Provide access to their geometry toolkits. Tell students to draw a different triangle with the same area if they finish their first one early.

During partner discussion, each partner should convince the other that the triangle drawn is indeed 12 square units.

Anticipated Misconceptions

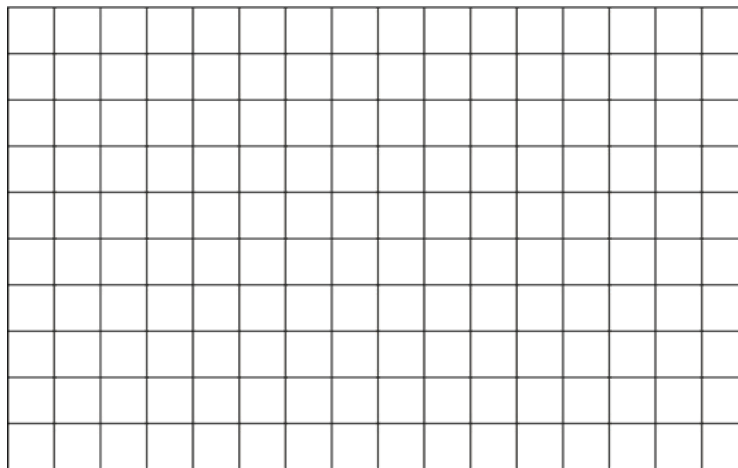
If students have trouble getting started, ask:

- “Can you draw a quadrilateral with an area of 12?”
- “Can you use what you know about parallelograms to help you?”
- “Can you use any of the area strategies—decomposing, rearranging, enclosing, subtracting—to arrive at an area of 12?”

Students who start by drawing rectangles and other parallelograms may use factors of 12, instead of factors of 24, for the base and height. If this happens, ask them what the area of the their quadrilateral is and how it relates to the triangle they are trying to draw.

Student Task Statement

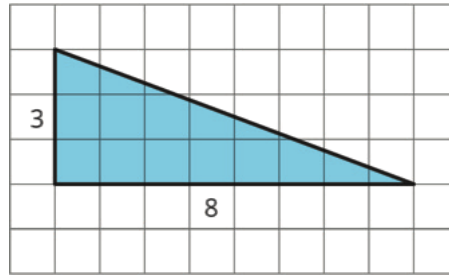
On the grid, draw a triangle with an area of 12 square units. Try to draw a non-right triangle. Be prepared to explain how you know the area of your triangle is 12 square units.



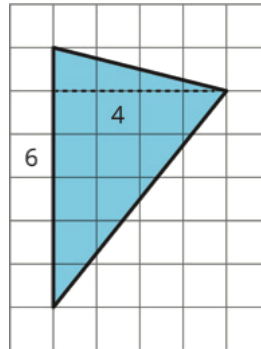
Student Response

Drawings and explanations vary. Sample responses:

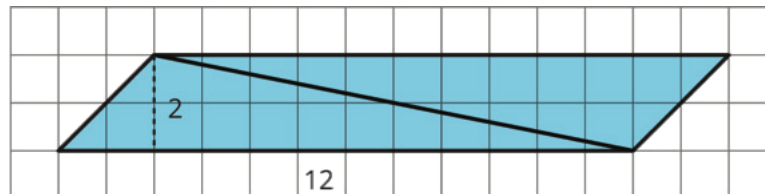
- This right triangle has a base of 8 units and a height of 3 units. The area is half of $3 \cdot 8$ or half of 24, which is 12.



- This triangle has a side of 6 units. This can be the base. Draw a height segment that is perpendicular to the base and is 4 units long. The area of the triangle is $b \cdot h \div 2$, so it is $6 \cdot 4 \div 2$, which is 12.



- Draw a parallelogram with a base of 12 and a height of 2, and then draw a diagonal line to create two identical triangles. Each of the triangles has an area of 12 because it is half of a parallelogram with an area of 24.



Activity Synthesis

Invite a few students to share their drawings and ways of reasoning with the class. For each drawing shared, ask the creator for the base and height and record them for all to see. Ask the class:

- “Did anyone else draw an identical triangle?”
- “Did anyone draw a different triangle but with the same base and height measurements?”

To reinforce the relationship between base, height, and area, discuss:

- “Which might be a better way to draw a triangle: by starting with the base measurement or with the height? Why?”
- “Can you name other base-height pairs that would produce an area of 12 square units without drawing? How?”

10.2 Hunting for Heights

25 minutes

Students may be able to recognize a measurement that can be used for height when they see it, but identifying and drawing an appropriate segment is more challenging. This activity, and the demonstration needed to launch it, gives students a concrete strategy for identifying a height accurately. When students use a strategy of drawing an auxiliary line to solve problems, they are looking for and making use of structure (MP7). Explicit instruction, as in this activity, is often needed before students can be expected to use this strategy spontaneously.

Addressing

- 6.G.A.1

Instructional Routines

- MLR2: Collect and Display

Launch

Explain to students that they will try to draw a height that corresponds to each side of a triangle. Arrange students in groups of 2. Give each student an index card and 1–2 minutes to complete the first question. Remind them that there is more than one correct way to draw the corresponding height for a base. Ask them to pause after the first question. As students work, notice how students are using the index cards (if at all).

Afterwards, solicit a few quick comments on the exploration. Ask questions such as:

- “How did you know where to draw the segments?”
- “How did you draw them?”
- “Why were you given index cards? How might they help?”

Explain that you will now demonstrate a way to draw heights effectively. (If any students used the index card correctly, acknowledge that they were on the right track.)

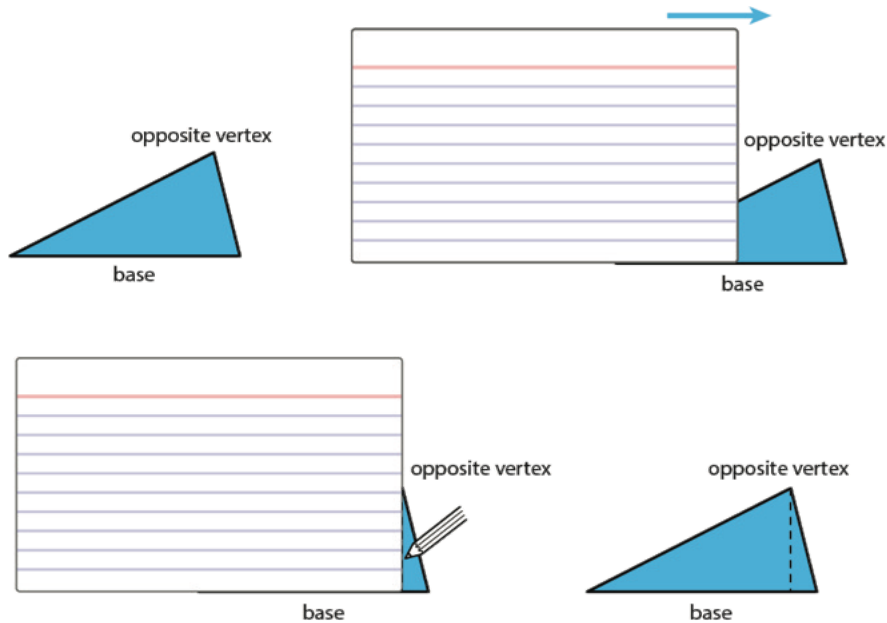
Remind students that any line we draw to show the height of a triangle must be drawn *perpendicular* to the base. Having a tool with a right angle and with straight edges can help us make sure the line we draw is both straight and perpendicular to the base. This is what the index card is for.

Ask: “How do we know where to stop this line we are drawing? How long should it be?”

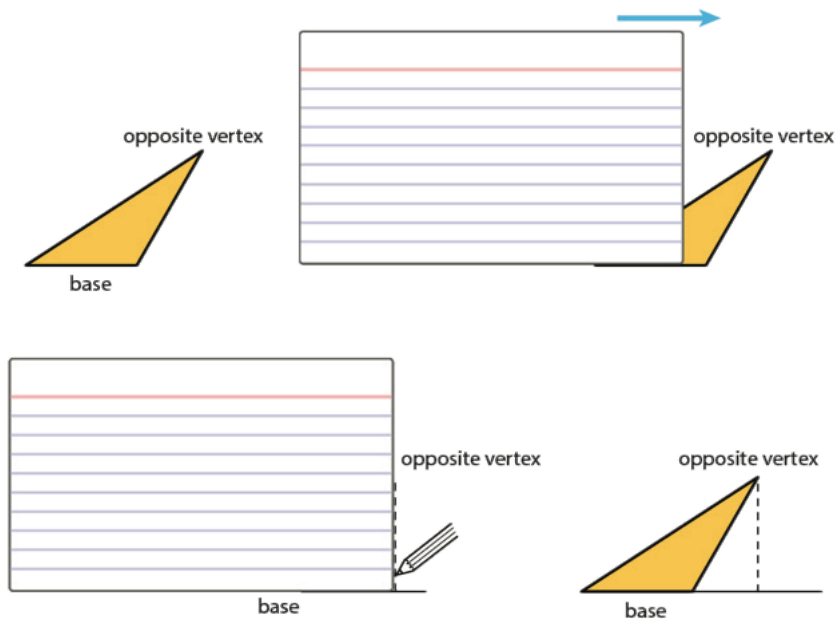
Explain that the easiest way is to draw the line so it would pass through the vertex opposite of the chosen base. Draw or display a triangle for all to see. Demonstrate the following.

- Choose one side of the triangle as the base. Identify the opposite vertex.
- Line up one edge of the index card with that base.

- Slide the card along the base until a perpendicular edge of the card meets the opposite vertex.
- Use that edge to draw a line segment from that vertex to the base. The measure of that segment is the height.



Ask: What if the opposite vertex is not directly over the base? Explain that sometimes we need to extend the line of the base and demonstrate the process.



Demonstrate the process with another example in which the card needs to slide from right to left (e.g., by rotating the obtuse triangle above clockwise). Left-handed students may find this particularly helpful.

Prompt students to use this method to check the heights they drew in the first question, revise the drawings if they were incorrect, and share their revisions with their partners. Circulate and support students as they draw. Those who finish verifying the heights in the first question can move on to complete the rest of the activity with their partners.

Support for Students with Disabilities

Action and Expression: Provide Access for Physical Action. Support access to tools and assistive technologies. Monitor for students who may need an additional demonstration, or assistance lining up one edge of their index card with the chosen base. Consider displaying the images from the Launch as students complete this task.

Supports accessibility for: Visual-spatial processing; Fine-motor skills

Support for English Language Learners

Representing, Speaking, Listening: MLR2 Collect and Display. While students work, circulate and collect examples of student drawings of line segments showing the base and height. To do this, take digital pictures, or sketch students' drawings onto a display. Look for examples that show the height inside and outside of the triangle, as well as bases that are horizontal or vertical. Then, in the whole-class discussion, display the various examples and ask students to compare the diagrams. For example, ask students, "Do any two diagrams have similar methods for determining the base or height?" Listen for and amplify the mathematical language students use to support their reasoning.

Design Principle(s): Support sense-making; Maximize meta-awareness

Anticipated Misconceptions

Some students may use the index card simply as a straightedge and therefore draw heights that are not perpendicular to the given base. Remind them that a height needs to be perpendicular or at a right angle to the base.

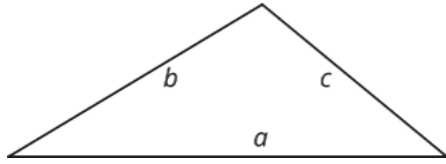
Students may mistakenly think that a base must be a horizontal side of a triangle (or one closest to being horizontal) and a height must be drawn inside of the triangle. Point to some examples from earlier work to remind students that neither is true. Remind them to align their index card to the side labeled "base."

Some students may find it awkward to draw height segments when the base is not horizontal. Encourage students to rotate their paper as needed to make drawing easier.

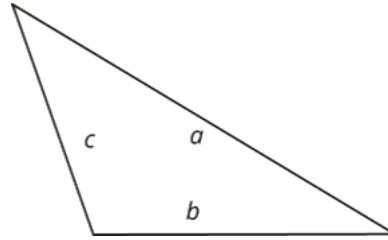
Student Task Statement

1. Here are three copies of the same triangle. The triangle is rotated so that the side chosen as the base is at the bottom and is horizontal. Draw a height that corresponds to each base. Use an index card to help you.

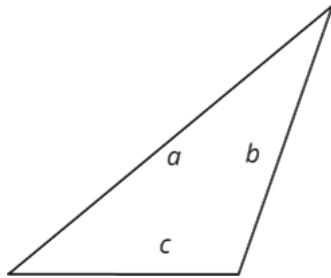
Side a as the base:



Side b as the base:



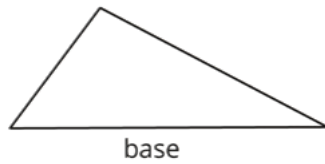
Side c as the base:



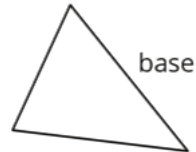
Pause for your teacher's instructions before moving to the next question.

2. Draw a line segment to show the height for the chosen base in each triangle.

A



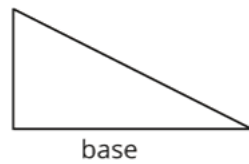
B



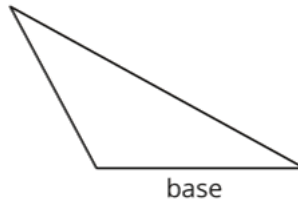
C



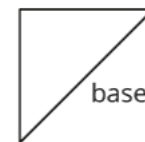
D



E

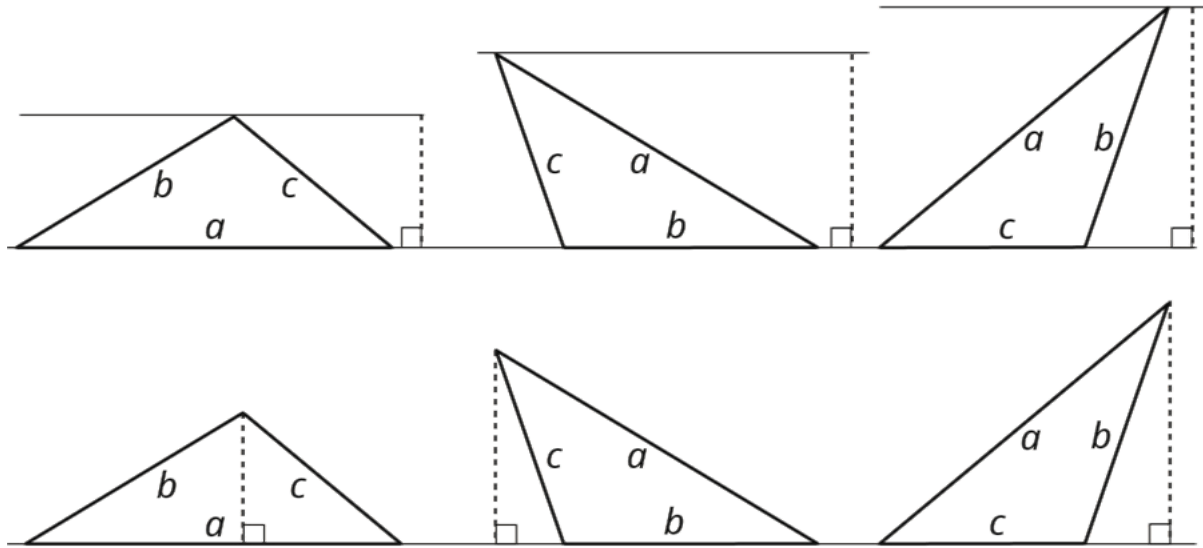


F

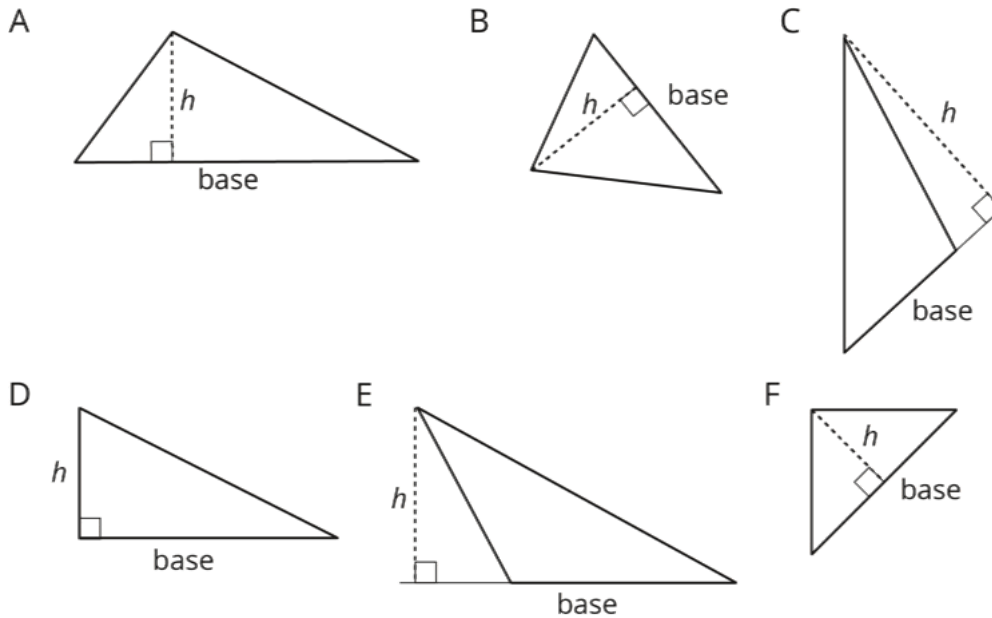


Student Response

1. Drawings vary. Sample drawings:



2.



Activity Synthesis

If time permits, consider selecting one student to share the height drawing for each triangle, or display the solutions in the Student Response for all to see. To help students reflect on their work, discuss questions such as:

- “For which triangles was it easy to find the corresponding height for the given base?”
- “For which triangles was it harder?”
- “How was the process of finding the height of triangle D different from that of the others?” (The height of a right triangle is already drawn: it is the other segment framing the right angle.)

- “When might we need to extend the line of the base, or draw a height line outside of the triangle?” (When dealing with obtuse triangles, or when the opposite vertex is not directly over the base.)

10.3 Some Bases Are Better Than Others

Optional: 15 minutes

This activity allows students to practice identifying the base and height of triangles and using them to find areas.

Because there are no directions on which base or height to use, and because not all sides would enable them to calculate area easily, students need to think structurally and choose strategically. All triangles in the problems have either a vertical or a horizontal side. Choosing such a side as the base makes it easier to identify the corresponding height.

In some cases, students may opt to use a combination of area-reasoning strategies rather than finding the base and height of the shaded triangles and applying the formula. For instance, they may enclose a shaded triangle with a rectangle and subtract the areas of extra triangles (with or without using the formula on those extra triangles). Notice students who use such strategies so they could share later.

Addressing

- 6.EE.A.2.c
- 6.G.A.1

Instructional Routines

- MLR8: Discussion Supports

Launch

Keep students in groups of 2. Explain that they will now practice locating or drawing heights and using them to find area of triangles. Give students 8–10 minutes of quiet think time and time to share their responses with a partner afterwards. Provide access to their geometry toolkits (especially index cards).

If time is limited, consider asking students find the area of two or three triangles instead of all four.

Anticipated Misconceptions

Students may think that a vertical side of a triangle is the height regardless of the segment used as the base. If this happens, have them use an index card as a straightedge to check if the two segments they are using as base and height are perpendicular.

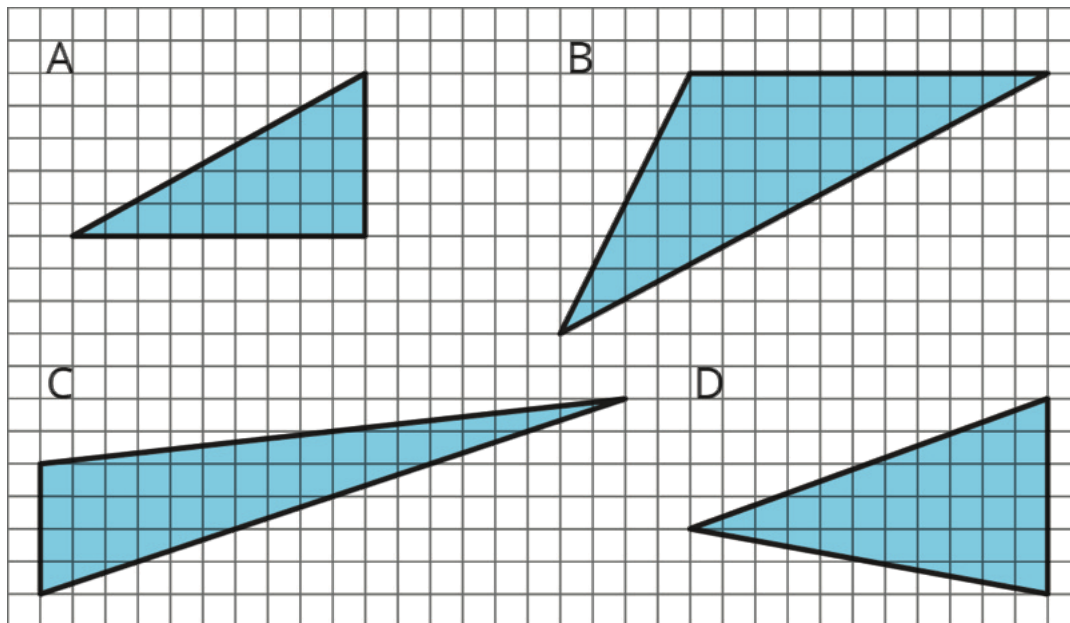
Some students may not immediately see that choosing a side that is either vertical or horizontal would enable them to find the corresponding height very easily. They may choose a non-vertical or

non-horizontal side and not take advantage of the grid. Ask if a different side might make it easier to determine the base-height lengths without having to measure.

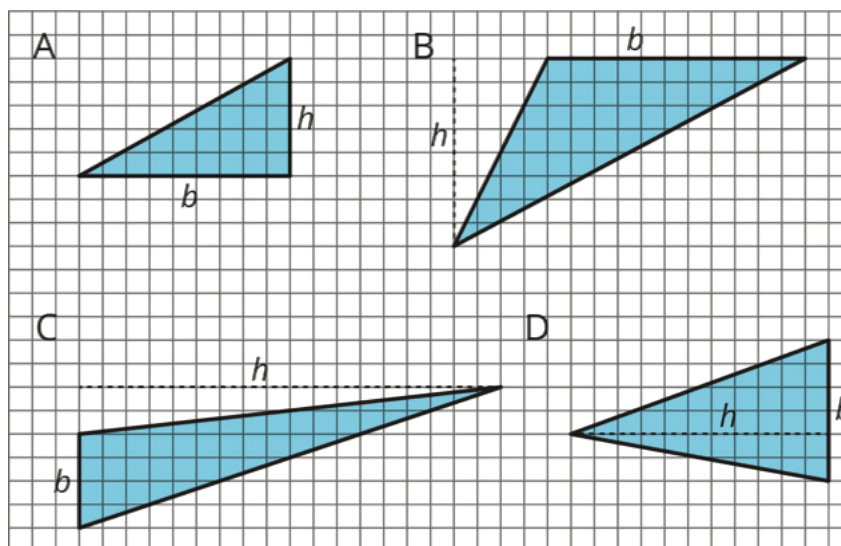
Student Task Statement

For each triangle, identify and label a base and height. If needed, draw a line segment to show the height.

Then, find the area of the triangle. Show your reasoning. (The side length of each square on the grid is 1 unit.)



Student Response



Triangle A: $b = 9$ and $h = 5$, $9 \cdot 5 \div 2 = 22.5$, area: 22.5 square units

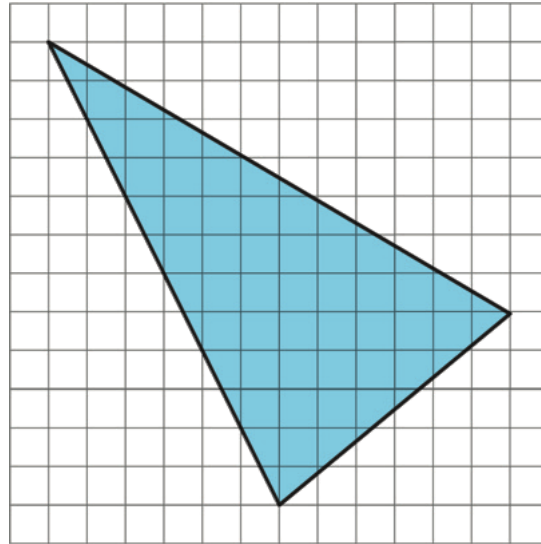
Triangle B: $b = 11$ and $h = 8$, $11 \cdot 8 \div 2 = 44$, area: 44 square units

Triangle C: $b = 4$ and $h = 18$, $4 \cdot 18 \div 2 = 36$, area: 36 square units.

Triangle D: $b = 6$ and $h = 11$, $6 \cdot 11 \div 2 = 33$, area: 33 square units.

Are You Ready for More?

Find the area of this triangle. Show your reasoning.



Student Response

51, since we can enclose the given triangle in a square that has an area of 144 ($12 \cdot 12 = 144$), then subtract away the area from right triangles in each corner.

Activity Synthesis

Focus the whole-class discussion how students went about identifying bases and heights. Discuss:

- "Which side did you choose as the base for triangle B? C? Why?"
- "Aside from choosing a vertical or horizontal side as the base, is there another way to find the area of the shaded triangles without using their bases and heights?" (Invite a couple of students who use the enclose-and-subtract method to find the area of B, C, or D to share.)
- "Which strategy do you prefer or do you think is more efficient?"
- "Can you think of an example where it might be preferable to find the base and height of the triangle of interest?" (Students may point to any of the triangles in the task.)
- "Can you think of an example where it might be preferable to enclose the triangle of interest and subtract other areas?" (Students may point to the triangle shown in "Are you ready for more?", where none of the shaded triangle's sides are horizontal or vertical.)

Support for English Language Learners

Speaking, Representing: MLR8 Discussion Supports. Use this routine to support whole-class discussion. Provide sentence frames to help students produce statements that describe the strategies they use to identify bases and heights, and to find area. For example, "For triangle ___, I chose side ___ as the base because . . ." or "The next time I need to find the area of a triangle, the strategy I will use is ___, because . . ."

Design Principle(s): Support sense-making

Lesson Synthesis

In this lesson, we looked closely at the heights of a triangle. We located or drew a height for any side of a triangle. We also considered which pair of base and height to use to find area.

- "What must we remember about the relationship between a base of a triangle and its corresponding height?" (The height must be perpendicular to the base.)
- "What tools might help us draw a height segment? What is it about an index card or a ruler that helps us?" (A tool with straight edges and a right angle can help us draw perpendicular segments.)

When we have a base and a corresponding height, we can find the area quite simply, but for every triangle there are multiple base-height pairs.

- "Does it matter which side we choose as the base? How do we decide?" (For the base, we need a side with a known length. For the height, we need a segment that is perpendicular to that base and whose length we can determine.)

10.4 Stretched Sideways

Cool Down: 5 minutes

Addressing

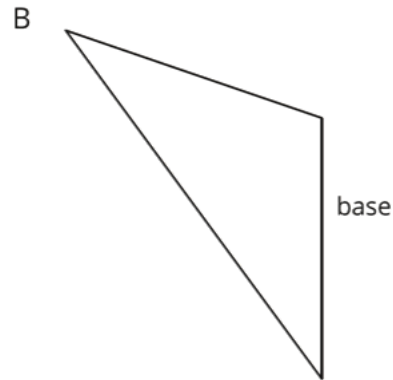
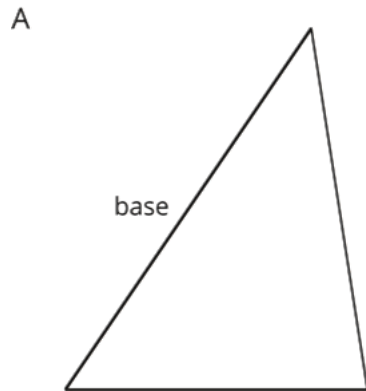
- 6.G.A.1

Launch

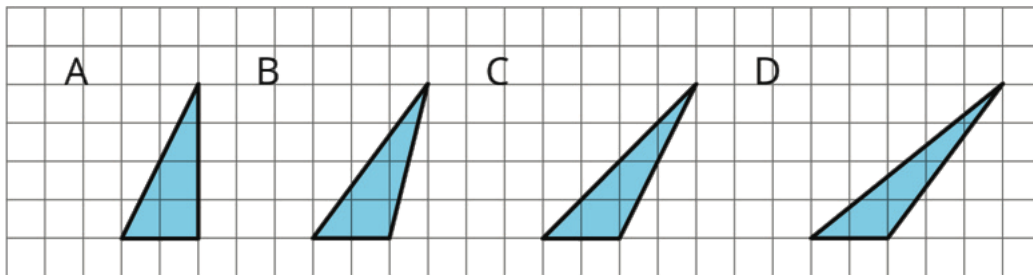
Provide access to geometry toolkits.

Student Task Statement

1. For each triangle, draw a height segment that corresponds to the given base, and label it h . Use an index card if needed.

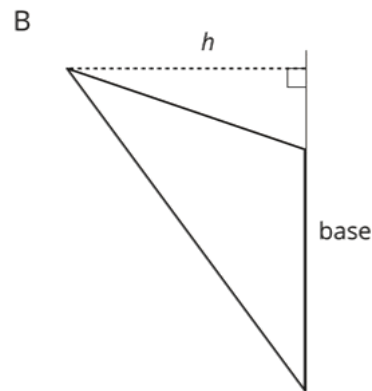
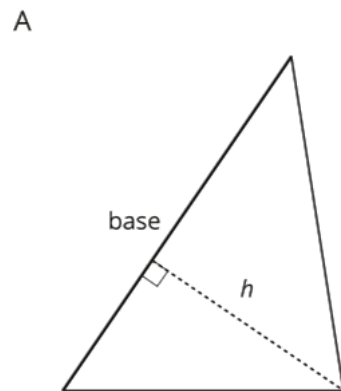


2. Which triangle has the greatest area? The least area? Explain your reasoning.



Student Response

1. Answers vary. There are many possible locations for a height segment. The segments shown are the most straightforward.



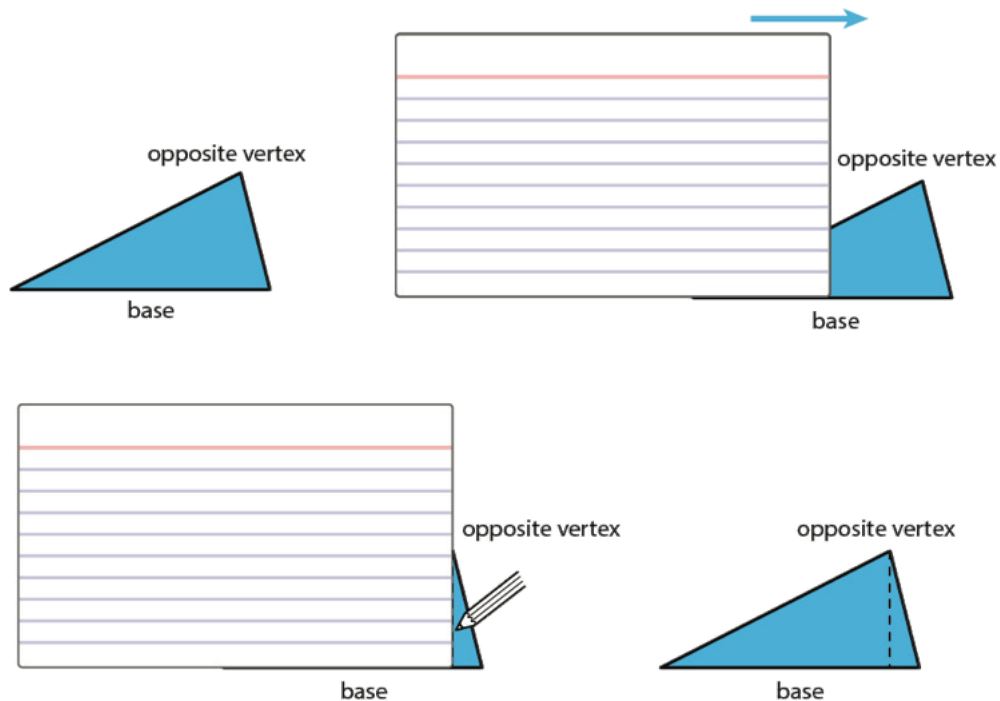
2. All of the triangles have the same area: 4 square units. They all have a base of 2 units and a height of 4 units.

Student Lesson Summary

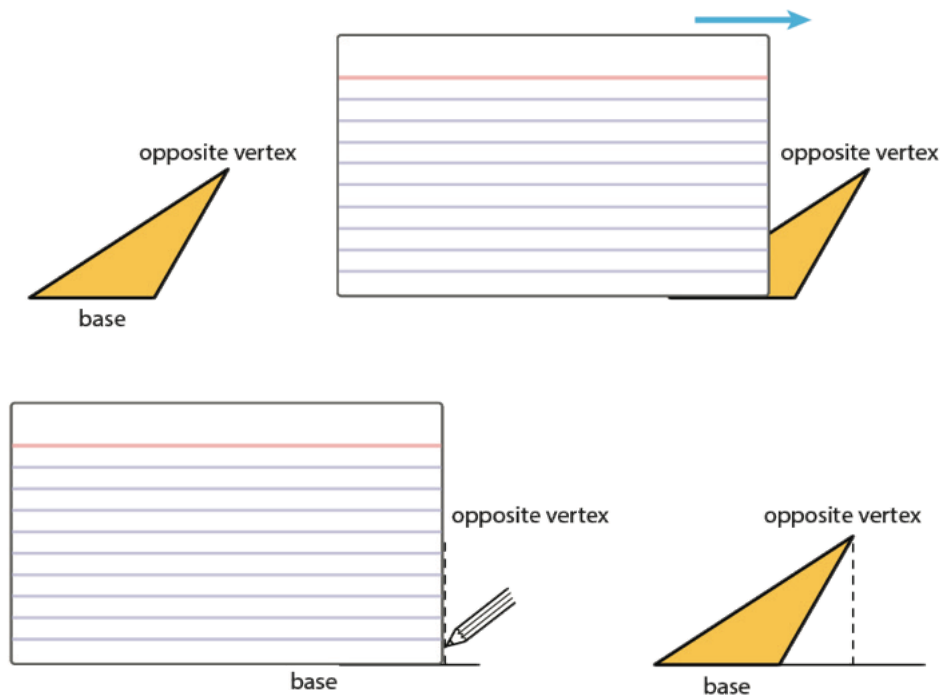
A height of a triangle is a perpendicular segment between the side chosen as the base and the opposite vertex. We can use tools with right angles to help us draw height segments.

An index card (or any stiff paper with a right angle) is a handy tool for drawing a line that is perpendicular to another line.

1. Choose a side of a triangle as the base. Identify its opposite vertex.
2. Line up one edge of the index card with that base.
3. Slide the card along the base until a perpendicular edge of the card meets the opposite vertex.
4. Use the card edge to draw a line from the vertex to the base. That segment represents the height.



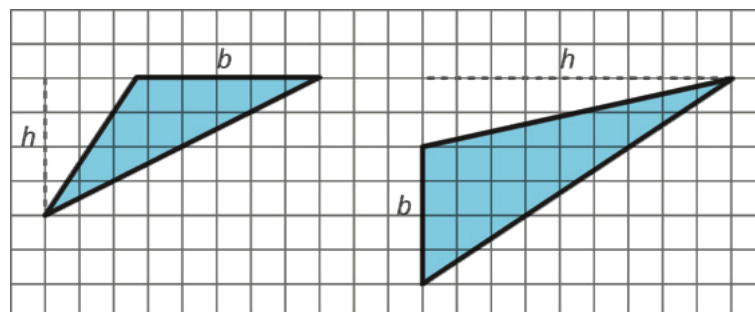
Sometimes we may need to extend the line of the base to identify the height, such as when finding the height of an obtuse triangle, or whenever the opposite vertex is not directly over the base. In these cases, the height segment is typically drawn *outside* of the triangle.



Even though any side of a triangle can be a base, some base-height pairs can be more easily determined than others, so it helps to choose strategically.

For example, when dealing with a right triangle, it often makes sense to use the two sides that make the right angle as the base and the height because one side is already perpendicular to the other.

If a triangle is on a grid and has a horizontal or a vertical side, you can use that side as a base and use the grid to find the height, as in these examples:



Glossary

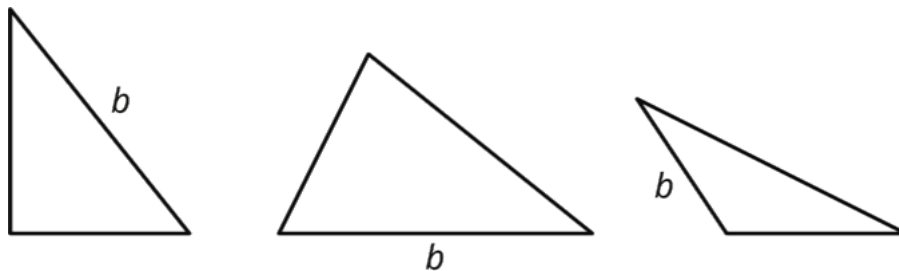
- edge
- vertex

Lesson 10 Practice Problems

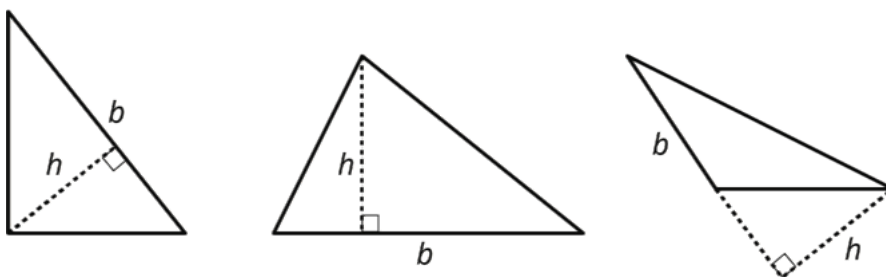
Problem 1

Statement

For each triangle, a base is labeled b . Draw a line segment that shows its corresponding height. Use an index card to help you draw a straight line.



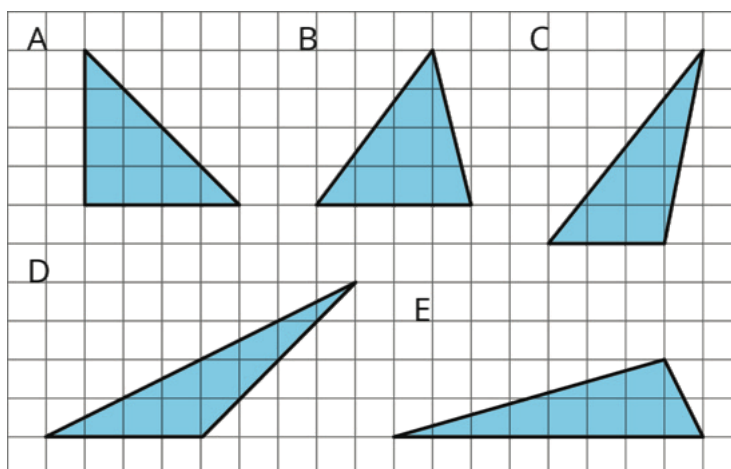
Solution



Problem 2

Statement

Select all triangles that have an area of 8 square units. Explain how you know.



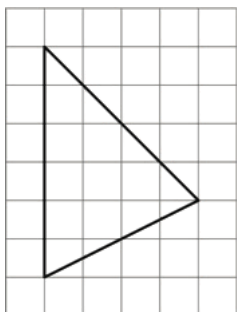
Solution

A, B, D, and E. Triangles A, B, and D all have a horizontal base of 4 units and a height of 4 units. $\frac{4 \cdot 4}{2} = 8$, so the area of each is 8 square units. Triangle C has a horizontal base of 3 units and a height of 5 units, so its area is 7.5 square units. Triangle E has a horizontal base of 8 units and a height of 2 units, so its area is 8 square units, since $\frac{8 \cdot 2}{2} = 8$.

Problem 3

Statement

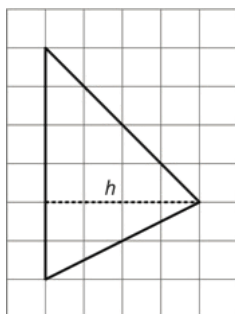
Find the area of the triangle. Show your reasoning.



If you get stuck, carefully consider which side of the triangle to use as the base.

Solution

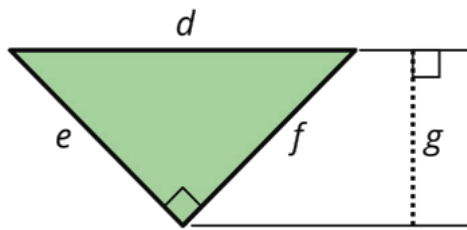
12 square units. Explanations vary. Sample response: The vertical side is 6 units long, and this side can be used as the base. The corresponding height, shown in the diagram, is 4 units. So the area is 12 square units. Another method is to surround the triangle with a rectangle then subtract the parts that are not in the triangle.



Problem 4

Statement

Can side d be the base for this triangle? If so, which length would be the corresponding height? If not, explain why not.



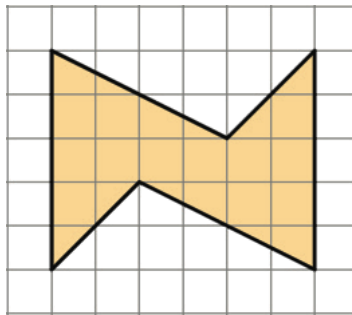
Solution

Yes, side d can be the base, because it is a side of the triangle. The corresponding height is g .

Problem 5

Statement

Find the area of this shape. Show your reasoning.



Solution

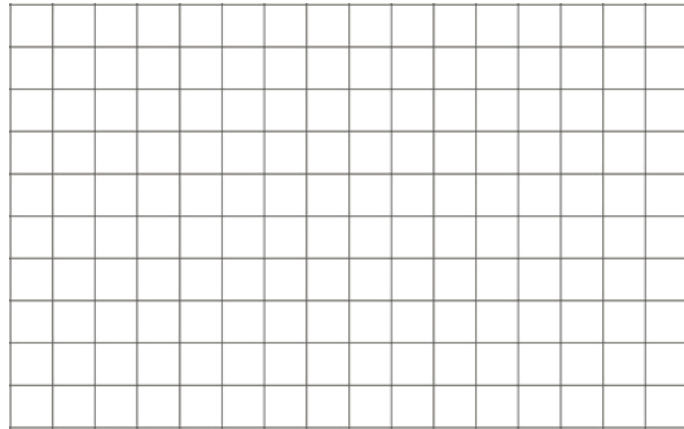
18 square units. Reasoning varies.

(From Unit 1, Lesson 3.)

Problem 6

Statement

On the grid, sketch two different parallelograms that have equal area. Label a base and height of each and explain how you know the areas are the same.



Solution

Answers vary.

(From Unit 1, Lesson 6.)

Section: Polygons

Lesson 11: Polygons

Goals

- Compare and contrast (orally) different strategies for finding the area of a polygon.
- Describe (orally and in writing) the defining characteristics of polygons.
- Find the area of a polygon, by decomposing it into rectangles and triangles, and present the solution method (using words and other representations).

Learning Targets

- I can describe the characteristics of a polygon using mathematical vocabulary.
- I can reason about the area of any polygon by decomposing and rearranging it, and by using what I know about rectangles and triangles.

Lesson Narrative

Students have worked with polygons in earlier grades and throughout this unit. In this lesson, students write a definition that characterizes polygons. There are many different accurate definitions for a **polygon**. The goal of this lesson is not to find the most succinct definition possible, but to articulate the defining characteristics of a polygon that makes sense to students.

Another key takeaway for this lesson is that the area of any polygon can be found by decomposing it into triangles. The proof that all polygons are *triangulable* (not a word students need to know) is fairly sophisticated, but students can just take it as a fact for now. In observing and using this fact students look for and make use of structure (MP7).

Alignments

Building On

- 4.G.A.2: Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
- 5.G.B: Classify two-dimensional figures into categories based on their properties.

Addressing

- 6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Building Towards

- 6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Instructional Routines

- Group Presentations
- MLR2: Collect and Display
- MLR8: Discussion Supports
- Think Pair Share
- Which One Doesn't Belong?

Required Materials

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Required Preparation

If doing the optional Pinwheel activity, prepare one copy of the Instructional master for every group of 4 students. If larger paper (and a photocopier that can accommodate it) is available, it would be helpful to have larger-format copies of this.

Student Learning Goals

Let's investigate polygons and their areas.

11.1 Which One Doesn't Belong: Bases and Heights

Warm Up: 5 minutes

This warm-up prompts students to consolidate what they learned in the past few lessons and make careful observations about triangles.

Expect students to describe the differences in the triangles in terms of:

- angles (acute, right, or obtuse)
- orientation of sides (vertical, horizontal)

- the side likely to be chosen as a base
- length of base or height

Building On

- 4.G.A.2
- 5.G.B

Instructional Routines

- Which One Doesn't Belong?

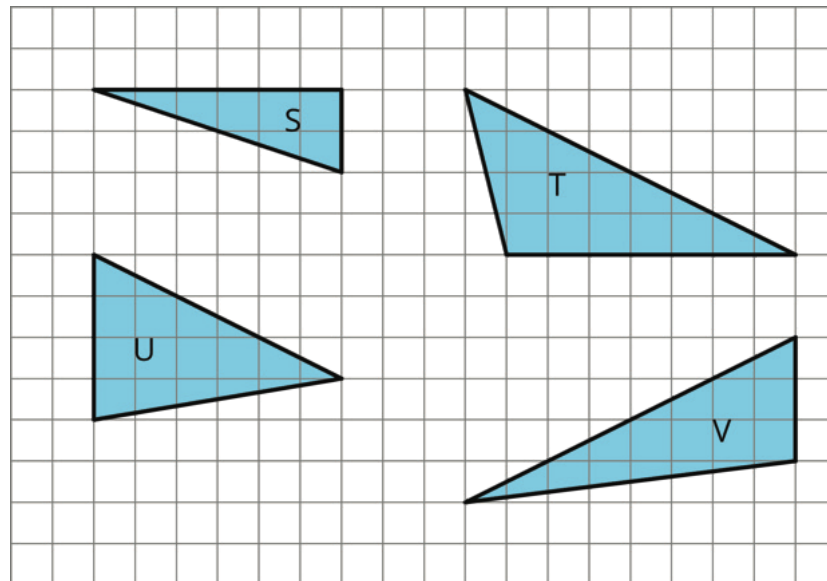
Launch

Arrange students in groups of 2–4. Display the image of triangles for all to see. Give students 1 minute of quiet think time and ask them to indicate when they have noticed one triangle that does not belong and can explain why. Encourage them to think of more than one possibility.

When the minute is up, give students 2 minutes to share their response with their group, and then together find at least one reason, if possible, that each triangle doesn't belong.

Student Task Statement

Which one doesn't belong?



Student Response

Answers vary. Sample responses:

- S: It is the only right triangle. It is the only one where two sides can be easily chosen as a base and used to find the area. It is the only one where the base and height are both sides of the triangle.
- T: It is the only triangle with no vertical side. It is the only triangle where the side most likely to be chosen as a base is horizontal.
- U: It is the only acute triangle. It is the only triangle that is most likely to have its height drawn inside the triangle.
- V: It is the only one with a height greater than 7 units.

Activity Synthesis

After students shared their observations in groups, invite each group to share one reason why a particular triangle might not belong. Record and display the responses for all to see. After each response, poll the rest of the class to see if others made the same observation.

Since there is no single correct answer to the question of which pattern does not belong, attend to students' explanations and ensure the reasons given are correct. Prompt students to explain the meaning of any terminology they use (parts of triangles, types of angles, etc.) and to substantiate their claims.

11.2 What Are Polygons?

20 minutes

Developing a useful and complete definition of a polygon is harder than it seems. A formal definition is often very wordy or hard to parse. Polygons are often referred to as “closed” figures, but if used, this term needs to be defined, as the everyday meaning of “closed” is different than its meaning in a geometric context.

This activity prompts students to develop a working definition of polygon that makes sense to them, but that also captures all of the necessary aspects that makes a figure a polygon (MP6). Here are some important characteristics of a polygon.

- It is composed of line segments. Line segments are always straight.
- Each line segment meets one and only one other line segment at each end.
- The line segments never cross each other except at the end points.
- It is two-dimensional.

One consequence of the definition of a polygon is that there are always as many vertices as edges. Students may observe this and want to include it in their definition, although technically it is a result of the definition rather than a defining feature.

As students work, monitor for both correct and incorrect definitions of a polygon. Listen for clear and correct descriptions as well as common but inaccurate descriptions (so they can be discussed and refined later). Notice students with accurate explanations so they could share later.

Building On

- 5.G.B

Building Towards

- 6.G.A.1

Instructional Routines

- MLR2: Collect and Display

Launch

Arrange students in groups of 2–4. Give students 3–4 minutes of quiet think time. Afterwards, ask them to share their responses with their group and complete the second question together. If there is a disagreement about whether a figure is a polygon, ask them to discuss each point of view and try to come to an agreement. Follow with a whole-class discussion.

Support for English Language Learners

Conversing, Representing, Writing: MLR2 Collect and Display. As students work, listen for, collect, and display terms and phrases students use to describe key characteristics of polygons (e.g., polygon, edge, vertices). Remind students to borrow language from the display as needed. This will help students use mathematical language when describing polygons.

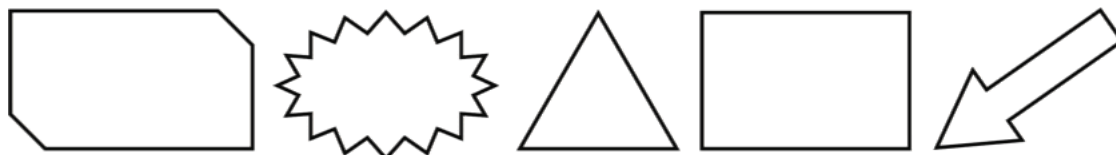
Design Principle(s): Support sense-making; Maximize meta-awareness

Anticipated Misconceptions

Students may think that Figures C and I are polygons because they can see several triangles or quadrilaterals in each figure. Ask students to look closely at the examples and non-examples and see there is a figure composed of multiple triangles or quadrilaterals, and if so, to see in which group it belongs.

Student Task Statement

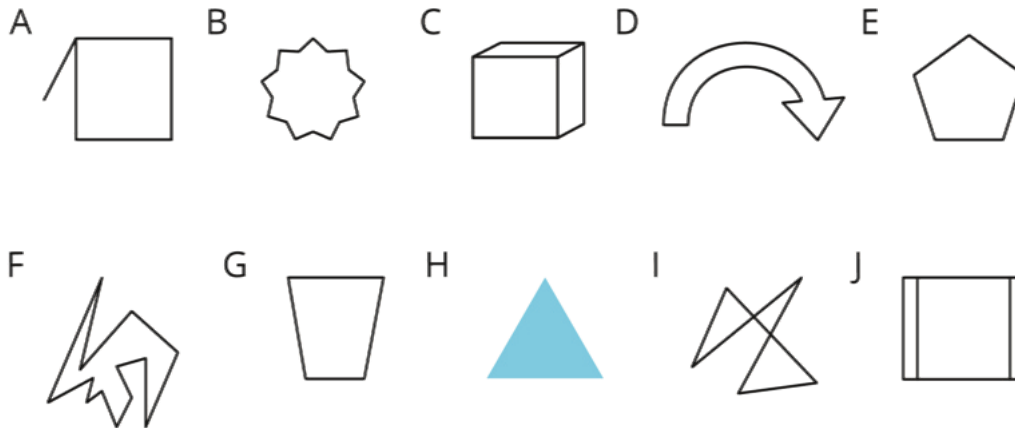
Here are five **polygons**:



Here are six figures that are *not* polygons:



1. Circle the figures that are polygons.



2. What do the figures you circled have in common? What characteristics helped you decide whether a figure was a polygon?

Student Response

1. B, E, F, and G are polygons.
2. Answers vary. Characteristics that the polygons have in common: They are two-dimensional, composed of line segments that never cross each other, and each line segment meets one and only one other line segment at each end.

Activity Synthesis

Display the figures in the first question for all to see. For each figure, ask at least one student to explain why they think it is or is not a polygon. (It is fine if students' explanations are not precise at this point.) Then, circle the figures that are polygons on the visual display.

Next, ask students to share their ideas about the characteristics of polygons. Record them for all to see. For each one, ask the class if they agree or disagree. If they generally agree, ask if there is anything they would add or elaborate on to make the description clearer or more precise. If they disagree, ask for an explanation.

If a key characteristic listed in the Activity Narrative is not mentioned by students, bring it up and revisit it at the end of the lesson.

Tell students we call the line segments in a polygon the **edges** or **sides**, and we call the points where the edges meet the **vertices**. Point to the sides and vertices in a few of the identified polygons.

Point out that polygons always enclose a region, but the region is not technically part of the polygon. When we talk about finding the area of a polygon, we are in fact finding the area of the region it encloses. So “the area of a triangle,” for example, is really shorthand for “area of the region enclosed by the triangle.”

Support for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding. Include the following terms and maintain the display for reference throughout the unit: vertex, vertices, edge, polygon.

Supports accessibility for: Language; Conceptual processing

11.3 Quadrilateral Strategies

15 minutes

This activity has several aims. It prompts students to apply what they learned to find the area of quadrilaterals that are not parallelograms, encourages them to plan before jumping into a problem, and urges them to reflect on the merits of different methods.

Students begin by thinking about the moves they would make to find the area of a quadrilateral and explaining their preference to their partners. They then consider and discuss the different strategies taken by other students. Along the way, they may notice that some strategies are more direct or efficient than others. Students reflect on these strategies and use their insights to plan the work of finding the area of polygons in this activity and beyond.

Note that it is unnecessary for students to take the most efficient path. It is more important that they choose an approach that makes sense to them but have the chance to see the pros and cons of various paths.

Addressing

- 6.G.A.1

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Ask students to recall the definition of **quadrilateral** from earlier grades, or tell students that a quadrilateral is a polygon with 4 sides. Tell students that we will now think about how to find the area of quadrilaterals.

Arrange students in groups of 4. Display the image of quadrilaterals A–F for all to see. Direct their attention to Quadrilateral D.

Give students a minute of quiet time to think about the first 2–3 moves they would make to find the area of D. Offer some sentence starters: "First, I would . . . Next I would . . ., and then I would . . ." Encourage them to show their moves on the diagram in their material. Emphasize that we are interested only in the plan for finding area and not in the area itself, so no calculation is expected. Then, give them 1–2 minutes to share their moves with their group.

Ask students to indicate what their first move was. Did their very first involve:

- decomposing the quadrilateral?
- enclosing the quadrilateral?
- another move?

Ask the students whose first move is to decompose the figure:

- "How many pieces resulted from the decomposition? 2 pieces? 3 pieces? 4 pieces? More?"
- "What is the next move? Rearrange? Duplicate a piece? Calculate the area of a piece? Something else?"

Ask the students whose first move is to enclose the figure:

- "How many rectangles did you create? 1 rectangle? 2 rectangle? More?"
- "What is the next move? Rearrange the extra pieces? Calculate the area of an extra piece? Something else?"

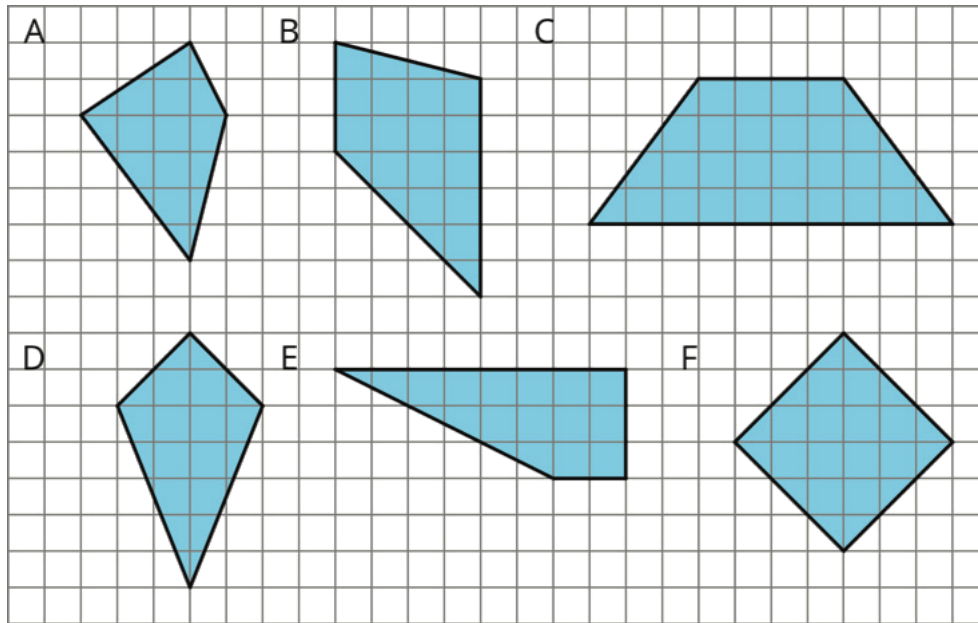
For each sequence that students mentioned, draw a quick diagram to illustrate it for all to see.

Once students have a chance to see a variety of approaches, ask students to revisit their sequence of moves. Give students 1–2 minutes to think about the pros and cons of their original plan, and if there was another strategy that they found productive. Invite a few students to share their reflections.

Then, give students quiet time to complete the activity and access to their geometry toolkits. Ask students to keep in mind the merits of the different strategies they have seen as they plan their work.

Student Task Statement

Find the area of *two* quadrilaterals of your choice. Show your reasoning.



Student Response

Reasoning varies. Students could decompose the quadrilateral into parallelograms and triangles to find the area, decompose and rearrange the pieces into a shape of which they can easily find the area, or enclose the figure in a rectangle and subtract the area of the extra pieces.

Figure A: 12 square units

Figure B: 18 square units

Figure C: 28 square units

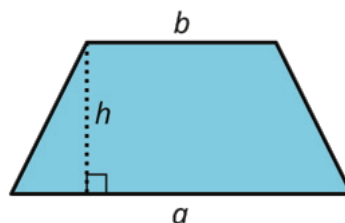
Figure D: 14 square units

Figure E: 15 square units

Figure F: 18 square units

Are You Ready for More?

Here is a trapezoid. a and b represent the lengths of its bottom and top sides. The segment labeled h represents its height; it is perpendicular to both the top and bottom sides.

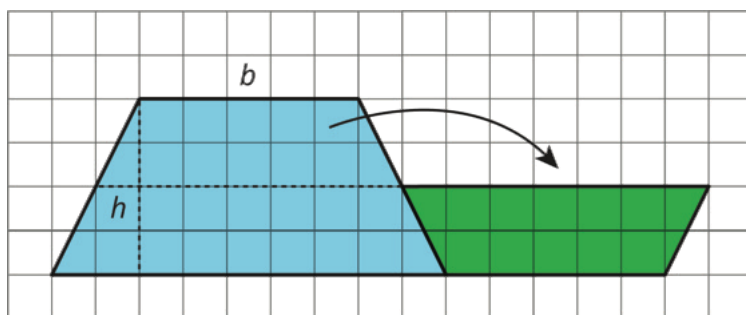


Apply area-reasoning strategies—decomposing, rearranging, duplicating, etc.—on the trapezoid so that you have one or more shapes with areas that you already know how to find. Use the shapes to help you write a formula for the area of a trapezoid. Show your reasoning.

Student Response

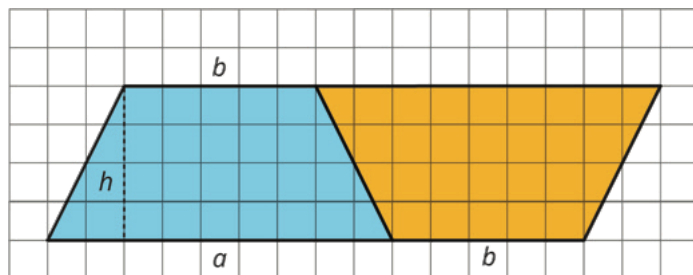
Students may cut the trapezoid in half horizontally, rotate the top piece and attach it to the bottom piece. They see that they can add the top and bottom side lengths and multiply that by half of the original height. This is built on prior understanding of finding the area of a parallelogram.

$$\frac{1}{2} \cdot h \cdot (a + b)$$



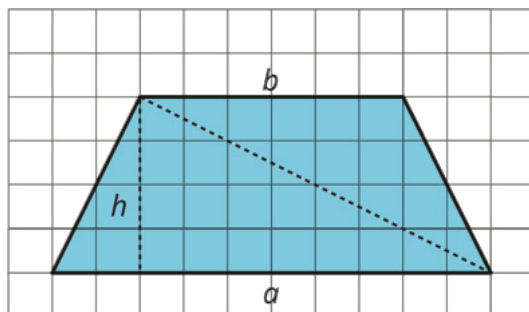
Students may also put an identical copy of the same trapezoid next to the original to make a parallelogram. Then, find the area of the parallelogram and divide that by 2.

$$(a + b) \cdot h \div 2$$



Another way to approach this is to draw a diagonal and add the areas of the two resulting triangles.

$$a \cdot h \div 2 + b \cdot h \div 2$$



Activity Synthesis

To conclude the activity, ask students to choose one quadrilateral they worked on (other than D) and tell their group the first couple of moves they made for finding its area and why. Encourage other group members to listen carefully, check that the reasoning is valid, and offer feedback.

Students may have noticed that all the approaches involved decomposing one or more regions into triangles, rectangles, or both. If not mentioned by students, point this out. Emphasize that we can decompose any polygon into triangles and rectangles to find its area.

Support for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine before groups share the first couple of moves they made to find the area of one of the quadrilaterals. Ask the class, "What are some important words or phrases you can use when you describe the moves you made?" Record and display student responses. In addition to the relevant mathematical terms, call students' attention to the language that helps to communicate order within the approaches. Remind students to use the display as a resource during their group discussions.

Design Principle(s): Optimize output (for explanation); Maximize meta-awareness

11.4 Pinwheel

Optional: 30 minutes

In this activity, students determine the area of an unfamiliar polygon and think about various ways for doing so. The task prepares students to find the areas of other unfamiliar shapes in real-world contexts. It also reinforces the practice of sense-making, planning, and persevering when solving a problem (MP1). Students reason independently before discussing and recording their strategies in groups.

Because the shape of the polygon is more complex than what students may have seen so far, expect students to experiment with one or more strategies. Consider preparing extra copies of the diagram for students to use, if needed.

As students work, monitor the paths taken by different groups and make note of the variations and complexities. If there is limited variation in strategies, look for groups who recorded the same strategy in different ways. Also check whether students make use of the structure of the pinwheel in their reasoning (MP7). Do they notice it could be decomposed into four identical pieces (or sets of pieces)? Or, if enclosing the pinwheel with a square, do they make use the fact that the extra regions are identical?

Addressing

- 6.G.A.1

Instructional Routines

- Group Presentations
- MLR2: Collect and Display

Launch

Arrange students in groups of 4. Give students access to their geometry toolkits and 5 minutes of quiet time to plan an approach for finding the area of the pinwheel. Then, ask them to share their plan with their group.

The group then decides on one or more strategies to pursue, works together to find the area, and creates a visual display of the strategy (or strategies) used. Give each group one or more copies of the Instructional master for the visual display.

Support for Students with Disabilities

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. Remind students that triangles can be decomposed, rearranged, enclosed, or duplicated to determine area.

Supports accessibility for: Social-emotional skills; Conceptual processing

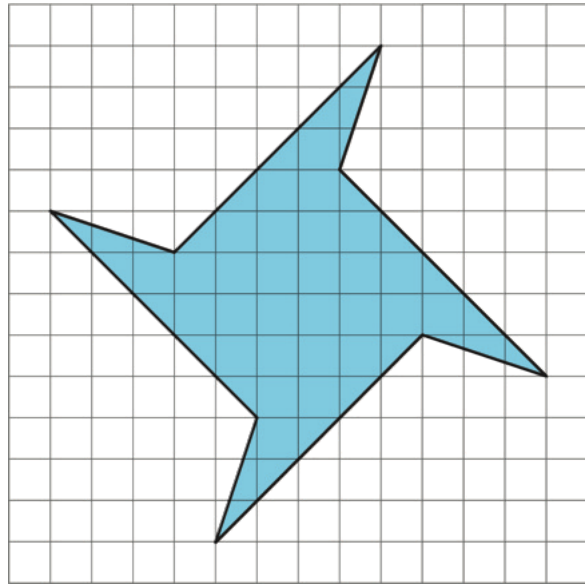
Anticipated Misconceptions

Students who overlay a rotated square over the figure such that the four pins are shown as four right triangles may use incorrect side lengths for the triangles (e.g. assuming that one of the side lengths are 2 units instead of $2\sqrt{2}$ units) or the square. Help them see that the diagonals of the unit squares are longer than the side length by measuring them.

If students struggle, suggest they use one or more of the tools in their toolkits to assist with solving the problem.

Student Task Statement

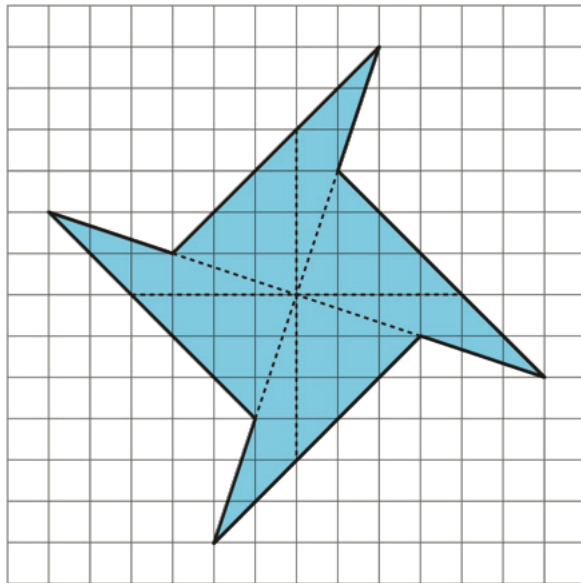
Find the area of the shaded region in square units. Show your reasoning.



Student Response

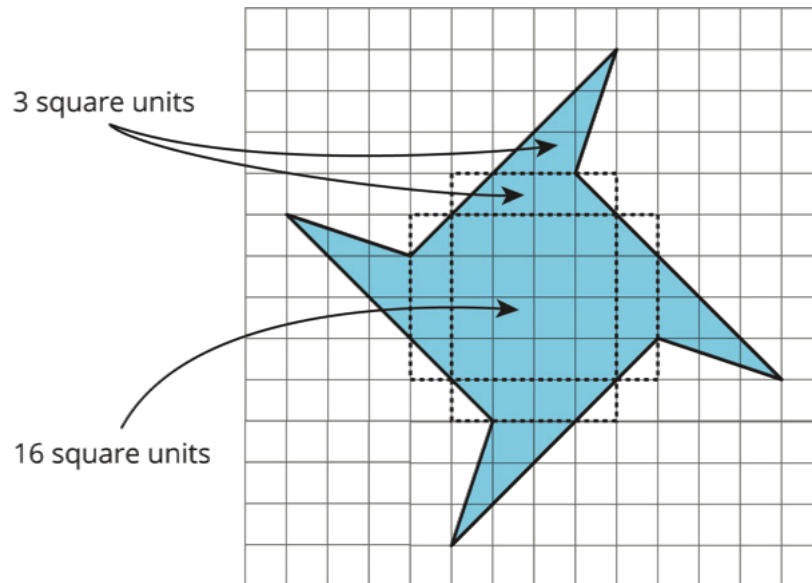
The area is 40 square units.

There are many ways to decompose this figure. Sample responses:



There are two different triangles duplicated 4 times. Both can be found by enclosing in rectangles and subtracting the areas of right triangles, or by using the formula for the area of a triangle.

$$4 \cdot \frac{1}{2}(3 \cdot 4 + 2 \cdot 4) = 40$$



The middle of the pinwheel is a square with the area of 16 square units. Around the square are 4 trapezoids that students can find the area of by enclosing and subtracting the area of the extra pieces or decomposing and rearranging to a rectangle with an area of 3 square units. Around each trapezoid is a triangle that students can find the area of using the formula for the area of a triangle to find each one is 3 square units.

Activity Synthesis

Ask the groups to visit one another's visual display and discuss the following questions as they observe others' work:

- Did this group arrive at the same area as our group? If not, why?
- How is their strategy like our strategy?
- How is their strategy different than ours?

After the gallery walk, ask a few students to comment on how their group's strategy compares to that of another group. Highlight similarities in students' work in broader terms. For example, all groups have likely used one of the following approaches:

- decomposed the pinwheel into triangles and used the formula to find the area of the triangles
- decomposed the pinwheel into triangles and rectangles, rearranged the pieces into rectangles or parallelograms, and found the areas of those regions
- enclosed the pinwheel with a square, decomposed the extra regions into triangles and rectangles, found their area of the extra regions, and subtracted them from that of the square

Reinforce that all approaches involve decomposing a polygon into triangles and rectangles to find area.

Support for English Language Learners

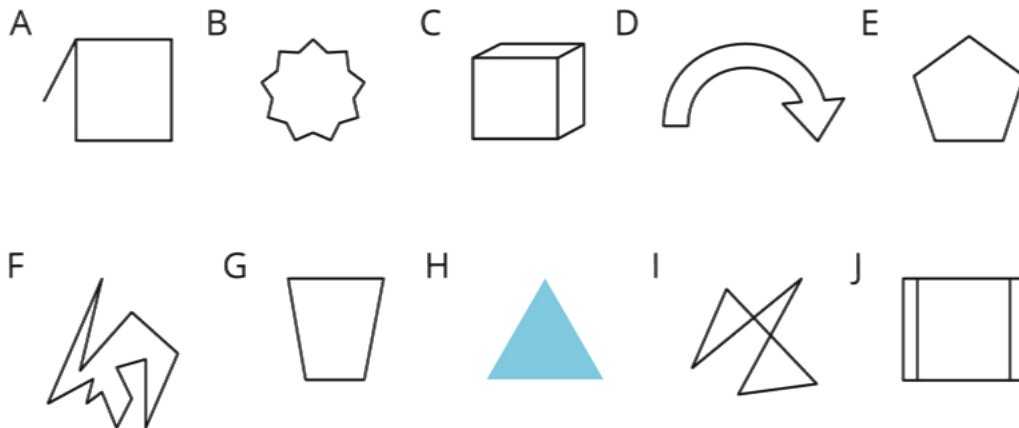
Conversing, Representing: MLR2 Collect and Display. During the gallery walk, circulate and listen to students talk about the similarities and differences between the different strategies. Listen for common phrases you hear students say about each strategy, and record students' words onto a visual display of the pinwheel. This will help students make connections between the strategies, and also provide them with mathematical language to use during the whole-class discussion.

Design Principle(s): Support sense-making

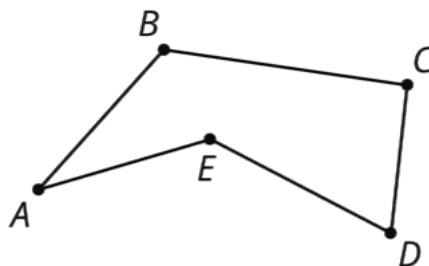
Lesson Synthesis

To review the defining characteristics of a **polygon**, return to the image in the first activity (What are Polygons?) and display the list of defining features students generated in that activity.

Revisit each figure that is *not* a polygon and ask students to explain why it is not a polygon. Encourage students to use their list to support their explanations, as well as to suggest revisions to their working definition.



Here is a polygon with 5 sides.



Ask students:

- “How do we know this figure is a polygon?” (It is composed of line segments. Each segment meets only one other segment at each end. The segments do not cross one another. It is two-dimensional.)
- “What does it mean to find the area of this polygon?” (It means finding the area of the region inside it.)
- “How can we find the area of this polygon?” (We can decompose the region inside it into triangles and rectangles.)

11.5 Triangulation

Cool Down: 5 minutes

This cool-down assesses students’ understanding of the defining characteristics of a polygon and the ways it can be decomposed.

Addressing

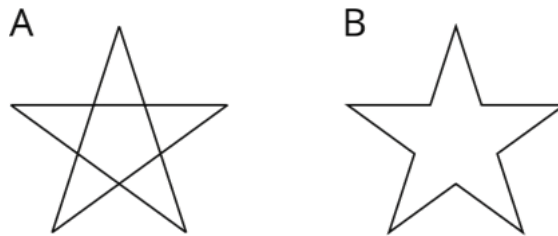
- 6.G.A.1

Launch

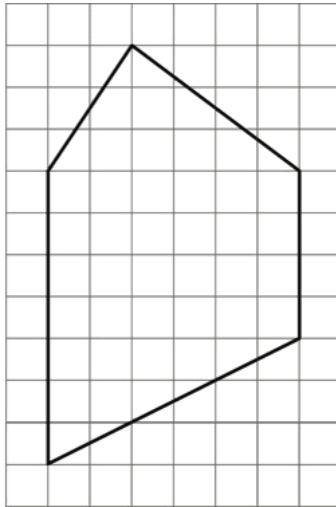
Give students access to their geometry toolkits. Tell students that they need to show only how the area could be found; they do not have to actually calculate the area.

Student Task Statement

1. Here are two five-pointed stars. A student said, “Both figures A and B are polygons. They are both composed of line segments and are two-dimensional. Neither have curves.” Do you agree with the statement? Explain your reasoning.

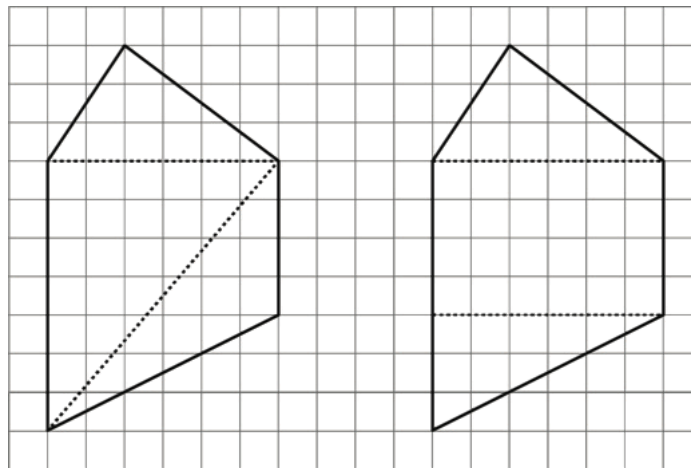


2. Here is a five-sided polygon. Describe or show the strategy you would use to find its area. Mark up and label the diagram to show your reasoning so that it can be followed by others. (It is not necessary to actually calculate the area.)



Student Response

1. Disagree. Only Figure B is a polygon. Explanations vary. Sample explanation: Every segment in Figure A meets or cross more than two segments at its ends, so it is not a polygon. Each segment in Figure B meets only one other segment at each end.
2. Answers vary. Sample diagrams and responses:



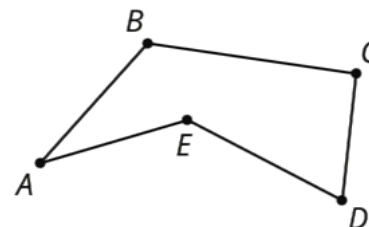
- The polygon can be decomposed into three triangles: one with a base of 6 units and a height of 3, a second one with a base of 7 and a height of 6, and a third with a base of 4 and a height of 6. All areas can be calculated using the area formula.
- The polygon can be decomposed into two triangles and a rectangle. One triangle has a base of 6 and a height 3, and the second has a base of 6 and a height of 1. Their areas can be calculated with the area formula. The rectangle is 6 by 4, so its area is the product of 6 and 4.

Student Lesson Summary

A **polygon** is a two-dimensional figure composed of straight line segments.

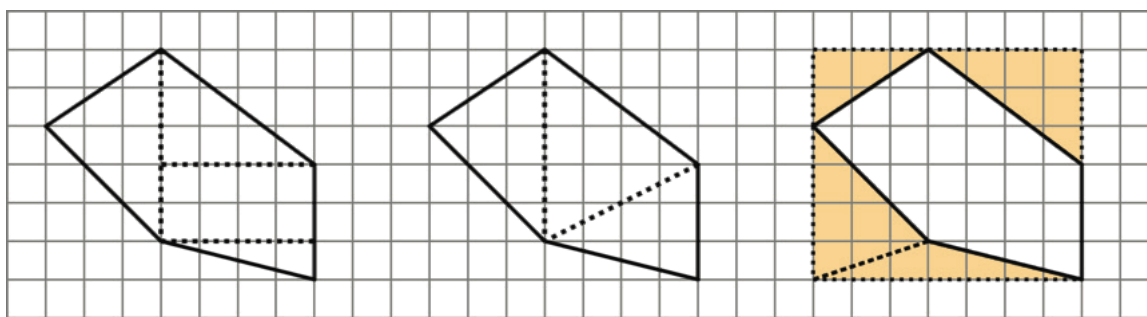
- Each end of a line segment connects to one other line segment. The point where two segments connect is a **vertex**. The plural of vertex is vertices.
- The segments are called the **edges** or **sides** of the polygon. The sides never cross each other. There are always an equal number of vertices and sides.

Here is a polygon with 5 sides. The vertices are labeled A , B , C , D , and E .



A polygon encloses a region. To find the area of a polygon is to find the area of the region inside it.

We can find the area of a polygon by decomposing the region inside it into triangles and rectangles.



The first two diagrams show the polygon decomposed into triangles and rectangles; the sum of their areas is the area of the polygon. The last diagram shows the polygon enclosed with a rectangle; subtracting the areas of the triangles from the area of the rectangle gives us the area of the polygon.

Glossary

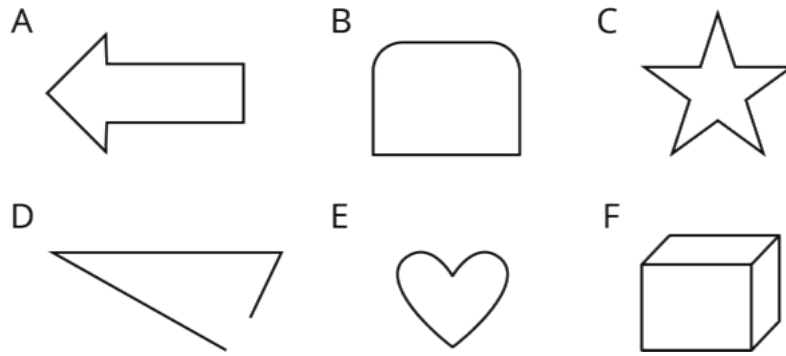
- polygon

Lesson 11 Practice Problems

Problem 1

Statement

Select all the polygons.



- A. A
- B. B
- C. C
- D. D
- E. E
- F. F

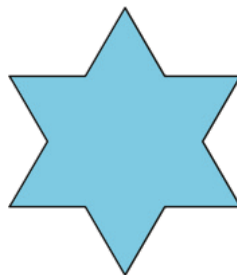
Solution

["A", "C"]

Problem 2

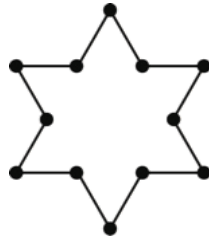
Statement

Mark each vertex with a large dot. How many edges and vertices does this polygon have?



Solution

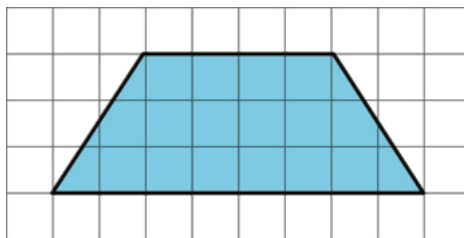
12 edges and 12 vertices



Problem 3

Statement

Find the area of this trapezoid. Explain or show your strategy.



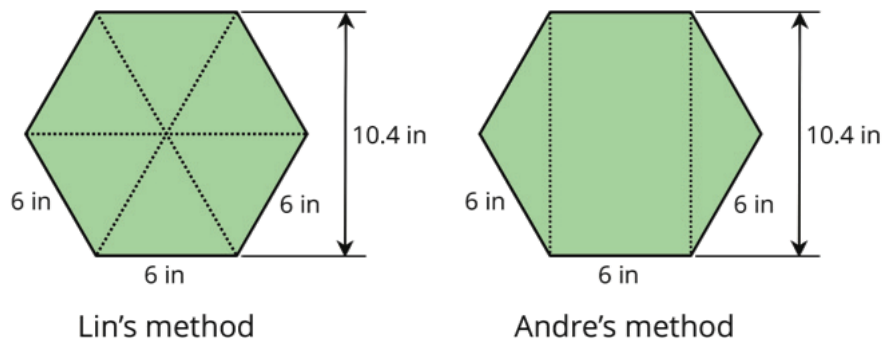
Solution

18 square units. Strategies vary. Possible strategy: Enclose the trapezoid inside a 3-unit-by-8-unit rectangle. The area of the rectangle is 24 square units because $8 \cdot 3 = 24$. The area of each unshaded triangle within the rectangle is 3 square units because $(2 \cdot 3) \div 2 = 3$. The sum of areas of the two triangles is 6 square units. $24 - 6 = 18$, so the area of the trapezoid is 18 square units.

Problem 4

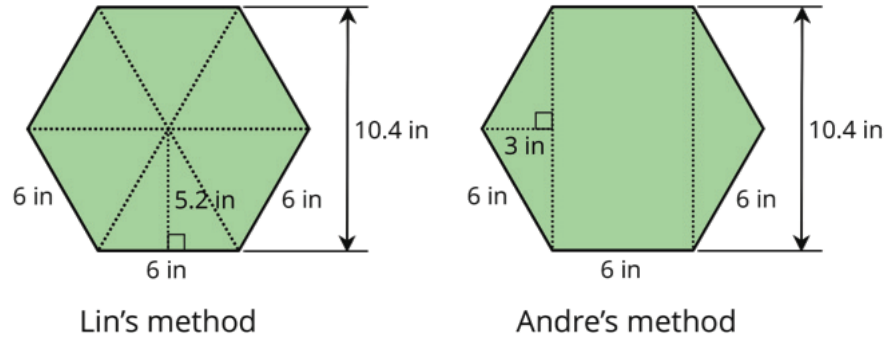
Statement

Lin and Andre used different methods to find the area of a regular hexagon with 6-inch sides. Lin decomposed the hexagon into six identical, equilateral triangles. Andre decomposed the hexagon into a rectangle and two triangles.



Find the area of the hexagon using each person's method. Show your reasoning.

Solution



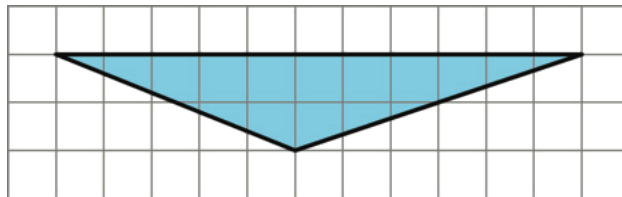
The height of each triangle in Lin's diagram is half of 10.4 inches or 5.2 inches. The area of each triangle is 15.6 square inches. $\frac{1}{2} \cdot 6 \cdot (5.2) = 15.6$. The hexagon is composed of 6 triangles, so its area is $6 \cdot (15.6)$ or 93.6 square inches.

The rectangle in Andre's diagram is $(10.4) \cdot 6$ or 62.4 square inches. Each triangle has a base of 10.4 inches and a height of 3 inches. (The horizontal distance across the middle of the hexagon is composed of two 6-inch segments. The vertical line that Andre drew cuts one 6-inch segment in half, so the segment on one side is 3 inches long.) The area of each triangle is $\frac{1}{2} \cdot 10.4 \cdot 3$ or 15.6 square inches. The area of the hexagon is therefore $62.4 + 15.6 + 15.6$ or 93.6 square inches.

Problem 5

Statement

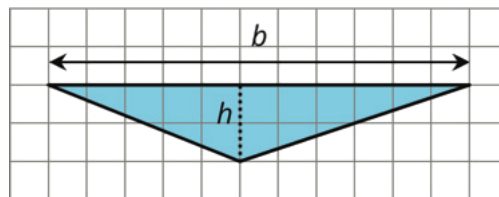
- a. Identify a base and a corresponding height that can be used to find the area of this triangle. Label the base b and the corresponding height h .



- b. Find the area of the triangle. Show your reasoning.

Solution

- a.



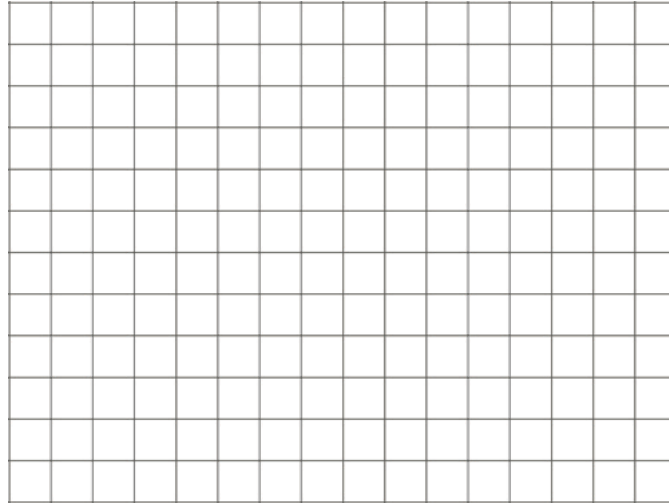
- b. 11 square units. $\frac{1}{2} \cdot 11 \cdot 2 = 11$.

(From Unit 1, Lesson 9.)

Problem 6

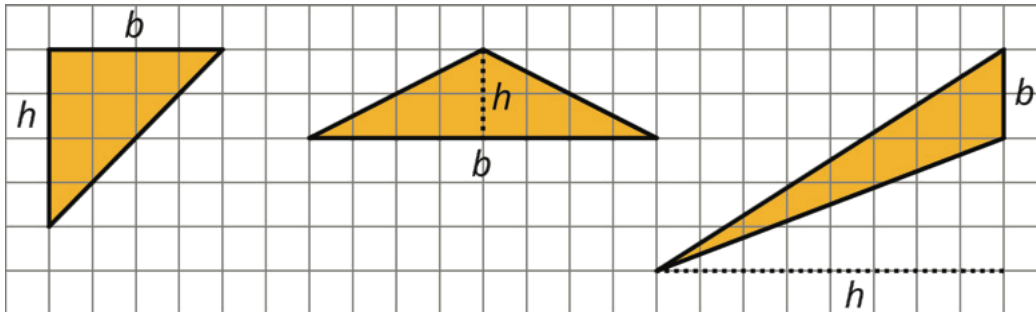
Statement

On the grid, draw three different triangles with an area of 8 square units. Label the base and height of each triangle.



Solution

Answers vary. Drawings should show triangles with a base and a height that multiply to be 24 square units (i.e., each triangle is half of a parallelogram with an area of 24 square units).



(From Unit 1, Lesson 10.)

Section: Surface Area

Lesson 12: What is Surface Area?

Goals

- Calculate the surface area of a rectangular prism and explain (orally and in writing) the solution method.
- Comprehend that the term “surface area” (in written and spoken language) refers to how many square units it takes to cover all the faces of a three-dimensional object.

Learning Targets

- I know what the surface area of a three-dimensional object means.

Lesson Narrative

This lesson introduces students to the concept of **surface area**. They use what they learned about area of rectangles to find the surface area of prisms with rectangular **faces**.

Students begin exploring surface area in concrete terms, by estimating and then calculating the number of square sticky notes it would take to cover a filing cabinet. Because students are not given specific techniques ahead of time, they need to make sense of the problem and persevere in solving it (MP1). The first activity is meant to be open and exploratory. In the second activity, they then learn that the surface area (in square units) is the number of unit squares it takes to cover all the surfaces of a three-dimensional figure without gaps or overlaps (MP6).

Later in the lesson, students use cubes to build rectangular prisms and then determine their surface areas.

Alignments

Addressing

- 6.G.A.4: Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Building Towards

- 6.G.A.4: Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Instructional Routines

- MLR7: Compare and Connect
- MLR8: Discussion Supports

- Notice and Wonder
- Poll the Class
- Think Pair Share

Required Materials

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Snap cubes

Required Preparation

- Prepare 12 cubes per student and extra copies of isometric dot paper for Building with Snap Cubes activity.
- Build several rectangular prisms that are each 2 cubes by 3 cubes by 5 cubes for the cool-down.

Student Learning Goals

Let's cover the surfaces of some three-dimensional objects.

12.1 Covering the Cabinet (Part 1)

Warm Up: 5 minutes

This activity prepares students to think about surface area, which they explore in this lesson and upcoming lessons. Students watch a video of a cabinet being gradually tiled with non-overlapping sticky notes. The cabinet was left only partially tiled, which raises the question of the number of sticky notes it takes to cover the entire rectangular prism. Students estimate the answer to this question.

This activity was inspired by Andrew Stadel. Media used with permission. <http://www.estimated180.com/filecabinet.html>.

Building Towards

- 6.G.A.4

Instructional Routines

- Notice and Wonder
- Poll the Class

Launch

Arrange students in groups of 2. Show the video of a teacher beginning to cover a large cabinet with sticky notes or display the following still images for all to see. Before starting the video or displaying the image, ask students be prepared to share one thing they notice and one thing they wonder.

Video 'File Cabinet - Act 1' available here: <https://player.vimeo.com/video/304136534>.



Give students a minute to share their observation and question with a partner. Invite a few students to share their questions with the class. If the question “How many sticky notes would it take to cover the entire cabinet?” is not mentioned, ask if anyone wondered how many sticky notes it would take to cover the entire cabinet.

Give students a minute to make an estimate.

Student Task Statement

Your teacher will show you a video about a cabinet or some pictures of it.

Estimate an answer to the question: How many sticky notes would it take to cover the cabinet, excluding the bottom?

Student Response

Estimates vary. The actual number of sticky notes is 935. Good estimates are in the 800–1,200 range.

Activity Synthesis

Poll the class for students' estimates, and record them for all to see. Invite a couple of students to share how they made their estimate. Explain to students that they will now think about how to answer this question.

12.2 Covering the Cabinet (Part 2)

20 minutes

After making an estimate of the number of sticky notes on a cabinet in the warm-up, students now brainstorm ways to find that number more accurately and then go about calculating an answer. The activity prompts students to transfer their understandings of the area of polygons to find the **surface area** of a three-dimensional object.

Students learn that the surface area of a three-dimensional figure is the total area of all its faces. Since the area of a region is the number of square units it takes to cover the region without gaps and overlaps, surface area can be thought of as the number of square units that needed to cover all sides of an object without gaps and overlaps. The square sticky notes illustrate this idea in a concrete way.

As students work, notice the varying approaches taken to determine the number of sticky notes needed to tile the faces of the cabinet (excluding the bottom). Identify students with different strategies to share later.

Addressing

- 6.G.A.4

Instructional Routines

- MLR7: Compare and Connect

Launch

Arrange students in groups of 2–4. Give students 1 minute of quiet time to think about the first question and another minute to share their responses with their group. Ask students to pause afterwards.

Select some students to share how they might go about finding out the number of sticky notes and what information they would need. Students may ask for some measurements:

- The measurements of the cabinet in terms of sticky notes: Tell students that the cabinet is 24 by 12 by 6.
- The measurements of the cabinet in inches or centimeters: Tell students that you don't have that information and prompt them to think of another piece of information they could use.
- The measurements of each sticky note: Share that it is 3 inches by 3 inches.

If no students mention needing the edge measurements of the cabinet in terms of sticky notes, let them begin working on the second question and provide the information when they realize that it is needed. Give students 8–10 minutes for the second question.

Anticipated Misconceptions

Students may treat all sides as if they were congruent rectangles. That is, they find the area of the front of the cabinet and then just multiply by 5, or act as if the top is the only side that is not congruent to the others. If there is a real cabinet (or any other large object in the shape of a rectangular prism) in the classroom, consider showing students that only the sides opposite each other can be presumed to be identical.

Students may neglect the fact that the bottom of the cabinet will not be covered. Point out that the bottom is inaccessible because of the floor.

Student Task Statement

Earlier, you learned about a cabinet being covered with sticky notes.

1. How could you find the actual number of sticky notes it will take to cover the cabinet, excluding the bottom? What information would you need to know?
2. Use the information you have to find the number of sticky notes to cover the cabinet. Show your reasoning.

Student Response

1. Find the area of each side of the cabinet, excluding the bottom, and add them together.
Needed information: measurements of the cabinet edge lengths in sticky notes.
2. Answers vary. Strategies may be a combination of the following two strategies:
 - Multiply the number of sticky notes along each edge of each side. Add all of the products.
 - Multiply the edge lengths of each side of the cabinet to find the area of each side. Add all of the areas.

Are You Ready for More?

How many sticky notes are needed to cover the outside of 2 cabinets pushed together (including the bottom)? What about 3 cabinets? 20 cabinets?

Student Response

Two cabinets: 1,582 sticky notes. Three cabinets: 2,229 sticky notes. Twenty cabinets: 13,228 sticky notes.

Activity Synthesis

Invite previously identified students or groups to share their answer and strategy. On a visual display, record each answer and each distinct process for determining the surface area (i.e. multiplying the side lengths of each rectangular face and adding up the products). After each presentation, poll the class on whether others had the same answer or process.

Play the video that reveals the actual number of sticky notes needed to cover the cabinet. If students' answers vary from that shown on the video, discuss possible reasons for the differences.

(For example, students may not have accounted for the cabinet's door handles. Some may have made a calculation error.)

Tell students that the question they have been trying to answer is one about the surface area of the cabinet. Explain that the **surface area** of a three-dimensional figure is the total area of all its surfaces. We call the flat surfaces on a three-dimensional figure its **faces**.

The surface area of a rectangular prism would then be the combined area of all six of its faces. In the context of this problem, we excluded the bottom face, since it is sitting on the ground and will not be tiled with sticky notes. Discuss:

- “What unit of measurement are we using to represent the surface area of the cabinet?” (Square sticky notes)
- “Would the surface area change if we used larger or smaller sticky notes? How?” (Yes, if we use larger sticky notes, we would need fewer. If we use smaller ones, we would need more.)

Support for English Language Learners

Speaking, Listening: MLR7 Compare and Connect. As students share their strategies for determining the number of sticky notes that cover the cabinet, ask students to make connections between the various strategies. Some students will calculate the number of sticky notes that will cover each of the five faces of the cabinet and add them together. Other students may realize that opposite faces of the cabinet are congruent so it is only necessary to calculate the area of three faces of the cabinet. Encourage students to explain why both methods result in the same answer. This will promote students' use of mathematical language as they make sense of the various methods for finding the surface area of a rectangular prism.

Design Principle(s): Cultivate conversation; Maximize meta-awareness

12.3 Building with Snap Cubes

20 minutes (there is a digital version of this activity)

This activity encourages students to apply strategies for finding the area of polygons to finding the *surface area* of rectangular prisms. Students use 12 cubes to build a prism, think about its surface area, and use isometric dot paper to draw their prism.

As students build their prisms, notice those with different designs and those with the same design but different approaches to finding surface area (e.g. by counting individual square, by multiplying the edge lengths of rectangular faces, etc.).

Addressing

- 6.G.A.4

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Read the first two lines of the task statement together. Remind students that we refer to the flat surfaces of a three-dimensional figure as **faces**.

Give students a minute to think about how we know the surface area of the shown prism is 32 square units. Ask 1–2 students to explain their reasoning to the class. Use students' explanations to highlight the meaning of surface area, i.e., that the area of all the faces need to be accounted for, including those we cannot see when looking at a two-dimensional drawing.

Tell students they will use 12 cubes to build a different prism, draw it, and find its surface area. Consider doing a quick demonstration on how to draw a simple prism on isometric dot paper. (Start with one cube and then add a cube in each dimension.) Tell students that in this activity, we call each face of a single cube, "1 square unit."

Give each student 12 cubes to build a prism and 6–8 minutes of quiet work time. If students are using snap cubes, say that we will pretend all of the faces are completely smooth, so they do not need to worry about the "innies and outies" of the snap cubes.

As students work, consider arranging two students with contrasting designs or strategies as partners. Ask partners to share their answers, explanations, and drawings. Stress that each partner should focus their explanation on how they went about finding surface area. The listener should think about whether the explanation makes sense or if anything is amiss in the reasoning.

For students in digital classrooms, an applet can be used to build and draw prisms. Physical cubes are still recommended and preferred for the building of the figures, however.

Support for Students with Disabilities

Action and Expression: Develop Expression and Communication. Support multiple forms of communication. Some students may benefit from an explicit demonstration and additional practice to learn how to draw cubes using isometric dot paper. Invite students who are unable to represent their figures using isometric dot paper to explain their reasoning orally, using virtual or concrete manipulatives.

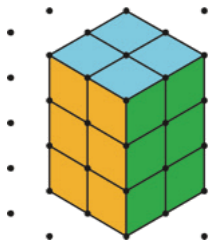
Supports accessibility for: Visual-spatial processing; Conceptual processing; Fine-motor skills

Anticipated Misconceptions

Students may count the faces of the individual snap cubes rather than faces of the completed prism. Help them understand that the faces are the visible ones on the outside of the figure.

Student Task Statement

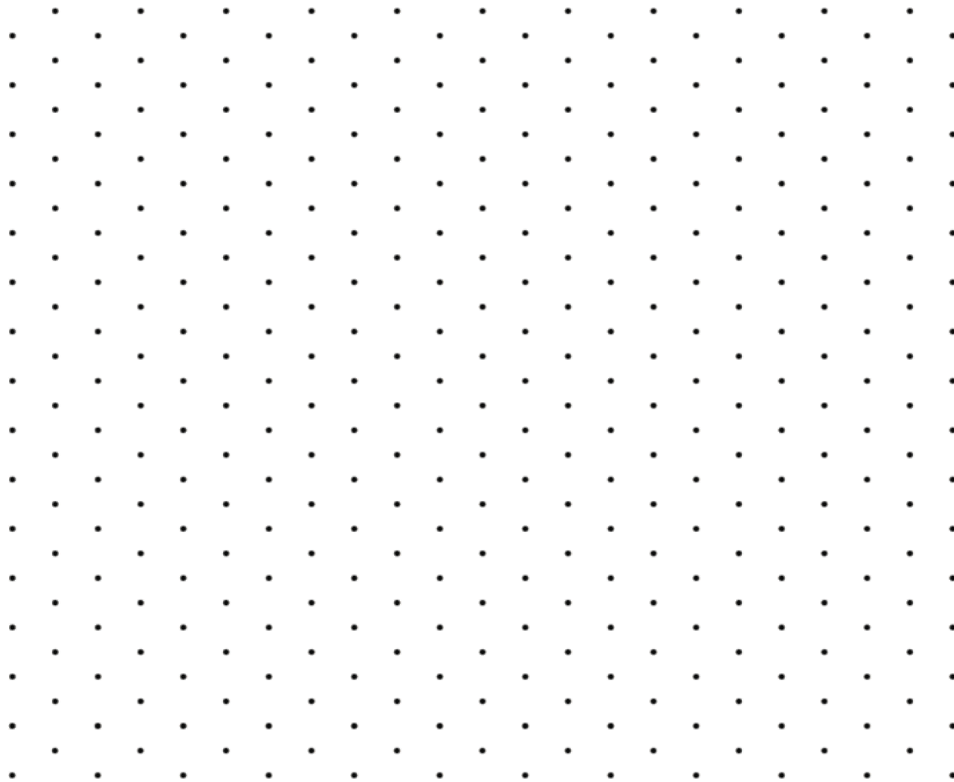
Here is a sketch of a rectangular prism built from 12 cubes:



It has six **faces**, but you can only see three of them in the sketch. It has a **surface area** of 32 square units.

Your teacher will give you 12 snap cubes. Use all of your snap cubes to build a different rectangular prism (with different edge lengths than the prism shown here).

1. How many faces does your figure have?
2. What is the surface area of your figure in square units?
3. Draw your figure on isometric dot paper. Color each face a different color.



Student Response

1. There are 6 faces—front, back, left, right, top, and bottom.
2. Answers vary based on design. Sample responses:
 - For a prism that is 12 unit by 1 unit by 1 unit, the surface area is 50 square units.
 $(4 \cdot 12) + (2 \cdot 1) = 50$
 - For a prism that is 6 units by 2 units by 1 unit, the surface area is 40 square units.
 $(2 \cdot 12) + (2 \cdot 6) + (2 \cdot 2) = 40$

- For a prism that is 4 units by 3 units by 1 unit, the surface area is 38 square units. $(2 \cdot 12) + (2 \cdot 4) + (2 \cdot 3) = 38$

3. Drawings vary, but all prisms should have one edge length that is 1 unit.

Activity Synthesis

After partner discussions, select a student to highlight for the class the strategy (or strategies) for finding surface area methodically. Point out that in this activity each face of their prism is a rectangle, and that we can find the area of each rectangle by multiplying its side lengths and then add the areas of all the faces.

Explain that later, when we encounter non-rectangular prisms, we can likewise reason about the area of each face the way we reasoned about the area of a polygon.

Support for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion. Call on students to use mathematical language (e.g., cubes, faces, surface area, square units, etc.), to restate and/or revoice the strategy (or strategies) presented. Consider providing students time to restate what they hear to a partner, before selecting one or two students to share with the class. This will provide more students with an opportunity to produce language that describes strategies for finding surface area.

Design Principle(s): Support sense-making; Maximize meta-awareness

Lesson Synthesis

In this lesson, we found the surface areas of a cabinet and of rectangular prisms built out of cubes.

- "What does it mean to find the **surface area** of a three-dimensional figure?" (It means finding the number of unit squares that cover the entire surface of the object without gaps or overlaps.)
- "How can we find the number of unit squares that cover the entire surface of an object?" (We can count them, or we can find the area of each **face** of the object and add the areas of all faces.)
- "How are finding surface area and finding area alike? How are they different?" (They both involve finding the number of unit squares that cover a region entirely without gaps and overlaps. Both have to do with two-dimensional regions. Finding area involves a single polygon. Finding surface area means finding the sum of the areas of multiple polygons (faces) of which a three-dimensional figure is composed.)

12.4 A Snap Cube Prism

Cool Down: 5 minutes

Addressing

- 6.G.A.4

Launch

Prepare several rectangular prisms that are each 2 cubes by 3 cubes by 5 cubes. Display one for all to see and pass the rest around for students to examine, if needed.

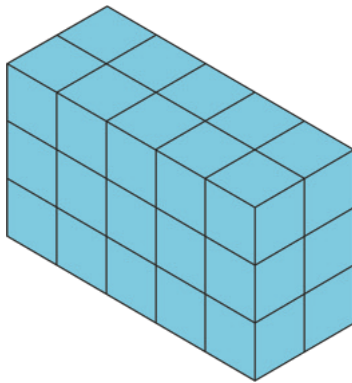
Anticipated Misconceptions

Students may not include the bottom face as it is not visible when the prism is sitting on a table.

Students may find the number of cubes instead of the surface area due to their previous volume work with rectangular prisms in grade 5.

Student Task Statement

A rectangular prism made is 3 units high, 2 units wide, and 5 units long. What is its surface area in square units? Explain or show your reasoning.



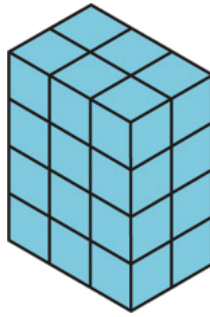
Student Response

The surface area is $2 \cdot [(3 \cdot 5) + (2 \cdot 5) + (2 \cdot 3)] = 62$ (or 62 square units).

Student Lesson Summary

- The **surface area** of a figure (in square units) is the number of unit squares it takes to cover the entire surface without gaps or overlaps.
- If a three-dimensional figure has flat sides, the sides are called **faces**.
- The surface area is the total of the areas of the faces.

For example, a rectangular prism has six faces. The surface area of the prism is the total of the areas of the six rectangular faces.



So the surface area of a rectangular prism that has edge-lengths 2 cm, 3 cm, and 4 cm has a surface area of

$$(2 \cdot 3) + (2 \cdot 3) + (2 \cdot 4) + (2 \cdot 4) + (3 \cdot 4) + (3 \cdot 4)$$

or 52 square centimeters.

Glossary

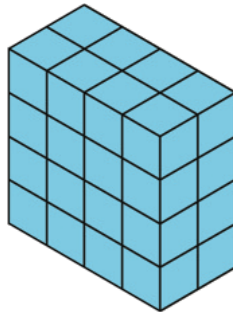
- face
- surface area

Lesson 12 Practice Problems

Problem 1

Statement

What is the surface area of this rectangular prism?



- A. 16 square units
- B. 32 square units
- C. 48 square units
- D. 64 square units

Solution

D

Problem 2

Statement

Which description can represent the surface area of this trunk?



- A. The number of square inches that cover the top of the trunk.
- B. The number of square feet that cover all the outside faces of the trunk.
- C. The number of square inches of horizontal surface inside the trunk.
- D. The number of cubic feet that can be packed inside the trunk.

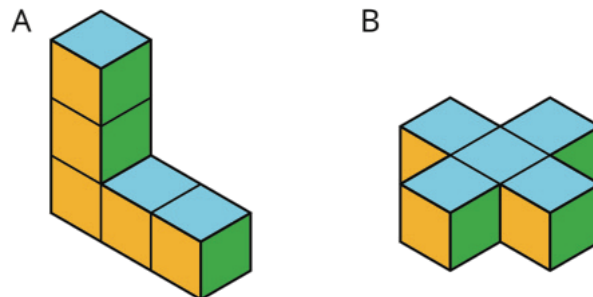
Solution

B

Problem 3

Statement

Which figure has a greater surface area?



Solution

Figure A and Figure B have the same surface area of 22 square units.

Problem 4

Statement

A rectangular prism is 4 units high, 2 units wide, and 6 units long. What is its surface area in square units? Explain or show your reasoning.

Solution

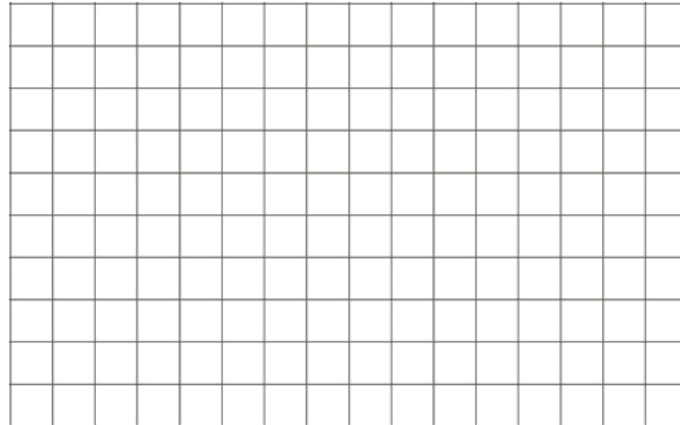
88 square units. Two faces are 4 units by 2 units, amounting to 16 square units. Two faces are 4 units by 6 units, amounting to 48 square units. Two faces are 2 units by 6 units, amounting to 24 square units. $16 + 48 + 24 = 88$.

Problem 5

Statement

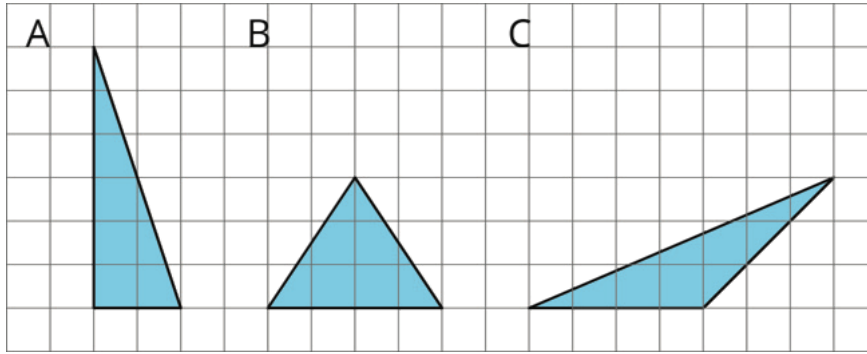
Draw an example of each of these triangles on the grid.

- A right triangle with an area of 6 square units.
- An acute triangle with an area of 6 square units.
- An obtuse triangle with an area of 6 square units.



Solution

Answers vary. Sample response:

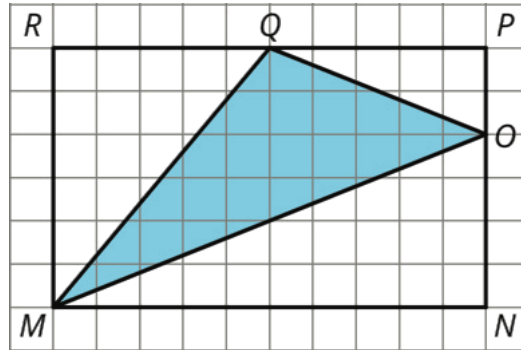


(From Unit 1, Lesson 9.)

Problem 6

Statement

Find the area of triangle MOQ in square units. Show your reasoning.



Solution

20 square units. Reasoning varies. Sample reasoning: The area of triangle MOQ can be found by subtracting the areas of the three right triangles from the area of rectangle $MNPR$.

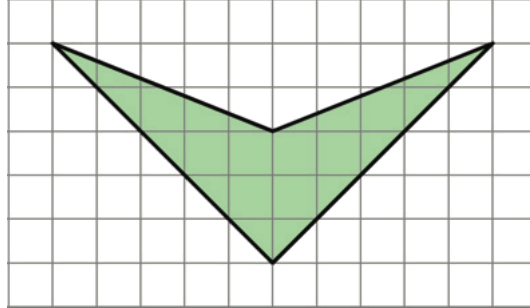
- The area of rectangle $MNPR$ is $10 \cdot 6$ or 60 square units.
- The area of triangle QRM is $\frac{1}{2} \cdot 6 \cdot 5$ or 15 square units.
- The area of triangle MNO is $\frac{1}{2} \cdot 10 \cdot 4$ or 20 square units.
- The area of triangle OPQ is $\frac{1}{2} \cdot 2 \cdot 5$ or 5 square units. $60 - (15 + 20 + 5) = 20$.

(From Unit 1, Lesson 10.)

Problem 7

Statement

Find the area of this shape. Show your reasoning.



Solution

15 square units. Reasoning varies. Sample reasoning:

- The shape can be decomposed into two identical triangles with a vertical cut down the middle. Each triangle has base 3 units and height 5 units, so its area is $\frac{1}{2} \cdot 3 \cdot 5$ or 7.5 square units. $2 \cdot (7.5) = 15$.
- The shape can be decomposed into two identical triangles and rearranged into a parallelogram with base 3 units and height 5 units. $3 \cdot 5 = 15$.

(From Unit 1, Lesson 3.)

Lesson 13: Polyhedra

Goals

- Compare and contrast (orally and in writing) features of prisms and pyramids.
- Comprehend and use the words “face”, “edge”, “vertex”, and “base” to describe polyhedra (in spoken and written language).
- Understand that the word “net” refers to a two-dimensional figure that can be assembled into a polyhedron, and create a net for a given polyhedron.

Learning Targets

- I can describe the features of a polyhedron using mathematical vocabulary.
- I can explain the difference between prisms and pyramids.
- I understand the relationship between a polyhedron and its net.

Lesson Narrative

In this lesson, students learn about **polyhedra** and their **nets**. They also study **prisms** and **pyramids** as types of polyhedra with certain defining features.

Polyhedra can be thought of as the three-dimensional analog of polygons.

Here are some important aspects of polygons:

- They are made out of line segments called *edges*.
- Edges meet at a *vertex*.
- The edges only meet at vertices.
- Polygons always enclose a two-dimensional region.

Here is an analogous way to characterize polyhedra:

- They are made out of filled-in polygons called *faces*.
- Faces meet at an edge.
- The faces only meet at edges.
- Polyhedra always enclose a three-dimensional region.

Students do not need to memorize a formal definition of a polyhedron, but help them make sense of nets and surface area.

Alignments

Addressing

- 6.G.A.4: Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Building Towards

- 6.G.A.4: Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Instructional Routines

- MLR2: Collect and Display
- Notice and Wonder

Required Materials

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this.

If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Glue or glue sticks

Nets of polyhedra

Pre-assembled or commercially produced polyhedra

Scissors

Tape

Required Preparation

- Assemble collections of geometric figures that each contains at least 2 familiar polyhedra, 2 unfamiliar polyhedra, and 2 non-polyhedra. Prepare one collection for each group of 3–4 students. If pre-made polyhedra are unavailable, assemble some from the nets in the Instructional master for the warm-up.
- Print and pre-cut the nets and polygons in the Instructional master for Prisms and Pyramids. Prepare 1 set per group of 3–4 students, along with tape to join the polygons into a net.
- For the optional Assembling Polyhedra activity, print the nets from the same Instructional master as for the warm-up. Prepare 2 copies of one net per student, and tape or glue to assemble the net.

Student Learning Goals

Let's investigate polyhedra.

13.1 What are Polyhedra?

Warm Up: 10 minutes

In this warm-up, students analyze examples and counterexamples of polyhedra, observe their defining characteristics, and use their insights to sort objects into polyhedra and non-polyhedra. They then start developing a working definition of polyhedron.

Prepare physical examples of polyhedra and non-polyhedra for students to sort. These examples should be geometric figures rather than real-world objects such as shoe boxes or canisters. If such figures are not available, make some ahead of time using the nets in the Instructional master.

As students work and discuss, notice those who can articulate defining features of a polyhedron. Invite them to share later.

Building Towards

- 6.G.A.4

Instructional Routines

- Notice and Wonder

Launch

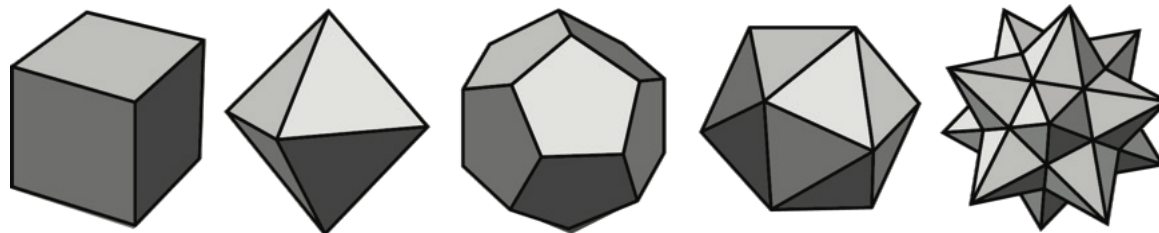
Arrange students in groups of 3–4. Give students 1 minute of quiet time to study the examples and non-examples in the task statement. Ask them to be ready to share at least one thing they notice and one thing they wonder. Give the class a minute to share some of their observations and questions.

Next, give each group a physical set of three-dimensional figures. The set should include some familiar polyhedra, some unfamiliar ones, and some non-polyhedra.

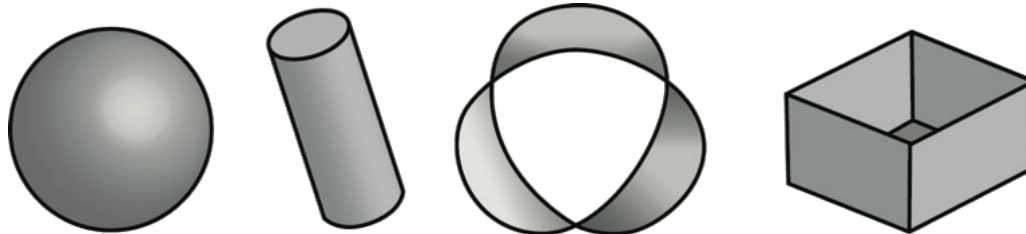
Ask groups to sort the figures into polyhedra and non-polyhedra (the first question). If groups members disagree about whether a figure is a polyhedron, discuss the disagreements. When the group has come to an agreement, give them 2–3 minutes of quiet time to complete the second question.

Student Task Statement

Here are pictures that represent **polyhedra**:



Here are pictures that do *not* represent polyhedra:



1. Your teacher will give you some figures or objects. Sort them into polyhedra and non-polyhedra.
2. What features helped you distinguish the polyhedra from the other figures?

Student Response

1. No response required.
2. Answers vary. Sample responses:
 - Polyhedra are made from polygons.
 - Polyhedra don't have any unattached edges.
 - Non-polyhedra sometimes have curved or round surfaces.
 - Some non-polyhedra have a face that is not a polygon.

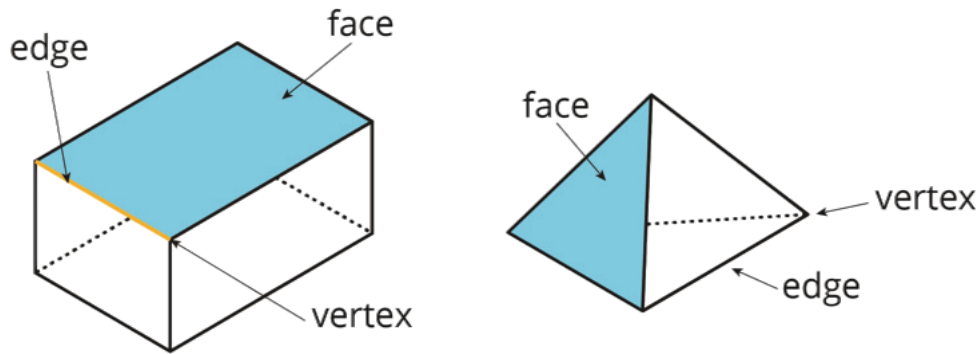
Activity Synthesis

Invite students to share the features that they believe characterize polyhedra. Record their responses for all to see. For each one, ask the class if they agree or disagree. If they generally agree, ask if there is anything they would add or elaborate to make the description clearer or more precise. If they disagree, ask for an explanation or a counterexample.

Students will have a chance to refine their definition of polyhedra later in the lesson—after exploring prisms and pyramids and learning about nets, so it is not important to compile a complete or precise set of descriptions or features.

Use a sample polyhedron or a diagram as shown here to introduce or reinforce the terminology surrounding polyhedra.

- The polygons that make up a polyhedron are called *faces*.
- The places where the sides of the faces meet are called **edges**.
- The “corners” are called **vertices**. (Clarify that the singular form is "vertex" and the plural form is "vertices.")



13.2 Prisms and Pyramids

30 minutes

This activity serves two goals: to uncover the defining features of **prisms** and **pyramids** as well as to introduce **nets** as two-dimensional representations of polyhedra.

Students first analyze prisms and pyramids and try to define their characteristics. Next, they learn about nets and think about the polygons needed to compose the nets of given prisms and pyramids. They then use their experience with the nets of prisms and pyramids to sharpen and refine their definitions of these polyhedra.

Ask students discuss the features of prisms and pyramids, encourage them to use the terms face, edge, and vertex (vertices) in their descriptions.

Addressing

- 6.G.A.4

Instructional Routines

- MLR2: Collect and Display

Launch

Arrange students in groups of 3–4.

For the first question:

- Tell students that Polyhedra A–F are all prisms and Polyhedra P–S are all pyramids. (Display and pass around the prisms and pyramids in the task statement, if available.)
- Give students 2–3 minutes of quiet time for the first question and 2–3 minutes to discuss their observations in their groups. Ask them to pause for a class discussion before moving on.
- Solicit students' ideas about features that distinguish prisms and pyramids. Record students' responses in a two columns—one for prisms and the other for pyramids. It is not important that the lists are complete at this point.

Next, tell students that we are going to use nets to better understand prisms and pyramids. Explain that a **net** is a two-dimensional representation of a polyhedron.

Display a cube assembled from the net provided in the Instructional master for the warm-up, as well as a cutout of an unfolded net (consider removing the flaps). Demonstrate how the net with squares could be folded and assembled into a cube, or use this book of digital applets, <https://ggbm.at/rcu3Ka3j>, created in GeoGebra by the [GeoGebra DocuTeam](#). Point out how the number and the shape of the faces on the cube correspond to the number and the shape of the polygons in the net.

For the second question:

- Give groups a minute of quiet think time and a minute to discuss their response.
- To verify their answer, give each group one of the three nets from the first page of the Instructional master. Ask them to try to assemble a triangular pyramid from their net.
- Invite groups to share with the class whether it can be done. Discuss why net 3 cannot be assembled into Pyramid P (two of the triangles would overlap).

For the last question, tell students that they will create a net of another prism or pyramid:

- Assign each group a prism or a pyramid from the task statement (except for Prism B and Pyramid P).
- Give each group a set of pre-cut polygons from the last two pages of the Instructional master.
- Tell students to choose the right kind and number of polygons that make up their polyhedron. Then, arrange the polygons so that, when taped and folded, the arrangement is a net and could be assembled into their prism or pyramid. Encourage them to think of more than one net, if possible.

Support for Students with Disabilities

Representation: Access for Perception. Provide access to concrete manipulatives. Provide prisms and pyramids for students to view or manipulate. These hands-on models will help students identify characteristics or features, and support net building for each polyhedra.

Supports accessibility for: Visual-spatial processing; Conceptual processing

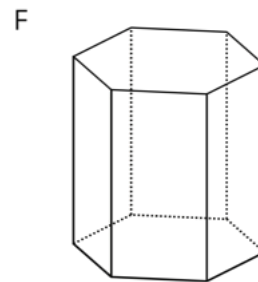
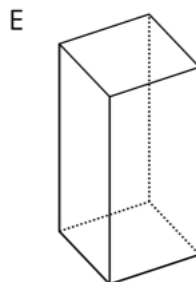
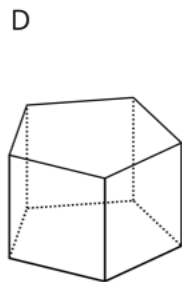
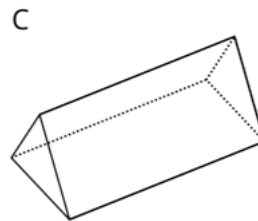
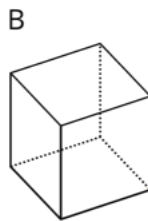
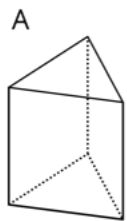
Support for English Language Learners

Speaking, Conversing: MLR2 Collect and Display. As students work on creating a net of their assigned polyhedron, circulate and listen to the language students use when talking about the polygons that make up their polyhedra, as well as the characteristics of their polyhedra (e.g., triangle, rectangle, square, hexagon, pentagon, vertex, edge, face). Collect this language, with corresponding drawings, and display it for all students to see. Remind students to borrow language from the display as they describe the features of prisms and pyramids. This will help students produce mathematical language to describe and define characteristics of polyhedra.

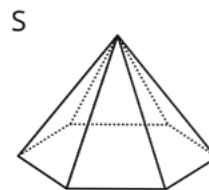
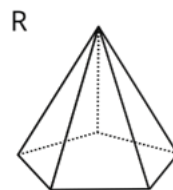
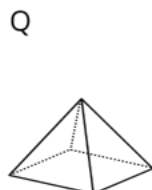
Design Principle(s): Support sense-making

Student Task Statement

1. Here are some polyhedra called **prisms**.



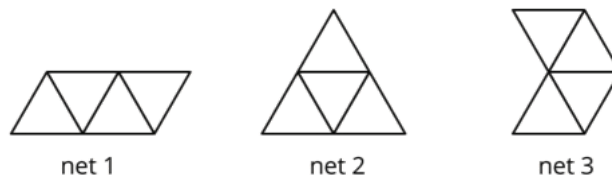
Here are some polyhedra called **pyramids**.



a. Look at the prisms. What are their characteristics or features?

b. Look at the pyramids. What are their characteristics or features?

2. Which of these **nets** can be folded into Pyramid P? Select all that apply.



3. Your teacher will give your group a set of polygons and assign a polyhedron.
 - a. Decide which polygons are needed to compose your assigned polyhedron. List the polygons and how many of each are needed.
 - b. Arrange the cut-outs into a net that, if taped and folded, can be assembled into the polyhedron. Sketch the net. If possible, find more than one way to arrange the polygons (show a different net for the same polyhedron).

Student Response

1. Answers vary. Sample responses:
 - a. A prism has rectangular faces. Some of the faces are parallel to one another. A prism may have two faces that are not rectangles.
 - b. A pyramid has a triangle for all or all but one of its faces. That one face might be a different polygon, and all triangles share an edge with it. All triangles also meet at a single vertex.
2. Nets 1 and 2 can be assembled into Pyramid P, but net 3 cannot.
3.
 - a. Answers vary. Possible responses:
 - Prism A: 2 triangles, 3 squares
 - Prism C: 2 triangles, 3 rectangles
 - Prism D: 2 pentagons, 5 rectangles
 - Prism E: 2 squares, 4 rectangles
 - Prism F: 2 hexagons, 6 rectangles
 - Pyramid Q: 1 square, 4 triangles
 - Pyramid R: 1 pentagon, 5 triangles
 - Pyramid S: 1 hexagon, 6 triangles
 - b. Drawings vary.

Are You Ready for More?

What is the smallest number of faces a polyhedron can possibly have? Explain how you know.

Student Response

Four faces (a triangular pyramid, also known as a tetrahedron). Explanations vary. Sample explanation: The triangle is the polygon with the fewest number of sides. If there is one triangular face, there must be another polygon attached at each edge of it, so it is impossible to use fewer

than four faces. It is possible to create a polyhedron with four triangle faces. (Pyramid P in this activity is an example.)

Activity Synthesis

Select groups to share their arrangements of polygons. If time permits and if possible, have students tape their polygons and fold the net to verify that it could be assembled into the intended polyhedron. Discuss:

- “What do the nets of prisms have in common?” (They all have rectangles. They have a pair of polygons that may not be rectangles.)
- “What do the nets of pyramids have in common?” (They all have triangles. They have one polygon that may not be a triangle.)
- “Is there only one possible net for a prism or a pyramid?” (No, the polygons can be arranged in different ways and still be assembled into the same prism or pyramid.)

Explain the following points about prisms and pyramids:

- A prism has two parallel, identical faces called **bases** and a set of rectangles connecting the bases.
- Prisms are named for the shape of the bases. For example, if the base of a prism is a pentagon, then the prism is called a “pentagonal prism.”
- A pyramid has one face called the base that can be any polygon and a set of faces that are all triangles. Each edge of the base is shared with an edge of a triangle. All of these triangles meet at a single vertex.
- Pyramids are named for the shape of their base. For example, if the base of a pyramid is a square, then the pyramid is called a “square pyramid.”

13.3 Assembling Polyhedra

Optional: 20 minutes

This optional activity gives students the experience of assembling polyhedra from nets. Counting the vertices, edges, and faces of a polyhedra helps to reinforce their understanding of the vocabulary. You will need the same Instructional master as the one provided for the warm-up.

Students are likely to need assistance in assembling their polyhedra. Circulate and support students as needed.

Addressing

- 6.G.A.4

Launch

Tell the class that they are going to assemble polyhedra from nets. Point out that the net has shaded and unshaded polygons. Display an example and explain that only the shaded polygons in the nets will show once the net is assembled. The unshaded polygons are "flaps" to make it easier to glue or tape the polygons together; they will get tucked behind the shaded polygons and are not really part of the polyhedron. Tell students that creasing along all of the lines first will make it easier to fold the net up and attach the various polygons together. A straightedge can be very helpful for making the creases.

Give each student two copies of a net so they can compare the assembled version with the unfolded net. Provide access to geometry toolkits and glue or tape. Ask students to build their figures and complete the question, and then to discuss their responses with another student with the same polyhedron.

Anticipated Misconceptions

Students may have trouble getting an accurate count of faces, edges, and vertices. Suggest that they set the figure on the table and then separately count the amount on the top, bottom, and lateral sides of the figure. Or recommend that they label each face with a number or a name and keep track of the parts associated with each face, taking care not to double count edges and vertices.

Student Task Statement

1. Your teacher will give you the net of a polyhedron. Cut out the net, and fold it along the edges to assemble a polyhedron. Tape or glue the flaps so that there are no unjoined edges.
2. How many **vertices**, **edges**, and **faces** are in your polyhedron?

Student Response

1. No answer required.
2. Answers vary. Possible responses:
 - A: 6 vertices, 9 edges, 5 faces
 - B: 8 vertices, 12 edges, 6 faces
 - C: 8 vertices, 12 edges, 6 faces
 - D: 8 vertices, 12 edges, 6 faces
 - E: 4 vertices, 6 edges, 4 faces
 - F: 5 vertices, 8 edges, 5 faces
 - G: 6 vertices, 10 edges, 6 faces
 - H: 8 vertices, 14 edges, 8 faces
 - J: 10 vertices, 20 edges, 12 faces
 - K: 9 vertices, 15 edges, 8 faces

Activity Synthesis

After students have conferred with another student and agreed on the number of vertices, edges, and faces of their polyhedron, tell the class they will now share their completed polyhedra and the unfolded version of the net with the class. Consider either asking students to pass their two items around, or to leave their the polyhedra and nets displayed while students circulate around the room to view others' work.

Support for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding and memory. Include the following terms and maintain the display for reference throughout the unit: polyhedra, prism, pyramid, face, edge, vertices, and net.

Supports accessibility for: Conceptual processing; Language; Memory

Lesson Synthesis

Review the features of prisms and pyramids by selecting 1–2 polyhedra used in the warm-up. Ask students to explain (using the terminology they learned, if possible) why each one is or is not a prism or a pyramid. If it is a prism or pyramid, ask students to name it.

Revisit the working definition of polyhedra generated earlier in the lesson and ask students to see if or how it might be refined. Ask if there is anything they should add, remove, or adjust given their work with prisms, pyramids, and nets.

Highlight the following points about polyhedra. Ask students to illustrate each point using a figure or a net.

- A **polyhedron** is a three-dimensional figure built from filled-in polygons. We call the polygons *faces*. (The plural of polyhedron is polyhedra.)
- All *edges* of polygons meet another polygon along a complete edge.
- Each polygon meets one and only one polygon on each of the edges.
- The polygons enclose a three-dimensional region.

Consider displaying in a visible place the key ideas from students' list and from this discussion so that they can serve as a reference later.

13.4 Three-Dimensional Shapes

Cool Down: 5 minutes

Addressing

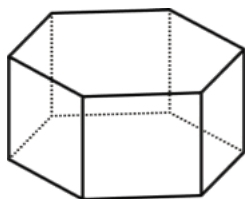
- 6.G.A.4

Anticipated Misconceptions

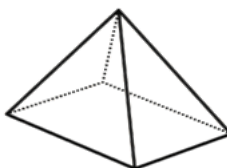
Some students may mistake a triangular prism (especially one that is not sitting on one of its bases) as a pyramid because it has triangular faces.

Student Task Statement

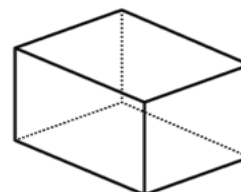
1. Write your best definition or description of a polyhedron. If possible, use the terms you learned in this lesson.
2. Which of these five polyhedra are prisms? Which are pyramids?



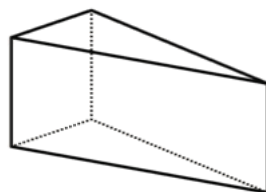
A



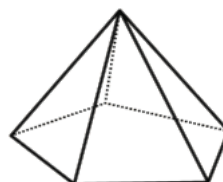
B



C



D



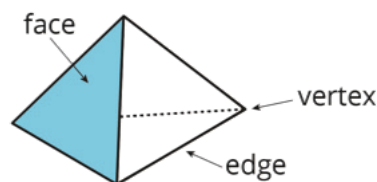
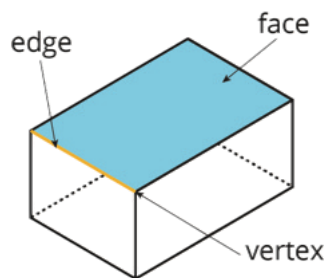
E

Student Response

1. Answers vary but might include one or more of these elements: A polyhedron is a three-dimensional figure made from faces that are filled-in polygons. Each face meets one and only one other face along a complete edge. The points where edges meet are called vertices.
2. A, C, and D are prisms. B and E are pyramids.

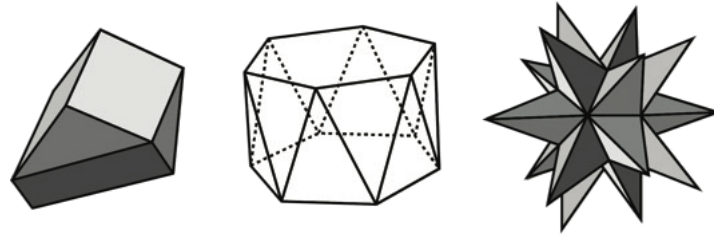
Student Lesson Summary

A **polyhedron** is a three-dimensional figure composed of faces. Each face is a filled-in polygon and meets only one other face along a complete edge. The ends of the edges meet at points that are called vertices.



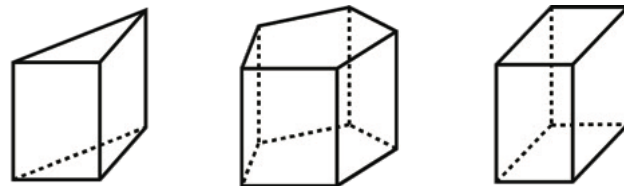
A polyhedron always encloses a three-dimensional region.

The plural of polyhedron is polyhedra. Here are some drawings of polyhedra:



A **prism** is a type of polyhedron with two identical faces that are parallel to each other and that are called **bases**. The bases are connected by a set of rectangles (or sometimes parallelograms).

A prism is named for the shape of its bases. For example, if the base is a pentagon, then it is called a “pentagonal prism.”



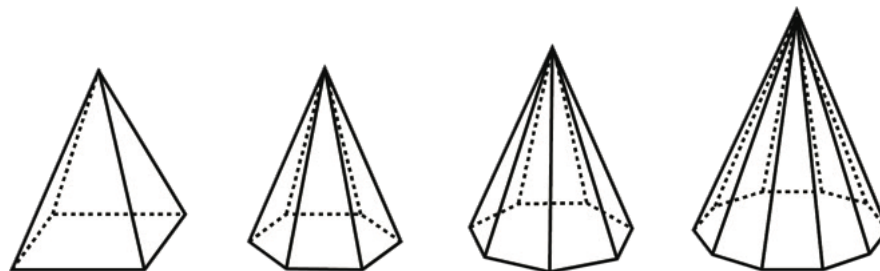
triangular
prism

pentagonal
prism

rectangular
prism

A **pyramid** is a type of polyhedron that has one special face called the base. All of the other faces are triangles that all meet at a single vertex.

A pyramid is named for the shape of its base. For example, if the base is a pentagon, then it is called a “pentagonal pyramid.”



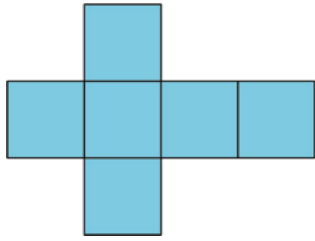
rectangular
pyramid

hexagonal
pyramid

heptagonal
pyramid

decagonal
pyramid

A **net** is a two-dimensional representation of a polyhedron. It is composed of polygons that form the faces of a polyhedron.



A cube has 6 square faces, so its net is composed of six squares, as shown here.

A net can be cut out and folded to make a model of the polyhedron.

In a cube, every face shares its edges with 4 other squares. In a net of a cube, not all edges of the squares are joined with another edge. When the net is folded, however, each of these open edges will join another edge.

It takes practice to visualize the final polyhedron by just looking at a net.

Glossary

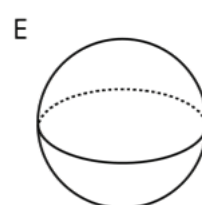
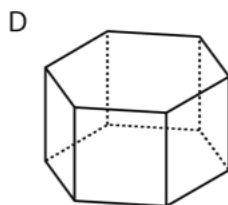
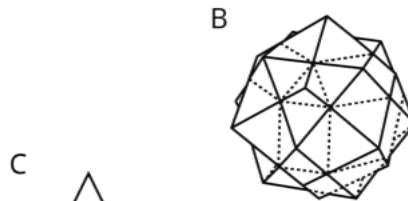
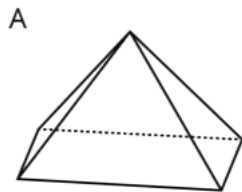
- base (of a prism or pyramid)
- net
- polyhedron
- prism
- pyramid

Lesson 13 Practice Problems

Problem 1

Statement

Select all the polyhedra.



- A. A
- B. B
- C. C
- D. D
- E. E

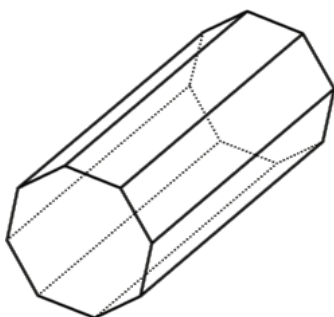
Solution

["A", "B", "D"]

Problem 2

Statement

- a. Is this polyhedron a prism, a pyramid, or neither? Explain how you know.



- b. How many faces, edges, and vertices does it have?

Solution

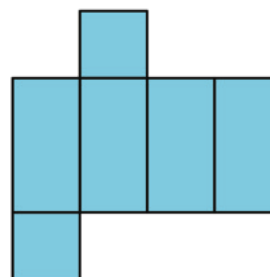
- a. Prism. It has two parallel octagonal bases that match up exactly.
b. 10 faces, 24 edges, 16 vertices

Problem 3

Statement

Tyler said this net cannot be a net for a square prism because not all the faces are squares.

Do you agree with Tyler? Explain your reasoning.



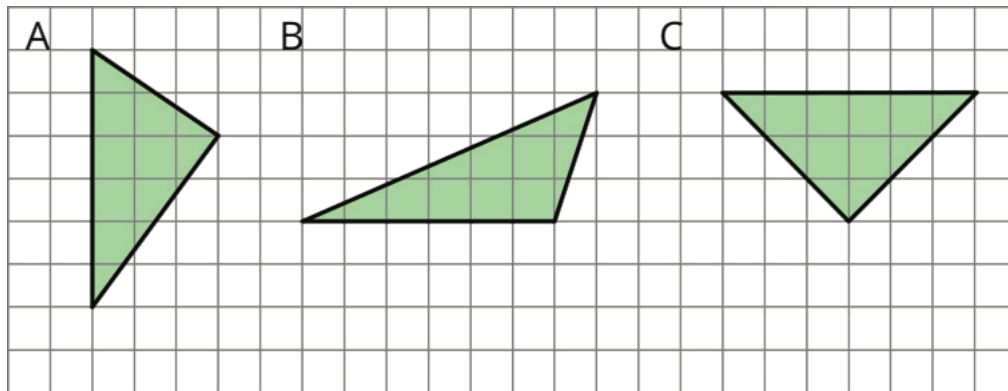
Solution

Disagree. Sample reasoning: A square prism must have two bases that are squares, but the other faces can be non-square rectangles. There are two squares in the net, and the net can be folded into a square prism.

Problem 4

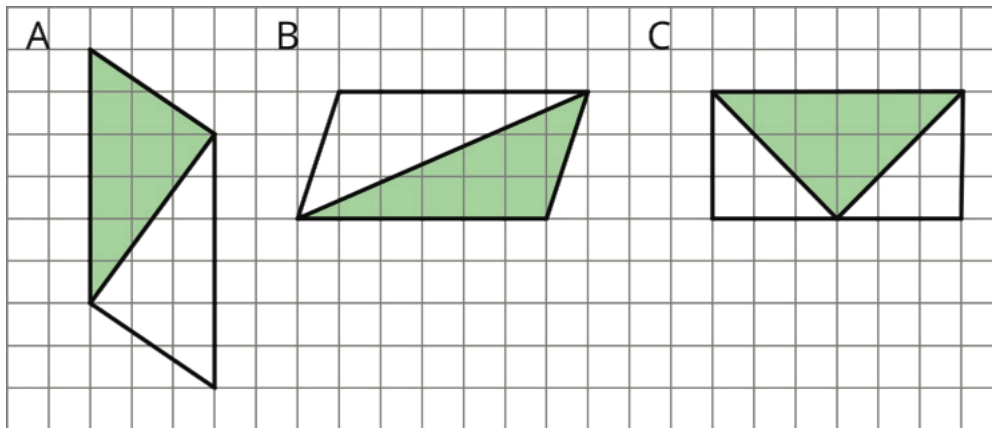
Statement

Explain why each of these triangles has an area of 9 square units.



Solution

Answers vary. Sample explanation: Each triangle is half of a parallelogram with an area of 18 square units (i.e., with a base of 6 units and a height of 3 units), as shown in these diagrams.



(From Unit 1, Lesson 8.)

Problem 5

Statement

- A parallelogram has a base of 12 meters and a height of 1.5 meters. What is its area?
- A triangle has a base of 16 inches and a height of $\frac{1}{8}$ inches. What is its area?

- c. A parallelogram has an area of 28 square feet and a height of 4 feet. What is its base?
- d. A triangle has an area of 32 square millimeters and a base of 8 millimeters. What is its height?

Solution

- a. 18 square meters
- b. 1 square inch
- c. 7 feet
- d. 8 millimeters

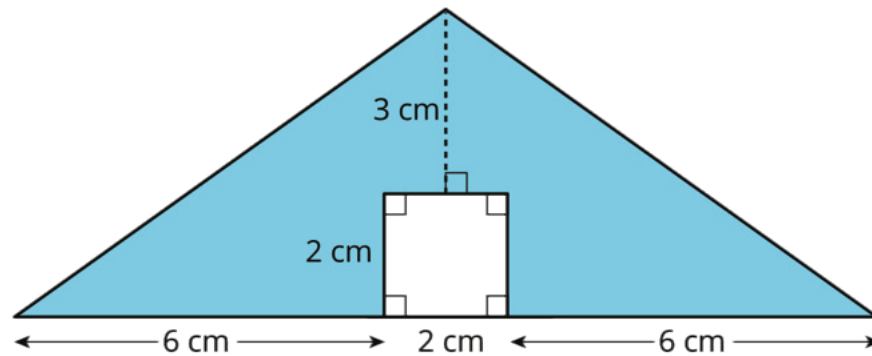
(From Unit 1, Lesson 9.)

Problem 6

Statement

Find the area of the shaded region. Show or explain your reasoning.

E



Solution

31 sq cm. Sample reasoning: The two right triangles can be put together to make a 7 cm-by-5 cm rectangle whose area is 35 sq cm. However, a 2 cm-by-2 cm square is removed. The shaded area is 31 sq cm.

(From Unit 1, Lesson 3.)

Lesson 14: Nets and Surface Area

Goals

- Match polyhedra with their nets and justify (orally) that they match.
- Use a net with gridlines to calculate the surface area of a prism or pyramid and explain (in writing) the solution method.
- Visualize and identify the polyhedron that can be assembled from a given net.

Learning Targets

- I can match polyhedra to their nets and explain how I know.
- When given a net of a prism or a pyramid, I can calculate its surface area.

Lesson Narrative

Previously, students learned about polyhedra, analyzed and defined their features, and investigated their physical representations. Students also identified the polygons that compose a polyhedron; they recognized a net as an arrangement of these polygons and as a two-dimensional representation of a three-dimensional figure.

This lesson extends students' understanding of polyhedra and their nets. They practice visualizing the polyhedra that could be assembled from given nets and use nets to find the **surface area** of polyhedra.

Alignments

Addressing

- 6.G.A.4: Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports
- Think Pair Share

Required Materials

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty

paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Glue or glue sticks

Nets of polyhedra

Tape

Required Preparation

Prepare physical copies of the nets in the warm-up, in case needed to support students with visualization. The Instructional master contains a larger version of these nets.

Make copies of the nets in the Instructional master for the main activity. Prepare one set of 3 nets (A, B, and C) and some glue or tape for each group of 3 students.

Student Learning Goals

Let's use nets to find the surface area of polyhedra.

14.1 Matching Nets

Warm Up: 10 minutes

This warm-up prompts students to match nets to polyhedra. It invites them to think about the polygons that make up a polyhedron and to mentally manipulate nets, which helps develop their visualization skills.

Addressing

- 6.G.A.4

Instructional Routines

- Think Pair Share

Launch

Give students 3 minutes of quiet think time to match nets to polyhedra and then another 2 minutes to discuss their response and reasoning with a partner. Encourage students to use the terminology they learned in prior lessons.

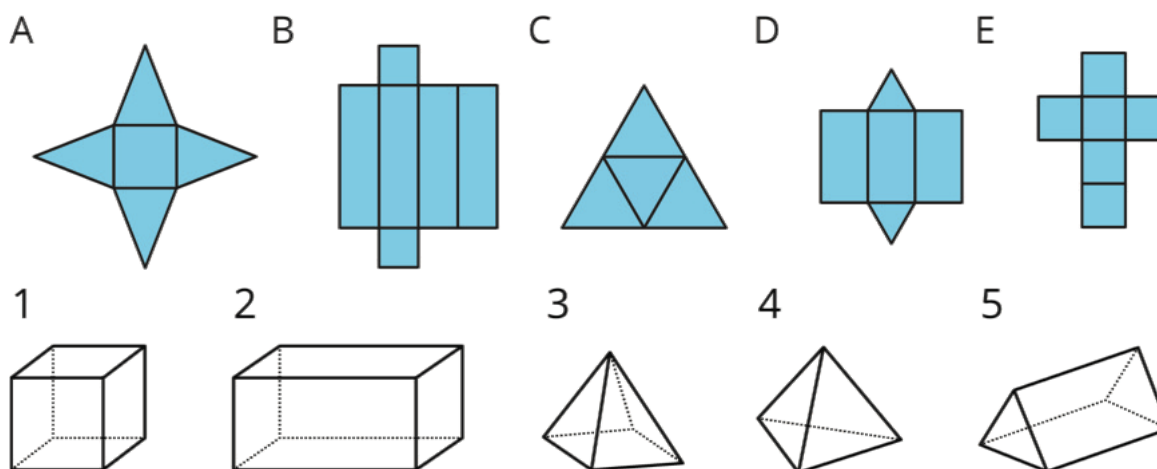
To support students who need more time or help in visualization, prepare physical models of the polyhedra and copies of the nets from the Instructional master. Pre-cut the nets or have scissors available so that students can assemble the nets and test their ideas.

Anticipated Misconceptions

If students have trouble distinguishing between figures A, C, and D, remind them that prisms and pyramids can both contain faces that are triangles. In a pyramid, all triangular faces that are not the base meet at a one vertex and have shared edges. In a prism, there can be a triangular base, but the other faces are quadrilaterals.

Student Task Statement

Each of the nets can be assembled into a polyhedron. Match each net with its corresponding polyhedron, and name the polyhedron. Be prepared to explain how you know the net and polyhedron go together.



Student Response

Net A is a square pyramid (3). It has five faces: one square and four triangles, just like the square pyramid (similar reasoning for each figure).

Net B is a rectangular prism (2).

Net C is a triangular pyramid (4).

Net D is a triangular prism (5).

Net E is a cube or square prism (1).

Activity Synthesis

Invite a few students to share their matching decisions and reasoning with the class. Ask students: "What clues did you use to help you match? How did you check if you were right?" If there is not unanimous agreement on any of the nets, ask students with differing opinions to explain their reasoning. Discuss to come to an agreement.

14.2 Using Nets to Find Surface Area

25 minutes

In this activity, students cut and assemble nets into polyhedra and learn to use nets to find surface area. The presence of a grid supports students in their calculations. It also reinforces the idea of area as the number of unit squares in a region and the connection between area and surface area. Students apply what they learned earlier about areas of triangles and parallelograms to find surface area.

As students make calculations, monitor their processes. Note those who work systematically to find surface area (e.g., by organizing the measurements of each face, calculating the area of each face, and adding the areas together) and those who don't. Encourage students with disorganized or scattered work to take a more systematic approach. Demonstrate strategies such as labeling both the polygons on the net and portions of their work that pertain to those faces.

Also notice students who look for and use structure (MP7), for instance, by grouping certain polygons together and finding the area of the composite shape (e.g., a group of rectangles that have a common side length), or by identifying multiple copies of the same polygon and calculating the area once. Select them to share their work later.

Addressing

- 6.G.A.4

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports

Launch

Arrange students in groups of 3. Give each group one of each net (A, B, and C), tape, and access to their geometry toolkits (especially scissors). Explain to students that they will cut some nets, assemble them into polyhedra, and calculate their surface areas. Remind students that the surface area of a three-dimensional figure is the sum of the areas of all of its faces. Ask students to complete the first question before cutting anything.

Point out that the net has shaded and unshaded polygons. Explain that only the shaded polygons in the nets will show once the net is assembled. The unshaded polygons are “flaps” to make it easier to glue or tape the polygons together. They will get tucked behind the shaded polygons and are not really part of the polyhedron. Tell students that creasing along all of the lines first will make it easier to fold the net up and attach the various polygons together. A straightedge can be very helpful for making the creases.

Tell students that it is easy to miss or double-count the area of a face when finding surface area. Ask them to think carefully about how to record their calculations to ensure that all faces are accounted for, correct measurements are used, and errors are minimized.

When students have completed their calculations, ask them to compare and discuss their work with another student with the same polyhedron.

Support for Students with Disabilities

Representation: Develop Language and Symbols. Eliminate barriers and provide concrete manipulatives to connect symbols to concrete objects or values. Provide students with access to pre-cut nets and the polyhedra.

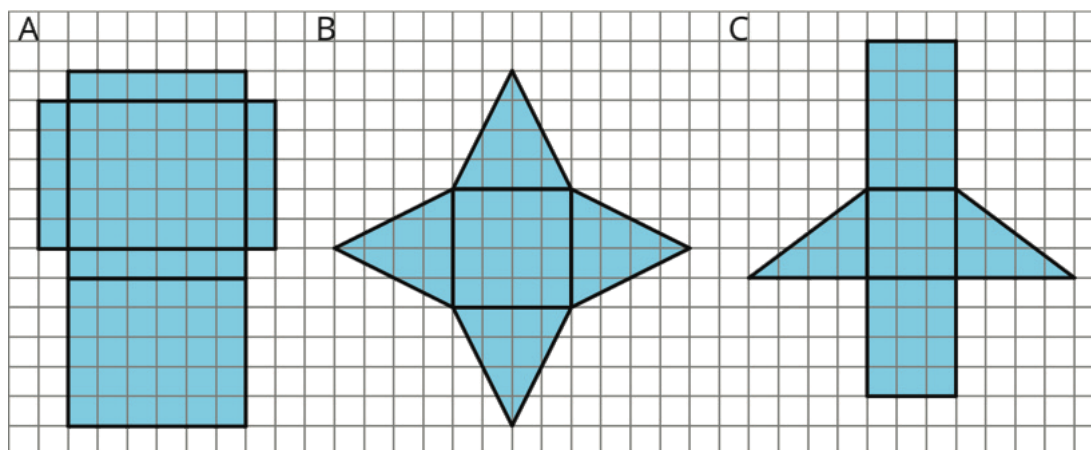
Supports accessibility for: Visual-spatial processing; Fine-motor skills

Anticipated Misconceptions

If students do not identify the specific type of prism or pyramid, remind them that they should also name each figure by the shape of their base.

Student Task Statement

1. Name the polyhedron that each net would form when assembled.



2. Your teacher will give you the nets of three polyhedra. Cut out the nets and assemble the three-dimensional shapes.
3. Find the **surface area** of each polyhedron. Explain your reasoning clearly.

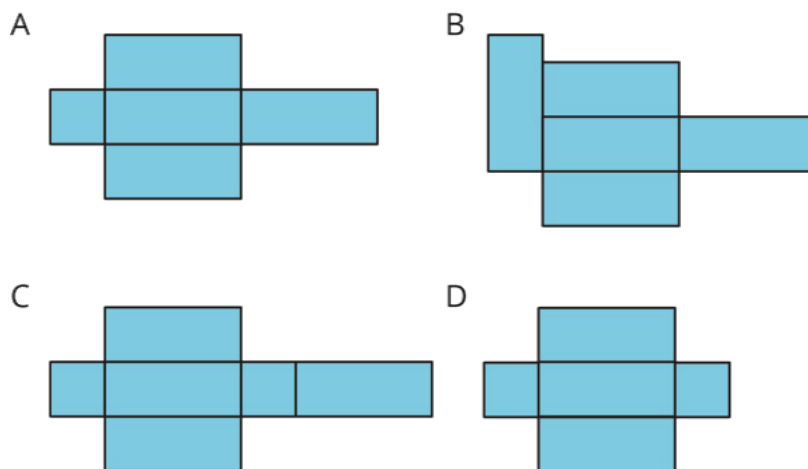
Student Response

1. A: rectangular prism, B: square pyramid, C: triangular prism
2. No answer required.
3. Explanations vary. Sample responses:
 - A: The surface area is 82 square units. $2(6 \cdot 1) + 2(5 \cdot 1) + 2(6 \cdot 5) = 82$
 - B: The surface area is 48 square units. $(4 \cdot 4) + 4\left(\frac{1}{2} \cdot 4 \cdot 4\right) = 48$

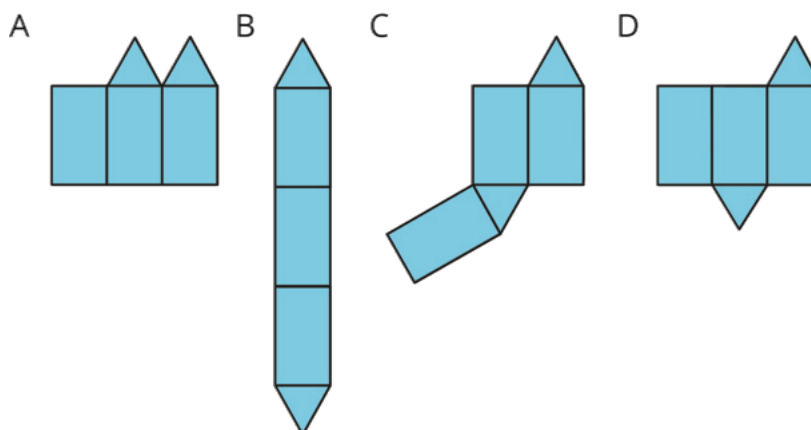
- C: The surface area is 48 square units. $(3 \cdot 5) + (3 \cdot 3) + (3 \cdot 4) + 2(\frac{1}{2} \cdot 3 \cdot 4) = 48$
- C: The combined area of the three rectangular faces is 36 square units. $3 \cdot 12 = 36$. The combined area of the two right triangles is 12 square units. $2(\frac{1}{2} \cdot 3 \cdot 4) = 12$. The surface area is 48 square units because $36 + 12 = 48$.

Are You Ready for More?

1. For each net, decide if it can be assembled into a rectangular prism.



2. For each net, decide if it can be folded into a triangular prism.



Student Response

1. Only C can be folded into a rectangular prism.
2. C and D can be folded into triangular prisms.

Activity Synthesis

For each polyhedron, select at least 2 students with correct calculations but different approaches to share their work, if possible.

For Polyhedron A, select students who took the following approaches, in this sequence:

- Found the area of each rectangle separately
- Found the areas of pairs of identical rectangles (3 pairs total)
- Calculated the area of a group of connected rectangles with the same length or width (e.g., the four rectangles on the net with side length 6 units)

For Polyhedron B, select students who:

- Found the area of each of the 5 polygons separately
- Found the area of the square, rearranged the 4 triangles into 2 parallelograms, and calculated the area of each parallelogram
- Calculated the area of the square and the area of 1 triangle, and multiplying the area of the triangle by 4

For Polyhedron C, select students who:

- Found the area of each of the 5 polygons separately
- Rearranged the 2 right triangles into a rectangle, and then found the area of each rectangle separately
- Calculated the area of each right triangle and doubled it, and found the area of the group of connected rectangles with a width of 4 units

Point out that the reasoning strategies we used earlier in the unit still apply here. Even though we are working with three-dimensional figures, surface area is a two-dimensional measure.

Highlight the benefits of approaching the problems systematically, e.g., by labeling parts, listing measurements and computations in order, etc.

Support for English Language Learners

Speaking: MLR8 Discussion Supports. Display sentence frames to support students when they explain their strategy. For example, "First, I ____ because . . ." or "I noticed ____ so I" Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Design Principle(s): Optimize output (for explanation)

Lesson Synthesis

In this lesson, we matched nets to the polyhedra, assembled polyhedra from nets, and used nets to find **surface area**. Discuss:

- “How do we use a net to find surface area?” (We calculate the area of each polygon on the net and add all the areas.)
- “How is finding surface area using a net different from finding surface area by looking at a picture of a polyhedron—as we had done with the filing cabinet, or by studying the actual object—as we had done with the snap cubes?” (A net allows us to see all the faces of a polyhedron at once. When working from a picture or drawing, we need to visualize the hidden faces. Working with an actual polyhedron could help, but again we are not looking at all the faces at once; we have to rotate the object and might miss or double-count a face.)
- “When using a net, how do we keep track of your calculations or make sure all faces are accounted for?” (We can label all the polygons and the calculations.)
- “Are there ways to simplify the calculations? Or is it best to find the area of each polygon one at a time?” (Sometimes we can simplify the process by combining polygons and finding the area of the combined region—e.g., a group of rectangles with the same side length. If there are several polygons that are identical, we can find the area of one polygon and multiply it by the number of identical polygons in the net.)

14.3 Unfolded

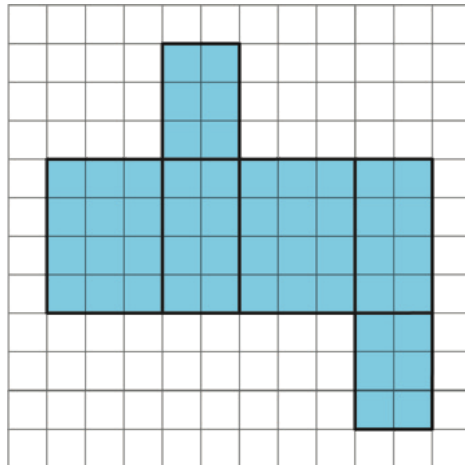
Cool Down: 5 minutes

Addressing

- 6.G.A.4

Student Task Statement

1. What kind of polyhedron can be assembled from this net?



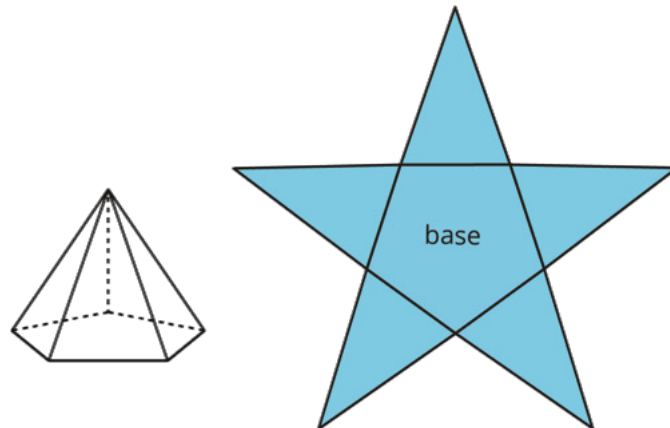
2. Find the surface area (in square units) of the polyhedron. Show your reasoning.

Student Response

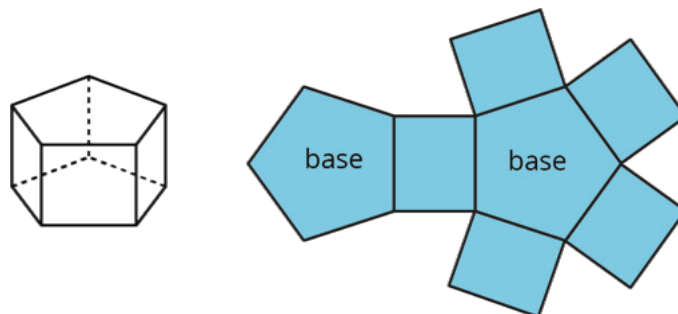
1. It would assemble into a rectangular prism.
2. The surface area would be 52 square units. $2(3 \cdot 4) + 2(2 \cdot 4) + 2(2 \cdot 3) = 52$

Student Lesson Summary

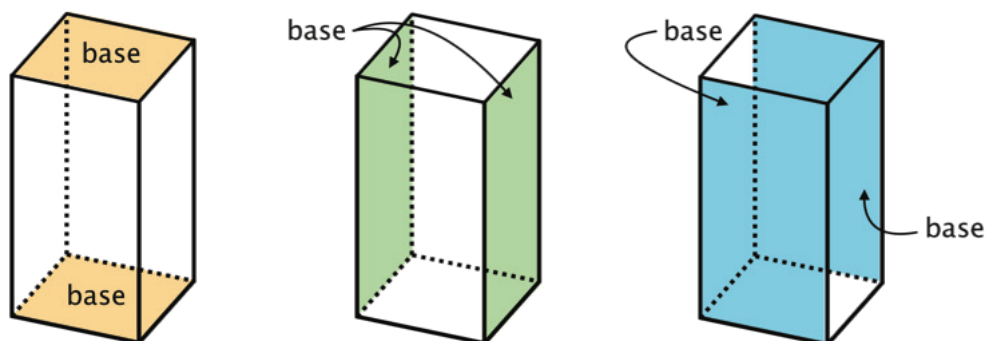
A net of a *pyramid* has one polygon that is the base. The rest of the polygons are triangles. A pentagonal pyramid and its net are shown here.



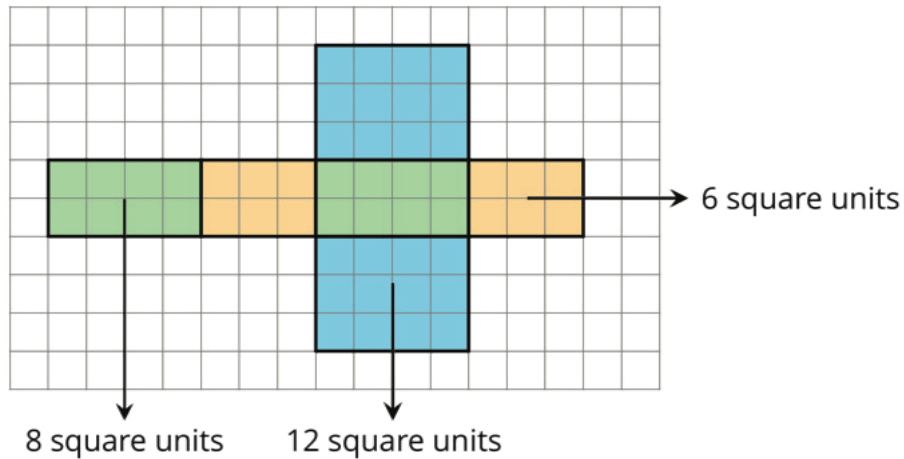
A net of a *prism* has two copies of the polygon that is the base. The rest of the polygons are rectangles. A pentagonal prism and its net are shown here.



In a rectangular prism, there are three pairs of parallel and identical rectangles. Any pair of these identical rectangles can be the bases.



Because a net shows all the faces of a polyhedron, we can use it to find its surface area. For instance, the net of a rectangular prism shows three pairs of rectangles: 4 units by 2 units, 3 units by 2 units, and 4 units by 3 units.



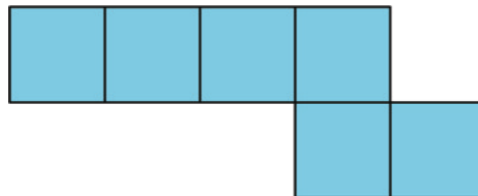
The **surface area** of the rectangular prism is 52 square units because $8 + 8 + 6 + 6 + 12 + 12 = 52$.

Lesson 14 Practice Problems

Problem 1

Statement

Can this net be assembled into a cube? Explain how you know. Label parts of the net with letters or numbers if it helps your explanation.



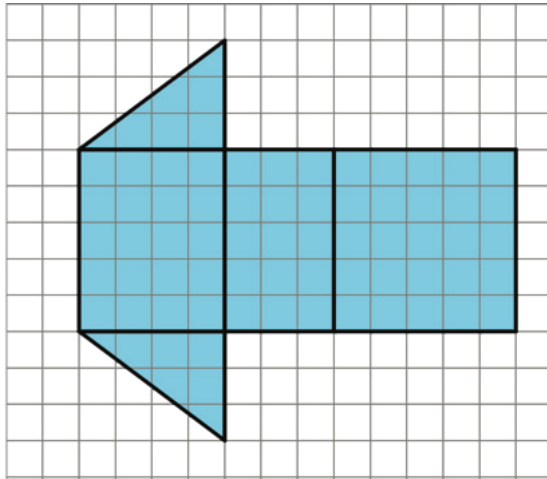
Solution

No. Sample explanation: The four squares placed side by side can only be folded in one way to meet up with one another, making a cube without a top and bottom. One of the remaining two squares can be folded to make the top or bottom, but the other one cannot be used.

Problem 2

Statement

- What polyhedron can be assembled from this net? Explain how you know.



- b. Find the surface area of this polyhedron. Show your reasoning.

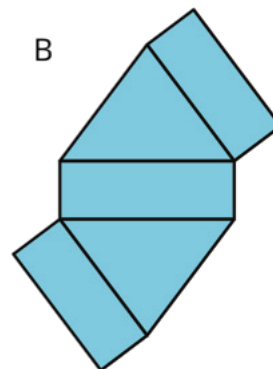
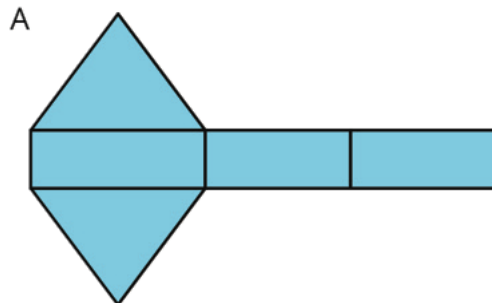
Solution

- a. A triangular prism. Sample explanation: There are two identical triangles that are the bases. The rest of the faces are rectangles.
- b. 72 square units. Sample reasoning: The area of the three rectangles are 20, 15, and 25 square units. The area of the two triangles are $2\left(\frac{1}{2} \cdot 4 \cdot 3\right)$ or 12 square units.
 $20 + 15 + 25 + 2(6) = 72$.

Problem 3

Statement

Here are two nets. Mai said that both nets can be assembled into the same triangular prism. Do you agree? Explain or show your reasoning.



Solution

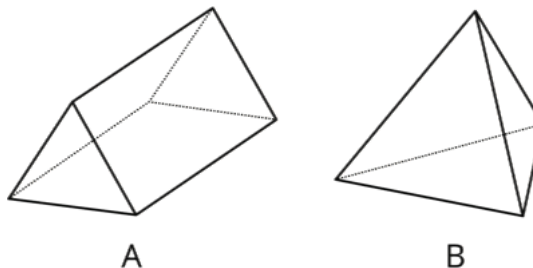
Agree. Sample reasoning: Both nets are composed of the same set of polygons. The positions of the one rectangular face are different, but when assembled, that face will meet the same edge of three other polygons.

Problem 4

Statement

Here are two three-dimensional figures.

Tell whether each of the following statements describes Figure A, Figure B, both, or neither.



- This figure is a polyhedron.
- This figure has triangular faces.
- There are more vertices than edges in this figure.
- This figure has rectangular faces.
- This figure is a pyramid.
- There is exactly one face that can be the base for this figure.
- The base of this figure is a triangle.
- This figure has two identical and parallel faces that can be the base.

Solution

- Both
- Both
- Neither
- Figure A
- Figure B
- Neither
- Both
- Figure A

(From Unit 1, Lesson 13.)

Problem 5

Statement

Select **all** units that can be used for surface area.

- A. square meters
- B. feet
- C. centimeters
- D. cubic inches
- E. square inches
- F. square feet

Solution

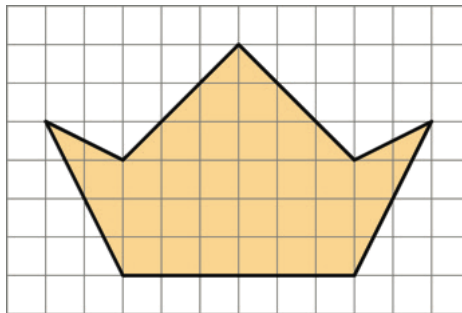
["A", "E", "F"]

(From Unit 1, Lesson 12.)

Problem 6

Statement

Find the area of this polygon. Show your reasoning.



Solution

33 square units. Reasoning varies.

(From Unit 1, Lesson 11.)

Lesson 15: More Nets, More Surface Area

Goals

- Draw and assemble a net for the prism or pyramid shown in a given drawing.
- Interpret (using words and other representations) two-dimensional representations of prisms and pyramids.
- Use a net without gridlines to calculate the surface area of a prism or pyramid and explain (in writing) the solution method.

Learning Targets

- I can calculate the surface area of prisms and pyramids.
- I can draw the nets of prisms and pyramids.

Lesson Narrative

This lesson further develops students' ability to visualize the relationship between nets and polyhedra and their capacity to reason about surface area.

Previously, students started with nets and visualized the polyhedra that could be assembled from the nets. Here they go in the other direction—from polyhedra to nets. They practice mentally unfolding three-dimensional shapes, drawing two-dimensional nets, and using them to calculate surface area. Students also have a chance to compare and contrast surface area and volume as measures of two distinct attributes of a three-dimensional figure.

Alignments

Building On

- 5.MD.C.5: Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

Addressing

- 6.G.A.2: Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
- 6.G.A.4: Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Instructional Routines

- MLR7: Compare and Connect

- MLR8: Discussion Supports
- Notice and Wonder
- Think Pair Share

Required Materials

Demonstration nets with and without flaps Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty

paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Glue or glue sticks
Pre-printed slips, cut from copies of the Instructional master
Tape

Required Preparation

Copy and cut the Instructional master for the Building Prisms and Pyramids activity. Make one copy for every 9 students, so that each student gets one drawing of a polyhedron. Consider assignments of polyhedra in advance.

Student Learning Goals

Let's draw nets and find the surface area of polyhedra.

15.1 Notice and Wonder: Wrapping Paper

Warm Up: 5 minutes

This warm-up prompts students to think about a prism and its measurements in context and to consider potential questions that could be asked and answered.

Given their recent work, students are likely to notice and wonder about surface area, nets, and the missing height of the box. Students may also wonder about the volume of the box, given their geometric work in grade 5.

When prompted about how to find the surface area or the volume of the box, students will likely ask about the missing measurement. This is an opportunity for them to practice making a reasonable estimate.

Addressing

- 6.G.A.2
- 6.G.A.4

Instructional Routines

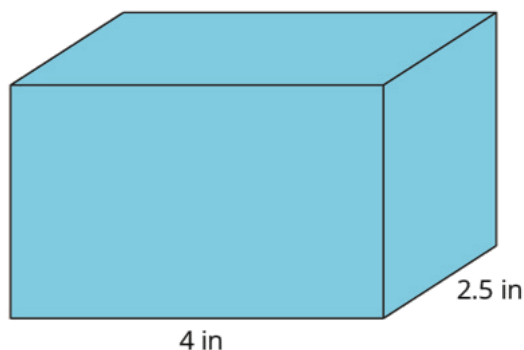
- Notice and Wonder

Launch

Arrange students in groups of 2. Give students a minute of quiet time to observe the image. Ask students to be prepared to share at least one thing they notice and one thing they wonder about the picture. Ask them to give a signal when they have noticed or wondered about something.

Student Task Statement

Kiran is wrapping this box of sports cards as a present for a friend.



What do you notice? What do you wonder?

Student Response

Answers vary. Possible responses:

- Notice: the given side lengths of the box, the height looks like the same length as the width, the box being covered in wrapping paper, the box being a rectangular prism, the area of the top and bottom faces being 10 square inches each
- Wonder: the missing side length, how many cards are in the box, the volume of the box, the surface area of the box, how much wrapping paper it will take to cover the box, how would you count the overlapped paper on the edges that are wrapped

Activity Synthesis

Invite students to share their observations and questions. Record the responses for all to see. If no students wonder about the surface area, the amount of wrapping paper needed, or the volume of the box, bring these questions up.

Tell students to choose either a question about surface area or one about volume and give them a minute to discuss with a partner *how* they would find the answer to the question. If students suggest that it cannot be done because of missing information, ask them to estimate the missing information.

Select a couple of students to share how they would find the surface area or the volume of the given box. After each response, poll the class on whether they agree or disagree.

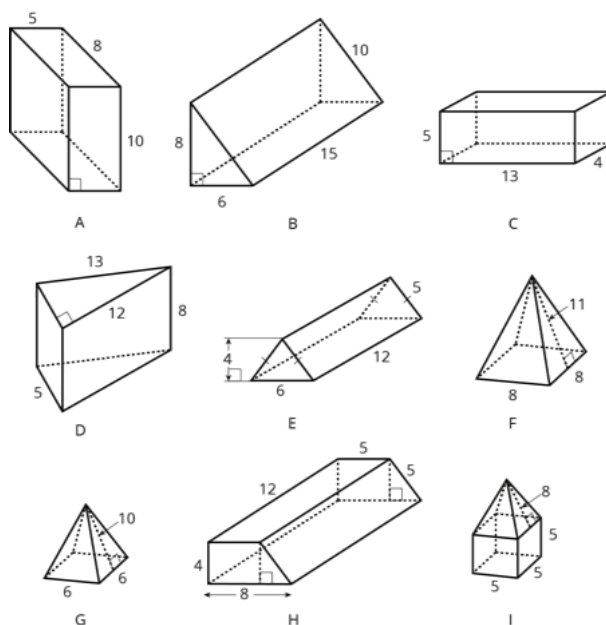
15.2 Building Prisms and Pyramids

30 minutes

Previously, students used a given net of a polyhedron to find its surface area. Here they use a given polyhedron to draw a net and then calculate its surface area.

Use the provided polyhedra to differentiate the work for students with varying degrees of visualization skills. Rectangular prisms (A and C), triangular prisms (B and D), and square pyramids (F and G) can be managed by most students. Triangular Prism E requires a little more interpretive work (i.e., the measurements of some sides may not be immediately apparent to students). Trapezoidal Prism H and Polyhedron I (a composite of a cube and a square pyramid) require additional interpretation and reasoning.

As students work, remind them of the organizational strategies discussed in previous lessons, i.e., labeling polygons, showing measurements on the net, etc.



Addressing

- 6.G.A.4

Instructional Routines

- MLR7: Compare and Connect

Launch

Arrange students in groups of 2–3. Give each student in the group a different polyhedron from the Instructional master and access to their geometry toolkits. Students need graph paper and a straightedge from their toolkits.

Explain to students that they will draw a net, find its surface area, and have their work reviewed by a peer. Give students 4–5 minutes of quiet time to draw their net on graph paper and then 2–3 minutes to share their net with their group and get feedback. When the group is sure that each net makes sense and all polygons of each polyhedron are accounted for, students can proceed and use the net to help calculate surface area.

If time permits, prompt students to cut and assemble their net into a polyhedron. Demonstrate how to add flaps to their net to accommodate gluing or taping. There should be as many flaps as there are edges in the polyhedron. (Remind students that this is different than the number of edges in the polygons of the net.)

Support for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge about calculating surface area. Some students may benefit from watching a physical demonstration of how to draw the net for a sample prism. Invite students to engage in the process by offering suggested directions as you demonstrate.

Supports accessibility for: Visual-spatial processing; Organization

Anticipated Misconceptions

Students may know what polygons make up the net of a polyhedron but arrange them incorrectly on the net (i.e., when cut and assembled the faces overlap instead of meeting at shared edges, or the faces are oriented incorrectly or are in the wrong places). Suggest that students label some faces of the polyhedron drawing and transfer the adjacencies they see to the net. If needed, demonstrate the reasoning (e.g., “Face 1 and face 5 both share the edge that is 7 units long, so I can draw them as two attached rectangles sharing a side that is 7-unit long.”)

It may not occur to students to draw each face of the polyhedron to scale. Remind them to use the grid squares on their graph paper as units of measurement.

If a net is inaccurate, this becomes more evident when it is being folded. This may help students see which parts need to be adjusted and decide the best locations for the flaps. Reassure students that a few drafts of a net may be necessary before all the details are worked out, and encourage them to persevere (MP1).

Student Task Statement

Your teacher will give you a drawing of a polyhedron. You will draw its net and calculate its surface area.

1. What polyhedron do you have?
2. Study your polyhedron. Then, draw its net on graph paper. Use the side length of a grid square as the unit.

3. Label each polygon on the net with a name or number.
4. Find the surface area of your polyhedron. Show your thinking in an organized manner so that it can be followed by others.

Student Response

1. Answers vary depending on polyhedron received. A and C are rectangular prisms. B, D and E are triangular prisms. F and G are square pyramids. H is a trapezoidal prism. I is a composite of a cube and a square pyramid.
2. Net drawings vary. A and C should have 6 rectangles. B, D, and E should have 5 polygons: 2 right triangles and 3 rectangles. F and G should have 5 polygons: 1 square and 4 triangles. H should have 6 polygons: 2 trapezoids and 4 rectangles. I should have 9 polygons: 5 squares and 4 triangles.
3. Answers vary.
4. Answers vary.
 - A: 340 square units. $2(5 \cdot 8) + 2(5 \cdot 10) + 2(8 \cdot 10) = 340$
 - B: 408 square units. $2(\frac{1}{2} \cdot 6 \cdot 8) + (6 \cdot 15) + (8 \cdot 15) + (10 \cdot 15) = 408$
 - C: 274 square units. $2(13 \cdot 4) + 2(13 \cdot 5) + 2(4 \cdot 5) = 274$
 - D: 300 square units. $2(\frac{1}{2} \cdot 5 \cdot 12) + (5 \cdot 8) + (12 \cdot 8) + (13 \cdot 8) = 300$
 - E: 216 square units. $2(\frac{1}{2} \cdot 6 \cdot 4) + (6 \cdot 12) + 2(5 \cdot 12) = 216$
 - F: 240 square units. $4(\frac{1}{2} \cdot 8 \cdot 11) + (8 \cdot 8) = 240$
 - G: 156 square units. $4(\frac{1}{2} \cdot 6 \cdot 10) + (6 \cdot 6) = 156$
 - H: 316 square units. The trapezoidal base can be decomposed into a 5-by-4 rectangle and a right triangle with a base of 3 units and a height of 4.
 $2(5 \cdot 4) + 2(\frac{1}{2} \cdot 3 \cdot 4) + (8 \cdot 12) + 2(5 \cdot 12) + (4 \cdot 12) = 316$
 - I: 205 square units. $5(5 \cdot 5) + 4(\frac{1}{2} \cdot 5 \cdot 8) = 205$

Activity Synthesis

Ask students who finish their calculation to find another person in the class with the same polyhedron and discuss the following questions (displayed for all to see):

- Do your calculations match? Should they?
- Do your nets result in the same polyhedra? Should they?
- Do your models match the picture you were given? Why or why not?

If time is limited, consider having the answer key posted somewhere in the classroom so students could quickly check their surface area calculations.

Reconvene briefly for a whole-class discussion. Invite students to reflect on the process of drawing a net and finding surface area based on a picture of a polyhedron. Ask questions such as:

- How did you know that your net show all the faces of your polyhedron?
- How did you know where to put each polygon or how to arrange all polygons so that, if folded, they can be assembled into the polyhedron in the drawing?
- How did the net help you find surface area?

Support for English Language Learners

Representing, Conversing: MLR7 Compare and Connect. Use this routine to help students consider their audience when preparing a visual display of their work. Ask students to prepare a visual display that shows their net drawings and surface area calculations. Students should consider how to display their calculations so that another student can interpret them. Some students may wish to add notes or details to their drawings to help communicate their thinking. When students find another person in the class with the same polyhedron, provide 2–3 minutes of quiet think time for students to read and interpret each other's drawings before they discuss the questions on display.

Design Principle(s): Cultivate conversation; Maximize meta-awareness

15.3 Comparing Boxes

Optional: 15 minutes

In this activity, students compare the surface areas and volumes of three rectangular prisms given nets that are not on a grid. To do this, they need to be able to visualize the three-dimensional forms that the two-dimensional nets would take when folded.

In grade 5, students had learned to distinguish area and volume as measuring different attributes. This activity clarifies and reinforces that distinction.

Building On

- 5.MD.C.5

Addressing

- 6.G.A.4

Instructional Routines

- MLR8: Discussion Supports

- Think Pair Share

Launch

Keep students in the same groups of 2–3. Tell students that this activity involves working with both volume and surface area. To refresh students' understanding of volume from grade 5, ask students:

- “When we find the volume of a prism, what are we measuring?”
- “How is volume different than surface area?”
- “How might we find the volume of a rectangular prism?”

Reiterate that volume measures the number of unit cubes that can be packed into a three-dimensional shape and that we can find the number of unit cubes in a rectangular prism by multiplying the side lengths of a prism.

Give students 1–2 minutes to read the task statement and questions. Ask them to think about how they might go about answering each question and to be prepared to share their ideas. Give students a minute to discuss their ideas with their group. Then, ask groups to collaborate: each member should perform the calculations for one prism (A, B, or C). Give students 5–7 minutes of quiet time to find the surface area and volume for their prism and then additional time to compare their results and answer the questions.

Support for Students with Disabilities

Action and Expression: Internalize Executive Functions. Provide students with a graphic organizer for recording measurements and calculations of surface areas and volumes of the boxes.

Supports accessibility for: Language; Organization

Anticipated Misconceptions

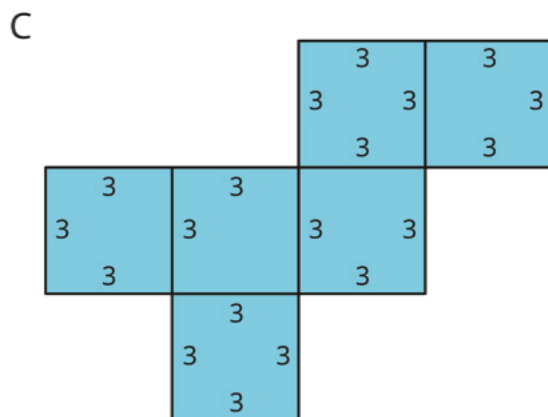
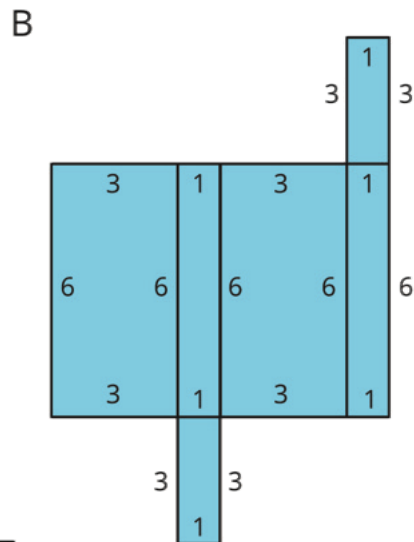
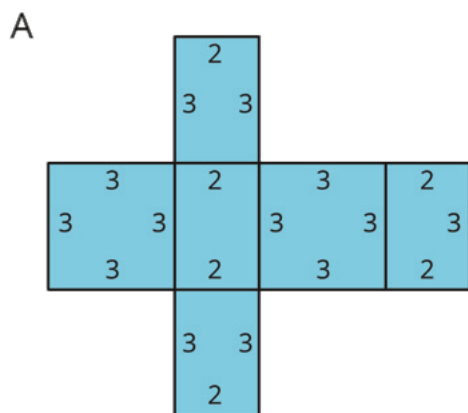
Students should have little trouble finding areas of rectangles but may have trouble keeping track of pairs of measurements to multiply and end up making calculation errors. Suggest that they label each polygon in the net and the corresponding written work and double-check their calculations to minimize such errors.

If students struggle to find the volume of their prism using information on a net, suggest that they sketch the prism that can be assembled from the net and label the edges of the prism.

Students may need a reminder that area is measured in square units and volume is measured in cubic units.

Student Task Statement

Here are the nets of three cardboard boxes that are all rectangular prisms. The boxes will be packed with 1-centimeter cubes. All lengths are in centimeters.



1. Compare the surface areas of the boxes. Which box will use the least cardboard? Show your reasoning.
2. Now compare the volumes of these boxes in cubic centimeters. Which box will hold the most 1-centimeter cubes? Show your reasoning.

Student Response

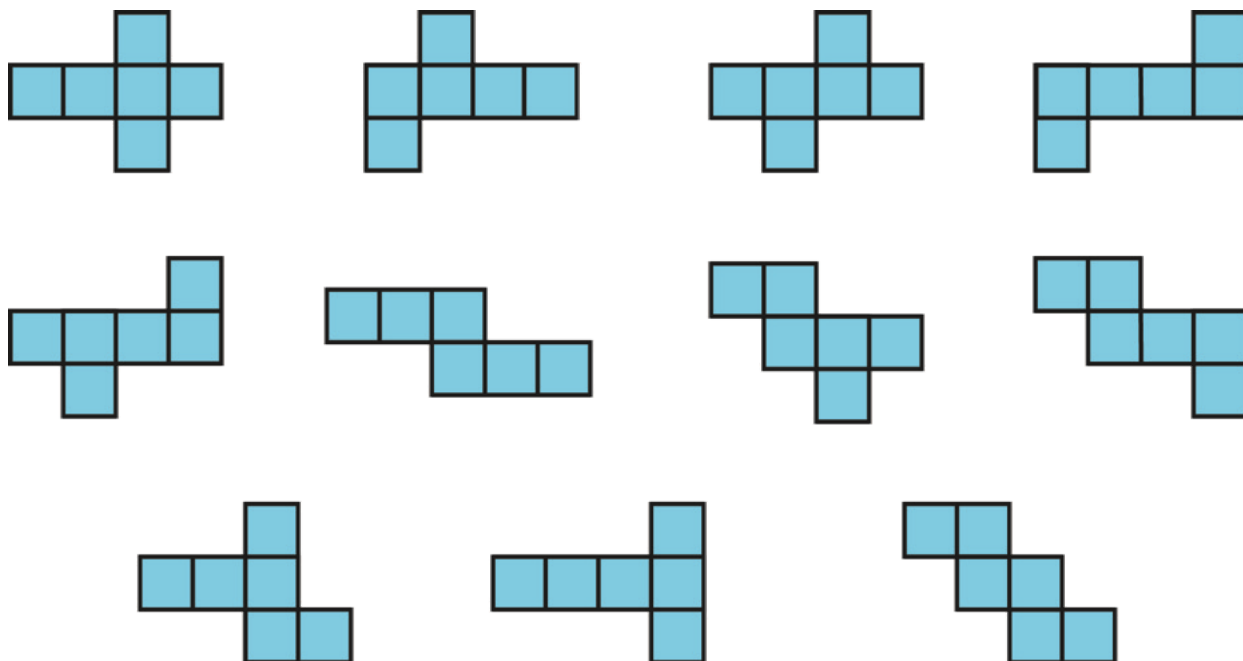
1. The surface area of A is 42 square centimeters. A: $4(2 \cdot 3) + 2(3 \cdot 3) = 42$. The surface area of B and C is 54 square centimeters. B: $2(3 \cdot 6) + 2(3 \cdot 1) + 2(6 \cdot 1) = 54$. C: $6(3 \cdot 3) = 54$. Box A uses the least cardboard. Boxes B and C require the same amount of cardboard, both more cardboard than A.
2. The volume of A and B is 18 cubic centimeters. A: $3 \cdot 2 \cdot 3 = 18$. B: $6 \cdot 1 \cdot 3 = 18$. The volume of C is 27 cubic centimeter. C: $3 \cdot 3 \cdot 3 = 27$. Box C fits the most 1-centimeter cubes. A and B fit the same number of cubes, but fewer than C.

Are You Ready for More?

Figure C shows a net of a cube. Draw a different net of a cube. Draw another one. And then another one. How many different nets can be drawn and assembled into a cube?

Student Response

There are 11 different nets for a cube. Any other net would be congruent to one of these.



Activity Synthesis

Select a few students to share the surface area and volume of each prism. After each person shares, poll those who worked on the same prism for agreement or disagreement. Record the results on the board.

Invite students to share a few quick observations about the relationship between the surface areas and volumes for these three prisms, or between the amounts of material needed to build the boxes and the number of cubes that they can contain. Discuss questions such as:

- “If these prisms are boxes, which prism—B or C—would take more material to build? Which can fit more unit cubes?” (B and C would likely take the same amount of material to build since their surface areas are the same. C has a greater volume than B, so it can fit more unit cubes.)
- “Which prism—A or B—would take more material to build? Which can fit more unit cubes?” (A and B can fit the same number of unit cubes but, B would require more material to build.)
- “If two prisms have the same surface area, would they also have the same volume? How do you know?” (No, prisms A, B, and C are examples of how two figures with the same volume may not have the same surface area, and vice versa.)

Students will gain more insights into these ideas as they explore squares, cubes, and exponents in upcoming lessons. If students could benefit from additional work on distinguishing area and volumes as different measures, do the optional lesson *Distinguishing Between Surface Area and Volume*.

Support for English Language Learners

Representing, Speaking: MLR8 Discussion Supports. Use this routine to support the use of mathematical language when comparing surface area and volume. Give students 3–4 minutes to write a response to the following prompt: “If two prisms have the same surface area, their volume will *always/sometimes/never* be the same because . . .” Invite students to discuss their responses with a partner before selecting 1–2 students to share with the whole class. Listen for and call students’ attention to how the use of examples and counterexamples can help justify their reasoning.

Design Principle(s): Optimize output (for justification); Support sense-making

Lesson Synthesis

To highlight some key points from the lesson, display a picture of a prism or a pyramid and a drawing of its net. Discuss these questions:

- “Can you find the surface area of a simple prism or pyramid from a picture, if all the necessary measurements are given?”
- “Can you find the surface area from a net, if all the measurements are given?”
- “Which might be more helpful for calculating surface area—a picture of a polyhedron or a net?” (If the polyhedron is simple—e.g., a cube, a square pyramid, etc.—and does not involve hidden faces with different measurements or require a lot of visualizing, either a picture or a net can work. Otherwise, a net may be more helpful because we can see all of the faces at once and can find the area of each polygon more easily. A net may also help us keep track of our calculations and notice missing or extra areas.)

15.4 Surface Area of a Triangular Prism

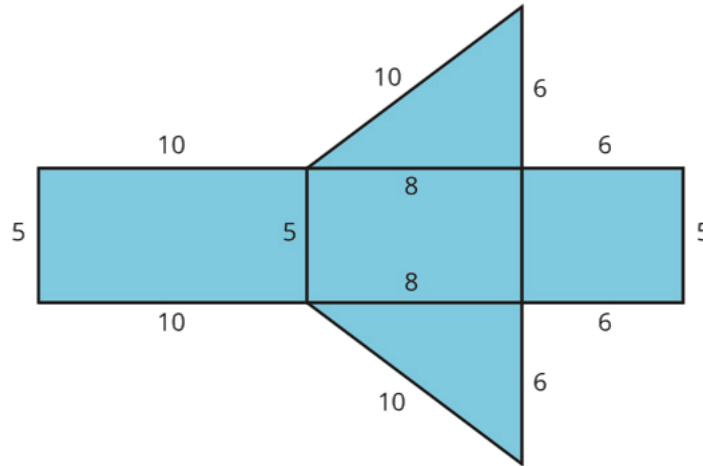
Cool Down: 5 minutes

Addressing

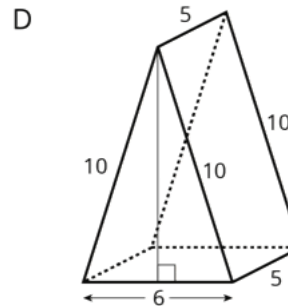
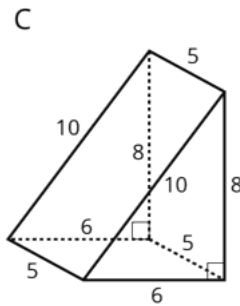
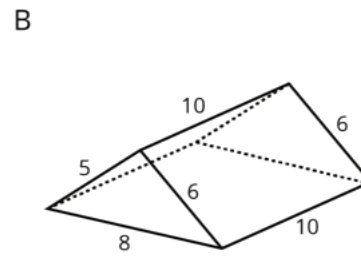
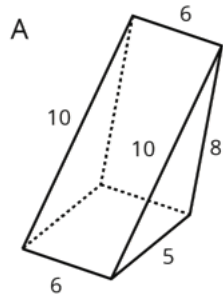
- 6.G.A.4

Student Task Statement

1. In this net, the two triangles are right triangles. All quadrilaterals are rectangles. What is its surface area in square units? Show your reasoning.



2. If the net is assembled, which of the following polyhedra would it make?



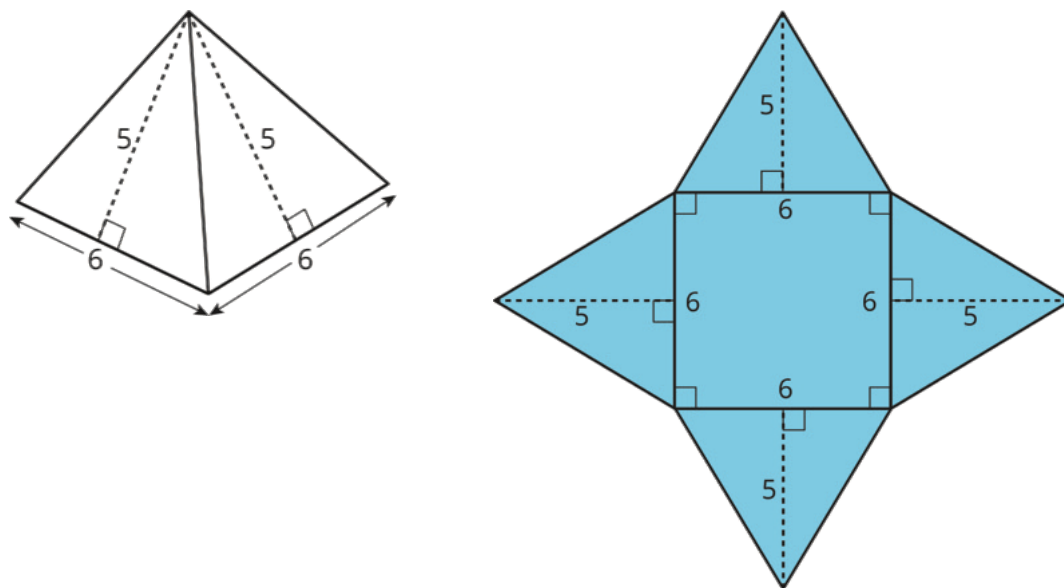
Student Response

1. The surface area is 168 square units. Explanations vary. Sample response: There are two triangular faces with area of 24 square units each. $\frac{1}{2} \cdot 6 \cdot 8 = 24$. There is a rectangular face with area of 50 square units. $10 \cdot 5 = 50$. There is one rectangular face with area of 40 square units. $5 \cdot 8 = 40$. There is one rectangular face with area $5 \cdot 6 = 30$ square units.
 $2 \cdot 24 + 50 + 40 + 30 = 168$

2. Prism C

Student Lesson Summary

The surface area of a polyhedron is the sum of the areas of all of the faces. Because a net shows us all faces of a polyhedron at once, it can help us find the surface area. We can find the areas of all polygons in the net and add them.



A square pyramid has a square and four triangles for its faces. Its surface area is the sum of the areas of the square base and the four triangular faces:

$$(6 \cdot 6) + 4 \cdot \left(\frac{1}{2} \cdot 5 \cdot 6\right) = 96$$

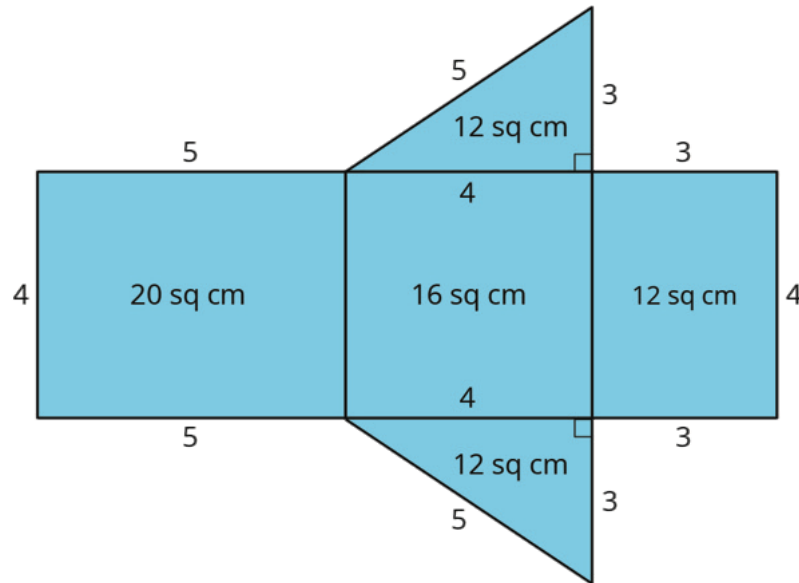
The surface area of this square pyramid is 96 square units.

Lesson 15 Practice Problems

Problem 1

Statement

Jada drew a net for a polyhedron and calculated its surface area.



- What polyhedron can be assembled from this net?
- Jada made some mistakes in her area calculation. What were the mistakes?
- Find the surface area of the polyhedron. Show your reasoning.

Solution

- Triangular prism
- She calculated the areas of the two triangular faces incorrectly. The right triangles have a base of 4 cm and a height of 3 cm, so the area of each should be $\frac{1}{2} \cdot 4 \cdot 3$ or 6 sq cm. Jada wrote "12 sq cm" for the area of each triangle.
- 60 sq cm. The triangular faces should be 6 sq cm each, so the surface area is $20 + 16 + 12 + 6 + 6$, or 60.

Problem 2

Statement

A cereal box is 8 inches by 2 inches by 12 inches. What is its surface area? Show your reasoning. If you get stuck, consider drawing a sketch of the box or its net and labeling the edges with their measurements.

Solution

272 square inches. Sample reasoning:

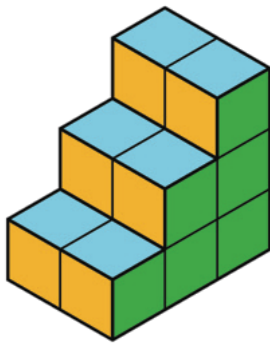
- The top and bottom faces are 2 inches by 8 inches each, so their combined area is $2(2 \cdot 8)$ or 32 square inches.

- The front and back faces are 8 inches by 12 inches each, so their combined area is $2(8 \cdot 12)$ or 192 square inches.
- The side faces are 2 inches by 12 inches each, so their combined area is $2(2 \cdot 12)$ or 48 square inches.
- The surface area is $32 + 192 + 48$ or 272 square inches.

Problem 3

Statement

Twelve cubes are stacked to make this figure.



- What is its surface area?
- How would the surface area change if the top two cubes are removed?

Solution

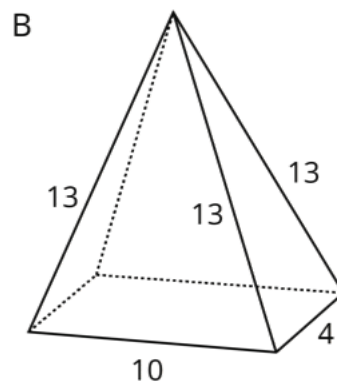
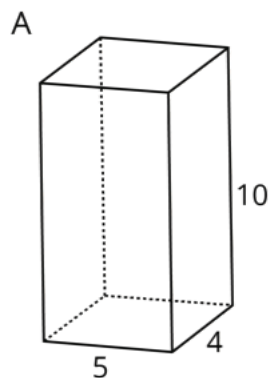
- 36 square units
- The surface area would decrease by 6 square units.

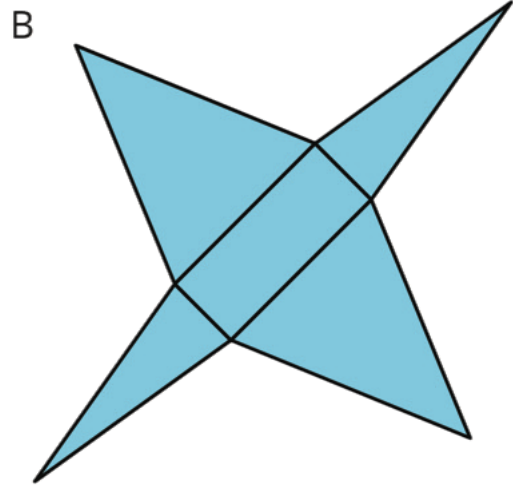
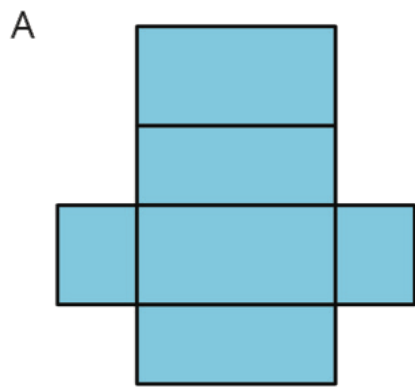
(From Unit 1, Lesson 12.)

Problem 4

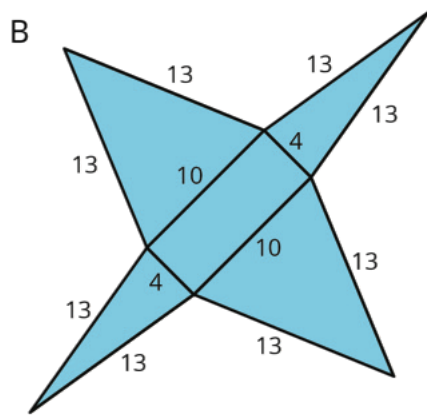
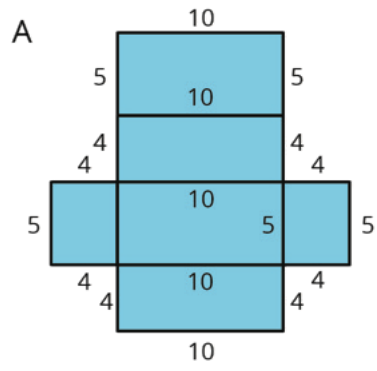
Statement

Here are two polyhedra and their nets. Label all edges in the net with the correct lengths.





Solution

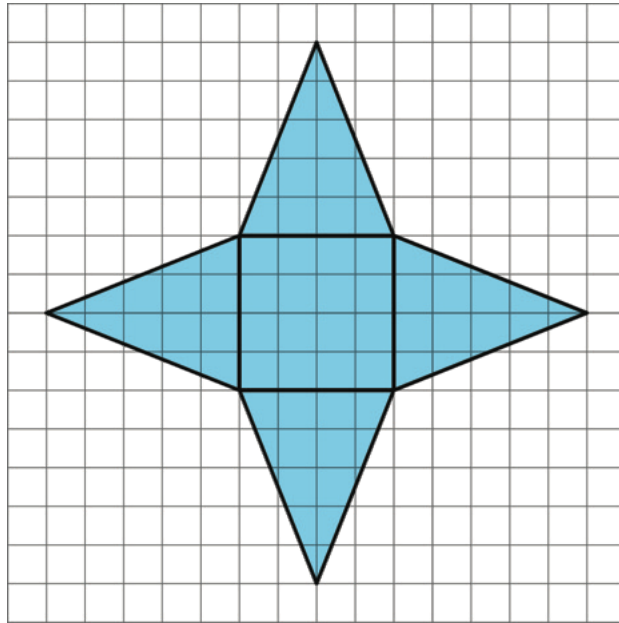


Problem 5



Statement

- a. What three-dimensional figure can be assembled from the net?



b. What is the surface area of the figure? (One grid square is 1 square unit.)

Solution

- a. Square pyramid
- b. 56 square units. The area of the base is 16 square units. Each triangular face has a base of 4 units and a height of 5 units. This means each triangular face has an area of 10 square units. The total surface area is 56 square units, because $16 + 10 + 10 + 10 + 10 = 56$.

(From Unit 1, Lesson 14.)

Lesson 16: Distinguishing Between Surface Area and Volume

Goals

- Comprehend that surface area and volume are two different attributes of three-dimensional objects and are measured in different units.
- Describe (orally and in writing) shapes built out of cubes, including observations about their surface area and volume.
- Determine the surface area and volume of shapes made out of cubes.

Learning Targets

- I can explain how it is possible for two polyhedra to have the same surface area but different volumes, or to have different surface areas but the same volume.
- I know how one-, two-, and three-dimensional measurements and units are different.

Lesson Narrative

In this optional lesson, students distinguish among measures of one-, two-, and three-dimensional attributes and take a closer look at the distinction between surface area and volume (building on students' work in earlier grades). Use this lesson to reinforce the idea that length is a one-dimensional attribute of geometric figures, surface area is a two-dimensional attribute, and volume is a three-dimensional attribute.

By building polyhedra, drawing representations of them, and calculating both surface area and volume, students see that different three-dimensional figures can have the same volume but different surface areas, and vice versa. This is analogous to the fact that two-dimensional figures can have the same area but different perimeters, and vice versa. Students must attend to units of measure throughout the lesson.

Note: Students will need to bring in a personal collection of 10–50 small objects ahead of time for the first lesson of the next unit. Examples include rocks, seashells, trading cards, or coins.

Alignments

Building On

- 3.MD.C.5: Recognize area as an attribute of plane figures and understand concepts of area measurement.
- 4.MD.A.1: Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length

of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...

- 5.MD.C: Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.
- 5.MD.C.3.b: A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.
- 5.MD.C.4: Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.
- 5.MD.C.5.a: Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

Addressing

- 6.G.A.4: Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports
- Notice and Wonder
- Think Pair Share

Required Materials

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Snap cubes

Sticky notes

Required Preparation

- Prepare solutions to the first question of 1-2-3 Dimensional Attributes activity on a large visual display.

- Prepare sets of 16 snap cubes and two sticky notes for each student.

Student Learning Goals

Let's contrast surface area and volume.

16.1 Attributes and Their Measures

Warm Up: 10 minutes

This activity strengthens students' awareness of one-, two-, and three-dimensional attributes and the units commonly used to measure them. Students decide on the units based on the attributes being measured and the size of the units and how appropriate they would be for describing given quantities.

As students work, select a few students to share their responses to the last two questions of the activity (on the quantities that could be measured in miles and in cubic meters).

Building On

- 3.MD.C.5
- 4.MD.A.1
- 5.MD.C

Instructional Routines

- Think Pair Share

Launch

Consider a quick review of metric and standard units of measurement before students begin work. Include some concrete examples that could help illustrate the size of each unit.

Then, pick an object in the classroom for which surface area and volume could be measured (e.g. a desk). Ask students, "What units might we use to measure the surface area of the desktop? What units might we use to measure the volume of a drawer?"

Clarify the relative sizes of the different units that come up in the conversation. For instance, discuss how a meter is a little over three feet, a yard is three feet, a kilometer is about two-thirds of a mile, a millimeter is one tenth of a centimeter, etc.

Give students 4–5 minutes of quiet think time and then a couple of minutes to share their responses with a partner. Prepare to display the answers to the first six questions for all to see.

Anticipated Misconceptions

Depending on the students' familiarity with metric and standard units, there may be some confusion about the size of each unit. Consider displaying measuring tools or a reference sheet that shows concrete examples of items measured in different-sized units.

Student Task Statement

For each quantity, choose one or more appropriate units of measurement.

For the last two, think of a quantity that could be appropriately measured with the given units.

Quantities

1. Perimeter of a parking lot:
2. Volume of a semi truck:
3. Surface area of a refrigerator:
4. Length of an eyelash:
5. Area of a state:
6. Volume of an ocean:
7. _____: miles
8. _____: cubic meters

Units

- millimeters (mm)
- feet (ft)
- meters (m)
- square inches (sq in)
- square feet (sq ft)
- square miles (sq mi)
- cubic kilometers (cu km)
- cubic yards (cu yd)

Student Response

1. Meters, feet
2. Cubic yards
3. Square inches, square feet
4. Millimeters
5. Square miles
6. Cubic kilometers, cubic yards
7. Answers vary. Sample responses: distance between home and school, length of a river
8. Answers vary. Sample responses: volume of a room, volume of a swimming pool

Activity Synthesis

Display the solutions to the first six questions for all to see and to use for checking. Then, select a few students to share their responses to the last two questions.

Ask students what they notice about the units for area and the units for volume. If not already mentioned by students, highlight that area is always measured in square units and volume in cubic units.

16.2 Building with 8 Cubes

Optional: 25 minutes (there is a digital version of this activity)

This activity clarifies the distinction between volume and surface area and illustrates that two polyhedra can have the same volume but different surface areas.

Students build shapes using two sets of eight cubes and determine their volumes and surface areas. Since all of the designs are made of the same number of cubes, they all have the same volume. Students then examine all of the designs and discuss what distinguishes shapes with smaller surface areas from those with greater ones.

As students work, monitor the range of surface areas for the shapes that students built. Select several students whose designs collectively represent that range.

Building On

- 5.MD.C.3.b
- 5.MD.C.4

Addressing

- 6.G.A.4

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports
- Notice and Wonder

Launch

Give each student (or group of 2 students) 16 snap cubes, two sticky notes, and 8–10 minutes of work time.

Explain that their job is to design and build two figures—using 8 cubes for each—and find the volume and surface area of each figure. Ask them to give each figure a name or a label and then record the name, surface area, and volume on a sticky note.

Anticipated Misconceptions

Even though students are dealing with only 8 cubes at a time, they may make counting errors by inadvertently omitting or double-counting squares or faces. This is especially likely for designs that are non-prisms. Encourage students to think of a systematic way to track the number of square units they are counting.

Some students may associate volume only with prisms and claim that the volume of non-prism designs cannot be determined. Remind them of the definition of volume.

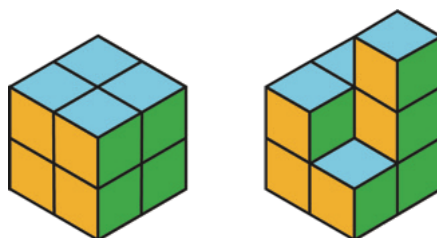
Student Task Statement

Your teacher will give you 16 cubes. Build two different shapes using 8 cubes for each. For each shape:

1. Give it a name or a label (e.g., Mai's First Shape or Diego's Steps).
2. Determine the **volume**.
3. Determine the surface area.
4. Record the name, volume, and surface area on a sticky note.

Student Response

Designs vary. Here are two possible shapes:



They both have a volume of 8 cubic units. The first has a surface area of 24 square units. The second has a surface area of 28 square units. The smallest possible surface area for an 8-cube construction is 24 square units, and the largest is 34 square units.

Activity Synthesis

Ask all students to display their designs and their sticky notes and give them a couple of minutes to circulate and view one another's work.

Then, ask previously identified students to arrange their designs in the order of their surface area, from least to greatest, and display their designs for all to see. Record the information about the designs in a table, in the same sequence. Display the table for all to see. Here is an example.

shapes	volume	surface area
Andre's cube	8	24
Lin's steps	8	28
Jada's first shape	8	28
...
Noah's tower	8	34

Give students a minute to notice and wonder about the designs and the information in the table. Ask students to give a signal when they have noticed and wondered about at least one thing. Invite a few students to share their observations and questions. Then, discuss the following questions (if not already mentioned by students):

- “What do all of the shapes have in common?” (Their volume)
- “Why are all the volumes the same?” (Volume measures the number of unit cubes that can be packed into a figure. All the designs are built using the same number of cubes.)
- “Why do some shapes have larger surface areas than others? What do shapes with larger surface areas look like?” (The cubes are more spread out and have more of their faces exposed.)
- “What about those with smaller surface areas?” (They are more compact and have more of their faces hidden or shared with another cube.)
- “Is it possible to build a shape with a different volume? How?” (Yes, but it would involve using fewer or more cubes.)

If students have trouble visualizing how surface area changes when the design changes, demonstrate the following:

- Make a cube made of 8 smaller cubes. Point to one cube and ask how many of its faces are exposed (3).
- Pop that cube off and move it to another place.
- Point out that, in the “hole” left by the cube that was moved, 3 previously interior faces now contribute to the surface area. At the same time, the relocated cube now has 5 faces exposed.

Support for English Language Learners

Speaking, Listening: MLR8 Discussion Supports. To help students describe and explain their comparisons, provide language students can use (e.g., spread out vs. compact, exposed/visible vs. hidden/covered). Demonstrate the comparisons using the visuals of the cube designs. Offer sentence frames for students to help with their explanations (e.g., “The surface area of shape ____ is larger (or smaller) than shape ____ because. . .”

Design Principle(s): Optimize output (for explanation)

16.3 Comparing Prisms Without Building Them

Optional: 20 minutes

Previously, students studied shapes with the same volume but different surface areas. Here they see that it is also possible for shapes to have the same surface area but different volumes. Students think about how the appearance of these shapes might compare visually.

Students are given the side lengths of three rectangular prisms and asked to find the surface area and the volume of each. Some students can visualize these, but others may need to draw nets, sketch the figures on isometric grid paper, or build physical prisms. Prepare cubes for students to use. Each of the three prisms can be built with 15 or fewer cubes, but 40 cubes are needed to build all three simultaneously. (If the cubes are not centimeter cubes, ask students to treat them as if the edge length of each cube was 1 cm.)

As students work, look out for errors in students' calculations in the first question, which will affect the observations they make in the second question. Select a few students who notice that the volumes of the prisms are all different but the surface areas are the same.

Building On

- 5.MD.C.5.a

Addressing

- 6.G.A.4

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. Provide access to snap cubes and geometry toolkits. Give students 6–7 minutes of quiet think time and then 2–3 minutes to discuss their responses with their partner. Ask partners to agree upon one key observation to share with the whole class.

Support for Students with Disabilities

Representation: Develop Language and Symbols. Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Provide snap-cubes or blocks to help students analyze the volumes and surface areas of the prisms.

Supports accessibility for: Conceptual processing

Support for English Language Learners

Conversing: MLR8 Discussion Supports. Use this routine to help students describe what they noticed about the volume and surface areas of their prisms. Students should take turns stating an observation and their reasoning with their partner. Invite Partner A to begin with this sentence frame: "I noticed ____, so . . ." Invite the listener, Partner B, to press for additional details referring to specific features of the prisms. Students should switch roles until they have listed all observations.

Design Principle(s): Support sense-making; Cultivate conversation

Anticipated Misconceptions

Students may miss or double-count one or more faces of the prisms and miscalculate surface areas. Encourage students to be systematic in their calculations and to use organizational strategies they learned when finding surface area from nets.

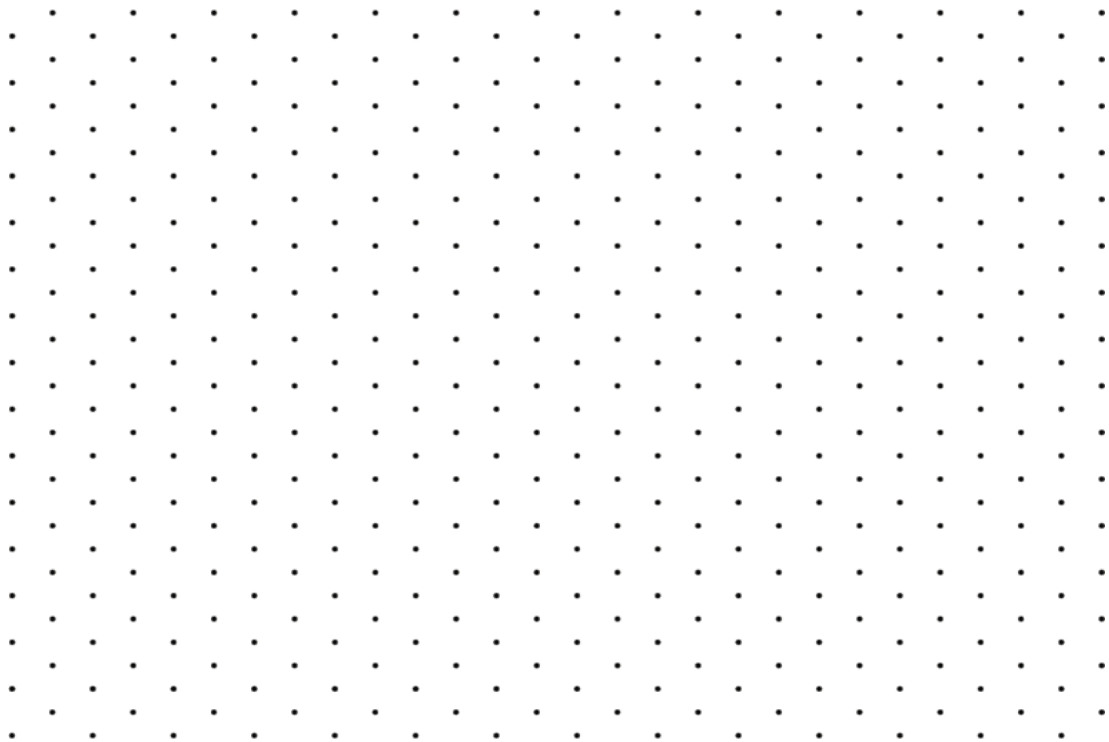
Students may need reminders to use square units for area and cubic units for volume.

Student Task Statement

Three rectangular prisms each have a height of 1 cm.

- Prism A has a base that is 1 cm by 11 cm.
- Prism B has a base that is 2 cm by 7 cm.
- Prism C has a base that is 3 cm by 5 cm.

1. Find the surface area and volume of each prism. Use the dot paper to draw the prisms, if needed.



2. Analyze the volumes and surface areas of the prisms. What do you notice? Write 1 or 2 observations about them.

Student Response

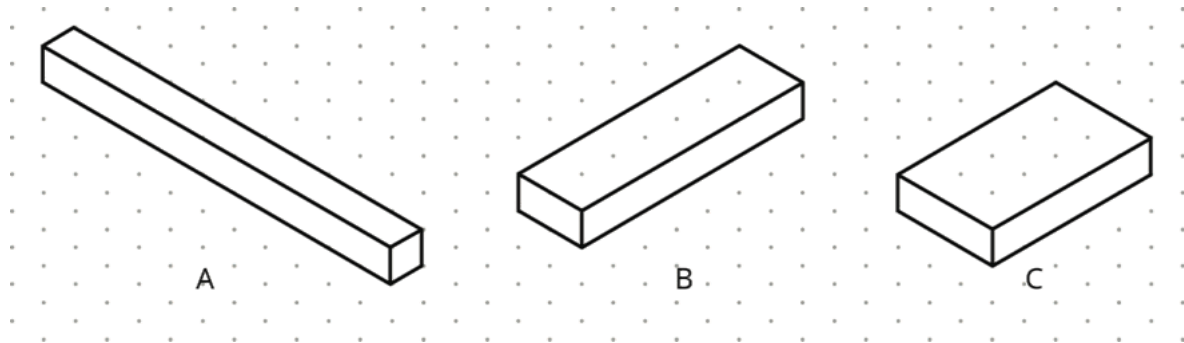
1. Surface areas:

- Prism A: $4(11 \cdot 1) + 2(1 \cdot 1) = 46$ square centimeters
- Prism B: $2(7 \cdot 2) + 2(7 \cdot 1) + 2(2 \cdot 1) = 46$ square centimeters

- Prism C: $2(5 \cdot 3) + 2(5 \cdot 1) + 2(3 \cdot 1) = 46$ square centimeters

Volumes:

- Prism A: 11 cubic centimeters ($11 \cdot 1 \cdot 1 = 11$)
- Prism B: 14 cubic centimeters ($7 \cdot 2 \cdot 1 = 14$)
- Prism C: 15 cubic centimeters ($5 \cdot 3 \cdot 1 = 15$)



2. Answers vary. Sample responses:

- The surface areas of the prisms are all the same, but the volumes are all different.
- The polygons that make up the faces of each prism are different-sized rectangles, but their areas all add up to the same total of square centimeters.
- Prism C can fit the most centimeter cubes, but because the cubes would fit together in a compact way, some of the cubes would only have two square centimeters of exposed faces.
- Prism A can fit the fewest centimeter cubes, but because the cubes would be more spread out, more of their faces would be exposed.

Are You Ready for More?

Can you find more examples of prisms that have the same surface areas but different volumes? How many can you find?

Student Response

Answers vary. Sample response: A prism that is 4 units by 5 units by 1 unit and one that is 2 units by 9 units by 1 unit have the same surface area but different volumes.

Generate examples by finding different pairs of factors of the same number and subtracting 1 from each factor. However, there are other ways. For example, $60 = 6 \cdot 10$ and $60 = 5 \cdot 12$. The 5-by-9-by-1 and 4-by-11-by-1 prisms have the same surface areas but different volumes.

Activity Synthesis

Ask students to share their observations in response to the second question. Record them for all to see. For each unique observation, poll the class to see if others noticed the same thing. Highlight the following observations, or point them out if not already mentioned by students:

- The volumes of the prisms are all different, but the surface areas are the same.
- Volume is described in terms of unit cubes and surface area in terms of the exposed faces of those unit cubes.

Explain that, in an earlier activity, we saw how different shapes could have the same volume (i.e. being made up of the same number of unit cubes) but different surface areas. Now we see that it is also possible for shapes with different volumes (i.e. consisting of different numbers of unit cubes) to have the same surface area.

If students have trouble conceptualizing the idea of figures with different volume having the same surface area, refer to the filing cabinet activity in the first lesson on surface area. The number of square sticky notes needed to cover all of the faces of the filing cabinet was its surface area. If we use all of those square notes (no more, no less) to completely cover (without overlapping sticky notes) a cabinet that has a different volume, we can say that the two pieces of furniture have the same surface area and different volumes.

Lesson Synthesis

In this lesson, we refreshed our memory of measures of one-, two-, and three-dimensional attributes. Reiterate that length is a one-dimensional attribute of geometric figures, area is a two-dimensional attribute, and volume is a three-dimensional attribute. Revisit a few examples of units for length, area, and volume.

We also explored the surface areas and volumes of polyhedra and noticed that two shapes can have the same volumes but different surface areas, and vice versa.

- "How could two figures with a volume of 4 cubic units have a surface area of 16 square units and 18 square units?" (Surface area and volume measure different attributes of a three-dimensional shape.)
- "What kind of measure is surface area? What kind of measure is volume?" (Surface area is a two-dimensional attribute; we measure it in square units. Volume is a three-dimensional attribute; we measure it in cubic units.)
- "Are the two measures related? Does a greater volume necessarily mean a greater surface area, and vice versa?" (No, one measure does not affect the other. A figure that has a greater volume than another may not necessarily have a greater surface area.)

16.4 Same Surface Area, Different Volumes

Cool Down: 5 minutes

Addressing

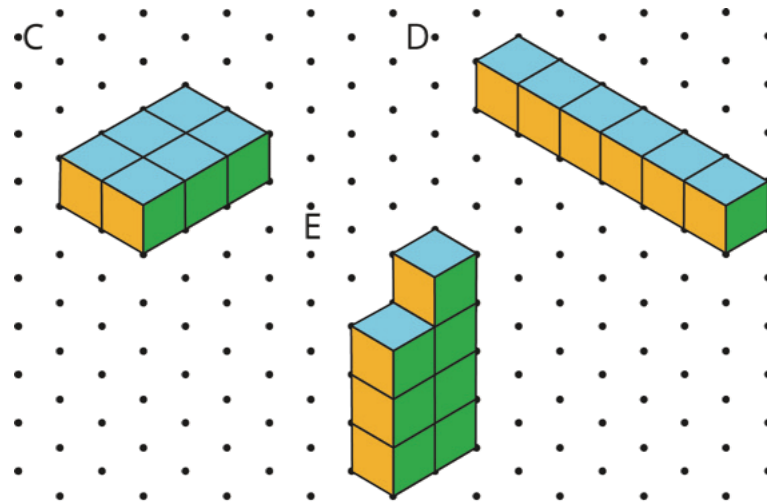
- 6.G.A.4

Launch

Encourage students to refer to the class list of observations from the previous activity

Student Task Statement

Choose two figures that have the same surface area but different volumes. Show your reasoning.



Student Response

Figure D and E both have a surface area of 26 square units, but D has a volume of 6 cubic units, and E has a volume of 7 cubic units.

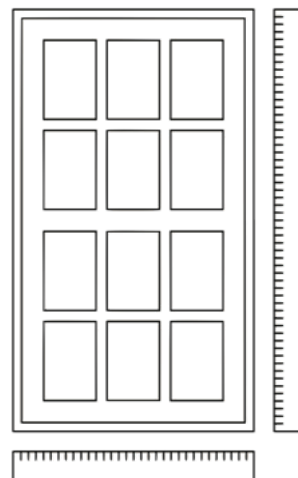
Student Lesson Summary

Length is a one-dimensional attribute of a geometric figure.

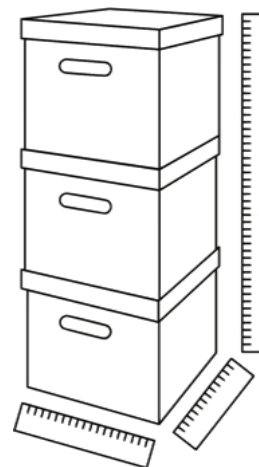
We measure lengths using units like millimeters, centimeters, meters, kilometers, inches, feet, yards, and miles.



Area is a two-dimensional attribute. We measure area in square units. For example, a square that is 1 centimeter on each side has an area of 1 square centimeter.



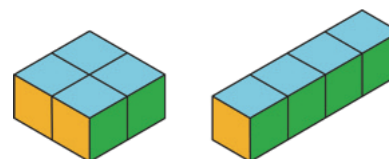
Volume is a three-dimensional attribute. We measure volume in cubic units. For example, a cube that is 1 kilometer on each side has a volume of 1 cubic kilometer.



Surface area and volume are different attributes of three-dimensional figures. Surface area is a two-dimensional measure, while volume is a three-dimensional measure.

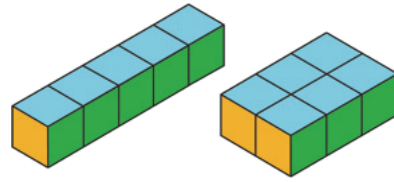
Two figures can have the same volume but different surface areas. For example:

- A rectangular prism with side lengths of 1 cm, 2 cm, and 2 cm has a volume of 4 cu cm and a surface area of 16 sq cm.
- A rectangular prism with side lengths of 1 cm, 1 cm, and 4 cm has the same volume but a surface area of 18 sq cm.



Similarly, two figures can have the same surface area but different volumes.

- A rectangular prism with side lengths of 1 cm, 1 cm, and 5 cm has a surface area of 22 sq cm and a volume of 5 cu cm.
- A rectangular prism with side lengths of 1 cm, 2 cm, and 3 cm has the same surface area but a volume of 6 cu cm.



Glossary

- volume

Lesson 16 Practice Problems

Problem 1

Statement

Match each quantity with an appropriate unit of measurement.

- | | |
|----------------------------------------|-----------------------|
| A. The surface area of a tissue box | 1. Square meters |
| B. The amount of soil in a planter box | 2. Yards |
| C. The area of a parking lot | 3. Cubic inches |
| D. The length of a soccer field | 4. Cubic feet |
| E. The volume of a fish tank | 5. Square centimeters |

Solution

- A: 5
- B: 3
- C: 1
- D: 2
- E: 4

Problem 2

Statement

Here is a figure built from snap cubes.



- Find the volume of the figure in cubic units.
- Find the surface area of the figure in square units.
- True or false: If we double the number of cubes being stacked, both the volume and surface area will double. Explain or show how you know.

Solution

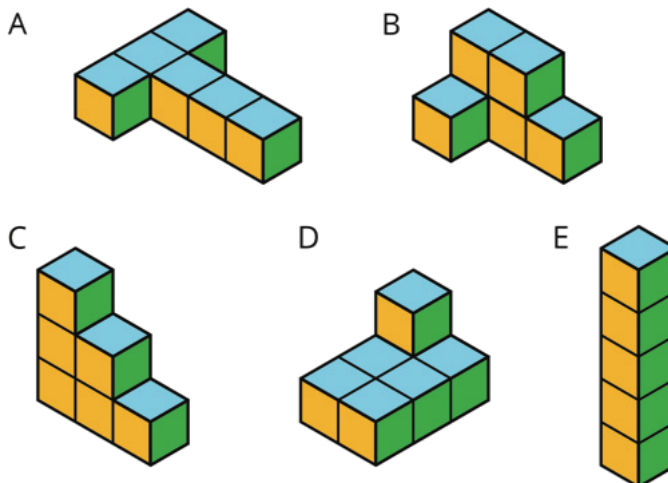
- 4 cubic units. $(1 \cdot 1 \cdot 4) = 4$.
- 18 square units. $(4 \cdot 4) + (2 \cdot 1) = 18$.
- False. Sample reasoning: The volume will double to 8 cubic units, but the surface area will not. Only the side faces will double in area, to $(4 \cdot 8)$ or 32 square units, but the top and bottom faces will not double, so the surface area will be 34, not 36, square units.

Problem 3

Statement

Lin said, "Two figures with the same volume also have the same surface area."

- Which two figures suggest that her statement is true?
- Which two figures could show that her statement is *not* true?



Solution

- B and C

b. A and B, or A and C

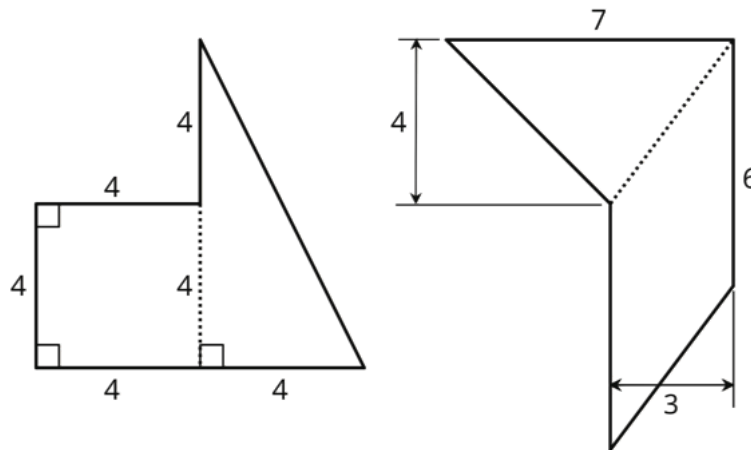
Problem 4

Statement

Draw a pentagon (five-sided polygon) that has an area of 32 square units. Label all relevant sides or segments with their measurements, and show that the area is 32 square units.

Solution

Answers vary. Sample responses:



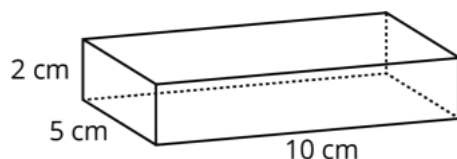
- The first pentagon is composed of a square and a right triangle. The square has an area of 16 square units. The triangle has a base of 4 and a height of 8, so its area is 16 square units. The combined area is $16 + 16$ or 32 square units.
- The second pentagon is composed of a parallelogram with a base of 6 and a height of 3, and a triangle with a base of 7 and a height of 4. The area of the parallelogram is $6 \cdot 3$ or 18 square units. The area of the triangle is $\frac{1}{2} \cdot 7 \cdot 4$ or 14 square units. The combined area is $18 + 14$ or 32 square units.

(From Unit 1, Lesson 11.)

Problem 5

Statement

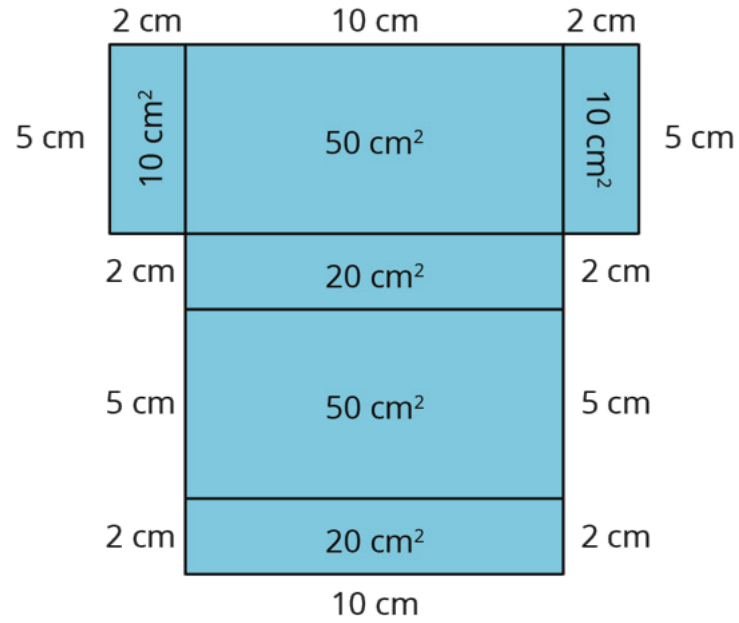
a. Draw a net for this rectangular prism.



- b. Find the surface area of the rectangular prism.

Solution

- a. Diagrams vary. Here is a sample net.



- b. 160 square units. (There are two faces with an area of 50 square cm, two faces with an area of 20 square cm, and two faces with an area of 10 square cm.)

(From Unit 1, Lesson 15.)

Section: Squares and Cubes

Lesson 17: Squares and Cubes

Goals

- Generalize a process for finding the volume of a cube, and justify (orally) why this can be abstracted as s^3 .
- Include appropriate units (orally and in writing) when reporting lengths, areas, and volumes, e.g. cm, cm^2 , cm^3 .
- Interpret and write expressions with exponents 2 and 3 to represent the area of a square or the volume of a cube.

Learning Targets

- I can write and explain the formula for the volume of a cube, including the meaning of the exponent.
- When I know the edge length of a cube, I can find the volume and express it using appropriate units.

Lesson Narrative

In this lesson, students learn about perfect squares and perfect cubes. They see that these names come from the areas of squares and the volumes of cubes with whole-number side lengths. Students find unknown side lengths of a square given the area or unknown edge lengths of a cube given the volume. To do this, they make use of the structure in expressions for area and volume (MP7).

Students also use **exponents** of 2 and 3 and see that in this geometric context, exponents help to efficiently express multiplication of the side lengths of squares and cubes. Students learn that expressions with exponents of 2 and 3 are called **squares** and **cubes**, and see the geometric motivation for this terminology. (The term “exponent” is deliberately *not* defined more generally at this time. Students will work with exponents in more depth in a later unit.)

In working with length, area, and volume throughout the lesson, students must attend to units. In order to write the formula for the volume of a cube, students look for and express regularity in repeated reasoning (MP8).

Note: Students will need to bring in a personal collection of 10–50 small objects ahead of time for the first lesson of the next unit. Examples include rocks, seashells, trading cards, or coins.

Alignments

Building On

- 4.MD.A.3: Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.
- 5.MD.C.5.a: Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

Addressing

- 6.EE.A: Apply and extend previous understandings of arithmetic to algebraic expressions.
- 6.EE.A.1: Write and evaluate numerical expressions involving whole-number exponents.

Building Towards

- 6.EE.A.1: Write and evaluate numerical expressions involving whole-number exponents.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports
- Think Pair Share

Required Materials

Snap cubes

Required Preparation

Prepare sets of 32 snap cubes for each group of 2 students.

Student Learning Goals

Let's investigate perfect squares and perfect cubes.

17.1 Perfect Squares

Warm Up: 5 minutes

This activity introduces the concept of perfect squares. It also includes opportunities to practice using units of measurement, which offers insights on students' knowledge from preceding lessons.

Provide access to square tiles, if available. Some students may benefit from using physical tiles to reason about perfect squares.

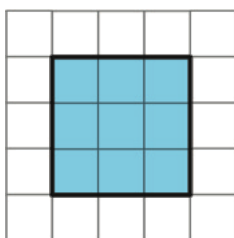
As students work, notice whether they use appropriate units for the second and third questions.

Building On

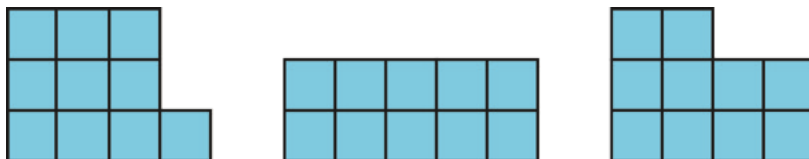
- 4.MD.A.3

Launch

Tell students, "Some numbers are called perfect **squares**. For example, 9 is a perfect square. Nine copies of a small square can be arranged into a large square." Display a square like this for all to see:



Explain that 10, however, is not a perfect square. Display images such as these below, emphasizing that 10 small squares can not be arranged into a large square (the way 9 small squares can).



Tell students that in this warm-up they will find more numbers that are perfect squares. Give students 2 minutes of quiet think time to complete the activity.

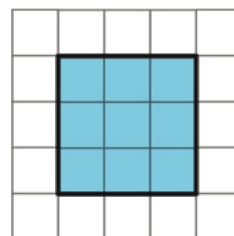
Anticipated Misconceptions

If students do not recall what the abbreviations km, cm, and sq stand for, provide that information.

Students may divide 64 by 2 for the third question. If students are having trouble with this, ask them to check by working backwards, i.e. by multiplying the side lengths to see if the product yields the given area measure.

Student Task Statement

1. The number 9 is a perfect **square**. Find four numbers that are perfect squares and two numbers that are not perfect squares.
2. A square has side length 7 in. What is its area?
3. The area of a square is 64 sq cm. What is its side length?



Student Response

1. Answers vary. For example, here are some squares: 9, 25, 4, 49, 100 and non-squares: $\frac{1}{2}$, 2, 3, 10.
2. The square has an area of 49 square inches.
3. The side length is 8 centimeters.

Activity Synthesis

Invite students to share the examples and non-examples they found for perfect squares. Solicit some ideas on how they decided if a number is or is not a perfect square.

If a student asks about 0 being a perfect square, wait until the end of the lesson, when the exponent notation is introduced. 0 is a perfect square because $0^2 = 0$.

Briefly discuss students' responses to the last two questions, the last one in particular. If not already uncovered in discussion, highlight that because the area of a square is found by multiplying side lengths to each other, finding the side lengths of a square with a known area means figuring out if that area measure is a product of two of the same number.

17.2 Building with 32 Cubes

Optional: 15 minutes (there is a digital version of this activity)

This activity gives students a concrete way to review the work on volume from grade 5. It prompts students to recall that the volume of a rectangular prism can be calculated in two different ways: by counting unit cubes that can be packed into the prism, and by multiplying the edge lengths of the prism. Students also become familiar with two perfect cubes, 27 and 64, before the next activity introduces this term.

As students work, monitor the different routes they take to find the volume of the built cube. They may count all of the snap cubes individually, count the number of snap cubes per layer and then multiply that by the number of layers, or simply multiply edge lengths. Select a student who uses each method to share later.

Building On

- 5.MD.C.5.a

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

Launch

Arrange students in groups of 2. Give 32 snap cubes to each group. If centimeter cubes are available, have students work in centimeters instead of the generic units listed here. Give students 8–10 minutes to build the largest cube they can with 32 cubes and to answer the questions.

For groups who finish early, consider asking them to combine their cubes and build the largest single cube they can with 64 cubes. Then, ask them to answer the same four questions as shown in the problem statement.

Students in digital classrooms can use the applet to build the cube with 32 cubes. For students who finish early, another applet with 64 cubes can be found in Digital Extension.

Anticipated Misconceptions

Students may neglect to write units for length or area and may need a reminder to do so.

When determining area, students may multiply a side by two instead of squaring it. When determining volume, they may multiply a side by three instead of cubing it. If this happens, ask them to count individual squares so that they can see that there is an error in their reasoning.

Student Task Statement

Your teacher will give you 32 snap cubes. Use them to build the largest single cube you can. Each small cube has an edge length of 1 unit.

1. How many snap cubes did you use?
2. What is the edge length of the cube you built?
3. What is the area of each face of the built cube? Be prepared to explain your reasoning.
4. What is the volume of the built cube? Be prepared to explain your reasoning.

Student Response

1. 27
2. 3 units
3. 9 square units
4. 27 cubic units

Activity Synthesis

Focus the whole-class discussion on the ways students calculated the volumes of the two cubes they built. Select previously identified students to share their approaches starting from the less efficient (counting individual cubes) to the most efficient (multiplying side lengths).

Highlight how the side length of a cube determines its volume, and specifically that the number 27 is $3 \cdot 3 \cdot 3$. If any group built a cube with 64 snap cubes, point out that the number 64 is $4 \cdot 4 \cdot 4$. These observations prepare students to think about perfect cubes in the next activity and about a general expression for the volume of a cube later in the lesson.

17.3 Perfect Cubes

10 minutes

Earlier, students looked at examples and non-examples of perfect squares. In this activity, they think about examples and non-examples of perfect **cubes** and find the volumes of cubes given their edge lengths. Students see that the edge length of a cube determines its volume, notice the numerical expressions that can be written when calculating volumes, and write a general expression for finding the volume of a cube (MP8).

Some students may feel uncomfortable writing the answer to the last question symbolically because it involves a variable and may prefer writing a verbal explanation. This is fine; the exponential notation that follows will help greatly.

Building On

- 5.MD.C.5.a

Addressing

- 6.EE.A

Building Towards

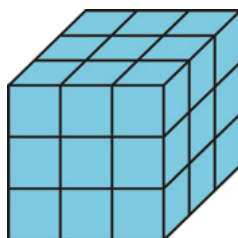
- 6.EE.A.1

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Tell students, “Some numbers are called perfect **cubes**. For example, 27 is a perfect cube.” Display a cube like this for all to see:



Arrange students in groups of 2. Give students a few minutes of quiet think time, and another minute to discuss their responses with their partner.

Support for Students with Disabilities

Representation: Develop Language and Symbols. Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Provide snap-cubes or blocks to help students analyze examples and nonexamples of perfect cubes, paying close attention to the relationships between side lengths and volume.

Supports accessibility for: Conceptual processing

Support for English Language Learners

Conversing, Representing: MLR8 Discussion Supports. As students work in pairs to make sense of perfect cubes, encourage students to press for details as peers share their ideas. Provide sentence frames for students to use, such as “How do you know. . .?”, “Tell me more about. . .”, and “I agree/disagree because. . .”

Design Principle(s): Support sense-making; Maximize meta-awareness

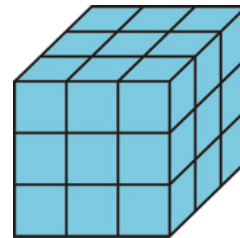
Anticipated Misconceptions

Watch for students using square units instead of cubic units. Remind them that volume is a measure of the space inside the cube and is measured in cubic units.

Students may multiply by 3 when finding the volume of a cube instead of multiplying three edge lengths (which happen to be the same number). Likewise, they may think a perfect cube is a number times 3. Suggest that they sketch or build a cube with that edge length and count the number of unit cubes. Or ask them to think about how to find the volume of a prism when the edge lengths are different (e.g., a prism that is 1 unit by 2 units by 3 units).

Student Task Statement

1. The number 27 is a perfect **cube**. Find four other numbers that are perfect cubes and two numbers that are *not* perfect cubes.
2. A cube has side length 4 cm. What is its volume?
3. A cube has side length 10 inches. What is its volume?
4. A cube has side length s units. What is its volume?



Student Response

1. Answers vary: 1, 8, 64, 125, 216, 1,000. Non-cubes: 2, 3, 4.
2. $4 \cdot 4 \cdot 4$ or 64 cubic cm
3. $10 \cdot 10 \cdot 10$ or 1,000 cubic inches

4. $s \cdot s \cdot s$ cubic units

Activity Synthesis

After partner discussions, invite students to share how they thought about the first question and decided if a number is or is not a perfect cube. Highlight the idea that multiplying three edge lengths allows us to determine volume efficiently, and that determining if a number is a perfect cube involves thinking about whether it is a product of three of the same number.

If a student asks about 0 being a perfect cube, wait until the end of the lesson, when exponent notation is introduced. 0 is a perfect cube because $0^3 = 0$.

Make sure students see that the answers to the last three questions written as expressions:

$$\begin{array}{c} 4 \cdot 4 \cdot 4 \\ 10 \cdot 10 \cdot 10 \\ s \cdot s \cdot s \end{array}$$

17.4 Introducing Exponents

15 minutes

This activity introduces students to the exponents of 2 and 3 and the language we use to talk about them. Students use and interpret this notation in the context of geometric squares and their areas, and geometric cubes and their volumes. Students are likely to have seen exponent notation for 10^3 in their work on place values in grade 5. That experience would be helpful but is not necessary.

Note that the term “exponent” is deliberately *not* defined more generally at this time. Students will work with exponents in more depth in a later unit.

As students work, observe how they approach the last two questions. Identify a couple of students who approach the fourth question differently so they can share later. Also notice whether students include appropriate units, written using exponents, in their answers.

Addressing

- 6.EE.A.1

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Ask students if they have seen an expression such as 10^3 before. Tell students that in this expression, the 3 is called an **exponent**. Explain the use of exponents of 2 and 3:

- “When we multiply two of the same number together, such as $5 \cdot 5$, we say we are **squaring** the number. We can write the expression as:

$$5^2$$

Because $5 \cdot 5$ is 25, we can write $5^2 = 25$, and we say, '5 squared is 25.' We can also say that 25 is a perfect square. The raised 2 in 5^2 is called an exponent."

- "When we multiply three of the same number together like $4 \cdot 4 \cdot 4$, we say we are **cubing** the number. We can write it like this:

$$4^3$$

Because $4 \cdot 4 \cdot 4$ is 64, we can write $4^3 = 64$, and we say, '4 cubed is 64.' We also say that 64 is a perfect cube. The raised 3 in 4^3 is called an exponent."

Explain that we can also use exponents as a shorthand for the units used for area and volume:

- A square with side length 5 inches has area of 25 square inches, which we can write as 25 in^2 .
- A cube with edge length 4 centimeters has a volume of 64 cubic centimeters, which we can write as 64 cm^3 .

Ask students to read a few areas and volumes in different units (e.g. 100 ft^2 is read "100 square feet" and 125 yd^3 is read "125 cubic yards").

Keep students in groups of 2. Give students 3–4 minutes of quiet time to complete the activity and a minute to discuss their response with their partner. Ask partners to note any disagreements so they can be discussed.

Anticipated Misconceptions

Upon seeing 6^3 in the fourth question, some students may neglect to interpret the question, automatically calculate $6 \cdot 6 \cdot 6$, and conclude that the edge length is 216 cm. Ask them to check their answer by finding the volume of a cube with edge length 216 cm.

Student Task Statement

Make sure to include correct units of measure as part of each answer.

1. A square has side length 10 cm. Use an **exponent** to express its area.
2. The area of a square is 7^2 sq in. What is its side length?
3. The area of a square is 81 m^2 . Use an exponent to express this area.
4. A cube has edge length 5 in. Use an exponent to express its volume.
5. The volume of a cube is 6^3 cm^3 . What is its edge length?
6. A cube has edge length s units. Use an exponent to write an expression for its volume.

Student Response

1. 10^2 cm^2
2. 7 inches

3. 9^2 m^2
4. 5^3 in^3
5. 6 cm
6. $s^3 \text{ units}^3$ or $s^3 \text{ cubic units}$

Are You Ready for More?

The number 15,625 is both a perfect square and a perfect cube. It is a perfect square because it equals 125^2 . It is also a perfect cube because it equals 25^3 . Find another number that is both a perfect square and a perfect cube. How many of these can you find?

Student Response

The smallest examples are 0, 1, 64, 729, and 4,096.

Activity Synthesis

Ask partners to share disagreements in their responses, if any. Then, focus the whole-class discussion on the last two questions. Select a couple of previously identified students to share their interpretations of the fourth question.

Highlight that a cube with a volume of 6^3 cubic units has an edge length of 6 units, because we know there are $6 \cdot 6 \cdot 6$ unit cubes in a cube with that edge length.

In other words, we can express the volume of a cube using a number (216), a product of three numbers ($6 \cdot 6 \cdot 6$), or an expression with exponent (6^3). This idea can be extended to all cubes. The volume of a cube with edge length s is:

$$\begin{array}{c} s \cdot s \cdot s \\ s^3 \end{array}$$

Students will have more opportunities to generalize the expressions for the volume of a cube in the next lesson.

Support for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding and memory: squaring, cubing, exponent.

Supports accessibility for: Conceptual processing; Language; Memory

Support for English Language Learners

Reading, Writing, Speaking: MLR3 Clarify, Critique, Correct. Use this routine before the whole-class discussion of the last two questions. Display an incorrect response for students to consider.

For example, “If the volume of a cube is 6^3 cm^3 , then the edge length is 216 cm because $6 \cdot 6 \cdot 6$ is 216.” Ask students to identify the error, critique the reasoning, and write a correct explanation. Listen for students who clarify that the volume of a cube can be represented as an exponent or a value and that neither of these represent the edge length. Invite students to share their critiques and corrected explanations with the class. Listen for and amplify the language students use to describe ways to generalize the relationship between the three representations of volume: a number, a product of three numbers, or an expression with exponent. This helps students evaluate, and improve upon, the written mathematical arguments of others, as they clarify their understanding of exponents.

Design Principle(s): Optimize output (for explanation); Maximize meta-awareness

Lesson Synthesis

Review the language and notation for **squaring** and **cubing** a number. Remind students we use this notation for square and cubic units, too.

- When we multiply two of the same number together like $10 \cdot 10$ we say we are **squaring** the number. We write, for example, $10^2 = 100$ and say, “Ten squared is one hundred.”
- When we multiply three of the same number together like $10 \cdot 10 \cdot 10$, we say we are **cubing** the number. We write, for example, $10^3 = 1,000$ and say, “Ten cubed is one thousand.”
- Exponents are used to write square and cubic units. The area of a square with side length 7 km is 7^2 km^2 . The volume of a cube with side length 2 millimeters is 2^3 mm^3 .

17.5 Exponent Expressions

Cool Down: 5 minutes

Addressing

- 6.EE.A.1

Anticipated Misconceptions

Students may perform calculations on the second question, which is not necessary since the target is an expression with an exponent.

Student Task Statement

1. Which is larger, 5^2 or 3^3 ?

2. A cube has an edge length of 21 cm. Use an exponent to express its volume.

Student Response

1. $3^3 = 27$ and $5^2 = 25$, so 3^3 is larger than 5^2 .
2. 21^3 cm^3 or 21^3 cubic centimeters

Student Lesson Summary

When we multiply two of the same numbers together, such as $5 \cdot 5$, we say we are **squaring** the number. We can write it like this:

$$5^2$$

Because $5 \cdot 5 = 25$, we write $5^2 = 25$ and we say, "5 squared is 25."

When we multiply three of the same numbers together, such as $4 \cdot 4 \cdot 4$, we say we are **cubing** the number. We can write it like this:

$$4^3$$

Because $4 \cdot 4 \cdot 4 = 64$, we write $4^3 = 64$ and we say, "4 cubed is 64."

We also use this notation for square and cubic units.

- A square with side length 5 inches has area 25 in^2 .
- A cube with edge length 4 cm has volume 64 cm^3 .

To read 25 in^2 , we say "25 square inches," just like before.

The area of a square with side length 7 kilometers is 7^2 km^2 . The volume of a cube with edge length 2 millimeters is 2^3 mm^3 .

In general, the area of a square with side length s is s^2 , and the volume of a cube with edge length s is s^3 .

Glossary

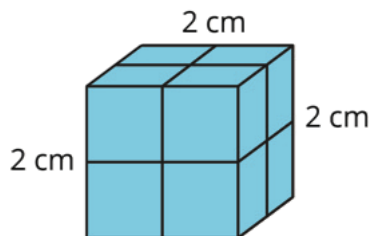
- cubed
- exponent
- squared

Lesson 17 Practice Problems

Problem 1

Statement

What is the volume of this cube?



Solution

8 cu cm ($2 \cdot 2 \cdot 2 = 8$)

Problem 2

Statement

a. Decide if each number on the list is a perfect square.

16

125

20

144

25

225

100

10,000

b. Write a sentence that explains your reasoning.

Solution

a. All of these numbers, except 20 and 125, are perfect squares.

b. Answers vary. Sample response: Perfect squares can be found by multiplying a whole number by itself.

Problem 3

Statement

a. Decide if each number on the list is a perfect cube.

1	27
3	64
8	100
9	125

b. Explain what a perfect cube is.

Solution

- a. All of the numbers except 3, 9, and 100 are perfect cubes.
- b. Answers vary. Sample response: Perfect cubes can be found by using a whole number as a factor three times.

Problem 4

Statement

- a. A square has side length 4 cm. What is its area?
- b. The area of a square is 49 m^2 . What is its side length?
- c. A cube has edge length 3 in. What is its volume?

Solution

- a. 16 cm^2
- b. 7 m
- c. 27 in^3

Problem 5

Statement

Prism A and Prism B are rectangular prisms.

- Prism A is 3 inches by 2 inches by 1 inch.
- Prism B is 1 inch by 1 inch by 6 inches.

Select all statements that are true about the two prisms.

- A. They have the same volume.
- B. They have the same number of faces.
- C. More inch cubes can be packed into Prism A than into Prism B.
- D. The two prisms have the same surface area.
- E. The surface area of Prism B is greater than that of Prism A.

Solution

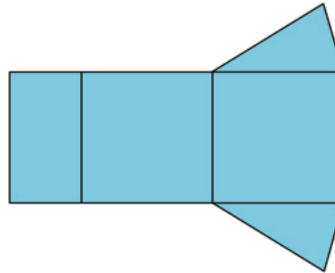
["A", "B", "E"]

(From Unit 1, Lesson 16.)

Problem 6

Statement

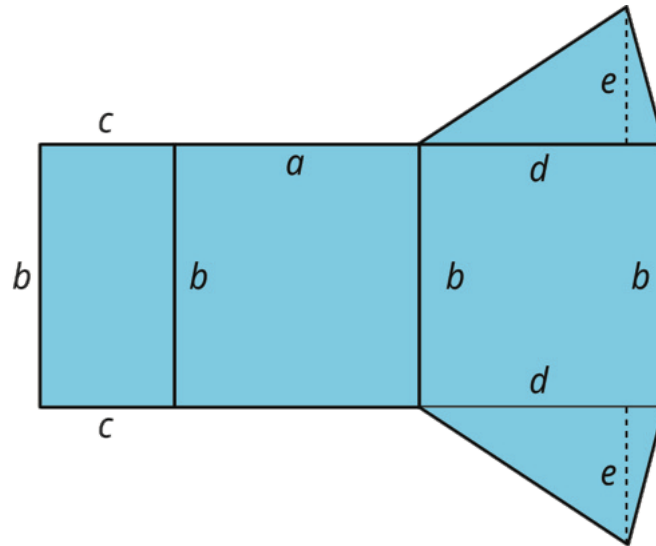
- a. What polyhedron can be assembled from this net?



- b. What information would you need to find its surface area? Be specific, and label the diagram as needed.

Solution

- a. Triangular prism
- b. Length and width of each rectangular face (as shown in the diagram), as well as the height of the triangular faces

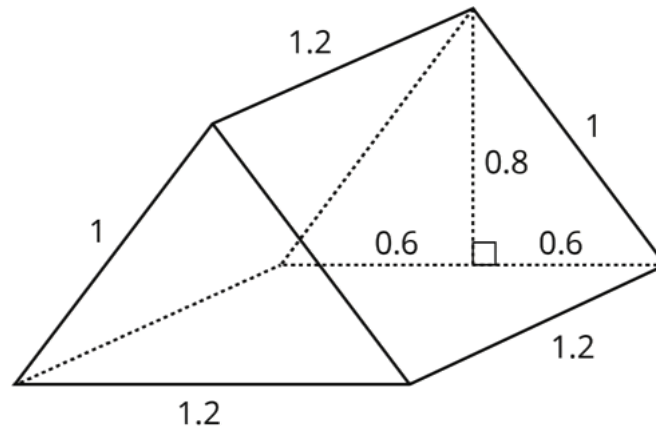


(From Unit 1, Lesson 14.)

Problem 7

Statement

Find the surface area of this triangular prism. All measurements are in meters.



Solution

4.8 square meters. Sample reasoning:

- There are two triangular faces with an area of 0.48 square meters each.
 $\frac{1}{2} \cdot (1.2) \cdot (0.8) = 0.48.$
- There are two rectangular faces with area of 1.2 square meters each. $1 \cdot (1.2) = 1.2.$
- There is one rectangular face with an area of $(1.2) \cdot (1.2) = 1.44$ square meters.
- $2 \cdot (0.48) + 2 \cdot (1.2) + (1.44) = 4.8$, or 4.8 square meters.

(From Unit 1, Lesson 15.)

Lesson 18: Surface Area of a Cube

Goals

- Generalize a process for finding the surface area of a cube, and justify (orally) why this can be abstracted as $6 \cdot s^2$.
- Interpret (orally) expressions that include repeated addition, multiplication, repeated multiplication, or exponents.
- Write expressions, with or without exponents, to represent the surface area of a given cube.

Learning Targets

- I can write and explain the formula for the surface area of a cube.
- When I know the edge length of a cube, I can find its surface area and express it using appropriate units.

Lesson Narrative

In this lesson, students practice using exponents of 2 and 3 to express products and to write square and cubic units. Along the way, they look for and make use of structure in numerical expressions (MP7). They also look for and express regularity in repeated reasoning (MP8) to write the formula for the surface area of a cube. Students will continue this work later in the course, in the unit on expressions and equations.

Note: Students will need to bring in a personal collection of 10–50 small objects ahead of time for the first lesson of the next unit. Examples include rocks, seashells, trading cards, or coins.

Alignments

Addressing

- 6.EE.A.1: Write and evaluate numerical expressions involving whole-number exponents.
- 6.EE.A.2.a: Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation “Subtract y from 5” as $5 - y$.
- 6.G.A.4: Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect
- Think Pair Share

Required Materials

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Student Learning Goals

Let's write a formula to find the surface area of a cube.

18.1 Exponent Review

Warm Up: 5 minutes

In this warm-up, students compare pairs of numerical expressions and identify the expression with the greater value. The task allows students to review what they learned about exponents and prompts them to look for and make use of structure in numerical expressions (MP7).

Students should do these without calculators and without calculating, although it is fine for them to check their answers with a calculator.

Addressing

- 6.EE.A.1

Launch

Give students 1–2 minutes of quiet think time. Ask them to answer the questions without multiplying anything or using a calculator, and to give a signal when they have an answer for each question and can explain their reasoning.

Anticipated Misconceptions

When given an expression with an exponent, students may misinterpret the base and the exponent as factors and multiply the two numbers. Remind them about the meaning of the exponent notation. For example, show that $5 \cdot 3 = 15$, which is much smaller than $5 \cdot 5 \cdot 5$, which equals 125.

Student Task Statement

Select the greater expression of each pair without calculating the value of each expression. Be prepared to explain your choices.

- $10 \cdot 3$ or 10^3
- 13^2 or $12 \cdot 12$

- $97 + 97 + 97 + 97 + 97 + 97$ or $5 \cdot 97$

Student Response

- 10^3 is greater because it is 1,000.
- 13^2 is greater because it is $13 \cdot 13$, and this will be greater than $12 \cdot 12$.
- $97 + 97 + 97 + 97 + 97 + 97$ is greater because it is $6 \cdot 97$, which is greater than $5 \cdot 97$.

Activity Synthesis

Ask one or more students to explain their reasoning for each choice. If not mentioned in students' explanations, highlight the structures in the expressions that enable us to evaluate each one without performing any calculations.

Point out, for example, that since we know that 10^3 means $10 \cdot 10 \cdot 10$, we can tell that it is much larger than $10 \cdot 3$.

For the last question, remind students that we can think of repeated addition in terms of multiple groups (i.e., that the sum of six 97s can be seen as six groups of 97 or $6 \cdot 97$). The idea of using groups to write equivalent expressions will support students as they write expressions for the surface area of a cube later in the lesson (i.e., writing the areas of all square faces of a cube as $6s^2$).

18.2 The Net of a Cube

20 minutes

This activity contains two sets of problems. The first set involves computations with simple numbers and should be solved numerically. Use students' work here to check that they are drawing a net correctly.

The second set encourages students to write expressions rather than to simplify them through calculations. The goal is to prepare students for the general rules s^3 and $6s^2$, which are more easily understood through an intermediate step involving numbers.

Note that students will be introduced to the idea that $5 \cdot x$ means the same as $5x$ in a later unit, so expect them to write $6 \cdot 17^2$ instead of $6(17^2)$. It is not critical that they understand that a number and a variable (or a number and an expression in parentheses) placed next to each other means they are being multiplied.

As students work on the second set, monitor the ways in which they write their expressions for surface area and volume. Identify those whose expressions include :

- products (e.g., $17 \cdot 17$ or $17 \cdot 17 \cdot 17$),
- sums of products (e.g., $(17 \cdot 17) + (17 \cdot 17) + \dots$),
- combination of like terms (e.g., $6 \cdot (17 \cdot 17)$),

- exponents (e.g., $17^2 + 17^2 + \dots$) or 17^3), and
- completed calculation (e.g., 289).

Select these students to share their work later. Notice the lengths of the expressions and sequence their explanations in order—from the longest expression to the most succinct.

Addressing

- 6.EE.A.1
- 6.G.A.4

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect
- Think Pair Share

Launch

Arrange students in groups of 2. Give students access to their geometry toolkits and 8-10 minutes of quiet work time. Tell students to try to answer the questions without using a calculator. Ask them to share their responses with their partner afterwards.

Support for Students with Disabilities

Representation: Develop Language and Symbols. Activate or supply background knowledge about calculating surface area and volume. Share examples of expressions for a cube in a few different forms to illustrate how surface area and volume can be expressed. Allow continued access to concrete manipulatives such as snap cubes for students to view or manipulate.

Supports accessibility for: Visual-spatial processing; Conceptual processing

Anticipated Misconceptions

Students might think the surface area is $(17 \cdot 17)^6$. Prompt students to write down how they would compute surface area step by step, before trying to encapsulate their steps in an expression. Dissuade students from using calculators in the last two problems and assure them that building an expression does not require extensive computation.

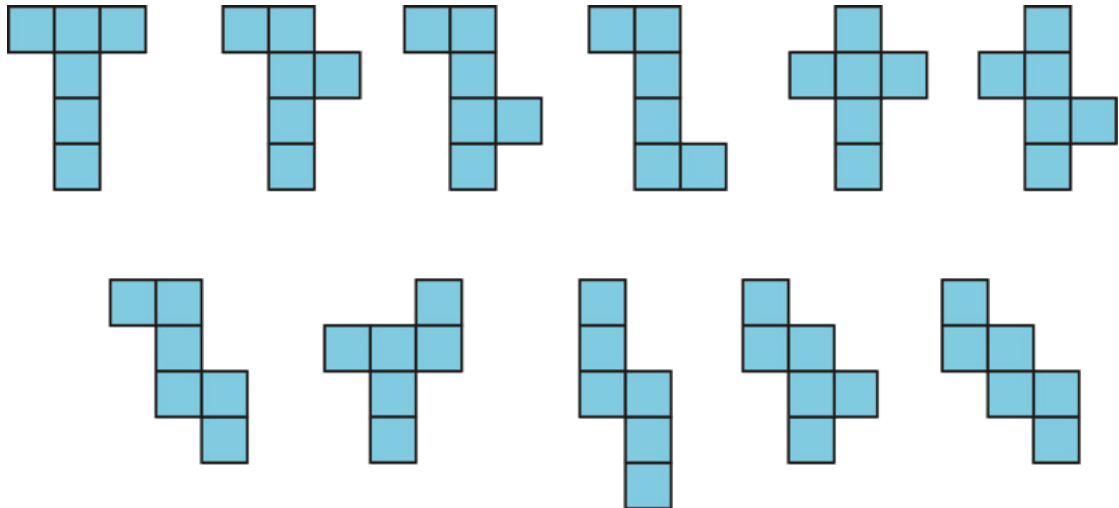
Students may think that refraining from using a calculator meant performing all calculations—including those of larger numbers—on paper or mentally, especially if they are unclear about the meaning of the term “expression.” Ask them to refer to the expressions in the warm-up, or share examples of expressions in a few different forms, to help them see how surface area and volume can be expressed without computation.

Student Task Statement

1. A cube has edge length 5 inches.
 - a. Draw a net for this cube, and label its sides with measurements.
 - b. What is the shape of each face?
 - c. What is the area of each face?
 - d. What is the surface area of this cube?
 - e. What is the volume of this cube?
2. A second cube has edge length 17 units.
 - a. Draw a net for this cube, and label its sides with measurements.
 - b. Explain why the area of each face of this cube is 17^2 square units.
 - c. Write an expression for the surface area, in square units.
 - d. Write an expression for the volume, in cubic units.

Student Response

1. For the cube that has edge length 5:
 - a. Drawings vary. 11 unique nets are possible:



- b. 25 square inches
 - c. 150 square inches
 - d. 125 cubic inches
2. For the cube that has edge length 17:
 - a. Drawings vary, but should be one of the 11 nets shown in the previous problem.

b. Answers vary. Sample explanation: The side length of each square face is 17 units, so its area is $17 \cdot 17$ or 17^2 square units.

c. $6 \cdot 17^2$ (or equivalent)

d. 17^3 (or equivalent)

Activity Synthesis

After partner discussions, select a couple of students to present the solutions to the first set of questions, which should be straightforward.

Then, invite previously identified students to share their expressions for the last two questions. If possible, sequence their presentation in the following order. If any expressions are missing but needed to illustrate the idea of writing succinct expressions, add them to the lists.

Surface area:

- $(17 \cdot 17) + (17 \cdot 17) + (17 \cdot 17) + (17 \cdot 17) + (17 \cdot 17) + (17 \cdot 17)$
- $17^2 + 17^2 + 17^2 + 17^2 + 17^2 + 17^2$
- $6 \cdot (17 \cdot 17)$
- $6 \cdot (17^2)$
- $6 \cdot (289)$
- 1,734

Volume:

- $17 \cdot 17 \cdot 17$
- 17^3
- 4,913

Discuss how multiplication can simplify expressions involving repeated addition and exponents can do the same for repeated multiplication. While the last expression in each set above is the simplest to write, getting there requires quite a bit of computation. Highlight $6 \cdot 17^2$ and 17^3 as efficient ways to express the surface area and volume of the cube.

As the class discusses the different expressions, consider directing students' attention to the units of measurements. Remind students that, rather than writing $6 \cdot (17^2)$ *square units*, we can write $6 \cdot (17^2)$ *units²*, and instead of 17^3 *cubic units*, we can write 17^3 *units³*. Unit notations will appear again later in the course, so it can also be reinforced later.

If students are not yet ready for the general formula, which comes next, offer another example. For instance, say: "A cube has edge length 38 cm. How can we express its surface area and volume?"

Help students see that its surface area is $6 \cdot (38^2)$ cm² and its volume is 38^3 cm³. The large number will discourage calculation and focus students on the form of the expressions they are building and the use of exponents.

Support for English Language Learners

Representing, Conversing: MLR7 Compare and Connect. Use this routine to prepare students for the whole-class discussion. At the appropriate time, invite groups to create a visual display showing their strategy and calculations for the surface area and volume of a cube with an edge length of 17 units. Allow students time to quietly circulate and analyze the strategies in at least 2 other displays in the room. Give students quiet think time to consider what is the same and what is different. Next, ask students to return to their original group to discuss what they noticed. Listen for and amplify observations that highlight the advantages and disadvantages to each method and their level of succinctness. This will help students make connections between calculations of cubes, regardless of the edge length.

Design Principle(s): Optimize output; Cultivate conversation

18.3 Every Cube in the Whole World

10 minutes

In this activity, students build on what they learned earlier and develop the formulas for the surface area and the volume of a cube in terms of a variable edge length s .

Encourage students to refer to their work in the preceding activity as much as possible and to generalize from it. As before, monitor for different ways of writing expressions for surface area and volume. Identify students whose work includes the following:

- products (e.g., $s \cdot s$, or $s \cdot s \cdot s$),
- sums of products (e.g., $(s \cdot s) + (s \cdot s) + \dots$),
- combination of like terms (e.g., $6 \cdot (s \cdot s)$), and
- exponents (e.g., $s^2 + s^2 + \dots$, or s^3).

Select these students to share their work later. Again, notice the lengths of the expressions and sequence their explanations in order—from the longest expression to the most succinct.

Addressing

- 6.EE.A.2.a
- 6.G.A.4

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

Launch

Give students access to their geometry toolkits and 7–8 minutes of quiet think time. Tell students they will be answering on the same questions as before, but with a variable for the side length. Encourage them to use the work they did earlier to help them here.

Anticipated Misconceptions

If students are unclear or unsure about using the variable s , explain that we are looking for an expression that would work for any edge length, and that a variable, such as s , can represent any number. The s could be replaced with any edge length in finding surface area and volume.

To connect students' work to earlier examples, point to the cube with edge length 17 units from the previous activity. Ask: "If you wrote the surface area as $6 \cdot 17^2$ before, what should it be now?"

As students work, encourage those who may be more comfortable using multiplication symbols to instead use exponents whenever possible.

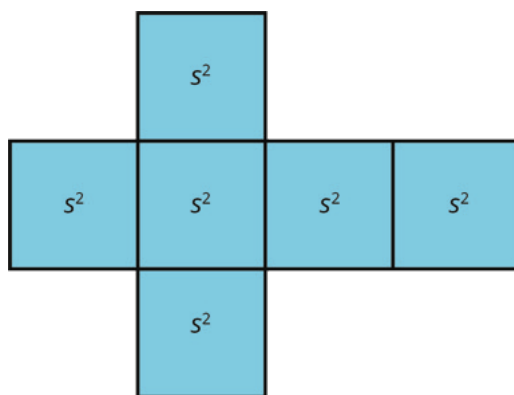
Student Task Statement

A cube has edge length s .

1. Draw a net for the cube.
2. Write an expression for the area of each face. Label each face with its area.
3. Write an expression for the surface area.
4. Write an expression for the volume.

Student Response

1. Drawings vary. Here is one possible labeled net (each face is a square whose side lengths are s):



2. The area of each face is s^2 .
3. The surface area is $6 \cdot s^2$.
4. The volume is s^3 .

Activity Synthesis

Discuss the problems in as similar a fashion as was done in the earlier activity involving a cube with edge length 17 units. Doing so enables students to see structure in the expressions (MP7) and to generalize through repeated reasoning (MP8).

Select previously identified students to share their responses with the class. If possible, sequence their presentation in the following order to help students see how the expressions $6 \cdot s^2$ and s^3 come about. If any expressions are missing but needed to illustrate the idea of writing succinct expressions, add them to the lists.

Surface area:

- $(s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s) + (s \cdot s)$
- $s^2 + s^2 + s^2 + s^2 + s^2 + s^2$
- $6(s \cdot s)$
- $6 \cdot (s^2)$ or $6 \cdot s^2$

Volume

- $s \cdot s \cdot s$
- s^3

Refer back to the example involving numerical side length (a cube with edge length 17 units) if students have trouble understanding where the most concise expression of surface area comes from.

Present the surface area as $6 \cdot s^2$. You can choose to also write it as $6s^2$.

Lesson Synthesis

Review the formulas for volume and surface area of a cube.

- The volume of a cube with edge length s is s^3 .
- A cube has 6 faces that are all identical squares. The surface area of a cube with edge length s is $6 \cdot s^2$.

18.4 From Volume to Surface Area

Cool Down: 5 minutes

Addressing

- 6.EE.A.1
- 6.G.A.4

Student Task Statement

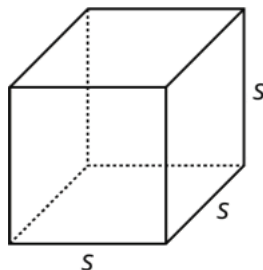
1. A cube has edge length 11 inches. Write an expression for its volume and an expression for its surface area.
2. A cube has a volume of 7^3 cubic centimeters. What is its surface area?

Student Response

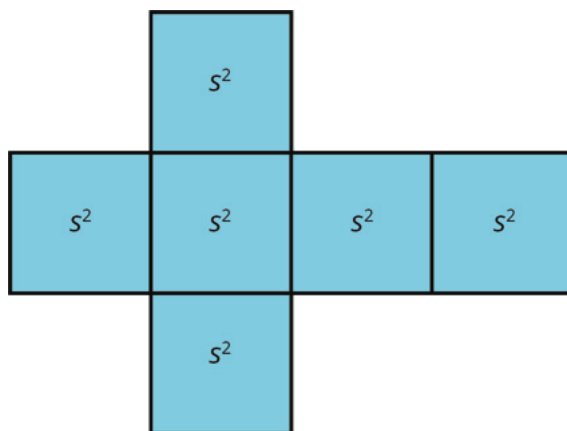
1. Volume: 11^3 or $11 \cdot 11 \cdot 11$. Surface area: $6 \cdot (11 \cdot 11)$ (or equivalent).
2. The surface area is $6 \cdot 7^2$, which is 294 square centimeters.

Student Lesson Summary

The volume of a cube with edge length s is s^3 .



A cube has 6 faces that are all identical squares. The surface area of a cube with edge length s is $6 \cdot s^2$.



Lesson 18 Practice Problems

Problem 1

Statement

- a. What is the volume of a cube with edge length 8 in?
- b. What is the volume of a cube with edge length $\frac{1}{3}$ cm?

c. A cube has a volume of 8 ft^3 . What is its edge length?

Solution

a. 512 cu in ($8 \cdot 8 \cdot 8 = 512$)

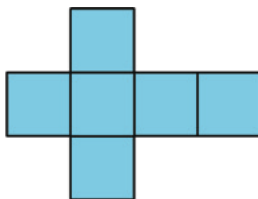
b. $\frac{1}{27} \text{ cu cm}$ ($\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{27}$)

c. 2 ft ($2 \cdot 2 \cdot 2 = 8$)

Problem 2

Statement

a. What three-dimensional figure can be assembled from this net?



b. If each square has a side length of 61 cm , write an expression for the surface area and another for the volume of the figure.

Solution

a. Cube

b. The surface area is $6 \cdot 61^2 \text{ sq cm}$, and the volume is 61^3 cu cm .

Problem 3

Statement

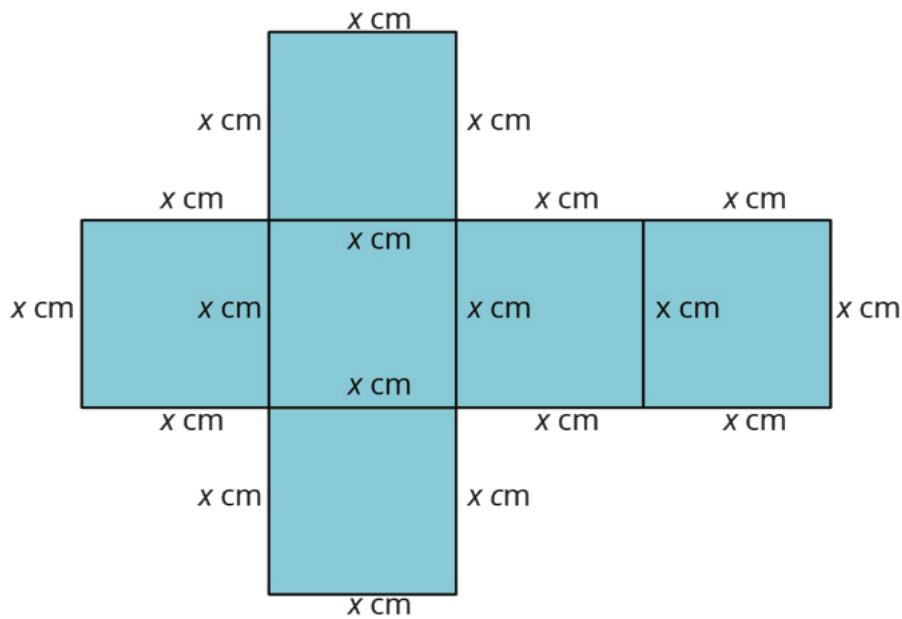
a. Draw a net for a cube with edge length $x \text{ cm}$.

b. What is the surface area of this cube?

c. What is the volume of this cube?

Solution

a.



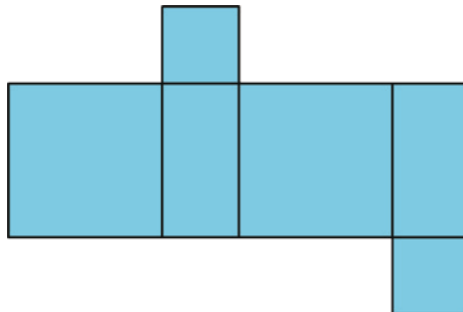
b. $6x^2$ sq cm (or equivalent)

c. $x \cdot x \cdot x$ cu cm (or equivalent)

Problem 4

Statement

Here is a net for a rectangular prism that was not drawn accurately.

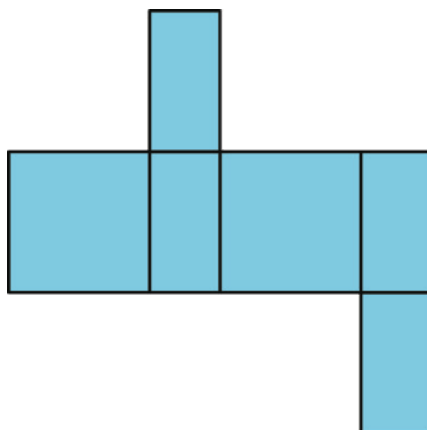


- Explain what is wrong with the net.
- Draw a net that can be assembled into a rectangular prism.
- Create another net for the same prism.

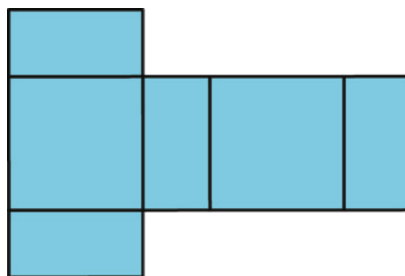
Solution

- When the shape is folded, the two small squares are not the right size to close the three-dimensional figure. The small squares can be replaced with rectangles as in the picture, or the large squares can be the same size and shape as the two (non-square) rectangles in the net.

b.



c.

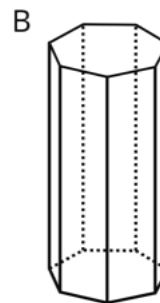
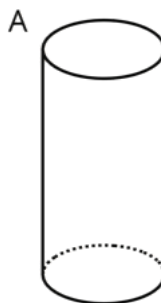


(From Unit 1, Lesson 14.)

Problem 5

Statement

State whether each figure is a polyhedron.
Explain how you know.



Solution

Figure A is not a polyhedron. It has a curved surface and there are faces that are not polygons.

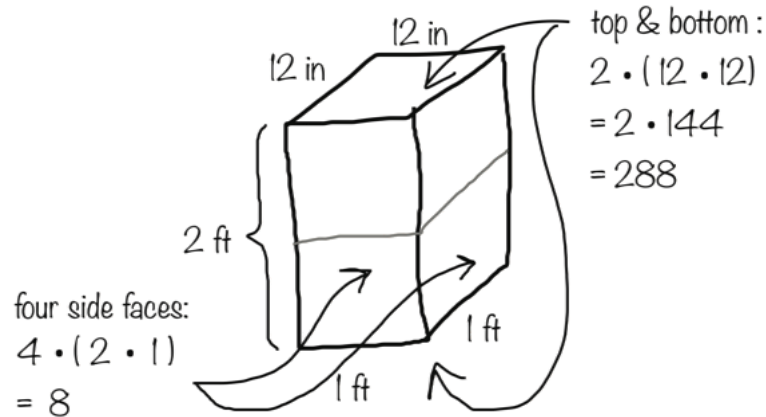
Figure B is a polyhedron. It is composed of polygons and each side of every polygon joins a side of another polygon.

(From Unit 1, Lesson 13.)

Problem 6

Statement

Here is Elena's work for finding the surface area of a rectangular prism that is 1 foot by 1 foot by 2 feet.



She concluded that the surface area of the prism is 296 square feet. Do you agree with her? Explain your reasoning.

Solution

Disagree. Sample reasoning: Elena calculated the area of the top and bottom faces in square inches but the area of the side faces in square feet. The combined area of the top and bottom faces is 2 square feet, so the correct surface area is 10 square feet.

(From Unit 1, Lesson 12.)

Section: Let's Put it to Work

Lesson 19: Designing a Tent

Goals

- Apply understanding of surface area to estimate the amount of fabric in a tent, and explain (orally and in writing) the estimation strategy.
- Compare and contrast (orally) different tent designs.
- Interpret information (presented in writing and through other representations) about tents and sleeping bags.

Learning Targets

- I can apply what I know about the area of polygons to find the surface area of three-dimensional objects.
- I can use surface area to reason about real-world objects.

Lesson Narrative

In this culminating lesson, students use what they learned in this unit to design a tent and determine how much fabric is needed for the tent. The task prompts students to model a situation with the mathematics they know, make assumptions, and plan a path to solve a problem (MP4). It also allows students to choose tools strategically (MP5) and to make a logical argument to support their reasoning (MP3).

The lesson has two parts. In the first part, students learn about the task, gather information, and begin designing. The introduction is important to ensure all students understand the context. Then, after answering some preparatory questions in groups and as a class, students work individually to design and draw their tents. They use their knowledge of area and surface area to calculate and justify an estimate of the amount of fabric needed for their design.

The second part involves reflection and discussion on students' work. Students explain their work to a partner or small group, discuss and compare their designs, and consider the impact of design decisions on the surface areas of their tents.

Depending on instructional choices made, this lesson could take one or more class meetings. The time estimates are intentionally left blank, as the time needed will vary based on instructional decisions made. It may depend on:

- whether students use the provided information about tents and sleeping bags or research this information.
- whether the Tent Design Planning Sheet is provided or students organize their work with more autonomy.

- expectations around drafting, revising, and the final product.
- how student work is ultimately shared with the class (not at all, informally, or with formal presentations).

Note: Students will need to bring in a personal collection of 10–50 small objects ahead of time for the first lesson of the next unit. Examples include rocks, seashells, trading cards, or coins.

Alignments

Addressing

- 6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
- 6.G.A.4: Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Instructional Routines

- MLR7: Compare and Connect

Required Materials

Copies of Instructional master

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6–8 they are listed as a separate Required Material.

Required Preparation

Prepare one copy of the Instructional master for each student.

Student Learning Goals

Let's design some tents.

19.1 Tent Design - Part 1

This activity has two parts: an introduction to the task and individual work time. In the first part, students read the design problem and ask clarifying questions, and then work with a partner or two to look at tent designs and specifications. Then, they work individually to design a tent, create

necessary representations of it, calculate its surface area, and estimate of the amount of fabric needed to construct it.

As students work individually, circulate and focus your observations on two main goals:

1. Notice the strategies and mathematical ideas students use to complete the task. Are students:
 - decomposing or rearranging parts of their tent design to find the area? How?
 - drawing a net of their design?
 - labeling their drawings with measurements?
 - calculating area precisely?
 - using formulas they learned in this unit? How?
 - accounting for the areas of all surfaces of their tent design?
 - using square units for area measures?
2. To record the sizes (in terms of numbers of people accommodated) and shapes of individual tent designs. Use this information to arrange students into groups—by tent size—in the next activity.

Collect student work at the end of the session. Arrange for 2–3 students who have tents that accommodate the same number of people but different designs to work together (e.g., two students design tents for three people, but one designed a triangular prism and the other a pentagonal prism). Put their papers together to begin the second session.

Addressing

- 6.G.A.1
- 6.G.A.4

Instructional Routines

- MLR7: Compare and Connect

Launch

Give students 1–2 minutes to read the task statement individually and ask any clarifying questions. At this point, students only need to understand that the tents need to accommodate same-sized sleeping bags and that there is not one right way to design them.

Next, arrange students in groups of 2. Give groups about 15 minutes to look at and discuss potential tent designs, tent specifications, and sleeping bag information. Tell students that the designs are provided for inspiration and reference, but students are not limited to them.

After partner discussions, give each student a copy of the Tent Design Planning Sheet from the Instructional master. Give students quiet think time to sketch out their tent design, create necessary

drawings, calculate surface area, and justify their estimate. Provide blank paper for students to use to draw their designs and access to their geometry toolkits. (Note that a scale drawing is not an expectation; scale factor is a grade 7 standard.)

Support for Students with Disabilities

Engagement: Internalize Self-Regulation. Provide a project checklist that chunks the various steps of the project into a set of manageable tasks.

Supports accessibility for: Organization; Attention

Anticipated Misconceptions

Some students may find it challenging to develop and represent a three-dimensional object on paper. Ask them what might help them create or convey their design. Some may find it useful to think in two-dimensional terms and start by drawing a net. Others may wish to build a physical model of their design from paper or other flexible material, or to use a digital drawing tool. Encourage students to consider the tools at their disposal and choose those that would enable them to complete the task (MP5).

Student Task Statement

Have you ever been camping?

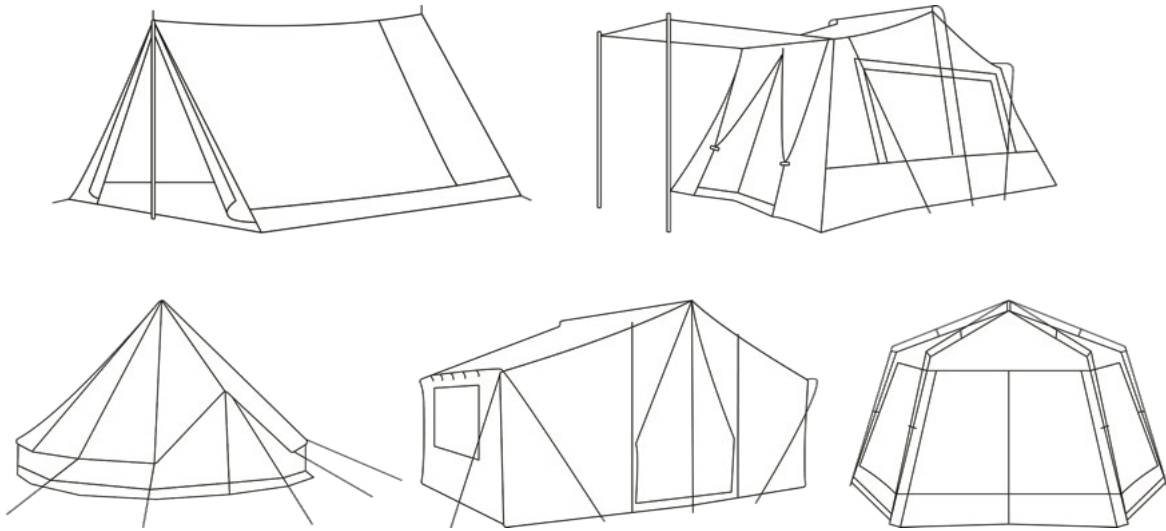
You might know that sleeping bags are all about the same size, but tents come in a variety of shapes and sizes.

Your task is to design a tent to accommodate up to four people, and estimate the amount of fabric needed to make your tent. Your design and estimate must be based on the information given and have mathematical justification.

First, look at these examples of tents, the average specifications of a camping tent, and standard sleeping bag measurements. Talk to a partner about:

- Similarities and differences among the tents
- Information that will be important in your designing process
- The pros and cons of the various designs

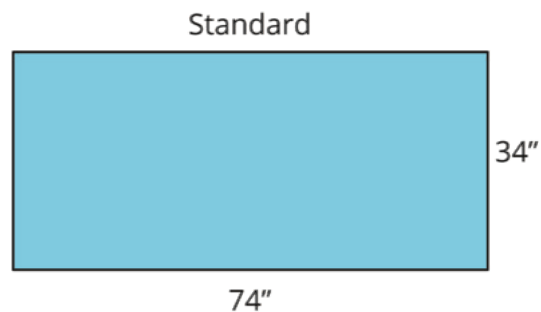
Tent Styles



Tent Height Specifications

height description	height of tent	notes
sitting height	3 feet	Campers are able to sit, lie, or crawl inside tent.
kneeling height	4 feet	Campers are able to kneel inside tent. Found mainly in 3-4 person tents.
stooping height	5 feet	Campers are able to move around on their feet inside tent, but most campers will not be able to stand upright.
standing height	6 feet	Most adult campers are able to stand upright inside tent.
roaming height	7 feet	Adult campers are able to stand upright and walk around inside tent.

Sleeping Bag Measurements



1. Create and sketch your tent design. The tent must include a floor.

2. What decisions were important when choosing your tent design?
3. How much fabric do you estimate will be necessary to make your tent? Show your reasoning and provide mathematical justification.

Student Response

Answers vary.

Activity Synthesis

After students complete the task, engage students in a whole-class discussion. Ask students: “What were important things you had to think about in your design?”

Collect student work at the end of the session. Tell students they will continue to think about the problem and their proposed solution in the next activity.

Arrange for 2–3 students who have tents that accommodate the same number of people but have different designs to work together (e.g., two students designed tents for three people, but one designed a triangular prism and the other a pentagonal prism). Put their papers together to begin the second session.

Support for English Language Learners

Representing, Conversing: MLR7 Compare and Connect. Use this routine to help students consider audience when preparing to display their work and prepare students for the discussion in the next activity. At the appropriate time, invite students to create a visual display showing their tent design and response to the task questions. Display the list of items that should be included on the display and ask students, “What kinds of details could you include on your display to help a reader understand your tent design, what decisions you made in your design, and how much fabric you will need?” Record ideas and display for all to see. Examples of these types of details or annotations include: the order in which responses are organized on the display, the clarity of any drawn diagrams, written notes or details to clarify diagrams, use of specific vocabulary or phrases, or color or arrows to show connections between representations. If time allows, after the gallery walk, ask students to describe specific examples of additional details that other groups used that helped them to interpret and understand their displays.

Design Principle(s): Maximize meta-awareness; Optimize output

19.2 Tent Design - Part 2

This activity gives students a chance to explain and reflect on their work. In groups of 2–3, they share drawings of their tent design, an estimate of the amount of fabric needed, and the justification. They compare their creations with one or more peers. Students discuss not only the amount of fabric required, but also the effects that different designs have on that amount.

Prior to the session, identify 2–3 students who have tents that accommodate the same number of people but different designs (e.g., two students each design a 3-person tent, but one designed a triangular prism and the other a pentagonal prism). Put their papers from Part 1 together.

As students discuss in groups, notice how they reason about and communicate their work. Do they:

- provide justification for their measurements and choices?
- explain clearly their process of calculating surface area?
- see how the type of design affects the amount of fabric?
- compare their tents in terms of the differences in the measurements at the base and the height of tent?

Addressing

- 6.G.A.1
- 6.G.A.4

Launch

Tell students that they will now reflect on and discuss their tents with another student who designed a tent for the same number of people but in a different way. Arrange students in the predetermined groups of 2–3 and return the presorted sets of papers to them.

Student Task Statement

1. Explain your tent design and fabric estimate to your partner or partners. Be sure to explain why you chose this design and how you found your fabric estimate.
2. Compare the estimated fabric necessary for each tent in your group. Discuss the following questions:
 - Which tent design used the least fabric? Why?
 - Which tent design used the most fabric? Why?
 - Which change in design most impacted the amount of fabric needed for the tent? Why?

Student Response

Answers vary.

Activity Synthesis

Much of the discussion will take place within the groups. Once groups have had an opportunity to share their designs, reconvene as a class. One idea would be to display tent designs that used the most and the least amount of fabric. Also consider asking students to reflect on the following prompts:

- What design choices lead to using less fabric?
 - What design choices lead to using more fabric?
 - What are some ways that tents designed to accommodate the same number of people could use very different amounts of fabric?
 - When calculating the surface area of your tent, what kinds of techniques from this unit did you find useful?
-

Support for Students with Disabilities

Action and Expression: Develop Expression and Communication. Provide sentence frames to support student explanations. Display sentence frames such as: "We chose our tent design because . . .", "This tent design uses the least/most fabric because . . ."

Supports accessibility for: Language; Organization

Lesson Synthesis

This culminating lesson could be wrapped up in a number of ways, depending on the time available and your goals and expectations. You could choose a simple wrap-up discussion, or assign students to develop a more elaborate presentation of their tent design involving posters or three-dimensional models of their tents.

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Family Support
Materials

Family Support Materials

Area and Surface Area

Reasoning to Find Area

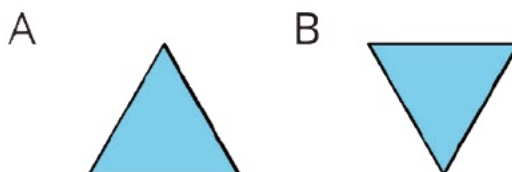
Family Support Materials 1

Before grade 6, your student learned to measure the area of a shape by finding the number of unit squares that cover the shape without gaps or overlaps. For example, the orange and blue shapes each have an area of 8 square units.

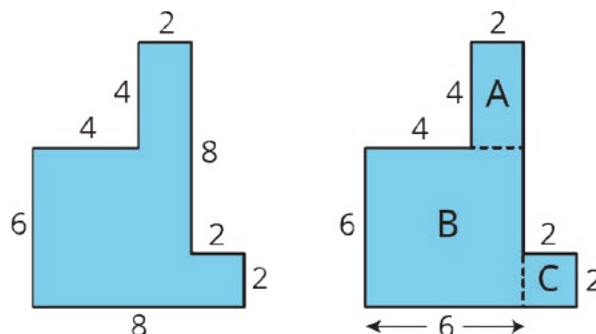


In grade 6, students learn to find the areas of more complicated shapes using two ideas:

- Two shapes that “match up exactly” have the same area. For example, triangles A and B have the same area because Triangle A can be placed on Triangle B so they match up exactly.

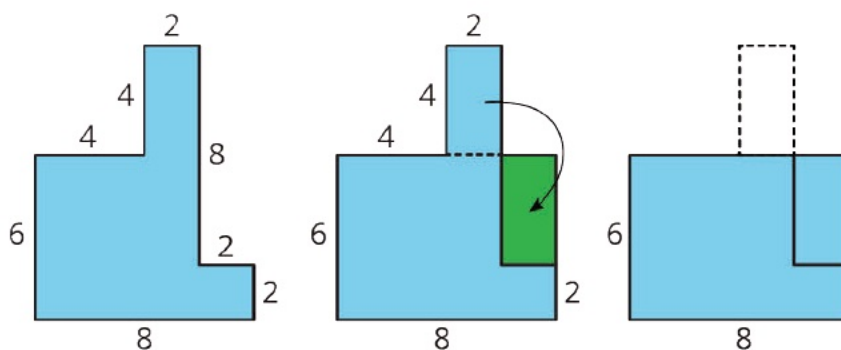


- We can decompose (break) a shape into smaller pieces and find its area by adding the areas of the pieces. For example, the area of the shape on the left is equal to the area of Rectangle A, plus the area of Rectangle B, plus the area of Rectangle C.



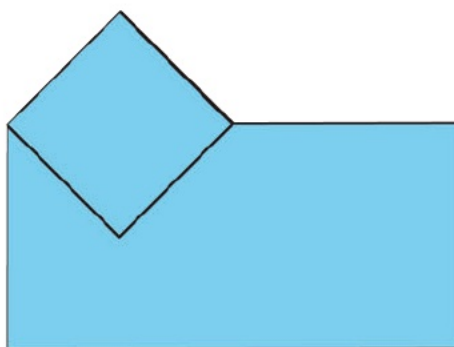
It is sometimes helpful to rearrange the pieces of a shape in order to find its area. For example, the rectangular piece that is 2 units by 4 units at the top of the shape can be

broken and rearranged to make a simple rectangle that is 8 units and 6 units. We can easily find the area of this rectangle (48 square units, because $8 \times 6 = 48$).



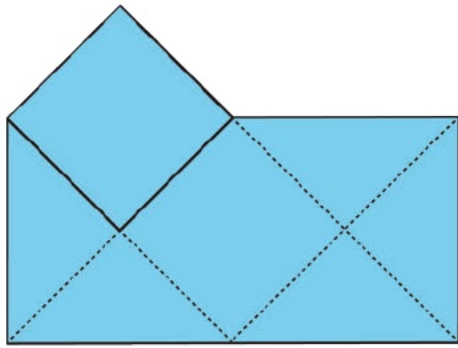
Here is a task to try with your student:

The area of the square is 1 square unit. Find the area of the entire shaded region. Show your reasoning.



Solution:

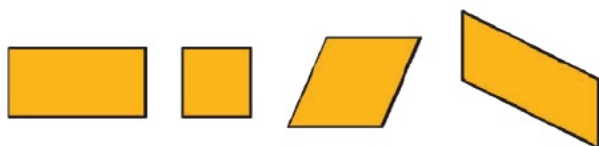
$4\frac{1}{2}$ square units. Sample reasoning: The rest of the region can be decomposed into a square and several triangles. Two triangles can be arranged to match up perfectly with a square, so each triangle has half the area of the square ($\frac{1}{2}$ square units). In the entire shape, there is a total of 2 squares (2 square units) and 5 triangles ($5 \times \frac{1}{2}$ or $2\frac{1}{2}$ square units). $2 + 2\frac{1}{2} = 4\frac{1}{2}$.



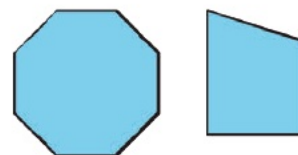
Parallelograms

Family Support Materials 2

This week, your student will investigate **parallelograms**, which are four-sided figures whose opposite sides are parallel.

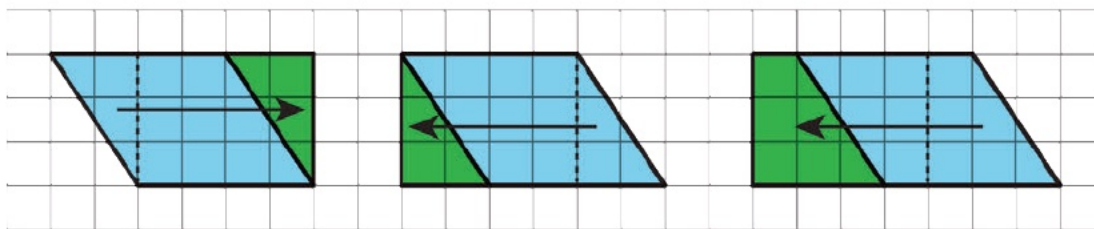


Parallelograms



Not Parallelograms

We can find the **area of a parallelogram** by breaking it apart and rearranging the pieces to form a rectangle. The diagram shows a few ways of rearranging pieces of a parallelogram. In each one, the result is a rectangle that is 4 units by 3 units, so its area is 12 square units. The area of the original parallelogram is also 12 square units.

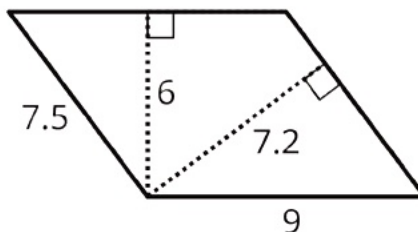


Using these strategies allows students to notice pairs of measurements that are helpful for finding the area of any parallelogram: a **base** and a corresponding **height**. The length of any side of a parallelogram can be used as a base. The height is the distance from the base to the opposite side, measured at a right angle. In the parallelogram shown here, we can say that the horizontal side that is 4 units long is the base and the vertical segment that is 3 units is the height that corresponds to that base.

The area of any parallelogram is $base \times height$.

Here is a task to try with your student:

Elena and Noah are investigating this parallelogram.



Elena says, "If the side that is 9 units is the base, the height is 7.2 units. If the side that is 7.5 units is the base, the corresponding height is 6 units."

Noah says, "I think if the base is 9 units, the corresponding height is 6 units. If the base is 7.5 units, the corresponding height is 7.2 units."

Do you agree with either one of them? Explain your reasoning.

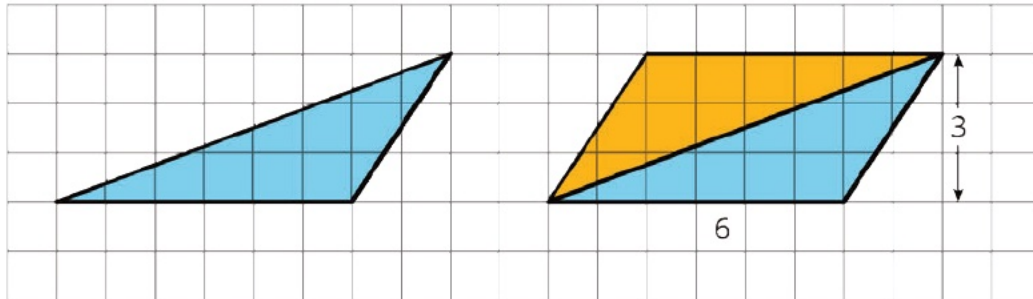
Solution:

Agree with Noah. Explanations vary. Sample explanation: A corresponding height must be perpendicular (drawn at a right angle) to the side chosen as the base. The dashed segment that is 6 units is perpendicular to the two parallel sides that are 9 units long. The dashed segment that is 7.2 units long is perpendicular to the two sides that are 7.5 units.

Triangles

Family Support Materials 3

Your student will now use their knowledge of the area of parallelograms to find the area of triangles. For example, to find the area of the blue triangle on the left, we can make a copy of it, rotate the copy, and use the two triangles to make a parallelogram.



This parallelogram has a base of 6 units, a height of 3 units, and an area of 18 square units. So the area of each triangle is half of 18 square units, which is 9 square units.

A triangle also has **bases** and corresponding **heights**. Any side of a triangle can be a base. Its corresponding height is the distance from the side chosen as the base to the opposite corner, measured at a right angle. In this example, the side that is 6 units long is the base and the height is 3 units.

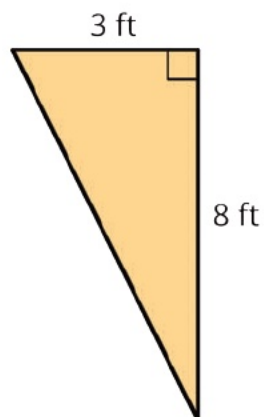
Because two copies of a triangle can always be arranged to make a parallelogram, the area of a triangle is always half of the area of a parallelogram with the same pair of base and height. We can use this formula to find the area of any triangle:

$$\frac{1}{2} \times \text{base} \times \text{height}$$

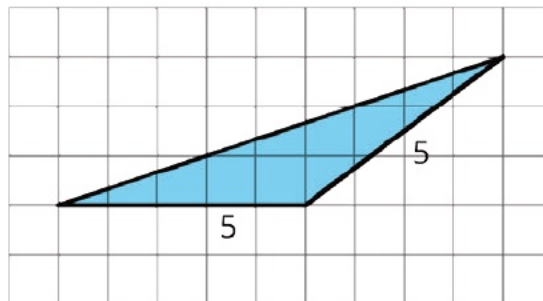
Here is a task to try with your student:

Find the area of each triangle. Show your reasoning.

1.



1.



Solution:

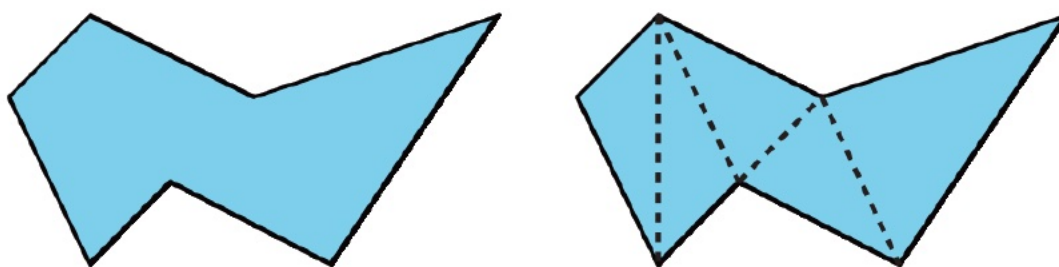
1. 12 square feet. Sample reasoning: The triangle is half of a rectangle that is 3 feet by 8 feet, which has an area of 24 square feet.
2. $\frac{15}{2}$ square units. Sample reasoning: The triangle is half of a parallelogram with a base of 5 units and a height of 3 units. $\frac{1}{2} \cdot 5 \cdot 3 = \frac{15}{2}$.

Polygons

Family Support Materials 4

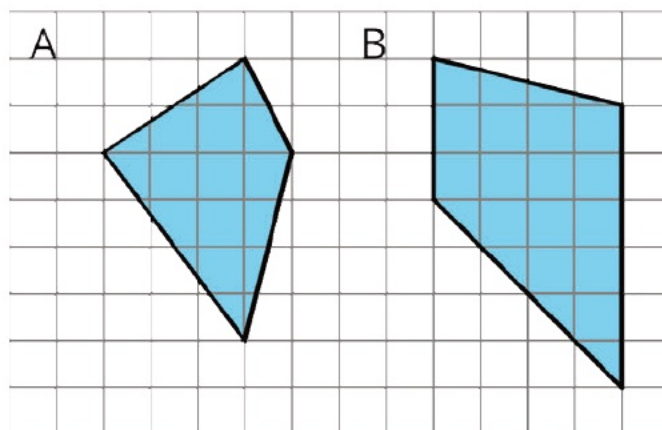
Knowing how to find the area of triangles allows your student to find the area of polygons, which are two-dimensional shapes made up of line segments. The line segments meet one another only at their end points. Triangles, quadrilaterals, pentagons, and hexagons are all polygons.

To find the area of *any* polygon, we can break it apart into rectangles and triangles. Here is a polygon with 7 sides and one way to break it apart into triangles. Finding the areas of all triangles and adding them gives the area of the original polygon.



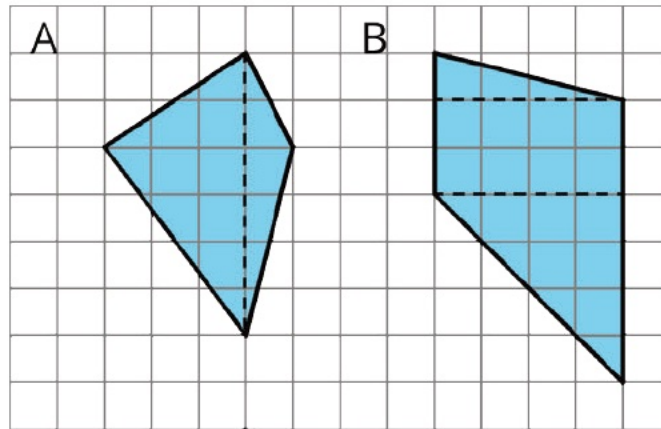
Here is a task to try with your student:

Find the area of polygons A and B. Explain or show your reasoning.



Solution:

A: 12 square units, B: 18 square units. Sample diagram and explanations:



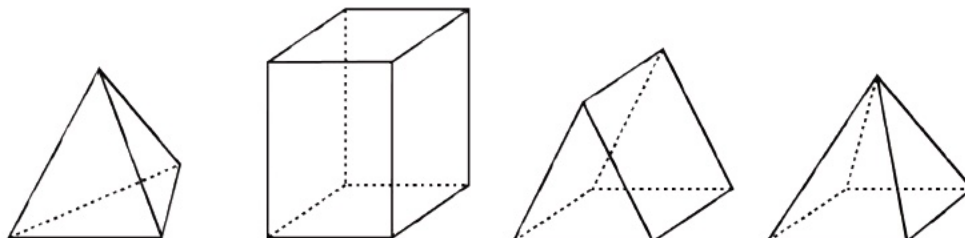
Polygon A can be broken into two triangles. The one on the left has base 6 units and height 3 units, so its area is 9 square units ($\frac{1}{2} \cdot 6 \cdot 3 = 9$). The one on the right has base 6 units and height 1 unit, so its area is 3 square units ($\frac{1}{2} \cdot 6 \cdot 1 = 3$). The total area is $9 + 3$ or 12 square units.

Polygon B can be broken into a rectangle and two triangles. The area of the top triangle is $\frac{1}{2} \cdot 4 \cdot 1$ or 2 square units. The rectangle is 8 square units. The area of the bottom triangle is $\frac{1}{2} \cdot 4 \cdot 4$ or 8 square units. $2 + 8 + 8 = 18$

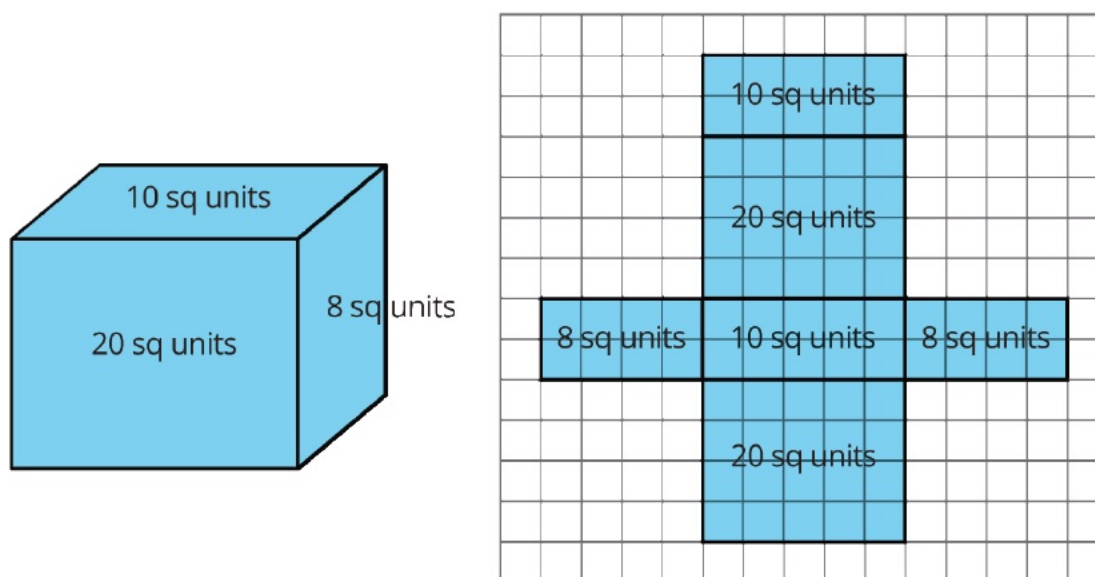
Surface Area

Family Support Materials 5

Imagine painting all of the sides of a box. The amount of surface to be covered with paint is the surface area of the box. Your student will focus on finding the surface areas of different three-dimensional objects such as the prisms and pyramids shown here.

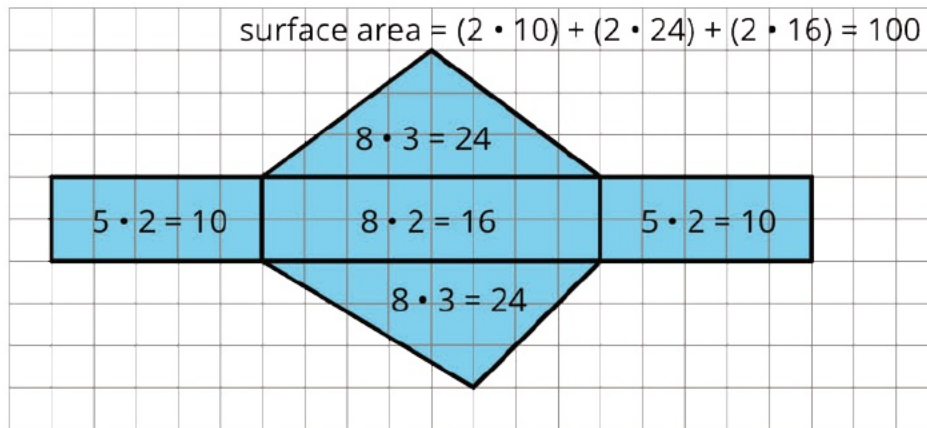


One way to find the surface area of a three-dimensional object is to draw its net, which shows all the faces of the object as a two-dimensional drawing. A net can be cut out and folded to make the object. To find the surface area of the object, we can find the area of each face (as shown on the net) and add them. The areas of the six rectangular faces shown add up to 76 square units because $10 + 20 + 10 + 20 + 8 + 8 = 76$, so the surface area of this box is 76 square units.



Here is a task to try with your student:

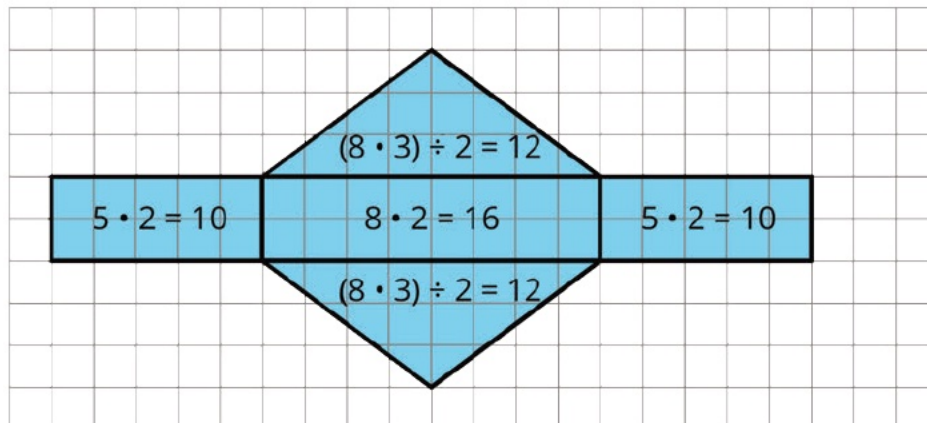
Andre drew a net of a triangular prism and calculated its surface area. He made an error in both the net drawing and in the calculation.



1. Identify Andre's errors.
2. Find the correct surface area for the prism. Show your reasoning.

Solution:

1. Net: The triangles in a triangular prism should be identical, but the net shows two different triangles. Calculation: There are a few errors. The area of each triangle should be $\frac{1}{2} \cdot 8 \cdot 3$ or 12 square units. Andre did not multiply the base and height by half. The wrong calculation is repeated for both triangles. In the calculation for the surface area, Andre doubled the area of the largest rectangle (which is 16 square units) while there is only one rectangle with that area.
2. The surface area should be 60 square units. The combined area of the two triangles should be $2(\frac{1}{2} \cdot 8 \cdot 3)$ or 24 square units. $10 + 10 + 16 + 24 = 60$. Sample corrected net:

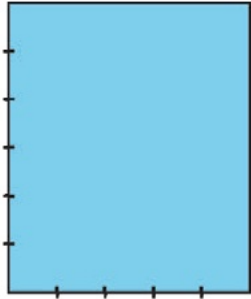


Unit Assessments

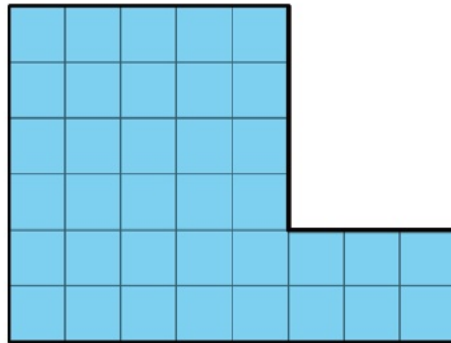
Check Your Readiness A and B
Mid-Unit Assessment A and B
Eod-of-Unit Assessment A and B

Area and Surface Area: Check Your Readiness (A)

1. The rectangle has sides measuring 6 cm and 5 cm. What is the area of this rectangle?
Explain your reasoning.



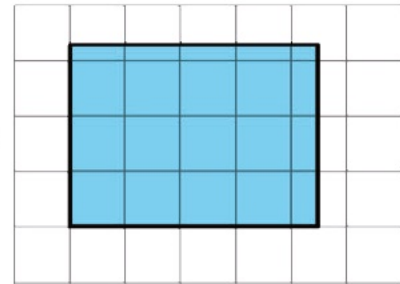
2. Select all the expressions that give the area of the figure in square units.



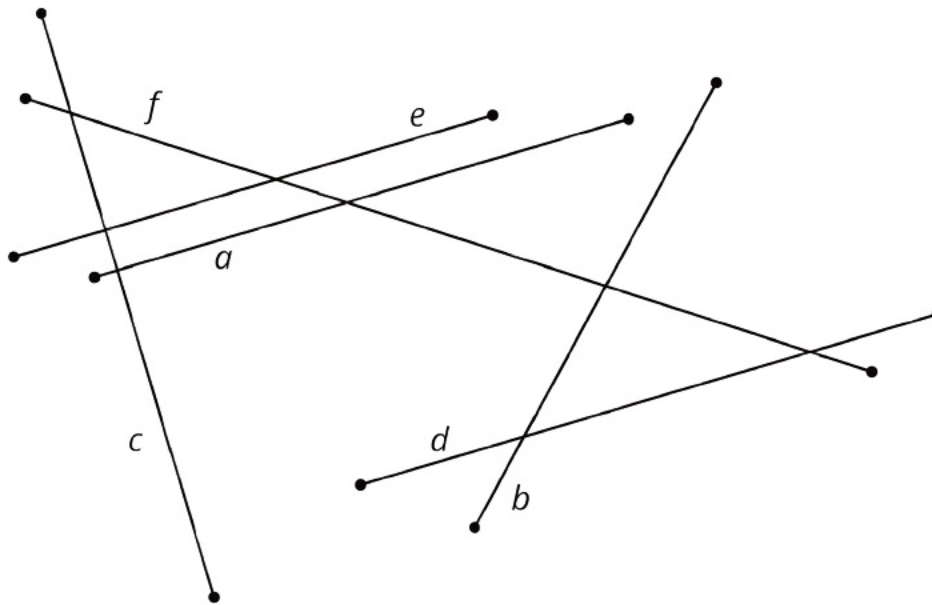
- A. $(5 + 6) \times (3 + 2)$
- B. $(8 \times 6) - (3 \times 4)$
- C. $6 \times 8 \times 2 \times 3 \times 4 \times 5$
- D. $(5 \times 6) + (3 \times 2)$
- E. 8×6

3. Each small square in the graph paper represents 1 square unit.

Which expression is closest to the area of the rectangle, in square units?



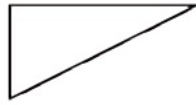
- A. $5\frac{1}{2} \times 4\frac{1}{4}$
B. $4\frac{1}{2} \times 3\frac{1}{4}$
C. 4×3
D. 5×4
4. Select all the line segments that appear to be parallel to a .



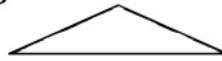
- A. a
B. b
C. c
D. d
E. e
F. f

5. Select all the figures that have sides that appear to be perpendicular.

A



B



C



D



E



A. A

B. B

C. C

D. D

E. E

6. Select all the expressions that are equal to 10^3 .

A. 30

B. 3,000

C. $1,000 \times 3$

D. 10×3

E. $10 \times 10 \times 10$

F. 100

G. 1,000

7. Draw two triangles:

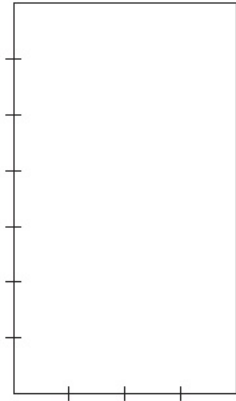
- One that is a right triangle

- One that is *not* a right triangle

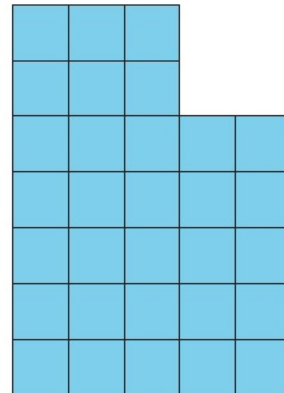
Then, explain what makes a triangle a right triangle.

Area and Surface Area: Check Your Readiness (B)

1. The rectangle has sides measuring 7 cm and 4 cm. What is the area of this rectangle?
Explain your reasoning.



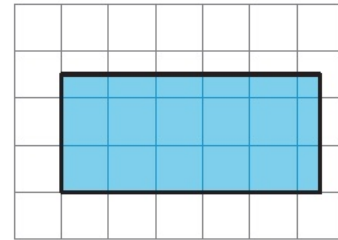
2. Select all the expressions that give the area of the figure in square units.



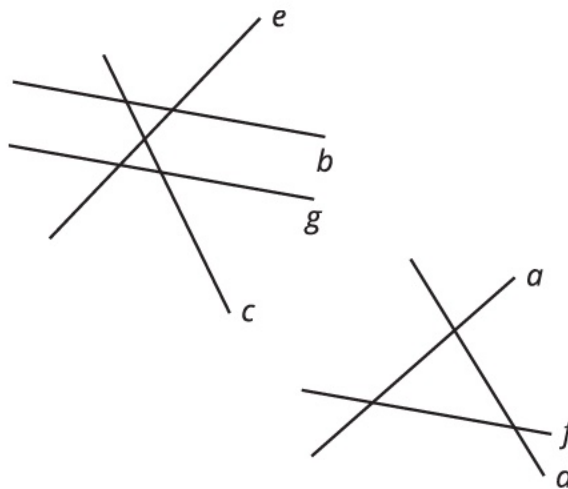
- A. 7×5
B. $(7 + 5) \times (5 + 2)$
C. $(7 \times 3) + (5 \times 2)$
D. $7 \times 5 \times 5 \times 2 \times 2 \times 3$
E. $(7 \times 5) - (2 \times 2)$

3. Each small square in the graph paper represents 1 square unit.

Which expression is closest to the area of the shaded rectangle, in square units?

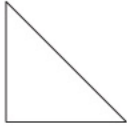



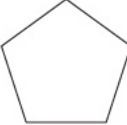


- A. 6×3
B. 5×2
C. $5\frac{1}{2} \times 2\frac{1}{2}$
D. $6\frac{1}{2} \times 3\frac{1}{4}$
4. Select all the line segments that appear to be parallel to g .



- A. a
B. b
C. c
D. d
E. e
F. f

5. Select all the figures that have sides that appear to be perpendicular.

- A. A right-angled triangle with the right angle at the bottom-left corner.
- B. A rectangle.
- C. A parallelogram that is not a rectangle.
- D. A rhombus (diamond shape).
- E. A regular pentagon.

6. Select all the expressions that are equal to 10^4 .

- A. 10,000
- B. 4,000
- C. $1,000 \times 4$
- D. 10×4
- E. $10 \times 10 \times 10 \times 10$
- F. 1,000
- G. 40

7. Select all triangles that appear to be a right triangle.

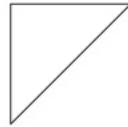
A.



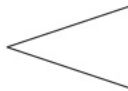
B.



C.



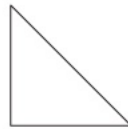
D.



E.



F.

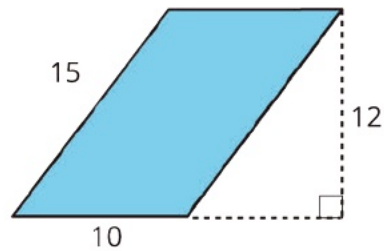


Area and Surface Area: Mid-Unit Assessment (A)

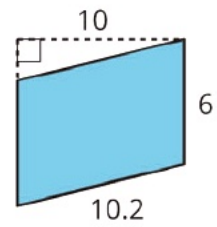
For all grids, each small square on the grid has an area of one square unit unless otherwise indicated.

1. Which parallelogram has an area of 60 square units?

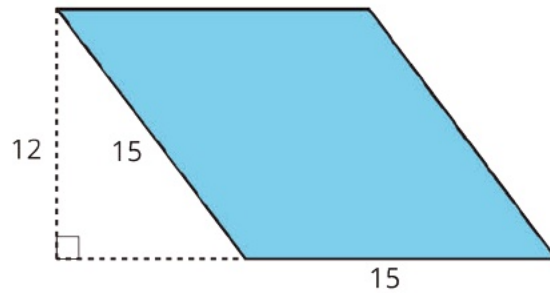
A



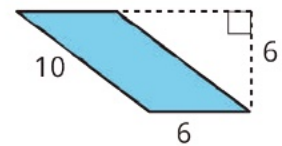
B



C



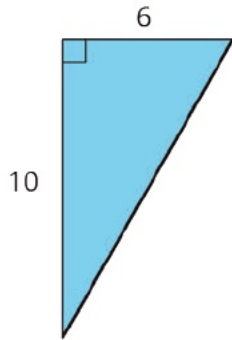
D



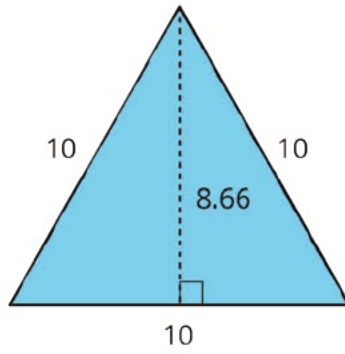
- A. A
- B. B
- C. C
- D. D

2. Select all the triangles that have an area of 30 square units.

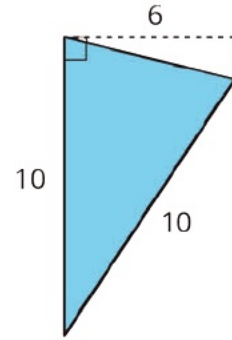
A



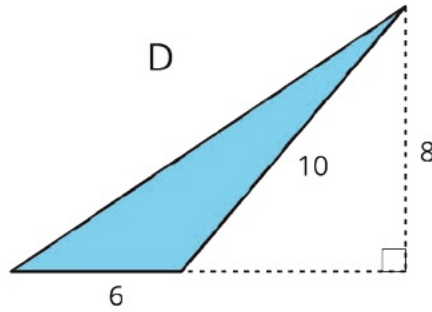
B



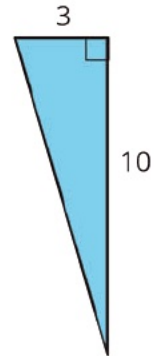
C



D



E



A. A

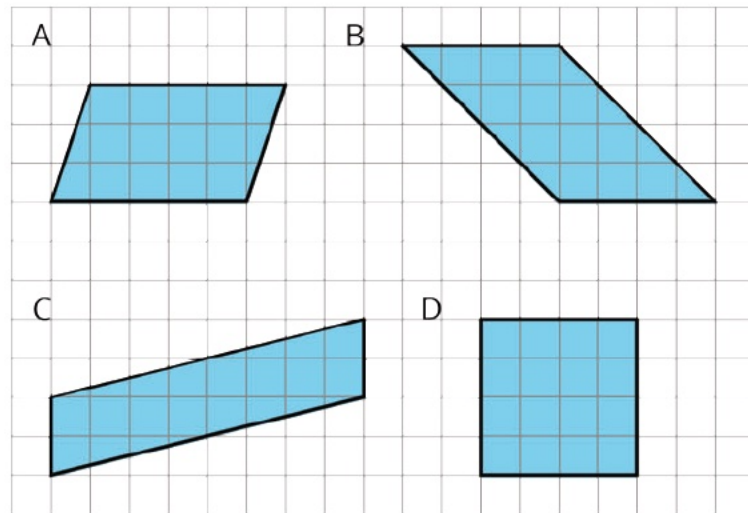
B. B

C. C

D. D

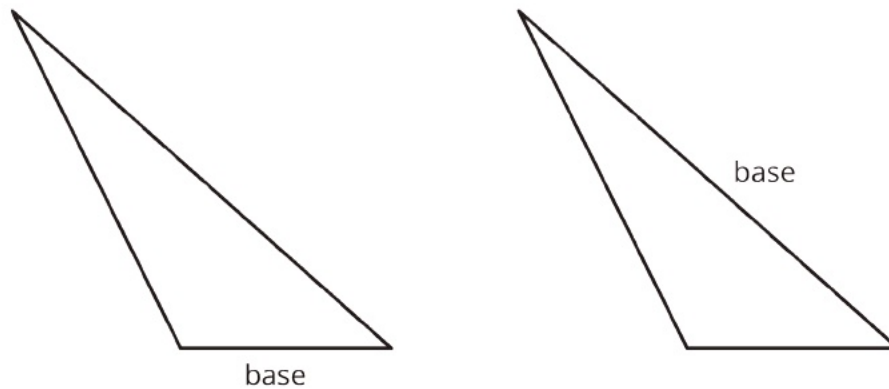
E. E

3. Select all the parallelograms that have an area of 16 square units.

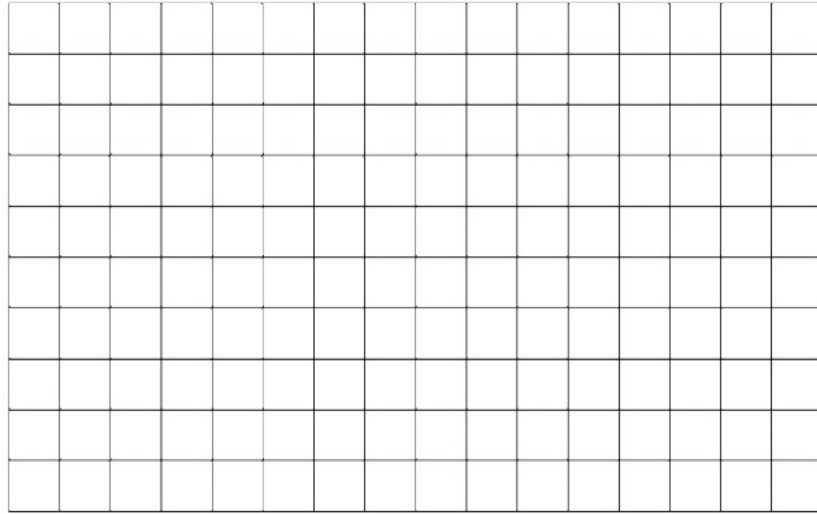


- A. A
- B. B
- C. C
- D. D

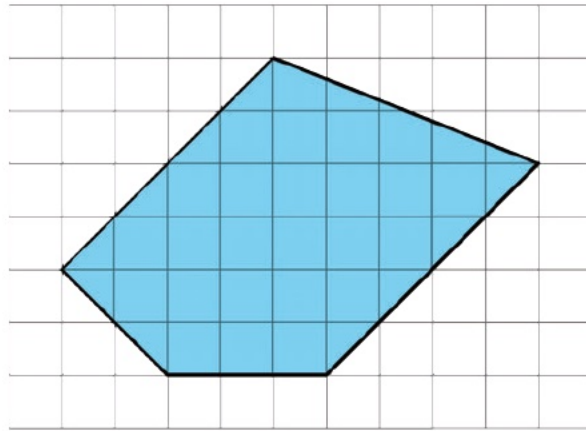
4. On each triangle, draw a segment to represent the height that corresponds to the given base. Label each height with the word "height."



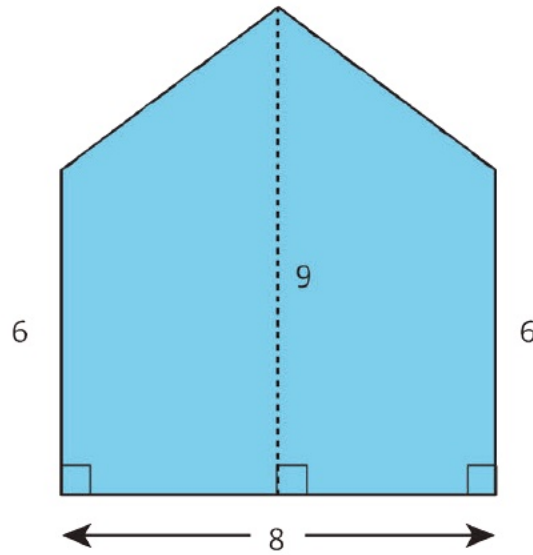
5. Draw two distinct parallelograms, both with areas of 18 square units. The two parallelograms should not be identical copies of each other.



6. Find the area of the given figure. Explain your reasoning.



7. The figure is a diagram of a wall. Lengths are given in feet.



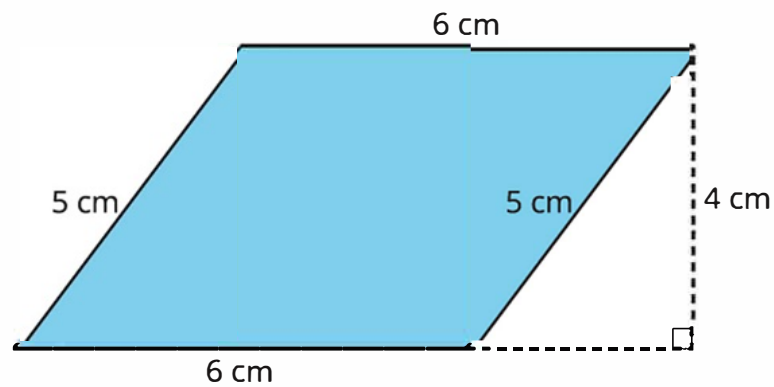
a. How many square feet of wallpaper would be needed to cover the wall? Explain your reasoning.

b. Wallpaper is sold in rolls that are 2 feet wide. What is the minimum length you would need to purchase to cover the wall?

Area and Surface Area: Mid-Unit Assessment (B)

For all grids, each small square on the grid has an area of one square unit unless otherwise indicated.

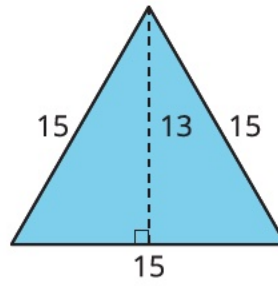
1. What is the area of this parallelogram?



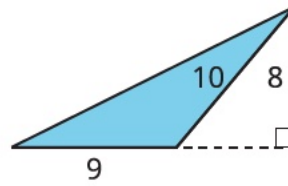
- A. 30 cm^2
- B. 24 cm^2
- C. 22 cm^2
- D. 12 cm^2

2. Select all the triangles that have an area of 45 square units.

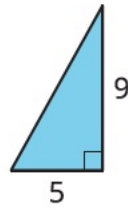
A.



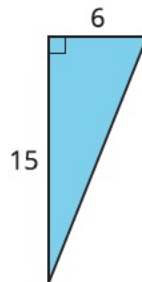
B.



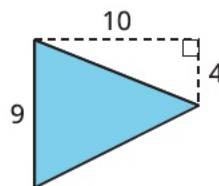
C.



D.

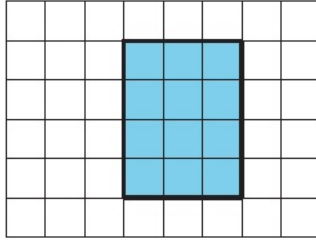


E.

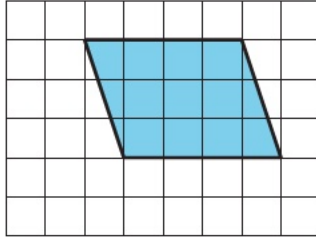


3. Select all the parallelograms that have an area of 12 square units.

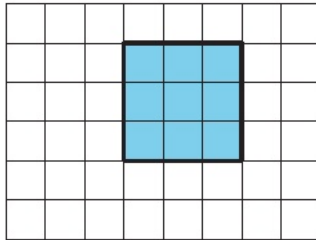
A.



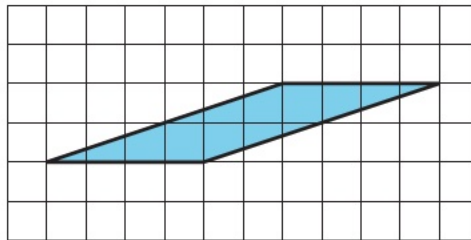
B.



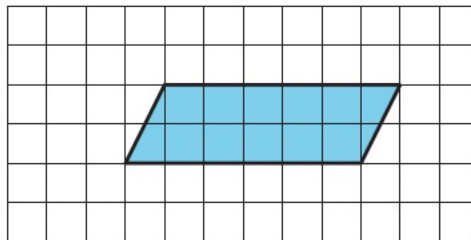
C.



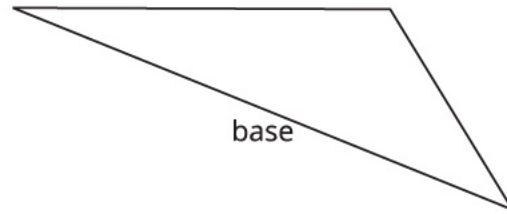
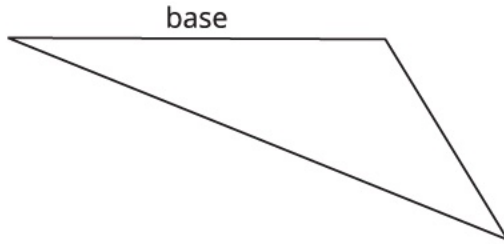
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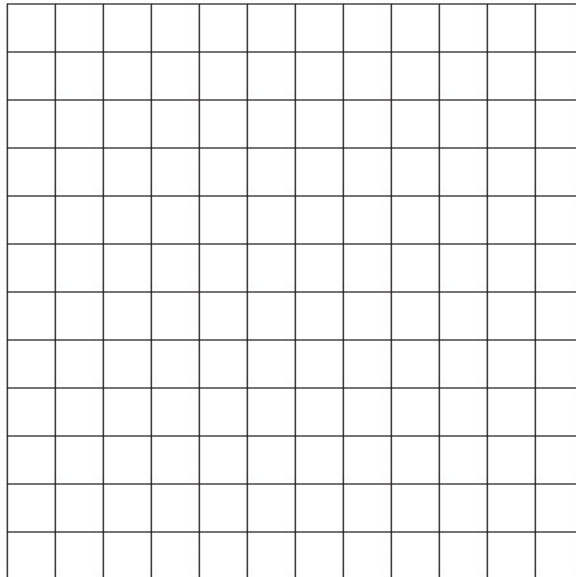
E.



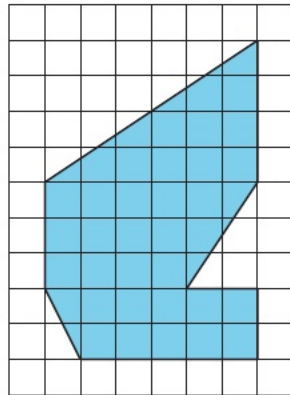
4. On each triangle, draw a segment to represent the height that corresponds to the given base. Label each height with the word "height."



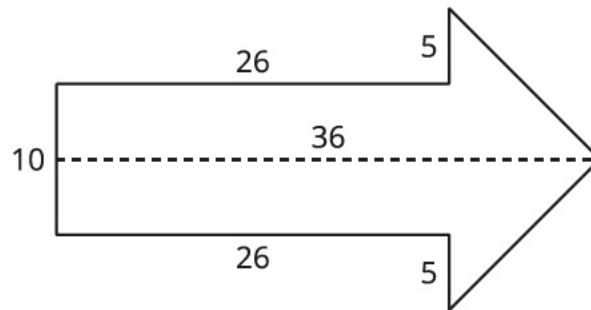
5. Draw two parallelograms, each with an area of 16 square units. The two parallelograms should not be identical copies of each other.



6. Find the area of the figure. Explain your reasoning.



7. The figure is a diagram of a sign. Lengths are given in inches.

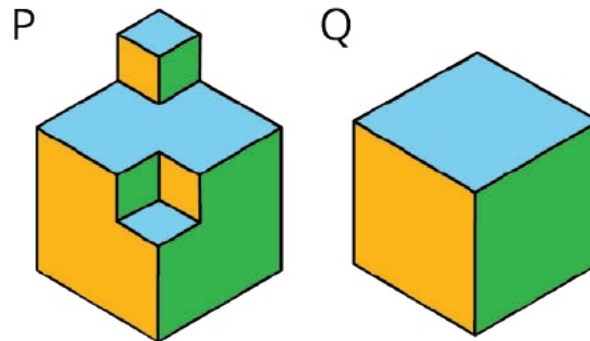


a. What is the area of the sign? Explain your reasoning.

b. A person is going to paint the sign, including both the front and back. How many square inches will they need to cover?

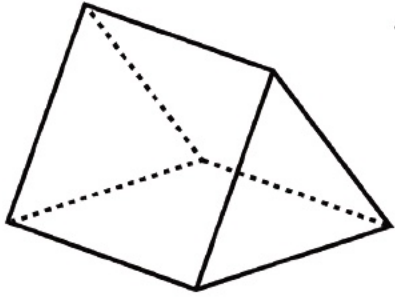
Area and Surface Area: End-of-Unit Assessment (A)

1. Polyhedron P is a cube with a corner removed and relocated to the top of P. Polyhedron Q is a cube with the same size base as Polyhedron P. How do their surface areas compare?

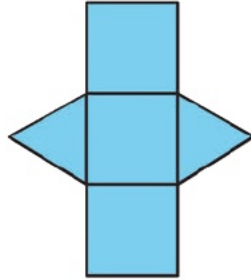


- A. P's surface area is less than Q's surface area.
- B. P's surface area is equal to Q's surface area.
- C. P's surface area is greater than Q's surface area.
- D. There is not enough information given to compare their surface areas.

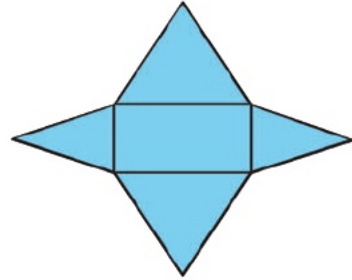
2. Select all of the nets that can be folded and assembled into a triangular prism like this one.



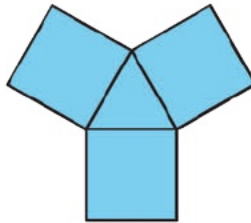
A



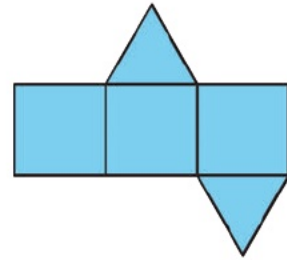
B



C



D



A. A

B. B

C. C

D. D

3. A cube has a side length of 8 inches.

Select all the values that represent the cube's volume in cubic inches.

A. 8^2

B. 8^3

C. $6 \cdot 8^2$

D. $6 \cdot 8$

E. $8 \cdot 8 \cdot 8$

4. a. A square has a side length 9 cm. What is its area?

b. A square has an area of 9 cm^2 . What is its side length?

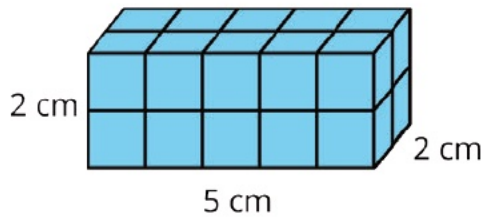
5. For each pair of expressions, circle the expression with the greater value.

a. 13^2 or 15^2

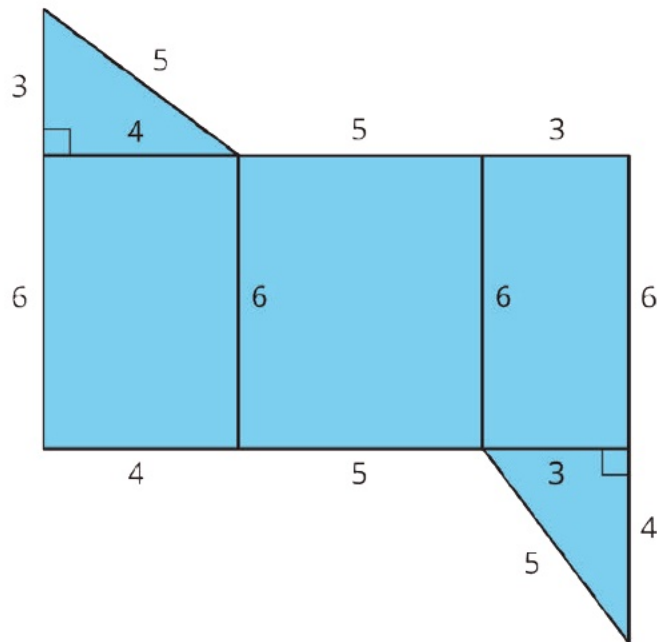
b. $7 \cdot 6^2$ or 6^3

c. 10^3 or 30^2

6. A rectangular prism has dimensions of 2 cm by 2 cm by 5 cm. What is its surface area? Explain or show your reasoning.



7. Here is a net made of right triangles and rectangles. All measurements are given in centimeters.

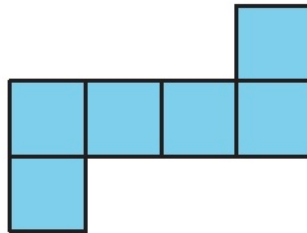


- a. If the net were folded and assembled, what type of polyhedron would it make?
- b. What is the surface area of the polyhedron? Explain your reasoning.

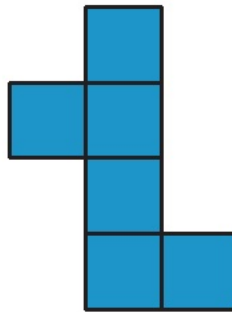
Area and Surface Area: End-of-Unit Assessment (B)

1. Select all of the nets that can be folded and assembled into a cube.

A.



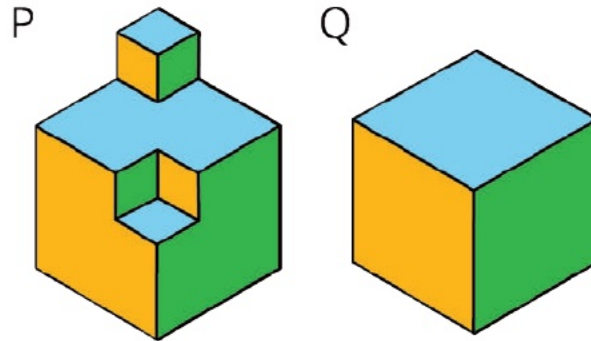
B.



C.



2. Polyhedron P is a cube with a corner removed and relocated to the top of P. Polyhedron Q is a cube with the same size base as Polyhedron P. How do their surface areas compare?



- A. Q's surface area is less than P's surface area.
- B. Q's surface area is equal to P's surface area.
- C. Q's surface area is greater than P's surface area.
- D. There is not enough information given to compare their surface areas.
3. For a cube whose side length is 4 inches, the expression 4^3 could represent . . .
- A. The length of 3 cubes lined up, in inches
- B. The cube's surface area in square inches
- C. The cube's volume in cubic inches
- D. The total volume of 3 cubes, in cubic inches

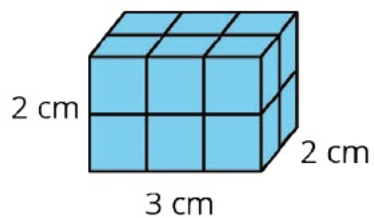
4. a. A square has an area of 16 cm^2 . What is the length of each of its sides?

b. A square has a side length of 8 cm. What is its area?

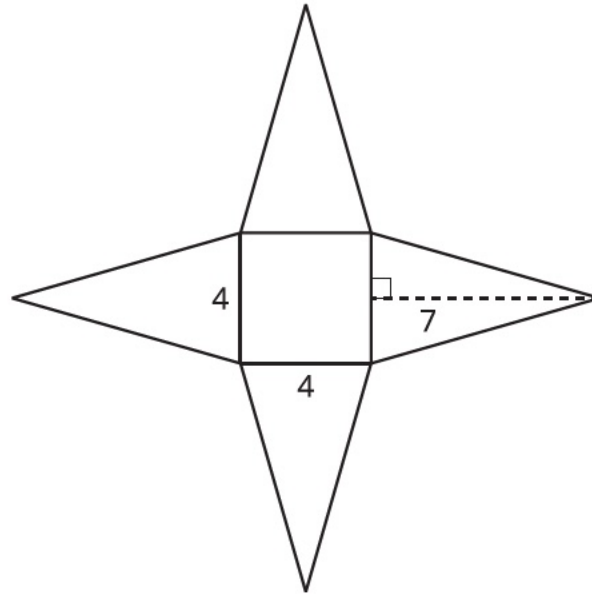
5. Here is a list of expressions. Order them from least to greatest.

$$40^2 \quad 8^3 \quad 10^3 \quad 7 \cdot 8^2 \quad 10^4$$

6. A rectangular prism has dimensions of 2 cm by 2 cm by 3 cm. What is its surface area? Explain or show your reasoning.



7. Here is a net made of a square and four identical triangles. All measurements are given in centimeters.



- a. If the net were folded and assembled, what type of polyhedron would it make?
- b. What is the surface area of the polyhedron? Explain your reasoning.

CKMath™
Core Knowledge **MATHEMATICS™**

Assessment Answer Keys

Check Your Readiness A and B
Mid-Unit Assessment A and B
Eod-of-Unit Assessment A and B

Assessment Answer Keys

Assessment : Check Your Readiness (A)

Problem 1

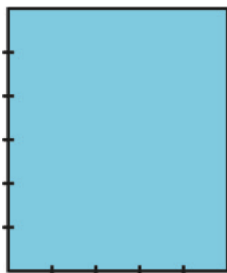
The content assessed in this problem is first encountered in Lesson 1: Tiling the Plane.

This item assesses how students approach finding the area of a rectangle with whole-number side lengths. The purpose of including tick marks is to give assistance to students who wish to draw a grid of unit squares. Responses that show drawings with the incorrect number of unit squares, irregular rows, or irregular columns may indicate that students have not yet learned to structure two-dimensional space, that is, to see a rectangle with whole-number side lengths as composed of unit squares, or composed of iterated rows or columns of unit squares.

If most students struggle with this item, plan to use the activities in Lesson 1 to support their understanding of area. The Practice Problems in Lesson 1 can be used for extra practice in calculating area. In Lesson 2 they will calculate area as they decompose and rearrange shapes to find area. Plan to emphasize tiling and square units in the warm-up of Lesson 2 if students struggle to make sense of tiling the rectangle with 30 squares to find its area.

Statement

The rectangle has sides measuring 6 cm and 5 cm. What is the area of this rectangle? Explain your reasoning.



Solution

30 square centimeters. Possible strategies:

- Use a formula like $l \times w$.
- Draw and count unit squares.
- Think in terms of tiling, but multiply to find the area.

Aligned Standards

3.MD.C.6, 3.MD.C.7.a, 3.MD.C.7.b, 4.MD.A.3

Problem 2

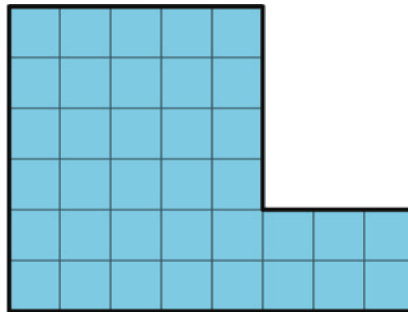
The content assessed in this problem is first encountered in Lesson 2: Finding Area by Decomposing and Rearranging.

This problem assesses two different prerequisite skills. To reason about the area of the shape, students will need to decompose the shape into two rectangles. Interpreting the expressions in the answer choices may also pose a challenge for some students. Note that choices B and D represent two different methods: surrounding with a larger rectangle and decomposing into smaller rectangles, respectively. It is worth discussing these two techniques with a class, particularly if many students selected only one of the two methods.

If most students struggle with this item, plan to use a few examples of decomposing rectangles to find area to begin this lesson. The Practice Problems in Lesson 1 can be used for this. Students will also get many opportunities in the first several lessons of this unit to decompose shapes and compare different ways to decompose the same shape.

Statement

Select all the expressions that give the area of the figure in square units.



- A. $(5 + 6) \times (3 + 2)$
- B. $(8 \times 6) - (3 \times 4)$
- C. $6 \times 8 \times 2 \times 3 \times 4 \times 5$
- D. $(5 \times 6) + (3 \times 2)$
- E. 8×6

Solution

["B", "D"]

Aligned Standards

3.MD.C.7.d, 5.OA.A.2

Problem 3

The content assessed in this problem is first encountered in Lesson 9: Formula for the Area of a Triangle.

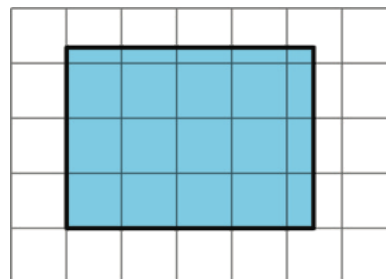
This problem requires students to calculate the area of a rectangle with sides that do not fit nicely in a lattice grid. In fifth grade, students learned to calculate the area of a rectangle by multiplying fractional side lengths. Watch for students who are having trouble estimating fractional units visually.

If most students struggle with this item, plan to use this problem to draw out and support any misconceptions during the synthesis of Lesson 2 Activity 1.

Statement

Each small square in the graph paper represents 1 square unit.

Which expression is closest to the area of the rectangle, in square units?



- A. $5\frac{1}{2} \times 4\frac{1}{4}$
- B. $4\frac{1}{2} \times 3\frac{1}{4}$
- C. 4×3
- D. 5×4

Solution

B

Aligned Standards

5.NF.B.4.b

Problem 4

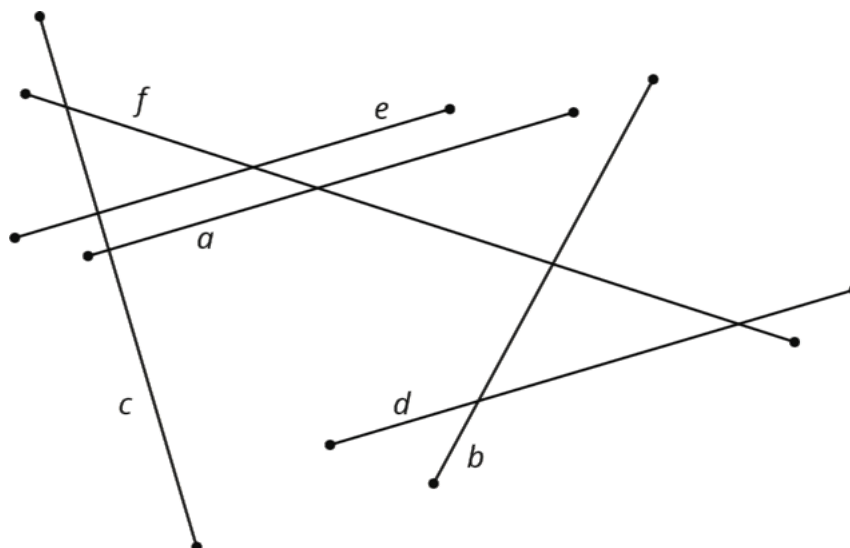
The content assessed in this problem is first encountered in Lesson 4: Parallelograms.

Students will need to be comfortable recognizing parallel lines before beginning their work with parallelograms later in the unit. Some students may correctly select e, but not notice d since that line is farther away.

If most students struggle with this item, plan to start with Lesson 4 Activity 1 Launch with an emphasis on defining the term parallel.

Statement

Select all the line segments that appear to be parallel to a .



- A. a
- B. b
- C. c
- D. d
- E. e
- F. f

Solution

["D", "E"]

Aligned Standards

4.G.A.1

Problem 5

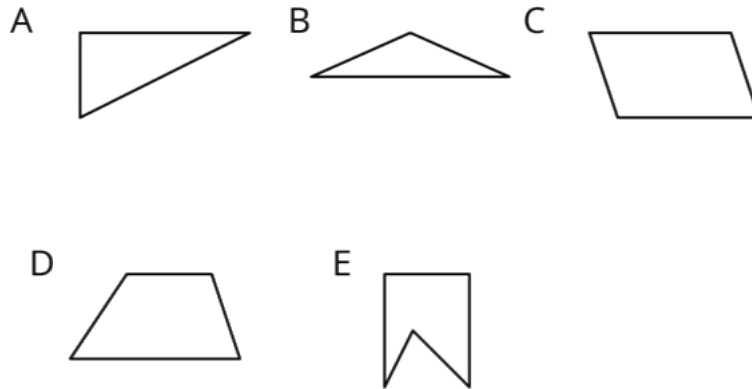
The content assessed in this problem is first encountered in Lesson 5: Bases and Heights of Parallelograms.

In this unit, students will find the area of parallelograms and triangles by decomposing them into shapes with perpendicular sides and rearranging the pieces. Students will need to be familiar with perpendicular lines in order to make sense of the “height” of a parallelogram or triangle.

If most students struggle with this item, plan to start Lesson 5 Activity 2 by amplifying the term perpendicular for the students. Students may need some visual cues to support this concept.

Statement

Select all the figures that have sides that appear to be perpendicular.



- A. A
- B. B
- C. C
- D. D
- E. E

Solution

["A", "E"]

Aligned Standards

4.G.A.2

Problem 6

The content assessed in this problem is first encountered in Lesson 17: Squares and Cubes.

Exponential notation is introduced in Lesson 17 of this unit, in the context of calculating surface area and volume of cubes. Students may have prior knowledge of exponents from their work with place value in fifth grade.

If most students struggle with this item, plan to review exponents using this problem as part of the Launch into Activity 4. This problem can further emphasize the concept of cubing, as it is described in the Launch and then used throughout the activity.

Statement

Select all the expressions that are equal to 10^3 .

- A. 30
- B. 3,000
- C. $1,000 \times 3$
- D. 10×3
- E. $10 \times 10 \times 10$
- F. 100
- G. 1,000

Solution

["E", "G"]

Aligned Standards

5.NBT.A.2

Problem 7

The content assessed in this problem is first encountered in Lesson 3: Reasoning to Find Area.

This problem assesses whether students understand the term “right triangle.” This problem will also reveal whether students can picture a 90° angle well enough to draw one freehand or to reach for an appropriate tool, like the corner of a piece of paper.

If most students do well with this item, plan to focus on this concept during Lesson 3 Activity 3, Off the Grid. In the Launch of this activity, include a discussion on the term right angle and point out the symbol used to identify it in the shapes. This concept will continue to be reinforced in the next several lessons.

Statement

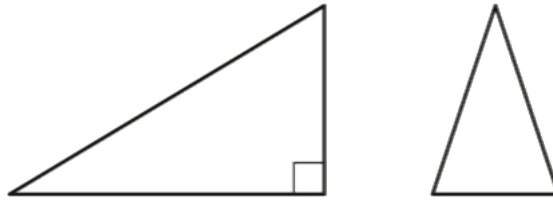
Draw two triangles:

- One that is a right triangle
- One that is *not* a right triangle

Then, explain what makes a triangle a right triangle.

Solution

Answers vary. Sample response:



A right triangle has a right (or 90°) angle.

Aligned Standards

4.G.A.2

Assessment : Check Your Readiness (B)

Problem 1

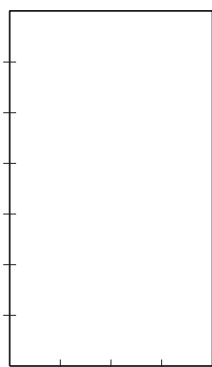
The content assessed in this problem is first encountered in Lesson 1: Tiling the Plane.

This item assesses how students approach finding the area of a rectangle with whole-number side lengths. The purpose of including tick marks is to give assistance to students who wish to draw a grid of unit squares. Responses that show drawings with the incorrect number of unit squares, irregular rows, or irregular columns may indicate that students have not yet learned to structure two dimensional space, that is, to see a rectangle with whole-number side lengths as composed of unit squares, or composed of iterated rows or columns of unit squares.

If most students struggle with this item, plan to use the activities in Lesson 1 to support their understanding of area. The Practice Problems in Lesson 1 can be used for extra practice in calculating area. In Lesson 2 they will calculate area as they decompose and rearrange shapes to find area. Plan to emphasize tiling and square units in the warm-up of Lesson 2 if students struggle to make sense of tiling the rectangle with 30 squares to find its area.

Statement

The rectangle has sides measuring 7 cm and 4 cm. What is the area of this rectangle? Explain your reasoning.



Solution

28 square centimeters. Possible strategies:

- Use a formula like $l \cdot w$
- Draw and count unit squares.
- Think in terms of tiling, but multiply to find the area.

Aligned Standards

3.MD.C.6, 3.MD.C.7.a, 3.MD.C.7.b, 4.MD.A.3

Problem 2

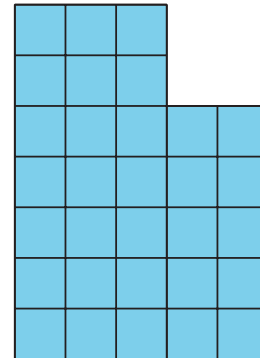
The content assessed in this problem is first encountered in Lesson 2: Finding Area by Decomposing and Rearranging.

This problem assesses two different prerequisite skills. To reason about the area of the shape, students will need to decompose the shape into two rectangles. Interpreting the expressions in the answer choices may also pose a challenge for some students. Note that choices C and E represent two different methods: surrounding with a larger rectangle and decomposing into smaller rectangles, respectively. It is worth discussing these two techniques with a class, particularly if many students selected only one of the two methods.

If most students struggle with this item, plan to use a few examples of decomposing rectangles to find area to begin this lesson. The Practice Problems in Lesson 1 can be used for this. Students will also get many opportunities in the first several lessons of this unit to decompose shapes and compare different ways to decompose the same shape.

Statement

Select all the expressions that give the area of the figure in square units.



- A. 7×5
- B. $(7 + 5) \times (5 + 2)$
- C. $(7 \times 3) + (5 \times 2)$
- D. $7 \times 5 \times 5 \times 2 \times 2 \times 3$
- E. $(7 \times 5) - (2 \times 2)$

Solution

["C", "E"]

Aligned Standards

3.MD.C.7.d, 5.OA.A.2

Problem 3

The content assessed in this problem is first encountered in Lesson 9: Formula for the Area of a Triangle.

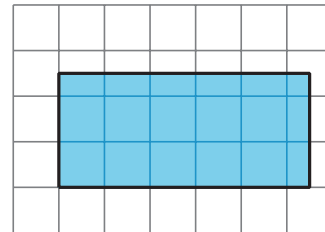
This problem requires students to calculate the area of a rectangle with sides that do not fit nicely in a lattice grid. In fifth grade, students learned to calculate the area of a rectangle by multiplying fractional side lengths. Watch for students who are having trouble estimating fractional units visually.

If most students struggle with this item, plan to use this problem to draw out and support any misconceptions during the synthesis of Lesson 2 Activity 1.

Statement

Each small square in the graph paper represents 1 square unit.

Which expression is closest to the area of the shaded rectangle, in square units?



- A. 6×3
- B. 5×2
- C. $5\frac{1}{2} \times 2\frac{1}{2}$
- D. $6\frac{1}{2} \times 3\frac{1}{4}$

Solution

C

Aligned Standards

5.NF.B.4.b

Problem 4

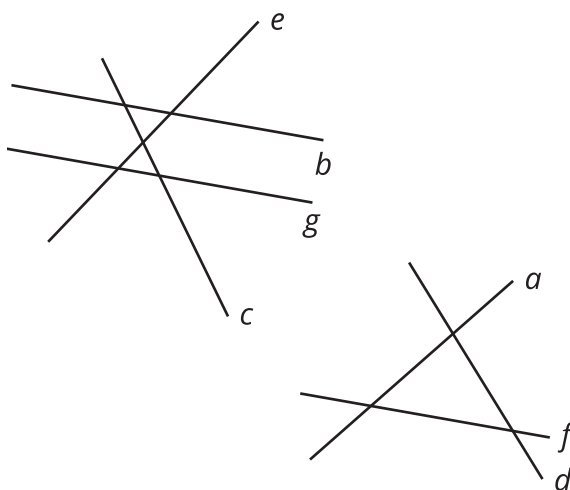
The content assessed in this problem is first encountered in Lesson 4: Parallelograms.

Students will need to be comfortable recognizing parallel lines before beginning their work with parallelograms later in the unit. Some students may correctly select b, but not notice g since that line is farther away.

If most students struggle with this item, plan to start with Lesson 4 Activity 1 Launch with an emphasis on defining the term parallel.

Statement

Select **all** the line segments that appear to be parallel to g .



- A. a
- B. b
- C. c
- D. d
- E. e
- F. f

Solution

["B", "F"]

Aligned Standards

4.G.A.1

Problem 5

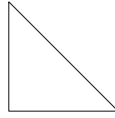
The content assessed in this problem is first encountered in Lesson 5: Bases and Heights of Parallelograms.

In this unit, students will find the area of parallelograms and triangles by decomposing them into shapes with perpendicular sides and rearranging the pieces. Students will need to be familiar with perpendicular lines in order to make sense of the “height” of a parallelogram or triangle.

Statement

Select all the figures that have sides that appear to be perpendicular.

A.



B.



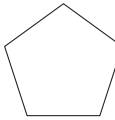
C.



D.



E.



Solution

["A", "B"]

Aligned Standards

4.G.A.2

Problem 6

The content assessed in this problem is first encountered in Lesson 17: Squares and Cubes.

Exponential notation is introduced in Lesson 17 of this unit, in the context of calculating surface area and volume of cubes. Students may have prior knowledge of exponents from their work with place value in fifth grade.

Statement

Select all the expressions that are equal to 10^4 .

- A. 10,000
- B. 4,000
- C. $1,000 \times 4$
- D. 10×4
- E. $10 \times 10 \times 10 \times 10$
- F. 1,000
- G. 40

Solution

["A", "E"]

Aligned Standards

5.NBT.A.2

Problem 7

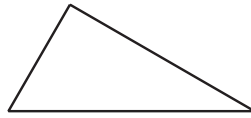
The content assessed in this problem is first encountered in Lesson 3: Reasoning to Find Area.

This problem assesses whether students understand the term “right triangle.” This problem will also reveal whether students can picture a 90 degree angle.

Statement

Select **all** triangles that appear to be a right triangle.

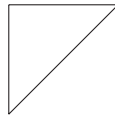
A.



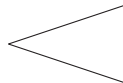
B.



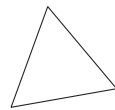
C.



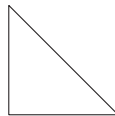
D.



E.



F.



Solution

["A", "C", "F"]

Aligned Standards

4.G.A.2

Assessment : Mid-Unit Assessment (A)

Teacher Instructions

Give this assessment after lesson 11.

Student Instructions

For all grids, each small square on the grid has an area of one square unit unless otherwise indicated.

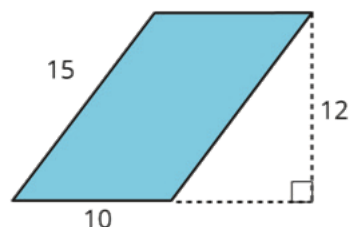
Problem 1

Students selecting A are likely applying the formula for the area of a triangle, $\frac{1}{2} \cdot b \cdot h$. Students failing to select B may be multiplying one base of the parallelogram by the other base, rather than multiplying one of the bases by the height. They may also be confused by the fact that the height is horizontal. Students selecting C picked the parallelogram with perimeter 60, and may need some further work on the conceptual differences between area and perimeter. Students selecting D may be multiplying the two bases together, or they may be multiplying the height by the incorrect base.

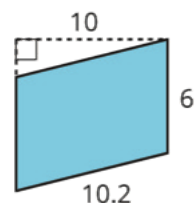
Statement

Which parallelogram has an area of 60 square units?

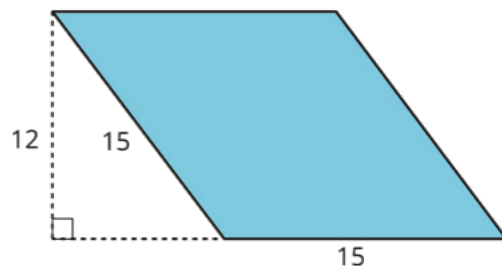
A



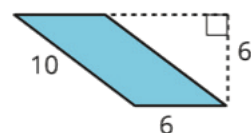
B



C



D



A. A

B. B

C. C

D. D

Solution

B

Aligned Standards

6.G.A.1

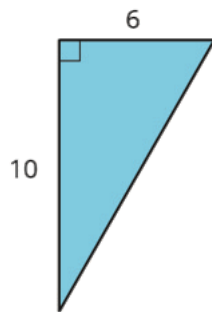
Problem 2

Students selecting B have calculated perimeter rather than area. Students selecting D are treating the side of length 10 as the height. Students selecting E have multiplied the base and height but have not multiplied by $\frac{1}{2}$. Students failing to select A have a major misconception about the area of a triangle. Students failing to select C may not have recognized the external height.

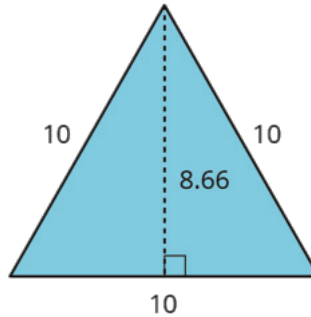
Statement

Select all the triangles that have an area of 30 square units.

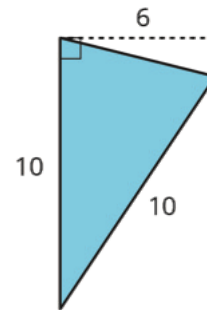
A



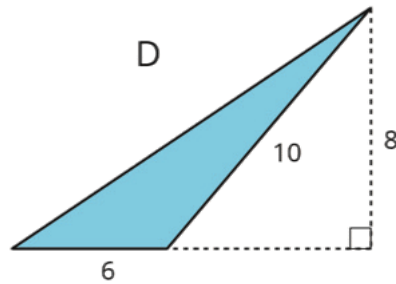
B



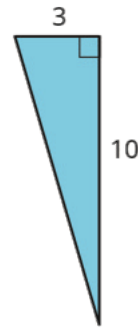
C



D



E



A. A

B. B

C. C

D. D

E. E

Solution

["A", "C"]

Aligned Standards

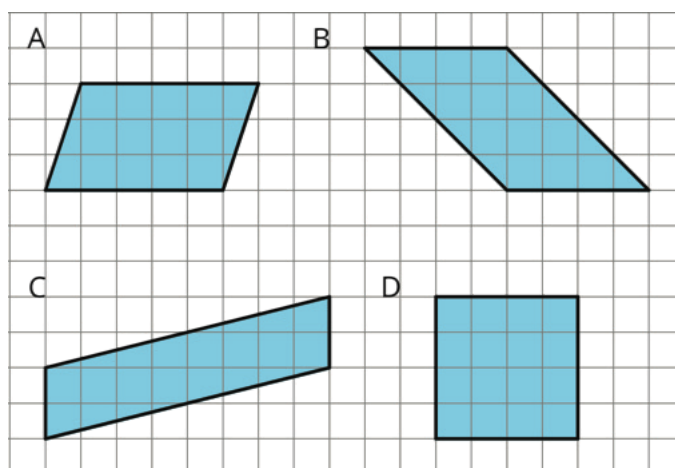
6.G.A.1

Problem 3

Students selecting A may be using a strategy of decomposing and rearranging the parallelogram, but miscounting the number of unit squares: the area is 15. Or they may be estimating the non-horizontal side length to be about 4, and using this value for the height. Students failing to select B may also be confusing the side length with the height. Students failing to select C may have chosen to use the area formula instead of decomposing and rearranging, then failed to use the vertical side length as the base. Students failing to select D may be thinking that a square is not a parallelogram.

Statement

Select all the parallelograms that have an area of 16 square units.



A. A

B. B

C. C

D. D

Solution

["B", "C", "D"]

Aligned Standards

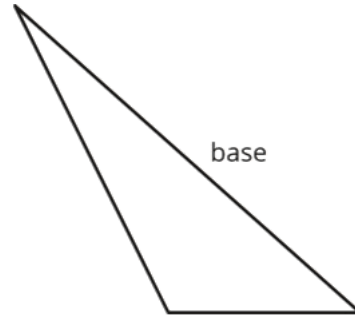
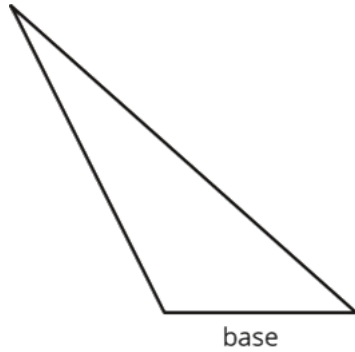
6.G.A.1

Problem 4

Identifying the height for a chosen base is an important tool for calculating the area of a triangle.

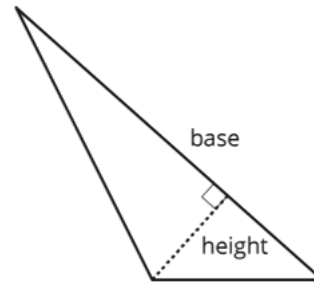
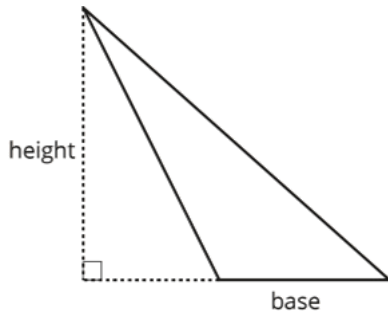
Statement

On each triangle, draw a segment to represent the height that corresponds to the given base. Label each height with the word "height."



Solution

Answers vary. Sample response:



Aligned Standards

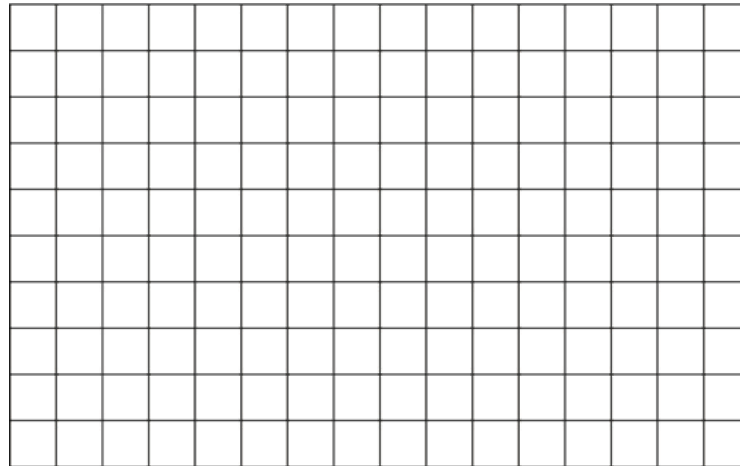
6.G.A.1

Problem 5

This question purposely admits the possibility that students might draw rectangles if they know that a rectangle is a particular kind of parallelogram. The question could be modified to instruct students to draw parallelograms that are not rectangles.

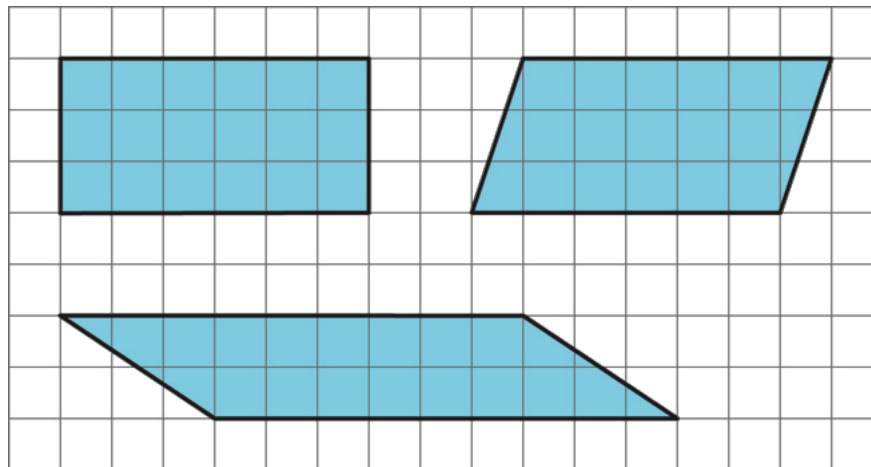
Statement

Draw two distinct parallelograms, both with areas of 18 square units. The two parallelograms should not be identical copies of each other.



Solution

Answers vary. Sample responses:



Minimal Tier 1 response:

- Work is complete and correct.
- The two parallelograms may have the same base and height as long as they are not congruent.
- Sample: Two parallelograms (rectangles allowed) with base/height pairs that multiply together to 18.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.

- Sample errors: Only one parallelogram is drawn; only one parallelogram has the correct area; an expression like “ $6 \cdot 3 = 18$ ” is written, but the base or height of the parallelogram is slightly off.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: Work indicates that student believes that a slanted side is the base or height; shapes drawn are not parallelograms.

Aligned Standards

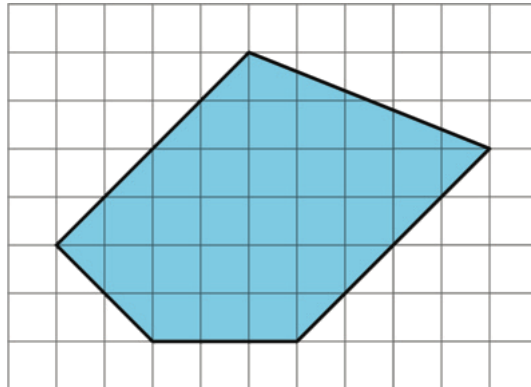
6.G.A.1

Problem 6

Watch for students breaking the shape into parts in unusual ways. These students may not be recalling well the methods developed in the lessons, and are likely to create parts for which they cannot determine the area.

Statement

Find the area of the given figure. Explain your reasoning.



Solution

31 square units. Possible strategies:

- Decompose into parallelograms and triangles with known bases and heights. Add the area of each component.
- Enclose in a 9-by-6 rectangle. From 54 square units, subtract the areas of the right triangles that are not part of the figure.

Minimal Tier 1 response:

- Work is complete and correct.

- A diagram is included.
- Acceptable errors: no units are included.
- Sample: A box around the shape has area 54. The triangles on the outside have area 8, 5, 8, and 2. The area of the box minus the triangles is 31.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: incorrectly calculating the area of a small number of the partitioned shapes; incorrectly adding or subtracting to find the area of the polygon; the area of one of the partitioned shapes cannot be correctly calculated because the base or height does not lie on a vertical or horizontal line; the partitioned shapes are not shown on the diagram.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: The partitioned shapes are not shapes for which students can find the area; pervasive use of incorrect area formulas for triangles or rectangles; many calculation errors.

Aligned Standards

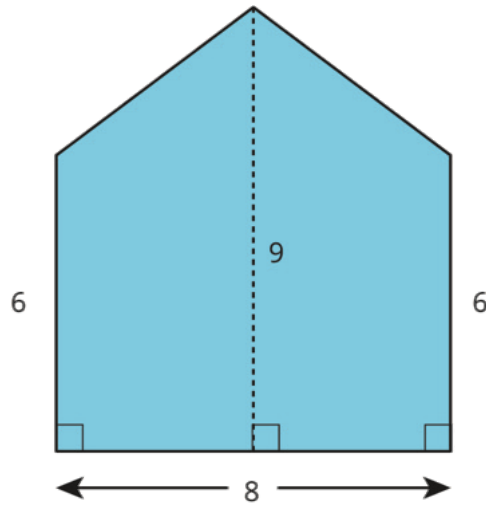
6.G.A.1

Problem 7

Strategies for this problem range from cutting out and rearranging the two trapezoids, to partitioning the wall into squares and rectangles, to “boxing in” the wall and subtracting the negative space.

Statement

The figure is a diagram of a wall. Lengths are given in feet.



1. How many square feet of wallpaper would be needed to cover the wall? Explain your reasoning.
2. Wallpaper is sold in rolls that are 2 feet wide. What is the minimum length you would need to purchase to cover the wall?

Solution

1. 60 square feet. Strategies vary. Sample strategy: Rearrange the halves of the wall to form a 4-by-15 foot rectangle. $4 \times 15 = 60$
2. 30 feet

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample:
 1. The wall looks like a 6-by-8 rectangle with a triangle of base 8 and height 3 on top.
 $6 \cdot 8 + \frac{1}{2} \cdot 8 \cdot 3 = 60$ square feet.
 2. Divide by 2 and buy 30 feet of wallpaper.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: units omitted; arithmetic errors in calculating area. Acceptable errors: correct work in part b based on an incorrect answer to part a.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.

- Sample errors: Plausible but flawed strategy for partitioning or boxing in the wall; incorrect area formulas with correct partitioning strategy.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: No reasonable strategy for partitioning or boxing in the wall.

Aligned Standards

6.G.A.1

Assessment : Mid-Unit Assessment (B)

Teacher Instructions

Give this assessment after lesson 11.

Student Instructions

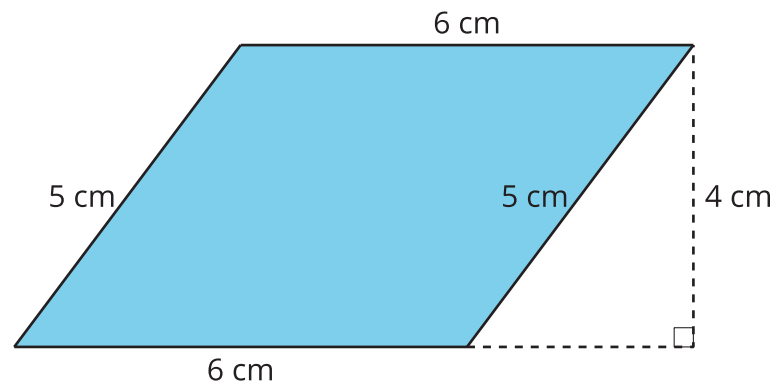
For all grids, each small square on the grid has an area of one square unit unless otherwise indicated.

Problem 1

Students selecting A may be multiplying the one side of the parallelogram by the other side, rather than multiplying one of the bases by the height. Students failing to select B may be confused about which measurements to use to calculate the area of a parallelogram. Students selecting C picked the perimeter of the parallelogram and may need some further work on the conceptual differences between area and perimeter. Students selecting D are likely applying the formula for the area of a triangle.

Statement

What is the area of this parallelogram?



A. 30 cm^2

B. 24 cm^2

C. 22 cm^2

D. 12 cm^2

Solution

B

Aligned Standards

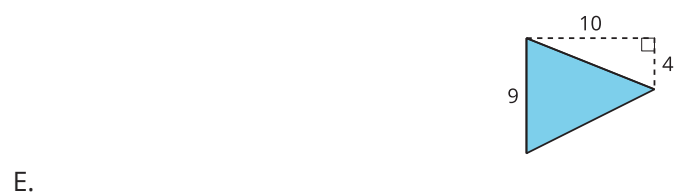
6.G.A.1

Problem 2

Students selecting A have calculated the perimeter rather than the area. Students selecting B may have treated the side of length 10 as the height. Students selecting C multiplied the base and the height but did not divide by two. Students failing to select D may have forgotten to divide by two. Students failing to select E may have used the incorrect base or height to calculate the area.

Statement

Select all the triangles that have an area of 45 square units.



Solution

["D", "E"]

Aligned Standards

6.G.A.1

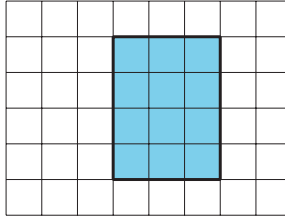
Problem 3

Students selecting C may have confused finding the area with finding the perimeter. Students selecting D are not applying the formula for finding the area of a parallelogram.

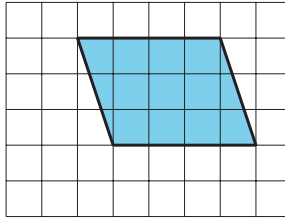
Statement

Select all the parallelograms that have an area of 12 square units.

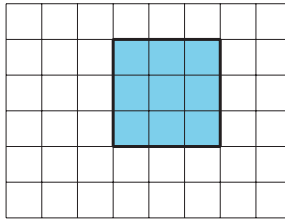
A.



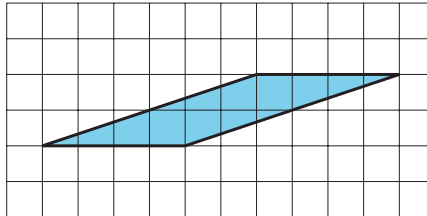
B.



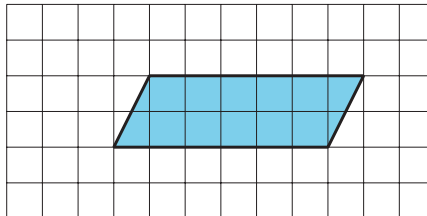
C.



D.



E.



Solution

["A", "B", "E"]

Aligned Standards

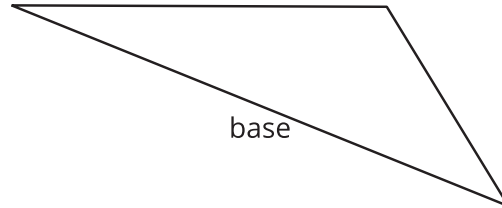
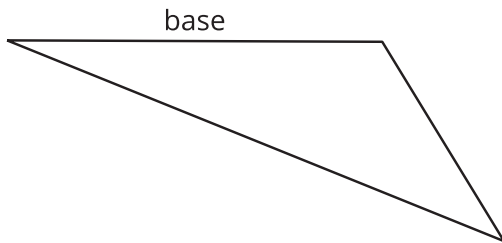
6.G.A.1

Problem 4

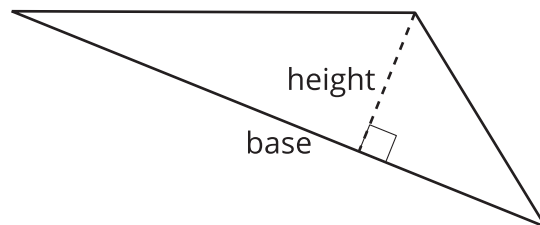
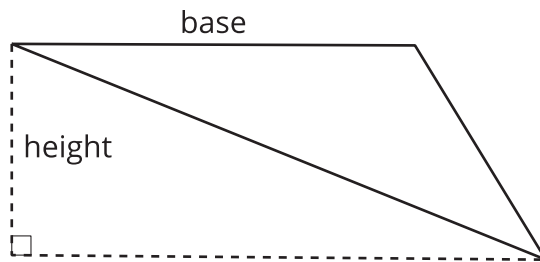
Identifying the height for a chosen base is an important tool for calculating the area of a triangle.

Statement

On each triangle, draw a segment to represent the height that corresponds to the given base. Label each height with the word "height."



Solution



Aligned Standards

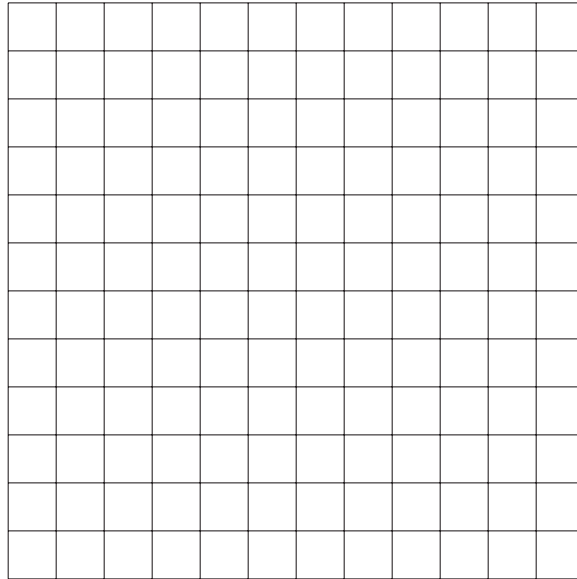
6.G.A.1

Problem 5

This question purposely admits the possibility that students might draw rectangles if they know that a rectangle is a particular kind of parallelogram. The question could be modified to instruct students to draw parallelograms that are not rectangles.

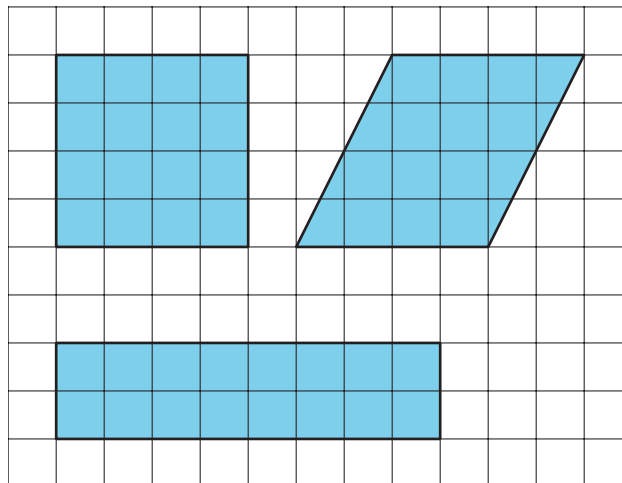
Statement

Draw two parallelograms, each with an area of 16 square units. The two parallelograms should not be identical copies of each other.



Solution

Answers vary. Sample responses:



Minimal Tier 1 response:

- Work is complete and correct.
- The two parallelograms may have the same base and height as long as they are not congruent.
- Sample: Two parallelograms (rectangles allowed) with base/height pairs that multiply together to 16.

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: Only one parallelogram is drawn; only one parallelogram has the correct area.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: Work indicates that student believes that a slanted side is the base or height; shapes drawn are not parallelograms.

Aligned Standards

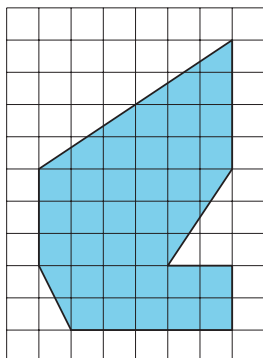
6.G.A.1

Problem 6

Watch for students breaking the shape into parts in unusual ways. These students may not be recalling the methods developed in the lessons, and may create parts for which they cannot determine the area.

Statement

Find the area of the figure. Explain your reasoning.



Solution

38 square units. Possible strategies:

- Decompose into rectangles and triangles with known bases and heights. Add the area of each component.
- Enclose in a 6-by-9 rectangle. From 54 square units, subtract the areas of the right triangles that are not part of the figure.

Minimal Tier 1 response:

- Work is complete and correct.
- A diagram is included.
- Acceptable errors: No units are included.
- Sample: A box around the shape has area 54. The triangles on the outside have area 12, 3, and 1. The area of the box minus the triangles is 38.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: Incorrectly calculating the area of one of the partitioned shapes; incorrectly adding or subtracting to find the area of the polygon; the area of one of the partitioned shapes cannot be correctly calculated because the base or height does not lie on a vertical or horizontal line; the partitioned shapes are not shown on the diagram.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: The partitioned shapes are not shapes for which students can find the area; pervasive use of incorrect area formulas for triangles or rectangles; many calculation errors.

Aligned Standards

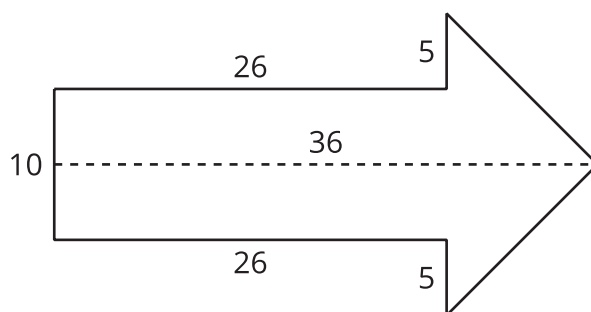
6.G.A.1

Problem 7

Strategies for this problem range from partitioning the sign into rectangles and triangles, to “boxing in” the sign and subtracting the negative space. Watch for students who struggle to find the dimensions of the tip of the arrow. Others may struggle with doubling the area in the second part.

Statement

The figure is a diagram of a sign. Lengths are given in inches.



1. What is the area of the sign? Explain your reasoning.
2. A person is going to paint the sign, including both the front and back. How many square inches will they need to cover?

Solution

1. 360 square inches. Strategies vary. Sample strategy: Divide the sign into a rectangle and a triangle. The area of the triangle is 100 square inches because $\frac{1}{2} \cdot 20 \cdot 10 = 100$. The area of the rectangle is 260 square inches because $10 \cdot 26 = 260$. $260 + 100 = 360$.
2. 720 square inches.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample:
 - The base of the triangle is 20 inches because $5 + 10 + 5 = 20$ and the height is 10 inches because $36 - 26 = 10$. $\frac{1}{2} \cdot 20 \cdot 10 = 100$.
 - The area of the rectangle is 260 square inches because $10 \cdot 26 = 260$ and the area of the triangle is 100 square inches because $\frac{1}{2} \cdot 20 \cdot 10 = 100$. Combined these two areas equal 360. For both sides, double that to 720.

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: units omitted; arithmetic errors in calculating area. Acceptable errors: correct work in part b based on an incorrect answer to part a.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.

- Sample errors: plausible but flawed strategy for partitioning; incorrect area formulas with correct partitioning strategy

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: no reasonable strategy for partitioning

Aligned Standards

6.G.A.1

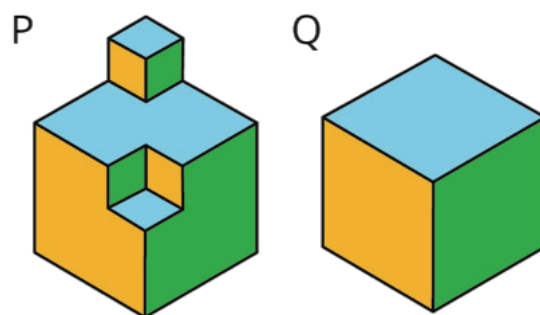
Assessment : End-of-Unit Assessment (A)

Problem 1

Students selecting A may think that removing a piece of a solid always decreases its surface area (or may just be guessing). Students selecting B are correct that the large cube has the same surface area before and after the small cube is removed, but are forgetting that putting the small cube back on top also contributes to the surface area. Students selecting D may not believe the problem can be solved without specifics, such as a side length.

Statement

Polyhedron P is a cube with a corner removed and relocated to the top of P. Polyhedron Q is a cube with the same size base as Polyhedron P. How do their surface areas compare?



- A. P's surface area is less than Q's surface area.
- B. P's surface area is equal to Q's surface area.
- C. P's surface area is greater than Q's surface area.
- D. There is not enough information given to compare their surface areas.

Solution

C

Aligned Standards

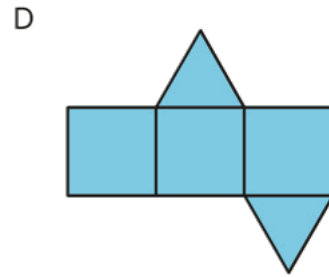
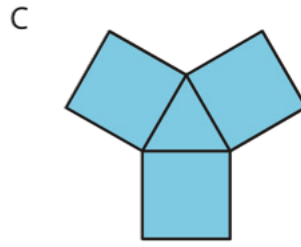
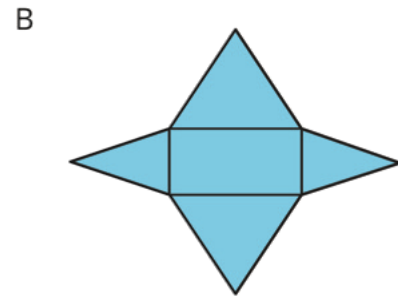
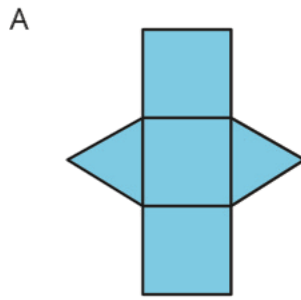
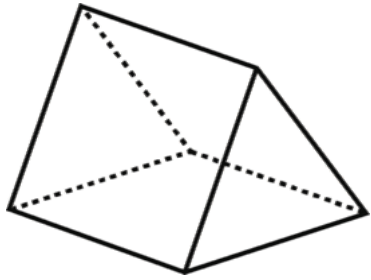
6.G.A

Problem 2

Students failing to select A or D may be having trouble visualizing how the nets can be folded to form the prism. Students selecting B or C do not understand that each face of the net corresponds to a face of the prism: there are 2 triangular faces and 3 rectangular faces in the prism, but that is not true of the nets in B or C.

Statement

Select **all** of the nets that can be folded and assembled into a triangular prism like this one.



A. A

B. B

C. C

D. D

Solution

["A", "D"]

Aligned Standards

6.G.A.4

Problem 3

Students selecting A are thinking of the area of a square. Students failing to select B may not understand the meaning of the exponent. Students selecting C are thinking of the surface area of a cube. Students selecting D may also be thinking of surface area, treating each face of the cube as if it has area 8 square inches. Students selecting B but failing to select E may have memorized that the volume of a cube is the cube of its side length, without thinking about what it means to cube a number.

Statement

A cube has a side length of 8 inches.

Select all the values that represent the cube's volume in cubic inches.

- A. 8^2
- B. 8^3
- C. $6 \cdot 8^2$
- D. $6 \cdot 8$
- E. $8 \cdot 8 \cdot 8$

Solution

["B", "E"]

Aligned Standards

6.EE.A.1

Problem 4

This problem has students find the area of a square given a side length, then the side length of a square given the area. Watch for students treating 9 as a side length in the second part of the problem.

Statement

1. A square has a side length 9 cm. What is its area?
2. A square has an area of 9 cm^2 . What is its side length?

Solution

1. 81 cm^2
2. 3 cm

Aligned Standards

6.EE.A.1

Problem 5

The first two parts of this problem can be solved without direct computation, by comparing factors. For the third part, students are very likely to compute each value, directly comparing the results. Some students will say 30^2 is larger because 30 is larger than 10.

Statement

For each pair of expressions, circle the expression with the greater value.

1. 13^2 or 15^2

2. $7 \cdot 6^2$ or 6^3

3. 10^3 or 30^2

Solution

1. 15^2

2. $7 \cdot 6^2$

3. 10^3

Aligned Standards

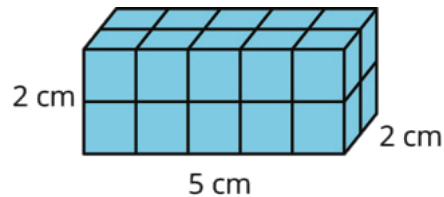
6.EE.A.1

Problem 6

Students use an understanding of area in rectangles to find the surface area of a rectangular prism.

Statement

A rectangular prism has dimensions of 2 cm by 2 cm by 5 cm. What is its surface area?
Explain or show your reasoning.



Solution

48 square centimeters. Sample explanation: the left and right faces have area $2 \cdot 2 = 4 \text{ cm}^2$. The top, bottom, front, and back faces have area $2 \cdot 5 = 10 \text{ cm}^2$. $2 \cdot 4 + 4 \cdot 10 = 48$.

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: $2 \cdot 2 + 2 \cdot 2 + 5 \cdot 2 + 5 \cdot 2 + 5 \cdot 2 + 5 \cdot 2 = 48$, so 48 cm^2 .

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.

- Sample errors: correct surface area without explanation or with no units included; work contains arithmetic mistakes but still indicates an intent to add up the areas of the six faces; response (with work shown) is the sum of the areas of the three visible faces only.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: student calculates a different quantity, such as volume or the area of only one face; incorrect answer with no work shown.

Aligned Standards

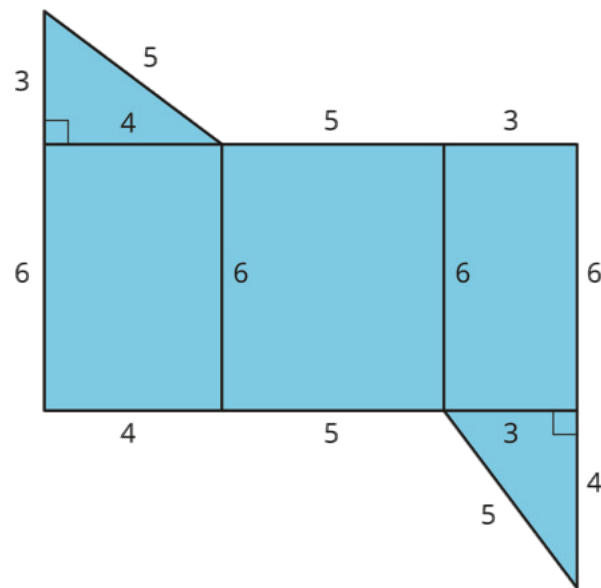
6.G.A.4

Problem 7

Students identify the polyhedron associated with a net, then calculate its surface area.

Statement

Here is a net made of right triangles and rectangles. All measurements are given in centimeters.



1. If the net were folded and assembled, what type of polyhedron would it make?
2. What is the surface area of the polyhedron? Explain your reasoning.

Solution

1. Triangular prism
2. 84 square centimeters. Sample reasoning: The net is made of two triangles with base 4 and height 3 as well as one big rectangle with base 12 and height 6. The two triangles have area 12

cm^2 because they can be put together to make a rectangle of area $4 \cdot 3 = 12$. The big rectangle has area 72 cm^2 because $12 \cdot 6 = 72$. So the total area is 84 cm^2 .

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.

- Sample:

1. Triangular prism

2. $\frac{1}{2} \cdot 4 \cdot 3 + \frac{1}{2} \cdot 4 \cdot 3 + 4 \cdot 6 + 5 \cdot 6 + 3 \cdot 6 = 84$, so 84 cm^2 .

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: arithmetic errors with otherwise correct work shown; area of one face is missing or incorrect; units omitted; surface area is correct and well-justified but the polyhedron is incorrect yet somewhat reasonable, like triangular pyramid or rectangular prism.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: correct identification of the shape as triangular prism but little progress on the surface area; no reasonable attempt at identifying the polyhedron (or an answer like "trapezoid"); surface area calculations involve serious mistakes like not knowing how to calculate the area of a right triangle; a badly incorrect answer to one part with a correct answer to the other part.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: Omissions or Tier 3 errors across both problem parts.

Aligned Standards

6.G.A.1, 6.G.A.4

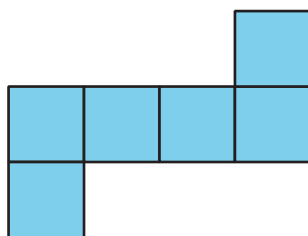
Assessment : End-of-Unit Assessment (B)

Problem 1

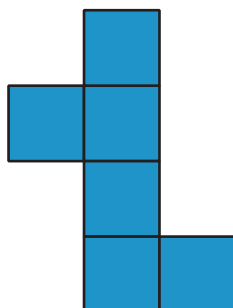
Statement

Select **all** of the nets that can be folded and assembled into a cube.

A.



B.



C.



Solution

["A", "B"]

Aligned Standards

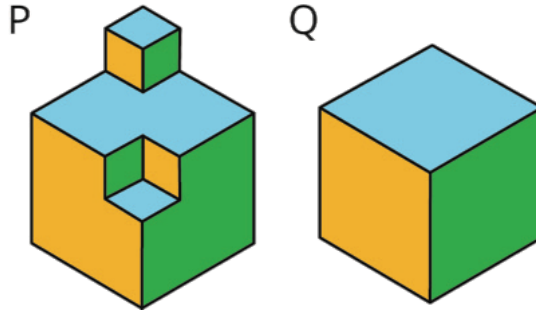
6.G.A.4

Problem 2

Students selecting B are correct that the large cube has the same surface area before and after the small cube is removed, but are forgetting that putting the small cube back on top also contributes to the surface area. Or, students selecting B may be confusing surface area for volume. Students selecting D may not believe the problem can be solved without specifics, such as a side length.

Statement

Polyhedron P is a cube with a corner removed and relocated to the top of P. Polyhedron Q is a cube with the same size base as Polyhedron P. How do their surface areas compare?



- A. Q's surface area is less than P's surface area.
- B. Q's surface area is equal to P's surface area.
- C. Q's surface area is greater than P's surface area.
- D. There is not enough information given to compare their surface areas.

Solution

A

Aligned Standards

6.G.A

Problem 3

Students selecting A may be confusing 4^3 with $3 \cdot 4$. Students selecting B may be confusing surface area with volume. Students selecting D may think that 4 could represent the volume of a single cube.

Statement

For a cube whose side length is 4 inches, the expression 4^3 could represent . . .

- A. The length of 3 cubes lined up, in inches
- B. The cube's surface area in square inches
- C. The cube's volume in cubic inches
- D. The total volume of 3 cubes, in cubic inches

Solution

C

Aligned Standards

6.EE.A.1

Problem 4

This problem has students find the side length when the area of square is given, and the area of a square when its side length is given.

Statement

1. A square has an area of 16 cm^2 . What is the length of each of its sides?
2. A square has a side length of 8 cm. What is its area?

Solution

1. 4 cm
2. 64 cm^2

Aligned Standards

6.EE.A.1

Problem 5

Students will evaluate the expressions involving exponents. Students failing to be able to rearrange these expressions in the correct order may not have a clear understanding of the definition of bases and exponents.

Statement

Here is a list of expressions. Order them from least to greatest.

$$40^2 \quad 8^3 \quad 10^3 \quad 7 \cdot 8^2 \quad 10^4$$

Solution

$$7 \cdot 8^2, 8^3, 10^3, 40^2, 10^4$$

Aligned Standards

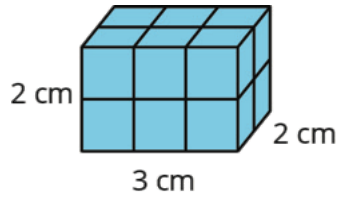
6.EE.A.1

Problem 6

Students use an understanding of area in rectangles to find the surface area of a rectangular prism.

Statement

A rectangular prism has dimensions of 2 cm by 2 cm by 3 cm. What is its surface area? Explain or show your reasoning.



Solution

32 square centimeters. Sample explanation: the front and back faces have area 4 square centimeters. The top, bottom, left, and right faces have area 6 square centimeters.

$$2 \cdot 4 + 4 \cdot 6 = 32.$$

Minimal Tier 1 response:

- Work is complete and correct.
- Sample: $2 \cdot 2 + 2 \cdot 2 + 3 \cdot 2 + 3 \cdot 2 + 3 \cdot 2 + 3 \cdot 2 = 32$, so 32 cm^2 .

Tier 2 response:

- Work shows general conceptual understanding and mastery, with some errors.
- Sample errors: correct surface area without explanation or with no units included; work contains arithmetic mistakes but still indicates an intent to add up the areas of the six faces; response (with work shown) is the sum of the areas of the three visible faces only.

Tier 3 response:

- Significant errors in work demonstrate lack of conceptual understanding or mastery.
- Sample errors: student calculates a different quantity, such as volume or the area of only one face; incorrect answer with no work shown.

Aligned Standards

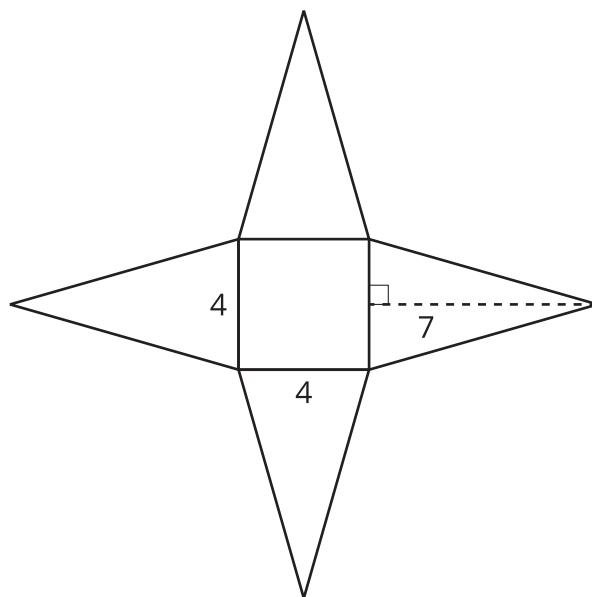
6.G.A.4

Problem 7

Students identify the polyhedron associated with a net, then calculate its surface area.

Statement

Here is a net made of a square and four identical triangles. All measurements are given in centimeters.



1. If the net were folded and assembled, what type of polyhedron would it make?
2. What is the surface area of the polyhedron? Explain your reasoning.

Solution

1. Square pyramid
2. 72 square centimeters. Sample reasoning: The net is made of four triangles with base 4 and height 7 as well as a square with side lengths of 4. The four triangles each have area 14 cm^2 . The square has an area of 16 cm^2 . $4 \cdot 14 + 16 = 72$.

Minimal Tier 1 response:

- Work is complete and correct, with complete explanation or justification.
- Sample: A. Square pyramid B. $(.5)(4)(7) = 14 \times 4 = 56$ area of total triangles and the base has an area of . Therefore $56 + 16 = 72$

Tier 2 response:

- Work shows good conceptual understanding and mastery, with either minor errors or correct work with insufficient explanation or justification.
- Sample errors: arithmetic errors with otherwise correct work shown; area of one face is missing or incorrect; units omitted; surface area is correct and well-justified but the polyhedron is incorrect yet somewhat reasonable, like triangular pyramid or rectangular prism.

Tier 3 response:

- Work shows a developing but incomplete conceptual understanding, with significant errors.
- Sample errors: correct identification of the shape as triangular prism but little progress on the surface area; no reasonable attempt at identifying the polyhedron (or an answer like “trapezoid”); surface area calculations involve serious mistakes like not knowing how to calculate the area of a triangle; a badly incorrect answer to one part with a correct answer to the other part.

Tier 4 response:

- Work includes major errors or omissions that demonstrate a lack of conceptual understanding and mastery.
- Sample errors: Omissions or Tier 3 errors across both problem parts.

Aligned Standards

6.G.A.1, 6.G.A.4

Lesson
Cool Downs

Lesson 1: Tiling the Plane

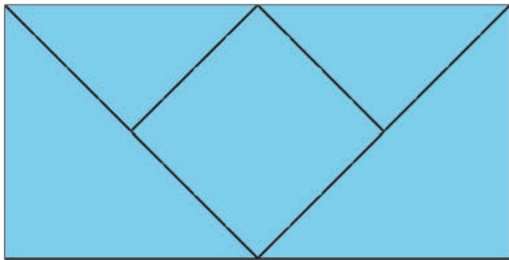
Cool Down: What is Area?

Think about your work today, and write your best definition of area.

Lesson 2: Finding Area by Decomposing and Rearranging

Cool Down: Tangram Rectangle

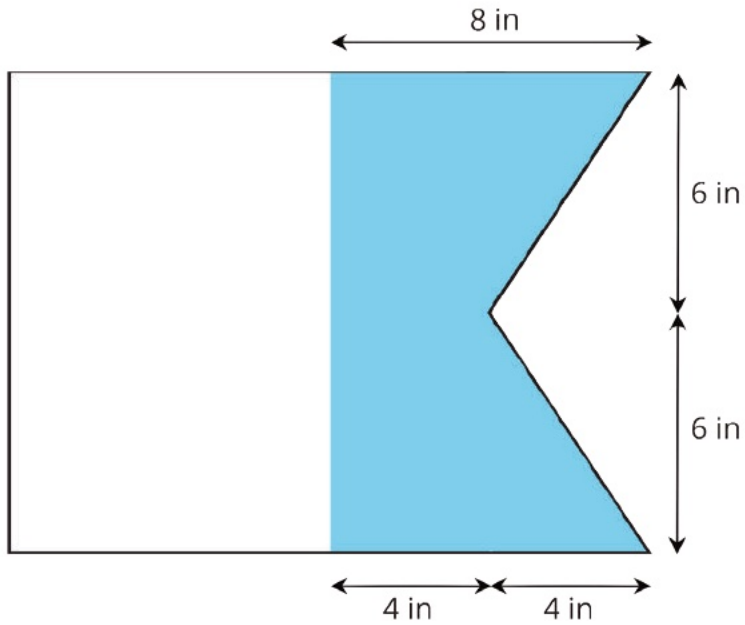
The square in the middle has an area of 1 square unit. What is the area of the entire rectangle in square units? Explain your reasoning.



Lesson 3: Reasoning to Find Area

Cool Down: Maritime Flag

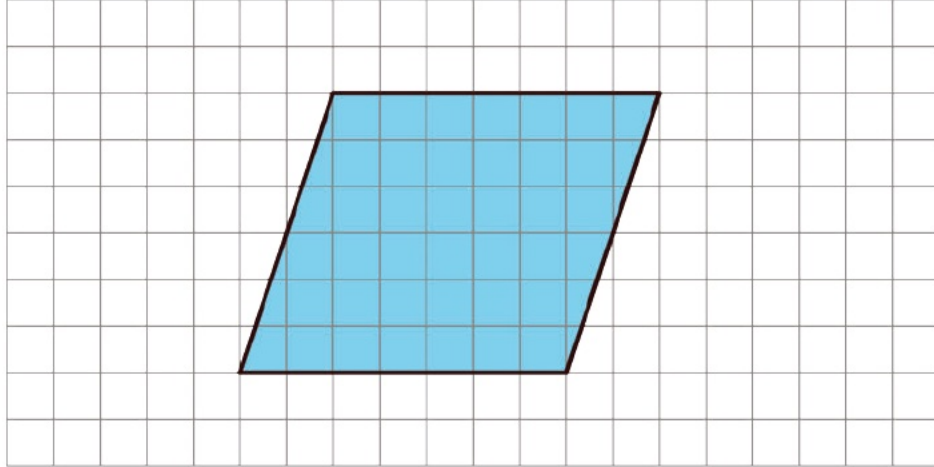
A maritime flag is shown. What is the area of the shaded part of the flag? Explain or show your reasoning.



Lesson 4: Parallelograms

Cool Down: How Would You Find the Area?

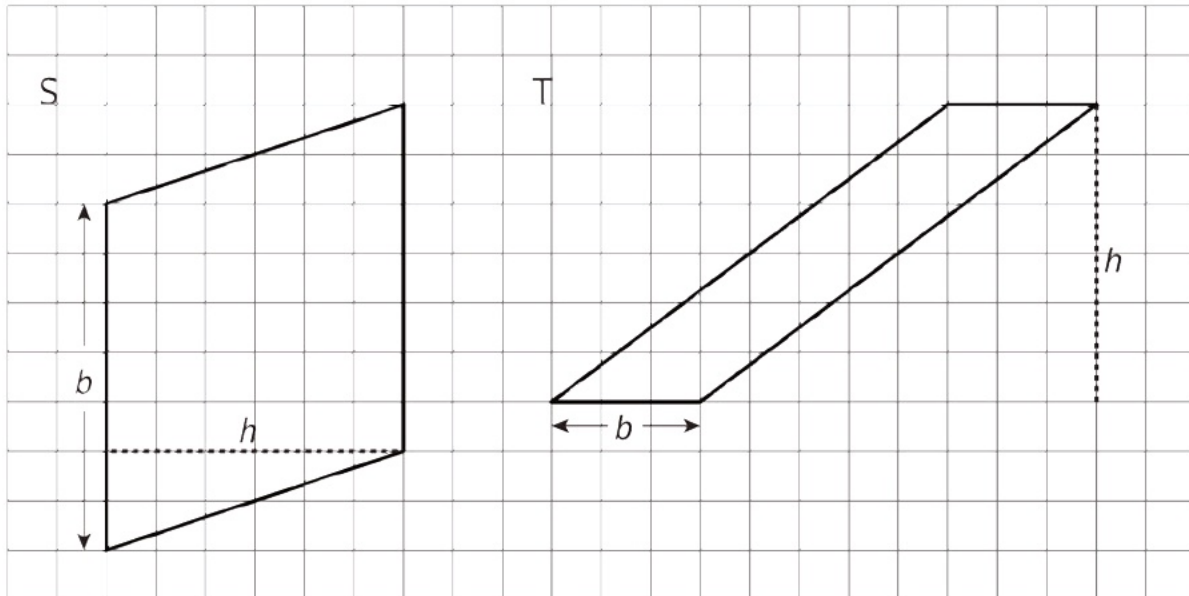
How would you find the area of this parallelogram? Describe your strategy.



Lesson 5: Bases and Heights of Parallelograms

Cool Down: Parallelograms S and T

Parallelograms S and T are each labeled with a base and a corresponding height.



1. What are the values of b and h for each parallelogram?

○ Parallelogram S: $b = \underline{\hspace{2cm}}$, $h = \underline{\hspace{2cm}}$

○ Parallelogram T: $b = \underline{\hspace{2cm}}$, $h = \underline{\hspace{2cm}}$

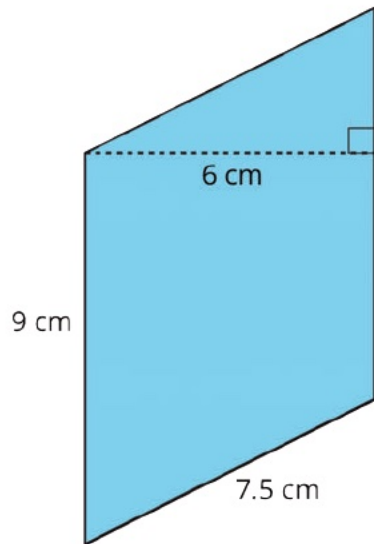
2. Use the values of b and h to find the area of each parallelogram.

○ Area of Parallelogram S:

○ Area of Parallelogram T:

Lesson 6: Area of Parallelograms

Cool Down: One More Parallelogram



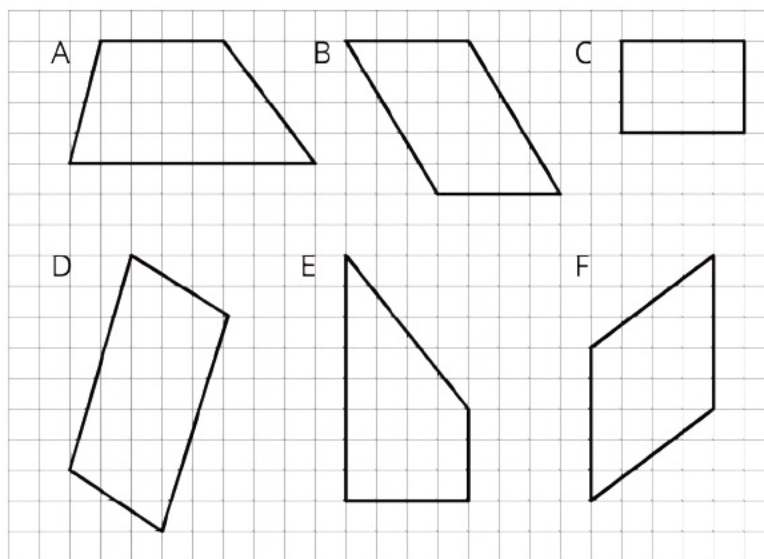
1. Find the area of the parallelogram. Explain or show your reasoning.

2. Was there a length measurement you did not use to find the area? If so, explain why it was not used.

Lesson 7: From Parallelograms to Triangles

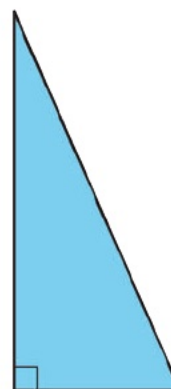
Cool Down: A Tale of Two Triangles (Part 3)

1. Here are some quadrilaterals.



- Circle all quadrilaterals that you think can be decomposed into two identical triangles using only one line.
- What characteristics do the quadrilaterals that you circled have in common?

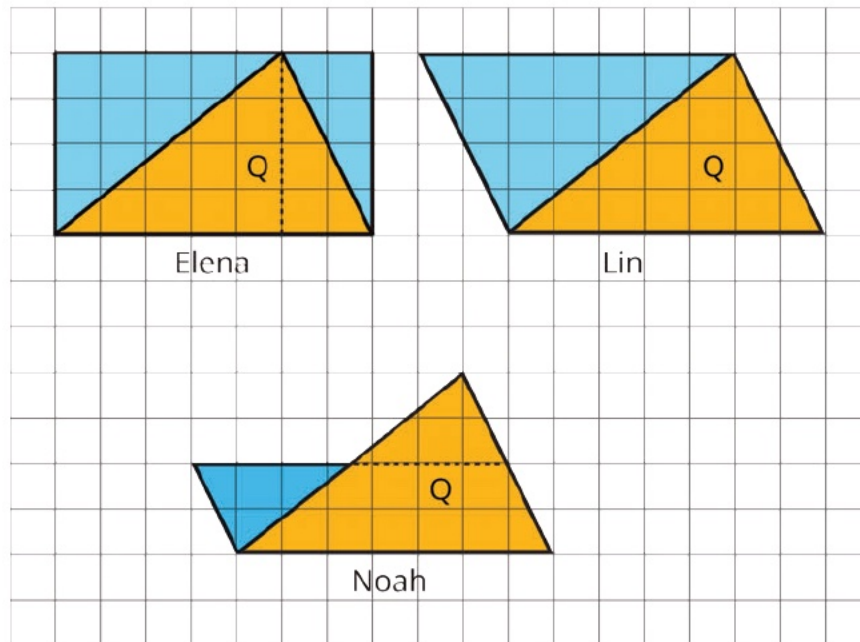
2. Here is a right triangle. Show or briefly describe how two copies of it can be composed into a parallelogram.



Lesson 8: Area of Triangles

Cool Down: An Area of 14

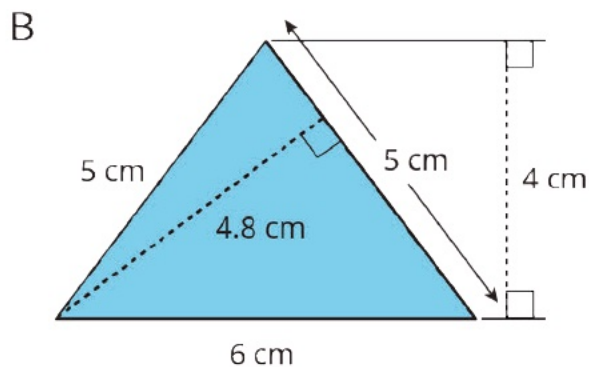
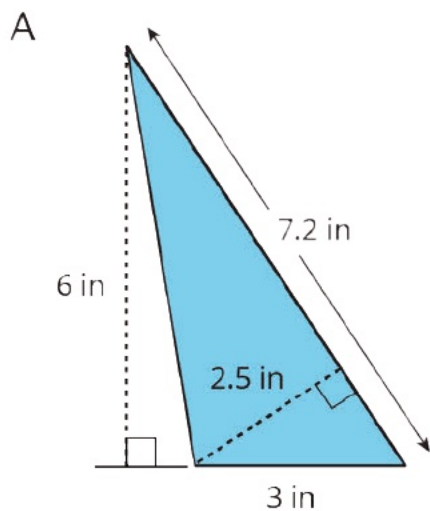
Elena, Lin, and Noah all found the area of Triangle Q to be 14 square units but reasoned about it differently, as shown in the diagrams. Explain *at least one* student's way of thinking and why his or her answer is correct.



Lesson 9: Formula for the Area of a Triangle

Cool Down: Two More Triangles

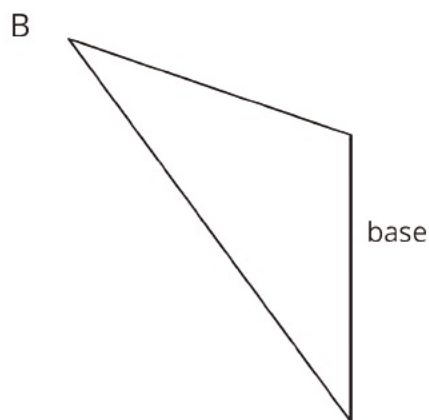
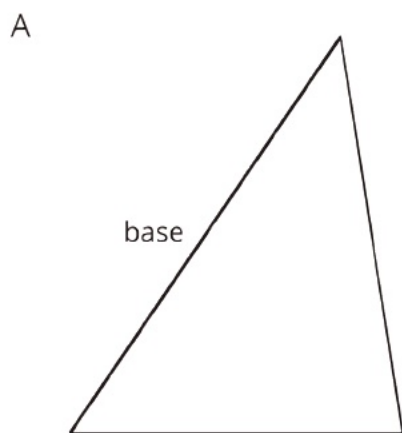
For each triangle, identify a base and a corresponding height. Use them to find the area. Show your reasoning.



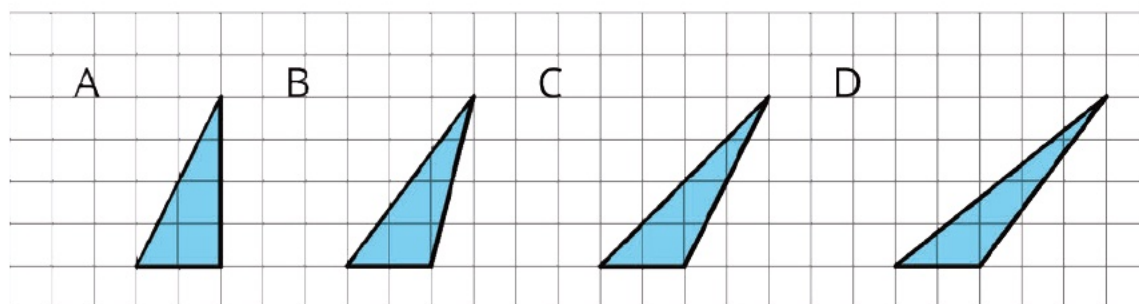
Lesson 10: Bases and Heights of Triangles

Cool Down: Stretched Sideways

1. For each triangle, draw a height segment that corresponds to the given base, and label it h . Use an index card if needed.



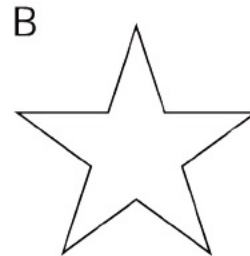
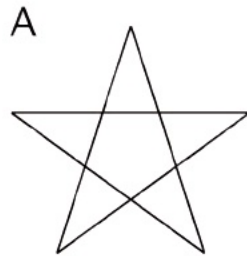
2. Which triangle has the greatest area? The least area? Explain your reasoning.



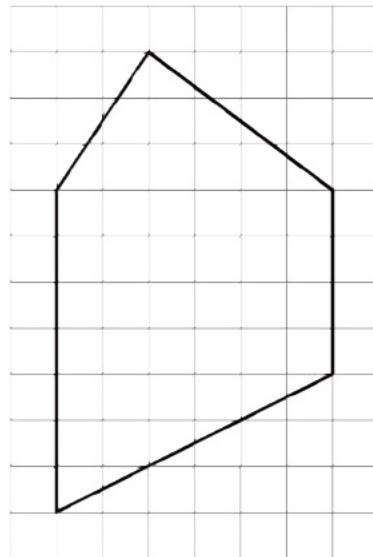
Lesson 11: Polygons

Cool Down: Triangulation

1. Here are two five-pointed stars. A student said, "Both figures A and B are polygons. They are both composed of line segments and are two-dimensional. Neither have curves." Do you agree with the statement? Explain your reasoning.



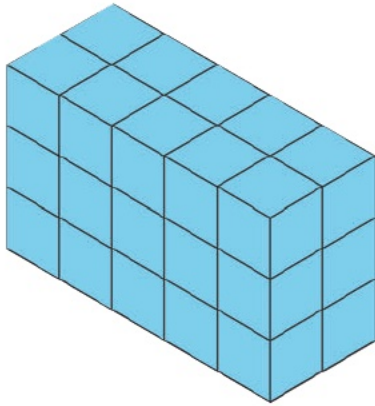
2. Here is a five-sided polygon. Describe or show the strategy you would use to find its area. Mark up and label the diagram to show your reasoning so that it can be followed by others. (It is not necessary to actually calculate the area.)



Lesson 12: What is Surface Area?

Cool Down: A Snap Cube Prism

A rectangular prism made is 3 units high, 2 units wide, and 5 units long. What is its surface area in square units? Explain or show your reasoning.

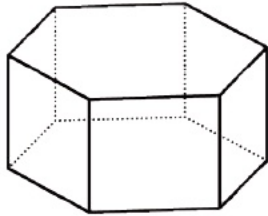


Lesson 13: Polyhedra

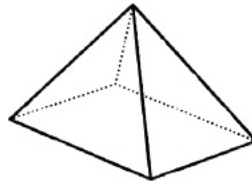
Cool Down: Three-Dimensional Shapes

1. Write your best definition or description of a polyhedron. If possible, use the terms you learned in this lesson.

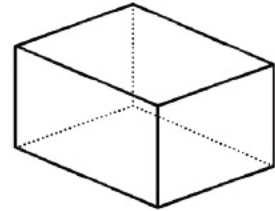
2. Which of these five polyhedra are prisms? Which are pyramids?



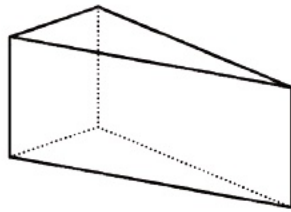
A



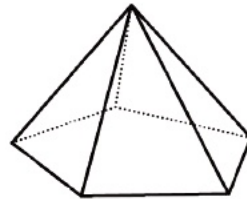
B



C



D

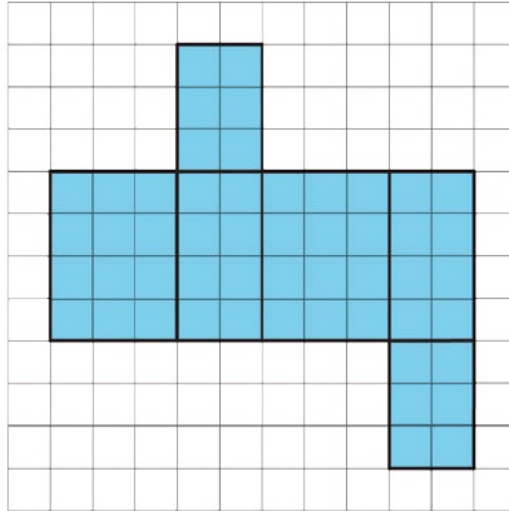


E

Lesson 14: Nets and Surface Area

Cool Down: Unfolded

1. What kind of polyhedron can be assembled from this net?

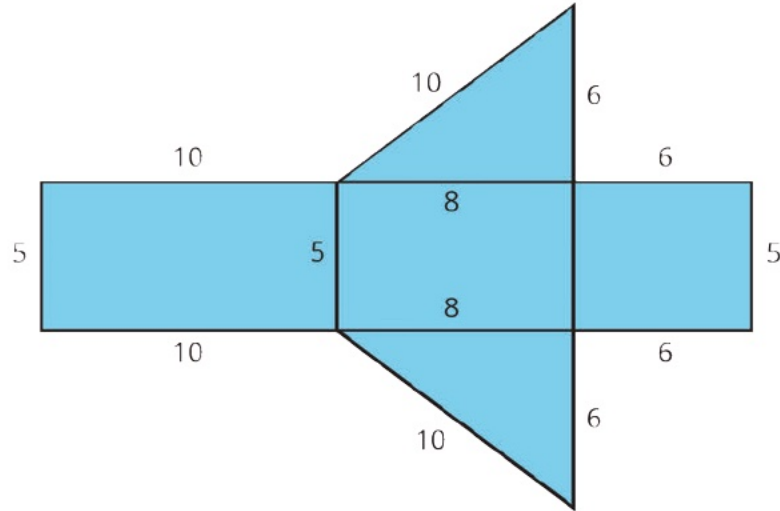


2. Find the surface area (in square units) of the polyhedron. Show your reasoning.

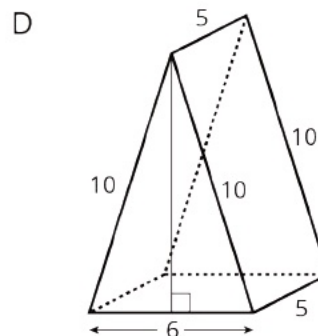
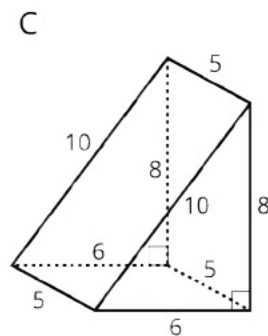
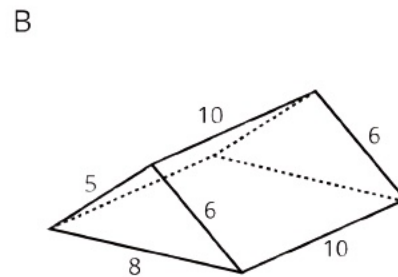
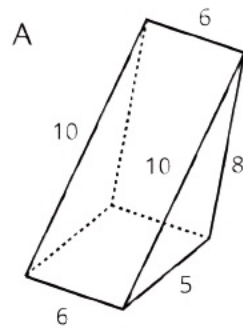
Lesson 15: More Nets, More Surface Area

Cool Down: Surface Area of a Triangular Prism

1. In this net, the two triangles are right triangles. All quadrilaterals are rectangles. What is its surface area in square units? Show your reasoning.



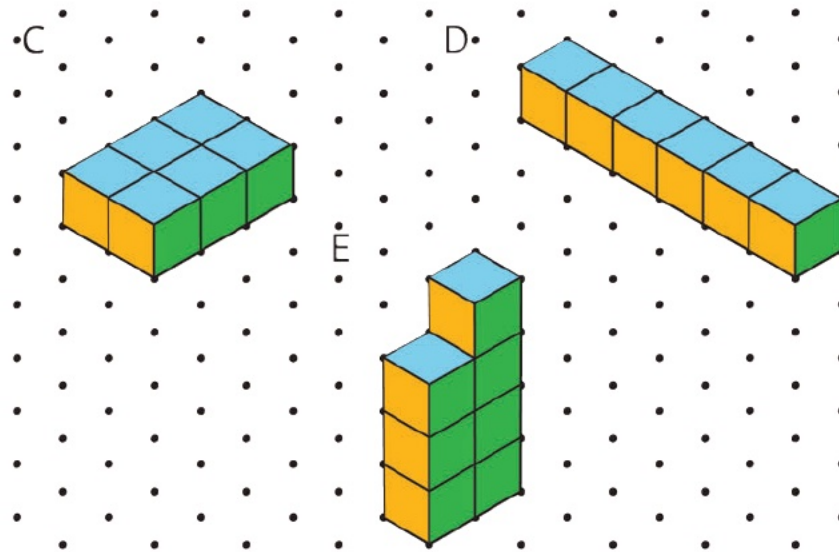
2. If the net is assembled, which of the following polyhedra would it make?



Lesson 16: Distinguishing Between Surface Area and Volume

Cool Down: Same Surface Area, Different Volumes

Choose two figures that have the same surface area but different volumes. Show your reasoning.



Lesson 17: Squares and Cubes

Cool Down: Exponent Expressions

1. Which is larger, 5^2 or 3^3 ?

2. A cube has an edge length of 21 cm. Use an exponent to express its volume.

Lesson 18: Surface Area of a Cube

Cool Down: From Volume to Surface Area

1. A cube has edge length 11 inches. Write an expression for its volume and an expression for its surface area.

2. A cube has a volume of 7^3 cubic centimeters. What is its surface area?

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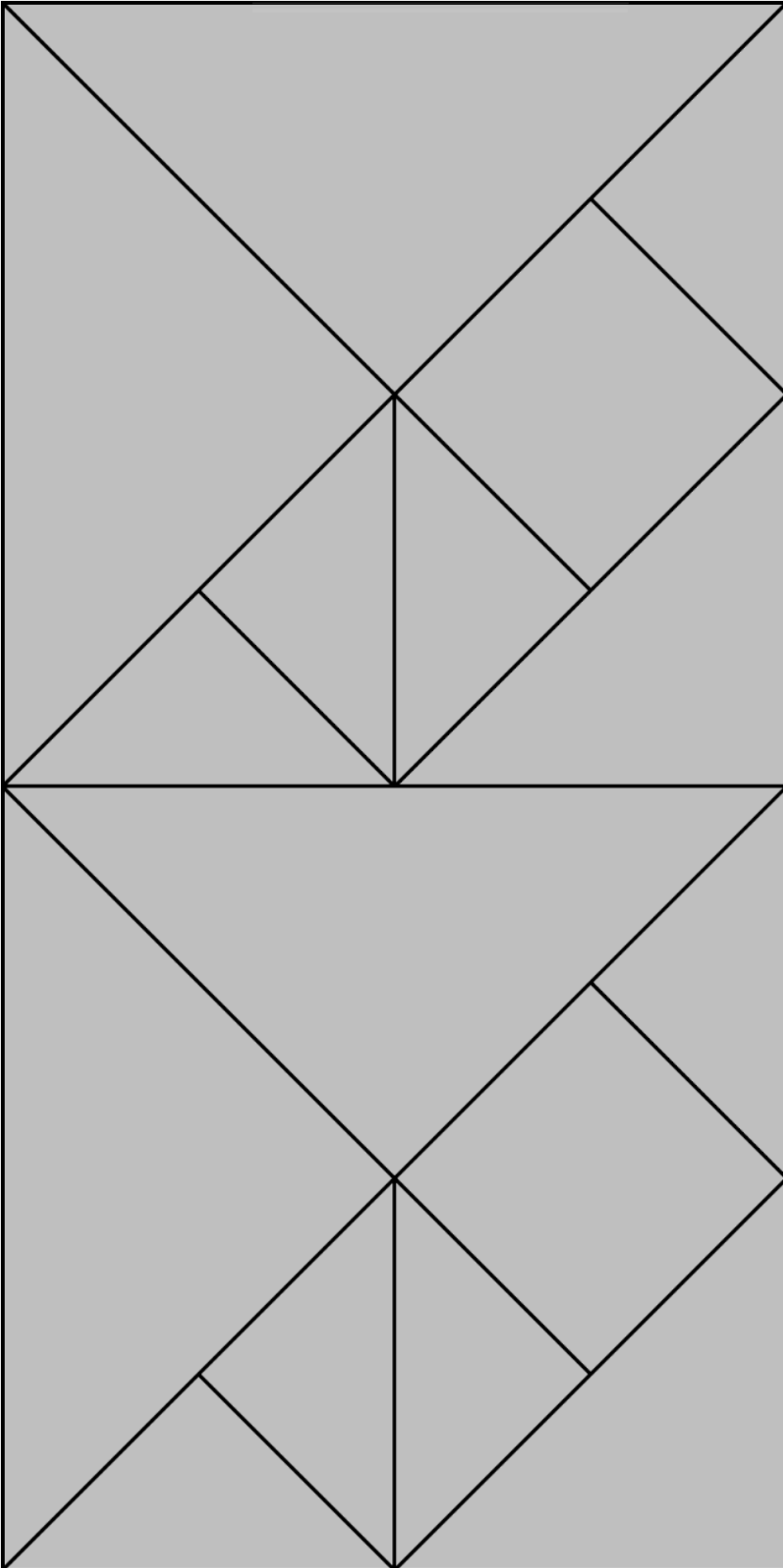


Instructional
Masters

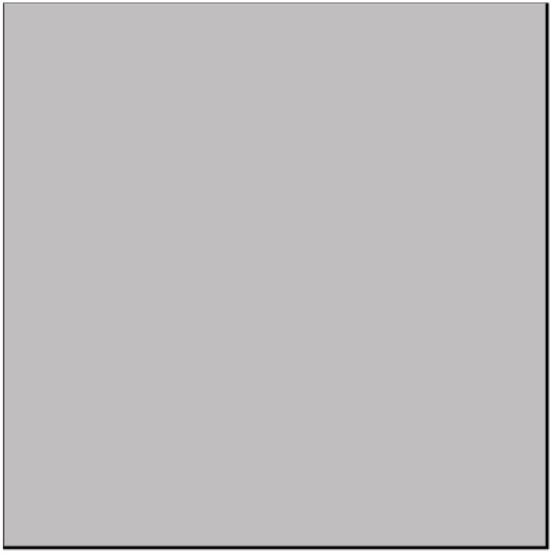
Instructional Masters for Area and Surface Area

address	title	students per copy	written on?	requires cutting?	card stock recommended?	color paper recommended?
Activity Grade6.1.7.3	A Tale of Two Triangles (Part 2)	3	no	yes	yes	no
Activity Grade6.1.2.2	Composing Shapes	2	no	yes	yes	no
Activity Grade6.1.13.3	Assembling Polyhedra	6	yes	no	no	no
Activity Grade6.1.8.3	Decomposing a Parallelogram	4	yes	yes	no	no
Activity Grade6.1.11.4	Pinwheel	4	yes	no	no	no
Activity Grade6.1.14.2	Using Nets to Find Surface Area	3	yes	no	no	no
Activity Grade6.1.19.1	Tent Design - Part 1	1	yes	no	no	no
Activity Grade6.1.15.2	Building Prisms and Pyramids	9	yes	no	no	no
Activity Grade6.1.3.1	Comparing Regions	1	yes	yes	no	no
Activity Grade6.1.13.1	What are Polyhedra?	12	no	yes	yes	no
Activity Grade6.1.14.1	Matching Nets	2	no	yes	no	no
Activity Grade6.1.13.2	Prisms and Pyramids	4	no	yes	yes	no

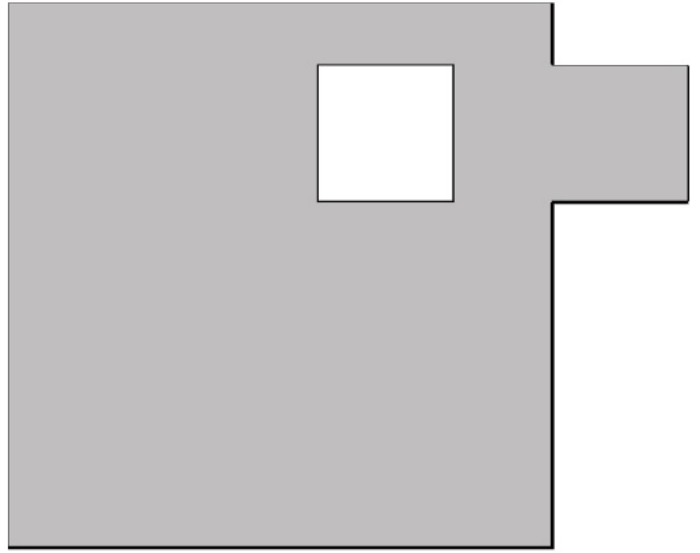
6.1.2.2 Composing Shapes.



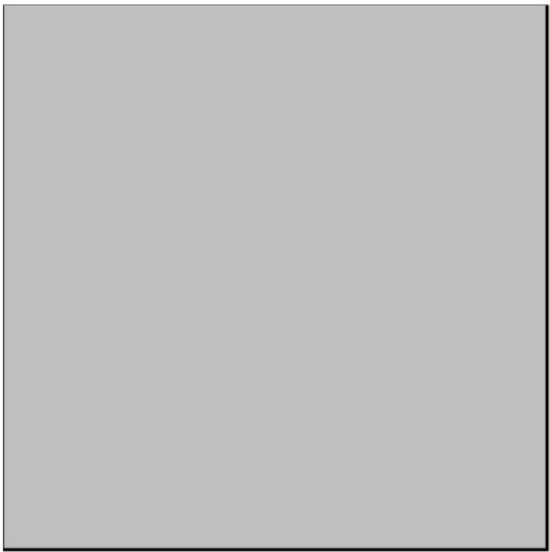
6.1.3.1 Comparing Regions.



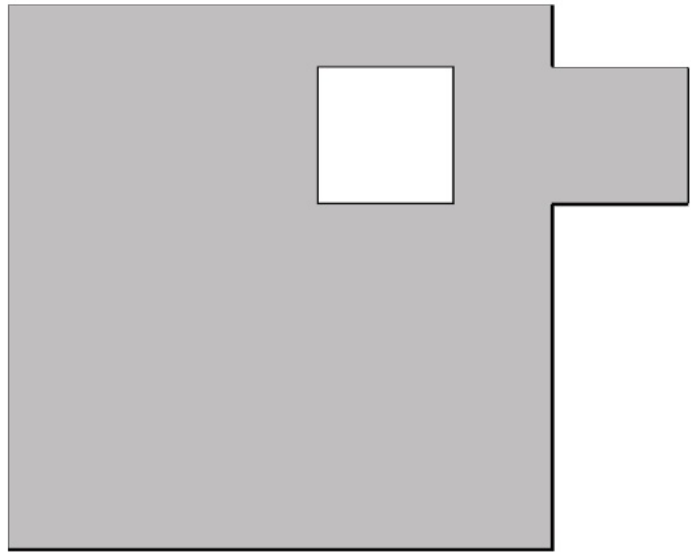
A



B

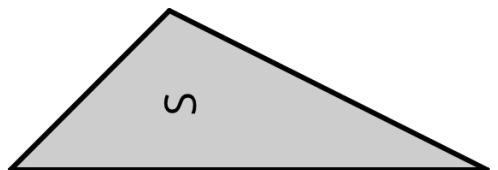
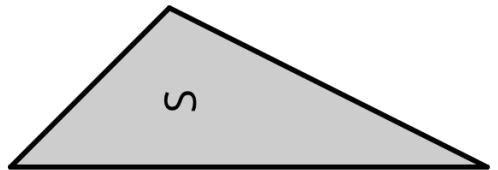
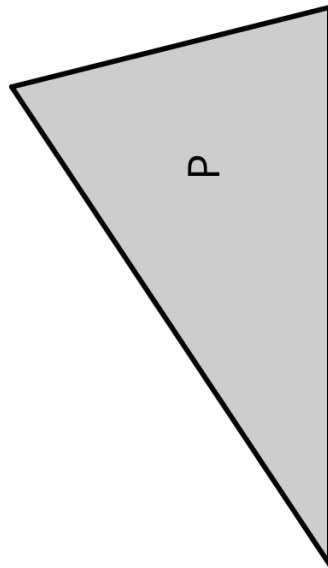
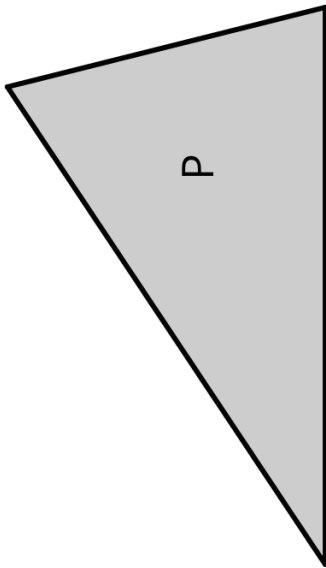
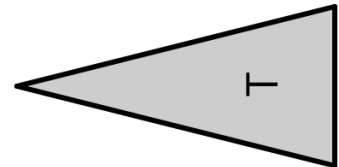
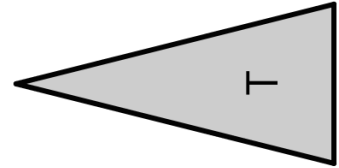
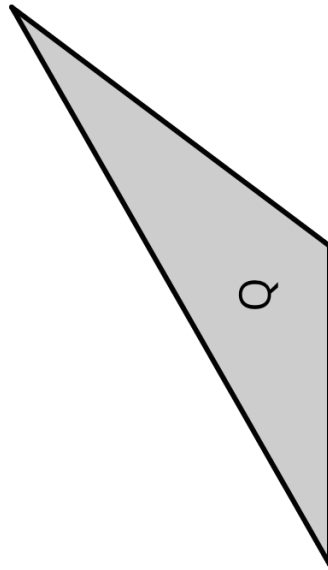
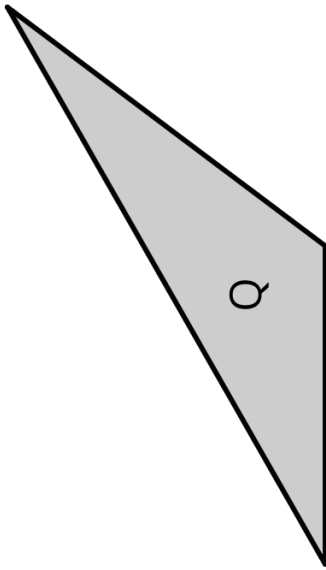
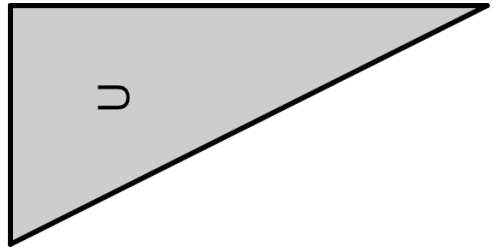
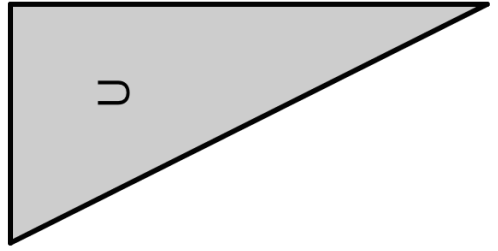
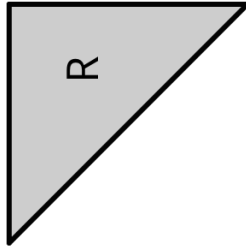
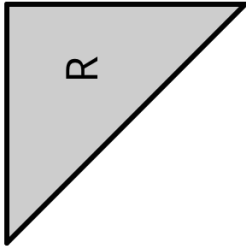


A



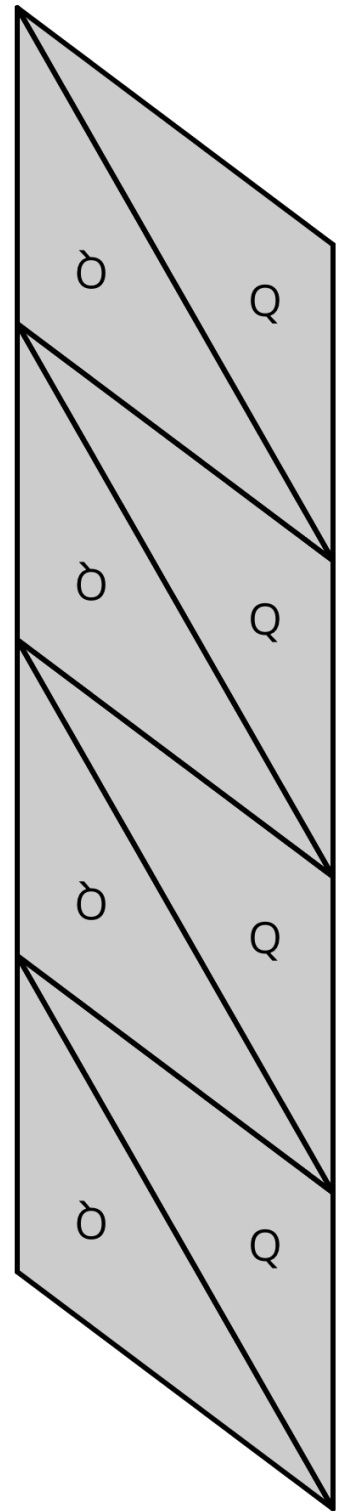
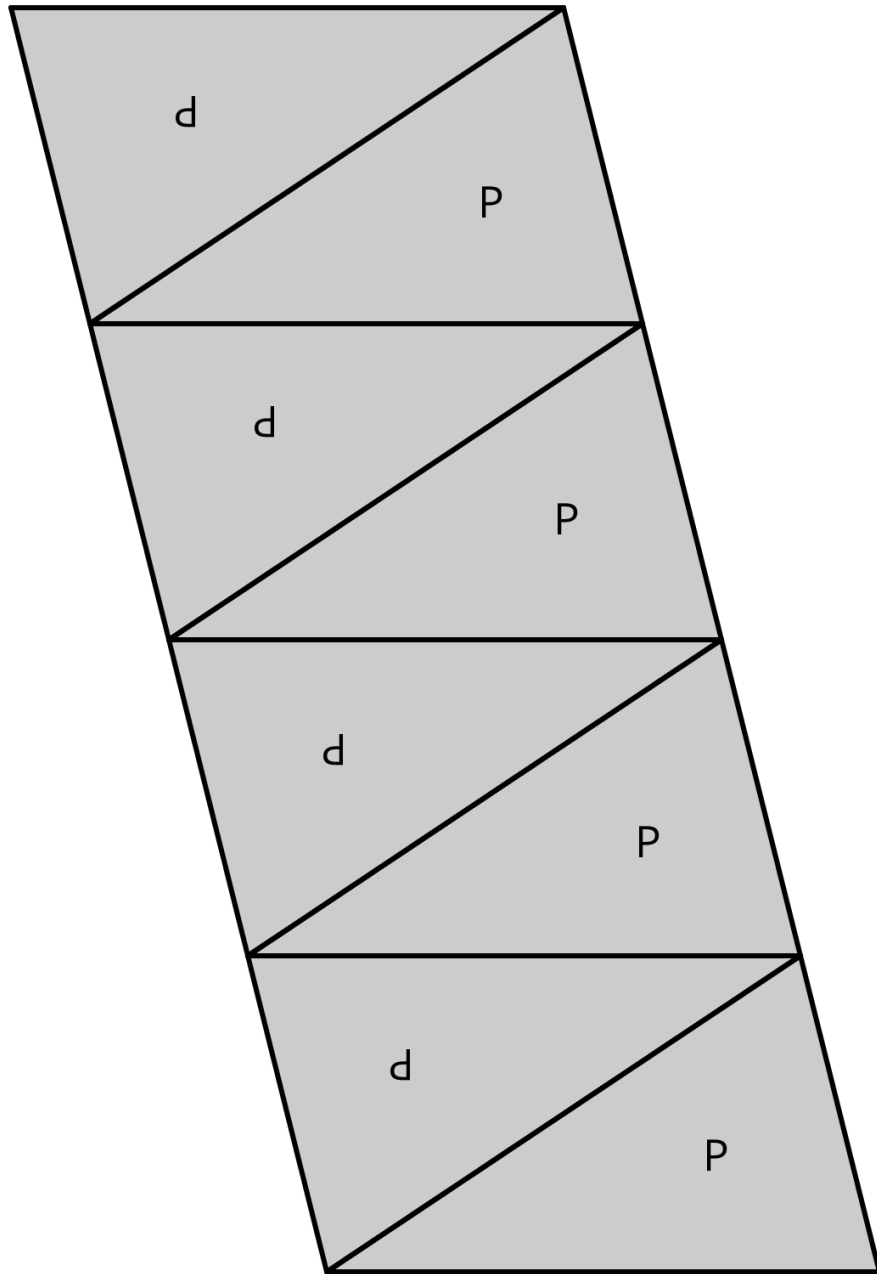
B

Use this Instructional master if triangles are to be cut by students.



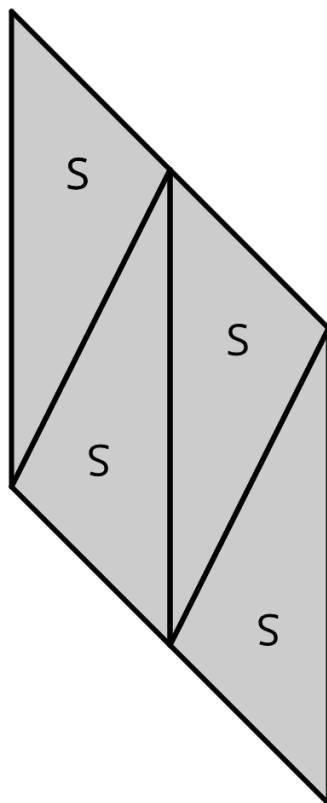
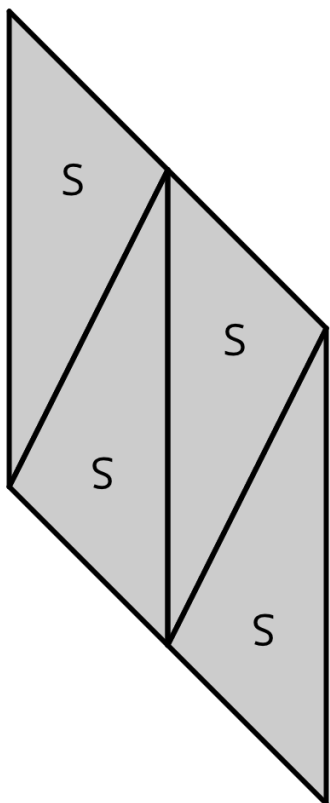
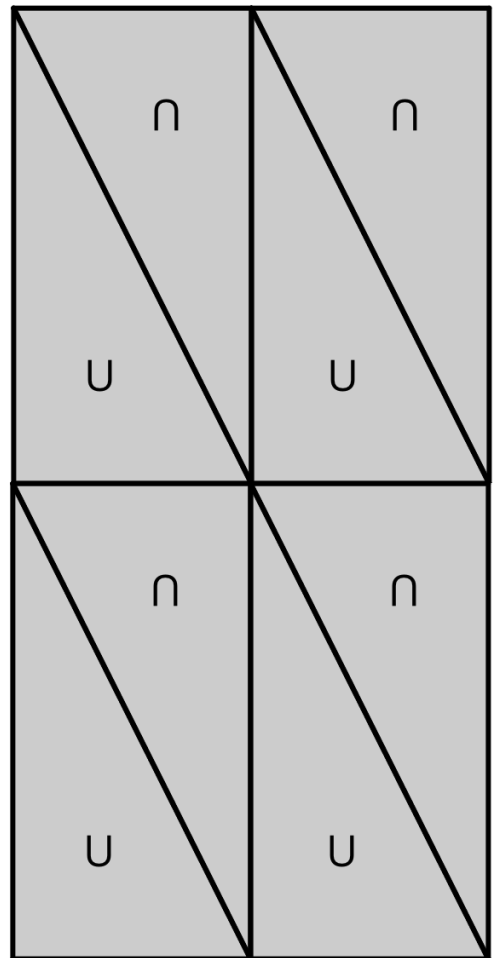
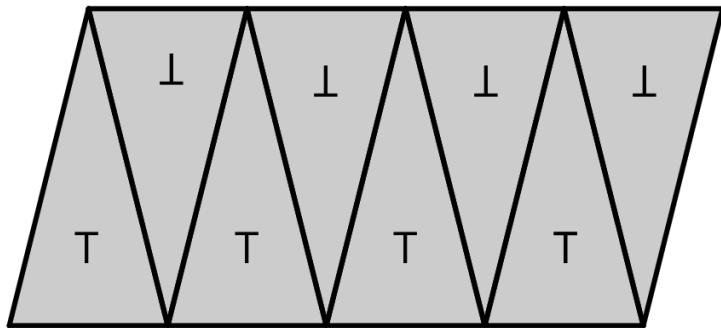
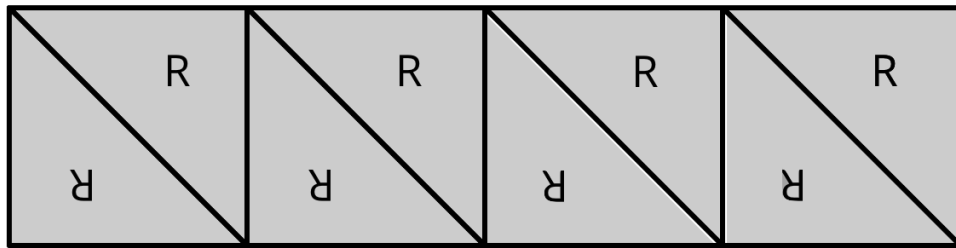
6.1.7.3 A Tale of Two Triangles (Part 2).

Use this Instructional master if triangles are pre-cut by teacher.



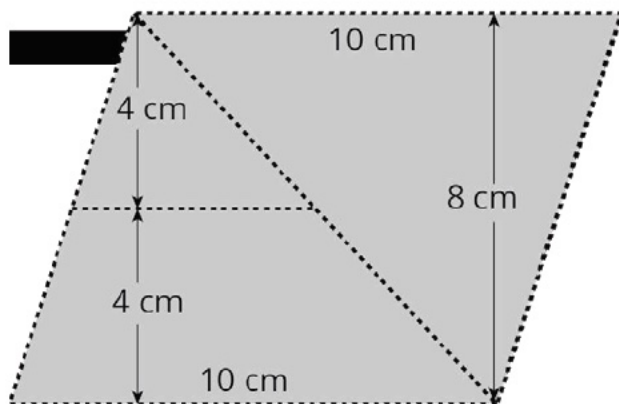
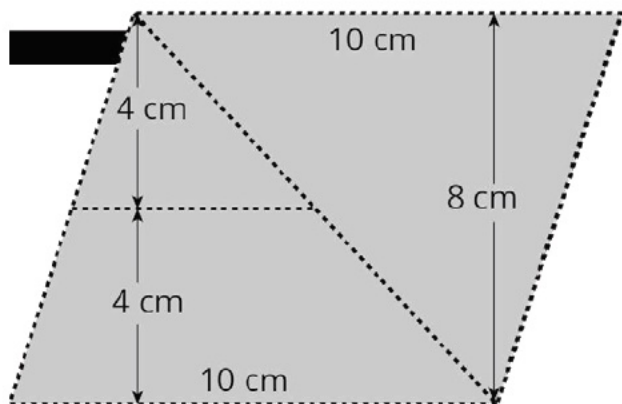
6.1.7.3 A Tale of Two Triangles (Part 2).

Use this Instructional master if triangles are pre-cut by teacher.

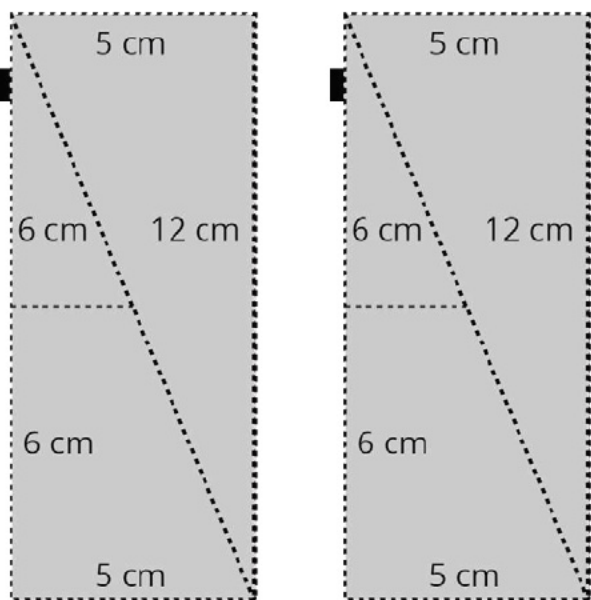


6.1.8.3 Decomposing a Parallelogram.

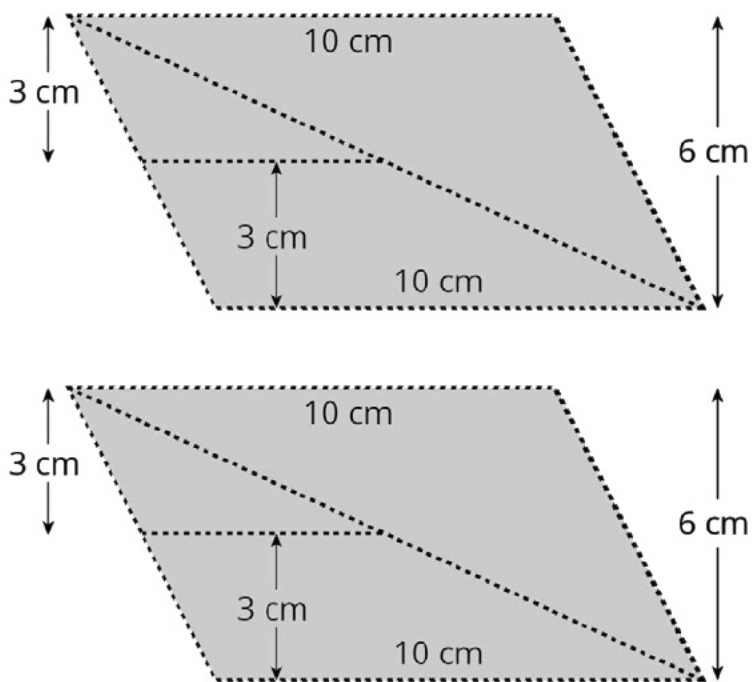
Parallelogram A



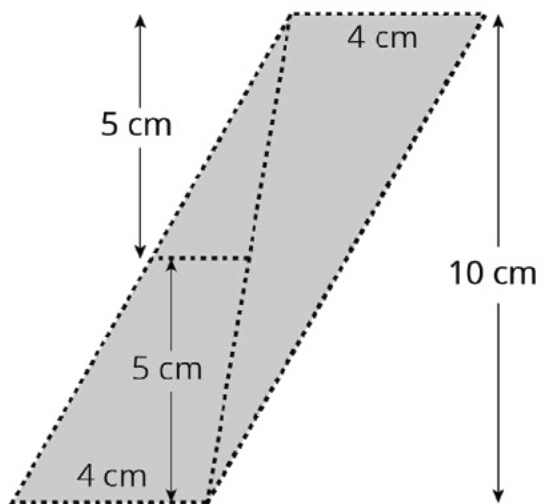
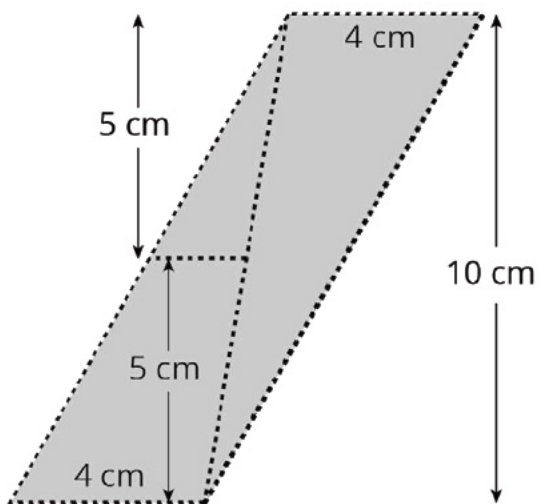
Parallelogram B



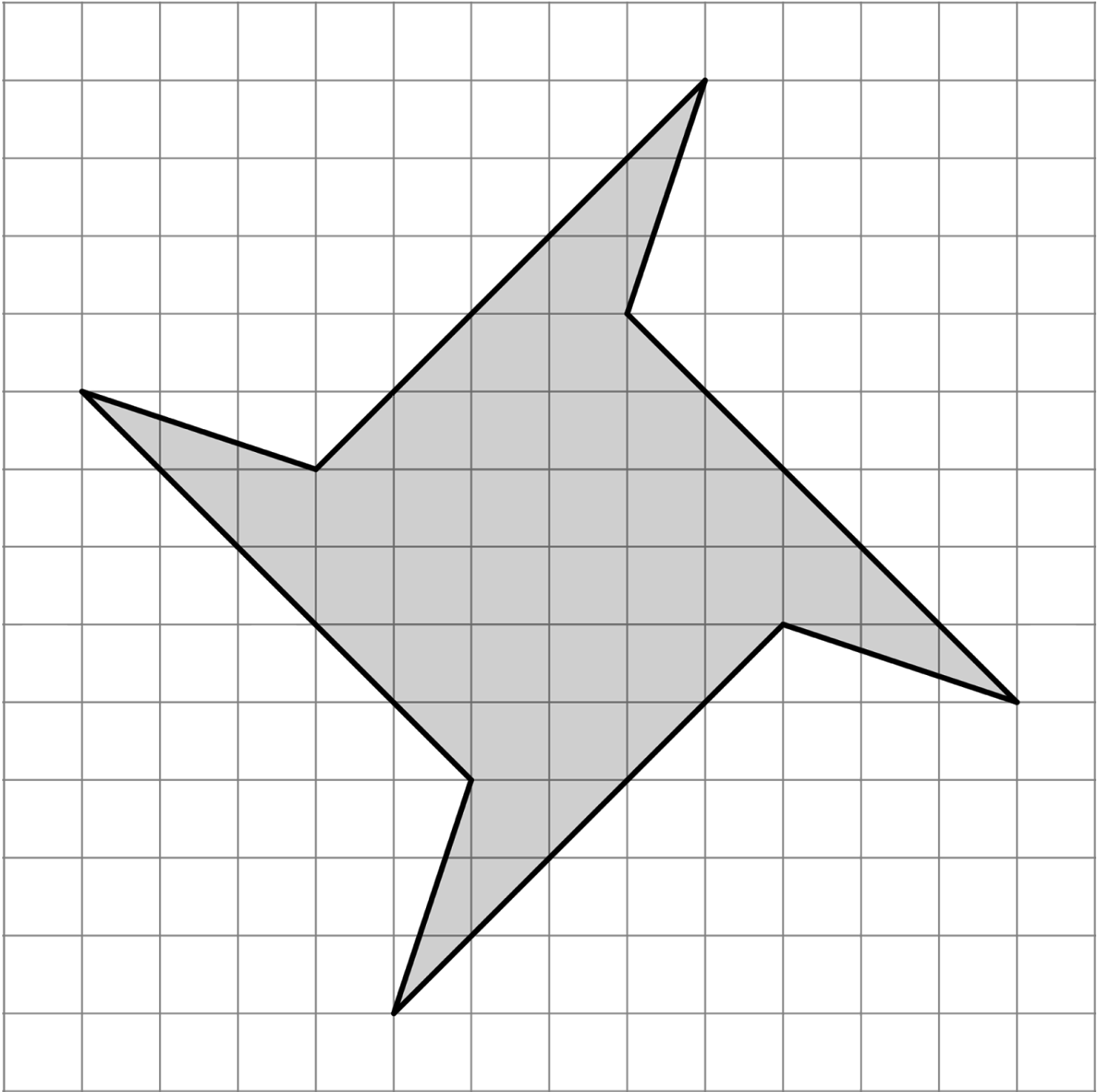
Parallelogram C

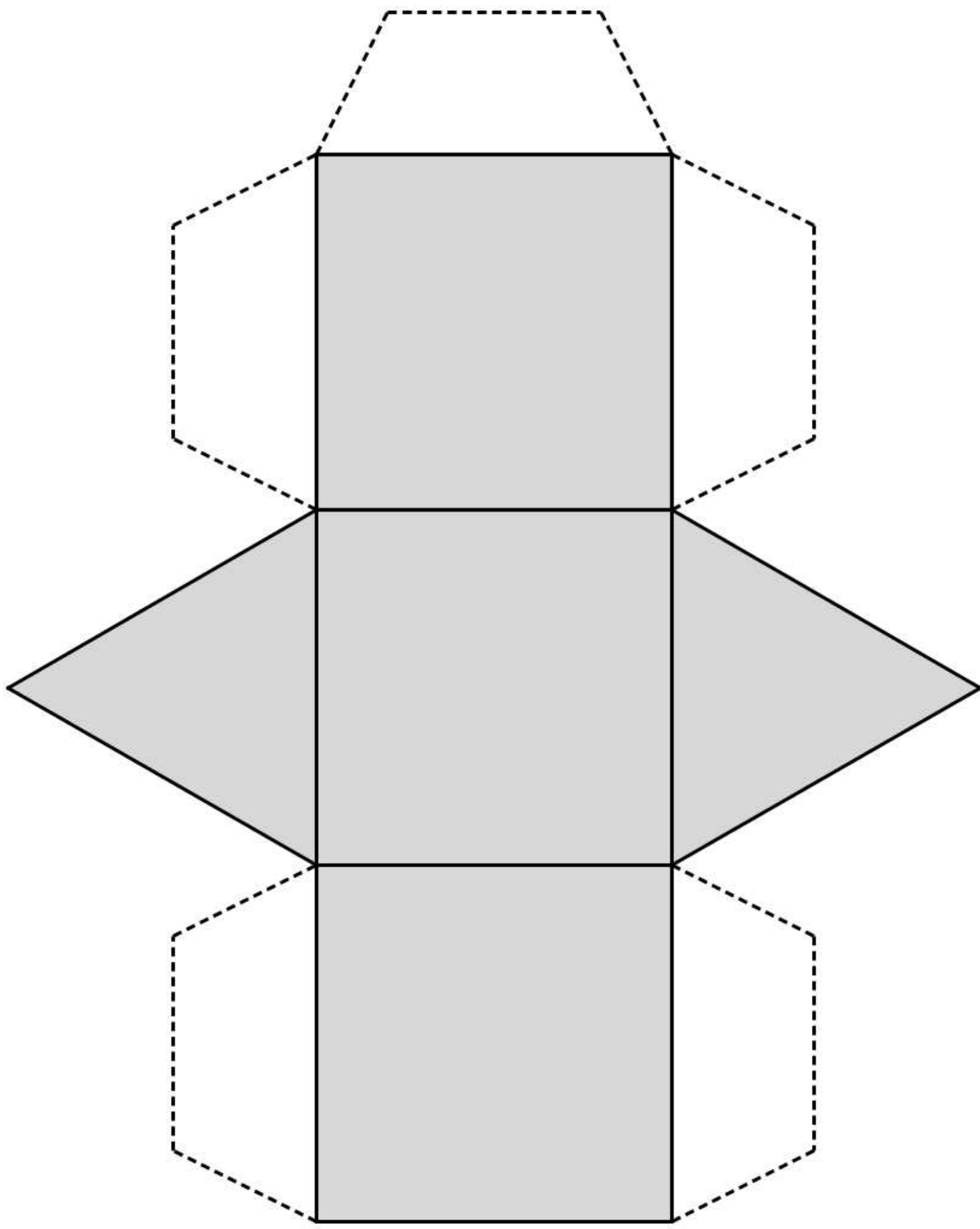


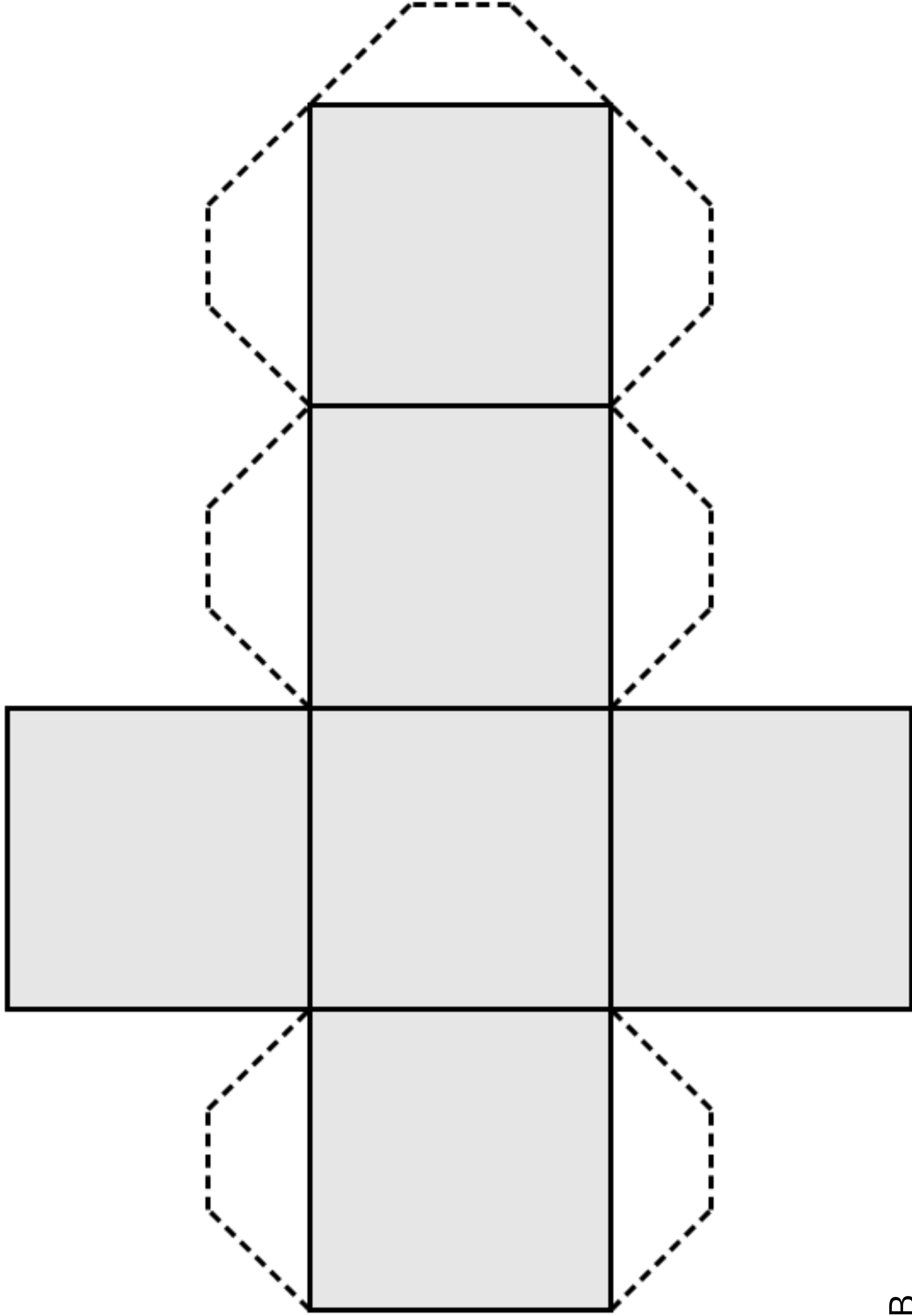
Parallelogram D

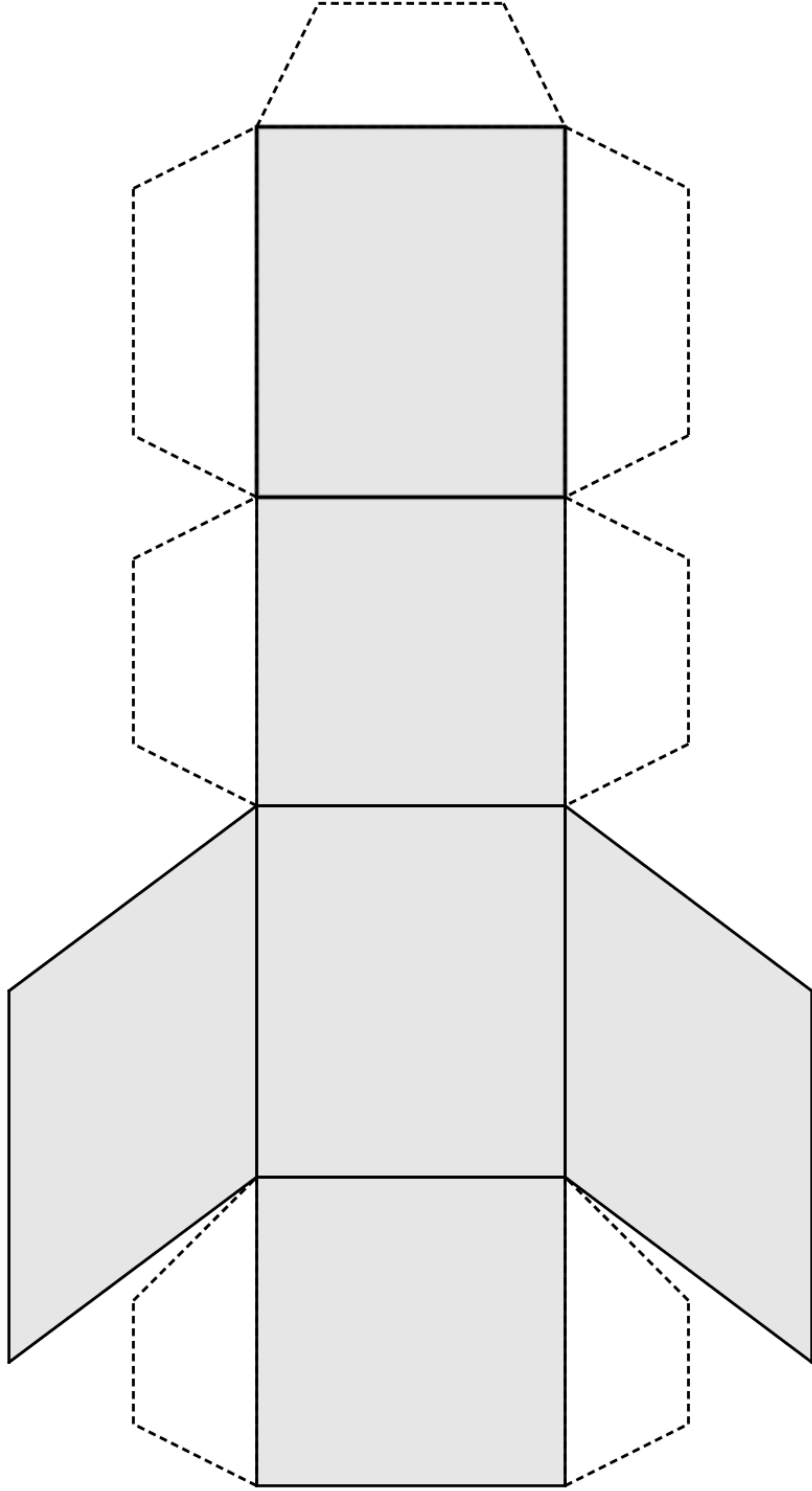


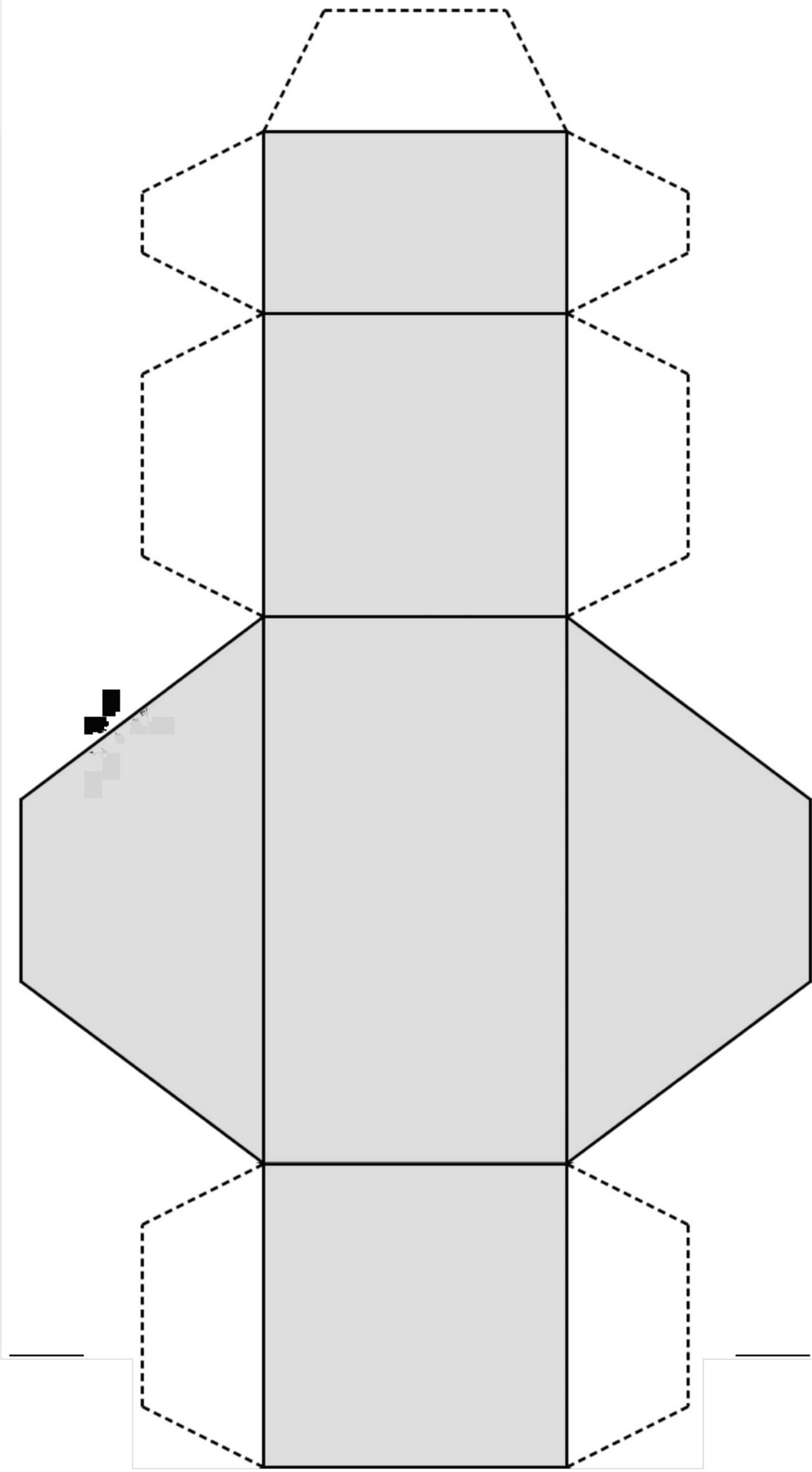
6.1.11.4 Pinwheel.

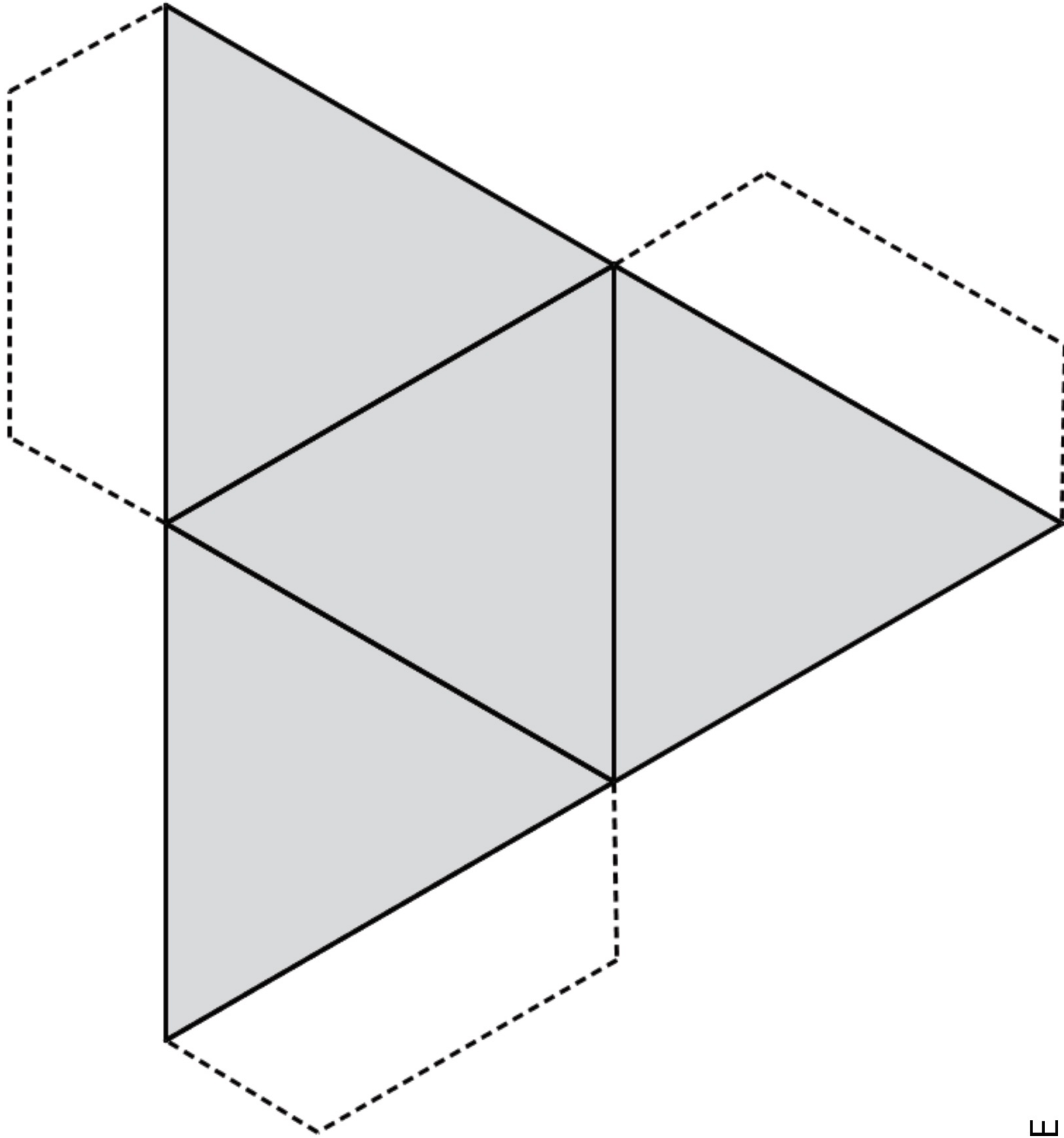


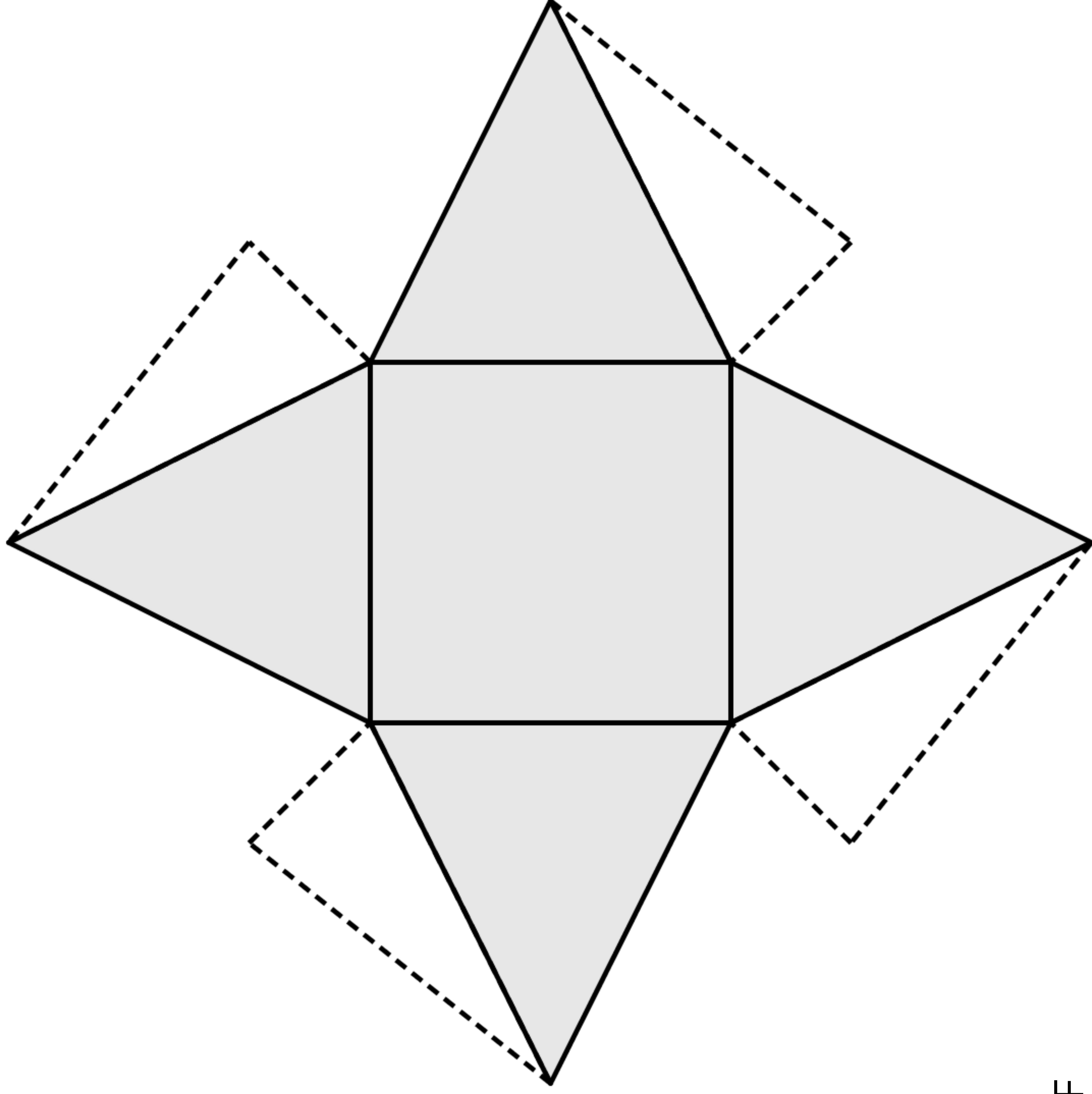


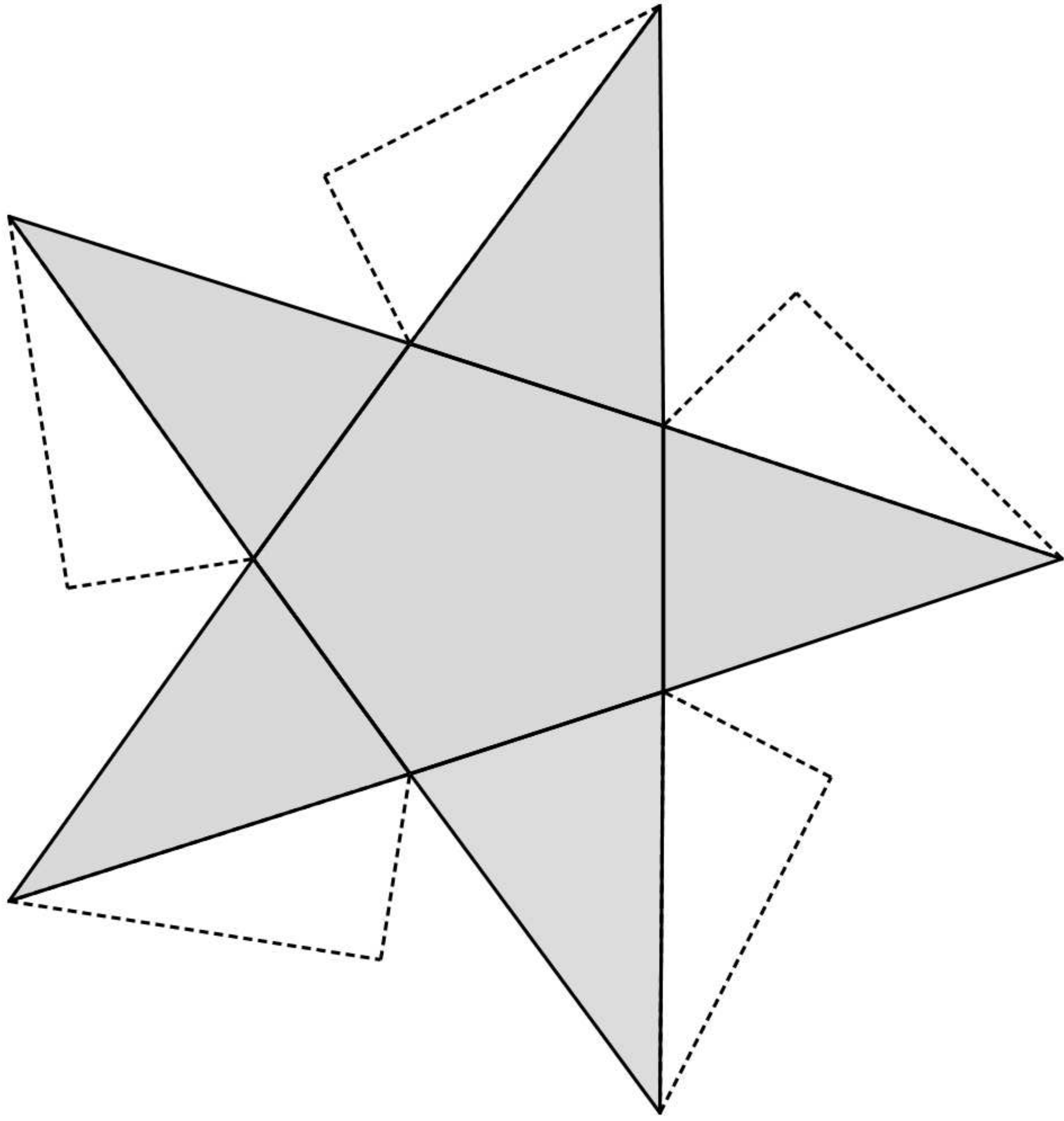


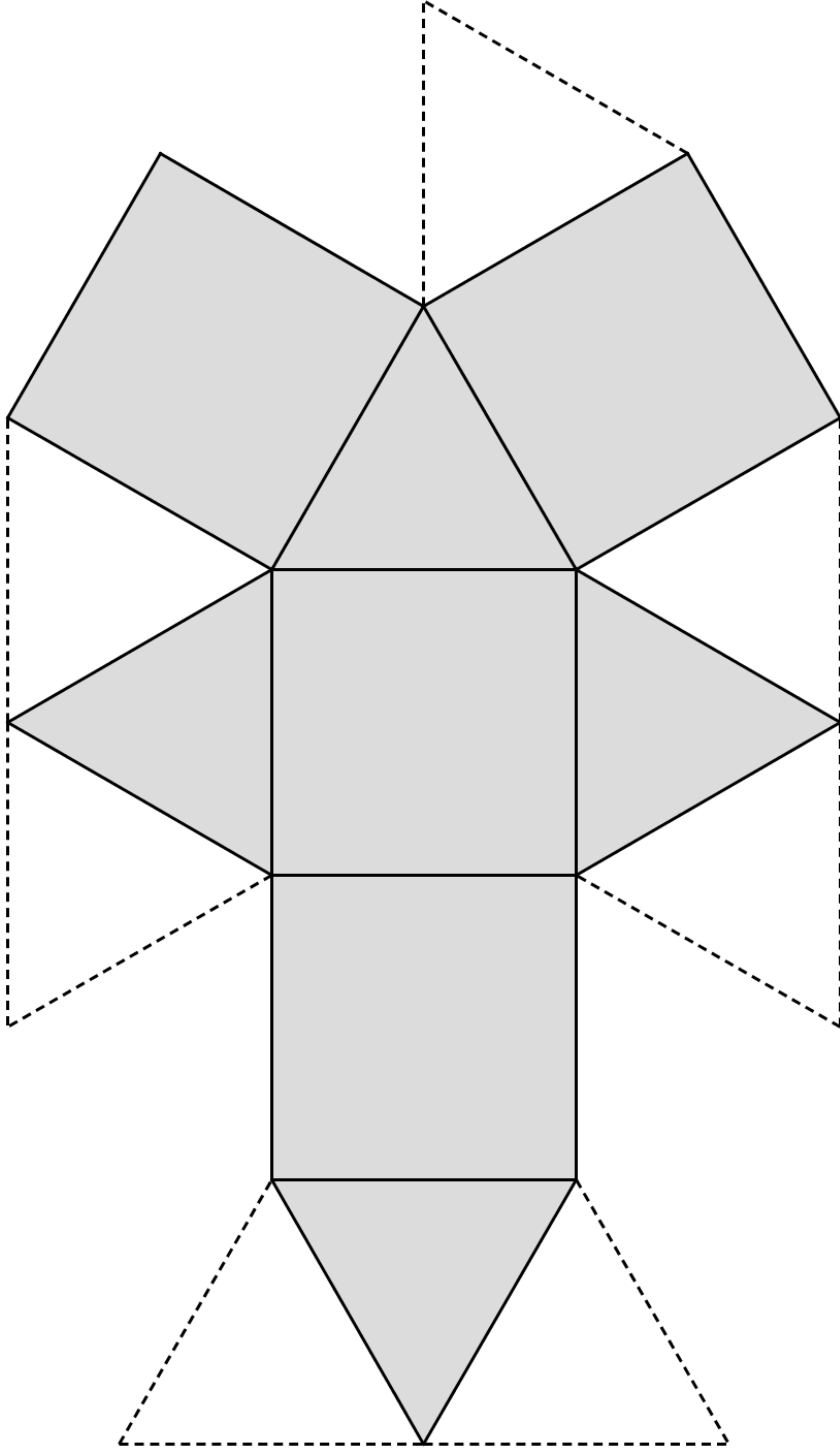


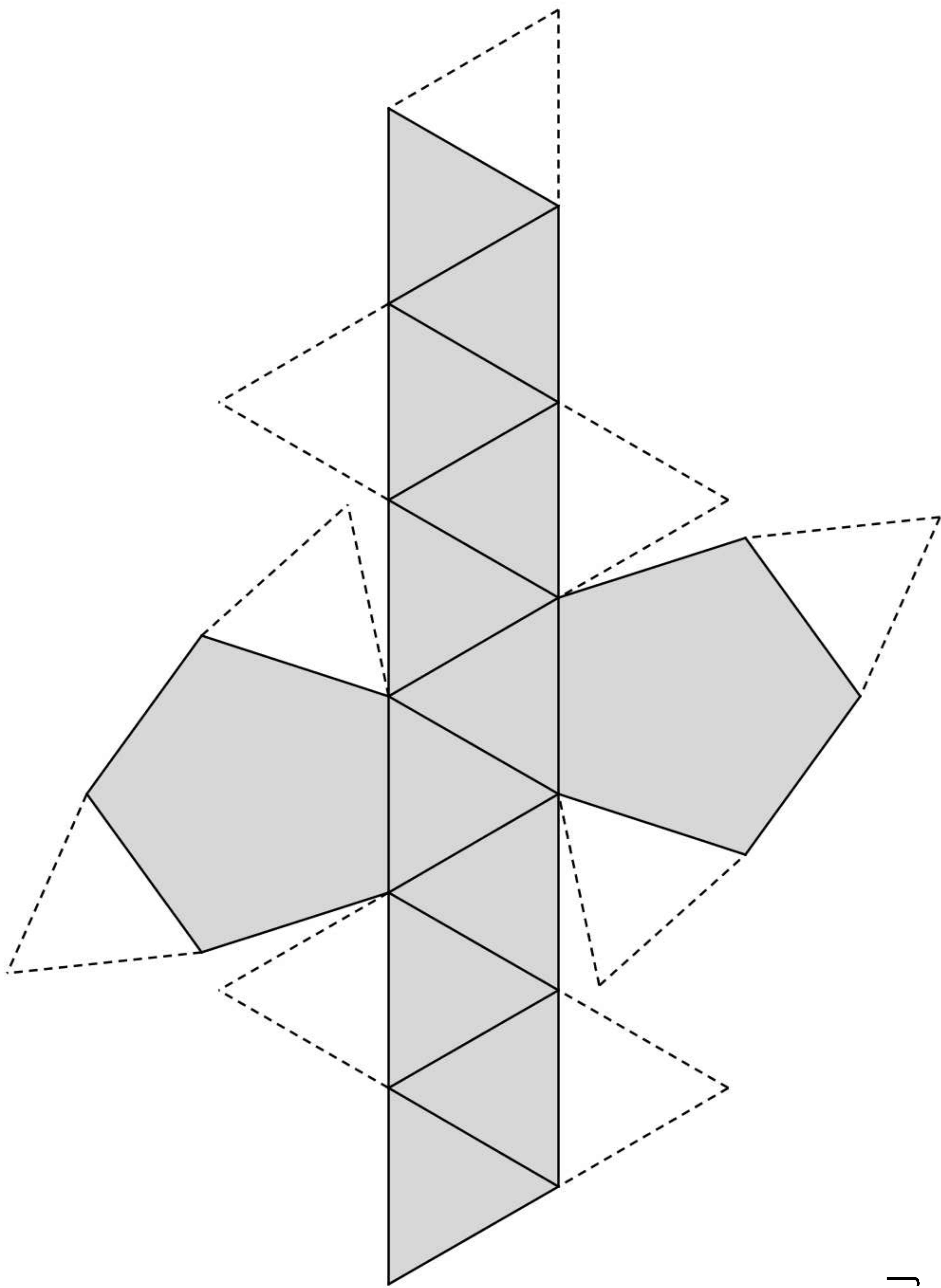


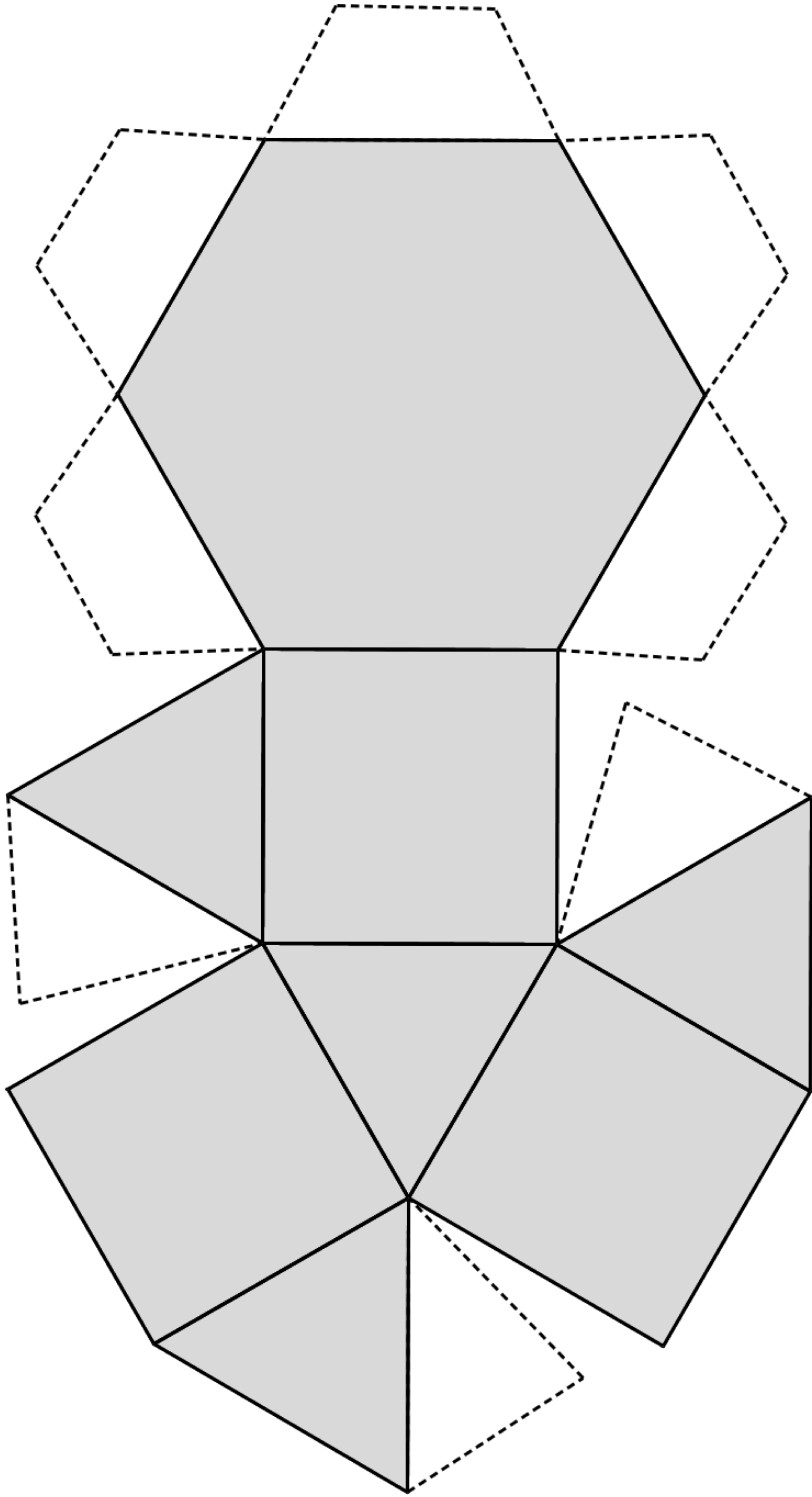


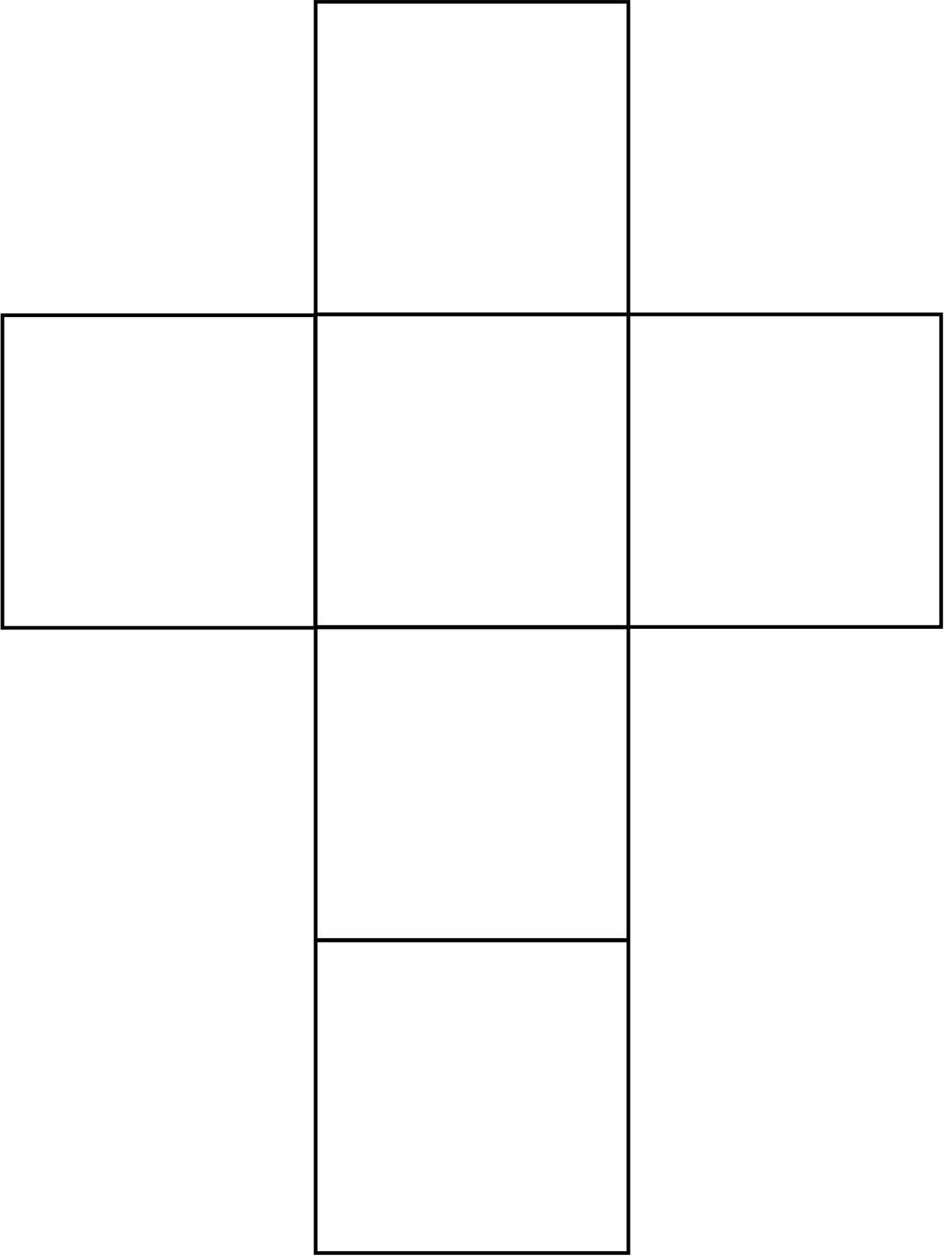


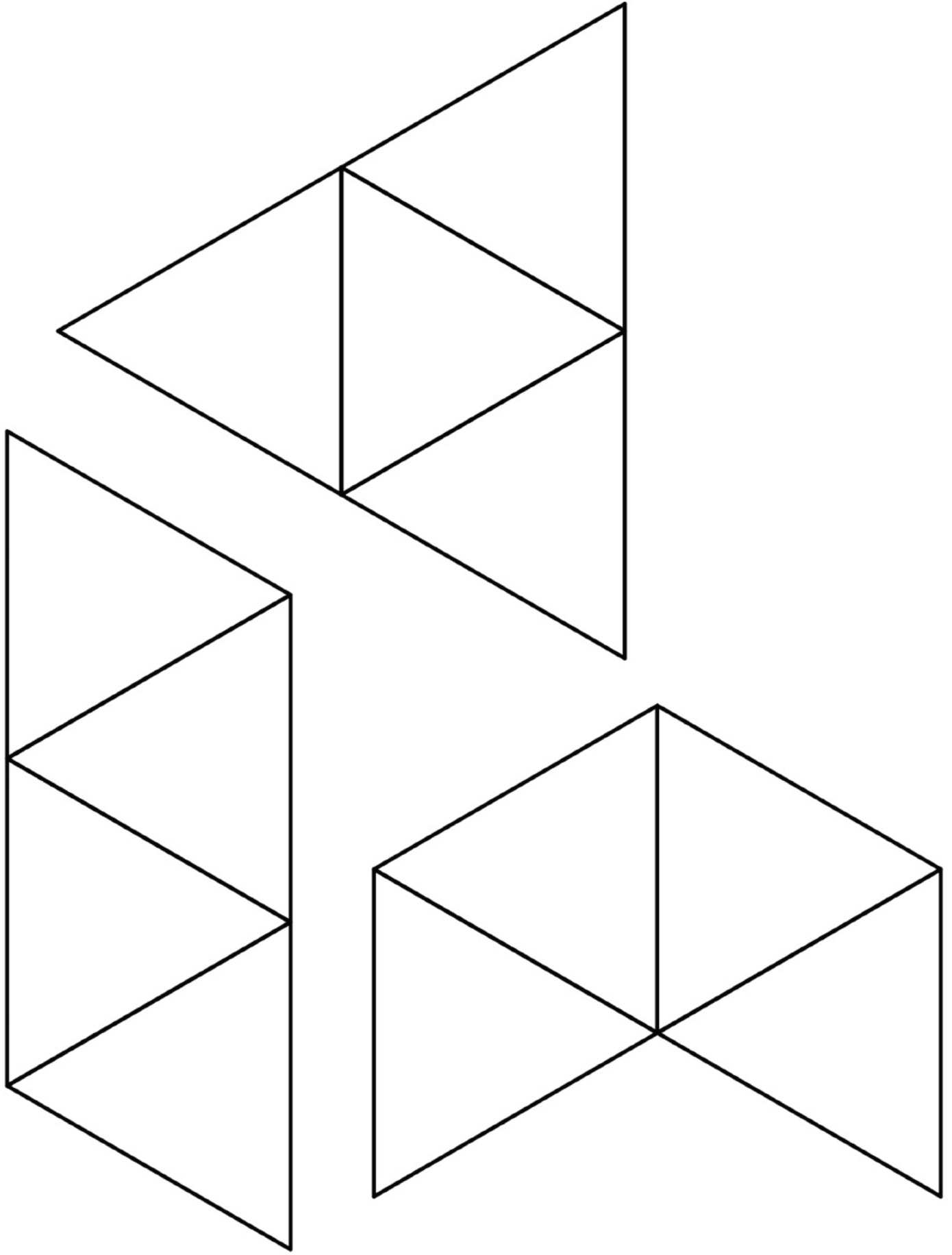


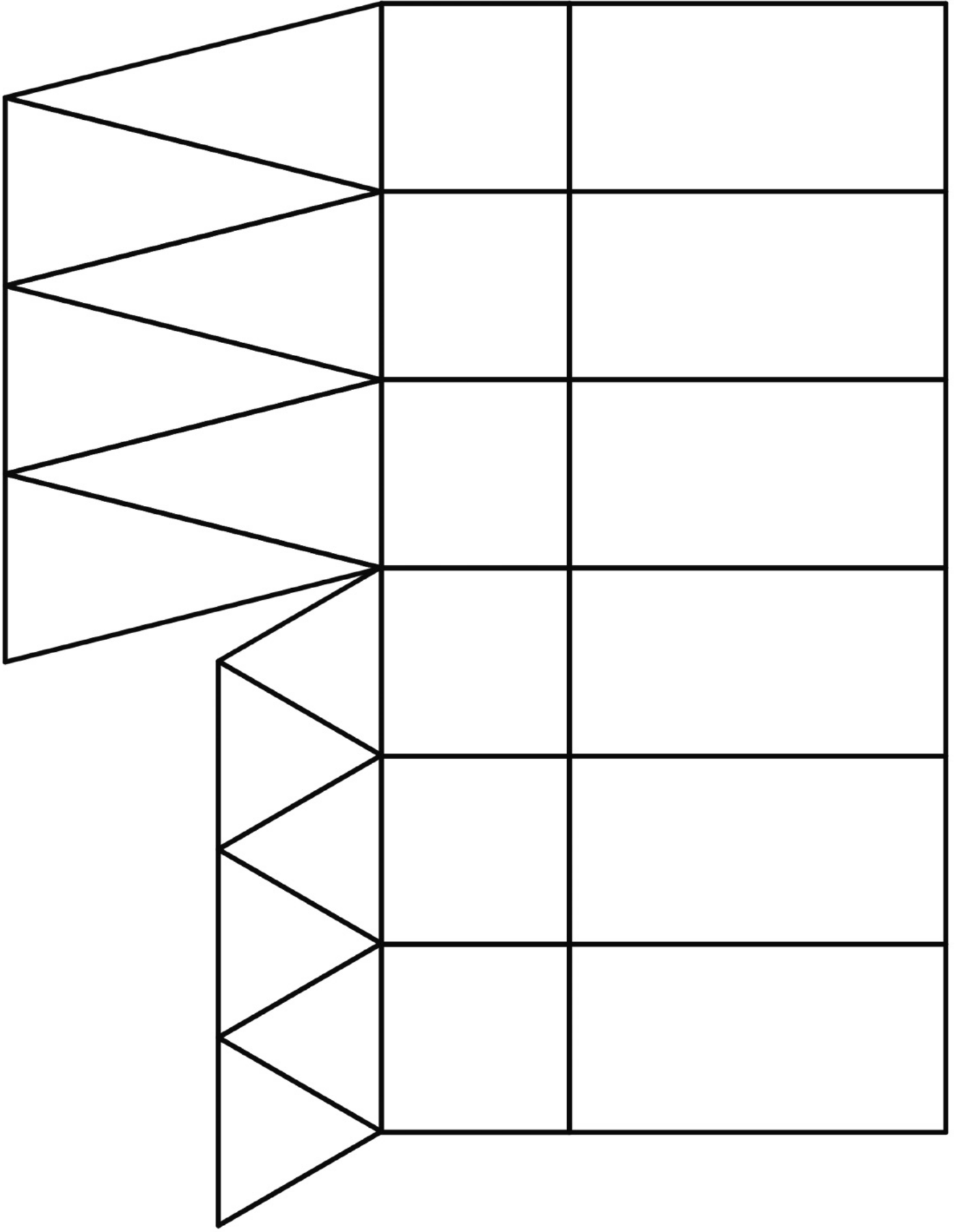


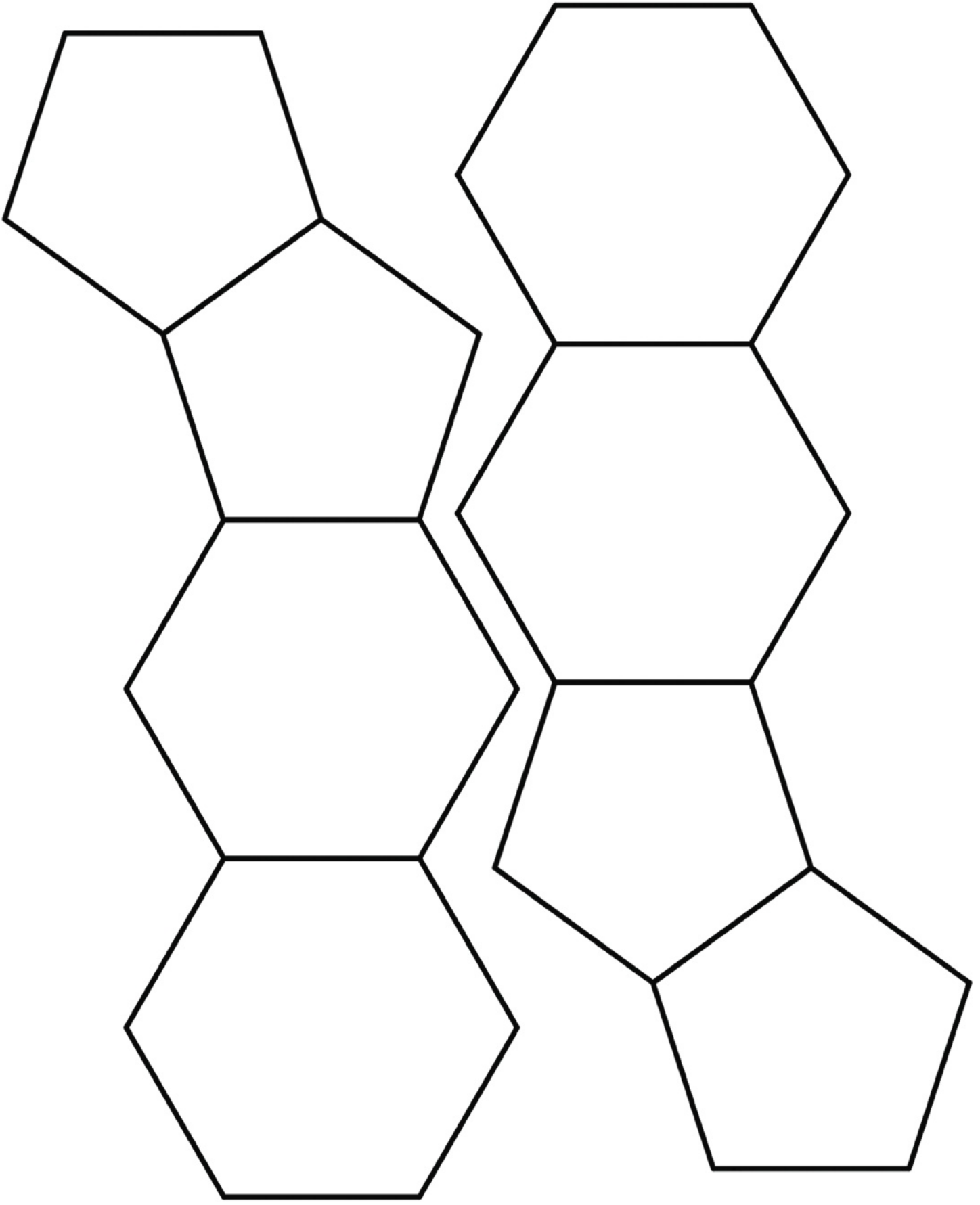


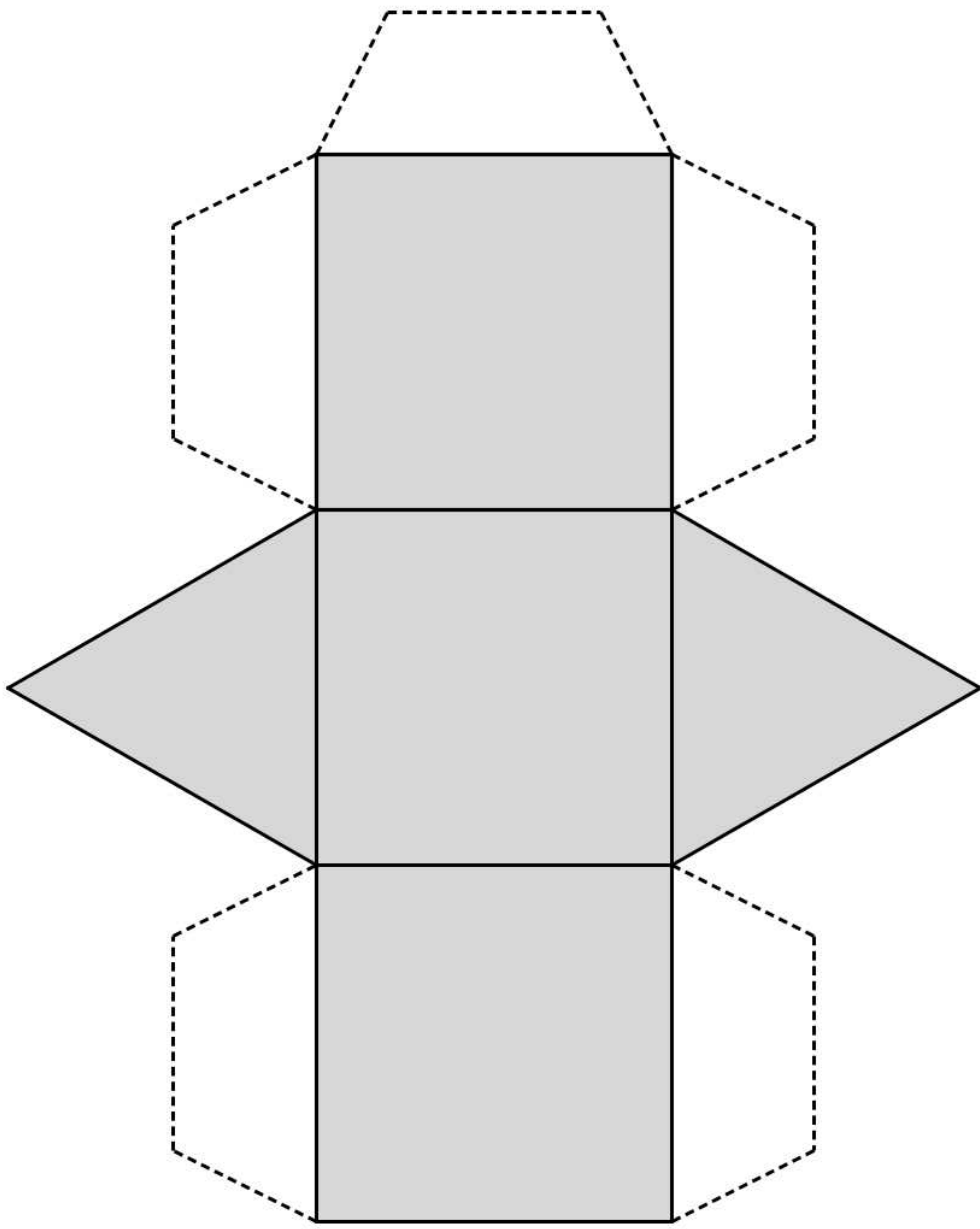


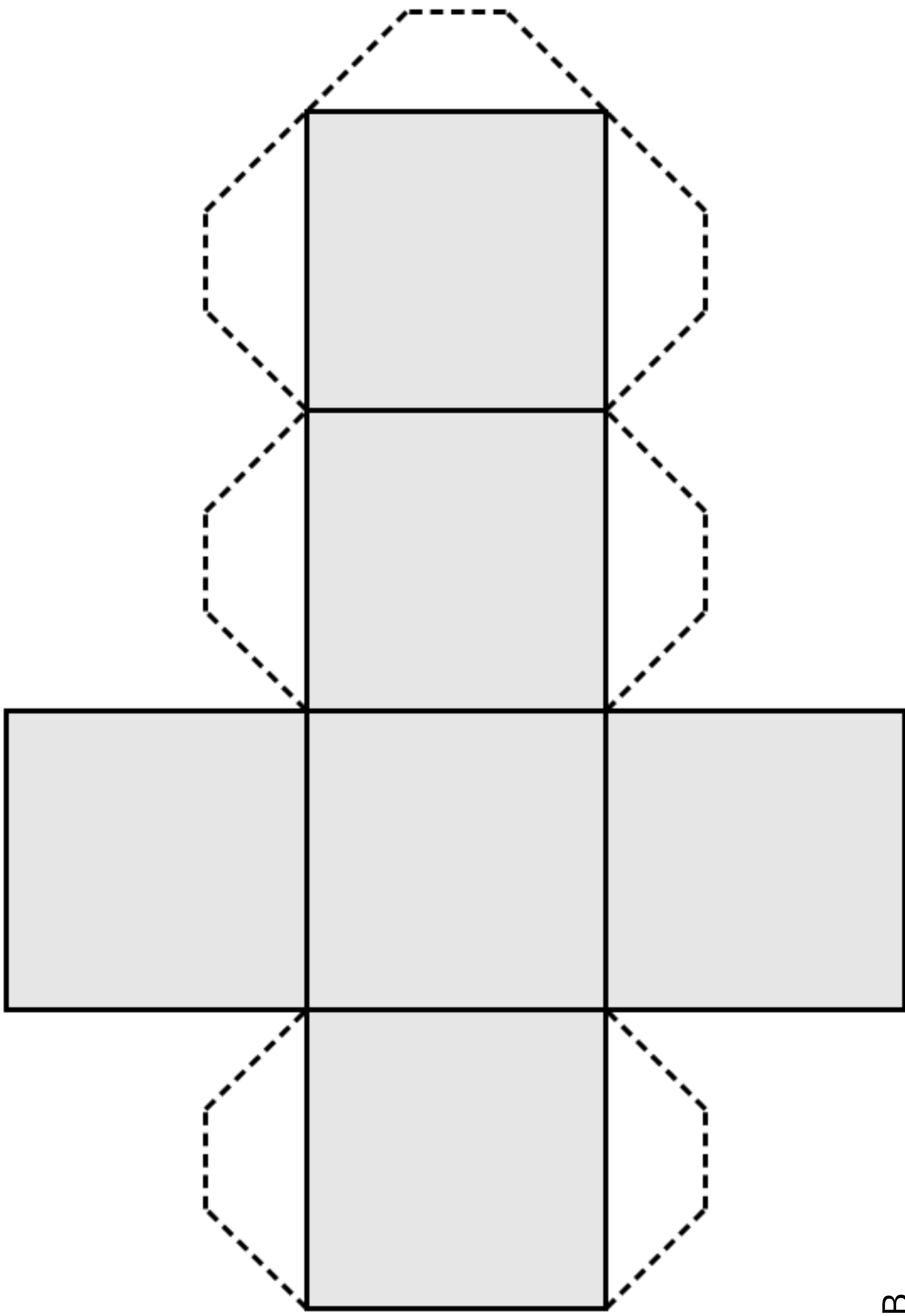


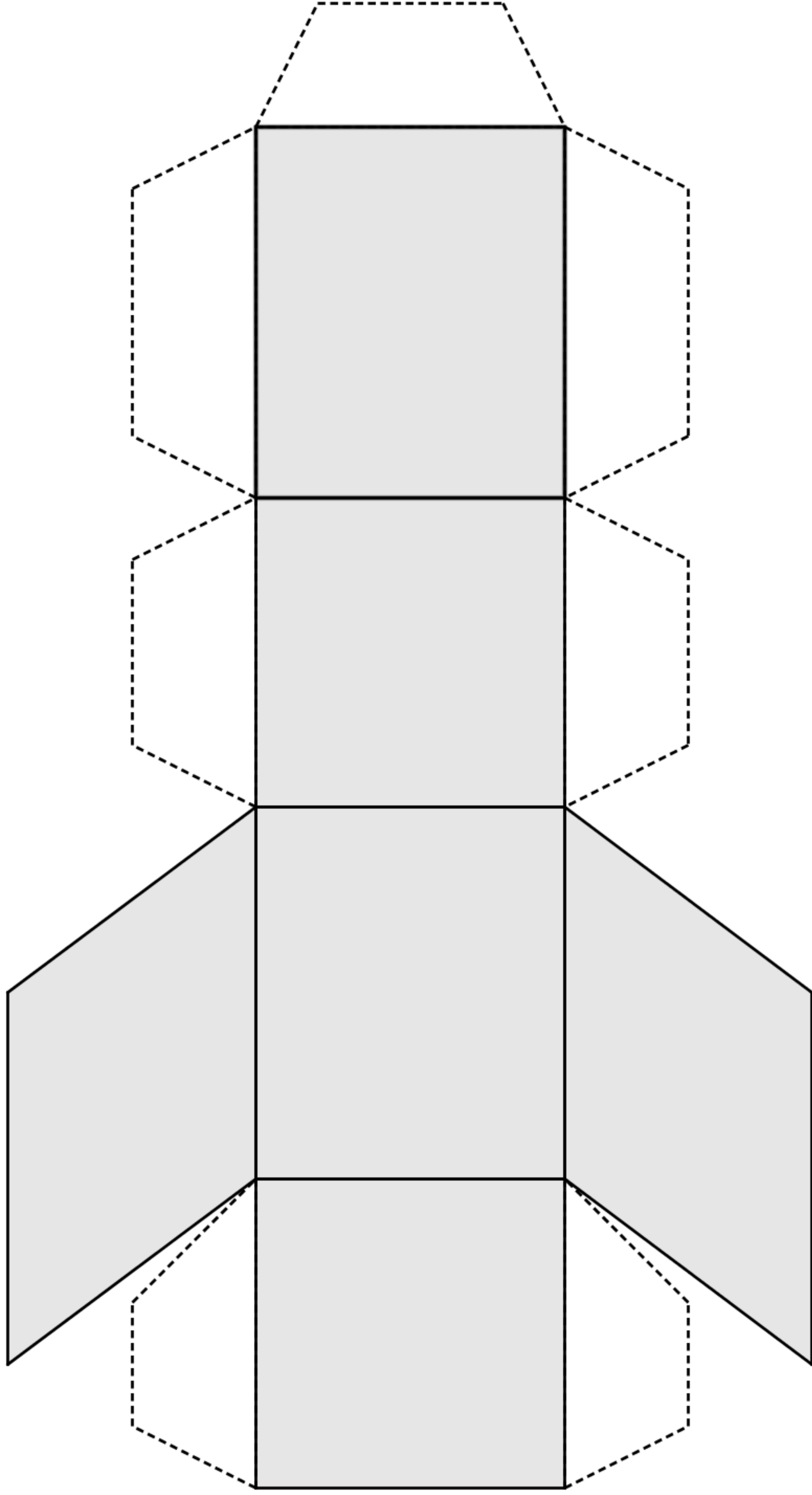


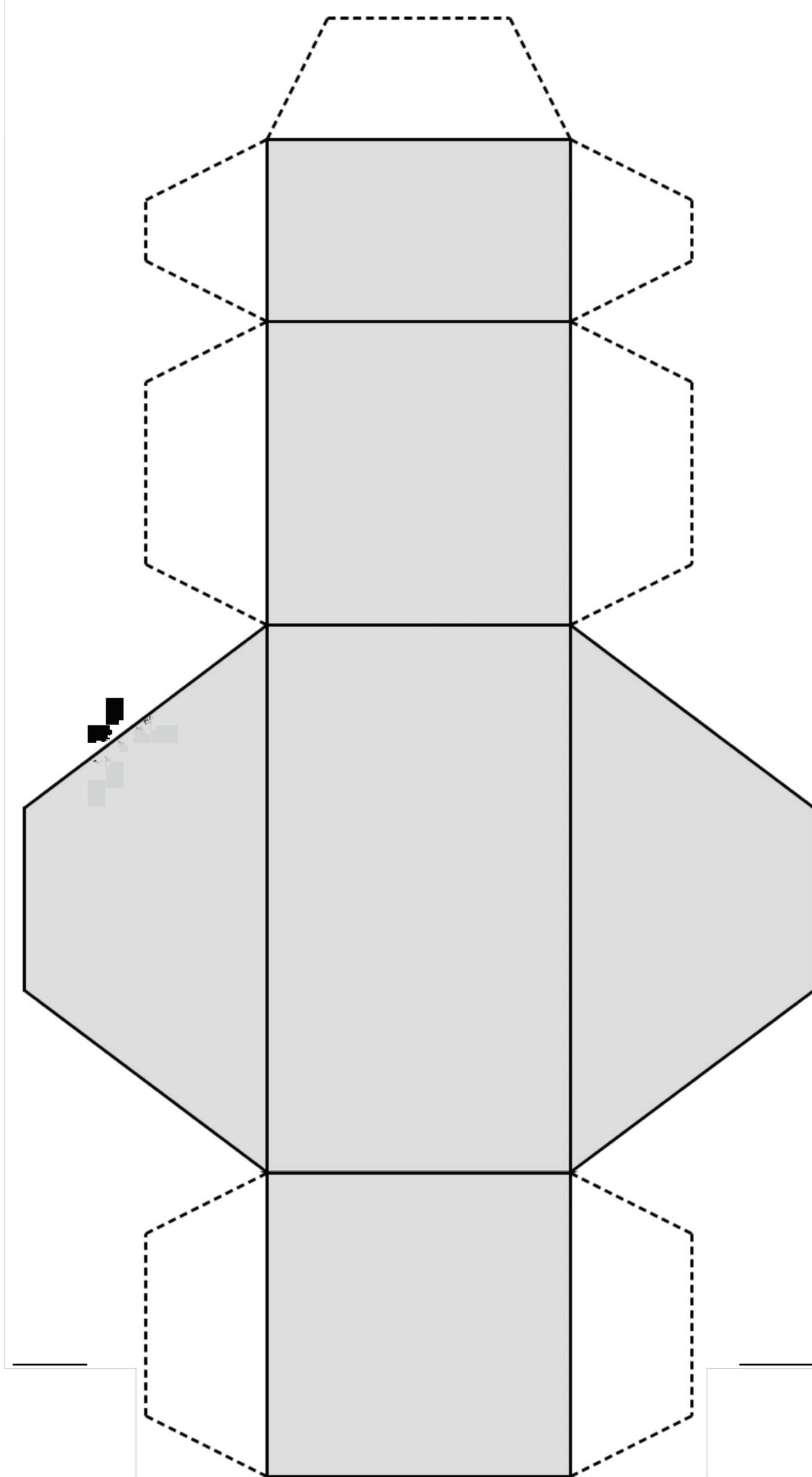


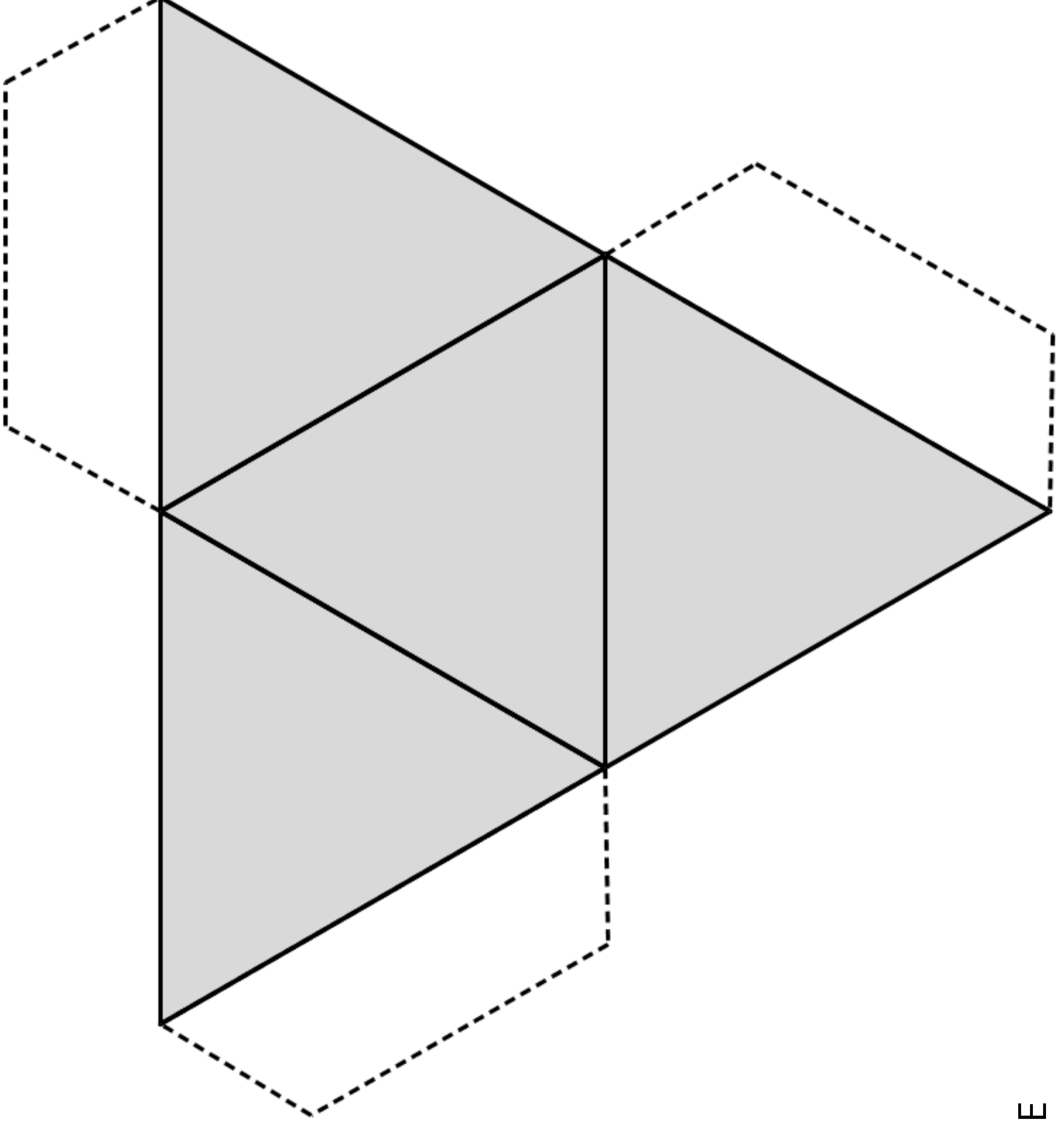


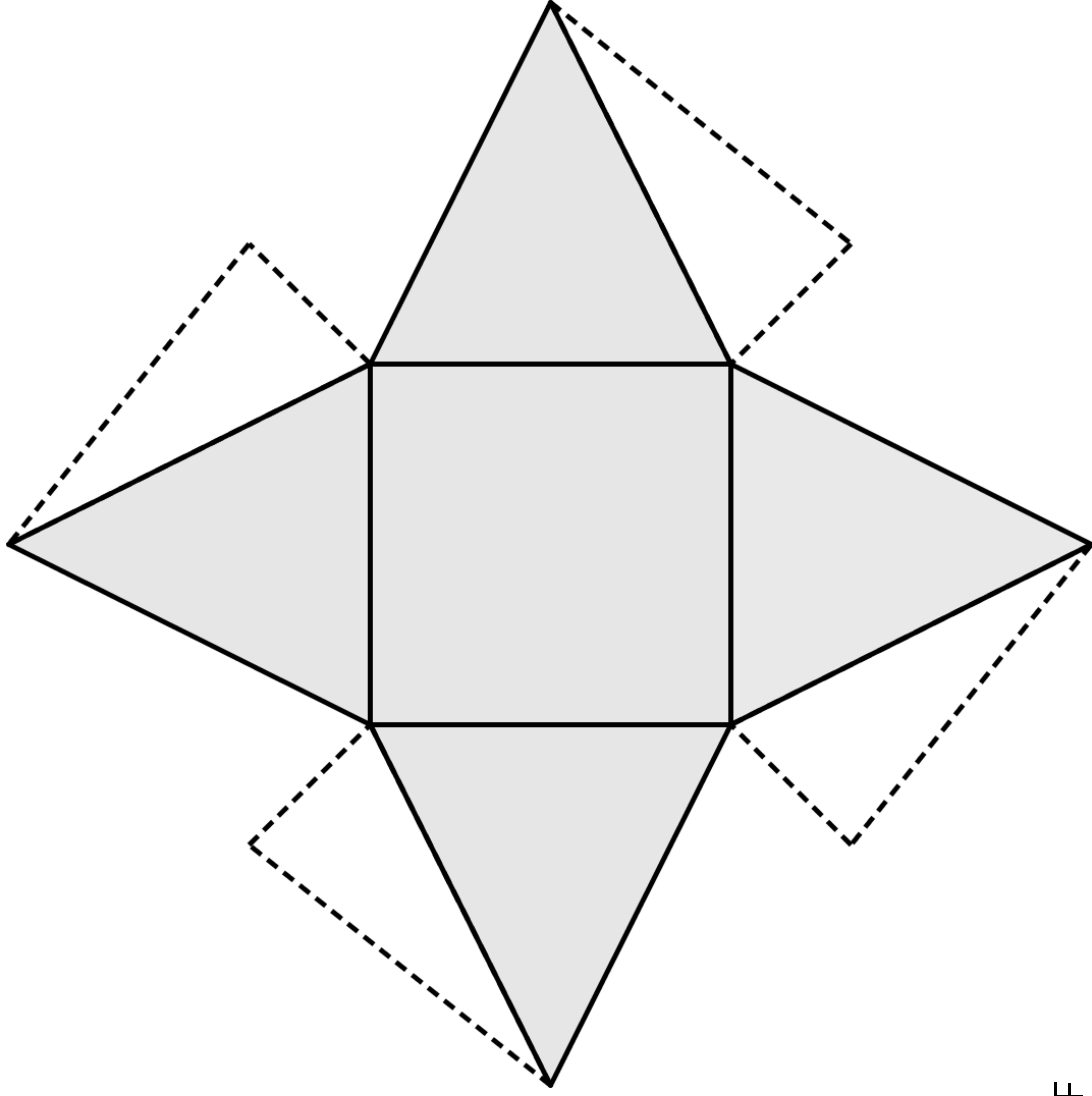


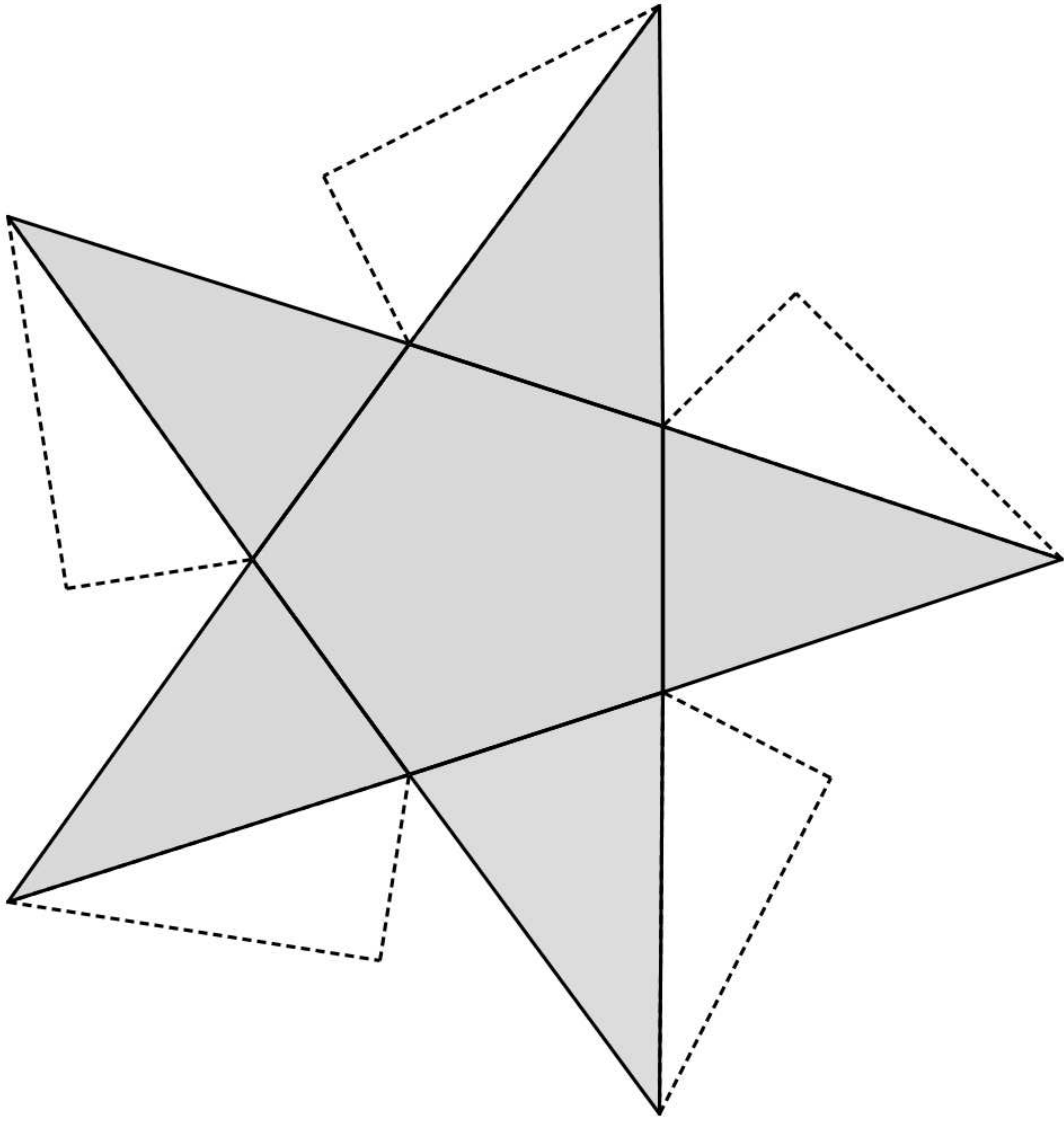


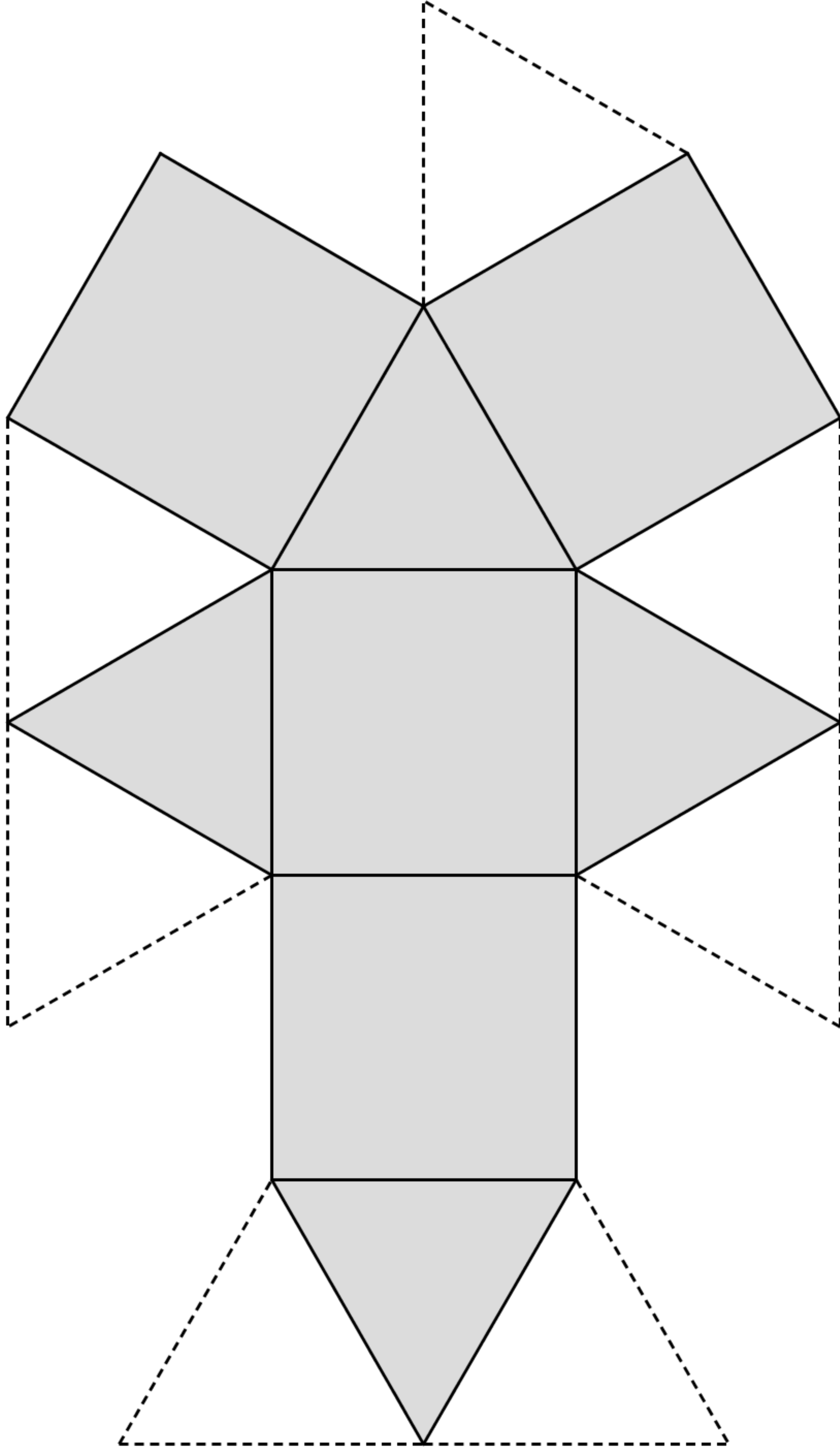


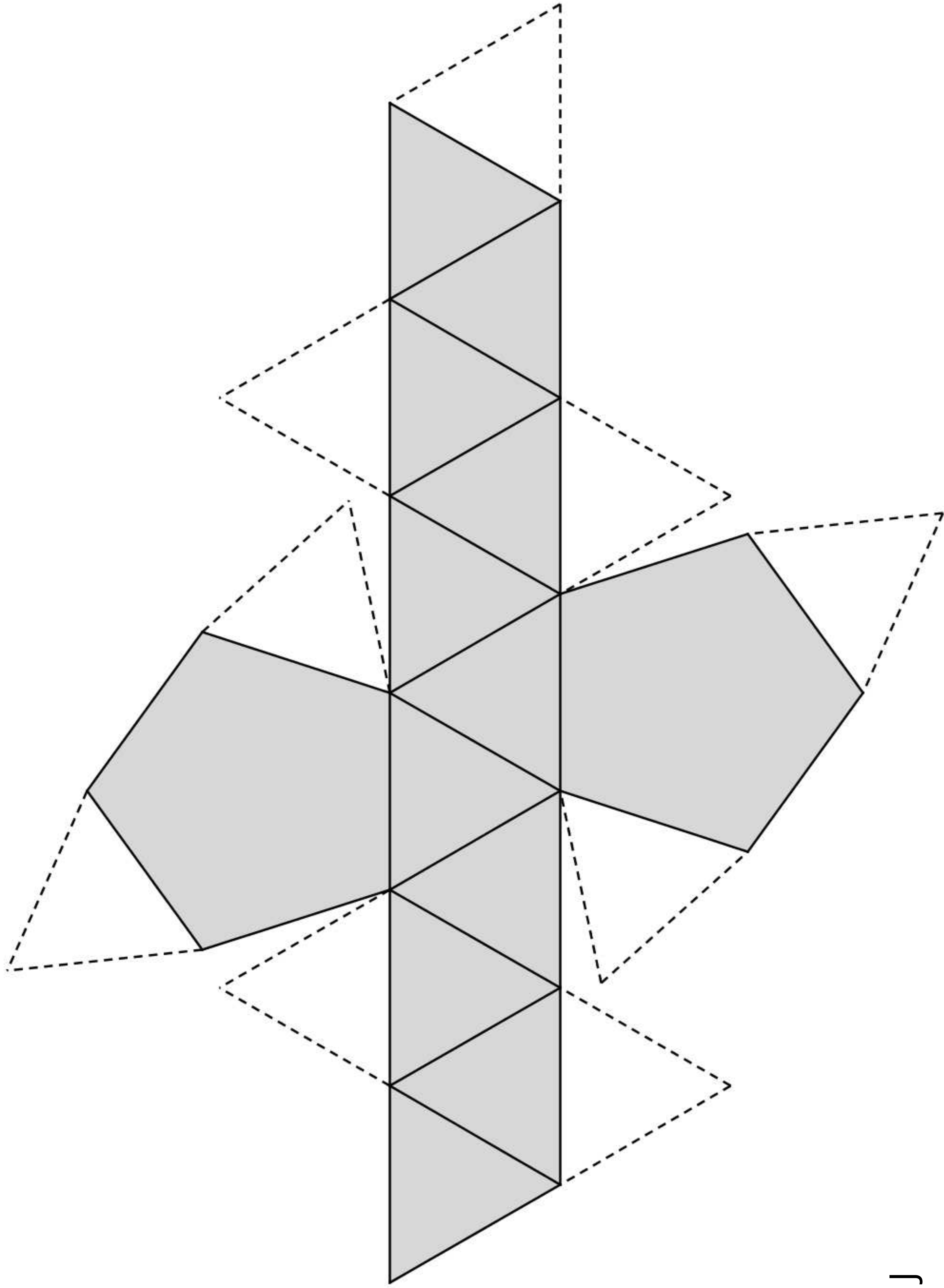


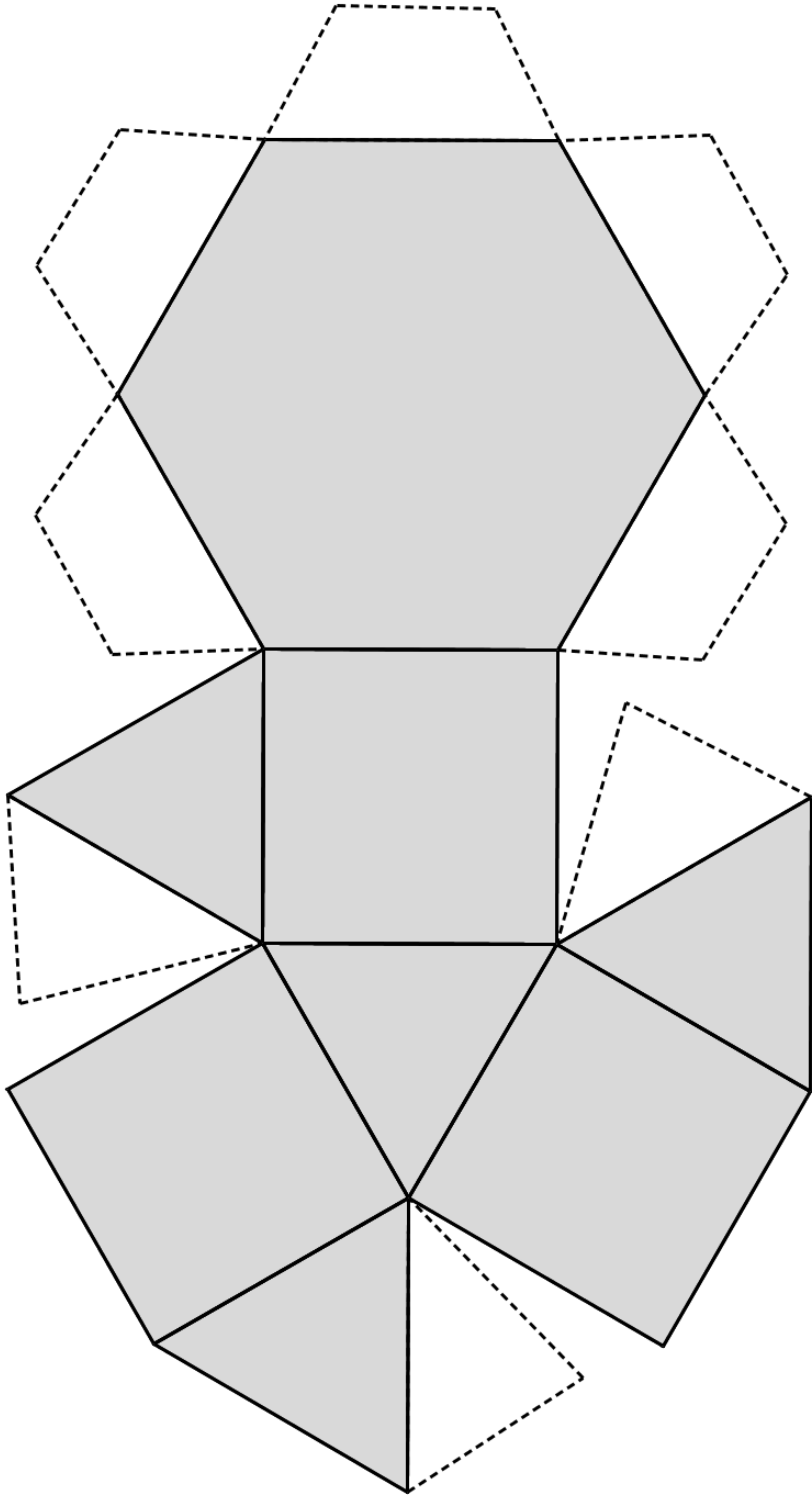


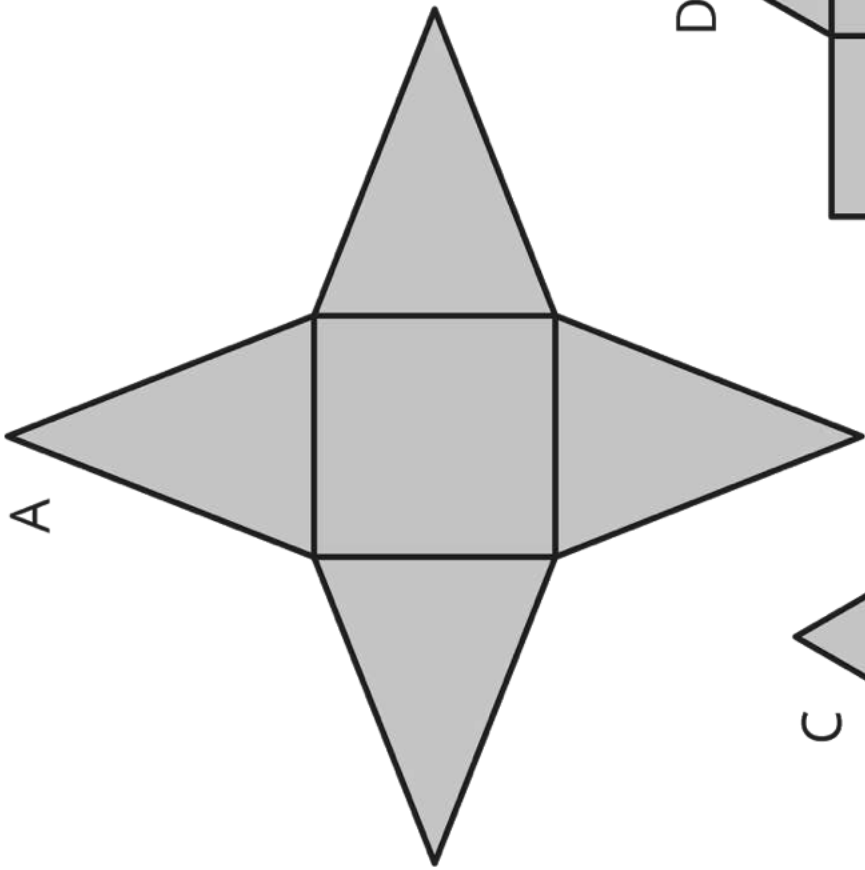




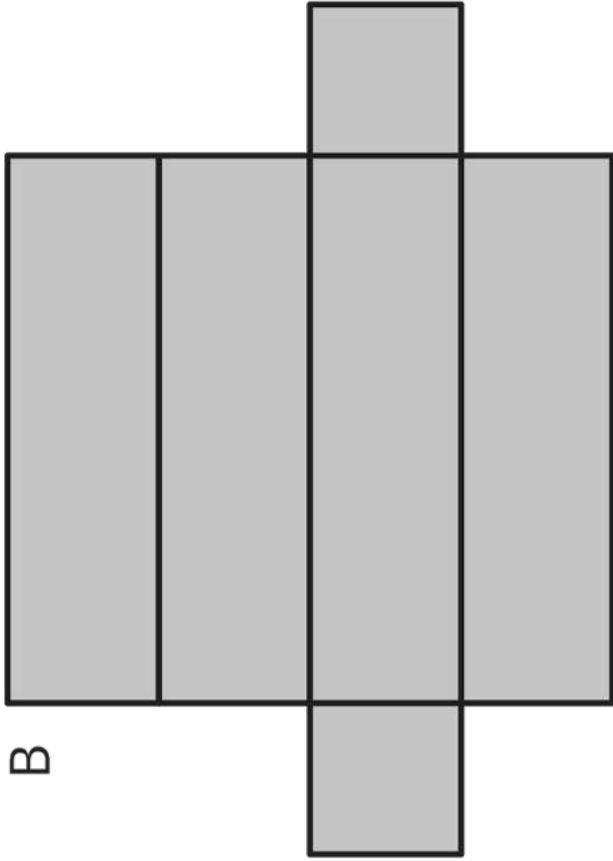




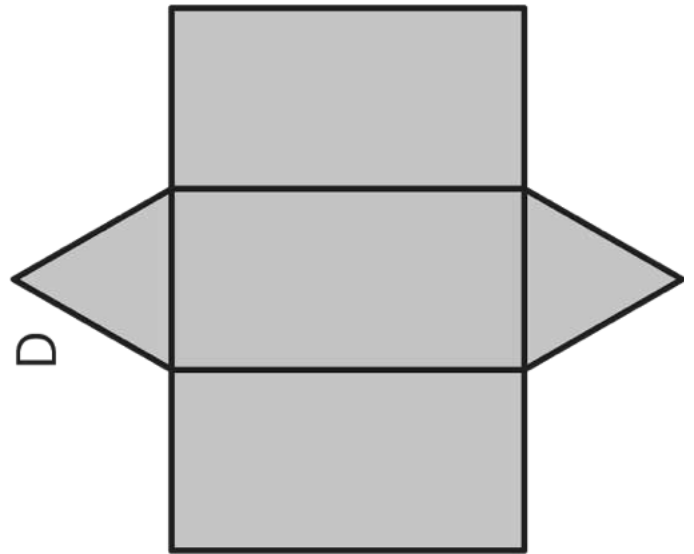




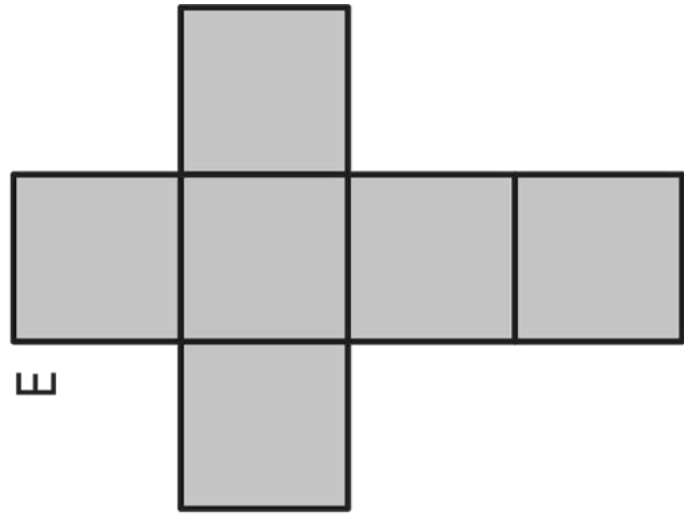
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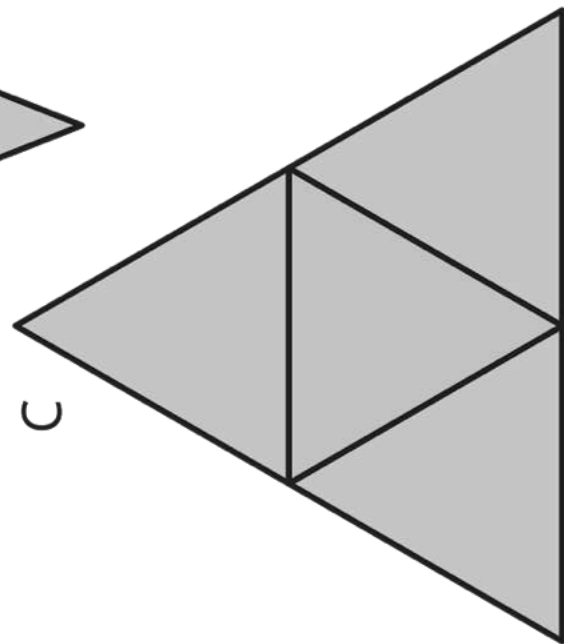
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D

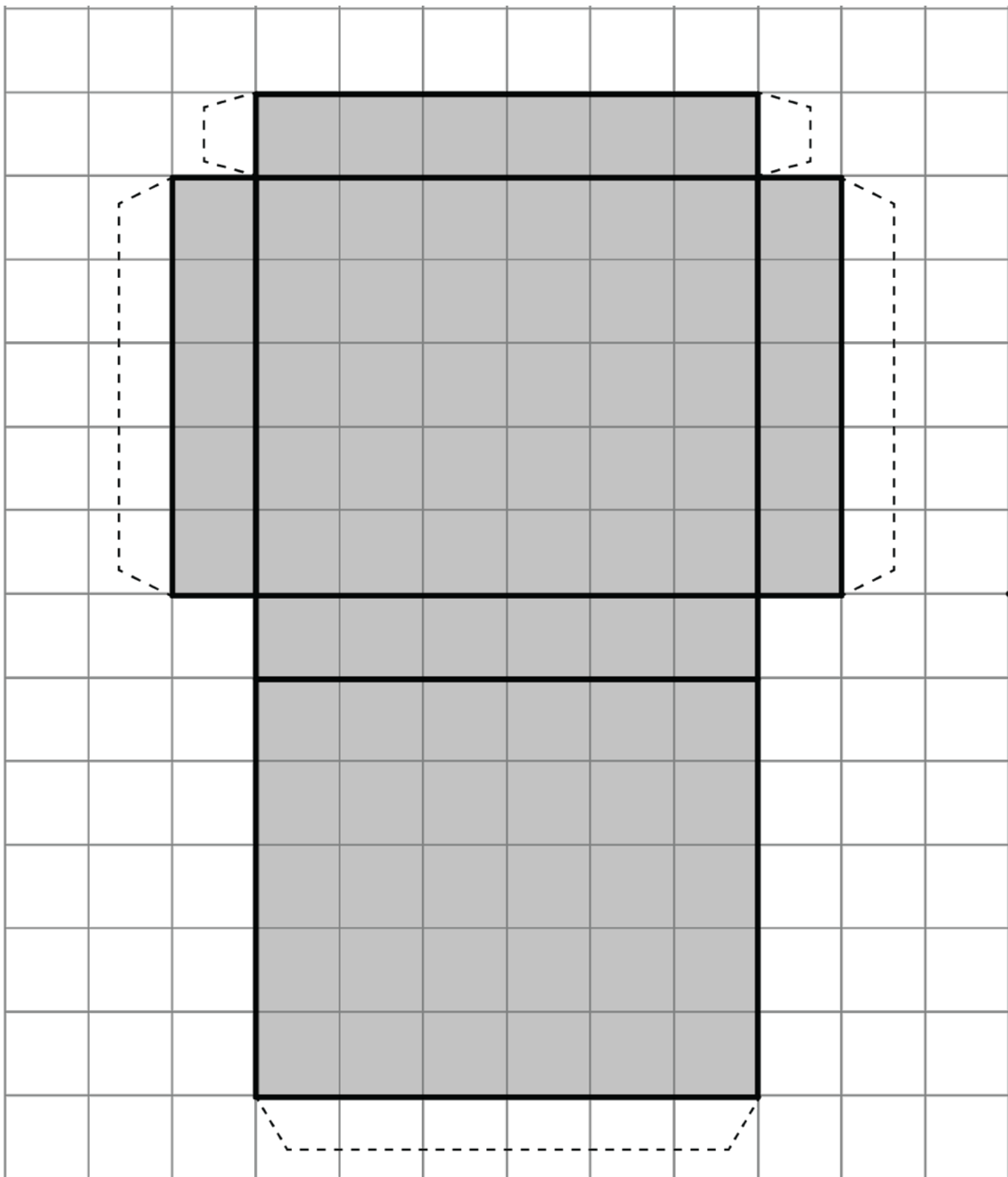


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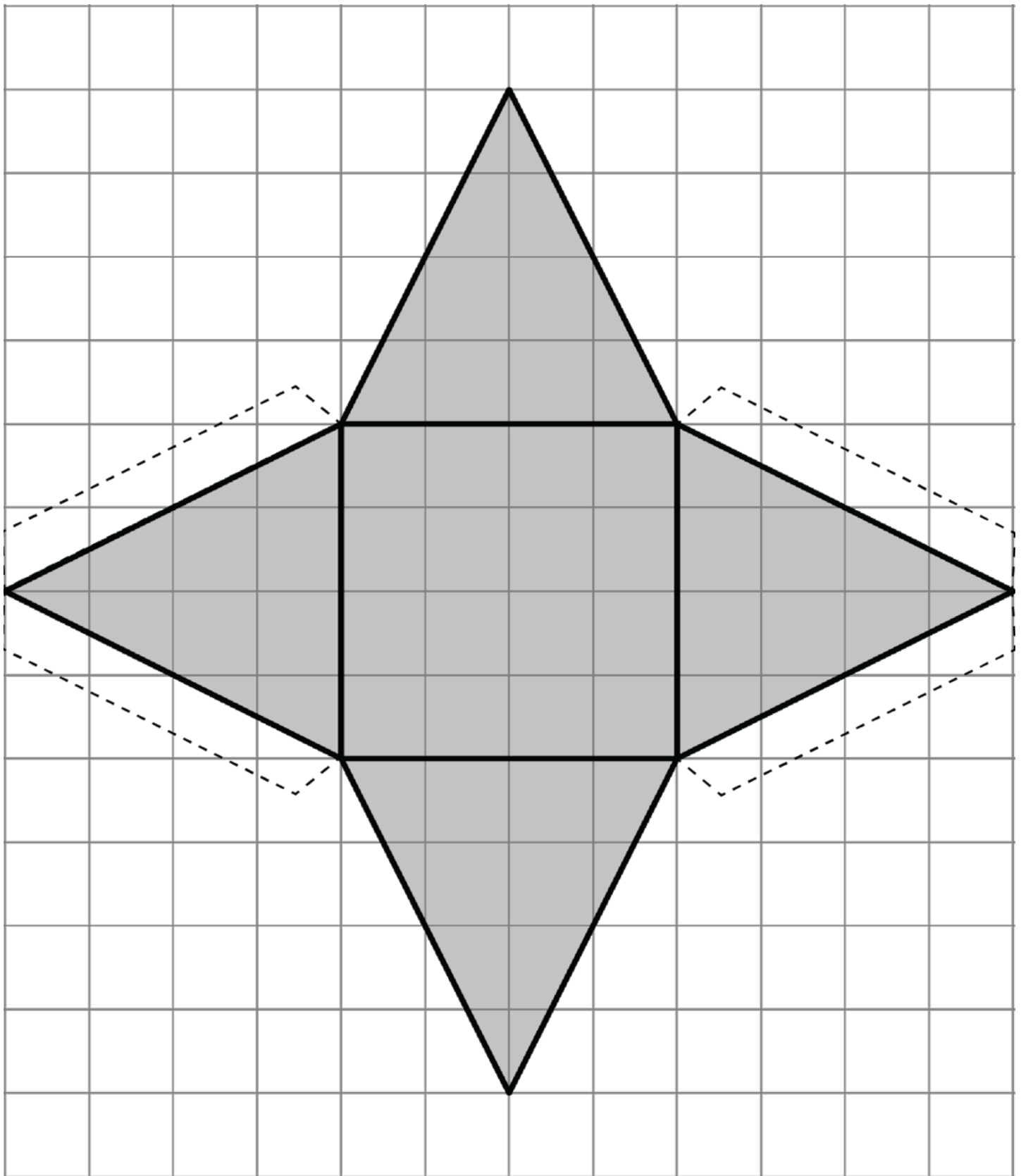


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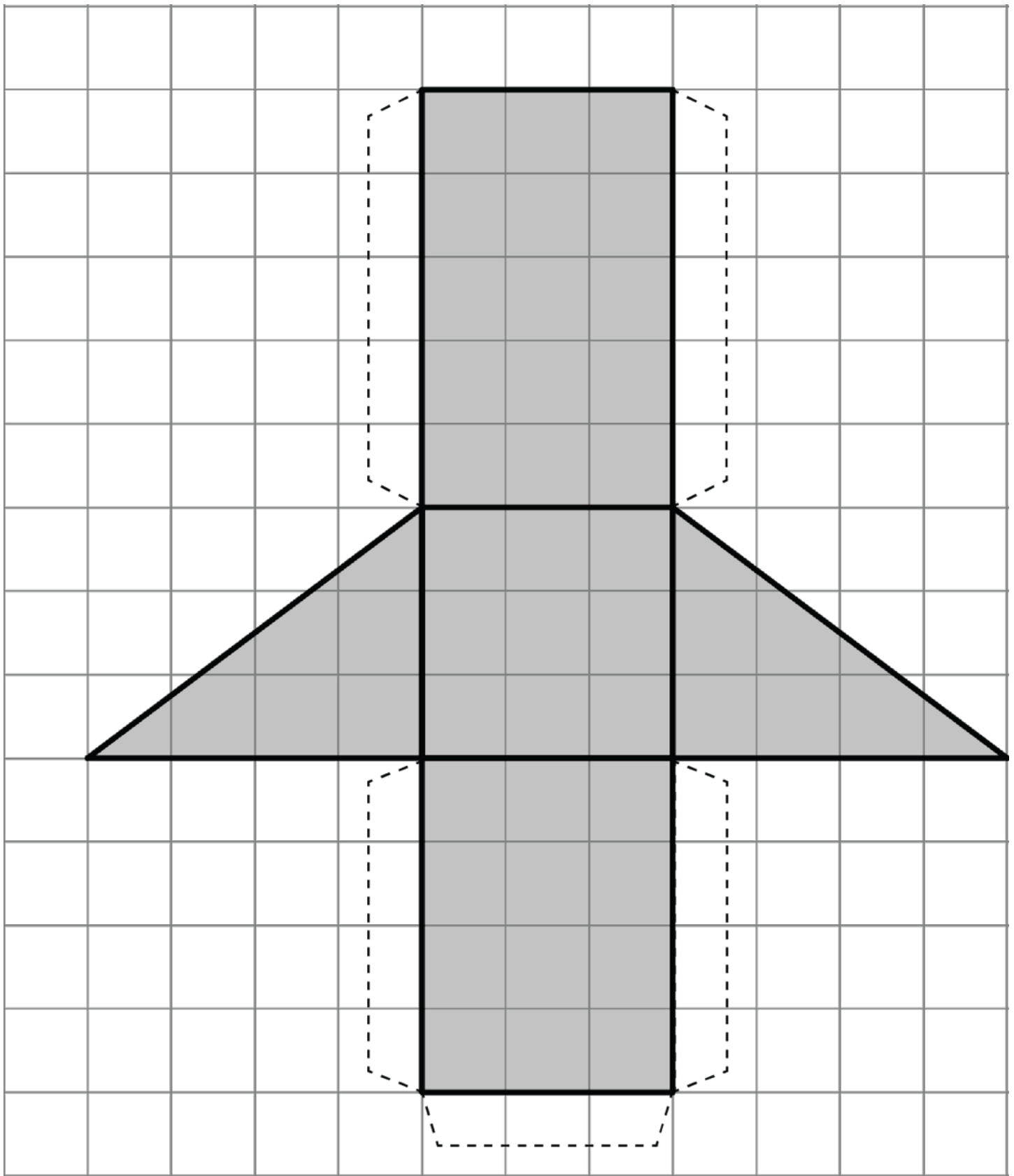
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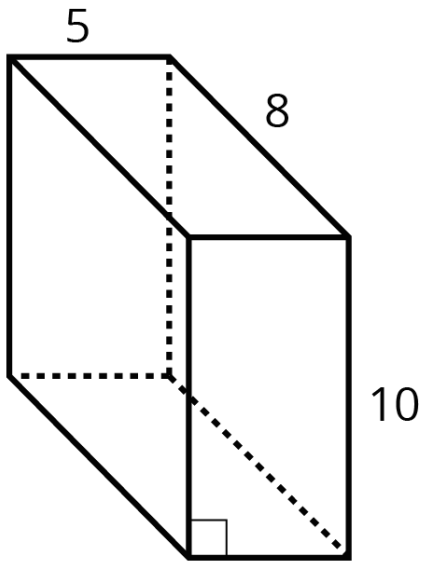


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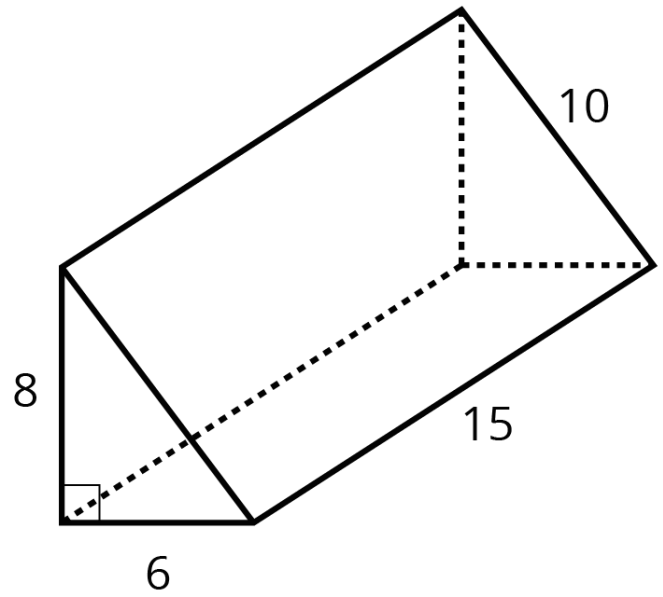


6.1.15.2 Building Prisms and Pyramids.

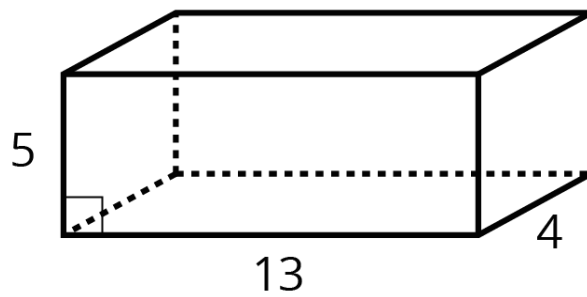
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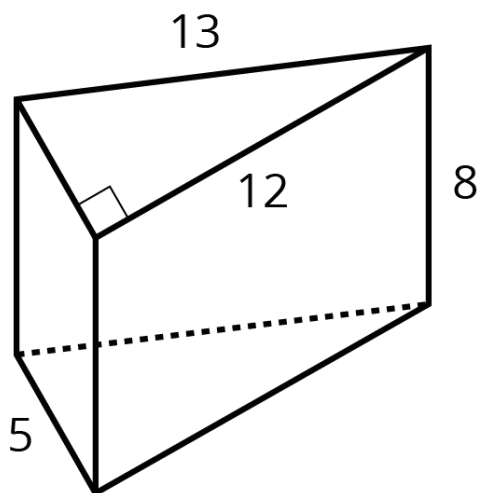
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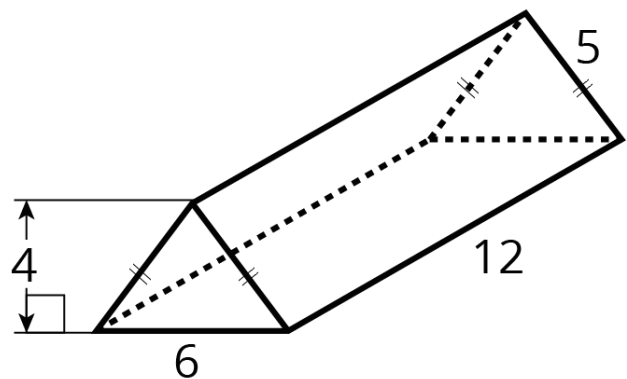
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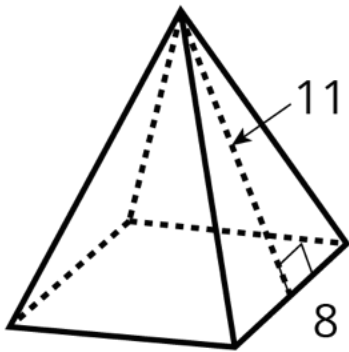
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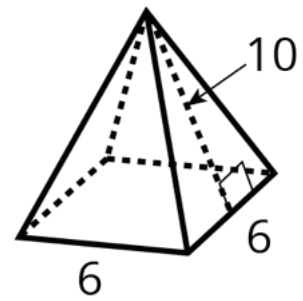
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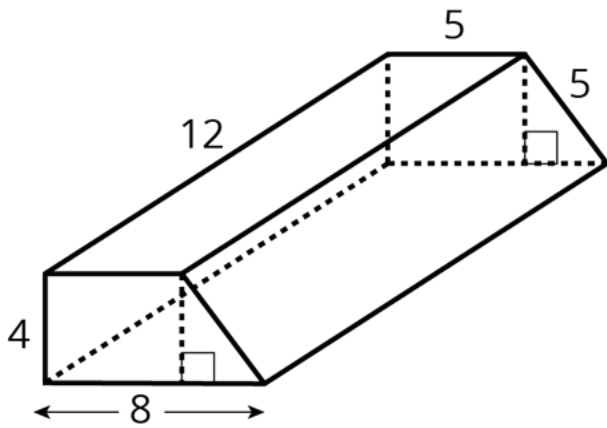
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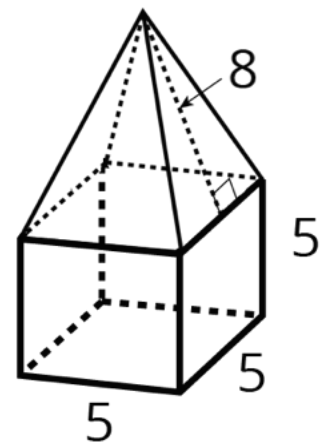
G



H



I



How many people can sleep in your tent? _____ What is the height of your tent? _____

Sketch the bottom panel of your tent and the locations where sleeping bags will go.	Sketch the overall design of your tent.

What decisions are important when choosing a tent design?

Use the remaining space to show any work (sketches of side panels, calculations, etc) needed to estimate the amount of fabric that will be necessary to make your tent.

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Editorial Director
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