



Core Knowledge[®] MATHEMATICS

Putting It All Together



Teacher Guide



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Putting It All Together

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Putting It All Together
Teacher Guide
Core Knowledge Mathematics™

Unit 8: Putting It All Together

At a Glance

Unit 8 is estimated to be completed in 19-20 days including 2 days for assessment.

This unit is divided into four sections including 17 lessons and 1 optional lesson.

- Section A—Multiply and Divide Whole Numbers (Lessons 1-5)
- Section B—Apply Volume Concepts (Lessons 6-9)
- Section C—Fraction and Decimal Operations (Lessons 10-13)
- Section D—Creation and Design (Lessons 14-18)

On pages 7-8 of this Teacher Guide is a chart that identifies the section each lesson belongs in and the materials needed for each lesson.

There are no new centers in this unit. Students choose from centers that have been introduced throughout the year. Students can work at any previously introduced stages of the centers.

Unit 8: Putting It All Together

Unit Learning Goals

- Students consolidate and solidify their understanding of various concepts and skills related to major work of the grade. They also continue to work toward fluency goals of the grade.

In this unit, students revisit major work and fluency goals of the grade, applying their learning from the year.

In section A, students deepen their understanding of the standard algorithm for multiplication and practice using it to find the value of products. They also revisit algorithms that use partial quotients to divide whole numbers. In Section B, students solve real-world problems about volume and have opportunities to model with mathematics.

*The base of the Great Pyramid of Egypt is a square.
One side length of the base is 230 meters.
The pyramid is 140 meters tall.*

*If the pyramid was shaped like a rectangular prism,
what would be the volume of the prism?*



Section C focuses on operation with decimals and fractions. In the final section, students review major work of the grade as they create activities in the format of the warm-ups routines they have encountered throughout the year (Notice and Wonder, Estimation Exploration, Number Talk, True or False, and Which One Doesn't Belong?).

The sections in this unit are standalone sections, not required to be completed in order. Within a section, lessons can also be completed selectively and without completing prior lessons. The goal is to offer ample opportunities for students to integrate the knowledge they have gained and to practice skills related to the expected fluencies of the grade.

Addressing 5.G.B.3, 5.G.B.4, 5.NBT.B.5, 5.NBT.B.6

Section Learning Goals

- Divide multi-digit whole numbers using place value strategies and the properties of operations.
- Fluently multiply multi-digit whole numbers using the standard algorithm.

In this section, students reinforce their understanding of the standard algorithm for multiplication and practice using it. They use estimation to determine the reasonableness of their answers, recognize and explain place-value patterns when multiplying multi-digit numbers, and learn how to use the algorithm when one or more of the factors has several zeros.

Here is how Kiran found the value of 650×27 .
Is the answer reasonable?

$$\begin{array}{r} 1 \\ 3 \\ 650 \\ \times 27 \\ \hline 4500 \\ + 1300 \\ \hline 5850 \end{array}$$

Find the value of each product.

$$\begin{array}{r} 260 \\ \times 35 \\ \hline \end{array}$$

$$\begin{array}{r} 2,600 \\ \times \quad 35 \\ \hline \end{array}$$

What's the relationship between 260×35 and $2,600 \times 35$?

Students also practice dividing multi-digit whole numbers using an algorithm involving partial quotients, which they learned in unit 4.

 PLC: Lesson 1, Activity 1, Talk About it

Section B: Apply Volume Concepts

Standards Alignments

Addressing 5.MD.C, 5.MD.C.5, 5.NBT.B.5, 5.NBT.B.6

Section Learning Goals

- Solve multi-step problems involving volume.

In this section, students revisit the meaning of volume and apply their understanding to solve problems. In each lesson, students estimate and calculate the volume of rectangular prisms in various contexts. The numbers used in this section are larger than the numbers students used in the opening unit, when they first learned how to calculate the volume of rectangular prisms.



A company packages 126 sugar cubes in each box.

The box is a rectangular prism.

What are some possible ways they could pack the cubes?

The side lengths of the box are about $1\frac{7}{8}$ inches by $3\frac{3}{4}$ inches by $4\frac{3}{8}$ inches.

What can we say about how the sugar cubes are packed?

The work here prompts students to make reasonable estimates, consider appropriate sizes of units in a given context, and take unit conversion into account in solving problems about volume.



Use the picture of the wagon to make an estimate of the length, width, and height of the wagon bed.

Then, improve your estimate for the volume of the wagon.

   PLC: Lesson 6, Warm-up, Estimation Exploration: Sugar Cubes

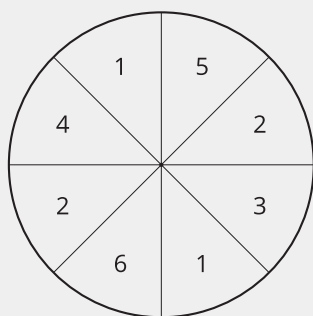
Section C: Fraction and Decimal Operations

Standards Alignments

Addressing 5.NBT.B.7, 5.NF.A.1, 5.NF.B.4

In this section, students strengthen their understanding of operations with fractions and decimals by playing a variety of games. Each lesson is structured as a game day.

Students begin with games that involve adding and subtracting fractions and in which the goal is to find the largest or the smallest sums or differences.



Round 1

$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} - \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} =$$

- Spin the spinner.
- Write the number in an empty box. Keep the chosen box hidden from view.
- Once a number is written down, it cannot be changed.
- Continue spinning and writing numbers until all 4 boxes have been filled.
- Find the difference. The person with the greatest difference wins the round.

Next, students practice adding and subtracting decimals. The games here likewise prompt students to meet certain goals, such as finding the largest decimal or reaching 1, 0.1, or 0.01.

At the end of the section, students play a game that involves multiplying fractions. All the games about fractions invite students to consider the meaning of the numerator and the denominator and to make strategic choices about the numbers they use in those positions.

 PLC: Lesson 13, Activity 2, Fraction Multiplication Compare Round 2

Section D: Creation and Design

Standards Alignments

Addressing 5.G, 5.MD, 5.MD.C.3, 5.NBT, 5.NBT.B.5, 5.NBT.B.6, 5.NF, 5.NF.A.1, 5.NF.B.3, 5.OA

Section Learning Goals

- Review the major work of the grade by creating and designing instructional routines.

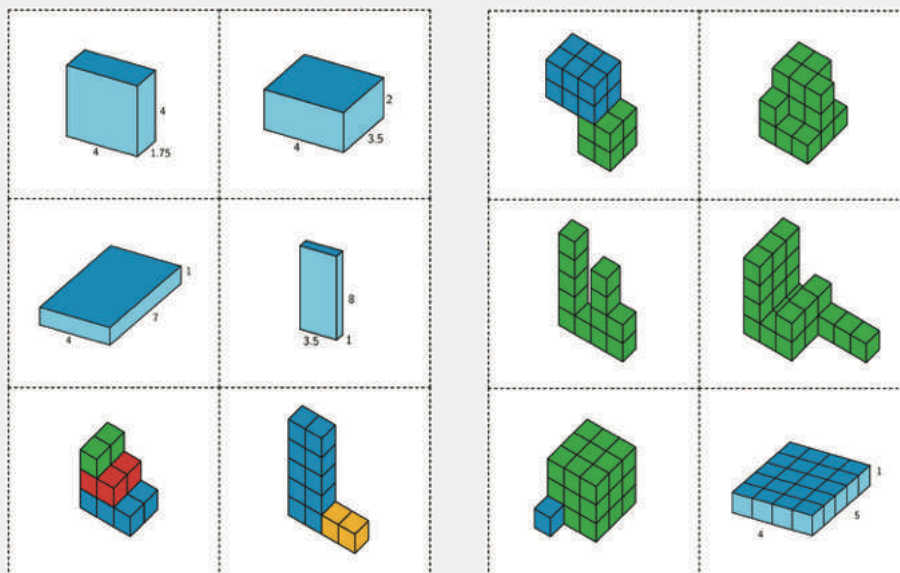
Throughout the course, students have engaged in warm-up routines such as Notice and Wonder, Exploration Estimation, True or False, Number Talk, and Which One Doesn't Belong? This section enables them to apply the mathematics they have learned to design warm-ups that incorporate these routines.

Each lesson is devoted to a particular routine. Students begin by completing at least two partially created tasks, each with more missing parts to complete than the previous one. They practice anticipating responses that others might give to the prompts they pose.

Choose 3 shapes from the set of cards.

Draw a fourth shape to complete the Which One Doesn't Belong?

For each shape, discuss one reason why each shape does not belong.



Which one doesn't belong?

A B

C D

PLC: Lesson 16, Warm-up, Number Talk: Division

Throughout the Unit

The warm-ups throughout the unit invite students to think about the topics addressed in each section. In Section A, the warm-ups invite students to think about important ideas related to multiplying and dividing multi-digit whole numbers that are revisited throughout the section. In section B, the warm-ups are related to applying multiplication of multi-digit whole numbers to volume concepts and extending concepts of volume to include further study of units used to measure volume. In section C, the warm-ups continue to push towards fluently adding and subtracting fractions and operating with decimals. In section D, each warm-up reflects the type of activity that students design that day.

Here is a sampling of the warm-ups in the unit.

lesson 4	lesson 7	lesson 10	lesson 17
Notice and Wonder $\underline{\quad} \div \underline{\quad} = 136$	Notice and Wonder 	Number Talk $\frac{2}{12} + \frac{1}{6}$ $\frac{2}{6} + \frac{1}{2}$ $\frac{1}{3} + \frac{1}{2}$ $\frac{1}{3} + \frac{3}{2}$	True or False $\frac{3}{4} + \frac{3}{8} = \frac{3}{8} + \frac{3}{8}$ $\frac{7}{5} + \frac{2}{3} = \frac{21}{15} + \frac{8}{15}$ $\frac{8}{9} + \frac{5}{12} = \frac{32}{36} + \frac{15}{36}$

Materials Needed

LESSON	GATHER	COPY
A.1	<ul style="list-style-type: none"> • none 	<ul style="list-style-type: none"> • none
A.2	<ul style="list-style-type: none"> • none 	<ul style="list-style-type: none"> • none
A.3	<ul style="list-style-type: none"> • none 	<ul style="list-style-type: none"> • none
A.4	<ul style="list-style-type: none"> • Tools for creating a visual display 	<ul style="list-style-type: none"> • none
A.5	<ul style="list-style-type: none"> • none 	<ul style="list-style-type: none"> • none
B.6	<ul style="list-style-type: none"> • none 	<ul style="list-style-type: none"> • none
B.7	<ul style="list-style-type: none"> • Rulers • Yardsticks 	<ul style="list-style-type: none"> • none
B.8	<ul style="list-style-type: none"> • none 	<ul style="list-style-type: none"> • none
B.9	<ul style="list-style-type: none"> • none 	<ul style="list-style-type: none"> • none
C.10	<ul style="list-style-type: none"> • Paper clips • Paper clips 	<ul style="list-style-type: none"> • none
C.11	<ul style="list-style-type: none"> • Paper clips 	<ul style="list-style-type: none"> • none
C.12	<ul style="list-style-type: none"> • Number cubes • Paper clips 	<ul style="list-style-type: none"> • none
C.13	<ul style="list-style-type: none"> • Paper clips 	<ul style="list-style-type: none"> • none
D.14	<ul style="list-style-type: none"> • Chart paper • Colored pencils, crayons, or markers 	<ul style="list-style-type: none"> • none
D.15	<ul style="list-style-type: none"> • Chart paper • Colored pencils, crayons, or markers 	<ul style="list-style-type: none"> • none

D.16	● none	● none
D.17	● none	● none
D.18	● none	● Shape Cards for WODB Design (groups of 2)

Section A: Multiply and Divide Whole Numbers

Lesson 1: Find the Largest Product

Standards Alignments

Addressing 5.NBT.B.5

Teacher-facing Learning Goals

- Fluently multiply multi-digit whole numbers using the standard algorithm.

Student-facing Learning Goals

- Let's look for patterns when we multiply multi-digit numbers.

Lesson Purpose

The purpose of this lesson is for students to recognize and explain place value patterns as they find products using the standard algorithm they learned in Unit 4.

In previous units, students learned the standard algorithm for multiplying whole numbers. In this lesson, students compare and contrast different products that can be made with the same set of 3 or 4 digits. They look for patterns in the arrangement of the digits and explain those patterns in terms of place value. For example, students might notice that the product of 2 two-digit numbers results in a greater product than that of a three-digit number and 1 one-digit number. If students need additional support with the concepts in this lesson, refer back to Unit 4, Section A in the curriculum materials.

Access for:

Students with Disabilities

- Action and Expression (Activity 1)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Notice and Wonder (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	15 min

Teacher Reflection Question

What did you learn about students' understanding of the standard algorithm for multiplication during the lesson today? How can

🕒 5 min

Standards Alignments

Student-facing Task Statement

1. 35×47
2. 37×45

1. 1,645

$$\begin{array}{r} 23 \\ \times 47 \\ \hline 235 \\ + 1410 \\ \hline 1645 \end{array}$$

2. 1,665

$$\begin{array}{r} 1 \\ 3 \\ 45 \\ \times 37 \\ \hline 315 \\ + 1,350 \\ \hline 1,665 \end{array}$$

 Begin Lesson

Warm-up

⌚ 10 min

Notice and Wonder: Digits

Standards Alignments

Addressing 5.NBT.B.5

The purpose of this warm-up is for students to discuss the location of digits in the products, which will be useful when students try to find the greatest product in a later activity. While students may notice and wonder many things, the location of the digits 6, 4, 1, and 8 is the important discussion point.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?

$$\begin{array}{r} 841 \\ \times \quad 6 \\ \hline \end{array} \qquad \begin{array}{r} 641 \\ \times \quad 8 \\ \hline \end{array}$$

Student Responses

Students may notice:

- Both products have the same digits, but some of the digits are in different places.
- Both products have factors with the number 41 in them.
- The 8 and the 6 switch places.

Students may wonder:

- What are the values of the products?
- Are they equal?
- Which one is larger?

Launch

- Groups of 2
- Display the image.

Activity

- “What do you notice? What do you wonder?”
- 1 minute: quiet think time
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- “Without finding the values, which product do you think will be greater? Explain your reasoning.” (I think 841×6 will be greater because 841 is a lot more than 641. I think 641×8 will be larger because there is 4,800 in each and the second product has more 41s.)
- “We are going to revisit these problems in the lesson synthesis.”

Activity 1

🕒 15 min

Talk About it

👤 ↔ 👤 PLC Activity

Standards Alignments

Addressing 5.NBT.B.5

The purpose of this activity is for students to practice using the standard algorithm and explain how the placement of the digits in factors impacts the value of the product when multiplying a two-digit number by a one-digit number. Students multiply different factors which use the same 3 digits and determine which combination yields the greatest product. While the problems were intentionally structured to encourage students to use an efficient strategy, such as the algorithm, students should use whatever strategy makes sense to them when solving these problems.

Students critically analyze a claim about the largest product that can be made with 3 digits and discuss their reasoning with several partners (MP3).

🌐 Access for English Learners

MLR8 Discussion Supports. During group work, invite students to take turns sharing their responses. Ask students to restate what they heard using precise mathematical language and their own words. Display the sentence frame: “I heard you say” Original speakers can agree or clarify for their partner.

Advances: Listening, Speaking

♿ Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Before they begin, invite students to verbalize a strategy they can use to determine whether they agree or disagree with each statement. Students can speak quietly to themselves, or share with a partner.

Supports accessibility for: Organization, Conceptual Processing, Language

Student-facing Task Statement

1. Consider the statement below. Decide whether you agree, disagree, or are unsure. Be prepared to explain your reasoning.

Launch

- Groups of 2
- “You will discuss a talking point in 2 rounds. In the first round, read the statement and take turns explaining whether you agree, disagree, or are unsure. Then we will switch

	agree	disagree	unsure
Given the digits 7, 5, and 2, the largest product you can make is 75×2 because 75 is the largest number you can make.			
After round 1: Given the digits 7, 5, and 2, the largest product you can make is 75×2 because 75 is the largest number you can make.			

Write about something new that you learned from your group or something you still wonder about:

- Use the digits 6, 3, and 1 to make the largest possible product. Be prepared to explain your reasoning.

Student Responses

- Sample response: 52×7 is the largest product you can make with the digits 5, 2, and 7. I learned that 52×7 is 4 greater than 72×5 because both products have 350, but making 7 the second factor gives a larger partial product than making 5 the second factor.
- 31×6 is the largest product you can make with the digits 6, 3, and 1 because $61 \times 3 = 183$, but $31 \times 6 = 186$. Putting the 1 in the tens place or multiplying by 1 gives a much smaller product.

groups and complete Round 2."

- Partner work time: 2–3 minutes

Activity

- After partner work time, rearrange students into groups of 4. New groups of 4 should be formed where partners are not in the same group.
- "Now, you will complete Round 2 in your new group. Each person in the group will be given time to restate their reasoning."
- 1–2 minutes: group discussion
- "Decide if you want to revise your thinking and be prepared to explain why you changed your thinking. Each person in the group will say whether or not they changed their answer and explain why or why not."
- 1–2 minutes: small-group discussion
- "Think about something new that you learned from your group or something that you are still wondering about."
- 1–2 minutes: independent work time
- 1–2 minutes: partner discussion
- "Now, use what you learned to complete the second problem."
- 1–2 minutes: independent work time

Synthesis

- Display: $72 \times 5 = (70 \times 5) + (2 \times 5)$

$$52 \times 7 = (50 \times 7) + (2 \times 7)$$

- "Why is 52×7 greater than 72×5 ?" (The product of ones and tens is the same in these products but 52×7 has a larger product of ones and ones.)
- "What did you learn about placement of digits when multiplying a two-digit number by a one-digit number?"

Activity 2

🕒 20 min

More Digits

Standards Alignments

Addressing 5.NBT.B.5

The purpose of this activity is for students to practice using the standard algorithm and to use place value reasoning to explain how the placement of the digits in factors impacts the value of the product when multiplying (MP7).

Student-facing Task Statement

1. Use the digits 7, 3, 2, and 5 to make the greatest product.
2. Explain or show how you know you have made the greatest product.

Student Responses

Sample responses:

1. 72×53
2. I figured out that the two-digit by two-digit multiplication problems made greater products and then I tried a few different problems until I found the one that made the greatest product.

Launch

- Groups of 2
- Display:

7, 3, 2, 5

- "Using only these digits, what multiplication expressions could we write?" (723×5 , 32×57 , $7 \times 3 \times 2 \times 5$, $73 \times 5 \times 2$.)
- 1 minute: quiet think time
- Record answers for all to see.
- "Which of these expressions do you think would make the greatest product? Be prepared to explain your reasoning." (I think the three-digit by one-digit expression would make the greatest product because you can put the 7 in the hundreds place.)
- "Use the digits 7, 3, 2, and 5 to make the greatest product."

Activity

- 5–7 minutes: partner work time

Synthesis

- “Which multiplication expression will have the greatest product?”
- Poll the class.
- Record responses for all to see.
- Display or write these expressions for all to see:

$$\begin{array}{r} \overset{1}{7} \overset{1}{3} 2 \\ \times 5 \\ \hline 3, 6 6 0 \end{array} \qquad \begin{array}{r} \overset{2}{5} \overset{1}{3} 2 \\ \times 7 \\ \hline 3, 7 2 4 \end{array}$$

- “Why does switching the placement of the 5 and the 7 increase the value of the product?” (Both products have 35 hundreds, but when 7 is the second factor, the product has 7 groups of 32 instead of 5 groups.)
- Display or write these expressions for all to see:

$$\begin{array}{r} 7 3 \\ \times 5 2 \\ \hline 6 \\ 1 4 0 \\ 1 5 0 \\ + 3, 5 0 0 \\ \hline 3, 7 9 6 \end{array} \qquad \begin{array}{r} 7 2 \\ \times 5 3 \\ \hline 6 \\ 2 1 0 \\ 1 0 0 \\ + 3, 5 0 0 \\ \hline 3, 8 1 6 \end{array}$$

- “Why does switching the placement of the digits 2 and 3 increase the value of the product?” (Both products have partial products 50×70 and 2×3 , but when we switch the digits 2 and 3, the partial products in 72×53 are greater.)

Advancing Student Thinking

If students need help getting started, refer to the products generated during the launch and ask students to evaluate them. Ask them to order the products from least to greatest and explain what they notice. Next, ask them to generate other products using the same digits. Continue to check in with them and ask them to explain any patterns they notice.

Lesson Synthesis

🕒 10 min

Display or write these products for all to see.

$$\begin{array}{r} 841 \\ \times \quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 641 \\ \times \quad 8 \\ \hline \end{array}$$

"Here are the problems from the warm-up. Does anyone want to revise their thinking about which one is the greater product?" (641×8 because both products will have 4,800, but there will be two more groups of 41 in 641×8 .)

"Today we explored ways to arrange digits to make the greatest product. We had to solve a lot of multiplication problems. What is something new that you learned about multiplication today?" (I never realized how many different problems you could create with the same digits. I was surprised by some of the largest products. I thought 841×6 would be larger than 641×8 .)

----- Complete Cool-Down -----

Response to Student Thinking

Students do not evaluate the product correctly.

Next Day Support

- Launch warm-up or Activity 1 by highlighting important notation from previous lessons.

Lesson 2: More Multiplication

Standards Alignments

Addressing 5.NBT.B.5

Teacher-facing Learning Goals

- Fluently multiply multi-digit whole numbers using the standard algorithm.

Student-facing Learning Goals

- Let's practice using the multiplication algorithm.

Lesson Purpose

The purpose of this lesson is for students to practice using the standard algorithm to multiply multi-digit numbers.

In previous units, students learned how to use the standard algorithm to multiply multi-digit whole numbers. In this lesson, they practice using the standard algorithm when one or more of the factors includes several zeros. The zeros are at the end of the number so students may identify each zero as representing a factor of 10 and work with smaller numbers to find the product. For example, they may interpret 350×74 as $10 \times 35 \times 74$. They may also use the standard algorithm for multiplication and pay close attention to the place value of each digit in the product. If students need additional support with the concepts in this lesson, refer back to Unit 4, Section A in the curriculum materials.

Access for:

Students with Disabilities

- Representation (Activity 2)

English Learners

- MLR1 (Activity 1)

Instructional Routines

Estimation Exploration (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min

Teacher Reflection Question

As students have discussions in their small groups today, take note of whose voices are heard and ideas are valued. How has that changed over the course of the school year? What structures would you attribute to this shift?

Cool-down

5 min

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

What is Important?

Standards Alignments

Addressing 5.NBT.B.5

Student-facing Task Statement

What is important to remember when using the standard algorithm to multiply large numbers?

Student Responses

Sample responses:

- It is important to estimate first so you know your answer is reasonable.
- It is important to keep track of the places as you multiply digits.
- It is important to know your basic math facts because you can use them to solve problems with bigger numbers.

----- **Begin Lesson** -----**Warm-up**

⌚ 10 min

Estimation Exploration: Large Product

Standards Alignments

Addressing 5.NBT.B.5

The purpose of an Estimation Exploration is for students to practice the skill of estimating a reasonable answer based on experience and known information.

Instructional Routines

Estimation Exploration

Student-facing Task Statement

$$9,999 \times 896$$

Record an estimate that is:

too low	about right	too high

Student Responses

- too low: 7,000,000–8,000,000
- about right: 8,000,000–10,000,000
- too high: 10,000,000–11,000,000

Launch

- Groups of 2
- Display the expression.
- “What is an estimate that’s too high? Too low? About right?”

Activity

- 1 minute: quiet think time
- 1 minute: partner discussion
- Record responses.
- Monitor for students who estimate by using $10,000 \times 900$.

Synthesis

- Invite students to share estimates.
- “Why is $10,000 \times 900$ a good estimate for the product?” (10,000 is just 1 more than 9,999 and 896 is close to 900.)
- “What is the value of $10,000 \times 900$? How do you know?” (9,000,000 because the two numbers have 6 factors of 10 combined.)

Activity 1

🕒 20 min

Kiran’s Work

Standards Alignments

Addressing 5.NBT.B.5

The purpose of this activity is for students to consider possible mistakes when multiplying large

numbers. Monitor for students who:

- revise their answer after examining Kiran's mistake.
- recognize that $650 \times 10 = 6,500$ so 650×27 has to be much greater than 5,850.
- can explain why Kiran should be multiplying $650 \times 2 \times 10$.
- recognize that $20 \times 50 = 1,000$ so there should be three zeros in the second partial product.

When students determine Kiran's error and make sense of his work, they interpret and critique the work of others (MP3).

Access for English Learners

MLR1 Stronger and Clearer Each Time. Synthesis: Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their response to “what parts of Kiran’s work do you agree and disagree with?” Invite listeners to ask questions, to press for details and to suggest mathematical language. Give students 2–3 minutes to revise their written explanation based on the feedback they receive.

Advances: Writing, Speaking, Listening

Student-facing Task Statement

- Find the value of the product.
- Below is Kiran’s work finding the value of the product 650×27 . Is his answer reasonable? Explain your reasoning.

$$\begin{array}{r} 650 \\ \times 27 \\ \hline \end{array}$$

$$\begin{array}{r} 13 \\ 650 \\ \times 27 \\ \hline 4,550 \\ + 1,300 \\ \hline 5,850 \end{array}$$

- What parts of the work do you agree with? Be prepared to explain your reasoning.
- What parts of the work do you disagree with? Be prepared to explain your reasoning.

Launch

- Display or write for all to see.
 650×27
- Display each number in a different corner of the room:
14,000
18,000
13,000
19,000
- “When I say go, stand in the corner with the number that you think is the most reasonable estimate for 650×27 . Be prepared to explain your reasoning.”
- 1 minute: quiet think time
- Ask a representative from each corner to explain their reasoning.
- “Does anyone want to switch corners?”
- Ask a student who switched corners to explain their reasoning.

- “Now you are going to find this product and analyze some work.”

Activity

- Groups of 2
- 5–7 minutes: partner work time

Synthesis

- Ask previously identified students to share their thinking.
- Display Kiran's work.
- "Why doesn't 5,850 make sense?"
($650 \times 10 = 6,500$ so 650×27 should be a lot larger than 6,500.)
- "What makes sense about Kiran's work?"
($650 \times 2 = 1,300$, but he needs to multiply $650 \times 2 \times 10$.)
- Display a student's solution or the image from the student solution.
- "How do we know that 17,550 is a reasonable value for the product?"
(Because $600 \times 30 = 18,000$.)

2. No. The answer is not reasonable. The second partial product should be a lot larger because we are multiplying $650 \times 10 \times 2$.
3. I agree that $650 \times 7 = 4,550$.
4. I agree with the digits in the second partial product, but they should be moved over one place to the left. The second partial product should be 13,000.

🕒 15 min

Standards Alignments

The purpose of this activity is for students to practice multiplying multi-digit numbers that have one or more digits of 0 at the end. Monitor for students who:

- Unit 8 Lesson 2

- multiply the product 67×89 by 10 to find the value of the product 670×89 .
- multiply the product of 670×89 by 10 to find the value of the product $6,700 \times 89$.

Students who observe that $670 = 10 \times 67$ and $6,700 = 10 \times 670$ and use these relationships to find the values of the products are observing regularity in repeated reasoning and using their knowledge of how to multiply a whole number by 10 (MP7, MP8).

Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Invite students to identify which details were important to find products of multi-digit numbers that have zero digits. Display the sentence frame, “The next time I multiply a number that contains zero digits, I will pay attention to”

Supports accessibility for: Conceptual Processing, Memory

Student-facing Task Statement

Find the value of each product.

1.

$$\begin{array}{r} 260 \\ \times 35 \\ \hline \end{array}$$

2.

$$\begin{array}{r} 2,600 \\ \times 35 \\ \hline \end{array}$$

3.

$$\begin{array}{r} 670 \\ \times 89 \\ \hline \end{array}$$

4.

$$\begin{array}{r} 6,700 \\ \times 89 \\ \hline \end{array}$$

Student Responses

- 9,100
- 91,000
- 59,630. Sample response: I found $67 \times 89 = 5,963$ and then multiplied that by 10.
- 596,300. Sample response:

Launch

- Groups of 2

Activity

- 5–7 minutes: independent work time
- 5–7 minutes: partner discussion

Synthesis

- Display the product $6,700 \times 89$:
- Ask previously identified students to share their solutions.
- Display student work or the image from the sample response:

$$\begin{array}{r} 5 \\ 6 \\ \times 6,700 \\ \times 89 \\ \hline 60,300 \\ + 536,000 \\ \hline 596,300 \end{array}$$

- “What is the relationship between 670×89 and $6,700 \times 89$?” (The product $6,700 \times 89$ is ten times larger because one of the

$$\begin{array}{r}
 5 \\
 6 \\
 6,700 \\
 \times 89 \\
 \hline
 60,300 \\
 + 536,000 \\
 \hline
 596,300
 \end{array}$$

factors is ten times greater.)

- “What is the relationship between 67×89 and $6,700 \times 89$?” (The product $6,700 \times 89$ is 100 times as large as 67×89 since 6,700 is 100×67 .)

Lesson Synthesis

🕒 10 min

“Today we multiplied multi-digit numbers using the standard algorithm. What was challenging about the problems we solved today?” (It was hard to keep track of the numbers as we multiplied. I wasn’t sure how many zeroes to write in the second partial product.)

“What is important to remember when using a standard algorithm to multiply 350×74 ?” (Estimate first so you know if your answer is reasonable. Pay attention to which place each digit is in.)

Display or write the product for all to see.

$$\begin{array}{r}
 350 \\
 \times 74 \\
 \hline
 \end{array}$$

“What is a reasonable estimate for 350×74 ?” (Sample responses: 21,000, 24,000, 28,000.)

Ask students to describe to a partner how they would use the standard algorithm to find the value of the product.

Complete Cool-Down

Response to Student Thinking

Students do not identify information that will help them reason about their solution when using the standard algorithm for multiplication.

Next Day Support

- Launch Activity 1 with a discussion about this cool-down.

Lesson 3: Factors as a Factor in Our Strategy Choice

Standards Alignments

Addressing 5.NBT.B.5

Teacher-facing Learning Goals

- Fluently multiply multi-digit whole numbers using the standard algorithm.

Student-facing Learning Goals

- Let's reason about strategies we use to multiply.

Lesson Purpose

The purpose of this lesson is for students to reason about how the factors in a problem can influence the strategy they use to find the product, in particular identifying cases where the standard algorithm would be useful.

In previous units, students learned to use the standard algorithm to multiply multi-digit numbers and practiced using it in prior lessons. In this lesson, students reason about how the factors in a problem can influence the multiplication strategy they use. They recognize that the standard algorithm can be more useful in some cases than others. Each activity launches with an opportunity for students to review the problems before solving and the syntheses focus on student reasoning about their choices. It is important to note that there is not one correct strategy for each problem. If students need additional support with the concepts in this lesson, refer back to Unit 4, Section A in the curriculum materials.

Access for:

Students with Disabilities

- Action and Expression (Activity 2)

English Learners

- MLR7 (Activity 1)

Instructional Routines

Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min

Teacher Reflection Question

Through teaching, we have the opportunity to learn about our own understandings as well as those of the students. What is one thing you learned today about your own understandings

Lesson Synthesis

10 min

and what is one thing you learned about the students?

Cool-down

5 min

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

Reflect on Multiplication

Standards Alignments

Addressing 5.NBT.B.5

Student-facing Task Statement

Describe something new, interesting, or challenging you learned today about multiplication.

Student Responses

Sample responses: I learned a new strategy. It was interesting to use the factors to choose my strategy. The algorithm was helpful as the size of the factor increased.

----- **Begin Lesson** -----**Warm-up**

⌚ 10 min

Number Talk: Increasing Factors

Standards Alignments

Addressing 5.NBT.B.5

The purpose of this Number Talk is for students to demonstrate strategies and understandings they have for multiplying multi-digit whole numbers. These understandings help students develop fluency. The products in the number talk get increasingly more cumbersome to keep track of mentally so students can identify times when products are more easily found mentally and when the algorithm, with pencil and paper, might be preferable.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- 230×10
- 230×12
- 230×15
- 232×15

Student Responses

- 2,300: $230 \times 10 = 2,300$
- 2,760: $230 \times 10 = 2,300$
 $230 \times 2 = 460$
 $2,300 + 460 = 2,760$
- 3,450: $230 \times 10 = 2,300$
 $230 \times 5 = 1,150$
 $2,300 + 1,150 = 3,450$
- 3,480: $3,450 + 30 = 3,480$

Launch

- Display one problem.
- “Give me a signal when you have an answer and can explain how you got it.”

Activity

- 1 minute: quiet think time
- Record answers and strategy.
- Keep problems and work displayed.
- Repeat with each problem.

Synthesis

- “Which problem was the most challenging to solve mentally? Why do you think so?”
 (Answers vary, but students will likely mention one of the last three because there are different products to keep track of and add.)

Activity 1

🕒 20 min

Choose a Multiplication Strategy

Standards Alignments

Addressing 5.NBT.B.5

The purpose of this activity is for students to consider the numbers when they choose a strategy to find the value of a product. Some students might choose to use the same strategy for any multiplication problem. In this activity, the numbers were chosen to encourage students to

choose different strategies in order to generate discussion about why certain multiplication problems lend themselves to different strategies.

When students choose a multiplication strategy based on the factors, using the associative and commutative properties of multiplication and the distributive property, along with known facts, they make use of structure to facilitate their calculations (MP7).

Access for English Learners

MLR7 Compare and Connect. Lead a discussion comparing, contrasting, and connecting the different strategies students chose to solve the problems. Ask, “Are there any benefits or drawbacks to one strategy compared to another?” and “Why did the different approaches lead to the same outcome?”

Advances: Conversing

Student-facing Task Statement

Find the value of each expression. Explain or show your reasoning.

1. 14×3
2. 14×101
3. 14×25
4. 14×9
5. 14×136

Student Responses

1. 42. Sample response: I multiplied 10×3 and 4×3 and added $30 + 12$
2. 1,414. Sample response: $100 \times 14 = 1,400$ and $1,400 + 14 = 1,414$
3. 350. Sample response: $14 \times 25 = 7 \times 50$ and $7 \times 50 = 350$
4. 126. Sample response: $14 \times 10 = 140$ and $140 - 14 = 126$
5. 1,904. Sample response:

Launch

- Groups of 2
- “Sometimes we use different strategies to solve problems, depending on the numbers in the problem. Without solving, look at the numbers in each expression and think about the strategy you would use to find the value.”
- 1–2 minutes: quiet think time
- “Choose 2 problems you would solve using a different strategy and describe the strategy to your partner. Explain why you chose different strategies.”
- 1–2 minutes: partner work time

Activity

- “Find the value of each expression. If you find the value mentally, record the steps you used in your head to arrive at the product.”
- 5–10 minutes: independent work time
- Monitor for students who used different strategies for the same problem. For example, for 14×25 , some students might

$$\begin{array}{r}
 \begin{array}{r}
 \textcolor{blue}{1} \text{ } \textcolor{blue}{2} \\
 1 \ 3 \ 6 \\
 \times \quad \quad 1 \ 4 \\
 \hline
 \textcolor{blue}{1} \\
 5 \ 4 \ 4 \\
 + \ 1, \ 3 \ 6 \ 0 \\
 \hline
 1, \ 9 \ 0 \ 4
 \end{array}
 \end{array}$$

multiply 14×20 and 14×5 and some students might double 25 and halve 14 and find 7×50 .

- 8–10 minutes: partner work time

Synthesis

- Invite students to share their strategies for finding the value of 14×101 .
- “Is there a mental strategy you can use to find the value of this product?” (Yes. I know 14×100 is 1,400 and then I can add 14 to that.)
- Invite students to share their strategies for finding the value of 14×136 .
- “Which strategy did you use to find the value of 14×136 ? Why?” (I used the standard algorithm because I did not see a quick mental strategy for finding the value.)

Activity 2

🕒 15 min

Compare Strategies

Standards Alignments

Addressing 5.NBT.B.5

The purpose of this activity is for students to try new ideas from the previous activity and practice multiplying using the standard algorithm.

🕒 Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Invite students to verbalize their strategy for finding the value of each expression before they begin. Students can speak quietly to themselves, or share with a partner.

Supports accessibility for: Organization, Conceptual Processing, Language

Student-facing Task Statement

Find the value of each expression.

1. 29×7
2. 12×45
3. 15×199
4. 24×154

Student Responses

1. 203. Sample response: $30 \times 7 = 210$, $210 - 7 = 203$.
2. 540. Sample response: $2 \times 45 = 90$, $6 \times 90 = 540$.
3. 2,985. Sample response: $15 \times 2 = 30$, $15 \times 200 = 3,000$, $3,000 - 15 = 2,985$.
4. 3,696. Sample response:

$$\begin{array}{r}
 1 \\
 2 1 \\
 1 5 4 \\
 \times 2 4 \\
 \hline
 6 1 6 \\
 + 3, 0 8 0 \\
 \hline
 3, 6 9 6
 \end{array}$$

Launch

- Groups of 4

Activity

- 10 minutes: independent work time
- “Compare your strategies with your small group. Where did you use the same strategy? Where did you use a different strategy?”
- 3–5 minutes: small-group discussion

Synthesis

- Ask students to share their strategies for the first 3 problems.
- “Did anyone use a strategy other than the standard algorithm on any of the problems? Why did you choose that strategy?” (For 15×199 I knew 199 is just 1 away from 200 and I could find 15×200 in my head. I just needed to take away 15 and I could do that in my head also.)
- “Does the standard algorithm also work for these problems?” (Yes. The standard algorithm always works.)
- “What strategy did you use to find the product 24×154 ? Why?” (I used the standard algorithm. The numbers are complex and I could not see a good mental strategy.)

Lesson Synthesis

🕒 10 min

“Today we reasoned about how the factors in a problem can influence the multiplication strategy we use. Can someone describe specific numbers that made you choose one strategy over another?” (Sometimes the numbers help me use a mental strategy. For 100×15 I just know it’s 1,500. Or for 99×15 that would be 15 less than 100×15 or 1,485.)

“What are some examples of problems in which we would use the standard algorithm to find a

product?” (If the numbers are complicated like 573×86 . I don’t see a mental approach so the standard algorithm would be a good method.)

----- Complete Cool-Down -----

Response to Student Thinking

Students do not list something they understand really well.

Next Day Support

- Launch the lesson by asking students to recap the important points of the previous lessons.

Lesson 4: Dive Back Into Division

Standards Alignments

Addressing 5.G.B.3, 5.G.B.4, 5.NBT.B.6

Teacher-facing Learning Goals

- Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors.

Student-facing Learning Goals

- Let's estimate and divide.

Lesson Purpose

The purpose of this lesson is for students to estimate and find whole-number quotients with up to four-digit dividends and two-digit divisors.

In previous units students learned how to find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors. They used strategies based on place value, properties of operations, and the relationship between multiplication and division. In this lesson, students apply these understandings as they decide if a quotient is reasonable and write division expressions for a given quotient. If students need additional support with the concepts in this lesson, refer back to Unit 4, Section B in the curriculum materials.

Access for:

Students with Disabilities

- Action and Expression (Activity 2)

Instructional Routines

MLR7 Compare and Connect (Activity 2), Notice and Wonder (Warm-up)

Materials to Gather

- Tools for creating a visual display: Activity 2

Lesson Timeline

Warm-up

10 min

Teacher Reflection Question

In tomorrow's lesson, students revisit an algorithm for division that records partial quotients. How did the mathematics in today's

Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

lesson prepare students for using the partial quotients algorithm?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

Estimate and Evaluate

Standards Alignments

Addressing 5.NBT.B.6

Student-facing Task Statement

1. Estimate the value of $540 \div 15$.
2. Find the value of the quotient.

$$540 \div 15$$

Student Responses

1. Sample responses: 30 or 40 or a number in between.
2. 36

----- Begin Lesson -----

Warm-up

🕒 10 min

Notice and Wonder: Blank Spaces

Standards Alignments

Addressing 5.NBT.B.6

The purpose of this warm-up is to elicit the relationship between multiplication and division, which will be useful when students identify missing dividends and divisors in a later activity. While students may notice and wonder many things about this equation, possible numbers to fill in the blanks are the important discussion points.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?

$$\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = 136$$

Student Responses

Students may notice:

- There is a division sign.
- There are two blanks.
- The division expression has to be equal to 136.
- 136 times a number in the second blank will be the number in the first blank.

Students may wonder:

- What numbers go in the blanks?
- Can there be more than one answer?
- How can I figure out what numbers go in the blanks?
- Could the numbers be fractions or decimals?

Launch

- Groups of 2
- Display the equation.
- “What do you notice? What do you wonder?”

Activity

- 1 minute: quiet think time
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- “What numbers could go in the blanks? How do you know?” (I could put in 136 and 1 or 272 and 2. The first number has to be 136 times the second number.)

Activity 1

 15 min

Reasonable Estimates

Standards Alignments

Addressing 5.NBT.B.6

The purpose of this activity is for students to estimate quotients of multi-digit numbers and reason about multiplication expressions that are helpful when dividing. In previous units, students learned a partial quotients algorithm to divide multi-digit whole numbers. This lesson prepares them to revisit this algorithm in the next lesson.

Student-facing Task Statement

1. Circle the most reasonable estimate. Show your reasoning.

a. $364 \div 13$ 20
 30
 40

b. $540 \div 12$ 40
 50
 60

c. $1,008 \div 14$ 70
 80
 90

2. Find the value of each quotient.

a. b.
 $13 \overline{)364}$ $12 \overline{)540}$

c.
 $14 \overline{)1,008}$

Student Responses

1. Sample responses:
 - a. 30 is the most reasonable because 2 thirteens is less than 30, so 20×13 is too small and 4 thirteens is more than 50, so 40×13 is too big.

Launch

- Groups of 2

Activity

- 5–10 minutes: independent work time
- 5 minutes: partner discussion

Synthesis

- “How did multiplication help you with your estimates?” (Multiplying by tens is easy to do in my head so I could find the product that was closest to the dividend.)
- “How did your estimates help you find the quotient?” (My estimates gave me a good idea what multiple of the divisor to subtract to begin my calculation. They helped me check that my answer was reasonable.)

- b. 40 or 50 is the most reasonable estimate because 4 twelves is 48 and 5 twelves is 60, so 40×12 is a bit less than 540 and 50×12 is a bit more.
- c. 70 is the most reasonable estimate because $14 \times 7 = 98$ so 14×70 is very close to 1,000 and the others are too large.
2. a. $364 \div 13 = 28$
- b. $540 \div 12 = 45$
- c. $1,008 \div 14 = 72$

Activity 2

🕒 20 min

Missing Dividends and Divisors

Standards Alignments

Addressing 5.G.B.3, 5.G.B.4

The purpose of this activity is for students to use the relationship between multiplication and division to determine possible dividends and divisors that have a given value for their quotient (MP7). Monitor for students who:

- multiply the quotient by friendly numbers such as 1, 2, or 5.
- multiply the quotient by powers of ten and use place value understanding.
- can explain the relationship between multiplication and division.

This activity uses *MLR7 Compare and Connect*. Advances: Representing, Conversing

♿ Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Invite students to verbalize their strategy to determine which whole number makes the equation true before they begin. Students can speak quietly to themselves or share with a partner.

Supports accessibility for: Organization, Conceptual Processing, Language

Instructional Routines

MLR7 Compare and Connect

Materials to Gather

Tools for creating a visual display

Student-facing Task Statement

- Write different numbers in the blanks that make the equations true.

$$\begin{array}{rcl} \underline{\hspace{1cm}} \div \underline{\hspace{1cm}} & = & 700 \quad \underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = 78 \\ \underline{\hspace{1cm}} \div \underline{\hspace{1cm}} & = & 700 \quad \underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = 78 \\ \underline{\hspace{1cm}} \div \underline{\hspace{1cm}} & = & 700 \quad \underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = 78 \\ \underline{\hspace{1cm}} \div \underline{\hspace{1cm}} & = & 700 \quad \underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = 78 \\ \underline{\hspace{1cm}} \div \underline{\hspace{1cm}} & = & 700 \quad \underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = 78 \end{array}$$

- What strategy did you use to choose numbers to write in the blanks?

Student Responses

Sample responses:

- $700 \div 1 = 700$, $7,000 \div 10 = 700$,
 $1,400 \div 2 = 700$, $14,000 \div 20 = 700$,
 $2,100 \div 3 = 700$
 - $156 \div 2 = 78$, $7,800 \div 100 = 78$,
 $312 \div 4 = 78$, $78 \div 1 = 78$,
 $780 \div 10 = 78$
- I multiplied the quotient by different numbers. I thought about halving and doubling relationships.

Launch

- Groups of 2

Activity

- 3–5 minutes: independent work time
- “Share your responses with your partner. If you have any equations that are the same, write a new equation that is different. Together, work to find 10 different equations.”
- 3–5 minutes: partner discussion
- Give each group tools for creating a visual display.

MLR7 Compare and Connect

- “Work with your partner to create a visual display that shows your thinking about problems 1 and 2.”
- 2–5 minutes: independent or group work
- 3–5 minutes: gallery walk

Synthesis

- Display: $\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = 700$
- Invite students to share the numbers they used to make the equation true.
- “How did you find numbers that make the equation true?” (I multiplied 700 by different numbers. I chose simple quotients, like dividing by 1 or 2 or 10.)
- Display: $\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = 78$
- “Did the same strategies work to find

numbers that make this equation true?"
(Yes, but 78 is a more difficult number to multiply by in my head. So, 1 worked and 2 and 10, but it was harder to find a variety of numbers.)

Lesson Synthesis

🕒 10 min

Display or write for all to see.

$$\underline{\hspace{1cm}} \div \underline{\hspace{1cm}} = 25$$

"How can you use multiplication to find numbers that make the equation true?" (I know the number in the first blank has to be 25 times the number in the second blank.)

"What are some examples of numbers that make the equation true?" (25 and 1, 50 and 2, 100 and 4, 250 and 10)

"How did you choose the numbers?" (I looked for numbers that are easy to calculate in my head. I know lots of multiples of 25 that I can find in my head.)

----- Complete Cool-Down -----

Response to Student Thinking

Students do not determine a reasonable estimate for the quotient.

Next Day Support

- Before the warm-up, have students work in partners to discuss a correct response to this cool-down.

Lesson 5: More Division

Standards Alignments

Addressing 5.NBT.B.6

Teacher-facing Learning Goals

- Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and the relationship between multiplication and division.

Student-facing Learning Goals

- Let's divide.

Lesson Purpose

The purpose of this lesson is for students to find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and the relationship between multiplication and division.

In previous units, students learned to find whole number quotients by successively subtracting and recording partial quotients.

The purpose of this lesson is for students to practice using this method to find whole number quotients. Students can choose which multiples of the divisor to subtract from the dividend. They reflect on the advantages and disadvantages of their choices when dividing. If students need additional support with the concepts in this lesson, refer back to Unit 4, Section B in the curriculum materials.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Engagement (Activity 2)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Estimation Exploration (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What do your students think it means to be good at math? How are you helping them change negative impressions they might have about their ability to reason mathematically?

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

Partial Quotients

Standards Alignments

Addressing 5.NBT.B.6

Student-facing Task Statement

- Find the value of the quotient.

$$24 \overline{) 2,976}$$

Student Responses

$$\begin{array}{r}
 \boxed{124} \\
 4 \\
 20 \\
 100 \\
 24 \overline{) 2,976} \\
 - 2,400 \\
 \hline
 576 \\
 - 480 \\
 \hline
 96 \\
 - 96 \\
 \hline
 0
 \end{array}$$

 Begin Lesson

Warm-up

⌚ 10 min

Estimation Exploration: Large Quotient

Standards Alignments

Addressing 5.NBT.B.6

The purpose of an Estimation Exploration is for students to practice the skill of estimating a reasonable answer based on experience and known information.

Instructional Routines

Estimation Exploration

Student-facing Task Statement

$$9,953 \div 37$$

Record an estimate that is:

too low	about right	too high

Student Responses

Sample responses

- too low: 100–200
- about right: 200–300
- too high: 300–400

Launch

- Groups of 2
- Display the expression.
- “What is an estimate that’s too high? Too low? About right?”

Activity

- 1 minute: quiet think time
- 1 minute: partner discussion
- Record responses.

Synthesis

- “How do you know 100 is too low?” (Because $100 \times 37 = 3,700$ and that’s less than 9,953.)
- “How can you use the value of the product 100×37 to estimate the value of $9,953 \div 37$?” (I know that 9,953 is more than $2 \times 3,700$ but less than $3 \times 3,700$ so the quotient is more than 200 but less than 300.)

Activity 1

🕒 20 min

Elena's Work

Standards Alignments

Addressing 5.NBT.B.6

The purpose of this activity is for students to revisit the partial quotients method to find whole number quotients. Students compare their strategy with Elena's strategy and reason about the similarities and differences using their understanding of place value. They may use estimation to identify that Elena's answer is not reasonable while they may also use parts of their own calculation to identify Elena's error (MP3).

Student-facing Task Statement

- Find the value of the quotient.

$$13 \overline{)6,773}$$

- Here is how Elena found the quotient. Is her answer reasonable?

Explain or show your reasoning.

53	
1	
2	
50	
13	6773
- 650	(50 × 13)
273	
- 26	(2 × 13)
13	
- 13	(1 × 13)
0	

- What parts of the work do you agree with? Be prepared to explain your reasoning.
- What parts of the work do you disagree with? Be prepared to explain your reasoning.

Launch

- Groups of 2
- "Complete the first problem."
- 1–2 minutes: independent work time

Activity

- "Work with your partner to complete the second, third, and fourth problems."
- 5–7 minutes: partner work time
- "Now you will have a chance to revisit your work from the first problem."
- 1–2 minutes: independent work time
- Monitor for students who:
 - revised their original solution
 - used different partial quotients

Synthesis

- Invite students to share different partial products.
- Keep student work displayed or display provided solutions.

5. Look at your solution to problem 1. Is there anything you want to revise? Be prepared to explain.

Student Responses

Sample responses:

1. **A**

$$\begin{array}{r}
 \boxed{521} \\
 1 \\
 20 \\
 500 \\
 13 \overline{)6,773} \\
 \underline{- 6,500} \quad (500 \times 13) \\
 273 \\
 \underline{- 260} \quad (20 \times 13) \\
 13 \\
 \underline{- 13} \quad (1 \times 13) \\
 0
 \end{array}$$

B

$$\begin{array}{r}
 \boxed{521} \\
 1 \\
 20 \\
 100 \\
 400 \\
 13 \overline{)6,773} \\
 \underline{- 5,200} \quad (400 \times 13) \\
 1,573 \\
 \underline{- 1,300} \quad (100 \times 13) \\
 273 \\
 \underline{- 260} \quad (20 \times 13) \\
 13 \\
 \underline{- 13} \quad (1 \times 13) \\
 0
 \end{array}$$

2. Elena's answer is not reasonable because $1,300 \div 13 = 100$ so the answer is going to be more than 100.
3. She found the products of 13 correctly, like $50 \times 13 = 650$
4. 650 is written so it represents 6,500.
5. Answers vary.

- "How are the solutions the same? How are they different?" (They both subtract some groups of 100, 10, and 1. They both show that the value of the quotient is 521. The first strategy takes out all the hundreds at once while the second strategy takes out 400 and then 100 more.)
- "Why are multiples of 100 good to subtract?" (I can calculate them quickly and they make the number I'm dividing get smaller quickly.)

Activity 2

🕒 15 min

Partial Quotients Practice

Standards Alignments

Addressing 5.NBT.B.6

The purpose of this activity is for students to practice using partial quotients. Students compare their strategy with the strategies of their classmates. They reason about the similarities and differences using their understanding of place value, balancing the complexity of calculations versus subtracting a large amount quickly.

Access for English Learners

MLR8 Discussion Supports. During group work, invite students to take turns sharing their responses. Ask students to restate what they heard using precise mathematical language and their own words. Display the sentence frame: “I heard you say . . .” Original speakers can agree or clarify for their partner.

Advances: Listening, Speaking

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Students may benefit from feedback that emphasizes effort, and time on task. For example, invite students who used one method to use a different method and compare the time it took for each approach.

Supports accessibility for: Conceptual Processing, Social-Emotional Functioning

Student-facing Task Statement

1. Use partial quotients to find the value of one of the quotients. Be prepared to explain how you found the quotient.

Partner A:

$$37 \overline{)2,516}$$

Partner B:

$$32 \overline{)2,272}$$

2. Explain to your partner how you found the quotient in your problem.

Student Responses

1. Partner A

Launch

- Groups of 2, then 4
- “You and your partner will each find a quotient independently. After you’re done, discuss your work with your partner.”

Activity

- 3–5 minutes: independent work time
- 1–3 minutes: partner discussion
- “Now, find another group of 2 and compare your work. How is it the same? How is it different?”

Synthesis

- Ask selected students who used different partial quotients to share or display or write work as shown below for all to see.
- Display:

$$\begin{array}{r}
 \boxed{68} \\
 4 \\
 4 \\
 20 \\
 40 \\
 37 \overline{)2,516} \\
 - 1,480 \\
 \hline
 1,036 \\
 - 740 \\
 \hline
 296 \\
 - 148 \\
 \hline
 148 \\
 - 148 \\
 \hline
 0
 \end{array}$$

Partner B

$$\begin{array}{r}
 \boxed{71} \\
 1 \\
 10 \\
 60 \\
 32 \overline{)2,272} \\
 - 1,920 \\
 \hline
 352 \\
 - 320 \\
 \hline
 32 \\
 - 32 \\
 \hline
 0
 \end{array}$$

2. Answers vary.

$$\begin{array}{r}
 \boxed{71} \\
 1 \\
 70 \\
 32 \overline{)2,272} \\
 - 2,240 \\
 \hline
 32 \\
 - 32 \\
 \hline
 0
 \end{array}$$

$$\begin{array}{r}
 \boxed{71} \\
 1 \\
 10 \\
 20 \\
 20 \\
 20 \\
 32 \overline{)2,272} \\
 - 640 \\
 \hline
 1,632 \\
 - 640 \\
 \hline
 992 \\
 - 640 \\
 \hline
 352 \\
 - 320 \\
 \hline
 32 \\
 - 32 \\
 \hline
 0
 \end{array}$$

- “How are the calculations the same? How are they different?” (They both show that the quotient is 71. One of them finishes very quickly because it starts by taking out 70 groups of 32. The other one takes longer because it subtracts smaller groups of 32.)
- “Which calculation do you prefer?” (The one that takes out all of the thirty-twos at once because it is fast. The one that takes more steps because each calculation is easier to do in my head and I know that I can subtract each multiple of 32, that is I am not trying to take out too much.)

Lesson Synthesis

⌚ 10 min

“Today we compared different ways to find whole number quotients. What questions do you still have about finding whole number quotients?” (Will our method work with larger numbers? Do you always get the same answer no matter which groups you choose to subtract? Is there another way so I don't have to choose the groups at each step?)

Student Section Summary

We investigated some different ways to find products and quotients, making sure to estimate the value before calculating. For example, the product 49×68 is about 50×70 or 3,500. We looked at two different ways to show the newly composed units.

We also found quotients using partial products and saw that there are many different ways to do this.

$$\begin{array}{r}
 \boxed{66} \\
 6 \\
 60 \\
 27 \overline{)1,782} \\
 \underline{- 1,620} \\
 162 \\
 \underline{- 162} \\
 0
 \end{array}$$

$$\begin{array}{r}
 \boxed{66} \\
 1 \\
 5 \\
 30 \\
 30 \\
 27 \overline{)1,782} \\
 \underline{- 810} \\
 972 \\
 \underline{- 810} \\
 162 \\
 \underline{- 135} \\
 27 \\
 \underline{- 27} \\
 0
 \end{array}$$

The first calculation uses only 2 products but the products are more challenging to calculate. The second calculation uses 4 products but they are easier to calculate.

----- Complete Cool-Down -----

Response to Student Thinking

Students do not find the correct value.

Next Day Support

- Before the warm-up, pass back the cool-down and work in small groups to make corrections.

Section B: Apply Volume Concepts

Lesson 6: Revisit Volume

Standards Alignments

Addressing 5.MD.C, 5.MD.C.5

Teacher-facing Learning Goals

- Solve real world and mathematical problems involving volume.

Student-facing Learning Goals

- Let's solve problems about volume.

Lesson Purpose

The purpose of this lesson is for students to solve real world problems about volume.

In this lesson, students calculate the volume of different objects. The first activity recalls the meaning of volume as the number of cubic units required to fill a space. Students experiment with different ways to build a rectangular prism using a fixed number of cubes and relate this to finding factors of the number of cubes. In the second activity they estimate the volume of some very large structures, the great pyramid of Egypt and the Empire State Building. Neither shape is a rectangular prism though they are each made up of smaller shapes that are rectangular prisms. Students combine the skills of making reasonable estimates with finding products of very large numbers. If students need additional support with the concepts in this lesson, refer back to Unit 1, Section A in the curriculum materials.

Access for:



Students with Disabilities

- Action and Expression (Activity 1)



English Learners

- MLR7 (Activity 1)

Instructional Routines

Estimation Exploration (Warm-up)

Lesson Timeline

Warm-up

10 min

Teacher Reflection Question

As students worked together today, where did you see evidence of the mathematical community established over the course of the

Activity 1	15 min	school year?
Activity 2	15 min	
Lesson Synthesis	10 min	
Cool-down	5 min	

Cool-down (to be completed at the end of the lesson)

 5 min

Reflection: Volume

Standards Alignments

Addressing 5.MD.C

Student-facing Task Statement

What are some big ideas about volume that you have learned this year?

Student Responses

Sample response: Volume is how we measure the space inside a rectangular prism. We measure volume in cubic units.

----- Begin Lesson -----

Warm-up

 10 min

Estimation Exploration: Sugar Cubes

   PLC Activity

Standards Alignments

Addressing 5.MD.C.5

The purpose of an Estimation Exploration is for students to practice the skill of estimating a reasonable answer based on experience and known information. In this activity, students estimate the number of sugar cubes in the bowl. During the synthesis, revisit what students know about rectangular prisms and volume and connect it to the image of the sugar cubes.

Instructional Routines

Estimation Exploration

Student-facing Task Statement



How many cubes are in the bowl?

Record an estimate that is:

too low	about right	too high

Student Responses

Sample responses:

- Too low: 3 to 6
- About right: 30 to 40
- Too high: 60 or more

Launch

- Groups of 2
- Display the image.
- “What is an estimate that’s too high?” “Too low?” “About right?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Record responses.

Synthesis

- “What would make it easier to find the exact number of cubes?” (If the cubes were organized in a way that we could count groups. If we could see them all.)

Activity 1

🕒 15 min

126 Cubes

Standards Alignments

Addressing 5.MD.C.5

The purpose of this activity is for students to review the concept that volume is the number of

unit cubes required to fill a space without gaps or overlaps. Students are asked to find all of the different ways that they can arrange 126 sugar cubes to create a rectangular prism. This provides practice with factoring since the side lengths will be factors of 126. If students struggle to find factors of 126, it may be worthwhile to pause early on in the task and discuss the different strategies students are using to find factors of 126.

A variety of different suggestions for how to pack the cubes should be anticipated and encouraged with the focus on how students decide on a particular shape of rectangular prism. In practice, many concerns influence the actual choice such as the amount of packaging material needed and how the package fits on a store shelf. When students interpret the meaning of the numbers in the context, they reason abstractly and quantitatively (MP2).

The goal of the synthesis is to share ideas about predictions for how the cubes are packaged and how students decided they should be packaged.

Access for English Learners

MLR7 Compare and Connect. Synthesis: Invite partners to prepare a visual display that shows the strategy they used to pack the sugar cubes. Encourage students to include details that will help others interpret their thinking. For example, using different colors, shading, arrows, labels, notes, diagrams or drawings. Give students time to investigate each others' work. During the whole-class discussion, ask students, "Did anyone solve the problem the same way, but would explain it differently?"

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Synthesis: Give students access to cubes to build smaller rectangular prisms and invite students to make connections to the task. *Supports accessibility for: Visual-Spatial Processing, Attention, Organization*

Student-facing Task Statement

A company packages 126 sugar cubes in each box. The box is a rectangular prism.



1. What are some possible ways they could pack the cubes?
2. How would you choose to pack the cubes? Explain or show your reasoning.
3. The side lengths of the box are about $1\frac{7}{8}$

Launch

- Groups of 2
- Display the image from the student workbook:



- "These are sugar cubes. They are used to

inches by $3\frac{3}{4}$ inches by $4\frac{3}{8}$ inches. What can we say about how the sugar cubes are packed?

Student Responses

1. Sample responses: 1 by 1 by 126, 1 by 2 by 63, 2 by 7 by 9, 3 by 6 by 7.
2. Sample responses: 3 by 6 by 7 as this has the smallest overall side lengths and will be the easiest to handle. 2 by 7 by 9 will be relatively small and thin (in the direction where there are only two layers).
3. Sample responses:
 - They can't be packed $1 \times 1 \times 126$ because there would be two short sides of the same length and one really long side. The cubes must be less than 1 inch by 1 inch by 1 inch.
 - If the cubes were $\frac{5}{8}$ inch by $\frac{5}{8}$ inch by $\frac{5}{8}$ inch, they are packed in 3 layers of 6 rows with 7 cubes in each row because $3 \times 6 \times 7 = 126$. If the height of the box is $\frac{15}{8}$ then there could be 3 layers of cubes that are each $\frac{5}{8}$ of an inch tall. If the box is 6 cubes long, $6 \times \frac{5}{8} = \frac{30}{8}$ or $3\frac{3}{4}$, and if the width of the box is $4\frac{3}{8}$ it is 7 cubes wide since $7 \times \frac{5}{8} = \frac{35}{8}$ or $4\frac{3}{8}$.

sweeten coffee and tea. Has anyone ever seen or tasted a sugar cube?"

- "You are going to investigate different ways you can arrange 126 cubes to make a rectangular prism and then look at a particular example."

Activity

- 2 minutes: independent work time
- 5–7 minutes: partner work time

Synthesis

- Invite students to share different ways they suggested packing the cubes and their reasoning for the choice.
- "How did you find the different side lengths?" (I divided 126 by different numbers and used multiplication facts I knew to find other combinations.)
- "Why is a 1 by 1 by 126 arrangement not useful for packaging the sugar cubes?" (It would be too long. It would break easily. It would be difficult to handle.)

Activity 2

🕒 15 min

Colossal Structures Old and New

Standards Alignments

Addressing 5.MD.C.5

The purpose of this activity is for students to solve problems about the volume of different buildings. While students can find products of the given numbers, those products do not represent the volume of the structure. In both cases, the Great Pyramid of Egypt and the Empire State Building, neither structure is a rectangular prism. The pyramid steadily decreases in size as it gets taller while the Empire State Building also decreases in size at higher levels but not in the same regular way as the pyramid. With not enough information to make a definitive conclusion, students can see that both structures are enormous and that their volumes are roughly comparable, close enough that more studying would be needed for a definitive conclusion (MP1).

Student-facing Task Statement

1. The base of the Great Pyramid of Egypt is a square. One side length of the base is 230 meters. The pyramid is 140 meters tall. If the pyramid was shaped like a rectangular prism, what would the volume of the prism be?



2. The Empire State Building is in New York City. The base is 129 meters by 59 meters. The building is 373 meters tall. Estimate the volume of the Empire State Building.



3. Which do you think is larger, the Great Pyramid or the Empire State Building? Explain or show your reasoning.

Launch

- Groups of 2
- To help understand how large the Great Pyramid and the Empire State Building are, consider estimating the size of the classroom. Estimates will vary but should be a few hundred cubic meters (versus several million for these huge structures).

Activity

- 5 minutes: independent work time
- 5 minutes: partner work time
- Monitor for students who
 - use the standard algorithm for multiplication
 - make estimates rather than using the standard algorithm for multiplication
 - identify that it is not possible with the given information to find the exact volume of either structure

Synthesis

- Invite students to share their calculations

Student Responses

1. The area of the base is 230×230 or 52,900 square meters. Multiplying by the height gives 7,406,000 cubic meters.
 2. I made an estimate. I first found 130×60 since that was easier than 129×59 and it is 7,800. Then I found $7,800 \times 400$ and it is 3,120,000. So the volume of the Empire State Building is near 3,000,000 cubic meters.
 3. I'm not sure. The 7,406,000 cubic meters for the Great Pyramid is too much because it does not take up all of that space. The 3,000,000 cubic meters for the Empire State Building is too much because I rounded 374 meters to 400 meters to make the calculation simpler. The Empire State Building is not exactly a rectangular prism either since it gets skinnier the farther up you go.
- "Why is it hard to find the exact volume of the Great Pyramid?" It's not a rectangular prism. It has slanted sides.)
 - "Is the product of the area of the base and the height larger than the volume of the pyramid or smaller? How do you know?" (Larger because the pyramid does not fill all of that space. It gets more and more narrow toward the top.)
 - "Why is it hard to find the exact volume of the Empire State Building?" (It's also not a rectangular prism. It also gets narrower toward the top.)
 - "Which do you think has greater volume?" (I think it's too close to tell. I think the Great Pyramid is bigger because it looks like the base of the Empire State Building does not go up very far. It gets a lot narrower quickly. The Great Pyramid gets narrower more gradually.)

Lesson Synthesis

🕒 10 min

"We started the year by exploring volume. What do you remember about the work we did in unit 1?"
(We used cubes. We learned formulas.)

"How did you apply what you learned in unit 1 in today's lesson?" (I knew that the volume of a rectangular prism is the product of length, width, and height and that it is the product of the area of a base and the height. I used those formulas to calculate prism volumes.)

"The blocks used in the Great Pyramid in Egypt have a volume of about 1 cubic meter. About how many blocks were used to build the Great Pyramid?" (There are probably more than one million. There are about 2 million.)

----- Complete Cool-Down -----

Response to Student Thinking

Students do not write big ideas.

Next Day Support

- Add this cool-down to Activity 1 to review.

Lesson 7: Estimating the Volume of the World's Largest Wagon

Standards Alignments

Addressing 5.MD.C.5, 5.NBT.B.5

Teacher-facing Learning Goals

- Multiply and divide multi-digit whole numbers.
- Solve problems involving volume.

Student-facing Learning Goals

- Let's solve problems about volume.

Lesson Purpose

The purpose of this lesson is for students to solve problems involving volume. Students multiply and divide multi-digit whole numbers using the algorithms learned in the previous sections.

The purpose of this lesson is to make an estimate of the volume of the world's largest toy wagon based on an image. In order to make reasoned estimates for the length, width, and height of the wagon students will need to:

- choose an appropriate unit of measure so that the estimates are whole numbers of the right size to visualize
- use the people in the wagon which are the one point of reference allowing to make an accurate estimate of the length, width, and height of the wagon

When students choose an appropriate unit of measure and use the picture to reason about the length, width, and height of the wagon, they are applying abstract mathematical reasoning to make conclusions about a real world object that they most likely have not actually seen (MP4).

For actual images of the wagon, search "World's Largest Toy Wagon 2019."

Access for:



Students with Disabilities

- Engagement (Activity 1)



English Learners

- MLR7 (Activity 2)

Instructional Routines

Notice and Wonder (Warm-up)

Materials to Gather

- Rulers: Activity 2
- Yardsticks: Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

As students worked in their small groups today, whose ideas were heard, valued, and accepted? How can you adjust the group structure tomorrow to ensure each student's ideas are part of the collective learning?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

The Volume of the Wagon

Standards Alignments

Addressing 5.MD.C.5

Student-facing Task Statement

If the Radio Flyer wagon is 27 feet long 13 feet wide and 2 feet deep, what is the volume of the wagon?

Student Responses

Sample response: $27 \times 13 \times 2$ or 702 cubic feet.

----- Begin Lesson -----

Warm-up

🕒 10 min

Notice and Wonder: Radio Flyer

Standards Alignments

Addressing 5.MD.C.5

The purpose of this warm-up is to elicit the idea that the wagon is shaped like a rectangular prism, which will be useful when students find the volume of the wagon in a later activity. While students may notice and wonder many things about this image, the size of the wagon is the important discussion point.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?



Student Responses

Students may notice:

- That is a giant wagon.
- There are 3 people.
- The wheels are bigger than the people.

Students may wonder:

- How big is that wagon?
- Is that wagon real or is it made-up?
- Where is the wagon?
- Can the wagon actually move?
- How many people could fit in that wagon?

Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- "What shapes do you notice in the wagon?"
(The wheels are circles, the body looks like a rectangular prism or box except that the edges are curved a little bit.)

Activity 1

🕒 15 min

Anatomy of an Estimate

Standards Alignments

Addressing 5.MD.C.5, 5.NBT.B.5

The purpose of this activity is for students to make a reasonable estimate for the volume of the wagon which they saw in the warm-up. Since no units are provided, students are also invited to think about which unit of measure is most appropriate for the size of the wagon. In order to compare their estimates, it is preferable if students all choose the same unit of measure, but this is not essential and it is one of the focuses of the activity synthesis. In particular there are 3 unit choices students could make:

- A small unit such as millimeters, centimeters, or inches which will require using large numbers.
- A unit that's just right, for example feet, which will yield more manageable numbers.
- A large unit, such as yards, which gives reasonable numbers to calculate but the height will likely be a fraction or decimal.

Choosing an appropriate unit of measure for an estimation and understanding how that choice affects both the calculations and the meaning of the estimate are important aspects of applying mathematics to solve real world problems (MP4).

The other focus of the activity synthesis is how to use different parts of the image to estimate the volume of the wagon. In particular, the size and number of the people in the wagon are the key points of reference in the image whose size students can reasonably estimate. Make rulers and meter or yardsticks available so that students can measure the length of their legs or the teachers' legs to help make a reasonable estimate.

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Synthesis: Revisit math community norms to prepare students for the whole-class discussion.

Supports accessibility for: Social-Emotional Functioning

Student-facing Task Statement

1. What measurements would you take of the

Launch

- Groups of 2

wagon to accurately estimate its volume?



- What units would you use to measure the wagon? Explain your reasoning.
- Record an estimate for the volume of the wagon that is:

too low	about right	too high

- What can you use in the picture to refine your estimate?

Student Responses

Sample responses:

- I would want to know the length, width, and height of the wagon.
- I would use feet, yards, or meters because the wagon is so big.

too low	about right	too high
100 to 200 cubic feet	400 to 1,000 cubic feet	2,500 cubic feet or greater

- I can use the height of the people to estimate the height of the wagon. I can use the number of people to estimate how long it is. The width is the hardest to estimate because of how the picture is taken.

- "We are going to estimate how large the wagon is and think about how to use the image to refine our estimates."

Activity

- 3–5 minutes: quiet work time
- 5 minutes: partner discussion time
- Monitor for students who:
 - notice that the wagon has a rectangular prism shape, roughly, and recognize that we need to know the side lengths of the wagon in order to make a reasonable estimate about its volume
 - use references, such as the size and number of people in the wagon, to help estimate the wagon's volume
 - choose different units of length and volume for their estimates

Synthesis

- Invite students to share the measurements they would take to make an accurate estimate of the wagon's volume.
- "Why are the length, width, and height important?" (The volume of a rectangular prism is the product of the length, width, and height.)
- Invite students to share their volume estimates, including how they chose their units and made their estimates.
- "How did you choose a unit of measure to estimate the volume?" (I wanted a longer unit because the wagon is big. Inches, centimeters, and millimeters are too small. Feet are good. Yards or meters might work too.)
- "Did anyone use millimeters or centimeters for their estimate?" (No, there would be so many millimeters and centimeters, so finding the product would be challenging)

and estimating the length, width and height would be hard too.)

- “Did anyone use yards or meters for their estimate?” (I tried to but I think that the height of the wagon is less than a yard or meter because the yardstick comes up way above my waist.)
 - “How can you compare an estimate in cubic feet and an estimate in cubic yards?” (I can find out how many cubic feet are in a cubic yard.)
 - “In the next activity you will refine your estimate, looking more carefully at the picture to make the estimate as accurate as possible.”
-

Activity 2

🕒 20 min

Estimating the Size of the Radio Flyer

Standards Alignments

Addressing 5.MD.C.5, 5.NBT.B.5

The goal of this activity is to use multiplication and estimation to improve the estimates from the previous activity for the volume of the wagon. Students will likely count the number of people in the wagon and use this to estimate its length. They can similarly use the people in the wagon to estimate its height. Make rulers or yardsticks available in case students wish to measure how much space they take up when they stand side by side or to measure their legs or waist to estimate how high the wagon is.

While students have the tools available to work with fractional measurements, encourage them to make whole-number estimates so they can use what they learned earlier in the course about the relationship between the (whole number) measurements of a rectangular prism and its volume.

Access for English Learners

MLR7 Compare and Connect. Synthesis: After the Gallery Walk, lead a discussion comparing, contrasting, and connecting the different approaches. Ask, “What did the approaches have in common?”, “How were they different?”, and “Did anyone solve the problem the same way, but would explain it differently?” To amplify student language, and illustrate connections, follow along and point to the relevant parts of the displays as students speak.

Advances: Representing, Conversing

Materials to Gather

Rulers, Yardsticks

Student-facing Task Statement

1. Use the picture of the wagon to make a better estimate of the length, width, and height of the wagon bed.

Make sure to:

- explain how you estimated each measurement
- include how accurate you think each estimate is



2. Then improve your estimate for the volume of the wagon.

Student Responses

Sample response:

1. I counted 10 people along the side of the

Launch

- Groups of 2

Activity

- 3 minutes: individual work time
- 5 minutes: partner work time
- 5–7 minutes: gallery walk
- During the gallery walk, invite students to leave notes with questions for their classmates.
- Invite students to read the questions from their classmates and revise their estimates, if necessary.
- 1–2 minutes: independent work time

Synthesis

- Invite students to share their estimates.
- “Why is there a range of estimates?” (We can’t calculate any of the side lengths exactly. Our estimates for the length, width, and height of the wagon were different.)
- “What would help you make a better estimate?” (Seeing the wagon in person would let us see how high the wagon is and how the length and width compare. More

wagon. Probably each person takes up about 2 feet of space, but there is still some empty space at the end of the wagon. I estimate that the wagon is about 30 feet long. The width is hard to estimate because the angle of the photo makes it look small. It looks like no more than $\frac{1}{3}$ of the length, so I used 10 feet for an estimate. The height is also hard to estimate because of the angle of the photo. The height is below the waist of most of the people, so I used 2 feet for an estimate. I think my estimates are reasonable but the length and width could easily be off by a several feet.

2. My overall estimate for the volume is $30 \times 10 \times 2$ or 600 cubic feet for the volume of the wagon.

photos might help.)

- “What parts of the picture were most helpful in making your estimates?” (The people, because I can estimate how tall they are and see how many of them are lined up along the side of the wagon.)

Lesson Synthesis

🕒 10 min

“Today we made and refined estimates for the volume of the world’s largest wagon.”

“What other question do you have about the wagon?” (Where is it located? How big are the wheels? How heavy is the handle? How many people can it hold altogether?)

“We are going to continue to investigate the wagon tomorrow.”

----- Complete Cool-Down -----

Response to Student Thinking

Students do not write the correct volume.

Next Day Support

- Create a poster with a diagram that represents the cool-down from previous lessons.

Lesson 8: Filling up the World's Largest Wagon

Standards Alignments

Addressing 5.MD.C.5, 5.NBT.B.5, 5.NBT.B.6

Teacher-facing Learning Goals

- Multiply and divide multi-digit whole numbers.
- Solve problems involving volume.

Student-facing Learning Goals

- Let's solve more problems about volume.

Lesson Purpose

The purpose of this lesson is for students to solve problems involving volume. Students multiply and divide multi-digit whole numbers using the algorithms learned in the previous sections.

The purpose of this lesson is for students to solve problems about filling the world's largest toy wagon. In the previous lesson students estimated the volume of the Radio Flyer using information from an image. In this lesson, students are given the dimensions of the wagon and they calculate its volume as the first step in solving problems about how many bags of sand and how many boxes of a given dimension it takes to fill the wagon. Students use what they have learned about volume, the standard algorithm for multiplication, and partial quotients to solve these problems.

Access for:

Students with Disabilities

- Representation (Activity 2)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Notice and Wonder (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	15 min
Lesson Synthesis	10 min

Teacher Reflection Question

How effective were your questions in supporting students' thinking today? What did students say or do that showed they were effective?

Cool-down

5 min

Cool-down (to be completed at the end of the lesson)

🕒 5 min

Multiplication and Division

Standards Alignments

Addressing 5.MD.C.5, 5.NBT.B.5

Student-facing Task Statement

How did you use multiplication and division to solve problems about volume?

Student Responses

Sample response: I used multiplication to figure out how many boxes would fit in the wagon and I used division to figure out how many trips the wagon would need to make to deliver 4,000 boxes. I also used division to figure out how many bags of sand it would take to fill the Radio Flyer and multiplication to figure out the wagon's volume.

----- **Begin Lesson** -----**Warm-up**

🕒 10 min

Notice and Wonder: Toy Boxes

Standards Alignments

Addressing 5.MD.C.5, 5.NBT.B.5, 5.NBT.B.6

The purpose of this warm-up is to elicit the idea that filling the wagon with cubic cardboard boxes is different than filling the wagon with people. While students may notice and wonder many things about this image, the size of the boxes is an important discussion point.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?



Student Responses

Students may notice:

- There are 4 boxes.
- The boxes are smaller than the wagon.
- The boxes have toys in them.

Students may wonder:

- Is that the same wagon as in the last lesson?
- How many boxes will fit in the wagon?

Launch

- Groups of 2
- Display the image.
- “What do you notice? What do you wonder?”

Activity

- 1 minute: quiet think time
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- Focus question:
 - “How is filling the wagon with boxes different than filling the wagon with people?”

Activity 1

🕒 15 min

Sand Wagon

Standards Alignments

Addressing 5.MD.C.5, 5.NBT.B.5, 5.NBT.B.6

The purpose of this activity is for students to apply what they know about multiplication and division to solve problems involving the volume of the Radio Flyer. Students estimated the dimensions and volume of the wagon in the previous lesson and now they learn the actual dimensions and solve problems with those dimensions. The context in this activity is filling the wagon with sand. Students will use multiplication and division to find how many bags of sand it will take to fill the wagon and then they find the cost and weight of all of that sand (MP2). The activity synthesis focuses on a strategic way to calculate the number of bags of sand it takes to fill the Radio Flyer.

To add movement to this activity, students could create a poster for the problems and do a gallery walk to look for similarities and differences in the strategies used to multiply and divide.

Student-facing Task Statement

The Radio Flyer wagon is 27 feet long, 13 feet wide, and 2 feet deep.

1. A 150-pound bag of sand will fill about 9 cubic feet. How many bags of sand will it take to fill the wagon with sand?
2. A 150-pound bag of sand costs about \$12. About how much will it cost to fill the wagon with sand? Explain or show your reasoning.
3. How many pounds of sand does the Radio Flyer hold when it is full? Explain or show your reasoning.

Student Responses

1. 78. Sample response: I first found the volume of the Radio Flyer which is $27 \times 13 \times 2$ cubic feet. That's 702 cubic feet. Then I divided by 9 since each bag of sand can fill 9 cubic feet: $702 \div 9 = 78$.
2. \$936. Sample response: Each bag of sand costs \$12 so I found the product 78×12 and that's 936.
3. 11,700. Sample response: I found 78×150 since each bag of sand weighs 150 pounds and there are 78 of them.

Launch

- Groups of 2
- Display:
27 feet long
13 feet wide
2 feet deep
- "These are the approximate dimensions of the actual Radio Flyer. How do they compare to the estimates you made in the previous lesson?" (We were close for the length and depth but the actual wagon is wider than what we guessed.)
- "Imagine the wagon was being filled with sand. Would you want to buy large bags of sand or small bags of sand? Why?" (I would want large bags because it would take fewer of them.)

Activity

- 2–3 minutes: quiet think time
- 7–8 minutes: partner work time

Synthesis

- Invite students to share their responses for the number of bags of sand it takes to fill the Radio Flyer.

- Display expression: $(27 \times 13 \times 2) \div 9$
- “How does this expression represent the number of bags of sand it takes to fill the Radio Flyer?”(The product is the number of cubic feet the wagon holds and then each bag of sand fills 9 cubic feet so dividing by 9 gives the number of bags of sand you need to fill the wagon.)
- “Jada says that she can find the value of the expression quickly by first finding the value of $27 \div 9$. Will her strategy work? How do you know?” (Rather than finding the product of all 3 numbers and then dividing by 9, Jada notices that she can just divide 27 by 9 and that’s 3. Then she just needs to multiply 3 by 13 and 2 which is easier since they are smaller numbers.)

Activity 2

🕒 15 min

More Boxes

The purpose of this activity is for students to solve another problem about the Radio Flyer using multiplication and division. Instead of filling the wagon with sand, they consider filling the wagon with boxes and determine how many boxes will fill the wagon. Unlike with the sand, the boxes do not fill the wagon completely and the number of boxes that do fit is not a divisor of the total number of boxes. Accounting for these considerations will be the focus of the synthesis. When students account for these constraints of the situation, they persevere in solving the problem (MP1).

Access for English Learners

MLR8 Discussion Supports. Prior to solving the problem, invite students to make sense of the situation and take turns sharing their understanding with their partner. Listen for and clarify any questions about the context.

Advances: Reading, Representing

Access for Students with Disabilities

Representation: Develop Language and Symbols. Provide students with access to the work and notes from the previous activity.

Supports accessibility for: Memory, Attention.

Student-facing Task Statement

The Radio Flyer wagon is 27 feet long 13 feet wide and 2 feet deep.

The wagon is being used to deliver 4,000 boxes that each have the side lengths 2 feet by 2 feet by 2 feet. How many trips will the wagon have to make? Explain or show your reasoning.

Student Responses

52. Sample response: First I needed to find how many boxes the Radio Flyer will hold. The boxes have the same depth as the Radio Flyer so I can only fit one layer of boxes inside. I can make rows of 13 boxes along the long side and I can fit 6 of those rows. So that's 78 boxes I can fit. So I need to find $4,000 \div 78$. I got 51 with 22 boxes leftover so it takes 52 trips altogether to deliver all of the boxes.

Launch

- Groups of 2

Activity

- 2–3 minutes: independent work time
- 7–8 minutes: partner work
- Monitor for students who use partial quotients to find the quotient and interpret the leftover boxes.

Synthesis

- Invite students to share their responses for the number of trips.
- “How did you find how many boxes will fit in the wagon?” (I counted how many rows of boxes I could fit and how many were in each row.”
- “Did the boxes fill the wagon?” (No. The length and width of the wagon are odd and the boxes are all 2 feet wide and long so there is 1 foot of empty space left on both sides.)
- “Do you think you could fit more boxes and make fewer trips?” (Yes. I could definitely fit those extra 22 boxes along the side where

$$\begin{array}{r}
 \boxed{51} \\
 1 \\
 20 \\
 30 \\
 \hline
 78 \overline{)4,000} \\
 \underline{-2,340} \\
 1,660 \\
 \underline{-1,560} \\
 100 \\
 \underline{-78} \\
 22
 \end{array}$$

it is not full in all of those other trips. I would just need to be careful so they don't fall over the side.)

Lesson Synthesis

🕒 10 min

"What strategies for multiplication and division did you find most helpful today? Why were they helpful?" (I used the standard algorithm to multiply because some of the numbers were large and I could not see a mental strategy that would work. I used partial quotients for division. It took time but it helped me keep track of my calculations.)

----- Complete Cool-Down -----

Response to Student Thinking

Students do not reference multiplication or division in their response.

Next Day Support

- Create a poster with important terms or vocabulary from this cool-down.

Lesson 9: Problem Solving with Volume: Water (Optional)

Standards Alignments

Addressing 5.MD.C.5

Teacher-facing Learning Goals

- Solve real world and mathematical problems involving volume.

Student-facing Learning Goals

- Let's solve problems about volume of water.

Lesson Purpose

The purpose of this lesson is for students to solve real world problems about volume.

In this optional lesson, students apply their understanding of volume to relate the amount of water that falls on a house roof to the amount of water a family might use for everyday activities. This lesson is optional because it requires conversions between different measurement systems. The calculations, especially those for family water use, require detailed estimates. Students can make their own estimates to increase the modeling aspect of the activity or they can use provided estimates. Students may consider a variety of uses of water beyond those highlighted in the materials such as the water they drink or the water used for plants.

When they compare the volume of water that falls on a house roof to the amount of water they use each month, students will use an important feature of the metric system, namely that length and volume units are naturally related to one another and the context gives students a chance to practice multiple conversions.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR5 (Activity 1)

Instructional Routines

Notice and Wonder (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What do your students think it means to be good at math? How are you helping them change negative impressions they might have about their ability to reason mathematically?

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

Reflection: Volume

Standards Alignments

Addressing 5.MD.C.5

Student-facing Task Statement

What questions do you still have about measuring volume?

Student Responses

Sample responses:

- How can we find the volume of something that isn't a rectangular prism?
- Is it possible to use the same strategy to find the volume for liquid?
- I know 2 numerical ways of finding the volume of a rectangular prism. Is there another way?

----- **Begin Lesson** -----**Warm-up**

⌚ 10 min

Notice and Wonder: Cubic Centimeters and Grams

Standards Alignments

Addressing 5.MD.C.5

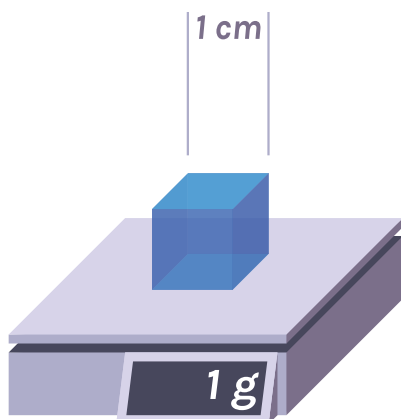
The purpose of this warm-up is for students to observe the relationship between the different types of units in the metric system. By contrast, in the standard system, it is not easy to see the relationship between inches, cups, and pounds. While students may notice and wonder many things about this image, conversions between liquid volume units (cups, gallons, liters) and regular volume units (cubic centimeters, cubic inches, cubic feet) are the important discussion points.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?



Student Responses

Students may notice

- There is a scale.
- The weight shows 1 gram.
- There is a cube.

Students may wonder

- What does the cube represent?
- Is that a cube?
- Does g stand for grams?

Launch

- Groups of 2
- Display the image.
- “What do you notice? What do you wonder?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- Highlight that the cube represents 1 cubic centimeter or 1 mL of water and one mL of water weighs 1 g.
- The picture shows the relationships between length, capacity (or volume), and weight in the metric system.

Activity 1

🕒 15 min

Catching Rainfall

Standards Alignments

Addressing 5.MD.C.5

The purpose of this activity is for students to estimate how much water falls on the roof of a house, given a particular amount of rainfall. For this calculation, standard units work well as the area of the roof could be given in square feet, for example, and the rain in inches. A conversion would readily give the volume in cubic feet or inches. But, the standard units used to measure volume are cups, pints, quarts, and gallons so more work would need to be done in order to figure out how many gallons, for example, there are in a cubic foot. With the metric system, liquid volume units (liters) and regular volume units (cubic centimeters) are naturally connected.

🌐 Access for English Learners

MLR5 Co-Craft Questions. Keep books or devices closed. Display only the image, without revealing the questions, and ask students to write down possible mathematical questions that could be asked about the situation. Invite students to compare their questions before revealing the task. Ask, “What do these questions have in common? How are they different?” Reveal the intended questions for this task and invite additional connections.

Advances: Reading, Writing

♿ Access for Students with Disabilities

Representation: Access for Perception. Read statements aloud. Students who both listen to and read the information will benefit from extra processing time.

Supports accessibility for: Conceptual Processing, Attention

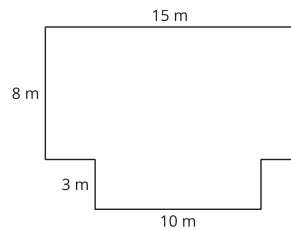
Student-facing Task Statement

Here is a diagram showing the roof of a house.

1. What is the area of the roof?

Launch

- Groups of 2
- “About how big is a square meter?” (It's about the size of my desk top.)
- “About how many square meters do you think there are in the classroom floor?” (maybe 100)



- Each month an average of 5 cm of rain falls on the house. How many cubic cm of rain is that?
- There are 1,000 cubic cm in 1 liter. How many liters of water fall on the house?
- You want to build a reservoir to catch the rain that falls so you can use the water. What side lengths would you suggest for the reservoir? Explain or show your reasoning.

Student Responses

- 150 square meters. Sample response: $(8 \times 15) + (3 \times 10) = 120 + 30$
- 7,500,000. Sample response: since there are 100 centimeters in a meter 150 square meters is $150 \times 100 \times 100$ or 1,500,000 square centimeters. To get the volume that needs to be multiplied by the height or depth of the water which is 5 cm, $5 \times 1,500,000 = 7,500,000$.
- 7,500. Sample response: There are 1,000 cubic centimeters in a liter so that's $7,500,000 \div 1,000$ or 7,500 liters of water.
- A cube with side length 2 meters would be 200 cm by 200 cm by 200 cm and its volume would be 8,000,000 cubic centimeters. That would be enough to hold the water.

Activity

- 5 minutes: independent work time
- 5 minutes: partner work time

Synthesis

- "When did you have to convert units during the activity?" (I needed to convert meters to centimeters since the roof is given in meters and the rain is given in centimeters. I needed to convert cubic centimeters to liters.)
- "How did you make the calculations for the conversions?" (the conversions required multiplying or dividing by powers of 10. I multiplied by 100 to convert m to cm and I divided by 1,000 to convert cubic centimeters to liters.)
- "Do you think that you could make a container to capture all of the rainfall?" (Yes, I think that a 2 meter cube is big but it's not so big that it would not fit near the house. Or we could use several containers that are a little smaller.)

Activity 2

How Much Water?

🕒 20 min

Standards Alignments

Addressing 5.MD.C.5

The purpose of this activity is for students to find out if the amount of water that falls on the house is sufficient for many of the daily household chores that use water. This will require a lot of estimation and will vary from house to house. How much of the calculations to leave up to the students is an individual teacher choice and this lesson could easily be extended for another day if the students make well reasoned estimates (some values are given in parentheses) for how much water is used for different activities such as:

- taking baths or showers (150 liters or 80 liters)
- washing clothes (100 liters)
- washing dishes (100 liters)
- washing hands (1 liter)
- flushing the toilet (10 liters)

More estimation comes into play for how often each of these activities happens and this will vary greatly depending on the student. When students make a list of the different things they do in the house that use water and then estimate how much water is used they model with mathematics (MP4).

Consider inviting students to check their estimates by looking at one of their monthly water bills. The bill will usually give the number of gallons of water used and there are almost 4 liters in a gallon.

Student-facing Task Statement

1. What are some of the ways you use water at home?
2. Estimate how much water you use at your home in a month.
3. How much rain would need to fall on your home each month to supply all of your water needs?
4. What challenges might come up if you tried to use the rainwater that falls on the roof of your home? Do you think it makes sense to try to capture the rain that falls on your home?

Launch

- Groups of 2
- After students work on the first problem, pause the class and make a list of the main daily uses of water.
- Depending on how much time is available and the modeling demand level desired, consider estimating together or providing estimates for how much water is used for each purpose.

Activity

- 5 minutes: independent work time

Student Responses

Sample responses:

1. Taking a shower, washing hands, using the toilet, washing clothes, and washing dishes.
2. For each person, 30 showers in a month makes 3,000 liters. I wash my hands 10 times a day so that's 10 liters in a day or 300 liters in a month. I use the toilet 5 times a day for 50 liters a day or 1,500 liters in a month. We wash clothes 10 times a month so that's 1,000 liters and we wash dishes once a day so that's 3,000 more liters. The clothes and dishes make 4,000 liters and there are 5 people in the house. The rest adds up to 4,800 liters for 1 person so 24,000 for all of us. In all, that's 28,000 liters of water.
3. If we get 1 cm of rain in a month that would be 1,500,000 cubic centimeters of water which is 1,500 liters of water. We need almost 20 times that amount so about 20 cm of rain a month. That's a lot of rain and I'm not sure we always get that much.
4.
 - I live in an apartment building so would need to share the water with others.
 - I live in a place where it does not rain very much.
 - I'm not sure if rainwater is safe to use in the house.

- 10 minutes: partner work time

Synthesis

- "How can you visualize the volume of water that it uses to take a bath?" (I can picture the bathtub filled up partway with water.)
- "How could you measure the volume of water in the bathtub?" (I could measure the length and width of the tub and the height of the water and multiply them.)
- "Are any of the amounts of water used for different things surprising to you? Why?" (I am surprised by how much water it takes to wash the dishes. It's almost as much as when you take a bath.)

Lesson Synthesis

🕒 10 min

"How is measuring the volume of water the same as measuring the volume of the Empire State Building?" (If I know the length, width, and height that the water takes up, then I can multiply them to get the volume, just like the building.)

"How is measuring the volume of water different than measuring the volume of a building?" (Water does not have a simple shape like a building. It needs to be put in a container in order to measure.)

"What is important to remember when measuring volume?" (It's the amount of space something can hold or that something takes up. I can measure it in cubic units or in liters for a liquid.)

Student Section Summary

We investigated several different complex volume questions. For the ancient pyramids of Egypt we gave an estimate of a couple million cubic meters. Since these pyramids are not rectangular prisms, an estimate is the best we could hope for. Then we estimated the volume of the world's largest wagon, using information from a photograph. Lastly, we investigated the amount of rain that falls on a house and the amount of water our families use in a year.

In each case, we could only make estimates because the situations are all complex. In the previous section we used estimation to check the reasonableness of calculations. In this section we saw that making reasoned estimates is a vital part of applying mathematics to many real world situations.

----- Complete Cool-Down -----

Response to Student Thinking

Students have ideas they could share with a partner.

Next Day Support

- After the warm-up in the next lesson, pair students up to discuss their responses.

Section C: Fraction and Decimal Operations

Lesson 10: Here Comes the Sum

Standards Alignments

Addressing 5.NF.A.1

Teacher-facing Learning Goals

- Add fractions with unlike denominators.

Student-facing Learning Goals

- Let's play some games to practice adding fractions.

Lesson Purpose

The purpose of this lesson is for students to practice adding fractions with unlike denominators.

In previous units, students learned how to add and subtract fractions with unlike denominators. In this lesson, students play games in which they must apply their previous understanding of addition of fractions.

Access for:

Students with Disabilities

- Engagement (Activity 1)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Number Talk (Warm-up)

Materials to Gather

- Paper clips: Activity 1
- Paper clips: Activity 2

Lesson Timeline

Warm-up

10 min

Teacher Reflection Question

Which strategy did students use today that you anticipated? Which did you not anticipate?

Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Cool-down (to be completed at the end of the lesson)

🕒 5 min

Reflect on Fraction Addition

Standards Alignments

Addressing 5.NF.A.1

Student-facing Task Statement

What is important to remember when adding fractions with unlike denominators?

Student Responses

Sample response: I need to find a common denominator and then I can add the numerators. One way to find a common denominator that always works is to take the product of the two denominators.

Begin Lesson

Warm-up

🕒 10 min

Number Talk: Adding Fractions

Standards Alignments

Addressing 5.NF.A.1

The purpose of this Number Talk is for students to demonstrate strategies they have for adding fractions with unlike denominators. These understandings help students develop fluency and will be helpful later in this lesson when students will need to be able to add and subtract fractions with unlike

denominators.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $\frac{2}{12} + \frac{1}{6}$
- $\frac{2}{6} + \frac{1}{2}$
- $\frac{1}{3} + \frac{1}{2}$
- $\frac{1}{3} + \frac{3}{2}$

Student Responses

- $\frac{2}{6}$ or $\frac{4}{12}$: $\frac{2}{12} = \frac{1}{6}$ so I doubled the number of sixths.
- $\frac{5}{6}$: $\frac{2}{6} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6}$
- $\frac{5}{6}$: It is the same as the one before because $\frac{1}{3} = \frac{2}{6}$.
- $1\frac{5}{6}$: It is just adding $\frac{2}{2}$ to the problem before it.

Launch

- Display one expression.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- “How do equivalent fractions help us add fractions with unlike denominators?” (It lets me find fractions with the same denominators and then I can just add the numerators.)
- “How do you decide on the denominator to use for your equivalent fractions?” (I either use a common multiple of the 2 denominators that I know or I can multiply the 2 denominators to find a common multiple.)

Activity 1

Greatest Sum

⌚ 15 min

Standards Alignments

Addressing 5.NF.A.1

The purpose of this activity is for students to practice adding fractions with unlike denominators and to reason about how the size of the numerators and denominators impacts the value of a fraction (MP7). Monitor for students who:

- place any large numbers like 4, 5, and 6 in the numerator, if possible, and smaller numbers like 1, 2, and 3 in the denominator
- notice that there are many 1's and 2's on the spinner and try to wait for these and use them as denominators

Access for English Learners

MLR8 Discussion Supports. Prior to solving the problems, invite students to make sense of the situations and take turns sharing their understanding with their partner. Listen for and clarify any questions about the context.

Advances: Reading, Representing

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Leverage choice around perceived challenge. Invite students to select the order of problems in which they complete. Invite students to explain why they found which sum before others.

Supports accessibility for: Organization, Attention, Social-emotional skills

Materials to Gather

Paper clips

Required Preparation

- Each group of 2 needs 1 paper clip for their spinner.

Student-facing Task Statement

Use the directions to play Greatest Sum with a partner.

1. Spin the spinner.
2. Each player writes the number that was spun in an empty box for Round 1. Be sure

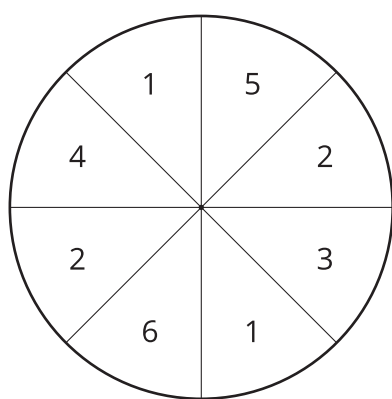
Launch

- Groups of 2

• Display: $\frac{\boxed{}}{\boxed{}} + \frac{\boxed{}}{\boxed{}}$

your partner cannot see your paper.

3. Once a number is written down, it cannot be changed.
4. Continue spinning and writing numbers in the empty boxes until all 4 boxes have been filled.
5. Find the sum.
6. The person with the greater sum wins the round.
7. After all 4 rounds, the player who won the most rounds wins the game.
8. If there is a tie, players add the sums from all 4 rounds and the highest total sum wins the game.



Round 1

$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} + \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} =$$

Round 2

$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} + \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} =$$

Round 3

$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} + \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} =$$

Round 4

$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} + \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} =$$

Total sum of all 4 rounds:

Student Responses

Answers vary.

- Spin the spinner.
- Write the number in one of the four boxes.
- Repeat until all four boxes are filled.
- Ask students to compute the sum.
- "Is it possible to get a larger sum by placing the 4 digits in different boxes?"
- 1 minute: quiet think time
- Ask students to share.
- Give students paper clips.
- "Now you will play 4 rounds of the Greatest Sum with your partner."

Activity

- 10–12 minutes: partner work time

Synthesis

- "What strategies were helpful as you played Greatest Sum?" (I tried to make fractions that have a larger numerator than denominator so they would be greater than one. I tried to make sure the ones and twos were in the denominator and put bigger numbers in the numerator.)
- "How did you add your fractions?" (My denominators were 1, 2, 3, and 4 so I used 12 as a common denominator for all of them.)

Activity 2

🕒 20 min

Smallest Sum

Standards Alignments

Addressing 5.NF.A.1

The purpose of this activity is for students to practice adding fractions with unlike denominators. Monitor for students who notice that the overall strategy in this game is the same as in the previous game except that the numbers that they placed in the numerator in the first game go in the denominator in this game and similarly the numbers that went in the denominator in the first game go in the numerator in this game (MP8).

Materials to Gather

Paper clips

Required Preparation

- Each group of 2 needs a paper clip.

Student-facing Task Statement

Use the directions to play Smallest Sum with a partner.

1. Spin the spinner.
2. Each player writes the number that was spun in an empty box for Round 1. Be sure your partner cannot see your paper.
3. Once a number is written down, it cannot be changed.
4. Continue spinning and writing numbers in the empty boxes until all 4 boxes have been filled.
5. Find the sum.
6. The person with the lesser sum wins the round.

Launch

- Groups of 2
- "Take a minute to read over the directions for Smallest Sum."
- 1 minute: quiet think time
- Give students paper clips.
- "Play Smallest Sum with your partner."

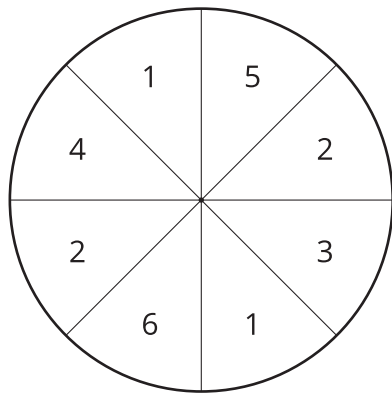
Activity

- 10–15 minutes: partner work time

Synthesis

- "What strategies were helpful as you played Smallest Sum?" (I tried to make unit fractions with large denominators. I used

7. After all 4 rounds, the player who won the most rounds wins the game.
8. If there is a tie, players add the sums from all 4 rounds and the lesser total sum wins the game.



Round 1

$$\frac{\boxed{}}{\boxed{}} + \frac{\boxed{}}{\boxed{}} =$$

Round 2

$$\frac{\boxed{}}{\boxed{}} + \frac{\boxed{}}{\boxed{}} =$$

Round 3

$$\frac{\boxed{}}{\boxed{}} + \frac{\boxed{}}{\boxed{}} =$$

Round 4

$$\frac{\boxed{}}{\boxed{}} + \frac{\boxed{}}{\boxed{}} =$$

Total sum of all 4 rounds:

Student Responses

Answers vary.

the opposite strategy to the previous game, trying to put the smallest numbers in the numerator and the largest numbers in the denominator.)

Lesson Synthesis

⌚ 10 min

"Today, we played some games that helped us practice adding fractions. How did the games help you think about adding fractions?" (I had to add the fractions that I made from the numbers I got with the spinner. The fractions in the sums had small denominators so it was not hard to find a common denominator.)

Complete Cool-Down

Response to Student Thinking

Students have ideas they could share with a partner.

Next Day Support

- After the warm-up in the next lesson, pair students up to discuss their responses.

Lesson 11: What's the Difference?

Standards Alignments

Addressing 5.NF.A.1

Teacher-facing Learning Goals

- Subtract fractions and mixed numbers.

Student-facing Learning Goals

- Let's subtract fractions.

Lesson Purpose

The purpose of this lesson is for students to practice subtracting fractions with unlike denominators.

This lesson complements the previous lesson except that students are looking to make the greatest or smallest differences rather than sums. Trying to make the greatest difference supports strategies like students used in the previous lesson, looking to choose a large numerator and small denominator for one fraction and a small numerator and large denominator for the other (MP7). Trying to make a small difference brings a new feature into play, namely that the difference can be 0 if the fractions are equivalent. Other than looking for equivalent fractions, students may also realize that if both fractions are small, then their difference will also be small.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Number Talk (Warm-up)

Materials to Gather

- Paper clips: Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min

Teacher Reflection Question

How effective were your questions in supporting students' thinking today? What did students say or do that showed they were effective?

Lesson Synthesis	10 min
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Cool-down	5 min
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Cool-down (to be completed at the end of the lesson)

🕒 5 min

Reflect on Subtracting Fractions

Standards Alignments

Addressing 5.NF.A.1

Student-facing Task Statement

What is important to remember when subtracting fractions with unlike denominators?

Student Responses

Sample response: You have to find common denominators before you can subtract.

Begin Lesson

Warm-up

🕒 10 min

Number Talk: Subtracting Fractions

Standards Alignments

Addressing 5.NF.A.1

The purpose of this Number Talk is for students to demonstrate strategies and understandings they have for subtracting fractions with unlike denominators. These understandings help students develop fluency and will be helpful later in this lesson when they will need to be able to subtract fractions with unlike denominators.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each difference mentally.

- $\frac{2}{3} - \frac{1}{6}$
- $\frac{2}{3} - \frac{1}{2}$
- $\frac{2}{3} - \frac{4}{6}$
- $\frac{2}{3} - \frac{1}{4}$

Student Responses

- $\frac{3}{6}$ or $\frac{1}{2}$: $\frac{2}{3} - \frac{1}{6} = \frac{4}{6} - \frac{1}{6}$
- $\frac{1}{6}$: $\frac{2}{3} - \frac{1}{2} = \frac{4}{6} - \frac{3}{6}$
- 0: They are equivalent.
- $\frac{5}{12}$: $\frac{2}{3} - \frac{1}{4} = \frac{8}{12} - \frac{3}{12}$

Launch

- Display one expression.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- Display first two differences.
- “How are the differences related?” ($\frac{1}{6}$ and $\frac{1}{2}$ are $\frac{2}{3}$ so I can take away $\frac{1}{6}$ and that gives me $\frac{1}{2}$ or I can take away $\frac{1}{2}$ and that gives me $\frac{1}{6}$.)

Activity 1

🕒 15 min

Greatest Difference

Standards Alignments

Addressing 5.NF.A.1

The purpose of this activity is for students to practice subtracting fractions with unlike denominators. The structure of this activity is identical to the first activity in the previous lesson except that students are calculating differences instead of sums. Monitor for students who:

- try to make one fraction in each pair as large as they can and the other fraction as small as they can

- find a common denominator for all 4 differences in order to add them together

While playing the game, students may find that they have a smaller fraction as the minuend and a larger fraction as the subtrahend. There are several ways students could navigate this situation, one of which is to switch the order of the fractions.

Access for English Learners

MLR8 Discussion Supports. Prior to solving the problems, invite students to make sense of the situations and take turns sharing their understanding with their partner. Listen for and clarify any questions about the context.

Advances: Reading, Representing

Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Invite students to identify which details were most useful to solve the problem. Display the sentence frame, “The next time I subtract fractions, I will look for/pay attention to . . .”

Supports accessibility for: Conceptual Processing, Attention, Memory

Materials to Gather

Paper clips

Required Preparation

- Each group of 2 needs a paper clip.

Student-facing Task Statement

Use the directions to play Greatest Difference with a partner.

1. Spin the spinner.
2. Each player writes the number that was spun in an empty box for Round 1. Be sure your partner cannot see your paper.
3. Once a number is written down, it cannot be changed.
4. Continue spinning and writing numbers in the empty boxes until all 4 boxes have been filled.
5. Find the difference.

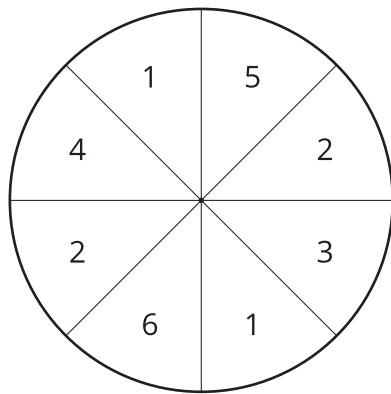
Launch

- Groups of 2
- “Take a minute to read over the directions for Greatest Difference.”
- 1 minute: quiet think time
- “Play Greatest Difference with your partner.”

Activity

- 10–12 minutes: partner work time

6. The person with the greatest difference wins the round.
7. After all 4 rounds, the player who won the most rounds, wins the game.
8. If there is a tie, players add the differences from all 4 rounds and the highest total wins the game.



Round 1

$$\frac{\boxed{}}{\boxed{}} - \frac{\boxed{}}{\boxed{}} =$$

Round 2

$$\frac{\boxed{}}{\boxed{}} - \frac{\boxed{}}{\boxed{}} =$$

Round 3

$$\frac{\boxed{}}{\boxed{}} - \frac{\boxed{}}{\boxed{}} =$$

Round 4

$$\frac{\boxed{}}{\boxed{}} - \frac{\boxed{}}{\boxed{}} =$$

Synthesis

- “What strategies were helpful as you played Greatest Difference?” (I tried to make the first fraction in each pair as large as possible and the second fraction in each pair as small as possible.)
- “What is the biggest difference possible between fractions in this game? How do you know?” ($\frac{6}{1} - \frac{1}{6}$ since $\frac{6}{1}$ is the biggest number and $\frac{1}{6}$ is the smallest.)
- “Did anyone get $\frac{6}{1} - \frac{1}{6}$ as one of their fractions?” (Answers vary.)

Student Responses

Answers vary.

Activity 2

🕒 20 min

What is the Smallest Difference?

Standards Alignments

Addressing 5.NF.A.1

The purpose of this activity is for students to practice subtracting fractions with unlike denominators. This activity has the same structure as the previous activity except that students are looking for the smallest difference rather than the largest difference and this time they are given all of the numbers at once rather than spinning them one at a time. Some strategies to monitor for include:

- trying to get differences that are 0 using equivalent fractions
- trying to make all of the fractions as small as possible, that is using a similar strategy to the sum game when they tried to get the smallest possible sum

While playing the game, students may find that they have a smaller fraction as the minuend and a larger fraction as the subtrahend. There are several ways students could navigate this situation, one of which is to switch the order of the fractions.

Student-facing Task Statement

Use the numbers below to fill in the squares. Find each difference. Add the 2 differences together.

1	1	2	2
3	4	5	6

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Student Responses

Sample response: $\frac{1}{2} - \frac{1}{2} = 0$, $\frac{4}{6} - \frac{3}{5} = \frac{2}{30}$,

Launch

- Groups of 2
- "Now, let's see how close you can get to a difference of 0 using a given set of numbers."

Activity

- 5 minutes: independent work time
- "Compare your total with your partner to see who is closest to 0. Describe your strategy to your partner."
- 10 minutes: partner discussion

Synthesis

- "What strategies were helpful as you tried to find the smallest difference?" (I tried to choose equivalent fractions so that I could get a difference of 0. I tried to choose small fractions so then the difference would also be small.)

$$0 + \frac{2}{30} = \frac{2}{30}$$

- “Did anyone choose numbers that made the second fraction larger than the first?” (Yes.)
- “What did you do?” (I switched the order of the fractions so that I could subtract.)
- “Was anyone able to get a total of 0?” (No. I could get some equivalent fractions with sixths, or fourths, thirds, or halves but there was nothing I could do with fifths. If I put the 5 in the numerator I can’t make an equivalent fraction.)

Lesson Synthesis

🕒 10 min

“Today, we played some games that involved adding and subtracting fractions. What advice would you give to someone who was learning how to add and subtract fractions with unlike denominators?” (When the denominators are small like the ones we worked on today, you can usually see a common denominator. Even if you have to use the product of the denominators, it’s not bad for fractions like fifths and sixths since 5 times 6 is 30.)

Complete Cool-Down

Response to Student Thinking

Students have ideas they could share with a partner.

Next Day Support

- After the warm-up in the next lesson, pair students up to discuss their responses.

Lesson 12: Decimal Game Day

Standards Alignments

Addressing 5.NBT.B.7

Teacher-facing Learning Goals

- Add, subtract, multiply, and divide decimals to hundredths.

Student-facing Learning Goals

- Let's play some games and practice adding and subtracting decimals.

Lesson Purpose

The purpose of this lesson is for students to practice adding and subtracting decimals.

In a previous unit, students learned how to add and subtract decimals.

In this lesson, students play games where they practice adding and subtracting decimals. The first game resembles the games students played in the previous two lessons. The second game has more complex rules but a simpler goal which is to make the largest number possible and then continue to add those numbers until reaching a given goal.

Access for:

Students with Disabilities

- Action and Expression (Activity 1)

English Learners

- MLR8 (Activity 1)

Instructional Routines

True or False (Warm-up)

Materials to Gather

- Number cubes: Activity 1
- Paper clips: Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	15 min

Teacher Reflection Question

When did students listen to one another's ideas today in class? What norms would help each student better attend to their classmates' ideas in future lessons?

Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Cool-down (to be completed at the end of the lesson)

🕒 5 min

Reflect on Operating with Decimals

Standards Alignments

Addressing 5.NBT.B.7

Student-facing Task Statement

What is important to remember when adding decimal numbers?

Student Responses

Sample response: Make sure to add digits with the same place value.

----- Begin Lesson -----

Warm-up

🕒 10 min

True or False: Adding Decimals

Standards Alignments

Addressing 5.NBT.B.7

The purpose of this True or False is for students to demonstrate strategies and understandings they have for adding decimals. These understandings help students deepen their understanding of the properties of operations and will be helpful later in the lesson when students will need to be able to add decimals.

Instructional Routines

True or False

Student-facing Task Statement

Decide if each statement true or false. Be prepared to explain your reasoning.

- $0.99 + 0.1 = 0.9 + 0.1 + 0.09$
- $0.99 + 0.01 = 0.9 + 0.1$
- $0.99 + 0.1 = 1.99$

Student Responses

- True.
 $0.99 + 0.1 = 0.9 + (0.09 + 0.1) = 0.9 + (0.1 + 0.09)$
- True.
 $0.99 + 0.01 = 0.9 + (0.09 + 0.01) = 0.9 + 0.1$
- False.
 $(0.9 + 0.09) + 0.1 = (0.9 + 0.1) + 0.09 = 1 + 0.09$

Launch

- Display one equation.
- “Give me a signal when you know whether the equation is true and can explain how you know.”
- 1 minute: quiet think time

Activity

- Share and record answers and strategy.
- Repeat with each equation.

Synthesis

- Display the first and third equations.
- “How can you justify your answer without evaluating both sides?” (For the first problem, I noticed that there are 9 tenths on the right and left and also 10 hundredths. For the third problem, I saw that the right hand side is almost 2 so it's much larger than the left hand side.)

Activity 1

🕒 15 min

Race to One or One Tenth

Standards Alignments

Addressing 5.NBT.B.7

The purpose of this activity is for students to practice adding decimals. There are two versions of the game, Race to One and Race to One Tenth. Students can choose which version to play, or, if there is time, play both versions of the game. Consider playing a round of the game with students

during the launch to demonstrate how it is played.

Access for English Learners

MLR8 Discussion Supports. Prior to solving the problems, invite students to make sense of the situations and take turns sharing their understanding with their partner. Listen for and clarify any questions about the context.

Advances: Reading, Representing

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Check for understanding by inviting students to rephrase directions in their own words.

Supports accessibility for: Memory, Organization

Materials to Gather

Number cubes

Required Preparation

- Each group of 2 needs a number cube.

Student-facing Task Statement

Use the directions to play Race to One or One Tenth with your partner. If there is time, play both versions of the game.

Race to One

1. Roll the number cube.
2. Decide if you want the number to represent tenths or hundredths.
3. Add the number to the last sum on your score sheet. If it is your first turn, you will add the number you roll to zero.
4. Take turns continuing to roll the number cube, decide the value, and add the number to your previous sum.
5. The first player to reach exactly 1 is the winner.
6. If you go over one, you lose your turn. For

Launch

- Groups of 2
- “Take a minute to read over the directions for Race to One or Race to One Tenth.”
- 1 minute: quiet think time
- “Are there any questions before we get started?”
- Give students a set of cards.

Activity

- “Play Race to One or One Tenth.”
- 10–12 minutes: partner work time

Synthesis

- “How are the two games, Race to One and Race to One Tenth the same?” (In both cases, I ended up close to the goal value

example, if your last sum was .95 and you roll a 6, you cannot go.

- You may not need to use all the blank spaces on your score sheet or you may need to write more spaces.

	number rolled	0.1	0.01	equation to represent the total
1				
2				
3				
4				
5				
6				

Race to One Tenth

- Roll the number cube.
- Decide if you want the number to represent hundredths or thousandths.
- Add the number to the last sum on your score sheet. If it is your first turn, you will add the number you roll to zero.
- Take turns continuing to roll the number cube, decide the value, and add the number to your previous sum.
- The first player to reach exactly 0.1 is the winner.
- If you go over 0.1, you lose your turn. For example, if your last sum was .095 and you roll a 6, you cannot go.
- You may not need to use all the blank spaces on your score sheet or you may need to write more spaces.

and then had to keep rolling, hoping I got the right number.)

- “How are the games different?” (In Race to One, I am aiming for 1 and in Race to One Tenth I am trying to get 0.1. When I get close to 1 in Race to One I have to choose hundredths and when I get close to 0.1 in Race to One Tenth I have to choose thousandths.)
- “How did you decide whether to have a number represent tenths or hundredths in Race to One?” (If I wasn’t close to one, I would have it represent the larger value.)

	number rolled	0.01	0.001	equation to represent the total
1				
2				
3				
4				
5				
6				

Student Responses

Answers vary.

Activity 2

🕒 20 min

Decimal Race to 500

Standards Alignments

Addressing 5.NBT.B.7

The purpose of this activity is for students to practice adding decimals.

Materials to Gather

Paper clips

Required Preparation

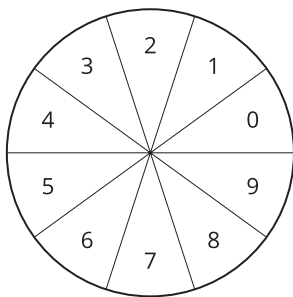
- Each group of 2 needs a paper clip.

Student-facing Task Statement

Use the directions to play Decimal Race to 500 with a partner.

Launch

- Groups of 2
- "Take a minute to read over the directions for Decimal Race to 500."
- 1 minute: quiet think time



1. Spin the spinner three times.
2. Arrange the digits to make a decimal number that follows this rule:
 - Odd numbers can only be used in the tenths, hundredths, or thousandths place.
 - Even numbers can only be used in the ones, tens, and hundreds places.

For example, if you spin the numbers 2, 3, and 9, these are some of the possible numbers you could make: 2.39 or 2.93.
3. Add your number to your previous sum. If it is your first turn, you will add your number to zero.
4. Continue taking turns until one person has reached 500 or more.

Student Responses

Answers vary.

- Give students paper clips.

Activity

- “Play Decimal Race to 500 with your partner.”
- 10–15 minutes: partner work time

Synthesis

- “What number would you choose to make if you spun 1, 8, and 4? Why?” (84.1 because that’s the biggest number I can make with those digits.)
- “Is it possible to win the game in one turn? How?” (Yes, I can spin 6, 2, and 8 and then can make 862 and that’s the end of the game.)
- “What numbers are the worst numbers to spin if you want to win the game quickly?” (Odd numbers because they only count for tenths, hundredths, or thousandths. Also 0 is no good unless there are some other even numbers.)
- “Can you think of a way to change this game to make it more challenging?” (Make it so you have to land exactly on 500.)

Lesson Synthesis

🕒 10 min

“Today, we practiced adding decimals. How is adding and subtracting with decimals the same as adding and subtracting with whole numbers? How is it different?” (If I add digits with the same place value, I can use the same method to add decimals as I use to add whole numbers. I have to make sure to add digits with the same place value so I can’t always line up the digits to the right, like I do with whole numbers.)

Complete Cool-Down

Response to Student Thinking

Students have ideas they could share with a partner.

Next Day Support

- After the warm-up in the next lesson, pair students up to discuss their responses.

Lesson 13: Multiply Fractions Game Day

Standards Alignments

Addressing 5.NF.B.4

Teacher-facing Learning Goals

- Multiply a fraction or whole number by a fraction.

Student-facing Learning Goals

- Let's multiply a fraction or whole number by a fraction.

Lesson Purpose

The purpose of this lesson is for students to practice multiplying fractions.

In previous lessons, students learned how to multiply fractions by whole numbers and fractions.

In this lesson, students practice multiplying fractions. Students spin numbers and use them as the numerators and denominators of two fractions. Students gain practice multiplying fractions and they need to think about the meaning of the numerator and denominator of the fractions in order to make strategic choices.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Engagement (Activity 2)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Number Talk (Warm-up)

Materials to Gather

- Paper clips: Activity 1, Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	15 min

Teacher Reflection Question

What question do you wish you had asked today? When and why should you have asked it?

Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

Reflect on Multiplication

Standards Alignments

Addressing 5.NF.B.4

Student-facing Task Statement

What is important to remember when multiplying fractions?

Student Responses

Sample responses:

- I can multiply numerators to get the numerator of the product and multiply denominators to get the denominator of the product.
- Sometimes the product will be larger than the fraction you started with and sometimes it will be smaller.

----- Begin Lesson -----

Warm-up

⌚ 10 min

Number Talk: Multiply One Third

The purpose of this Number Talk is for students to demonstrate strategies and understandings students have for multiplying fractions. These understandings help students develop fluency and will be helpful later in this lesson when students will need to multiply fractions.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $\frac{1}{3} \times 3$
- $\frac{1}{3} \times 4$
- $\frac{1}{3} \times \frac{6}{3}$
- $\frac{1}{3} \times \frac{1}{4}$

Student Responses

- 1: I just knew it.
- $1\frac{1}{3}: \frac{3}{3} = 1$ and there is one more third.
- $\frac{2}{3}$ or $\frac{6}{9}: \frac{1}{3} \times 2 = \frac{2}{3}$
- $\frac{1}{12}$: I multiplied the denominators.

Launch

- Display one expression.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- Display: $\frac{1}{3} \times \frac{6}{3}$
- “How did you find this product?” (I took the products of the numerators and denominators. I knew $\frac{6}{3}$ is 2, so it's $\frac{2}{3}$.)

Activity 1

🕒 15 min

Fraction Multiplication Compare

Standards Alignments

Addressing 5.NF.B.4

The purpose of this activity is for students to practice multiplying fractions. Students spin a spinner and use the numbers to generate fractions with a goal of making the largest product. Monitor for students who use their understanding of fractions (MP7) to:

- put larger numbers in the numerator and smaller numbers in the denominator
- use the Wild possibility to their advantage

Access for English Learners

MLR8 Discussion Supports. Prior to solving the problems, invite students to make sense of the situations and take turns sharing their understanding with their partner. Listen for and clarify any questions about the context.

Advances: Reading, Representing

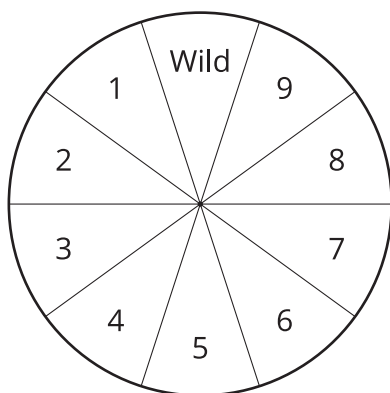
Materials to Gather

Paper clips

Required Preparation

- Each group of 2 needs a paper clip.

Student-facing Task Statement



- Use the directions to play Fraction Multiplication Compare with your partner.
 - Spin the spinner.
 - Write the number you spun in one of the empty blank boxes. Once you write a number, you cannot change it.
 - Player two spins and writes the number on their game board.
 - Continue taking turns until all four blank boxes are filled.
 - Multiply your fractions.

Launch

- Groups of 2
- "Take a minute to read over the directions for Fraction Multiplication Compare."
- 1 minute: quiet think time
- Give each group a paper clip.
- "Play Fraction Multiplication Compare with your partner."

Activity

- 10–12 minutes: partner work time

Synthesis

- "Which numbers were easiest to choose where to put on the game board? Why?" (When I got an 8 or 9 I knew to put it in the numerator to make a bigger fraction. When I got a 1 or 2 I knew to put it in the denominator to get the biggest possible fraction.)
- "Which numbers were hardest to choose where to put on the game board? Why?" (The middle numbers like 4, 5, and 6. I did not want them in the denominator because

- The player with the greatest product wins.
- Play again.

Round 1

$$\frac{\boxed{}}{\boxed{}} \times \frac{\boxed{}}{\boxed{}} = \underline{\hspace{2cm}}$$

Round 2

$$\frac{\boxed{}}{\boxed{}} \times \frac{\boxed{}}{\boxed{}} = \underline{\hspace{2cm}}$$

2. What strategy do you use to decide where to write the numbers?

they make the fraction pretty small. But I did not want them in the numerator because they won't give a very big fraction unless the denominator is 1 or 2.)

- "How did you use the 'wild' when you spun it?" (I put a 1 in the denominator where I had an 8 in the numerator.)

Student Responses

1. Answers vary.
2. I tried to put larger numbers in the numerator and smaller numbers in the denominator. When I got the Wild I made it a 1 and put it in the denominator.

Activity 2

⌚ 20 min

Fraction Multiplication Compare Round 2

 PLC Activity

Standards Alignments

Addressing 5.NF.B.4

The purpose of this activity is for students to practice multiplying fractions. The structure of the activity is identical to the previous one except that the goal is to have the smallest product. Monitor for students who identify the common structure with the previous game and place the larger numbers in the denominator and the smaller ones in the numerator (MP8).

Access for Students with Disabilities

Engagement: Internalize Self-Regulation. Synthesis: Provide students an opportunity to self-assess and reflect on their own progress. For example, ask students to compare their calculated products to their partner's. Encourage students to include how each factor impacted the product in their comparisons.

Supports accessibility for: Conceptual Understanding, Language

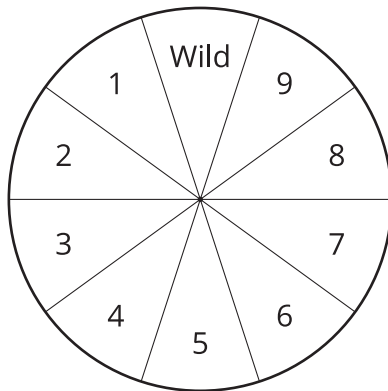
Materials to Gather

Paper clips

Required Preparation

- Each group of 2 needs a paper clip.

Student-facing Task Statement



- Use the directions to play Fraction Multiplication Compare with your partner.
 - Spin the spinner.
 - Write the number you spun in one of the four blank boxes.
 - Player two spins and writes the number on their game board.
 - Continue taking turns until all four blank boxes are filled.
 - Multiply your fractions.
 - The player with the smallest product

Launch

- Groups of 2
- "We are going to play another round of Fraction Multiplication Compare, but this time the person with the smallest product is the winner."
- "Will you use the same strategy that you used when trying to make the greatest product?" (No because there the goal was to get the biggest product.)
- 1 minute: quiet think time
- 1 minute: partner discussion
- Give each group a paper clip.
- "Play Fraction Multiplication Compare with your partner."

Activity

- 10–15 minutes: partner work time

Synthesis

- "How was this game the same as the earlier version of Fraction Multiplication Compare?" (I knew where to put big

wins.

- Play again.

Round 1

$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \times \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} = \underline{\hspace{2cm}}$$

Round 2

$$\begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \times \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} = \underline{\hspace{2cm}}$$

2. What strategy did you use to choose where to write the numbers?

Student Responses

1. Answers vary.
2. Sample response: I tried to put the larger numbers in the denominator boxes and the smaller numbers in the numerator boxes to make the fractions small. But sometimes it was hard to decide because I did not know what other numbers I would spin.

numbers like 8 and 9 and small numbers like 1 and 2. It was hard to know where to put the middle numbers like 4 or 5.)

- “How did your strategy change when trying to make the smallest product?” (It was the opposite of the first game. I tried to put the biggest numbers in the denominator and the smallest numbers in the numerator).

Lesson Synthesis

🕒 10 min

“Today, we practiced multiplying fractions. How is multiplying fractions the same as multiplying whole numbers? How is multiplying fractions different from multiplying whole numbers?” (When I multiply fractions, I have both numerators and denominators to multiply. So I have to use what I know about multiplying whole numbers but I need to do it twice.)

Student Section Summary

We played games with fractions and decimals, trying to make the largest or smallest number with given digits. Let's use the numbers 1, 3, 5, and 6. What is the smallest sum of two fractions we can make with these numbers? We want to use the smaller numbers, 1 and 3, for the numerators and the larger

numbers, 5 and 6, for the denominators. This gives two possibilities, $\frac{1}{6} + \frac{3}{5}$ and $\frac{1}{5} + \frac{3}{6}$. The expression $\frac{1}{5} + \frac{3}{6}$ has the smaller value which makes sense since we want the larger numerator, which means more equal pieces, to go with the larger denominator which makes those pieces smaller.

The smallest difference we can make with these numbers is $\frac{3}{6} - \frac{1}{5}$ which is a little smaller than $\frac{3}{5} - \frac{1}{6}$. Finally, the largest product we can make is $\frac{6}{3} \times \frac{5}{1}$ or $\frac{5}{1} \times \frac{6}{3}$ which both have the value $\frac{30}{3}$ or 10.

Complete Cool-Down

Response to Student Thinking

Students have ideas they could share with a partner.

Next Day Support

- After the warm-up in the next lesson, pair students up to discuss their responses.

Section D: Creation and Design

Lesson 14: Notice and Wonder

Standards Alignments

Addressing 5.G, 5.MD, 5.NBT, 5.NF, 5.NF.B.3, 5.OA

Teacher-facing Learning Goals

- Interpret a fraction as division of the numerator by the denominator.

Student-facing Learning Goals

- Let's create a Notice and Wonder.

Lesson Purpose

The purpose of this lesson is for students to apply their understanding of fractions as division to create a Notice and Wonder activity.

This lesson offers teachers the opportunity to listen to ways in which students notice and describe sharing and understanding fractions as division. After the warm-up, students create their own Notice and Wonder activity and then facilitate their Notice and Wonder with other students in the class. Students can draw pictures or find images to use for their Notice and Wonder from books or other sources as the teacher determines.

Access for:



Students with Disabilities

- Engagement (Activity 2)



English Learners

- MLR8 (Activity 2)

Instructional Routines

Notice and Wonder (Warm-up)

Materials to Gather

- Chart paper: Activity 2
- Colored pencils, crayons, or markers: Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Activity 3	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What do you love most about math? How are you sharing that joy with your students and encouraging them to think about what they love about math?

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

Reflection

Standards Alignments

Addressing 5.NF.B.3

Student-facing Task Statement

Describe something you really understand well after today's lesson or describe something that was confusing or challenging.

Student Responses

Sample response: I understand how a fraction can be understood as the numerator divided by the denominator. ($a \div b = \frac{a}{b}$)

----- **Begin Lesson** -----**Warm-up**

⌚ 10 min

Notice and Wonder: Sharing Bread

Standards Alignments

Addressing 5.NF.B.3

The purpose of this warm-up is for students to discuss how sharing situations can be represented using division and fractions, which will be useful when students create their own Notice and Wonder in a later activity. In the synthesis it is important to discuss things the writer had to pay attention to when they designed this activity.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?



Student Responses

Students may notice:

- There is garlic bread that is cut into slices.
- Each slice is equally sized.
- There are other pieces of garlic bread that we can partially see in the picture.

Students may wonder:

- How much garlic bread is there?
- How many pieces or slices are there?
- How many people will be eating the bread?

Launch

- Groups of 2
- Display the image.
- “What do you notice? What do you wonder?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- “What did the writer of this activity have to pay attention to when they designed this activity?”
- “Where do we see those things in what we noticed and wondered?” (something that can easily be shared, something that is divided or cut up, equal pieces)
- Record and display responses for all to see.

Activity 1

🕒 20 min

Design Your Notice and Wonder

Standards Alignments

Addressing 5.NF.B.3

The purpose of this activity is for students to work in groups to create a Notice and Wonder activity that focuses on viewing quotients as fractions. Students find an image in a book or from another source and fill in what other students might notice and wonder about the image.

Required Preparation

- Gather books, magazines, or other print materials with images for each group of 2 students.

Student-facing Task Statement

1. Find an image that would encourage your classmates to notice and wonder about sharing and interpreting the result as a fraction.
2. Fill in the possible things students might notice and wonder about your image.

Students may notice:

-
-
-

Students may wonder:

-
-
-

Student Responses

Answers vary.

Launch

- Groups of 2
- “You will create a Notice and Wonder activity. Use the magazines or books to find an image that shows sharing or can be used to represent division as a fraction.”

Activity

- 15 minutes: partner work time

Synthesis

- “What questions do you still have about creating a Notice and Wonder?” (How do I know what someone will notice or wonder? What if they don't think of division when they see the picture? What if they focus on a part of the picture I was not thinking about?)
- Give students a few minutes to make adjustments based on questions, if needed.

Activity 2

🕒 15 min

Facilitate Your Notice and Wonder

Standards Alignments

Addressing 5.NF.B.3

The purpose of this activity is for students to facilitate the Notice and Wonder they created in the previous activity for another group in the class. Each group should be paired with another group and they will take turns facilitating their Notice and Wonder for the other group. If time allows, students could facilitate their Notice and Wonder with more than 1 group.

🌐 Access for English Learners

MLR8 Discussion Supports. At the appropriate time, give students 2–3 minutes to make sure that everyone in their group can explain their Notice and Wonder activity. Invite groups to rehearse what they will say when they share with the whole class.

Advances: Speaking, Conversing, Representing

♿ Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Invite students to generate a list of shared expectations for group work. Record responses on a display and keep visible during the activity.

Supports accessibility for: Attention, Social-Emotional Functioning

Materials to Gather

Chart paper, Colored pencils, crayons, or markers

Student-facing Task Statement

1. Display your image for your classmates.
2. Ask them, “What do you notice? What do you wonder?”
3. Give them a minute to discuss together.
4. Have them share what they notice and wonder.
5. Record their ideas.

Launch

- Groups of 4
- Match partners from the previous activity with another pair.
- “Now you will do your Notice and Wonder with another group.”
- Give each pair a piece of chart paper and a marker to record responses.

Student Responses

Answers vary.

Activity

- 6 minutes: small-group work time
- Switch roles.
- 6 minutes: small-group work time

Synthesis

- “What did you learn as you presented your Notice and Wonder?” (I learned that people might notice or wonder things we did not think of. I learned that recording the ideas helps people understand what is being shared. I learned that the picture that I thought showed division can also be seen as showing multiplication.)

Activity 3

🕒 15 min

Design Your Notice and Wonder, Part 2

Standards Alignments

Addressing 5.G, 5.MD, 5.NBT, 5.NF, 5.OA

The purpose of this activity is for students to work in groups to create a Notice and Wonder activity that focuses on finding meaningful mathematics in images or photographs. Students are encouraged to make connections to a mathematical topic they have become familiar with this past school year. No specific content is specified, unlike in the previous activity which focused on viewing fractions as division of the numerator by the denominator.

Required Preparation

- Each group of 2 needs a few books, magazines, or other print materials with images.

Student-facing Task Statement

1. Find an image that you find interesting and would encourage your classmates to notice and wonder about a mathematical topic you have learned this year.
2. Fill in the possible things students might notice and wonder about your image.

Students may notice:

-
-
-

Students may wonder:

-
-
-

Student Responses

Answers vary.

Launch

- Groups of 2-4

Activity

- 15 minutes: small-group work time

Synthesis

- “What mathematical questions does your image make you think about? Why? What are some specific things in the image that will help your peers see or wonder about these mathematical concepts?”
- “What questions do you still have about creating your Notice and Wonder?”
- Give students a few minutes to make adjustments based on questions, if needed.

Lesson Synthesis

🕒 10 min

“What were the most important things about your image you had to consider as you created your Notice and Wonder? Why were these things important?” (I wanted an image that would show equal parts for division, but it was hard to find things that were equal. Sometimes the equal groups, like groups of windows on each floor of a building, made me think of multiplication, not division or fractions.)

----- Complete Cool-Down -----

Response to Student Thinking

Students have ideas they could share with a partner.

Next Day Support

- After the warm-up in the next lesson, pair students up to discuss their responses.

Lesson 15: Estimation Exploration

Standards Alignments

Addressing 5.NBT.B.5

Teacher-facing Learning Goals

- Fluently multiply multi-digit whole numbers using the standard algorithm.

Student-facing Learning Goals

- Let's create an Estimation Exploration.

Lesson Purpose

The purpose of this lesson is for students to apply their understanding of multi-digit multiplication and the standard algorithm of multiplication to create an Estimation Exploration activity.

This lesson offers teachers the opportunity to listen to ways in which students reason about multi-digit multiplication. After the warm-up, students create their own Estimation Exploration activity and then facilitate their Estimation Exploration with other students in the class. Students can draw pictures or find images to use for their Estimation Exploration from books or other sources as the teacher determines or use actual objects from around the classroom.

Access for:



Students with Disabilities

- Action and Expression (Activity 1)



English Learners

- MLR8 (Activity 2)

Instructional Routines

Estimation Exploration (Warm-up)

Materials to Gather

- Chart paper: Activity 2
- Colored pencils, crayons, or markers: Activity 2

Lesson Timeline

Warm-up

10 min

Teacher Reflection Question

Think about who volunteered to share their thinking with the class today. Are the same

Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

students always volunteering, while some students never offer to share? What can you do to help the class understand the value of hearing the ideas of every mathematician in the classroom?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

Reflection

Standards Alignments

Addressing 5.NBT.B.5

Student-facing Task Statement

In math class, it's important to listen to other people's ideas. During class today, what is something you learned by listening carefully to someone?

Student Responses

Sample response: I learned that thinking about other students' ideas helps me clarify my own understanding. When we were writing possible responses, I learned about how other students might think differently about the problem than I did.

----- Begin Lesson -----

Warm-up

🕒 10 min

Estimation Exploration: Umbrellas

Standards Alignments

Addressing 5.NBT.B.5

The purpose of an Estimation Exploration is for students to practice the skill of estimating a reasonable answer based on experience and known information. In the synthesis it is important to discuss things

the writer had to pay attention to when they designed this activity.

Instructional Routines

Estimation Exploration

Student-facing Task Statement

How many umbrellas are there?



Record an estimate that is:

too low	about right	too high

Student Responses

Sample responses:

- Too low: 50
- About right: 300–500
- Too high: 5,000+

Launch

- Groups of 2
- Display the image.
- “What is an estimate that’s too high?” “Too low?” “About right?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Record responses.

Synthesis

- “What did the writer of this activity have to pay attention to when they designed this activity?” (Something in the image that shows how many things fit in one line or a small part of the image. An image where it is hard to see or count the exact number of objects.)
- Record and display responses for all to see.

Activity 1

🕒 20 min

Design Your Estimation Exploration

Standards Alignments

Addressing 5.NBT.B.5

The purpose of this activity is for students to work in groups to create an Estimation Exploration activity that focuses on multi-digit multiplication. Students use an object from the classroom, their own drawing, or find an image in a book or from another source and fill in how other students might estimate the quantity of items.

🕒 Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Provide students with alternatives to writing on paper: students can share their learning using a picture, orally, a video, a skit, or another appropriate form of expression to create an estimation Exploration activity.

Supports accessibility for: Language, Attention, Conceptual Processing

Student-facing Task Statement

1. Find an image that would encourage your classmates to estimate the total number of an item using strategies for multi-digit multiplication.
2. Fill in the possible estimates students might make.

Record an estimate that is:

too low	about right	too high

Student Responses

Answers vary.

Launch

- Groups of 3 or 4
- “Work with your group to create an Estimation Exploration activity about counting a large number of objects.”

Activity

- 15 minutes: small-group work time

Synthesis

- “What questions do you still have about creating your Estimation Exploration?”
(What if there are so many things that it's hard to estimate? Like people in a stadium or leaves on a tree or trees in a forest? What if there are not so many things and estimating is simpler, like the number of

kids in a classroom or the number of pattern blocks in a container?)

- Give students a few minutes to make adjustments based on questions, if needed.

Activity 2

🕒 15 min

Facilitate Your Estimation Exploration

Standards Alignments

Addressing 5.NBT.B.5

The purpose of this activity is for students to facilitate the Estimation Exploration they created in the previous activity for another group in the class. Each group should be paired with another group and they will take turns facilitating their Estimation Exploration for the other group. If time allows, students could facilitate their Estimation Exploration with more than 1 group.

🌐 Access for English Learners

MLR8 Discussion Supports. At the appropriate time, give students 2–3 minutes to make sure that everyone in their group can explain their Estimation Exploration activity. Invite groups to rehearse what they will say when they share with the whole class.

Advances: Speaking, Conversing, Representing

Materials to Gather

Chart paper, Colored pencils, crayons, or markers

Student-facing Task Statement

1. Display your image for your classmates.
2. Ask them, “What is an estimate that’s too high?” “Too low?” “About right?”
3. Give them a minute of quiet think time.

Launch

- Groups of 3 or 4
- Give each group a piece of chart paper and a marker to record responses.
- “Pair up with another group in the class. Each group will present their Estimation

4. Give them a minute to discuss together.
5. Have them share estimates.
6. Record their ideas.

Student Responses

Answers vary.

Exploration.”

Activity

- 10–12 minutes: small-group work time
- Remind students to switch roles halfway through the time allowed.

Synthesis

- “What did you learn as you presented your Estimation Exploration?” (I learned that ranges of estimates might be bigger than you expect. I learned that it’s important to understand the size of the items you are trying to estimate or be familiar with it, otherwise you cannot make good estimates.)

Lesson Synthesis

🕒 10 min

“What were the most important things that you had to consider as you created your Estimation Exploration? Why were these things important?” (I needed to find things that were organized so that it was possible to make some estimates. There needed to be a lot, so it wasn’t possible to just count them. But if there were too many, then making an estimate was really hard.)

----- Complete Cool-Down -----

Response to Student Thinking

Students have ideas they could share with a partner.

Next Day Support

- After the warm-up in the next lesson, pair students up to discuss their responses.

Lesson 16: Number Talk

Standards Alignments

Addressing 5.NBT.B.6

Teacher-facing Learning Goals

- Find quotients of whole numbers multi-digit dividends and divisors without any remainders.

Student-facing Learning Goals

- Let's create a Number Talk.

Lesson Purpose

The purpose of this lesson is for students to apply their understanding of dividing multi-digit whole numbers to create a Number Talk activity.

This lesson offers teachers the opportunity to listen to ways in which students make use of structure and repeated reasoning to design a Number Talk. After the warm-up, three activities are given, but it is not expected that students do all three. As the activities progress, there is one additional problem missing from each Number Talk. The choice of which activities to use is left to the teacher based on how much scaffolding the students may need. This lesson can take 1–2 days if students facilitate their creations with other groups.

Access for:

Students with Disabilities

- Engagement (Activity 1)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	15 min
Activity 3	15 min

Teacher Reflection Question

Which students came up with an unexpected strategy in today's lesson? What are some ways you can be more open to the ideas of each and every student?

Lesson Synthesis

10 min

Cool-down

5 min

Cool-down (to be completed at the end of the lesson)

🕒 5 min

Reflection

Standards Alignments

Addressing 5.NBT.B.6

Student-facing Task Statement

As mathematicians we use patterns and structure in problems we solve to reason about new problems. Describe a time today when you did that.

Student Responses

Sample response: Today when we were designing a number talk, I was trying to think of how to make the problems similar enough so you could use the same strategy for all the problems.

----- **Begin Lesson** -----**Warm-up**

🕒 10 min

Number Talk: Division

👤 ↔ 👤 PLC Activity

Standards Alignments

Addressing 5.NBT.B.6

The purpose of this Number Talk is for students to demonstrate strategies and understandings they have for finding whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors. These understandings help students develop fluency and will be helpful later in this lesson when students will develop their own number talk activity.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $28 \div 14$
- $70 \div 14$
- $98 \div 14$
- $350 \div 14$

Student Responses

- 2: $2 \times 14 = 28$
- 5: $14 \times 10 = 140$ so $14 \times 5 = 70$
- 7: $98 = 28 + 70$ so
 $98 \div 14 = (28 \div 14) + (70 \div 14)$
- 25: $14 \times 20 = 280$ and $14 \times 5 = 70$ and
 $280 + 70 = 350$

Launch

- Display one problem.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep problems and work displayed.
- Repeat with each problem.

Synthesis

- “What did the writer of this activity have to pay attention to when they designed this activity?” (The expressions need to be done mentally so they can't be too complex.)
- “Where do we see those things in how the expressions change during the Number Talk?” (The first problem helps to do the second one and the first 3 help to do the last one.)
- “Imagine this number talk continued with a fifth expression. How does $700 \div 28$ fit in with this number talk?” (It doubles the dividend and the divisor from the previous expression, which means the quotient is the same, 25.)

Activity 1

🕒 15 min

Number Talk: Design 1

Standards Alignments

Addressing 5.NBT.B.6

The purpose of this activity is for students to reason about strategies for finding whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors. Students add one expression to a partially-completed Number Talk activity. If there is time, students can facilitate their Number Talk with another group.

Access for English Learners

MLR8 Discussion Supports: Prior to solving the problems, invite students to make sense of the situations and take turns sharing their understanding with their partner. Listen for and clarify any questions about the context.

Advances: Reading, Representing

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Synthesis: Optimize meaning and value. Invite students to share how they chose the last expression to complete the number talk with their favorite math teacher.

Supports accessibility for: Conceptual Processing, Language

Student-facing Task Statement

Write an expression to complete the number talk. Be prepared to explain how you chose the last expression.

- $30 \div 15$
- $45 \div 15$
- $300 \div 15$
- _____

Student Responses

Sample responses: $3 \div 15$, $375 \div 15$, $345 \div 15$

Launch

- Groups of 2 or 4
- “Now you will work with your group to complete a Number Talk activity. This activity has one expression missing. Decide on an expression that would complete the Number Talk and write it on the blank line.”

Activity

- 10 minutes: small-group work time
- As students work, monitor for groups who discuss and design an expression based on some of the following:
 - They keep the same divisor.
 - They add the dividends, but keep the same divisor.
 - They subtract the dividends and keep the same divisor.
 - They make the dividend smaller to get $15 \div 15$ or $3 \div 15$.

- They use a combination of multiples, half, double, or triple the dividend and divisor to change both the dividend and divisor. For example, $15 \div 60$.

Synthesis

- Choose small groups to share who had different reasons for their fourth expression.
- Ask students to share their completed Number Talk and ask the class to share reasons for the last expression.
- As each group shares, continually ask others in the class if they agree or disagree and the reasons why.

Activity 2

🕒 15 min

Number Talk: Design 2

Standards Alignments

Addressing 5.NBT.B.6

The purpose of this activity is for students to reason about strategies for finding whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors. Students add two expressions to a partially-completed Number Talk activity. If there is time, students can facilitate their Number Talk with another group.

Student-facing Task Statement

Choose one of the number talks to complete. Be prepared to share your reasoning for the expressions you chose.

Launch

- Groups of 2 or 4
- “Now you will work with your group to complete a Number Talk activity. This activity has two options that each have two

Option 1:

- $220 \div 22$
- $66 \div 22$
- _____
- _____

Option 2:

- $260 \div 26$
- $260 \div 13$
- _____
- _____

Student Responses

Sample response, Option 2:

$$130 \div 13$$

$$130 \div 26$$

expressions missing. Decide on expressions that would complete the Number Talk and write them in the blank lines."

Activity

- 10 minutes: small-group work time
- As students work, monitor for groups who discuss and design expressions based on some of the following:
 - They adjust the dividend but keep the divisor.
 - They adjust the divisor and keep the dividend.
 - Use partial quotients either through addition or subtraction.
 - Use multiplicative relationships such as halving and doubling and reason how that impacts the quotient.
 - Use multiplicative relationships such as multiplying by 2 or 10 and dividing by 2 or 10 and reason how that impacts the quotient.

Synthesis

- Choose small groups to share that had different reasons for their expressions.
- Ask students to share their completed Number Talk and ask the class to share reasons for their expressions.
- As each group shares, continually ask others in the class if they agree or disagree and the reasons why.

Activity 3

⌚ 15 min

Number Talk: Design 3

Standards Alignments

Addressing 5.NBT.B.6

The purpose of this activity is for students to reason about strategies for finding whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors. Students add three expressions to a partially-completed Number Talk activity. If there is time, students can facilitate their Number Talk with another group.

Student-facing Task Statement

Write expressions to complete the Number Talk.
Be prepared to share your reasoning for the expressions.

- $430 \div 43$
- _____
- _____
- _____

Student Responses

Sample response:

$$860 \div 43$$

$$8,600 \div 43$$

$$9,460 \div 43$$

Launch

- Groups of 2 or 4
- “Now you will work with your group to complete a Number Talk activity. This activity has three expressions missing. Decide on expressions that would complete the Number Talk and write them in the blank lines.”

Activity

- 10 minutes: small-group work time
- As students work, monitor for groups who discuss and design expressions based on some of the following:
 - They adjust the dividend but keep the divisor.
 - They adjust the divisor and keep the dividend.
 - Use partial quotients either through addition or subtraction.
 - Use multiplicative relationships such as multiplying by 2 or 10 and dividing by 2 or 10 and reason how that impacts the quotient.

Synthesis

- Choose small groups to share that had different reasons for their expressions.

- Ask students to share their completed Number Talk and ask the class to share reasons for their expressions.
- As each group shares, continually ask others in the class if they agree or disagree and the reasons why.

Lesson Synthesis

🕒 10 min

“What were the most important things about your expressions you had to consider as you created your Number Talk? Why were these things important?” (I needed to make sure my expressions could be evaluated mentally. I also needed to make sure my reasoning to connect the expressions was also visible to others.)

----- Complete Cool-Down -----

Response to Student Thinking

Students have ideas they could share with a partner.

Next Day Support

- After the warm-up in the next lesson, pair students up to discuss their responses.

Lesson 17: True or False?

Standards Alignments

Addressing 5.NF.A.1

Teacher-facing Learning Goals

- Add and subtract fractions and mixed numbers with unlike denominators.

Student-facing Learning Goals

- Let's create a True or False.

Lesson Purpose

The purpose of this lesson is for students to apply their understanding of adding and subtracting fractions with unlike denominators to create a True or False activity.

This lesson offers teachers the opportunity to listen to ways in which students make use of structure and repeated reasoning to design a True or False. After the warm-up, three activities are given, but it is not expected that students do all three. As the activities progress, there is one additional problem missing from each True or False. The choice of which activities to use is left to the teacher based on how much scaffolding the students may need. This lesson can take 1–2 days if students facilitate their creations with other groups.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR8 (Activity 1)

Instructional Routines

True or False (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	15 min
Activity 3	15 min

Teacher Reflection Question

If you were to teach this lesson over again, what activity would you redo? How would your proposed changes support student learning?

Lesson Synthesis

10 min

Cool-down

5 min

Cool-down (to be completed at the end of the lesson)

🕒 5 min

Reflection

Standards Alignments

Addressing 5.NF.A.1

Student-facing Task Statement

In math class, it's important to listen to other people's ideas. What was something you learned today by listening to someone else's ideas?

Student Responses

Sample response: Today when we were discussing an equation to include to finish a true or false I learned a different strategy to use when I'm multiplying by 7.

----- **Begin Lesson** -----**Warm-up**

🕒 10 min

True or False: Fraction Addition

Standards Alignments

Addressing 5.NF.A.1

The purpose of this True or False is for students to demonstrate strategies and understandings they have for adding fractions with unlike denominators. These understandings help students deepen their understanding of the properties of operations and will be helpful later in this lesson when students will develop their own true or false activity.

In the synthesis it is important to discuss things the writer had to pay attention to when they designed this activity.

Instructional Routines

True or False

Student-facing Task Statement

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $\frac{3}{4} + \frac{3}{8} = \frac{3}{8} + \frac{3}{8}$
- $\frac{7}{5} + \frac{2}{3} = \frac{21}{15} + \frac{8}{15}$
- $\frac{8}{9} + \frac{5}{12} = \frac{32}{36} + \frac{15}{36}$

Student Responses

- False: $\frac{3}{4}$ is not equal to $\frac{3}{8}$.
- False: $\frac{2}{3} = \frac{10}{15}$
- True: $\frac{8}{9} = \frac{32}{36}$ and $\frac{2}{3} = \frac{10}{15}$

Launch

- Display one equation.
- “Give me a signal when you know whether the equation is true and can explain how you know.”
- 1 minute: quiet think time

Activity

- Share and record answers and strategy.
- Repeat with each equation.

Synthesis

- “What did the writer of this activity have to pay attention to when they designed this activity?” (Some equations are true and some are false. Some terms on both sides are equal. Pay attention to the unlike and like denominators.)
- “Where do we see those things in how the equations change during the True or False?” (The first two equations are false, but they use an appropriate common denominator.)

Activity 1

🕒 15 min

True or False: Design 1

Standards Alignments

Addressing 5.NF.A.1

The purpose of this activity is for students to reason about subtracting fractions with unlike

denominators to add one equation to a partially-completed True or False activity. If there is time, students can facilitate their True or False with another group.

Access for English Learners

MLR8 Discussion Supports. Prior to solving the problems, invite students to make sense of the situations and take turns sharing their understanding with their partner. Listen for and clarify any questions about the context.

Advances: Reading, Representing

Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Use multiple examples and non-examples to emphasize the importance of finding like units in order to subtract fractions.

Supports accessibility for: Conceptual Processing, Memory

Student-facing Task Statement

Write an equation to complete the True and False task. Be prepared to share your reasoning for the last equation.

- $\frac{5}{6} - \frac{4}{9} = \frac{45}{54} - \frac{24}{54}$

- $\frac{5}{6} - \frac{4}{9} = \frac{15}{36}$

- _____

Student Responses

Sample response:

$$\frac{5}{6} - \frac{4}{9} = \frac{15}{18} - \frac{8}{18}$$

Launch

- Groups of 2 or 4
- “Now you will work with your group to complete a True or False activity. This activity has one equation missing. Decide on an equation that would complete the True or False and write it on the blank line.”

Activity

- 10 minutes: small-group work time
- As students work, monitor for groups who discuss and design an equation based on some of the following:
 - They create equations where the fractions on one side of the equation have unlike denominators and the other side consists of fractions with like denominators.
 - They create an equation where the common denominator is appropriate, but the corresponding fraction used is not equivalent.
 - They create an equation that is

similar to a previous equation. For example: $\frac{5}{6} - \frac{4}{9} = \frac{30}{36} - \frac{16}{36}$

- They create an equation where a similar strategy can be used to determine if it's true.

Synthesis

- Choose small groups to share that had different reasons for their fourth equation.
- Ask students to share their completed True or False and ask the class to share reasons for the last equation.
- As each group shares, continually ask others in the class if they agree or disagree and the reasons why.

Activity 2

🕒 15 min

Design 2

Standards Alignments

Addressing 5.NF.A.1

The purpose of this activity is for students to reason about adding fractions with unlike denominators to add two equations to a partially-completed True or False activity. If there is time, students can facilitate their True or False with another group.

Student-facing Task Statement

Write two equations to complete the True or False task. Be prepared to share your reasoning for the equations.

- $\frac{8}{14} + \frac{3}{7} = \frac{4}{7} + \frac{3}{7}$

Launch

- Groups of 2 or 4
- "Now you will work with your group to complete a True or False activity. This activity has two equations missing. Decide on equations that would complete the True

- _____
- _____

Student Responses

Sample response:

$$\frac{8}{14} + \frac{3}{7} = \frac{8}{14} + \frac{3}{14}$$

$$\frac{8}{14} + \frac{3}{7} = \frac{8}{14} + \frac{6}{14}$$

or False and write them on the blank lines.”

Activity

- 10 minutes: small-group work time
- As students work, monitor for groups who discuss and design equations based on some of the following:
 - They create equations where the fractions on one side of the equation have unlike denominators and the other side consists of fractions with like denominators.
 - They create an equation where the common denominator is appropriate, but the corresponding fraction used is not equivalent.
 - They create an equation that is similar to a previous equation.
 - They create an equation where a similar strategy can be used to determine if it's true.

Synthesis

- Choose small groups to share that had different reasons for their equations.
- Ask students to share their completed True or False and ask the class to share reasons for the equations.
- As each group shares, continually ask others in the class if they agree or disagree and the reasons why.

Activity 3

🕒 15 min

Design 3

Standards Alignments

Addressing 5.NF.A.1

The purpose of this activity is for students to reason about adding and subtracting fractions with unlike denominators. Students create equations to complete a True or False activity. If there is time, students can facilitate their True or False with another group.

Student-facing Task Statement

Write three equations to complete the True or False task. Be prepared to share your reasoning for the equations.

- _____
- _____
- _____

Student Responses

Answers vary.

Launch

- Groups of 2 or 4
- “Now you will work with your group to complete a True or False activity. This activity has all three equations missing. Decide on equations that would complete the True or False and write them on the blank lines.”

Activity

- 10 minutes: small-group work time
- As students work, monitor for groups who discuss and design equations based on some of the following:
 - They create equations where the fractions on one side of the equation have unlike denominators and the other side consists of fractions with like denominators.
 - They create an equation where the common denominator is appropriate, but the corresponding fraction used is not equivalent.
 - They create an equation that is similar to a previous equation.
 - They create an equation where a similar strategy can be used to determine if it's true.

Synthesis

- Choose small groups to share that had different reasons for their equations.
- Ask students to share their completed True or False and ask the class to share reasons for the equations.
- As each group shares, continually ask others in the class if they agree or disagree and the reasons why.

Lesson Synthesis

🕒 10 min

“What were the most important things about your equations you had to consider as you created your True or False? Why were these things important?” (I needed to find equations where you could figure out whether or not it was true with a mental strategy. So the numbers had to be related in a nice way and there needed to be some structure to help see if the expressions were equal or not, without finding the sums or differences.)

----- Complete Cool-Down -----

Response to Student Thinking

Students have ideas they could share with a partner.

Next Day Support

- After the warm-up in the next lesson, pair students up to discuss their responses.

Lesson 18: Which One Doesn't Belong?

Standards Alignments

Addressing 5.MD.C.3

Teacher-facing Learning Goals

- Categorize shapes by their shared attributes.

Student-facing Learning Goals

- Let's create a Which One Doesn't Belong.

Lesson Purpose

The purpose of this lesson is for students to apply their understanding of volume to create a Which One Doesn't Belong activity.

This lesson offers teachers the opportunity to listen to ways in which students describe and calculate volume. After the warm-up, three activities are given, but it is not expected that students do all three. As the activities progress, there is one additional element missing from the Which One Doesn't Belong. The choice of which activities to use is left to the teacher based on how much scaffolding the students may need. This lesson can take 1–2 days if students facilitate their creations with other groups.

Access for:



Students with Disabilities

- Action and Expression (Activity 1)



English Learners

- MLR8 (Activity 1)

Instructional Routines

Which One Doesn't Belong? (Warm-up)

Materials to Copy

- Shape Cards for WODB Design (groups of 2): Activity 1
- Shape Cards for WODB Design (groups of 2): Activity 2

Lesson Timeline

Warm-up

10 min

Teacher Reflection Question

Reflect on your experience with the instructional

Activity 1	15 min
Activity 2	15 min
Activity 3	15 min
Lesson Synthesis	10 min
Cool-down	5 min

routines in the curriculum. What moves or questions have improved the learning for each or your students during this routine? What improvements would you make next time?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

Reflection

Standards Alignments

Addressing 5.MD.C.3

Student-facing Task Statement

As mathematicians, it is important to justify our thinking and listen to the reasoning of others. Describe a time when you learned something new or thought differently about something based on what someone else in the class said today.

Student Responses

Sample response: When we were creating a Which One Doesn't Belong, I thought we could use a square for the final shape, but we couldn't because we needed a quadrilateral that didn't have right angles.

----- Begin Lesson -----

Warm-up

🕒 10 min

Which One Doesn't Belong? Volume

Standards Alignments

Addressing 5.MD.C.3

This warm-up prompts students to compare four images. It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the items in comparison to one another.

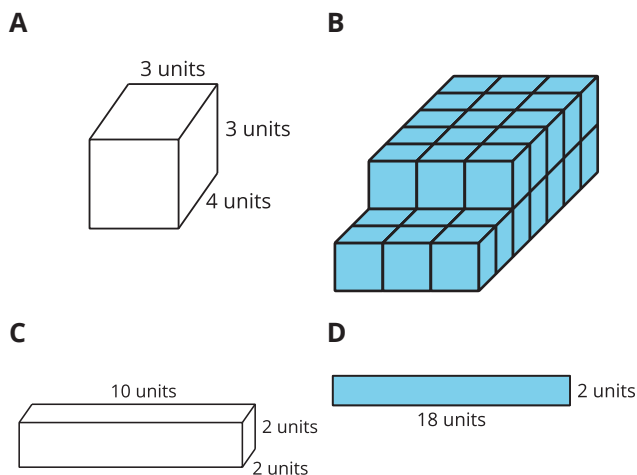
In the synthesis it is important to discuss things the writer had to pay attention to when they designed this activity.

Instructional Routines

Which One Doesn't Belong?

Student-facing Task Statement

Which one doesn't belong?



Student Responses

Sample responses:

A doesn't belong because:

- Only one that does not have a side length of 2 units.

B doesn't belong because:

- Only one where side lengths are not labeled.

C doesn't belong because:

- Only one that does not have an area or volume of 36.

Launch

- Groups of 2
- Display the image.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."

Activity

- 1 minute: quiet think time
- 2–3 minutes: partner discussion
- Record responses.

Synthesis

- "What did the writer of this activity have to pay attention to when they designed this activity?" (the number of sides, the types of angles or corners, the way they were oriented, how many shapes there were in each)
- "Where do we see those things in our reasons for which one didn't belong?"
- Record and display responses for all to see.

D doesn't belong because:

- Only one that is not a 3-D shape.
- Only one not made up of cubes.

Activity 1

🕒 15 min

Which One Doesn't Belong? Design 1

Standards Alignments

Addressing 5.MD.C.3

The purpose of this activity is for students to reason about attributes of three-dimensional shapes including side lengths and volume. Students select a few shapes from premade cards and create their own to complete a Which One Doesn't Belong activity. If there is time, students can facilitate their Which One Doesn't Belong with another group.

🌐 Access for English Learners

MLR8 Discussion Supports. Display sentence frames to support small-group discussion: “____ and ____ are the same/alike because . . .”, “____ and ____ are different because . . .”, and “I agree/disagree because . . .”

Advances: Conversing, Representing

♿ Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support working memory, provide students with access to sticky notes or mini whiteboards.

Supports accessibility for: Memory, Organization

Materials to Copy

Shape Cards for WODB Design (groups of 2)

Required Preparation

- Create a set of cards from the Instructional master for each group of 2.

Student-facing Task Statement

1. Choose 3 shapes from the set of cards.
2. Draw a fourth shape to complete the Which One Doesn't Belong.
3. For each shape, discuss one reason why it does not belong.

Which one doesn't belong?

A

B

C

D

Student Responses

Answers vary.

Launch

- Groups of 2 or 4
- "Now you will work with your group to complete a Which One Doesn't Belong activity. You will select three shapes from the set of cards and create one shape on your own. Discuss why the shapes you select and draw do not belong."

Activity

- 10 minutes: small-group work time
- As students work, monitor for groups who discuss and design an image based on some of the following:
 - the volume of the shape: 12, 20, or 27 cubic units
 - if the shape is a rectangular prism or a figure composed of rectangular prisms
 - the side lengths of the shape
 - if the cubes are visible or not

Synthesis

- Choose small groups to share the different reasons the shapes they chose or drew do not belong.
- Ask students to share their completed Which One Doesn't Belong and ask the class to share reasons the images do not belong.
- As each group shares, continually ask others in the class if they agree or disagree and the reasons why.

Activity 2

🕒 15 min

Which One Doesn't Belong? Design 2

Standards Alignments

Addressing 5.MD.C.3

The purpose of this activity is for students to reason about attributes of three-dimensional shapes including side lengths and volume. Students select two shapes from premade cards and create their own to complete a Which One Doesn't Belong activity. If there is time, students can facilitate their Which One Doesn't Belong with another group.

Materials to Copy

Shape Cards for WODB Design (groups of 2)

Student-facing Task Statement

1. Choose 2 shapes from the set of cards.
2. Draw a third and fourth shape to complete the Which One Doesn't Belong.
3. For each shape, discuss one reason why it does not belong.

Which one doesn't belong?

A	B
C	D

Student Responses

Answers vary.

Launch

- Groups of 2 or 4
- "Now you will work with your group to complete a Which One Doesn't Belong activity. You will select two shapes from the set of cards and create two shapes on your own. Discuss why the shapes you select and draw do not belong."

Activity

- 10 minutes: small-group work time
- As students work, monitor for groups who discuss and design an image based on some of the following:
 - the volume of the shape: 12, 20, or 27 cubic units
 - if the shape is a rectangular prism or a composite
 - the side lengths of the shape
 - if the cubes are visible or not

Synthesis

- Choose small groups to share the different reasons the shapes they chose or drew do not belong.

- Ask students to share their completed Which One Doesn't Belong and ask the class to share reasons the images do not belong.
- As each group shares, continually ask others in the class if they agree or disagree and the reasons why.

Activity 3

🕒 15 min

Which One Doesn't Belong? Design 3

Standards Alignments

Addressing 5.MD.C.3

The purpose of this activity is for students to design their own Which One Doesn't Belong and describe the mathematical ideas they wanted others to notice in their images. If students struggle to decide on a focus of their design, it may be helpful to give them a topic such as "properties of shapes" or "fractions."

Student-facing Task Statement

Create your own Which One Doesn't Belong about any mathematical idea you want others to notice.

Which one doesn't belong?

A	B
C	D

Student Responses

Answers vary.

Launch

- Groups of 2 or 4
- "Now you will work with your group to create your own Which One Doesn't Belong. After you create your images, record the mathematical ideas you wanted others to notice."

Activity

- 10 minutes: small-group work time
- As students work, monitor for groups who choose different areas of focus to share in the whole-group discussion.

Synthesis

- “What mathematical ideas did you want others to notice about the images you created?”
- As each group shares, continually ask others in the class if they agree or disagree and the reasons why.

Lesson Synthesis

⌚ 10 min

“What were the most important things about your images you had to consider as you created your Which One Doesn’t Belong? Why were these things important?” (I was able to get three different shapes where each one did not belong but when I tried to add the fourth, it didn’t work any more. Then I tried changing one of the other shapes and each time I made a change, it created some other problem.)

----- Complete Cool-Down -----

Response to Student Thinking

Students have ideas they could share with a partner.

Next Day Support

- After the warm-up in the next lesson, pair students up to discuss their responses.



Family Support Materials

Family Support Materials

Putting It All Together

In this unit, students put together their understanding from throughout the year to cap off major work and fluency goals of the grade.

Section A - Multiply and Divide Whole Numbers

Students deepen their understanding of the standard algorithm for multiplication and practice using it. They recognize and explain place value patterns when multiplying multi-digit numbers, learn how to use the algorithm when one or more of the factors has several zeroes, and examine an alternative recording method for newly composed units.

Section B - Apply Volume Concepts

Students investigate volume by calculating the volume of rectangular prisms and estimating the number of books in a library. Students solve problems in a real-world context and consider what it means to measure volume using different units. They estimate the amount of water that falls on the roof of their home and use it to reason whether or not that amount is sufficient for many of the daily household chores that use water.

Section C - Fraction and Decimal Operations

Students operate with fractions and decimals. Each lesson is structured as a game day where students learn games for adding and subtracting fractions, adding and subtracting decimals, and multiplying fractions.

Section D - Creation and Design

Students have the opportunity to apply what they've learned about instructional routines throughout the year to create their own Notice and Wonder, Exploration Estimation, Number Talk, True or False, and Which One Doesn't Belong. Students design and create routines in small-groups and can facilitate their routine with another group in the class. Each lesson has a content focus provided, so students review major work of the grade through their routines.

Try it at home!

Near the end of the unit, ask your student to share the instructional routines they created. Questions that may be helpful as they share:

- How did you design the routine?
- How does the routine relate to what you learned this year?
- How could we modify the routine?

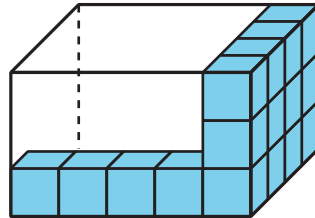


Unit Assessments

End-of-Course Assessment and Resources

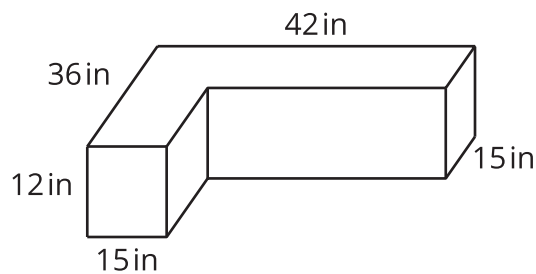
Putting It All Together: End-of-Course Assessment and Resources

1. Select **all** expressions that represent the volume of this rectangular prism in cubic units.



- A. $5 \times 4 \times 3$
- B. $(3 \times 4) + 4$
- C. $5 \times (4 + 3)$
- D. 3×20
- E. 4×15

2. The image shows the measurements of a toy container.



What is the volume of the container? Explain or show your reasoning.

3. Select **all** expressions whose value is **less** than $\frac{1}{2}$.

A. $\frac{1}{3} + \frac{1}{4}$

B. $\frac{5}{11} - \frac{2}{15}$

C. $\frac{10}{20} - \frac{1}{12}$

D. $1 + \frac{7}{12}$

E. $1 - \frac{7}{12}$

4. Select **all** expressions whose value is greater than 1.

A. $10 \div 12$

B. $115 \div 73$

C. $\frac{9}{8} \times \frac{7}{6}$

D. $\frac{3}{2} \times \left(\frac{4}{10} + \frac{3}{5} \right)$

E. $\frac{7}{8} \times \frac{9}{13}$

5. A cooler contains 5 gallons of water.

a. One bottle of water is $\frac{1}{8}$ of a gallon. How many bottles of water does the cooler contain? Explain or show your reasoning.

b. The running team drinks $2\frac{3}{4}$ coolers of water during practice. How many gallons is that? Explain or show your reasoning.

6. Jada's math book is $\frac{5}{16}$ of an inch thick. Her science book is $\frac{1}{4}$ of an inch thick.

a. Which book is thicker? How much thicker? Explain or show your reasoning.

b. How thick are the math and science books together?

7. Select **all** expressions that have the value one million.

A. 10^3

B. 10^6

C. 10^7

D. 1,000,000

E. $10 \times 10 \times 10 \times 10 \times 10 \times 10$

F. 100×100

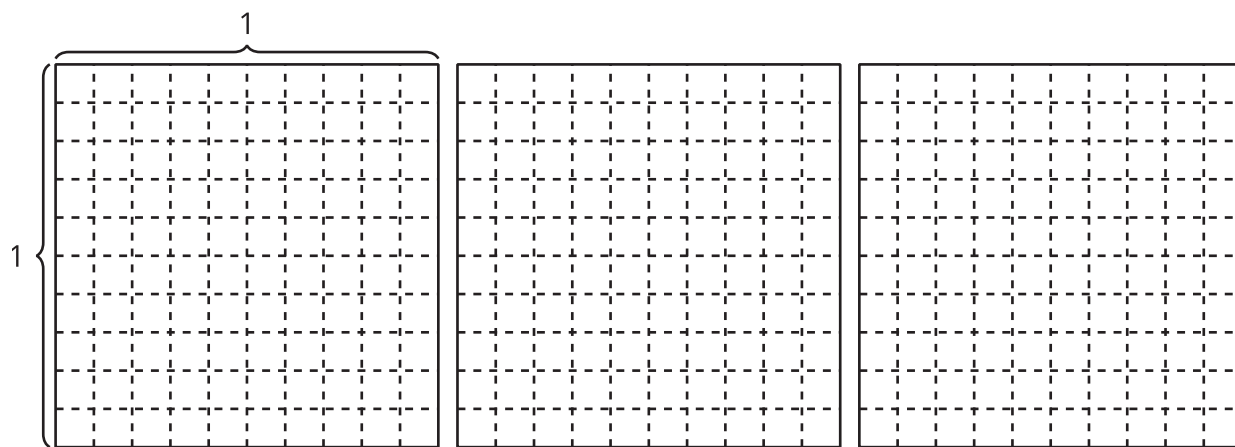
G. $100 \times 100 \times 100$

8. Find the value of each quotient. Explain or show your reasoning.

a. $1,512 \div 21$

b. $7,776 \div 36$

9. Find the value of each expression. Explain or show your reasoning. Use the hundred grids if they are helpful.



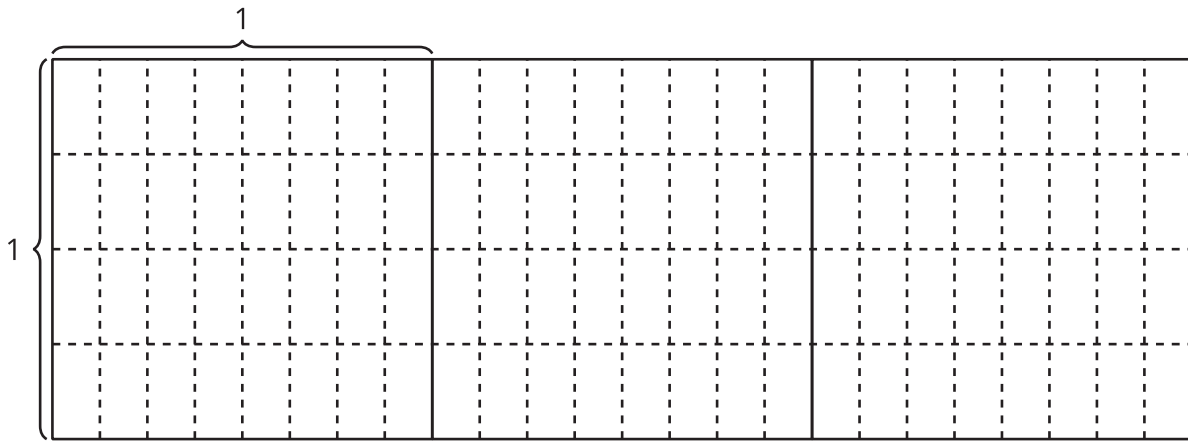
a. $2.4 + 0.02$

b. $2.4 - 0.02$

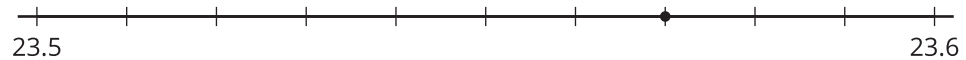
c. 2.4×0.2

d. $2.4 \div 0.2$

10. A rectangular cornfield is $\frac{3}{4}$ mile wide and $2\frac{1}{8}$ miles long. What is the area of the field?
Explain or show your reasoning. Use the diagram if it is helpful.



11. a. Label the point with a number it represents. Explain or show your reasoning.



- b. Locate 23.513 and 23.531 on the number line.

- c. Fill in the blank with $<$, $>$, or $=$ to make the statement true. Explain or show your reasoning.

$$23.513 \text{ ___ } 23.531$$

12. Find the value of each expression. Explain or show your reasoning.

a. 100×314

b. $314 \div 100$

c. 10×35.2

d. $35.2 \div 10$

13. Your goal is to use all of the numbers on the card and the operations of addition, subtraction, multiplication, and division to get as close as you can to the goal number.

14. Find each product using a strategy that makes sense to you.

a. 516×100

b. 473×36

c. 55×281

d. 999×67

e. 625×50

f. 83×315

g. 99×472

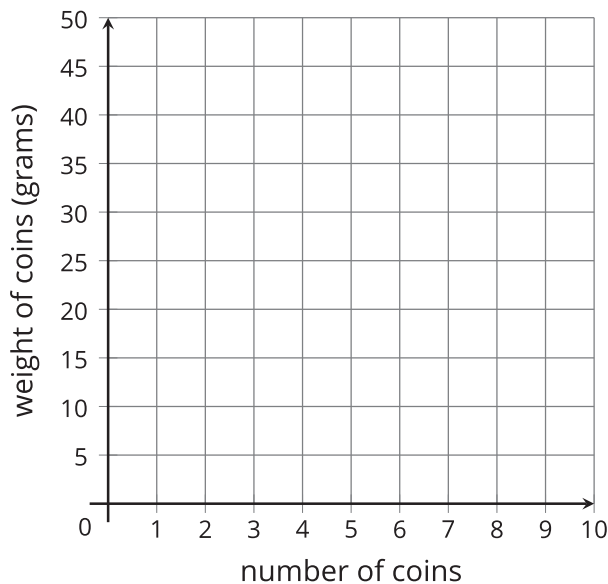
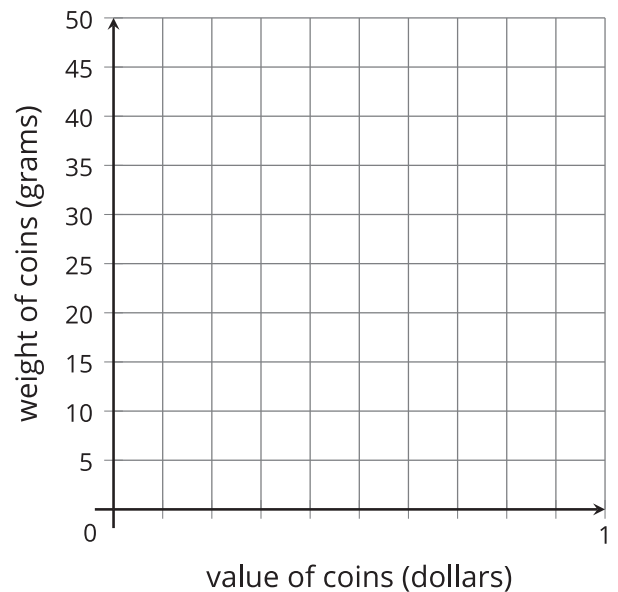
h. 20×358

i. 687×58

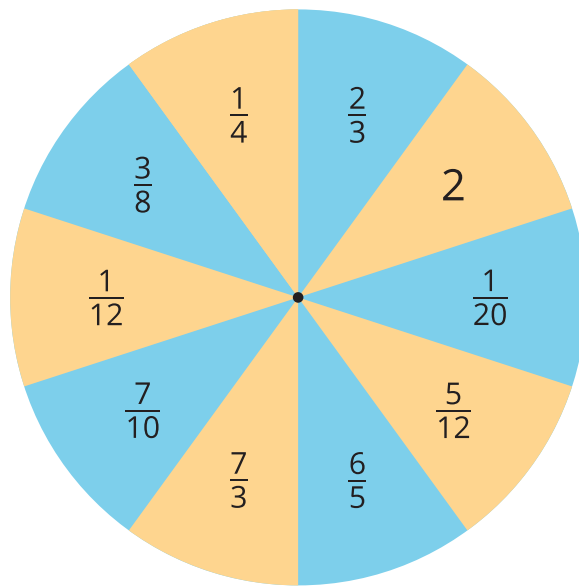
15. The table shows the weights of different coins.

coin	weight (grams)
penny	3.11
nickel	5
dime	2.27
quarter	5.67
half dollar	11.34

- Select a card that will tell you which coins you have. Plot a point on the "number of coins" graph for your coins.
- Try to figure out which coins each of your partners has based on the location of their point. Then check with each of them to see if you are correct.
- Select a new card telling you which coins you have. Plot a point on the "value of coins" graph for your coins.
- Try to figure out which coins each of your partners has based on the location of their point. Then check with each of them to see if you are correct.



16. Your goal is to make 1 with the numbers that you spin. You can add, subtract, or multiply.
You do not have to use all of the numbers you spin. Continue to spin until you can make 1.



17. Clare read that a bath can use between 120 and 200 liters of water. She wants to check if this is reasonable and makes some measurements of her bathtub.

length of bathtub	1.5 meters
width of bathtub	0.6 meters

Clare estimates that the depth of water in her bath is about 20 cm.

- What is the area of the base of the bathtub in square meters? Explain your reasoning.
- What is the area of the base of the bathtub in square centimeters?
- What is the volume of the water in Clare's bath in cubic centimeters?
- There are 1,000 cubic centimeters in 1 liter. Does the volume of water in Clare's bath agree with what she read? Explain your reasoning.



Assessment Answer Keys

End-of-Course Assessment and Resources

Assessment: End-of-Course Assessment and Resources

Teacher Instructions

The items here focus on major work of the grade, fluencies of the grade, and also include at least one in-depth problem that provides a context where students apply key ideas they have learned over the year. The items included here can be used prior to the final unit to focus remaining time in the year or to assess student understanding at the end of the year. It is not recommended that these resources be used all at once.

Students need card sets for two activities, number fluency cards and coin weight cards (Instructional masters are provided).

Problem 1

Standards Alignments

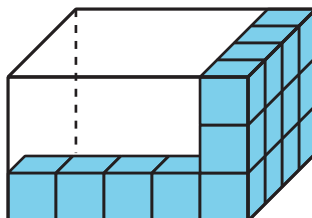
Addressing 5.MD.C.5.a, 5.MD.C.5.b, 5.OA.A

Narrative

Students select expressions that represent the volume of a rectangular prism. Students may select B if they are only counting the cubes that are shown in the image but which do not entirely fill the prism. Students may select C if they confuse the operations of addition and multiplication and what they mean in the context of finding the volume of a figure. Students may not select D if they do not see that the prism can be divided horizontally into 3 layers of 20. Students may not select E if they do not see the vertical slices that give 4 groups of 15 cubes.

Students can also solve this problem by calculating the volume of the prism in a way that makes sense to them and then comparing this to the value of each given expression.

Select **all** expressions that represent the volume of this rectangular prism in cubic units.



A. $5 \times 4 \times 3$

- B. $(3 \times 4) + 4$
- C. $5 \times (4 + 3)$
- D. 3×20
- E. 4×15

Solution

["A", "D", "E"]

Problem 2

Standards Alignments

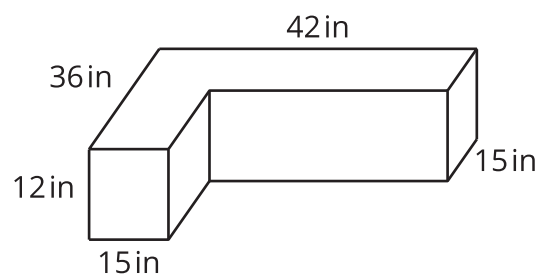
Addressing 5.MD.C.5.c, 5.NBT.B.5

Narrative

Students find the volume of a composite rectangular prism. They may divide it as in the solution or they may divide it into a 12 by 15 by 36 prism and a 12 by 15 by 27 prism. They could also cut off the 15 by 15 by 12 overlapping piece and find the volume of the remaining two pieces but this would require more calculation.

The numbers for the multiplication calculations have been chosen to be relatively friendly depending on how students divide the shape and the order in which they multiply the factors. Students may make small calculation errors but still demonstrate good understanding of how volume works. The arithmetic part and the volume understanding part of the task are distinct and are both vital to a thorough understanding of grade 5 material.

The image shows the measurements of a toy container.



What is the volume of the container? Explain or show your reasoning.

Solution

11,340 cubic inches

I can cut the shape into a 42 inch by 15 inch by 12 inch rectangular prism and a 21 inch by 15 inch by 12 inch prism. The volume of the first prism is $42 \times 15 \times 12$ cubic inches. First I found $15 \times 12 = 180$. Then I found $42 \times 180 = 7,560$. The second prism I found is 3,780 cubic inches. I added those and found 11,340 cubic inches.

Problem 3

Standards Alignments

Addressing 5.NF.A.1

Narrative

Students compare the value of sums and differences of fractions with $\frac{1}{2}$. In each case, there is a way to make the comparison without calculating but students may also find the value of an expression and then compare directly $\frac{1}{2}$. Students who select A have probably misread the question or made a miscalculation. They can reason that $\frac{1}{3} > \frac{1}{4}$ and $\frac{2}{4} = \frac{1}{2}$. Students who select B have probably made a calculation error or misread the $-$ sign for a $+$ sign. Since $\frac{5}{11} < \frac{1}{2}$, the result of the subtraction must also be less than $\frac{1}{2}$. Students who fail to select C have probably made a calculation error. Here, they can observe that $\frac{10}{20}$ is equivalent to $\frac{1}{2}$ so the value of the expression is less than $\frac{1}{2}$. Students can find the value of expressions D and E more readily than the others or reason that expression D is larger than 1 and expression E is less than $\frac{1}{2}$ because $\frac{7}{12} > \frac{1}{2}$.

Select **all** expressions whose value is **less** than $\frac{1}{2}$.

- A. $\frac{1}{3} + \frac{1}{4}$
- B. $\frac{5}{11} - \frac{2}{15}$
- C. $\frac{10}{20} - \frac{1}{12}$
- D. $1 + \frac{7}{12}$
- E. $1 - \frac{7}{12}$

Solution

["B", "C", "E"]

Problem 4

Standards Alignments

Addressing 5.NF.B.3, 5.NF.B.4, 5.NF.B.5

Narrative

This item complements the previous item. In this case, students compare expressions with 1 rather than with $\frac{1}{2}$ and the expressions all involve multiplication or division. Students can approach the division expressions using their understanding of the relationship between division and fractions or thinking about the meaning of division. For example if 10 sandwiches are split equally by 12 people then each person will get less than 1 sandwich because there are more people than sandwiches.

The problems involving expressions with multiplication assess a different skill. In each case, students can compare the expression with 1 using what they learned about how to compare a product with one of the factors based on the size of the other factor. The numbers are chosen to be complex so finding the value of the expression and comparing with 1, while it will always work, takes time and can lead to calculation errors.

Select **all** expressions whose value is greater than 1.

- A. $10 \div 12$
- B. $115 \div 73$
- C. $\frac{9}{8} \times \frac{7}{6}$
- D. $\frac{3}{2} \times \left(\frac{4}{10} + \frac{3}{5} \right)$
- E. $\frac{7}{8} \times \frac{9}{13}$

Solution

["B", "C", "D"]

Problem 5

Standards Alignments

Addressing 5.NF.B.4, 5.NF.B.7.c

Narrative

Students answer two questions about amounts of water. One is represented by the expression $5 \div \frac{1}{8}$. Students may not think of it this way and may also solve the problem using the expression 5×8 . They also may draw a diagram such as a tape diagram or a number line. The second problem is represented by the product $2\frac{3}{4} \times 5$. Here too, students could draw a picture or use the distributive property or rewrite $2\frac{3}{4}$ as a fraction.

A cooler contains 5 gallons of water.

- One bottle of water is $\frac{1}{8}$ of a gallon. How many bottles of water does the cooler contain?
Explain or show your reasoning.
- The running team drinks $2\frac{3}{4}$ coolers of water during practice. How many gallons is that?
Explain or show your reasoning.

Solution

40. There are 8 bottles in each gallon so that's 5×8 or 40 bottles of water in one cooler.
- $13\frac{3}{4}$ gallons. There are 10 gallons in 2 coolers and then $\frac{3}{4}$ of a cooler is $\frac{3}{4} \times 5$ or $\frac{15}{4}$ more gallons. That's $3\frac{3}{4}$ so there are $13\frac{3}{4}$ gallons altogether.

Problem 6

Standards Alignments

Addressing 5.NF.A.2

Narrative

Students compare two fractions in context where one of the denominators divides the other. Then they find the sum and difference of those fractions. Students do not need to use $\frac{1}{16}$ s of an inch for their calculations (some students may use a common denominator of 64, the product of 4 and 16) but will probably notice that by rewriting $\frac{1}{4}$ as $\frac{4}{16}$ they can make the comparison and perform the arithmetic operations.

Jada's math book is $\frac{5}{16}$ of an inch thick. Her science book is $\frac{1}{4}$ of an inch thick.

- Which book is thicker? How much thicker? Explain or show your reasoning.
- How thick are the math and science books together?

Solution

- The math book is thicker because $\frac{1}{4} = \frac{4}{16}$ and that's $\frac{1}{16}$ of an inch less thick than the math book.
- $\frac{9}{16}$ of an inch since $\frac{4}{16} + \frac{5}{16} = \frac{9}{16}$.

Problem 7

Standards Alignments

Addressing 5.NBT.A.2, 5.NBT.A.3

Narrative

Students identify different expressions that have the value one million. They may select A if they confuse thousands and millions. They may select C if they are not sure how to write one million. They may not select E or G, and may select F, if they do not understand place value well and do not know how to multiply a number by a power of 10.

Select **all** expressions that have the value one million.

- 10^3
- 10^6
- 10^7
- 1,000,000
- $10 \times 10 \times 10 \times 10 \times 10 \times 10$
- 100×100
- $100 \times 100 \times 100$

Solution

["B", "D", "E", "G"]

Problem 8

Standards Alignments

Addressing 5.NBT.B.6

Narrative

Students find two quotients, both are a 4-digit number divided by a 2-digit number. The first problem has simpler numbers and can be found readily by adding up multiples of 21, for example, and using the relationship between multiplication and division. The numbers for the second problem are more complex and partial quotients is a good strategy here, although students can use the same strategy and multiply up or draw a diagram to help them work through the division procedure.

Find the value of each quotient. Explain or show your reasoning.

- a. $1,512 \div 21$
- b. $7,776 \div 36$

Solution

- a. $50 \times 21 = 1,050$
 $20 \times 21 = 420$
 $2 \times 21 = 42$
 $72 \times 21 = 1,512$

- b. 216. I subtracted multiples of 36 until there was no remainder and then added up how many 36s I subtracted.

$$\begin{array}{r}
 \boxed{216} \\
 6 \\
 10 \\
 200 \\
 36 \overline{)7,776} \\
 \underline{-7,200} \\
 576 \\
 \underline{-360} \\
 216 \\
 \underline{-216} \\
 0
 \end{array}$$

Problem 9

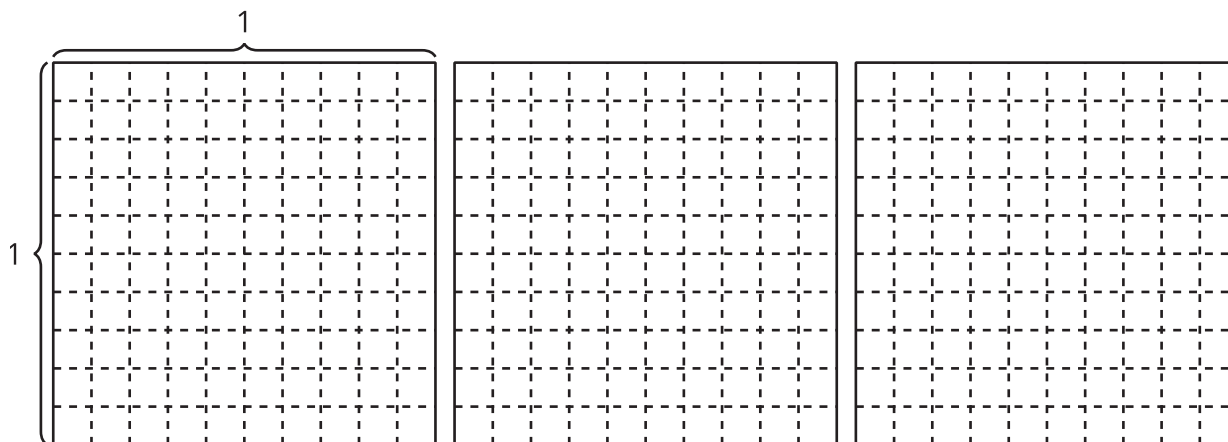
Standards Alignments

Addressing 5.NBT.B.7

Narrative

Students perform all 4 operations with decimal numbers. The addition and subtraction problems require students to identify that 2.4 can also be thought of as 2.40, that is as 2 and 40 hundredths. Then they can add or subtract the 2 hundredths from the 40. The multiplication problem can be solved by multiplying by place value. In order to evaluate 0.4×0.2 students can draw an area diagram or they can multiply both factors by 10, take the product of whole numbers, and then divide the result by 100. For the division problem, students may use an area diagram or reason about how many 0.2s there are in one whole.

Find the value of each expression. Explain or show your reasoning. Use the hundred grids if they are helpful.

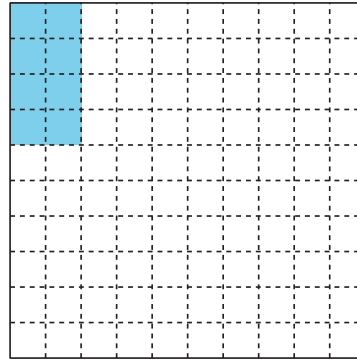


- a. $2.4 + 0.02$
- b. $2.4 - 0.02$
- c. 2.4×0.2
- d. $2.4 \div 0.2$

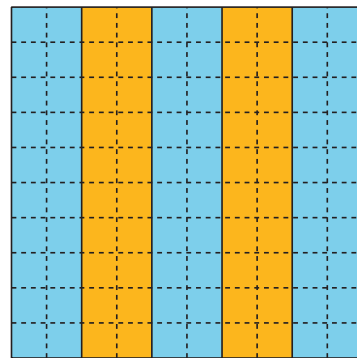
Solution

- a. 2.42 because I have 2 ones, 4 tenths, and 2 hundredths.
- b. 2.38 because I have 2 ones and 4 tenths and that's 2 tens, 3 tenths and 10 hundredths and I am left with 8 hundredths after taking away 2 hundredths.
- c. 0.48. I know $2 \times 0.2 = 0.4$ and I found

$0.4 \times 0.2 = 0.08$ using the diagram.



- d. 12. I drew a diagram and found that $5 \times 0.2 = 1$ so that means $10 \times 0.2 = 2$ and then 2 more makes 2.4.



Problem 10

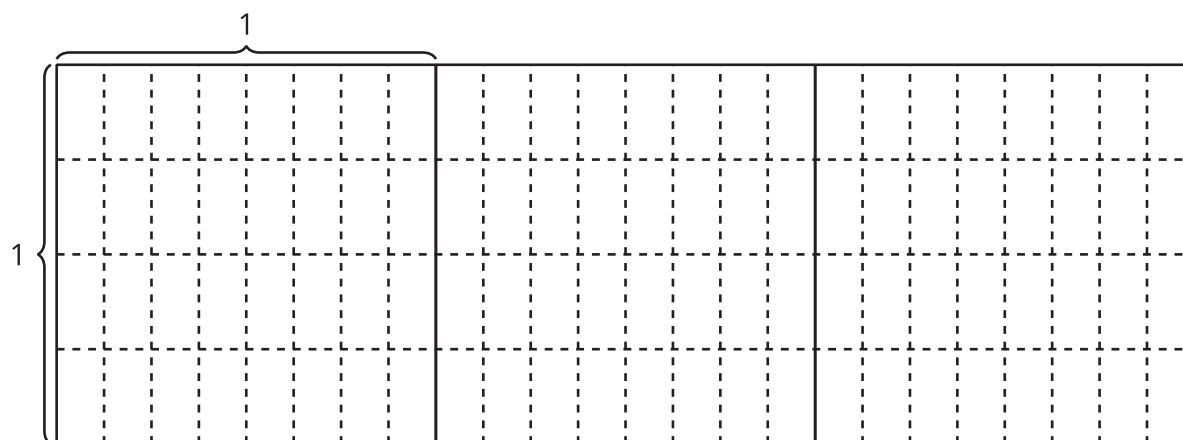
Standards Alignments

Addressing 5.NF.B.4, 5.NF.B.6

Narrative

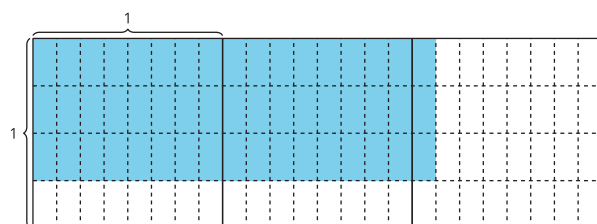
Students find the product of a fraction and a mixed number in an area context. They are given a diagram which they may use to represent the situation and help them solve the problem. Students may also use what they know about products of fractions and find the product without using the diagram.

A rectangular cornfield is $\frac{3}{4}$ mile wide and $2\frac{1}{8}$ miles long. What is the area of the field? Explain or show your reasoning. Use the diagram if it is helpful.



Solution

The area of the field is $\frac{3}{4} \times \frac{17}{8}$ square miles. That's $\frac{51}{32}$ square miles. The diagram shows that the number of small rectangles is 3×17 and each one is $\frac{1}{32}$ of a full square or square mile.



Problem 11

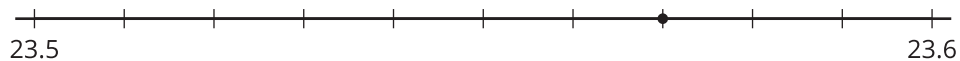
Standards Alignments

Addressing 5.NBT.A.3.b

Narrative

Students interpret a point on the number line and locate two other numbers, neither of which lies on a tick mark. Students will need to first identify that the tick marks represent hundredths and then estimate where 1 thousandth and 3 thousandths are in between two tick marks. It is not important that students plot 23.513 and 23.531 exactly. They should be relatively close to 23.51 and 23.53 respectively.

- Label the point with a number it represents. Explain or show your reasoning.

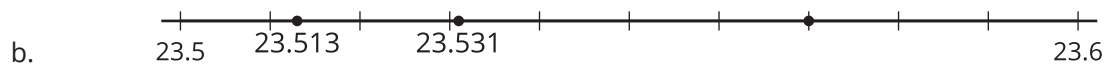


- b. Locate 23.513 and 23.531 on the number line.
- c. Fill in the blank with $<$, $>$, or $=$ to make the statement true. Explain or show your reasoning.
 23.513 23.531

Solution

Sample responses:

- a. 23.57 because the tick marks are hundredths and it is on the 7th tick after 23.5 so I need 7 hundredths.



- b.
- c. $<$ because 23.513 is to the left of 23.531 on the number line. It has only 1 tenth while 23.531 has 3 tenths.

Problem 12

Standards Alignments

Addressing 5.NBT.A.1, 5.NBT.A.2, 5.NBT.B.7

Narrative

Students multiply and divide whole numbers and decimals by powers of ten. As they find the value of each expression they relate the value of each place to the value of the place to its left and right by multiplication or division by 10.

Find the value of each expression. Explain or show your reasoning.

- a. 100×314
- b. $314 \div 100$
- c. 10×35.2
- d. $35.2 \div 10$

Solution

- a. 31,400 because each place value to the left has 10 times the value of a given place value. So multiplying by 100 shifts each digit two places to the left, once for each factor of 10.
- b. 3.14 because ones become hundredths when divided by 100, tens become tenths and

hundreds become ones.

- c. 352 because moving each digit one place to the left multiplies its value by 10.
- d. 3.52 because moving each digit one place to the right multiplies its value by $\frac{1}{10}$ or divides its value by 10.

Problem 13

Standards Alignments

Addressing 5.NBT.B.5, 5.OA.A.1, 5.OA.A.2

Narrative

The goal of this game is to provide practice applying the four operations to whole numbers. This is a variant of the game 24 where the goal is always to make 24 using 4 given numbers and the operations of addition, subtraction, multiplication, and division. Here the cards sometimes contain more than 4 numbers and the strategies will differ substantially depending how large the goal number is.

The important part of this activity is less on how close a student gets to the goal number, though that is supposed to serve as motivation and guidance for their calculations, than on their thinking and the practice they get developing fluency with the 4 operations.

It is recommended that the class play a round of this game together to communicate the rules. The cards vary in terms of difficulty and the size of the numbers and can be selected accordingly.

Use the Number Fluency cards attached as a blacklinemaster to this assessment.

Your goal is to use all of the numbers on the card and the operations of addition, subtraction, multiplication, and division to get as close as you can to the goal number.

Solution

For card Q

Sample response 1: $((5 \times 8) + 9) \times 3 \times 10 = 1,470$

Sample response 2: $((10 \times 9) + 8) \times 3 \times 5 = 1,470$

Sample response 3: $((8 + 9) \times 10) \times (5 + 3) = 1,360$

Problem 14

Standards Alignments

Addressing 5.NBT.B.5

Narrative

Students find products of a two-digit and three-digit number using a strategy that makes sense to them. Some of the problems call for the standard algorithm as there is no additional structure that would help make the calculation more efficiently. Other problems can be done using place value understanding and using the distributive property. Students may choose to do all of the problems with the standard algorithm and this will provide them with ample practice. It is not essential for students to do all of these problems at once. These problems can be used as needed for practice or to evaluate student fluency with multiplication.

Find each product using a strategy that makes sense to you.

- a. 516×100
- b. 473×36
- c. 55×281
- d. 999×67
- e. 625×50
- f. 83×315
- g. 99×472
- h. 20×358
- i. 687×58

Solution

Sample responses

- a. 51,600 because multiplying by 100 shifts each digit one place to the left
- b. 17,028

$$\begin{array}{r}
 2 \\
 4 1 \\
 4 7 3 \\
 \times 3 6 \\
 \hline
 1 1 \\
 2, 8 3 8 \\
 + 1 4, 1 9 0 \\
 \hline
 1 7, 0 2 8
 \end{array}$$

c. 15,455

$$\begin{array}{r}
 4 \\
 4 \\
 2 8 1 \\
 \times 5 5 \\
 \hline
 1, 4 0 5 \\
 + 1 4, 0 5 0 \\
 \hline
 1 5, 4 5 5
 \end{array}$$

d. 66,933. $1,000 \times 67 = 67,000$, $67,000 - 67 = 66,933$ e. 31,250. I first found 625×5 and then multiplied that by 10.

f. 26,145

$$\begin{array}{r}
 1 4 \\
 1 \\
 3 1 5 \\
 \times 8 3 \\
 \hline
 1 \\
 9 4 5 \\
 + 2 5, 2 0 0 \\
 \hline
 2 6, 1 4 5
 \end{array}$$

g. 46,728. I found $100 \times 472 - 472$ or $47,200 - 472$.h. 7,160. I found 2×358 then multiplied that by 10.

i. 39,846

$$\begin{array}{r}
 4 3 \\
 6 5 \\
 6 8 7 \\
 \times 5 8 \\
 \hline
 1 \\
 5, 4 9 6 \\
 + 3 4, 3 5 0 \\
 \hline
 3 9, 8 4 6
 \end{array}$$

Problem 15

Standards Alignments

Addressing 5.G.A.2, 5.NBT.B.7, 5.NF.A.1

Narrative

Students plot and interpret points in the coordinate plane representing sets of coins. Three different aspects of the coins are considered

- their value
- the number of coins
- the weight of the coins

Students worked with this context in an earlier unit so they are familiar with it. Here they do more arithmetic and interpret points on the graphs. In both cases, trying to figure out which set of coins a point on the graph corresponds to can lead to different solutions. This is clearest for the weight and value graph as 2 quarters have the same value and weight as 1 half dollar. For the number of coins and weight, four quarters and a dime have almost the same weight as 5 nickels and on the graph these two points will be indistinguishable.

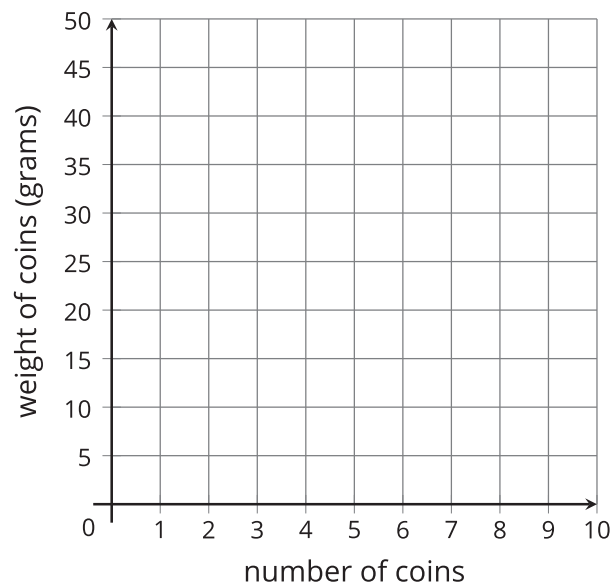
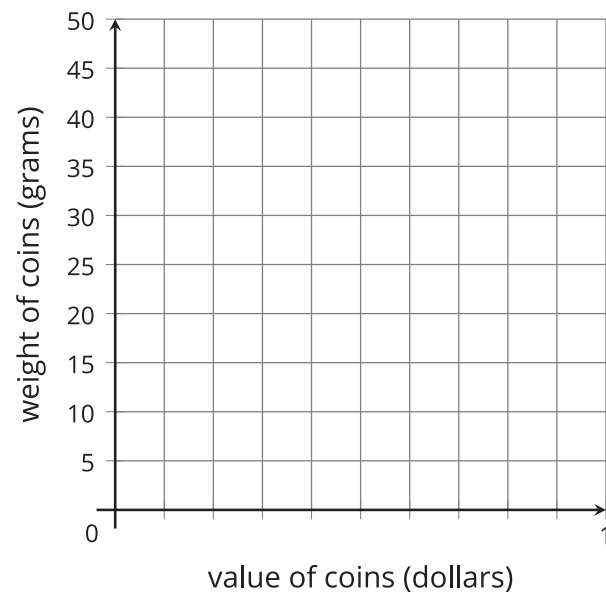
Students have not formally worked with the relationship between cents and dollars and so they may represent the value, in dollars, of the coins as fractions or as decimals. Either is acceptable and for this reason the value axis on the graph has not been labeled except for 0 and 1. It is divided into 10 equal parts but these could be labeled as decimals or fractions.

The Coin Weight Cards blacklinemaster attached to the assessment is required for this activity.

The table shows the weights of different coins.

coin	weight (grams)
penny	3.11
nickel	5
dime	2.27
quarter	5.67
half dollar	11.34

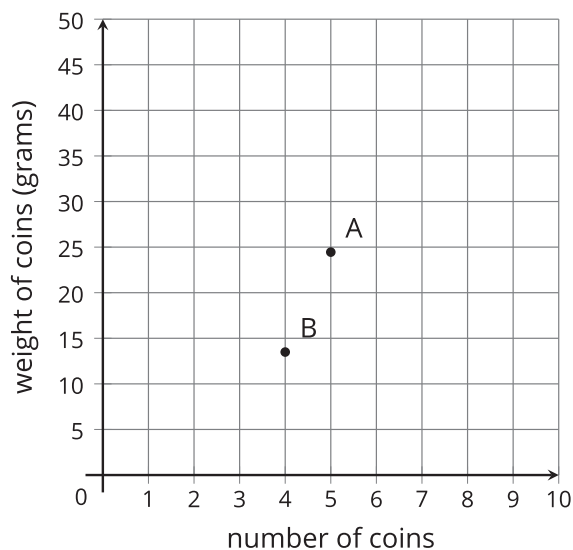
- Select a card that will tell you which coins you have. Plot a point on the “number of coins” graph for your coins.
- Try to figure out which coins each of your partners has based on the location of their point. Then check with each of them to see if you are correct.
- Select a new card telling you which coins you have. Plot a point on the “value of coins” graph for your coins.
- Try to figure out which coins each of your partners has based on the location of their point. Then check with each of them to see if you are correct.



Solution

Sample responses:

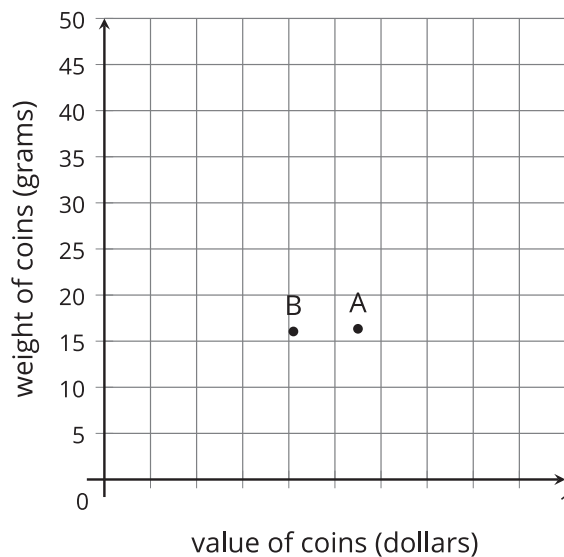
- a. I had 2 quarters, 2 nickels, and 1 penny (card G) so that is 5 coins and they weigh 24.45 grams so my point A is at 5 on the horizontal axis and between 24 and 25 on the vertical axis.



- b. My partner had 4 coins and they look like they weigh close to 14 grams. I tried 1 nickel, 2 pennies, and 1 dime and that is 4 coins that weigh 13.49 grams. That looks like it is about right but I am not sure. If I replace the nickel with a quarter, it is a little over 14 grams and that looks to be too heavy. But I could replace nickel with a quarter and a penny with a dime

and the change is very little. I'm not completely sure.

- c. I have 2 quarters and a nickel (Card A) so that is 55 cents or 0.55 dollars and they weigh 16.34 grams. I plotted the point A.



- d. My partner plotted point B and it also looks like she has about 16 grams and the value of her coins is a little more than 0.4 dollars or 40 cents. She could have 4 dimes and 2 pennies but that would be closer to 15 grams. Otherwise she must have a quarter and then nickels and pennies would weigh too much so she must have a dime. I think she has a quarter, a dime, a nickel, and a penny (Card E) which is 41 cents with weight between 16 and 17 grams .

Problem 16

Standards Alignments

Addressing 5.NF.A.1, 5.NF.B.4

Narrative

Students try to make 1 out of fractions that they get from a spinner. The goal here is to develop fluency with addition, subtraction, and multiplication of fractions with different denominators. The longer students play the game they will start to see patterns and likely be more creative in how they find ways to make 1. The first few tries may take longer as they need to experiment with the fractions to see how to combine them in an effort to get to 1.

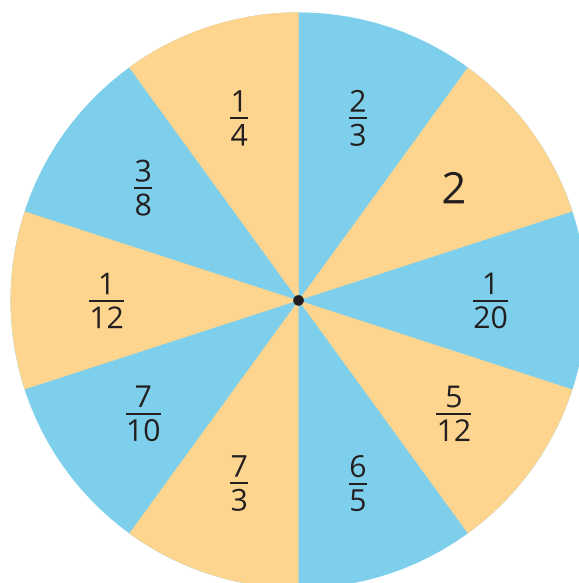
The numbers have been designed so that

- there is no way to make 1 with just two numbers
- there are many ways to make 1 with three (or more) numbers

The game can be beneficially played repeatedly until students develop a sense of the numbers and

their combinations and at that point it has served its purpose and starts to become more routine.

Your goal is to make 1 with the numbers that you spin. You can add, subtract, or multiply. You do not have to use all of the numbers you spin. Continue to spin until you can make 1.



Solution

Sample response: I got $\frac{1}{4}$ and $\frac{5}{12}$ which I could add to get $\frac{8}{12}$ or $\frac{2}{3}$ or subtract to get $\frac{2}{12}$ or $\frac{1}{6}$. Then I got $\frac{6}{5}$ which did not look like it was going to help. Next I got $\frac{5}{3}$ and then I was done because I could take my $\frac{2}{3}$ away from $\frac{5}{3}$ to get 1.

Problem 17

Standards Alignments

Addressing 5.MD.A.1, 5.MD.C.5, 5.NBT.A, 5.NBT.B.7

Narrative

Students multiply decimal and whole numbers to find a volume. They also perform two unit conversions, each of which requires either multiplying or dividing by a power of 10, giving students an opportunity to use what they have learned about place value.

Implicit in the problem is that Clare's bathtub is generally shaped like a rectangular prism. The teacher may wish to highlight this natural modeling assumption. This problem can also be made more hands-on by asking students to take measurements of a bathtub. The numbers students are

likely to get with their own measurements will not be as nice as the ones provided in the problem, making the arithmetic more difficult and possibly beyond the standards but it will make the problem more meaningful and they might be instructed to round their numbers to facilitate further calculations.

Clare read that a bath can use between 120 and 200 liters of water. She wants to check if this is reasonable and makes some measurements of her bathtub.

length of bathtub	1.5 meters
width of bathtub	0.6 meters

Clare estimates that the depth of water in her bath is about 20 cm.

- What is the area of the base of the bathtub in square meters? Explain your reasoning.
- What is the area of the base of the bathtub in square centimeters?
- What is the volume of the water in Clare's bath in cubic centimeters?
- There are 1,000 cubic centimeters in 1 liter. Does the volume of water in Clare's bath agree with what she read? Explain your reasoning.

Solution

- 0.9 square meters, I multiplied 1.5 and 0.6.
- There are 100 centimeters in a meter so there are 100×100 centimeters in a square meter. That's 10,000 square centimeters in a square meter so 0.9 square meters is $0.9 \times 10,000$ square centimeters or 9,000 square centimeters.
- I multiply the area of the base by the height of the water which is 20 centimeters to get 180,000 cubic centimeters of water Clare uses in her bath.
- Since 180 liters is 180,000 cubic centimeters that means Clare uses 180 liters of water in her bath and this is within the range specified.

Lesson
Cool Downs

Lesson 1: Find the Largest Product

Cool Down: Multiply 2 Digits by 2 Digits

Find the value of each product. Explain or show your reasoning.

1. 35×47

2. 37×45

Lesson 2: More Multiplication

Cool Down: What is Important?

What is important to remember when using the standard algorithm to multiply large numbers?

Lesson 3: Factors as a Factor in Our Strategy Choice

Cool Down: Reflect on Multiplication

Describe something new, interesting, or challenging you learned today about multiplication.

Lesson 4: Dive Back Into Division

Cool Down: Estimate and Evaluate

1. Estimate the value of $540 \div 15$.

2. Find the value of the quotient.

$$540 \div 15$$

Lesson 5: More Division

Cool Down: Partial Quotients

1. Find the value of the quotient.

$$24 \overline{) 2,976}$$

Lesson 6: Revisit Volume

Cool Down: Reflection: Volume

What are some big ideas about volume that you have learned this year?

Lesson 7: Estimating the Volume of the World's Largest Wagon

Cool Down: The Volume of the Wagon

If the Radio Flyer wagon is 27 feet long 13 feet wide and 2 feet deep, what is the volume of the wagon?

Lesson 8: Filling up the World's Largest Wagon

Cool Down: Multiplication and Division

How did you use multiplication and division to solve problems about volume?

Lesson 9: Problem Solving with Volume: Water

Cool Down: Reflection: Volume

What questions do you still have about measuring volume?

Lesson 10: Here Comes the Sum

Cool Down: Reflect on Fraction Addition

What is important to remember when adding fractions with unlike denominators?

Lesson 11: What's the Difference?

Cool Down: Reflect on Subtracting Fractions

What is important to remember when subtracting fractions with unlike denominators?

Lesson 12: Decimal Game Day

Cool Down: Reflect on Operating with Decimals

What is important to remember when adding decimal numbers?

Lesson 13: Multiply Fractions Game Day

Cool Down: Reflect on Multiplication

What is important to remember when multiplying fractions?

Lesson 14: Notice and Wonder

Cool Down: Reflection

Describe something you really understand well after today's lesson or describe something that was confusing or challenging.

Lesson 15: Estimation Exploration

Cool Down: Reflection

In math class, it's important to listen to other people's ideas. During class today, what is something you learned by listening carefully to someone?

Lesson 16: Number Talk

Cool Down: Reflection

As mathematicians we use patterns and structure in problems we solve to reason about new problems. Describe a time today when you did that.

Lesson 17: True or False?

Cool Down: Reflection

In math class, it's important to listen to other people's ideas. What was something you learned today by listening to someone else's ideas?

Lesson 18: Which One Doesn't Belong?

Cool Down: Reflection

As mathematicians, it is important to justify our thinking and listen to the reasoning of others. Describe a time when you learned something new or thought differently about something based on what someone else in the class said today.

Instructional Masters

Instructional Masters for Putting It All Together

address	title	students per copy	written on?	requires cutting?	card stock recommended?	color paper recommended?
Assessment Grade5.8	Coin Weight Cards	2	no	yes	no	no
Activity Grade5.8.18.1	Shape Cards for WODB Design	2	no	yes	no	no
Activity Grade5.8.18.2	Shape Cards for WODB Design	2	no	yes	no	no
Assessment Grade5.8	Number Fluency Cards	2	no	yes	no	no

Coin Weight Cards

Coin Weight Cards

A

- 1 nickel
- 2 quarters

Coin Weight Cards

B

- 5 pennies

Coin Weight Cards

C

- 3 quarters

Coin Weight Cards

D

- 2 nickels
- 2 dimes

Coin Weight Cards

E

- 1 penny
- 1 nickel
- 1 dime
- 1 quarter

Coin Weight Cards

F

- 3 dimes
- 2 quarters

Coin Weight Cards

G

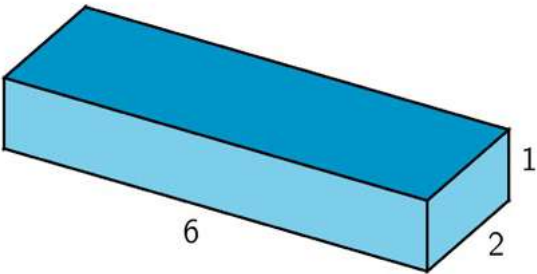
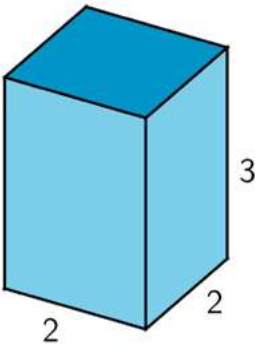
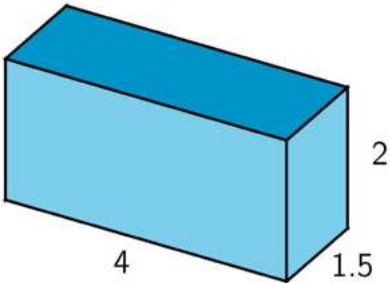
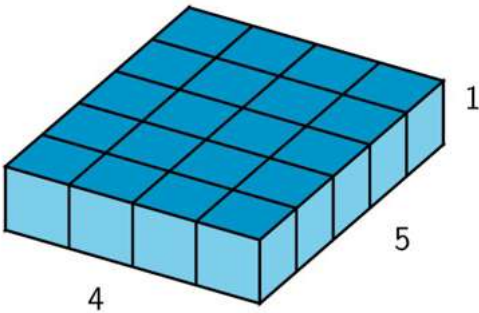
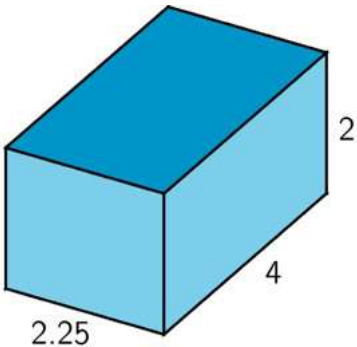
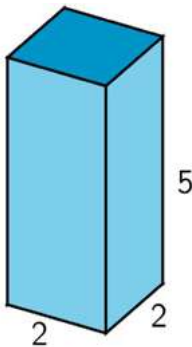
- 1 penny
- 2 nickels
- 2 quarters

Coin Weight Cards

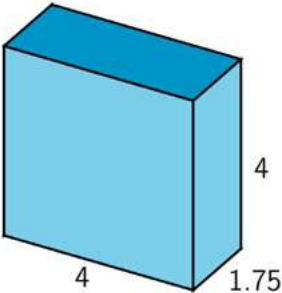
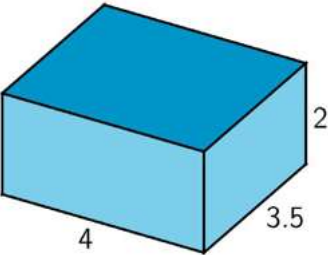
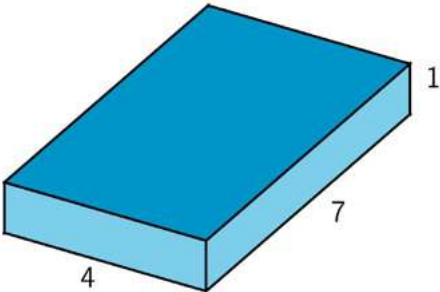
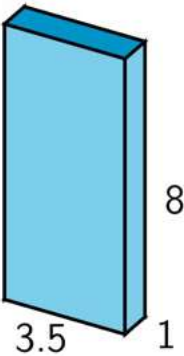
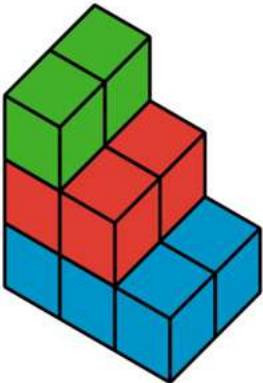
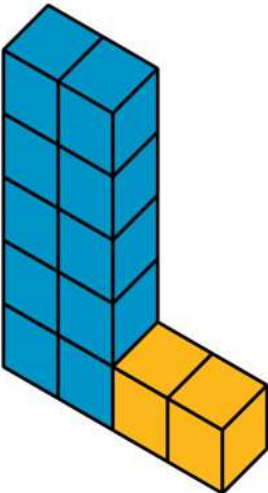
H

- 2 pennies
- 1 nickel
- 1 dime

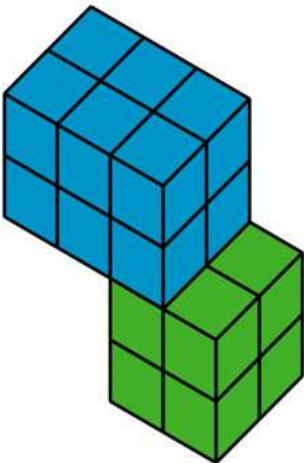
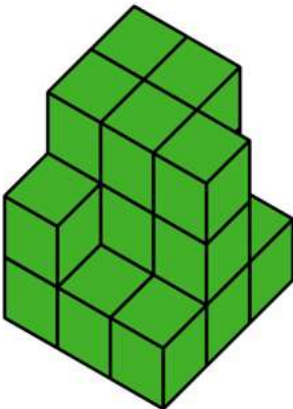
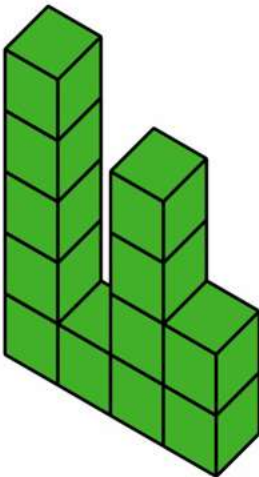
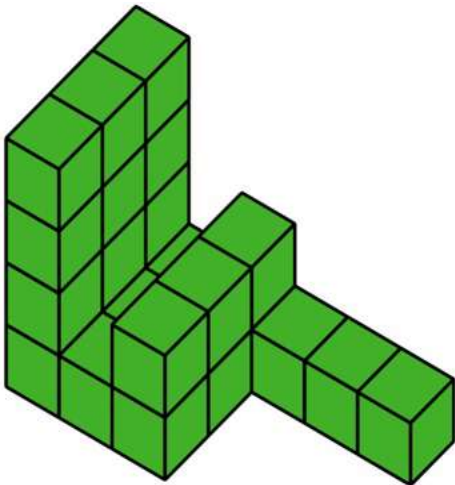
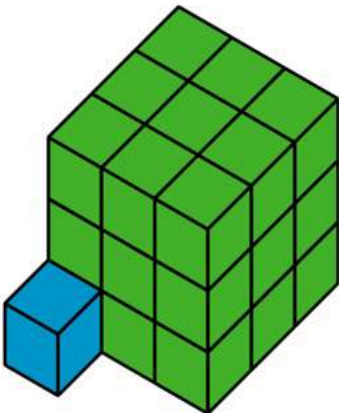
Shape Cards for WODB Design



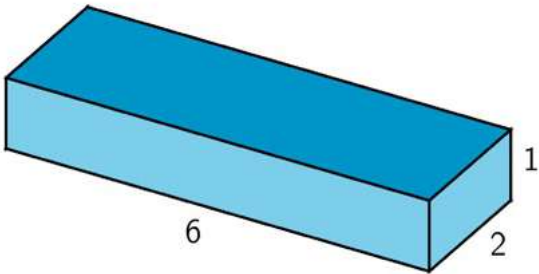
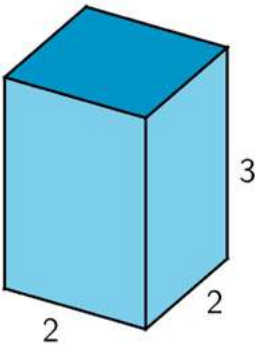
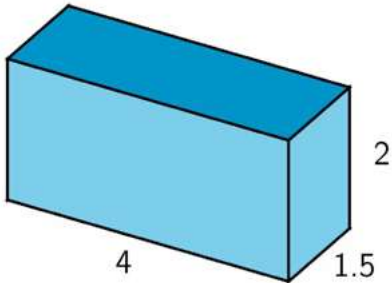
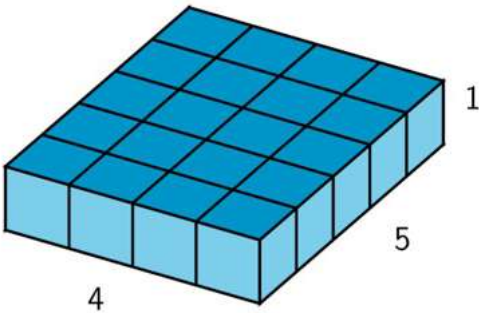
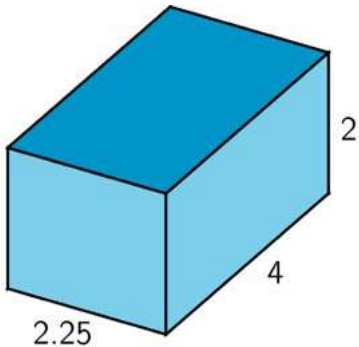
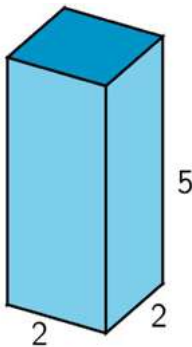
Shape Cards for WODB Design



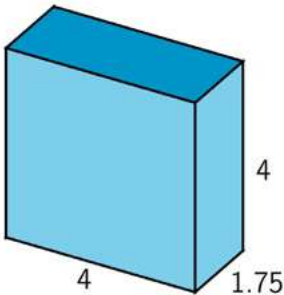
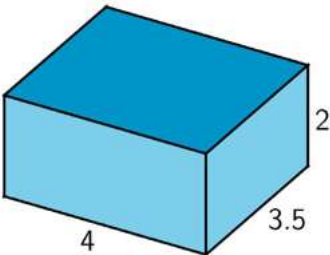
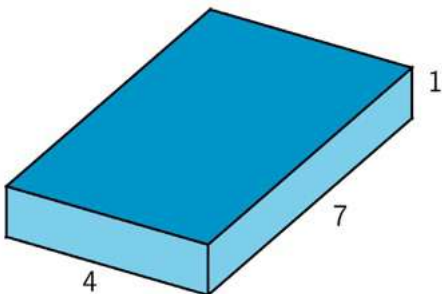
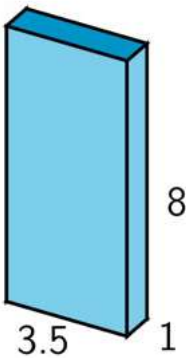
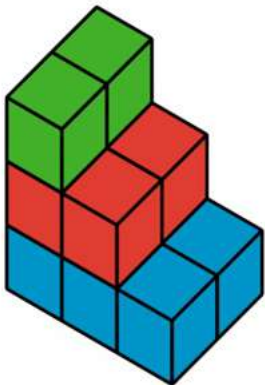
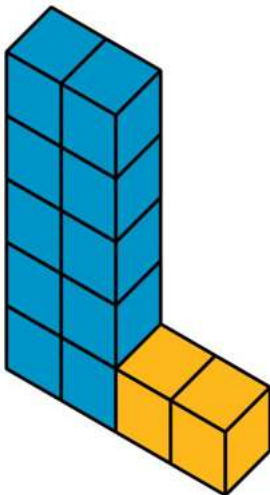
Shape Cards for WODB Design



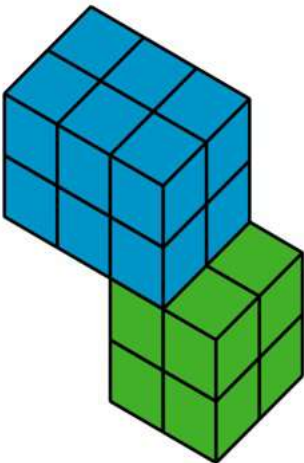
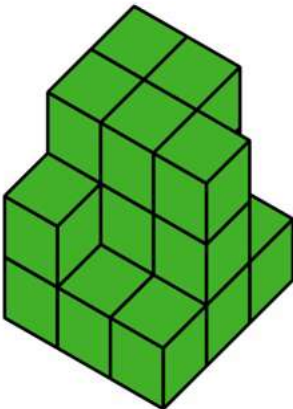
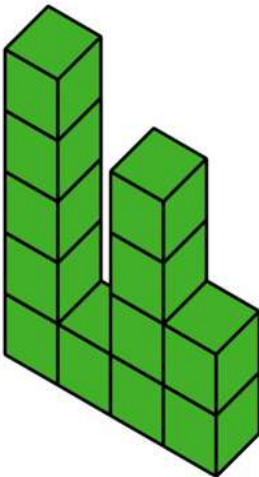
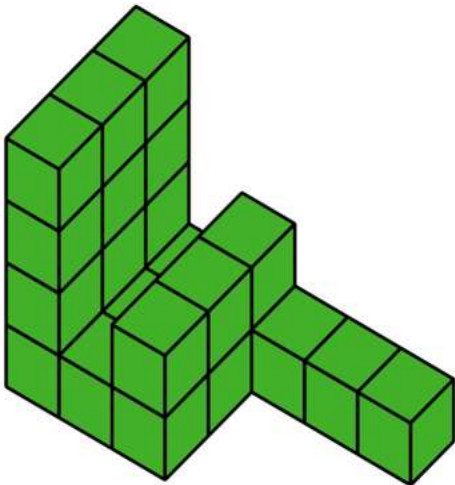
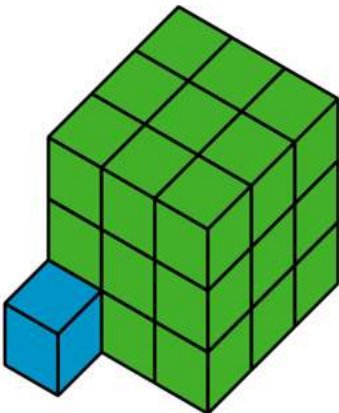
Shape Cards for WODB Design



Shape Cards for WODB Design



Shape Cards for WODB Design



Number Fluency Cards

Number Fluency Cards

A

- Numbers: 1, 2, 3, 4
- Goal Number: 24

Number Fluency Cards

B

- Numbers: 1, 2, 3, 4
- Goal Number: 25

Number Fluency Cards

C

- Numbers: 1, 2, 3, 4
- Goal Number: 26

Number Fluency Cards

D

- Numbers: 1, 2, 3, 4
- Goal Number: 27

Number Fluency Cards

E

- Numbers: 1, 2, 3, 4
- Goal Number: 17

Number Fluency Cards

F

- Numbers: 1, 2, 3, 4
- Goal Number: 32

Number Fluency Cards

G

- 1, 2, 3, 4, 5
- 100

Number Fluency Cards

H

- 1, 2, 3, 4, 5
- 150

Number Fluency Cards

Number Fluency Cards

I

- Numbers: 1, 4, 5, 6, 10
- Goal Number: 1,000

Number Fluency Cards

J

- Numbers: 2, 4, 8, 10, 10
- Goal Number: 2,000

Number Fluency Cards

K

- Numbers: 2, 4, 10, 12, 20
- Goal Number: 10,000

Number Fluency Cards

L

- Numbers: 5, 5, 5, 5, 5
- Goal Number: 500

Number Fluency Cards

M

- Numbers: 10, 10, 10, 10, 10, 10
- Goal Number: 1,000,000

Number Fluency Cards

N

- Numbers: 10, 10, 10, 10, 10, 10
- Goal Number: 190,000

Number Fluency Cards

O

- 10, 10, 10, 10, 10, 10
- 990,000

Number Fluency Cards

P

- 100, 100, 100, 100, 100, 100
- 203

Number Fluency Cards

Number Fluency Cards

Q

- Numbers: 3, 5, 8, 9, 10
- Goal Number: 1,500

Number Fluency Cards

R

- Numbers: 2, 6, 7, 10, 10
- Goal Number: 2,300

Number Fluency Cards

S

- Numbers: 5, 5, 6, 8, 10
- Goal Number: 3,500

Number Fluency Cards

T

- Numbers: 1, 5, 10, 20, 100
- Goal Number: 8,900

Number Fluency Cards

U

- Numbers: 3, 6, 9, 10, 100
- Goal Number: 570,000

Number Fluency Cards

V

- Numbers: 4, 7, 10, 10, 10
- Goal Number: 25,000

Number Fluency Cards

W

- Numbers: 3, 5, 10, 25, 50
- Goal Number: 240,000

Number Fluency Cards

X

- Numbers: 4, 5, 6, 7, 8, 9
- Goal Number: 7,500

Credits

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