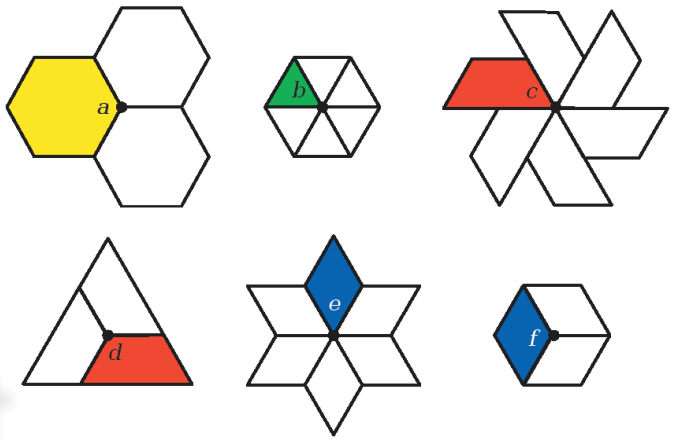
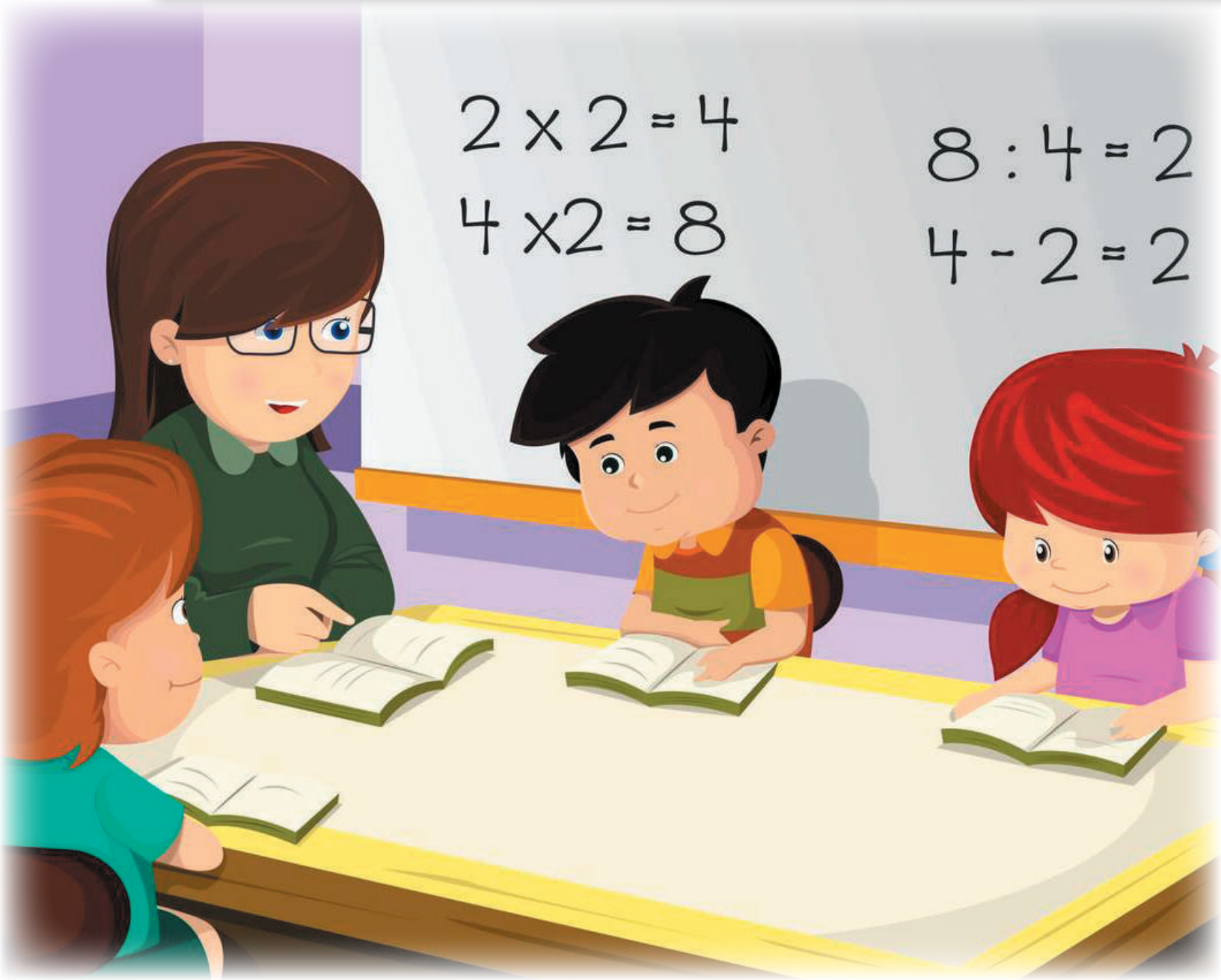
Core Knowledge® MATHEMATICS

Angles and Angle Measurement



Teacher Guide



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Angles and Angle Measurement

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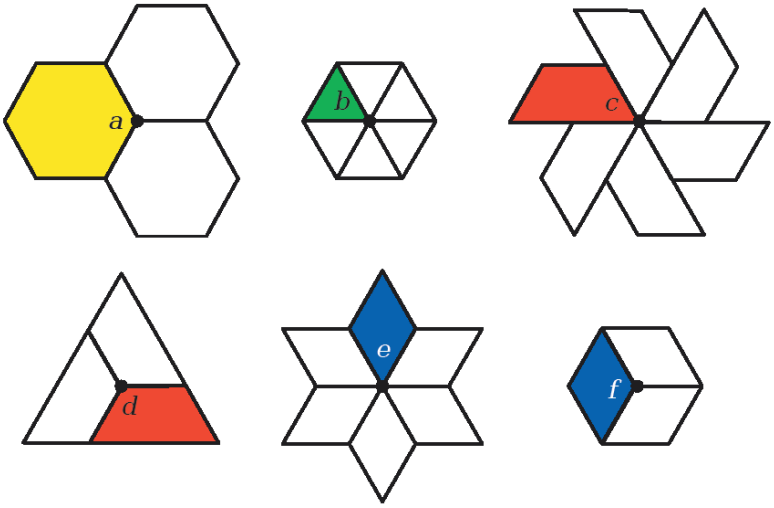


Angles and Angle Measurement

Teacher Guide

Core Knowledge Mathematics™

Unit 7: Angles and Angle Measurement

At a Glance

Unit 7 is estimated to be completed in 17-18 days including 2 days for assessment.

This unit is divided into three sections including 15 lessons and 1 optional lesson.

- Section A—Points, Lines, Segments, Rays, and Angles (Lessons 1-5)
- Section B—The Size of Angles (Lessons 6-11)
- Section C—Angle Analysis (Lessons 12-16)

On pages 8-9 of this Teacher Guide is a chart that identifies the section each lesson belongs in and the materials needed for each lesson.

This unit uses five new student centers.

- Rolling for Fractions
- Compare
- Target Measurements
- Which One?
- Can You Draw It?

Unit 7: Angles and Angle Measurement

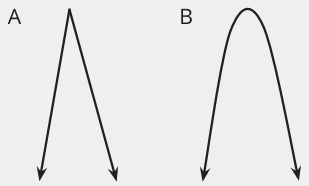
Unit Learning Goals

- Students learn to draw and identify points, rays, segments, angles, and lines, including parallel and perpendicular lines. Students also learn how to use a protractor to measure angles and draw angles of given measurements, and identify acute, obtuse, right, and straight angles in two-dimensional figures.

In this unit, students deepen and refine students' understanding of geometric figures and measurement.

In earlier grades, students learned about two-dimensional shapes and their attributes, which they described informally early on but with increasing precision over time. Here, students formalize their intuitive knowledge about geometric features and draw them. They identify and define some building blocks of geometry (points, lines, rays, and line segments), and develop concepts and language to more precisely describe and reason about other geometric figures.

Jada says figure A shows an angle, but figure B does not. Do you agree?



Students analyze cases where lines intersect and where they don't, as in the case of parallel lines. They learn that an angle as a figure composed of two rays that share an endpoint.

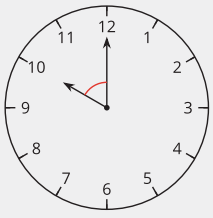
Later, students compare the size of angles and consider ways to quantify it. They learn that angles can be measured in terms of the amount of turn one ray makes relative to another ray that shares the same vertex.

Students come to see that a 1-degree angle is $\frac{1}{360}$ of a full turn or full circle and can be used to measure angles. They use a protractor to measure angles in whole-number degrees.

Students also learn that angles are additive. When an angle is composed of multiple non-overlapping parts, the measure of the whole is the sum of the angle measures of the parts. These insights enable students to classify angles (as acute, obtuse, right, or straight) and to solve problems about missing angle measurements in concrete and abstract contexts.

How many degrees is each marked angle on the clock? Show your reasoning.

A



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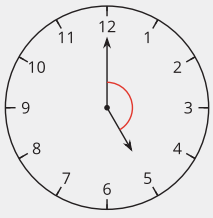


Section A: Points, Lines, Segments, Rays, and Angles

Standards Alignments

Addressing	4.G.A.1, 4.MD.C.5, 4.NBT.B.4, 4.NBT.B.5
Building Towards	4.G.A.2, 4.MD.C.5

Section Learning Goals

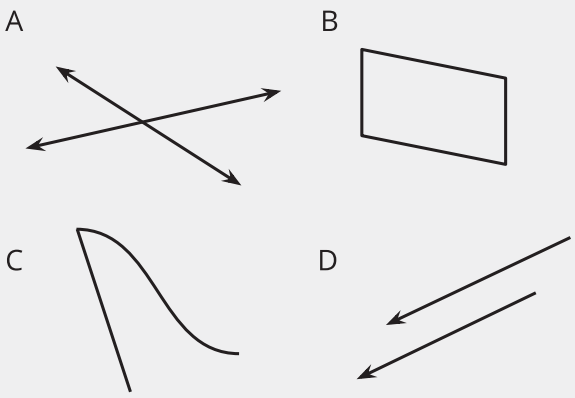
- Draw and identify points, lines, rays, segments, and parallel and intersecting lines in geometric figures.
- Recognize that angles are formed wherever two rays share a common endpoint and identify angles in two-dimensional figures.

This section introduces students to some building blocks of geometric figures and the language to describe them. Students start by describing images that contain lines for others to draw and drawing images relying only on others' descriptions. The experience motivates a need for more precise vocabulary to describe geometric parts. They learn to distinguish points as locations in space, rays as lines that are bounded by one point, and line segments as lines that are bounded by two points.

Students are familiar with lines that cross or intersect. Here, they identify and then draw parallel lines, lines that never intersect.

Students also learn that an angle is a figure that is made up of two rays that share the same endpoint, called the vertex of the angle. They then practice identifying angles, noticing that angles are ubiquitous around us and can have different sizes.

Decide if each figure shows at least one angle.



PLC: Lesson 3, Activity 2, To Cross or Not to Cross

Suggested Centers

- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Supporting)
- Compare (1–5), Stage 7: Multi-digit Operations (Supporting)
- Target Measurements (2–5), Stage 4: Degrees (Addressing)
- Compare (1–5), Stage 5: Fractions (Supporting)

Section B: The Size of Angles

Standards Alignments

Building On	4.NBT.A.1
Addressing	4.G.A.1, 4.MD.C.5, 4.MD.C.5.a, 4.MD.C.5.b, 4.MD.C.6, 4.MD.C.7, 4.NBT.B.5, 4.NBT.B.6
Building Towards	4.MD.C.5, 4.MD.C.5.a, 4.MD.C.6, 4.MD.C.7

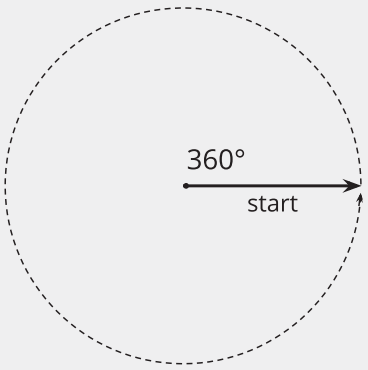
Section Learning Goals

- Recognize that angles can be measured in degrees, and can be found using addition and subtraction.
- Use a protractor to measure and draw angles, and recognize that perpendicular lines meet or cross at a right angle.

In this section, students learn two main ideas: that angles can be measured, with degrees (°) as the unit of measurement, and that angles can be composed and decomposed, and are therefore additive. They also learn to use a protractor to measure and draw angles.

Students begin by comparing angles visually and exploring ways to describe their size. They then try to describe angles made by the hour and minute hands of an analog clock, using the numbers and tick marks on the clock or units of time to quantify the size of angles. This experience reinforces the idea of an angle as a figure formed when a ray rotates around a vertex shared with another ray. It also motivates the need for a more precise unit for measuring angles.

Students learn that a ray that makes a full turn around a point makes a 360° angle. Decomposing this angle into halves gives a 180° angle. Half of that angle is a 90° angle or a right angle. Composing three 90° angles gives a 270° angle.

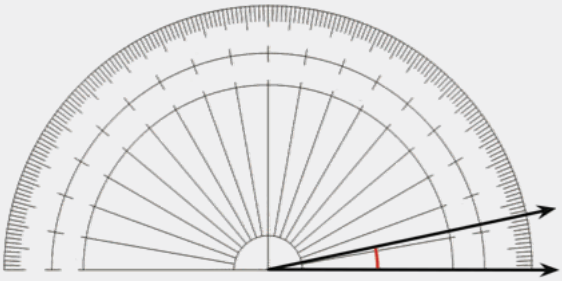


Students then use these benchmark angles to estimate and measure the sizes of other angles. For example, decomposing a right angle into halves gives 45° angles. Composing three copies of a 45° angle makes a 135° angle, and so on.

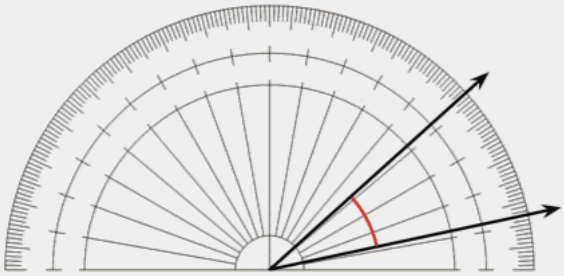
Students also learn that 90° angles are formed by perpendicular lines.

Later, students make sense of a 1° angle and see that it is $\frac{1}{360}$ of a full turn. They use a protractor and 1° as a unit for measuring and drawing angles of all sizes.

How many degrees is this angle? Explain how you know.



An angle contains thirty 1° angles, as shown. How many degrees is this angle?



Throughout the section, students build their understanding of angles of different sizes using tactile tools such as paper cutouts and patty paper, and by folding, cutting, marking, and assembling pieces of paper.

Suggested Centers

- Compare (1–5), Stage 5: Fractions (Supporting)
- Target Measurements (2–5), Stage 4: Degrees (Addressing)

Section C: Angle Analysis

Standards Alignments

Building On	4.G.A.1, 4.NF.B.4.b
Addressing	4.G.A.1, 4.G.A.2, 4.MD.C.6, 4.MD.C.7
Building Towards	4.G.A.1

Section Learning Goals

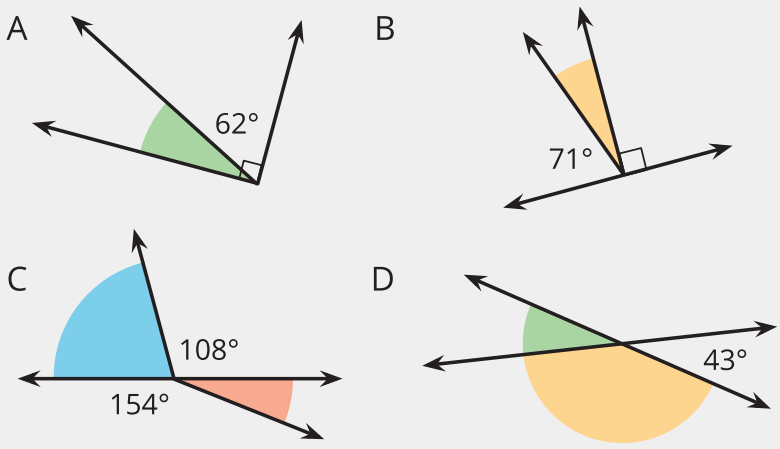
- Draw and identify acute, obtuse, right, and straight angles in two-dimensional figures.
- Write equations to represent angle relationships and reason about and find unknown measurements.

In this section, students continue to draw and analyze angles and reason about their measurements.

They first classify angles by their size and identify acute, obtuse, and straight angles. Then, they further develop the idea that angle is additive by composing and decomposing angles, using tactile tools and drawings, and writing expressions or equations to support their reasoning.

Students solve problems about angles in different contexts, both concrete and abstract. They use their understanding of right angle and straight angle to reason about unknown angle measurements.

Find the measurement of each shaded angle. Show how you know.



PLC: Lesson 14, Activity 2, Tick Tock

Suggested Centers

- Which One? (K–5), Stage 4: Grade 3 Shapes (Supporting)
- Can You Draw It? (1–5), Stage 4: Area and Perimeter (Supporting)
- Target Measurements (2–5), Stage 4: Degrees (Addressing)
- Compare (1–5), Stage 5: Fractions (Supporting)

Throughout the Unit

The Number Talk routines in this unit offer opportunities for students to look for structure to mentally perform subtraction, division, and multiplication. The numbers are chosen to reinforce students' familiarity with some benchmark angles and with factors of 180 and 360, which support their work as they reason about angle measurements.

Here is a sampling of Number Talk warm-ups in this unit.

lesson 2	lesson 10	lesson 12
$90 - 45$	$180 \div 2$	$12 \times \frac{1}{12}$
$270 - 45$	$180 \div 4$	$120 \times \frac{1}{12}$
$270 - 135$	$360 \div 8$	$360 \times \frac{1}{12}$
$360 - 135$	$360 \div 16$	$360 \times \frac{3}{12}$

Materials Needed

LESSON	GATHER	COPY
A.1	• Chart paper • Index cards • Rulers or straightedges	• Do You See What I See? (groups of 4)
A.2	• Rulers or straightedges	• Card Sort: Who Am I? (groups of 2)
A.3	• Rulers or straightedges	• Illustrated Word Wall (groups of 1)
A.4	• Materials from a previous lesson • Rulers or straightedges	• none
A.5	• Rulers or straightedges	• Tricky Figures (groups of 8)
B.6	• Materials from a previous activity • Patty paper	• Card Sort: Angles (groups of 2)
B.7	• Patty paper • Rulers or straightedges	• none
B.8	• Paper • Rulers or straightedges	• Making a Measuring Tool (groups of 3)
B.9	• Protractors	• none
B.10	• Colored pencils • Paper • Protractors • Rulers or straightedges	• none
B.11	• Index cards • Protractors • Rulers or straightedges	• none

C.12	• Materials from a previous lesson • Pattern blocks • Protractors	• none
C.13	• Origami paper • Patty paper	• How Big Are These Angles? (groups of 2)
C.14	• Protractors • Rulers or straightedges	• none
C.15	• none	• Info Gap: Whole Bunch of Angles (groups of 2)
C.16	• Rulers or straightedges	• Make a Change (groups of 2)

Center: Rolling for Fractions (3–5)

Stage 2: Multiply a Fraction by a Whole Number

Lessons

- Grade4.7.A1 (supporting)
- Grade4.7.A2 (supporting)
- Grade4.7.A3 (supporting)
- Grade4.7.A4 (supporting)

Stage Narrative

Students roll 3 number cubes to generate a multiplication expression with a whole number and a fraction and compare the value of the expression to 1 in order to determine how many points are earned. Two recording sheets are provided, one where the fraction is a unit fraction and one where it can be any fraction.

Variation:

Students may choose a different target number to compare the value of their expression to.

Stage Description

Each group of 2 needs 3 number cubes.

Standards Alignments

Addressing 4.NF.B.4

Materials to Gather

Number cubes

Materials to Copy

Rolling for Fractions Stage 2 Recording Sheet (groups of 1)

Stages used in Grade 3

Stage 1

Addressing

- Grade3.5.C
- Grade3.5.D

Center: Compare (1–5)

Stage 5: Fractions

Lessons

- Grade4.7.A5 (supporting)
- Grade4.7.B6 (supporting)
- Grade4.7.B7 (supporting)
- Grade4.7.B8 (supporting)
- Grade4.7.B9 (supporting)
- Grade4.7.B10 (supporting)
- Grade4.7.B11 (supporting)
- Grade4.7.C13 (supporting)

Stage Narrative

Students use cards with fractions. They may use either deck of fraction cards or combine them together to play.

Standards Alignments

Addressing 4.NF.A.2

Materials to Copy

Compare Stage 3-8 Directions (groups of 2), Fraction Cards Grade 3 (groups of 2), Fraction Cards Grade 4 (groups of 2)

Stage 7: Multi-digit Operations

Lessons

- Grade4.7.A1 (supporting)
- Grade4.7.A2 (supporting)
- Grade4.7.A3 (supporting)
- Grade4.7.A4 (supporting)

Stage Narrative

Students use cards with expressions with all 4 operations resulting in numbers over 1,000.

Standards Alignments

Addressing 4.NBT.B.4, 4.NBT.B.5, 4.NBT.B.6

Materials to Copy

Compare Stage 3-8 Directions (groups of 2), Compare Stage 7 Cards (groups of 2)

Stages used in Grade 3

Stage 2

Supporting

- Grade3.4.C

Stage 3

Addressing

- Grade3.4.C

Supporting

- Grade3.6.D

Stage 4

Addressing

- Grade3.4.D

Supporting

- Grade3.7.C
- Grade3.7.D

Center: Target Measurements (2–5)

Stage 4: Degrees

Lessons

- Grade4.7.A5 (addressing)
- Grade4.7.B6 (addressing)
- Grade4.7.B7 (addressing)
- Grade4.7.B8 (addressing)
- Grade4.7.B9 (addressing)
- Grade4.7.B10 (addressing)
- Grade4.7.B11 (addressing)
- Grade4.7.C13 (addressing)

Stage Narrative

Students estimate angle measurement and then use a protractor to find the exact measurement.

Standards Alignments

Addressing 4.G.A.1

Materials to Gather

Protractors, Scissors

Materials to Copy

Target Measurement Stage 4 Homemade Protractor (groups of 2), Target Measurement Stage 4 Recording Sheet (groups of 2)

Stages used in Grade 3

Stage 1

Supporting

- Grade3.6.A

Stage 2

Addressing

- Grade3.6.A
- Grade3.6.B
- Grade3.6.C

Center: Which One? (K–5)

Stage 4: Grade 3 Shapes

Lessons

- Grade4.7.C12 (supporting)
- Grade4.7.C14 (supporting)
- Grade4.7.C15 (supporting)
- Grade4.7.C16 (supporting)

Stage Narrative

Students lay out the shape cards face up in rows. One partner chooses a shape. The other partner asks questions to figure out what shape they chose. Students work with polygons, with a focus on quadrilaterals.

Standards Alignments

Addressing 3.G.A.1

Materials to Copy

Quadrilateral Cards Grade 3 (groups of 2), Shape Cards Grade 3 (groups of 2), Triangle Cards Grade 3 (groups of 2)

Stages used in Grade 3

Stage 3

Supporting

- Grade3.7.A

Stage 4

Addressing

- Grade3.7.A
- Grade3.7.B
- Grade3.7.C

Center: Can You Draw It? (1–5)

Stage 4: Area and Perimeter

Lessons

- Grade4.7.C12 (supporting)
- Grade4.7.C14 (supporting)
- Grade4.7.C15 (supporting)
- Grade4.7.C16 (supporting)

Stage Narrative

Partner A draws a rectangle and tells Partner B either the area or the perimeter of their shape. Partner B tries to draw the rectangle. They earn two points if their rectangle matches the one Partner A drew exactly and one point if it doesn't match exactly, but matches the clue given. This game encourages students to use multiplication fluency to think about rectangles that could be more difficult for their partner to draw.

Standards Alignments

Addressing 3.MD.C, 3.MD.D.8

Materials to Gather

Folders

Materials to Copy

Can You Draw It Stage 4 Recording Sheet (groups of 1)

Stages used in Grade 3

Stage 2

Supporting

- Grade3.4.D
- Grade3.7.A

Stage 3

Addressing

- Grade3.7.B
- Grade3.7.C

Stage 4

Addressing

- Grade3.7.C
- Grade3.7.D

Section A: Points, Lines, Segments, Rays, and Angles

Lesson 1: How Would You Describe These Figures?

Standards Alignments

Addressing 4.G.A.1

Teacher-facing Learning Goals

- Draw points, lines, and line segments, and identify them in geometric figures.

Student-facing Learning Goals

- Let's draw and describe geometric figures.

Lesson Purpose

The purpose of this lesson is to motivate the need for geometric language for describing and drawing images and to introduce points, lines, and segments.

In earlier grades, students examined, described, compared, and contrasted attributes of two-dimensional figures. They may have used geometric terms such as **point**, **line**, and **segment** intuitively and informally.

This lesson serves two goals. The first is to elicit the language students have for talking about geometric figures, motivating a need to develop more precision in using geometric terminology (MP6). The second is to enable students to see that a line segment is a part of a line, so it has a start point and an endpoint. The cool-down for this lesson is designed to determine the types of words students use to describe figures. Students may choose to use informal or formal language.

In the next lesson, students will encounter the formal definitions of these geometric terms, but for now, the aim is simply to engage them in noticing and drawing these geometric figures. It is not necessary to expect students to distinguish between line segments and lines in this lesson.

Access for:

Students with Disabilities

- Action and Expression (Activity 1)

Instructional Routines

MLR2 Collect and Display (Activity 1), Notice and Wonder (Warm-up)

Materials to Gather

- Chart paper: Activity 1
- Index cards: Activity 1
- Rulers or straightedges: Activity 1, Activity 2

Materials to Copy

- Do You See What I See? (groups of 4): Activity 1

Required Preparation

Lesson Timeline

Warm-up	5 min
Activity 1	25 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

This is the first time students have formally encountered geometry in grade 4. What vocabulary did students bring to the lesson? How might you use their informal language to make formal connections in the next lesson?

Cool-down (to be completed at the end of the lesson)

5 min

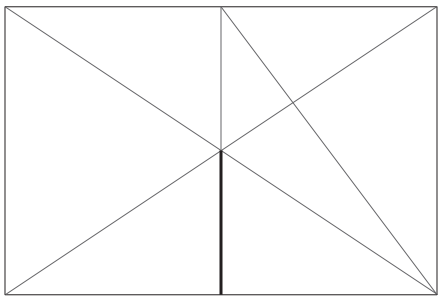
Lines and More

Standards Alignments

Addressing 4.G.A.1

Student-facing Task Statement

Here is a drawing on a card:



Write a description of the drawing that could be used by a classmate to make a copy.

Student Responses

Sample response: Draw two diagonal lines: one from the top left corner to the bottom right, and another from the bottom left corner to the top right. Draw a line that goes up and down through the point where the two diagonal lines cross. From the top of that line, draw a line to the bottom right corner. The bottom segment of the up-and-down line is thicker than the rest of the lines. The lines make a lot of triangles of different sizes.

Begin Lesson

Warm-up

5 min

Notice and Wonder: String Art

Standards Alignments

Addressing 4.G.A.1

This warm-up prompts students to generate formal and informal geometric language (lines, points, straight, curved) that will be used in an upcoming task by familiarizing themselves with a context and the mathematics that might be involved.

When students articulate what they notice and wonder, they have an opportunity to attend to precision in the language they use to describe what they see (MP6). They might first propose less formal or imprecise language, and then restate their observation with more precise language in order to communicate more clearly.

Instructional Routines

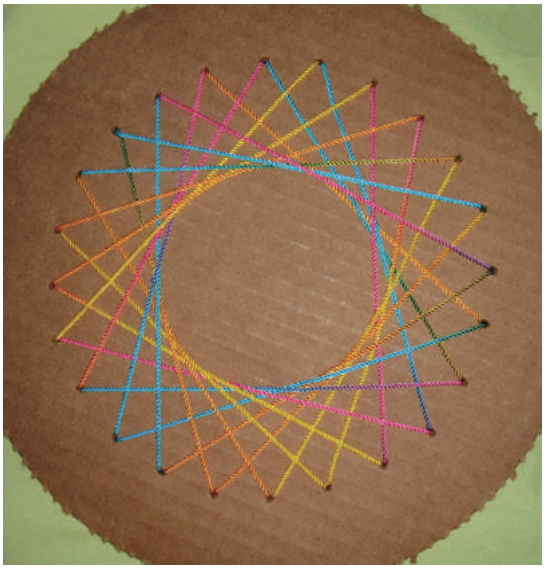
Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?

Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time



Student Responses

Students may notice:

- There are different colored lines.
- All the points form a circle.
- The lines start at one point and end at another point.
- The lines are straight but go in different directions.
- The lines make a circle in the middle.

Students may wonder:

- How are the straight lines making a circle?
- Are any of the lines curved?

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- "Which words or terms came up multiple times in the observations you made and in the questions you had?" (point, line, straight)
- "We'll work with these geometric features in this lesson and in upcoming ones. Let's see what we can find out about them."

Activity 1

25 min

Do You See What I See?

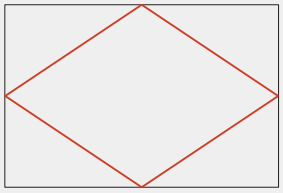
Standards Alignments

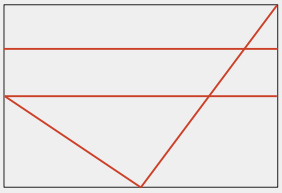
Addressing 4.G.A.1

The purpose of this activity is to motivate a need for more precise geometric language. Students work with a partner to replicate given geometric images—one partner describes the images and the other draws them solely based on the verbal descriptions from their partner. Students do this over several rounds, switching roles after two rounds. As students attempt to produce more accurate drawings, they try to fine-tune their descriptions. They notice that more specific language or terminology is needed to better describe the features in the images (MP6).

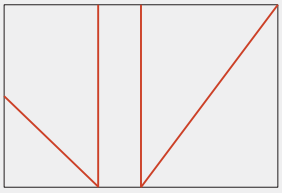
Here are the two sets of images:

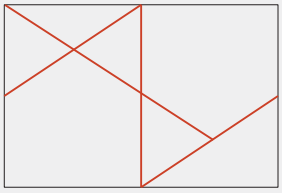
Set 1





Set 2





As students are working, listen for and collect the terms or descriptions that come up often or that effectively help the drawer replicate the image (for example: horizontal, vertical, point, lines, segment, top, bottom, and so on).

This activity uses *MLR2 Collect and Display. Advances: conversing, reading, writing*.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Invite students to plan and verbalize a strategy for how they will describe the images to their partners. Invite Partner As to consult another Partner A before describing each image to Partner B. Offer sentence frames to support collaboration, such as "How are you planning to describe this part?" and "I think I'm going to say . . . Do you think that will work?"
Supports accessibility for: Organization, Language, Social-Emotional Functioning

Instructional Routines

MLR2 Collect and Display

Materials to Gather

Chart paper, Index cards, Rulers or straightedges

Materials to Copy

Do You See What I See? (groups of 4)

Required Preparation

- Create a set of 4 cards from the Instructional master for each group of 2.

- Create a poster with the two images shown in activity synthesis.

Student-facing Task Statement

Work with a partner in this activity. Sit back to back or use a divider to block each other's view.

Partner A:

- Your teacher will give you a card. Don't show it to your partner.
- Describe the image on the card—as clearly and precisely as possible—so that your partner can draw it on a blank card.

Partner B:

- Your teacher will give you a blank card.
- Your partner will describe an image. Listen carefully to the descriptions. On your card, create a drawing as described.

1. When done, compare the given image and the drawn image. Discuss:
    - Which parts were accurate? Which were off?
    - How could the descriptions be improved so the drawing could be more accurate?
    - Which words or descriptions are useful?
2. Try again with a second card from your teacher. When done, compare and discuss the images again.
3. Switch roles and repeat the exercise. Your teacher will give you 2 new cards (1 for each round).

Student Responses

Sample description for each image:

- Set 1, image 1: Four slanted lines. Each line starts in the middle of one side of the

Launch

- Groups of 2
- Read the instructions together as a class. Demonstrate or clarify the process as needed.
- Give each student a ruler or a straightedge. Give each group four blank (unlined) index cards.

Activity

- Give 4 blank cards to each group.
- Give one partner the first image card from Set 1. When the group is done with the first round and has discussed the drawing, give the second card.
- 3–4 minutes per round.

MLR2 Collect and Display

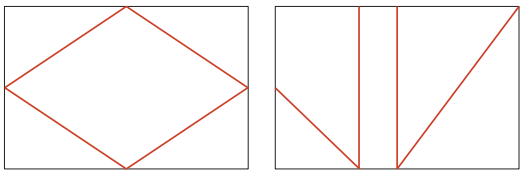
- Circulate, listen for, and collect the language students use to describe the first and second images. Listen for: line, point, straight, corner, quadrilateral, triangle, rhombus, vertical, horizontal, middle, top, bottom, left, right.
- Record students' words and phrases on a visual display and update it throughout the lesson.
- Repeat for the second partner, using the cards from Set 2.

Synthesis

- "Are there any other words or phrases that are important to include on our display?"
- Display the chart of terms and these images to facilitate discussion. Annotate on them to support students with mathematical terms.

paper and stretches to the middle of the side next to it. The four lines make a rhombus.

- Set 1, image 2: Two straight lines go left to right, one on top of the other, with a triangle underneath them with one side cutting across both lines.
- Set 2, image 1: Two Vs, side by side. The bottom tip of each V is at the bottom edge of the paper. The left arm of the first V (on the left) is on the left side of the paper and the right arm is at the top edge. Both arms of the second V (on the right) are at the top edge of the paper. The right arm of the first V and the left arm of the second V are both vertical.
- Set 2, image 2: This card has 4 lines. The first line goes from the middle of the top edge of the card to the middle of the bottom edge. The second line goes from the middle of the left edge to the middle of the top edge. The third goes from the middle of the right edge of the card to the middle of the bottom edge. The three lines look a bit like a tilted Z. The last line goes from the middle of the third line to the upper left corner of the card.



- As students share responses, update the display, by adding (or replacing) language, diagrams, or annotations.
- Remind students to borrow language from the display as needed.
- "How did you describe what you saw?" (By describing where the lines start and where they end, their directions, the distances between them, what they look like—if they look like a familiar shape or letter.)
- "What words or descriptions were more helpful when describing the figures and which were less helpful?" (Helpful examples: line, point, straight, corner, triangle, rhombus, vertical, horizontal, middle, top, bottom, left, right. Less helpful examples: here, there, a little bit, pointy, slanted.)
- "What was easy to describe?" (Lines that start a corner or the middle of an edge, lines that go left and right or up and down.)
- "What was not?" (Lines that cross other lines or that stop at hard-to-describe points.)
- "Did anyone measure something or use measurements? When might measurements have been helpful?" (When describing distances between lines or the position of a starting point.)

Advancing Student Thinking

Students may recognize familiar shapes in the figures, but may not provide details to describe its size, orientation, or location (for example, "First, draw a big triangle.") Consider asking:

- "What words could you use to help your partner draw that part of the figure exactly how you want it?"
- "How could you break the _____ you see into parts? How could you describe how to draw each part?"

Activity 2

15 min

Lines and Line Segments

Standards Alignments

Addressing 4.G.A.1

The purpose of this activity is to enable students to notice segments as parts of lines and motivate a need for a term to describe them.

Students are asked to draw multiple lines and to notice shapes that intersecting lines might have created. As they look for familiar shapes or figures in their drawing—polygons, letters, or numbers—their attention shifts from the lines to portions of the lines that make up those figures. Certain sections of the lines now have new significance apart from the lines that contain them. The observations here prepare students to better understand the mathematical definition of line segments.

The synthesis introduces the term **line segment** informally. In the next lesson, the meaning of the term, as well as of the meanings of lines and points, will be formalized.

Materials to Gather

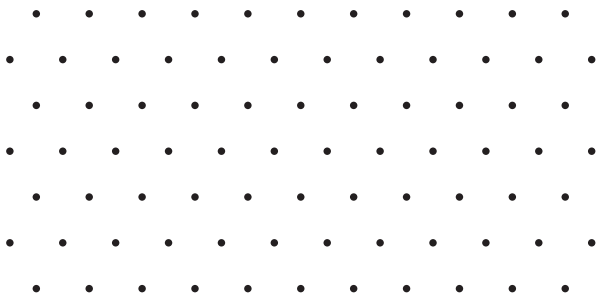
Rulers or straightedges

Student-facing Task Statement

Here is a field of dots.

Launch

- Groups of 3–4
- Give each student a ruler or a straightedge.
- Display the image of dots.
- "What do you notice?" (There are lots of dots or points. If we connect them, we can

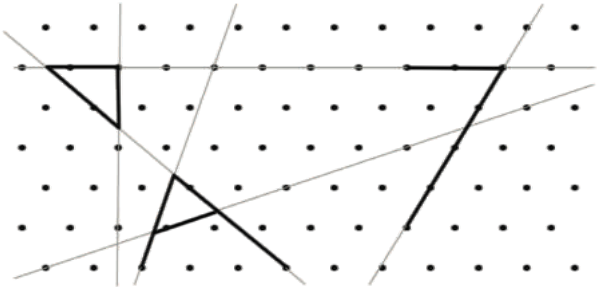


1. Draw 5 lines, each one connecting at least 2 dots and extending as far as possible.
2. Did your lines make some familiar shapes or figures—perhaps a triangle, quadrilateral, a letter, or a number?

   Identify at least one familiar shape or figure in your drawing. Trace the shape with a heavier mark or use a color pencil to make it obvious.
3. Share your drawing with your group members. Discuss:
   - How are the drawings alike? What do all the shapes have in common?
   - How are the drawings different?

Student Responses

Sample response for 1 and 2:



make shapes. I see triangles, rhombuses, and hexagons. The dots make straight lines left to right, and they are the same distance apart.)

- "What do you wonder?" (Why are the points in one row not lined up with the points in the rows above or below it? Will we be connecting the dots?)
- "In this activity, we will use the dots in the image to make different figures."
- Demonstrate drawing a line through a set of dots in the isometric dot image, extending them as far as possible.

Activity

- 3 minutes: independent work time
- 2 minutes: partner discussion
- As students discuss, listen for informal descriptions of points, lines, and line segments.

Synthesis

- Display the image on the task statement. Invite students to share their drawings and their responses to the discussion questions.
- "How are your drawings alike?" (They all have long lines. The shapes are made of shorter pieces of the lines. Some of those pieces end in dots. Others are cut off by another line.)
- "How are your drawings different?" (The lines made different shapes and letters. Some shapes are made of parts that always end in dots. Others have parts that don't end in dots.)
- Display the annotated chart from the previous activity.
- "Each part of a line that makes up your shape is called a **line segment** or a **segment**. We can see where a segment

starts and where it stops."

- Ask students to point out to their partners some segments in their drawings.
- "Some of you have used the word '**points**' to describe the dots or certain parts of the figures you drew."
- "How many points do you see marked on this line or line segment? Where are they?" (I see 3 points marking the top of my 7.)
- "Can you show your partner some points in the figures you drew?"

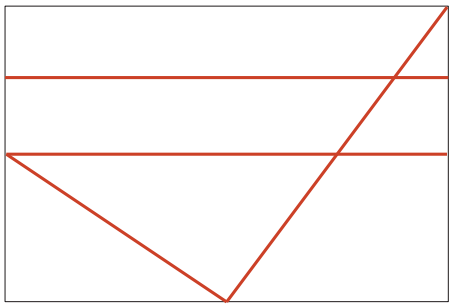
Lesson Synthesis

10 min

"Today we described some drawings and created some drawings based on descriptions. We noticed how **points**, **lines**, and **segments** are present in the drawings, and that those terms were handy for describing what we were seeing."

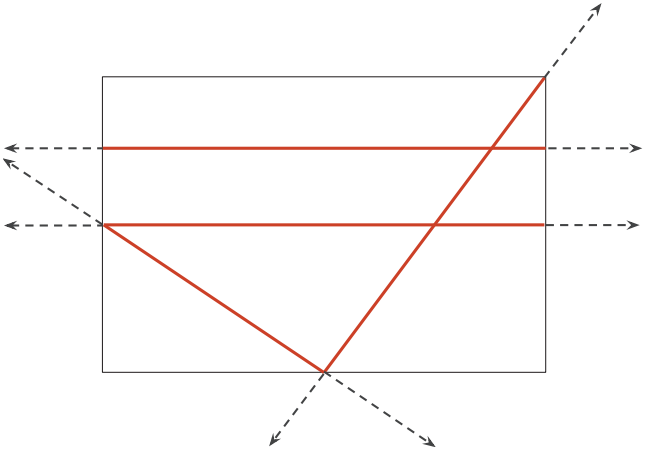
"Let's look at image 2 of set 1 from the first activity."

Draw a quick sketch of this image to annotate during discussion.



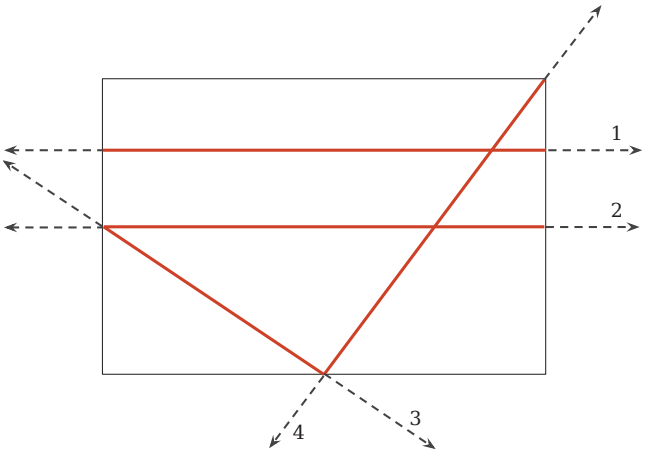
"Let's imagine that the lines that reach the edge of the card actually go on if the card was larger."

Annotate sketch to demonstrate extended lines.



"How many lines do you see?" (four)

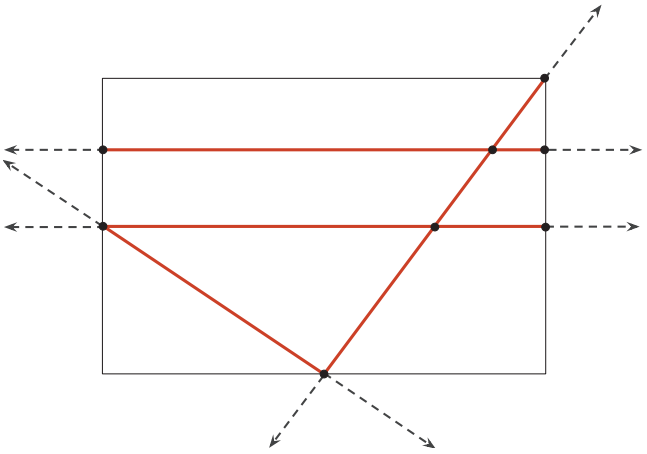
Label each line as students identify.



"Where do you see points in this drawing?" (Students are likely to identify vertices of the triangle and also places where lines intersect.)

As students identify points, add each to the sketch for reference.

"Which points were most helpful when describing the drawing to a friend?" (the points where the lines cross one another, or the points where the lines cross the edges of the card)



"Where are the line segments in this image?" (any part of a line with a starting point and an ending point)

"How many segments do you see in this drawing?" (Students are likely to say 8, but there are more, including pieces that go from one edge of the card to another edge.)

Suggested Centers

- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Supporting)
- Compare (1–5), Stage 7: Multi-digit Operations (Supporting)

Complete Cool-Down

Response to Student Thinking

Students describe the image only in terms of real-world objects (such as "there are 2 kite-like shapes") or symbols (such as "two lines make a big X and one more line makes a letter A on the right side").

The work of this lesson builds on the geometry concepts developed in a prior unit.

Next Day Support

- Launch warm-up or Activity 1 by highlighting key vocabulary from previous lessons.

Prior Unit Support

Grade 3, Unit 7, Section A: Reason with Shapes

Lesson 2: Points, Lines, Rays, and Segments

Standards Alignments

Addressing 4.G.A.1, 4.NBT.B.4, 4.NBT.B.5

Teacher-facing Learning Goals

- Draw points, lines, rays, and segments.
- Identify points, lines, rays, and segments in geometric drawings.

Student-facing Learning Goals

- Let's draw points, lines, line segments, and rays.

Lesson Purpose

The purpose of this lesson is for students to identify and draw lines, segments, and rays.

In the previous lesson, students created drawings and used informal language to describe their drawings. They began to see segments as pieces contained within lines.

In this lesson, students begin to connect points, lines, and line segments with formal definitions. They also encounter rays and learn that a ray is different from a line segment, but, like a segment, it is also a part of a line. The first activity, a Card Sort, encourages students to look for these distinctions.

In the second activity, students draw segments and rays that form other shapes and figures. An isometric dot paper is used for drawing to reinforce the idea of segments and rays having endpoints.

To support students with the vocabulary in this unit, consider making time for them to build a personal illustrated "word wall" at the end of each lesson in which new terms are introduced. Allow a few minutes for students to add new terms, illustrations, and definitions in their own words to an organizer as shown in the Instructional master.

Access for:

Students with Disabilities

- Engagement (Activity 1)

English Learners

- MLR8 (Activity 1)

Instructional Routines

MLR7 Compare and Connect (Activity 2), Number Talk (Warm-up)

Materials to Gather

- Rulers or straightedges: Activity 1, Activity 2

Materials to Copy

- Card Sort: Who Am I? (groups of 2): Activity 1

Required Preparation

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

Who participated in math class today? Who did not participate and why might this be? How can you leverage each of your student's ideas to support them in being seen and heard in tomorrow's class?

Cool-down (to be completed at the end of the lesson)

5 min

True or False: What's the Point?

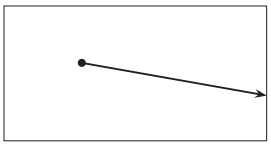
Standards Alignments

Addressing 4.G.A.1

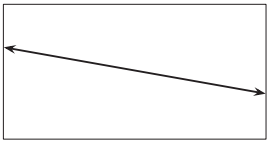
Student-facing Task Statement

Decide if each statement is true or false. If it is false, correct it.

1. A point marks a place.
2. This is a drawing of a ray.



3. A line can be curved or straight.
4. This is a drawing of a segment.



5. The length of a ray can be measured.

Student Responses

1. True
2. True
3. False. A line is always straight.
4. False. Sample corrections:
   - A line segment is a part of a line and has two endpoints.
   - The drawing shows a line or two rays pointing in opposite directions.
   - This is a drawing of a segment:

5. false. A ray goes on forever in one direction so the length cannot be measured.

Begin Lesson

Warm-up

10 min

Number Talk: Finding Differences

Standards Alignments

Addressing 4.NBT.B.4, 4.NBT.B.5

This Number Talk encourages students to look for and make use of structure to mentally evaluate a series of subtraction expressions. The numbers preview some benchmark angle measurements students will see in upcoming lessons.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $90 - 45$
- $270 - 45$
- $270 - 135$
- $360 - 135$

Student Responses

- 45: $90 = 45 + 45$
- 225: 270 is 180 more than 90, so $270 - 45$ is 180 more than $90 - 45$.
- 135: 135 is 90 more than 45, so $270 - 135$ means subtracting an additional 90 from $270 - 45$.
- 225: 360 is 90 more than 270, so $360 - 135$ is 90 more than $270 - 135$.

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "What do these expressions have in common?" (The first number in each sequence is a multiple of 90.)
- "How did this observation—that the first numbers are all multiples of 90—help you find the value of the differences?"
- Consider asking:
  - "Who can restate _____'s reasoning in a different way?"
  - "Did anyone have the same strategy but would explain it differently?"
  - "Did anyone approach the expression in a different way?"
  - "Does anyone want to add on to _____'s strategy?"

Activity 1

20 min

Card Sort: Who Am I?

Standards Alignments

Addressing 4.G.A.1

In this activity, students are given cards that contain illustrations, definitions, and descriptions of **points**, **lines**, **rays**, and **segments**. They sort the cards into groups so that each group describes one of the four geometric figures.

When students sort the cards, they begin to connect the terms to their formal definitions and attributes (MP6).

Students may have trouble making sense of a point having "no size." It is not necessary to discuss this in depth at this point, but during the synthesis, clarify that a point marks a location, and we need a symbol or a mark to represent it. The symbol or the mark has size, but the location itself doesn't. It is important that students recognize that points make up lines even though we do not always identify them or label them with a dot. If needed, revisit the isometric grid from the previous activity as a reference.

Access for English Learners

MLR8 Discussion Supports. Students should take turns finding a match and explaining their reasoning to their partner. Display the following sentence frames for all to see: "I noticed _____, so I matched . . . ." Encourage students to challenge each other when they disagree.
Advances: Conversing.

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Revisit math community norms to prepare students for group work and the whole-class discussion.
Supports accessibility for: Memory, Social-Emotional Functioning

Materials to Gather

Rulers or straightedges

Materials to Copy

Card Sort: Who Am I? (groups of 2)

Required Preparation

- Create a set of cards from the Instructional master for each group of 2–4 students.

Student-facing Task Statement

Your teacher will give you a set of cards that describe or illustrate **points**, **lines**, **rays**, and **line segments**.

Launch

- Groups of 2–4
- Give each group a set of cards from the

Sort the cards into 4 groups. Each group should represent the attributes or characteristics of one of the geometric figures.

Pause for directions from your teacher before completing the graphic organizer.

point	line
ray	line segment

Student Responses

- Point:
  - I am usually represented by a dot or a circle.
  - I have no size (no width, no length, and thickness).
  - I am a location.
  -
- Line:
  - My length can't be measured. I have no start and no end.
  - I am a set of points arranged in a straight way. I extend infinitely in two directions.
  - I often have arrows to show that I keep going in two directions.
  - 

Instructional master and access to rulers or straightedges.

- Display the list of words and phrases collected during the previous lesson.
- "We used many different words to describe figures. We learned how to identify points, lines, and line segments."
- Ask students to act out each term with arms and hand gestures. (For example, "Show me a point using only your arms or hands or both." Students may makes fists to represent points or identify a spot along their arm as a point.)
- "We are going to continue to define points, lines, and line segments in this next activity. You are also going to use the cards to define another figure, a **ray**."

Activity

- "Work together to sort the cards into four groups. The cards in each group should describe a particular geometric figure."
- "When your group finishes, compare your results with another group's."
- "Set aside cards that were hard to place. Be prepared to explain why they were more challenging."
- Monitor for reasoning students use to sort cards. (Examples: Points are parts of lines so the description of a point will not use the word point in it. Line segments are parts of lines.)

Synthesis

- Invite students to discuss their sorting decisions.
- "Which cards did you spend the most time debating as a group?" ("I have no size." or "My length cannot be measured.")
- "A point might be tricky to think about. It is often represented by a dot or a circle,

- Ray:
  - I am a line that is bounded by one point but goes on in the other direction.
  - My length can't be measured. I have a start but no end.
  - I often have one dot and one arrow to show that I go on in one direction.
  - 
- Line segment:
  - I am a part of a line that is bounded by two points.
  - I have a specific length.
  - I am often shown with two dots or with no dots.
  - 

which could be large or small. But the point itself cannot be large or small since it only marks a location."

- "What about a line? Why can it not be measured?" (It just keeps going in both directions so we don't know where to start or stop measuring.)
- "Why is a ray also impossible to measure?" (There is a starting point for measurement but there is no endpoint.)
- Consider displaying a graphic organizer (as shown in the activity statement) for all to see and placing the cards in the right boxes along the way.
- Ask students to write a sentence and draw an image to represent each figure in the graphic organizer in the student material.

Advancing Student Thinking

Students may determine that the clue "I have no size" is not a match for any of the other clues. Consider asking, "If we had to place this clue with some of the others, which would you place it with and why?"

Activity 2

15 min

Make Some Shapes

Standards Alignments

Addressing 4.G.A.1

The purpose of this activity is for students to use line segments and rays to draw familiar two-dimensional figures, letters, and numerals. Drawing on dot paper helps to reinforce the idea that segments have a defined endpoint on both ends and to distinguish rays from segments. As they describe and compare figures, students use vocabulary from the previous activity.

The activity also enables the teacher to hear the geometric vocabulary students are bringing from earlier grades. Consider displaying a chart with an image of each shape listed in the first problem during the launch.

This activity uses *MLR7 Compare and Connect. Advances: representing, conversing*.

Instructional Routines

MLR7 Compare and Connect

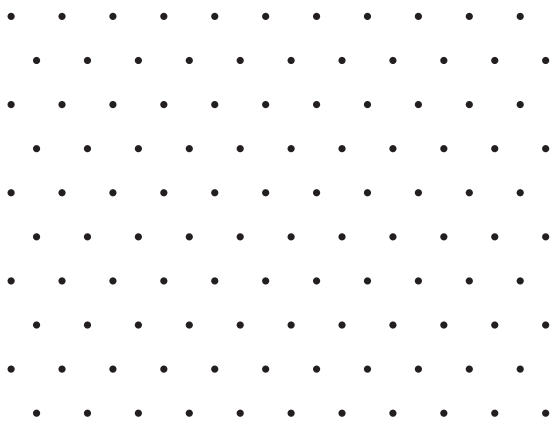
Materials to Gather

Rulers or straightedges

Student-facing Task Statement

1. Each dot on the grid represents a point. Draw line segments to create:

a triangle	a trapezoid	a pentagon
a rhombus	a hexagon	a rectangle



2. Draw a combination of rays and line segments to create:

an uppercase letter	a number	a lowercase letter

Launch

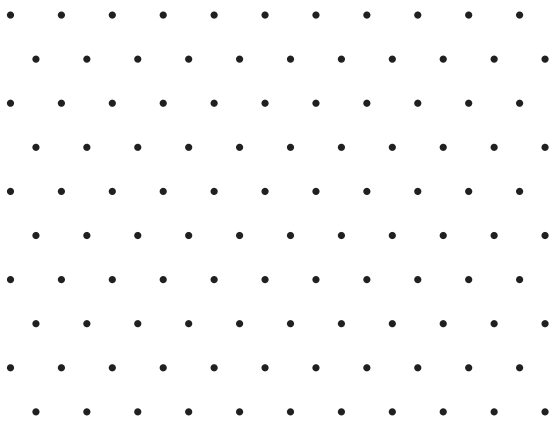
- Groups of 2
- "We are going to use the field of dots to create different figures."
- Give students access to rulers or straightedges.

Activity

- 3–5 minutes: independent work time

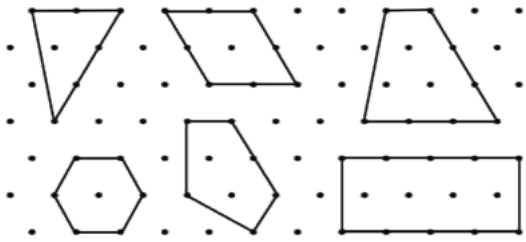
MLR7 Compare and Connect

- "How are your shapes and figures the same? How are they different?"
- 3–5 minutes: partner discussion.
- As students discuss, consider asking:
  - "Are the shapes you drew for the first question the same or very different? For example, are your triangles alike? Your trapezoids?" (Some are taller and wider, and others are shorter and narrower.)

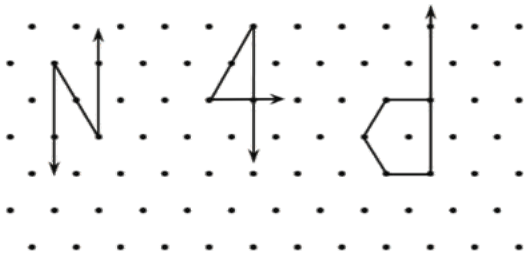


Student Responses

1. Sample response:



2. Sample response:



- "How many line segments did you use to make your letters or numbers?" (3, 4, 5, 6)
- "Did you make the same letters or numbers? If so, did you have the same reasons for choosing those letters or numbers?" (Some numbers have curves in them and are not as easy to draw.)

Synthesis

- "How many possible triangles can we draw on the dot paper? How many possible trapezoids? Hexagons?" (There are countless ways to create and connect a specified number of line segments to make a certain type of shape.)
- "Did anyone not start or end their line segments or rays on a dot? Do the results still count as segments or rays?" (Yes. A line segment stops at 2 endpoints, but the endpoints don't have to be on a dot of the paper.)
- "How did you distinguish line segments and rays when drawing numbers and letters in the second question?" (Line segments stop on both ends. Rays go on on one end, the one marked with an arrow.)

Lesson Synthesis

10 min

Use student examples to reference during the synthesis questions, or invite students to illustrate their explanations for each question.

"Today we learned the meanings of **points**, **lines**, **line segments**, and **rays**, and we used those geometric parts to create drawings."

"How might we explain to a new student how lines, rays, and line segments are different?" (A line is

straight and goes on forever in both directions. A line segment is a part of a line that ends on both ends. A ray is also a part of a line, but it goes on forever in one direction and ends in the other direction. We can use drawings to show how they are different.)

"Are the dots on the paper we used today the only points that could be in the shapes and figures?" (No. Each shape we drew had many points, not just the ones that were already there.)

Draw a capital A. "The tip of the letter A and the ends of the horizontal segment don't have any dots. Can we call these parts of the 'A' line segments?" (Yes. There doesn't have to be dots at the end. They have a starting point and an endpoint. Dots are just what we use to label points.)

Draw a capital L. "Is the bottom left corner of the letter L a point? Why or why not?" (Yes. A point is a location. It doesn't have to be marked by a dot. Any location on the line segments that make up the L are points on that letter.)

Suggested Centers

- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Supporting)
- Compare (1–5), Stage 7: Multi-digit Operations (Supporting)

Complete Cool-Down

Response to Student Thinking

Students recognize that some statements are false, but do not correct the statements.

Next Day Support

- Launch warm-up or Activity 1 by highlighting key vocabulary from previous lessons.

The work in this lesson builds from the geometry concepts developed in a prior unit.

Prior Unit Support

Grade 3, Unit 7, Section A: Reason with Shapes

Lesson 3: Two or More Lines

Standards Alignments

Addressing 4.G.A.1

Teacher-facing Learning Goals

- Draw parallel and intersecting lines.
- Identify parallel and intersecting lines.

Student-facing Learning Goals

- Let's look at lines that cross and lines that don't.

Lesson Purpose

The purpose of this lesson is for students to identify and draw parallel and intersecting lines.

In previous lessons, students identified and drew rays, lines, and segments. In this lesson, they turn their attention to lines that **intersect**, or cross, and those that never do.

In the first activity, students draw two kinds of quadrilaterals, one of which is a rectangle. The task motivates them to consider the relationship between lines—whether they would intersect at some point or never would. Students are introduced to **parallel lines** in this context. Lines typically have arrows at both ends, which represents that they go on in each direction. In this unit, arrows are at the ends of lines when it is important that students distinguish lines from segments and other features.

In the second activity, students draw parallel and intersecting lines. They also practice constructing an argument for how they know that two lines are parallel (MP3). In an upcoming unit, students will classify shapes based on whether they have parallel and perpendicular sides.

To support students with the new vocabulary in this lesson, consider making time for them to add to their "word wall" during the lesson synthesis. Allow a few minutes for students to add new terms, illustrations, and definitions in their own words to an organizer as shown in the Instructional master.

Access for:

Students with Disabilities

- Representation (Activity 2)

English Learners

- MLR2 (Activity 1)

Instructional Routines

How Many Do You See? (Warm-up)

Materials to Gather

- Rulers or straightedges: Activity 1, Activity 2

Materials to Copy

- Illustrated Word Wall (groups of 1): Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

In the next unit, students will analyze and categorize two-dimensional figures based on whether they have parallel and perpendicular as attributes. How does this lesson prepare that upcoming work?

Cool-down (to be completed at the end of the lesson)

5 min

Parallel and Not Quite Parallel

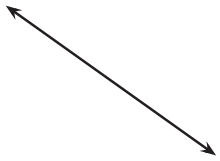
Standards Alignments

Addressing 4.G.A.1

Student-facing Task Statement

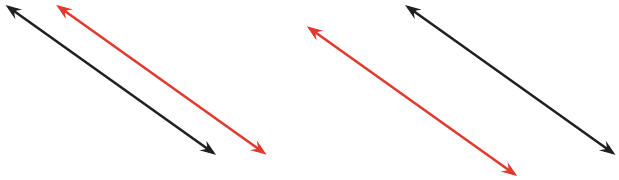
1. Explain why these lines are not parallel.

2. Sketch a line that is parallel to this line.



Student Responses

1. Sample responses:
   - The two lines get closer to each other in one direction. If we extend them, they will eventually intersect.
   - The two lines are not the same distance apart everywhere. The gap between the two lines is noticeably wider on one side and narrower on the other, so the two lines will cross if they're extended.
2. Sample drawings:



Begin Lesson

Warm-up

10 min

How Many Do You See: A Curious Figure

Standards Alignments

Addressing 4.G.A.1

The purpose of this How Many Do You See is to allow students to use subitizing or grouping strategies to describe the image they see. Listen for the language students use to describe how they count and define the line segments in the image.

Instructional Routines

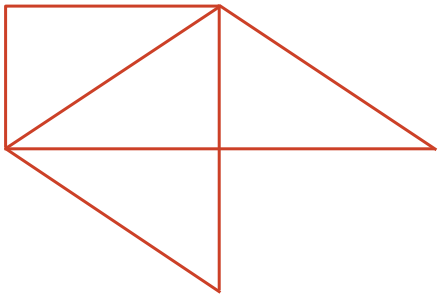
How Many Do You See?

Student-facing Task Statement

How many line segments do you see? How do you see them?

Launch

- Groups of 2



Student Responses

- 7 line segments. Sample reasoning:
  - There are 2 long segments that make a plus sign, 1 horizontal segment, and 1 vertical segment in the upper left, and 3 slanted segments.
- 9 line segments. Sample reasoning:
  - There are 4 triangles with 3 segments each, which means 12 sides. The triangle in the middle shares all 3 sides with the other triangles, so we need to subtract 3 to avoid double counting those segments.
  - There are 3 vertical segments, 3 horizontal segments, and 3 slanted segments.
- 11 line segments. Sample reasoning:
  - There are 4 horizontal segments (3 short, 1 long), 4 vertical segments (3 short, 1 long), and 3 slanted segments.

- "How many do you see? How do you see them?"
- Display the image.
- 1 minute: quiet think time

Activity

- Display the image.
- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Record responses.

Synthesis

- "How many line segments does this image have?" (9)
- "How do we determine where line segments start and end?" (They are between pairs of points or vertices in the figure.)
- Consider asking:
  - "Who can restate the way _____ saw the figure in different words?"
  - "Did anyone see the figure the same way but would explain it differently?"
  - "Does anyone want to add an observation to the way _____ saw the figure?"

Activity 1

15 min

Four Lines

Standards Alignments

Addressing 4.G.A.1

The purpose of this activity is to draw students' attention to intersecting lines and parallel lines. Students may not have had a reason to consider how to describe the characteristics of lines that cross. To motivate them to do so, they are given three intersecting lines and asked to add a fourth one—first to make any quadrilateral, and then to make a rectangle.

Students notice that creating a quadrilateral is not a problem, but creating a rectangle is. They analyze the given lines and consider the attributes of a rectangle that make the second task difficult. Along the way, students are likely to recognize that, to form a rectangle, they would need two pairs of lines where each pair is always pointing in the same direction and never converging, and where the first pair make square corners when they cross the second pair (MP7).

Access for English Learners

MLR2 Collect and Display. Collect the language students use to form lines to create a quadrilateral and a rectangle. Display words and phrases such as: a pair of lines that never cross have no intersection. During the synthesis, invite students to suggest ways to update the display: "What are some other words or phrases we should include?" Invite students to borrow language from the display as needed.
Advances: Conversing

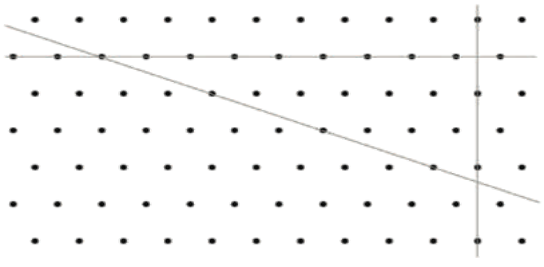
Materials to Gather

Rulers or straightedges

Student-facing Task Statement

1. Three lines on a field of dots **intersect** (cross), forming a triangle. Can you draw a fourth line so that the four lines form a quadrilateral?

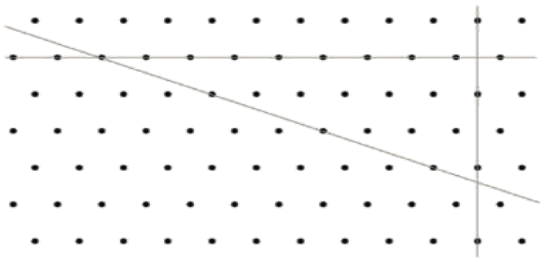
   Use the drawing to show your reasoning, or explain why it cannot be done.



2. Here is a copy of the same drawing. Can you draw a fourth line to form a rectangle?

Launch

- Groups of 2–4
- Give access to rulers or straightedges.
- "What do you notice about the lines in the first problem?" (There are three of them. They cross each other. Some are straight and some are slanted. The segments make a triangle.)
- "The three lines form a triangle. Describe the line segments that make up the triangle. Where do they start? Where do they end?" (The line segment starts and ends where it crosses the other lines.)
- 1–2 minutes: partner discussion
- Share responses.
- Record student observations and say "lines that cross are **intersecting** lines."

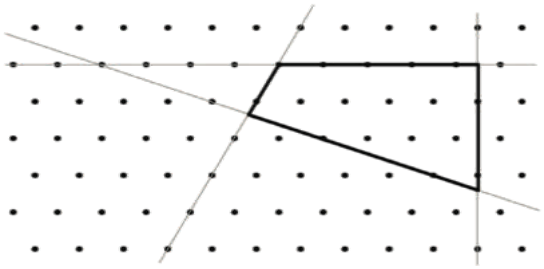


Use the drawing to show your reasoning, or explain why it cannot be done.

3. Discuss your drawings with your group. Check if they agree with your conclusions on both questions.

Student Responses

1. Sample drawing:



2. It can't be done. Sample reasoning: The horizontal and vertical lines make a square corner, but the third line that is slanted downward doesn't make a square corner with either of the other two lines. Adding a fourth line won't produce a rectangle. If adding a fifth line was allowed, then we could make it work.

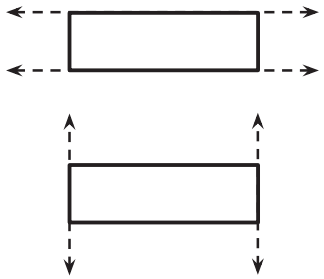
- "Where else have you heard the word **intersect**?" (Streets have intersections or places where they cross each other.)

Activity

- 4–5 minutes: independent work time
- 3 minutes: group discussion
- Monitor for students who notice that:
    - the slanted line can't be a side of a rectangle because it intersects the horizontal and vertical lines, adding another line won't help
    - to make a rectangle, we need two pairs of lines, where the lines in each pair never cross, but the first pair make square corners with the second pair when they cross

Synthesis

- Display the image on the task statement. Invite students to share their responses and reasoning.
- "What must be true about the four lines for them to form a rectangle?" (Sample responses:
    - The lines need to make square corners whenever they cross.
    - We need 2 pairs of lines, each pair going in the same direction and never coming together, and where they cross, they need to make square corners.)
- Record examples to match student explanations or invite students to draw examples of their thinking for all to see.
- "Lines that never **intersect**—like those on opposite sides of a rectangle—are called **parallel lines**."
- Display or add to student examples to emphasize lines and line segments:



- If time permits, consider asking, "Can you find a pair of parallel lines here in our classroom?" (Lines on either side of a row of floor tiles, the top and bottom sides of the board, or the left and right sides of the door.)

Advancing Student Thinking

Students may add a line to the drawing, but not yet identify how it creates (or does not create) a quadrilateral. Consider asking:

- "Where did you add your new line? Where does you new line cross the lines that were there?"
- "What new shapes do you see because of that extra line?"
- "Where else could you draw your line so that it would make a new shape?"

Activity 2

20 min

To Cross or Not to Cross

PLC Activity

Standards Alignments

Addressing 4.G.A.1

In this activity, students are prompted to draw intersecting and parallel lines, and to explain how they know a pair of parallel lines would never intersect.

Students are not expected to formally justify that two lines are parallel. They are expected to

make a case that goes beyond appearance (such as "it looks like they would never cross") and notice that the parallel lines maintain the same distance apart (MP3). Students are also introduced to the convention of naming lines with letters to support precision when describing and comparing lines. They are not expected to formally name lines or line segments with letters.

Access for Students with Disabilities

Representation: Access for Perception. Synthesis: Invite students to use their hands to represent parallel and intersecting lines. For example, hold your hands straight up and parallel to each other. Invite students to mimic you. Say, "Imagine our hands are lines. Are they parallel or will they intersect?" Move your hands in different ways (bring them closer together, tilt both but keep them parallel, tilt one, tilt both toward each other), repeating the question each time.
Supports accessibility for: Conceptual Processing, Visual-Spatial Processing, Memory

Materials to Gather

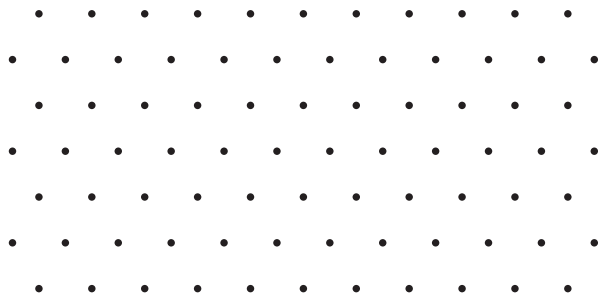
Rulers or straightedges

Materials to Copy

Illustrated Word Wall (groups of 1)

Student-facing Task Statement

Here is another field of dots. Each dot represents a point.



1. Draw a line through at least 2 points. Label it line h.
2. Draw another line that goes through at least 2 points and intersects your first line. Label it line g.
3. Can you draw a new line that you think would never intersect:
   a. line h?
   b. line g?

   If so, draw the line. Be prepared to explain

Launch

- Groups of 2
- Give each student access to a ruler or a straightedge.
- Display a field of dots.
- Select a student to draw a line in the field.
- "Sometimes we label lines to help communicate about different parts of a figure."
- Demonstrate labeling the line with a letter.
- "We can call this 'line a' because we labeled it with an 'a'."

Activity

- 5 minutes: independent work time
- 5 minutes: Discuss differences in drawings and make revisions.

Synthesis

- Discuss the first three questions. Select a few students to show or explain how they

or show how you know the lines would never cross. If not, explain or show why it can't be done.

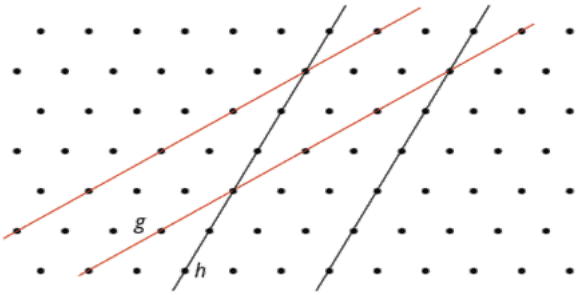
4. Here is a trapezoid.

Do you think its top and bottom sides are parallel? What about its left and right sides? Explain or show how you know.

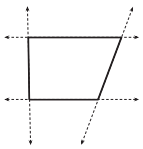
If you have time: Can you draw a new line that you think would never intersect either line h or line g? If so, draw the line and be prepared to explain or show how you know the lines would never cross. If not, explain why it can't be done.

Student Responses

Sample response for questions 1–3b:



Sample response for question 4: The top and bottom sides seem parallel. They are the same distance apart throughout. The left and right sides are not. If we extend the sides, the lines will eventually intersect.



If you have time: No, it cannot be done. Sample reasoning: If a line is parallel to h, then it will cross g at some point. If a line is parallel to g, then it will cross h at some point. This means it's not possible for a line to not cross both lines.

know their lines would never cross.

- If students suggested that they extended the lines and the lines didn't cross, ask if there is a chance that the lines would cross if they continue to be extended.
- "How did you know that your lines would never cross?" (Sample reasoning:
  - I lined up one side of a ruler with line h or line g and traced the other side of the ruler to make a new line. The two sides of a ruler never cross.
  - The distance between the dots is always the same, so if one line is always, for instance, 2 dots away horizontally from the other line, then they would never cross.)
- If students discovered after extending lines that the lines would cross, ask: "What do you notice about the distance between the two lines that would eventually cross?" (They are getting closer.)
- "What do you notice about the distance between two parallel lines?" (The distance doesn't change.)
- Discuss students' responses to the last question. Ask students to support their predictions by extending the sides of the trapezoid (or display the image in the student response).

Advancing Student Thinking

Students may say that lines are intersecting only if they cross in the field of dots or that they are parallel if they do not cross in the field of dots. Consider asking:

- "Can you show me what your lines would look like if you extended them outside the field of dots?"
- "What do you notice? If you kept extending the lines, would they ever intersect?"

Lesson Synthesis

10 min

"Today we learned about lines that cross—**intersecting lines**—and lines that never do—**parallel lines**."

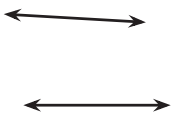
Display:

Draw extensions to each segment if suggested by students.

"Are these two line segments parallel?" (They appear to be.)

"How do we know for sure?" (We could extend the segments to see if the lines that contain the segments intersect. We could check the distance between them and see if they are the same distance apart.)

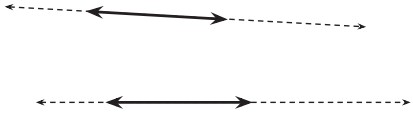
Display:



"Are these lines parallel?" (Students will likely be split on the answer.)

"How might we check to see if the lines are parallel or intersecting?" (Extend the lines to see that they will intersect, or check if the distance between them stays the same throughout.)

Display the following image and ask students if they could better tell if the lines are parallel.



"Look around the classroom and find a pair of parallel lines."

"Explain to a partner how you know those lines are parallel."

Provide students with a blank graphic organizer (from the Instructional master) for creating an illustrated word wall.

"Add the new vocabulary words from the past two lessons to your word wall—one box for each word. Write the definition of each term in your own words, and add a sketch to illustrate its meaning."

"Take a few minutes to do so now."

Suggested Centers

- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Supporting)
- Compare (1–5), Stage 7: Multi-digit Operations (Supporting)

Complete Cool-Down

Response to Student Thinking

Students say the lines are parallel because they do not cross.

Next Day Support

- Before the warm-up, have students work in groups to discuss a correct response to this cool-down.

Lesson 4: Points and Lines All Around

Standards Alignments

Addressing	4.G.A.1
Building Towards	4.G.A.2

Teacher-facing Learning Goals

- Draw figures with parallel and intersecting lines.
- Identify parallel and intersecting lines in figures and drawings.

Student-facing Learning Goals

- Let's find parallel and intersecting lines all around us.

Lesson Purpose

The purpose of this lesson is for students to practice identifying parallel and intersecting lines and drawing them.

In this lesson, students identify parallel and intersecting lines in the world around them—in a map of a neighborhood, in the letters of the alphabet, in some part of their classroom, and in familiar logos. They apply their understanding to represent and draw a part of their environment that shows such lines and to create a new logo with these types of lines. The synthesis of this lesson further highlights the presence and necessity of parallel and intersecting lines in real life.

In future lessons, students will use their understanding of lines that intersect to talk about angles.

Access for:

Students with Disabilities

- Action and Expression (Activity 2)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Which One Doesn't Belong? (Warm-up)

Materials to Gather

- Materials from a previous lesson: Activity 1
- Rulers or straightedges: Activity 1, Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What strategy did most students use in their work today? What strategy did you anticipate today? Which did you not anticipate?

Cool-down (to be completed at the end of the lesson)

5 min

Word Fun

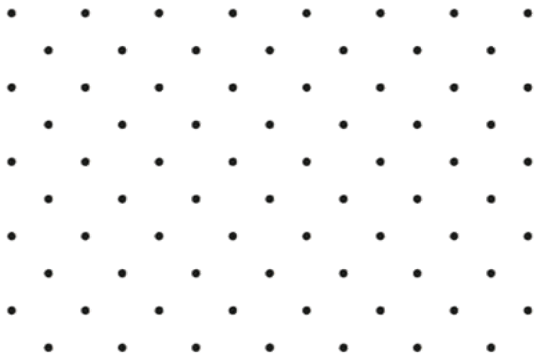
Standards Alignments

Addressing 4.G.A.1

Student-facing Task Statement

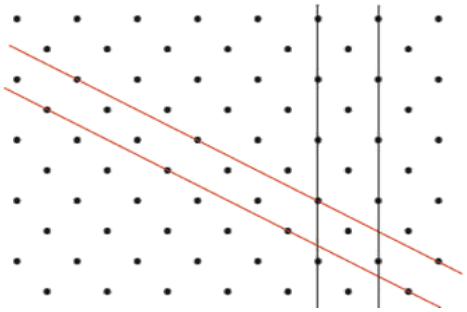
FUN KITES

1. Which letters in the phrase FUN KITES have:
   a. parallel segments
   b. no parallel segments
2. Here is a field of dots. Use it to draw 2 pairs of parallel lines, each pair pointing in a different direction.



Student Responses

1. a. F, U, N, and E
   b. K, T, S and I
2. Sample response:



Begin Lesson

Warm-up

10 min

Which One Doesn't Belong: Four-sided Shapes

Standards Alignments

Addressing	4.G.A.1
Building Towards	4.G.A.2

This warm-up prompts students to carefully analyze and compare quadrilaterals and their sides. When students make comparisons, they have a reason to use geometric language precisely (MP6). The activity also enables the teacher to hear the terminology students know and how they talk about characteristics of two-dimensional figures. The knowledge and ideas that students show here may also be insightful to teachers in the next lesson, when students learn about angles.

Instructional Routines

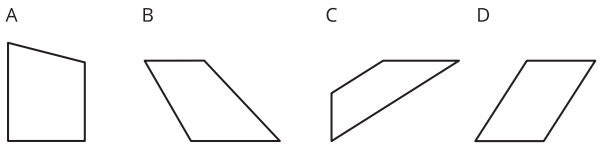
Which One Doesn't Belong?

Student-facing Task Statement

Which one doesn't belong?

Launch

- Groups of 2
- Display the image.



Student Responses

- A is the only one that doesn't have a horizontal segment across the top. It is the only one that doesn't have two "pointy" corners and two "wide" corners.
- B is the only one whose longest side is not parallel to another side.
- C is the only one without a horizontal segment across the bottom.
- D is the only one that doesn't have exactly one pair of parallel lines. It is the only one that is not a trapezoid. It is the only one where the parallel lines are not of different lengths.

- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 2–3 minutes: partner discussion
- Record responses.

Synthesis

- "How might we know if the sides of a figure are parallel?" (We could extend the lines or measure to see if the two sides are always the same distance apart.)
- "All four quadrilaterals have at least one pair of parallel sides. Do all quadrilaterals have at least one pair of parallel sides?" (No) "Can you draw one with no pairs of parallel sides?"

Activity 1

15 min

Spot Lines and Line Segments

Standards Alignments

Addressing 4.G.A.1

In this activity, students practice identifying line segments and both intersecting and parallel lines. First, students find these figures on a map and then in the alphabet. In both contexts, they encounter marks that may appear to be segments, but are not actually perfectly straight, or pairs of lines that appear to be parallel, but are not exactly so. Students have an opportunity to attend to precision when analyzing the given images (MP6).

When analyzing some letters in the alphabet, students may say that J and O have lines or segments that turn. Remind students that we had defined a line as being straight, so a line segment is also straight. In the letter J, the segment can be distinguished from the curve.

Access for English Learners

MLR8 Discussion Supports. Synthesis: To support the transfer of new vocabulary to long-term memory, invite students to chorally repeat the ending statements in unison 1–2 times: All intersecting lines cross each other, some intersecting lines form square corners, and all parallel lines never touch.
Advances: Speaking, Conversing.

Materials to Gather

Materials from a previous lesson, Rulers or straightedges

Required Preparation

- Gather Collect and Display charts from previous lessons.
- Each student will need access to their personal word walls created in previous lessons.

Student-facing Task Statement

1. Here is a map of a neighborhood in Staten Island, New York.

On the map, find and label the following items:

- 4 line segments of different lengths

Launch

- Groups of 2
- Display the map of Staten Island, NY.
- Give students access to rulers or straightedges.
- Display the chart of phrases collected during Collect and Display in a previous lesson.
- "Use your personal word walls or this chart of words and phrases to describe what you notice about this map of Staten Island, New York."
- "What do you notice about the streets on the map?" (Some are short and some are long. Some end and some cross others.)

Activity

- 5 minutes: quiet think time
- 3–4 minutes: partner work time
- Monitor for students who:
  - notice that some marks on the map

- 3 pairs of parallel lines
- 2 pairs of lines that are not parallel

(Consider using different colors for the different types of lines.)

2. In the words WHALE and JOY,

WHALE JOY

which letter has:

a. No parallel segments? _____

b. Exactly one pair of parallel segments? _____

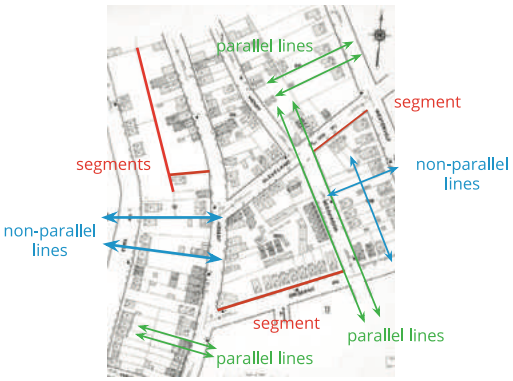
c. More than one pair of parallel segments? _____

d. Only one segment? _____

If you have time: Which does the uppercase alphabet use more: parallel segments or intersecting segments?

Student Responses

1. Sample response:



2. a. A, L, J, O, Y
   b. H
   c. W, E
   d. J (It has 1 segment and 1 curved

are not straight (even if they might appear to be) and are therefore neither lines nor segments

- recognize that some pairs of lines or segments might appear to be parallel but are not

Synthesis

- Select students to share their responses to the first question. Display their work (using a document camera or projection), or display the map and ask them to show their lines on it.
- To elicit the use of precise vocabulary and encourage more participation, consider asking:
  - "Were there any pairs of lines that you had assumed to be parallel at first but then realized that they are not? How did you find out?"
  - "Can someone else show a different pair of parallel lines? A different pair of lines that are not parallel?"
- "Do two segments have to be the same length to be parallel? For example, the top horizontal segment in E and the middle horizontal segment have different lengths. Are they parallel?" (Yes. The segments are part of lines that are parallel. The length of the segments does not determine if they are parallel.)
- "How might we finish these statements?"
  - "All intersecting lines _____." (cross each other)
  - "Some intersecting lines _____." (form square corners, make an "X" shape)
  - "All parallel lines _____." (never cross)

piece.)

If you have time: There are more letters with intersecting segments than parallel segments in the alphabet.

Advancing Student Thinking

Students may describe letters as having parallel segments (or as having no intersecting segments) because the segments do not cross. Ask students to explain and show what they mean. If students show segments that meet, but do not cross, consider asking,

- "What do you think it means for segments to be parallel?"
- "If you draw the lines that these segments are on, would the lines ever cross?"

Activity 2

20 min

Draw and Design with Lines

Standards Alignments

Addressing 4.G.A.1

In this activity, students look for parallel and intersecting lines in their environment and record them in a drawing. Students notice that parallel and intersecting segments can be found in logos and symbols and use these figures to design their own logo. When students recognize mathematical features of objects in their classroom and design a logo with intersecting and parallel line segments they model with mathematics (MP4).

If time permits, ask students to display their drawings and logos and do a gallery walk.

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Offer students an alternative to drawing on paper, such as using pipe cleaners to create a representation of lines in the classroom or using painter's tape to physically mark lines around the classroom.
Supports accessibility for: Visual-Spatial Processing, Fine Motor Skills

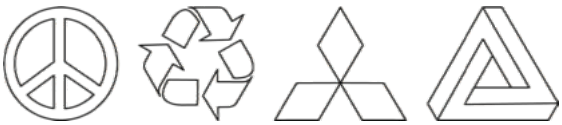
Materials to Gather

Rulers or straightedges

Student-facing Task Statement

1. Draw a sketch of a part of our classroom and be sure to include:
    a. at least 3 pairs of parallel line segments
    b. intersecting line segments that make square corners
    c. intersecting line segments that don't make square corners

    Trade sketches with a partner and find the specified lines in each other's sketches.
2. Here are some symbols and logos that you may recognize. All of them have intersecting and parallel line segments.

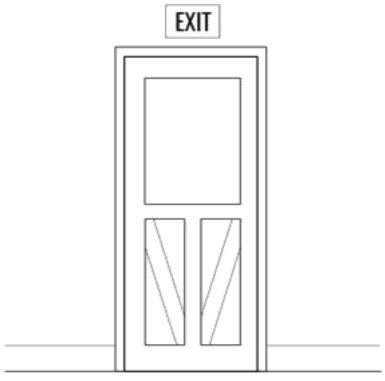


Design a logo with at least 8 parallel segments and 8 intersecting line segments.

Use a ruler for any straight parts of your logo.

Student Responses

1. Sample response: Door and exit sign:



Launch

- Groups of 2
- Give students access to a ruler or a straightedge.
- "For this activity, look around the classroom, find parts that have certain geometric parts we've been studying, and draw a sketch of them."

Activity

- 5–7 minutes: independent work time on the first question
- 2–3 minutes: partners trade sketches and verify that the required lines are shown
- Monitor for students who:
    - represent the thickness of objects (of a frame or a countertop, for example) with two parallel lines
    - represent objects abstractly, showing only essential shapes and lines
    - attend to precision by using tools like rulers or straightedges
    - choose to draw freehand
- For students who draw freehand, consider asking:
    - "How can you be sure that what you are sketching are lines or segments and not curves?"
    - "How do you know if you are drawing parallel segments?"
- 5 minutes: independent work time on the second question

2. Any design that contains 8 parallel and 8 intersecting line segments.

Synthesis

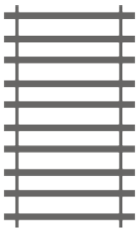
- "Where did you find parallel lines in our classroom?" (windows, doors, floor tiles, cubbies, desks)
- "Where did you find lines that make square corners?" (windows, doors, floor tiles, cubbies, desks)
- "Where did you find lines or segments that do not make square corners when they intersect?" (design on the doors, slant on the ceiling or floor, railings of stairs, braces or brackets of desks or chairs, hands on the clock)
- If time is limited, ask partners to trade their logo designs, look for the required lines in each other's work, and share feedback.
- If more time is available, ask students to display their designs and visit others' work in a gallery walk. As they look at each design, students should look for parallel and intersecting lines and line segments.

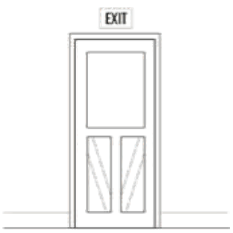
Lesson Synthesis

10 min

"Today we saw various examples of parallel and intersecting lines and line segments."

Display:



"Where do you see parallel lines in the images?" (The left and right sides of the ladder. The outside pieces of the drawing of the track.)

"What's the difference between the lines you see in the photos of the track and the drawing of the track?" (The horizontal lines look parallel in both. The vertical lines of the track do not look parallel in

the photo, but do look parallel in the drawing.)

"Why do you think there's a difference?" (Maybe some things in the real-world look parallel, but are not really parallel. I think it depends on how you look at it. If we looked at the track from above, maybe the lines would look parallel. When you draw things, you might make some lines parallel to make it look nicer or simpler.)

"When you were creating your sketch or logo today, how did you make sure that the segments that need to be parallel are actually parallel?" (I measured the distance between them, I used a ruler or another rectangular object as a guide.)

"Take 1–2 minutes to add any new words from today's lesson to your word wall. Share your new entries with a neighbor and add any new ideas you learn from your conversation."

Suggested Centers

- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Supporting)
- Compare (1–5), Stage 7: Multi-digit Operations (Supporting)

Complete Cool-Down

Response to Student Thinking

Students identify lines that are not parallel as being parallel.

Next Day Support

- Launch warm-up or Activity 1 by highlighting important ideas from previous lessons.

Lesson 5: What is an Angle?

Standards Alignments

Addressing	4.G.A.1, 4.MD.C.5
Building Towards	4.MD.C.5

Teacher-facing Learning Goals

- Identify angles in two-dimensional figures.
- Recognize angles as geometric figures that are formed wherever two rays share a common endpoint.

Student-facing Learning Goals

- Let's look for angles and find out ways to describe them.

Lesson Purpose

The purpose of this lesson is to introduce angles and to motivate a need for vocabulary to describe what they are and their size.

In this lesson, students are introduced to angles. They learn that an angle can be defined in terms of the geometric parts they have been working with in this unit.

In previous grades, students have used "square corners" to describe right angles within two-dimensional shapes. They may have considered an angle as the space within a square corner or the "pointy" corner itself. Here, students learn that an **angle** is a geometric figure made up of two rays that share the same endpoint, which we refer to as the **vertex** of the angle.

Throughout the lesson, students use the vocabulary they have developed to describe other geometric figures to identify and describe angles. Monitor for the ways students reason about how to describe the size of angles. Students will compare and measure angles in future lessons.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Engagement (Activity 1)

Instructional Routines

MLR1 Stronger and Clearer Each Time (Activity 2), MLR2 Collect and Display (Activity 1), Notice and Wonder (Warm-up)

Materials to Gather

- Rulers or straightedges: Activity 1, Activity 3

Materials to Copy

- Tricky Figures (groups of 8): Activity 1

Required Preparation

Lesson Timeline

Warm-up	5 min
Activity 1	15 min
Activity 2	15 min
Activity 3	10 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What was the best question you asked students today? Why would you consider it the best one based on what students said or did?

Cool-down (to be completed at the end of the lesson)

5 min

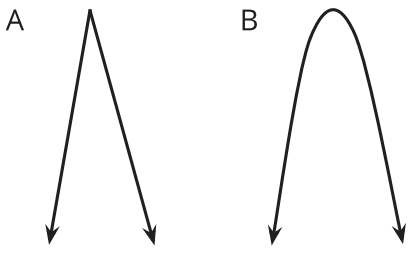
Spot the Angles

Standards Alignments

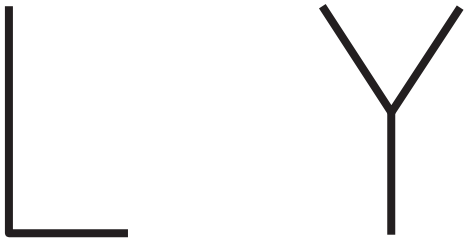
Addressing 4.G.A.1, 4.MD.C.5

Student-facing Task Statement

1. Jada says Figure A shows an angle, but Figure B does not. Do you agree? Explain your reasoning.

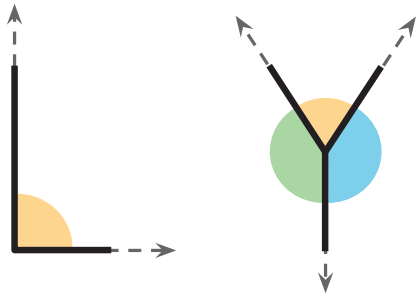


2. Identify the angles in each letter and draw the rays to show each angle.



Student Responses

1. Yes. Sample reasoning: Figure A is made up of two rays with the same endpoint. Figure B is a continuous curve, so it doesn't count as an angle.
2. Sample response:



Begin Lesson

Warm-up

5 min

Notice and Wonder: A Wall of Clocks

Standards Alignments

Building Towards 4.MD.C.5

The purpose of this warm-up is to draw students' attention to the figures formed by pairs of segments that are joined at a point in preparation for an exploration of angles. Students may notice and wonder many things about the clocks, but describing how the figures formed by the hands and how they are the same and different are the important discussion points.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?

Student Responses

Students may notice:

- The clocks show many different times.
- There are 3 hands that intersect on each clock.
- The hands look like line segments.
- The clocks have no numbers.
- On two of the clocks, the hour hand and minute hand overlap and make a straight segment.

Students may wonder:

- Why does each clock have a different time?
- Are these clocks from different places?
- Are we going to learn more about time?

Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- "Besides the time, what else changes when the long and short hands of the clocks turn?" (The directions of the hands, the figures created by the two hands, the appearance of the clock.)
- "Some of you may wonder what time and clocks have to do with lines, points, rays, and segments. Let's keep thinking about this as we work on our first activity."

Activity 1

15 min

Tricky Figures

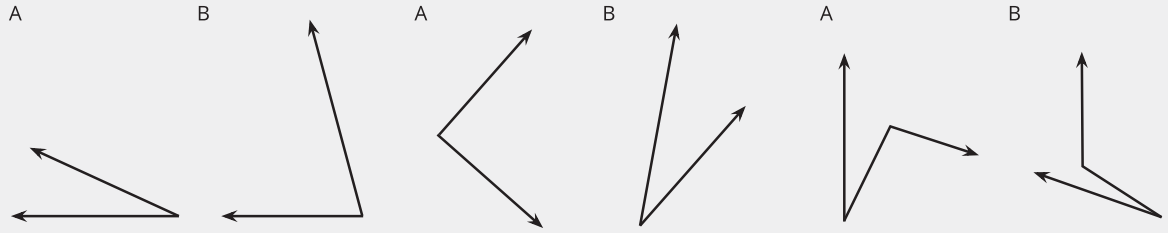
Standards Alignments

Addressing 4.G.A.1, 4.MD.C.5

In this activity, students work with a partner to replicate images of angles. One partner describes the figure and the other draws based on the verbal descriptions. The purpose of the activity is to draw students' attention to how they use the vocabulary they have learned from previous lessons to describe the figures (MP6). In this synthesis, students learn that an angle is a geometric figure that is made up of two rays that share the same endpoint. Students may also become aware that they need a clear way to describe the size of the figure. Angle measurement is not addressed in this lesson, but in the process of describing or drawing the figures, students are likely to use terms such as "narrower," "wider," or the like. Save the chart that shows the words students use to describe angles during the activity to revisit in future lessons.

Here are the two sets of images for the activity and one set for the extension:

Set 1 Set 2 Set 3 (If time permits)



This activity uses MLR2 Collect and Display. Advances: conversing, reading, writing.

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Synthesis: Optimize meaning and value. Ask, "How might thinking about angles be useful in our lives?" Consider making a connection to sports. For example, it might be easier to score in soccer if the ball is in front of the goal rather than off to the side, because of the angles involved. Show pictures if applicable and possible. (Consider drawing or labeling a picture in which the soccer ball is the vertex and the posts are points along the rays.)
Supports accessibility for: Conceptual Processing, Attention, Social-Emotional Functioning

Instructional Routines

MLR2 Collect and Display

Materials to Gather

Rulers or straightedges

Materials to Copy

Tricky Figures (groups of 8)

Required Preparation

- Create a set of cards (4 cards total) for each group of 2 from the Instructional master.
- Each group of 2 needs 2 cards (sets 1 and 2). Additional cards (sets 3A and 3B) can be used for extension.

Student-facing Task Statement

Work with a partner in this activity. Choose a role: A or B. Sit back to back, or use a divider to keep one person from seeing the other person's work.

Partner A:

- Your teacher will give you a card. Don't show it to your partner.
- Describe both images on the card—as clearly and precisely as possible—so that your partner can draw the same images.

Partner B:

- Your partner will describe two images. Listen carefully to the descriptions.
- Create the drawings as described. Follow the instructions as closely as possible.

1. When done, compare the drawings to the original images. Discuss:
    - Which parts were accurate? Which were off?
    - How could the descriptions be improved so the drawing could be more accurate?
2. Switch roles and repeat the exercise.

Launch

- Groups of 2
- Read the instructions together as a class. Demonstrate or clarify the process as needed.

Activity

- Give one partner the card for set 1. When the partners have discussed the drawings, give the other partner the card for set 2 and repeat the exercise.
- Provide one ruler or straightedge per student.
- 3–4 minutes per round

MLR2 Collect and Display

- Monitor for geometry terms from this section and phrases students use to describe the pace between the rays on each card. (spread apart, slide, moved over)
- Record students' words and phrases on a visual display and update it throughout the lesson.
- If groups are ready for more, give them a third card to describe and draw.

Compare the drawings to the original images afterwards.

If you have time: Request two new cards from your teacher (one card at a time). Take turns describing and drawing the geometric figure on each card.

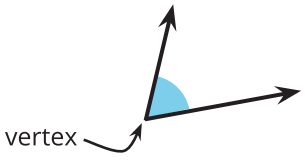
Student Responses

Sample description of each image:

- Set 1, figure 1: Two rays. A horizontal ray pointing to the left and another one pointing up and to the left, similar to where the long hand is at around 9:45.
- Set 1, figure 2: Two rays like in Figure A. This time the second ray is longer and points up higher, similar to where the long hand points at 11 o'clock.
- Set 2, figure 1: Two rays pointing to the right, like the letter K without the vertical segment.
- Set 2, figure 2: Two rays pointing to the upper right, one longer than the other. The longer ray is just a little to the right of vertical. The shorter ray is pointing to where the minute hand is about 8 minutes after the hour.

Synthesis

- "How are the two drawings on each card the same?" (They each have 2 rays. The rays start at the same point. One ray is pointing in the same direction in both drawings.)
- "How are they different?" (The rays are pointing in different directions on some cards. The rays are farther apart in some cards.)
- "How did you describe what you saw? What terms did you use to help you describe the directions of the rays?" (We tried to explain by describing the hands on a clock. We tried using words like north, south, east, and west. We described them in relation to vertical and horizontal.)
- As students share responses, update the display, by adding (or replacing) language, diagrams, or annotations.
- Remind students to borrow language from the display as needed.
- "Did anyone use the term 'angle?' Did anyone measure something or use measurements?"
- "The figures that you drew are angles. An **angle** is a figure that is made up of two rays that share the same endpoint."
- "The point where the two rays meet is called the **vertex** of the angle."



Display:

- "Where around us do we see angles?"

Activity 2

15 min

Angles or Not Angles?

Standards Alignments

Addressing 4.MD.C.5

In the previous activity, students learned what constitutes an angle. In this activity, they identify angles within geometric figures and explain their reasoning. Listen for the ways students show their understanding of rays even when they are not explicitly labeled in a given geometric figure.

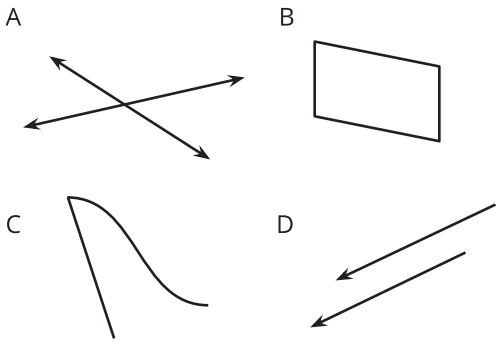
This activity uses MLR1 Stronger and Clearer Each Time. Advances: reading, writing.

Instructional Routines

MLR1 Stronger and Clearer Each Time

Student-facing Task Statement

1. Decide if each figure shows at least one angle. Explain or show your reasoning for each.



2. Clare and Kiran are looking at this diagram. Clare says there are no angles because the rays do not meet at a point. Kiran says he sees two angles.

Do you agree with either of

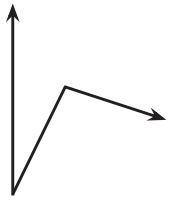
Launch

- Groups of 2
- Read the prompt to students.

Activity

MLR1 Stronger and Clearer Each Time

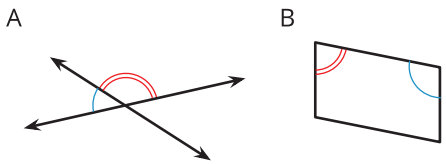
- 5 minutes: independent work time
- "Share your response to the first problem with your partner. Take turns being the speaker and the listener. If you are the speaker, share your ideas and writing so far. If you are the listener, ask questions and give feedback to help your partner improve their work."
- 3–5 minutes: partner discussion
- Repeat with 2–3 different partners.
- Monitor for students who hypothesize that there are no angles in figure B or C



them? How many angles do you see?

Student Responses

1. A and B have at least one angle, as shown. C and D have no angles.



2. Agree with Kiran. I count 2 angles.

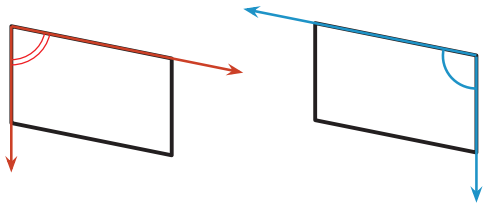
because no rays are drawn. Address this idea in the synthesis.

- "Revise your initial draft based on the feedback you got from your partners."
- 2–3 minutes: independent work time

Synthesis

- "How did you decide if there's at least one angle in each figure? What did you look for?" (I looked for two lines or segments that intersect.)
- "How might you use the word 'angle' in our description of figure A? If so how?" (Two intersecting lines make 4 angles.)
- "In figure B, the sides of the quadrilateral are segments rather than rays. Does that mean there are no angles in the figure?" (No, there are angles. We can think of the segments as being parts of longer rays.)
- "Can you show the rays that make the angles in figure B?"

  Display:



- "Can we show an angle in figure C by drawing rays, just as we have done with B?" (No, because one of the marks is not straight and can't be a part of a ray.)
- "In Clare and Kiran's argument, where are the two angles that Kiran saw?" (Kiran saw angles where each ray meets the line segment. The line segment is part of the rays that start at either of its endpoints.)

Activity 3

10 min

Discover Angles

Standards Alignments

Addressing 4.G.A.1, 4.MD.C.5

In this activity, students identify and sketch angles in their environment—in the text, graphics, and shapes in their physical surroundings—and reinforce the idea of an angle as a figure made up of two rays that share an endpoint.

In future lessons, students will look more closely at the properties of angles and consider how they can be measured and whether the length of the segments that form them impacts the size of the angles.

Materials to Gather

Rulers or straightedges

Student-facing Task Statement

Here are two figures.

1. Find 2–3 angles in each figure. Draw pairs of rays to show the angles.
2. Sketch a part of your classroom that has 2–3 angles. Draw pairs of rays to show the angles.

Student Responses

1. Sample response:

Launch

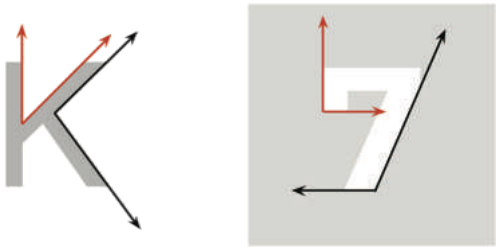
- Groups of 2
- Give each student a ruler or a straightedge.

Activity

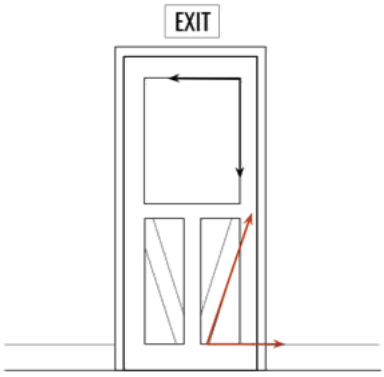
- 5 minutes: independent work time
- 2 minutes: partner discussion

Synthesis

- "How did you find the angles?" (We looked for intersections of segments. When the segments meet or cross, angles are formed.)
- "Where is the vertex of each angle you found?"



2. Sample response:



Advancing Student Thinking

Students may say they have identified an angle because they have found a vertex. Ask these students to explain what they mean and to describe the angle they see. Consider asking:

- "How do you see the rays that share this vertex? Can you draw them?"
- "Could more than one angle share a vertex? How could you use figure A from the previous activity to explain?"

Lesson Synthesis

10 min

"Today we learned that an **angle** is a figure made up of two rays that share the same endpoint, and that the shared point is the **vertex** of the angle."

"Use the words 'sometimes', 'always', or 'never' to respond to each statement about angles and lines:"

- "Intersecting lines form angles." (Always, because they make rays that share the same endpoint.)
- "Parallel lines can form angles." (Never, because they will never meet or share a point.)
- "Angles can be formed by curves." (Never, because a ray is a part of a line, which is always straight.)

"In upcoming lessons, we'll learn more about how to describe angles and how to measure them."

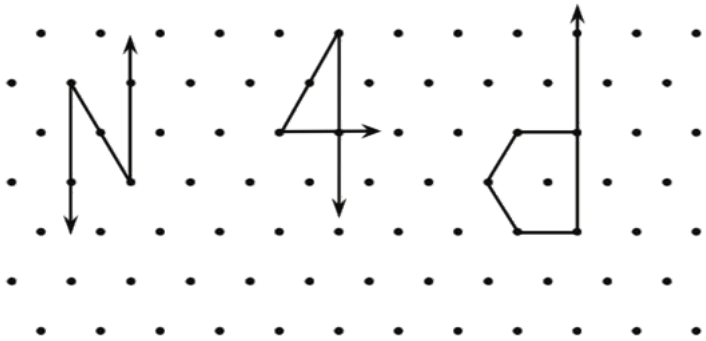
"Take 1–2 minutes to add the new words from today's lesson to your word wall. Share your new entries with a neighbor and add any new ideas you learn from your conversation."

Suggested Centers

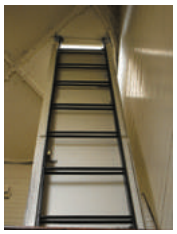
- Target Measurements (2–5), Stage 4: Degrees (Addressing)
- Compare (1–5), Stage 5: Fractions (Supporting)

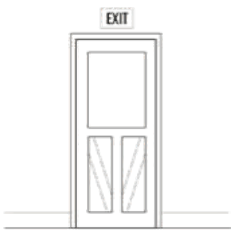
Student Section Summary

In this section, we learned the meanings of **points**, **lines**, **line segments**, and **rays.** We used these terms to describe figures and used these geometric parts to create drawings.

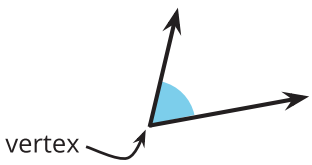


We learned about lines that cross—**intersecting lines**—and lines that never do—**parallel lines**, and we looked for examples of intersecting lines and parallel lines and segments in life.





Finally, we learned that an **angle** is a figure made up of two rays that share the same endpoint, and that the shared point is the **vertex** of the angle.



Complete Cool-Down

Response to Student Thinking

Students say figure B is an angle or identify some, but not all of the angles in the letter Y.

Next Day Support

- Launch warm-up or activities by highlighting important vocabulary from previous lessons.

Section B: The Size of Angles

Lesson 6: Compare and Describe Angles

Standards Alignments

Addressing	4.G.A.1, 4.MD.C.5
Building Towards	4.MD.C.5.a

Teacher-facing Learning Goals

- Compare angles in ways that make sense to students.
- Reason about how to describe the size of angles.

Student-facing Learning Goals

- Let's think about how to compare and describe angles.

Lesson Purpose

The purpose of this lesson is for students to consider ways to compare angles and describe the size of angles.

In previous lessons, students identified and described lines, line segments, rays, and angles in geometric and real-world contexts. The purpose of the exploratory work in this lesson is for students to make sense of which attributes define an angle (rays at a common endpoint) and which attributes do not (length of the line segments). The activities in the lesson intentionally elicit students' ideas for how to describe an angle's size.

Throughout the lesson, monitor for the ways students distinguish visible line segments from the rays that compose the angles and the language they use to describe the size of angles.

Instructional Routines

MLR2 Collect and Display (Activity 1), Which One Doesn't Belong? (Warm-up)

Materials to Gather

- Materials from a previous activity: Activity 2
- Patty paper: Activity 2

Materials to Copy

- Card Sort: Angles (groups of 2): Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

In today's lesson, students had the opportunity to compare angles using the language that makes sense to them. What language did they use to describe the size of angles? How can you build on this language in the next lesson when the size of an angle is described as a turn from one ray from the other?

Cool-down (to be completed at the end of the lesson)

5 min

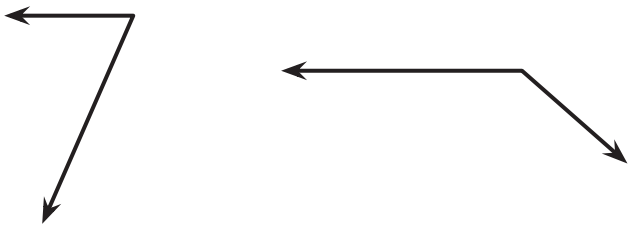
Compare Two Angles

Standards Alignments

Addressing 4.MD.C.5

Student-facing Task Statement

Here are two angles.



1. Describe at least one way they are alike.
2. Describe at least one way they are different.

Student Responses

Sample response:

1. Alike:
    - They both show 2 rays that share a common endpoint.
    - They both have one ray that is pointing in the same direction.

2. Different:
    - They each have one ray that is pointing in different directions.
    - One angle has a line segment that is longer than the other.
    - The angle on the right looks wider than the other angle.

Begin Lesson

Warm-up

10 min

Which One Doesn't Belong: Angles

Standards Alignments

Addressing 4.G.A.1, 4.MD.C.5

This warm-up prompts students to compare four angles. It gives the teacher an opportunity to hear how students use terminology and talk about characteristics of the angles in comparison to one another. This language will be important as students sort and order angles in the lesson activities. During the synthesis, ask students to explain the meaning of any terminology they use, such as angle, ray, point, smaller, larger, flat, upside-down, and so on.

Instructional Routines

Which One Doesn't Belong?

Student-facing Task Statement

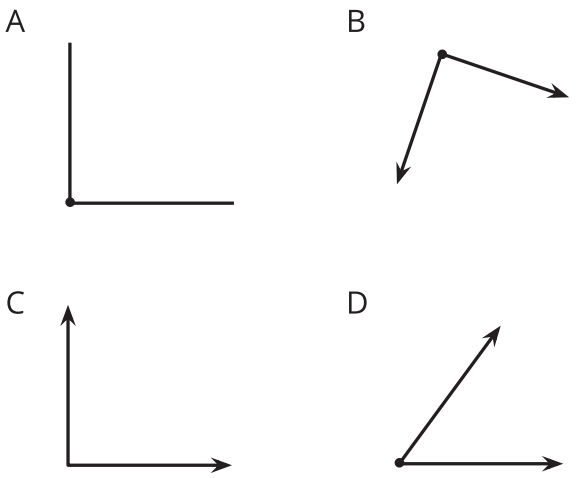
Which one doesn't belong?

Launch

- Groups of 2
- Display the image.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."



Student Responses

Sample responses:

- A is the only one that doesn't show arrows.
- B is the only one that doesn't have a ray that is horizontal and doesn't open upward.
- C is the only one that doesn't show a point where the rays meet (at the vertex).
- D is the only angle that looks smaller than the others and doesn't look like a square corner. The rays aren't as far apart.

- 2–3 minutes: partner discussion
- Share and record responses.

Synthesis

- "Do the images each show angles? How do you know?" (They each have two rays that share an endpoint. The angle in A doesn't show arrows, but the line segments are on rays that start at the vertex.)
- "Let's find at least one reason why each one doesn't belong."

Activity 1

20 min

Card Sort: Angles

Standards Alignments

Addressing 4.G.A.1, 4.MD.C.5

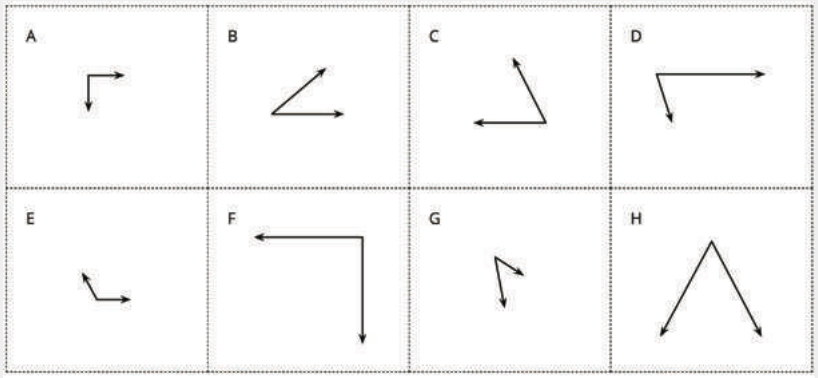
Previously, students learned that angles are geometric figures made up of two rays that share a common endpoint. The purpose of this activity is for students to compare angles using the language that makes sense to them. As students look for ways to sort the cards into different categories, they have a reason to look for and describe the parts of the angles that make them

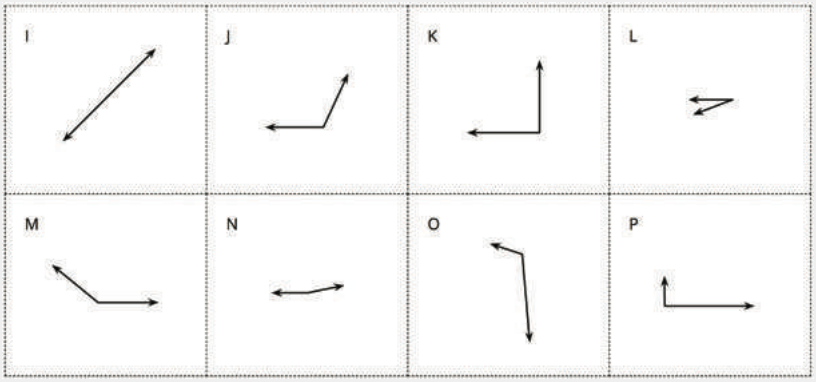
different (MP7). Students may describe:

- the orientation of the rays
- the length of the segments used to represent the rays
- the point that the rays share
- the distance or "space" between the rays or other invented descriptions of the size of the angles

MLR2 Collect and Display is used during the synthesis of the lesson to record and organize the language students use to describe parts of angles and their size. Encourage students to notice differences in the ways they attempt to compare the angles. This work helps elicit the need for more precise vocabulary to describe angle size and ways to measure angles.

Here are images of the cards for reference:





This activity uses *MLR2 Collect and Display*. Advances: conversing, reading, writing.

Instructional Routines

MLR2 Collect and Display

Materials to Copy

Card Sort: Angles (groups of 2)

Required Preparation

- Create one set of cards from the Instructional master for each group of 2 students.

Student-facing Task Statement

Your teacher will give you a set of cards with angles on them.

- Sort the angles into 3 or more categories and in a way that makes sense to you.
- Record your sorted angles here. Write words or phrases to describe each category. Be prepared to explain how you sorted the angles.

Student Responses

Sample responses:

- Angles O, M, J are “more open.”
- Angles L, G, and B are “almost closed.”
- Angles I and N are “almost straight.”
- Angles A, F, K, and P are "perfect square corners.”

Launch

- Groups of 2
- Give each group a set of cards from the Instructional master.

Activity

MLR2 Collect and Display

- “With your partner, sort these angles into different categories. Explain the categories you made.”
- Display the chart of words used to describe angles from the previous lesson.
- “As you think about how to sort the angles, it may be helpful to use the chart we created.”
- 3–5 minutes; group work time
- As students work, listen for and collect the language they use to add to the chart.
- Record students’ words and phrases on a visual display and update it throughout the lesson.
- “Now, you’ll compare how you sorted the angles and how another group did it.”
- “When you have partnered up with another group, take a minute to look at their cards without talking. Then ask them questions you have about their categories.”
- Consider posting and encouraging students to use sentence starters such as:
    - “I noticed you . . . .”
    - “How did you . . . ?”

    - "Why did you . . . ?"
- Arrange for each group of 2 to work with another group.
- 1 minute: quiet think time
- 2–3 minutes: group discussion
- Monitor for a variety of ways students sort and explain their sorting decisions.

Synthesis

- "How was others' way of sorting like your group way of sorting?" (We all looked at the length of the drawn rays.)
- "How was their way of sorting different from yours?" (They sorted based on if a ray was pointing horizontally or not. We sorted based on if the rays were open wide or whether they looked like they were closing.)
- "What words did you use to name the categories you created? What words did you hear others use?"
- As students share, update the display of words students use to describe the angles.
- Remind students to borrow language from the display as needed.

Activity 2

15 min

Order Angles

Standards Alignments

Addressing	4.MD.C.5
Building Towards	4.MD.C.5.a

Previously, students sorted angles in ways that made sense to them. In this activity, students reason about how to compare angles based on a measurable attribute. They are asked to sort the angles from smallest to largest. Students may interpret this prompt in many ways, but all students must begin to reason about how to describe the size of an angle. As students discuss and justify their decisions, they share a mathematical claim and the thinking behind it (MP3).

Accept any way that students compare the angles so long as they can explain how they determine whether an angle is smaller or larger than another. Students may use informal language to describe the size of the angles. For example, they may use "less wide", "more pointy", or "less space" to describe smaller angles. They may also reason that an angle that "can fit inside" another angle is smaller. Continue to collect student language during the activity and to offer formal language when appropriate.

Materials to Gather

Materials from a previous activity, Patty paper

Student-facing Task Statement

You will need cards A–P from an earlier activity.

Order the angles on the cards from smallest to largest.

Record your ordered angles. Explain or show how you decided which angle was the smallest and which was the largest.

Student Responses

Sample responses:

- By length of rays or overall image size: N, L, G, E, and A are the smallest. They have the shortest ray lengths and take up the least amount of space on the cards. B, C, J, and M are larger because they have longer ray lengths but not the longest. D, P, and O are the next larger size because they show a longer length for one ray. H, F, and I are the largest because they have the longest lengths for both rays.
- By space between the rays or how "close" or "open" the rays are: L, G, B, and D are

Launch

- Groups of 2
- Make sure each group has the angle cards from the previous activity.
- Make patty paper available, if requested.

Activity

- "Now let's compare the angles we used in the last activity in a different way."
- "Order the angles from smallest to largest. Discuss with your partner what you think that might mean. Then, order the cards."
- 5–7 minutes: partner work time
- As students work, monitor for the language they use and the features of the angles they focus on when ordering their size.

Synthesis

- Invite 2–3 groups who ordered the angles in different ways to share.
- For each, consider asking:

the smallest because there isn't much space between the two rays. A, F, K, and P are the next larger size because there is enough space to fit a square corner between the rays. J, E, O, M, and N are larger than the others because they are more "open" and there is more space between the rays. Angle I is the largest because the two rays make a straight line.

    - "What features of the angle did you focus on?"
    - "How did you determine whether an angle was smaller or larger?"
- "There is a standard way of measuring the size of angles."
- Display angles organized from smallest to largest: L, B, G, H, C, D, A (and F, K, P), O, J, E, M, N, I.
- "These angles are ordered from the smallest angle measure to the largest angle measure."
- "How is this the same as how you ordered the angles? How is it different?" (The smallest is the same in both. The largest are different, and the ones in middle are also different.)

Advancing Student Thinking

Some students may focus on the length of the line segments used to represent the rays. This is acceptable for the purposes of this activity, but encourage students to explain what they mean. Check that they understand that rays extend forever in one direction.

Lesson Synthesis

10 min

"Today we compared different angles. We sorted them in different ways and ordered them based on their size. We saw that we can describe angles and interpret their size in many different ways."

Display the chart used to collect the words students have used to describe angles.

"What questions do you have about describing and comparing angles? What do you need to know more about to describe and compare the size of angles?" (How do we measure angles? Can we use a ruler? What do we measure? Do we use inches or something else?)

Record student responses.

"In the next lessons, we'll learn more about angle measurement and answer some of these questions."

Suggested Centers

- Compare (1–5), Stage 5: Fractions (Supporting)
- Target Measurements (2–5), Stage 4: Degrees (Addressing)

Complete Cool-Down

Response to Student Thinking

Students may describe one way the angles are the same or different but not both. Students may compare the angles without describing the rays.

Next Day Support

- After the warm-up, ask students to work with a partner to discuss and add to their responses to this cool-down.

Lesson 7: The Size of Angles on a Clock

Standards Alignments

Addressing	4.MD.C.5, 4.MD.C.5.a
Building Towards	4.MD.C.5.a

Teacher-facing Learning Goals

- Describe the size of an angle as a turn of one ray from the other.
- Use the features of an analog clock to describe and compare the size of angles.

Student-facing Learning Goals

- Let's describe angles using the hands of a clock.

Lesson Purpose

The purpose of this lesson is for students to describe the size of an angle as a turn of one ray to the other.

In this lesson, students use an analog clock as a tool to describe the size of angles. They begin by using the clock to help describe how to draw a given angle, which involves describing the positions of the two hands of the clock. This work encourages students to relate the turning of the hands in a circular arc to the turning of rays of an angle around their shared endpoint.

Students then use language that suggests rotational movement to describe and compare the size of angles on a clock. To describe whether an angle is greater or smaller than another, they reference the amount of turn made by one or both rays. Students will connect the ideas developed in this lesson to the standard measurement of an angle (in degrees) in subsequent lessons.

Instructional Routines

Notice and Wonder (Warm-up)

Materials to Gather

- Patty paper: Warm-up
- Rulers or straightedges: Activity 1

Lesson Timeline

Warm-up	10 min

Teacher Reflection Question

What student questions about angles or angle measurement were addressed in this lesson? What can you do to help students refine their

Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

emerging understanding of angle measurement or resolve any questions that were not answered?

Cool-down (to be completed at the end of the lesson)

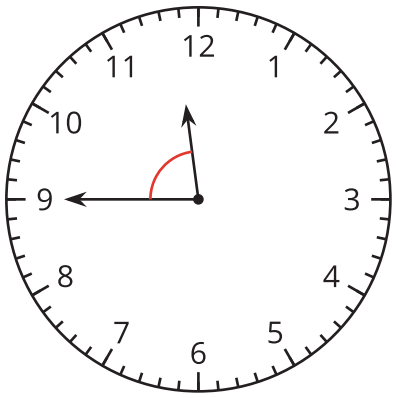
5 min

Which Angle is Larger? By How Much?

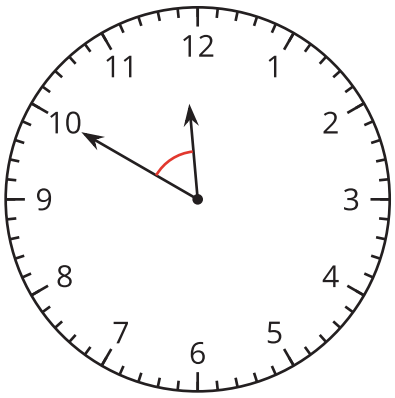
Student-facing Task Statement

The hands on each clock form an angle.

A



B



Which angle is larger? How much larger is it than the other angle? Explain how you know.

Student Responses

The angle on Clock A is larger by about 5 minutes. Sample response: In Clock A, the minute hand would have to turn 13 or 14 minutes to get to where the hour hand is. In Clock B, the minute hand would only have to turn 8 or 9 minutes.

Begin Lesson

Warm-up

10 min

Notice and Wonder: Two Sets of Angles

Standards Alignments

Addressing 4.MD.C.5, 4.MD.C.5.a

Students commonly think that angles formed by longer segments are greater in size than those formed by shorter segments. The purpose of this warm-up is to bring up and address this likely misconception. The diagrams prompt students to observe the lengths of segments forming the angles and consider how they affect our perception of the size of the angles.

While students may notice and wonder many things about these sets of angles, it is important to discuss the relative sizes of the angles in the two sets. Make sure students see that the two sets of angles are identical in size even though the segments that form them seem to suggest otherwise.

Consider using patty paper to demonstrate equal-size angles during the synthesis.

Instructional Routines

Notice and Wonder

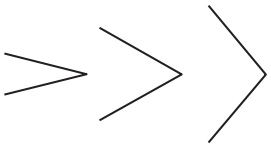
Materials to Gather

Patty paper

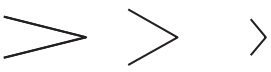
Student-facing Task Statement

What do you notice? What do you wonder?

Set 1



Set 2



Student Responses

Students may notice:

Launch

- Groups of 2
- Display the two sets of angles.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- "Are the angles getting larger or getting smaller in the top set? What about in the bottom set?" (They are getting larger as you

- There are two sets of angles. Each set has 3 angles.
- The lengths of the segments that make the angles are different—some are short, some are long.
- In set 1, the segments seem to be the same length but the angles get larger.
- In set 2, the segments seem to get shorter.

Students may wonder:

- Are the angles in the second set getting larger or getting smaller?
- Are the sizes of the angles the same in the two sets?
- Can the last angle (formed by the shortest pair of segments) be measured?

move from left to right.)

- "How does the first angle in set 1 compare to that in set 2?" (They look very similar and almost the same size.)
- "How does the second angle in set 1 compare to that in set 2?" (They look the same size but the rays that create the angles are longer in set 1 than in set 2.)
- "What about the third angle in each set?" (Set 2 has shorter lines than set 1 but the angle is the same size.)
- "How can we find out if one is larger, smaller, or the same size as the other?" (Measure them by laying them on top of each other.)
- Consider using patty paper to trace corresponding angles in the two sets and show that they are the same size even if the segments in the second set are shorter.
- Highlight that the size of an angle is not determined by the length of the segments that frame it.

Activity 1

15 min

Draw Angles Andre's Way

Standards Alignments

Addressing	4.MD.C.5
Building Towards	4.MD.C.5.a

In a previous lesson, students described an angle to a peer so that they might draw the angle accurately without seeing it. Students learned what an angle is and have reasoned about how to describe the size of an angle.

In this activity, students use the features of an analog clock (minute hand, hour hand, and position of numbers) to explain how to draw a given angle. In doing so, they use language related to turning one or both hands on the clock. In future activities, students will relate this idea to turning a ray around a fixed point.

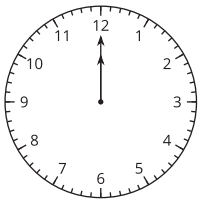
The hands on an actual clock are designed so that moving the minute moves the hour hand. The examples here disregard this constraint. For the purposes of this activity, it is assumed that both hands can be moved freely, or that one hand can be held in place while the other moved freely.

Materials to Gather

Rulers or straightedges

Student-facing Task Statement

Andre used the hands of a clock to explain how to draw an angle to his partner.

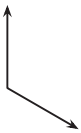


"Imagine both hands are pointing at the 12. Turn the minute hand so it's pointing at the 3."

1. Draw Andre's angle.
2. What is another way to describe how to draw the same angle using the clock?
3. Use Andre's way to explain how to draw these angles:

a.

b.



c.

Launch

- Groups of 2
- Give students access to rulers or straightedges.

Activity

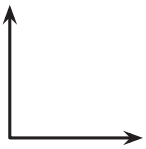
- "In an earlier lesson, we played a game where you described an angle to your partner to help them draw it without seeing it."
- "Read Andre's explanation for how to draw an angle. Try to draw his angle, and then think of another way to describe how to draw the same angle."
- 3 minutes: independent work time on the first two problems
- "Share your drawing and description with your partner."
- "Discuss how you would explain how to draw the other angles."
- 3–5 minutes: partner discussion and work time
- Monitor for students who clearly describe turning a hand around the clock and who connect the hands to the rays of the angles.

Synthesis

- "Let's listen to some different directions for how to draw these angles. As you listen, try

Student Responses

1. Sample response:



2. Sample responses:
    - Imagine both hands are pointed at the 3. Turn the hour hand so it's pointing at the 12.
    - Imagine both hands are pointed at the 12. Turn the hour hand so it's pointing at the 3.
3. Sample responses:
    a. Imagine both hands are pointed at the 12. Turn the hour hand so it's pointing at the 2.
    b. Imagine both hands are pointed at the 12. Turn the hour hand so it's pointing at the 4.
    c. Imagine both hands are pointed at the 4. Turn the minute hand so it's pointing at the 10.

to sketch the angle and guess which one they are describing how to draw."

- Invite 2–3 students to share the directions they wrote for drawing one of the angles. Choose students to share in a different order than the angles are presented in the task.
- For each, consider asking:
    - "Which angle did they describe (a, b, or c) and how do you know?"
    - "What words help you picture how to draw the angle?" (turn right, turn left, point at 4)
- "We can describe the size of an angle by explaining how much one ray has turned from the other."

Activity 2

20 min

Compare Angles on the Clock

Standards Alignments

Addressing 4.MD.C.5.a

In this activity, students compare the size of angles by thinking in terms of a turn from one ray from the other ray. They continue to use the clock as a tool for reasoning and for talking about "how much" of a turn. This work helps to elicit a need for more formal units and tools for measuring degrees while building the foundation for understanding angle measurement as additive. Students may describe the unit of measurement as "minutes," "hours," or other informal names (turn-units). They will learn about degrees as a unit of measurement in the upcoming lessons.

The final question gives students an opportunity to describe the size of an angle in as many ways as they can, using the clockface, benchmark angles, or other informal descriptions (MP6) preparing them for the introduction of a numerical way to measure angles.

Student-facing Task Statement

1. Here are some angles formed by the two hands of a clock.

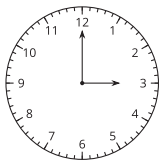
   In each pair of angles, which angle is larger? Explain or show your reasoning.

   a.

   5:00

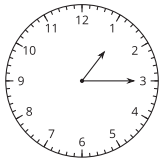
   

   3:00

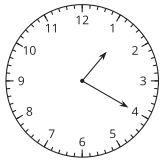
   

   b.

   1:15

   1:20

   c.

Launch

- Groups of 2
- "Earlier, we used the hands of a clock to describe how to draw an angle."
- "Now, let's think about how a clock might help us talk about the size of an angle."

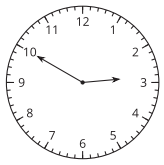
Activity

- "Use the clocks to compare the angles formed by each clock's hands."
- 3–4 minutes: independent work time
- 2–3 minutes: partner discussion
- Monitor for different ways students compare the same pairs of angles.

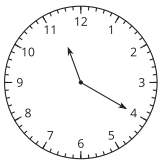
Synthesis

- Invite 2–3 previously identified students to share their responses.
- "How did each student see the angle formed by the hands in different ways?" (Some thought about moving the _____

2:50

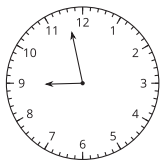


11:20

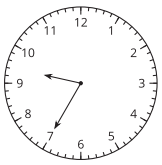


d.

8:58

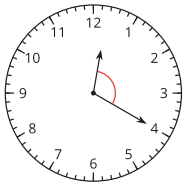


9:35



2. How large is this angle?

   Describe its size in as many ways as you can.



Student Responses

1. a. The angle showing 5:00 is larger. Sample response: The hour hand makes a bigger turn to go from 12 to 5 than from 12 to 3.

   b. The angle showing 1:20 is larger. Sample response: The minute hand travels farther from where the hour hand is (around 1) to go to 4.

   c. Sample responses:

   - The angle showing 2:50 is larger. The space between the two hands is how far the minute hand would turn in 24 minutes. The space in the 11:20 angle looks a little smaller, maybe 23 minutes.
   - The angle showing 11:20 is larger. If the hour hand stays where it is, the minute hand has to turn more than half a hand toward the _____ hand, like the direction it really moves on the clock. Others thought about the minute hand starting at the same spot as the hour hand and moved it in either direction.)

- "How did this change which angle they thought was larger?" (It changed how far they thought one ray turned.)
- "When we describe an angle to others, we often draw an arc, or part of a circle, between the rays to show which turn we are talking about."

circle to get to it. For the 2:50 angle, the minute hand would turn less than half a circle to get to the hour hand.

d. Sample responses:
   - The angles look the same size. One clock hand has to turn two and a half numbers over (or $2\frac{1}{2}$ hours or 12 minutes) to meet the other hand.
   - The 8:58 angle is larger. The minute hand has to turn almost all the way around to reach the hour hand.

2. Sample responses: The angle is:
   - a little larger than a square corner
   - the space between 3 tick marks plus a little more
   - about a third of a turn around a circle
   - the size of 3-and-a-half hours
   - the same amount of space traveled by the minute hand in 18 minutes

Lesson Synthesis

10 min

"Today we used a clock to compare angles. We described the size of an angle as the amount of turn one ray makes from another ray that is fixed at the shared endpoint."

"What were some different ways you described 'how much' a ray turned from another, or 'how much' bigger or smaller one angle was than another?" (We used the tick marks on the clock. We know they represent minutes on a clock, so we just called them minutes. For some angles, we just used the large numbers to describe the turn. We know those mean the hours on a clock, so we called them hours.)

As needed, prompt students to use examples from the last activity.

Suggested Centers

- Compare (1–5), Stage 5: Fractions (Supporting)

- Target Measurements (2–5), Stage 4: Degrees (Addressing)

Complete Cool-Down

Response to Student Thinking

Students determine the larger angle but do not use precise language to describe how much larger.

Next Day Support

- Before the next day's warm-up, pair students up to discuss their responses.

Lesson 8: The Size of Angles in Degrees

Standards Alignments

Building On	4.NBT.A.1
Addressing	4.MD.C.5.a, 4.MD.C.7
Building Towards	4.MD.C.5, 4.MD.C.6, 4.MD.C.7

Teacher-facing Learning Goals

- Understand that the measure of a full rotation of a ray at a fixed point is 360 degrees.
- Use benchmark angle measurementes (such as 90°, 180°, 270°, 360°) to reason about and estimate the size of angles in degrees.

Student-facing Learning Goals

- Let's describe the size of angles using degrees.

Lesson Purpose

The purpose of this lesson is for students to understand that angles can be measured in degrees and use benchmark angle measurements to make sense of the new unit.

Previously, students used the hands of an analog clock to describe and compare the size of angles. In this lesson, students learn about degrees as a unit for measuring angles.

In the first activity, students are introduced to 360 degrees as the measurement of a full rotation of a ray about a fixed point. They use this to interpret and describe other benchmark angle measurements (90, 180, 270). They then use these benchmarks to estimate and sketch new angles with given measurements in degrees.

Next, students use these reference angles to create an angle measurement tool from paper. They partition the straight angle of a semi-circle into into smaller angles by folding. In doing so, they draw from their experience with the clock, where each hour or each minute can be thought of as equal-size parts around the center point of the clock.

Throughout the lesson, listen for the way students make connections to their work with clocks and to their understanding of fractions of a circle as they reason about how to estimate and sketch angles in degrees using an understanding that a full rotation is 360 degrees.

Instructional Routines

What Do You Know About _____? (Warm-up)

Materials to Gather

- Paper: Activity 2
- Rulers or straightedges: Activity 2

Materials to Copy

- Making a Measuring Tool (groups of 3): Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

How did students connect the angles they created in the second activity to the fractions of a circle? How can you help students make connections between degrees and a fraction of a circle in upcoming lessons?

Cool-down (to be completed at the end of the lesson)

5 min

Estimate Angle Size in Degrees

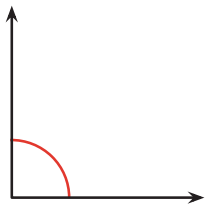
Standards Alignments

Addressing 4.MD.C.5.a

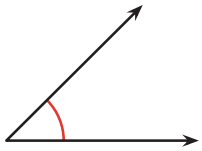
Student-facing Task Statement

Use the tool you created to estimate the size of each angle in degrees.

a



b



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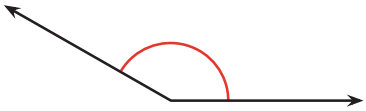


Student Responses

1. 90 degrees
2. 45 degrees

3. 150 degrees

Begin Lesson

Warm-up

10 min

What Do You Know about 360?

Standards Alignments

Building On	4.NBT.A.1
Building Towards	4.MD.C.5, 4.MD.C.7

The purpose of this warm-up is to invite students to think about 360 in terms of related numbers—as a result of addition, subtraction, multiplication, or division. The reasoning done here will be helpful when students compose angles into a sum of 360° or decompose a 360° angle into smaller angles, particularly benchmark angles such as 30°, 60°, 90°, and 180°.

Instructional Routines

What Do You Know About _____?

Student-facing Task Statement

What do you know about 360?

Student Responses

Sample responses:

- It is 40 less than 400, and 60 more than 300.
- It is 10 times 36, twice 180, and half of 720.
- It is a multiple of 10, 20, 30, 40, 60, 90, 120, 180.
- It is 2×180, 3×120, 4×90, 6×60, 9×40, 10×36, 12×30, 15×24, 18×20.

Launch

- Display the number.
- "What do you know about 360?"

Activity

- 1 minute: quiet think time
- Record responses.
- If no students mentioned different ways to express 360, ask: "How could we express the number 360?" and "What do you know about the factors of 360?"

Synthesis

- Draw students' attention to the factors of 360. "What are the factors of 360? How many are there?"
- "The number 360 and its factors are important when describing angles. Let's find out why they show up again and again as we look at a new way to describe and measure the size of angles."

Activity 1

15 min

A Full Turn

Standards Alignments

Addressing 4.MD.C.5.a, 4.MD.C.7

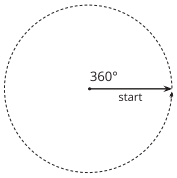
In previous activities, students used the features of a clock to describe and compare angles. This activity introduces students to **degree** as a unit of measure.

Because one degree is much more abstract than one inch or one square inch tile, students are first introduced to 360 degrees as a full rotation of a ray around its endpoint. Students use this information to reason about other angle measurements. They may use their understanding of fractions of a circle to determine the sizes (MP7).

In the synthesis, students describe angle measurement as additive. Students will continue to build this understanding and reason about the size of 1 degree in subsequent lesson activities.

Student-facing Task Statement

A ray that turns all the way around its endpoint and back to its starting place has made a full turn.

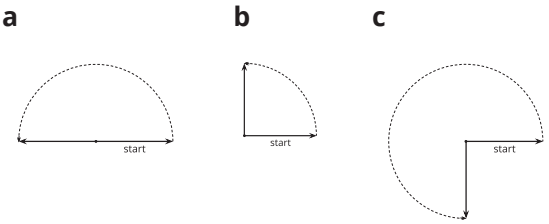


We say that the ray has turned 360 **degrees**.

Launch

- Groups of 2
- "Just like length and area can be measured in standard units, angles have standard units."
- "One standard unit for measuring the size of angles is **degree**."
- Display the image of the ray turning 360

1. How many degrees has the ray turned from where it started?



2. Sketch two angles:
   a. an angle where a ray has turned 50°
   b. an angle where a ray has turned 130°

Student Responses

1. a. 180 degrees
   b. 90 degrees
   c. 270 degrees
2. Students' sketches need not be precise, but should show reasonable estimates of an angle that is between 40 and 60 degrees for the 50-degree angle and between 120 and 140 degrees for 130-degree angle.

degrees and read the opening task statement.

Activity

- "Use this information to determine how many degrees each ray has turned from where it started. Then sketch some angles that are about the size given in degrees."
- 3–4 minutes: independent work time
- "Compare your thinking with your partner. Explain how you made your estimates."
- 2–3 minutes: partner discussion
- Monitor for students who:
  - describe the 180 degree turn as half a full turn or turning half a circle
  - describe the 90 degree angle as half of half a turn or as a turning a fourth of the way around the circle
  - use their estimate for the 90 degree angle to estimate the 270 degree angle by adding ($90 + 90 + 90$ or $180 + 90$) or by subtracting ($360 - 90$)

Synthesis

- Invite previously identified students to share how they estimated the turn of each ray in degrees.
- "How were your methods the same? How were they different?" (We all used the first diagram to figure out how much of a turn. Some of us used one measure to find the next measure. Some of us thought about addition and some thought about division).
- Consider displaying the equation $90 + 90 + 90 = 270$ and discussing:
  - "How does this match a way to estimate the third angle?"
  - "What's another equation we could use to describe that angle?"

- "An angle that measures 90 degrees is called a **right angle**."
- "Where have you seen right angles before?" (Corners of squares or rectangles. Corners of paper. Angle made by the hands of a clock when it is 3 o'clock or 9 o'clock.)

Activity 2

20 min

Make a Measuring Tool

Standards Alignments

Addressing	4.MD.C.5.a, 4.MD.C.7
Building Towards	4.MD.C.6

In this activity, students construct a protractor-like tool that shows some benchmark angles. They do so by halving given angles—120° and 180°—and then of subsequent angles identified along the way.

The activity serves several goals. The first is to familiarize students with the structure of a protractor using tactile processes (folding paper and aligning lines or edges). The second goal is to develop students' intuition for thinking of a larger angle as composed of smaller angles, preparing them to see (in future lessons) that a 1° angle is $\frac{1}{360}$ of a full turn. A final goal is to motivate the need for a tool that can measure angles more precisely.

Some students may use a square corner of a sheet of paper to find a 90° angle on their semi-circle and others may choose to fold their semi-circle in half. Expect most students to fold their paper to find all subsequent angles.

Materials to Gather

Paper, Rulers or straightedges

Materials to Copy

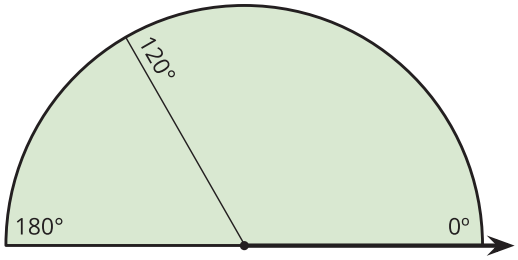
Making a Measuring Tool (groups of 3)

Required Preparation

- Create a paper half-circle from the Instructional master for each student.

Student-facing Task Statement

Your teacher will give you a sheet of paper in the shape of half a circle. It shows a 120° angle and a 180° angle from the ray on the bottom right.



On the half-a-circle paper:

1. Draw a line segment to show a 90° angle from the same ray. Label it with the measurement. Be as precise as possible.
2. Draw lines to show the following angles (measured from the same ray). Label each line with the measurement.

60° 45°

30° 150°

135°

3. Can you find a 1° angle from that same ray? Explain or show how you might do it.
4. You just made a measuring tool!

   How can it be used to estimate the size of an angle? Discuss your ideas with your group. Then, use your tool to estimate the sizes of at least two angles.

Launch

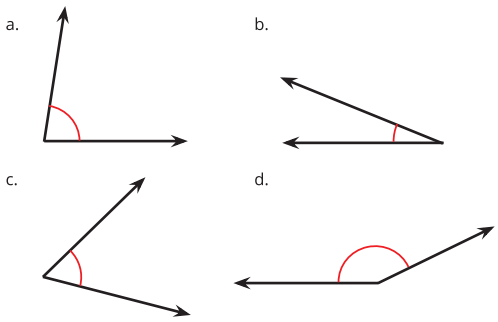
- Groups of 2–4
- Give each student one paper half-circle and access to rulers or straightedges.
- "Your sheet of paper is in the shape of half a circle. It shows a ray on the bottom right and two angles (120° and 180°) measured from the ray."
- "We see the 120° label. Where is the 120° angle? Where are the two rays that make this angle?"
- 1 minute: quiet think time for the first problem
- 1 minute: group discussion
- "Where do you think the second ray of a 90° angle would be?" (Between 0 and 120, but closer to 120.)

Activity

- 5–7 minutes: independent work time
- As students work on the last problem, monitor their ideas for using their tool to estimate angle measurements.

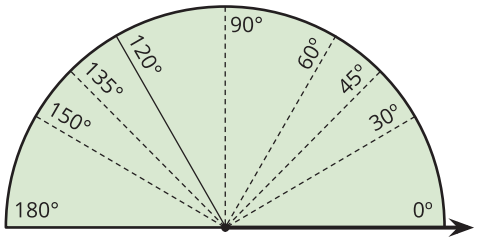
Synthesis

- "How did you find a 90° angle?" (I folded the semi-circle into halves.)
- "How did you find all the other angles?" (For 60°, fold to line up the thick ray with the 120° line, splitting 120 into two. For 45°, fold to line up the ray with the 90° line, splitting 90 into two. Repeat in a similar fashion to find the others.)
- Invite students to share how they might find a 1° angle on their half circle. Highlight explanations that involve finding some fraction of increasingly smaller and smaller angles.
- Solicit some estimates of the angle measurements in the last problem. Record



Student Responses

1. See image.
2. See image.



3. Sample responses:
   a. Split 30° into three equal parts to get 10° each. Split the 10° into two parts of 5° each. Break each 5° angle into five parts of 1°.
   b. It can't really be done—not precisely, anyway. The angle is too small to find by folding.
4. Sample estimates:
   a. 80°
   b. 25°
   c. 60°
   d. 155°

and display them for comparison later, when the same four angles are measured using a protractor.

- Students are likely to notice that their tool is imprecise and is not reliable or practical for measuring angles beyond estimations. Explain that we will look at another tool in the next activity.

Advancing Student Thinking

Students may create angles that are not precise when they estimate where to draw a new line segment. Ask the students to explain how they determine where to draw a line segment and suggest folding as a strategy. Consider asking:

- "Do you think the angle you need to create is smaller or larger than those that you have already drawn? How much smaller or larger?"
- "How might folding help you create more precise angles during this task?"

Lesson Synthesis

10 min

"In earlier lessons, we used clocks to help us compare angles and talk about their size. Today, we learned that angles can be measured in degrees and that some angles are helpful for estimating the size of other angles."

"What did you learn about 360°?" (It's the measurement of the angle made by a ray that makes a full turn around a point.)

"How would you describe a 180° angle?" (It's half of a full turn. The two rays make a straight line or point in opposite directions.)

"How would you describe a 90° angle?" (It's a half of a half turn. It's the size of an angle made when a ray makes a fourth of a full turn around a point. It is called a **right angle**. It's the size of angle at the corners of a piece of paper.)

Suggested Centers

- Compare (1–5), Stage 5: Fractions (Supporting)
- Target Measurements (2–5), Stage 4: Degrees (Addressing)

Complete Cool-Down

Response to Student Thinking

Students find angle measurements that are more than 5 degrees greater or less than 90, 45, or 150.

Next Day Support

- Launch Activity 1 with a discussion about this cool-down.

Lesson 9: Use a Protractor to Measure Angles

Standards Alignments

Addressing 4.MD.C.5.a, 4.MD.C.5.b, 4.MD.C.6, 4.MD.C.7, 4.NBT.B.5

Teacher-facing Learning Goals

- Recognize that 1 degree is a measurement of a $\frac{1}{360}$ turn through a full circle.
- Use tools to measure angles.

Student-facing Learning Goals

- Let's use some tools to measure angles.

Lesson Purpose

The purpose of this lesson is for students to understand 1° as a measurement of a turn through $\frac{1}{360}$ of a circle and to use a protractor to measure angles.

Before this lesson, students have compared and measured angles using informal tools (analog clocks) and reference angles, all of which were multiples of 5° or multiples of 10°. In this lesson, students transition to measuring angles in units of 1°.

Students then make sense of one-degree angles in terms of a fraction of a 360° turn and are introduced to the **protractor** as a tool of measurement. They make sense of the numbers on the tool and how 1° angles are shown. They learn to read the measurement of angles whose vertices have been pre-aligned to the center point of a protractor.

Students will continue to add new vocabulary to their personal word walls. In the next lesson, students will further develop their ability to use a protractor by measuring a variety of angles with less support.

Access for:

Students with Disabilities

- Action and Expression (Activity 2)

English Learners

- MLR2 (Activity 1)

Instructional Routines

MLR1 Stronger and Clearer Each Time (Activity 2), True or False (Warm-up)

Materials to Gather

- Protractors: Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

The first two activities offered opportunities for students to take multiple solution paths. Were all unique perspectives or strategies heard? Which students were able to share their ideas? Which didn't get a chance? How can their voices be added into the conversation tomorrow?

Cool-down (to be completed at the end of the lesson)

5 min

Measure the Angles

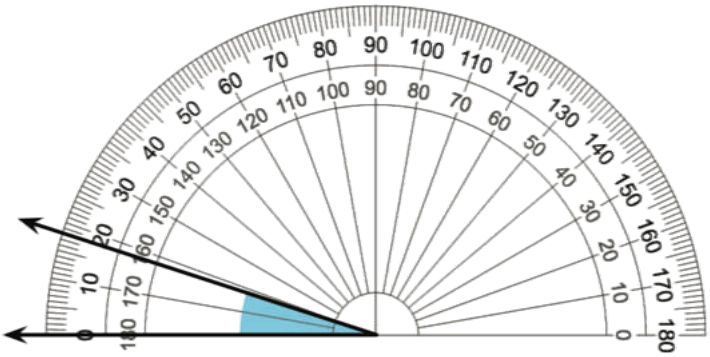
Standards Alignments

Addressing 4.MD.C.5.b, 4.MD.C.6, 4.MD.C.7

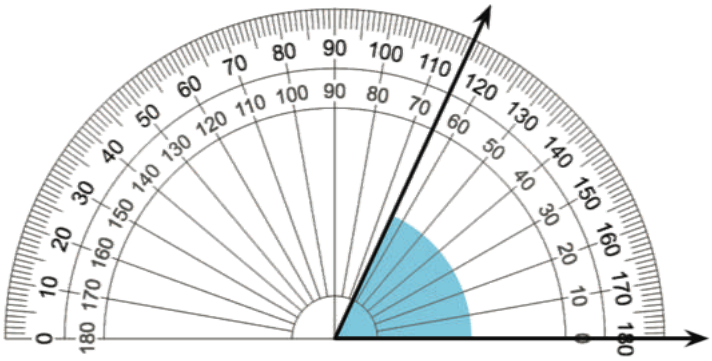
Student-facing Task Statement

1. An angle is composed of seventeen 1° angles. How many degrees is the angle?
2. What is the measurement of each angle?

a.



b.



Student Responses

1. 17°
2. a. 18°
   b. 65°

Begin Lesson

Warm-up

10 min

True or False: There's Something about 45

Standards Alignments

Addressing 4.NBT.B.5

The purpose of this warm-up is to draw students' attention to the first few multiples of 45, which will be helpful as students continue to work with benchmark angles and use a protractor to measure angles. Students have the skills to perform the multiplication in each equation, but computing each product may be time-consuming. Students can more efficiently tell if the equations are true or false if they consider properties of operations and look for and make use of structure.

Instructional Routines

True or False

Student-facing Task Statement

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $2 \times 45 = 6 \times 15$
- $4 \times 45 = 2 \times 90$
- $3 \times 45 = 180 - 90$
- $6 \times 45 = 45 + 90 + 135$

Student Responses

- True: the 45 on the left side is 3×15 and the 6 on the right is 2×3. Both sides are $2 \times 3 \times 15$.
- True: 2×90 is $2 \times 2 \times 45$, which is equal to 4×45.
- False: the right side is 90, which is 2×45, so the 3×45 on the left side cannot also be 90.
- True: the right side is $(1 \times 45) + (2 \times 45) + (3 \times 45)$, which is equal to 6×45.

Launch

- Display one equation.
- "Give me a signal when you know whether the equation is true and can explain how you know."
- 1 minute: quiet think time

Activity

- Share and record answers and strategy.
- Repeat with each statement.

Synthesis

- Some students may notice that it is handy to think in terms of 2×45 because it would mean dealing with multiples of 90 rather than multiples of 45. Highlight their explanations.
- If no students decomposed expressions such as 3×45, 4×45, and 6×45 into sums of 1×45 and 2×45, discuss how this could be done. (See sample responses.)

Activity 1

15 min

How Large is a 1° Angle?

Standards Alignments

Addressing 4.MD.C.5.a, 4.MD.C.5.b, 4.MD.C.6, 4.MD.C.7

This activity develops an understanding of the degree as a unit used to measure angles and introduces students to the protractor.

By now students have encountered many angle measurements and have some intuitive awareness of the sizes relative to a full turn (360°), half of a full turn (180°), and a quarter of a full turn (90°). In this activity, students learn that a 1° angle is $\frac{1}{360}$ of a full turn and that an angle that is composed of n 1° angles has a measurement of n°. For example, a 7° angle is made up of seven 1° angles.

Access for English Learners

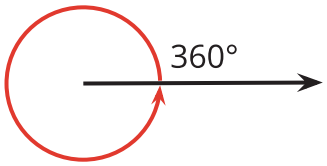
MLR2 Collect and Display. Synthesis: Direct attention to words collected and displayed from the previous lesson. Invite students to borrow language from the display as needed, and update it throughout the lesson.
Advances: Conversing, Reading

Materials to Gather

Protractors

Student-facing Task Statement

1. A ray that turns all the way around its endpoint and back to its starting place has made a full turn or has turned 360°.



What fraction of a full turn is each of the following angle measurements?

   a. 120°
   b. 60°
   c. 45°
   d. 30°
   e. 10°
   f. 1°

2. Your teacher will give you a **protractor**, a tool for measuring the number of degrees in an angle.

   a. How is 1° shown on the protractor?
   b. How many 1° measurements do you see?

3. A protractor with no numbers has been placed over an angle.

Launch

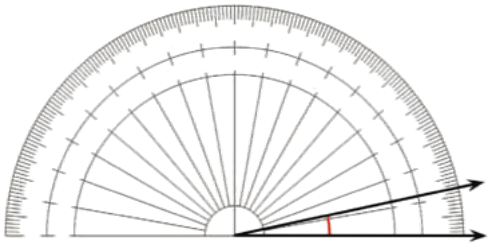
- Groups of 2
- 2 minutes: independent work time to find the fractions of a full turn
- Record students' responses. Solicit some ideas on how large a 1° angle is. Emphasize that a 1° angle is $\frac{1}{360}$ of a turn and that it's the size of the angle formed by the sides of the pieces of the circle created if we cut a full circle into 360 equal parts.
- "To measure angles in degrees, we can use a **protractor**." (Consider displaying different types of protractors.)
- Give each student a protractor.
- "Compare this tool to the one you used in the previous lesson. How are they the same? How are they different?" (They are both semi-circles. They both show angles like 0°, 30°, 45°, and 90°. The protractor is transparent or has a hole, while the paper version is solid. The protractor shows many more lines or tick marks and more numbers around the curve.)

Activity

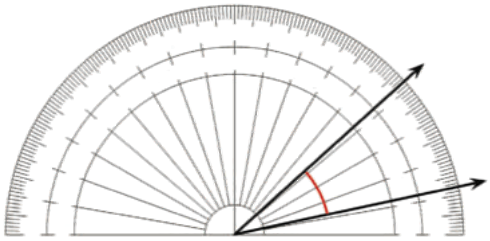
- 2–3 minutes: group work time on the second set of questions.
- Monitor for students who look for structure

- The center of the protractor is lined up with the vertex of the angle.
- The straight edge of the protractor is lined up with a ray of the angle.

How many degrees is this angle? Explain how you know.



4. An angle contains thirty 1° angles, as shown. How many degrees is this angle?



Student Responses

1. a. $\frac{1}{3}$ of a turn
   b. $\frac{1}{6}$ of a turn
   c. $\frac{1}{8}$ of a turn
   d. $\frac{1}{12}$ of turn
   e. $\frac{1}{36}$ of a turn
   f. $\frac{1}{360}$ of a turn
2. Sample response:
   a. It is the space between two neighboring tick marks, which is pretty small.
   b. 180 (or 360, if using a circular protractor)

to find the number of 1° increments on the protractor (for example, noticing that every group of ten 1° increments are marked and counting those instead of counting individual tick marks).

- Pause for a discussion. Make sure students see that a 1° angle on the protractor results when we draw rays through a pair of neighboring tick marks.
- 2–3 minutes: individual or group work time on the remaining questions

Synthesis

- "How did you find out the size of the two angles when the protractors show no numbers or scales?" (We know that the turn from one tick mark to the next is 1°. We can use the tick marks to count the number of 1° turns. We can imagine the tick marks split the angle into a number of 1° angles. We can count each 1°angle to find the measurement.)

3. 12°. Sample response: I counted the number of spaces between tick marks and there are 12.
4. 30°

Activity 2

20 min

Use a Protractor

Standards Alignments

Addressing 4.MD.C.5.b, 4.MD.C.6

In this activity, students learn how to use a protractor. They align a protractor to the vertex and a ray of an angle so that its measurement can be read. The given angles are oriented in different ways, drawing students' attention to the two sets of scales on a protractor. Students need to consider which set of numbers to pay attention to and think about a possible explanation for when or how to use each scale. Moreover, one of the scales is only marked in increments of 10 degrees so if students use this scale they need to reason carefully about the precise angle measure (MP6).

This activity uses *MLR1 Stronger and Clearer Each Time. Advances: reading, writing.*

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Invite students to estimate the size of the angle before finding each precise measurement. Offer the sentence frame: "This angle will be greater than _____ and less than _____. It will be closer to _____."
Supports accessibility for: Conceptual Processing, Visual-Spatial Processing, Attention

Instructional Routines

MLR1 Stronger and Clearer Each Time

Student-facing Task Statement

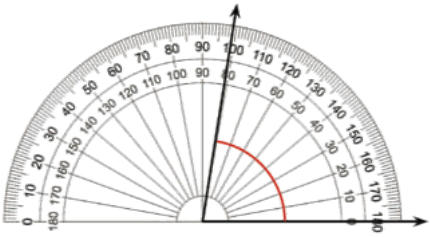
1. Here are four angles whose sizes you may have estimated earlier. A protractor has

Launch

- Groups of 2–4

been placed over each angle. Measure the size of each angle in degrees.

a.



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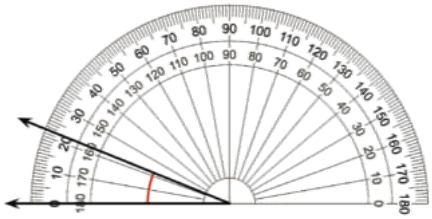
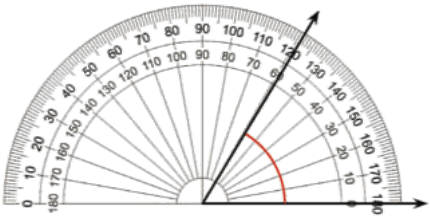
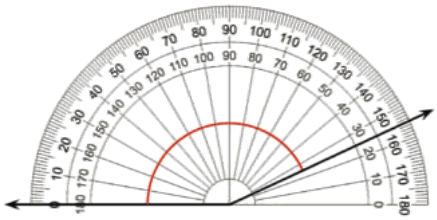


c.



d.



2. Elena and Kiran are measuring an angle with a protractor. Elena says the angle is 80°. Kiran says it shows 100°. Why might they end up with different measurements? Which one is correct? Explain your reasoning.

Activity

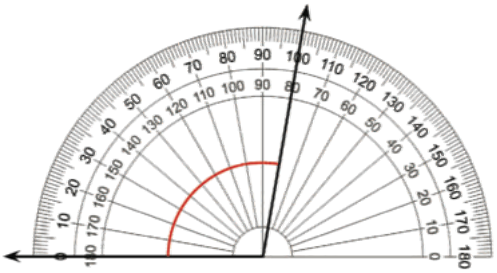
- 5 minutes: independent work time
- 2–3 minutes: group discussion
- Monitor for students who find each measurement by:
  - looking for the scale with a ray on 0° and counting up on that scale
  - subtracting the two numbers (on the same scale) that the rays pass through

MLR1 Stronger and Clearer Each Time

- "Share your response to the last problem with your partner. Take turns being the speaker and the listener. If you are the speaker, share your ideas and writing so far. If you are the listener, ask questions and give feedback to help your partner improve their work."
- 3–5 minutes: partner discussion
- Repeat with 2–3 different partners.
- "Revise your initial draft based on the feedback you got from your partners."
- 2–3 minutes: independent work time

Synthesis

- Select students who used different strategies to share their responses and reasoning.
- Display the image of Elena and Kiran's angle. If no students mentioned that 80° is not possible because the angle clearly appears greater than a right angle (90°), consider asking how Elena can use what she knows about right angles to think about her measurement.



Student Responses

1. a. 81°
   b. 22°
   c. 59°
   d. 155°
2. There are two sets of numbers on the protractor. Elena and Kiran used different scales. The angle is 100°. Sample response:
   - One ray of the angle is aligned with 0° of the outside scale, so we count up from 0 to 100, using the numbers on the outside scale. If we use the inside scale, we need to count down from 180 to 80, which is a difference of 100.
   - The angle is greater than a right angle, so it cannot be 80°.

Lesson Synthesis

10 min

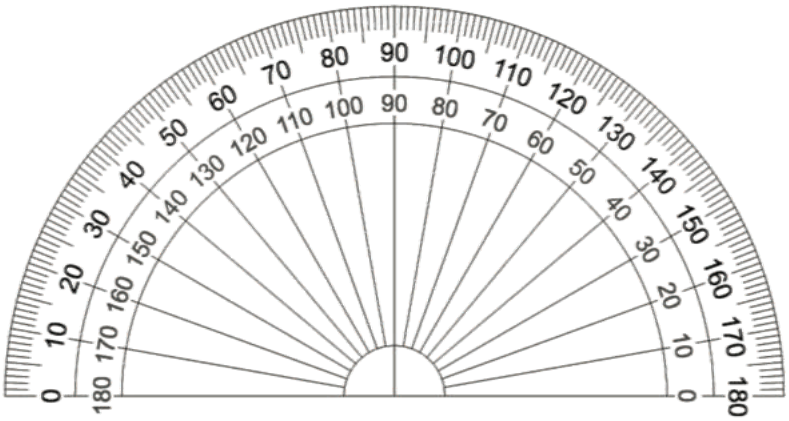
"Today, we used 1° angles and a protractor to measure the size of angles."

"What do you know about a 1° angle?" (It's $\frac{1}{360}$ of a full turn of a ray through a circle. It's a small angle compared to the other angles we have seen. It's $\frac{1}{180}$ of a half turn of a ray through a circle, like on a protractor.)

"How can 1° angles tell us the size of other angles?" (It's a smaller angle, so we can use it to be more precise when we measure or compare angles. We can count or find the number of 1° angles in an angle to find its measurement.)

"How would we know how many 1° angles are in another angle?" (We can use a protractor, which is marked with 180 or 360 one-degree angles.)

Display:



"We saw that a **protractor** has two sets of numbers. How do you know which set of numbers to use when measuring an angle?" (Either set could be used, but it is easier to use the set that counts up from 0 rather than count down from 180.)

"Take 1–2 minutes to add any new words from today's lesson to your word wall. Share your new entries with a neighbor and add any new ideas you learn from your conversation."

Suggested Centers

- Target Measurements (2–5), Stage 4: Degrees (Addressing)
- Compare (1–5), Stage 5: Fractions (Supporting)

Complete Cool-Down

Response to Student Thinking

Students find a measure other than 17°, 18°, or 65° degrees.

Next Day Support

- Launch the lesson by reviewing the use of a protractor and how to read it.

Lesson 10: Angle Measurement and Perpendicular Lines

Standards Alignments

Addressing 4.G.A.1, 4.MD.C.5.b, 4.MD.C.6, 4.NBT.B.6

Teacher-facing Learning Goals

- Recognize that perpendicular lines meet or cross at a right angle.
- Use a protractor to measure angles.

Student-facing Learning Goals

- Let's measure all kinds of angles.

Lesson Purpose

The purpose of this lesson is for students to practice using a protractor to measure angles and to learn that lines that intersect at a right angle are **perpendicular lines**.

In a previous lesson, students learned that a 1-degree angle measures a turn through $\frac{1}{360}$ of a circle and that a protractor could be used to measure angles in degrees.

In this lesson, students practice using a protractor to measure a variety of angles—angles formed by rays or line segments and those that are in other two-dimensional figures. Students consider how to position the tool, which set of numbers to use, and whether their measurements make sense.

Next, students are prompted to fold paper to create two lines that form right angles. They learn that intersecting lines that form 90° angles are **perpendicular**. They then practice identifying perpendicular lines and segments.

Access for:

Students with Disabilities

- Representation (Activity 2)

English Learners

- MLR8 (Activity 1)

Instructional Routines

MLR2 Collect and Display (Activity 2), Number Talk (Warm-up)

Materials to Gather

- Colored pencils: Activity 2
- Paper: Activity 2

- Protractors: Activity 1
- Rulers or straightedges: Activity 1, Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

In the past few lessons, students may have made assumptions (correct or incorrect) about angle sizes based on their appearance. In this lesson, they may have done the same when looking for perpendicularity. What questions can you ask, or what instructional moves can you consider, to encourage students to check their assumptions?

Cool-down (to be completed at the end of the lesson)

5 min

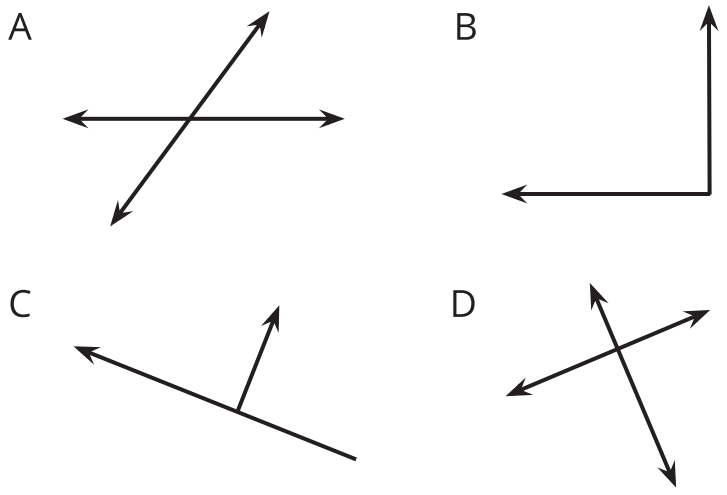
Size Up Angles

Standards Alignments

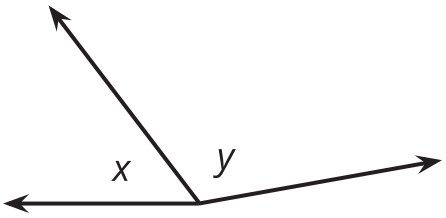
Addressing 4.G.A.1, 4.MD.C.5.b, 4.MD.C.6

Student-facing Task Statement

1. Which figures show perpendicular lines or rays?



2. Use a protractor to measure the labeled angles in the figure.



Student Responses

1. B, C, and D
2. Angle x is 53°. Angle y is 117°.

Begin Lesson

Warm-up

10 min

Number Talk: Quotients

Standards Alignments

Addressing 4.NBT.B.6

The purpose of this Number Talk is to encourage students to make use of structure to perform division involving increasingly larger dividends and divisors (MP7). The reasoning here helps students to develop fluency in finding quotients of multi-digit numbers. It also reinforces students' familiarity with factors of 180 and 360, which will be helpful as they continue to work with angle measurements.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

Launch

- Display one expression.

- $180 \div 2$
- $180 \div 4$
- $360 \div 8$
- $360 \div 16$

Student Responses

- 90: 2×90 is 180.
- 45: $180 \div 4$ is $(180 \div 2) \div 2$ or $90 \div 2$.
- 45: $360 \div 4$ is 90 and $90 \div 2$ is 45.
- $22\frac{1}{2}$: if the divisor is doubled (from $360 \div 8$ to $360 \div 16$), then the quotient is halved.

- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "How are the expressions related?"
- "How did finding the value of one expression help you find the value of the next expression?"
- Consider asking:
  - "Who can restate _____'s reasoning in a different way?"
  - "Did anyone have the same strategy but would explain it differently?"
  - "Did anyone approach the expression in a different way?"
  - "Does anyone want to add on to _____'s strategy?"

Activity 1

15 min

Angles Here, There, Everywhere

Standards Alignments

Addressing 4.MD.C.6

In previous lessons, students learned to read the measurement of an angle with a protractor already in position. In this activity, students practice using a protractor to measure angles. They decide where to place the tool, how to align it with the vertex and rays of the angle, and which set of numbers on the protractor to use.

Some of the figures in the activity explicitly show angles formed by two rays. In others, students are asked to find and measure the angles within polygons. In both cases, students may find it necessary to extend one or both rays of an angle so that it can be measured more effectively or precisely (MP6). Doing so reinforces the idea that the size of an angle is not determined by the length of the segments that frame it, but by the rays that compose the angle.

Access for English Learners

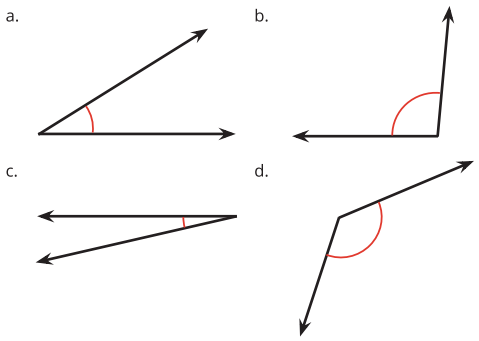
MLR8 Discussion Supports. Synthesis: For each strategy that is shared using the protractor, invite students to turn to a partner and restate what they heard using precise mathematical language. *Advances: Listening, Speaking*

Materials to Gather

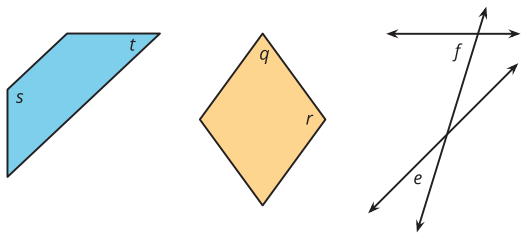
Protractors, Rulers or straightedges

Student-facing Task Statement

1. Use a protractor to find the value of each angle measurement in degrees.



2. Use a protractor to measure the labeled angles in each figure.



Launch

- Groups of 2
- Give each student a protractor and access to rulers or straightedges.

Activity

- 5 minutes: independent work time
- 1–2 minutes: partner discussion
- Monitor for students who:
  - align the rays of the angle to tick marks on the protractor and count from one ray to the other
  - don't align either ray of an angle to 0° or 180° on the protractor and instead find the difference of the numbers where the two rays land on the protractor
  - always align one ray of an angle with the 0° or 180° line on the protractor and always read from the scale that starts with 0°

Student Responses

1. a. 32°
   b. 95°
   c. 13°
   d. 131°
2. Angle s is 134°. Angle t is 43°. Angle q is 72°. Angle r is 108°. Angle e is 28°. Angle f is 73°.

Synthesis

- Display the angles and figures. Invite previously identified students to share their methods for measuring angles, in the order shown in the monitoring notes.
- "What are some benefits of aligning one ray of an angle to 0° on the protractor?" (The measurement can be identified right away: it's the number where the second ray lands on the protractor. Aligning the first ray to a non-zero number means having to count or subtract two numbers before finding the measurement.)
- "How was measuring the second set of angles like measuring the first set of angles?" (They both involve aligning the center point of a protractor to the vertex of the angle, and matching the 0° line on the protractor to one ray or segment of the angle. It can be helpful to extend one or both lines framing the angle.)
- "How was it different than measuring the first set?" (The two-dimensional shapes have other segments and angles nearby, so more attention was needed when placing the protractor and reading the measurement.)
- "How can you tell if your measurement was reasonable? How can you make sure your measurements make sense?" (Compare them against a familiar angle like 90°. If an angle looks larger than a right angle, it can't be less than 90°.)

Advancing Student Thinking

Students may carefully line up rays on the protractor, but find angle measurements that are unreasonable. Ask students to explain how they used the protractor to measure. Consider asking:

- "Based on the angles you have measured in previous lessons, does _____ degrees make sense as the measure of this angle? Why or why not?"

- "What is another way you could use the protractor to check your measurement?"

Activity 2

20 min

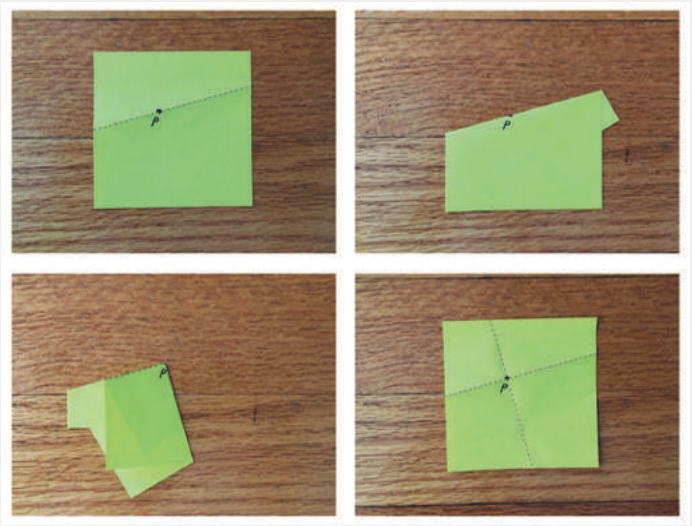
A Folding Challenge

Standards Alignments

Addressing 4.G.A.1, 4.MD.C.6

In this activity, students fold paper to form right angles and learn that intersecting lines that form 90° angles are **perpendicular lines**. They then identify perpendicular segments in two-dimensional figures and explain how they know the segments are perpendicular.

To create four right angles that share the same vertex by folding generally means making two folds through the same point. The first fold, which can be done in any way (as long as it goes through point P, in this case), creates two straight angles. The second involves folding through the point again such that the edges formed by the crease of the first fold match up exactly, creating two equal halves or two 90° angles.



While students have experience with folding paper to partition a shape or an angle, some may need support in folding precisely. Consider providing a straightedge to facilitate the folding.

This activity uses *MLR2 Collect and Display. Advances: conversing, reading, writing*

Access for Students with Disabilities

Representation: Internalize Comprehension. After students have had some time to infer the meaning of "perpendicular lines," pause the activity. Invite them to look around the room and share examples and non-examples of perpendicular lines.
Supports accessibility for: Visual-Spatial Processing, Memory, Attention

Instructional Routines

MLR2 Collect and Display

Materials to Gather

Colored pencils, Paper, Rulers or straightedges

Required Preparation

- Prepare at least 2 pieces of paper (or sticky notes) for each student.

Student-facing Task Statement

Tyler gave Lin a challenge: "Without using a protractor, draw four 90° angles. All angles have their vertex at point P."

Lin folded the paper twice, making sure each fold goes through point P. Then, she traced the creases.

1. Your teacher will give you a sheet of paper. Draw a point on it. Then, show how Lin might have met the challenge.
2. When Lin folded the paper, the creases formed a pair of **perpendicular lines**. What do you think "perpendicular lines" mean?
3. Use Lin's method to create a new pair of perpendicular lines through the same point. Trace the creases with a different color. Be prepared to explain how you know the lines you created are perpendicular.
4. Which shapes have sides that are

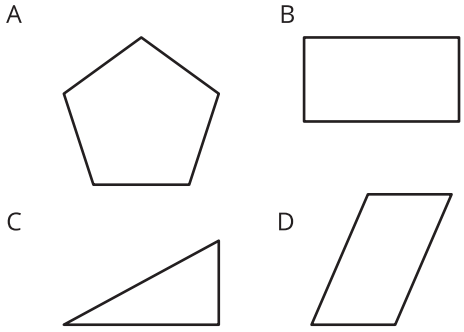
Launch

- Groups of 2–4
- Give each student 2 pieces of paper and colored pencils. Provide access to straightedges or rulers, in case requested.
- Read the opening prompts and the first question.
- "What do you think Lin did with her paper? Mark a point on a piece of paper and try folding it as Lin might have done."
- 2–3 minutes: quiet think time on the first problem
- Pause for a discussion. Invite a couple of students to share how they think Lin met the challenge.

Activity

- 6–7 minutes: group work on the remaining questions
- Circulate, listen for, and collect the

perpendicular to one another?

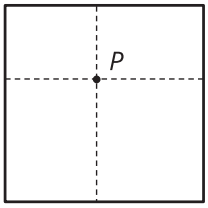


Mark the perpendicular sides. Be prepared to explain how you know the sides are perpendicular.

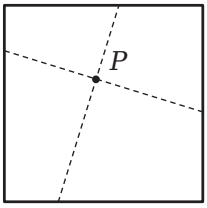
Student Responses

Sample responses:

1. Lin made two folds through point P. For the second fold, she made sure the folded edges on either side of point P match up exactly.



2. Lines that cross at a right angle.
3.



4. Figures B and C. Sample reasoning: I measured the angles in the figures using a protractor (or using the square corner of a sheet of paper).

language students use to define **perpendicular lines**.

- Record students' words and phrases on a visual display and update it throughout the lesson.
- Monitor for students who:
  - reason that their folded lines form right angles because the first fold makes two 180° angles through the point and the second fold splits each into halves, making 90° angles
  - use a protractor (or a square corner) to verify perpendicularity of the sides of shapes in the last problem (rather than relying on appearance)

Synthesis

- "What statement could we write to explain to another student what **perpendicular lines** are?" (Lines that intersect and create right angles.)
- Remind students that they can use words or phrases from their personal word walls in their responses.
- Invite students to share the perpendicular lines they created by folding.
- "How can you be sure that the creases from your folding are perpendicular or created 90° angles?" (My first fold makes two 180° angles. My second fold splits each of those into two equal angles, so each one must be 90°. We can measure each angle with a protractor.)
- "In the last problem, how did you know which shapes have perpendicular sides?" (By measuring the angles with a protractor, or by comparing them with a square corner.)

Advancing Student Thinking

When making their second fold, students may not align the two edges formed by the crease of the first fold, resulting in a pair of angles of one size and another pair of a different size (instead of four right angles). Consider asking: "How might you adjust your folding to create two equal angles?"

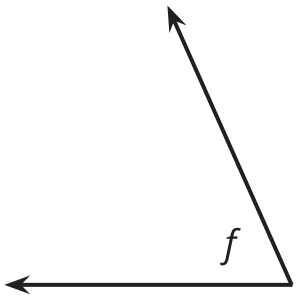
Students may say they do not see perpendicular sides in the last problem because, unlike the intersecting lines in the folded paper, the sides of the shapes do not cross. Consider asking:

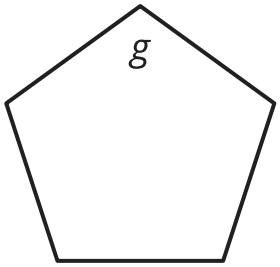
- "Where do the sides meet? What do you notice about where the sides meet in this figure?"
- "What if you extend each side to show the line that each segment is a part of? What do you notice?"

Lesson Synthesis

10 min

"Today you used a protractor to measure different angles. We also learned that intersecting lines that form right angles are **perpendicular.** Suppose you are to show a classmate who is absent today how to measure the angles labeled f and g." Display:





"How would you describe the process of measuring the angles so that it is clear to them?"

"Would your description for measuring angle g be different from that for angle f?"

"Are there any perpendicular lines in either of the diagrams? How can we tell?" (No, none of the angles are right angles.)

"Take 1–2 minutes to add the new words from today's lesson to your word wall. Share your new entries with a neighbor and add any new ideas you learn from your conversation."

Suggested Centers

- Target Measurements (2–5), Stage 4: Degrees (Addressing)
- Compare (1–5), Stage 5: Fractions (Supporting)

Complete Cool-Down

Response to Student Thinking

Students identify figure A as an example of perpendicular lines. Students find angle measurements other than 53° and 117°.

Next Day Support

- Launch warm-up or Activity 1 by highlighting key vocabulary from previous lessons.

Lesson 11: Use a Protractor to Draw Angles

Standards Alignments

Addressing 4.G.A.1, 4.MD.C.5.a, 4.MD.C.6, 4.MD.C.7

Teacher-facing Learning Goals

- Use a protractor to draw angles of given measurements.

Student-facing Learning Goals

- Let's draw some angles.

Lesson Purpose

The purpose of this lesson is for students to use a protractor to draw angles of given measurements.

In earlier lessons, students reasoned about angle measurements, learned to use a protractor, and measured given angles. They have sketched angles by referring to clock faces and sketched estimates of angles of a given size based on benchmark angles.

In this lesson, students use a protractor to draw angles of specified measurements (not limited to benchmark angles) and to verify the size of angles in their peers' drawings. They begin to use known angle measurements to reason about unknown measurements and notice relationships between the measurements of angles that share a common endpoint.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Action and Expression (Activity 1)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Estimation Exploration (Warm-up)

Materials to Gather

- Index cards: Activity 2
- Protractors: Activity 1, Activity 2
- Rulers or straightedges: Activity 1, Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What were some of the challenges students encountered when measuring and drawing angles? What support might help students overcome those hurdles?

Cool-down (to be completed at the end of the lesson)

5 min

A Ray or Two

Standards Alignments

Addressing 4.G.A.1, 4.MD.C.6, 4.MD.C.7

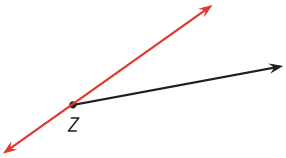
Student-facing Task Statement

1. Draw a new ray starting from point Z to create a 25° angle.

2. Draw two rays to create an angle that is 165°.

Student Responses

1. Sample responses:



2. Sample response:

Begin Lesson

Warm-up

10 min

Estimation Exploration: Long Hand and Short Hand

Standards Alignments

Addressing 4.MD.C.5.a

In this warm-up, students practice estimating a reasonable angle measurement based on their knowledge of angles so far and their familiarity with clocks. Later in the unit, students will take a closer look at the angles in an analog clock and apply their understanding of angles to solve more sophisticated problems.

Instructional Routines

Estimation Exploration

Student-facing Task Statement

How many degrees is the angle formed by the long hand and the short hand of the clock?

Launch

- Groups of 2
- Display the image.
- "What is an estimate that's too high? Too low? About right?"
- 1 minute: quiet think time

Activity

- 1 minute: partner discussion

Make an estimate that is:

too low	about right	too high

Student Responses

- Too low: 0°–90°
- About right: 130°–160°
- Too high: 170° or more

- Record responses.
- Draw an arc to show the clockwise turn of the minute hand from the hour hand (label the 143° measurement with an arc).
- "Your estimate should show the size of this angle in degrees. If you need to, revise your estimates."
- As needed, record any revisions.

Synthesis

- Consider asking:
  - "Is anyone's estimate less than _____? Is anyone's estimate greater than _____?"
  - "How did you go about making an estimate? How did you know that _____ must be too low and _____ must be too high?"
  - "Based on this discussion does anyone want to revise their estimate?"
- Consider revealing the actual measurement: 143°.

Activity 1

15 min

Draw These Angles

Standards Alignments

Addressing 4.MD.C.6

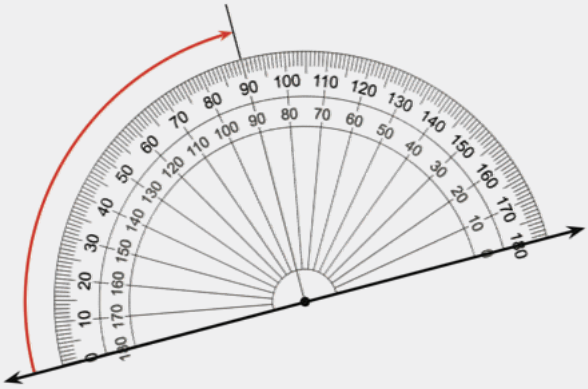
In this activity, students follow directions for drawing lines, rays, and angles. To create angles precisely and as specified, students need to use a protractor and a ruler or straightedge (MP6).

Each step in the drawing process involves one or more decisions for students to make. In some cases, the resulting drawing will be the same.

For example, in the first question, students could use the protractors in different ways to create perpendicular lines.

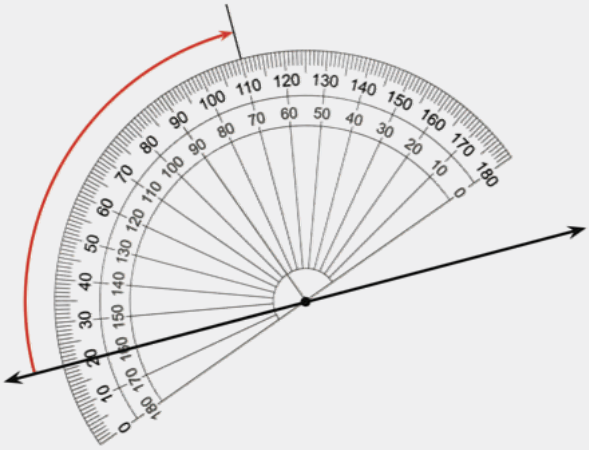
Going from 0° to 90°

(outer set of numbers):



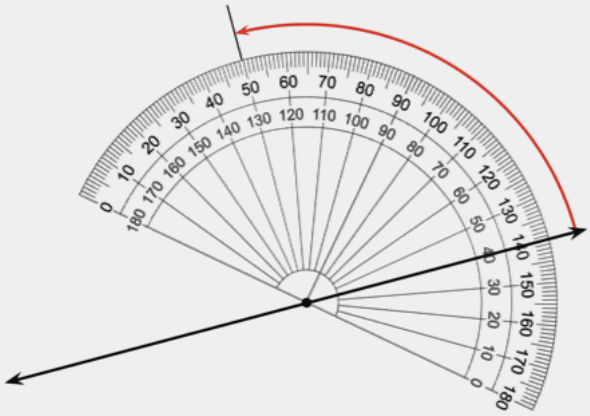
Going from 20° to 110°

(outer set of numbers):



Going from 40° to 130°

(inner set of numbers):



In other cases, the resulting drawings will vary depending on the decisions made. For example, in the second question, students could choose to draw the first angle (40°) above or below the given ray. When drawing the second angle (20°), they could choose to draw it inside the 40° angle or adjacent to the 40° angle (and choosing one side or the other)—in both cases meeting the specifications. The answer to the last question will thus also vary.

Access for English Learners

MLR8 Discussion Supports. Synthesis: Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Provide alternative options for expression. For example, invite students to work with a partner. One partner can draw and the other partner tells them what to do (for example, exactly how to move the protractor and where to draw points and lines).
Supports accessibility for: Visual-Spatial Processing, Language, Fine Motor Skills

Materials to Gather

Protractors, Rulers or straightedges

Student-facing Task Statement

1. Draw a line that is neither vertical nor horizontal. Put a point somewhere on that line. Use your protractor to draw a perpendicular line through that point. Be as precise as possible. (No folding this time!)
2. Here is a ray that starts at point M.

Use a protractor to draw:

a. A ray starting at point M to create a 40° angle.
b. Another ray starting at point M to create a 20° angle.
c. One more ray starting at point M to create a 95° angle. Label each angle

Launch

- Groups of 2
- Give each student a protractor and access to rulers or straightedges.
- 2 minutes: independent work time on the first question
- Pause for class discussion. Ask 1–2 students to share how they drew their perpendicular lines.

Activity

- 5–6 minutes: independent work time on the remaining questions
- 2 minutes: partner discussion
- Identify students with different-looking drawings to share later.

Synthesis

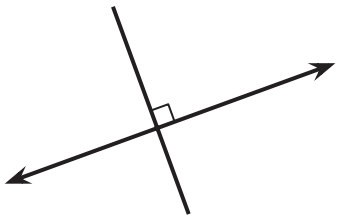
- Select students to share their drawings and their reasoning for the last question.
- "What decisions did you have to make

with its measurement.

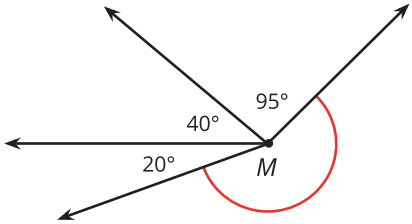
3. In your drawing, there should be one angle that is not labeled with a measurement and is larger than 180°. Label the angle with an arc. How many degrees is this angle? Be prepared to explain how you know.

Student Responses

1. Sample response:



2. Sample response:



3. Sample response: The unlabeled angle is 205°. The sum of the three known angles are $20 + 40 + 95$, which is 155°. Subtracting 155 from 360 gives 205.

when creating the drawing?"

- "Many of you placed the 20° angle next to the 40° angle. Some of you placed them inside the 40° angle. How did the different choices affect the size of the angle in the last question?" (Putting the 20° inside the 40° angle made the last angle 20° larger.)

Activity 2

20 min

Angles Made to Order

Standards Alignments

Addressing 4.G.A.1, 4.MD.C.6

In the first activity, students drew angles with some scaffolding in place: a line and a point were given, each step was described, and the vertex and measurements of each angle were specified.

In this activity, students continue to draw angles but with less guidance. For each drawing, students are given only a range of angle measurements and no other criteria, prompting them to make additional decisions about how to draw the angles (for instance, where the vertex of an angle should be, how the first ray or line should be oriented, and so on). After drawing, students trade their cards and use a protractor to measure and check one another's angles.

The drawings created here will be used in the next lesson. Consider collecting the cards from each group or otherwise supporting students in keeping the cards until then.

Materials to Gather

Index cards, Protractors, Rulers or straightedges

Student-facing Task Statement

Your teacher will give you some blank cards. Label them a–d.

1. On each card, draw an angle that meets one requirement. Use a ruler and a protractor.
    a. an angle that is less than 35°
    b. an angle that is between 35° and 80°
    c. an angle that is greater than 80° but less than 120°
    d. an angle that is greater than 120° but less than 180°
2. Trade cards with your partner.
    a. Measure and record each angle your partner drew. Check to make sure each angle meets the requirement.
    b. If a requirement is not met, return it to your partner so it can be corrected. Save the cards for the next lesson.

If you have time:

1. Create a drawing that shows several angles.

Launch

- Groups of 2
- Give each student one protractor and 4 blank (unlined) index cards.
- Give students access to rulers or straightedges.

Activity

- 7–8 minutes: independent work time on the first question, then switch cards and complete the second set of questions

Synthesis

- Invite students to share 1–2 examples of an angle that meets each requirement.
- Consider asking:
    - "Can you tell just by looking that this angle is _____?"
    - "If you say yes, explain."
    - "If you say no, what would you need to make sure it is _____?"

Then, write some descriptions of your drawing. Be as specific as possible.

2. Ask a partner to recreate the drawing based on your descriptions. Does their drawing turn out as you had drawn? If not, adjust your descriptions and ask them to try again.

Student Responses

1. Drawings vary.
2. No response required.

Lesson Synthesis

10 min

"Today we used protractors to draw angles of different sizes and to check one another's drawings."

"What were some challenges in drawing angles precisely?" (Some possible challenges:

- The distance between the closest tick marks, showing 1° angles, is very small. It's easy to misread the marks.
- If the first ray is not lined up correctly at 0° or 180°, or if the vertex is not lined up exactly at the center point of the protractor, then the created angle would be off.)

"In the last activity, you drew a bunch of angles, some smaller, some larger. Did you find some sizes of angles easier to draw than others? Why or why not?"

"If we were explaining to a classmate how to use a protractor to measure angles, what should we say?"

Suggested Centers

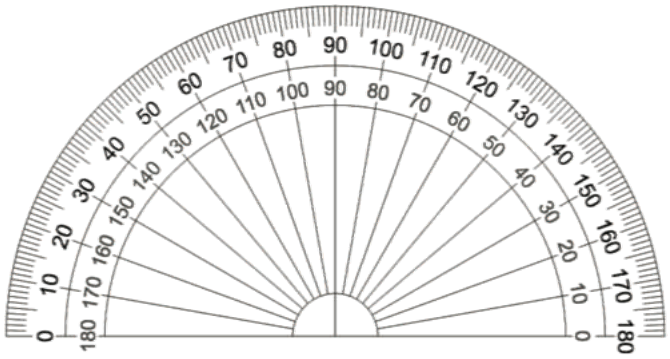
- Target Measurements (2–5), Stage 4: Degrees (Addressing)
- Compare (1–5), Stage 5: Fractions (Supporting)

Student Section Summary

In this section, we learned about ways to describe and measure the size of angles.

We used clocks to describe angles as a turn of one away from the other. We learned that a degree is a measure of the turn around a circle and that 1 degree is $\frac{1}{360}$ of a full turn of a ray through a circle.

Finally, we learned that a **protractor** is a tool used to measure angles and can also be used to create angles of a certain measure.



A protractor has two sets of numbers and that either set of numbers could be used, but it is helpful to use the set that counts up from 0 rather than count down from 180. We used a protractor to measure and draw different angles.

Complete Cool-Down

Response to Student Thinking

Students draw an angle that is larger or smaller than required.

Next Day Support

- Before the warm-up, pass back the cool down and work in small groups to make corrections.

Section C: Angle Analysis

Lesson 12: Types of Angles

Standards Alignments

Building On	4.NF.B.4.b
Addressing	4.G.A.1, 4.MD.C.7

Teacher-facing Learning Goals

- Draw acute and obtuse angles.
- Identify acute, obtuse, right, and straight angles in two-dimensional figures.

Student-facing Learning Goals

- Let's look at different types of angles.

Lesson Purpose

The purpose of this lesson is for students to classify angles as acute, obtuse, and straight, and to identify and draw acute and obtuse angles.

In previous lessons, students learned to measure angles and draw angles of given measurements. They saw that a 90° angle is called a right angle and is formed by two perpendicular lines. Students are also familiar with some benchmark angle measurements.

In this lesson, students classify angles by their size and identify angles as **acute** or **obtuse** in a variety of contexts. Students also learn that a 180° is called a **straight angle**.

Access for:

Students with Disabilities

- Representation (Activity 1)

Instructional Routines

MLR2 Collect and Display (Activity 1), Number Talk (Warm-up)

Materials to Gather

- Materials from a previous lesson: Activity 1

- Pattern blocks: Activity 3
- Protractors: Activity 2, Activity 3

Lesson Timeline

Warm-up	10 min
Activity 1	10 min
Activity 2	10 min
Activity 3	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

Before this point, students have had experiences of sorting mathematical objects relative to some benchmarks. In this lesson, angles are categorized by their size and relative to two benchmarks: 90° and 180°. How readily did students conceptualize angles this way? Which past experiences of classifying objects would help to make this idea more intuitive?

Cool-down (to be completed at the end of the lesson)

5 min

Obtuse, Acute, and Straight Angles

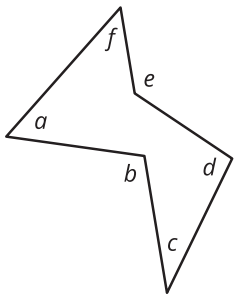
Standards Alignments

Addressing 4.G.A.1, 4.MD.C.7

Student-facing Task Statement

1. Here is a ray. Draw another ray from point P to make an acute angle.

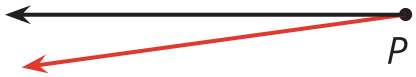
2. Here are some labeled angles. Identify all angles that are obtuse.



3. An angle is formed by four 35° angles. Is that angle a straight angle? Explain how you know.

Student Responses

1. Sample response:



2. Angles b, e, and d
3. No. Sample response: A straight angle is 180°. Four 35° angles make 140° ($4 \times 35 = 140$).

Begin Lesson

Warm-up

10 min

Number Talk: Fractions of 120 and 360

Standards Alignments

Building On 4.NF.B.4.b

This Number Talk elicits strategies that students have for multiplying a fraction by a whole number, and what they know about the size of fractions and equivalent fractions. Students have learned that a fraction $\frac{a}{b}$ is a multiples of $\frac{1}{b}$. They use these insights and properties of operations to find the products of a whole number and a fraction. The work here helps students develop fluency and will be helpful later in the next lesson, when students find angle measurements formed by the hands of a clock.

The progression of expressions encourages students to look for and make use of structure (MP7) in

each expression and across expressions. Noticing the connections between the whole-number and fractional factors can help students find each product efficiently.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $12 \times \frac{1}{12}$
- $120 \times \frac{1}{12}$
- $360 \times \frac{1}{12}$
- $360 \times \frac{3}{12}$

Student Responses

- 1: there are 12 twelfths in 1 whole.
- 10: I know that 120 is 10 times 12, so $120 \times \frac{1}{12}$ is 10 times $12 \times \frac{1}{12}$ or 10 times 1.
- 30: there are 3 groups of 120 in 360, so $360 \times \frac{1}{12}$ is 3 times the previous product or 3×10.
- 90:
  - $\frac{3}{12}$ is 3 times $\frac{1}{12}$, so the product is 3×30.
  - $\frac{3}{12}$ is equivalent to $\frac{1}{4}$, which is half of $\frac{1}{2}$. I know $360 \times \frac{1}{2} = 180$, so $360 \times \frac{1}{4}$ is half of 180, which is 90.

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "What connections did you see between the factors in the four expressions?" (The whole numbers are multiples of 12. The fractions are all twelfths. 360 is a multiple of 120.)
- "How did those observations help you find the value of the last expression?"

Activity 1

10 min

Sorting Angles

Standards Alignments

Addressing 4.G.A.1

In this activity, students revisit the angles they drew in the preceding lesson and sort them into two groups based on the features of the angles. While there is no single correct way to sort the angles (as long as the categories are reasonable or can be explained), the goal is to highlight classification of angles by their size—as greater or less than a right angle. Students learn the terms **acute angles** and **obtuse angles** in the activity synthesis. Consider creating a poster of the graphic organizer from the student activity book in advance to support the lesson synthesis discussion.

When students sort the angles, they look for important features or properties of the angles including their measure but also perhaps looking at the orientation of the rays making the angles (MP7).

This activity uses *MLR2 Collect and Display*. Advances: Conversing, Reading, Writing.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Synthesis: Support students in internalizing new vocabulary. Invite them to use their bodies to create each type of angle, perhaps hinging at the hips to represent the vertex there. Ask how each angle feels in their bodies. Which is the easiest to form? The most difficult?
Supports accessibility for: Language, Memory, Attention

Instructional Routines

MLR2 Collect and Display

Materials to Gather

Materials from a previous lesson

Required Preparation

- Students need their angle cards from the previous lesson.

Student-facing Task Statement

In an earlier lesson, you and your partner drew some angles on cards.

Put the cards together and sort the angles into

Launch

- Groups of 2 (same partners as in the previous lesson)
- "What types of angles did you and your

two groups. Be prepared to explain why you sort them the way you do.

Student Responses

Answers vary. Sample categories:

- angles made by segments and angles made by rays
- angles that are pointy and angles that are wide or broad
- angles greater than 90° and angles less than 90°

partner draw in the last lesson? Sort them into two groups and be ready to explain how you categorize the angles."

Activity

- 2–3 minutes: group work time
- Monitor for the different categories that students use to sort their angles, especially categories based on angle size.

MLR2 Collect and Display

- Circulate, listen for and collect the language and features students use to sort their angles, especially those related to angle size. Listen for descriptions such as "pointy (or wide)," "smaller (or wider) than a right angle," and "close to a right angle (or straight line)."
- Record students' words and phrases on a visual display and update it throughout the lesson.

Synthesis

- Select groups to share their categories, saving the categories that are based on the size of angles (for example "pointy" and "wide") to the end.
- "Are there any other words or phrases that are important to include on our display?"
- As students share responses, update the display, by adding (or replacing) language, diagrams, or annotations. Refer to relevant visual displays from earlier lessons, if possible.
- Remind students to borrow language from the display as needed.
- Introduce **acute angles** as angles that are less than 90° and **obtuse angles** as those greater than 90° and add these words to the vocabulary display from the lesson.
- "Is a 90° angle acute or obtuse?" (Neither.

It's a right angle—a category of its own.)

- "Let's try to use these words in the next activity as we explain our reasoning."

Advancing Student Thinking

Students may sort their angle drawings based on non-mathematical features. For instance, they might classify them based on whether the angles are labeled or shaded, the color in which they're drawn, the line type used, and so on. Consider asking students: "If all the angles are labeled (or shaded, or drawn with the same color), would it still be possible to sort the angles? How might you sort them then?"

Activity 2

10 min

What is It, Really?

Standards Alignments

Addressing 4.G.A.1

In this activity, students classify a 180° angle as a **straight angle** and further develop their understanding of acute and obtuse angles. Given a pair of intersecting segments, they explain why the figure contains both an acute angle and an obtuse angle.

Some students may wish to measure the angles in the letter Y in the second question. Provide access to protractors if requested.

Materials to Gather

Protractors

Student-facing Task Statement

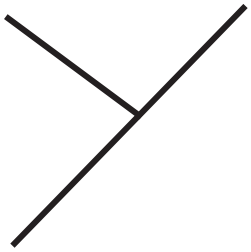
1. Mai and Jada are looking at this drawing. Jada says it is just a line. Mai says it is an angle.

Launch

- Provide access to protractors, if requested.
- Remind students to reference the chart of helpful words, phrases, and vocabulary terms, collected in Activity 1.

With whom do you agree? Explain your reasoning.

2. Tyler and Andre were measuring an angle in a letter Y.



Andre said the angle he measured is obtuse. Tyler said the angle is acute.

Explain why they could both be right.

Student Responses

1. Agree with both. Sample reasoning: The figure shows a line that goes on in two opposite directions, but it also shows an angle that is 180° with the point as the vertex (or two angles that are each 180° on either side of the line and share the point as the vertex).
2. Sample response: Tyler and Andre were looking at different angles formed by the two segments. Andre measured the angle at the top part of the Y, which is obtuse. Tyler measured the angle on the left side of the Y, which is acute.

- "In your own words, how do you explain the difference between **acute** and **obtuse** angles?"

Activity

- 5 minutes: independent work time

Synthesis

- Invite students to share responses to the first question.
- Highlight that when two rays that meet at the same point extend out in opposite directions, they form a 180° angle and a straight line.
- Introduce **straight angle** as an angle that is 180°.
- Select students to share responses to the second question using the chart from the previous activity as a reference to reinforce acute and obtuse angles.
- Emphasize the idea introduced earlier in the unit—that when two rays share a point, two or more angles are formed.

Activity 3

15 min

Small Angles, Large Angles

Standards Alignments

Addressing 4.G.A.1

In this activity, students encounter angles in various forms—in diagrams, descriptions, a construction drawing, and pattern blocks—and practice classifying them by size.

Students should have no trouble distinguishing angles without a protractor, but some students may wish to use a protractor to verify their classification.

The second question prompts students to label the angles in a drawing of a roof structure, which contains many angles. It is not essential that they catch all acute, right, and obtuse angles in the drawing as long as they identify some of each.

For the last question, provide access to pattern blocks if requested.

Materials to Gather

Pattern blocks, Protractors

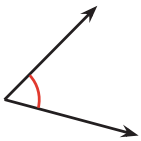
Student-facing Task Statement

1. Identify each angle as acute, right, obtuse, or straight.

a.

b.

c.



d. An angle formed by two 45° angles

e. A 91° angle

f. An angle that is in a rectangle

Launch

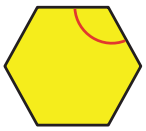
- Groups of 2–4
- Give students access to protractors and pattern blocks.

Activity

- 7–8 minutes: independent work time
- Make the chart from the first activity visible to all students and invite students to use the chart to support their reasoning.
- 3 minutes: group discussion

Synthesis

- Select students to share their responses to the first question.
- Select other students to share their labeled drawings for the second question, or display the drawing of the roof structure for them to mark up.

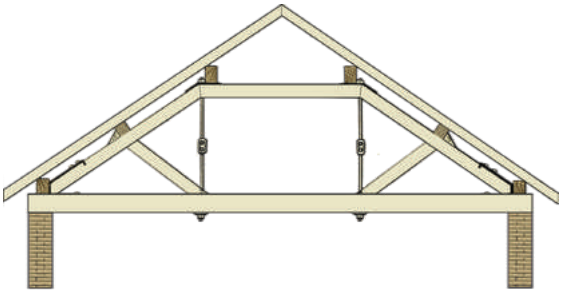
g. 

h. 

i. An angle composed of two right angles

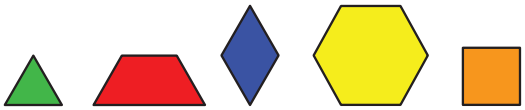
j. An angle composed of five 12° angles

2. Here is a drawing of the structure of a roof.



Find as many acute and obtuse angles as you can in the drawing.

Use an "A" to label acute angles, a square (□) for right angles, and an "O" for obtuse angles.

3. 

a. Diego is holding a pattern block that has 2 acute angles and 2 obtuse angles. Which pattern block could it be?

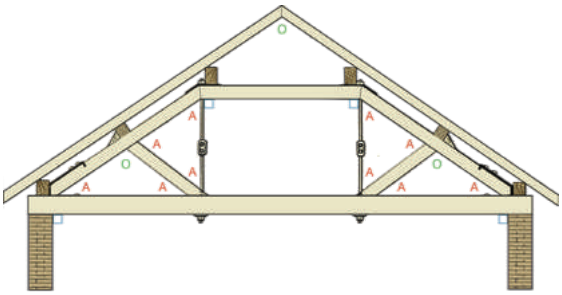
b. He then picks up a pattern block with no obtuse angles. Which pattern block could he be picking up? Explain your reasoning.

- As students share, add images and labels to the chart with language describing **obtuse** and **acute angles** from the first activity.
- Invite students who haven't shared before to discuss their responses to the last set of questions. Ask them to point to the angles in the pattern blocks to support their responses.
- If students mention only one pattern block, ask: "Are there other shapes Diego could be holding? Which one(s)? How do you know?"

Student Responses

1. a. obtuse b. acute c. acute d. right
   e. obtuse f. right g. obtuse h. acute
   i. straight j. acute

2. Sample response (not exhaustive):



3. a. The pattern block could be a rhombus or a trapezoid.
   b. The pattern block could be a triangle (all the angles are acute) or a square (all the angles are right).

Lesson Synthesis

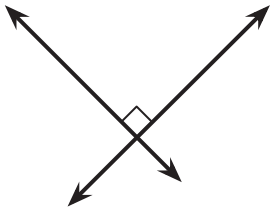
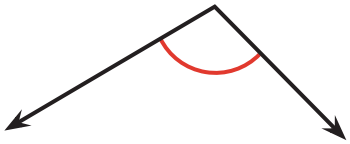
10 min

"Earlier in this unit, we categorized lines by their attributes or characteristics. Today, we categorized angles by their size. Let's recall the types of lines and the types of angles we've come across so far."

Create on chart paper a graphic organizer such as the one shown here. Give each student 1–2 blank sticky notes. Ask students to help complete the missing terms, definitions, and diagrams by writing the missing information on the sticky notes. Consider assigning 1–2 blank cells for each student to complete, or arranging students into groups, and ask each group to complete the missing information for each geometric figure.

Use students' input as a way to check for understanding and use the completed graphic organizer as a reference in future lessons.

	attribute (if they…)	type (we call them…)	diagram (they look like…)
lines	have one endpoint		
		line segments	

	never meet or cross		
			
angles	measure less than 90°		
		right angle	
			
	measure 180°		

Suggested Centers

- Which One? (K–5), Stage 4: Grade 3 Shapes (Supporting)
- Can You Draw It? (1–5), Stage 4: Area and Perimeter (Supporting)

Complete Cool-Down

Response to Student Thinking

Students may recognize that "obtuse" and "acute" refer to angles that are greater than and less than 90º but get the terms reversed.

Next Day Support

- Launch warm-up or Activity 1 by highlighting key vocabulary from previous lessons.

Lesson 13: Find Angle Measurements

Standards Alignments

Addressing 4.MD.C.7

Teacher-facing Learning Goals

- Compose and decompose angles to determine their measurements.

Student-facing Learning Goals

- Let's compose and decompose angles to find their measurements.

Lesson Purpose

The purpose of this lesson is for students to find unknown angle measurements by composing or decomposing known measurements, and to see that an angle is not determined by the length of the segments that form it.

In this lesson, students use tactile tools to find angle measurements and observe more clearly that angles are additive. They compose and decompose angles by arranging paper cutouts, by folding paper or tracing, and by drawing diagrams. Students arrange smaller angles whose sizes are unknown into larger angles with familiar sizes and features (90°, 180°, and 360°). Once the measurement of an angle is known, they use it to find those of other angles. For example, if two copies of angle x form a right angle, angle x must be 45°. If another angle, z, can be decomposed into three of these 45° angles, then z must be 135°.

Encourage students to continue to collect, define, and illustrate new terms to support communication and reasoning at the end of each lesson.

Access for:

Students with Disabilities

- Representation (Activity 2)

Instructional Routines

MLR5 Co-craft Questions (Activity 1), Notice and Wonder (Warm-up)

Materials to Gather

- Origami paper: Activity 2
- Patty paper: Activity 1

Materials to Copy

- How Big Are These Angles? (groups of 2): Activity 1

Required Preparation

Lesson Timeline

Warm-up	10 min
Activity 1	25 min
Activity 2	10 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

The work of finding angle measurements in this lesson offered opportunities to reason about equal groups. Did you hear students use this type of reasoning? What were some other ways students reasoned about the angle sizes?

Cool-down (to be completed at the end of the lesson)

5 min

Sets of Three Angles

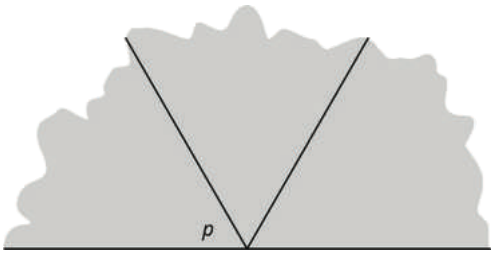
Standards Alignments

Addressing 4.MD.C.7

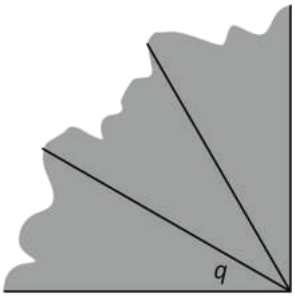
Student-facing Task Statement

Noah cuts out 3 copies of angle p and 3 copies of angle q. He arranges them side by side.

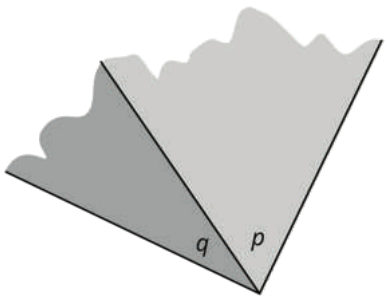
1. Three copies of angle p make a straight line. How many degrees is angle p? Explain or show your reasoning.



2. Three copies of angle q make a right angle. How many degrees is angle q? Explain or show your reasoning.



3. Noah puts angle p and angle q together. How many degrees is the resulting angle? Explain or show your reasoning.



Student Responses

1. 60°.Sample response: Three times the measure of p is 180°, so p must measure 60°.
2. 30°. Sample response: $90 \div 3 = 30$.
3. 90°. Sample response: $60 + 30 = 90$.

Begin Lesson

Warm-up

10 min

Notice and Wonder: Corner Pieces

Standards Alignments

Addressing 4.MD.C.7

This warm-up prompts students to visualize the idea of arranging angles around a point and adding their measurements as more pieces are added. The angles are familiar angles: 90°, 180°, and 270°. Students previously arrived at these benchmarks by decomposing a full turn or 360°. Here, they compose a full turn from 90° angles.

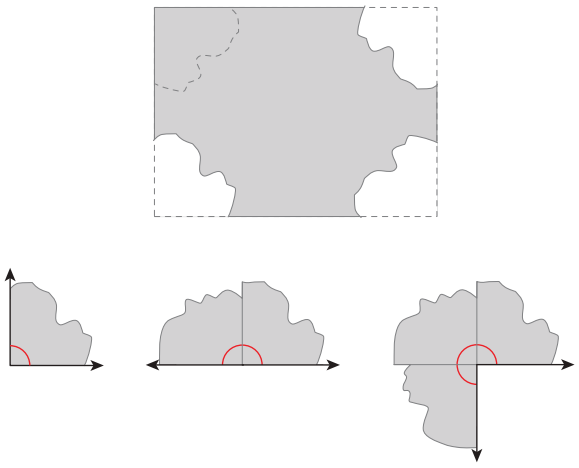
The work here familiarizes students with the context and mathematics that might be involved later in the lesson. In the subsequent activities, students will compose and decompose paper cutouts of angles to determine angle measurements.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?



Student Responses

Students may notice:

- The rectangle looks like a piece of paper with 3 corners torn out.
- There are 3 angles.
- The torn corner pieces fit in each angle.
- In the last two angles, the straight edge of the pieces line up with one another and the pieces share the same corner point.

Students may wonder:

- Why is the last corner not torn out?
- Would the last piece of paper fit in the right angle in the bottom right?
- What are the angle measurements?

Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- "What angles do you think are the measurement of each angle?" (They look like 90°, 180°, and 270° angles.)
- "How do you know?" (If the paper is a rectangle, then the corner pieces are right angles or 90° each. Two of the corner pieces would be $90 + 90$. Three pieces would be $90 + 90 + 90$.)
- "What angle would we get if we add the last corner piece?" (360°)

Activity 1

25 min

How Big Are These Angles?

Standards Alignments

Addressing 4.MD.C.7

In this activity, students use their knowledge of 90°, 180°, and 360° and paper cutouts of some acute angles to determine the measurements of those angles. They then use those measurements to compose and find the measurements of larger angles.

No explicit directions for finding the angles are given, so students have an opportunity make sense of the problem and use what they know about the additivity of angles to find the angle measures (MP7). If requested, give students access to rectangular sheets of paper whose square corners could be torn off.

This activity uses *MLR5 Co-Craft Questions.* Advances: Reading, Writing.

Instructional Routines

MLR5 Co-craft Questions

Materials to Gather

Patty paper

Materials to Copy

How Big Are These Angles? (groups of 2)

Required Preparation

- Create 4 copies of each angle (p, q, r, and s) from the Instructional master for each group of 2–4 students.
- Cut out the angles in advance, or prepare scissors and extra time for students to cut out the angles.
- If using patty paper instead of cutouts of the angles, each student needs 1–2 sheets of patty paper.

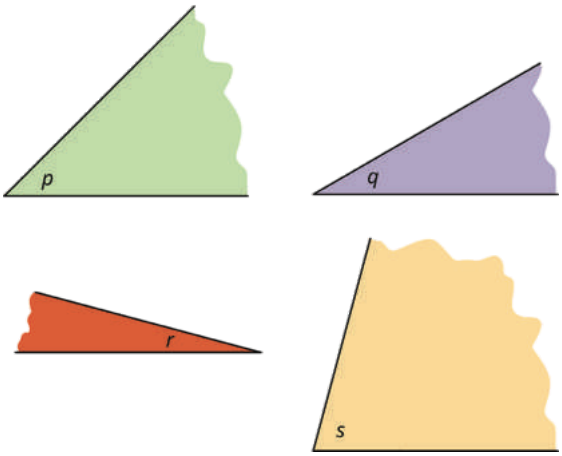
Student-facing Task Statement

Your teacher will give you materials that can help you find angle measurements.

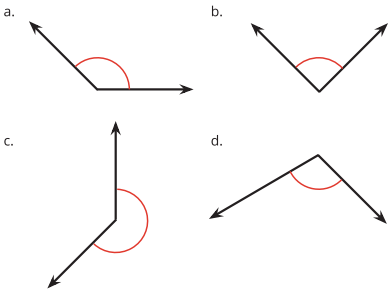
Launch

- Groups of 2–4
- Give each group the cutouts of the four angles, 4 copies of each angle per group, or

1. Use the materials and what you know about a right angle to find the size of angles p, q, r, and s. Be prepared to explain or show your reasoning.



2. Next, use the measurements of angles p, q, r, and s to find the measurements of the following angles.



Student Responses

1. Sample responses:
   - Angle p measures 45° because 2 copies of it make 90°.
   - Angle q measures 30° because 3 copies of it make 90°.
   - Angle r measures 15° because 2 copies of it make 30°.
   - Angle s measures 75°. It is the same size as angle p and angle q combined. When angle s is combined with angle r (sharing their endpoint and one

if using patty paper, give 1–2 sheets to each student.

- If using patty paper, demonstrate that it can be used for tracing the angles.

Activity

MLR5 Co-Craft Questions

- Display only the image to the first problem without revealing the question.
- "Write a list of mathematical questions that could be asked about this image." (What is the size of each of the angles? Could we put angles together to make right angles? Would all of the angles make a straight line? Which angle is closest to 90 degrees and how far away from 90 degrees is it?)
- 2 minutes: independent work time
- 2–3 minutes: partner discussion
- Invite several students to share one question with the class. Record responses.
- "How are these questions alike? How are they different?" (Most questions are related to the size of the angles.)
- Reveal the task (students open books), and invite additional connections.
- Monitor for groups who:
  - compose 2 copies of angle p or 3 copies of angle q into a right angle
  - compose 2 copies of angle r into q
  - compose angles p and q into angle s
- Pause for a whole-class discussion. Select students or groups who reasoned as previously outlined to share their reasoning.
- 3–4 minutes: independent work time on the second question
- 2 minutes: group discussion
- Monitor for the different ways that smaller angles are used to compose the angles in

edge), the combined angle is 90°.

2. a. 135° b. 90° c. 225° d. 105°

parts a–d.

Synthesis

- Invite students to share their responses and reasoning for the second question. Display the angles.
- For each angle in parts a–d, record the different compositions of angles students use to find its measurement. For example:
  - To compose the angle in part a, students may use angles p, r, and s, or they may use 3 copies of p.
  - To find the angle in part c, students may use 3 copies of angle s, or they may draw a line to separate 180° of the angle and fit angle p in the remaining space.

Advancing Student Thinking

Offer patty paper to students who may need support using smaller angles to find the size of larger angles. Consider asking: "How might tracing the angles help you compare the angles you know and those you don't know?"

Activity 2

10 min

Angles in a Kite

Standards Alignments

Addressing 4.MD.C.7

In this activity, students find the size of angles created by folding paper several times and reasoning about the resulting angles (MP7).

The first fold decomposes the paper into two congruent shapes whose edges line up exactly, and students could see how the folding split two of the angles into equal halves. The subsequent folds

decompose an angle into two equal angles, but this may not be obvious to students because the shapes of the two resulting parts are different. (The edges or creases that form the angles are of different lengths.) Students need to remember that angles are not determined by the length of the segments that form it and reason accordingly.

Some students may need support in folding paper precisely. Consider providing a larger sheet of paper or a straightedge to facilitate the folding.

Access for Students with Disabilities

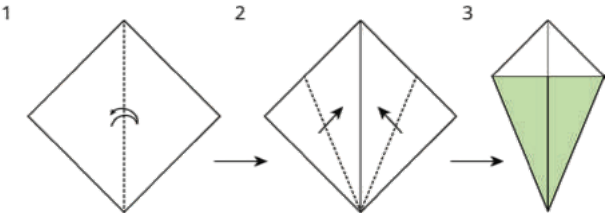
Representation: Access for Perception. Walk students through the steps to fold the paper into a kite, demonstrating with your own paper. Before beginning, and then after each step, invite students to share what they notice about the angles on the paper.
Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Materials to Gather

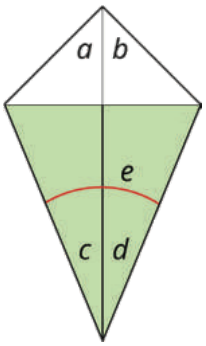
Origami paper

Student-facing Task Statement

Your teacher will give you a square sheet of paper. Follow the steps to fold the paper into a kite. Fold as precisely as possible.



Can you find the measurement of each labeled angle on the kite? If so, show your reasoning. If not, explain why not.



Launch

- Groups of 2–4
- Give each student a sheet of origami paper or square paper.
- Display the paper folding diagrams.

Activity

- 2–3 minutes: independent work time
- 2–3 minutes: group discussion
- Monitor for students who recognize that two angles are equal if the edges or creases that form them line up exactly (even if one crease or edge is longer than the other).

Synthesis

- Invite students to share their responses and reasoning.
- "How can we tell if the first fold resulted in pairs of equal-size angles?" (The two

Student Responses

The measurement of angles a and b is 45° each. Angles c and d each measure $22\frac{1}{2}^\circ$. Angles e measures 45°.

Sample reasoning:

- Angles a and b are each half of 90°.
- Angles c and d are the results of folding a right angle in half and then folding each half in half again. This means the four angles at the bottom of the paper are all equal. Each one is $90 \div 4$, which is 22.5°.
- The measure of angle e is the sum of the measures of c and d, so it is $22\frac{1}{2} + 22\frac{1}{2}$.

triangles are identical and match up exactly, which means the angles in the two halves are the same.)

- "The shapes that result from the second and third folds are not the same. Do those folds produce equal-size angles as well?" (Yes. The edges of the resulting angles match up exactly and meet at the same endpoint, so the angles are the same size.)
- "Which angle is larger, b or e?" (They are the same, both are 45°. Angle b is half of 90. Angle e is twice 22.5°.)

Advancing Student Thinking

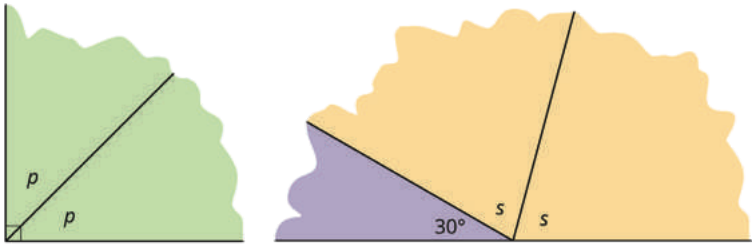
Students may benefit from reasoning about angle sizes concretely. Consider providing a piece of paper and asking after the first fold: "How might we tell the size of the angles we have created at this point?" and "How can we check our thinking or show that it is correct?"

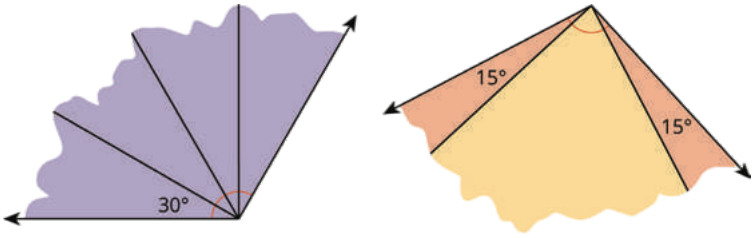
Lesson Synthesis

10 min

"Today we used different operations to find the measurement of different angles."

Display:





"Here are some angles whose measurements we tried to find: angle p, angle s, and some angles composed of smaller angles. We used different operations to find the unknown measurements."

"Which of these angles can we find by using division?" (Angle p: If we know that 2 copies of p make a right angle, which is 90°, then dividing 90° by 2 gives us the measure of p.)

"Which unknown angle can we find by multiplication?" (The angle made up of four 30° angles has a measurement of 4×30.)

"Which unknown angle can we find by subtracting one angle from another?" (Angle s: We can subtract 30° from 180° and divide by 2 to find the measure of s, which is 75°.)

"Which unknown angle can we find by adding known angles?" (Once we know the measure of angle s, we can find the last angle: $15 + 75 + 15$, which is 105°.)

Suggested Centers

- Target Measurements (2–5), Stage 4: Degrees (Addressing)
- Compare (1–5), Stage 5: Fractions (Supporting)

Complete Cool-Down

Response to Student Thinking

Students may compose angles in ways that do not result in the correct answer.

Next Day Support

- During the activity syntheses, connect diagrams to expressions or equations.

Lesson 14: Reasoning about Angles (Part 1)

Standards Alignments

Building On	4.G.A.1
Addressing	4.MD.C.6, 4.MD.C.7

Teacher-facing Learning Goals

- Draw angles of given measurements.
- Reason about angle measurements within a circle.

Student-facing Learning Goals

- Let's find the size of angles on the clock.

Lesson Purpose

The purpose of this lesson is twofold: for students to use known angles to reason about unknown angles on a clock, and for them to practice drawing angles of given measurements.

Angles are ever present on an analog clock. In this lesson, students investigate and solve problems about the angles formed by a clock's hour and minute hands. Students reason about the number of degrees between the two hands or the number of degrees the minute hand has turned over some specified time. To do so, students rely on their understanding of fractional parts (for example, a round clock can be divided into 12 and 60 equal parts), their ability to tell time and elapsed time, and their knowledge of angle types and measurements.

Access for:

Students with Disabilities

- Engagement (Activity 2)

English Learners

- MLR8 (Activity 1)

Instructional Routines

MLR1 Stronger and Clearer Each Time (Activity 2), Which One Doesn't Belong? (Warm-up)

Materials to Gather

- Protractors: Activity 1, Activity 2
- Rulers or straightedges: Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

How did students' understandings about time—how to tell time and find elapsed time—help their work with angles on the clock?

Cool-down (to be completed at the end of the lesson)

5 min

One Angle at a Time

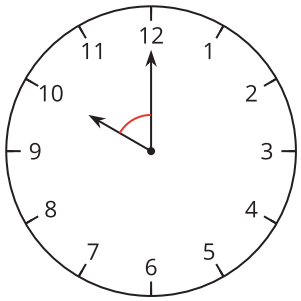
Standards Alignments

Addressing 4.MD.C.6, 4.MD.C.7

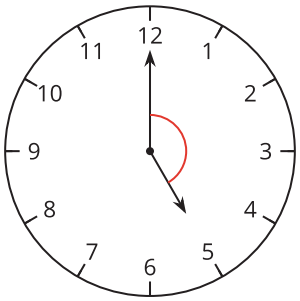
Student-facing Task Statement

How many degrees is each marked angle on the clock? Explain or show your reasoning.

A



B



Student Responses

A. 60°. Sample response: Every time the minute hand moves from one number to the next, it turns 30°. The angle between the hands is 2 numbers apart, so it is 2×30, which is 60.

B. 150°. Sample responses:

- $5 \times 30 = 150$
- If it was 6 o'clock, the angle would be 180°. The angle for 5 o'clock is 30° less than 180°.

Begin Lesson

Warm-up

10 min

Which One Doesn't Belong: Time After Time

Standards Alignments

Building On 4.G.A.1

This warm-up prompts students to carefully analyze and compare geometric features of four clock faces. Students may compare the times being represented, but because no numbers are shown, they are likely to compare the hands of the clocks and the angles they form.

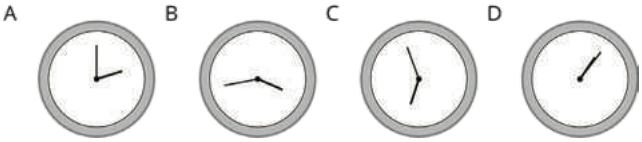
In making comparisons, students have a reason to use language precisely (MP6). Teachers have a chance to hear the terminologies students use and how they talk about characteristics of angles.

Instructional Routines

Which One Doesn't Belong?

Student-facing Task Statement

Which one doesn't belong?



Student Responses

- A doesn't belong because:
  - The smaller angle is not obtuse.
  - The number of minutes in the time doesn't have non-zero digits.
- B doesn't belong because:
  - The time is not within a few minutes of the hour.
  - The minute hand is not pointing up.
- C doesn't belong because it is the only one that could not show a time in the afternoon.
- D doesn't belong because:

Launch

- Groups of 2
- Display the image.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."

Activity

- 1 minute: quiet think time
- 2–3 minutes: partner discussion
- Record responses.
- As students explain, find opportunities to reinforce the terms "acute" and "obtuse."

Synthesis

- Consider stating: "Let's find at least one reason why each one doesn't belong."
- Display the following sentences and ask students to complete each sentence with

- It doesn't have two obvious segments.
- Its hands do not point in different directions.

"always," "sometimes," or "never:"

- "The hour and minute hands of a clock _____ form one angle."
- "The hour and minute hands of a clock _____ form two angles."

- Ask students to explain their choice or use counterexamples to disagree with a classmate's choice.

Activity 1

15 min

Draw a Clock

Standards Alignments

Addressing 4.MD.C.6, 4.MD.C.7

In an earlier lesson, students had folded paper and used supplemental tools to form and draw some benchmark angles (30°, 45°, 60°, and so on). In this activity, they apply their ability to measure and draw angles with a protractor to create a reasonably accurate clock face. The measuring and drawing here prepare students to reason about the angles formed by the hands of a clock in the next activity.

Students may notice that lines that give the positions of 1 and 2 on the clock can be extended through the center of the clock to give the positions of 7 and 8. Students who use these observations to create the drawing practice making use of structure (MP7).

The clock that students draw in this activity can be a helpful reference in the next activity.

Access for English Learners

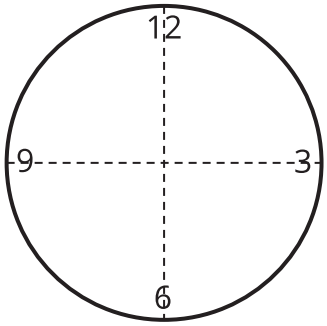
MLR8 Discussion Supports. Use multimodal examples to show the meaning of the angles and numbers on a clock. Use verbal descriptions along with gestures, drawings, or concrete objects to show how to precisely complete a clock.
Advances: Listening, Representing

Materials to Gather

Protractors, Rulers or straightedges

Student-facing Task Statement

Kiran is drawing a clock. He draws a pair of perpendicular lines to find the placement of the numbers 3, 6, 9, and 12 around the circle.



1. How many degrees is each angle he has drawn so far? Explain how you know.
2. Help Kiran find the exact placement of the numbers "1" and "2" on the clock.
    a. How many new lines does he need to draw?
    b. What angles should be formed between the two lines he has already drawn and the new ones?
    c. Draw the lines precisely and place the numbers "1" and "2" on the drawing.
3. Measure and draw as many lines as needed to complete the clock drawing so that all the numbers are precisely placed where they should be.

Student Responses

1. 90°, because he drew a pair of perpendicular lines.
2. a. two lines
    b. 30° between each pair
    c. See drawing.
3.

Launch

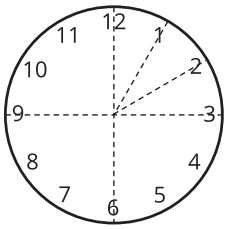
- Groups of 2
- Give one protractor and a straightedge or a ruler to each student.

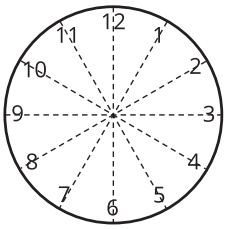
Activity

- 5 minutes: independent work time
- 1–2 minutes: partner discussion
- Monitor for students who:
    - measure the angle needed to place each number (one at a time)
    - draw the lines to position the numbers 1, 2, 4, and 5 and then try to find a way to mirror them vertically to locate 11, 10, 8, and 7
    - extend the lines they drew to find the numbers 1 and 2 and use them to find 7 and 8, and do the same with 3 and 4

Synthesis

- Display the incomplete clock face. Select students to share their completed drawing and their drawing process. Sequence the presentation in the order shown in the monitoring notes.
- "How did you find the size of the angle formed between the number 1 and 2?" (Divide 90 by 3, or divide 180 by 6, or divide 360 by 12.)
- "Is the angle formed by any two consecutive rays always 30°? How do you know?" (Yes, because the angle is always $\frac{1}{12}$ of 360.)





Activity 2

20 min

Tick Tock

PLC Activity

Standards Alignments

Addressing 4.MD.C.7

In grade 3, students learned to tell and write time to the nearest minute and measure time intervals in minutes. They understand that moving from one number on the clock to the next means 5 minutes have elapsed. In this activity, students build on those understandings to solve problems about angles formed by the hands of a clock.

Many students would benefit from having a visual reference of a clock as they are solving these problems. Encourage them to use their clock drawing from the previous activity for support.

Some students may try to answer each question by drawing each indicated time and then measuring the angles formed by the hands. Ask them to consider finding the size of the angles by reasoning and without measuring. For example, ask: "What do you know about the angle that is formed when a hand goes from 12 to 3? From 12 to 1?" This encourages students to use the structure of the clock and the equal parts the clock face is divided into by the numbers on the clock (MP7).

This activity uses *MLR1 Stronger and Clearer Each Time*. Advances: Reading, Writing.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Students may benefit from feedback that emphasizes effort, time on task, and continuous learning. For example, invite students to choose which part of the first two questions to start with, and let them know that they will have the opportunity to share and revise their thinking throughout the lesson. Share examples of students who revised their drafts after discussing with a partner.
Supports accessibility for: Social Emotional Functioning

Instructional Routines

MLR1 Stronger and Clearer Each Time

Materials to Gather

Protractors

Student-facing Task Statement

1. What angles are formed by the hour and minute hands of the clock at these times?
    a. 6 o'clock
    b. 8 o'clock
    c. 9 o'clock
    d. 11 o'clock
    e. 12 o'clock
2. How many degrees has the minute hand turned when it moves from 2:00 to 2:05?

    What about from 2:05 to 2:30? Explain how you know.
3. The minute hand of the clock is vertical at 7 p.m. Sometime later, it makes an angle that is 120° from where it was at 7 p.m. What time could it be?
4. How many degrees does the minute hand turn in:
    a. 10 minutes?
    b. 1 minute?

Launch

- Groups of 4
- Give students access to protractors.

Activity

- 5 minutes: independent work time on the first two sets of questions

MLR1 Stronger and Clearer Each Time

- "Share your response to the second question with a partner. Take turns being the speaker and the listener. If you are the speaker, share your ideas and writing so far. If you are the listener, ask questions and give feedback to help your partner improve their work."
- 1–2 minutes: structured partner discussion
- Repeat with 2 different partners (other members of the group).
- "Revise your initial draft based on the feedback you got from your partners."
- 2 minutes: independent work time

c. 4 minutes?

Student Responses

1. a. 180°
   b. 120° and 240°
   c. 90° and 270°
   d. 30° and 330°
   e. 360°
2. From 2:00 to 2:05, it turned 30°. From 2:05 to 2:30, it turned 150°. Sample reasoning: The long hand is at 1 when it is 2:05 and at 6 when it is 2:30. There are five 30° angles between 1 and 6.
3. 7:20 p.m. or 7:40 p.m. When the long hand is at 4 or 8 it makes a 120° angle with a vertical line from the center of the clock.
4. a. 60°
   b. 6°
   c. 24°

- 3–4 minutes: quiet work time on the last two sets of questions

Synthesis

- Display a clock face. Invite students to share their responses to the first question and the last two.
- When discussing the first set of questions, highlight that—except at 12 o'clock— the positions of the hour and minute hands produce two angles—a larger angle and a smaller angle.
- Likewise, when discussing the third question, if no students mentioned that there are two possible times that meet the described constraint, bring it up.
- "How did you find out the number of degrees the minute hand turns in 10 minutes and 1 minute?" (Ten minutes is twice 5 minutes, so it is twice 30°, or 60°. One minute is 10 minutes divided by 10, so it is 60° divided by 10, which is 6°.)

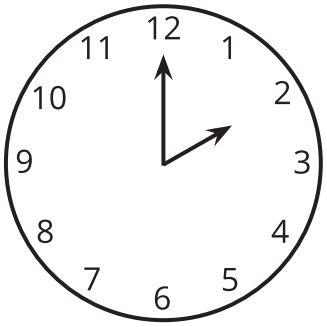
Lesson Synthesis

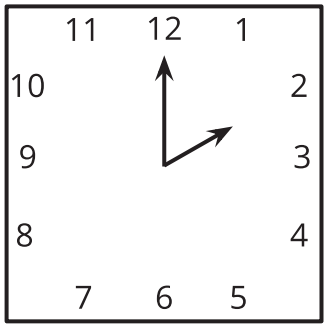
10 min

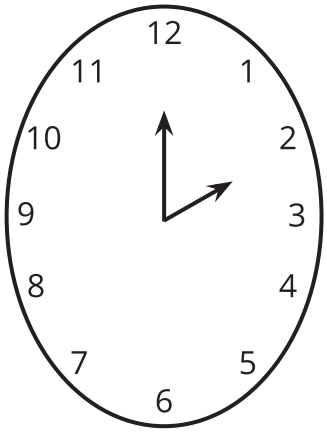
"Today we learned about angle measurements on a clock. We looked at the angles formed by the two hands, and we also thought about the number of degrees that a minute hand turns over time."

"Which is more useful for finding the size of angles on a clock: thinking in terms of number of minutes, the number of 5 minutes, or the numbers 1–12?" (It depends on the situation.)

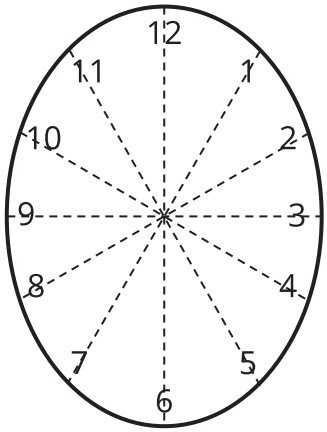
Display the following images of clocks:







"Does the minute hand on a square clock or an oval clock turn the same number of degrees every minute as it does on a round clock? Explain or show how you know." (Yes. The minute hand still travels a full turn or 360° in an hour or 60 minutes, so each minute it still travels 6°, regardless of the outer shape of the clock or how far away the numbers are spread out from the center point.)



Consider displaying an image of the oval clock showing 12 equal angles. Reinforce the idea that the size of an angle is not determined by the length of segments or rays that form the angle.

"Take 1–2 minutes to add the new words from the past two lessons to your word wall. Share your new entries with a neighbor and add any new ideas you learn from your conversation."

Suggested Centers

- Which One? (K–5), Stage 4: Grade 3 Shapes (Supporting)
- Can You Draw It? (1–5), Stage 4: Area and Perimeter (Supporting)

Complete Cool-Down

Response to Student Thinking

Students may mistake the angle between two consecutive numbers to be 5° because that space represents 5 minutes of elapsed time.

Next Day Support

- Before the warm-up, select a student's cool-down from the previous lesson (name anonymous). Ask students to identify what the student did well and what the student needs to do to improve the cool-down.

Lesson 15: Reasoning About Angles (Part 2)

Standards Alignments

Addressing 4.G.A.1, 4.MD.C.7

Teacher-facing Learning Goals

- Represent angle relationships and solve for unknown angle measurements.

Student-facing Learning Goals

- Let's figure out missing angle measurements.

Lesson Purpose

The purpose of this lesson is to use known angles to reason about unknown angles and write equations to reason about unknown angles.

In this lesson, students use what they know about angles and their measurements to solve problems that are increasingly more complex and abstract. To find measurements of unknown angles, students need to look for structure in the diagrams, reason about the relationships of the angles (including writing equations to represent the relationships), and perform addition, subtraction, and sometimes division.

The problems in the lesson can be solved in more than one way and in different orders, but a small handful of the angles can only be quantified after the values of some other angles are known. Students pay attention to the process and explain why sometimes a certain sequence is necessary.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR8 (Activity 1)

Instructional Routines

How Many Do You See? (Warm-up), MLR4 Information Gap (Activity 2)

Materials to Copy

- Info Gap: Whole Bunch of Angles (groups of 2): Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

As you finish up this unit, reflect on the norms and activities that have supported students' development as thinkers. How have students grown in their ability to reason more abstractly and logically? What instructional strategies or routines worked well in supporting this development? What will you continue to do and improve on in Unit 7, and what will you adjust?

Cool-down (to be completed at the end of the lesson)

5 min

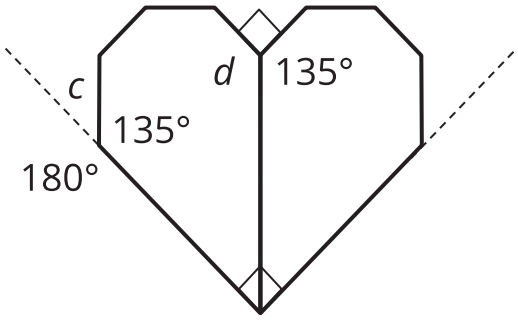
Heart to Heart

Standards Alignments

Addressing 4.G.A.1, 4.MD.C.7

Student-facing Task Statement

Find the measurement of each labeled angle. Show your reasoning.



Student Responses

The measure of angle c is 45° and that of angle d is 135°. Sample reasoning:

- Angle c and the 135° angle together make a straight angle, which is 180°, so the measure of c is $180 - 135$, which is 45.
- Angle d, the right angle, and the 135° angle make 360°. $90 + 135 = 225$ and $360 - 225 = 135$.

Begin Lesson

Warm-up

10 min

How Many Do You See: Obtuse Angles

Standards Alignments

Addressing 4.G.A.1

In this warm-up, students practice identifying obtuse angles in an image. They may, for instance, rely on the symmetry of the figure or on a grouping strategy, or otherwise scan the figure in a methodical way.

Instructional Routines

How Many Do You See?

Student-facing Task Statement

How many angles do you see in the folded paper heart?

Student Responses

10. Sample responses:

- I see 5 obtuse angles in each half of the heart.
- I see 2 acute angles at the bottom of the heart.
- I see 8 obtuse angles around the outer edge of the heart (4 on the left and 4 on the right) and 2 in the middle (on either side of the center line).
- Starting from the center "dip" of the heart and

Launch

- Groups of 2
- "How many angles do you see? How do you see them?"
- Display the image.
- 1 minute: quiet think time

Activity

- Display the image.
- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Record responses.

Synthesis

- "How did you make sure all the angles are accounted for?"(I put a mark through them or numbered them.)
- "How many obtuse angles are in this image?" (10)
- Label each obtuse angle with reasoning from students.

moving around the outside, I see 5 left-right pairs of obtuse angles.

- Consider asking:
  - "Who can restate in different words the way _____ saw the angles?"
  - "Did anyone see the angles the same way but would explain it differently?"
  - "Does anyone want to add an observation to the way _____ saw the angles?"

Activity 1

15 min

Shaded and Unshaded Angles

Standards Alignments

Addressing 4.MD.C.7

Previously, students found numerous angle sizes by reasoning and without using a protractor. They have done so with problems with and without context. In this activity, students consolidate various skills and understandings gained in the unit and apply them to solve problems that are more abstract and complex. They rely, in particular, on their knowledge of right angles and straight angles to reason about unknown measurements. (Students may need a reminder that an angle marked with a small square is a right angle.)

The angles with unknown measurements are shaded but not labeled, motivating students to consider representing them (or their values) with symbols or letters for easier reference. Students may also choose to write equations to show how they are thinking about the problems.

When students use the fact that angles making a line add up to 180° and that angles making a right angle add up to 90° they make use of structure to find the unknown angle measures (MP7).

Access for English Learners

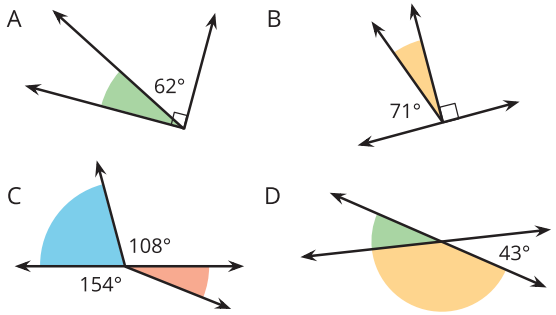
MLR8 Discussion Supports. Display sentence frames to support small-group discussion: "First, I _____ because . . .", "I noticed _____ so I . . .", and "Why did you . . . ?"
Advances: Conversing, Representing

Access for Students with Disabilities

Representation: Internalize Comprehension. Synthesis: Invite students to identify what they had to look for in the pictures to solve each problem. Display the sentence frame: "The next time I am finding the measurement of an angle without a protractor, I will look for . . . ." Record responses and invite students to refer to them in the next activity.
Supports accessibility for: Conceptual Processing, Memory, Attention

Student-facing Task Statement

Find the measurement of each shaded angle. Show how you know.



Student Responses

A. 28°. Adding this angle and 62 make 90, or $62 + ? = 90$.

B. 19°. The sum of 71, 90, and this angle gives 180, or $71 + 90 + ? = 180$. This means the ? is $180 - (71 + 90)$ or $180 - 161$, which is 19.

C. The smaller angle is 26° because $26 + 154 = 180$. The larger angle is 72° because $72 + 108 = 180$.

D. The larger angle at the bottom is 137° because $137 + 43 = 180$. The smaller angle on the left is 43° because $43 + 137 = 180$.

Launch

- Groups of 2

Activity

- 5 minutes: independent work time
- 2 minutes: partner discussion
- Monitor for students who:
  - use symbols or letters to represent unknown angles
  - write equations to help them reason about the angle measurements

Synthesis

- Display the angles. Select students to share their responses. Record and display their reasoning.
- Highlight equations that illuminate the relationship between the known angle, the unknown angle, and the reference angle (90°, 180°, or 360°). For instance: $62 + p = 90$, $71 + r + 90 = 180$, $x + 154 = 180$, and so on.
- Label the diagrams with letters or symbols as needed to facilitate equation writing.
- When discussing the last question, highlight that finding unknown values sometimes involve multiple steps, and some steps may need to happen before others.

Advancing Student Thinking

To find the size of the shaded acute angle in the last diagram, students will need to reason indirectly by first finding the size of the shaded obtuse angle. (At this stage they are not expected to know that vertical angles are always equal.) Students who try to find the measurement of the acute angle first will likely get stuck. Consider asking students what they know about straight angles and how they can use what they know to pick which unknown angle to find first.

Activity 2

20 min

Info Gap: A Whole Bunch of Angles

Standards Alignments

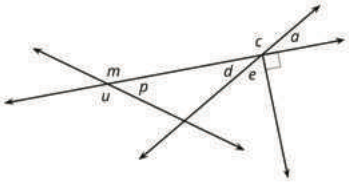
Addressing 4.MD.C.7

In this Info Gap activity, students solve abstract multi-step problems involving an arrangement of angles with several unknown measurements. By now students have the knowledge and skills to find each unknown value, but the complexity of the diagram and the Info Gap structure demand that students carefully make sense of the visual information and look for entry points for solving the problems. They need to determine what information is necessary, ask for it, and persevere if their initial requests do not yield the information they need (MP1). The process also prompts them to refine the language they use and ask increasingly more precise questions until they get useful input (MP6).

Here is an image of the cards for reference:

Info Gap: A Whole Bunch of Angles
Problem Card 1

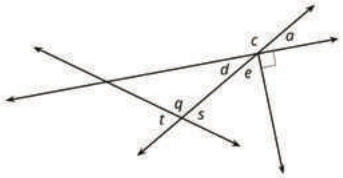
Three lines and a ray cross and make a bunch of angles.



1. What is the size of angle p?
2. Which angle is greater: p or d?

Info Gap: A Whole Bunch of Angles
Problem Card 2

Three lines and a ray cross and make a bunch of angles.



1. What is the size of angle q?
2. Which angle is greater: t or e?

Info Gap: A Whole Bunch of Angles
Data Card 1

- Angle u is 128°.
- Angle d is the same size as angle a.
- Angle c is 143°.

Info Gap: A Whole Bunch of Angles
Data Card 2

- Angle t is smaller than a right angle.
- Angle s is 79°.
- Angle a is 37°.

Instructional Routines

MLR4 Information Gap

Materials to Copy

Info Gap: Whole Bunch of Angles (groups of 2)

Required Preparation

- Create a set of cards from the Instructional master for each group of 2.

Student-facing Task Statement

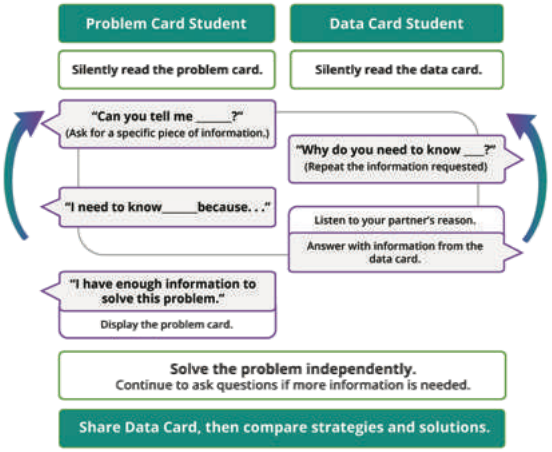
Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

Launch

- Groups of 2

MLR4 Information Gap

- Display the task statement, which shows a diagram of the Info Gap structure.
- 1–2 minutes: quiet think time
- Read the steps of the routine aloud.
- "I will give you either a problem card or a



Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Student Responses

Problem Card 1:

1. Angle p measures 52°. Sample reasoning: $128 + p = 180$, so $p = 180 - 128$, which is 52.
2. Angle p is greater. Sample reasoning: $d + 143 = 180$, so $d = 180 - 143$, which is 37.

Problem Card 2:

1. Angle q measures 101°. Sample reasoning: $u + q = 180$. We know u is 79, so q is $180 - 79$, which is 101.
2. Angle t is greater. Sample reasoning: $t + 101 = 180$, so $t = 79$. We know that $d + e + 90 = 180$ and that d is 37, so $37 + e + 90 = 180$, which means e is $180 - 90 - 37$, which is 53.

data card. Silently read your card. Do not read or show your card to your partner."

- Distribute the cards.
- "The diagram is not drawn accurately, so using a protractor to measure is not recommended."
- 1–2 minutes: quiet think time
- Remind students that after the person with the problem card asks for a piece of information, the person with the data card should respond with "Why do you need to know (restate the information requested)?"

Activity

- 5 minutes: partner work time
- After students solve the first problem, distribute the next set of cards. Students switch roles and repeat the process with Problem Card 2 and Data Card 2.

Synthesis

- Select students to share how they found each angle measure. Record their reasoning and highlight equations that clearly show the relationships between angles.
- "Which angle measurements were easy to find? What made them easy?" (Sample response: p and d, because it was fairly easy to see that each of them and a neighboring angle make a straight angle.)
- "Which ones were a bit more involved? Why?" (Sample response: e, because there are 5 angles that meet at that point. We needed to find a or d before finding e.)

Advancing Student Thinking

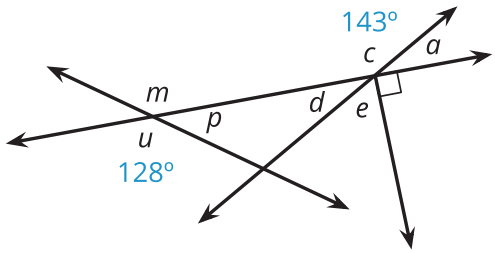
Some students may be overwhelmed by the visual information. Ask them to try isolating a part of the diagram at a time, covering other parts that are not immediately relevant.

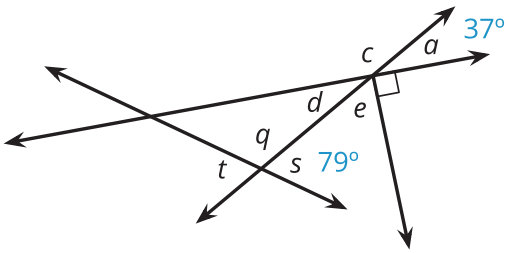
Lesson Synthesis

10 min

"Today we solved angle problems involving multiple steps, all without measuring with a protractor."

Display the two diagrams on the problem cards of the Info Gap activity. Label the angles whose measurements are given on the data cards. (128° for u, 143° for c, 79° for s, and 37° for a.)





Focus the discussion on how equations could be used to represent students' reasoning process and to help find the unknown angle measurements.

"What equations can we write to help us find the value of p? What about d?" (See sample equations in student responses.)

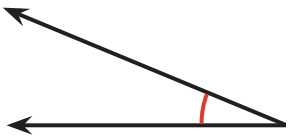
Suggested Centers

- Which One? (K–5), Stage 4: Grade 3 Shapes (Supporting)
- Can You Draw It? (1–5), Stage 4: Area and Perimeter (Supporting)

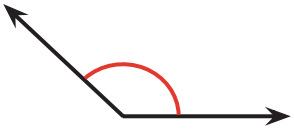
Student Section Summary

Earlier in the unit, we learned that a right angle measures exactly 90°. In this section, we learned other ways to name angles based on their measurements.

- **Acute angles** are less than 90°.



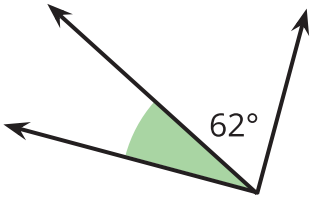
- **Obtuse angles** are greater than 90° but less than 180°.



- **Straight angles** are exactly 180º.

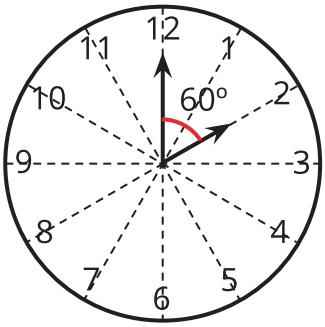
We also solved problems about angles. For example, if two angles make a right angle or a straight angle, we can use the size of one angle to find the other.

The shaded angle here must be 28° because it makes a right angle when combined with the 62° angle.



Another example: Knowing that a full turn measures 360°, we reasoned that the long hand of a clock makes:

- a 360° angle every hour
- a 180° angle every one-half hour
- a 90° angle every 15 minutes
- a 60° angle every 10 minutes



Complete Cool-Down

Response to Student Thinking

Students may not find the values of c or d if they don't recall that a full turn around a point makes a 360° angle.

Next Day Support

- Before the warm-up, have students work in partners to discuss a correct response to this cool-down.

Lesson 16: Guess the Figure (Optional)

Standards Alignments

Addressing	4.G.A.1, 4.G.A.2
Building Towards	4.G.A.1

Teacher-facing Learning Goals

- Draw and identify acute, obtuse, right, and straight angles in two-dimensional figures.
- Draw and identify points, lines, rays, segments, and parallel and intersecting lines in geometric figures.

Student-facing Learning Goals

- Let's draw and identify all kinds of two-dimensional figures.

Lesson Purpose

The purpose of this lesson is for students to apply their understanding of geometric figures and measurements to draw, describe, and identify two-dimensional figures.

This lesson is optional because it does not address any new mathematical content standards. This lesson does provide students with an opportunity to apply precursor skills of mathematical modeling.

In the first activity, students practice using the geometric vocabulary they acquired during the unit to describe changes they notice in a given figure. In the second activity, students first draw a two-dimensional figure and then describe it to a partner without showing it to them. The partner attempts to draw the figure based solely on the description. Students have an opportunity to compare the original and drawn figure.

Access for:

Students with Disabilities

- Representation (Activity 2)

English Learners

- MLR2 (Activity 1)

Instructional Routines

Notice and Wonder (Warm-up)

Materials to Gather

- Rulers or straightedges: Activity 2

Materials to Copy

- Make a Change (groups of 2): Activity 1

Required Preparation

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min

Teacher Reflection Question

Which students had opportunities to share their diagrams and thinking during the whole-class discussion? How did you select these students?

Begin Lesson

Warm-up

10 min

Notice and Wonder: Two Figures

Standards Alignments

Addressing	4.G.A.2
Building Towards	4.G.A.1

The purpose of this warm-up is to elicit the differences students notice between the two figures, which will be useful when students describe the changes their partners make in a later activity.

Instructional Routines

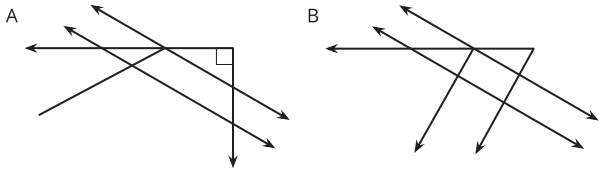
Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?

Launch

- Groups of 2
- Display the images.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time



Student Responses

Students may notice:

- Both images have rays and parallel lines.
- Figure A has a right angle labeled and B does not.
- Figure B has 2 sets of parallel lines.

Students may wonder:

- What are the measures of all the angles?
- What are the different shapes made by the intersecting lines and segments?

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- "What similarities and differences do you notice?"
- "We are going to keep thinking about the differences that we notice in figures in our first activity today."

Activity 1

15 min

Make a Change

Standards Alignments

Addressing 4.G.A.1, 4.G.A.2

The purpose of this activity is for students to use the vocabulary they have learned to describe changes they notice in a two-dimensional figure (MP6).

In this activity, students select a two-dimensional figure and draw it in their workbook while making one change. Their partner describes the change they notice using appropriate vocabulary from this unit.

Access for English Learners

MLR2 Collect and Display. Circulate, listen for and collect the language students use as they describe the change their partner made. On a visible display, record words and phrases such as: points, lines, line segments, rays, angles, perpendicular and parallel. Invite students to borrow language from the display as needed, and update it throughout the lesson.
Advances: Conversing, Reading

Materials to Copy

Make a Change (groups of 2)

Required Preparation

- Create a set of cards from the Instructional master for each group of 2 students.

Student-facing Task Statement

Your teacher will give you a set of cards. Each partner picks one of the cards.

1. Draw the figure you selected, but make at least one change.
2. Look at your partner's card and drawing. What change did your partner make?

If you have time, repeat with another card.

Student Responses

Sample responses:

1. A student adds a line that is parallel to a given line in the original figure.
2. There is one more line in this figure and it is parallel to this line.

Launch

- Groups of 2
- Give each pair of students a set of cards.

Activity

- 5 minutes: independent work time
- 5 minutes: partner work time
- Monitor for students who use geometric vocabulary to describe the changes they notice.

Synthesis

- Display student drawings.
- Invite previously selected students to share.

Activity 2

20 min

Guess My Figure

Standards Alignments

Addressing 4.G.A.1, 4.G.A.2

The purpose of this activity is for students to practice drawing and describing figures with points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines.

Students draw two-dimensional figures and then describe it to their partner. Their partner attempts to draw the figure without seeing it, making it important to use precise language to communicate important features of the figure (MP6).

Access for Students with Disabilities

Representation: Develop Language and Symbols. Provide students with access to definitions or reminders of recently learned terms, such as: ray, line segment, right angle, acute angle, obtuse angle, perpendicular lines, parallel lines.
Supports accessibility for: Memory, Conceptual Processing, Attention

Materials to Gather

Rulers or straightedges

Student-facing Task Statement

1. Create a two-dimensional shape that has at least 3 of the following:
    a. ray
    b. line segment
    c. right angle
    d. acute angle
    e. obtuse angle
    f. perpendicular lines
    g. parallel lines
2. Without showing your partner, describe the figure so that your partner is able to draw it as best as possible.
3. Switch roles, and draw your partner's shape based on their description.

Launch

- Groups of 2

Activity

- 5 minutes: independent work time
- 8–10 minutes: partner work time
- Monitor for diagrams that reflect a variety of geometric features.
- Monitor for students who consider both geometric features and measurement in the description.

Synthesis

- Invite previously selected students to share.
- "What would have helped your partner make a more accurate drawing?" (If I

Student Responses

Sample responses:

1. Student creates a two-dimensional shape that uses at least 3 features from the list.
2. Student describes the figure to their partner and the partner draws the figure based on the given description.

described the measure of the angle, measure of the line segment, the location, a grid, using a straight edge)

Lesson Synthesis

10 min

"Today, we described and drew two-dimensional figures."

"In this unit, we learned about different geometric features and measurement of two-dimensional figures. What are the most important things to remember about what we learned?" (Students may mention different vocabulary, how to measure angles with a protractor, and types of angles they learned in this unit.)

Suggested Centers

- Which One? (K–5), Stage 4: Grade 3 Shapes (Supporting)
- Can You Draw It? (1–5), Stage 4: Area and Perimeter (Supporting)

CKMath™

Core Knowledge MATHEMATICS™

Family Support Materials

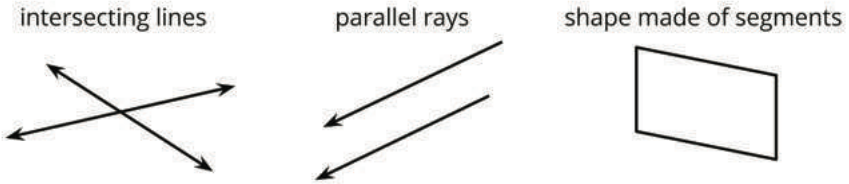
Family Support Materials

Angles and Angle Measurement

In this unit, students learn new language for describing parts of geometric figures and practice identifying and drawing them. They also learn to talk about angles, measure their size, and draw angles of different measurements.

Section A: Points, Lines, Segments, Rays, and Angles

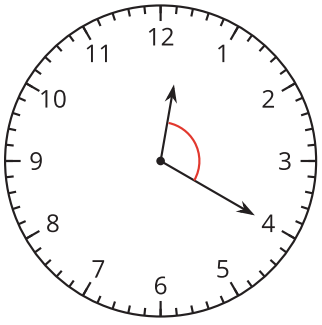
This section introduces students to some building blocks of geometric figures—points, rays, segments, angles, and lines. Students learn about parallel lines (lines that never intersect) and perpendicular lines (lines that meet or intersect at a right angle).

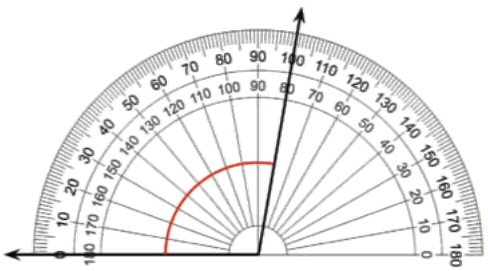


They also learn that an angle is a figure that is made up of two rays that share the same endpoint, called the vertex of the angle. Students practice identifying angles, noticing that angles are all around us and can have different sizes.

Section B: The Size of Angles

In this section, students compare and describe the size of angles. They begin by comparing angles visually, for example, by considering ways to describe the size of angles on a clock. The hands of a clock helps to show that an angle is formed when one ray rotates around a point shared with another ray.



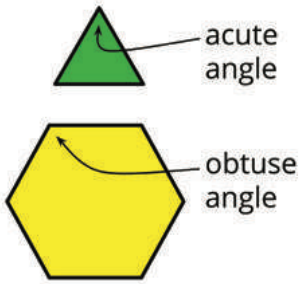


Students then learn that angles can be measured, with degrees (°) as the unit of measurement, and that a ray that makes a full turn around a point makes a 360-degree angle.

Later in the section, students learn to use a protractor to measure angles and to draw angles.

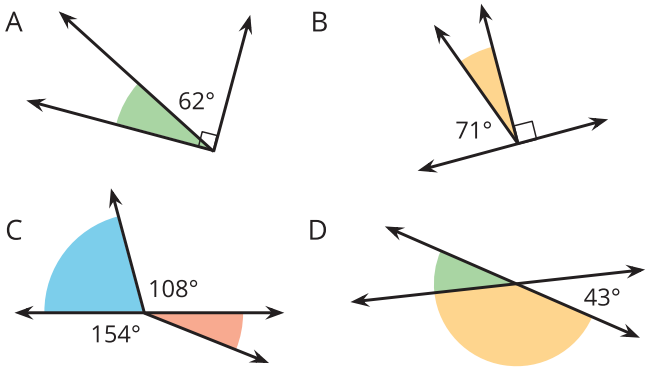
Section C: Angle Analysis

In this section, students continue to draw and analyze angles and to reason about their measurement. They classify angles by their size and identify angles as right, acute, obtuse, and straight.



Students learn that angles can be added. To investigate this idea, they use paper cutouts, patty paper, and drawings. Students fold, cut, mark, and assemble pieces of paper to see how angles can be composed (put together) and decomposed (broken apart).

Later, students solve problems and find unknown angle measurements in different contexts.



Try it at home!

Near the end of the unit, ask your student to:

- Find an acute angle, obtuse angle, straight angle, right angle, and parallel and perpendicular lines around the house.
- Describe and measure some angles found around the house.

Questions that may be helpful as they work:

- How would you describe that figure? How do you know it is a _____ ?
- How does that angle compare to a right angle (or a straight angle)?

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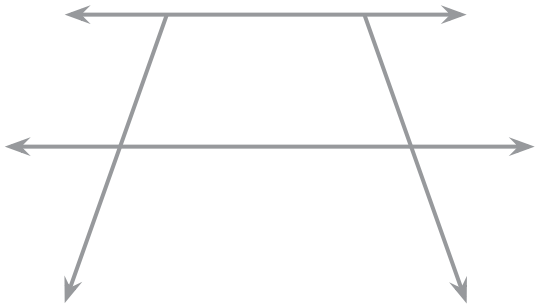
Unit Assessments

Check Your Readiness A, B and C
End-of-Unit Assessment

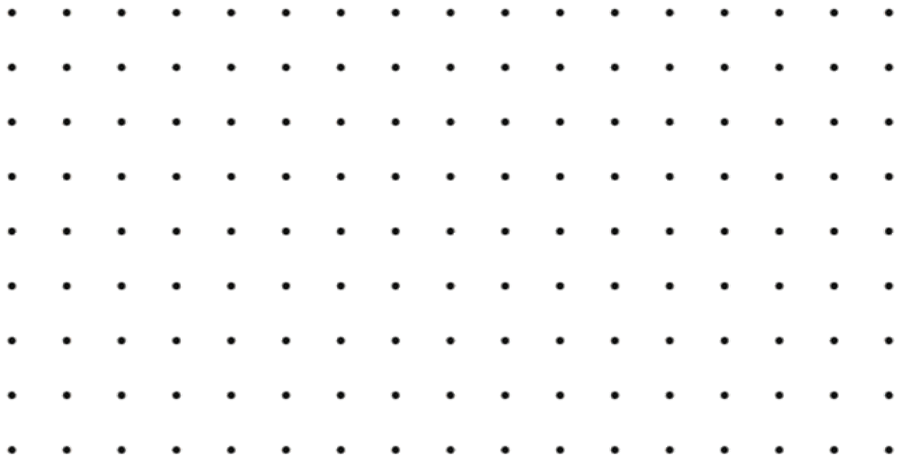
Angles and Angle Measurement: Section A Checkpoint

1. Identify a line, a ray, and a line segment in the figure by tracing those parts and labeling them.

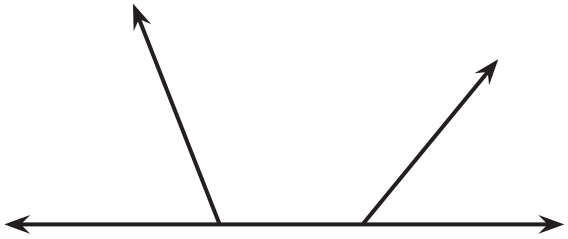


2. a. On the grid, draw a number or a letter that has at least two line segments that are parallel and two line segments that intersect.

   b. Draw a shape that has at least two pairs of line segments that are parallel.



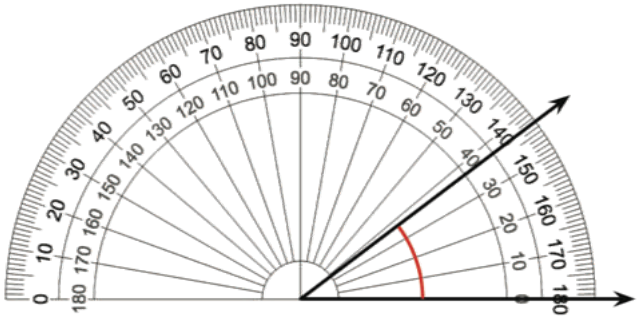
3. Mark as many angles as you can find in the diagram.



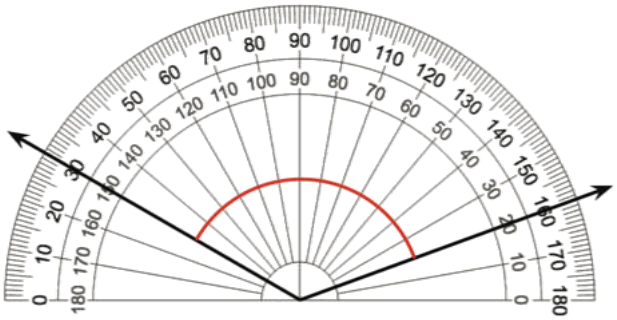
Angles and Angle Measurement: Section B Checkpoint

1. What is the measurement of each angle?

a.



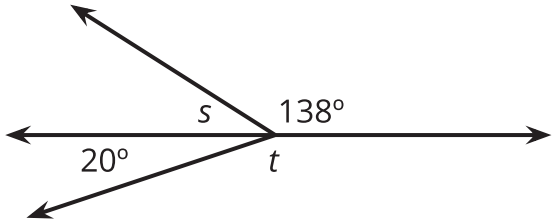
b.



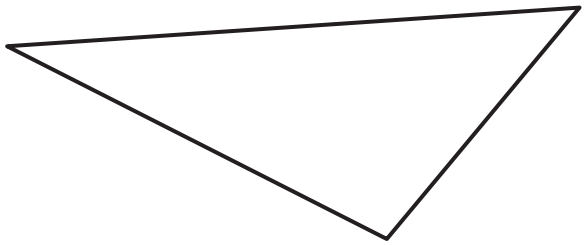
2. Use a protractor to draw a 135 degree angle.

Angles and Angle Measurement: Section C Checkpoint

1. Angle s and the angle that is 138° make a straight angle. What are the measurements of angles s and t? Explain or show your reasoning.

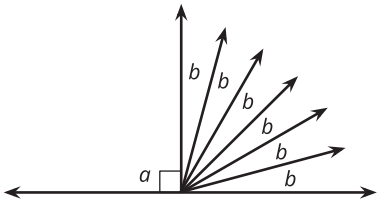


2. a. Decide if each angle in the triangle is acute, right, or obtuse.



b. Draw a triangle with one right angle. Label the right angle B.

3. What are the measurements of angles a and b if all of the angles add up to 180 degrees? Explain or show your reasoning.



Angles and Angle Measurement: End-of-Unit Assessment

1. Here is Andre's drawing of two line segments.

   a. Circle Andre's two segments.

   b. If the segments are extended in both directions, are the lines parallel? Are they perpendicular? Explain how you know.

2. Select **all** correct statements.

   A. There are 360 one-degree angles in a circle.

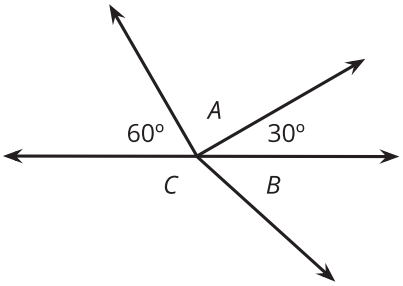
   B. There are 180 one-degree angles in a circle.

   C. There are 90 one-degree angles in a right angle.

   D. There are 180 one-degree angles in a right angle.

   E. There are 4 right angles in a circle.

3. The 60° angle, the 30° angle, and angle A together make a straight angle.



Select **all** correct statements.

A. Angle A is acute.

B. Angle B is acute.

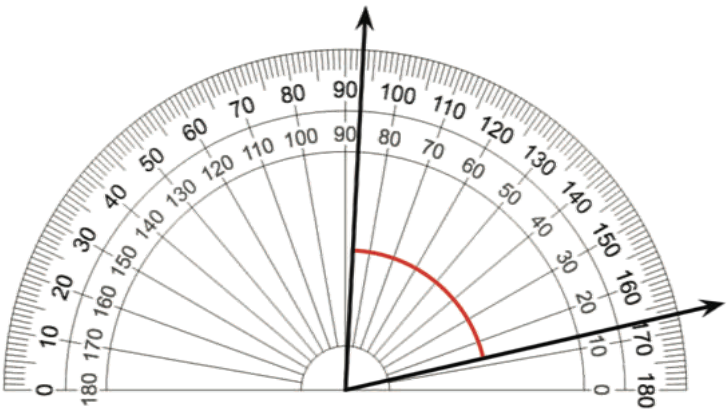
C. Angle C is acute.

D. Angle A is obtuse.

E. Angle B is obtuse.

F. Angle C is obtuse.

4. What is the measure of the angle?



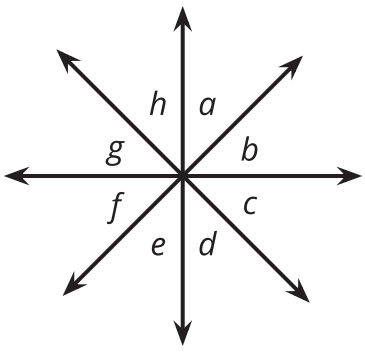
A. 74 degrees

B. 87 degrees

C. 93 degrees

D. 167 degrees

5. Angles a–h each have the same measurement. What is that measurement? Explain or show your reasoning.



6. Use a protractor to complete the following:

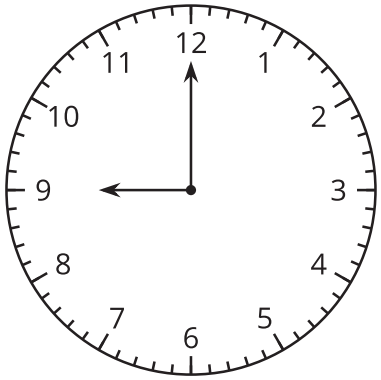
   a. Draw a ray that makes a 25 degree angle with the given ray.

   b. Draw a ray that makes a 60 degree angle with the given ray.

   c. What is the size of the angle made by the two rays you drew? Explain how you know.

7. a. The clock shows when Diego's spelling class begins.

What angle do the hour and minute hand on the clock make?



b. During the spelling class, the hour hand turns 15 degrees. How long is the spelling class? Explain or show your reasoning.

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Assessment Answer Keys

Check Your Readiness A, B and C
End-of-Unit Assessment

Assessment Answer Keys
Assessment: Section A Checkpoint

Teacher Instructions

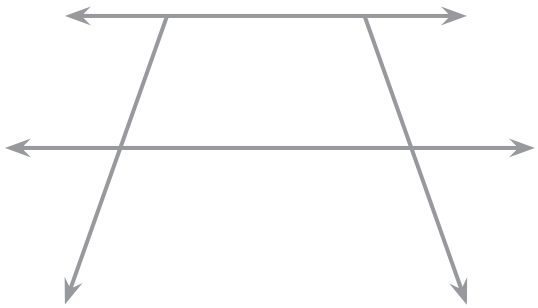
Give students access to straight edges or rulers.

Problem 1

Goals Assessed

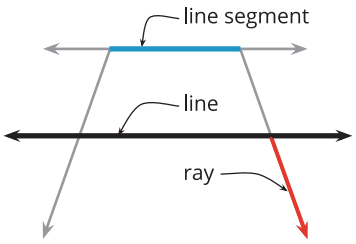
- Draw and identify points, lines, rays, segments, and parallel and intersecting lines in geometric figures.

Identify a line, a ray, and a line segment in the figure by tracing those parts and labeling them.



Solution

Sample response:

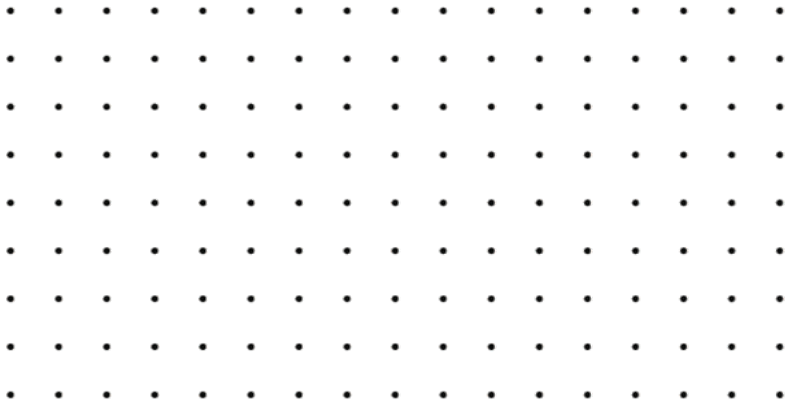


Problem 2

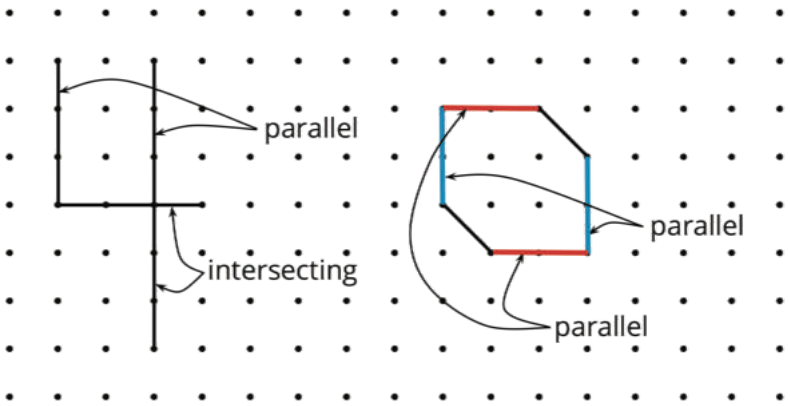
Goals Assessed

- Draw and identify points, lines, rays, segments, and parallel and intersecting lines in geometric figures.

a. On the grid, draw a number or a letter that has at least two line segments that are parallel and two line segments that intersect.
b. Draw a shape that has at least two pairs of line segments that are parallel.



Solution



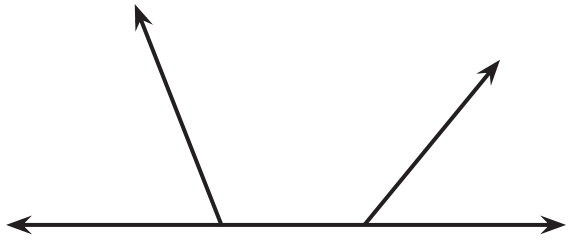
Problem 3

Goals Assessed

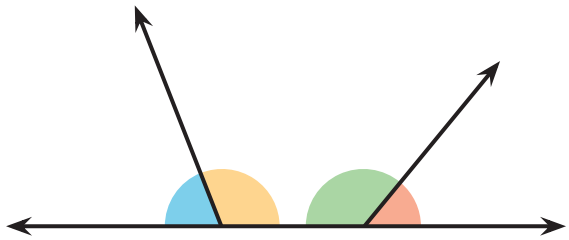
- Recognize that angles are formed wherever two rays share a common endpoint and identify

angles in two-dimensional figures.

Mark as many angles as you can find in the diagram.



Solution



Assessment: Section B Checkpoint

Teacher Instructions

Give students access to protractors.

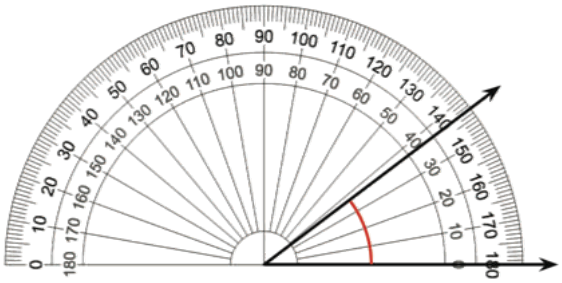
Problem 1

Goals Assessed

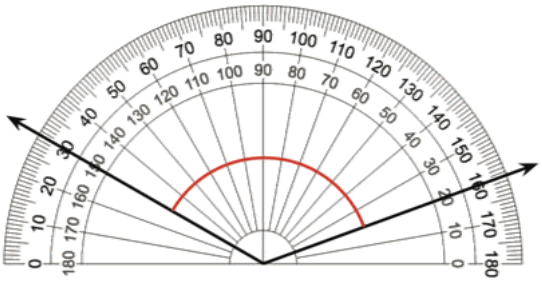
- Recognize that angles can be measured in degrees and can be found using addition and subtraction.
- Use a protractor to measure and draw angles and recognize that perpendicular lines meet or cross at a right angle.

What is the measurement of each angle?

a.



b.



Solution

a. 37°, because $180 - 143 = 37$.

b. 130°, because $160 - 30 = 130$.

Problem 2

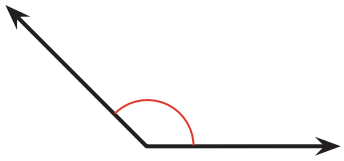
Goals Assessed

- Use a protractor to measure and draw angles and recognize that perpendicular lines meet or cross at a right angle.

Use a protractor to draw a 135 degree angle.

Solution

Sample response:



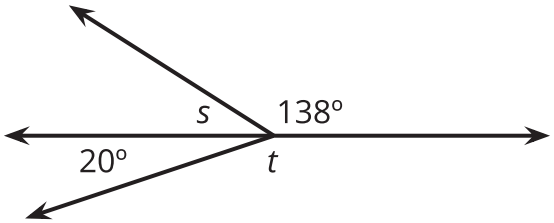
Assessment: Section C Checkpoint

Problem 1

Goals Assessed

- Write equations to represent angle relationships and reason about and find unknown measurements.

Angle s and the angle that is 138° make a straight angle. What are the measurements of angles s and t? Explain or show your reasoning.



Solution

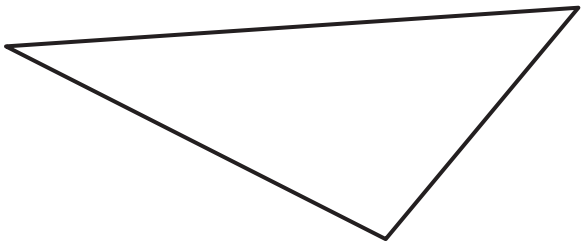
- Angle s is 42°. Sample reasoning: Angle s and the 138° angle make a straight angle, and $180 - 138 = 42$.
- Angle t is 160°. Sample reasoning: $180 - 20 = 160$

Problem 2

Goals Assessed

- Draw and identify acute, obtuse, right, and straight angles in two-dimensional figures.

a. Decide if each angle in the triangle is acute, right, or obtuse.

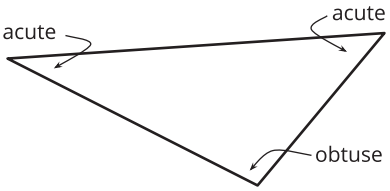


b. Draw a triangle with one right angle. Label the right angle B.

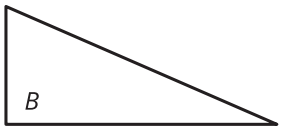
Solution

Sample responses:

a.



b.

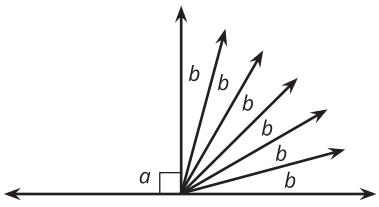


Problem 3

Goals Assessed

- Recognize that angles can be measured in degrees and can be found using addition and subtraction.

What are the measurements of angles a and b if all of the angles add up to 180 degrees? Explain or show your reasoning.



Solution

- Angle a is 90° because the rays that create the angle are perpendicular.
- Angle b is 15° because six of them make a 90° angle ($180 - 90 = 90$), so each of them is $90 \div 6$ or 15 degrees.

Assessment: End-of-Unit Assessment

Teacher Instructions

Give students access to protractors.

Problem 1

Standards Alignments

Addressing 4.G.A.1

Narrative

Students identify two segments in a drawing and analyze the angles made by the lines containing those segments. The lines are not parallel because they meet at the center of the X and they are not perpendicular either. Students may use a protractor to check that the angles do not measure 90 degrees or they may visually identify that the lines are not perpendicular.

Here is Andre's drawing of two line segments.

a. Circle Andre's two segments.
b. If the segments are extended in both directions, are the lines parallel? Are they perpendicular? Explain how you know.

Solution

a. Student circles or labels the two segments making the X.

b. No, the lines are not parallel because they meet in the middle of the X. They are also not perpendicular because the angles where the lines meet are not right angles.

Problem 2

Standards Alignments

Addressing 4.MD.C.5, 4.MD.C.7

Narrative

Students evaluate claims about the angles that make right angles and full circles. Students who do not select A or C may understand that the measure of a full circle is 360 degrees and the measure of a right angle is 90 degrees but have not connected that idea to the notion of many one-degree angles making up these larger angles. Students who select B or D need more practice measuring angles and in particular practice with problems that use the fact that there are 180 degrees in a line to find the value of different angles. Students can reason about E using either the additivity of angles or by thinking about a pair of perpendicular lines.

Select **all** correct statements.

A. There are 360 one-degree angles in a circle.

B. There are 180 one-degree angles in a circle.

C. There are 90 one-degree angles in a right angle.

D. There are 180 one-degree angles in a right angle.

E. There are 4 right angles in a circle.

Solution

["A", "C", "E"]

Problem 3

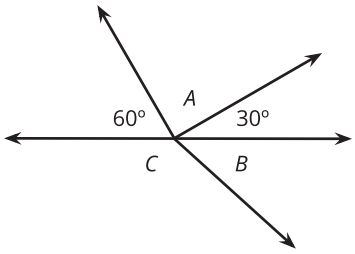
Standards Alignments

Addressing 4.MD.C.7

Narrative

Students identify different types of angles in a diagram. They can visually identify that angle B is acute and angle C is obtuse. They can calculate that angle A is a right angle since it makes a 180 degree angle along with the angles labeled as 30 degrees and 60 degrees. Students who select E probably do not understand what acute and obtuse angles are. Students who select A or D may be measuring inaccurately with a protractor rather than using the additivity of angles. These students may understand what acute and obtuse angles are.

The 60° angle, the 30° angle, and angle A together make a straight angle.



Select **all** correct statements.

A. Angle A is acute.

B. Angle B is acute.

C. Angle C is acute.

D. Angle A is obtuse.

E. Angle B is obtuse.

F. Angle C is obtuse.

Solution

["B", "F"]

Problem 4

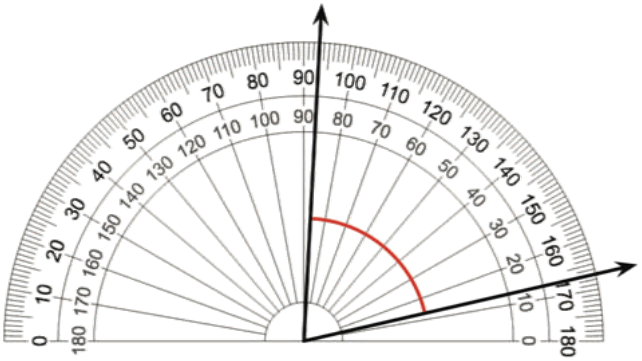
Standards Alignments

Addressing 4.MD.C.6

Narrative

Students find the measure of an angle using an image of a protractor. Students who select C or D are likely using the protractor to identify one of the two rays but are not performing arithmetic with those numbers to find the measure of the angle. Students who select B have either made an arithmetic error or possibly subtracted 93 from 180 which would be another misuse of the protractor.

What is the measure of the angle?



A. 74 degrees

B. 87 degrees

C. 93 degrees

D. 167 degrees

Solution

A

Problem 5

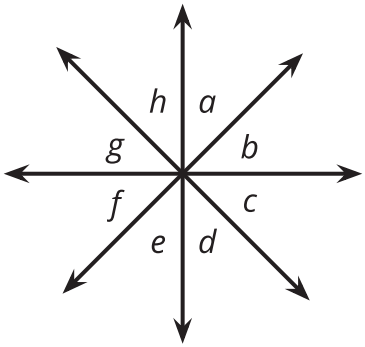
Standards Alignments

Addressing 4.MD.C.5, 4.MD.C.7

Narrative

Students find the measure of an angle using the facts that there are 360 degrees in a full circle and that angle measure is additive. Students may reason that 8 of these angles make 360 degrees, 4 of them make 180 degrees, or 2 of them make 90 degrees. It is not given that 2 of the angles make a 90 degree angle or that 4 of them make a line but students can find this by measuring or use their visual intuition for 90 degree and 180 degree angles.

Angles a–h each have the same measurement. What is that measurement? Explain or show your reasoning.



Solution

45 degrees. The full circle is 360 degrees and it is divided into 8 equal angles and $360 \div 8 = 45$.

Problem 6

Standards Alignments

Addressing 4.MD.C.6, 4.MD.C.7

Narrative

Students construct two angles of given measures and then use these angles to define a third angle and find its measure. For the third part, depending on how students construct the two angles, the answer can be 85 degrees or 35 degrees. If students use a protractor to measure this new angle, they may get a slightly different answer while they will get an exact answer if they use angle additivity.

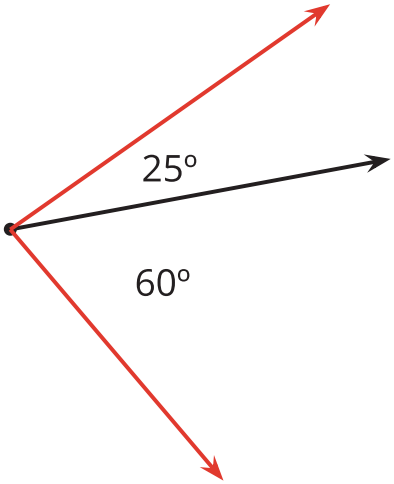
Use a protractor to complete the following:

a. Draw a ray that makes a 25 degree angle with the given ray.
b. Draw a ray that makes a 60 degree angle with the given ray.
c. What is the size of the angle made by the two rays you drew? Explain how you know.

Solution

Sample response:

a. See drawing.
b. See drawing.
c. The angle on top measures 25° and the angle on bottom is 60°. The two rays make an angle of 85° ($60 + 25 = 85$).



Problem 7

Standards Alignments

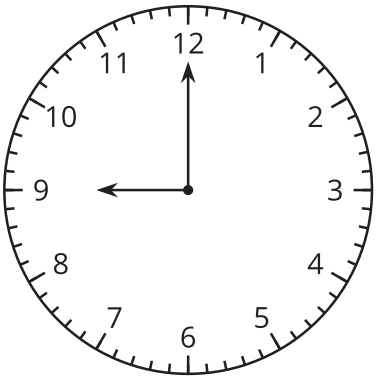
Addressing 4.MD.C.5, 4.MD.C.7

Narrative

Students find the measure of angles made by clock hands. For the first problem, they will likely recognize that the hands make a right angle. For the second problem, they need to use the additivity of angles and the fact that the hours on the clock face divide the entire circle into 12 equal angles, each of which measures 30 degrees. Combined with the fact that half of 30 is 15 this is the key idea that will help students find how long it takes for the hour hand to move through a 15 degree angle.

a. The clock shows when Diego's spelling class begins.

   What angle do the hour and minute hand on the clock make?



b. During the spelling class, the hour hand turns 15 degrees. How long is the spelling class? Explain or show your reasoning.

Solution

a. 90 degrees or 270 degrees

b. Half an hour. Sample response:

   - From 9 to 12 is a 90 degree angle and so from 9 to 10 is a third of that or a 30 degree angle. Half of a 30 degree angle is a 15 degree angle so the class lasts half an hour.
   - I know that successive hours are 30 degree angles since there are 12 of them in the 360 degree circle. And two 15 degree angles make a 30 degree angle so this is half of an hour.

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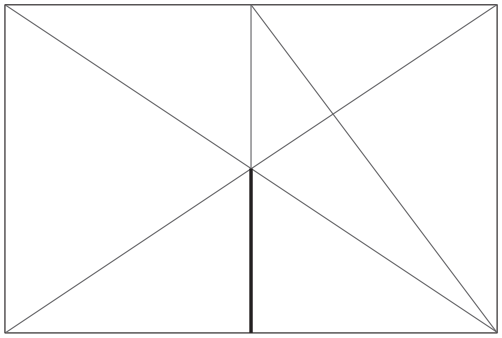
Lesson Cool Downs

Lesson 1: How Would You Describe These Figures?

Cool Down: Lines and More

Here is a drawing on a card:



Write a description of the drawing that could be used by a classmate to make a copy.

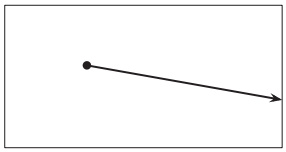
Lesson 2: Points, Lines, Rays, and Segments

Cool Down: True or False: What's the Point?

Decide if each statement is true or false. If it is false, correct it.

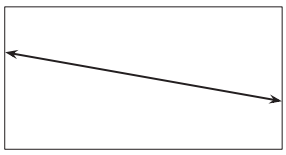
1. A point marks a place.

2. This is a drawing of a ray.



3. A line can be curved or straight.

4. This is a drawing of a segment.

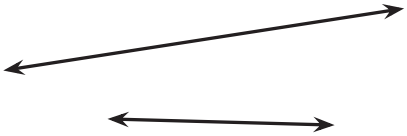


5. The length of a ray can be measured.

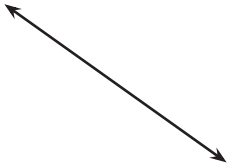
Lesson 3: Two or More Lines

Cool Down: Parallel and Not Quite Parallel

1. Explain why these lines are not parallel.



2. Sketch a line that is parallel to this line.



Lesson 4: Points and Lines All Around

Cool Down: Word Fun

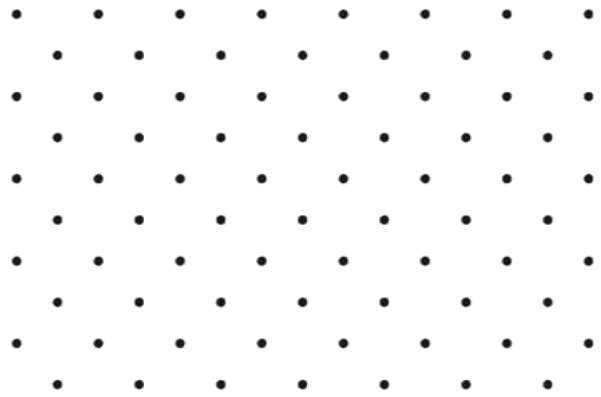
FUN KITES

1. Which letters in the phrase FUN KITES have:

    a. parallel segments

    b. no parallel segments

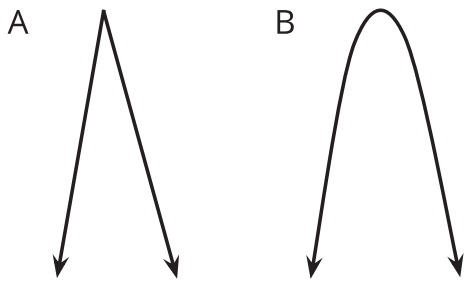
2. Here is a field of dots. Use it to draw 2 pairs of parallel lines, each pair pointing in a different direction.



Lesson 5: What is an Angle?

Cool Down: Spot the Angles

1. Jada says Figure A shows an angle, but Figure B does not. Do you agree? Explain your reasoning.

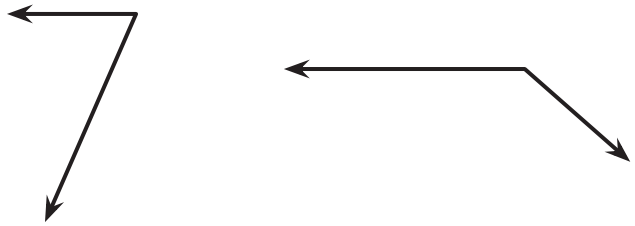


2. Identify the angles in each letter and draw the rays to show each angle.

Lesson 6: Compare and Describe Angles

Cool Down: Compare Two Angles

Here are two angles.



1. Describe at least one way they are alike.

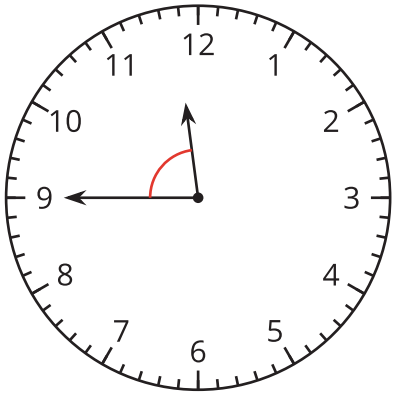
2. Describe at least one way they are different.

Lesson 7: The Size of Angles on a Clock

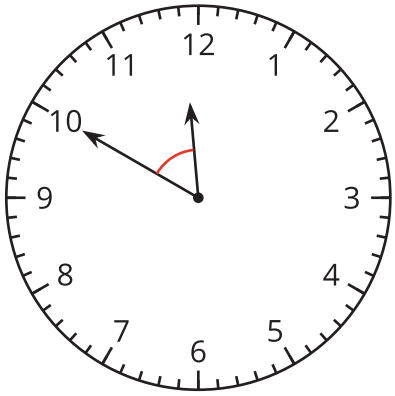
Cool Down: Which Angle is Larger? By How Much?

The hands on each clock form an angle.

A



B



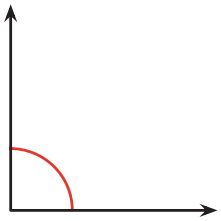
Which angle is larger? How much larger is it than the other angle? Explain how you know.

Lesson 8: The Size of Angles in Degrees

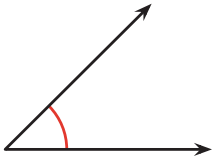
Cool Down: Estimate Angle Size in Degrees

Use the tool you created to estimate the size of each angle in degrees.

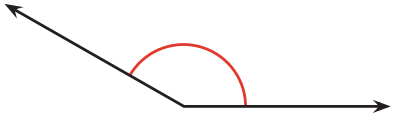
a



b



c



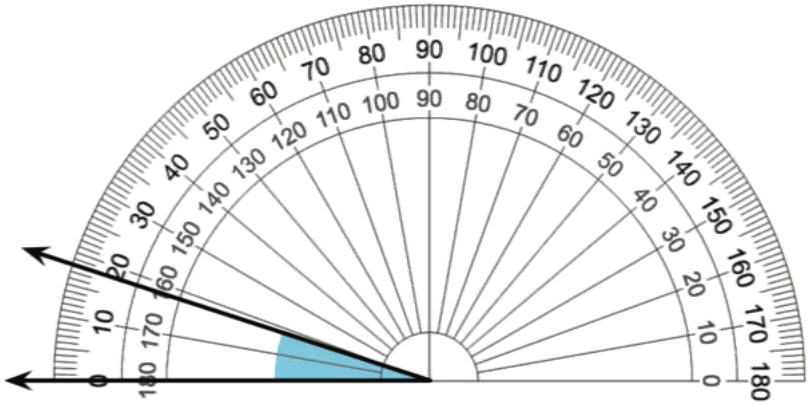
Lesson 9: Use a Protractor to Measure Angles

Cool Down: Measure the Angles

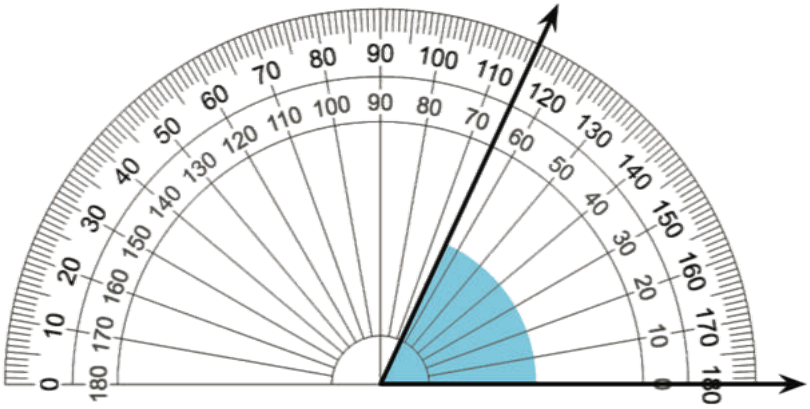
1. An angle is composed of seventeen 1° angles. How many degrees is the angle?

2. What is the measurement of each angle?

   a.



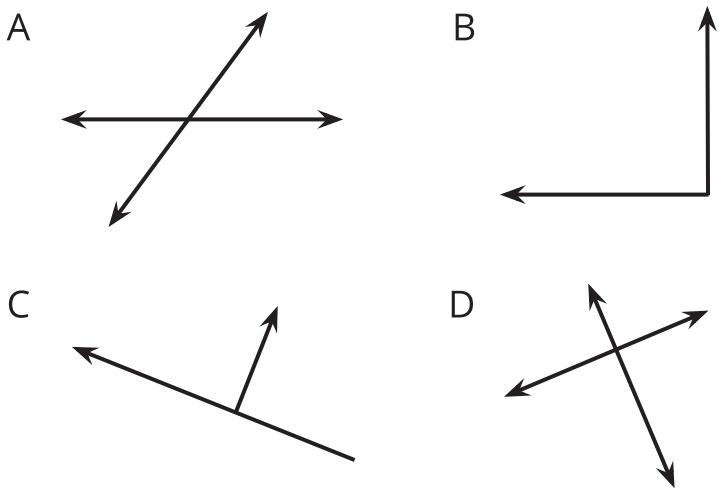
   b.



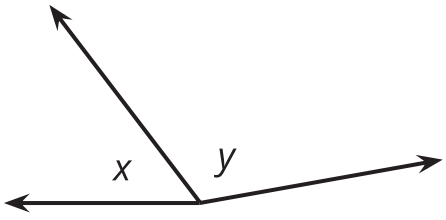
Lesson 10: Angle Measurement and Perpendicular Lines

Cool Down: Size Up Angles

1. Which figures show perpendicular lines or rays?



2. Use a protractor to measure the labeled angles in the figure.



Lesson 11: Use a Protractor to Draw Angles

Cool Down: A Ray or Two

1. Draw a new ray starting from point Z to create a 25° angle.

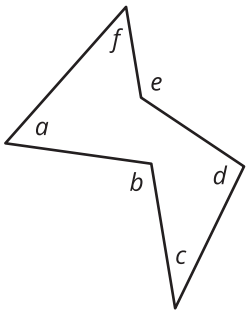
2. Draw two rays to create an angle that is 165°.

Lesson 12: Types of Angles

Cool Down: Obtuse, Acute, and Straight Angles

1. Here is a ray. Draw another ray from point P to make an acute angle.

2. Here are some labeled angles. Identify all angles that are obtuse.



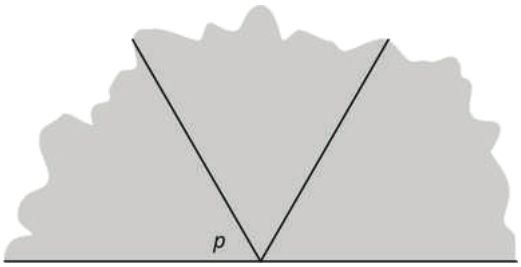
3. An angle is formed by four 35° angles. Is that angle a straight angle? Explain how you know.

Lesson 13: Find Angle Measurements

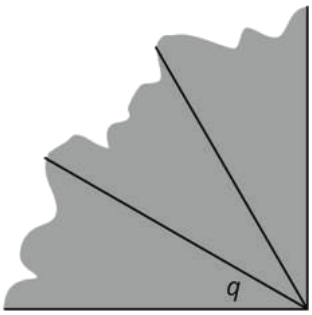
Cool Down: Sets of Three Angles

Noah cuts out 3 copies of angle p and 3 copies of angle q. He arranges them side by side.

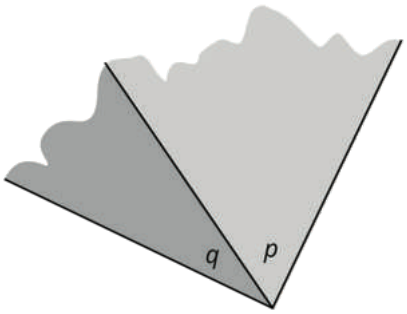
1. Three copies of angle p make a straight line. How many degrees is angle p? Explain or show your reasoning.



2. Three copies of angle q make a right angle. How many degrees is angle q? Explain or show your reasoning.



3. Noah puts angle p and angle q together. How many degrees is the resulting angle? Explain or show your reasoning.

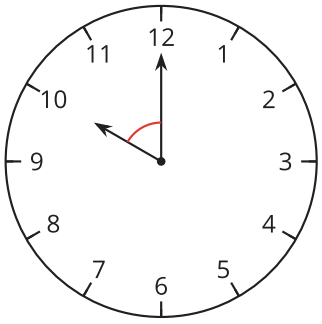


Lesson 14: Reasoning about Angles (Part 1)

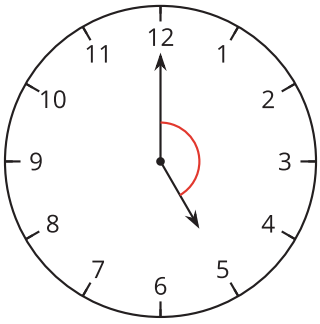
Cool Down: One Angle at a Time

How many degrees is each marked angle on the clock? Explain or show your reasoning.

A



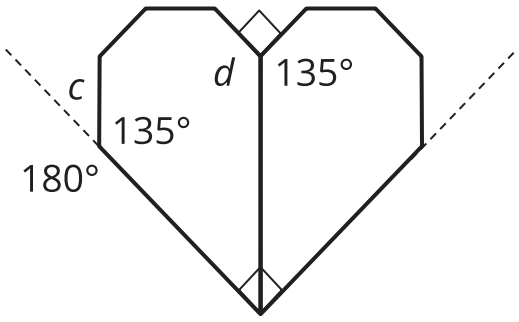
B



Lesson 15: Reasoning About Angles (Part 2)

Cool Down: Heart to Heart

Find the measurement of each labeled angle. Show your reasoning.



CKMath™

Core Knowledge MATHEMATICS™

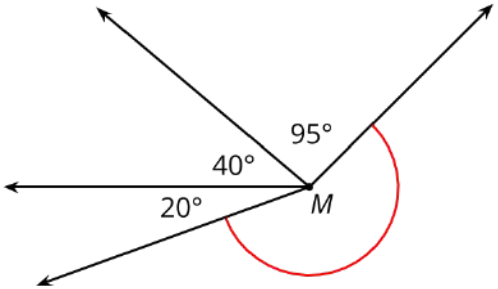
Instructional Masters

Instructional Masters for Angles and Angle Measurement

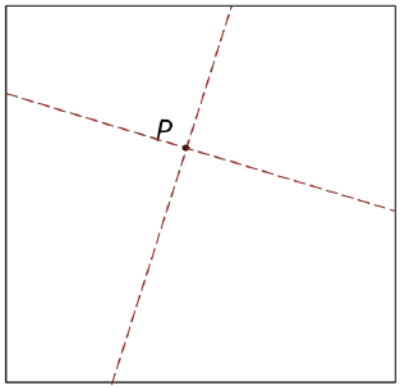
address	title	students per copy	written on?	requires cutting?	card stock recommended?	color paper recommended?
Activity Grade4.7.16.1	Make a Change	2	no	yes	no	no
Activity Grade4.7.1.1	Do You See What I See?	4	no	yes	yes	no
Activity Grade4.7.5.1	Tricky Figures	8	no	yes	no	no
Activity Grade4.7.15.2	Info Gap: Whole Bunch of Angles	2	no	yes	no	no
Activity Grade4.7.3.2	Illustrated Word Wall	1	yes	no	no	no
Activity Grade4.7.2.1	Card Sort: Who Am I?	2	no	yes	yes	no
Activity Grade4.7.13.1	How Big Are These Angles?	2	no	yes	no	no
Activity Grade4.7.6.1	Card Sort: Angles	2	no	yes	yes	no
Activity Grade4.7.8.2	Making a Measuring Tool	3	yes	yes	no	no
Center	Rolling for Fractions Stage 2 Recording Sheet	1	yes	no	no	no
Center	Compare Stage 7 Cards	2	no	yes	no	no
Center	Compare Stage 3-8 Directions	2	yes	no	no	no
Center	Compare Stage 3-8 Directions	2	yes	no	no	no

Center	Target Measurement Stage 4 Recording Sheet	2	yes	no	no	no
Center	Target Measurement Stage 4 Homemade Protractor	2	no	yes	yes	no
Center	Fraction Cards Grade 3	2	no	no	yes	no
Center	Fraction Cards Grade 4	2	no	no	yes	no
Center	Shape Cards Grade 3	2	no	yes	no	no
Center	Triangle Cards Grade 3	2	no	yes	no	no
Center	Quadrilateral Cards Grade 3	2	no	yes	no	no
Center	Can You Draw It Stage 4 Recording Sheet	1	yes	no	no	no

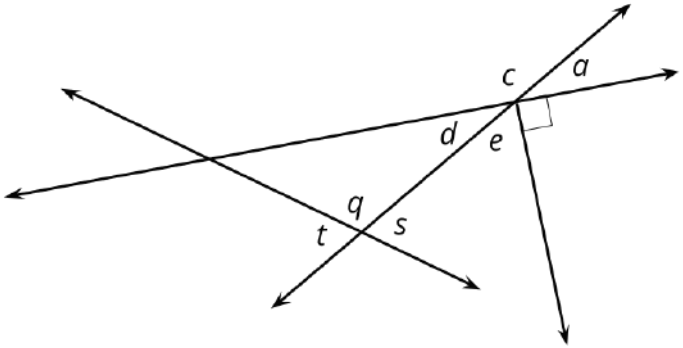
Make a Change
Card 1



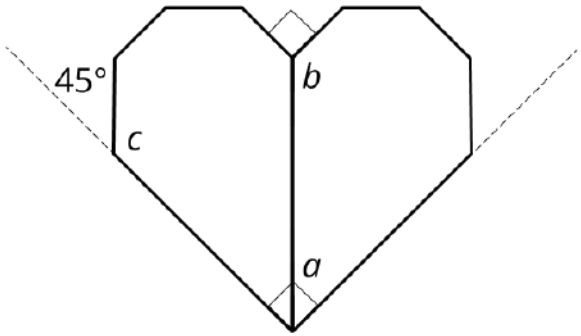
Make a Change
Card 2



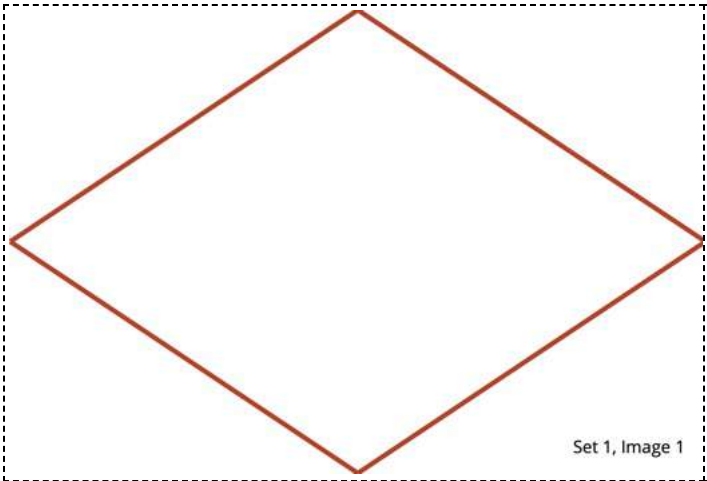
Make a Change
Card 3



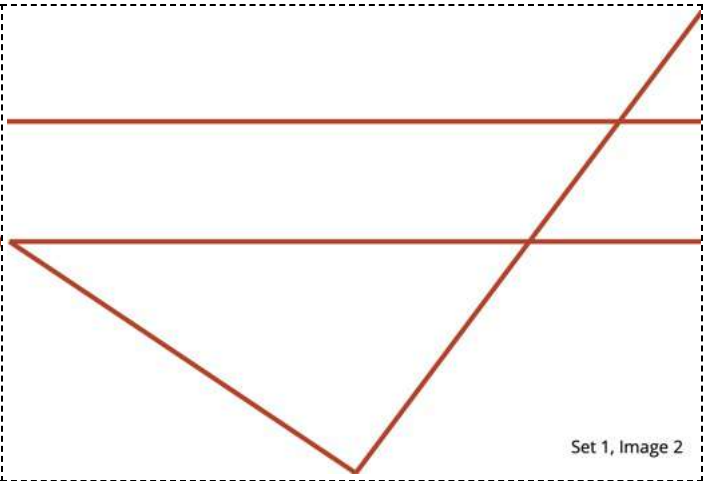
Make a Change
Card 4



Do You See What I See?



Set 1, Image 1



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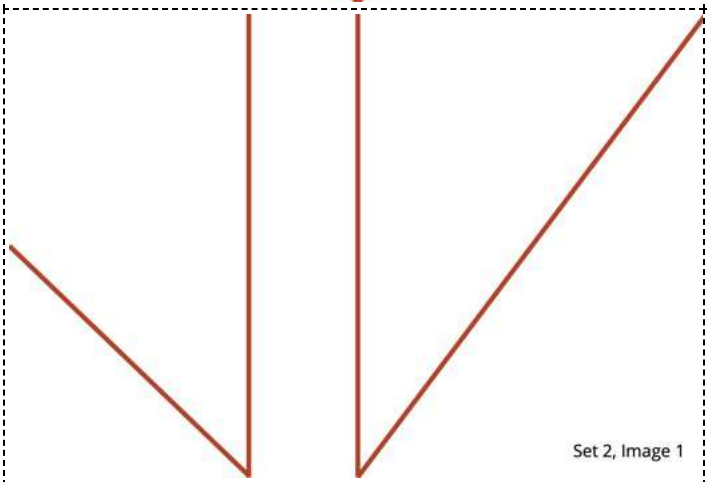
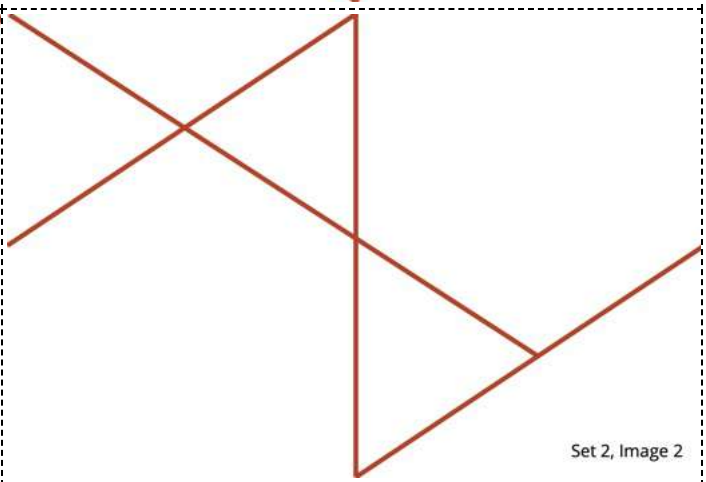


Set 2, Image 1



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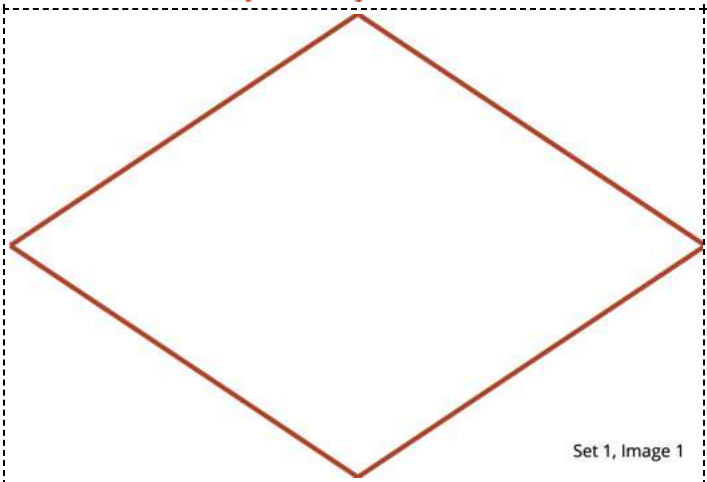

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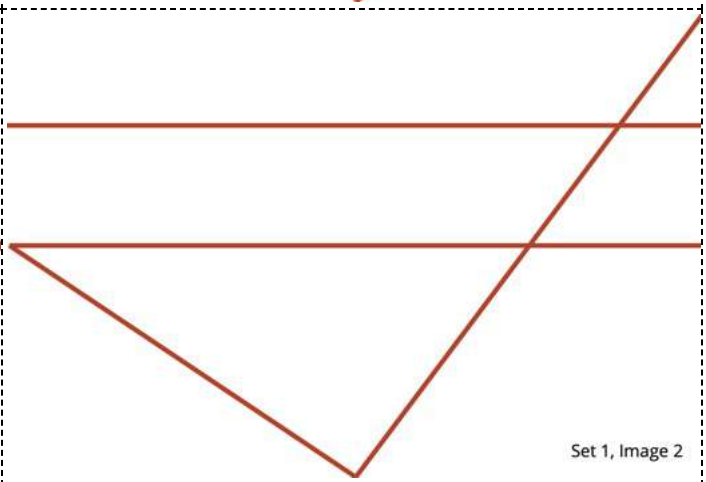

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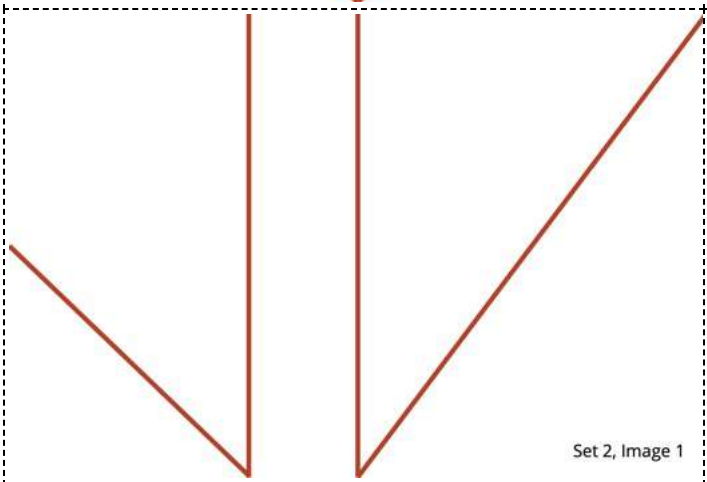

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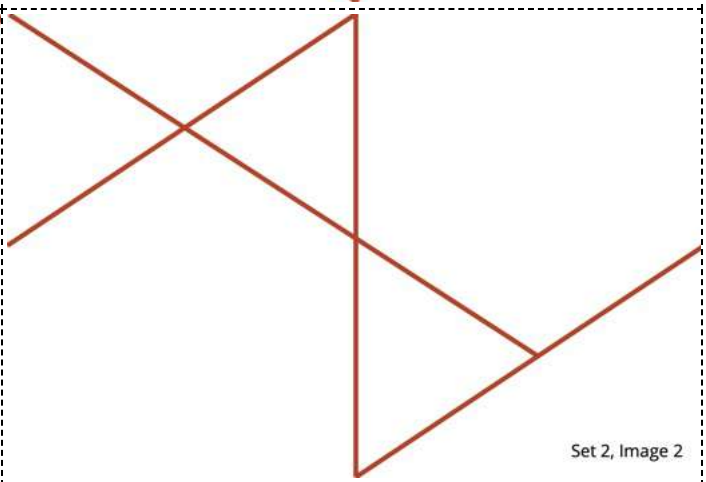
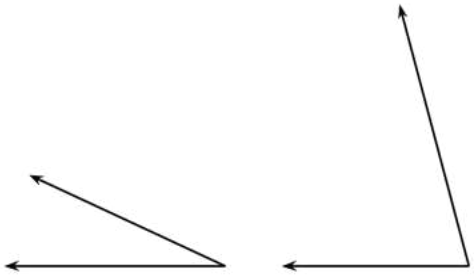


Set 2, Image 2

Tricky Figures

Set 1



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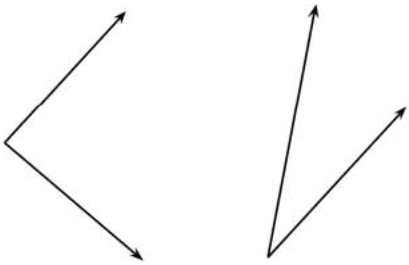

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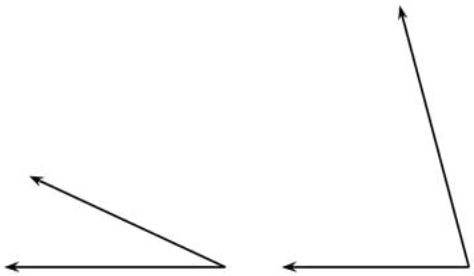

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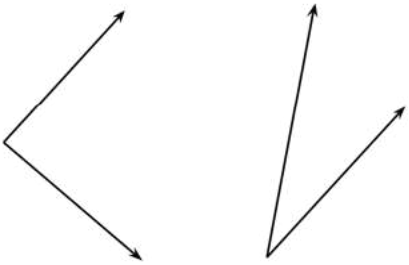

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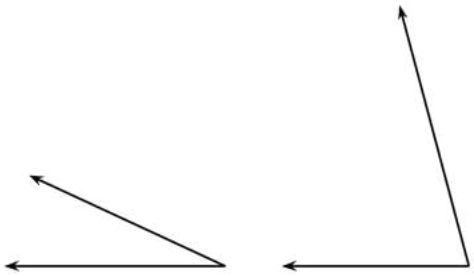

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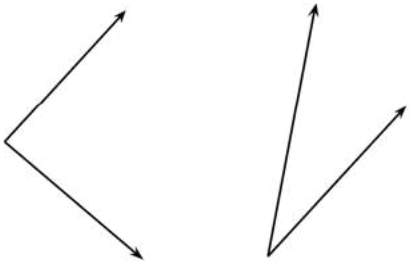

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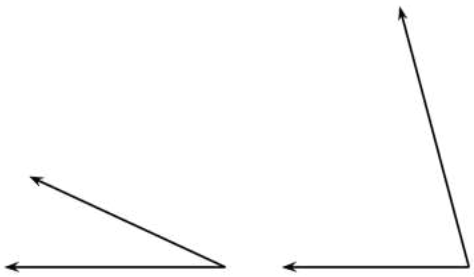

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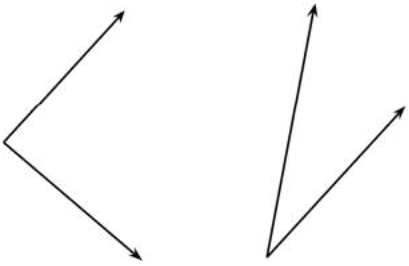
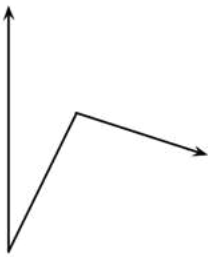


Tricky Figures

Set 3A (if you have time)



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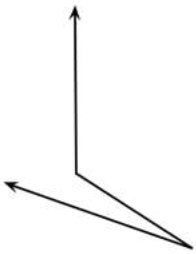
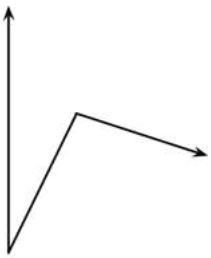


Set 3A (if you have time)



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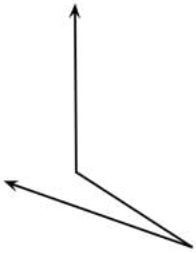
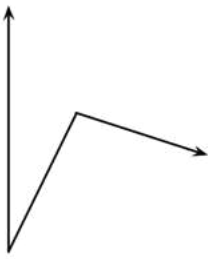
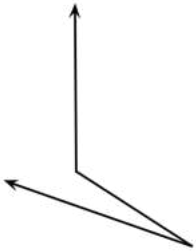


Set 3A (Are You Ready for More?)



Set 3B (Are You Ready for More?)



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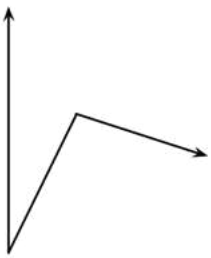

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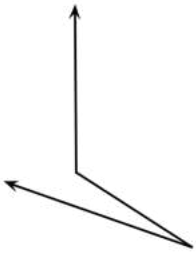
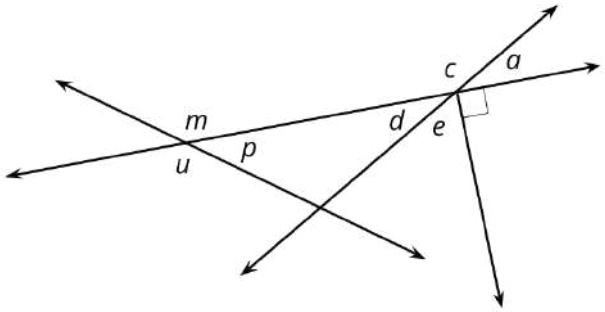


Info Gap: A Whole Bunch of Angles

Problem Card 1

Three lines and a ray cross and make a bunch of angles.

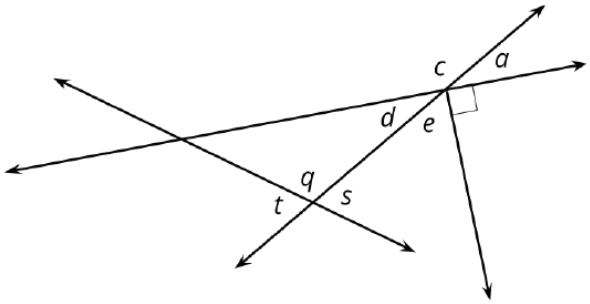


1. What is the size of angle *p*?
2. Which angle is greater: *p* or *d*?

Info Gap: A Whole Bunch of Angles

Problem Card 2

Three lines and a ray cross and make a bunch of angles.



1. What is the size of angle *q*?
2. Which angle is greater: *t* or *e*?

Info Gap: A Whole Bunch of Angles

Data Card 1

- Angle *u* is 128°.
- Angle *d* is the same size as angle *a*.
- Angle *c* is 143°.

Info Gap: A Whole Bunch of Angles

Data Card 2

- Angle *t* is smaller than a right angle.
- Angle *s* is 79°.
- Angle *a* is 37°.

Illustrated Word Wall

Add unfamiliar words and phrases and illustrations to your word wall.

A.	B.	C.	D.
E.	F.	G.	H.
I.	J.	K.	L.

Illustrated Word Wall

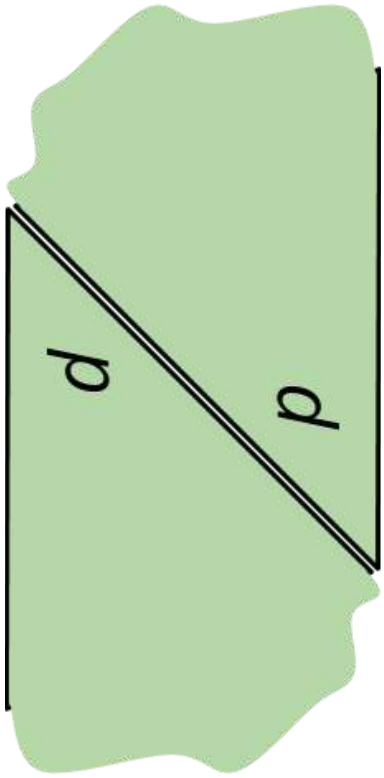
Add unfamiliar words and phrases and illustrations to your word wall.

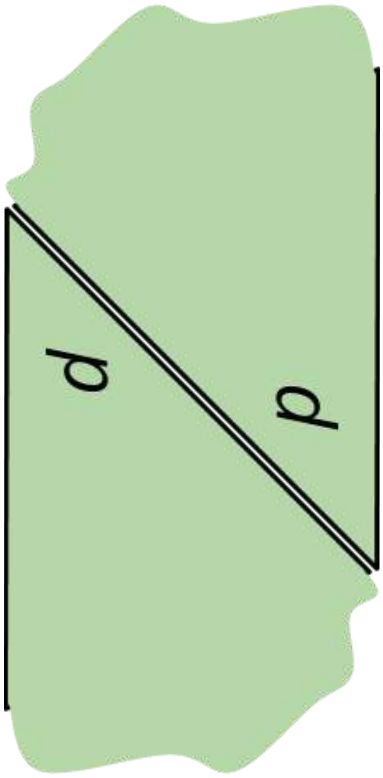
M.	N	O.	P.
Q.	R.	S.	T.
U.	V.	W.	XYZ.

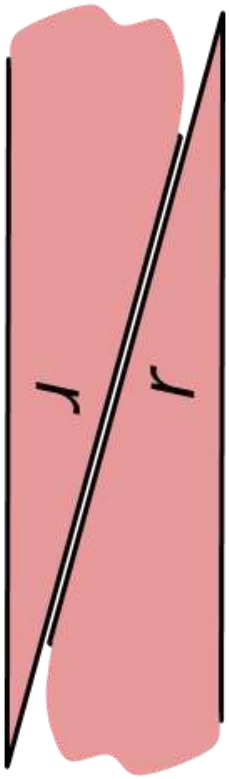
Card Sort: Who Am I?

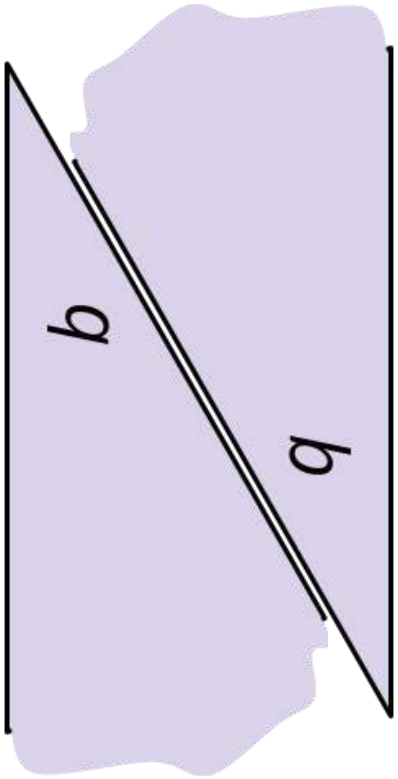
	My length can't be measured. I have a start but no end.
I have no size (no width, length, or thickness).	I am a part of a line that is bounded by two points.
I am a line that is bounded by one point but goes on in the other direction.	I am a set of points arranged in a straight way. I extend infinitely in two directions.
I often have one dot and one arrow to show that I go on in one direction.	My length can't be measured. I have no start and no end.
	I have a specific length.
I am usually represented by a dot or a circle.	
I often have arrows to show that I keep going in two directions.	I am a location.
	I am often shown with two dots or with no dots.

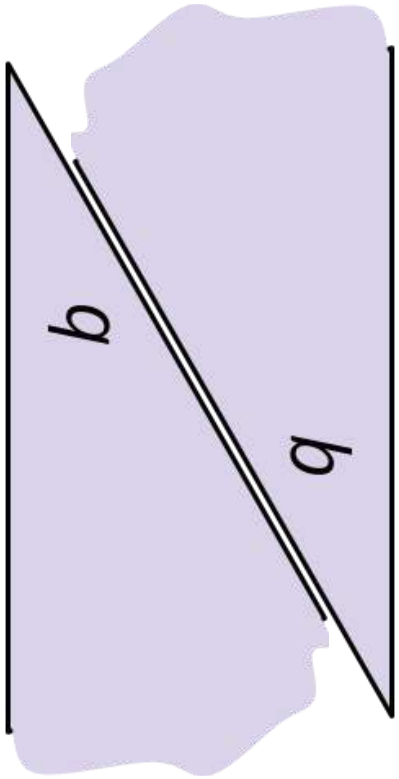
How Big Are These Angles?

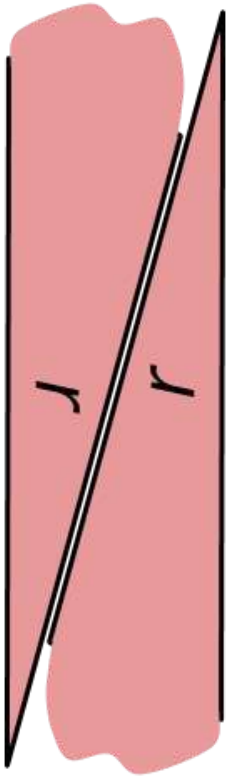




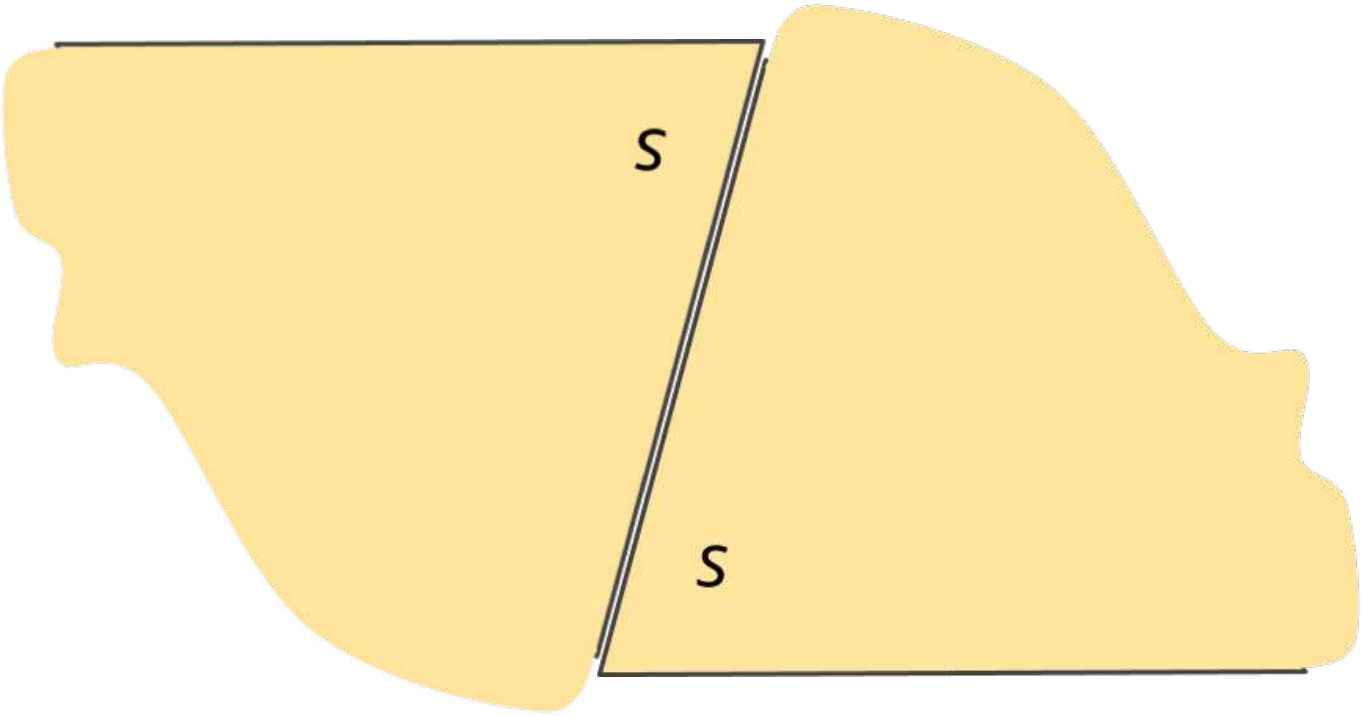


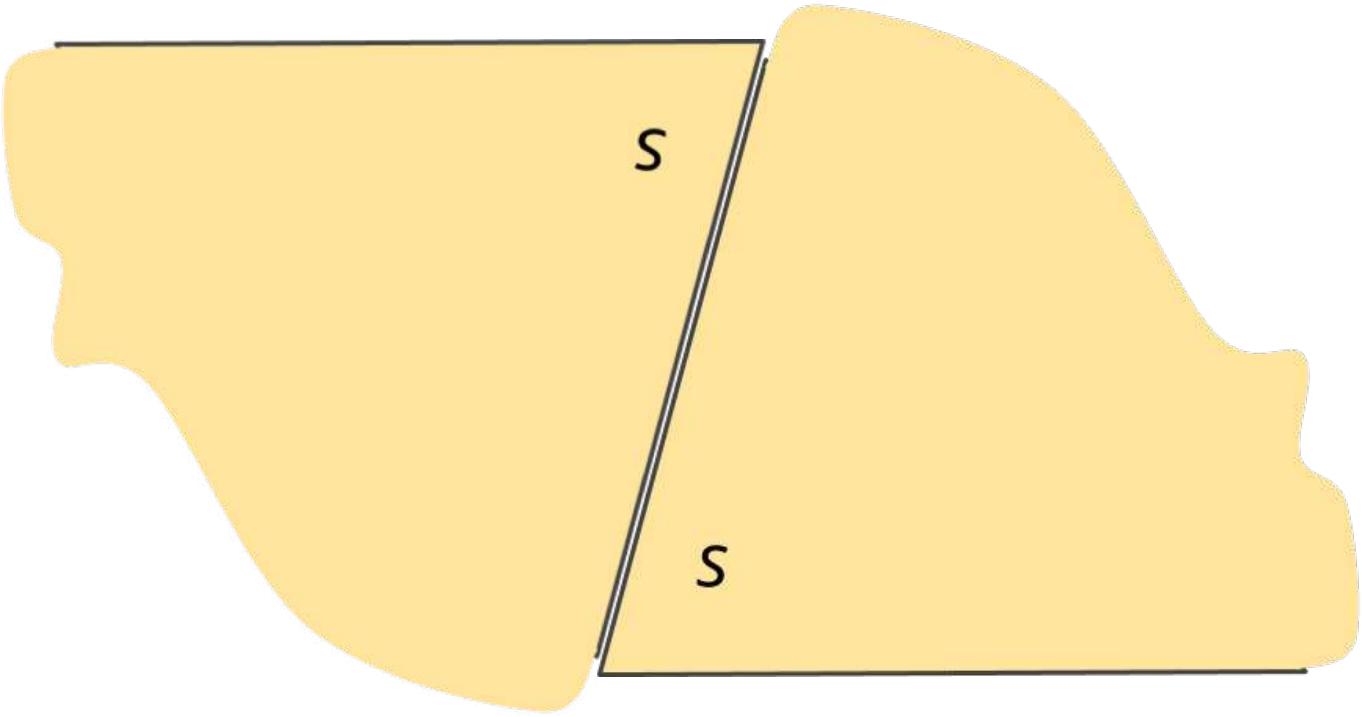






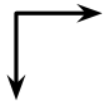
How Big Are These Angles?



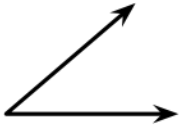


Card Sort: Angles

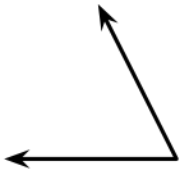
Card Sort: Angles
A



Card Sort: Angles
B

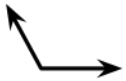


Card Sort: Angles
C

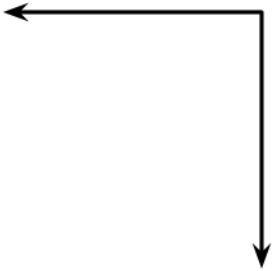


Card Sort: Angles
D

Card Sort: Angles
E

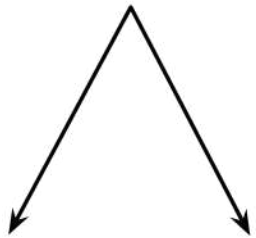


Card Sort: Angles
F



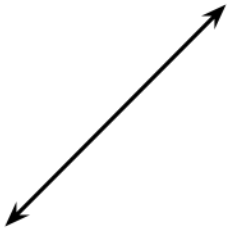
Card Sort: Angles
G

Card Sort: Angles
H

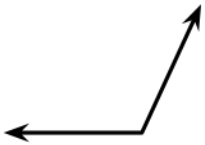


Card Sort: Angles

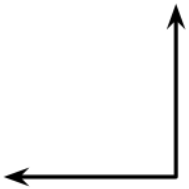
Card Sort: Angles
I



Card Sort: Angles
J

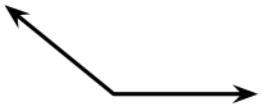


Card Sort: Angles
K



Card Sort: Angles
L

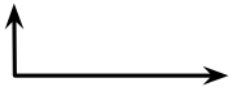
Card Sort: Angles
M



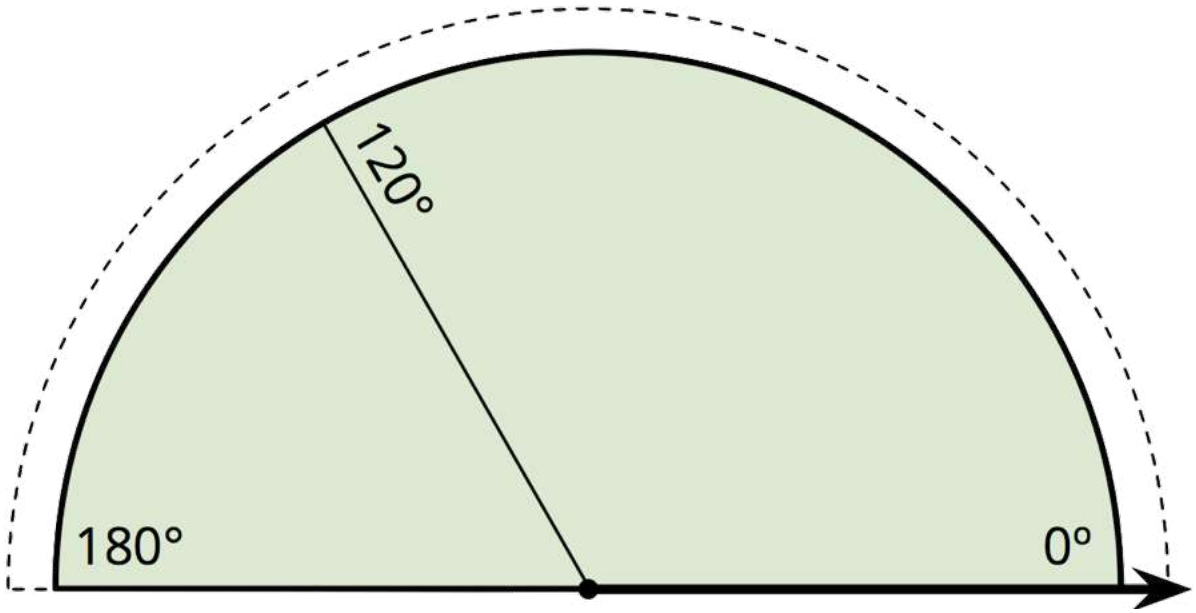
Card Sort: Angles
N

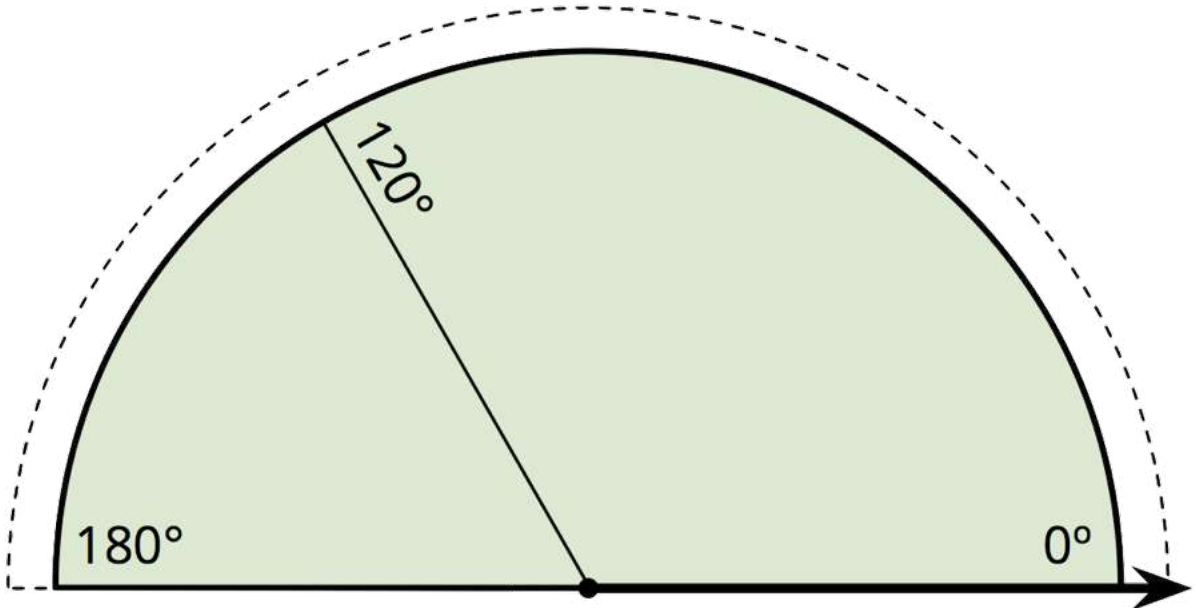
Card Sort: Angles
O

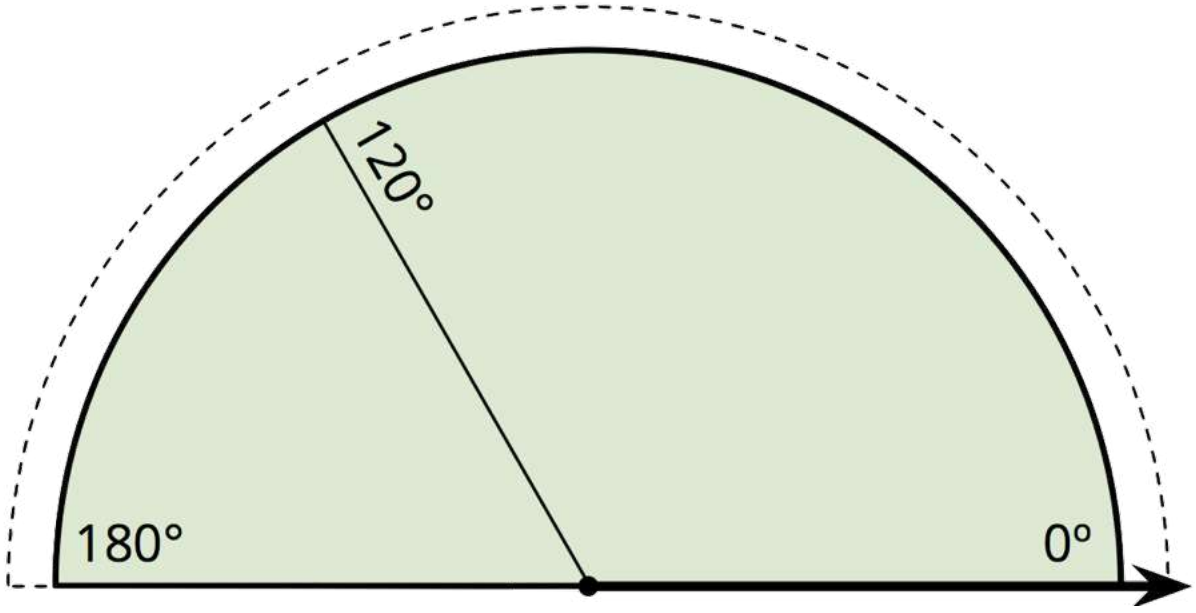
Card Sort: Angles
P



Making a Measuring Tool







Rolling for Fractions Stage 2 Recording Sheet

- Each partner:
  - Roll 3 number cubes. Use the numbers to complete the expression. Write the product.
  - Check your partner's work to make sure you agree.
  - Determine the number of points each partner gets:
    - 2 points for creating an expression less than 1
    - 5 points for creating an expression greater than 1
    - 10 points for creating an expression that is equal to 1
- Repeat for the next round. The partner who has the most points once the recording sheet is full wins the game.

round	equation	points
1	$\square \times \frac{\square}{\square} =$	
2	$\square \times \frac{\square}{\square} =$	
3	$\square \times \frac{\square}{\square} =$	
4	$\square \times \frac{\square}{\square} =$	
5	$\square \times \frac{\square}{\square} =$	
6	$\square \times \frac{\square}{\square} =$	

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4	$\square \times \frac{\square}{\square} =$	
5	$\square \times \frac{\square}{\square} =$	
6	$\square \times \frac{\square}{\square} =$	

Compare Stage 7 Cards

Compare Stage 7 $23{,}446 + 12{,}802$	Compare Stage 7 $43{,}921 + 102{,}392$
Compare Stage 7 $27{,}301 + 821{,}821$	Compare Stage 7 $91{,}234 + 89{,}001$
Compare Stage 7 $912{,}245 + 81{,}928$	Compare Stage 7 $82{,}391 + 28{,}319$
Compare Stage 7 $27{,}392 - 16{,}121$	Compare Stage 7 $86{,}954 - 42{,}321$
Compare Stage 7 $30{,}204 - 8{,}659$	Compare Stage 7 $100{,}000 - 72{,}734$

Compare Stage 7 Cards

Compare Stage 7 $182{,}492 - 18{,}652$	Compare Stage 7 $109{,}203 - 73{,}928$
Compare Stage 7 $8{,}354 \times 5$	Compare Stage 7 $5{,}294 \times 8$
Compare Stage 7 $9{,}263 \times 4$	Compare Stage 7 $4{,}826 \times 9$
Compare Stage 7 $7{,}934 \times 6$	Compare Stage 7 $6{,}839 \times 7$
Compare Stage 7 36×24	Compare Stage 7 28×42

Compare Stage 7 Cards

Compare Stage 7 54×25	Compare Stage 7 68×29
Compare Stage 7 74×56	Compare Stage 7 47×32
Compare Stage 7 $2{,}287 \div 3$	Compare Stage 7 $1{,}244 \div 4$
Compare Stage 7 $5{,}286 \div 6$	Compare Stage 7 $2{,}534 \div 5$
Compare Stage 7 $6{,}972 \div 3$	Compare Stage 7 $8{,}728 \div 4$

Compare Stage 3-8 Directions

Directions:

- Split the deck between the players.
- Each player turns over a card.
- Compare the values. The player with the greater value keeps both cards.
- If the values are the same, each player turns over one more card. The player with the greater value keeps all four cards.
- Play until you run out of cards. The player with the most cards at the end of the game wins.

Record any sets of cards that are challenging to compare:

Compare Stage 3-8 Directions

Directions:

- Split the deck between the players.
- Each player turns over a card.
- Compare the values. The player with the greater value keeps both cards.
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- Play until you run out of cards. The player with the most cards at the end of the game wins.

Record any sets of cards that are challenging to compare:

Target Measurement Stage 4 Recording Sheet

Directions:

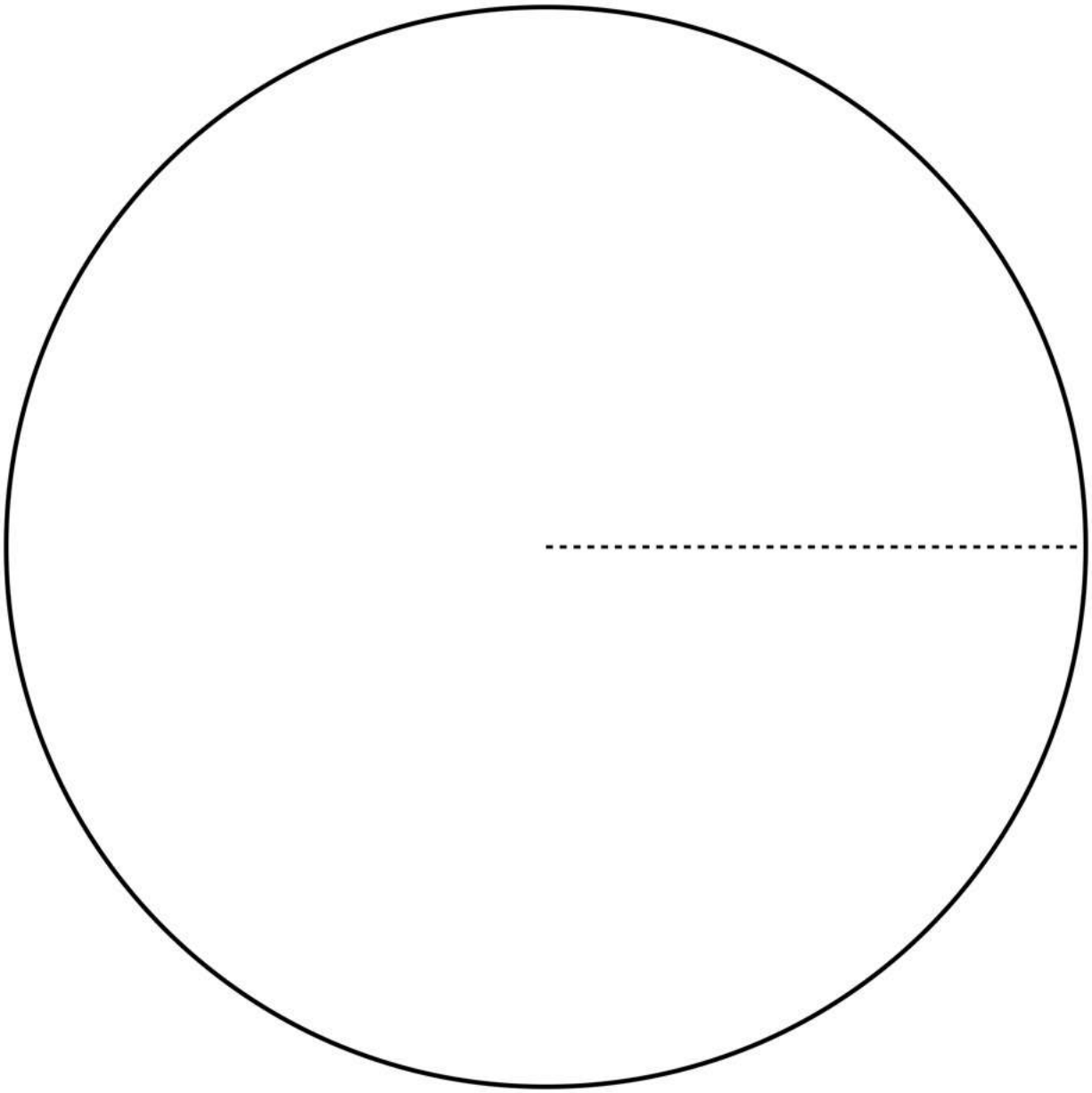
- Partner A:
  - Choose a target angle measure.
  - Begin to turn the handmade protractor.
- Partner B:
  - Say “Stop!” when you think the measure of the angle is equal to the target measurement.
- Both partners measure the line and find the difference between the actual measurement and the target measurement. The difference is Partner B’s score for the round.
- Take turns. After 8 rounds, the player with the lowest total score wins.

round	Partner A			Partner B		
	target degrees	actual degrees	points	target degrees	actual degrees	points
1						
2						
3						
4						
5						
6						
7						
8						

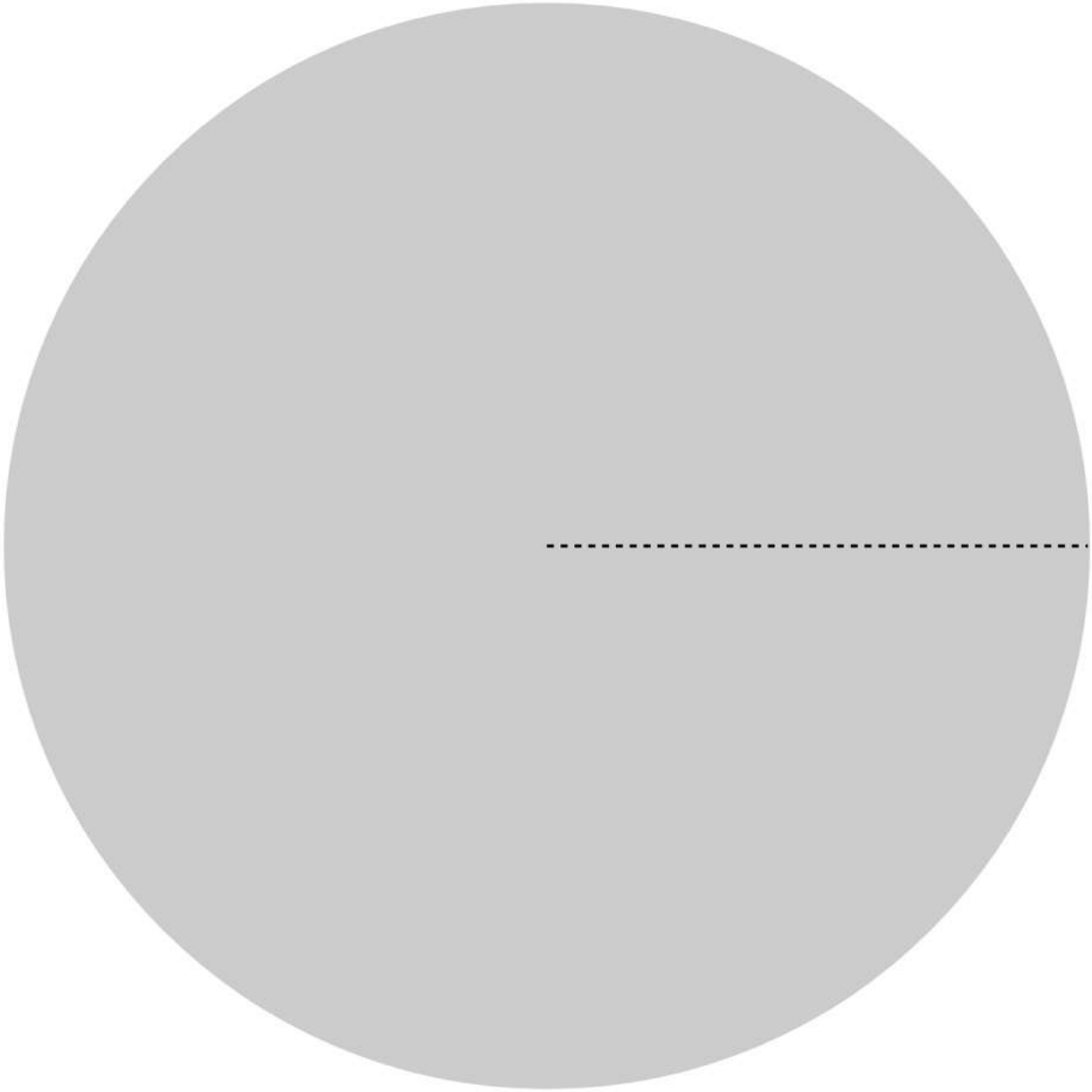
Target Measurement Stage 4 Homemade Protractor

To make a handmade protractor:

- Cut out each circle and cut a slit along the dashed segment (creating two flaps).
- Place the circles on top of one another and with the slit lined up.
- Rotate the top circle counterclockwise so that the flaps of the two circles interlock.



Target Measurement Stage 4 Homemade Protractor



Fraction Cards Grade 3

Fraction Cards Grade 3

$$\frac{1}{4}$$

Fraction Cards Grade 3

$$\frac{2}{4}$$

Fraction Cards Grade 3

$$\frac{3}{4}$$

Fraction Cards Grade 3

$$\frac{4}{4}$$

Fraction Cards Grade 3

$$\frac{5}{4}$$

Fraction Cards Grade 3

$$\frac{1}{6}$$

Fraction Cards Grade 3

$$\frac{2}{6}$$

Fraction Cards Grade 3

$$\frac{3}{6}$$

Fraction Cards Grade 3

Fraction Cards Grade 3 $\frac{4}{6}$	Fraction Cards Grade 3 $\frac{5}{6}$
Fraction Cards Grade 3 $\frac{6}{6}$	Fraction Cards Grade 3 $\frac{7}{6}$
Fraction Cards Grade 3 $\frac{1}{2}$	Fraction Cards Grade 3 $\frac{2}{2}$
Fraction Cards Grade 3 $\frac{1}{3}$	Fraction Cards Grade 3 $\frac{2}{3}$

Fraction Cards Grade 3

$\frac{3}{3}$

Fraction Cards Grade 3

$\frac{6}{3}$

Fraction Cards Grade 3

$\frac{4}{2}$

Fraction Cards Grade 3

$\frac{16}{6}$

Fraction Cards Grade 3

$\frac{6}{2}$

Fraction Cards Grade 3

$\frac{8}{2}$

Fraction Cards Grade 3

$\frac{5}{3}$

Fraction Cards Grade 3

$\frac{13}{4}$

Fraction Cards Grade 4

Fraction Cards Grade 4

$\frac{1}{8}$

Fraction Cards Grade 4

$\frac{2}{8}$

Fraction Cards Grade 4

$\frac{3}{8}$

Fraction Cards Grade 4

$\frac{4}{8}$

Fraction Cards Grade 4

$\frac{5}{8}$

Fraction Cards Grade 4

$\frac{6}{8}$

Fraction Cards Grade 4

$\frac{7}{8}$

Fraction Cards Grade 4

$\frac{8}{8}$

Fraction Cards Grade 4

$\frac{1}{5}$

Fraction Cards Grade 4

$\frac{2}{5}$

Fraction Cards Grade 4

$\frac{3}{5}$

Fraction Cards Grade 4

$\frac{4}{5}$

Fraction Cards Grade 4

$\frac{5}{5}$

Fraction Cards Grade 4

$\frac{6}{5}$

Fraction Cards Grade 4

$\frac{1}{10}$

Fraction Cards Grade 4

$\frac{2}{10}$

Fraction Cards Grade 4

Fraction Cards Grade 4

$$\frac{3}{10}$$

Fraction Cards Grade 4

$$\frac{4}{10}$$

Fraction Cards Grade 4

$$\frac{5}{10}$$

Fraction Cards Grade 4

$$\frac{6}{10}$$

Fraction Cards Grade 4

$$\frac{7}{10}$$

Fraction Cards Grade 4

$$\frac{8}{10}$$

Fraction Cards Grade 4

$$\frac{9}{10}$$

Fraction Cards Grade 4

$$\frac{10}{10}$$

Fraction Cards Grade 4

Fraction Cards Grade 4

$$\frac{11}{10}$$

Fraction Cards Grade 4

$$\frac{19}{10}$$

Fraction Cards Grade 4

$$\frac{1}{12}$$

Fraction Cards Grade 4

$$\frac{3}{12}$$

Fraction Cards Grade 4

$$\frac{4}{12}$$

Fraction Cards Grade 4

$$\frac{7}{12}$$

Fraction Cards Grade 4

$$\frac{9}{12}$$

Fraction Cards Grade 4

$$\frac{10}{12}$$

Fraction Cards Grade 4

$$\frac{13}{12}$$

Fraction Cards Grade 4

$$\frac{15}{12}$$

Fraction Cards Grade 4

$$\frac{1}{100}$$

Fraction Cards Grade 4

$$\frac{5}{100}$$

Fraction Cards Grade 4

$$\frac{10}{100}$$

Fraction Cards Grade 4

$$\frac{20}{100}$$

Fraction Cards Grade 4

$$\frac{49}{100}$$

Fraction Cards Grade 4

$$\frac{50}{100}$$

Fraction Cards Grade 4

Fraction Cards Grade 4

$$\frac{51}{100}$$

Fraction Cards Grade 4

$$\frac{75}{100}$$

Fraction Cards Grade 4

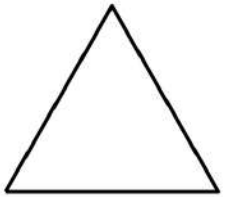
$$\frac{99}{100}$$

Fraction Cards Grade 4

$$\frac{200}{100}$$

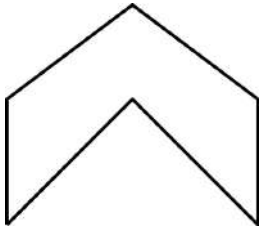
Shape Cards Grade 3

Shape Cards Grade 3
A

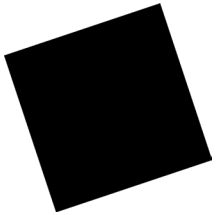


Shape Cards Grade 3
B

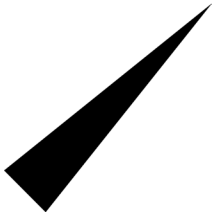
Shape Cards Grade 3
C



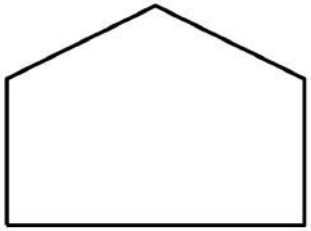
Shape Cards Grade 3
D



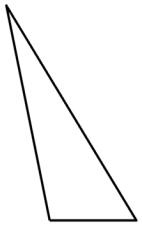
Shape Cards Grade 3
E



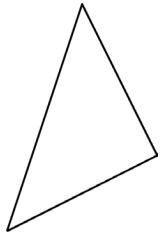
Shape Cards Grade 3
F



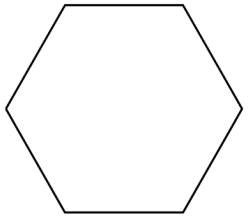
Shape Cards Grade 3
G



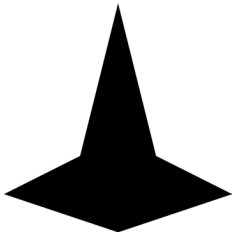
Shape Cards Grade 3
H



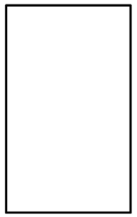
Shape Cards Grade 3
I



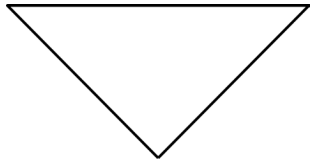
Shape Cards Grade 3
J



Shape Cards Grade 3
K

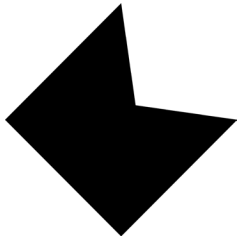


Shape Cards Grade 3
L



Shape Cards Grade 3

Shape Cards Grade 3
M



Shape Cards Grade 3
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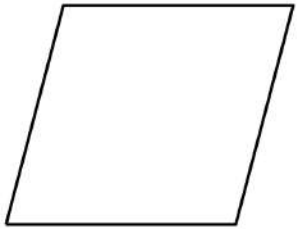

Shape Cards Grade 3
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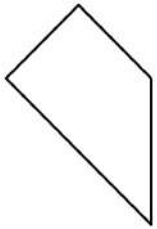

Shape Cards Grade 3
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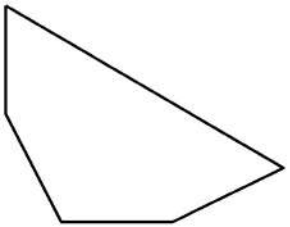

Shape Cards Grade 3
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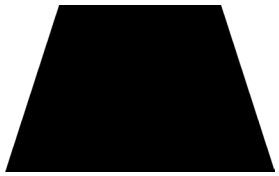

Shape Cards Grade 3
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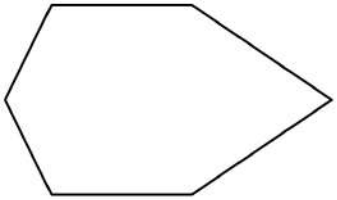

Shape Cards Grade 3
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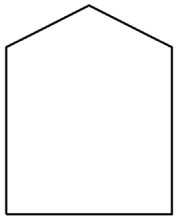
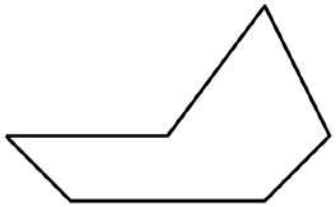


Shape Cards Grade 3
T

Shape Cards Grade 3
U



Shape Cards Grade 3
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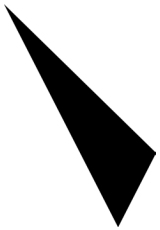
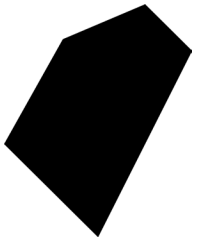


Shape Cards Grade 3
W

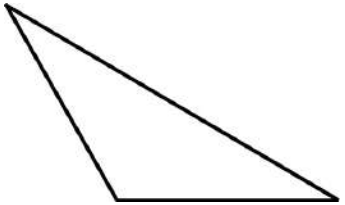
Shape Cards Grade 3
X



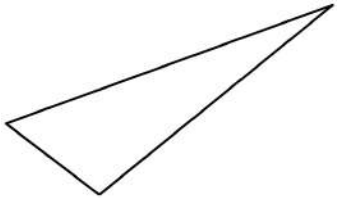
Triangle Cards Grade 3

Triangle Cards Grade 3
A

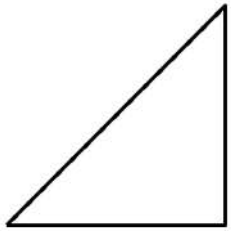
Triangle Cards Grade 3
B



Triangle Cards Grade 3
C



Triangle Cards Grade 3
D



Triangle Cards Grade 3
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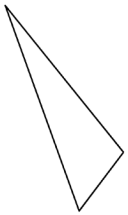
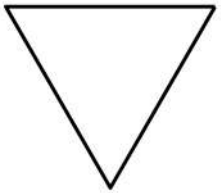
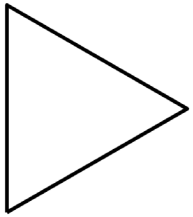


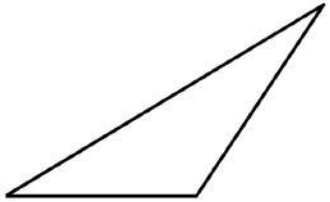
Triangle Cards Grade 3
F



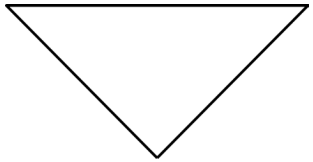
Triangle Cards Grade 3
G



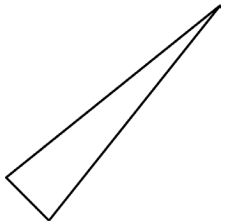
Triangle Cards Grade 3
H



Triangle Cards Grade 3
I



Triangle Cards Grade 3
J



Triangle Cards Grade 3
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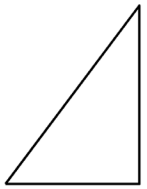
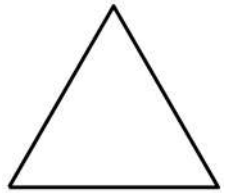
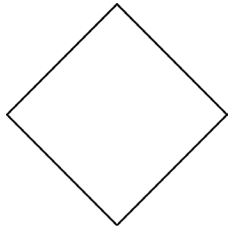


Triangle Cards Grade 3
L



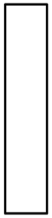
Quadrilateral Cards Grade 3

Quadrilateral Cards Grade 3
M

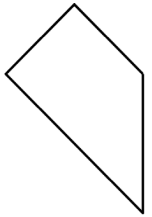


Quadrilateral Cards Grade 3
N

Quadrilateral Cards Grade 3
O



Quadrilateral Cards Grade 3
P



Quadrilateral Cards Grade 3
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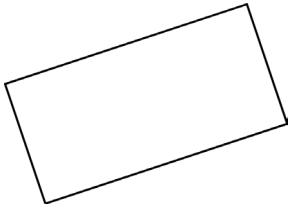

Quadrilateral Cards Grade 3
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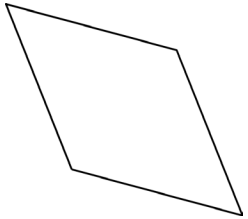

Quadrilateral Cards Grade 3
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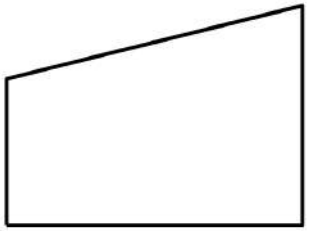

Quadrilateral Cards Grade 3
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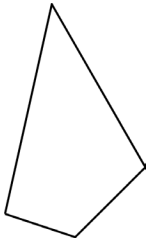
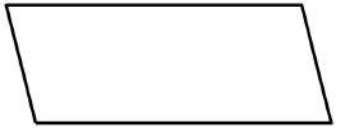


Quadrilateral Cards Grade 3
U

Quadrilateral Cards Grade 3
V



Quadrilateral Cards Grade 3
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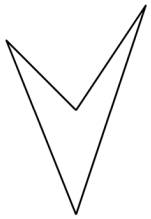
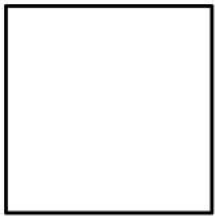
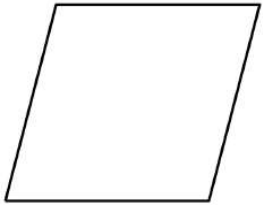


Quadrilateral Cards Grade 3
X



Quadrilateral Cards Grade 3

Quadrilateral Cards Grade 3
Y



Quadrilateral Cards Grade 3
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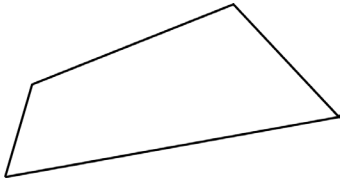
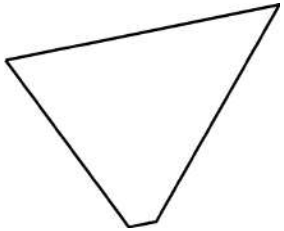


Quadrilateral Cards Grade 3
AA



Quadrilateral Cards Grade 3
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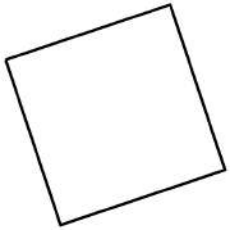

Quadrilateral Cards Grade 3
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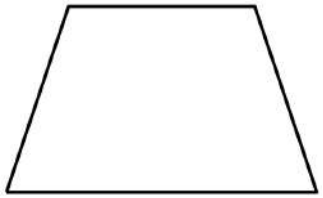
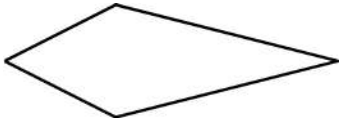


Quadrilateral Cards Grade 3
DD

Quadrilateral Cards Grade 3
EE



Quadrilateral Cards Grade 3
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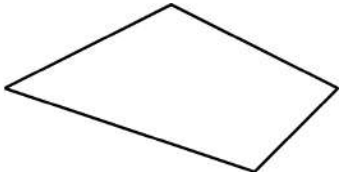

Can You Draw It Stage 4 Recording Sheet

Directions:

- Partner A: Draw a rectangle and tell your partner either the area or the perimeter of your shape.
- Partner B:
  - Draw the rectangle you think your partner drew.
  - Earn 2 points if your rectangle matches your partners.
  - Earn 1 point if it doesn't match but has the correct area or perimeter.
- Take turns. The partner with the highest score at the end of 8 rounds wins.

round	drawing	points
1		
2		
3		
4		

Can You Draw It Stage 4 Recording Sheet

5		
6		
7		
8		

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