



Core Knowledge[®] MATHEMATICS

Fractions as Quotients and Fraction Multiplication



Teacher Guide



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Fractions as Quotients and Fraction Multiplication

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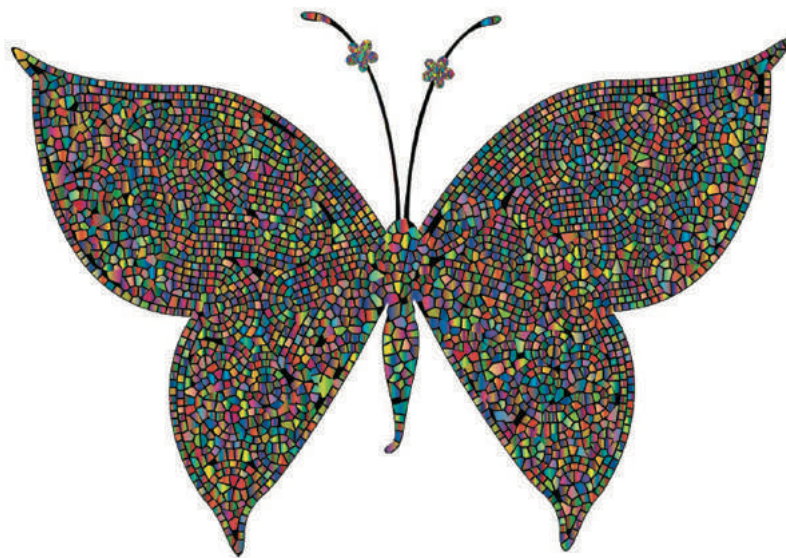
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Fractions as Quotients and Fraction Multiplication

Teacher Guide

Core Knowledge Mathematics™

Unit 2: Fractions as Quotients and Fraction Multiplication

At a Glance

Unit 2 is estimated to be completed in 17-19 days including 2 days for assessment.

This unit is divided into three sections including 15 lessons and 2 optional lessons.

- Section A—Fractions as Quotients (Lessons 1-5)
- Section B—Fractions of Whole Numbers (Lessons 6-8)
- Section C—Area and Fractional Side Lengths (Lessons 9-17)

On pages 10-11 of this Teacher Guide is a chart that identifies the section each lesson belongs in and the materials needed for each lesson.

This unit uses six new student centers.

- Rolling for Fractions
- Compare
- Target Measurements
- How Close?
- Rectangle Rumble
- Can You Build It

Unit 2: Fractions as Quotients and Fraction Multiplication

Unit Learning Goals

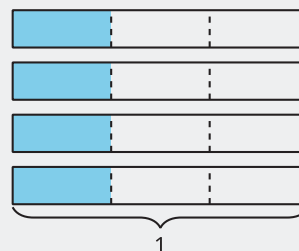
- Students develop an understanding of fractions as the division of the numerator by the denominator, that is $a \div b = \frac{a}{b}$, and solve problems that involve the multiplication of a whole number and a fraction, including fractions greater than 1.

In this unit, students learn to interpret a fraction as a quotient and extend their understanding of multiplication of a whole number and a fraction.

In grade 3, students made sense of multiplication and division of whole numbers in terms of equal-size groups. In grade 4, they used multiplication to represent equal-size groups with a fractional amount in each group and to express comparison.

For instance, $4 \times \frac{1}{3}$ can represent “4 groups of $\frac{1}{3}$ ” or “4 times as much as $\frac{1}{3}$.”

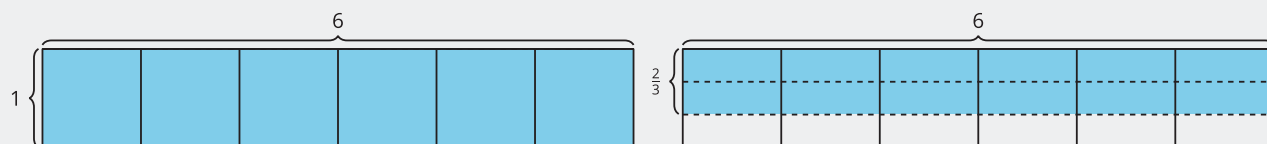
The amount in both situations can be represented by the shaded parts of a diagram like this:



Here, students learn that a fraction like $\frac{4}{3}$ can also represent:

- a division situation, where 4 objects are being shared by 3 people, or $4 \div 3$
- a fraction of a group, in this case, $\frac{1}{3}$ of a group of 4 objects, or $\frac{1}{3} \times 4$

Students also interpret the product of a whole number and a fraction in terms of the side lengths of a rectangle. The expression 6×1 represents the area of a rectangle that is 6 units by 1 unit. In the same way, $6 \times \frac{2}{3}$ represents one that is 6 units by $\frac{2}{3}$ unit.



The commutative and associative properties become evident as students connect different expressions to the same diagram. The distributive property comes into play as students multiply a whole number and a fraction written as a mixed number, for instance: $2 \times 3\frac{2}{5} = (2 \times 3) + (2 \times \frac{2}{5})$.

Throughout this unit, it is assumed that the sharing is always equal sharing, whether explicitly stated or not. For example, in the situation above, 4 objects are being shared equally by 3 people.

Section A: Fractions as Quotients

Standards Alignments

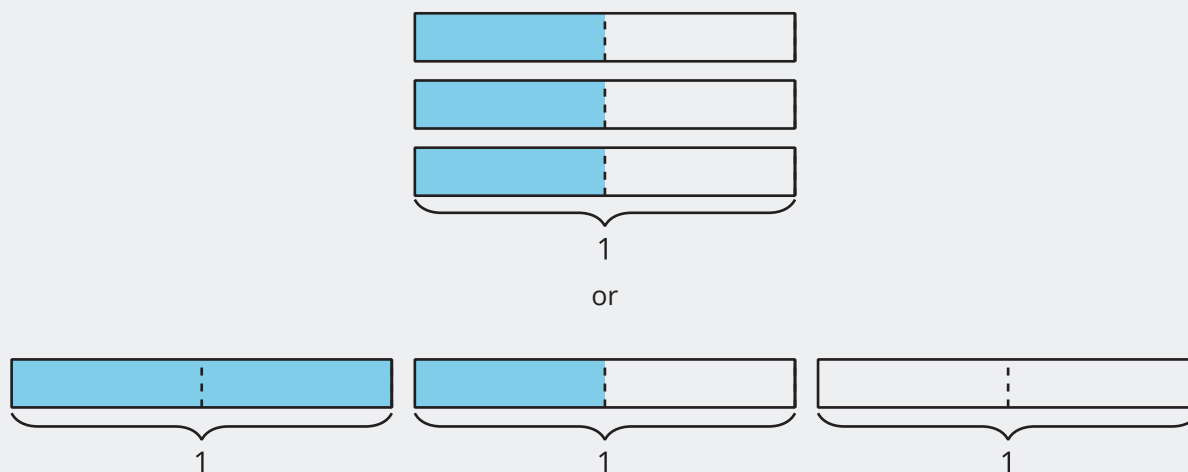
Building On	3.NF.A.1, 3.OA.A.2
Addressing	5.NF.B.3
Building Towards	5.NF.B.3

Section Learning Goals

- Represent and explain the relationship between division and fractions.
- Solve problems involving division of whole numbers leading to answers that are fractions.

In this section, students learn to see a fraction as a quotient, a result of dividing the numerator by the denominator. They solve a sequence of problems about situations that involve sharing a whole number of objects equally. Through repeated reasoning, they notice regularity in the result of division (MP8) and generalize that $\frac{a}{b} = a \div b$.

For example, 3 objects being shared equally by 2 people can be represented by the expression $3 \div 2$ and by a diagram. Each person's share can be shown by the shaded parts in a diagram such as:



Each person would get half of the 3 objects, or 3 groups of $\frac{1}{2}$ an object. The value of this expression is $\frac{3}{2}$ or $1\frac{1}{2}$.

 PLC: Lesson 3, Activity 2, Interpret Expressions

Suggested Centers

- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Supporting)
- Compare (1–5), Stage 4: Divide within 100 (Supporting)
- Rolling for Fractions (3–5), Stage 3: Divide Whole Numbers (Addressing)
- Target Measurements (2–5), Stage 4: Degrees (Supporting)

Section B: Fractions of Whole Numbers

Standards Alignments

Building On	4.NF.B.4
Addressing	5.NF.B, 5.NF.B.3, 5.NF.B.4, 5.NF.B.4.a, 5.OA.A.2
Building Towards	5.NF.B.4

Section Learning Goals

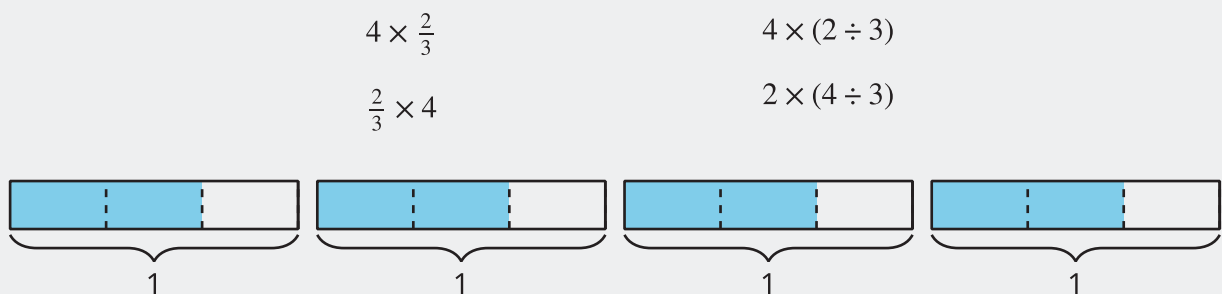
- Connect division to multiplication of a whole number by a non-unit fraction.
- Connect division to multiplication of a whole number by a unit fraction.
- Explore the relationship between multiplication and division.

In grade 4, students saw that a non-unit fraction can be expressed as a product of a whole number and a unit fraction, or a whole number and a non-unit fraction with the same denominator. For instance, $\frac{8}{3}$ can be expressed as $8 \times \frac{1}{3}$, as $4 \times \frac{2}{3}$, or as $2 \times \frac{4}{3}$. In the previous section, students interpreted a fraction like $\frac{8}{3}$ as a quotient: $8 \div 3$.

This section allows students to connect these two interpretations of $\frac{8}{3}$ and relate $8 \times \frac{1}{3}$ and $8 \div 3$.

Students use diagrams and contexts to make sense of division situations that result in a fractional quotient. As they interpret and write expressions that represent the quantities, students observe the commutative property of multiplication. For example, they interpret $8 \times \frac{1}{3}$ and $\frac{1}{3} \times 8$ as 8 groups of a third and a third of 8, respectively, and recognize that both are equal to $\frac{8}{3}$.

These understandings then help students make sense of other multiplication and division expressions that can be represented by the same diagram and have the same value:



PLC: Lesson 7, Activity 1, How Far Did They Run?

Suggested Centers

- Target Measurements (2–5), Stage 4: Degrees (Supporting)
- Compare (1–5), Stage 5: Fractions (Supporting)
- Target Measurements (2–5), Stage 5: Fractions of Angles (Addressing)

Section C: Area and Fractional Side Lengths

Standards Alignments

Addressing 5.NF.B, 5.NF.B.3, 5.NF.B.4, 5.NF.B.4.a, 5.NF.B.4.b, 5.OA.A, 5.OA.A.1

Building Towards 5.NF.B.4

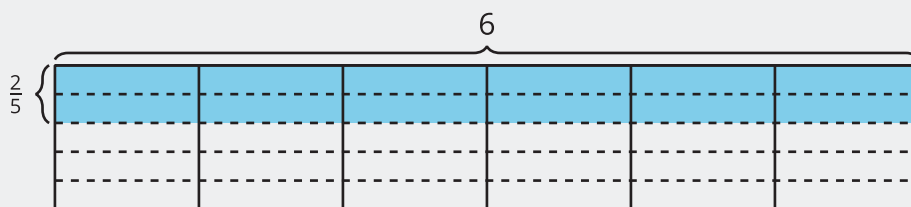
Section Learning Goals

- Find the area of a rectangle when one side length is a whole number and the other side length is a fraction or mixed number.
- Represent and solve problems involving the multiplication of a whole number by a fraction or mixed number.
- Write, interpret and evaluate numerical expressions that represent multiplication of a whole number by a fraction or mixed number.

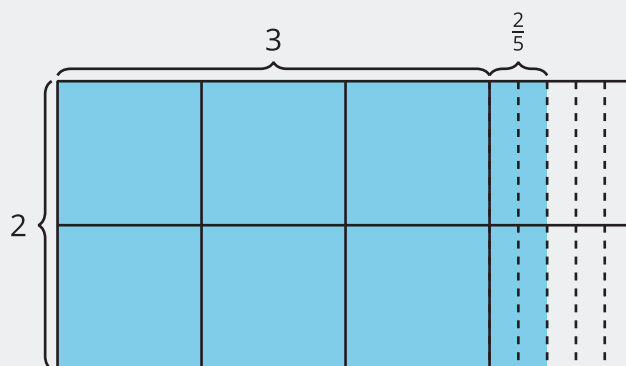
In this section, students learn that they can reason about the area of a rectangle with a fractional side length the same way they had with rectangles with whole-number side lengths: using diagrams and multiplication.

To find the area of such rectangles, students work through a progression of fractional side lengths: a unit fraction ($\frac{1}{3}$), a non-unit fraction ($\frac{2}{3}$), a fraction greater than 1 ($\frac{5}{3}$), and a mixed number ($1\frac{2}{3}$). They write and interpret multiplication expressions, such as $6 \times \frac{1}{3}$ and $6 \times \frac{5}{3}$, that represent the area of such rectangles. Students use shaded diagrams and their understanding of fractions to reason about the value of the expressions.

Along the way, the associative property of multiplication becomes evident. For instance, students see that the expressions $6 \times \frac{2}{5}$, $6 \times 2 \times \frac{1}{5}$, and $12 \times \frac{1}{5}$ can all describe the area of the shaded region in this diagram.



The distributive property is illustrated as students reason about the area of a rectangle where the side lengths are a whole number and a mixed number. To find $2 \times 3\frac{2}{5}$, for example, students may decompose the rectangle by grouping the whole-number units and the fractional units and multiply them separately before combining them, resulting in an expression such as $(2 \times 3) + (2 \times \frac{2}{5})$.



PLC: Lesson 11, Activity 1, Greater Than One

Suggested Centers

- Target Measurements (2–5), Stage 5: Fractions of Angles (Addressing)
- How Close? (1–5), Stage 6: Multiply to 3,000 (Supporting)
- How Close? (1–5), Stage 7: Multiply Fractions and Whole Numbers to 5 (Addressing)
- Rectangle Rumble (3–5), Stage 3: Factors 1–10 (Supporting)
- Rectangle Rumble (3–5), Stage 4: Whole Number and Fraction Factors (Addressing)
- Can You Build It? (3–5), Stage 2: Multiple Rectangles (Supporting)

Throughout the Unit

The warm-up activities support the development of concepts in the unit. The Number Talks enable students to revisit the distributive property with whole numbers, in preparation for multiplying of a whole number and a mixed number. The True or False warm-ups allow students to recognize equivalent expressions and explore the relationship between multiplication and division.

Here is a sampling of Number Talk warm-ups in the unit.

lesson 4	lesson 6	lesson 12	lesson 13	lesson 14	lesson 15
$35 \div 7$	$3 \times \frac{1}{2}$	3×20	$5 \times (7 + 4)$	$3 \times (10 \div 2)$	$6 \times \frac{3}{8}$
$1 \div 7$	$3 \times \frac{2}{2}$	3×24	$(5 \times 7) + (5 \times 4)$	$\frac{3}{2} \times 10$	$6 \times 2\frac{3}{8}$
$36 \div 7$	$3 \times \frac{3}{2}$	5×2	$(5 \times 7) + (5 \times \frac{1}{4})$	$(\frac{14}{7}) \times 10$	$7 \times \frac{9}{10}$
$37 \div 7$	$5 \times \frac{3}{2}$	$5 \times 2\frac{1}{2}$	$(5 \times 7) - (5 \times \frac{1}{4})$	$14 \times (\frac{10}{7})$	$7 \times 3\frac{9}{10}$

Here is a sampling of True or False warm-ups in the unit.

lesson 5	lesson 8	lesson 11
$5 \div 2 = \frac{5}{2}$	$2 \times (\frac{1}{3} \times 6) = \frac{2}{3} \times 6$	$10 \div 3 = 10 \times \frac{1}{3}$
$\frac{5}{2} = 5\frac{1}{2}$	$2 \times (\frac{1}{3} \times 6) = 2 \times (6 \div 3)$	$10 \div 3 = 10\frac{1}{3}$
$\frac{6}{2} = 3$	$\frac{2}{3} \times 6 = 2 \times (\frac{1}{4} \times 6)$	$\frac{10}{3} = 5 \times \frac{2}{3}$

Materials Needed

LESSON	GATHER	COPY
A.1	• none	• none
A.2	• none	• Sandwich Match (groups of 2)
A.3	• none	• none
A.4	• none	• none
A.5	• none	• none
B.6	• none	• none
B.7	• none	• Match the Situation (groups of 2)
B.8	• Materials from a previous lesson	• none
C.9	• none	• Grid Paper 5 (groups of 2)
C.10	• none	• none
C.11	• none	• none
C.12	• none	• none
C.13	• none	• Card Sort: Diagrams and Expressions (groups of 2)
C.14	• none	• Info Gap: Area (groups of 2)
C.15	• none	• none
C.16	• none	• none

C.17

- Colored paper
 - Glue
 - Rulers
 - Scissors
- none

Center: Rolling for Fractions (3–5)

Stage 2: Multiply a Fraction by a Whole Number

Lessons

- Grade5.2.A1 (supporting)
- Grade5.2.A2 (supporting)

Stage Narrative

Students roll 3 number cubes to generate a multiplication expression with a whole number and a fraction and compare the value of the expression to 1 in order to determine how many points are earned. Two recording sheets are provided, one where the fraction is a unit fraction and one where it can be any fraction.

Variation:

Students may choose a different target number to compare the value of their expression to.

Stage Description

Each group of 2 needs 3 number cubes.

Standards Alignments

Addressing 4.NF.B.4

Materials to Gather

Number cubes

Materials to Copy

Rolling for Fractions Stage 2 Recording Sheet
(groups of 1)

Stage 3: Divide Whole Numbers

Lessons

- Grade5.2.A2 (addressing)
- Grade5.2.A3 (addressing)
- Grade5.2.A4 (addressing)
- Grade5.2.A5 (addressing)

Stage Narrative

Students roll 2 number cubes to generate a division expression, write the quotient as a fraction, and then compare the value of the expression to 1 in order to determine how many points are earned.

Variation:

Students may choose a different target number to compare the value of their expression to.

Standards Alignments

Addressing 5.NF.B.3

Materials to Gather

Number cubes

Materials to Copy

Rolling for Fractions Stage 3 Recording Sheet
(groups of 1)

Additional Information

Each group of 2 needs 2 number cubes.

Stages used in Grade 4

Stage 1

Supporting

- Grade4.3.A
- Grade4.4.A

Stage 2

Addressing

- Grade4.3.A
- Grade4.3.B
- Grade4.3.C

Supporting

- Grade4.6.C
- Grade4.7.A

Center: Compare (1–5)

Stage 4: Divide within 100

Lessons

- Grade5.2.A1 (supporting)
- Grade5.2.A2 (supporting)
- Grade5.2.A3 (supporting)

Stage Narrative

Students use cards with division expressions within 100.

This stage of the Compare center is used in grades 3, 4, and 5. When used in grade 3 or 4, remove the cards with two-digit divisors.

Standards Alignments

Addressing 3.OA.C.7

Materials to Copy

Compare Stage 3-8 Directions (groups of 2), Compare Stage 4 Division Cards (groups of 2)

Stage 5: Fractions

Lessons

- Grade5.2.B6 (supporting)
- Grade5.2.B7 (supporting)
- Grade5.2.B8 (supporting)

Stage Narrative

Students use cards with fractions. They may use either deck of fraction cards or combine them together to play.

Standards Alignments

Addressing 4.NF.A.2

Materials to Copy

Compare Stage 3-8 Directions (groups of 2), Fraction Cards Grade 3 (groups of 2), Fraction Cards Grade 4 (groups of 2)

Stages used in Grade 4

Stage 3

Supporting

- Grade4.2.C
- Grade4.3.C
- Grade4.5.A
- Grade4.5.B
- Grade4.6.B

Stage 4

Supporting

- Grade4.6.C

Stage 5

Addressing

- Grade4.2.C

Supporting

- Grade4.3.A
- Grade4.7.A
- Grade4.7.B
- Grade4.7.C
- Grade4.8.A
- Grade4.8.B

Stage 6

Addressing

- Grade4.3.B
- Grade4.3.C

Stage 7

Addressing

- Grade4.6.D

Supporting

- Grade4.7.A
- Grade4.8.A
- Grade4.8.B

Center: Target Measurements (2–5)

Stage 4: Degrees

Lessons

- Grade5.2.A4 (supporting)
- Grade5.2.A5 (supporting)
- Grade5.2.B6 (supporting)

Stage Narrative

Students estimate angle measurement and then use a protractor to find the exact measurement.

Standards Alignments

Addressing 4.G.A.1

Materials to Gather

Protractors, Scissors

Materials to Copy

Target Measurement Stage 4 Homemade Protractor (groups of 2), Target Measurement Stage 4 Recording Sheet (groups of 2)

Stage 5: Fractions of Angles

Lessons

- Grade5.2.B7 (addressing)
- Grade5.2.B8 (addressing)
- Grade5.2.C9 (addressing)
- Grade5.2.C10 (addressing)

Stage Narrative

Students spin a spinner to get a denominator for their target fraction. They choose a fraction less than one with that denominator and estimate an angle measurement that is the target fraction amount of 180. (For example, if a student gives a target fraction of $\frac{5}{6}$, they estimate when they think the angle is 150 degrees.) Then students use a protractor to find the exact measurement.

Standards Alignments

Addressing 5.NF.B.4.a

Materials to Gather

Protractors

Materials to Copy

Target Measurement Stage 5 Recording Sheet
(groups of 2), Target Measurement Stage 5 Spinner
(groups of 2)

Stages used in Grade 4

Stage 2

Supporting

- Grade4.3.A
- Grade4.3.B

Stage 3

Addressing

- Grade4.3.B

Stage 4

Addressing

- Grade4.7.A
- Grade4.7.B
- Grade4.7.C

Center: How Close? (1–5)

Stage 6: Multiply to 3,000

Lessons

- Grade5.2.C9 (supporting)
- Grade5.2.C10 (supporting)
- Grade5.2.C11 (supporting)

Stage Narrative

Before playing, students remove the cards that show 10 and set them aside.

Each student picks 6 cards and chooses 4 of them to create a multiplication expression. Each student multiplies the numbers and the student whose product is closest to 3,000 wins a point for the round. Students pick new cards so that they have 6 cards in their hand and then start the next round.

Variation:

Students can choose a different number as the goal.

Standards Alignments

Addressing 3.OA.B.5

Materials to Gather

Number cards 0–10

Materials to Copy

How Close? Stage 6 Recording Sheet (groups of 1)

Stage 7: Multiply Fractions and Whole Numbers to 5

Lessons

- Grade5.2.C11 (addressing)
- Grade5.2.C12 (addressing)
- Grade5.2.C13 (addressing)

Stage Narrative

Before playing, students remove the cards that show 10 and set them aside.

Each student picks 6 cards and chooses 3 of them to create a multiplication expression with a fraction and a whole number. Each student multiplies their numbers and the student whose product is closest to 5 wins a point for the round. Students pick new cards so that they have 6 cards in their hand and then start the next round.

Variation:

Students can choose a different number as the goal.

Standards Alignments

Addressing 5.NBT.B.7

Materials to Gather

Number cards 0–10

Materials to Copy

How Close? Stage 7 Recording Sheet (groups of 1)

Stages used in Grade 4

Stage 5

Supporting

- Grade4.5.A

Stage 6

Addressing

- Grade4.5.A
- Grade4.5.B

Supporting

- Grade4.2.C

Center: Rectangle Rumble (3–5)

Stage 3: Factors 1–10

Lessons

- Grade5.2.C12 (supporting)
- Grade5.2.C13 (supporting)
- Grade5.2.C14 (supporting)

Stage Narrative

Students generate factors with two spinners, one that shows 1–5 and one that shows 6–10. Students use a 20×20 grid.

Standards Alignments

Addressing 3.MD.C.7

Materials to Gather

Colored pencils, crayons, or markers, Paper clips

Materials to Copy

Rectangle Rumble Stage 3 Grid (groups of 2),
Rectangle Rumble Stage 3 Spinners (groups of 2)

Additional Information

Each group of students need two paper clips and two different color writing utensils.

Stage 4: Whole Number and Fraction Factors

Lessons

- Grade5.2.C14 (addressing)
- Grade5.2.C15 (addressing)
- Grade5.2.C16 (addressing)
- Grade5.2.C17 (addressing)

Stage Narrative

Students generate factors with a number cube and a spinner with fractions with denominators of 3, 6, or 12. Students use a 24×24 grid that represents 16 square units.

Standards Alignments

Addressing 5.NF.B.4

Materials to Gather

Colored pencils, crayons, or markers, Number cubes, Paper clips

Materials to Copy

Rectangle Rumble Stage 4 Grid (groups of 2),
Rectangle Rumble Stage 4 Spinner (groups of 2)

Additional Information

Each group of students need a paper clip, a number cube, and two different color writing utensils.

Stages used in Grade 4**Stage 3****Supporting**

- Grade4.5.C

Center: Can You Build It? (3–5)

Stage 2: Multiple Rectangles

Lessons

- Grade5.2.C15 (supporting)
- Grade5.2.C16 (supporting)
- Grade5.2.C17 (supporting)

Stage Narrative

Before playing, students remove the cards that show 6 or higher and set them aside.

Students flip two number cards to get a number of tiles and then each partner tries to create as many rectangles as possible for that area. If both students each found all the rectangles, they get one point. A student gets two points for any rectangle they built that their partner did not. The player who gets the most points after eight rounds is the winner. Students may choose to draw their rectangle on grid paper, rather than use inch tiles.

This center stage is the first time Number Cards 0–10 are used in Grade 4, so they are provided as a Instructional master. Students will continue to use these throughout the year. Consider copying them on cardstock or laminating them and keeping them organized to be used repeatedly.

Standards Alignments

Addressing 4.OA.B.4

Materials to Gather

Folders, Grid paper, Inch tiles

Materials to Copy

Can You Build It Stage 2 Directions (groups of 6),
Number Cards (0-10) (groups of 2)

Additional Information

Each group of 2 needs at least 120 inch tiles and a set of number cards.

Stages used in Grade 4

Stage 1

Supporting

- Grade4.1.A

Stage 2

Addressing

- Grade4.1.A
- Grade4.1.B

Section A: Fractions as Quotients

Lesson 1: Share Sandwiches

Standards Alignments

Building On	3.NF.A.1, 3.OA.A.2
Addressing	5.NF.B.3
Building Towards	5.NF.B.3

Teacher-facing Learning Goals

- Interpret and represent contexts relating division and fractions in a way that makes sense to them.

Student-facing Learning Goals

- Let's share sandwiches.

Lesson Purpose

The purpose of this lesson is for students to relate equal shares of objects to division and to fractions.

In previous grades, students learned to interpret products of whole numbers, such as 3×5 , as the total number of objects in 3 groups each containing 5 objects. They interpreted division, such as $15 \div 3$, to be either the number of groups when 15 things are put in groups of 3 or as the number of things in each group when 15 things are put in 3 equal groups. They also solve word problems posed with whole numbers and having whole-number answers, including problems in which remainders must be interpreted. The goal of the next several lessons is to extend this understanding of division to quotients like $15 \div 6$ where the result is not a whole number. Students learned to interpret fractions such as $\frac{15}{6}$ in a previous grade and this unit will establish that $\frac{15}{6}$ is the value of the quotient $15 \div 6$.

In this lesson, students use what they know about division to make sense of situations where people equally share sandwiches. This lesson is meant to be an invitation to explore the relationships between division and fractions. The problems were written so students can revisit the meaning of division and be curious about how division applies to situations when the quotient represents a fractional quantity without having to name the quantity. Although students discuss how the situations in the lesson can be represented with division expressions, they do not need to write them or formally explain them, as that will be the focus of upcoming lessons. Throughout this unit, it is assumed that the sharing is always equal sharing, whether explicitly stated or not.

Access for:** Students with Disabilities**

- Representation (Activity 2)

** English Learners**

- MLR1 (Activity 1)

Instructional Routines

5 Practices (Activity 1), MLR2 Collect and Display (Activity 2), Which One Doesn't Belong? (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What evidence did you see that each student felt they belonged in math class today?

Cool-down (to be completed at the end of the lesson)
 5 min

How Much?

Standards Alignments

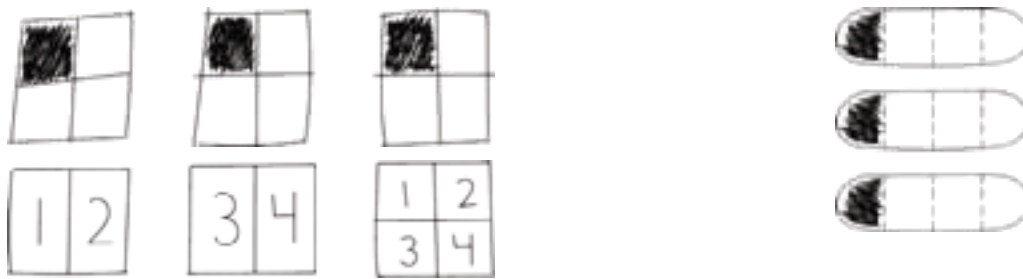
Addressing 5.NF.B.3

Student-facing Task Statement

1. Draw a diagram to show how much sandwich each person will get.
3 sandwiches are equally shared by 4 people.
2. Explain or show how you know that each person gets the same amount of sandwich.

Student Responses

1. Sample pictures:



2. Sample responses

- Each person gets one fourth of each sandwich.
- Each person gets one half plus one fourth of each sandwich.
- Each person gets $\frac{3}{4}$ of a sandwich.

Begin Lesson

Warm-up

🕒 10 min

Which One Doesn't Belong: Sandwiches

Standards Alignments

Building Towards 5.NF.B.3

This purpose of this warm-up is for students to compare four images. It introduces the context of sandwiches which will be used in the lesson to examine equal sharing situations, giving students a chance to engage with the context in an informal way before they interpret division situations about sharing sandwiches.

Instructional Routines

Which One Doesn't Belong?

Student-facing Task Statement

Which one doesn't belong?

Launch

- Groups of 2

A**B****C****D**

Student Responses

Sample responses:

- A is the only one that is not whole. It is divided into smaller pieces and the rest are whole.
- B is the only one that is not on a plate or tray. It is floating and the rest are resting on something.
- C is the only one that is not a sandwich. The rest have bread.
- D is the only one that has ingredients that have not been combined. The rest have ingredients that have been mixed together.

- Display the image.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 2-3 minutes: partner discussion
- Share and record responses

Synthesis

- "What kind of sandwich do you like to eat? Are there special occasions when you eat sandwiches? Describe a time when you have shared food with your family or friends."

Activity 1

🕒 20 min

Share Sandwiches

Standards Alignments

Building On	3.NF.A.1, 3.OA.A.2
Addressing	5.NF.B.3

The purpose of this activity is for students to equally divide sandwiches in situations where the number of portions does not evenly divide the number of sandwiches. Students select the number of people sharing and the number of sandwiches from a small set of deliberately chosen numbers. Depending on their choice, each share may be less than a full sandwich or more than a full sandwich. This encourages students to make sense of the problem and persevere in solving it (MP1). The focus of the synthesis is on different ways students choose to make equal shares and how they know that the shares are equal. Many of the strategies for dividing the sandwiches equally are examined in detail in the next activity. In this activity, students explain how they know the shares are equal. Student representations may include coloring to show the equal shares. Monitor and select students who use the following strategies to share in the synthesis:

- choose numbers so each person gets one or two whole sandwiches and a fraction of a sandwich
- choose numbers so each person gets less than a full sandwich

Access for English Learners

MLR1 Stronger and Clearer Each Time. Synthesis: Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their response to how much sandwich each person gets. Invite listeners to ask questions, to press for details and to suggest mathematical language. Give students 2–3 minutes to revise their written explanation based on the feedback they receive.

Advances: Writing, Speaking, Listening

Instructional Routines

5 Practices

Student-facing Task Statement

_____ sandwiches are shared equally by _____ people.

1. Choose numbers to fill in the blanks. You can only use each number once: 2, 3, 5.
2. Represent the situation with a diagram or drawing.
3. Explain or show how you know that each person will get the same amount of sandwich.

Launch

- Groups of 2
- Display the prompt without the questions. _____ sandwiches are shared equally by _____ people.
- “What do you notice? What do you wonder?” (There are no numbers. How many sandwiches are there? How many people are there? What kind of sandwiches are they?)
- “What are some numbers that would make sense? Why do those numbers make

Student Responses

1. Sample responses: 2 sandwiches are shared equally by 3 people, 3 sandwiches are shared equally by 2 people, 2 sandwiches are shared equally by 5 people, 5 sandwiches are shared equally by 2 people, 3 sandwiches are shared equally by 5 people, 5 sandwiches are shared equally by 3 people.

2. Sample responses:

3 sandwiches shared by 2 people



2 sandwiches shared by 5 people



3. If 2 people share 3 sandwiches, each person will get 1 whole sandwich and one half of a sandwich. If 2 sandwiches are shared by 5 people, each sandwich will be cut into 5 equal pieces and each person will get $\frac{1}{5}$ of each sandwich.

sense?" (One sandwich per person because that is a familiar serving size. Many people and many sandwiches because there is a party or event. More people than sandwiches because the people are sharing big sandwiches.)

Activity

- 2 minutes: independent think time
- 5–8 minutes: partner work time
- As students work, consider asking:
 - "Where are the sandwiches in your drawing or diagram?"
 - "How does your diagram or drawing show how much sandwich each person will get?"

Synthesis

- Ask the selected students to display their responses side by side for all to see.
- "Does each person get more than a full sandwich or less? How do you know?" (Less because there are more people than sandwiches or more because there are more sandwiches than people.)
- "This situation is about sharing. We can represent equal sharing situations with division expressions. What division expressions can we write to represent each of the situations we represented?" (Answers vary depending on number choices: $2 \div 3$, $3 \div 2$, $2 \div 5$, $5 \div 2$, $3 \div 5$, $5 \div 3$)

Advancing Student Thinking

Students may recognize that the natural shape of their chosen sandwich impacts the size of certain pieces and, therefore, some people are getting a little bit more sandwich if they get a certain piece. When they recognize the constraints of the shape of a sandwich in real life, students are making sense of problems. Ask these students, "If you were going to share these sandwiches with your friends, which pieces would you want? Why would you want those pieces? Why do you

think someone might choose the end pieces? Is it fair to say that each of you got about the same amount of sandwich?"

Activity 2

🕒 15 min

The Same Amount

Standards Alignments

Building On 3.NF.A.1, 3.OA.A.2

Addressing 5.NF.B.3

In the previous activity students chose numbers to represent the sandwiches and the people sharing them and then represented those equal shares. The purpose of this activity is for students to explain how different representations can show equal shares when a division situation results in a quotient that is a fraction. Students have an opportunity to use their own language to describe equal shares in a division situation. Students may name the numerical amount of sandwich that each person gets, but that is not the focus of this activity. In future lessons, students will focus on numerical solutions for division situations that result in a quotient that is a fraction.

This activity uses *MLR2 Collect and Display*. Advances: Reading, Writing.

🕒 Access for Students with Disabilities

Representation: Access for Perception. Begin by enacting a physical demonstration of sharing a sandwich with multiple people in different ways to support understanding of the context.

Supports accessibility for: Conceptual Processing, Memory

Instructional Routines

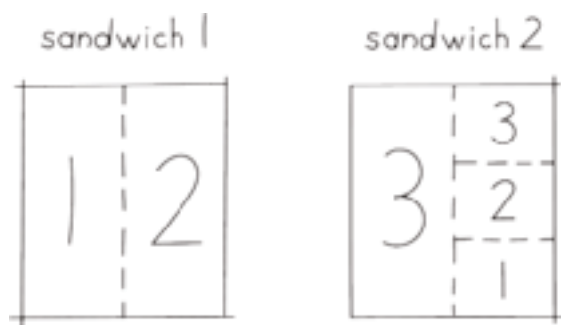
MLR2 Collect and Display

Student-facing Task Statement

1. Han's work shows how 3 people could equally share 2 sandwiches.

Launch

- Groups of 2



How do you know that each person gets about the same amount of sandwich? Explain or show your thinking. Organize it so it can be followed by others.

2. Draw a diagram to show a different way that 3 people could share 2 sandwiches so each person got about the same amount of sandwich.

Student Responses

1. Sample response: Each person gets one half of a sandwich and one third of one half of a sandwich. The pieces that each person gets are about the same size and each person gets the same number of pieces.
2. Sample responses include ones that show each sandwich divided into 3 equal pieces or diagrams that show different shaped sandwiches.

Activity

- 8–10 minutes: partner work

MLR2 Collect and Display

- Circulate, listen for, and collect the language students use to describe how they know each person gets the same amount.
- Listen for these words and phrases: divide, same, equal, fair, size of the piece, number of pieces, and one third of one half.
- Record students' words, phrases, and expressions on a visual display and update it throughout the lesson.

Synthesis

- Refer to the display.
- "These are the words, phrases, and expressions that you used when you were describing how you knew each person got the same amount of sandwich."
- "Are there any other words or phrases that are important to include on our display?"
- As students share responses, update the display by adding (or replacing) language, diagrams, or annotations.
- Remind students to borrow language from the display as needed.
- Display: $2 \div 3$
- "How does this expression represent the situation that Han drew?" (There are 2 sandwiches being shared by 3 people.)

Advancing Student Thinking

If students do not explain how they know that each person got the same amount of sandwich, consider asking: "How does the diagram represent the situation?"

Lesson Synthesis

🕒 10 min

"Today, we represented division situations about people sharing sandwiches."

Display the problems the students solved today.

Consider asking students to respond to these questions in their journal.

- "What was the same about all the problems we solved today?" (They were all about sandwiches. There was always a fraction of some sort. Both problems had the same numbers.)
- "What was different about the problems?" (The diagrams were different. Sometimes people got more than one whole sandwich and sometimes they got less than one whole sandwich. Sometimes the number represented sandwiches and sometimes it represented people.)
- "What is something you are still wondering about division?" (Can the answer be a fraction? What would happen if they were sharing a fraction of a sandwich?)

Suggested Centers

- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Supporting)
- Compare (1–5), Stage 4: Divide within 100 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students do not draw a diagram that shows equal shares.

Next Day Support

- During Activity 1, encourage students to draw a diagram to represent each situation in the table and explain where they see the number of people sharing the sandwich in each diagram.

Lesson 2: Share More Sandwiches

Standards Alignments

Building On 3.NF.A.1, 3.OA.A.2

Addressing 5.NF.B.3

Teacher-facing Learning Goals

- Represent the relationship between division and fractions with diagrams and expressions.

Student-facing Learning Goals

- Let's use diagrams and expressions to represent division situations.

Lesson Purpose

The purpose of this lesson is for students to relate equal shares to division expressions and visual representations of fractions.

In the previous lesson, students explored the relationship between fractions and division by representing situations where some people shared some sandwiches. They used informal language to describe how they knew each person got about the same amount of sandwich.

In this lesson, students recognize the relationship between a fraction and a division expression. For example, $\frac{1}{5} = 1 \div 5$. Students interpret $1 \div 5$ as the amount in one group when a single whole is divided into 5 equal portions. They see that the quantity in that portion is $\frac{1}{5}$ of a whole.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Card Sort (Activity 2), Estimation Exploration (Warm-up)

Materials to Copy

- Sandwich Match (groups of 2): Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What have you noticed about the language students use to describe the relationship between fractions and division? How can you continue to recognize and honor their authentic language and also connect it to more formal math vocabulary?

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

How Much Sandwich?

Standards Alignments

Addressing 5.NF.B.3

Student-facing Task Statement

- 4 sandwiches are equally shared by 5 students. How much sandwich does each student get? Show or explain your reasoning.
- Write a division expression to represent the situation.

Student Responses

- $\frac{4}{5}$ sandwich: Students may draw different shaped sandwiches and partition them in a variety of ways. They may also write an equation.
- $4 \div 5$

----- **Begin Lesson** -----**Warm-up**

⌚ 10 min

Estimation Exploration: Name that Fraction

Standards Alignments

Addressing 5.NF.B.3

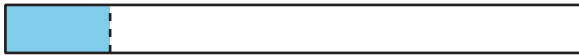
The purpose of this estimation exploration is for students to use what they know about fractions to estimate how much of the tape is shaded. Students use what they know about division to determine about how much of the bar is shaded.

Instructional Routines

Estimation Exploration

Student-facing Task Statement

The large rectangle represents 1. What fraction of the large rectangle is shaded?



Record an estimate that is:

too low	about right	too high

Student Responses

too low	about right	too high
Greater than 0 and less than $\frac{1}{10}$	$\frac{1}{6}$ to $\frac{1}{5}$	$\frac{1}{3}$ to $\frac{1}{2}$

Launch

- Groups of 2
- Display the image.
- “What is an estimate that’s too high? Too low? About right?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Record responses.

Synthesis

- “How did you make your about right estimate?” (It looks like there are about 5 of those shaded pieces in the whole rectangle so that’s about $\frac{1}{5}$.)

Advancing Student Thinking

If students do not have an estimate, encourage them to draw on the diagram or cut it out and fold it. Ask students: “How can drawing on or folding the diagram help you figure out how much is shaded?”

Activity 1

🕒 15 min

One Sandwich

Standards Alignments

Building On 3.NF.A.1, 3.OA.A.2

Addressing 5.NF.B.3

The purpose of this activity is for students to connect their understanding of unit fractions with their understanding of division. Students understand a unit fraction such as $\frac{1}{3}$, as 1 piece where 3 of those equal pieces make the whole. Students also understand division, $1 \div 3$, as 1 thing divided into 3 equal shares.

During the activity synthesis, connect the two expressions, $1 \div 3$ and $\frac{1}{3}$, to a common diagram to show the relationship between the operation of division and the fraction as a quotient. Students relate diagrams, fractions, and division expressions with one another and interpret them within the context of sandwiches (MP2).

🕒 Access for Students with Disabilities

Representation: Access for Perception. Use a rectangular shaped piece of paper to demonstrate what is happening in the task.

Supports accessibility for: Conceptual Processing, Memory

Student-facing Task Statement

Jada's family made sandwiches to share at a family celebration. Complete the table to show how much sandwich each person gets.



Launch

- Groups of 2

Activity

- 5–8 minutes: partner work time
- Monitor for students who:
 - notice that the size of each piece is getting smaller as more people share it
 - notice that the denominator in the amount of sandwich each person gets is the number of people

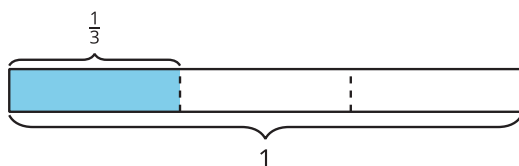
sandwiches being shared	number of people sharing sandwiches	amount of sandwich each person gets	division expression
1	2		
1	3		
1	4		
1	5		

1. Choose one row from the table and represent your thinking with a diagram.
2. What patterns do you notice in the table?

Student Responses

sandwiches being shared	number of people sharing sandwiches	amount of sandwich each person gets	division expression
1	2	$\frac{1}{2}$	$1 \div 2$
1	3	$\frac{1}{3}$	$1 \div 3$
1	4	$\frac{1}{4}$	$1 \div 4$
1	5	$\frac{1}{5}$	$1 \div 5$

1. Sample response for 3 people:



2. Sample response: As the number of people sharing increases, the amount each person gets decreases. There is a 1 in all of the division expressions but the other number changes as the number of people changes.

sharing the sandwich

Synthesis

- Invite selected students to share the patterns they noticed in the table.
- Display student work that shows a diagram of one sandwich shared by 3 people or display the diagram from the student solutions.
- “How does the diagram you drew represent the expression $1 \div 3$?” (Each rectangle is divided into 3 equal pieces so that’s $1 \div 3$.)
- Highlight that the division sign means that the whole is divided into equal pieces.
- “How does the fraction $\frac{1}{3}$ represent the shaded amount?” (One of the 3 equal-sized pieces in the rectangle is shaded in so that’s $\frac{1}{3}$.)

Advancing Student Thinking

If students do not write the correct amount of sandwich each person gets or the correct division

expression, encourage them to draw their own diagram of the situation and ask:

- “How does your diagram represent the number of sandwiches being shared?”
- “How does your diagram represent the number of people sharing the sandwiches?”
- “How does your diagram represent the amount of sandwich each person will get? What number represents the amount of sandwich each person will get?”

Activity 2

🕒 20 min

Card Sort: Sandwich Match

Standards Alignments

Addressing 5.NF.B.3

This sorting task gives students opportunities to analyze and connect representations, situations, and expressions (MP2, MP7). As students work, encourage them to refine their descriptions of how the diagrams represent the situations and expressions using more precise language and mathematical terms (MP6).

🌐 Access for English Learners

MLR8 Discussion Supports. Students should take turns finding a match and explaining their reasoning to their partner. Display the following sentence frames for all to see: “I noticed ___, so I matched . . .” Encourage students to challenge each other when they disagree.

Advances: Reading, Conversing

Instructional Routines

Card Sort

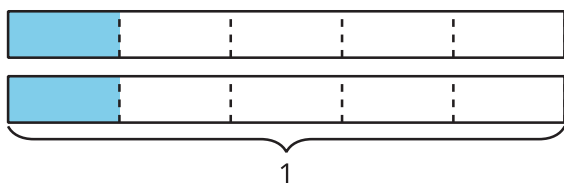
Materials to Copy

Sandwich Match (groups of 2)

Required Preparation

- Create a set of cards from the Instructional master for each group of 2.

Student-facing Task Statement



Your teacher will give you a set of cards. Match each representation with a situation and expression. Some situations and expressions will have more than one matching representation.

Choose one set of matched cards.

1. Show or explain how the diagram(s) and expression represent the number of sandwiches being shared.
2. Show or explain how the diagram(s) and expression represent the number of people sharing the sandwiches.
3. How much sandwich does each person get in the situation?

Student Responses

Matches:

A, H, L B, J, N, D C, G, E, K F, I, M

1. Sample response: The sandwiches are the number of big rectangles. They are the first number in the expressions.
2. The number of people is represented by the number of equal pieces in each sandwich. The number of people is the second number in the expressions.
3. A, H, L: Each person gets $\frac{3}{4}$ of a sandwich.

B, J, N, D: Each person gets $\frac{3}{2}$ or $1\frac{1}{2}$ of a sandwich.

C, G, E, K: Each person gets $\frac{4}{3}$ or $1\frac{1}{3}$ of a sandwich.

Launch

- Groups of 2
- Display image from student workbook.
- “This representation shows how 2 sandwiches can be shared by 5 people equally. How much sandwich does each person get? Be prepared to share your thinking.” ($\frac{2}{5}$ since each piece is $\frac{1}{5}$ of one whole and there are two of them.)
- 1 minute: quiet think time
- Share responses.
- Distribute one set of pre-cut cards to each group of students.

Activity

- “This set of cards includes diagrams, expressions, and situations. Match each diagram to a situation and an expression. Some situations and expressions will match more than one diagram. Work with your partner to justify your choices. Then, answer the questions in your workbook.”
- 5–8 minutes: partner work time
- Monitor for students who:
 - notice that the number of large rectangles in the picture and the dividend in the expressions represent the number of sandwiches
 - notice that the number of pieces in each whole and the divisor in the expressions represent the number of people sharing the sandwiches

Synthesis

- Have students share the matches they made and how they know those cards go together.
- Attend to the language that students use to

F, I, M: Each person gets $\frac{2}{3}$ of a sandwich.

describe their matches, giving them opportunities to describe how the diagrams and expressions represent the situation more precisely.

- Highlight the use of terms like divide, dividend, divisor, number of pieces, and size of each piece.
- Display cards B, D, J, and N.
- “How does each diagram represent 3 sandwiches being shared by 2 people?” (Each of the large rectangles is a sandwich and the shaded part shows how much each person gets.)
- “How much sandwich does each person get? How do you know?” ($\frac{3}{2}$ or $1\frac{1}{2}$ because each rectangle is cut into halves and 3 of them are shaded.)
- Display:

$$3 \div 2$$
- “How does this expression represent the situation?” (3 sandwiches are being shared equally by 2 people.)

Advancing Student Thinking

If students do not match all of the diagrams to a situation or did not match the diagrams correctly, point to one of the diagrams that they did match correctly, and ask: “How does this diagram represent some people sharing some sandwiches?”

Lesson Synthesis

🕒 10 min

“Today we matched division situations with representations and division expressions.”

Display expression: $1 \div 6$

“What does the expression mean in terms of the problems we were solving about people sharing sandwiches?” (It means that 1 sandwich is being shared by 6 people.)

"How much of the sandwich will each person get?" ($\frac{1}{6}$)

"Describe how you would figure out the amount of sandwich each person gets if any amount of people share 1 sandwich." (I would divide the sandwich into however many people there are so the amount each person gets is going to be 1 piece and the size of the piece will be based on however many people there are.)

"What did you learn about the relationship between division and fractions today?"

Consider having students respond in their journals.

Suggested Centers

- Rolling for Fractions (3–5), Stage 3: Divide Whole Numbers (Addressing)
- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Supporting)
- Compare (1–5), Stage 4: Divide within 100 (Supporting)

----- Complete Cool-Down -----

Response to Student Thinking

Students do not write $\frac{4}{5}$ as the amount of sandwich each person gets.

Students do not recognize representations of unit fractions.

Next Day Support

- Before the warm-up of the next lesson, create a display with the students to show the connection between $4 \div 5$, $\frac{4}{5}$, and 4 sandwiches being shared by 5 people.

Prior Unit Support

Grade 3, Unit 5, Section A: Introduction to Fractions

Lesson 3: Interpret Equations

Standards Alignments

Addressing 5.NF.B.3

Teacher-facing Learning Goals

- Represent the relationship between division and fractions with equations.

Student-facing Learning Goals

- Let's use equations to show the relationship between division and fractions.

Lesson Purpose

The purpose of this lesson is for students to write equations to represent division situations and relate each part of the equation to the situation.

In previous lessons, students developed the understanding that $1 \div b = \frac{1}{b}$. In this lesson, students deepen their understanding of the relationship between division and fractions. They write equations and explain how each part of the equation corresponds to a situation.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR2 (Activity 2)

Instructional Routines

MLR3 Clarify, Critique, Correct (Activity 1), What Do You Know About ____? (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

In tomorrow's lessons, students will be writing situations to match division expressions. In what ways is division used by your students' families and communities? Be prepared to share a few division situations that will be familiar to students during tomorrow's lesson.

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

Share Water

Standards Alignments

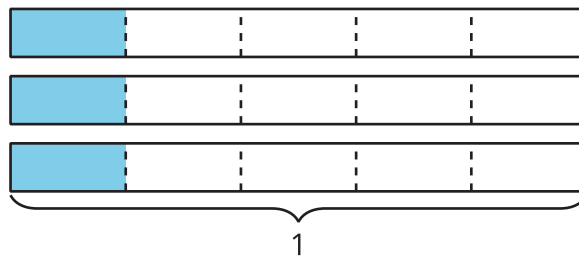
Addressing 5.NF.B.3

Student-facing Task Statement

3 liters of water are shared equally by 5 people. How much water does each person get? Write a division equation to represent the situation. Draw a diagram if it is helpful.

Student Responses

Each person gets $\frac{3}{5}$ liters of water, $3 \div 5 = \frac{3}{5}$.



----- Begin Lesson -----

Warm-up

⌚ 10 min

What Do You Know About $\frac{3}{2}$?**Standards Alignments**

Addressing 5.NF.B.3

The purpose of this What Do You Know About ____? is for students to share what they know and how they can represent the number $\frac{3}{2}$. This will be useful when students write equations to represent the relationship between a division expression and a fraction in a later activity. Record answers on a poster because students will revisit their answers to the warm-up during the lesson synthesis.

Instructional Routines

What Do You Know About ____?

Student-facing Task Statement

What do you know about $\frac{3}{2}$?

Student Responses

Sample responses:

- It is more than 1.
- It is equal to $1\frac{1}{2}$.
- It is 3 groups of $\frac{1}{2}$.
- It is $3 \div 2$.

Launch

- Display the number.
- “What do you know about $\frac{3}{2}$?”
- 1 minute: quiet think time

Activity

- Record responses.
- “How could we represent the number $\frac{3}{2}$?”

Synthesis

- “What diagrams can I draw to represent $\frac{3}{2}$?”
(Sample responses may include a number line, shaded rectangles, or a tape diagram.)
- Draw the diagrams.
- “How are the diagrams the same and different?”

Activity 1

🕒 20 min

Dehydrated Dancers

Standards Alignments

Addressing 5.NF.B.3

The purpose of this activity is for students to write and interpret division expressions and equations that represent equal sharing situations. They explain the relationships between the dividend and the numerator and divisor and the denominator. Students may draw diagrams to help them make sense of these relationships (MP1).

The last problem provides an opportunity for students to think critically about a proposed

solution to a problem (MP3). Different ways to think about the proposed solution include:

- estimation: with 3 friends sharing 2 liters, each friend gets less than 1 liter
- thinking about the meaning of the numerator (how many liters are being shared) and denominator (how many people are sharing the water)

This activity uses *MLR1 Stronger and Clearer Each Time*. Advances: Reading, Writing.

Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Invite students to identify which details were needed to solve the problem. Display the sentence frame, “The next time I write division equations, I will pay attention to . . .”

Supports accessibility for: Conceptual Processing, Memory, Language

Instructional Routines

MLR3 Clarify, Critique, Correct

Student-facing Task Statement



1. Three dancers share 2 liters of water. How much water does each dancer get? Write a division equation to represent the situation.
2. Mai said that each dancer gets $\frac{3}{2}$ of a liter of water because 3 divided into 2 equal groups is $\frac{3}{2}$. Do you agree with Mai? Show or explain your reasoning.

Student Responses

1. $\frac{2}{3}$ liter of water. $2 \div 3 = \frac{2}{3}$
2. $3 \div 2$ does equal $\frac{3}{2}$, but it doesn't represent the situation, $3 \div 2 = \frac{3}{2}$ would mean 3 dancers are divided into 2 equal groups and

Launch

- Groups of 2

Activity

- 5 minutes: independent work time
- 5 minutes: partner discussion
- As students work, monitor for students who:
 - draw a diagram to determine the amount of water each dancer drinks if 3 dancers share 2 liters of water.
 - revise their solution for how much water each dancer gets after explaining why Mai's answer doesn't make sense.

Synthesis

- Invite previously selected students to share.
- If no student mentions it, ask students to

that doesn't make sense. $2 \div 3$ means 2 liters of water divided into 3 equal groups and each dancer gets $\frac{2}{3}$ liter of water.

explain which part of Mai's solution doesn't make sense and why. (It doesn't make sense for each dancer to get $\frac{3}{2}$ of a liter of water because $\frac{3}{2}$ is equal to $1\frac{1}{2}$ and if the dancers are only sharing 2 liters of water, they don't have enough water for each person to get $1\frac{1}{2}$ liters of water.)

MLR3 Clarify, Critique, Correct

- Display the following partially correct answer and explanation:
- Mai said that each dancer gets $\frac{3}{2}$ liter water because 3 divided into 2 equal groups is $\frac{3}{2}$.
- Read the explanation aloud.
- "What do you think Mai means?" (She thinks $3 \div 2 = \frac{3}{2}$ represents the situation.)
- "Is anything unclear?" (The 3 represents the number of dancers, not the amount of water so Mai would be dividing dancers instead of liters of water.)
- 1 minute: quiet think time
- 2 minute: partner discussion
- "With your partner, work together to write a revised explanation."
- Display and review the following criteria:
 - Specific words and phrases, such as dancers and liters of water
 - Labeled equation or diagram
- 3–5 minute: partner work time
- Select 1–2 groups to share their revised explanation with the class. Record responses as students share.
- "What is the same and different about the explanations?" (Some people labeled the diagram to show what is being divided and some people labeled the numbers in the equation.)

Advancing Student Thinking

If students write the numbers in the division equation in the wrong order, ask “Can you describe how your equation represents the situation?”

Activity 2

🕒 15 min

Interpret Expressions

👤 ↔ 👤 PLC Activity

Standards Alignments

Addressing 5.NF.B.3

In previous activities, students interpreted diagrams and expressions that represented equal sharing situations. The purpose of this activity is for students to interpret division expressions without the support of diagrams in order to deepen their understanding about the relationship between fractions and division expressions. The activity is designed to highlight the relationship between the number of objects being shared and the numerator, on the one hand, and the number of people sharing and the denominator on the other (MP7).

🌐 Access for English Learners

MLR2 Collect and Display. Circulate, listen for and collect the language students use as they discuss the problem. On a visible display, record words and phrases such as: “divide,” “numerator,” “denominator,” “part of,” “fraction,” “whole.” Invite students to borrow language from the display as needed, and update it throughout the lesson.

Advances: Conversing, Reading

Student-facing Task Statement

1. Complete the table. Draw a diagram if it is helpful.

Launch

- Groups of 2

Activity

- 5-8 minutes: independent work time
- 5-8 minutes: partner work time
- Monitor for students who:

number of dancers	liters of water	division expression	amount of water each dancer drank in liters
4	2		
		$3 \div 4$	
		$3 \div 5$	
4			
	5		

2. What patterns do you notice in the table?

Student Responses

1.

number of dancers	liters of water	division expression	amount of water each dancer drank in liters
4	2	$2 \div 4$	$\frac{2}{4}$
4	3	$3 \div 4$	$\frac{3}{4}$
5	3	$3 \div 5$	$\frac{3}{5}$
4	(any whole number)	liters of water $\div 4$	$\frac{\text{liters of water}}{4}$
(any whole number)	5	$5 \div$ number of dancers	$\frac{5}{\text{number of dancers}}$

2. Sample responses: The number of dancers is always the denominator in the fraction that shows how much each dancer drank. The number of liters of water is always the numerator in the fraction that shows how much each dancer drank. The dividend is the same as the numerator of the fraction because it represents how much water is being shared and the divisor is the same as the denominator of the fraction because it shows how many dancers are sharing the

- notice and can explain the relationship between the numerator and the number of liters of water and the denominator and the number of dancers.

Synthesis

- Display the table from the activity.
- "What are some of the numbers you used for the last two rows?"
- Record the answers as additional rows to the table.
- "What are some patterns that you notice in the table?"
- Ask previously selected students to share their solutions.

water.

Advancing Student Thinking

If students fill in the table according to the order of the dividend and divisor, encourage them to draw a diagram to represent each situation in the table. Ask “How do your diagrams represent the division expressions?”

Lesson Synthesis

🕒 10 min

Display poster of responses from the warm-up.

“What can we add to our answers from the warm-up question based on what we learned today?”

(Sample responses: $\frac{3}{2}$ can describe a division situation, $\frac{3}{2}$ can represent 2 dancers sharing 3 liters of water, $3 \div 2 = \frac{3}{2}$ or $1\frac{1}{2}$)

If no student mentions it, say, “If 2 dancers share 3 liters of water, how much water does each dancer get? Write an equation that represents the situation.” Invite students to share their equation.

Display equation: $3 \div 2 = \frac{3}{2}$.

“How does this equation represent the situation?” (3 liters of water are being shared by 2 dancers. Each of the 2 dancers gets $\frac{3}{2}$ liter of water.)

Suggested Centers

- Rolling for Fractions (3–5), Stage 3: Divide Whole Numbers (Addressing)
- Compare (1–5), Stage 4: Divide within 100 (Supporting)

----- Complete Cool-Down -----

Response to Student Thinking

Students do not write the division expression in the correct order.

Next Day Support

- During the synthesis of the warm up, ask students to describe the meaning of the dividend and divisor in the expressions.

Lesson 4: Division Situations

Standards Alignments

Addressing 5.NF.B.3

Teacher-facing Learning Goals

- Solve problems involving division of whole numbers leading to answers in the form of fractions.

Student-facing Learning Goals

- Let's solve and represent division problems.

Lesson Purpose

The purpose of this lesson is for students to solve division problems when the quotient is a fraction or mixed number.

In previous lessons students solved equal sharing problems involving division of whole numbers with answers in the form of mixed numbers and fractions using diagrams. They noticed patterns and made generalizations about the relationship between the expression $a \div b$ and the fraction $\frac{a}{b}$ for specific values of a and b . Students have observed that the numerator of the fraction is the number of objects being shared, while the denominator is the number of equal shares. This lesson brings all of these ideas together through contexts, equations, and diagrams. Students continue to notice patterns across these contexts and build flexibility with interpreting fractions in terms of division by creating their own situations. Fluently moving between representations gives students the ability to choose an appropriate representation to solve a problem (MP1).

Consider what division situations students or their families might be familiar with. Measurement contexts often present situations when division results in a fraction or mixed number. For example, making something with fabric, dividing large amounts of food in to smaller containers, or measuring and cutting wood for a project.

Access for:

Students with Disabilities

- Engagement (Activity 2)
- Representation (Activity 1)

Instructional Routines

Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What strategies are students using to divide numbers that result in a quotient that is a fraction or mixed number? What questions have you asked that encourage students to see the relationship between the dividend and the numerator and the divisor and the denominator?

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

How Much Milk?**Standards Alignments**

Addressing 5.NF.B.3

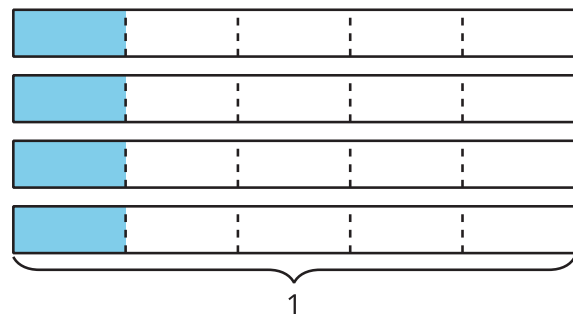
Student-facing Task Statement

Complete the table below.

Equation	Situation
	5 children share 4 cups of milk so each child gets the same amount of milk. How many cups of milk will each child get?
Diagram	

Student Responses

$$4 \div 5 = \frac{4}{5}$$



Begin Lesson

Warm-up

🕒 10 min

Number Talk: Division

Standards Alignments

Addressing 5.NF.B.3

The purpose of this Number Talk is for students to interpret a fraction as division of the numerator by the denominator. The strategies elicited here will be helpful later in the lesson when students match division situations, expressions, and diagrams. In this activity, students have an opportunity to notice and make use of structure (MP7) because the divisor stays the same.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $35 \div 7$
- $1 \div 7$
- $36 \div 7$
- $37 \div 7$

Student Responses

- 5; I just know it, $7 \times 5 = 35$
- $\frac{1}{7}$; I know that there are seven $\frac{1}{7}$'s in 1
- $5\frac{1}{7}$; I added the answers to the two previous problems
- $5\frac{2}{7}$; I added one more $\frac{1}{7}$ because 37 is one more than 36

Launch

- Display one problem.
- "Give me a signal when you have an answer and can explain how you got it."

Activity

- 1 minute: quiet think time
- Record answers and strategy.
- Keep problems and work displayed.
- Repeat with each problem.

Synthesis

- "What patterns do you notice?" (The solutions for $35 \div 7$, $36 \div 7$, $37 \div 7$ increase by $\frac{1}{7}$)
- "If we kept increasing the dividend by 1, what would be the next whole number quotient?" (6)

Activity 1

🕒 20 min

Pounds of Blueberries

Standards Alignments

Addressing 5.NF.B.3

The purpose of this activity is for students to move back and forth between equations, situations, and diagrams. Each member of a group is assigned one of the representations to begin with and all of the representations can represent the same situation. This is the first time in this unit students have been asked to write situations that represent division equations or diagrams. As students work, ask them to explain how the diagram represents the number of objects being shared and the number of equal shares.

Students go back and forth between equations, situations, and diagrams, interpreting the diagrams and equations and creating situations that these diagrams and equations represent (MP2).

🌐 Access for English Learners

Representation: Access for Perception. Synthesis: Invite students to identify correspondences between representations. Make connections between representations visible through gestures and labeled displays.

Supports accessibility for: Visual-Spatial Processing, Conceptual Processing

Student-facing Task Statement

1. Complete the missing parts of the table. Be prepared to explain your thinking.
2. Discuss both your solutions with your group. What is the same? What is different?

Partner A

Equation	Situation
$4 \div 6 = \frac{4}{6}$	
Diagram	

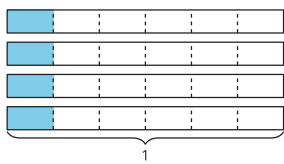
Partner B

Launch

- Display image:



- "Where do you see division situations in this picture?" (The berries are split into 6 groups.)

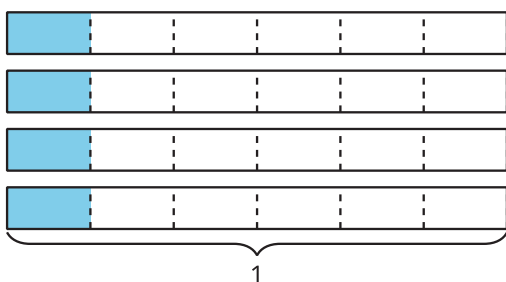
Equation	Situation
Diagram	
	

Partner C

Equation	Situation
	<p>Six students share 4 pounds of blueberries.</p> <p>How many pounds of blueberries did each student get?</p>
Diagram	

Student Responses

- Sample response for partner A: There are 4 bags of popcorn being shared equally with 6 people. How much popcorn does each person get?



- Sample response for partner B: There are 6 students sharing 4 sandwiches equally. How much sandwich does each student get? $4 \div 6 = \frac{4}{6}$
- Sample response for partner C: $4 \div 6 = \frac{4}{6}$

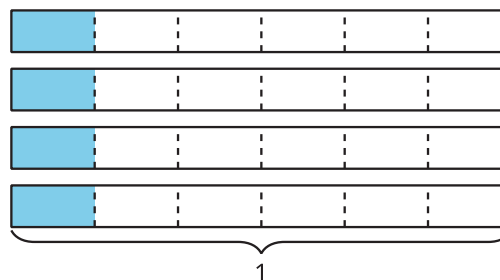
- Groups of 3
- Assign groups. Make sure each student knows who is Partner A, Partner B, and Partner C.

Activity

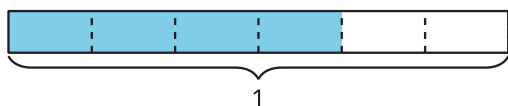
- 6–8 minutes: independent work time (problems 1, 2, 3)
- 4–5 minutes: group discussion
- Monitor for students who write division situations, for $a \div b$, where a represents the amount to be shared and b represents the number of equal shares.

Synthesis

- Select 2–3 students to share their problems and solutions. Show a variety of accurate student representations.
- Display equation: $4 \div 6 = \frac{4}{6}$.
- “How did you use the equation to help determine the number of objects and equal shares in your story?” (The numerator is the number of objects. The denominator is the number of shares. This story needs 4 objects and 6 equal shares.)
- Display image:



- “How did you use the diagram to help determine the number of objects and equal shares in your story?” (The number of objects is the number of wholes or large rectangles and the number of shares is the number of equal pieces each rectangle is divided into. There are 4 objects and 6



equal shares.)

- “How were your stories the same? How were they different?” (There were 4 objects in each and there are divided into 6 equal shares, but our contexts were different.)

Advancing Student Thinking

If students do not think of a situation that is represented by the equation, show them the image from the launch and ask, “How can the equation represent some people sharing some pounds of blueberries?”

Activity 2

🕒 15 min

Grams of Gold

Standards Alignments

Addressing 5.NF.B.3

The purpose of this activity is for students to solve problems about the same context. The context for both problems is equally splitting some gold. There are three related quantities:

- the total amount of gold collected
- the total number of friends who collect the gold
- the amount of gold each person gets

In previous lessons, students solved problems where the unknown was the amount each person gets after some people shared something equally. In this activity, the unknowns are the amount of gold the friends share and the number of friends sharing the gold. Students may need to draw diagrams to interpret the unknown. During the synthesis, connect the meaning of division to the meaning of a fraction.

🕒 Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Synthesis: Invite students to generate a list of additional examples of amounts that they can split that connect to their personal backgrounds and interests.

Supports accessibility for: Conceptual Processing, Attention

Student-facing Task Statement



1. A group of 3 friends spent the afternoon panning for gold. They shared the gold equally. If each friend got $\frac{4}{3}$ grams of gold, how much gold did they collect together? Explain or show your reasoning.
2. A group of friends spent the afternoon panning for gold. They shared the gold equally. If they collected 5 grams of gold together and each friend got $\frac{5}{6}$ grams of gold after they shared it, how many friends shared the gold? Explain or show your reasoning.

Student Responses

1. There were 3 friends and each friend got $\frac{4}{3}$ grams of gold so that's $3 \times \frac{4}{3}$ grams of gold altogether. They collected $\frac{12}{3}$ or 4 grams of gold.
2. There were 6 friends. Since $5 \div 6 = \frac{5}{6}$ that means that if 6 friends collect 5 grams of gold then each friend will get $\frac{5}{6}$ grams of gold.

Launch

- Display first image.
- "Tell a story about what is happening in this image."
- 1 minute: quiet think time
- 1 minute: partner discussion
- Display second image.
- "This is gold dust."
- "We are going to solve some problems about panning for gold."

Activity

- 5-8 minutes: partner work time
- Monitor for students who:
 - draw a diagram to explain or show the relationship between the number of friends and the denominator.
 - use words to explain or show the relationship between the number of friends and the denominator.

Synthesis

- Invite previously selected students to share their responses.
- Display the first situation that describes 3 friends sharing gold and each friend gets $\frac{4}{3}$ grams of gold.
- "What do we know about the situation?" (There are 3 friends and each friend gets $\frac{4}{3}$ grams of gold.)
- Display: $\frac{4}{3}$
- "How does $\frac{4}{3}$ represent the situation of friends sharing gold?" (It is the amount of gold each friend got.)

- “What does the 3 represent in $\frac{4}{3}$?” (Thirds. The amount they collected was split into 3 equal parts because there are 3 friends.)
- “What does the 4 represent in $\frac{4}{3}$?” (It represents how many thirds there are and also how many friends there are.)

Advancing Student Thinking

If students don’t have a way to start the problem, ask: “What do you know about the problem? What are you still trying to figure out?”

Lesson Synthesis

🕒 10 min

Display equation: $5 \div ? = \frac{5}{6}$

“How does this equation represent the situation about friends sharing gold?” (5 grams of gold are being shared by some friends and each friend gets $\frac{5}{6}$ grams of gold.)

“What number makes this equation true? How do you know?” ($5 \div 6 = \frac{5}{6}$; The first number in the division equation is the numerator, the number of objects being divided, and the second number is the denominator, the number of equal shares.)

Suggested Centers

- Rolling for Fractions (3–5), Stage 3: Divide Whole Numbers (Addressing)
- Target Measurements (2–5), Stage 4: Degrees (Supporting)

----- Complete Cool-Down -----

Response to Student Thinking

Students do not write the correct equation.

Next Day Support

- Before the warm-up, pass back the cool down and work in small groups to make corrections.

Lesson 5: Relate Division and Fractions

Standards Alignments

Addressing 5.NF.B.3

Teacher-facing Learning Goals

- Explain the relationship between division and fractions.

Student-facing Learning Goals

- Let's explain the relationship between division and fractions.

Lesson Purpose

The purpose of this lesson is for students to explain why $a \div b = \frac{a}{b}$ and apply their understanding to flexibly interpret division situations and equations where the unknown is the numerator, denominator, or the value of the quotient.

In this lesson, students generalize their understanding that a fraction can be interpreted as division of the numerator by the denominator. They interpret situations where a certain amount of pounds of blueberries is shared with a certain number of people when the pounds of blueberries each person gets is equal to 1, greater than 1, and less than 1. Then, they construct arguments about why an equation would make sense for any numerator and for any denominator. As they do so, they have a chance to use language precisely (MP6), explaining that the numerator a represents the number of objects being shared and the denominator b represents the number of equal shares.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Engagement (Activity 2)

Instructional Routines

MLR7 Compare and Connect (Activity 2), True or False (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min

Teacher Reflection Question

How has your thinking about division changed since the beginning of the unit? What evidence did you see during this section of the unit that

Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

each of your students extended their understanding of the meaning of division?

Cool-down (to be completed at the end of the lesson)

 5 min

Explain It.

Standards Alignments

Addressing 5.NF.B.3

Student-facing Task Statement

Explain why $8 \div 5 = \frac{8}{5}$.

Student Responses

Sample response: I can divide 8 into 5 equal parts. This is $8 \div 5$. Each of the parts is $\frac{1}{5}$ of one whole and since there are 8 wholes, $8 \div 5 = \frac{8}{5}$.

----- Begin Lesson -----

Warm-up

 10 min

True or False: Interpret Fractions

Standards Alignments

Addressing 5.NF.B.3

The purpose of this True or False is for students to demonstrate strategies and understandings they have for interpreting a fraction as division of the numerator by the denominator and vice versa. These strategies help students deepen their understanding of the relationship between division and fractions where the unknown is the numerator, denominator, or the value of the quotient.

Instructional Routines

True or False

Student-facing Task Statement

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $5 \div 2 = \frac{5}{2}$
- $\frac{5}{2} = 5\frac{1}{2}$
- $\frac{6}{2} = 3$

Student Responses

- True. If I divided 5 into two groups, there will be 5 halves in each group.
- False. $\frac{5}{2} = 5 \div 2$ and if I divide 5 into two equal groups there will be less than 5 in each group.
- True. Six halves is equal to 3 wholes.

Launch

- Display one statement.
- “Give me a signal when you know whether the statement is true and can explain how you know.”
- 1 minute: quiet think time

Activity

- Share and record answers and strategy.
- Repeat with each statement.

Synthesis

- Display:
 - $5\frac{1}{2}$
 - $\frac{5}{2}$
- “How are these expressions the same? How are they different?” (They both have 5 and 2 in them. They both show halves. $5\frac{1}{2}$ means 5 wholes plus one half and $\frac{5}{2}$ means 5 groups of one half.)

Activity 1

🕒 20 min

Relate Pounds to People

Standards Alignments

Addressing 5.NF.B.3

The purpose of this activity is for students to analyze several different situations about the same context of sharing pounds of blueberries and generalize what they have learned about the

relationship between fractions and quotients. Students rely on their understanding of the relationship between division and fractions to choose numbers that make sense based on the constraints listed in the table. During the synthesis, students generalize their understanding of the relationship between division situations and fractions greater than 1, less than 1, and equal to one whole.

Student-facing Task Statement

	Each person gets _____ pound(s) of blueberries.			
	more than 1	exactly 1	less than 1	$\frac{1}{2}$
_____ people share 7 pounds of blueberries	☒			
_____ people share _____ pounds of blueberries		☒		
Three people share _____ pounds of blueberries			☒	
_____ people share _____ pounds of blueberries				☒

1. Fill in the blanks to match the rules in the table.
2. How many pounds of blueberries did each person get when they got more than 1 pound of blueberries?
3. How many pounds of blueberries did each person get when they got less than 1 pound of blueberries?

Launch

- Groups of 2
- Display table from student workbook. Refer to the corresponding parts of the table and read, “each person gets exactly 1 pound of blueberries” and “_____ people share _____ pounds of blueberries.”
- “What numbers can we write in the blanks so that each person will get exactly 1 pound of blueberries?” (Sample responses: 3 and 3, 4 and 4, etc.)
- Record responses for all to see.
- “What is true about all of the pairs of numbers we used?” (Each blank has the same number in it.)
- “Why do the numbers in the blanks have to be the same?” (In order for each person to get exactly one pound of blueberries, the number of people sharing has to be the same as the number of pounds of fruit.)
- “We are going to solve more problems like this one. For each row in the table, write numbers in the blanks to fit the rule that is checked.”

Activity

- 5 minutes: partner work time
- “As you walk, notice how the numbers in the tables are the same and different.”
- 5 minutes: gallery walk
- Match each group of 2 with another group

(Pause for teacher directions.)

- Work with your group to make a poster that shows or explains your thinking about the questions below.
 - What is true about all of the pairs of numbers that were used when each person got less than 1 pound of blueberries?
 - What is true about all of the pairs of numbers that were used when each person got more than 1 pound of blueberries?
 - What is true about all of the pairs of numbers that were used when each person gets exactly $\frac{1}{2}$ pound of blueberries?

Student Responses

1. Sample responses:

- Each person gets more than 1 pound of blueberries: 5 people share 7 pounds of blueberries
- Each person gets exactly 1 pound of blueberries: 5 people share 5 pounds of blueberries.
- Each person gets less than 1 pound of blueberries: 3 people share 2 pounds of blueberries.
- Each person gets $\frac{1}{2}$ pound of blueberries: 4 people share 2 pounds of blueberries.

1. Sample response: $\frac{7}{5}$ pounds of blueberries

2. Sample response: $\frac{2}{3}$ pound of blueberries

- See activity synthesis for sample responses for what is true about all pairs of numbers.

of 2 so there are groups of 4.

- “Work with your group to make a poster about the numbers that were used in the tables.”
- 5 minute: group work time
- Monitor for groups who:
 - show or explain why the number of people is always less than the number of pounds of blueberries when each person gets more than one pound of blueberries.
 - show or explain why the number of people is always more than the number of pounds of blueberries when each person gets less than one pound of blueberries.
 - show or explain why the number of people is always $\frac{1}{2}$ the number of pounds of blueberries when each person gets $\frac{1}{2}$ pound of blueberries.

Synthesis

- Ask previously selected groups to share their posters.
- “What is the same about the pairs of numbers that represent each person getting more than one pound of blueberries?” (There are always more pounds of blueberries than there are people sharing the blueberries.)
- “What is the same about the pairs of numbers that represent each person getting less than one pound of blueberries?” (There are always less pounds of blueberries than there are people.)
- “What is the same about the pairs of numbers that represent each person getting exactly $\frac{1}{2}$ pound of blueberries?” (The number of people sharing the blueberries is always double the number of

pounds of blueberries.)

- “We are going to think more about these ideas in the next activity.”

Activity 2

🕒 15 min

Why Does It Work?

Standards Alignments

Addressing 5.NF.B.3

The purpose of this activity is for students to explain why $a \div b = \frac{a}{b}$ for any whole numbers a and b when b is not 0. Students may use words, equations, or diagrams to explain why this is true. In order to see a wide variety of interpretations, students take a gallery walk to observe their classmates' work. Then they discuss how words and diagrams help show the equation $a \div b = \frac{a}{b}$ for different values of a and b .

Constructing an argument that works for any pair of numbers requires thinking carefully about the meaning of the dividend, a , and the divisor, b . Students may use diagrams or situations to help communicate their thinking but will need to explain why these make sense for any numbers a and b (MP3).

This activity uses *MLR7 Compare and Connect*. Advances: Representing, Conversing.

♿ Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Synthesis: Optimize meaning and value. Invite students to share why every division expression can be written as a fraction with another teacher.
Supports accessibility for: Attention, Conceptual Processing

Instructional Routines

MLR7 Compare and Connect

Student-facing Task Statement

1. What numbers can replace the question

Launch

- Groups of 2

marks in each equation? Explain your reasoning.

$$? \div 2 = \frac{?}{2} \qquad 2 \div ? = \frac{2}{?}$$

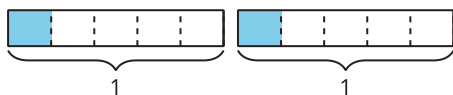
(Pause for teacher directions.)

- Work with your partner to explain why any division expression can be interpreted as a fraction. You can use diagrams, expressions, equations, and words.

Student Responses

- Sample responses:

- When 2 objects are divided into some equal shares I get 2 of those equal shares.
- When 2 people share some number of objects each person gets that number of halves.
- Diagram for $2 \div 5$



- Diagram for $3 \div 2$



- $1 \div 2 = \frac{1}{2}, 2 \div 2 = \frac{2}{2}, 3 \div 2 = \frac{3}{2}$
 - $2 \div 3 = \frac{2}{3}, 2 \div 4 = \frac{2}{4}, 2 \div 5 = \frac{2}{5}$.
- Sample response: Whenever you divide something into equal groups, you can use a fraction to show how much is in each group. The numerator in the fraction shows how much is being shared and it also shows how many parts each person gets. The denominator shows the number of people sharing and it also shows the size of each part.

Activity

- “Complete the first problem on your own.”
- 1-2 minutes: independent work time
- 2-3 minutes: partner discussion

MLR7 Compare and Connect

- “Create a visual display that shows your thinking about why every division expression can be interpreted as a fraction. You may want to include details such as words, diagrams, expressions, etc. to help others understand your thinking.”
- 3-5 minutes: partner work time
- 5 minutes: gallery walk
- “What is the same and what is different between the different explanations?”
- 30 seconds quiet think time
- 1 minute: partner discussion

Synthesis

- Display the image from the first problem in Student Responses or use student work.
- “How do diagrams help see that $? \div 2 = \frac{?}{2}$?” (They show a set of objects divided in half and also show the same number of halves as there are objects.)
- “How do the diagrams help see that $2 \div ? = \frac{2}{?}$?” (They show 2 things divided into equal parts and that’s the same as 2 of those equal parts.)
- Invite students to share contexts that they used to help understand the relationship between division and fractions.

Advancing Student Thinking

If students need more opportunities to explain the relationship between division and fractions, refer to work that was displayed during the gallery walk and ask students to explain the

representations in their own words.

Lesson Synthesis

🕒 10 min

Display and read: “What do you know about the relationship between division and fractions?” (Both can represent fair sharing situations. A fraction can mean division, for example, $2 \div 3$ can mean 3 people shared 2 things and each person gets $\frac{2}{3}$ of the thing.)

Record responses for all to see.

If not mentioned by students, ask, “How can we represent the relationship between division and fractions?” (We can use diagrams, situations, and equations to represent the relationship.)

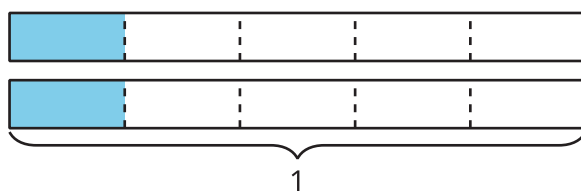
Suggested Centers

- Rolling for Fractions (3–5), Stage 3: Divide Whole Numbers (Addressing)
- Target Measurements (2–5), Stage 4: Degrees (Supporting)

✍ Student Section Summary

We learned that there is a relationship between division and fractions.

We can see this relationship in diagrams, situations, and equations. This diagram represents 2 sandwiches being shared equally by 5 people. Each person will get $\frac{2}{5}$ of a sandwich. The equation, $2 \div 5 = \frac{2}{5}$ also represents the situation.



----- Complete Cool-Down -----

Response to Student Thinking

Students do not explain why $8 \div 5 = \frac{8}{5}$.

Next Day Support

- Create a poster with important terms or vocabulary from this cool-down.

Section B: Fractions of Whole Numbers

Lesson 6: Relate Division and Multiplication

Standards Alignments

Building On	4.NF.B.4
Addressing	5.NF.B.3, 5.OA.A.2
Building Towards	5.NF.B.4

Teacher-facing Learning Goals

- Explore the relationship between multiplication and division.

Student-facing Learning Goals

- Let's explore the relationship between multiplication and division.

Lesson Purpose

The purpose of this lesson is for students to understand that dividing an amount into a whole number of equal parts can be interpreted as multiplying the same amount by a unit fraction.

In previous lessons, students interpreted a fraction as division of the numerator by the denominator, and equivalently, as a whole number divided into equal sized pieces. In this lesson, students relate division of two whole numbers to multiplying a whole number by a unit fraction. In the first activity, students are given an opportunity to solve a division problem using any strategy and, in the synthesis, they examine how the solution can be interpreted in terms of multiplication or division. In the second activity, students continue to explore the relationship between a fraction, a division expression, and a multiplication expression.

In grade 4, students multiplied a unit fraction by a whole number and in this lesson they begin to explore how to interpret a whole number multiplied by a unit fraction.

Access for:

English Learners

- MLR1 (Activity 2)

Instructional Routines

MLR2 Collect and Display (Activity 1), Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What was the best question you asked today?
Why was it the best?

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

A Different Relay Race**Standards Alignments**

Addressing 5.NF.B.3

Student-facing Task Statement

1. Lin and Han ran a 5 mile relay race as a team. They each ran the same distance. Draw a diagram to represent the situation.
2. How far did each student run?

Student Responses

1. Sample response:



2. $2\frac{1}{2}$ miles or $\frac{5}{2}$ mile. Sample response: The diagram shows 2 whole miles and $\frac{1}{2}$ of another mile.

----- **Begin Lesson** -----

Warm-up

🕒 10 min

Number Talk: Multiply and Divide

Standards Alignments

Building On 4.NF.B.4

Building Towards 5.NF.B.4

The purpose of this Number Talk is for students to demonstrate strategies and understandings they have for multiplying a fraction by a whole number. These understandings help students develop fluency and will be helpful later in the lesson when students explore the relationship between multiplication and division.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $3 \times \frac{1}{2}$
- $3 \times \frac{2}{2}$
- $3 \times \frac{3}{2}$
- $5 \times \frac{3}{2}$

Student Responses

- $\frac{3}{2}$: 3 groups of $\frac{1}{2}$ is $\frac{3}{2}$
- $\frac{6}{2}$: 3 groups of $\frac{2}{2}$ is $\frac{6}{2}$
- $\frac{9}{2}$: 3 groups of $\frac{3}{2}$ is $\frac{9}{2}$
- $\frac{15}{2}$ or $7\frac{1}{2}$: 5 groups of $\frac{3}{2}$ is 15 halves

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- Display equation: $\frac{15}{2} = 7\frac{1}{2}$
- "How do we know this is true?" (They are both equal to $15 \div 2$ or 7 is $\frac{14}{2}$ so $7\frac{1}{2}$ is $\frac{15}{2}$).

Activity 1

⌚ 20 min

The Race

Standards Alignments

Addressing 5.NF.B.3

Building Towards 5.NF.B.4

The purpose of this activity is for students to apply what they learned in earlier lessons to represent a division situation. During the synthesis, students connect what they know about division to multiplication when they see that the situation of 2 people equally sharing the distance in a 3 mile race can also be described as each person running one-half of the three mile race. Students connect the language they used to describe the situation to multiplication and division expressions. This relationship between multiplication and division is the focus of the next several lessons.

This activity uses *MLR2 Collect and Display*. Advances: Conversing, Reading, Writing.

Instructional Routines

MLR2 Collect and Display

Student-facing Task Statement



1. Lin and Han ran a 3 mile relay race as a team. They each ran the same distance. Draw a diagram to represent the situation.
2. Take turns describing to your partner how your diagrams represent the situation.

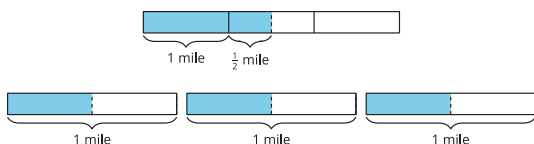
Launch

- Display the image from the student workbook.
- "Tell a story about this image to your partner."
- "This image shows two children who are running in a relay race. They are on the same team. They each have to run the same distance. The boy holding the baton is finishing his turn. When he hands the baton to his teammate, his teammate will take her turn."

3. How far did each person run?

Student Responses

1. Sample responses:



2. Answers vary.

3. $\frac{3}{2}$ miles or $1\frac{1}{2}$ miles

Activity

- Groups of 2
- 2 minutes: independent think time
- 5–8 minutes: partner work time

MLR2 Collect and Display

- Circulate, listen for and collect the language students use to describe how to represent the situation and how far each person ran.
- Listen for:
 - 3 miles divided into 2 parts
 - half of 3 miles
 - one and one half miles
 - three halves
- Record students' words and phrases on a visual display and update it throughout the lesson.

Synthesis

- "These are the words and phrases that you used to describe the situation. Are there any other words or phrases that are important to include on our display?"
- As students share responses, update the display, by adding (or replacing) language, diagrams, or annotations.
- Display: $3 \div 2$, $1\frac{1}{2}$, $\frac{3}{2}$, and $3 \times \frac{1}{2}$
- "How does each of these expressions represent the situation?" (3 miles was divided into 2 equal parts, each person ran $1\frac{1}{2}$ miles or $\frac{3}{2}$ miles which can be thought of as $3 \times \frac{1}{2}$.)
- Consider giving students time to record their answers in their journal.
- Display: "one half of 3 miles"
- "This is another way to describe how far each person ran."

- Display: $\frac{1}{2} \times 3$
- “We can also use this multiplication expression to represent one-half of 3.”
- “In the next activity, you will describe how different diagrams represent each of these expressions.”

Activity 2

🕒 15 min

Where Do You See It?

Standards Alignments

Addressing 5.NF.B.3, 5.OA.A.2

Building Towards 5.NF.B.4

The purpose of this activity is for students to build on the previous activity to interpret diagrams as representing different expressions. Both diagrams in the activity represent solutions to the running situation in the previous activity. In this activity, the diagrams are interpreted without the running context. As students use the diagrams to interpret expressions, they begin to see relationships between the expressions $\frac{3}{2}$, $1\frac{1}{2}$, $3 \times \frac{1}{2}$, $\frac{1}{2} \times 3$, and $3 \div 2$. In this activity, students make sense of different ways to interpret a given diagram and relate the operations of division and multiplication (MP7).

🌐 Access for English Learners

MLR1 Stronger and Clearer Each Time. Synthesis: Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their response to “How is each expression represented in the diagrams?” Invite listeners to ask questions, to press for details and to suggest mathematical language. Give students 2–3 minutes to revise their written explanation based on the feedback they receive.

Advances: Writing, Speaking, Listening

Student-facing Task Statement

Diagram A

Diagram B

Launch

- Groups of 2



For each expression, choose one of the diagrams and describe how the diagram represents the expression. Be prepared to explain why you chose that diagram.

1. $3 \times \frac{1}{2}$
2. $3 \div 2$
3. $\frac{1}{2} \times 3$

Student Responses

Sample responses:

1. (Diagram A) There are 3 shaded pieces and each is $\frac{1}{2}$ of a whole.
(Diagram B) There are three shaded pieces and each one is $\frac{1}{2}$ of one whole rectangle.
2. (Diagram A) There are three rectangles and one of 2 equal parts is shaded in each rectangle.
(Diagram B) There are three rectangles and the shaded part is equal to the unshaded part.
3. (Diagram A) There are three rectangles and $\frac{1}{2}$ of each one is shaded.
(Diagram B) One half of the three rectangles is shaded.

Activity

- 5 minutes: independent work time
- 5–8 minutes: partner discussion
- Monitor for students who choose different diagrams to represent the same expression.

Synthesis

- “What is the same about the diagrams?” (They both show 3 rectangles that are divided into 2 equal pieces.)
- “What is different about the diagrams?” (The shaded pieces are next to each other in one of the diagrams.)
- Ask previously selected students to share their explanations.
- Display expression: $3 \div 2$
- “Which diagram did you choose for the expression $3 \div 2$? Why?” (I chose diagram A because I see the 3 miles and the 2 equal parts. I chose diagram B because I can see the 2 equal parts that make 3 miles and can see that they make $1\frac{1}{2}$ miles.)
- Display expressions: $3 \times \frac{1}{2}$, $3 \div 2$, $\frac{1}{2} \times 3$
- “How do we know all the expressions are equal?” (Because the same diagram represents all of them or because they all equal $\frac{3}{2}$ or $1\frac{1}{2}$.)

Advancing Student Thinking

If students do not explain where they see each expression in one of the diagrams, refer to the expressions and ask: “How are these expressions the same? How are they different?” Then, refer to one of the numbers in one of the expressions and ask students to describe how the diagrams represent the number.

Lesson Synthesis

🕒 10 min

Record answers to the questions below for all to see and save responses to revisit during the synthesis of an upcoming lesson.

“When did we use multiplication today?” (In the first activity, we found one half of three miles and we can show that with $\frac{1}{2} \times 3$. The diagrams from the second activity represent $3 \times \frac{1}{2}$ and $\frac{1}{2} \times 3$.)

“When did we use division today?” (During the first activity, we divided 3 miles into 2 equal parts to figure out how far each person ran. The diagram from the second activity represents $3 \div 2$, and $\frac{3}{2}$.)

“What did we learn about the relationship between multiplication and division?” (We can use both of them to solve and represent the same problem.)

“What do you still wonder about the relationship between multiplication and division?” (Can we always use multiplication or division to solve a division problem?)

Suggested Centers

- Target Measurements (2–5), Stage 4: Degrees (Supporting)
- Compare (1–5), Stage 5: Fractions (Supporting)

----- Complete Cool-Down -----

Response to Student Thinking

Students do not respond that each person ran $2\frac{1}{2}$ miles or $\frac{5}{2}$ mile.

Next Day Support

- During the Activity 1 Synthesis, connect diagrams to expressions or equations.

Lesson 7: Divide to Multiply Unit Fractions

Standards Alignments

Addressing 5.NF.B.3, 5.NF.B.4.a

Building Towards 5.NF.B.4

Teacher-facing Learning Goals

- Connect division to multiplication of a whole number by a unit fraction.

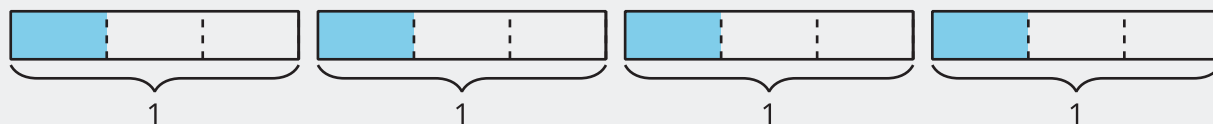
Student-facing Learning Goals

- Let's solve problems about multiplying whole numbers by unit fractions.

Lesson Purpose

The purpose of this lesson is for students to solve problems involving multiplication of whole numbers by unit fractions and represent the problems with equations and diagrams.

In this lesson students interpret situations and solve problems that involve products of a whole number and a fraction. Students solve story problems in a way that makes sense to them and match stories with diagrams and expressions. They work with expressions and flexibly interpret a diagram to extend their understanding of the relationship between fractions and multiplication and division. For example, consider this image:



It shows $4 \div 3$ because there are 4 whole squares divided into 3 equal parts with 1 of those parts shaded. It also shows $\frac{4}{3}$ as there are 4 pieces shaded and each is $\frac{1}{3}$ of a unit rectangle. It shows the multiplication expression $4 \times \frac{1}{3}$ because there are 4 groups of $\frac{1}{3}$ shaded. It shows the multiplication expression $\frac{1}{3} \times 4$ since there is a total of 4, and $\frac{1}{3}$ of that is shaded.

Access for:

Students with Disabilities

- Engagement (Activity 2)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Estimation Exploration (Warm-up), MLR7 Compare and Connect (Activity 1)

Materials to Copy

- Match the Situation (groups of 2): Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

Which student responses did you anticipate from today's lesson? Which student responses surprised you in today's lesson?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

Another Race

Standards Alignments

Addressing 5.NF.B.4.a

Student-facing Task Statement

Together, 6 children run a 5 mile relay race. They each run the same distance.

Select **all** the expressions that represent this situation.

- A. $\frac{1}{6} \times 5$
- B. $\frac{1}{5} \times 6$
- C. $5 \div 6$
- D. $\frac{5}{6}$

Student Responses

A, C, D

----- **Begin Lesson** -----

Warm-up

🕒 10 min

Estimation Exploration: Number Line

Standards Alignments

Addressing 5.NF.B.4.a

The purpose of this Estimation Exploration is for students to practice estimating a given length on a number line. Students are given the length of a longer segment as a point of reference and apply their understanding of equal parts to the number line to estimate a shorter length.

Instructional Routines

Estimation Exploration

Student-facing Task Statement

What number is marked on the number line?



Record an estimate that is:

too low	about right	too high

Student Responses

Sample responses:

- Too low: $\frac{1}{2}$ to 1
- About right: $1\frac{1}{4}$ to 2
- Too high: $2\frac{1}{2}$ to 3

Launch

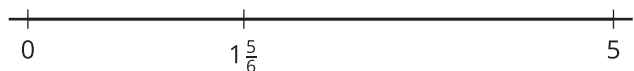
- Groups of 2
- Display the image.
- “What number could go in the box? What is an estimate that’s too high? Too low? About right?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Record responses.

Synthesis

- “Is the number that goes in the box more or less than half of 5? How do you know?” (Less, since it is less than half the way to 5.)
- “How does this help you estimate the value?” (I know half of 5 is $2\frac{1}{2}$, so it is less than that.)
- Optional: Reveal the actual value and add it to the display.



Advancing Student Thinking

If students do not explain why the number in the box is going to be less than half of 5, ask them to identify the approximate location of the numbers 1, 2, 3, and 4 on the number line.

Activity 1

🕒 20 min

How Far Did They Run?

👤 ↔ 👤 PLC Activity

Standards Alignments

Addressing 5.NF.B.3

Building Towards 5.NF.B.4

The purpose of this activity is for students to use the structure they noticed in the previous lesson to solve real world problems in which a whole number is multiplied by a unit fraction. Students may use a variety of strategies to solve these problems. They may relate the situations to multiplication of a whole number by a fraction or division of two whole numbers. During the synthesis, connect the different interpretations of the situations.

Students share their different representations and expressions and explain to each other how they relate to the running situation (MP3).

This activity uses *MLR7 Compare and Connect*. Advances: Representing, Conversing.

Instructional Routines

MLR7 Compare and Connect

Student-facing Task Statement

Solve each problem. Draw a diagram if it is helpful.

1. Mai ran $\frac{1}{4}$ the length of her road, which is 9 miles long. How far did Mai run?
2. Han ran $\frac{1}{4}$ the length of his road, which is 7 miles long. How far did Han run?

Launch

- Groups of 2

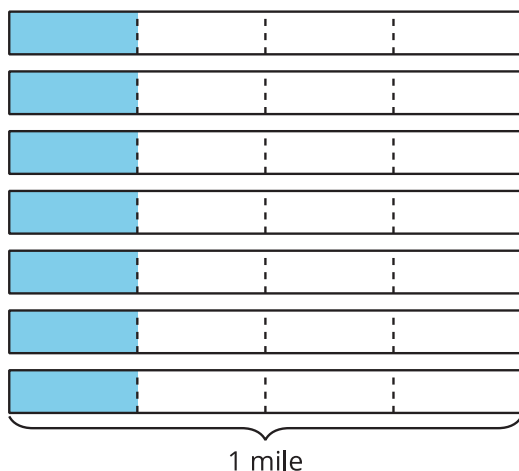
Activity

- 1–2 minutes: independent think time
- 5 minutes: partner work time
- Monitor for:
 - students who draw diagrams

Student Responses

Sample responses:

1. $\frac{9}{4}$ or $2\frac{1}{4}$: Sample response: $9 \div 4 = \frac{9}{4}$ or $2\frac{1}{4}$
2. $\frac{7}{4}$ or $1\frac{3}{4}$: Sample response: The diagram shows $\frac{1}{4}$ of 7 miles and there are 7 parts that are each $\frac{1}{4}$ of a mile so that's $\frac{7}{4}$ miles.



including continuous number line representations and discrete rectangular representations like the ones used in earlier lessons

- students who use different multiplication or division expressions
- students who write their final answer as a fraction or as a mixed number

MLR7 Compare and Connect

- “Create a visual display that shows your thinking about the second problem. You may want to include details such as notes, diagrams, drawings, etc., to help others understand your thinking.”
- 5–7 minutes: gallery walk
- “What is the same and what is different between the different approaches to solving the problem?”
- 30 seconds quiet think time
- 1 minute: partner discussion
- Additional questions could include:
 - “Does anyone have a question they would like to ask about a strategy or solution?”
- Consider asking students if they would like to revise their work before the synthesis.

Synthesis

- Ask previously selected students to share their solutions.
- “What are some expressions that represent the distance Han ran?” ($7 \div 4$, $1\frac{3}{4}$, $\frac{7}{4}$, $\frac{1}{4} \times 7$)
- “How does each of these expressions represent the distance Han ran?” (He ran $\frac{1}{4}$ of 7 miles and we can write that as $\frac{1}{4} \times 7$. We can figure out how many miles that is if we divide 7 into 4 equal parts. That’s $7 \div 4$

or $\frac{7}{4}$ or $1\frac{3}{4}$.)

Advancing Student Thinking

If students do not know how to figure out what $\frac{1}{4}$ of 7 or 9 miles is, ask them to figure out how far Mai would have run if she ran $\frac{1}{4}$ of an 8 mile road.

Activity 2

🕒 15 min

Match the Situation

Standards Alignments

Addressing 5.NF.B.3, 5.NF.B.4.a

The purpose of this activity is to match different expressions and diagrams with one situation. Some students may match the expressions, diagrams, and situation by finding the solution to the problem and the value of each expression. Some may match the representations without finding the value of the expressions. During the activity synthesis, highlight how the different expressions relate both to the situation and to the diagrams, and connect the relationships to the meaning of the expressions and diagrams.

Students reason abstractly and quantitatively (MP2) when they relate the story to the diagrams and expressions. All of the diagrams and expressions involve the same set of numbers so students need to carefully analyze the numbers in the story, the diagrams, and the expressions in order to choose the correct matches.

Access for English Learners

MLR8 Discussion Supports. Students should take turns finding a match and explaining their reasoning. Display the following sentence frames for all to see: “I noticed _____, so I matched . . .” and “_____ and _____ are the same/different because . . .” Encourage students to challenge each other when they disagree.

Advances: Speaking, Listening

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Synthesis: Invite students to generate a list of additional situations the images could represent that connect to their personal backgrounds and interests.

Supports accessibility for: Attention, Conceptual Processing

Materials to Copy

Match the Situation (groups of 2)

Required Preparation

- Create a set of cards from the Instructional master for each group of 2.

Student-facing Task Statement

Han, Lin, Kiran, and Jada together ran a 3 mile relay race. They each ran the same distance.

1. Find the expressions and diagrams that match this situation. Be prepared to explain your reasoning.
2. How far did each person run?

Student Responses

1. The matching expressions are: B, D, G, and H
The matching diagrams are: J, K, and M
2. Each person ran $\frac{3}{4}$ mile.

Launch

- Groups of 2

Activity

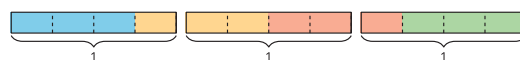
- 8 minutes: partner work time
- Monitor for students who match the expressions and diagrams by thinking about the meaning in each case.

Synthesis

- Display cards J and K.
- “How are these diagrams the same? How are they different?” (They both show $\frac{3}{4}$ or $3 \div 4$. The first one shows $\frac{1}{4}$ of each whole. In the second one, the shaded parts are all together, so they show $\frac{3}{4}$ of a single

whole.)

- Display the expression: $\frac{1}{4} \times 3$.
- “How does this expression relate to the situation?” (The race is 3 miles long and each person will run $\frac{1}{4}$ of the race.)
- “How do the diagrams represent the expression?” (In the first diagram $\frac{1}{4}$ of each whole is shaded. It is harder to see in the second diagram because I can't tell that the shaded parts are $\frac{1}{4}$ of the 3 rectangles.)
- “How can we adapt card K to show there are 4 equal sections of $\frac{3}{4}$?” (We could show the sections.)
- Mark card K to show the 4 equal sections of $\frac{3}{4}$. For example, use different colors to show the other 3 sections of $\frac{3}{4}$.



Advancing Student Thinking

If students do not match the diagrams and expressions correctly, ask students to explain how each diagram and expression represents each part of the situation.

Lesson Synthesis

🕒 10 min

“Today we learned how to find fractions of a whole number.”

“What does it mean to run $\frac{1}{4}$ of a 10 mile road?” (If you break up the 10 miles into 4 equal pieces, you ran 1 of those pieces.)

“What expressions can you write to represent $\frac{1}{4}$ of 10? ($\frac{1}{4} \times 10$, $10 \div 4$, $10 \times \frac{1}{4}$)

“Which expression helps you calculate how far $\frac{1}{4}$ of 10 miles is?” ($10 \div 4$ because I know it is $\frac{10}{4}$.)

Suggested Centers

- Target Measurements (2–5), Stage 5: Fractions of Angles (Addressing)
- Compare (1–5), Stage 5: Fractions (Supporting)

Complete Cool-Down

Response to Student Thinking

Students do not select the correct expressions.

Next Day Support

- Create a poster with a diagram that represents the cool-down from this lesson.

Lesson 8: Divide to Multiply Non-unit Fractions

Standards Alignments

Addressing 5.NF.B, 5.NF.B.4, 5.NF.B.4.a, 5.OA.A.2

Teacher-facing Learning Goals

- Connect division to multiplication of a whole number by a non-unit fraction.

Student-facing Learning Goals

- Let's solve problems about multiplying whole numbers by fractions.

Lesson Purpose

The purpose of this lesson is for students to represent and solve problems involving a non-unit fraction.

In this lesson, students to make sense of the product of a whole number and a non-unit fraction. Students relate the product of a whole number and a non-unit fraction to the product of a whole number and a unit fraction. They will have more opportunities to multiply a whole number by a fraction in the next section, systematically using the idea of area. This lesson continues to focus on the relationship between multiplication and division and encourages students to solve and interpret the problems in ways that make sense to them.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Engagement (Activity 2)

Instructional Routines

MLR2 Collect and Display (Activity 2), True or False (Warm-up)

Required Preparation

Gather the chart from the synthesis of a previous lesson that describes what students know and wonder about the relationship between multiplication and division.

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

If you were to teach this lesson again what would you do the same? What would you change?

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

Two Thirds

Standards Alignments

Addressing 5.NF.B.4

Student-facing Task Statement

Find the value of each expression. Explain or show your reasoning.

1. $\frac{1}{3} \times 4$
2. $\frac{2}{3} \times 4$

Student Responses

1. $\frac{4}{3}$ or equivalent: Sample response: $4 \div 3 = \frac{4}{3}$
2. $\frac{8}{3}$ or equivalent: Sample response: I doubled the answer to the first question.

----- **Begin Lesson** -----**Warm-up**

⌚ 10 min

True or False: A Fraction by a Whole Number

Standards Alignments

Addressing 5.NF.B, 5.OA.A.2

The purpose of this True or False is to elicit the strategies and insights students have for multiplying fractions by whole numbers. Students do not need to find the value of any of the expressions but rather can reason about properties of operations and the relationship between multiplication and division. In this lesson, they will see some ways to find the value of an expression like $\frac{2}{3} \times 6$.

Instructional Routines

True or False

Student-facing Task Statement

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $2 \times \left(\frac{1}{3} \times 6\right) = \frac{2}{3} \times 6$
- $2 \times \left(\frac{1}{3} \times 6\right) = 2 \times (6 \div 3)$
- $\frac{2}{3} \times 6 = 2 \times \left(\frac{1}{4} \times 6\right)$

Student Responses

- True: I can first multiply 2 and $\frac{1}{3}$ and that's $\frac{2}{3}$.
- True: Multiplying $\frac{1}{3}$ by 6 is the same as $6 \div 3$.
- False: $\frac{2}{3} \times 6 = 2 \times \frac{1}{3} \times 6$ so it can't be the same as $2 \times \frac{1}{4} \times 6$.

Launch

- Display one statement.
- "Give me a signal when you know whether the statement is true and can explain how you know."
- 1 minute: quiet think time

Activity

- Share and record answers and strategy.
- Repeat with each statement.

Synthesis

- "How can you explain why $\frac{2}{3} \times 6 = 2 \times \left(\frac{1}{4} \times 6\right)$ is false without finding the value of both sides?" (It can't be true because $\frac{2}{3} \times 6 = 2 \times \frac{1}{3} \times 6$.)

Activity 1

 15 min

Multiply a Whole Number by a Fraction

Standards Alignments

Addressing 5.NF.B.4.a

The purpose of this activity is for students to relate multiplying a non-unit fraction by a whole number to multiplying a unit fraction by the same whole number. After finding the value of $\frac{1}{5} \times 3$ in a way that makes sense to them, they then consider the value of the products $\frac{2}{5} \times 3$ and $\frac{3}{5} \times 3$. In the synthesis students address how they can use the value of $\frac{1}{5} \times 3$ to find the value other expressions.

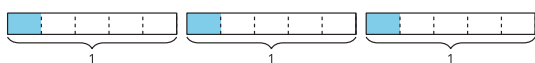
Student-facing Task Statement

Find the value of each expression. Explain or show your reasoning. Draw a diagram if it is helpful.

1. $\frac{1}{5} \times 3$
2. $\frac{2}{5} \times 3$
3. $\frac{3}{5} \times 3$

Student Responses

1. $\frac{3}{5}$ or equivalent. Sample response: There are 3 parts shaded and each one is $\frac{1}{5}$ of a whole.



2. $\frac{6}{5}$ or equivalent. Sample response: It's $\frac{2}{5}$ instead of $\frac{1}{5}$ so I doubled the previous answer.
3. $\frac{9}{5}$ or equivalent. Sample response: I added one more $\frac{1}{5} \times 3$.

Launch

- Groups of 2

Activity

- 8 minutes: independent work time
- Monitor for students who:
 - draw a diagram
 - use division to solve
 - recognize a relationship between $\frac{1}{5} \times 3$, $\frac{2}{5} \times 3$, and $\frac{3}{5} \times 3$.

Synthesis

- Ask previously selected students to share their solutions.
- Display: $\frac{1}{5} \times 3$, $\frac{2}{5} \times 3$, $\frac{3}{5} \times 3$
- “How are the expressions the same?” (They all have a 3. They all have some fifths and there is a product.)
- “How are the expressions different?” (The number of fifths is different. There is 1 and then 2 and then 3.)
- “How can you use the value of $\frac{1}{5} \times 3$ to help find the value of $\frac{2}{5} \times 3$?” (I can just double the result because it's $\frac{2}{5}$ instead of

$$\frac{1}{5}.)$$

- “What about $\frac{3}{5} \times 3$?” (That’s just another $\frac{1}{5} \times 3$.)
- Display diagram from student solution or a student generated diagram like it.
- “How does the diagram show $\frac{1}{5} \times 3$?” (There is 3 total and $\frac{1}{5}$ of it is shaded.)
- Display: $\frac{2}{5} \times 3$.
- “How could you adapt the diagram to show $\frac{2}{5} \times 3$?” (I could fill in 2 of the fifths in each whole instead of 1.)
- “In the next activity we will study a diagram for $\frac{2}{5} \times 3$ more.”

Activity 2

🕒 20 min

Match Expressions to Diagrams

Standards Alignments

Addressing 5.NF.B.4

The purpose of this activity is to interpret diagrams in multiple ways, focusing on different multiplication and division expressions. The repeating structure in the diagrams allows for many different ways to find the value and interpret the meaning of the expressions. Encourage students to use words, diagrams, or expressions to explain how the diagram represents each of the expressions.

Monitor for students who:

- can explain that the diagram represents the multiplication expression $3 \times \frac{2}{5}$ because it shows 3 groups of $\frac{2}{5}$
- can explain that the diagram represents $2 \times (3 \div 5)$ because there are 3 wholes divided into 5 equal pieces and 2 of the pieces in each whole are shaded

- can explain how the diagram represents the relationship between $\frac{6}{5}$ and $2 \times (3 \div 5)$

This activity gives students an opportunity to generalize their learning about fractions, division and multiplication. Students see shaded diagrams in different ways, representing different operations, and begin to see the operations as a convenient way to represent complex calculations (MP8).

This activity uses *MLR2 Collect and Display*. Advances: Conversing, Reading, Writing.

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Provide choice. Invite students to decide which expression to start with.

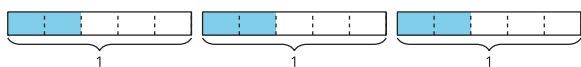
Supports accessibility for: Visual-Spatial Processing, Conceptual Processing, Attention

Instructional Routines

MLR2 Collect and Display

Student-facing Task Statement

Explain how each expression represents the shaded region.



1. $2 \times (3 \div 5)$
2. $\frac{6}{5}$
3. $3 \times \frac{2}{5}$
4. $3 \times 2 \times \frac{1}{5}$

Student Responses

Sample responses:

1. Each of the three rectangles is divided into 5 equal pieces and two of those pieces are shaded in.
2. There are 6 shaded rectangles and each one is $\frac{1}{5}$ so that's $\frac{6}{5}$ total.

Launch

- Groups of 2

Activity

- 5–10 minutes: partner work time

MLR2 Collect and Display

- Circulate, listen for, and collect the language students use to describe how each part of the expression represents each part of the diagram.
- Listen for language described in the narrative.
- Look for notes, labels, and markings on the diagrams that connect the parts of the diagram to the parts of the expressions.
- Record students' words and phrases on a visual display and update it throughout the lesson.

3. There are 3 groups of 2 shaded rectangles and the 2 shaded rectangles are $\frac{2}{5}$ of a unit so that's $3 \times \frac{2}{5}$.
4. There are 3 groups of 2 shaded rectangles and each is $\frac{1}{5}$ so that's $3 \times 2 \times \frac{1}{5}$.

Synthesis

- Display the expression: $3 \times \frac{2}{5}$
- “How does the diagram represent the expression?” (It shows 3 groups of $\frac{2}{5}$.)
- Display the expression: $2 \times (3 \div 5)$
- “How does the diagram represent the expression?”
- Display: $2 \times (3 \div 5) = \frac{6}{5}$
- “How do we know this is true?” (We can see both of them in the diagram. $3 \div 5$ is the same as $\frac{3}{5}$ and $2 \times \frac{3}{5} = \frac{6}{5}$)
- “Are there any other words, phrases, or diagrams that are important to include on our display?”
- As students share responses, update the display, by adding (or replacing) language, diagrams, or annotations.
- Remind students to borrow language from the display as needed.

Advancing Student Thinking

If students do not choose any expressions that represent the diagram, ask them to describe the diagram. Write down the words and phrases they use and ask, “Which expressions represent the words you used to describe the diagram?”

Lesson Synthesis

🕒 10 min

Revisit the chart about the relationship between multiplication and division created in an earlier lesson.

“What would you add to or revise about the relationship between multiplication and division?”

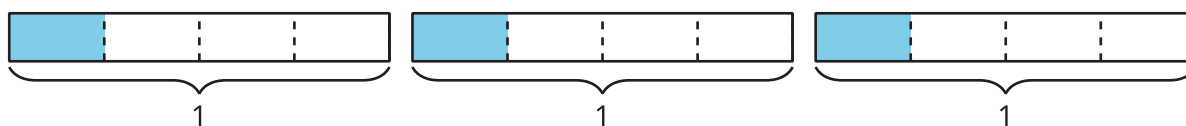
Revise chart as necessary.

Suggested Centers

- Target Measurements (2–5), Stage 5: Fractions of Angles (Addressing)
- Compare (1–5), Stage 5: Fractions (Supporting)

Student Section Summary

In this section, we explored the relationship between multiplication and division. We learned that 1 diagram can represent different multiplication and division expressions. For example, we can interpret this diagram with 4 different expressions:



- $\frac{3}{4}$ because each rectangle is divided into 4 equal parts and three of them are shaded.
- $3 \times \frac{1}{4}$ because there are 3 parts shaded and each one is $\frac{1}{4}$ of the rectangle.
- $3 \div 4$ because there are 3 rectangles and each one is divided into 4 equal parts.
- $\frac{1}{4} \times 3$ because there are 3 rectangles and $\frac{1}{4}$ of each one is shaded.

We know that all of these expressions are equal because they all represent the same diagram. We can use any of these expressions to represent and solve this problem:

- Mai ate $\frac{1}{4}$ of a 3 pound bag of blueberries. How many pounds of blueberries did Mai eat?

----- Complete Cool-Down -----

Response to Student Thinking

Students do not find the value of the expressions.

Next Day Support

- Before the warm-up, invite students to work in small groups to discuss a correct response to this cool-down.

Section C: Area and Fractional Side Lengths

Lesson 9: Relate Area to Multiplication

Standards Alignments

Addressing 5.NF.B.4.b

Teacher-facing Learning Goals

- Find the area of a rectangle with a unit fraction side length in a way that makes sense to them.

Student-facing Learning Goals

- Let's explore the area of rectangles with one side length that is a unit fraction.

Lesson Purpose

The purpose of this lesson is for students to calculate the area of a rectangle whose side lengths are a unit fraction and a whole number in a way that makes sense to them.

Students build on their understanding of multiplication and area from grade 3 as they work with areas with fractional side lengths. Students may count the number of smaller parts within a rectangle and will need to recognize and consider the size of these parts, which is a fraction of a square unit. It is important to be precise in the units used to describe the area and teachers should make sure to refer to "square units" rather than "squares."

As with the area work in grade 3, the commutative property may come up. The commutative property allows students to find products in a way that makes sense to them. For example a student might find the area of a rectangle with side lengths $\frac{1}{2}$ and 4 by thinking of 4 groups of $\frac{1}{2}$.

In Unit 6, students have another opportunity to interpret the product of a fraction and a whole number in the context of multiplication as scaling. This section is about area.

Access for:



Students with Disabilities

- Action and Expression (Activity 1)



English Learners

- MLR8 (Activity 1)

Instructional Routines

Which One Doesn't Belong? (Warm-up)

Materials to Copy

- Grid Paper 5 (groups of 2): Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

As students shared their ideas today, how did you ensure all students' voices were heard and valued as an important part of the collective learning?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

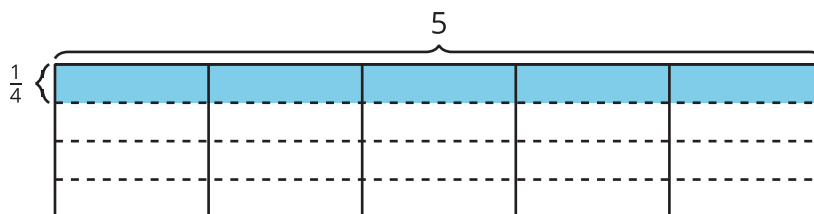
Fractional Pieces

Standards Alignments

Addressing 5.NF.B.4.b

Student-facing Task Statement

Find the area of the shaded region. Explain or show your reasoning.



Student Responses

The area is $\frac{5}{4}$ or $1\frac{1}{4}$ square units.

Sample response: I counted the shaded pieces which are fourths and figured out that I had enough to fill one unit square and $\frac{1}{4}$ of a second unit square.

 Begin Lesson

Warm-up

⌚ 10 min

Which One Doesn't Belong: Area

Standards Alignments

Addressing 5.NF.B.4.b

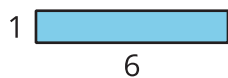
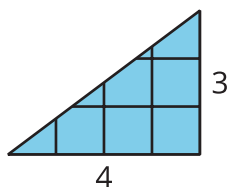
This warm-up prompts students to compare four images. It gives students a reason to use language precisely (MP6). It gives the teacher an opportunity to hear how students use terminology to describe the characteristics of area.

Instructional Routines

Which One Doesn't Belong?

Student-facing Task Statement

Which one doesn't belong?

A**B****C****D**

Launch

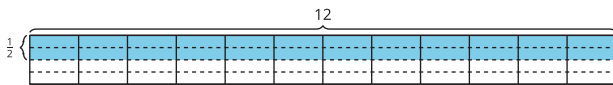
- Groups of 2
- Display the image.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 2–3 minutes: partner discussion
- Share and record responses.

Synthesis

- "Let's find at least one reason why each one doesn't belong."



Student Responses

- A doesn't belong because it doesn't have grid lines.
- B doesn't belong because it doesn't show any side lengths. It also isn't wider than it is tall.
- C doesn't belong because it is not a rectangle.
- D doesn't belong because it is not all shaded.

Activity 1

🕒 15 min

Find the Area

Standards Alignments

Addressing 5.NF.B.4.b

The purpose of this activity is for students to find the area of rectangles with one fractional side length and one whole number side length. Students begin by considering a rectangle with whole number side lengths and then look at a series of rectangles with unit fraction side length. All of the rectangles have the same whole number width to help students see how the area changes when the fractional width changes. Students should use a strategy that makes sense to them. These strategies might include counting the individual shaded parts in the diagram or thinking about moving them to fill in unit squares. Some students may use multiplication or division. These ideas will be brought out in future lessons. During discussion, connect the different strategies students use to calculate the areas. As they choose a strategy, they have an opportunity to use appropriate tools, whether it be expressions that represent the shaded area or physical manipulations of the diagrams, strategically (MP5).

Access for English Learners

MLR8 Discussion Supports. Prior to solving the problems, invite students to make sense of the situations and take turns sharing their understanding with their partner. Listen for and clarify any questions about the context.

Advances: Reading, Representing

Access for Students with Disabilities

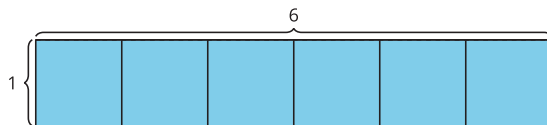
Action and Expression: Internalize Executive Functions. Invite students to plan a strategy, including the tools they will use, for finding the area of a rectangle that has one side length that is a fraction. If time allows, invite students to share their plan with a partner before they begin.

Supports accessibility for: Conceptual Processing, Memory, Language

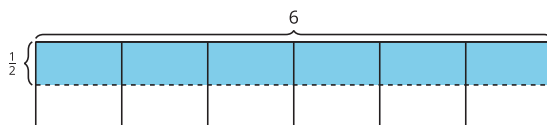
Student-facing Task Statement

Find the area of the shaded region. Explain or show your reasoning.

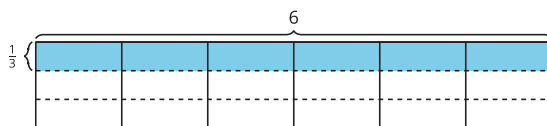
1.



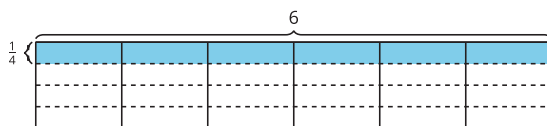
2.



3.



4.



Launch

- Groups of 2
- Display the images of the shaded rectangles.
- “What is the same about all of the rectangles? What is different?” (They are all shaded. They have different amounts shaded. They have different widths.)
- “We are going to figure out how much of each rectangle is shaded. We call this finding the area of the shaded region. What are some strategies we could use to find the area of each of the shaded regions?” (Move the pieces around to make full squares, count the number of blue pieces and multiply the number of pieces by their size.)

Activity

- 5–7 minutes: partner work time
- Encourage students to find the area in a way that makes sense to them.
- Monitor for students who:
 - count the number of shaded parts

Student Responses

1. 6 square units. Sample response: There are six shaded squares.
2. 3 square units. Sample response: Each unit square is half shaded and I pictured moving three of the shaded pieces to be in the same unit square as the other three pieces so that 3 whole squares would be shaded.
3. 2 square units. Sample response: Three groups of $\frac{1}{3}$ is 1 and 6 groups of $\frac{1}{3}$ is 2.
4. $\frac{6}{4}$ square units or equivalent. Sample response: There are 6 pieces and each one is $\frac{1}{4}$ of a square unit.

and multiply the total number of parts by their fractional area.

- visualize moving the shaded parts to fill whole unit squares.

Synthesis

- Ask previously selected students to share their reasoning.
- “What is the same about the strategies? What is different?” (They all counted the number of shaded parts, but they counted them in different ways. Some people multiplied and some people moved the parts to make whole unit squares.)
- Display image from final problem.
- “How does the expression $6 \times \frac{1}{4}$ represent the shaded area in square units?” (There are 6 shaded parts and each one has an area of $\frac{1}{4}$ square unit.)
- “How does the expression $\frac{1}{4} \times 6$ represent the shaded area in square units?” (There is a rectangle whose area is 6 square units and $\frac{1}{4}$ of the rectangle is shaded.)

Advancing Student Thinking

If students do not find the area of the shaded region, ask “How can you use the rectangle that has 6 unit squares shaded in to help you find the area of the other shaded regions?”

Activity 2

🕒 20 min

Draw Rectangles

Standards Alignments

Addressing 5.NF.B.4.b

The purpose of this activity is for students draw and shade rectangles with a unit fraction side length and a whole number side length. Then they find the areas of the shaded regions. The tactile experience of drawing and shading encourages students to count the number of shaded parts and then either reason about their size or think about moving them to make full unit squares. They also consider a diagram where not all of the unit squares are shown. Students estimate how many of the unit squares are hidden. This helps to highlight that finding the total area can be done with multiplication where one factor is the area of each shaded part and the other factor is the total number of shaded parts (MP7).

Materials to Copy

Grid Paper 5 (groups of 2)

Student-facing Task Statement

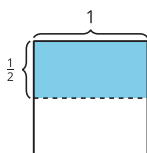
1. Represent each rectangle on grid paper:
 - $\frac{1}{2}$ unit by 1 unit
 - $\frac{1}{2}$ unit by 2 units
 - $\frac{1}{2}$ unit by 3 units
 - $\frac{1}{2}$ unit by 4 units
2. Find the area of each rectangle that you drew.
3. What information do you need to find the area of the shaded region?



4. What might the area of the shaded region be? Explain or show your reasoning.

Student Responses

1. Sample responses:



Launch

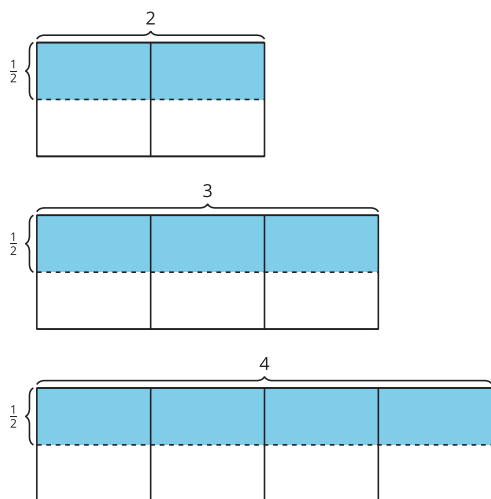
- Groups of 2
- Give students grid paper.

Activity

- 5–7 minutes: independent work time
- 5 minutes: partner work time
- Monitor for students who:
 - use the grid structure on the paper to draw their rectangles
 - count the number of unit squares that might be hidden under the yellow rectangle

Synthesis

- Display a student generated image of the $\frac{1}{2}$ unit by 4 unit rectangle.
- Ask previously selected students to describe how they drew they 4 by $\frac{1}{2}$ rectangle.
- Display image from student workbook of the rectangle that is partly covered.
- “What do you need to know to determine the area of the shaded region?” (We need to know how many unit squares are under



2. $\frac{1}{2}$ square unit, 1 square unit, $\frac{3}{2}$ square units, 2 square units
3. Sample response: I would need to know how many unit squares are under the yellow paper.
4. Sample response: 3 square units because I can see 4 half square units which I could put together to make 2 square units and then there are some more that I can't see. I think there might be 2 more so that would make 3 square units.

the yellow rectangle.)

- "How many unit squares do you think make up the rectangle?" (6 or 7)
- "How did you use the number of these unit squares to make an estimate for the shaded region?" (I know that each shaded blue rectangle is $\frac{1}{2}$ of a square so if there are 6 or 7 of those, that would be $\frac{6}{2}$ or $\frac{7}{2}$ square units.)

Advancing Student Thinking

If students do not draw the rectangles correctly, show them the shaded region from the previous activity with side lengths 6 units and $\frac{1}{2}$ unit and ask "How could you adapt this diagram to show a rectangle that is 4 units by $\frac{1}{2}$ unit?"

Lesson Synthesis

🕒 10 min

Display the shaded rectangle that has an area of 6 whole units:

"What strategies do we use to find the area of rectangles with 2 whole number side lengths?" (We can count the number of squares. We can multiply the side lengths.)

"What strategies did we use today to find the area of rectangles with a whole number side length and a

unit fraction side length?” (We counted the number of unit squares and multiplied by the size of the shaded region in each unit square.)

“How are the strategies we used to find the area of rectangles with whole number side lengths the same as and different from the strategies we use to find the area of rectangles with a whole number side length and a fractional side length?” (We can use the same strategies, but we count area that is less than one unit square. We are still multiplying, but one of the numbers is a fraction.)

Suggested Centers

- Target Measurements (2–5), Stage 5: Fractions of Angles (Addressing)
- How Close? (1–5), Stage 6: Multiply to 3,000 (Supporting)

Complete Cool-Down

Response to Student Thinking

The work in this lesson builds from relating multiplication to area addressed in a prior unit.

Prior Unit Support

Grade 3, Unit 2, Section B: Relate Area to Multiplication

Lesson 10: Fractional Side Lengths Less Than 1

Standards Alignments

Addressing 5.NF.B.3, 5.NF.B.4.a, 5.NF.B.4.b
 Building Towards 5.NF.B.4

Teacher-facing Learning Goals

- Find the area of a rectangle with one non-unit fractional side length.
- Represent the area of a rectangle with a multiplication expression.

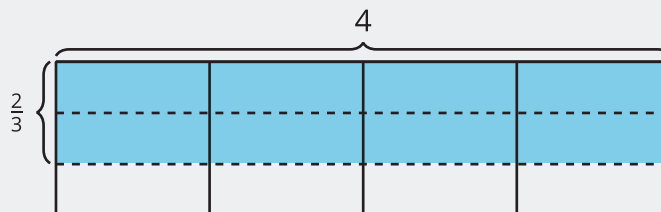
Student-facing Learning Goals

- Let's find the area of rectangles with a fractional side length.

Lesson Purpose

The purpose of this lesson is for students to find the area of rectangles with one non-unit fractional side length and one whole number side length.

In the previous lesson, students extended their understanding of multiplication to find the area of rectangles with a side length that is a unit fraction. In this lesson, students will find the area of rectangles with a whole number side length and a non-unit fraction side length. Students will apply what they learned in earlier lessons to area representations and recognize that a side length of $\frac{a}{b}$ is equivalent to a side length of $a \times \frac{1}{b}$. This allows them to find areas by counting the number of pieces covering the area and then multiplying this by the unit fractional area of each piece. For example, in the image below, there are 8 shaded pieces and each piece has an area of $\frac{1}{3}$ square unit.



Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR2 (Activity 2)

Instructional Routines

5 Practices (Activity 1), Estimation Exploration (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

In the next lesson, students will find the area of a rectangle where one of the side lengths is a fraction greater than 1. Try finding the area of a rectangle that is $\frac{5}{4}$ by 6. How do the understandings in today's lesson support how you found the area of that rectangle?

Cool-down (to be completed at the end of the lesson)

🕒 5 min

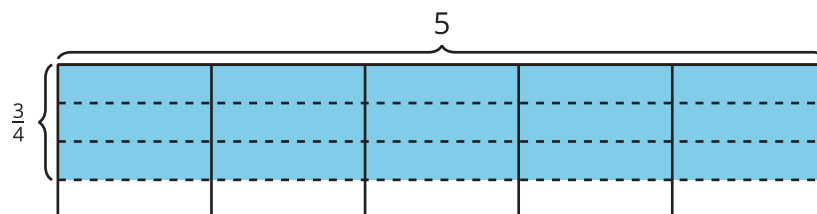
A Fractional Side Length

Standards Alignments

Addressing 5.NF.B.4.b

Student-facing Task Statement

1. Write a multiplication expression to represent the area of the shaded region.



2. Find the area of the shaded region.

Student Responses

1. $\frac{3}{4} \times 5$ or $5 \times \frac{3}{4}$
2. $\frac{15}{4}$ or $3\frac{3}{4}$ square units

Begin Lesson

Warm-up

🕒 10 min

Estimation Exploration: What is the Area?

Standards Alignments

Addressing 5.NF.B.4.b

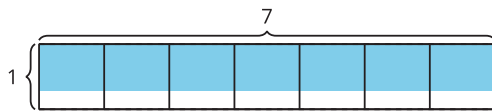
The purpose of this Estimation Exploration is for students to practice the skill of estimating a reasonable answer based on experience and known information. It gives students a low-stakes opportunity to estimate the area of a rectangle when one side is not a unit fraction.

Instructional Routines

Estimation Exploration

Student-facing Task Statement

What is the area of the shaded region?



Record an estimate that is:

too low	about right	too high

Student Responses

too low	about right	too high
2-3	4-6	$6\frac{1}{2} - 7$

Launch

- Groups of 2
- Display the image.
- “What might be the area of the shaded region?”

Activity

- 1 minute: quiet think time
- 1 minute: partner discussion
- Record responses.

Synthesis

- “Is more than half or less than half of the rectangle shaded?” (More than half)
- “How can you use this to help make your estimate?” (Half of 7 is $3\frac{1}{2}$ so it’s a little more than that.)
- “Based on this discussion does anyone want to revise their estimate?”

Activity 1

🕒 20 min

Rectangle With a Fractional Side Length

Standards Alignments

Addressing 5.NF.B.3

Building Towards 5.NF.B.4

The purpose of this activity is for students to find the area of rectangles with a side length that is a non-unit fraction. Students may use a variety of strategies to find the areas of the shaded region. Monitor for students who are noticing and using the structure of the rectangle and expressions to determine the area (MP7) by:

- grouping the shaded pieces with a fractional area to make whole unit squares
- multiplying the whole number of units by the numerator of the fractional side length and dividing the result by the denominator

🕒 Access for Students with Disabilities

Representation: Internalize Comprehension. Begin by asking, “Does this problem/situation remind anyone of something we have seen/read/done before?”

Supports accessibility for: Memory, Conceptual Processing

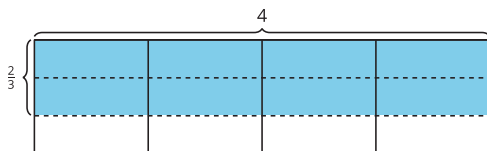
Instructional Routines

5 Practices

Student-facing Task Statement

Write a multiplication expression to represent the area of each shaded region. Then find the area.

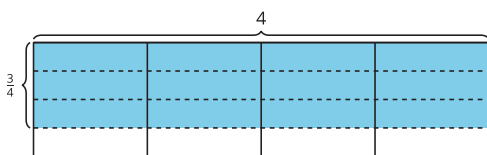
1.



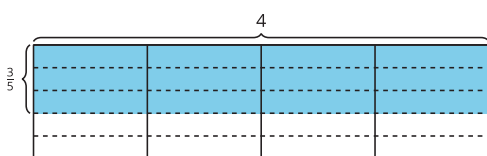
Launch

- Display image from warm-up.
- “If the height of the shaded region was $\frac{5}{6}$ of a square unit, what expression could you write to represent the area of the shaded region?” ($\frac{5}{6} \times 7$ or $7 \times \frac{5}{6}$)
- Groups of 2

2.



3.



Student Responses

1. $\frac{2}{3} \times 4$ or $4 \times \frac{2}{3}$ or $8 \times \frac{1}{3}$; the area is $\frac{8}{3}$ or $2\frac{2}{3}$ square units
2. $\frac{3}{4} \times 4$ or $4 \times \frac{3}{4}$ or $12 \times \frac{1}{4}$; the area is $\frac{12}{4}$ or 3 square units
3. $\frac{3}{5} \times 4$ or $4 \times \frac{3}{5}$ or $12 \times \frac{1}{5}$; the area is $\frac{12}{5}$ or $2\frac{2}{5}$ square units

Activity

- 1–2 minutes: quiet think time
- 5–8 minutes: partner work time

Synthesis

- Select previously identified students to share their strategies.
- “How are the second and third shaded regions the same? How are they different?” (They are each 4 units long. There are 12 shaded pieces in each. The 12 pieces in the second example are $\frac{1}{4}$ of a unit square. The 12 shaded pieces in the third shaded region are $\frac{1}{5}$ of a unit square.)
- Display the expression: 4×3
- “How does this expression relate to the second and third shaded regions?” (In both of them, the number of shaded pieces is 4×3 .)
- “Why are the areas of these two shaded regions different?” (The small pieces are not the same size. They are $\frac{1}{4}$ of a unit square in one and $\frac{1}{5}$ in the other.)
- Display the expressions: $12 \times \frac{1}{4}$ and $12 \times \frac{1}{5}$
- “The expressions for the area show that there are 12 shaded pieces in both but they are different sizes.”

Activity 2

⌚ 15 min

What Are the Side Lengths?

Standards Alignments

Addressing 5.NF.B.4.a, 5.NF.B.4.b

The purpose of this activity is for students to find the area of rectangles with a fractional side length. The side lengths are not labeled, so students will have to determine them by considering the number and size of the shaded pieces.

In the second problem, students are given expressions and they determine whether or not each expression represents the shaded area in a given diagram. The numbers in the expressions are similar, so students need to consider the structure of the expressions and the shaded regions. While they may match the expressions with the diagrams based on their value, students are encouraged to look for and clearly express how the diagrams represent the different expressions (MP6, MP7).

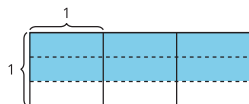
Access for English Learners

MLR2 Collect and Display. Circulate, listen for and collect the language students use as they determine the area. On a visible display, record words and phrases such as: length, shaded, fraction, pieces, width, multiply, area, whole, part, expression. Invite students to borrow language from the display as needed, and update it throughout the lesson.

Advances: Conversing, Representing

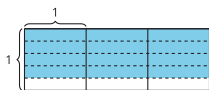
Student-facing Task Statement

- Write a multiplication expression to represent the area of the shaded region.
What is the area?

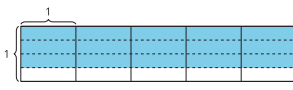


- Here are two diagrams. Consider each expression and decide whether it represents the shaded region in one of the diagrams. Be prepared to share your thinking.

X



Y



- $\frac{3}{4} \times 5$
- $3 \times \frac{3}{5}$
- $3 \times 4 \times \frac{1}{5}$

Launch

- Groups of 2

Activity

- 2–3 minutes: quiet think time
- 5–6 minutes: partner work time
- Monitor for students who:
 - correctly determine the side lengths of the shaded region.
 - use the side lengths to determine a correct multiplication expression.
 - refer to the number of shaded pieces and the size of the shaded pieces when determining the area.

Synthesis

- "How did you determine the side lengths for shaded region in diagram X?" (I looked at the side lengths of the squares and the

d. $4 \times \frac{3}{4}$

e. $3 \times 3 \times \frac{1}{4}$

3. For each diagram, what is the area?

Student Responses

1. $\frac{2}{3} \times 3$ or $3 \times \frac{2}{3}$; the area is $\frac{6}{3}$ or 2 square units

2. Matches:

a. Y

b. neither

c. X

d. neither

e. neither

3. The area of figure Y is $\frac{15}{4}$ or $3\frac{3}{4}$. The area of figure X is $\frac{12}{5}$ or $2\frac{2}{5}$.

partitions, then I counted how many are shaded.)

- Display the expression: $3 \times 4 \times \frac{1}{5}$
- "How does this expression represent the shaded region in diagram X?" (The vertical rectangle has 3 rows of 4 shaded pieces, so that's 3×4 shaded pieces. Each one has an area of $\frac{1}{5}$ square unit.)

Advancing Student Thinking

If students do not write the correct expression, show them the correct expression and ask, "How does the expression represent the area of the corresponding diagram?"

Lesson Synthesis

🕒 10 min

"Today we found the area of rectangles with a whole number side length and a fractional side length."

Display image from first problem in the last activity.

"What are some different expressions that represent the shaded region?" ($\frac{2}{3} \times 3$, $2 \times \frac{1}{3} \times 3$, $\frac{6}{3}$)

"Pick one of the expressions and explain to your partner how it represents the shaded region." (There are 3 unit squares and $\frac{2}{3}$ of them are shaded. There are two shaded rows and each one is $\frac{1}{3}$ of 3 squares. There are 6 shaded parts and each one is $\frac{1}{3}$.)

Suggested Centers

- Target Measurements (2–5), Stage 5: Fractions of Angles (Addressing)
- How Close? (1–5), Stage 6: Multiply to 3,000 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students do not find the correct area of the shaded region.

The work in this lesson builds from relating multiplication to area addressed in a prior unit.

Next Day Support

- Throughout the next lesson, ask: “How does the multiplication expression represent the area of the shaded region?”

Prior Unit Support

Grade 3, Unit 2, Section B: Relate Area to Multiplication

Lesson 11: Fractional Side Lengths Greater Than 1

Standards Alignments

Addressing 5.NF.B.3, 5.NF.B.4.b

Teacher-facing Learning Goals

- Find the area of a rectangle with one fractional side length greater than 1 in a way that makes sense to them.
- Represent the area of a rectangle with a multiplication expression.

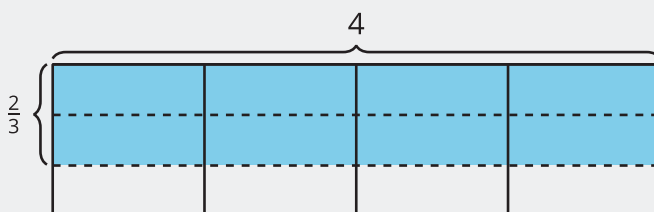
Student-facing Learning Goals

- Let's find the area of more rectangles.

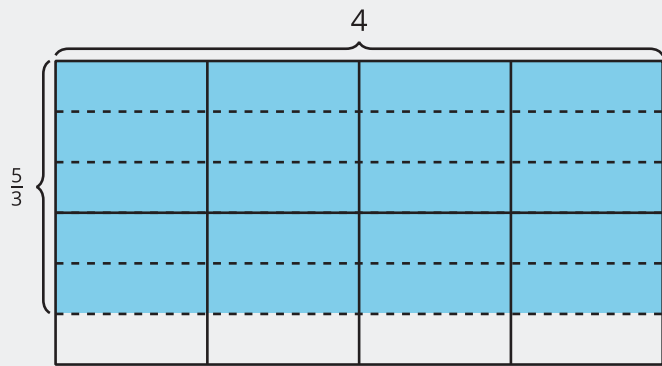
Lesson Purpose

The purpose of this lesson is for students to find the area of a rectangle where one of the side lengths is a fraction greater than 1.

In previous lessons, students multiplied fractions by whole numbers and found the area of rectangles with one fractional side length when the fraction was less than 1. They used visual representations to support their reasoning. For example, students use this picture to explain why $\frac{2}{3} \times 4 = (2 \times 4) \times \frac{1}{3}$.



In this lesson, students apply these strategies to find the area of a rectangle with a fractional side length greater than 1.



Using an area diagram like this, the same reasoning shows that $\frac{5}{3} \times 4 = (5 \times 4) \times \frac{1}{3}$.

Access for:

 Students with Disabilities

- Engagement (Activity 2)

 English Learners

- MLR8 (Activity 1)

Instructional Routines

5 Practices (Activity 1), True or False (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

- Why is it important for students to be able to write and interpret different expressions to represent and find the area of rectangles with fractional side lengths?

Cool-down (to be completed at the end of the lesson)

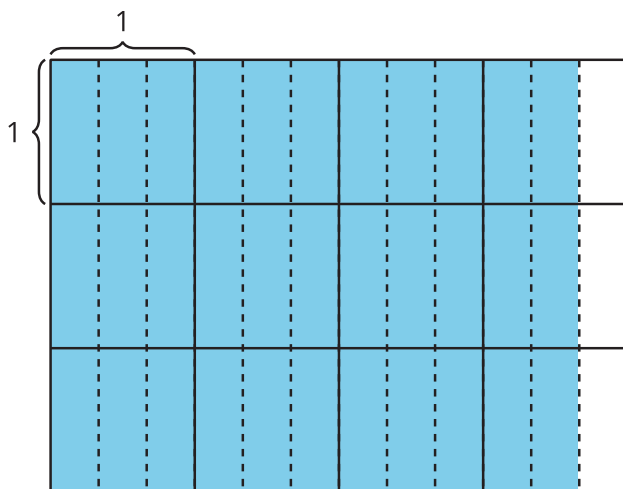
 5 min

Find the Area

Standards Alignments

Addressing 5.NF.B.4.b

Student-facing Task Statement



1. Write a multiplication expression to represent the area of the shaded region.
2. What is the area of the shaded region?

Student Responses

1. $3 \times \frac{11}{3}$ or $\frac{11}{3} \times 3$ or $\frac{11 \times 3}{3}$ or $\frac{3 \times 11}{3}$
2. 11 square units or equivalent

Begin Lesson

Warm-up

🕒 10 min

True or False: Thirds

Standards Alignments

Addressing 5.NF.B.3

The purpose of this True or False is for students to demonstrate strategies they have for relating division of two whole numbers to multiplication of a fraction by a whole number. The reasoning students use here helps to deepen their understanding of the relationship between multiplication and division. It will also be helpful later when students find the area of rectangles with mixed number side lengths.

Instructional Routines

True or False

Student-facing Task Statement

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $10 \div 3 = 10 \times \frac{1}{3}$
- $10 \div 3 = 10\frac{1}{3}$
- $\frac{10}{3} = 5 \times \frac{2}{3}$

Student Responses

- True: There are 10 groups of $\frac{1}{3}$ in $\frac{10}{3}$.
- False: $10\frac{1}{3}$ is greater than 10 and much greater than $10 \div 3$.
- True: 5 groups of $\frac{2}{3}$ is $\frac{10}{3}$.

Launch

- Display one equation.
- "Give me a signal when you know whether the statement is true and can explain how you know."
- 1 minute: quiet think time

Activity

- Share and record answers and strategy.
- Repeat with each statement.

Synthesis

- "How can you explain your answer to the last statement without finding the value of both sides?"

Activity 1

Greater Than One

🕒 20 min

👤 ↔ 👤 PLC Activity

Standards Alignments

Addressing 5.NF.B.4.b

The purpose of this activity is for students to multiply a whole number by a fraction greater than 1 in a way that makes sense to them. Monitor for students who:

- can explain why the area of the shaded region is $18\frac{2}{3}$ or $16\frac{8}{3}$ by counting the number of shaded whole square units and then counting the number of shaded third of a square units
- can explain how to use the expression $4\frac{2}{3} \times 4$ to find the area of the shaded region
- can explain how to use the expression $\frac{14}{3} \times 4$ to find the area of the shaded region

As students work with fractions greater than 1, they may choose to write or rewrite them as mixed numbers. Students may also relate the expressions to the diagrams in different ways. Encourage them to interpret the diagram and find the area of the shaded region in whatever way makes sense to them. During the synthesis, show the relationships between the different ways of finding and representing the area.

Identifying which expressions represent the area of the rectangle requires careful analysis of the expressions and the figure and the correspondences between them (MP7).

Access for English Learners

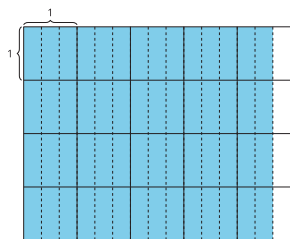
MLR8 Discussion Supports. Display sentence frames to support small-group discussion: “I wonder if . . .”, “_____ and _____ are the same because. . .”, and “_____ and _____ are different because . . .”
Advances: Conversing, Representing

Instructional Routines

5 Practices

Student-facing Task Statement

1. Find the area of the shaded region in square units. Explain or show your reasoning.



2. Select **all** the expressions which represent the area of the shaded region in square

Launch

- Groups of 2

Activity

- 1–2 minutes: quiet think time
- 5 minutes: partner work time
- As students work, consider asking:
 - “How did you calculate the area of the shaded region?”
 - “How do you know your answer makes sense?”

units. For each correct expression, explain your reasoning.

- A. $4\frac{2}{3} \times 4$
- B. $16 \times \frac{8}{3}$
- C. $\frac{14}{3} \times 4$
- D. $\frac{56}{3}$
- E. $4 \times \frac{5}{3}$

Student Responses

1. Sample responses:

- The area is $16\frac{8}{3}$ square units because there is a 4 by 4 array that has an area of 16 square units and then I see 2, 4, 6, 8 thirds.
- The area is $18\frac{2}{3}$ square units because there is a 4 by 4 array that has an area of 16, and then I combined the leftover thirds to make 2 more whole units and two thirds of a square unit.
- The area is $\frac{56}{3}$ because there are 56 little pieces and each one has an area of $\frac{1}{3}$ square unit.

2.

A. Sample response: $4\frac{2}{3} \times 4$ represents the area because those are the side lengths of the rectangle.

C. Sample response: $\frac{14}{3} \times 4$ because the length of the rectangle is $\frac{14}{3}$ units and the width is 4 units.

D. Sample response: $\frac{56}{3}$ because that is how many thirds of a square unit are shaded.

Synthesis

- Ask selected students to share in the given order.
- “How does $\frac{56}{3}$ represent the area of the shaded region?” (There are 56 pieces shaded in and each piece has an area of $\frac{1}{3}$ of a unit square.)
- Display: $4\frac{2}{3} \times 4 = \frac{14}{3} \times 4$
- “How do we know these expressions are equal?” (They both represent the shaded area in the diagram or I know that 3, 6, 9, 12 thirds is 4 wholes and then there are two thirds left.)

Advancing Student Thinking

If students do not interpret the factors as side lengths of rectangles, encourage them to listen to a partner explain where they see the multiplication expression and then describe what their partner says in their own words.

Activity 2

🕒 15 min

Diagrams and Expressions for Area

Standards Alignments

Addressing 5.NF.B.4.b

The purpose of this activity is for students to find areas of rectangles where one side is a whole number and the other side is a fraction that is greater than 1. Students should solve the problems in a way that makes sense to them. Ask students to explain how the diagrams show the multiplication expressions.

🕒 Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Synthesis: Optimize meaning and value. Invite students to share how they use the diagram to calculate the area of the desk or the garden and the method they used to do so with a classmate.

Supports accessibility for: Conceptual Processing, Attention

Student-facing Task Statement

1. a. Write a multiplication expression to represent the area of the shaded region.

Launch

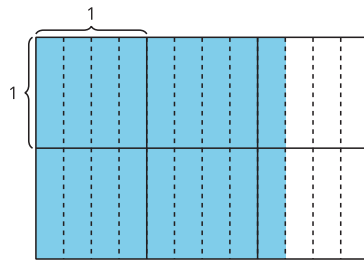
- Groups of 2

Activity

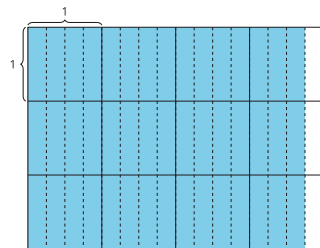
- 5 minutes: individual work time
- 5 minutes: partner work time

Synthesis

- Ask several students to share their



- b. What is the area of the shaded region?
2. a. Write a multiplication expression to represent the area of the shaded region.



- b. What is the area of the shaded region?

Student Responses

1. a. Sample response: $2\frac{1}{4} \times 2$
- b. The shaded region is 2 units by $2\frac{1}{4}$ units so I need to find $2 \times 2\frac{1}{4}$. That is $4\frac{1}{2}$ square units.
2. a. Sample response: $3 \times 3\frac{3}{4}$
- b. The shaded region has a part that is 3 units by 3 units which is 9 square units and a part that is $\frac{3}{4}$ units by 3 units and that has an area of $\frac{9}{4}$ square units or $2\frac{1}{4}$ square units. The total area is $11\frac{1}{4}$ square units.

responses to the second problem.

- Display:
 - $3 \times 3\frac{3}{4}$
 - $3 \times \frac{15}{4}$
- "How do these expressions represent the area of the shaded region?" (They are the width and length of the shaded region. In one, the length is a whole number and some fourths, and in the other, it is all fourths.)
- "How is finding the value of these two expressions different?" (For the first one, I can multiply 3 by 3 and then 3 by $\frac{3}{4}$ and add them together. For the second one, I can see how many fourths I have using multiplication.)

Advancing Student Thinking

If students do not write multiplication expressions to represent the area of the shaded region,

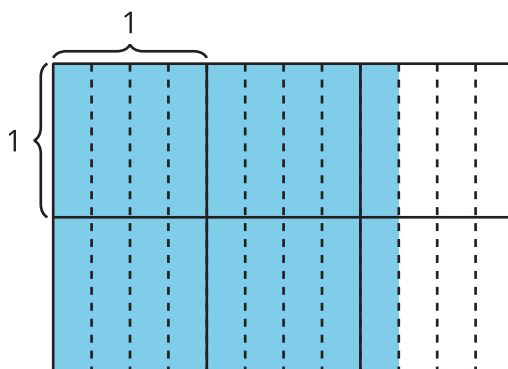
prompt them to explain how they found the area of the shaded region. Then, write expressions to represent the student's strategy and ask, "How do these expressions represent your strategy?"

Lesson Synthesis

⌚ 10 min

"Today we learned that we can apply our understanding of multiplication to find the area of a rectangle with a side length that is a fraction greater than 1."

Display the image.



Display the expression: $2 \times 9 \times \frac{1}{4}$

"How does the diagram represent the expression?" (There are 2×9 small pieces and each one has an area of $\frac{1}{4}$ square unit.)

Display the expression: $2 \times \frac{9}{4}$

"How does the diagram represent the expression?" (There are two rows of shaded pieces and each row has area $\frac{9}{4}$ square units.)

Display the expression: $(2 \times 2) + (2 \times \frac{1}{4})$

"How does the diagram represent the expression?" (There's a 2 by 2 array of whole square units and then there are 2 shaded pieces each having area $\frac{1}{4}$ square unit.)

Suggested Centers

- How Close? (1–5), Stage 7: Multiply Fractions and Whole Numbers to 5 (Addressing)
- How Close? (1–5), Stage 6: Multiply to 3,000 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students write an expression that does not match the area.

The work in this lesson builds from relating multiplication to area addressed in a prior unit.

Next Day Support

- During the warm up of the next lesson, draw a diagram to represent the last expression in the number talk.

Prior Unit Support

Grade 3, Unit 2, Section B: Relate Area to Multiplication

Lesson 12: Decompose Area

Standards Alignments

Addressing 5.NF.B.4, 5.NF.B.4.b

Teacher-facing Learning Goals

- Decompose a rectangle to find its area.

Student-facing Learning Goals

- Let's decompose rectangles to find their area.

Lesson Purpose

The purpose of this lesson is for students to apply what they know about multiplication of whole numbers and fractions to decompose a rectangle to find its area.

In previous lessons, students found the area of rectangles with one whole number side length and one fractional side length.

The purpose of this lesson is for students to apply what they know about decomposing rectangles with whole number side lengths to represent and find the area of rectangles with a mixed number side length. Students first find the area in a way that makes sense to them and then analyze different strategies for finding area.

Access for:



Students with Disabilities

- Engagement (Activity 2)



English Learners

- MLR1 (Activity 1)

Instructional Routines

Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min

Teacher Reflection Question

What question do you wish you had asked today? When and why should you have asked it?

Cool-down

5 min

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

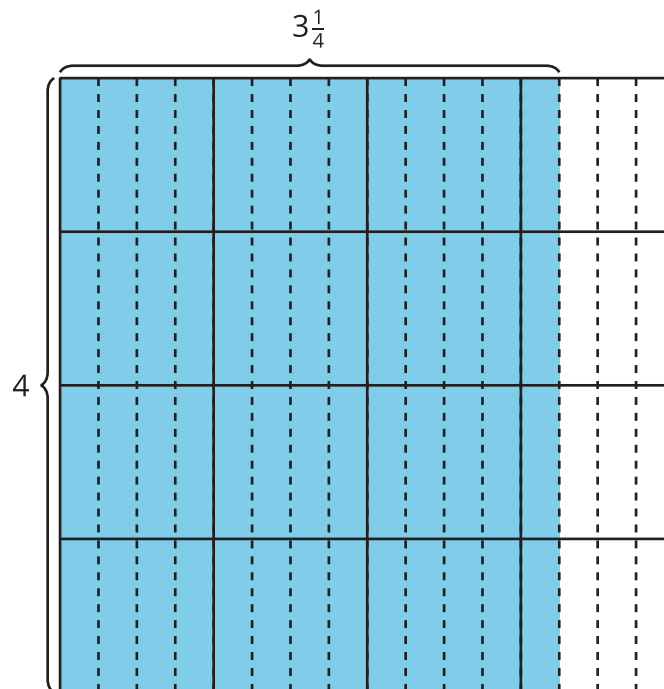
Decompose Rectangles

Standards Alignments

Addressing 5.NF.B.4.b

Student-facing Task Statement

Find the area of the shaded region.

**Student Responses**

Sample responses:

- $4 \times 3\frac{1}{4}$ square units
- 13 square units

Begin Lesson

Warm-up

🕒 10 min

Number Talk: Partial Products**Standards Alignments**

Addressing 5.NF.B.4

The purpose of this Number Talk is for students to demonstrate strategies and understandings they have for multiplying a mixed number by a whole number. These understandings help students develop fluency and will be helpful later in this lesson when students decompose rectangles with fractional side lengths to find the area.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- 3×20
- 3×24
- 5×2
- $5 \times 2\frac{1}{2}$

Student Responses

- 60: $3 \times 2 = 6$, so $3 \times 20 = 60$
- 72: $3 \times 20 = 60$, $3 \times 4 = 12$
- 10: I just know it.
- $12\frac{1}{2}$: I added half of 5 to 10 from the previous problem.

Launch

- Display one problem.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep problems and work displayed.
- Repeat with each problem.

Synthesis

- “How does breaking apart the numbers help us find the product?” (It is easier to find the product of big numbers when we decompose the numbers and add the smaller partial products.)

Activity 1

🕒 20 min

Which Garden Is Larger?

Standards Alignments

Addressing 5.NF.B.4

The purpose of this activity is for students to find the area of a rectangle with a whole number side length and a side length that is a mixed number. Students draw diagrams to represent the area of the gardens in the problem. Students should draw diagrams and find the area in a way that makes sense to them. As students work, ask them to explain their strategy for finding the area. In the activity synthesis, students consider multiplication expressions that use the distributive property to represent decomposing the rectangle into two smaller rectangles.

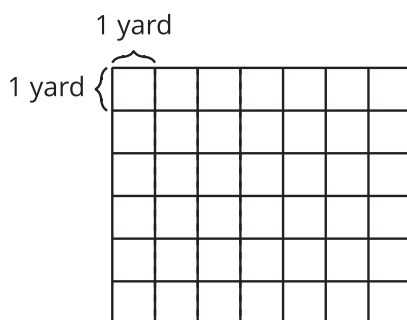
🌐 Access for English Learners

MLR1 Stronger and Clearer Each Time. Synthesis: Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their response to “Who’s garden do you think is larger, Noah’s or Priya’s?”. Invite listeners to ask questions, to press for details and to suggest mathematical language. Give students 2–3 minutes to revise their written explanation based on the feedback they receive.

Advances: Writing, Speaking, Listening

Student-facing Task Statement

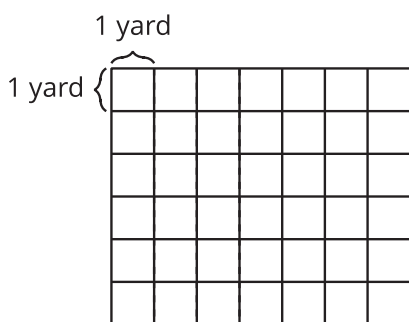
- Noah’s garden is 5 yards by $6\frac{1}{4}$ yards. Draw a diagram of Noah’s garden on the grid.



- Priya’s garden is 6 yards by $5\frac{1}{4}$ yards. Draw a diagram of Priya’s garden on the grid.

Launch

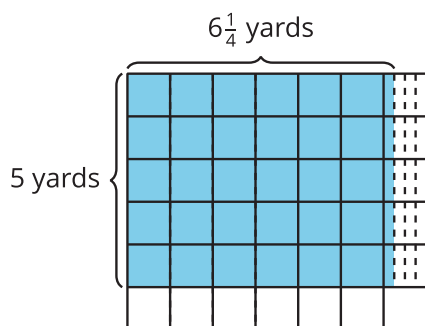
- Groups of 2
- Display: Noah’s garden is 5 yards by $6\frac{1}{4}$ yards. Priya’s garden is 6 yards by $5\frac{1}{4}$ yards.
- “Whose garden do you think is larger? Why?”
- 1 minute: quiet think time
- 1-2 minutes: partner discussion
- “We are going to draw the diagrams of each garden and determine which garden has a larger area.”



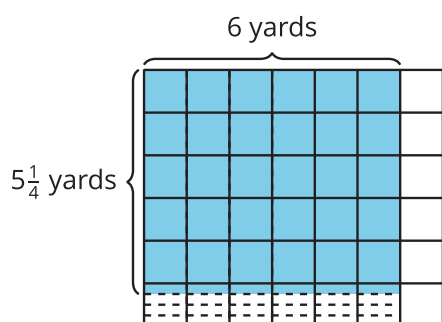
3. Whose garden covers a larger area? Be prepared to explain your reasoning.

Student Responses

1. Sample response:



2. Sample response:



3. Priya's garden covers more area because she has $\frac{1}{4}$ of a square unit more area than Noah.

Activity

- 1–2 minutes: independent work time
- 8–10 minutes: partner work
- Monitor for students who:
 - label the side lengths with mixed numbers
 - multiply the whole number side lengths, then add the fractional parts
 - decompose the larger rectangles into two smaller rectangles and multiply to find the area

Synthesis

- Select 2–3 students to share their responses and reasoning about how they determined which garden had a greater area.
- Display diagrams in sample student responses and the expressions $(5 \times 6) + (5 \times \frac{1}{4})$ and $(6 \times 5) + (6 \times \frac{1}{4})$.
- “How are these expressions the same? How are they different?” (Each expression is a sum of two products. The first part of each expression is 30. The second part is different because one expression has a 5 and the other one has a 6.)
- “How can we determine which garden has a larger area without evaluating the expressions?” (The 5×6 is the same as 6×5 but one garden has 5 one-fourths and the other has 6 one-fourths.)

Activity 2

🕒 15 min

Different Ways to Find the Area

Standards Alignments

Addressing 5.NF.B.4

The purpose of this activity is for students to interpret different strategies for multiplying whole numbers and fractions greater than 1 to find the area of a rectangle. Students use a diagram to describe different ways to determine the area of a shaded region. Consider having multiple copies of the diagrams available if students want to use a separate diagram for each strategy. Encourage students to draw on the diagram to show how they decomposed the rectangle. Students use what they have learned about area to construct different reasonable arguments for effective calculations of the area (MP3).

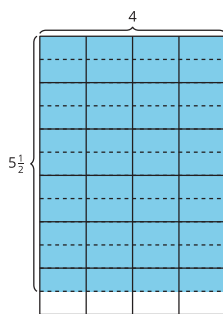
🕒 Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Provide choice. Invite students to decide which student to start with when completing the task.

Supports accessibility for: Organization, Attention

Student-facing Task Statement

Partner A

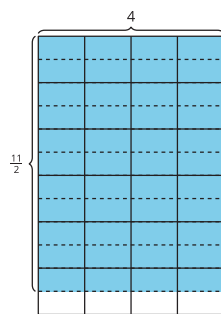


Jada: 4×5

Priya: $4 \times \frac{1}{2}$

Tyler: 6×4

Partner B



Clare: $\frac{10}{2} \times 4$

Diego: 4×6

Elena: 4×11

Launch

- Groups of 2
- “Decide with your partner who will be Partner A and who will be Partner B. You’ll each look at how some students started a problem and how they could finish their work. Then, you’ll share your work with your partner.”

Activity

- 5–8 minutes: independent work time
- 3–5 minutes: partner discussion
- Monitor for students who:
 - complete Tyler's and Diego's work by calculating the missing or excess

1. Each problem shows the first step a student used to find the area of the shaded region. Explain how each student could finish their work to find the area and show your thinking on the diagram.
2. Share your response with your partner. What is the same? What is different?

Student Responses

Sample responses:

- Jada: $4 \times 5 = 20$ and $4 \times \frac{1}{2} = 2$ and $20 + 2 = 22$
- Priya: $4 \times \frac{1}{2} = 2$ and $4 \times 5 = 20$ and $20 + 2 = 22$
- Tyler: $6 \times 4 = 24$ and $\frac{1}{2} \times 4 = 2$ and $24 - 2 = 22$
- Clare: $\frac{10}{2} \times 4 = \frac{40}{2}$ and $\frac{40}{2} = 20$ and $\frac{1}{2} \times 4 = 2$ and $20 + 2 = 22$
- Diego: $4 \times 6 = 24$ and $4 \times \frac{1}{2} = 2$ and $24 - 2 = 22$
- Elena: $4 \times 11 = 44$ and $44 \div 2 = 22$

area

Synthesis

- Select previously identified students to share their solutions.
- "What is the same and what is different between the two diagrams?" (They have the same area and the side lengths are equal, but they are written in different ways. The expressions for the side lengths are equal, but they are written in different ways.)
- "Which student's strategy do you prefer and why?"

Advancing Student Thinking

If students do not complete the steps that were started in the problem, refer to one of the diagrams and corresponding expressions and ask "How does the diagram show this expression? What else do we need to find out in order to determine the area of the shaded region?"

Lesson Synthesis

🕒 10 min

"Today we tried several different strategies for decomposing rectangles with a fractional side length to find the area of a rectangle. How can we describe the strategies we used today?" (We can decompose the rectangle into two smaller rectangles and add the areas. We can find the area of a larger rectangle and then subtract the area of a smaller rectangle.)

Record answers for all to see. Keep the display visible so students can refer to it in future lessons.

Suggested Centers

- How Close? (1–5), Stage 7: Multiply Fractions and Whole Numbers to 5 (Addressing)
- Rectangle Rumble (3–5), Stage 3: Factors 1–10 (Supporting)

Lesson 13: Area and Properties of Operations

Standards Alignments

Addressing 5.NF.B.4, 5.NF.B.4.b, 5.OA.A, 5.OA.A.1

Teacher-facing Learning Goals

- Represent the decomposition of a rectangle with diagrams and expressions.

Student-facing Learning Goals

- Let's write expressions to represent the area of rectangles.

Lesson Purpose

The purpose of this lesson is for students to analyze area diagrams and use the properties of operations to represent the area of rectangles.

In previous lessons, students applied what they know about multiplication of whole numbers and fractions to decompose a rectangle to find its area. In this lesson, students use the properties of operations to represent the area of rectangles with expressions. As students go through the activities, they will apply concepts they have seen and used in earlier grades, units, and lessons, such as:

- the distributive property
- the relationship between fractions and division
- the relationship between multiplication and division
- the relationship between addition and subtraction

Students use diagrams and expressions to investigate the distributive property, with both addition and subtraction. The diagrams provide a way to visualize how the different expressions represent the area of a given figure. This generalizes work that students have done in earlier grades with whole numbers. Understanding how the distributive property can be represented with diagrams helps students understand the structure of expressions representing products (MP7) and generalizes what they learned in an earlier grade using whole numbers (MP8).

Access for:

Students with Disabilities

- Action and Expression (Activity 2)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Card Sort (Activity 1), Number Talk (Warm-up)

Materials to Copy

- Card Sort: Diagrams and Expressions (groups of 2): Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

Why is it important for students to be able to write and use different expressions to represent and find the area of rectangles?

Cool-down (to be completed at the end of the lesson)

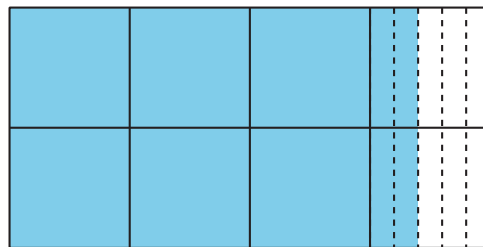
🕒 5 min

Equivalent Expressions

Standards Alignments

Addressing 5.NF.B.4.b

Student-facing Task Statement



Select **all** the expressions that represent the area of the shaded region.

- A. $(2 \times 3) + (2 \times \frac{2}{5})$
- B. $6\frac{2}{5}$
- C. $2 \times (3 + \frac{2}{5})$

D. $(2 \times 4) - (2 \times \frac{3}{5})$

E. $(2 \times 3) + \frac{2}{5}$

F. $2 \times \frac{17}{5}$

Student Responses

A, C, D, F

----- **Begin Lesson** -----**Warm-up**

🕒 10 min

Number Talk: Parentheses

Standards Alignments

Addressing 5.NF.B.4, 5.OA.A.1

This Number Talk encourages students to think about equivalent expressions and to rely on the properties of operations to mentally solve problems. The strategies elicited here will be helpful later in the lesson when students match diagrams to expressions.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $5 \times (7 + 4)$
- $(5 \times 7) + (5 \times 4)$
- $(5 \times 7) + (5 \times \frac{1}{4})$
- $(5 \times 7) - (5 \times \frac{1}{4})$

Launch

- Display one expression.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.

Student Responses

- 55: $5 \times 11 = 55$
- 55: $35 + 20 = 55$
- $36\frac{1}{4}$: $5 \times 7 = 35$, $5 \times \frac{1}{4} = 1\frac{1}{4}$, and $35 + 1\frac{1}{4} = 36\frac{1}{4}$
- $33\frac{3}{4}$: $5 \times 7 = 35$ and $5 \times \frac{1}{4} = 1\frac{1}{4}$, and $35 - 1\frac{1}{4} = 33\frac{3}{4}$

- Repeat with each expression.

Synthesis

"What is the same about the last two expressions? What is different?" (They both have the same two products but one of them is the sum and the other is the difference.)

Activity 1

🕒 20 min

Card Sort: Diagrams and Expressions

Standards Alignments

Addressing 5.NF.B.4

The purpose of this activity is for students to analyze area diagrams and use the properties of operations to interpret expressions. The diagrams are decomposed in different ways and the expressions all have a fractional part but sometimes it is written as a mixed number and sometimes the whole number and fraction are separated using the distributive property. The numbers in the diagrams, both the whole number part and the fractional part, are deliberately chosen to resemble one another so students need to analyze the expressions carefully to make matches.

🌐 Access for English Learners

MLR8 Discussion Supports. Students should take turns finding a match and explaining their reasoning to their partner. Display the following sentence frame for all to see: "I noticed __, so I matched" Encourage students to challenge each other when they disagree.

Advances: Conversing, Representing

Instructional Routines

Card Sort

Materials to Copy

Card Sort: Diagrams and Expressions (groups of

2)

Required Preparation

- Create a set of cards from the Instructional master for each group of 2.

Student-facing Task Statement

Your teacher will give you and your partner a set of cards.

1. Sort the cards in a way that makes sense to you.
2. Match each expression to an appropriate diagram. Some diagrams match more than one expression.
3. Work with your partner to find the area of each shaded region. Explain or show your reasoning.

Student Responses

1. Answers vary.
2. Diagram A: E, F, K, I
Diagram B: L
Diagram C: J, O
Diagram D: G, H, M, N
3. Sample responses:
Diagram A: 17 square units. $5 \times 3 = 15$,
 $5 \times \frac{2}{5} = 2$, $15 + 2 = 17$
Diagram B: 13 square units. $5 \times 2 = 10$,
 $5 \times \frac{3}{5} = \frac{15}{5}$ or 3, $10 + 3 = 13$
Diagram C: $16\frac{1}{5}$ square units. $5 \times 3 = 15$,
 $3 \times \frac{2}{5} = \frac{6}{5}$, $15 + \frac{6}{5} = 16\frac{1}{5}$
Diagram D: 12 square units. $5 \times \frac{12}{5} = \frac{60}{5}$,
 $\frac{60}{5} = 12$

Launch

- Groups of 2
- Give each group a set of cards from the Instructional master.

Activity

- 1–2 minutes: independent work time
- 8–10 minutes: partner work

Synthesis

- Display Diagram C and Expressions J and O.

$$3 \times 5\frac{2}{5}$$

$$(3 \times 5) + (3 \times \frac{2}{5})$$

- “How does each expression represent the area of the shaded region?” (There are 3 rows and each row has an area of $5\frac{2}{5}$ square units.)
- Display Diagram B and Expression L.

$$(5 \times 3) - (5 \times \frac{2}{5})$$

- “How does the expression represent the area of the shaded region?” (5×3 is the area of the full rectangle and then I take away the unshaded part, $5 \times \frac{2}{5}$.)

Activity 2

🕒 15 min

Write Expressions

Standards Alignments

Addressing 5.NF.B.4, 5.OA.A

The purpose of this activity is for students to write expressions that represent the area of shaded regions. Monitor for students who are writing a variety of expressions that represent the distributive property.

🕒 Access for Students with Disabilities

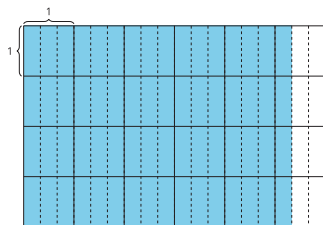
Action and Expression: Internalize Executive Functions. Invite students to verbalize their strategy for writing an expression to match the shaded area in each diagram before they begin. Students can speak quietly to themselves, or share with a partner.

Supports accessibility for: Organization, Conceptual Processing, Language

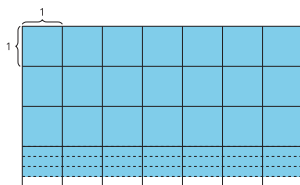
Student-facing Task Statement

Write as many expressions as you can to match the area of the shaded region in each diagram.

1.



2.



Launch

- Groups of 2

Activity

- 1–2 minutes: quiet think time
- 5–7 minutes: partner work time

Synthesis

- Select 2–3 students to share their expressions for the first problem.
- If not mentioned by students, display:

$$4 \times 5\frac{1}{3}$$

$$4 \times \frac{16}{3}$$

- “How does each expression represent the first problem?” (There are 4 rows and each row has $5\frac{1}{3}$ square units or $\frac{16}{3}$ square units.)

Student Responses

- Sample responses: $4 \times 5\frac{1}{3}$,
 $(4 \times 5) + (4 \times \frac{1}{3})$, $(4 \times 6) - (4 \times \frac{2}{3})$,
 $4 \times \frac{16}{3}$
- Sample responses: $7 \times 3\frac{3}{4}$,
 $(7 \times 3) + (7 \times \frac{3}{4})$, $(7 \times 4) - (7 \times \frac{1}{4})$, $7 \times \frac{15}{4}$

- “Did your approach change when writing expressions for the last diagram? If yes, how?” (It was easier to see the square units and apply the distributive property.)
- If not mentioned by students, display these expressions for the area of the last diagram in square units:

$$(7 \times 3) + (7 \times \frac{3}{4})$$

$$(7 \times 4) - (7 \times \frac{1}{4})$$

Advancing Student Thinking

If students only write one expression for a diagram, partner them with a student who found a different expression and ask:

- “How does this expression represent the shaded region?”
- “How can two different expressions represent the same shaded region?”
- “What other expressions can we write to represent the shaded region?”

Lesson Synthesis

🕒 10 min

“What’s your favorite way to find the area of a shaded region? What is a new way that you are excited to try or learn more about?”

Consider having students write their responses in their journals.

Suggested Centers

- How Close? (1–5), Stage 7: Multiply Fractions and Whole Numbers to 5 (Addressing)
- Rectangle Rumble (3–5), Stage 3: Factors 1–10 (Supporting)

----- Complete Cool-Down -----

Response to Student Thinking

Students do not select all the correct expressions.

Next Day Support

- Launch activity 1 by reviewing the cool down from this lesson.

Lesson 14: Area Situations

Standards Alignments

Addressing 5.NF.B, 5.NF.B.4

Teacher-facing Learning Goals

- Solve problems involving the multiplication of a whole number by a fraction, including fractions greater than 1.

Student-facing Learning Goals

- Let's apply what we've learned about fraction multiplication.

Lesson Purpose

The purpose of this lesson is for students to apply their understanding of multiplying a whole number by a fraction to solve mathematical and real-world problems.

In previous lessons, students used a range of skills to solve problems involving the multiplication of a whole number by a fraction. They represented situations with diagrams and expressions, and they solved problems using a variety of strategies.

In this lesson, students use their conceptual understanding to build procedural fluency with multiplication of whole numbers by fractions. Students also apply these skills to solve problems in an Info Gap activity. Students use what they know about interpreting multiplication as area to ask questions of their peers about the missing information in the problems and explain their reasoning for needing that information.

As in the previous lessons, students will encounter problems with fractions greater than 1. Students are encouraged to rewrite fractions greater than 1 as whole numbers when possible. They are not required to rewrite fractions greater than 1 as mixed numbers.

Access for:

Students with Disabilities

- Representation (Activity 2)

Instructional Routines

MLR4 Information Gap (Activity 1), Number Talk (Warm-up)

Materials to Copy

- Info Gap: Area (groups of 2): Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

If you were to teach this lesson over again, what activity would you redo? How would your proposed changes support student learning?

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

Evaluate Expressions

Standards Alignments

Addressing 5.NF.B.4

Student-facing Task Statement

Evaluate the expressions. Show your thinking.

1. $\frac{5}{3} \times 15$
2. $1\frac{3}{4} \times 8$
3. $\frac{10}{25} \times 10$

Student Responses

1. $\frac{75}{3}$ or 25 or equivalent. Sample response: I multiplied 15 and 5 and have that many $\frac{1}{3}$ s.
2. 14. Sample response: $8 \times 1 = 8$ and $\frac{3}{4} \times 8 = 6$ and $8 + 6 = 14$
3. $\frac{100}{25}$ or 4 or equivalent. Sample response: I multiplied 10 and 10 and have that many $\frac{1}{25}$ s.

Warm-up

🕒 10 min

Number Talk: Multiply Fractions

Standards Alignments

Addressing 5.NF.B.4

This Number Talk encourages students to think about the relationship between division and fractions and the order of operations in order to strategically multiply whole numbers by fractions. The strategies elicited here will be helpful later in the lesson when students find the missing value in multiplication equations for a whole number and a fraction.

To use the properties of operations, students need to look for and make use of structure (MP7). In explaining strategies, students need to be precise in their word choice and use of language (MP6).

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $3 \times (10 \div 2)$
- $\frac{3}{2} \times 10$
- $\left(\frac{14}{7}\right) \times 10$
- $14 \times \frac{10}{7}$

Student Responses

- 15: $10 \div 2 = 5$ and $3 \times 5 = 15$
- 15: This is the same as $\frac{3 \times 10}{2}$ or $3 \times (10 \div 2)$.
- 20: $\frac{14}{7} = 2$ and $2 \times 10 = 20$
- 20: This is equivalent to the last one.

Launch

- Display one problem.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep problems and work displayed.
- Repeat with each problem.

Synthesis

- "How can rearranging the numbers and operations help find the value of the last expression?" (I know that $\frac{14}{7}$ is 2 so finding this first and then multiplying by 10 is easier than trying to work with the fraction $\frac{10}{7}$.)

Activity 1

🕒 20 min

Info Gap: Area

Standards Alignments

Addressing 5.NF.B.4

This Info Gap activity gives students an opportunity to determine and request the information needed to solve multi-step problems involving multiplication of a whole number by a fraction. In each case, there are multiple ways to solve the problem but the information card does not have the one piece of information—the missing side length—that the student with the problem card could use to find the area. The cards are designed this way to encourage the students to discuss and communicate further.

The Info Gap structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need (MP1). Moreover, they are prompted by their partner to explain why they need each piece of information so they need to articulate their strategy and reasoning. The Info Gap structure also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need (MP6).

Instructional Routines

MLR4 Information Gap

Materials to Copy

Info Gap: Area (groups of 2)

Required Preparation

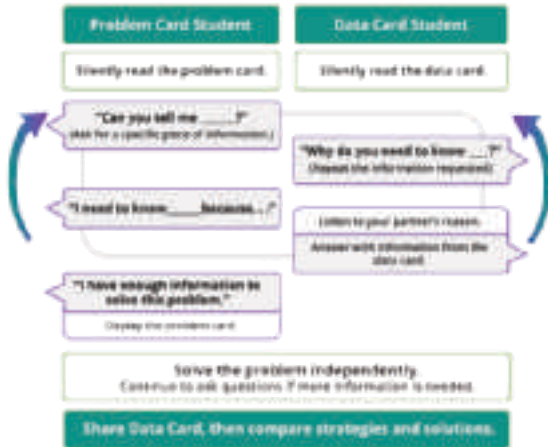
Create a set of cards from the Instructional master for each group of 2.

Student-facing Task Statement

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

Launch

- Groups of 2
- Distribute a problem card to one student and a data card to the other student.



Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Student Responses

Problem 1: The width of the paper towel is $\frac{1}{2} \times 11$ which is $\frac{11}{2}$ inches. The area is length times width or $11 \times \frac{11}{2}$ square inches. That's $\frac{121}{2}$ square inches.

Problem 2: The corn field is 3 times as long as it is wide so that's $3 \times 1\frac{2}{3}$. That's 5 miles. The area of the field is $5 \times 1\frac{2}{3}$ and that's $8\frac{1}{3}$ square miles.

Activity

- Explain the Info Gap structure, and consider demonstrating the protocol if students are unfamiliar with it.
- 10 minutes: partner work time
- After you review their work on the first problem, give them the cards for a second problem and instruct them to switch roles.

Synthesis

- Select 2–3 students to share the correct answers and discuss the process of solving the problems.
- “What questions did you ask to get the information you needed to solve the problems?” (I was missing just one piece of information—the width of the paper towel or the length of the corn field. I asked for that but my partner did not have it. I was stuck and then my partner said they had information comparing the length and width. I was able to use that to find the width or length and then the area.)

Activity 2 (optional)

🕒 15 min

Fill in the Blank

Standards Alignments

Addressing 5.NF.B

The purpose of this activity is for students to find the missing value in equations involving a whole number and a fraction. They find missing factors or products to make equations true. Many of the problems encourage students to think in steps, interpreting a fraction $\frac{a}{b}$ in terms of division by the denominator and multiplication by the numerator.

Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Invite students to identify which details were most important to solve the problems. Display the sentence frame: “The next time I solve multiplication problems of a whole number by a fraction, I will pay attention to”

Supports accessibility for: Conceptual Processing, Memory

Student-facing Task Statement

Fill in the blanks to make each equation true. Be prepared to explain your reasoning.

1. $\frac{1}{3} \times 18 = \underline{\hspace{2cm}}$
2. $\frac{7}{9} \times \underline{\hspace{2cm}} = \frac{21}{9}$
3. $\frac{1}{15} \times \underline{\hspace{2cm}} = 2$
4. $9 \times 6\frac{2}{3} = \underline{\hspace{2cm}}$
5. $14\frac{99}{100} \times 10 = \underline{\hspace{2cm}}$
6. $7\frac{3}{5} \times 6 = \underline{\hspace{2cm}}$
7. $4 \times 6\frac{9}{10} = \underline{\hspace{2cm}}$

Student Responses

1. 6
2. 3
3. 30
4. 60
5. $149\frac{9}{10}$
6. $45\frac{3}{5}$
7. $27\frac{6}{10}$

Launch

- Groups of 2

Activity

- 5–8 minutes: independent work time
- 1–2 minutes: partner discussion
- Monitor for:
 - partners who use different strategies.
 - students who are challenged but persevere and arrive at a viable solution.

Synthesis

- Ask previously selected students to share their responses.
- “What was different about you and your partner’s strategies?”
- “Which problem did you find most challenging and why?”
- “Which problem made the most sense to you and why?”

Advancing Student Thinking

If students do not get the correct solution for the missing factor problems, ask “How would you represent this expression with a diagram?”

Lesson Synthesis

 10 min

“Today we multiplied whole numbers by fractions greater than one in different forms.”

“We have learned a lot about multiplying a whole number by a fraction or mixed number. What have you learned? What do you still wonder?”

Consider giving students time to respond in their journals before sharing.

Suggested Centers

- Rectangle Rumble (3–5), Stage 4: Whole Number and Fraction Factors (Addressing)
- Rectangle Rumble (3–5), Stage 3: Factors 1–10 (Supporting)

Lesson 15: Multiply More Fractions

Standards Alignments

Addressing 5.NF.B.3, 5.NF.B.4, 5.NF.B.4.a, 5.NF.B.4.b

Teacher-facing Learning Goals

- Multiply whole numbers and fractions using the properties of operations.

Student-facing Learning Goals

- Let's multiply mixed numbers.

Lesson Purpose

The purpose of this lesson is for students to apply their understanding of the properties of operations to multiply whole numbers and fractions greater than 1 written as mixed numbers.

In previous lessons, students used the properties of operations to write equivalent expressions in order to find products of fractions greater than 1 written as mixed numbers. In this lesson, students apply their understanding of the properties of operations to multiply whole numbers and fractions greater than 1 written as mixed numbers. In each activity, encourage students to think flexibly, using everything they have learned about fraction decomposition, multiplication, and the properties of operations.

Access for:

Students with Disabilities

- Engagement (Activity 2)

English Learners

- MLR7 (Activity 2)

Instructional Routines

Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min

Teacher Reflection Question

Who has been sharing their ideas in class lately? Make a note of students whose ideas have not been featured in class and look for an opportunity for them to share their thinking in tomorrow's lesson.

Cool-down

5 min

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

Mixed Number Multiplication

Standards Alignments

Addressing 5.NF.B.4

Student-facing Task Statement

Find the value of each expression. Explain or show your reasoning.

1. $12 \times 9\frac{2}{3}$

2. $3\frac{5}{9} \times 18$

Student Responses

1. 116: $12 \times 9 + (12 \times \frac{2}{3}) = 108 + 8 = 116$

2. 64: $(3 \times 18) + (\frac{5}{9} \times 18) = 54 + 10 = 64$

----- **Begin Lesson** -----**Warm-up**

⌚ 10 min

Number Talk: Multiply Mixed Numbers

Standards Alignments

Addressing 5.NF.B.4

The purpose of this Number Talk is for students to demonstrate strategies and understandings they have for using the properties of operations when multiplying whole numbers and mixed numbers. These understandings help students develop fluency and will be helpful later in this lesson when

students will flexibly multiply.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $6 \times \frac{3}{8}$
- $6 \times 2\frac{3}{8}$
- $7 \times \frac{9}{10}$
- $7 \times 3\frac{9}{10}$

Student Responses

- $\frac{18}{8}$ or $2\frac{2}{8}$: I multiplied 6 and 3 and had that many eighths.
- $14\frac{2}{8}$: 6×2 is 12 so I added that to the product from the first problem.
- $\frac{63}{10}$ or $6\frac{3}{10}$: I multiplied 7 and 9 and had that many tenths.
- $27\frac{3}{10}$: 7×3 is 21 so I added 21 to the product previous problem.

Launch

- Display one problem.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep problems and work displayed.
- Repeat with each problem.

Synthesis

- Display the last problem.
- "If someone found 7×4 first, what might they do next?" (Subtract $7 \times \frac{1}{10}$ since $3\frac{9}{10}$ is $\frac{1}{10}$ less than 4.)

Activity 1

 20 min

Multiply Your Way

Standards Alignments

Addressing 5.NF.B.4.b

The purpose of this activity is for students to consider situations and the operations involved in

order to select reasonable numbers for each situation. Students are given a set of numbers which must each be used once in the statements. Students may find that their initial thinking does not work with the given constraints and may need to revise their work (MP1). They also need to think carefully about the units involved and consider whether a number is representing a linear unit or a square unit. Students may solve these problems using several different strategies.

Student-facing Task Statement

Write numbers from the list in the blank spaces so the situations make sense. Each number will be used only one time. Be prepared to explain your thinking.

4	5	$5\frac{1}{2}$
3	$5\frac{3}{4}$	2

1. The area of the rug is $16\frac{1}{2}$ square feet. The length of the rug is _____ feet. The width of the rug is _____ feet.
2. The puzzle is $2\frac{1}{2}$ feet wide. It is _____ feet long. It has an area of _____ square feet.
3. The area of the whiteboard is 23 square feet. The length of the whiteboard is _____ feet. The width of the whiteboard is _____ feet.

Share your solutions with your partner. Explain what choices you made and why.

Student Responses

1. The area of the rug is $16\frac{1}{2}$ square feet. The length of the rug is $5\frac{1}{2}$ feet. The width of the rug is 3 feet.
2. The puzzle is $2\frac{1}{2}$ feet wide. It is 2 feet long. It has an area of 5 square feet.
3. The area of the whiteboard is 23 square

Launch

- Groups of 2

Activity

- 3–5 minutes: independent work time
- 8–10 minutes: partner discussion
- Monitor for students who:
 - draw an area diagram.
 - write multiplication equations.
 - revise their thinking and can explain why their original solution didn't work, but their revised solution does work.

Synthesis

- Ask previously selected students to share their solutions.
- “Which numbers did you think made sense for the length of the rug in feet?” (I thought any of the numbers 5, $5\frac{1}{2}$, or $5\frac{3}{4}$ made sense.)
- “How did you decide which number to use for the length of the rug?” (The product had to be $16\frac{1}{2}$. So I tried a width of 2 feet and that was too small. Then I tried a width of 3 feet and that worked with $5\frac{1}{2}$ feet for the length.)

feet. The length of the whiteboard is $5\frac{3}{4}$ feet. The width of the whiteboard is 4 feet.

Advancing Student Thinking

If students need more of an invitation to enter the task, display the first problem with no numbers. Ask students to name some numbers that would make sense in this situation and explain why.

Activity 2

🕒 15 min

Equivalent Expressions

Standards Alignments

Addressing 5.NF.B.3, 5.NF.B.4.a, 5.NF.B.4.b

The purpose of this activity is for students to match different diagrams and expressions representing the same product. In the previous several lessons, students have studied different ways to find products of a whole number and a fraction using arithmetic properties such as the distributive property. Earlier in the unit they learned about the connection between fractions and division. They combine these skills as the expressions and diagrams they work with incorporate both the distributive property and the interpretation of a fraction as division (MP7). Students may match diagrams to expressions differently than the ways that are listed in the sample responses. Encourage students to match the expressions and diagrams in a way that makes sense to them, as long as they can accurately explain how the expression is represented in the diagram they chose.

MLR7 Compare and Connect. Invite students to prepare a visual display that shows their thinking about their favorite diagram and expression. Encourage students to include details that will help others understand what they see, such as using different colors, arrows, labels, or notes. If time allows, invite students time to investigate each others' work.

Advances: Representing

Access for Students with Disabilities

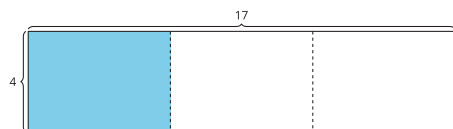
Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. Give students a subset of options to start with and introduce the remaining numbers once students have completed their initial set of matches.

Supports accessibility for: Organization, Social-Emotional Functioning

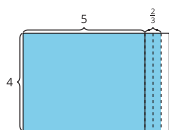
Student-facing Task Statement

Each diagram represents a way to calculate $4 \times 5\frac{2}{3}$. Each expression is equivalent to $4 \times 5\frac{2}{3}$. Match the diagrams and expressions. Show or explain your reasoning.

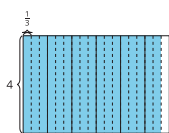
A



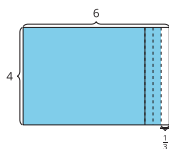
B



C



D



1. $(4 \times 5) + (4 \times \frac{2}{3})$

2. $(4 \times 6) - (4 \times \frac{1}{3})$

3. $4 \times \frac{17}{3}$

4. $(4 \times 17) \div 3$

Choose your favorite diagram and expression to find the value of $4 \times 5\frac{2}{3}$. Explain why it is your favorite.

Launch

- Groups of 2
- “You are going to match expressions and diagrams that show different ways to find the value of the product $4 \times 5\frac{2}{3}$.”

Activity

- 5–7 minutes: independent work time
- 2–3 minutes: partner discussion

Synthesis

- Display the diagrams B and D.
- “How are the diagrams the same? How are they different?” (They both have the same shaded area and the same divisions inside. The top one shows the length of the shaded rectangle. The bottom one does not though it has enough information to find the shaded area.)
- Invite students to share their favorite ways to find the product.

Student Responses

Sample responses:

1. $(4 \times 5) + (4 \times \frac{2}{3})$ matches diagram B because the rectangle has one part that has area 4×5 and another part whose area is $4 \times \frac{2}{3}$.
2. $(4 \times 6) - (4 \times \frac{1}{3})$ matches diagram D because the diagram shows a 4 by 6 rectangle and a 4 by $\frac{1}{3}$ rectangle that needs to be removed to get the shaded area.
3. $4 \times \frac{17}{3}$ matches diagram C because the diagram shows a rectangle that is 4 units wide and has a length of $\frac{17}{3}$.
4. $(4 \times 17) \div 3$ matches diagram A because the full rectangle has area 4×17 and $\frac{1}{3}$ is shaded so the shaded area is $(4 \times 17) \div 3$.

I like the diagram showing $(4 \times 17) \div 3$ because it's simple. The area is $\frac{68}{3}$ or $22\frac{2}{3}$ because $4 \times 17 = 68$.

Lesson Synthesis

🕒 10 min

"Today we used what we have learned to find the value of expressions involving multiplication of whole numbers and fractions greater than 1 written as mixed numbers."

Display: $7\frac{3}{5} \times 6$

"Tell me everything you know about this expression." (It is equal to a number between 42 and 48. It is equal to $\frac{38}{5} \times 6$ and to $(7 + \frac{3}{5}) \times 6$. We can draw an area diagram to represent $7\frac{3}{5} \times 6$.)

Record student responses for all to see.

If not mentioned by students, record an area diagram and equivalent expressions.

Suggested Centers

- Rectangle Rumble (3–5), Stage 4: Whole Number and Fraction Factors (Addressing)
- Can You Build It? (3–5), Stage 2: Multiple Rectangles (Supporting)

Complete Cool-Down

Response to Student Thinking

Students do not find the correct product.

Next Day Support

- During the next lesson, ask students “What 2 whole numbers is the product between?”

Lesson 16: Estimate Products (Optional)

Standards Alignments

Addressing 5.NF.B.4, 5.NF.B.4.b
Building Towards 5.NF.B.4

Teacher-facing Learning Goals

- Use estimation and the properties of operations to reason about the product of a whole number and a fraction greater than 1.

Student-facing Learning Goals

- Let's estimate products of a whole number and a fraction.

Lesson Purpose

The purpose of this lesson is for students to reason about the value of the product of a whole number and a fraction greater than 1 and use the properties of operations to find the product.

In previous lessons, students represented the decomposition of a rectangle with diagrams, expressions, and equations and found the product of a whole number and a fraction. In this optional lesson, students will practice multiplying fractions by using their understanding of the properties of operations. This time, they will not be provided with a diagram to represent each product. They will also apply what they have learned about multiplying fractions to reason about the proximity of fractional areas to whole number areas.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR5 (Activity 1)

Instructional Routines

Notice and Wonder (Warm-up)

Lesson Timeline

Warm-up	10 min
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Teacher Reflection Question

What evidence did you see in today's lesson that your students are extending their understanding of multiplication?

Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Cool-down (to be completed at the end of the lesson)

⌚ 5 min

Estimate and Solve

Standards Alignments

Addressing 5.NF.B.4.b

Student-facing Task Statement

Jada says the value of each product is about 20. For each problem, explain why Jada's estimate is too high, just right, or too low.

1. $5\frac{5}{6} \times 4 = \underline{\hspace{2cm}}$

20 is...

too low

too high

about right

2. $3 \times 6\frac{5}{8} = \underline{\hspace{2cm}}$

20 is...

too low

too high

about right

Student Responses

1. 20 is too low. $5\frac{5}{6}$ is very close to 6, and $6 \times 4 = 24$. $5\frac{5}{6} \times 4 = 23\frac{2}{6}$

2. 20 is about right. $3 \times 6 = 18$ and $\frac{5}{8}$ is a little more than $\frac{1}{2}$ so it's a little more than $18 + \frac{3}{2}$.

----- Begin Lesson -----

Warm-up

⌚ 10 min

Notice and Wonder: Garden Size

Standards Alignments

Building Towards 5.NF.B.4

The purpose of this warm-up is for students to reason about the side lengths of a garden with a given area in preparation for an upcoming activity. If students do not name any fractional side lengths, ask the synthesis question to prompt that discussion.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?



Student Responses

Students may notice:

- There are 3 sections in the garden.
- There are bushes next to the garden.

Students may wonder:

- What are the poles that are standing up?
- What is the area of the garden?
- What is growing in the garden?

Launch

- Groups of 2
- Display the image.
- “What do you notice? What do you wonder?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Share and record responses.
- “If the garden is between 30 square feet and 40 square feet, what are some possible side lengths?” (Sample responses: 8 feet by 4 feet, 7 feet by 5 feet)
- Share and record responses.

Synthesis

- “Would side lengths of 10 feet and $3\frac{1}{8}$ feet be possible based on the area?” (Yes because 10 times $3\frac{1}{8}$ is 30 and since $10 \times \frac{1}{8}$ is more than 1 but less than 2, so the area would be more than 30 but less than 40.)

Activity 1

🕒 20 min

Priya's Garden

Standards Alignments

Addressing 5.NF.B.4.b

The purpose of this activity is for students to notice and use the structure in multiplication expressions to represent the area of rectangles. Students estimate products to see if they are greater than or less than a given amount to solve a problem. Remind students they can draw a diagram if it is helpful.

🌐 Access for English Learners

MLR5 Co-Craft Questions. After students discuss what they know about gardens, read the problem statement and ask, "What mathematical questions could be asked about this situation?" Give students 2–3 minutes to write a list of mathematical questions, before comparing their questions with a partner. Invite students to make comparisons about the subject and language of their questions.

Advances: Reading, Writing

♿ Access for Students with Disabilities

Representation: Develop Language and Symbols. Synthesis: Invite students to explain their thinking orally, using a picture or diagram to show possible side lengths.

Supports accessibility for: Conceptual Processing, Organization

Student-facing Task Statement

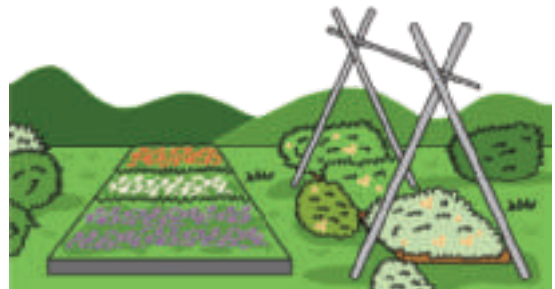


Priya has enough materials to build a garden that is 36 square feet.

Choose **all** the side lengths that are reasonable for her garden. Be prepared to explain your thinking to your partner.

Launch

- Groups of 2
- Display image from student book:



1. 9 feet by $4\frac{2}{3}$ feet
2. 9 feet by $3\frac{8}{9}$ feet
3. 12 feet by $2\frac{11}{12}$ feet
4. 9 feet by $2\frac{2}{3}$ feet

Student Responses

Sample responses:

1. 9 feet by $4\frac{2}{3}$ feet is not reasonable because it is bigger than 36 square feet and she only has enough materials for a 36 square foot garden.
2. 9 feet by $3\frac{8}{9}$ feet is reasonable because it is going to be less than 36 square feet, but close to it.
3. 12 feet by $2\frac{11}{12}$ feet is reasonable because it is going to be a little bit less than 36 square feet.
4. 9 feet by $2\frac{2}{3}$ feet is not reasonable because it is less than 27 square feet and she will have left over materials. It is reasonable because she can build something else with her extra materials.

- “What do you know about gardens?”

Activity

- 2–3 minutes: independent work time
- 6–7 minutes: partner discussion
- Monitor for students who:
 - draw diagrams.
 - estimate the approximate area.
 - reason which gardens are less than 36 square feet and which ones are bigger than 36 square feet.

Synthesis

- Ask previously selected students to share how they know that two of the expressions will have values a little bit less than 36. (I know that $9 \times 4 = 36$ and $3\frac{8}{9}$ is a little less than 4. I know that $12 \times 3 = 36$ and $2\frac{11}{12}$ is a little less than 3.)

Advancing Student Thinking

If students don't estimate, ask them to think of some whole-number side lengths that would work for Priya's garden and explain why they would work. Then, ask them to consider the side lengths in the problem.

Activity 2

🕒 15 min

Too High, Too Low, Just About Right

Standards Alignments

Addressing 5.NF.B.4

In the previous activity, students reasoned about the value of each product by thinking about the decomposition of the mixed number factor, and how close the mixed number is to the nearest whole number. The purpose of this activity is for students to reason about the value of products by rounding either the whole number or mixed number factors and multiplying.

When students try to make a product close to 20 using the given digits, they will rely on number sense but also may need to experiment and refine their choices and strategy after finding the value of the product (MP1).

Student-facing Task Statement

- Write a whole number product that is slightly less than, slightly greater than, or about equal to the value of $7 \times 12\frac{8}{9}$.
 - slightly less:
 - slightly greater:
 - just right:
- Write a whole number product that is slightly less than, slightly greater than, or about equal to the value of $9 \times 4\frac{2}{29}$.
 - slightly less:
 - slightly greater:
 - just right:
- Without calculating, use the numbers 2, 3, 5, 6, and 7, to complete the expression with a value close to 20.

$$\underline{\quad} \times \underline{\quad} \frac{\boxed{\quad}}{\boxed{\quad}}$$

- Explain how you know your expression represents a value close to 20.

Launch

- Groups of 2

Activity

- 5–8 minutes: independent work time
- 1–2 minutes: partner discussion
- Monitor for students who:
 - Multiply the whole numbers in problem 1 to find their product that is too low.
 - Round the mixed number factor to the nearest whole number before multiplying to find the just right estimate.

Synthesis

- Ask selected students to share.
- Consider asking:
 - “What strategies did you determine the just right product?” (I rounded the mixed number to a whole number based on the size of the fraction and multiplied.)
 - “In the third problem, is your

Student Responses

1. Sample responses:
 - a. 7×12 , 7×10
 - b. 7×13 , 10×12 , 10×13
 - c. It's slightly less than $7 \times 13 = 91$
2. Sample responses:
 - a. 9×4
 - b. 9×5 , 10×5
 - c. It's slightly more than $9 \times 4 = 36$
3. Sample responses: $3 \times 6\frac{5}{7}$ and $3 \times 7\frac{2}{6}$
4. Sample responses: I chose 2 numbers that multiplied to be just less than 20 and made a fraction that was close to 1.

product more, less, or equal to 20?
How do you know?"

Lesson Synthesis

🕒 10 min

"Today we made reasonable estimates for the value of multiplication expressions and used what we know about the properties of operations to find the value of the expressions."

Display:

$$(5 \times 4) - (5 \times \frac{1}{4})$$

- 20
- 19
- 16

"What is a reasonable estimate for the value of this expression?" (19 because we are subtracting a little bit more than one from 20. This means 20 is too high and 16 is too low.)

Display:

$$(5 \times 4) - (5 \times \frac{1}{4}) = (5 \times 3) + (5 \times \frac{3}{4})$$

"How do we know this equation is true?"

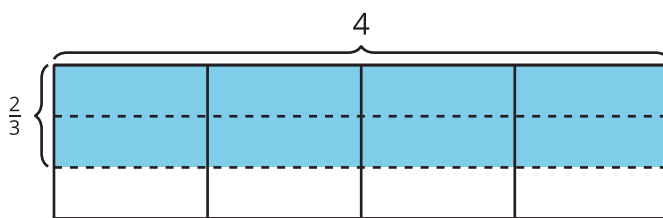
(Both expressions are equivalent to $18\frac{3}{4}$. Some will notice both can be rewritten as $5 \times 3\frac{3}{4}$.)

Suggested Centers

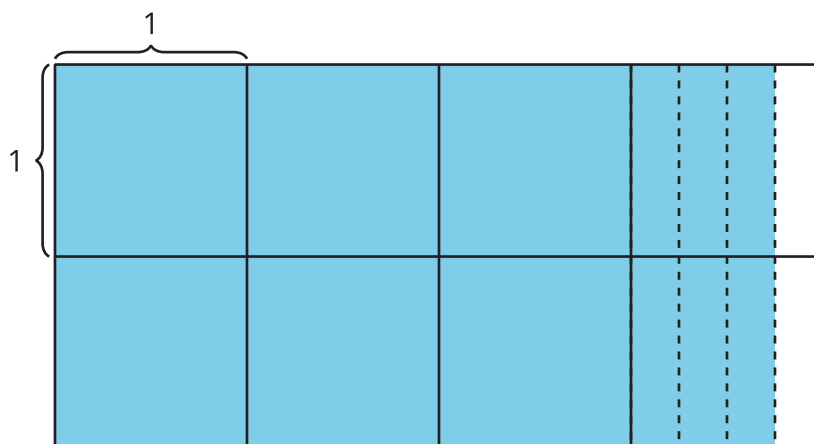
- Rectangle Rumble (3–5), Stage 4: Whole Number and Fraction Factors (Addressing)
- Can You Build It? (3–5), Stage 2: Multiple Rectangles (Supporting)

Student Section Summary

In this section, we learned how to find the area of a rectangle with a fractional side length. The shaded region has an area of $4 \times \frac{2}{3}$ because there are 4 groups of $\frac{2}{3}$ of a square unit shaded. The area is $\frac{8}{3}$ or $2\frac{2}{3}$ because there are 8 shaded parts and each one is $\frac{1}{3}$ of a square unit.



We also learned to multiply a mixed number by a whole number. We used area diagrams and expressions to see why our strategies work. For example, to solve $3\frac{3}{4} \times 2$, we can use the expression $(3 \times 2) + (\frac{3}{4} \times 2)$. We can see both of these expressions in the diagram.



Lesson 17: Mosaic Pictures (Optional)

Standards Alignments

Addressing	5.NF.B.4
Building Towards	5.NF.B.4

Teacher-facing Learning Goals

- Multiply fractions by whole numbers to find areas of rectangles.

Student-facing Learning Goals

- Let's make a mosaic.

Lesson Purpose

The purpose of this lesson is for students to use multiplication of fractions to create and analyze a mosaic of rectangles.

This lesson is optional because it does not address any new mathematical content standards. This lesson does provide students with an opportunity to apply precursor skills of mathematical modeling. In previous lessons, students found products of whole numbers and fractions, including fractions greater than 1. In this lesson, they apply what they learned about multiplying whole numbers and fractions to make mosaic art pieces out of rectangles and use area to determine how much it costs to recreate the mosaic with hard material like stone, tile, and glass. Throughout the activity, students make sense of problems and persevere in solving them (MP1).

In the first activity, students create rectangles from colored paper. Each rectangle has a side that is a fraction greater than 1 and a side that is a whole number. Students multiply whole numbers by fractions to find the area of one rectangle and then find the combined area of all of their rectangles. In the second activity, students exchange their different sized and colored rectangles and make a mosaic. They analyze and compare their mosaics by area. Finally in the synthesis, students sort selected mosaics from different groups. For example, they sort from smallest to largest area covered.

When students make decisions and choices, analyze contextual objects with mathematical ideas, and translate a mathematical answer back into the context of a situation, they model with mathematics (MP4).

This lesson is allocated more than 60 minutes, but it can be adjusted to meet the needs of the students. The activities can be modified or cut to fit within 1 day or extended to span over 2 days.

Access for:

Students with Disabilities

- Action and Expression (Activity 1)

Instructional Routines

MLR1 Stronger and Clearer Each Time (Activity 2), Notice and Wonder (Warm-up)

Materials to Gather

- Colored paper : Activity 1
- Glue: Activity 1
- Rulers: Activity 1
- Scissors: Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	40 min
Activity 2	30 min
Lesson Synthesis	10 min

Teacher Reflection Question

How does connecting rectangles to art help students to engage with the mathematics of the unit?

----- Begin Lesson -----

Warm-up

🕒 10 min

Notice and Wonder: Mosaic

Standards Alignments

Building Towards 5.NF.B.4

The purpose of this warm-up is for students to discuss the context of mosaics and the mathematics that might be involved, which will be useful when students create their own mosaics in a later activity. While students may notice and wonder many things about this image, the different smaller shapes used to create the mosaic are the important discussion points.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?



Student Responses

Students may notice:

- The image shows a butterfly.
- The butterfly is made up of many smaller shapes.
- There are a lot of different colors.

Students may wonder:

- How many pieces are all together?
- How many of each color are used?
- How long did this take to make?

Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- "A mosaic is a pattern or picture created using small pieces of ceramic, stone, or glass to cover a surface."
- "Ancient Greek and Roman artists decorated important buildings with mosaics, and Jewish and Muslim artists in the Middle East created mosaics to decorate religious buildings."
- "The pieces that create a mosaic can be cut in different shapes. What shapes do you notice in this mosaic?"
- "In the next activity, you'll create your own mosaics with rectangles."

Activity 1

Create a Mosaic

⌚ 40 min

Standards Alignments

Addressing 5.NF.B.4

The purpose of this activity is for students to create rectangles from colored paper. Students make identical rectangles that have one side length that is a whole number and one side length that is a fraction greater than 1. Then, as a group they create one mosaic. It is not necessary for students to use all of the rectangle pieces. It is important that all students in each group measure their rectangles using the same unit, either inches or centimeters.

Access for Students with Disabilities

Action and Expression: Provide Access for Physical Action. Provide access to pre-cut materials to reduce barriers for students who need support with fine motor skills and students who benefit from extra processing time.

Supports accessibility for: Fine Motor Skills, Organization, Visual-Spatial Processing

Materials to Gather

Colored paper , Glue, Rulers, Scissors

Required Preparation

- Each student needs a few sheets of the same colored paper.
- Each group of 4 needs a ruler, scissors, glue and one large piece of poster paper.

Student-facing Task Statement

1. Use the colored paper and scissors to cut identical rectangles. Make sure the measurement of one side of the rectangle is a whole number and the other is a fraction greater than 1.
2. What is the area of one of your rectangles? Show your reasoning.
3. Use the rectangles from your group to make a group mosaic by arranging some of the different colored rectangles on a blank piece of paper.

Launch

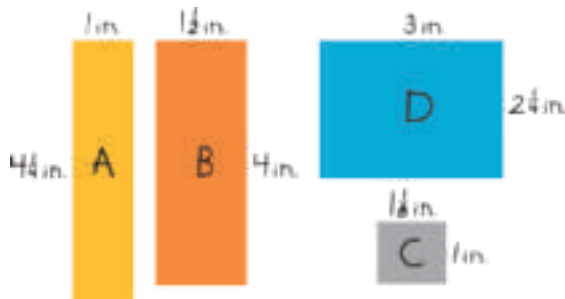
- Groups of 4
- Distribute materials. Make sure each student in the group gets a different color paper.

Activity

- 10 minutes: independent work time
- 5 minutes: group work time
- Monitor for students who:
 - write numerical expressions to represent the area of the rectangles.
 - use a diagram or write on the

Student Responses

1. Sample response:



2. Sample response: Students may choose to solve using a diagram or write an expression and find the product.

Partner A: $1 \times 4\frac{1}{4} = 4\frac{1}{4}$

Partner B:

$$4 \times 1\frac{1}{2} = 4 \times 1 + 4 \times \frac{1}{2} = 4 + 2 = 6$$

Partner C: $1 \times 1\frac{1}{8} = 1\frac{1}{8}$

Partner D: $3 \times 2\frac{1}{4} = 2\frac{1}{4} + 2\frac{1}{4} + 2\frac{1}{4} = 6\frac{3}{4}$



3.

physical rectangles.

Synthesis

- Invite a few students to share and describe their mosaic.
- “What are some questions we can ask about the mosaics?” (Which color rectangle did we use the most of? What are some other ways we can arrange the same pieces? What are some other pieces that can make the same pattern? How much would this cost if we actually made this using hard materials?)

Activity 2

Cost of Mosaic

🕒 30 min

Standards Alignments

Addressing 5.NF.B.4

The purpose of this activity is for students to apply their understanding of multiplying whole numbers by fractions greater than 1 to determine the cost of making a mosaic with different materials. It is not necessary for students to find the exact cost for the mosaic. Encourage students to use estimation strategies. Some students may find the exact cost as a fraction of a dollar, for example $131\frac{3}{5}$ dollars. Encourage students to make sense of the fraction with respect to money.

This activity uses *MLR1 Stronger and Clearer Each Time*. Advances: Reading, Writing.

Instructional Routines

MLR1 Stronger and Clearer Each Time

Student-facing Task Statement

About how much would it cost to create your mosaic with your preferred material? Explain or show your reasoning.

Material	Cost per square unit
Stone	\$5
Tile	\$3
Glass	\$2

Student Responses

Sample response:

We will use stone. In our mosaic we used:

- 3 rectangles with an area of $4\frac{1}{4}$ square inches so that's a total area of $12\frac{3}{4}$ square inches ($3 \times 4\frac{1}{4} = 12\frac{3}{4}$). This will cost 5 times that which is \$63.75.
- 1 rectangle with an area of 6 square inches. This will cost 5 times that which is \$30.

Launch

- Group of 4
- Display table from student task.
- "Traditionally mosaics are made of stone, tile, or glass pieces. Imagine you are making your mosaic as art for the school. In this activity, you will figure out the cost of your mosaic."

Activity

- 8-10 minutes: Group work time
- Monitor for students who:
 - Determine the cost for each color rectangle needed and then determine the total cost.
 - Find the approximate total area of the materials needed for the mosaic and then determine the cost.
 - Write clear expressions that show multiplication of whole numbers by fractions greater than 1.

- 1 rectangle with an area of $1\frac{1}{8}$ square inches. This will cost 5 times that so the cost is $5\frac{5}{8}$ dollars. That is a little more than \$5.50, so about \$5.60.
- 1 rectangle with an area of $6\frac{3}{4}$ inches. This will cost 5 times that which is \$33.75.

The total cost is around \$133.10.

Synthesis

MLR1 Stronger and Clearer Each Time

- “Share your response with a partner. Take turns being the speaker and the listener. If you are the speaker, share your ideas and writing so far. If you are the listener, ask questions and give feedback to help your partner improve their work.”
- 3–5 minutes: structured partner discussion.
- Repeat with 2–3 different partners.
- If needed, display question starters and prompts for feedback.
 - “Can you add an expression to show . . .?”
 - “The part I understood best was . . .”
 - “I liked how you . . .”
- “Revise your initial draft based on the feedback you got from your partners.”
- 2–3 minutes: independent work time

Lesson Synthesis

🕒 10 min

“Today we made mosaics with rectangles. Why is it important for an artist to know the area of the pieces they use for the mosaic?” (It affects how much the mosaic will cost. They need to know if their pieces will fit in the space they have.)

“In this lesson, many of us multiplied whole numbers by fractions greater than 1. Explain to a partner, what strategies you have to multiply whole numbers by fractions.”

Suggested Centers

- Rectangle Rumble (3–5), Stage 4: Whole Number and Fraction Factors (Addressing)
- Can You Build It? (3–5), Stage 2: Multiple Rectangles (Supporting)



Family Support Materials

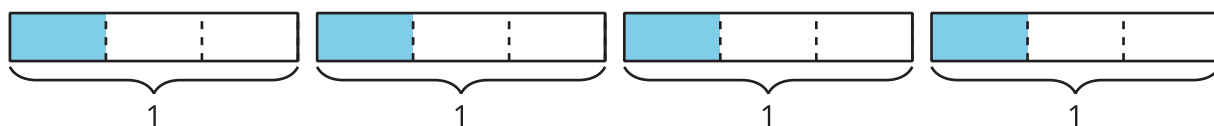
Family Support Materials

Fractions as Quotients and Fraction Multiplication

In this unit, students solve problems involving division of whole numbers with answers that are fractions (which could be in the form of mixed numbers). They develop an understanding of fractions as the division of the numerator by the denominator, that is $a \div b = \frac{a}{b}$. They then solve problems that involve the multiplication of a whole number by a fraction or mixed number.

Section A: Fractions as Quotients

In this section, students learn that fractions are quotients and can be interpreted as division of the numerator by the denominator. Students draw and analyze tape diagrams that represent sharing situations. Through the context of first sharing 1, then sharing more than 1, then sharing a number of things with increasingly more people, students notice patterns and begin to understand that in general $\frac{a}{b} = a \div b$. For example, students use the diagram below to show 4 objects being shared equally by 3 people, or $4 \div 3$, which can also be written as a fraction, $\frac{4}{3}$.



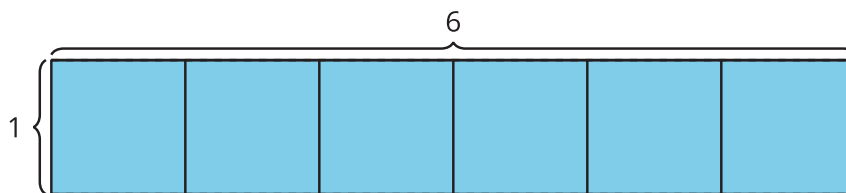
Section B: Fractions of Whole Numbers

In this section, students make connections between multiplication and division and use visual representations that can show both operations. For example, the diagram above can also represent 4 groups of $\frac{1}{3}$, or $4 \times \frac{1}{3}$. Students discover ways of finding the product of a fraction and whole number that make sense to them and connect the product to the context and diagrams. They multiply a whole number by a fraction, $\frac{a}{b} \times q$.

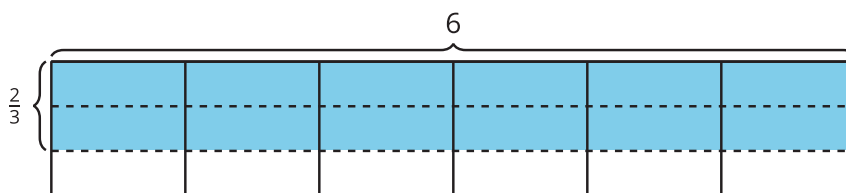
Section C: Area and Fractional Side Lengths

In this section, students use what they know about the area of rectangles with whole number side lengths to find the area of rectangles that have one whole number side length and one fractional side length.

The expression 6×1 represents the area of a rectangle that is 6 units by 1 unit.

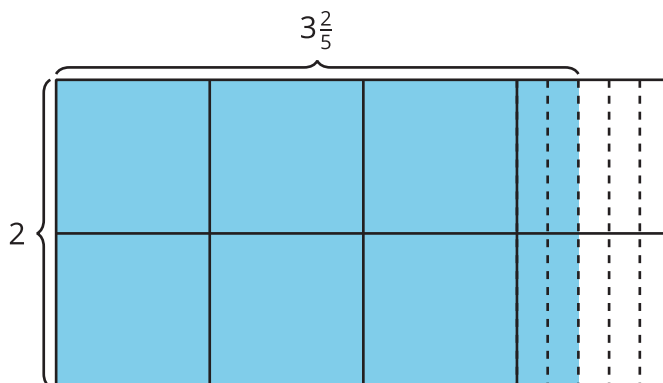


In the same way, $6 \times \frac{2}{3}$ represents the area of a rectangle that is 6 units by $\frac{2}{3}$ unit.



In addition, students see that the expressions $6 \times \frac{2}{3}$, $6 \times 2 \times \frac{1}{3}$, and $12 \times \frac{1}{3}$ can all represent the area of this same diagram.

Students analyze diagrams where one side length is a mixed number, for example a rectangle that is 2 by $3\frac{2}{5}$. They decompose the shaded region to show the whole units and the fractional units.



To find the area represented by this diagram, students may see two rectangles: a rectangle that is 2 units by 3 units and a rectangle that is 2 units by $\frac{2}{5}$ unit. While they may recognize that the area can be represented as $2 \times 3\frac{2}{5}$, students who see the decomposed rectangle may write $(2 \times 3) + (2 \times \frac{2}{5})$ to find the area.

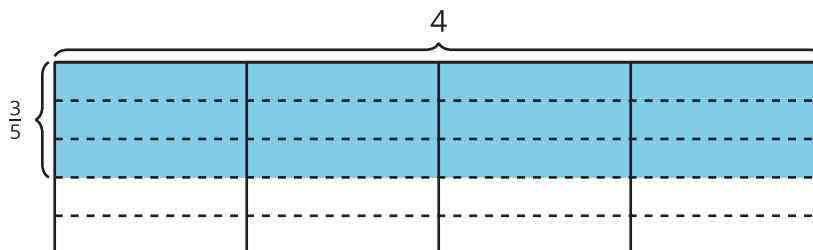
Grade 5 Unit 2

Fractions as Quotients and Fraction
Multiplication

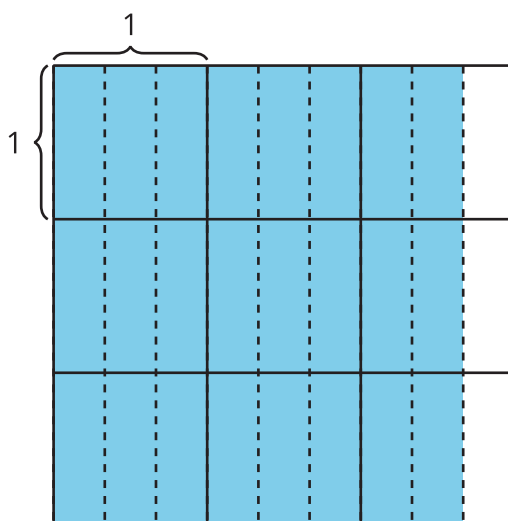
Try it at home!

Near the end of the unit, ask your student the following questions:

1. Write as many expressions as you can that represent this diagram:



2. What is the area of the following rectangle?



Questions that may be helpful as they work:

- How are the two problems similar? How are they different?
- How does your expression represent the diagram?
- How did you break up the rectangle to help you solve for the entire area?
- What are the side lengths of the rectangle?

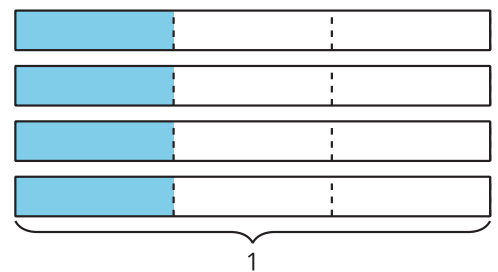
Unit Assessments

Check Your Readiness A, B and C
End-of-Unit Assessment

Fractions as Quotients and Fraction Multiplication: Section A Checkpoint

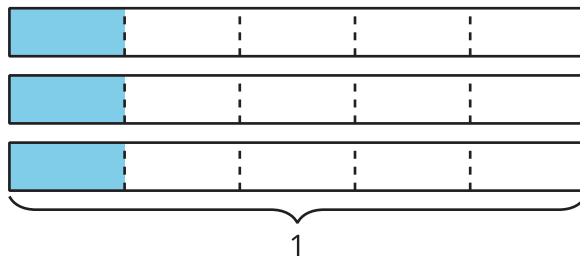
1. Five friends equally share 3 liters of water. How many liters of water does each person get? Explain or show your reasoning.

2. Write a division equation that matches the diagram. Explain or show your reasoning.



3. Explain why $10 \div 4 = \frac{10}{4}$.

Fractions as Quotients and Fraction Multiplication: Section B Checkpoint

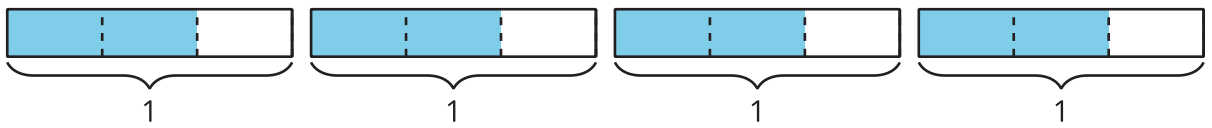


1. a. Explain how the diagram shows $3 \div 5$.

- b. Explain how the diagram shows $3 \times \frac{1}{5}$.

- c. What is the value of $3 \div 5$? Explain or show your reasoning.

2. Explain or show how each expression represents the shaded parts of the diagram.



a. $2 \times (4 \div 3)$

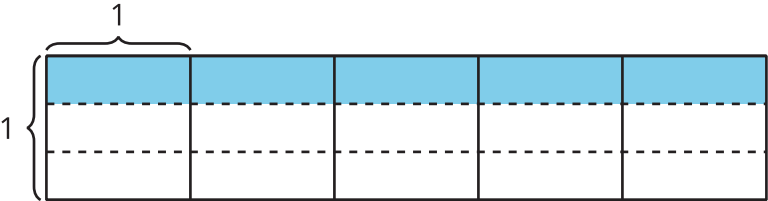
b. $4 \times \frac{2}{3}$

c. $4 \times 2 \times \frac{1}{3}$

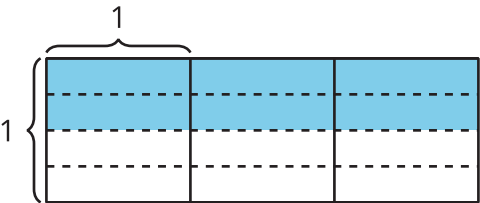
Fractions as Quotients and Fraction Multiplication: Section C Checkpoint

1. For each diagram, write an expression for the area of the shaded region. Then, find the area.

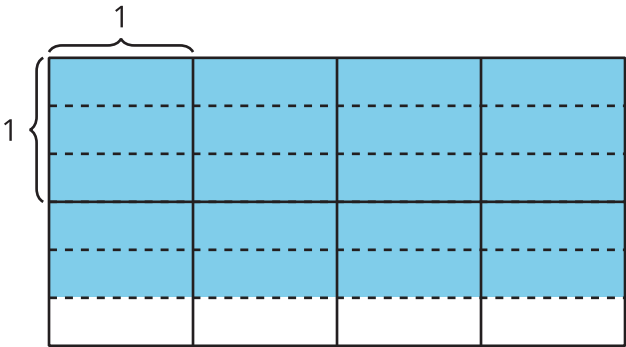
a.



b.



c.



2. A bottle holds 2 liters of water. Clare drank $\frac{3}{5}$ of the bottle of water. How many liters of water is that? Explain or show your reasoning.

3. Find the value of each expression.

a. $\frac{1}{5} \times 10$

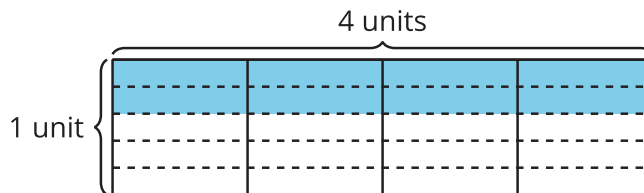
b. $5\frac{2}{3} \times 4$

c. $\frac{13}{4} \times 5$

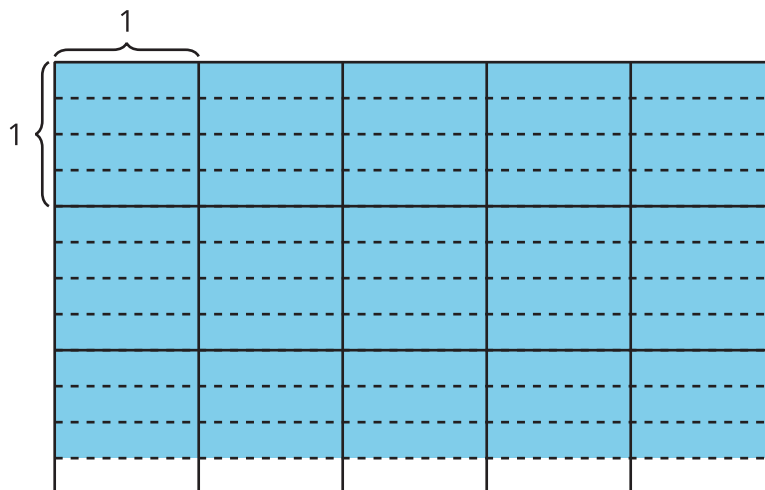
Fractions as Quotients and Fraction Multiplication: End-of-Unit Assessment

1. Find the area of the shaded region.

Explain or show your reasoning.



2. Select **all** of the expressions that represent the area of the shaded region.



- A. $(5 \times 3) - (5 \times \frac{1}{4})$
- B. 5×11
- C. $\frac{11}{4} \times 5$
- D. $55 \times \frac{1}{4}$
- E. $2\frac{3}{4} \times 5$
- F. $5 \times 2 + \frac{5}{4}$

3. There are 8 ounces of pasta in the package. Jada cooks $\frac{2}{3}$ of the pasta. How many ounces of pasta did Jada cook?

A. $2\frac{2}{3}$

B. $5\frac{1}{3}$

C. $7\frac{1}{3}$

D. 12

4. A piece of string is 18 inches long. Jada cuts it into 4 equal parts. What is the length of each part in inches? Select **all** that apply.

A. $\frac{4}{18}$

B. $4 \div 18$

C. $\frac{18}{4}$

D. $18 \div 4$

E. $4\frac{1}{2}$

5. A hiking trail is 7 miles long. Han hikes $\frac{1}{2}$ of the trail and then stops for water. Jada hikes $\frac{2}{3}$ of the trail and then stops for water.

a. How many miles did Han hike before stopping for water? Explain or show your reasoning.

b. How many miles did Jada hike before stopping for water? Explain or show your reasoning.

6. Find the value of each expression.

a. $\frac{1}{7} \times 28$

b. $15 \times 3\frac{2}{5}$

c. $\frac{2}{5} \times 6$

d. $3\frac{9}{10} \times 4$

7. A farm is rectangular in shape. It is 2 km long and 3 km wide.

a. What is the area of the farm? Explain or show your reasoning.

b. The farm is divided into 5 equal parts. Corn is grown in one of the parts. Draw a diagram to show where the corn is grown.

c. What is the area of the part of the farm where corn is grown? Explain or show your reasoning.

Assessment Answer Keys

Check Your Readiness A, B and C
End-of-Unit Assessment

Assessment Answer Keys

Assessment: Section A Checkpoint

Problem 1

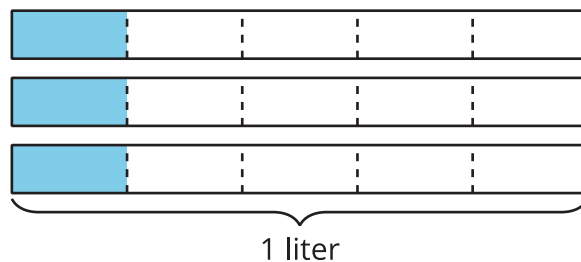
Goals Assessed

- Solve problems involving division of whole numbers leading to answers that are fractions.

Five friends equally share 3 liters of water. How many liters of water does each person get? Explain or show your reasoning.

Solution

$\frac{3}{5}$ liter. Sample representation:

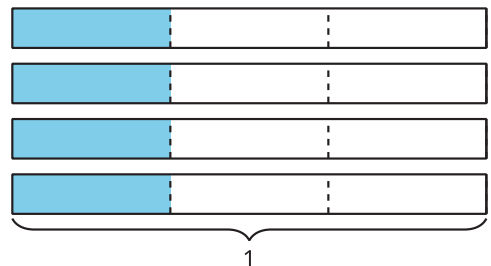


Problem 2

Goals Assessed

- Represent and explain the relationship between division and fractions.

Write a division equation that matches the diagram. Explain or show your reasoning.



Solution

$4 \div 3 = \frac{4}{3}$. There are 4 wholes total and they are divided into three equal shares. One of those shares is shaded and that's $\frac{4}{3}$ total shaded.

Problem 3

Goals Assessed

- Represent and explain the relationship between division and fractions.

Explain why $10 \div 4 = \frac{10}{4}$.

Solution

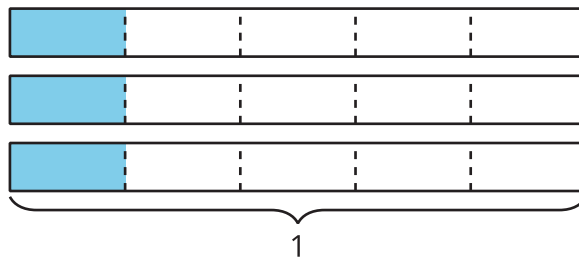
If I divide 10 things each into 4 equal shares and take one of each, that's $10 \div 4$ since there are 4 equal shares in 10. Since each share is $\frac{1}{4}$ and there are 10 of those shares, one for each thing, that's also $\frac{10}{4}$.

Assessment: Section B Checkpoint

Problem 1

Goals Assessed

- Connect division to multiplication of a whole number by a unit fraction.
- Explore the relationship between multiplication and division.



- Explain how the diagram shows $3 \div 5$.
- Explain how the diagram shows $3 \times \frac{1}{5}$.
- What is the value of $3 \div 5$? Explain or show your reasoning.

Solution

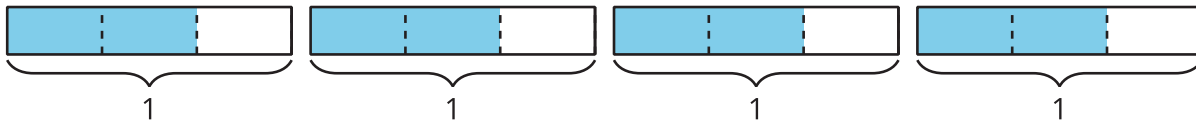
- There are 3 whole rectangles and 1 out of 5 equal shares of the rectangles is shaded so that's $3 \div 5$.
- There are 3 shaded parts and each is $\frac{1}{5}$ of a whole rectangle so that's $3 \times \frac{1}{5}$.
- $\frac{3}{5}$ because there are 3 shaded pieces and each is $\frac{1}{5}$ of a whole rectangle.

Problem 2

Goals Assessed

- Connect division to multiplication of a whole number by a non-unit fraction.
- Explore the relationship between multiplication and division.

Explain or show how each expression represents the shaded parts of the diagram.



- a. $2 \times (4 \div 3)$
- b. $4 \times \frac{2}{3}$
- c. $4 \times 2 \times \frac{1}{3}$

Solution

Sample responses:

- a. There are 2 groups of 4 rectangles divided into 3 equal parts so that's $2 \times (4 \div 3)$.
- b. There are 4 groups of $\frac{2}{3}$ of a rectangle so that's $4 \times \frac{2}{3}$.
- c. There are 4 groups of 2 small parts and each one is $\frac{1}{3}$ of a rectangle so that's $4 \times 2 \times \frac{1}{3}$.

Assessment: Section C Checkpoint

Problem 1

Goals Assessed

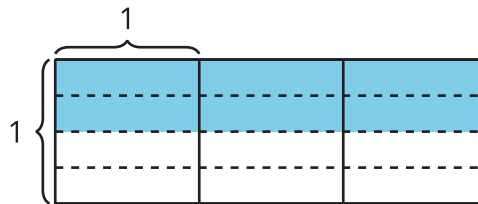
- Find the area of a rectangle when one side length is a whole number and the other side length is a fraction or mixed number.
- Write, interpret and evaluate numerical expressions that represent multiplication of a whole number by a fraction or mixed number.

For each diagram, write an expression for the area of the shaded region. Then, find the area.

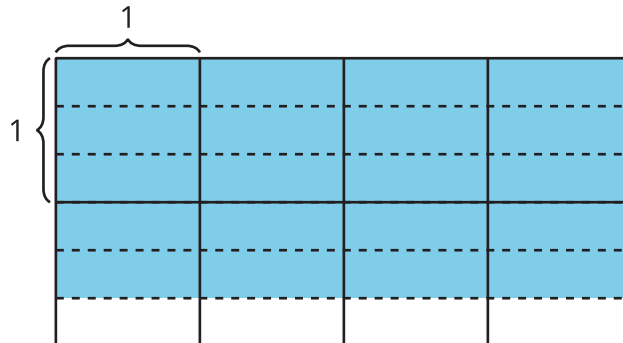
a.



b.



c.



Solution

- $\frac{1}{3} \times 5$, $5 \times \frac{1}{3}$, or equivalent. The area is $\frac{5}{3}$ square units.
- $\frac{2}{4} \times 3$, $\frac{1}{2} \times 3$, or equivalent. Each shaded row is $\frac{1}{4}$ of the 3 squares and there are two of them so that is $\frac{2}{4}$ of the 3 squares or $\frac{2}{4} \times 3$. The area is $\frac{6}{4}$ square units.
- $\frac{5}{3} \times 4$ or equivalent. Each shaded row is $\frac{1}{3}$ of 4 squares and there are 5 of these shaded so that's $\frac{5}{3} \times 4$. The area is $\frac{20}{3}$ square units.

Problem 2

Goals Assessed

- Represent and solve problems involving the multiplication of a whole number by a fraction or mixed number.

A bottle holds 2 liters of water. Clare drank $\frac{3}{5}$ of the bottle of water. How many liters of water is that? Explain or show your reasoning.

Solution

$\frac{6}{5}$ liters. There are 2 liters of water and she had $\frac{3}{5}$ of the water so that's $\frac{3}{5} \times 2$ or $\frac{6}{5}$ liters.

Problem 3

Goals Assessed

- Write, interpret and evaluate numerical expressions that represent multiplication of a whole number by a fraction or mixed number.

Find the value of each expression.

- $\frac{1}{5} \times 10$
- $5\frac{2}{3} \times 4$
- $\frac{13}{4} \times 5$

Solution

- a. $\frac{10}{5}$ or 2 or equivalent
- b. $\frac{68}{3}$ or equivalent
- c. $\frac{65}{4}$ or equivalent

Assessment: End-of-Unit Assessment

Problem 1

Standards Alignments

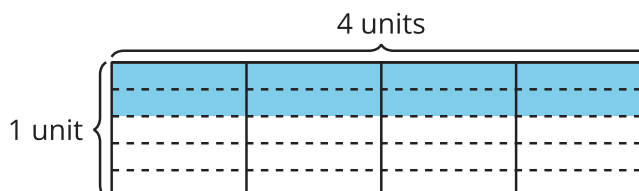
Addressing 5.NF.B.4.b

Narrative

Students find the area of a rectangle with one side of integer length and the other side of fractional length. The numbers are small enough in this item that students can count that there are 8 shaded parts so the main work here is identifying that each of those parts represents $\frac{1}{5}$ square unit.

Students could also say that the rectangle is 4 units long and $\frac{2}{5}$ units wide and find the product but the numbers are deliberately made small to encourage thinking concretely about the meaning of each shaded part.

Find the area of the shaded region.
Explain or show your reasoning.



Solution

$\frac{8}{5}$ square units since there are 8 shaded parts and each is $\frac{1}{5}$ of a square unit.

Problem 2

Standards Alignments

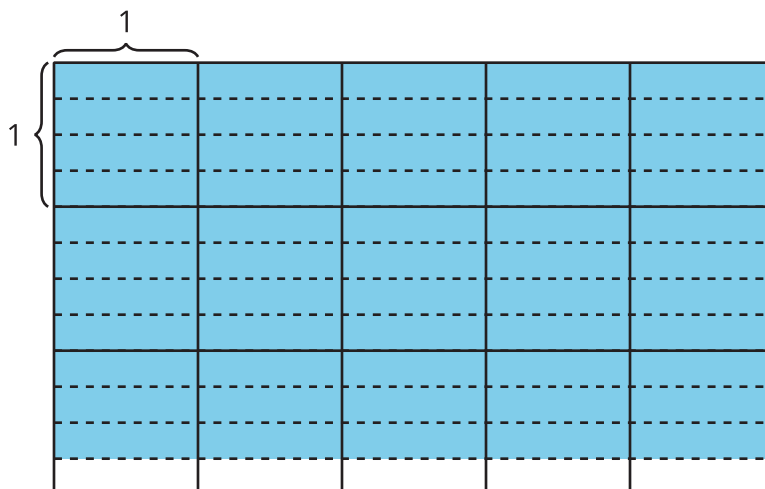
Addressing 5.NF.B.4.b, 5.OA.A.2

Narrative

Students identify expressions that represent a shaded area with one fractional side length and one whole-number side length. Answers C and E are the essential ones for students to see to test their understanding of the standard 5.NF.B.4.b. Answers A and D assess their understanding of expressions and they will have many opportunities over the year to further develop these skills.

Students who select B or F need more work finding fractional areas as B represents the number of parts while F represents only some of the area.

Select **all** of the expressions that represent the area of the shaded region.



- A. $(5 \times 3) - (5 \times \frac{1}{4})$
- B. 5×11
- C. $\frac{11}{4} \times 5$
- D. $55 \times \frac{1}{4}$
- E. $2\frac{3}{4} \times 5$
- F. $5 \times 2 + \frac{5}{4}$

Solution

["A", "C", "D", "E"]

Problem 3

Standards Alignments

Addressing 5.NF.B.4.a, 5.NF.B.6

Narrative

Students solve a problem involving a product of a whole number and a fraction. Students may select A if they correctly find $\frac{1}{3}$ of 8 ounces and do not multiply by 2. Students may select C if they subtract $\frac{2}{3}$ from 8 rather than finding the product. They may select D if they perform multiplication incorrectly and find $\frac{3}{2} \times 8$.

Students can also solve this problem using general number sense. Response A is too small as it is less than $\frac{1}{2}$ of 8. Response C is too large as it is very close to 8. Response D is larger than 8 so it is definitely too large. This kind of number sense is a valuable skill and students who solve this problem via a process of elimination and this line of reasoning will have opportunities to address their understanding of a fraction of a whole number directly in other items.

There are 8 ounces of pasta in the package. Jada cooks $\frac{2}{3}$ of the pasta. How many ounces of pasta did Jada cook?

- A. $2\frac{2}{3}$
- B. $5\frac{1}{3}$
- C. $7\frac{1}{3}$
- D. 12

Solution

B

Problem 4

Standards Alignments

Addressing 5.NF.B.3

Narrative

Students represent the result of division of two whole numbers in multiple ways: a fraction, a mixed number, and a division expression. Students who select A or B (and fail to select C or D) need more practice interpreting division situations and understanding the relationship between division and fractions. Students may fail to select E if they do not write the answer to the question as a mixed number or if they do so but do not recall that $\frac{2}{4}$ and $\frac{1}{2}$ are equivalent. These students

may need more practice writing fractions in different forms.

A piece of string is 18 inches long. Jada cuts it into 4 equal parts. What is the length of each part in inches? Select **all** that apply.

- A. $\frac{4}{18}$
- B. $4 \div 18$
- C. $\frac{18}{4}$
- D. $18 \div 4$
- E. $4\frac{1}{2}$

Solution

["C", "D", "E"]

Problem 5

Standards Alignments

Addressing 5.NF.B.4.a, 5.NF.B.6

Narrative

Students multiply a whole number by a fraction to solve a story problem. No representation for the problem is requested so students may draw a tape diagram (or discrete diagram), or an area diagram, or they may reason about the quantities without a picture.

A hiking trail is 7 miles long. Han hikes $\frac{1}{2}$ of the trail and then stops for water. Jada hikes $\frac{2}{3}$ of the trail and then stops for water.

- a. How many miles did Han hike before stopping for water? Explain or show your reasoning.
- b. How many miles did Jada hike before stopping for water? Explain or show your reasoning.

Solution

- a. Han hiked $3\frac{1}{2}$ or $\frac{7}{2}$ miles. Half of 7 is $3\frac{1}{2}$ or $7 \div 2 = \frac{7}{2}$.

- b. Jada hiked $\frac{14}{3}$ miles or $4\frac{2}{3}$ miles since $\frac{2}{3}$ of 7 is $\frac{14}{3}$.

Problem 6

Standards Alignments

Addressing 5.NF.B.4

Narrative

Students multiply a whole number by a fraction with no context. Some of the fractions are listed as mixed numbers and some are listed as fractions. While students are not expected to convert between these forms, the way they write their answer may help reveal how they are thinking about the products.

Find the value of each expression.

- a. $\frac{1}{7} \times 28$
- b. $15 \times 3\frac{2}{5}$
- c. $\frac{2}{5} \times 6$
- d. $3\frac{9}{10} \times 4$

Solution

- a. 4 or equivalent
- b. 51 or equivalent
- c. $\frac{12}{5}$ or $2\frac{2}{5}$ or equivalent
- d. $15\frac{6}{10}$ or equivalent

Problem 7

Standards Alignments

Addressing 5.NF.B.3, 5.NF.B.4.a, 5.NF.B.6

Narrative

Students solve a problem about area. There are different ways students might draw a diagram.

- They might divide one side of the rectangular farm into 5 equal pieces, using the relationship between division and fractions.
- They could also shade $\frac{1}{5}$ of each square kilometer.

The provided solution uses the previous approach but either solution will have $\frac{6}{5}$ of a square kilometer shaded.

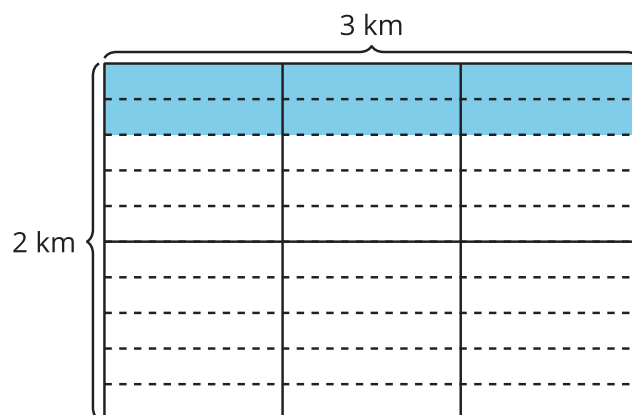
If students draw an incorrect diagram for the second question and answer the third question correctly based on the diagram, they still demonstrate an understanding of fractions. A common error for the drawing may be to shade $\frac{1}{5}$ of a kilometer for the width or length of the part of the farm where corn is grown. Note that students can use the meaning of division to answer the last question, independent of their diagram: one out of 5 equal parts of 6 square kilometers is $6 \div 5$ or $\frac{6}{5}$ square kilometers.

A farm is rectangular in shape. It is 2 km long and 3 km wide.

- What is the area of the farm? Explain or show your reasoning.
- The farm is divided into 5 equal parts. Corn is grown in one of the parts. Draw a diagram to show where the corn is grown.
- What is the area of the part of the farm where corn is grown? Explain or show your reasoning.

Solution

- 6 square kilometers because $2 \times 3 = 6$.
- Sample response: I decided to cut the side that is 2 km into 5 equal parts and each of those is $\frac{2}{5}$ of a kilometer wide since $2 \div 5 = \frac{2}{5}$.



- Sample response: The area where the corn is grown is $\frac{6}{5}$ square kilometers. There are 6 shaded parts and each one represents $\frac{1}{5}$ square kilometer.

Lesson Cool Downs

Lesson 1: Share Sandwiches

Cool Down: How Much?

1. Draw a diagram to show how much sandwich each person will get.

3 sandwiches are equally shared by 4 people.

2. Explain or show how you know that each person gets the same amount of sandwich.

Lesson 2: Share More Sandwiches

Cool Down: How Much Sandwich?

1. 4 sandwiches are equally shared by 5 students. How much sandwich does each student get? Show or explain your reasoning.

2. Write a division expression to represent the situation.

Lesson 3: Interpret Equations

Cool Down: Share Water

3 liters of water are shared equally by 5 people. How much water does each person get? Write a division equation to represent the situation. Draw a diagram if it is helpful.

Lesson 4: Division Situations

Cool Down: How Much Milk?

Complete the table below.

Equation	Situation
	5 children share 4 cups of milk so each child gets the same amount of milk. How many cups of milk will each child get?
Diagram	

Lesson 5: Relate Division and Fractions

Cool Down: Explain It.

Explain why $8 \div 5 = \frac{8}{5}$.

Lesson 6: Relate Division and Multiplication

Cool Down: A Different Relay Race

1. Lin and Han ran a 5 mile relay race as a team. They each ran the same distance. Draw a diagram to represent the situation.

2. How far did each student run?

Lesson 7: Divide to Multiply Unit Fractions

Cool Down: Another Race

Together, 6 children run a 5 mile relay race. They each run the same distance.

Select **all** the expressions that represent this situation.

A. $\frac{1}{6} \times 5$

B. $\frac{1}{5} \times 6$

C. $5 \div 6$

D. $\frac{5}{6}$

Lesson 8: Divide to Multiply Non-unit Fractions

Cool Down: Two Thirds

Find the value of each expression. Explain or show your reasoning.

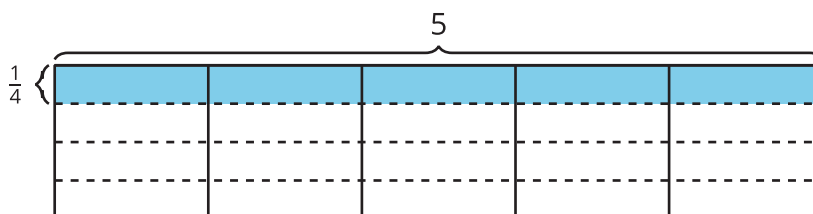
1. $\frac{1}{3} \times 4$

2. $\frac{2}{3} \times 4$

Lesson 9: Relate Area to Multiplication

Cool Down: Fractional Pieces

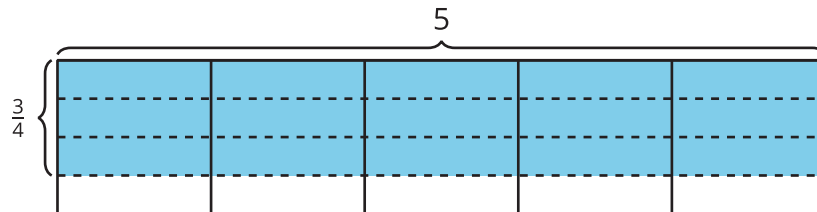
Find the area of the shaded region. Explain or show your reasoning.



Lesson 10: Fractional Side Lengths Less Than 1

Cool Down: A Fractional Side Length

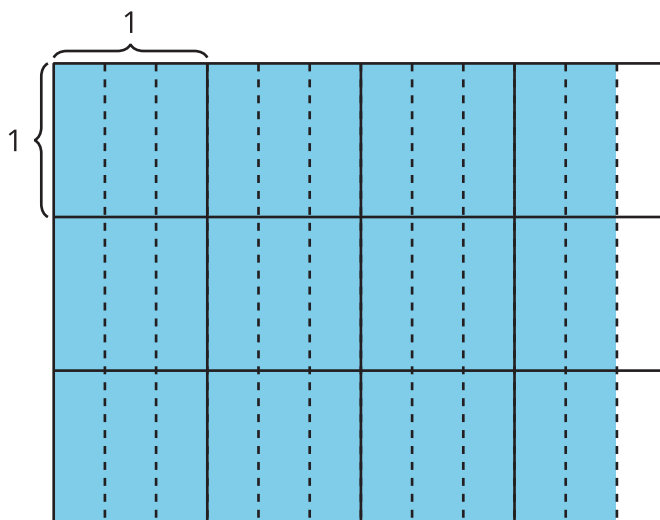
1. Write a multiplication expression to represent the area of the shaded region.



2. Find the area of the shaded region.

Lesson 11: Fractional Side Lengths Greater Than 1

Cool Down: Find the Area



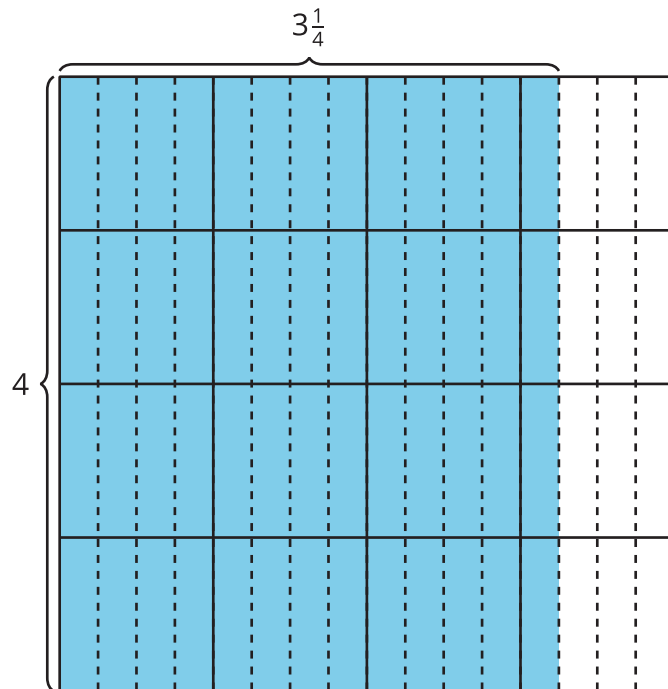
1. Write a multiplication expression to represent the area of the shaded region.

2. What is the area of the shaded region?

Lesson 12: Decompose Area

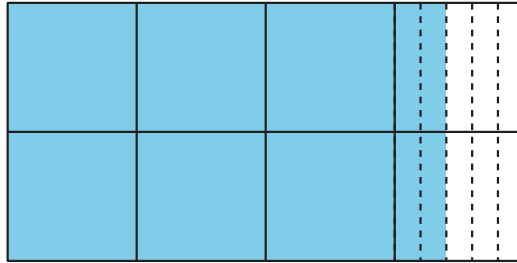
Cool Down: Decompose Rectangles

Find the area of the shaded region.



Lesson 13: Area and Properties of Operations

Cool Down: Equivalent Expressions



Select **all** the expressions that represent the area of the shaded region.

A. $(2 \times 3) + (2 \times \frac{2}{5})$

B. $6\frac{2}{5}$

C. $2 \times (3 + \frac{2}{5})$

D. $(2 \times 4) - (2 \times \frac{3}{5})$

E. $(2 \times 3) + \frac{2}{5}$

F. $2 \times \frac{17}{5}$

Lesson 14: Area Situations

Cool Down: Evaluate Expressions

Evaluate the expressions. Show your thinking.

1. $\frac{5}{3} \times 15$

2. $1\frac{3}{4} \times 8$

3. $\frac{10}{25} \times 10$

Lesson 15: Multiply More Fractions

Cool Down: Mixed Number Multiplication

Find the value of each expression. Explain or show your reasoning.

1. $12 \times 9\frac{2}{3}$

2. $3\frac{5}{9} \times 18$

Lesson 16: Estimate Products

Cool Down: Estimate and Solve

Jada says the value of each product is about 20. For each problem, explain why Jada's estimate is too high, just right, or too low.

1. $5\frac{5}{6} \times 4 = \underline{\hspace{2cm}}$

20 is...

too low

too high

about right

2. $3 \times 6\frac{5}{8} = \underline{\hspace{2cm}}$

20 is...

too low

too high

about right

Instructional Masters

Instructional Masters for Fractions as Quotients and Fraction Multiplication

address	title	students per copy	written on?	requires cutting?	card stock recommended?	color paper recommended?
Activity Grade5.2.7.2	Match the Situation	2	no	yes	no	no
Activity Grade5.2.9.2	Grid Paper 5	2	yes	no	no	no
Activity Grade5.2.13.1	Card Sort: Diagrams and Expressions	2	no	yes	no	no
Activity Grade5.2.2.2	Sandwich Match	2	no	yes	no	no
Activity Grade5.2.14.1	Info Gap: Area	2	no	yes	no	no
Center	Rolling for Fractions Stage 2 Recording Sheet	1	yes	no	no	no
Center	Compare Stage 4 Division Cards	2	no	yes	no	no
Center	Compare Stage 3-8 Directions	2	yes	no	no	no
Center	Compare Stage 3-8 Directions	2	yes	no	no	no
Center	Rolling for Fractions Stage 3 Recording Sheet	1	yes	no	no	no
Center	Target Measurement Stage 4 Recording Sheet	2	yes	no	no	no
Center	Target Measurement Stage 4 Homemade Protractor	2	no	yes	yes	no
Center	Fraction Cards Grade 3	2	no	no	yes	no
Center	Fraction Cards Grade 4	2	no	no	yes	no

Center	Target Measurement Stage 5 Recording Sheet	2	yes	no	no	no
Center	Target Measurement Stage 5 Spinner	2	no	no	no	no
Center	How Close? Stage 6 Recording Sheet	1	yes	no	no	no
Center	How Close? Stage 7 Recording Sheet	1	yes	no	no	no
Center	Rectangle Rumble Stage 3 Grid	2	yes	no	no	no
Center	Rectangle Rumble Stage 3 Spinners	2	no	no	no	no
Center	Rectangle Rumble Stage 4 Spinner	2	no	no	no	no
Center	Rectangle Rumble Stage 4 Grid	2	yes	no	no	no
Center	Can You Build It Stage 2 Directions	6	no	yes	no	no
Center	Number Cards (0-10)	2	no	yes	yes	no

Match the Situation

Match the Situation

A

$$4 \div 3$$

Match the Situation

B

$$3 \times \frac{1}{4}$$

Match the Situation

C

$$4 \times \frac{1}{3}$$

Match the Situation

D

$$3 - 4$$

Match the Situation

E

$$4 - 3$$

Match the Situation

F

$$\frac{1}{3} \times 4$$

Match the Situation

G

$$\frac{1}{4} \times 3$$

Match the Situation

H

$$3 \div 4$$

Match the Situation

Match the Situation

J



Match the Situation

K



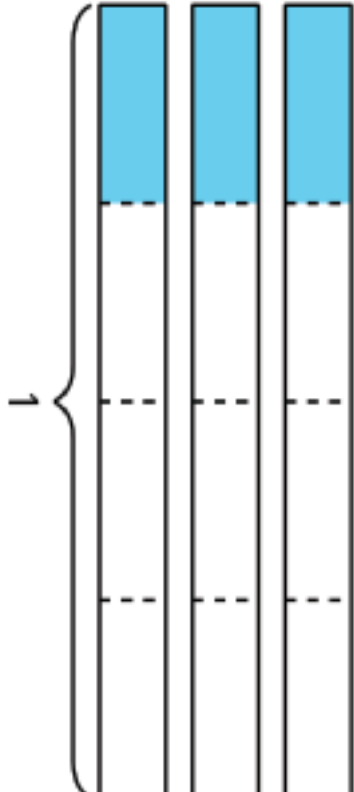
Match the Situation

L



Match the Situation

M





Card Sort: Diagrams and Expressions

Diagrams and Expressions

E

$$5 \times 3 \frac{2}{5}$$

Diagrams and Expressions

K

$$(5 \times 3) + \left(5 \times \frac{2}{5}\right)$$

Diagrams and Expressions

F

$$5 \times \left(3 + \frac{2}{5}\right)$$

Diagrams and Expressions

L

$$(5 \times 3) - \left(5 \times \frac{2}{5}\right)$$

Diagrams and Expressions

G

$$5 \times 2 \frac{2}{5}$$

Diagrams and Expressions

M

$$5 \times \frac{12}{5}$$

Card Sort: Diagrams and Expressions

Diagrams and Expressions

H

$$5 \times \left(2 + \frac{2}{5}\right)$$

Diagrams and Expressions

N

$$(5 \times 2) + \left(5 \times \frac{2}{5}\right)$$

Diagrams and Expressions

I

$$5 \times \frac{17}{5}$$

Diagrams and Expressions

O

$$3 \times 5\frac{2}{5}$$

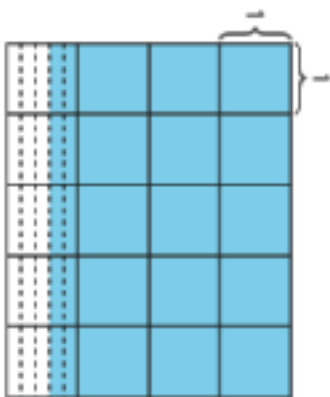
Diagrams and Expressions

J

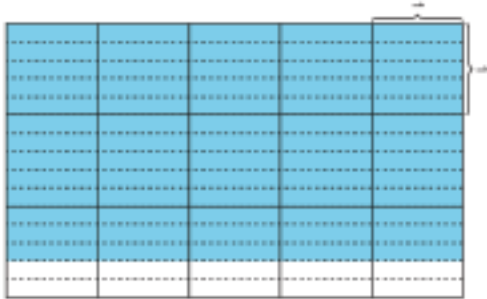
$$(3 \times 5) + \left(3 \times \frac{2}{5}\right)$$

Card Sort: Diagrams and Expressions

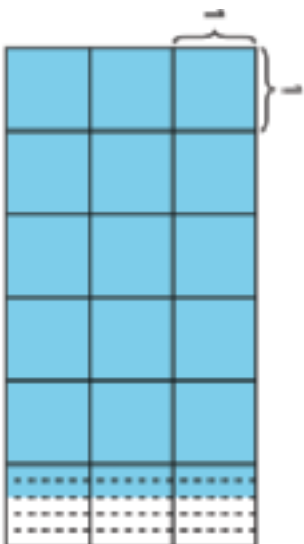
Diagrams and Expressions
A



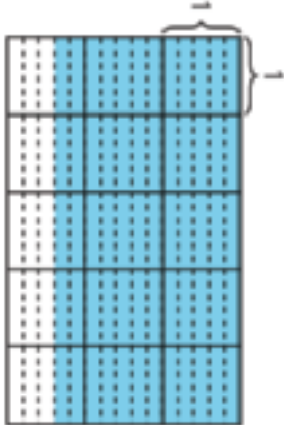
Diagrams and Expressions
B



Diagrams and Expressions
C

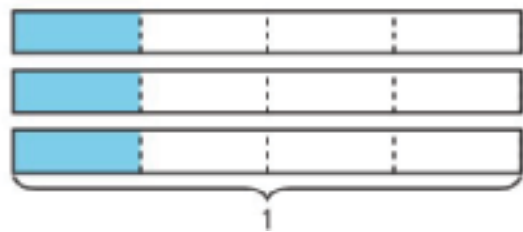


Diagrams and Expressions
D

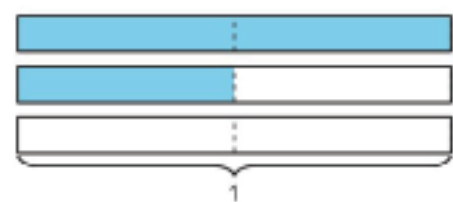


Sandwich Match

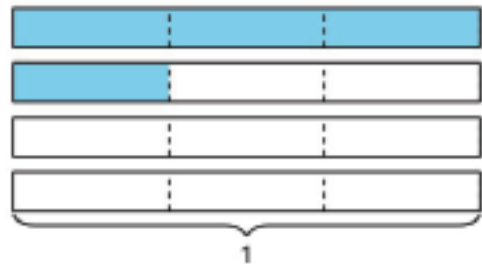
Sandwich Match
A.



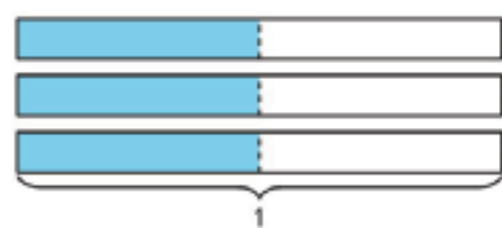
Sandwich Match
B.



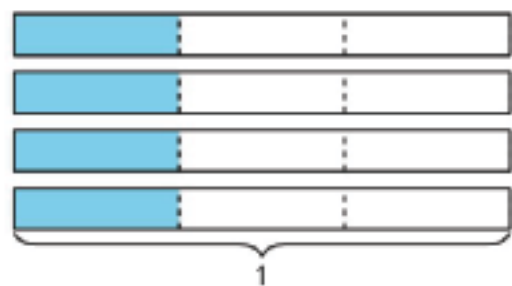
Sandwich Match
C.



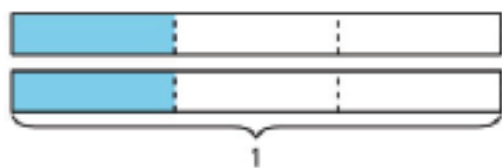
Sandwich Match
D.



Sandwich Match
E.



Sandwich Match
F.



Sandwich Match

Sandwich Match

G.

4 sandwiches are equally shared by 3 people

Sandwich Match

H.

3 sandwiches are equally shared by 4 people

Sandwich Match

I.

2 sandwiches are equally shared by 3 people

Sandwich Match

J.

3 sandwiches are equally shared by 2 people

Sandwich Match

K.

$$4 \div 3$$

Sandwich Match

L.

$$3 \div 4$$

Sandwich Match

M.

$$2 \div 3$$

Sandwich Match

N.

$$3 \div 2$$

Info Gap: Area

Info Gap: Area

Problem Card 1

A paper towel is rectangular. The length of the paper towel is 11 inches.

What is the area of the paper towel?

Info Gap: Area

Data Card 1

- The perimeter of the paper towel is 33 inches.
- The paper towel is half as wide as it is long.

Info Gap: Area

Problem Card 2

A corn field is rectangular. The width of the corn field is $1\frac{2}{3}$ miles.

What is the area of the corn field?

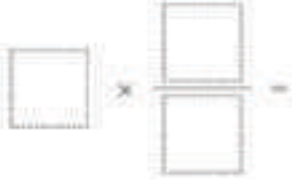
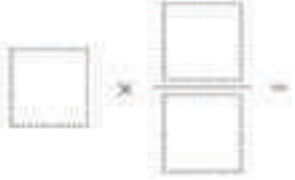
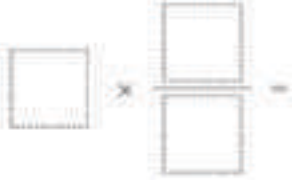
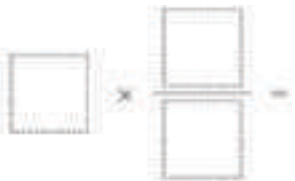
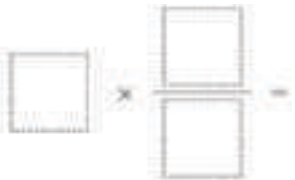
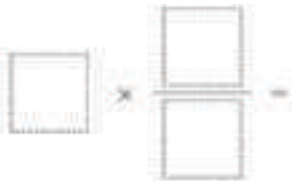
Info Gap: Area

Data Card 2

- The corn field is 3 times as long as it is wide.
- The corn field is $3\frac{1}{3}$ miles longer than it is wide.

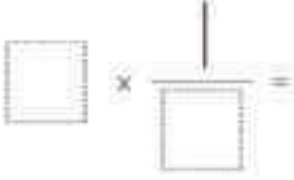
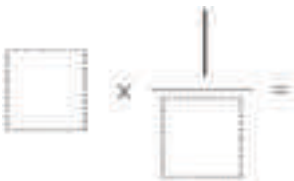
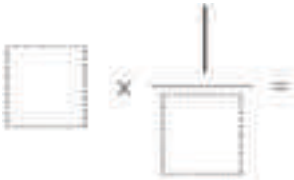
Rolling for Fractions Stage 2 Recording Sheet

- Each partner:
 - Roll 3 number cubes. Use the numbers to complete the expression. Write the product.
 - Check your partner's work to make sure you agree.
 - Determine the number of points each partner gets:
 - 2 points for creating an expression less than 1
 - 5 points for creating an expression greater than 1
 - 10 points for creating an expression that is equal to 1
- Repeat for the next round. The partner who has the most points once the recording sheet is full wins the game.

round	equation	points
1		
2		
3		
4		
5		
6		

Rolling for Fractions Stage 2 Recording Sheet

- Each partner:
 - Roll 3 number cubes. Use the numbers to complete the expression. Write the product.
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round	equation	points
1		
2		
3		
4		
5		
6		

Compare Stage 4 Division Cards

Compare Stage 4

$$78 \div 6$$

Compare Stage 4

$$84 \div 7$$

Compare Stage 4

$$68 \div 4$$

Compare Stage 4

$$65 \div 5$$

Compare Stage 4

$$90 \div 6$$

Compare Stage 4

$$45 \div 15$$

Compare Stage 4

$$57 \div 19$$

Compare Stage 4

$$72 \div 18$$

Compare Stage 4 Division Cards

Compare Stage 4

$$52 \div 13$$

Compare Stage 4

$$84 \div 12$$

Compare Stage 4

$$42 \div 7$$

Compare Stage 4

$$56 \div 8$$

Compare Stage 4

$$72 \div 9$$

Compare Stage 4

$$64 \div 8$$

Compare Stage 4

$$81 \div 9$$

Compare Stage 4

$$72 \div 3$$

Compare Stage 4 Division Cards

Compare Stage 4

$$92 \div 4$$

Compare Stage 4

$$69 \div 3$$

Compare Stage 4

$$84 \div 4$$

Compare Stage 4

$$63 \div 3$$

Compare Stage 3-8 Directions

Directions:

- Split the deck between the players.
- Each player turns over a card.
- Compare the values. The player with the greater value keeps both cards.
- If the values are the same, each player turns over one more card. The player with the greater value keeps all four cards.
- Play until you run out of cards. The player with the most cards at the end of the game wins.

Record any sets of cards that are challenging to compare:

Compare Stage 3-8 Directions

Directions:

- Split the deck between the players.
- Each player turns over a card.
- Compare the values. The player with the greater value keeps both cards.
- If the values are the same, each player turns over one more card. The player with the greater value keeps all four cards.
- Play until you run out of cards. The player with the most cards at the end of the game wins.

Record any sets of cards that are challenging to compare:

Rolling for Fractions Stage 3 Recording Sheet

- Each partner:
 - Roll 2 number cubes. Use the numbers to complete the expression. Write the quotient.
 - Check your partner’s work to make sure you agree.
 - Determine the number of points each partner gets:
 - 2 points for creating an expression less than 1
 - 5 points for creating an expression greater than 1
 - 10 points for creating an expression that is equal to 1
- Repeat for the next round. The partner who has the most points once the recording sheet is full wins the game.

round	equation	points
1	<div><div></div> ÷ <div></div> = <div><div></div><div></div></div></div>	
2	<div><div></div> ÷ <div></div> = <div><div></div><div></div></div></div>	
3	<div><div></div> ÷ <div></div> = <div><div></div><div></div></div></div>	
4	<div><div></div> ÷ <div></div> = <div><div></div><div></div></div></div>	
5	<div><div></div> ÷ <div></div> = <div><div></div><div></div></div></div>	
6	<div><div></div> ÷ <div></div> = <div><div></div><div></div></div></div>	

Target Measurement Stage 4 Recording Sheet

Directions:

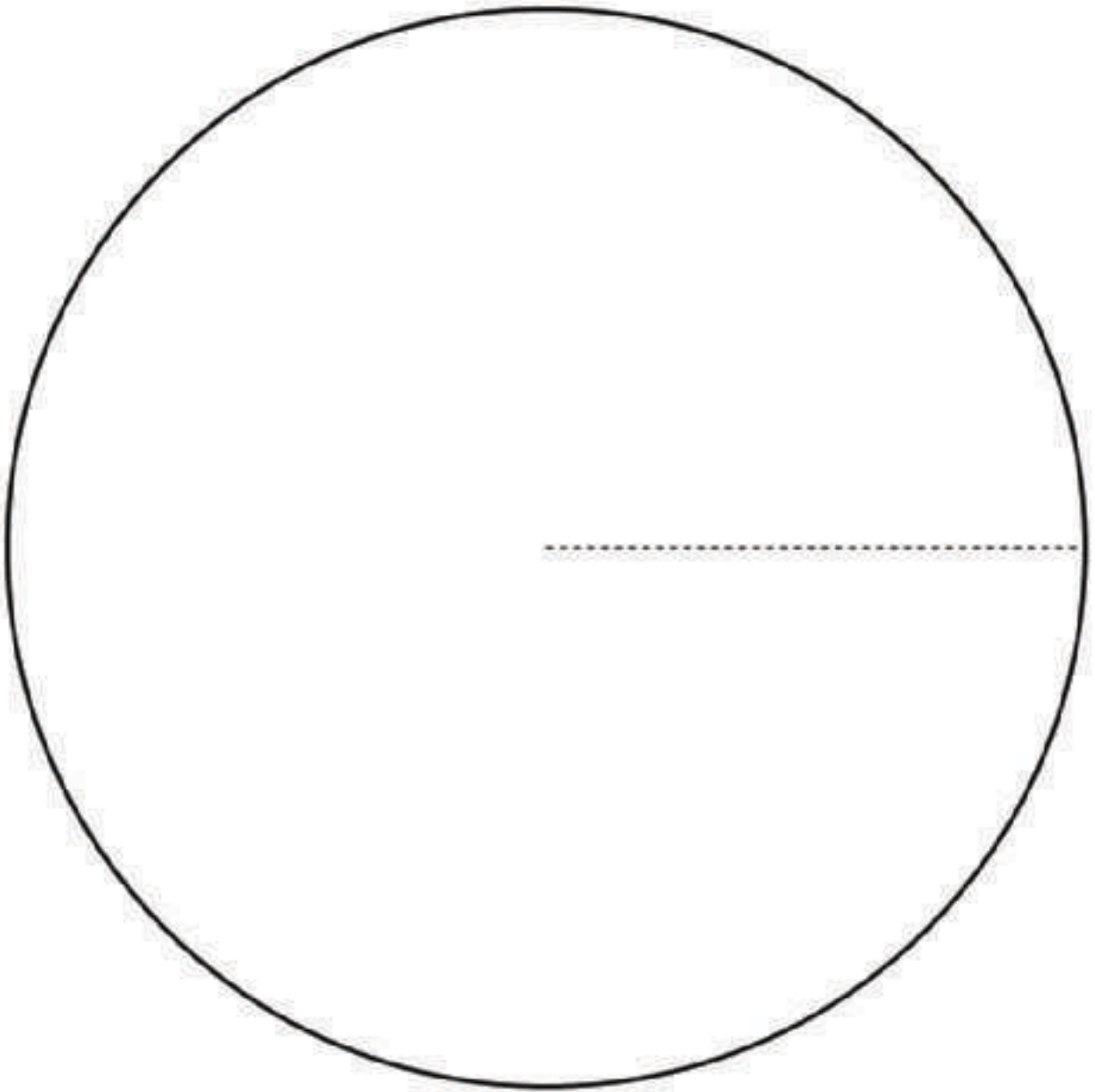
- Partner A:
 - Choose a target angle measure.
 - Begin to turn the handmade protractor.
- Partner B:
 - Say "Stop!" when you think the measure of the angle is equal to the target measurement.
- Both partners measure the line and find the difference between the actual measurement and the target measurement.
The difference is Partner B's score for the round.
- Take turns. After 8 rounds, the player with the lowest total score wins.

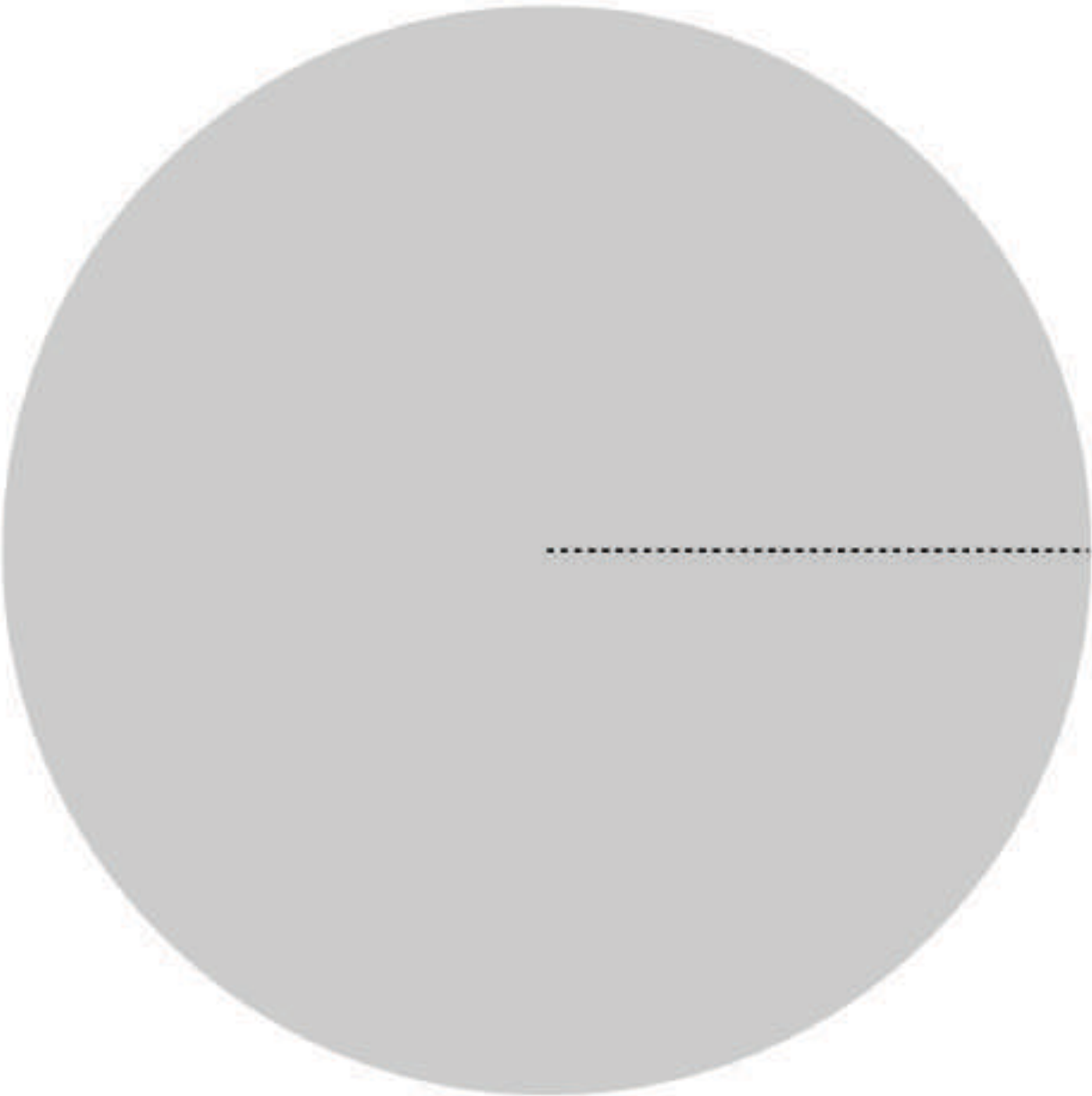
round	Partner A			Partner B		
	target degrees	actual degrees	points	target degrees	actual degrees	points
1						
2						
3						
4						
5						
6						
7						
8						

Target Measurement Stage 4 Homemade Protractor

To make a handmade protractor:

- Cut out each circle and cut a slit along the dashed segment (creating two flaps).
- Place the circles on top of one another and with the slit lined up.
- Rotate the top circle counterclockwise so that the flaps of the two circles interlock.





Fraction Cards Grade 3

Fraction Cards Grade 3

$$\frac{1}{4}$$

Fraction Cards Grade 3

$$\frac{2}{4}$$

Fraction Cards Grade 3

$$\frac{3}{4}$$

Fraction Cards Grade 3

$$\frac{4}{4}$$

Fraction Cards Grade 3

$$\frac{5}{4}$$

Fraction Cards Grade 3

$$\frac{1}{6}$$

Fraction Cards Grade 3

$$\frac{2}{6}$$

Fraction Cards Grade 3

$$\frac{3}{6}$$

Fraction Cards Grade 3

Fraction Cards Grade 3

$$\frac{4}{6}$$

Fraction Cards Grade 3

$$\frac{5}{6}$$

Fraction Cards Grade 3

$$\frac{6}{6}$$

Fraction Cards Grade 3

$$\frac{7}{6}$$

Fraction Cards Grade 3

$$\frac{1}{2}$$

Fraction Cards Grade 3

$$\frac{2}{2}$$

Fraction Cards Grade 3

$$\frac{1}{3}$$

Fraction Cards Grade 3

$$\frac{2}{3}$$

Fraction Cards Grade 3

Fraction Cards Grade 3

$$\frac{3}{3}$$

Fraction Cards Grade 3

$$\frac{6}{3}$$

Fraction Cards Grade 3

$$\frac{4}{2}$$

Fraction Cards Grade 3

$$\frac{16}{6}$$

Fraction Cards Grade 3

$$\frac{6}{2}$$

Fraction Cards Grade 3

$$\frac{8}{2}$$

Fraction Cards Grade 3

$$\frac{5}{3}$$

Fraction Cards Grade 3

$$\frac{13}{4}$$

Fraction Cards Grade 4

Fraction Cards Grade 4

$$\frac{1}{8}$$

Fraction Cards Grade 4

$$\frac{2}{8}$$

Fraction Cards Grade 4

$$\frac{3}{8}$$

Fraction Cards Grade 4

$$\frac{4}{8}$$

Fraction Cards Grade 4

$$\frac{5}{8}$$

Fraction Cards Grade 4

$$\frac{6}{8}$$

Fraction Cards Grade 4

$$\frac{7}{8}$$

Fraction Cards Grade 4

$$\frac{8}{8}$$

Fraction Cards Grade 4

Fraction Cards Grade 4

$$\frac{1}{5}$$

Fraction Cards Grade 4

$$\frac{2}{5}$$

Fraction Cards Grade 4

$$\frac{3}{5}$$

Fraction Cards Grade 4

$$\frac{4}{5}$$

Fraction Cards Grade 4

$$\frac{5}{5}$$

Fraction Cards Grade 4

$$\frac{6}{5}$$

Fraction Cards Grade 4

$$\frac{1}{10}$$

Fraction Cards Grade 4

$$\frac{2}{10}$$

Fraction Cards Grade 4

Fraction Cards Grade 4

$$\frac{3}{10}$$

Fraction Cards Grade 4

$$\frac{4}{10}$$

Fraction Cards Grade 4

$$\frac{5}{10}$$

Fraction Cards Grade 4

$$\frac{6}{10}$$

Fraction Cards Grade 4

$$\frac{7}{10}$$

Fraction Cards Grade 4

$$\frac{8}{10}$$

Fraction Cards Grade 4

$$\frac{9}{10}$$

Fraction Cards Grade 4

$$\frac{10}{10}$$

Fraction Cards Grade 4

Fraction Cards Grade 4

$$\frac{11}{10}$$

Fraction Cards Grade 4

$$\frac{19}{10}$$

Fraction Cards Grade 4

$$\frac{1}{12}$$

Fraction Cards Grade 4

$$\frac{3}{12}$$

Fraction Cards Grade 4

$$\frac{4}{12}$$

Fraction Cards Grade 4

$$\frac{7}{12}$$

Fraction Cards Grade 4

$$\frac{9}{12}$$

Fraction Cards Grade 4

$$\frac{10}{12}$$

Fraction Cards Grade 4

Fraction Cards Grade 4

$$\frac{13}{12}$$

Fraction Cards Grade 4

$$\frac{15}{12}$$

Fraction Cards Grade 4

$$\frac{1}{100}$$

Fraction Cards Grade 4

$$\frac{5}{100}$$

Fraction Cards Grade 4

$$\frac{10}{100}$$

Fraction Cards Grade 4

$$\frac{20}{100}$$

Fraction Cards Grade 4

$$\frac{49}{100}$$

Fraction Cards Grade 4

$$\frac{50}{100}$$

Fraction Cards Grade 4

Fraction Cards Grade 4

$$\frac{51}{100}$$

Fraction Cards Grade 4

$$\frac{75}{100}$$

Fraction Cards Grade 4

$$\frac{99}{100}$$

Fraction Cards Grade 4

$$\frac{200}{100}$$

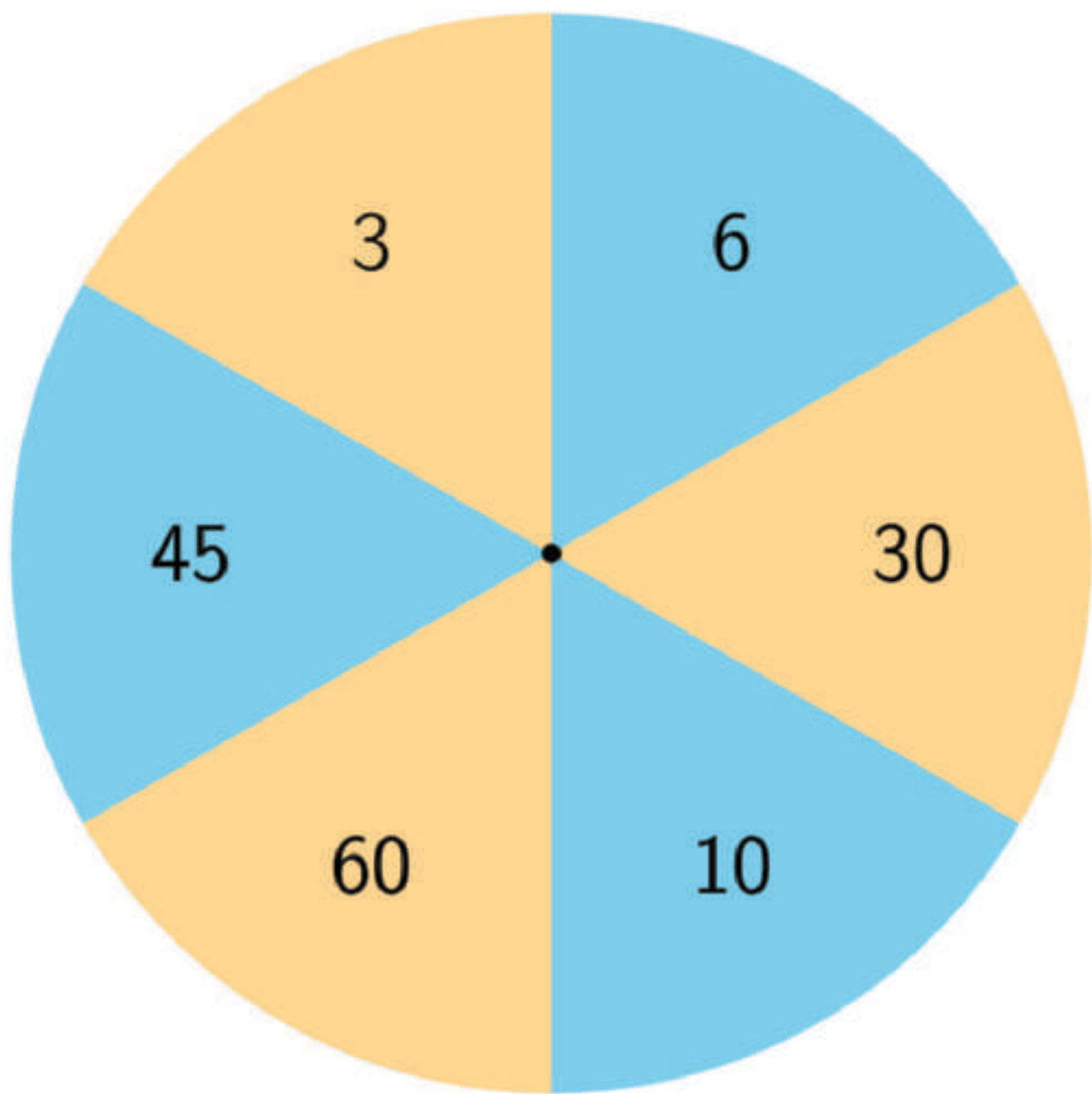
Target Measurement Stage 5 Recording Sheet

Directions:

- Partner A:
 - Spin the spinner to get a denominator for your fraction.
 - Choose a target fraction less than 1 with that denominator.
 - Begin to turn the handmade protractor.
- Partner B:
 - Say "Stop!" when you think the measure of the angle is that fraction of 180 degrees.
- Both partners measure the angle and calculate the difference between the target number of degrees and the actual number of degrees. That is Partner B's score for the round.
- Take turns. After 8 rounds, the player with the lowest total score wins.

round	Partner A				Partner B			
	target fraction	target fraction in degrees	actual measure	points	target fraction	target fraction in degrees	actual measure	points
1								
2								
3								
4								
5								
6								
7								
8								

Target Measurement Stage 5 Spinner



How Close? Stage 6 Recording Sheet

Directions:

- Each partner:
 - Take 6 cards.
 - Choose 4 cards to make a multiplication expression. You can multiply a one-digit number by a three-digit number or a two-digit number by a two-digit number.
 - Write an equation to show the product of the numbers you made.
 - Your score for each round is the difference between your product and 3,000.
- Take new cards so that you have 6 cards to start the next round.
- At the end of the game, add your score for each round. The player with the lowest score wins.

round	multiplication equation	points for the round
1		
2		
3		
4		
5		
6		
7		
8		

How Close? Stage 7 Recording Sheet

Directions:

- Each partner:
 - Take 6 cards.
 - Choose 3 cards to make a multiplication expression.
 - Write an equation to show the product of the numbers you made.
 - Your score for each round is the difference between your product and 5.
- Take new cards so that you have 6 cards to start the next round.
- At the end of the game, add your score for each round. The player with the lowest score wins.

round	multiplication expression	points for the round
1	<div><div><div></div><div></div></div><div>×</div><div></div><div>=</div><div></div></div>	
2	<div><div><div></div><div></div></div><div>×</div><div></div><div>=</div><div></div></div>	
3	<div><div><div></div><div></div></div><div>×</div><div></div><div>=</div><div></div></div>	
4	<div><div><div></div><div></div></div><div>×</div><div></div><div>=</div><div></div></div>	

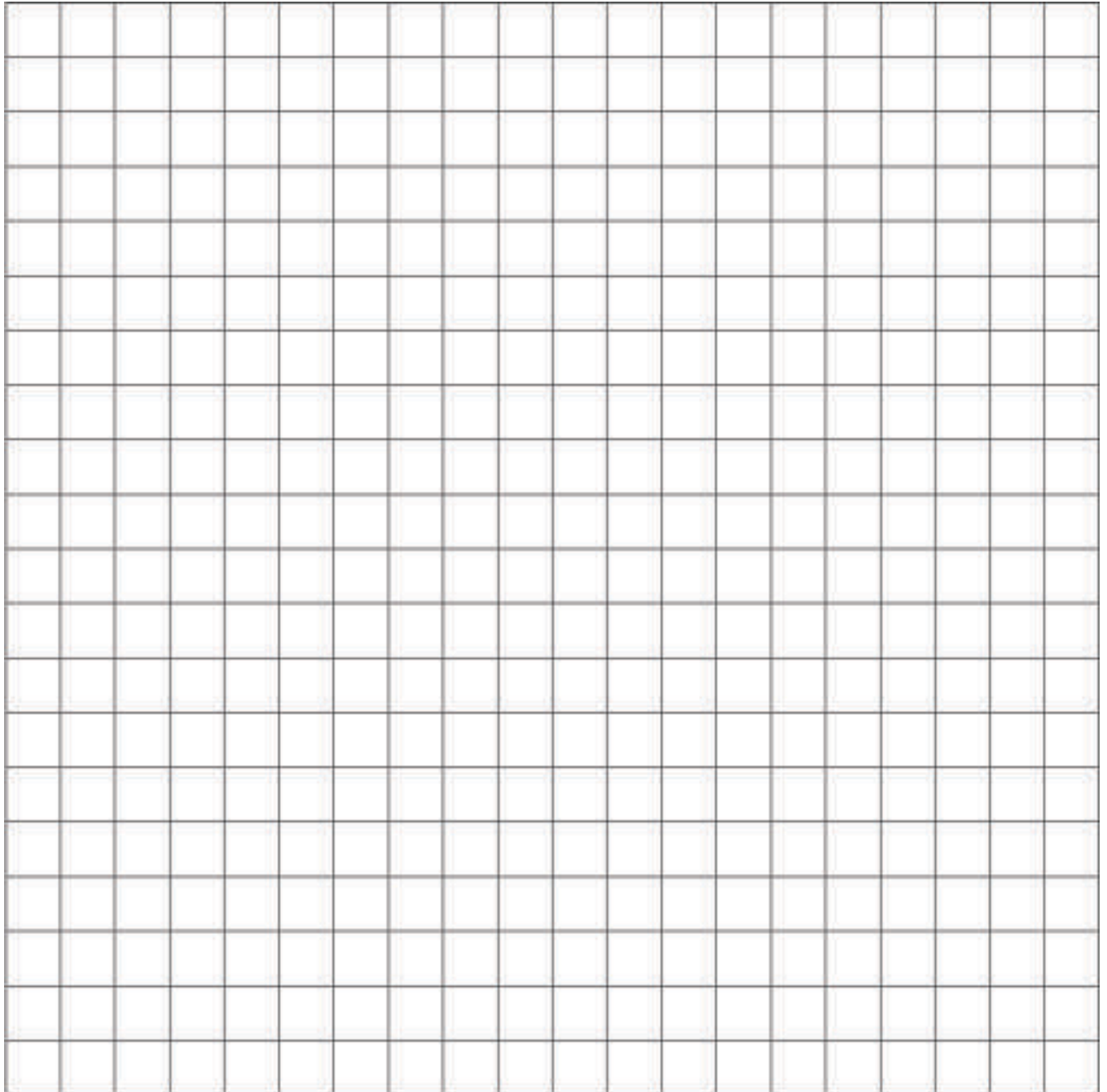
How Close? Stage 7 Recording Sheet

5	<div><div><div></div><div></div></div><div>×</div><div></div><div>=</div><div></div></div>	
6	<div><div><div></div><div></div></div><div>×</div><div></div><div>=</div><div></div></div>	
7	<div><div><div></div><div></div></div><div>×</div><div></div><div>=</div><div></div></div>	
8	<div><div><div></div><div></div></div><div>×</div><div></div><div>=</div><div></div></div>	

Rectangle Rumble Stage 3 Grid

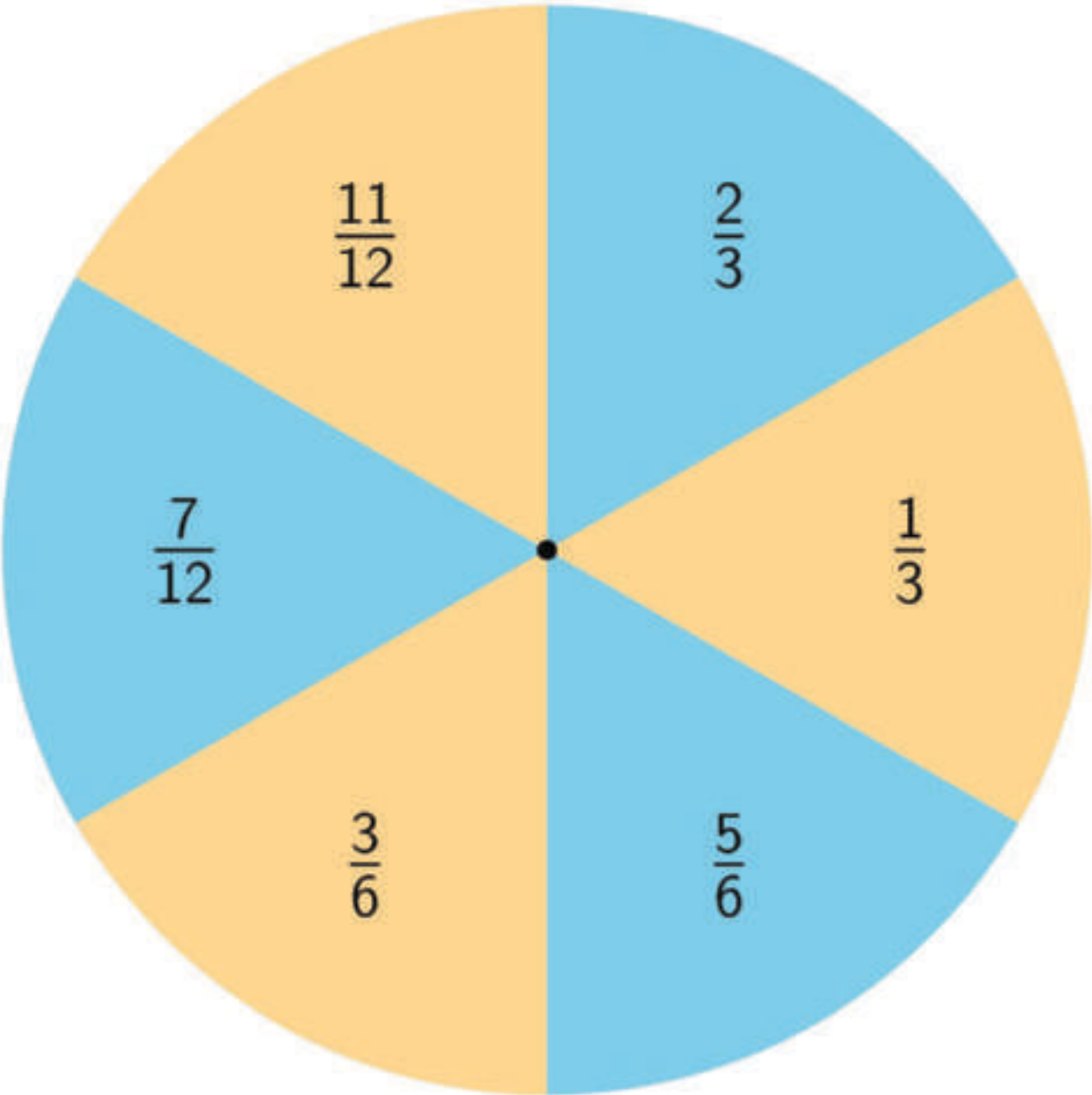
Directions:

- Choose a color for your rectangles different from your partner.
- On your turn:
 - Spin each spinner.
 - Shade in a rectangular area to represent the product of the two numbers.
- Take turns until the grid can't fit any more rectangles.
- Each partner adds up their total area, the partner with the greatest total square units wins.





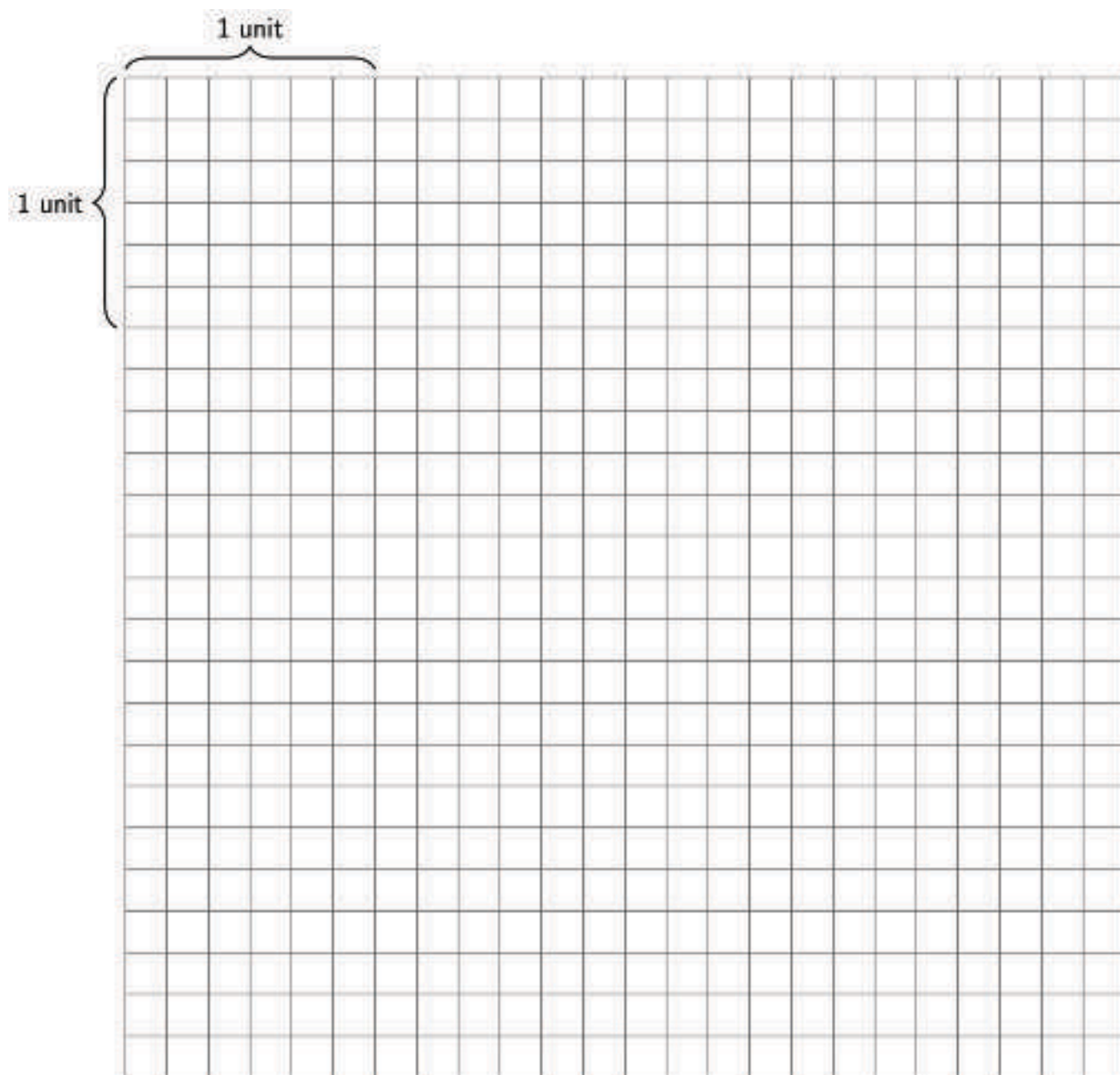




Rectangle Rumble Stage 4 Grid

Directions:

- Choose a color for your rectangles different from your partner.
- On your turn:
 - Spin the spinner and roll the number cube.
 - Shade in a rectangular area to represent the product of the two numbers.
- Take turns until the grid can't fit any more rectangles.
- Each partner adds up their total area, the partner with the greatest total square units wins.



Can You Build It Stage 2 Directions

Directions:

- Take 2 number cards to make a two-digit number.
- Both partners build as many rectangles as they can with that area.
- When both players are finished, compare rectangles.
- Each player gets 1 point if they both have all the same rectangles.
- A player gets 2 points if they build a rectangle with the given area that their partner does not have.
- The player with the most points after 8 rounds wins the game.



Directions:

- Take 2 number cards to make a two-digit number.
- Both partners build as many rectangles as they can with that area.
- When both players are finished, compare rectangles.
- Each player gets 1 point if they both have all the same rectangles.
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Directions:

- Take 2 number cards to make a two-digit number.
- Both partners build as many rectangles as they can with that area.
- When both players are finished, compare rectangles.
- Each player gets 1 point if they both have all the same rectangles.
- A player gets 2 points if they build a rectangle with the given area that their partner does not have.
- The player with the most points after 8 rounds wins the game.



Number Cards (0-10)

1

2

3

4

5

6

Number Cards (0-10)

7

8

9

1

2

3

Number Cards (0-10)

4

5

6

7

8

9

Number Cards (0-10)

0

0

10

10

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