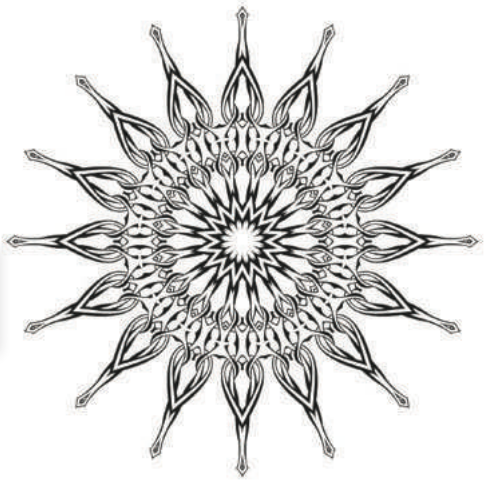
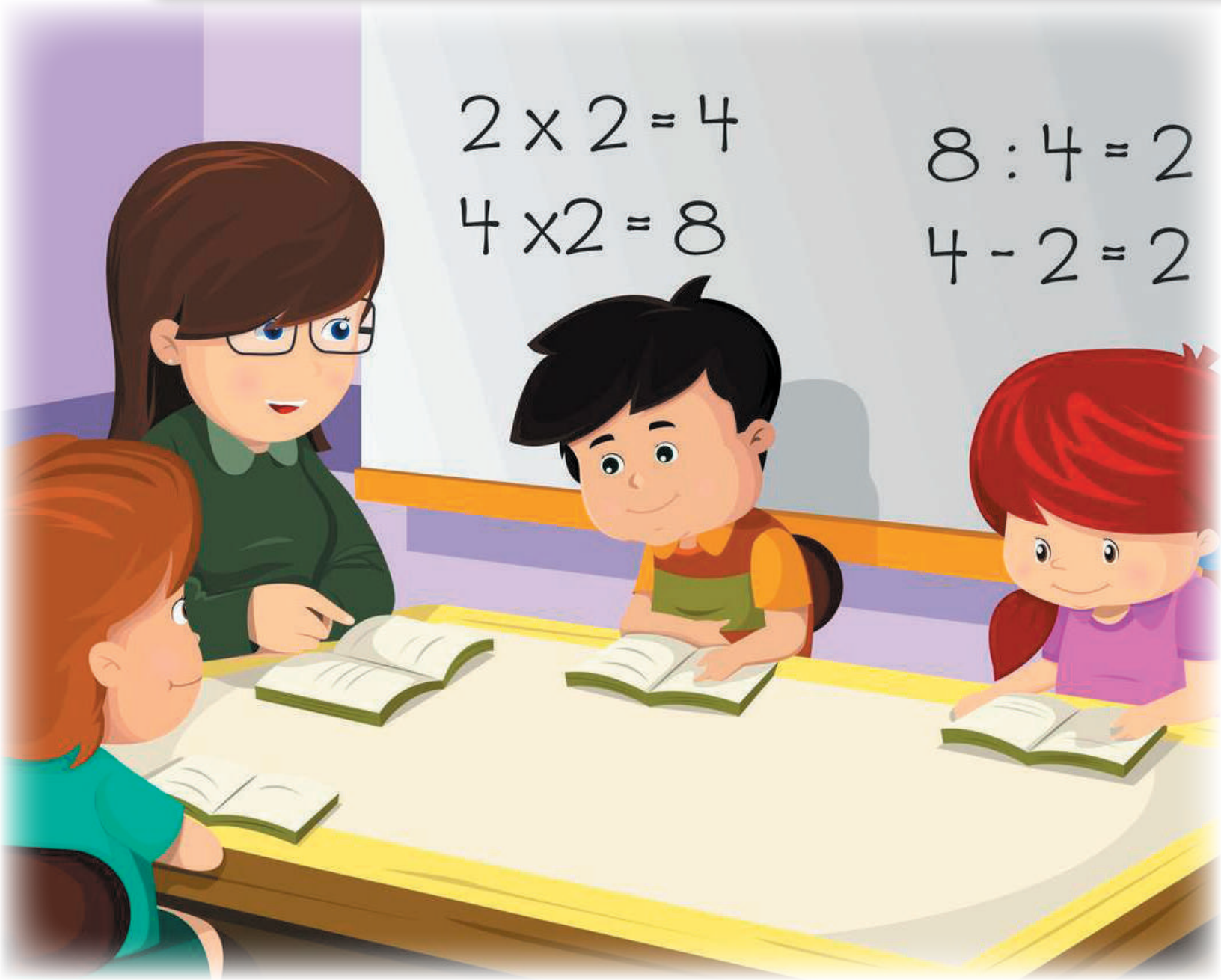
Core Knowledge® MATHEMATICS

Properties of Two-dimensional Shapes



Student Workbook



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Properties of Two-dimensional Shapes

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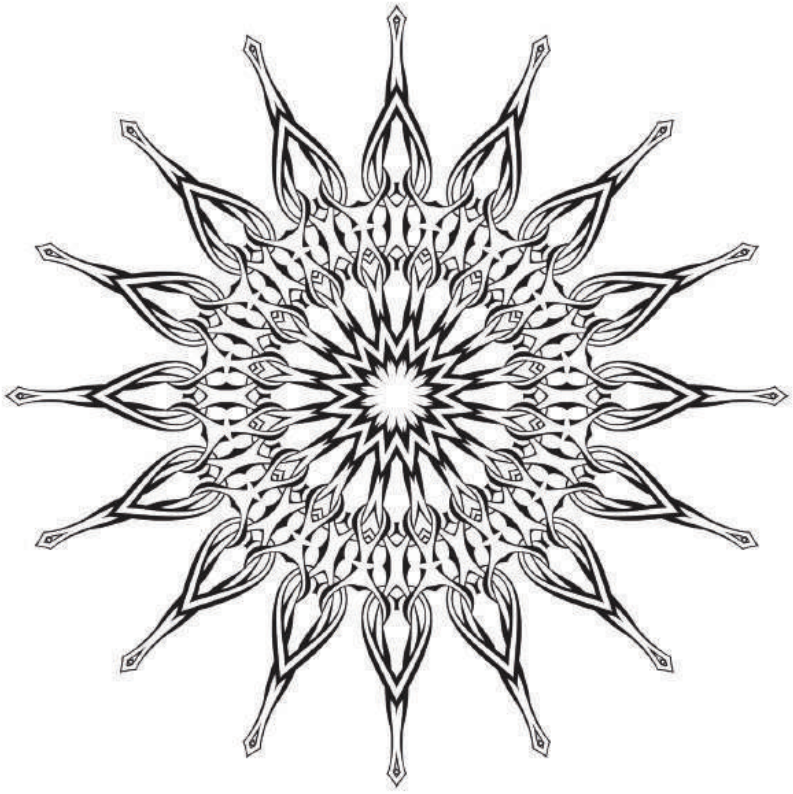


Properties of Two-dimensional Shapes

Student Workbook

Core Knowledge Mathematics™

Lesson 1: Ways to Look at Figures

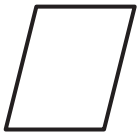
- Let's sort two-dimensional figures.

Warm-up: Which One Doesn't Belong: Geometric Figures

Which one doesn't belong?

A

B



C

D

1.1: Card Sort: Shapes

1. Sort the shapes from your teacher into 3–5 categories. For each category, write a title on a sticky note.

2. Share your categories with another group. Take turns listening to each other's explanations.
    - Do your categories make sense to them?
    - Do their categories make sense to you?
    - Any suggestions or corrections?

3. Cover or hide the titles of your categories. Trade places with another group. Study their sorted cards while they study yours.

   Guess their categories and how they sorted the shapes.

1.2: Guess the Category

Partner A:

- Write down a category from the first activity (or think of a new one). Don't show it to your partner.
- Find 3 shapes that fit the category and 3 shapes that don't. Place them in the columns of the table.

Partner B:

- Study the shapes chosen by your partner.
- Pick another shape from the set. Ask: "Does this shape fit in your category?"
- Find 2 shapes that fit the category and 2 shapes that don't.
- Guess the category. If your guess is off, ask more questions before guessing again.

Switch roles after the category is guessed correctly.

- Partner A's category: _____

fit the category	do not fit the category

- Partner B's category: _____

fit the category	do not fit the category

Lesson 2: Ways to Look at Triangles

- Let's sort and analyze triangles.

Warm-up: Number Talk: Sums and Products

Find the value of each expression mentally.

- $12 + 12 + 75$

- $12\frac{1}{2} + 12\frac{1}{2} + 75$

- $(2 \times 12\frac{1}{2}) + (4 \times 12\frac{1}{2})$

- $7 \times 12\frac{1}{2}$

2.1: Triangle Hunt

1. From the set of triangle cards, find all the triangles that have each attribute. Record their letter names here.

a. No right angles	b. Parallel sides	c. Perpendicular sides
d. The same length for all sides	e. The same size for all angles	f. More than one right angle
g. More than one obtuse angle	h. More than one acute angle	i. Foldable into two equal halves

2. Choose one sentence to complete based on your work.

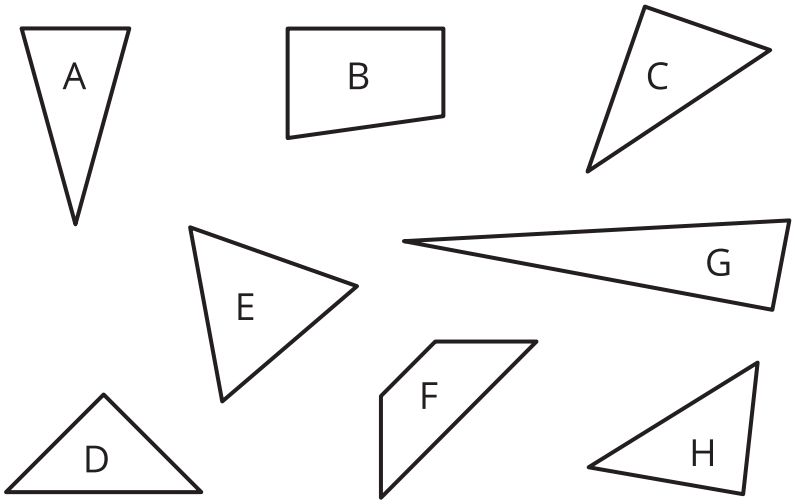
   a. I noticed that some triangles . . .

   b. I noticed that all triangles . . .

   c. I noticed that no triangles . . .

2.2: The Right Kind of Triangle

1. Identify all shapes that are right triangles. For each right triangle, mark the right angle with a small square.



2. Explain why the other shapes are not right triangles.

Lesson 3: Ways to Look at Quadrilaterals

- Let's sort and identify quadrilaterals.

Warm-up: How Many Do You See: Brick Pattern

How many bricks have 2 pairs of parallel sides?

3.1: Quadrilateral Hunt

1. Find the quadrilaterals that have each of the following attributes. Record their letter names here.

attribute	quadrilaterals with the attribute
a. no right angles	
b. one pair of parallel sides	
c. one pair of perpendicular sides	
d. same length for all sides	
e. same size for all angles	
f. same length for only two sides	
g. no parallel sides	
h. two obtuse angles	

2. Choose one sentence to complete based on your work.

   a. I noticed some quadrilaterals . . .

   b. I noticed that all quadrilaterals . . .

   c. I noticed that no quadrilaterals . . .

If you have time: Do you think it is possible for a quadrilateral to have:

- More than 2 acute angles?

- More than 2 obtuse angles?

- Exactly 3 right angles?

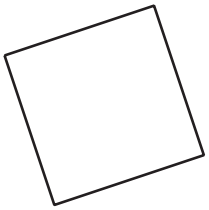
If you think so, sketch an example. If you don't think so, explain or show why you think it is impossible.

3.2: What's True about These Quadrilaterals?

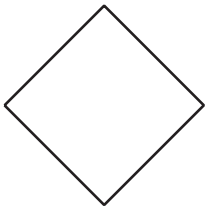
Here are four sets of quadrilaterals.

Quadrilaterals D and AA are squares.

D

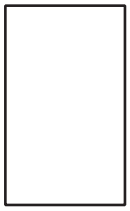


AA

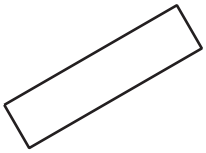


Quadrilaterals K, Z, and AA are rectangles.

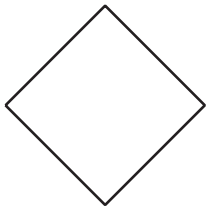
K



Z



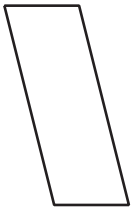
AA



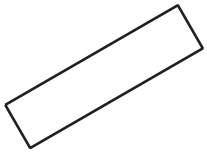
Quadrilaterals N, U, and Z are parallelograms.

N

U

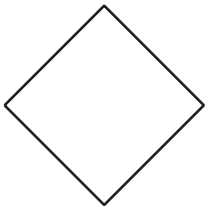


Z

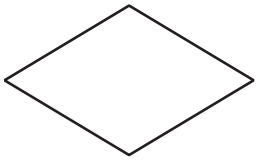


Quadrilaterals AA, EE, and JJ are rhombuses.

AA



EE



JJ

Write 4–5 statements about the sides and angles of the quadrilaterals in each set. Each statement must be true for all the shapes in the set.

square	rectangle
rhombus	parallelogram

3.3: Guess Again

Partner A:

- Write down an attribute that a quadrilateral could have. Don't show it to your partner.
- Find 3 quadrilaterals that have that attribute and 3 that don't. Place them in the columns of the table.

Partner B:

- Study the quadrilaterals chosen by your partner.
- Pick another quadrilateral from the set. Ask: "Does this quadrilateral have the attribute?"
- Find at least 1 quadrilateral that has the attribute and 1 that doesn't.
- Guess the attribute. If your guess is off, ask more questions before guessing again.

Switch roles after the attribute is guessed correctly.

- Partner A's attribute: _____

have the attribute	do not have the attribute

- Partner B's attribute: _____

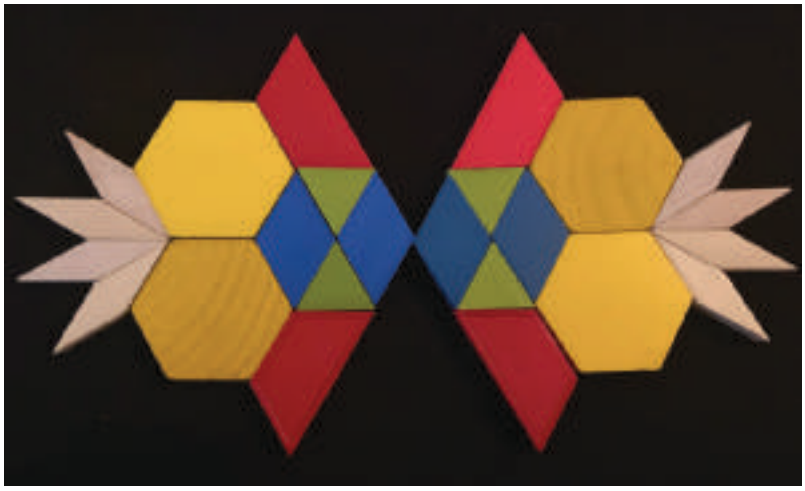
have the attribute	do not have the attribute

Lesson 4: Symmetry in Figures (Part 1)

- Let's describe symmetry in two-dimensional figures.

Warm-up: Notice and Wonder: Seeing Double

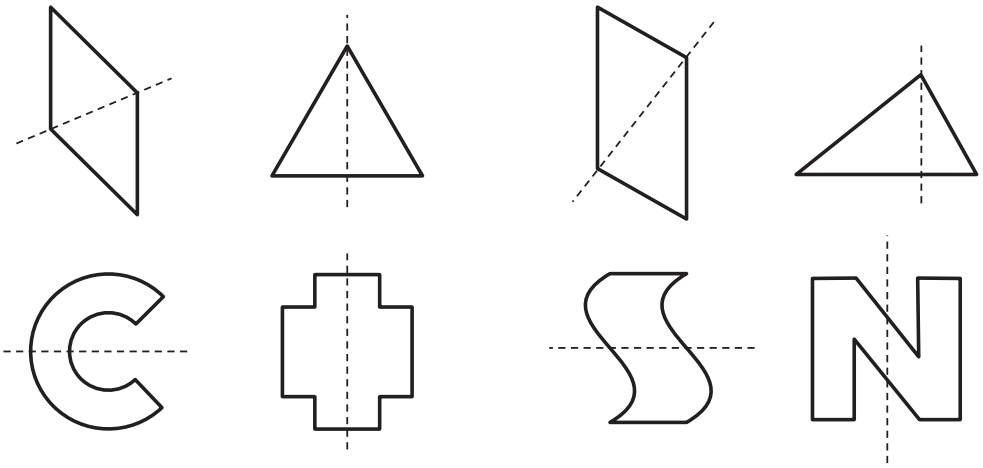
What do you notice? What do you wonder?



4.1: Perfect Matches

1. Lin has pieces of paper in different shapes. She folds each piece of paper once, creating two smaller parts.

   She then sorts the pieces into two categories based on the folding lines.



folding line is a line of symmetry

folding line is not a line of symmetry

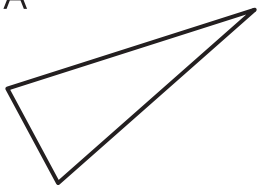
Study the figures in each category. What do you think a **line of symmetry** means?

Complete this sentence:

A line of symmetry is . . .

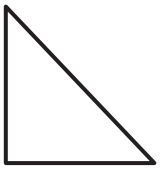
2. Do the following figures have a line of symmetry? If so, draw the line. If not, explain how you know.

A

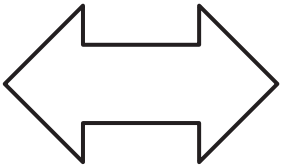


B

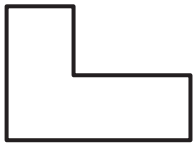
C



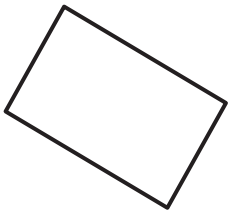
D



E



F



3. Are there any figures with more than one line of symmetry? If you think so, draw all the lines of symmetry.

4.2: In Search of Symmetry

Your teacher will give your group a set of cards.

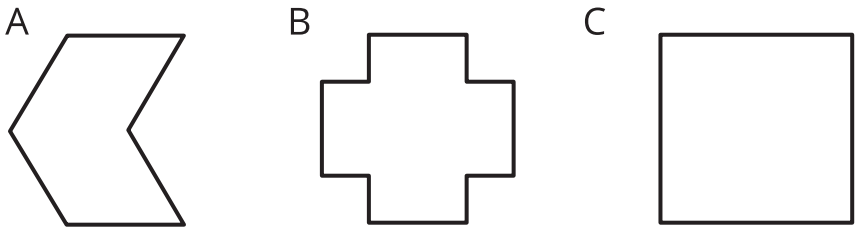
1. Sort the figures on the cards by the number of lines of symmetry they have.

0 lines of symmetry	1 line of symmetry	2 lines of symmetry	3 lines of symmetry

2. Find another group that has the same set of cards. Compare how you sorted the figures. Did you agree with how their figures are sorted? If not, discuss any disagreement.

4.3: Just Keep Folding

Priya is folding paper of different shapes along their lines of symmetry. She keeps folding each one until the folded shape has no more lines of symmetry.



1. How many times can she fold each shape before she can no longer continue?

2. What do you notice about each folded shape when it can no longer be folded?

Lesson 5: Symmetry in Figures (Part 2)

- Let's draw some figures that have lines of symmetry.

Warm-up: Number Talk: Keeping Track

Find the value of each expression mentally.

- $43 + 57 + 50 + 7 + 3 + 40$

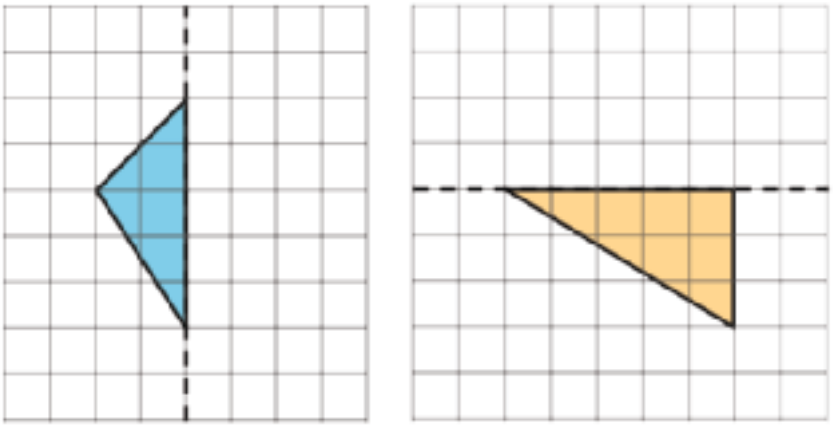
- $243 + 57 + 43 + 257$

- $1{,}043 + 257 + 57 + 200 + 43 + 1{,}000$

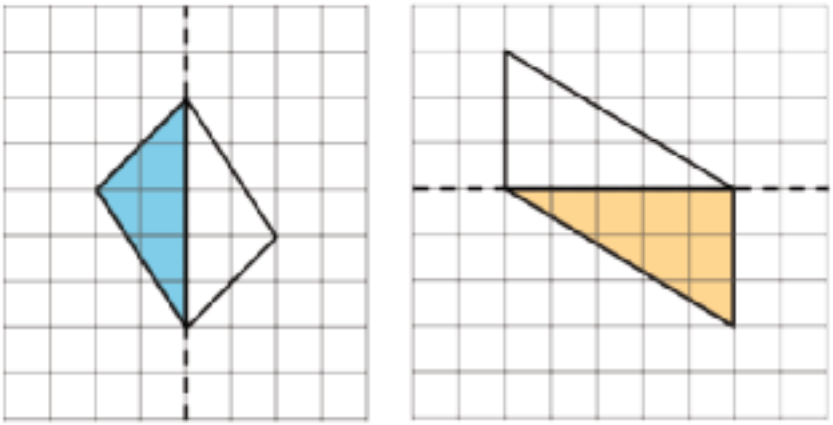
- $1{,}943 + 257 + 1 + 257 + 1{,}000 + 943$

5.1: Half-drawn Figures

Each shaded triangle is half of a whole figure that has a line of symmetry shown by the dashed line.



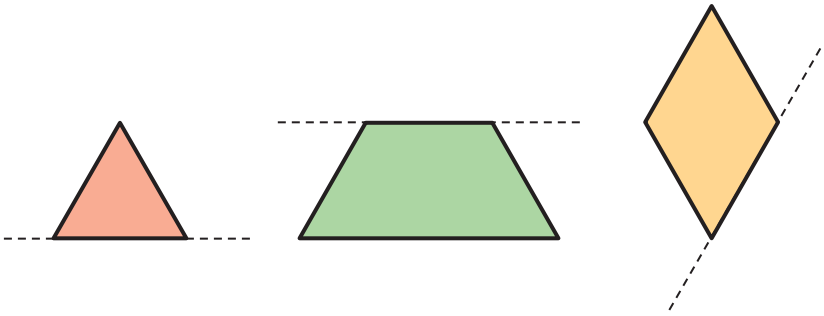
Clare drew in some segments to show the missing half of each figure.



Do you agree that the dashed line is a line of symmetry for each figure Clare completed? Explain your reasoning. If you disagree with Clare's work, show a way to complete the drawing so the dashed line is a line of symmetry.

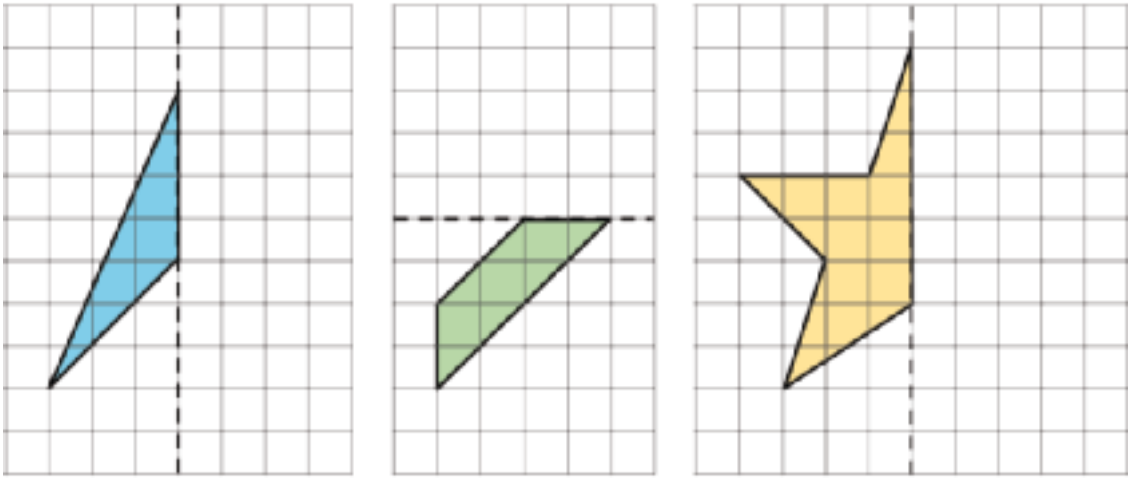
5.2: What's the Whole Picture?

1. Here are three figures. Each figure is half of a whole figure. The dashed line is a line symmetry of that figure.

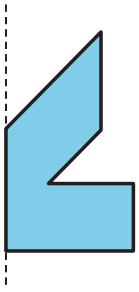


Use patty paper to help you draw the whole figure.

2. Each figure on the grid is half of a whole figure that has a line of symmetry. The dashed line shows the line of symmetry. Use the grid to help you draw the whole figure. Be as precise as possible.



3. Here is another figure that is half of a whole figure with a vertical line of symmetry. Draw the whole figure. Be as precise as possible.



5.3: What Could the Whole Figure Be?

Trace a triangle cutout from your teacher.

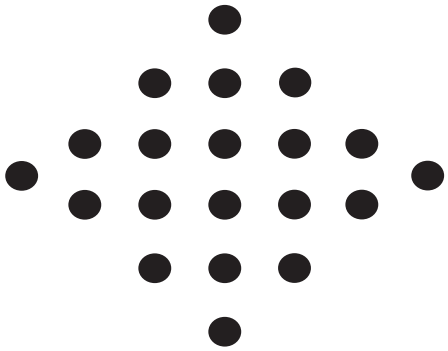
If the triangle is half of a whole figure that has line symmetry, what could the whole figure look like? Can you show two possibilities? Three possibilities?

Lesson 6: All Kinds of Attributes

- Let's use what we know about attributes of figures to create drawings.

Warm-up: How Many Do You See: Dot after Dot

How many do you see? How do you see them?



6.1: You're Gonna Draw It: It's Symmetric

1. Here is a pair of parallel segments that have the same length.

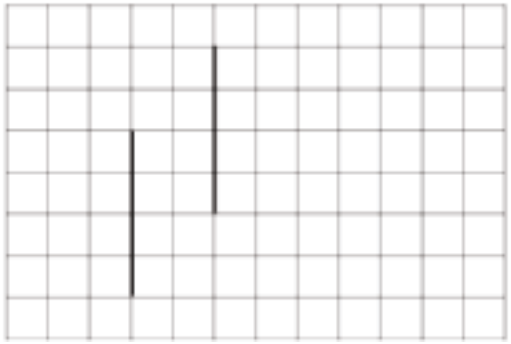
Add one or more segments to create a figure with only 1 line of symmetry.

2. Here are two more pairs of parallel segments. Add more segments to make:

   a. a figure with 2 lines of symmetry

   b. a figure with no lines of symmetry

If you have time: Here are some other pairs of parallel lines. Add more segments to create a figure with 1 line of symmetry.



6.2: Hidden Shapes

Here is a field of dots.

Can you connect the dots to create each of the following shapes? If so, draw the shapes. If not, be prepared to explain your reasoning.

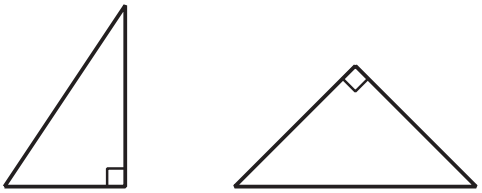
1. A triangle with only one line of symmetry
2. A quadrilateral with only one line of symmetry
3. A quadrilateral with two pairs of parallel sides
4. A quadrilateral with one pair of perpendicular sides
5. A rectangle
6. A six-sided shape with only one line of symmetry

Section Summary

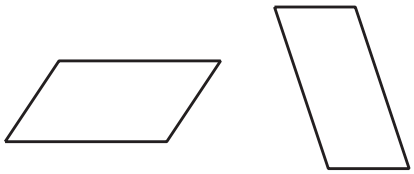
Section Summary
In this section, we looked at different attributes of shapes, such as the number and length of sides, the measurements of sides and angles, and whether the shapes had parallel and perpendicular sides.

We then used these attributes to classify quadrilaterals and triangles.

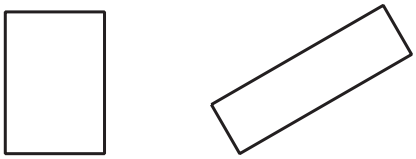
Triangles with a right angle are **right triangles**.



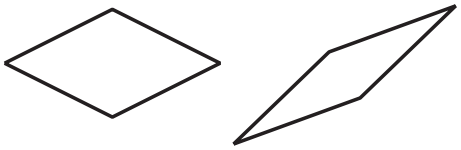
Quadrilaterals with two pairs of parallel sides are **parallelograms**.



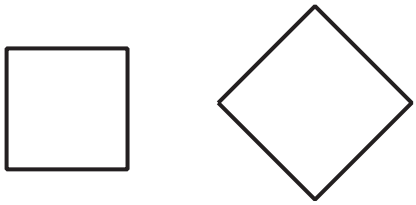
Quadrilaterals with two pairs of parallel sides and four right angles are **rectangles**.



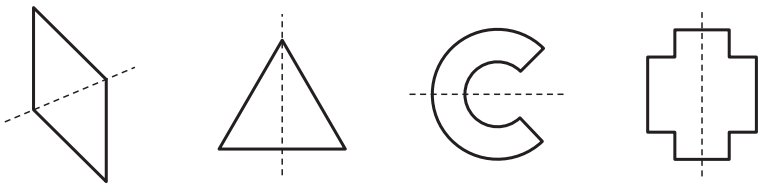
Quadrilaterals with four equal sides are **rhombuses**.



Quadrilaterals with four equal sides and four right angles are **squares**.



We also learned about **lines of symmetry**. A figure that has a line of symmetry can be folded along that line to create two halves that match up exactly.



Lesson 7: Ways to Find Unknown Length (Part 1)

- Let's find the perimeter of different shapes.

Warm-up: Number Talk: Multiple Thirds

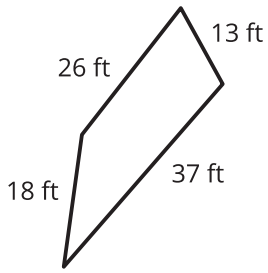
Find the value of each expression mentally.

- $6 \times \frac{1}{3}$
- $30 \times \frac{1}{3}$
- $60 \times \frac{2}{3}$
- $90 \times \frac{2}{3}$

7.1: All the Way Around

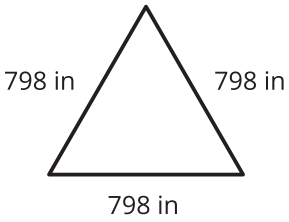
1. Find the perimeter of each shape. Write an expression that shows how you find the perimeter.

a.

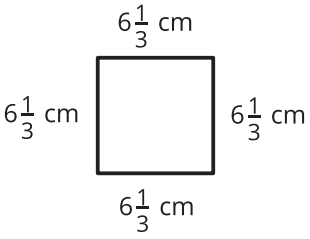


b.

c.



d.

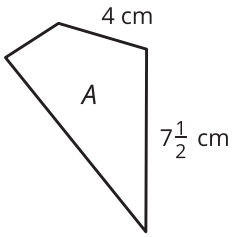


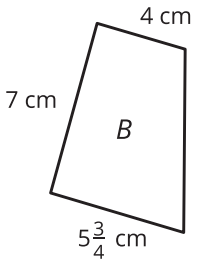
2. Compare your expressions with your partners' expressions. Make 1–2 observations.

7.2: Ponder Perimeter

Here are four shapes and what we know about them:

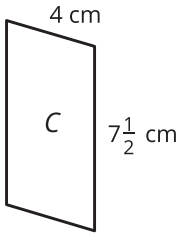
- A, B, and C have no lines of symmetry.
- A has no parallel sides.
- B has 1 pair of parallel sides.
- C has 2 pairs of parallel sides.
- D has 1 pair of parallel sides and 1 line of symmetry.

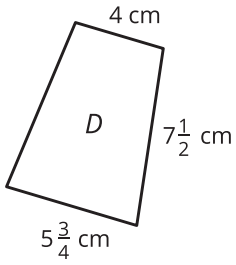




Mai says, "We can't find the perimeter of any quadrilateral because each one is missing one or more side lengths."

Andre disagrees. He says, "We can find the perimeters for C and D but not for A and B."





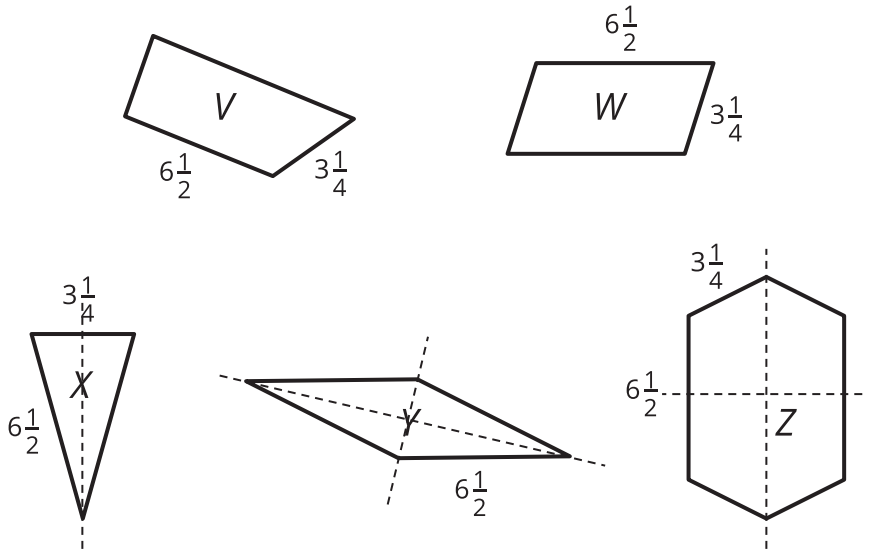
1. Do you agree with either one of them? Explain or show your reasoning.

2. Find the perimeters that could be found, if any.

7.3: Perimeter Expressions

Here are five shapes and what we know about them.

- Not all the sides are labeled.
- The lines of symmetry are shown.
- Only the triangle has no parallel sides.



1. For which shapes is it possible to find the perimeter? For which shapes is it not possible? Be prepared to explain how you know.

2. Here are four expressions. Each one represents the perimeter of one shape. The $6\frac{1}{2}$ and $3\frac{1}{4}$ in each represent side lengths. Can you tell which expression represents which shape?

   a. $(2 \times 6\frac{1}{2}) + 3\frac{1}{4}$

   b. $4 \times 6\frac{1}{2}$

   c. $(2 \times 6\frac{1}{2}) + (4 \times 3\frac{1}{4})$

   d. $(2 \times 6\frac{1}{2}) + (2 \times 3\frac{1}{4})$

Lesson 8: Ways to Find Unknown Length (Part 2)

- Let's find the unknown lengths in figures.

Warm-up: True or False: Equations with Fractions

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $1\frac{1}{5} + 2\frac{2}{5} + 3\frac{3}{5} + 4\frac{4}{5} = 12$

- $10 - \frac{1}{2} - \frac{2}{2} - \frac{3}{2} - \frac{4}{2} = 5$

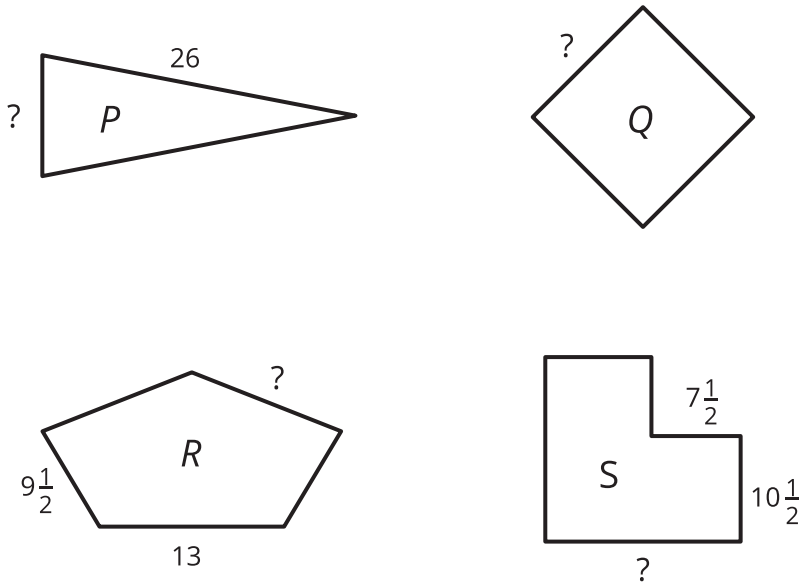
- $1\frac{1}{6} + 2\frac{2}{6} + 3\frac{3}{6} + 4\frac{4}{6} + 5\frac{5}{6} = 15\frac{3}{6}$

- $\frac{1}{3} + \frac{2}{3} + \frac{3}{3} = 3 \times \frac{2}{3}$

8.1: Unknown Lengths

Here are four shapes.

- Each shape has a perimeter of 64 inches.
- P, R, and S each have 1 line of symmetry.
- Q has 4 lines of symmetry.



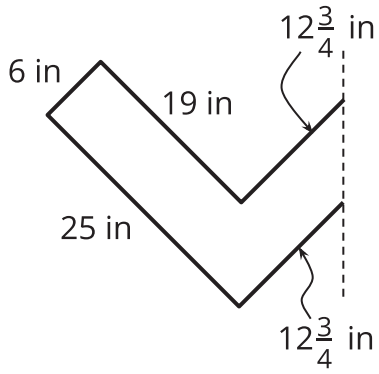
1. Draw the lines of symmetry of each shape.

2. Find the unknown side length in each shape. Show your reasoning.

8.2: Lin's Design

Lin is using 145 inches of fancy tape for the outline of a design with line symmetry.

Here is half of the design. The dashed line is the line of symmetry.



1. Sketch Lin's entire design.

2. Does she have enough tape for the entire outline? Show your reasoning.

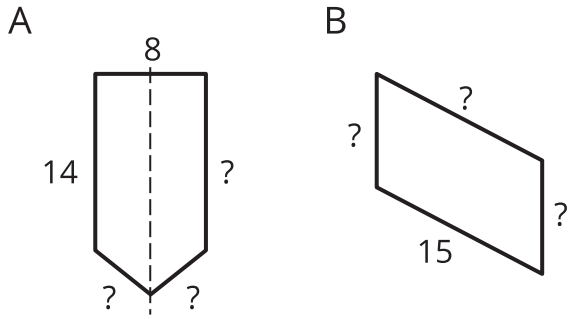
If you have time: Lin has a sheet of fancy paper that she can cut up to cover the inside of the design. The paper is a rectangle that is 30 inches by 18 inches. If the angles in the design are right angles, does Lin have enough paper to cover the inside of the design? Show your reasoning.

Section Summary

Section Summary
In this section, we used the attributes such as side lengths, angles, lines of symmetry, and parallel sides to solve problems about perimeter of shapes.

We learned that, if a shape has certain attributes, we can use them to find its perimeter, even if we don't have all of its side lengths. Or, if we know the perimeter of a shape, we can find its side lengths if there is enough information about their attributes.

For example, here are two shapes:



If we know the perimeter of each shape is 48 units and the dashed line through shape A is a line of symmetry, we can find the missing side lengths.

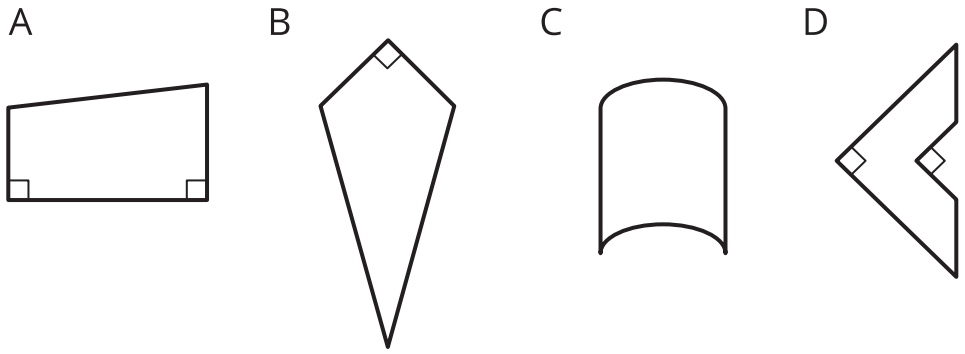
Shape B doesn't have a line of symmetry, but if we were told that its opposite sides have equal lengths, then we can also reason about the three missing side lengths.

Lesson 9: Symmetry in Action

- Let's investigate symmetry and perimeter in folded figures.

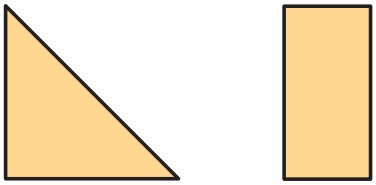
Warm-up: Which One Doesn't Belong: Figures

Which one doesn't belong?



9.1: Before and After

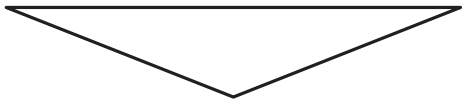
1. Mai has a piece of paper. She can get two different shapes by folding the paper along a line of symmetry. What is the shape of the paper before it was folded?



2. Diego folded a piece of paper once along a line of symmetry and got this right triangle.

Which shapes could the paper have before it was folded? Explain or show how you know.

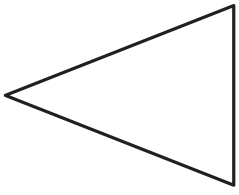
A



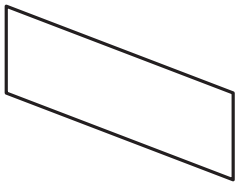
B

C

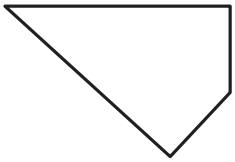
D



E

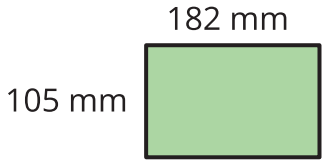


F



9.2: Before and After, Perimeter Edition

1. Jada folded a piece of paper along a line of symmetry and got this rectangle.



a. What could the paper look like before being folded? Draw one or more sketches.

b. Write an expression for the perimeter of the unfolded paper.

2. Kiran folded a piece of paper twice—each time along a line of symmetry—and got the same rectangle as Jada did.

   Show that each expression could represent the perimeter of the paper Kiran folded.

   a. $(4 \times 182) + (4 \times 105)$

   b. $(2 \times 182) + (8 \times 105)$

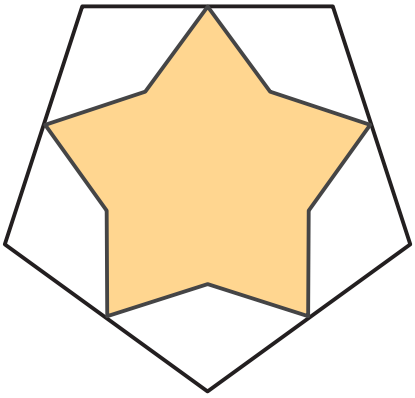
   c. $(8 \times 182) + (2 \times 105)$

Lesson 10: Ways to Find Angle Measurements

- Let's find angle measurements in figures with line symmetry.

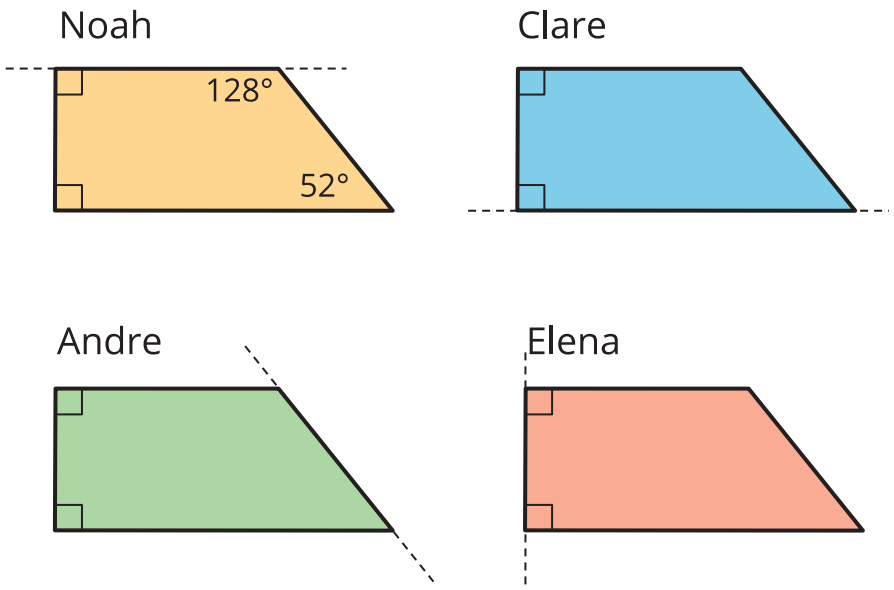
Warm-up: How Many Do You See: Symmetry in a Star

How many lines of symmetry do you see? How do you see them?



10.1: Before and After, Angle Edition

Noah, Clare, Andre, and Elena each have a sheet of paper with one line of symmetry. When they folded their paper along the line of symmetry, they all produced the same shape. The dashed line represents the folding line.

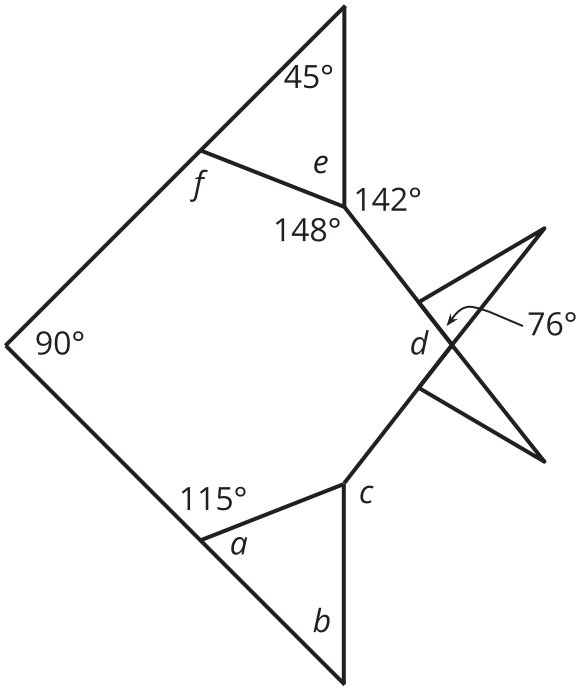


1. Draw the shape of the unfolded paper that each student received. Be as precise as possible.

2. Without measuring, find the measurement of all angles within the shape (of the unfolded paper) that you drew.

10.2: Angular Fish

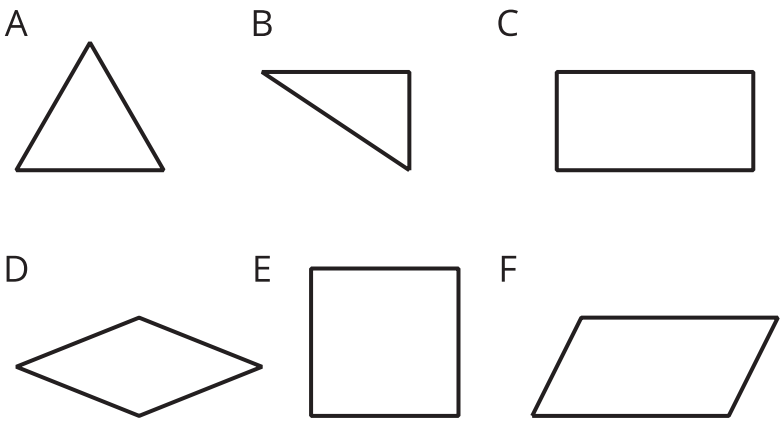
Here is a diagram of an origami fish, which has one line of symmetry.



1. Draw the line of symmetry.
2. Without measuring, find the measurement of angles labeled a–f. Be prepared to explain your reasoning.

Section A: Practice Problems

1. **Pre-unit**



a. Which shapes are quadrilaterals?

b. Which shapes are rhombuses?

c. Which shapes are rectangles?

2. **Pre-unit**

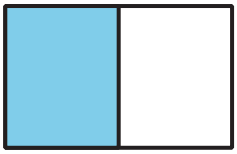
Find the perimeter and area of the rectangle. Explain or show your reasoning.

3. **Pre-unit**

Select **all** images that show half of the shaded rectangle.

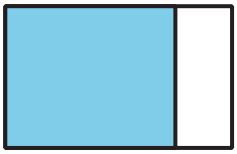
A.

B.

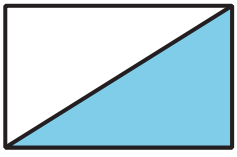


C.

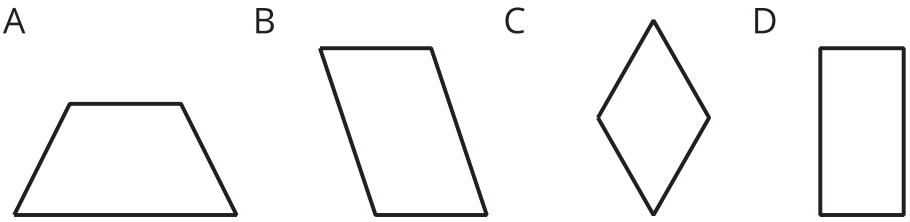
D.



E.



4.

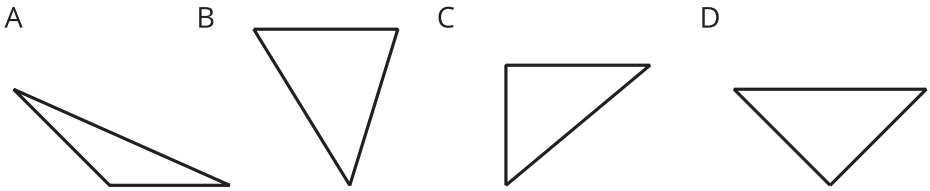


a. Name some attributes that these shapes share.

b. Name some attributes that the shapes do not share.

(From Unit 8, Lesson 1.)

5.



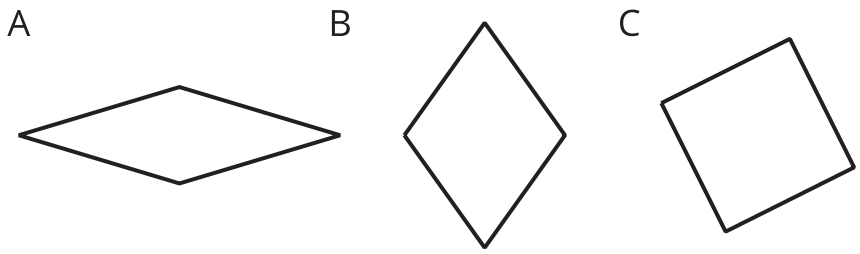
a. Which of the triangles are right triangles?

b. Which of the triangles have an obtuse angle?

c. Which of the triangles have 3 acute angles?

(From Unit 8, Lesson 2.)

6. Here are 3 rhombuses:



a. What attributes do the rhombuses share?

b. What attributes are different in the three rhombuses?

(From Unit 8, Lesson 3.)

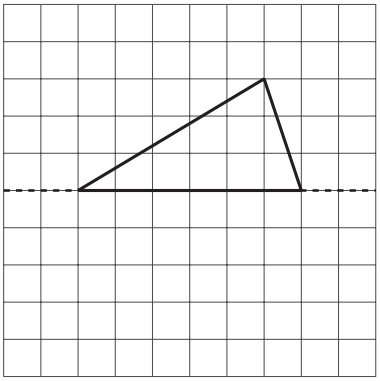
7. Draw the lines of symmetry for these letters:

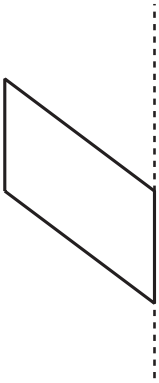
A C G

N O Z

(From Unit 8, Lesson 4.)

8. Complete each figure so that the dashed line is a line of symmetry for the new figure.





(From Unit 8, Lesson 5.)

9. **Exploration**

Draw all the lines of symmetry you can find in this snowflake. How many can you find?

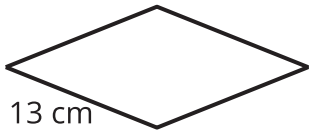
10. **Exploration**

Draw each shape and all the lines of symmetry you can find in it.

- rectangle
- rhombus
- square

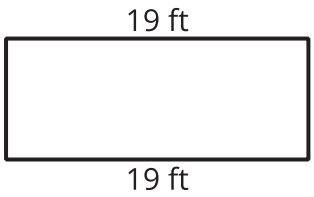
Section B: Practice Problems

1. a. What is the perimeter of the rhombus? Explain or show your reasoning.



b. Diego says he can find the area of this rectangle because he knows two side lengths.

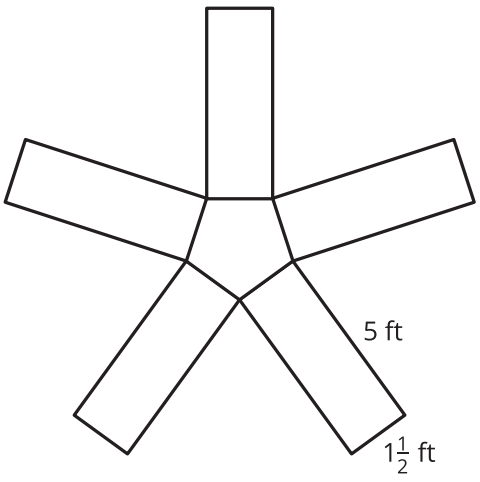
Do you agree with Diego? Explain your reasoning.



(From Unit 8, Lesson 7.)

2. a. Draw the lines of symmetry for the windmill blades.

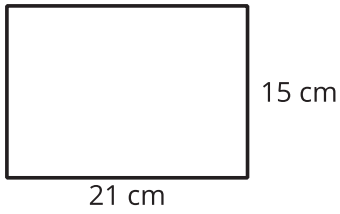
b. Each blade is 5 feet long and $1\frac{1}{2}$ feet wide. What is the perimeter of the windmill? Explain or show your reasoning.



(From Unit 8, Lesson 8.)

3. Here is a rectangle R.

   a. What shape can be folded along a line of symmetry to give R? What are the side lengths of that shape?

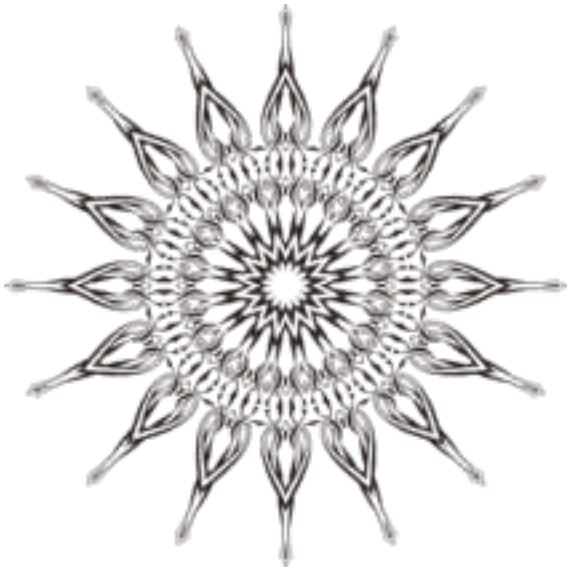


   b. What shape can be folded twice along lines of symmetry to give R? What are its side lengths?

(From Unit 8, Lesson 9.)

4. **Exploration**

   How many lines of symmetry are there in this design? Explain or show how you know.



5. **Exploration**

   Make a shape or design with one or more lines of symmetry. Trade shapes with a partner and find all of the lines of symmetry of your partner's shape. You may find pattern blocks helpful to make your shape or design.

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