



Core Knowledge® MATHEMATICS

More Decimal and Fraction Operations

Student Workbook



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More Decimal and Fraction Operations

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More Decimal and Fraction Operations Student Workbook

Core Knowledge Mathematics™

Lesson 1: Place Value Patterns

- Let's observe place value patterns.

Warm-up: Notice and Wonder: Same Digits

What do you notice? What do you wonder?

8,200

820

82

8.2

0.82

0.082

1.1: Many True Equations

Use the numbers and symbols to write as many different true equations as you can.
You may use each number and symbol more than once.

600	0.06	100
60	×	10
6	÷	0.1
0.6	=	0.01

1.2: Describe Multiplicative Relationships

600
60
6
0.6
0.06

1. Explain or show how the value of the 6 changes in the different numbers.

2. Which numbers would come before 600 if the list continued? Explain your reasoning.

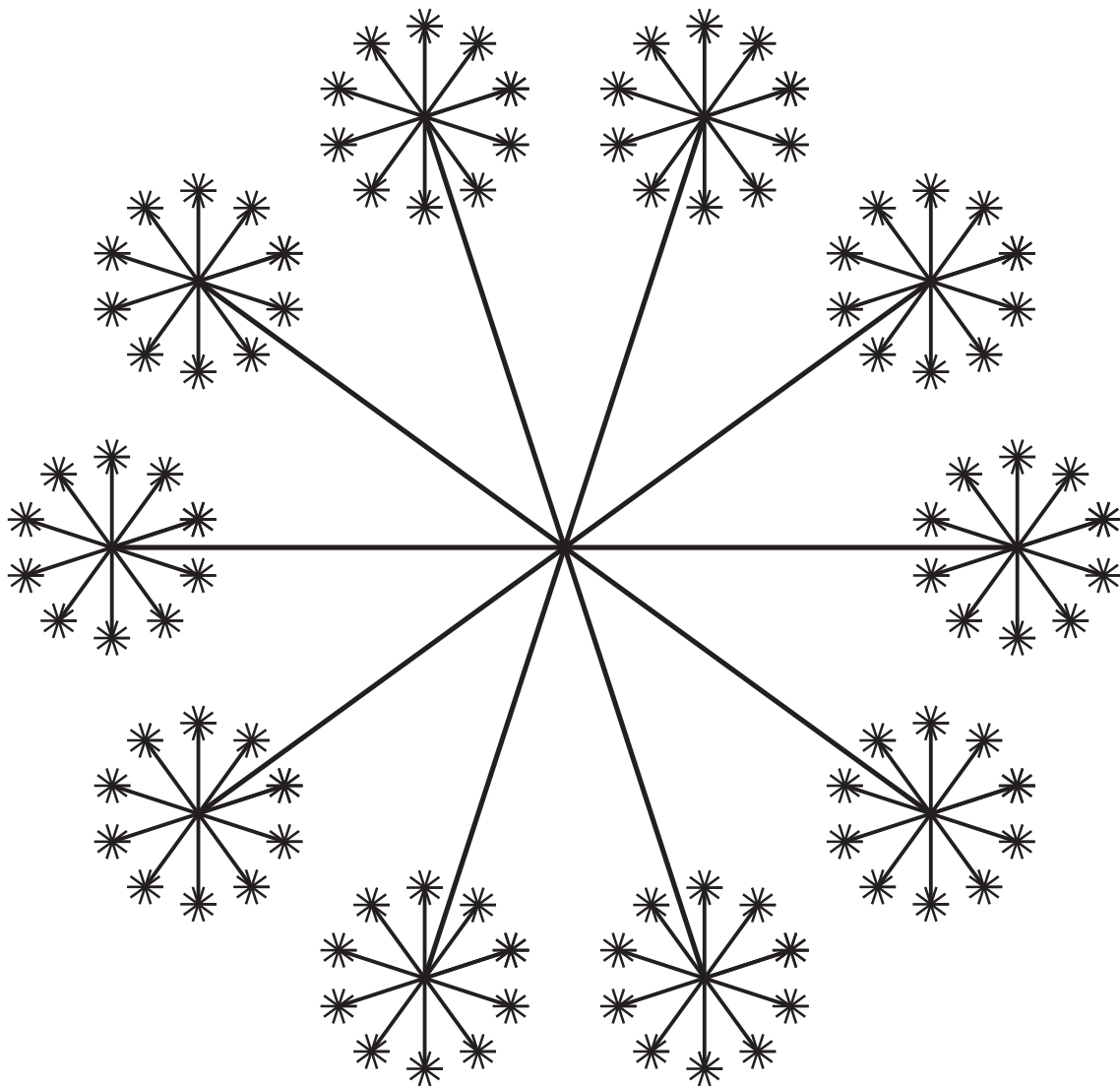
3. Which numbers would come after 0.06 if the list continued? Explain your reasoning.

Lesson 2: Powers of 10

- Let's use exponents to show powers of 10.

Warm-up: How Many Do You See: Starburst

How many do you see? How do you see them?



2.1: Population of Delaware and India

1. About 1,000,000 people live in Delaware.

a. How do you say this number?

b. How many thousands is this? Explain or show your reasoning.

c. Write the number using powers of 10.

d. How many times would you need to extend the diagram from the warm-up to get 1,000,000 tiny segments? Explain or show your reasoning.

2. In 1997, the population of India was about 1,000,000,000.

a. How would you say this number?

b. How many millions is this? How many thousands is it? Explain or show your reasoning.

c. Write the number using powers of 10.

d. How many times would you need to extend the diagram from the warm-up to get 1,000,000,000 tiny segments? Explain or show your reasoning.

2.2: Powers of 10

1. Find the missing number that makes each equation true. Show your reasoning.

a. $2,000 = \underline{\hspace{2cm}} \times 20$

b. $20 \times 10 \times \underline{\hspace{2cm}} = 20,000$

c. $\underline{\hspace{2cm}} \times 10 = 100,000$

d. $1,000 \times 10,000 = \underline{\hspace{2cm}}$

2. How were products of 10s useful in solving these problems?

3. Write each power of 10 as a number.

a. 10^3

b. 10^4

c. 10^7

2.3: Beyond a Billion

1. How would you say the number 1,000,000,000,000?
2. How many billions is that? How many millions is it? Explain or show your reasoning.
3. Write the number using powers of 10.
4. Describe an example of something that there are 1,000,000,000,000 of in the world.

Lesson 3: Metric Conversion and Multiplication by Powers of Ten

- Let's notice patterns in metric measurements.

Warm-up: Number Talk

Find the value of each expression mentally.

- 100×1.5

- $1,000 \times 1.5$

- $15 \div 10$

- $15 \div 100$

3.1: How Tall? How Long? How Far?



1. Complete the table.

meters	centimeters	millimeters
1		
10		
10^2		

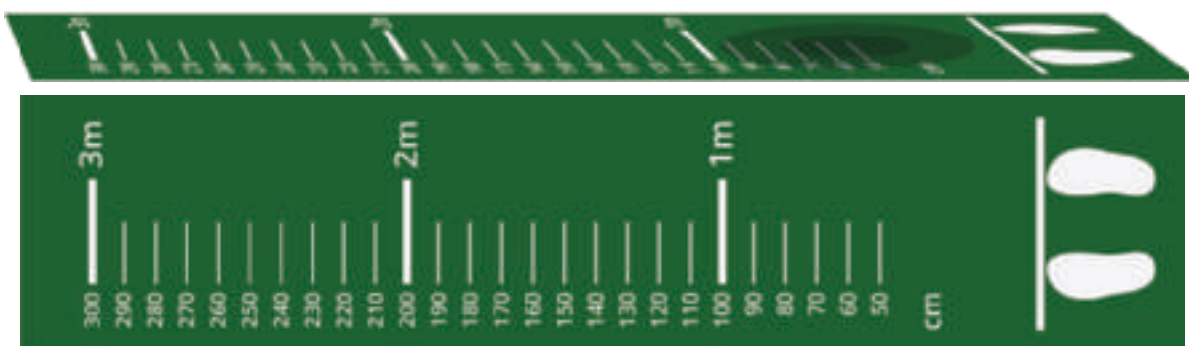
2. What patterns do you notice in the table?

3. Three long-distance races are 10 kilometers, 100 kilometers, and 1,000 kilometers. How many meters are there in these races?

distance in kilometers	distance in meters
1	1,000
10	
100	
10^3	

4. What patterns do you notice in the table?

3.2: Broad Jump



Here are the distances that each student jumped.

student	distance
Mai	1.61 meters
Tyler	1.43 meters
Clare	1.57 meters

1. The average standing broad jump distance for 5th graders is 148 centimeters. Are each of the students in the table below, at, or above the average? Explain or show your reasoning.

2. The world record for the standing broad jump is 337 centimeters. Jada says that's more than Mai and Clare jumped combined. Do you agree with Jada? Explain or show your reasoning.

3. Tyler says his jump sounds more impressive if he reports it in millimeters.
a. How far is Tyler's jump in millimeters? What about Mai's and Clare's jumps?

b. Which unit do you think is best for reporting the jumps? Explain your reasoning.

Lesson 4: Metric Conversion and Division by Powers of Ten

- Let's convert units.

Warm-up: True or False: Divide by a Hundred and a Thousand

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $5 \div 1,000 = 0.05$
- $36 \div 100 = 0.36$
- $1,328 \div 1,000 = 1.328$

4.1: Long Jump, Javelin Throw, and Shot Put

athlete	long jump	javelin throw	shot put
Jackie Joyner-Kersey, USA	727 cm	4,566 cm	1,580 cm
Sabine John, Germany	671 cm	4,256 cm	1,623 cm
Anke Behmer, Germany	678 cm	4,454 cm	1,420 cm

1. Below are some results Jackie Joyner-Kersey recorded in different events in 1988. Complete the table.

event	centimeters	meters
long jump	727	
javelin throw	4,566	
shot put	1,580	

2. Which unit of measure is most helpful when you picture each distance, centimeters or meters? Explain or show your reasoning.

3. Why do you think that the distances are measured to the nearest centimeter?



4.2: Hurdles

1. The table shows how many meters some students ran during a week. Complete the table to show how many kilometers each student ran.

student	distance (meters)	distance (kilometers)
Diego	9,513	
Clare	11,018	
Priya	8,210	
Andre	10,000	

2. What patterns do you notice in the table?

3. Below is Tyler's strategy to divide a whole number by 10, 100, or 1,000.

I find the quotient by shifting the digits to the right — once when I divide by 10, twice when I divide by 100, 3 times when I divide by 1,000.

$$5,632 / 10 = 563.2$$

$$5,632 / 100 = 56.32$$

$$5,632 / 1,000 = 5.632$$

Describe to your partner what Tyler means.

(Pause for teacher direction.)

4. Why does Tyler's strategy work? Will Tyler's strategy always work? Explain or show your reasoning.

Lesson 5: Multi-step Conversion Problems: Metric Length

- Let's solve multi-step problems about metric length.

Warm-up: True or False: Powers of 10

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $5,423 \times 10 = 50,423$

- $5,423 \div 10 = 542.3$

- $5,423 \div 100 = 54.23$

5.1: Walk All Day

Lin has a watch that counts the number of steps she takes during the day and displays those steps in centimeters, meters, or kilometers.

- Here is a list of activities Lin did on Monday. Next to each activity, write whether it would make sense to display the distance in cm, m, or km.
 - walked to her friend's desk
 - walked to the front of the classroom
 - walked from her classroom to the bus
 - ran twice around the playground
- The table shows the amount of steps Lin's watch displayed for each activity. If each of her steps is 50 centimeters, how many centimeters and meters did Lin walk for each activity?

activity	number of steps	distance (cm)	distance (m)
walked to her friend's desk	5		
walked to the front of the classroom	12		
walked from her classroom to the bus	250		
ran twice around the playground	1,000		

3. At the end of the day, Lin's watch displayed 8,500 steps. Would it make sense for her watch to record the distance in centimeters, meters, or kilometers? Why?

4. How many kilometers did Lin walk that day?



5.2: Who Ran Farther?

1. Use the table to find the total distance Tyler ran during the week. Explain or show your reasoning.

day	distance (km)
Monday	8.5
Tuesday	6.25
Wednesday	10.3
Thursday	5.75
Friday	9.25

2. Use the table to find the total distance Clare ran during the week. Show your reasoning.

day	distance (m)
Monday	5,400
Tuesday	7,500
Wednesday	8,250
Thursday	6,750
Friday	7,250

3. Who ran farther, Clare or Tyler? How much farther? Explain or show your reasoning.

Lesson 6: Multi-step Conversion Problems: Metric Liquid Volume

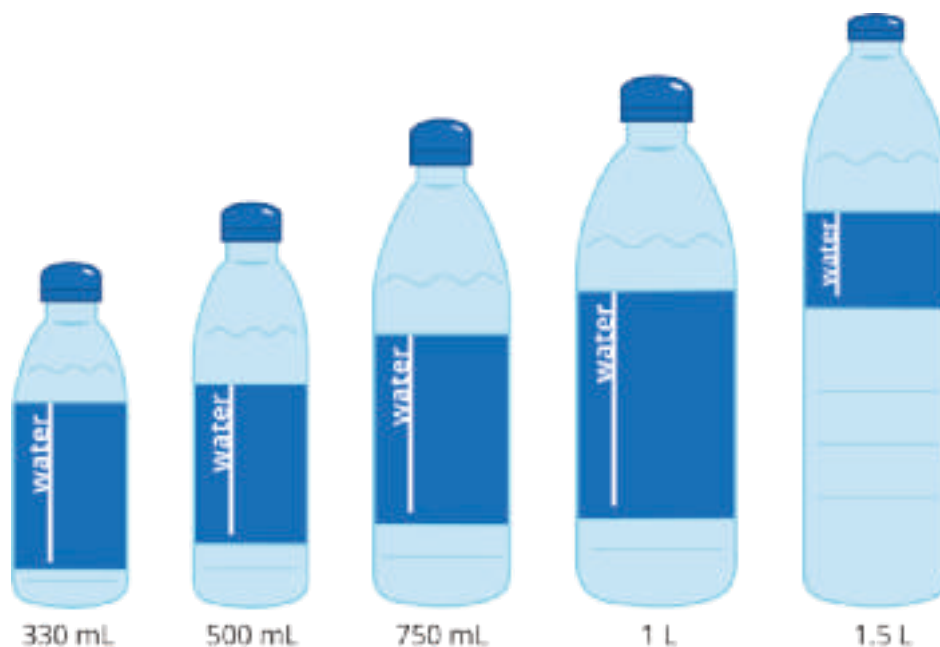
- Let's solve multi-step problems about metric liquid volume.

Warm-up: Number Talk: Divide by Powers of 10

Find the value of each expression mentally.

- $1,400 \div 10$
- $1,400 \div 100$
- $1,400 \div 1,000$
- $1,401 \div 1,000$

6.1: Liquid Volume Conversions



1. Complete the table.

L	mL
5	
6.3	
0.95	
10^2	
	800,000
	10^6
	65

2. Decide if the two measurements are equal. If not, choose which one is greater.
Explain or show your reasoning.

a. 15 mL and 0.15 L

b. 2,500 mL and 2.5 L

c. 200 mL and $\frac{1}{4}$ L

d. 1 mL and $\frac{1}{1,000}$ L

e. 15,600 mL and 15.5 L

6.2: Rehydrating Dancers

There are 25 dancers in the performance group. During practice, each dancer drinks $1\frac{1}{2}$ bottles of water.

1. Each bottle holds 500 mL of water. How many liters of water do the dancers drink? Explain or show your reasoning.

2. Each cooler holds 15 L of water. How many coolers does the team need? How much water will they have left over after practice? Explain or show your reasoning.



3. The dancers can make a sports drink by mixing 30 mL of drink mix with each 500 mL of water. How many liters of drink mix does the team need for their practice? Explain or show your reasoning.

Lesson 7: Multi-step Conversion Problems: Customary Length

- Let's solve multi-step problems about customary length.

Warm-up: Number Talk: Multiples of 12

Find the value of each expression mentally.

- 45×10

- 45×2

- 45×12

- 46×12

7.1: Card Sort: Customary Measurements

1. Your teacher will give you a set of cards that show different measurements. Sort the cards into 2 categories of your choosing. Be prepared to explain the meaning of your categories.

(Pause for teacher directions.)

2. Match the cards with equal measurements. Then, list the groups of matching measurements in increasing order.

7.2: Run a Mile or Two

1. A rectangular field is 90 yards long and $42\frac{1}{4}$ yards wide. Priya says that 6 laps around the field is more than a mile. Do you agree with Priya? Explain or show your reasoning.



2. A different rectangular field is $408\frac{1}{2}$ feet long and $240\frac{1}{4}$ feet wide. How many laps around this field would Priya need to run if she wants to run at least 2 miles?

Section Summary

Section Summary

In this section we studied powers of 10 and conversions between units. We learned that we can write a product of 10s like

$$10 \times 10 \times 10 \times 10$$

as 10^4 . The number 4 is an exponent and it means that there are 4 factors of 10.

We also converted between different measurement units, mostly metric lengths. For example, there are 1,000 millimeters in a meter and 1,000 meters in a kilometer. This means that there are $1,000 \times 1,000$ or 1,000,000 millimeters in a kilometer. We could also say that there are 10^6 millimeters in a kilometer. We also used our understanding of decimals to make conversions. For example, since there are 1,000 meters in a

kilometer that means that each meter is $\frac{1}{1,000}$ or 0.001 kilometers. So 853 meters can also be written as 0.853 kilometers.

Lesson 8: Add and Subtract Fractions

- Let's add and subtract fractions.

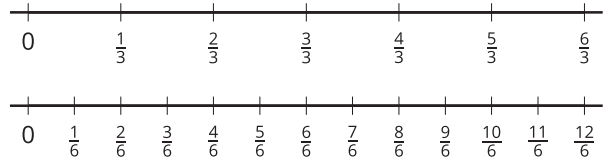
Warm-up: Which One Doesn't Belong: Fraction Representations

Which one doesn't belong?

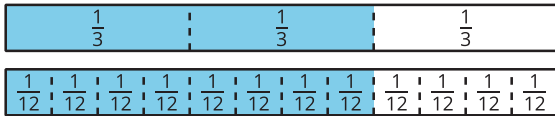
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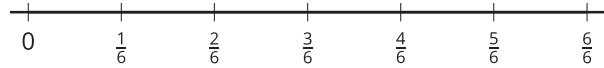
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8.1: Card Sort: Fraction Sums and Differences

Your teacher will give you a set of cards that show expressions.

1. Sort the cards into 2 categories of your choosing. Be prepared to explain the meaning of your categories.
2. Sort the cards into 2 categories in a different way. Be prepared to explain the meaning of your new categories.

8.2: Add and Subtract

Find the value of each expression. Explain or show your thinking.

1. $\frac{2}{3} + \frac{2}{3}$

2. $\frac{2}{3} - \frac{1}{6}$

3. $\frac{2}{3} + \frac{1}{2}$

Lesson 9: Use Equivalent Expressions

- Let's use equivalent expressions to add and subtract fractions with unlike denominators.

Warm-up: True or False: Fraction Addition and Subtraction

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$

- $\frac{1}{2} + \frac{1}{4} = \frac{2}{4}$

- $\frac{3}{4} - \frac{1}{2} = \frac{2}{4}$

9.1: Equal Sums

1. Explain or show why each expression is equivalent to $\frac{2}{3} + \frac{10}{12}$.

○ $\frac{8}{12} + \frac{10}{12}$

○ $\frac{4}{6} + \frac{5}{6}$

2. Find the value of the expression $\frac{2}{3} + \frac{10}{12}$. Explain or show your reasoning.

9.2: Find the Value of the Difference

1. Find the value of the expression $\frac{16}{12} - \frac{3}{6}$. Explain or show your reasoning.
2. Compare your strategy with your partner's strategy. What is the same? What is different?

9.3: Grow Plants

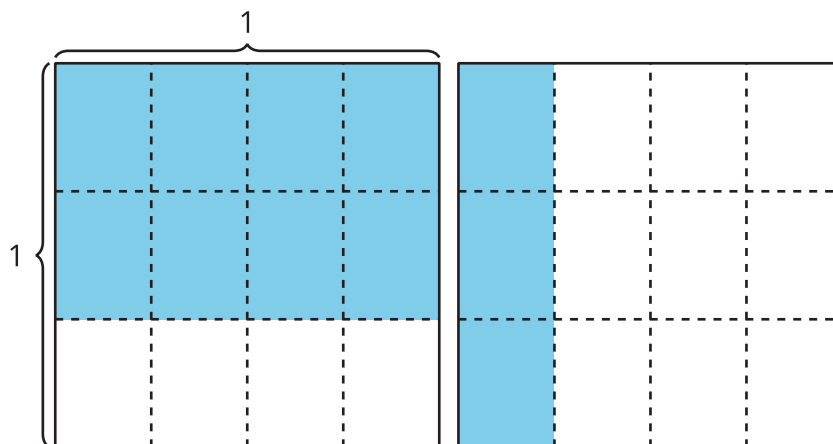
Jada and Andre compare the growth of their plants. Jada's plant grew $1\frac{3}{4}$ inches since last week. Andre's plant grew $\frac{7}{8}$ inches. How much more did Jada's plant grow? Explain or show your reasoning.

Lesson 10: All Sorts of Denominators

- Let's find common denominators.

Warm-up: How Many Do You See: Fraction Sum

How many do you see? How do you see them?



10.1: Different Denominators

Find the value of each expression. Explain or show your thinking.

1. $\frac{3}{4} + \frac{7}{8}$

2. $\frac{3}{4} + \frac{4}{6}$

3. $\frac{3}{4} - \frac{2}{5}$

10.2: Multiply Denominators

1. Here is Lin's strategy for finding the value of $\frac{2}{5} + \frac{4}{9}$: "I know 5×9 is a common denominator so I'll use that." Does Lin's strategy for finding a common denominator work? Explain or show your thinking and then find the value of $\frac{2}{5} + \frac{4}{9}$.

2. Find the value of each expression using a method that makes sense to you.

a. $\frac{3}{8} + \frac{1}{5}$

b. $\frac{7}{10} - \frac{2}{3}$

c. $\frac{7}{20} + \frac{41}{50}$

d. $\frac{2}{9} - \frac{1}{6}$

Lesson 11: Different Ways to Subtract

- Let's subtract fractions and mixed numbers.

Warm-up: Number Talk: Mixed Number Addition and Subtraction

Find the value of each expression mentally.

- $3 + \frac{7}{8}$

- $3 - \frac{7}{8}$

- $1\frac{5}{8} + \frac{6}{8}$

- $1\frac{5}{8} - \frac{6}{8}$

11.1: Challenging Differences

1. Circle all of the expressions that are equivalent to $3\frac{5}{8}$. Explain or show your reasoning.

☐ $\frac{20}{8}$

☐ $2\frac{13}{8}$

☐ $3\frac{10}{16}$

2. Find the value of each expression. Explain or show your reasoning.

☐ $3\frac{5}{8} - \frac{3}{16}$

☐ $3\frac{5}{8} - 1\frac{15}{16}$

☐ $3\frac{5}{8} - 1\frac{12}{16}$

11.2: Find the Difference

Find the value of each difference. Explain or show your reasoning.

1. $9\frac{1}{8} - 8\frac{8}{9}$

2. $3\frac{1}{2} - \frac{10}{4}$

3. $4\frac{3}{5} - 1\frac{2}{3}$

Lesson 12: Solve Problems

- Let’s solve more problems by adding and subtracting fractions with unlike denominators.

Warm-up: Estimation Exploration: Large Denominators

What is the value of the sum?

$$\frac{3}{17} + \frac{17}{19}$$

Record an estimate that is:

too low	about right	too high

12.1: Priya's Salad Dressing

Priya's Salad Dressing Recipe

- $\frac{3}{4}$ cup olive oil
- $\frac{1}{3}$ cup lemon juice
- $\frac{1}{2}$ cup mustard
- Pinch of salt and pepper

1. Priya has $\frac{2}{3}$ cup of olive oil. She is going to borrow some more from her neighbor. How much olive oil does she need to borrow to have enough to make the dressing?

2. 1 tablespoon is equal to $\frac{1}{16}$ of a cup. Priya decides that 1 tablespoon of olive oil is close enough to what she needs to borrow from her neighbor. Do you agree with Priya? Explain or show your reasoning.

3. Priya says her recipe will make about $1\frac{1}{2}$ cups of dressing. Do you agree? Explain or show your reasoning.

12.2: More Problems to Solve

1. Choose a problem to solve.

Problem A:

Jada is baking protein bars for a hike. She adds $\frac{1}{2}$ cup of walnuts and then decides to add another $\frac{1}{3}$ cup. How many cups of walnuts has she added altogether?

If the recipe requires $1\frac{1}{3}$ cups of walnuts, how many more cups of walnuts does Jada need to add? Explain or show your reasoning.

Problem B:

Kiran and Jada hiked $1\frac{1}{2}$ miles and took a rest. Then they hiked another $\frac{4}{10}$ mile before stopping for lunch. How many miles have they hiked so far?

If the trail they are hiking is a total of $2\frac{1}{2}$ miles, how much farther do they have to hike? Explain or show your reasoning.

2. Discuss the problems and solutions with your partner. What is the same about your strategies and solutions? What is different?
3. Revise your work if necessary.

Lesson 13: Put It All Together: Add and Subtract Fractions

- Let's add and subtract fractions with unlike denominators.

Warm-up: Number Talk: Sums with $\frac{1}{8}$

Find the value of each expression mentally.

- $\frac{1}{8} + \frac{5}{8}$

- $\frac{1}{8} + \frac{6}{16}$

- $\frac{1}{8} + \frac{1}{3}$

- $\frac{1}{8} + \frac{5}{12}$

13.1: Common Denominators

$$\frac{4}{6} + \frac{5}{8}$$

Tyler says: "To find the sum, I can use 18 as a common denominator."

Han says: "To find the sum, I can use 24 as a common denominator."

Clare says: "To find the sum, I can use 48 as a common denominator."

1. Whom do you agree with? Explain or show your reasoning.

2. What is the value of $\frac{4}{6} + \frac{5}{8}$?

3. Are there other common denominators you could use to find the sum? Explain or show your reasoning.

13.2: Unlike Denominators

Find the value of each expression. Explain or show your reasoning.

1. $\frac{2}{5} + \frac{13}{15}$

2. $\frac{6}{5} - \frac{1}{3}$

3. $\frac{11}{12} + 3\frac{5}{9}$

4. $\frac{6}{10} - \frac{9}{25}$

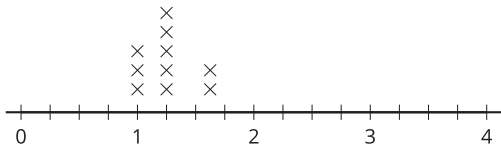
Lesson 14: Representing Fractions on a Line Plot

- Let's make a line plot and analyze the data we collect.

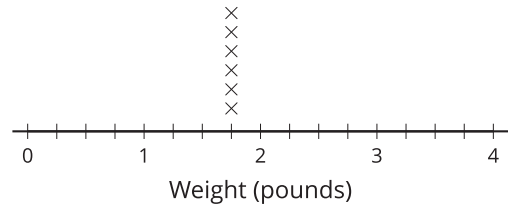
Warm-up: Which One Doesn't Belong: Line Plot

Which one doesn't belong?

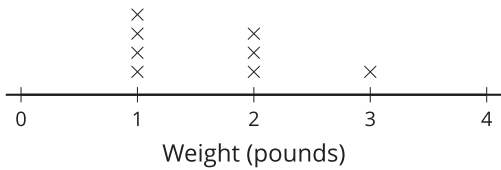
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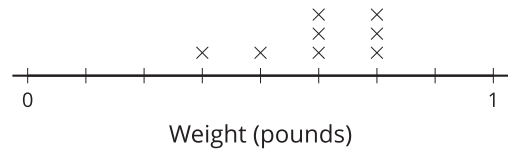
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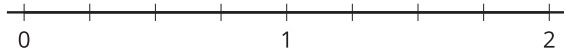
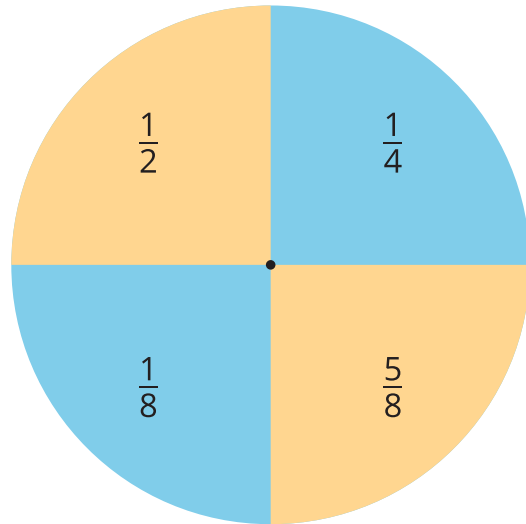
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14.1: Sums of Fractions

1. Play Sums of Fractions with your partner.

- Take turns with your partner.
- Spin the spinner twice.
- Add the two fractions.
- Record the sum on the line plot.
- Play the game until you and your partner together have 12 data points.



2. How did you know where to plot the sums of eighths?

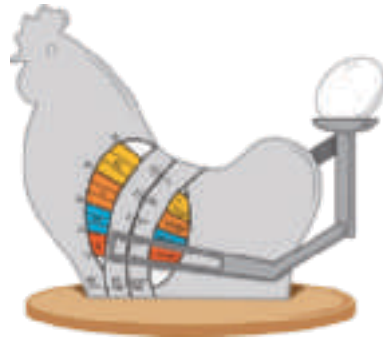
3. What is the difference between your highest and lowest number?

4. What do you notice about the data you collected?

14.2: Lots of Eggs

1. Here are the weights of some eggs, in ounces. Use them to make a line plot.

$1\frac{7}{8}$, $2\frac{1}{2}$, $2\frac{3}{8}$, $1\frac{3}{4}$, $2\frac{1}{4}$, $2\frac{4}{8}$, $2\frac{1}{8}$, $1\frac{7}{8}$, $2\frac{1}{4}$, $1\frac{6}{8}$, $2\frac{1}{8}$, $1\frac{7}{8}$



-
2. Jada said that $\frac{1}{4}$ of the eggs weigh $1\frac{7}{8}$ ounces. Do you agree? Explain or show your reasoning.
 3. How much heavier is the heaviest egg than the lightest egg? Explain or show your reasoning.

Lesson 15: Problem Solving with Line Plots

- Let's solve problems using a line plot.

Warm-up: Number Talk: Multiply by 18

Find the value of each expression mentally.

- $\frac{1}{3} \times 18$

- $\frac{2}{3} \times 18$

- $\frac{4}{3} \times 18$

- $\frac{5}{3} \times 18$

15.1: Info Gap: Picking Fruit

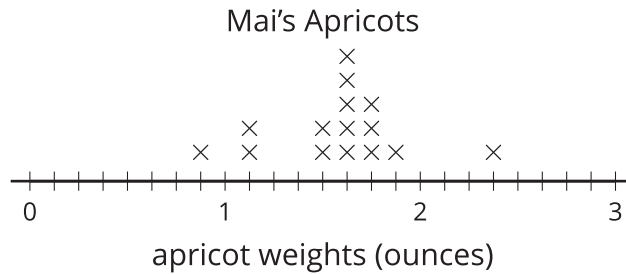
Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

Pause here so your teacher can review your work.

Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

15.2: Mathematical Questions

This line plot shows the weights of some apricots that Mai picked.



1. What fraction of the apricots weigh less than $1\frac{1}{2}$ ounces? Explain or show your reasoning.
2. Write a multiplication equation that represents the total weight of the apricots that each weigh $1\frac{5}{8}$ ounces.
3. Do all of Mai's apricots together weigh more or less than a pound? Explain or show your reasoning.

Section Summary

Section Summary

In this section we learned to add and subtract fractions. When the denominators are the same, such as $\frac{7}{10} + \frac{4}{10}$, we can just add the tenths and see that there are 11 of them so $\frac{7}{10} + \frac{4}{10} = \frac{11}{10}$. When the denominators are not the same, such as $\frac{1}{6} + \frac{3}{8}$, we look for a common denominator so that we can add parts of the same size. One way to find a common denominator is to use the product of the two denominators, 6×8 , because that's always a multiple of both denominators. Using 48 as a denominator we find $\frac{1}{6} + \frac{3}{8} = \frac{1 \times 8}{6 \times 8} + \frac{3 \times 6}{8 \times 6}$. This means $\frac{1}{6} + \frac{3}{8} = \frac{26}{48}$. For the expression $\frac{1}{6} + \frac{3}{8}$ we can

also use a smaller common denominator. Since 24 is a multiple of 6 and 8 we can also rewrite $\frac{1}{6} + \frac{3}{8}$ as $\frac{4}{24} + \frac{9}{24}$ which is $\frac{13}{24}$.

Lesson 16: Compare Products

- Let's compare products.

Warm-up: True or False: Compare Products

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $\frac{4}{5} \times 100 = 120$

- $\frac{4}{5} \times 100 < 100$

- $\frac{4}{5} \times 100 = 80$

16.1: Go the Distance

Kiran, Noah, and Elena each ran as far as they could in one hour.

- Elena ran $\frac{3}{4}$ of a 5 mile trail.
- Noah ran $\frac{1}{2}$ of a 5 mile trail.
- Kiran ran $1\frac{1}{4}$ of a 5 mile trail.

1. List the distances the students ran in increasing order. Be prepared to explain your reasoning.

2. Fill in the blanks to make each statement true. Be prepared to explain your reasoning.

a. Diego ran farther than Noah, but not as far as Kiran.

Diego ran _____ of a 5 mile trail.

b. Lin ran farther than Kiran, but not twice as far as Kiran.

Lin ran _____ of a 5 mile trail.

c. Tyler ran farther than Noah, but not as far as Elena.

Tyler ran _____ of a 5 mile trail.

16.2: Compare Expressions

1. Write $<$ or $>$ in each blank to make the statement true. Explain or show your reasoning.

a. $\frac{5}{4} \times 100$ ____ 100

b. $\frac{5}{7} \times 2$ ____ 2

c. $\frac{1}{3} \times 50$ ____ 100

2. Write a number in each box to make the statement true. Explain or show your reasoning.

a. $\frac{\boxed{}}{9} \times 50 < 50$

b. $\frac{\boxed{}}{9} \times 50 = 50$

c. $\frac{\boxed{}}{9} \times 50 > 50$

3. Write a number in each box to make the statement true. Explain or show your reasoning.

a. $\frac{9}{\boxed{}} \times 50 < 50$

b. $\frac{9}{\boxed{}} \times 50 = 50$

c. $\frac{9}{\boxed{}} \times 50 > 50$

Lesson 17: Interpret Diagrams

- Let’s compare products without multiplying.

Warm-up: Estimation Exploration: Fraction of a Whole Number

$$\frac{5}{3} \times 9,625$$

Record an estimate that is:

too low	about right	too high

17.1: Match the Diagram

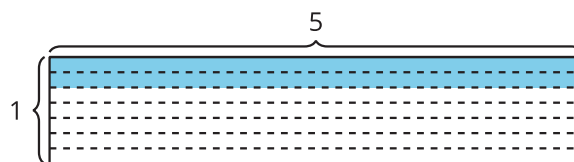
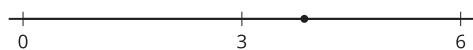
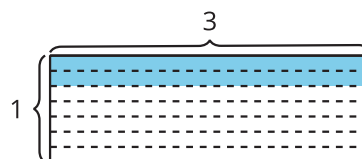
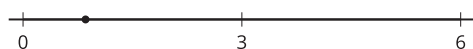
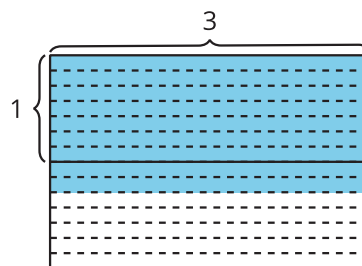
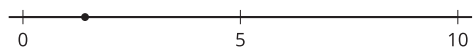
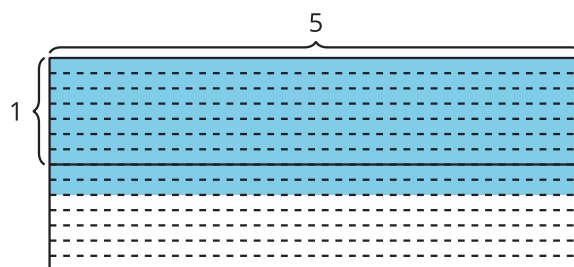
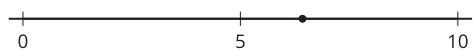
1. Match the expressions and diagrams.

$$\frac{2}{7} \times 3$$

$$\frac{9}{7} \times 3$$

$$\frac{2}{7} \times 5$$

$$\frac{9}{7} \times 5$$



2. Write $<$ or $>$ in each blank to make the inequality true.

a. $\frac{2}{7} \times 3$ ____ 3

b. $\frac{9}{7} \times 3$ ____ 3

c. $\frac{2}{7} \times 5$ ____ 5

d. $\frac{9}{7} \times 5$ ____ 5

17.2: Who Ran Farther?

- Priya ran to her grandmother's house.
- Jada ran twice as far as Priya.
- Han ran $\frac{6}{7}$ as far as Priya.
- Clare ran $\frac{14}{8}$ as far as Priya.
- Mai ran $\frac{3}{5}$ times as far as Priya.

1. Which students ran farther than Priya? _____
2. Which students did not run as far as Priya? _____
3. List the runners in order from shortest distance run to longest. Explain or show your reasoning.
4. The point P represents how far Priya ran. Write the initial of each student in the blank that shows how far they ran. One of the students will be missing.



5. Label the distance for the missing student on the number line above.

Lesson 18: Compare Without Multiplying

- Let's compare expressions, without evaluating them.

Warm-up: Notice and Wonder: Expressions and Number Lines

What do you notice? What do you wonder?



$$\frac{2}{3} \times 5$$

18.1: Approximate Location

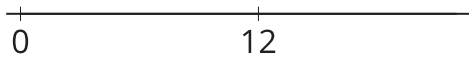
1. Label each expression at its approximate location on the number line.

Partner A

a. $\frac{2}{5} \times 12$

b. $\frac{5}{3} \times 12$

c. $\frac{7}{7} \times 12$

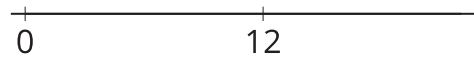


Partner B

a. $\frac{4}{7} \times 12$

b. $\frac{8}{5} \times 12$

c. $\frac{9}{9} \times 12$



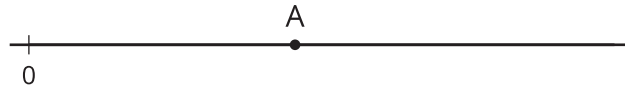
2. Choose a number to put in each box to make the statement true.

a. $\frac{\boxed{}}{11} \times 12 > 12$

b. $\frac{\boxed{}}{15} \times 12 = 12$

c. $\frac{13}{\boxed{}} \times 12 < 12$

18.2: An Unknown Number



1. The number A is shown on the number line. Label the approximate location of the value of each expression. Explain or show your reasoning.

◦ $\frac{1}{4} \times A$

◦ $2 \times A$

◦ $\frac{13}{8} \times A$

◦ $\frac{2}{3} \times A$

2. Is $\frac{13}{8} \times \frac{11}{39}$ less than, greater than, or equal to $\frac{11}{39}$? Explain or show your reasoning.

3. Is $\frac{2}{3} \times \frac{17}{53}$ less than, greater than, or equal to $\frac{17}{53}$? Explain or show your reasoning.

Lesson 19: Compare to 1

- Let's explain what happens when we multiply a fraction by a fraction greater than, less than, or equal to 1.

Warm-up: What Do You Know About $\frac{15}{14} \times \frac{23}{30}$?

What do you know about $\frac{15}{14} \times \frac{23}{30}$?

19.1: Compare Fraction Products on the Number Line

1. Match the expressions and number lines that show the same value.

○ $\frac{2}{5} \times \frac{4}{3}$

○ $(1 + \frac{1}{3}) \times \frac{5}{2}$

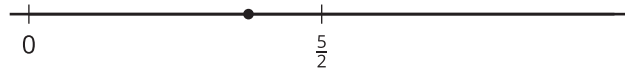
○ $\frac{3}{4} \times \frac{5}{2}$

○ $(1 - \frac{3}{5}) \times \frac{4}{3}$

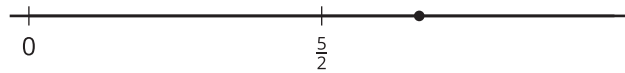
○ $\frac{4}{3} \times \frac{5}{2}$

○ $(1 - \frac{1}{4}) \times \frac{5}{2}$

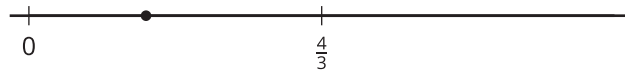
A



B



C



2. Choose one of the expressions from each set and explain whether the value is greater than or less than the second factor.

19.2: True Statement

1. Rewrite each expression as a sum or difference of 2 products.

a. $(1 - \frac{2}{5}) \times \frac{4}{7}$

b. $(1 + \frac{1}{5}) \times \frac{4}{7}$

c. $(1 - \frac{3}{8}) \times \frac{4}{7}$

d. $(1 + \frac{1}{8}) \times \frac{4}{7}$

2. Fill in each blank with $<$ or $>$ to make the inequality true.

a. $(1 - \frac{2}{5}) \times \frac{4}{7}$ _____ $\frac{4}{7}$

b. $(1 + \frac{1}{5}) \times \frac{4}{7}$ _____ $\frac{4}{7}$

c. $(1 - \frac{3}{8}) \times \frac{4}{7}$ _____ $\frac{4}{7}$

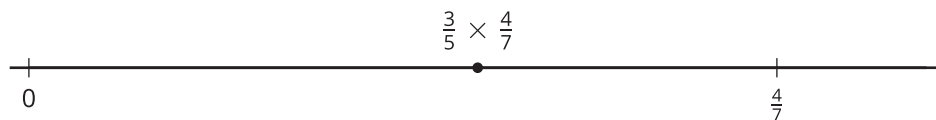
d. $(1 + \frac{1}{8}) \times \frac{4}{7}$ _____ $\frac{4}{7}$

3. Describe the value of the product when $\frac{4}{7}$ is multiplied by a fraction greater than 1. Explain your reasoning.

4. Describe the value of the product when $\frac{4}{7}$ is multiplied by a fraction less than 1. Explain your reasoning.

Section Summary

Section Summary



In this section, we learned how to compare the size of a product to the size of the factors. To compare $\frac{3}{5} \times \frac{4}{7}$ with $\frac{4}{7}$, for example, we can put them on a number line. Since $\frac{3}{5}$ is 3 equal parts with 5 parts in the whole, it is to the left of $\frac{4}{7}$, only part of the way there. We can also see this by writing $\frac{3}{5}$ as $1 - \frac{2}{5}$.

$$\left(1 - \frac{2}{5}\right) \times \frac{4}{7} = \frac{4}{7} - \left(\frac{2}{5} \times \frac{4}{7}\right)$$

The product is less than $\frac{4}{7}$ because it is $\frac{4}{7}$ minus a fraction.

Lesson 20: Will it Always Work?

- Let's make generalizations about multiplying a whole number by a fraction.

Warm-up: True or False: Distributing

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $\frac{3}{4} = 1 - \frac{1}{4}$

- $(1 - \frac{1}{4}) \times 9 = 9 - (\frac{1}{4} \times 9)$

- $(1 + \frac{1}{4}) \times 7 = (1 \times 7) + \frac{1}{4}$

20.1: True Statements

Write $<$, $>$, or $=$ in each blank to make true statements.

Choose one problem and explain or show your reasoning.

1. 567 ____ 345×567

2. $\frac{4}{5} \times 851$ ____ 851

3. $\frac{1}{4}$ ____ $\frac{5}{5} \times \frac{1}{4}$

4. $\frac{103}{104}$ ____ $\frac{103}{104} \times \frac{103}{104}$

5. $\frac{99}{8} \times \frac{23}{22}$ ____ $\frac{99}{8}$

6. $\frac{10}{10} \times \frac{1}{2}$ ____ $\frac{1}{2}$

7. $\frac{100}{7} \times \frac{9}{13}$ ____ $\frac{9}{13}$

20.2: Andre's Rules

Andre says:

- When you multiply any fraction by a number less than 1, the product will be less than the fraction.
- When you multiply any fraction by a number greater than 1, the product will be greater than the fraction.

Each partner choose one of the statements and describe why it is true. You may want to include details such as notes, diagrams, and drawings to help others understand your thinking.

Lesson 21: Weekend Investigation

- Let's find out about how students spend free time on the weekend.

Warm-up: Number Talk: Fractions of 60

Find the value of each expression mentally.

- $\frac{1}{4} \times 60$

- $\frac{3}{4} \times 60$

- $\frac{5}{4} \times 60$

- $\frac{9}{4} \times 60$

21.1: Data Collection

Imagine on the weekend you have 2 hours of free time that you can spend any way you like.

1. How would you spend it? Record your answer in fractions of an hour. Show your reasoning.
2. Record the time for each activity from your list on the appropriate poster, if there is a category for it.

21.2: Data Analysis

Your teacher will assign a poster with a data set for one of the categories from the previous activity.

1. Create a line plot that represents the data. Make sure to label the line plot.

2. Analyze the data and tell the story of your data. Choose at least 3 things. Use the following questions if they are helpful.
 - What is the total number of hours the class spends on this activity?
 - What is the difference between greatest and least time?
 - Is there something surprising?
 - How many data points are there? What does that tell you?
 - What fraction of your classmates spend less than an hour on this activity?
More than an hour?

Be prepared to share the story with the class.

Section A: Practice Problems

1. Pre-unit

Find the number that makes each equation true. Explain or show your reasoning.

a. $\frac{\boxed{}}{12} = \frac{2}{3}$

b. $\frac{5}{6} = \frac{\boxed{}}{12}$

2. Pre-unit

a. The road around a lake is 15 kilometers long. How many meters is that?

b. The length of an alligator is 4 meters. How many centimeters is that?

3. Pre-unit

The value of the 6 in 618,204 has how many times the value as the 6 in 563?
Explain or show your reasoning.

4. Pre-unit

Find the value of each sum.

a. $\frac{3}{8} + \frac{9}{8}$

b. $3\frac{1}{5} + \frac{3}{5}$

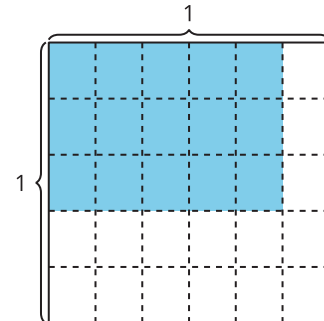
c. $2\frac{4}{10} + 1\frac{7}{10}$

5. Pre-unit

Lin spent 5 minutes reading the story. Noah spent 3 times as long as Lin. How long did Noah spend reading the story? Explain or show your reasoning.

6. Pre-unit

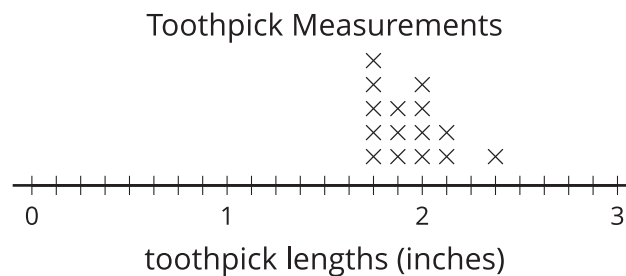
- a. Write a multiplication expression for the shaded area and find the value of the expression. Explain or show your reasoning.



- b. Find the value of $\frac{5}{7} \times \frac{10}{3}$.

7. Pre-unit

The line plot shows the lengths of some toothpicks in inches.



- a. How many measurements are there?
- b. What is the difference between the longest and shortest toothpicks?

8. a. Write a multiplication equation relating the values of 0.5 and 0.05.

b. Write a division equation relating the values of 0.5 and 0.05.

(From Unit 6, Lesson 1.)

9. Write each number using exponential notation.

a. $10 \times 10 \times 10$

b. 100×100

c. 100,000

d. 1,000,000,000

(From Unit 6, Lesson 2.)

10. a. How many centimeters are in each measurement?

0.12 m

3.5 m

19 m

- b. How many millimeters are in each measurement?

3 m

37 m

1,915 m

- c. How does a whole number of meters change when it is converted to millimeters?

(From Unit 6, Lesson 3.)

11. a. How many meters are in each measurement?

16 millimeters

1,375 millimeters

57 millimeters

- b. How does a whole number of millimeters change when you express the measurement in meters?

(From Unit 6, Lesson 4.)

12. A track is 366 meters around. An athlete runs 15 laps. How many kilometers is that? Explain or show your reasoning.

(From Unit 6, Lesson 5.)

13. Clare drinks 8 glasses of water each day. There are 235 milliliters in each glass. How many liters of water does Clare drink each day? Explain or show your reasoning.

(From Unit 6, Lesson 6.)

14. A track is 400 yards around. How many full laps does Tyler need to run if he wants to run at least 2 miles?

(From Unit 6, Lesson 7.)

15. Exploration

a. Write each of these numbers using exponential notation.

- i. 1,000,000,000 (the approximate population of Africa in 2009)

- ii. 100,000,000,000 (estimated number of stars in Milky Way)

- iii. 1,000,000,000,000 (amount of dollars added to United States debt each year recently)

- iv. 100,000,000,000,000 (denomination of a bill in Zimbabwe)



- v. 1,000,000,000,000,000,000 (estimated number of stars in universe)

- vi. 10,000,000,000,000,000,000 (estimated number of grains of sand on the earth)

- [illegible]

b. How is exponential notation helpful for writing these numbers?

16. Exploration

You have a piggy bank with 1 kg of coins inside. Which coins would you like to be in the piggy bank? Explain or show your reasoning.

coin	approximate weight (grams)
penny	2.5
nickel	5
dime	2.3
quarter	5.7

Section B: Practice Problems

1. a. Find the value of each sum. Explain or show your reasoning.

i. $\frac{5}{6} + \frac{2}{6}$

ii. $\frac{5}{6} + \frac{2}{3}$

- b. How were the calculations the same? How were they different?

(From Unit 6, Lesson 8.)

2. a. Explain why the expressions $\frac{2}{3} - \frac{7}{12}$ and $\frac{8}{12} - \frac{7}{12}$ are equivalent.

- b. How is the expression $\frac{8}{12} - \frac{7}{12}$ helpful to find the value of $\frac{2}{3} - \frac{7}{12}$?

(From Unit 6, Lesson 9.)

3. Find the value of each expression. Explain or show your reasoning.

a. $\frac{1}{4} + \frac{1}{5}$

b. $\frac{10}{9} - \frac{3}{4}$

(From Unit 6, Lesson 10.)

4. a. Find the value of $2\frac{3}{4} - \frac{1}{3}$. Explain or show your reasoning.

b. Find the value of $3\frac{2}{7} - \frac{4}{5}$. Explain or show your reasoning.

(From Unit 6, Lesson 11.)

5. Jada picked $4\frac{2}{3}$ cups of blackberries. Andre picked $3\frac{5}{8}$ cups of blackberries.

a. How many cups of blackberries did Jada and Andre pick together? Explain or show your reasoning.

b. How many more cups of blackberries did Jada pick than Andre? Explain or show your reasoning.

(From Unit 6, Lesson 12.)

6. Find the value of each expression. Explain or show your reasoning.

a. $\frac{7}{8} + \frac{4}{13}$

b. $\frac{7}{8} - \frac{3}{20}$

(From Unit 6, Lesson 13.)

7. Here are the lengths of some pieces of ribbon measured in inches:

$$\begin{array}{ccccccc} 3\frac{1}{4} & 4\frac{1}{8} & 3\frac{6}{8} & 3\frac{1}{8} & 2\frac{5}{8} & 3\frac{2}{4} & 3\frac{1}{4} \\ 3\frac{7}{8} & 4\frac{1}{8} & 3\frac{1}{2} & 2\frac{7}{8} & 4\frac{1}{8} & 3\frac{3}{4} & 3\frac{2}{8} \end{array}$$

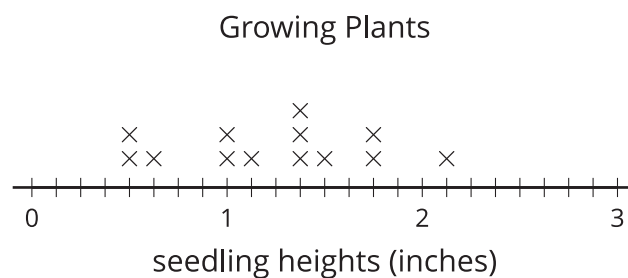
a. Complete the line plot with the ribbon lengths.



b. What is the sum of the lengths of the ribbons that measure more than 4 inches? Explain or show your reasoning.

(From Unit 6, Lesson 14.)

8. Han is making a line plot of the seedlings his class grew. This is what he has done so far.



Use this information to complete the line plot. Explain or show your reasoning.

- There are 15 seedlings altogether.
- The tallest seedling is $2\frac{1}{8}$ taller than the shortest seedling.
- There are 3 seedlings of the shortest height.

(From Unit 6, Lesson 15.)

9. Exploration

- a. Put the numbers 2, 3, 4, and 5 in the four boxes so that the expression is as

close to 1 as possible. $\frac{\boxed{}}{\boxed{}} + \frac{\boxed{}}{\boxed{}}$

- b. Put the numbers 2, 3, 4, and 5 in the four boxes so that the expression is as

close to 1 as possible. $\frac{\boxed{}}{\boxed{}} - \frac{\boxed{}}{\boxed{}}$

10. **Exploration**

Make a line plot of seedling heights so that each of these statements is true.

- There are 12 measurements.
- The largest measurement is $2\frac{3}{8}$ inches more than the smallest measurement.
- The sum of the measurements is $18\frac{3}{8}$ inches.

Explain how you made the line plot.

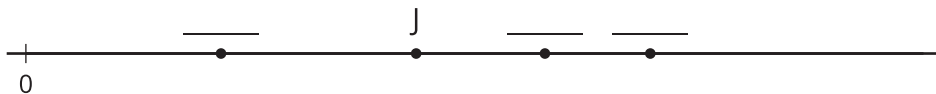
Section C: Practice Problems

1. a. Andre ran $\frac{4}{5}$ of a 7 mile trail. Did Andre run more or less than 7 miles?
Explain or show your reasoning.

- b. Clare ran $\frac{\boxed{}}{10}$ of a 7 mile trail. She ran more than 7 miles. Choose a number that could go in the box. Explain or show your reasoning.

(From Unit 6, Lesson 16.)

2. The point J on the number line shows how many miles Jada ran. Label the points on the number line to show how far each of these students ran.
 - a. Clare ran $\frac{8}{5}$ as far as Jada.
 - b. Tyler ran $\frac{4}{3}$ as far as Jada.
 - c. Lin ran $\frac{1}{2}$ as far as Jada.



(From Unit 6, Lesson 17.)

3. The point A is labeled on the number line.



Label each of these points on the number line.

○ $\frac{2}{5} \times A$

○ $\frac{13}{10} \times A$

○ $\frac{7}{4} \times A$

(From Unit 6, Lesson 18.)

4. Use the equation $\frac{5}{7} = \left(1 - \frac{2}{7}\right)$ to explain why $\frac{5}{7} \times \frac{11}{3} < \frac{11}{3}$.

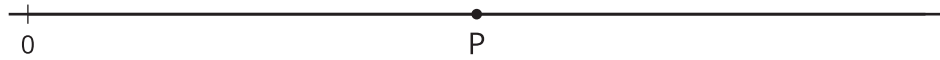
(From Unit 6, Lesson 19.)

5. Explain why multiplying a fraction by a number less than 1 makes the fraction smaller.

(From Unit 6, Lesson 20.)

6. Exploration

A point P is labeled on the number line.



a. P is $\frac{3}{4}$ of a number A. Plot A on the number line. Explain or show your reasoning.

b. P is $\frac{5}{9}$ of a number B. Plot B on the number line. Explain or show your reasoning.

7. Exploration

a. About 10^6 people live in Michigan. About 10^4 of the people in Michigan live in Flint.

i. How many times as many people live in Michigan as in Flint?

ii. How many times as many people live in Flint as in Michigan?

b. There are about 10^{11} stars in the Milky Way. There are about 10^{21} stars in the universe.

i. How many times as many stars are there in the universe than in the Milky Way?

ii. How many times as many stars are there in the Milky Way than in the universe?

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