Core Knowledge® MATHEMATICS

More Decimal and Fraction Operations

Teacher Guide

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More Decimal and Fraction Operations

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Teacher Guide

Core Knowledge Mathematics™

Unit 6: More Decimal and Fraction Operations

At a Glance

Unit 6 is estimated to be completed in 21-23 days including 2 days for assessment.

This unit is divided into three sections including 19 lessons and 2 optional lessons.

- Section A—Measurement Conversions and Powers of 10 (Lessons 1-7)
- Section B—Add and Subtract Fractions with Unlike Denominators (Lessons 8-15)
- Section C—The Size of Products (Lessons 16-21)

On pages 9-10 of this Teacher Guide is a chart that identifies the section each lesson belongs in and the materials needed for each lesson.

This unit uses eight student centers.

- Number Puzzles: Multiplication and Division
- Five in a Row: Multiplication
- Would You Rather?
- Compare
- How Close?
- Creating Line Plots
- Rectangle Rumble
- Rolling for Fractions

Unit 6: More Decimal and Fraction Operations

Unit Learning Goals

- Students solve multi-step problems involving measurement conversions, line plots, and fraction operations, including addition and subtraction of fractions with unlike denominators. They also explain patterns when multiplying and dividing by powers of 10 and interpret multiplication as scaling by comparing products with factors.

In this unit, students deepen their understanding of place-value relationships of numbers in base ten, unit conversion, operations on fractions with unlike denominators, and multiplicative comparison. The work here builds on several important ideas from grade 4.

In grade 4, students learned the value of each digit in a whole number is 10 times the value of the same digit in a place to its right. Here, they extend that insight to include decimals to the thousandths. Students recognize that the value of each digit in a place (including decimal places) is $\frac{1}{10}$ the value of the same digit in the place to its left.

This idea is highlighted as students perform measurement conversions in metric units.

Previously, students learned to convert from a larger unit to a smaller unit. Here, they learn to convert from a smaller unit to a larger unit. They observe how the digits shift when multiplied or divided by a power of 10 and learn to use exponential notation for powers of 10 to represent large numbers.

L	mL
5	
6.3	
0.95	
10^2	
	800,000
	10^6
	65

Next, students turn their attention to fractions. In earlier grades, students made sense of equivalent fractions, added and subtracted fractions with the same denominator, and added tenths and hundredths. In this unit, they add and subtract fractions with different denominators. They see that the key is to find a common denominator and analyze different techniques for doing so.

Students then solve problems that involve measurement data (in halves, fourths, and eighths) that are displayed on line plots.

In the final section, students reason about the size of a product of fractions and that of the factors. This work builds on the multiplicative comparison work in grade 4, in which students compared a whole number as "_____ times as many (or as much) as" another whole number. Here, students reason about products of a whole number and a fraction without finding the value of each product. They use diagrams and expressions to support their reasoning.

Write <, >, or = in each blank to make true statements.

$\frac{4}{5} \times 851$ ____ 851 $\qquad$ $\frac{99}{8} \times \frac{23}{22}$ ____ $\frac{99}{8}$

$\frac{1}{4}$ ____ $\frac{5}{5} \times \frac{1}{4}$ $\qquad$ $\frac{100}{7} \times \frac{9}{13}$ ____ $\frac{9}{13}$

Section A: Measurement Conversions and Powers of 10

Standards Alignments

Building On	5.MD.A.1, 5.NBT.A.2
Addressing	5.MD.A.1, 5.NBT.A, 5.NBT.A.1, 5.NBT.A.2
Building Towards	5.MD.A.1, 5.NBT.A.2

Section Learning Goals

- Explain patterns when multiplying and dividing by powers of 10.
- Solve multi-step problems involving measurement conversions.

In this section, students extend their understanding of place value and apply it to perform conversions between different, mostly metric, units.

Students begin by observing that the value of the digit in each place is 10 times the value of the same digit in the place to its right and $\frac{1}{10}$ the value of the same digit in the place to its left. They see that this applies not only to whole-number places but also to decimal places. Students then learn to use exponential notation for powers of 10 and use this notation to represent very large numbers such as 1 million or 1 billion.

Next, students reason about measurement conversions in metric and customary units. Conversion in metric units further highlights place-value relationships in numbers in base ten. For example, this table shows some distances in centimeters, meters, and kilometers.

centimeters (cm)	meters (m)	kilometers (km)
1,500	15	0.015
15,000	150	0.15
150,000	1,500	1.5

Students notice that multiplying or dividing by a power of 10 shifts the position of the digits in a decimal number to the right or left.

As they perform conversions from a larger unit to a smaller unit and the other way around, students apply what they learned about performing operations on whole numbers and decimals.

PLC: Lesson 2, Activity 1, Population of Delaware and India

Suggested Centers

- Number Puzzles: Multiplication and Division (4–5), Stage 2: Multi-digit Factors (Supporting)
- Five in a Row: Multiplication (3–5), Stage 3: Two-digit Factors (Supporting)
- Would You Rather? (2–5), Stage 2: Compare to Smaller Units (Supporting)
- Compare (1–5), Stage 5: Fractions (Supporting)
- Would You Rather? (2–5), Stage 3: Compare Units in a Given System (Addressing)
- Compare (1–5), Stage 6: Add and Subtract Fractions (Supporting)

Section B: Add and Subtract Fractions with Unlike Denominators

Standards Alignments

Addressing	5.MD.B.2, 5.NF.A.1, 5.NF.A.2, 5.NF.B.4
Building Towards	5.MD.B.2

Section Learning Goals

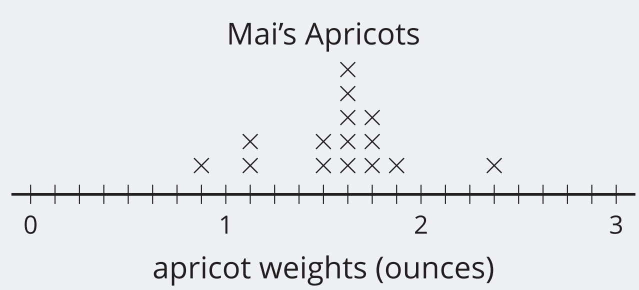
- Add and subtract fractions with unlike denominators.
- Create line plots to display fractional measurement data, and use the information to solve problems.
- Solve problems involving addition and subtraction of fractions.

In this section, students learn to add and subtract fractions (including mixed numbers) with unlike denominators and apply this learning to solve problems.

Students begin to add and subtract fractions using strategies and diagrams that make sense to them, relying what they know about adding and subtracting fractions with like denominators and with equivalent fractions. They then consider ways to write equivalent fractions so that the fractions in an expression have the same denominator. Later, they analyze and then use numerical strategies for finding common denominators, such as multiplying the denominators and finding a common multiple.

At the end of the section, students create line plots to display measurement data in fractional units (halves, fourths, and eighths), interpret the data on line plots, and use all four fraction operations to solve problems involving fractional measurements.

Do all of Mai's apricots together weigh more or less than a pound?



PLC: Lesson 8, Activity 2, Add and Subtract

Suggested Centers

- Would You Rather? (2–5), Stage 3: Compare Units in a Given System (Addressing)
- Compare (1–5), Stage 6: Add and Subtract Fractions (Supporting)
- How Close? (1–5), Stage 9: Add Fractions to 5 (Addressing)
- Creating Line Plots (2–5), Stage 3: Eighth Inches, Add and Subtract (Supporting)
- Creating Line Plots (2–5), Stage 4: Eighth Inches, Add, Subtract, and Multiply (Addressing)
- Rectangle Rumble (3–5), Stage 4: Whole Number and Fraction Factors (Supporting)

Section C: The Size of Products

Standards Alignments

Addressing	5.MD.B.2, 5.NF.A.2, 5.NF.B.4, 5.NF.B.5, 5.NF.B.5.a, 5.NF.B.5.b, 5.OA.A
Building Towards	5.NF.B.5.a, 5.NF.B.5.b

Section Learning Goals

- Interpret multiplication as scaling (resizing).
- Make generalizations about multiplying a whole number by a fraction greater than, less than and equal to 1.

In this section, students build on their understanding of multiplication to include the concept of scaling. They interpret multiplication expressions as a quantity that is resized or scaled by a factor. This idea builds on the multiplicative comparison work students did with whole numbers in grade 4.

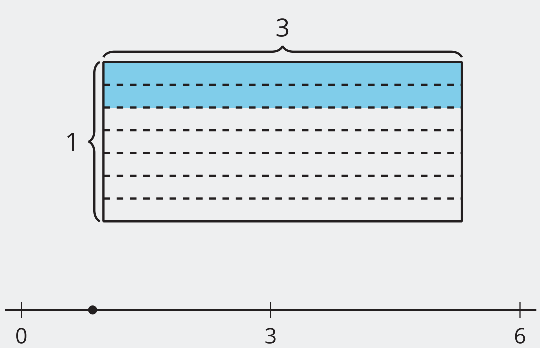
To develop an understanding of this concept, students compare the value of multiplication expressions without performing the multiplication. Early in the section, the expressions are such that one factor is the same and the other one is different.

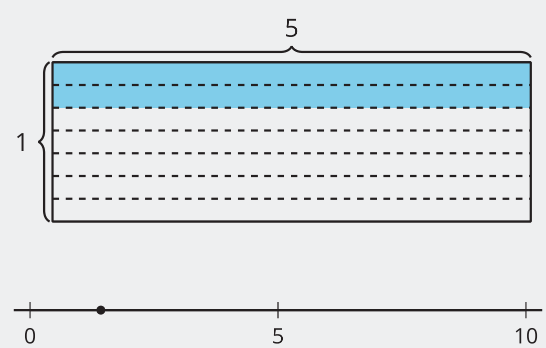
Which expression represents the largest product?

$\frac{5}{8} \times 4$ $\qquad$ $\frac{7}{6} \times 4$ $\qquad$ $\frac{1}{2} \times 4$

For example, they reason that $\frac{7}{6} \times 4$ is greater than $\frac{5}{8} \times 4$ and $\frac{1}{2} \times 4$ because in each expression, 4 is being multiplied by a fraction, and $\frac{7}{6}$ is the largest of the three.

Students use visual representations to help them compare products. For instance, the following diagrams can represent $\frac{2}{7} \times 3$ and $\frac{2}{7} \times 5$.





Students also reason about products with one unknown factor, which prompts them to make the comparisons based on the size of the other factor.

PLC: Lesson 19, Activity 1, Compare Fraction Products on the Number Line

Suggested Centers

- Creating Line Plots (2–5), Stage 4: Eighth Inches, Add, Subtract, and Multiply (Addressing)
- Rectangle Rumble (3–5), Stage 4: Whole Number and Fraction Factors (Supporting)
- Rectangle Rumble (3–5), Stage 5: Fraction Factors (Addressing)
- Rolling for Fractions (3–5), Stage 4: Multiply Fractions (Supporting)

Throughout the Unit

The Number Talk routine is used throughout the unit to support students' developing fluency and to see the multiplicative structures present in the base-ten system, adding and subtracting of fractions, and multiplication of fractions. Students use benchmark fractions and equivalent fractions to reason about the value of the expressions.

Here is a sampling of Number Talk warm-ups in the unit.

lesson 3	lesson 6	lesson 7	lesson 11	lesson 13	lesson 15
100×1.5 $1,000 \times 1.5$ $15 \div 10$ $15 \div 100$	$1,400 \div 10$ $1,400 \div 100$ $1,400 \div 1,000$ $1,401 \div 1,000$	45×10 45×2 45×12 46×12	$3 + \frac{7}{8}$ $3 - \frac{7}{8}$ $1\frac{5}{8} + \frac{6}{8}$ $1\frac{5}{8} - \frac{6}{8}$	$\frac{1}{8} + \frac{5}{8}$ $\frac{1}{8} + \frac{6}{16}$ $\frac{1}{8} + \frac{1}{3}$ $\frac{1}{8} + \frac{5}{12}$	$\frac{1}{3} \times 18$ $\frac{2}{3} \times 18$ $\frac{4}{3} \times 18$ $\frac{5}{3} \times 18$

The True or False routine is used to bring to the forefront some of the concepts students are learning in this unit, such as converting metric units, adding fractions, and determining how factors impact the value of a product.

Here is a sampling of True or False warm-ups in the unit.

lesson 4	lesson 9	lesson 16	lesson 20
$5 \div 1,000 = 0.05$ $36 \div 100 = 0.36$ $1,328 \div 1,000 = 1.328$	$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$ $\frac{1}{2} + \frac{1}{4} = \frac{2}{4}$ $\frac{3}{4} - \frac{1}{2} = \frac{2}{4}$	$\frac{4}{5} \times 100 = 120$ $\frac{4}{5} \times 100 < 100$ $\frac{4}{5} \times 100 = 80$	$\frac{3}{4} = 1 - \frac{1}{4}$ $\left(1 - \frac{1}{4}\right) \times 9 = 9 - \left(\frac{1}{4} \times 9\right)$ $\left(1 + \frac{1}{4}\right) \times 7 = (1 \times 7) + \frac{1}{4}$

Materials Needed

LESSON	GATHER	COPY
A.1	• Tools for creating a visual display	• none
A.2	• none	• none
A.3	• Metersticks	• none
A.4	• none	• none
A.5	• Metersticks	• none
A.6	• none	• none
A.7	• Yardsticks	• Customary Measurement Card Sort (groups of 1)
B.8	• none	• Fraction Add and Subtract Sort (groups of 2)
B.9	• none	• none
B.10	• none	• none
B.11	• none	• none
B.12	• none	• none
B.13	• none	• none
B.14	• Paper clips • Pencils	• none
B.15	• none	• Info Gap: Picking Fruit (groups of 2)
C.16	• none	• none
C.17	• none	• none
C.18	• none	• none

C.19	• none	• none
C.20	• Tools for creating a visual display	• none
C.21	• none	• none

Center: Number Puzzles: Multiplication and Division (4–5)

Stage 2: Multi-digit Factors

Lessons

- Grade5.6.A1 (supporting)
- Grade5.6.A2 (supporting)
- Grade5.6.A3 (supporting)

Stage Narrative

Students use the digits 0–9 to make multiplication equations with multi-digit factors true. Each digit may only be used one time.

Standards Alignments

Addressing	5.NBT.B.5, 5.NBT.B.6

Materials to Copy

Number Puzzles Mult Stage 2 Recording Sheet (groups of 1)

Stages used in Grade 4

Stage 1

Addressing

- Grade4.6.B

Center: Five in a Row: Multiplication (3–5)

Stage 3: Two-digit Factors

Lessons

- Grade5.6.A1 (supporting)
- Grade5.6.A2 (supporting)
- Grade5.6.A3 (supporting)

Stage Narrative

Students multiply using two-digit factors. Partner A chooses two numbers and places a paper clip on each number. They multiply the numbers and place a counter on the product. Partner B moves one of the paper clips to a different number, multiplies the numbers, and places a counter on the product. Students take turns moving one paper clip, finding the product, and covering it with a counter.

Standards Alignments

Addressing 4.NBT.B.5

Materials to Gather

Paper clips, Two-color counters

Materials to Copy

Five in a Row Multiplication and Division Stage 3 Gameboard (groups of 2)

Additional Information

Each group of 2 needs 25 two-color counters and 2 paper clips.

Stages used in Grade 4

Stage 1

Supporting

- Grade4.1.A

Stage 2

Addressing

- Grade4.1.A
- Grade4.1.B

Supporting

- Grade4.5.A
- Grade4.6.A
- Grade4.6.B

Stage 3

Addressing

- Grade4.6.B
- Grade4.6.C

Center: Would You Rather? (2–5)

Stage 2: Compare to Smaller Units

Lessons

- Grade5.6.A4 (supporting)
- Grade5.6.A5 (supporting)
- Grade5.6.A6 (supporting)

Stage Narrative

The first partner spins to get a measurement and a unit. They write a question that compares the amount they spun to a quantity reported in a smaller unit of measurement.

Standards Alignments

Addressing 4.MD.A.1

Materials to Copy

Would You Rather Stage 2 Recording Sheet (groups of 2), Would You Rather Stage 2 Spinner (groups of 2)

Stage 3: Compare Units in a Given System

Lessons

- Grade5.6.A7 (addressing)
- Grade5.6.B8 (addressing)
- Grade5.6.B9 (addressing)

Stage Narrative

The first partner spins to get a measurement and a unit. They write a question that compares the amount they spun to a quantity reported in the given measurement system.

Standards Alignments

Addressing 5.MD.A.1

Materials to Copy

Would You Rather Stage 3 Recording Sheet (groups of 2), Would You Rather Stage 3 Spinner (groups of 2)

Stages used in Grade 4

Stage 2

Addressing

- Grade4.5.B

Center: Compare (1–5)

Stage 5: Fractions

Lessons

- Grade5.6.A4 (supporting)
- Grade5.6.A5 (supporting)
- Grade5.6.A6 (supporting)

Stage Narrative

Students use cards with fractions. They may use either deck of fraction cards or combine them together to play.

Standards Alignments

Addressing 4.NF.A.2

Materials to Copy

Compare Stage 3-8 Directions (groups of 2), Fraction Cards Grade 3 (groups of 2), Fraction Cards Grade 4 (groups of 2)

Stage 6: Add and Subtract Fractions

Lessons

- Grade5.6.A7 (supporting)
- Grade5.6.B8 (supporting)
- Grade5.6.B9 (supporting)
- Grade5.6.B10 (supporting)

Stage Narrative

Students use cards with expressions with addition and subtraction of fractions with the same denominator.

Standards Alignments

Addressing 4.NF.B.3

Materials to Copy

Compare Stage 3-8 Directions (groups of 2), Compare Stage 6 Cards (groups of 2)

Stages used in Grade 4

Stage 3

Supporting

- Grade4.2.C
- Grade4.3.C
- Grade4.5.A
- Grade4.5.B
- Grade4.6.B

Stage 4

Supporting

- Grade4.6.C

Stage 5

Addressing

- Grade4.2.C

Supporting

- Grade4.3.A
- Grade4.7.A
- Grade4.7.B
- Grade4.7.C
- Grade4.8.A
- Grade4.8.B

Stage 6

Addressing

- Grade4.3.B
- Grade4.3.C

Stage 7

Addressing

- Grade4.6.D

Supporting

- Grade4.7.A
- Grade4.8.A
- Grade4.8.B

Center: How Close? (1–5)

Stage 9: Add Fractions to 5

Lessons

- Grade5.6.B10 (addressing)
- Grade5.6.B11 (addressing)
- Grade5.6.B12 (addressing)
- Grade5.6.B13 (addressing)

Stage Narrative

Before playing, students remove the cards that show 10 and set them aside.

Each student picks 6 cards and chooses 4 of them to create an addition expression with fractions. Each student adds their numbers and the student whose sum is closest to 5 wins a point for the round. Students pick new cards so that they have 6 cards in their hand and then start the next round.

Variation:

Students can choose a different number as the goal.

Standards Alignments

Addressing 5.NBT.B.7

Materials to Gather

Number cards 0–10

Materials to Copy

How Close? Stage 9 Recording Sheet (groups of 1)

Stages used in Grade 4

Stage 5

Supporting

- Grade4.5.A

Stage 6

Addressing

- Grade4.5.A
- Grade4.5.B

Supporting

- Grade4.2.C

Center: Creating Line Plots (2–5)

Stage 3: Eighth Inches, Add and Subtract

Lessons

- Grade5.6.B11 (supporting)
- Grade5.6.B12 (supporting)

Stage Narrative

Students measure up to eight objects to the nearest $\frac{1}{8}$ inch. They work with a partner to create a line plot to represent their measurement data. Then, they ask their partner two questions that can be answered based on the data in their line plot that uses addition or subtraction.

Variation:

If students completed the Estimate and Measure Center, they may choose to use their length measurements to represent on the line plot.

Standards Alignments

Addressing	4.MD.B.4

Materials to Gather

Objects of various lengths, Rulers (inches)

Materials to Copy

Creating Line Plots Stage 3 Recording Sheet (groups of 1)

Additional Information

Gather or identify objects of various lengths (pencils, markers, books, glue, scissors, shoe, tape dispenser, side of desk, length of bulletin board).

Stage 4: Eighth Inches, Add, Subtract, and Multiply

Lessons

- Grade5.6.B14 (addressing)
- Grade5.6.B15 (addressing)
- Grade5.6.C16 (addressing)

Stage Narrative

Students measure up to eight objects to the nearest $\frac{1}{8}$ inch. They work with a partner to create a line plot to represent their measurement data. Then, they ask their partner two questions that can be answered based on the data in their line plot that uses addition, subtraction, or multiplication.

Standards Alignments

Addressing 5.MD.B.2

Materials to Gather

Objects of various lengths, Rulers (inches)

Materials to Copy

Creating Line Plots Stage 4 Recording Sheet (groups of 1)

Additional Information

Gather or identify objects of various lengths (pencils, markers, books, glue, scissors, shoe, tape dispenser, side of desk, length of bulletin board).

Stages used in Grade 4

Stage 2

Supporting

- Grade4.3.B

Stage 3

Addressing

- Grade4.3.B

Center: Rectangle Rumble (3–5)

Stage 4: Whole Number and Fraction Factors

Lessons

- Grade5.6.B14 (supporting)
- Grade5.6.B15 (supporting)
- Grade5.6.C16 (supporting)

Stage Narrative

Students generate factors with a number cube and a spinner with fractions with denominators of 3, 6, or 12. Students use a 24×24 grid that represents 16 square units.

Standards Alignments

Addressing 5.NF.B.4

Materials to Gather

Colored pencils, crayons, or markers, Number cubes, Paper clips

Materials to Copy

Rectangle Rumble Stage 4 Grid (groups of 2), Rectangle Rumble Stage 4 Spinner (groups of 2)

Additional Information

Each group of students need a paper clip, a number cube, and two different color writing utensils.

Stage 5: Fraction Factors

Lessons

- Grade5.6.C17 (addressing)
- Grade5.6.C18 (addressing)
- Grade5.6.C19 (addressing)
- Grade5.6.C20 (addressing)
- Grade5.6.C21 (addressing)

Stage Narrative

Students generate factors with a number cube and a spinner with the numbers $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{20}$, and wild, where students can choose their own number. They find the products and fill a 20×20 grid that represents 1 square unit.

Stage Description

Each group of students need a paper clip, a number cube, and two different color writing utensils.

Standards Alignments

Addressing 5.NF.B.4

Materials to Gather

Colored pencils, crayons, or markers, Number cubes, Paper clips

Materials to Copy

Rectangle Rumble Stage 5 Grid (groups of 2), Rectangle Rumble Stage 5 Spinners (groups of 2)

Stages used in Grade 4

Stage 3

Supporting

- Grade4.5.C

Center: Rolling for Fractions (3–5)

Stage 4: Multiply Fractions

Lessons

- Grade5.6.C17 (supporting)
- Grade5.6.C18 (supporting)
- Grade5.6.C19 (supporting)
- Grade5.6.C20 (supporting)
- Grade5.6.C21 (supporting)

Stage Narrative

Students roll 4 number cubes to generate a multiplication expression involving 2 fractions and compare the value of the expressions. Two recording sheets are provided, one where one fraction is a unit fraction and one with no numerators given.

Standards Alignments

Addressing 5.NF.B.4.a

Materials to Gather

Number cubes

Materials to Copy

Rolling for Fractions Stage 4 Recording Sheet (groups of 1)

Additional Information

Each group of 2 needs 4 number cubes.

Stages used in Grade 4

Stage 1

Supporting

- Grade4.3.A
- Grade4.4.A

Stage 2

Addressing

- Grade4.3.A
- Grade4.3.B
- Grade4.3.C

Supporting

- Grade4.6.C
- Grade4.7.A

Section A: Measurement Conversions and Powers of 10

Lesson 1: Place Value Patterns

Standards Alignments

Addressing 5.NBT.A, 5.NBT.A.1

Teacher-facing Learning Goals

- Observe place value patterns when multiplying and dividing.

Student-facing Learning Goals

- Let's observe place value patterns.

Lesson Purpose

The purpose of this lesson is for students to observe place value patterns when multiplying and dividing.

In previous grades, students recognized that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. In the previous unit, students saw that this same pattern continues to the right of the decimal. In this lesson, students apply what they know about multiplication, division, and place value to express that each digit in a decimal represents ten times as much as it represents in the place to its right and one tenth as much as it represents in the place to its left (MP7).

Access for:

Students with Disabilities

- Representation (Activity 2)

Instructional Routines

MLR7 Compare and Connect (Activity 1), Notice and Wonder (Warm-up)

Materials to Gather

- Tools for creating a visual display: Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	10 min

Teacher Reflection Question

Describe the relationships between place value, multiplication, and division that your students made sense of today.

Cool-down (to be completed at the end of the lesson)

10 min

Multiplication and Division Equations

Standards Alignments

Addressing 5.NBT.A.1

Student-facing Task Statement

Fill in the blank to make each equation true.

1. $0.06 \times 10 =$ ______
2. $60 =$ ______ $\times 0.6$
3. ______ $= 6 \div 100$

Student Responses

1. 0.6
2. 100
3. 0.06

Begin Lesson

Warm-up

10 min

Notice and Wonder: Same Digits

Standards Alignments

Addressing 5.NBT.A

The purpose of this warm-up is for students to discuss the multiplicative relationships between the place values of the digits in two numbers. This will be useful when students write multiplication and division expressions to represent place value relationships in a later activity. While students may notice and wonder many things about these numbers, the place value relationships between the digits in the numbers and the numbers themselves are the important discussion points.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?

8,200

820

82

8.2

0.82

0.082

Student Responses

Students may notice:

- The numbers have the same digits.
- The digits are in different places.
- The numbers are getting smaller.
- The value of each number is one tenth the value of the number above.

Students may wonder:

- Is it a pattern?
- What other numbers can we write with those digits?

Launch

- Groups of 2
- Display the image.
- “What do you notice? What do you wonder?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- “How does the value of 8,200 compare to the value of 820?” (It’s 10 times as much.)
- “How does the value of 0.82 compare to the value of 0.082? How do you know?” (It’s also ten times as much since there are ten thousandths in a hundredth.)

- What number comes next?

Activity 1

20 min

Many True Equations

The purpose of this activity is for students to express place value relationships using multiplication and division. Students examined decimal place values in depth in the previous unit and used the relationships between the values when they performed arithmetic with decimals. Here they focus on expressing these relationships using multiplication and division. This will be helpful throughout the next several lessons as students examine powers of ten and then use them for measurement conversions.

This activity uses MLR7 Compare and Connect. Advances: representing, conversing.

Instructional Routines

MLR7 Compare and Connect

Materials to Gather

Tools for creating a visual display

Student-facing Task Statement

Use the numbers and symbols to write as many different true equations as you can. You may use each number and symbol more than once.

600	0.06	100
60	×	10
6	÷	0.1
0.6	=	0.01

Student Responses

Sample solutions:

- $600 \div 100 = 6$

Launch

- Groups of 2
- Display the numbers: 60, 6
- “How many times the value of 6 is 60? How do you know?” (10 times because it’s 6 tens)
- Display the equation: $60 = 10 \times 6$.
- “What division equation shows that 60 is ten times the value of 6?” ($60 \div 6 = 10$ is another way of saying that 60 is ten 6s or ten times 6.)
- Display the equation: $60 \div 6 = 10$.
- “You are going to write equations like these relating different numbers.”

- $600 \div 10 = 60$
- $600 \div 0.1 = 6{,}000$
- $6 \times 10 = 60$
- $6 \times 0.1 = 0.6$

Activity

- 5 minutes: partner work time

MLR7 Compare and Connect

- “Create a visual display that shows your equations. You may want to include details such as notes, diagrams or drawings to help others understand your thinking.”
- Monitor for students who:
  - identify an equation that is incorrect during the gallery walk
  - notice place value patterns during the gallery walk
- 2–5 minutes: independent or group work
- 5–7 minutes: gallery walk

Synthesis

- Invite students to share equations they made.
- Display the equation: $0.6 \div 10 = 0.06$
- “How do you know this equation is true?” (When I divide tenths into ten equal pieces I get hundredths so if I divide 6 tenths into 10 equal pieces that's 6 hundredths.)
- “Can you express the relationship between 0.6 and 0.06 using multiplication?” (Yes. $0.6 = 10 \times 0.06$.)
- Display the equation: $600 \times 0.01 = 6$
- “How do you know this equation is true?” (I know 100 hundredths is 1 so 600 hundredths is 6.)
- “Can you express the relationship between 600 and 6 using division?” (Yes. $600 \div 100 = 6$.)
- Invite students to describe any patterns they noticed.

Activity 2

15 min

Describe Multiplicative Relationships

In the previous activity, students wrote multiplication and division equations relating numbers with a single non-zero digit. The purpose of this activity is to focus on the same set of numbers and describe how the value of the non-zero digit changes when it moves one place to the left or right. This serves to highlight two important patterns that came out in some of the equations of the previous activity:

- The value of a digit is multiplied by 10 when it shifts one place to the left (MP7).
- The value of a digit is multiplied by 0.1 or $\frac{1}{10}$ when it shifts one place to the right (MP7).

The former idea will be further developed in the next lesson where students examine large numbers and exponential notation and the latter idea will be developed when students examine conversions from a smaller metric unit to a larger metric unit.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Synthesis: Make connections between representations visible, such as between information provided in the task statement and equations from the previous activity.
Supports accessibility for: Conceptual Processing, Visual-Spatial Processing, Organization

Student-facing Task Statement

$$\begin{array}{r} 600 \\ 60 \\ 6 \\ 0.6 \\ 0.06 \end{array}$$

1. Explain or show how the value of the 6 changes in the different numbers.
2. Which numbers would come before 600 if the list continued? Explain your reasoning.
3. Which numbers would come after 0.06 if the list continued? Explain your reasoning.

Launch

- Groups of 2
- “We are going to continue to work with the numbers from the previous activity to explore more patterns.”

Activity

- 5 minutes: individual work time
- 5 minutes: partner work time

Synthesis

- “What happens to the value of the 6 when it shifts one place to the left?” (It is

Student Responses

1. Sample response: The value of the 6 in each number is one tenth the value of the number above it. The value of the 6 is ten times greater in each number than it is in the number below it.
2. 6,000, 60,000, 600,000. Sample response: To get each previous number I am shifting the 6 one place to the left.
3. 0.006. Sample response: To get each successive number I am shifting the 6 one place to the right. I am not sure if I can keep shifting to the right because I only know thousandths.

multiplied by 10.)

- "What happens to the value of the 6 when it shifts one place to the right?" (It is multiplied by $\frac{1}{10}$ or 0.1. It is divided by 10.)
- Invite students to share the numbers that they listed that come before 600 on the list.
- "Do you think you can keep listing bigger and bigger numbers with more and more zeros?"
    - Yes, I can always add more zeros.
    - I don't know. After 600,000, I don't know if I can keep going.
- "In the next lesson we will look at some really big numbers and how they relate to multiplying over and over by 10."

Lesson Synthesis

10 min

"Today we looked at place values and expressed relationships between them using division and multiplication."

Display: 0.1 and 0.01

"What multiplication equation can I write to describe the relationship between a tenth and a hundredth?" ($0.1 = 10 \times 0.01$, $0.01 = 0.1 \times 0.1$)

"What division equation can I write to describe the relationship between a tenth and a hundredth?" ($0.1 \div 10 = 0.01$.)

Display: 10,000 and 1,000

"Can you also compare the value of these two numbers using multiplication and division?" (Yes. I know $1{,}000 = 10{,}000 \div 10$ and $10{,}000 = 10 \times 1{,}000$.)

"In the next several lessons we will multiply and divide whole numbers and decimals by 10."

Suggested Centers

- Number Puzzles: Multiplication and Division (4–5), Stage 2: Multi-digit Factors (Supporting)

- Five in a Row: Multiplication (3–5), Stage 3: Two-digit Factors (Supporting)

Complete Cool-Down

Response to Student Thinking

Students don't write the correct solutions.

Next Day Support

- Before the next day’s warm-up, pair students up to discuss their responses.

Lesson 2: Powers of 10

Standards Alignments

Addressing	5.NBT.A.1, 5.NBT.A.2
Building Towards	5.NBT.A.2

Teacher-facing Learning Goals

- Use whole-number exponents to denote powers of 10.

Student-facing Learning Goals

- Let’s use exponents to show powers of 10.

Lesson Purpose

The purpose of this lesson is for students to recognize exponential notation for powers of 10 and use exponential notation to represent large numbers.

In the previous lesson, students multiplied and divided numbers by 10 or 100 and noticed place value patterns and relationships. This lesson continues to look at patterns when several factors of 10 are multiplied together. It also introduces a convenient strategy for recording these numbers, exponential notation for positive powers of 10. Students represent numbers up to 1,000,000,000 and apply their understanding of multiplication by 10 and its powers to see why exponential notation is a convenient way to represent certain large numbers. This lesson includes an optional activity that allows students to explore the number 1 trillion.

Access for:

Students with Disabilities

- Engagement (Activity 2)

English Learners

- MLR2 (Activity 1)

Instructional Routines

How Many Do You See? (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min

Teacher Reflection Question

What unfinished learning or misunderstandings do you have about powers of 10 and exponential notation? How can you leverage those partial understandings in a positive way to further the understanding of the class?

Activity 3	10 min
Lesson Synthesis	10 min
Cool-down	5 min

Cool-down (to be completed at the end of the lesson)

5 min

Exponential Notation

Standards Alignments

Addressing 5.NBT.A.2

Student-facing Task Statement

1. Write 10,000 and 100,000 using exponential notation. Explain or show your reasoning.
2. Write 10^6 as a number.

Student Responses

1. 10^4 and 10^5 because 10,000 is $10 \times 10 \times 10 \times 10$ and 100,000 has one more factor of 10.
2. 1,000,000

Begin Lesson

Warm-up

10 min

How Many Do You See: Starburst

Standards Alignments

Building Towards 5.NBT.A.2

The goal of this warm-up is for students to visualize 10^n for different exponents before they learn exponential notation in this lesson. Monitor for students who use the symmetry of the diagram to estimate how many line segments there are of each size. For example, the picture can be rotated 10 times around the center and each arm is the same so that means the number of each size segment has one factor of 10. This idea can be applied at a smaller scale to get a second and third factor of 10.

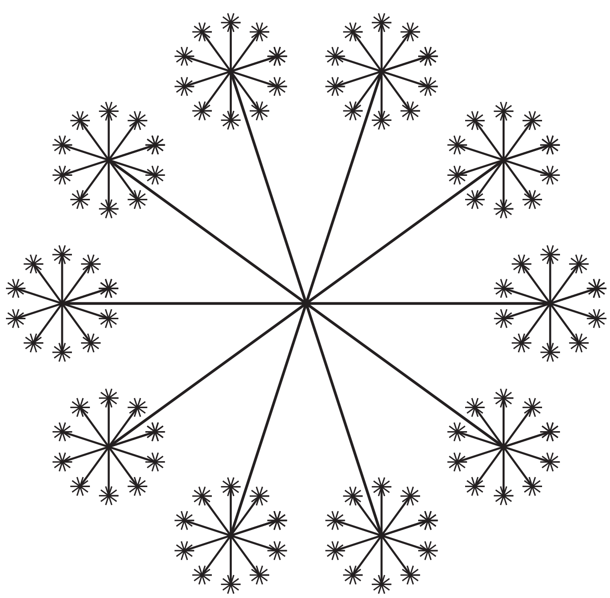
When students analyze the diagram and determine how many segments there are of each length, they are observing and making use of the repeated structure of ten segments joining at the different vertices (MP7, MP8).

Instructional Routines

How Many Do You See?

Student-facing Task Statement

How many do you see? How do you see them?



Student Responses

Sample responses:

- 10. Big snowflakes, long spokes, medium spokes on the end of each long spoke.
- 100. Medium spokes because there are 10 at the end of each long spoke.
- 1,000. Little spokes because there are 10 little spokes that make up each little snowflake and there are ten little snowflakes in each

Launch

- Groups of 2
- "How many do you see? How do you see them?"
- Display the image.
- 1 minute: quiet think time

Activity

- Display the image.
- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Record responses.

Synthesis

- Invite students to share their estimates for how many of the smallest line segments there are in the diagram.
- "How can you find out exactly how many there are?" (I can count the number of long segments and then the number of medium size segments on one long segment and then the number of tiny segments on one medium size one. Then I multiply those numbers.)
- Invite students to count and then display the expression: $10 \times 10 \times 10$.
- "How does the expression relate to the diagram?" (It's the total number of tiny segments.)

medium snowflake and there are 10 medium snowflakes in the one big snowflake.

- "Another way to write $10 \times 10 \times 10$ is 10^3. This is called a **power of ten**. The number 3 tells us how many factors of 10 there are, or how many times we multiply 10 to get the number."

Activity 1

20 min

Population of Delaware and India

PLC Activity

Standards Alignments

Addressing 5.NBT.A.1, 5.NBT.A.2

The purpose of this activity is for students to make sense of and then use exponential notation to represent large numbers, namely 1 million and 1 billion. Students should be encouraged to say the names of the numbers in a way that makes sense to them. Contexts, in the form of human populations, are provided for each of the large numbers to help students conceptualize the magnitude of the number. Students recognize that the purpose of exponential notation is to write large numbers efficiently and recognize how many factors of ten are in a given number.

When students relate 1 million and 1 billion to products of 10 and powers of 10 they look for and make use of base-ten structure (MP7).

Access for English Learners

MLR2 Collect and Display. Circulate, listen for and collect the language students use as they use exponential notation to represent large numbers. On a visible display, record words and phrases such as: million, thousands, billion, powers of 10, exponential notation, represent, times, multiply by ten, number of zeros. Invite students to borrow language from the display as needed, and update it throughout the lesson.
Advances: Conversing, Reading

Student-facing Task Statement

1. About 1,000,000 people live in Delaware.
    a. How do you say this number?
    b. How many thousands is this? Explain

Launch

- Groups of 2

Activity

- 3–5 minutes: independent work time.

or show your reasoning.

c. Write the number using powers of 10.

d. How many times would you need to extend the diagram from the warm-up to get 1,000,000 tiny segments? Explain or show your reasoning.

2. In 1997, the population of India was about 1,000,000,000.

   a. How would you say this number?

   b. How many millions is this? How many thousands is it? Explain or show your reasoning.

   c. Write the number using powers of 10.

   d. How many times would you need to extend the diagram from the warm-up to get 1,000,000,000 tiny segments? Explain or show your reasoning.

Student Responses

1. a. one million

   b. It's 1,000 thousands. Sample response: Because $10 \times 1{,}000 = 10{,}000$, $10 \times 10{,}000 = 100{,}000$ and $10 \times 100{,}000 = 1{,}000{,}000$. I multiplied it by 10 three times and $10 \times 10 \times 10 = 1{,}000$.

   c. 10^6. Sample response: Because one thousand is $10 \times 10 \times 10$ and then we multiplied by another 3 tens.

   d. 3 more than in the picture. Sample response: Each new set of segments multiplies the previous number by 10.

2. a. Sample response: I would call it a thousand millions.

   b. It is a thousand millions. Sample response: You have to multiply by 10 three more times. The first one makes 10 million, the second makes 100 million, and the third makes 1,000 million.

- 3–5 minutes: partner discussion
- Monitor for students who:
  - call 1,000,000,000 a thousand million or a million thousands
  - make up names such as a zillion
  - know that 1,000,000,000 is called a billion

Synthesis

- Ask previously selected students to share their names for 1,000,000,000 in the given order.
- “How many millions are in this number? How do you know?” (A thousand because there are 3 more zeros and that means multiplying by 10 three times.)
- “How do you write this number using powers of 10? Why?” (10^9 because there are 9 factors of 10.)
- “This number is called a billion.”
- “Using powers of 10, can you write a number that is bigger than a billion?” (Yes, 10^{10}, 10^{11}, 10^{100}.)
- “Why are powers of 10 useful for representing really big numbers?” (I would not want to write 100 zeros. I would also have to count all those zeros to see how big the number is.)

c. 10^9. Sample response: Because there are 9 zeros.

d. 6 more than in the picture. Sample response: 3 more than for 10^6 since that means writing 3 more zeros at the end of the number.

Activity 2

15 min

Powers of 10

Standards Alignments

Addressing 5.NBT.A.1, 5.NBT.A.2

The purpose of this activity is for students to find the missing number that makes multiplication equations true where that value is a power of 10. The numbers in this activity were chosen to build toward numbers that are larger than what students have worked with before.

The synthesis highlights that these powers of 10 are represented by a 1 followed by some zeros. The power of 10 tells us how many zeros the number has.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Invite students to generate a list of shared expectations for group work. Record responses on a display and keep visible during the activity.
Supports accessibility for: Organization, Social-Emotional Functioning

Student-facing Task Statement

1. Find the missing number that makes each equation true. Show your reasoning.
   a. $2{,}000 = \underline{\hspace{3cm}} \times 20$
   b. $20 \times 10 \times \underline{\hspace{3cm}} = 20{,}000$
   c. $\underline{\hspace{3cm}} \times 10 = 100{,}000$
   d. $1{,}000 \times 10{,}000 = \underline{\hspace{3cm}}$

Launch

- Groups of 2

Activity

- 1–2 minutes: quiet think time
- 6–8 minutes: partner work time
- Monitor for students who:

2. How were products of 10s useful in solving these problems?
3. Write each power of 10 as a number.

   a. 10^3

   b. 10^4

   c. 10^7

Student Responses

1. a. 100. Sample response: Because 10 twenties is 200 and 100 twenties is 2,000.

   b. 100. Sample response: Because 10 twenties is 200 and then 100 twenties is 2,000 and 1,000 twenties is 20,000 so I need two more factors of 10.

   c. 10,000. Sample response: Because 10 ten thousands is a hundred thousand.

   d. 10,000,000. Sample response: because I know that a thousand thousands is a million and then this is 10 times as much so that's 10 million.
2. Products of 10 were helpful because I can solve each of these problems by taking successive products of 10 and keeping track of how many factors of 10 I used.
3. a. 1,000

   b. 10,000

   c. 10,000,000

   - use multiplication equations, such as $10 \times 10 = 100$, $10 \times 100 = 1{,}000$, $10 \times 1{,}000 = 10{,}000$, to help them find the solution to ________________ $\times 10 = 100{,}000$
   - name or write the number 10,000,000 in a way that makes sense to them to represent the product of $1{,}000 \times 10{,}000$. For example, they may write "one hundred one hundred thousands, 100 100,000, or $100 \times 100{,}000$."
   - write the number 10,000,000

Synthesis

- Ask selected students to share how they solve the equation________________ $\times 10 = 100{,}000$.
- "How did thinking about factors of 10 help to solve the equation?" (I started with 1,000 and 10 times that is 10,000 and then I knew I needed another factor of 10 to get to 100,000.)
- Ask selected students to share their responses for the value of $1{,}000 \times 10{,}000$ in the given order.
- "How did you find the value?" (I know 1,000 is $10 \times 10 \times 10$ so I started multiplying by 10. I got 100,000 and then 1,000,000 which is 1 million. I put one more zero in for the last 10.)
- "How do you say this number?" (10 million)
- Display the equations:
  $10^3 = 1{,}000$
  $10^4 = 10{,}000$
  $10^7 = 10{,}000{,}000$
- "What do you notice about the equations?" (The number of zeros on each number is the same as the power of 10.)

Advancing Student Thinking

If students do not write a power of ten correctly, display each power of ten as a product of tens and ask, “What is the same about these expressions? What is different?”

Activity 3 (optional)

10 min

Beyond a Billion

Standards Alignments

Addressing 5.NBT.A.2

The goal of this optional activity is to introduce one more number, a trillion, which is 1,000 billions. These large numbers become more and more difficult to conceptualize and the goal of the synthesis is to share and provide some ideas of things in the world of which there might be a trillion or more.

Relating huge numbers to things in the world, as students do in the synthesis, is a part of modeling with mathematics (MP4).

Student-facing Task Statement

1. How would you say the number 1,000,000,000,000?
2. How many billions is that? How many millions is it? Explain or show your reasoning.
3. Write the number using powers of 10.
4. Describe an example of something that there are 1,000,000,000,000 of in the world.

Student Responses

1. a thousand billions, a million millions
2. a thousand billion. Sample response: Because there are 3 more zeros. It’s a

Launch

- “We are going to investigate a big number. What is the biggest number you can think of? How do you say it? How do you write it in number form?”
- Consider asking students to write in their journal and then share with a partner.

Activity

- 3 minutes: individual work time
- 3 minutes: partner discussion
- Monitor for different ideas students have for what things there may be a trillion of in the world.

million millions because there are 6 more zeros than a million.

3. 10^{12}
4. Sample responses: blades of grass in a lawn, grains of sand at the beach, animals, plants on the Earth

Synthesis

- Invite students to share things they think there may be a trillion of in the world.
- “How did you choose?” (I picked something that there were too many to count.)
- “Why is it hard to guess whether there are a trillion of something, like grains of sand?” (It’s too big a number to count or imagine.)
- Consider sharing some things that there are around a trillion of:
    - The total number of pennies made in U.S. is about 500,000,000,000.
    - The total number of fish in the oceans is about 3,500,000,000,000.
    - The total number of trees on earth is about 3,000,000,000,000.
    - The total number of insects on earth greatly exceeds a trillion: 10,000,000,000,000,000,000.
    - The total number of ants on earth also greatly exceeds a trillion: 10,000,000,000,000,000.

Lesson Synthesis

10 min

“Today we looked at some really big numbers that are powers of 10 and represented them using exponents.”

Display the image from the warm-up.

“How many of the medium sized segments are there?” (100) “What expression could you write to represent the number of these segments?” (10×10)

“How does the image represent the expression 10×10?” (There are 10 groups of these segments and 10 segments in each group.)

Display: $10 \times 10 = 10^2$

"We can also represent this expression with a power of ten."

Refer to the smallest line segments in the warm-up image.

"What expression could you write to represent the number of small segments? How do you know?" ($10 \times 10 \times 10$ or 100×10 because there are ten more of these for each of the medium size segments.)

Display: $10 \times 10 \times 10 =$ ______

"What power of ten can we write to make this equation true?" (10^3)

"If you kept going and drew 10 more tiny segments, how many of these would there be?" (10,000)

"How can we write the number as a power of ten?" (10^4 since there would be another factor of 10.)

"If I kept going a total of 6 times, how many of the smallest segments would there be?" (One million or 10^6.)

Suggested Centers

- Number Puzzles: Multiplication and Division (4–5), Stage 2: Multi-digit Factors (Supporting)
- Five in a Row: Multiplication (3–5), Stage 3: Two-digit Factors (Supporting)

Complete Cool-Down

Response to Student Thinking

Students do not use correct exponential notation or do not correctly write the power of ten in standard form.

Next Day Support

- Create a poster with a diagram that represents the cool-down from this lesson.

Students do not read, represent, and describe the relative magnitude of multi-digit whole numbers up to 1 million.

Prior Unit Support

Grade 4, Unit 4, Section B: Place-value Relationships through 1,000,000

Lesson 3: Metric Conversion and Multiplication by Powers of Ten

Standards Alignments

Addressing 5.MD.A.1, 5.NBT.A.2

Teacher-facing Learning Goals

- Convert from larger units to smaller units within a given system of measurement.
- Explain patterns in the number of zeros of the product when multiplying a number by powers of 10.

Student-facing Learning Goals

- Let's notice patterns in metric measurements.

Lesson Purpose

The purpose of this lesson is for students to convert from a larger metric length unit to a smaller unit. Students observe patterns when different numbers are multiplied by 10, 100, or 1,000.

In this lesson, students apply place value reasoning to convert between different metric lengths. Students notice patterns in whole number and decimal products when a number is multiplied by a power of 10 (MP7). This builds on the previous lesson where students studied powers of 10. This is the first of several lessons that focus on metric units, specifically length, which all differ by powers of 10. The contexts for the first two lessons come from different events in track and field. This lesson focuses on the standing broad jump and longer distance runs.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Number Talk (Warm-up)

Materials to Gather

- Metersticks: Activity 1, Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

Reflect on your experience with the Number Talks in the curriculum. What questions have you asked to connect student thinking during this routine? What new questions could you ask next time?

Cool-down (to be completed at the end of the lesson)

5 min

Kilometers

Standards Alignments

Addressing 5.MD.A.1, 5.NBT.A.2

Student-facing Task Statement

Complete the table. Explain or show your reasoning.

meters	centimeters	millimeters
6.5		

Student Responses

meters	centimeters	millimeters
6.5	650	6,500

Sample response: $100 \times 6.5 = 650$, $10 \times 650 = 6{,}500$

Begin Lesson

Warm-up

10 min

Number Talk

Standards Alignments

Addressing 5.NBT.A.2

In this number talk, students find products of a decimal number and a power of 10 and quotients of a whole number and a power of 10 whose value is a decimal. This skill will be useful throughout the next several lessons as students convert between different metric units of measurement and also specifically address how a whole or decimal number changes when it is multiplied or divided by a power of 10.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- 100×1.5
- $1{,}000 \times 1.5$
- $15 \div 10$
- $15 \div 100$

Student Responses

- 150. It’s 100 and half of another 100.
- 1,500. It’s 1,000 and half of 1,000.
- 1.5. I pictured it as a fraction, $\frac{15}{10}$, which is equal to 1 and 5 tenths.
- 0.15. $15 \div 100 = 15 \times \frac{1}{100}$ and that is equal to 0.15.

Launch

- Display one expression.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- “What is the same about the values of the expressions?” (There is a 1 and a 5 in each value.)
- “What is different about the values of the expressions?”(Some of them are whole numbers and some are decimals.)

Activity 1

20 min

How Tall? How Long? How Far?

Standards Alignments

Addressing 5.MD.A.1, 5.NBT.A.2

The goal of this activity is to convert meters to centimeters and millimeters and to convert kilometers to meters. This metric measurement context gives students an opportunity to observe several patterns (MP7).

- When students convert from meters to centimeters or multiply by 100, the digits shift 2 places to the left.
- When students convert from kilometers to meters or multiply by 1,000, the digits shift 3 places to the left.

Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Use multiple examples and non-examples to reinforce the relationship between meters, centimeters, and millimeters.
Supports accessibility for: Conceptual Processing, Memory

Materials to Gather

Metersticks

Student-facing Task Statement

1. Complete the table.

meters	centimeters	millimeters
1		
10		
10^2		

Launch

- Groups of 2
- Give students access to meter sticks.
- Display image from student workbook.
- "What do you notice? What do you wonder?"
- Display additional information about track and field events:
    - The height of a hurdle is 1 meter.
    - The approximate distance between hurdles in 110 meter races is 10 meters.

2. What patterns do you notice in the table?
3. Three long-distance races are 10 kilometers, 100 kilometers, and 1,000 kilometers. How many meters are there in these races?

distance in kilometers	distance in meters
1	1,000
10	
100	
10^3	

4. What patterns do you notice in the table?

Student Responses

1.

meters	centimeters	millimeters
1	100	1,000
10	1,000	10,000
10^2	10^4 or 10,000	10^5 or 100,000

2. The number in each column is ten times greater than the number above it. There is one more 0 in each number. The numbers in the rows are multiplied by 100 to get centimeters and by 1,000 to get millimeters.

3.

distance in kilometers	distance in meters
1	1,000
10	10,000
100	100,000
10^3	10^6 or 1,000,000

4. The numbers in each row are multiplied by 1,000 because there are 1,000 meters in 1 kilometer.

    - The shortest race in many track competitions is 100 meters.
- "Work with you partner to complete the problems."

Activity

- 1–2 minutes: quiet think time
- 8–10 minutes: partner work time
- Monitor for students who observe that:
    - the digits shift 2 places to the left when converting from meters to centimeters or multiplying by 100
    - the digits shift 3 places to the left when converting from kilometers to meters or multiplying by 1,000

Synthesis

- Display completed tables for all to see.
- Invite students to share patterns they observed in the tables.
- "How are the tables the same? How are they different?" (Some of the numbers in the tables are the same. Some of the relationships are the same. Going from meters to millimeters is the same as going from kilometers to meters because they are both 1,000 times greater. The second table only has two columns but has a bigger number, 1,000,000.)
- "What do you notice about the location of the 1 and the number of zeros after converting from kilometers to meters?" (Each number in the meters column has three more zeros than the number in the kilometers column. The one in each number in the meters column is three places to the left of the corresponding number in the kilometers column.)

Advancing Student Thinking

If students do not fill in the tables show them a meter stick and ask, “How can you use the meter stick to fill in the first row of the table?”

Activity 2

15 min

Broad Jump

Standards Alignments

Addressing 5.MD.A.1, 5.NBT.A.2

The purpose of this activity is for students to convert from meters to centimeters using the context of a standing broad jump, a common test for physical fitness. Students may multiply by 100 to convert from meters to centimeters or divide by 100 to convert from centimeters to meters. Students should be encouraged to use whatever strategy makes sense to them. Future lessons will focus specifically on conversion from centimeters to meters. Give students access to meter sticks.

Access for English Learners

MLR8 Discussion Supports. Synthesis: Provide students with the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking

Materials to Gather

Metersticks

Student-facing Task Statement

Launch

- Groups of 2
- Display the image from student workbook.
- Give students access to meter sticks.
- “What do you notice? What do you wonder?” (The girl is jumping. There are numbers on the mat. Did she start where

Here are the distances that each student jumped.

student	distance
Mai	1.61 meters
Tyler	1.43 meters
Clare	1.57 meters

1. The average standing broad jump distance for 5th graders is 148 centimeters. Are each of the students in the table below, at, or above the average? Explain or show your reasoning.
2. The world record for the standing broad jump is 337 centimeters. Jada says that's more than Mai and Clare jumped combined. Do you agree with Jada? Explain or show your reasoning.
3. Tyler says his jump sounds more impressive if he reports it in millimeters.
    a. How far is Tyler's jump in millimeters? What about Mai's and Clare's jumps?
    b. Which unit do you think is best for reporting the jumps? Explain your reasoning.

Student Responses

1. Sample response: Mai's jump is 161 cm,

the footprints are? How far did she jump?)

Activity

- 1–2 minutes: quiet think time
- 6–8 minutes: partner work time
- Monitor for students who:
    - use their result for Tyler's jump in centimeters to find his jump in millimeters
    - use the result in meters to find how far Tyler jumped in millimeters

Synthesis

- Display the expression: 1.43×100.
- "How does this expression represent Tyler's jump in centimeters?" (There are 100 centimeters in a meter.)
- Invite students to share how they found Tyler's jump in millimeters.
- Display the expression: $(1.43 \times 100) \times 10$.
- "In what unit of measurement does this expression represent Tyler's jump? How do you know?" (It represents Tyler's jump in millimeters. There are 10 millimeters in a centimeter and 1.43×100 represents the centimeters, so if we multiply the number of centimeters by 10, we will get the number of millimeters.)
- Display the expression: $1.43 \times 1{,}000$.
- "How does this expression represent Tyler's jump in millimeters?" (There are 1,000 millimeters in a meter so I have to multiply by 1,000 to get millimeters from meters.)
- Display the equation: $1.43 \times 1{,}000 = (1.43 \times 100) \times 10$.
- Invite students to share which unit they prefer to express the distances that the students jumped.
- "It's important to choose a unit that makes

Tyler's is 143 cm, and Clare's is 157 cm since there are 100 cm in a meter. Mai and Clare have jumps longer than the average but Tyler's is shorter than average.

2. Yes. Sample response: Mai and Clare jump a combined distance of 3.18 m which is the same as 318 cm. This is less than 337 cm.
3. Sample responses:
    a. Tyler's jump is 1,430 mm. There are 10 mm in a centimeter and so I need to multiply 143 by 10. Mai's jump is 1,610 mm, and Clare's jump is 1,570 mm.
    b. I like meters the best because it's a smaller number and I can visualize it best. I like cm because there is no decimal and I can still imagine how far it is.

sense to you."

Advancing Student Thinking

If students do not know the relationship between millimeters, centimeters, and meters, show them a meter stick and ask "How can you use the meter stick to figure out how many millimeters are in 1 meter?"

Lesson Synthesis

10 min

"Today we converted between metric units to express distances related to track events in different ways."

Display a table that looks like this:

kilometers	meters	centimeters	millimeters
2.5			

As students respond to the questions, record their responses in the table.

"Jada ran 2.5 kilometers. How many meters is that?" (2,500)

"How many centimeters is that?" (250,000)

"How many millimeters is that?" (2,500,000)

"Would you use kilometers, meters, centimeters, or millimeters to report how far Jada ran?" (Sample response: kilometers or meters because I can imagine how long those distances are and the numbers of centimeters and millimeters are too big to visualize.)

Suggested Centers

- Number Puzzles: Multiplication and Division (4–5), Stage 2: Multi-digit Factors (Supporting)
- Five in a Row: Multiplication (3–5), Stage 3: Two-digit Factors (Supporting)

Complete Cool-Down

Response to Student Thinking

Students do not recognize or explain a relationship.

Next Day Support

- Give students access to meter sticks during activity 1 of the next lesson.

The work in this lesson builds from the measurement concepts developed in a prior unit.

Prior Unit Support

Grade 4, Unit 5, Section B: Measurement Conversion

Lesson 4: Metric Conversion and Division by Powers of Ten

Standards Alignments

Addressing 5.MD.A.1, 5.NBT.A.2

Teacher-facing Learning Goals

- Convert metric lengths from a smaller unit to a larger unit.
- Recognize and explain patterns in the placement of the decimal point when a decimal is divided by a power of 10.

Student-facing Learning Goals

- Let’s convert units.

Lesson Purpose

The purpose of this lesson is for students to convert from a smaller metric length unit to a larger unit using a context of track and field. Students observe patterns when different numbers are divided by 10, 100, or 1,000.

In this lesson, students extend their understanding of metric length conversions. In the previous lesson, students converted from a larger unit to a smaller unit, often resulting in very large numbers because they multiplied by powers of 10. In this lesson, students convert from a smaller unit to a larger unit, resulting in a smaller number, and sometimes, a decimal. Students notice how the digits in a number shift to the right when dividing by powers of 10 and consequently whole numbers often become decimals (MP7).

Access for:

Students with Disabilities

- Representation (Activity 1)

Instructional Routines

MLR1 Stronger and Clearer Each Time (Activity 2), True or False (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

If you were to teach this lesson over again, what would you do differently? How would your proposed changes support student learning?

Cool-down (to be completed at the end of the lesson)

5 min

Han’s Run

Standards Alignments

Addressing 5.MD.A.1, 5.NBT.A.2

Student-facing Task Statement

Han ran 12,500 meters last week. How many kilometers is that? Explain or show your reasoning.

Student Responses

12.500 or equivalent

There are 1,000 meters in a kilometer so I need to divide by 1,000. $12{,}000 \div 1{,}000 = 12$ and $500 \div 1{,}000 = 0.500$.

Begin Lesson

Warm-up

10 min

True or False: Divide by a Hundred and a Thousand

Standards Alignments

Addressing 5.NBT.A.2

The purpose of this True or False is for students to demonstrate the strategies and understandings they have for dividing by powers of 10. In this lesson they will convert from a smaller metric unit to a larger unit which means dividing by an appropriate power of 10. The problems here are selected so that students can begin to see how the value of the digits in a number change when that number is divided by 100 or 1,000.

Instructional Routines

True or False

Student-facing Task Statement

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $5 \div 1{,}000 = 0.05$
- $36 \div 100 = 0.36$
- $1{,}328 \div 1{,}000 = 1.328$

Student Responses

- False. 5 divided by 1,000 is $\frac{5}{1{,}000}$ or 0.005.
- True. 36 divided by 10 is 3.6 and 3.6 divided by 10 is 0.36.
- True. There is 1 group of 1,000 and then 328 thousandths.

Launch

- Display one equation.
- “Give me a signal when you know whether the equation is true and can explain how you know.”
- 1 minute: quiet think time

Activity

- Share and record answers and strategy.
- Repeat with each equation.

Synthesis

- “How did you find the value of $1{,}328 \div 1{,}000$?” (I thought of it as 1,328 thousandths and then wrote that as a decimal.)

Activity 1

15 min

Long Jump, Javelin Throw, and Shot Put

Standards Alignments

Addressing 5.MD.A.1, 5.NBT.A.2

The purpose of this activity is for students to convert from a smaller metric length unit to a larger metric length unit. In this activity, students divide a three- or four-digit number by 100. The measurements in centimeters are whole numbers, but the converted measurements in meters are decimals. To find the value of the quotients, students may reason about the meaning of each place value and how that changes when the value is divided by 100. They may also reason that each place value has $\frac{1}{10}$ the value of the place to its left so shifting two places to the right gives a value of $\frac{1}{100}$ the value for each place (MP2, MP7).

Access for Students with Disabilities

Representation: Develop Language and Symbols. Synthesis: Invite students to explain their thinking orally, using a place value chart.
Supports accessibility for: Conceptual Processing, Language, and Memory

Student-facing Task Statement

athlete	long jump	javelin throw	shot put
Jackie Joyner-Kersee, USA	727 cm	4,566 cm	1,580 cm
Sabine John, Germany	671 cm	4,256 cm	1,623 cm
Anke Behmer, Germany	678 cm	4,454 cm	1,420 cm

1. Below are some results Jackie Joyner-Kersee recorded in different events in 1988. Complete the table.

event	centimeters	meters
long jump	727	
javelin throw	4,566	
shot put	1,580	

2. Which unit of measure is most helpful when you picture each distance, centimeters or meters? Explain or show your reasoning.
3. Why do you think that the distances are measured to the nearest centimeter?

Launch

- Groups of 2
- Display the table shown in the student workbook.
- “What do you notice? What do you wonder?” (Who are those people? What is a javelin? What is a shot put? What is the long jump? Some of the numbers are close. Why are the long jump numbers so much smaller?)
- Record responses for all to see.
- “This table shows the results from track and field events.”

Activity

- 2 minutes: quiet think time
- 10 minutes: partner work time
- Monitor for students who observe that when they convert the whole number distances in centimeters to meters they get decimals to the hundredth.

Student Responses

1.

event	centimeters	meters
long jump	727	7.27
javelin throw	4,566	45.66
shot put	1,580	15.80

2. Sample responses: Meters because there are too many centimeters to imagine, but there are fewer meters and they are longer so it is easier for me to picture how far it is.
3. They might need to measure to the nearest centimeter to see who comes in first place, second place, and third place. A meter is so big that measuring these distances to the nearest meter is not very accurate.

Synthesis

- Display:
  $727 \div 100 = 7.27$
  $4,566 \div 100 = 45.66$
  $1,580 \div 100 = 15.80$
- "What patterns do you notice?" (The measurements in centimeters are whole numbers but the measurements in meters are in hundredths. All the digits in the numbers are the same but they shifted 2 places to the right. So the 7 hundreds in centimeters is 7 ones in meters.)
- "Why does dividing by 100 represent how many meters are in the given centimeters?" (I know there are 100 centimeters in each meter so each centimeter is $\frac{1}{100}$ of a meter.)
- "How did you find the result of 727 divided by 100?" (I went place by place. 7 hundreds divided by 100 is 7 ones. 2 tens divided by 100 is 2 tenths. 7 ones divided by 100 is 7 hundredths.)
- Invite students to share which unit of measurement they found most informative for these distances and why.

Advancing Student Thinking

If students do not have a strategy, ask:

- "How many centimeters are in a meter? How many meters is 200 centimeters?"
- "What expression can you write to represent this relationship?"

Activity 2

15 min

Hurdles

Standards Alignments

Addressing 5.MD.A.1, 5.NBT.A.2

This activity continues the work of the previous activity as students convert from a smaller metric length unit to a larger one. Students are given several different conversions and multiple numbers for each set of conversions. They observe that dividing by 100 shifts each digit two places to the right while dividing by 1,000 shifts each digit three places to the right when they convert different units. This allows students to solidify their understanding that dividing by powers of 10 shifts each digit one place to the right for every power of 10 in the divisor (MP8).

This activity uses MLR1 Stronger and Clearer Each Time. Advances: Reading, Writing.

Instructional Routines

MLR1 Stronger and Clearer Each Time

Student-facing Task Statement

1. The table shows how many meters some students ran during a week. Complete the table to show how many kilometers each student ran.

student	distance (meters)	distance (kilometers)
Diego	9,513	
Clare	11,018	
Priya	8,210	
Andre	10,000	

2. What patterns do you notice in the table?
3. Below is Tyler's strategy to divide a whole number by 10, 100, or 1,000.

Launch

- Groups of 2
- "Work with your partner on the first three problems."

Activity

- 5–7 minutes: partner work time
- "Now, you are going to work independently to show or explain whether you think Tyler's strategy will always work."
- 3–5 minutes: independent work time

Synthesis

MLR1 Stronger and Clearer Each Time

- "Share with your partner your response as to whether or not Tyler's strategy will always work. Take turns being the speaker and the listener. If you are the speaker, share your ideas and writing so far. If you are the listener, ask questions and give feedback to help your partner improve

I find the quotient by shifting the digits to the right — once when I divide by 10, twice when I divide by 100, 3 times when I divide by 1,000.

5,632 / 10 = 563.2
5,632 / 100 = 56.32
5,632 / 1,000 = 5.632

Describe to your partner what Tyler means.

(Pause for teacher direction.)

4. Why does Tyler's strategy work? Will Tyler's strategy always work? Explain or show your reasoning.

Student Responses

1.

student	distance (meters)	distance (kilometers)
Diego	9,513	9.513
Clare	11,018	11.018
Priya	8,210	8.210
Andre	10,000	10

2. Sample response: When I go from meters to kilometers I divide by 1,000. Each digit in the number shifts three places to the right.
3. Sample response: When Tyler divides by 10, he shifts all the digits 1 place to the right. When he divides by 100, he shifts the digits 2 places to the right. When he divides by 1,000, he shifts the digits 3 places to the right.
4. Sample responses:
    a. I think it will always work because I tried it with some other numbers.
    b. I think it always works because when I move one place to the right the value of a digit is multiplied by $\frac{1}{10}$.

their work."

- 3–5 minutes: structured partner discussion
- Repeat with 1–2 different partners.
- If needed, display question starters and prompts for feedback.
    - "Can you give an example to help show . . . ?"
    - "Can you add a diagram, table, or representation to show . . . ?"
- "Revise your initial draft based on the feedback you got from your partners."
- 2–3 minutes: independent work time

Lesson Synthesis

10 min

"Today we converted from smaller metric distance units to larger metric distance units. We noticed and explained patterns in numbers when they are divided by 10, 100, and 1,000."

Display a table like this:

kilometers	meters
	7,864
2.037	

"What are the missing values in the table? How do you know?" (7.864 kilometers and 2,037 meters. To get from meters to kilometers, I divide by 1,000 and to get from kilometers from meters I multiply by 1,000 because there are 1,000 meters in a kilometer.)

"How is converting from smaller meters to larger kilometers the same and how is it different from converting from larger kilometers to smaller meters?" (In both cases you keep the same digits, only their place values change and they change by 3 place values ;in both cases. When I go from meters to kilometers the digits shift 3 places to the right and when I go from kilometers to meters they shift 3 places to the left.)

Suggested Centers

- Would You Rather? (2–5), Stage 2: Compare to Smaller Units (Supporting)
- Compare (1–5), Stage 5: Fractions (Supporting)

Complete Cool-Down

Response to Student Thinking

Students do not divide correctly by 1,000 when they convert meters to kilometers.

Next Day Support

- After the warm-up in the next lesson, pair students up to discuss their responses.

Lesson 5: Multi-step Conversion Problems: Metric Length

Standards Alignments

Addressing 5.MD.A.1, 5.NBT.A.1

Teacher-facing Learning Goals

- Solve multi-step problems involving metric length measurement conversions.

Student-facing Learning Goals

- Let's solve multi-step problems about metric length.

Lesson Purpose

The purpose of this lesson is for students to solve multi-step conversion problems about distance in metric units.

In this lesson, students convert different metric distance measurements and perform arithmetic with those measurements in order to solve problems (MP2). The values of the measurements are mostly decimals so students practice performing arithmetic with decimals. They have opportunities to use all four operations and to select whether to convert from the larger unit to a smaller unit or from the smaller unit to a larger unit. Using a smaller unit requires dealing with larger numbers while using a larger unit requires dealing with decimals. Students are invited to compare the two strategies while using a strategy that makes sense to them.

Access for:

Students with Disabilities

- Engagement (Activity 1)

English Learners

- MLR1 (Activity 2)

Instructional Routines

True or False (Warm-up)

Materials to Gather

- Metersticks: Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What evidence did you see that each of your students applied understanding of decimals from a previous unit?

Cool-down (to be completed at the end of the lesson)

5 min

Compare Lengths

Standards Alignments

Addressing 5.MD.A.1

Student-facing Task Statement

Jada ran 15.25 kilometers. Han ran 8,500 meters. Who ran farther? How much farther? Explain or show your reasoning.

Student Responses

Jada ran 6.75 kilometers farther. Sample response: 8,500 meters is 8.5 kilometers. So Jada ran $15.25 - 8.5$ kilometers farther and that's 6.75 kilometers.

Begin Lesson

Warm-up

10 min

True or False: Powers of 10

Standards Alignments

Addressing 5.NBT.A.1

The purpose of this True or False is for students to demonstrate strategies and understandings they have for multiplying and dividing by powers of 10. They will use these operations when they convert measurements between different metric length units.

Instructional Routines

True or False

Student-facing Task Statement

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $5{,}423 \times 10 = 50{,}423$
- $5{,}423 \div 10 = 542.3$
- $5{,}423 \div 100 = 54.23$

Student Responses

- False. Only the 5 is ten times greater. It should be 54,230.
- True. Each digit in 542.3 has $\frac{1}{10}$ the value of the corresponding digit in 5,423.
- True. Each digit in 54.23 has $\frac{1}{100}$ the value of the corresponding digit in 5,423.

Launch

- Display one statement.
- "Give me a signal when you know whether the statement is true and can explain how you know."
- 1 minute: quiet think time

Activity

- Share and record answers and strategy.
- Repeat with each statement.

Synthesis

- "How did you decide if the equation $5{,}423 \div 100 = 54.23$ is true?" (It's like dividing by 10 twice. $5{,}423 \div 10 = 542.3$ and $542.3 \div 10 = 54.23$. Each number has digits 5, 4, 2, 3 in the same order. The values of the digits' places in 54.23 are $\frac{1}{100}$ the values of the corresponding digits' places in 5,423.)

Activity 1

15 min

Walk All Day

Standards Alignments

Addressing 5.MD.A.1, 5.NBT.A.1

The purpose of this activity is for students to solve multi-step distance problems using centimeters, meters, and kilometers. This gives students an opportunity to think about which units are most helpful for communicating a distance (MP6). When the distance is short, like the length of a single footstep, centimeters or meters both work well. For a longer distance, like the distance a person walks in a day, it is reasonable to use meters or kilometers, but the number of centimeters is very large and more difficult to visualize.

To add movement or make this activity interactive, consider providing groups of 2 or 4 with a centimeter ruler or meter stick to measure their or a classmate's step before working on the task. While students could use the measurements of their own steps to complete the table, the arithmetic may be more complex and, as a result, it may be harder to observe patterns.

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Optimize meaning and value. Invite students to share the places they could walk in the classroom, at home, in the community, and so on, that would be equivalent to the distances presented in the chart.
Supports accessibility for: Conceptual Processing; Visual-Spatial Processing

Materials to Gather

Metersticks

Student-facing Task Statement

Lin has a watch that counts the number of steps she takes during the day and displays those steps in centimeters, meters, or kilometers.

1. Here is a list of activities Lin did on Monday. Next to each activity, write whether it would make sense to display the distance in cm, m, or km.
    - walked to her friend's desk
    - walked to the front of the classroom
    - walked from her classroom to the bus
    - ran twice around the playground
2. The table shows the amount of steps Lin's watch displayed for each activity. If each of her steps is 50 centimeters, how many centimeters and meters did Lin walk for

Launch

- Groups of 2 or 4
- "If someone wants to measure the length of one of their footsteps, which unit do you think they should use? Millimeters, centimeters, meters, or kilometers?"
- Highlight that millimeters are too small and kilometers are too large. Both centimeters and meters make sense and the first part of this activity will use those two units of measure.
- "There are 100 centimeters in a meter. About how many steps are in 1 meter?" (2 or 3)
- "There are 1,000 meters in a kilometer. About how many steps are in a kilometer?" (2,000 or 3,000)
- Give students access to a meter stick.

each activity?

activity	number of steps	distance (cm)	distance (m)
walked to her friend's desk	5		
walked to the front of the classroom	12		
walked from her classroom to the bus	250		
ran twice around the playground	1,000		

3. At the end of the day, Lin's watch displayed 8,500 steps. Would it make sense for her watch to record the distance in centimeters, meters, or kilometers? Why?
4. How many kilometers did Lin walk that day?

Student Responses

1. Sample responses:
    - walked to her friend's desk: centimeters or meters
    - walked to the front of the classroom: centimeters or meters
    - walked from her classroom to the bus: meters
    - ran twice around the playground: meters or kilometers
2.

activity	number of steps	distance (cm)	distance (m)
walked to her friend's desk	5	250	2.5
walked to the front of the	12	600	6.0

Activity

- 5 minutes: independent work time
- 5 minutes: small-group work time
- Monitor for students who use the following strategies when determining how many kilometers Lin walks during the day:
    - multiply 50 centimeters by 8,500 steps to determine the distance in centimeters that Lin walked and divide 425,000 by 100,000
    - multiply 0.5 meters by 8,500 steps to determine the distance in meters that Lin walked and then divide 4,250 by 1,000

Synthesis

- "How did you determine how many kilometers Lin walked during the day?"
- Ask previously selected students to share their solutions.
- Display student work or write these equations for all to see:
    - $8{,}500 \times 50 = 425{,}000$
    - $8{,}500 \times 0.5 = 4{,}250$
- "How does each of these equations represent the situation?" ($8{,}500 \times 50 = 425{,}000$ represents 8,500 steps that are each 50 centimeters long so that would be a total of 425,000 centimeters.$8{,}500 \times 0.5 = 4{,}250$ represents 8,500 steps that are each 0.5 meter long so that would be a total of 4,250 meters.)
- "What is the same about these equations?" (They have the same number of steps. They are both multiplication equations.)
- "What is different about these equations?" (One multiplies the number of steps by 50 centimeters and one multiplies the number of steps by 0.5 meter. The products are

activity	number of steps	distance (cm)	distance (m)
classroom			
walked from her classroom to the bus	250	12,500	125
ran twice around the playground	1,000	50,000	500

3. Meters or kilometers because centimeters would be too big of a number.
4. 4.25 km: 1 step is 50 cm long so 8,500 steps is going to be $8{,}500 \times 50$ and $8{,}500 \times 50 = 425{,}000$ and 425,000 cm is 4,250 m because $425{,}000 \div 100 = 4{,}250$ and 4,250 m is 4.25 km because $4{,}250 \div 1{,}000 = 4.25$.

different. 4,250 is 100 times smaller than 425,000 because it represents meters instead of centimeters.)

- Display:
    - $425{,}000 \div 100{,}000 = 4.25$
    - $4{,}250 \div 1{,}000 = 4.25$
- “How does each of these equations represent the situation?” (They both represent the number of kilometers that Lin walked. 425,000 is the distance she walked in centimeters so if you divide it by 100,000, you get the number of kilometers she walked. 4,250 is the distance in meters that she walked so we only have to divide by 1,000 to figure out how many kilometers she walked.)
- Display: 4.25 km, 4,250 m, 425,000 cm
- “Which of these do you think best communicates how far Lin walked this day?” (4.25 kilometers because I can picture how long a kilometer is.)

Activity 2

20 min

Who Ran Farther?

Standards Alignments

Addressing 5.MD.A.1

The purpose of this activity is for students to convert between meters and kilometers to decide which of two measurements is larger. Monitor for students who convert from kilometers to meters, which will give two large whole-number measurements, and for students who convert from meters to kilometers, which will give two decimal numbers. The goal of the activity synthesis is to connect these two different solution strategies.

Access for English Learners

MLR1 Stronger and Clearer Each Time. Synthesis: Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their response to who ran farther, Tyler or Clare. Invite listeners to ask questions, to press for details and to suggest mathematical language. Give students 2–3 minutes to revise their written explanation based on the feedback they receive.
Advances: Writing, Speaking, Listening

Student-facing Task Statement

1. Use the table to find the total distance Tyler ran during the week. Explain or show your reasoning.

day	distance (km)
Monday	8.5
Tuesday	6.25
Wednesday	10.3
Thursday	5.75
Friday	9.25

2. Use the table to find the total distance Clare ran during the week. Show your reasoning.

day	distance (m)
Monday	5,400
Tuesday	7,500
Wednesday	8,250
Thursday	6,750
Friday	7,250

3. Who ran farther, Clare or Tyler? How much farther? Explain or show your reasoning.

Launch

- Groups of 2

Activity

- 5 minutes: independent work time
- 5 minutes: partner discussion
- For the third problem, monitor for students who:
    - convert from kilometers to meters to find the difference between Clare and Tyler’s runs
    - convert from meters to kilometers to find the difference between Clare and Tyler’s runs

Synthesis

- Invite students to share strategies for how they added the two sets of numbers (that is, looking for “friendly” pairs of numbers to combine first or adding by place value).
- Invite a student who converted from kilometers to meters to share their solution to the last problem.
- “What worked well in this solution?” (All of the numbers are whole numbers.)
- “What was difficult?” (The numbers were all big.)
- Invite a student who converted from

Student Responses

1. Tyler ran 40.05 kilometers during the week. I first added 5.75 and 9.25 since they make 15 together. Then I added the whole number part of the other three decimals and finally added the decimal part.
2. Clare ran 35,150 meters during the week. I lined up all the numbers and added the digits in each place.
3. Tyler ran 4.9 kilometers farther. Clare ran 35,150 meters which is the same as 35.15 kilometers. $40.05 - 35.15 = 4.9$.

meters to kilometers to share.

- "What worked well?" (The numbers were a good size to visualize.)
- "What was difficult?" (I needed to add and subtract decimals to find out how much farther Tyler ran than Clare.)

Advancing Student Thinking

If a student does not realize that they have to convert the units to compare them, refer to the letters next to the word "distance" in the table headers and ask the student to explain what they mean.

Lesson Synthesis

10 min

"Today we solved problems and converted length measurements in metric units. We solved problems about how far students walked or ran."

"How far do you think you walk in a day?" (2 or 3 kilometers because I walk to and from school each day and I think that's a kilometer and then I run around on the playground a lot during recess. 5 kilometers because I walk to and from school every day and I also usually take my dog out for a walk once or twice a day.)

Consider giving students time to respond in their journals.

Collect some responses and ask students to explain how they know their answers are reasonable.

Suggested Centers

- Would You Rather? (2–5), Stage 2: Compare to Smaller Units (Supporting)
- Compare (1–5), Stage 5: Fractions (Supporting)

Complete Cool-Down

Response to Student Thinking

This lesson builds on work students did converting larger measurement units to smaller measurement units in a previous unit.

Prior Unit Support

Grade 4, Unit 5, Section B: Measurement Conversion

Lesson 6: Multi-step Conversion Problems: Metric Liquid Volume

Standards Alignments

Building On	5.MD.A.1, 5.NBT.A.2
Addressing	5.MD.A.1, 5.NBT.A.1, 5.NBT.A.2

Teacher-facing Learning Goals

- Solve multi-step problems involving metric liquid measurement conversions.

Student-facing Learning Goals

- Let’s solve multi-step problems about metric liquid volume.

Lesson Purpose

The purpose of this lesson is for students to solve conversion problems using metric volume units.

In this lesson, students solve conversion problems involving metric liquid volume measurements. The first activity in this lesson focuses on base-ten structure and conversions and also gives students a chance to work with decimals, fractions, and powers of 10 in exponential form. The second activity is contextual and also involves work with fractions and decimals. It gives students a chance to practice multiplication (by numbers that are not powers of ten) either with whole numbers or a whole number and a decimal depending how they solve the problem.

Access for:

Students with Disabilities

- Engagement (Activity 1)

English Learners

- MLR1 (Activity 2)

Instructional Routines

Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min

Teacher Reflection Question

What strategy did most students use for the second activity? How can you encourage students to be more flexible with their use of multiplication or division when converting metric units?

Lesson Synthesis	10 min
Cool-down	5 min

Cool-down (to be completed at the end of the lesson)

5 min

Dance Team

Standards Alignments

Addressing 5.MD.A.1

Student-facing Task Statement

A dance team used 60 bottles of water during their practices last week. Each bottle holds 750 mL. How many liters of water did the dance team drink during their practices?

Student Responses

45 liters. Sample response: First I found how many mL there are in 60 bottles. That's 60×750 or 45,000 mL. That's the same as 45 liters.

Begin Lesson

Warm-up

10 min

Number Talk: Divide by Powers of 10

Standards Alignments

Addressing 5.NBT.A.1

This Number Talk encourages students to use place value structure to mentally solve problems. The strategies elicited here will be helpful later in the lesson when students convert from milliliters to liters. When they divide by powers of 10, students need to look for and make use of place value structure (MP7).

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $1,400 \div 10$
- $1,400 \div 100$
- $1,400 \div 1,000$
- $1,401 \div 1,000$

Student Responses

- 140. The value of each digit in 140 is $\frac{1}{10}$ the value of the corresponding digit in 1,400.
- 14. I divided 140 by 10 since dividing 1,400 by 100 is the same as dividing by 10 and dividing by 10 again.
- 1.4. I divided the last product by 10.
- 1.401. The value of each digit is $\frac{1}{1,000}$ the value of the corresponding digit in 1,401.

Launch

- Display one problem.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep problems and work displayed.
- Repeat with each problem.

Synthesis

- Display: 1,401 and 1.401
- "How are these numbers the same?" (They both have a 1, then a 4, then a 0, then a 1.)
- "How are they different?" (The place values of the digits are different. The value of each digit in 1.401 is $\frac{1}{1,000}$ the value of that digit in 1,401.)

Activity 1

20 min

Liquid Volume Conversions

Standards Alignments

Addressing 5.MD.A.1, 5.NBT.A.2

The purpose of this activity is for students to convert between measurements in milliliters and liters, providing practice multiplying and dividing by 1,000. Students work with numbers in many forms including whole numbers, decimals, fractions, and numbers in exponential form.

Access for Students with Disabilities

Engagement: Internalize Self-Regulation. Synthesis: Provide students an opportunity to self-assess and reflect on their own progress. For example, provide students with a partially completed chart to compare their answers and talk about the similarities and differences with their partners. *Supports accessibility for: Attention; Memory; Social-Emotional Functioning*

Student-facing Task Statement

1. Complete the table.

L	mL
5	
6.3	
0.95	
10^2	
	800,000
	10^6
	65

2. Decide if the two measurements are equal. If not, choose which one is greater. Explain or show your reasoning.

   a. 15 mL and 0.15 L

   b. 2,500 mL and 2.5 L

   c. 200 mL and $\frac{1}{4}$ L

   d. 1 mL and $\frac{1}{1,000}$ L

Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?" (There are different sized water bottles. There are numbers underneath the water bottles. How much more water does the big one hold than the little one? How many milliliters are in a liter?)
- Display a table like this one:

L	mL
1	1,000
10	
0.1	
	100,000
	10

- "What numbers go in the empty boxes in the table?"
- 30 seconds quiet think time
- "Explain your thinking to a partner."
- Fill in the table and leave it displayed for students to refer to during the lesson.

L	mL
1	1,000
10	10,000
0.1	100
100	100,000
0.01	10

e. 15,600 mL and 15.5 L

Student Responses

1.

L	mL
5	5,000
6.3	6,300
0.95	950
10^2	10^5 or equivalent
800	800,000
10^3 or equivalent	10^6
0.065	65

2.
   a. 0.15 L is 150 mL so that's greater than 15 mL.
   b. 2500 mL is the same as 2.5 L.
   c. Since 1 L is 1,000 mL, $\frac{1}{4}$ L is 250 mL and that's greater than 200 mL.
   d. They are the same.
   e. 15,600 mL is the same as 15.6 liters and that's greater than 15.5 L.

Activity

- 3–5 minutes: independent work time
- 1–2 minutes: partner discussion
- Monitor for students who:
  - compare the quantities by converting milliliters to liters
  - compare by converting liters to milliliters

Synthesis

- Invite selected students to share how they compared the measurements.
- Display: 15,600 mL and 15.5 L
- "How many liters are 15,600 milliliters? How do you know?" (15.6 since I divide by 1,000)
- "How many milliliters are 15.5 liters? How do you know?" (15,500 since there are 1,000 milliliters in each liter so that's 15,000 and half of a thousand which is 500.)
- "Which is greater? 15,600 milliliters or 15.5 liters? How do you know?" (15,600 mL because I can compare them using either liters or milliliters.)
- "When you solved this problem, did you convert from milliliters to liters or from liters to milliliters? Why?" (I converted from liters to milliliters because multiplying by 1,000 is more comfortable than dividing by 1,000.)

Advancing Student Thinking

If students confuse the operations they need to use to convert milliliters to liters or liters to milliliters, refer to the table from the launch and ask, "What patterns do you notice?"

If necessary, add rows to the table and ask them to explain how many milliliters of water are in 2 liters of water and 3 liters of water, and 0.001 liter of water.

Activity 2

15 min

Rehydrating Dancers

Standards Alignments

Building On 5.MD.A.1, 5.NBT.A.2

The purpose of this activity is for students to solve multi-step problems involving metric units of liquid volume (MP2). The given quantities involve fractions. One of the quantities involves the fraction $\frac{1}{2}$ which students may convert to a decimal or they may perform the needed arithmetic with fractions. Students also have a choice of converting to milliliters or liters and there are different points in the calculations when they may choose to make the conversion.

Different approaches students may use to solve the problems include:

- convert the volume of the bottle to liters (as a decimal or fraction) and work in liters
- find out how much each dancer drinks in milliliters and then convert to liters
- find out how much all the dancers drink in milliliters and then convert to liters
- find out how much all the dancers drink in milliliters and then convert the cooler volume to milliliters

The purpose of the lesson synthesis is to compare some of these different approaches.

Access for English Learners

MLR1 Stronger and Clearer Each Time. Synthesis: Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their response to “How many liters of water did the dancers drink?” Invite listeners to ask questions, to press for details and to suggest mathematical language. Give students 2–3 minutes to revise their written explanation based on the feedback they receive.
Advances: Writing, Speaking, Listening

Student-facing Task Statement

There are 25 dancers in the performance group. During practice, each dancer drinks $1\frac{1}{2}$ bottles of water.

1. Each bottle holds 500 mL of water. How

Launch

- Groups of 2
- Display the image.
- “What do you notice? What do you wonder?” (The orange thing is a lot bigger

many liters of water do the dancers drink? Explain or show your reasoning.

2. Each cooler holds 15 L of water. How many coolers does the team need? How much water will they have left over after practice? Explain or show your reasoning.

3. The dancers can make a sports drink by mixing 30 mL of drink mix with each 500 mL of water. How many liters of drink mix does the team need for their practice? Explain or show your reasoning.

Student Responses

1. 18.75 L. Sample response: Each dancer drinks 750 mL of water. The 25 dancers will drink 25×750 mL or 18,750 mL. There are 1,000 mL in one L so that is the same as 18.75 L.
2. 11.25 L. Sample response: One cooler of water is not enough. Two coolers are enough, and if they are full, that leaves $30 - 18.75$ or 11.25 liters of water.
3. 1.125 L. Sample response. Each dancer needs 45 mL of drink mix to mix with $1\frac{1}{2}$ bottles of water. There are 25 dancers so that's 25×45 or 1,125 mL.

than the water bottle. What is that orange thing? How many bottles of water will fill up the orange thing?)

- "This is an illustration of a water bottle and a water cooler. The orange water cooler can hold a lot of water. We are going to solve some problems about the water in the cooler."

Activity

- 5–8 minutes partner work time
- Monitor for students who:
    - convert from milliliters to liters at different steps in the calculations for the first problem
    - convert from liters to milliliters at different steps in the calculations for the first problem

Synthesis

- Invite a student who found how many milliliters all of the dancers drank to share their reasoning.
- "How did you figure out how many milliliters of water one dancer drinks?" (I took 500 and then half of 500, or 250, more.)
- "How did you figure out how many milliliters all of the dancers drink?" (I multiplied 750 by 25.)
- "How did you find how many coolers the dancers need?" (I multiplied the number of liters in the cooler by 1,000.)
- Invite a student who found how many liters all of the dancers drank to share their reasoning.
- "How are the methods different?" (One calculates in milliliters and the other one in liters. The numbers with milliliters are much bigger. The numbers with liters are smaller. They are decimals or fractions.)

Advancing Student Thinking

If a student mixes up the units as they are solving the problem, display a table like the one from the previous activity and ask them to use it to record the number of liters and milliliters of water in a bottle and a cooler.

Lesson Synthesis

10 min

"Today we converted between liters and milliliters and used these conversions to solve problems. We multiplied or divided."

"We saw two ways to solve the water cooler problem."

Display student work from the lesson that shows multiplication and division.

"Which strategy do you prefer? Why?" (I liked working in milliliters because then I could use whole numbers. I like using liters because I can visualize a liter and that helps me make sense of the calculations.)

Suggested Centers

- Would You Rather? (2–5), Stage 2: Compare to Smaller Units (Supporting)
- Compare (1–5), Stage 5: Fractions (Supporting)

Complete Cool-Down

Response to Student Thinking

This lesson builds on liquid capacity concepts addressed in an earlier unit.

Prior Unit Support

Grade 4, Unit 5, Section B: Measurement Conversion

Lesson 7: Multi-step Conversion Problems: Customary Length

Standards Alignments

Addressing	5.MD.A.1
Building Towards	5.MD.A.1

Teacher-facing Learning Goals

- Solve multi-step problems involving customary length measurement conversions.

Student-facing Learning Goals

- Let's solve multi-step problems about customary length.

Lesson Purpose

The purpose of this lesson is for students to solve problems involving customary length units.

In this lesson, students solve multi-step conversion problems with standard length units. These conversions can be more challenging because they involve multiplying by numbers other than 10 and powers of 10. This means that students will use the skills they developed in previous units where they learned to multiply whole numbers and fractions. Students continue to work with whole numbers and fractions and to think strategically about whether to convert from the larger unit to the smaller unit or from the smaller unit to the larger unit.
This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Engagement (Activity 1)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Card Sort (Activity 1), Number Talk (Warm-up)

Materials to Gather

- Yardsticks: Activity 1

Materials to Copy

- Customary Measurement Card Sort (groups of 1): Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

When students were sorting the lengths, what question did you ask to help students make connections?

Cool-down (to be completed at the end of the lesson)

5 min

Whiteboard Width

Standards Alignments

Addressing 5.MD.A.1

Student-facing Task Statement

The whiteboard is 4.5 feet in width.

1. How many inches wide is the whiteboard? Explain or show your reasoning.
2. How many yards wide is the whiteboard? Explain or show your reasoning.

Student Responses

1. 54 inches. Sample response: $4.5 \times 12 = 54$
2. 1.5 yards. Sample response: $4.5 \div 3 = 1.5$

Begin Lesson

Warm-up

10 min

Number Talk: Multiples of 12

Standards Alignments

Building Towards 5.MD.A.1

This warm-up helps students develop strategies to find multiples of 12 mentally using place value strategies and the distributive property. This prepares students for converting measurements in feet to measurements in inches.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- 45×10
- 45×2
- 45×12
- 46×12

Student Responses

- 450. Every digit shifts to the left one place value because I multiplied by 10.
- 90. I doubled 40 and added 10.
- 540. I added the previous two products.
- 552. I just added another 12.

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "How are the values of the products 45×12 and 46×12 related?" (There is one more 12 in 46×12.)

Activity 1

15 min

Card Sort: Customary Measurements

Standards Alignments

Addressing 5.MD.A.1

The goal of this activity is to compare measurements in the customary length units of inches, feet, and yards. Students first sort the measurements in a way that makes sense to them. Monitor for students who:

- sort by the unit of measure
- sort by the way the quantity is written (whole number, mixed number, fraction)

Then, students find the equivalent lengths and list the sets of equivalent lengths in increasing order. Four different lengths have been chosen and each one is presented in inches, feet, and yards. The activity synthesis highlights why expressing all the measurements using one unit is a convenient way to identify common measures and list them in increasing order. Give students access to yard sticks.

Access for English Learners

MLR8 Discussion Supports. Students should take turns finding a match and explaining their reasoning to their partner. Display the following sentence frames for all to see: “I noticed _____, so I matched . . . .” Encourage students to challenge each other when they disagree.
Advances: Conversing, Representing

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. Give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.
Supports accessibility for: Conceptual Processing; Memory

Instructional Routines

Card Sort

Materials to Gather

Yardsticks

Materials to Copy

Customary Measurement Card Sort (groups of 1)

Required Preparation

- Create a set of cards from the Instructional master for each group of 2.

Student-facing Task Statement

1. Your teacher will give you a set of cards that show different measurements. Sort the

Launch

- Groups of 2
- Give each group of students one set of pre-

cards into 2 categories of your choosing. Be prepared to explain the meaning of your categories.

(Pause for teacher directions.)

2. Match the cards with equal measurements. Then, list the groups of matching measurements in increasing order.

Student Responses

1. Students may sort by:
    - the unit of measure
    - the way the quantity is written (whole number, mixed number, fraction)
2. These four sets of measurements are listed in increasing order of size.
    - 6 inches, $\frac{1}{2}$ foot, and $\frac{1}{6}$ yard are the same.
    - 18 inches, $1\frac{1}{2}$ feet, and $\frac{1}{2}$ yard are the same.
    - 54 inches, $4\frac{1}{2}$ feet, and $1\frac{1}{2}$ yards are the same.
    - 1,800 inches, 150 feet, and 50 yards are the same.

cut cards.

- Display a yardstick.
- “What do you notice? What do you wonder?” (It shows feet and inches. It shows 36 inches. I wonder if it’s the same length as a meterstick.)

Activity

- “In this activity, you will sort some cards into categories of your choosing. When you sort the measurements, you should work with your partner to come up with categories.”
- 4 minutes: partner work time
- Select groups to share their categories and how they sorted their cards.
- Choose as many different types of categories as time allows, but ensure that one set of categories identifies the way the quantity is written (whole number, mixed number, fraction).
- “Now work with your partner to match the cards with equal measurements. Then, list the groups of matching measurements in increasing order.”
- 3 minutes: partner work time

Synthesis

- Invite students to share the matches they made and how they know those cards go together.
- Attend to the language that students use to describe their matches, giving them opportunities to describe how they know the measurements are equal.
- Highlight the use of phrases, such as:
    - There are 12 inches in one foot.
    - There are 3 feet in one yard.
    - To find (a fraction) of a foot, I multiplied 12 by the fraction.

- “How did you compare the sets of measurements? Why?” (I chose one of the units, feet, and compared all of the measurements in that unit.)
- “Why was it important for all of the measurements to be in the same unit in order to compare?” (That way I can just compare the numbers because they all have the same unit of measure.)

Activity 2

20 min

Run a Mile or Two

Standards Alignments

Addressing 5.MD.A.1

The goal of this activity is to solve multi-step conversion problems using customary length units. The given information for both problems is in fraction form. Students need to find the perimeter of the different fields and then investigate how many times around each field is a given number of miles. The first problem involves 3 steps:

- find the perimeter of the rectangular field
- find the total distance of 6 laps around the field
- convert from yards to feet or feet to yards

The second problem has only two steps, but the number of laps is unknown and students need to find how many laps make at least 2 miles.

When students critically analyze Priya's claim that six laps of the soccer field is more than a mile, they critique the reasoning of others (MP3).

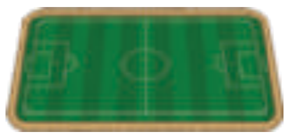
Required Preparation

- Before the lesson, figure out a location that students would be familiar with that is about 1 mile away from the school. You will share this location in the launch to help students understand how far 1 mile is.

Student-facing Task Statement

1. A rectangular field is 90 yards long and $42\frac{1}{4}$ yards wide. Priya says that 6 laps around the field is more than a mile. Do you agree with Priya?

   Explain or show your reasoning.

2. A different rectangular field is $408\frac{1}{2}$ feet long and $240\frac{1}{4}$ feet wide. How many laps around this field would Priya need to run if she wants to run at least 2 miles?

Student Responses

1. I disagree with Priya. Sample response: The perimeter of the field is $180 + 84\frac{1}{2}$ yards. That's $264\frac{1}{2}$ yards. If Priya runs 6 laps that's $6 \times 264\frac{1}{2}$ yards. That's $1{,}200 + 360 + 27$ or 1,587 yards. There are 3 feet in a yard so that's 4,761 feet. That's less than a mile.
2. 9 laps. Sample response: The perimeter of this field is twice the length, 817 feet, and twice the width, $480\frac{1}{2}$ feet. That's $1{,}297\frac{1}{2}$ feet. Four laps is a little less than $4 \times 1{,}300$ feet or 5,200 feet. So 8 full laps is a little less than 2 miles and 9 laps is more than 2 miles.

Launch

- "About how far is a mile?"
- 1–2 minutes: quiet think time
- Record responses for all to see.
- Describe to students a location that is about a mile away from the school.
- "About how many feet are in a mile?"
- "What is an estimate that is too low? Too high? About right?"
- Record student responses in a table like this one:

too low	about right	too high

- Display: There are 5,280 feet in one mile.
- Leave the display up throughout the lesson.
- "We are going to solve some problems about miles."

Activity

- 7-10 minutes: partner work time
- Monitor for students who:
  - find the number of yards in a mile
  - convert yards to feet for the first problem

Synthesis

- Invite students to share their solutions to the first problem.
- "How did you find how far 1 lap around the field is?" (I multiplied the length and width by 2 and added them.)
- "How far is one lap?" ($264\frac{1}{2}$ yards)
- "How far is 6 laps? How do you know?" (1,587 yards, I multiplied $264\frac{1}{2}$ by 6.)
- "How did you find the product $6 \times 264\frac{1}{2}$?" (I

multiplied 264 by 6. I know that $6 \times \frac{1}{2}$ is 3 so I added that.)

- "Is 6 laps more or less than a mile?" (Less, because it's less than 5,000 feet. Less, because a mile is more than 1,700 yards.)

Advancing Student Thinking

If students don't have a strategy to solve the first problem, consider asking:

- "Can you represent a person running one lap around the field?"
- "How can you figure out the distance of one lap around the field?"

Lesson Synthesis

10 min

"Today we converted between distances in customary units."

Display: 10 feet

"How many inches are in 10 feet? How many yards?" (120 inches, $3\frac{1}{3}$ yards)

Display: 10 meters

"How many centimeters are in 10 meters? How many kilometers?" (1,000 centimeters, 0.01 kilometer)

"How is converting between metric length units the same as converting between customary length units?" (In each case I multiply by a number when going from a bigger unit to a smaller unit and I divide by a number when going from a bigger unit to a smaller unit.)

"How is converting between metric length units different than converting between customary length units?" (When we convert in metric units we multiply or divide by a power of 10 so the digits in the measurement stay the same. In customary units the division or multiplication is not by a power of 10 so it takes more work, the digits change, and I may need to use fractions instead of decimals.)

Suggested Centers

- Would You Rather? (2–5), Stage 3: Compare Units in a Given System (Addressing)
- Compare (1–5), Stage 6: Add and Subtract Fractions (Supporting)

Student Section Summary

In this section we studied powers of 10 and conversions between units. We learned that we can write a product of 10s like

$$10 \times 10 \times 10 \times 10$$

as 10^4. The number 4 is an exponent and it means that there are 4 factors of 10.

We also converted between different measurement units, mostly metric lengths. For example, there are 1,000 millimeters in a meter and 1,000 meters in a kilometer. This means that there are $1{,}000 \times 1{,}000$ or $1{,}000{,}000$ millimeters in a kilometer. We could also say that there are 10^6 millimeters in a kilometer. We also used our understanding of decimals to make conversions. For example, since there are 1,000 meters in a kilometer that means that each meter is $\frac{1}{1{,}000}$ or 0.001 kilometers. So 853 meters can also be written as 0.853 kilometers.

Complete Cool-Down

Response to Student Thinking

Students do not write the correct number of inches or yards.

Next Day Support

- Before the warm-up, pass back the cool down and work in small groups to make corrections.

Section B: Add and Subtract Fractions with Unlike Denominators

Lesson 8: Add and Subtract Fractions

Standards Alignments

Addressing 5.NF.A.1

Teacher-facing Learning Goals

- Add and subtract fractions with unlike denominators in a way that makes sense to them.

Student-facing Learning Goals

- Let's add and subtract fractions.

Lesson Purpose

The purpose of this lesson is for students to add fractions with unlike denominators in a way that makes sense to them.

In this lesson, students add and subtract fractions in a way that makes sense to them. They consider several important cases:

- The denominators of the two fractions are the same, which is review of work from a previous grade.
- One denominator is a multiple of the other so the fractions can be added by replacing only one of the fractions with an equivalent fraction.
- Neither denominator is a multiple of the other so a third new common denominator is needed to add the fractions.

Students describe how the situations are different and find the sums and differences in a way that makes sense to them. The denominators of the fractions used in this lesson are familiar from grade 3, inviting students to use a variety of different familiar representations.

Access for:

Students with Disabilities

- Action and Expression (Activity 2)

English Learners

- MLR8 (Activity 1)

Instructional Routines

5 Practices (Activity 2), Which One Doesn't Belong? (Warm-up)

Materials to Copy

- Fraction Add and Subtract Sort (groups of 2): Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

Which students had opportunities to share their diagrams and thinking during whole-class discussion? How did you select these students?

Cool-down (to be completed at the end of the lesson)

5 min

Sum of Fractions

Standards Alignments

Addressing 5.NF.A.1

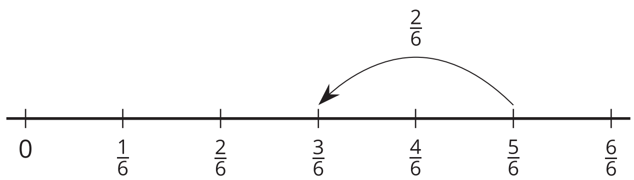
Student-facing Task Statement

Find the value of each expression. Explain or show your reasoning.

1. $\frac{5}{6} - \frac{1}{3}$
2. $\frac{3}{4} + \frac{1}{2}$

Student Responses

1. $\frac{3}{6}$ or $\frac{1}{2}$. Sample response:



2. $1\frac{1}{4}$ or $\frac{5}{4}$. Sample response: I know that $\frac{3}{4} = \frac{1}{2} + \frac{1}{4}$ so I added the two halves to make 1 and then I added $\frac{1}{4}$.

Begin Lesson

Warm-up

10 min

Which One Doesn’t Belong: Fraction Representations

Standards Alignments

Addressing 5.NF.A.1

The purpose of this Which One Doesn’t Belong is for students to recall representations of fractions they have seen in an earlier course. Two of the representations are fraction strips and the other two are number lines. These representations will be useful to students in this and future lessons as they think about representing equivalent fractions.

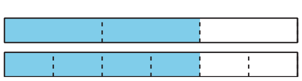
Instructional Routines

Which One Doesn’t Belong?

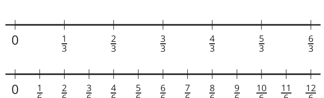
Student-facing Task Statement

Which one doesn’t belong?

A



B

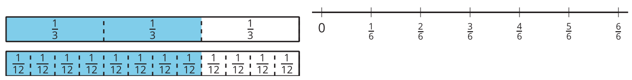


C

D

Launch

- Groups of 2
- Display the image.
- “Pick one that doesn’t belong. Be ready to share why it doesn’t belong.”
- 1 minute: quiet think time



Student Responses

Sample responses:

- A does not belong because it has no numbers. It does not belong because it does not show what the whole is.
- B does not belong because it does not stop at 1.
- C does not belong because it does not show any sixths.
- D does not belong because it only shows 1 representation of fractions.

Activity

- “Discuss your thinking with your partner.”
- 2–3 minutes: partner discussion
- Share and record responses.

Synthesis

- “How do the diagrams in B and C help us see the relationship between thirds, sixths, and twelfths?” (We can see that $\frac{1}{3} = \frac{4}{12}$ and $\frac{1}{3} = \frac{2}{6}$.)

Activity 1

15 min

Card Sort: Fraction Sums and Differences

Standards Alignments

Addressing 5.NF.A.1

The purpose of this activity is for students to sort expressions showing sums and differences of fractions. The cards include mixed numbers and expressions with the same denominator or with different denominators. Students are not expected to find the value of the expressions as that will be the work of the next activity. One way of sorting, however, may be based on whether or not they know how to find the value of the expression.

Access for English Learners

MLR8 Discussion Supports. Students should take turns sorting cards and explaining their reasoning to their partner. Display the following sentence frames for all to see: “I noticed _____, so I matched . . . .” Encourage students to challenge each other when they disagree.
Advances: Speaking, Conversing

Materials to Copy

Fraction Add and Subtract Sort (groups of 2)

Required Preparation

- Create a set of cards from the Instructional master for each group of 2.

Student-facing Task Statement

Your teacher will give you a set of cards that show expressions.

1. Sort the cards into 2 categories of your choosing. Be prepared to explain the meaning of your categories.
2. Sort the cards into 2 categories in a different way. Be prepared to explain the meaning of your new categories.

Student Responses

Students may sort by:

- addition expressions and subtraction expressions
- expressions with the same denominator and expressions with different denominators
- expressions with and without mixed numbers

Launch

- Groups of 2 or 4
- Distribute one set of pre-cut cards to each group of students.
- "In this activity, you will sort some cards into categories of your choosing. When you sort the expressions, you should work with your partner to come up with categories."

Activity

- 8 minutes: partner work time
- Monitor for students who sort the expressions according to whether the denominators of the fractions are the same or different.

Synthesis

- Select groups to share their categories and how they sorted their cards.
- Choose as many different types of categories as time allows, but ensure that one set of categories distinguishes between expressions that have the same denominator and expressions that have different denominators.
- Display: $\frac{2}{3} - \frac{1}{3}$
- "How could you find the value of this expression?" (I can just take 1 from 2 since they are both thirds.)
- Display: $\frac{2}{3} - \frac{1}{6}$
- "Why is finding the value of this expression

different?" (It's thirds and sixths so I can't just take the sixth away.)

- "In the next activity we will find the values of expressions like these."

Activity 2

20 min

Add and Subtract

PLC Activity

Standards Alignments

Addressing 5.NF.A.1

The purpose of this activity is for students to add and subtract fractions in a way that makes sense to them. Students may use strategies such as drawing tape diagrams or number lines, or they may use computations to find a common denominator. Monitor for and select students with the following strategies to share in the synthesis:

- use the meaning of fractions to explain why $\frac{2}{3} + \frac{2}{3} = \frac{4}{3}$
- use a diagram like a number line to find the value of $\frac{2}{3} - \frac{1}{6}$ and $\frac{2}{3} + \frac{1}{2}$
- use equivalent fractions and arithmetic to find the value of $\frac{2}{3} - \frac{1}{6}$ and $\frac{2}{3} + \frac{1}{2}$

Students who choose to use the number line or tape diagrams use appropriate tools strategically (MP5).

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Invite students to verbalize their strategy for finding the value of each expression before they begin. Students can speak quietly to themselves or share with a partner.
Supports accessibility for: Organization, Conceptual Processing, Language

Instructional Routines

5 Practices

Student-facing Task Statement

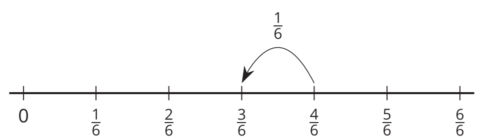
Find the value of each expression. Explain or show your thinking.

1. $\frac{2}{3} + \frac{2}{3}$
2. $\frac{2}{3} - \frac{1}{6}$
3. $\frac{2}{3} + \frac{1}{2}$

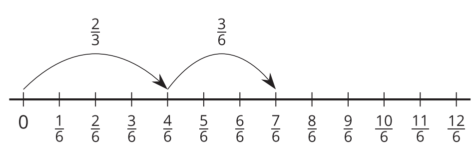
Student Responses

Sample responses:

1. $\frac{4}{3}$. Sample response: They are both thirds so I just added the numerators and kept the denominator the same.
2. $\frac{3}{6}$ or $\frac{1}{2}$. Sample responses:
    - The tape diagrams from the warm up show that $\frac{2}{3} = \frac{4}{6}$ and then taking away $\frac{1}{6}$ leaves $\frac{3}{6}$.
    - I used a number line. I know $\frac{2}{3} = \frac{4}{6}$ and then taking $\frac{1}{6}$ away leaves $\frac{3}{6}$.



    - $\frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$ and $\frac{4}{6} - \frac{1}{6} = \frac{3}{6}$
3. $1\frac{1}{6}$ or $\frac{7}{6}$. Sample responses:



    - I showed $\frac{2}{3} + \frac{3}{6}$ on the number line. Since each $\frac{1}{3}$ is $\frac{2}{6}$ I know $\frac{2}{3}$ is $\frac{4}{6}$ and then I added $\frac{3}{6}$ to get $\frac{7}{6}$.
    - $\frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$ and
      $\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$ and $\frac{4}{6} + \frac{3}{6} = \frac{7}{6}$

Launch

- Groups of 4

Activity

- 5 minutes: independent work time
- 5 minutes: small-group discussion
- As students work, consider asking:
    - “What is the same about these expressions? What is different?”
    - “How did you decide which strategy to use?”

Synthesis

- Ask selected students to share their response for how to add or subtract fractions with the same denominator.
- “How was this sum different than the other 2 sums?” (It was thirds and thirds so I could just add them. I did not need to find any equivalent fractions to make the denominators the same.)
- Ask selected students to display their responses how to add and subtract fractions with unlike denominators, such as $\frac{2}{3} + \frac{1}{2}$.
- “How was your strategy for finding this sum different than the other problems?” (For the first one, I had thirds and thirds so could just add them. For the second one, it was thirds and sixths so I just had to change the thirds to sixths. Here I had to change both the thirds and the half to sixths to get parts of the same size.)
- “Why is having a common denominator helpful when adding or subtracting fractions?” (When the parts all have the same size I can just add or subtract the number of parts.)

Advancing Student Thinking

If students do not have a strategy to add or subtract fractions with unlike denominators, refer to one of the expressions and ask, "How could you use one number line to show both of these fractions?"

Lesson Synthesis

10 min

"Today we compared different strategies for adding and subtracting fractions."

Display: $\frac{2}{5} + \frac{3}{10}$

"Describe how you would find the value of this sum." (I would use tape diagrams for fifths and tenths. I would use a number line using 10 as a common denominator. I would break each fifth into two equal pieces which are tenths and then add $\frac{4}{10}$ and $\frac{3}{10}$ to make $\frac{7}{10}$.)

Consider giving students time to record their answers in a math journal before they share their thinking.

Suggested Centers

- Would You Rather? (2–5), Stage 3: Compare Units in a Given System (Addressing)
- Compare (1–5), Stage 6: Add and Subtract Fractions (Supporting)

Complete Cool-Down

Response to Student Thinking

Students do not find the correct value of the sum.	**Next Day Support** • During the warm-up of the next lesson, use diagrams to represent student thinking.
The work in this lesson builds from the equivalent fractions concepts developed in a prior unit.	**Prior Unit Support** Grade 4, Unit 2, Section B: Equivalent Fractions

Lesson 9: Use Equivalent Expressions

Standards Alignments

Addressing 5.NF.A.1, 5.NF.A.2

Teacher-facing Learning Goals

- Use equivalent expressions to add and subtract fractions with unlike denominators.

Student-facing Learning Goals

- Let's use equivalent expressions to add and subtract fractions with unlike denominators.

Lesson Purpose

The purpose of this lesson is for students to add and subtract fractions with unlike denominators by replacing the given expressions with equivalent expressions with common denominators.

In a previous lesson, students saw that having a common denominator is useful for adding or subtracting fractions. In this lesson students add and subtract fractions using equivalent expressions where the fractions have the same denominator. Students work with denominators where one is a multiple of the other so they only need to change the denominator in one of the 2 fractions. In each case the numerators are chosen so that either denominator works as a common denominator and students compare different strategies for finding the sum or difference.

Access for:

Students with Disabilities

- Action and Expression (Activity 1)

English Learners

- MLR8 (Activity 3)

Instructional Routines

True or False (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	15 min
Activity 3	10 min

Teacher Reflection Question

How effective were your questions in advancing students' thinking today? What did students say or do that showed they were effective?

Lesson Synthesis	10 min
Cool-down	5 min

Cool-down (to be completed at the end of the lesson)

5 min

Write an Expression

Standards Alignments

Addressing 5.NF.A.1

Student-facing Task Statement

Find the value of $\frac{9}{12} - \frac{1}{4}$.

Student Responses

$\frac{2}{4}$ or equivalent

Begin Lesson

Warm-up

10 min

True or False: Fraction Addition and Subtraction

Standards Alignments

Addressing 5.NF.A.1

The purpose of this True or False is for students to demonstrate strategies they have for using equivalent fractions to add and subtract fractions with different denominators. These mental calculations prepare students for working with more complex common denominators during this lesson.

Instructional Routines

True or False

Student-facing Task Statement

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$
- $\frac{1}{2} + \frac{1}{4} = \frac{2}{4}$
- $\frac{3}{4} - \frac{1}{2} = \frac{2}{4}$

Student Responses

- True. I just know it.
- False. $\frac{2}{4} = \frac{1}{2}$, so $\frac{1}{2} + \frac{1}{4}$ has to be more than $\frac{1}{2}$.
- False. $\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$ and $\frac{1}{2}$ is more than $\frac{1}{4}$.

Launch

- Display one statement.
- “Give me a signal when you know whether the statement is true and can explain how you know.”
- 1 minute: quiet think time

Activity

- Share and record answers and strategy.
- Repeat with each statement.

Synthesis

- “How can we find the correct value of $\frac{3}{4} - \frac{1}{2}$?” ($\frac{1}{2} = \frac{2}{4}$ so $\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$.)

Activity 1

15 min

Equal Sums

Standards Alignments

Addressing 5.NF.A.1

The purpose of this activity is for students to identify equivalent sums of fractions and use them to find the value of sums of fractions with different denominators. In a previous course, students learned to use factors and multiples to generate and identify equivalent fractions. They recall that technique here and then use those equivalent fractions to find sums. This helps reinforce the idea that it is helpful, when adding two fractions, if the fractions have the same denominator while also recalling how to find an equivalent fraction with a different denominator.
When students identify that equivalent fractions with the same denominator help to find the value of a sum they notice and take advantage of the meaning and structure of fractions (MP7).

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Invite students to verbalize their strategy for explaining each equivalent expression before they begin. Students can speak quietly to themselves, or share with a partner.
Supports accessibility for: Organization, Conceptual Processing, Language

Student-facing Task Statement

1. Explain or show why each expression is equivalent to $\frac{2}{3} + \frac{10}{12}$.
    - $\frac{8}{12} + \frac{10}{12}$
    - $\frac{4}{6} + \frac{5}{6}$
2. Find the value of the expression $\frac{2}{3} + \frac{10}{12}$. Explain or show your reasoning.

Student Responses

1.
    - $\frac{8}{12} + \frac{10}{12} = \frac{2}{3} + \frac{10}{12}$ because $\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$ or because $8 \div 4 = 2$ and $12 \div 4 = 3$.
    - $\frac{4}{6} + \frac{5}{6} = \frac{2}{3} + \frac{10}{12}$ because $\frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$ and $\frac{5}{6} \times \frac{2}{2} = \frac{10}{12}$ or $10 \div 2 = 5$ and $12 \div 2 = 6$.
2. $\frac{9}{6}$ or equivalent. Sample response: I added $\frac{4}{6}$ and $\frac{5}{6}$.

Launch

- Groups of 2

Activity

- 5–8 minutes: independent work time
- 1–2 minutes: partner discussion
- Monitor for students who:
    - use multiplication to explain why the expressions are equivalent. For example, multiply $\frac{2 \times 4}{3 \times 4}$ to show why $\frac{2}{3} = \frac{8}{12}$
    - use division to explain why the expressions are equivalent. For example, divide $\frac{10 \div 2}{12 \div 2}$ to show why $\frac{10}{12} = \frac{5}{6}$

Synthesis

- Invite previously selected students to share how they know the expressions $\frac{8}{12} + \frac{10}{12}$ and $\frac{4}{6} + \frac{5}{6}$ are equivalent to $\frac{2}{3} + \frac{10}{12}$.
- "How do you know that $\frac{8}{12} + \frac{10}{12} = \frac{2}{3} + \frac{10}{12}$?" (I can divide each $\frac{1}{3}$ into 4 equal parts. Those parts are $\frac{1}{12}$s and there are 8 of them.)
- "Why is the expression $\frac{8}{12} + \frac{10}{12}$ helpful for finding the sum?" (It's all twelfths. I have 8 of them and 10 more so that's $\frac{18}{12}$.)

- “Which expression did you choose to find the sum?” (Sample response: I used $\frac{4}{6} + \frac{5}{6}$ because the numbers were smaller.)

Advancing Student Thinking

If a student needs help getting started, suggest they draw 2 different number lines to represent $\frac{2}{3}$. Then, for each number line, ask, “How can you adapt the diagram to show $\frac{8}{12}$? $\frac{4}{6}$?”

Activity 2

15 min

Find the Value of the Difference

Standards Alignments

Addressing 5.NF.A.1

This activity builds on the previous activity where students saw how equivalent expressions can be a valuable tool to add or subtract fractions. The purpose of this activity is for students to generate an equivalent expression in order to find the value of a difference of fractions. Monitor for students who:

- find equivalent fractions with smaller numerators and denominators than the given fractions
- find equivalent fractions with larger numerators and denominators than the given fractions

Student-facing Task Statement

1. Find the value of the expression $\frac{16}{12} - \frac{3}{6}$. Explain or show your reasoning.
2. Compare your strategy with your partner’s strategy. What is the same? What is different?

Launch

- Groups of 2

Activity

- 5 minutes: independent work time
- 5 minutes: partner discussion

Student Responses

1. $\frac{5}{6}$, $\frac{10}{12}$ or equivalent. Sample reasoning:
    - $\frac{16}{12} - \frac{3}{6} = \frac{16}{12} - \frac{6}{12}$ because $\frac{3}{6} \times \frac{2}{2} = \frac{6}{12}$ and $\frac{16}{12} - \frac{6}{12} = \frac{10}{12}$.
    - $\frac{16}{12} - \frac{3}{6} = \frac{8}{6} - \frac{3}{6}$ because $\frac{16 \div 2}{12 \div 2} = \frac{8}{6}$ and $\frac{8}{6} - \frac{3}{6} = \frac{5}{6}$.
2. Sample responses:
    - We used different common denominators.
    - I drew a diagram and my partner made calculations to find a common denominator.

Synthesis

- Invite previously selected students to share how they found the value of $\frac{16}{12} - \frac{3}{6}$.
- "How are the strategies for finding the value of the expression the same?" (They both change one of the fractions to an equivalent fraction so the fractions have the same denominator.)
- "How are the strategies for finding the value of the expression different?" (To make the denominator bigger I multiply by a whole number. To make the denominator smaller I divide by a whole number.)
- "Why is it important to have the same denominator?" (Then I can add or subtract the number of parts because they are the same size.)

Advancing Student Thinking

If students try to use $1\frac{2}{6} - \frac{3}{6}$ to find the value of $\frac{16}{12} - \frac{3}{6}$ and do not get the correct value, ask, "How can you use a number line to represent the expression $1\frac{2}{6} - \frac{3}{6}$?"

Activity 3 (optional)

10 min

Grow Plants

Standards Alignments

Addressing 5.NF.A.1, 5.NF.A.2

The purpose of this activity is for students to solve a problem that involves finding the difference of fractions. Students may use addition or subtraction to solve the problem. Either way they will need to find a common denominator for the fractions. One of the numbers is a mixed number so students may:

- convert the mixed number to a fraction
- find the difference in steps, adding on or subtracting

When students recognize mathematical features of objects in the real world, they model with mathematics (MP4).

Access for English Learners

MLR8 Discussion Supports. Students who are working toward verbal output may benefit from access to mini-whiteboards, sticky notes, or spare paper to write down and show their responses to their partner.
Advances: Writing, Representing

Student-facing Task Statement

Jada and Andre compare the growth of their plants. Jada's plant grew $1\frac{3}{4}$ inches since last week. Andre's plant grew $\frac{7}{8}$ inches. How much more did Jada's plant grow? Explain or show your reasoning.

Student Responses

Jada's plant grew $\frac{7}{8}$ inch more than Andre's plant. Sample responses:

- $\frac{7}{8} + \frac{1}{8} = \frac{8}{8}$ and $\frac{8}{8} = 1$, $1 + \frac{3}{4} = 1\frac{3}{4}$, and $\frac{1}{8} + \frac{3}{4} = \frac{1}{8} + \frac{6}{8}$
- $1\frac{3}{4} - \frac{7}{8} = \frac{4}{4} + \frac{3}{4} - \frac{7}{8}$, $\frac{7}{4} - \frac{7}{8} = \frac{14}{8} - \frac{7}{8}$

Launch

- Groups of 2

Activity

- 5 minutes: independent work time
- 1–2 minutes: partner discussion

Synthesis

- Continue to lesson synthesis.

Lesson Synthesis

10 min

"Today we used equivalent expressions to add and subtract fractions with unlike denominators."

Display: $\frac{15}{12} - \frac{3}{4}$

"Describe to your partner how you would find the value of this expression." (I need to find a common denominator so I would figure out how many twelfths are equal to $\frac{3}{4}$. $\frac{3}{4} = \frac{9}{12}$. Then, I would find the difference between $\frac{15}{12}$ and $\frac{9}{12}$. I can use fourths as a common denominator because $\frac{15}{12} = \frac{5}{4}$ so the difference is $\frac{2}{4}$.)

"How do you decide which common denominator to use when you are adding or subtracting fractions with unlike denominators?" (Here one denominator is 3 times the other. So I can use that as my common denominator by splitting the fourths into 3 equal pieces or combining the twelfths to make fourths.)

Suggested Centers

- Would You Rather? (2–5), Stage 3: Compare Units in a Given System (Addressing)
- Compare (1–5), Stage 6: Add and Subtract Fractions (Supporting)

Complete Cool-Down

Response to Student Thinking

Students do not find the value of the expression.

Next Day Support

- During the warm-up of the next lesson, use expressions and equations to represent student thinking.

Lesson 10: All Sorts of Denominators

Standards Alignments

Addressing 5.NF.A.1

Teacher-facing Learning Goals

- Recognize that when adding or subtracting fractions with unlike denominators, a common denominator can be found by multiplying the denominators.

Student-facing Learning Goals

- Let’s find common denominators.

Lesson Purpose

The purpose of this lesson is for students to explain why it is possible to find a common denominator for two given fractions by multiplying the denominators.

In this lesson students begin to develop formal strategies for adding and subtracting fractions. They deal with all cases, including situations where the two denominators share no common factor and cases where the denominators share a large factor. Students show why the product of the two denominators is always a common denominator but are encouraged to use the common denominator that makes sense to them. The discussions focus on different choices students make for a common denominator and how their calculations are the same and different.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

How Many Do You See? (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min

Teacher Reflection Question

Reflect on a time your thinking changed about something in class recently. How will you alter your teaching practice to incorporate your new understanding?

Lesson Synthesis	10 min
Cool-down	5 min

Cool-down (to be completed at the end of the lesson)

5 min

Sums of Fractions

Standards Alignments

Addressing 5.NF.A.1

Student-facing Task Statement

Find the value of $\frac{4}{5} + \frac{2}{7}$.

Student Responses

$\frac{38}{35}$ or equivalent

Begin Lesson

Warm-up

10 min

How Many Do You See: Fraction Sum

Standards Alignments

Addressing 5.NF.A.1

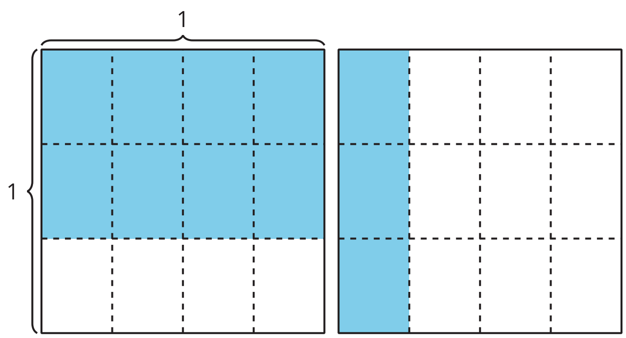
The purpose of this How Many do You See is for students to visualize a common denominator for two fractions. The diagram can be seen as showing $\frac{8}{12} + \frac{3}{12}$ but it can also be seen as showing $\frac{2}{3} + \frac{1}{4}$. The area diagram provides a way to visualize why the product of two denominators works as a common denominator for two fractions.

Instructional Routines

How Many Do You See?

Student-facing Task Statement

How many do you see? How do you see them?



Student Responses

Sample responses:

- I see $\frac{8}{12}$ and $\frac{2}{3}$ in the left square.
- I see $\frac{3}{12}$ and $\frac{1}{4}$ in the right square.
- I see 11 parts which are each $\frac{1}{12}$ so that's $\frac{11}{12}$.

Launch

- Groups of 2
- "How many do you see? How do you see them?"
- Display the image.
- 1 minute: quiet think time

Activity

- Display the image.
- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Record responses.

Synthesis

- "How does the diagram show $\frac{2}{3} + \frac{1}{4}$?" (There are $\frac{2}{3}$ of the left square and $\frac{1}{4}$ of the right square.)
- "What is the value of $\frac{2}{3} + \frac{1}{4}$? How do you know?" ($\frac{11}{12}$ because there are 11 shaded pieces and each one is $\frac{1}{12}$.)

Activity 1

15 min

Different Denominators

Standards Alignments

Addressing 5.NF.A.1

The purpose of this activity is for students to apply what they have learned about using common denominators to add and subtract fractions with unlike denominators. In a previous lesson, students added two fractions where neither denominator was a multiple of the other, $\frac{1}{2} + \frac{1}{3}$, using a strategy that made sense to them. In this activity students see more complex examples. Having built an understanding that they need to find equivalent fractions with a common denominator students will develop strategies for finding a common denominator (MP7, MP8). Monitor for these students who:

- look at multiples of the denominators and pick a common one
- notice that the product of the denominators is a common denominator for the two fractions

Access for Students with Disabilities

Representation: Internalize Comprehension. Begin by asking, "Do these expressions remind anyone of something we have done before?"
Supports accessibility for: Conceptual Processing, Memory

Student-facing Task Statement

Find the value of each expression. Explain or show your thinking.

1. $\frac{3}{4} + \frac{7}{8}$
2. $\frac{3}{4} + \frac{4}{6}$
3. $\frac{3}{4} - \frac{2}{5}$

Student Responses

1. $\frac{13}{8}$ or $1\frac{5}{8}$ or equivalent. Sample response: I used $\frac{6}{8}$ instead of $\frac{3}{4}$ since they are equivalent.
2. $\frac{17}{12}$ or $1\frac{5}{12}$ or equivalent. Sample response: $\frac{3}{4} + \frac{4}{6} = \frac{9}{12} + \frac{8}{12}$ and $\frac{9}{12} + \frac{8}{12} = \frac{17}{12}$.
3. $\frac{7}{20}$ or equivalent. Sample response: $\frac{3}{4} - \frac{2}{5} = \frac{15}{20} - \frac{8}{20}$ and $\frac{15}{20} - \frac{8}{20} = \frac{7}{20}$.

Launch

- Groups of 2

Activity

- 5 minutes: individual work time
- 5 minutes: partner discussion
- monitor for students who:
    - use twelfths as a common denominator to find the value of $\frac{3}{4} + \frac{4}{6}$
    - use twenty-fourths as a common denominator to find the value of $\frac{3}{4} + \frac{4}{6}$

Synthesis

- Ask previously selected students to share their responses.
- "How did you decide which common denominator to use?" (I know that 4 and 6 are both factors of 12 or I know that 4

and 6 are factors of 24 because 24 is 4×6.)

- "How did you use the common denominator to find the sum?" (I found equivalent expressions with 12 or 24 as a denominator and then I could add the fractions since they had the same denominator.)
- "In the next activity, we are going to see a general strategy to find a common denominator for two fractions."

Advancing Student Thinking

If students don't use equivalent fractions to find the value of the expressions, ask, "How can you write an equivalent sum (or difference) of fractions with the same denominator?"

Activity 2

20 min

Multiply Denominators

Standards Alignments

Addressing 5.NF.A.1

The purpose of this activity is for students to explain why the product of the denominators of two fractions is always a common denominator for the two fractions. Students noticed in the previous activity that there are several possible common denominators. Sometimes it is possible to just see a common denominator. For example, for $\frac{2}{3} + \frac{5}{9}$ students might notice that 9 is a common denominator because it is a multiple of 3. It can be convenient, however, to have a strategy that always works, especially for more challenging denominators. After explaining why the product of two denominators is always a common denominator for a pair of fractions (MP3), students practice finding sums and differences of fractions in any way that makes sense to them. This may include

- using the product of the denominators
- thinking about each pair of fractions individually

Both strategies are important. For example, $\frac{3}{50} + \frac{11}{100} = \frac{17}{100}$ since $\frac{3}{50}$ is equivalent to $\frac{6}{100}$. The number $\frac{17}{100}$ is probably easier to grasp mentally than the number $\frac{850}{50{,}000}$ which is what you get if you use the product of the denominators.

Access for English Learners

MLR8 Discussion Supports. Synthesis: Provide students with the opportunity to rehearse with a partner what they will say before they share with the whole class.
Advances: Speaking

Student-facing Task Statement

1. Here is Lin's strategy for finding the value of $\frac{2}{5} + \frac{4}{9}$: "I know 5×9 is a common denominator so I'll use that." Does Lin's strategy for finding a common denominator work? Explain or show your thinking and then find the value of $\frac{2}{5} + \frac{4}{9}$.
2. Find the value of each expression using a method that makes sense to you.
   a. $\frac{3}{8} + \frac{1}{5}$
   b. $\frac{7}{10} - \frac{2}{3}$
   c. $\frac{7}{20} + \frac{41}{50}$
   d. $\frac{2}{9} - \frac{1}{6}$

Student Responses

1. Lin says 5×9 is a common denominator which means it needs to be a multiple of 5 and a multiple of 9. I can see that it is a multiple of 5 since it's 9×5 and I can see that it is a multiple of 9 because it's 5×9. Using this denominator I get $\frac{2}{5} = \frac{18}{45}$ and $\frac{4}{9} = \frac{20}{45}$. So $\frac{2}{5} + \frac{4}{9} = \frac{38}{45}$.
2. a. $\frac{23}{40}$ or equivalent. Sample response: $\frac{15}{40} + \frac{8}{40} = \frac{23}{40}$

Launch

- Groups of 2

Activity

- 5 minutes: individual work time
- 5 minutes: partner work time
- Monitor for students who use different denominators for the last two sums and differences.

Synthesis

- Invite students to share how they found the value of $\frac{3}{8} + \frac{1}{5}$
- "How did your strategy compare to Lin's method?" (I used the product of the denominators for a common denominator just like Lin.)
- Invite students to share how they found the value of $\frac{7}{10} - \frac{2}{3}$.
- "Does Lin's strategy work here too?" (Yes, I used 30 which is 10×3.)
- Invite selected students to share their responses for $\frac{7}{20} + \frac{41}{50}$.
- Display: $\frac{117}{100}$ and $\frac{1{,}170}{1{,}000}$
- "Are these fractions equivalent? How do

b. $\frac{1}{30}$ or equivalent. Sample response: $\frac{7}{10} - \frac{2}{3} = \frac{21}{30} - \frac{20}{30} = \frac{1}{30}$

c. $\frac{117}{100}$ or equivalent. Sample response: $\frac{35}{100} + \frac{82}{100} = \frac{117}{100}$

d. $\frac{1}{18}$ or equivalent. Sample response: $\frac{4}{18} - \frac{3}{18} = \frac{1}{18}$.

you know?" (Yes, the numerator and denominator for the second one are 10 times the numerator and denominator of the first.)

- "Which of these denominators do you prefer?" (I like using hundredths because I'm used to them. I like using thousandths because I did not have to think about finding the common denominator. I just took the product of 20 and 50.)

Lesson Synthesis

10 min

"Today we investigated different ways to add and subtract fractions."

Display: $\frac{2}{9} - \frac{1}{6}$

"How can we find the value of this expression?" (We can find a common denominator for the two fractions.)

"What are some common denominators that you used?" (18, 36, 54)

"What do you notice about these common denominators?" (They are all multiples of 6. They are all multiples of 9. 36 is double 18 and 54 is triple 18.)

"Which denominator did you use to help you find the value of $\frac{2}{9} - \frac{1}{6}$? Why did you choose that one?" (I chose 18 because it is the smallest. I chose 54 because I know that $9 \times 6 = 54$.)

Suggested Centers

- How Close? (1–5), Stage 9: Add Fractions to 5 (Addressing)
- Compare (1–5), Stage 6: Add and Subtract Fractions (Supporting)

Complete Cool-Down

Response to Student Thinking

Students do not find the value of $\frac{4}{5} + \frac{2}{7}$.

Next Day Support

- During the warm-up of the next lesson, highlight notation to write equivalent expressions with common denominators.

Lesson 11: Different Ways to Subtract

Standards Alignments

Addressing 5.NF.A.1, 5.NF.A.2

Teacher-facing Learning Goals

- Subtract fractions and mixed numbers.

Student-facing Learning Goals

- Let’s subtract fractions and mixed numbers.

Lesson Purpose

The purpose of this lesson is for students to subtract fractions with unlike denominators including mixed numbers.

In this lesson, students continue to find differences of fractions with a focus on mixed numbers. There are many ways to find these differences including

- finding equivalent fractions with a common denominator and finding their difference
- adding on, exploiting the whole number parts of the mixed numbers
- using equivalent expressions which help to find a common denominator and deal both with the whole number and fractional parts of the numbers

The second and third strategies have close analogies in arithmetic with whole numbers. One way to find a difference, such as $135 - 28$, is to add on, first 2, then 5, then 100, finding that the difference is 107. For a fraction difference such as $2\frac{3}{8} - \frac{3}{4}$ the corresponding reasoning would be to add $\frac{1}{4}$, then 1, then $\frac{3}{8}$ and find that the difference is $1\frac{5}{8}$. Students could also rewrite the expression $135 - 28$ as $(100 + 20 + 15) - (20 + 8)$ and then find the differences $100 - 0 = 100$, $20 - 20 = 0$ and $15 - 8 = 7$. With the fraction difference, they can rewrite $2\frac{3}{8}$ as $1 + \frac{11}{8}$ and then subtract $\frac{3}{4}$ or $\frac{6}{8}$ from $\frac{11}{8}$, again getting a result of $1\frac{5}{8}$. Students are not expected to bring out these connections but it is important to see that the techniques students used for finding whole number differences can also be used, with appropriate modification, for finding mixed number differences.

Access for:

Students with Disabilities

- Action and Expression (Activity 2)

Instructional Routines

MLR7 Compare and Connect (Activity 1), Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What connections did students make between the different strategies shared? What questions did you ask to help make the connections more visible?

Cool-down (to be completed at the end of the lesson)

5 min

Mixed Differences

Standards Alignments

Addressing 5.NF.A.1, 5.NF.A.2

Student-facing Task Statement

Find the value of each expression. Explain or show your reasoning.

1. $2\frac{4}{5} - \frac{3}{10}$
2. $1\frac{2}{3} - \frac{3}{4}$

Student Responses

1. $2\frac{5}{10}$ or equivalent. Sample response: I rewrote $2\frac{4}{5}$ as $2\frac{8}{10}$ and then subtracted $\frac{3}{10}$.
2. $\frac{11}{12}$ or equivalent. I added $\frac{1}{4}$ to $\frac{3}{4}$ to get 1 and then $\frac{2}{3}$ more to get $1\frac{2}{3}$. Then $\frac{2}{3} = \frac{8}{12}$ and $\frac{1}{4} = \frac{3}{12}$.

Begin Lesson

Warm-up

10 min

Number Talk: Mixed Number Addition and Subtraction

Standards Alignments

Addressing 5.NF.A.1

The purpose of this Number Talk is for students to demonstrate strategies and understandings they have for adding and subtracting a fraction and a whole number. For these problems, students do not need to focus on a common denominator as the numbers either have the same denominator or one of the numbers in the sum is a whole number. Their strategies for thinking about the sums and differences will be helpful throughout the lesson as they calculate more complex differences involving mixed numbers.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $3 + \frac{7}{8}$
- $3 - \frac{7}{8}$
- $1\frac{5}{8} + \frac{6}{8}$
- $1\frac{5}{8} - \frac{6}{8}$

Student Responses

- $3\frac{7}{8}$ or equivalent. I just put the two numbers together.
- $2\frac{1}{8}$ or equivalent. I thought of 3 as 2 an 1 and took $\frac{7}{8}$ from 1.
- $2\frac{3}{8}$ or equivalent. I added $\frac{3}{8}$ to get 2 and then $\frac{3}{8}$ more.
- $\frac{7}{8}$ or equivalent. I took away $\frac{5}{8}$ to get 1 and then $\frac{1}{8}$ more.

Launch

- Display one problem.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep problems and work displayed.
- Repeat with each problem.

Synthesis

- “How did you find the value of $1\frac{5}{8} + \frac{6}{8}$?” (I made a fraction from the mixed number and then added the numerators. I added on to get 2 and then added the rest of the eighths.)

Activity 1

20 min

Challenging Differences

Standards Alignments

Addressing 5.NF.A.1

The purpose of this activity is for students to subtract a fraction or mixed number from a mixed number. There are multiple strategies available and the differences are selected in order to highlight these strategies:

- subtracting the whole number and fraction parts of the numbers separately
- rewriting the mixed number to facilitate subtraction
- adding on to find the difference

In each case, students will need to choose a common denominator in their calculation. Students first identify expressions that are equivalent to the mixed number that appears in all of the differences they calculate. These expressions are deliberately chosen to support the listed techniques to find the value of the subtraction expressions. The goal of the activity synthesis is to compare and connect several different strategies and consider the benefits and challenges of each strategy.

When students adapt their subtraction strategy to the numbers, they look for and make use of structure (MP7).

This activity uses *MLR7 Compare and Connect.* Advances: Conversing.

Instructional Routines

MLR7 Compare and Connect

Student-facing Task Statement

1. Circle all of the expressions that are equivalent to $3\frac{5}{8}$. Explain or show your reasoning.
    - $\frac{20}{8}$

Launch

- Groups of 2

Activity

- 10 minutes: independent or group work
- monitor for students who:

- $2\frac{13}{8}$
- $3\frac{10}{16}$

2. Find the value of each expression. Explain or show your reasoning.
   - $3\frac{5}{8} - \frac{3}{16}$
   - $3\frac{5}{8} - 1\frac{15}{16}$
   - $3\frac{5}{8} - 1\frac{12}{16}$

Student Responses

1.
   - $\frac{20}{8}$ is not equivalent to $3\frac{5}{8}$ because it's less than 3.
   - equivalent since $\frac{13}{8} = 1\frac{5}{8}$
   - equivalent since $\frac{5}{8} = \frac{10}{16}$
2.
   - $3\frac{5}{8} - \frac{3}{16} = 3\frac{7}{16}$ or equivalent. Sample response: I replaced $3\frac{5}{8}$ with $3\frac{10}{16}$ and then subtracted $\frac{3}{16}$.
   - $3\frac{5}{8} - 1\frac{15}{16} = 1\frac{11}{16}$ or equivalent. Sample response: I added $\frac{1}{16}$ to $1\frac{15}{16}$ to get 2 and then $1\frac{10}{16}$ more to get $3\frac{10}{16}$ which is equivalent to $3\frac{5}{8}$.
   - $3\frac{5}{8} - 1\frac{12}{16} = 1\frac{7}{8}$ or equivalent. Sample response: I saw that $1\frac{12}{16}$ is equivalent to $1\frac{6}{8}$ and so I thought of $3\frac{5}{8}$ as $2\frac{13}{8}$ and then I could subtract 1 from 2 and $\frac{6}{8}$ from $\frac{13}{8}$.

- add on to find the value of $3\frac{5}{8} - 1\frac{15}{16}$
- use an equivalent expression that has a fraction greater than 1, such as $2\frac{13}{8}$, to find the value of $3\frac{5}{8} - 1\frac{12}{16}$

MLR7 Compare and Connect

- "Create a visual display that shows your thinking about $3\frac{5}{8} - 1\frac{15}{16}$. You may want to include details such as notes, diagrams or drawings to help others understand your thinking."
- 2 minutes: independent or group work
- 5 minutes: gallery walk

Synthesis

- "What is the same and what is different between the strategies?" (Some people added on or used a number line. Some people changed the mixed number to a whole number plus a fraction greater than one. Some people changed the mixed number to a fraction. Some people got different, equivalent answers.)
- Display: $3\frac{5}{8} = 2\frac{13}{8}$
- "How do we know this is true?" (There are 8 eighths in 1 so I can break up the 3 as 2 and 1 and put the 1 with the $\frac{5}{8}$ to make $\frac{13}{8}$.)
- Invite previously selected students to share how they found the value of $3\frac{5}{8} - 1\frac{12}{16}$.
- "How was rewriting $3\frac{5}{8}$ as $2\frac{13}{8}$ helpful in this calculation?" (I could take 1 from 2 and take $\frac{12}{16}$ from $\frac{13}{8}$ after rewriting it as $\frac{6}{8}$.)
- Invite previously selected students to share how they found the value of $3\frac{5}{8} - 1\frac{15}{16}$.

- "Why did you decide to use that strategy?" (I noticed that $\frac{15}{16}$ was really close to 1 so it was easy for me to count up.)
- "How was rewriting $3\frac{5}{8}$ as $3\frac{10}{16}$ helpful in this calculation?" (I needed to add a sixteenth to get to 2 and it's easy to combine sixteenths and sixteenths.)
- "We are going to find the values of more differences of mixed numbers and fractions in the next activity."
- Keep displays available for students to refer to during activity 2.

Activity 2

15 min

Find the Difference

Standards Alignments

Addressing 5.NF.A.1

The purpose of this activity is for students to find the value of differences of mixed numbers. The numbers are chosen to encourage a variety of strategies that were highlighted in the previous activity. Students should be encouraged to find the differences in a way that makes sense to them. This may mean choosing a different strategy depending on the problem but it could also mean writing each difference as a difference of fractions and then finding a common denominator.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Invite students to verbalize their strategy for finding the difference of each expression before they begin. Students can speak quietly to themselves, or share with a partner.
Supports accessibility for: Organization, Conceptual Processing, Language

Student-facing Task Statement

Find the value of each difference. Explain or show your reasoning.

1. $9\frac{1}{8} - 8\frac{8}{9}$
2. $3\frac{1}{2} - \frac{10}{4}$
3. $4\frac{3}{5} - 1\frac{2}{3}$

Student Responses

Sample responses:

1. $\frac{17}{72}$. Sample response: $8\frac{8}{9} + \frac{1}{9} = 9$ and $9 + \frac{1}{8} = 9\frac{1}{8}$ so $9\frac{1}{8} - 8\frac{8}{9} = \frac{1}{9} + \frac{1}{8}$. That's $\frac{8}{72} + \frac{9}{72}$ or $\frac{17}{72}$.
2. 1. Sample response: $\frac{10}{4} = 2\frac{1}{2}$ and $3\frac{1}{2} - 2\frac{1}{2} = 1$.
3. $2\frac{14}{15}$. Sample response: $4\frac{3}{5} = 3\frac{24}{15}$, $3 - 1 = 2$, $\frac{24}{15} - \frac{10}{15} = \frac{14}{15}$.

Launch

- Groups of 2

Activity

- 5 minutes: independent think time
- 5 minutes: small-group work time
- Monitor for students who:
    - add on to find the value of $9\frac{1}{8} - 8\frac{8}{9}$
    - rewrite $\frac{10}{4}$ as $2\frac{1}{2}$ to find the value of $3\frac{1}{2} - \frac{10}{4}$
    - use an equivalent expression with a fraction greater than one, such as $3\frac{24}{15} - 1\frac{10}{15}$, to find the value of $4\frac{3}{5} - 1\frac{2}{3}$

Synthesis

- Ask previously selected students to share in the given order.
- "Why did you decide to add on to $8\frac{8}{9}$?" ($\frac{8}{9}$ is really close to 1.)
- "Can you also subtract in 2 steps to find the value of $9\frac{1}{8} - 8\frac{8}{9}$?" (Yes, I can subtract $\frac{1}{8}$ from $9\frac{1}{8}$ to get 9 and then take away $\frac{1}{9}$ more to get $8\frac{8}{9}$. That's the same idea as adding on.)
- "Why did you decide to use $2\frac{1}{2}$ instead of $\frac{10}{4}$?" (It is easy to subtract $3\frac{1}{2} - 2\frac{1}{2}$.)

Advancing Student Thinking

If students do not have a strategy to find the value of $4\frac{3}{5} - 1\frac{2}{3}$, write some expressions that are equivalent to $4\frac{3}{5}$, such as $4\frac{9}{15}$ and $3\frac{24}{15}$ and ask, "How do you know these expressions are equivalent?"

Lesson Synthesis

10 min

"Today we found differences of mixed numbers and fractions."

Display the differences.

- $3\frac{5}{8} - 1\frac{15}{16}$
- $3\frac{5}{8} - \frac{3}{16}$
- $3\frac{5}{8} - 1\frac{12}{16}$
- $9\frac{1}{8} - 8\frac{8}{9}$
- $3\frac{1}{2} - \frac{10}{4}$
- $4\frac{3}{5} - 1\frac{2}{3}$

"How can we sort these expressions based on the strategies we used?" Sample responses:

- We could put $9\frac{1}{8} - 8\frac{8}{9}$ and $3\frac{5}{8} - 1\frac{15}{16}$ together because they both have fractions that are close to 1 and adding on was a good strategy to find these differences.
- We could put $3\frac{1}{2} - \frac{10}{4}$ and $3\frac{5}{8} - \frac{3}{16}$ together because we just had to find a common denominator for the fractional part and write $\frac{10}{4}$ as a mixed number to find the values. We could take the whole number from the whole number and the fraction from the fraction.
- We could put $3\frac{5}{8} - 1\frac{12}{16}$ and $4\frac{3}{5} - 1\frac{2}{3}$ together because they were the most challenging or they required the most steps.

"How do you decide which strategy to use when finding the difference of mixed numbers or a mixed number and a fraction?" (I look at the numbers and think about which strategy would be easy to use and accurate.)

Suggested Centers

- How Close? (1–5), Stage 9: Add Fractions to 5 (Addressing)
- Creating Line Plots (2–5), Stage 3: Eighth Inches, Add and Subtract (Supporting)

Complete Cool-Down

Response to Student Thinking

Students don't find the value of each expression.

Next Day Support

- Create a poster with the steps to solving the cool-down problem from the previous lesson.

Lesson 12: Solve Problems

Standards Alignments

Addressing 5.NF.A.1, 5.NF.A.2

Teacher-facing Learning Goals

- Solve problems involving addition and subtraction of fractions with unlike denominators.

Student-facing Learning Goals

- Let's solve more problems by adding and subtracting fractions with unlike denominators.

Lesson Purpose

The purpose of this lesson is for students to solve problems that involve adding and subtracting fractions with unlike denominators.

In this lesson, students apply their understanding of fraction addition and subtraction to solve multi-step problems. They work with fractions and mixed numbers and make estimates as well as finding the value of sums and differences. Students find the sums and differences in a way that makes sense to them. They will choose common denominators and can also use techniques they have seen in earlier lessons such as adding on to make a whole number.

Access for:

Students with Disabilities

- Representation (Activity 2)

English Learners

- MLR6 (Activity 1)

Instructional Routines

Estimation Exploration (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min

Teacher Reflection Question

What strategies did most students use to add and subtract fractions today? What strategies did you anticipate? Which did you not anticipate?

Cool-down	5 min

Cool-down (to be completed at the end of the lesson) 5 min

Evaluate Expressions

Standards Alignments

Addressing 5.NF.A.1, 5.NF.A.2

Student-facing Task Statement

1. Priya hiked $1\frac{2}{3}$ miles. Diego hiked $\frac{1}{2}$ mile. How much farther did Priya hike than Diego? Explain or show your reasoning.
2. On Monday, Andre hiked $\frac{3}{4}$ mile in the morning and $1\frac{1}{3}$ miles in the afternoon. How far did Andre hike on Monday? Explain or show your reasoning.

Student Responses

1. $1\frac{1}{6}$ miles or equivalent. $1\frac{2}{3} - \frac{1}{2} = \frac{5}{3} - \frac{1}{2}, \frac{5}{3} - \frac{1}{2} = \frac{10}{6} - \frac{3}{6} = \frac{7}{6} = 1\frac{1}{6}$
2. $2\frac{1}{12}$ miles or equivalent. $\frac{3}{4} + 1\frac{1}{3} = \frac{9}{12} + \frac{16}{12} = \frac{25}{12} = 2\frac{1}{12}$

Begin Lesson

Warm-up 10 min

Estimation Exploration: Large Denominators

Standards Alignments

Addressing 5.NF.A.1

The purpose of this estimation exploration is for students to reason about the size of a complex fraction sum with large denominators. Students can see that 1 is a good estimate because one fraction is small and the other is close to 1. In the synthesis they refine this estimate to explain why the value of the sum is a little larger than 1.

Instructional Routines

Estimation Exploration

Student-facing Task Statement

What is the value of the sum?

$\frac{3}{17} + \frac{17}{19}$

Record an estimate that is:

too low	about right	too high

Student Responses

Sample responses:

- Too low: less than 1
- About right: 1 to $1\frac{1}{5}$
- Too high: more than $1\frac{1}{5}$

Launch

- Groups of 2
- Display the expression.
- "What is an estimate that's too high?" "Too low?" "About right?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Record responses.

Synthesis

- "How do you know that the sum is greater than 1?" ($\frac{17}{19}$ is $\frac{2}{19}$ short of a whole. Since 17ths are bigger than 19ths, adding $\frac{3}{17}$ makes it greater than 1.)

Activity 1

20 min

Priya's Salad Dressing

Standards Alignments

Addressing 5.NF.A.2

The purpose of this activity is for students to add and subtract fractions and estimate sums and differences of fractions using the context of a recipe. Students may have different responses and reasoning for the estimation questions. In both cases, they can calculate and compare fractions but they may have different thoughts about how these differences would affect the recipe or what exactly it means for the recipe to make "about $1\frac{1}{2}$ cups." In the synthesis, students discuss the reasonableness of the estimates and how to make precise calculations (MP6). When students relate their calculations to Priya's salad dressing they reason abstractly and quantitatively (MP2).

Access for English Learners

Reading: MLR6 Three Reads. Keep books or devices closed. Display only the problem stem, without revealing the questions. "We are going to read this question 3 times." After the 1st Read: "Tell your partner what this situation is about." After the 2nd Read: "List the quantities. What can be counted or measured?" Reveal the question(s). After the 3rd Read: "What strategies can we use to solve this problem?"
Advances: Reading, Representing

Student-facing Task Statement

Priya's Salad Dressing Recipe

- $\frac{3}{4}$ cup olive oil
- $\frac{1}{3}$ cup lemon juice
- $\frac{1}{2}$ cup mustard
- Pinch of salt and pepper

1. Priya has $\frac{2}{3}$ cup of olive oil. She is going to borrow some more from her neighbor. How much olive oil does she need to borrow to have enough to make the dressing?
2. 1 tablespoon is equal to $\frac{1}{16}$ of a cup. Priya decides that 1 tablespoon of olive oil is close enough to what she needs to borrow from her neighbor. Do you agree with Priya? Explain or show your reasoning.
3. Priya says her recipe will make about $1\frac{1}{2}$

Launch

- Groups of 2
- "What kind of ingredients do you like to put in your salad?" (lettuce, cabbage, beans, seeds, beets, tomatoes, cheese)
- "What kinds of dressings do you put on your salad?" (homemade, Italian, blue cheese, tamari)

Activity

- 1–2 minutes: quiet think time
- 6–8 minutes: small-group work time
- Monitor for students who:
    - estimate to determine that Priya's recipe will make about $1\frac{1}{2}$ cups of dressing
    - add $\frac{3}{4} + \frac{1}{3} + \frac{1}{2}$ to determine the precise amount of dressing Priya's recipe will make

cups of dressing. Do you agree? Explain or show your reasoning.

Student Responses

1. Priya needs $\frac{1}{12}$ cup of olive oil.
2. Sample responses: $\frac{1}{12} - \frac{1}{16} = \frac{1}{48}$. It won't make a difference because the difference is so small or it might taste more lemony or more mustardy.
3. Sample responses:
    a. I added all of the fractions and $\frac{3}{4} + \frac{1}{3} + \frac{1}{2} = 1\frac{7}{12}$. That's just $\frac{1}{12}$ cup more than $1\frac{1}{2}$ cups.
    b. I think Priya's recipe will make about $1\frac{1}{2}$ cups of dressing because $\frac{3}{4} + \frac{1}{4} + \frac{1}{2} = 1\frac{1}{2}$ and $\frac{1}{3}$ is close to $\frac{1}{4}$.

Synthesis

- "If Priya borrows a tablespoon of olive oil from her neighbor and uses it to make dressing, will she be putting in more or less olive oil than the recipe calls for?" ($\frac{1}{16}$ is smaller than $\frac{1}{12}$ so she will be putting in less olive oil.)
- "Do you think 1 tablespoon is close enough?"
- Poll the class.
- "How might Priya's decision to use 1 tablespoon of olive oil change the salad dressing?" (It won't make a difference because the difference is so small. It might taste more lemony or more mustardy because there is not as much oil. It might affect the consistency of the dressing a little.)
- Ask previously selected students to share their estimates for the amount of salad dressing in the given order.
- "Why might Priya estimate that the recipe makes $1\frac{1}{2}$ cups of salad dressing?" ($\frac{3}{4}$ is $\frac{1}{4}$ away from 1 and $\frac{1}{3}$ is close to $\frac{1}{4}$.)
- "Does the recipe make more or less than $1\frac{1}{2}$ cups? How do you know?" (More because $\frac{1}{3}$ is more than $\frac{1}{4}$.)
- "How many cups does Priya's recipe make? How do you know?" ($1\frac{7}{12}$, I added $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{1}{2}$.)

Activity 2

15 min

More Problems to Solve

Standards Alignments

Addressing 5.NF.A.2

The purpose of this activity is for students to solve multi-step problems involving the addition and subtraction of fractions with unlike denominators. Students work with both fractions and mixed numbers and can use strategies they have learned such as adding on to make a whole number. When students connect the quantities in the story problem to an equation, they reason abstractly and quantitatively (MP2).

Access for Students with Disabilities

Representation: Access for Perception. Read both problems aloud. Students who both listen to and read the information will benefit from extra processing time.
Supports accessibility for: Conceptual Processing, Language

Student-facing Task Statement

1. Choose a problem to solve.

   Problem A:

   Jada is baking protein bars for a hike. She adds $\frac{1}{2}$ cup of walnuts and then decides to add another $\frac{1}{3}$ cup. How many cups of walnuts has she added altogether?

   If the recipe requires $1\frac{1}{3}$ cups of walnuts, how many more cups of walnuts does Jada need to add? Explain or show your reasoning.

   Problem B:

   Kiran and Jada hiked $1\frac{1}{2}$ miles and took a rest. Then they hiked another $\frac{4}{10}$ mile before stopping for lunch. How many miles have they hiked so far?

   If the trail they are hiking is a total of $2\frac{1}{2}$ miles, how much farther do they have to hike? Explain or show your reasoning.

Launch

- Groups of 2
- "You and your partner will each choose a different problem to solve and then you will discuss your solutions."

Activity

- 3–5 minutes: independent work time
- 3–5 minutes: partner discussion

Synthesis

- "How were the problems the same? How were they different?" (For both problems I had to add fractions first and then subtract that total from another number. There were mixed numbers in both problems.)
- "How did you use equivalent fractions to solve these problems?" (All the fractions we worked with had different denominators so we had to find equivalent fractions with the same denominators in order to add or subtract.)

2. Discuss the problems and solutions with your partner. What is the same about your strategies and solutions? What is different?
3. Revise your work if necessary.

Student Responses

1.
   A. $\frac{5}{6}$ cup and $\frac{1}{2}$ cup. Sample response: $\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6}$. Jada has already added $\frac{1}{2}$ cup and $\frac{1}{3}$ cup and $\frac{1}{2} + \frac{1}{2} = 1$ so $\frac{1}{2} + \frac{1}{3} + \frac{1}{2} = 1\frac{1}{3}$.

   B. $1\frac{9}{10}$ miles and $\frac{6}{10}$ mile. Sample response:
   $1\frac{1}{2} + \frac{4}{10} = 1\frac{5}{10} + \frac{4}{10}$ and
   $1\frac{5}{10} + \frac{4}{10} = 1\frac{9}{10}$.
   Then $1\frac{9}{10} + \frac{1}{10} = 2$ and
   $2 + \frac{5}{10} = 2\frac{1}{2}$ and $\frac{1}{10} + \frac{5}{10} = \frac{6}{10}$.
2. Sample response: We both added on to one of the numbers to make a whole number and then added on again to get the target number.
3. Answers vary.

Lesson Synthesis

10 min

“Today we solved problems that required adding and subtracting fractions.”

Display Priya’s salad dressing recipe.

“What strategy did you use to find out how much salad dressing Priya’s recipe makes?” (The denominators for the fractions are 2, 3 and 4 so I used 12 because I know that it is a multiple of 2, 3, and 4. I put the half and fourths together first since I could use 4 as a common denominator and then I used 12 to add the fourths and third.

Display: $\frac{1}{12} - \frac{1}{16}$

“What strategy did you use to find this difference for the olive oil?” (I knew that 48 is 4×12 and 3×16

so I used that as a common denominator. I used 12×16 as a common denominator.)

"How do you decide which strategy to use?" (It depends on the numbers. If I know a small common multiple of the denominators, I use that. If I don't, I can always use the product of the denominators.)

Suggested Centers

- How Close? (1–5), Stage 9: Add Fractions to 5 (Addressing)
- Creating Line Plots (2–5), Stage 3: Eighth Inches, Add and Subtract (Supporting)

Complete Cool-Down

Response to Student Thinking

Students do not use equivalent fractions to create an equivalent expression with like denominators.

Next Day Support

- Launch the lesson by asking students to recap the important points of the previous lessons.

Lesson 13: Put It All Together: Add and Subtract Fractions

Standards Alignments

Addressing 5.NF.A.1

Teacher-facing Learning Goals

- Add and subtract fractions with unlike denominators.

Student-facing Learning Goals

- Let's add and subtract fractions with unlike denominators.

Lesson Purpose

The purpose of this lesson is for students to consider different denominators they can use to add or subtract fractions.

In this lesson, students examine and implement different strategies to find a common denominator when adding or subtracting fractions. For any pair of fractions, the product of the denominators will be a common denominator and in many cases this is also the smallest choice of common denominator. In other cases, however, there is a smaller choice than the product and sometimes this can be a useful choice. For example, the sum $\frac{3}{20} + \frac{9}{50}$ can be found using 1,000 as a common denominator but it can also be found with 100 as a common denominator. Either choice will work and there are reasons for preferring each strategy. In this lesson, students think about these ideas as they investigate different common denominators for finding sums and differences of fractions.

Access for:

Students with Disabilities

- Engagement (Activity 2)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min

Teacher Reflection Question

As students shared their ideas today, how did you ensure all students' voices were heard and valued as an important part of the collective

Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

learning?

Cool-down (to be completed at the end of the lesson)

5 min

Fraction Addition and Subtraction

Standards Alignments

Addressing 5.NF.A.1

Student-facing Task Statement

Find the value of each expression. Explain or show your reasoning.

1. $\frac{8}{7} - \frac{2}{3}$
2. $\frac{5}{6} + \frac{2}{9}$

Student Responses

1. $\frac{10}{21}$ or equivalent. Sample reasoning: $\frac{8}{7} - \frac{2}{3} = \frac{24}{21} - \frac{14}{21} = \frac{10}{21}$
2. $\frac{19}{18}$ or equivalent. Sample reasoning: $\frac{5}{6} + \frac{2}{9} = \frac{15}{18} + \frac{4}{18} = \frac{19}{18}$

Begin Lesson

Warm-up

10 min

Number Talk: Sums with $\frac{1}{8}$

Standards Alignments

Addressing 5.NF.A.1

The purpose of this Number Talk is for students to use different strategies to add fractions. Each pair of fractions has $\frac{1}{8}$ and the difference between the expressions is the denominator of the second fraction which is chosen to suggest different strategies for finding a common denominator. Students will explore these strategies in depth in this lesson.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $\frac{1}{8} + \frac{5}{8}$
- $\frac{1}{8} + \frac{6}{16}$
- $\frac{1}{8} + \frac{1}{3}$
- $\frac{1}{8} + \frac{5}{12}$

Student Responses

Sample responses:

- $\frac{6}{8}$ or equivalent. I just found $1 + 5$ and knew it was eighths.
- $\frac{4}{8}$ or equivalent. I know $\frac{6}{16}$ is $\frac{3}{8}$ and then I combined the eighths.
- $\frac{11}{24}$ or equivalent. I used 24 as a common denominator.
- $\frac{13}{24}$ or equivalent. I used 24 as a common denominator.

Launch

- Groups of 2
- Display one problem.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep problems and work displayed.
- Repeat with each problem.
- “Discuss your thinking with your partner.”

Synthesis

- “How did you decide which denominator to use when you found the sums of the fractions?” (The first two were the easiest because I could use eighths. For the others I used the product of the denominators for 8 and 3 and then I knew 24 would work for 8 and 12.)

Activity 1

20 min

Common Denominators

Standards Alignments

Addressing 5.NF.A.1

The purpose of this activity is for students to recognize that different strategies can be used to find common denominators. The two strategies highlighted in this activity are the ones students have used throughout the last several lessons.

- Use the product of the two denominators.
- Use a smaller, recognizable common multiple of the two denominators.

Both strategies are valuable and students can consider using these strategies in the next activity when they practice finding a variety of sums and differences.

When students evaluate the student claims about common denominators they critique the reasoning of others (MP3).

Access for English Learners

MLR8 Discussion Supports. During group work, invite students to take turns sharing their responses. Ask students to restate what they heard using precise mathematical language and their own words. Display the sentence frame: “I heard you say . . . .” Original speakers can agree or clarify for their partner.
Advances: Listening, Speaking

Student-facing Task Statement

$\frac{4}{6} + \frac{5}{8}$

Tyler says: “To find the sum, I can use 18 as a common denominator.”

Han says: “To find the sum, I can use 24 as a common denominator.”

Clare says: “To find the sum, I can use 48 as a common denominator.”

1. Whom do you agree with? Explain or show your reasoning.
2. What is the value of $\frac{4}{6} + \frac{5}{8}$?
3. Are there other common denominators you

Launch

- Groups of 2

Activity

- 2 minutes: independent work time
- 8 minutes: small-group work time
- Monitor for students who:
    - can explain why 18 is not a common denominator
    - can explain why 48 can be used as a common denominator
    - can explain why 24 can be used as a common denominator

could use to find the sum? Explain or show your reasoning.

Student Responses

1. Han and Clare are correct because 24 and 48 are multiples of 6 and 8. Tyler is incorrect because 18 is not a multiple of 8 though it is a multiple of 6.
2. $\frac{31}{24}$ or $\frac{62}{48}$ or equivalent. Sample responses:
   - $\frac{4}{6}+\frac{5}{8}=\frac{16}{24}+\frac{15}{24}=\frac{31}{24}$
   - $\frac{4}{6}+\frac{5}{8}=\frac{32}{48}+\frac{30}{48}=\frac{62}{48}$
3. Yes, 72 is a common multiple and so is 96. Since 24 is a common multiple I can just keep adding 24 and I get other common multiples.

Synthesis

- Invite previously selected students to share.
- "How do you know that 18 will not work as a common denominator?" (It is not a multiple of 8.)
- "How do you know that 24 will work as a common denominator?" (It is a multiple of 6 and 8.)
- "How do you know that 48 will work as a common denominator?" ($6 \times 8 = 48$)
- "Which denominator do you prefer to use and why?" (I like 24 because it's smaller so the arithmetic is easier. I like 48 because I know right away which multiples to take to get the common denominator.)
- Invite students to share other common denominators that they found.
- "Would you use any of these denominators to find the sum? Why or why not?" (No, it makes the numbers bigger and I need to figure out which multiple to use to get these common denominators.)

Activity 2

15 min

Unlike Denominators

Standards Alignments

Addressing 5.NF.A.1

The purpose of this activity is for students to consider which common denominators will be most helpful to add and subtract fractions. The numbers for each problem are chosen to highlight different strategies. The first problem has denominators where one is a multiple of the other. Students will likely recognize they can use one of them as a common denominator, reducing the number of computations needed. The second problem does not have one denominator that is a multiple of the other. The numbers are small and the product is a good choice for a common denominator. The other problems have larger denominators that share common factors. For these problems, some students may prefer to find a smaller common denominator as it can make the arithmetic simpler. Other students may prefer taking the product of the denominators because that way they don't need to work to find a common multiple of the two denominators.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Check in and provide each group with feedback that encourages collaboration and community. For example, ask students to identify the similarities and differences between their choices of denominators.
Supports accessibility for: Conceptual Processing; Social-Emotional Functioning

Student-facing Task Statement

Find the value of each expression. Explain or show your reasoning.

1. $\frac{2}{5} + \frac{13}{15}$
2. $\frac{6}{5} - \frac{1}{3}$
3. $\frac{11}{12} + 3\frac{5}{9}$
4. $\frac{6}{10} - \frac{9}{25}$

Student Responses

1. $\frac{19}{15}$ or equivalent. Sample response: $\frac{2}{5} + \frac{13}{15} = \frac{6}{15} + \frac{13}{15} = \frac{19}{15}$
2. $\frac{13}{15}$ or equivalent. Sample response: $\frac{6}{5} - \frac{1}{3} = \frac{18}{15} - \frac{5}{15} = \frac{13}{15}$
3. $4\frac{17}{36}$ or equivalent. Sample response: $\frac{11}{12} + 3\frac{5}{9} = \frac{33}{36} + 3\frac{20}{36} = 3\frac{53}{36} = 4\frac{17}{36}$
4. $\frac{12}{50}$ or equivalent. Sample response: $\frac{6}{10} - \frac{9}{25} = \frac{30}{50} - \frac{18}{50} = \frac{12}{50}$

Launch

- Groups of 2

Activity

- 5 minutes: independent work time
- 5 minutes: small group work time
- Monitor for students who use different common denominators including the product of the denominators.

Synthesis

- Display expression: $\frac{11}{12} + 3\frac{5}{9}$
- Invite students to share the denominators they used to find the value of the expression.
- "Why are there different choices for a common denominator?" (Any number that is a multiple of both 9 and 12 works.)
- Display expression: $\frac{6}{10} - \frac{9}{25}$
- "Which common denominator did you use for these fractions?" (50 or 100 because I could see that they are common multiples and know what factors to multiply 10 and

25 by to get those numbers.)

- “Which strategy do you prefer to use to find a common denominator?” (Using the product always works, but the multiplication can be challenging sometimes. Finding a smaller common denominator can be helpful because I might not have to multiply large numbers, but it can be time consuming.)

Lesson Synthesis

10 min

“Today we added and subtracted fractions with unlike denominators. We found common denominators including the product of the denominators.”

Display: $\frac{6}{10} + \frac{9}{25}$

“Why might someone use 250 as a common denominator to add these fractions?” (They can just multiply the two denominators.)

“Why might someone use 50 as a common denominator?” (They want to use a smaller common denominator to simplify the arithmetic or visualize the answer more easily.)

Highlight the idea that when we add and subtract fractions with unlike denominators, we replace the given fractions with equivalent fractions that have the same denominator, whether that common denominator is found by multiplying the original denominators or finding a smaller common multiple of the two.

Suggested Centers

- How Close? (1–5), Stage 9: Add Fractions to 5 (Addressing)

Complete Cool-Down

Response to Student Thinking

Students don’t use 18 as a common denominator to find the value of $\frac{5}{6} + \frac{2}{9}$.

Next Day Support

- Launch Activity 1 with a discussion about this cool-down.

Lesson 14: Representing Fractions on a Line Plot

Standards Alignments

Addressing	5.MD.B.2, 5.NF.A.1
Building Towards	5.MD.B.2

Teacher-facing Learning Goals

- Create line plots and use the information to solve problems.

Student-facing Learning Goals

- Let's make a line plot and analyze the data we collect.

Lesson Purpose

The purpose of this lesson is for students to make and interpret line plots displaying fractions in eighths.

In this lesson, students use fraction arithmetic to solve problems about line plots. Students make line plots by generating data and using given data and then they solve problems about the line plots. Students have worked with these line plots in a previous course. The new part of the work in this lesson and the next is the level of complexity of the questions they answer. In this lesson they perform arithmetic with the fractions and answer questions that involve thinking about all the data as a whole and certain measurements as a fraction of that whole (MP2).

Access for:

Students with Disabilities

- Engagement (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Which One Doesn't Belong? (Warm-up)

Materials to Gather

- Paper clips: Activity 1
- Pencils: Activity 1

Lesson Timeline

Warm-up	10 min

Teacher Reflection Question

How did the student work that you selected,

Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

particularly in the second activity, impact the direction of the discussion? What student work might you pick next time if you taught the lesson again?

Cool-down (to be completed at the end of the lesson)

5 min

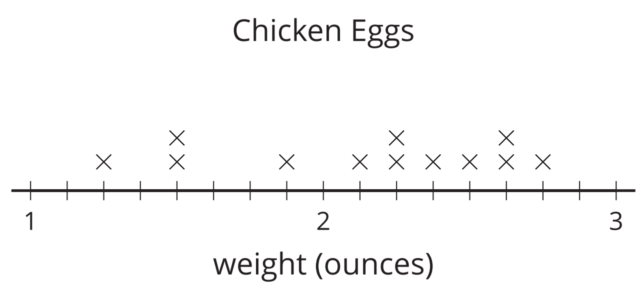
A Dozen Eggs

Standards Alignments

Addressing 5.MD.B.2

Student-facing Task Statement

Here are the weights of a different collection of chicken eggs.



What is the combined weight of all the eggs that weigh more than $2\frac{1}{2}$ ounces? Explain or show your reasoning.

Student Responses

8 ounces or equivalent. There are 2 eggs that weigh $2\frac{5}{8}$ ounces and 1 egg that weighs $2\frac{3}{4}$ ounces or $2\frac{6}{8}$ ounces. If I add them up, I get $6\frac{16}{8}$ ounces which is the same as 8 ounces.

Begin Lesson

Warm-up

10 min

Which One Doesn’t Belong: Line Plot

Standards Alignments

Addressing 5.MD.B.2

The purpose of this warm-up is for students to recall the line plots with fractional measurements which they have studied in prior courses. This prepares them to do more arithmetic with fractions using the data from line plots in the next two lessons.

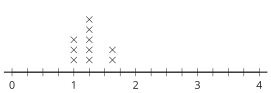
Instructional Routines

Which One Doesn’t Belong?

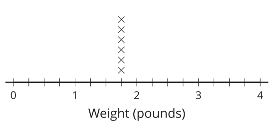
Student-facing Task Statement

Which one doesn’t belong?

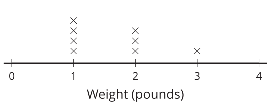
A



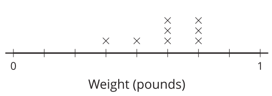
B



C



D



Student Responses

Sample responses

- A is the only line plot that doesn’t have a title.
- B is the only line plot that doesn’t have multiple measurements.
- C is the only line plot that has no fractional measurements or tick marks.
- D is the only line plot where all of the data have values less than 1.

Launch

- Groups of 2
- Display the image.
- “Pick one that doesn’t belong. Be ready to share why it doesn’t belong.”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 2–3 minutes: partner discussion
- Share and record responses.

Synthesis

- “Why doesn’t B belong?” (There are no fractions marked and there is no data with fractional values.)
- “Today we are going to work with line plots with fractional data and use what we have learned about fractions to solve problems.”

Activity 1

15 min

Sums of Fractions

Standards Alignments

Addressing	5.NF.A.1
Building Towards	5.MD.B.2

The purpose of this activity is for students to make a line plot and answer questions about the data collected. The numbers that students plot come from spinning a spinner twice and adding the fractions on the spinner. The denominators are chosen so that 8 can be used as a common denominator. Students observe and think about patterns and then discuss them during the synthesis.

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Synthesis: Revisit math community norms to prepare students for the whole-class discussion.
Supports accessibility for: Attention, Social-Emotional Functioning

Materials to Gather

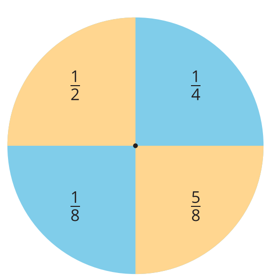
Paper clips, Pencils

Required Preparation

- Each group of 2 needs 1 paper clip and one pencil.

Student-facing Task Statement

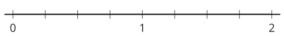
1. Play Sums of Fractions with your partner.
    - Take turns with your partner.
    - Spin the spinner twice.
    - Add the two fractions.



Launch

- Groups of 2
- Display the number line image from student workbook.
- "You are going to play a game with your partner. You will use a paper clip and a pencil to make a spinner out of the image in your workbook. Let's practice."
- Demonstrate how to use the pencil and paper clip as a spinner. Spin twice and

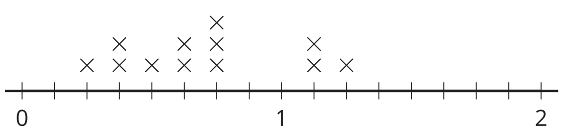
- Record the sum on the line plot.
- Play the game until you and your partner together have 12 data points.



2. How did you know where to plot the sums of eighths?
3. What is the difference between your highest and lowest number?
4. What do you notice about the data you collected?

Student Responses

1. Sample response:



2. Sample response: I partitioned the number line plot into a total of 8 parts and used the numerator of each sum to decide where to plot each sum. Sometimes, I had to think about equivalent fractions.
3. Sample response: $1\frac{2}{8} - \frac{2}{8} = 1$
4. Sample responses:
   a. Some sums were more common than others.
   b. Most of the sums were less than 1.
   c. All the sums could be written as some number of eighths. I never got $\frac{7}{8}$.

record the fractions you landed on for all to see.

- "I need to find the value of the sum of these two fractions."
- Demonstrate how to record the sum on the number line with an X.
- "Which number do you think will have the most Xs if you spin the spinner a lot of times? Why?"
- 1–2 minutes: partner discussion

Activity

- 1–2 minutes: quiet think time
- 6–8 minutes: partner work time
- Monitor for students who:
  - partition the number line into eighths
  - use common denominators to convert fractions with unlike denominators to fractions with like denominators

Synthesis

- Ask previously identified students to share their thinking.
- "Did anyone record a one? What did you spin?" (I got $\frac{1}{2}$ on both spins.)
- "Is there any other way to get 1 as a sum?" (No. I would need to add $\frac{3}{4}$ to $\frac{1}{4}$, $\frac{7}{8}$ to $\frac{1}{8}$, and $\frac{3}{8}$ to $\frac{5}{8}$ and none of those is possible.)
- "What is the largest number you recorded?" (Sample responses: $1\frac{2}{8}$, $1\frac{1}{8}$)
- "Is it possible to get more than $1\frac{2}{8}$?" (No, the biggest number is $\frac{5}{8}$ and two of those is $1\frac{2}{8}$.)
- "Name a fraction that would have made the game more challenging if it were on the

number mat. Why would this have made the game more challenging?" ($\frac{2}{3}$, $\frac{4}{5}$, or any other fraction with a denominator that is not a factor or multiple of 8. It would be more challenging because we could not use 8 as a common denominator to easily add the fractions.)

Activity 2

20 min

Lots of Eggs

Standards Alignments

Addressing 5.MD.B.2

The purpose of this activity is for students to use measurement data to make a line plot and then solve problems about the data presented in the line plot (MP2). The line plot is blank so students will choose which whole numbers to label and which fractions to label in between. They will use their understanding of equivalent fractions (halves, quarters, and eighths) to accurately make the line plot. Jada's statement about the eggs that weigh $1\frac{7}{8}$ ounces is interesting because it uses two fractions referring to different quantities: $\frac{1}{4}$ is a fraction of the eggs and $1\frac{7}{8}$ is their weight in ounces. The focus of the synthesis is on how students reason about Jada's statement. As students reason through Jada's statement, they critique the reasoning of others (MP3).

Access for English Learners

MLR8 Discussion Supports. Prior to solving the problems, invite students to make sense of the situations and take turns sharing their understanding with their partner. Listen for and clarify any questions about the context.
Advances: Reading, Representing

Student-facing Task Statement

1. Here are the weights of some

Launch

- Groups of 2
- Display the image.

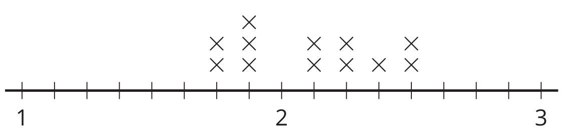
eggs, in ounces. Use them to make a line plot.

$1\frac{7}{8}, 2\frac{1}{2}, 2\frac{3}{8}, 1\frac{3}{4}, 2\frac{1}{4}, 2\frac{4}{8}, 2\frac{1}{8}, 1\frac{7}{8}, 2\frac{1}{4}, 1\frac{6}{8}, 2\frac{1}{8}, 1\frac{7}{8}$

2. Jada said that $\frac{1}{4}$ of the eggs weigh $1\frac{7}{8}$ ounces. Do you agree? Explain or show your reasoning.
3. How much heavier is the heaviest egg than the lightest egg? Explain or show your reasoning.

Student Responses

1. Chicken Eggs



2. Jada is correct. Sample response: There are 3 eggs that weigh $1\frac{7}{8}$ ounce. $\frac{1}{4} \times 12 = 3$ or $12 \div 4 = 3$.
3. $\frac{6}{8}$ ounce or equivalent. Sample reasoning: The heaviest egg is $2\frac{4}{8}$ ounces and the lightest is $1\frac{6}{8}$ ounces. I added $\frac{2}{8}$ to $1\frac{6}{8}$ to get 2 and then $\frac{4}{8}$ more to get $2\frac{4}{8}$.

- “What do you notice?” (It is a chicken. There is an egg. There are numbers and units.)
- “What do you wonder?” (What is it for? Is it a scale? Why does it say small, medium, large, X-Large?)
- “This is an egg scale. It is used to weigh eggs. What can you say about the egg on the scale?” (It's small. It weighs less than 2 ounces.)
- “Now you will make a line plot and answer questions about the egg weights.”

Activity

- 5 minutes: independent work time
- 5 minutes: partner work time
- Monitor for students who use either of these expressions to determine if Jada is correct:
  - $\frac{1}{4} \times 12 = 3$
  - $12 \div 4 = 3$

Synthesis

- Ask previously selected students to share their response to Jada's statement.
- “How did you know how many eggs Jada weighed?” (I could count the measurements or count the marks on the graph.)
- “How did you decide if her statement was correct?” (I found $\frac{1}{4}$ of 12 and then counted the number eggs that weighed $1\frac{7}{8}$ ounces. They were both 3 so Jada is correct.)
- If students do not write an equation matching Jada’s response, consider displaying the equations $\frac{1}{4} \times 12 = 3$ and $12 \div 4 = 3$.
- “How do these equations represent Jada’s reasoning?” (They both say that $\frac{1}{4}$ of the 12 egg measurements is 3 eggs. One uses

multiplication and the other one uses division.)

Lesson Synthesis

10 min

"Today we made line plots and answered questions about the data in line plots."

Display line plots from the student solutions for the 2 activities or use student-generated examples.

"How are the line plots the same?" (They both show data that is measured in fractions. The fractions are all eighths. There are 12 measurements in each line plot.)

"How are the line plots different?" (The numbers for the game are just numbers. The other ones are weights. The numbers for the game only go up to $\frac{10}{8}$. There is more variation in the weights of the eggs.)

Suggested Centers

- Creating Line Plots (2–5), Stage 4: Eighth Inches, Add, Subtract, and Multiply (Addressing)
- Rectangle Rumble (3–5), Stage 4: Whole Number and Fraction Factors (Supporting)

Complete Cool-Down

Response to Student Thinking

The work in this lesson builds from the line plot concepts developed in a prior unit.

Prior Unit Support

Grade 4, Unit 3, Section B: Addition and Subtraction of Fractions

Lesson 15: Problem Solving with Line Plots

Standards Alignments

Addressing 5.MD.B.2, 5.NF.A.2, 5.NF.B.4

Teacher-facing Learning Goals

- Create line plots to display fractional measurement data, and use the information to solve problems.

Student-facing Learning Goals

- Let’s solve problems using a line plot.

Lesson Purpose

The purpose of this lesson is for students to make line plots and solve problems using the data.

In this lesson, students use line plots to solve multi-step problems about the data presented in line plots. They add and subtract fractions and work together to solve problems about data using the Info Gap routine. The data for line plots can all be expressed using eighths, making the arithmetic accessible with the challenge being on partner communication to share the information needed to solve the problems. The second optional activity also uses data from a line plot and here students relate repeated addition of fractions to multiplication and estimate the sum of all of the data presented in the line plot.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Representation (Activity 1)

Instructional Routines

MLR4 Information Gap (Activity 1), Number Talk (Warm-up)

Materials to Copy

- Info Gap: Picking Fruit (groups of 2): Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	25 min
Activity 2	10 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

As you finish up this section, reflect on the norms and activities that have supported each student in learning math. List ways you have seen each student grow as a young mathematician throughout this work. List ways you have seen yourself grow as a teacher. What will you continue to do and what will you improve on in the next section?

Cool-down (to be completed at the end of the lesson)

5 min

Reflect

Standards Alignments

Addressing 5.MD.B.2

Student-facing Task Statement

In this section, you added and subtracted fractions and worked with data on line plots. What did you get better at during this section?

Student Responses

Sample response: I can add fractions that don’t have the same denominator.

Begin Lesson

Warm-up

10 min

Number Talk: Multiply by 18

Standards Alignments

Addressing 5.NF.B.4

The purpose of this Number Talk is to for students to demonstrate strategies and understandings they have for multiplying whole numbers by fractions. These understandings help students develop fluency and will be helpful later in this lesson when students will need to be able to solve problems involving multiplication of a whole number by a fraction.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $\frac{1}{3} \times 18$
- $\frac{2}{3} \times 18$
- $\frac{4}{3} \times 18$
- $\frac{5}{3} \times 18$

Student Responses

Sample responses:

- 6. $18 \div 3 = 6$
- 12. I doubled the product from the first problem.
- 24. I doubled the product again.
- 30. I found 5 times the value of the first product.

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "What patterns do you notice in the products?" (There is an 18 in one factor for all of them and it's always some number of thirds. Once I know the value of $\frac{1}{3} \times 18$, I can find the rest by multiplying by the number of thirds.)

Activity 1

25 min

Info Gap: Picking Fruit

Standards Alignments

Addressing 5.MD.B.2, 5.NF.A.2

This Info Gap activity gives students an opportunity solve problems about data represented on line plots. In both sets of cards, there is a partially complete line plot and some missing data.

For the first set of cards, the problem card has the missing data and the data card has a partially complete line plot. Monitor for students who:

- request all the information on the data card and create a complete line plot which they may use to answer the question
- only request the information they need to answer the question about the heaviest apricot and the most common weight

For the second set of cards, the problem card has the partially complete line plot and the data card has information to determine the missing data. Here students will likely need to communicate with each other as the information about the most common weight is vital to solve the problem, but the student with the problem card may not think to ask about this.

The Info Gap allows students to refine the language they use and ask increasingly more precise questions until they get the information they need (MP6).

Access for Students with Disabilities

Representation: Access for Perception. Provide appropriate reading accommodations and supports to ensure student access to written directions, word problems, and other text-based content. *Supports accessibility for: Conceptual Processing, Memory*

Instructional Routines

MLR4 Information Gap

Materials to Copy

Info Gap: Picking Fruit (groups of 2)

Required Preparation

- Create a set of cards from the Instructional master for each group of 2.

Student-facing Task Statement

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

Pause here so your teacher can review your work.

Launch

- Groups of 2

MLR4 Information Gap

- Recall, if necessary, the steps of the info gap routine.

Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Student Responses

1. Problem Card 1: First I found out that $2\frac{5}{8}$ ounces, a weight on my card, was the heaviest of the apricot weights. Then I found out that the most common weight was $1\frac{1}{4}$ ounces and my other apricot weight did not change this. The difference is $1\frac{3}{8}$ ounces since $\frac{1}{4} = \frac{2}{8}$.
2. Problem Card 2: First I counted the number of weights on the line plot and there were 12. So I needed 3 more. I asked about the lightest and heaviest apricots and that gave me one new weight for the heaviest. I asked if my partner had any more weights and they said they only know the most common weight. I asked for the most common weight and it was $1\frac{1}{2}$ ounces. That told me the other two apricots must have weighed $1\frac{1}{2}$ ounces and then I was able to complete the line plot.

- "I will give you either a problem card or a data card. Silently read your card. Do not read or show your card to your partner."
- Distribute the first set of cards.
- Remind students that after the person with the problem card asks for a piece of information, the person with the data card should respond with: "Why do you need to know (restate the information requested)?"

Activity

- 8–10 minutes: partner work time
- After students solve the first problem, distribute the next set of cards. Students switch roles and repeat the process with Problem Card 2 and Data Card 2.

Synthesis

- "What kinds of questions were the most useful to ask?"
    - (Card 1: I asked for the heaviest apricot and then realized it was one of the ones on my card. Then I asked for the most common weight and could solve the problem.
    - Card 2: I needed to find out what the rest of the apricot weights were. I tried asking for that but my partner did not have that information. My partner told me she had some information about the heaviest apricot and the most common weight and once I found that out I was able to solve the problem.)
- Invite students to share their strategy for solving one of the problems.
- Consider asking "Did anyone solve the problem in a different way?"

Activity 2 (optional)

10 min

Mathematical Questions

Standards Alignments

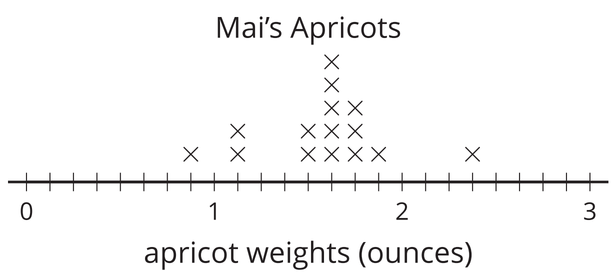
Addressing 5.MD.B.2, 5.NF.A.2

The purpose of this optional activity is for students to answer questions about a line plot using the same context as the previous activity. Students relate repeated addition of the same fraction to multiplication which they studied in a previous unit. They also address a question about the sum of all of the data. Because there is a lot of data, there are many viable strategies to answer this question and the synthesis focuses on sharing these strategies.

When students solve problems about the apricot weights using the line plot, they reason abstractly and quantitatively (MP2).

Student-facing Task Statement

This line plot shows the weights of some apricots that Mai picked.



1. What fraction of the apricots weigh less than $1\frac{1}{2}$ ounces? Explain or show your reasoning.
2. Write a multiplication equation that represents the total weight of the apricots that each weigh $1\frac{5}{8}$ ounces.
3. Do all of Mai's apricots together weigh more or less than a pound? Explain or show your reasoning.

Launch

- Groups of 2

Activity

- 1–2 minutes: quiet think time
- 5–6 minutes: partner work time
- When students decide whether or not the apricots altogether weigh more than 1 pound, monitor for these strategies:
    - adding up all the weights, in ounces, to see if they weigh more than 1 pound
    - estimating the sum but not calculating it exactly
    - calculating the total weight of the apricots that weigh $1\frac{5}{8}$ ounces and then using the structure of the graph

Student Responses

1. $\frac{3}{15}$ or $\frac{1}{5}$. Sample response: Because there are 15 measurements total and 3 of them are less than $1\frac{1}{2}$ ounces.
2. $5 \times 1\frac{5}{8} = 8\frac{1}{8}$
3. More. Sample response: There are 16 ounces in a pound, so the 5 apricots that each weigh $1\frac{5}{8}$ ounces weigh more than half a pound. Then there are 5 more apricots each heavier than the $1\frac{5}{8}$ ounce apricots, so the total is more than a pound.

Synthesis

- Invite previously selected students to share their equation for the total weight of the $1\frac{5}{8}$ ounce apricots.
- Display the equation: $5 \times 1\frac{5}{8} = 8\frac{1}{8}$.
- "Can you use this information to help decide whether or not the apricots weigh more than a pound?"
    - (Yes. There are 10 more apricots, and except for one, they all weigh more than an ounce so that will be more than 16 ounces for sure.
    - Yes. I added up all the weights and it was more than 16 ounces.
    - Yes. The 5 apricots that weigh $1\frac{5}{8}$ ounces together are more than $\frac{1}{2}$ pound. So the 5 heavier apricots are also more than $\frac{1}{2}$ pound so together they weigh more than 1 pound.)

Lesson Synthesis

10 min

"We have added, subtracted, and multiplied fractions to solve problems about line plots."

"In what ways did we use these operations to help us solve problems about line plots?" (Line plots have a lot of different data and the data had fractions so when we answered questions about the data we had to add, subtract, or multiply.)

"Which was your favorite problem about line plots?" (The eggs because I thought the picture was really interesting.)

Suggested Centers

- Creating Line Plots (2–5), Stage 4: Eighth Inches, Add, Subtract, and Multiply (Addressing)
- Rectangle Rumble (3–5), Stage 4: Whole Number and Fraction Factors (Supporting)

Student Section Summary

In this section we learned to add and subtract fractions. When the denominators are the same, such as $\frac{7}{10} + \frac{4}{10}$, we can just add the tenths and see that there are 11 of them so $\frac{7}{10} + \frac{4}{10} = \frac{11}{10}$. When the denominators are not the same, such as $\frac{1}{6} + \frac{3}{8}$, we look for a common denominator so that we can add parts of the same size. One way to find a common denominator is to use the product of the two denominators, 6×8, because that's always a multiple of both denominators. Using 48 as a denominator we find $\frac{1}{6} + \frac{3}{8} = \frac{1 \times 8}{6 \times 8} + \frac{3 \times 6}{8 \times 6}$. This means $\frac{1}{6} + \frac{3}{8} = \frac{26}{48}$. For the expression $\frac{1}{6} + \frac{3}{8}$ we can also use a smaller common denominator. Since 24 is a multiple of 6 and 8 we can also rewrite $\frac{1}{6} + \frac{3}{8}$ as $\frac{4}{24} + \frac{9}{24}$ which is $\frac{13}{24}$.

Complete Cool-Down

Response to Student Thinking

Students have something they want to share with a partner

Next Day Support

- After the warm-up in the next lesson, pair students up to discuss their responses.

Section C: The Size of Products

Lesson 16: Compare Products

Standards Alignments

Addressing	5.NF.B.5.a
Building Towards	5.NF.B.5.a

Teacher-facing Learning Goals

- Compare products in a way that makes sense to them.

Student-facing Learning Goals

- Let’s compare products.

Lesson Purpose

The purpose of this lesson is for students to compare the size of a product to the size of one factor using a strategy that makes sense to them.

In previous lessons students have found products of whole numbers, decimals, and fractions. The goal of this lesson is for students to examine the size of the product compared to the size of its factors. For example, students know that if they find a product of two whole numbers greater than 1, such as 5×7, the value of the product is greater than the value of either factor. They also know that the value of a product of fractions, such as $\frac{2}{3} \times \frac{5}{8}$, is less than the value of either factor. In this lesson, students study the situation where one of the factors is a fraction and the other is a whole number. They make the comparison using any strategy that makes sense to them. This might include calculating the value of the product, thinking about the meaning of fractions, or using a diagram.

Access for:

Students with Disabilities

- Action and Expression (Activity 2)

Instructional Routines

MLR2 Collect and Display (Activity 1), True or False (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

Identify ways the math community you are working to foster is going well. What aspects would you like to work on? What actions can you take to improve those areas?

Cool-down (to be completed at the end of the lesson)

5 min

Greater Than or Less Than

Standards Alignments

Addressing 5.NF.B.5.a

Student-facing Task Statement

1. Is $\frac{1}{8} \times 20$ greater than or less than 20? Explain or show your reasoning.
2. Is $\frac{10}{8} \times 20$ greater than or less than 20? Explain or show your reasoning.

Student Responses

1. Less than. Sample response: It takes eight $\frac{1}{8}$s to make a whole so $\frac{1}{8}$ of 20 is less than 20.
2. Greater than. Sample response: Since $\frac{10}{8}$ is more than 1 whole $\frac{10}{8} \times 20$ is more than 1 group of 20.

Begin Lesson

Warm-up

10 min

True or False: Compare Products

Standards Alignments

Building Towards 5.NF.B.5.a

The purpose of this True or False is for students to demonstrate strategies they have to estimate the size of a product. Students can find the value of $\frac{4}{5} \times 100$ and thereby solve all of the problems but the exact value is not needed to make the comparisons. Throughout the next several lessons, students will investigate different ways to compare a product like this to one of the factors (100 in this case).

Instructional Routines

True or False

Student-facing Task Statement

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $\frac{4}{5} \times 100 = 120$
- $\frac{4}{5} \times 100 < 100$
- $\frac{4}{5} \times 100 = 80$

Student Responses

Sample responses:

- False. $\frac{4}{5}$ of 100 is less than 100 so it can't be 120.
- True. $\frac{4}{5}$ is less than a whole so $\frac{4}{5}$ of 100 is less than 100.
- True. $\frac{1}{5}$ of 100 is 20 and $4 \times 20 = 80$.

Launch

- Display one statement.
- "Give me a signal when you know whether the equation or inequality is true and can explain how you know."
- 1 minute: quiet think time

Activity

- Share and record answers and strategy.
- Repeat with each equation.

Synthesis

- Display: $\frac{4}{5} \times 100 < 100$
- "How do we know this is true, without finding the value of $\frac{4}{5} \times 100$?" (Since $\frac{4}{5}$ is $\frac{1}{5}$ less than a whole, $\frac{4}{5} \times 100$ is less than 1×100. I know that $\frac{4}{5} \times 100$ is 80 and that's less than 100.)

Activity 1

15 min

Go the Distance

Standards Alignments

Addressing 5.NF.B.5.a

The purpose of this activity is for students to compare the size of different products where one factor stays the same, allowing students to focus on the size of the varying factor. Students should be encouraged to use whatever strategies and representations make sense to them. Monitor for students who

- use number lines
- use multiplication to compute the total distances
- reason about the relationship between the fraction of the trail and the total distance without performing any computations

This activity uses *MLR2 Collect and Display*. Advances: Reading, Speaking.

Instructional Routines

MLR2 Collect and Display

Student-facing Task Statement

Kiran, Noah, and Elena each ran as far as they could in one hour.

- Elena ran $\frac{3}{4}$ of a 5 mile trail.
- Noah ran $\frac{1}{2}$ of a 5 mile trail.
- Kiran ran $1\frac{1}{4}$ of a 5 mile trail.

1. List the distances the students ran in increasing order. Be prepared to explain your reasoning.
2. Fill in the blanks to make each statement true. Be prepared to explain your reasoning.
   a. Diego ran farther than Noah, but not as far as Kiran.

      Diego ran __________ of a 5 mile trail.
   b. Lin ran farther than Kiran, but

Launch

- Groups of 2

Activity

- 3–5 minutes: independent work time
- 3–5 minutes: partner discussion

MLR2 Collect and Display

- Circulate, listen for and collect the language students use to explain their reasoning for each of the problems.
- When students describe the order of the distances that Elena, Noah, and Kiran ran, listen and look for:
  - "_____ of the trail is longer than _____ of the trail."
  - "_____ of the trail is shorter than _____ of the trail."

not twice as far as Kiran.

Lin ran __________ of a 5 mile trail.

c. Tyler ran farther than Noah, but not as far as Elena.

Tyler ran __________ of a 5 mile trail.

Student Responses

1. Noah, Elena, Kiran. Sample response: Noah's distance is the least because he ran $\frac{1}{2}$ of a 5 mile trail and Elena ran $\frac{3}{4}$ of a 5 mile trail. Since $\frac{1}{2}$ is less than $\frac{3}{4}$, Elena ran further. Kiran's distance is the greatest because he ran more than the complete 5-mile trail.
2. Sample responses:
    - Diego ran $\frac{3}{4}$ of a mile trail.
    - Lin ran $1\frac{1}{2}$ of a 5 mile trail.
    - Tyler ran $\frac{5}{8}$ of a 5 mile trail.

    - gestures or diagrams that represent comparisons of the fractions
- When students describe the numbers that could go in the blanks, listen for:
    - "The number has to be between _____ and _____."
    - "The number has to be larger than ______ because . . . ."
    - "The number has to be smaller than _____ because . . . ."
    - gestures or diagrams that represent comparisons of the fractions
- Record students' words and phrases on a visual display and update it throughout the lesson.

Synthesis

- "Are there any other words or phrases that are important to include on our display?"
- As students share responses, update the display by adding (or replacing) language, diagrams, or annotations.
- Remind students to borrow language from the display as needed.
- Display: $1\frac{1}{4} \times 5$ miles
- "Whose distance does this expression represent?" (Kiran)
- "What multiplication expression represents the number of miles Noah ran?" ($\frac{1}{2} \times 5$)
- "How did you decide how to order the lengths that each student ran?" (The length of the trail is always 5 so we can just compare the fraction factors.)
- Display the problem about Tyler.
- "What numbers make sense? Why?" (I can use any fraction that is bigger than $\frac{1}{2}$ but less than $\frac{3}{4}$. It has to be bigger than $\frac{1}{2}$ so that Tyler runs farther than Noah. It has to be less than $\frac{3}{4}$ so that Elena runs farther

than Tyler.)

Advancing Student Thinking

If students do not start to solve the problems during the independent work time, draw a diagram to represent the trail and ask students to label the distance that each student ran.

Activity 2

20 min

Compare Expressions

Standards Alignments

Building Towards 5.NF.B.5.a

The purpose of this activity is for students to compare a fractional amount of a whole number with that same whole number. Students may calculate, draw a diagram, or reason about the size of the factor. When students choose their own numerator or denominator to make equations and inequalities true, monitor for students who:

- experiment with different numerators and multiply the fraction by the whole number to see if it makes the statement true
- choose a numerator of 1 and use their understanding of unit fractions as part of a whole
- explain why more than one answer makes sense

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Invite students to verbalize their strategy for comparing fractional amounts before they begin. Students can speak quietly to themselves, or share with a partner.
Supports accessibility for: Organization, Conceptual Processing, Language

Student-facing Task Statement

1. Write $<$ or $>$ in each blank to make the statement true. Explain or show your reasoning.

Launch

- Groups of 2

a. $\frac{5}{4} \times 100$ ____ 100

b. $\frac{5}{7} \times 2$ ____ 2

c. $\frac{1}{3} \times 50$ ____ 100

2. Write a number in each box to make the statement true. Explain or show your reasoning.

   a. $\frac{\square}{9} \times 50 < 50$

   b. $\frac{\square}{9} \times 50 = 50$

   c. $\frac{\square}{9} \times 50 > 50$

3. Write a number in each box to make the statement true. Explain or show your reasoning.

   a. $\frac{9}{\square} \times 50 < 50$

   b. $\frac{9}{\square} \times 50 = 50$

   c. $\frac{9}{\square} \times 50 > 50$

Student Responses

Sample responses:

1. a. $\frac{5}{4} \times 100 > 100$ because $\frac{5}{4}$ is greater than 1.

   b. $\frac{5}{7} \times 2 < 2$ because $\frac{5}{7}$ is less than 1.

   c. $\frac{1}{3} \times 50 < 100$ because $\frac{1}{3} \times 50$ is less than 50 and 50 is less than 100.

2. a. $\frac{1}{9} \times 50 < 50$ because it's just one part

Activity

- 6–8 minutes: independent work time
- "Check in with your partner and compare solutions. Revise your thinking, if necessary."
- 3–5 minutes: partner work time

Synthesis

- Display the inequality: $\frac{\square}{9} \times 50 < 50$
- Invite students to share their responses.
- "How do you know $\frac{1}{50} \times 50 < 50$?" (Because $\frac{1}{50} \times 50 = 1$.)
- "How do you know $\frac{10}{50} \times 50 < 50$?" (Because it is 10 of 50 parts, there are 40 other parts.)
- Display inequality: $\frac{9}{\square} \times 50 < 50$
- Invite students to share their responses.
- "How do you know $\frac{9}{90} \times 50 < 50$?" (Because $\frac{9}{90}$ is less than 1. $\frac{9}{90} \times 50$ is equal to 5.)
- Display the equation: $\frac{\square}{9} \times 50 = 50$
- "How did you find the number that makes this equation true?" (It has to be equal to 1 and $\frac{9}{9} = 1$.)

out of 9.

b. $\frac{9}{9} \times 50 = 50$ because $\frac{9}{9}$ is 1.

c. $\frac{18}{9} \times 50 > 50$ because $\frac{18}{9} = 2$.

3. a. $\frac{9}{10} \times 50 < 50$ because $\frac{9}{10} \times 50 = 45$.

b. $\frac{9}{9} \times 50 = 50$ because $\frac{9}{9}$ is 1.

c. $\frac{9}{1} \times 50 > 50$ because it's several 50s.

Advancing Student Thinking

If students multiply to determine whether an expression is greater or less than a given number, draw a number line diagram with the whole number labeled and ask them to explain the approximate location of the expression.

Lesson Synthesis

10 min

"Today we compared the value of a product to the value of one of the factors."

"What patterns did you notice?" (I noticed that if I multiply a number by a fraction less than 1, the product gets smaller. If the fraction is greater than 1 then the product gets bigger.)

"Do you think these patterns will always be true?" (Yes, if I multiply a number by $\frac{1}{2}$ it will be smaller. It will just be a half. If I multiply a number by 2 it will be bigger, it will be double.)

Suggested Centers

- Creating Line Plots (2–5), Stage 4: Eighth Inches, Add, Subtract, and Multiply (Addressing)
- Rectangle Rumble (3–5), Stage 4: Whole Number and Fraction Factors (Supporting)

Complete Cool-Down

Response to Student Thinking

Students do not reason about the size of the product based on the size of factors in relation to 1 to determine whether the expression is greater than or less than 20.

Next Day Support

- Throughout the lesson, ask, “What diagram would be helpful in making sense of this problem?”

Lesson 17: Interpret Diagrams

Standards Alignments

Addressing 5.NF.B.5.a, 5.NF.B.5.b

Teacher-facing Learning Goals

- Compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

Student-facing Learning Goals

- Let's compare products without multiplying.

Lesson Purpose

The purpose of this lesson is for students to compare the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

In the previous lesson, students compared products in a way that made sense to them, including finding the value of the product. In this lesson, students focus on other ways to compare a product to one of the factors. First, they match products with diagrams. This helps them to think about the meaning of the product as well as giving them a visual representation which they can use to help see the comparison. In the second activity, students use this general understanding to compare products where one of the factors is not known, requiring them to make the comparisons based on the size of the other factor. It is important that students connect their products to the corresponding situations and representations (MP2).

Access for:

Students with Disabilities

- Representation (Activity 2)

English Learners

- MLR6 (Activity 2)

Instructional Routines

Estimation Exploration (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min

Teacher Reflection Question

What question do you wish you had asked today? When and why should you have asked it?

Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Cool-down (to be completed at the end of the lesson)

5 min

Read Books

Standards Alignments

Addressing 5.NF.B.5.a

Student-facing Task Statement

Diego, Kiran, Elena, and Mai were reading a book.

- Diego read 40 pages.
- Elena read $\frac{7}{8}$ times as many pages as Diego.
- Mai read $2\frac{1}{2}$ times as many pages as Diego.
- Kiran read $\frac{4}{5}$ times as many pages as Diego.

Write the 4 names in order of how many pages they read from least to greatest.

Student Responses

Kiran, Elena, Diego, Mai

Begin Lesson

Warm-up

10 min

Estimation Exploration: Fraction of a Whole Number

Standards Alignments

Addressing 5.NF.B.5.a

The purpose of this Estimation Exploration is to estimate the product of a fraction and a large whole number. Students know how to find the exact answer but it would require many calculations. Making an estimate will help develop the intuition that because $\frac{5}{3}$ is greater than 1, the product has to be greater than the other factor. Students can make a better estimate by replacing the whole number 9,625 with a friendlier number that they can find $\frac{1}{3}$ of mentally. Throughout this lesson, students will continue to compare the size of products to the size of one of the factors.

Instructional Routines

Estimation Exploration

Student-facing Task Statement

$$\frac{5}{3} \times 9{,}625$$

Record an estimate that is:

too low	about right	too high

Student Responses

Sample responses

- too low: 3,000 to 12,000
- about right: 12,000 to 16,000
- too high: 18,000

Launch

- Groups of 2
- Display the expression.
- "What is an estimate that's too high? Too low? About right?"
- 1 minute: quiet think time

Activity

- 1 minute: partner discussion
- Record responses.

Synthesis

- "How do we know the product is going to be greater than 9,625?" ($\frac{5}{3}$ is more than 1 so the product is greater than the other factor, 9,625.)
- "How do we know the product is going to be greater than 15,000?" ($\frac{1}{3} \times 9{,}000 = 3{,}000$ so $\frac{5}{3} \times 9{,}000 = 15{,}000$ and we are trying to figure out what $\frac{5}{3}$ of more than 9,000 is.)

Activity 1

20 min

Match the Diagram

Standards Alignments

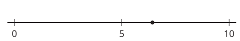
Addressing 5.NF.B.5.b

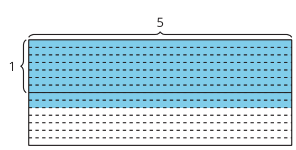
The goal of this activity is for students to match expressions and diagrams and then compare the value of each expression with one of the factors. To match the expressions with the diagrams students will likely use the meaning of multiplication. For example, $\frac{2}{7} \times 3$ means 2 of 7 equal parts of 3 wholes. The area diagram shows the 7 parts with 2 shaded whereas the number line only shows the relative locations of $\frac{2}{7} \times 3$ and 3, requiring students to understand the relationship between $\frac{2}{7} \times 3$ and 3 in order to pick the right match. Once they have made the matches, the diagrams help to visualize that $\frac{2}{7} \times 3$ is less than 3 and the activity synthesis highlights this. When students match diagrams and expressions they look for and identify structure in the number line and area diagrams (MP7).

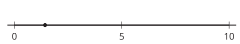
Student-facing Task Statement

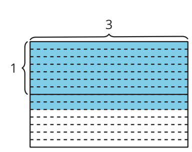
1. Match the expressions and diagrams.

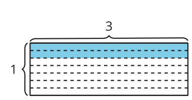
$\frac{2}{7} \times 3$ $\frac{9}{7} \times 3$ $\frac{2}{7} \times 5$ $\frac{9}{7} \times 5$









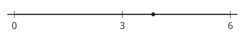


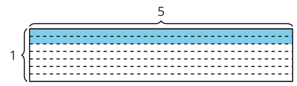
Launch

- Groups of 2

Activity

- 1–2 minutes: quiet think time
- 8–10 minutes: partner work time
- Monitor for students who compare the numbers by:
    - finding the value of the expressions
    - using the number lines which show how the value of each expression compares to 3 or 5
    - using the area diagrams which also show how the value of each expression compares to 3 or 5





2. Write < or > in each blank to make the inequality true.

   a. $\frac{2}{7} \times 3$ ____ 3

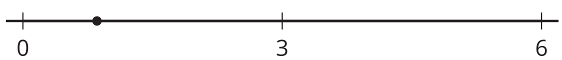
   b. $\frac{9}{7} \times 3$ ____ 3

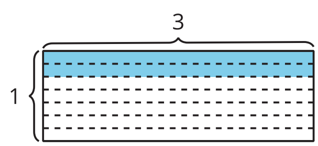
   c. $\frac{2}{7} \times 5$ ____ 5

   d. $\frac{9}{7} \times 5$ ____ 5

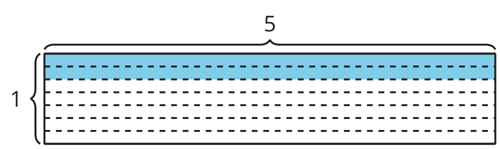
Student Responses

1. $\frac{2}{7} \times 3$ matches:

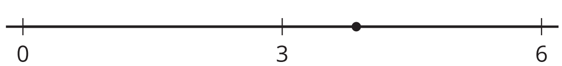




$\frac{2}{7} \times 5$ matches:

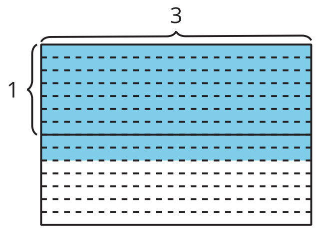


$\frac{9}{7} \times 3$ matches:

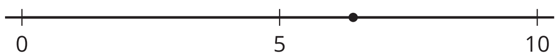


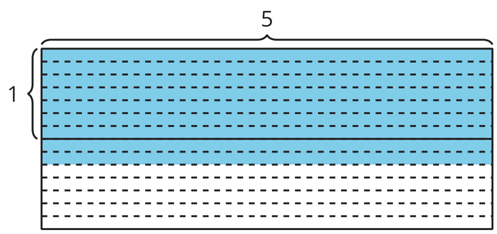
Synthesis

- Display the expression: $\frac{2}{7} \times 3$
- "How did you decide which diagrams match the expression?" (For the area diagram I took the rectangle with length 3 and width less than 1. For the number line, I picked the one with 3 and a point that was less than 3.)
- Invite students to share how they compared $\frac{2}{7} \times 3$ with 3, highlighting these strategies:
  - reasoning about the size of $\frac{2}{7}$
  - using the number line
  - using the area diagram



$\frac{9}{7} \times 5$ matches:





2. a. <
   b. >
   c. <
   d. >

Activity 2

15 min

Who Ran Farther?

Standards Alignments

Addressing 5.NF.B.5.b

The purpose of this activity is for students to compare a product to an unknown factor based on the size of the other factor. In this case, students cannot calculate the values of the products to compare but instead rely on their understanding of fractions and the meaning of multiplication.

Students also use a number line to help them visualize the different distances after listing them in order. For this part of the activity the expectation is that they will use what they already know about the order of the distances to determine which point corresponds to each student. They might, however, also reason about the quantities. For example twice Priya's distance can be found by marking off Priya's position on the number line a second time (MP2).

Access for English Learners

Reading: MLR6 Three Reads. Keep books or devices closed. Display only the problem stem, without revealing the questions. "We are going to read this question 3 times." After the 1st Read: "Tell your partner what this situation is about." After the 2nd Read: "List the quantities. What can be counted or measured?" Reveal the question(s). After the 3rd Read: "What strategies can we use to solve this problem?"
Advances: Reading, Representing

Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Use multiple examples and non-examples to emphasize how a factor that is greater than one, equal to one, and less than 1 impacts a product.
Supports accessibility for: Memory, Conceptual Processing, Attention

Student-facing Task Statement

- Priya ran to her grandmother's house.
- Jada ran twice as far as Priya.
- Han ran $\frac{6}{7}$ as far as Priya.
- Clare ran $\frac{14}{8}$ as far as Priya.
- Mai ran $\frac{3}{5}$ times as far as Priya.

1. Which students ran farther than Priya?____________________
2. Which students did not run as far as Priya?____________________
3. List the runners in order from shortest distance run to longest. Explain or show your reasoning.
4. The point P represents how far Priya ran. Write the initial of each student in the blank that shows how far they ran. One of the students will be missing.

Launch

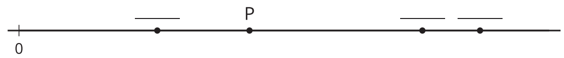
- Groups of 2
- 1–2 minutes: quiet think time

Activity

- 6–8 minutes: partner work time

Synthesis

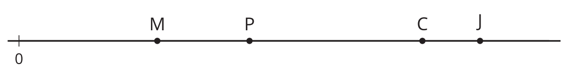
- Display:
  $\frac{3}{5} \times 2$
  $\frac{6}{7} \times 2$
  $\frac{14}{8} \times 2$
  2×2
- "What do you notice about these expressions?" (They represent the amount that each person ran, if Priya's distance is 2. They are all a number multiplied by 2. They

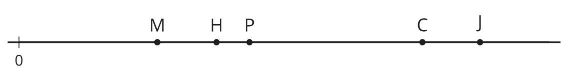


5. Label the distance for the missing student on the number line above.

Student Responses

1. Jada and Clare ran farther than Priya.
2. Han and Mai did not run as far as Priya.
3. Mai, Han, Priya, Clare, Jada. I know Mai ran the shortest distance because $\frac{3}{5}$ is smaller than the other multiples of Priya's distance. The next smallest is Han at $\frac{6}{7}$. Priya comes next and then the two students who ran farther than Priya, Clare and Jada. Jada ran the farthest because twice as far is more than $\frac{14}{8}$ as far.
4. I knew to label the points to the right of P Clare and Jada since they ran farther than Priya. I decided the other point shows Mai's distance because she ran $\frac{3}{5}$ as far as Priya which is a little more than half. Han's distance is missing since $\frac{6}{7}$ is close to 1 and less than 1.



5. 

are listed in increasing order.)

- "What if Priya ran 4 miles? What multiplication expressions can we write to represent how many miles each of the other students ran?" (It would be just like the expressions above except that the 2 would be replaced with a 4.)
- Record expressions for all to see:
  $\frac{3}{5} \times 4$
  $\frac{5}{7} \times 4$
  $\frac{14}{8} \times 4$
  2×4
- "Does the order of the distances change when Priya's distance changes? Why or why not?" (No, the order of the products is the same as the order of the other factor, the multiple of Priya's distance.)

Lesson Synthesis

10 min

"Today we compared products without calculating their values."

Display: Han ran $\frac{6}{7}$ as far as Priya.

"How do you know Priya ran farther than Han?" ($\frac{6}{7}$ of Priya's distance is just a fraction of her distance. It's $\frac{1}{7}$ short of the full distance Priya ran. So Priya ran farther.)

Display image showing all student distances in activity 2 or a student generated solution.

"How can you tell who ran farther than Priya?" (Clare and Jada are to the right of Priya on the number line so they ran farther.)

"In the next lesson we are going to continue to use the number line to locate and compare the values of multiplication expressions with fractions."

Suggested Centers

- Rectangle Rumble (3–5), Stage 5: Fraction Factors (Addressing)
- Rolling for Fractions (3–5), Stage 4: Multiply Fractions (Supporting)

Complete Cool-Down

Response to Student Thinking

Students multiply to determine the order of the numbers of pages.

Next Day Support

- Launch warm-up or activities by highlighting important representations from previous lessons.

Lesson 18: Compare Without Multiplying

Standards Alignments

Addressing 5.NF.B.5.a

Teacher-facing Learning Goals

- Recognize that the product of a fraction and a whole number is less than, equal to, or greater than the whole number when the fraction is correspondingly less than, equal to, or greater than 1.

Student-facing Learning Goals

- Let's compare expressions, without evaluating them.

Lesson Purpose

The purpose of this lesson is for students to compare the size of a product to the size of one factor on the basis of the size of the other factor.

In this lesson, students continue to compare the size of a product to the size of one of the factors based on the size of the other factor. They also continue to represent these comparisons on the number line. In this lesson there is no longer a context and students interpret the number line diagram in situations where both factors are fractions. Just as they observed in previous lessons, the size of a number becomes greater when multiplied by a fraction greater than 1 and smaller when multiplied by a fraction less than 1. In the next lesson, students will work toward giving a general explanation for these patterns.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR7 (Activity 2)

Instructional Routines

Notice and Wonder (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	15 min

Teacher Reflection Question

It is important that students convince themselves that mathematics makes sense. Today, students were noticing patterns and

Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

determining whether or not the patterns were generalizable. In what ways did each of your students convince themselves that mathematics makes sense?

Cool-down (to be completed at the end of the lesson)

5 min

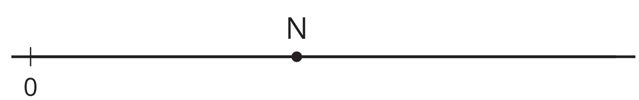
Comparison Statements

Standards Alignments

Addressing 5.NF.B.5.a

Student-facing Task Statement

1. The number N is shown on the number line.

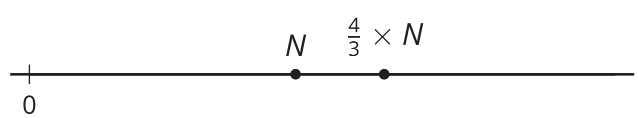


   a. Locate and label $\frac{4}{3} \times \mathrm{N}$ on the number line.

   b. Is $\frac{4}{3} \times \mathrm{N}$ less than, equal to, or greater than N? Explain how you know.

Student Responses

Sample responses:

1.
   a. 

   b. $\frac{4}{3} \times \mathrm{N} > \mathrm{N}$ because it is to the right on the number line. It is N and then an extra $\frac{1}{3}$ of N.

Begin Lesson

Warm-up

10 min

Notice and Wonder: Expressions and Number Lines

Standards Alignments

Addressing 5.NF.B.5.a

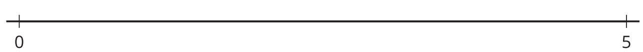
The purpose of this warm-up is for students to interpret a multiplication expression as a location on the number line. This builds on work students did in the previous lesson with an emphasis now on precisely locating the expression using the meaning of multiplication. Students will build on this idea and locate the value of more complex expressions throughout the lesson.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?



$$\frac{2}{3} \times 5$$

Student Responses

Students may notice:

- There is a number line.
- Zero and five are the only numbers on the number line.
- There is a 5 on the number line and in the expression, but there is no $\frac{2}{3}$ on the number line.

Students may wonder:

- Why are there no other numbers on the number line?
- What is $\frac{2}{3} \times 5$?
- Can we put $\frac{2}{3} \times 5$ on the number line?

Launch

- Groups of 2
- Display the image.
- “What do you notice? What do you wonder?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Share and record responses

Synthesis

- “How can you find the approximate location of $\frac{2}{3} \times 5$ on the number line?” (I can find the value and add more tick marks to find it exactly. I can divide the number line between 0 and 5 into 3 equal parts and $\frac{2}{3} \times 5$ will be the second of those tick marks.)

Activity 1

15 min

Approximate Location

Standards Alignments

Addressing 5.NF.B.5.a

The purpose of this activity is for students to understand, using complex numbers and no context, the relationship between the size of a product and the size of one of the factors. They begin by using a number line to locate such products and then choose the numerator or denominator of a fraction in order to make a product smaller, the same, or greater. To choose the number correctly students need to understand both:

- the relationship between the numerator and denominator of a fraction and the size of the fraction
- the relationship between the size of a factor and the size of the product

When students locate the expressions on the number line they use their understanding of multiplication, fractions, and the structure of the number line (MP7).

Access for Students with Disabilities

Representation: Access for Perception. Synthesis: Use gestures during the discussion to emphasize if the product was greater than, less than, or equal to twelve.
Supports accessibility for: Conceptual Processing, Fine Motor Skills

Student-facing Task Statement

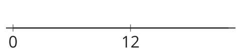
1. Label each expression at its approximate location on the number line.

Partner A

a. $\frac{2}{5} \times 12$

b. $\frac{5}{3} \times 12$

c. $\frac{7}{7} \times 12$

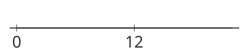


Partner B

a. $\frac{4}{7} \times 12$

b. $\frac{8}{5} \times 12$

c. $\frac{9}{9} \times 12$



Launch

- Groups of 2

Activity

- 3–5 minutes: independent work time
- 3–5 minutes: partner discussion
- Monitor for students who notice patterns as they solve the last problem. For example, they notice that to make $\frac{\boxed{}}{11} \times 12 > 12$ true, the numerator in the

2. Choose a number to put in each box to make the statement true.

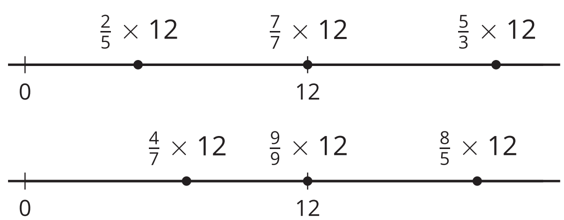
   a. $\frac{\square}{11} \times 12 > 12$

   b. $\frac{\square}{15} \times 12 = 12$

   c. $\frac{13}{\square} \times 12 < 12$

Student Responses

1.



2. a. any number greater than 11
   b. 15
   c. any number greater than 13

fraction must be larger than the denominator.

Synthesis

- Display the equation: $\frac{\square}{15} \times 12 = 12$
- “What solution(s) did you find for this statement?” (Just 15.)
- “Why is there only one solution?” (Because the only multiple of 12 that’s 12 is 1×12.)
- Display the inequality: $\frac{13}{\square} \times 12 < 12$
- “What solutions did you find for this statement?” (14, 15, 16, and so on.)
- “What do the solutions all have in common?” (They are all more than 13.)
- “Why?” (Because the product will only be less than 12 if the fraction is less than 1. That means the numerator has to be smaller than the denominator.)
- “How do the number lines help us understand the comparison?” (They show the relationship between the size of the fraction and the value of the product.)

Advancing Student Thinking

If students don’t use the number line to consider the relationship between the factors, ask them to consider where $\frac{1}{2}$ of 12 would be located. What about $\frac{1}{4}$ of 12? Ask them to explain how they can identify the location of the value of these expressions without multiplying.

Activity 2

20 min

An Unknown Number

Standards Alignments

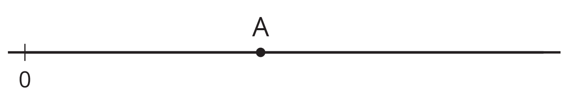
Addressing 5.NF.B.5.a

In the previous activity students located numerical expressions on the number line, noticing that $\frac{2}{5} \times 12$, for example, is less than 12 because it is only 2 out of 5 equal parts making 12. The goal of this activity is for students to extend this reasoning to all numbers, including 12 but also including fractions which is new. Students continue to use a number line to support their reasoning and the reasoning is identical to what students did in the previous lesson comparing different distances students ran to Priya's (unknown) distance. If P is how far Priya ran in miles then $\frac{1}{2} \times \mathrm{P}$ is halfway between 0 and P on the number line whether P is a whole number or a fraction.

Access for English Learners

MLR7 Compare and Connect. Synthesis: After all strategies have been presented, lead a discussion comparing, contrasting, and connecting the different approaches. Ask, “What did the approaches have in common?”, “How were they different?”, and “Did anyone solve the problem the same way, but would explain it differently?”
Advances: Representing, Conversing

Student-facing Task Statement



1. The number A is shown on the number line. Label the approximate location of the value of each expression. Explain or show your reasoning.
    - $\frac{1}{4} \times A$
    - $2 \times A$
    - $\frac{13}{8} \times A$
    - $\frac{2}{3} \times A$
2. Is $\frac{13}{8} \times \frac{11}{39}$ less than, greater than, or equal to $\frac{11}{39}$? Explain or show your reasoning.
3. Is $\frac{2}{3} \times \frac{17}{53}$ less than, greater than, or equal to $\frac{17}{53}$? Explain or show your reasoning.

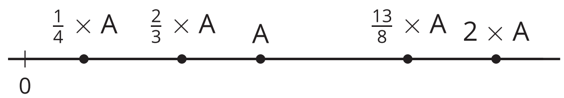
Launch

- Groups of 2

Activity

- 6–8 minutes: independent work time
- 3–5 minutes: partner discussion
- Monitor for students who:
    - label the number line with tick marks to show the location of the value of each expression in relation to A
    - refer to $\frac{8}{8}$ to explain why $\frac{13}{8} \times \frac{11}{39}$ is greater than $\frac{11}{39}$
    - draw a number line or use the given number line to show the relationship between $\frac{17}{53}$ and $\frac{2}{3} \times \frac{17}{53}$

Student Responses



1. To find $\frac{1}{4} \times \text{A}$ I took half of half the distance to A. For $\frac{2}{3} \times \text{A}$. I divided the distance from 0 to A into thirds and took 2 of them. For $2 \times \text{A}$, I just doubled the distance from A to 0. The hardest was $\frac{13}{8} \times \text{A}$. I know that is A and $\frac{5}{8}$ more A so I put it a little more than half of the way between A and $2 \times \text{A}$.
2. It's greater. I can use the number line. If A is $\frac{11}{39}$ then it shows $\frac{13}{8} \times \frac{11}{39}$ is to the right on the number line and so it is greater than A.
3. It's less. I can use the number line. If A is $\frac{17}{53}$ then it shows $\frac{2}{3} \times \frac{17}{53}$ is to the left on the number line and so it is less than A.

Synthesis

- Invite students to share how they found the location of $\frac{2}{3} \times \text{A}$ on the number line.
- "Why is $\frac{2}{3} \times \text{A}$ to the left of A?" (It's less than A since it's missing $\frac{1}{3}$ of A. So it's to the left.)
- "How do you know $\frac{13}{8} \times \text{A}$ is to the right of A on the number line?" (Because $\frac{13}{8}$ is more than 1. It's an extra $\frac{5}{8}$ of A.)
- Invite students to share how they compared $\frac{13}{8} \times \frac{11}{39}$ with $\frac{11}{39}$.
    - I know $\frac{13}{8}$ is more than 1 so that means the product is bigger.
    - I can use the number line and imagine that A is $\frac{11}{39}$.

Lesson Synthesis

10 min

"Today we compared products to the size of one factor when both factors were fractions."

Display: $\underline{\qquad} \times 6 < 6$

"What numbers make this statement true?" ($\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{8}$, any number that's less than 1)

Display: $\underline{\qquad} \times 6 > 6$

"What numbers make this statement true?" (2, 5, 10, $\frac{8}{5}$, any number that's greater than 1)

Record the numbers so students see them.

"What if I replace 6 with $\frac{3}{8}$? Do your numbers still make the statements true?" (Yes, half of $\frac{3}{8}$ is still less than $\frac{3}{8}$ and twice $\frac{3}{8}$ is still more than $\frac{3}{8}$.)

Record student explanations and keep a copy to refer to during future lessons.

Suggested Centers

- Rectangle Rumble (3–5), Stage 5: Fraction Factors (Addressing)
- Rolling for Fractions (3–5), Stage 4: Multiply Fractions (Supporting)

Complete Cool-Down

Response to Student Thinking

Students do not write fractions that make true statements.

Next Day Support

- Throughout the lesson, ask, “What diagram would be helpful in making sense of this problem?”

Lesson 19: Compare to 1

Standards Alignments

Addressing	5.NF.B.5.b
Building Towards	5.NF.B.5.b

Teacher-facing Learning Goals

- Explain what happens to a given fraction when multiplied by a fraction greater than or less than 1.

Student-facing Learning Goals

- Let's explain what happens when we multiply a fraction by a fraction greater than, less than, or equal to 1.

In previous lessons, students have compared the size of a product to the size of one factor by reasoning about the size of the other factor. They have done this using calculation, area diagrams, and number line diagrams. The goal of this lesson is to use the distributive property to explain why the comparisons work in all cases without calculating. The key observation is that a number greater than 1, such as $\frac{5}{4}$, can be written as $1 + \frac{1}{4}$ so multiplying by $\frac{5}{4}$ increases any number by $\frac{1}{4}$ of that number. In the same way multiplying by $\frac{3}{4}$ or $1 - \frac{1}{4}$ decreases any number by $\frac{1}{4}$ of that number.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Engagement (Activity 2)

English Learners

- MLR8 (Activity 1)

Instructional Routines

What Do You Know About _____? (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

During the last two lessons, students have noticed and explained patterns and generalizations about multiplying by numbers greater than, less than, and equal to 1. What are some ways that you honored student language while strategically incorporating more precise academic language?

Cool-down (to be completed at the end of the lesson)

5 min

Compare without Calculating

Standards Alignments

Addressing 5.NF.B.5.b

Student-facing Task Statement

1. Is $\left(1 - \frac{16}{33}\right) \times \frac{11}{14}$ greater than, equal to, or less than $\frac{11}{14}$? Explain or show your reasoning.
2. Is $\frac{49}{33} \times \frac{11}{14}$ greater than, equal to, or less than $\frac{11}{14}$? Explain or show your reasoning.

Student Responses

1. Less than $\frac{11}{14}$, because it's $\frac{11}{14}$ minus some amount.
2. Greater than $\frac{11}{14}$, because it's $\frac{11}{14}$ plus some amount as I can see by rewriting $\frac{49}{33}$ as $1 + \frac{16}{33}$.

Begin Lesson

Warm-up

10 min

What Do You Know About $\frac{15}{14} \times \frac{23}{30}$?

Standards Alignments

Building Towards 5.NF.B.5.b

The purpose of this What Do You Know About _____ is for students to share what they know about and how they can represent the product $\frac{15}{14} \times \frac{23}{30}$. The numbers were intentionally chosen to make finding the exact value of the product challenging.

Instructional Routines

What Do You Know About _____?

Student-facing Task Statement

What do you know about $\frac{15}{14} \times \frac{23}{30}$?

Student Responses

Sample responses:

- It would be hard to multiply those fractions.
- $\frac{15}{14}$ is a little bit bigger than 1.
- It represents a number that is a little bit bigger than $\frac{23}{30}$.

Launch

- Display the expression.
- "What do you know about $\frac{15}{14} \times \frac{23}{30}$?"
- 1 minute: quiet think time

Activity

- Record responses.
- "How could we find the value of the product $\frac{15}{14} \times \frac{23}{30}$?" (Find the product of the numerators and the product of the denominators.)

Synthesis

- "Is $\frac{15}{14} \times \frac{23}{30}$ less than, equal to, or greater than $\frac{23}{30}$? Why?" (It is greater since $\frac{15}{14}$ is greater than 1.)

Activity 1

15 min

Compare Fraction Products on the Number Line

PLC Activity

Standards Alignments

Addressing 5.NF.B.5.b

The goal of this activity is to continue to compare the size of a product of fractions to the size of the second factor. In addition to the number line representation which students have worked with in the last few lessons, they also see a different expression that represents the product. In the next activity, this expression will be combined with the distributive property to explain in all cases why multiplying a number by a fraction less than one results in a smaller number while multiplying by a fraction greater than one results in a larger number (MP8).

Access for English Learners

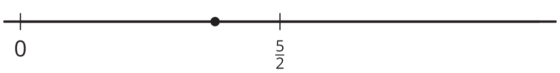
MLR8 Discussion Supports. Students should take turns finding a match and explaining their reasoning to their partner. Display the following sentence frames for all to see: “I noticed _____, so I matched . . . .” Encourage students to challenge each other when they disagree.
Advances: Representing, Conversing

Student-facing Task Statement

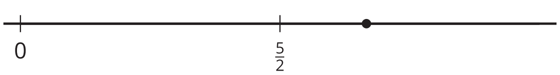
1. Match the expressions and number lines that show the same value.

   - $\frac{2}{5} \times \frac{4}{3}$
   - $\frac{3}{4} \times \frac{5}{2}$
   - $\frac{4}{3} \times \frac{5}{2}$
   - $\left(1 + \frac{1}{3}\right) \times \frac{5}{2}$
   - $\left(1 - \frac{3}{5}\right) \times \frac{4}{3}$
   - $\left(1 - \frac{1}{4}\right) \times \frac{5}{2}$

   A

   B

   C

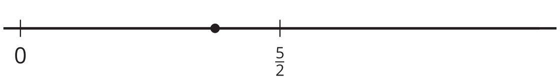
   

2. Choose one of the expressions from each set and explain whether the value is greater than or less than the second factor.

Student Responses

1. $\frac{3}{4} \times \frac{5}{2}$, $\left(1 - \frac{1}{4}\right) \times \frac{5}{2}$, and

   A

   $\frac{4}{3} \times \frac{5}{2}$, $\left(1 + \frac{1}{3}\right) \times \frac{5}{2}$, and

Launch

- Groups of 2

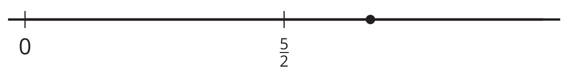
Activity

- 1–2 minutes: quiet think time
- 6–8 minutes: partner work time

Synthesis

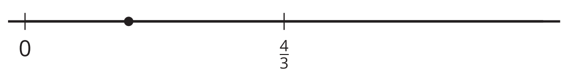
- Invite students to share their matches.
- “How did you find the matching number line for $\frac{3}{4} \times \frac{5}{2}$?” (I saw that two of the number lines have $\frac{5}{2}$ on them and looked for the one that showed $\frac{3}{4}$ of $\frac{5}{2}$. I knew which one it was because $\frac{3}{4}$ of $\frac{5}{2}$ is less than $\frac{5}{2}$.)
- “How did you find the matching expression for $\frac{3}{4} \times \frac{5}{2}$?” (I looked for an expression with $\frac{5}{2}$ and only one of them had another factor with the value $\frac{3}{4}$.)
- “How did you know whether the value of $\frac{3}{4} \times \frac{5}{2}$ was greater than or less than $\frac{5}{2}$?” (I knew it was less because $\frac{3}{4}$ is less than 1. That was what helped me find the right number line.)

B



$\frac{2}{5} \times \frac{4}{3}$, $\left(1 - \frac{3}{5}\right) \times \frac{4}{3}$, and

C



2. Sample responses:
    - $\frac{3}{4} \times \frac{5}{2} < \frac{5}{2}$ because it is to the left of $\frac{5}{2}$ on the number line.
    - $\frac{4}{3} \times \frac{5}{2} > \frac{5}{2}$ because it is to the right of $\frac{5}{2}$ on the number line.
    - $\frac{2}{5} \times \frac{4}{3} < \frac{4}{3}$ because it is to the left of $\frac{4}{3}$ on the number line.

Activity 2

20 min

True Statement

Standards Alignments

Addressing 5.NF.B.5.b

The goal of this activity is to use the distributive property to explain why multiplying a number by a fraction greater than one increases the size of the number while multiplying by a fraction less than one decreases the size of the number. Expressions are particularly useful here because they show explicitly how the size of the number relates to the product. For example writing $\frac{3}{5}$ as $1 - \frac{2}{5}$ and then multiplying by $\frac{4}{7}$ gives:

$$\left(1 - \frac{2}{5}\right) \times \frac{4}{7} = \frac{4}{7} - \left(\frac{2}{5} \times \frac{4}{7}\right)$$

The revealing part of this calculation is that the structure of the right hand side shows that it is less than $\frac{4}{7}$ without calculating the exact value (MP7). It must be less than $\frac{4}{7}$ because it is $\frac{4}{7}$ minus some other number.

Access for Students with Disabilities

Engagement: Internalize Self-Regulation. Provide students an opportunity to self-assess and reflect on their own progress. For example, provide students with questions that relate to the size of the factors for them to reflect on once the activity is complete.
Supports accessibility for: Conceptual Processing, Attention, Memory

Student-facing Task Statement

1. Rewrite each expression as a sum or difference of 2 products.
   a. $\left(1 - \frac{2}{5}\right) \times \frac{4}{7}$
   b. $\left(1 + \frac{1}{5}\right) \times \frac{4}{7}$
   c. $\left(1 - \frac{3}{8}\right) \times \frac{4}{7}$
   d. $\left(1 + \frac{1}{8}\right) \times \frac{4}{7}$
2. Fill in each blank with $<$ or $>$ to make the inequality true.
   a. $\left(1 - \frac{2}{5}\right) \times \frac{4}{7}$ _____ $\frac{4}{7}$
   b. $\left(1 + \frac{1}{5}\right) \times \frac{4}{7}$ _____ $\frac{4}{7}$
   c. $\left(1 - \frac{3}{8}\right) \times \frac{4}{7}$ _____ $\frac{4}{7}$
   d. $\left(1 + \frac{1}{8}\right) \times \frac{4}{7}$ _____ $\frac{4}{7}$
3. Describe the value of the product when $\frac{4}{7}$ is multiplied by a fraction greater than 1. Explain your reasoning.
4. Describe the value of the product when $\frac{4}{7}$ is multiplied by a fraction less than 1. Explain your reasoning.

Student Responses

1. a. $\left(1 - \frac{2}{5}\right) \times \frac{4}{7} = \frac{4}{7} - \left(\frac{2}{5} \times \frac{4}{7}\right)$
   b. $\left(1 + \frac{1}{5}\right) \times \frac{4}{7} = \frac{4}{7} + \left(\frac{1}{5} \times \frac{4}{7}\right)$

Launch

- Groups of 2

Activity

- 1–2 minutes: quiet think time
- 8–10 minutes: partner work time
- Monitor for students who use the expressions in the first problem to make the comparisons and then generalize about what happens when you multiply a number by any fraction greater than 1 or less than 1.

Synthesis

- Invite students to share their expressions for the products in the first problem.
- Display the equation: $\left(1 - \frac{2}{5}\right) \times \frac{4}{7} = \frac{4}{7} - \left(\frac{2}{5} \times \frac{4}{7}\right)$
- "How can you see that the value of the expression is less than $\frac{4}{7}$?" (It's $\frac{4}{7}$ minus something.)
- "Does this reasoning also work for $\left(1 - \frac{3}{8}\right) \times \frac{4}{7}$?" (Yes, it's again $\frac{4}{7}$ minus some other number.)
- "Will this reasoning work whenever you multiply a number less than 1 by $\frac{4}{7}$?" (Yes, I'll always get $\frac{4}{7}$ minus an amount so that's

c. $\left(1 - \frac{3}{8}\right) \times \frac{4}{7} = \frac{4}{7} - \left(\frac{3}{8} \times \frac{4}{7}\right)$

d. $\left(1 + \frac{1}{8}\right) \times \frac{4}{7} = \frac{4}{7} + \left(\frac{1}{8} \times \frac{4}{7}\right)$

2. a. $\left(1 - \frac{2}{5}\right) \times \frac{4}{7} < \frac{4}{7}$ because it's $\frac{4}{7}$ minus a fraction.

   b. $\left(1 + \frac{1}{5}\right) \times \frac{4}{7} > \frac{4}{7}$ because it's $\frac{4}{7}$ and another fraction.

   c. $\left(1 - \frac{3}{8}\right) \times \frac{4}{7} < \frac{4}{7}$ because it's $\frac{4}{7}$ minus a fraction.

   d. $\left(1 + \frac{1}{8}\right) \times \frac{4}{7} > \frac{4}{7}$ because it's $\frac{4}{7}$ and another fraction.

3. It's greater than $\frac{4}{7}$ because I can write the number as 1 plus some amount and when I multiply by $\frac{4}{7}$ I get $\frac{4}{7}$ and some more.

4. It's less than $\frac{4}{7}$ because I can write the number as 1 minus some amount and when I multiply by $\frac{4}{7}$ I get $\frac{4}{7}$ minus some.

less than $\frac{4}{7}$.)

Lesson Synthesis

10 min

"Today we compared the value of a product of fractions to the value of one of the factors without calculating the product."

Display product: $\frac{7}{9} \times \frac{15}{13}$.

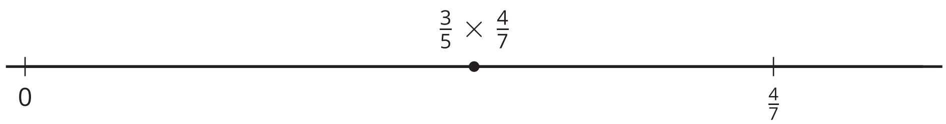
"What are some ways you can compare the value of the product with $\frac{15}{13}$?" (I can calculate the value, but the numbers are complicated. I can make a number line diagram and see that it is to the left of $\frac{15}{13}$. I can rewrite $\frac{7}{9}$ as $1 - \frac{2}{9}$ and see that it is less.)

"What are some ways you can compare the value of the product with $\frac{7}{9}$?" (I can calculate the value. I can make a number line diagram and see that it is to the right of $\frac{7}{9}$. I can rewrite $\frac{15}{13}$ as $1 + \frac{2}{13}$ and see that it is more.)

Suggested Centers

- Rectangle Rumble (3–5), Stage 5: Fraction Factors (Addressing)
- Rolling for Fractions (3–5), Stage 4: Multiply Fractions (Supporting)

Student Section Summary



In this section, we learned how to compare the size of a product to the size of the factors. To compare $\frac{3}{5} \times \frac{4}{7}$ with $\frac{4}{7}$, for example, we can put them on a number line.

Since $\frac{3}{5}$ is 3 equal parts with 5 parts in the whole, it is to the left of $\frac{4}{7}$, only part of the way there. We can also see this by writing $\frac{3}{5}$ as $1 - \frac{2}{5}$.

$$\left(1 - \frac{2}{5}\right) \times \frac{4}{7} = \frac{4}{7} - \left(\frac{2}{5} \times \frac{4}{7}\right)$$

The product is less than $\frac{4}{7}$ because it is $\frac{4}{7}$ minus a fraction.

Complete Cool-Down

Response to Student Thinking

Students do not explain why the solution will be greater or less than 1.

Next Day Support

- Create a poster with a diagram that represents the cool-down from this lesson.

Lesson 20: Will it Always Work? (Optional)

Standards Alignments

Addressing 5.NF.B.5, 5.NF.B.5.b, 5.OA.A

Teacher-facing Learning Goals

- Make generalizations about multiplying a whole number by a fraction greater than, less than, or equal to 1.

Student-facing Learning Goals

- Let's make generalizations about multiplying a whole number by a fraction.

Lesson Purpose

The purpose of this lesson is for students to explain how the product of two numbers compares to one factor based on the size of the other factor.

The goal of this optional lesson is to compare the size of a product to the size of one factor, applying what students have learned in the last several lessons. They make the comparisons and describe in general how to make comparisons using a strategy that makes sense to them. No representation is provided or suggested so this gives students an opportunity to think about the different methods they have learned, namely calculation, number lines and the distributive property, and choose the one that makes sense in each situation.

Access for:

Students with Disabilities

- Representation (Activity 2)

English Learners

- MLR1 (Activity 1)

Instructional Routines

True or False (Warm-up)

Materials to Gather

- Tools for creating a visual display: Activity 2

Lesson Timeline

Warm-up	10 min

Teacher Reflection Question

What questions did you ask today that helped students reason about the size of a product without performing multiplication?

Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Cool-down (to be completed at the end of the lesson)

5 min

Compare

Standards Alignments

Addressing 5.NF.B.5.b

Student-facing Task Statement

Write <, =, or > in each blank to make the statements true.

1. $\frac{13}{18} \times \frac{11}{3}$ ______ $\frac{11}{3}$
2. $\frac{19}{16} \times \frac{22}{3}$ ______ $\frac{22}{3}$
3. $\frac{8}{8} \times \frac{1}{5}$ ______ $\frac{1}{5}$

Student Responses

1. <
2. >
3. =

Begin Lesson

Warm-up

10 min

True or False: Distributing

Standards Alignments

Addressing 5.OA.A

The purpose of this True or False is for students to demonstrate strategies they have for comparing expressions. The reasoning students use here helps to deepen their understanding of the properties of operations. It will also be helpful later when students compare expressions and generalize their understanding of how the size of a number changes when multiplied by a fraction that is less than 1, equal to 1, and greater than 1.

Instructional Routines

True or False

Student-facing Task Statement

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $\frac{3}{4} = 1 - \frac{1}{4}$
- $\left(1 - \frac{1}{4}\right) \times 9 = 9 - \left(\frac{1}{4} \times 9\right)$
- $\left(1 + \frac{1}{4}\right) \times 7 = (1 \times 7) + \frac{1}{4}$

Student Responses

- True. 1 is 4 fourths so taking away $\frac{1}{4}$ leaves $\frac{3}{4}$.
- True. 1 is multiplied by 9 and I take away $\frac{1}{4} \times 9$.
- False. The 1 is multiplied by 7 but the $\frac{1}{4}$ is not.

Launch

- Display one equation.
- "Give me a signal when you know whether or not the equation is true and can explain how you know."
- 1 minute: quiet think time

Activity

- Share and record answers and strategy.
- Repeat with each equation.

Synthesis

- Display the expression: $(1 + \frac{1}{4}) \times 7$
- "How can I rewrite this expression as a sum?" (You can multiply 1 and 7 and that's 7 and then add $\frac{1}{4} \times 7$.)
- Display equation: $\left(1 + \frac{1}{4}\right) \times 7 = (1 \times 7) + \left(\frac{1}{4} \times 7\right)$

Activity 1

20 min

True Statements

Standards Alignments

Addressing 5.NF.B.5.b

The purpose of this activity is for students to apply what they have learned in this section to compare a number to the product of that number with a fraction. There are two cases where the fraction has a value of 1. Students may identify that the value of the fraction is 1 and use what they know about multiplying a number by 1. They may also use their knowledge of how to multiply fractions and the calculations for these products have been made simpler so that students can find the product to make the comparison. For the other problems, the numbers are sufficiently complex that the most efficient way to compare is to think about the size of the factors.

Access for English Learners

MLR1 Stronger and Clearer Each Time. Synthesis: Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their response to whether the expressions are <, >, or =. Invite listeners to ask questions, to press for details and to suggest mathematical language. Give students 2–3 minutes to revise their written explanation based on the feedback they receive.
Advances: Writing, Speaking, Listening

Student-facing Task Statement

Write <, >, or = in each blank to make true statements.

Choose one problem and explain or show your reasoning.

1. $567 \_\_\_\_ 345 \times 567$
2. $\frac{4}{5} \times 851 \_\_\_\_ 851$
3. $\frac{1}{4} \_\_\_\_ \frac{5}{5} \times \frac{1}{4}$
4. $\frac{103}{104} \_\_\_\_ \frac{103}{104} \times \frac{103}{104}$

Launch

- Groups of 2

Activity

- 1–2 minutes: quiet think time
- 10–12 minutes: partner work time
- For the expressions $\frac{1}{4} \times \frac{5}{5}$ and $\frac{10}{10} \times \frac{1}{2}$ monitor for students who calculate the products and for students who reason about the size of the factors.

5. $\frac{99}{8} \times \frac{23}{22}$ ____ $\frac{99}{8}$
6. $\frac{10}{10} \times \frac{1}{2}$ ____ $\frac{1}{2}$
7. $\frac{100}{7} \times \frac{9}{13}$ ____ $\frac{9}{13}$

Student Responses

1. $<$, 345×567 is more than 100,000.
2. $<$, because $\frac{4}{5} < 1$.
3. $=$, because they are the same. They are equivalent fractions.
4. $<$, because $\frac{103}{104} < 1$.
5. $>$, because $\frac{23}{22} > 1$.
6. $=$, because they are the same. They are equivalent fractions.
7. $>$, $\frac{100}{7} \times \frac{9}{13}$ is more than 10 times greater.

Synthesis

- Invite students to share their solution for comparing $\frac{10}{10} \times \frac{1}{2}$ and $\frac{1}{2}$.
- “How do you know that $\frac{10}{20} = \frac{1}{2}$?” (If you cut the half into 10 equal pieces then you get 10 pieces and there are 20 in the whole.)
- “Can you use the equation $\frac{10}{10} = 1$ to see that $\frac{10}{10} \times \frac{1}{2} = \frac{1}{2}$?” (Yes, because 1 times any number is that same number.)
- Invite students to share their solution for comparing $\frac{103}{104}$ and $\frac{103}{104} \times \frac{103}{104}$.
- “Did you find the product to compare?” (No, the numbers are big so it would have taken a long time.)
- “What did you do to compare the numbers?” (I know that $\frac{103}{104}$ is less than 1, it’s $1 - \frac{1}{104}$. So when I multiply it by any number I get less than that number.)

Activity 2

15 min

Andre’s Rules

Standards Alignments

Addressing 5.NF.B.5

The purpose of this activity is for students to reflect on different ways to compare a product of fractions to one of the factors. Students have seen multiple strategies that will always work, including calculating the product, thinking about the product on the number line, and using the distributive property to explain how the size of a product compares to the size of the factors. Students must use language precisely in their explanation (MP6).

Access for Students with Disabilities

Representation: Develop Language and Symbols. Synthesis: Make connections between representations visible. Provide students access to a blank card or sticky note for the gallery walk to list similarities between the representations and create a class list.
Supports accessibility for: Attention, Language, Organization

Materials to Gather

Tools for creating a visual display

Student-facing Task Statement

Andre says:

- When you multiply any fraction by a number less than 1, the product will be less than the fraction.
- When you multiply any fraction by a number greater than 1, the product will be greater than the fraction.

Each partner choose one of the statements and describe why it is true. You may want to include details such as notes, diagrams, and drawings to help others understand your thinking.

Student Responses

Sample response: If I multiply a number by a fraction less than 1 then I have less than that number. Since the fraction is less than 1 that means I only get some of the number, not all of it. I divide it into some equal pieces but I have fewer pieces than there are in the whole number so that's less than the number.

Launch

- Groups of 2

Activity

- 3–5 minutes: independent work time
- "Work with your partner to create a visual display that shows your thinking about Andre's statements. You may want to include details such as notes, diagrams or drawings to help others understand your thinking."
- 5–7 minutes: independent or small-group work time
- 3–5 minutes: gallery walk

Synthesis

- Invite students to share explanations for how to compare the size of a product to the size of one of the factors.
- "What is challenging about giving an explanation that works for all products?" (When I have specific numbers I can find the products and compare them or I can put them on the number line or write expressions. Without specific numbers it's hard to write or reason about the product.)

Lesson Synthesis

10 min

"Today we generalized some rules to compare products of fractions."

"What is your favorite way to compare products of fractions?" (I like to use the number line to help me visualize and think of fractions as parts in a whole. I like to calculate and compare numbers. I like using the distributive property so I can see if the product is greater or less than one factor without finding the value.)

Consider asking students to write their response in their journal.

Suggested Centers

- Rectangle Rumble (3–5), Stage 5: Fraction Factors (Addressing)
- Rolling for Fractions (3–5), Stage 4: Multiply Fractions (Supporting)

Complete Cool-Down

Response to Student Thinking

Students multiply to determine if the expression is greater than, less than, or equal to the fraction.

Next Day Support

- Create a poster with a diagram that represents the cool-down from previous lessons.

Lesson 21: Weekend Investigation (Optional)

Standards Alignments

Addressing 5.MD.B.2, 5.NF.A.2, 5.NF.B.4

Teacher-facing Learning Goals

- Create line plots and use the information to solve problems.
- Solve problems involving addition and subtraction of fraction with unlike denominators.

Student-facing Learning Goals

- Let's find out about how students spend free time on the weekend.

Lesson Purpose

The purpose of this lesson is for students to apply their understanding of making line plots and using operations with fractions to analyze data.

This lesson is optional because it does not address any new mathematical content standards. This lesson does provide students with an opportunity to apply precursor skills of mathematical modeling. In this lesson, students brainstorm and define categories of how to spend time. Then they collect and represent data on a line plot. They analyze and describe the data to tell a story about the time use.

When students define categories, choose and ask questions, collect and analyze data, and tell a story about the situation based on data, they model with mathematics (MP4).

Access for:

Students with Disabilities

- Engagement (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min

Teacher Reflection Question

As students worked together today, where did you see evidence of the mathematical

Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min

community established over the course of the school year?

Begin Lesson

Warm-up

10 min

Number Talk: Fractions of 60

Standards Alignments

Addressing 5.NF.B.4

The purpose of this Number Talk is for students to demonstrate strategies and understandings they have for fraction multiplication. The whole number 60 is intentionally selected to represent the minutes in an hour. These understandings help students develop fluency and will be helpful in this lesson when students need to be able to interpret the number of minutes in fractional parts of an hour.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $\frac{1}{4} \times 60$
- $\frac{3}{4} \times 60$
- $\frac{5}{4} \times 60$
- $\frac{9}{4} \times 60$

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time.

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Student Responses

- 15. $60 \div 4 = 15$
- 45. $3 \times 15 = 45$
- 75. $5 \times 15 = 75$
- 135. Add all of the products from the earlier expressions. $15 + 45 = 60$, $60 + 75 = 135$

Synthesis

- "What is $\frac{1}{8} \times 60$? How do you know?" ($7\frac{1}{2}$ because that is half of 15. $\frac{1}{8}$ is half of $\frac{1}{4}$.)

Activity 1

15 min

Data Collection

Standards Alignments

Addressing 5.NF.A.2

The purpose of this activity is for students to think about different activities they might participate in during free time, to define a smaller set of categories, and to collect data. A significant part of this activity is the definition of the categories, which is an important aspect of mathematical modeling (MP4). Encourage students to convert the time for their activities from minutes to hours. Students may notice that some fractions are more meaningful in the context of time such as half, third, and quarter of an hour. Students will further explore the relationship between time and fractions in the activity synthesis.

If a student sees that a specific activity is not on the poster, have them consider if there is an umbrella activity. For example, "What category, that is on a poster, might talking to your friends go under?"

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Invite students to generate a list of shared expectations for group work. Record responses on a display and keep visible during the activity.
Supports accessibility for: Social-Emotional Functioning, Attention

Student-facing Task Statement

Imagine on the weekend you have 2 hours of free time that you can spend any way you like.

Launch

- Groups of 4
- "What are some things you do or like to do if you have free time on the weekend?"

1. How would you spend it? Record your answer in fractions of an hour. Show your reasoning.
2. Record the time for each activity from your list on the appropriate poster, if there is a category for it.

Student Responses

Sample response:
watch tv: $\frac{3}{4}$ hour, read: $\frac{3}{8}$ hour, play games: $\frac{1}{2}$ hour, communication: $\frac{3}{8}$ hour,
$\frac{3}{4} + \frac{3}{8} + \frac{1}{2} + \frac{3}{8} = \frac{6}{8} + \frac{3}{8} + \frac{4}{8} + \frac{3}{8}$ which is $\frac{16}{8} = 2$ hours

(sleep, play video games, play sports, spend time with family, social media, read, go to the movies)

Activity

- "For this activity, you'll think about how you might spend 2 hours of free time and then we'll record the time we spend for some of those activities."
- "As you work, I will write down the different categories of your responses on the posters hanging around the room. You will record your times on the posters."
- As students work, monitor for 4–6 popular activities and write the titles of these activities on the large pieces of poster paper. If necessary, list some examples of the umbrella category.
- 8–10 minutes: independent work time
- Monitor for students who record using multiples of $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$ of an hour.

Synthesis

- "Look at the data of the different categories around the room. What do you notice?" (Most people recorded using halves and fourths. More students play games than read books. There is more data on one poster than on another. There are more small numbers on one poster, it's hard to compare because there are so many numbers and they are not in any order.)
- If it doesn't come up, ask, "Why did most people record time as multiples of $\frac{1}{2}$ and $\frac{1}{4}$?" (Because half of 60 minutes is 30 and half of that is 15. I can easily see that on the clock.)
- "Does it make sense to talk about time using $\frac{1}{8}$ or $\frac{1}{7}$ of an hour?" (Yes but it's not a whole number of minutes like $\frac{1}{2}$ hour or $\frac{1}{4}$

hour.)

- “What are some ways we can organize the data?” (Bar graph, line plot, chart, list the numbers in order.)

Activity 2

20 min

Data Analysis

Standards Alignments

Addressing 5.MD.B.2, 5.NF.A.2

The purpose of this activity is for students to make and analyze line plots. In this activity, students analyze the free time data collected in the previous activity. They make observations and comparisons to tell the story of their data set.

Access for English Learners

MLR8 Discussion Supports. Synthesis: During group presentations, invite the student(s) who are not speaking to follow along and point to the corresponding parts of the display.
Advances: Speaking, Representing

Student-facing Task Statement

Your teacher will assign a poster with a data set for one of the categories from the previous activity.

1. Create a line plot that represents the data. Make sure to label the line plot.
2. Analyze the data and tell the story of your data. Choose at least 3 things. Use the following questions if they are helpful.
    - What is the total number of hours the class spends on this activity?
    - What is the difference between

Launch

- Groups of 4
- Give each group one of the data sets from the previous activity.

Activity

- 10 minutes: small-group work time
- Monitor for groups who:
    - label the line plot with fractions such as fourths
    - use operations with fractions

greatest and least time?
- Is there something surprising?
- How many data points are there? What does that tell you?
- What fraction of your classmates spend less than an hour on this activity? More than an hour?

Be prepared to share the story with the class.

Student Responses

1. Students create an appropriate line plot that is most likely scaled with fourths (eighths if some of the data is given in eighths) and goes from 0 to 2 hours.
2. Sample response: There are 25 students who watch videos. The shortest time is $\frac{1}{4}$ hour and the longest time is $1\frac{1}{2}$ hours. The total amount of time spent is $18\frac{3}{4}$ hours. More than $\frac{2}{3}$ of the class spend an hour or less watching videos. Most students watch videos for around $\frac{1}{2}$ hour.

- come up with their own question to analyze the data

Synthesis

- Invite previously selected groups to tell their line plot and data stories.
- "What other information can we tell about our classmates from the line plots?" (More students like this than that. People would like to spend more time on _____, and less time on _____.)

Lesson Synthesis

10 min

"Today, we used fractions of hours to talk about how we might spend free time. In the warm up, we saw these expressions."

Display:

$\frac{5}{4} \times 60$

$\frac{9}{4} \times 60$

"What connections do you notice between these expressions and fractions of an hour?" (There are 60 minutes in an hour so we can see that $\frac{5}{4}$ of an hour is an hour and 15 minutes or $\frac{5}{4} \times 60$ minutes.)

"Today, we also made and analyzed line plots."

“Who might be interested in collecting and analyzing data like this? Why?” (Makers of children’s toys, stores, and advertisers want to know how kids spend their time so that they can make money or sell the things kids want. Parents and educators want to know how kids spend time and see if it is what they should be doing.)

Suggested Centers

- Rectangle Rumble (3–5), Stage 5: Fraction Factors (Addressing)
- Rolling for Fractions (3–5), Stage 4: Multiply Fractions (Supporting)

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Family Support Materials

Family Support Materials

More Decimal and Fraction Operations

In this unit, students solve multi-step problems involving measurement conversions, line plots, and fraction operations, including addition and subtraction of fractions with unlike denominators. They also explain patterns when multiplying and dividing by powers of 10. Students interpret multiplication as scaling by comparing products with factors.

Section A: Measurement Conversions and Powers of 10

In this section, students convert smaller units to larger units (for example, centimeters to kilometers), and describe the patterns they notice when multiplying and dividing by powers of 10. Students work with the metric and customary system (for example. feet, quarts, pounds, and so on) and develop an understanding of the relative sizes of units of length, volume, and weight. Students use the four operations with whole numbers, decimals, and fractions to solve multi-step word problems involving measurement conversions.

Section B: Add and Subtract Fractions with Unlike Denominators

In this section, students add and subtract fractions and mixed numbers with unlike denominators, and apply this learning to problem solving. Students first encounter problems where one denominator is a factor of the other (for example, $\frac{1}{4}$s and $\frac{1}{8}$s), so that they will only need to change one denominator. Then, students solve problems where the denominators are not related (for example, $\frac{1}{3}$s and $\frac{1}{4}$s). Students conclude that multiplying the denominators or finding a common multiple are helpful ways to create common denominators.

Students also extend their understanding of line plots. They create line plots using measurement data in fractional units (halves, fourths, and eighths), and interpret the data on line plots to solve problems involving the four fraction operations like this one.

Jada says $\frac{3}{4}$ of the students spend less than 2 hours on a screen. Is she correct?

Explain how you know your answer is correct.

Section C: The Size of Products

In this section, students build on their understanding of multiplication to include the concept of scaling. Students interpret multiplication expressions as a quantity that is resized or scaled by a factor.

Students compare multiplication expressions without performing the multiplication. In the example shown, students reason that $\frac{7}{6} \times 4$ is greater than the other two expressions because in each expression, 4 is being multiplied by a fraction, and $\frac{7}{6}$ is the largest fraction of the three.

Which of these expressions represents the largest product?

$\frac{5}{8} \times 4$ $\qquad$ $\frac{7}{6} \times 4$ $\qquad$ $\frac{1}{2} \times 4$

Students locate multiplication expressions on a number line, and analyze expressions to determine if the product is greater than, less than, or equal to one of its factors. Students make sense of their learning by recognizing that if a given number is multiplied by:

- a fraction greater than 1, then the product will be greater than the given number
- a fraction less than 1, then the product will be less than the given number
- a fraction equal to 1, then the product will be equal to the given number

Try it at home!

Near the end of the unit, ask your student to solve the following problems:

- How many kilometers is equal to 200 centimeters?
- $\frac{2}{3} + \frac{2}{9}$
- $\frac{2}{3} + \frac{5}{8}$
- Will $\frac{4}{3} \times 5$ be greater than, less than, or equal to 5? How do you know?

Questions that may be helpful as they work:

- What strategy are you going to use to help you solve the problem?
- Could you have solved the problem in a different way?
- Which problem was easier to solve? Why?

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Unit Assessments

Check Your Readiness A, B and C

End-of-Unit Assessment

More Decimal and Fraction Operations: Section A Checkpoint

1. Complete the table with equivalent measurements.

kilometers	meters	centimeters
1.7		
		15,900
	23	

2. Choose **all** representations of the number 100,000,000.

   A. 10^8

   B. 10 million

   C. $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$

   D. 100 thousand

   E. 100 million

3. It is 325 meters around a track. Jada ran around the track 12 times. How many kilometers did Jada run?

More Decimal and Fraction Operations: Section B Checkpoint

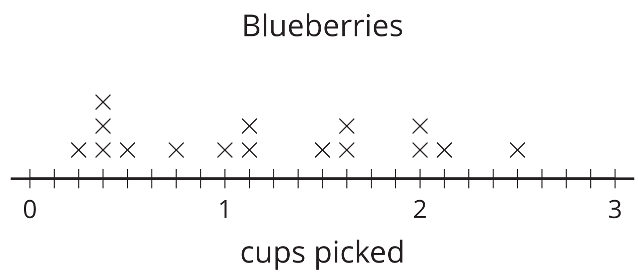
1. Elena ran $2\frac{7}{10}$ miles. Diego ran $2\frac{3}{4}$ miles. How much farther did Diego run than Elena? Explain or show your reasoning.

2. Find the value of each expression:

   a. $2\frac{11}{12} - 1\frac{3}{8}$

   b. $\frac{3}{4} + \frac{2}{9}$

3. The line plot shows the amount of blueberries Lin picked on different days during harvesting season.



a. What is the difference between the greatest number of cups and the least number of cups of blueberries?

b. How many days did Lin pick more than $1\frac{1}{2}$ cups of blueberries?

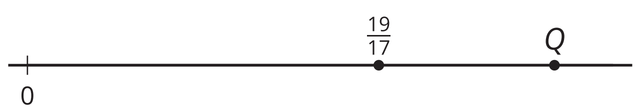
More Decimal and Fraction Operations: Section C Checkpoint

1. Write $<$, $=$, or $>$ in the blanks to make each statement true.

   a. $\frac{9}{7} \times 187$ ______ 187

   b. $\frac{19}{19} \times \frac{11}{13}$ ______ $\frac{11}{13}$

   c. $\frac{19}{19} \times \frac{11}{13}$ ______ $\frac{19}{19}$



2. What could be the value of the number labeled Q?

   A. $\frac{2}{3} \times \frac{19}{17}$

   B. $\frac{19}{17} \times \frac{7}{7}$

   C. $\frac{13}{13} \times \frac{19}{17}$

   D. $\frac{3}{2} \times \frac{19}{17}$

More Decimal and Fraction Operations: End-of-Unit Assessment

1. Select **all** expressions that represent the number of millimeters in a kilometer.

A. 10^3

B. 10^5

C. 10^6

D. 1,000

E. 100,000

F. 1,000,000

2. Which fraction is equivalent to $\frac{7}{10}$?

A. $\frac{7 \times 10}{9 \times 10}$

B. $\frac{10 \times 10}{10 \times 7}$

C. $\frac{7 \times 41}{10 \times 41}$

D. $\frac{7+10}{10+10}$

3. Select **all** expressions with a value larger than 1.

A. $\frac{4}{5} + \frac{1}{6}$

B. $\frac{3}{4} + \frac{1}{3}$

C. $\frac{7}{5} - \frac{1}{10}$

D. $2\frac{1}{8} - 1\frac{1}{7}$

E. $\frac{5}{4} + \frac{1}{9}$

F. $1\frac{1}{2} - \frac{3}{5}$

4. Find the value of each expression.

a. $\frac{7}{4} + \frac{3}{5}$

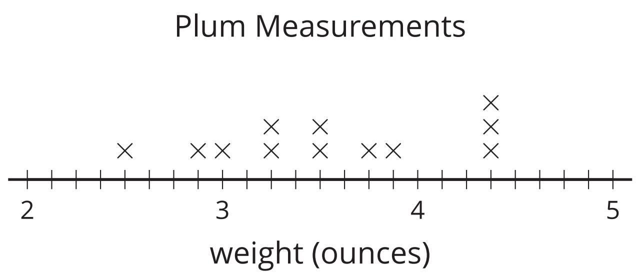
b. $\frac{3}{4} - \frac{2}{5}$

c. $6\frac{1}{5} - 4\frac{5}{6}$

5. Han's backpack weighs $\frac{5}{4}$ as much as Lin's backpack. Clare's backpack weighs $\frac{2}{3}$ as much as Lin's backpack. Whose backpack weighs the most? Whose backpack weighs the least? Explain or show how you know.

6. Elena drinks 9 glasses of water during the day. Each glass is 250 milliliters. How many liters of water does Elena drink during the day? Explain or show your reasoning.

7. Use the line plot to answer the questions.



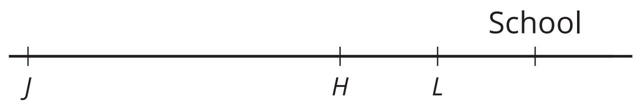
a. How much heavier is the heaviest plum than the lightest plum? Explain or show your reasoning.

b. How many plums were weighed?

c. Noah says that half the plums weigh more than $3\frac{1}{2}$ ounces. Do you agree with Noah? Explain your reasoning.

8. Jada lives $1\frac{3}{10}$ miles from school. Han lives $\frac{1}{2}$ mile farther from school than Jada. Lin lives $\frac{1}{4}$ mile closer to school than Jada.

a. Jada drew this diagram to represent the situation. Explain why the diagram is not accurate.



b. Locate on the diagram an estimate of where you think Lin's home and Han's home might be.

c. How far do Han and Lin live from the school? Explain or show reasoning.

d. Does your diagram of how far Han and Lin are from the school agree with your calculations? Explain or show your reasoning.

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Assessment Answer Keys

Check Your Readiness A, B and C

End-of-Unit Assessment

Assessment Answer Keys

Assessment: Section A Checkpoint

Teacher Instructions

Give students access to meter sticks.

Problem 1

Goals Assessed

- Solve multi-step problems involving measurement conversions.

Complete the table with equivalent measurements.

kilometers	meters	centimeters
1.7		
		15,900
	23	

Solution

kilometers	meters	centimeters
1.7	1,700	170,000
0.159	159	15,900
0.023	23	2,300

Problem 2

Goals Assessed

- Explain patterns when multiplying and dividing by powers of 10.

Choose **all** representations of the number 100,000,000.

A. 10^8

B. 10 million

C. $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$

D. 100 thousand

E. 100 million

Solution

["A", "E"]

Problem 3

Goals Assessed

- Solve multi-step problems involving measurement conversions.

It is 325 meters around a track. Jada ran around the track 12 times. How many kilometers did Jada run?

Solution

3.900. Jada ran 325×12 meters. That's $3{,}250 + 650$ or $3{,}900$ meters. There are 1,000 meters in a kilometer so each digit moves three places to the right.

Assessment: Section B Checkpoint

Problem 1

Goals Assessed

- Solve problems involving addition and subtraction of fractions

Elena ran $2\frac{7}{10}$ miles. Diego ran $2\frac{3}{4}$ miles. How much farther did Diego run than Elena? Explain or show your reasoning.

Solution

$\frac{1}{20}$ mile or equivalent. I used 20 as a common denominator and $\frac{3}{4} = \frac{15}{20}$ and $\frac{7}{10} = \frac{14}{20}$ so Diego ran $\frac{1}{20}$ mile farther.

Problem 2

Goals Assessed

- Add and subtract fractions with unlike denominators.

Find the value of each expression:

a. $2\frac{11}{12} - 1\frac{3}{8}$

b. $\frac{3}{4} + \frac{2}{9}$

Solution

a. $1\frac{13}{24}$

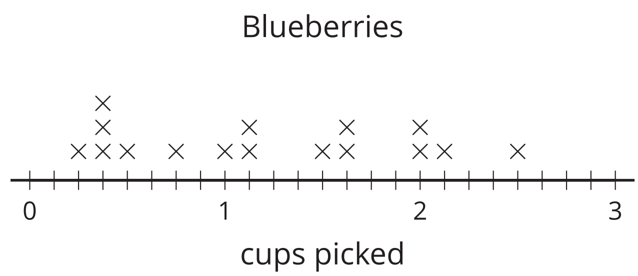
b. $\frac{35}{36}$

Problem 3

Goals Assessed

- Create line plots to display fractional measurement data and use the information to solve problems.

The line plot shows the amount of blueberries Lin picked on different days during harvesting season.



a. What is the difference between the greatest number of cups and the least number of cups of blueberries?

b. How many days did Lin pick more than $1\frac{1}{2}$ cups of blueberries?

Solution

a. $2\frac{2}{8}$ or equivalent

b. 6 days

Assessment: Section C Checkpoint

Problem 1

Goals Assessed

- Make generalizations about multiplying a whole number by a fraction greater than, less than and equal to 1.

Write $<$, $=$, or $>$ in the blanks to make each statement true.

a. $\frac{9}{7} \times 187$ ______ 187

b. $\frac{19}{19} \times \frac{11}{13}$ ______ $\frac{11}{13}$

c. $\frac{19}{19} \times \frac{11}{13}$ ______ $\frac{19}{19}$

Solution

a. $>$

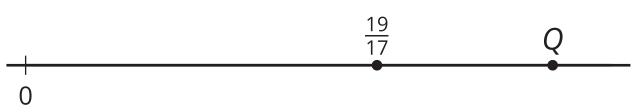
b. $=$

c. $<$

Problem 2

Goals Assessed

- Interpret multiplication as scaling (resizing).



What could be the value of the number labeled Q?

A. $\frac{2}{3} \times \frac{19}{17}$

B. $\frac{19}{17} \times \frac{7}{7}$

C. $\frac{13}{13} \times \frac{19}{17}$

D. $\frac{3}{2} \times \frac{19}{17}$

Solution

D

Assessment: End-of-Unit Assessment

Teacher Instructions

Give students access to meter sticks.

Problem 1

Standards Alignments

Addressing 5.MD.A.1, 5.NBT.A.2

Narrative

Students find how many millimeters are in a kilometer and express it both as a number and using exponential notation. Students may select A and D if they think about the number of millimeters in a meter or the number of meters in a kilometer. They may select B and E if they recall the number of centimeters in a kilometer which was covered in the unit. If students select a number and expression that do not agree then they may need to review exponential notation.

Select **all** expressions that represent the number of millimeters in a kilometer.

A. 10^3

B. 10^5

C. 10^6

D. 1,000

E. 100,000

F. 1,000,000

Solution

["C", "F"]

Problem 2

Standards Alignments

Addressing 5.NF.B.5.b

Narrative

Students choose a fraction equivalent to $\frac{7}{10}$. Response A is not correct because $\frac{7}{10}$ has been multiplied by a fraction different than 1 and response B has the numerator and denominator of $\frac{7}{10}$ switched. If students select A or B then they have probably made an arithmetic error in their calculations and are not using the structure of the expressions. Students who select D are probably thinking that adding the same amount to the numerator and denominator does not change the value of the fraction. The numbers were chosen so that they can readily see that this is incorrect since $\frac{7}{10}$ is equivalent to $\frac{14}{20}$.

Which fraction is equivalent to $\frac{7}{10}$?

A. $\frac{7\times10}{9\times10}$

B. $\frac{10\times10}{10\times7}$

C. $\frac{7\times41}{10\times41}$

D. $\frac{7+10}{10+10}$

Solution

C

Problem 3

Standards Alignments

Addressing 5.NF.A.1

Narrative

Students compare the value of addition and subtraction expressions with fractions and mixed numbers to the benchmark 1. In each case, there is a way to make the comparison by reasoning about the size of the numbers without finding a common denominator and calculating. Students who do express the sums and differences as a fraction or mixed number in order to make the comparisons will show fluency in that work. Students who select D or F likely need more work with mixed numbers.

Select **all** expressions with a value larger than 1.

A. $\frac{4}{5} + \frac{1}{6}$

B. $\frac{3}{4} + \frac{1}{3}$

C. $\frac{7}{5} - \frac{1}{10}$

D. $2\frac{1}{8} - 1\frac{1}{7}$

E. $\frac{5}{4} + \frac{1}{9}$

F. $1\frac{1}{2} - \frac{3}{5}$

Solution

["B", "C", "E"]

Problem 4

Standards Alignments

Addressing 5.NF.A.1

Narrative

Students find sums and differences of fractions. Though no explanation is requested, students will likely find a common denominator in order to perform the operations. The numbers, particularly in the last problem, are relatively complex so students may make a calculation error even though they are performing the correct calculations.

Find the value of each expression.

a. $\frac{7}{4} + \frac{3}{5}$

b. $\frac{3}{4} - \frac{2}{5}$

c. $6\frac{1}{5} - 4\frac{5}{6}$

Solution

a. $\frac{47}{20}$ or equivalent
b. $\frac{7}{20}$ or equivalent
c. $1\frac{11}{30}$ or equivalent

Problem 5

Standards Alignments

Addressing 5.NF.B.5.a

Narrative

Students compare numbers which are given as fractional multiples of the same number. Since they are not given the weight of Lin's backpack, students are expected to order the weights by reasoning about the size of the factors rather than by calculation. If students choose a weight for Clare's backpack they will still find the correct order assuming that they calculate and compare correctly but have not shown mastery of the standard which calls for comparing without calculating.

Han's backpack weighs $\frac{5}{4}$ as much as Lin's backpack. Clare's backpack weighs $\frac{2}{3}$ as much as Lin's backpack. Whose backpack weighs the most? Whose backpack weighs the least? Explain or show how you know.

Solution

Clare's backpack weighs the least, then Lin's, and then Han's. Since $\frac{2}{3}$ is less than 1, Clare's backpack weighs less than Lin's. And since $\frac{5}{4}$ is greater than 1, Han's backpack weighs more than Lin's.

Problem 6

Standards Alignments

Addressing 5.MD.A.1

Narrative

Students solve a problem that requires expressing a volume given in a smaller unit in terms of a larger unit. In doing so, they will demonstrate an understanding of place value since the conversion factor is $\frac{1}{1,000}$. They may express their answer as a decimal or as a fraction.

Elena drinks 9 glasses of water during the day. Each glass is 250 milliliters. How many liters of water does Elena drink during the day? Explain or show your reasoning.

Solution

2.25 or equivalent. She drinks 9×250 or 2,250 milliliters of water. There are 1,000 milliliters in a liter so that's $2,250 \div 1,000$ liters or 2.25 liters.

Problem 7

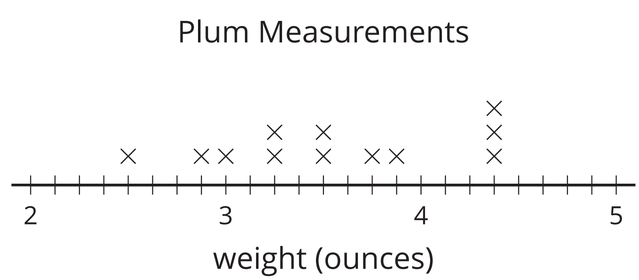
Standards Alignments

Addressing 5.MD.B.2, 5.NF.A

Narrative

Students read a line plot of weights given in ounces. Only the whole numbers are labeled so students will need to determine that the weights are given in eighths of an ounce.

Use the line plot to answer the questions.



a. How much heavier is the heaviest plum than the lightest plum? Explain or show your reasoning.
b. How many plums were weighed?
c. Noah says that half the plums weigh more than $3\frac{1}{2}$ ounces. Do you agree with Noah? Explain your reasoning.

Solution

a. $\frac{15}{8}$ ounces or equivalent. Sample response: The heaviest plum is $4\frac{3}{8}$ ounces and the lightest is $2\frac{4}{8}$ ounces. Since $4\frac{3}{8}$ is equivalent to $\frac{35}{8}$ and $2\frac{4}{8}$ is equivalent to $\frac{20}{8}$ the heaviest plum is $\frac{15}{8}$ ounces heavier than the lightest plum.

b. 12

c. No. There are 12 plums and only 5 of them weigh more than $3\frac{1}{2}$ ounces. Half of 12 is 6 so less than half of the plums weigh more than $3\frac{1}{2}$ ounces.

Problem 8

Standards Alignments

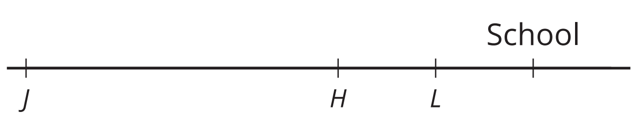
Addressing 5.NF.A.2

Narrative

Students solve a story problem about distances and reason about how to represent these distances on a diagram resembling a number line. They first analyze an incorrect representation, allowing them to think about the given quantities before making any calculations. They then sketch a new diagram to represent the relationship between the given distances before calculating the actual distances. It is not expected that students will plot the locations of Lin's house and Han's house precisely. The most important point is that Han is farther from the school than Jada and that Lin is closer. The relative distances between Jada and Lin, Jada and Han, and all three to the school do not need to be accurate. The final question gives students a chance to check and possibly correct their work placing the locations of Lin, Jada, and Han on the number line. While the exact locations of Lin and Han are complex, students can see that Lin is close to 1 mile from the school, Han is close to 2 miles, and Jada is in between, closer to 1 mile than to 2 miles.

Jada lives $1\frac{3}{10}$ miles from school. Han lives $\frac{1}{2}$ mile farther from school than Jada. Lin lives $\frac{1}{4}$ mile closer to school than Jada.

a. Jada drew this diagram to represent the situation. Explain why the diagram is not accurate.

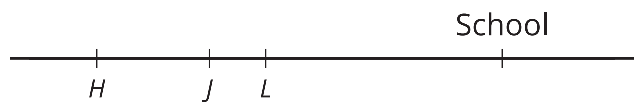


b. Locate on the diagram an estimate of where you think Lin's home and Han's home might be.

c. How far do Han and Lin live from the school? Explain or show reasoning.
d. Does your diagram of how far Han and Lin are from the school agree with your calculations? Explain or show your reasoning.

Solution

a. Han lives farther from the school than Jada and the diagram shows Han is closer. It also shows Lin closer to the school than to Jada and Lin should be closer to Jada than to the school.
b. Sample response: I put Han farther from the school and Lin closer to the school. The distance from Han to Jada is greater than the distance from Lin to Jada.



c. Han lives $1\frac{8}{10}$ miles from the school and Lin lives $\frac{42}{40}$ miles from the school. Han is $\frac{1}{2}$ or $\frac{5}{10}$ mile farther than Jada from school. Adding $\frac{5}{10}$ to $1\frac{3}{10}$ gives $1\frac{8}{10}$. Jada lives $\frac{13}{10}$ miles from school. This is the same as $\frac{52}{40}$ miles. Lin is $\frac{1}{4}$ mile closer which is $\frac{10}{40}$ mile. Subtracting $\frac{10}{40}$ from $\frac{52}{40}$ gives $\frac{42}{40}$.
d. Sample response: Yes, Han is further than Jada and Lin is closer. Lin is close to 1 mile from school and Han is almost 2 times as far as Lin from school.

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Lesson Cool Downs

Lesson 1: Place Value Patterns

Cool Down: Multiplication and Division Equations

Fill in the blank to make each equation true.

1. $0.06 \times 10 = \underline{\qquad}$

2. $60 = \underline{\qquad} \times 0.6$

3. $\underline{\qquad} = 6 \div 100$

Lesson 2: Powers of 10

Cool Down: Exponential Notation

1. Write 10,000 and 100,000 using exponential notation. Explain or show your reasoning.

2. Write 10^6 as a number.

Lesson 3: Metric Conversion and Multiplication by Powers of Ten

Cool Down: Kilometers

Complete the table. Explain or show your reasoning.

meters	centimeters	millimeters
6.5		

Lesson 4: Metric Conversion and Division by Powers of Ten

Cool Down: Han’s Run

Han ran 12,500 meters last week. How many kilometers is that? Explain or show your reasoning.

Lesson 5: Multi-step Conversion Problems: Metric Length

Cool Down: Compare Lengths

Jada ran 15.25 kilometers. Han ran 8,500 meters. Who ran farther? How much farther? Explain or show your reasoning.

Lesson 6: Multi-step Conversion Problems: Metric Liquid Volume

Cool Down: Dance Team

A dance team used 60 bottles of water during their practices last week. Each bottle holds 750 mL. How many liters of water did the dance team drink during their practices?

Lesson 7: Multi-step Conversion Problems: Customary Length

Cool Down: Whiteboard Width

The whiteboard is 4.5 feet in width.

1. How many inches wide is the whiteboard? Explain or show your reasoning.

2. How many yards wide is the whiteboard? Explain or show your reasoning.

Lesson 8: Add and Subtract Fractions

Cool Down: Sum of Fractions

Find the value of each expression. Explain or show your reasoning.

1. $\frac{5}{6} - \frac{1}{3}$

2. $\frac{3}{4} + \frac{1}{2}$

Lesson 9: Use Equivalent Expressions

Cool Down: Write an Expression

Find the value of $\frac{9}{12} - \frac{1}{4}$.

Lesson 10: All Sorts of Denominators

Cool Down: Sums of Fractions

Find the value of $\frac{4}{5} + \frac{2}{7}$.

Lesson 11: Different Ways to Subtract

Cool Down: Mixed Differences

Find the value of each expression. Explain or show your reasoning.

1. $2\frac{4}{5} - \frac{3}{10}$

2. $1\frac{2}{3} - \frac{3}{4}$

Lesson 12: Solve Problems

Cool Down: Evaluate Expressions

1. Priya hiked $1\frac{2}{3}$ miles. Diego hiked $\frac{1}{2}$ mile. How much farther did Priya hike than Diego? Explain or show your reasoning.

2. On Monday, Andre hiked $\frac{3}{4}$ mile in the morning and $1\frac{1}{3}$ miles in the afternoon. How far did Andre hike on Monday? Explain or show your reasoning.

Lesson 13: Put It All Together: Add and Subtract Fractions

Cool Down: Fraction Addition and Subtraction

Find the value of each expression. Explain or show your reasoning.

1. $\frac{8}{7} - \frac{2}{3}$

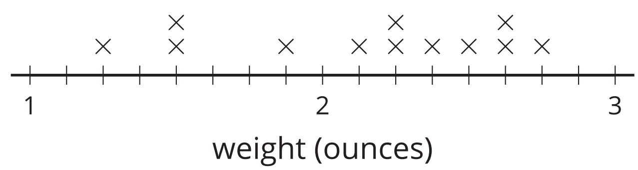
2. $\frac{5}{6} + \frac{2}{9}$

Lesson 14: Representing Fractions on a Line Plot

Cool Down: A Dozen Eggs

Here are the weights of a different collection of chicken eggs.

Chicken Eggs



What is the combined weight of all the eggs that weigh more than $2\frac{1}{2}$ ounces? Explain or show your reasoning.

Lesson 15: Problem Solving with Line Plots

Cool Down: Reflect

In this section, you added and subtracted fractions and worked with data on line plots. What did you get better at during this section?

Lesson 16: Compare Products

Cool Down: Greater Than or Less Than

1. Is $\frac{1}{8} \times 20$ greater than or less than 20? Explain or show your reasoning.

2. Is $\frac{10}{8} \times 20$ greater than or less than 20? Explain or show your reasoning.

Lesson 17: Interpret Diagrams

Cool Down: Read Books

Diego, Kiran, Elena, and Mai were reading a book.

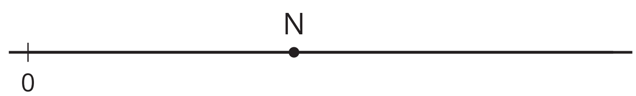
- Diego read 40 pages.
- Elena read $\frac{7}{8}$ times as many pages as Diego.
- Mai read $2\frac{1}{2}$ times as many pages as Diego.
- Kiran read $\frac{4}{5}$ times as many pages as Diego.

Write the 4 names in order of how many pages they read from least to greatest.

Lesson 18: Compare Without Multiplying

Cool Down: Comparison Statements

1. The number N is shown on the number line.



a. Locate and label $\frac{4}{3} \times \text{N}$ on the number line.

b. Is $\frac{4}{3} \times \text{N}$ less than, equal to, or greater than N? Explain how you know.

Lesson 19: Compare to 1

Cool Down: Compare without Calculating

1. Is $\left(1 - \frac{16}{33}\right) \times \frac{11}{14}$ greater than, equal to, or less than $\frac{11}{14}$? Explain or show your reasoning.

2. Is $\frac{49}{33} \times \frac{11}{14}$ greater than, equal to, or less than $\frac{11}{14}$? Explain or show your reasoning.

Lesson 20: Will it Always Work?

Cool Down: Compare

Write $<$, $=$, or $>$ in each blank to make the statements true.

1. $\frac{13}{18} \times \frac{11}{3}$ ______ $\frac{11}{3}$

2. $\frac{19}{16} \times \frac{22}{3}$ ______ $\frac{22}{3}$

3. $\frac{8}{8} \times \frac{1}{5}$ ______ $\frac{1}{5}$

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Instructional Masters

Instructional Masters for More Decimal and Fraction Operations

address	title	students per copy	written on?	requires cutting?	card stock recommended?	color paper recommended?
Activity Grade5.6.15.1	Info Gap: Picking Fruit	2	no	yes	no	no
Activity Grade5.6.7.1	Customary Measurement Card Sort	1	no	yes	no	no
Activity Grade5.6.8.1	Fraction Add and Subtract Sort	2	no	yes	no	no
Center	Number Puzzles Mult Stage 2 Recording Sheet	1	yes	no	no	no
Center	Five in a Row Multiplication and Division Stage 3 Gameboard	2	no	no	no	no
Center	Would You Rather Stage 2 Spinner	2	no	no	no	no
Center	Would You Rather Stage 2 Recording Sheet	2	no	no	no	no
Center	Fraction Cards Grade 3	2	no	no	yes	no
Center	Fraction Cards Grade 4	2	no	no	yes	no
Center	Compare Stage 3-8 Directions	2	yes	no	no	no
Center	Compare Stage 3-8 Directions	2	yes	no	no	no
Center	Would You Rather Stage 3 Spinner	2	no	no	no	no
Center	Would You Rather Stage 3 Recording Sheet	2	yes	no	no	no
Center	Compare Stage 6 Cards	2	no	yes	no	no

Center	How Close? Stage 9 Recording Sheet	1	yes	no	no	no
Center	Creating Line Plots Stage 3 Recording Sheet	1	yes	no	no	no
Center	Creating Line Plots Stage 4 Recording Sheet	1	yes	no	no	no
Center	Rectangle Rumble Stage 4 Spinner	2	no	no	no	no
Center	Rectangle Rumble Stage 4 Grid	2	yes	no	no	no
Center	Rectangle Rumble Stage 5 Spinners	2	no	no	no	no
Center	Rectangle Rumble Stage 5 Grid	2	yes	no	no	no
Center	Rolling for Fractions Stage 4 Recording Sheet	1	yes	no	no	no

Picking Fruit

Problem Card 1

Han still has two apricot weights to add to complete his line plot.

First weight: $2\frac{5}{8}$ ounces

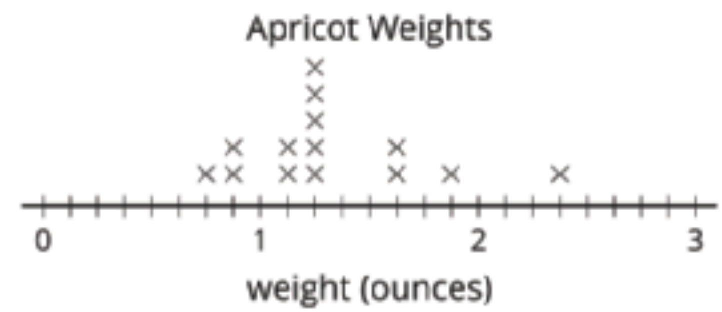
Second weight: $1\frac{1}{8}$ ounces

What is the difference between Han's heaviest apricot and the most common apricot weight?

Picking Fruit

Data Card 1

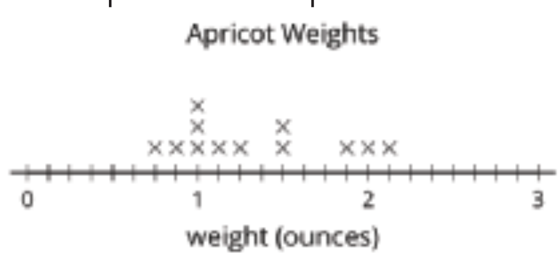
Here are some of Han's apricot weights.



Picking Fruit

Problem Card 2

Here are some of the weights of Mai's apricots. There are 15 apricot measurements total. Find the missing weights to complete the line plot.



Picking Fruit

Data Card 2

- The lightest apricot is $\frac{3}{4}$ ounces.
- The heaviest apricot is $2\frac{3}{8}$ ounces.
- The most common apricot weight is $1\frac{1}{2}$ ounces.

Customary Measurements Card Sort

$1,800$ inches

Customary Measurements Card Sort

18 inches

Customary Measurements Card Sort

6 inches

Customary Measurements Card Sort

$\frac{1}{6}$ yard

Customary Measurements Card Sort

150 feet

Customary Measurements Card Sort

$\frac{1}{2}$ foot

Customary Measurements Card Sort $1\frac{1}{2}$ feet	Customary Measurements Card Sort 50 yards
Customary Measurements Card Sort 54 inches	Customary Measurements Card Sort $1\frac{1}{2}$ yards
Customary Measurements Card Sort $4\frac{1}{2}$ feet	Customary Measurements Card Sort $\frac{1}{2}$ yard

Fraction Add and Subtract Sort

Fraction Add and Subtract Sort
A

$$\frac{2}{3} - \frac{1}{6}$$

Fraction Add and Subtract Sort
B

$$\frac{2}{3} + \frac{1}{2}$$

Fraction Add and Subtract Sort
C

$$3\frac{5}{8} - 1\frac{15}{16}$$

Fraction Add and Subtract Sort
D

$$4\frac{3}{5} - 1\frac{2}{5}$$

Fraction Add and Subtract Sort
E

$$\frac{3}{4} + \frac{7}{8}$$

Fraction Add and Subtract Sort
F

$$\frac{5}{7} + \frac{4}{7}$$

Fraction Add and Subtract Sort
G

$$\frac{2}{3} - \frac{1}{3}$$

Fraction Add and Subtract Sort
H

$$\frac{2}{3} + \frac{2}{3}$$

Puzzle 1
Fill in digits to make each equation true.
You may only use each digit (0–9) once.

$19 \times \boxed{3}\,\boxed{}\,\boxed{} = 6{,}802$
$\boxed{}\,\boxed{1} \times \boxed{1}\,\boxed{}\,\boxed{0} = 11{,}830$
$\boxed{4}\,\boxed{} \times \boxed{1}\,\boxed{5}\,\boxed{} = 6{,}240$
$\boxed{}\,\boxed{0}\,\boxed{1} \times \boxed{}\,\boxed{1} = 8{,}421$
$\boxed{}\,\boxed{2}\,\boxed{7} \times \boxed{1}\,\boxed{2}\,\boxed{} = 16{,}129$

Puzzle 2
Fill in digits to make each equation true.
You may only use each digit (0–9) once.

$15 \times \boxed{2}\,\boxed{}\,\boxed{} = 3{,}510$
$\boxed{}\,\boxed{1} \times \boxed{1}\,\boxed{}\,\boxed{0} = 10{,}650$
$\boxed{7}\,\boxed{} \times \boxed{1}\,\boxed{1}\,\boxed{} = 8{,}330$
$\boxed{}\,\boxed{3}\,\boxed{5} \times \boxed{}\,\boxed{1} = 19{,}035$
$\boxed{}\,\boxed{5}\,\boxed{2} \times \boxed{2}\,\boxed{4}\,\boxed{} = 37{,}392$

Puzzle 3

Fill in digits to make each equation true.
You may only use each digit (0–9) once.

$52 \times \boxed{3}\,\boxed{}\,\boxed{} = 17{,}212$
$\boxed{}\,\boxed{1} \times \boxed{1}\,\boxed{}\,\boxed{0} = 3{,}990$
$\boxed{4}\,\boxed{} \times \boxed{5}\,\boxed{2}\,\boxed{} = 23{,}144$
$\boxed{}\,\boxed{2}\,\boxed{5} \times \boxed{}\,\boxed{1} = 37{,}275$
$\boxed{}\,\boxed{1}\,\boxed{1} \times \boxed{3}\,\boxed{2}\,\boxed{} = 259{,}520$

Puzzle 4
Fill in digits to make each equation true.
You may only use each digit (0–9) once.

$12 \times \boxed{3}\,\boxed{}\,\boxed{} = 4{,}548$
$\boxed{}\,\boxed{1} \times \boxed{1}\,\boxed{}\,\boxed{0} = 7{,}380$
$\boxed{2}\,\boxed{} \times \boxed{4}\,\boxed{9}\,\boxed{} = 12{,}250$
$\boxed{}\,\boxed{7}\,\boxed{4} \times \boxed{}\,\boxed{5} = 9{,}590$
$\boxed{}\,\boxed{5}\,\boxed{1} \times \boxed{2}\,\boxed{6}\,\boxed{} = 169{,}911$

Five in a Row Multiplication and Division Stage 3 Gameboard

Directions:

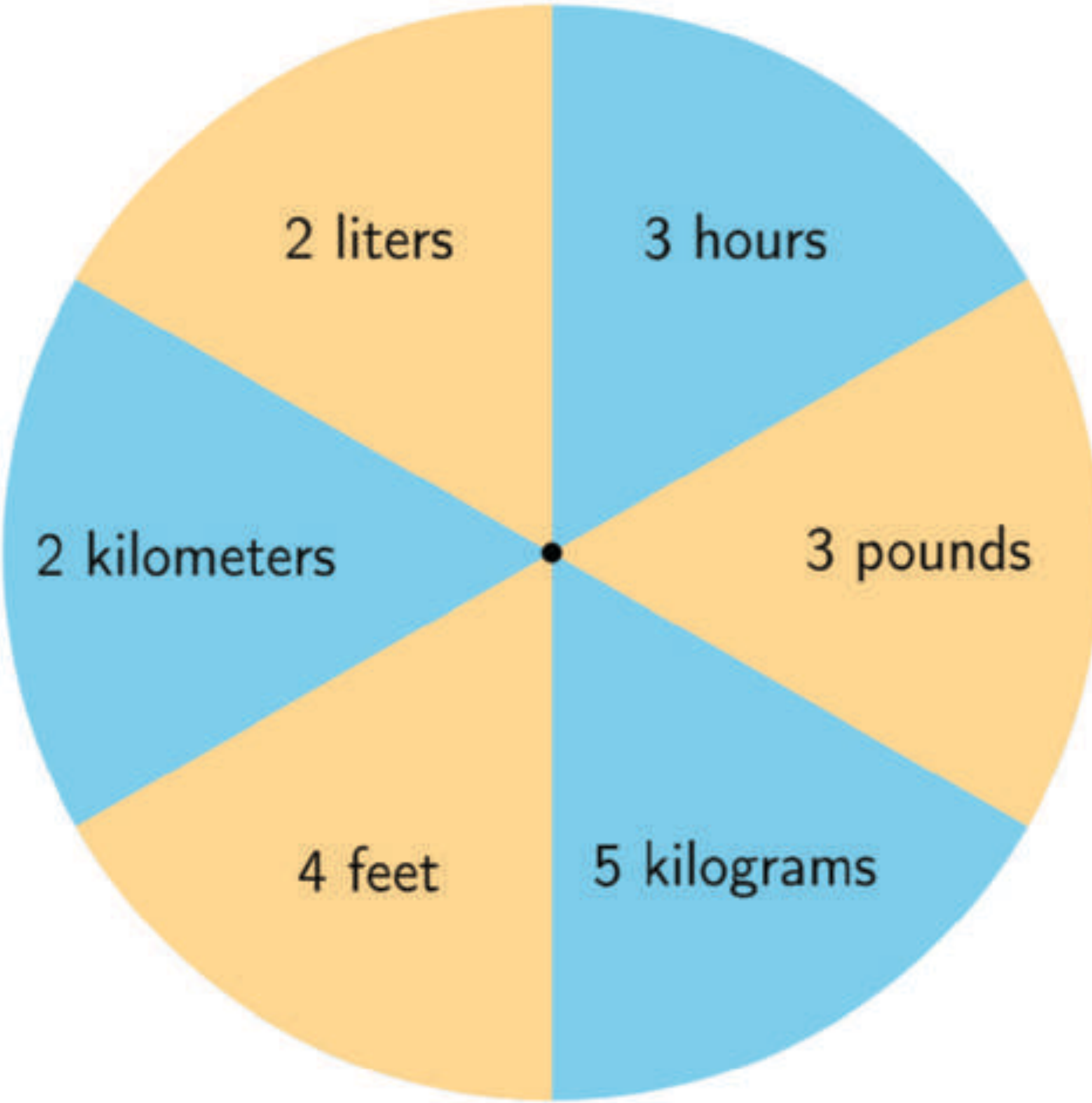
- Partner A:
  - Put a paper clip on 2 numbers in the grey rows. Multiply the numbers. Cover the product of the 2 numbers with a counter.
- Partner B:
  - Move 1 of the paper clips, multiply the numbers, and cover the product with a counter.
- Take turns. The first partner to cover 5 squares in a row wins.

252	294	450	351	360
378	252	312	336	405
315	405	288	273	351
390	273	378	324	450
360	450	294	360	252

12	13	14	15

21	24	27	30

Would You Rather Stage 2 Spinner



Would You Rather Stage 2 Recording Sheet

Directions:

- Partner A:
  - Spin to get a measurement.
  - Ask your partner a question comparing that measurement to an amount in a smaller unit of measurement.

hours - minutes - seconds	liters - milliliters
kilometers - meters- centimeters	pounds - ounces
feet - inches	kilograms - grams

- Partner B:
  - Answer your partner's question.
  - Explain your choice.
- Switch roles and repeat.

Would you rather ____________________ ____________________
verb measurement you spun

or ____________________ ________ ____________________?
verb number measurement unit you chose

Fraction Cards Grade 3

Fraction Cards Grade 3 $\frac{1}{4}$	Fraction Cards Grade 3 $\frac{2}{4}$
Fraction Cards Grade 3 $\frac{3}{4}$	Fraction Cards Grade 3 $\frac{4}{4}$
Fraction Cards Grade 3 $\frac{5}{4}$	Fraction Cards Grade 3 $\frac{1}{6}$
Fraction Cards Grade 3 $\frac{2}{6}$	Fraction Cards Grade 3 $\frac{3}{6}$

Fraction Cards Grade 3

Fraction Cards Grade 3

$$\frac{4}{6}$$

Fraction Cards Grade 3

$$\frac{5}{6}$$

Fraction Cards Grade 3

$$\frac{6}{6}$$

Fraction Cards Grade 3

$$\frac{7}{6}$$

Fraction Cards Grade 3

$$\frac{1}{2}$$

Fraction Cards Grade 3

$$\frac{2}{2}$$

Fraction Cards Grade 3

$$\frac{1}{3}$$

Fraction Cards Grade 3

$$\frac{2}{3}$$

Fraction Cards Grade 3 $\frac{3}{3}$	Fraction Cards Grade 3 $\frac{6}{3}$
Fraction Cards Grade 3 $\frac{4}{2}$	Fraction Cards Grade 3 $\frac{16}{6}$
Fraction Cards Grade 3 $\frac{6}{2}$	Fraction Cards Grade 3 $\frac{8}{2}$
Fraction Cards Grade 3 $\frac{5}{3}$	Fraction Cards Grade 3 $\frac{13}{4}$

Fraction Cards Grade 4

Fraction Cards Grade 4

$$\frac{1}{8}$$

Fraction Cards Grade 4

$$\frac{2}{8}$$

Fraction Cards Grade 4

$$\frac{3}{8}$$

Fraction Cards Grade 4

$$\frac{4}{8}$$

Fraction Cards Grade 4

$$\frac{5}{8}$$

Fraction Cards Grade 4

$$\frac{6}{8}$$

Fraction Cards Grade 4

$$\frac{7}{8}$$

Fraction Cards Grade 4

$$\frac{8}{8}$$

Fraction Cards Grade 4

Fraction Cards Grade 4 $\frac{1}{5}$	Fraction Cards Grade 4 $\frac{2}{5}$
Fraction Cards Grade 4 $\frac{3}{5}$	Fraction Cards Grade 4 $\frac{4}{5}$
Fraction Cards Grade 4 $\frac{5}{5}$	Fraction Cards Grade 4 $\frac{6}{5}$
Fraction Cards Grade 4 $\frac{1}{10}$	Fraction Cards Grade 4 $\frac{2}{10}$

Fraction Cards Grade 4

$$\frac{3}{10}$$

Fraction Cards Grade 4

$$\frac{4}{10}$$

Fraction Cards Grade 4

$$\frac{5}{10}$$

Fraction Cards Grade 4

$$\frac{6}{10}$$

Fraction Cards Grade 4

$$\frac{7}{10}$$

Fraction Cards Grade 4

$$\frac{8}{10}$$

Fraction Cards Grade 4

$$\frac{9}{10}$$

Fraction Cards Grade 4

$$\frac{10}{10}$$

Fraction Cards Grade 4

$$\frac{11}{10}$$

Fraction Cards Grade 4

$$\frac{19}{10}$$

Fraction Cards Grade 4

$$\frac{1}{12}$$

Fraction Cards Grade 4

$$\frac{3}{12}$$

Fraction Cards Grade 4

$$\frac{4}{12}$$

Fraction Cards Grade 4

$$\frac{7}{12}$$

Fraction Cards Grade 4

$$\frac{9}{12}$$

Fraction Cards Grade 4

$$\frac{10}{12}$$

Fraction Cards Grade 4

$$\frac{13}{12}$$

Fraction Cards Grade 4

$$\frac{15}{12}$$

Fraction Cards Grade 4

$$\frac{1}{100}$$

Fraction Cards Grade 4

$$\frac{5}{100}$$

Fraction Cards Grade 4

$$\frac{10}{100}$$

Fraction Cards Grade 4

$$\frac{20}{100}$$

Fraction Cards Grade 4

$$\frac{49}{100}$$

Fraction Cards Grade 4

$$\frac{50}{100}$$

Fraction Cards Grade 4

$$\frac{51}{100}$$

Fraction Cards Grade 4

$$\frac{75}{100}$$

Fraction Cards Grade 4

$$\frac{99}{100}$$

Fraction Cards Grade 4

$$\frac{200}{100}$$

Compare Stage 3-8 Directions

Directions:

- Split the deck between the players.
- Each player turns over a card.
- Compare the values. The player with the greater value keeps both cards.
- If the values are the same, each player turns over one more card. The player with the greater value keeps all four cards.
- Play until you run out of cards. The player with the most cards at the end of the game wins.

Record any sets of cards that are challenging to compare:

Compare Stage 3-8 Directions

Directions:

- Split the deck between the players.
- Each player turns over a card.
- Compare the values. The player with the greater value keeps both cards.
- If the values are the same, each player turns over one more card. The player with the greater value keeps all four cards.
- Play until you run out of cards. The player with the most cards at the end of the game wins.

Record any sets of cards that are challenging to compare:

Would You Rather Stage 3 Spinner

Would You Rather Stage 3 Recording Sheet

Directions:

- Partner A:
  - Spin to get a measurement.
  - Ask your partner a question comparing that measurement to a measurement in a different unit in the same system.

kilometers - meters- centimeters	pounds - ounces
liters - milliliters	inches - feet - yards
gallons - cups - pints - quarts	seconds - minutes - hours - days

- Partner B:
  - Answer your partner's question.
  - Explain your choice.
- Switch roles and repeat.

Would you rather ____________________ ______ ____________________
verb number unit you spun

or ______ ________________________ ?
number unit you chose

Compare Stage 6 Cards

Compare Stage 6

$$\frac{4}{6}+\frac{1}{6}$$

Compare Stage 6

$$\frac{2}{4}-\frac{1}{4}$$

Compare Stage 6

$$\frac{2}{5}+\frac{4}{10}$$

Compare Stage 6

$$\frac{3}{6}-\frac{1}{3}$$

Compare Stage 6

$$\frac{4}{6}+\frac{4}{12}$$

Compare Stage 6

$$\frac{5}{8}-\frac{1}{2}$$

Compare Stage 6

$$\frac{7}{10}+\frac{35}{100}$$

Compare Stage 6

$$\frac{8}{10}-\frac{64}{100}$$

Compare Stage 6 Cards

Compare Stage 6

$$\frac{8}{10} + \frac{26}{100}$$

Compare Stage 6

$$\frac{7}{10} - \frac{59}{100}$$

Compare Stage 6

$$\frac{5}{10} + \frac{43}{100}$$

Compare Stage 6

$$\frac{9}{10} - \frac{72}{100}$$

Compare Stage 6

$$2\frac{2}{5} + 3\frac{3}{5}$$

Compare Stage 6

$$1\frac{4}{6} + 4\frac{1}{6}$$

Compare Stage 6

$$3\frac{1}{4} - \frac{2}{4}$$

Compare Stage 6

$$4\frac{3}{5} - 2\frac{4}{5}$$

Compare Stage 6 Cards

Compare Stage 6

$$5\frac{2}{12} - \frac{7}{12}$$

Compare Stage 6

$$2 + \frac{3}{6} + \frac{4}{6}$$

Compare Stage 6

$$3\frac{1}{4} - 1\frac{3}{4}$$

Compare Stage 6

$$5\frac{2}{7} + \frac{4}{7} + \frac{3}{7}$$

Compare Stage 6

$$1 - \frac{3}{12} - \frac{5}{12}$$

Compare Stage 6

$$1 - \frac{5}{8}$$

Compare Stage 6

$$\frac{9}{10} + \frac{4}{10} + \frac{5}{10}$$

Compare Stage 6

$$8 + \frac{6}{9}$$

Compare Stage 6 Cards

Compare Stage 6

$$\frac{3}{4} + \frac{6}{4} + 1$$

Compare Stage 6

$$1 - \frac{2}{9} - \frac{6}{9}$$

Compare Stage 6

$$1\frac{3}{8} - \frac{6}{8}$$

Compare Stage 6

$$6\frac{5}{8} - 2\frac{7}{8}$$

How Close? Stage 9 Recording Sheet

Directions:

- Each partner:
  - Take 6 cards.
  - Choose 4 cards to make an addition expression.
  - Write an equation to show the sum of the numbers you made.
  - Your score for each round is the difference between your sum and 5.
- Take new cards so that you have 6 cards to start the next round.
- At the end of the game, add your score for each round. The player with the lowest score wins.

round	addition equation	points for the round
1	$\frac{\square}{\square} + \frac{\square}{\square} =$ ______	
2	$\frac{\square}{\square} + \frac{\square}{\square} =$ ______	
3	$\frac{\square}{\square} + \frac{\square}{\square} =$ ______	
4	$\frac{\square}{\square} + \frac{\square}{\square} =$ ______	

How Close? Stage 9 Recording Sheet

5	$\dfrac{\square}{\square} + \dfrac{\square}{\square} =$ ________	
6	$\dfrac{\square}{\square} + \dfrac{\square}{\square} =$ ________	
7	$\dfrac{\square}{\square} + \dfrac{\square}{\square} =$ ________	
8	$\dfrac{\square}{\square} + \dfrac{\square}{\square} =$ ________	

Creating Line Plots Stage 3 Recording Sheet

Directions:

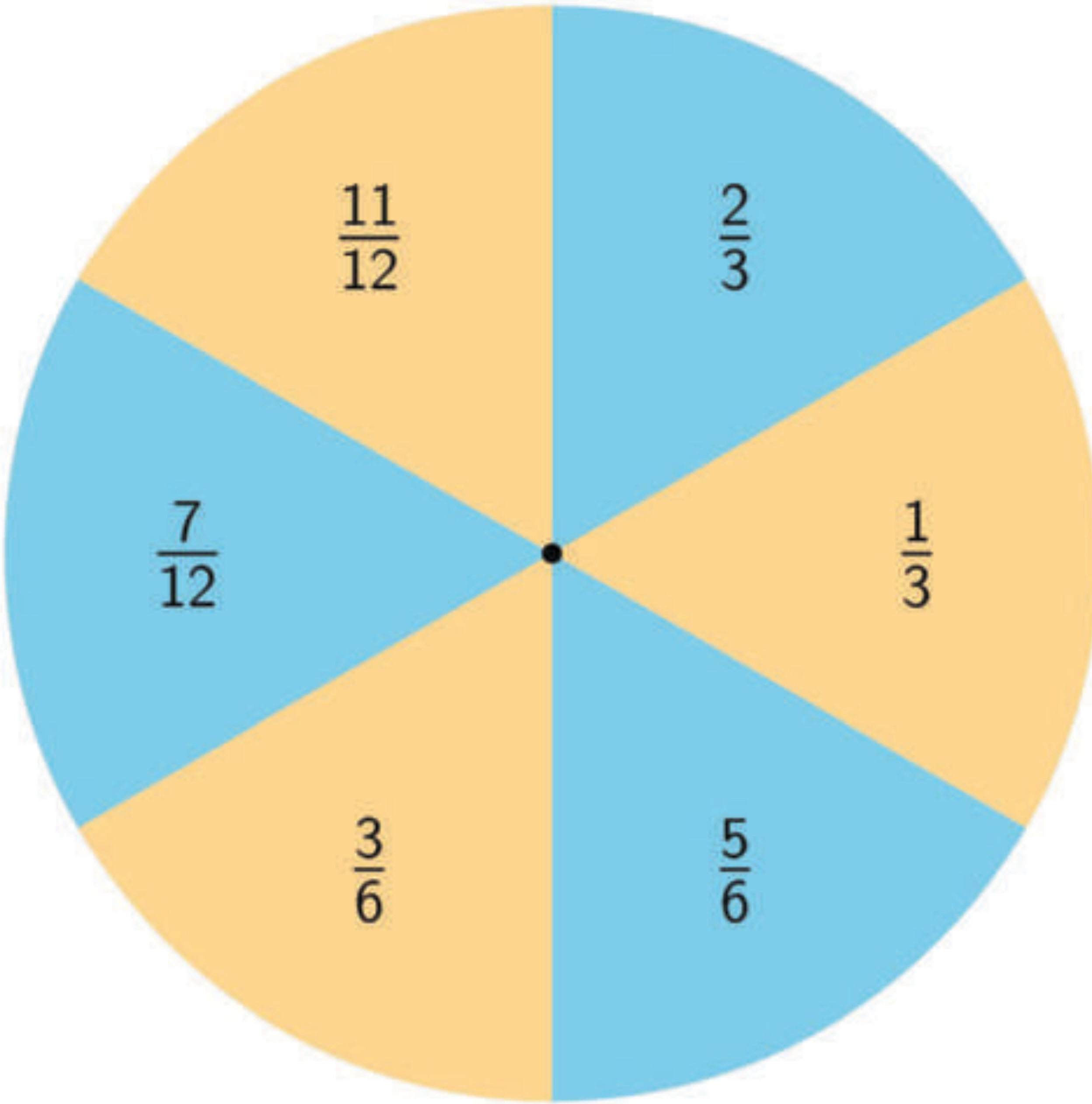
- Measure up to 8 objects to the nearest $\frac{1}{8}$ inch.
- Create a line plot of your measurement data. Don't forget to add a title and label.
- Ask your partner 2 questions that can be answered based on the data in your line plot using addition or subtraction.

Creating Line Plots Stage 4 Recording Sheet

Directions:

- Measure up to 8 objects to the nearest $\frac{1}{8}$ inch.
- Create a line plot of your measurement data. Don’t forget to add a title and label.
- Ask your partner 2 questions that can be answered based on the data in your line plot that use addition, subtraction, or multiplication.

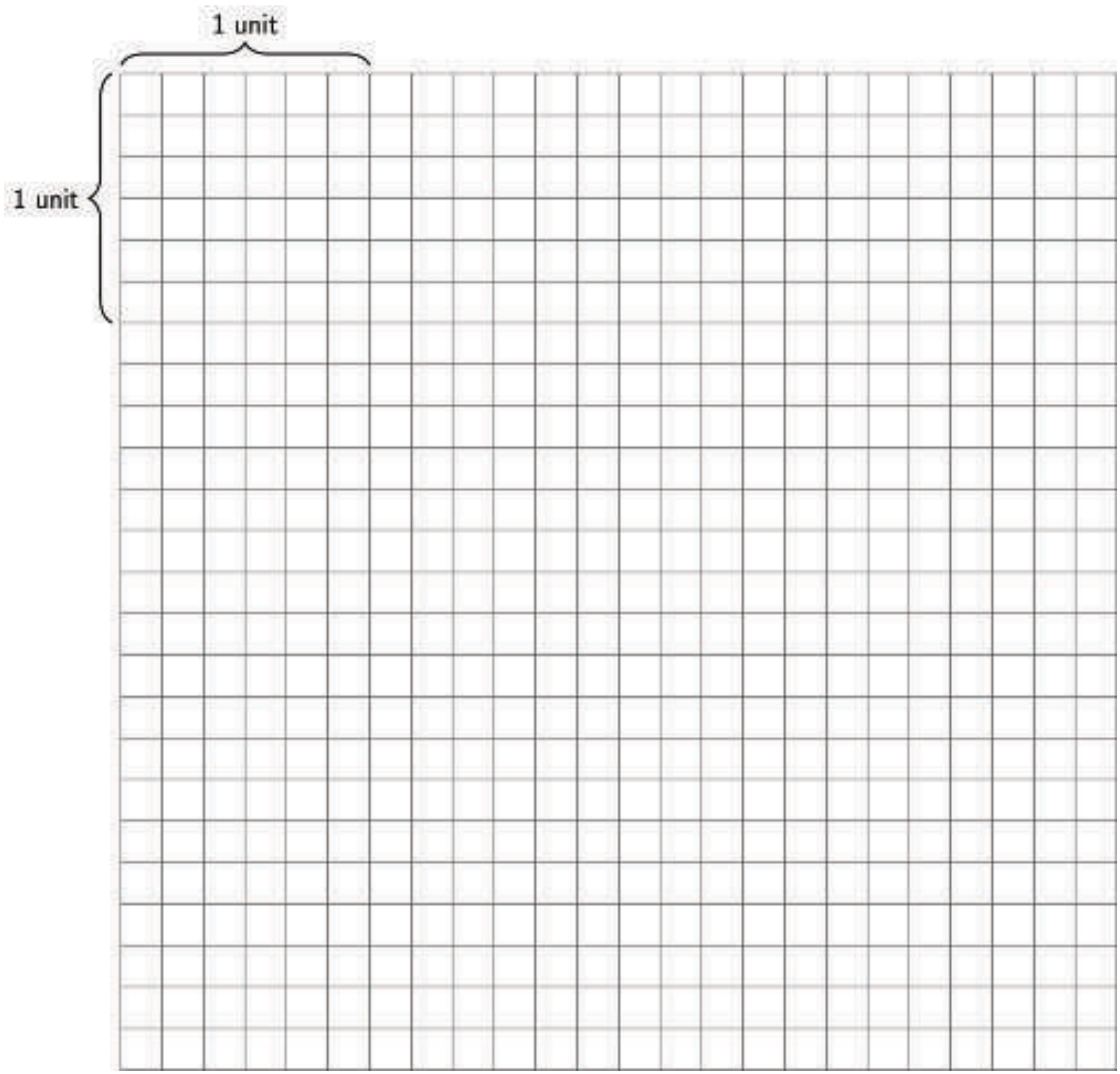
Rectangle Rumble Stage 4 Spinner



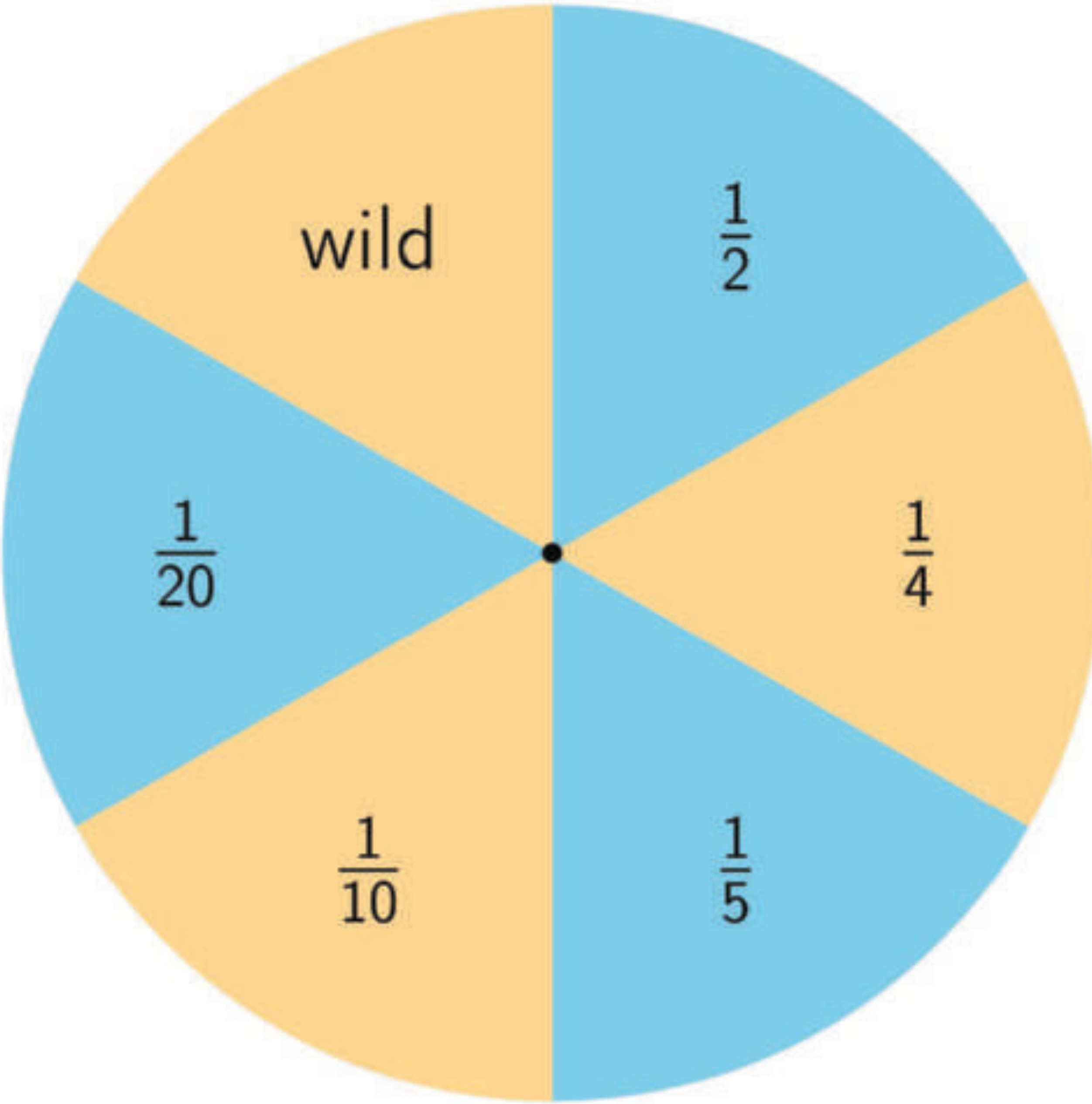
Rectangle Rumble Stage 4 Grid

Directions:

- Choose a color for your rectangles different from your partner.
- On your turn:
  - Spin the spinner and roll the number cube.
  - Shade in a rectangular area to represent the product of the two numbers.
- Take turns until the grid can't fit any more rectangles.
- Each partner adds up their total area, the partner with the greatest total square units wins.



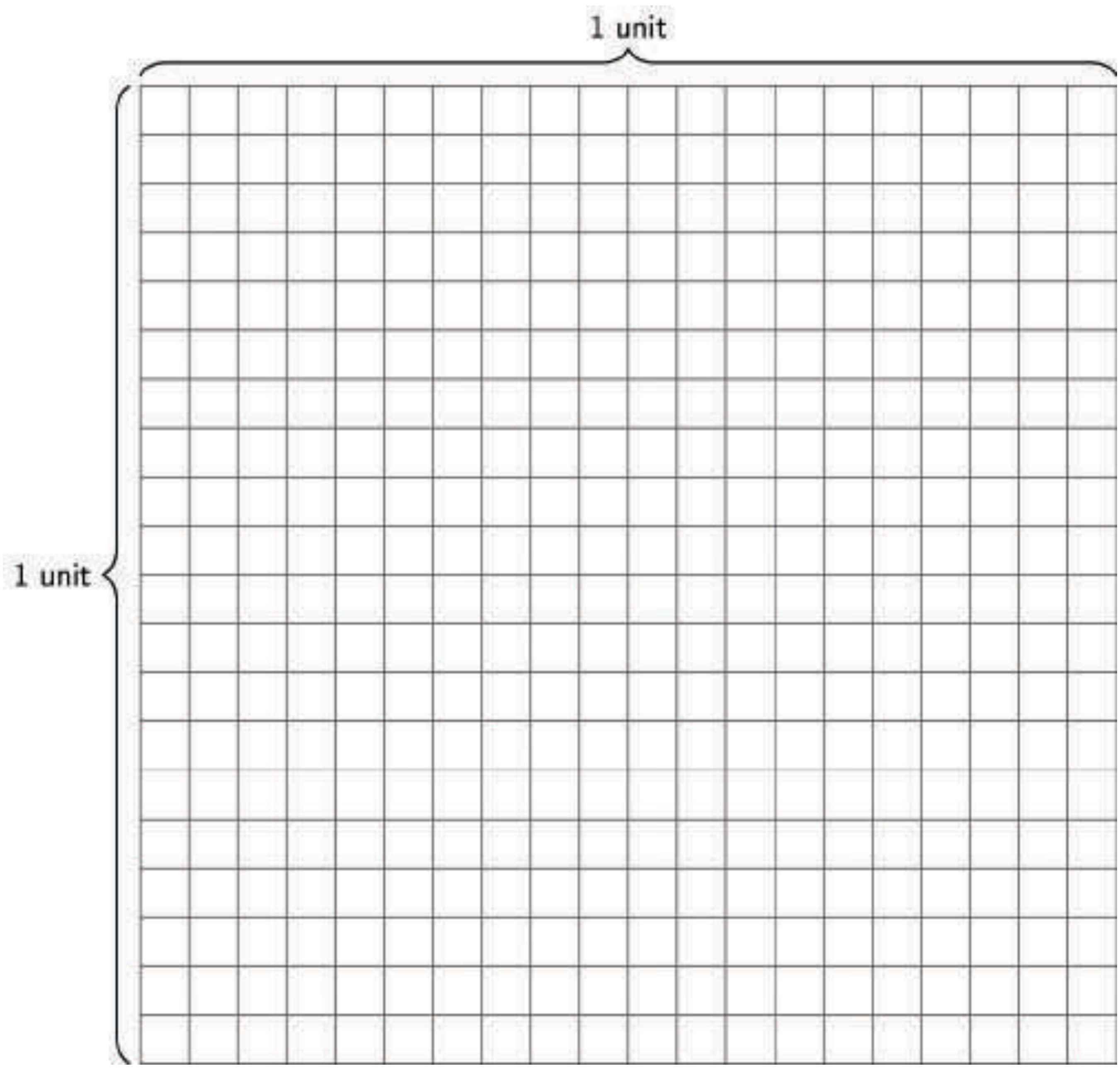
Rectangle Rumble Stage 5 Spinners



Rectangle Rumble Stage 5 Grid

Directions:

- Choose a color for your rectangles different from your partner.
- On your turn:
    - Spin the spinner and roll the number cube.
    - Shade in a rectangular area to represent the product of the two numbers.
- Take turns until the grid can't fit any more rectangles.
- Each partner adds up their total area, the partner with the greatest total square units wins.



Rolling for Fractions Stage 4 Recording Sheet

Each partner:

- ○ Roll 4 number cubes. Use the numbers to complete the expression and write the product.
- ○ Check your partner's work to make sure you agree.
- ○ Determine the number of points each partner gets:
  - 5 points for the largest product
  - 3 points for the smallest product

Repeat for the next round. The partner who has the most points once the recording sheet is full wins the game.

round	**equation**	**points**
1	$\frac{\square}{\square} \times \frac{\square}{\square} =$	
2	$\frac{\square}{\square} \times \frac{\square}{\square} =$	
3	$\frac{\square}{\square} \times \frac{\square}{\square} =$	
4	$\frac{\square}{\square} \times \frac{\square}{\square} =$	
5	$\frac{\square}{\square} \times \frac{\square}{\square} =$	
6	$\frac{\square}{\square} \times \frac{\square}{\square} =$	

Rolling for Fractions Stage 4 Recording Sheet

Each partner:

- Roll 3 number cubes. Use the numbers to complete the expression and write the product.
- Check your partner’s work to make sure you agree.
- Determine the number of points each partner gets:
  - 5 points for the largest product
  - 3 points for the smallest product

Repeat for the next round. The partner who has the most points once the recording sheet is full wins the game

round	**equation**	**points**
1	$\frac{1}{\square} \times \frac{\square}{\square} =$	
2	$\frac{1}{\square} \times \frac{\square}{\square} =$	
3	$\frac{1}{\square} \times \frac{\square}{\square} =$	
4	$\frac{1}{\square} \times \frac{\square}{\square} =$	
5	$\frac{1}{\square} \times \frac{\square}{\square} =$	
6	$\frac{1}{\square} \times \frac{\square}{\square} =$	

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Putting it All Together

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