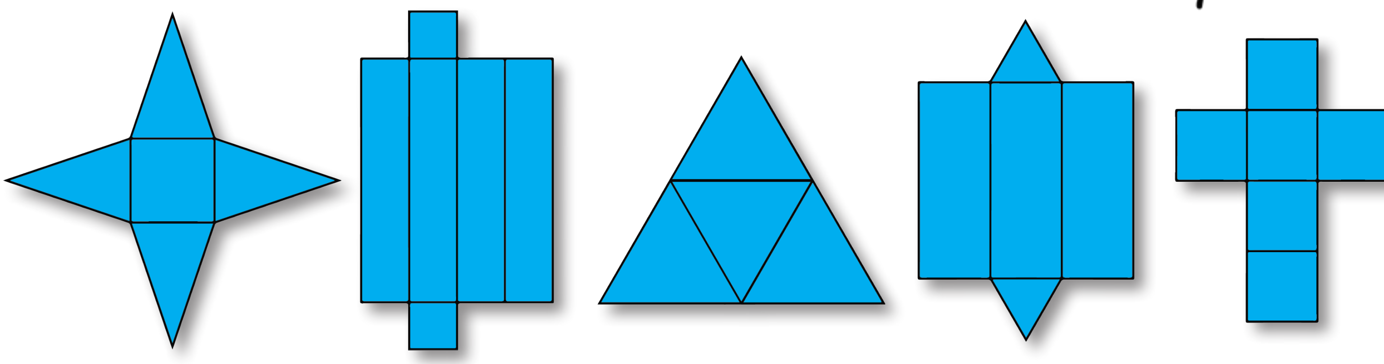
Core Knowledge® MATHEMATICS

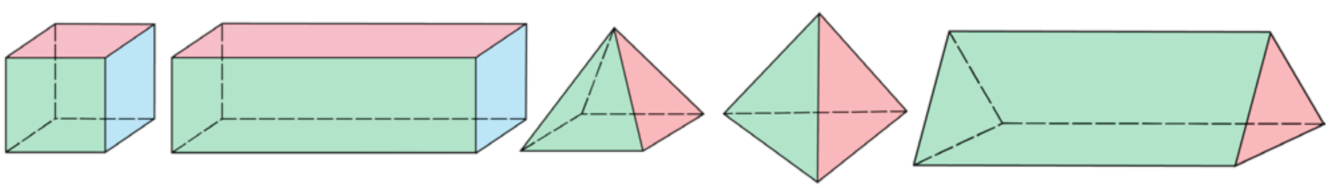
Area and Surface Area

Student Workbook

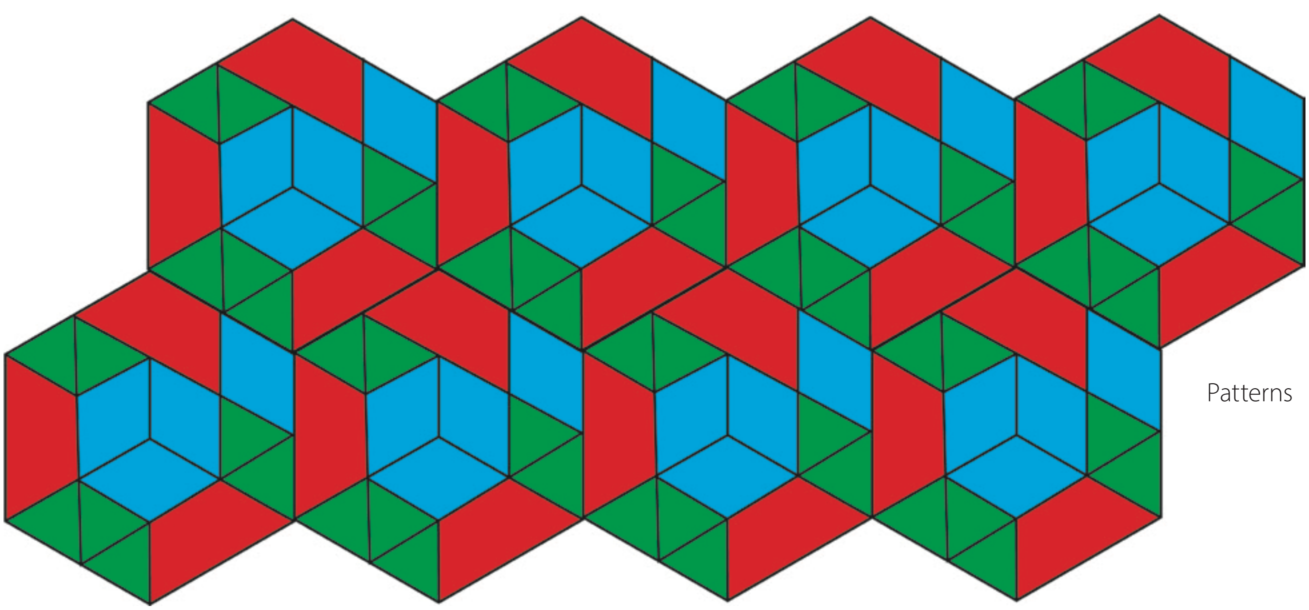
Designing a Tent



Nets and Surface Area



Prisms



Patterns

ISBN: 978-1-68380-633-2

Area and Surface Area

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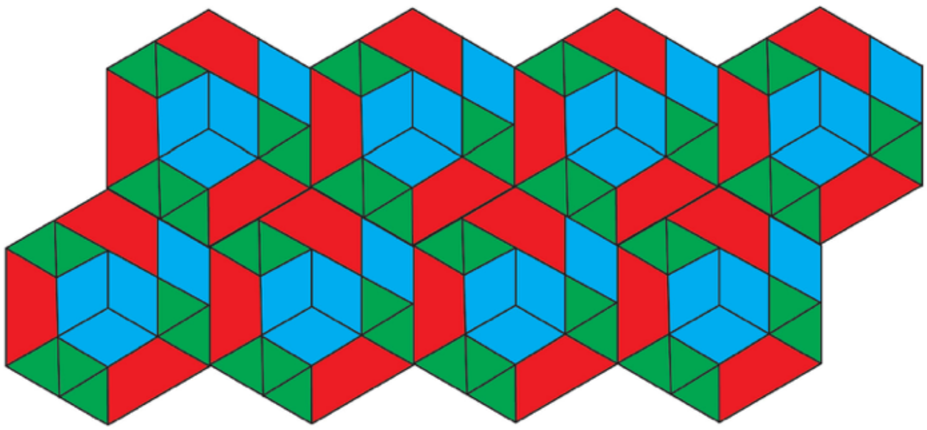


Area and Surface Area

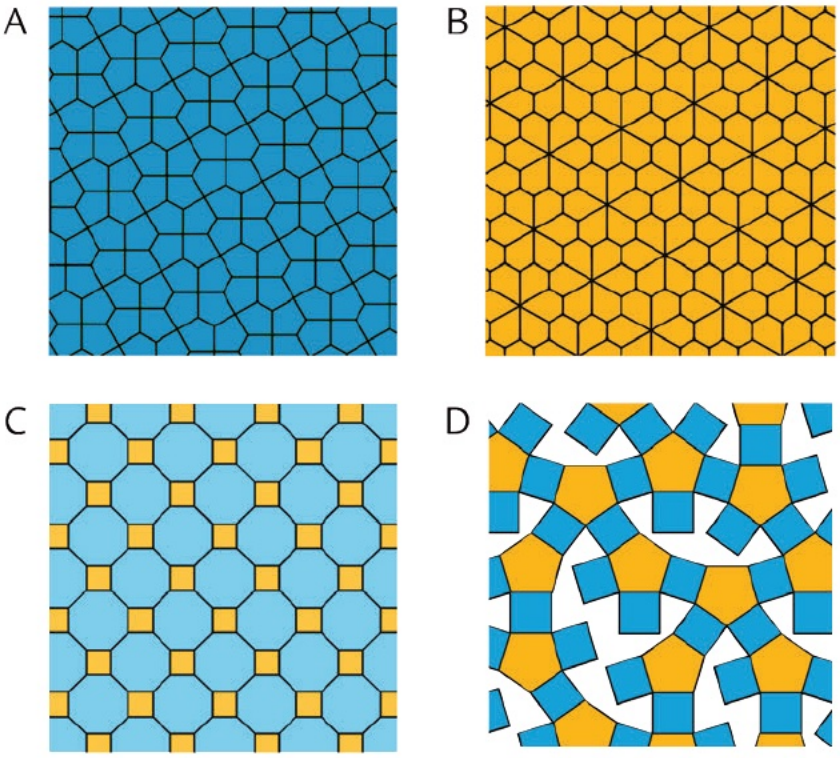
Student Workbook

Core Knowledge Mathematics™

Lesson 1: Tiling the Plane

1.1: Which One Doesn't Belong: Tilings

Which pattern doesn't belong?

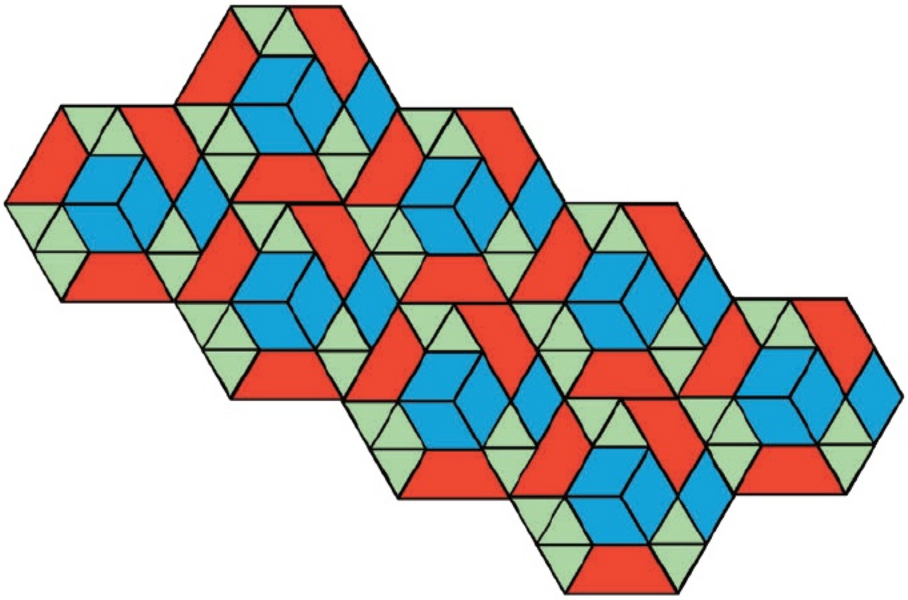


1.2: More Red, Green, or Blue?

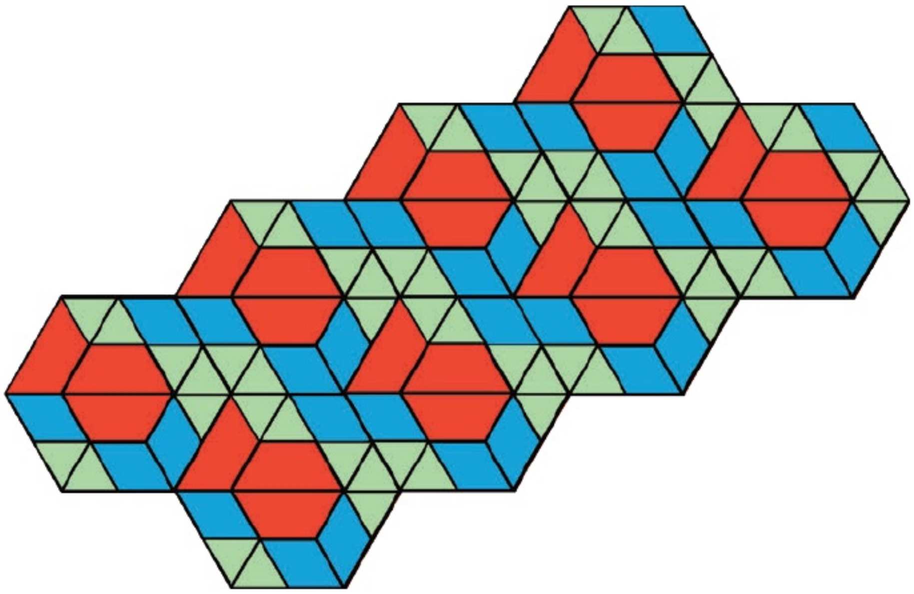
Your teacher will assign you to look at Pattern A or Pattern B.

In your pattern, which shape covers more of the plane: blue rhombuses, red trapezoids, or green triangles? Explain how you know.

Pattern A



Pattern B



Are you ready for more?

On graph paper, create a tiling pattern so that:

- The pattern has at least two different shapes.
- The same amount of the plane is covered by each type of shape.

Lesson 1 Summary

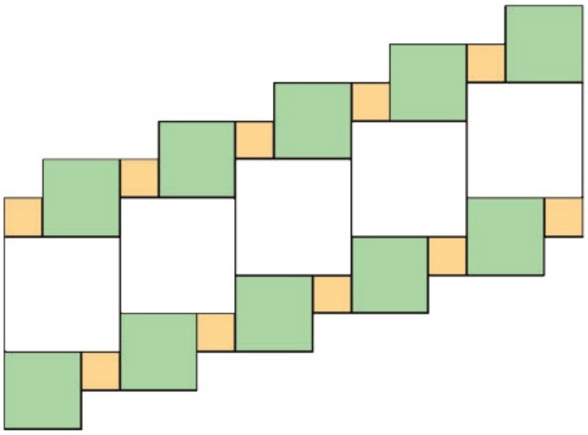
In this lesson, we learned about *tiling* the plane, which means covering a two-dimensional region with copies of the same shape or shapes such that there are no gaps or overlaps.

Then, we compared tiling patterns and the shapes in them. In thinking about which patterns and shapes cover more of the plane, we have started to reason about **area**.

We will continue this work, and to learn how to use mathematical tools strategically to help us do mathematics.

Unit 1 Lesson 1 Cumulative Practice Problems

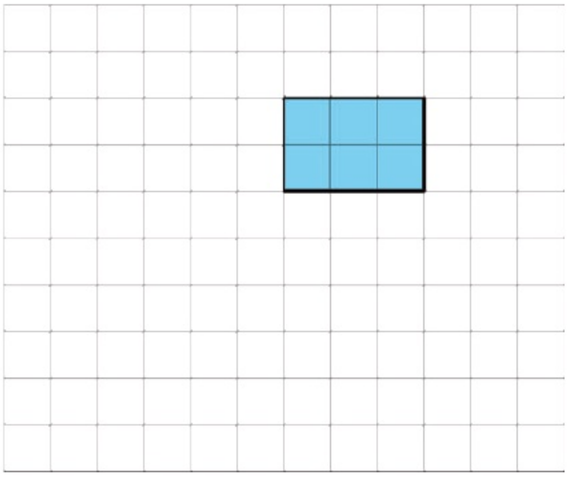
1. Which square—large, medium, or small—covers more of the plane? Explain your reasoning.



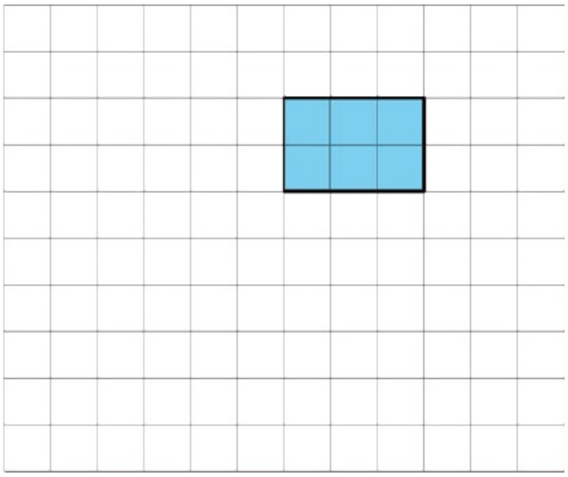
2. Draw three different quadrilaterals, each with an area of 12 square units.

3. Use copies of the rectangle to show how a rectangle could:

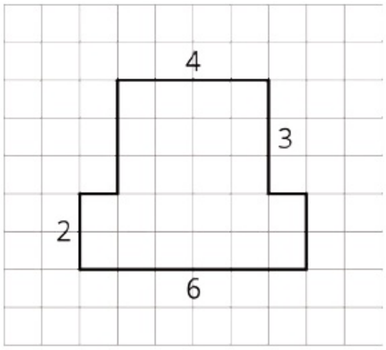
a. tile the plane.



b. *not* tile the plane.



4. The area of this shape is 24 square units. Which of these statements is true about the area? Select **all** that apply.



A. The area can be found by counting the number of squares that touch the edge of the shape.

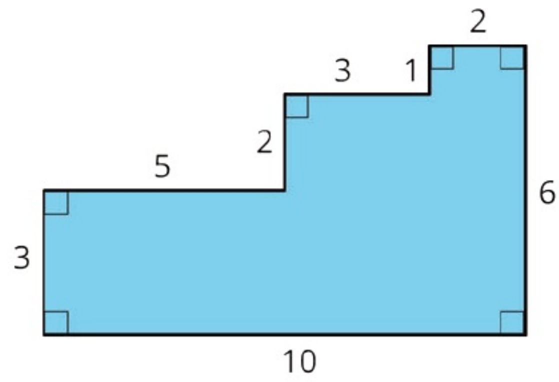
B. It takes 24 grid squares to cover the shape without gaps and overlaps.

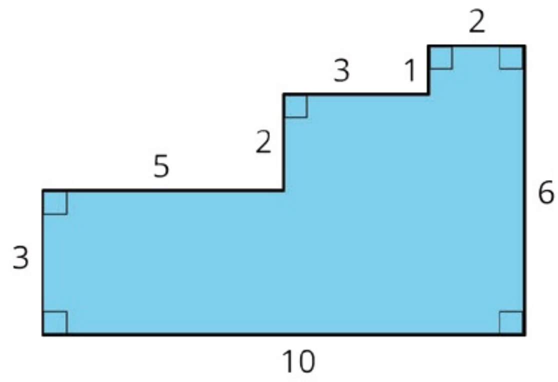
C. The area can be found by multiplying the sides lengths that are 6 units and 4 units.

D. The area can be found by counting the grid squares inside the shape.

E. The area can be found by adding 4×3 and 6×2.

5. Here are two copies of the same figure. Show two different ways for finding the area of the shaded region. All angles are right angles.





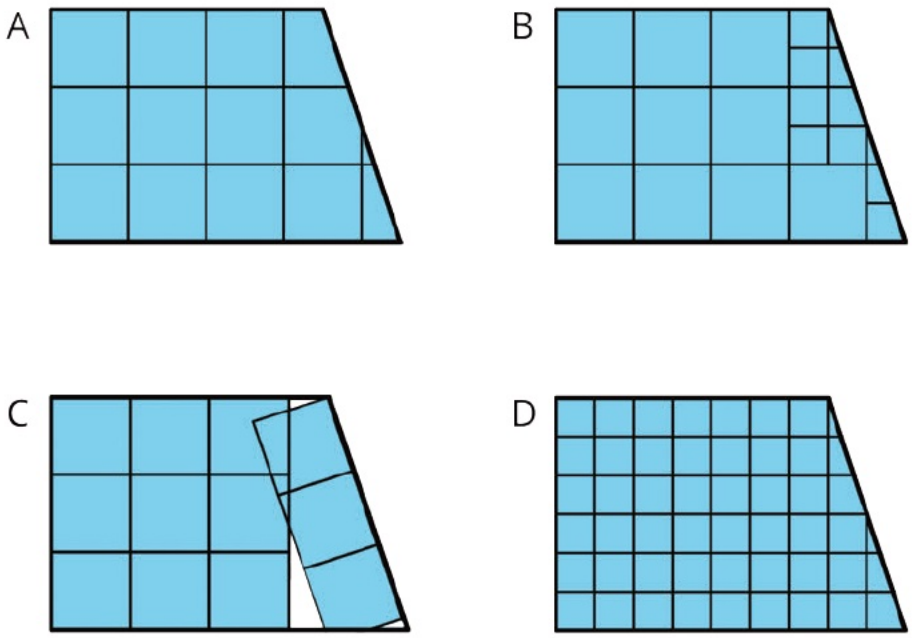
6. Which shape has a larger area: a rectangle that is 7 inches by $\frac{3}{4}$ inch, or a square with side length of $2\frac{1}{2}$ inches? Show your reasoning.

Lesson 2: Finding Area by Decomposing and Rearranging

2.1: What is Area?

You may recall that the term **area** tells us something about the number of squares inside a two-dimensional shape.

1. Here are four drawings that each show squares inside a shape. Select **all** drawings whose squares could be used to find the area of the shape. Be prepared to explain your reasoning.



2. Write a definition of area that includes all the information that you think is important.

2.2: Composing Shapes

Your teacher will give you one square and some small, medium, and large right triangles. The area of the square is 1 square unit.

1. Notice that you can put together two small triangles to make a square. What is the area of the square composed of two small triangles? Be prepared to explain your reasoning.

2. Use your shapes to create a new shape with an area of 1 square unit that is not a square. Trace your shape.

3. Use your shapes to create a new shape with an area of 2 square units. Trace your shape.

4. Use your shapes to create a *different* shape with an area of 2 square units. Trace your shape.

5. Use your shapes to create a new shape with an area of 4 square units. Trace your shape.

Are you ready for more?

Find a way to use all of your pieces to compose a single large square. What is the area of this large square?

2.3: Tangram Triangles

Recall that the area of the square you saw earlier is 1 square unit. Complete each statement and explain your reasoning.

1. The area of the small triangle is ________ square units. I know this because . . .

2. The area of the medium triangle is ________ square units. I know this because . . .

3. The area of the large triangle is ________ square units. I know this because . . .

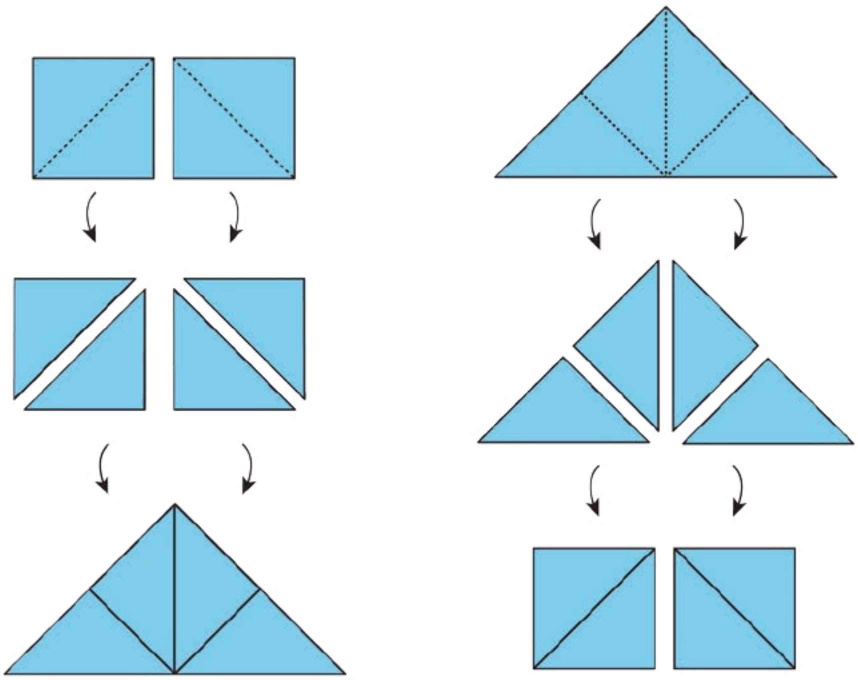
Lesson 2 Summary

Here are two important principles for finding **area**:

1. If two figures can be placed one on top of the other so that they match up exactly, then they have the *same area*.

2. We can **decompose** a figure (break a figure into pieces) and **rearrange** the pieces (move the pieces around) to find its area.

Here are illustrations of the two principles.

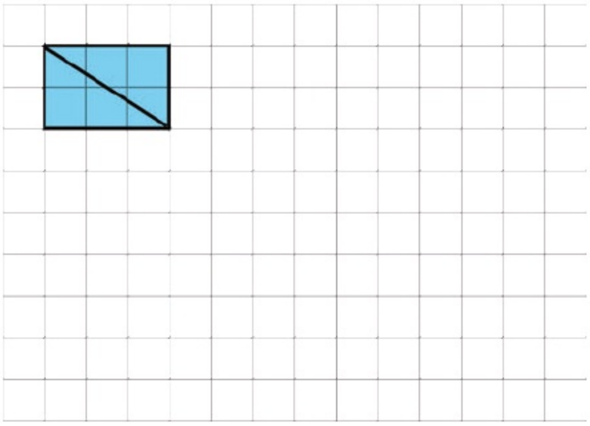


- Each square on the left can be decomposed into 2 triangles. These triangles can be rearranged into a large triangle. So the large triangle has the *same area* as the 2 squares.

- Similarly, the large triangle on the right can be decomposed into 4 equal triangles. The triangles can be rearranged to form 2 squares. If each square has an area of 1 square unit, then the area of the large triangle is 2 square units. We also can say that each small triangle has an area of $\frac{1}{2}$ square unit.

Unit 1 Lesson 2 Cumulative Practice Problems

1. The diagonal of a rectangle is shown.

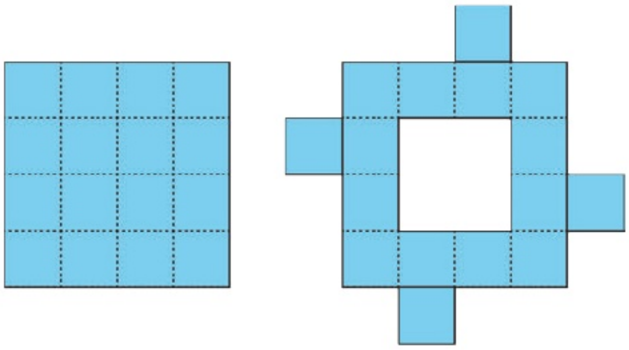


   a. Decompose the rectangle along the diagonal, and recompose the two pieces to make a *different* shape.

   b. How does the area of this new shape compare to the area of the original rectangle? Explain how you know.

2. Priya decomposed a square into 16 smaller, equal-size squares and then cut out 4 of the small squares and attached them around the outside of original square to make a new figure.

   How does the area of her new figure compare with that of the original square?



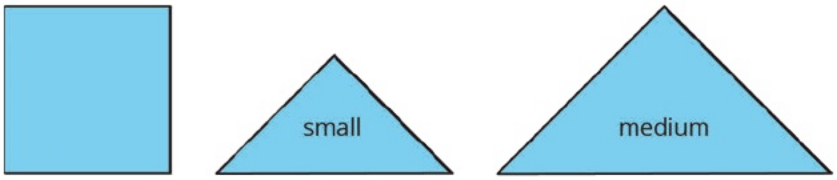
A. The area of the new figure is greater.

B. The two figures have the same area.

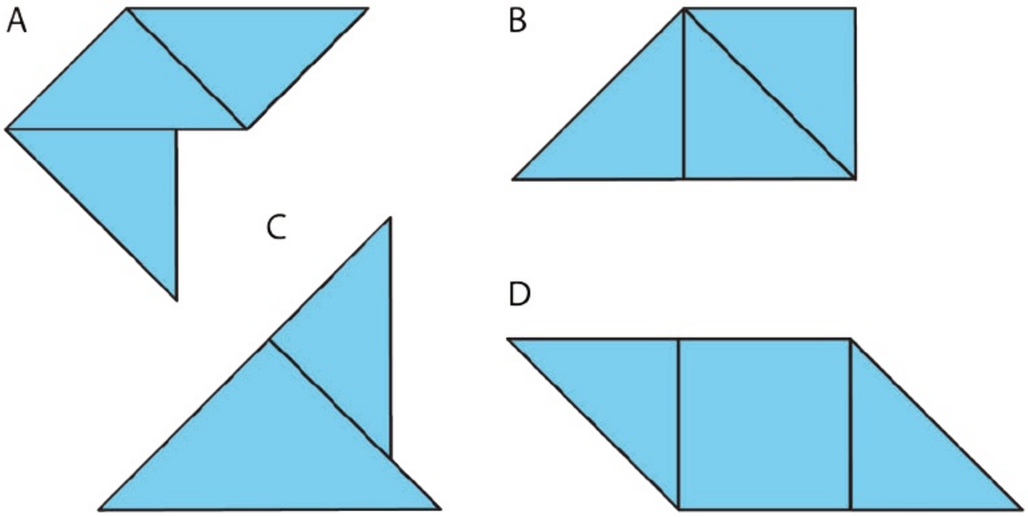
C. The area of the original square is greater.

D. We don't know because neither the side length nor the area of the original square is known.

3. The area of the square is 1 square unit. Two small triangles can be put together to make a square or to make a medium triangle.



Which figure also has an area of $1\frac{1}{2}$ square units? Select **all** that apply.



A. Figure A

B. Figure B

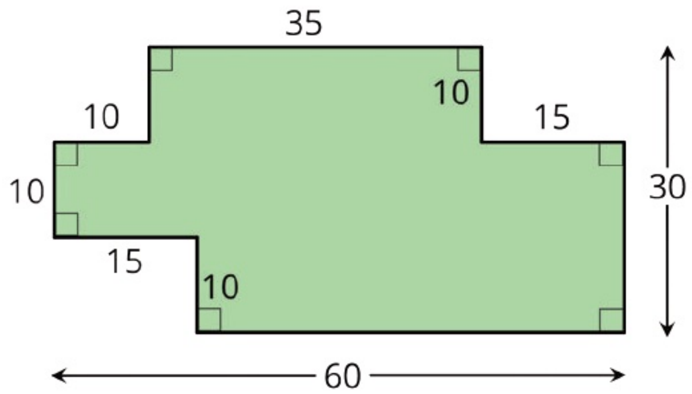
C. Figure C

D. Figure D

4. The area of a rectangular playground is 78 square meters. If the length of the playground is 13 meters, what is its width?

(From Unit 1, Lesson 1.)

5. A student said, “We can’t find the area of the shaded region because the shape has many different measurements, instead of just a length and a width that we could multiply.”



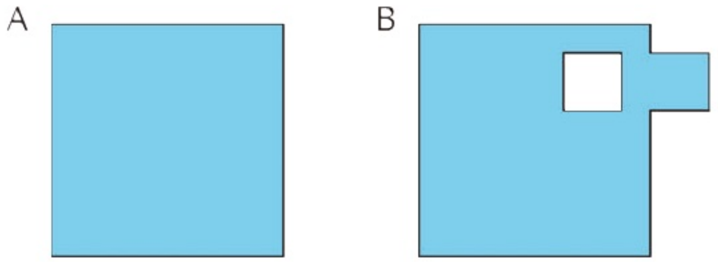
Explain why the student’s statement about area is incorrect.

(From Unit 1, Lesson 1.)

Lesson 3: Reasoning to Find Area

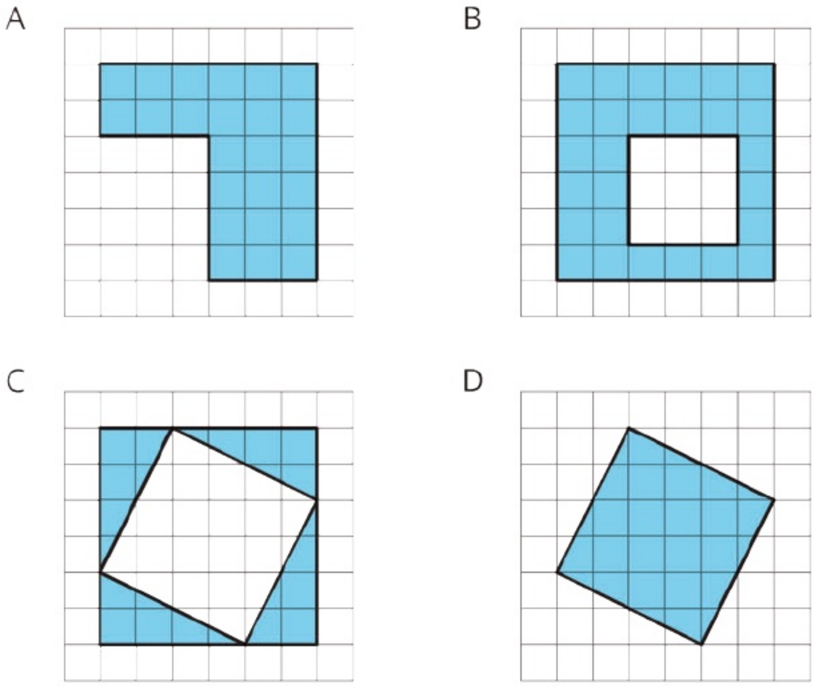
3.1: Comparing Regions

Is the area of Figure A greater than, less than, or equal to the area of the shaded region in Figure B? Be prepared to explain your reasoning.



3.2: On the Grid

Each grid square is 1 square unit. Find the area, in square units, of each shaded region without counting every square. Be prepared to explain your reasoning.



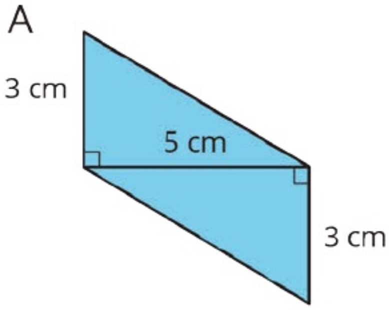
Are you ready for more?

Rearrange the triangles from Figure C so they fit inside Figure D. Draw and color a diagram of your work.

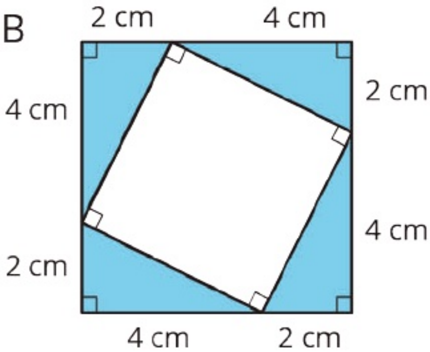
3.3: Off the Grid

Find the area of the shaded region(s) of each figure. Explain or show your reasoning.

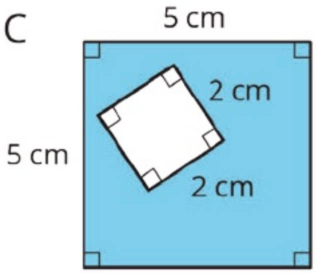
A



B



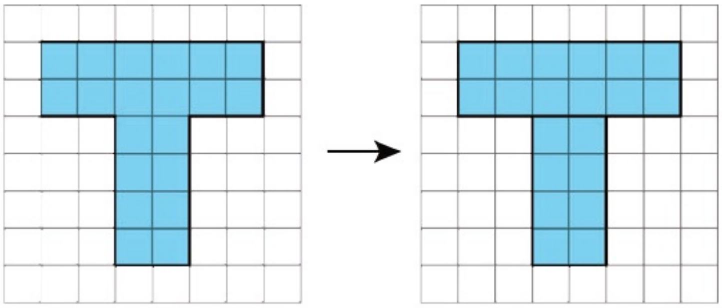
C



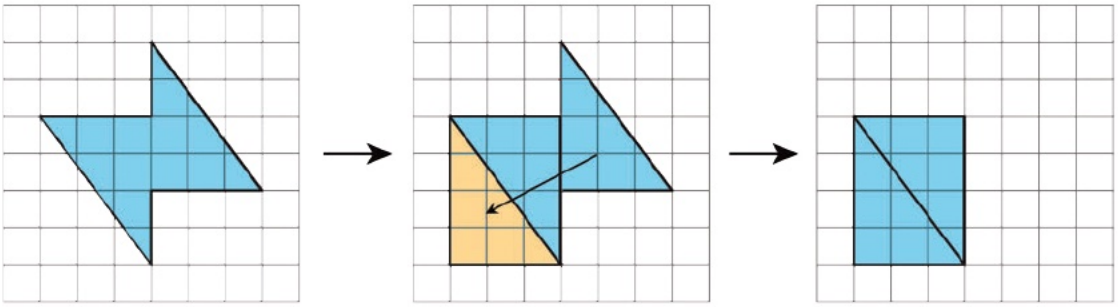
Lesson 3 Summary

There are different strategies we can use to find the area of a region. We can:

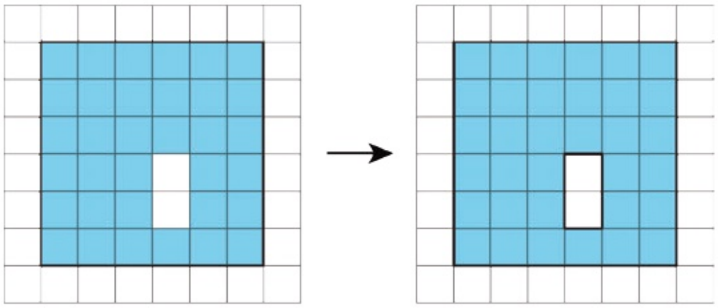
- Decompose it into shapes whose areas you know how to calculate; find the area of each of those shapes, and then add the areas.



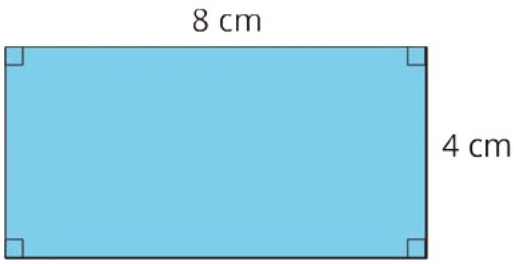
- Decompose it and rearrange the pieces into shapes whose areas you know how to calculate; find the area of each of those shapes, and then add the areas.



- Consider it as a shape with a missing piece; calculate the area of the shape and the missing piece, and then subtract the area of the piece from the area of the shape.

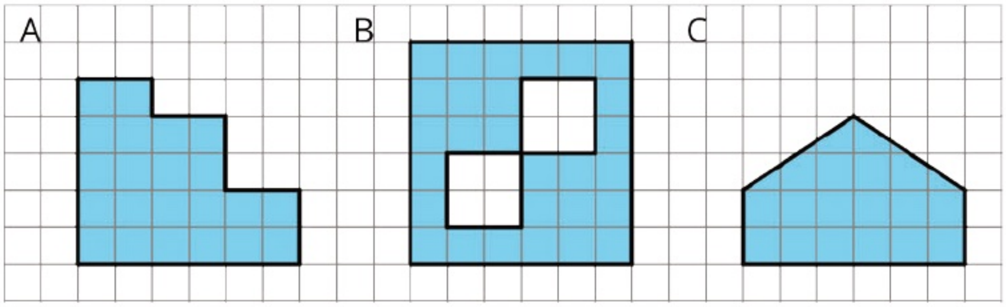


The area of a figure is always measured in square units. When both side lengths of a rectangle are given in centimeters, then the area is given in square centimeters. For example, the area of this rectangle is 32 square centimeters.



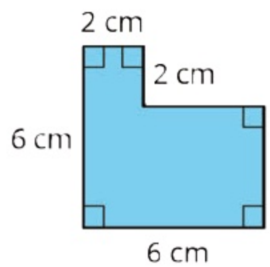
Unit 1 Lesson 3 Cumulative Practice Problems

1. Find the area of each shaded region. Show your reasoning.

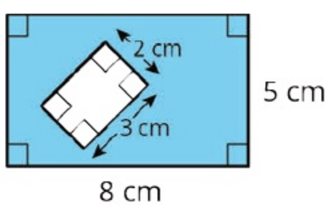


2. Find the area of each shaded region. Show or explain your reasoning.

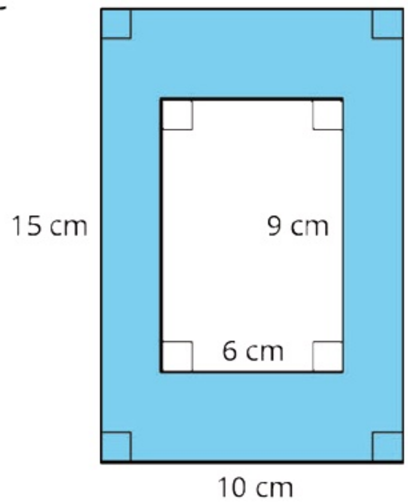
A



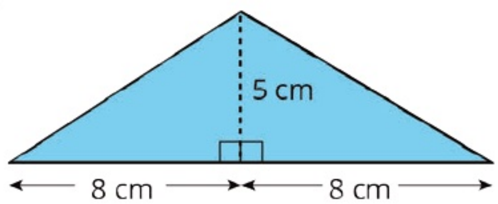
B



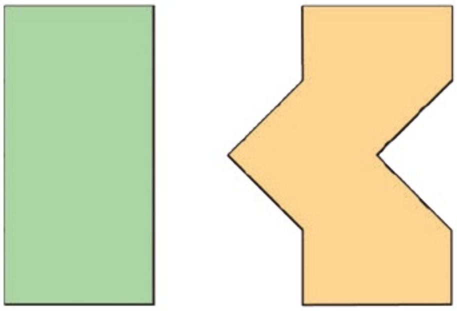
C



D



3. Two plots of land have very different shapes. Noah said that both plots of land have the same area.



plot A   plot B

Do you agree with Noah? Explain your reasoning.

4. A homeowner is deciding on the size of tiles to use to fully tile a rectangular wall in her bathroom that is 80 inches by 40 inches. The tiles are squares and come in three side lengths: 8 inches, 4 inches, and 2 inches. State if you agree with each statement about the tiles. Explain your reasoning.

    a. Regardless of the size she chooses, she will need the same number of tiles.

    b. Regardless of the size she chooses, the area of the wall that is being tiled is the same.

    c. She will need two 2-inch tiles to cover the same area as one 4-inch tile.

    d. She will need four 4-inch tiles to cover the same area as one 8-inch tile.

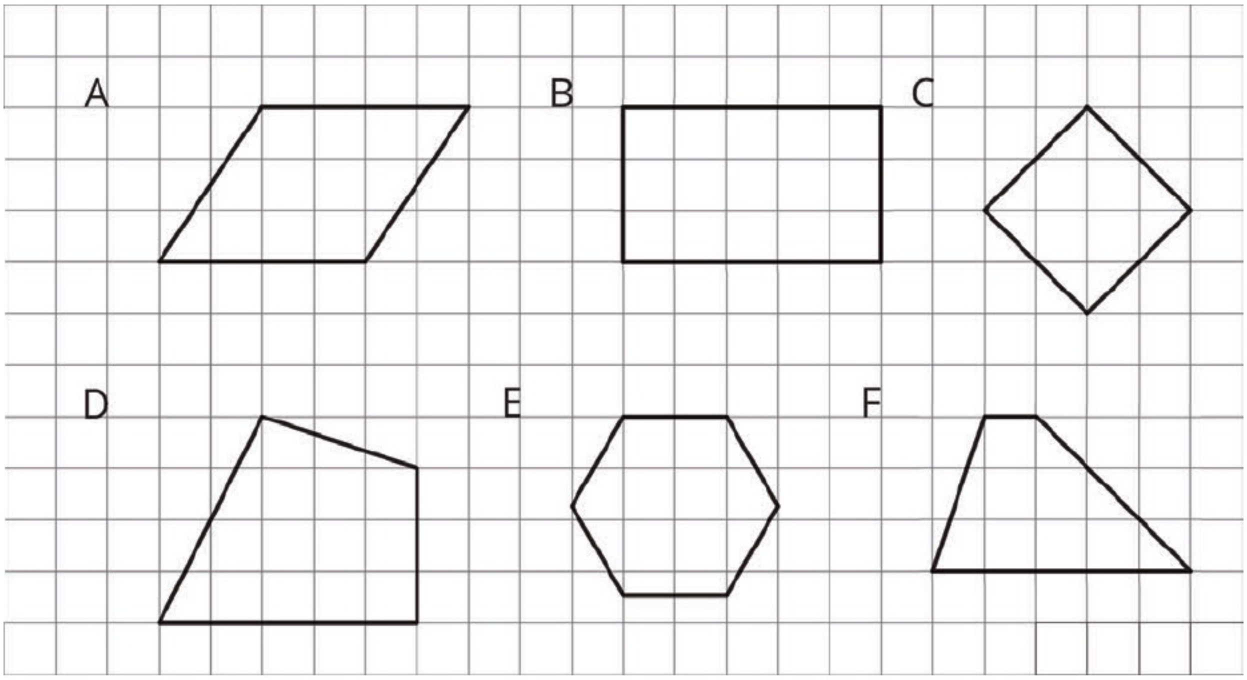
    e. If she chooses the 8-inch tiles, she will need a quarter as many tiles as she would with 2-inch tiles.

(From Unit 1, Lesson 2.)

Lesson 4: Parallelograms

4.1: Features of a Parallelogram

Figures A, B, and C are **parallelograms**. Figures D, E, and F are *not* parallelograms.



Study the examples and non-examples. What do you notice about:

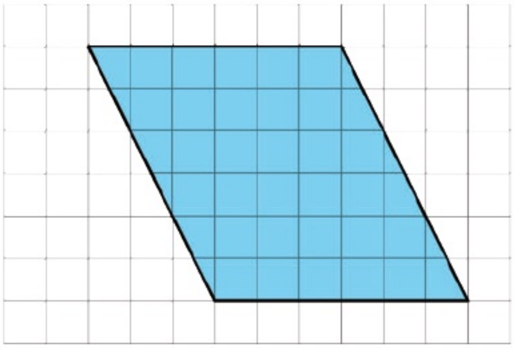
1. the number of sides that a parallelogram has?

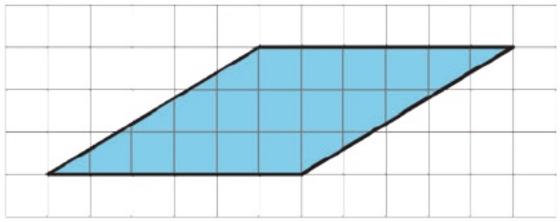
2. opposite sides of a parallelogram?

3. opposite angles of a parallelogram?

4.2: Area of a Parallelogram

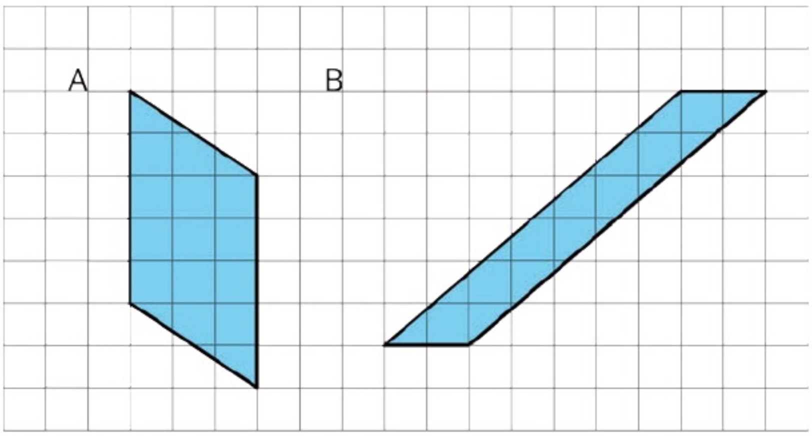
Find the area of each parallelogram. Show your reasoning.



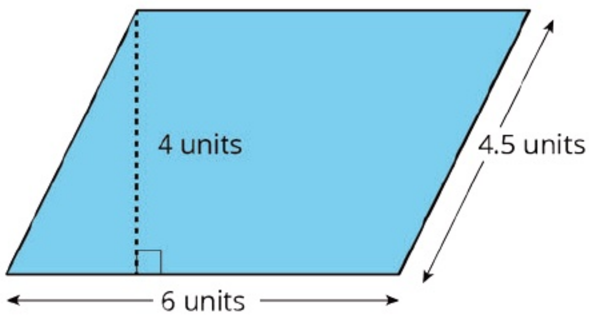


4.3: Lots of Parallelograms

Find the area of each parallelogram. Show your reasoning.

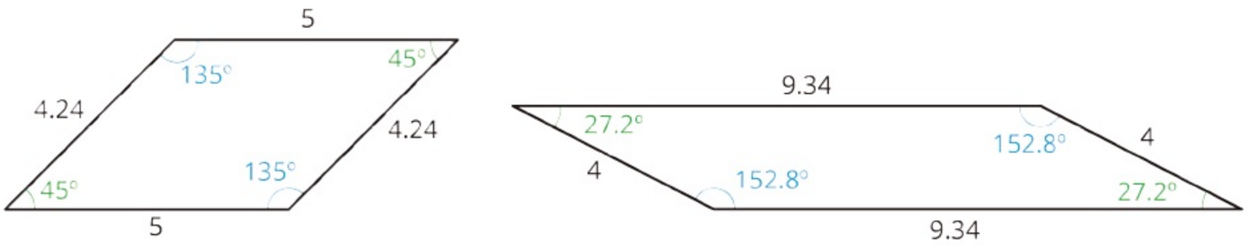


C



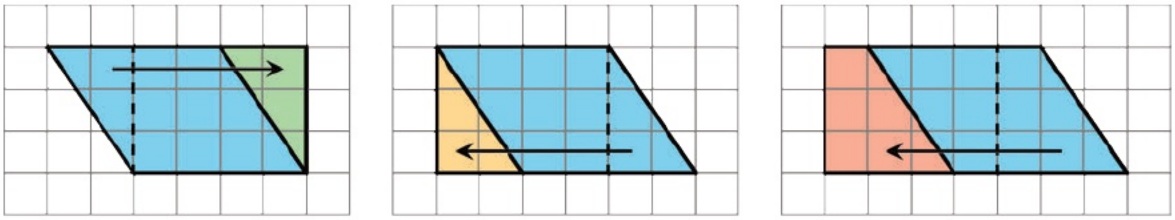
Lesson 4 Summary

A **parallelogram** is a quadrilateral (it has four sides). The opposite sides of a parallelogram are parallel. It is also true that the opposite sides of a parallelogram have equal length, and the opposite angles of a parallelogram have equal measure.

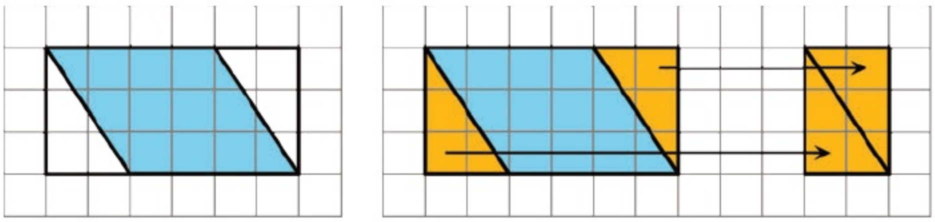


There are several strategies for finding the area of a **parallelogram**.

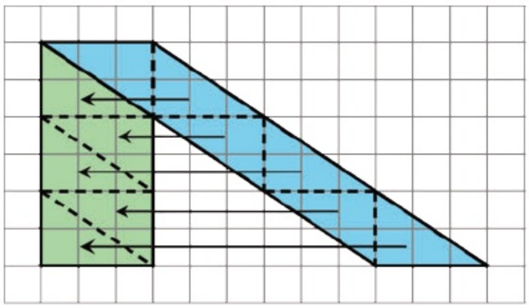
- We can decompose and rearrange a parallelogram to form a rectangle. Here are three ways:



- We can enclose the parallelogram and then subtract the area of the two triangles in the corner.

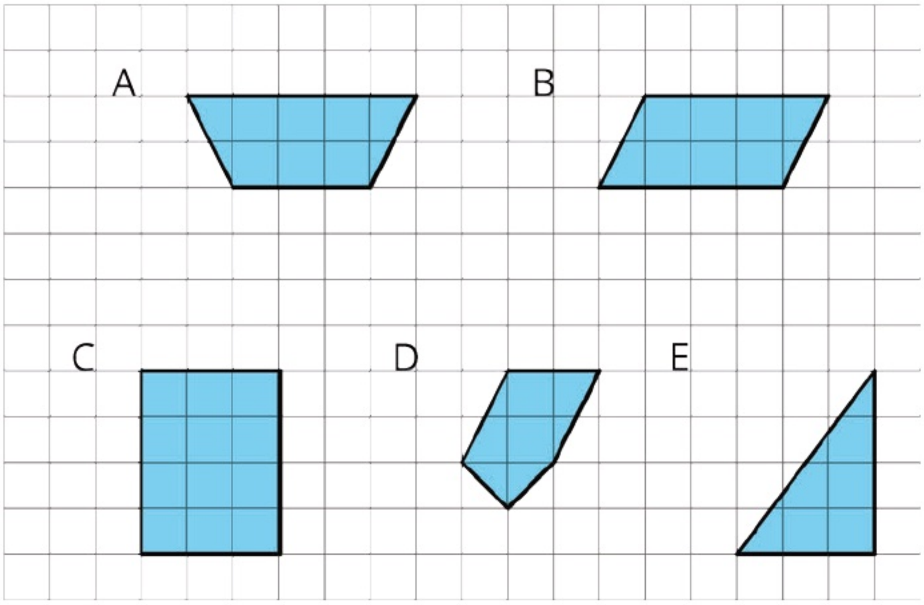


Both of these ways will work for any parallelogram. However, for some parallelograms the process of decomposing and rearranging requires a lot more steps than if we enclose the parallelogram with a rectangle and subtract the combined area of the two triangles in the corners.

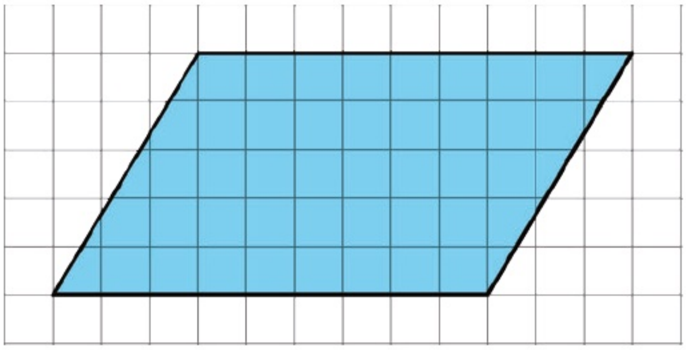


Unit 1 Lesson 4 Cumulative Practice Problems

1. Select **all** of the parallelograms. For each figure that is *not* selected, explain how you know it is not a parallelogram.

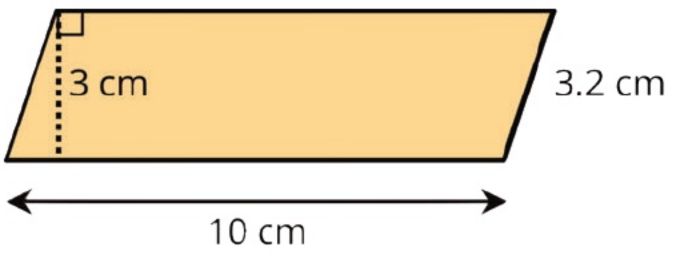


2. a. Decompose and rearrange this parallelogram to make a rectangle.

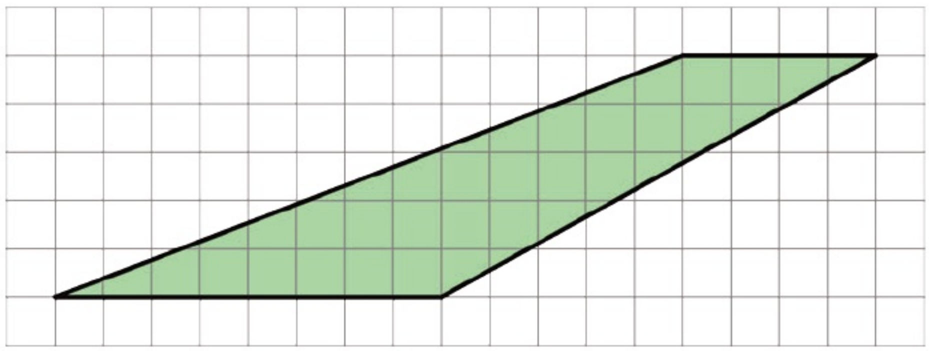


b. What is the area of the parallelogram? Explain your reasoning.

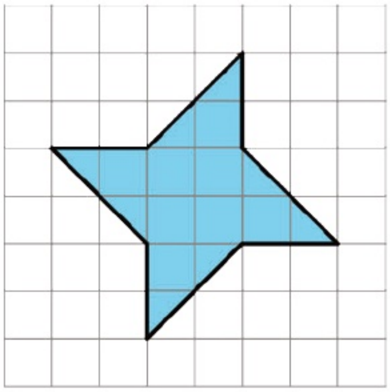
3. Find the area of the parallelogram.

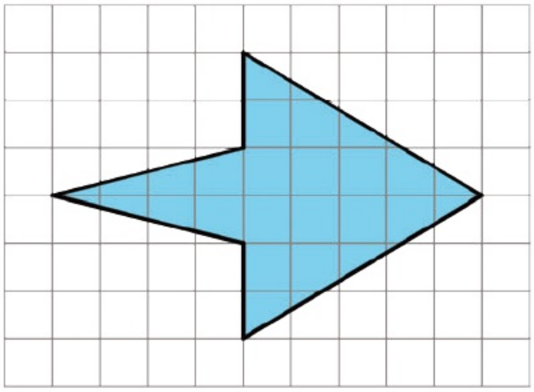


4. Explain why this quadrilateral is *not* a parallelogram.



5. Find the area of each shape. Show your reasoning.





(From Unit 1, Lesson 3.)

6. Find the area of the rectangle with each set of side lengths.

a. 5 in and $\frac{1}{3}$ in

b. 5 in and $\frac{4}{3}$ in

c. $\frac{5}{2}$ in and $\frac{4}{3}$ in

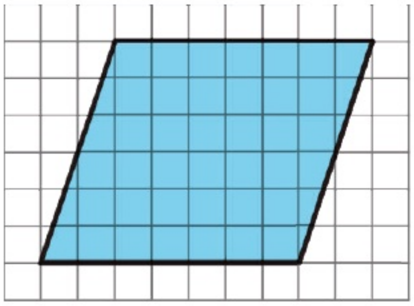
d. $\frac{7}{6}$ in and $\frac{6}{7}$ in

(From Unit 1, Lesson 1.)

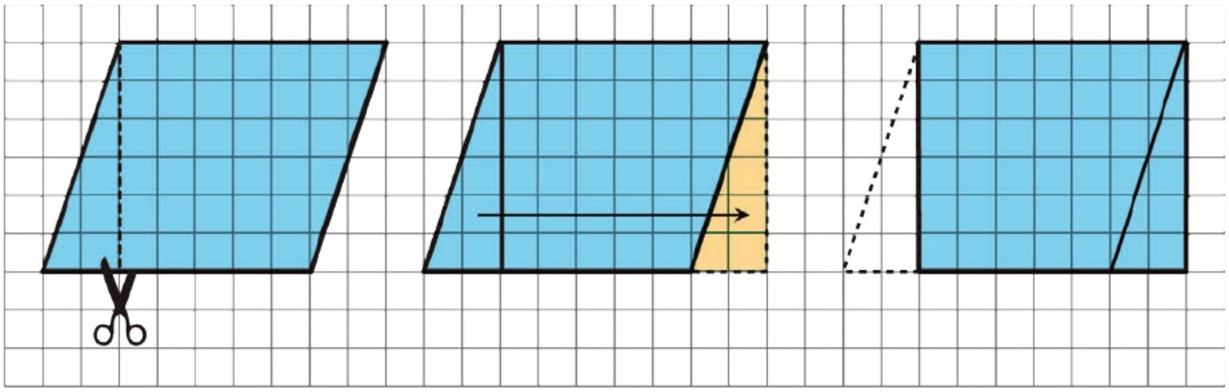
Lesson 5: Bases and Heights of Parallelograms

5.1: A Parallelogram and Its Rectangles

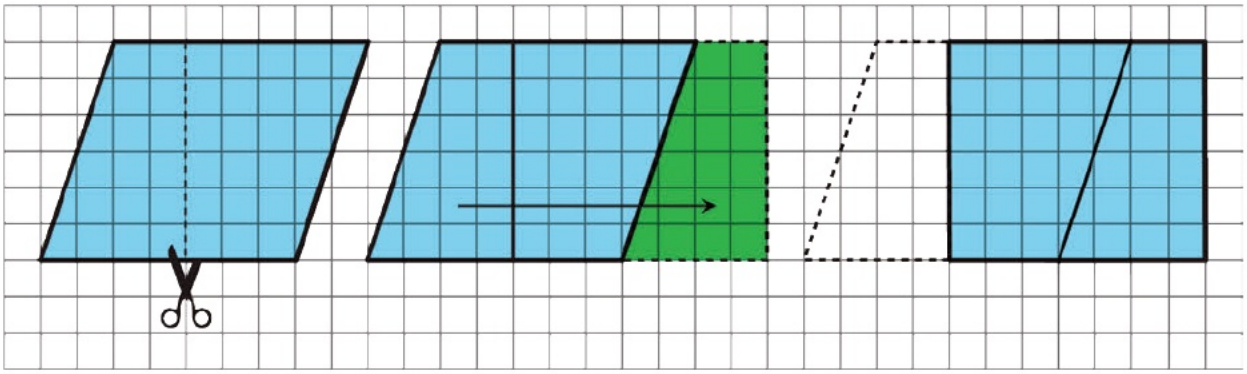
Elena and Tyler were finding the area of this parallelogram:



Here is how Elena did it:



Here is how Tyler did it:

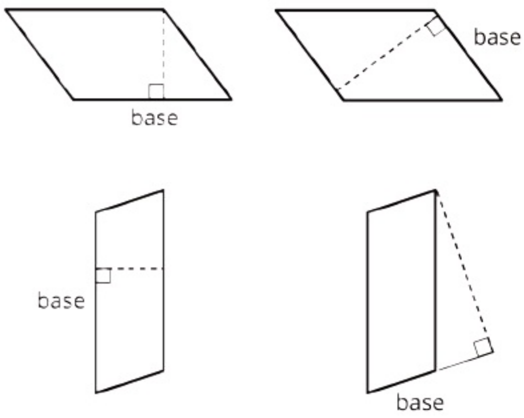


How are the two strategies for finding the area of a parallelogram the same? How they are different?

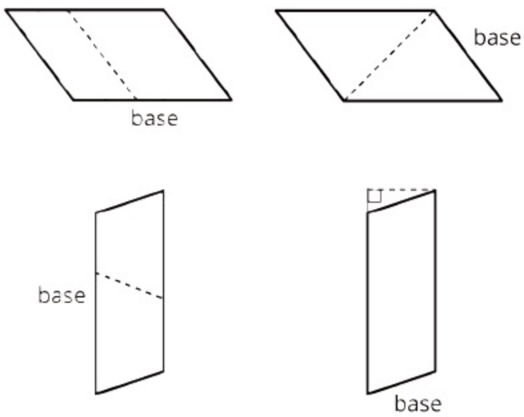
5.2: The Right Height?

Study the examples and non-examples of **bases** and **heights** of parallelograms.

- Examples: The dashed segments in these drawings represent the corresponding height for the given base.



- Non-examples: The dashed segments in these drawings do *not* represent the corresponding height for the given base.



1. Select **all** the statements that are true about bases and heights in a parallelogram.

    a. Only a horizontal side of a parallelogram can be a base.

    b. Any side of a parallelogram can be a base.

    c. A height can be drawn at any angle to the side chosen as the base.

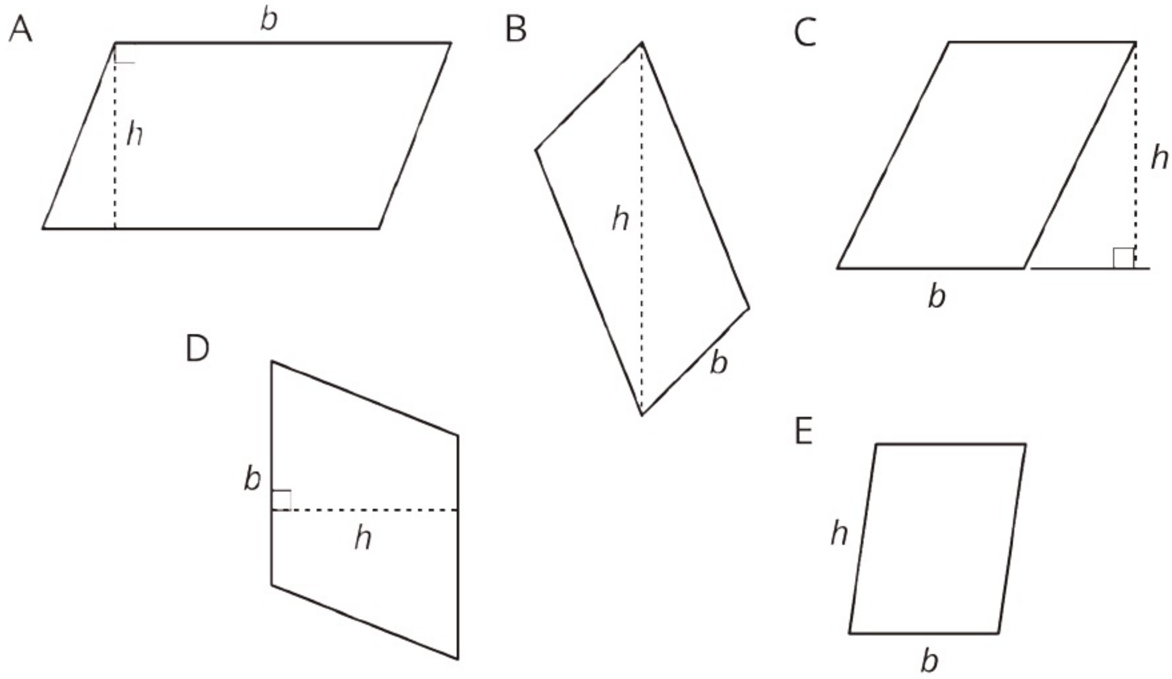
    d. A base and its corresponding height must be perpendicular to each other.

    e. A height can only be drawn inside a parallelogram.

    f. A height can be drawn outside of the parallelogram, as long as it is drawn at a 90-degree angle to the base.

    g. A base cannot be extended to meet a height.

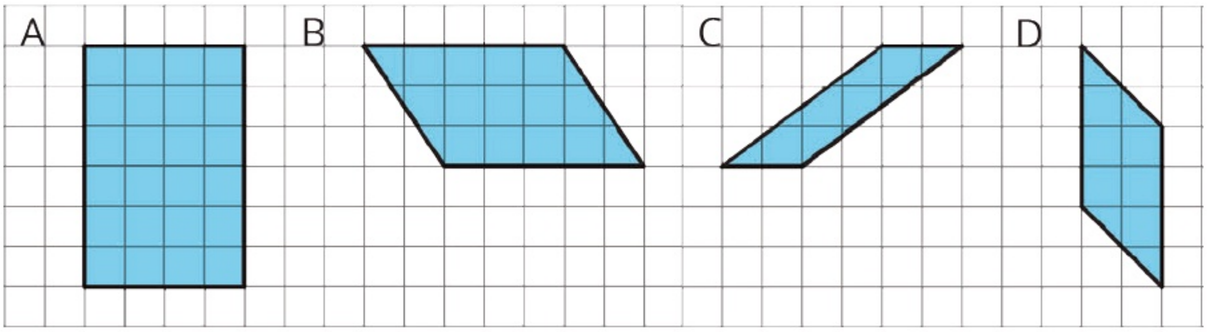
2. Five students labeled a base b and a corresponding height h for each of these parallelograms. Are all drawings correctly labeled? Explain how you know.



5.3: Finding the Formula for Area of Parallelograms

For each parallelogram:

- Identify a base and a corresponding height, and record their lengths in the table.
- Find the area of the parallelogram and record it in the last column of the table.



parallelogram	base (units)	height (units)	area (sq units)
A			
B			
C			
D			
any parallelogram	b	h	

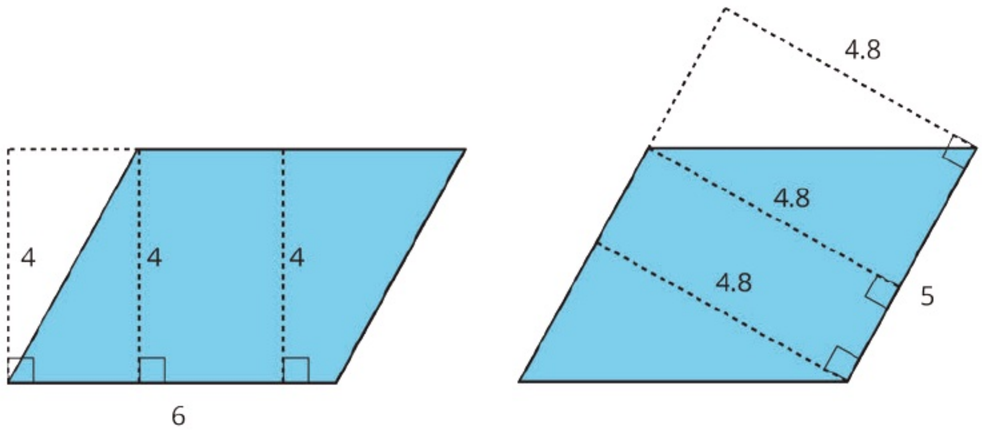
In the last row, write an expression for the area of any parallelogram, using b and h .

Are you ready for more?

1. What happens to the area of a parallelogram if the height doubles but the base is unchanged? If the height triples? If the height is 100 times the original?
2. What happens to the area if both the base and the height double? Both triple? Both are 100 times their original lengths?

Lesson 5 Summary

- We can choose any of the four sides of a parallelogram as the **base**. Both the side (the segment) and its length (the measurement) are called the base.
- If we draw any perpendicular segment from a point on the base to the opposite side of the parallelogram, that segment will always have the same length. We call that value the **height**. There are infinitely many segments that can represent the height!

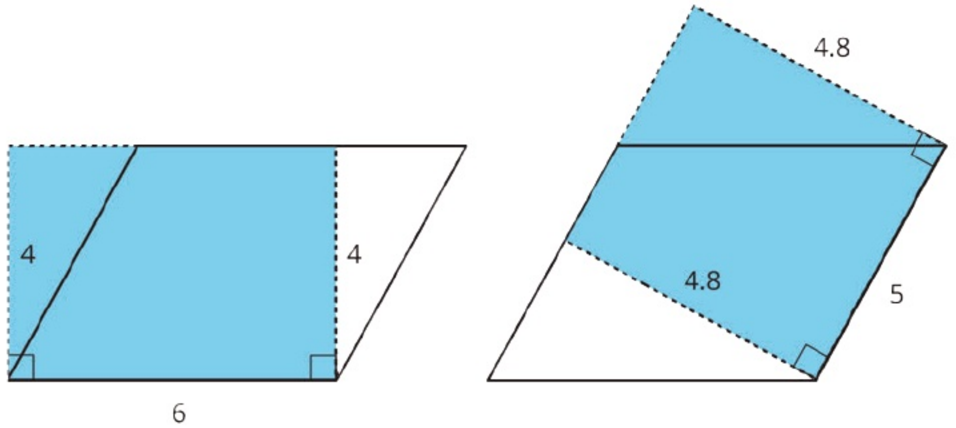


Here are two copies of the same parallelogram. On the left, the side that is the base is 6 units long. Its corresponding height is 4 units. On the right, the side that is the base is 5 units long. Its corresponding height is 4.8 units. For both, three different segments are shown to represent the height. We could draw in many more!

No matter which side is chosen as the base, the area of the parallelogram is the product of that base and its corresponding height. We can check this:

$4 \times 6 = 24$ and $4.8 \times 5 = 24$

We can see why this is true by decomposing and rearranging the parallelograms into rectangles.



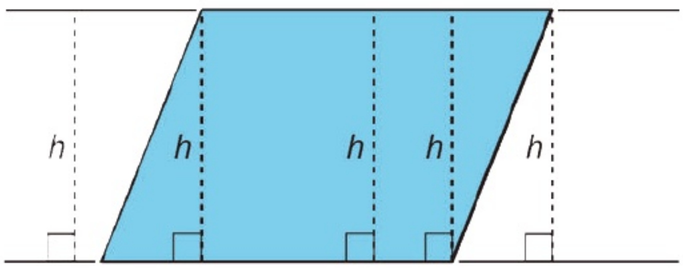
Notice that the side lengths of each rectangle are the base and height of the parallelogram. Even though the two rectangles have different side lengths, the products of the side lengths are equal, so they have the same area! And both rectangles have the same area as the parallelogram.

We often use letters to stand for numbers. If b is base of a parallelogram (in units), and h is the corresponding height (in units), then the area of the parallelogram (in square units) is the product of these two numbers.

$$b \cdot h$$

Notice that we write the multiplication symbol with a small dot instead of a $\times$ symbol. This is so that we don't get confused about whether $\times$ means multiply, or whether the letter x is standing in for a number.

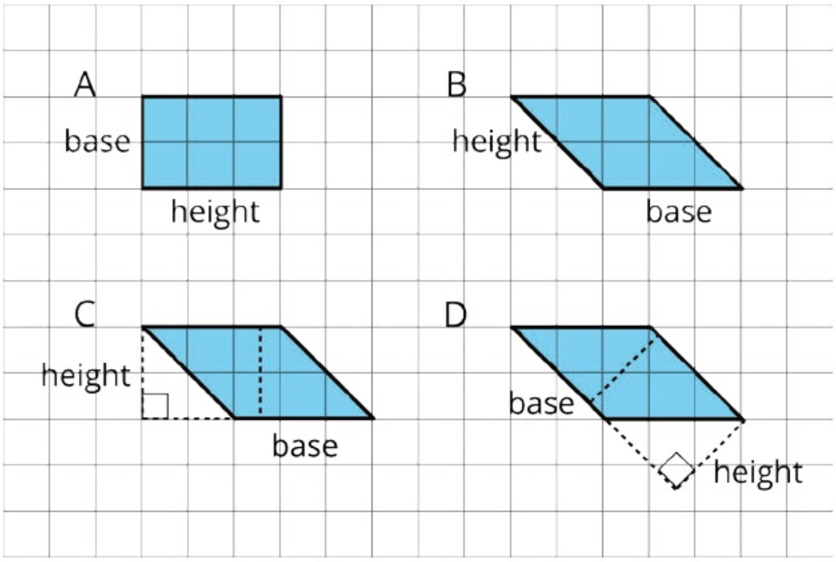
In high school, you will be able to prove that a perpendicular segment from a point on one side of a parallelogram to the opposite side will always have the same length.



You can see this most easily when you draw a parallelogram on graph paper. For now, we will just use this as a fact.

Unit 1 Lesson 5 Cumulative Practice Problems

1. Select **all** parallelograms that have a correct height labeled for the given base.



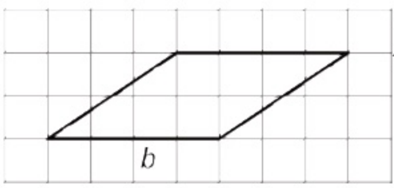
A. A

B. B

C. C

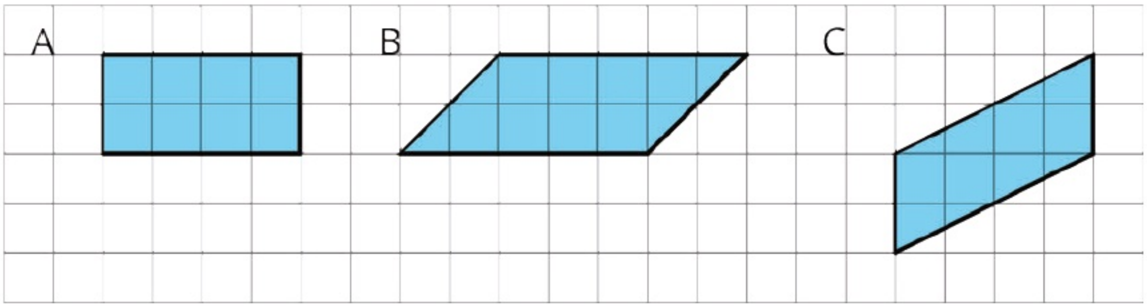
D. D

2. The side labeled b has been chosen as the base for this parallelogram.

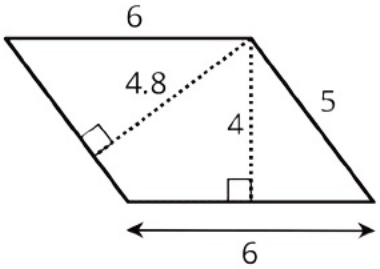


Draw a segment showing the height corresponding to that base.

3. Find the area of each parallelogram.



4. If the side that is 6 units long is the base of this parallelogram, what is its corresponding height?



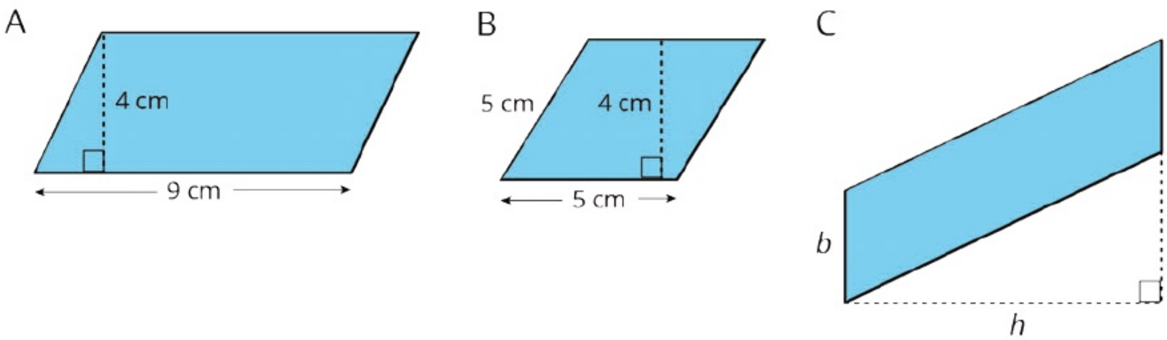
A. 6 units

B. 4.8 units

C. 4 units

D. 5 units

5. Find the area of each parallelogram.



6. Do you agree with each of these statements? Explain your reasoning.

   a. A parallelogram has six sides.

   b. Opposite sides of a parallelogram are parallel.

   c. A parallelogram can have one pair or two pairs of parallel sides.

   d. All sides of a parallelogram have the same length.

   e. All angles of a parallelogram have the same measure.

(From Unit 1, Lesson 4.)

7. A square with an area of 1 square meter is decomposed into 9 identical small squares. Each small square is decomposed into two identical triangles.

   a. What is the area, in square meters, of 6 triangles? If you get stuck, consider drawing a diagram.

   b. How many triangles are needed to compose a region that is $1\frac{1}{2}$ square meters?

(From Unit 1, Lesson 2.)

Lesson 6: Area of Parallelograms

6.1: Missing Dots

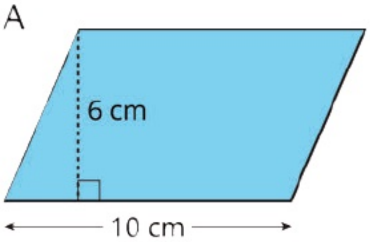
How many dots are in the image?

How do you see them?

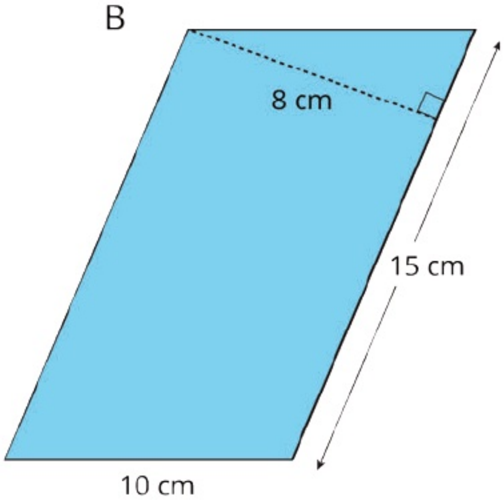
6.2: More Areas of Parallelograms

1. Find the area of each parallelogram. Show your reasoning.

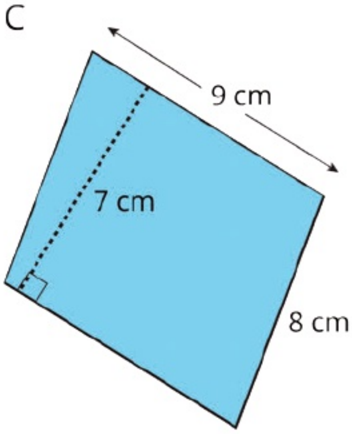
A



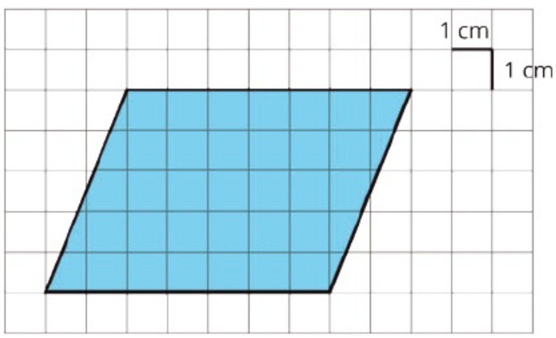
B



C



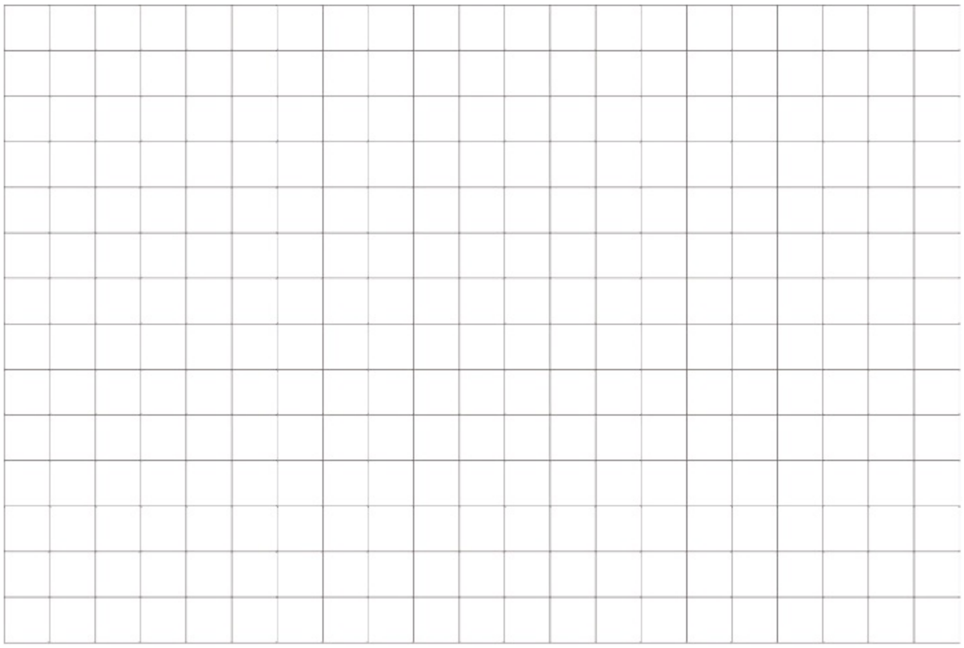
D



2. In Parallelogram B, what is the corresponding height for the base that is 10 cm long? Explain or show your reasoning.

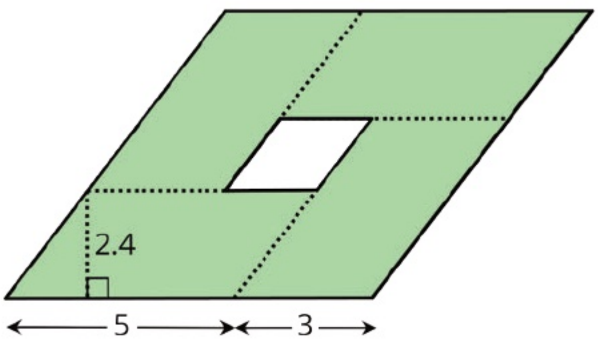
3. Two different parallelograms P and Q both have an area of 20 square units. Neither of the parallelograms are rectangles.

   On the grid, draw two parallelograms that could be P and Q.



Are you ready for more?

Here is a parallelogram composed of smaller parallelograms. The shaded region is composed of four identical parallelograms. All lengths are in inches.

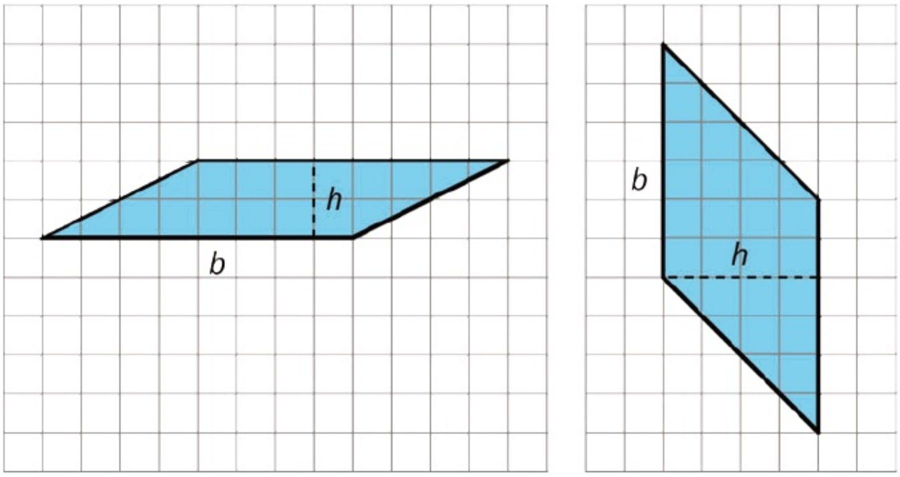


What is the area of the unshaded parallelogram in the middle? Explain or show your reasoning.

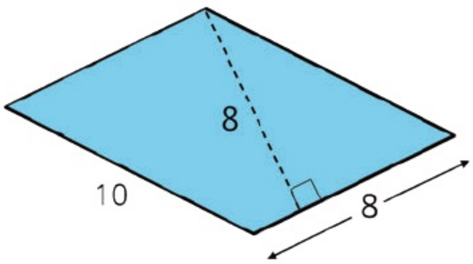
Lesson 6 Summary

Any pair of base and corresponding height can help us find the area of a parallelogram, but some base-height pairs are more easily identified than others.

When a parallelogram is drawn on a grid and has *horizontal* sides, we can use a horizontal side as the base. When it has *vertical* sides, we can use a vertical side as the base. The grid can help us find (or estimate) the lengths of the base and of the corresponding height.

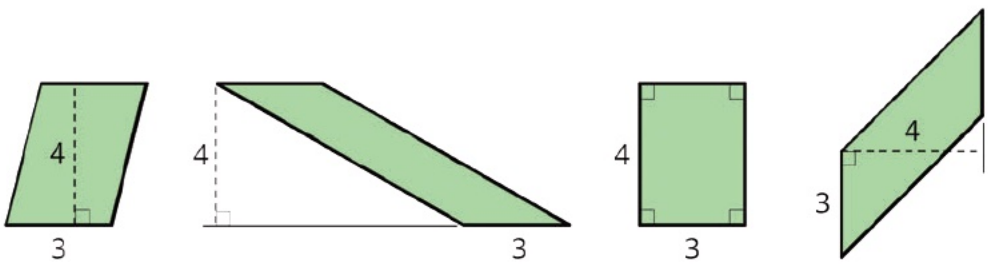


When a parallelogram is *not* drawn on a grid, we can still find its area if a base and a corresponding height are known.



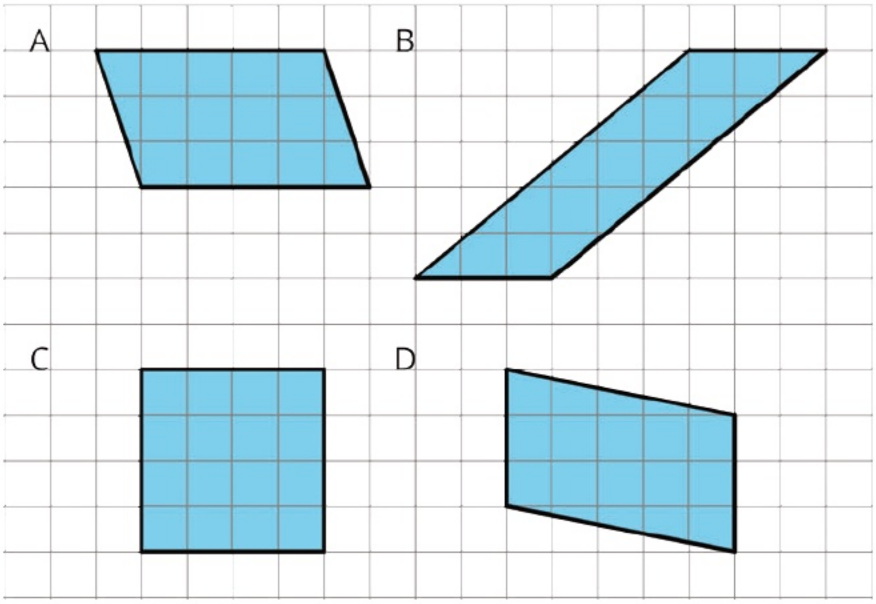
In this parallelogram, the corresponding height for the side that is 10 units long is not given, but the height for the side that is 8 units long is given. This base-height pair can help us find the area.

Regardless of their shape, parallelograms that have the same base and the same height will have the same area; the product of the base and height will be equal. Here are some parallelograms with the same pair of base-height measurements.



Unit 1 Lesson 6 Cumulative Practice Problems

1. Which three of these parallelograms have the same area as each other?



A. A

B. B

C. C

D. D

2. Which pair of base and height produces the greatest area? All measurements are in centimeters.

A. $b = 4$, $h = 3.5$

B. $b = 0.8$, $h = 20$

C. $b = 6$, $h = 2.25$

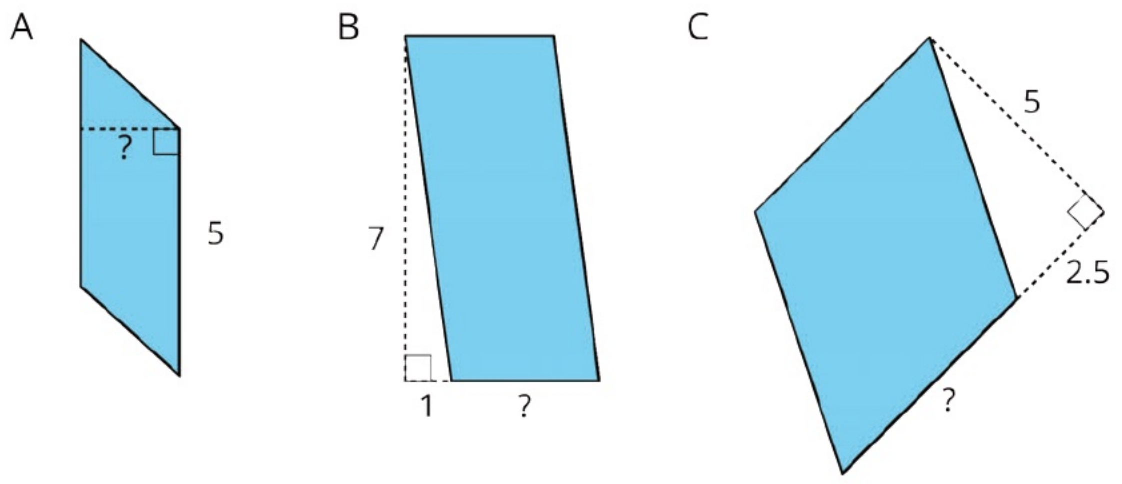
D. $b = 10$, $h = 1.4$

3. Here are the areas of three parallelograms. Use them to find the missing length (labeled with a "?") on each parallelogram.

A: 10 square units

B: 21 square units

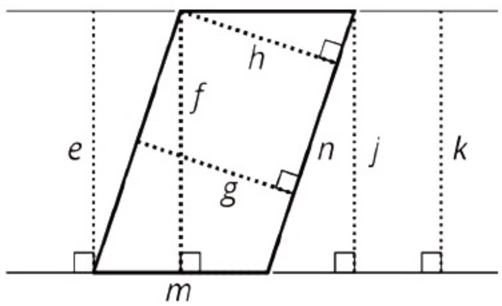
C: 25 square units



4. The Dockland Building in Hamburg, Germany is shaped like a parallelogram.

If the length of the building is 86 meters and its height is 55 meters, what is the area of this face of the building?

5. Select **all** segments that could represent a corresponding height if the side m is the base.



A. e

B. f

C. g

D. h

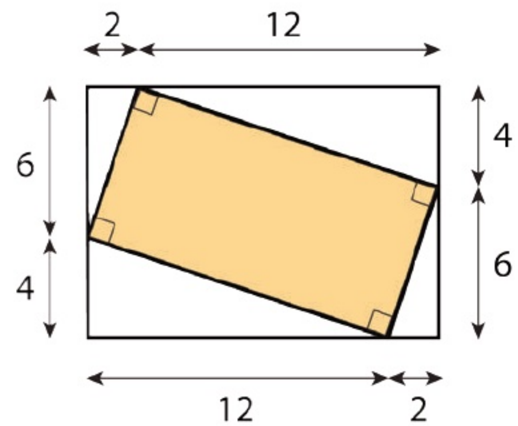
E. j

F. k

G. n

(From Unit 1, Lesson 5.)

6. Find the area of the shaded region. All measurements are in centimeters. Show your reasoning.

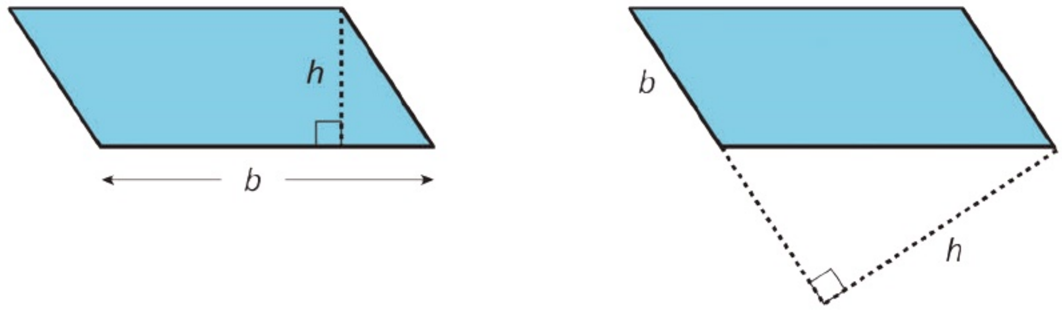


(From Unit 1, Lesson 3.)

Lesson 7: From Parallelograms to Triangles

7.1: Same Parallelograms, Different Bases

Here are two copies of a parallelogram. Each copy has one side labeled as the base b and a segment drawn for its corresponding height and labeled h.



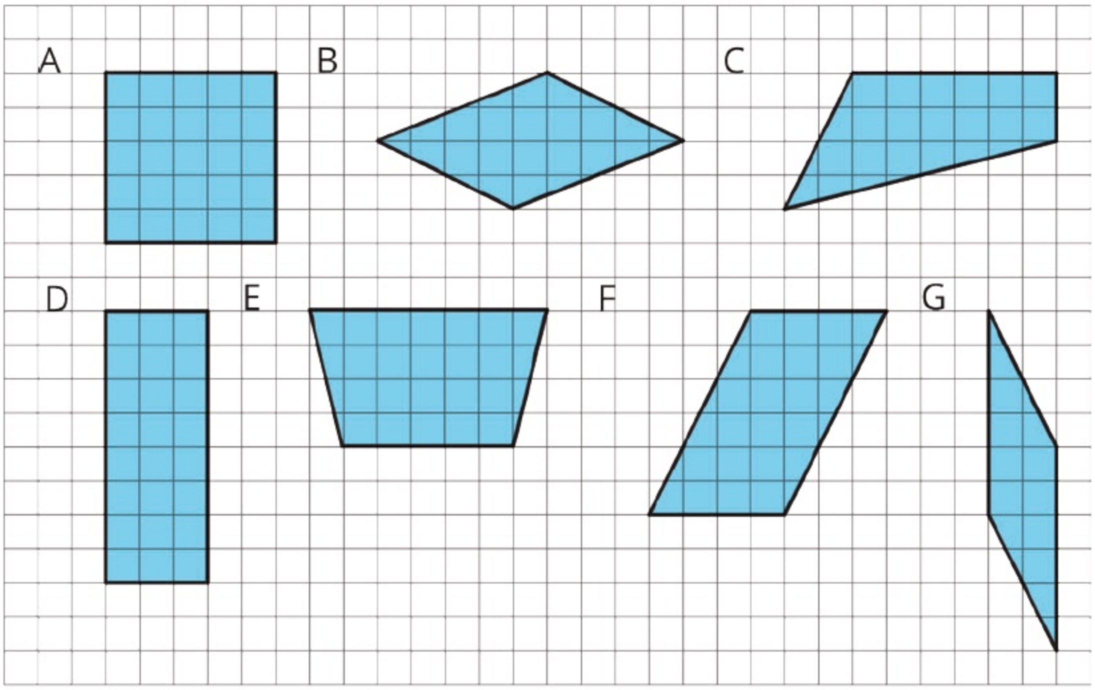
1. The base of the parallelogram on the left is 2.4 centimeters; its corresponding height is 1 centimeter. Find its area in square centimeters.

2. The height of the parallelogram on the right is 2 centimeters. How long is the base of that parallelogram? Explain your reasoning.

7.2: A Tale of Two Triangles (Part 1)

Two polygons are identical if they match up exactly when placed one on top of the other.

1. Draw *one* line to decompose each polygon into two identical triangles, if possible. Use a straightedge to draw your line.



2. Which quadrilaterals can be decomposed into two identical triangles?

   Pause here for a small-group discussion.

3. Study the quadrilaterals that can, in fact, be decomposed into two identical triangles. What do you notice about them? Write a couple of observations about what these quadrilaterals have in common.

Are you ready for more?

On the grid, draw some other types of quadrilaterals that are not already shown. Try to decompose them into two identical triangles. Can you do it?

Come up with a rule about what must be true about a quadrilateral for it to be decomposed into two identical triangles.

7.3: A Tale of Two Triangles (Part 2)

Your teacher will give your group several pairs of triangles. Each group member should take 1 or 2 pairs.

1. a. Which pair(s) of triangles do you have?

   b. Can each pair be composed into a rectangle? A parallelogram?

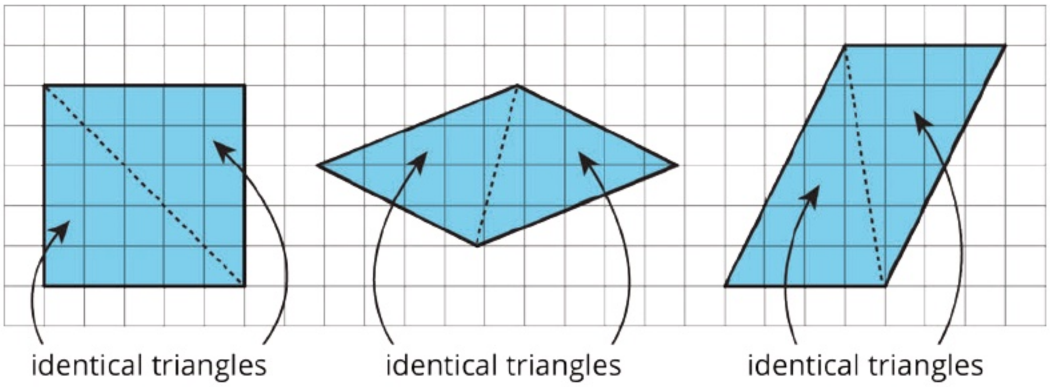
2. Discuss with your group your responses to the first question. Then, complete each statement with *All*, *Some*, or *None*. Sketch 1 or 2 examples to illustrate each completed statement.

   a. _______________ of these pairs of identical triangles can be composed into a *rectangle*.

   b. _______________ of these pairs of identical triangles can be composed into a *parallelogram*.

Lesson 7 Summary

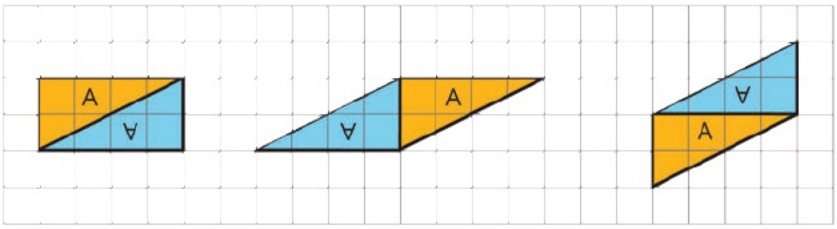
A parallelogram can always be decomposed into two identical triangles by a segment that connects opposite vertices.

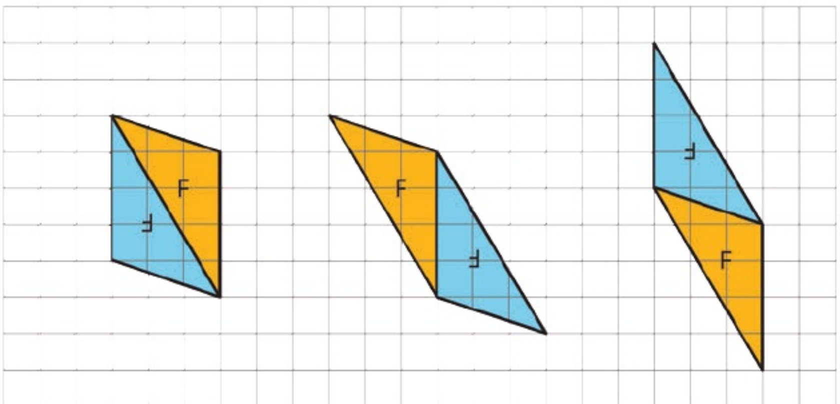


Going the other way around, two identical copies of a triangle can always be arranged to form a parallelogram, regardless of the type of triangle being used.

To produce a parallelogram, we can join a triangle and its copy along any of the three sides, so the same pair of triangles can make different parallelograms.

Here are examples of how two copies of both Triangle A and Triangle F can be composed into three different parallelograms.

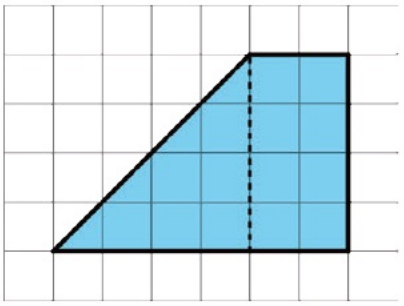




This special relationship between triangles and parallelograms can help us reason about the area of any triangle.

Unit 1 Lesson 7 Cumulative Practice Problems

1. To decompose a quadrilateral into two identical shapes, Clare drew a dashed line as shown in the diagram.



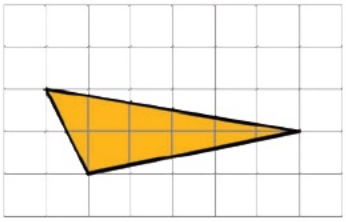
   a. She said the that two resulting shapes have the same area. Do you agree? Explain your reasoning.

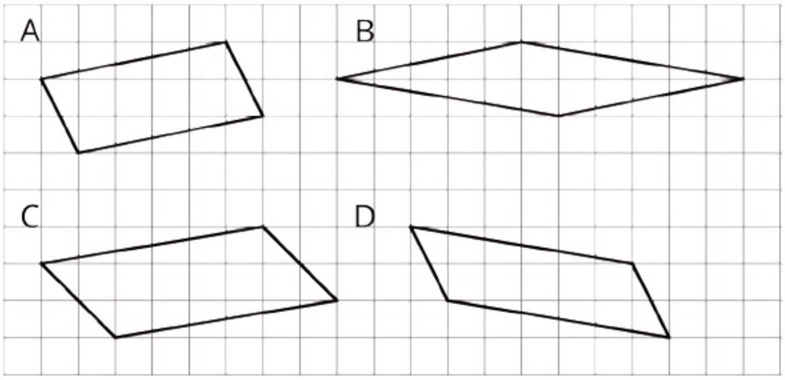
   b. Did Clare partition the figure into two identical shapes? Explain your reasoning.

2. Triangle R is a right triangle. Can we use two copies of Triangle R to compose a parallelogram that is not a square?

   If so, explain how or sketch a solution. If not, explain why not.

3. Two copies of this triangle are used to compose a parallelogram. Which parallelogram *cannot* be a result of the composition? If you get stuck, consider using tracing paper.





A. A

B. B

C. C

D. D

4. a. On the grid, draw at least three different quadrilaterals that can each be decomposed into two identical triangles with a single cut (show the cut line). One or more of the quadrilaterals should have non-right angles.

b. Identify the type of each quadrilateral.

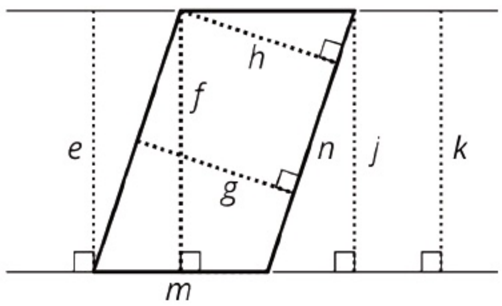
5. a. A parallelogram has a base of 9 units and a corresponding height of $\frac{2}{3}$ units. What is its area?

   b. A parallelogram has a base of 9 units and an area of 12 square units. What is the corresponding height for that base?

   c. A parallelogram has an area of 7 square units. If the height that corresponds to a base is $\frac{1}{4}$ unit, what is the base?

(From Unit 1, Lesson 6.)

6. Select **all** the segments that could represent the height if side n is the base.



A. e

B. f

C. g

D. h

E. m

F. n

G. j

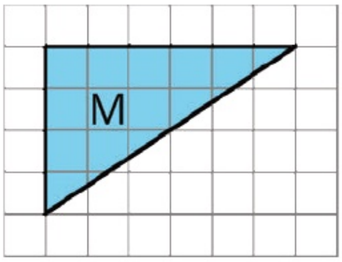
H. k

(From Unit 1, Lesson 5.)

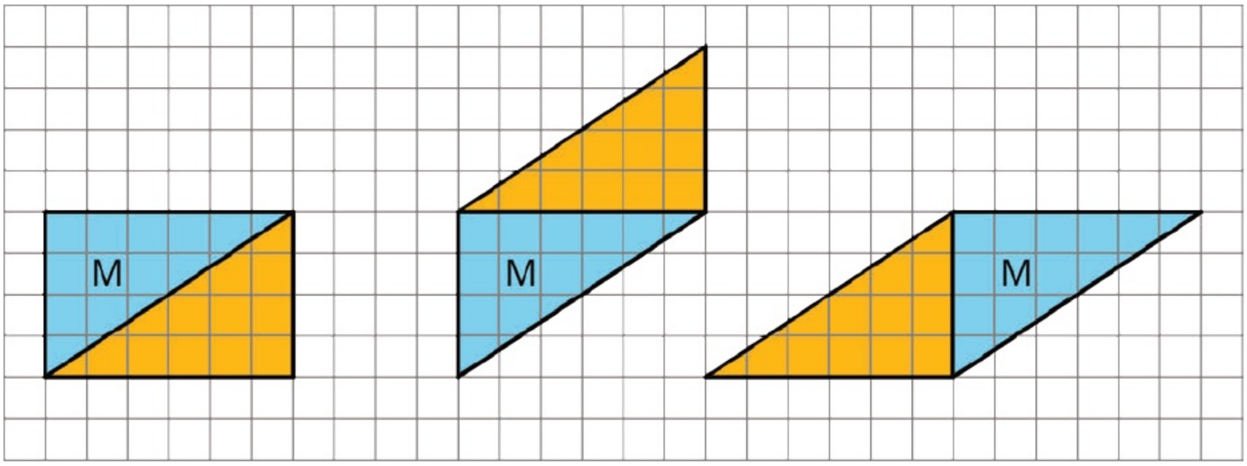
Lesson 8: Area of Triangles

8.1: Composing Parallelograms

Here is Triangle M.



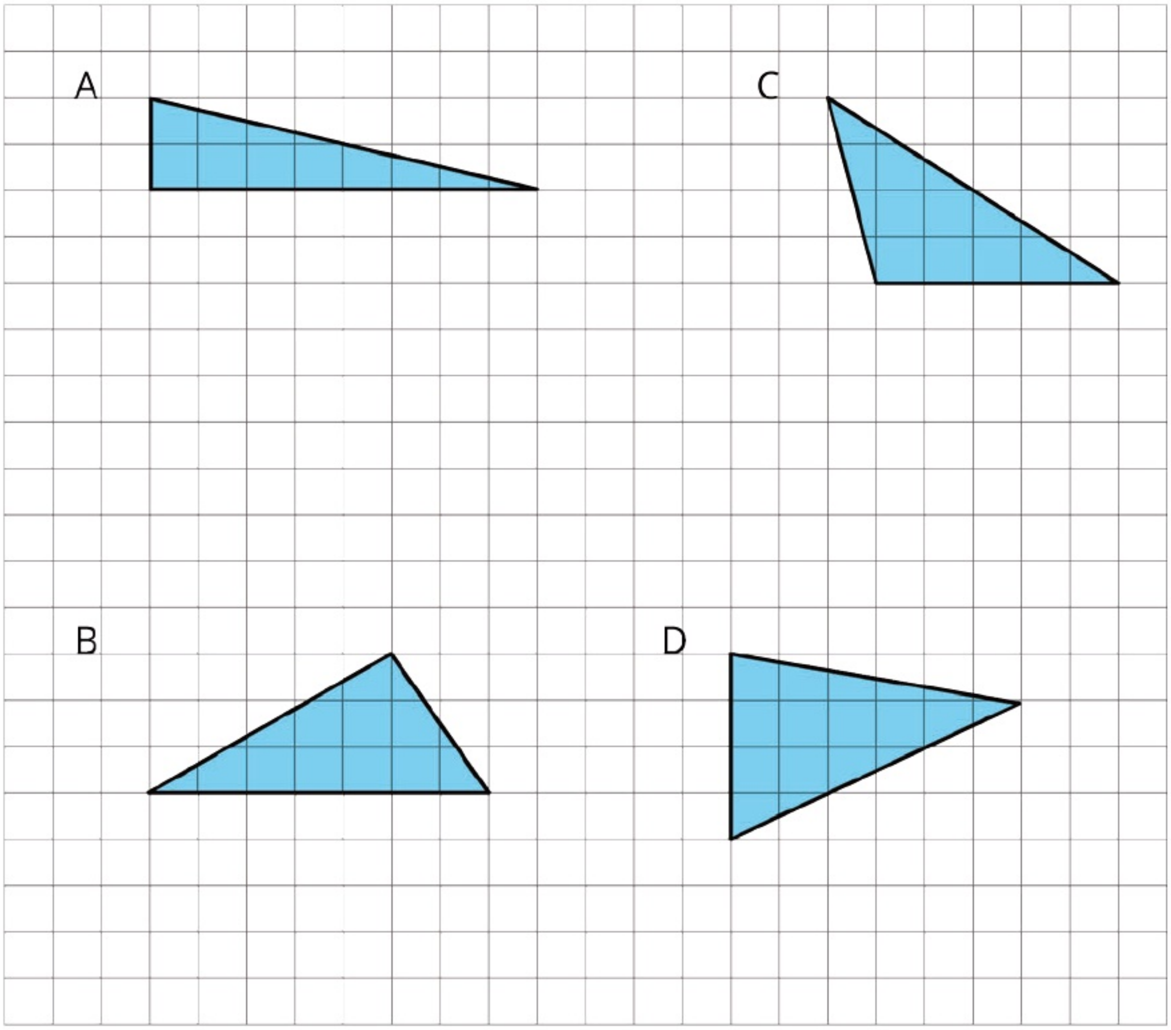
Han made a copy of Triangle M and composed three different parallelograms using the original M and the copy, as shown here.



1. For each parallelogram Han composed, identify a base and a corresponding height, and write the measurements on the drawing.
2. Find the area of each parallelogram Han composed. Show your reasoning.

8.2: More Triangles

Find the areas of at least two of these triangles. Show your reasoning.



8.3: Decomposing a Parallelogram

1. Your teacher will give you two copies of a parallelogram. Glue or tape *one* copy of your parallelogram here and find its area. Show your reasoning.

2. Decompose the second copy of your parallelogram by cutting along the dotted lines. Take *only* the small triangle and the trapezoid, and rearrange these two pieces into a different parallelogram. Glue or tape the newly composed parallelogram on your paper.

3. Find the area of the new parallelogram you composed. Show your reasoning.

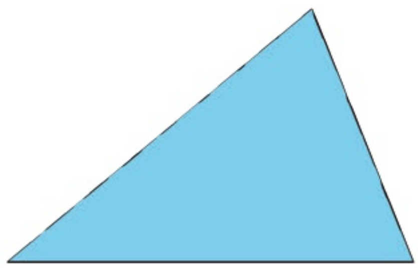
4. What do you notice about the relationship between the area of this new parallelogram and the original one?

5. How do you think the area of the large triangle compares to that of the new parallelogram: Is it larger, the same, or smaller? Why is that?

6. Glue or tape the remaining large triangle to your paper. Use any part of your work to help you find its area. Show your reasoning.

Are you ready for more?

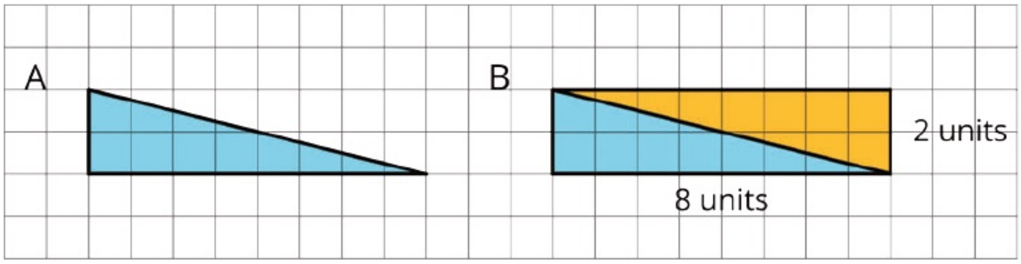
Can you decompose this triangle and rearrange its parts to form a rectangle? Describe how it could be done.

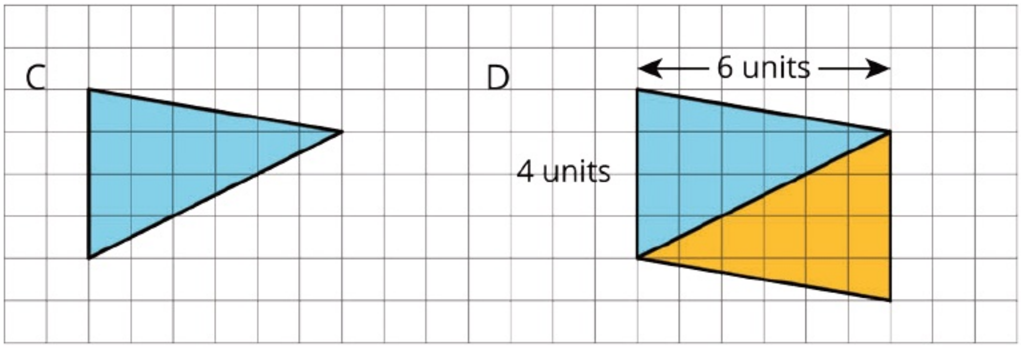


Lesson 8 Summary

We can reason about the area of a triangle by using what we know about parallelograms. Here are three general ways to do this:

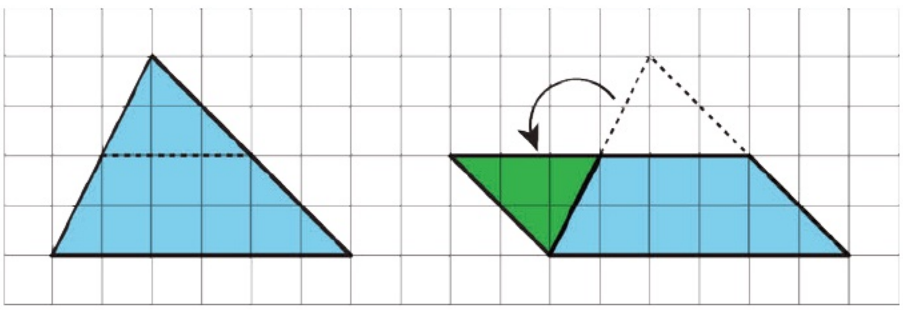
- Make a copy of the triangle and join the original and the copy along an edge to create a parallelogram. Because the two triangles have the same area, one copy of the triangle has half the area of that parallelogram.





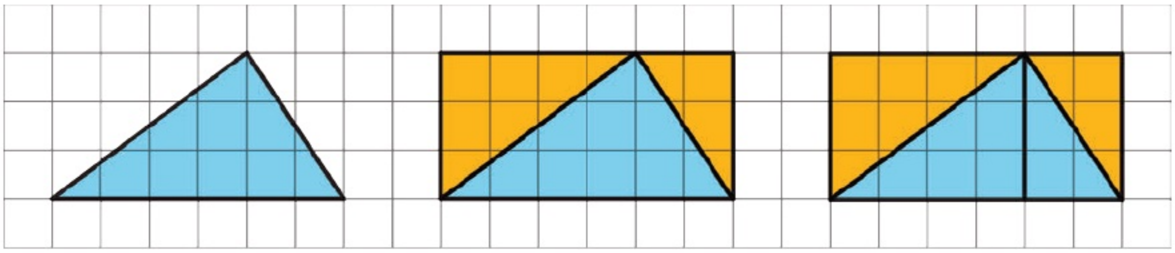
The area of Parallelogram B is 16 square units because the base is 8 units and the height 2 units. The area of Triangle A is half of that, which is 8 square units. The area of Parallelogram D is 24 square units because the base is 4 units and the height 6 units. The area of Triangle C is half of that, which is 12 square units.

- Decompose the triangle into smaller pieces and compose them into a parallelogram.



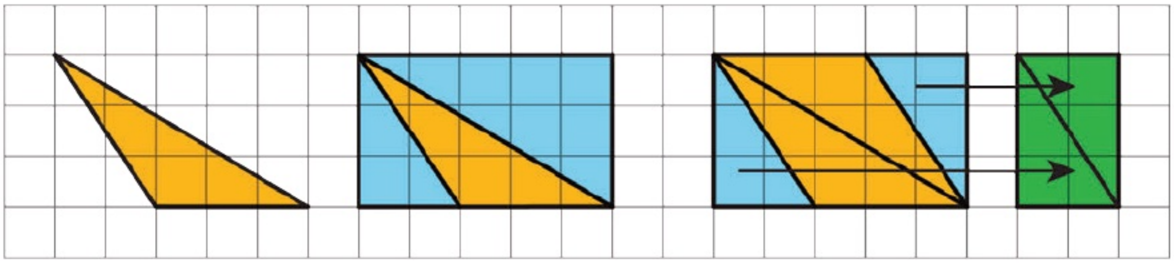
In the new parallelogram, $b = 6$, $h = 2$, and $6 \cdot 2 = 12$, so its area is 12 square units. Because the original triangle and the parallelogram are composed of the same parts, the area of the original triangle is also 12 square units.

- Draw a rectangle around the triangle. Sometimes the triangle has half of the area of the rectangle.



The large rectangle can be decomposed into smaller rectangles. The one on the left has area $4 \cdot 3$ or 12 square units; the one on the right has area $2 \cdot 3$ or 6 square units. The large triangle is also decomposed into two right triangles. Each of the right triangles is half of a smaller rectangle, so their areas are 6 square units and 3 square units. The large triangle has area 9 square units.

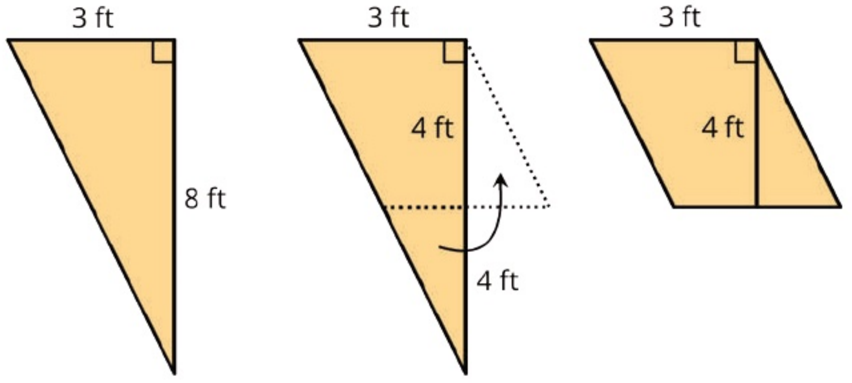
Sometimes, the triangle is half of what is left of the rectangle after removing two copies of the smaller right triangles.



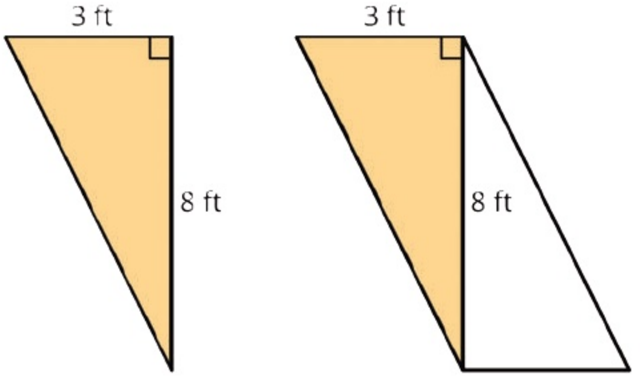
The right triangles being removed can be composed into a small rectangle with area $(2 \cdot 3)$ square units. What is left is a parallelogram with area $5 \cdot 3 - 2 \cdot 3$, which equals $15 - 6$ or 9 square units. Notice that we can compose the same parallelogram with two copies of the original triangle! The original triangle is half of the parallelogram, so its area is $\frac{1}{2} \cdot 9$ or 4.5 square units.

Unit 1 Lesson 8 Cumulative Practice Problems

1. To find the area of this right triangle, Diego and Jada used different strategies. Diego drew a line through the midpoints of the two longer sides, which decomposes the triangle into a trapezoid and a smaller triangle. He then rearranged the two shapes into a parallelogram.



Jada made a copy of the triangle, rotated it, and lined it up against one side of the original triangle so that the two triangles make a parallelogram.

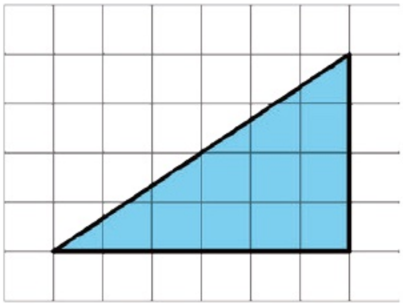


a. Explain how Diego might use his parallelogram to find the area of the triangle.

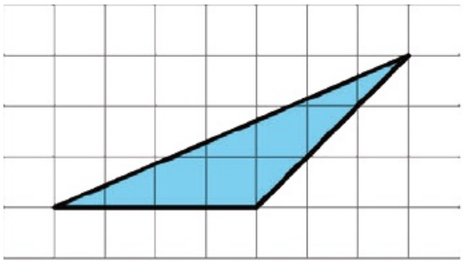
b. Explain how Jada might use her parallelogram to find the area of the triangle.

2. Find the area of the triangle. Explain or show your reasoning.

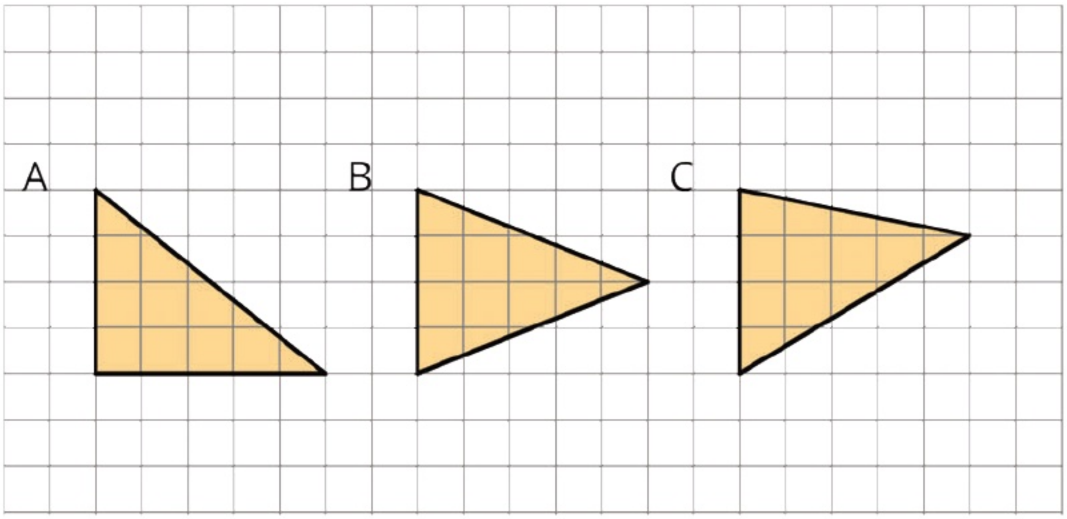
a.



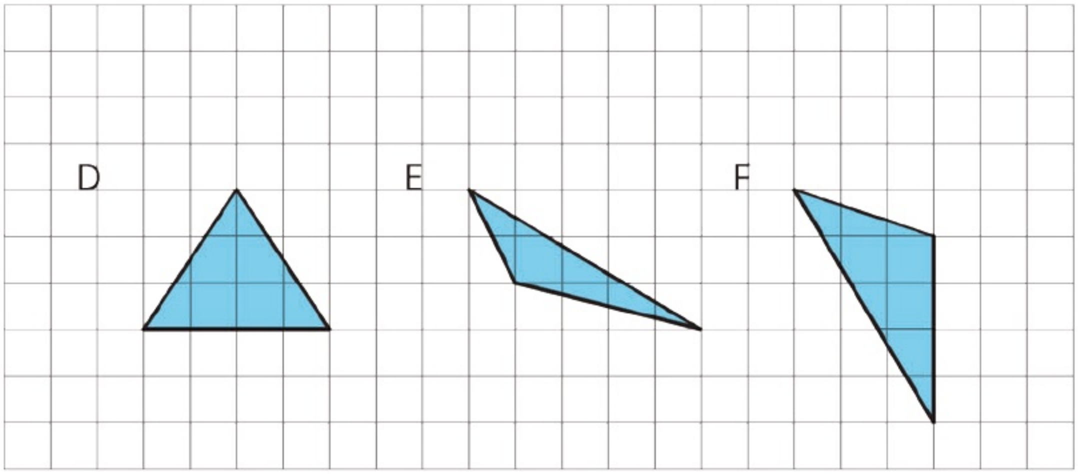
b.



3. Which of the three triangles has the greatest area? Show your reasoning. If you get stuck, try using what you know about the area of parallelograms.



4. Draw an identical copy of each triangle such that the two copies together form a parallelogram. If you get stuck, consider using tracing paper.



(From Unit 1, Lesson 7.)

5. a. A parallelogram has a base of 3.5 units and a corresponding height of 2 units. What is its area?

   b. A parallelogram has a base of 3 units and an area of 1.8 square units. What is the corresponding height for that base?

   c. A parallelogram has an area of 20.4 square units. If the height that corresponds to a base is 4 units, what is the base?

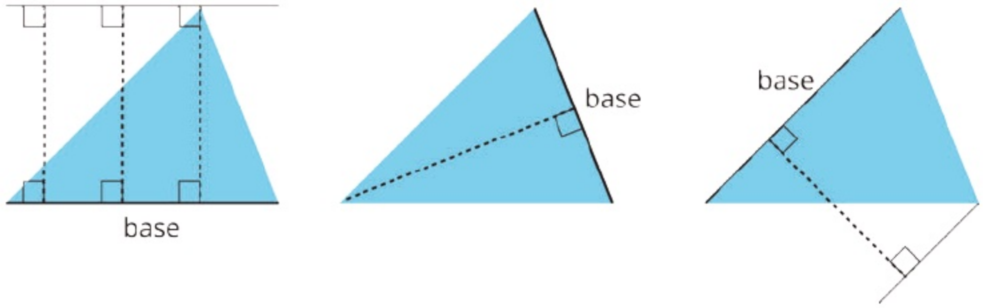
(From Unit 1, Lesson 6.)

Lesson 9: Formula for the Area of a Triangle

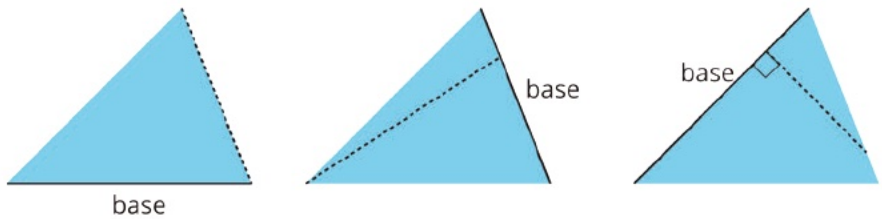
9.1: Bases and Heights of a Triangle

Study the examples and non-examples of **bases** and **heights** in a triangle.

- Examples: These dashed segments represent heights of the triangle.



- Non-examples: These dashed segments do *not* represent heights of the triangle.



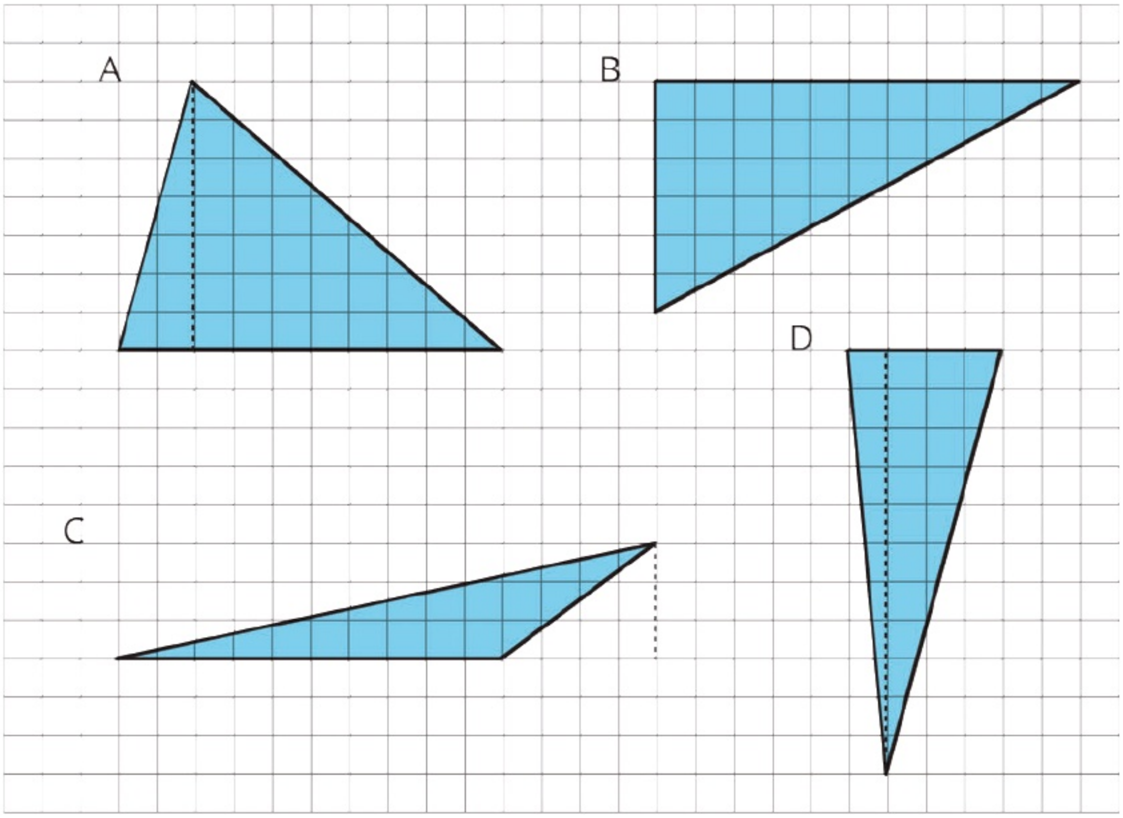
Select **all** the statements that are true about bases and heights in a triangle.

1. Any side of a triangle can be a base.
2. There is only one possible height.
3. A height is always one of the sides of a triangle.
4. A height that corresponds to a base must be drawn at an acute angle to the base.
5. A height that corresponds to a base must be drawn at a right angle to the base.
6. Once we choose a base, there is only one segment that represents the corresponding height.
7. A segment representing a height must go through a vertex.

9.2: Finding a Formula for Area of a Triangle

For each triangle:

- Identify a base and a corresponding height, and record their lengths in the table.
- Find the area of the triangle and record it in the last column of the table.



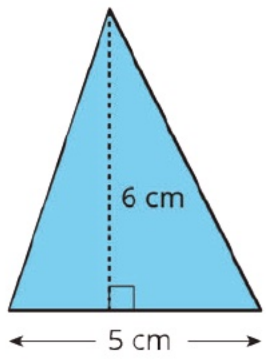
triangle	base (units)	height (units)	area (square units)
A			
B			
C			
D			
any triangle	b	h	

In the last row, write an expression for the area of any triangle, using b and h.

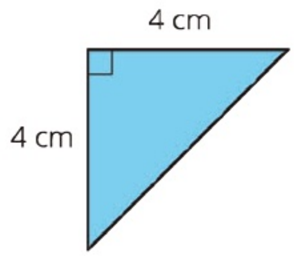
9.3: Applying the Formula for Area of Triangles

For each triangle, circle a base measurement that you can use to find the area of the triangle. Then, find the area of any *three* triangles. Show your reasoning.

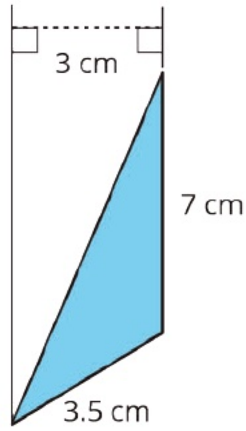
A



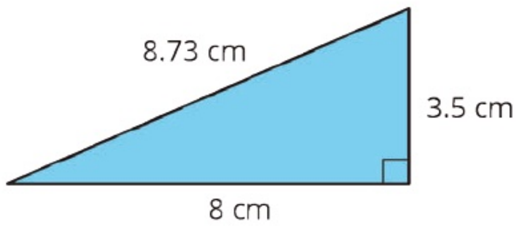
B



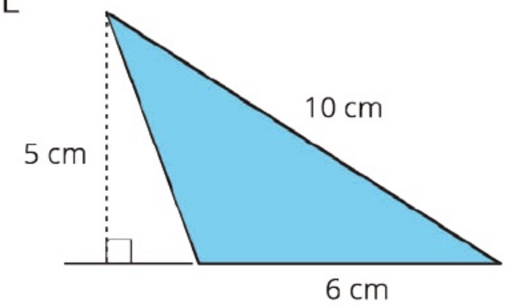
C



D



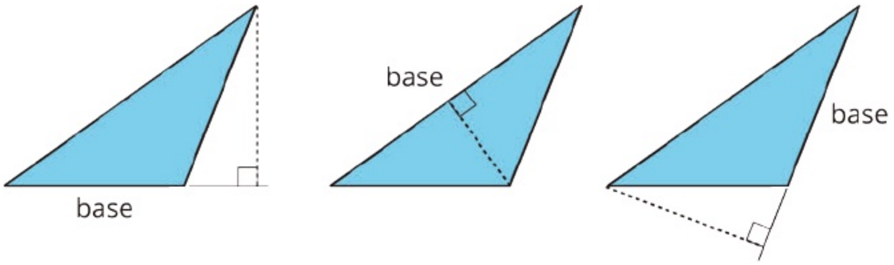
E



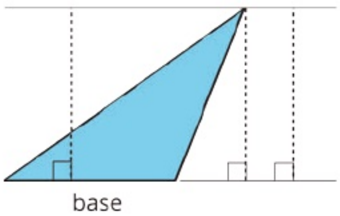
Lesson 9 Summary

- We can choose any of the three sides of a triangle to call the **base**. The term “base” refers to both the side and its length (the measurement).
- The corresponding **height** is the length of a perpendicular segment from the base to the vertex opposite of it. The **opposite vertex** is the vertex that is *not* an endpoint of the base.

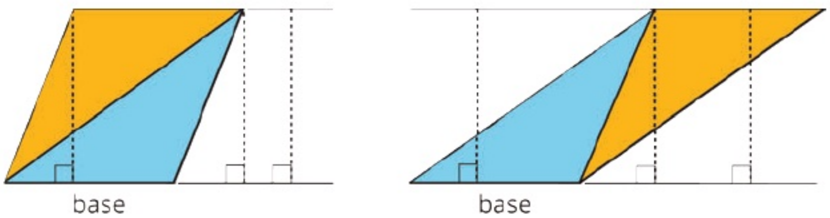
Here are three pairs of bases and heights for the same triangle. The dashed segments in the diagrams represent heights.



A segment showing a height must be drawn at a right angle to the base, but it can be drawn in more than one place. It does not have to go through the opposite vertex, as long as it connects the base and a line that is parallel to the base and goes through the opposite vertex, as shown here.



The base-height pairs in a triangle are closely related to those in a parallelogram. Recall that two copies of a triangle can be composed into one or more parallelograms. Each parallelogram shares at least one base with the triangle.

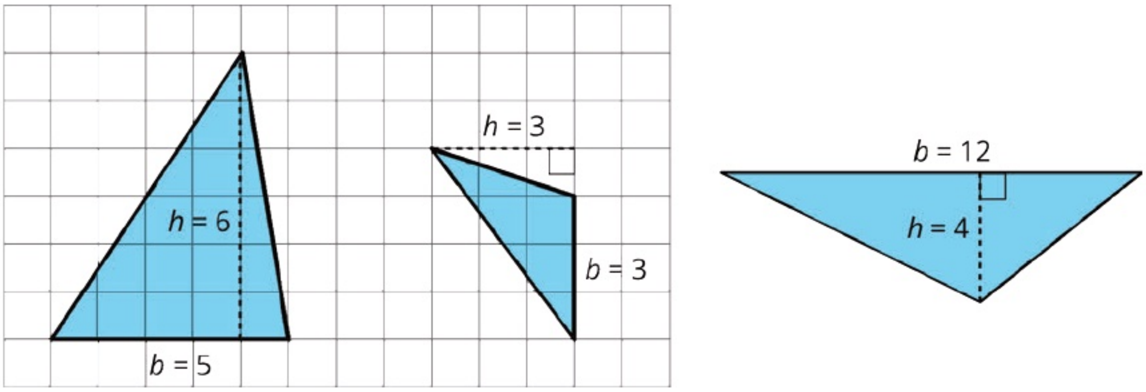


For any base that they share, the corresponding height is also shared, as shown by the dashed segments.

We can use the base-height measurements and our knowledge of parallelograms to find the area of any triangle.

- The formula for the area of a parallelogram with base b and height h is $b \cdot h$.
- A triangle takes up half of the area of a parallelogram with the same base and height. We can therefore express the area A of a triangle as:

$$A = \frac{1}{2} \cdot b \cdot h$$

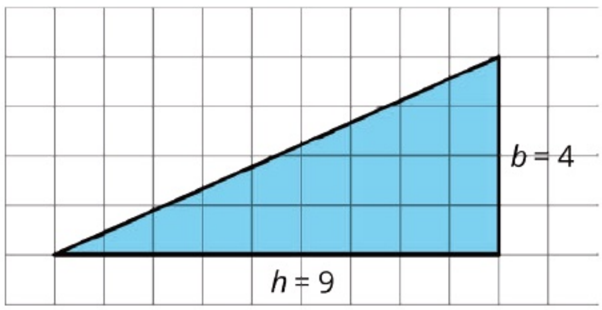


- The area of Triangle A is 15 square units because $\frac{1}{2} \cdot 5 \cdot 6 = 15$.
- The area of Triangle B is 4.5 square units because $\frac{1}{2} \cdot 3 \cdot 3 = 4.5$.
- The area of Triangle C is 24 square units because $\frac{1}{2} \cdot 12 \cdot 4 = 24$.

In each case, one side of the triangle is the base but neither of the other sides is the height. This is because the angle between them is not a right angle.

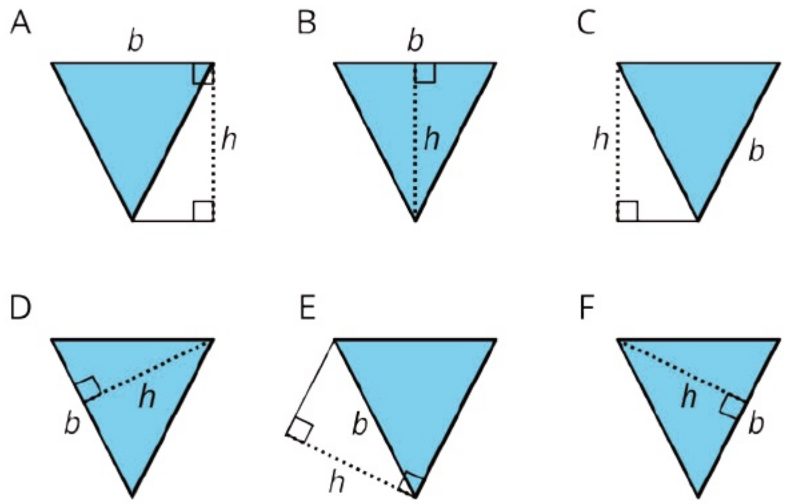
In right triangles, however, the two sides that are perpendicular can be a base and a height.

The area of this triangle is 18 square units whether we use 4 units or 9 units for the base.



Unit 1 Lesson 9 Cumulative Practice Problems

1. Select **all** drawings in which a corresponding height h for a given base b is correctly identified.



A. A

B. B

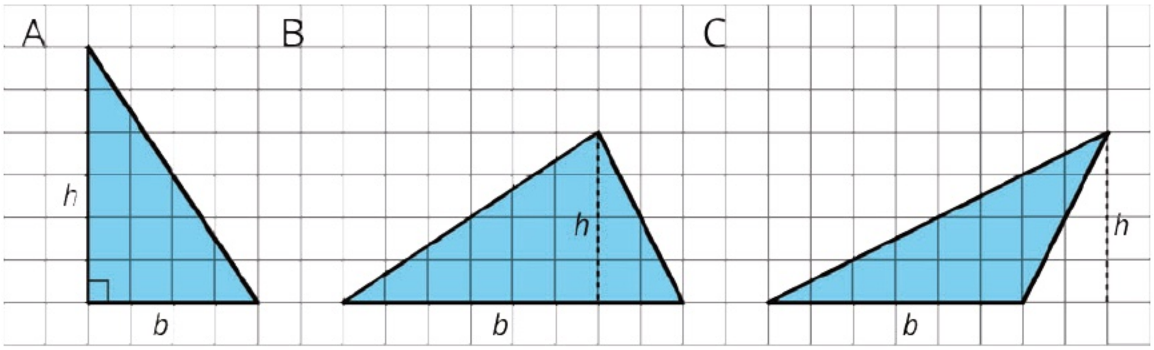
C. C

D. D

E. E

F. F

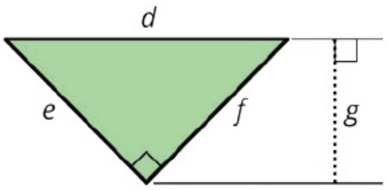
2. For each triangle, a base and its corresponding height are labeled.



a. Find the area of each triangle.

b. How is the area related to the base and its corresponding height?

3. Here is a right triangle. Name a corresponding height for each base.

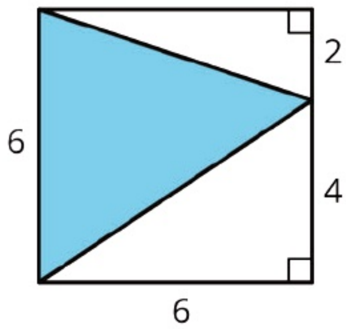


a. Side d

b. Side e

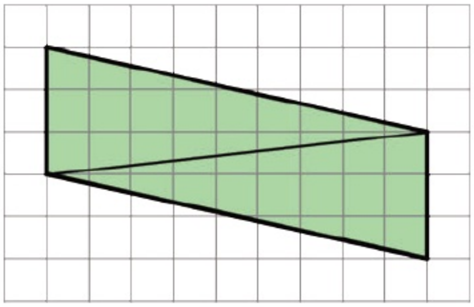
c. Side f

4. Find the area of the shaded triangle. Show your reasoning.



(From Unit 1, Lesson 8.)

5. Andre drew a line connecting two opposite corners of a parallelogram. Select **all** true statements about the triangles created by the line Andre drew.



A. Each triangle has two sides that are 3 units long.

B. Each triangle has a side that is the same length as the diagonal line.

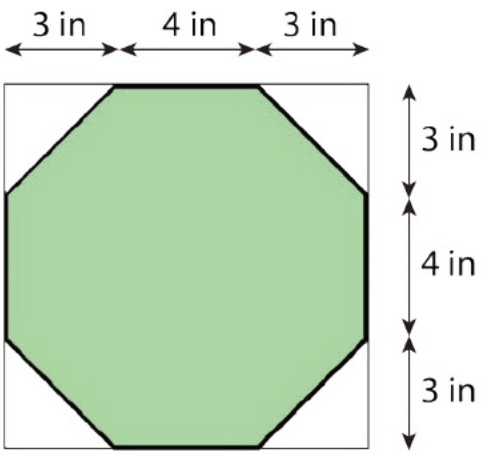
C. Each triangle has one side that is 3 units long.

D. When one triangle is placed on top of the other and their sides are aligned, we will see that one triangle is larger than the other.

E. The two triangles have the same area as each other.

(From Unit 1, Lesson 7.)

6. Here is an octagon. (Note: The diagonal sides of the octagon are *not* 4 inches long.)



a. While estimating the area of the octagon, Lin reasoned that it must be less than 100 square inches. Do you agree? Explain your reasoning.

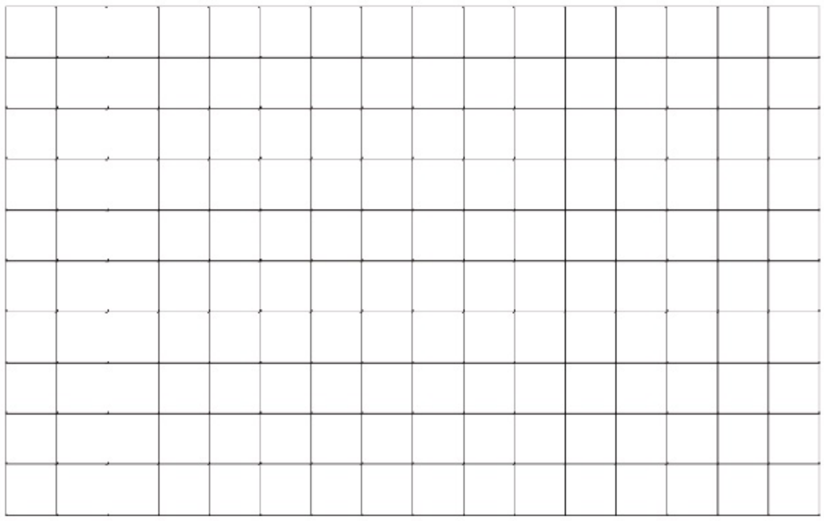
b. Find the exact area of the octagon. Show your reasoning.

(From Unit 1, Lesson 3.)

Lesson 10: Bases and Heights of Triangles

10.1: An Area of 12

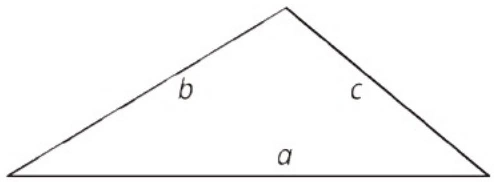
On the grid, draw a triangle with an area of 12 square units. Try to draw a non-right triangle. Be prepared to explain how you know the area of your triangle is 12 square units.



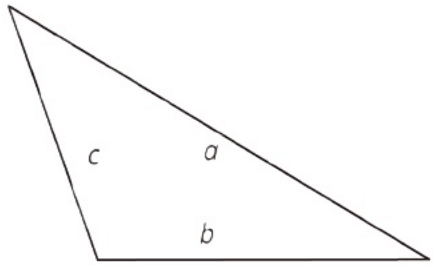
10.2: Hunting for Heights

1. Here are three copies of the same triangle. The triangle is rotated so that the side chosen as the base is at the bottom and is horizontal. Draw a height that corresponds to each base. Use an index card to help you.

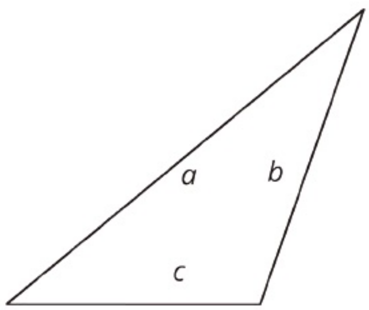
Side a as the base:



Side b as the base:



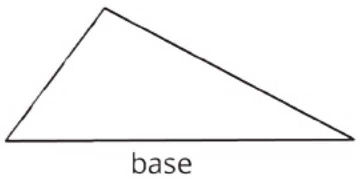
Side c as the base:



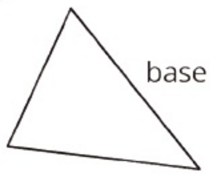
Pause for your teacher's instructions before moving to the next question.

2. Draw a line segment to show the height for the chosen base in each triangle.

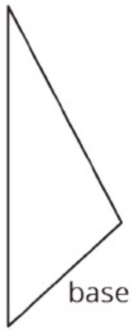
A



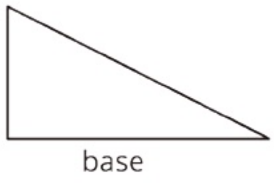
B



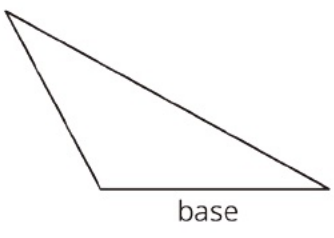
C



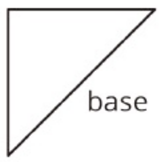
D



E



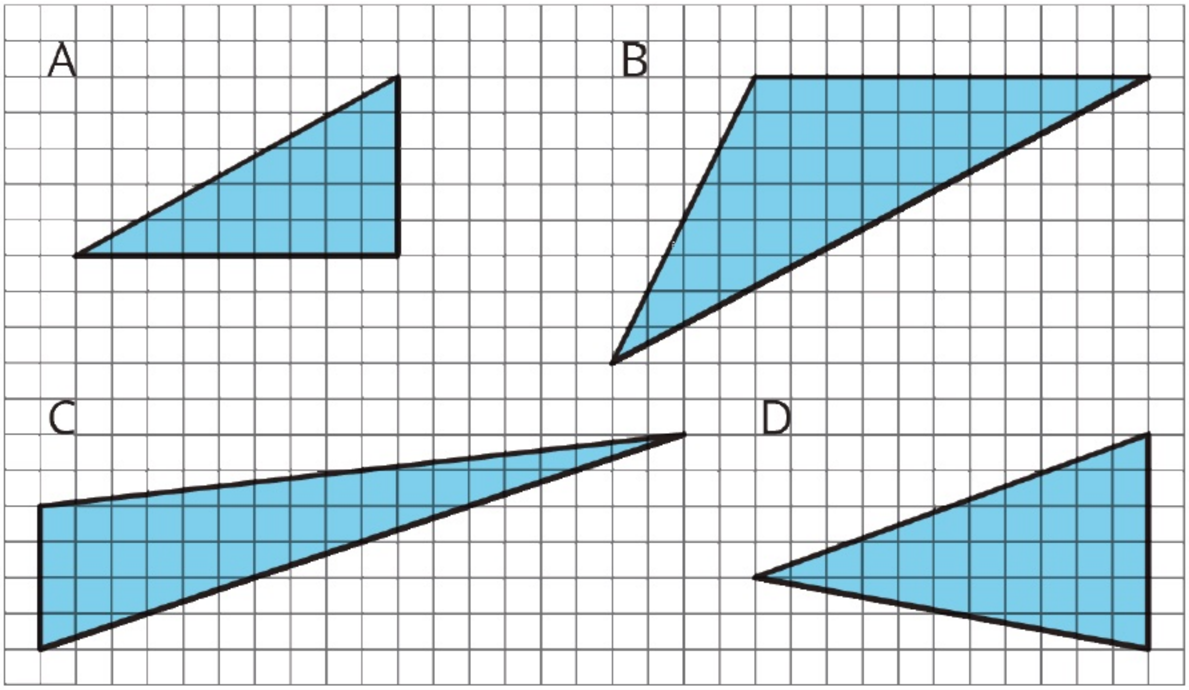
F



10.3: Some Bases Are Better Than Others

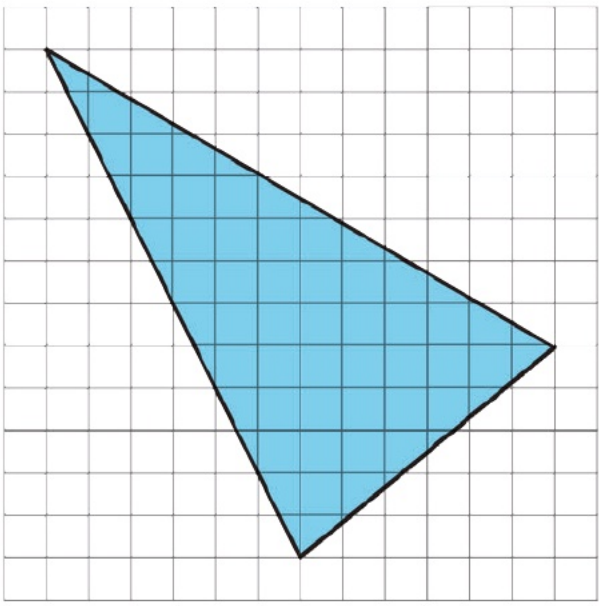
For each triangle, identify and label a base and height. If needed, draw a line segment to show the height.

Then, find the area of the triangle. Show your reasoning. (The side length of each square on the grid is 1 unit.)



Are you ready for more?

Find the area of this triangle. Show your reasoning.

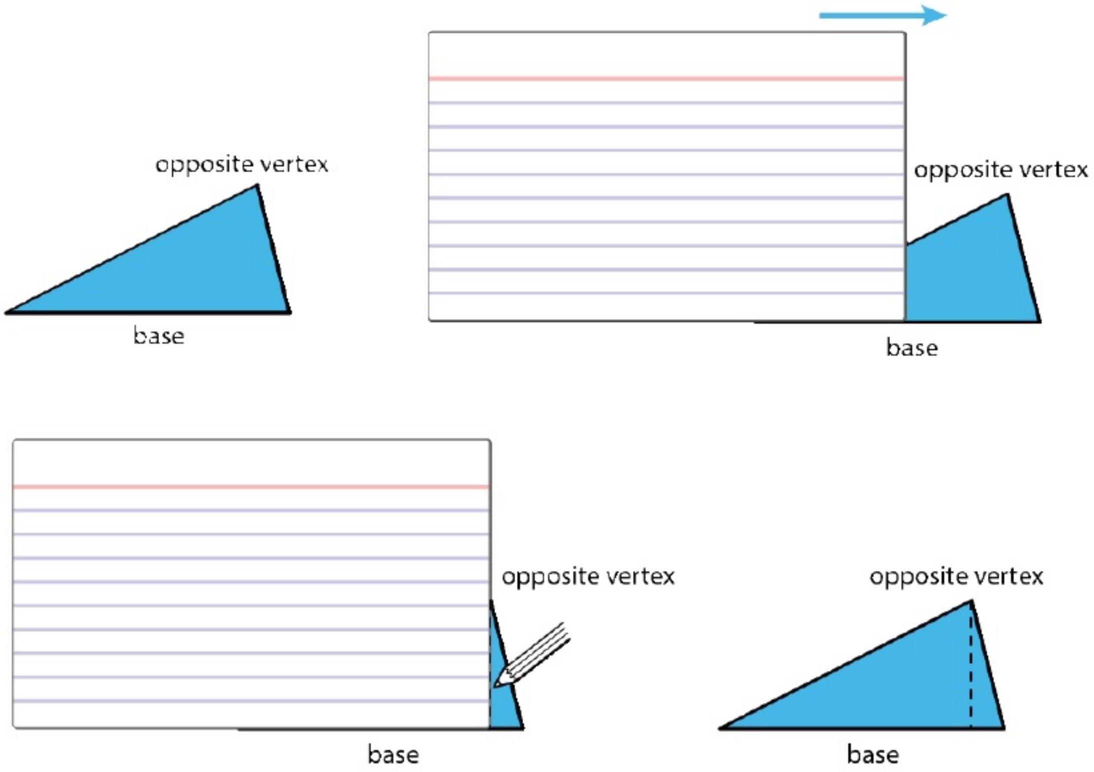


Lesson 10 Summary

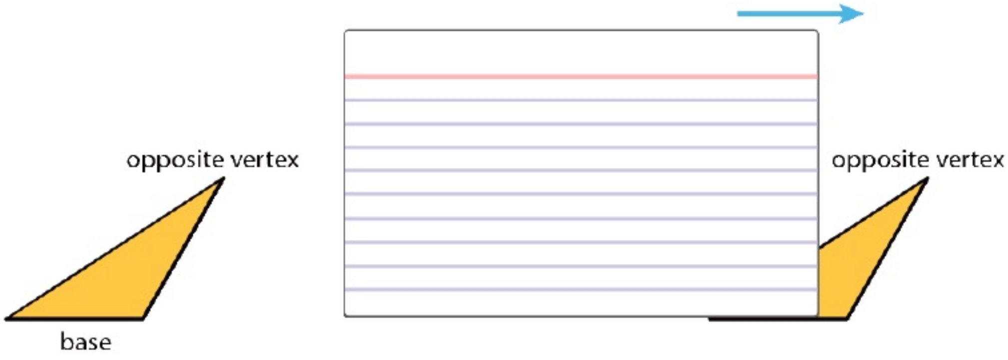
A height of a triangle is a perpendicular segment between the side chosen as the base and the opposite vertex. We can use tools with right angles to help us draw height segments.

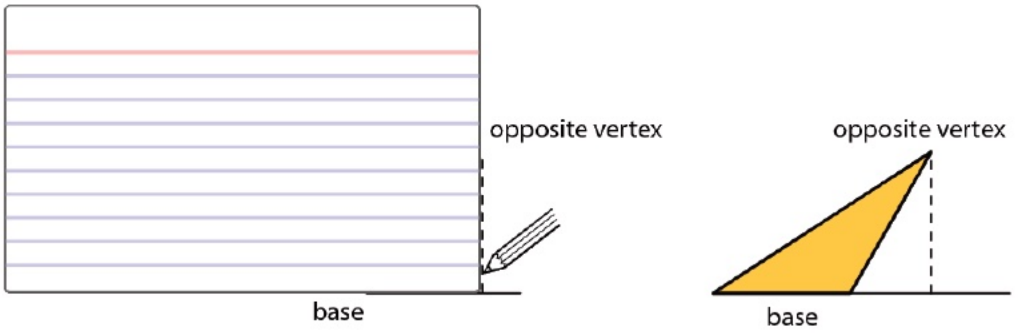
An index card (or any stiff paper with a right angle) is a handy tool for drawing a line that is perpendicular to another line.

1. Choose a side of a triangle as the base. Identify its opposite vertex.
2. Line up one edge of the index card with that base.
3. Slide the card along the base until a perpendicular edge of the card meets the opposite vertex.
4. Use the card edge to draw a line from the vertex to the base. That segment represents the height.



Sometimes we may need to extend the line of the base to identify the height, such as when finding the height of an obtuse triangle, or whenever the opposite vertex is not directly over the base. In these cases, the height segment is typically drawn *outside* of the triangle.

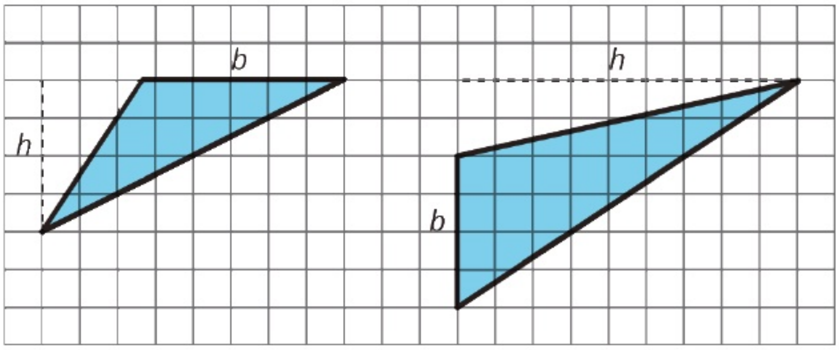




Even though any side of a triangle can be a base, some base-height pairs can be more easily determined than others, so it helps to choose strategically.

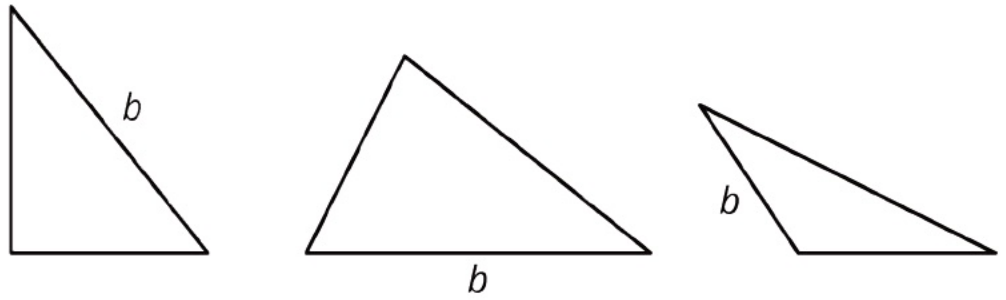
For example, when dealing with a right triangle, it often makes sense to use the two sides that make the right angle as the base and the height because one side is already perpendicular to the other.

If a triangle is on a grid and has a horizontal or a vertical side, you can use that side as a base and use the grid to find the height, as in these examples:

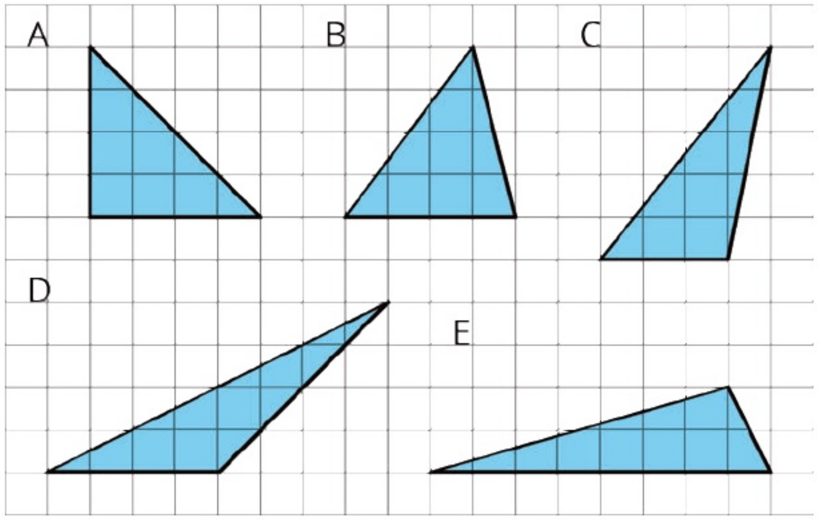


Unit 1 Lesson 10 Cumulative Practice Problems

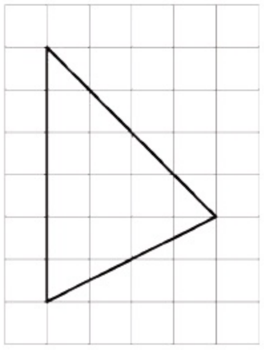
1. For each triangle, a base is labeled b. Draw a line segment that shows its corresponding height. Use an index card to help you draw a straight line.



2. Select all triangles that have an area of 8 square units. Explain how you know.

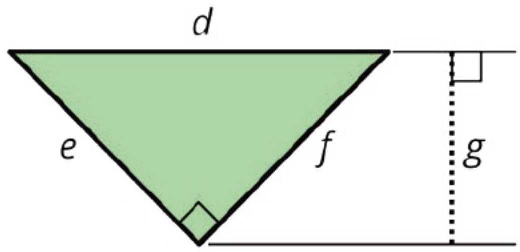


3. Find the area of the triangle. Show your reasoning.

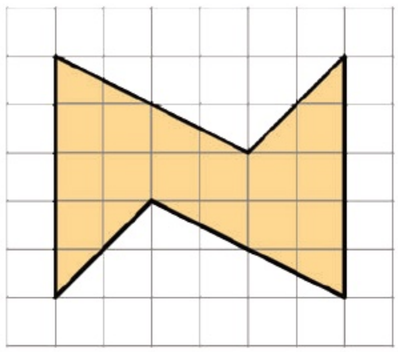


If you get stuck, carefully consider which side of the triangle to use as the base.

4. Can side d be the base for this triangle? If so, which length would be the corresponding height? If not, explain why not.



5. Find the area of this shape. Show your reasoning.



(From Unit 1, Lesson 3.)

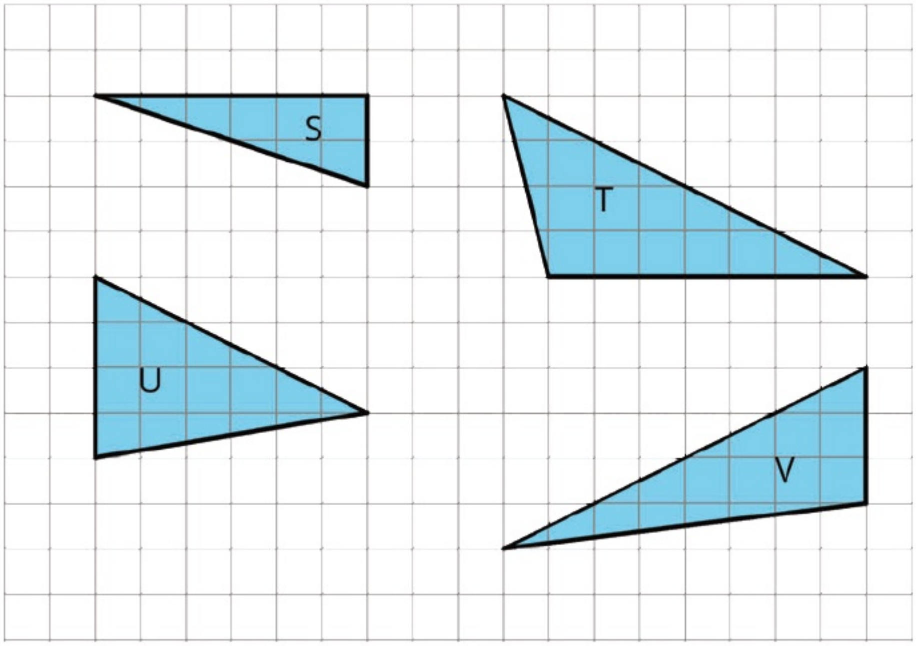
6. On the grid, sketch two different parallelograms that have equal area. Label a base and height of each and explain how you know the areas are the same.

(From Unit 1, Lesson 6.)

Lesson 11: Polygons

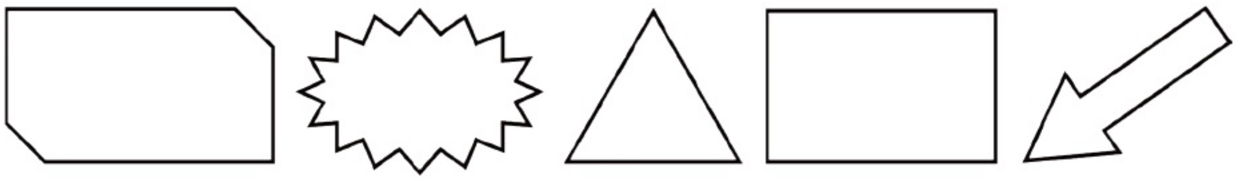
11.1: Which One Doesn't Belong: Bases and Heights

Which one doesn't belong?

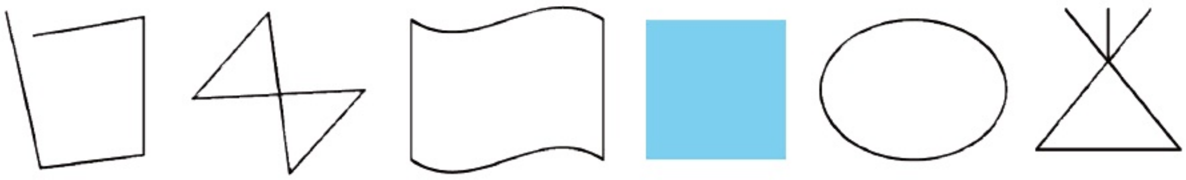


11.2: What Are Polygons?

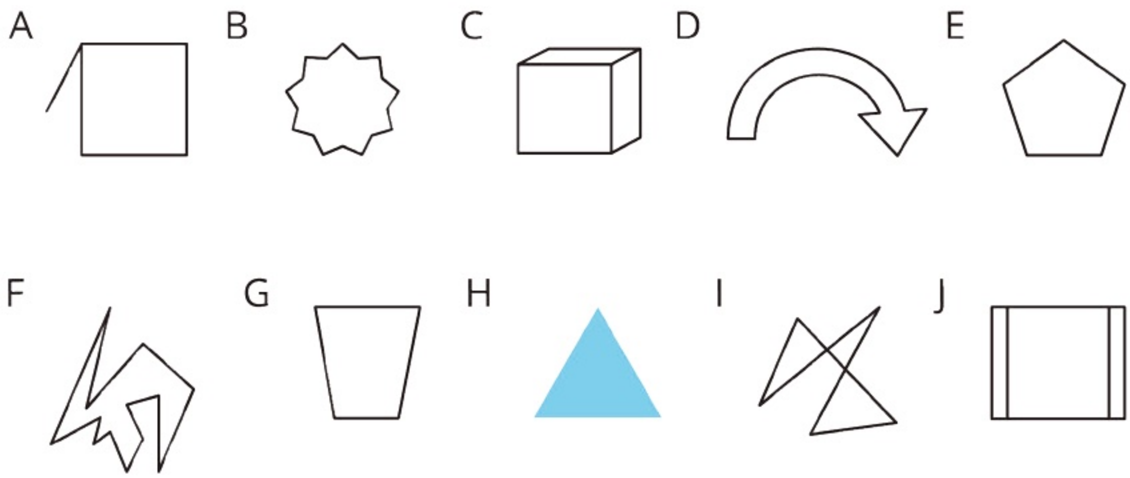
Here are five **polygons**:



Here are six figures that are *not* polygons:



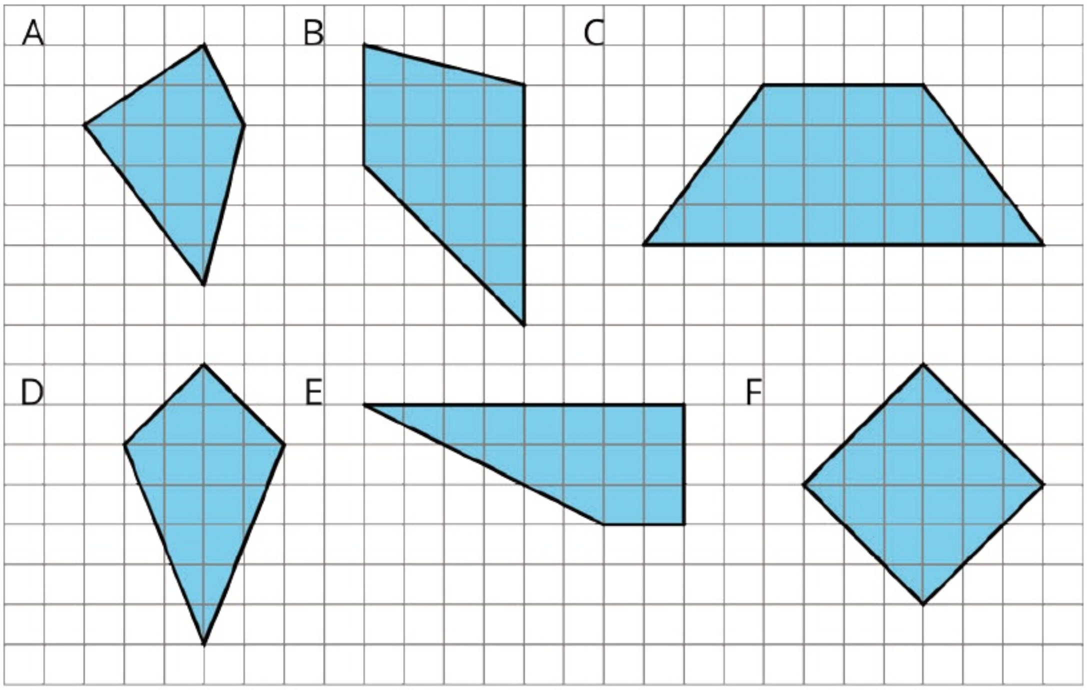
1. Circle the figures that are polygons.



2. What do the figures you circled have in common? What characteristics helped you decide whether a figure was a polygon?

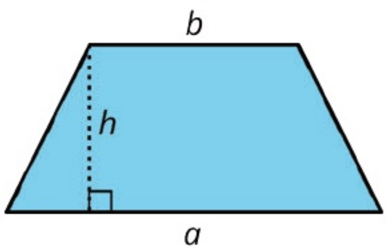
11.3: Quadrilateral Strategies

Find the area of *two* **quadrilaterals** of your choice. Show your reasoning.



Are you ready for more?

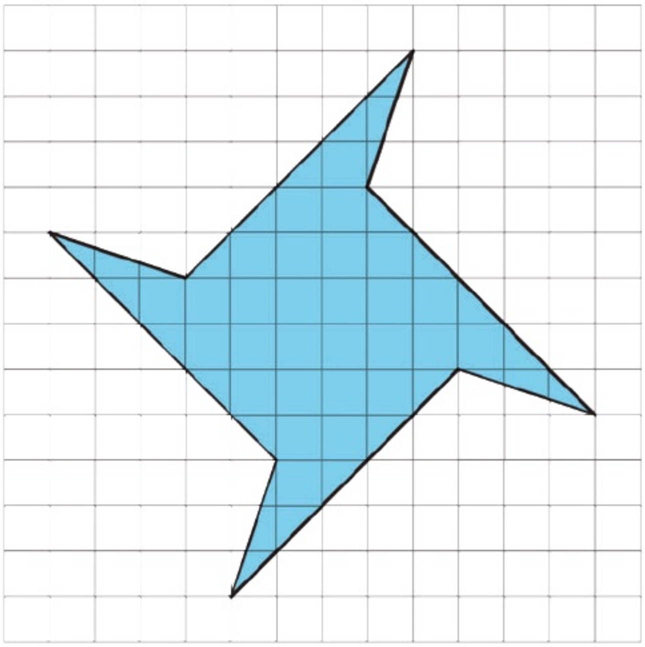
Here is a trapezoid. a and b represent the lengths of its bottom and top sides. The segment labeled h represents its height; it is perpendicular to both the top and bottom sides.



Apply area-reasoning strategies—decomposing, rearranging, duplicating, etc.—on the trapezoid so that you have one or more shapes with areas that you already know how to find. Use the shapes to help you write a formula for the area of a trapezoid. Show your reasoning.

11.4: Pinwheel

Find the area of the shaded region in square units. Show your reasoning.



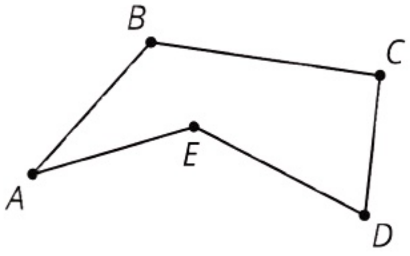
Lesson 11 Summary

A **polygon** is a two-dimensional figure composed of straight line segments.

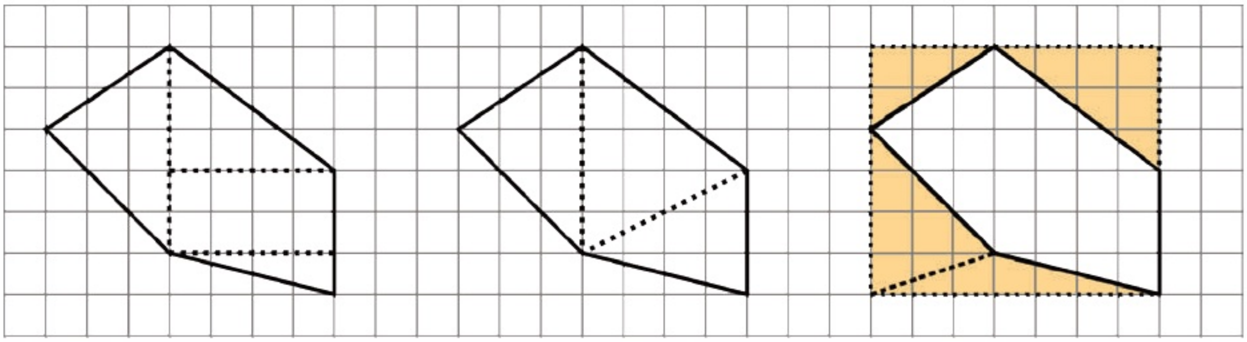
- Each end of a line segment connects to one other line segment. The point where two segments connect is a **vertex**. The plural of vertex is vertices.
- The segments are called the **edges** or **sides** of the polygon. The sides never cross each other. There are always an equal number of vertices and sides.

Here is a polygon with 5 sides. The vertices are labeled A, B, C, D, and E.

A polygon encloses a region. To find the area of a polygon is to find the area of the region inside it.



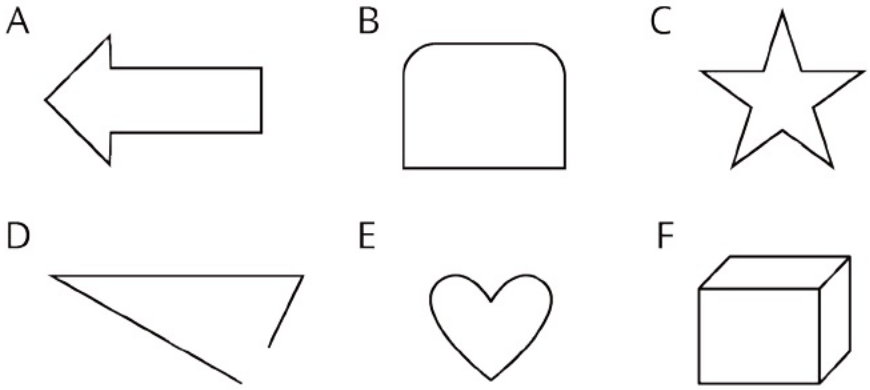
We can find the area of a polygon by decomposing the region inside it into triangles and rectangles.



The first two diagrams show the polygon decomposed into triangles and rectangles; the sum of their areas is the area of the polygon. The last diagram shows the polygon enclosed with a rectangle; subtracting the areas of the triangles from the area of the rectangle gives us the area of the polygon.

Unit 1 Lesson 11 Cumulative Practice Problems

1. Select **all** the polygons.



   A. A

   B. B

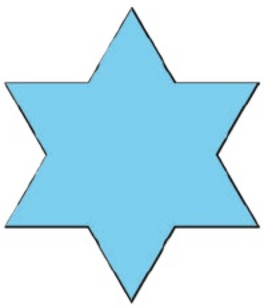
   C. C

   D. D

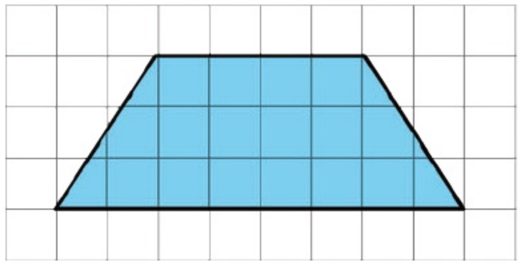
   E. E

   F. F

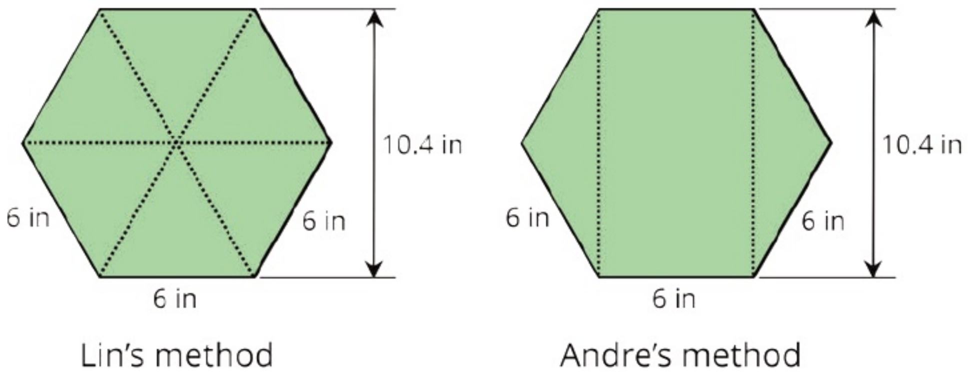
2. Mark each vertex with a large dot. How many edges and vertices does this polygon have?



3. Find the area of this trapezoid. Explain or show your strategy.

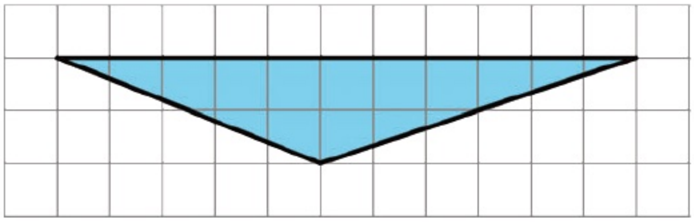


4. Lin and Andre used different methods to find the area of a regular hexagon with 6-inch sides. Lin decomposed the hexagon into six identical, equilateral triangles. Andre decomposed the hexagon into a rectangle and two triangles.



Find the area of the hexagon using each person's method. Show your reasoning.

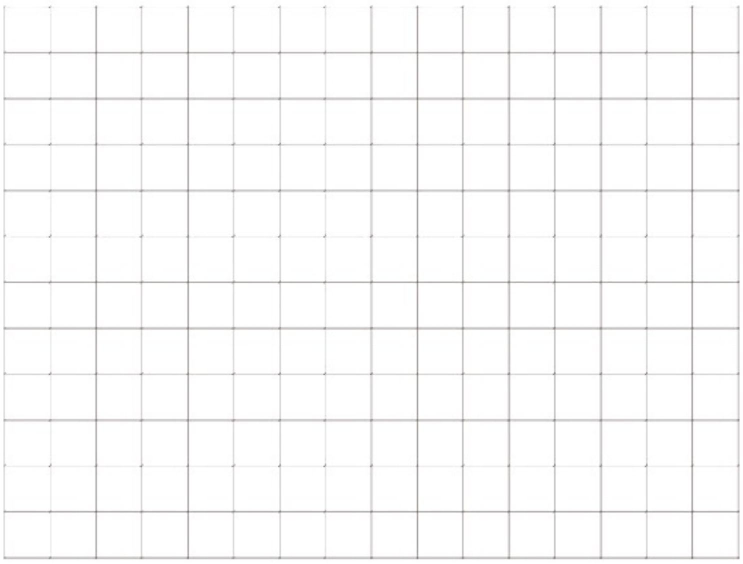
5. a. Identify a base and a corresponding height that can be used to find the area of this triangle. Label the base b and the corresponding height h.



b. Find the area of the triangle. Show your reasoning.

(From Unit 1, Lesson 9.)

6. On the grid, draw three different triangles with an area of 8 square units. Label the base and height of each triangle.



(From Unit 1, Lesson 10.)

Lesson 12: What is Surface Area?

12.1: Covering the Cabinet (Part 1)

Your teacher will show you a video about a cabinet or some pictures of it.

Estimate an answer to the question: How many sticky notes would it take to cover the cabinet, excluding the bottom?

12.2: Covering the Cabinet (Part 2)

Earlier, you learned about a cabinet being covered with sticky notes.

1. How could you find the actual number of sticky notes it will take to cover the cabinet, excluding the bottom? What information would you need to know?

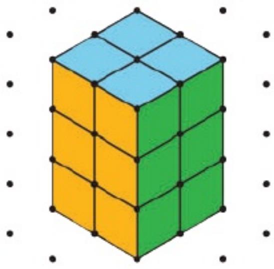
2. Use the information you have to find the number of sticky notes to cover the cabinet. Show your reasoning.

Are you ready for more?

How many sticky notes are needed to cover the outside of 2 cabinets pushed together (including the bottom)? What about 3 cabinets? 20 cabinets?

12.3: Building with Snap Cubes

Here is a sketch of a rectangular prism built from 12 cubes:



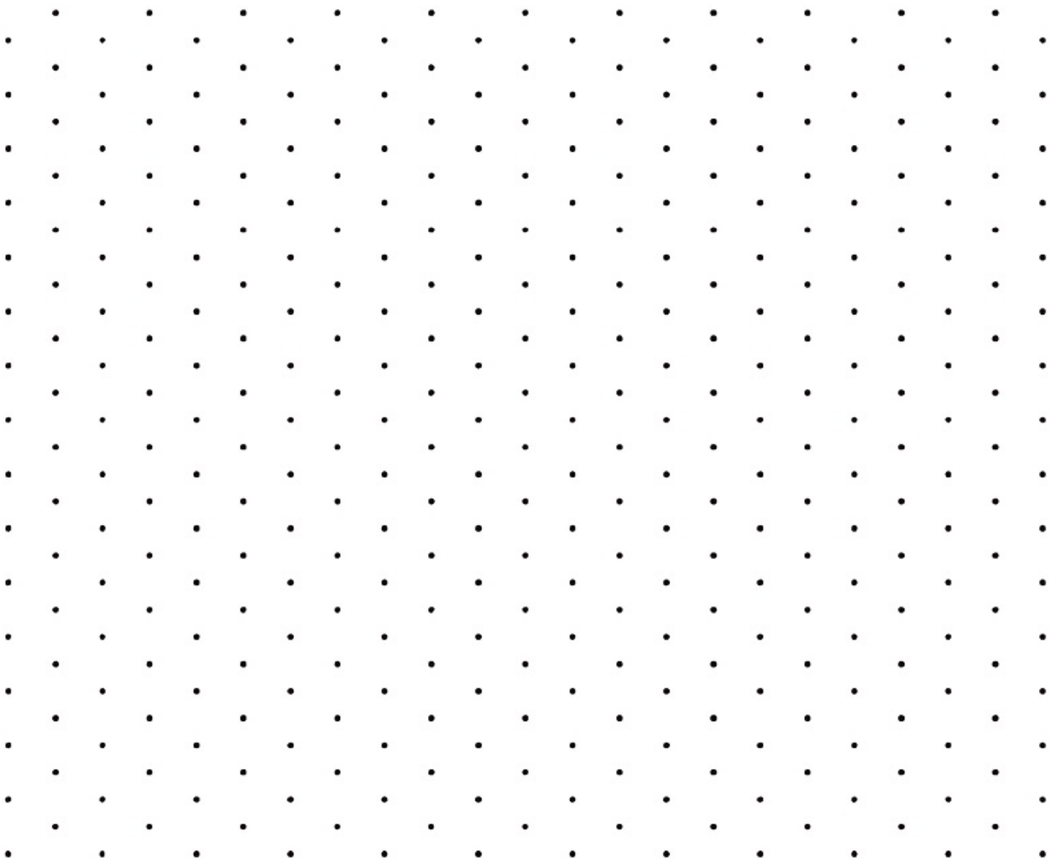
It has six **faces**, but you can only see three of them in the sketch. It has a **surface area** of 32 square units.

Your teacher will give you 12 snap cubes. Use all of your snap cubes to build a different rectangular prism (with different edge lengths than the prism shown here).

1. How many faces does your figure have?

2. What is the surface area of your figure in square units?

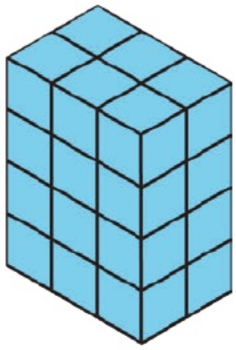
3. Draw your figure on isometric dot paper. Color each face a different color.



Lesson 12 Summary

- The **surface area** of a figure (in square units) is the number of unit squares it takes to cover the entire surface without gaps or overlaps.
- If a three-dimensional figure has flat sides, the sides are called **faces**.
- The surface area is the total of the areas of the faces.

For example, a rectangular prism has six faces. The surface area of the prism is the total of the areas of the six rectangular faces.



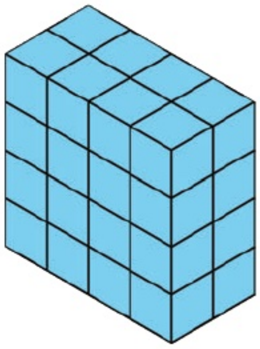
So the surface area of a rectangular prism that has edge-lengths 2 cm, 3 cm, and 4 cm has a surface area of

$$(2 \cdot 3) + (2 \cdot 3) + (2 \cdot 4) + (2 \cdot 4) + (3 \cdot 4) + (3 \cdot 4)$$

or 52 square centimeters.

Unit 1 Lesson 12 Cumulative Practice Problems

1. What is the surface area of this rectangular prism?



A. 16 square units

B. 32 square units

C. 48 square units

D. 64 square units

2. Which description can represent the surface area of this trunk?

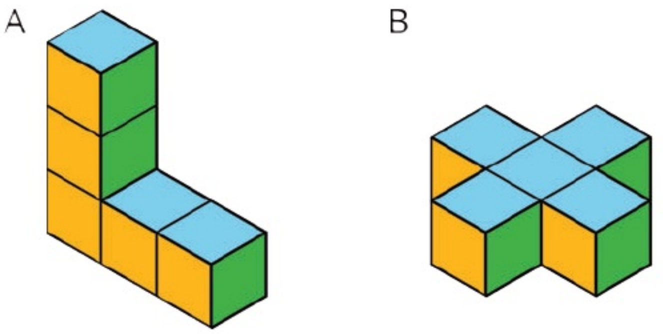
A. The number of square inches that cover the top of the trunk.

B. The number of square feet that cover all the outside faces of the trunk.

C. The number of square inches of horizontal surface inside the trunk.

D. The number of cubic feet that can be packed inside the trunk.

3. Which figure has a greater surface area?



4. A rectangular prism is 4 units high, 2 units wide, and 6 units long. What is its surface area in square units? Explain or show your reasoning.

5. Draw an example of each of these triangles on the grid.

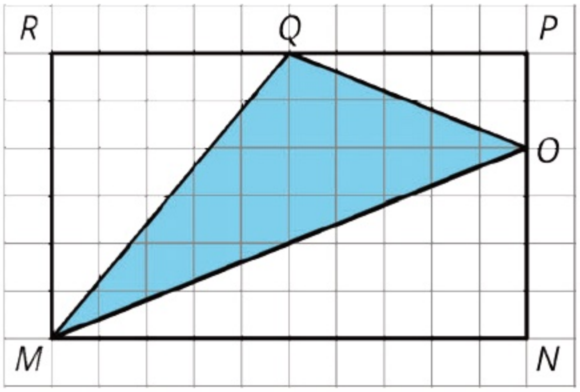
   a. A right triangle with an area of 6 square units.

   b. An acute triangle with an area of 6 square units.

   c. An obtuse triangle with an area of 6 square units.

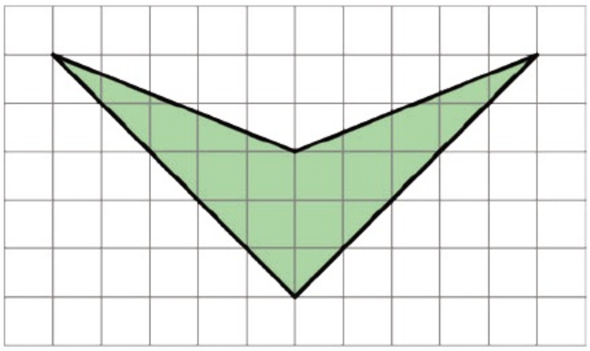
(From Unit 1, Lesson 9.)

6. Find the area of triangle MOQ in square units. Show your reasoning.



(From Unit 1, Lesson 10.)

7. Find the area of this shape. Show your reasoning.

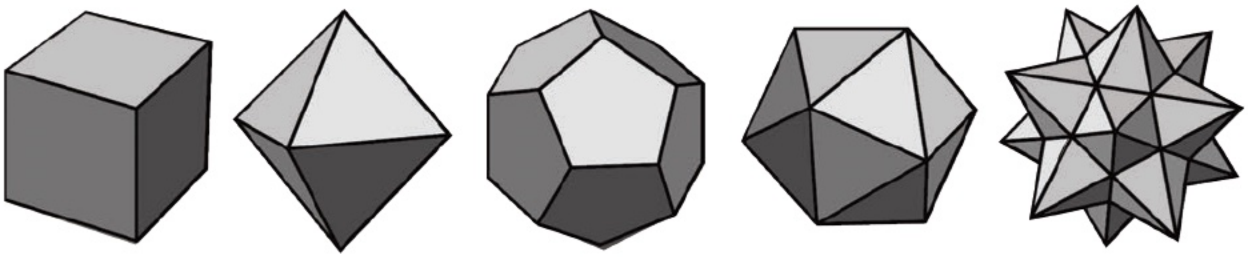


(From Unit 1, Lesson 3.)

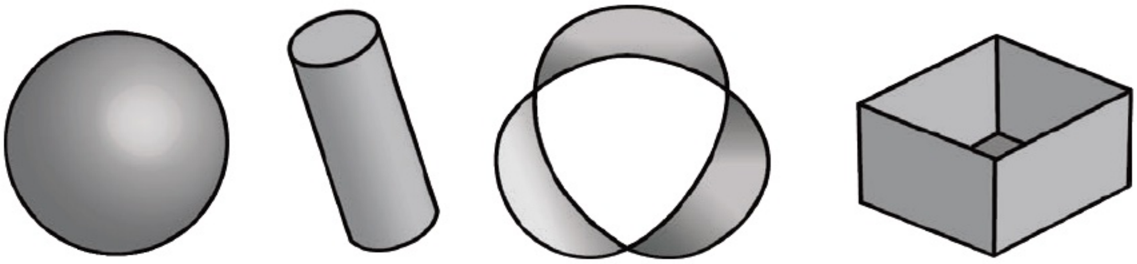
Lesson 13: Polyhedra

13.1: What are Polyhedra?

Here are pictures that represent **polyhedra**:



Here are pictures that do *not* represent polyhedra:

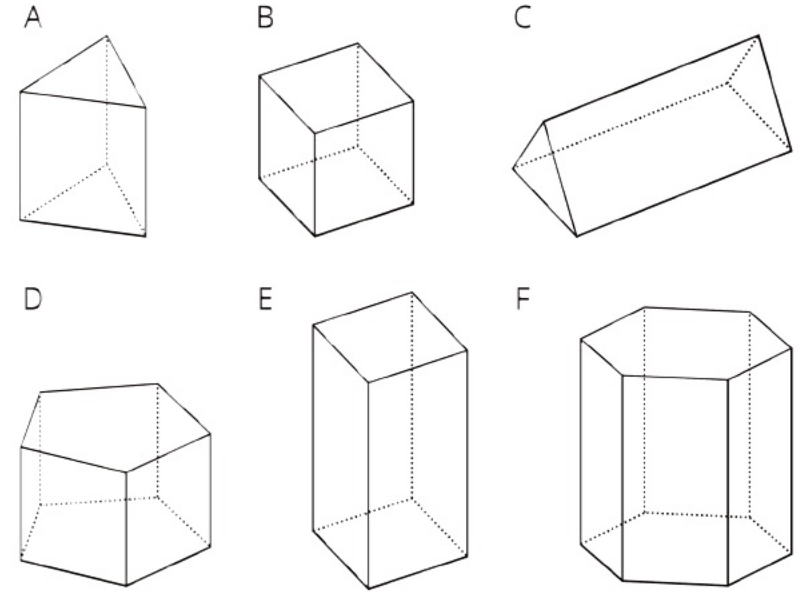


1. Your teacher will give you some figures or objects. Sort them into polyhedra and non-polyhedra.

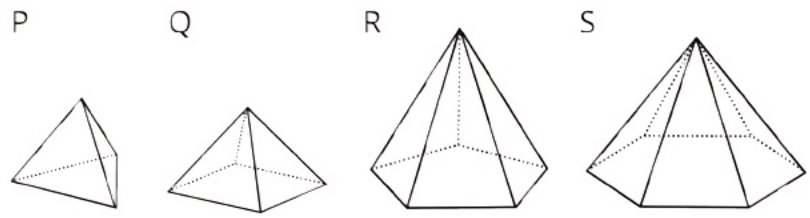
2. What features helped you distinguish the polyhedra from the other figures?

13.2: Prisms and Pyramids

1. Here are some polyhedra called **prisms**.



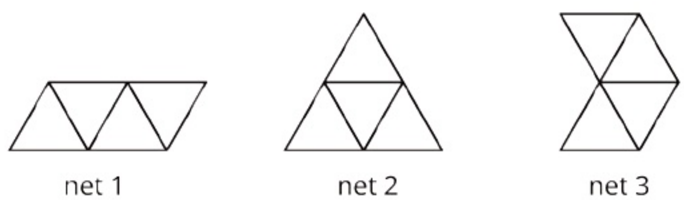
Here are some polyhedra called **pyramids**.



a. Look at the prisms. What are their characteristics or features?

b. Look at the pyramids. What are their characteristics or features?

2. Which of these **nets** can be folded into Pyramid P? Select all that apply.



3. Your teacher will give your group a set of polygons and assign a polyhedron.

    a. Decide which polygons are needed to compose your assigned polyhedron. List the polygons and how many of each are needed.

    b. Arrange the cut-outs into a net that, if taped and folded, can be assembled into the polyhedron. Sketch the net. If possible, find more than one way to arrange the polygons (show a different net for the same polyhedron).

Are you ready for more?

What is the smallest number of faces a polyhedron can possibly have? Explain how you know.

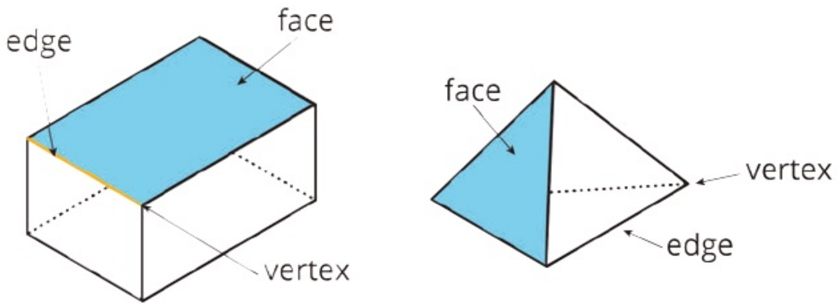
13.3: Assembling Polyhedra

1. Your teacher will give you the net of a polyhedron. Cut out the net, and fold it along the edges to assemble a polyhedron. Tape or glue the flaps so that there are no unjoined edges.

2. How many **vertices**, **edges**, and **faces** are in your polyhedron?

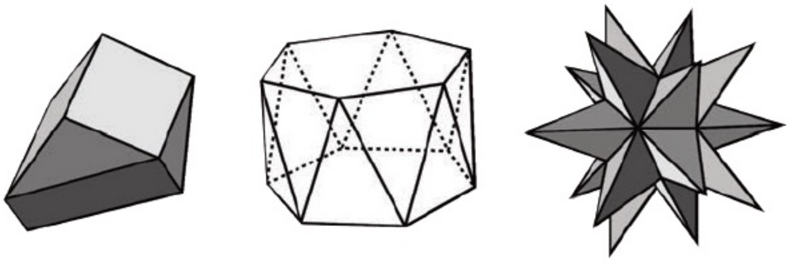
Lesson 13 Summary

A **polyhedron** is a three-dimensional figure composed of faces. Each face is a filled-in polygon and meets only one other face along a complete edge. The ends of the edges meet at points that are called vertices.



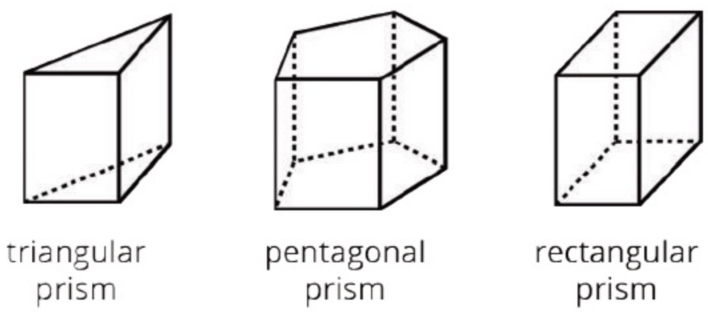
A polyhedron always encloses a three-dimensional region.

The plural of polyhedron is polyhedra. Here are some drawings of polyhedra:



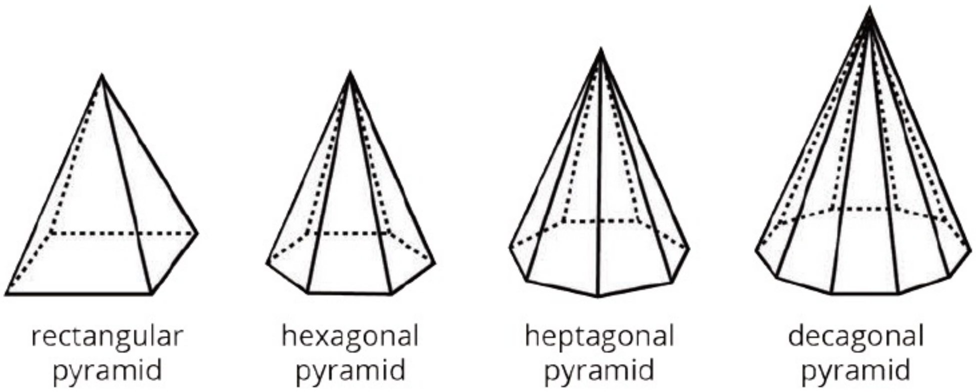
A **prism** is a type of polyhedron with two identical faces that are parallel to each other and that are called **bases**. The bases are connected by a set of rectangles (or sometimes parallelograms).

A prism is named for the shape of its bases. For example, if the base is a pentagon, then it is called a "pentagonal prism."

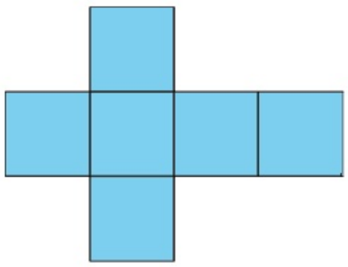


A **pyramid** is a type of polyhedron that has one special face called the base. All of the other faces are triangles that all meet at a single vertex.

A pyramid is named for the shape of its base. For example, if the base is a pentagon, then it is called a “pentagonal pyramid.”



A **net** is a two-dimensional representation of a polyhedron. It is composed of polygons that form the faces of a polyhedron.



A cube has 6 square faces, so its net is composed of six squares, as shown here.

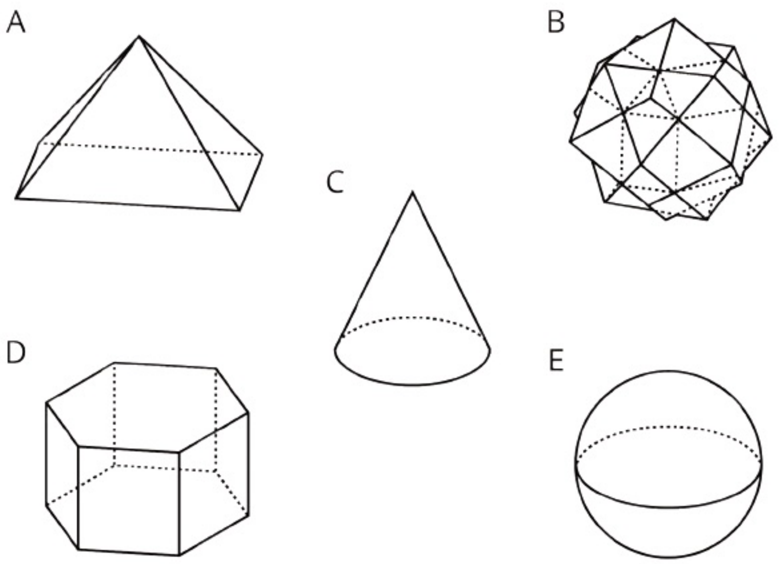
A net can be cut out and folded to make a model of the polyhedron.

In a cube, every face shares its edges with 4 other squares. In a net of a cube, not all edges of the squares are joined with another edge. When the net is folded, however, each of these open edges will join another edge.

It takes practice to visualize the final polyhedron by just looking at a net.

Unit 1 Lesson 13 Cumulative Practice Problems

1. Select **all** the polyhedra.



A. A

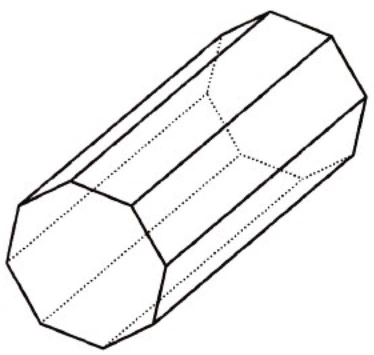
B. B

C. C

D. D

E. E

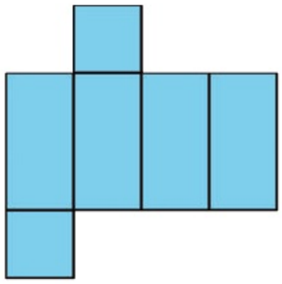
2. a. Is this polyhedron a prism, a pyramid, or neither? Explain how you know.



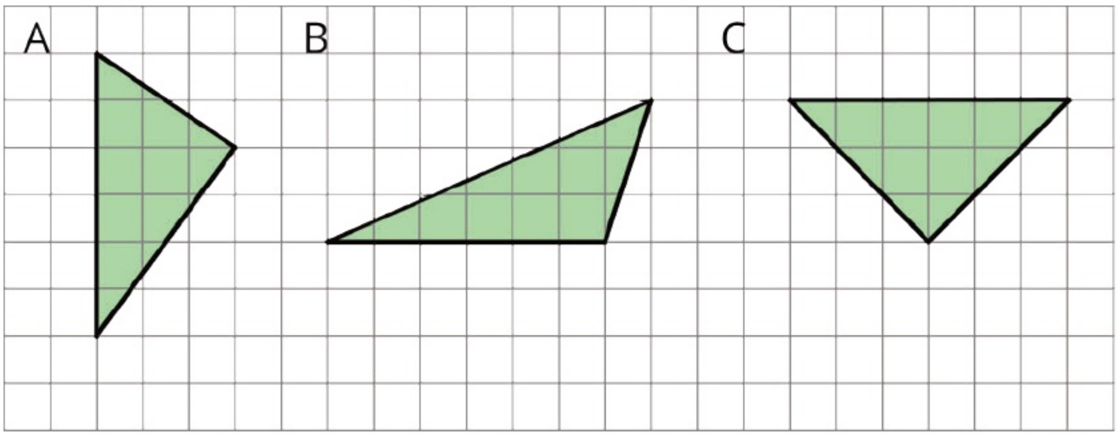
b. How many faces, edges, and vertices does it have?

3. Tyler said this net cannot be a net for a square prism because not all the faces are squares.

   Do you agree with Tyler? Explain your reasoning.



4. Explain why each of these triangles has an area of 9 square units.



(From Unit 1, Lesson 8.)

5. a. A parallelogram has a base of 12 meters and a height of 1.5 meters. What is its area?

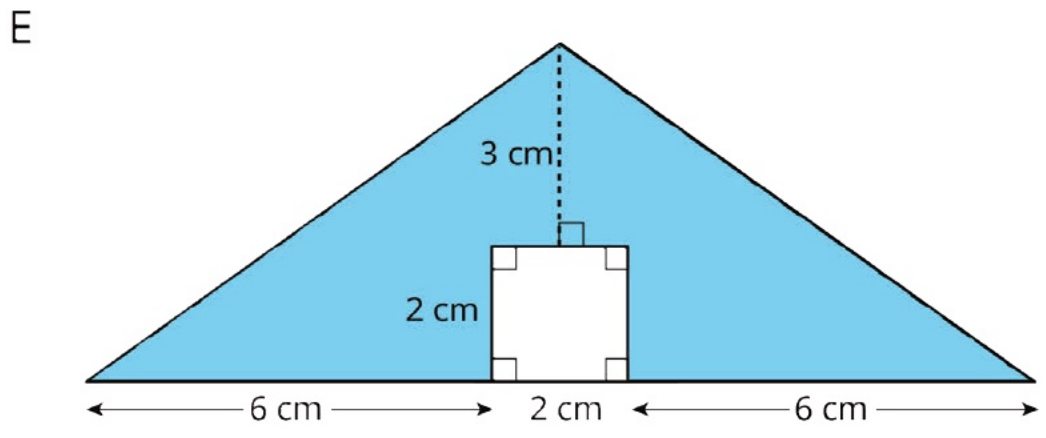
b. A triangle has a base of 16 inches and a height of $\frac{1}{8}$ inches. What is its area?

c. A parallelogram has an area of 28 square feet and a height of 4 feet. What is its base?

d. A triangle has an area of 32 square millimeters and a base of 8 millimeters. What is its height?

(From Unit 1, Lesson 9.)

6. Find the area of the shaded region. Show or explain your reasoning.

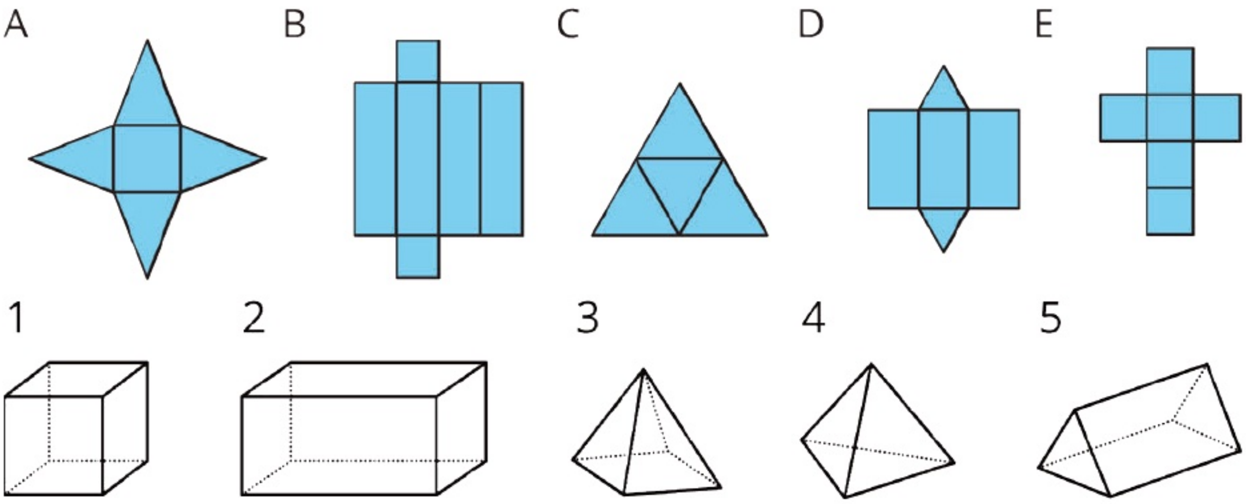


(From Unit 1, Lesson 3.)

Lesson 14: Nets and Surface Area

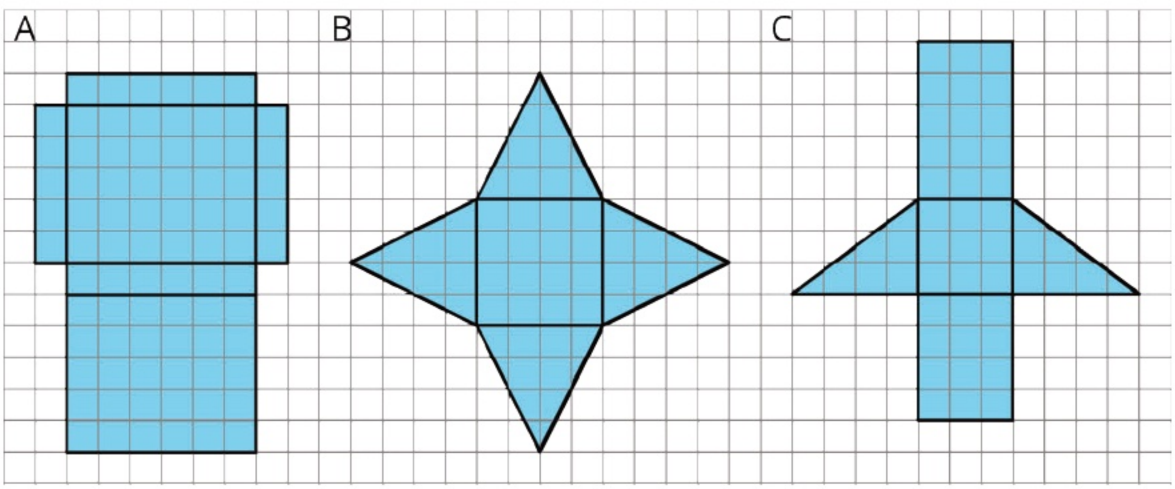
14.1: Matching Nets

Each of the nets can be assembled into a polyhedron. Match each net with its corresponding polyhedron, and name the polyhedron. Be prepared to explain how you know the net and polyhedron go together.



14.2: Using Nets to Find Surface Area

1. Name the polyhedron that each net would form when assembled.



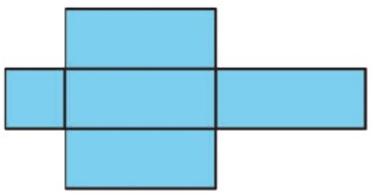
2. Your teacher will give you the nets of three polyhedra. Cut out the nets and assemble the three-dimensional shapes.

3. Find the **surface area** of each polyhedron. Explain your reasoning clearly.

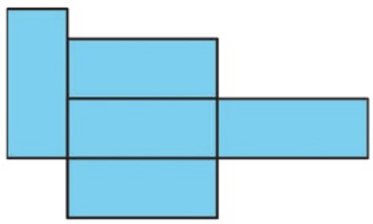
Are you ready for more?

1. For each net, decide if it can be assembled into a rectangular prism.

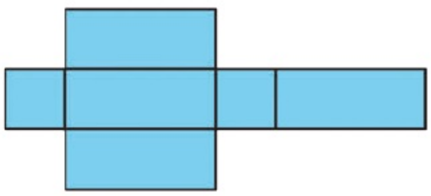
A



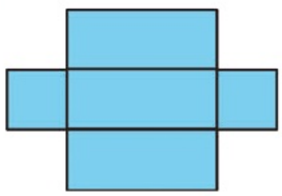
B



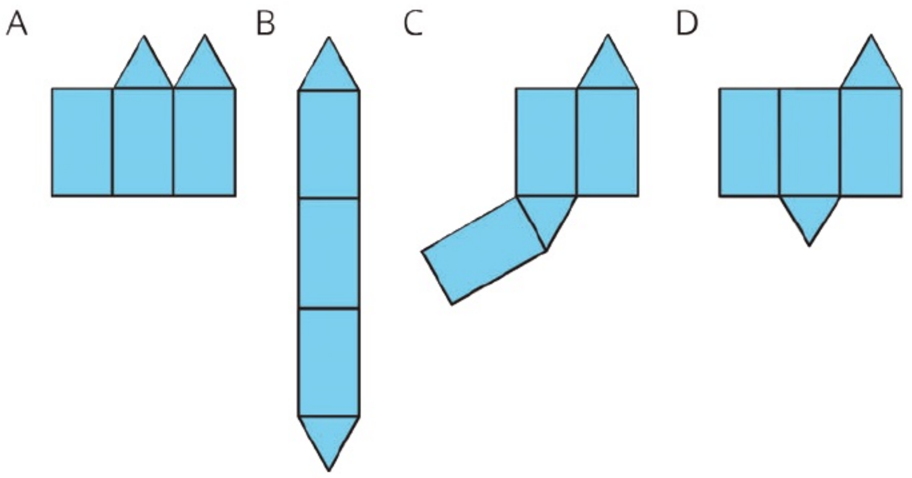
C



D

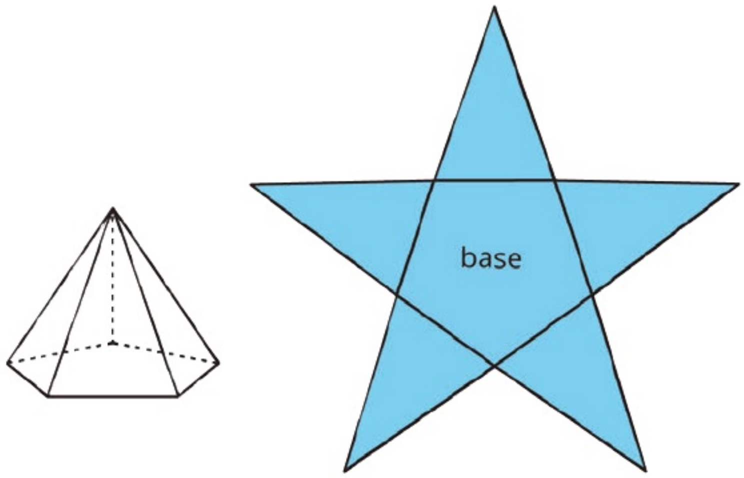


2. For each net, decide if it can be folded into a triangular prism.

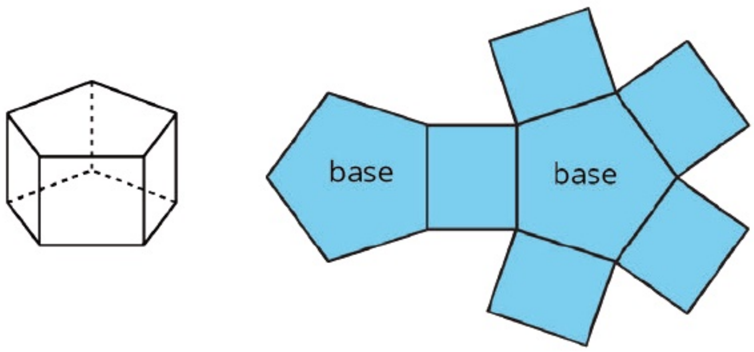


Lesson 14 Summary

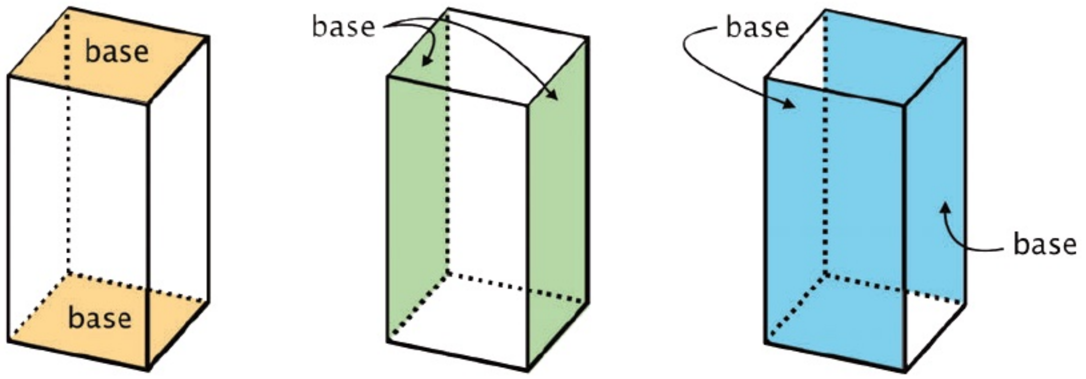
A net of a *pyramid* has one polygon that is the base. The rest of the polygons are triangles. A pentagonal pyramid and its net are shown here.



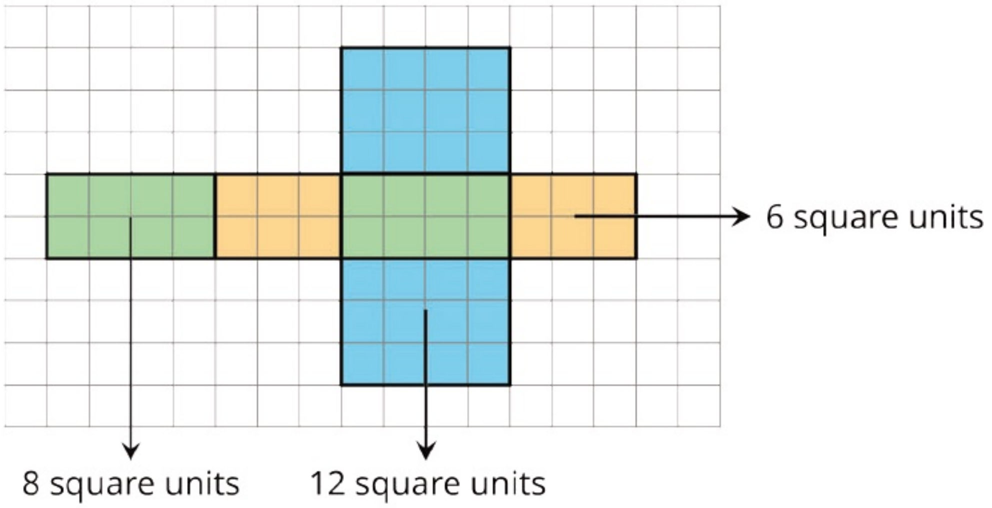
A net of a *prism* has two copies of the polygon that is the base. The rest of the polygons are rectangles. A pentagonal prism and its net are shown here.



In a rectangular prism, there are three pairs of parallel and identical rectangles. Any pair of these identical rectangles can be the bases.



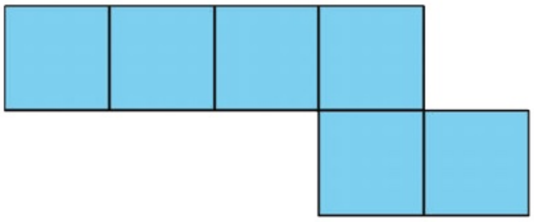
Because a net shows all the faces of a polyhedron, we can use it to find its surface area. For instance, the net of a rectangular prism shows three pairs of rectangles: 4 units by 2 units, 3 units by 2 units, and 4 units by 3 units.



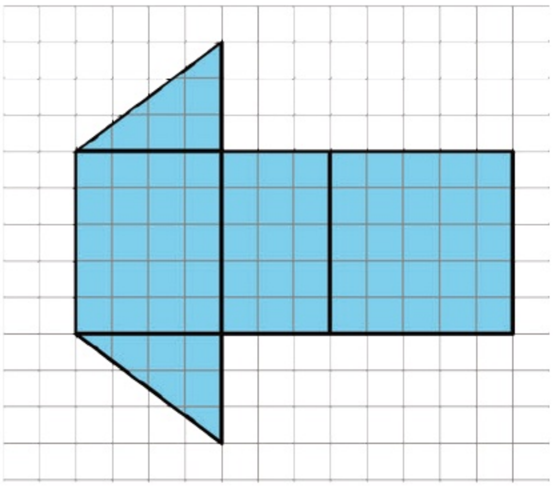
The **surface area** of the rectangular prism is 52 square units because $8+8+6+6+12+12=52$.

Unit 1 Lesson 14 Cumulative Practice Problems

1. Can this net be assembled into a cube? Explain how you know. Label parts of the net with letters or numbers if it helps your explanation.

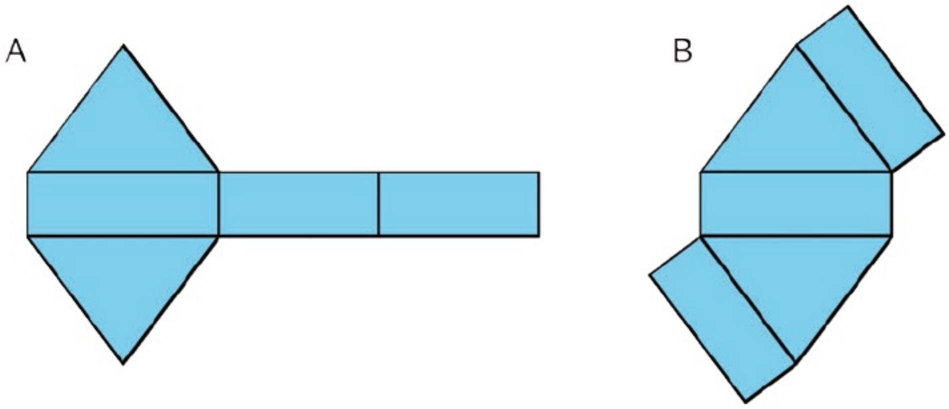


2. a. What polyhedron can be assembled from this net? Explain how you know.



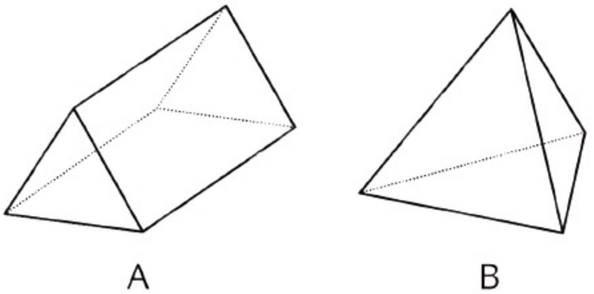
b. Find the surface area of this polyhedron. Show your reasoning.

3. Here are two nets. Mai said that both nets can be assembled into the same triangular prism. Do you agree? Explain or show your reasoning.



4. Here are two three-dimensional figures.

   Tell whether each of the following statements describes Figure A, Figure B, both, or neither.



   a. This figure is a polyhedron.

   b. This figure has triangular faces.

   c. There are more vertices than edges in this figure.

   d. This figure has rectangular faces.

   e. This figure is a pyramid.

   f. There is exactly one face that can be the base for this figure.

   g. The base of this figure is a triangle.

   h. This figure has two identical and parallel faces that can be the base.

(From Unit 1, Lesson 13.)

5. Select **all** units that can be used for surface area.

A. square meters

B. feet

C. centimeters

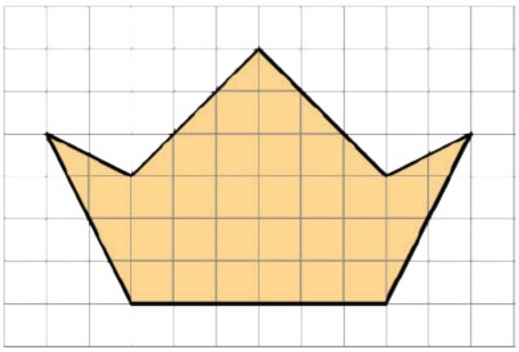
D. cubic inches

E. square inches

F. square feet

(From Unit 1, Lesson 12.)

6. Find the area of this polygon. Show your reasoning.

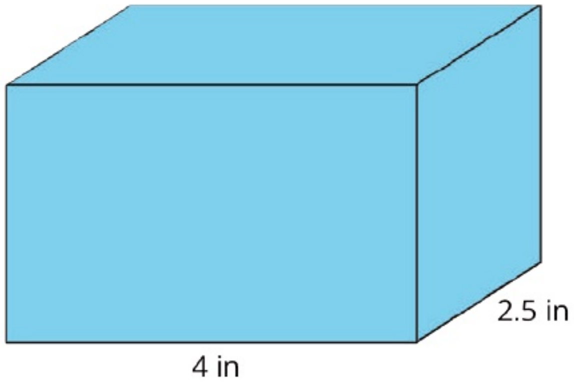


(From Unit 1, Lesson 11.)

Lesson 15: More Nets, More Surface Area

15.1: Notice and Wonder: Wrapping Paper

Kiran is wrapping this box of sports cards as a present for a friend.



What do you notice? What do you wonder?

15.2: Building Prisms and Pyramids

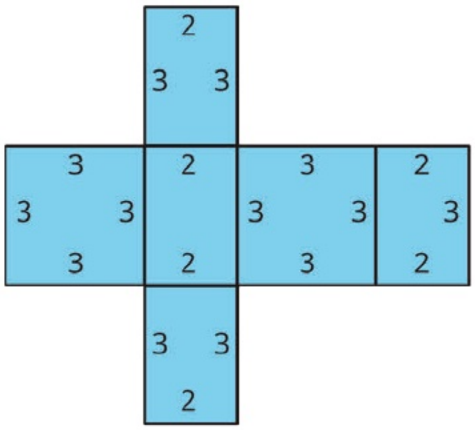
Your teacher will give you a drawing of a polyhedron. You will draw its net and calculate its surface area.

1. What polyhedron do you have?
2. Study your polyhedron. Then, draw its net on graph paper. Use the side length of a grid square as the unit.
3. Label each polygon on the net with a name or number.
4. Find the surface area of your polyhedron. Show your thinking in an organized manner so that it can be followed by others.

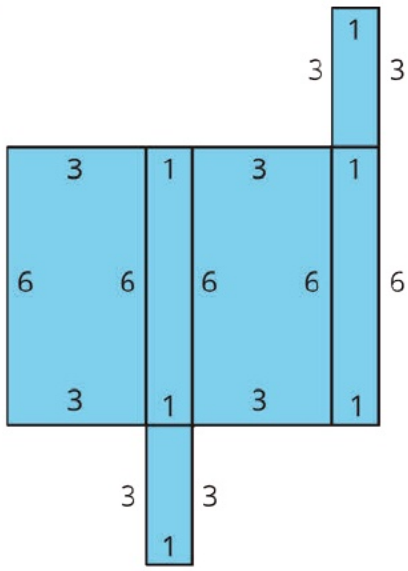
15.3: Comparing Boxes

Here are the nets of three cardboard boxes that are all rectangular prisms. The boxes will be packed with 1-centimeter cubes. All lengths are in centimeters.

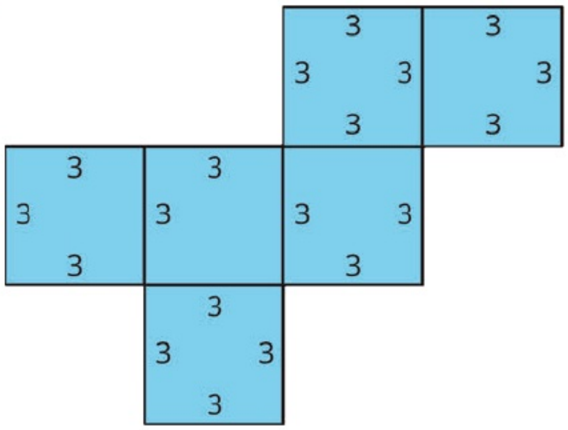
A



B



C



1. Compare the surface areas of the boxes. Which box will use the least cardboard? Show your reasoning.

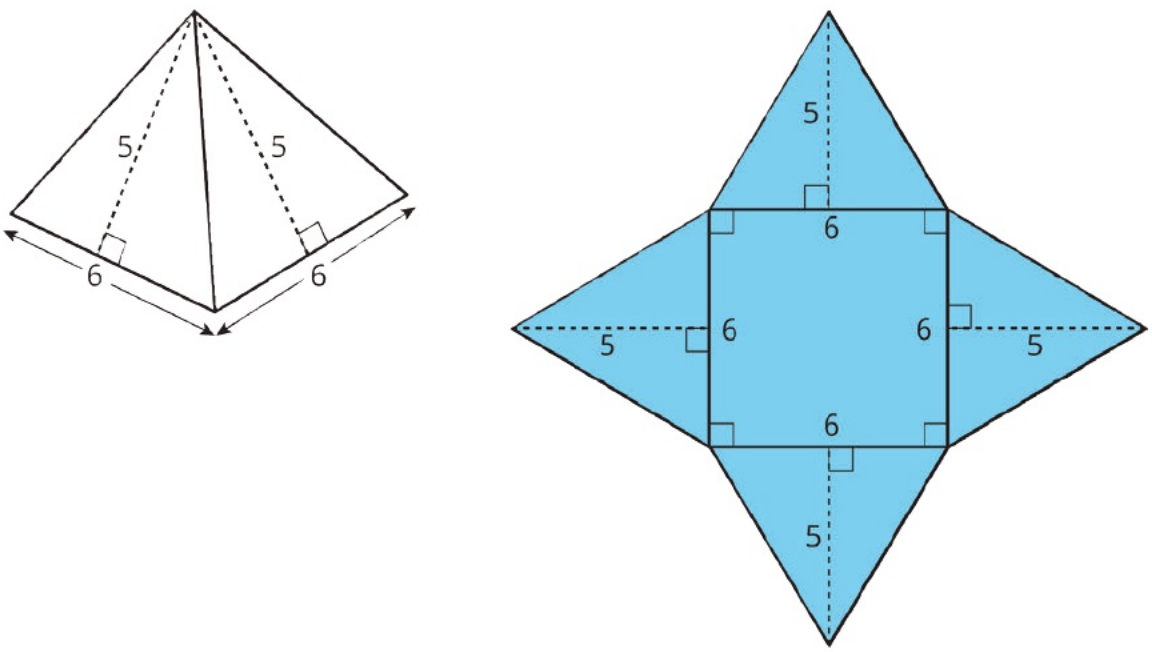
2. Now compare the volumes of these boxes in cubic centimeters. Which box will hold the most 1-centimeter cubes? Show your reasoning.

Are you ready for more?

Figure C shows a net of a cube. Draw a different net of a cube. Draw another one. And then another one. How many different nets can be drawn and assembled into a cube?

Lesson 15 Summary

The surface area of a polyhedron is the sum of the areas of all of the faces. Because a net shows us all faces of a polyhedron at once, it can help us find the surface area. We can find the areas of all polygons in the net and add them.



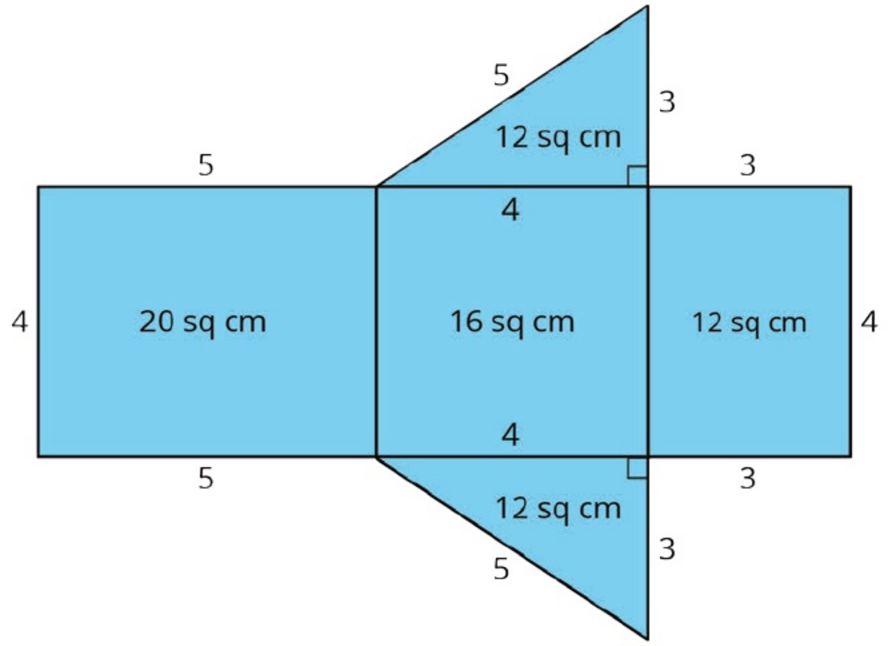
A square pyramid has a square and four triangles for its faces. Its surface area is the sum of the areas of the square base and the four triangular faces:

$$(6 \cdot 6) + 4 \cdot \left(\frac{1}{2} \cdot 5 \cdot 6\right) = 96$$

The surface area of this square pyramid is 96 square units.

Unit 1 Lesson 15 Cumulative Practice Problems

1. Jada drew a net for a polyhedron and calculated its surface area.



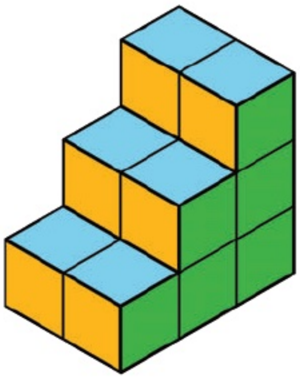
a. What polyhedron can be assembled from this net?

b. Jada made some mistakes in her area calculation. What were the mistakes?

c. Find the surface area of the polyhedron. Show your reasoning.

2. A cereal box is 8 inches by 2 inches by 12 inches. What is its surface area? Show your reasoning. If you get stuck, consider drawing a sketch of the box or its net and labeling the edges with their measurements.

3. Twelve cubes are stacked to make this figure.

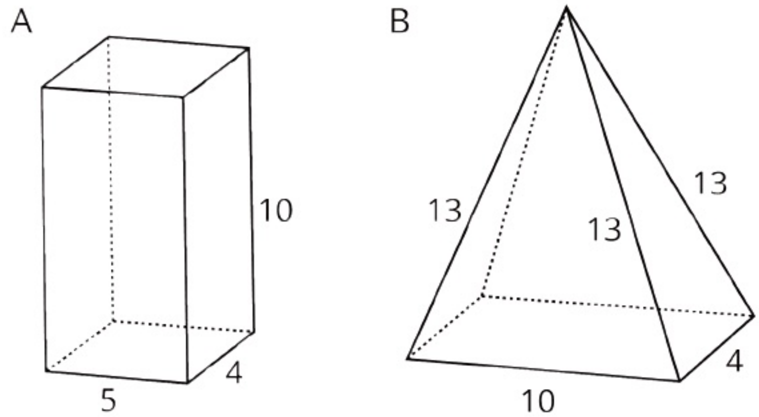


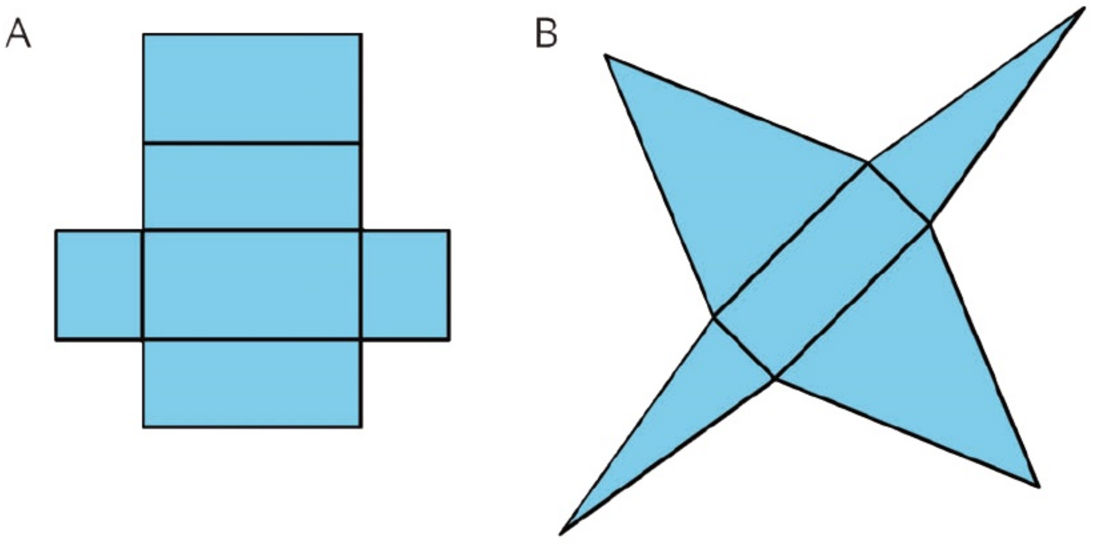
a. What is its surface area?

b. How would the surface area change if the top two cubes are removed?

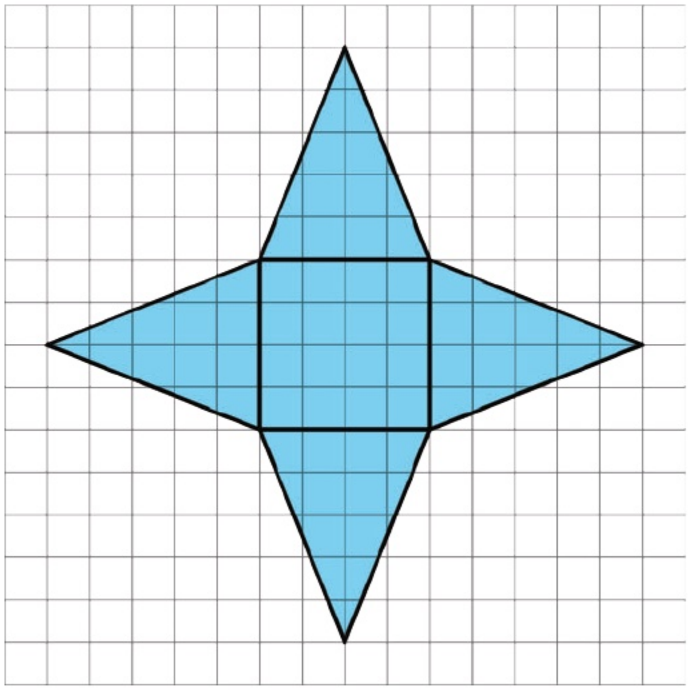
(From Unit 1, Lesson 12.)

4. Here are two polyhedra and their nets. Label all edges in the net with the correct lengths.





5. a. What three-dimensional figure can be assembled from the net?



b. What is the surface area of the figure? (One grid square is 1 square unit.)

(From Unit 1, Lesson 14.)

Lesson 16: Distinguishing Between Surface Area and Volume

16.1: Attributes and Their Measures

For each quantity, choose one or more appropriate units of measurement.

For the last two, think of a quantity that could be appropriately measured with the given units.

Quantities

1. Perimeter of a parking lot:
2. Volume of a semi truck:
3. Surface area of a refrigerator:
4. Length of an eyelash:
5. Area of a state:
6. Volume of an ocean:
7. ______________________: miles
8. ______________________: cubic meters

Units

- millimeters (mm)
- feet (ft)
- meters (m)
- square inches (sq in)
- square feet (sq ft)
- square miles (sq mi)
- cubic kilometers (cu km)
- cubic yards (cu yd)

16.2: Building with 8 Cubes

Your teacher will give you 16 cubes. Build two different shapes using 8 cubes for each. For each shape:

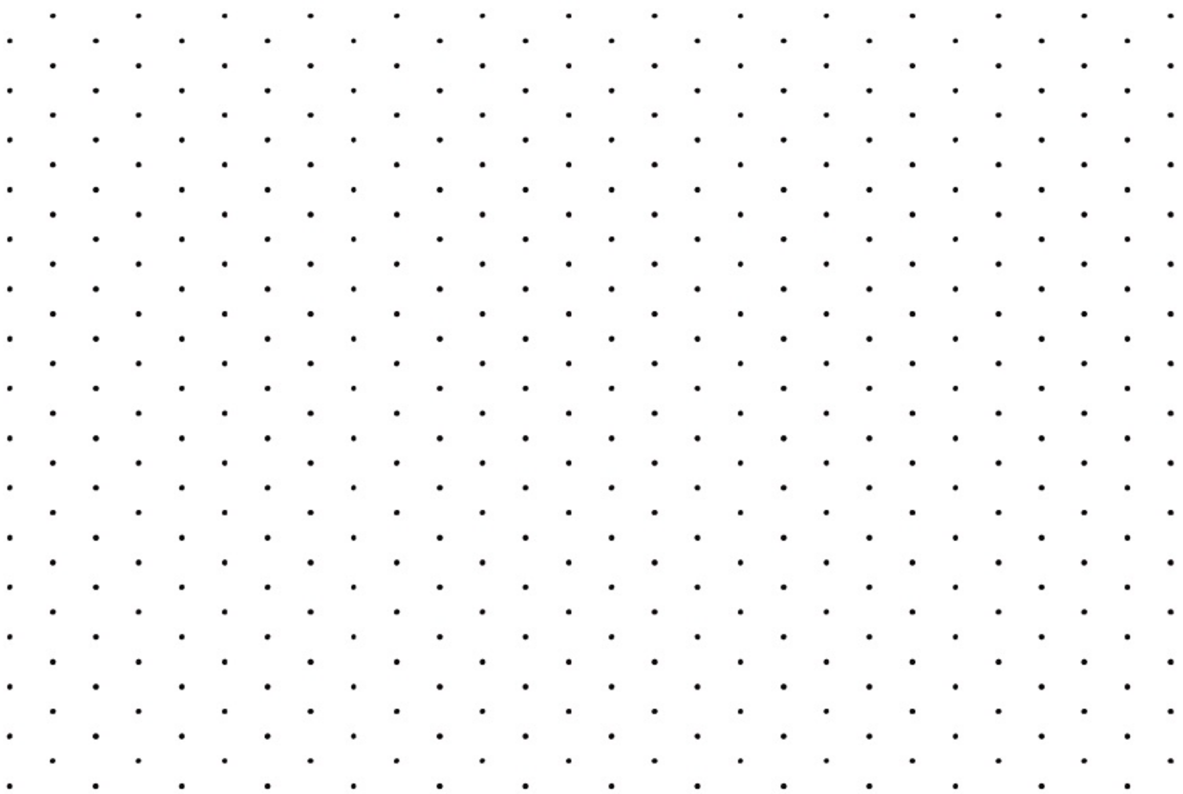
1. Give it a name or a label (e.g., Mai’s First Shape or Diego’s Steps).
2. Determine the **volume**.
3. Determine the surface area.
4. Record the name, volume, and surface area on a sticky note.

16.3: Comparing Prisms Without Building Them

Three rectangular prisms each have a height of 1 cm.

- Prism A has a base that is 1 cm by 11 cm.
- Prism B has a base that is 2 cm by 7 cm.
- Prism C has a base that is 3 cm by 5 cm.

1. Find the surface area and volume of each prism. Use the dot paper to draw the prisms, if needed.



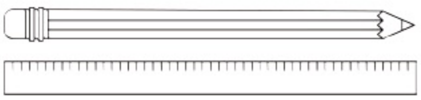
2. Analyze the volumes and surface areas of the prisms. What do you notice? Write 1 or 2 observations about them.

Are you ready for more?

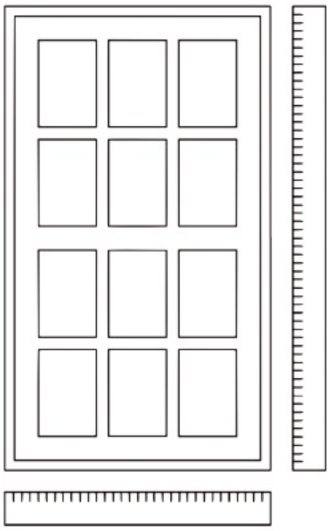
Can you find more examples of prisms that have the same surface areas but different volumes? How many can you find?

Lesson 16 Summary

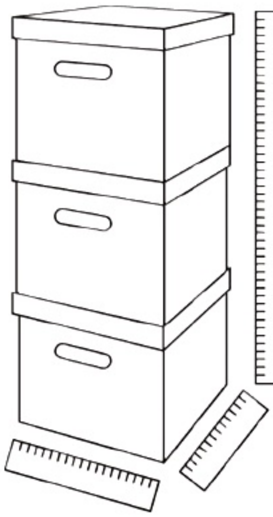
Length is a one-dimensional attribute of a geometric figure. We measure lengths using units like millimeters, centimeters, meters, kilometers, inches, feet, yards, and miles.



Area is a two-dimensional attribute. We measure area in square units. For example, a square that is 1 centimeter on each side has an area of 1 square centimeter.



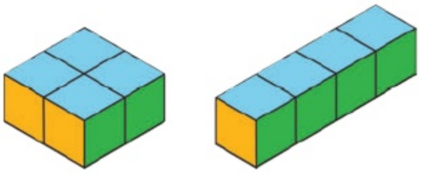
Volume is a three-dimensional attribute. We measure volume in cubic units. For example, a cube that is 1 kilometer on each side has a volume of 1 cubic kilometer.



Surface area and volume are different attributes of three-dimensional figures. Surface area is a two-dimensional measure, while volume is a three-dimensional measure.

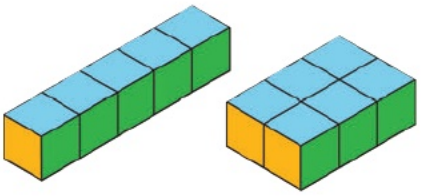
Two figures can have the same volume but different surface areas. For example:

- A rectangular prism with side lengths of 1 cm, 2 cm, and 2 cm has a volume of 4 cu cm and a surface area of 16 sq cm.
- A rectangular prism with side lengths of 1 cm, 1 cm, and 4 cm has the same volume but a surface area of 18 sq cm.



Similarly, two figures can have the same surface area but different volumes.

- A rectangular prism with side lengths of 1 cm, 1 cm, and 5 cm has a surface area of 22 sq cm and a volume of 5 cu cm.
- A rectangular prism with side lengths of 1 cm, 2 cm, and 3 cm has the same surface area but a volume of 6 cu cm.

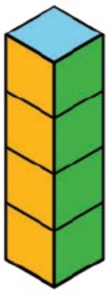


Unit 1 Lesson 16 Cumulative Practice Problems

1. Match each quantity with an appropriate unit of measurement.

A. The surface area of a tissue box	1. Square meters
B. The amount of soil in a planter box	2. Yards
C. The area of a parking lot	3. Cubic inches
D. The length of a soccer field	4. Cubic feet
E. The volume of a fish tank	5. Square centimeters

2. Here is a figure built from snap cubes.



a. Find the volume of the figure in cubic units.

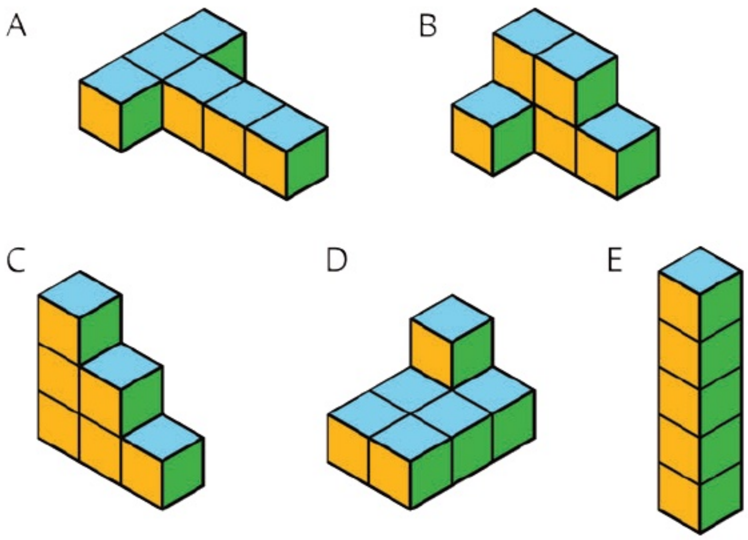
b. Find the surface area of the figure in square units.

c. True or false: If we double the number of cubes being stacked, both the volume and surface area will double. Explain or show how you know.

3. Lin said, "Two figures with the same volume also have the same surface area."

   a. Which two figures suggest that her statement is true?

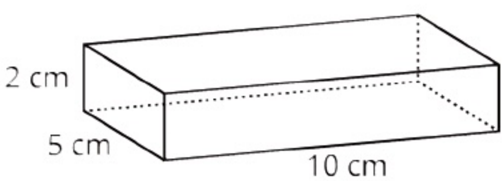
   b. Which two figures could show that her statement is *not* true?



4. Draw a pentagon (five-sided polygon) that has an area of 32 square units. Label all relevant sides or segments with their measurements, and show that the area is 32 square units.

(From Unit 1, Lesson 11.)

5. a. Draw a net for this rectangular prism.



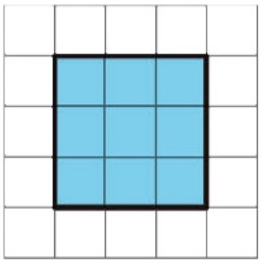
   b. Find the surface area of the rectangular prism.

(From Unit 1, Lesson 15.)

Lesson 17: Squares and Cubes

17.1: Perfect Squares

1. The number 9 is a perfect **square.** Find four numbers that are perfect squares and two numbers that are not perfect squares.



2. A square has side length 7 km. What is its area?

3. The area of a square is 64 sq cm. What is its side length?

17.2: Building with 32 Cubes

Your teacher will give you 32 snap cubes. Use them to build the largest single cube you can. Each small cube has an edge length of 1 unit.

1. How many snap cubes did you use?

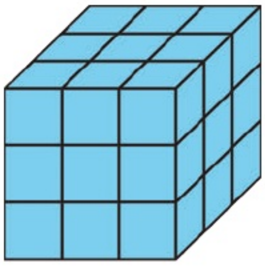
2. What is the edge length of the cube you built?

3. What is the area of each face of the built cube? Be prepared to explain your reasoning.

4. What is the volume of the built cube? Be prepared to explain your reasoning.

17.3: Perfect Cubes

1. The number 27 is a perfect **cube**. Find four other numbers that are perfect cubes and two numbers that are *not* perfect cubes.



2. A cube has side length 4 cm. What is its volume?

3. A cube has side length 10 inches. What is its volume?

4. A cube has side length s units. What is its volume?

17.4: Introducing Exponents

Make sure to include correct units of measure as part of each answer.

1. A square has side length 10 cm. Use an **exponent** to express its area.
2. The area of a square is 7^2 sq in. What is its side length?
3. The area of a square is 81 m^2. Use an exponent to express this area.
4. A cube has edge length 5 in. Use an exponent to express its volume.
5. The volume of a cube is 6^3 cm^3. What is its edge length?
6. A cube has edge length s units. Use an exponent to write an expression for its volume.

Are you ready for more?

The number 15,625 is both a perfect square and a perfect cube. It is a perfect square because it equals 125^2. It is also a perfect cube because it equals 25^3. Find another number that is both a perfect square and a perfect cube. How many of these can you find?

Lesson 17 Summary

When we multiply two of the same numbers together, such as $5 \cdot 5$, we say we are **squaring** the number. We can write it like this:

$$5^2$$

Because $5 \cdot 5 = 25$, we write $5^2 = 25$ and we say, "5 squared is 25."

When we multiply three of the same numbers together, such as $4 \cdot 4 \cdot 4$, we say we are **cubing** the number. We can write it like this:

$$4^3$$

Because $4 \cdot 4 \cdot 4 = 64$, we write $4^3 = 64$ and we say, "4 cubed is 64."

We also use this notation for square and cubic units.

- A square with side length 5 inches has area 25 in^2.
- A cube with edge length 4 cm has volume 64 cm^3.

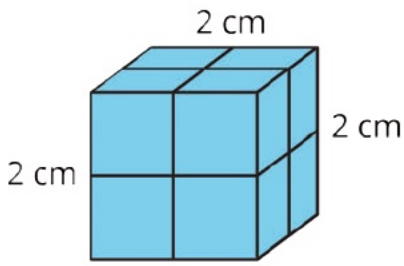
To read 25 in^2, we say "25 square inches," just like before.

The area of a square with side length 7 kilometers is 7^2 km^2. The volume of a cube with edge length 2 millimeters is 2^3 mm^3.

In general, the area of a square with side length s is s^2, and the volume of a cube with edge length s is s^3.

Unit 1 Lesson 17 Cumulative Practice Problems

1. What is the volume of this cube?



2. a. Decide if each number on the list is a perfect square.

16	125
20	144
25	225
100	10,000

b. Write a sentence that explains your reasoning.

3. a. Decide if each number on the list is a perfect cube.

1	27
3	64
8	100
9	125

b. Explain what a perfect cube is.

4. a. A square has side length 4 cm. What is its area?

b. The area of a square is 49 m^2. What is its side length?

c. A cube has edge length 3 in. What is its volume?

5. Prism A and Prism B are rectangular prisms.

- Prism A is 3 inches by 2 inches by 1 inch.
- Prism B is 1 inch by 1 inch by 6 inches.

Select **all** statements that are true about the two prisms.

A. They have the same volume.

B. They have the same number of faces.

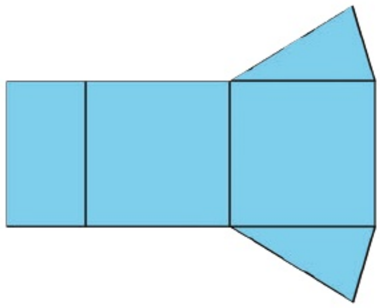
C. More inch cubes can be packed into Prism A than into Prism B.

D. The two prisms have the same surface area.

E. The surface area of Prism B is greater than that of Prism A.

(From Unit 1, Lesson 16.)

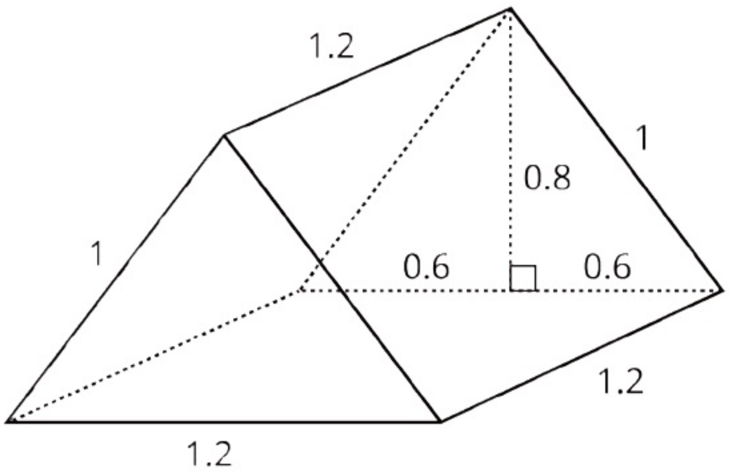
6. a. What polyhedron can be assembled from this net?



b. What information would you need to find its surface area? Be specific, and label the diagram as needed.

(From Unit 1, Lesson 14.)

7. Find the surface area of this triangular prism. All measurements are in meters.



(From Unit 1, Lesson 15.)

Lesson 18: Surface Area of a Cube

18.1: Exponent Review

Select the greater expression of each pair without calculating the value of each expression. Be prepared to explain your choices.

- $10 \cdot 3$ or 10^3

- 13^2 or $12 \cdot 12$

- $97 + 97 + 97 + 97 + 97 + 97$ or $5 \cdot 97$

18.2: The Net of a Cube

1. A cube has edge length 5 inches.

   a. Draw a net for this cube, and label its sides with measurements.

   b. What is the shape of each face?

   c. What is the area of each face?

   d. What is the surface area of this cube?

   e. What is the volume of this cube?

2. A second cube has edge length 17 units.

   a. Draw a net for this cube, and label its sides with measurements.

   b. Explain why the area of each face of this cube is 17^2 square units.

   c. Write an expression for the surface area, in square units.

   d. Write an expression for the volume, in cubic units.

18.3: Every Cube in the Whole World

A cube has edge length s.

1. Draw a net for the cube.

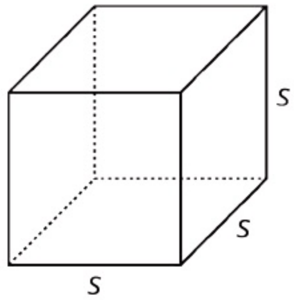
2. Write an expression for the area of each face. Label each face with its area.

3. Write an expression for the surface area.

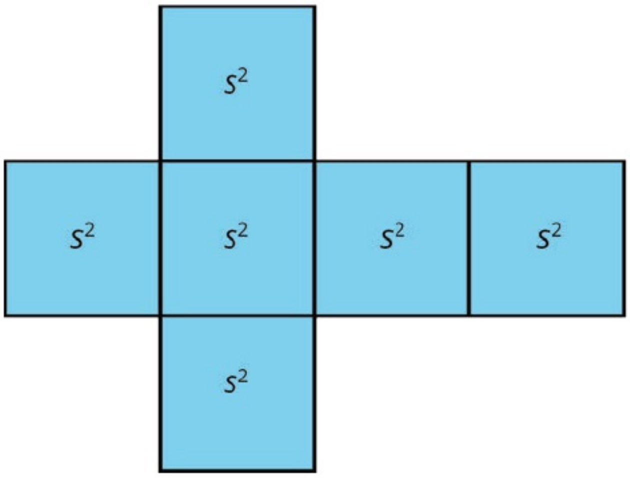
4. Write an expression for the volume.

Lesson 18 Summary

The volume of a cube with edge length s is s^3.



A cube has 6 faces that are all identical squares. The surface area of a cube with edge length s is $6 \cdot s^2$.



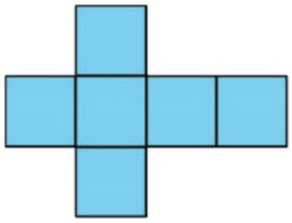
Unit 1 Lesson 18 Cumulative Practice Problems

1. a. What is the volume of a cube with edge length 8 in?

   b. What is the volume of a cube with edge length $\frac{1}{3}$ cm?

   c. A cube has a volume of 8 ft^3. What is its edge length?

2. a. What three-dimensional figure can be assembled from this net?



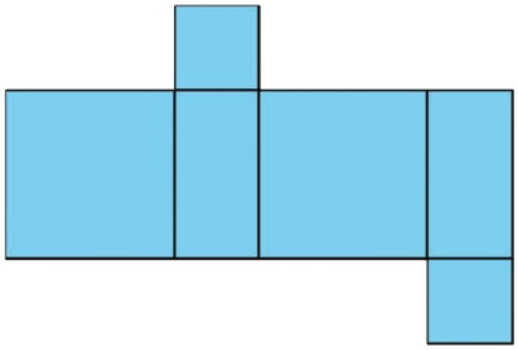
   b. If each square has a side length of 61 cm, write an expression for the surface area and another for the volume of the figure.

3. a. Draw a net for a cube with edge length x cm.

b. What is the surface area of this cube?

c. What is the volume of this cube?

4. Here is a net for a rectangular prism that was not drawn accurately.



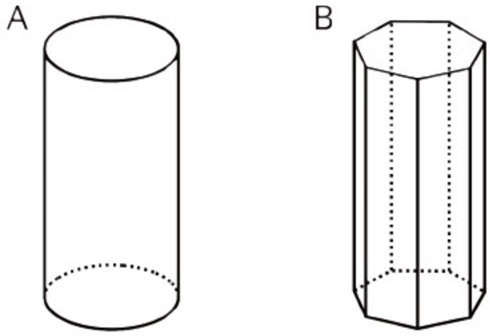
a. Explain what is wrong with the net.

b. Draw a net that can be assembled into a rectangular prism.

c. Create another net for the same prism.

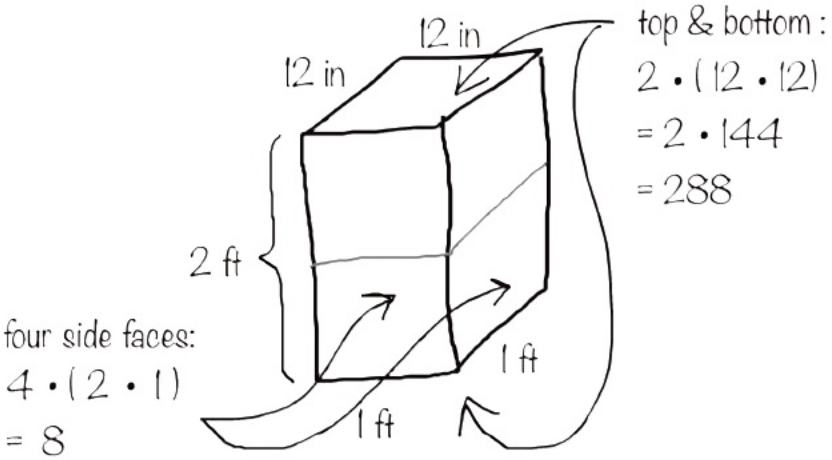
(From Unit 1, Lesson 14.)

5. State whether each figure is a polyhedron. Explain how you know.



(From Unit 1, Lesson 13.)

6. Here is Elena's work for finding the surface area of a rectangular prism that is 1 foot by 1 foot by 2 feet.



She concluded that the surface area of the prism is 296 square feet. Do you agree with her? Explain your reasoning.

(From Unit 1, Lesson 12.)

Lesson 19: Designing a Tent

19.1: Tent Design - Part 1

Have you ever been camping?

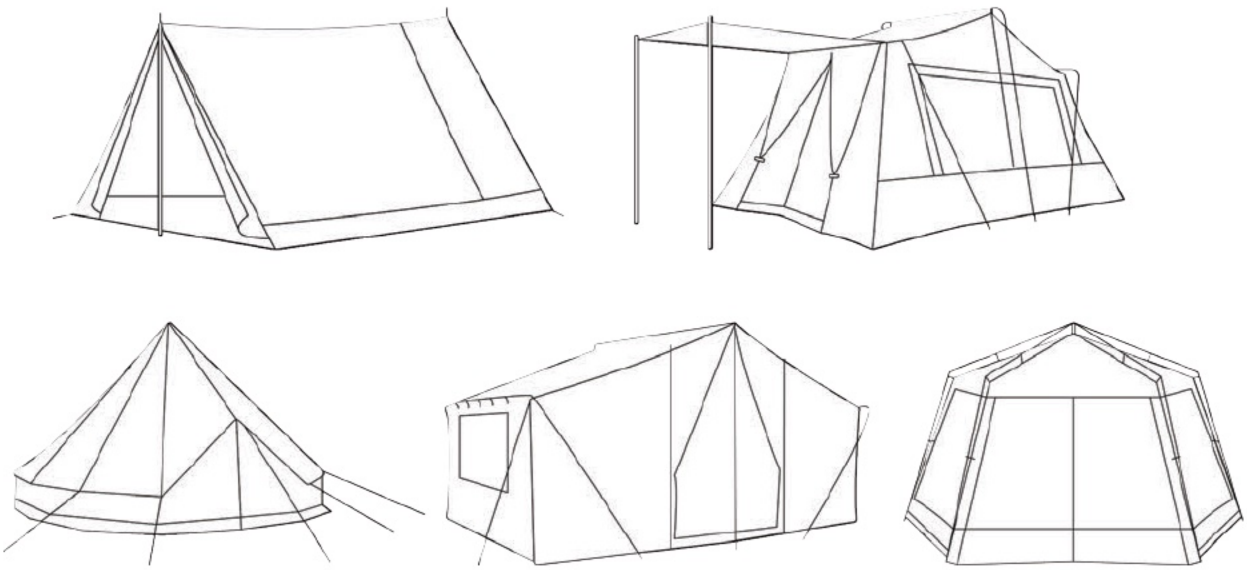
You might know that sleeping bags are all about the same size, but tents come in a variety of shapes and sizes.

Your task is to design a tent to accommodate up to four people, and estimate the amount of fabric needed to make your tent. Your design and estimate must be based on the information given and have mathematical justification.

First, look at these examples of tents, the average specifications of a camping tent, and standard sleeping bag measurements. Talk to a partner about:

- Similarities and differences among the tents
- Information that will be important in your designing process
- The pros and cons of the various designs

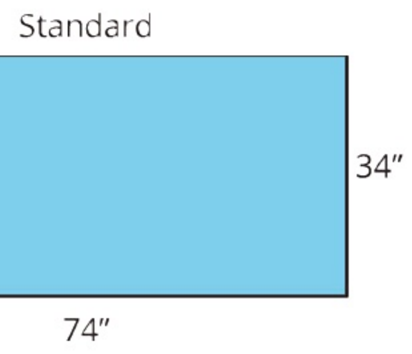
Tent Styles



Tent Height Specifications

height description	height of tent	notes
sitting height	3 feet	Campers are able to sit, lie, or crawl inside tent.
kneeling height	4 feet	Campers are able to kneel inside tent. Found mainly in 3–4 person tents.
stooping height	5 feet	Campers are able to move around on their feet inside tent, but most campers will not be able to stand upright.
standing height	6 feet	Most adult campers are able to stand upright inside tent.
roaming height	7 feet	Adult campers are able to stand upright and walk around inside tent.

Sleeping Bag Measurements



1. Create and sketch your tent design. The tent must include a floor.

2. What decisions were important when choosing your tent design?

3. How much fabric do you estimate will be necessary to make your tent? Show your reasoning and provide mathematical justification.

19.2: Tent Design - Part 2

1. Explain your tent design and fabric estimate to your partner or partners. Be sure to explain why you chose this design and how you found your fabric estimate.
2. Compare the estimated fabric necessary for each tent in your group. Discuss the following questions:
    - Which tent design used the least fabric? Why?
    - Which tent design used the most fabric? Why?
    - Which change in design most impacted the amount of fabric needed for the tent? Why?

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