Core Knowledge® MATHEMATICS

Putting It All Together

$2 \times 2 = 4$

$4 \times 2 = 8$

$8 : 4 = 2$

$4 - 2 = 2$

Putting It All Together

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Teacher Guide
Core Knowledge Mathematics™

Unit 9: Putting It All Together

At a Glance

Unit 9 is estimated to be completed in 14 days including 2 days for assessment.

This unit is divided into four sections including 12 lessons.

- Section A—Reason with Fractions (Lessons 1-3)
- Section B—Whole-number Operations (Lessons 4-6)
- Section C—Solve Problems with Multiplication and Division (Lessons 7-9)
- Section D—Creation and Design (Lessons 10-12)

On page 7 of this Teacher Guide is a chart that identifies the section each lesson belongs in and the materials needed for each lesson.

There are no new centers in this unit. Students choose from centers that have been introduced throughout the year. Students can work at any previously introduced stages of the centers.

Unit 9: Putting It All Together

Unit Learning Goals

- Students consolidate and solidify their understanding of various concepts and skills related to major work of the grade. They also continue to work toward fluency goals of the grade.

In this unit, students revisit major work and fluency goals of the grade, applying their learning from the year.

In Section A, students reinforce what they learn about comparing fractions, adding and subtracting fractions, and multiplying fractions and whole numbers. In Section B, they strengthen their ability to add and subtract multi-digit numbers fluently using the standard algorithm. They also multiply and divide numbers by reasoning about place value and practice doing so strategically.

Here are the times of the runners for two teams.
Which team won the relay race?

runner	Diego's team, time (seconds)	Jada's team, time (seconds)
1	$10\frac{25}{100}$	$11\frac{9}{10}$
2	$11\frac{40}{100}$	$9\frac{8}{10}$
3	$9\frac{7}{10}$	$9\frac{84}{100}$
4	$10\frac{5}{100}$	$10\frac{60}{100}$

In Section C, students practice making sense of situations and solving problems that involve reasoning with multiplication and division, including multiplicative comparison and interpreting remainders. In the final section, students review major work of the grade as they create activities in the format of the warm-ups routines they have encountered throughout the year (Estimation Exploration, Number Talk, and Which One Doesn't Belong?).

The sections in this unit are standalone sections, not required to be completed in order. Within a section, lessons can also be completed selectively and without competing prior lessons. The goal is to offer ample opportunities for students to integrate the knowledge they have gained and to practice skills related to the expected fluencies of the grade.

Section A: Reason with Fractions

Standards Alignments

Addressing   4.NF.A.1, 4.NF.A.2, 4.NF.B.3, 4.NF.B.3.a, 4.NF.B.3.b, 4.NF.B.3.c, 4.NF.B.3.d, 4.NF.B.4, 4.NF.C.5, 4.NF.C.6, 4.NF.C.7

Building
Towards   5.NF.A.1

Section Learning Goals

- Solve problems involving fraction equivalence and operating with fractions.

In this section, students solve problems that require multiplying fractions by whole numbers and adding and subtracting fractions with the same denominator.

They apply the reasoning strategies developed in the course and their understanding of fractions and equivalence to compare fractions, add and subtract whole numbers and fractions (including mixed numbers), and find sums and differences of tenths and hundredths.

The lessons also prompt students to reason about fractional quantities in a variety of contexts that invite them to share their own cultural experiences and learn about the experiences of others.

Jada and Lin are making head wraps from African wax print fabric.

Jada stitches together 5 pieces of fabric that each have a length of $\frac{2}{6}$ yard.

Lin stitched together 3 pieces of fabric that are each $\frac{2}{3}$ yard long.

Who used more fabric?

👤↔👤 PLC: Lesson 1, Activity 1, Let's Make Head Wraps!

Section B: Whole-number Operations

Standards Alignments

Addressing   4.NBT.B, 4.NBT.B.4, 4.NBT.B.5, 4.NBT.B.6

Section Learning Goals

- Add, subtract, multiply, and divide multi-digit numbers using place value understanding.

In this section, students deepen their understanding of place value and build their fluency in performing operations on multi-digit numbers.

Students begin by practicing the standard algorithm for addition and subtraction. They also attend to potential errors in using the algorithm, particularly when it is necessary to decompose or compose a base-ten unit multiple times, as in the case when subtracting from a number with zeros. Students consider different strategies for approaching multi-digit subtraction, including by leveraging the relationship between addition and subtraction.

To find the value of $20{,}000 - 472$, Priya and Han set up their calculations differently.

Use both methods to find the difference of 20,000 and 472.

Priya

$$
\begin{array}{r}
2\ 0,\ 0\ 0\ 0 \\
-\ \ \ \ \ \ \ \ 4\ 7\ 2 \\
\hline
\end{array}
$$

Han

$$
\begin{array}{r}
4\ 7\ 2 \\
+\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \\
\hline
2\ 0,\ 0\ 0\ 0 \\
\end{array}
$$

Next, they practice multiplying and dividing multi-digit numbers using algorithms that involve partial products and partial quotients. In both cases, students analyze and make connections across different methods of recording the process of multiplication and division. The work here prepares students to study the standard algorithm for multiplication and for division more closely in grade 5.

Here are two ways to find 34×21.

In method A, where do the 4, 30, 80, and 600 come from?

In method B, which two numbers are multiplied to get 34? 680?

A

$$
\begin{array}{r}
3\ 4 \\
\times\ \ \ 2\ 1 \\
\hline
1\ \ \ \ \ \\
4 \\
3\ 0 \\
8\ 0 \\
+\ 6\ 0\ 0 \\
\hline
7\ 1\ 4 \\
\end{array}
$$

B

$$
\begin{array}{r}
3\ 4 \\
\times\ \ \ 2\ 1 \\
\hline
1\ \ \ \ \ \\
3\ 4 \\
+\ 6\ 8\ 0 \\
\hline
7\ 1\ 4 \\
\end{array}
$$

👤↔👤 PLC: Lesson 4, Activity 1, Lots of Zeros

Section C: Solve Problems with Multiplication and Division

Standards Alignments

Addressing   4.NBT.B, 4.NBT.B.4, 4.NBT.B.5, 4.NBT.B.6, 4.OA.A.2, 4.OA.A.3

Section Learning Goals

- Solve problems involving measurement comparison.

In this section, students practice solving real-world problems using multiplication and division. Throughout the section, students reason with mathematics in different ways. They look for ways to compare quantities with addition or multiplication. They make estimates to simplify a problem or to assess the reasonableness of a statement or value before and after performing calculations. They also continue to reason with diagrams and equations, connecting these representations and the solution to a problem back to the context of the problem.

	Bermuda	cost in Bermuda is _____ as in India	India
a meal with drink (1 person)		12 times as much	$2
gasoline (1 gallon)	$8	2 times as much	
brand-name jeans		2.5 times as much	$31
men's leather shoes	$143	4 times as much	
internet connection		14 times as much	$13

Students encounter problems that involve division and multiplication with large numbers, but are not expected to divide by multi-digit divisors. All problems can be reasoned and estimated by multiplication, by rounding, and by relating the quantities to nearby multiples of 10 or 100. In one lesson, students have the opportunity to formulate their own problems given a context and some parameters about the situation.

👤↔👤 PLC: Lesson 7, Activity 1, The Most and Least Expensive

Section D: Creation and Design

Standards Alignments
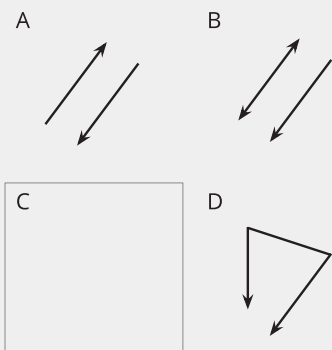
Addressing   4.G.A.1, 4.NBT.A, 4.NBT.A.1, 4.NBT.B, 4.NBT.B.5, 4.NF.B, 4.NF.B.4, 4.OA, 4.OA.A, 4.OA.A.3, 4.OA.B.4, 4.OA.C, 4.OA.C.5

Section Learning Goals

- Review the major work of the grade by creating and designing instructional routines.

Throughout the course, students have engaged in warm-up routines such as How Many Do You See, Exploration Estimation, Which One Doesn't Belong, True or False, and Number Talk. This section enables them to apply the mathematics they have learned to design warm-ups that incorporate some of these routines

Each lesson is devoted to a particular routine. Students begin by completing at least two partially created tasks, each with more missing parts to complete than the previous one. They practice anticipating responses that others might give to the prompts they pose.

a. Find at least one reason that all items belong in the set.

b. Find at least one reason that each item doesn't belong.

c. Add an item to complete each set. Make sure there is at least one reason it belongs and one reason it doesn't belong.

Along the way, students gain the skills and insights needed to create an activity from scratch or with minimal scaffolding. In each lesson, students have the option to facilitate their activity with another group in the class.

👤↔👤 PLC: Lesson 11, Activity 2, Add Two That Don't Belong

Throughout the Unit

The warm-ups throughout the unit develop students' fluency in using the four operations with whole numbers and promote reasoning about the structure of place value. They also promote flexibility with addition and subtraction of fractions, and with multiplication of fractions and whole numbers.

Here is a sampling of the warm-ups in the unit:

lesson 1	lesson 4	lesson 8	lesson 11
Number Talk	Number Talk	Number Talk	Which One Doesn't Belong?
$5 \times \frac{10}{5}$	$87 - 24$	$848 \div 8$	A. 0, 4, 8, 12, 16
$8 \times \frac{11}{4}$	$387 - 124$	$4,848 \div 8$	B. 3, 6, 9, 12, 15
$9 \times \frac{6}{3}$	$6,387 - 129$	$4,852 \div 8$	C. 5, 105, 205, 305, 405
$6 \times \frac{12}{10}$	$6,387 - 4,329$	$5,848 \div 8$	D. 6, 60, 600, 6,000, 60,000

Materials Needed

LESSON	GATHER	COPY
A.1	• none	• none
A.2	• none	• none
A.3	• none	• none
B.4	• Grid paper	• none
B.5	• Grid paper	• none
B.6	• none	• none
C.7	• none	• none
C.8	• none	• none
C.9	• none	• none
D.10	• Tools for creating a visual display	• none
D.11	• none	• none
D.12	• none	• none

Section A: Reason with Fractions

Lesson 1: Add, Subtract, and Multiply Fractions

Standards Alignments

Addressing   4.NF.B.3, 4.NF.B.4, 4.NF.C.5

Teacher-facing Learning Goals

- Solve problems involving addition and subtraction of fractions.
- Solve problems involving multiplication of a fraction by a whole number.

Student-facing Learning Goals

- Let's practice solving problems involving fractions.

Lesson Purpose

The purpose of this lesson is for students to represent and solve problems involving fraction operations. Students also reason about equivalence to compare fractions to whole numbers.

In this lesson, students practice multiplying a fraction and a whole number and adding and subtracting fractions, including mixed numbers. They rely on their understanding of equivalence and the properties of operations to decompose fractions, whole numbers, and mixed numbers to enable comparison, addition, subtraction, and multiplication (MP7).

If students need additional support with the concepts in this lesson, refer back to Unit 3, Section B in the curriculum materials.

Access for:

🛧 Students with Disabilities

- Representation (Activity 2)

🌐 English Learners

- MLR8 (Activity 1)

Instructional Routines

Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	10 min
Activity 3	10 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What evidence from today's lesson indicates students are thinking flexibly as they add, subtract, and multiply fractions?

Cool-down (to be completed at the end of the lesson) ⏱ 5 min

Compare to 2

Standards Alignments

Addressing   4.NF.B.3

Student-facing Task Statement

Here are some fractions: $\frac{15}{10}$ $\frac{13}{10}$ $\frac{53}{100}$ $\frac{9}{10}$

1. Select two fractions that have a sum greater than 2. Explain or show your reasoning.
2. Use all four fractions to write an expression that has a value greater than 1 but less than 2.

Student Responses

1. Sample response:
   a. $\frac{13}{10} + \frac{9}{10} = \frac{22}{10} = \frac{10}{10} + \frac{10}{10} + \frac{2}{10} = 2\frac{2}{10}$
   b. I know $\frac{15}{10}$ is the same as $1\frac{1}{2}$. All the other choices are more than $\frac{1}{2}$, so I could pick $\frac{15}{10}$ and any of the others.
   c. $\frac{15}{10} + \frac{53}{100} = \frac{203}{100} = \frac{100}{100} + \frac{100}{100} + \frac{3}{100} = 2\frac{3}{100}$
2. $\frac{15}{10} + \frac{53}{100} - \frac{9}{10} + \frac{13}{10}$ or $\frac{15}{10} + \frac{9}{10} - \frac{53}{100} - \frac{13}{100}$

- Begin Lesson -

Warm-up

🕐 10 min

Number Talk: Fluency and Fractions

Standards Alignments

Addressing   4.NF.B.4

This Number Talk encourages students to think flexibly about numbers to multiply. The understandings elicited here will be helpful throughout this unit as students build toward fluency with multiplying fractions and whole numbers.

Students use what they know about fractions and equivalent fractions to apply the properties of operations to find the products (MP7).

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $5 \times \frac{10}{5}$
- $9 \times \frac{6}{3}$
- $8 \times \frac{11}{4}$
- $6 \times \frac{12}{10}$

Student Responses

- $\frac{50}{5}$ or 10.
  - I know that $\frac{10}{5}$ is 2 and 5×2 is 10.
  - If 5 fifths is 1, then 50 fifths is 10.
- $\frac{54}{3}$ or 18.
  - Six thirds is 2 and 9×2 is 18.
  - If 3 groups of 6 thirds is 18 thirds, which is 6, then 9 groups of 6 thirds is 3 times as much, which is 18.
- $\frac{88}{4}$ or 22.
  - 8×11 is 88, so there are 88 fourths. I

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."

Activity

- 1 minute: quiet think time
- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "How is the last expression different from the others?" (Sample responses:
  - The denominator in the fraction is not a factor of the whole number.
  - The whole number is not a multiple of the denominator in the fraction.
  - The value of the product is not a whole number.)
- Consider asking:

know 80 fourths is 20 and 8 fourths is 2, so 88 fourths is $20 + 2$.

- $\frac{11}{4}$ is $11 \times \frac{1}{4}$. To find $8 \times 11 \times \frac{1}{4}$, I multiplied 8 and $\frac{1}{4}$ first, which is $\frac{8}{4}$ or 2, and $2 \times 11 = 22$.

- $\frac{72}{10}$ or $7\frac{2}{10}$ or $7\frac{1}{5}$.

  - Six times 10 is 60 and 6 times 2 is 12, so 6 times 12 tenths is 72 tenths.

  - I know that $\frac{12}{10}$ is $1\frac{2}{10}$. I found 6×1, which is 6, and $6 \times \frac{2}{10}$, which is $\frac{12}{10}$ or $1\frac{2}{10}$ and added them.

- "Who can restate _____'s reasoning in a different way?"

- "Did anyone have the same strategy but would explain it differently?"

- "Did anyone approach the problem in a different way?"

- "Does anyone want to add on to _____'s strategy?"

Activity 1

🕐 15 min

Let's Make Head Wraps!

👤↔👤 PLC Activity

Standards Alignments

Addressing   4.NF.B.4

In this activity, students multiply fractions by whole numbers and compare fractions to solve problems (MP2). They make comparisons by reasoning about the denominator of fractions or about equivalence.

🌐 Access for English Learners

MLR8 Discussion Supports. Prior to solving the problems, invite students to make sense of the situations and take turns sharing their understanding with their partner. Listen for and clarify any questions about the context.
Advances: Reading, Representing

Student-facing Task Statement

Jada and Lin saw a picture of head wraps made of African wax print fabric and

Launch

- "Look at the picture of the two women with head wraps. What do you notice? What do you wonder?"

would like to make their own.

1. Jada stitches together 5 pieces of fabric that each have a length of $\frac{2}{6}$ yard. Write an equation to show the total length of fabric Jada used.

2. Lin stitches together 3 pieces of fabric that are each $\frac{2}{3}$ yard long. Write an equation to show the total length of fabric Lin used.

3. Who used more fabric? Explain or show your reasoning.

Student Responses

1. $5 \times \frac{2}{6} = \frac{10}{6}$
2. $3 \times \frac{2}{3} = \frac{6}{3}$
3. Lin used more fabric because her head wrap is $\frac{6}{3}$ yards or exactly 2 yards and Jada's is $\frac{10}{6}$ yards, which is less than 2 yards.

- Collect observations and questions from 1–2 students.
- "In many African cultures, women wrap their hair with colorful fabric when they dress for the day."
- "Have you seen a similar practice such as this one? What is your routine for dressing for the day?"
- Allow 1–2 students to share.
- "We will be thinking about the length of head wraps in this activity."

Activity

- 5 minutes: independent work time
- 5 minutes: partner work
- Monitor for diagrams and multiplication equations that represent each situation.

Synthesis

- Select 2–3 students to share their equations and reasoning.

Advancing Student Thinking

If students use repeated addition to find the product of a whole number and a fraction, consider asking:

- "How did you find the length of the fabric?"
- "What is another way you could have found the length?"
- "How could you find the length of the fabric using multiplication?"

Activity 2

⏱ 10 min

Make 2 Yards of Fabric

Standards Alignments

Addressing   4.NF.B.3, 4.NF.B.4

The purpose of this activity is to practice adding and subtracting fractions. Students reason about different combinations of fractions to make 2. Students can find many combinations by looking for ways to add fractions that have the same denominator. Although not required by the standards for grade 4, the activity also invites students to use their understanding of equivalence to combine fractions with unlike denominators in preparation for grade 5. In the synthesis, emphasize the ways students used addition and subtraction and reasoned about equivalent fractions.

♿ Access for Students with Disabilities

Representation: Develop Language and Symbols. Before showing the problem, activate or supply background knowledge. Tell students that it will be helpful to think flexibly about different ways to make the number 2. Offer the examples $1\frac{1}{2} + \frac{1}{2}$ and $\frac{6}{3}$. Invite students to write down at least three additional different ways to represent 2 and explain their reasoning to a partner.
Supports accessibility for: Conceptual Processing, Organization, Memory

Student-facing Task Statement

Jada and Lin's moms taught the fourth-grade class how to combine and use fabric pieces for head wraps. The lengths of each piece of fabric are listed here.

$\frac{2}{6}$ yard	$\frac{2}{6}$ yard	$\frac{2}{6}$ yard	$\frac{11}{10}$ yard
$1\frac{2}{5}$ yards	$\frac{9}{10}$ yard	$\frac{2}{6}$ yard	$\frac{6}{12}$ yard
$\frac{3}{6}$ yard	$\frac{2}{6}$ yard	$\frac{2}{6}$ yard	$\frac{12}{12}$ yard
$\frac{2}{6}$ yard	$\frac{3}{5}$ yard	$\frac{2}{6}$ yard	

Launch

- Groups of 2

Activity

- 5 minutes: independent work time
- 5 minutes: partner discussion
- Monitor for students who:
    - use multiplication to show combining multiples of the same length
    - use equations with addition and multiplication

Find as many different combinations of fabric that would have a length of 2 yards as you can. Each piece of fabric can only be used one time. Write an equation for each combination.

Student Responses

Sample responses:

- $6 \times \frac{2}{6} = 2$
- $1\frac{2}{5} + \frac{3}{5} = 2$
- $\frac{6}{12} + \frac{3}{6} + \frac{12}{12} = 2$
- $\frac{11}{10} + \frac{9}{10} = 2$

- ○ share how they thought of fractions that were equivalent to $\frac{1}{2}$, 1, or 2 when adding fractions

Synthesis

- Invite 2–3 previously identified students to share their equations and their reasoning.
- "How did you know when your fraction was equivalent to 2?" (I looked for ways to make 1 first. When the numerator is twice as big as the denominator, the fraction is equivalent to 2.).
- "Why can we use multiplication to represent the combination $\frac{2}{6} + \frac{2}{6} + \frac{2}{6} + \frac{2}{6} + \frac{2}{6} + \frac{2}{6}$?" (There are 6 groups of $\frac{2}{6}$ or $6 \times \frac{2}{6}$. Both expressions are equal to $\frac{12}{6}$ or 2.)

Advancing Student Thinking

If students find fewer than 4 combinations, consider asking:

- "How might you use a combination you have already found to help you think of another?"
- "What do you know about the fractions you have left?"
- "How much would you need to add to one fraction to make 1 yard? How much to make 2 yards?"

Activity 3

⏱ 10 min

Play by the Rules

Standards Alignments

Addressing   4.NF.B.3, 4.NF.C.5

The purpose of this activity is to practice adding and subtracting fractions. Students reason about different combinations of fractions, including fractions greater than 1, and the relationship between addition and subtraction. Students also reason about equivalent decimal fractions to add and subtract fractions with unlike denominators (MP7).

Student-facing Task Statement

1. Here are four fractions: $\frac{15}{12}$ $\frac{7}{12}$ $\frac{21}{12}$ $\frac{18}{12}$

   a. What is the sum of all the fractions?

   b. Select two fractions with a difference that is less than $\frac{1}{3}$. Show or explain your reasoning.

   c. Select two fractions with a sum greater than 3. Show or explain your reasoning.

2. Here are four new fractions: $\frac{5}{12}$ $\frac{8}{12}$ $\frac{3}{12}$ $\frac{2}{12}$

   Use them to make the value 1, following these rules:

   o Use addition, subtraction, or both.

   o Use all four fractions.

   o Use each fraction only one time.

3. Try to make the value of 1 again using the following fractions and the same rules.

   $\frac{15}{10}$ $\frac{13}{100}$ $\frac{53}{100}$ $\frac{9}{10}$

Student Responses

Sample response:

1. a. $\frac{61}{12}$

    b. $\frac{18}{12} - \frac{15}{12} = \frac{3}{12}$, and $\frac{3}{12}$ is equivalent to $\frac{1}{4}$, which is less than $\frac{1}{3}$.

    c. $\frac{21}{12} + \frac{18}{12} = \frac{39}{12}$. The number 3 is $\frac{36}{12}$, so

Activity

- 5 minutes: independent work time

- 5 minutes: partner work time

- Monitor for students who use benchmarks such as $\frac{1}{2}$ and whole numbers to reason about how to add and subtract fractions when working with the decimal fractions to share in the synthesis.

Synthesis

- Invite previously identified students to share how they used the decimal fractions to make 1.

- "How did these students use equivalence to help make a total of 1 using these fractions?" (They thought about ways to make all the fractions have the same denominator. They thought about how the fractions compared to 1 to decide how to add or subtract.)

$\frac{39}{12}$ is greater than 3.

2. $\frac{5}{12} - \frac{3}{12} + \frac{2}{12} + \frac{8}{12} = \frac{12}{12} = 1$

3. $\frac{15}{10} - \frac{9}{10} + \frac{53}{100} - \frac{13}{100} = \frac{113}{100} - \frac{13}{100} = \frac{100}{100} = 1$

Lesson Synthesis

🕐 10 min

"Today we added, subtracted, and multiplied fractions to solve problems."

"Why is it important to understand fraction equivalence while operating with fractions?" (Sometimes we will need to compare products, sums, and differences to whole numbers. If we understand when a fraction is equivalent to a whole number, we can determine which ones are greater or less than that number. We can also use benchmarks such as $\frac{1}{2}$ and $\frac{1}{3}$ to help us reason about our responses.)

- - - - - - - - - - - - - - - **Complete Cool-Down** - - - - - - - - - - - - - - -

Response to Student Thinking

Students select two fractions that do not have a sum greater than 2.

Students create an expression that does not have a value between 1 and 2.

Next Day Support

- Add this cool-down to Activity 1 to review.

Lesson 2: Sums and Differences of Fractions

Standards Alignments

Addressing	4.NF.A.1, 4.NF.A.2, 4.NF.B.3.a, 4.NF.B.3.b, 4.NF.B.3.c, 4.NF.B.3.d
Building Towards	5.NF.A.1

Teacher-facing Learning Goals

- Add and subtract fractions and mixed numbers with like denominators.
- Compare fractions and mixed numbers by reasoning about equivalence.

Student-facing Learning Goals

- Let's practice solving problems involving fractions.

Lesson Purpose

The purpose of this lesson is for students to represent and solve problems involving the addition and subtraction of fractions. Students also reason about equivalence to compare fractions and make sense of problems.

In this lesson, students apply what they know about equivalence and addition and subtraction of fractions to solve problems. Throughout the lesson, students have opportunities to reason quantitatively and abstractly as they connect their representations, including equations, to the situations (MP2) and to compare their reasoning with others' (MP3).

The work of this lesson helps prepare students for adding and subtracting with unlike denominators in grade 5. If students need additional support with the concepts in this lesson, refer back to Unit 3, Section B in the curriculum materials.

Access for:

♿ Students with Disabilities

- Action and Expression (Activity 2)

🌐 English Learners

- MLR8 (Activity 1)

Instructional Routines

Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min

Teacher Reflection Question

What strategies were students using to solve

Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

problems? What evidence did students show that they were reasoning about equivalence as they solved the problems?

Cool-down (to be completed at the end of the lesson)   🕐   5 min

The Flagpole

Standards Alignments

Addressing   4.NF.B.3.c, 4.NF.B.3.d

Student-facing Task Statement

The school flagpole is placed about $3\frac{2}{6}$ feet into the ground. Students can see $12\frac{4}{6}$ feet of the flagpole.

How long is the entire flagpole? Show your reasoning.

Student Responses

16 feet, because $12\frac{4}{6} + 3\frac{2}{6} = 16$.

-------------------- **Begin Lesson** --------------------

Warm-up

🕐 10 min

Number Talk: Wholes and Units

Standards Alignments

Addressing   4.NF.B.3.c

This Number Talk encourages students to think about sums that make 10, 100, and one whole and look for ways to use these sums to mentally find the value of different expressions with whole numbers and fractions. The understandings elicited here will be helpful throughout this unit as students add and subtract whole numbers fluently and add and subtract fractions with the same denominator.

When students identify ways to make 1, 10, or 100, they look for and make use of the properties of operations and the structure of whole numbers and fractions (MP7).

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $38 + 62$
- $38\frac{2}{6} + 62\frac{3}{6}$
- $38\frac{2}{6} + 62\frac{3}{6} + 17\frac{1}{6}$
- $138\frac{2}{6} + 162\frac{3}{6} + 17\frac{2}{6}$

Student Responses

Sample reasoning:

- 100. I know $38 + 62$ is the same as $40 + 60$ if we add 2 to 38 and take 2 from 62.
- $100\frac{5}{6}$. The whole numbers add up to 100 and the fractions $\frac{2}{6}$ and $\frac{3}{6}$ add up $\frac{5}{6}$.
- 118 or $117\frac{6}{6}$. I know $100\frac{5}{6} + 17\frac{1}{6}$ is $117 + \frac{6}{6}$, which is $117 + 1$ or 118.
- $318\frac{1}{6}$ or $317\frac{7}{6}$, because $138 + 162 + 17$ is

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."

Activity

- 1 minute: quiet think time
- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "How were these expressions the same? How were they different?" (They each have $38 + 62$, so you can look for ways to make 100 in each. The last three expressions all have a mixed number with sixths. Some expressions did not have fractions. One expression had fractions that had a sum that was equivalent to 1 whole.)

317 and $\frac{2}{6} + \frac{3}{6} + \frac{2}{6} = \frac{7}{6}$ or $1\frac{1}{6}$.	• As needed, "How could we look for ways to make 100 in each expression? How could we look for ways to make 1?"

Activity 1

⏱ 15 min

Straws for A Roller Coaster

Standards Alignments

Addressing	4.NF.A.1, 4.NF.A.2, 4.NF.B.3.a, 4.NF.B.3.d
Building Towards	5.NF.A.1

The purpose of this activity is to represent and solve a measurement problem with fractions. Students may approach this activity in multiple ways and are invited to apply what they know about operations with fractions, comparing fractions, and fraction equivalence to make sense of and solve the problems (MP2). Throughout the activity, listen for the ways students use what they know about comparing fractions and fraction equivalence as they make sense of the problem. Although students may consider ways to subtract fractions with unlike denominators, this is not a requirement for grade 4. Focus the conversation during the activity and the synthesis on how students can solve the problem by reasoning about equivalent fractions and the representation they use to make sense of the problem.

🌐 Access for English Learners

MLR8 Discussion Supports. Synthesis: Create a visual display of the diagrams. As students share their strategies, annotate the display to illustrate connections. For example, next to each representation, write how the diagram relates to the situation.
Advances: Speaking, Representing

Student-facing Task Statement

In science class, Noah, Tyler, and Jada are building a model of a roller coaster out of 1-foot-long paper straws.

- Noah needs a piece that is $\frac{7}{12}$ foot long.

Launch

- Groups of 2

Activity

- 5 minutes: independent work time
- 5 minutes: partner work
- Monitor for different representations of

- Tyler needs one that is $\frac{1}{4}$ foot long.
- Jada needs one that is shorter than the other two.

Jada says, "We can just use one straw for all these pieces."

1. Draw a diagram to represent this situation and explain to your partner how it matches the situation. Then, find the length of the piece of straw that could be Jada's piece.

2. Did Noah use more than $\frac{1}{2}$ foot or less than $\frac{1}{2}$ foot of straw? Explain or show your reasoning.

3. Tyler says, "If Jada uses a piece that is $\frac{1}{6}$ foot long, there would be a piece of straw that is $\frac{1}{12}$ foot left."

   Do you agree or disagree with Tyler? Explain your reasoning.

Student Responses

1. Sample response:

   The diagram represents 1 foot partitioned into pieces that are $\frac{1}{12}$ foot each. Seven of those pieces are for Noah, and 3 for Tyler, because $\frac{1}{4} = \frac{3}{12}$. The left over piece for Jada is $\frac{2}{12}$ foot long, because $1 - \frac{7}{12} - \frac{3}{12} = \frac{2}{12}$.

2. Noah's piece was greater than $\frac{1}{2}$ foot because $\frac{1}{2} = \frac{6}{12}$ and Noah's piece is $\frac{7}{12}$ foot long.

3. I disagree. Sample reasoning: If Jada uses a piece that is $\frac{1}{6}$ foot long, there will be no straw left, because $\frac{1}{6} = \frac{2}{12}$.

the situation (for example, equations, drawings, and number lines).

Synthesis

- Invite previously selected students to share their arguments and allow peers to extend and add ideas to the conversation.

- "How does each representation match this situation about the straw?" (They each show the total length of the straw and ways it could be broken into smaller parts. They each show the length of Noah and Tyler's pieces. They show how long Jada's piece could be.)

Advancing Student Thinking

If students create representations that do not match the quantities in the problem, consider asking:

- "How does your representation show the length of the straw? How does it show Noah and Tyler's pieces?"
- "How does your representation show the action in the situation?"

Activity 2

🕐 20 min

Tall Enough for a Ride?

Standards Alignments

Addressing   4.NF.B.3.b, 4.NF.B.3.c, 4.NF.B.3.d

In this activity, students practice solving word problems that involve adding and subtracting mixed numbers. Students interpret fractions in the context of comparing heights and use what they know about decomposing whole numbers and equivalent fractions to make sense of and solve each problem (MP2). Look for the different ways students use what they know about the structure of whole numbers and fractions as they reason about how to solve each problem and share their thinking with others.

♿ Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Provide access to a variety of tools, such as fraction strips and meter sticks (or if possible, tape measures).
Supports accessibility for: Conceptual Processing, Visual-Spatial Processing, Attention

Student-facing Task Statement

Lin's class is on a trip to the amusement park. Visitors must be at least a certain height to get on rides. Use the table to answer questions about four students' height.

Launch

- Groups of 2
- "Have you ever ridden a ride at an amusement park or a fair? Have you ever not been able to ride a ride because you

ride	height requirement
tilt and spin	52 inches
roller coaster	54 inches
bumper cars	44 inches

1. Andre is $3\frac{3}{8}$ inches shorter than the height requirement for the roller coaster. How tall is Andre?

2. Lin is $\frac{18}{8}$ inches taller than Andre. How tall is Lin?

3. Elena was $1\frac{3}{4}$ inches too short to ride the bumper cars last year. Since then she grew $4\frac{1}{2}$ inches. How tall was Elena last year? How tall is she now?

4. Mai is tall enough to ride all the rides this year. Mai was $51\frac{7}{8}$ inches tall last year. At least how many inches did Mai grow?

Student Responses

1. Andre is $50\frac{5}{8}$ inches tall. Sample reasoning: $54 - 3\frac{3}{8} = 50\frac{5}{8}$

2. Lin is $52\frac{7}{8}$ inches tall. Sample reasoning: $50\frac{5}{8} + \frac{18}{8} = 52\frac{7}{8}$

3. Elena was $42\frac{1}{4}$ inches tall last year. She is $46\frac{3}{4}$ inches tall now. Sample reasoning: $44 - 1\frac{3}{4} = 42\frac{1}{4}$ and $42\frac{1}{4} + 4\frac{2}{4} = 46\frac{3}{4}$

4. Mai grew at least $2\frac{1}{8}$ inches. Sample reasoning: $54 - 51\frac{7}{8} = 2\frac{1}{8}$

were too young or not tall enough?"

- As needed, explain that some rides require the riders to be a certain height for their safety.

- "Let's solve some problems about height and amusement park rides."

Activity

- 10 minutes: independent work time

- 5 minutes: partner discussion

- "Compare your strategy with your partner's strategy."

- Monitor for students who use the relationship between addition and subtraction to represent the situations.

Synthesis

- Invite students to share their equations and explain their solution for Lin's height.

- Display: $50\frac{5}{8} + \frac{18}{8}$

- "How might you reason about how to find the value of Lin's height?" (Think about adding $\frac{3}{8}$ to $\frac{5}{8}$ to make 51, then it'd be $51\frac{15}{8}$. We are adding more than 1 because $\frac{18}{8}$ is more than $\frac{8}{8} \cdot \frac{18}{8} = \frac{8}{8} + \frac{8}{8} + \frac{2}{8}$ or $2\frac{2}{8}$, and $50\frac{5}{8} + 2\frac{2}{8} = 52\frac{7}{8}$)

  $(50\frac{5}{8} + \frac{18}{8} = 50\frac{5}{8} + 2\frac{2}{8} = 52\frac{7}{8})$

- Select students to share both addition and subtraction equations for Mai's current height and discuss how both equations $(54 - 51\frac{7}{8} = 2\frac{1}{8}$ and $54 = 51\frac{7}{8} + 2\frac{1}{8})$ could be used to represent this situation.

Advancing Student Thinking

If students add or subtract quantities in each problem in ways that do not match the situation, consider asking:

- "Who should be taller _____ or _____? How do you know?"
- "Does your answer match the heights in the situation? How do you know?"
- "How could you use a diagram to help you represent the different heights in this problem?"

Lesson Synthesis   🕐 10 min

"Problem solving is about reasoning. Today we solved problems involving addition and subtraction of fractions and mixed numbers."

"What did you use to make sense of the problems? What helped you make sense of the strategies that others shared?" (The diagrams and representations helped me visualize the situation and make sense of the math in the problems. It helped to see others' equations and compare them to what I wrote.)

"How did understanding fraction equivalence help you solve the problems?" (It helped me write equations that made it easier to add or subtract. It helped me compare fractions and make sense of what the problems were asking.)

- - - - - - - - - - - - - - - - **Complete Cool-Down** - - - - - - - - - - - - - - - -

Response to Student Thinking

Students find a length that is not equivalent to 16 feet as the length of the flagpole.

Next Day Support

- During the warm-up of the next lesson review representations used to solve the cool down. Consider asking students to identify parts of the problem in the representation.

Lesson 3: Stories with Fractions

Standards Alignments

Addressing   4.NF.B.3.c, 4.NF.B.3.d, 4.NF.C.5, 4.NF.C.6, 4.NF.C.7

Teacher-facing Learning Goals

- Solve and create word problems involving addition and subtraction of fractions referring to the same whole.

Student-facing Learning Goals

- Let's add and subtract mixed numbers.

Lesson Purpose

The purpose of this lesson is to use understanding of equivalence to solve addition and subtraction problems with decimal fractions. Students also create their own word problems that involve the addition and subtraction of fractions.

In a previous lesson, students solved word problems that involved adding and subtracting fractions and using equivalent fractions. Here, students continue to apply their understanding of fractions to solve problems that require adding or subtracting decimal fractions. Students also show their understanding of the structure of fractions and word problems to create their own situations given a value or equation and some constraints. In doing so, they practice reasoning quantitatively and abstractly (MP2).

Invite students to use the Three Reads routine as needed to solve problems. If students need additional support with the concepts in this lesson, refer back to Unit 3, Section B in the curriculum materials.

Access for:

♿ Students with Disabilities

- Representation (Activity 1)

🌐 English Learners

- MLR8 (Activity 2)

Instructional Routines

Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min

Teacher Reflection Question

Think about a time in today's lesson when you asked questions to address a misconception or

Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

to clarify a student's understanding. What questions were effective in surfacing a misunderstanding or the student's thinking about the math? What questions would you ask differently and why?

Cool-down (to be completed at the end of the lesson)   🕐   5 min

Mai's Milky Cereal

Standards Alignments

Addressing   4.NF.B.3.c, 4.NF.B.3.d

Student-facing Task Statement

There were 7 cups of milk before Mai made breakfast. Now there are $2\frac{2}{8}$ cups of milk. How much milk did Mai use for breakfast?

Student Responses

$4\frac{6}{8}$ or $4\frac{3}{4}$ cups. Sample reasoning:

- $7 - 2 = 5$ and $5 - \frac{2}{8} = 4 + \frac{8}{8} - \frac{2}{8} = 4\frac{6}{8}$
- I know $\frac{2}{8} = \frac{1}{4}$. So $7 - 2 = 5$ and $5 - \frac{1}{4} = 4\frac{3}{4}$

- - - - - - - - - - - - - - - - - **Begin Lesson** - - - - - - - - - - - - - - - - - -

Warm-up   🕐 10 min

Number Talk: One Whole, Many Names

Standards Alignments

Addressing   4.NF.B.3.c

This Number Talk encourages students to think about equivalent forms of whole numbers and decomposing fractions in order to subtract. When students consider equivalent fractions, look for ways to decompose fractions, or use the structure of mixed numbers to find the value of each difference, they look for and make use of structure (MP7).

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $1 - \frac{8}{10}$
- $1\frac{4}{10} - \frac{8}{10}$
- $2\frac{4}{10} - \frac{8}{10}$
- $10\frac{5}{10} - \frac{8}{10}$

Student Responses

- $\frac{2}{10}$. 1 is $\frac{10}{10}$ and $\frac{10}{10} - \frac{8}{10} = \frac{2}{10}$.
- $\frac{6}{10}$. The number being subtracted is the same but the first number here is $\frac{4}{10}$ more than the first number earlier, so the difference here is $\frac{4}{10}$ more than $\frac{2}{10}$, which is $\frac{6}{10}$.
- $1\frac{6}{10}$. $1\frac{4}{10}$ is 1 more than the first number in the problem before this, so the difference here is 1 more than $\frac{6}{10}$, which is $1\frac{6}{10}$.
- $9\frac{7}{10}$. We're subtracting the same number ($\frac{8}{10}$) but the first number is $8\frac{1}{10}$ more than that in the problem before this. $8\frac{1}{10} + 1\frac{6}{10} = 9\frac{7}{10}$.

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."

Activity

- 1 minute: quiet think time
- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "How are these expressions alike?" (They all involve subtracting $\frac{8}{10}$ from a number that is at least 1. To subtract, it's helpful or necessary to decompose a 1 or to write an equivalent fraction.)
- "How did you use earlier expressions to help you find the value of later expressions?"

Activity 1

⏱ 20 min

Relay Race at Recess

Standards Alignments

Addressing   4.NF.B.3.c, 4.NF.B.3.d, 4.NF.C.5, 4.NF.C.6, 4.NF.C.7

In previous lessons, students have used their understanding of fraction equivalence to compare fractions and solve problems. The purpose of this activity is to practice solving addition and subtraction problems involving decimal fractions (MP2). Students use what they know about equivalent fractions and the relationship between 10 and 100 to add tenths and hundredths.

♿ Access for Students with Disabilities

Representation: Develop Language and Symbols. Represent the problem in multiple ways to support understanding of the situation. For example, show a picture of children working with clay and invite students to draw a comic strip or storyboard to represent the problem.
Supports accessibility for: Conceptual Processing, Language, Attention

Student-facing Task Statement

Students in the fourth-grade class had a relay race during recess. Each team had four runners. Each runner ran the length of the school playground.

Here are the times of the runners for two teams.

runner	Diego's team, time (seconds)	Jada's team, time (seconds)
1	$10\frac{25}{100}$	$11\frac{9}{10}$
2	$11\frac{40}{100}$	$9\frac{8}{10}$
3	$9\frac{7}{10}$	$9\frac{84}{100}$
4	$10\frac{5}{100}$	$10\frac{60}{100}$

1. Which team won the relay race? Show your reasoning.

2. How much faster is the winning team than the other team? Show your reasoning.

3. The record time for the playground relay race was 40.27 seconds. Did the

Launch

- Groups of 2

Activity

- 1–2 minutes: independent work time
- "Compare your strategies with your partner's."
- 5 minutes: partner discussion
- Monitor for expressions, strategies, and representations students use to determine connections between strategies and evidence of reasoning about equivalence.

Synthesis

- Invite previously identified students to share how they solved the problems.
- "How was solving these problems the same as the problems we solved in previous lessons? How was it different?" (It was the same because we were adding and subtracting mixed numbers. We still looked for ways to make a new whole number

winning team beat this record time? Show your reasoning.

when we could. We had to add up fractions that had different denominators in these problems. We had to compare with a decimal fraction written with decimal notation.)

Student Responses

1. Diego's team won because they completed the race in a shorter amount of time. Sample reasoning:

   - Diego's team's time:
   $$10\tfrac{25}{100} + 11\tfrac{40}{100} + 9\tfrac{7}{10} + 10\tfrac{5}{100}$$
   $$= (10 + 11 + 9 + 10) +$$
   $$(\tfrac{25}{100} + \tfrac{40}{100} + \tfrac{70}{100} + \tfrac{5}{100})$$
   $$= 40 + \tfrac{140}{100}$$
   $$= 40 + \tfrac{100}{100} + \tfrac{40}{100}$$
   $$= 40 + 1 + \tfrac{40}{100}$$
   $$= 41\tfrac{40}{100}$$

   - Jada's team's time:
   $$11\tfrac{9}{10} + 9\tfrac{8}{10} + 9\tfrac{84}{100} + 10\tfrac{60}{100}$$
   $$= (11 + 9 + 9 + 10) +$$
   $$(\tfrac{9}{10} + \tfrac{8}{10} + \tfrac{84}{100} + \tfrac{60}{100})$$
   $$= 39 + \tfrac{17}{10} + \tfrac{144}{100}$$
   $$= 39 + \tfrac{170}{100} + \tfrac{144}{100}$$
   $$= 39 + \tfrac{314}{100}$$
   $$= 39 + \tfrac{300}{100} + \tfrac{14}{100}$$
   $$= 39 + 3 + \tfrac{14}{100}$$
   $$= 42\tfrac{14}{100}$$

2. $\tfrac{74}{100}$ seconds faster. Sample reasoning: $42\tfrac{14}{100}$ is the same as $41 + 1 + \tfrac{14}{100}$ or $41 + \tfrac{114}{100}$. Subtracting 41 from 41 gives 0. Subtracting $\tfrac{4}{10}$ or $\tfrac{40}{100}$ from $\tfrac{114}{100}$ gives $\tfrac{74}{100}$.

3. No, Diego's team did not beat the record.

Sample reasoning:

- ○ His team's time is 41.4, which is greater than 40.27.

- ○ The record is $40\frac{27}{100}$, which is less than $41\frac{40}{100}$.

Activity 2

🕐 15 min

You Be the Author

Standards Alignments

Addressing   4.NF.B.3.d

The purpose of this activity is to create and solve addition and subtraction problems with fractions. Students first create stories to match a given value or equation and some given constraints.

🌐 Access for English Learners

MLR8 Discussion Supports. Display sentence frames to support small-group discussion: "_____ and _____ are the same/alike because . . ." or "_____ and _____ are different because . . . ."
Advances: Speaking, Conversing, Representing

Student-facing Task Statement

Think of three situations as described here. After each problem is written, trade papers with a partner to compare your problems and check your solutions.

1. A problem that can be solved by addition and has $9\frac{2}{5}$ as an answer

2. A problem that can be solved by subtraction and has $\frac{32}{100}$ as an answer

Launch

- Groups of 2
- "Think of a situation with a problem that could be solved by finding the value of $3\frac{4}{10} + \frac{2}{10} + \frac{1}{2}$."
- 1–2 minutes: partner discussion
- Share responses.

3. A problem that could be solved by writing the equation: $9 - \underline{\hspace{1cm}} = 3\frac{3}{5}$

Student Responses

Sample response:

1. Priya biked $5\frac{1}{5}$ miles on Monday and $4\frac{1}{5}$ miles on Tuesday. How many total miles did she bike?

2. Noah has a piece of tape that is 1 meter long. He measures and cuts a piece that is $\frac{68}{100}$ meter long. How long is the piece that is left?

3. Lin's bus ride is 9 miles. The bus makes a stop $3\frac{3}{5}$ miles before reaching the destination. How far did the bus travel before making the stop?

Activity

- 5–6 minutes: independent work time
- 4–5 minutes: compare with a partner
- Monitor for students who create situations that involve different problem types. For example, for the problem that can be solved with addition, look for students who create an Add to, Result Unknown problem and a student who creates a Compare, Difference Unknown problem.

Synthesis

- Invite 1–2 previously identified students to share their situations for each problem.
- "How are these situations the same? How are they different? How does each one match the directions?"

Lesson Synthesis

🕐 10 min

"In this section, we have solved many problems that involved adding, subtracting, multiplying, and comparing fractions."

"What are two things that you have learned from listening to the ideas of other students in these lessons?"

"What is one thing you want to continue to practice when solving problems with fractions?"

------------ **Complete Cool-Down** ------------------

Response to Student Thinking

Students find a quantity that is not equivalent to $4\frac{6}{8}$ cups.

Next Day Support

- Review the cool-down in partners and compare reasoning.

Section B: Whole-number Operations

Lesson 4: Another Look at the Standard Algorithm

Standards Alignments

Addressing   4.NBT.B, 4.NBT.B.4

Teacher-facing Learning Goals

- Compare different methods for subtracting multi-digit numbers.
- Subtract multi-digit numbers using the standard algorithm.

Student-facing Learning Goals

- Let's subtract from numbers with zeros.

Lesson Purpose

The purpose of this lesson is to practice the standard algorithm for subtraction and to compare this method to other methods. Students choose a subtraction method based on the relationship between the numbers in the expression and explain their choices.

In previous units, students used the standard algorithm to subtract numbers. They interpreted and practiced ways to record 1 larger unit being decomposed and 10 being added to the unit to its right.

In this lesson, students encounter problems where they would need to decompose a series of larger units in order to subtract using the standard algorithm, such as when subtracting a number with non-zero digits from a number with multiple zeros (for example, $5,000 - 741$). Students consider the merits (the efficiency, likelihood of error, or reliability) of different ways to reason about such differences. They recognize that the standard algorithm may not always be the most efficient strategy for subtracting multi-digit numbers. Students explain how they can use the relationship between the numbers in an expression to select a strategy.

If students need additional support with the concepts in this lesson, refer back to Unit 4, Section D in the curriculum materials.

Access for:

♿ Students with Disabilities

- Engagement (Activity 1)

🌐 English Learners

- MLR7 (Activity 1)

Instructional Routines

Number Talk (Warm-up)

Materials to Gather

- Grid paper: Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What evidence did students give that they use their understanding of place value when using the standard algorithm? What evidence did students give that they use their number sense to select a subtraction method?

Cool-down (to be completed at the end of the lesson)   ⏱ 5 min

A Couple of Differences

Standards Alignments

Addressing   4.NBT.B, 4.NBT.B.4

Student-facing Task Statement

Find the value of each difference. Show your reasoning.

1. $8{,}050 - 213$
2. $60{,}000 - 1{,}984$

Student Responses

1. 7,837. Sample reasoning:
   - $8{,}000 - 200 = 7{,}800$ and $50 - 13 = 37$. Adding 7,800 and 37 gives 7,837.

$$\begin{array}{r} {\scriptstyle 7\ 10\ 4\ 10} \\ \not{8},\ \not{0}\ \not{5}\ 0 \\ -\quad\ 2\ 1\ 3 \\ \hline 7,\ 8\ 3\ 7 \end{array}$$

2. 58,016. Sample reasoning: $1{,}984 + 16 = 2{,}000$ and $2{,}000 + 58{,}000 = 60{,}000$. Adding 58,000 and 16 gives 58,016.

- **Begin Lesson** -

Warm-up
🕐 10 min

Number Talk: Differences

Standards Alignments

Addressing   4.NBT.B

This Number Talk encourages students to think about the base-ten structure of whole numbers and properties of operations to mentally solve subtraction problems. The reasoning elicited here will be helpful later in the lesson when students find differences of multi-digit numbers.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each difference mentally.

- $87 - 24$
- $387 - 124$
- $6{,}387 - 129$
- $6{,}387 - 4{,}329$

Student Responses

- 63. $80 - 20 = 60$, $7 - 4 = 3$, and $60 + 3 = 63$.

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."

Activity

- 1 minute: quiet think time
- Record answers and strategy.
- Keep expressions and work displayed.

- 263. $300 - 100 = 200$, and we already know that $87 - 24 = 63$.

- 6,258. We know that $387 - 124 = 263$. 129 is 5 more than 124, so $387 - 129$ would be 5 less than 263, which is 258.

- 2,058. 4,329 is 4,200 more than 129, so the difference is 4,200 less than the previous difference. $6,258 - 4,200 = 2,058$

- Repeat with each expression.

Synthesis

- "How is each expression related to the one before it?"

- "How might the first expression help us find the value of the last expression?" (Sample response: Knowing $87 - 24$ can help us find $87 - 29$. The latter is 5 less than the former. Then we can just find $6,300 - 4,300$.)

Activity 1

Lots of Zeros

🕐 15 min

👤↔👤 PLC Activity

Standards Alignments

Addressing   4.NBT.B, 4.NBT.B.4

In this activity, students subtract multi-digit numbers. They do so in two ways: by using the standard algorithm for subtraction and by finding unknown addends. Students find the value of a string of related differences that encourage them to look for and express regularity in repeated reasoning (MP8).

Students may notice that when a subtraction problem requires them to decompose multiple units to subtract in one place when using the standard algorithm—as is the case when the minuend has multiple zeros and the subtrahend has mostly non-zero digits—the standard algorithm for subtraction may not be the most practical. Students use their work in the lesson activity to discuss alternatives to the standard algorithm in these cases, including methods based on the relationship between addition and subtraction and reasoning about sums and differences that are easier to calculate (MP7).

🌐 Access for English Learners

MLR7 Compare and Connect. Synthesis: After all strategies have been presented, lead a discussion comparing, contrasting, and connecting the different approaches. Ask, "What did the approaches have in common?", "How were they different?", or "Did anyone solve the problem the same way, but would explain it differently?"
Advances: Representing, Conversing

⊕ Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Leverage choice around perceived challenge while activating background knowledge. Provide access to materials that students found helpful in Unit 4, such as base-ten blocks, 10×10 grids, digit cards, and colored pencils. As the value of the numbers increase, invite students to consider how they might use mental pictures of these materials instead of the materials themselves.
Supports accessibility for: Conceptual Processing, Attention, Social-Emotional Functioning

Materials to Gather

Grid paper

Student-facing Task Statement

1. Find the value of each difference.

   a.

   $$\begin{array}{r} 7\ 0\ 0 \\ -\ \ \ \ 1\ 6 \\ \hline \end{array}$$

   b.

   $$\begin{array}{r} 7,0\ 0\ 0 \\ -\ \ \ \ \ \ 1\ 6 \\ \hline \end{array}$$

   c.

   $$\begin{array}{r} 7\ 0,0\ 0\ 0 \\ -\ \ \ \ \ \ \ 1\ 6 \\ \hline \end{array}$$

   d.

   $$\begin{array}{r} 7\ 0\ 0,0\ 0\ 0 \\ -\ \ \ \ \ \ \ \ \ 1\ 6 \\ \hline \end{array}$$

2. Find the number that makes each expression true.

   a.

   $$\begin{array}{r} 4\ 3 \\ +\ \ \ \ \ \ \ \ \ \\ \hline 2\ 0\ 0 \end{array}$$

   b.

   $$\begin{array}{r} 4\ 3 \\ +\ \ \ \ \ \ \ \ \ \ \ \\ \hline 2,0\ 0\ 0 \end{array}$$

   c.

   $$\begin{array}{r} 4\ 3 \\ +\ \ \ \ \ \ \ \ \ \ \ \ \\ \hline 2\ 0,0\ 0\ 0 \end{array}$$

   d.

   $$\begin{array}{r} 4\ 3 \\ +\ \ \ \ \ \ \ \ \ \ \ \ \ \\ \hline 2\ 0\ 0,0\ 0\ 0 \end{array}$$

Student Responses

1. a. 684
   b. 6,984

Launch

- Groups of 2
- Display the first set of equations.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time
- Share responses.

Activity

- 6–8 minutes: independent work time
- 3–4 minutes: partner discussion
- Monitor students who:
  - use the standard algorithm and record how they decompose each place
  - think flexibly about how to decompose a larger place one time (for example, for $7,000 - 16$, rather than thinking about decomposing by starting with 7 thousands, they think of decomposing 700 tens and record to show 7,000 as 699 tens and 10 ones)
  - use the results of solved problems to help solve new problems (by

c. 69,984

d. 699,984

2. a. 157

b. 1,957

c. 19,957

d. 199,957

using known differences or by making use of a pattern)

○ use additive reasoning (for example, thinking about what number to add to 16 to find 100 and adding on to 100 to reach the total)

Synthesis

- Select previously identified students to share their strategies they used to find the value of $7,000 - 16$.

- Ask students who didn't use the standard algorithm to subtract why they chose another way.

- "How was finding the value of the differences in the first set of equations like finding the unknown addend in the second set of equations?" (They both had totals that kept adding a new place. The two-digit number in each did not have any zeros, but the total number in each had many zeros. There was a pattern in each. You could think about adding on and adding to a number like 100 first for each set.)

Advancing Student Thinking

If students appear to subtract or add to find each expression in a string without making connections between expressions, consider asking:

- "How did you find the unknown value in the previous expression?"

- "What do you notice about the value you found in each expression?"

- "Before you start in any steps on the next expression, what do you think the unknown value will be?"

Activity 2   ⏱ 20 min

Ways of Finding Differences

Standards Alignments

Addressing   4.NBT.B, 4.NBT.B.4

Previously, students used the standard algorithm for subtraction to find the differences involving minuends with multiple zeros. They also used the standard algorithm of addition to find a missing addend that gives a sum with multiple zeros for its digits. In this activity, students make connections between these two ways of reasoning about differences. They also analyze another way to find differences between multi-digit numbers.

Student-facing Task Statement

To find the value of $20{,}000 - 472$, Priya and Han set up their calculations differently.

Priya   **Han**

$$\begin{array}{r} 2\ 0,\ 0\ 0\ 0 \\ -\qquad 4\ 7\ 2 \\ \hline \end{array}$$

$$\begin{array}{r} 4\ 7\ 2 \\ +\qquad\qquad \\ \hline 2\ 0,\ 0\ 0\ 0 \end{array}$$

1. Use both methods to find the difference of 20,000 and 472.

2. Kiran uses another method. Explain how Kiran found the value of $20{,}000 - 472$.

$$472 + 8 = 480$$
$$480 + 20 = 500$$
$$500 + 500 = 1{,}000$$
$$1{,}000 + 19{,}000 = 20{,}000$$

$$19{,}000 + 500 + 20 + 8 = 19{,}528$$

3. Which method do you prefer? Or do you prefer another way? Explain your reasoning.

4. Find the value of $50{,}400 - 1{,}389$. Show your reasoning.

Student Responses

1. Sample response:

Launch

- "Look at Priya and Han's calculations. How are their setups alike? How are they different?"
- 1 minute: quiet think time
- 1 minute: share responses
- Highlight that Priya is subtracting 472 from 20,000, while Han is finding a number to add to 472 to get 20,000, but both are finding the same missing number.

Activity

- 6–8 minutes: independent work time on the first three questions
- Pause for a discussion before the last question.
- Select students to share their calculations, or display:

Priya   **Han**

$$\begin{array}{r} {}^{9}\ {}^{10} \\ {}^{9}\ \cancel{1}0 \\ {}^{9}\ \cancel{1}0 \\ 1\ \cancel{1}0 \\ \cancel{2}\ \cancel{0},\ 0\ 0\ 0 \\ -\qquad 4\ 7\ 2 \\ \hline 1\ 9,\ 5\ 2\ 8 \end{array}$$

$$\begin{array}{r} 1\ 1\ 1\ 1 \\ 4\ 7\ 2 \\ +\ 1\ 9,\ 5\ 2\ 8 \\ \hline 2\ 0,\ 0\ 0\ 0 \end{array}$$

Priya   **Han**

$$
\begin{array}{r}
\;9\;\;10 \\
9\;\cancel{10} \\
9\;\cancel{10} \\
1\;\cancel{10} \\
\cancel{2}\;\cancel{0},\,0\;\;0\;\;0 \\
-\qquad\;\;4\;\;7\;\;2 \\
\hline
1\;9,\,5\;\;2\;\;8
\end{array}
$$

$$
\begin{array}{r}
1\;\;1\;\;1\;\;1 \\
4\;\;7\;\;2 \\
+\;\;1\;9,\,5\;2\;8 \\
\hline
2\;0,\,0\;0\;0
\end{array}
$$

Kiran

$$472 + 8 = 480$$
$$480 + 20 = 500$$
$$500 + 500 = 1,000$$
$$1,000 + 19,000 = 20,000$$
$$19,000 + 500 + 20 + 8 = 19,528$$

- Ask a student to explain what is happening in each calculation.
- "Did anyone think of another way to find the difference?" (See sample student response.)
- Poll the class on their preferred strategy. Ask a student from each camp to explain their reasoning.
- 3 minutes: independent work time on the last question
- Monitor for students who try Priya's way, Han's way, Kiran's way, or a different way for sharing in the lesson synthesis.

Synthesis

- See lesson synthesis.

2. Sample response: Kiran found the numbers to add to 472 to get to the next ten, then the next hundred, thousand, and ten thousand, up to 20,000. He added all those numbers to get the difference of 20,000 and 472.

$$472 + 8 = 480$$
$$480 + 20 = 500$$
$$500 + 500 = 1,000$$
$$1,000 + 19,000 = 20,000$$
$$19,000 + 500 + 20 + 8 = 19,528$$

3. Sample responses:

   ○ I prefer Kiran's method, because it is easier to try to find the nearest ten, hundred, thousand, and so on, than find the differences of all the digits.

   ○ I prefer Han's method, because it's easier to find a digit to add to make 10 in each place than to subtract from a bunch of zeros, which requires regrouping each time.

   ○ Another way: 472 is 28 from 500 and 500 is 19,500 from 20,000. The difference is $19,500 + 28$ or 19,528.

4. 49,011. Sample reasoning: See examples in the lesson synthesis.

Lesson Synthesis   ⏱ 10 min

"Today we used different ways to subtract a number with non-zero digits from a number with zeros."

"What strategy did you use to find the difference between 50,400 and 1,389?" (Display the strategies used in the last question. Or, display the three ways here and any additional methods. Ask students to explain each method.)

Using Kiran's method:

$$1{,}389 + 11 = 1{,}400$$
$$1{,}400 + 49{,}000 = 50{,}400$$
$$49{,}000 + 11 = 49{,}011$$

Using Han's method:

```
    1      1   1
        1,  3   8   9
  +   4  9,  0   1   1
      5  0,  4   0   0
```

Using Priya's method (standard algorithm):

```
                      9
          4   10  3  1̶0  10
          5̶   0̶,  4̶   0   0
      −           1,  3   8   9
          4   9,  0   1   1
```

"When might it be convenient to use the standard algorithm to subtract two multi-digit numbers?" (When most digits in the second number are smaller than those in the same place in the first number.)

"When might it be inconvenient to use the standard algorithm to subtract?" (When most digits in the second number are greater than those in the same place in the first number, making it necessary to do multiple rounds of regrouping.)

- - - - - - - - - - - - - - - - **Complete Cool-Down** - - - - - - - - - - - - - - - - -

Response to Student Thinking

Students find differences other than 7,837 and 58,016.

Next Day Support

- Before the next day's warm-up, pair students up to discuss their responses.

The work of this lesson builds on previous understandings developed in Unit 4 Section C.

Prior Unit Support

Grade 4, Unit 4, Section D: Add and Subtract

Lesson 5: Multiplication of Multi-digit Numbers

Standards Alignments

Addressing   4.NBT.B.5

Teacher-facing Learning Goals

- Multiply multi-digit numbers using strategies based on place value and the properties of operations.

Student-facing Learning Goals

- Let's multiply multi-digit numbers.

Lesson Purpose

The purpose of this lesson is to deepen students' understanding of the connections between algorithms that use partial products and the standard algorithm when multiplying a pair of two-digit numbers.

In an earlier unit, students analyzed and experimented with different ways to multiply multi-digit numbers by single-digit whole numbers. This included analyzing diagrams and series of equations, including those that represented algorithms that use partial products. Students were introduced to the standard algorithm of multiplication and its connections to other ways of reasoning about products, particularly algorithms that use partial products.

This lesson reinforces students' awareness of the connections between the two algorithms and the role that place value plays in both (MP7). Students practice using the algorithms to multiply a one-digit number by another number up to four digits, building their fluency on multi-digit multiplication. If students need additional support with the concepts in this lesson, refer back to Unit 6, Section B in the curriculum materials.

As in the earlier unit, students are not expected to use the standard algorithm for multiplication without support, or to independently choose it to find products. They will continue to develop their facility with the standard algorithm in grade 5.

Access for:

♿ Students with Disabilities

- Representation (Activity 1)

🌐 English Learners

- MLR8 (Activity 1)

Instructional Routines

Estimation Exploration (Warm-up)

Materials to Gather

- Grid paper: Activity 1, Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

Identify who has been sharing their ideas in class lately. Make a note of students whose ideas have not been shared and look for an opportunity for them to share their thinking in tomorrow's lesson.

Cool-down (to be completed at the end of the lesson) 🕐 5 min

Four by One and Two by Two

Standards Alignments

Addressing   4.NBT.B.5

Student-facing Task Statement

1. Find the value of $2{,}617 \times 4$. Show your reasoning.
2. Find the value of 52×14. Show your reasoning.

Student Responses

1. 10,468. Sample reasoning:

   $(2{,}000 \times 4) + (600 \times 4) + (17 \times 4)$
   $= 8{,}000 + 2{,}400 + 68$
   $= 10{,}468$

$$
\begin{array}{r}
2{,}\;6\;\;1\;\;7 \\
\times \qquad\quad 4 \\
\hline
{\scriptstyle 1} \qquad\qquad \\
2\;\;8 \\
4\;\;0 \\
2{,}\;4\;\;0\;\;0 \\
+\;\;\;8{,}\;0\;\;0\;\;0 \\
\hline
1\;\;0{,}\;4\;\;6\;\;8 \\
\end{array}
$$

2. 728. Sample reasoning:

    ○ $52 \times 10 = 520$ and $52 \times 4 = 208$. The sum of 520 and 208 is 728.

    ○ $14 \times 50 = 700$ and $14 \times 2 = 28$. The sum of 700 and 28 is 728.

    ○ Using algorithms:

$$
\begin{array}{r}
5\ 2 \\
\times\quad 1\ 4 \\
\hline
8 \\
2\ 0\ 0 \\
2\ 0 \\
+\ 5\ 0\ 0 \\
\hline
7\ 2\ 8
\end{array}
\qquad
\begin{array}{r}
5\ 2 \\
\times\quad 1\ 4 \\
\hline
2\ 0\ 8 \\
+\ 5\ 2\ 0 \\
\hline
7\ 2\ 8
\end{array}
$$

--------------- **Begin Lesson** ---------------

Warm-up

◷ 10 min

Estimation Exploration: A Silly Riddle

Standards Alignments

Addressing   4.NBT.B.5

In this warm-up, students practice estimating a reasonable answer using known information, rounding, and multiplicative reasoning strategies. For example, students may create a diagram and arrive at $7 \times 7 \times 7 \times 7$ as an estimate of the number of fish. Then, they may approximate $(7 \times 7) \times (7 \times 7)$ with 50×50, or estimate $(7 \times 7 \times 7) \times 7$ with $(50 \times 7) \times 7$ or 350×7. Some students may notice that the question is vague and asks, "How many are going to the park?" rather than "How many people are going to the park?" and account only for the number of children and teachers in their estimation. If students ask for clarification, ask them to make their own assumptions and explain why they made those assumptions when they share their estimates.

Instructional Routines

Estimation Exploration

Student-facing Task Statement

- Seven teachers are going to the park.
- Each teacher is taking 7 students.
- Each student is bringing 7 fishbowls.
- Each fishbowl has 7 fish.

How many are going to the park?

Record an estimate that is:

too low	about right	too high

Student Responses

- Too low: 1,500–2,000
- About right: 2,400–2,500
- Too high: 2,600–3,000

Launch

- Groups of 2
- Display the description.

Activity

- "What is an estimate that's too high? Too low? About right?"
- 2 minutes: quiet think time
- 1 minute: partner discussion
- Record responses.

Synthesis

- Consider asking:
    - "Is anyone's estimate less than 500? Greater than 5,000?"
    - "Did anyone include the number of teachers and students in their estimate?"
- Invite students to share their estimation strategies. After each explanation, ask if others reasoned the same way.
- Consider revealing the actual value of 2,457 teachers, students, and fish.

Activity 1

🕐 20 min

Two Methods Revisited

Standards Alignments

Addressing   4.NBT.B.5

In this activity, students revisit two algorithms for multiplying numbers. They recall that, in the standard algorithm, the digit in one factor is multiplied by each digit in the other factor, but the partial products are not recorded on separate lines. Rather, the standard algorithm condenses multiple partial products into a single product.

🌐 **Access for English Learners**

MLR8 Discussion Supports. Synthesis: For each strategy that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language.
Advances: Listening, Speaking

♿ **Access for Students with Disabilities**

Representation: Develop Language and Symbols. Activate or supply background knowledge. Display 416×2 without the solutions. Ask students to tell you what each digit in 416 represents (for example, the 1 represents 10).
Supports accessibility for: Conceptual Processing, Memory, Language

Materials to Gather

Grid paper

Student-facing Task Statement

1. Earlier in the course, we used these two ways to multiply numbers:

   A

   ```
       4  1  6
   ×         2
   ─────────────
       1  2
       2  0
   + 8  0  0
   ─────────────
     8  3  2
   ```

   B

   ```
             1
       4  1  6
   ×         2
   ─────────────
     8  3  2
   ```

   a. In method A, where do the 12, 20, and 800 come from?

   b. In method B, where does the 1 above 416 come from?

2. Diego used both methods to find the value of 215×3 but ended up with very different results.

   ```
       2  1  5
   ×         3
   ─────────────
       1  5
       3  0
   + 6  0  0
   ─────────────
     6  4  5
   ```

   ```
       2  1  5
   ×         3
   ─────────────
   6, 3  1  5
   ```

   a. Without calculating anything, can you

Launch

- Groups of 2
- Give students access to grid paper, if needed for aligning the digits in a multiplication algorithm.

Activity

- 2 minutes: independent work time
- Pause to discuss the first set of questions. Display the two algorithms in the first question. Ask students to share responses.
- "How are the two algorithms alike? How are they different?"
- Highlight student responses to emphasize:
  - In method A, each partial product is listed separately before being added at the end.
  - In method B, only one digit is recorded at a time. The values for any place value unit are added and only one digit is recorded. Any new

tell which method shows the correct product? How do you know the other one is not correct?

b. For the incorrect result, explain what was correct and what was incorrect in his steps. Then, show the correct calculation using method B.

3. Use either way to find the value of each product. Show your reasoning.

    a. 521×3

    b. $6{,}121 \times 4$

    c. 305×9

Student Responses

1.   a. 12 is 2×6, 20 is 2×10, and 800 is 2×400.

    b. It is the 1 ten from the result of 6×2, which is 12.

2. Sample response:

    a. Method B shows the wrong product. Three times 215 should be in the 600's, not in the 6,000's.

    b. Correct: 5×3 is 15, 1×3 is 3, and 2×3 is 6. All those partial products—15, 3, and 6—are in the product.

    Incorrect: The digits are not in the right places. The 1 ten of the 15 should've been in the tens place, along with the 3 tens. The 6 should've been in the hundreds place.

    Or, Diego may have mistaken 3×10 to be 300 (instead of 30) and 3×200 to be 6,000 (instead of 600).

$$\begin{array}{r} 1 \\ 2\;\;1\;\;5 \\ \times\;\;\;3 \\ \hline 6\;\;4\;\;5 \end{array}$$

units are recorded in the next highest place.

- 6–10 minutes: independent work time
- 2–4 minutes: partner discussion

Synthesis

- Select students to share their responses to the second set of questions.
- If not mentioned in students' explanations, highlight that:
  - The result of 3×5 is 15, a two-digit number, so the 1 ten should be carried over to the tens place and added to the 3 tens that result from 3×10.
  - One ten and 3 tens make 4 tens.
- Poll the class on whether their preferred method is A, B, or is dependent on the problem. Select a student from each camp to explain their reason.

3.
   a. 1,563

   b. 24,484

   c. 2,745

Activity 2

🕐 15 min

Two by Two

Standards Alignments

Addressing   4.NBT.B.5

Earlier, students compared and made connections between two algorithms for multiplying a multi-digit number and a single-digit number. In this activity, students compare an algorithm that uses partial products with the standard algorithm for multiplying 2 two-digit numbers. As students analyze and critique each method, they practice looking for and making use of base-ten structure of whole numbers (MP7).

Materials to Gather

Grid paper

Student-facing Task Statement

Here are two ways to find the value of 34×21.

     A   **B**

```
      3  4              3  4
   ×  2  1           ×  2  1
      1                 1
         4                3  4
      3  0           +  6  8  0
      8  0              7  1  4
+  6  0  0
   7  1  4
```

1. In method A, where do the 4, 30, 80, and 600 come from?

2. In method B, which two numbers are

Launch

- Groups of 2
- Access to grid paper, in case needed to align digits when multiplying

Activity

- 3–4 minutes: independent work time on the first two problems
- 1–2 minutes: partner discussion
- Monitor for students who can explain the numbers in the standard algorithm.
- 5 minutes: independent work time on the remaining questions

multiplied to get:

   a. 34?

   b. 680?

3. Use the two methods to show that each equation is true.

a. $44 \times 12 = 528$ **b.** $63 \times 21 = 1{,}323$

Synthesis

- Invite students to share how they used each method to show that $44 \times 12 = 528$ and $63 \times 21 = 1{,}323$

Student Responses

1. $1 \times 4 = 4$, $1 \times 30 = 30$, $20 \times 4 = 80$, and $20 \times 30 = 600$.

2. a. 1×34 makes 34.

    b. 20×34 makes 680.

3. a. 528. Sample reasoning:

   b. 1,323. Sample reasoning:

Lesson Synthesis   ⏱ 10 min

"Today we looked at several methods for multiplying a multi-digit number by a single-digit number and also multiplying 2 two-digit numbers."

"Here are some reasoning or calculation strategies we have seen for multiplying 2 two-digit numbers."

$$33 \times 2 = 66$$

$$33 \times 10 = 330$$

$$66 + 330 = 396$$

"What connections do you see among these strategies? Point out as many as you can."

"Which of these strategies makes the most sense or is clearest to you?"

— — — — — — — — — — — — — **Complete Cool-Down** — — — — — — — — — — — — — —

Response to Student Thinking

Students find a product other than 10,468.

Next Day Support

- Partner students to compare solutions for the cool-down. Review two different strategies for multiplying multi digit numbers.

Lesson 6: What's the Quotient?

Standards Alignments

Addressing   4.NBT.B.6

Teacher-facing Learning Goals

- Divide up to four-digit numbers by single digit numbers using place value strategies.

Student-facing Learning Goals

- Let's find some quotients of multi-digit numbers.

Lesson Purpose

The purpose of this lesson is to reinforce students' understanding of division algorithms that use partial quotients and build their fluency in using it to divide multi-digit numbers by a single-digit divisor. Students also consider different strategies for dividing and their merits.

In an earlier unit, students learned to use partial quotients to divide whole numbers up to four digits by single-digit divisors. In this lesson, students deepen their understanding of algorithms that use partial quotients and continue to build their fluency with multiplication and division. Students also analyze different ways to divide whole numbers and consider how to improve their efficiency.

If students need additional support with the concepts in this lesson, refer back to Unit 6, Section C in the curriculum materials.

Access for:

⊛ Students with Disabilities

- Engagement (Activity 1)

🌐 English Learners

- MLR2 (Activity 2)

Instructional Routines

Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min

Teacher Reflection Question

How effective were your questions in supporting students to compare and connect different methods for division? What did students say or do that showed they were effective?

Lesson Synthesis   10 min

Cool-down   5 min

Cool-down (to be completed at the end of the lesson)   ○ 5 min

Divide Like a Pro

Standards Alignments

Addressing   4.NBT.B.6

Student-facing Task Statement

1. Here are two different ways to start finding the value of $8{,}435 \div 7$. Choose one way and complete the calculation.

$$\begin{array}{r} 5 \\ 7\,)\overline{8{,}435} \\ -\quad 35 \\ \hline 8{,}400 \end{array} \qquad \begin{array}{r} 1{,}000 \\ 7\,)\overline{8{,}435} \\ -\quad 7{,}000 \\ \hline 1{,}435 \end{array}$$

2. Find the value of $1{,}038 \div 6$. Try to use as few steps as possible.

Student Responses

1. 1,205. See sample response.
2. 173. See sample response.

$$\begin{array}{r} \boxed{1{,}205} \\ 5 \\ 200 \\ 1{,}000 \\ 7\,)\overline{8{,}435} \\ -\quad 7{,}000 \\ \hline 1{,}435 \\ -\quad 1{,}400 \\ \hline 35 \\ -\quad 35 \\ \hline 0 \end{array} \qquad \begin{array}{r} \boxed{173} \\ 3 \\ 70 \\ 100 \\ 6\,)\overline{1{,}038} \\ -\quad 600 \\ \hline 438 \\ -\quad 420 \\ \hline 18 \\ -\quad 18 \\ \hline 0 \end{array}$$

Warm-up

🕐 10 min

Number Talk: Divide by 3 and by 6

Standards Alignments

Addressing   4.NBT.B.6

This Number Talk encourages students to look for and use the structure of base-ten numbers and properties of operations to mentally find the value of division expression (MP7). The reasoning elicited here will be helpful later in the lesson when students find quotients of multi-digit numbers.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $48 \div 3$
- $480 \div 3$
- $528 \div 3$
- $5{,}280 \div 3$

Student Responses

Sample reasoning:

- 16. 48 is $30 + 18$. Dividing 30 by 3 gives 10, and dividing 18 by 3 gives 6, and $10 + 6 = 16$.
- 160. 480 is 10 times 48, so the result of $480 \div 3$ is 10 times the result of $48 \div 3$.
- 176. 528 is $480 + 48$, so $528 \div 3$ is $(480 \div 3) + (48 \div 3)$ or $160 + 16$.
- 1,760. 5,280 is 10 times 528, so the result of $5{,}280 \div 3$ is 10 times the result of $528 \div 3$.

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."

Activity

- 1 minute: quiet think time
- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "How is each expression related to the one before it?"
- Consider asking:
  - "Who can restate _____'s reasoning in a different way?"
  - "Did anyone have the same strategy but would explain it differently?"
  - "Did anyone approach the problem in a different way?"

○ "Does anyone want to add on to _____'s
    strategy?"

Activity 1

⏱ 15 min

Unfinished Divisions

Standards Alignments

Addressing   4.NBT.B.6

In a previous lessons, students saw that there are many ways to find products of multi-digit numbers. In this activity, students analyze and connect different ways to divide a multi-digit whole number by a single-digit whole number, and complete calculations to find the value of the quotient. In the synthesis, students compare the different methods and explain their preference.

♿ Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Provide choice. Tell students they will be finding the value of $7,465 \div 5$, and that there are four unfinished strategies to look at. Invite students to choose whether they want to solve it in their own way or look at the unfinished strategies first. *Supports accessibility for: Organization, Attention, Social-Emotional Functioning*

Student-facing Task Statement

Here are four calculations to find the value of $7,465 \div 5$, but each one is unfinished.

Complete at least two of the unfinished calculations. Be prepared to explain how you know what to do to complete the work.

Launch

- Groups of 2–4
- "Choose at least two calculations to finish. Make sure each calculation is completed by someone in your group."

Activity

- 3–4 minutes: independent work time
- 2 minutes: small-group discussion

Synthesis

- "How are the four strategies the same? How are they different?" (The first three are

the same because they involve partial quotients. Each one records the partial quotients in different ways. The last one involves estimation.)

- Consider asking:
  - "Which method or methods do you find easy to follow? Which did you find hard to follow?"
  - "Which method uses the most steps? Which uses the fewest steps?"
  - "Which methods would work to find the value of any quotient? Which might work for this expression, but might be less useful for others?"

$$
\begin{array}{r}
200 \\
80 \\
13 \\
5\overline{)7{,}465} \\
-65 \\
\hline
7{,}400 \\
-400 \\
\hline
7{,}000 \\
-1{,}000 \\
\hline
\end{array}
$$

$5{,}000 \div 5 = 1{,}000$
$60 \div 5 = 12$
$5 \div 5 = 1$

$$
\begin{array}{r}
400 \\
1{,}000 \\
5\overline{)7{,}465} \\
-5{,}000 \\
\hline
2{,}465 \\
-2{,}000 \\
\hline
\end{array}
$$

7,465 is a little less than 7,500.

$7{,}500 \div 5 = 1{,}500$
$35 \div 5 = 7$

Student Responses

$$
\begin{array}{r}
\boxed{1{,}493} \\
200 \\
1{,}000 \\
200 \\
80 \\
13 \\
5\overline{)7{,}465} \\
-65 \\
\hline
7{,}400 \\
-400 \\
\hline
7{,}000 \\
-1{,}000 \\
\hline
6{,}000 \\
-5{,}000 \\
\hline
1{,}000 \\
-1{,}000 \\
\hline
0
\end{array}
$$

$$
\begin{array}{r}
\boxed{1{,}493} \\
13 \\
80 \\
400 \\
1{,}000 \\
5\overline{)7{,}465} \\
-5{,}000 \\
\hline
2{,}465 \\
-2{,}000 \\
\hline
465 \\
-400 \\
\hline
65 \\
-65 \\
\hline
0
\end{array}
$$

$$5{,}000 \div 5 = 1{,}000$$
$$60 \div 5 = 12$$
$$5 \div 5 = 1$$
$$2{,}000 \div 5 = 400$$
$$400 \div 5 = 80$$

$$1{,}000 + 400 + 80 + 12 + 1 = 1{,}493$$

7,465 is a little less than 7,500.

$$7{,}500 \div 5 = 1{,}500$$
$$35 \div 5 = 7$$

$$7{,}465 \div 5 = 1{,}493$$

Advancing Student Thinking

Students may determine quotients other than 1,493. Consider asking:

- "How did you make sense of this method? How would you explain the numbers in it?"
- "How is this method like the others? How is it different?"
- "How could you use multiplication to check the value of the quotient you found?"

Activity 2

🕐 20 min

Where Do We Begin?

Standards Alignments

Addressing   4.NBT.B.6

This activity serves two goals. First, it prompts students to consider whether the order in which parts of the dividend are divided makes a difference in the process or in the result. Second, it deepens students' understanding of the structure of algorithms that use partial quotients.

Students first explain why different initial steps could be equally productive for starting a division process. Next, they analyze and complete some partial-quotients calculations with missing numbers. The missing numbers could be partial quotients, parts of the dividend being removed, or results of subtraction. To find the unknown numbers, students need to recognize and make use of the structure of the algorithm (MP7). Lastly, students use the algorithm to find a quotient, being mindful of their starting move and of the efficiency of their process.

MLR2 Collect and Display. Collect the language students use to explain how they found the quotient. Display words and phrases such as: "quotient," "partial quotient," and "dividend." During the synthesis, invite students to suggest ways to update the display: "What are some other words

or phrases we should include?" Invite students to borrow language from the display as needed.
Advances: Conversing, Reading

Student-facing Task Statement

1. Jada and Noah are finding the value of $3,681 \div 9$. Jada says to start by dividing 81 by 9. Noah says start by dividing 3,600 by 9.

   a. Explain why each suggestion is helpful for finding the quotient.

   b. Find the value of $3,681 \div 9$. Show your reasoning.

2. Find the missing numbers such that each calculation shows a correct division calculation.

3. Consider the expression $5,016 \div 8$.

   a. What would you do to start finding the value of the quotient?

   b. Show how you would find the value with as few steps as possible.

Student Responses

1. Sample response:

Launch

- Groups of 2

Activity

- 6–8 minutes: independent work time on the first two sets of questions

- 2–3 minutes: partner discussion

- Monitor for students who:

  ○ can clearly explain why Jada and Noah's initial steps are both effective

  ○ recognize the structure of the partial quotients method and can articulate how it helps to find the missing numbers

- Pause for a discussion before the last question. Select students to share responses and reasoning.

- When discussing the second set of questions, ask: "How do you determine what the missing numbers were?" Display the incomplete calculations to facilitate students' explanations.

- Consider annotating the calculations to clarify the structure (for instance, by drawing arrows between partial quotients and the corresponding parts of the dividend being subtracted, labeling the parts, and so on).

a. Jada's suggestion is helpful because 81 is a familiar multiple of 9 ($9 \times 9 = 81$) and removing it from 3,681 leaves 3,600, which is also a familiar multiple of 9. Noah's suggestion means removing the largest partial quotient first, which is helpful because then only 81 is left to divide, and it is a familiar multiple of 9.

b. 409. Sample reasoning: $3,600 \div 9 = 400$ and $81 \div 9 = 9$, and $400 + 9 = 409$.
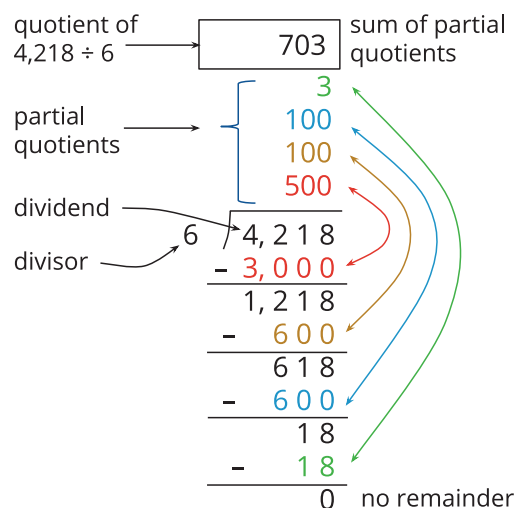
2.

```
   703          139          276
     3            4            6
   100           10           70
   100           25          100
   500          100          100
 6)4,218      4)556        7)1,932
 − 3,000       − 400        − 700
   1,218        156          1,232
 −   600       − 100        − 700
     618         56          532
 −   600       −  40        − 490
      18         16           42
 −    18       −  16        −  42
       0          0            0
```

quotient of 4,218 ÷ 6 → 703 sum of partial quotients

partial quotients →
```
              3
            100
            100
            500
dividend  ──────
        6 ⟌4, 2 1 8
divisor  − 3, 0 0 0
          1, 2 1 8
        −   6 0 0
            6 1 8
        −   6 0 0
              1 8
        −     1 8
               0   no remainder
```

- 3–4 minutes: independent work time on the last question

- Monitor for students who take different first steps to divide 5,016 by 8.

Synthesis

- See lesson synthesis.

3. a. Sample response: Divide 16 by 8 to get a partial quotient of 2.

b. Sample response:
```
            627
             25
            600
              2
        8)5,016
        −    16
          5,000
        − 4,800
            200
        −   200
              0
```

Advancing Student Thinking

Students may find some, but not all of the missing numbers in the algorithms. Consider asking:

- "Which missing numbers are you sure are accurate? How do you know?"
- "Could you use multiplication to find the missing numbers? How might that work?"
- "Could you work backwards to find the missing numbers? How would that help?"

Lesson Synthesis

🕐 10 min

"Today we studied different ways to divide multi-digit numbers and single-digit divisors."

Select students who took different initial steps to find $5,016 \div 8$ to share their calculations. Discuss:

"Why did you decide to start with that number?"

"How did you determine the next chunk to divide and remove?"

"Can you think of a way to find the quotient with fewer steps?"

- - - - - - - - - - - - - - - - - - **Complete Cool-Down** - - - - - - - - - - - - - - - - - - -

Response to Student Thinking

Students determine a quotient other than 1,205.

Next Day Support

- Before the warm-up, review strategies and solutions for the cool-down.

Section C: Solve Problems with Multiplication and Division

Lesson 7: Solve Multiplicative Comparison Problems

Standards Alignments

Addressing   4.NBT.B.5, 4.NBT.B.6, 4.OA.A.2, 4.OA.A.3

Teacher-facing Learning Goals

- Use the four operations to solve word problems involving multiplicative comparison.

Student-facing Learning Goals

- Let's solve real-world multiplicative comparison problems.

Lesson Purpose

The purpose of this lesson is for students to solve problems involving multiplicative comparison.

This lesson allows students to solve problems that involve multiplicative comparisons in the context of cost of living. Students are presented with different cost information and asked to make comparisons in different ways. For instance, they may be given the cost in one country and told that the cost in a second country is "9 times as much", and asked to find the dollar cost in the second country. They may be given the costs in two countries and then asked to compare the costs using a comparison statement that uses multiplication. Students also reason about how many of an item could be purchased in a country given a certain dollar amount (for example, how many months of rent in Ghana can one afford with $2,000?).

The work requires students to use several operations and to consider estimates where the operations would go beyond grade level (MP1, MP2). In many questions, it is not important that students find exact products, quotients, or answers. The emphasis is on reasoning flexibly about relative sizes of quantities and solving problems multiplicatively.

If students need additional support with the concepts in this lesson, refer back to Unit 5, Sections A and C in the curriculum materials.

Access for:

♿ Students with Disabilities

- Action and Expression (Activity 2)

🌐 English Learners

- MLR8 (Activity 1)

Instructional Routines

Notice and Wonder (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What unfinished learning or misunderstandings do your students have about multiplicative comparison? How did you leverage those misconceptions in a positive way to further the understanding of the class?

Cool-down (to be completed at the end of the lesson)

⏱ 5 min

Restaurant Budget

Standards Alignments

Addressing   4.NBT.B.5, 4.NBT.B.6, 4.OA.A.2, 4.OA.A.3

Student-facing Task Statement

In the United States, the cost of a meal for two people in a mid-range restaurant is about $50. A similar meal in Ghana is about $25.

1. Write a statement that compares the cost of a meal at a restaurant in the United States to a meal in Ghana and uses the phrase ". . . times as much as . . ."

2. A couple only wants to spend $240 at restaurants each month. How many more times could the couple go out to a restaurant each month if they eat in Ghana than in the United States? Show or explain your reasoning.

Student Responses

1. Sample response: A meal at a restaurant in the United States costs about 2 times (or twice) as much as a meal in Ghana.

2. 5 more times. Sample response: In the United States: $4 \times 50 = 200$, which is less than 240. They could eat at a restaurant 4 times. There will be $40 extra. In Ghana: $9 \times 25 = 225$. They could eat out 9 times and have $15 left.

--------------- **Begin Lesson** ---------------

Warm-up
⏱ 10 min

Notice and Wonder: Two Cities

Standards Alignments

Addressing   4.OA.A.2

The purpose of this warm-up is to elicit observations about differences in costs of living in different parts of the United States. It prepares students to look for and make sense of multiplicative comparisons later in the lesson. It also helps elicit what students know about how the different costs may be different depending on where you live.

Students may notice and wonder many things about the data in the table and both additive and multiplicative comparisons are likely to come up. Some students may note that an item costs "some dollars more" in San Francisco than in Fort Wayne, while others may say it costs "some number of times as much" based on the quantities or their estimation of the quantities. In the synthesis, focus student discussion on additive and multiplicative comparison.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?

	San Francisco, CA	Fort Wayne, IN
population	805,235	253,691
milk (1 gallon)	$4.45	$2.14

Launch

- Groups of 2
- Display the table.

Activity

- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

	San Francisco, CA	Fort Wayne, IN
bread (1 loaf)	$3.54	$1.82
gasoline (1 gallon)	$3.70	$2.42
movie ticket	$15	$12
internet connection (1 month)	$70	$50
rent for a 3 bedroom apartment in the city center (1 month)	$6,000	$1,500
cost of a house	$1,400,000	$160,000

Student Responses

Students may notice:
- The table shows a price list of different things in two cities.
- San Francisco has more people than Fort Wayne.
- Milk costs about 2 times as much in San Francisco as Fort Wayne.
- An apartment is about 4 times as much in San Francisco as in Fort Wayne.
- A house costs almost 10 times as much in San Francisco.

Students may wonder:
- Why is everything more expensive in San Francisco?
- Why is the difference in price smaller for some things than other things?
- Why is it so much cheaper to buy a house in Fort Wayne than in San Francisco?

- 1 minute: partner discussion
- Share and record responses.

Synthesis

- "How could you compare the cost for renting an apartment in San Francisco to the cost in Fort Wayne? How could you use multiplication to compare the costs?"
- As needed, "About how many more times as much is the cost of a house in San Francisco compare to the cost of a house in Fort Wayne?"
- "The prices of the things you need to live, like food, clothing, and shelter, are called the cost of living. The cost of living can be very different depending on where in the world you live. Today we are going to compare the cost of living in different places around the world."

Activity 1

The Most and Least Expensive

🕐 20 min

👤↔👤 PLC Activity

Standards Alignments

Addressing   4.NBT.B.5, 4.NBT.B.6, 4.OA.A.2, 4.OA.A.3

This activity prompts students to use multiplicative comparison to determine the cost of different living expenses in Bermuda and in India. Students use these costs to interpret and solve a multi-step problem. The multi-step problem can be solved in more than one way. Students might use multiplication or division. They may also choose to estimate rather than perform each calculation. Encourage students to use the Three Reads routine as needed to solve problems.

When finding the missing prices in the table, students also have an opportunity to reason about how to find a value that is 2.5 times as much. Students are not expected to multiply whole numbers by decimals in grade 4, but may use their understanding of decimal notation and decimal fractions to estimate the cost by doubling the price in India and adding half the price or by reasoning that 2.5 is close to 3.

🌐 Access for English Learners

MLR8 Discussion Supports. Prior to solving the problems, invite students to make sense of the situations. Monitor and clarify any questions about the context.
Advances: Reading, Representing.

Student-facing Task Statement

Bermuda is the most expensive country in the world and India is one of the least expensive.

1. The table shows how prices of some things in the two countries compare. Estimate or calculate the missing costs and complete the table.

Launch

- Groups of 2
- "We saw one example of how the same items can vary a lot in cost depending on where you live in the United States. Let's look at the costs of some other things in two different countries—one known to be very expensive and the other known to be inexpensive."

Activity

- 5 minutes: independent work time on the first question
- Monitor for students who use the multiplication or division algorithm, draw diagrams, write equations, or use estimation to find the missing values.

	Bermuda	cost in Bermuda is ____ as in India	India
a meal with drink (1 person)		12 times as much	$2
gasoline (1 gallon)	$8	2 times as much	
brand-name jeans		2.5 times as much	$31
men's leather shoes	$143	4 times as much	
internet connection		14 times as much	$13

2. In India, rent for a 1-bedroom apartment outside of a city center costs about $76 per month. A similar apartment in Bermuda costs 23 times as much.

   Utilities (electric, gas, water, and heating) for a small apartment costs about $27 per month in India and 7 times as much in Bermuda.

   If a person earns $2,000 per month, can they afford to pay rent and utilities in Bermuda?

Student Responses

	Bermuda	cost in Bermuda is ____ as in India	India
a meal with drink (1 person)	$24	12 times as much	$2
gasoline (1 gallon)	$8	2 times as much	$4
brand-name jeans	$77.50 (or about $78)	2.5 times as much	$31
men's leather shoes	$143	4 times as much	$35.75 (or about $36)
internet connection	$182	14 times as much	$13

1.

2. Yes (though there won't be much left for other expenses). Sample reasoning: Rent in Bermuda costs 23×76 or $1,748. Utilities cost 7×27 or $189. The combined cost is $1,748 + 189$ or $1,937, which is less than $2,000. By estimation, the sum of rent and utilities is $1,750 + 200$ or $1,950.

- Pause for a discussion. Select students to share their responses and reasoning for finding the costs in the table.

- Highlight that to find the missing cost in Bermuda, we can multiply the cost in India by the number in the middle column, and to find the cost in India, we divide the cost in Bermuda by the number in the middle column.

- 4 minutes: independent work time on the second question

- 4 minutes: partner discussion

Synthesis

- Select students to share their responses and strategies for how they determined if they can afford to live a month in Bermuda with $2,000.

Activity 2

🕐 15 min

The Cost of Living

Standards Alignments

Addressing   4.NBT.B.5, 4.NBT.B.6, 4.OA.A.2, 4.OA.A.3

In this activity, students continue to solve multiplicative comparison problems in the context of cost of living. In addition to using multiplicative comparison to find a greater or smaller unknown value, students also find an unknown multiplier.

Students also work with larger numbers. Some questions (about the costs of utilities, for instance) could be answered by dividing one multi-digit number by another (for instance, $153 \div 73$), but students are not expected to do this in grade 4. Instead, they may reason in terms of multiplication and by using estimation. For example, they may reason that 73×2 is 146 or 75×2 is 150, so the missing multiplier must be close to 2. Encourage students to use the Three Reads routine as needed to solve problems.

♿ Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Provide access to a variety of tools, such as base-ten blocks, counters, pre-formatted tape diagrams, grid paper, and mini whiteboards.
Supports accessibility for: Conceptual Processing, Organization, Attention

Student-facing Task Statement

The cost of living in the United States is higher than in Ghana.

1. The table shows how the prices of some other things in the two countries compare. Estimate or calculate the missing information in the table.

Launch

- Groups of 2
- "Have you ever traveled to another place far away? Do you know a friend or family member who has?"
- Share responses.
- "In this activity, you will compare the typical cost for things in the United States to the cost for the same things in Ghana."

Activity

- 4 minutes: independent work time on the

monthly cost	United States	cost in the US is ____ as in Ghana	Ghana
1-bedroom apartment in city center	$1,300	2 times as much	
1-bedroom apartment outside of city center		9 times as much	$112
utilities for a 915 square foot apartment	$152		$73
transportation pass	$70	3 times as much	
private preschool for 1 child	$906		$162

2. Suppose a family has $3,000 for housing and wants a 1-bedroom apartment outside of a city center. With that amount of money, how many months of rent can they afford in:

    a. the United States? Explain or show your reasoning.

    b. Ghana? Explain or show your reasoning.

Student Responses

monthly cost	United States	cost in the US is _____ as in Ghana	Ghana
apartment in city center	$1,300	2 times as much	$650
apartment outside of city center	$1,008 (or about $1,000)	9 times as much	$112
utilities	$152	about 2 times as much	$73
transportation pass	$70	3 times as much	about $23
private preschool	$906	about 5.5 times as much	$162

1.

2.   a. 2 months of rent in the US

    b. 26 months of rent in Ghana

---

first question

- 2 minutes: partner discussion
- Monitor for the different ways students make estimates, for example, whether they:
  - opt to multiply or divide
  - use exact or rounded numbers
  - round before computing or compute before rounding
- Pause for a brief discussion. Select students to share their responses and reasoning for finding the costs in the table.
- 4 minutes: independent work time on the second question
- 2 minutes: partner discussion
- Monitor for different strategies students use to find how many months of rent the family could afford in each country, including strategies based on estimation.

Synthesis

- See lesson synthesis.

Lesson Synthesis

⏱ 10 min

"Today we compared the cost of living in several countries."

"What are the different ways you compared the prices and the cost of living between two different places in the activities today?" (We used multiplication to describe how many times as much the cost of living was. We used multiplication and division to determine a price in one country. We also used addition and subtraction to check to see how much of something we could buy in one country compared to another.)

"When would it be important to compare the cost of living in one place compared to another?" (If you were planning a trip, you might want to plan for how much things will cost. If you are moving to a new place.)

"What questions do you have about cost of living that you would like to learn more about?"

- - - - - - - - - - - - - - - - - - - **Complete Cool-Down** - - - - - - - - - - - - - - - - - - -

Response to Student Thinking

Students write a multiplicative comparison statement that is not true. Students find a difference other than 5 visits when comparing the number of times the couple could visit a restaurant.

Next Day Support

- Before the warm-up review strategies used to solve the cool-down of today's lesson.

Lesson 8: Solve Problems with Multiplication and Division

Standards Alignments

Addressing   4.NBT.B.5, 4.NBT.B.6, 4.OA.A.3

| Teacher-facing Learning Goals | Student-facing Learning Goals |

Teacher-facing Learning Goals

- Determine if a solution to a word problem is reasonable using mental strategies and estimation.
- Interpret remainders in word problems involving division.
- Solve multi-step word problems using the four operations.

Student-facing Learning Goals

- Let's make sense of situations and solve word problems.

Lesson Purpose

The purpose of this lesson is for students to practice solving multi-step problems using all operations. Students interpret their solutions (including remainders in division situations) and determine the reasonableness of their answer for a given situation.

In the previous lesson, students solved word problems involving multiplicative comparison. In this lesson, they practice solving a wider variety of problems, with a focus on the relationships among multiple quantities in a situation. Students think about how to represent the relationships with one or multiple equations and using multiple operations. They also interpret their solutions and the solutions of others in context, including interpreting remainders in situations that involve division (MP2). Students also have opportunities to make estimates and to assess their reasonableness when solving problems.

 If students need additional support with the concepts in this lesson, refer back to Unit 6, Section D in the curriculum materials.

Access for:

⊕ Students with Disabilities

- Engagement (Activity 1)

🌐 English Learners

- MLR8 (Activity 2)

Instructional Routines

Number Talk (Warm-up)

<table>
<tr><td colspan="2">**Lesson Timeline**</td><td>**Teacher Reflection Question**</td></tr>
</table>

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What evidence do you have from student discussions that students used estimation strategies to make sense of problems and explain their thinking? How did students explain their estimates and how did they critique the estimates of others?

Cool-down (to be completed at the end of the lesson)   🕐   5 min

To and Fro

Standards Alignments

Addressing   4.NBT.B.5, 4.NBT.B.6, 4.OA.A.3

Student-facing Task Statement

In one week, a train made 8 round trips between its home station and Union Station. At the end of the week, it traveled a few more miles from the home station to a repair center. That week, the train traveled a total of 1,564 miles.

1. Which statement is true for this situation? Explain or show your reasoning.

    a. The distance traveled for each round trip is 200 miles. The distance to the repair station is 26 miles.

    b. The distance traveled for each round trip is 195 miles. The distance to the repair station is 4 miles.

    c. The distance traveled for each round trip is 8 miles. The distance to the repair station is 1,500 miles.

    d. The distance traveled for each round trip is 193 miles. The distance to the repair station is 8 miles.

2. Explain why one of the choices could not be true.

Student Responses

1. B. Sample responses:
    - $1,560 - 4 = 1,560, 1,560 \div 8 = 195$
    - $195 \times 8 = 1,560, \ 1,560 + 4 = 1,564$

2. Sample responses:
    - I know A could not be true because I know $200 \times 8 = 1,600$ and that's more than the total distance the train traveled.
    - I know C could not be true because the situation says its just a few more miles to the repair center. 1,500 miles is not a few more miles. $(8 \times 8) + 1,500$ does match the total distance, but it doesn't match the situation.

------------------------------ **Begin Lesson** ------------------------------

Warm-up

🕐 10 min

Number Talk: Divide by 8

Standards Alignments

Addressing   4.NBT.B.6

This Number Talk encourages students to think flexibly about numbers to divide. The understandings elicited here will be helpful throughout this unit as students divide whole numbers and build toward fluent multiplication and division.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $848 \div 8$
- $4,848 \div 8$

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."

- $4,852 \div 8$
- $5,848 \div 8$

Student Responses

- 106, because $800 \div 8$ is 100 and $48 \div 8 = 6$, and $100 + 6$ is 106
- 606, because:
  - $4,800 \div 8 = 600$ and $48 \div 8 = 6$, and $600 + 6$ is 606
  - $4,000 \div 8 = 500$ and $848 \div 8 = 106$, and $500 + 106$ is 606
- 606 with a remainder of 4, because 4,852 is only 4 more than 4,848, so it's not enough to make another group of 8.
- 731, because 5,848 is 1,000 more than 4,848 and $1,000 \div 8 = 125$. ($800 \div 8 = 100$ and $200 \div 8 = 25$.) $606 + 125 = 731$

Activity

- 1 minute: quiet think time
- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "How did you use the first few expressions to help you find the value of the last expression?"
- "How might you use multiplication to find the value of each quotient?"

Activity 1

🕐 15 min

Two Truths and a Lie, or Two Lies and a Truth?

Standards Alignments

Addressing   4.NBT.B.5, 4.NBT.B.6, 4.OA.A.3

In this activity, students are given three situations and asked to determine which ones could be true and which are not. To do so they need to carefully make sense of the quantities in each story and how they are related (MP2). Students may explain why a situation is true by writing one or more expressions or equations to represent the relationships and perform the calculations to check. Students may also reason using estimation and mental computation when explaining why a situation must be false.

⊙ **Access for Students with Disabilities**

Engagement: Develop Effort and Persistence. Chunk this task into more manageable parts. For example, invite students to perform the following steps: select one situation to start with, read the problem out loud and visualize the situation, represent the situation with a mathematical drawing or a tape diagram, write an equation to represent the problem, solve or estimate the solution to the equation. Check in with students to provide feedback and encouragement after each chunk.

Supports accessibility for: Conceptual Processing, Organization, Social-Emotional Functioning

Student-facing Task Statement

Here are three situations. Which ones are true? Which ones are not true?

2 Truths and I Lie?

2 Lies and I Truth?

Show how you know.

- Situation A: A high-rise building has 53 stories. The first floor is 17 feet tall, but all other stories are each 11 feet tall. The building is 610 feet tall.

- Situation B: A window washer has 600 seconds to wash 17 windows of a building. It takes 54 seconds to wash each window. The washer will finish washing all the windows and have 11 seconds to spare.

- Situation C: Eleven students set a goal to raise at least $600 for charity. Each student raised $17 each day. After 3 days of fundraising, the group will still be short by $54.

Student Responses

Situation A is the only one that is true. Sample reasoning:

- A: $(53 \times 11) + 17 = 583 + 17 = 600$

- B: $17 \times 54 = 918$. The window washer will need at least 918 seconds to wash all the windows.

Launch

- Groups of 2–4
- 5 minutes: independent work time

Activity

- 5 minutes: group discussion
- Monitor for different ways students represent and prove how each situation could be true or false, including using reasoning based on estimation and mental math.

Synthesis

- Select students to share their responses and reasoning.
- Record the expressions or equations they wrote to represent the situations. Highlight different ways of representing the same situation.
- "For which situations did you need to find the actual values in order to tell if they were true or not true? Why is that?" (I tried to estimate on Situation A and I knew it would be close, so I did the multiplication and added to find out if the total height was really 610 feet. I needed to write equations to make sense of Situation C. I just did the multiplication and subtraction

- B: 600 seconds is 10 minutes. Each window takes almost 1 minute to wash, so the window washer will need close to 17 minutes, not 10 minutes.

- C: $(17 \times 11) \times 3 = 187 \times 3 = 561$ and $600 - 561 = 39$. The group will be short by only $39 after 3 days.

to check to see if it could really be $54.)

- "For which stories was it possible to tell by estimation and mental math?" (I could do some estimation for all of them, but for Situation B I could tell that that even if he washed 10 windows the window washer would almost be out of time and couldn't do 17.)

Activity 2

🕐 20 min

Buses for a Field Trip

Standards Alignments

Addressing   4.NBT.B.5, 4.NBT.B.6, 4.OA.A.3

In this activity, students interpret situations that involve equal groups and require making sense of a remainder. Students solve problems using their understanding of multiplication and by connecting their solutions to the quantities in the situation (MP2). Although parts of the task could be solved by dividing a multi-digit number by a two-digit number, students are not expected to perform division with these numbers until grade 5. Students may access this task using multiplication and addition.

Encourage students to use the Three Reads routine as needed to solve problems.

🌐 Access for English Learners

MLR8 Discussion Supports. During group work, invite students to take turns sharing their responses. Ask students to restate what they heard using precise mathematical language and their own words. Display the sentence frame: "I heard you say . . . ." Original speakers can agree or clarify for their partner.
Advances: Listening, Speaking

Student-facing Task Statement

A school is taking everyone on a field trip. It

Launch

- Groups of 2

needs buses to transport 375 people.

Bus Company A has small buses with 27 seats in each.

Bus Company B has large buses with 48 seats in each.

1. What is the smallest number of buses that will be needed if the school goes with:

   ○ Bus Company A? Show your reasoning.

   ○ Bus Company B? Show your reasoning.

2. Which bus company should the school choose? Explain your reasoning.

3. Bus Company C has large buses that can take up to 72 passengers.

   Diego says, "If the school chooses Bus Company C, it will need only 6 buses, but the buses will have more empty seats."

   Do you agree? Explain your reasoning.

Student Responses

1. a. 14 buses. Sample reasoning: $27 \times 10 = 270$. $27 \times 4 = 108$. Fourteen small buses can fit $270 + 108$ or 378 people, which is more than 375.

    b. 8 buses. Sample reasoning: $48 \times 8 = 384$, and 384 is more than 375.

2. Sample response: Company A, because there will be fewer empty seats. $27 \times 14 = 378$ and 378 is 3 more than 375, so there will be 3 empty seats. $48 \times 8 = 384$ and 384 is 9 more than 375, so there will be 9 empty seats.

3. Agree. Sample reasoning:

   ○ $6 \times 72 = 432$, which is 57 more than

Activity

- 5 minutes: independent work time
- 5 minutes: group work time
- Monitor for the different ways students represent and solve the problem, including how they discuss how to treat any unfilled buses (the remainder).

Synthesis

- Select students to share their responses and reasoning. Record the different representations students used to solve the problems.
- Consider asking:
  - "How did you decide how many buses you would need from each company?"
  - "Do all the buses carry the same amount of passengers? How can you see that in your representations or equations?"
  - "How did you decide what operations to use to answer each question?"

375. There will be almost 60 empty seats.

○ 375 divided by 6 is 62 with a remainder of 3. This means each bus will have about 9 or 10 open seats.

Lesson Synthesis

🕐 10 min

"Today we analyzed and solved many kinds of word problems."

"What are some strategies we should use when solving problems to make sure we understand what the problem is asking?" (Read it carefully, think about what the numbers tell us and how they are related to one another.)

"What are some ways to figure out the relationships between the numbers?" (Create a representation—an equation or a diagram—and check to see if the representation matches the problem being solved.)

"How would we know if our answer makes sense?" (Check to see if it's reasonable in the situation, double check our calculations, check to see if it answers the question.)

- **Complete Cool-Down** -

Response to Student Thinking

Students explain only the answer they found and do not offer justification for the answer.

Next Day Support

- Before the first activity, pair students up to discuss their responses.

Lesson 9: Create Word Problems

Standards Alignments

Addressing   4.NBT.B, 4.NBT.B.4, 4.NBT.B.5, 4.NBT.B.6, 4.OA.A.3

Teacher-facing Learning Goals

- Write and solve multi-step word problems using the four operations.

Student-facing Learning Goals

- Let's write our own word problems.

Lesson Purpose

The purpose of this lesson is for students to create their own mathematical questions and word problems given constraints (specific values, possible operations).

In this lesson, students reason about solutions in order to create multi-step problems. Each problem structure is open, allowing students to reason flexibly about numbers and the questions being asked. The intent is for them to exercise the skill of discerning the question when solving problems. As students work, allow them the space to make sense of problems, without providing strategies or steps. After each activity, students reflect on the skills and practices they used to create their questions.

If students need additional support with the concepts in this lesson, refer back to Unit 6, Section D in the curriculum materials.

Access for:

ⓣ Students with Disabilities

- Representation (Activity 1)

🌐 English Learners

- MLR8 (Activity 1)

Instructional Routines

Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min

Teacher Reflection Question

As students worked together today, where did you see evidence of the mathematical community established over the course of the school year?

Lesson Synthesis	10 min
Cool-down	5 min

Cool-down (to be completed at the end of the lesson)   🕐 5 min

A Music Festival

Standards Alignments

Addressing   4.NBT.B.4, 4.NBT.B.5, 4.NBT.B.6, 4.OA.A.3

Student-facing Task Statement

The school band wants to raise $1,700 for a music festival. They have raised $175 each week for the past 6 weeks.

Write a question that could be asked about this situation and answer it. Show your reasoning.

Student Responses

Sample response:

- Question: If the band keeps raising $175 each week, how many more weeks before it reaches its goal?
- Answer: 4 weeks. In 6 weeks it raises $1,050, so it still needs to raise $650. Three more weeks mean an additional 3×175 or $525. So one more week would mean $525 + 175$ or $700.

- **Begin Lesson** -

Warm-up   🕐 10 min

Number Talk: Difference and Thousands

Standards Alignments

Addressing   4.NBT.B, 4.NBT.B.5

The purpose of this Number Talk is to elicit strategies and understandings students have for subtracting multi-digit numbers when the minuend is a multiple of 100. These understandings help students develop fluency and will be helpful later in this lesson when students will need to be able to interpret student work and solve word problems that involve multi-digit numbers.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $5{,}000 - 403$
- $5{,}300 - 473$
- $25{,}300 - 493$
- $26{,}000 - 1{,}493$

Student Responses

- 4,597. I know that $5{,}000 - 400 = 4{,}600$ and $4{,}600 - 3 = 4{,}597$.
- 4,827.
  - We can add 300 to our result before (5,300 is 300 more than 5,000) to get 4,897 and subtract 70 from it (473 is 70 more than 403) to get 4,827.
  - $5{,}300 - 400 = 4{,}900$. Subtracting 70 from 4,900 gives 4,830 and subtracting 3 from 4,830 gives 4,827.
- 24,807. We can add 20,000 to the previous result (25,300 is 20,000 more than 5,300), and then subtract 20 (493 is 20 more than 473).$4{,}827 + 20{,}000 - 20 = 24{,}807$
- 24,507. I know 1,493 is 7 away from 1,500 and $26{,}000 - 1{,}500$ is 24,500, so $26{,}000 - 1{,}493$ is 7 more than 24,500.

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."

Activity

- 1 minute: quiet think time
- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "Did you find yourself always using an earlier expression to help you find the value of a new expression? Did you find yourself switching strategies at times?"
- "Which expression did you find most challenging to reason about?" (Sample response: $26{,}000 - 1{,}493$ because there were many zeros in the number being subtracted from.)

Activity 1

🕐 15 min

What's the Question?

Standards Alignments

Addressing   4.NBT.B.4, 4.OA.A.3

In this activity, students analyze a situation and given solutions and think about what questions were asked. They also write their own question that can be answered with the quantities in the situation. As they connect equations to a context, students reason quantitatively and abstractly (MP2) and engage in aspects of modeling (MP4).

🌐 Access for English Learners

MLR8 Discussion Supports: Display sentence frames to support small-group discussion: "First, I _____ because . . ." or "Why did you . . . ?"
Advances: Speaking, Conversing

♿ Access for Students with Disabilities

Representation: Develop Language and Symbols. Help students make sense of the situation by seeing it represented in multiple ways. For example, before showing students Kiran's answers and reasoning, invite them to draw labeled tape diagrams, number lines, or comic strips to represent the situation. Give students the opportunity to see their classmates' representations before working independently on the task.
Supports accessibility for: Conceptual Processing, Organization, Memory

Student-facing Task Statement

George Meegan walked 19,019 miles between 1977 and 1983. He finished at age 31. He wore out 12 pairs of hiking boots.

Jean Beliveau walked 46,900 miles between 2000 and 2011 and finished at age 56.

Here are the responses Kiran gave to answer

Launch

- Groups of 2
- "Have you ever gone on a long hike? What is the longest distance you ever traveled just by walking?"
- "Let's look at the work a student did to answer questions about two men who set world records for traveling by walking."

some questions about the situation.

Write the question that Kiran might be answering. In the last row, write a new question about the situation and show the answer, along with your reasoning.

question	response and reasoning
1.	$1983 - 1977 = 6$ $12 \div 6 = 2$ 2 pairs of hiking boots
2.	$56 - 31 = 23$ 23 years
3.	$2011 - 2000 = 11$ 11 years $11 \times \underline{\hspace{1cm}} = 46,900$ $11 \times 4,000 = 44,000$ $11 \times 200 = 2,200$ $11 \times 70 = 770$ $44,000 + 2,200 + 770 = 46,970$ $4,000 + 200 + 70 = 4,270$ 4,270 miles
4.	

Student Responses

1. How many pairs of hiking boots did George Meegan use a year between 1977 and 1983?

2. How much older was Jean Beliveau than George Meegan when he finished his walk?

3. About how many miles did Beliveau walk each year (if he walked roughly the same number of miles per year)?

4. How many more miles did Beliveau walk than Meegan?

Activity

- 5 minutes: independent work time
- 5 minutes: partner work
- "Share solutions with your partner and share strategies for solving the task."
- 5 minutes: Trade problems with a partner, solve, and review strategies for solving.

Synthesis

- "How did you figure out the question that was asked just by looking at the equations?" (We can look at the numbers they used and look back at the story to see what they meant. For example, subtracting the years lets us know something about the difference between the years. Dividing 12 by 6 lets us know that the years and the pairs of boots were used to find an answer.)

Activity 2

🕐 20 min

What's the Problem?

Standards Alignments

Addressing   4.NBT.B.4, 4.NBT.B.5, 4.NBT.B.6, 4.OA.A.3

In this task, students create their own word problems that must have a specific answer and follow additional constraints involving the type of operations that could be used to solve the problem or the size of the numbers they can use (MP2, MP4).

Student-facing Task Statement

Elena, Noah, and Han each created a problem with an answer of 1,564.

- Elena used multiplication.
- Noah used multi-digit numbers and addition only.
- Han used multiplication and subtraction.

Write a problem that each student could have written. Show that the answer to the question is 1,564.

- Elena's
  problem:

   Solution:

- Noah's
  problem:

   Solution:

- Han's
  problem:

   Solution:

Student Responses

Sample responses:

Launch

- Groups of 2

Activity

- 5 minutes: independent work time
- 10 minutes: partner work time

Synthesis

- Select 1–2 students to share the problem they created for each constraint.
- Consider asking:
    - "How did you start?"
    - "How did you make sure 1,564 would be the answer? How did you make sure your problem would match what each student used?"

Elena:

- Dog walking pays $391 dollars each year. How much does a dog walker earn in 4 years?
- $391 \times 4 = 1{,}564$, because $300 \times 4 = 1{,}200$ and $91 \times 4 = 364$, and $1{,}200 + 364 = 1{,}564$.

Noah:

- Fourth grade students collected 752 cans and third grade students collected 812 cans. How many cans were collected by students in both grades?
- $752 + 812 = 1{,}564$, because $700 + 800 = 1{,}500$ and $52 + 12 = 64$, and $1{,}500 + 64 = 1{,}564$.

Han:

- Kindergarten, first, and second grades raised $500 each. Third, fourth, and fifth grades also raised $500 each. The school donated $1,436 to a local charity. How much money was left over?
- $(6 \times 500) - 1{,}436 = 3{,}000 - 1{,}436 = 1{,}564$

Lesson Synthesis

🕐 10 min

"Today we determined the questions for different problems and created new problems that involve different operations."

"How was the thinking process different when coming up with questions than when answering questions?"

"What was challenging about creating questions when a situation and some information about it are given? What about when no information other than the answer was given?"

-------------------- **Complete Cool-Down** --------------------

Response to Student Thinking

Students may recognize the problem structure and need support developing questions of their own.

Next Day Support

- Before the next day's warm-up, pass back the cool-down and brainstorm strategies for solving the problem.

Section D: Creation and Design

Lesson 10: Estimation Exploration

Standards Alignments

Addressing   4.OA, 4.OA.A, 4.OA.A.3

Teacher-facing Learning Goals

- Analyze and write estimation problems.

Student-facing Learning Goals

- Let's design an Estimation Exploration activity.

Lesson Purpose

The purpose of this lesson is for students to analyze different kinds of estimation problems and ways to make estimates, and then write their own estimation problems.

Earlier in the course, students developed their ability to reason multiplicatively and to find products of multi-digit numbers. Throughout the course, students have also been prompted to make estimates. In this lesson, they apply these understandings and skills to create their own Estimation Exploration activity.

The Estimation Exploration routine encourages students to anticipate multiple ways others might make an estimate based on a given image or description, and to revise their thinking accordingly. Later in the lesson, students will facilitate their activity in small groups. The lesson may take more than one class period if students take turns facilitating their estimation problem.

Access for:

⚙ Students with Disabilities

- Engagement (Activity 1)

🌐 English Learners

- MLR8 (Activity 3)

Instructional Routines

Estimation Exploration (Warm-up)

Materials to Gather

- Tools for creating a visual display: Activity 3

Lesson Timeline

Warm-up	10 min
Activity 1	10 min
Activity 2	15 min
Activity 3	10 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

Today's lesson invited students to contribute in different ways and settings. Reflect on who participated in math class today. What assumptions are you making about those who did not participate? How can you leverage each of your student's ideas to support them in being seen and heard in tomorrow's math class?

Cool-down (to be completed at the end of the lesson)   🕐 5 min

Reflection

Standards Alignments

Addressing   4.OA.A, 4.OA.A.3

Student-facing Task Statement

Describe something mathematical that you understand better after completing today's activities, or something that you find confusing or challenging.

Student Responses

Sample response: After multiplying several numbers multiple times to make estimates, I understand much more that there are different ways to break apart and rearrange the factors to make it easier to find products. Usually I'd just multiply the numbers in the order they were given.

- **Begin Lesson** -

Warm-up   🕐 10 min

Estimation Exploration: No Driver Required

Standards Alignments

Addressing   4.OA.A

The purpose of an Estimation Exploration is for students to practice the skill of estimating a reasonable answer based on experience and known information. Students are given some information about a parking garage and pictures of the interior and exterior.

Invite students to share the assumptions they used when they make and share their estimates. For example, students may share how they used the photos to estimate how many floors there were and how many cars could fit on each floor. They may share the assumptions they made about whether each floor held the same amount of cars or how the cars were arranged on the floors based on the picture that they used to estimate that there are 10 floors and each floor holds the same amount of cars.

Instructional Routines

Estimation Exploration

Student-facing Task Statement

Here are pictures showing the exterior and interior of a parking tower in Wolfstadt, Germany. The parking is automated: each car goes up on a lift and is then placed in a parking space.

How many cars can fit in the tower?

Record an estimate that is:

too low	about right	too high

Launch

- Groups of 2
- Display image.

Activity

- "What is an estimate that's too high? Too low? About right?"
- 1 minute: quiet think time
- 1 minute: partner discussion
- Record responses.

Synthesis

- "What assumptions did you make about the parking garage to make your estimate?"
- "What information would help you make a more accurate estimate?"
- Consider revealing the actual number of cars in the structure, which is 400. The parking structure has 20 stories and 20 cars per story.

Student Responses

Sample responses:

- Too low: 100–200
- About right: 400–500
- Too high: 700–800

Activity 1   ⏱ 10 min

Dental Care

Standards Alignments

Addressing   4.OA.A.3

In this activity, students use given descriptions and their knowledge of multiplication to make some estimates of the cost and time associated with brushing teeth. To complete the estimation, students will need to make some assumptions about what constitutes "a lifetime" and the cost of a single toothbrush. The activity gives students another example before they write their own Estimation Exploration activity.

♿ Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Revisit math community norms to prepare students for working with an uncertain situation. Remind students about expectations for handling disagreements and provide sentence frames to support possible ways to come to a resolution.
Supports accessibility for: Social-Emotional Functioning

Student-facing Task Statement

Jada brushes for 2 minutes, twice a day. She changes her toothbrush every 3 months.

Based on this information, make some estimates and show your reasoning:

Launch

- Groups of 2
- Read the first paragraph of the task statement.
- Solicit some ideas from students: "What kinds of things can we estimate in this situation?"

1. In a lifetime, approximately:

   a. How many toothbrushes will she use?

   b. How many dollars will she spend on toothbrushes?

2. About how many minutes would she spend brushing her teeth:

   a. in a year?

   b. in a lifetime?

3. Think of another estimation question you could ask about this situation.

Student Responses

1. Sample response: Assuming 80 years of brushing, 4 toothbrushes a year, and $2 per toothbrush:

   a. 320 toothbrushes in a lifetime $(80 \times 4 = 320)$

   b. $640 $(320 \times 2 = 640)$

2. Sample response:

   a. 1,460 minutes in a year $(365 \times 2 \times 2 = 1,460)$

   b. 116,800 a lifetime $(1,460 \times 80 = 116,800)$, assuming 80 years of having teeth that need brushing

3. Sample responses:
   - If she puts all her used toothbrushes in shoeboxes, how many shoeboxes would she need for a lifetime's number of used toothbrushes?
   - How many days in a lifetime would she spend brushing her teeth?

Activity

- 4–5 minutes: independent work time
- 2 minutes: partner discussion
- Monitor for the different assumptions that students make to answer the questions.

Synthesis

- Select students who made different assumptions about cost and the length of a lifetime to share their responses and reasoning. Record their estimates.
- If time permits, invite students to share the new estimation questions they thought of for this situation. "How might you make those estimates?"

Activity 2

🕐 15 min

Get Your Classmates to Estimate

Standards Alignments

Addressing   4.OA.A, 4.OA.A.3

In this activity, students create an Estimation Exploration activity that focuses on multi-digit multiplication. Students may choose to present a situation (as in the toothbrush example), something observed in the classroom (such as a large shelf filled with books), or an image (as in the warm-up). They need to anticipate the reasoning strategies others might use to make an estimate.

Required Preparation

- Gather two magazines or other sources of images for each group of 3–4 students.

Student-facing Task Statement

It's your turn to create an estimation problem.

1. Think of situations or look around for images that would make interesting estimation problems. Write down 4–5 ideas or possible topics.

2. Choose your favorite idea. Then:

   - Write an estimation question that would encourage others to use multiplication of multi-digit numbers to answer.

   - Record an estimate that is:

too low	about right	too high

   Be prepared to explain your reasoning.

Student Responses

Answers vary.

Launch

- Groups of 3–4
- Provide access to magazines or other sources of images.

Activity

- "Work with your group to create an Estimation Exploration activity that involves a large number of objects or a large quantity."
- "Be sure to consider different strategies that others might use to make an estimate."
- 11–12 minutes: small-group work time

Synthesis

- Encourage students to ask lingering questions about designing their activity.
- Clear up any confusion and allow them a few minutes to revise their design, if needed.

Activity 3

🕐 10 min

Facilitate Your "Estimation Exploration" Activity

Standards Alignments

Addressing   4.OA, 4.OA.A.3

In this activity, students facilitate the Estimation Exploration activity they created for another group in the class. If time allows, encourage students to run the activity for multiple groups.

🌐 Access for English Learners

MLR8 Discussion Supports. Synthesis: During group presentations, invite the student(s) who are not speaking to follow along and point to the corresponding parts of the display.
Advances: Speaking, Representing

Materials to Gather

Tools for creating a visual display

Student-facing Task Statement

Follow these steps to facilitate your Estimation Exploration activity for another group:

- Display your image or present your scenario.
- Ask your classmates: "What is an estimate that's too high? Too low? About right?"
- Give them 1 minute of quiet think time.
- Give them 1–2 minutes to discuss together.
- Invite them to share their estimates and how they made them. Record their ideas.
- If you know the actual number or quantity, consider revealing it.

Student Responses

No response required.

Launch

- Combine every two groups of 3–4.
- Give each group a piece of poster paper to record their written scenario (if they have one) and to record responses.

Activity

- "Take turns facilitating your estimation activity with the group you're partnered with."
- Remind groups to switch roles halfway through the allotted time.

Synthesis

- See lesson synthesis.

Lesson Synthesis

🕐 10 min

Invite students to reflect on their process. Discuss questions such as:

"How was the process of creating an Estimation Exploration activity different from answering one?" (Creating the activity is more involved. It requires researching, thinking of possible ways to estimate, actually making estimates, and maybe revising the question. Doing the activity only involves finding a way to estimate and explaining it.)

"What did you learn as you facilitated your Estimation Exploration?" (The estimates that others find could be close or far from our estimates or the actual number. Others might use other strategies that we did not think of or make assumptions that are very different than ours.)

"If you had a chance to revise your activity, would you change anything? What would you change?"

- **Complete Cool-Down** -

Response to Student Thinking

Students do not share all of their reflections in writing.

Next Day Support

- Before the warm-up consider sharing reflections from the previous day and inviting students to think about similar ideas when reflecting today.

Lesson 11: Which One Doesn't Belong?

Standards Alignments

Addressing   4.G.A.1, 4.NBT.A, 4.NBT.B, 4.NBT.B.5, 4.OA.B.4, 4.OA.C, 4.OA.C.5

Teacher-facing Learning Goals

- Analyze numbers, expressions, geometric figures, and computations, and identify their shared and unique features.
- Create a Which One Doesn't Belong set of items with both shared and unique features.

Student-facing Learning Goals

- Let's complete and create Which One Doesn't Belong sets.

Lesson Purpose

The purpose of this lesson is for students to analyze numbers, expressions, geometric figures, and computations, identify their shared features and unique features, and use their analyses to complete or create Which One Doesn't Belong sets.

This lesson allows students to apply their knowledge of various mathematical ideas from this course in generative ways. Students examine sets of numbers, expressions, figures, or calculations, identify both shared and unique features, and create new items to complete Which One Doesn't Belong sets.

In the warm-up, students are given a full set of items. In the three subsequent activities, they see sets with fewer given items and create more new items. Not all three activities are required. Consider the amount of scaffolding that students may need and decide accordingly. If possible, give students time to facilitate their Which One Doesn't Belong activity for another group. Expect all three activities to span two class periods, especially if students present their creations.

Note that each Which One Doesn't Belong warm-up in this curriculum is designed to meet a rigid set of criteria. Any 3 out of the 4 choices must share a common feature that is not shared by the other choice. This makes it possible to say for each item: "___ doesn't belong because it's the only one that is *not* (or does *not* have) . . . ". The negation here makes it particularly difficult to design a set of objects and common characteristics.

Students are not expected to follow these criteria. While they still look for common and distinguishing features of 4 objects, which is the hallmark of this routine, the structure is loosened to be more engaging. Students are invited to:

- find a reason why each of the 4 objects belongs
- find a reason why each of the 4 objects does not belong

In the first activity, students are given 3 items and are not asked to revisit the reasons why the first 3 items belong or do not belong after they introduce the fourth item. For the other activities, they do make this extra step to ensure that all 4 items still have a reason to belong and a reason not to belong.

Access for:

♿ Students with Disabilities

- Representation (Activity 1)

🌐 English Learners

- MLR8 (Activity 1)

Instructional Routines

Which One Doesn't Belong? (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Activity 3	30 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

In today's lesson, students honed their ability describe and compare different mathematical structures and create their own Which One Doesn't Belong activities. In what ways did you see students apply what they have learned from the Which One Doesn't Belong routine from their work this year? What aspects of your facilitation of this routine improved over the year and what would you like to continue to improve for next year?

Cool-down (to be completed at the end of the lesson)   🕐 5 min

Reflection

Standards Alignments

Addressing   4.G.A.1, 4.NBT.A, 4.NBT.B.5, 4.OA.B.4, 4.OA.C.5

Student-facing Task Statement

As mathematicians, it is important to explain our thinking and listen to the reasoning of others.

Describe a time today when you learned something new, or thought differently about something, based on what a classmate said.

Student Responses

Sample response: In one of the Which One Doesn't Belong activities, one set showed only numbers. I was sure that the only reason they belong or did not belong was based on the digits being used. My groupmates helped me see that we could also look at place value, whole number versus decimals, and whether the digits are odd or even.

- **Begin Lesson** -

Warm-up

🕐 10 min

Which One Doesn't Belong: Strings of Numbers

Standards Alignments

Addressing   4.OA.C

This warm-up prompts students to compare four strings of numbers. To identify a string that doesn't belong, students may use what they know about properties and kinds of numbers—factors and multiples, odd and even numbers, numbers in base ten, and so on.

This warm-up also prepares students for a series of design activities that prepare students to create their own Which One Doesn't Belong activity.

Instructional Routines

Which One Doesn't Belong?

Student-facing Task Statement

Which one doesn't belong?

A. 0, 4, 8, 12, 16

B. 3, 6, 9, 12, 15

C. 5, 105, 205, 305, 405

D. 6, 60, 600, 6,000, 60,000

Launch

- Groups of 2
- Display the image.

Activity

- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time
- 2–3 minutes: partner discussion

Student Responses

Sample response:

- A is the only one without any multiples of 5.
- B is the only one that doesn't have all odd or all even numbers.
- C is the only one without any two-digit numbers and without any even numbers.
- D is the only one in which the numbers are not increasing by the same amount each time.

- Record responses.

Synthesis

- "Let's find at least one reason why each one doesn't belong."
- "What did you look for when deciding which one doesn't belong, or when finding a reason why each one doesn't belong?"
- "What did the writer of this activity have to consider when designing this activity?" (Sample responses: All items must have something in common, but each item must have a feature that the other three don't have.)

Activity 1

🕐 20 min

Add One That Doesn't Belong

Standards Alignments

Addressing   4.G.A.1, 4.NBT.A

In this activity, students use their knowledge of numbers in base ten and of geometric figures to complete two Which One Doesn't Belong sets. Students first study the attributes of the given items. They articulate why the items all belong to a set and why each one has a reason not to belong. Next, they propose a new item that shares a feature with others in the set but also has a unique feature that excludes it from the set.

🌐 Access for English Learners

MLR8 Discussion Supports. Display sentence frames to support small-group discussion: "I agree because . . ." and "I disagree because . . . ."
Advances: Speaking, Conversing.

♿ Access for Students with Disabilities

Representation: Internalize Comprehension. Activate background knowledge. Display the sets, and begin by asking, "What language might we use to talk about the math you see here?"
Supports accessibility for: Language, Attention, Memory

Student-facing Task Statement

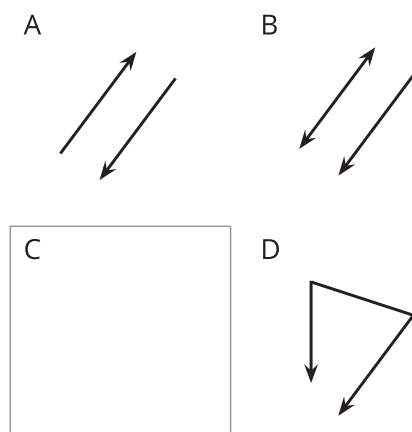Here are two incomplete Which One Doesn't Belong sets, each with one item missing. For each set:

- Find at least one reason that all items belong in the set.
- Find at least one reason that each item doesn't belong.
- Add an item to complete each set. Make sure there is at least one reason it belongs and one reason it doesn't belong.

1. Set 1

A	B	C	D
4.8		13.05	5

   a. A, C, and D all belong because . . .

   b. A doesn't belong because . . .

   C doesn't belong because . . .

   D doesn't belong because . . .

   c. Add a new item B. It belongs because . . .

   It doesn't belong because . . .

2. Set 2

Launch

- Groups of 3–4
- 7–8 minutes: small-group work time on the first set

Activity

- Pause for a whole-class discussion. Select a group to share their reasoning and new item. Record their responses.
- After their explanation, ask if others:
    - reasoned the same way but created a different fourth item. Invite them to share their addition.
    - saw other reasons why the given items belong in a set and why each one doesn't.
- 4 minutes: independent work time on the second set
- 4 minutes: small-group discussion on the second set

Synthesis

- Select another group to share their responses, reasoning, and new item for the second set. Record their responses.
- Ask others in the class if they reasoned about the properties of the given figures the same way as the presenting group but produced a different item to complete the set. Invite them to share their thinking and creation.

a. A, B, and D all belong because . . .

b. A doesn't belong because . . .

B doesn't belong because . . .

D doesn't belong because . . .

c. Add a new item C. It belongs because . . .

It doesn't belong because . . .

Student Responses

1. Set 1: Sample responses:

a. They all belong because they are all numbers.

b. A reason each doesn't belong:

■ A is the only one that doesn't have an odd-numbered digit.

■ C is the only one that doesn't have a number less than 10.

■ D is the only one that isn't a decimal with tenths or hundredths.

c. 0.25 belongs because it is a number in base ten. It doesn't belong because it doesn't have a non-zero digit in the ones place.

A	B	C	D
4.8	0.25	13.05	5

2. Set 2: Sample responses:

a. They all belong because they all have a ray and one or more parts.

b. A reason each doesn't belong:

■ A is the only one that doesn't show 2 different geometric features. It shows 2 rays.

■ B is the only one that has a line and a ray, instead of two rays.

■ D is the only one that doesn't have only two parts. It has a segment and two angles. It is the only one that doesn't have parallel parts.

c. C belongs because it has a ray and another geometric part. It doesn't belong because it's the only one in which only two rays are connected.
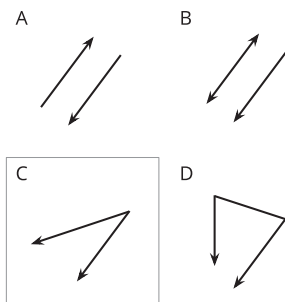
Activity 2

🕐 15 min

Add Two That Don't Belong

👤↔👤 PLC Activity

Standards Alignments

Addressing   4.NBT.A

In the second round of analysis and design, students use their knowledge of operations and expressions to complete a Which One Doesn't Belong set with two missing items. Students first study the attributes of the given items. They articulate why the items all belong to a set and why each one has a reason not to belong. Next, they propose a new item that shares a feature with others in the set but also has a unique feature that excludes it from the set.

Student-facing Task Statement

Here is an incomplete Which One Doesn't

Launch

● Groups of 2

Belong set. It has two missing items.

A
$$(4 + 3) \times (2 - 1)$$

B
$$(4 + 3) + (2 - 1)$$

C
D

1. Find at least one reason that the first two items, A and B, belong in the set.

2. Add two items to complete the set. Make sure there is at least one reason that each new item belongs and at least one reason it doesn't belong.

   ○ C and D both belong because . . .

   ○ C doesn't belong because . . .

   ○ D doesn't belong because . . .

3. After you've completed the set, check items A and B. Does each one still have a reason not to belong? If not, adjust your new items so that A and B are each still unique in some way.

Student Responses

1. Sample responses:

   ○ They all belong because each is an expression with the numbers 1, 2, 3, and 4, and three operations.

   ○ They all belong because each has the numbers placed into two groups with parentheses. The first group shows $4 + 3$ and the second group shows $2 - 1$.

2. Sample response:

- 5 minutes: independent work time

Activity

- "Work with your partner to make sure each new item added to the set has a reason to belong and not to belong."

- 5 minutes: partner work time

Synthesis

- Invite students to share their analyses and new items.

- "Why do both A and B belong to the set?"

- "What were some features of the expressions that you looked at or experimented with to complete your set?" (The kind of numbers used, how many numbers are used, the order of the numbers, the symbols, the value of the expression.)

- If no students considered the value of the expression, ask them about it.

- "After adding C and D, did A and B still have a reason not to belong in your set? Did you have to revise your items?"

A

$(4 + 3) \times (2 - 1)$

B

$(4 + 3) + (2 - 1)$

C

$(4 + 3) + (2 \times 1)$

D

$(4 \times 3) + (2 - 1)$

- ○ D is the only one where the 4 and 3 are multiplied and not added. It is the only one in which the value of the expression is not less than 10.
- ○ C is the only one in which the 1 is not subtracted from 2.

3. Sample response: Yes, A and B are each still unique. A is the only one that doesn't have an addition between $(4 + 3)$ and $(2 - 1)$. B is the only expression that doesn't have an odd-numbered value. It's also the only one that doesn't use multiplication.

Activity 3 (optional)

🕐 30 min

Add Three That Don't Belong

Standards Alignments

Addressing   4.NBT.B

In this optional third round of analysis and design activity, students are given only one item showing multi-digit multiplication using an algorithm with partial products. Students apply their understanding of operations on multi-digit numbers and algorithms to add three new items that would complete a Which One Doesn't Belong set.

As before, students begin by examining what's given, but because only one item is shown, they now have considerably more freedom to define the attributes of other items in the set. Expect the increased openness to be a greater cognitive lift for students and to yield more varied sets. For instance, students might add to the set an area diagram, a calculation in standard algorithm, and

> a division algorithm.
>
> For the activity synthesis, ask each group to test their set by presenting it to another group, and then to make note of one or more ways to improve their set based on their peers' feedback.

Student-facing Task Statement

Here is an incomplete Which One Doesn't Belong set. It has three missing items.

A	B	C	D
$\begin{array}{r} 4\ 2\ 3 \\ \times\qquad 6 \\ \hline 1\ 8 \\ 1\ 2\ 0 \\ +\ 2,\ 4\ 0\ 0 \\ \hline 2,\ 5\ 3\ 8 \end{array}$			

Add three items to complete the set. Make sure there is at least one reason that all items belong and at least one reason each item doesn't belong.

- They all belong in the set because . . .
- A doesn't belong because . . .
- B doesn't belong because . . .
- C doesn't belong because . . .
- D doesn't belong because . . .

Student Responses

Sample responses:

A	B	C	D
$\begin{array}{r} 4\ 2\ 3 \\ \times\quad 6 \\ \hline 1\ 8 \\ 1\ 2\ 0 \\ +\ 2,\ 4\ 0\ 0 \\ \hline 2,\ 5\ 3\ 8 \end{array}$	$\begin{array}{r} 4\ 2\ 3 \\ \times\quad 1 \\ \hline 4\ 2\ 3 \end{array}$	$\begin{array}{r} 5 \\ 4\ 2\ 3 \\ \times\quad 1\ 5 \\ \hline 1\ 5 \\ 1\ 0\ 0 \\ +\ 2,\ 0\ 0\ 0 \\ \hline 2,\ 1\ 1\ 5 \end{array}$	$\begin{array}{r} 1,\ 4\ 2\ 3 \\ \times\quad 9 \\ \hline 2\ 7 \\ 1\ 8\ 0 \\ 3,\ 6\ 0\ 0 \\ +\ 9,\ 0\ 0\ 0 \\ \hline 1\ 2,\ 8\ 0\ 7 \end{array}$

- They all belong because they all show an algorithm for multiplying two numbers: a

Launch

- Groups of 3–4
- "Look at the only item in the given Which One Doesn't Belong set. What do you notice? What do you wonder?"

Activity

- 1 minute: quiet think time
- 2 minutes: Invite students to share as many observations as they can. Record their responses.
- "Work with your group to add three more items to make a Which One Doesn't Belong set. Just like before, each item has to have at least one reason to belong and one reason not to."
- 12–15 minutes: group work time
- 2–3 minutes: Record the completed set on poster paper.

Synthesis

- "Partner with another group and take turns presenting your set. Give your audience 1 minute of quiet think time and 2 minutes to discuss their observations. Record their responses."
- "Based on your peers' feedback, make a note of changes your group could make to improve the set."
- "What were some mathematical features you paid attention to when creating and checking the new items?" (Sample responses: the operation, number of digits,

multi-digit number and a single-digit number.

- A is the only one that doesn't have an odd single-digit factor and doesn't have an odd-numbered product.
- B is the only one that doesn't show partial quotients and doesn't have a 2 or a thousand separator in the product.
- C is the only one whose multi-digit factor is not listed first.
- D is the only one that doesn't have 423 as a factor.

algorithm used, odd- or even-numbered result, whether carrying was involved, whether the numbers represent certain quantities)

Lesson Synthesis

🕐 10 min

"Today (or the past couple of days) you've used your mathematical understanding to complete and improve several Which One Doesn't Belong sets. You've also created an original set."

"As you studied more and more sets, did you find yourself getting better at identifying reasons that items belong or do not belong together? If so, in what ways? If not, what were some challenges?"

"What did you pay attention to at the end that you hadn't in the beginning?"

- - - - - - - - - - - - - - - - - - - **Complete Cool-Down** -

Response to Student Thinking

Students do not share all of their reflections in writing.

Next Day Support

- Before the warm-up consider sharing reflections from the previous day and inviting students to think about similar ideas when reflecting today.

Lesson 12: Number Talk

Standards Alignments

Addressing   4.NBT.A.1, 4.NBT.B, 4.NF.B, 4.NF.B.4

Teacher-facing Learning Goals

- Apply understanding of addition, subtraction, multipilcation, and division to create a Number Talk activity.

Student-facing Learning Goals

- Let's create our own Number Talks.

Lesson Purpose

The purpose of this lesson is for students to use patterns, structure, and understanding of properties of operations to design a series of expressions for a Number Talk activity.

This lesson provides an opportunity to listen to ways in which students make use of structure and repeated reasoning to design a Number Talk. In the warm-up, students are given a typical Number Talk with four expressions and discuss how the expressions are related. In the first activity, students practice anticipating the different ways someone might reason about addition and subtraction for a group of expressions and create expressions that would fit with the given string of expressions. In the final two activities, students are given incomplete sets with a decreasing number of expressions and asked to write new expressions to create a Number Talk activity. This lesson can take 1–2 days if students facilitate their creations with other groups.

Access for:

🧑‍🦽 Students with Disabilities

- Action and Expression (Activity 1)

🌐 English Learners

- MLR7 (Activity 2)

Instructional Routines

Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min

Teacher Reflection Question

Think about two questions you asked students today: a question that yielded insight into student thinking and one that you wish you had framed differently. How do the questions

Activity 3	30 min	
Lesson Synthesis	10 min	
Cool-down	5 min	

compare? How would you ask the latter differently to elicit the desired reasoning or insight?

Cool-down (to be completed at the end of the lesson)   ⏱ 5 min

Reflection

Standards Alignments

Addressing   4.NBT.B, 4.NF.B

Student-facing Task Statement

As mathematicians, we often use patterns to help us reason about new problems. Observing something that repeats over and over can also help us solve problems.

Describe a time, during today's lesson or recently, when you noticed a pattern or a repetition and used it to help you think through a problem. How did the pattern or repetition help you?

Student Responses

Sample response: Recently, I noticed that one way to find the product of a whole number and a fraction like $\frac{1}{8}$ is to divide the whole number by the denominator of the fraction. When I multiply 16 by $\frac{1}{8}$, the result is the same as the result of dividing 16 by 8. I have used this observation to check my answers to multiplication problems like this.

- - - - - - - - - - - - - - - - - **Begin Lesson** - - - - - - - - - - - - - - - - -

Warm-up   ⏱ 10 min

Number Talk: A Whole Number and a Fraction

Standards Alignments

Addressing   4.NF.B.4

This Number Talk encourages students to rely on properties of operations and what they know about multiplication of a fraction and a whole number to mentally solve problems. The understandings elicited here will be helpful later in the lesson when students complete or create Number Talk activities.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $6 \times \frac{1}{4}$
- $6 \times \frac{3}{4}$
- $18 \times \frac{3}{4}$
- $180 \times \frac{3}{4}$

Student Responses

- $\frac{6}{4}$, $1\frac{2}{4}$, or $1\frac{1}{2}$. $4 \times \frac{1}{4} = 1$ and $2 \times \frac{1}{4} = \frac{1}{2}$, so $6 \times \frac{1}{4} = 1\frac{1}{2}$.

- $\frac{18}{4}$, $4\frac{2}{4}$ or $4\frac{1}{2}$. $\frac{3}{4}$ is 3 times $\frac{1}{4}$, so it is 3 times the last product or $3 \times 1\frac{1}{2}$, which is $4\frac{1}{2}$.

- $\frac{54}{4}$, $13\frac{2}{4}$, or $13\frac{1}{2}$. 18 is 3 times as much as 6, so this product is 3 times as much as the last product. $3 \times 4\frac{1}{2} = 12\frac{3}{2} = 13\frac{1}{2}$

- $\frac{540}{4}$, $130\frac{20}{4}$, $130\frac{10}{2}$ or 135. 180 is 10 times as much as 18, so this product must be 10 times as much as the last product. $10 \times 13 = 130$, $10 \times \frac{1}{2} = \frac{10}{2} = 5$, $130 + 5 = 135$

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."

Activity

- 1 minute: quiet think time
- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "Today we are going to write our own Number Talks. Imagine the writer of this Number Talk started with $6 \times \frac{1}{4}$. What did you notice about how each expression changed? What do you think the writer was trying to get you to notice?" (Each expression had a factor that changed by multiplying by 3 for the first two expressions, then by 10 for the last expression. I think they were getting us to notice how you can look for ways to make each expression easier to solve by using products we've already found.)

- As needed, "How is each expression like the expression before it? How can you use that change to find each new value?" (Each expression has one factor that changes. If you think about how many times greater the factor is than the factor before it, you can use that to find the product.)

Activity 1

🕐 20 min

Related Numbers, Related Expressions

Standards Alignments

Addressing   4.NBT.A.1, 4.NBT.B

A Number Talk activity encourages students to look for structure and relationships that can help them reason about expressions. This activity puts students in the mindset of a Number Talk writer. It prompts students to anticipate some ways that others might decompose, rearrange, and regroup numbers, or to otherwise make use of structure to find the value of expressions. The work here will be helpful as students consider the numbers and numerical relationships to use as they write their own expressions later.

♿ Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Provide students with alternatives to writing on paper: students can explain their strategies orally or using manipulatives, such as base-ten blocks.
Supports accessibility for: Language, Organization, Memory

Student-facing Task Statement

1. Here are two addition expressions. Think of at least two different ways to find the value of each sum mentally.

    a. $15 + 29$

    b. $30 + 58$

2. Here are three subtraction expressions. Think of at least two different ways to find the value of each difference mentally.

    a. $91 - 11$

    b. $91 - 16$

    c. $391 - 86$

3. Can you write a fourth subtraction expression that uses the same strategy you

Launch

- Groups of 2
- "Find at least two ways to mentally find the value of each expression, without writing."

Activity

- 7–8 minutes: independent work time
- 2–3 minutes: partner discussion

Synthesis

- Invite students to share their mental computation strategies. Collect as many strategies as time permits and record them.
- If no students mentioned making use of

used to find the value of the other differences?

Student Responses

1. a. Some ways to find $15 + 29$:
    - Think of 29 as $30 - 1$ and find $15 + 30 - 1$, which is $45 - 1$ or 44.
    - Think of 29 as $20 + 9$ and find $15 + 20 + 9$, which is $35 + 9$ or 44.
    - Think of 15 as $10 + 5$ and find $29 + 10 + 5$, which is $39 + 5$ or 44.
    - Add tens and ones separately: $10 + 20 = 30$ and $5 + 9 = 14$, and $30 + 14 = 44$.

    b. Some ways to find $30 + 58$:
    - Think of 58 as $50 + 8$ and find $30 + 50 + 8$, which is $80 + 8$ or 88.
    - See 58 as $60 - 2$: $30 + 60 - 2 = 90 - 2 = 88$
    - 30 is twice 15 and 58 is twice 29, so the sum must be twice the sum of $15 + 29$ or 2×44, which is 88.

2. a. Some ways to find $91 - 11$:
    - Subtract tens and ones separately: $90 - 10 = 80$ and $1 - 1 = 0$, and $80 + 0 = 80$.
    - Subtract 10, then 1: $91 - 10 = 81$ and $81 - 1 = 80$.

    b. Some ways to find $91 - 16$:
    - $91 - 16$ is 5 less than $91 - 11$, so it is 5 less than 80, which is 75.
    - Subtract 10, then 6: $91 - 10 = 81$, and $81 - 6 = 75$.
    - Subtract 6, then 10: $91 - 6 = 85$,

one or more preceding expressions in their mental computation, ask them how it could be done. (See student response for examples.)

- Select a couple of students to share their new expressions. Ask the class to find the value of the expressions mentally.

and $85 - 10$ is 75.

 c. Some ways to find $391 - 86$:

  ■ $91 - 86$ is 5, and $391 - 86$ is 300 more, so it is $5 + 300$ or 305.

  ■ $91 - 16 = 75$, so $391 - 16 = 375$. Because $391 - 86$ is 70 less than $391 - 16$, its value is $375 - 70$ or 305.

 3. Sample responses:

  ○ $1{,}391 - 86$

  ○ $3{,}910 - 860$

Activity 2

⏰ 15 min

Add One New Expression, Then Two

Standards Alignments

Addressing 4.NBT.A.1, 4.NBT.B

In this activity, students use their understanding of place value and knowledge of operations on numbers to write new multiplication and division expressions. Students study the given expressions and their relationships and consider how they could find the value of each expression mentally. Next, they propose new expressions to complete each Number Talk activity.

🌐 Access for English Learners

MLR7 Compare and Connect. Synthesis: After all strategies have been presented, lead a discussion comparing, contrasting, and connecting the different approaches. Ask, "Why did the different approaches lead to the same (or different) outcome(s)?" or "What did the approaches have in common? How were they different?"
Advances: Representing, Conversing

Student-facing Task Statement

1. Here are three division expressions. Find the value of each quotient mentally and think about how they might be related.

   ○ $35 \div 5$

   ○ $70 \div 5$

   ○ $210 \div 5$

   ○ _____

   Write a new division expression whose value can be found more easily after working through the first three.

2. Here are two multiplication expressions. Analyze them and think about how they might be related.

   ○ 21×7

   ○ 42×7

   ○ _____

   ○ _____

   Write two new expressions. Be prepared to explain your reasoning for each expression.

one EXPRESSION $\div$
$\times$ **TWO EXPRESSIONS**

Student Responses

Sample response and reasoning:

1. The first expression is 7, the second is 14 (twice $35 \div 5$), and the third is 42 (or 3 times $70 \div 5$).

   New expression: $2{,}100 \div 5$. Its value is 420. Because the dividend is 10 times as much as in $210 \div 5$ and the divisor is the same, then the quotient is 10×42.

2. The first expression is 147 and the second is twice as much, 294.

   New expressions:

Launch

- Groups of 2
- "You'll see two incomplete Number Talk sets, each with one or more missing expressions. Find the value of each expression mentally and think about their relationship."

Activity

- "Write one or more new expressions to complete the sets. Be prepared to explain your reasoning for the expression."
- 7–8 minutes: group work time
- Monitor for students who:
  ○ adjust the dividend but keep the divisor
  ○ adjust the divisor but keep the dividend
  ○ use place value to multiply or divide
  ○ use multiplicative relationships such as halving and doubling and reason how it impacts the quotient

Synthesis

- Select students who reasoned differently to share their expressions. Record each set.
- Ask the class to determine the reasoning behind the new expression(s) and ask the writers to respond to the feedback.
- "How might the second expression help someone find the value of the third expression?"
- "How might the third expression help someone find the value of the fourth?"

○ 84×7. It is twice 42×7, so it is 2×294 or 588.

○ 84×70. It is 10 times 84×7, so it is 5,880.

Activity 3 (optional)

○ 30 min

Add Three New Expressions

Standards Alignments

Addressing 4.NBT.A.1, 4.NBT.B

In this optional activity students are given four expressions—three expressions involve operations of two whole numbers and one involves multiplication of a whole number and a fraction. Students choose one expression and write three new ones to create a Number Talk activity.

Students have considerably more freedom to decide the direction of subsequent expressions, but should be just as prepared to explain the rationale behind their expressions. Expect the increased openness to be a greater cognitive lift for students.

For the activity synthesis, ask each group of 2 to test their set by presenting it to another group and attending to how others reason about their expressions.

Student-facing Task Statement

Here are four expressions you could use to start a Number Talk activity.

$$75 + 30 \qquad 160 - 51 \qquad 24 \div 8 \qquad 3 \times \tfrac{1}{6}$$

1. Choose one starting expression. Think of at least two different ways to find its value mentally.
2. Write three equations to create a Number Talk activity. Be prepared to explain your reasoning for writing each expression.

Launch

- Groups of 2

Activity

- "Choose a starting expression. Then, work with your group to write three new expressions to make a complete Number Talk activity. Be prepared to explain the reasoning for each expression."

- "Each new expression may be more challenging to evaluate on its own, but

_____ (starting expression)

_____

_____

_____

3. Create an answer key for your Number Talk. Include at least one way to find the value of each expression mentally.

Student Responses

Answers vary.

easier to evaluate after working through the ones before it."

- 15 minutes: group work time

Synthesis

- "Work with another group and take turns presenting your Number Talk activity."

- "Show one expression at a time. Move on only after an answer and an explanation are given. Record their reasoning."

- After students have a chance to test their Number Talk, discuss questions such as:

    - "Did others reason in a way you anticipated?"

    - "Were there expressions that you'd revise?"

Lesson Synthesis

🕐 10 min

"Today (or the past couple of days) you've used your mathematical understanding to write expressions for Number Talk activities. You've also created an original set."

"What were important things you considered as you wrote expressions for a Number Talk? Why were these things important?"

"What were some challenges in creating a brand new Number Talk, or in completing a partial set?"

- **Complete Cool-Down** -

Response to Student Thinking

Students may generate a pattern in response to this question instead of considering a way that patterns were used to help reason about a solution.

Next Day Support

- Before the next day's warm-up, pair students up to discuss their responses.

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Family Support
Materials

Family Support Materials

Putting It All Together

In this unit, students apply what they have learned throughout the year to strengthen major concepts and fluency goals of the grade.

Section A: Fraction Fun

In this section, students practice multiplying fractions and whole numbers, as well as adding and subtracting fractions with the same denominator. They also solve problems that involve comparing fractions and adding and subtracting tenths and hundredths.

Here are the times of the runners for two teams.
Which team won the relay race?

runner	Diego's team, time (seconds)	Jada's team, time (seconds)
1	$10\frac{25}{100}$	$11\frac{9}{10}$
2	$11\frac{40}{100}$	$9\frac{8}{10}$
3	$9\frac{7}{10}$	$9\frac{84}{100}$
4	$10\frac{5}{100}$	$10\frac{60}{100}$

Section B: Whole-number Operations

In this section, students deepen their understanding of place value and build their fluency in performing operations on multi-digit numbers.

Students begin by using the standard algorithm to add and subtract numbers within 1 million. They recall when to compose (or "carry") a new place-value unit (a ten, a hundred, a thousand, and so on) when adding, and when to decompose a unit (or "regroup") when subtracting.

Students learn to pay attention to potential errors, especially when subtracting a number with non-zero digits from a number with zeros, and to be more strategic in choosing a method.

Use both Priya and Han's methods to find the difference of 20,000 and 472.

Priya

$$
\begin{array}{r}
2\,0,0\,0\,0 \\
-\quad\ 4\,7\,2 \\
\hline
\end{array}
$$

Han

$$
\begin{array}{r}
4\,7\,2 \\
+ \\
\hline
2\,0,0\,0\,0
\end{array}
$$

Next, students practice multiplying and dividing multi-digit numbers using algorithms that involve partial products and partial quotients. In both cases, students make connections across the different methods they see or use.

Section C: Multiplicative Comparison and World Travel

In this section, students use multiplication and division to make comparisons and solve real-world problems. They make estimates to simplify a problem, help with calculations, or assess whether a statement or a number is reasonable.

A school needs buses to take 375 people on a field trip.

- *Bus Company A has small buses with 27 seats in each.*
- *Bus Company B has large buses with 48 seats in each.*

Which bus company should the school choose?

Section D: Creation and Design

Throughout the course, students have participated in warm-up routines such as How Many Do You See, Exploration Estimation, Which One Doesn't Belong, True or False, and Number Talk.

In this section, they apply the mathematics they have learned to design warm-ups that use some of these routines.

Add an item to complete the set.

A   B

C   D
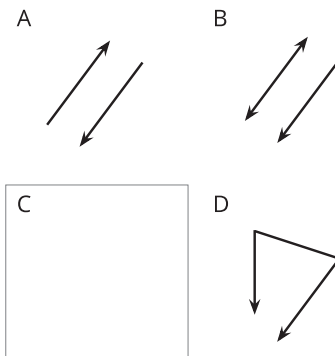
Make sure there is at least one reason it belongs and one reason it doesn't belong.

Try it at home!

Near the end of the unit, ask your student to share the warm-up routines they created. Questions that may be helpful as they share:

- How did you design the routine?
- How does the routine relate to what you learned this year?
- What might you change to improve the routine?

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Unit Assessments

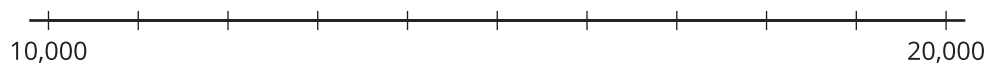End-of-Course Assessment and Resources

Putting It All Together: End-of-Course Assessment and Resources

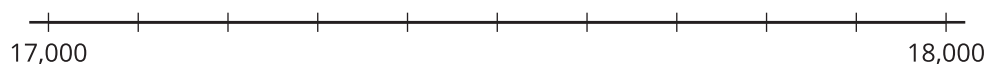1. Select the number where the value of 6 is 1,000 times the value of the 6 in 463.

   A. 643

   B. 6,118

   C. 63,479

   D. 627,385

2. Locate the number 17,258 on each number line.

   a.

   |————|————|————|————|————|————|————|————|————|————|
   10,000   20,000

   b.

   |————|————|————|————|————|————|————|————|————|————|
   17,000   18,000

3. Complete each blank with $<$, $=$, or $>$ to make the statement true.

   a. $200{,}000 + 5{,}000 + 700 + 6$ _____ $250{,}706$

   b. $62{,}318$ _____ $60{,}000 + 2{,}000 + 300 + 10 + 8$

   c. $251{,}864$ _____ $251{,}846$

   d. $68{,}173$ _____ $103{,}012$

   e. $357{,}928$ _____ $358{,}715$

4.   a. Round 73,526 to the nearest ten-thousand. Use the number line if it is helpful.

$$\vdash\quad\vdash\quad\vdash\quad\vdash\quad\vdash\quad\vdash\quad\vdash\quad\vdash\quad\vdash\quad\vdash\quad\vdash$$

      b. Round 73,526 to the nearest thousand. Use the number line if it is helpful.

$$\vdash\quad\vdash\quad\vdash\quad\vdash\quad\vdash\quad\vdash\quad\vdash\quad\vdash\quad\vdash\quad\vdash\quad\vdash$$

      c. Round 73,526 to the nearest hundred. Use the number line if it is helpful.

$$\vdash\quad\vdash\quad\vdash\quad\vdash\quad\vdash\quad\vdash\quad\vdash\quad\vdash\quad\vdash\quad\vdash\quad\vdash$$

5. Andre ran 1,270 meters. Clare ran 3 times as far as Andre. How many meters did Clare run? Explain or show your reasoning.

6. A school has raised $3,000 for a class trip. There are 26 children in the class. The trip costs $95 per student. Transportation costs are an additional $350.

Has the school raised enough money for the trip? Explain or show your reasoning.

7. There are 5,760 fourth-grade students in City A. That's 4 times as many fourth-grade students as there are in City B. How many fourth-grade students are there in City B? Explain or show your reasoning.

8. Select **all** fractions that are equivalent to $\frac{3}{5}$.

    A. $\frac{6}{10}$

    B. $\frac{6}{100}$

    C. $\frac{60}{100}$

    D. $\frac{4}{6}$

    E. $\frac{15}{25}$

9. Select **all** correct statements.

    A. $0.49 < \frac{4}{7}$

    B. $0.08 = \frac{8}{10}$

    C. $\frac{5}{6} = \frac{20}{24}$

    D. $\frac{15}{8} < \frac{15}{11}$

    E. $\frac{3}{7} > \frac{3}{9}$

    F. $0.08 > 0.4$

    G. $0.15 < 0.3$

10. Locate the following numbers on the number line:
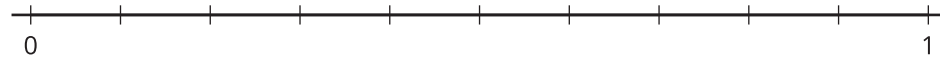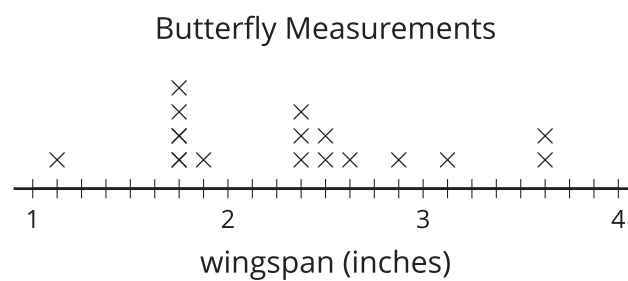
$$0.53 \qquad \frac{7}{10} \qquad \frac{35}{100} \qquad 0.08$$

11. The line plot shows the wingspans of some butterflies in inches.

Butterfly Measurements

wingspan (inches)

   a. How much greater is the longest wingspan than the shortest wingspan? Explain or show your reasoning.

   b. How much greater is the longest wingspan than the most common wingspan? Explain or show your reasoning.

12. Select **all** expressions equivalent to $\frac{9}{8}$.

   A. $\frac{6}{8} + \frac{2}{8} + \frac{1}{8}$

   B. $\frac{3}{8} + \frac{3}{8}$

   C. $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{6}{8}$

   D. $\frac{9}{4} + \frac{9}{4}$

   E. $1 + \frac{1}{8}$

   F. $3 \times \frac{3}{8}$

   G. $2 \times \frac{9}{4}$

13.   a. Mai's house is $\frac{5}{8}$ mile from school. She walked to school all 5 days of the week. How many miles did Mai walk altogether from home to school? Explain or show your reasoning.

   b. Mai wants to walk 6 miles total for the week. How much farther does she need to walk? Explain or show your reasoning.

14. Find the value of each expression.

    a. $\frac{2}{5} + \frac{6}{5} + \frac{1}{5}$

    b. $\frac{3}{10} + \frac{17}{100}$

    c. $2\frac{5}{8} - 1\frac{7}{8}$

15. Find the value of each expression using the standard addition or subtraction algorithm.

    a. $52{,}166 + 61{,}735$

    b. $18{,}275 - 9{,}516$

    c. $247{,}853 + 175{,}329$

d. $247,853 - 175,329$

e. $603,117 - 358,245$

16. a. Find the value of each expression.

   i. $52,506 + 99,999$

   ii. $571,382 - 299,999$

   iii. $1,000,000 - 255,372$

b. In each blank, write a number of your choice that you think will make a simple
   calculation. Then find the value of the expression.

   i. $374{,}815 +$ _____

   ii. $374{,}815 -$ _____

   iii. _____ $- 374{,}815$

c. In each blank, write a number of your choice that you think will make a challenging
   calculation. Then find the value of the expression.

   i. $374{,}815 +$ _____

   ii. $374{,}815 -$ _____

   iii. _____ $- 374{,}815$

17. There are 21 students in Clare's class and she wants to bring enough sparkling juice so each student can have $1\frac{1}{2}$ cups of sparkling juice.

    a. How many cups of sparkling juice does Clare need to make? Explain or show your reasoning.

    b. Each bottle of sparkling juice contains $3\frac{1}{5}$ cups. How many students will one bottle serve? How much juice will be left over? Explain or show your reasoning.

    c. Clare thinks she should buy 10 bottles of sparkling juice. Would that be enough? Explain or show your reasoning.

_____

_____

_____

_____

18. As the moon goes around the earth, the closest it gets is 225,623 miles. The farthest the moon gets is 252,088 miles.

   a. What is the difference between the farthest and closest distances from the moon to the earth?

   b. A book says that distance from the moon to the earth is 238,855 miles. What is the difference between this distance and the closest distance? What about the farthest distance?

   c. A car on the highway travels 70 miles each hour. How far would the car travel in:

      i. 24 hours

      ii. 10 days

      iii. 100 days

   d. If it were possible, about how long do you think it would take to drive to the moon? Explain your reasoning.

   _____

   _____

   _____

   _____

CKMath™
Core Knowledge Mathematics™

Assessment
Answer Keys

End-of-Course Assessment and Resources

Assessment Answer Keys
Assessment: End-of-Course Assessment and Resources

Teacher Instructions

The items here focus on major work of the grade, fluencies of the grade, and also include at least one in-depth problem that provides a context where students apply key ideas they have learned over the year. The items included here can be used prior to the final unit to focus remaining time in the year or to assess student understanding at the end of the year. It is not recommended that these resources be used all at once.

Problem 1

Standards Alignments

Addressing   4.NBT.A.1

Narrative

Students use their understanding of place value to compare the value of the same digit in different places in a number. Students may select B if they see that the 6 in 6,118 has value 6 thousand. Students who select A or D need more work with place value.

Select the number where the value of 6 is 1,000 times the value of the 6 in 463.

     A.   643

     B.   6,118

     C.   63,479

     D.   627,385

Solution

C

Problem 2

Standards Alignments

Addressing   4.NBT.A.1
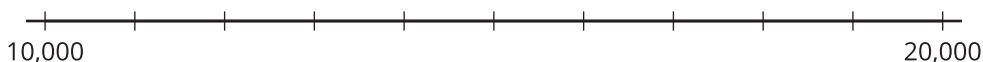
Narrative

Students locate a number on two number lines where the outer tick marks are labeled. Understanding of place value plays a key role in this item as the tick marks on the first number line increase by thousands and on the second number line they increase by hundreds. Accurately labeling the number lines and placing the given number demonstrate an understanding of place value.
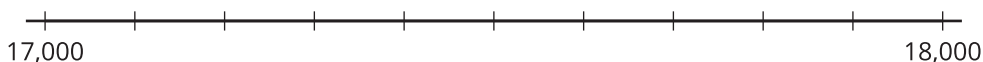
Since the number does not lie exactly on a tick mark, students need to estimate its location. They are not expected to place the number in its precise position but it should be between the correct two tick marks and, in the first case, closer to 17,000 than to 18,000.

Locate the number 17,258 on each number line.

a.

```
+----+----+----+----+----+----+----+----+----+----+
10,000                                        20,000
```

b.

```
+----+----+----+----+----+----+----+----+----+----+
17,000                                        18,000
```

Solution

a.

```
+----+----+----+----+----+----+----+●---+----+----+
10,000                              17,258      20,000
```

b.

```
+----+----●----+----+----+----+----+----+----+----+
17,000    17,258                               18,000
```
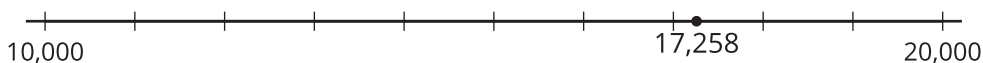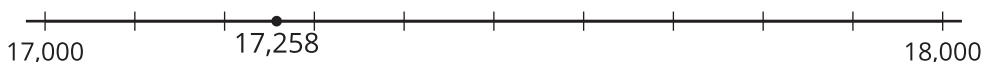
Problem 3

Standards Alignments

Addressing   4.NBT.A.2

Narrative

Students identify numbers in expanded form and compare numbers using $<$ and $>$. Students who do not select $<$ for the first problem have probably not paid sufficiently close attention to the place value of each digit. Students who do not select $=$ for the second problem may need more work with expanded form.

The remaining 3 problems compare numbers that are not equal. Students who miss one or more of these problems may have misread the numbers or may need more work with place value.

Complete each blank with $<$, $=$, or $>$ to make the statement true.

    a. $200{,}000 + 5{,}000 + 700 + 6$ _____ $250{,}706$

    b. $62{,}318$ _____ $60{,}000 + 2{,}000 + 300 + 10 + 8$

    c. $251{,}864$ _____ $251{,}846$

    d. $68{,}173$ _____ $103{,}012$

    e. $357{,}928$ _____ $358{,}715$

Solution

    a. $<$

    b. $=$

    c. $>$

    d. $<$

    e. $<$

Problem 4

Standards Alignments

Addressing   4.NBT.A.3

Narrative

Students round a number to the nearest ten-thousand, thousand, and hundred. No method is suggested so students may use their understanding of place value or they may label the number lines and use them.

    a. Round 73,526 to the nearest ten-thousand. Use the number line if it is helpful.

    b. Round 73,526 to the nearest thousand. Use the number line if it is helpful.
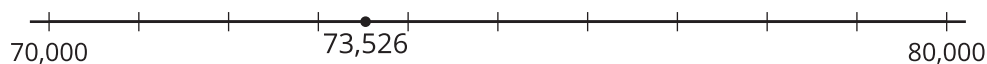
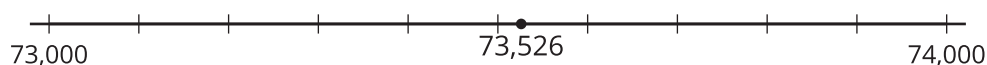   c. Round 73,526 to the nearest hundred. Use the number line if it is helpful.
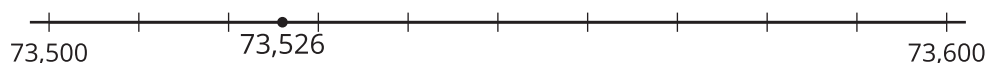
Solution

a. 70,000 since it is closer to 70,000 than to 80,000

```
  ┝━━┿━━┿━━┿━━●━┿━━┿━━┿━━┿━━┿━━┿━━┥
70,000          73,526                      80,000
```

b. 74,000 since it is closer to 74,000 than to 73,000.

```
  ┝━━┿━━┿━━┿━━┿━━●━┿━━┿━━┿━━┿━━┥
73,000              73,526                  74,000
```

c. 73,500 since it is closer to 73,500 than to 73,600.

```
  ┝━━┿━━┿━●━┿━━┿━━┿━━┿━━┿━━┿━━┥
73,500      73,526                          73,600
```

Problem 5

Standards Alignments

Addressing   4.NBT.B.5, 4.OA.A.2

Narrative

Students solve a multiplicative comparison problem that requires multiplying a four-digit number by a one-digit number. Students may draw a diagram or may decompose 1,270 by place value and use equations.

Andre ran 1,270 meters. Clare ran 3 times as far as Andre. How many meters did Clare run? Explain or show your reasoning.

Solution

3,810 meters. Sample response: $3 \times 1,000 = 3,000$ $3,000 + 600 + 210 = 3,810$
$3 \times 200 = 600$
$3 \times 70 = 210$

Problem 6

Standards Alignments

Addressing   4.NBT.B.5, 4.OA.A.3

Narrative

Students solve a two-step story problem which requires multiplying 2 two-digit numbers and then adding a number to the product. Students may calculate the exact amount of money the school has raised or they could use estimation. Since $26 \times 100 = 2,600$ and $2,600 + 350 = 2,950$, this means they have raised enough money.

A school has raised $3,000 for a class trip. There are 26 children in the class. The trip costs $95 per student. Transportation costs are an additional $350.

Has the school raised enough money for the trip? Explain or show your reasoning.

Solution

Yes. Sample response: First I found $26 \times 95 = 2,470$. Then I added 2,470 and 350 to get 2,820.

	90	5
20	1,800	100
6	540	30

Problem 7

Standards Alignments

Addressing   4.NBT.B.6, 4.OA.A.2

Narrative

Students divide a four-digit number by a one-digit number to solve a word problem. They may use partial products or draw a diagram or they could take multiples of 4 and add them until they reach 5,760.

There are 5,760 fourth-grade students in City A. That's 4 times as many fourth-grade students as there are in City B. How many fourth-grade students are there in City B? Explain or show your reasoning.

Solution

1,440. Sample response:

$$
\begin{array}{r}
\boxed{1,440} \\
40 \\
400 \\
1,000 \\
4\,)\overline{5,760} \\
-\;\;4,000 \\
\hline
1,760 \\
-\;\;1,600 \\
\hline
160 \\
-\;\;\;\;160 \\
\hline
0
\end{array}
$$

Problem 8

Standards Alignments

Addressing   4.NF.A.1

Narrative

Students identify fractions equivalent to a given fraction. Because the given fraction has a denominator of 5, this also gives students an opportunity to identify decimal fractions though they are not asked here to write them as decimals. Students who select B have made an arithmetic mistake and the answer is too small to be reasonable. Students who select D have probably added 1 to the numerator and denominator which does not give an equivalent fraction. Students who do not select C may need more work with decimal fractions and students who do not select A or E need more work finding equivalent fractions.

Select **all** fractions that are equivalent to $\frac{3}{5}$.

A. $\frac{6}{10}$

B. $\frac{6}{100}$

C. $\frac{60}{100}$

D. $\frac{4}{6}$

E. $\frac{15}{25}$

Solution

["A", "C", "E"]

Problem 9

Standards Alignments

Addressing   4.NF.A.2, 4.NF.C.6, 4.NF.C.7

Narrative

Students compare fractions and decimal numbers. For item A, they can make the comparison using the benchmark fraction $\frac{1}{2}$ which is also 0.50. Students may select B if they see the 8 in the decimal but fail to identify the 0 before the 8. Students may select D (and not select E) if they see the 15s in the numerators and reason that $8 < 11$ so $\frac{15}{8} < \frac{15}{11}$. Students may select F and not G if they compare the decimals as if they were whole numbers, that is not paying attention to the decimal or the 0's.

Select **all** correct statements.

A. $0.49 < \frac{4}{7}$

B. $0.08 = \frac{8}{10}$

C. $\frac{5}{6} = \frac{20}{24}$

D. $\frac{15}{8} < \frac{15}{11}$

E. $\frac{3}{7} > \frac{3}{9}$

F. $0.08 > 0.4$

G. $0.15 < 0.3$

Solution

["A", "C", "E", "G"]

Problem 10

Standards Alignments

Addressing   4.NF.C.5, 4.NF.C.6

Narrative

Students plot decimals and decimal fractions on the number line. Other than $\frac{7}{10}$, it is not important that the numbers be plotted exactly though they should be between the correct tick marks and generally in the right location between those tick marks. The number line is not labeled so students can think of the tick marks in terms of decimals, fractions, or both.

Locate the following numbers on the number line:

$$0.53 \qquad \frac{7}{10} \qquad \frac{35}{100} \qquad 0.08$$
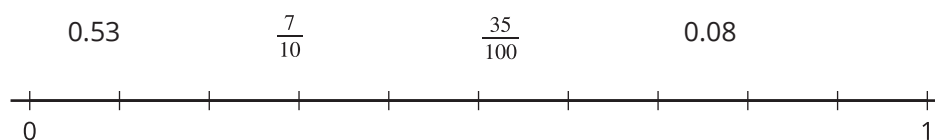
Solution
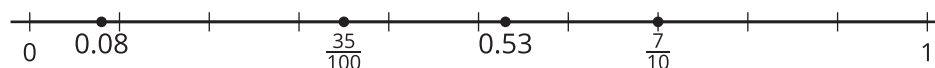
Problem 11

Standards Alignments
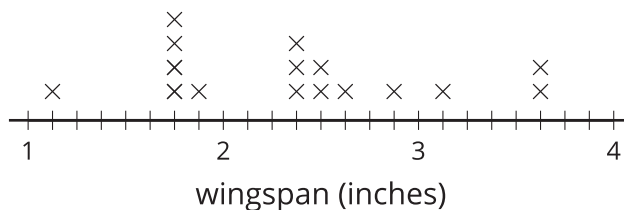
Addressing   4.MD.B.4, 4.NF.B.3.c

Narrative

Students subtract mixed numbers which they read from a line plot. The line plot is a convenient way of presenting the information and also a situation where mixed numbers occur naturally. Students can reason about the differences abstractly or they may use the horizontal axis which can play the role of a number line.

The line plot shows the wingspans of some butterflies in inches.

Butterfly Measurements

wingspan (inches)

a. How much greater is the longest wingspan than the shortest wingspan? Explain or show your reasoning.

b. How much greater is the longest wingspan than the most common wingspan? Explain or show your reasoning.

Solution

a. The shortest wingspan is $1\frac{1}{8}$ inches and the longest is $3\frac{5}{8}$ inches. I can take 1 from 3 and $\frac{1}{8}$ from $\frac{5}{8}$ and that leaves $2\frac{4}{8}$.

b. The longest wingspan is $3\frac{5}{8}$ inches and the most common is $1\frac{6}{8}$.

$$3 - 1 = 2$$
$$2 - \frac{6}{8} = 1 + \frac{8}{8} - \frac{6}{8} = 1\frac{2}{8}$$
$$1\frac{2}{8} + \frac{5}{8} = 1\frac{7}{8}$$

Problem 12

Standards Alignments

Addressing   4.NF.B.3.b, 4.NF.B.4.b

Narrative

Students identify sums of fractions that are equal to a given fraction. Students who select B may be applying the wrong operation, multiplication, to add fractions. Students who select D are likely adding fractions in the wrong way. Students may not select E if they forget that 1 is equivalent to $\frac{8}{8}$. Response F is the correct form of the distractor B. Students may select G, and perhaps not select F, if they multiply incorrectly.

Select **all** expressions equivalent to $\frac{9}{8}$.

A. $\frac{6}{8} + \frac{2}{8} + \frac{1}{8}$

B. $\frac{3}{8} + \frac{3}{8}$

C. $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{6}{8}$

D. $\frac{9}{4} + \frac{9}{4}$

E. $1 + \frac{1}{8}$

F. $3 \times \frac{3}{8}$

G. $2 \times \frac{9}{4}$

Solution

["A", "C", "E", "F"]

Problem 13

Standards Alignments

Addressing   4.NF.B.3.d, 4.NF.B.4.c

Narrative

Students find a whole number multiple of a fraction and a difference of a whole number and a fraction in context. While students can draw a number line to help visualize the problem, the numbers are large enough to make this an inconvenient approach to solve the entire problem. For the second problem, students may choose to write $\frac{25}{8}$ as a mixed number or may choose to write 6 as a fraction. They can also use the number line if they have the patience to put in the tick marks.

a. Mai's house is $\frac{5}{8}$ mile from school. She walked to school all 5 days of the week. How many miles did Mai walk altogether from home to school? Explain or show your reasoning.

b. Mai wants to walk 6 miles total for the week. How much farther does she need to walk? Explain or show your reasoning.

Solution

a. $\frac{25}{8}$ miles since $5 \times 5 = 25$.

b. $2\frac{7}{8}$ miles or equivalent. $\frac{24}{8}$ is 3 miles so she needs another $\frac{7}{8}$ to get to 4 miles and then 2 more miles to reach 6.

Problem 14

Standards Alignments

Addressing   4.NF.B.3.b, 4.NF.B.3.c, 4.NF.C.5

Narrative

Students add and subtract fractions with no explanation required. Students have evaluated expressions like these in the process of solving other problems on this assessment. Some students may have difficulties interpreting the contexts in those items. This item gives an insight into their ability to work noncontextually with fractions.

Find the value of each expression.

a. $\frac{2}{5} + \frac{6}{5} + \frac{1}{5}$

b. $\frac{3}{10} + \frac{17}{100}$

c. $2\frac{5}{8} - 1\frac{7}{8}$

Solution

a. $\frac{9}{5}$ or equivalent

b. $\frac{47}{100}$ or equivalent

c. $\frac{6}{8}$ or equivalent

Problem 15

Standards Alignments

Addressing   4.NBT.B.4

Narrative

This item assesses student ability to perform addition and subtraction within 1,000,000 using the

standard algorithm. The numbers are not special so the standard algorithm is a good approach for all of the problems. Each problem requires multiple compositions or decompositions.

It is not important for students to do all of these problems. They are intended to check fluency and can be used for students who need or want to practice these skills.

Find the value of each expression using the standard addition or subtraction algorithm.

  a. $52{,}166 + 61{,}735$

  b. $18{,}275 - 9{,}516$

  c. $247{,}853 + 175{,}329$

  d. $247{,}853 - 175{,}329$

  e. $603{,}117 - 358{,}245$

Solution

a.

```
              1  1
      5 2, 1  6  6
  +   6 1, 7  3  5
    1 1 3, 9  0  1
```

b.

```
          17
      0  7 12 6 15
      1  8, 2  7  5
  −      9, 5  1  6
         8, 7  5  9
```

c.

```
    1 1 1     1
    2 4 7, 8  5  3
  + 1 7 5, 3  2  9
    4 2 3, 1  8  2
```

d.

```
    1 14        4 13
    2 4 7, 8  5  3
  − 1 7 5, 3  2  9
    7 2, 5  2  4
```

e.

$$
\begin{array}{r}
91210 \\
510\cancel{10}\cancel{0}11 \\
\cancel{6}\cancel{0}\cancel{3},\cancel{1}\cancel{1}7 \\
-358,245 \\
\hline
244,872
\end{array}
$$

Problem 16

Standards Alignments

Addressing   4.NBT.B.4

Narrative

Students continue to perform addition and subtraction with multi-digit numbers. These numbers are chosen to encourage alternative methods, other than the standard algorithms. The standard algorithms will work in all cases and students who choose to use them will gain valuable practice and test their fluency. Students who use other methods show a different kind of fluency, namely choosing a strategy appropriate to the problem.

   a. Find the value of each expression.

     i. $52{,}506 + 99{,}999$

     ii. $571{,}382 - 299{,}999$

     iii. $1{,}000{,}000 - 255{,}372$

   b. In each blank, write a number of your choice that you think will make a simple calculation. Then find the value of the expression.

     i. $374{,}815 +$ _____

     ii. $374{,}815 -$ _____

     iii.   _____ $- 374{,}815$

   c. In each blank, write a number of your choice that you think will make a challenging calculation. Then find the value of the expression.

     i. $374{,}815 +$ _____

     ii. $374{,}815 -$ _____

     iii.   _____ $- 374{,}815$

Solution

a.
  i. 152,505. I added 100,000 and then subtracted 1.
  ii. 271,383. I subtracted 300,000 and then added 1.
  iii. 744,628. I chose numbers to add up to 9 in each place and then added 1 at the end.

b.
  i. Sample response: $100{,}000 + 374{,}815 = 474{,}815$
  ii. Sample response: $374{,}815 - 100{,}000 = 274{,}815$
  iii. Sample response: $474{,}815 - 374{,}815 = 100{,}000$

c.
  i. Sample response $374{,}815 + 248{,}596 = 623{,}411$
  ii. Sample response $374{,}815 - 186{,}928 = 187{,}887$
  iii. Sample response $513{,}206 - 374{,}815 = 138{,}391$

Problem 17

Standards Alignments

Addressing   4.MD.A.2, 4.NF.B.3, 4.NF.B.4

Narrative

Students solve a word problem about liquid volume using fractions. They will need to multiply mixed numbers by whole numbers and then find differences of the results. Students can directly solve the final question without doing the first two questions but the calculations on these questions, in addition to building familiarity with the context, give them some numbers that they can use when they solve the final problem.

There are 21 students in Clare's class and she wants to bring enough sparkling juice so each student can have $1\frac{1}{2}$ cups of sparkling juice.

a. How many cups of sparkling juice does Clare need to make? Explain or show your reasoning.

b. Each bottle of sparkling juice contains $3\frac{1}{5}$ cups. How many students will one bottle serve? How much juice will be left over? Explain or show your reasoning.

c. Clare thinks she should buy 10 bottles of sparkling juice. Would that be enough? Explain or show your reasoning.

Solution

a. $21 \times 1\frac{1}{2}$. Since $1\frac{1}{2} = \frac{3}{2}$, each student drinks 3 one-half cups. So Clare needs to make 21×3 half cups of sparkling juice. That's 63 half-cups which is 31 cups and 1 half-cup.

b. 2 students because that's 2 cups and 2 half-cups which makes 3 cups total. There would be just $\frac{1}{5}$ cup left over.

c. Yes. Each bottle can serve 2 students with a little left over so 10 bottles will serve 20 and I just need to see if the left over is enough for one more serving. There is $\frac{1}{5}$ cup left after serving 2 from each bottle so that's $\frac{10}{5}$ cups left from all 10 bottles. That is 2 cups which means I can serve 21 students and have $\frac{1}{2}$ cup left.

Problem 18

Standards Alignments

Addressing   4.MD.A.1, 4.MD.A.2, 4.NBT.A.1, 4.NBT.B.5

Narrative

Students solve problems about the distance from the earth to the moon. The orbit of the moon is not a circle and so the distance varies over the year. The usual "distance" listed in resources is almost halfway between the greatest and least distances. This makes sense because it is simpler to report one number but reporting the greatest or least distances is misleading so instead the number usually reported is almost exactly halfway between.

After thinking about this, students next work with unit conversions of time to estimate how long it would take to travel to the moon by car, if there were a road that went there. This involves multiplying a pair of two-digit numbers and then using place value understanding to continue to multiply by factors of 10.

As the moon goes around the earth, the closest it gets is 225,623 miles. The farthest the moon gets is 252,088 miles.

a. What is the difference between the farthest and closest distances from the moon to the earth?

b. A book says that distance from the moon to the earth is 238,855 miles. What is the difference between this distance and the closest distance? What about the farthest distance?

c. A car on the highway travels 70 miles each hour. How far would the car travel in:

    i. 24 hours

    ii. 10 days

    iii. 100 days

d. If it were possible, about how long do you think it would take to drive to the moon? Explain your reasoning.

Solution

a. 26,465 miles

$$
\begin{array}{r}
\;\;\;11 \\
\;\;4\;\;\cancel{1}\;\;10 \\
2\;\;\cancel{5}\;\;\cancel{2},\;\cancel{0}\;\;8\;\;8 \\
-\;\;2\;\;2\;\;5,\;6\;\;2\;\;3 \\
\hline
2\;\;6,\;4\;\;6\;\;5
\end{array}
$$

b. The two differences are almost the same.

$$
\begin{array}{r}
\;\;\;11 \\
\;\;4\;\;\cancel{1}\;\;10 \\
2\;\;\cancel{5}\;\;\cancel{2},\;\cancel{0}\;\;8\;\;8 \\
-\;\;2\;\;3\;\;8,\;8\;\;5\;\;5 \\
\hline
1\;\;3,\;2\;\;3\;\;3
\end{array}
$$

$$
\begin{array}{r}
2\;\;3\;\;8,\;8\;\;5\;\;5 \\
-\;\;2\;\;2\;\;5,\;6\;\;2\;\;3 \\
\hline
1\;\;3,\;2\;\;3\;\;2
\end{array}
$$

c.   i. 1,680 miles. First I found 24×7 and that's 168 and then I multiplied that by 10 which makes 1,680.

    ii. 16,800 miles

    iii. 168,000 miles

d. From my calculations in the previous problem, I can see that in 200 days I could travel 336,000 miles so that would get me to the moon. But that is driving all day long. So a more realistic estimate would be 600 days, with 8 hours of driving each day, or almost 2 years.

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Lesson
Cool Downs

Lesson 1: Add, Subtract, and Multiply Fractions

Cool Down: Compare to 2

Here are some fractions:
$\frac{15}{10}$ $\frac{13}{10}$ $\frac{53}{100}$ $\frac{9}{10}$
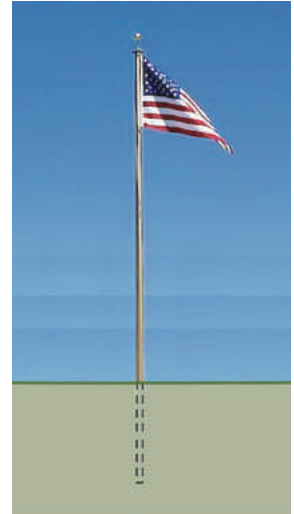
1. Select two fractions that have a sum greater than 2. Explain or show your reasoning.

2. Use all four fractions to write an expression that has a value greater than 1 but less than 2.

Lesson 2: Sums and Differences of Fractions

Cool Down: The Flagpole

The school flagpole is placed about $3\frac{2}{6}$ feet into the ground.

Students can see $12\frac{4}{6}$ feet of the flagpole.

How long is the entire flagpole? Show your reasoning.

Lesson 3: Stories with Fractions

Cool Down: Mai's Milky Cereal

There were 7 cups of milk before Mai made breakfast. Now there are $2\frac{2}{8}$ cups of milk. How much milk did Mai use for breakfast?

Lesson 4: Another Look at the Standard Algorithm

Cool Down: A Couple of Differences

Find the value of each difference. Show your reasoning.

1. $8{,}050 - 213$

2. $60{,}000 - 1{,}984$

Lesson 5: Multiplication of Multi-digit Numbers

Cool Down: Four by One and Two by Two

1. Find the value of $2{,}617 \times 4$. Show your reasoning.

2. Find the value of 52×14. Show your reasoning.

Lesson 6: What's the Quotient?

Cool Down: Divide Like a Pro

1. Here are two different ways to start finding the value of $8{,}435 \div 7$. Choose one way and complete the calculation.

$$
\begin{array}{r}
5 \\
7\,\overline{)8{,}435} \\
-35 \\
\hline
8{,}400
\end{array}
\qquad\qquad
\begin{array}{r}
1{,}000 \\
7\,\overline{)8{,}435} \\
-\;7{,}000 \\
\hline
1{,}435
\end{array}
$$

2. Find the value of $1{,}038 \div 6$. Try to use as few steps as possible.

Lesson 7: Solve Multiplicative Comparison Problems

Cool Down: Restaurant Budget

In the United States, the cost of a meal for two people in a mid-range restaurant is about $50. A similar meal in Ghana is about $25.

1. Write a statement that compares the cost of a meal at a restaurant in the United States to a meal in Ghana and uses the phrase ". . . times as much as . . ."

2. A couple only wants to spend $240 at restaurants each month. How many more times could the couple go out to a restaurant each month if they eat in Ghana than in the United States? Show or explain your reasoning.

Lesson 8: Solve Problems with Multiplication and Division

Cool Down: To and Fro

In one week, a train made 8 round trips between its home station and Union Station. At the end of the week, it traveled a few more miles from the home station to a repair center. That week, the train traveled a total of 1,564 miles.

1. Which statement is true for this situation? Explain or show your reasoning.

    a. The distance traveled for each round trip is 200 miles. The distance to the repair station is 26 miles.

    b. The distance traveled for each round trip is 195 miles. The distance to the repair station is 4 miles.

    c. The distance traveled for each round trip is 8 miles. The distance to the repair station is 1,500 miles.

    d. The distance traveled for each round trip is 193 miles. The distance to the repair station is 8 miles.

2. Explain why one of the choices could not be true.

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Lesson 9: Create Word Problems

Cool Down: A Music Festival

The school band wants to raise $1,700 for a music festival. They have raised $175 each week for the past 6 weeks.

Write a question that could be asked about this situation and answer it. Show your reasoning.

Lesson 10: Estimation Exploration

Cool Down: Reflection

Describe something mathematical that you understand better after completing today's activities, or something that you find confusing or challenging.

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Lesson 11: Which One Doesn't Belong?

Cool Down: Reflection

As mathematicians, it is important to explain our thinking and listen to the reasoning of others.

Describe a time today when you learned something new, or thought differently about something, based on what a classmate said.

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Lesson 12: Number Talk

Cool Down: Reflection

As mathematicians, we often use patterns to help us reason about new problems. Observing something that repeats over and over can also help us solve problems.

Describe a time, during today's lesson or recently, when you noticed a pattern or a repetition and used it to help you think through a problem. How did the pattern or repetition help you?

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