



Core Knowledge<sup>®</sup> MATHEMATICS



# Wrapping Up Multiplication and Division with Multi-Digit Numbers

Teacher Guide



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# Wrapping Up Multiplication and Division with Multi-Digit Numbers

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# Wrapping Up Multiplication and Division with Multi-Digit Numbers Teacher Guide

Core Knowledge Mathematics™



# **Unit 4: Wrapping Up Multiplication and Division with Multi-Digit Numbers**

## **At a Glance**

Unit 4 is estimated to be completed in 21-23 days including 2 days for assessment.

This unit is divided into three sections including 19 lessons and 2 optional lessons.

- Section A—Multi-digit Multiplication Using the Standard Algorithm (Lessons 1-9)
- Section B—Multi-digit Division Using Partial Quotients (Lessons 10-17)
- Section C—Let's Put it to Work (Lessons 18-21)

On pages 8-9 of this Teacher Guide is a chart that identifies the section each lesson belongs in and the materials needed for each lesson.

This unit uses three new student centers.

- Number Puzzles: Multiplication and Division
- Watch Your Remainder
- Mystery Number



## Unit 4: Wrapping Up Multiplication and Division with Multi-Digit Numbers

### Unit Learning Goals

- Students use the standard algorithm to multiply multi-digit whole numbers. They divide whole numbers up to four-digits by two-digits divisors using strategies based on place value and properties of operations.

In this unit, students multiply multi-digit whole numbers using the standard algorithm and begin working toward end-of-grade expectation for fluency. They also find whole-number quotients with up to four-digit dividends and two-digit divisors.

In grade 4, students used strategies based on place value and properties of operations to multiply a one-digit whole number and a whole number of up to four digits, and to multiply a pair of two-digit numbers. They decomposed the factors by place value, and used diagrams and algorithms using partial products to record their reasoning.

Here, students build on those strategies to make sense of the standard algorithm for multiplication. They recognize that it is also based on place value but records the partial products in a condensed way.

*Han and Elena used different algorithms to find the value of  $3 \times 318$ .*

**Han**

$$\begin{array}{r} 318 \\ \times 3 \\ \hline 954 \end{array}$$

**Elena**

$$\begin{array}{r} 2 \\ 318 \\ \times 3 \\ \hline 954 \end{array}$$

*Explain to your partner what Han and Elena did. What does the 2 represent in Elena's algorithm?*

In grade 4, students also found whole-number quotients using place-value strategies and the relationship between multiplication and division. They decomposed dividends in various ways and found partial quotients. The numbers they encountered then were limited to four-digit dividends and one-digit divisors. In this unit, they extend that work to include two-digit divisors.

As they build their facility with multi-digit multiplication and division, students solve problems about area and volume and reinforce their understanding of these concepts.

## Section A: Multi-digit Multiplication Using the Standard Algorithm

### Standards Alignments

Building On	4.NBT.B.5
Addressing	5.MD.C.3, 5.MD.C.5, 5.NBT, 5.NBT.B, 5.NBT.B.5, 5.NF.B.4, 5.OA.A.2
Building Towards	5.NBT.B.5

### Section Learning Goals

- Multiply multi-digit whole numbers using the standard algorithm.

This section introduces the standard algorithm for multiplication, extending students' earlier work on multiplication. In grade 4, students used diagrams and partial-products algorithms to find the product of a one-digit number and a number up to four digits, and the product of 2 two-digit numbers. They attended to the role of place value along the way.

Students revisit these strategies and representations here, but work with factors with more digits than encountered in grade 4. They make connections between the partial products in diagrams and previous algorithms to the numbers in the standard algorithm. They also learn the notation for recording new place-value units that result from finding partial products.

	100	20	3
20	2,000	400	60
3	300	60	9

$$\begin{array}{r}
 123 \\
 \times 23 \\
 \hline
 369 \\
 2460 \\
 \hline
 2829
 \end{array}$$

	123
20	2,460
3	369

When using the standard algorithm to multiply a two-digit number and a three-digit number, students account for the place value of the digits being multiplied, as they had done before.

For example, the 3 in 23 represents 3 ones, so  $3 \times 123$  is 369.


The 2 in 23, however, represents 2 tens, so the partial product is  $2 \times 10 \times 123$  or 2,460, instead of  $2 \times 123$  or 246.

The partial products 369 and 2,460 can be seen in a diagram as well.

$$\begin{array}{r}
 123 \\
 \times 23 \\
 \hline
 369 \\
 + 2,460 \\
 \hline
 2,829
 \end{array}$$

Once students have practiced recording products this way, they learn to multiply factors that require composing new units, such as  $264 \times 38$ .

$$\begin{array}{r}
 11 \\
 53 \\
 264 \\
 \times \quad 38 \\
 \hline
 1712 \\
 + 7920 \\
 \hline
 10032
 \end{array}$$

   PLC: Lesson 6, Activity 1, Compose a New Unit

### Suggested Centers

- Number Puzzles: Multiplication and Division (4–5), Stage 1: Two-digit Factors (Supporting)
- Number Puzzles: Multiplication and Division (4–5), Stage 2: Multi-digit Factors (Addressing)

## Section B: Multi-digit Division Using Partial Quotients

### Standards Alignments

Building On	4.NBT.B.6, 5.NF.B.3
Addressing	5.NBT.B.5, 5.NBT.B.6, 5.NF.B.3, 5.OA.A.2
Building Towards	5.NBT.B.6

### Section Learning Goals

- Divide multi-digit whole numbers using strategies based on place value, properties of operations, and the relationship between multiplication and division.

In grade 4, students found whole-number quotients and remainders with up to four-digit dividends and one-digit divisors using strategies based on place value and partial quotients. In grade 5, they extend this work to include quotients involving two-digit divisors.

Students begin with an exploration that relates division of large numbers to a real-world context. They use strategies based on place value and the relationship between multiplication and division to estimate how the world's longest noodle could be shared. Then, they analyze and use different ways to decompose a dividend.

For instance, here are two ways to divide 448 by 16:

$$\begin{array}{r}
 160 \div 16 = 10 \\
 160 \div 16 = 10 \\
 80 \div 16 = 5 \\
 48 \div 16 = 3 \\
 \hline
 448 \div 16 = 28
 \end{array}$$

$$\begin{array}{r}
 \boxed{28} \\
 3 \\
 5 \\
 20 \\
 16 \overline{)448} \\
 \underline{-320} \quad (20 \times 16) \\
 128 \\
 \underline{-80} \quad (5 \times 16) \\
 48 \\
 \underline{-48} \quad (3 \times 16) \\
 0
 \end{array}$$

Students see that some decompositions may be more helpful than others for finding whole-number quotients. They use this insight to make sense of algorithms using partial quotients that are more complex.

Note that use of the standard algorithm for division is not an expectation in grade 5, but students can begin to develop the conceptual understanding needed to do so. The algorithms using partial quotients seen here are based on place value, which will allow students to make sense of the logic of the standard algorithm they'll learn in grade 6.

## **Suggested Centers**

- Number Puzzles: Multiplication and Division (4–5), Stage 2: Multi-digit Factors (Addressing)
- Watch Your Remainder (4–5), Stage 1: One-digit Divisors (Supporting)
- Mystery Number (1–4), Stage 6: Decimals (Supporting)

## Section C: Let's Put it to Work

### Standards Alignments

Building On	4.MD.A.1, 5.NBT.B
Addressing	5.MD.C.5, 5.NBT.B, 5.NBT.B.5, 5.NBT.B.6
Building Towards	5.NBT.B.5

### Section Learning Goals

- Multiply and divide to solve real-world and mathematical problems involving area and volume.

The final section invites students to use multiplication and division of whole numbers to estimate large quantities and solve real-world and mathematical problems.

Students encounter area and volume problems in the context of geography—the area of states—and everyday consumption—the volume of milk consumed, the area of plastic waste in the Pacific Ocean, and the volume of recyclable plastic shipped abroad for processing.

*New Mexico is about 596 km long and 552 km wide.  
Which is larger, the garbage patch or New Mexico?*



The section ends with an additional opportunity for mathematical modeling. Students estimate and calculate the weight of food waste produced in the United States per year, using an average per-person amount. They also estimate and reflect on the amount of their own food waste.

 PLC: Lesson 19, Activity 1, Square Kilometers

### Suggested Centers

- Mystery Number (1–4), Stage 6: Decimals (Supporting)

### Throughout the Unit

Throughout the unit, students engage in warm-up activities that support the work of the current unit,

while allowing them to revisit concepts from previous grades and units. The Number Talk routines in this unit supports students to multiply powers of 10, and encourages them to think about partial products and partial quotients.

Here is a sampling of Number Talk warm-ups in the unit.

lesson 1	lesson 4	lesson 5	lesson 6	lesson 8	lesson 13	lesson 20
$50 \times 6$	$3 \times 3$	$20 \times 3$	$(2 \times 3) \times 10$	$6 \times 15$	$110 \div 10$	$8 \times 4$
$50 \times 60$	$3 \times 20$	$24 \times 3$	$(2 \times 40) \times 10$	$6 \times 17$	$121 \div 11$	$8 \times 8$
$50 \times 600$	$3 \times 600$	$120 \times 3$	$(2 \times 200) \times 10$	$6 \times 2\frac{1}{3}$	$132 \div 12$	$8 \times 8 \times 2$
$600 \times 500$	$3 \times 623$	$140 \times 3$	$(2 \times 243) \times 10$	$6 \times 2\frac{2}{3}$	$154 \div 14$	$8 \times 8 \times 20$

## Materials Needed

LESSON	GATHER	COPY
A.1	<ul style="list-style-type: none"> <li>● none</li> </ul>	<ul style="list-style-type: none"> <li>● none</li> </ul>
A.2	<ul style="list-style-type: none"> <li>● none</li> </ul>	<ul style="list-style-type: none"> <li>● none</li> </ul>
A.3	<ul style="list-style-type: none"> <li>● none</li> </ul>	<ul style="list-style-type: none"> <li>● Partial Product Expressions (groups of 2)</li> </ul>
A.4	<ul style="list-style-type: none"> <li>● none</li> </ul>	<ul style="list-style-type: none"> <li>● none</li> </ul>
A.5	<ul style="list-style-type: none"> <li>● none</li> </ul>	<ul style="list-style-type: none"> <li>● none</li> </ul>
A.6	<ul style="list-style-type: none"> <li>● none</li> </ul>	<ul style="list-style-type: none"> <li>● none</li> </ul>
A.7	<ul style="list-style-type: none"> <li>● none</li> </ul>	<ul style="list-style-type: none"> <li>● Greatest Product (groups of 1)</li> <li>● Number Cards (0-10) (groups of 2)</li> </ul>
A.8	<ul style="list-style-type: none"> <li>● Materials from a previous lesson</li> <li>● Materials from previous centers</li> </ul>	<ul style="list-style-type: none"> <li>● none</li> </ul>
A.9	<ul style="list-style-type: none"> <li>● none</li> </ul>	<ul style="list-style-type: none"> <li>● none</li> </ul>
B.10	<ul style="list-style-type: none"> <li>● none</li> </ul>	<ul style="list-style-type: none"> <li>● none</li> </ul>
B.11	<ul style="list-style-type: none"> <li>● none</li> </ul>	<ul style="list-style-type: none"> <li>● Partial Quotient Expressions (groups of 2)</li> </ul>
B.12	<ul style="list-style-type: none"> <li>● none</li> </ul>	<ul style="list-style-type: none"> <li>● none</li> </ul>
B.13	<ul style="list-style-type: none"> <li>● none</li> </ul>	<ul style="list-style-type: none"> <li>● none</li> </ul>
B.14	<ul style="list-style-type: none"> <li>● none</li> </ul>	<ul style="list-style-type: none"> <li>● none</li> </ul>
B.15	<ul style="list-style-type: none"> <li>● none</li> </ul>	<ul style="list-style-type: none"> <li>● none</li> </ul>
B.16	<ul style="list-style-type: none"> <li>● none</li> </ul>	<ul style="list-style-type: none"> <li>● none</li> </ul>
B.17	<ul style="list-style-type: none"> <li>● none</li> </ul>	<ul style="list-style-type: none"> <li>● none</li> </ul>
C.18	<ul style="list-style-type: none"> <li>● none</li> </ul>	<ul style="list-style-type: none"> <li>● none</li> </ul>



# Center: Number Puzzles: Multiplication and Division (4–5)

## Stage 1: Two-digit Factors

### Lessons

- Grade5.4.A1 (supporting)
- Grade5.4.A2 (supporting)
- Grade5.4.A3 (supporting)

### Stage Narrative

Students use the digits 0–9 to make multiplication equations with two-digit factors true. Each digit may only be used one time.

### Standards Alignments

Addressing 4.NBT.B.5, 4.NBT.B.6

### Materials to Copy

Number Puzzles Mult Stage 1 Recording Sheet  
(groups of 1)

## Stage 2: Multi-digit Factors

### Lessons

- Grade5.4.A4 (addressing)
- Grade5.4.A5 (addressing)
- Grade5.4.A6 (addressing)
- Grade5.4.A7 (addressing)
- Grade5.4.A8 (addressing)
- Grade5.4.A9 (addressing)
- Grade5.4.B10 (addressing)
- Grade5.4.B11 (addressing)
- Grade5.4.B12 (addressing)
- Grade5.4.B13 (addressing)
- Grade5.4.B14 (addressing)
- Grade5.4.B15 (addressing)

## Stage Narrative

Students use the digits 0–9 to make multiplication equations with multi-digit factors true. Each digit may only be used one time.

## Standards Alignments

Addressing 5.NBT.B.5, 5.NBT.B.6

## Materials to Copy

Number Puzzles Mult Stage 2 Recording Sheet  
(groups of 1)

## Stages used in Grade 4

### Stage 1

#### Addressing

- Grade4.6.B

## Center: Watch Your Remainder (4–5)

### Stage 1: One-digit Divisors

#### Lessons

- Grade5.4.B10 (supporting)
- Grade5.4.B11 (supporting)
- Grade5.4.B12 (supporting)

#### Stage Narrative

Before playing, students remove the cards that show 10 and set them aside.

Students spin the spinner to get the divisor for the round. Each student picks 6 cards and chooses 3–4 of them to create a dividend. Each student finds their quotient. The score for the round is the remainder from each expression. Students pick new cards so that they have 6 cards in their hand and then start the next round. The player with the lowest score after 6 rounds wins.

#### Standards Alignments

Addressing 4.NBT.B.6

#### Materials to Gather

Number cards 0–10, Paper clips

#### Materials to Copy

Watch Your Remainder Stage 1 Recording Sheet (groups of 1), Watch Your Remainder Stage 1 Spinner (groups of 2)

### Stages used in Grade 4

#### Stage 1

##### Addressing

- Grade4.6.C
- Grade4.6.D

## Center: Mystery Number (1–4)

### Stage 6: Decimals

#### Lessons

- Grade5.4.B16 (supporting)
- Grade5.4.B17 (supporting)
- Grade5.4.C18 (supporting)
- Grade5.4.C19 (supporting)
- Grade5.4.C20 (supporting)

#### Stage Narrative

Students choose a mystery number (up to nine digits) from the gameboard. Students give clues using the given vocabulary.

#### Standards Alignments

Addressing 4.NBT.A

#### Materials to Copy

Mystery Number Stage 6 Gameboard (groups of 2)

### Stages used in Grade 4

#### Stage 3

##### Supporting

- Grade4.2.A

#### Stage 4

##### Addressing

- Grade4.2.B
- Grade4.2.C

##### Supporting

- Grade4.4.B

#### Stage 5

##### Addressing

- Grade4.4.C

# Section A: Multi-digit Multiplication Using the Standard Algorithm

## Lesson 1: Estimate and Find Products

### Standards Alignments

Building On	4.NBT.B.5
Addressing	5.NBT
Building Towards	5.NBT.B.5

### Teacher-facing Learning Goals

- Multiply multi-digit numbers in a way that makes sense to them.

### Student-facing Learning Goals

- Let's estimate and calculate products.

### Lesson Purpose

The purpose of this lesson is for students to make estimates and calculations of products.

In grade 4, students multiplied two-digit by two-digit and one-digit by up to four-digit numbers using strategies based on place value understanding and the properties of operations. Students learned a partial product strategy for multiplication and represented it with diagrams and equations. This lesson gives teachers an opportunity to see how students apply previous work to estimate and find products. Expect to see a wide range of strategies from students and be sure to listen to them as they explain their reasoning. In the synthesis, the strategies highlighted involve place value and properties of operations which will be built on in future lessons.

### Access for:

#### Students with Disabilities

- Engagement (Activity 1)

### Instructional Routines

MLR7 Compare and Connect (Activity 2), Number Talk (Warm-up)

**Lesson Timeline**

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

**Teacher Reflection Question**

What unfinished learning or misunderstandings do your students have about multiplication? How did you leverage those misconceptions in a positive way to further the understanding of the class?

**Cool-down** (to be completed at the end of the lesson)

🕒 5 min

Fifteen

**Standards Alignments**

Building Towards 5.NBT.B.5

**Student-facing Task Statement**

Find the value of each expression. Explain or show your reasoning.

- $15 \times 20$
- $15 \times 120$
- $15 \times 121$

**Student Responses**

- 300 since  $2 \times 15 = 30$  and I did 10 times that.
- 1,800 since  $100 \times 15 = 1,500$  and I added that to 300.
- 1,815 since there is one more in each of the 15 groups.

----- **Begin Lesson** -----**Warm-up**

🕒 10 min

Number Talk: A Multiple of 10

## Standards Alignments

Building On 4.NBT.B.5

The purpose of this Number Talk is to elicit strategies and understandings students have for multiplication of two- and three-digit numbers that are multiples of 10 or 100. These understandings help students develop fluency and will be helpful later in this lesson when students use multiplication to estimate products. This work also prepares them for the work of the standard algorithm for multiplication in which each product is a product of single-digit multiples of powers of ten.

In this activity, students have an opportunity to look for and make use of structure (MP7) because the basic fact they are using is  $5 \times 6 = 30$ , and each successive product is ten times larger.

## Instructional Routines

Number Talk

### Student-facing Task Statement

Find the value of each product mentally.

- $50 \times 6$
- $50 \times 60$
- $50 \times 600$
- $600 \times 500$

### Student Responses

- 300:  $(5 \times 6) \times 10 = 300$
- 3,000:  $(50 \times 6) \times 10 = 3,000$
- 30,000:  $(50 \times 60) \times 10 = 30,000$
- 300,000:  $(50 \times 600) \times 10 = 300,000$

### Launch

- Display one expression.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

### Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

### Synthesis

- “How can we use  $5 \times 6$  to find the value of each product?” ( $50 \times 6 = 5 \times 6 \times 10$ ,  $50 \times 60 = 5 \times 6 \times 10 \times 10$ ,  $50 \times 600 = 5 \times 6 \times 10 \times 100$ ,  $600 \times 500 = 6 \times 5 \times 100 \times 100$ )
- “Why does each product in the number talk have one more 0 in it than the previous product?” (Because one of the factors has an additional 0.)

## Activity 1

🕒 15 min

### Reasonable Estimates

#### Standards Alignments

Addressing 5.NBT

The purpose of this activity is for students to make a reasonable estimate for a given product. In addition to estimating the product, students also decide whether the estimate is too large or too small. In the activity synthesis, students consider about how far their estimate is from the actual product. In the next activity, students will evaluate the expressions using a strategy of their choice.

Students choose between several different possible estimates and justify their choice before they calculate the product (MP3).

#### 🕒 Access for Students with Disabilities

*Engagement: Provide Access by Recruiting Interest.* Synthesis: Revisit math community norms to prepare students for the whole-class discussion.

*Supports accessibility for: Social Emotional Functioning, Language*

#### Student-facing Task Statement

- Which estimate for the product  $18 \times 149$  is most reasonable? Explain or show your reasoning.
  - 2,000
  - 4,000
  - 3,000
  - 1,500
- Are any of the estimates unreasonable? Explain or show your reasoning.
- Do you think the actual product will be more or less than your estimate? Explain or show your reasoning.

#### Launch

- Groups of 2

#### Activity

- 5–7 minutes: independent work time
- 2–3 minutes: partner discussion
- Monitor for students who:
  - relate the given expression to each proposed answer by rounding or changing one or both factors.
  - estimate by rounding the factors.
  - use benchmark numbers.
  - use place value reasoning or the properties of operations to explain

## Student Responses

1. 3,000. Sample response: 3,000 is a little too large because  $3,000 = 20 \times 150$  and both factors in that product are bigger than the factors of  $18 \times 149$ . This means 4,000 is definitely too big. 1,500 is too small as  $10 \times 150$  is 1,500. The actual answer is between 2,000 and 3,000. Since neither factor was rounded very far to get  $20 \times 150$ , I think the real answer is closer to 3,000 than to 2,000.
2. Sample response: 1,500 is definitely too small as it is  $10 \times 150$  and 4,000 is definitely too big as it is  $20 \times 200$ .
3. The actual product is less than 3,000 because  $150 \times 20 = 3,000$  and both of the factors are smaller in  $18 \times 149$  so the product has to be smaller.

why their estimate is reasonable.

## Synthesis

- Invite students to share their estimates for the product and their reasoning.
- “Why do you think 3,000 is a good estimate?” (18 is close to 20 and 149 is very close to 150 and I know  $20 \times 150 = 3,000$ .)
- “Is the value of  $18 \times 149$  greater than or less than 3,000? How do you know?” (Less, because 18 is less than 20 and 149 is less than 150.)
- “Can you estimate how much less?” (About 300, because I added 2 to 18 to get 20 and  $2 \times 149$  is about 300.)
- Display:  $18 \times 149$  is about 2,700.
- “In the next activity, we’ll check to see how good our estimate is.”

## Advancing Student Thinking

If students do not choose a reasonable estimate for  $18 \times 149$ , ask, “Can you explain how you chose your estimate?”

## Activity 2

🕒 20 min

Multiply by 18

### Standards Alignments

Building Towards 5.NBT.B.5

The purpose of this activity is for students to multiply a three-digit number by a two-digit number using a strategy that makes sense to them. The expressions are scaffolded so that students can use one calculation to help with the next, particularly when they look for the final product, which is  $18 \times 149$ .

Students may:

- draw diagrams to help visualize the calculations.
- use a form of partial products and the distributive property.
- round 49 to 50 or 149 to 150 and then compensate.

This activity encourages students to compare the strategy they used with the strategies that their classmates used, and to discuss the similarities and differences. The intent of this activity is not to create a list of strategies for students to choose from. Instead, students have an opportunity to think about how the properties of operations and place value understanding were used in each strategy.

This activity uses *MLR7 Compare and Connect*. Advances: representing, conversing

## Instructional Routines

MLR7 Compare and Connect

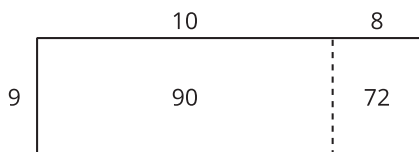
### Student-facing Task Statement

Find the value of each expression. Explain or show your reasoning.

1.  $18 \times 9$
2.  $18 \times 49$
3.  $18 \times 149$

### Student Responses

1. 162. Sample responses:
  - a. I found  $18 \times 10$  and then took away 18.
  - b. I drew a diagram



2. 882. Sample response: I found  $18 \times 50$  and took away 18.
3. 2,682. Sample response: I added  $18 \times 100$  to

### Launch

- Groups of 2

### Activity

- 1–2 minutes: quiet think time
- 5–8 minutes: partner work time

### MLR 7: Compare and Connect

- “Create a visual display that shows your thinking about each of the problems. You may want to include details such as notes, diagrams, drawings, and so on, to help others understand your thinking.”
- 2–5 minutes: independent or group work
- Monitor for students who:
  - use diagrams to show and keep track of their calculations
  - use their solution from one problem to solve a different problem
  - find a related product like  $18 \times 10$  or  $18 \times 50$  and use that to find the

the result of  $18 \times 49$ .

value of  $18 \times 9$  or  $18 \times 49$

### Synthesis

- 5–7 minutes: gallery walk
- “What is the same and what is different between the approaches to solve the problems?”
- 30 seconds: quiet think time
- 1 minute: partner discussion
- Invite students to share their diagrams.
- Highlight at least one of the diagrams or use the diagram from the student solutions. “How does the diagram help keep track of the calculations?” (For  $18 \times 9$  I know I need to find  $10 \times 9$  and then  $8 \times 9$  and add them.)

### Advancing Student Thinking

If students do not use the value of the product  $18 \times 9$  to help them find the value of  $18 \times 49$ , ask, “How could you use the value of the product  $18 \times 9$  to help you find the value of  $18 \times 49$ ?”

## Lesson Synthesis

🕒 10 min

“Today, we multiplied numbers.”

“What did you already know about multiplication that you used in today’s lesson? What questions do you have about multiplying large numbers?” (I knew that I can break a number up by place value and find products of the pieces. Then I add them up to get the full product. I knew how to draw a diagram to help organize the products.)

Consider having students respond to the questions in writing and then sharing them.

### Suggested Centers

- Number Puzzles: Multiplication and Division (4–5), Stage 1: Two-digit Factors (Supporting)

---

**Complete Cool-Down**

---

**Response to Student Thinking**

Students do not find all the correct values of the expressions.

**Next Day Support**

- After the next lesson, refer to the completed cool-down from today's lesson and ask, "What diagram would be helpful to make sense of these problems?"

## Lesson 2: Partial Products in Diagrams

### Standards Alignments

Addressing	5.NBT.B
Building Towards	5.NBT.B.5

### Teacher-facing Learning Goals

- Interpret partial products diagrams.
- Multiply a three-digit number and a two-digit number.

### Student-facing Learning Goals

- Let's interpret diagrams that can help us find products.

### Lesson Purpose

The purpose of this lesson is for students to multiply multi-digit whole numbers using strategies based on place value and the properties of operations.

This lesson builds on the partial products representation students saw in grade 4 to help organize calculations of whole number products. This becomes even more important for the product of a two-digit and three-digit number as the number of partial products is larger, depending on the diagram that is used. Part of the value of the standard algorithm which students will also see in later lessons is that it condenses the calculations and the number of partial products.

### Access for:

#### Students with Disabilities

- Engagement (Activity 2)

#### English Learners

- MLR8 (Activity 1)

### Instructional Routines

Which One Doesn't Belong? (Warm-up)

### Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min

### Teacher Reflection Question

Think about which students haven't shared their strategies in class lately. Were there missed opportunities to highlight their thinking during recent lessons? How can you take advantage of those opportunities when they arise?

Cool-down

5 min

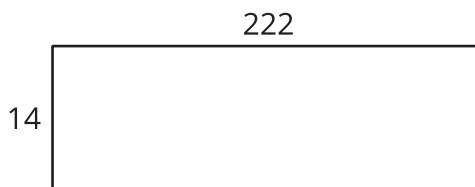
**Cool-down** (to be completed at the end of the lesson)

🕒 5 min

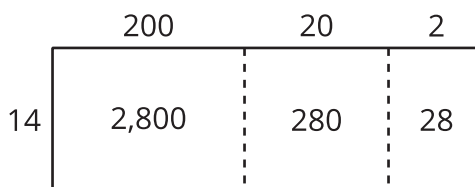
$222 \times 14$

**Standards Alignments**

Building Towards 5.NBT.B.5

**Student-facing Task Statement**Here is a diagram that represents  $222 \times 14$ .Find the value of  $222 \times 14$ . Use the diagram if it is helpful. Explain or show your reasoning.**Student Responses**

3,108. Sample response: I broke 222 down into hundreds, tens, and ones, and multiplied each by 14.

The product is  $2,800 + 280 + 28$ , which is 3,108.----- **Begin Lesson** -----

## Warm-up

🕒 10 min

### Which One Doesn't Belong: Diagrams to Find Products

#### Standards Alignments

Building Towards 5.NBT.B.5

The purpose of this warm-up is for students to compare and contrast different diagrams that can be used to represent and calculate products of two-digit numbers. Students used these partial products diagrams in Grade 4. They will extend them to represent the product of a three-digit number and a two-digit number later in the lesson.

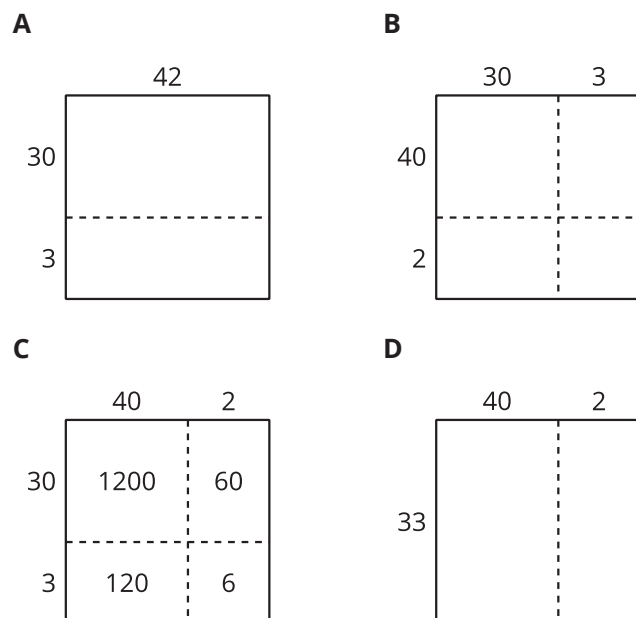
These rectangular diagrams use the intuition and properties of area to support representing multiplication. But, a genuine area diagram would be difficult to read, so the individual pieces are not drawn to scale.

#### Instructional Routines

Which One Doesn't Belong?

#### Student-facing Task Statement

Which one doesn't belong?



#### Launch

- Groups of 2
- Display the image.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time

#### Activity

- "Discuss your thinking with your partner."
- 2–3 minutes: partner discussion
- Share and record responses.

#### Synthesis

- "How might Diagram C be helpful for calculating the product  $42 \times 33$ ?" (I can add those numbers to get the value of  $42 \times 33$ .)
- Highlight that this is the type of diagram that will be used throughout the next several lessons. The purpose of the diagram is to help

## Student Responses

Sample responses:

- A doesn't belong because it is not divided vertically.
- B doesn't belong because the large number is not on the horizontal side.
- C doesn't belong because it is the only one that shows all the partial products; it's not empty inside.
- D doesn't belong because it is not divided horizontally.

see different ways to calculate products of numbers.

## Activity 1

🕒 15 min

Decompose in Many Ways

### Standards Alignments

Building Towards 5.NBT.B.5

The purpose of this activity is for students to use a diagram to help calculate the product of a three-digit number and a two-digit number. The diagram helps to organize the individual products that can be used to find the larger product. During the activity synthesis, students connect the diagram to the distributive property when they explain how the sum of the individual products gives the larger product (MP7).

### 🌐 Access for English Learners

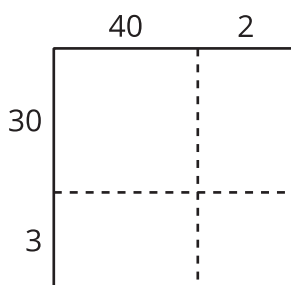
*MLR8 Discussion Supports.* For each observation that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language.

*Advances: Listening, Speaking*

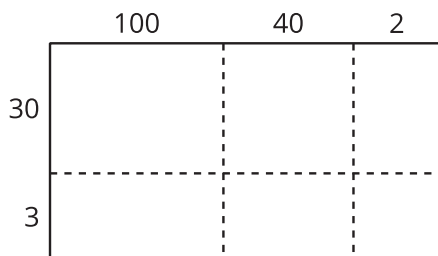
### Student-facing Task Statement

### Launch

- Groups of 2
- Display the image from the student workbook.



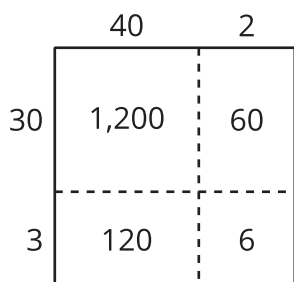
1. Write the value of each product inside the rectangles.
2. Find the value of  $42 \times 33$ .
3. This diagram represents  $142 \times 33$ .



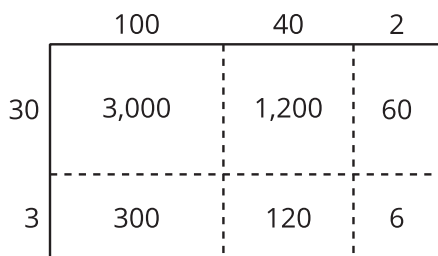
Write the value of each product inside the rectangles.

4. Find the value of  $142 \times 33$ .

### Student Responses



- 1.
2. 1,386



- 3.

- "In these problems, write each product inside the part of the diagram that represents that product."
- Demonstrate by writing 1,200 inside the rectangle with sides marked 30 and 40.

### Activity

- 1-2 minutes: quiet think time
- 6-8 minutes: partner work time
- Monitor for students who use their work for the first product to find the second product.

### Synthesis

- Invite students to share their work for finding the product  $42 \times 33$ .
- Display:  $42 \times 33 = (40 + 2) \times (30 + 3)$
- "How does the diagram represent this equation?" (It shows 42 broken up into 40 and 2 and 33 broken up into 30 and 3.)
- Display:  $(40 + 2) \times (30 + 3) = (40 \times 30) + (2 \times 30) + (40 \times 3) + (2 \times 3)$
- "How do you know this equation is true?" (The diagram shows  $42 \times 33$  broken up into those 4 partial products.)
- "How is finding the product  $142 \times 33$  related to finding the product  $42 \times 33$ ?" (The products and partial products are the same, except that I also have  $100 \times 33$  in  $142 \times 33$ .)

4. 4,686

## Activity 2

⌚ 20 min

Calculate in Many Ways

### Standards Alignments

Addressing 5.NBT.B

The purpose of this activity is for students to write expressions to represent different ways to decompose a product. Then they choose one of the decompositions to find the product. Students consider how certain decompositions are more helpful than others, depending on the specific numbers in the problem. The diagrams used here relate to the partial products and standard algorithm methods which students will learn in future lessons.

### 🕒 Access for Students with Disabilities

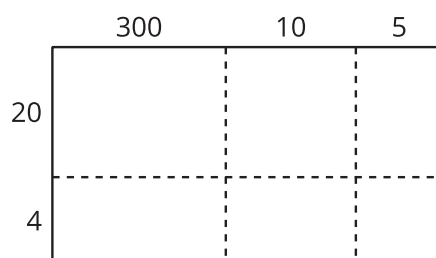
*Engagement: Provide Access by Recruiting Interest.* Provide choice. Invite students to decide which problem to start with and how they want to write the expressions.

*Supports accessibility for: Attention*

### Student-facing Task Statement

Here are some different diagrams that represent  $315 \times 24$ . For each diagram, write a multiplication expression inside each rectangle to represent the product.

1.



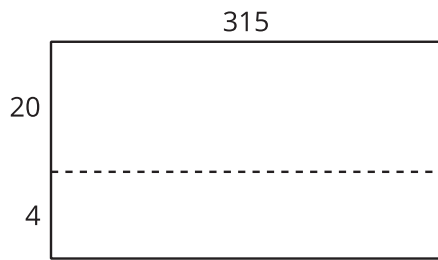
### Launch

- Groups of 2
- Give students time to read the task statement.
- “This time, you will write an expression in each piece of the diagram, rather than a number.”

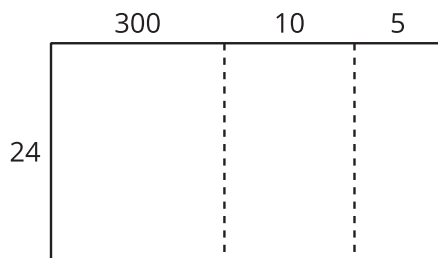
### Activity

- 1 minute: independent think time
- 7–8 minutes: partner work time
- Monitor for students who:

2.



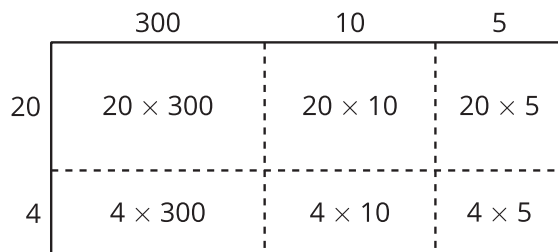
3.

4. Use one of the diagrams to find the value of  $315 \times 24$ .

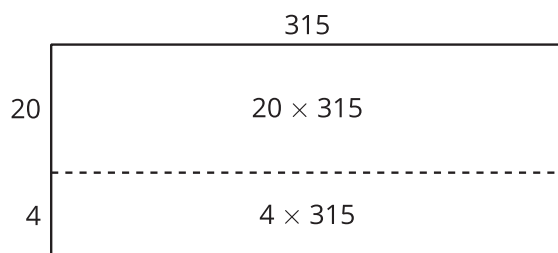
5. Explain why you chose that diagram to find the product.

### Student Responses

1.



2.

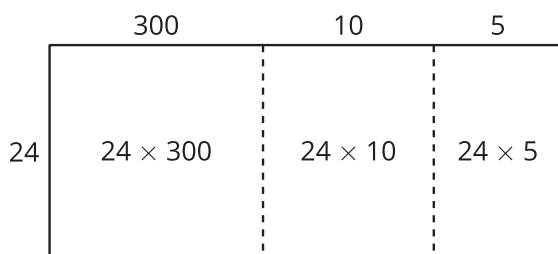


3.

- use the first diagram to help calculate the values for the other two diagrams.
- choose different diagrams for their calculations.

### Synthesis

- Display:  
 $20 \times 300$
- “How does this expression relate to the product  $315 \times 24$ ?” (It represents one of the products in the first diagram.)
- “Why isn’t this expression written in any of the other diagrams?” (Because the other diagrams are decomposed differently.)
- Invite students to share the diagram they chose to find the product and how it was helpful. As students share, record equations to represent each partial product.
- “What are the advantages or disadvantages of this way to calculate  $315 \times 24$ ?” (For full partial products, each product is simple to calculate. I do have 6 different numbers to add up at the end. When I broke the full product into two products, the calculations I used to find each product were harder, but once I had them, there were only two things to add. When I broke the full product into 3 products, this was a good compromise. The products were not too hard to calculate and there were just 3 of them to add.)



4. Sample response:  $24 \times 300 = 7,200$ ,  
 $24 \times 10 = 240$ ,  $24 \times 5 = 120$ ,  
 $7,200 + 240 + 120 = 7,560$ .
5. The first way leaves 6 products to find but they are all products of one-digit numbers with some factors of 10. In the second way, there are just two products to find. With the third way, there are 3 products to find but each one is simpler.

### Advancing Student Thinking

If students do not write the correct partial product in the diagram, ask, “What is a reasonable estimate for the product of  $315 \times 24$ ?”

## Lesson Synthesis

🕒 10 min

“Today we multiplied numbers and thought about how diagrams could help.”

Display the first image from the last activity.

“How can the diagram help us find the value of the product  $315 \times 24$ ?” (It helps me break up the product by place value. I take the hundreds, tens, and ones of one number and multiply them by the tens and ones of the other number.)

“Tomorrow we are going to work with partial products and organize them in a different way.”

### Suggested Centers

- Number Puzzles: Multiplication and Division (4–5), Stage 1: Two-digit Factors (Supporting)

---

**Complete Cool-Down**

---

**Response to Student Thinking**

Students do not draw a diagram that represents the product.

**Next Day Support**

- Launch the warm-up by drawing a diagram to represent the product in the estimation exploration. Ask, “How does the diagram represent the product?”

## Lesson 3: Partial Products in Algorithms

### Standards Alignments

Addressing 5.OA.A.2  
Building Towards 5.NBT.B.5

### Teacher-facing Learning Goals

- Multiply a three-digit number and a two-digit number.
- Represent a partial products algorithm.

### Student-facing Learning Goals

- Let's find partial products.

### Lesson Purpose

The purpose of this lesson is for students to multiply multi-digit whole numbers using partial products.

In previous lessons, students multiplied three-digit numbers and two-digit numbers, using strategies based on place value and the properties of operations. Students used diagrams to illustrate and explain their partial products calculations.

In this lesson, students move from diagrams to an algorithm that records partial products which they used in an earlier course when multiplying a two-digit number and a two-digit number. Students relate partial product expressions to diagrams and then analyze a systematic way to record partial products. This work sets students up to learn the standard algorithm in the next lesson.

### Access for:

#### Students with Disabilities

- Action and Expression (Activity 2)

#### English Learners

- MLR8 (Activity 1)

### Instructional Routines

Which One Doesn't Belong? (Warm-up)

### Materials to Copy

- Partial Product Expressions (groups of 2): Activity 1

**Lesson Timeline**

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

**Teacher Reflection Question**

In a future lesson, students will learn how to use the standard algorithm for multiplication to multiply multi-digit numbers. How do the diagrams and expressions used in today's lesson support this work?

**Cool-down** (to be completed at the end of the lesson)

🕒 5 min

Using Partial Products

**Standards Alignments**

Building Towards 5.NBT.B.5

**Student-facing Task Statement**Find the value of  $415 \times 43$  using partial products.**Student Responses**

Sample response:

$$\begin{array}{r}
 415 \\
 \times 43 \\
 \hline
 1245 \\
 1660 \\
 \hline
 17845
 \end{array}$$

----- **Begin Lesson** -----

## Warm-up

🕒 10 min

### Which One Doesn't Belong: Multiplying Large Numbers

#### Standards Alignments

Building Towards 5.NBT.B.5

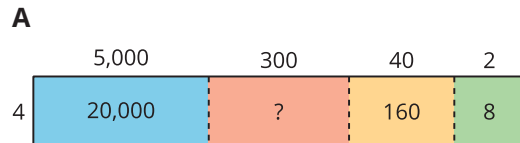
This warm-up prompts students to compare four representations of multiplication. Students compare diagrams and equations that represent multi-digit multiplication. This prepares them for the work of the lesson where they compare different ways to represent products as sums of partial products.

#### Instructional Routines

Which One Doesn't Belong?

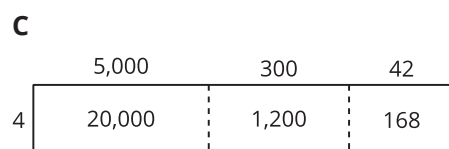
#### Student-facing Task Statement

Which one doesn't belong?



**B**

$$(4 \times 5,000) + (4 \times 300) + (4 \times 40) + (4 \times 2)$$



#### Student Responses

Sample responses:

- A is the only one that doesn't show all of the partial products, it has a question mark.

#### Launch

- Groups of 2
- Display the image.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time

#### Activity

- "Discuss your thinking with your partner."
- 2–3 minutes: partner discussion
- Share and record responses.

#### Synthesis

- "Why doesn't B belong?" (It's not a diagram. It's an expression.)
- "Does the value of expression B match the value represented in any of the diagrams?" (Yes, diagrams A and C both represent the product  $4 \times 5,342$  and that's the same as B.)

- B is the only one that is not a diagram and it does not show any values of partial products.
- C is the only one that doesn't show 4 partial products.
- D is the only one that doesn't represent the product  $4 \times 5,342$ .

## Activity 1

🕒 20 min

### Partial Products Everywhere

#### Standards Alignments

Addressing 5.OA.A.2

The goal of this activity is for students to examine different ways to write the product of a three-digit number and a two-digit number as a sum of partial products. Students match sets of partial products which can be put together to make the full product. Students are provided blank diagrams, familiar from the previous lesson, that they may choose to use to support their reasoning. In the activity synthesis, students relate the expressions and diagrams to equations to prepare them to analyze symbolic notation for partial products in the next activity.

When students relate partial products and diagrams to the product  $245 \times 35$  they look for and identify structure (MP7).

#### Access for English Learners

*MLR8 Discussion Supports.* Display the following sentence frame to support small-group discussion: "I noticed \_\_\_\_\_, so I matched . . ." Encourage students to challenge each other when they disagree.

*Advances: Speaking, Conversing*

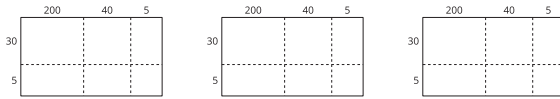
#### Materials to Copy

Partial Product Expressions (groups of 2)

#### Required Preparation

- Create a set of cards from the Instructional master for each group of 2.

## Student-facing Task Statement



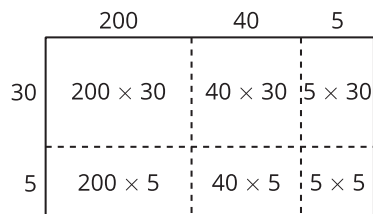
1. Take turns picking out a set of expressions that are equal to  $245 \times 35$  when added together. Use the diagrams if they are helpful.
2. Explain how you know the sum of your expressions is equal to  $245 \times 35$ .
3. What is the value of  $245 \times 35$ ? Explain or show your reasoning.

## Student Responses

Sample responses:

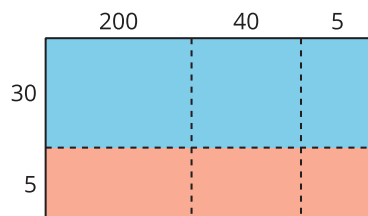
1.
  - $200 \times 30, 40 \times 30, 5 \times 30, 200 \times 5, 40 \times 5, 5 \times 5$

Sample diagram:



- $200 \times 35, 40 \times 35, 5 \times 35$
- $30 \times 245, 5 \times 245$

Sample diagram:



The top part shows  $30 \times 245$  and the bottom part shows  $5 \times 245$ .

- $240 \times 30, 5 \times 35, 240 \times 5$

Sample diagram:

## Launch

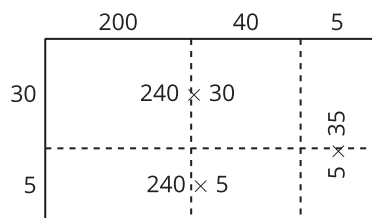
- Groups of 2
- Display first image from student book.
- “What product does this rectangle represent?” ( $245 \times 35$ )
- “Today, you are going to take turns with your partner picking expressions that can be added together to give the product  $245 \times 35$ . You can use the diagrams to explain your reasoning, if they are helpful.”

## Activity

- 10 minutes: partner work time
- Monitor for students who:
  - use the diagram to determine which expressions they will use.
  - look at the expressions and think about how they could be used to find the full product.
  - compute the full product in different ways.

## Synthesis

- Invite previously selected students to share their strategies. As students share, record their reasoning with equations.
- Display:  $245 \times 30 + 245 \times 5 = 245 \times 35$
- “How do you know this equation is true?” (I can put the 30 and 5 together since they are both multiplied by 245. I see that  $245 \times 30$  is the top part of the diagram and  $245 \times 5$  is the bottom part. Together that’s the whole diagram.)



- $205 \times 5$ ,  $205 \times 30$ ,  $40 \times 5$ ,  $30 \times 40$
  - $200 \times 35$ ,  $40 \times 30$ ,  $40 \times 5$ ,  $5 \times 35$
2. I made sure that my expressions included all of the partial products making up  $245 \times 35$ . I used the diagram to see that I had everything.
  3. 8,575. I used the 6 partial products in the first diagram and added them.

### Advancing Student Thinking

If students do not choose correct expressions to represent a sum that is equal to  $245 \times 35$ , refer to one of the empty boxes in the diagram and ask, “Which multiplication expression represents this partial product?”

## Activity 2

🕒 15 min

### Record Partial Products

#### Standards Alignments

Building Towards 5.NBT.B.5

The purpose of this activity is for students to consider 2 different ways of recording partial products in an algorithm that they worked with in a previous course. The numbers are the same as in the previous activity to allow students to make connections between the diagram and the written strategies. Students examine two different ways to list the partial products in vertical calculations, corresponding to working from left to right and from right to left. Regardless of the order, the key idea behind the algorithm is to multiply the values of each digit in one factor by the values of each digit in the other factor.

## Access for Students with Disabilities

*Action and Expression: Develop Expression and Communication.* Provide access to a variety of tools. Provide access to colored pencils or highlighters they can use to identify the partial products. *Supports accessibility for: Visual-Spatial Processing, Conceptual Processing*

### Student-facing Task Statement

Andre	Clare
$\begin{array}{r} 245 \\ \times 35 \\ \hline 6,000 \\ 1,200 \\ 150 \\ 1,000 \\ 200 \\ + \phantom{000} \\ \hline 8,575 \end{array}$	$\begin{array}{r} 245 \\ \times 35 \\ \hline 25 \\ 200 \\ 1,000 \\ 1,200 \\ + 6,000 \\ \hline 8,575 \end{array}$

- How are Andre's and Clare's strategies the same? How are they different?
- Create a list of equations to match the partial products Andre and Clare found.

### Student Responses

- Andre and Clare both find 6 partial products. The partial products are the same in both calculations but they are listed in different order.
- $30 \times 200 = 6,000$   
 $30 \times 40 = 1,200$   
 $30 \times 5 = 150$   
 $5 \times 200 = 1,000$   
 $5 \times 40 = 200$   
 $5 \times 5 = 25$

### Launch

- Groups of 2
- "We're going to look at two ways students recorded partial products for multiplying 245 by 35."
- Display the image of Andre's and Clare's calculations.
- "How does this relate to what you just did?" (You can see they split it up into different partial products and listed the results to add them up.)

### Activity

- 3 minutes: independent work time
- 5 minutes: partner work time
- Monitor for students who identify a pattern for how Andre and Clare list the partial products

### Synthesis

- "Both of these strategies use an algorithm that lists the partial products. An algorithm is a set of steps that works every time as long as the steps are carried out correctly."
- "How are both the approaches the same?" (They both multiply ones and tens by hundreds, tens, and ones.)
- "How are the approaches different?" (One starts with the hundreds and the other starts with the ones. One goes from left to right and the other goes from right to left.)

- “Why is it important to list the products in an organized way?” (That way I know I found all the partial products. I did not leave some out or take some twice.)
- Display:  
 $245 \times 35$
- Display student work to show the list of equations from the second problem or use the list in the student responses.
- “How does each expression relate to the product  $245 \times 35$ ?” ( $30 \times 200$  is the product of the 3 in the tens place of 35 and the 2 in the hundreds place of 245.)

### Advancing Student Thinking

If students do not write the correct equations, refer to the individual partial products and ask, “Where is this partial product represented in the multiplication expression  $245 \times 35$ ?”

## Lesson Synthesis

🕒 10 min

“Today we found products of two-digit and three-digit numbers using partial products. We saw how diagrams can help us make sure we found all the partial products. We also saw we could list partial products using an algorithm.”

“How do you know that all the different ways to find the product give the same answer?” (You’re always adding up the same partial products, just calculating them and putting them together in different ways.)

“What is helpful to remember when you are using partial products to determine a full product?” (You have to make sure to find all of the partial products. You have to make sure you add them. Sometimes I can add them mentally and then don’t need to list all of them.)

### Suggested Centers

- Number Puzzles: Multiplication and Division (4–5), Stage 1: Two-digit Factors (Supporting)

---

----- Complete Cool-Down -----

**Response to Student Thinking**

Students do not find the value of  $415 \times 43$ .

**Next Day Support**

- Before the warm-up, invite students to work in small groups to discuss a correct response to this cool-down.

# Lesson 4: Standard Algorithm: One-digit and Multi-digit Numbers with Composing

## Standards Alignments

Addressing 5.NBT.B.5

### Teacher-facing Learning Goals

- Use the standard algorithm to multiply up to five-digit numbers by one-digit factors, including composing new units.

### Student-facing Learning Goals

- Let's use the standard algorithm to multiply one-digit numbers and multi-digit numbers.

## Lesson Purpose

The purpose of this lesson is for students to use the standard algorithm to multiply up to five-digit numbers and one-digit numbers.

In grade 4, students interpreted the standard algorithm for multiplication and compared it to a partial products algorithm to multiply up to four-digit numbers and one-digit numbers. In this lesson, students extend their understanding of the standard algorithm to multiply up to five-digit numbers and one-digit numbers, including problems where one or more new units are composed. This is the first in a series of lessons to support students in developing fluency using the standard algorithm to multiply multi-digit numbers.

## Access for:

### Students with Disabilities

- Representation (Activity 2)

## Instructional Routines

MLR1 Stronger and Clearer Each Time (Activity 1), Number Talk (Warm-up)

## Lesson Timeline

Warm-up	10 min
Activity 1	20 min

## Teacher Reflection Question

What evidence do you see that your students are applying what they learned about partial products to make sense of the standard algorithm?

Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

## Cool-down (to be completed at the end of the lesson)

 5 min

Standard Algorithm Calculation

### Standards Alignments

Addressing 5.NBT.B.5

### Student-facing Task Statement

Use the standard algorithm to find the value of  $3,514 \times 7$ .

### Student Responses

Sample response:

$$\begin{array}{r}
 \phantom{3,} \overset{3}{5} \phantom{1} \overset{2}{4} \\
 3, \phantom{5} \phantom{1} \phantom{4} \\
 \times \phantom{3,} \phantom{5} \phantom{1} \phantom{4} \phantom{7} \\
 \hline
 2 \phantom{4,} \phantom{5} \phantom{9} \phantom{8}
 \end{array}$$

----- Begin Lesson -----

## Warm-up

 10 min

Number Talk: Partial Product

### Standards Alignments

Addressing 5.NBT.B.5

The purpose of this Number Talk is to highlight the calculations that students will make when they use the standard algorithm. The first three calculations are partial products. The fourth calculation is the

sum of the first three and this is the number that is recorded when performing the standard multiplication algorithm to which students will be introduced in this lesson.

## Instructional Routines

Number Talk

### Student-facing Task Statement

Find the value of each product mentally.

- $3 \times 3$
- $3 \times 20$
- $3 \times 600$
- $3 \times 623$

### Student Responses

Sample responses:

- 9: I just knew it.
- 60: It's 6 tens.
- 1,800: It's 18 hundreds.
- 1,869: I added the previous three numbers.

### Launch

- Display one problem.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

### Activity

- Record answers and strategy.
- Keep problems and work displayed.
- Repeat with each problem.

### Synthesis

- "How is the last product related to the first three?" (It is the sum of the first three.)
- "Did the first three calculations help you find the last product?" (Yes, I was able to add them together to find  $3 \times 623$ .)

## Activity 1

🕒 20 min

Compose with the Standard Algorithm

### Standards Alignments

Addressing 5.NBT.B.5

The goal of this activity is for students to understand how to record newly composed units when

using the standard algorithm for multiplication. Students compare the familiar partial products algorithm to the standard algorithm. Students may draw on their experience with the standard algorithm for addition to make sense of the new units being composed.

When students discuss their interpretation of Elena's calculation and improve their explanations they construct viable arguments and critique the reasoning of others (MP3).

## Instructional Routines

MLR1 Stronger and Clearer Each Time

### Student-facing Task Statement

Here is how Han calculated  $318 \times 3$  using partial products.

$$\begin{array}{r} 318 \\ \times 3 \\ \hline 24 \\ 30 \\ + 900 \\ \hline 954 \end{array}$$

Here is how Elena calculated  $318 \times 3$  using the standard algorithm.

$$\begin{array}{r} 2 \\ 318 \\ \times 3 \\ \hline 954 \end{array}$$

1. What does the 2 in Elena's calculation represent? Explain or show your reasoning.
2. What does the 5 in Elena's solution represent? Explain or show your reasoning.

### Student Responses

1. The 2 represents two tens because  $3 \times 8 = 24$ .
2. The 5 represents 50 because  $3 \times 10 = 30$  and  $30 + 20 = 50$  or the 5 represents 5 tens because 3 times 1 ten is 3 tens plus 2 more

### Launch

- Groups of 2
- Display the problems.
- "Take a moment to look at how Elena and Han calculated  $318 \times 3$ . Explain to your partner what each student did."
- 2 minutes: quiet think time
- 2 minutes: partner discussion

### Activity

- 5–6 minutes: independent work time
- Monitor for students who:
  - explain that the 2 represents the 20 in 24, which is the product of  $3 \times 8$ .
  - explain that the 5 represents 5 tens because 3 times 1 ten is 3 tens, plus 2 more tens is 5 tens.

### MLR1 Stronger and Clearer Each Time

- "Share your response with your partner. Take turns being the speaker and the listener. If you are the speaker, share your ideas and writing so far. If you are the listener, ask questions and give feedback to help your partner improve their work."
- 3–5 minutes: structured partner discussion
- Repeat with 2–3 different partners.

tens is 5 tens.

- “Revise your initial draft based on the feedback you got from your partners.”
- 2–3 minutes: independent work time

### Synthesis

- Invite previously selected students to share their revised explanations.
- “Elena used the standard algorithm for multiplication to find the product. When we compose a new unit in the standard algorithm, we record the new number of new units over the place value to the left of the digit we are multiplying.”

### Advancing Student Thinking

If students do not explain what the 2 or 5 represents, ask, “How can we use partial products to figure out what the 2 or 5 represents?”

## Activity 2

🕒 15 min

Use the Standard Algorithm

### Standards Alignments

Addressing 5.NBT.B.5

The purpose of this activity is for students to use the standard algorithm to multiply a multi-digit number by a one-digit number. Students find products that involve one or more compositions of a new unit.

### ♿ Access for Students with Disabilities

*Representation: Develop Language and Symbols.* Provide students with access to a chart of Elena’s method so students can refer to it as they complete the task.

*Supports accessibility for: Memory, Organization*

## Student-facing Task Statement

Calculate each product using Elena’s strategy.

- $3,615 \times 4$
- $16,023 \times 3$
- $27,326 \times 3$
- $10,215 \times 6$

## Student Responses

$$\begin{array}{r}
 1. \quad \quad \quad 2 \quad \quad 2 \\
 \quad \quad \quad 3, 6 \ 1 \ 5 \\
 \times \quad \quad \quad \quad \quad 4 \\
 \hline
 1 \ 4, 4 \ 6 \ 0
 \end{array}$$

$$\begin{array}{r}
 2. \quad \quad \quad 1 \\
 \quad \quad \quad 1 \ 6, 0 \ 2 \ 3 \\
 \times \quad \quad \quad \quad \quad 3 \\
 \hline
 4 \ 8, 0 \ 6 \ 9
 \end{array}$$

$$\begin{array}{r}
 3. \quad \quad \quad 2 \quad \quad \quad 1 \\
 \quad \quad \quad 2 \ 7, 3 \ 2 \ 6 \\
 \times \quad \quad \quad \quad \quad 3 \\
 \hline
 8 \ 1, 9 \ 7 \ 8
 \end{array}$$

$$\begin{array}{r}
 4. \quad \quad \quad 1 \quad \quad 3 \\
 \quad \quad \quad 1 \ 0, 2 \ 1 \ 5 \\
 \times \quad \quad \quad \quad \quad 6 \\
 \hline
 6 \ 1, 2 \ 9 \ 0
 \end{array}$$

## Launch

- Groups of 2
- “You are going to practice Elena’s multiplication strategy, the standard algorithm.”

## Activity

- 5-6 minutes: independent work time
- 2-3 minutes: partner discussion time

## Synthesis

- Invite students to share their solutions and reasoning for the first and second problems.
- “Which new units did you compose in the first problem?” (New tens because  $4 \times 5 = 20$  and ten-thousands because  $3,000 \times 4 = 12,000$ ).
- “Which new units did you compose in the second problem?” (New ten-thousands because  $3 \times 6,000 = 18,000$ .)
- Invite students to share their solutions for the third and fourth problems.

## Lesson Synthesis

🕒 10 min

“Today we learned the standard algorithm to multiply whole numbers.”

Display student work for  $27,326 \times 3$ .

“Which new units were composed here? How do you know?” (They composed new tens and new ten-thousands. I see a 1 above the 2 tens and a 2 above the 2 ten thousands.)

“How did they keep track of the new units?” (They wrote them above the first factor in the correct place value.)

## Suggested Centers

- Number Puzzles: Multiplication and Division (4–5), Stage 2: Multi-digit Factors (Addressing)

---

## Complete Cool-Down

---

## Response to Student Thinking

Students do not use the standard algorithm correctly.

## Next Day Support

- Launch activity 1 with a discussion about this cool-down.

# Lesson 5: Standard Algorithm: Multi-digit Numbers without Composing

## Standards Alignments

Addressing	5.NBT.B.5
Building Towards	5.NBT.B.5

## Teacher-facing Learning Goals

- Use the standard algorithm to multiply up to three-digit numbers and two-digit numbers, without composing new units.

## Student-facing Learning Goals

- Let's use the standard algorithm to multiply two-digit numbers and three-digit numbers.

## Lesson Purpose

The purpose of this lesson is for students to use the standard algorithm to multiply up to three-digit numbers and two-digit numbers without composing a new unit.

In a previous lesson, students used the standard algorithm to multiply up to five-digit numbers by one-digit numbers. They connected the standard algorithm to a different algorithm that uses partial products. In this lesson, students find products of a three-digit number and a two-digit number when composing is not required. The numbers in this lesson do not require composing so that students can make sense of where to record the second partial when using the standard algorithm to multiply a three-digit number by a two-digit number. Students will work with products of a two-digit number and a three-digit number when composing is required in the next lesson.

## Access for:

### Students with Disabilities

- Representation (Activity 2)

### English Learners

- MLR8 (Activity 1)

## Instructional Routines

Number Talk (Warm-up)

**Lesson Timeline**

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

**Teacher Reflection Question**

How did the student work that you selected impact the direction of the discussion? What student work might you pick next time if you taught the lesson again?

**Cool-down** (to be completed at the end of the lesson)

🕒 5 min

Standard Algorithm without Composition of a New Unit

**Standards Alignments**

Addressing 5.NBT.B.5

**Student-facing Task Statement**

Use the standard algorithm to find the value of  $203 \times 23$ .

**Student Responses**

4,669

$$\begin{array}{r}
 203 \\
 \times 23 \\
 \hline
 609 \\
 + 4,060 \\
 \hline
 4,669
 \end{array}$$

----- **Begin Lesson** -----

**Warm-up**

🕒 10 min

Number Talk: Partial Products

## Standards Alignments

Building Towards 5.NBT.B.5

The purpose of this Number Talk is to elicit strategies and understandings students have for mentally multiplying numbers that require composing a new unit. Students apply this understanding in the lesson when they compose a new unit using the standard algorithm.

## Instructional Routines

Number Talk

### Student-facing Task Statement

Find the value of each product mentally.

- $20 \times 3$
- $24 \times 3$
- $120 \times 3$
- $140 \times 3$

### Student Responses

Sample responses:

- 60 because  $3 \times 2$  is 6 and then I multiplied by 10.
- 72 because  $4 \times 3$  is 12 and then I added that to the 60.
- 360 because  $3 \times 100$  is 300 and  $3 \times 20$  is 60.
- 420 because  $3 \times 100$  is 300 and  $3 \times 40$  is 120, and then I added 300 and 120.

### Launch

- Display one problem.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

### Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

### Synthesis

- “How do the tens in  $20 \times 3$  compare to the tens in  $24 \times 3$ ?” (There is one more ten in  $24 \times 3$  that came from  $3 \times 4$ .)
- “How do the hundreds in  $120 \times 3$  compare to the hundreds in  $140 \times 3$ ?” (There is one more hundred in  $140 \times 3$  that came from  $40 \times 3$ .)

## Activity 1

🕒 20 min

Compare Two Algorithms

## Standards Alignments

Addressing 5.NBT.B.5

The purpose of this activity is for students to connect an algorithm that uses partial products to the standard algorithm when multiplying a three-digit and a two-digit number. The standard algorithm shows 2 partial products while the other algorithm shows 6 partial products. While the products are recorded differently, the same 6 partial products are still part of both calculations, and this activity gives students a chance to see this common structure while also appreciating the different way the standard algorithm records the calculations.

Students use the common structure in the two algorithms (MP7) to make sense of the standard algorithm before they use it themselves in the next activity.

### Access for English Learners

*MLR8 Discussion Supports.* Display sentence frames to support small-group discussions: “\_\_\_\_ and \_\_\_\_ are the same/alike because . . .,” “\_\_\_\_ and \_\_\_\_ are different because . . . .”

Advances: Listening, Speaking

### Student-facing Task Statement

Two algorithms for finding the value of  $413 \times 21$  are shown.

$$\begin{array}{r}
 413 \\
 \times 21 \\
 \hline
 3 \text{ step 1} \\
 10 \text{ step 2} \\
 400 \text{ step 3} \\
 60 \text{ step 4} \\
 200 \text{ step 5} \\
 + 8,000 \text{ step 6} \\
 \hline
 8,673 \text{ step 7}
 \end{array}$$

step 1

$$\begin{array}{r}
 413 \\
 \times 21 \\
 \hline
 3
 \end{array}$$

step 4

step 2

$$\begin{array}{r}
 413 \\
 \times 21 \\
 \hline
 10
 \end{array}$$

step 5

step 3

$$\begin{array}{r}
 413 \\
 \times 21 \\
 \hline
 400
 \end{array}$$

step 6

### Launch

- Groups of 2
- Display the algorithms.
- “We are going to learn about a new algorithm today.”

### Activity

- 1–2 minutes: quiet think time
- 8–10 minutes: partner work time
- Monitor for students who notice that:
  - the two algorithms show the same products in the same right to left order.
  - the two algorithms record the results of the products differently.

### Synthesis

- Invite students to share what was alike in

$$\begin{array}{r} 413 \\ \times 21 \\ \hline 413 \\ 8260 \\ \hline \end{array}$$

$$\begin{array}{r} 413 \\ \times 21 \\ \hline 413 \\ 260 \\ \hline \end{array}$$

$$\begin{array}{r} 413 \\ \times 21 \\ \hline 413 \\ 8260 \\ \hline 8,260 \end{array}$$

step 7

$$\begin{array}{r} 413 \\ \times 21 \\ \hline 413 \\ + 8,260 \\ \hline 8,673 \end{array}$$

1. How are the two algorithms the same? How are they different?
2. Explain or show where you see each step from the first algorithm in the second algorithm.
3. How do the final steps in the two algorithms compare?

## Student Responses

Sample responses:

1. For both algorithms, each place value in one number is multiplied by each place value in the other number. Numbers are multiplied in the same order, from right to left. In the first algorithm, each number is recorded on its own line. In the second algorithm, several numbers are recorded on the same line. Both have their totals added at the end.
2. In the second algorithm, each digit in 21 is multiplied by each digit in 413, like is done in the first algorithm, but instead of recording these numbers on separate lines, all the multiples of 1 are recorded on a line and all the multiples of 20 are recorded on a line. When 200 is recorded in step 5, that is  $20 \times 10$ , but it is put with the 60 from the  $20 \times 3$ .
3. Both algorithms have partial products that need to be added. There are 6 different products that have to be added in the first algorithm, while there are only 2 to add in the second algorithm.

the two algorithms, highlighting:

- all six partial products are calculated in both.
- they are calculated in the same order.
- they both need to add up their partial products at the end.
- Invite students to share what was different in the two algorithms highlighting:
  - one algorithm lists each partial product on a separate line while the standard algorithm lists some of them on the same line
- Circle the first partial product (413) in the standard algorithm.
- "What does the first partial product 413 represent?" (It's  $1 \times 413$ .)
- Circle the second partial product (8,260) in the standard algorithm.
- "What does the second partial product 8,260 represent?" (It's  $20 \times 413$ .)

## Advancing Student Thinking

If students do not explain where each of the partial products are in the standard algorithm, ask, “How can we use multiplication expressions to show where each partial product is in the standard algorithm?”

## Activity 2

🕒 15 min

Use the Standard Algorithm

### Standards Alignments

Addressing 5.NBT.B.5

The purpose of this activity is for students to practice multiplying a two-digit and a three-digit number using the standard algorithm. The problems do not involve composing new units so that students can practice the procedure of multiplying each place in one factor by each place in the other factor. In the last problem, students look at incorrect work where the value of the digit in the tens place is not accounted for. This problem encourages them to use estimation to assess the reasonableness of their answers and is the focus of the lesson synthesis.

### ♿ Access for Students with Disabilities

*Representation: Internalize Comprehension.* Synthesis: Invite students to identify which details were needed to solve the problems using the standard algorithm. Display the sentence frame, “The next time I use the standard algorithm, I will pay attention to . . . .”

*Supports accessibility for: Attention, Conceptual Processing*

### Student-facing Task Statement

Use the standard algorithm to find the value of each expression.

1.  $202 \times 12$
2.  $122 \times 33$
3.  $321 \times 24$

### Launch

- Groups of 2

### Activity

- 8-10 minutes: independent work time
- 2-3 minutes: partner discussion

4. Diego found the value of  $301 \times 24$ . Here is his work. Why doesn't Diego's answer make sense?

$$\begin{array}{r} 301 \\ \times 24 \\ \hline 1,204 \\ + 6,020 \\ \hline 1,806 \end{array}$$

### Student Responses

1. 
$$\begin{array}{r} 202 \\ \times 12 \\ \hline 404 \\ + 2,020 \\ \hline 2,424 \end{array}$$

2. 
$$\begin{array}{r} 122 \\ \times 33 \\ \hline 11 \\ 366 \\ + 3,660 \\ \hline 4,026 \end{array}$$

3. 
$$\begin{array}{r} 321 \\ \times 24 \\ \hline 1 \\ 1,284 \\ + 6,420 \\ \hline 7,704 \end{array}$$

4. It's too small.  $10 \times 300 = 3,000$  and Diego's answer is smaller than that.

### Synthesis

- Invite students to share their solution for  $122 \times 33$ .
- "How is multiplying 122 by the 3 in the ones place of 33 the same as multiplying 122 by the 3 in the tens place?" (In both cases I get 366.)
- "How is it different?" (The 3 in the tens place represents 30 and so the 366 needs to shift one place to the left because it is really 366 tens or 3660.)

## Lesson Synthesis

🕒 10 min

"Today, we used the standard algorithm to multiply a two-digit number and a three-digit number."

Display Diego's work from the last problem.

"Why doesn't Diego's answer make sense?" (The product is too small.  $300 \times 20$  is 6,000, so the product is greater than that.)

"What advice would you give Diego to revise his thinking?" (Remember that the 2 in 24 is 2 tens. So 2 tens times 1 should be 20, so you need to write the 2 in the tens place.)

## Suggested Centers

- Number Puzzles: Multiplication and Division (4–5), Stage 2: Multi-digit Factors (Addressing)

---

## Complete Cool-Down

---

## Response to Student Thinking

Students do not use the standard algorithm correctly.

## Next Day Support

- Create a poster with the steps to solving the cool-down problem from the previous lesson.

# Lesson 6: Standard Algorithm: Multi-digit Numbers with Composing

## Standards Alignments

Addressing 5.NBT.B.5

### Teacher-facing Learning Goals

- Use the standard algorithm to multiply up to three-digit numbers and two-digit numbers, including composing new units.

### Student-facing Learning Goals

- Let's multiply with the standard algorithm and compose new units.

## Lesson Purpose

The purpose of this lesson is for students to understand and apply the standard algorithm for multiplication when multiplying two- and three-digit numbers with composition of new units.

In previous lessons, students learned to use the standard algorithm for multiplication with up to five-digit numbers and one-digit numbers when a new unit is composed. They also multiplied three-digit and two-digit numbers without composing a new unit. In this lesson, students combine these two skills. They use the standard algorithm for multiplication of a three-digit and a two-digit number and record the composition of new units.

### Access for:

#### Students with Disabilities

- Engagement (Activity 2)

#### English Learners

- MLR8 (Activity 1)

## Instructional Routines

Number Talk (Warm-up)

### Lesson Timeline

Warm-up	10 min
Activity 1	25 min
Activity 2	10 min

### Teacher Reflection Question

As students worked in their small groups today, whose ideas were heard, valued, and accepted? How can you adjust the group structure tomorrow to ensure each student's ideas are a part of the collective learning?

Lesson Synthesis 10 min

Cool-down 5 min

**Cool-down** (to be completed at the end of the lesson)

⌚ 5 min

Use the Standard Algorithm

**Standards Alignments**

Addressing 5.NBT.B.5

**Student-facing Task Statement**Use the standard algorithm to find the product  $251 \times 34$ .**Student Responses**

8,534

$$\begin{array}{r}
 1 \\
 2 \\
 251 \\
 \times 34 \\
 \hline
 1,004 \\
 + 7,530 \\
 \hline
 8,534
 \end{array}$$

----- Begin Lesson -----

**Warm-up**

⌚ 10 min

Number Talk: Three Factors

**Standards Alignments**

Addressing 5.NBT.B.5

The purpose of this Number Talk is to elicit strategies and understandings students have for multiplying three factors, one of which is ten. These understandings help students develop fluency and will be helpful when students apply the standard algorithm to find the product of a three-digit and a two-digit number.

Students have an opportunity to look for and make use of structure (MP7) because they can use the distributive property to find a product using previous calculations.

## Instructional Routines

### Number Talk

#### Student-facing Task Statement

Find the value of each product mentally.

- $(2 \times 3) \times 10$
- $(2 \times 40) \times 10$
- $(2 \times 200) \times 10$
- $(2 \times 243) \times 10$

#### Student Responses

Sample responses:

- 60: I know  $2 \times 3 = 6$  and 10 times as much is 60.
- 800: I know  $2 \times 4 = 8$  and 100 times as much is 800.
- 4,000: I know  $2 \times 2 = 4$  and 1,000 times as much is 4,000.
- 4,860: I added up the three numbers from the other calculations.

#### Launch

- Display one expression.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

#### Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

#### Synthesis

- “How did multiplying all the products by 10 influence the result?” (It made the result ten times as big, so the digits all shift one place to the left and it has a zero at the end.)
- “How are the products  $2 \times 243$  and  $20 \times 243$  related?” (The second one is ten times as big, so the digits shift one place to the left and it has a 0 at the end.)
- “You can use this idea today when we apply the standard algorithm to find products of a 3-digit number and a 2-digit number.”

## Activity 1

🕒 25 min

Compose a New Unit

👤 ↔ 👤 PLC Activity

### Standards Alignments

Addressing 5.NBT.B.5

The goal of this activity is to use the standard algorithm to find products in which composition of a new unit happens once. Students first calculate a 3-digit and 2-digit example using a strategy of their choice and then analyze the same example done with composition recorded above the product. Students may use different strategies when they try on their own including

- partial products
- mentally accounting for the hundred that is composed when finding the product  $3 \times 40$

After students discuss how composing new units is recorded in the algorithm, they find the value of two multiplication expressions using the standard algorithm.

When students interpret a new way of multiplying a 3-digit and 2-digit number, they use their understanding of place value to make sense of the method (MP7).

### 🌐 Access for English Learners

*MLR8 Discussion Supports.Synthesis:* At the appropriate time, give groups 2–3 minutes to plan what they will say when they present to the class. “Practice what you will say when you share what each number represents in Lin’s problem with the class. Talk about what is important to say, and decide who will share each part.”

*Advances: Speaking, Conversing, Representing*

### Student-facing Task Statement

1. Find the value of  $241 \times 23$ .
2. Lin used the standard algorithm to find the value of  $241 \times 23$ . Here is her work:

### Launch

- Groups of 2
- “You are now going to learn how to compose and record new units for a three-digit and two-digit product.”

### Activity

- “Work with your partner on the first 2

$$\begin{array}{r}
 1 \\
 241 \\
 \times 23 \\
 \hline
 723 \\
 + 4,820 \\
 \hline
 5,543
 \end{array}$$

- Where do you see  $241 \times 3$  in Lin's work?
  - Where do you see  $241 \times 20$  in Lin's work?
  - What does the 1 above 241 represent in Lin's calculation?
- Use the standard algorithm to find the value of  $182 \times 41$ .
  - Use the standard algorithm to find the value of  $304 \times 23$ .

### Student Responses

- Sample response:

$$\begin{array}{r}
 241 \\
 \times 23 \\
 \hline
 4,000 \\
 800 \\
 20 \\
 600 \\
 120 \\
 + 3 \\
 \hline
 5,543
 \end{array}$$

- This is the number 723 that Lin wrote.
  - This is the number 4,820 that Lin wrote..
  - When she multiplies 40 and 3 this is 120. She writes down the 20 and the 1 is for the 1 hundred that will be added to the other hundreds of the product.

3.

4.

problems."

- 2-3 minutes: independent work time
- 5-7 minutes: partner work time

### Synthesis

- Invite students to share how they interpret Lin's work finding  $241 \times 23$ .
- Display the image of Lin's calculation.
- Circle the 2 in the number 723 in Lin's calculation.
- "What does the 2 in the tens place represent?" (It's 2 of the tens from  $3 \times 40$ .)
- "What does Lin do with the other 10 tens?" (She makes a hundred out of them and puts them together with the other hundreds when she multiplies 200 by 3.)
- Circle the 1 above 241 in Lin's work.
- "What does this 1 represent?" (It's the hundred from  $3 \times 40$ .)
- Circle the partial product 4,820.
- "What does 4,820 represent in the calculation?" ( $20 \times 241$ . The 2 from the factor 23 is in the tens place and so it represents 20.)
- "Now take a few minutes to solve the last two problems."
- 4-5 minutes independent work time
- Invite students to share the products and ask students what questions they have about the standard algorithm with composition.

$$\begin{array}{r}
 \phantom{0}3 \\
 182 \\
 \times \phantom{0}41 \\
 \hline
 182 \\
 + 7,280 \\
 \hline
 7,462
 \end{array}$$

$$\begin{array}{r}
 \phantom{0}1 \\
 304 \\
 \times \phantom{0}23 \\
 \hline
 912 \\
 + 6,080 \\
 \hline
 6,992
 \end{array}$$

## Activity 2

🕒 10 min

All the Products

### Standards Alignments

Addressing 5.NBT.B.5

The goal of this activity is to multiply numbers with no restrictions on the number of new units composed. Students first multiply a 3-digit number by a 1-digit number and a 3-digit number by a 2-digit number with no ones. They can then put these two results together to find the product of a 3-digit and 2-digit number with many carries. They then solve one more 3-digit and 2-digit example with no scaffold. Because these calculations have new units composed in almost every place value, students will need to locate and use the composed units carefully. It gives students a reason to attend to the features of their calculation and to use language precisely (MP6).

### 🕒 Access for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Chunk this task into more manageable parts. Check in with students to provide feedback and encouragement after each chunk.

*Supports accessibility for: Organization, Conceptual Processing*

### Student-facing Task Statement

Find the value of each product using the standard algorithm.

1.  $647 \times 9$

### Launch

- “You are going to find products with many new composed units. As you work, think carefully about where you place these values.”

2.  $647 \times 50$
3.  $647 \times 59$
4.  $264 \times 38$

### Student Responses

$$\begin{array}{r}
 \phantom{0}4 \phantom{0}6 \\
 6 \phantom{0}4 \phantom{0}7 \\
 \times \phantom{0}9 \\
 \hline
 5, \phantom{0}8 \phantom{0}2 \phantom{0}3
 \end{array}
 \qquad
 \begin{array}{r}
 \phantom{0}2 \phantom{0}3 \\
 6 \phantom{0}4 \phantom{0}7 \\
 \times \phantom{0}5 \phantom{0}0 \\
 \hline
 0 \phantom{0}0 \phantom{0}0 \\
 + \phantom{0}3 \phantom{0}2, \phantom{0}3 \phantom{0}5 \phantom{0}0 \\
 \hline
 3 \phantom{0}2, \phantom{0}3 \phantom{0}5 \phantom{0}0
 \end{array}$$
  

$$\begin{array}{r}
 \phantom{0}2 \phantom{0}3 \\
 6 \phantom{0}4 \phantom{0}7 \\
 \times \phantom{0}5 \phantom{0}9 \\
 \hline
 1 \\
 5, \phantom{0}8 \phantom{0}2 \phantom{0}3 \\
 + \phantom{0}3 \phantom{0}2, \phantom{0}3 \phantom{0}5 \phantom{0}0 \\
 \hline
 3 \phantom{0}8, \phantom{0}1 \phantom{0}7 \phantom{0}3
 \end{array}
 \qquad
 \begin{array}{r}
 \phantom{0}1 \phantom{0}1 \\
 2 \phantom{0}6 \phantom{0}4 \\
 \times \phantom{0}3 \phantom{0}8 \\
 \hline
 1 \\
 2, \phantom{0}1 \phantom{0}1 \phantom{0}2 \\
 + \phantom{0}7, \phantom{0}9 \phantom{0}2 \phantom{0}0 \\
 \hline
 1 \phantom{0}0, \phantom{0}0 \phantom{0}3 \phantom{0}2
 \end{array}$$

### Activity

- 8–10 minutes: independent work time
- 3–5 minutes: partner discussion
- Monitor for students who use the results of the first two calculations to find the third, and for students who correctly compose all the new place values.

### Synthesis

- Display the expression:  $647 \times 59$ .
- “How did you use the first two calculations to help with the third problem?” (They gave me the two partial products for the product  $647 \times 59$ , so I just had to add them up.)
- Invite students to share their responses for the last product, focusing on the newly composed units.

## Lesson Synthesis

🕒 10 min

“Today, we practiced using the standard algorithm to multiply multi-digit numbers with new units composed.”

“What do you have to think about when you are multiplying and a lot of new units are composed?” (You have to keep track of how you record the units. You can make an estimate to see if your answer is reasonable.)

Display student work for  $264 \times 38$  from activity 2 or use the example from the student responses.

$$\begin{array}{r}
 \phantom{0}1 \phantom{0}1 \\
 2 \phantom{0}6 \phantom{0}4 \\
 \times \phantom{0}3 \phantom{0}8 \\
 \hline
 1 \\
 2, \phantom{0}1 \phantom{0}1 \phantom{0}2 \\
 + \phantom{0}7, \phantom{0}9 \phantom{0}2 \phantom{0}0 \\
 \hline
 1 \phantom{0}0, \phantom{0}0 \phantom{0}3 \phantom{0}2
 \end{array}$$

"Where did we compose new units when we solved this problem?" (When we multiplied to find the two partial products, we had to compose new units above the 2 and 6 in 264. When we added the partial products, we composed a new one thousand above the 2.)

"How is composing new units when we multiply the same as composing new units when we add?" (When I am multiplying or adding numbers sometimes I get a value that's too much for the place I'm in. The composed units are recorded separately and then I add them.)

"How is composing new units when we multiply different from composing new units when we add?" (When we multiply, we multiply and then add the new units. When we add, we are adding the whole time, there is no multiplication.)

---

### **Suggested Centers**

- Number Puzzles: Multiplication and Division (4–5), Stage 2: Multi-digit Factors (Addressing)

## Lesson 7: Build Multiplication Fluency

### Standards Alignments

Addressing 5.NBT.B.5

### Teacher-facing Learning Goals

- Use the standard algorithm to find products with any number of newly composed units.

### Student-facing Learning Goals

- Let's multiply multi-digit whole numbers using the standard algorithm.

### Lesson Purpose

The purpose of this lesson is to multiply two-digit and three-digit numbers with any number of newly composed units.

In previous lessons students have used the standard algorithm to find products of 3-digit and 2-digit numbers including products with newly composed units. In this lesson, students continue to find products, building fluency in situations with any number of newly composed units. Students learn a center game where they try to make the greatest product using 5 digits which they successively select. Continuing the theme of large products, students examine the number of new units that can be composed using the standard algorithm for multiplication and see that it is possible to compose up to 8 new units but not 9.

### Access for:

#### Students with Disabilities

- Action and Expression (Activity 1)

#### English Learners

- MLR8 (Activity 2)

### Instructional Routines

Notice and Wonder (Warm-up)

### Materials to Copy

- Greatest Product (groups of 1): Activity 1
- Number Cards (0-10) (groups of 2): Activity 1





in these ways?

- Which way of finding these products is faster?

## Activity 1

🕒 15 min

### Greatest Product

#### Standards Alignments

Addressing 5.NBT.B.5

Students use their understanding of place value to generate expressions that have the greatest product. They take turns selecting number cards 0 through 9 and place them strategically to make the largest product of a 2-digit number and a 3-digit number. Students will need to think about how the different place values influence the value of the product and choose where to put their digits accordingly. There is also a large element of chance since they do not know in advance which numbers they will select.

#### 🕒 Access for Students with Disabilities

*Action and Expression: Develop Expression and Communication.* Give students access to grid paper for finding the product.

*Supports accessibility for: Visual-Spatial Processing, Organization*

#### Materials to Copy

Greatest Product (groups of 1), Number Cards (0-10) (groups of 2)

#### Student-facing Task Statement

$$\begin{array}{r} \square \square \square \\ \times \quad \square \square \\ \hline \end{array}$$

Directions:

- Partner A chooses a number card and

#### Launch

- Groups of 2
- Give each group a set of number cards and 2 copies of the Instructional master.
- “Remove the cards that show 10 and set them aside.”
- “We’re going to play a game called Greatest Product. Let’s read through the directions and play one round together.”

writes the number in one of the blanks for Round 1.

- Partner B does the same.
- Repeat until each partner has a two-digit by three-digit multiplication problem.
- Find the product.
- The partner with the greater product wins a point.
- The partner with the most points after 5 rounds wins the game.

### Student Responses

Answers vary.

- Read through the directions with the class and play a round against the class using the diagram in the student workbook:
- Display each number card.
  - Think through your choices aloud.
  - Record your move and score for all to see.

### Activity

- “Now, play the game with your partner.”
- 8–10 minutes: partner work time

### Synthesis

- Display a blank image from game board.
- “If this is your game board and you pick an 8, where would you write the 8? Why?” (I would either put it in the hundreds place of the top number or the tens place of the bottom number because it’s a big number and those are the biggest place values.)
- Write the 8 in the hundreds place of the top number.
- “What if you next select a 1? Where would you write the 1? Why?” (I would put it in the ones place of one of the numbers because I want to put a bigger number in the tens place.)
- “What did you find challenging about the game?” (Since I didn’t know what numbers I was going to get on later picks, I sometimes wasn’t sure where to put a number because I didn’t know if I would get a bigger number on a later pick.)

## Activity 2 (optional)

🕒 20 min

Desperately Seeking 9 New Units

## Standards Alignments

Addressing 5.NBT.B.5

The purpose of this activity is for students to explore the number of new units that can be composed when using the standard algorithm. As students experiment with the given numbers, they will find examples of 1, 2, 3, 4, 5, 6, 7, or 8 new units composed. Here they address the question of whether or not it is possible to compose 9 or more new units when using the standard algorithm. In order to make sense of and persevere in solving this problem (MP1), there are several types of arguments students may make, all of which highlight how the standard algorithm works:

- they may calculate  $999 \times 9$  and see that while they get very close to composing 9 new units, they fall 1 short
- the biggest product that can be made from multiplying a pair of 1-digit numbers is 81, which would mean that 8 units are composed
- this 81 has to be combined with whatever new units were composed before, but since that's 8, that means the largest number you can form at each step in the standard algorithm is 89, which is 1 short of the 90 you would need to compose 9 new units

### Access for English Learners

*MLR8 Discussion Supports.* Synthesis: At the appropriate time, give groups 2–3 minutes to plan what they will say when they present to the class. “Practice what you will say when you share your solutions with the class. Talk about what is important to say, and decide who will share each part.”  
*Advances: Speaking, Conversing*

### Student-facing Task Statement

Tyler notices that when he uses the standard algorithm and composes a new unit, sometimes there is 1 new unit, sometimes 2, all the way up to 8. He has not seen an example with 9 of the new unit.

1. For each of these products, how many of each new unit do you compose?
  - a.  $256 \times 5$
  - b.  $587 \times 8$
  - c.  $809 \times 9$

### Launch

- Groups of 2

### Activity

- 1–2 minutes: quiet think time
- 3–5 minutes: partner work time

### Synthesis

- Invite students to share the number of units they composed in the calculations of problem 1.
- “Was anything missing from 1 to 8?” (No,

2. Do you think it is possible to compose 9 of a new unit with the standard multiplication algorithm?

### Student Responses

- 3 tens, 2 hundreds, 1 thousand
  - 5 tens, 6 hundreds, 4 thousands
  - 8 tens, 7 thousands
- No. Possible responses: in order to have 9 of a new unit, 10 would have to be one of the factors and that is not possible because 10 is a two digit number. For example, even if we multiply  $999 \times 99$ , 8 is the highest number of new units that is composed.

we composed everything from 1 to 8 new units.)

- “Was anyone able to compose 9 new tens in a product? Why?” (No. The new tens come when I take the product of the ones. The biggest that can be is  $9 \times 9$  which gives me 8 tens.)
- “Was anyone able to compose 9 new hundreds? Why?” (No. I could get 8 new hundreds if I have  $9 \times 90$ , giving me 810. I might also have to combine that with the new tens from the product of the ones, but that would give me at most 890, not 900.)

### Advancing Student Thinking

If students are not fluent with their multiplication facts, offer them a multiplication table. As they use the standard algorithm to find the products in problem 1, ask them to circle each new unit. If they are not sure if it is possible to compose 9 new units, ask them to use the algorithm to find the product of  $99 \times 9$  and  $999 \times 99$ .

## Lesson Synthesis

🕒 10 min

“Today we used the standard algorithm to find products of numbers with no restriction on the number of newly composed units and we examined how many new units can be composed.”

“Do you think the algorithm for multiplying whole numbers will work for any and all whole numbers? Why or why not? Discuss with a partner.” (I think so but if the numbers are big it will take up a lot of space and there could be a lot more new units to compose. I think it might work but it would take a long time if the numbers are big and I think it would be easy to make a mistake.)

Ask students to share their thinking.

“What do you still wonder about the standard algorithm for multiplying whole numbers?” (When is it a good strategy to use? Are there other ways that work well or better?)

## Suggested Centers

- Number Puzzles: Multiplication and Division (4–5), Stage 2: Multi-digit Factors (Addressing)

---

## Complete Cool-Down

---

## Response to Student Thinking

Students do not use the standard algorithm correctly.

## Next Day Support

- Launch activity 2 with a discussion about this cool-down.

# Lesson 8: Multiplication Fluency

## Standards Alignments

Addressing 5.NBT.B.5, 5.NF.B.4

### Teacher-facing Learning Goals

- Solve problems that involve the multiplication of multi-digit numbers.

### Student-facing Learning Goals

- Let's practice multiplication.

## Lesson Purpose

The purpose of this lesson is for students to play games to continue to develop fluency with multiplying multi-digit numbers with the standard algorithm.

In this lesson, students continue to find products of a 2-digit number and a 3-digit number with a strategic goal. They continue to play Greatest Product and they also use 5 given digits to try to make a product that is as close as possible to a given number. This is more challenging than making the greatest product as rather than putting the largest digits in the largest place values students will need to experiment or reflect about how each digit in the numbers influences the value of the product. This lesson provides an opportunity for teachers to observe students as they use the standard algorithm and offer support, as needed.

### Access for:

#### Students with Disabilities

- Action and Expression (Activity 2)

## Instructional Routines

Number Talk (Warm-up)

### Materials to Gather

- Materials from a previous lesson: Activity 1
- Materials from previous centers: Activity 1

## Lesson Timeline

Warm-up

10 min

## Teacher Reflection Question

Reflect on a time your thinking changed about

Activity 1	20 min	something in class recently. How will you alter your teaching practice to incorporate your new understanding?
Activity 2	15 min	
Lesson Synthesis	10 min	
Cool-down	5 min	

## Cool-down (to be completed at the end of the lesson)

 5 min

### Multiplication Reflection

#### Standards Alignments

Addressing 5.NBT.B.5

#### Student-facing Task Statement

Describe something you really understand well from this section on multiplying multi-digit numbers, or describe something that was confusing or challenging.

#### Student Responses

Sample response: I like using the standard algorithm because it's faster than writing all the partial products. I have to remember to estimate to check to make sure my answer makes sense.

## ----- Begin Lesson -----

## Warm-up

 10 min

### Number Talk: The Distributive Property

#### Standards Alignments

Addressing 5.NF.B.4

The purpose of this Number Talk is to elicit and connect strategies and understandings students have for multiplication of whole numbers and mixed numbers. These understandings help students develop fluency through reasoning about the distributive property.

## Instructional Routines

### Number Talk

#### Student-facing Task Statement

Find the value of each expression mentally.

- $6 \times 15$
- $6 \times 17$
- $6 \times 2\frac{1}{3}$
- $6 \times 2\frac{2}{3}$

#### Student Responses

- 90:  $6 \times 10 + 6 \times 5 = 60 + 30 = 90$
- 102:  $6 \times 15 + 6 \times 2 = 90 + 12 = 102$
- 14:  $6 \times 2 + 6 \times \frac{1}{3} = 12 + 2$
- 16: I can add  $\frac{1}{3} \times 6$  to the previous problem.

#### Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

#### Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

#### Synthesis

- "How were your strategies for the first 2 problems the same as your strategies for the last 2 problems?" (I decomposed the 15 into 10 and 5 and I also decomposed 2 and  $\frac{1}{3}$  in the third problem. I also used what I knew about problem 1 to help with problem 2 and used problem 3 to help with problem 4.)

## Activity 1

🕒 20 min

### Fluency Practice Choice Time

#### Standards Alignments

Addressing 5.NBT.B.5

The purpose of this activity is for students to choose from activities that offer practice multiplying multi-digit numbers, with and without composing.

- Greatest Product

- Number Puzzles Stage 2

## Materials to Gather

Materials from a previous lesson, Materials from previous centers

## Required Preparation

- Gather materials from:
  - Greatest Product
  - Number Puzzles Stage 2

## Student-facing Task Statement

Choose a game.

- Greatest Product
- Number Puzzles: Multiplication and Division

## Student Responses

Answers vary.

## Launch

- “Today we are going to choose from games we have already learned.”
- Display the choices in the student book.
- “Think about what you would like to do first.”
- 30 seconds: quiet think time

## Activity

- Invite students to work at the game of their choice.
- 10 minutes: partner work time
- “Choose what you would like to do next.”
- 10 minutes: partner work time

## Synthesis

- See lesson synthesis.

## Activity 2

🕒 15 min

Targeted Products

## Standards Alignments

Addressing 5.NBT.B.5

The purpose of this activity is for students to make products of a 2-digit number and a 3-digit number using 5 given digits with a goal of getting as close as they can to a given target number. This builds on their work from the previous activity and lesson where they tried to make the largest possible product. It is not important that students select the best possible products and the activity synthesis focuses on the strategies that they use. Fundamental to these strategies will be understanding place value and making estimates of products of a 2-digit number and a 3-digit number (MP7).

### Access for English Learners

*MLR8 Discussion Supports.* Prior to solving the problems, invite students to make sense of the situations. Monitor and clarify any questions about the context.

*Advances: Reading, Representing*

### Access for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Invite students to verbalize their strategy for finding a 3-digit and a 2-digit number before they begin. Students can speak quietly to themselves, or share with a partner.

*Supports accessibility for: Organization, Conceptual Processing, Language*

## Student-facing Task Statement

- Using the digits 3, 5, 6, 8, and 9 make a product whose value is close to 50,000.

$$\begin{array}{r} \square \square \square \\ \times \quad \square \square \\ \hline \end{array}$$

- Using the digits 3, 5, 6, 8, and 9 make a product whose value is close to 20,000.

## Launch

- Groups of 2.
- “You are going to choose a 3-digit and a 2-digit number to get as close as possible to a target product.”

## Activity

- 5 minutes: independent work time
- 3 minutes: partner discussion time
- Monitor for students who change their product after finding the value and comparing to the target number.



---

----- **Complete Cool-Down** -----

**Response to Student Thinking**

Students have ideas they could share with a partner.

**Next Day Support**

- After the warm-up in the next lesson, pair students up to discuss their responses.

## Lesson 9: The Birds

### Standards Alignments

Addressing 5.MD.C.3, 5.MD.C.5, 5.NBT.B.5

### Teacher-facing Learning Goals

- Solve problems that involve the multiplication of multi-digit numbers.

### Student-facing Learning Goals

- Let's solve multiplication problems.

### Lesson Purpose

The purpose of this lesson is for students to use whole-number multiplication to solve problems.

In previous lessons, students learned to use the standard algorithm to multiply multi-digit numbers. In this lesson, they solve problems that involve multiplication. Students are not asked to use a particular algorithm when they solve these problems. Some of the numbers in the problems are large and cumbersome and lend themselves well to using the standard algorithm for multiplication. Other products have factors of 10 and other smaller factors and these lend themselves well to mental calculations or use of the associative and commutative properties of multiplication. Use this lesson as an opportunity to observe the strategies your students are applying. This lesson has a Student Section Summary.

### Access for:

#### Students with Disabilities

- Action and Expression (Activity 2)

#### English Learners

- MLR1 (Activity 2)

### Instructional Routines

Notice and Wonder (Warm-up)

### Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min

### Teacher Reflection Question

If you were to teach this lesson over again, what activity would you redo? How would your proposed changes support student learning?

Cool-down

5 min

**Cool-down** (to be completed at the end of the lesson)

🕒 5 min

On Screech

**Standards Alignments**

Addressing 5.MD.C.5, 5.NBT.B.5

**Student-facing Task Statement**

To make a birdhouse for a screech owl, the recommended area of the floor is 8 inches by 8 inches and the recommended height is 12 inches to 15 inches. What is the recommended range of volumes for a screech owl birdhouse? Explain or show your thinking.

**Student Responses**

768 to 960 cubic inches.  $8 \times 8 = 64$ ,  $64 \times 12 = 768$  and  $64 \times 15 = 960$ .

----- **Begin Lesson** -----**Warm-up**

🕒 10 min

Notice and Wonder: For the Birds

**Standards Alignments**

Addressing 5.MD.C.3

The purpose of this warm-up is to elicit the idea that the shape of a birdhouse can be modeled by a rectangular prism, which will be useful when students solve problems about the volume of birdhouses in a later activity. While students may notice and wonder many things about the photograph, the shape of the birdhouse is the important discussion point.

## Instructional Routines

Notice and Wonder

### Student-facing Task Statement

What do you notice? What do you wonder?



### Student Responses

Students may notice:

- There are two birds and only one house.
- The house is tilted.
- One of the birds is flying.

Students may wonder:

- Is there a nest inside the house?
- Who built the house?
- Do both birds live in the house?

### Launch

- Groups of 2
- Display the image.
- “What do you notice? What do you wonder?”
- 1 minute: quiet think time

### Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Share and record responses.

### Synthesis

- “How would you describe the shape of the birdhouse?” (It looks like the sides are rectangles. It could be a rectangular prism.)

## Activity 1

🕒 15 min

Home is Where the Bird Lives

### Standards Alignments

Addressing 5.MD.C.5, 5.NBT.B.5

The purpose of this activity is for students to estimate whole number products in the context of volume. In the next activity students will calculate the smallest and largest volumes within the range recommended for each type of bird. The estimates here may or may not fall within the range, depending on the numbers students pick. When making reasoned estimates, there is always some tension between accuracy and using the most friendly numbers. During the synthesis, students explain the different strategies they use to make reasonable estimates with calculations that they can perform as simply as possible, often mentally (MP3).

Students may need a quick reminder of how to find the volume of a rectangular prism. If needed, remind students that the volume of a rectangular prism is the product of the length, width, and height, or alternatively, the product of the area of a base and the height for that base.

### Student-facing Task Statement

Different types of birds use different types of houses. The table gives you the recommended side lengths for birdhouses of various species.

type of bird	side lengths of floor	height	volume estimate
chickadee	4 in by 4 in	6 to 10 in	
wood duck	10 in by 18 in	10 to 24 in	
barn owl	10 in by 18 in	15 to 18 in	
red-headed woodpecker	6 in by 6 in	12 to 15 in	
bluebird	5 in by 5 in	6 to 12 in	
swallow	6 in by 6 in	6 to 8 in	

Estimate a possible volume for each birdhouse. Be prepared to explain your reasoning.

### Student Responses

Sample responses:

### Launch

- Display table from task.
- “What do you notice and wonder about the table?” (There are different kinds of birds listed. There are side lengths for the floor but the height is a range. What do the numbers mean? Do smaller birds have smaller houses?)
- 1 minute: quiet think time
- Share and record student responses.
- If needed, display images of different kinds of birds.

### Activity

- 2 minutes: quiet think time
- 5 minutes: partner work time
- Monitor for students who consider friendly numbers to use for the height and also for estimating the product of the area of the floor and the height.

### Synthesis

- “How did you estimate the volume of a house for a wood duck?”
  - I know  $10 \times 18$  is 180 and I multiplied by 10 since that just adds

type of bird	side lengths of floor	height	volume estimate
chickadee	4 in by 4 in	6 to 10 in	160 cubic inches
wood duck	10 in by 18 in	10 to 24 in	1,800 cubic inches
barn owl	10 in by 18 in	15 to 18 in	3,000 cubic inches
red-headed woodpecker	6 in by 6 in	12 to 15 in	500 cubic inches
bluebird	5 in by 5 in	6 to 12 in	250 cubic inches
swallow	6 in by 6 in	6 to 8 in	240 cubic inches

- a zero.
- I also used  $10 \times 18 = 180$  but multiplied by 20 since that is between 10 and 24. I got 3,600 cubic inches.
- “How did you estimate the volume of a house for a red-headed woodpecker?”
  - I found that the area of the floor is 36 square inches. I multiplied this by 10 to get 360 and by 20 to get 720 and then picked a number in between, 500 cubic inches.
  - I found the area of the floor was 36 square inches. I rounded that to 40 since it is a nicer number and then found  $40 \times 12$  which is 480.
- “Which numbers are the friendliest for estimating products? Why?” (10 is the friendliest because I can use place value. Multiples of 10, like 20, are also friendly as I can multiply by 10 and then double.)

## Activity 2

🕒 20 min

What is the Volume?

### Standards Alignments

Addressing 5.MD.C.5, 5.NBT.B.5

The purpose of this activity is for students to find the range of recommended volumes for the birdhouses introduced in the first activity. This means finding the value of products of 3 numbers. Students will be able to choose which two factors to multiply first and may do so strategically so they can find the value mentally. Monitor for students who change their strategy based on the numbers they are multiplying. Also monitor for students who are using the standard algorithm to multiply three-digit numbers by two-digit numbers.

When students interpret the meaning of the products they find in the volume context, they reason abstractly and quantitatively (MP2).

### Access for English Learners

*MLR1 Stronger and Clearer Each Time.* Synthesis: Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their response to “What is the possible range of volumes for each type of birdhouse?”. Invite listeners to ask questions, to press for details, and to suggest mathematical language. Give students 2–3 minutes to revise their written explanation based on the feedback they receive.

*Advances: Writing, Speaking, Listening*

### Access for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Invite students to verbalize their strategy for determining the possible range of volumes for each type of birdhouse before they begin.

*Supports accessibility for: Organization, Conceptual Processing, Language*

## Student-facing Task Statement

Use the criteria from the table to determine the recommended range of volumes for each type of birdhouse.

type of bird	side lengths of floor	height	range of volume
chickadee	4 in by 4 in	6 to 10 in	
wood duck	10 in by 18 in	10 to 24 in	
barn owl	10 in by 18 in	15 to 18 in	
red-headed woodpecker	6 in by 6 in	12 to 15 in	
bluebird	5 in by 5 in	6 to 12 in	
swallow	6 in by 6 in	6 to 8 in	

## Launch

- Groups of 2
- “You are now going to find the full range of recommended volumes for each of the birdhouses.”

## Activity

- 5 minutes: individual work time
- 5 minutes: partner work time
- Monitor for students who use different strategies including:
  - mental calculations for smaller products
  - place value understanding when multiplying by 10
  - the standard algorithm

## Synthesis

- “How did you find the recommended volumes of the bluebird

## Student Responses

type of bird	side lengths of floor	height	range of volume
chickadee	4 in by 4 in	6 to 10 in	96 to 160 cubic inches
wood duck	10 in by 18 in	10 to 24 in	1,800 to 4,320 cubic inches
barn owl	10 in by 18 in	15 to 18 in	2,700 to 3,240 cubic inches
red-headed woodpecker	6 in by 6 in	12 to 15 in	432 to 540 cubic inches
bluebird	5 in by 5 in	6 to 12 in	150 to 300 cubic inches
swallow	6 in by 6 in	6 to 8 in	216 to 288 cubic inches

house?"

- I knew  $5 \times 5 = 25$  and  $25 \times 6 = 150$ . I used an algorithm to find  $25 \times 12$ .
- I knew  $5 \times 6 = 30$  and  $30 \times 5 = 150$  and then doubled that to get  $5 \times 5 \times 12$ .
- "How did you find the recommended volumes for the wood duck?"
  - The smallest one is  $10 \times 10 \times 18$  so I used place value to find the volume of 1,800.
  - The biggest one I multiplied 10 by  $18 \times 24$ , which I found with the standard algorithm.

## Lesson Synthesis

🕒 10 min

"Today we used different strategies to solve multiplication problems."

"When is it most helpful to use the standard algorithm for multiplication?" (I like to use it when the numbers are complicated. I always like to use it because it's reliable and I know how it works.)

"Take a minute to think about which of these problems you would use the standard algorithm to solve. Then share your strategy with your partner."

$$45 \times 6$$

$$20 \times 200$$

$$143 \times 67$$

$$125 \times 9$$

"Different problems call for different strategies, and we each might choose a different way to solve each of these problems. We could use the standard algorithm to solve all these problems, but we don't have to."

## Suggested Centers

- Number Puzzles: Multiplication and Division (4–5), Stage 2: Multi-digit Factors (Addressing)

## Student Section Summary

In this unit we found products of a three-digit number and a two-digit number. We first

represented the products with diagrams that help us break down the product by place value.

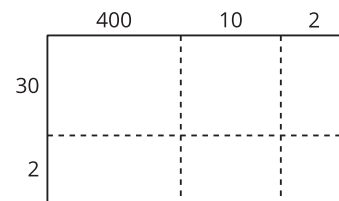
This diagram breaks up the product  $412 \times 32$  by place value. If we find and add up all of the partial products, we will get the product of  $412 \times 32$ .

Then we learned a new algorithm to multiply numbers, the standard algorithm for multiplication.

We can see the partial products are organized in a different way. 824 represents the partial product for  $2 \times 412$  and 12,360 represents the partial product for  $30 \times 412$ .

We noticed that sometimes we need to compose a new unit when we use the standard algorithm, and we represent that unit with notation. Sometimes, we may have to compose more than one new unit.

The 1 above the 1 in 216 represents the ten from the product  $3 \times 6$  and the 2 represents 2 hundreds from the product  $40 \times 6$ .



$$\begin{array}{r}
 412 \\
 \times 32 \\
 \hline
 824 \\
 + 12,360 \\
 \hline
 13,184
 \end{array}$$

$$\begin{array}{r}
 2 \\
 1 \\
 216 \\
 \times 43 \\
 \hline
 648 \\
 + 8,640 \\
 \hline
 9,288
 \end{array}$$

## ----- Complete Cool-Down -----

### Response to Student Thinking

Students do not calculate the correct range of volumes.

### Next Day Support

- Before the warm-up in the next lesson, invite students to work in partners and discuss a correct response to the cool-down from this lesson.

## Section B: Multi-digit Division Using Partial Quotients

### Lesson 10: World's Record Folk Dance

#### Standards Alignments

Building On 4.NBT.B.6

Building Towards 5.NBT.B.6

#### Teacher-facing Learning Goals

- Divide multi-digit whole numbers in a way that makes sense to them.

#### Student-facing Learning Goals

- Let's explore division with multi-digit numbers.

#### Lesson Purpose

The purpose of this lesson is for students to estimate and solve multi-digit division problems in a way that makes sense to them.

In this lesson, students explore a context to make sense of division with multi-digit numbers (MP1). This builds on work students did in grade 4 where they divided with up to 4-digit dividends and single-digit divisors. Students used place value understanding, the relationship between multiplication and division and partial quotients to divide. The work in this lesson gives teachers an opportunity to see how students apply their prior understanding, including multiplying multi-digit numbers in the last section. In future lessons, students work toward using more efficient methods to divide multi-digit numbers, including partial quotients.

The context is a world record event for making the largest Peruvian folk dance. The mathematically important part of the context is that there were 4,704 people at the record breaking event and they were in groups of 8. For more information about this event, follow the link:

<https://www.guinnessworldrecords.com/world-records/largest-peruvian-folk-dance>

#### Access for:

##### Students with Disabilities

- Engagement (Activity 2)

##### English Learners

- MLR8 (Activity 2)

#### Instructional Routines

5 Practices (Activity 1), How Many Do You See? (Warm-up)

**Lesson Timeline**

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

**Teacher Reflection Question**

How did the student work that you selected impact the direction of the discussion? What student work might you pick next time if you taught the lesson again?

**Cool-down** (to be completed at the end of the lesson)

🕒 5 min

## Another Dance

**Standards Alignments**

Building On 4.NBT.B.6

**Student-facing Task Statement**

A different group of 4,632 dancers make groups of 8.

1. Write a division expression to represent the situation.
2. How many groups of 8 will there be? Explain or show your thinking.

**Student Responses**

1.  $4,632 \div 8$
2. 579. Sample response:  $8 \times 500 = 4,000$ ,  $8 \times 50 = 400$ ,  $8 \times 20 = 160$ ,  $8 \times 9 = 72$  and  $500 + 50 + 20 + 9 = 579$ .

----- **Begin Lesson** -----**Warm-up**

🕒 10 min

## How Many Do You See: World Record Event

## Standards Alignments

Building Towards 5.NBT.B.6

The purpose of this warm-up is to introduce the context of a world record event about the largest Peruvian folk dance, which will be useful when students solve problems about this event in the lesson. While students may count many things in the image, the number of groups of 8 people is the important discussion point.

## Instructional Routines

How Many Do You See?

### Student-facing Task Statement

How many do you see? How do you see them?



### Student Responses

Sample responses:

- 17: 4 groups of 4 light blue dresses and 1 more
- 18: groups of 8 dancers and then lots of parts of other groups of dancers
- 144: dancers in full circles of 8

### Launch

- Groups of 2
- "How many do you see? How do you see them?"
- Display image.
- 1 minute: quiet think time

### Activity

- Display image.
- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Record responses.

### Synthesis

- "How could we figure out how many people are in the picture?" (There are 8 people in each circle so we could count the number of circles we can see and multiply it by 8 and then add in the extra people that are parts of circles on the edges of the image.)
- "This picture shows the largest Peruvian folk dance. Today we are going to solve some problems about this event."

## Activity 1

🕒 20 min

How Many Groups of 8 Dancers?

### Standards Alignments

Building Towards 5.NBT.B.6

The purpose of this activity is for students to use a strategy that makes sense to them to solve a division problem. Students may apply understanding developed in a previous course about the relationship between multiplication and division and place value, including using partial quotients to divide. They may also apply work from the previous section where they multiplied using the standard algorithm.

Students determine the number of groups of 8 people that participated in the record breaking folk dance. The numbers and context were chosen to encourage students to consider what they know about the meaning of division and to use multiplication to solve the problem. Monitor for and select students with the following strategies to share in the synthesis:

- Students do not get the correct solution and can explain the mistake they made.
- Students multiply 8 by 100, or multiples of 100 until they close get 4,704 and then multiply 8 by multiples of tens and single digit numbers to find the solution.
- Students use a partial quotients strategy like the one in the student responses.

When students connect the quantities in the story problem to calculations, including the operations of multiplication and division, they reason abstractly and quantitatively (MP2).

### Instructional Routines

5 Practices

#### Student-facing Task Statement

There were 4,704 people at the record breaking folk dance in Peru. How many groups of 8 dancers were there? Explain or show your thinking.

#### Launch

- Groups of 2
- “What do you know about dancing?” (There are lots of different kinds of dances. Sometimes people dance in pairs.)
- 30 seconds: quiet think time
- 1 minute: partner discussion

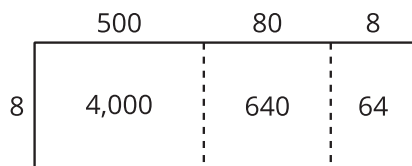
## Student Responses

588.

- Sample response 1:

$$\begin{array}{r}
 \boxed{588} \\
 8 \\
 30 \\
 50 \\
 500 \\
 8 \overline{)4,704} \\
 \underline{- 4,000} \\
 704 \\
 \underline{- 400} \\
 304 \\
 \underline{- 240} \\
 64 \\
 \underline{- 64} \\
 0
 \end{array}$$

- Sample response 2:



- Sample response 3:

$$\begin{aligned}
 500 \times 8 &= 4,000 \\
 50 \times 8 &= 400 \\
 30 \times 8 &= 240 \\
 8 \times 8 &= 64 \\
 500 + 50 + 30 + 8 &= 588
 \end{aligned}$$

## Activity

- 10 minutes: partner work time
- As students work, consider asking:
  - “How do your diagrams or expressions represent the problem?”
  - “Why did you decide to multiply?”
  - “What do the numbers in your calculations mean, in terms of the situation?”

## Synthesis

- Ask selected students to share in the given order.
- “Why did you decide to multiply 8 by a multiple of 100?” (There are 8 people in a group and there 4,704 people so I need to multiply 8 by a large number to get the total number of groups.)
- Display student work or use work from student responses:
 
$$\begin{aligned}
 500 \times 8 &= 4,000 \\
 50 \times 8 &= 400 \\
 30 \times 8 &= 240 \\
 8 \times 8 &= 64 \\
 500 + 50 + 30 + 8 &= 588
 \end{aligned}$$
- “Where are the groups of 8 people represented in this work? Where are the 4,704 people represented in this work?” (The 8 represents each group of 8 and the other factor tells how many groups of 8 dancers there are. If we add up the partial products, it will equal 4,704.)
- “What division expression can represent this problem?” ( $4,704 \div 8$ )
- Ask a student to share who used division or display the following diagram:

$$\begin{array}{r}
 \boxed{588} \\
 8 \\
 30 \\
 50 \\
 500 \\
 8 \overline{)4,704} \\
 - 4,000 \\
 \hline
 704 \\
 - 400 \\
 \hline
 304 \\
 - 240 \\
 \hline
 64 \\
 - 64 \\
 \hline
 0
 \end{array}$$

- “How does this division work relate to the multiplication work?” (They both show the number of groups of 8 people. Both show the partial products but the division shows them being subtracted to see how many people are left after some groups of 8 were made.)

### Advancing Student Thinking

If students do not have an entry point to determine how many groups of 8 people were at the record breaking event, ask, “How could you use multiplication to solve this problem?”

## Activity 2

🕒 15 min

More Groups of Dancers

### Standards Alignments

Building On 4.NBT.B.6

The purpose of this activity is for students to solve division problems, using the context from the first activity, in a way that makes sense to them. The sample student solutions for the problems in

this activity highlight certain numbers to multiply and divide by, but students may use multiplication or division in various ways. During the synthesis, highlight the different ways students used multiplication and division to solve the problems and focus on the relationship between multiplication and division. When students relate the different number of groups of dancers to the number of dancers in each group they observe structure in the relationship of the quotient to the size of the divisor (MP7).

### Access for English Learners

*MLR8 Discussion Supports.* Encourage students to begin partner discussions by reading their written responses aloud. If time allows, invite students to revise or add to their responses based on the conversation that follows.

*Advances: Conversing, Speaking*

### Access for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Check in and provide each group with feedback that encourages collaboration and community. For example, invite students to share the connections between their method of solving and their partner's.

*Supports accessibility for: Social-Emotional Functioning*

## Student-facing Task Statement

1. 4,704 people participate in the Peruvian folk dance. They need to be organized into groups of 4.
  - a. Write a division expression to represent the situation.
  - b. How many groups of 4 will there be? Explain or show your thinking.
  - c. Compare your work with your partner's. What is the same? What is different?
2. 4,704 people participate in the Peruvian folk dance. They need to be organized into groups of 2.
  - a. Write a division expression to represent the situation.
  - b. How many groups of 2 will there be? Explain or show your thinking.

## Launch

- Groups of 2

## Activity

- 3–5 minutes: independent work time
- 3–5 minutes: partner discussion
- Monitor for students who:
  - use multiplication to solve the problems.
  - use division to solve the problems.
  - use the solution from one problem to solve another problem.

## Synthesis

- Display expressions:  $4,704 \div 8$ ,  $4,704 \div 4$  and  $4,704 \div 2$
- “How are the expressions the same? How

- c. Compare your work with your partner's. What is the same? What is different?

### Student Responses

1.
  - a.  $4,704 \div 4$
  - b. 1,176. Sample responses: I doubled my answer from  $4,704 \div 8$  because there will be twice as many groups since there are half as many people in each group.  $1,000 \times 4 = 4,000$ ,  $100 \times 4 = 400$ ,  $70 \times 4 = 280$ ,  $6 \times 4 = 24$ , and  $1,000 + 100 + 70 + 6 = 1,176$
  - c. Sample response: My partner used a partial quotients method and I used multiplication.
2.
  - a.  $4,704 \div 2$
  - b. 2,352. Sample response: I doubled my answer from  $4,704 \div 4$  because there will be twice as many groups since there are half as many people in each group.
  - c. Sample response: My partner multiplied and I used the answer to the previous problem.

are they different?" (They all have 4,704 in them, but the divisor is 8, then 4, then 2.)

- "How does each expression represent the situation?" (There are always 4704 dancers, but the size of the groups changes.)
- "How are the values of the expressions related? Why?" (When there are 4 dancers in each group there are twice as many dancers as when there are 8 in each group because each group of 8 makes two groups of 4. When there are 2 dancers in each group there are twice as many groups as when there are 4 in each group because each group of 4 makes two groups of 2.)

## Lesson Synthesis

 10 min

"Today, we solved problems using division. We used the relationship between multiplication and division."

Display equation from the last activity:

$$4,704 \div 4 = 1,176$$

"How does this equation relate to the Peruvian dancers?" (It shows that there were 4,704 altogether and they made 1,176 groups of 4 dancers.)

Display equation:

$$1,176 \times 4 = 4,704$$

"How does this equation relate to the Peruvian dancers?" (It also shows that there were 1,176 groups of 4 dancers and 4,704 dancers altogether.)

"In the next several lessons we will continue to see the close relationship between multiplication and division."

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## Suggested Centers

- Number Puzzles: Multiplication and Division (4–5), Stage 2: Multi-digit Factors (Addressing)
- Watch Your Remainder (4–5), Stage 1: One-digit Divisors (Supporting)

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## Complete Cool-Down

### Response to Student Thinking

If students do not find the correct amount of the number of groups of 8 dancers.

### Next Day Support

- Before the warm-up, pass back the cool down and work in small groups to make corrections.

# Lesson 11: Different Partial Quotients

## Standards Alignments

Addressing 5.NBT.B.6, 5.OA.A.2

### Teacher-facing Learning Goals

- Divide multi-digit whole numbers using place value understanding and the relationship between multiplication and division.

### Student-facing Learning Goals

- Let's use what we know about multiplication and place value to find quotients.

## Lesson Purpose

The purpose of this lesson is for students to use the relationship between multiplication and division and place value understanding to divide multi-digit numbers.

In the previous lesson, students found quotients in a way that makes sense to them. In this lesson, students consider notation to record a partial quotients strategy, which they have used with one-digit divisors in a prior course. Students use the notation to record how dividends can be decomposed in different ways to make different partial quotients. Students consider more efficient ways to make partial quotients based on place value understanding and calculations they are able to do mentally.

### Access for:

#### Students with Disabilities

- Action and Expression (Activity 1)

## Instructional Routines

MLR2 Collect and Display (Activity 1), Notice and Wonder (Warm-up)

### Materials to Copy

- Partial Quotient Expressions (groups of 2): Activity 1

## Lesson Timeline

Warm-up	10 min
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## Teacher Reflection Question

Reflect on how comfortable your students are asking questions of you and of each other. What

Activity 1	20 min	can you do to encourage students to ask questions?
Activity 2	15 min	
Lesson Synthesis	10 min	
Cool-down	5 min	

## Cool-down (to be completed at the end of the lesson)

 5 min

Find the Value

### Standards Alignments

Addressing 5.NBT.B.6

### Student-facing Task Statement

Find the value of  $465 \div 15$ . Explain or show your reasoning.

### Student Responses

31. Sample responses:

- $30 \times 15 = 450$ ,  $1 \times 15 = 15$ ,  $450 + 15 = 465$
- $450 \div 15 = 30$  and  $15 \div 15 = 1$  and  $30 + 1 = 31$

## ----- Begin Lesson -----

## Warm-up

 10 min

Notice and Wonder: Ways to Record

### Standards Alignments

Addressing 5.OA.A.2

The purpose of this warm-up is to elicit the idea that multiplication and division can both be used to

represent partial quotients. Students compare two representations, both familiar from grade 4, of partial quotients. This will be helpful when students record partial quotients with division expressions later in the lesson.

## Instructional Routines

Notice and Wonder

### Student-facing Task Statement

What do you notice? What do you wonder?

Clare's strategy:

$$\begin{array}{r}
 364 \div 13 \\
 13 \times 10 = 130 \\
 13 \times 20 = 260 \\
 13 \times 5 = 65 \\
 13 \times 3 = 39 \\
 \hline
 \end{array}$$

Jada's strategy:

$$\begin{array}{r}
 130 \div 13 = 10 \\
 130 \div 13 = 10 \\
 65 \div 13 = 5 \\
 39 \div 13 = 3 \\
 \hline
 364 \div 13 = 28
 \end{array}$$

### Student Responses

Students may notice:

- They used some of the same numbers, but Clare multiplied and Jada divided.
- Clare used subtraction to figure out how much she had left after she found part of the quotient.
- They both have numbers whose sum is 28.

Students may wonder:

- What is the answer?
- Did they do the same thing?
- Why did Clare write  $13 \times 10 = 130$  but she did not circle the 10?

### Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

### Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

### Synthesis

- "How are these strategies the same and different as the way you found quotients in the previous lesson?" (They use multiplication and division. They break the problem into smaller parts with numbers that are easier to calculate.)
- "Multiplication can help us think about partial quotients."

## Activity 1

 20 min

Division Expressions

 PLC Activity

### Standards Alignments

Addressing 5.NBT.B.6, 5.OA.A.2

The purpose of this activity is to see that some equivalent ways of rewriting a division expression are more helpful than others for finding its value. In particular expressions whose value can be calculated mentally are most helpful. This supports students when they choose their own ways of breaking up the dividend when they use an algorithm that uses partial products in future lessons.

When they identify expressions to find the value of a quotient, students notice that dividends that are readily identifiable as multiples of 14 are most useful for finding the value of the quotient (MP7).

This activity uses *MLR2 Collect and Display*. Advances: Conversing, Reading, Writing.

### Access for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Invite students to verbalize their strategy for choosing the set of expressions before they begin. Students can speak quietly to themselves, or share with a partner.

*Supports accessibility for: Organization, Conceptual Processing, Language*

### Instructional Routines

MLR2 Collect and Display

### Materials to Copy

Partial Quotient Expressions (groups of 2)

### Required Preparation

- Create a set of cards from the Instructional master for each group of 2.

### Student-facing Task Statement

Take turns:

### Launch

- Groups of 2
- Display cards for the activity.

1. Choose a set of expressions that, when added together, is equal to  $308 \div 14$ . Not all expressions will be used.
2. Explain to your partner how you know that your cards represent a sum that is equal to  $308 \div 14$ .

(Pause for teacher directions.)

3. Choose one of the sets of expressions whose sum is equal to  $308 \div 14$  and use it to find the value of  $308 \div 14$ .

### Student Responses

1. Sample responses:
  - $280 \div 14$  and  $28 \div 14$
  - $288 \div 14$  and  $20 \div 14$
  - $300 \div 14$  and  $8 \div 14$
  - $200 \div 14$  and  $108 \div 14$
  - $140 \div 14$ ,  $140 \div 14$ , and  $28 \div 14$
  - $188 \div 14$  and  $120 \div 14$
  - $70 \div 14$ ,  $70 \div 14$ ,  $70 \div 14$ ,  $70 \div 14$ , and  $28 \div 14$
2. Possible response: I added the dividends and the sum equals 308.
3. Sample response:  $280 \div 14 = 20$  and  $28 \div 14 = 2$  so  $308 \div 14 = 22$

- “What do you notice? What do you wonder?” (They all have a 14 in them. Some of the expressions are repeated. All of them show a number being divided by 14. A lot of the expressions show multiples of 10. A lot of the expressions show multiples of 14. I wonder what we are going to do with these expressions. I wonder why they all divide a number by 14?)
- “You are going to choose expressions that represent a sum that is equal to  $308 \div 14$ .”
- “Keep each set of expressions that you choose.”

### Activity

#### MLR2 Collect and Display

- 3–5 minutes: partner work time
- As students play the game, circulate, listen for, and collect the language students use to explain how they know the cards they chose represent a sum that is equal to  $308 \div 14$ . Listen for: add, plus, divide, dividend, quotient, equals, “the same as”, and groups of.
- Record students’ words and phrases on a visual display and update it throughout the lesson.
- “Choose one set of expressions and use it to find the value of  $308 \div 14$ .”
- 3–5 minutes: independent work time
- Monitor for students who use these different combinations to find the value of  $308 \div 14$ :
  - $280 \div 14$ ,  $28 \div 14$
  - $140 \div 14$ ,  $140 \div 14$ ,  $28 \div 14$

### Synthesis

- Refer to the visual display of the words students used during the activity.
- “These are the words you used to explain

how you knew the cards you chose represent a sum that is equal to  $308 \div 14$ .”

- “Are there any other words or phrases that are important to include on our display?”
- As students share responses, update the display, by adding (or replacing) language, diagrams, or annotations.
- Remind students to borrow language from the display as needed.
- Display:  
 $308 \div 14$
- Ask previously selected students to share.
- “Why did you choose that set of expressions to find the value of  $308 \div 14$ ?” (I could find the values of the expressions mentally.)
- Display:  
 $280 \div 14$
- “Why is  $280 \div 14$  a helpful expression to start with?” ( $280 \div 14$  is a helpful expression to start with because  $20 \times 14 = 280$  and 280 is close to 308.)
- “How do we know that  $28 \div 14$  is the expression that matches  $280 \div 14$ ?” ( $280 \div 28 = 10$ )

### Advancing Student Thinking

If students do not recognize which expressions created by the cards would be helpful to find the value of the quotient  $308 \div 14$ , ask, “Which of these expressions have a value that is a whole number? How could you use the expression to find the value of  $308 \div 14$ ?”

## Activity 2

🕒 15 min

Choose Your Own Partial Quotients

## Standards Alignments

Addressing 5.NBT.B.6

The purpose of this activity is for students to consider efficient ways to use the partial quotients strategy. The strategy that is most efficient for a student will depend on which products or quotients they can find mentally. Encourage students to try to use larger partial quotients as they work, but to try to find partial quotients which they can calculate mentally.

### Student-facing Task Statement

For each expression, choose one of the partial quotients and, beginning with that expression, find the value of the quotient.

1.  $360 \div 15$ 
  - $150 \div 15$
  - $300 \div 15$
  - $60 \div 15$
2.  $945 \div 45$ 
  - $45 \div 45$
  - $450 \div 45$
  - $900 \div 45$
3.  $992 \div 31$ 
  - $62 \div 31$
  - $341 \div 31$
  - $310 \div 31$
4. How did you decide which partial quotient to use to begin finding the quotient? Did you change your mind with any of the problems?

### Student Responses

1. 24. Sample response:
 
$$\begin{array}{r} 60 \div 15 = 4 \\ 300 \div 15 = 20 \\ \hline 360 \div 15 = 24 \end{array}$$

### Launch

- Groups of 2
- "For these problems, choose one of the division expressions to use to begin finding the quotient. Then use the expressions you chose to find the value of each quotient."

### Activity

- 6-8 minutes: independent work time
- 2-3 minutes: partner discussion
- Monitor for students who
  - use different expressions for  $992 \div 31$ .
  - wrote about changing their mind in the last question.

### Synthesis

- Invite selected students to share their reasoning for  $992 \div 31$  and explain why they started with their expression.
- Invite students to share how they changed their mind about which expressions to use during the activity.

2. 21. Sample response
- $$\begin{array}{r} 900 \div 45 = 20 \\ 45 \div 45 = 1 \\ \hline 945 \div 45 = 21 \end{array}$$
3. 32. Sample response:
- $$\begin{array}{r} 310 \div 31 = 10 \\ 62 \div 31 = 2 \\ \hline 992 \div 31 = 32 \end{array}$$
4. Sample response: I tried to use the largest quotient that I could figure out quickly. I started by working only with expressions that have a value of 10, but then I saw I can combine these and use an expression whose value is multiple of 10.

## Lesson Synthesis

🕒 10 min

“Today we thought strategically about which partial quotients are most helpful for finding the value of a division expression.”

Display the last problem from the second activity:  $945 \div 45$ .

“Diego said, ‘In order to solve division problems, you can use all the operations.’ What does Diego mean? When did we use addition, subtraction, and multiplication to divide?” (We can multiply in order to find how many 45s are in 945. After we find those multiples, we can add them up to get 945. We can also use division. I know that  $90 \div 45 = 2$  and  $900 \div 45 = 20$  so I can subtract 900 from 945 and that leaves just 1 more 45.)

### Suggested Centers

- Number Puzzles: Multiplication and Division (4–5), Stage 2: Multi-digit Factors (Addressing)
- Watch Your Remainder (4–5), Stage 1: One-digit Divisors (Supporting)

----- Complete Cool-Down -----

**Response to Student Thinking**

Students find the correct partial quotients or products but do not combine them and record them as the final quotient.

**Next Day Support**

- During the launch of activity 1 in the next lesson, discuss a correct solution to the cool-down from this lesson.

## Lesson 12: An Algorithm Using Partial Quotients

### Standards Alignments

Addressing 5.NBT.B.6

### Teacher-facing Learning Goals

- Make sense of an algorithm using partial quotients.

### Student-facing Learning Goals

- Let's make sense of an algorithm using partial quotients.

### Lesson Purpose

The purpose of this lesson is for students to learn an algorithm using partial quotients to divide multi-digit numbers by two-digit numbers.

In previous lessons, students used estimation strategies, properties of operations, and the relationship between multiplication and division to divide multi-digit numbers by two-digit divisors. They analyzed different strategies for recording partial quotients.

In this lesson, students move from strategies to algorithms in which the connections to place value, properties of operations, and the relationship between multiplication and division are clear. Students interpret an algorithm using partial quotients with a two-digit divisor for the first time. They used this algorithm with one-digit divisors in grade 4. They practice using this algorithm with less and less scaffolding. Throughout the lesson, the emphasis is on making sense of each step and the different operations used in each step. Students also see that there are many different ways to correctly find a quotient with this algorithm and they are encouraged to find partial quotients that make sense to them.

### Access for:

#### Students with Disabilities

- Representation (Activity 1)

### Instructional Routines

MLR1 Stronger and Clearer Each Time (Activity 1), Notice and Wonder (Warm-up)

### Lesson Timeline

Warm-up

10 min

### Teacher Reflection Question

Think about a recent time from class when your students were confused. What did you do to

Activity 1	20 min	support them in reasoning about their confusion together as a community of learners?
Activity 2	15 min	
Lesson Synthesis	10 min	
Cool-down	5 min	

## Cool-down (to be completed at the end of the lesson)

 5 min

What's Next?

### Standards Alignments

Addressing 5.NBT.B.6

### Student-facing Task Statement

Han started finding the value of a quotient.

$$\begin{array}{r}
 300 \\
 15 \overline{)5,400} \\
 \underline{-4,500} \quad (300 \times 15) \\
 900
 \end{array}$$

- Write the division expression that represents the quotient Han is finding.
- Complete the algorithm that Han started.

### Student Responses

- $5,400 \div 15$

$$\begin{array}{r}
 \boxed{360} \\
 60 \\
 300 \\
 15 \overline{)5,400} \\
 \underline{-4,500} \quad (300 \times 15) \\
 900 \\
 \underline{-900} \quad (60 \times 15) \\
 0
 \end{array}$$

-

---

**Begin Lesson**


---

**Warm-up**

🕒 10 min

Notice and Wonder: Incomplete Solution

**Standards Alignments**

Addressing 5.NBT.B.6

The purpose of this warm-up is for students to analyze the structure of an algorithm that uses partial quotients, which will be useful when students use this method to divide multi-digit numbers in a later activity. Students have seen algorithms that use partial quotients in grade 4. The new aspect to these calculations in grade 5 is that the dividend is now a 2-digit number. While students may notice and wonder many things about this image, the relationship between multiplication and division and the purpose of subtraction are the important discussion points.

**Instructional Routines**

Notice and Wonder

**Student-facing Task Statement**

What do you notice? What do you wonder?

$$\begin{array}{r}
 20 \\
 16 \overline{)448} \\
 \underline{-320} \quad (20 \times 16) \\
 128 \\
 \quad \quad (5 \times 16)
 \end{array}$$

**Student Responses**

Students may notice:

- There are multiplication expressions on the side.
- $5 \times 16$  doesn't have anything written next to it.
- They multiplied 16 by 20 and then wrote the 20 above the 448.
- This looks like how we divided in grade 4.

Students may wonder:

**Launch**

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

**Activity**

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

**Synthesis**

- "This is an algorithm used to divide whole numbers. The algorithm is not complete. What might you do next?" (Find  $5 \times 16$  and subtract it from 128.)

- Are they finished solving the problem?
- What will they do next?

## Activity 1

⌚ 20 min

Elena's Work

### Standards Alignments

Addressing 5.NBT.B.6

The purpose of this activity is for students to interpret a partial quotients calculation with a two-digit divisor. Before interpreting the partial products calculation, students find the value of the quotient in a way that makes sense to them. This will help them understand the partial quotients calculation by familiarizing themselves with the numbers and likely some of the steps in the calculation. In explaining both their answers and strategies and Elena's, students need to be precise in their word choice and use of language (MP6) and they also have an opportunity to improve their argument and critique the reasoning of others (MP3).

This activity uses MLR1 *Stronger and Clearer Each Time*. Advances: Writing, Speaking, Listening.

### 🕒 Access for Students with Disabilities

*Representation: Develop Language and Symbols.* Provide students with access to Jada's and Clare's methods from the Notice and Wonder routine in the previous lesson so that students can reference their methods when solving the division problem.

*Supports accessibility for: Memory, Organization*

### Instructional Routines

MLR1 Stronger and Clearer Each Time

### Student-facing Task Statement

1. Find the value of  $448 \div 16$ . Show your thinking. Organize it so it can be followed by others.

(Pause for teacher directions.)

### Launch

- Groups of 2
- "Pause your work after you find the value of  $448 \div 16$ ."

2. This is Elena's work. Describe the steps Elena took to find the value of  $448 \div 16$ .

$$\begin{array}{r}
 \boxed{28} \\
 3 \\
 5 \\
 20 \\
 16 \overline{)448} \\
 \underline{-320} \quad (20 \times 16) \\
 128 \\
 \underline{-80} \quad (5 \times 16) \\
 48 \\
 \underline{-48} \quad (3 \times 16) \\
 0
 \end{array}$$

### Student Responses

1. Sample response:
- $$\begin{array}{r}
 320 \div 16 = 20 \\
 64 \div 16 = 4 \\
 64 \div 16 = 4 \\
 \hline
 448 \div 16 = 28
 \end{array}$$
2. Elena records all of the operations. She shows the multiples of 16 she found and then subtracts them from the total. Then she adds up the multiples.

### Activity

- 3–5 minutes: independent work time
- "Discuss how you found the value of  $448 \div 16$  with your partner."
- 1–2 minutes: partner discussion
- "Describe the steps Elena took to find the value of  $448 \div 16$ ."
- 3–5 minutes: independent work time

### Synthesis

#### MLR1 Stronger and Clearer Each Time

- "Share your description of how Elena found the value of  $448 \div 16$  with your partner. Take turns being the speaker and the listener. If you are the speaker, share your ideas and writing so far. If you are the listener, ask questions and give feedback to help your partner improve their work."
- 3–5 minutes: structured partner discussion
- Repeat with 2–3 different partners.
- (Optional) If needed, display question starters and prompts for feedback.
  - "Can you give an example to help show . . . ?"
  - "Can you use the word \_\_\_\_ in your explanation?"
- "Revise your initial draft based on the feedback you got from your partners."
- 2–3 minutes: independent work time
- "How was Elena's strategy the same as and different from your strategy?" (She used the same calculations, but organized her work differently.)
- "How does the method Elena used help her organize her work?" (She records the multiples of 16 in one place and subtracts them in another.)

## Advancing Student Thinking

If students do not find the correct value of  $448 \div 16$ , ask, “How is your work similar to and different from Elena’s work?”

## Activity 2

🕒 15 min

Complete the Solution

### Standards Alignments

Addressing 5.NBT.B.6

The purpose of this activity is for students to deepen their understanding of an algorithm that uses partial quotients and practice using it. Students use an algorithm that uses partial quotients to find quotients with a three-digit dividend and a two-digit divisor. Different levels of scaffolding are provided as some of the calculations are partly completed. If students struggle to decide what multiple of the divisor to subtract, encourage them to pick a multiple they can calculate easily and that is less than or equal to what remains of the dividend.

### Student-facing Task Statement

Use Elena’s strategy to complete the following problems:

1.

$$\begin{array}{r} 20 \\ 20 \\ 12 \overline{)492} \\ -240 \quad (20 \times 12) \\ \hline 252 \\ -240 \quad (20 \times 12) \\ \hline \end{array}$$

2.

$$\begin{array}{r} 40 \\ 15 \overline{)630} \quad (40 \times 15) \end{array}$$

3.

$$14 \overline{)364}$$

### Launch

- Groups of 2

### Activity

- 8–10 minutes: independent work time
- 1–2 minutes: partner discussion
- Monitor for students who:
  - multiply by 10 to find the value of  $364 \div 14$ .
  - multiply by multiples of 10 to find the value of  $364 \div 14$ .

### Synthesis

- Ask students to share the steps in the

## Student Responses

Sample responses:

1.

$$\begin{array}{r}
 \boxed{41} \\
 1 \\
 20 \\
 20 \\
 12 \overline{)492} \\
 \underline{-240} \quad (20 \times 12) \\
 252 \\
 \underline{-240} \quad (20 \times 12) \\
 12 \\
 \underline{-12} \quad (1 \times 12) \\
 0
 \end{array}$$

2.

$$\begin{array}{r}
 \boxed{42} \\
 2 \\
 40 \\
 15 \overline{)630} \\
 \underline{-600} \quad (40 \times 15) \\
 30 \\
 \underline{-30} \quad (2 \times 15) \\
 0
 \end{array}$$

3.

$$\begin{array}{r}
 \boxed{26} \\
 1 \\
 5 \\
 20 \\
 14 \overline{)364} \\
 \underline{-280} \quad (20 \times 14) \\
 84 \\
 \underline{-70} \quad (5 \times 14) \\
 14 \\
 \underline{-14} \quad (1 \times 14) \\
 0
 \end{array}$$

algorithm for the first problem and complete the problem.

- Ask previously identified students to share their work for  $364 \div 14$ .
- "Do you have any questions about your classmates' work?"
- If needed, clarify any of the steps for using the algorithm.

## Advancing Student Thinking

If students do not complete the steps correctly, refer to the partially completed problems and ask them to describe the steps that are shown.

## Lesson Synthesis

🕒 10 min

Display student work for the quotient  $364 \div 14$  in the last activity or use the work in the student responses.

---

“What was the first multiple of 14 that you subtracted from 364? How did you choose that multiple?” (I started with 10 because I knew that’s 140 and that I could subtract 140 from 364. I started with 20 because I knew that was 280 which was less than 364 but not too far from 364.)

“Why are multiples of 10 good choices for an algorithm using partial quotients?” (I know how to find them in my head.)

---

### Suggested Centers

- Number Puzzles: Multiplication and Division (4–5), Stage 2: Multi-digit Factors (Addressing)
- Watch Your Remainder (4–5), Stage 1: One-digit Divisors (Supporting)

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### Complete Cool-Down

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### Response to Student Thinking

Students do not finish the algorithm correctly.

### Next Day Support

- Before the launch of activity 1 of the next lesson, review the cool-down from this lesson and record the multiplication expressions that were used to find each of the partial quotients in the solution.

# Lesson 13: Divide Using Partial Quotients

## Standards Alignments

Addressing 5.NBT.B.6

### Teacher-facing Learning Goals

- Divide three-digit and four-digit dividends by two-digit divisors using an algorithm using partial quotients.

### Student-facing Learning Goals

- Let's use an algorithm using partial quotients to divide three-digit and four-digit dividends by two-digit divisors.

## Lesson Purpose

The purpose of this lesson is for students to deepen their understanding of an algorithm using partial quotients and use it to divide up to four-digit dividends by two-digit divisors.

In previous lessons, students compared different strategies to divide multi-digit numbers and learned to use an algorithm using partial quotients for division. In this lesson they refine strategies for using an algorithm using partial quotients. In the first activity, students compare their strategies, giving them an opportunity both to explain why they chose their partial quotients and to see that different choices are possible. In the second activity, they begin by estimating the value of the quotient and then use the estimate to help guide the partial quotients choices in the algorithm.

### Access for:

#### Students with Disabilities

- Engagement (Activity 2)

#### English Learners

- MLR7 (Activity 2)

## Instructional Routines

Number Talk (Warm-up)

### Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min

### Teacher Reflection Question

Reflect on who participated in math class today. What assumptions are you making about those who did not participate? How can you leverage each of your students' ideas to support them in being seen and heard in tomorrow's math class?

Cool-down

5 min

**Cool-down** (to be completed at the end of the lesson)

🕒 5 min

Divide Using Partial Quotients

**Standards Alignments**

Addressing 5.NBT.B.6

**Student-facing Task Statement**

Find the value of the quotient.

$$27 \overline{)405}$$

**Student Responses**

15. Sample response:

$$\begin{array}{r} \boxed{15} \\ 5 \\ 10 \\ 27 \overline{)405} \\ -270 \\ \hline 135 \\ -135 \\ \hline 0 \end{array}$$

----- Begin Lesson -----

**Warm-up**

🕒 10 min

Number Talk: Divide

**Standards Alignments**

Addressing 5.NBT.B.6

The purpose of this Number Talk is for students to demonstrate strategies and understandings they have for dividing a three-digit number by a two-digit number. These understandings help students develop fluency and will be helpful later in this lesson when students use an algorithm that uses partial quotients to divide larger three-digit numbers by two-digit numbers.

## Instructional Routines

### Number Talk

#### Student-facing Task Statement

Find the value of each expression mentally.

- $110 \div 10$
- $121 \div 11$
- $132 \div 12$
- $154 \div 14$

#### Student Responses

- 11: I just knew it.
- 11:  $10 \times 11 = 110$  and  $1 \times 11 = 11$ ,  $10 + 1 = 11$
- 11:  $10 \times 12 = 120$  and  $1 \times 12 = 12$ ,  $10 + 1 = 11$
- 11:  $10 \times 14 = 140$  and  $1 \times 14 = 14$ ,  $10 + 1 = 11$

#### Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

#### Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

#### Synthesis

- "What stays the same with each problem? What changes?" (The quotient is always 11. There are always 11 groups of the divisor. The dividend and the divisor change.)

## Activity 1

🕒 20 min

### Compare Solutions

#### Standards Alignments

Addressing 5.NBT.B.6

The purpose of this activity is for students to explain the steps for using an algorithm that uses partial quotients to divide a three-digit number by a two-digit number. Because the size of the dividends is larger and the numbers are less friendly, the problems encourage students to reflect about which partial quotients will be most efficient for making the calculations. Monitor for students who begin with partial quotients that are multiples of 10. This strategy helps make the calculations simpler and students have seen and used multiples of 10 for these calculations throughout the last several lessons.

When students share and compare their methods with their partner and with other groups, they explain and improve their calculations (MP3).

### Student-facing Task Statement

1. Use an algorithm using partial quotients to find the value of one of the quotients. Be prepared to explain how you found the quotient.

Partner 1:

Partner 2:

$$32 \overline{)608} \quad 19 \overline{)589}$$

2. Explain to your partner how you found the value of the quotient.
3. Pair up with another group and compare your work.

### Student Responses

1. Sample response for Partner A:

$$\begin{array}{r} \boxed{19} \\ 2 \\ 2 \\ 5 \\ 10 \\ 32 \overline{)608} \\ \underline{-320} \\ 288 \\ \underline{-160} \\ 128 \\ \underline{-64} \\ 64 \\ \underline{-64} \\ 0 \end{array}$$

### Launch

- Groups of 2, then 4
- “You are going to use the partial quotients algorithm to solve some problems and compare the steps you used with other students.”

### Activity

- “Now, decide who will solve each problem, solve your problem, and explain it to your partner.”
- 3–5 minutes: independent work time
- 1–3 minutes: partner discussion
- “Now, find another group of 2 and compare your solutions. How are they the same? How are they different?”
- 3–5 minutes: small group work time
- Monitor for students who use different multiples for the quotients.

### Synthesis

- Ask selected students who used different strategies to divide.
- “What questions do you have about algorithms that use partial quotients?”
- Display solutions from student solutions

Sample responses for Partner B:

$\boxed{31}$	$\boxed{31}$	$\boxed{31}$
1	1	1
10	30	10
20	$19 \overline{)589}$	10
$19 \overline{)589}$	$\underline{-570}$	10
$\underline{-380}$	$\underline{\quad 19}$	$19 \overline{)589}$
209	$\underline{- 19}$	$\underline{-190}$
$\underline{-190}$	$\underline{\quad 0}$	$\underline{\quad 399}$
19		$\underline{-190}$
$\underline{- 19}$		$\underline{\quad 209}$
$\underline{\quad 0}$		$\underline{-190}$
		$\underline{\quad 19}$
		$\underline{- 19}$
		$\underline{\quad 0}$

2. No response required.

3. No response required.

for the quotient  $589 \div 19$ .

- “How are the strategies the same? How are they different?” (They all subtract 1 group of 19 at the end. Before that, they all subtract multiples of 10.)
- “How is the 30 in the solution C represented in the partial quotients in solutions A and B?” (Solution A uses 20 and 10 and solution B uses 10, 10, and 10 more. These add up to 30.)

## Activity 2

🕒 15 min

Estimate and Solve

### Standards Alignments

Addressing 5.NBT.B.6

The purpose of this activity is for students to practice using an algorithm that uses partial quotients to divide multi-digit numbers by two-digit divisors. Before finding the quotient, students estimate the value of the quotient which both helps students decide which partial quotients to use and helps them evaluate the reasonableness of their solution (MP8). For example, if students estimate that the value of  $529 \div 23$  is a little more than 20, that means that a good choice for first partial quotient is 20. Whenever possible, ask students to explain the steps they are taking.

To add movement to the activity, after students have solved each problem, they can partner with other students who used different partial quotients or got a different solution. This gives students the opportunity to explain their reasoning to one another and make adjustments to their work, as needed.

### Access for English Learners

*MLR7 Compare and Connect.* Synthesis: After all strategies have been presented, lead a discussion comparing, contrasting, and connecting the different approaches. Ask, “What did the approaches have in common?”, “How were they different?” and “Why did the different approaches lead to the same outcome?”.

*Advances: Representing, Conversing*

### Access for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Chunk this task into more manageable parts. Check in with students to provide feedback and encouragement after each chunk. Provide feedback on their use of algorithms using partial quotients.

*Supports accessibility for: Social-Emotional Functioning*

## Student-facing Task Statement

Estimate the value of each quotient. Then, use an algorithm using partial quotients to find the value.

1. A reasonable estimate for  $612 \div 34$  is:  $34 \overline{)612}$

2. A reasonable estimate for  $529 \div 23$  is:  $23 \overline{)529}$

3. A reasonable estimate for  $1,044 \div 29$  is:  $29 \overline{)1,044}$

## Student Responses

- Sample response: It's about 20 since  $600 \div 30 = 20$ .

## Launch

- Groups of 2

## Activity

- 5 minutes: independent work time
- 5 minutes: partner work time
- Monitor for students who use different sets of partial quotients for the same problem.

## Synthesis

- Ask 2–3 students to share their work for the same problem that shows different partial quotients.
- “How can you make sure that the whole number quotient you got at the end is reasonable?” (It should be close to my estimate. I can multiply the quotient and divisor and that should give me the dividend.)
- If students pair and share with other partners, ask, “How did explaining your work to others help you today?” or “What did someone say today that helped you in

$$\begin{array}{r}
 \boxed{18} \\
 3 \\
 5 \\
 10 \\
 34 \overline{)612} \\
 \underline{-340} \\
 272 \\
 \underline{-170} \\
 102 \\
 \underline{-102} \\
 0
 \end{array}$$

2. Sample response: It's close to 25 since  $500 \div 20 = 25$ .

$$\begin{array}{r}
 \boxed{23} \\
 3 \\
 20 \\
 23 \overline{)529} \\
 \underline{-460} \\
 69 \\
 \underline{-69} \\
 0
 \end{array}$$

3. Sample response: It's close to 40 since  $1,000 \div 25 = 40$ .

$$\begin{array}{r}
 \boxed{36} \\
 1 \\
 5 \\
 10 \\
 20 \\
 29 \overline{)1044} \\
 \underline{-580} \\
 464 \\
 \underline{-290} \\
 174 \\
 \underline{-145} \\
 29 \\
 \underline{-29} \\
 0
 \end{array}$$

your understanding of division?" (I learned that it's ok to take more steps because I was comfortable with the multiples I used.)

---

“Today we used partial quotients to divide whole numbers.”

“What makes sense to you about this procedure?” (I get to remove multiples I’m comfortable with. The way the numbers are recorded makes it clear what is happening at each step.)

“What questions do you still have about using this procedure?” (What do I do if the numbers are bigger and more difficult? What if I try to subtract too much?)

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### Suggested Centers

- Number Puzzles: Multiplication and Division (4–5), Stage 2: Multi-digit Factors (Addressing)

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### Complete Cool-Down

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### Response to Student Thinking

Students do not find the correct solution.

### Next Day Support

- During the launch of activity 1 in the next lesson, discuss a correct solution to the cool-down from this lesson.

# Lesson 14: Practice an Algorithm Using Partial Quotients

## Standards Alignments

Addressing 5.NBT.B.6

### Teacher-facing Learning Goals

- Divide four-digit dividends by two-digit divisors using an algorithm using partial quotients.

### Student-facing Learning Goals

- Let's practice using an algorithm using partial quotients.

## Lesson Purpose

The purpose of this lesson is for students to practice using an algorithm using partial quotients.

In previous lessons, students learned to use an algorithm using partial quotients to evaluate division expressions involving two-digit divisors. In this lesson, students extend their work in two ways. First, using partial quotients requires care in finding multiples and subtracting those multiples or partial quotients. Students identify and correct errors in multiplication and subtraction. Then they find quotients of larger numbers. Here the multiplication and subtraction demands increase and students see the first cases where they can subtract 100 or a multiple of 100 of the divisor from the dividend.

### Access for:

#### Students with Disabilities

- Action and Expression (Activity 2)

#### English Learners

- MLR8 (Activity 1)

## Instructional Routines

Which One Doesn't Belong? (Warm-up)

### Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min

### Teacher Reflection Question

It is important for students to build procedural skills from conceptual understanding. How will you connect student thinking to support each student in using increasingly more efficient multiples of the divisor when they use an

Lesson Synthesis

10 min

algorithm using partial quotients?

Cool-down

5 min

**Cool-down** (to be completed at the end of the lesson)

🕒 5 min

Divide Four-digit Numbers

**Standards Alignments**

Addressing 5.NBT.B.6

**Student-facing Task Statement**Find the value of  $1,736 \div 28$ .

$$28 \overline{)1,736}$$

**Student Responses**

62. Sample response:

$$\begin{array}{r} \boxed{62} \\ 2 \\ 10 \\ 50 \\ 28 \overline{)1,736} \\ \underline{-1,400} \\ 336 \\ \underline{-280} \\ 56 \\ \underline{-56} \\ 0 \end{array}$$

----- **Begin Lesson** -----

## Warm-up

🕒 10 min

### Which One Doesn't Belong: Different Ways

#### Standards Alignments

Addressing 5.NBT.B.6

The purpose of this warm-up is to examine a variety of ways to use partial products to find a single quotient. As students use partial products to find more complex quotients, they need to be strategic about which multiples of the dividend to subtract. Small multiples may be easier for finding the partial products but it takes more of them to give a sum equal to the dividend.

#### Instructional Routines

Which One Doesn't Belong?

#### Student-facing Task Statement

Which one doesn't belong?

A

$$\begin{array}{r} \boxed{16} \\ 82 \overline{)1,312} \end{array}$$

B

$$\begin{array}{r} \boxed{16} \\ 1 \\ 5 \\ 5 \\ 5 \\ 82 \overline{)1,312} \end{array}$$

C

$$\begin{array}{r} 1 \\ 5 \\ 10 \\ 82 \overline{)1,312} \end{array}$$

D

$$\begin{array}{r} \boxed{16} \\ 10 \\ 5 \\ 1 \\ 82 \overline{)1,312} \end{array}$$

#### Student Responses

- A is the only one that doesn't show any work, it just has the answer.
- B is the only one that doesn't ever subtract 10 or more 82's.
- C is the only one that doesn't add up the partial quotients. It doesn't show the value of the quotient.

#### Launch

- Groups of 2
- Display the image.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time

#### Activity

- 2–3 minutes: partner discussion
- Record responses.

#### Synthesis

- "Which strategy do you prefer for finding the value of  $1,312 \div 82$ ?" (I like method C because it begins by taking out a pretty big multiple of 82.)

- D is the only one that does not start by subtracting lots of 82's.

## Activity 1

🕒 20 min

Find the Mistake

### Standards Alignments

Addressing 5.NBT.B.6

The purpose of this activity is for students to identify and correct common errors in using an algorithm that uses partial quotients. One of the errors involves subtraction and two of them involve multiplication. Students may choose to correct the errors and continue the work that is there or they may choose to find the quotient in different way that makes sense to them. When students determine where the errors are and explain their reasoning, they critique and construct viable arguments(MP3).

### 🌐 Access for English Learners

*MLR8 Discussion Supports.* Display sentence frames to support partner discussion: "I noticed \_\_\_\_ so I . . . ." and "I agree/disagree because . . . ."

*Advances: Conversing, Representing*

### Student-facing Task Statement

For each problem, describe where you see an error in the calculation. Then find the correct whole number quotient.

- 1.
- 2.

### Launch

- Groups of 2

### Activity

- 1-2 minutes: quiet think time
- 3-5 minutes: partner work time
- Monitor for students who:
  - are able to clearly identify and describe the errors.
  - are able to use the algorithm to calculate the quotients correctly.

$$\begin{array}{r}
 \boxed{29} \\
 4 \\
 5 \\
 20 \\
 46 \overline{)1,656} \\
 \underline{-920} \\
 436 \\
 \underline{-230} \\
 206 \\
 \underline{-184} \\
 22
 \end{array}$$

3.

$$\begin{array}{r}
 \boxed{211} \\
 1 \\
 10 \\
 200 \\
 24 \overline{)744} \\
 \underline{-480} \\
 264 \\
 \underline{-240} \\
 24
 \end{array}$$

### Student Responses

Sample responses:

1. Error: Subtraction of 1,656 and 920 is wrong.

$$\begin{array}{r}
 \boxed{36} \\
 1 \\
 5 \\
 10 \\
 20 \\
 46 \overline{)1,656} \\
 \underline{-920} \\
 736 \\
 \underline{-460} \\
 276 \\
 \underline{-230} \\
 46 \\
 \underline{-46} \\
 0
 \end{array}$$

2. Error: The multiplication is wrong.

### Synthesis

- Invite students to share where they notice errors and describe the errors.
- "What can you do to make sure you are not making the same errors in your calculation?" (Before I calculate, I can estimate. While I work on the problem, I can double check when I subtract or multiply. After I calculate, I can multiply the quotient by the divisor and make sure I get the dividend.)
- Encourage students to consider what they can do before, during, and after they calculate.

$$\begin{array}{r}
 \boxed{54} \\
 4 \\
 10 \\
 40 \\
 18 \overline{)972} \\
 \underline{-720} \\
 252 \\
 \underline{-180} \\
 72 \\
 \underline{-72} \\
 0
 \end{array}$$

3. Error: The first group of 24's that is taken away is 20, not 200. It also does not show subtracting the final 24.

$$\begin{array}{r}
 \boxed{31} \\
 1 \\
 10 \\
 20 \\
 24 \overline{)744} \\
 \underline{-480} \\
 264 \\
 \underline{-240} \\
 24 \\
 \underline{-24} \\
 0
 \end{array}$$

### Advancing Student Thinking

If students do not identify the mistakes, prompt them to evaluate the division expressions and ask, "How does your solution compare with the one in the activity?"

## Activity 2

🕒 15 min

Practice Problems

### Standards Alignments

Addressing 5.NBT.B.6

The purpose of this activity is for students to divide three- and four-digit dividends by two-digit divisors. As the size of the dividend increases, students have an option to subtract multiples of 100 of the divisor. In order to calculate efficiently, this becomes essential for a quotient like  $8,715 \div 21$  as it will take a lot of partial quotients that are multiples of 10 to reach the full quotient. Using an algorithm that uses partial quotients for larger numbers also requires fluency with subtraction.

### Access for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* To support working memory, provide students with access to sticky notes or mini whiteboards.

*Supports accessibility for: Memory, Organization*

### Student-facing Task Statement

Find the value of each expression. Then check in with a partner to review your work.

- |                        |                        |
|------------------------|------------------------|
| 1.                     | 2.                     |
| $16 \overline{)768}$   | $29 \overline{)1,305}$ |
| 3.                     | 4.                     |
| $21 \overline{)8,715}$ | $53 \overline{)6,572}$ |

### Student Responses

- |   |  |
|---|--|
| 1.  | 2.   |
| $\begin{array}{r} \boxed{48} \\ 3 \\ 5 \\ 40 \\ 16 \overline{)768} \\ \underline{-640} \\ 128 \\ \underline{-80} \\ 48 \\ \underline{-48} \\ 0 \end{array}$ | $\begin{array}{r} \boxed{45} \\ 3 \\ 2 \\ 20 \\ 20 \\ 29 \overline{)1,305} \\ \underline{-580} \\ 725 \\ \underline{-580} \\ 145 \\ \underline{-58} \\ 87 \\ \underline{-87} \\ 0 \end{array}$ |
| 3.  | 4.   |

### Launch

- Groups of 2

### Activity

- 8-10 minutes: independent work time
- "When you check in with your partner, consider the following questions."
- Display:
  - "How do the calculations represent each partial quotient?"
  - "What suggestions can you give your partner to improve their work for next time?"
- 2-3 minutes: partner discussion

### Synthesis

- Ask a few students to share their work.
- "How was your work similar to or different from your partner's?" (We both tried to subtract multiples of 10 or 100 and then when the remaining amount was small we subtracted smaller multiples. My partner used fewer steps.)
- "What did you notice in your partner's work that shows they understand how to use an algorithm that uses partial quotients?" (My partner picked multiples that could mostly be calculated in their head and worked on subtracting the partial quotients.)

$  \begin{array}{r}  \boxed{415} \\  5 \\  10 \\  400 \\  21 \overline{)8,715} \\  \underline{- 8,400} \\  315 \\  \underline{- 210} \\  105 \\  \underline{- 105} \\  0  \end{array}  $	$  \begin{array}{r}  \boxed{124} \\  4 \\  20 \\  100 \\  53 \overline{)6,572} \\  \underline{- 5,300} \\  1,272 \\  \underline{- 1,060} \\  212 \\  \underline{- 212} \\  0  \end{array}  $
--	--

## Lesson Synthesis

🕒 10 min

“Today, we practiced using an algorithm using partial quotients to divide multi-digit numbers.”

“Tell your partner how to use an algorithm using partial quotients to find the value of  $935 \div 85$ .” (Look for the biggest multiple of 10 and 85 that I can subtract from 935. Find  $10 \times 85$  and subtract the product from 935 to see how much more I have left to divide. Keep doing this until I get zero as a difference. To check my answer, multiply the quotient by the divisor to make sure I get back to the original dividend.)

### Suggested Centers

- Number Puzzles: Multiplication and Division (4–5), Stage 2: Multi-digit Factors (Addressing)

## ----- Complete Cool-Down -----

### Response to Student Thinking

Students need support using an algorithm that uses partial quotients.

### Next Day Support

- During the activity 1 synthesis, invite students to describe which multiplication expressions they used to find each of the

partial quotients in division problems.

## Lesson 15: Find Missing Side Lengths

### Standards Alignments

Addressing 5.NBT.B.5, 5.NBT.B.6

### Teacher-facing Learning Goals

- Solve problems involving area and volume using the relationship between multiplication and division.

### Student-facing Learning Goals

- Let's use the relationship between multiplication and division to solve problems.

### Lesson Purpose

The purpose of this lesson is for students to find missing side lengths of rectangles and rectangular prisms using their understanding of area, volume, and multi-digit division.

In an earlier unit, students learned to find the volume of rectangular prisms either by multiplying the length, width, and height or choosing a base and multiplying the area of that base by the corresponding height. In this unit, students learned to use division strategies to divide multi-digit numbers. The purpose of this lesson is to use multiplication and division to solve problems about area and volume. Throughout, students apply multiplication and division and what they know about the relationship between the length, width, and height of a rectangular prism and its volume to solve problems (MP2).

This lesson has a Student Section Summary.

### Access for:

#### Students with Disabilities

- Engagement (Activity 2)

#### English Learners

- MLR8 (Activity 2)

### Instructional Routines

Estimation Exploration (Warm-up)

### Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min

### Teacher Reflection Question

As you finish up this unit, reflect on the norms and activities that have supported each student in learning math. List ways you have seen each student grow as a young mathematician

Lesson Synthesis 10 min

Cool-down 5 min

throughout this work. List ways you have seen yourself grow as a teacher. What will you continue to do and what will you improve on in the next unit?

## Cool-down (to be completed at the end of the lesson)

⌚ 5 min

### The Area of the Garden

#### Standards Alignments

Addressing 5.NBT.B.6

#### Student-facing Task Statement

The area of a rectangular garden is 832 square feet and its length is 16 feet. What is its width?

#### Student Responses

52 feet. Sample response:

$$\begin{array}{r}
 \boxed{52} \\
 2 \\
 10 \\
 40 \\
 16 \overline{)832} \\
 \underline{-640} \\
 192 \\
 \underline{-160} \\
 32 \\
 \underline{-32} \\
 0
 \end{array}$$

----- Begin Lesson -----

## Warm-up

⌚ 10 min

Estimation Exploration: The Garden

## Standards Alignments

Addressing 5.NBT.B.6

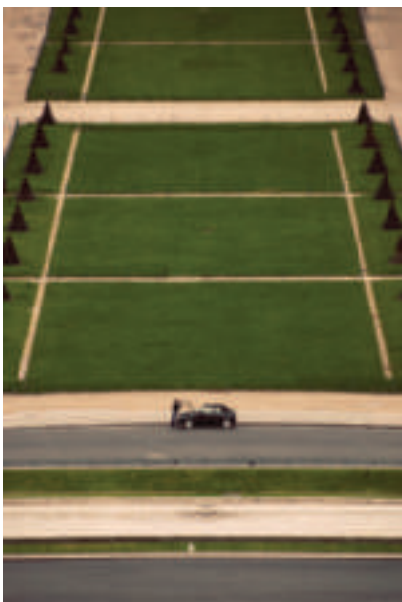
The purpose of this Estimation Exploration is to recall the concept of area. Students need to think strategically because the one point of reference for the size of the grassy area in the image is the car and the road. In order to facilitate mental calculation, expect students to choose multiples of ten for the length and width of the rectangle.

## Instructional Routines

Estimation Exploration

### Student-facing Task Statement

What is the area of one of the large rectangles in the garden?



Record an estimate that is:

too low	about right	too high

### Student Responses

- Too low: range: 1,000 to 3,000 sq ft
- About right: 5,000 to 10,000 sq ft

### Launch

- Groups of 2
- Display the image.
- “What is an estimate that’s too high?” “Too low?” “About right?”
- 1 minute: quiet think time

### Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Record responses.

### Synthesis

- “What did you use in the image to make an estimate for the area of the garden?” (The car and the person gave me an idea of how big it is.)

- Too high: 20,000 sq ft or more

## Activity 1

🕒 15 min

Find the Missing Side Length, Part 1

### Standards Alignments

Addressing 5.NBT.B.5, 5.NBT.B.6

The purpose of this activity is to use multiplication and division to solve area problems. In most cases the area and one side length are given and students can use division to find the missing side length. In one case the two side lengths are given and students find their product which is the area.

### Student-facing Task Statement

Complete the table.

area (square feet)	length (feet)	width (feet)
816	24	
1,248		48
	23	253
5,796		36

### Student Responses

area (square feet)	length (feet)	width (feet)
816	24	34
1,248	26	48
5,819	23	253
5,796	161	36

### Launch

- Display garden from warm-up.
- “The area of one of the large rectangular pieces is 9,175 square feet and the length is 75 feet.”
- Display:  
Area: 9,175 square feet  
One side length: 75 feet
- “What is a reasonable estimate for the width?” (100 because  $100 \times 75 = 7,500$  and  $7,500 \div 75 = 100$ .)
- 2 minutes: partner discussion
- “How can we find the exact width of the garden?” (Divide the area by the length.)

### Activity

- 1–2 minutes: quiet think time
- 5–8 minutes: partner work time
- Monitor for students who use a partial quotients algorithm to divide to share

during the synthesis.

- If students try to divide to find the missing area, consider asking, “How do we find the area of a rectangle given the length and width?”

### Synthesis

- Ask selected students to share their work and explain their steps.
- “How can we make sure that we found the correct missing side lengths?” (Multiply the length and width together to make sure it gives the area or divide the area by one of the side lengths and make sure it gives the other side length.)
- “How was finding the area of the rectangle using the length and width different than finding one of the side lengths using the area and the other side length?” (I had to multiply to find the area from the length and width. I used division to get one side length from the area and the other side length.)

---

## Activity 2

🕒 20 min

Find the Missing Side Length, Part 2

### Standards Alignments

Addressing 5.NBT.B.5, 5.NBT.B.6

The purpose of this activity is for students to apply what they learned about dividing multi-digit numbers to find the missing side length(s) of rectangular prisms given the volume and at least one other side length. As in the previous activity, both multiplication and division are important to solve the problems. This is true both because finding partial quotients uses multiplication and because they have different choices how to find a missing side length when two side lengths and the volume are given. Monitor for students who

- find the product of the two given side lengths and then use division to find the third side length.
- divide successively by the two given side lengths.

### Access for English Learners

*MLR8 Discussion Supports.* Prior to solving the problems, invite students to make sense of the situations and take turns sharing their understanding with their partner. Listen for and clarify any questions about the context.

*Advances: Reading, Representing*

### Access for Students with Disabilities

*Engagement: Provide Access by Recruiting Interest.* Provide choice. Invite students to decide in which order to complete the task and choose what strategy they want to use.

*Supports accessibility for: Organization, Social-Emotional Functioning*

## Student-facing Task Statement

1. Complete the table.

volume (cubic feet)	base (square feet)	height (feet)
375	15	
1,176		28

2. Clare wants to find the height of a rectangular prism with the following measurements:

volume (cubic feet)	length (feet)	width (feet)	height (feet)
882	6	7	

- a. First, Clare finds the quotient  $882 \div 6$ . What could she do next to find the height?
  - b. Find the missing height to finish the problem for Clare.
3. Complete the table.

## Launch

- Groups of 2

## Activity

- 5-8 minutes: independent work time
- 5 minutes: partner work time
- Monitor for students who:
  - use a partial quotients algorithm to divide.
  - find the missing side length in the third table by dividing twice, similar to Clare's steps in the next problem.
  - find the missing length in the third table by first multiplying the given side lengths and then dividing the volume by the product.
  - notice there are multiple possible lengths and widths for the last rectangular prism in the third table

volume (cubic feet)	length (feet)	width (feet)	height (feet)
936	8		9
1,536		48	2
1,008	36		

### Student Responses

1.

volume (cubic feet)	base (square feet)	height (feet)
375	15	25
1,176	42	28

2. a. Clare could divide the quotient she got by 7 to get the missing height.  
b. 21. Sample response:

$$\begin{array}{r}
 \boxed{147} \\
 7 \\
 40 \\
 100 \\
 6 \overline{)882} \\
 \underline{-600} \\
 282 \\
 \underline{-240} \\
 42 \\
 \underline{-42} \\
 0
 \end{array}
 \qquad
 \begin{array}{r}
 \boxed{21} \\
 1 \\
 20 \\
 7 \overline{)147} \\
 \underline{-140} \\
 7 \\
 \underline{-7} \\
 0
 \end{array}$$

c.

volume (cubic feet)	length (feet)	width (feet)	height (feet)
936	8	13	9
1,536	16	48	2

The width and height of the last row should multiply to 28.

### Synthesis

- Ask students to share their work and reasoning.
- Highlight multiple approaches to find the missing side length in the second and third tables.
- Display second row of third table.
- “What are some different ways you can find the length of this rectangular prism?” (I can first divide the volume by 2 and then divide by 48 or I can multiply 2 and 48 and then divide the volume by that product.)
- “Which method did you prefer? Why?” (I divided the volume by 2 and then by 48 as that was quick and kept the numbers smaller.)

## Lesson Synthesis

🕒 10 min

“Today we found missing side lengths of rectangles and rectangular prisms using division.”

Display last row of the table from the last problem of the last activity.

Invite students to share different responses for the width and height.

"What is the value of  $1,008 \div 36$ ?" (28)

"Why is there more than one solution for the width and height of this rectangular prism?" (I only know that the product of the width and the height is 28. But there are different factors whose product is 28.)

## Suggested Centers

- Number Puzzles: Multiplication and Division (4–5), Stage 2: Multi-digit Factors (Addressing)

## Student Section Summary

In this section, we learned how to divide multi-digit whole numbers. To find a quotient like  $448 \div 16$  we broke 448 down into multiples of 16 and then added these partial quotients.

$$320 \div 16 = 20$$

$$80 \div 16 = 5$$

$$48 \div 16 = 3$$

---


$$448 \div 16 = 28$$

Then, we worked with a way to record these calculations that we first saw in an earlier course.

$$\begin{array}{r}
 \boxed{28} \\
 3 \\
 5 \\
 20 \\
 16 \overline{)448} \\
 \underline{-320} \quad (20 \times 16) \\
 128 \\
 \underline{-80} \quad (5 \times 16) \\
 48 \\
 \underline{-48} \quad (3 \times 16) \\
 0
 \end{array}$$

## Complete Cool-Down

### Response to Student Thinking

Students do not correctly use a strategy that uses partial quotients when calculating.

### Next Day Support

- Create a poster with the steps to solving the cool-down problem from the previous lesson.

## Lesson 16: World's Record Noodle Soup

### Standards Alignments

Addressing 5.NBT.B.6, 5.NF.B.3

### Teacher-facing Learning Goals

- Solve problems that involve the division of multi-digit numbers.

### Student-facing Learning Goals

- Let's solve a problem using division with large numbers.

### Lesson Purpose

The purpose of this lesson is for students to estimate and solve multi-digit division with mixed number quotients.

In previous lessons, students use strategies based on place value, the properties of operations, and/or the relationship between multiplication and division to find whole-number quotients with up to four-digit dividends and two-digit divisors. In previous units, students interpreted fractions as division of the numerator by the denominator.

In this lesson, students explore a real world context and apply what they have learned about division to solve problems related to the context. The context is a world record event for making the longest noodle. The mathematically important part of the context is that the noodle cannot be pieced together from separate smaller noodles. Instead, it must be rolled out as one continuous noodle. It took the employees 17 hours to roll out the approximately 10,119 ft noodle and it fed 400 people. For more information about this event, follow the link: <https://www.guinnessworldrecords.com/news/commercial/2017/12/video-worlds-longest-noodle-is-more-than-3-km-long-502606>.

When students recognize mathematical features in the real world, they model with mathematics (MP4).

### Access for:

#### Students with Disabilities

- Engagement (Activity 2)

#### English Learners

- MLR8 (Activity 2)

### Instructional Routines

Notice and Wonder (Warm-up)

**Lesson Timeline**

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

**Teacher Reflection Question**

How did the student work that you selected impact the direction of the discussion? What student work might you pick next time if you taught the lesson again?

**Cool-down** (to be completed at the end of the lesson)

🕒 5 min

## Division Reflection

**Standards Alignments**

Addressing 5.NBT.B.6

**Student-facing Task Statement**

Describe something you really understand well from this section on dividing multi-digit numbers, or describe something that was confusing or challenging.

**Student Responses**

Sample response: I understand how to check my subtraction and multiplication while I am solving the problem. I like using a partial quotients algorithm because I can double check my work. I have to remember to estimate to check to make sure my answer makes sense. I get confused with all of the steps.

----- **Begin Lesson** -----**Warm-up**

🕒 10 min

## Notice and Wonder: World Record Event

## Standards Alignments

Addressing 5.NBT.B.6

The purpose of this warm-up is to introduce the context of a world record event about the longest continuous noodle, which will be useful when students solve problems about this event in a later activity. While students may notice and wonder many things about this text and image, the noodle's length and the number of people sharing the noodle are the important discussion points.

## Instructional Routines

Notice and Wonder

### Student-facing Task Statement

What do you notice? What do you wonder?

A Chinese food company holds the Guinness World Record for making the longest noodle. The noodle measured about 10,119 ft.



### Student Responses

Sample responses:

Students may notice:

- That is a really long noodle.
- The noodle is in a big bowl.
- There are several people making the noodle.

Students may wonder:

- How long did it take them to make the noodle?
- Why are they making the noodle?
- How did they cook it?

### Launch

- Groups of 2
- Consider showing students a Guinness Book world record image or video of the world's longest noodle.
- Display the image.

### Activity

- "What do you notice? What do you wonder?"
- 1 minute: quiet think time
- 1 minute: partner discussion
- Share and record responses.

### Synthesis

- "What is something else that is about 10,000 feet long?" (That is about how high people are when they skydive. It is about 2 miles.)
- "These pictures show the world's longest noodle being made. We are going to solve some problems about this event."

## Activity 1

 20 min

### How Many Feet in One Serving?

#### Standards Alignments

Addressing 5.NBT.B.6, 5.NF.B.3

The purpose of this activity is for students to use a method of their choice, likely multiplication or division, to solve a contextual problem about equal sharing of the longest noodle ever made. The numbers in this activity are larger than the numbers students have worked with in previous lessons on division. Students estimate the number of feet of noodle each person ate at the record breaking event. The numbers and context were chosen to encourage students to consider what they know about the meaning of division, to make a reasonable estimate, and to reason about the meaning of the quotient in the context of the situation presented (MP2).

Monitor and select students with the following strategies to share in the synthesis:

- Students use multiplication or division to estimate that each person will get about 25 feet of noodle.
- Students can explain why 25 feet of noodle for each person is a low estimate.

#### Student-facing Task Statement

A Chinese food company cooked a single noodle measuring about 10,119 ft. It served 400 people.

1. If the noodle was shared equally, estimate how many feet of noodle each person was served.
2. Is your estimate lower or higher than the actual length of noodle each person ate? Explain your reasoning without calculating the actual length.

#### Student Responses

1. About 25 feet.

#### Launch

- “What kind of noodles do you like to eat?” (ramen, spaghetti, fettucini, chicken noodle soup)
- 30 seconds: partner discussion
- “About how long is one of the noodles you like to eat?” (about 1 foot long)
- Groups of 2

#### Activity

- 5 minutes: independent work time
- 5 minutes: partner discussion
- As students work, consider asking “What do the numbers in your calculations mean, in

Sample response 1:      Sample response 2:

$$\begin{array}{r}
 10 \times 400 = 4,000 \\
 20 \times 400 = 8,000 \\
 5 \times 400 = 2,000 \\
 25 \times 400 = 10,000.
 \end{array}$$

$$\begin{array}{r}
 \boxed{25} \\
 5 \\
 10 \\
 10 \\
 400 \overline{)10,119} \\
 \underline{- 4,000} \\
 6,119 \\
 \underline{- 4,000} \\
 2,119 \\
 \underline{- 2,000} \\
 119
 \end{array}$$

2. Sample response: Lower, because there are 119 feet of noodle left to divide.

terms of the situation?"

### Synthesis

- Ask selected students to share in the given order (or use the provided student solutions if needed).
- "How are the methods for estimating the amount of noodle each person gets the same?" (They both start by giving each person 10 feet of noodle. Then they give more until they have both given 25 feet to each person. They both find multiples of 400.)
- "How are they different?" (One thinks of the process as division and one uses just multiplication.)
- "How do you know the estimate of 25 feet is too low?" (Because there was still some of the noodle left. There are 119 feet left over.)

## Activity 2

🕒 15 min

Han's Estimate

### Standards Alignments

Addressing 5.NBT.B.6, 5.NF.B.3

The purpose of this activity is to consider a more precise estimate for the length of noodle each person would get if 400 people equally shared a 10,119 foot noodle. This estimate includes a fractional part and encourages students to connect division to what they know about fractions. In the next lesson students will continue to examine fractions and how they relate to partial quotients.

Making an estimate or a range of reasonable answers with incomplete information is a part of modeling with mathematics (MP4).

### Access for English Learners

*MLR8 Discussion Supports.* Activity: During group work, invite students to take turns sharing their responses. Ask students to restate what they heard using precise mathematical language and their own words. Display the sentence frame: "I heard you say . . ." Original speakers can agree or clarify for their partner.

*Advances: Listening, Speaking*

### Access for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Check in and provide each group with feedback that encourages collaboration and community. For example, encourage students to use sentence frames to agree or disagree with each other and take turns sharing their ideas.

*Supports accessibility for: Social-Emotional Functioning.*

## Student-facing Task Statement

Han said that each person will get about  $25\frac{1}{4}$  feet of noodle. Do you agree with Han? Explain or show your reasoning.

## Student Responses

Yes. Sample response: Each person gets 10, 119  $\div$  400 feet of the noodle. I know that this is 25 feet for each person and then there are 119 feet of noodle left. If I divide that evenly between 400 people that would be  $\frac{119}{400}$  feet for each person and that is about  $\frac{100}{400}$  or  $\frac{1}{4}$ .

## Launch

- Groups of 2

## Activity

- 3–5 minutes: independent work time
- 3–5 minutes: partner discussion

## Synthesis

- Display:  $25\frac{119}{400}$
- "What does  $25\frac{119}{400}$  mean in this situation?" (Each person gets 25 feet of the noodle and then the 119 feet leftover would be divided into 400 equal pieces.)
- Display:  $25\frac{1}{4}$
- "Why is Han's estimate reasonable?" (Because is  $\frac{119}{400}$  really close to  $\frac{100}{400}$  and  $\frac{100}{400} = \frac{1}{4}$ )
- "Do you think they actually measured and cut the noodle into equal pieces when they served it?" (No, because it would take too long and be too difficult. Yes, because if

long noodles represent long life they probably want to serve the noodle soup with sections that are one piece of the original noodle.)

## Lesson Synthesis

🕒 10 min

"Today, we solved problems about a real life context. We also discussed solutions that were mixed numbers. In what ways did we use division today?" (We estimated and divided the number of feet of noodle by the number of servings. We thought about fractions as division to help us make more precise estimates.)

"In what ways did we use fractions?" (We used what we know about fractions to make our estimates more precise.)

---

### Suggested Centers

- Mystery Number (1–4), Stage 6: Decimals (Supporting)

---

### Complete Cool-Down

#### Response to Student Thinking

Students have ideas they could share with a partner.

#### Next Day Support

- After the warm-up in the next lesson, pair students up to discuss their responses.

## Lesson 17: Fractions as Partial Quotients (Optional)

### Standards Alignments

Building On 5.NF.B.3  
Addressing 5.OA.A.2

### Teacher-facing Learning Goals

- Make sense of partial quotients using fractions.

### Student-facing Learning Goals

- Let's use fractions to help us divide whole numbers.

### Lesson Purpose

The purpose of this lesson is for students to interpret sums of fractions as partial quotients.

Most of the quotients students have worked with in this unit have had whole number values. As seen in the previous lesson, sometimes the quotient has a remainder and fractions can be used to express the meaning of that remainder. The purpose of this optional lesson is to evaluate division expressions using fractions. Although the quotients students work with here are whole numbers, the fractions still highlight an important part of the partial quotients algorithm, namely that working with nice multiples of the divisor facilitates the calculation process. The new part of this lesson is that some of the multiples are fractions.

### Access for:

#### Students with Disabilities

- Representation (Activity 2)

### Instructional Routines

MLR1 Stronger and Clearer Each Time (Activity 2), What Do You Know About \_\_\_\_? (Warm-up)

### Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min

### Teacher Reflection Question

What opportunities are you giving students to reflect on their understanding of the mathematical content?

Cool-down

5 min

**Cool-down** (to be completed at the end of the lesson)

⌚ 5 min

Choose One Expression

**Standards Alignments**

Building On 5.NF.B.3

**Student-facing Task Statement**Choose one expression and use it to find the value of  $154 \div 14$ . Explain or show your thinking.

$154 \div 14 = \underline{\hspace{2cm}}$

- $\frac{140}{14} + \frac{14}{14}$
- $\frac{150}{14} + \frac{4}{14}$
- $\frac{70}{14} + \frac{70}{14} + \frac{14}{14}$

**Student Responses**11. Sample response:  $\frac{140}{14} + \frac{14}{14} = 10 + 1$ ----- **Begin Lesson** -----**Warm-up**

⌚ 10 min

What Do You Know About  $\frac{60}{6} + \frac{6}{6}$ ?**Standards Alignments**

Building On 5.NF.B.3

The purpose of this What Do You Know About \_\_\_\_? is for students to share what they know about a sum of fractions. The fractions are selected because they represent whole numbers and the whole number values are visible. Students will work with expressions like these throughout this lesson.

## Instructional Routines

What Do You Know About \_\_\_\_?

### Student-facing Task Statement

What do you know about  $\frac{60}{6} + \frac{6}{6}$ ?

### Student Responses

- There are two fractions being added.
- One fraction is greater than 1 and one fraction is equal to 1.
- Both fractions are sixths.
- It equals  $\frac{66}{6}$ .
- $\frac{60}{6} = 10$  and  $\frac{6}{6} = 1$ , so it equals 11.

### Launch

- Display the number.
- “What do you know about  $\frac{60}{6} + \frac{6}{6}$ ?”
- 1 minute: quiet think time


### Activity

- Record responses.

### Synthesis

- “What are some expressions that are equal to  $\frac{60}{6} + \frac{6}{6}$ ?” (10 + 1, 11,  $(60 \div 6) + (6 \div 6)$ )

## Activity 1

 15 min

Select Expressions

### Standards Alignments

Building On 5.NF.B.3

Addressing 5.OA.A.2

The purpose of this activity is for students to relate their understanding of fractions as representing division to think about decomposing a quotient into partial quotients in a way that simplifies the calculation. To find the value of  $78 \div 6$ , students may

- use their understanding of division.
- use multiplication and find how many groups of 6 there are in 78.
- use the fraction expressions from the first part of the problem.

### Student-facing Task Statement

- Select **all** the expressions that are equivalent to  $\frac{78}{6}$ . Explain or show your reasoning.
  - $78 \div 6$
  - $\frac{66}{6} + \frac{12}{6}$
  - $\frac{60}{6} + \frac{18}{6}$
  - $(60 \div 6) + (18 \div 6)$
  - $\frac{77}{6} + \frac{8}{6}$
  - $(60 \div 6) + 18$
- What is the value of  $78 \div 6$ ? Explain or show your thinking.

### Student Responses

- $78 \div 6 = \frac{78}{6}$  because when I divide each whole into 6 equal pieces those pieces are sixths. I have 78 of those pieces or  $\frac{78}{6}$ .
  - $\frac{66}{6} + \frac{12}{6} = \frac{78}{6}$  because  $66 + 12 = 78$  and that is how many sixths there are.
  - $\frac{60}{6} + \frac{18}{6} = \frac{78}{6}$  because I can add the numerators.
  - $(60 \div 6) + (18 \div 6) = \frac{78}{6}$  because it is  $\frac{60}{6} + \frac{18}{6}$  written in terms of division expressions.
  - $\frac{77}{6} + \frac{8}{6}$  does not equal  $\frac{78}{6}$  because it equals  $\frac{85}{6}$ .
  - $(60 \div 6) + 18$  is not equal to  $\frac{78}{6}$  because the 60 is divided by 6 but 18 is not.
- Sample responses:
  - $10 \times 6 = 60$  and  $3 \times 6 = 18$  and  $10 + 3 = 13$ .
  - $60 \div 6 = 10$  and  $18 \div 6 = 3$  and

### Launch

- Groups of 2

### Activity

- 5–8 minutes: partner work time
- Monitor for students who:
  - use multiplication to find the value of  $78 \div 6$ .
  - use the expression  $\frac{60}{6} + \frac{18}{6}$  to find the value of  $78 \div 6$ .
  - use the expression  $\frac{66}{6} + \frac{12}{6}$  to find the value of  $78 \div 6$ .

### Synthesis

- Invite students to share the expressions that match  $78 \div 6$ .
- Display:  $78 \div 6 = \frac{78}{6}$
- “How do we know this is true?” (A fraction shows you are dividing the numerator by the denominator.)
- Display:  $78 \div 6 = \frac{60}{6} + \frac{18}{6}$
- “How can you use this equation to find the value of  $\frac{78}{6}$ ?” (I know  $\frac{60}{6}$  is 10 and  $\frac{18}{6}$  is 3 so  $\frac{78}{6}$  is 13.)
- “In the next activity we will use expressions with fractions to find values of other quotients.”

$$10 + 3 = 13.$$

## Advancing Student Thinking

If students do not identify all of the expressions whose value is equal to  $\frac{78}{6}$ , ask, "How did you decide which expressions were equal to  $\frac{78}{6}$ ?"

## Activity 2

🕒 20 min

Choose One Expression

### Standards Alignments

Building On 5.NF.B.3

The purpose of this activity is for students to find the whole number value of quotients using sums of fractions and to think about which sums were most helpful. They may notice that it is helpful to decompose the dividend into a multiple of the divisor and multiples of 10 are particularly helpful. This is closely related to how students found quotients using partial products which requires strategically choosing the number of groups of the divisor to subtract.

This activity uses *MLR1 Stronger and Clearer Each Time*. Advances: Reading, Writing.

### 🕒 Access for Students with Disabilities

*Representation: Internalize Comprehension.* Invite students to identify which details were most useful to solve the problem. Display the sentence frame: "The next time the dividend is not divisible by the divisor, I will look for multiples of 10 or multiples of the divisor to help me divide more efficiently."

*Supports accessibility for: Conceptual Processing, Memory, Organization*

### Instructional Routines

MLR1 Stronger and Clearer Each Time

## Student-facing Task Statement

- Use each expression to find the value of  $165 \div 15$ . Explain or show your thinking.
  - $\frac{75}{15} + \frac{80}{15} + \frac{10}{15}$
  - $\frac{30}{15} + \frac{30}{15} + \frac{30}{15} + \frac{60}{15} + \frac{15}{15}$
  - $\frac{150}{15} + \frac{15}{15}$
- Choose one expression and use it to find the value of  $540 \div 18$ . Explain or show your thinking.
  - $\frac{180}{18} + \frac{180}{18} + \frac{180}{18}$
  - $\frac{500}{18} + \frac{40}{18}$
  - $\frac{360}{18} + \frac{180}{18}$
- Which expressions were most helpful? Which expressions were least helpful? Explain or show your thinking.

## Student Responses

Sample responses:

- $75 \div 15 = 5$ ,  $80 \div 15 = 5\frac{5}{15}$ ,  
 $\frac{5}{15} + \frac{10}{15} = 1$ ,  $5 + 5 + 1 = 11$
  - $\frac{30}{15} = 2$ ,  $3 \times 2 = 6$ ,  $\frac{60}{15} = 4$ ,  $\frac{15}{15} = 1$ ,  
 $6 + 4 + 1 = 11$
  - $\frac{150}{15} = 10$ ,  $\frac{15}{15} = 1$ ,  $10 + 1 = 11$
- $\frac{180}{18} = 10$ ,  $\frac{180}{18} = 10$ ,  $\frac{180}{18} = 10$ ,  
 $3 \times 10 = 30$
  - $\frac{500}{18} = 27\frac{14}{18}$ ,  $\frac{40}{18} = 2\frac{4}{18}$ ,  $27 + 2 = 29$ ,  
 $\frac{14}{18} + \frac{4}{18} = 1$ ,  $29 + 1 = 30$
  - $\frac{360}{18} = 20$ ,  $\frac{180}{18} = 10$ ,  $20 + 10 = 30$
- Sample response: The most helpful expressions were ones where I could find the quotient mentally. For example  $\frac{180}{18}$  was nice because I knew it was 10. The least helpful expressions were ones whose value

## Launch

- Groups of 2
- Display:  
 $\frac{60}{6} + \frac{18}{3}$   
 $\frac{55}{6} + \frac{13}{6}$
- “Which of these expressions would you use to find the value of  $\frac{78}{6}$ ?” (The first one because the fractions have nice whole number values.)
- 1–2 minutes: partner discussion
- “You are going to choose expressions like this one that are helpful for finding quotients.”

## Activity

- 5–8 minutes: independent work time
- 1–2 minutes: partner discussion
- Monitor for students who:
  - explain that numerators that are multiples of the divisor are helpful to divide.
  - explain that numerators that are not multiples of the divisor require working with fractions.

## Synthesis

### MLR1 Stronger and Clearer Each Time

- “Share your response as to why some expressions were helpful and others were not with your partner. Take turns being the speaker and the listener. If you are the speaker, share your ideas and writing so far. If you are the listener, ask questions and give feedback to help your partner improve their work.”
- 3–5 minutes: structured partner discussion.
- Repeat with 2–3 different partners.
- (Optional) If needed, display question

was not a whole number like  $\frac{80}{15}$ .

starters and prompts for feedback.

- “Can you give an example to help show . . . ?”
- “Can you use the word \_\_\_\_ in your explanation?”
- “The part I understood best was . . .”
- “Revise your initial draft based on the feedback you got from your partners.”
- 2–3 minutes: independent work time

## Lesson Synthesis

🕒 10 min

Display:

$$\frac{180}{18} + \frac{180}{18} + \frac{180}{18}$$

“How do we know this expression is equal to  $\frac{540}{18}$ ?” ( $180 + 180 + 180 = 540$  and they are 18ths.)

“How can we use this expression to find the value of  $540 \div 18$ ?” ( $\frac{180}{18} = 10$  and there are three of them so the value of  $540 \div 18$  is 30.)

Display:

$$\frac{360}{18} + \frac{180}{18}$$

“How can we use this expression to help us find the value of  $540 \div 18$ ?” ( $36 \div 18 = 2$  so  $360 \div 18 = 20$  and  $180 \div 18 = 10$  and  $20 + 10 = 30$ .)

Display:

$$\frac{500}{18} + \frac{40}{18}$$

“How do we know this expression is equal to  $540 \div 18$ ?” ( $500 + 40 = 540$  and they're 18ths)

“Why is this expression not as helpful as the others?” (The values of those fractions are not whole numbers so we have to calculate with fractions.)

## Suggested Centers

- Mystery Number (1–4), Stage 6: Decimals (Supporting)

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**Complete Cool-Down**

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**Response to Student Thinking**

Students don't choose an expression that results in whole number partial quotients.

**Next Day Support**

- After the warm-up, pair students up to discuss their cool-down from this lesson and make revisions.

## Section C: Let's Put it to Work

### Lesson 18: Lots of Milk

#### Standards Alignments

Building On 5.NBT.B  
Addressing 5.MD.C.5, 5.NBT.B

#### Teacher-facing Learning Goals

- Estimate products and quotients of whole numbers.

#### Student-facing Learning Goals

- Let's make estimates with big numbers.

#### Lesson Purpose

The purpose of this lesson is to estimate products and quotients using a volume context.

The purpose of this lesson is for students to use their understanding of multiplication and division to estimate products and quotients. This is the first of several lessons where students use multiplication and division to make estimates of large quantities. All of the lessons except the next one use a context of volume.

Students make estimates about the amount of milk consumed by different groups and the number of days it would take these groups to consume 1,000,000 cubic inches of milk. The estimates are structured to build on one another allowing students to use the associative property of multiplication.

#### Access for:

##### Students with Disabilities

- Representation (Activity 1)

##### English Learners

- MLR2 (Activity 1)

#### Instructional Routines

Estimation Exploration (Warm-up)

#### Lesson Timeline

Warm-up	10 min
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#### Teacher Reflection Question

What connections did students make between the different strategies shared? What questions

Activity 1	15 min	did you ask to help make the connections more visible?
Activity 2	20 min	
Lesson Synthesis	10 min	
Cool-down	5 min	

## Cool-down (to be completed at the end of the lesson)

 5 min

### So Much Milk

#### Standards Alignments

Addressing 5.NBT.B

#### Student-facing Task Statement

There are 17,566 students in a city who drink a carton of milk at lunch one day. Each carton of milk is about 20 cubic inches. About how many cubic inches of milk do the students drink altogether? Explain or show your reasoning.

#### Student Responses

Sample responses:

- about 400,000 since  $20 \times 20 = 400$  and there are close to 20,000 students
- about 350,000 since  $17 \times 20 = 340$  and  $18 \times 20 = 360$ , so I took the number in the middle and multiplied by 1,000 since it's between 17 and 18 thousand

## ----- Begin Lesson -----

### Warm-up

 10 min

Estimation Exploration: How Big is the Milk Carton?

#### Standards Alignments

Addressing 5.MD.C.5

The purpose of this Estimation Exploration is for students to estimate a volume based on an image and on their own personal experience with cartons of milk. Students recall the meaning of volume as the number of cubic inches, in this case, it would take to fill the milk carton without gaps or overlaps. Because the carton is relatively small, students can formulate a reasoned, accurate estimate of the milk carton's volume. They will then use this estimate throughout the lesson.

## Instructional Routines

### Estimation Exploration

### Student-facing Task Statement

What is the volume of the milk carton in cubic inches?



Record an estimate that is:

too low	about right	too high

### Student Responses

Sample responses:

- too low: 4–8
- about right: 10–20
- too high: 30 or more

### Launch

- Groups of 2
- Display the image.
- “What is an estimate that’s too high?” “Too low?” “About right?”
- 1 minute: quiet think time

### Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Record responses.

### Synthesis

- “How can you use what you know about volume to estimate the volume of the milk container?” (I can measure to see how many cubic inches it would take to fill the carton. I can measure the length, width, and height and multiply them.)
- “What units do you usually use to measure liquids?” (Liters, quarts, cups)
- “We learned in an earlier unit that cubic centimeters or cubic inches are also units for measuring a volume.”

## Activity 1

🕒 15 min

### Milk for Everyone

#### Standards Alignments

Building On 5.NBT.B

The purpose of this activity is for students to estimate products using the context of volume introduced in the warm-up. Students estimate how many cubic inches of milk different-sized groups of students might consume. For example, at first, students multiply the amount of milk they consume by the number of students in the class. Next, students multiply the amount consumed by one class by the number of classes. Because these are all estimates, the fact that not every student in one class drinks the same amount of milk or that different classes or grades or schools have different numbers of students can be overlooked. When students make simplifying hypotheses like this, they model with mathematics (MP4).

As currently structured, the activity is quite open-ended so that students can use their own school to make their estimates. There is a lot of variation in school size. The average size of an elementary school in Montana, for example, is less than 200, while in California, it is 600. Some large elementary schools in New York City have close to 2,000 students. The important mathematical part of this activity does not depend on the exact numbers for a particular school. The key is which numbers students choose as they make estimates, focusing on multiples of powers of 10.

#### Access for English Learners

*MLR2 Collect and Display.* Circulate, listen for, and collect the language students use as they estimate the volume. On a visible display, record words and phrases such as: estimate, guess, predict, multiply, times, and product. Invite students to borrow language from the display as needed, and update it throughout the lesson.

*Advances: Conversing, Reading*

#### Access for Students with Disabilities

*Representation: Access for Perception.* Use centimeter cubes to demonstrate how many cubic centimeters can fit inside the milk carton so that students understand the size of a cubic centimeter.

*Supports accessibility for: Conceptual Processing, Visual-Spatial Processing*

## Student-facing Task Statement

In each situation, estimate the volume of milk, in cubic inches, that you or the group would drink in one day. Explain your reasoning.

1. you
2. your class
3. your grade
4. your school
5. 10 schools

## Student Responses

Sample responses based on 25 students in class, 3 fifth-grade classes at the school, 6 grades at school with same number of students each:

1. 20
2. 500
3. 1,500
4. 9,000
5. 90,000

## Launch

- “What kind of milk do you like to drink?”
- Partner discussion
- “You are going to estimate the amount of milk that different groups of students drink in one day.”
- “You can use the estimate of 20 cubic inches for one carton of milk.”

## Activity

- Monitor for students who select round numbers for their estimates and who use multiplication to go from each estimate to the next estimate.

## Synthesis

- Invite students to share responses and estimates.
- “How did you use your estimates from each question to help answer the next question?” (Once I knew how much milk I drank, I multiplied by the number of students in our class. Then I multiplied that by the number of fifth-grade classes.)
- “How did you make an estimate for your class?” (I think there are between 20 and 30 students in the class but not everyone likes milk. So I estimated that 20 students drink milk with lunch.)

## Advancing Student Thinking

If students do not like milk and, therefore, do not have a connection to the problem, suggest they survey a few classmates to find out what their estimates were for how much milk they drink in one day.

## Activity 2

🕒 20 min

How Big is 1,000,000?

### Standards Alignments

Building On 5.NBT.B

The purpose of this activity is for students to make estimates about how long it would take different groups of students to drink 1,000,000 cubic inches of milk. Unlike the previous activity in which students multiplied the 20 cubic inches of milk by larger and larger numbers, in this activity, students divide 1,000,000 cubic inches of milk by smaller and smaller numbers to find out how long it would take each group to drink 1,000,000 cubic inches of milk. If students attempt to calculate exact answers remind them that they are only looking for an estimate and the amount of milk consumed by each group in the previous activity is also only an estimate. Making an estimate or a range of reasonable answers with incomplete information is a part of modeling with mathematics (MP4).

### Student-facing Task Statement

Estimate the number of days it would take each group to drink 1,000,000 cubic inches of milk. Explain your reasoning.

1. 10 local schools
2. your school
3. your grade
4. your class
5. you

### Student Responses

Sample responses (based on the answers given in the previous activity)

1. 10
2. 100
3. 600
4. 1,800

### Launch

- Groups of 2
- “How much do you think 1,000,000 cubic inches of milk is? Could you drink it?” (No, that’s a lot of milk. I don’t like milk that much.)
- 1 minute: quiet think time
- 1 minute: partner discussion

### Activity

- 2-3 minutes individual work time
- 7-8 minutes partner work time
- Monitor for students who use the estimates from the previous activity and who base each successive calculation on the previous one, dividing by an appropriate number at each step.

5. 50,000

### Synthesis

- “How did you estimate the number of days it takes 10 schools to drink 1,000,000 cubic inches of milk?” (We estimated that they drink close to 100,000 cubic inches a day, so in 10 days that’s 1,000,000.)
- “How did you use this estimate to estimate how long it takes your school to drink 1,000,000 cubic inches of milk?” (I multiplied by 10 because it takes 1 class 10 times as long as it takes 10 classes.)
- “Do you think that you will ever drink 1,000,000 cubic inches of milk?” (No, 50,000 days is a lot. There are only 365 days in a year, so that would be more than 100 years.)

### Advancing Student Thinking

Students may need support with initiating the task. Ask them to explain how they can use the solutions from the previous activity to help them solve the problems.

## Lesson Synthesis

 10 min

“In this lesson we estimated products and quotients.”

“How can you use multiplication to estimate how many days it would take your school to drink 1,000,000 cubic inches of milk?” (In 2 days we drink twice as much milk, in 3 days we drink 3 times as much. So I needed to estimate what to multiply the amount for one day by to get about 1,000,000.)

“Could you also make this estimate using division?” (Yes, our school drinks about 10,000 cubic inches of milk each day, so I can find how many 10,000s there are in 1,000,000. That’s  $1,000,000 \div 10,000$ .)

### Suggested Centers

- Mystery Number (1–4), Stage 6: Decimals (Supporting)

## Response to Student Thinking

Students do not respond with reasonable estimates.

Students need support multiplying by multiples of 10.

## Next Day Support

- Launch warm-up or activity 1 by highlighting important notation from previous lessons.

## Prior Unit Support

Grade 4, Unit 4, Section B: Place-value Relationships through 1,000,000

## Lesson 19: Trash Talk

### Standards Alignments

Building On	4.MD.A.1
Addressing	5.NBT.B.5
Building Towards	5.NBT.B.5

### Teacher-facing Learning Goals

- Use multiplication to solve problems about the area of the Great Garbage Patch.

### Student-facing Learning Goals

- Let's multiply to solve problems about the area of the Great Garbage Patch.

### Lesson Purpose

The purpose of this lesson is to find areas by multiplying side lengths in situations where the side lengths are two- or three-digit numbers.

In previous lessons, students have learned to use the standard algorithm to multiply whole numbers including a three-digit and a two-digit number. The purpose of this lesson is to apply those techniques to a situation involving areas related to the Great Garbage Patch. The measurements are given in kilometers so students review, in the first activity, how long a kilometer is. Then, students calculate the area of states to get a sense for the size of the Great Garbage Patch. One of these calculations is a product of two three-digit numbers. After this brief interlude solving problems about area, students return to volume for the final two lessons. The area calculations for the states showed the importance of estimation since finding the exact products requires careful calculations whereas a quick estimate shows that the area of New Mexico is less than the area of the Great Garbage Patch. This estimation builds on student learning from the previous activity and then next two lessons combine estimation, volume, and trash.

### Access for:

#### Students with Disabilities

- Representation (Activity 1)

#### English Learners

- MLR8 (Activity 2)

### Instructional Routines

Notice and Wonder (Warm-up)



## Warm-up

🕒 10 min

### Notice and Wonder: Trash and Ice

#### Standards Alignments

Building Towards 5.NBT.B.5

The purpose of this Notice and Wonder is to introduce the idea of large amounts of trash floating on the water. In an earlier unit, students saw that large amounts of trash are shipped around the world. Before reconsidering trash in a volume context, this lesson focuses on the area that trash covers in some parts of the ocean. Students may notice and wonder many things about the image. The key discussion points are that the debris seen in the image is mostly trash and it covers a large area.

#### Instructional Routines

Notice and Wonder

#### Student-facing Task Statement

What do you notice? What do you wonder?



#### Student Responses

Students may notice:

- There is a lot of stuff floating in the water.
- I see cans, bottles, and paper.
- There are some darker and lighter splotches in the water.

Students may wonder:

- Is that all trash?

#### Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

#### Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

#### Synthesis

- "The image shows what it looks like in some parts of the oceans. The small things floating on the water are trash."
- "How do you think trash gets into the ocean?" (People throw things away near the ocean. People throw things into the ocean when they are on boats.)

- Is this a lake or the ocean?
- Where is this?
- Where did the trash come from?
- What is that gray stuff in the middle?

## Activity 1

 10 min

Square Kilometers

 PLC Activity

### Standards Alignments

Building On 4.MD.A.1

In previous grades, students worked with metric units of length. The purpose of this activity is to reason about the size of a kilometer. Students consider the distance they might walk or run, perhaps to and from school or during sports practice, and they also consider other lengths that they might be familiar with such as going across town or taking a bike ride or the distance to a friend's house. Building an intuition for large numbers or distances or other measurements takes a lot of time and practice. The intent of this activity is to begin this work informally. A wide variety of answers should be expected and the goal of the synthesis is to share this variety and come to a general agreement of approximately how far a kilometer, or some number of kilometers, is.

### Access for Students with Disabilities

*Representation: Internalize Comprehension.* Activate or supply background knowledge. Provide students with a visual representation of a kilometer for students who are unfamiliar with the distance.

*Supports accessibility for: Conceptual Processing, Memory, Attention*

### Materials to Gather

Metersticks

### Student-facing Task Statement

1. Mai walked around a soccer field 2 times. She thinks she walked about a kilometer. Do you agree with Mai? Show or explain your reasoning.

### Launch

- Groups of 2
- Lay a meter stick on the ground.
- “Tyler walked from his classroom to the cafeteria. He said, ‘I think that’s about a

2. Decide whether each distance is less than a kilometer, about a kilometer, or more than a kilometer.
  - a. the distance across the state you live in
  - b. the distance from your home to school
  - c. the distance from your school room to the restroom
  - d. the distance you travel on a vacation in the car

### Student Responses

1. Yes, a soccer field is pretty big but it's still a distance you can walk in a pretty short time.
2.
  - a. more than a kilometer, this is a very long distance I would not walk
  - b. Sample response 1: less, I can see the school right down the street from my home.  
Sample response 2: I think it's about a kilometer, it's not close but I can walk.
  - c. less, this is something I would measure with meters, not kilometers
  - d. more than a kilometer, the car goes fast and we travel for a long time

kilometer.' Do you agree with Tyler?" (No, a kilometer is a long distance, it's 1,000 meters, and it is not that far from a classroom to the cafeteria.)

- 1 minute: quiet think time
- 1–2 minutes: partner discussion
- Give students access to meter sticks.

### Activity

- 2 minutes: independent work time
- 3–4 minutes: partner discussion
- Monitor for students who have different answers and explanations for the question about the distance between their home and school including:
  - less because I can walk there really quickly
  - less because it's no more than 1 or 2 lengths of a soccer field
  - about a kilometer because I sometimes walk, so it's not too far, but it's also not real close
  - more than a kilometer as I take a bus and it's a long trip

### Synthesis

- Invite students to share their responses for how far it is from school to where they live.
- "How many meters are there in a kilometer?" (1,000)
- "About how many steps does it take to go 1 meter?" (2)
- "About how many steps does it take to go 1000 meters or 1 kilometer?" (2,000)
- "Does 2,000 steps help communicate how far 1 km is?" (Yes. That's a lot but I can count to 1,000 and take that many steps so it's not too far. No. 1,000 steps is a lot, too much for me to imagine.)

## Advancing Student Thinking

If students do not recognize the distance of a kilometer, give them a meter stick and ask, "Estimate distances around the school that would be 1 meter, 10 meters, and 100 meters."

## Activity 2

🕒 25 min

### So Much Trash

#### Standards Alignments

Addressing 5.NBT.B.5

In the previous activity students developed an intuition for the size of a kilometer. The purpose of this activity is to calculate the area of different states in square kilometers and compare them to the area occupied by the Great Pacific Garbage Patch, a large collection of trash floating in the Pacific Ocean. This activity has several distinct goals:

- communicate the enormity of the Great Pacific Garbage Patch
- continue to build a sense of distances by looking at a map of the United States and thinking about the sizes of different states
- multiply large numbers, most likely using the standard algorithm
- calculate a product of two 3-digit numbers for the first time

Students calculate the areas of different states in order to grasp the enormity of a floating area of trash in the Pacific Ocean. It turns out that this area is substantially larger than most states! As they calculate areas of different states, students will use the standard multiplication algorithm and estimation. Estimation plays an important role because the area of New Mexico is a product of two 3-digit numbers. Students are prepared to make and understand this calculation but it also provides an opportunity to see the power of estimation because the area of New Mexico can be readily compared with the area of the Great Garbage Patch without calculating the exact area. For the last question, students use the area of the states they calculated and apply that to a map of the United States.

Students model with mathematics when they make assumptions about the states, for example that they are approximated by rectangles, in order to calculate or estimate their area and when they choose states whose area is approximately equal to the area of the Great Garbage Patch (MP4).

## Access for English Learners

*MLR8 Discussion Supports.* Prior to solving the problems, invite students to make sense of the situations and take turns sharing their understanding with their partner. Listen for and clarify any questions about the context.

*Advances: Reading, Representing*

### Student-facing Task Statement

The Great Pacific Garbage Patch is a large area in the Pacific Ocean where trash has accumulated. Some estimates indicate that the garbage covers about 1,000,000 square kilometers.

1. Rhode Island is the smallest state. It is about 77 km long and 60 km wide. Which is larger, the garbage patch or Rhode Island? Explain or show your reasoning.
2. Delaware is about 154 km long and 48 km wide. Which is larger, the garbage patch or Delaware? Explain or show your reasoning.
3. New Mexico is about 596 km long and 552 km wide. Which is larger, the garbage patch or New Mexico?
4. Circle an area on the map of the U.S. that you think estimates the area of the garbage patch. Explain your thinking.



### Student Responses

1. Rhode Island has area  $77 \times 60$  or 4,620 square kilometers. The Garbage Patch is much larger.

### Launch

- Groups of 2
- “The Great Pacific Garbage Patch is a large area in the Pacific Ocean where trash has accumulated and floats on top of the water. We are going to compare the size of this part of the ocean to different states in the United States.”
- Display an image of the state you live in.
- “What do you know about the size of our state?”
- Highlight Rhode Island, Delaware, and New Mexico on the map in the student workbook so students can visualize the states and their relative sizes.

### Activity

- 5–8 minutes: independent work time
- 5–8 minutes: partner discussion
- Monitor for students who:
  - use the standard algorithm to find the areas of Rhode Island and Delaware
  - use the standard algorithm to find the area of New Mexico
  - show the size of the garbage patch differently on the map of the United States

2. The area of Delaware is  $154 \times 48$  kilometers. That's 7,392 square kilometers so the garbage patch is much bigger.

$$\begin{array}{r} 21 \\ 43 \\ 154 \\ \times \quad 48 \\ \hline 1,232 \\ + 6,160 \\ \hline 7,392 \end{array}$$

3. The area of New Mexico is 328,992 square kilometers so it's also smaller than the garbage patch.

$$\begin{array}{r} 43 \\ 43 \\ 11 \\ 596 \\ \times \quad 552 \\ \hline 11 \\ 1,192 \\ 29,800 \\ + 298,000 \\ \hline 328,992 \end{array}$$

4. The garbage patch is about 3 times as big as New Mexico so I circled New Mexico, Arizona, and Colorado.

## Synthesis

- Invite students to share their calculations for Delaware.
- "Is Delaware close in size to the garbage patch? How do you know?" (No, it is not close at all. 1,000,000 is a lot more than 7,392.)
- "Is the garbage patch more or less than 10 times as large as Delaware?" (Much more, because that's only about 70,000 which is way less than a million.)
- "Is the garbage patch more or less than 100 times as large as Delaware?" (More, because that's about 700,000 square kilometers.)
- "How could you estimate the area of New Mexico?" (596 is real close to 600 which is a nice number. 552 is harder, but if I say that's about 500 then I get an estimate of  $600 \times 500$  or 300,000.)
- "How did you decide which area to circle on the map?" (New Mexico was the biggest and 3 New Mexicos would make about 1,000,000 square miles so I circled 3 states close in size to New Mexico.)

## Lesson Synthesis

🕒 10 min

"Today we looked at the Great Garbage Patch which has an area of about 1,000,000 square kilometers. We saw how big that is by comparing it to different states."

"If the great garbage patch were a rectangle, what could its side lengths be?" (It could be 1,000,000 kilometers by 1 kilometer but 1,000,000 kilometers is too long. It could be 1,000 kilometers by 1,000 kilometers. I could triple one of the lengths of New Mexico so it could be about 1,800 kilometers by 550 kilometers.)

"Much of the garbage in the Great Garbage Patch is plastic. Tomorrow we will investigate the amount of recyclable plastic we produce each year."

## Suggested Centers

- Mystery Number (1–4), Stage 6: Decimals (Supporting)

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## Complete Cool-Down

## Response to Student Thinking

Students do not calculate 271,200 as the area.

## Next Day Support

- Before the next day's warm-up, pair students up to discuss their responses.

## Lesson 20: Shipping Trash

### Standards Alignments

Addressing 5.MD.C.5, 5.NBT.B.5, 5.NBT.B.6

### Teacher-facing Learning Goals

- Estimate and calculate products and quotients of whole numbers.

### Student-facing Learning Goals

- Let's estimate volumes.

### Lesson Purpose

The purpose of this lesson is to estimate and calculate products and quotients of whole numbers in order to understand the volume of recyclable plastic the United States ships abroad each year.

This lesson uses the structure of the first lesson in this section where students estimated how much milk different groups of students drink in a day and then how many days it would take the students to drink specified amounts of milk. In this lesson, students make similar estimates and calculations, but now they are estimating the volume of recyclable garbage students at their school produce. Continuing the context of the previous lesson as well as the large numbers, students start to conceptualize how much recyclable garbage the United States produces and ships overseas. They first encountered this context at the end of the first unit and now can use the whole number multiplication and division strategies they learned in this unit to study the situation in greater depth. Because there are a lot of estimates involved, students also see a quotient of numbers much larger than they have seen to this point. They find the value of this quotient using known facts and reasoning about place value.

Throughout the lesson, students make estimates and simplifying assumptions in order to answer complex mathematical questions (MP4).

### Access for:

#### Students with Disabilities

- Engagement (Activity 1)

#### English Learners

- MLR1 (Activity 2)

### Instructional Routines

Number Talk (Warm-up)

## Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

## Cool-down (to be completed at the end of the lesson)

 5 min

Ship It

### Standards Alignments

Addressing 5.NBT.B.5, 5.NBT.B.6

### Student-facing Task Statement

- A different shipping container is 40 feet long, 9 feet wide, and 8 feet tall.
  - What is the volume of this container? Explain or show your thinking.
  - A school makes 24 cubic feet of recyclable plastic each day. How many days does it take the school to fill this container? Explain or show your thinking.

### Student Responses

- 2,880 cubic feet. I first found  $40 \times 9 = 360$  and then found  $360 \times 8$ .
  - 120.

$$\begin{array}{r}
 \boxed{120} \\
 20 \\
 100 \\
 24 \overline{)2,880} \\
 \underline{-2,400} \\
 480 \\
 \underline{-480} \\
 0
 \end{array}$$

---

**Begin Lesson**


---

**Warm-up**

🕒 10 min

## Number Talk: Three Factors

**Standards Alignments**

Addressing 5.NBT.B.5

The purpose of this Number Talk is for students to mentally calculate a product which students will work with in context in this lesson. The first two products students may know from memory but if not, the idea of doubling  $8 \times 4$  to find  $8 \times 8$  can be helpful both for finding the value of  $8 \times 8$  and for starting a pattern that continues with the next product,  $8 \times 8 \times 2$ , which is double  $8 \times 8$ . The factors 8, 8, and 20 turn out to be the side lengths, in feet, of a standard container used on cargo ships. Students will examine these containers and the ships that carry them throughout the lesson.

**Instructional Routines**

Number Talk

**Student-facing Task Statement**

Find the value of each expression mentally.

- $8 \times 4$
- $8 \times 8$
- $8 \times 8 \times 2$
- $8 \times 8 \times 20$

**Student Responses**

Sample responses

- 32: I just knew it.
- 64: I doubled 32.
- 128: I doubled 60 to get 120 and then doubled 4 and added that to 120.
- 1,280: I know that multiplying by 10 shifts all of the digits one place to the left.

**Launch**

- Display one expression.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

**Activity**

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

**Synthesis**

- “How did you find the value of  $8 \times 8 \times 20$ ?” (I multiplied  $8 \times 8 \times 2$  by 10 because 20 is  $10 \times 2$ .)

## Activity 1

🕒 15 min

### What a Waste

#### Standards Alignments

Addressing 5.MD.C.5, 5.NBT.B.5

The purpose of this activity is for students to make reasoned estimates about the volume of recyclable goods a school produces in a day, a week, a month, and a year. Students have seen cubic feet and cubic meters briefly in an earlier unit and it may be helpful to build a cubic foot to enable them to visualize this unit of measure and improve their estimates.

Answers to the questions here will vary widely based on the size of the school, the size of the recycling bins, and the exact estimates students make. Mathematically, the important point is that students use multiplication appropriately in their estimates and develop a sense for the staggering size of the waste that they will consider in the second activity.

#### 🕒 Access for Students with Disabilities

*Engagement: Provide Access by Recruiting Interest.* Optimize meaning and value. Invite students to share how they contribute to the recyclable materials in the classroom.

*Supports accessibility for: Social-Emotional Functioning, Language*

### Required Preparation

- Gather a small recycle bin or trash can and a large recycle bin or trash can.

#### Student-facing Task Statement

1. Estimate the value of each quantity.
  - a. The number of cubic feet that the class recycling bin holds.
  - b. The number of cubic feet that the school recycling bins hold.
2. About how many cubic feet of recyclable materials do you think your school produces in each amount of time? Explain or show your reasoning.

#### Launch

- “The United States ships recyclable goods like plastic to other countries for processing. We are going to estimate the volume of recyclable materials our school produces.”
- Show a recycling bin or trash can from class.
- Show a large recycling bin or large trash can from school (if possible).
- “How can we estimate the number of cubic

- a. a day
  - b. a week
  - c. a month
  - d. a year
3. Do you think all of the recyclable materials your school produces in a year could fit in your classroom? Show or explain your reasoning.

### Student Responses

Sample responses

1.
  - a. 3 cubic feet
  - b. 12 cubic feet
2.
  - a. 100 cubic feet. Sample response: There are about 20 classrooms and if they fill up a recycling bin each day that's about 40 cubic feet. There are also several larger bins around the school and if those are filled each day that's another 36 cubic feet, assuming there are 3 of them. I decided to round this to 100 cubic feet.
  - b. 500 cubic feet. I multiplied 100 cubic feet by 5 school days in a week.
  - c. 2,000 cubic feet. I multiplied 100 cubic feet each day by about 20 school days for the month.
  - d. 20,000 cubic feet. I estimated that there are about 200 school days a year and 100 cubic feet each day.
3. Sample response: I'm not sure. I estimate our room is 10 feet high, 40 feet long, and 30 feet wide. That would be 12,000 cubic feet since  $40 \times 30 = 1,200$  and then  $1,200 \times 10 = 12,000$ . But I'm not sure about my estimates so it might fit.

feet the class recycling bin holds?" (We can measure the length, width, and height. We can build a cubic foot and put it inside the recycling bin to see how much space it takes up.)

- 1 minute: small-group discussion

### Activity

- 5 minutes: individual think time
- 5 minutes: partner discussion time
- Monitor for students who
  - make different estimates for the size of the recycling bins and how often they are filled
  - make round estimates for the volume of recyclables the school produces and for the classroom

### Synthesis

- Invite students to share their estimates for the volume of the different recycling bins.
- "How did you estimate the volume of the small bins?" (It looks like it's about 2 feet tall and I think the base is maybe a square foot. So that's 2 cubic feet.)
- "How did you estimate the volume of the large recycling bins?" (I think they're about 3 feet tall with a base of about 4 square feet so that would be 12 cubic feet.)

## Activity 2

🕒 20 min

### Plastic Palooza

#### Standards Alignments

Addressing 5.NBT.B.5, 5.NBT.B.6

The purpose of this activity is to use estimation to compare the amount of recyclable plastic produced by all elementary schools in the United States to the amount of recyclable plastics the United States exports every year for processing. In this task, an estimate for each school is provided. The new parts of this activity are considering all of the schools in the country, for which an estimate is provided, and the total amount of plastics exported by the United States each year, for which an estimate is also provided.

The numbers in this activity go beyond those expected for the grade 5 division standard but they are friendly and the quotient is small enough that students could find it by repeated subtraction or addition. It is also possible that they will use their place value understanding and their understanding of single-digit multiplication facts.

#### 🌐 Access for English Learners

*MLR1 Stronger and Clearer Each Time.* Synthesis: Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their response to the final question. Invite listeners to ask questions, to press for details and to suggest mathematical language. Give students 2–3 minutes to revise their written explanation based on the feedback they receive.

*Advances: Writing, Speaking, Listening*

#### Student-facing Task Statement

Your goal is to decide, by estimating, whether it is possible for all of the elementary schools in the country to produce enough recyclable plastic to fill the cargo containers that the United States ships each year.

1. A standard cargo container for a ship measures 20 feet long, 8 feet wide, and 8 feet tall. What is the volume of the container?

#### Launch

- Groups of 2
- “You are going to compare an estimate for the amount of recyclable plastic produced by all elementary schools in the country with the amount of plastic the United States shipped in 2018 to other countries where it is recycled.”

- Each school makes about 40 cubic feet of recyclable plastic each day. How many days would it take for a school to fill one cargo container?
- In 2018 the United States exported about 210,000 cargo containers of plastic. There are about 70,000 elementary schools in the United States. How many cargo containers does each school need to fill in order to fill all of these containers?
- Do you think all the schools in the country produce enough plastic recyclables to fill all the cargo containers that the United States ships? Show or explain your reasoning.

### Student Responses

- 1,280 cubic feet,  $8 \times 8 \times 20 = 1,280$
- 32,  $1,280 \div 40 = 32$

$$\begin{array}{r}
 \boxed{32} \\
 2 \\
 30 \\
 40 \overline{)1,280} \\
 \underline{-1,200} \\
 80 \\
 \underline{-80} \\
 0
 \end{array}$$

- 3, I know  $3 \times 7 = 21$  so  $3 \times 70,000 = 210,000$  because I am just adding 4 zeros to the numbers.
- Sample response 1: If it takes each school 32 days to fill 1 cargo container then it will take 96 days to fill 3 cargo containers. So the 70,000 different schools could fill 210,000 cargo containers with plastic in a school year.  
Sample response 2: I'm not sure. The calculations say that it's possible but the schools are not all the same size and some may make a lot less plastic than others. I don't know how good the estimates are so I am not sure.

### Activity

- 5 minutes: individual work time
- 10 minutes: partner work time
- Monitor for students who
  - recall that they made the calculation for the first problem in the warm-up
  - use the division algorithm to find the number of days it takes a school to fill one cargo container
  - use the fact that  $3 \times 7 = 21$  and place value to find the value of  $210,000 \div 70,000$

### Synthesis

- Invite students to share their solutions for finding the number of days it would take a school to fill one of the cargo containers with plastic.
- "How can you use the equation  $128 \div 4 = 32$  to find the value of  $1,280 \div 40$ ?" (They have the same value because if 32 groups of 4 is 128, putting ten times as many in each group will give 10 times as much for the total.)
- Invite students to share their response for how many cargo containers each school will fill.
- "How did you find the value of  $210,000 \div 70,000$ ?" (I know that  $3 \times 70 = 210$  and so this is also true for thousands.)
- Invite students to share their reasoning for the last question about all elementary school plastic recyclables.
- "Much of the debris that is in the Great Pacific Garbage Patch is plastic. Today, we got a sense of how much recyclable plastic all of the schools in our country produce."

## Lesson Synthesis

🕒 10 min

“Today we made estimates for the amount of recyclable plastic elementary schools might produce and compared this with the amount of plastic that the United States ships abroad.”

“What are some of the different estimates you made or worked with today?” (the volume of recycling bins, the amount of things we put in the bins each day, the number of schools and amount of recyclable plastics shipped)

“How is calculating with estimates the same as using exact values? How is it different?” (I still need to know what operation to use. But the round numbers are easier to calculate with.)

“If you knew that there were 68,372 schools rather than 70,000 and the U.S. shipped 207,364 cargo containers of plastic, would that change your answer to the question whether the schools could fill all of those containers? Why?” (No. I don't think I could find the value of the quotient but it should still be close to 3.)

“Estimation is important not only to check the reasonableness of answers but also we sometimes don't need an exact calculation to answer a question.”

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### Suggested Centers

- Mystery Number (1–4), Stage 6: Decimals (Supporting)

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### Complete Cool-Down

#### Response to Student Thinking

Students do not determine the correct number of days it would take to fill the shipping container.

#### Next Day Support

- Before the first activity, pair students up to discuss their responses.

## Lesson 21: Food Waste Journal (Optional)

### Standards Alignments

Addressing 5.MD.C.5, 5.NBT.B.5, 5.NBT.B.6

### Teacher-facing Learning Goals

- Estimate and calculate products and quotients of whole numbers.

### Student-facing Learning Goals

- Let's use multiplication and division to think about food waste.

### Lesson Purpose

The purpose of this lesson is to estimate and calculate products and quotients of whole numbers in order to understand the amount of food waste that is produced in the United States each year.

This lesson is optional because it does not address any new mathematical content standards. This lesson does provide students with an opportunity to apply precursor skills of mathematical modeling. In previous lessons, students made estimates about the volume of recyclable garbage students at their school produce. In this lesson, students make similar estimates and calculations, but now they estimate the weight of food waste produced. In the first activity students estimate the amount of food waste they produce based on average production by individuals in the United States. In the second activity, students are introduced to a food waste journal and make some initial calculations about their food waste.

The last activity is optional and can be used after students use the journal to record their food waste for a week. Students share what they notice and compare the amount of food waste they produce to the national average. Students use this sample data to estimate how much food waste they produce monthly and annually.

When students recognize the mathematical features of familiar real world situations and use those features to solve problems, they model with mathematics (MP4).

### Access for:

#### Students with Disabilities

- Engagement (Activity 2)

#### English Learners

- MLR7 (Activity 1)

### Instructional Routines

Notice and Wonder (Warm-up)

**Lesson Timeline**

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Activity 3	20 min
Lesson Synthesis	10 min

**Teacher Reflection Question**

What question do you wish you had asked today? When and why should you have asked it?

---

----- **Begin Lesson** -----

**Warm-up**

🕒 10 min

**Notice and Wonder: Food Waste**

The purpose of this warm-up is to introduce students to the context of food waste, which will be useful when students keep a journal of food waste in a later activity. While students may notice and wonder many things about this illustration, what students consider to be food waste in their neighborhood or community is the important discussion point.

**Instructional Routines**

Notice and Wonder

**Student-facing Task Statement**

What do you notice? What do you wonder?

**Launch**

- Groups of 2
- Display the image.
- “What do you notice? What do you wonder?”
- 1 minute: quiet think time

**Activity**

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Share and record responses.

## Student Responses

Students may notice:

- There is a trash dumpster filled with food.
- There are bones and fish skeletons.
- Some food is eaten, but other food looks uneaten.
- There are two whole eggplants.

Students may wonder:

- How much more food is in the dumpster?
- How much does this all weigh?
- Why was the food thrown out?

## Synthesis

- "What kinds of things do you think we throw out when we cook or eat at home or outside?" (food scraps, leftover food, spoiled ingredients or food, packaging for food)
- "What are some reasons we throw out food?" (It's spoiled. It smells or looks bad. We don't want it anymore. It's too old. It fell on the floor.)
- Record responses and keep it displayed.

## Activity 1

🕒 15 min

### Food Waste in the United States

#### Standards Alignments

Addressing 5.MD.C.5, 5.NBT.B.5

The purpose of this activity is to make reasoned estimates about the weight of food waste produced by individuals, families, and the school community each week and each month. Answers will vary widely based on the size of families, the class, and the school. Students use multiplication and division appropriately in their estimates.

#### Access for English Learners

*MLR7 Compare and Connect.* Synthesis: After all strategies have been presented, lead a discussion comparing, contrasting, and connecting the different approaches. Ask, "Did anyone solve the problem the same way, but would explain it differently?" and "How did the average amount of food waste show up in each method?"

*Advances: Representing, Conversing*

#### Student-facing Task Statement

The average person produces 219 pounds of

#### Launch

- Groups of 2

food waste per year.

1. If each person produces the average amount, about how many pounds of food waste is produced by each group in each amount of time? Explain or show your reasoning.
  - a. a person in 1 month
  - b. a person in 1 week
  - c. your family in 1 year
  - d. your class in 1 year
  - e. everyone in your school in 1 year
2. There are 16 ounces in a pound. How many ounces of food waste does the average person produce in 1 year?

### Student Responses

1. Sample responses:
  - a. About 18 pounds because  $219 \div 12$  is almost the same as  $216 \div 12$  and  $216 \div 12 = 18$
  - b. About 4 or 5 pounds because each month has either 4 or 5 weeks.  $18 \div 4$  is between 4 and 5.
  - c. There are 5 people in my family and that's 1,095 pounds.  $5 \times 219 = 1,095$
  - d. There are 24 people in our class and that's 5,256 pounds ( $24 \times 219$ ).
  - e. There are 24 classes in this school that each have about 24 students. So that's 126,144 pounds per year ( $24 \times 5,256$ ).
2. 3,504 ounces per year ( $16 \times 219$ ).

- "The average person in the United States throws out about 219 pounds of trash related to food waste each year. We are going to estimate the amount of food waste we produce based on this data."

### Activity

- 5 minutes: independent work time
- 5 minutes: partner discussion
- Monitor for students who:
  - use their result for the month to get their result for the week
  - divide 219 by 52 to get the weekly result
  - make reasoned estimates

### Synthesis

- Invite students to share their strategies for determining how much food waste they produce per week.
- "Were you surprised at the estimate of how much food waste everyone in your school produces in a year?"

## Activity 2

Food Journal

🕒 20 min

## Standards Alignments

Addressing 5.NBT.B.5, 5.NBT.B.6

The purpose of this activity is to introduce the food journal that students will use to track the food waste they or their families produce for a week. In this activity students practice filling out the journal and estimate the weight of what they throw out. To help students develop a sense for the weight of the food waste, consider bringing in a few common food items or images and display their weights in ounces. A few examples are provided below.

Banana: 4 ounces

Cucumber: 7 ounces

Chicken thigh: 2–3 ounces

Small lemon: 2–3 ounces

Carrot: 2 ounces

Medium apple: 6–7 ounces

### Access for Students with Disabilities

*Engagement: Develop Effort and Persistence.* Invite students to generate a list of shared expectations for group work. Record responses on a display and keep visible during the activity.

*Supports accessibility for: Organization, Attention*

## Student-facing Task Statement

Complete the table for the food waste you produced today. Be prepared to share your reasoning for the estimated weight.

name/ type	reason thrown away	estimated weight (ounces)

- About how many pounds of food waste have you produced so far today?
- Look back at the weekly estimated pounds of food waste based on the national average in the last activity. Do you think the estimate is more or less than what you actually

## Launch

- Groups of 2
- “Think about one thing related to the food that you threw out today. Share with your partner. Why did you throw it out? How much do you think it weighed?”
- 1 minute: partner discussion
- Display food journal.
- Record responses on the food journal.
- “Try your best to think about all the food waste you produced today to complete the table.”

## Activity

- 5 minutes: independent work time

produce in a week? Explain your reasoning.

## Student Responses

Sample response:

name/type	reason thrown away	estimated weight (ounces)
2 banana peels	peel is inedible	2
apple	half was bad	3
milk (quarter of cup)	didn't finish	2
grapes	rotten package	4

- 11 ounces is less than a pound, about  $\frac{3}{4}$  of a pound.
- By the end of the day I might produce a total of a pound, which is 7 pounds for the week. This is a lot more than the national average.

- 5 minutes: partner work time
- Monitor for students who:
  - divide the total number of ounces of food waste by 16 to determine the pounds of food waste
  - multiply appropriately to compare the estimated pounds of food waste from the previous activity

## Synthesis

- Invite a few students to share how they found the total pounds of food waste produced.
- Give each student a food waste journal.
- “For the next week, you will keep track of the food waste you produced. Record what you throw out, why you throw it out, and the estimated weight in ounces.”
- “Scientists collect this kind of information for cities and countries all over the world. Why is this information helpful to them?” (to see why people are throwing things out and see how to reduce it, to see how much food waste is going to landfills, to learn what is in food waste and see how that affects the environment)

## Activity 3 (optional)

🕒 20 min

Analyze Food Journals

### Standards Alignments

Addressing 5.NBT.B.5

The purpose of this activity is for students to analyze their completed food waste journals. Students begin the activity by sharing their food journal with a partner and discuss a few things

they notice. Then they use the data from the journal and their experience with multiplying and dividing large numbers in this unit to estimate how much food waste they might produce in a month and in a year. They compare those results to the national average and discuss some possible reasons for the difference.

### Student-facing Task Statement

Use your food waste journal to answer the questions. Be prepared to share.

1. How many pounds of food waste did you or your family produce in a week?
2. If you produce about the same food waste each week as recorded in the food waste journal, how much would you produce in a month? In a year?
3. The average person produces 219 pounds of food waste per year in the United States. With your partner, discuss reasons for the differences between your data and the national average.

### Student Responses

1. My family produced about 26 pounds of food waste during the week.
2. In a month, my family will produce 104 pounds of food waste ( $4 \times 26$ ). In a year my family will produce 1,352 pounds of food waste ( $52 \times 26$ ).
3. There are 4 people in my family. This means each of us produces about 338 pounds per year which is more than the national average of 219 pounds. It is possible that my estimates of the weights were more than the actual weights. Or my family buys a lot of fruits and vegetables and they sometimes go bad faster if we don't eat them in time.

### Launch

- Groups of 2
- "Share your food waste journal with a partner. As you look over your journals, what do you notice?"
- "What is the same? What is different?"

### Activity

- 10 minutes: independent work time
- 2 minutes: partner discussion

### Synthesis

- See lesson synthesis

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## Lesson Synthesis

🕒 10 min

“This week, we collected information about the food waste we produce. We used that information to make estimates about how much food waste we might produce over a whole month or a whole year.”

Invite a few students to share how much food waste they will produce in a year and some reasons for any difference between their figure and the national average.

“As you looked over your food waste journal, what information surprised you?”

“What other questions can we explore about food waste?” (Where does it all go? How can we reduce food waste? Is there a difference in the amount of food waste produced by vegetarians and those that eat meat? Does eating out or eating at home produce more food waste?)

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Family Support  
Materials

# Family Support Materials

## Wrapping Up Multiplication and Division with Multi-Digit Numbers

In this unit, students multiply and divide multi-digit whole numbers using place value understanding, properties of operations, and the relationship between multiplication and division. They use the standard algorithm to multiply multi-digit whole numbers and partial quotients algorithms to divide whole numbers up to four digits by two digits. They then apply these skills as they solve problems involving volume.

### Section A: Multi-digit Multiplication Using the Standard Algorithm

Students begin this unit by estimating products and quotients in a real-world context. Students use their understanding of place value, and their understanding of powers of 10 to make reasonable estimates. Students connect multiplication strategies, like partial products, to the standard multiplication algorithm. This is the partial products area diagram for  $412 \times 32$ .

	400	10	2
30	$30 \times 400$	$30 \times 10$	$30 \times 2$
2	$2 \times 400$	$2 \times 10$	$2 \times 2$

They find partial products using area diagrams, and then translate that to a series of equations. These equations are compared against the steps in the standard algorithm to learn how the steps are based on place value reasoning and why the algorithm works. This table shows the connection between an algorithm using partial products and the standard algorithm.

Partial Products Area Diagram

	400	10	2
30	$30 \times 400$	$30 \times 10$	$30 \times 2$
2	$2 \times 400$	$2 \times 10$	$2 \times 2$

Standard Algorithm

$$\begin{array}{r}
 412 \\
 \times 32 \\
 \hline
 824 \\
 + 12360 \\
 \hline
 13184
 \end{array}$$

Area Diagram Aligned to Standard Algorithm

	412
30	12,360
2	824

### Section B: Multi-digit Division Using Partial Quotients

Students begin the work on whole number division by deepening their understanding of division expressions and the effect that changing the divisor or dividend has on the value of the

quotient. In a progression that leads to students engaging in algorithms using partial quotients, students estimate quotients and write partial quotient equations that match their own methods for finding the value of the quotient. Once students understand that they can find the value of the quotient by decomposing the dividend into multiples of the divisor, students learn to express this decomposition using equations and then an algorithm using partial quotients.

#### Decomposition of the Dividend

$$448 \div 16 = (320 \div 16) + (80 \div 16) + (48 \div 16)$$

$$448 \div 16 = 20 + 5 + 3$$

$$448 \div 16 = 28$$

#### An Algorithm Using Partial Quotients

$$\begin{array}{r}
 \boxed{28} \\
 3 \\
 5 \\
 20 \\
 16 \overline{)448} \\
 \underline{-320} \quad (20 \times 16) \\
 128 \\
 \underline{-80} \quad (5 \times 16) \\
 48 \\
 \underline{-48} \quad (3 \times 16) \\
 0
 \end{array}$$

## Section C: Let's Put it to Work

Students practice their multiplication and division skills as they solve problems involving volume. Students are using the volume formulas ( $V = l \times w \times h$  and  $V = b \times h$ ) to practice the multiplication and division work of the previous sections. Students engage with relatively large numbers to multiply and divide using these volume formulas, developing fluency with the standard algorithm for multiplication and the algorithm using partial quotients.

### Try it at home!

Near the end of the unit, ask your student to solve the following problems:

- $219 \times 52$
- $868 \div 14$

Questions that may be helpful as they work:

- Can you draw a diagram to help you solve the problem?
- Can you explain the steps of your algorithm?

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# Unit Assessments

Check Your Readiness A, B and C  
End-of-Unit Assessment

# Wrapping Up Multiplication and Division with Multi-Digit Numbers: Section A Checkpoint

1. Use the standard algorithm to find the value of each product. Explain or show your reasoning.

a.  $628 \times 25$

b.  $359 \times 63$

2. a. Lin says the value of  $257 \times 63$  is about 1,500. Do you agree with Lin? Explain or show your reasoning.

b. Use the standard algorithm to find the value of the product.

$$\begin{array}{r} 257 \\ \times 63 \\ \hline \end{array}$$

# Wrapping Up Multiplication and Division with Multi-Digit Numbers: Section B Checkpoint

1. Find the value of  $966 \div 23$ . Explain or show your reasoning.

2. A toolbox is shaped like a rectangular prism. The length is 14 inches and the height is 7 inches. If the volume of the toolbox is 1,176 cubic inches, what is its width? Explain or show your reasoning.

# Wrapping Up Multiplication and Division with Multi-Digit Numbers: Section C Checkpoint

1. Kansas and South Dakota both have rectangular shapes. Kansas is 660 km long and 343 km wide. South Dakota is 610 km long and 340 km wide.

a. Is the area of Kansas greater than or less than 200,000 square kilometers? Explain or show your reasoning.

b. Explain why 200,000 square kilometers is a good estimate for the area of South Dakota.

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2. The back of a garbage truck is 23 feet long, 8 feet wide, and 12 feet tall. How many loads of trash from the garbage truck will it take to fill a 40 foot by 9 foot by 8 foot storage container? Explain or show your reasoning.

# Wrapping Up Multiplication and Division with Multi-Digit Numbers: End-of-Unit Assessment

1. Find the value of each product. Explain or show your reasoning.

a.

$$\begin{array}{r} 213 \\ \times 54 \\ \hline \end{array}$$

b.

$$\begin{array}{r} 375 \\ \times 47 \\ \hline \end{array}$$

2. The equation  $83 \times 27 = 2,241$  is true. Select **all** true equations.

A.  $2,241 \div 83 = 27$

B.  $27 \div 2,241 = 83$

C.  $83 \div 2,241 = 27$

D.  $2,241 \div 27 = 83$

E.  $27 \times 83 = 2,241$

3. Choose the number that is closest to the value of the expression  $8,745 \div 3$ .

A. 30

B. 300

C. 3,000

D. 30,000

4. Find the value of  $1,530 \div 34$ . Explain or show your reasoning.

5. The area of a rectangular yard is 5,063 square feet and its length is 61 feet. What is its width? Explain or show your reasoning.

6. The area of the base of a box is 825 square centimeters. It has a height of 26 centimeters.  
a. The box is 25 cm wide. How long is the box?

b. Explain why the volume of the box is less than 30,000 cubic centimeters.

---

---

c. Find the volume of the box in cubic centimeters. Explain or show your reasoning.

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# Assessment Answer Keys

Check Your Readiness A, B and C  
End-of-Unit Assessment



## Problem 2

**Goals Assessed**

- Fluently multiply multi-digit whole numbers using the standard algorithm.

- Lin says the value of  $257 \times 63$  is about 1,500. Do you agree with Lin? Explain or show your reasoning.
- Use the standard algorithm to find the value of the product.

$$\begin{array}{r} 257 \\ \times 63 \\ \hline \end{array}$$

## Solution

Sample responses:

- Lin is not correct.  $200 \times 10 = 2,000$  and this product has a much greater value than  $200 \times 10$ .
- 16,191

$$\begin{array}{r} 34 \\ 12 \\ 257 \\ \times 63 \\ \hline 1 \\ 771 \\ + 15,420 \\ \hline 16,191 \end{array}$$

## Assessment: Section B Checkpoint

### Problem 1

#### Goals Assessed

- Divide multi-digit whole numbers using strategies based on place value, properties of operations, and relationship between multiplication and division.

Find the value of  $966 \div 23$ . Explain or show your reasoning.

### Solution

42

Sample response:

$$\begin{array}{r}
 \boxed{42} \\
 2 \\
 20 \\
 20 \\
 23 \overline{)966} \\
 \underline{-460} \\
 506 \\
 \underline{-460} \\
 46 \\
 \underline{-46} \\
 0
 \end{array}$$

### Problem 2

#### Goals Assessed

- Divide multi-digit whole numbers using strategies based on place value, properties of operations, and relationship between multiplication and division.

A toolbox is shaped like a rectangular prism. The length is 14 inches and the height is 7 inches. If the volume of the toolbox is 1,176 cubic inches, what is its width? Explain or show your reasoning.

## Solution

12 inches

Sample response:

$$14 \times 7 = 98$$

$$\begin{array}{r} \boxed{12} \\ 2 \\ 10 \\ 98 \overline{)1,176} \\ \underline{-980} \\ 196 \\ \underline{-196} \\ 0 \end{array}$$

## Assessment: Section C Checkpoint

### Problem 1

#### Goals Assessed

- Multiply and divide to solve real-world and mathematical problems involving area and volume.

Kansas and South Dakota both have rectangular shapes. Kansas is 660 km long and 343 km wide. South Dakota is 610 km long and 340 km wide.

- Is the area of Kansas greater than or less than 200,000 square kilometers? Explain or show your reasoning.
- Explain why 200,000 square kilometers is a good estimate for the area of South Dakota.

### Solution

Sample responses:

- Greater. I know  $600 \times 300 = 180,000$  and then  $600 \times 40$  is more than 20,000 so that's already more than 200,000.
- South Dakota is a little more than 600 km long and 300 km wide. I know  $3 \times 600 = 1,800$  and then 100 times more is 180,000 so 200,000 is a good estimate.

### Problem 2

#### Goals Assessed

- Multiply and divide to solve real-world and mathematical problems involving area and volume.

The back of a garbage truck is 23 feet long, 8 feet wide, and 12 feet tall. How many loads of trash from the garbage truck will it take to fill a 40 foot by 9 foot by 8 foot storage container? Explain or show your reasoning.

### Solution

2

Sample response: Each set of side lengths has an 8 so I can compare  $23 \times 12$  to  $40 \times 9$ . I know  $23 \times 12 = 276$  and  $40 \times 9 = 360$ . So one load of trash is not enough but two loads are.

## Assessment: End-of-Unit Assessment

### Problem 1

#### Standards Alignments

Addressing 5.NBT.B.5

#### Narrative

Students multiply a 3-digit number and a 2-digit number using a method of their choice. The numbers are arranged for the standard algorithm but students could choose to use the partial products algorithm or a different method.

Find the value of each product. Explain or show your reasoning.

a. 
$$\begin{array}{r} 213 \\ \times 54 \\ \hline \end{array}$$

b. 
$$\begin{array}{r} 375 \\ \times 47 \\ \hline \end{array}$$

### Solution

a. Sample response:

$$\begin{array}{r} 213 \\ \times 54 \\ \hline 11 \\ 10,000 \\ 500 \\ 150 \\ 800 \\ 40 \\ + 12 \\ \hline 11,502 \end{array}$$

b. Sample response:

$$\begin{array}{r}
 32 \\
 53 \\
 375 \\
 \times 47 \\
 \hline
 2,625 \\
 + 15,000 \\
 \hline
 17,625
 \end{array}$$

## Problem 2

**Standards Alignments**

Addressing 5.NBT.B.6

**Narrative**

Students select equations that represent different ways of expressing the value of a product. Since multiplication is commutative, the order of the factors can be reversed. Each multiplication equation is also equivalent to two division equations. Students who select B or C do not understand the meaning of division as the value of each of these expressions is less than 1. The relationship between multiplication and division is essential for all the different ways students have learned to find whole number quotients.

The equation  $83 \times 27 = 2,241$  is true. Select **all** true equations.

- A.  $2,241 \div 83 = 27$
- B.  $27 \div 2,241 = 83$
- C.  $83 \div 2,241 = 27$
- D.  $2,241 \div 27 = 83$
- E.  $27 \times 83 = 2,241$

**Solution**

["A", "D", "E"]

## Problem 3

**Standards Alignments**

Addressing 5.NBT.A.1, 5.NBT.B.6

**Narrative**

Students estimate the value of a quotient. Because the answers differ by powers of 10, students can answer the question by noticing that 8,745 is a little less than 3,000 threes. Students may select answer D if they multiply 8,745 by 3 rather than divide. If students answer A or B then they need more work with estimation, general number sense, and place value.

Choose the number that is closest to the value of the expression  $8,745 \div 3$ .

- A. 30
- B. 300
- C. 3,000
- D. 30,000

## Solution

C

## Problem 4

**Standards Alignments**

Addressing 5.NBT.B.6

**Narrative**

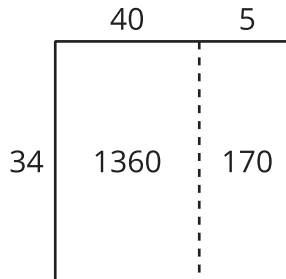
Students find a quotient of a four-digit number by a two-digit number using a method of their choice. Many options are available, including:

- a diagram
- partial quotients
- multiplication

Find the value of  $1,530 \div 34$ . Explain or show your reasoning.

## Solution

Sample response 1:



Sample response 2:

$$\begin{array}{r}
 \boxed{45} \\
 5 \\
 40 \\
 34 \overline{)1530} \\
 \underline{-1360} \\
 170 \\
 \underline{-170} \\
 0
 \end{array}$$

Sample response 3:

$$\begin{aligned}
 30 \times 34 &= 1020 \\
 10 \times 34 &= 340 \\
 5 \times 34 &= 170 \\
 1020 + 340 + 170 &= 1530 \\
 30 + 10 + 5 &= 45
 \end{aligned}$$

## Problem 5

**Standards Alignments**

Addressing 5.NBT.B.6

**Narrative**

Students find a quotient of a four-digit number by a two-digit number with a context using a method of their choice. As for the previous non-contextual division problem, the same methods are available, including:

- a diagram
- partial quotients
- partial products of 61

The area of a rectangular yard is 5,063 square feet and its length is 61 feet. What is its width?  
Explain or show your reasoning.

### Solution

83 feet. Sample response:

$$\begin{array}{r}
 \boxed{83} \\
 3 \\
 30 \\
 50 \\
 61 \overline{)5,063} \\
 \underline{-3,050} \\
 2,013 \\
 \underline{-1,830} \\
 183 \\
 \underline{-183} \\
 0
 \end{array}$$

### Problem 6

#### Standards Alignments

Addressing 5.MD.C.5, 5.NBT.B.5

#### Narrative

Students perform multiplication and division with an area and volume context. The quotient is a 3-digit number divided by a 2-digit number with friendly numbers. Students may use an algorithm or they may use multiplication to find the quotient. The rest of the problem does not depend on the value they get for the width unless they were to multiply length, width, and height to find the volume and perform the multiplication incorrectly. Students also explain an overestimate of the volume. This serves as a way of checking the reasonableness of their answer for the volume.

The area of the base of a box is 825 square centimeters. It has a height of 26 centimeters.

- The box is 25 cm wide. How long is the box?
- Explain why the volume of the box is less than 30,000 cubic centimeters.
- Find the volume of the box in cubic centimeters. Explain or show your reasoning.



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Lesson  
Cool Downs

# Lesson 1: Estimate and Find Products

## Cool Down: Fifteen

Find the value of each expression. Explain or show your reasoning.

1.  $15 \times 20$

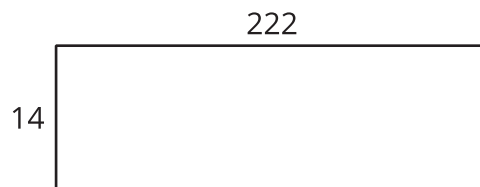
2.  $15 \times 120$

3.  $15 \times 121$

## Lesson 2: Partial Products in Diagrams

### Cool Down: $222 \times 14$

Here is a diagram that represents  $222 \times 14$ .



Find the value of  $222 \times 14$ . Use the diagram if it is helpful. Explain or show your reasoning.

# Lesson 3: Partial Products in Algorithms

## Cool Down: Using Partial Products

Find the value of  $415 \times 43$  using partial products.

# Lesson 4: Standard Algorithm: One-digit and Multi-digit Numbers with Composing

## Cool Down: Standard Algorithm Calculation

Use the standard algorithm to find the value of  $3,514 \times 7$ .

# **Lesson 5: Standard Algorithm: Multi-digit Numbers without Composing**

## **Cool Down: Standard Algorithm without Composition of a New Unit**

Use the standard algorithm to find the value of  $203 \times 23$ .

# Lesson 6: Standard Algorithm: Multi-digit Numbers with Composing

## Cool Down: Use the Standard Algorithm

Use the standard algorithm to find the product  $251 \times 34$ .

# Lesson 7: Build Multiplication Fluency

## Cool Down: Calculating a Product

Use the standard algorithm to find the product  $372 \times 83$ .

# Lesson 8: Multiplication Fluency

## Cool Down: Multiplication Reflection

Describe something you really understand well from this section on multiplying multi-digit numbers, or describe something that was confusing or challenging.

# Lesson 9: The Birds

## Cool Down: On Screech

To make a birdhouse for a screech owl, the recommended area of the floor is 8 inches by 8 inches and the recommended height is 12 inches to 15 inches. What is the recommended range of volumes for a screech owl birdhouse? Explain or show your thinking.

# Lesson 10: World's Record Folk Dance

## Cool Down: Another Dance

A different group of 4,632 dancers make groups of 8.

1. Write a division expression to represent the situation.
2. How many groups of 8 will there be? Explain or show your thinking.

# Lesson 11: Different Partial Quotients

## Cool Down: Find the Value

Find the value of  $465 \div 15$ . Explain or show your reasoning.

# Lesson 12: An Algorithm Using Partial Quotients

## Cool Down: What's Next?

Han started finding the value of a quotient.

$$\begin{array}{r} 300 \\ 15 \overline{) 5,400} \\ \underline{-4,500} \quad (300 \times 15) \\ 900 \end{array}$$

1. Write the division expression that represents the quotient Han is finding.
2. Complete the algorithm that Han started.

# Lesson 13: Divide Using Partial Quotients

## Cool Down: Divide Using Partial Quotients

Find the value of the quotient.

$$27 \overline{)405}$$

# Lesson 14: Practice an Algorithm Using Partial Quotients

## Cool Down: Divide Four-digit Numbers

Find the value of  $1,736 \div 28$ .

$$28 \overline{) 1,736}$$

# Lesson 15: Find Missing Side Lengths

## Cool Down: The Area of the Garden

The area of a rectangular garden is 832 square feet and its length is 16 feet. What is its width?

# Lesson 16: World's Record Noodle Soup

## Cool Down: Division Reflection

Describe something you really understand well from this section on dividing multi-digit numbers, or describe something that was confusing or challenging.

# Lesson 17: Fractions as Partial Quotients

## Cool Down: Choose One Expression

Choose one expression and use it to find the value of  $154 \div 14$ . Explain or show your thinking.

$$154 \div 14 = \underline{\hspace{2cm}}$$

- $\frac{140}{14} + \frac{14}{14}$
- $\frac{150}{14} + \frac{4}{14}$
- $\frac{70}{14} + \frac{70}{14} + \frac{14}{14}$

# Lesson 18: Lots of Milk

## Cool Down: So Much Milk

There are 17,566 students in a city who drink a carton of milk at lunch one day. Each carton of milk is about 20 cubic inches. About how many cubic inches of milk do the students drink altogether? Explain or show your reasoning.

# Lesson 19: Trash Talk

## Cool Down: Wyoming

Wyoming is 600 km wide and 452 km long. What is the area of Wyoming?

# Lesson 20: Shipping Trash

## Cool Down: Ship It

1. A different shipping container is 40 feet long, 9 feet wide, and 8 feet tall.
  - a. What is the volume of this container? Explain or show your thinking.

- b. A school makes 24 cubic feet of recyclable plastic each day. How many days does it take the school to fill this container? Explain or show your thinking.

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# Instructional Masters

# Instructional Masters for Wrapping Up Multiplication and Division with Multi-Digit Numbers

address	title	students per copy	written on?	requires cutting?	card stock recommended?	color paper recommended?
Activity Grade5.4.7.1	Greatest Product	1	yes	no	no	no
Activity Grade5.4.3.1	Partial Product Expressions	2	no	yes	no	no
Activity Grade5.4.1.1.1	Partial Quotient Expressions	2	no	yes	no	no
Activity Grade5.4.7.1	Number Cards (0-10)	2	no	yes	yes	no
Center	Number Puzzles Mult Stage 1 Recording Sheet	1	yes	no	no	no
Center	Number Puzzles Mult Stage 2 Recording Sheet	1	yes	no	no	no
Center	Watch Your Remainder Stage 1 Recording Sheet	1	yes	no	no	no
Center	Watch Your Remainder Stage 1 Spinner	2	no	no	no	no
Center	Mystery Number Stage 6 Gameboard	2	no	no	no	no

## Greatest Product

Directions:

- Partner A chooses a number card and writes the number in one of the blanks for Round 1.
- Partner B does the same.
- Repeat until each partner has a two-digit by three-digit multiplication problem.
- Find the product.
- The partner with the greater product wins a point.
- The partner with the most points after 5 rounds wins the game.

round		workspace	points
1	$\begin{array}{r} \square\square\square \\ \times \quad \square\square \\ \hline \end{array}$		
2	$\begin{array}{r} \square\square\square \\ \times \quad \square\square \\ \hline \end{array}$		
3	$\begin{array}{r} \square\square\square \\ \times \quad \square\square \\ \hline \end{array}$		
4	$\begin{array}{r} \square\square\square \\ \times \quad \square\square \\ \hline \end{array}$		
5	$\begin{array}{r} \square\square\square \\ \times \quad \square\square \\ \hline \end{array}$		

# Partial Product Expressions

Partial Product Expressions $40 \times 30$	Partial Product Expressions $5 \times 30$	Partial Product Expressions $200 \times 5$	Partial Product Expressions $40 \times 5$
Partial Product Expressions $5 \times 5$	Partial Product Expressions $200 \times 35$	Partial Product Expressions $40 \times 35$	Partial Product Expressions $5 \times 35$
Partial Product Expressions $30 \times 245$	Partial Product Expressions $5 \times 245$	Partial Product Expressions $240 \times 30$	Partial Product Expressions $5 \times 35$
Partial Product Expressions $240 \times 5$	Partial Product Expressions $205 \times 5$	Partial Product Expressions $205 \times 30$	Partial Product Expressions $5 \times 40$

## Partial Product Expressions

Partial Product Expressions $30 \times 45$	Partial Product Expressions $200 \times 35$	Partial Product Expressions $40 \times 30$	Partial Product Expressions $40 \times 5$
Partial Product Expressions $5 \times 35$	Partial Product Expressions $30 \times 40$	Partial Product Expressions $5 \times 45$	Partial Product Expressions $5 \times 5$
Partial Product Expressions $5 \times 200$	Partial Product Expressions $30 \times 200$	Partial Product Expressions $200 \times 30$	Partial Product Expressions $30 \times 5$

## Partial Quotient Expressions

Partial Quotient Expressions

$$100 \div 14$$

Partial Quotient Expressions

$$200 \div 14$$

Partial Quotient Expressions

$$188 \div 14$$

Partial Quotient Expressions

$$140 \div 14$$

Partial Quotient Expressions

$$28 \div 14$$

Partial Quotient Expressions

$$70 \div 14$$

Partial Quotient Expressions

$$8 \div 14$$

Partial Quotient Expressions

$$70 \div 14$$

Partial Quotient Expressions

$$140 \div 14$$

Partial Quotient Expressions

$$8 \div 14$$

Partial Quotient Expressions

$$300 \div 14$$

Partial Quotient Expressions

$$108 \div 14$$

Partial Quotient Expressions

$$70 \div 14$$

Partial Quotient Expressions

$$120 \div 14$$

Partial Quotient Expressions

$$140 \div 14$$

Partial Quotient Expressions

$$14 \div 14$$

## Partial Quotient Expressions

Partial Quotient Expressions

$$20 \div 14$$

Partial Quotient Expressions

$$70 \div 14$$

Partial Quotient Expressions

$$300 \div 14$$

Partial Quotient Expressions

$$288 \div 14$$

Partial Quotient Expressions

$$100 \div 14$$

Partial Quotient Expressions

$$28 \div 14$$

Partial Quotient Expressions

$$280 \div 14$$

Partial Quotient Expressions

$$28 \div 14$$

Partial Quotient Expressions

$$300 \div 14$$

Partial Quotient Expressions

$$154 \div 14$$

Partial Quotient Expressions

$$100 \div 14$$

Partial Quotient Expressions

$$188 \div 14$$

Partial Quotient Expressions

$$28 \div 14$$

Partial Quotient Expressions

$$28 \div 14$$

Partial Quotient Expressions

$$28 \div 14$$

Partial Quotient Expressions

$$154 \div 14$$

Number Cards (0-10)

1

2

3

4

5

6

Number Cards (0-10)

7

8

9

1

2

3

Number Cards (0-10)

4

5

6

7

8

9

Number Cards (0-10)

0

0

10

10

**Puzzle 1**

Fill in digits to make each equation true.  
You may only use each digit (0-9) once.

$$\boxed{1} \boxed{\phantom{0}} \boxed{\phantom{0}} \times \boxed{\phantom{0}} \boxed{\phantom{0}} \boxed{\phantom{0}} = 230$$

$$\boxed{\phantom{0}} \boxed{\phantom{0}} \times \boxed{2} \boxed{5} = 425$$

$$\boxed{\phantom{0}} \boxed{0} \times 31 = 1,550$$

$$\boxed{\phantom{0}} \boxed{0} \times \boxed{\phantom{0}} \boxed{\phantom{0}} = 2,400$$

$$\boxed{1} \boxed{\phantom{0}} \times \boxed{2} \boxed{\phantom{0}} = 522$$

**Puzzle 2**

Fill in digits to make each equation true.  
You may only use each digit (0-9) once.

$$11 \times \square \square 2 = \square 3 \square \square 2$$

$$4 \square \square \times 20 = \square 9 \square 2 \square$$

$$\square \square \square \times 25 = 675$$

$$10 \times \square \square \square = 890$$

$$\square \square 1 \times 1 \square \square = 154$$

**Puzzle 3**

Fill in digits to make each equation true.  
You may only use each digit (0-9) once.

$$\square \square 1 \times \square 1 \square = 1,349$$

$$\square \square \square \times 30 = 1,800$$

$$\square \square 5 \times \square \square 1 = 775$$

$$4 \square \square \times \square 3 \square = 1,395$$

$$3 \square \square \times 23 = \square 8 \square 7 \square$$

**Puzzle 4**

Fill in digits to make each equation true.  
You may only use each digit (0-9) once.

$$\square\square 1 \times \square 1 \square = 610$$

$$\square\square \square \times 41 = 3,239$$

$$\square\square 7 \times \square\square 4 = 1,428$$

$$5\square\square \times \square 1 \square = 795$$

$$1\square\square \times 47 = \square 5 \square 6 \square$$

**Puzzle 1**

Fill in digits to make each equation true.

You may only use each digit (0-9) once.

$$19 \times \boxed{3} \boxed{\phantom{0}} \boxed{\phantom{0}} \boxed{\phantom{0}} = 6,802$$

$$\boxed{\phantom{0}} \boxed{1} \times \boxed{1} \boxed{\phantom{0}} \boxed{\phantom{0}} \boxed{0} = 11,830$$

$$4 \boxed{\phantom{0}} \boxed{\phantom{0}} \times \boxed{1} \boxed{5} \boxed{\phantom{0}} = 6,240$$

$$\boxed{\phantom{0}} \boxed{0} \boxed{1} \times \boxed{\phantom{0}} \boxed{\phantom{0}} \boxed{1} = 8,421$$

$$\boxed{\phantom{0}} \boxed{2} \boxed{7} \times \boxed{1} \boxed{2} \boxed{\phantom{0}} = 16,129$$

**Puzzle 2**

Fill in digits to make each equation true.  
You may only use each digit (0-9) once.

$$15 \times \boxed{2} \boxed{\phantom{0}} \boxed{\phantom{0}} \boxed{\phantom{0}} = 3,510$$

$$\boxed{\phantom{0}} \boxed{1} \times \boxed{1} \boxed{\phantom{0}} \boxed{\phantom{0}} \boxed{0} = 10,650$$

$$\boxed{7} \boxed{\phantom{0}} \boxed{\phantom{0}} \times \boxed{1} \boxed{1} \boxed{\phantom{0}} \boxed{\phantom{0}} = 8,330$$

$$\boxed{\phantom{0}} \boxed{3} \boxed{5} \times \boxed{\phantom{0}} \boxed{\phantom{0}} \boxed{1} = 19,035$$

$$\boxed{\phantom{0}} \boxed{5} \boxed{2} \times \boxed{2} \boxed{4} \boxed{\phantom{0}} \boxed{\phantom{0}} = 37,392$$

**Puzzle 3**

Fill in digits to make each equation true.

You may only use each digit (0-9) once.

$$52 \times \boxed{3} \boxed{\phantom{0}} \boxed{\phantom{0}} \boxed{\phantom{0}} = 17,212$$

$$\boxed{\phantom{0}} \boxed{1} \times \boxed{1} \boxed{\phantom{0}} \boxed{\phantom{0}} \boxed{0} = 3,990$$

$$4 \boxed{\phantom{0}} \boxed{\phantom{0}} \times \boxed{5} \boxed{2} \boxed{\phantom{0}} \boxed{\phantom{0}} = 23,144$$

$$\boxed{\phantom{0}} \boxed{2} \boxed{5} \times \boxed{\phantom{0}} \boxed{\phantom{0}} \boxed{1} = 37,275$$

$$\boxed{\phantom{0}} \boxed{1} \boxed{1} \times \boxed{3} \boxed{2} \boxed{\phantom{0}} = 259,520$$

**Puzzle 4**

Fill in digits to make each equation true.

You may only use each digit (0-9) once.

$$12 \times \boxed{3} \boxed{\phantom{0}} \boxed{\phantom{0}} \boxed{\phantom{0}} = 4,548$$

$$\boxed{\phantom{0}} \boxed{1} \times \boxed{1} \boxed{\phantom{0}} \boxed{\phantom{0}} \boxed{0} = 7,380$$

$$\boxed{2} \boxed{\phantom{0}} \times \boxed{4} \boxed{9} \boxed{\phantom{0}} = 12,250$$

$$\boxed{\phantom{0}} \boxed{7} \boxed{4} \times \boxed{\phantom{0}} \boxed{5} = 9,590$$

$$\boxed{\phantom{0}} \boxed{5} \boxed{1} \times \boxed{2} \boxed{6} \boxed{\phantom{0}} = 169,911$$

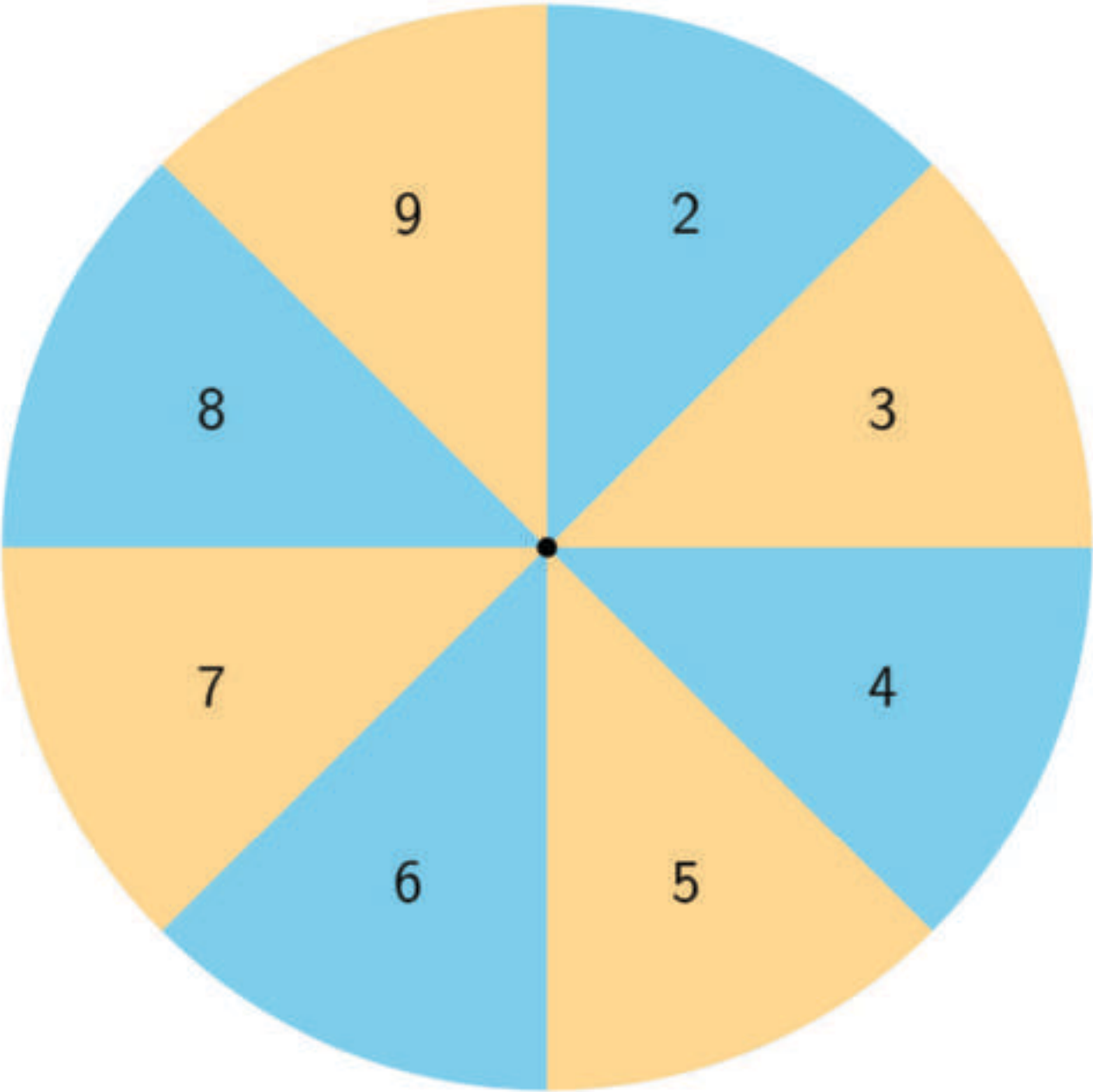
## Watch Your Remainder Stage 1 Recording Sheet

### Directions:

- Spin the spinner to get your one-digit divisor.
- Each partner:
  - Pick cards and use 3–4 of them to create a dividend.
  - Write a multiplication expression to represent the quotient. (For example, for  $109 \div 9$  you would write  $(9 \times 12) + 1$  and your score would be 1.)
  - Check your partner's work to make sure you agree.
  - Your score for the round is the remainder when you divide.
- Take new cards so that you have 4 cards to start the next round.
- The partner who has the fewest points once the recording sheet is full wins the game.

round	expression	points
1		
2		
3		
4		
5		
6		

Watch Your Remainder Stage 1 Spinner



## Mystery Number Stage 6 Gameboard

### Directions:

- Partner A:
  - Pick a number on the game board. Don't tell your partner!
  - Give your partner a clue about your mystery number. You can use the vocabulary below to help you give clues, or make up your own.
- Partner B:
  - Guess your partner's mystery number.
- If Partner B guesses the mystery number, switch roles.
- If Partner B does not guess the mystery number, Partner A gives another clue. Go back and forth guessing the number and giving clues until Partner B guesses the mystery number.

### Vocabulary:

thousandths, hundredths, tenths, ones, tens, hundreds, thousands, ten-thousands, hundred-thousands, greater than, less than, between, 10 times as much,  $\frac{1}{10}$  as much

5,055.05	2,384.901	329.192	8,788.30
7,404.361	263,059.1	14,874.58	2,139.825
38,513.92	262,987.5	6,535.803	784,936.6
49,281.74	919,675.3	60,146.09	76,350.68
31,452.34	1,345.03	591,230.3	35,621.72
892.508	47,592.98	137.04	98,670.88
9,067.213	72,540.11	3,517.27	154,239.6
754.402	66,293.41	3,762.893	82,415.86
123,456.4	1,938.823	58,678.99	1,109.222
83,751.72	40,820.6	9,999.98	6,537.28

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