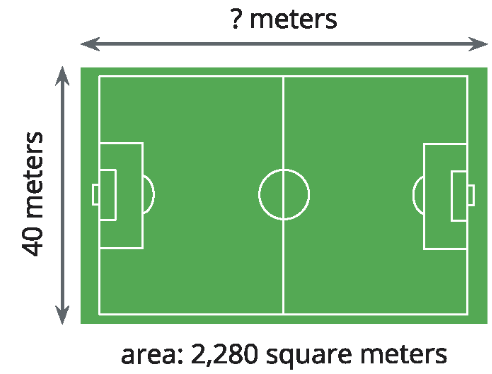
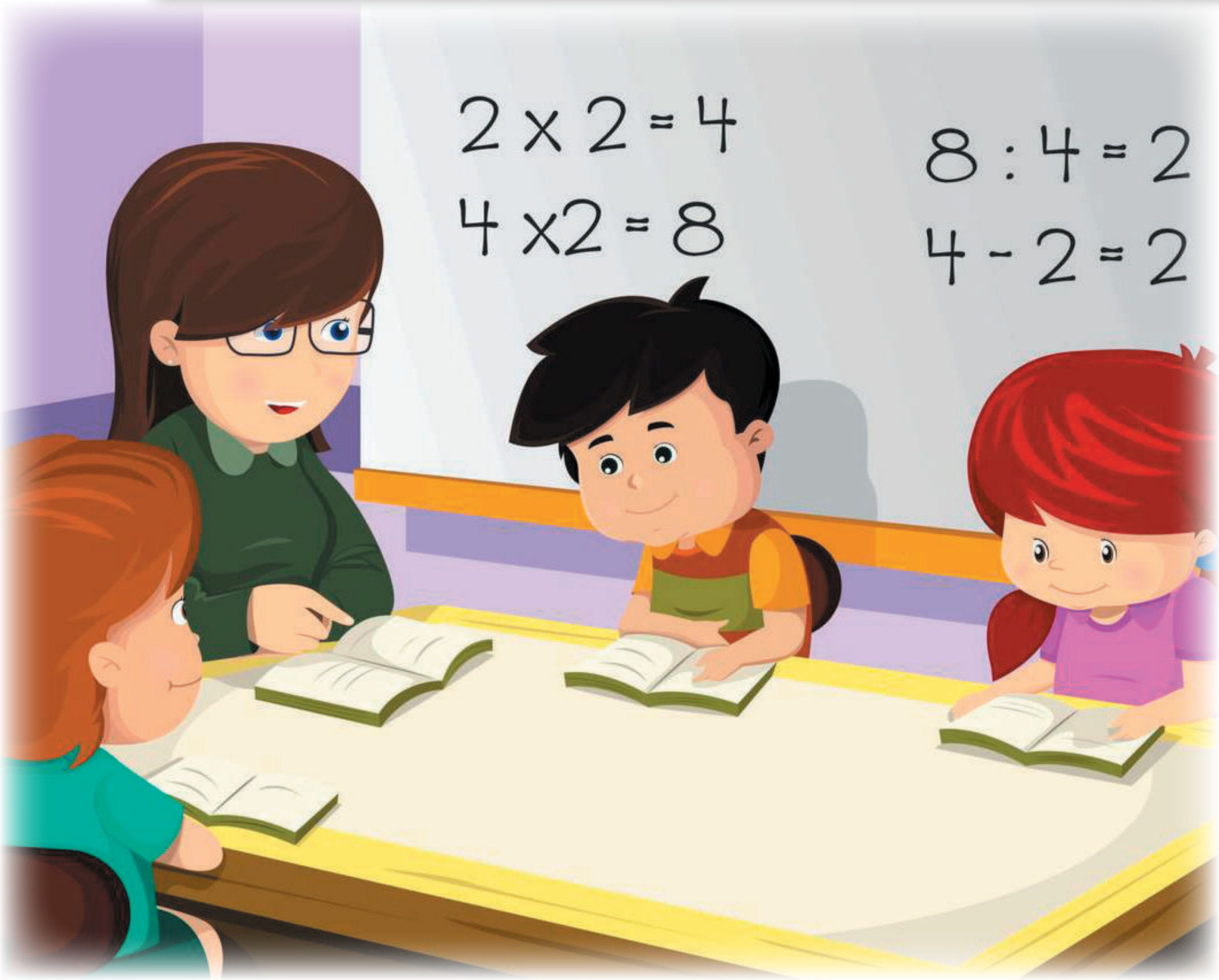
Core Knowledge® MATHEMATICS

Multiplying and Dividing Multi-digit Numbers



Teacher Guide



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ISBN: 979-8-88970-917-6

Multiplying and Dividing Multi-digit Numbers

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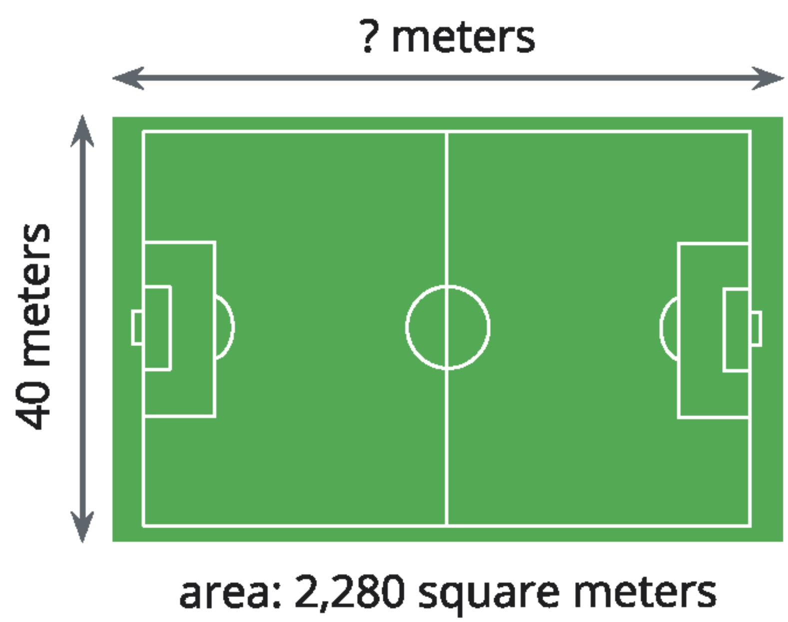


Multiplying and Dividing Multi-digit Numbers

Teacher Guide

Core Knowledge Mathematics™

Unit 6: Multiplying and Dividing Multi-digit Numbers

At a Glance

Unit 6 is estimated to be completed in 26-27 days including 2 days for assessment.

This unit is divided into four sections including 24 lessons and 1 optional lesson.

- Section A—Features of Patterns (Lessons 1-4)
- Section B—Multi-digit Multiplication (Lessons 5-12)
- Section C—Multi-digit Division (Lessons 13-20)
- Section D—Let’s Put It to Work: Problem Solving with Large Numbers (Lessons 21-25)

On pages 9-10 of this Teacher Guide is a chart that identifies the section each lesson belongs in and the materials needed for each lesson.

This unit uses six new student centers.

- Can You Draw It?
- Five in a Row: Multiplication
- Number Puzzles: Multiplication and Division
- Compare
- Rolling for Fractions
- Watch Your Remainder

Unit 6: Multiplying and Dividing Multi-digit Numbers

Unit Learning Goals

- Students multiply and divide multi-digit whole numbers using partial products and partial quotients strategies, and apply this understanding to solve multi-step problems using the four operations.

In this unit, students extend their knowledge of multiplication and division to find products and quotients of multi-digit numbers.

In grade 3, students learned that they could find the value of a product by decomposing one factor into smaller parts, finding partial products, and then combining them. To support this reasoning, they used base-ten diagrams (decomposing two-digit factors into tens and ones) and area diagrams (decomposing one side length into smaller numbers). Here, students use those understandings to multiply up to four digits by single-digit numbers, and to multiply a pair of two-digit numbers.

Students begin by describing features of geometric and numerical patterns using ideas and language related to multiplication and multiplicative relationships (such as factors, multiples, double, and triple).

Next, students reason about products of multi-digit numbers. They transition from using diagrams to using algorithms to record partial products.

Students learn that they can multiply the factors by place value, one digit at a time, and then organize the partial products vertically. Here are two ways to show partial products for $3{,}419 \times 8$.

$$\begin{array}{r} 3{,}419 \\ \times \quad\quad 8 \\ \hline 72 \\ 80 \\ 3{,}200 \\ +\ 24{,}000 \\ \hline \end{array} \qquad\qquad \begin{array}{r} 3{,}419 \\ \times \quad\quad 8 \\ \hline 24{,}000 \\ 3{,}200 \\ 80 \\ +\quad 72 \\ \hline \end{array}$$

Later, students divide dividends up to four-digit by single-digit divisors. Students see that it helps to decompose a dividend into smaller numbers and find partial quotients, just as it helped to decompose factors and find partial products.

They also recognize that sometimes it is most productive to decompose a dividend by place value. For instance, to find $465 \div 5$, we can divide each 400, 60, and 5 by 5.

$$\begin{array}{r} 400 \div 5 = 80 \\ 60 \div 5 = 12 \\ 5 \div 5 = \ \ 1 \\ \hline 465 \div 5 = 93 \end{array}$$

Students encounter various ways to record the division process, including an algorithm that records partial quotients in a vertical arrangement.

$$\begin{array}{rrl} & \boxed{93} & \\ & 1 & \\ & 12 & \\ & 80 & \\ 5 & \overline{)465} & \\ - & 400 & 5 \times 80 \\ \hline & 65 & \\ - & 60 & 5 \times 12 \\ \hline & 5 & \\ - & 5 & 5 \times 1 \\ \hline & 0 & \end{array}$$

At the end of the unit, students apply their expanded knowledge of operations to solve multi-step problems about measurement in various contexts—calendar days, distance, and population.

Section A: Features of Patterns

Standards Alignments

Building On	3.OA.D.9, 4.NBT.A.1, 4.OA.B.4
Addressing	4.OA.C.5
Building Towards	4.NBT.B.5, 4.OA.C.5

Section Learning Goals

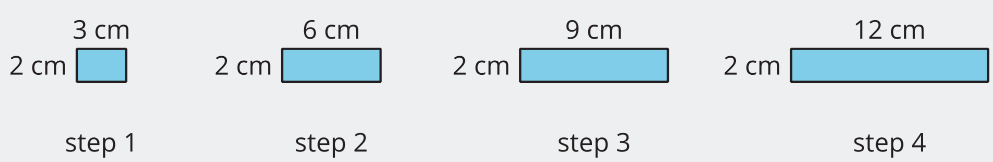
- Generate a number or shape pattern that follows a given rule.
- Identify apparent features of a number pattern that were not explicit in the rule itself.

In this section, students observe and describe features of geometric and numerical patterns. Given the rule of a pattern, they predict the values or features of future terms in a pattern sequence. To make predictions, students use their understanding of operations and place value.

The section begins with patterns that are more concrete—such as shapes with features that change quantitatively and thus elicit addition or multiplication. It then moves toward patterns with repeating objects or numbers, which require students to reason more abstractly.

Later, students explore patterns in the features of rectangles—side length, perimeter, and area—that change by a rule. Along the way, students apply their knowledge of factors and multiples.

If the pattern continues, could 50 represent the longer side length or the area of one of the rectangles? If so, which step? If not, why not?



PLC: Lesson 1, Activity 2, Taller and Taller

Suggested Centers

- Can You Draw It? (1–5), Stage 4: Area and Perimeter (Supporting)
- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)

Section B: Multi-digit Multiplication

Standards Alignments

Building On	3.OA.A.3
Addressing	4.MD.A.2, 4.NBT.B.4, 4.NBT.B.5, 4.OA.A.3
Building Towards	4.NBT.B.5, 4.OA.A.3

Section Learning Goals

- Multiply a whole number of up to four digits by a one-digit whole number, and 2 two-digit numbers using strategies based on place value and the properties of operations.

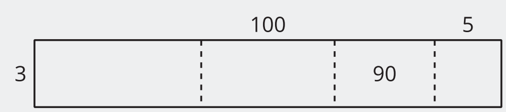
In this section, students use their knowledge of multiplication, place value, and area of rectangles to multiply one-digit numbers and numbers up to four digits, and to multiply pairs of two-digit numbers.

A key thread here is the idea of decomposing factors—particularly by place value—as a productive way of finding products. Students explore this idea with concrete and visual representations: arrays, base-ten diagrams, and rectangles with grids. As they decompose larger factors, they see the limits of these representations, motivating more efficient representations and strategies.

In grade 3, students saw that rectangles can help us reason about multiplication—the side lengths of a rectangle can represent the two factors and its area can represent the product. As the factors become larger (for instance, $3 \times 2{,}135$), it becomes necessary to draw rectangles whose side lengths are not to scale. When rectangles no longer accurately represent area, the term “area diagrams” is not used. Instead, “rectangular diagrams” is used in teacher materials and “diagrams” in student materials.

Students use such diagrams as a visual tool to decompose factors by place value and to organize partial products.

Lin drew a diagram to represent $3 \times 2{,}135$.

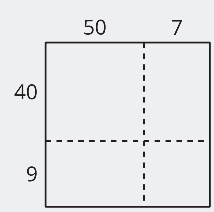


Complete the diagram.
Use it to find the value of $3 \times 2{,}135$.

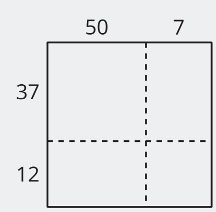
The benefits of decomposing factors by place value become more apparent as students multiply pairs of two-digit numbers.

They consider, for example, why diagram A may be more helpful than diagram B for finding the value of 49×57.

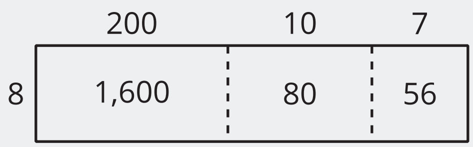
A



B



Later, students encounter an algorithm that uses partial products, a different way to record the reasoning they used with diagrams. They learn that the partial products can be listed vertically, instead of inside the boxes of a rectangular diagram.



1,600 + 80 + 56 = 1,736

$$\begin{array}{rrrrl} & 2 & 1 & 7 & \\ \times & & & 8 & \\ \hline & & 5 & 6 & \quad 8 \times 7 \\ & & 8 & 0 & \quad 8 \times 10 \\ + \; 1, & 6 & 0 & 0 & \quad 8 \times 200 \\ \hline 1, & 7 & 3 & 6 & \end{array}$$

Students use this algorithm to multiply two-digit numbers, likewise connecting the partial products to the values in a corresponding diagram.

Algorithms that use partial products prepare students to make sense of the standard algorithm for multiplication, which students preview in this unit but will study closely in grade 5.

PLC: Lesson 9, Activity 1, An Algorithm for Noah

Suggested Centers

- Can You Draw It? (1–5), Stage 4: Area and Perimeter (Supporting)
- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)
- Number Puzzles: Multiplication and Division (4–5), Stage 1: Two-digit Factors (Addressing)
- Compare (1–5), Stage 3: Multiply within 100 (Supporting)
- Five in a Row: Multiplication (3–5), Stage 3: Two-digit Factors (Addressing)

Section C: Multi-digit Division

Standards Alignments

Building On	3.MD.C.7, 3.OA.A.2, 3.OA.A.3, 4.NBT.B.5, 4.OA.B.4
Addressing	4.MD.A.3, 4.NBT.B.6, 4.OA.A.3, 4.OA.B.4
Building Towards	4.MD.A.3, 4.NBT.A, 4.NBT.B.6

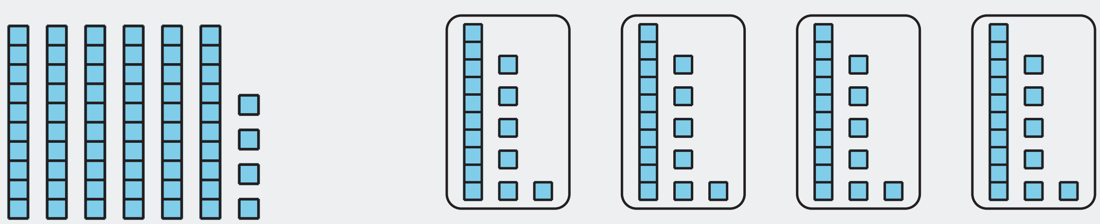
Section Learning Goals

- Divide numbers of up to four digits by one-digit divisors to find whole-number quotients and remainders, using strategies based on place value, properties of operations, and the relationship between multiplication and division.

In grade 3, students made sense of division in relation to multiplication and equal-size groups. They reasoned about division problems in context and found whole-number quotients from two-digit dividends and one-digit divisors. Here, students find quotients from larger dividends (up to four digits), investigate new division strategies and ways to represent them, and interpret division situations that involve remainders.

Students begin by solving problems in various situations, including those about equal-size groups, factors and multiples, and area of rectangles. These experiences reinforce students' understanding of the relationship between multiplication and division. They also build students' intuition for the kinds of situations that involve division (including those where a remainder may be involved), before focusing on finding the value of quotients.

Students first reason about division problems in any way that makes sense to them, and later use base-ten representations. They recall that to find the value of $64 \div 4$, for instance, they could first put 4 tens and 4 ones into 4 groups (1 ten and 1 one in each group), and then decompose the remaining 2 tens into 20 ones and put 5 ones in each group.



Students see that, just as they can distribute blocks of tens and ones into groups incrementally, they can decompose a dividend into parts and find partial quotients.

While there is not a single way to decompose a dividend, doing so by place value is often helpful, as was the case when finding partial products.

Students learn to use a series of equations and a vertical recording method to organize partial quotients.

$$\begin{array}{r} 720 \div 9 = 80 \\ 18 \div 9 = 2 \\ \hline 738 \div 9 = 82 \end{array}$$

$$\begin{array}{rrl} & \boxed{82} & \\ & 2 & \\ & 80 & \\ 9 & \overline{)738} & \\ - & 720 & 9 \times 80 \\ \hline & 18 & \\ - & 18 & 9 \times 2 \\ \hline & 0 & \end{array}$$

Later in the section, students take a closer look at division problems that do not have whole-number quotients and interpret the remainders in the context of the problem.

PLC: Lesson 15, Activity 1, Elena’s Mural

Suggested Centers

- Five in a Row: Multiplication (3–5), Stage 3: Two-digit Factors (Addressing)
- Compare (1–5), Stage 4: Divide within 100 (Supporting)
- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Supporting)
- Watch Your Remainder (4–5), Stage 1: One-digit Divisors (Addressing)

Section D: Let’s Put It to Work: Problem Solving with Large Numbers

Standards Alignments

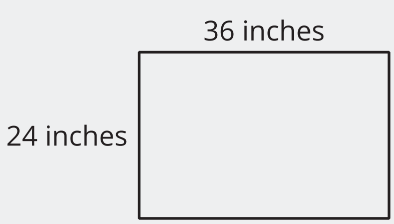
Addressing	4.MD.A.2, 4.MD.A.3, 4.NBT.B.4, 4.NBT.B.5, 4.NBT.B.6, 4.OA.A.2, 4.OA.A.3, 4.OA.C.5
Building Towards	4.OA.C.5

Section Learning Goals

- Use the four operations to solve problems that involve multi-digit whole numbers and assess the reasonableness of answers.

In the final section of this unit, students engage with a variety of contextual problems that involve multi-digit numbers and all four operations. The problems can be approached in many ways, presenting students with opportunities to choose their strategies and representations strategically. Many of them also involve multiple steps and justifications, prompting students to practice constructing logical reasoning and critiquing the reasoning of others (MP3).

Jada plans to cut up a sheet of poster paper, rearrange the pieces, and tape them to make a banner that is 8 inches tall and 8 feet long.



Does she have enough paper to make the banner?

Are there more people who only speak English or more people who speak a language other than English? Show how you know.

language	number of speakers
English only	1,224,539
Spanish	127,352
Other Indo-European	6,750
Asian	364

PLC: Lesson 22, Activity 1, Create a Class Banner

Suggested Centers

- Compare (1–5), Stage 7: Multi-digit Operations (Addressing)
- Watch Your Remainder (4–5), Stage 1: One-digit Divisors (Addressing)

Throughout the Unit

The Number Talk routines in this unit offer opportunities for students to look for structure in multiplication and division expressions. Their observations of structure in turn support their ability to operate on or otherwise work with larger multi-digit numbers. For instance:

- In lessons 3 and 18, students compose familiar multiples of a number to help them recognize other multiples that are less familiar or are much larger.
- In lessons 5 and 10, students use doubling and halving to find products of one- and two-digit numbers.
- In lesson 8, students decompose a factor in multiplication expressions and use the distributive property to multiply pairs of two-digit numbers.
- In lesson 14, students use what they know about familiar multiples of 7 to notice that a larger number is not a multiple of 7 (and will therefore have a remainder if divided by 7).

Here is a sampling of Number Talk warm-ups in the unit.

lesson 3	lesson 5	lesson 8
20×3	8×30	20×60
21×3	5×30	21×60
40×3	10×30	20×62
42×3	15×30	19×60

lesson 10	lesson 14	lesson 18
30×7	$21 \div 7$	$90 \div 3$
15×14	$35 \div 7$	$96 \div 3$
50×8	$140 \div 7$	$960 \div 3$
25×16	$196 \div 7$	$954 \div 3$

Materials Needed

LESSON	GATHER	COPY
A.1	• Pattern blocks	• none
A.2	• none	• none
A.3	• Graph paper	• none
A.4	• none	• none
B.5	• Tools for creating a visual display	• none
B.6	• none	• none
B.7	• none	• none
B.8	• none	• none
B.9	• none	• none
B.10	• none	• none
B.11	• none	• none
B.12	• Tools for creating a visual display	• none
C.13	• none	• none
C.14	• none	• none
C.15	• Grid paper • Sticky notes	• none
C.16	• Base-ten blocks • Tools for creating a visual display	• none
C.17	• Base-ten blocks	• none
C.18	• Base-ten blocks	• none

C.19	• none	• none
C.20	• none	• none
D.21	• none	• Going on a Field Trip (groups of 1)
D.22	• Grid paper • Inch tiles	• none
D.23	• Grid paper	• none
D.24	• Grid paper	• none
D.25	• none	• none

Center: Can You Draw It? (1–5)

Stage 4: Area and Perimeter

Lessons

- Grade4.6.A1 (supporting)
- Grade4.6.A2 (supporting)
- Grade4.6.A3 (supporting)
- Grade4.6.A4 (supporting)
- Grade4.6.B5 (supporting)
- Grade4.6.B6 (supporting)

Stage Narrative

Partner A draws a rectangle and tells Partner B either the area or the perimeter of their shape. Partner B tries to draw the rectangle. They earn two points if their rectangle matches the one Partner A drew exactly and one point if it doesn't match exactly, but matches the clue given. This game encourages students to use multiplication fluency to think about rectangles that could be more difficult for their partner to draw.

Standards Alignments

Addressing 3.MD.C, 3.MD.D.8

Materials to Gather

Folders

Materials to Copy

Can You Draw It Stage 4 Recording Sheet (groups of 1)

Stages used in Grade 3

Stage 2

Supporting

- Grade3.4.D
- Grade3.7.A

Stage 3

Addressing

- Grade3.7.B
- Grade3.7.C

Stage 4

Addressing

- Grade3.7.C
- Grade3.7.D

Center: Five in a Row: Multiplication (3–5)

Stage 2: Factors 1–9

Lessons

- Grade4.6.A1 (supporting)
- Grade4.6.A2 (supporting)
- Grade4.6.A3 (supporting)
- Grade4.6.A4 (supporting)
- Grade4.6.B5 (supporting)
- Grade4.6.B6 (supporting)

Stage Narrative

Students multiply using factors of 1–9. Partner A chooses two numbers and places a paper clip on each number. They multiply the numbers and place a counter on the product. Partner B moves one of the paper clips to a different number, multiplies the numbers, and places a counter on the product. Students take turns moving one paper clip, finding the product, and covering it with a counter.

Standards Alignments

Addressing 3.OA.C.7

Materials to Gather

Paper clips, Two-color counters

Materials to Copy

Five in a Row Multiplication and Division Stage 2 Gameboard (groups of 2)

Additional Information

Each group of 2 needs 25 two-color counters and 2 paper clips.

Stage 3: Two-digit Factors

Lessons

- Grade4.6.B10 (addressing)
- Grade4.6.B11 (addressing)
- Grade4.6.B12 (addressing)
- Grade4.6.C13 (addressing)

Stage Narrative

Students multiply using two-digit factors. Partner A chooses two numbers and places a paper clip on each number. They multiply the numbers and place a counter on the product. Partner B moves one of the paper clips to a different number, multiplies the numbers, and places a counter on the product. Students take turns moving one paper clip, finding the product, and covering it with a counter.

Standards Alignments

Addressing 4.NBT.B.5

Materials to Gather

Paper clips, Two-color counters

Materials to Copy

Five in a Row Multiplication and Division Stage 3 Gameboard (groups of 2)

Additional Information

Each group of 2 needs 25 two-color counters and 2 paper clips.

Stages used in Grade 3

Stage 1

Addressing

- Grade3.1.C

Supporting

- Grade3.2.A

Stage 2

Addressing

- Grade3.2.C

Supporting

- Grade3.3.B
- Grade3.3.D
- Grade3.4.A
- Grade3.4.B
- Grade3.5.D

Center: Number Puzzles: Multiplication and Division (4–5)

Stage 1: Two-digit Factors

Lessons

- Grade4.6.B7 (addressing)
- Grade4.6.B8 (addressing)
- Grade4.6.B9 (addressing)

Stage Narrative

Students use the digits 0–9 to make multiplication equations with two-digit factors true. Each digit may only be used one time.

Standards Alignments

Addressing	4.NBT.B.5, 4.NBT.B.6

Materials to Copy

Number Puzzles Mult Stage 1 Recording Sheet (groups of 1)

Center: Compare (1–5)

Stage 3: Multiply within 100

Lessons

- Grade4.6.B7 (supporting)
- Grade4.6.B8 (supporting)
- Grade4.6.B9 (supporting)
- Grade4.6.B10 (supporting)
- Grade4.6.B11 (supporting)
- Grade4.6.B12 (supporting)

Stage Narrative

Students use cards with multiplication expressions within 100.

Standards Alignments

Addressing 3.OA.C.7

Materials to Copy

Compare Stage 3-8 Directions (groups of 2), Compare Stage 3 Multiplication Cards (groups of 2)

Stage 4: Divide within 100

Lessons

- Grade4.6.C13 (supporting)
- Grade4.6.C14 (supporting)
- Grade4.6.C15 (supporting)
- Grade4.6.C16 (supporting)
- Grade4.6.C17 (supporting)
- Grade4.6.C18 (supporting)
- Grade4.6.C19 (supporting)
- Grade4.6.C20 (supporting)

Stage Narrative

Students use cards with division expressions within 100.

This stage of the Compare center is used in grades 3, 4, and 5. When used in grade 3 or 4, remove the cards with two-digit divisors.

Standards Alignments

Addressing 3.OA.C.7

Materials to Copy

Compare Stage 3-8 Directions (groups of 2), Compare Stage 4 Division Cards (groups of 2)

Stage 7: Multi-digit Operations

Lessons

- Grade4.6.D21 (addressing)
- Grade4.6.D22 (addressing)
- Grade4.6.D23 (addressing)
- Grade4.6.D24 (addressing)
- Grade4.6.D25 (addressing)

Stage Narrative

Students use cards with expressions with all 4 operations resulting in numbers over 1,000.

Standards Alignments

Addressing 4.NBT.B.4, 4.NBT.B.5, 4.NBT.B.6

Materials to Copy

Compare Stage 3-8 Directions (groups of 2), Compare Stage 7 Cards (groups of 2)

Stages used in Grade 3

Stage 2

Supporting

- Grade3.4.C

Stage 3

Addressing

- Grade3.4.C

Supporting

- Grade3.6.D

Stage 4

Addressing

- Grade3.4.D

Supporting

- Grade3.7.C
- Grade3.7.D

Center: Rolling for Fractions (3–5)

Stage 2: Multiply a Fraction by a Whole Number

Lessons

- Grade4.6.C14 (supporting)
- Grade4.6.C15 (supporting)
- Grade4.6.C16 (supporting)
- Grade4.6.C17 (supporting)
- Grade4.6.C18 (supporting)
- Grade4.6.C19 (supporting)

Stage Narrative

Students roll 3 number cubes to generate a multiplication expression with a whole number and a fraction and compare the value of the expression to 1 in order to determine how many points are earned. Two recording sheets are provided, one where the fraction is a unit fraction and one where it can be any fraction.

Variation:

Students may choose a different target number to compare the value of their expression to.

Stage Description

Each group of 2 needs 3 number cubes.

Standards Alignments

Addressing 4.NF.B.4

Materials to Gather

Number cubes

Materials to Copy

Rolling for Fractions Stage 2 Recording Sheet (groups of 1)

Stages used in Grade 3

Stage 1

Addressing

- Grade3.5.C
- Grade3.5.D

Center: Watch Your Remainder (4–5)

Stage 1: One-digit Divisors

Lessons

- Grade4.6.C20 (addressing)
- Grade4.6.D21 (addressing)
- Grade4.6.D22 (addressing)
- Grade4.6.D23 (addressing)
- Grade4.6.D24 (addressing)
- Grade4.6.D25 (addressing)

Stage Narrative

Before playing, students remove the cards that show 10 and set them aside.
Students spin the spinner to get the divisor for the round. Each student picks 6 cards and chooses 3–4 of them to create a dividend. Each student finds their quotient. The score for the round is the remainder from each expression. Students pick new cards so that they have 6 cards in their hand and then start the next round. The player with the lowest score after 6 rounds wins.

Standards Alignments

Addressing 4.NBT.B.6

Materials to Gather

Number cards 0–10, Paper clips

Materials to Copy

Watch Your Remainder Stage 1 Recording Sheet (groups of 1), Watch Your Remainder Stage 1 Spinner (groups of 2)

Section A: Features of Patterns

Lesson 1: Patterns that Grow

Standards Alignments

Addressing	4.OA.C.5
Building Towards	4.OA.C.5

Teacher-facing Learning Goals

- Analyze and describe number and shape patterns.

Student-facing Learning Goals

- Let's describe patterns and think about what might come next.

Lesson Purpose

The purpose of this lesson is for students to analyze, describe, and extend visual patterns in which one or more shapes grow by a rule.

In grade 3, students identified patterns in numbers and used mathematical operations to explain them. In this lesson and the next few, students continue the work of analyzing and describing shape and number patterns, looking for features of patterns that are not apparent in the rule.

The patterns in this lesson consist of shapes that use an increasing number of objects in each step. Students describe not only the rule of the pattern (that is, how the number of objects is changing), but also any features of the patterns that are not explicit in the rule. They also extend patterns and make predictions by looking for and making use of structure (MP7), rather than by drawing or writing out each step along the way.

Access for:

Students with Disabilities

- Action and Expression (Activity 1)

English Learners

- MLR2 (Activity 1)

Instructional Routines

5 Practices (Activity 2), MLR7 Compare and Connect (Activity 2), Notice and Wonder (Warm-up)

Materials to Gather

- Pattern blocks: Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

Reflect on whose thinking was heard today. Reflect on whose thinking was not heard but could have enriched the conversations. What prompts or structures might better enable the latter to share their voices and reasoning?

Cool-down (to be completed at the end of the lesson)

5 min

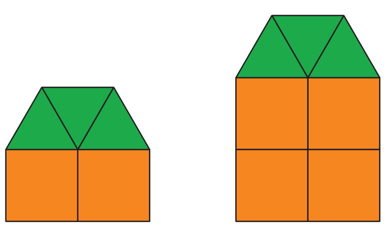
Andre's House Pattern

Standards Alignments

Addressing 4.OA.C.5

Student-facing Task Statement

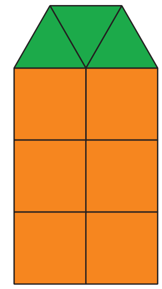
Andre used pattern blocks to make houses in a pattern. For each new step, he adds a new “floor” made of squares. The triangles are used for the roof of the house.



1. Draw the next step in Andre’s pattern.
2. If Andre continues the pattern:
    a. How many triangles will Andre use in the 15th house? Explain or show your reasoning.
    b. How many squares will Andre use in the 15th house? Explain or show your reasoning.

Student Responses

1. See drawing.
2. a. Andre will use 3 triangles. Sample response: I know because the number of triangles is not changing.
   b. Andre will use 30 squares. Sample response: He adds a row of 2 squares each time. The 15th house will have 15 rows or 15×2 squares.



Begin Lesson

Warm-up

10 min

Notice and Wonder: Sets of Circles

Standards Alignments

Building Towards 4.OA.C.5

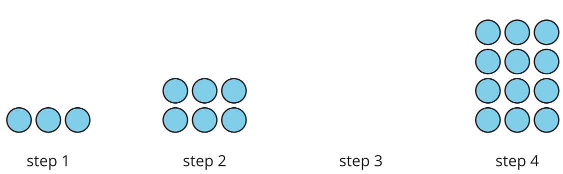
This warm-up prompts students to analyze a visual pattern and the mathematics involved in how each step in the pattern changes. They also familiarize themselves with a kind of pattern they will investigate closely later in the lesson.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?



Launch

- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

Student Responses

Students may notice:

- The number of circles is increased by 3 each time.
- The circles are organized in rows and columns.
- The third step has no circles

Students may wonder:

- Why are there no circles in the third step?
- What goes in the third step? Is it 3 rows of 3 circles?
- Is this a pattern?
- How many circles are there altogether?

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- "What could go in the third step? Why is that?" (Three rows of 3 circles. The step number seems to correspond to how many rows of 3 there are.)
- "Is there a rule this pattern follows?" (Add a row of 3 circles each time.)
- "If we follow the rule, what will the fifth step look like?" (Five rows of 3 circles)

Activity 1

15 min

Bottle Cap Patterns

Standards Alignments

Addressing 4.OA.C.5

This activity invites students to look for structure in visual diagrams and describe possible patterns in them (MP7). Because only the first two steps of the pattern are given, students could draw different conclusions about the rule the pattern follows and the subsequent steps. They may say, for instance, that the number of caps are increasing by 5 each time, doubling each time, or increasing by 5 the first time, by 6 the second time, and so on. In the last question, students represent the visual patterns numerically and begin to notice patterns in the numbers as well.

Students may choose to describe the pattern they see using expressions or in terms of operations but are not expected to do so. They may describe their observations using words, numbers, or diagrams.

Access for English Learners

MLR2 Collect and Display. Collect the language students use to describe the patterns they notice. Display words and phrases such as: "increase," "decrease," "same factor," and "doubling." During the synthesis, invite students to suggest ways to update the display: "What are some other words or phrases we should include?" etc. Invite students to borrow language from the display as needed.
Advances: Conversing, Reading

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Provide access to grid paper. Students may use it to draw the bottle caps in each step of Han's design. In this activity and throughout the section, students may also use grid paper to organize and record their thinking about numerical patterns. For example, in this case, they might create a two-row table, recording the step in the design in the top row and the number of bottle caps in the bottom row.
Supports accessibility for: Organization, Attention, Fine Motor Skills

Student-facing Task Statement

Han is arranging his bottle caps in a pattern. Here are the first two steps.

1. a. What might be the rule that Han has in mind? How do you think the pattern might continue?
   b. Describe or draw the next 2 steps.
2. Is there another possible rule?
3. For each rule that you found, write the numbers that represent the number of caps in step 1 through step 6.

Student Responses

1. Sample responses:
    - Five caps are added each time.

Launch

- Groups of 2–4
- "What are some patterns you see around your neighborhood, at home or on the way to school?"
- 30 seconds: quiet think time
- 1 minute: partner discussion

Activity

- "Take a few quiet minutes to look at Han's pattern and answer the first two problems."
- "Share your thinking with your group before continuing to the last problem."
- 5 minutes: independent work time
- Monitor for the different ideas students have about the rule that Han might have in mind.
- Identify students with different ideas and ways of representing or describing their

- Sample diagram:

- $5 + 5 + 5$ and $5 + 5 + 5 + 5$
- 3×5 and 4×5

2. Sample responses:
   - The caps are doubling each time. The third step would have 20 caps. The fourth step would have 40 caps.
   - There are 5 caps, then 10 caps and then the pattern repeats. So Step 3 is 5 caps and Step 4 is 10 caps and Step 5 is 5 caps and Step 6 is 10 caps.
   - The caps are growing by 5, then by 6, then by 7, and so on. The third and fourth steps would have 16 and 23 caps.
3. Sample responses:
   - 5, 10, 15, 20, 25, 30
   - 5, 10, 20, 40, 80, 160
   - 5, 10, 5, 10, 5, 10
   - 5, 10, 16, 23, 31, 40

ideas, to share during the activity synthesis.

Synthesis

- Select previously identified students to share their responses, including the numbers they wrote to represent their visual patterns.
- Display all of the numerical patterns students generated.
- "How can we tell from each numerical pattern what the rule is?" (We look at how the numbers change. They might:
  - increase or decrease by the same amount each time
  - be multiplied by the same factor each time
  - change by 1 more than the previous change.)
- "Besides the rule, what other interesting features do you notice about each number pattern?" (Alternating odd and even numbers, multiples of 5 and 10)

Advancing Student Thinking

If students find only one rule for the pattern, consider asking:

- "What do you know about the relationship between 5 and 10? Think of as many ways as you can about how they are related."
- "How can those relationships help you find another rule?"

Activity 2

20 min

Taller and Taller

PLC Activity

Standards Alignments

Addressing 4.OA.C.5

In this activity, students analyze a new visual pattern, describe its features, and make predictions about what they would see if the pattern continues (MP7). As in the first activity, students may show their reasoning using words, numbers, expressions, or equations. Unlike in the first activity, some elements in the steps remain constant, and students are given the rule the pattern follows.

Monitor for the ways students reason about the number of square blocks (partner A) or the number of all blocks (partner B) in the tenth step of the pattern. For the square blocks, students may:

- Start with only the number of square blocks in the legs of the giraffe (4) and reason:
  - We can skip-count by 2 ten times, starting from 4.
  - At each step, 2 square blocks are added to the 4 for the neck, so at the tenth step, there are 10 times 2, or 20, more square blocks.
  - $4 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2$
  - $4 + (10 \times 2)$
- Start with the number of square blocks in the first step (6) and reason:
  - We can add 2 to 6 nine times.
  - $6 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2$
  - $6 + (9 \times 2)$
- Start with the number in the fifth step (14) and reason:
  - We can add 2 to 14 five times.
  - $14 + 2 + 2 + 2 + 2 + 2$
  - $14 + (5 \times 2)$

In the activity synthesis, highlight the connections across the different representations (words, addition expressions, multiplication expressions) used to describe and extend the pattern.

This activity uses *MLR7 Compare and Connect*. Advances: representing, conversing

Instructional Routines

5 Practices, MLR7 Compare and Connect

Materials to Gather

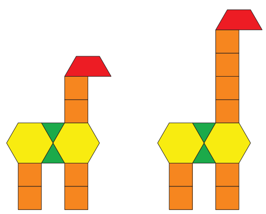
Pattern blocks

Required Preparation

- Consider preparing a set of pattern blocks for building the first two or three steps of the giraffe pattern. The set should include 6 hexagons, 6 triangles, 3 trapezoids, and 24 squares.

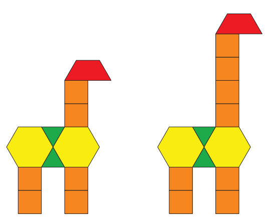
Student-facing Task Statement

Jada used pattern blocks to make giraffes. Here are the first two steps. She continued to add 2 square blocks for each step that followed.



Partner A:

1. List the number of square blocks in each of the first five steps. Write two observations about the numbers.
2. Without drawing the giraffe, predict how many square blocks the tenth step will have. Explain or show your reasoning.
3. Will a step ever have 25 square blocks? Explain or show your reasoning.



Partner B:

Launch

- Groups of 2
- "Work with your partner on this activity. One person should be partner A and the other person partner B."
- Give access to pattern blocks.

Activity

- "Take a few quiet minutes to work on your part. Afterwards, share your responses with your partner."
- 6–7 minutes: independent work time
- "When it's your turn to share, explain your thinking so that it is clear to your partner."
- "When it's your turn to listen, pay close attention to your partner's explanation. If you disagree or are unclear about a statement they make, ask questions or discuss the disagreement."
- 4–5 minutes: partner discussion
- Identify students who reason differently about the number of blocks in the tenth step (as noted in the Activity Narrative) and about whether 25 could be a number in each pattern.
- Consider asking them to create a visual display that shows their reasoning and include details to help others understand

1. List the total number of blocks in each of the first five steps. Write two observations about the numbers.
2. Predict how many total blocks the tenth step will have. Explain or show your reasoning.
3. Will a step ever have a total of 25 blocks? Explain or show your reasoning.

Student Responses

Partner A:

1. 6, 8, 10, 12, 14. Sample response: They are all even numbers. Each number is found by adding 2 to the previous number. The starting number is 6.
2. 24 square blocks. Sample response: I added 2 five more times to 14.
3. No. Sample response: The number of square blocks is always even.

Partner B:

1. 11, 13, 15, 17, 19. Sample response: They are all odd numbers. The numbers are increasing by 2.
2. 29 blocks. Sample response: I added 2 five more times to 19.
3. Yes. Sample response: Adding 2 to 19 three more times gives 25.

their thinking.

Synthesis

- Select previously identified students to share their visual displays or otherwise share their responses and reasoning to the second question (about the number of blocks in the tenth step).
- Sequence students' explanations to go from the more concrete (using words or lists of numbers) to the more abstract (using expressions and operations).

MLR7 Compare and Connect

- Keep the visual displays visible or record students' reasoning for all to see.
- "What is the same and what is different between the different strategies shared?"
- 2 minutes: partner discussion
- Highlight the similarities and differences across the various solution paths and representations used to reason about the same question. For example:
  - Alike: The pattern can be described by using skip-counting, words, addition, or multiplication. The pattern was expressed in terms of an increase by a fixed amount each time.
  - Different: We could use different numbers in the sequence as a starting point for figuring out the 10th term or to see if 25 is a value in the sequence.

Lesson Synthesis

10 min

"Today we looked at several patterns. Each of them shows steps that change according to a rule."

"What are some ways we used to describe and extend the patterns we saw?" (Using words, numbers, and expressions.)

Display:

5, 10, 15, 20, 25, 30

"These numbers represent a possible pattern for Han's bottle caps in the first activity."

"How might we find the number of caps in the eighth step?" (Add 5 to 30 two times, $30 + 5 + 5$, or $30 + (2 \times 5)$. Multiply 5 by 8 or 8×5.)

"Could 72 be a number of bottle caps in a step in the pattern? Why or why not?" (No, because the numbers in the patterns are all multiples of 5, and 72 is not a multiple of 5.)

Suggested Centers

- Can You Draw It? (1–5), Stage 4: Area and Perimeter (Supporting)
- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students start counting the 15th stage from the third one, leading to an incorrect number of triangles and squares.

Next Day Support

- During the warm-up of the next lesson, have students work with a partner to discuss a correct response to the cool-down.

Lesson 2: Patterns that Repeat

Standards Alignments

Addressing	4.OA.C.5
Building Towards	4.OA.C.5

Teacher-facing Learning Goals

- Analyze, describe, and generate patterns that follow a given rule.

Student-facing Learning Goals

- Let's look at shapes that repeat by a rule and make some predictions about the patterns they create.

Lesson Purpose

The purpose of this lesson is for students to analyze, describe, extend, and generate visual patterns in which a series of symbols or shapes repeat by a rule, using structure and mathematical reasoning to do so.

In an earlier lesson, students analyzed and described features of patterns that followed a rule. In this lesson, students do the same with designs with shapes that repeat according to a rule. Students begin by examining the patterns visually. They look for structure and make use of it to extend the patterns (MP7). Later, they represent each shape in the pattern with numbers and reason about the repetition mathematically—by using operations and observing the properties of the numbers (MP2). The third activity is optional as it provides an opportunity for extra practice.

Access for:

Students with Disabilities

- Action and Expression (Activity 3)

English Learners

- MLR8 (Activity 1)

Instructional Routines

How Many Do You See? (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min

Teacher Reflection Question

In an upcoming section, students will learn to multiply multi-digit numbers. What do you notice in their work from today's lesson that you might leverage in that future lesson?

Activity 2	15 min
Activity 3	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Cool-down (to be completed at the end of the lesson)

5 min

Happy Faces

Standards Alignments

Addressing 4.OA.C.5

Student-facing Task Statement

Diego created a pattern with smiley faces.

1. Extend Diego's pattern by drawing the next 5 shapes.
2. If Diego numbered the smiley faces, what numbers would he write for the first 5 large smiley faces?
3. Will the 42nd smiley face be a large one or a small one? Explain or show your reasoning.

Student Responses

1. Draws 1 large smiley face and 4 smaller faces added to the original pattern.
2. 5, 10, 15, 20, 25
3. Small smiley face. Sample response: The large ones are multiples of 5, and 42 is not a multiple of 5.

Begin Lesson

Warm-up

10 min

How Many Do You See: Colorful Tiles

Standards Alignments

Building Towards 4.OA.C.5

This warm-up encourages students to look for structure in the ways the symbols or colors repeat and to use grouping strategies or the structure they see (MP7) to quantify something that would be tedious to count individually. The work here prepares students to analyze and describe patterns formed by repetition later in the lesson.

Instructional Routines

How Many Do You See?

Student-facing Task Statement

How many tiles do you see? How do you see them?

Student Responses

22 tiles. Sample response:

- I see the colors (or symbols) repeating every four tiles. It repeats 6 times and there are 2 extra tiles. 5×4 is 20, and 2 more makes 22.
- I see 6 reds (*) and 6 blues (?), and 5 greens (!) and 5 yellows (#). $6 + 6 + 5 + 5 = 22$.
- I see multiple sets of red, blue, green, and yellow. Each set always starts with red and ends with yellow. There are 5 full sets and one incomplete set with only 2 tiles.

Launch

- Groups of 2
- "How many do you see? How do you see them?"
- Flash image.
- 30 seconds: quiet think time

Activity

- Display image.
- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Record responses.

Synthesis

- "What patterns helped you figure out how many tiles there were?" (Repeating colors, repeating symbols, repeating groups of 4 tiles)

Activity 1

20 min

Patterns that Repeat

Standards Alignments

Addressing 4.OA.C.5

In this activity, students analyze a pattern with repeating shapes and look for as many features of the pattern as they can find. They then extend the pattern based on their observations. Students also generate an original pattern of shapes that repeat by following a rule. The work here prepares students to reason about such patterns represented numerically in the next activity.

Access for English Learners

MLR8 Discussion Supports. During group work, invite students to take turns sharing their responses. Ask students to restate what they heard using precise mathematical language and their own words. Display the sentence frame: "I heard you say . . ." Original speakers can agree or clarify for their partner.
Advances: Listening, Speaking

Student-facing Task Statement

1. Here is a pattern made by arranging shapes.

   a. Look for as many features of patterns as you can and describe them to your partner.
   b. What rule might this pattern follow?
   c. Use the rule to extend the pattern so that it repeats one more time.
2. Create a new pattern that uses only a circle and one other shape, and that follows a new rule.

   a. Trade your pattern with your partner.

Launch

- Groups of 2

Activity

- "Take a few quiet minutes to find more than one feature of the pattern made with shapes in the first problem. Then, share your thinking with your partner."
- 2 minutes: quiet think time on the first part of the first problem
- 3–4 minutes: partner work time on the rest of the first problem
- Pause for a discussion before students proceed to the second problem. Invite students to share their responses.
- "Now, create a pattern of your own by

Look for as many features of patterns as you can and describe them.

b. What rule might your partner have followed to create their pattern?

c. Use the rule to extend their pattern so that it repeats one more time.

Student Responses

1. Sample responses:
    a. Every other shape is a triangle. Circles and squares are each 4 shapes apart. Every 4 shapes make a set and the set repeats.
    b. Triangle, circle, triangle, square. Repeat.
    c. Draws a sequence of 4 shapes added to the design.
2. Answers vary based on the pattern. Sample responses:

    ▲ ▲ ○ ▲ ▲ ○ ▲ ▲ ○ ▲ ▲ ○

    a. Two triangles, 1 circle, 2 triangles, 1 circle, and so on. Every third shape is a circle. There are half as many circles as there are triangles.
    b. Triangle, triangle, circle. Repeat.
    c. Draws 2 more triangles and 1 circle added to the design.

repeating a circle and another shape.” (Alternatively, students can create a pattern that uses only two colors.)

- “When you’re done, trade your pattern with your partner and complete the rest of the second problem.”

Synthesis

- If time permits, invite 1–2 groups of students to share their patterns.

Activity 2

15 min

Numbered Patterns

Standards Alignments

Addressing 4.OA.C.5

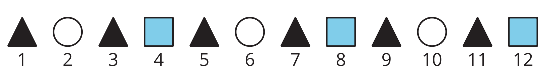
In this activity, students number the shapes in the pattern from the first activity, examine and use features of the numerical patterns to predict the shapes in particular spots.

Student-facing Task Statement

Here is the pattern of shapes you saw earlier.

1. Number the shapes 1 to 12.
2. Your teacher will assign you a shape. Write it in every blank space and answer the questions.
    a. What numbers were written for the ____________________s?
    b. If you extend the pattern, what numbers will be written for the next two ____________________s?
    c. What number will the tenth ____________________ have? Explain or show your reasoning.
    d. Will the 30th shape be a ____________________? Explain or show your reasoning.

Student Responses

1. 

2. Triangle:
    a. 1, 3, 5. 7, 9, 11
    b. 13, 15
    c. 19. Sample response: I added 2 to 11 four more times, which gives 19 ($11 + 4 \times 2 = 19$).
    d. No. Sample response: Triangles have odd numbers and 30 is an even

Launch

- Groups of 3–4
- Assign each group member one shape in the design.
- "Record your assigned shape in the four blanks in the second problem."

Activity

- "Number each shape in the pattern of shapes from 1 to 12, in order."
- "Then, work independently to answer the questions in the second problem, using the shape assigned to you."
- "Afterwards, share your findings with your group."
- 5 minutes: independent work time
- 5 minutes: small-group discussion
- Monitor for the different ways students answer the questions. They may, for instance:
    - use skip-counting (4, 8, 12, . . .)
    - reason additively (add 2 or 4 each time) or use "_____ more" language
    - reason multiplicatively or use the term "multiples" (multiples of 2 or 4, or groups of 2 or 4)
    - write addition or multiplication expressions or equations

Synthesis

- Invite one student who worked on each shape to share their responses and reasoning. Record the number sequences number.

Square:

a. 4, 8, 12
b. 16, 20
c. 40. Sample response: I saw that the tile numbers are multiples of 4, so I found 10×4.
d. No. Sample response: Because 30 is not a multiple of 4.

Circle:

a. 2, 6, 10
b. 14, 18
c. 38. Sample response: The numbers are increasing by 4 each time. I added 9 times 4 to the first number, 2, to get the tenth number.
$2 + (9 \times 4) = 2 + 36 = 38$
d. Yes. Sample response: When I added 4 to 18 three more times and I got 30.

for all to see.

- "How are the numerical patterns the same? How are they different?"

Activity 3

15 min

Clare's Pattern

Standards Alignments

Addressing 4.OA.C.5

This optional activity prompts students to generate a shape pattern given a rule and to describe the numerical patterns that are created when they number the shapes. They make predictions about whether a certain value or shape would appear in a particular position of the pattern (MP2).

While students may make predictions in a number of ways, during the synthesis, highlight reasoning that is based on the idea of multiples or adding a certain multiple to a number.

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Provide students with alternatives to writing on paper. Students can share their learning verbally.
Supports accessibility for: Language, Conceptual Processing

Student-facing Task Statement

Clare created a pattern using 3 shapes—a triangle, a circle, and a square—that repeat in that order.

1. Draw the first 10 shapes in Clare's pattern.
2. Clare numbered her shapes. What numbers are the first 5 squares?
3. What rule is the numerical pattern following?
4. What is the 31st shape in Clare's pattern? Explain or show your reasoning.
5. Clare wants to use 40 shapes in her pattern and the last shape to be a square. Is this possible? Explain or show your reasoning.

Student Responses

1. 
2. 3, 6, 9, 12, 15
3. Sample responses: Each number is a multiple of 3.
4. Triangle. Sample response: The 30th tile would be a square because $3 \times 10 = 30$. The next tile would be a triangle.
5. No. Sample response: The 39th shape would be a square, so the 40th would be a triangle. Forty is not a multiple of 3.

Launch

- Groups of 2
- "Clare has in mind a particular arrangement of shapes. Let's see what her design looks like and make some predictions about it."

Activity

- "Work independently for a few minutes, and then share your thinking with your partner."
- 5–7 minutes: independent work time
- 2–3 minutes: partner discussion
- Monitor for the ways students:
  - describe the rule
  - reason about the 31st shape in Clare's pattern
  - reason about whether a square could be the last one of 40 shapes

Synthesis

- Select students to share their responses and reasoning to the last two problems.
- If all students used the numerical pattern that represents squares to help them find the 31st shape or to reason about the last shape in a list of 40, ask:
  - "Is it possible to answer the last two questions using the numerical pattern that represents triangles?" (Yes. The numbers would be 1, 4, 7, .

. . We could keep adding 3 or multiples of 3 to one of these numbers to see if we get 31 at some point.)

- "Is it possible to use the numerical pattern to represent the circles?" (Yes. The numbers would be 2, 5, 8, . . . We could keep adding 3 or multiples of 3 to one of these numbers.)

- "Why might you want to use the numerical pattern that represents the squares?" (The numbers are all multiples of 3, which are familiar numbers. We can reason using only multiplication, instead of adding repeatedly.)

Advancing Student Thinking

If students extend the pattern one shape at a time until it reaches the 31st term, consider asking: "How can the numbers help us find the 31st shape?"

Lesson Synthesis

10 min

"Today we explored patterns created by shapes that repeat according to a rule. When we numbered the shapes, we created a numerical pattern."

Display numerical patterns that represent each shape in the last activity:

Triangles: 1, 3, 5, 7, 9

Circles: 2, 6, 10, 14, 18

Squares: 4, 8, 12, 16, 20

"Here are some numerical patterns we saw in this lesson. How would you find the 50th number in each pattern, without listing all 50 numbers?" (Sample responses:

- For triangles: Add 45×2 to 9, or find $9 + (45 \times 2)$
- For circles: Add 45×4 to 18, or find $18 + (45 \times 4)$
- For squares: Find the 50th multiple of 4, or 50×4.)

Suggested Centers

- Can You Draw It? (1–5), Stage 4: Area and Perimeter (Supporting)
- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students may find the 42nd shape by drawing all the smiley faces up to that point, rather than by using numerical reasoning.

Next Day Support

- Before the warm-up, display reasoning that mirrors a common misconception. Ask students to make sense of the reasoning and note any questions they have about it. Discuss possible revisions to the reasoning and allow students to review and revise their cool-downs.

Lesson 3: From Visual Patterns to Numerical Patterns

Standards Alignments

Building On	4.NBT.A.1, 4.OA.B.4
Addressing	4.OA.C.5
Building Towards	4.NBT.B.5

Teacher-facing Learning Goals

- Analyze patterns represented visually and numerically.
- Use numbers, words, and the idea of factors and multiples to describe and extend patterns in the features of rectangles.

Student-facing Learning Goals

- Let's look at numerical patterns we can write to describe patterns in rectangles.

Lesson Purpose

The purpose of this lesson is for students to analyze, describe, and extend numerical patterns that follow a rule.

Previously, students explored growing and repeating patterns and reasoned about the patterns using words, numbers, and operations. In this lesson, students investigate patterns in a geometric context and explore how the side lengths, area, perimeter, and other features of a rectangle change when the rectangle changes by a rule. In doing so, students practice looking for and making use of structure (MP7).

Students also practice reasoning quantitatively and abstractly (MP2) as they interpret the values in number sequences that represent geometric features of rectangles, and vice versa. (For example, 6, 8, 10, 12, . . . may represent the area, in square centimeters, of a series of rectangles whose width is 2 centimeters and whose length grows by 1 centimeter each time.)

The second activity in this lesson is optional as it allows students more time to work with the ideas from the first activity.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR2 (Activity 3)

Instructional Routines

MLR1 Stronger and Clearer Each Time (Activity 2), MLR3 Clarify, Critique, Correct (Activity 1), Number Talk (Warm-up)

Materials to Gather

- Graph paper: Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	20 min
Activity 3	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

Identify who has been sharing their ideas in class lately. Make a note of students whose ideas have not been shared and look for an opportunity for them to share their thinking in tomorrow's lesson.

Cool-down (to be completed at the end of the lesson)

5 min

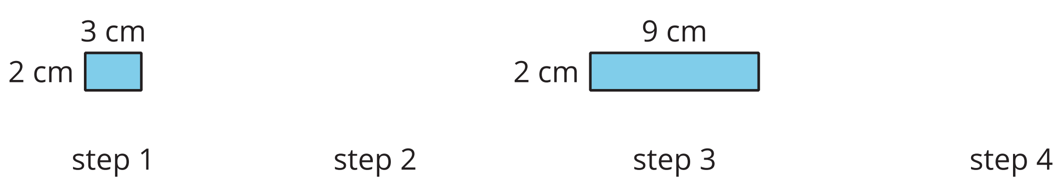
Another Set of Rectangles

Standards Alignments

Addressing 4.OA.C.5

Student-facing Task Statement

Here are steps 1 and 3 in a pattern of rectangles where a side length grows by 3 centimeters each time.



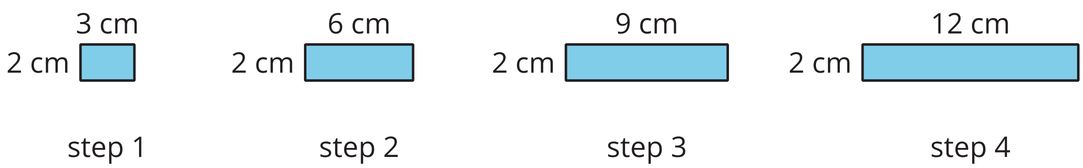
1. Draw the missing rectangles in steps 2 and 4. Label the sides with their lengths.
2. Write a numerical pattern to represent the pattern. Explain how your numerical pattern

represents the rectangles.

3. If the pattern continues, could 50 represent the side length or area of one of the rectangles? If so, which step? If not, why not? Explain or show your reasoning.

Student Responses

1. Completed drawing:



2. Sample responses:
    - Side length: 3, 6, 9, 12, 15, 18
    - Area: 6, 12, 18, 24, 30, 36
    - Perimeter: 10, 16, 22, 28, 34, 40
3. Sample response: No, 50 is not a multiple of 3 so it can’t represent the side length of a rectangle. It is not a multiple of 6, so it cannot represent the area of a rectangle.

Begin Lesson

Warm-up

10 min

Number Talk: Patterns in Multiplication

Standards Alignments

Building On	4.NBT.A.1
Building Towards	4.NBT.B.5

This Number Talk encourages students to rely on what they know about place value, multiples of 3 and 4, and properties of operations to find the value of products mentally. The reasoning elicited here will be helpful as students use multiplication to find the area and perimeter of rectangles and look for patterns in these measurements.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- 20×3
- 21×3
- 40×3
- 42×3

Student Responses

- 60: 20 is 2×10 and $3 \times 2 \times 10$ is 6×10 or 60.
- 63: 21×3 is one more group of 3 than 20×3, or is $(20 \times 3) + (1 \times 3)$.
- 120:
  - $(4 \times 3) \times 10 = 12 \times 10 = 120$
  - 40×3 is twice 20×3, so it is twice 60, which is 120.
- 126:
  - 42×3 is 2 more groups of 3 (or 6 more) than 40×3.
  - 42 is twice 21, so 42×3 is two times as much as 21×3 or two times 63, which is 126.

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "How can one of the previous expressions help us find the value of 42×3?" (We can take 21×3 and double it to get 42×3. We can take 40×3 and add 2 threes.)

Activity 1

20 min

Growing Rectangles

Standards Alignments

Building On	4.OA.B.4
Addressing	4.OA.C.5

In this activity, students examine a pattern of rectangles and consider different numerical patterns that could represent the rectangles. Students begin by analyzing claims about how the rectangles are growing and work to make the claims clearer and more precise (MP6). In doing so, they notice that the numerical patterns that represent side lengths, perimeter, or area of the rectangles each display a different rule. Students use the rules they observe to predict the value in later terms in the sequence.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Engage in a brief whole-group discussion after the partner discussion. Record language that students use while talking about the rectangles. Consider asking, "What can we count or measure in a rectangle?" so that all students have access to vocabulary like units, side lengths, area, and perimeter for the rest of the activity.
Supports accessibility for: Conceptual Processing, Language

Instructional Routines

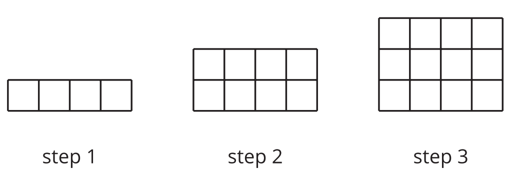
MLR3 Clarify, Critique, Correct

Materials to Gather

Graph paper

Student-facing Task Statement

Here is a pattern of rectangles that follows a rule.



- Priya says, "Each step increases by 1."
- Noah says, "Each step increases by 4."
- Lin says, "Each step increases by 2."

1. Can you think of possible reasons that all of them could be correct even though they describe the patterns differently?
2. Revise the statement made by each student so that what they mean is clearer and more precise.

Launch

- Groups of 2
- Display the image of the rectangles.
- "What do you notice? What do you wonder?"
- 30 seconds: quiet think time
- 30 seconds: partner discussion
- Provide access to graph paper, in case requested.

Activity

- Read aloud Priya, Noah, and Lin's claims about the rectangles and the first question.
- "Let's look at Priya's statement together."

MLR3 Clarify, Critique, Correct

3. Priya writes the number list 1, 2, 3, 4, 5, 6 to represent the first six steps of the pattern she sees. Write a list of numbers to represent the first six steps of the pattern that Noah and Lin see.
4. Predict what number Priya, Noah, and Lin will write for step 20 if the pattern of rectangles continue. Explain or show your reasoning.

Student Responses

1. Sample response: Priya is describing the number of rows in the arrays or a side length of the rectangles. Noah is describing the number of squares or the area. Lin is describing the perimeter.
2. Sample response:
    - Priya: One side length of the rectangles is increasing by 1 unit each time.
    - Noah: The area of the rectangles is increasing by 4 square units each time.
    - Lin: The perimeter of the rectangles is increasing by 2 units each time.
3. Noah: 4, 8, 12, 16, 20, 24
   Lin: 10, 12, 14, 16, 18, 20
4. Priya: 20, because the number starts with 1 and increases by 1 each time.

   Noah: 80, because every number is a multiple of 4, starting from 1×4. The 20th multiple of 4 is 20×4 or 80.

   Lin: 48, because, from the first number, the numbers are growing by 2 nineteen more times to get the 20th number. $19 \times 2 = 38$.

- Display and read aloud Priya's claim: "Each step increases by 1."
- "What do you think Priya is trying to say? Is anything unclear?"
- 1 minute: quiet think time
- 1 minute: partner discussion
- "With your partner, work together to write a revised statement that Priya could say so that her intention is clearer."
- Display and review the following criteria:
    - Write in a complete sentence.
    - Include mathematical vocabulary when possible.
    - Include an example, if possible.
- 2–3 minutes: partner work time
- Select 1–2 groups to share their revised explanation with the class. Record responses as students share.
- "What is the same and different about the revisions to Priya's claim?"
- "Take a few quiet minutes to analyze Noah and Lin's claims and revise them so that their intentions are clearer. Then work with your group to complete the activity."
- 3–4 minutes: independent work time
- 5–6 minutes: group work time
- Monitor for students who attend to precision and clarity as they revise Noah's and Lin's claims. Select them to share during the synthesis.

Synthesis

- Invite previously selected students to share their explanations and revisions of Noah and Lin's claims. Record the revised claims for all to see.
- Select other students to share the numerical patterns they wrote to represent the rectangles, and how they made

predictions for the 20th number in each pattern. Record their responses for all to see.

Advancing Student Thinking

Students may count individual squares within rectangles to find the area. Consider asking:

- “How might you find out the number of squares in the rectangle you just drew without counting?”
- “How might you find out the size of the next rectangle without drawing it?”

Activity 2 (optional)

20 min

More Growing Rectangles

Standards Alignments

Addressing 4.OA.C.5

This optional activity gives students an additional opportunity to reason about patterns in the side lengths, area, and perimeter of rectangles that follow a rule.

Instructional Routines

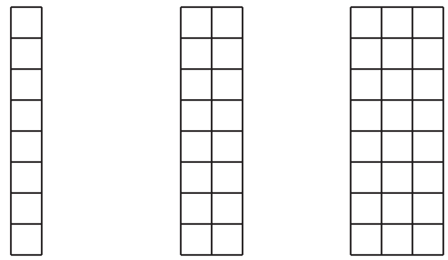
MLR1 Stronger and Clearer Each Time

Student-facing Task Statement

Here is another pattern of rectangles that also follows a rule.

Launch

- Groups of 2
- Display the image of the rectangles.
- “How many vertical columns do you see in the rectangles?” (One in step 1, two in step 2, and three in step 3.)
- “How is the number of columns changing?”



1. The number list 1, 2, 3, _____, _____, _____ represents the number of vertical columns in the first six steps of the pattern. Complete the number list.
2. Find another feature of the rectangles that can be represented with a number list and would show a pattern. Write at least one list of numbers for the first six steps of that feature.

   Feature: ____

   Number list: _____, _____, _____, _____, _____, _____
3. Without writing out all the numbers, predict the 30th number in your list. Explain your reasoning by completing this sentence frame:

   I know that the 30th number is _____ because . . .

Student Responses

1. 4, 5, 6
2. Sample responses:
   - Number of squares: 8, 16, 24, 32, 40, 48
   - Perimeter: 18, 20, 22, 24, 26, 28
   - Number of squares not on the perimeter of the rectangle: 0, 0, 6, 12, 18, 24
3. Sample responses:

(It grows by 1 each time.)

- "Besides vertical columns, what other features of the rectangles could we count or measure?" (Sample responses: Number of rows, area or number of square units, side lengths, perimeter)
- "Let's see what other patterns you can find in this set of rectangles."

Activity

- "Work on the activity independently for a few minutes."
- 5 minutes: independent work time

Synthesis

MLR1 Stronger and Clearer Each Time

- "Find a partner who wrote a different numerical pattern."
- "Take turns being the speaker and the listener. If you are the speaker, share your numerical pattern and your explanation for the last problem. If you are the listener, ask questions and give feedback to help your partner improve their explanation for the last problem."
- 3–5 minutes: structured partner discussion.
- Repeat with 1–2 other partners who chose a different feature than you did.
- "Revise your initial response to the last question based on the feedback you got from your partners."
- 2–3 minutes: independent work time

- Number of squares: 240, because $30 \times 8 = 240$.
- Perimeter: 76, because:
  - The 30th rectangle will be 30 by 8, so its perimeter will be $(2 \times 30) + (2 \times 8)$ or $60 + 16$.
  - From the first number, 18, the perimeter grows by 2 twenty-nine times, so I added 29×2 or 58 to 18, which gives 76.
- Number of squares not on the perimeter: 168, because:
  - From the sixth number, 24, the number of squares grows by 6 twenty-four more times. $24 \times 6 = 144$, so I found $24 + 144$, which is 168.
  - If we remove the outer layer of squares from the 30th rectangle, we'd have a rectangle that is 28 units wide and 6 units tall. $28 \times 6 = 168$.

Activity 3

15 min

No Grid This Time!

Standards Alignments

Addressing 4.OA.C.5

Students continue to analyze and describe patterns related to rectangles. In this activity, the rectangles show no grid. To reason about possible patterns in the features of the rectangles, students rely on what they know about the relationship between side lengths of a rectangle and its perimeter and area.

Access for English Learners

MLR2 Collect and Display. Synthesis: Direct attention to words collected and displayed from the previous lessons. Invite students to borrow language from the display as needed, and update it throughout the lesson.
Advances: Conversing, Reading

Student-facing Task Statement

Here are steps 1 and 4 in a pattern of rectangles. One side length of the rectangle increases by 5 units each time.

1. Sketch the missing rectangles in steps 2 and 3. Label the sides with their lengths.
2. Write two numerical patterns that each represent the rectangles, from step 1 to step 6.
    a. What are you representing? : ________________________

       Numerical pattern: _____, _____, _____, _____, _____, _____
    b. What are you representing? : ________________________

       Numerical pattern: _____, _____, _____, _____, _____, _____
3. For each of the following questions, if you answer yes, show how you know and state the step number. If you answer no, explain or show why not.

   If the pattern continues:

    a. Could 82 inches be a side length of a rectangle?
    b. Could 300 square inches be the area

Launch

- Groups of 2
- Read the opening paragraph and the first question as a class.
- "Take a quiet minute to sketch the missing rectangles. Label the sides."
- 1 minute: independent work time
- Share responses and display the completed sequence of four rectangles.
- Display this sequence of numbers: 3, 3, 3, 3
- Ask students: "How does this numerical pattern represent the rectangles?" (The length of the shorter side of the rectangle, which doesn't change.)
- "Let's think about other numerical patterns that can represent the rectangles."

Activity

- 8–10 minutes: independent work time
- 3 minutes: partner discussion
- Monitor for the different ways students reason about the last set of problems.

Synthesis

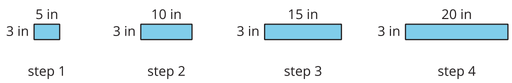
- See lesson synthesis.

of a rectangle in the pattern?

c. Could 100 inches be the perimeter of a rectangle in the pattern?

Student Responses

1. Completed drawing:



2. Sample responses:
    - Side length: 5, 10, 15, 20, 25, 30
    - Area: 15, 30, 45, 60, 75, 90
    - Perimeter: 16, 26, 36, 46, 56, 66
3. Sample responses:
    a. No, because the side length is either 3 or a multiple of 5.
    b. Yes, because the area is always a multiple of 15, and 300 is a multiple of 15. The rectangle in step 20 has an area of 300 square inches, because $20 \times 15 = 300$.
    c. No, because the perimeter always ends with a 6 (or has 6 in the ones place).

Lesson Synthesis

10 min

"Today we looked at a pattern of rectangles that follow a rule. We saw that we could write different numerical patterns to represent the rectangles."

Display the numerical patterns that represent the side length, area, and perimeter of the rectangles in the last activity.

Side length: 5, 10, 15, 20, 25, 30

Area: 15, 30, 45, 60, 75, 90

Perimeter: 16, 26, 36, 46, 56, 66

Invite students to share their responses and reasoning to the last set of questions in the last activity (on whether a number in the pattern for the longer side length, area, and perimeter of the rectangles in the sequence could have a certain value). Record their reasoning for all to see.

Highlight reasoning that reinforces the idea of multiples ("82 is not a multiple of 5"), multiplicative reasoning ("20 times 15 is 300"), additive reasoning ("the perimeter increases by 10 each time"), and place value ("the perimeter always has 6 in the ones place").

Suggested Centers

- Can You Draw It? (1–5), Stage 4: Area and Perimeter (Supporting)
- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students say that 50 could represent a side length or the perimeter of a rectangle in the pattern because they incorrectly recall multiples of 3 and 6.

Next Day Support

- Before the warm-up, invite students to work in small groups to discuss a correct response to this cool-down.

Lesson 4: Numerical Patterns

Standards Alignments

Building On	3.OA.D.9
Addressing	4.OA.C.5
Building Towards	4.NBT.B.5

Teacher-facing Learning Goals

- Analyze and describe patterns in numbers that follow a rule.
- Use understanding of place value and operations to explain and extend patterns of numbers.

Student-facing Learning Goals

- Let’s explore numerical patterns.

Lesson Purpose

The purpose of this lesson is for students to analyze numerical patterns and use their understanding of place value and operations to find a rule and explain features of the pattern.

Previously, students examined numerical patterns alongside visual patterns (diagrams of pattern blocks, arrangements of shapes, attributes of rectangles, and so on). In this lesson, they focus solely on numerical patterns, without a visual representation. Students use their understanding of operations and place value to make sense of and explain patterns in multiples of numbers. Along the way, students have multiple opportunities to look for and make use of structure and regularity (MP7, MP8) to solve problems.

The reasoning in this lesson helps to transition students to the work in the next section, in which students explore strategies for multiplying single-digit numbers and multi-digit numbers up to four digits, and for multiplying 2 two-digit numbers. The third activity is optional as it provides an opportunity for extra practice.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR8 (Activity 1)

Instructional Routines

MLR5 Co-craft Questions (Activity 3), Which One Doesn't Belong? (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Activity 3	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What connections did students make between the different strategies shared? What questions did you ask to help make the connections more visible?

Cool-down (to be completed at the end of the lesson)

5 min

Count by 8

Standards Alignments

Addressing 4.OA.C.5

Student-facing Task Statement

Kiran counted by 8 and recorded the numbers he counted:

8 16 24 32 40 48

Could 105 be a number that Kiran writes if he continued to count by 8? Explain or show your reasoning.

Student Responses

No. Sample response: 105 doesn't have an even digit in the ones place.

Begin Lesson

Warm-up

10 min

Which One Doesn't Belong: Stacked Squares

Standards Alignments

Addressing 4.OA.C.5

This warm-up prompts students to carefully analyze and compare representations of patterns. Listen for the language students use to describe and compare the elements of each pattern and give them opportunities to clarify what they mean when they use numbers to describe the patterns (MP6).

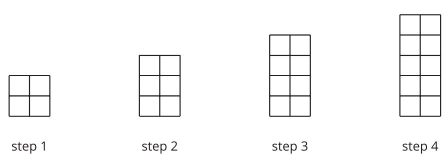
Instructional Routines

Which One Doesn't Belong?

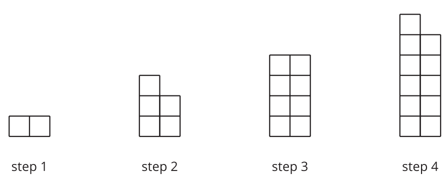
Student-facing Task Statement

Which one doesn't belong?

A



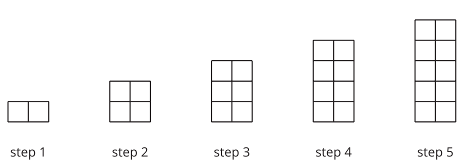
B



C

2, 4, 6, 8

D



Launch

- Groups of 2
- Display image.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 2–3 minutes: partner discussion
- Share and record responses.

Synthesis

- "What features of the diagrams did you look at when you tried to find which one doesn't belong?" (Sample response: the number of small squares in each shape)
- If time permits, ask students to think about which one doesn't belong if they pay attention to the number of rows or the perimeter rather than the number of squares.

Student Responses

Sample responses, assuming that the feature being quantified in A, B, and D is the number of small squares in each shape:

- A is the only pattern whose first value is not 2.
- B is the only pattern that doesn't increase by 2 each time, doesn't have 4 or 6 as a value, and doesn't include only even numbers.
- C is the only pattern that doesn't show shapes and doesn't include a value greater than 9.
- D is the only pattern that doesn't show only four steps.

Activity 1

20 min

Count by 10 and by 9

Standards Alignments

Building On	3.OA.D.9
Addressing	4.OA.C.5

This activity prompts students to examine patterns in multiples of 10 and 9, and to notice that the digits in the multiples of 9 can be reasoned in relation to the more-familiar multiples of 10. Students use what they know about the place value and operations to explain the patterns in these multiples (MP7). For instance, students may reason that, because 9 is 1 less than 10, to find 12×9 is to find the 12×10 and then subtract 1 twelve times (or subtract 12×1) from the product.

The reasoning in this activity prepares them to notice patterns in the multiples of 100 and 99 in the next lesson.

Access for English Learners

MLR8 Discussion Supports. Use multimodal examples to show the patterns of both columns. Use verbal descriptions along with gestures, drawings, or concrete objects to show the connection between the multiples of 9 and 10.
Advances: Listening, Representing

Access for Students with Disabilities

Representation: Access for Perception. Synthesis: Use pictures of the long rectangle base-ten blocks to help students visualize the patterns. For example, display a picture of eight long rectangle base-ten blocks. Count by 10 while pointing to each block. Then, cross out one unit in each block and discuss how this shows that counting by 9 is like multiplying by 10 and subtracting.
Supports accessibility for: Conceptual Processing, Visual Spatial Processing

Student-facing Task Statement

Andre's class is choral counting by 10 and then by 9. The left column shows the numbers they say when counting by 10.

1. Complete the right column with the first ten numbers the class will say when counting by 9.

   What patterns do you notice about the features of the numerical patterns? Make at least two observations about each list of numbers.

counting by 10	counting by 9
10	
20	
30	
40	
50	
60	
70	
80	
90	
100	

2. For the numbers in the "counting by 10" column, why do you think:
   a. the digits in the tens place change the way they do?
   b. the digits in the ones place are the

Launch

- Groups of 2
- Read the opening paragraph as a class.
- "Which do you prefer, counting by 10 or counting by 9? Why?" (Counting by 10 because I've been doing that since kindergarten.)
- 30 seconds: partner discussion
- "Let's look at numbers we get by counting by 9 and by 10 and see what patterns we can find."

Activity

- "Take a few quiet minutes to work on the first few problems. Then, share your thinking and complete the rest of the activity with your partner."
- 5–6 minutes: independent work time
- 5–6 minutes: partner discussion
- Monitor for students who:
  - can clearly explain the pattern of the digits in multiples of 10 in terms of place value

way they are?

3. For the numbers in the "counting by 9" column, why do you think the digits in the ones place change the way they do? Explain your reasoning.

Student Responses

1. Counting by 9: 9, 18. 27, 36, 45, 54, 63, 72, 81, 90. Sample responses:
    - Counting by 10
        - The ones are always 0.
        - The tens go up by 1 each time.
    - Counting by 9:
        - There are no digits in the tens place of the first number. After that, it increases by 1 each time.
        - The digit in the ones place starts at 9 and goes down by 1 each time.
2. Sample response:
    - The digit in the tens place goes up by 1 because we're adding 10 each time.
    - The digit in the ones place stays at 0 because adding a ten doesn't change the ones.
3. Sample response: Counting by 9 is the same as counting by 10 and then removing 1. If we add 10 each time, the digit in the ones place doesn't change, but because we're also removing 1, the ones go down by 1 each time.

    - notice connections between the values in the two columns and use them to explain the patterns in the digits in multiples of 9

Synthesis

- Display the completed table.
- Invite students to share the patterns they noticed in the numbers in each column. Record their observations by annotating the numbers in the table.
- If no students mentioned that the two sets of numbers are multiples of 10 and multiples of 9, ask them about it.
- Select previously identified students to share their responses to the last two problems.
- If no students reason that counting by 9 can be thought of as counting by 10 and subtracting 1 each time, bring this to their attention.
- In other words, counting by 9 once means $10 - 1$, which is 9. Counting by 9 again means adding another $10 - 1$ to 9, or $9 + 10 - 1$, which is 18. Counting by 9 a third time means $18 + 10 - 1$, which is 27. And so on.
- "Counting by 9 eight times is the same as counting by 10 eight times and subtracting 1 eight times, or $(8 \times 10) - (8 \times 1)$, which is $80 - 8$ or 72."

Advancing Student Thinking

Students may not see that the digit in the ones place decreases by 1 each time the count goes up by 9. Consider asking:

- "What patterns do you see in the pair of numbers in each row?"

- "How do you think the numbers in the 'counting by 9' column are related to those in the 'counting by 10' column?"

Activity 2

15 min

Count by 99

Standards Alignments

Addressing	4.OA.C.5
Building Towards	4.NBT.B.5

In this activity, students continue to analyze patterns in numbers. This time, they look at the relationship between multiples of 100 and multiples of 99. As in the previous activity, they rely on their understanding of place value and operations to explain the patterns in the digits of the numbers (MP7). Although the use of the distributive property is not expected or made explicit, the work in both activities in this lesson develops students' intuition for seeing, for instance, that $12 \times (10 - 1) = (12 \times 10) - (12 \times 1)$.

Student-facing Task Statement

Andre's class did a choral count by 99. Here are the first six numbers they said.

1. Study the list of numbers. Make at least 3 observations about features of the pattern.

counting by 99
99
198
297
396
495

Launch

- Groups of 2
- "Earlier we counted by 9 and found some patterns in the numbers. Now let's see what patterns we can find when we count by 99."

Activity

- "Work with your partner to complete the activity."
- 8–10 minutes: partner work time
- Monitor for students who:
    - Identify different patterns in the numbers

counting by 99
594

2. Extend the list with the next four multiples of 99. Be prepared to discuss how you know what numbers to write.
3. Why do you think the digits in the numbers change the way they do?

Student Responses

1. Sample responses:
    - The digit in the ones place decreases by 1 each time.
    - The digit in the tens place stays the same.
    - The digit in the hundreds place increases by 1 each time.
    - The numbers alternate between odd and even.
    - The sum of the digits in each number is always 18.
    - The digits in the hundreds place and in the ones place always add up to 9.
2. 693, 792, 891, 990. Sample reasoning:
    - I followed the pattern and added 1 to the hundreds place of the last number and subtracted 1 from the ones place.
    - I added 100 to the last number and then subtracted 1 each time.
3. Sample response: Adding 99 is the same as adding 100 and then removing 1. Adding 100 makes the digit in the hundreds place

    - reason about the numbers in the "counting by 99" column (multiples of 99), by reasoning about multiples of 100

Synthesis

- Select students to share the features of the patterns they noticed, making sure to highlight the digits in each the hundreds, tens, and ones place. Annotate the numbers as needed.
- Select other students to share how they extended the patterns and record their responses. If no students mentioned using multiples of 100 as a strategy, discuss this with students.
- "Counting by 99 five times is the same as counting by 100 five times and subtracting 1 five times, or $(5 \times 100) - (5 \times 1)$."
- "How can we use the pattern to find the 20th multiple of 99?" (Find 20×100 and subtract 20×1 from it.)

increase by 1. Removing 1 makes the digit in the ones place decrease by 1. Nothing is happening in the tens place.

Activity 3 (optional)

20 min

Count by 15

Standards Alignments

Addressing 4.OA.C.5

In this optional activity, students investigate patterns in multiples of 15 and analyze and describe features of the digits in the tens and ones place. The activity also prompts them to consider why those features exist and to predict whether a given number could be a multiple of 15. The goal here is not to elicit clear justifications, but rather to encourage students to use their understanding of place value and numbers in base-ten to reason more generally about patterns in numerical patterns.

This activity uses *MLR5 Co-craft Questions*. Advances: writing, reading, representing

Instructional Routines

MLR5 Co-craft Questions

Student-facing Task Statement

Elena counted by 15 and recorded the numbers she counted:

15	30	45	60
75	90		

1. Write the next four numbers she'd record if she kept going.
2. What patterns do you see? Describe as

Launch

- Groups of 2

MLR5 Co-Craft Questions

- Display only the opening sentence and list of numbers, without revealing the question(s).
- "Write a list of mathematical questions that could be asked about this situation."
- 2 minutes: independent work time

many as you can.
3. Choose one pattern that you noticed and explain why you think it happens.
4. Could 250 be a number that Elena calls out if she continued to count by 15? Explain or show your reasoning.

Student Responses

1. 105, 120, 135, 150
2. Sample responses:
    - The digit in the ones place alternates between 0 and 5.
    - The digit in the ones place alternates between odd and even.
    - Every other number is a multiple of 30.
    - The digit in the tens place alternates between increasing by 1 and by 2.
3. Sample responses:
    - The digit in the ones place alternates between 0 and 5: Adding 5 to 0 gives 5 in the ones place. Adding 5 to 5 gives 10, with 0 in the ones place.
    - The numbers alternate between even and odd: Adding 5 to a 0 in the ones place gives 5 and makes the number odd. Adding 5 to 5 in the ones place gives 10 and a 0 in the ones place, which makes the number even.
4. No. Sample responses:
    - Every number that ends in 0 is a multiple of 30, and 250 is not a multiple of 30.

- 2–3 minutes: partner discussion
- Invite several students to share one question with the class. Record responses.
- “What do these questions have in common? How are they different?”
- Reveal the task (students open books), and invite additional connections.
- “Let’s see what patterns we can find when we count by 15.”

Activity

- “Take a few quiet minutes to work on the activity. Afterwards, discuss your responses with your partner.”
- 5 minutes: independent work time
- 5 minutes: partner discussion
- Monitor for:
    - the different patterns students notice
    - the different ways they explain the patterns
    - the ways students reason about whether 250 could be a number being called out

Synthesis

- Invite students to share the patterns they noticed and their explanations for the patterns. Record them for all to see.
- Select other students to share their explanation on whether 250 could be a number that Elena calls said. Highlight explanations that make use of the structure in the numbers.

Advancing Student Thinking

To answer the last question, students may try to count up by 15 and see if they would reach 250. Encourage students to see if any of the patterns they noticed could help them answer the question.

Lesson Synthesis

10 min

"Today we saw different features of patterns in the numbers that we get when counting by 9, 10, 99, and 100." (Include 15, if students completed the optional activity).

"What new ideas did you have about patterns in this section?"

"What are you still wondering about patterns?"

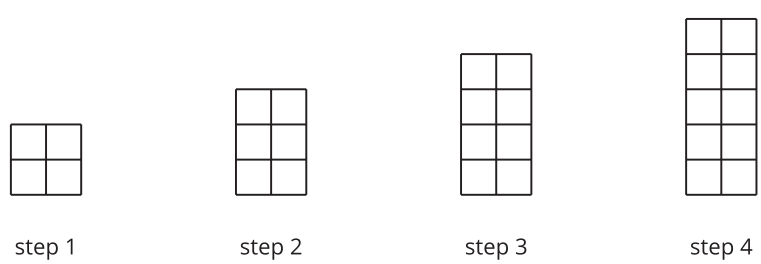
Suggested Centers

- Can You Draw It? (1–5), Stage 4: Area and Perimeter (Supporting)
- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)

Student Section Summary

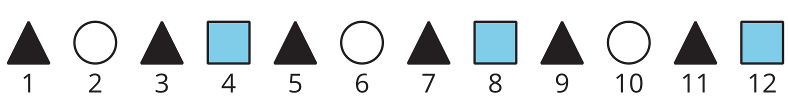
In this section, we looked at different patterns of shapes and patterns of numbers. We saw shapes that grew or repeated by certain rules, and we used numbers to help us see how the shapes changed. Here are some examples of the patterns:

- Shapes that grow by a rule: add 1 row of equal-size squares



Area of the rectangle: 4, 6, 8, 10, . . .

- Shapes that repeat by a rule: triangle, circle, triangle, square, repeat

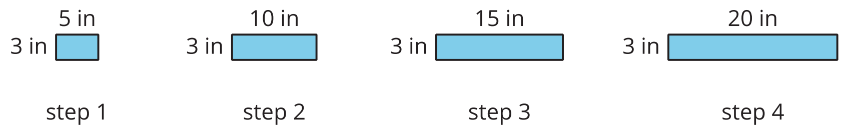


▲ : 1, 3, 5, 7, . .

○ : 2, 6, 10, . . .

▨ : 4, 8, 12, . . .

- Rectangles that change by a rule: increase the length of the rectangle by 5 inches



Side length:
5, 10, 15, 20, . . .

Area:
15, 30, 45, 60, . . .

Perimeter:
16, 26, 36, 46, . . .

- Numbers that change by a rule
    - Add 9: 9, 18, 27, 36, 45
    - Add 10: 10, 20, 30, 40, 50
    - Add 99: 99, 198, 297, 396, 495
    - Add 100: 100, 200, 300, 400, 500

We learned to extend the patterns by first finding their rule. Sometimes we can use addition and multiplication to represent a rule and then extend the pattern. Other times we can see how the digits in the numbers change to make predictions.

Complete Cool-Down

Response to Student Thinking

Students list every multiple of 8 rather than using a pattern.

Next Day Support

- Before the warm-up, have students work in small groups and compare their strategies for answering the question. Allow them to make revisions.

Section B: Multi-digit Multiplication

Lesson 5: Products Beyond 100

Standards Alignments

Building On	3.OA.A.3
Addressing	4.NBT.B.5
Building Towards	4.NBT.B.5

Teacher-facing Learning Goals

- Multiply two-digit by one-digit whole numbers in ways that make sense to them.

Student-facing Learning Goals

- Let’s find products beyond 100.

Lesson Purpose

The purpose of this lesson is for students to find the product of a one-digit number and a two-digit number in ways that make sense to them.

In grade 3, students learned about multiplication and learned to find products within 100. Earlier in this course, students identified factors and multiples, performed multiplicative comparison with whole numbers and fractions, and used the structure of base-ten numbers and properties of operations to find multiples of 10, 100, 1,000, and so on.

This lesson is the first in a series focused on finding whole-number products beyond 100. Here, students reason about equal-group situations involving one-digit and two-digit numbers in any way that makes sense to them. In the first activity, students work with an array of objects to build on a familiar representation. In the second activity, no visual representation is provided. Students may find products by creating arrays or diagrams, decomposing a factor into smaller numbers or place value, and using their understanding of properties of operations.

Access for:

Students with Disabilities

- Representation (Activity 2)

English Learners

- MLR2 (Activity 1)

Instructional Routines

MLR7 Compare and Connect (Activity 1), Number Talk (Warm-up)

Materials to Gather

- Tools for creating a visual display: Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What was the best question you asked students today? Why would you consider it the best one based on what students said or did?

Cool-down (to be completed at the end of the lesson)

5 min

Rows of Seats

Standards Alignments

Addressing 4.NBT.B.5

Student-facing Task Statement

A theater has 8 rows of seats and 27 seats in each row. How many seats are in the theater? Show your reasoning.

Student Responses

216 seats. Sample response:

- Eight rows of 20 is 160, and 8 rows of 7 is 56. $160 + 56 = 216$
- Eight rows of 30 is 8×30, which is 240. Because there are 27 seats per row and not 30 seats per row, I subtracted 8×3 or 24 from 240, which gives 216.
- I know 2×27 is 54, so 4×27 is twice 54 or 108, and 8×27 is twice 108, which is 216.

Begin Lesson

Warm-up

10 min

Number Talk: A Number Times Some Multiple of 10

Standards Alignments

Addressing 4.NBT.B.5

This Number Talk encourages students to decompose factors and to rely on the distributive property to mentally solve. The strategies elicited here will be helpful later in the lesson when students multiply up to four-digit numbers by one-digit numbers, and later in the section when they multiply 2 two-digit numbers by decomposing factors.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- 8×30
- 5×30
- 10×30
- 15×30

Student Responses

- 240: 8×30 is $8 \times 3 \times 10$ or 24×10, which is 240.
- 150: $5 \times 3 = 15$ so $5 \times 30 = 150$.
- 300: 10×30 is 5×30 doubled.
- 450: 15×30 is $15 \times 3 \times 10$ or 45×10, which is 450.

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "How did the first three expressions help you find the value of 15×30?" (I know that $(5 \times 30) + (10 \times 30)$ is 15×30. I doubled the value of 8×30 to get the value of 16×30 and then I subtracted 30 from it to get the value of 15×30.)

Activity 1

15 min

Elena’s Sticky Gift

Standards Alignments

Building On	3.OA.A.3
Building Towards	4.NBT.B.5

In this activity, students build on grade 3 work with arrays to consider how to find the total number in an array without counting by 1. Students are not asked to find the answer, but instead share their strategies for doing so. This allows teachers to observe how students make sense of multiplying larger numbers.

Students may decompose the larger array of stickers into two smaller arrays using the distributive property to determine the product (MP7). They may also use the idea of doubling and tripling to find the product. (For instance, they may start with 13×2 and triple the result to get 13×6.)

This activity uses *MLR7 Compare and Connect*. Advances: *representing, conversing*

Access for English Learners

MLR2 Collect and Display. Circulate, listen for, and collect the language students use as they create a display of their strategies. On a visible display, record words and phrases such as: decompose, partition, associative property, distributive property, and array. Invite students to borrow language from the display as needed, and update it throughout the lesson.
Advances: Conversing, Reading

Instructional Routines

MLR7 Compare and Connect

Materials to Gather

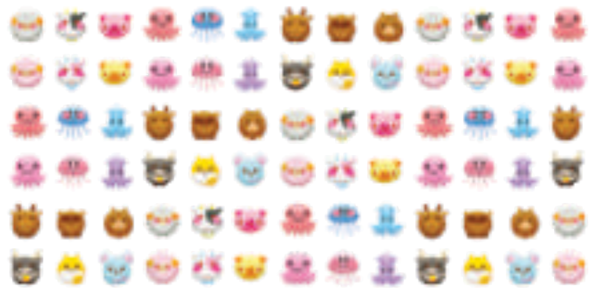
Tools for creating a visual display

Student-facing Task Statement

Elena receives a sheet of fancy stickers as a gift.

Launch

- Groups of 2
- Give each group tools for creating a visual display.



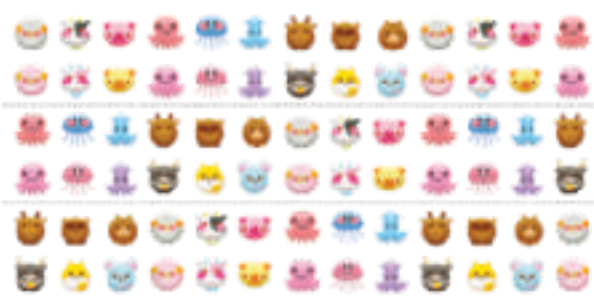
How many stickers are there? Explain or show how you would find out without counting every sticker.

Student Responses

Sample responses:

- $(6 \times 10) + (6 \times 3)$

- $(2 \times 13) + (2 \times 13) + (2 \times 13)$



Activity

- "Take a few quiet minutes to answer the question. Then, compare your strategy with your partner's."
- 3 minutes: independent work time
- 2 minutes: partner discussion

MLR7 Compare and Connect

- "Create a display that shows both of your ideas. Record your thinking so that it can be followed by others."
- 5 minutes: partner work time
- Monitor for students who:
    - decompose the two-digit factor by place value and use a drawing or an expression to show the decomposition (for example: partition the 13 columns in the array into 10 and 3 columns)
    - write expressions that involve the distributive or associative properties (as noted in Student Responses)
- 3 minutes: gallery walk

Synthesis

- "Which ideas did you see the most as you walked around today?"
- 30 seconds: quiet think time
- Invite students to share their strategies and reasoning.
- To highlight similarities and differences, consider comparing and contrasting the strategies using the term "decompose."

Advancing Student Thinking

If students choose to count the number in the first column or the first row and then skip-count by that number, consider asking them how they might record that strategy (other than writing "skip-count by 6" or "skip-count by 13").

Activity 2

20 min

More and More Stickers

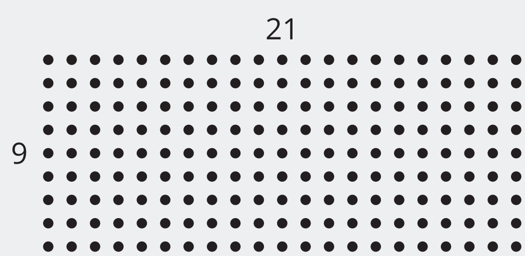
Standards Alignments

Addressing 4.NBT.B.5

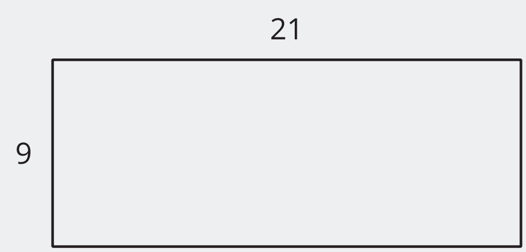
In this activity, students use strategies and representations that make sense to them to find products beyond 100. As before, the context of stickers lends itself to be represented with an array. The factors are large enough, however, that doing so would be inconvenient, motivating other representations or strategies (MP2). Look for the ways that students extend or generalize previously learned ideas or representations to find multiples of larger two-digit numbers. While many of the student responses are written with expressions, students are not expected to represent their reasoning using equations and expressions as this time. Teachers may choose to represent student reasoning using equations and expression so students can start connecting representations.

After students work on the first problem, pause to discuss some possible representations for finding the number of stickers. Each of the representations show different ways to represent the decomposition of a factor and students may decompose the factors in a variety of ways.

A. I created an array and decomposed it into smaller arrays.



B. I drew a diagram and decomposed it into smaller sections.



C. I decomposed the 21 and wrote one or more expressions or equations.

$(9 \times 20) + (9 \times 1)$
$(9 \times 10) + (9 \times 10) + (9 \times 1)$

D. I decomposed the 9 and wrote one or more expressions or equations.

$(3 \times 21) + (3 \times 21) + (3 \times 21)$
$(4 \times 21) + (4 \times 21) + (1 \times 21)$

Consider using the “four corners” structure to allow for movement and for interactions among students who might not typically interact. Post each of the four strategies in a different corner of the classroom. For the representations that use arrays or rectangular diagrams, it may help to give examples of decomposing based on 1–2 samples of student work that you observe during the activity. Then, ask students to move to a corner based on their reasoning strategy and representation.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Before displaying the four strategies shown in the Activity Narrative, activate background knowledge. Ask, “What does it mean to decompose a number?” While reviewing the strategies, ask students to engage with their classmate’s explanations by asking, “How did your classmate decompose 21 (or 9)?” For students who need extra support approaching question 2, begin by asking, “How might you decompose 48?”
Supports accessibility for: Memory, Language

Required Preparation

- Create 4 posters showing the 4 representations shown in the activity narrative.

Student-facing Task Statement

1. Elena has another sheet of stickers that has 9 rows and 21 stickers in each row. How many stickers does Elena have? Explain or show your reasoning.
2. Noah’s sticker sheet has 3 rows with 48 stickers in each row. Andre’s sticker sheet has 7 rows with 23 stickers in each row.

   Who has more stickers? Explain or show your reasoning.

Student Responses

1. Sample responses:
    - Ten rows of 21 is 10×21 or 210, so 9 rows of 21 is 21 fewer than 210 or $210 - 21$, which is 189.
    - Nine rows of 20 make 180 ($9 \times 20 = 180$) and 9 rows of 1 make 9, so altogether there are 189.
2. Andre has more stickers. Sample reasoning:
    - Noah: $3 \times 40 = 120$ and $3 \times 8 = 24$, and $120 + 24 = 144$.

Launch

- As a class, read the first problem about Elena’s stickers.
- “Make an estimate: Do you think Elena has fewer than 100 stickers, between 100 and 200, or more than 200?”
- 30 seconds: quiet think time
- Poll the class on their estimates (fewer than 100, between 100 and 200, or more than 200).
- “Turn to your partner and explain how you made your estimate.”
- 1 minute: partner discussion

Activity

- “Take a few quiet minutes to find the exact number of stickers Elena has and explain or show your reasoning.”
- 2–3 minutes: independent work time
- Display the four representations shown in the activity narrative.
- “Which representation best describes your

- Andre: 7 groups of 20 make 140 and 7 more groups of 3 make 21, and $140 + 21 = 161$.

approach? If none of them does, create a display that shows your thinking."

- Poll the class on their representations. Select a student who uses each strategy to explain more fully how they solved the problem.
- "Now answer the last question using any of these representations or another one that makes sense to you."
- 5–7 minutes: group work time
- Monitor for the strategies students use to find 3×48 and 7×23.

Synthesis

- See lesson synthesis.

Advancing Student Thinking

If students start to draw arrays to represent 3×48 and 7×23, ask them if they could represent the factors in another way that doesn't require drawing individual dots for the stickers.

Lesson Synthesis

10 min

"Today we multiplied a two-digit number by one-digit number."

Display 3×48 and 7×23 for all to see. Invite students to share their strategies for finding the value of each product.

"To find the value of 3×48, some of you started by finding 3×40—with or without drawing diagrams—and others started by finding 3×50. If you started with 3×40, what did you do next?" (Add 3×8.) "If you started with 3×50, what did you do next?" (Subtract 3×2.)

"To find the value of 7×23, some of you found 7×20 first and then 7×3. Why did you decide to decompose the 23 into 20 and 3?" (It makes it possible to multiply the 7 by a multiple of 10, which is easier than multiplying 7 by a number that is not a multiple of 10.)

Suggested Centers

- Can You Draw It? (1–5), Stage 4: Area and Perimeter (Supporting)
- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students find a number of seats other than 218.

Next Day Support

- After the warm-up, have students discuss with a partner their strategies for the cool-down and allow time to make revisions to their thinking.

The work of this lesson builds from the multiplication concepts developed in a previous unit.

Prior Unit Support

Grade 3, Unit 4, Section C: Multiplying Larger Numbers

Lesson 6: Multiply Two-digit Numbers and One-digit Numbers

Standards Alignments

Addressing 4.NBT.B.5

Teacher-facing Learning Goals

- Multiply two-digit and one-digit whole numbers using place value understanding and properties of operations.

Student-facing Learning Goals

- Let's multiply two-digit and one-digit numbers.

Lesson Purpose

The purpose of this lesson is for students to multiply a two-digit number and a one-digit number using place value understanding.

In the previous lesson, students solved two-digit multiplication problems in a way that made sense to them. They discussed decomposing factors and considered different representations of their strategy. In this lesson, students extend these ideas to find the value of products beyond 100, focusing on representations and strategies based on place value and the properties of operations, which are familiar from grade 3.

Students analyze base-ten diagrams and diagrams that involve rectangles, some of which are partitioned by place value. They explain how the diagrams represent multiplication and make connections between them, deepening their understanding of place value and properties of operations. At the end of the lesson, students consider a rectangular diagram that will be used through the rest of the section.

Access for:

Students with Disabilities

- Engagement (Activity 2)

English Learners

- MLR1 (Activity 2)

Instructional Routines

Notice and Wonder (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What strategy did most students use in their work today? What strategy did you anticipate today? Which did you not anticipate?

Cool-down (to be completed at the end of the lesson)

5 min

Represent the Product

Standards Alignments

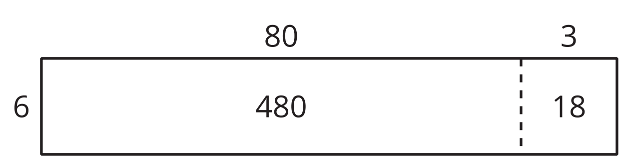
Addressing 4.NBT.B.5

Student-facing Task Statement

Find the value of 6×83. Use a diagram if it is helpful.

Student Responses

Sample response:



$6 \times 80 = 480$
$6 \times 3 = 18$
$480 + 18 = 498$

Begin Lesson

Warm-up

10 min

Notice and Wonder: With and Without a Grid

Standards Alignments

Addressing 4.NBT.B.5

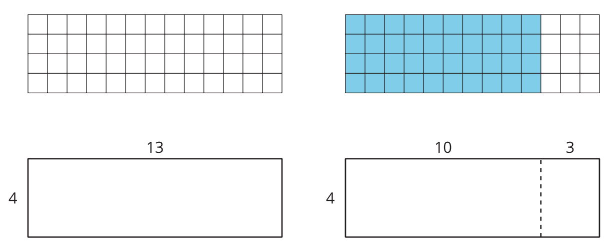
The purpose of this warm-up is to elicit students' prior knowledge of area and the idea that a rectangle can be decomposed into smaller rectangular regions. Students look at 4 different area diagrams they used in grade 3. The reasoning here will be useful when students use diagrams to multiply two- and one-digit numbers in a later activity. While students may notice and wonder many things about the number of units within the area of the gridded region, focus on the connections between the diagrams with a grid and those without.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?



Student Responses

Students may notice:

- Two rectangles are gridded and have no numbers. The other two have no grid and have numbers.
- In the first two diagrams, there are 13 squares in each row and 4 squares in each column.
- A portion of the gridded rectangle is shaded.
- Some rectangles have the side lengths marked.
- The last rectangle is partitioned into two sections.

Students may wonder:

Launch

- Groups of 2
- Display the image.
- "What do you notice? What do you wonder?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- "What do you know about the 13 and 4 in the first ungridded rectangle?" (They are side lengths. They correspond to the number of squares across and the number of squares down in the first rectangle.)
- "What about the 10, 3, and 4 in the second ungridded rectangle?" (The 10 and 3 are numbers that add up to 13. The 4 is the length of the shorter side.)
- "How are the four diagrams related? Name as many connections as you see." (Sample responses:

- Where are the numbers in the gridded rectangles?
- Why is the second rectangle shaded?
- Do the four diagrams represent the same rectangle?

  - They all represent the same rectangle. They have the same side lengths and the same area.
  - The first two show the number of square units that fit in the rectangle. The last two don't show it but we can tell by multiplying the side lengths.
  - The shaded portion in the second gridded rectangles shows the 4×10 portion in the fourth rectangle.)
- If no students mentioned the area of the rectangles in their analysis, ask: "How might we find the area of the rectangles?" (For the gridded rectangles, we can count the unit squares or multiply the number of units across and down. For the other two, we can multiply the side lengths.)

Activity 1

20 min

Tyler's Diagrams

Standards Alignments

Addressing 4.NBT.B.5

This activity prompts students to make sense of base-ten diagrams for representing multiplication. The representation supports students in grouping tens and ones and encourages them to use place value understanding and to apply the distributive property (MP7).

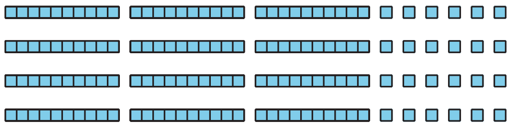
This activity is an opportunity for students to build conceptual understanding of partial products in a more concrete way. In the next activity, students will notice that working with these drawings can be cumbersome and transition to using rectangular diagrams, which are more abstract.

Student-facing Task Statement

1. To find the value of 4×36, Tyler uses a base-ten diagram, as shown here.

Launch

- Groups of 3–4

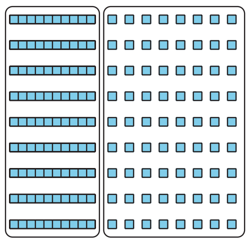


a. Where is the 36 in Tyler's diagram?

b. Where is the 4 in Tyler's diagram?

c. What is the value of 4×36?

2. Here is a diagram Tyler made to find the value of 9×18.



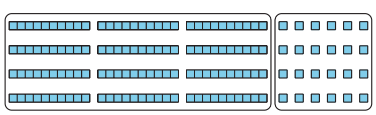
Explain or show how his diagram helps him find the value of 9×18.

Student Responses

1. a. Each row has 3 tens and 6 ones.

   b. There are 4 rows of 3 tens and 6 ones.

   c. 144. Sample responses:

      - There are 12 tens and 24 ones, or 120 and 24, which make 144.
      - $(4 \times 30) + (4 \times 6) = 120 + 24$, which is 144.
      - 

2. Sample responses:
   - The 9 tens make 90, and the 9 groups of 8 ones make 72, so the product is $90 + 72$, which is 162.
   - $(9 \times 10) + (9 \times 8) = 90 + 72 = 162$

Activity

- 5 minutes: independent work time on the first problem
- 2–3 minutes: partner discussion
- 3–4 minutes: independent work time on the second problem
- Monitor for students who use different methods to find the value of 4×36 in the first problem.

Synthesis

- Select 2 students who organized the diagram in the first problem in different ways and then compare to what Tyler did.
- Invite selected students to share their method for finding the value of 4×36.
- "How are these methods the same? How are they different?"
- "How are these methods like what Tyler did to find the value of 9×18? How are they different?"

Activity 2

15 min

Two Kinds of Diagrams

Standards Alignments

Addressing 4.NBT.B.5

This activity continues to encourage place value reasoning for finding the product of a two-digit factor and a one-digit factor.

Students make sense of two representations that show the two-digit factor decomposed by place value: a base-ten diagram and a rectangle. The latter looks like an area diagram that students have used in grade 3, where the side lengths of a rectangle represents two factors. As the factors become larger, however, it becomes necessary to draw rectangles whose side lengths are not proportional. When rectangles no longer accurately represent area, the term “area diagrams” is not used. Instead, “rectangular diagrams” is used in teacher materials and “diagrams” in student materials.

Students then choose a representation to use to find products and write corresponding expressions. In the synthesis, they learn that the results of multiplying a part of one factor by the other factor can be called “partial products.” In future lessons, students will use rectangular diagrams to represent multiplication of larger numbers.

Access for English Learners

MLR1 Stronger and Clearer Each Time. Synthesis: Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their response to which method they prefer when multiplying 6 by 53. Invite listeners to ask questions, to press for details, and to suggest mathematical language. Give students 2–3 minutes to revise their written explanation based on the feedback they receive.
Advances: Writing, Speaking, Listening

Access for Students with Disabilities

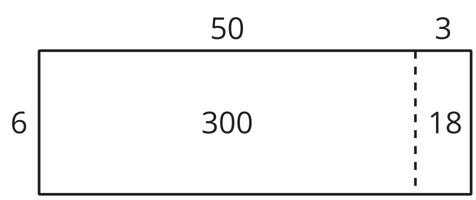
Engagement: Provide Access by Recruiting Interest. Optimize meaning and value. Invite students to share examples from their own lives in which they might need to multiply two-digit and one-digit numbers. Invite them to imagine and share why Han and Priya might be multiplying 6 times 53.
Supports accessibility for: Attention, Social-Emotional Functioning

Student-facing Task Statement

1. Priya drew a base-ten diagram to multiply 6×53. She said it shows that the product can be found by adding 300 and 18.

   a. Where do you see 6 and 53 in her diagram?
   b. Where do you see 300 and 18 in Priya's diagram? What do they represent?

2. Han drew this diagram to multiply 6×53:



   Where do you see 300 and 18 in his diagram? What do they represent?

3. Which diagram do you prefer for multiplying 6×53: Han's way or Priya's way? Explain your reasoning.
4. Find the value of 6×53.
5. Draw a diagram to represent each multiplication expression. Then, find the value of each product.

   a. 6×48
   b. 9×67

Student Responses

1. Sample response:
   - Each row has 5 tens and 3 ones, which represent 53. The six rows represent the 6.
   - The 300 is the 30 tens, which comes from 6 groups of 5 tens. The 18 is the

Launch

- Groups of 2

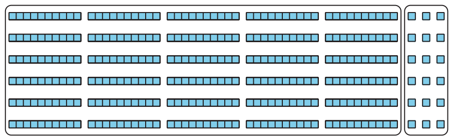
Activity

- "Take a few quiet minutes to work on the activity. Afterward, share your thinking with your partner."
- 6–7 minutes: independent work time
- 3–4 minutes: partner discussion
- As students work on the last problem, monitor for those who:
  - recognize that base-ten drawings are cumbersome and a less efficient way to find the product
  - draw diagrams that show the two-digit side length partitioned by tens and ones
  - write expressions that show place value reasoning
  - use the distributive property (find the partial products, then add to find the value of the product)

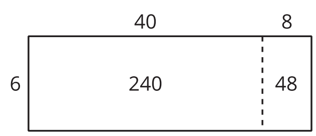
Synthesis

- Select 2–3 students to share their responses and reasoning.
- "How are Han's and Priya's diagrams similar?" (They both show the 53 as 50 and 3. They multiply the parts and add them to find the total value.)
- "How are they different?" (Han just drew a rectangle, and Priya drew all the tens and ones.)
- "What expressions can you write to show how to multiply 6×53? What is the product?" ($6 \times 50) + (6 \times 3)$. The product is 318.)
- Explain that the 300 and 18 in both Han's and Priya's diagrams are called partial

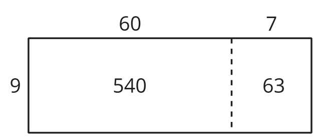
18 ones, which comes from 6 groups of 3.



2. Sample response: The 300 and 18 are areas of the smaller rectangles in the diagram. 300 is 6×50 and the 18 is 6×3.
3. Sample responses:
    - Han's way, because it is easier to see and multiply the numbers.
    - Priya's way, because it's easier to count the tens and ones.
4. $6 \times 53 = 318$
5. a. 288



b. 603



products. Each is found by multiplying a part of one of the factors by the other factor.

- For the last problem, if students do not partition their rectangular diagrams by place value, display the sample diagrams as shown in the student responses without products. Ask:
    - "What expressions can we write to represent each part of the diagram?" (6×40 and 6×8 or 9×60 and 9×7.)
    - "How does partitioning the two-digit number into tens and ones help to find the product?" (Finding multiples of ten and facts like 6×8 is easier than finding a product of say, 36 and 8. Then we can add the smaller products together to find the total product.)

Lesson Synthesis

10 min

"Today we used different diagrams and expressions to represent multiplication."

Display 8×79 and a blank rectangle.

"How could we use this rectangle to represent 8×79?" (Place 8 on one of the sides of the rectangle and partition the rectangle to show 79 across the other side.)

Label the rectangle and partition the rectangles to show 79 decomposed into 70 and 9.

"How does this diagram help us find the value of 8×79?" (8×79 is hard to do mentally, but I know that it is like finding the area of a rectangle with side lengths 8 and 79. I can decompose the rectangle into smaller rectangles and add the areas of the smaller rectangles to find the area of the large rectangle. Also, I can do 8×70 and 8×9 in my head, which makes the multiplication easier.)

"What is the value of the product?" (632, because 8×70 is 560 and 8×9 is 72, and $560 + 72 = 632$)

Suggested Centers

- Can You Draw It? (1–5), Stage 4: Area and Perimeter (Supporting)
- Five in a Row: Multiplication (3–5), Stage 2: Factors 1–9 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students use a diagram, but find a value other than 498 as the product.

Next Day Support

- Launch warm-up or activities by highlighting important representations from previous lessons.

Lesson 7: Multiply Three- and Four-digit Numbers by One-digit Numbers

Standards Alignments

Addressing 4.NBT.B.5

Teacher-facing Learning Goals

- Multiply three- and four-digit numbers using place value understanding and properties of operations.

Student-facing Learning Goals

- Let's multiply three- and four-digit numbers by one-digit numbers.

Lesson Purpose

The purpose of this lesson is for students to multiply a whole number of up to four digits by a one-digit number by decomposing factors by place value, finding partial products, and using properties of operations.

In the previous lesson, students represented multiplication using base-ten diagrams and rectangular diagrams, and used place value reasoning to multiply two-digit numbers by one-digit numbers. In this lesson, they use rectangular diagrams and expressions to multiply up to four-digit numbers by one-digit numbers. They continue to use place value reasoning to decompose the multi-digit factor and to use partial products in their computation.

Students should have multiple opportunities to hear the term "partial products" as referring to the results of multiplying a part of one factor and the other factor (or a part of one factor and a part of the other factor).

Access for:

Students with Disabilities

- Representation (Activity 2)

English Learners

- MLR2 (Activity 1)

Instructional Routines

Estimation Exploration (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

How can you leverage each of your student's ideas to support them in being seen and heard in tomorrow's math class?

Cool-down (to be completed at the end of the lesson)

5 min

The Value of the Product

Standards Alignments

Addressing 4.NBT.B.5

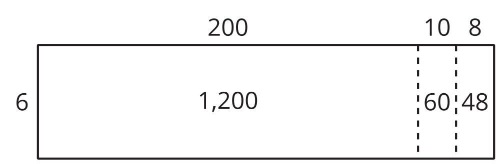
Student-facing Task Statement

Find the value of 6×218. Show your reasoning.

Student Responses

1,308. Sample response:

$$(6 \times 200) + (6 \times 10) + (6 \times 8)$$
$$= 1{,}200 + 60 + 48$$
$$= 1{,}308$$



Begin Lesson

Warm-up

10 min

Estimation Exploration: Mysterious Area

Standards Alignments

Addressing 4.NBT.B.5

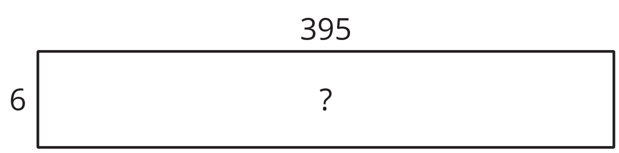
The purpose of an Estimation Exploration is to practice the skill of estimating a reasonable answer based on experience and known information. Listen for the different ways students use their understanding of place value to estimate the area and explain why an estimate is too low or too high.

Instructional Routines

Estimation Exploration

Student-facing Task Statement

What is the area of the rectangle?



Record an estimate that is:

too low	about right	too high

Student Responses

- Too low: 1,800 to 2,000.
- About right: 2,200 to 2,400
- Too high: 2,500 or higher

Launch

- Groups of 2
- Display the image.
- “What is an estimate that’s too high?” “Too low?” “About right?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Record responses.

Synthesis

- “Is your estimate greater or less than the actual product?” (greater)
- “How could we find the actual product?” ($6 \times 400 - 6 \times 5$, or subtract 6×5 from 2,400)

Activity 1

15 min

Larger Numbers to Multiply

Standards Alignments

Addressing 4.NBT.B.5

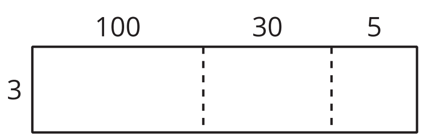
In this activity, students use rectangular diagrams to represent multiplication of three-digit and one-digit numbers. Though students may decompose the multi-digit factor in different ways, the activity is designed to encourage them to decompose it by place value—into hundreds, tens, and ones.

Access for English Learners

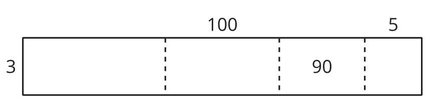
MLR2 Collect and Display. Synthesis: Direct attention to words collected and displayed from the previous activity. Invite students to borrow language from the display as needed, and update it throughout the lesson.
Advances: Conversing, Reading

Student-facing Task Statement

1. Clare drew this diagram.



   a. What multiplication expression can be represented by the diagram?
   b. Find the value of the expression. Show your reasoning.
2. Consider the expression 6×252.
   a. Draw a diagram to represent the expression.
   b. Find the value of the expression. Show your reasoning.
3. Lin drew a diagram to represent $3 \times 2{,}135$.



   a. Complete Lin's diagram.
   b. Write an expression to represent the

Launch

- Groups of 2

Activity

- "Work with your partner on the first problem. Then, try the rest of the activity independently."
- 2 minutes: group work time on the first problem
- 5–7 minutes: independent work time on the rest of the activity
- 3 minutes: partner discussion
- Monitor for students who:
  - write expressions that show the multi-digit factor decomposed by place value
  - draw diagrams that partition the multi-digit factor by place value

Synthesis

- Invite several students to share diagrams value of each part of the diagram.

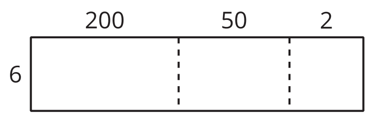
c. Find the value of $3 \times 2{,}135$. Show your reasoning.

Student Responses

1. a. 3×135

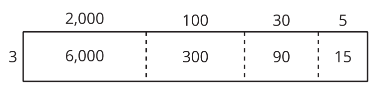
   b. $(3 \times 100) + (3 \times 30) + (3 \times 5) = 300 + 90 + 15 = 405$

2. a.



   b. $6 \times 200 = 1{,}200$
   $6 \times 50 = 300$
   $6 \times 2 = 12$
   $1{,}200 + 300 + 12 = 1{,}512$

3. a.



   b. $(3 \times 2{,}000) + (3 \times 100) + (3 \times 30) + (3 \times 5)$

   c. $3 \times 2{,}000 = 6{,}000$
   $3 \times 100 = 300$
   $3 \times 30 = 90$
   $3 \times 5 = 15$
   $6{,}000 + 300 + 90 + 15 = 6{,}405$

they drew to represent 6×252.

- As each student shares, consider asking the class:
  - "Where do you see the parts of the factor that was decomposed?"
  - "What expressions are represented in the diagram?"
- Record expressions as students share.

Activity 2

20 min

Jada's Errors

Standards Alignments

Addressing 4.NBT.B.5

This activity extends students' work with multiplication to include a factor with up to four digits. Students begin to generalize that they could decompose any number into parts and multiply the parts. In this activity, students analyze a common error when multiplying. The work they look at does not apply place value understanding and therefore represents a product that is unreasonable for the given expression. When students analyze Jada's work, find her errors and explain their reasoning, they critique the reasoning of others (MP3).

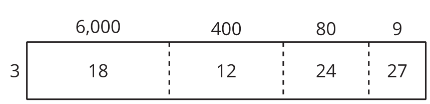
Students see partial products as a way to describe the sub-products when a factor is decomposed and multiplied using the distributive property. Continue to refer to partial products in students' diagrams and calculations by their mathematical name to build students' intuition for their meaning (though students are not expected to use them in their reasoning).

Access for Students with Disabilities

Representation: Access for Perception. Use base-ten blocks to demonstrate Jada's error and how her work should be corrected. Invite students to discuss how they might avoid such an error (for example, by estimating the product first, or by visualizing the value in base-ten blocks).
Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Student-facing Task Statement

1. Jada used a diagram to multiply $3 \times 6{,}489$ and made a few errors.



   a. Explain the errors Jada made.
   b. Find the value of $3 \times 6{,}489$. Show your reasoning.
2. Find the value of 5×699. Show your reasoning.
3. Find the value of $8 \times 4{,}973$. Show your reasoning.

Student Responses

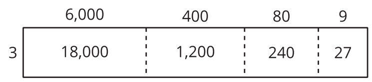
1. Sample response:
   a. Jada forgot to include the place values (1,000, 100, and 10) when multiplying each part. She only multiplied the non-zero digits.
   b. 19,467

Launch

- Groups of 2

Activity

- "Take a few quiet minutes to analyze Jada's errors. Then, share your thinking with your partner."
- 2 minutes: independent work time on the first problem about Jada's diagram
- 2 minutes: partner discussion
- Pause for a discussion.
- "What did Jada do correctly?" (She multiplied the non-zero digits correctly. She decomposed the 6,489 by place value, which is helpful.)
- "What did Jada miss?" (She multiplied only the non-zero digits. She didn't account for the place value of the digits being multiplied. For example: The partial product of $3 \times 6{,}000$ is 18,000, not 18.)
- Display and correct Jada's diagram as a class.



18,000 + 1,200 + 240 + 27 = 19,467

2. 3,495. Sample reasoning:
$$\begin{aligned} 5 \times 699 &= (5 \times 600) + (5 \times 90) + (5 \times 9) \\ &= 3{,}000 + 450 + 45 \\ &= 3{,}495 \end{aligned}$$
3. 39,784. Sample reasoning:
$$\begin{aligned} 8 \times 4{,}973 &= (8 \times 4{,}000) + (8 \times 900) + (8 \times 70) + (8 \times 3) \\ &= 32{,}000 + 7{,}200 + 560 + 24 \\ &= 39{,}784 \end{aligned}$$

- “Now complete the remaining problems independently.”
- 5 minutes: independent work time on the last two problems
- Monitor for students who:
    - draw diagrams that partition the multi-digit factor by place value
    - write expressions that show the multi-digit factor decomposed by place value

Synthesis

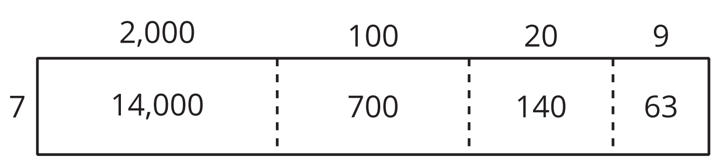
- Select 1–2 students to share their solutions and reasoning for finding the value of $8 \times 4{,}973$.
- Consider asking students to write expressions that represent the way they found each product. For example:
  $8 \times 4{,}973$
  $8 \times (4{,}000 + 900 + 70 + 3)$
  $(8 \times 4{,}000) + 8 \times 900 + 8 \times 70 + 8 \times 3$
  $32{,}000 + 7{,}200 + 560 + 24$
  $39{,}784$

Lesson Synthesis

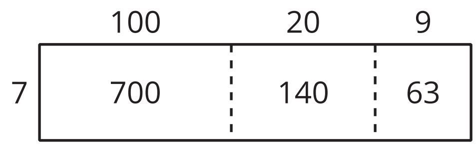
10 min

“Today we used diagrams to multiply three- and four-digit numbers by one-digit numbers. Let’s compare diagrams that represent $7 \times 2{,}129$ and 7×129.”

Display: $7 \times 2{,}129$ and 7×129



$$14{,}000 + 700 + 140 + 63 = 14{,}903$$



$$700 + 140 + 63 = 903$$

"How are the representations alike and how are they different?" (Sample responses:

- Alike: They both show the expanded form of the multi-digit factor, have 7 as one of the factors, and show partial products. They both show one factor decomposed by place value or written in expanded form.
- Different: One diagram is decomposed into four parts because the factor 2,129 has four digits. The diagram shows 4 partial products. The other diagram is decomposed into 3 parts, because 129 has three digits. The diagram shows 3 partial products.)

"How would you find the value of $7 \times 2{,}039$?" (Think of 2,039 as $2{,}000 + 30 + 9$ and find $(7 \times 2{,}000) + (7 \times 30) + (7 \times 9)$.)

Suggested Centers

- Number Puzzles: Multiplication and Division (4–5), Stage 1: Two-digit Factors (Addressing)
- Compare (1–5), Stage 3: Multiply within 100 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students may find the correct partial products but add them incorrectly.

Next Day Support

- Before the warm-up, invite students to work in small groups to discuss a correct response to this cool-down.

Lesson 8: Multiply 2 Two-digit Numbers

Standards Alignments

Addressing 4.NBT.B.4, 4.NBT.B.5

Teacher-facing Learning Goals

- Multiply 2 two-digit numbers using place value understanding and properties of operations.

Student-facing Learning Goals

- Let's multiply 2 two-digit numbers.

Lesson Purpose

The purpose of this lesson is for students to multiply 2 two-digit numbers.

Previously, students used place-value reasoning to decompose a factor in a multiplication expression to multiply numbers up to four-digit by one-digit numbers. In this lesson, they apply these ideas to multiply 2 two-digit numbers. They reason about why it is helpful to decompose both two-digit numbers by place value. As students analyze the connections between expressions and diagrams, they recognize that partial products in which the factors are either single-digit numbers or multiples of 10 can be found mentally, making the rectangular diagram a useful tool for multiplying two-digit numbers.

Access for:

Students with Disabilities

- Action and Expression (Activity 1)

English Learners

- MLR8 (Activity 1)

Instructional Routines

MLR5 Co-craft Questions (Activity 2), Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min

Teacher Reflection Question

In a future lesson, students will be analyzing partial products from rectangular diagrams and making connections to the traditional algorithm notation. How do rectangular diagrams support this thinking?

Cool-down	5 min

Cool-down (to be completed at the end of the lesson)

5 min

What's the Product?

Standards Alignments

Addressing 4.NBT.B.5

Student-facing Task Statement

Find the value of 24×17. Explain or show your reasoning. Use a diagram if it helpful.

Student Responses

408. Sample reasoning:

$200 + 140 + 40 + 28 = 408$

Begin Lesson

Warm-up

10 min

Number Talk: Extra Groups

Standards Alignments

Addressing 4.NBT.B.4

This Number Talk encourages students to use multiples of 10 to mentally multiply two-digit numbers that are close to multiples of 10. Students can use place value reasoning and the distributive property of multiplication over addition or subtraction to find the value of the products. The work here prompts students to think flexibly about how numbers can be decomposed strategically when multiplying.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- 20×60
- 21×60
- 20×62
- 19×60

Student Responses

- 1,200: I know 2×6 is 12, so $2 \times 10 \times 6 \times 10$ is 12×100, which is 1,200.
- 1,260: 21×60 is one more group of 60 than 20×60, so it is $1{,}200 + 60$ or 1,260 (or $(20 \times 60) + (1 \times 60) = 1{,}260$).
- 1,240: 20×62 is 20×2 more than 20×60, so it is 40 more than 1,200 (or $(20 \times 60) + (20 \times 2) = 1{,}240$).
- 1,140: 19 groups of 60 is 1 group of 60 less than 20 groups of 60, so it is $1{,}200 - 60$ or 1,140 (or $(20 \times 60) - (1 \times 60) = 1{,}140$).

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "How can 20×60 help us find the value of 19×60?" (19×60 is one group of 60 less than 20×60, so we can subtract 60 from 20×60.)

Activity 1

20 min

Two by Two

Standards Alignments

Addressing 4.NBT.B.5

In this activity, students use rectangular diagrams and similar reasoning as in earlier activities to represent the multiplication of 2 two-digit numbers. They analyze a progression of diagrams, starting with those that represent multiplication of two-digit and one-digit numbers (18 and 6), a two-digit number and a ten (18 and 10), and then 2 two-digit numbers (18 and 16).

Students may decompose factors in different ways. For example, those who are familiar with multiples of 25 may find it intuitive to decompose 25×46 as 25×40 and 25×6, rather than decomposing 25 into $20 + 5$.

Access for English Learners

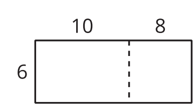
MLR8 Discussion Supports. Pair gestures with verbal directions to clarify the meaning of any unfamiliar terms such as partial product.
Advances: Listening, Representing

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Give students access to base-ten blocks. If students use the blocks for the last question, ask them to also draw a diagram that represents their work with the blocks.
Supports accessibility for: Conceptual Processing, Visual-Spatial Processing

Student-facing Task Statement

1. For each diagram, write a multiplication expression that the diagram can represent. Then, find the value of the expression. Use equations to show or explain your reasoning.

a.


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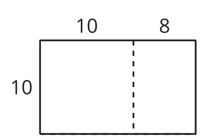
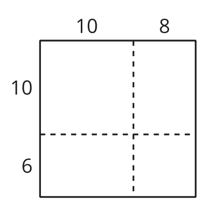


c.


2. How are the diagrams alike? How are they

Launch

- Display the three diagrams in the activity.
- “What do you notice? What do you wonder?”
- 1 minute: quiet think time
- 1 minute: partner discussion

Activity

- “Take a few quiet minutes to work on the first problem. Then, share your thinking with your partner.”
- 3 minutes: independent work time
- 3 minutes: partner discussion
- Invite students to share their responses: “What multiplication expression can be represented by each diagram?”
- “Complete the rest of the activity.”
- 3 minutes: independent or partner work time

different? Discuss with your partner.

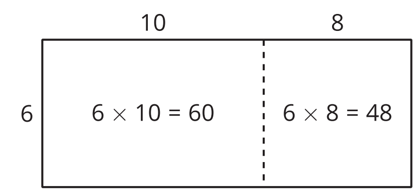
3. Use a diagram to find each product.

   a. 13×21

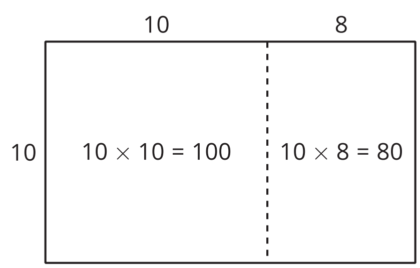
   b. 25×46

Student Responses

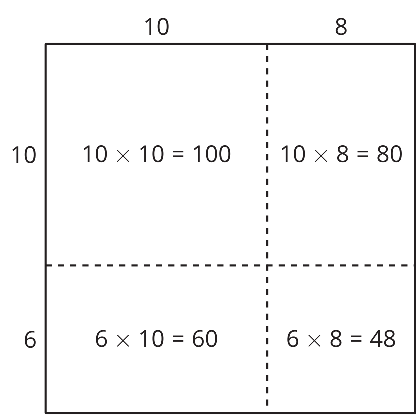
1. a. 18×6 or 6×18. The value is 108.
   Sample response:
   $(10 \times 6) + (8 \times 6) = 60 + 48 = 108$



   b. 18×10 or 10×18. The value is 180.
   Sample response:
   $(10 \times 10) + (10 \times 8) = 100 + 80 = 180$



   c. 18×16 or 16×18. The value is 288.
   Sample response:
   $(10 \times 10) + (10 \times 8) + (6 \times 10) + (6 \times 8) = 100 + 80 + 60 + 48 = 288$



2. Sample response: The diagrams are alike because they all show multiplication by 18,

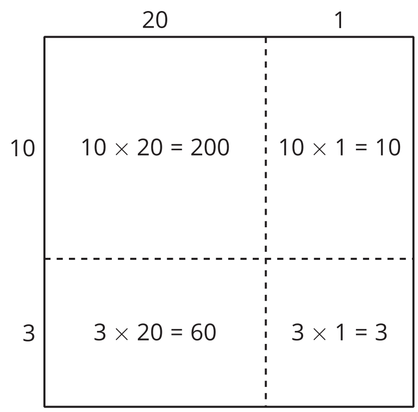
- Monitor for students who decompose both factors into tens and ones and those who choose to keep one of the factors intact.

Synthesis

- Select two students to share their diagrams, solutions, and reasoning for the last problem.
- "How did you decompose the factors and the diagram?" (I decomposed the 13 into 10 and 3 and the 21 into 20 and 1.)
- "What expression could we write to show the partial product represented by each part of the diagram?"
- "How do these partial products help us find the value of 13×21?" (Adding them gives us the value of 13×21: $200 + 10 + 3 + 60 = 273$)

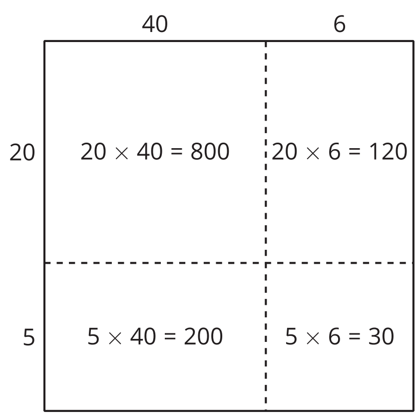
decomposed by place value. They are different because diagram A shows a one-digit factor, B shows 10 as a factor, and C shows a two-digit factor partitioned into two rows. Diagrams A and B are smaller parts of diagram C.

3. a. 13×21:



$200 + 10 + 60 + 3 = 273$

b. 25×46:



$800 + 120 + 200 + 30 = 1{,}150$

Activity 2

15 min

Decompose by Place Value

Standards Alignments

Addressing 4.NBT.B.5

In this activity, students analyze two ways of decomposing a factor: by place value and not by place value. As they write the corresponding partial products, they see more clearly why it is helpful to decompose each of the factors by place value (MP7). Students may notice that when the factors are decomposed by place value, they end up finding multiples of 10 and multiplying a number by single-digit factors—both of which they can do with some degree of fluency.

This activity uses *MLR5 Co-craft Questions.* Advances: writing, reading, representing

Instructional Routines

MLR5 Co-craft Questions

Student-facing Task Statement

These diagrams could be used to find the value of 49×57.

Diagram A

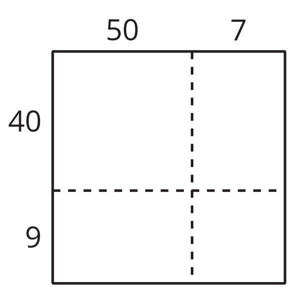
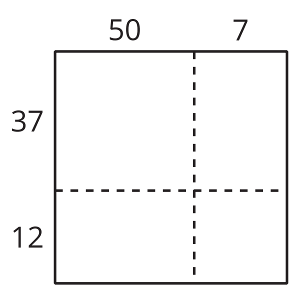


Diagram B



1. Which diagram is more helpful when finding the value of 49×57? Why?
2. Use a diagram to find each product.
   a. 49×57
   b. 29×55

Student Responses

1. Sample response: Diagram A, because 40×50 is easier than 37×50 to find or

Launch

- Groups of 2

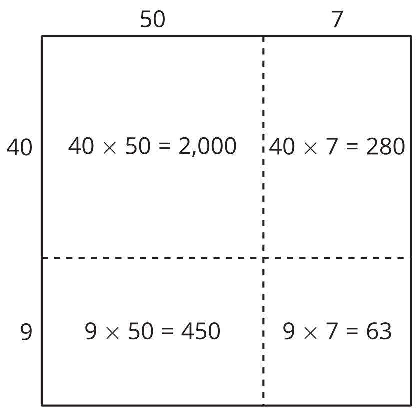
Activity

MLR5 Co-Craft Questions

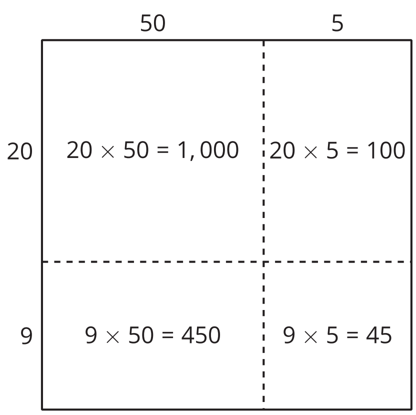
- Display only the two diagrams, without revealing the opening sentence or the question(s).
- "Write a list of mathematical questions that could be asked about this situation."
- 2 minutes: independent work time
- 2 minutes: partner discussion
- Invite several students to share one question with the class. Record responses.
- "What do these questions have in common? How are they different?"
- Reveal the task (students open books), and invite additional connections.
- "Take two quiet minutes to think about the first question. Then, share your thinking with your partner."
- 2 minutes: independent work time on the

know.

2. a. $2,000 + 280 + 450 + 63 = 2,793$



b. $1,000 + 100 + 450 + 45 = 1,595$



first problem

- 1 minute: partner discussion
- "Now complete the rest of the activity."
- Monitor for students who:
  - decompose the side lengths of their diagrams by place value
  - write expressions to show the sum of the partial products (This is not required, but it is helpful for the synthesis.)
- If extra time is available, add more two-digit by two-digit expressions to the last problem:
  - 83×39
  - 64×92

Synthesis

- "How can both diagrams be used to find the value of 49×57?" (We can partition each side of a rectangle in many ways without changing the total side lengths.)
- "Why was the partitioning in diagram A more helpful than in diagram B?" (40×50 is easier than 37×50 or 12×50 to find mentally, because we are multiplying multiples of 10.)

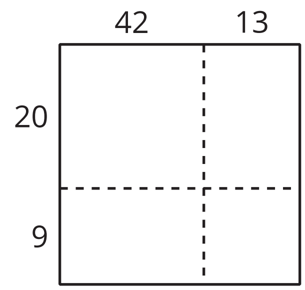
Lesson Synthesis

10 min

"Today we learned how to represent the multiplication of 2 two-digit numbers using a rectangular diagram. We learned that we can decompose each factor by place value and show the tens and ones on each side of the rectangle, and that doing this can help us to multiply efficiently."

Select students with different strategies to share their reasoning for finding the value of 29×55 (the last problem of the last activity).

Display the following diagram as an example of how decomposing can result in facts that are not helpful when multiplying to support using place value to decompose.



"Why might it be more helpful to decompose 55 into $50 + 5$ than into $42 + 13$?" (Multiplying by multiples of 10 and by single-digit numbers is easier than multiplying numbers like 42 and 9.)

Suggested Centers

- Number Puzzles: Multiplication and Division (4–5), Stage 1: Two-digit Factors (Addressing)
- Compare (1–5), Stage 3: Multiply within 100 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students may decompose factors in ways to result in partial products that are not useful.

Next Day Support

- Launch warm-up or activities by analyzing two different ways to decompose the factors to multiply. Discuss which is most useful and why.

Lesson 9: Recording Partial Products: One-digit and Three- or Four-digit Factors

Standards Alignments

Addressing 4.NBT.B.5

Teacher-facing Learning Goals

- Multiply multi-digit whole numbers by one-digit numbers using an algorithm that uses partial products.

Student-facing Learning Goals

- Let’s analyze and try an algorithm that uses partial products.

Lesson Purpose

The purpose of this lesson is for students to multiply a multi-digit number by a one-digit number using an algorithm that uses partial products. Students make connections between this algorithm, rectangular diagrams, and equations.

In previous lessons, students used diagrams to represent multiplication of a one-digit number and a whole number of up to four digits. They learned to decompose larger factors by place value and used diagrams and corresponding expressions to support them in finding partial products. In this lesson, students learn an algorithm for keeping track of partial products that come from multiplying the digits of the factors. This algorithm that uses partial products lays the foundation for the standard algorithm for multiplication.

Students engage in quantitative and abstract reasoning (MP2) as they relate the partial products in a diagram and in an algorithm. Because this lesson offers an initial exposure to the new notation, students are not required to use an algorithm that uses partial products to multiply. They can rely on other methods they have learned so far.

Access for:

Students with Disabilities

- Action and Expression (Activity 2)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Which One Doesn’t Belong? (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What part of the lesson went really well today in terms of students learning? What did you do that made that part go well?

Cool-down (to be completed at the end of the lesson)

5 min

Partial Products

Standards Alignments

Addressing 4.NBT.B.5

Student-facing Task Statement

Find the value of $5 \times 1{,}023$. Show your reasoning.

Student Responses

5,115. Sample responses:

- $5 \times 3 = 15$, $5 \times 20 = 100$, and $5 \times 1{,}000 = 5{,}000$. The sum of 15, 100, and 5,000 is 5,115.

$$\begin{array}{r} 1{,}023 \\ \times \quad 5 \\ \hline 15 \\ 100 \\ 0 \\ +\ 5{,}000 \\ \hline 5{,}115 \end{array}$$

Begin Lesson

Warm-up

10 min

Which One Doesn't Belong: Expressions Galore

Standards Alignments

Addressing 4.NBT.B.5

This warm-up prompts students to carefully analyze and compare four equivalent expressions. In making comparisons, students have a reason to attend closely to the features and the value of the expressions. The activity also enables the teacher to gain insight into students' understanding of properties of operations and how they talk about them.

Instructional Routines

Which One Doesn't Belong?

Student-facing Task Statement

Which one doesn't belong?

A. 7×50
B. $(3 \times 50) + (4 \times 50)$
C. $(5 \times 10) \times 7$
D. $50 + 50 + 50 + 50 + 50 + 50 + 50$

Student Responses

- A is the only one without two or more operations.
- B is the only one that doesn't use only one operation. It has two different operations (addition and multiplication).
- C is the only one that doesn't have the number 50.
- D is the only one that doesn't show multiplication or have the number 7.

Launch

- Groups of 2
- Display the expressions.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 2–3 minutes: partner discussion
- Share and record responses.

Synthesis

- "How are the expressions alike?" (They all have a value of 350.)
- "Were there expressions that you knew right away were equivalent? Which ones? How did you know?"

Activity 1

20 min

An Algorithm for Noah

PLC Activity

Standards Alignments

Addressing 4.NBT.B.5

The purpose of this activity is to analyze an algorithm that uses partial products. Students are not required to use a specific notation, but analyzing each algorithm deepens their understanding of the structure of place value in multiplication.

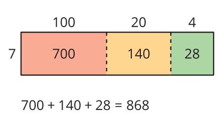
When students interpret and make sense of Noah's work, they construct viable arguments and critique the reasoning of others (MP3).

Access for English Learners

MLR8 Discussion Supports. Synthesis: Revoice student ideas to demonstrate and amplify mathematical language use. For example, revoice the student statement: “I added all the numbers together to find the answer” as “I added all the partial products to calculate the final product.” *Advances: Speaking, Representing*

Student-facing Task Statement

1. Noah drew a diagram and wrote expressions to show his thinking as he multiplied two numbers.

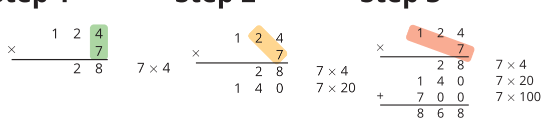
   

   7×124
   $7 \times (100 + 20 + 4)$
   $(7 \times 100) + (7 \times 20) + (7 \times 4)$
   $700 + 140 + 28$

   How does each expression represent Noah’s diagram? Be prepared to share your thinking with a partner.

2. Later, Noah learned another way to record the multiplication, as shown here.

   Step 1 **Step 2** **Step 3**

   Make sense of each step of the calculations and record your thoughts. Be prepared to

Launch

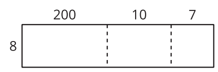
- Groups of 2
- Display the diagram in the activity.
- “What do you notice? What do you wonder?”
- 1 minute: partner discussion
- Share and record responses.

Activity

- 4 minutes: independent work time on the first problem about Noah’s diagram
- 4 minutes: partner discussion
- 5 minutes: group work time on the rest of the activity
- Monitor for students who include the place value of each digit in 124 in explaining what is happening in the algorithm.

explain Noah's steps to a partner.

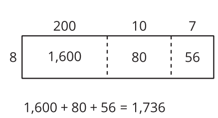
3. Complete the diagram to find the value of 217×8. Use Noah's recording method to check your work.



2 1 7	
× 8	
	8 × 7
	8 × 10
+	8 × 200

Student Responses

1. Sample response:
   - 7×124 represents the value of the whole diagram.
   - $7 \times (100 + 20 + 4)$ represents one factor (one side length of the rectangle) being decomposed by place value and being multiplied by 7.
   - $(7 \times 100) + (7 \times 20) + (7 \times 4)$ represents each decomposed part of 124 being multiplied by 7 and combined together.
   - $700 + 140 + 28$ represents the sum of the products in all the parts, which is the product of 7×124.
2. Sample response: Noah multiplies the single-digit factor by each digit of the multi-digit number at the top in order from right to left. He places the products below the line and adds them up to get the final product.
3. Completed diagram and calculation:



2 1 7	
× 8	
5 6	8 × 7
8 0	8 × 10
+ 1, 6 0 0	8 × 200
1, 7 3 6	

Synthesis

- Select students to share their interpretations of the steps in the second problem.
- Highlight these key ideas along the way:
  - Multiply the 7 by the 4 ones in 124 and write the product below the line.
  - Multiply the 7 by 2 tens in 124 and record this product below the first product.
  - Multiply the 7 by the 1 hundred and record this product below the previous product.
  - Add all the partial products to calculate the final product.

1 2 4
× 7
2 8
1 4 0
+ 7 0 0
8 6 8

- If needed, review these steps for 8×217.
- Reiterate that we typically begin by multiplying the single-digit factor by the digit in the ones place, and then repeat with then the digit in the tens place, and so on.
- "The recording strategy that Noah learned is an algorithm that uses partial products. We used an algorithm to add and subtract large numbers in our last unit."
- "How do we know if we have finished finding all the partial products?" (We keep track of all the partial products or smaller parts of the whole product as we go. We went in order from the smallest place value to the largest place value.)

Activity 2

🕒 15 min

Try an Algorithm with Partial Products

Standards Alignments

Addressing 4.NBT.B.5

In this activity, students continue to analyze an algorithm that uses partial products and learn that there are different ways to write the partial products. While students are not required to use a specific algorithm for multiplication, analyzing variations in the partial products notation deepens their understanding of how to use base-ten structure to multiply efficiently (MP7).

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Provide access to colored pencils and sticky notes. To support organization, invite students to color code each partial product as seen in the previous activity. To support working memory, invite students to use the sticky notes to record the expanded form of the multi-digit number they are multiplying by.
Supports accessibility for: Memory, Organization

Student-facing Task Statement

Noah and Mai want to find the value of $8 \times 3{,}419$. They recorded their steps in different ways, as shown.

Noah

	3,419
$\times$	8
	72
	80
	3,200
+	24,000

Mai

	3,419
$\times$	8
	24,000
	3,200
	80
+	72

1. How are Mai's and Noah's notation alike? How are they different?
2. Use a diagram to show what each of the partial products 72, 80, 3,200 and 24,000
3. represent. Then, find the value of $8 \times 3{,}419$.

Launch

- Groups of 2
- "What strategy would you use to find the value of $8 \times 3{,}419$?"
- 1 minute: partner discussion
- Share responses.
- Display Noah's and Mai's computations.
- "Take a quiet moment to make sense of Noah and Mai's work. How do you think they arrived at the last four numbers?"
- 1 minute: quiet think time
- 1 minute: partner discussion
- "Which representation is more like how you thought about it?"

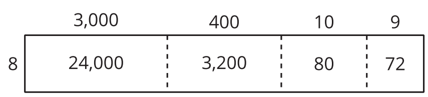
Find the value of each expression. For at least one expression, use the algorithm that Noah used. Show your reasoning.

a. $4 \times 5{,}342$

b. 7×983

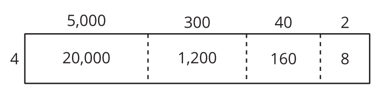
Student Responses

1. They both multiplied using an algorithm and they both used partial products, but Mai multiplied starting with the thousands and Han started with the ones.



2. 24,000 + 3,200 + 80 + 72 = 27,352

3. a. 21,368. Sample reasoning:



20,000 + 1,200 + 160 + 8 = 21,368

$$\begin{array}{r} 5{,}342 \\ \times \quad\quad 4 \\ \hline 8 \\ 160 \\ 1{,}200 \\ + \; 20{,}000 \\ \hline 21{,}368 \end{array}$$

b. 6,881

Activity

- “Work with your partner on the first two questions.”
- 5 minutes: partner work time on the first two problems
- Pause for a brief discussion.
- “How are Mai’s and Noah’s notation the same and how are they different?” (Both of these methods calculate partial products, but Noah’s goes in order from multiplying the ones, tens, hundreds, then the thousands. Mai’s goes in order the other way from the thousands, hundreds, tens, and then the ones.)
- “Are both methods correct? How do you know?” (Yes, because they both result in the same product when you calculate it.)
- “Work on the last set of problems independently before discussing with your partner.”
- 5 minutes: independent work time on the last set of problems

Synthesis

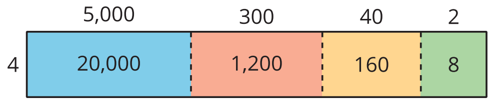
- See lesson synthesis.

Lesson Synthesis

10 min

“Today we learned different ways of recording partial products to multiply four-digit by one-digit numbers. We made connections between a diagram and using algorithms that use partial products.”

Display the expression $4 \times 5{,}342$ and corresponding diagram and computation.



20,000 + 1,200 + 160 + 8 = 21,368

$$\begin{array}{r} 5{,}342 \\ \times \quad\quad 4 \\ \hline 8 \\ 160 \\ 1{,}200 \\ +\ 20{,}000 \\ \hline 21{,}368 \end{array}$$

Invite students to take turns making connections between each part of the diagram to the algorithm.

Highlighting the following:

- Each number in the smaller rectangles in the diagram is a partial product—a result of multiplying a part of one factor by the second factor.
- When we use a diagram, we can find the partial products for different smaller rectangles and add the pieces together in any order. The product stays the same no matter how we decompose the diagram.
- When we use an algorithm that uses partial products, we multiply the ones, tens, hundreds, then thousands, and record the partial products vertically. Changing the order of multiplying doesn't change the final product.

Suggested Centers

- Number Puzzles: Multiplication and Division (4–5), Stage 1: Two-digit Factors (Addressing)
- Compare (1–5), Stage 3: Multiply within 100 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students decompose factors and multiply without considering place value, resulting in incorrect partial products.

Next Day Support

- After the warm-up, have students discuss with a partner their strategies for the cool-down and allow time to make revisions to their thinking.

Lesson 10: Using Algorithms with Partial Products: 2 Two-digit Numbers

Standards Alignments

Addressing 4.NBT.B.5

Teacher-facing Learning Goals

- Multiply 2 two-digit numbers using an algorithm that uses partial products.

Student-facing Learning Goals

- Let’s try to multiply two-digit numbers with an algorithm that uses partial products.

Lesson Purpose

The purpose of this lesson is for students to use partial products in an algorithm to multiply 2 two-digit numbers.

In the previous lesson, students learned to record the partial products from multiplying vertically, using an algorithm. They made connections between the new notation to the structure of the rectangular diagram on which they used earlier. In this lesson, students apply that work to 2 two-digit factors.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

MLR1 Stronger and Clearer Each Time (Activity 1), Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	25 min
Activity 2	10 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

How did understanding the cool-down of the lesson before you started teaching today help you synthesize that learning?

Cool-down (to be completed at the end of the lesson)

5 min

Choose Your Own Strategy

Standards Alignments

Addressing 4.NBT.B.5

Student-facing Task Statement

Find the value of 15×43. Show your reasoning.

Student Responses

645. Sample responses:

$15 \times 40 + 15 \times 3 = 600 + 45 = 645$

$$\begin{array}{r} 15 \\ \times \quad 43 \\ \hline 15 \\ 30 \\ 200 \\ + \quad 400 \\ \hline 645 \end{array}$$

Begin Lesson

Warm-up

10 min

Number Talk: Products

Standards Alignments

Addressing 4.NBT.B.5

This Number Talk encourages students to think about the strategies they can use to multiply 2 two-digit numbers. Students can decompose factors by place value to multiply by multiples of ten, or they can use a doubling and halving strategy to create an equivalent expression. The strategies elicited here will be helpful later in developing a flexible sense of numbers and using this sense to make decisions when multiplying mentally.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- 30×7
- 15×14
- 50×8
- 25×16

Student Responses

- 210: $3 \times 7 = 21$ and $21 \times 10 = 210$.
- 210:
    - I know that 15 is half of 30 and 14 is twice 7, so I took half of 210 and then multiplied it by 2, which is still 210.
    - Fourteen is $10 + 4$, so I found $(15 \times 10) + (15 \times 4)$, which is $150 + 60$ or 210.
- 400: 8 is 2×4, so 50×8 is $50 \times 2 \times 4$, which is 100×4 or 400.
- 400:
    - I know that 25 is half of 50 and 16 is twice 8, so I took half of 400 and then doubled it, which is still 400.
    - $25 \times 4 = 100$, so $25 \times 4 \times 4$ is 100×4, which is 400.

Launch

- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "How can 30×7 help you solve 15×14?" (30×7 is twice as many groups with half as much in each group, which results in the same product with easier factors to multiply mentally.)

Activity 1

25 min

Partial Products, Recorded

Standards Alignments

Addressing 4.NBT.B.5

In this activity, students analyze multiplication involving 2 two-digit factors. Students make sense of work where the partial products are recorded, but there is no indication of where each product came from. After making sense of work, students use the same strategy of finding and keeping track of partial products to evaluate 31×15.

This activity uses *MLR1 Stronger and Clearer Each Time*. Advances: reading, writing

Access for Students with Disabilities

Representation: Internalize Comprehension. Provide students with a partially completed version of the itemized steps as described in the Activity Synthesis. For example, offer students a page that shows four copies of 64×87. Each equation could have one of the partial products written in its corresponding color underneath. Invite students to use a green, yellow, red, and purple colored pencil to highlight the digits involved in each step and to write the corresponding multiplication expression next to each partial product.
Supports accessibility for: Visual Spatial Processing, Organization, Attention

Instructional Routines

MLR1 Stronger and Clearer Each Time

Student-facing Task Statement

1. Tyler used an algorithm to find the value of 64×87.

$$\begin{array}{rrrrr} & & & 6 & 4 \\ \times & & & 8 & 7 \\ \hline & & & 2 & 8 \\ & & 4 & 2 & 0 \\ & & 3 & 2 & 0 \\ + & 4, & 8 & 0 & 0 \\ \hline & 5, & 5 & 6 & 8 \end{array}$$

   How do you think he arrived at the last five numbers? Record your thinking. Be prepared to share it with a partner.

2. Use Tyler's method to find the value of 31×15. Then, draw a diagram to check your answer.

Launch

- Groups of 2
- "Find the value of 64×87 and share your reasoning with a partner."

Activity

- "Take a few minutes to make sense of Tyler's calculation. Be prepared to explain your thinking."
- 3–4 minutes: independent work time on the first problem

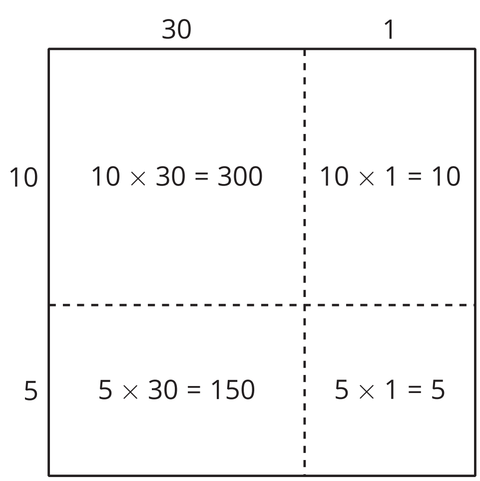
MLR1 Stronger and Clearer Each Time

- "Share your analysis of Tyler's calculation with a partner. Take turns being the speaker and the listener. If you are the speaker, share your ideas and writing so far. If you are the listener, ask questions and give feedback to help your partner

Student Responses

1. Sample response:
    - 28 is the product of 7 in 87 and 4 in 64.
    - 420 is the product of 7 in 87 and 60 in 64.
    - 320 is the product of 80 in 87 and 4 in 64.
    - 4,800 is the product of 80 in 87 and 60 in 64.
    - 5,568 is the sum of the four partial products above it.
2. Sample response:

$$\begin{array}{r} 3\ 1 \\ \times\ 1\ 5 \\ \hline 5 \\ 1\ 5\ 0 \\ 1\ 0 \\ +\ 3\ 0\ 0 \\ \hline 4\ 6\ 5 \end{array}$$



improve their work."

- 2 minutes: structured partner discussion
- Consider displaying these prompts to support students' conversations:
    - "Can you explain how multiplying _____ and _____ gives _____?"
    - "Can you use the phrase 'partial products' in your explanation?"
- Repeat with 1–2 different partners.
- "Revise your initial draft based on the feedback you got from your partners."
- 2–3 minutes: independent work time
- "Now try using Tyler's method to complete the last problem and use a diagram to check your work."
- 5 minutes: independent work time on the last problem

Synthesis

- To clarify the steps in Tyler's notation, consider itemizing each step and using color coding, as shown here:

$$\begin{array}{r} 6\ 4 \\ \times\ 8\ 7 \\ \hline 2\ 8 \end{array} \quad 7 \times 4$$

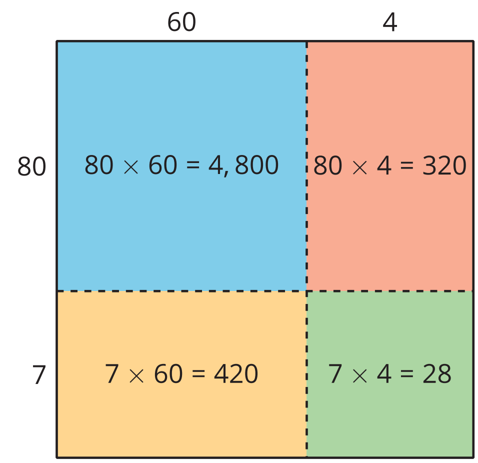
$$\begin{array}{r} 6\ 4 \\ \times\ 8\ 7 \\ \hline 4\ 2\ 0 \end{array} \quad 7 \times 60$$

$$\begin{array}{r} 6\ 4 \\ \times\ 8\ 7 \\ \hline 3\ 2\ 0 \end{array} \quad 80 \times 4$$

$$\begin{array}{r} 6\ 4 \\ \times\ 8\ 7 \\ \hline 4,8\ 0\ 0 \end{array} \quad 80 \times 60$$

			6	4	
×			8	7	
			2	8	7 × 4
		4	2	0	7 × 60
		3	2	0	80 × 4
+	4,	8	0	0	80 × 60
	5,	5	6	8	

- Also consider displaying a corresponding diagram for 64×87 and inviting students to make connections to an algorithm.



Activity 2

10 min

Han's Multiplication Mishap

Standards Alignments

Addressing 4.NBT.B.5

When using an algorithm that uses partial products, students may be inclined to pay attention only to single digits in each number and pay little attention to the value of the digits. For example, 32×19 might sound like "9 times 2 is 18 and 9 times 3 is 27." In this activity, students analyze this error (MP3) and also look at the commutativity of multiplication when finding partial products.

To help students see that the place value of the digits impacts each partial product, consider displaying a diagram that shows partial products alongside the vertical notation.

Access for English Learners

MLR8 Discussion Supports. Synthesis: Display sentence frames to support whole-class discussion: "_____ and _____ are the same/alike because . . ." and "Why did you . . .?"
Advances: Speaking, Conversing

Student-facing Task Statement

1. Decide with your partner who will find each product. Show your reasoning.

$$\begin{array}{rrr} & 1 & 9 \\ \times & 3 & 2 \\ \hline \end{array} \qquad \begin{array}{rrr} & 3 & 2 \\ \times & 1 & 9 \\ \hline \end{array}$$

2. Here is Han's computation of 51×47.

$$\begin{array}{rrrrl} & & 5 & 1 & \\ \times & & 4 & 7 & \\ \hline & & & 7 & 7 \times 1 \\ & & 3 & 5 & 7 \times 5 \\ & & 4 & 0 & 40 \times 1 \\ + & 2 & 0 & 0 & 40 \times 5 \\ \hline & 2 & 8 & 2 & \end{array}$$

   a. What error or errors did Han make?

   b. Show the correct computation for finding the value of 51×47.

$$\begin{array}{rrr} & 5 & 1 \\ \times & 4 & 7 \\ \hline \end{array}$$

Student Responses

1.

$$\begin{array}{rrrr} & & 1 & 9 \\ \times & & 3 & 2 \\ \hline & & 1 & 8 \\ & & 2 & 0 \\ & 2 & 7 & 0 \\ + & 3 & 0 & 0 \\ \hline & 6 & 0 & 8 \end{array} \qquad \begin{array}{rrrr} & & 3 & 2 \\ \times & & 1 & 9 \\ \hline & & 1 & 8 \\ & 2 & 7 & 0 \\ & & 2 & 0 \\ + & 3 & 0 & 0 \\ \hline & 6 & 0 & 8 \end{array}$$

Launch

- Groups of 2

Activity

- "Work with your partner on the first problem. Each partner should find the value of one product."
- 3–4 minutes: partner work time on the first problem
- Pause for a discussion. Select a group whose calculations yield the same product to display their work.
- "Should the calculations show the same result? Why or why not?" (Yes, because the same two numbers are being multiplied.)
- "How are the two calculations different?" (The partial products are written in different orders.)
- "Does it matter which number is listed first and which is listed second?" (No)
- "Complete the last problem independently."
- 3–4 minutes: independent work time

Synthesis

- See lesson synthesis.

2. a. Han found 7×5 instead of 7×50. He also found 40×5 instead of 40×50. The second and fourth partial products should be 350 and 2,000, not 35 and 200.

   b.

$$
\begin{array}{r}
51 \\
\times\ 47 \\
\hline
7 \\
350 \\
40 \\
+\ 2{,}000 \\
\hline
2{,}397
\end{array}
$$

Lesson Synthesis

10 min

"Today we used partial products in an algorithm to multiply pairs of two-digit numbers. We wrote expressions and equations to help us find and keep track of the partial products as we were multiplying."

Select 1–2 examples of student work for 31×15 for the focus of class discussion.

"Let's look at the strategies used by one student from our class to solve 31×15."

"Before we analyze this sample closely, can you tell whether his answer is correct or seems reasonable? Why or why not?"

Select students to explain each step in the computation and any questions the sample raises.

Display a different calculation for 31×15. Consider using a diagram to support students in making sense of the products of the decomposed factors in the vertical calculation.

Reiterate the importance of paying attention to the place value of each digit being multiplied as we find and record the partial products.

Suggested Centers

- Five in a Row: Multiplication (3–5), Stage 3: Two-digit Factors (Addressing)
- Compare (1–5), Stage 3: Multiply within 100 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students multiply the digits without accounting for their place value (for example, treating 4 and 1 as ones, rather than 4 tens and 1 ten), resulting in inaccurate partial products.

Next Day Support

- Before the warm-up, have students discuss in small groups the partial products for this problem and this specific error.

Lesson 11: Partial Products and the Standard Algorithm

Standards Alignments

Addressing 4.NBT.B.5

Teacher-facing Learning Goals

- Identify similarities and differences between algorithms that use partial-products and the standard algorithm for multiplication.
- Make sense of the standard algorithm for multiplication.

Student-facing Learning Goals

- Let's compare multiplication algorithms.

Lesson Purpose

The purpose of this lesson is for students to analyze the standard algorithm for multiplication and compare it to an algorithm that uses partial products they saw in earlier lessons.

In previous lessons, students analyzed and used an algorithm that uses partial products to multiply multi-digit whole numbers. They learned that an algorithm can represent the base-ten diagrams and rectangular diagrams, but it is more efficient for keeping track of and recording partial products.

This lesson extends students' analysis to include the standard algorithm for multiplication of multi-digit numbers. In grade 4, the standards focus on understanding place value and how it is represented in different methods for finding products. The work here serves to build the groundwork for making sense of the standard algorithm in grade 5, so students are not expected to use the standard algorithm at this time.

Access for:

Students with Disabilities

- Engagement (Activity 2)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

In grade 5, students will use the traditional algorithm. How does the way they analyzed two different algorithms in Activity 2 build toward this work?

Cool-down (to be completed at the end of the lesson)

5 min

Choose a Way to Multiply

Standards Alignments

Addressing 4.NBT.B.5

Student-facing Task Statement

Find the value of each product. Show your reasoning.

1. 4×798
2. $8 \times 2{,}864$

Student Responses

1. 3,192. Sample reasoning:
    - $(4 \times 700) + (4 \times 90) + (4 \times 8) = 2{,}800 + 360 + 32 = 3{,}192$
    - I know that 798 is 2 less than 800. So 4 groups of 798 is 4×2 less than 4×800 or 8 less than 3,200, which is 3,192.
2. 22,912. Sample reasoning:
    - $(8 \times 2{,}000) + (8 \times 800) + (8 \times 60) + (8 \times 4) = 16{,}000 + 6{,}400 + 480 + 32 = 22{,}912$

Begin Lesson

Warm-up

10 min

Number Talk: The Value of the Digits

Standards Alignments

Addressing 4.NBT.B.5

This Number Talk routine encourages students to think about decomposing factors by place value to multiply multi-digit numbers by one-digit numbers. As students look for and make use of structure, they may notice that multiplying the ones place has a result in the pattern of 5, 10, and 15. Students may use the partial products equations to multiply.

This routine helps students pay attention to the value of the digits when multiplying. This is important for setting up the conversation about the standard algorithm, in which students will find partial products mentally and use what they know about the value of the digits to condense the number of steps to multiply by multi-digit numbers.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- 5×101
- 5×102
- 5×203
- 5×404

Student Responses

- 505: $5 \times 100 = 500$ and $5 \times 1 = 5$, and $500 + 5 = 505$.
- 510: 5×102 is 5 more than 5×101, so it is 5 more than 505, which is 510.
- 1,015: 5×200 is 1,000 and 5×3 is 15, so 5×203 is 1,015.
- 2,020: 404 is 4 times 101, 5×404 is 4 times 5×101 or 4×505, which is 2,020.

Launch

- Display one expression.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expression and work displayed.
- Repeat with each expression.

Synthesis

- “What connections or relationships do you see between each expression?” (Each problem involves one more group of 5 or sets or 100 more.)

Activity 1

20 min

Two Algorithms to Multiply

Standards Alignments

Addressing 4.NBT.B.5

This activity introduces students to the standard algorithm for multiplication. Students make sense of it by comparing and contrasting it to an algorithm that uses partial products for multiplying three- and four-digit numbers by one-digit numbers where no regrouping is necessary. When they interpret the given student work showing the standard algorithm students construct a viable argument for what Kiran did in his calculation (MP3). They also have an opportunity to make use of the structure they notice to compute the value of other products.

Access for English Learners

MLR8 Discussion Supports. Synthesis: Create a visual display of Diego's algorithm. As students share their strategies, annotate the display to illustrate connections. For example, next to each step, write the multiplication expression used to find each partial product.
Advances: Speaking, Representing

Student-facing Task Statement

1. Here are two algorithms for finding the value of 3×713.

Kiran

$$\begin{array}{rrrrr} & & 7 & 1 & 3 \\ \times & & & & 3 \\ \hline & 2, & 1 & 3 & 9 \end{array}$$

Diego

$$\begin{array}{rrrrr} & & 7 & 1 & 3 \\ \times & & & & 3 \\ \hline & & & & 9 \\ & & & 3 & 0 \\ + & 2, & 1 & 0 & 0 \\ \hline & 2, & 1 & 3 & 9 \end{array}$$

Discuss with your partner:

   a. How are Kiran's algorithm and Diego's algorithm alike? How are they different?

Launch

- Groups of 2

Activity

- "Make sense of Kiran and Diego's algorithms. Talk to your partner about how they are alike and different, and how Kiran might have found his answer."
- 3 minutes: partner discussion
- Pause for a discussion.
- Invite students to share their conjectures on how Kiran might have reasoned about the product.
- Alternatively, consider displaying a few scenarios and polling students on which

b. How do you think Kiran found the product 2,139?

2. Find the value of each product.

   a. 212×4

   b. $3 \times 4{,}132$

Student Responses

1. Sample response:
   a. They both have the same result. Kiran's way doesn't show his steps, but you can see the partial products in Diego's way.
   b. Kiran was doing the same process as Diego in terms of finding partial products, but he added the products in his head after finding each partial product.
2. a. 848
   b. 12,396

one might be closest to their conjecture, if any. For example:

- A: Kiran found $(3 \times 700) + (3 \times 13)$ mentally and wrote down the result of $2{,}100 + 39$.
- B: Kiran used the same method as Diego but added the 9, 30, and 2,100 mentally, without writing them down.
- C: Kiran drew a diagram and did the computation on another sheet of paper and wrote the result here.
- D: Kiran multiplied the single-digit 3 with each digit in 713 and wrote each partial product in a single line.

- Explain that Kiran had multiplied 3 by each digit in 713, but instead of reasoning about 3×3, 3×10, and 3×700, Kiran reasoned about 3×3, 3×1, and 3×7, while paying attention to the place value of each digit.
- Demonstrate Kiran's process:
  - Because the 3 in 713 means 3 ones, he wrote the result of 3×3 or 9 in the ones place.
  - Because the 1 means 1 ten, he wrote the result of 3×1 in the tens place.
  - Because the 7 means 7 hundreds, he wrote the result of 3×7 in the hundreds place.
- "Try using Kiran's algorithm to find the value of the last two products."
- 3 minutes: independent work time
- 2 minutes: partner discussion

Synthesis

- Select students to share and explain their calculations of the last two products.
- Consider demonstrating the process of Kiran's strategy to find $3 \times 4{,}132$.

- "Kiran's strategy is called the 'standard algorithm for multiplication.' We'll take a closer look at this algorithm in the next activity."

Activity 2

15 min

Algorithm Comparison

Standards Alignments

Addressing 4.NBT.B.5

The purpose of this activity is for students to compare the standard algorithm for multiplication and an algorithm that uses partial products. The focus of the synthesis is on the convention used for composing a new unit and how it connects to their work with the standard algorithm for addition.

Access for Students with Disabilities

Engagement: Internalize Self-Regulation. Synthesis: Provide students an opportunity to self-assess and reflect on their own progress. Remind students what kinds of multiplication problems they were working on before this unit, and ask them to notice how the multiplication they are working on now is different. Invite them to list the new strategies they have learned in this section.
Supports accessibility for: Conceptual Processing, Social-Emotional Functioning

Student-facing Task Statement

1. Analyze the two algorithms used to find the value of 4×223.

Kiran **Diego**

$$\begin{array}{rccc} & & 1 & \\ & 2 & 2 & 3 \\ \times & & & 4 \\ \hline & 8 & 9 & 2 \end{array}$$

Launch

- Groups of 2

Activity

- "Work with your partner on the first problem."
- 3–4 minutes: partner work time
- Pause for brief discussion. Invite students to share how the two methods are alike

$$\begin{array}{r} 2\ 2\ 3 \\ \times \quad\quad 4 \\ \hline 1\ 2 \\ 8\ 0 \\ +\ 8\ 0\ 0 \\ \hline 8\ 9\ 2 \end{array}$$

a. How are Kiran and Diego's algorithms alike? How are they different?

b. Where is the 12 in Kiran's algorithm?

2. a. Try using Kiran's algorithm to find the value of 512×3.

   b. Check your work using a different method.

Student Responses

1. Sample response:
   a. Alike: They both multiply the same numbers and have the same answer. Different: One shows all the place values and the other has a small one on top.
   b. The 12 is decomposed into $10 + 2$. The 2 is in the ones place and the 10 is a 1 added to the tens place.
2. a. 1,536
   b. Students check using an algorithm showing partial products, a diagram, or another method.

and how they are different.

- "In Kiran's algorithm, why is a 1 written in above the 2 in the tens column?" (It represents 1 ten from the number 12. It helps us remember to add it to the tens place because we are doing a lot of calculations in our heads.)
- "Where have we seen this notation before?" (When we add using the standard algorithm, we use this notation to show that we have more than 9 ones, tens or hundreds, and so on, in a given place, we add each group of 10 units to the place value to the left.)
- "Kiran's used the standard algorithm for multiplication. Try using it to find 512×3."
- 3 minutes: independent work time
- 1–2 minutes: partner discussion

Synthesis

- See lesson synthesis.

Lesson Synthesis

10 min

"Today we compared the standard algorithm for multiplication to an algorithm that uses partial-products. Let's see how we'd find 3×512."

Display:

$$\begin{array}{r} 5\ 1\ 2 \\ \times \quad\quad 3 \\ \hline \end{array}$$

"How would you find the value of the product?" (Sample responses:

- I know that 500 x 3 is 1500 and 12 x 3 is 36 and 1500 + 36 = 1,536.
- Three times 2 is 6, so that goes in the ones place. 3 times 1 is 3, so that goes in the tens place. 3 times 5 is 15, so that goes in the thousands and hundreds place.)

Assure students that they are not expected to use a particular method for multi-digit multiplication in grade 4. Explain that they will study this algorithm more in grade 5. Invite them to try to use it to multiply as we continue to work through lessons.

Suggested Centers

- Five in a Row: Multiplication (3–5), Stage 3: Two-digit Factors (Addressing)
- Compare (1–5), Stage 3: Multiply within 100 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students may decompose the multi-digit factors by place value and correctly find the partial products but make computation errors when adding them.

Next Day Support

- Before the warm-up, invite students to work in small groups to discuss a correct response to this cool-down.

Lesson 12: Solve Problems Involving Multiplication

Standards Alignments

Addressing	4.MD.A.2, 4.NBT.B.5, 4.OA.A.3
Building Towards	4.OA.A.3

Teacher-facing Learning Goals

- Multiply multi-digit numbers using strategies based on place value and the properties of operations.

Student-facing Learning Goals

- Let's solve problems using what we learned about multiplication of whole numbers.

Lesson Purpose

The purpose of this lesson is for students to solve contextual problems that involve multiplication of a single-digit number and a whole number of up to four digits, or multiplication of 2 two-digit numbers.

This lesson gives students the opportunity to apply the multiplication strategies they have learned to solve various contextual problems involving measurement. The problems vary in format and complexity—some involve a single computation and others require multiple steps to solve. The work here prompts students to make sense of problems and persevere in solving them (MP1) and to reason quantitatively and abstractly (MP2).

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Action and Expression (Activity 1)

Instructional Routines

MLR7 Compare and Connect (Activity 1), What Do You Know About _____? (Warm-up)

Materials to Gather

- Tools for creating a visual display: Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	25 min
Activity 2	10 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

As students shared their ideas today, how did you ensure all students' voices were heard and valued as an important part of the collective learning?

Cool-down (to be completed at the end of the lesson)

5 min

Leap Year

Standards Alignments

Addressing 4.MD.A.2, 4.NBT.B.5, 4.OA.A.3

Student-facing Task Statement

In a leap year, the month of February has 29 days. How many hours are in that month? Show your reasoning.

Student Responses

696 hours. Sample response:

$$\begin{array}{r} 29 \\ \times\ 24 \\ \hline 36 \\ 80 \\ 180 \\ +\ 400 \\ \hline 696 \end{array}$$

Begin Lesson

Warm-up

10 min

What Do You Know About 1 Year?

Standards Alignments

Building Towards 4.OA.A.3

The purpose of this warm-up is to elicit students’ knowledge about 1 year as a measure of time and the ways it can be represented. The reasoning and conversations here will be helpful as students solve problems that involve time in years later in the lesson.

Instructional Routines

What Do You Know About _____?

Student-facing Task Statement

What do you know about 1 year?

Student Responses

Sample responses:

- It is 365 days long.
- It is 12 months long.
- Sometimes 1 year has an extra day.
- It is the time it takes the earth to make a rotation around the sun.

Launch

- Display: “1 year”
- “What do you know about 1 year?”
- 1 minute: quiet think time

Activity

- Record responses.

Synthesis

- “What does a year measure?” (time)
- “What are some aspects of time that are related to a year?” (seasons, months, weeks, days)

Activity 1

25 min

Time Flies When We Leap Years

Standards Alignments

Addressing 4.NBT.B.5, 4.OA.A.3

In this activity, students use what they learned about multiplication of multi-digit numbers and unit conversion to solve problems involving measurements. Students may choose to represent the situations in a number of ways—concretely or visually (by drawing diagrams) or abstractly (by writing expressions and equations). While some problems can be reasoned additively, students may opt to reason multiplicatively for practical reasons.

Regardless of their chosen representations and reasoning strategy, students reason quantitatively and abstractly when they interpret and solve the questions about different units of time (MP2).

This activity uses *MLR7 Compare and Connect*. Advances: representing, conversing

Access for English Learners

Action and Expression: Develop Expression and Communication. Provide access to a variety of tools, including colored pencils, grid paper, base-ten blocks, and a visual display that students can use as a reference showing a variety of strategies from throughout the section.
Supports accessibility for: Conceptual Processing, Organization, Memory

Instructional Routines

MLR7 Compare and Connect

Materials to Gather

Tools for creating a visual display

Student-facing Task Statement

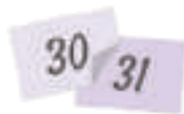
1. A baby elephant was born exactly 48 weeks ago. How many days old is she?
2. A leap year has 366 days. A non-leap year (or a common year) has 365 days. How many days are in 3 leap years?
3. In our calendar system, some months are 31 days long, some are 30 days long, and one month (February) is either 28 or 29 days long.

   What if the calendar system changed so that

Launch

- Groups of 2–4
- Give each group tools for creating a visual display.
- “Today we’ll solve some problems that involve measurements.”
- “Before we solve any problem, let’s do some estimation.”
- Read the first problem as a class.
- “Estimate: About how many days old is the baby elephant?”

each month has 31 days? How many more days would there be in a year?



Student Responses

1. $48 \times 7 = 336$
2. $366 \times 3 = 1{,}098$
3. 7 days more than a common year (or 6 days more than a leap year). Sample reasoning:
    - $12 \times 31 = 372$ and 372 is 7 more than 365 and 6 more than 366.
    - In a common year, there are four months that are 30 days long and one that is 28 days long. To make these five months 31 days each means adding $(4 \times 1) + 3$ or 7 days. In a leap year, it'd mean adding $(4 \times 1) + 2$ or 6 days.

- 1 minute: quiet think time
- Display the following ranges and poll the class on their estimate. (Alternatively, post each range in a different part of the classroom and ask students to stand by the range that reflects their estimate.)
    - 280 to 299
    - 300 to 349
    - 350 to 400
- Select a student who selected each range to explain their estimate. Then, ask students if they'd revise their estimate.

Activity

- "Take a few quiet minutes to solve the first two problems. Then, discuss your responses with your group and work on the last problem together."
- 5 minutes: independent work time
- 5 minutes: group work time
- Monitor for the different representations and strategies used to solve the problems.
- Assign one problem to each group.

MLR7 Compare and Connect

- "Create a visual display to show how you solved the problem assigned to your group. Organize your work so that it can be followed by others."
- 3 minutes: group work time
- 5 minutes: gallery walk
- Prompt students to make note of the different strategies they see and any solutions that they question.

Synthesis

- "Most of the problems can be solved by multiplication. What is the same between the solutions and multiplication strategies that you saw? What's different?"

- 30 seconds quiet think time
- 1 minute: partner discussion
- Record students' responses, or display students' diagrams and representations.
- "What are some common strategies for multiplying a multi-digit number by a one-digit number (48 and 7, or 366 and 3)?"
- "What are some strategies you saw for multiplying 2 two-digit numbers (12 and 31)?"

Activity 2

10 min

Coin Collection

Standards Alignments

Addressing 4.NBT.B.5, 4.OA.A.3

This activity offers students more practice with using multiplication to solve contextual problems (MP2), including situations in which at least one factor is four digits long, and to generate a new problem according to some parameters.

Student-facing Task Statement

1. Lin's family has collected 2,074 nickels over the years. How many pennies are worth the same amount?
2. If Lin's family saved 2,074 nickels each year for 4 years, how many nickels would her family have?
3. Create a situation that involves a problem that can be solved by finding the value of $8 \times 1{,}049$. Solve the problem and show your

Launch

- Groups of 2–4

Activity

- 6–7 minutes: independent work time
- 2–3 minutes: group discussion
- Monitor for the different ways students represent the situations and solve the problems.

reasoning.

Student Responses

1. $2{,}074 \times 5 = 10{,}370$
2. $2{,}074 \times 4 = 8{,}296$ (1 fewer group of 2,074 than in 10,370, or $10{,}370 - 2{,}074$)
3. Sample response: The local library had 1,049 books checked out each day for 8 days. How many books were checked out in total? $8 \times 1{,}000 + 8 \times 40 + 4 \times 9 = 8{,}392$

Synthesis

- See lesson synthesis.

Lesson Synthesis

10 min

"Today we used what we have learned about multiplication to solve problems involving measurement."

Select students to share their responses to the problems in the last activity. As each student shares, ask if others in the class solved it the same way and if they approached it differently.

Prompt students to explain what their numerical solutions represent in each situation.

"How would you know if your solutions were correct?" (Sample responses: I used another strategy to see if I got the same answer. I estimated first so that I had an idea how big or small the answer would be. I checked with my groupmates.)

Consider asking: "When you had to multiply numbers, which method did you rely on the most? What made you choose that method?"

Suggested Centers

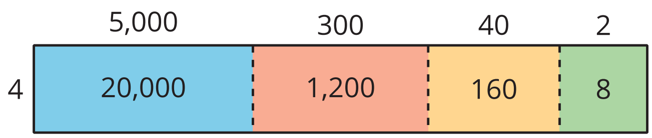
- Five in a Row: Multiplication (3–5), Stage 3: Two-digit Factors (Addressing)
- Compare (1–5), Stage 3: Multiply within 100 (Supporting)

Student Section Summary

In this section, we learned to multiply factors whose products are greater than 100, using different representations and strategies to do so.

When working with multi-digit factors, it helps to decompose them by place value before multiplying.

For example, to find the value of $4 \times 5{,}342$, we can decompose the 5,342 into its expanded form, $5{,}000 + 300 + 40 + 2$, and then use a diagram or an algorithm to help us multiply.



20,000 + 1,200 + 160 + 8 = 21,368

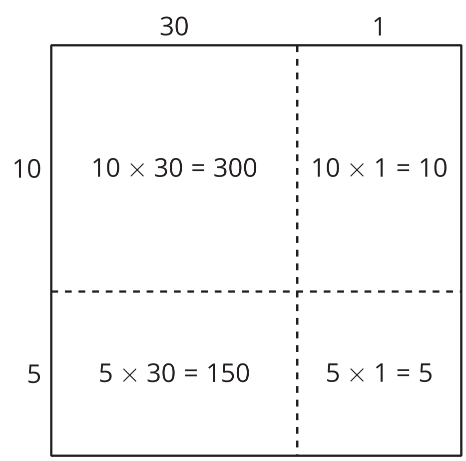
$$\begin{array}{r} 5{,}342 \\ \times \quad\quad 4 \\ \hline 8 \\ 160 \\ 1{,}200 \\ + \; 20{,}000 \\ \hline 21{,}368 \end{array}$$

In both the diagram and the algorithm, the 20,000, 1,200, 160, and 8 are called the partial products. They are the result of multiplying each decomposed part of 5,342 by 4.

We can do the same to multiply a two-digit number by another two-digit number.

For example, here are two ways to find the value of 31×15. The 31 is decomposed into $30 + 1$ and 15 is decomposed into $10 + 5$.

$$\begin{array}{r} 31 \\ \times \quad 15 \\ \hline 5 \\ 150 \\ 10 \\ + \; 300 \\ \hline 465 \end{array}$$



Complete Cool-Down

Response to Student Thinking

Students who use an algorithm that uses partial-products may neglect to consider the place value of the digits they are multiplying and end up with some incorrect partial products.

Next Day Support

- Before the warm-up, select a student's cool-down from the previous lesson (name anonymous). Ask students to identify what the student did well and what the student needs to do to improve the cool-down.

Section C: Multi-digit Division

Lesson 13: Situations Involving Equal-size Groups

Standards Alignments

Building On	3.OA.A.2, 3.OA.A.3
Addressing	4.NBT.B.6

Teacher-facing Learning Goals

- Reason about division of two- and three-digit numbers in situations involving equal-size groups.

Student-facing Learning Goals

- Let's interpret and solve division problems.

Lesson Purpose

The purpose of this lesson is for students to solve division problems in context, and to recall the two meanings of division: "how many in each group" and "how many groups."

In grade 3, students learned about the relationship between multiplication and division and reasoned about division in terms of equal groups. They also saw two interpretations of division: as a way to find the size of a group and as a way to find the number of groups. They used these understandings to find quotients and to reason about division problems in context, finding whole-number quotients from two-digit dividends and one-digit divisors.

In this lesson, students build from the skills and ideas learned in grade 3 as they recall strategies for reasoning about division problems in context (MP2). Students encounter situations that involve equal-size groups and that call for finding the size of a group and the number of groups. The work here prepares students to rely on the relationship between division and multiplication to solve problems involving three-digit dividends in the next lesson.

Access for:

Students with Disabilities

- Action and Expression (Activity 1)

English Learners

- MLR2 (Activity 2)

Instructional Routines

Estimation Exploration (Warm-up), MLR5 Co-craft Questions (Activity 1)

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

How readily did students see equal-size situations in terms of division (and see division expressions in terms of equal-size groups)? If they struggled to make a connection, what might be challenging?

Cool-down (to be completed at the end of the lesson)

5 min

After the Class Party

Standards Alignments

Addressing 4.NBT.B.6

Student-facing Task Statement

After the class party, 6 students offer to wash 96 pieces of utensils (spoons and forks). Each student is washing the same number of utensils.

How many pieces of utensils does each student wash? Explain or show your reasoning.

Student Responses

Each student washes 16 pieces. Sample responses:

- $6 \times 10 = 60$ and $6 \times 6 = 36$ so $6 \times 16 = 96$.
- $60 \div 6 = 10$, $36 \div 6 = 6$, and $10 + 6 = 16$.

Begin Lesson

Warm-up

10 min

Estimation Exploration: Lots of Paletas

Standards Alignments

Building On 3.OA.A.3

This Estimation Exploration prompts students to practice making a reasonable estimate based on experience and known information. In this case, it is not practical to count the paletas, but students could reason about groups of paletas by color, or estimate the complete rows and columns of paletas and extend their estimate to the whole set. Some students might also make an estimate based on their familiarity with how paletas are usually arranged in cases.

Instructional Routines

Estimation Exploration

Student-facing Task Statement

How many paletas are in the case?

Record an estimate that is:

too low	about right	too high

Student Responses

- Too low: less than 30
- About right: 31–80
- Too high: more than 80

Launch

- Groups of 2
- Display the image.
- "These are ice pops called paletas. They originated in Mexico and are typically made with many different fruits."
- Ask students to estimate without counting.
- "What is an estimate that's too high?" "Too low?" "About right?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Record responses.

Synthesis

- "Is anyone's estimate less than 30? Greater than 80?"
- "How did you know that 30 (or another number) would be too low and 80 (or another number) too high?"
- "Based on this discussion does anyone want to revise their estimate?"

Activity 1

15 min

Paletas for a Class Party

Standards Alignments

Building On 3.OA.A.2
Addressing 4.NBT.B.6

In this activity, students recall what they know about division from grade 3. The context allows students to connect lived experiences to the math of the activity. By inviting students to consider treats that they enjoy in their homes or neighborhoods, they share experiences and foster connections that build community.

The first question gives students an opportunity to co-craft mathematical questions based on a situation before answering a question based on a division equation. Students divide a two-digit number by a one-digit divisor, as they did in grade 3, in a way that makes sense to them. The activity synthesis highlights different representations students made and relates them to the situation. The term **dividend** is re-introduced in this lesson to describe a number being divided into equal groups.

This activity uses *MLR5 Co-craft Questions*. Advances: writing, reading, representing

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Give students access to concrete manipulatives (connecting cubes, counters, or square tiles) and grid paper. Invite students to use these to act out or draw $84 \div 7$.
Supports accessibility for: Conceptual Processing, Attention

Instructional Routines

MLR5 Co-craft Questions

Student-facing Task Statement

Diego's aunt is buying paletas, which are ice pops, for a class party. At the local market, paletas come in different flavors. She buys the same number of paletas of each flavor.

Launch

- Groups of 2
- "What are some of your favorite treats or snacks from home?"
- 30 seconds: quiet think time

1. What mathematical questions can we ask about this situation?
2. Here is an equation:

$$84 \div 7 = ?$$

   In the situation about the class party, what questions could the equation represent?

3. Find the answer to one of the questions you wrote. Show your reasoning.

Student Responses

1. Sample responses:
    - How many people are coming to the party?
    - How many flavors of paletas are there?
    - How many paletas of each flavor did Diego's aunt buy?
    - How many flavors of paletas did she buy?
    - How much money does she have to spend on paletas?
2. Sample responses:
    - Diego's aunt is buying 84 paletas in 7 flavors. How many paletas of each flavor is she buying?
    - Diego's aunt is buying 84 paletas. There are 7 paletas of each flavor. How many flavors are there?
3. $84 \div 7 = 12$. Sample reasoning:
    - I know that 7 groups of 10 is 70 and 7 groups of 2 is 14, so 7 groups of 12 is 84.
    - $10 \times 7 = 70$ and $2 \times 7 = 14$, so

- 1 minute: partner discussion

MLR 5: Co-craft Questions

- Display the opening paragraph and the first question.
- "Write a list of mathematical questions that could be asked about this situation."
- 2 minutes: independent work time
- 2–3 minutes: partner discussion
- Invite several students to share one question with the class. Record responses.
- "How are these questions alike?" (The questions involve multiplying or dividing.) "How are they different?" (The questions can be answered using different operations.)
- "Let's look at the next question in the activity."

Activity

- 3–4 minutes: quiet work time

Synthesis

- Display two questions that students wrote for the equation $84 \div 7 = ?$.
- "What does the 84 represent in both problems?" (The amount being divided into equal groups.)
- "In mathematics, the number being divided is known as the **dividend**."
- Invite students to share their strategies for the last question. Highlight strategies that show equal-size groups and reasoning that relates multiplication and division.

$12 \times 7 = 84$.

- $70 \div 7 = 10$ and $14 \div 7 = 2$, so $84 \div 7 = 12$.

Advancing Student Thinking

If students are unsure how to write a division question for $84 \div 7$, consider asking:

- "If you were to act out the meaning of this expression, what would you do?"
- "If you were to explain its meaning, what would you say?"
- "In what type of situations would these types of actions take place?"

Activity 2

20 min

More Snacks for a Class Party

Standards Alignments

Building On	3.OA.A.3
Addressing	4.NBT.B.6

In this activity, students continue to use any strategy to solve division problems in context and to recall the two interpretations of division. One problem involves finding how many in each group and the second involves finding the number of groups. Students work with two- and three-digit dividends and encounter division that results in a number with a remainder. They consider what the leftover means in the given context.

During the synthesis, highlight that both multiplication and division can be used to reason about the solutions, and elicit equations can be written to represent students' reasoning about equal-size groups. Introduce **remainders** as "leftovers" or quantities that remain after dividing a number into equal groups.

Student-facing Task Statement

1. Priya's mom made 85 gulab jamuns for the class to share. Priya gave 5 to each student in the class.

   How many students are in Priya's class? Explain or show your reasoning.

2. Han's uncle sent in 110 chocolate-covered breadsticks for a snack. The students in Han's class are seated at 6 tables. Han plans to give the same number of breadsticks to each table.

   How many breadsticks does each table get? Explain or show your reasoning.

Student Responses

1. 17 students. Sample response: I know that 50 is 10 groups of 5, and then there are 35 more gulab jamuns to give out. I know that 35 is 7 groups of 5. So Priya gave out gulab jamuns to $10 + 7$ kids.
2. 18 breadsticks with 2 breadsticks leftover. $108 = 60 + 48$. Sixty is 6×10, and 48 is 6×8. Adding 10 and 8 gives 18.

Launch

- Groups of 2
- Display images of snacks in the activity.
- "Take a look at the images. What do you notice? What do you wonder?"
- Share responses.
- "Gulab jamuns are sweet treats that are popular in India, Pakistan, and their neighboring countries in South Asia."
- "Breadsticks that are covered with chocolate, strawberry cream, or other flavors are popular snacks in Japan, Taiwan, and other East Asian countries."
- Ask a student to read the first problem aloud.
- "The problems in this activity involve treats that students enjoy from different places around the world."

Activity

- 6–8 minutes: independent work time
- 2–3 minutes: partner discussion
- Monitor for students who reason in terms of partial products or partial quotients and record their thinking as equations.
- Monitor for the different ways students describe and decide what to do with the 2 leftover breadsticks.

Synthesis

- Invite students to share their response for the first problem.
- If no students wrote equations to show their thinking, ask: "What equations can represent how you found the solution to the first problem?"
- Record both multiplication and division equations for all to see. For example:
  - $5 \times 10 = 50$ and $5 \times 7 = 35$, so

$5 \times 17 = 85$.
    - $50 \div 5 = 10$ and $35 \div 5 = 7$, so $85 \div 5 = 17$.
- Invite other students to discuss the second problem.
- “When we divide, sometimes we have leftovers that are not enough to make a new group or not enough to put an additional item to each group. We call the leftovers **remainders**.”
- “What was the remainder when 110 breadsticks were divided by 6?” (2 breadsticks were leftover)
- “How are the questions in the two situations—gulab jamuns and breadsticks—alike?” (They involved division into equal groups.)
- “How are they different?” (The first looks for the number of groups. The second looks for the size of a group. The second involves some leftovers.)

Lesson Synthesis

10 min

“Today we solved problems involving division of whole numbers. We thought about the kinds of division problems we were solving—whether we were trying to find the number of groups or the amount in each group.”

Display:

Eight students are sharing 96 breadsticks equally.
How many breadsticks would each student get?

“How is this situation related to division?” (It's about putting 96 breadsticks into equal-size groups.)

“What are we finding out when we divide 96 by 8?” (How many breadsticks each student would get.)

“What are some ways to find the answer?” (Multiply 8 by a number until we get 96. Divide smaller numbers by 8, until 96 are divided by 8.)

“What equations could we write to represent the problem and the solution, or how we find the solution?” ($8 \times 12 = 96$ or $96 \div 8 = 12$, or a series of equations such as $8 \times 10 = 80$ and $8 \times 2 = 16$, or

$80 \div 8 = 10$ and $16 \div 8 = 2$.)

Suggested Centers

- Five in a Row: Multiplication (3–5), Stage 3: Two-digit Factors (Addressing)
- Compare (1–5), Stage 4: Divide within 100 (Supporting)

Complete Cool-Down

Response to Student Thinking

Students show they understand the situation as a division problem, but find a quotient other than 16.

Next Day Support

- After the warm-up in the next lesson, pair students up to discuss their responses.

The work in this lesson builds from the division concepts developed in a prior unit.

Prior Unit Support

Grade 3, Unit 4, Section A: What is Division?

Lesson 14: Situations Involving Factors and Multiples

Standards Alignments

Building On	4.NBT.B.5, 4.OA.B.4
Addressing	4.NBT.B.6, 4.OA.A.3

Teacher-facing Learning Goals

- Reason about division of two- and three-digit numbers in situations involving factors and multiples.

Student-facing Learning Goals

- Let's interpret and solve division problems beyond 100.

Lesson Purpose

The purpose of this lesson is for students to solve division problems that involve finding unknown factors. They do so by reasoning with partial quotients and by decomposing a dividend into familiar multiples of the divisor. In the problems, the dividends are greater than 100 and the divisions result in whole numbers with and without a remainder.

In this lesson, students relate problems about factors and multiples to division. To solve the problems, they rely on the relationship between multiplication and division, and their understanding of division as a way to find an unknown factor.

Students continue to interpret division in terms of finding the number of groups ("If we write multiples of 5, how many numbers will we need to write to get to 105?") and the size of a group ("What number are we finding multiples of if we get to 112 after writing 7 numbers?"). They may solve the problems by multiplying in parts (finding partial products) or by dividing in parts (finding partial quotients). Through repeated reasoning, they notice that it helps to decompose a dividend into familiar multiples (MP2, MP8).

In these materials, division that results in a whole number with a remainder—for example $145 \div 7$—is not expressed with an expression such as "20 R 5." Instead, students will relate this result to a multiplication equation, in that $145 = 7 \times 20 + 5$.

In future lessons, students will more formally investigate partial quotients as a strategy dividing numbers.

Access for:

Students with Disabilities

- Engagement (Activity 1)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Number Talk (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

Which questions that you asked today would you rephrase to improve students' ability to make connections or to help them better consolidate what they did? How would you rephrase them?

Cool-down (to be completed at the end of the lesson)

5 min

Reaching 161 with Multiples

Standards Alignments

Addressing 4.NBT.B.6, 4.OA.A.3

Student-facing Task Statement

Mai is writing multiples of a number. When she reaches 161, she has written 7 numbers.

1. What number is Mai writing multiples of? Explain or show your reasoning.
2. What division equation can represent the question you just answered?

Student Responses

1. 23. Sample reasoning: I know that $7 \times 20 = 140$ and $7 \times 3 = 21$. Because $140 + 21 = 161$, the number must be $20 + 3$ or 23.
2. $161 \div 7 = ?$ or $161 \div ? = 7$

Begin Lesson

Warm-up

10 min

Number Talk: Dividing by 7

Standards Alignments

Addressing 4.NBT.B.6

This Number Talk encourages students to compose or decompose multiples of 7 and to rely on properties of operations to mentally solve problems. The ability to compose and decompose numbers will be helpful when students divide multi-digit numbers. It also promotes the reasoning that is useful when finding multiples of a number, or when deciding if a number is a multiple of another number.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $21 \div 7$
- $35 \div 7$
- $140 \div 7$
- $196 \div 7$

Student Responses

- 3: Three times 7 is 21.
- 5: Five times 7 is 35.
- 20: Two times 7 is 14, so 20 times 7 is 140.
- 28: Sample reasoning:
    - $196 = 140 + 56$, and $56 = 21 + 35$. This means $196 = 140 + 21 + 35$, so $196 = (20 \times 7) + (3 \times 7) + (5 \times 7)$.
    - Three times 7 is 21, so 30 times 7 is 210. 196 is 14 less than 210, which means it is 2×7 less than 30×7.

Launch

- Display one expression.
- “Give me a signal when you have an answer and can explain how you got it.”
- 1 minute: quiet think time

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- “What do the expressions have in common?” (They all involve division by 7. The dividends are all multiples of 7. The results have no remainders.)
- “How did the first three expressions help us find the value of the last expression?”
- Consider asking:
    - “Who can restate _______ 's reasoning in a different way?”
    - “Did anyone have the same strategy but would explain it differently?”

- "Did anyone approach the expression in a different way?"
- "Does anyone want to add on to____'s strategy?"

Activity 1

20 min

Write Multiples

Standards Alignments

Building On	4.NBT.B.5, 4.OA.B.4
Addressing	4.NBT.B.6

This activity prompts students to use the relationship between multiplication and division and their understanding of factors and multiples to solve problems about an unknown factor (MP7). Students recognize such problems as division situations. Here the dividends are three-digit numbers and the divisors are one-digit numbers. Students use the context of factors and multiples to interpret division that results in a remainder.

One approach for solving these problems is by decomposing the dividend into familiar multiples of 10 and then dividing the remaining number (which is now smaller) by the divisor. Another is to find increasingly greater multiples of the divisor until reaching the dividend. Both are productive and appropriate. In the synthesis, help students see the connections between the two paths.

Access for English Learners

MLR8 Discussion Supports. Synthesis: Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. If students don't recognize this as a division situation at first, they may do so when presented with a more accessible value. Consider offering this situation first: "Han starts writing multiples of a number. When he reaches 12, he has written 3 numbers." Invite students to identify what number Han is writing multiples of (4), and how they know. Then ask how they might apply that reasoning to the task as presented here.
Supports accessibility for: Conceptual Processing

Student-facing Task Statement

1. Han starts writing multiples of a number. When he reaches 104, he has written 8 numbers.

   For each of the following questions, show your reasoning.

   a. What number is Han writing multiples of?
   b. What is the 15th multiple of this number?
   c. Han gets to 286. How many numbers has he written at that point?

2. Kiran wants to know how many multiples of 7 are between 0 and 150.

   a. He thinks he can use division to find out. Do you agree? Explain your reasoning.
   b. How many multiples will he find? Show your reasoning.
   c. Is 150 a multiple of 7? Show how you know.

Student Responses

1. a. 13. Sample reasoning:
      - $8 \times 10 = 80$ and $80 + 24 = 104$. Since 24 is 8×3, Han must be finding multiples of $10 + 3$ or 13.
      - $80 \div 8 = 10$ and $24 \div 8 = 3$, so $104 \div 8 = 13$.

   b. 195, because $15 \times 13 = 195$.

   c. 22 numbers. Sample reasoning:
      - $(10 \times 13) + (10 \times 13) + (2 \times 13) = 286$
      - $20 \times 13 = 260$ and $2 \times 13 = 26$, so 22×13 is $260 + 26$, which is 286.

2. a. Agree. Sample reasoning: He can

Launch

- "Who can remind the class of the meaning of 'multiple?'"

Activity

- 4–5 minutes: quiet work time for the first set of questions
- Monitor for students who:
  - decompose 104 into a multiple of 8 and another number
  - compose 104 from increasingly larger multiples of 8
  - use multiplication or division equations to show their reasoning
- Pause for a discussion before the second set of questions.
- Select students who use different decomposition strategies to share responses. Record and display their reasoning.
- "Let's revisit each question we just answered. What equations could we write to represent them?"
- Display equations and highlight their connection to the questions:
  a. $8 \times ? = 104$ or $104 \div 8 = ?$
  b. $15 \times 13 = ?$ or $13 \times 15 = ?$
  c. $? \times 13 = 286$ or $286 \div 13 = ?$
- 4–5 minutes: quiet work time for the second set of questions

Synthesis

- Invite students to share responses for the last set of questions.
- Highlight that to divide a number by a smaller number—say, divide 150 by 7, we can:
  - Use familiar multiples or

divide 150 by 7 because he's trying to find how many 7s are in 150.

b. 21. Sample reasoning:
   - There are twenty 7s in 140, plus one more 7 makes 147. This is as high as we could get without going over 150.
   - $20 \times 7 = 140$ and $1 \times 7 = 7$, so $21 \times 7 = 147$, which is less than 150.

c. No. $7 \times 21 = 147$ and $7 \times 22 = 154$

multiplication facts to help us. For example, if we know $(20 \times 7) + (1 \times 7) = 147$, we know the result is 21 with a remainder of 3, or $150 = 21 \times 7 + 3$.

- Think of the dividend in smaller chunks. For example: We can see the 150 as $140 + 10$ and divide each 140 and 10 by 7 separately, which gives $20 + 1$ with a remainder of 3.

Activity 2

15 min

Jada's Mystery Number

Standards Alignments

Addressing 4.NBT.B.6

In this activity, students continue to use the relationship between multiplication and division to reason about situations that involve division. Students divide three-digit numbers by single-digit divisors and find results with and without a remainder.

Student-facing Task Statement

Jada is writing multiples of a mystery number. After writing a bunch of numbers, she writes out 126.

- Mai says 6 is the mystery number.
- Priya says 8 is the mystery number.
- Andre says 9 could be the mystery number.

1. Which student do you agree with? Show your reasoning and include equations.

Launch

- Groups of 2

Activity

- 5 minutes: independent work time on the first question
- 2 minutes: partner discussion
- 2–3 minutes: partner work time on the second question

2. Jada gives one more clue: "If I keep writing multiples, I'll get to 153."

   What is the mystery number? Explain or show your reasoning.

Student Responses

1. Sample responses:
   - I agree with Mai, because 126 is $120 + 6$, and 120 is a multiple of 6 $(6 \times 20 = 120)$ and 6 is the first multiple of 6 $(6 \times 1 = 6)$.
   - I agree with Andre and Mai because 126 is a multiple of both 9 and 6, because when counting by each number I say 126.
2. 9. Sample response: 153 and 126 are both multiples of 9.

- Monitor for students who clearly show how they use partial products or partial quotients to answer the questions.

Synthesis

- "How could we use division to help us find out the mystery number?" (If the result has a remainder, then the divisor could not be the mystery number. 153 divided by 6 has 3 as a remainder. 153 divided by 9 has no remainder, so 9 is the mystery number.)

Lesson Synthesis

10 min

"Today we tried to find out if a number is a multiple or a factor of another number. For instance: Is 267 a multiple of 8?"

"Is this question a division problem?" (Yes)

"Why?" (We're looking for how many 8s are in 267.)

"What are we dividing?" (267 by 8)

"One way to answer the question is by using familiar multiplication facts or by finding partial products. How would you start?" (One way is to start with 10 x 8 or its multiples, build the products up to 267 or close to it, and then try smaller multiples of 8.

$$10 \times 8 = 80$$
$$20 \times 8 = 160$$
$$30 \times 8 = 240$$
$$3 \times 8 = 24$$
$$33 \times 8 = 264$$

264 is 3 away from 267, not enough to make another 8, so 267 is not a multiple of 8.)

"Why might it be helpful to start with multiples of 10?" (They're easy to find and easy to add.)

"Can we use division to answer the question?" (We can start a number close to 267 that is a multiple of 8, divide it by 8, see what is left, and find a multiple of 8 that is close to that number. For example:

- $160 \div 8 = 20$. After taking 160 away, there's 107 left.
- $80 \div 8 = 10$. After taking 80 away, there's 27 left.
- $24 \div 8 = 3$. After taking 24 away, there's 3 left, which is not enough to make 8.)

"How are the two approaches alike?" (They involve using smaller multiples of a number to see if a larger number is a multiple of that number.)

Suggested Centers

- Compare (1–5), Stage 4: Divide within 100 (Supporting)
- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Supporting)

Complete Cool-Down

Response to Student Thinking

Students do not write a division equation to represent the situation.

Next Day Support

- Launch the warm-up or Activity 1 by highlighting important notation from previous lessons.

The work in this lesson builds from the concept of factors and multiples developed in a prior unit.

Prior Unit Support

Grade 4, Unit 1, Section A: Understand Factors and Multiples

Lesson 15: Situations Involving Area

Standards Alignments

Building On	3.MD.C.7, 4.NBT.B.5
Addressing	4.MD.A.3, 4.NBT.B.6, 4.OA.A.3
Building Towards	4.MD.A.3, 4.NBT.B.6

Teacher-facing Learning Goals

- Reason about division of two- and three-digit numbers in situations involving area of rectangles.

Student-facing Learning Goals

- Let's divide to find the side length of a rectangle.

Lesson Purpose

The purpose of this lesson is for students to use partial quotients to solve division problems that involve tiling squares and finding a side length of a rectangle with a known area.

In this lesson, students encounter division as they find a side length of a rectangle whose area is a three-digit number and one side is a one-digit number.

The context involves tiling rectangles with square tiles. This enables students to connect the dividend to the number of tiles in the rectangle and the divisor to the number of rectangles along one side. The grid provided in the first activity encourages students to partition the area (the dividend) into smaller parts, which in turn facilitates finding the unknown length (the quotient).

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR7 (Activity 2)

Instructional Routines

Estimation Exploration (Warm-up)

Materials to Gather

- Grid paper: Activity 2
- Sticky notes: Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

Which questions that you asked today would you rephrase to improve students' ability to make connections or to help them better consolidate what they did? How would you rephrase them?

Cool-down (to be completed at the end of the lesson)

5 min

Sticky Notes on the Door

Standards Alignments

Addressing 4.MD.A.3, 4.NBT.B.6, 4.OA.A.3

Student-facing Task Statement

Jada's class is decorating their door with square sticky notes for their teacher. Each sticky note has a drawing or a message from a student.

The class used 234 square sticky notes to cover their classroom door completely, leaving no gaps or overlaps between the notes. It takes 9 square notes to cover the width of the door.

How many square notes does it take to cover the full height of the door? Show how you know.

Student Responses

26 square notes. Sample reasoning: I know that $9 \times 20 = 180$ and $9 \times 6 = 54$. $180 + 54 = 234$, so it takes $20 + 6$ or 26 notes to cover the height of the door.

Begin Lesson

Warm-up

10 min

Estimation Exploration: Area of a Soccer Field

Standards Alignments

Building On	3.MD.C.7, 4.NBT.B.5
Building Towards	4.MD.A.3, 4.NBT.B.6

The purpose of this warm-up is to elicit students' understandings of the relationship between the side lengths of a rectangle and its area. These understandings prepare students to reason about an unknown length or width of rectangles in the activities.

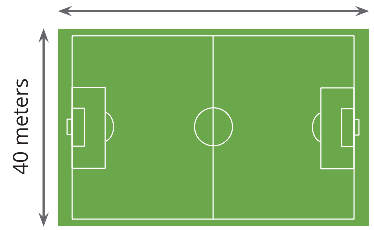
Students use their understanding about the relationship between multiplication and division, and their understanding of multiples of 10 to divide beyond 100.

Instructional Routines

Estimation Exploration

Student-facing Task Statement

Estimate: What is the length of the soccer field in meters?



area: 2,280 square meters

Record an estimate that is:

too low	about right	too high

Launch

- Groups of 2
- Display the image.
- "What is an estimate that's too high?" "Too low?" "About right?"
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 1 minute: partner discussion
- Record responses.

Synthesis

- "How did you make your estimate? How did you know it's reasonable?" (Sample response: I know that the area is about 2,300 and one of the side lengths is 40. I know that $40 \times 60 = 2{,}400$ and $40 \times 50 = 2{,}000$, so the

Student Responses

- Too low: 40–50 meters
- About right: 55–60 meters
- Too high: 60 meters

estimate is between 50 and 60, but closer to 60.)

Consider asking:

- “Is anyone’s estimate less than 40 meters? Is anyone’s estimate greater than 60 meters?”
- “Based on this discussion does anyone want to revise their estimate?”

Activity 1

Elena’s Mural

15 min

PLC Activity

Standards Alignments

Addressing 4.MD.A.3, 4.NBT.B.6

In this activity, students find the length of one side of a rectangle given the length of the other side and the area of the rectangle. This work builds on what students have done in grade 3, where the area was within 100 square units. In this lesson, the area is a three-digit number beyond 100.

The use of tiles as a context and the presence of a grid allow students to see more concretely the relationship between a product and a factor, but the size of the product discourages students to count the tiles to find the unknown factor. Instead, students are encouraged to find multiples of the known factor or to decompose the product into parts (MP2).

Access for Students with Disabilities

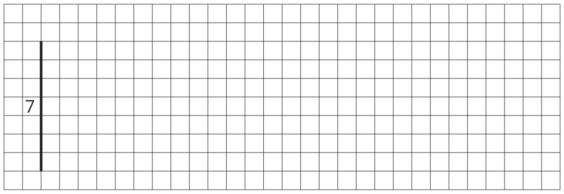
Representation: Internalize Comprehension. Synthesis: Invite students to identify what information and thinking was most helpful to solve the problem. Record their responses, including mathematical language and pictures, and encourage them to use this display as a reference for the next activity.
Supports accessibility for: Conceptual Processing, Memory

Student-facing Task Statement

Elena used 189 square tiles to create a rectangular mural for the art club. The mural is 7 tiles wide.

Launch

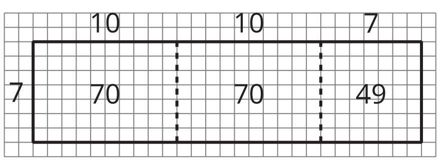
- Groups of 2
- Explain what a mural is or show an image.



1. How many tiles long is Elena's mural? Be prepared to explain or show how you know.
2. Write one or more equations that show how you solved this problem.

Student Responses

1. 27 tiles long. Sample reasoning:
    - If the length is 20 tiles, the mural would need 140 tiles, leaving 49 tiles. Seven times 7 is 49, so the length must be $20 + 7$ or 27 tiles.
    - I know that 10×7 is 70. I marked off two 10×7 rectangles, which are equal to 140 tiles. $189 - 140$ is 49 and $7 \times 7 = 49$, so I marked off a 7×7 rectangle. The length of the mural is $10 + 10 + 7$, which is 27.



2. Sample responses:
    - $(7 \times 10) + (7 \times 10) + (7 \times 7) = 189$
    - $140 \div 7 = 20$, $49 \div 7 = 7$, and $20 + 7 = 27$
    - $7 \times \ell = 189$, or $\ell = 189 \div 7$

Activity

- 5 minutes: independent work time
- 2 minutes: partner discussion
- Monitor for students who use multiples of 10 and 7 and then either add or subtract multiples of 7 to reach 189.

Synthesis

- Select students to share their responses and reasoning. Display their responses for all to see.
- Highlight strategies that use multiples of 10 and 7 to find the side length of the rectangle. If no students use an area diagram in their reasoning, display an example for all to see (as shown in Student Responses).
- Invite students to write the equations they wrote. If no division equations are included, display division equations and ask students if they could be used to answer the question. (For instance, $140 \div 7 = 20$ and $49 \div 7 = 7$, so $189 \div 7 = 27$.)

Activity 2

20 min

Tyler's Mural

Standards Alignments

Addressing 4.MD.A.3, 4.NBT.B.6, 4.OA.A.3

This activity continues the work in the first activity. It uses a similar context and prompts students to reason about division, but the result of the division has a remainder, which students will need to interpret.

After students have had some independent work time, consider a gallery walk of strategies.

- Post 3–4 posters around the room, each showing a likely strategy for the last problem, such as using partial products, partial quotients, pictures, or words.
- Provide some blank posters for students to show additional unique strategies.

This activity uses *MLR7 Compare and Connect*. Advances: representing, conversing

Access for English Learners

MLR7 Compare and Connect. Synthesis: After the Gallery Walk, lead a discussion comparing, contrasting, and connecting the different approaches. To amplify student language, and illustrate connections, follow along and point to the relevant parts of the displays as students speak.
Advances: Representing, Conversing

Materials to Gather

Grid paper, Sticky notes

Required Preparation

- If doing a gallery walk, create 3–4 posters to display during the activity that show or describe different strategies students are likely to use to solve the problem.

Student-facing Task Statement

Tyler is also creating a rectangular mural for the art club. He has 197 tiles for his mural. His mural is 6 tiles wide.

1. Will Tyler use all of his tiles in the mural? Explain your reasoning.
2. How many tiles long is Tyler's mural? Show your reasoning using numbers, pictures, or words.

Launch

- Groups of 2
- Give access to grid paper, in case students wish to use it to create an area diagram.

Activity

MLR7 Compare and Connect

- 2–5 minutes: independent or group work

Student Responses

1. No. Sample response:
    - 30 times 6 is 180, and 3 times 6 is 18. Adding the two gives 198, not 197.
2. 32 tiles long. Sample responses:
    - $180 \div 6 = 30$, $12 \div 6 = 2$, $180 + 12 = 192$, $197 = 6 \times 32 + 5$. Tyler will not use 5 of his tiles.
    - $30 \times 6 = 180$ and $2 \times 6 = 12$, so $32 \times 6 = 192$. There will be 5 extra tiles remaining.
    - $197 = 60 + 60 + 60 + 12 + 5$

- Give each student a sticky note.
- "Make one round to visit each poster. Place your sticky note on the poster with a strategy that matches your strategy or that makes the most sense to you."
- "After your first round, make another round to visit 1–2 other posters that you didn't select. Make sense of the strategy of the poster and be prepared to explain how it is different than yours."
- 5–7 minutes: gallery walk

Synthesis

- Discuss the results of the gallery walk: "Which strategy seems to be most common? The least common? Why might they be the most or least common?"
- "How many tiles did Tyler use for his mural? How do you know?" (Tyler uses 192 tiles because the mural is rectangular and there are 6 rows of tiles. 192 is the greatest multiple of 6 within 197.)
- "How many tiles were not used?" ($197 - 192 = 5$. Five tiles were not used.)
- Consider asking: "How are partial quotients a helpful strategy for finding the side length?" (Sample response: I can use multiples of 10, which are easy to divide in my head. For example, the greatest multiple of 10 that is also a multiple of 6 within 197 is 180. I try to choose the greatest number so that I can keep track of the dividend. $197 - 180 = 17$. 17 is left from the dividend to divide by 6, but 17 is not a multiple of 6, so I repeat the strategy. The greatest multiple of 6 within 17 is 12, and 5 is remaining.)

Lesson Synthesis

10 min

"Today we used division to find side lengths of rectangles. For each rectangle, we knew the area and the length of one side and we used division to find the length of the other."

"What is the relationship between the side lengths and the area of a rectangle?" (The area is the product of the two side lengths.)

"How do we find the missing side length?" (Divide the area by the side length that we do know, or multiply one side length by different numbers until we find the area.)

Suggested Centers

- Compare (1–5), Stage 4: Divide within 100 (Supporting)
- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Supporting)

Complete Cool-Down

Response to Student Thinking

Students show they understand the problem as a division or unknown side length problem, but find a solution other than 26 square sticky notes.

Next Day Support

- After the warm-up in the next lesson, pair students up to discuss their responses.

The work in this lesson builds from the area concepts developed in a prior unit.

Prior Unit Support

Grade 3, Unit 2, Section A: Concepts of Area Measurement

Lesson 16: Base-ten Blocks to Divide

Standards Alignments

Addressing	4.NBT.B.6
Building Towards	4.NBT.A

Teacher-facing Learning Goals

- Divide two-digit numbers by one-digit divisors using base-ten blocks.

Student-facing Learning Goals

- Let’s use base-ten blocks to divide.

Lesson Purpose

The purpose of this lesson is for students to make sense of base-ten representations for division.

In the previous lesson, students applied their understanding from grade 3 to divide two- and three-digit numbers by one-digit divisors. Students worked with dividends slightly beyond 100 and represented their thinking in a way that made sense to them.

In this lesson, students work with larger dividends and represent problems with base-ten blocks. This representation emphasizes place value, which supports the work with division in this section. Students are asked to represent their work with base-ten blocks on paper, but that is not the emphasis of this lesson. In the next lesson, students will make sense of and use base-ten diagrams. In future lessons, they will be able to choose a representation and method that makes sense to them as they go deeper into division work.

Access for:

Students with Disabilities

- Representation (Activity 2)

Instructional Routines

MLR7 Compare and Connect (Activity 1), What Do You Know About _____? (Warm-up)

Materials to Gather

- Base-ten blocks: Warm-up, Activity 1, Activity 2
- Tools for creating a visual display: Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	25 min
Activity 2	10 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

What surprised you about how students used base-ten blocks to find the value of quotients? How might you use this in tomorrow's lesson?

Cool-down (to be completed at the end of the lesson)

5 min

Division Reflection

Standards Alignments

Addressing 4.NBT.B.6

Student-facing Task Statement

How was using the base-ten blocks helpful in your work today? How was it not helpful?

Student Responses

Sample response: It was helpful when we were working with smaller numbers and we didn't have to decompose blocks. It wasn't helpful when I was trying to work with larger numbers.

Begin Lesson

Warm-up

10 min

What Do You Know About Base-ten Blocks?

Standards Alignments

Building Towards 4.NBT.A

The purpose of this What Do You Know About _____? is to invite students to share what they know about base-ten blocks in the context of division. Some students may choose to reflect on base-ten blocks and division while others may simply describe what they know about base-ten blocks. Any reflection offered by students is useful for activating prior knowledge for the lesson.

Instructional Routines

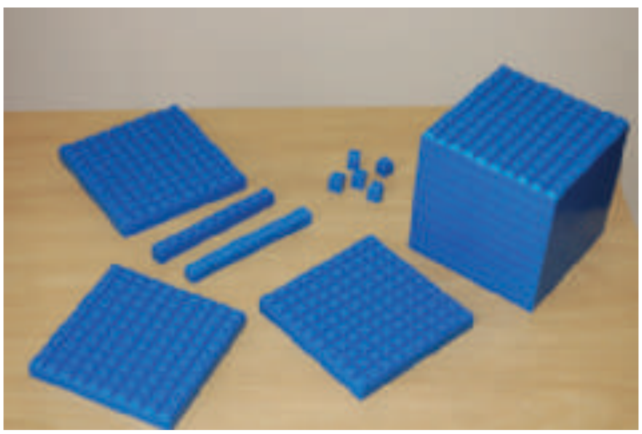
What Do You Know About _____?

Materials to Gather

Base-ten blocks

Student-facing Task Statement

What do you know about base-ten blocks?



Student Responses

Sample responses:

- They show thousands, hundreds, tens, and ones.
- They can represent different values.
- Each larger block is 10 times the size of the previous block.

Launch

- Display a pile of base-ten blocks.
- "What do you know about base-ten blocks?"
- 1 minute: quiet think time

Activity

- Record responses.

Synthesis

- "Which blocks would we use to represent the number 324?"
- "You have used base-ten blocks to represent large numbers. Today you are going to see how they can be helpful to divide larger numbers."

Activity 1

25 min

Blocks to Divide

Standards Alignments

Addressing 4.NBT.B.6

The purpose of this activity is for students to find quotients using base-ten blocks and represent their methods. Students solve two problems, one where decomposing a hundred or ten is not necessary and one where it is. Students use base-ten blocks to represent the problem and find the quotient, then they work in small groups to create a visual display of how they used the base-ten blocks. This builds on work students have done in previous grades representing operations with base-ten representations.

The base-ten blocks help highlight the important role place value plays in division (MP7). There are not enough hundreds in 104 to divide into 8 equal groups but there are enough tens to put 1 ten in 8 equal groups and then the 2 remaining tens can be broken into ones to complete the division.

Instructional Routines

MLR7 Compare and Connect

Materials to Gather

Base-ten blocks, Tools for creating a visual display

Required Preparation

- Each group of 3–4 students needs a set of base-ten blocks that includes 4 hundreds blocks, 10 ten blocks, and 25 ones blocks.

Student-facing Task Statement

Use the base-ten blocks to represent each expression. Then find the value of each expression.

1. $488 \div 4$
2. $104 \div 8$

Student Responses

1. 122. Sample response: Students split the 4 hundreds into 4 groups and then do the

Launch

- Groups of 3–4
- Give each group at least 4 hundreds blocks, 10 tens blocks and 25 ones blocks.

Activity

- “Work with your group to represent each expression with base-ten blocks, then find the quotient.”
- 5–6 minutes: group work time

same with the 8 tens and 8 ones. They see they have 1 hundred, 2 tens ,and 2 ones in each group.

2. 13. Sample response: Students trade the hundred block in for 10 tens. They split the 10 tens into 8 groups and are left with 2 tens. They split the 2 tens into 20 ones and then split the 24 ones into 8 groups.

MLR7 Compare and Connect

- Give each group tools for making a visual display.
- "Create a visual display that shows how you used the base-ten blocks to find the quotient, including details such as notes, diagrams, drawings, and so on, to help others understand your thinking."
- 2–5 minutes: group work
- 5–7 minutes: gallery walk
- "As you look at the displays from other groups, record things that are the same and things that are different."

Synthesis

- "What is the same and what is different between the representations?"
- 30 seconds quiet think time
- 1 minute: partner discussion

Advancing Student Thinking

Students may show their reasoning and computation only by writing expressions or equations. Consider asking:

- "How can you use base-ten blocks to represent the division and show your way of thinking about it?"
- "What does the answer you found mean in terms of the blocks?"

Activity 2

10 min

Show Us Your Blocks

Standards Alignments

Addressing 4.NBT.B.6

The purpose of this activity is for students to find quotients and represent their thinking with base-ten blocks. The numbers in the expressions are designed to encourage students to think about when the base-ten blocks may be helpful and when they become cumbersome. In future lessons, students will be asked to interpret base-ten representations but may use any method or representation that makes sense to them.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Synthesis: Use gestures or annotations such as labels or arrows to make connections between representations visible on displayed student work. *Supports accessibility for: Conceptual Processing, Visual-Spatial Processing*

Materials to Gather

Base-ten blocks

Student-facing Task Statement

Find the value of each expression. Explain or show how you used base-ten blocks to find the value.

1. $96 \div 4$
2. $86 \div 2$
3. $108 \div 9$

Student Responses

1. 24. Sample response: Draw 4 groups with 2 tens in each group. Divide remaining 16 and draw 4 ones in each group.
2. 43. Sample response: Draw 2 groups with 4 tens in each group. Draw 3 ones in each group.
3. 12. Sample response: Start with 10 tens and draw 1 ten in each of 9 groups.
   Divide remaining 18 and draw 2 ones in each group.

Launch

- Groups of 2
- Give students access to base-ten blocks.
- "You are going to solve some problems on your own. Use base-ten blocks and represent your thinking in your book."

Activity

- 5–6 minutes: independent work time
- 2 minutes: partner discussion
- Monitor for students who draw base-ten representations using small and large squares and rectangles labeled with numbers, to share in the lesson synthesis.

Synthesis

- See lesson synthesis.

Advancing Student Thinking

Students may use mental math to solve problems. Consider asking: "How might your use the blocks to check your mental math?”

Lesson Synthesis

10 min

Display student work showing base-ten representations for the last expression ($108 \div 9$).

“How did this student represent their reasoning?” (They drew the 10 tens and then crossed out 2 tens and drew 20.)

“Do you have any questions or suggestions that could help them make their work clearer?” (They could add numbers to label the parts.)

Repeat with other student’s work, as time allows.

“Take a minute, and if you’d like, revise your work to make it easier for someone else to understand.”

Suggested Centers

- Compare (1–5), Stage 4: Divide within 100 (Supporting)
- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Supporting)

Complete Cool-Down

Response to Student Thinking

This lesson builds on concepts of division developed in a previous unit.

Prior Unit Support

Grade 3, Unit 4, Section D: Dividing Larger Numbers

Lesson 17: Base-ten Diagrams to Represent Division

Standards Alignments

Addressing 4.NBT.B.6

Teacher-facing Learning Goals

- Divide two- and three-digit by one-digit numbers using base-ten diagrams.

Student-facing Learning Goals

- Let's divide using base-ten blocks or diagrams.

Lesson Purpose

The purpose of this lesson is for students to find the quotients of two-digit and three-digit dividends and one-digit divisors. They do so by decomposing the dividend by place value—decomposing a larger unit to 10 of a smaller unit—and by reasoning in terms of equal-size groups.

In grade 3, students used base-ten representations to help them reason about division of a two-digit number into equal-size groups. This lesson builds on that understanding and revisits it in the context of three-digit dividends. Students recall that they can exchange or decompose one or more units of a higher place value for 10 units of the next lower place value in order to have enough units to put into equal groups.

The work here sets the groundwork for students to later decompose a dividend by place value (even when not using base-ten blocks or diagrams). It is also the basis for dividing multi-digit numbers using the standard division algorithm (in grade 5), which relies on dividing by place value, one digit at a time.

Access for:

Students with Disabilities

- Engagement (Activity 2)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Which One Doesn't Belong? (Warm-up)

Materials to Gather

- Base-ten blocks: Activity 1, Activity 2

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

How did the representations in today's lesson support students in dividing multi-digit numbers?

Cool-down (to be completed at the end of the lesson)

5 min

Find the Value of a Quotient

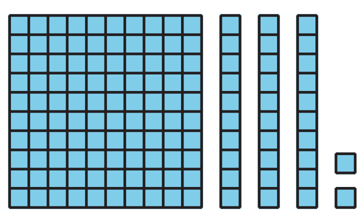
Standards Alignments

Addressing 4.NBT.B.6

Student-facing Task Statement

Find the value of $132 \div 4$. Show your reasoning.

The base-ten diagram represents 132. Use the diagram if you find it helpful.



Student Responses

33. Sample reasoning:

- I know that $132 = 100 + 32$. I also know that $100 \div 4 = 25$ and $32 \div 4 = 8$, so $132 \div 4$ is the sum of 25 and 8, which is 33.
- The large square represents 1 hundred and can be decomposed into 10 tens. Now we have 13 tens. Twelve of the tens can be put into 4 groups of 3 tens. The last ten can be decomposed into 10 ones. There are now 12 ones, or 4 groups of 3 ones. Three tens and 3 ones is 33.
- I know that $120 \div 4 = 30$ and $12 \div 4 = 3$, so $132 \div 4 = 33$.

Begin Lesson

Warm-up

10 min

Which One Doesn't Belong: Base-ten Diagrams

Standards Alignments

Addressing 4.NBT.B.6

This warm-up prompts students to carefully analyze and compare features of base-ten diagrams, looking not only at the number and types of shapes in each diagram, but also the value each diagram represents. The activity also enables students to recall what they know about representations of numbers in base-ten and enables the teacher to hear how they talk about these representations.

The analysis here prepares students for the activities in the lesson, in which they use base-ten diagrams to find whole-number quotients.

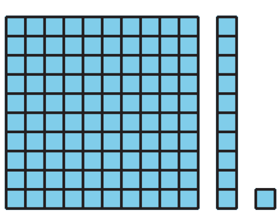
Instructional Routines

Which One Doesn't Belong?

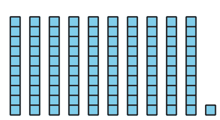
Student-facing Task Statement

Which one doesn't belong?

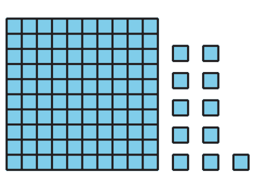
A



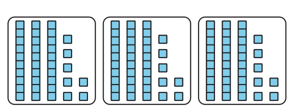
B



C



D



Student Responses

Sample responses:

Launch

- Groups of 2
- Display image.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 2–3 minutes: partner discussion
- Record responses.

Synthesis

- "How is it that A and C both show 111?" (If a small square represents 1, then a rectangle is 10 and a large square is 100. In A: $100 + 10 + 1 = 111$. In C: $100 + 11 = 111$.)

- A doesn't belong because it does not show more than 1 of any place value.
- B doesn't belong because it does not show 111.
- C doesn't belong because it has no tens.
- D doesn't belong because because it doesn't have a column with just a single square.

- "How do we know that D also shows 111?" (Each group in D represents (3×10) + 7 or 37. Three groups of 37 makes 111.)
- "Suppose we don't know what a small square represents except that it represents the same value in all diagrams. Can we tell if C and D represent the same value? How?" (Yes. We know that 10 small squares make 1 rectangle and 10 rectangles make 1 large square. In D, we'd have 21 small squares and 9 rectangles. Trading 10 small squares for a rectangle gives 10 rectangles and 11 small squares, which is equal to 1 large square and 11 small squares.)

Activity 1

15 min

Divide with Diagrams or Blocks

Standards Alignments

Addressing 4.NBT.B.6

In this activity, students use base-ten diagrams to find quotients of two-digit dividends and single-digit divisors. They think about distributing the pieces in the diagram into equal-size groups, decomposing a higher-value piece with 10 of the lower-value pieces as needed to divide.

Some students may benefit from manipulating physical blocks. Provide each group of students with access to a set of base-ten blocks.

Access for English Learners

MLR8 Discussion Supports. Use multimodal examples to show the meaning of place value. Use verbal descriptions along with gestures, drawings, or concrete objects to show how a base-ten block is equivalent to 10 one blocks and how they are interchangeable.
Advances: Listening, Representing

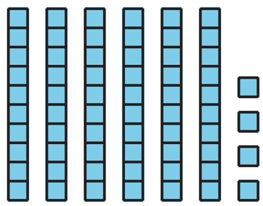
Materials to Gather

Base-ten blocks

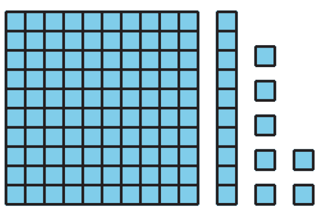
Student-facing Task Statement

1. Priya draws a base-ten diagram to find the value of $64 \div 4$. A rectangle represents 10. A small square represents 1.

   Use the diagram (or actual blocks) to help Priya complete the division. Explain or show your reasoning.

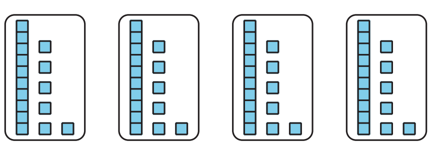
   

2. Use this base-ten diagram (or actual blocks) to find the value of $117 \div 3$.

Student Responses

1. 16. Sample response: Four of the 6 tens can be put into 4 groups of 1 ten. The other 2 tens can be decomposed into 20 ones. There are now 24 ones, which can be divided into 4 groups of 6. One ten and 6 ones make 16.

2. 39. Sample response: One hundred can be decomposed into 10 tens. Nine of the 11 tens can be put into 3 groups of 3 tens. There are now 2 tens which can be composed into 20 ones. There are now 27 ones, which can be divided into 3 groups of 9. Three tens and 9 ones make 39.

Launch

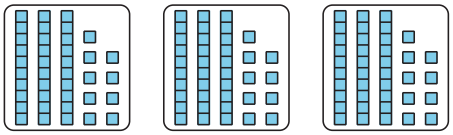
- Groups of 4
- Give students access to base-ten blocks.
- Display the first diagram. Make sure students can explain why it represents 64.

Activity

- 5 minutes: quiet think time
- 2 minutes: group discussion
- Monitor for students who see that a larger piece can be decomposed into 10 of the next smaller piece to help with distribution.

Synthesis

- Invite students to share their responses and reasoning.
- Make sure students see that:
  - We can think of $64 \div 4$ as putting 6 tens and 4 ones into 4 equal groups, and $117 \div 3$ as putting 1 hundred 1 ten and 7 ones into 3 equal groups.
  - To divide the base-ten pieces, we can decompose a piece representing a larger place value with 10 of the next smaller place value.



Activity 2

20 min

Help Noah Get Unstuck

Standards Alignments

Addressing 4.NBT.B.6

In this activity, students continue to use base-ten representations and to reason about equal-size groups to find whole-number quotients. The work reinforces the idea of decomposing a hundred into 10 tens as needed to perform division.

Students explicitly use place value understanding to decompose hundreds and tens (MP7) while making sense of a students' reason to help him complete the division problem (MP3).

Access for Students with Disabilities

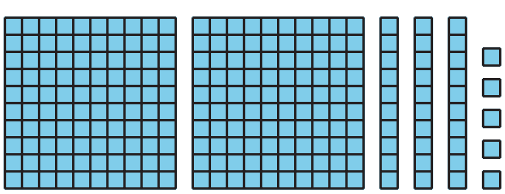
Engagement: Provide Access by Recruiting Interest. Optimize meaning and value. Invite students to share examples from their own lives in which they might need to divide three-digit numbers by one-digit numbers. Invite them to imagine and share why Noah might be dividing 235 by 5.
Supports accessibility for: Attention, Social-Emotional Functioning

Materials to Gather

Base-ten blocks

Student-facing Task Statement

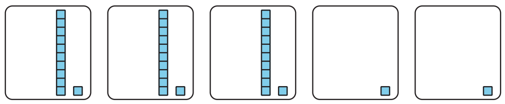
1. This diagram represents 235.



Launch

- Groups of 2
- Give students access to base-ten blocks.
- Display the first diagram.
- "How does the diagram represent 235?" (A large square represents 100. A rectangle

To find $235 \div 5$, Noah draws the following diagram but then gets stuck.



He says, "There are not enough of the hundreds or the tens pieces to put into 5 groups."

Explain or show how Noah could find $235 \div 5$ with his diagram.

2. Find the value of $432 \div 6$. Show your reasoning. Use base-ten diagrams or blocks if you find them helpful.

Student Responses

1. Sample response: Noah can exchange the 2 hundreds for 20 tens, which can make 5 groups of 4 tens. The other 3 tens can be exchanged for 30 ones. There are now 35 ones in total, which can make 5 groups of 7 ones with no remainder. Each group has 4 tens and 7 ones, which is 47.
2. Sample response: 432 can be represented by 4 hundreds, 3 tens, and 2 ones.
    - Exchange 4 hundreds for 40 tens. Now there are 43 tens.
    - Use 42 tens to make 6 groups of 7 tens. There is 1 ten left over.
    - Exchange the 1 ten for 10 ones. There are now 12 ones.
    - Put the 12 ones into 6 groups of 2 ones.
    - Each group has 7 tens and 2 ones, or 72.

represents 10. A small square represents 1.)

Activity

- 4–5 minutes: independent work time on the first question
- Monitor for students who see the 2 hundreds as 20 tens and those who see them as 200 ones.
- Pause after the first question. Make sure students see that the 2 hundreds can be decomposed into 20 tens (or 200 ones) and split into 5 equal groups, and the 3 tens can be decomposed into 30 ones and split into 5 groups. Complete the second diagram to illustrate this reasoning.
- 5 minutes: partner work time on the last question

Synthesis

- Select students to share their responses and reasoning for the last question. Display them for all to see.
- Highlight the idea that each unit can be decomposed into 10 units of a lower place value to make it possible to create equal-sized groups.

Lesson Synthesis

10 min

“In earlier lessons, we solved division problems such as ‘712 divided by 4’ in different ways. We used familiar multiples or multiplication facts, or divided a series of smaller numbers. We also used area diagrams to reason about division.”

“Today we used base-ten diagrams and blocks to find quotients such as $712 \div 4$. How is this approach like earlier ones?” (It also involves performing division in a series of steps, rather than all at once.)

“How is this approach different?” (It involves:

- using place values
- dividing the amount in each place value into equal-size groups
- thinking of a digit in a number as 10 times the value of the digit to the right of it (for example, thinking of 3 tens as 30 ones)

“Instead of drawing base-ten pieces or using blocks, suppose we represent 712 with numbers and words: 7 hundreds + 1 ten + 2 ones. Can we still find $712 \div 4$ by reasoning about place values?” (Yes, we can distribute the hundreds, tens, and ones into 4 equal groups.)

“Here is a student’s unfinished work for finding $712 \div 4$. How would you complete it?”

Display:

“$712 \div 4$ means putting 7 hundreds + 1 ten + 2 ones into 4 equal groups.”

1 hundred	1 hundred	1 hundred	1 hundred

(178. Sample reasoning: After putting 1 hundred in each group, there are 3 hundreds, 1 ten, and 2 ones left. The hundreds can be decomposed into tens and the tens can be decomposed into ones so that there’s enough to put into 4 groups.

$$\begin{aligned} &3 \text{ hundreds} + 1 \text{ ten} + 2 \text{ ones} \\ =\ &30 \text{ tens} + 1 \text{ ten} + 2 \text{ ones} \\ =\ &28 \text{ tens} + 3 \text{ tens} + 2 \text{ ones} \\ =\ &28 \text{ tens} + 30 \text{ ones} + 2 \text{ ones} \\ =\ &28 \text{ tens} + 32 \text{ ones} \end{aligned}$$

$28 \div 4 = 7$, so 7 tens in each group.

$32 \div 4 = 8$, so 8 ones in each group.)

1 hundred 7 tens 8 ones	1 hundred 7 tens 8 ones	1 hundred 7 tens 8 ones	1 hundred 7 tens 8 ones

Suggested Centers

- Compare (1–5), Stage 4: Divide within 100 (Supporting)

- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Supporting)

Complete Cool-Down

Response to Student Thinking

Students decompose the dividend into 100, 30, and 2, divide each part by 4 separately, and do not divide completely when working with 30 and 2. (For example, they see that dividing 30 by 4 gives 7 with a remainder of 2 but do not combine it with the 2 ones.)

Next Day Support

- Before the warm-up, select a student's cool-down from the previous lesson (name anonymous). Ask students to identify what the student did well and what the student needs to do to improve the cool-down.

Lesson 18: Divide with Partial Quotients

Standards Alignments

Addressing 4.NBT.B.6, 4.OA.A.3

Teacher-facing Learning Goals

- Analyze ways of using and recording partial quotients to divide multi-digit numbers.

Student-facing Learning Goals

- Let's analyze and use an algorithm that uses partial quotients.

Lesson Purpose

The purpose of this lesson is to introduce students to ways to record partial quotients when dividing multi-digit numbers.

Previously, students have found quotients by decomposing a dividend and finding the quotient for each decomposed part until all of the dividend is divided. They have also reasoned in terms of multiplication—adding partial products until they reach the value of the dividend—and in terms of place value. They have also used area diagrams and base-ten diagrams—among other representations—to support their reasoning.

In this lesson, students use partial quotients and a couple of ways to record them systematically—by writing a series of equations, and by using an algorithm that uses partial quotients.

Access for:

Students with Disabilities

- Action and Expression (Activity 2)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Number Talk (Warm-up)

Materials to Gather

- Base-ten blocks: Activity 1

Lesson Timeline

Warm-up	10 min

Teacher Reflection Question

Today's lesson encouraged small-group collaboration. How did students interact with

Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

each other's ideas today in the work? Who was heard in their group? Who was not heard? How can you ensure in future small-group collaborations that all student's voices are heard?

Cool-down (to be completed at the end of the lesson)

5 min

Subtract Groups

Standards Alignments

Addressing 4.NBT.B.6

Student-facing Task Statement

Priya and Tyler use different methods to find $430 \div 5$. Their work is incomplete. Complete Priya's and Tyler's work.

Priya's work

$$\begin{array}{r} 300 \div 5 = \\ 100 \div 5 = \\ 30 \div 5 = \\ \hline 430 \div 5 = \end{array}$$

Tyler's work

$$\begin{array}{r} 6 \\ 20 \\ 60 \\ 5\,\overline{)430} \\ -\ 300 \quad 5 \times 60 \\ \hline \end{array}$$

What is the value of $430 \div 5$?

Student Responses

86. Sample reasoning: 300 is 60 groups of 5, 100 is 20 groups of 5, and 30 is 6 groups of 5. Adding the groups of 5—the 60, 20, and 6—gives the quotient.

Priya's work

$$\begin{array}{r} 300 \div 5 = 60 \\ 100 \div 5 = 20 \\ 30 \div 5 = 6 \\ \hline 430 \div 5 = 86 \end{array}$$

Tyler's work

$$\begin{array}{r l} \boxed{86} & \\ 6 & \\ 20 & \\ 60 & \\ 5\overline{)430} & \\ -\ 300 & 5 \times 60 \\ \hline 130 & \\ -\ 100 & 5 \times 20 \\ \hline 30 & \\ -\ 30 & 5 \times 6 \\ \hline 0 & \end{array}$$

Begin Lesson

Warm-up

10 min

Number Talk: Divide by 3

Standards Alignments

Addressing 4.NBT.B.6

This Number Talk encourages students to look for and make use of the structure of numbers in base-ten to mentally solve division problems. The reasoning elicited here will be helpful later in the lesson when students divide large numbers using increasingly more abstract strategies.

Instructional Routines

Number Talk

Student-facing Task Statement

Find the value of each expression mentally.

- $90 \div 3$
- $96 \div 3$
- $960 \div 3$

Launch

- Groups of 2
- Display one expression.
- "Give me a signal when you have an answer and can explain how you got it."
- 1 minute: quiet think time

- $954 \div 3$

Student Responses

- 30: I know that 90 is 10×9 and $9 \div 3$ is 3, so $90 \div 3$ is $10 \times 9 \div 3$, which is 30.
- 32: Ninety is 30×3 and 6 is 2×3, so 96 is $(30 \times 3) + (2 \times 3)$, which is $(30 + 2) \times 3$ or 32×3.
- 320: I know that $96 \div 3$ is 32, so $960 \div 3$ is $10 \times 96 \div 3$ or 10×32, which is 320.
- 318. I know that 954 is $960 - 6$, so it is $(320 \times 3) - (2 \times 3)$, which is $(320 - 2) \times 3$ or 318×3.

Activity

- Record answers and strategy.
- Keep expressions and work displayed.
- Repeat with each expression.

Synthesis

- "How did each expression help you find the next one?"
- Consider asking:
    - "Who can restate _______ 's reasoning in a different way?"
    - "Did anyone have the same strategy but would explain it differently?"
    - "Did anyone approach the expression in a different way?"
    - "Does anyone want to add on to____'s strategy?"

Activity 1

20 min

Decompose Dividends

Standards Alignments

Addressing 4.NBT.B.6

In this activity, students encounter a way to divide a multi-digit number by using partial quotients and writing equations for them. They analyze and interpret the equations and consider how it is like and unlike finding quotients using base-ten representations. In the next activity, students will be introduced to a way to record partial quotients vertically.

Access for English Learners

MLR8 Discussion Supports. To support the transfer of new vocabulary to long term memory, invite students to chorally repeat the word in unison 1-2 times: partial quotient.
Advances: Conversing, Speaking, Representing

Materials to Gather

Base-ten blocks

Student-facing Task Statement

1. Find the value of $465 \div 5$. Show your reasoning. You may use base-blocks if you find them helpful.
2. Here's how Priya finds the value of $465 \div 5$.

$$\begin{aligned} 400 \div 5 &= 80 \\ 60 \div 5 &= 12 \\ 5 \div 5 &= 1 \\ \hline 465 \div 5 &= 93 \end{aligned}$$

   a. What has Priya done? Describe her steps.
   b. How is Priya's method similar to your method?
   c. Use Priya's method to find the value of $428 \div 4$.

Student Responses

1. 93. Sample response: There are 4 hundreds, 6 tens, and 5 ones, to be put into 5 equal groups. Sample reasoning:

   The 5 tens and 5 ones can be divided into 5 groups.

1 ten 1 one	1 ten 1 one	1 ten 1 one	1 ten 1 one	1 ten 1 one

   The 4 hundreds can be decomposed into 40 tens and then split across 5 groups, 8 tens in each.

1 ten 1 one 8 tens 2 ones	1 ten 1 one 8 tens 2 ones	1 ten 1 one 8 tens 2 ones	1 ten 1 one 8 tens 2 ones	1 ten 1 one 8 tens 2 ones

   The 1 ten can be decomposed into 10 ones, which means 2 ones per group.
   In each group, there are 9 tens and 3 ones.
2. Sample response:

Launch

- Groups of 4.
- Give students access to base-ten blocks.

Activity

- Pause after the first question and discuss students' responses. Record and display responses for all to see.

Synthesis

- Invite students to share their interpretations of Priya's work and compare it to their reasoning in the first question.
    - "How did Priya decompose the number 465?" (By place value. $400 + 60 + 5$)
    - "What does Priya do after writing the first three equations?" (She adds up the quotients.)
- "We can find a quotient in parts—dividing a portion of the dividend at a time—until there is no more (or until there is not enough) of the dividend to divide."
- "Each quotient is called a partial quotient."

a. Priya divided 465 into 5 groups in smaller parts. First she divided 400 by 5, which gave 80. Then, she divided 60 by 5, which gave 12. Lastly, she divided 5 by 5, which is 1. The quotient is $80 + 12 + 1$, which is 93.

b. They are similar in that the division into equal groups is done hundreds, tens, and ones.

c.

$$\begin{array}{r} 400 \div 4 = 100 \\ 20 \div 4 = 5 \\ 8 \div 4 = 2 \\ \hline 428 \div 4 = 107 \end{array}$$

Activity 2

15 min

Tyler's Method

Standards Alignments

Addressing 4.NBT.B.6, 4.OA.A.3

In this activity, students are introduced to an algorithm that uses partial quotients, a vertical method of recording partial quotients. They compare and contrast this approach with other ways of dividing numbers using partial quotients and try using it to divide multi-digit numbers.

When students analyze Priya and Tyler's work and explain their reasoning, they critique the reasoning of others (MP3).

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Invite students to verbalize their strategy for problem 2 before they begin. Students can speak quietly to themselves, or share with a partner. *Supports accessibility for: Conceptual Processing, Organization*

Student-facing Task Statement

Tyler uses a different method to find the value of $465 \div 5$. Let's compare Priya's and Tyler's work.

Priya's method

$$\begin{array}{r} 400 \div 5 = 80 \\ 60 \div 5 = 12 \\ 5 \div 5 = \ \ 1 \\ \hline 465 \div 5 = 93 \end{array}$$

Tyler's method

$$\begin{array}{rrl} & \boxed{93} & \\ & 1 & \\ & 12 & \\ & 80 & \\ 5 & \overline{)465} & \\ - & 400 & 5 \times 80 \\ \hline & 65 & \\ - & 60 & 5 \times 12 \\ \hline & 5 & \\ - & 5 & 5 \times 1 \\ \hline & 0 & \end{array}$$

1. How are Priya and Tyler's methods alike? How are they different? List as many similarities and differences as you can find.
2. Why do you think Tyler uses subtraction in his method?
3. Show how Tyler might record the process of finding the value of $428 \div 4$.

Student Responses

1. Alike:
    - The 465 is decomposed into parts and each part is divided by 5 separately.
    - The partial quotients are 80, 12, and 1.
    - The 80, 12, and 1 are added up to get 93.
    - Both methods show 5 groups of 80, 5 groups of 12, and 5 groups of 1.

   Different:
    - Priya wrote division equations to show the divisor and partial

Launch

- Groups of 4
- Display Priya and Tyler's methods.

Activity

- "Tyler used a different method to record $465 \div 5$. Analyze what is happening in his method. Think about how the two methods are alike and different."
- 3 minutes: independent work time on the first two questions
- 3 minutes: small-group discussion
- Invite students to share their analyses on the two methods.
- If not mentioned by students, highlight that both Priya and Tyler divided in parts, but reasoned and recorded differently.
    - Priya recorded the partial quotients with division equations.
    - The partial quotients in Tyler's work are recorded as factors being multiplied by 5, and also listed above the dividend.
    - Tyler kept dividing in parts and subtracting until there's nothing left of the dividend to divide.
- Clarify the meaning of the numbers in Tyler's method before students work on the last question.
- 3 minutes: independent work time to find the value of $428 \div 4$.

Synthesis

- Invite students to share their responses. Highlight the different ways to decompose 428 or the different partial quotients that could be used to find $428 \div 4$.
- Make sure students see that Priya's equations and Tyler's method are simply

quotients.

- Tyler wrote multiplication expressions. He stacked the partial quotients vertically.
- Tyler subtracted numbers several times. There is no subtraction in Priya's work.
- Priya showed 5 groups of 80 in $400 \div 5 = 80$. Tyler showed it in 5×80.

2. To keep track of what is left after he removed a partial amount that has been divided by 5. (For example, after dividing 400 by 5, he removed it from 465 to get 65.)
3. Sample response:

```
  [107]
      2
      5
    100
 4 )428
 −  400     4 × 100
     28
 −   20     4 × 5
      8
 −    8     4 × 2
      0
```

```
  [107]
      7
     50
     50
 4 )428
 −  200     4 × 50
    228
 −  200     4 × 50
     28
 −   28     4 × 7
      0
```

two ways to record partial quotients, but they are not fundamentally different.

- "Tyler's vertical recording method is another type of algorithm."

Lesson Synthesis

10 min

"Today we learned to use an algorithm that uses partial quotients to divide numbers."

"How would you explain 'partial quotients' to a classmate who might be absent today?" (We can find a quotient in parts—dividing a portion of the dividend at a time—until there is no more or until there is not enough of the dividend to divide. Each quotient is called a partial quotient.)

"Suppose we'd like to find the value of $738 \div 9$ and know we could decompose the 738 into parts. How would we know what numbers to choose?" (Look for multiples of 9. Try to start with the largest multiple of 9 and 10 within 738.)

"What are some ways to decompose 738 into multiples of 9?" ($720 + 18$, or $450 + 270 + 18$, among others.)

Display:

$$738 \div 9$$

$$\begin{array}{r}
720 \div 9 = 80 \\
18 \div 9 = 2 \\
\hline
738 \div 9 = 82
\end{array}$$

$$\begin{array}{rl}
\boxed{82} & \\
2 & \\
80 & \\
9\overline{)738} & \\
-\ 720 & \quad 9 \times 80 \\
\hline
18 & \\
-\ 18 & \quad 9 \times 2 \\
\hline
0 &
\end{array}$$

“We saw two ways of recording partial quotients—by writing a series of equations and by recording the steps of division vertically. Where can we see the partial quotients in each one?”

Suggested Centers

- Compare (1–5), Stage 4: Divide within 100 (Supporting)
- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Supporting)

Complete Cool-Down

Response to Student Thinking

Students find the correct partial quotients but don't find their sum, or don't connect the sum to the value of $430 \div 5$.

Students complete one student's work, but not both.

Next Day Support

- Launch the warm-up or Activity 1 by highlighting important notation from previous lessons.

Lesson 19: Division With and Without Remainders

Standards Alignments

Addressing 4.NBT.B.6, 4.OA.A.3, 4.OA.B.4

Teacher-facing Learning Goals

- Find whole-number quotients and remainders using an algorithm that uses partial quotients.

Student-facing Learning Goals

- Let's find quotients and remainders using an algorithm that uses partial quotients.

Lesson Purpose

The purpose of this lesson is for students to use an algorithm that uses partial quotients to find whole number quotients and remainders with up to four-digit dividends and one-digit divisors. Students also analyze some common errors when using an algorithm that uses partial quotients.

In this lesson, students deepen and apply what they learned about partial quotients to divide four-digit numbers by single-digit divisors. They also deepen their understanding of an algorithm that uses partial quotients—by noticing how the algorithm shows whether a division would result in a remainder, and by analyzing missteps that are commonly made in an algorithm like it.

Access for:

Students with Disabilities

- Representation (Activity 2)

English Learners

- MLR2 (Activity 1)

Instructional Routines

Notice and Wonder (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	10 min
Activity 3	10 min

Teacher Reflection Question

Students have encountered many different ways to reason about division in the past few lessons. They might have other unique ways to think about division. Identify one or more ways in which your students' thinking offered a new insight or a positive surprise today.

Lesson Synthesis	10 min
Cool-down	5 min

Cool-down (to be completed at the end of the lesson)

5 min

Find a Quotient

Standards Alignments

Addressing 4.NBT.B.6

Student-facing Task Statement

How many groups of 4 are in 1,865?

Use partial quotients to show your reasoning.

Student Responses

466 groups with a remainder of 1. Sample response:

$$\begin{array}{r} \boxed{466} \\ 6 \\ 60 \\ 400 \\ 4\overline{)1{,}865} \\ -1{,}600 \\ \hline 265 \\ -\ 240 \\ \hline 25 \\ -\ 24 \\ \hline 1 \end{array}$$

Begin Lesson

Warm-up

10 min

Notice and Wonder: Equations with Hundreds

Standards Alignments

Addressing 4.NBT.B.6, 4.OA.B.4

This warm-up prompts students to analyze patterns and look for structure in division equations (MP7), and to reinforce their understanding of factors and multiples.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice? What do you wonder?

$$100 = 33 \times 3 + 1$$

$$200 = 66 \times 3 + 2$$

$$300 = 100 \times 3$$

$$400 = 133 \times 3 + 1$$

$$500 = 166 \times 3 + 1$$

$$600 = 200 \times 3$$

Student Responses

Students may notice:

- A number is multiplied by 3 in each equation.
- The value on the left side of the equal sign increases by 100 each time.
- The first number is always multiplied by 3.
- The product of the first two numbers are added by 1, 2, and 0.
- The first number ends with 33, 66, and 00.

Students may wonder:

- Why does the first number on the right change the way it does (increasing by 33, another 33, then 34)?

Launch

- Groups of 2
- Display the equations.
- “What do you notice? What do you wonder?”
- 1 minute: quiet think time

Activity

- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Share and record responses.

Synthesis

- “If we continue the pattern through 1,000, what would the equations look like?”
- “In all the equations, there is multiplication by 3, which suggests we could think of them in terms of division by 3.”
- “How could we interpret $300 = 100 \times 3$ in terms of division by 3?” (Dividing 300 by 3 gives 100.)
- “How about $600 = 200 \times 3$?” (Dividing 600 by 3 gives 200.)
- “Can we interpret the equation $100 = 33 \times 3 + 1$ in terms of division by 3? What does it tell us?” (Yes. Dividing 100 by 3 gives 33 and a remainder of 1.)
- “What about $500 = 166 \times 3 + 2$?” (Dividing 500 by 3 gives 166 and a remainder of 2.)

- Why do the third and sixth equations not have a number added to the product?
- If we continue the string of equations, would the pattern on the right side continue?

Activity 1

15 min

A Stack of Partial Quotients

Standards Alignments

Addressing 4.NBT.B.6, 4.OA.A.3

This activity develops students' understanding of the vertical method of recording partial quotients and their ability to use it to perform division. One of the quotients here involves a remainder, prompting students to interpret it. Students are reminded that the term "remainder" is used to describe "leftovers" when dividing.

Access for English Learners

MLR2 Collect and Display. Synthesis: Direct attention to words collected and displayed from the previous lessons. Invite students to borrow language from the display as needed, and update it throughout the lesson.
Advances: Conversing, Reading

Student-facing Task Statement

Jada used partial quotients to find out how many groups of 7 are in 389.

Analyze Jada's steps in the algorithm.

```
     [55]
       8
       7
      40
  7 ) 389
   −  280
      109
   −   49
       60
   −   56
        4
```

1. a. Look at the three numbers above 389. What do they represent?

Launch

- Groups of 2

Activity

- 3 minutes: independent work time on the first 2 problems.
- Pause after problem 2 to discuss students' responses.
- Display the different ways that students decompose 389 to divide it by 7.
- "Most other calculations we've seen so far

   b. Look at the three subtractions below 389. What do they represent?
   c. What is another way you can decompose 389 to divide by 7?
2. Is 389 a multiple of 7? Explain your reasoning.
3. Use an algorithm that uses partial quotients to find out how many groups of 3 are in 702.
4. Is 702 a multiple of 3? Explain your reasoning.

Student Responses

1. Sample response:
   a. First we make 40 groups of 7, then 7 groups of 7, and finally 8 groups of 7, for a total of 55 groups of 7.
   b. First we subtract 40×7 (or 7×40) or 280 from the dividend, 389, leaving 109. Then we subtract, 7×7 or 49 from the 109, leaving 60. Finally we subtract, 8×7 (or 7×8) or 56, leaving 4. The quotient is $40 + 7 + 8$, which is 55, with a remainder of 4.
   c.

$$\begin{array}{rrl} & \boxed{55} & \\ & 5 & \\ & 50 & \\ 7 & \overline{)389} & \\ - & 350 & 7 \times 50 \\ \hline & 39 & \\ - & 35 & 7 \times 5 \\ \hline & 4 & \end{array}$$

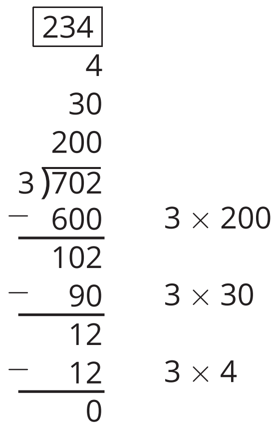
2. No, because there is a remainder of 4, and we cannot make another 7 groups (or another group of 7) with only 4 left. $389 = 7 \times 55 + 4$
3. 234. Sample reasoning:

end with a 0, but this one ends with a 4. What does the 4 tell us?" (We cannot make a group of 7 with 4 leftover. 389 is not a multiple of 7, and there are leftovers.)

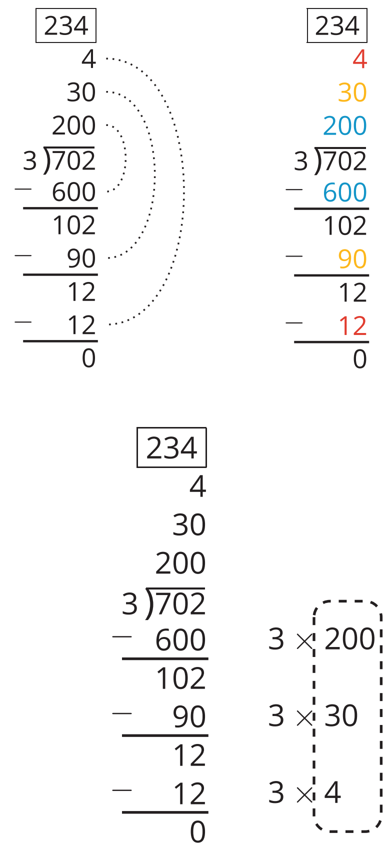
- "When we divide and end up with leftovers we call them remainders, because they represent what is remaining after we divide into equal groups."
- Display: $389 = 7 \times 55 + 4$
  - "How does this equation show that $389 \div 7$ has a remainder?" (It shows that 389 is not a multiple of 7. It also shows that 7 and 55 make a factor pair for 385, and 389 is 4 more than that.)
- 3 minutes: independent work time on the last 2 problems.
- As students work on the last two problems monitor for students who:
  - start with the largest multiple of 3 and 10 within 702 that they can think of to decompose the dividend (690, 600).
  - use the fewest steps to find the quotient.

Synthesis

- Display responses that show variation in how students decomposed 702 in problem 3.
- Highlight that there are countless ways of using partial quotients to divide a number, but it may be more efficient to divide larger groups than smaller groups (for example, removing 350 once, as opposed to removing 70 five times. Removing smaller groups is just as valid, however.)
- To help students see the relationship between corresponding partial products and partial quotients, consider illustrating them visually. Some examples:



4. Yes. 702 is a multiple of 3 because there is no remainder.



Activity 2

10 min

Andre and Elena’s Work

Standards Alignments

Addressing 4.NBT.B.6

In this activity, students apply their understanding of partial quotients and the vertical recording method to divide four-digit numbers. They also identify some errors that are common when finding quotients this way. When students determine where the errors are and correct them, they critique the reasoning of others and construct viable arguments (MP3).

Access for Students with Disabilities

Representation: Access for Perception. Provide access to colored pencils. As shown in the previous activity, invite students to shade corresponding partial products and partial quotients.
Supports accessibility for: Visual-Spatial Processing, Organization

Student-facing Task Statement

Andre and Elena are dividing 2,316 by 5. Before they begin, Andre says, “I can already tell that there will be a remainder.”

1. Without doing any calculations, decide if you agree with Andre. Explain your reasoning.
2. Here is Andre and Elena’s work. Each student made one or more errors. Identify the errors each student made. Then, show a correct computation.

Andre's Work

```
  [103]
      3
     60
     40
5 )2,316
 −2,000
    316
 −  300
     16
 −   15
      1
```

Elena's Work

```
  [400]
     60
    100
    300
5 )2,316
 −1,500
    816
 −  500
    316
 −  300
     16
```

Student Responses

1. Yes, I agree with Andre. Sample reasoning: All multiples of 5 end with a 5 or a 0. The number 2,316 doesn’t end with either 5 or 0, which means there will be a remainder when it is divided by 5.
2. Sample response:
    - Andre subtracted 2,000, which

Launch

- Groups of 2

Activity

- 1 minute: quiet think time for the first question
- Briefly discuss student responses. Highlight that multiples of 5 end with 0 and 5, so 2,316 is not a multiple of 5 and the division will result in a remainder.
- 3 minutes: independent work time for the second question.
- 1–2 minutes: partner discussion

Synthesis

- Select students to share their responses to the last question.
- “What are some ways to check our answers and avoid the mistakes Andre and Elena made? For example, how can we tell if dividing 2,316 by 5 gives a result in the 100s or in the 400s?” (We can estimate by multiplying the result by 5: 5×103 is a little over 500, and 5×400 is 2,000.)
- To reinforce the importance of keeping track of the partial quotients during division, consider annotating a corrected version of Elena's algorithm. Record the product that corresponds to each number being subtracted from the dividend (the 1,500, 500, 300, and 15).

should've been 5×400 but he wrote 40 instead. The quotient should've been $400 + 60 + 3$, or 463.
- Elena didn't finish the division. There is a remainder of 16, which can still be divided by 5 to get a whole-number partial quotient of 3. There's a remainder of 1.
- The quotient should've been $300 + 100 + 60 + 3$, or 463.

```
      [403]
          3
         60
        100
        300
   5 )2,316
    −1,500     5 × 300
        816
    −   500    5 × 100
        316
    −   300    5 × 60
         16
    −    15    5 × 3
          1
```

Activity 3

10 min

Incomplete Calculations

Standards Alignments

Addressing 4.NBT.B.6

The purpose of this activity is to reinforce the idea that there are many ways to use partial quotients to divide numbers and for students to see that some strategies are more practical or efficient than others.

Student-facing Task Statement

Here are four calculations to find the value of $3{,}294 \div 3$, but each one is unfinished.

Complete at least two of the unfinished calculations. Be prepared to explain why you chose them.

Launch

- Groups of 2–4
- "Choose at least two calculations to finish. Make sure each calculation is completed by someone in your group."

A

$$
\begin{array}{r l}
90 & \\
1{,}000 & \\
3\overline{)3{,}294} & \\
-3{,}000 & 3 \times 1{,}000 \\
\hline
294 & \\
-\ 270 & 3 \times 90 \\
\hline
\end{array}
$$

B

$$
\begin{array}{r l}
80 & \\
200 & \\
400 & \\
400 & \\
3\overline{)3{,}294} & \\
-1{,}200 & 3 \times 400 \\
\hline
2{,}094 & \\
-1{,}200 & 3 \times 400 \\
\hline
894 & \\
-\ 600 & 3 \times 200 \\
\hline
294 & \\
-\ 240 & 3 \times 80 \\
\hline
\end{array}
$$

C

$$
\begin{array}{l}
600 \div 3 = \\
600 \div 3 = \\
600 \div 3 = \\
600 \div 3 = \\
600 \div 3 = \\
270 \div 3 =
\end{array}
$$

D

$$
\begin{array}{r r}
 & 3{,}300 \div 3 = 1{,}100 \\
- & 6 \div 3 = \quad\quad 2 \\
\hline
\end{array}
$$

Student Responses

A

$$
\begin{array}{r l}
\boxed{1{,}098} & \\
8 & \\
90 & \\
1{,}000 & \\
3\overline{)3{,}294} & \\
-3{,}000 & 3 \times 1{,}000 \\
\hline
294 & \\
-\ 270 & 3 \times 90 \\
\hline
24 & \\
-\ 24 & 3 \times 8 \\
\hline
0 &
\end{array}
$$

B

$$
\begin{array}{r l}
\boxed{1{,}098} & \\
8 & \\
10 & \\
80 & \\
200 & \\
400 & \\
400 & \\
3\overline{)3{,}294} & \\
-1{,}200 & 3 \times 400 \\
\hline
2{,}094 & \\
-1{,}200 & 3 \times 400 \\
\hline
894 & \\
-\ 600 & 3 \times 200 \\
\hline
294 & \\
-\ 240 & 3 \times 80 \\
\hline
54 & \\
-\ 30 & 3 \times 10 \\
\hline
24 & \\
-\ 24 & 3 \times 8 \\
\hline
0 &
\end{array}
$$

Activity

- 3–4 minutes: independent work time
- 2 minutes: group discussion

Synthesis

- “How are the four strategies alike? How are they different?” (The first three are similar in that they involve partial quotients. The last one involves estimation.)
- “Which strategy or strategies do you find easy to follow? Hard to follow?”
- “Which strategy seems the most efficient? The least efficient?” (Calculations B and C seem lengthy and could be shortened by using larger multiples of 3. The last seems most efficient.)

C

$$
\begin{array}{r}
600 \div 3 = 200 \\
600 \div 3 = 200 \\
600 \div 3 = 200 \\
600 \div 3 = 200 \\
600 \div 3 = 200 \\
270 \div 3 = 90 \\
24 \div 3 = 8 \\
\hline
3{,}294 \div 3 = 1{,}098
\end{array}
$$

D

$$
\begin{array}{rr}
 & 3{,}300 \div 3 = 1{,}100 \\
- & 6 \div 3 = 2 \\
\hline
 & 3{,}294 \div 3 = 1{,}098
\end{array}
$$

Lesson Synthesis

10 min

"Today we looked at different ways to divide multi-digit numbers by one-digit divisors. Some divisions result in a number with a remainder and others result in no remainders."

"Can we always tell if there will be a remainder?" (No, not always, but sometimes we can.)

"How can we sometimes tell that there will be a remainder?" (We can use what we know about the multiples of a number. For example, all multiples of 2, 4, 6, and 8 have an even number for the last digit. All multiples of 5 end in 5 or in 0.)

"Some ways to divide are pretty lengthy. What are some ways to divide efficiently?" (Dividing larger portions of the dividend, or taking larger multiples of the divisor.)

"In the last activity, we saw estimating as a rather efficient way to find a quotient. How might we use estimation to find $5{,}970 \div 3$ or $6{,}986 \div 7$?" (Notice that:

- $5{,}970$ is 30 less than $6{,}000$, which is $3 \times 2{,}000$. Thirty is 3×10, so $5{,}970$ is $3 \times 1{,}990$.
- $6{,}986$ is close to $7{,}000$, which is $7 \times 1{,}000$, and $6{,}986$ is 14 or 7×2 less than $7{,}000$. So $6{,}986$ is 7×998.)

"How can we check the result of our division to make sure it's not off?" (We can multiply the result by the divisor, adding the remainder if there is one, and see if it gives the dividend.)

Suggested Centers

- Compare (1–5), Stage 4: Divide within 100 (Supporting)
- Rolling for Fractions (3–5), Stage 2: Multiply a Fraction by a Whole Number (Supporting)

Complete Cool-Down

Response to Student Thinking

Students identify partial quotients appropriately but make computation errors when multiplying a number by 4, or when subtracting numbers from the dividend.

Next Day Support

- Before the warm-up, invite students to work in small groups to discuss a correct response to this cool-down.

Lesson 20: Interpret Remainders in Division Situations

Standards Alignments

Building On	4.OA.B.4
Addressing	4.NBT.B.6, 4.OA.A.3

Teacher-facing Learning Goals

- Interpret the result and remainder of division in situations.
- Represent and solve problems that involve finding whole-number quotients and remainders.

Student-facing Learning Goals

- Let's solve problems involving division and interpret remainders.

Lesson Purpose

The purpose of this lesson is for students to represent and solve contextual problems that involve dividing a whole number of up to four-digits by a single-digit divisor, resulting in a number with or without a remainder. Students also interpret the result and remainder given a situation.

By now students have developed various strategies to divide multi-digit numbers by single-digit divisors and have used different representations along the way. In this lesson, students apply what they learned to solve a variety of word problems that involve division (MP2).

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Action and Expression (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

Choral Count (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	15 min

Teacher Reflection Question

What productive and unproductive beliefs did students show when they were solving problems today? How might you amplify the

Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

productive beliefs and address the unproductive ones?

Cool-down (to be completed at the end of the lesson)

5 min

Miscounting?

Standards Alignments

Addressing 4.NBT.B.6, 4.OA.A.3

Student-facing Task Statement

Mai is reciting multiples of 6. The last number she calls out is 194. Clare says, "I think you may have made a mistake."

Do you agree with Clare? Explain or show your reasoning.

Student Responses

Yes, I agree with Clare. Sample reasoning:

- 194 is not a multiple of 6. I know that $6 \times 30 = 180$, and 194 is 14 away from 180. Because 14 is not a multiple of 6, then 194 is also not a multiple of 6.
- Six is not a factor of 194. I divided 194 by 6 and got 32 with a remainder of 2. If Mai counted correctly, she would have called out 192 and then 198.

Begin Lesson

Warm-up

10 min

Choral Count: 2, 3, and 5

Standards Alignments

Building On 4.OA.B.4

Addressing 4.NBT.B.6

The purpose of this warm-up is to elicit strategies and understandings students have for identifying multiples of small numbers, primarily by looking for and making use of structure (MP7). These understandings will be helpful later when students solve division problems that involve distinguishing quotients with and without a remainder.

Instructional Routines

Choral Count

Student Responses

Counts by 2:

- 90 92 94 96
- 98 100 102 104
- 106 108 110 112

Counts by 3:

- 90 93 96
- 99 102 105
- 108 111 114

Counts by 5:

- 90 95 100
- 105 110 115

Launch

- “Count by 2 starting at 90.”
- Record as students count.
- Stop counting and recording at 112.

Activity

- “What patterns do you notice in each of the recorded counts?”
- Repeat with 3 and 5.
- “Count by 3 starting at 90.” Stop counting and recording at 114.
- “Count by 5 starting at 90.” Stop counting and recording at 115.

Synthesis

- “Is 105 a multiple of 2, 3, or 5? How do you know?”
- “Is 105 a multiple of 15?”

Activity 1

15 min

Muffins and Seats

Standards Alignments

Addressing 4.NBT.B.6, 4.OA.A.3

This activity encourages students to interpret the quantities in situations, represent them mathematically, use their representations to find solutions, and then interpret their solutions in context (MP2). The dividends here are limited to three-digit numbers.

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Develop fluency with multiplication by 4 and 9. Provide a partially completed two-column table for each factor, and suggest that students record times tables that might be helpful for this activity before getting started. For example, in the left column of one table, students should complete a list of equations showing 9 times 1–10 ($9 \times 1 = 9, 9 \times 2 = 18 \ldots 9 \times 10 = 90$). On the right, students should complete a list of equations showing 9 times multiples of 10 ($9 \times 10 = 90, 9 \times 20 = 180 \ldots 9 \times 100 = 900$). Repeat for 4, and invite students to use this reference they've made to complete the task.
Supports accessibility for: Conceptual Processing, Memory

Student-facing Task Statement

1. Two bakers at a bakery made 378 muffins. The muffins are put in boxes of 4.

   - The first baker says they will need 94 boxes for all the muffins.
   - The second baker says 95 boxes are needed.

   Who do you agree with? Explain or show your reasoning.
2. An auditorium seats 258 people. The seats are arranged in rows of 9, but there is one short row with fewer than 9 seats.

   How many rows of 9 seats are there? How many seats are in the shorter row?

Launch

- Groups of 2

Activity

- 5–6 minutes: independent work time
- 2–3 minutes: partner discussion
- Monitor for students who appropriately interpret remainders based on the situation.

Synthesis

- Invite students to share their responses and reasoning.
- For problem 1, highlight that both 94 and 95 boxes are plausible if they could be defended and the assumptions are made clear. (For example, students might say that the bakers could have two leftover muffins rather than trying to sell them in

Student Responses

1. Sample responses:
    - I agree with the first baker. $378 \div 4$ is 94 with a remainder of 2. The bakers can't just put the 2 extra muffins in a box and sell half a box, so they only need 94 boxes.
    - I agree with the second baker. If all the muffins are to be put in a box, there will be 94 boxes of 4 and 1 box with only 2 muffins. This means they will need 95 boxes.
2. There are 28 rows of 9 and 1 row of 6. Sample reasoning: 258 divided by 9 is 28 with a remainder of 6.

boxes, so 94 boxes are enough.)

- "What equation could we write to describe the relationship between the number of muffins and the number of full boxes?" (One possible equation: $(94 \times 4) + 2 = 378$.)
- For the last problem, ask: "What equation could we write to describe the relationship between the number of rows and the number of seats?" (One possible equation: $(28 \times 9) + 6 = 258$.)

Activity 2

20 min

Save for a Garden

Standards Alignments

Addressing 4.NBT.B.6, 4.OA.A.3

In this activity, students continue to solve contextual problems that involve division (MP2). Here, the dividends extend to four-digit numbers and the problems demand a greater lift.

In the second half of the activity, students are asked to reason in the opposite direction: given a division expression, they are to invent a situation that it can represent and interpret the value of the expression in context.

If time permits, consider asking students to create a visual display of the situation they invent for problem 2, so they can present the situation and their reasoning to the class.

Access for English Learners

MLR8 Discussion Supports. Synthesis: At the appropriate time, give groups 2–3 minutes to plan what they will say when they present to other groups. “Practice what you will say when you share your strategy with another group. Talk about what is important to say, and decide who will share each part.”
Advances: Speaking, Conversing, Representing

Student-facing Task Statement

1. A school needs $1,270 to build a garden. After saving the same amount each month for 8 months, the school is still short by $6.

   How much did they save each month? Explain or show your reasoning.

2. Choose one of the following division expressions.

   $711 \div 3$ $\qquad$ $3{,}128 \div 8$

   a. Write a situation to represent the expression.
   b. Find the value of the quotient. Show your reasoning.
   c. What does the value of the quotient represent in your situation?

Student Responses

1. $158 a month. Sample reasoning:
   a. $1{,}270 \div 8 = 158$ with a remainder of 6. This means the school has saved up $1,264.
   b. $1{,}270 - 6 = 1{,}264$ and $1{,}264 \div 8 = 158$.
2. Sample response for $711 \div 3$:

Launch

- Groups of 2

Activity

- 3–4 minutes: independent work time on the first problem
- Invite students to share responses and reasoning.
- “What equation(s) can we write to represent the relationship between the amount saved each month, the savings, the number of months of saving, and the amount needed for the garden?” (One possible equation: $(8 \times 158) + 6 = 1{,}270$)
- 5–7 minutes: independent work time on problem 2

Synthesis

- Students find a partner who chose a different expression and take turns presenting their work. The person listening should consider whether the response makes sense and check if the quotient is correct.
- If time permits, students share with a different student or partnership.

a. Three siblings collected 711 pennies and are dividing them equally. The quotient is the number of pennies each sibling gets.
b. $711 \div 3 = 237$
c. Each sibling gets 237 pennies.

Sample response for $3,128 \div 8$:

a. A family is making a road trip that is 3,128 miles long over 8 days. The quotient is the distance traveled each day if they travel the same distance per day.
b. $3,128 \div 8 = 391$
c. The family travels 391 miles a day.

Lesson Synthesis

10 min

"Today we solved problems that involved division. What strategies did you find yourself using to divide numbers? Did you:

- use partial products?
- use partial quotients?
- draw diagrams?
- divide by place value (thousands, hundreds, tens, and ones)?
- write a series of equations?
- estimate first?"

Suggested Centers

- Watch Your Remainder (4–5), Stage 1: One-digit Divisors (Addressing)
- Compare (1–5), Stage 4: Divide within 100 (Supporting)

Student Section Summary

In this section, we solved different problems that involve dividing whole numbers.

We recalled two ways of thinking about division. For example, suppose $274 \div 8$ represents a situation where 274 markers are put into equal groups. The value of $274 \div 8$ can tell us:

- how many markers are in each group if there were 8 groups, or
- how many groups can be made if there were 8 markers in each group.

We learned that the 274 in $274 \div 8$ is called the **dividend**. We then explored different ways to find the value of a quotient (or the result of the division). For $274 \div 8$, we can:

- Divide by place value and think about putting 2 hundred, 7 tens, and 4 ones into 8 equal groups.
- Divide in parts and find partial quotients. For example, we can first find $160 \div 8$ (which is 20), and then $80 \div 8$ (which is 10), and then $32 \div 8$ (which is 4).
- Think in terms of multiplication. For example, we can think of $8 \times 20 = 160$, $8 \times 10 = 80$, and so on.

Here is one way to record division using partial quotients:

$$
\begin{array}{rrl}
 & \boxed{34} & \\
 & 4 & \\
 & 10 & \\
 & 20 & \\
8\,\big) & \overline{274} & \\
- & 160 & 8 \times 20 \\
\hline
 & 114 & \\
- & 80 & 8 \times 10 \\
\hline
 & 34 & \\
- & 32 & 8 \times 4 \\
\hline
 & 2 &
\end{array}
$$

Sometimes a division results in a leftover that can't be put into equal groups or is not enough to make a new group. We call the leftover a **remainder**. Dividing 274 by 8 gives 34 and a remainder of 2.

Complete Cool-Down

Response to Student Thinking

Students disagree with Mai because of an error in computation or reasoning. (For example, they may think that 200 is a multiple of 6 and therefore 194, which is 6 less than 200, is also a multiple of 6.)

Next Day Support

- Before the warm-up, invite students to work in small groups to discuss a correct response to this cool-down.

Section D: Let’s Put It to Work: Problem Solving with Large Numbers

Lesson 21: Different Ways to Solve Problems

Standards Alignments

Addressing 4.NBT.B.5, 4.NBT.B.6, 4.OA.A.3

Teacher-facing Learning Goals

- Interpret products, quotients, and remainders in terms of a situation.
- Solve multi-step problems in ways that make sense to students.

Student-facing Learning Goals

- Let’s reason about and solve multi-step problems.

Lesson Purpose

The purpose of this lesson is for students to represent and solve multi-step contextual problems involving multiplication and division, including division with remainders.

In this lesson, students analyze and use various strategies and representations to reason about multi-step problems. They use their knowledge of multiplication and division, including the ideas of factors and multiples, to represent situations. Students also interpret products, quotients, and remainders in context (MP2).

Access for:

Students with Disabilities

- Representation (Activity 2)

English Learners

- MLR7 (Activity 1)

Instructional Routines

MLR5 Co-craft Questions (Activity 2), Which One Doesn’t Belong? (Warm-up)

Materials to Copy

- Going on a Field Trip (groups of 1): Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

In the first activity, students had to make sense of strategies, explanations, and representations that were not their own. What did students say or do that showed the exercise was effective in expanding their view of problem solving?

Cool-down (to be completed at the end of the lesson)

5 min

Big Weekend at the Movies

Standards Alignments

Addressing 4.NBT.B.5, 4.NBT.B.6, 4.OA.A.3

Student-facing Task Statement

A one-room movie theater has 278 seats. Its goal is to sell 2,600 tickets every weekend. The theater plays a movie 5 times each Saturday and 4 times each Sunday.

Last weekend, the movie theater was completely full for every movie played on Saturday and Sunday. Did the movie theater meet its goal?

Student Responses

No, the goal was not met. Sample reasoning: A ticket was sold for each seat 5 times on Saturday, and 4 times on Sunday: $278 \times 5 = 1{,}390$ and $278 \times 4 = 1{,}112$, and $1{,}390 + 1{,}112 = 2{,}502$, The goal was not met because $2{,}502$ is less than 2,600.

Begin Lesson

Warm-up

10 min

Which One Doesn’t Belong: Expressions with 5 or 90

Standards Alignments

Addressing 4.OA.A.3

This warm-up prompts students to carefully analyze and compare features of expressions. In making comparisons, students practice looking for structure (MP7). The work here prepares students to reason flexibly and to use multiple strategies (including writing different expressions) to solve word problems later in the lesson.

Instructional Routines

Which One Doesn't Belong?

Student-facing Task Statement

Which one doesn't belong?

A. 5×90

B. $90 + 90 + 90 + 90 + 90$

C. $(4 \times 90) + (1 \times 90)$

D. $3 \times 3 \times 10 \times 5$

Student Responses

Sample responses:

- A is the only one that doesn't have multiple operations.
- B is the only one that doesn't involve multiplication.
- C is the only one that doesn't use the same operation in the expression.
- D is the only one that doesn't have a 90. It is the only in which 450 is not shown in terms of groups of 90.

Launch

- Groups of 2
- Display expressions.
- "Pick one that doesn't belong. Be ready to share why it doesn't belong."
- 1 minute: quiet think time

Activity

- "Discuss your thinking with your partner."
- 2–3 minutes: partner discussion
- Record responses.

Synthesis

- "What do all of the expressions have in common?" (They all have a value of 450.)
- "Can you write another expression that has the same value as these expressions but that doesn't belong?"

Activity 1

20 min

Going on a Field Trip

Standards Alignments

Addressing 4.NBT.B.5, 4.OA.A.3

In this activity, students encounter a multiplication problem that can be reasoned in a number of ways. After finding a solution, they analyze several other strategies. As they make sense of alternative solution paths and representations, students practice reasoning abstractly and quantitatively (MP2).

Before the lesson, display the posters with the following five strategies (as shown in the Instructional master) around the classroom.

A. Clare:

If tickets were $20 each, the cost would be 45×20 or 900. Because $18 is $2 less than $20, we need to subtract 45×2 from 45×20, or subtract 90 from 900, which is 810.

B. Kiran:

$10 \times 18 = 180$
$20 \times 18 = 360$
$40 \times 18 = 720$
$5 \times 18 = 90$
$45 \times 18 = (40 \times 18) + (5 \times 18) = 720 + 90 = 810$

C. Han:

100 tickets cost 1,800. 50 tickets is half of 1,800, which is 900. 45 tickets is less than 50 tickets, so they will have enough money.

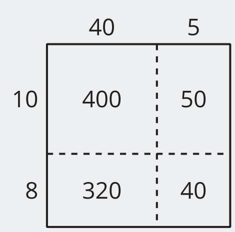
D. Tyler:

$2 \times 45 = 90$
$9 \times 2 = 18$

This means:

18×45
$= 9 \times 2 \times 45$
$= 9 \times 90$
$= 810$

E. Mai:



400 + 320 + 50 + 40 = 810

Access for English Learners

MLR7 Compare and Connect. Synthesis: Lead a discussion comparing, contrasting, and connecting the different strategies. Ask, “How are the strategies the same?”, “How are they different?” and “How do these different strategies show the same information?”
Advances: Representing, Conversing

Materials to Copy

Going on a Field Trip (groups of 1)

Student-facing Task Statement

1. Forty-five students are going on a field trip to a museum. Tickets for the museum are $18 each. Teachers have $900 to cover tickets for the trip. Will this be enough to cover tickets for every student?

   If yes, will there be any leftover money and how much?

   If no, how much more money is needed?

2. Your teacher will show five strategies for answering the previous question. Analyze the strategies.

   a. Which strategy is closest to yours? With a partner, take turns explaining how your strategy is close to the poster you chose.

   b. Discuss a different strategy with your partner. Try using this strategy to find the value of 14×35.

Student Responses

1. Yes. The teachers will need $810 to cover tickets, so $900 is enough and would leave $90 leftover.
2. a. Answers vary.

   b. Sample response: Mai used a diagram to find 45×18. She decomposed the length of the rectangle into $40 + 5$ and the width into $10 + 8$, partitioning the rectangle into four smaller ones. She then computed the areas of the four rectangles and found their sum.

Launch

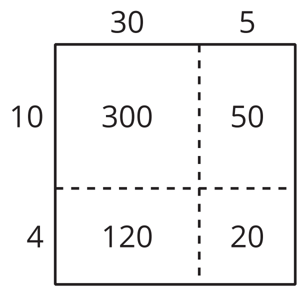
- Groups of 2

Activity

- 3 minutes: independent work time
- 1 minute: partners discuss responses to the first question
- 10 minutes: gallery walk to complete the second question, 1–2 minutes per poster
- As students answer the last question, monitor for the solution paths that students identify as sensible or understandable.

Synthesis

- Poll the class on which strategy most closely resembles their own.
- Poll the class on which strategy that doesn't resemble their own makes the most sense to them.
- Invite students to share their responses for the last question and why they found a particular strategy to make sense.



300 + 120 + 50 + 20 = 490

Advancing Student Thinking

Students may not see connections between their strategies and the ones shown in posters A–E. Consider offering students an option to stand at a poster labeled “a totally different strategy” and asking: “How is your strategy different from the ones shown?” During synthesis, ask the class if they see any connections between the “totally different” strategies and a strategy they have selected.

Activity 2

15 min

A Trip to the Movies

Standards Alignments

Addressing 4.OA.A.3

Students begin the activity by looking at the problem displayed, rather than in their books. At the end of the launch, students work on the problem. This activity prompts students to use what they know about multiplication, division, factors, and multiples to solve problems. The problem does not have a question, so students will need to make sense of the context and generate potential questions that might be answered (MP2). Students are encouraged in the task to attend to the details of the situation and to engage in genuine curiosity about the mathematics that is embedded within it.

This activity uses MLR5 Co-craft Questions. Advances: writing, reading, representing

Access for Students with Disabilities

Representation: Develop Language and Symbols. Represent the problem in multiple ways to support understanding. For example, invite students to represent the situation as a comic strip or a collage. Offer relevant images such as a movie theater, a sign showing admissions prices, a cash register, and a calendar. Alternatively, invite students to act out the situation. Consider using play money, signs with two days of the week written on them, and the action of "fast forwarding" through days and a night. Consider asking, "How might you represent the situation in a mathematical diagram?"
Supports accessibility for: Conceptual Processing, Visual-Spatial Processing, Attention

Instructional Routines

MLR5 Co-craft Questions

Student-facing Task Statement

Movie tickets are $9 each. The theater sold the same number of tickets two days in a row.

The theater made $3,132 from ticket sales on the first day.

1. Record and answer one question of your choice from the list the class generated. Discuss your strategy with your partner.
2. Use the given information about movie tickets to complete the following statement:

   __________ tickets were sold on the first and second days.
3. A medium drink is $7 and small popcorn is $5. If each ticket holder purchases popcorn and a drink, how much money will the theater collect from the sales of popcorn and drink?

Student Responses

1. Answers vary. Sample question and response:

Launch

- Groups of 2

MLR5 Co-Craft Questions

- Read only the first two paragraphs without revealing the question(s).
- "Write a list of mathematical questions that could be asked about this situation."
- 2 minutes: independent work time
- 2–3 minutes: partner discussion
- Invite several students to share one question with the class. Record responses for use later in the task.
- "What information from the situation can be used to answer this question?" (The number of days tickets were purchased, the total number of money earned by the theater, the price of movie tickets)
- Reveal the task (students open books), and invite additional connections.

Activity

- "Choose a question from the list to

- How many tickets were sold on the first day?
- 348 tickets were sold on the first day. $3,132 \div 9 = 348$

2. 696 tickets were sold on the first and second day. $348 \times 2 = 696$
3. $8,352. Sample reasoning: 696 people purchased tickets. Each person bought popcorn for $7 ($696 \times 7 = 4,872$) and a drink for $5 ($696 \times 5 = 3,480$), so the theater earned $4,872 + 3,480$ or $8,352.

answer."

- "Work with a partner to complete the activity."
- Remind students of the list of questions generated during the launch as a reference during the activity.

Synthesis

- Select 1–2 students to share their reasoning and responses.
- If not clarified in students' explanations, discuss a possible path for finding out the number of tickets sold over the two days using the given information. (For instance: Each ticket is $9 and we know the total amount of money earned by selling tickets in one day, $3,132. If we divide the total amount earned by the price of each ticket, we can find out how many tickets were sold on one day. $3,132 \div 9 = 348$. If 348 tickets were sold on one day, then 348×2 or 696 tickets were sold in the two days. We can also multiply $3,132 by 2 first then divide by $9 to get the total number of tickets.)

Advancing Student Thinking

Students may think of questions that cannot be answered using the information provided. Consider asking: "What would we need to know to be able to answer this question?" and "How might we find out this information?" The result of these questions may not enable the students to answer the questions, but will support them in making sense of the problem and identifying information necessary to solve the problems.

Lesson Synthesis

10 min

"Today we encountered problems with more than one step that can each be solved using different strategies. For instance, we saw at least five ways to think about the product of 45 and 18. Some of the strategies involve using multiplication and division equations, or multiplying and dividing mentally."

Display the five strategies from the first activity and students' reasoning from the second activity.

"Look back at your work today. Can you find an example in which you solved a problem by using more than one step?"

Record strategies and discuss how strategies were used to address different steps in the multi-step problem.

Suggested Centers

- Compare (1–5), Stage 7: Multi-digit Operations (Addressing)
- Watch Your Remainder (4–5), Stage 1: One-digit Divisors (Addressing)

Complete Cool-Down

Response to Student Thinking

Students estimate instead of calculating the number of tickets sold and draw an incorrect conclusion as a result. (For example, they may think that $5 + 4$ is 9, which is close to 10, find 10×278 instead, and reason that 2,780 is quite a bit more than 2,600.)

Next Day Support

- Launch Activity 1 with a discussion about whether to estimate or to calculate precisely when solving a problem such as in the cool-down. Invite students to consider the implications of each approach.

Lesson 22: Problems About Perimeter and Area

Standards Alignments

Addressing 4.MD.A.2, 4.MD.A.3, 4.NBT.B.5, 4.OA.A.3

Teacher-facing Learning Goals

- Solve multi-step problems involving measurement conversions, perimeter, and area.

Student-facing Learning Goals

- Let’s solve situations involving perimeter and area.

Lesson Purpose

The purpose of this lesson is for students to apply what they know about multiplication and division to convert units of measurement and solve multi-step problems involving perimeter and area.

This lesson prompts students to apply their reasoning skills and knowledge of all operations to solve problems about area and perimeter. Along the way, students also use multiplication and division to convert units of measurement. Most numbers used here are two- and three-digit numbers. The problems in the lesson may include more than one step and can be solved in multiple ways, offering students opportunities to construct logical arguments to communicate their thinking and to critique the reasoning of others (MP3). As students begin the lesson remind them of their past experiences with multi-step problems and explain that the problems in this lesson may involve more than one step.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

How Many Do You See? (Warm-up)

Materials to Gather

- Grid paper: Activity 1
- Inch tiles: Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Lesson Synthesis	10 min
Cool-down	5 min

Teacher Reflection Question

Some problems in the lesson can reveal the depth of students' understanding of multiplication and division, the flexibility of their thinking, and their ability to make use of structure. What evidence of flexible reasoning, structural thinking, or deep understanding did you see today?

Cool-down (to be completed at the end of the lesson)

5 min

Paper for a Banner

Standards Alignments

Addressing 4.MD.A.2, 4.MD.A.3, 4.NBT.B.5

Student-facing Task Statement

Han has a rectangular piece of paper that is 96 inches by 36 inches. He is using it to create a banner for Awards Day. Last year the banner measured 2,304 square inches.

1. Will the new banner fit in the same area that the old banner was? Show your reasoning.
2. What is the difference in square inches between the area of last year's banner and this year's banner?

Student Responses

1. No. Sample reasoning:
    - The paper for this year's banner has an area of 3,456 square inches, because $96 \times 36 = 3{,}456$. Last year's banner had an area of 2,304 square inches, because $48 \times 48 = 2{,}304$, so Han will need a bigger space to hang the new banner.
2. The difference is 1,152. $3{,}456 - 2{,}304 = 1{,}152$

Begin Lesson

Warm-up

10 min

How Many Do You See: Shaded Squares

Standards Alignments

Addressing 4.OA.A.3

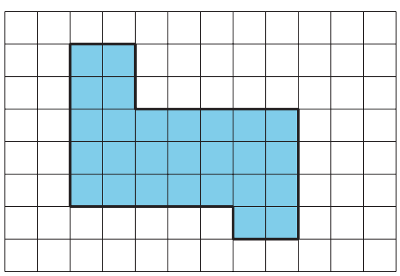
The purpose of this How Many Do You See routine is to prompt students to decompose a rectilinear figure to find its area and to recognize that there are many ways to do so. Students are also reminded that area is additive. The reasoning here prepares students to reason flexibly about the area of rectilinear figures later in the lesson.

Instructional Routines

How Many Do You See?

Student-facing Task Statement

How many shaded squares do you see? How do you see them?



Student Responses

27 squares. Sample responses:

- I see a 2-by-2 square, a 7-by-3 rectangle, and a 2-by-1 square.
  $(2 \times 2) + (7 \times 3) + (2 \times 1) = 27$
- I see a 2-by-5 rectangle, a 3-by-3 square, and a 2-by-4 rectangle.
  $(2 \times 5) + (3 \times 3) + (2 \times 4) = 27$
- I see a 6-by-7 rectangle with two corners cut out. The corners are rectangles, 5 by 1 and 5 by 2.
  $(6 \times 7) - (5 \times 1) - (5 \times 2) = 42 - 5 - 10 = 27$

Launch

- Groups of 2
- "How many do you see? How do you see them?"
- Flash the image.

Activity

- 30 seconds: quiet think time
- Display the image.
- 1 minute: partner discussion
- Record responses.

Synthesis

- "We've seen how helpful it is to decompose the figure into rectangles. In how many ways could we do that here?" (Many ways)
- As students share each way, record the thinking for all to see.
- "Are there ways to partition that are more helpful than others?" (Partitioning into larger rectangles is more efficient than smaller ones. The latter would mean more multiplication and more partial areas to add up.)

Activity 1

15 min

Create a Class Banner

PLC Activity

Standards Alignments

Addressing 4.MD.A.2, 4.MD.A.3, 4.OA.A.3

In this activity, students solve geometric problems by reasoning about length and area, decomposing and recomposing of rectangles, considering units of measurements, and performing operations.

Each question can be approached in a variety of ways. Consider asking students to create a visual display of their approach and to share it with the class.

The first problem offers students an opportunity to make sense of a problem and persevere in solving it (MP1). They may focus on the area of the banner and poster paper or start thinking about cutting up the poster paper into pieces that can be used for the banner. They will also need to convert between feet and inches at some point in their solution.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate background knowledge. Say, "We are going to solve problems related to measurement and rectangles today. What are some words that we might come across?" Prompt for students to include words such as length, area, perimeter, inches, feet, yards, and square units. Then ask, "What are some relationships between these words that might be helpful for us to remember?" Prompt for students to include both conceptual relationships (for example, area is measured in square units) and conversions (for example, there are 12 inches in 1 foot). Create a visual display to record responses and invite students to examine some tools for measuring length, such as a ruler, yardstick, or tape measure.
Supports accessibility for: Conceptual Processing, Language, Memory

Materials to Gather

Grid paper, Inch tiles

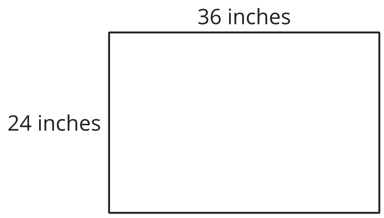
Student-facing Task Statement

Jada's teacher bought a poster paper that measures 36 inches by 24 inches. Her plan is to cut it into pieces, rearrange them, and tape

Launch

- Explain what a banner is or show an example, if needed.
- Give students access to grid paper and inch

them back together to create a welcome banner that is 8 inches tall and 8 feet long.



1. Does she have enough paper to make the banner? Show your reasoning.
2. How many square inches is the poster paper?

Student Responses

1. Yes. Sample reasoning:
    - 8 feet is 96 inches. Jada's teacher can cut the poster paper into three rectangles that are 36 inches by 8 inches each. If she puts them side by side (along the short end), she will have 108 inches for the banner, which only needs to be 96 inches $(3 \times 36 = 108)$.
    - Jada's teacher can cut the poster paper into four rectangles that are 24 inches by 8 inches, plus an extra rectangle that is 24 inches by 4 inches. If she puts the four rectangles side by side (along the shorter end), the banner will be 96 inches $(4 \times 24 = 96)$.
2. The area of the poster paper is 864 square inches. Sample reasoning:
    - $36 \times 24 = 864$
    - $108 \times 8 = 864$
    - $(36 \times 8) \times 3 = 288 \times 3 = 864$
    - 36 inches by 24 inches is 3 feet by 2 feet, so the area is 6 square feet. There are 144 square inches in 1 square foot. $6 \times 144 = 864$

tiles.

Activity

- 5–7 minutes: independent work time
- Monitor for different ways students reason about decomposing the 24 by 36 rectangle:
    - students who discuss the side length that is 8 feet long and consider how many inches long it would be
    - students who use tiles or a drawing to help reorganize the area

Synthesis

- Select students who use different strategies to share their reasoning. Record and display their strategies.
- If it does not come up as a strategy, consider asking, "How could Jada's teacher cut the paper up and rearrange it to make a banner?"
- Consider discussing any benefits or potential challenges of the different approaches. "Are some strategies more efficient or more prone to error than others?" (When cutting the paper into more pieces, there are more measurements to account for, making it more likely to miss something. Cutting the paper into more pieces is also less efficient for Jada's teacher, as it means more taping as well.)

Advancing Student Thinking

Students may see that there are several things to consider about the situation but may be unsure how to begin. Consider asking:

- "What do you know about the problem and the situation? What don't you know?"
- "What are some ways to visualize or show what Jada's teacher plans to do?"
- "How might the relationship between feet and inches help us think about this problem?"

Activity 2

20 min

Replace the Classroom Carpet

Standards Alignments

Addressing 4.MD.A.2, 4.MD.A.3, 4.OA.A.3

In this activity, students perform operations on multi-digit numbers to solve situations about perimeter and area. They use operations to convert units of measurements along the way. Converting inches to feet could be done by dividing by 12, but this is not an expectation at this point. Students could perform the conversion with multiplicative reasoning. To convert 180 inches into feet, for example, they could reason $12 \times ? = 180$, or $12 \times 10 = 120$ and $12 \times 5 = 60$.

In grade 3, students learned that area is additive, and that the area of rectilinear figures can be found by decomposing them into non-overlapping rectangles. Students apply that understanding here, after converting lengths in different units into the same unit.

Access for English Learners

MLR8 Discussion Supports. Prior to solving the problems, invite students to make sense of the situation. Monitor and clarify any questions about the context.
Advances: Reading, Representing

Student-facing Task Statement

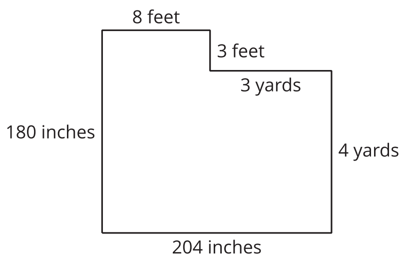
A classroom is getting new carpet and baseboards. Tyler and a couple of friends are

Launch

- Groups of 2

helping to take measurements.

Here is a sketch of the classroom and the measurements they recorded.



For each question, show your reasoning.

1. How many feet of baseboard will they need to replace in the classroom? How many inches is that?
2. 1,200 inches of baseboard material was delivered. Is that enough?
3. How many square feet of carpet will be needed to cover the floor area?

Student Responses

1. 61 feet, or 732 inches, not including the wall where the door is located. Sample response: The length of each wall, in feet:
    - 180 inches, which is 15 feet
    - 8 feet
    - 3 yards, which is 9 feet
    - 4 yards, which is 12 feet
    - 204 inches, which is 17 feet
    - The perimeter, not including the door, is $15 + 8 + 9 + 12 + 17$, which is 61 feet. $61 \times 12 = 732$, so the perimeter is 732 inches.
2. Yes, that's enough. That's 468 inches more than needed $(1{,}200 - 732 = 468)$.
3. 228 square feet. Sample reasoning:
    - The room can be decomposed into two rectangles: 8 feet by 3 feet and

- 1 minute: quiet time to read the opening paragraphs and look at the diagram in the task statement
- "What do you notice? What do you wonder?"
- Explain what a baseboard is or show an example.
- Make sure students recognize that all measurements need to be in the same unit before finding perimeter and area.

Activity

- 6–8 minutes: independent work time, problems 1–2
- 2 minutes: partner discussion
- Monitor for students who:
    - choose to consider all units in terms of inches, feet, or yards strategically
    - find a way to convert and keep track of the values systematically
- Pause for a whole-class discussion on the first two questions before students answer the last question.
    - "How did you decide which unit to use? How do we convert from _____ to _____?"
    - "How did you keep track of all the conversions?"
- As students work on the last question, monitor for:
    - different ways students decompose the diagram of the room to find its area
    - equations that show how the area is computed

Synthesis

- Select students to share how they reasoned about the area of the room.

17 feet by 12 feet (204 inches by 4 yards). The combined area is $(8 \times 3) + (17 \times 12)$ or $24 + 204$, which is 228 square feet.

- ○ The room can be decomposed into two rectangles: 8 feet by 15 feet (8 feet by 180 inches) and 9 feet by 12 feet (3 yards by 4 yards). The combined area is $(8 \times 15) + (9 \times 12)$ or $120 + 108$ or 228 square feet.

Record and display their reasoning.

- If some students found the area using different units, ask how they would find out if the two answers represent the same amount.

Lesson Synthesis

10 min

"Today we used all kinds of operations on large numbers to solve problems about measurements."

"Can you find examples where it was helpful or necessary to multiply, divide, add, or subtract multi-digit numbers?" (Multiplication and division were handy for converting from one unit to another. Multiplication was needed to find the area of rectangular shapes. Addition helped us find a total length or area. Subtraction was useful for finding a difference or amount left over.)

"What challenges did you come across when solving the problems? When multiplying or dividing?"

Suggested Centers

- Compare (1–5), Stage 7: Multi-digit Operations (Addressing)
- Watch Your Remainder (4–5), Stage 1: One-digit Divisors (Addressing)

Complete Cool-Down

Response to Student Thinking

Students find an area other than 3,456 square inches for the new banner, or a difference other than 1,152 square inches when comparing the area of the banners.

Next Day Support

- Before the warm-up, invite students to work in small groups to discuss a correct response to this cool-down.

Lesson 23: Solve Problems with Many Operations

Standards Alignments

Addressing 4.MD.A.2, 4.NBT.B.4, 4.NBT.B.5, 4.NBT.B.6

Teacher-facing Learning Goals

- Solve multi-step problems involving the four operations.

Student-facing Learning Goals

- Let's solve multi-step problems involving the four operations.

Lesson Purpose

The purpose of this lesson is for students to use the four operations to solve problems involving multi-digit numbers. Students also use the standard algorithm for addition and subtraction to solve problems. For each problem, they assess the reasonableness of their responses.

In the preceding lessons, students interpret situations and solve them using a variety of reasoning strategies. The computations focus mostly on multiplication and division, and the numbers are mainly two and three digits long.

In this lesson, students continue to engage in problem solving—this time in the context of finding distances. Students now use the four operations and work with numbers up to five digits. In the next lesson, they will work with numbers up to 1 million.

Access for:

Students with Disabilities

- Engagement (Activity 1)

English Learners

- MLR8 (Activity 2)

Instructional Routines

True or False (Warm-up)

Materials to Gather

- Grid paper: Activity 2

Lesson Timeline

Warm-up	10 min

Teacher Reflection Question

As you finish up this unit, reflect on the norms and activities that have supported each student

Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

in learning math. How have you seen each student grow as a young mathematician throughout this work? How have you seen yourself grow as a teacher? What will you continue to do and what will you improve on in Unit 7?

Cool-down (to be completed at the end of the lesson)

5 min

Long-distance Driving

Standards Alignments

Addressing 4.MD.A.2, 4.NBT.B.4

Student-facing Task Statement

A truck driver needs to deliver goods to a city that is 2,654 km away.

1. If she drives 285 km each day, could she get there in 8 days? Show your reasoning.
2. In the first three days, the driver traveled 1,087 km. At the end of the fourth day, she has 972 km to go. How many km did she travel on the fourth day?

Student Responses

1. No. Sample response:
    - Even if she drives 300 km a day, she'd only travel 2,400 km in 8 days, so she can't travel 2,654 km with less than 300 km per day.
    - $285 \times 8 = 2{,}280$. At 285 km per day, she'd only travel 2,280 km in 8 days.
    - $2{,}654 = 8 \times 337 + 6$. This means she'd need to travel at least 338 km a day to get to her destination in 8 days.
2. 595 km. Sample response: After the third day, she had $2{,}654 - 1{,}087$ or 1,567 km left. After the fourth day she has 972 km left, so she must have traveled 595 km, because $1{,}567 - 972 = 595$.

Begin Lesson

Warm-up

10 min

True or False: Differences

Standards Alignments

Addressing 4.NBT.B.4

The purpose of this True or False is to elicit strategies and understandings students have for finding differences between two numbers. These understandings help students build fluency in addition and subtraction, while preparing them to think about distances between two points.

Students may use estimation or place value understanding to solve the problems (MP7).

Instructional Routines

True or False

Student-facing Task Statement

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $50{,}000 - 999 = 49{,}001$
- $4{,}799 = 5{,}000 - 311$
- $3{,}005 = 4{,}000 - 1{,}995$
- $2{,}000 - 1{,}234 = 1{,}876$

Student Responses

- True: $49{,}001 + 999 = 50{,}000$.
- False: 4,800 is 200 from 5,000, so 4,799 is $200 + 1$ or 201 from 5,000.
- False: 3,000 is $4{,}000 - 1{,}000$, so 3,005 is just 5 less than 1,000 from 4,000. (It cannot be almost 2,000 away from 4,000.)
- False: $2{,}000 - 1{,}200$ is 800, so $2{,}000 - 1{,}234$ is a little less than 800 (instead of 1,000 more than 800).

Launch

- Display one statement.
- "Give me a signal when you know whether the statement is true and can explain how you know."

Activity

- 1 minute: quiet think time
- Share and record answers and strategy.
- Repeat with each statement.

Synthesis

- "How can we tell if each equation is true without calculating?"
- "We could use the standard algorithm to find each difference and then decide if the equation is true or false. Would that be a good idea? Why or why not?"

Activity 1

20 min

Back and Forth

Standards Alignments

Addressing 4.NBT.B.4, 4.NBT.B.5, 4.NBT.B.6

This activity prompts students to interpret and represent situations about distances and use multiple operations to solve problems. In problem 3, a piece of information is withheld. Students will need to make sense of what's missing and find out that information before the question could be answered. Throughout the activity, students reason abstractly and quantitatively as they interpret the diagram and use the information to solve problems (MP2).

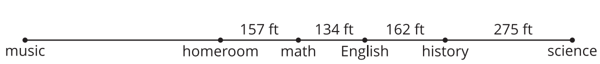
Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Some students may benefit from feedback that emphasizes effort and time on task. For example, look for opportunities to provide positive feedback to students who have not finished the task or gotten the right answer, but who have worked carefully, attempted a new strategy, or collaborated productively with their partner. *Supports accessibility for: Social-Emotional Functioning*

Student-facing Task Statement

Mai's cousin is in middle school. She travels from her homeroom to math, then English, history, and science. When she finishes her science class, she takes the same path back to her homeroom.

Mai's cousin makes the same trip 5 times each week. The distances between the classes are shown.



1. How far does Mai's cousin travel each round trip—from her homeroom to the four classes and back? Write one or more expressions or equations to show your

Launch

- Groups of 2
- Give students access to grid paper.
- Read opening paragraphs as a class. Display the diagram.
- "What mathematical questions can you ask about this situation?"
- 1–2 minutes: quiet think time
- 1–2 minutes: partners compare questions
- Share and record responses.

Activity

- 6–8 minutes: independent work time
- 3–4 minutes: partner discussion

reasoning.

2. Each week, Mai's cousin makes 3 round trips from her homeroom to her music class. The total distance traveled on those 3 round trips is 2,364 feet.

   How far away is the music room from her homeroom? Show your reasoning.

3. Mai thinks her cousin travels 2 miles each week just going between classes. Do you agree? Explain or show your reasoning.

Student Responses

1. 1,456 feet. Sample responses:
   - One way trip: $157 + 134 + 162 + 275 = 728$. Round trip: $728 \times 2 = 1{,}456$.
   - $2 \times (157 + 134 + 162 + 275)$ is 2×728, which is 1,456.
2. 394 feet. Sample responses: There are 6 one-way trips in 3 round trips. $2{,}364 \div 6 = 394$
3. Disagree. Sample responses: Each week, Mai's cousin travels: $(1{,}456 \times 5) + 2{,}364$ or $7{,}280 + 2{,}364$, which is 9,644 feet. There are 5,280 feet in a mile, or 10,560 in 2 miles. Mai's cousin travels less than (but close to) 2 miles going between classes each week.

- When requested, tell students that 1 mile is equal to 5,280 feet.
- Monitor for:
  - the different equations students write for the first question (see examples in Student Responses)
  - the assumptions students make when answering the last question (as noted in the narrative)
  - the different ways of reasoning (as noted in the narrative)

Synthesis

- Consider collecting and displaying the different expressions students wrote for the first question and discussing what they tell us about the way the writer reasoned about the problems. For example:
  - $2 \times (157 + 134 + 162 + 275)$ suggests that the writer found the distance one way then doubled it.
  - $(2 \times 157) + (2 \times 134) + (2 \times 162) + (2 \times 275)$ suggests that the distances between classes were doubled first before being added.
- "How did you find out if Mai's cousin traveled 2 miles each week? What did you do first? What did you do next?"
- Select students to share their responses and reasoning.
- Highlight how the strategies are alike and different.

Activity 2

15 min

Fitness Challenge

Standards Alignments

Addressing 4.NBT.B.4

This activity gives students another opportunity to use multiple operations to model the quantities in a situation and to solve problems involving large numbers. Students interpret the quantities in context, reason about them abstractly as they perform computations, and then return to the context to interpret the results. As they do so, students are reasoning quantitatively and abstractly (MP2).

Students may choose to answer the first problem by dividing a five-digit number by a one-digit divisor. Though finding a quotient of a five-digit dividend is not an expectation, this particular number ends in a 0. Students can use the division strategies they've learned so far and what they know about the structure of numbers in base ten to find the quotient (MP7).

Access for English Learners

MLR8 Discussion Supports. Synthesis: Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Advances: Speaking

Materials to Gather

Grid paper

Student-facing Task Statement

To motivate students to exercise, Han's school is holding a fitness challenge with prizes.

1. Han walked 32,550 steps in the first week. He walked the same number of steps every day. How many steps did Han walk each day? Show your reasoning.

Launch

- Groups of 2
- Give students access to grid paper.
- "Sometimes people compete to see who can walk the most steps in a day, week, or month."
- "These competitions are called challenges."
- "This activity involves a fitness challenge involving steps."

Activity

- 6–8 minutes: independent work time
- Monitor for the different ways students reason about each question.

2. The table shows the number steps Han took each week for the first three weeks. How much did the number of steps drop from the first week to the second week?

week 1	week 2	week 3	week 4
32,550	28,098	36,249	

3. If Han wants to meet the challenge, what is the fewest number of steps that he needs to take in week 4? Show your reasoning.
4. How do you know your answer to problem 3 is reasonable?

Student Responses

1. About 4,650 steps a day. Sample responses:
    - $3{,}255 \div 7 = 465$, so $32{,}550 \div 7 = 4{,}650$.
    - 32,550 is 2,450 from 35,000. I know that $35{,}000 \div 7 = 5{,}000$ and $2{,}450 \div 7 = 350$, so $32{,}550 \div 7 = (35{,}000 \div 7) - (2{,}450 \div 7)$, which is $5{,}000 - 350$ or 4,650.
2. 4,452. Sample responses:
    - $32{,}550 - 28{,}098 = 4{,}452$
    - 28,098 is 1,902 from 30,000, and 30,000 is 2,550 from 32,550, and $1{,}902 + 2{,}550 = 4{,}452$.
3. 23,103. Sample response: $32{,}550 + 28{,}098 + 36{,}249 = 96{,}897$ and $120{,}000 - 96{,}897 = 23{,}103$.
4. Sample responses:
    - I can estimate: 120,000 subtracted by $(30{,}000 + 30{,}000 + 40{,}000)$ is 20,000.
    - I can see if adding the answer to the other three numbers gives 120,000.

Synthesis

- Invite students to share their responses and reasoning.
- Focus the discussion on how students found the number of steps Han took each day and how they found the last number in the table.
- "How would you know if your answer to the first question is correct?" (Multiply 4,650 by 7 to see if it equals 32,550.)
- "How would you know if your answer to problem 3 is reasonable?" (Add it to the other three numbers and see if the sum is 120,000.)

Lesson Synthesis

10 min

"Today we solved problems that involved numbers with four or more digits. Some of those problems could be interpreted in more than one way."

"In the fitness challenge activity, how did you think about finding Han's steps each day? Did you think of it in terms of multiplication (what number times 7 is 32,550?) or in terms of division (what is 32,550 divided by 7?)? Is one way of thinking more convenient? Why or why not?"

To facilitate discussion, display equations such as:

$$7 \times n = 32{,}550$$

$$32{,}550 \div 7 = n$$

"How did you think about finding Han's steps in week 4? Did you think in terms of addition (what number must be added to 96,897 to make 120,000?) or subtraction (what is the difference between 120,000 and 96,897?)?"

Display equations such as:

$$32{,}550 + 28{,}098 + 36{,}249 + f = 120{,}000$$

$$120{,}000 - 96{,}897 = f$$

$$96{,}897 + f = 120{,}000$$

Suggested Centers

- Compare (1–5), Stage 7: Multi-digit Operations (Addressing)
- Watch Your Remainder (4–5), Stage 1: One-digit Divisors (Addressing)

Complete Cool-Down

Response to Student Thinking

Students only answer part of the problem.

Next Day Support

- Before the warm-up, select a student's cool-down from the previous lesson (name anonymous). Ask students to identify what the student did well and what the student needs to do to improve the cool-down.

Lesson 24: Assess the Reasonableness of Solutions

Standards Alignments

Addressing 4.NBT.B.4, 4.OA.A.2, 4.OA.A.3

Teacher-facing Learning Goals

- Assess the reasonableness of responses.
- Solve multi-step problems involving the four operations.

Student-facing Learning Goals

- Let's solve problems and assess the reasonableness of solutions.

Lesson Purpose

The purpose of this lesson is for students to solve multi-step word problems by analyzing data, estimating, reasoning, and performing multiple operations. It also helps students to build fluency in using the standard algorithm to add and subtract multi-digit numbers up to 1 million. In each activity, students assess the reasonableness of their responses.

In the final lesson of the unit, students apply their knowledge of numbers in base-ten and their estimation and computation skills to solve problems about languages and populations in the United States. The census data used here prompts students to work with large numbers and to interpret them carefully.

This lesson has a Student Section Summary.

Access for:

Students with Disabilities

- Representation (Activity 1)

English Learners

- MLR8 (Activity 1)

Instructional Routines

Notice and Wonder (Warm-up)

Materials to Gather

- Grid paper: Activity 1

Lesson Timeline

Warm-up	10 min
Activity 1	20 min
Activity 2	15 min
Lesson Synthesis	10 min
Cool-down	5 min

Cool-down (to be completed at the end of the lesson)

5 min

The Children and the Elderly

Standards Alignments

Addressing 4.NBT.B.4, 4.OA.A.3

Student-facing Task Statement

Here are the data on the numbers of children and senior citizens in Philadelphia as of 2017.

age	number of people
under 5 years	107,736
5–14 years	184,323
15–17 years	53,530
65 years and over	203,007

1. As of 2017, what is the number of people under the age of 18 in Philadelphia?
2. How do you know your answer to problem 1 is reasonable?

Student Responses

1. 345,599 children under $18 (107{,}736 + 184{,}323 + 53{,}530 = 345{,}599)$
2. Estimate: $110{,}000 + 180{,}000 + 50{,}000 = 340{,}000$. There are about 340,000 people under 18, which is close to the actual number 345,599 calculated.

Begin Lesson

Warm-up

10 min

Notice and Wonder: Native American Languages

Standards Alignments

Addressing 4.OA.A.2, 4.OA.A.3

This warm-up familiarizes students with a context of the first activity and elicits observations that will be useful when students analyze population data more closely in later activities. Students are likely to notice and wonder many things about the infographics, especially the great discrepancy between the first data point and the rest. They will have the chance to reason about this gap in multiplicative terms and in terms of difference.

Instructional Routines

Notice and Wonder

Student-facing Task Statement

What do you notice and what do you wonder?

Student Responses

Students may notice:

- One language, Navajo, has the highest number and the longest bar in the graph.

Launch

- Display the image.
- Explain that the names are Native American languages.

Activity

- "What do you notice? What do you wonder?"
- 1 minute: quiet think time
- Share and record responses.

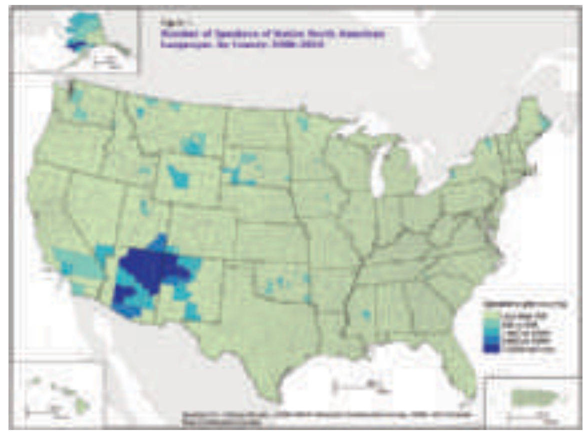
Synthesis

- Reveal that the infographic shows the top ten most widely spoken Native American languages in the United States, collected in 2009–2013. The numbers represent people over 5 who speak the language.
- Consider sharing that about 170 Native American languages are spoken in the U.S., but many are on the verge of disappearing.

- There are 10 Native American languages shown.
- Five of the languages have numbers between 10,000 and 20,000.
- Four languages have numbers in the thousands.

Students may wonder:

- What do the numbers represent? Do they represent the number of words in the language? The number of people speaking the language?
- Why does one language outnumber the other ones by so much?
- Where are all these languages spoken?
- Are there other Native American languages?
- "Based on what you see here, could you predict how many people speak the 20th most widely spoken language? The language in the 100th place?"
- Consider showing a census map showing the distribution of Native American languages. Here is one from a survey done by the U.S. Census in 2006–2010:



Activity 1

20 min

Do You Speak Navajo?

Standards Alignments

Addressing 4.NBT.B.4, 4.OA.A.3

The purpose of this activity is for students to practice using estimation and using the standard algorithm for addition and subtraction to solve problems involving large numbers. The same data from the warm-up is used as a context. The last two questions prompt students to first examine the data set and make some estimations before deciding which pairs of numbers to use to find the answers.

Students reason abstractly and quantitatively with several operations in order to estimate and solve the problems (MP2).

Access for English Learners

MLR8 Discussion Supports. During group work, invite students to take turns sharing their responses. Ask students to restate what they heard using precise mathematical language and their own words. Display the sentence frame: "I heard you say . . ." Original speakers can agree or clarify for their partner.
Advances: Listening, Speaking

Access for Students with Disabilities

Representation: Access for Perception. Read the problems aloud. Students who both listen to and read the information will benefit from extra processing time.
Supports accessibility for: Language, Memory

Materials to Gather

Grid paper

Student-facing Task Statement

1. In the list of the ten most widely spoken Native American languages in the U.S., Navajo and Yupik are the most widely spoken.

   a. How many more Navajo speakers are there than Yupik speakers? Show your reasoning.

language	number of speakers
Navajo	166,826
Yupik	19,750
Dakota	17,855
Apache	13,445
Keres	13,190
Cherokee	11,465
Ojibwa	9,735
Choctaw	9,635
Zuni	9,615
Pima	6,990

Launch

- Groups of 2
- Give students access to grid paper.
- "What languages are spoken in your family or your neighborhood?"
- "Do you or your friends use different languages in different places or occasions?"
- "For this activity, you'll need to refer to the data you saw in the warm-up."

Activity

- 7–8 minutes: quiet work time
- 3–5 minutes: partner discussion
- Monitor for students who:
  - use the bar graphs from the warm-up to help them estimate question 1b
  - use estimation or rounding to assess whether their response is reasonable
  - group some numbers selectively to

b. About how many times as many Navajo speakers as there are Yupik speakers? Show your reasoning.

2. Navajo, Apache, and Cherokee languages have been used during wartime to help the U.S. military keep its communications secure and incomprehensible to their enemies.

   a. Based on the data here, how many people might have been able to understand the communications? Show your reasoning.
   b. How do you know that your answer is reasonable?

3. Are there more Navajo speakers than the speakers of all the other nine languages combined? Explain or show how you know.

Student Responses

1. a. 147,076 more Navajo speakers. $166{,}826 - 19{,}750 = 147{,}076$
   b. About 8–9 times as many Navajo speakers as there are Yupik speakers. Sample responses:
      - $19{,}750 \times 8 = 158{,}000$ and $19{,}750 \times 9 = 177{,}750$
      - 19,750 could be rounded to 20,000. Eight times 20,000 is 160,000.
2. Sample responses:
   a. 191,736 people. $166{,}826 + 13{,}445 + 11{,}465 = 191{,}736$
   b. Estimate: $167{,}000 + 13{,}000 + 11{,}000 = 191{,}000$. About 191,000 people.
3. Yes. Sample responses:
   a. Adding the numbers of speakers of the nine languages gives 111,680 people, which is less than 166,826.
   b. The sum of Yupik and Cherokee make estimation or computation more efficient

Synthesis

- Invite students to share their responses and reasoning.
- “How did you know what operations to use to answer the first set of problems? The second problem? The last problem?”
- “How do you know that your answer is reasonable?” (Mentally use a standard algorithm to find the differences, round the numbers, or look only at certain digits to estimate.)

speakers is about 31,000. The sum of Dakota and Apache speakers is about 30,000. The sum of Keres and Pima speakers is about 20,000. The sum of Ojibwa, Choctaw, and Zuni speakers is about 29,000. The sum of all of these groups is only about 110,000.

Activity 2

15 min

Languages in Philadelphia and Chicago

Standards Alignments

Addressing 4.NBT.B.4, 4.OA.A.3

In this activity, students continue to analyze population data and to use addition and subtraction skills to solve problems. The first question, which prompts students to compare two population groups, could be answered by rounding and estimating rather than precise calculations. Students who look for structure may notice that the number of digits in each row in the table changes by one digit as we move down the table. They make use of this observation to simplify their estimation or calculation (MP7).

Note that the data shown here do not represent all language types spoken in the two major cities. If desired, consider finding local data or data that reflect the languages spoken in your students' community. Then, ask students to make comparisons as shown in the activity.

Student-facing Task Statement

Philadelphia, the birthplace of the United States, is a diverse city and home to people of different backgrounds.

The table shows 2017 data on some types of languages spoken in Philadelphia and the numbers of people who speak them.

Launch

- Display the two sets of data.
- “What do you notice? What do you wonder?”
- 1 minute: quiet think time
- Share and record responses.

language	number of speakers in Philadelphia
English only	1,224,539
Spanish	127,352
Other Indo-European	6,750
Asian	364

1. Based on the data, are there more people in Philadelphia who only speak English or more people who speak a language other than English? Show how you know.
2. What is the difference between the number of people who speak only English and those who speak another language? Show how you know.
3. Chicago is a city with a similar population to Philadelphia.

   This table shows data on some types of languages spoken by people in Chicago.

language	number of speakers in Chicago
English only	1,731,836
Spanish	422,568
Other Indo-European	25,777
Asian	1,005

   a. How many more speakers of Spanish and other Indo-European languages are in Chicago than in Philadelphia? Show your reasoning.
   b. How do you know your answer is reasonable?

Student Responses

1. More people who speak only English. Sample reasoning:
   a. $127{,}352 + 6{,}750 + 364 = 134{,}466$. The total number of people speaking a

Activity

- 5 minutes: independent work time
- Monitor for:
  - the different strategies students use to answer the first question
  - the different ways in which students decide to add numbers (for example, the largest numbers first, or three numbers at a time)

Synthesis

- Select students to share their responses and reasoning.
- If no students answered the first question by estimation, ask if it could be done and how.
- Discuss potential pros and cons of rounding. (Pro: It would be quicker. Con: The data for each group of languages have a different number of digits, so it’s hard to choose the place to which the numbers should be rounded. If we round to different places for each group, we’d end up over- or under-estimating some groups much more than others.)

non-English language is much less than 1 million.

b. Estimating: The numbers in the last three rows have six or fewer digits, and all are less than 130,000. Their sum will be much less than 1 million.

2. $1{,}224{,}539 - 134{,}466 = 1{,}090{,}073$
3. a. 314,243. Sample reasoning:
   - Philadelphia: $127{,}352 + 6{,}750 = 134{,}102$
   - Chicago: $422{,}568 + 25{,}777 = 448{,}345$
   - Difference between the two: $448{,}345 - 134{,}102 = 314{,}243$

   b. It is reasonable because if I estimate this group's population in Philadelphia: 135,000 and the group's population in Chicago: 450,000 and subtract, I get 315,000, which is close to 314,243.

Lesson Synthesis

10 min

"Today we performed addition and subtraction on very large numbers. Even though most of us used the standard algorithm, we didn't all add or subtract the numbers the same way."

"In the second activity, you had to add a series of numbers in a table. Which numbers did you add (or subtract) first? Does it matter?"

"When the numbers being added or subtracted have many digits, it is easy to make an error, to regroup or carry incorrectly, or to get the digits mixed up. What are some ways we could check our answers? What are some common errors, and how can we avoid them?"

Suggested Centers

- Compare (1–5), Stage 7: Multi-digit Operations (Addressing)
- Watch Your Remainder (4–5), Stage 1: One-digit Divisors (Addressing)

Student Section Summary

In this section, we encountered problems that involve large numbers from different contexts and that could be solved with different strategies.

In the beginning, we saw at least five ways to find the product of 45 and 18: by multiplying and dividing in parts, using a series of equations, drawing diagrams, and more.

Later, we explored problems about measurements, with numbers up to four digits. We found that, often, the same problem could be solved using different operations.

For example, in the fitness challenge activity, Han took 32,550 steps in 7 days. We can find the number of steps he took each day by thinking in terms of multiplication (what number times 7 is 32,550?) or in terms of division (what is 32,550 divided by 7?).

We can also write different equations.

$$7 \times n = 32{,}550 \qquad 32{,}550 \div 7 = n$$

To find out how many steps Han had to take to reach a goal of 120,000 steps if he had 96,897 steps, we can think in terms of addition (what number must be added to 96,897 to make 120,000?) or subtraction (what is the difference between 120,000 and 96,897?).

$$96{,}897 + n = 120{,}000 \qquad 120{,}000 - 96{,}897 = n$$

Complete Cool-Down

Response to Student Thinking

Students may determine the number of people under 18 as something other than 345,599.

Next Day Support

- Before the warm-up, review strategies and solutions for the cool-down.

Lesson 25: Paper Flower Decorations (Optional)

Standards Alignments

Addressing	4.OA.A.3, 4.OA.C.5
Building Towards	4.OA.C.5

Teacher-facing Learning Goals

- Generate a pattern of numbers or shapes that follows a given rule.
- Use the four operations to solve problems that involve multi-digit whole numbers and assess the reasonableness of responses.

Student-facing Learning Goals

- Let's make patterns with paper flowers.

Lesson Purpose

The purpose of this lesson is for students to create and analyze patterns in a real-world context and to solve multi-step problems.

This lesson is optional because it does not address any new mathematical content standards. This lesson does provide students with an opportunity to apply precursor skills of mathematical modeling. In this lesson, students build on their prior understanding and experiences with creating and analyzing patterns to solve multi-step problems in a real-world context.

In the first activity, students make different types of paper flowers. In the second activity, they consider patterns and solve problems involving paper flower garlands. In the third activity, students think of their own pattern and multi-step problems inspired by their process of making paper flowers.

When students ask and answer questions that arise from a given situation, use mathematical features of an object to solve a problem, make choices, analyze real-world situations with mathematical ideas, interpret a mathematical answer in context, and decide if an answer makes sense in the situation, they model with mathematics (MP4).

Access for:

Students with Disabilities

- Representation (Activity 3)

English Learners

- MLR7 (Activity 2)

Instructional Routines

How Many Do You See? (Warm-up)

Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Activity 3	15 min
Lesson Synthesis	10 min

Teacher Reflection Question

Think about times when students were able to make connections to and build on the ideas of their peers during discussions today. What norms or routines allowed students to engage with other students' ideas?

Begin Lesson

Warm-up

10 min

How Many Do You See: Paper Flowers

Standards Alignments

Addressing 4.OA.C.5

The purpose of this How Many Do You See is for students to subitize or use grouping strategies to describe the images they see. The paper flowers also set the stage for the subsequent activities in the lesson.

Instructional Routines

How Many Do You See?

Student-facing Task Statement

How many do you see? How do you see them?

Launch

- Groups of 2
- "How many paper flowers do you see? How do you see them?"
- Display image.
- 1 minute: quiet think time

Student Responses

Sample responses:

- 16 flowers. There are 4 rows of 4.
- 16 flowers. There are 4 columns of 4.
- 8 flowers of one color and 8 of the other color. There are 2 of each color in each of the 4 rows.
- 8 flowers. There are 4 of each color in the top half and 4 of each color in the bottom half.

Activity

- Display image.
- “Discuss your thinking with your partner.”
- 1 minute: partner discussion
- Record responses.

Synthesis

- “These flowers are sometimes used as decorations at a quinceañera.”
- “A quinceañera is a celebration of a girl’s 15th birthday. It is celebrated most widely in Mexico, but is also celebrated in Spain and other countries throughout the Caribbean, Central and South America. It is celebrated in the US and around the world by people who have cultural ties to these regions.”
- “What are some celebrations in your family or culture? What are some decorations used in the celebrations?”
- Consider asking:
    - “Who can restate the way ___ saw the paper flowers in different words?”
    - “Did anyone see the flowers the same way but would explain it differently?”
    - “Does anyone want to add an observation to the way ____ saw the flowers?”

Activity 1

15 min

Paper Flower Construction

Standards Alignments

Building Towards 4.OA.C.5

The purpose of this activity is for students to make paper flowers, use them to create patterns, and describe the patterns. The experience of making the flowers provides a concrete reference that will be helpful when students make sense of, solve, and create multi-step problems with the same context later in the lesson.

Required Preparation

- Gather rubber bands or pipe cleaners and 60 sheets of tissue paper that measure 18 inches by 24 inches.
- Cut the tissue paper in the following ways (measurements do not need to be exact):
  - 20 sheets cut into strips that are 4 inches by 9 inches
  - 40 sheets cut into strips that are 6 inches by 12 inches (length should be about 2 times the width)

Student-facing Task Statement

Follow these steps to make paper flowers:

- Place 6 pieces of tissue paper on top of each other.
- Starting at one side, fold over about 1 inch, then fold in the opposite direction. Repeat with this accordion fold (like a paper fan) until you have a strip that is 1 inch wide and the length of the original paper.
- Tie a rubber band around the middle of the folded paper strip. Then, open up the folds.

- Carefully, one layer at a time, fold the layers up into the middle to make the petals.

Launch

- Groups of 4
- Give students strips of tissue paper and rubber bands or pipe cleaners.
- “Today we are going to make paper flowers. Has anybody made this type of flower or seen them used for decorations before?”
- Demonstrate how to make a paper flower.
- “In your group, make some small and large paper flowers. Make as many paper flowers as you can in 10 minutes.”
- “As you’re working, try to keep in mind the different things you have to think about to make the flowers.”

Activity

- 10 minutes: small-group work time

Synthesis

- “How many flowers was your group able to make in 10 minutes?”
- Record responses on the board.
- “How many flowers did all of us make

Student Responses

Students make paper flowers.

together?"

- "What were some things you had to think about when you made the flowers?" (Sample responses: The number of sheets needed, the different colors to use, the size of sheets to use, the time it took to make each flower.)
- Record the responses and leave them displayed for the following activity.

Activity 2

20 min

Quinceañera Decorations

Standards Alignments

Addressing 4.OA.A.3, 4.OA.C.5

In this activity, students use the context of paper flowers to analyze patterns and solve multi-step problems. The patterns are fairly straightforward, but to use them to solve problems, students will need to represent or otherwise reason about them mathematically. Their earlier experience of making flowers supports students in making sense and visualizing each step of the problem.

Access for English Learners

MLR7 Compare and Connect. Synthesis: After all strategies have been presented, lead a discussion comparing, contrasting, and connecting the different approaches. Ask, "Did anyone solve the problem the same way, but would explain it differently?" and "Why did the different approaches lead to the same (or different) outcome(s)?"
Advances: Representing, Conversing

Student-facing Task Statement

Priya and Jada are making paper flower garlands for their friend's quinceañera. Each garland uses 12 flowers.

1. Priya wants 2 big flowers, followed by 2 small flowers. Jada wants 1 big flower, followed by 2 small flowers. Use their patterns to draw the garlands.
2. Priya and Jada make 25 garlands of each type. How many large and small flowers will they need altogether?
3. Diego and Kiran also made flowers. They made a total of 155 flowers for garlands that require 16 flowers each. How many garlands can they make?
4. It takes 1 minute to cut the strips for a flower and 2 minutes to finish it. How long did it take Diego and Kiran to make the 155 flowers, if they each make about the same number of flowers?

Student Responses

Sample response:

1. Priya's: Students draw a pattern that shows 2 big flowers and then 2 small flowers until there are 12 total. Jada's: Students draw a pattern that shows 1 big flower and then 2 small flowers until there are 12 total.
2. For Priya's pattern, they need 150 small flowers and 150 large ones. For Jada's pattern, they need 200 small flowers and 100 large ones. In total they need 350 small flowers and 250 large ones.
3. Dividing 155 by 16 gives 9 with remainder 11. They can make 9 garlands and they'll have 11 flowers left over.

Launch

- Groups of 2
- "The paper flowers are strung together to make a garland."

Activity

- 2 minute: independent work time
- 10 minutes: partner work time
- Monitor for students who:
    - Use equations to show their thinking.
    - Organize the multi-step process

Synthesis

- Display student work of selected students and invite them to share.
- "What specific part of the work makes their explanation clear?"
- Consider asking:
    - "What suggestion do you have to make it clearer?"
    - "How can you revise your own work to make it more clear or organized?"

4. It takes 3 minutes to make one flower, so it takes 465 minutes to make 155 flowers. One person makes 77 and the other 78 garlands. They will both be done after 234 minutes, which is 3 hours and 54 minutes.

Activity 3

15 min

Make Your Own Problems

Standards Alignments

Addressing 4.OA.A.3, 4.OA.C.5

The purpose of this activity is for students to use their experience of making flowers to create their own pattern and multi-step problems.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Ask students to generate a list of situations about making paper flowers to reduce barriers for students who need support writing a multi-step problem.
Supports accessibility for: Conceptual Processing, Language, Organization

Student-facing Task Statement

1. Write a multi-step problem about making paper flowers.
2. Exchange the problem with your partner and solve each other's problems.

Student Responses

Sample response: It takes 1 sheet of tissue paper to make a big flower and $\frac{1}{2}$ sheet to make a small flower. How much tissue paper is needed to make a garland that has 7 small and

Launch

- Groups of 2
- "You will write your own problems about making paper flowers. You can use similar ideas from the first activity, the things we listed in the last activity, or come up with new ones."

Activity

- 5 minutes: independent work time
- 5 minutes: partner work time

7 large garlands? (7 sheets for big flowers and 4 sheets for small flowers, where half a sheet will not be used. 11 sheets are needed, with $\frac{1}{2}$ a sheet left over).

- Monitor for different types of problems students write involving different operations and contexts.

Synthesis

- Invite previously identified students to share their problems.

Lesson Synthesis

10 min

"Today, we solved many different problems about paper flowers and created flowers and problems of our own."

"What are some ways making the flowers helped you write your own math problems?" (I was able to see what kinds of situations I could ask about in my story problem.)

Suggested Centers

- Compare (1–5), Stage 7: Multi-digit Operations (Addressing)
- Watch Your Remainder (4–5), Stage 1: One-digit Divisors (Addressing)

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Core Knowledge MATHEMATICS™

Family Support Materials

Family Support Materials

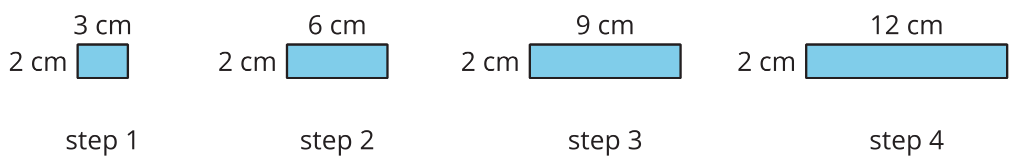
Multiplying and Dividing Multi-digit Numbers

In this unit, students deepen their understanding of multiplication and division and expand their ability to perform these operations on multi-digit numbers.

Section A: Features of Patterns

In this section, students analyze patterns. They use ideas related to multiplication (such as factors, multiples, double, and triple) to describe and extend the patterns.

If the pattern continues, could 50 represent the side length or the area of one of the rectangles? If so, which step? If not, why not?

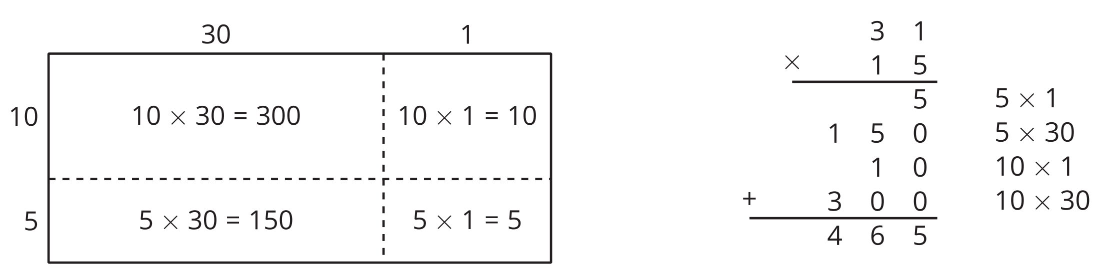


Section B: Multi-Digit Multiplication

In this section, students multiply one-digit numbers and numbers up to four digits, and pairs of two-digit numbers. They learn to use increasingly more efficient methods to multiply.

Students begin by using visual representations—arrays, base-ten diagrams, and grids—to help them find products. They recall that rectangles can be used to represent multiplication, with the side lengths representing the factors and the area representing the product.

Students see that it helps to decompose (break apart) the factors by place value. For example, to multiply 31 and 15, we can think of the 31 as $30 + 1$ and the 15 as $10 + 5$. We can then label these values on a diagram, multiply the parts separately, and add the partial products.



Later, students use an algorithm that lists partial products vertically. This work prepares them to make sense of the standard algorithm for multiplication, to be studied closely in grade 5.

Section C: Multi-Digit Division

In this section, students divide larger numbers (up to four digits), explore new division strategies, and interpret division situations that involve remainders.

Students begin by solving various problems that involve division, including those about equal groups, factors and multiples, and area of rectangles. They recall that an expression such as $96 \div 8$ can be used to find how many groups of 8 in 96, or to find the size of one group if 96 are put into 8 groups.

Students see that just as they can multiply two numbers by decomposing the factors and finding partial products, they can divide by decomposing the dividend (the number being divided) and finding partial quotients. Thinking about place value can help us as well.

Students then learn to organize partial quotients using equations and an algorithm that records division vertically.

$$
\begin{array}{r}
400 \div 5 = 80 \\
60 \div 5 = 12 \\
5 \div 5 = 1 \\
\hline
465 \div 5 = 93
\end{array}
$$

$$
\begin{array}{r l}
\boxed{93} & \\
1 & \\
12 & \\
80 & \\
5\,\overline{)465} & \\
-\ 400 & 5 \times 80 \\
\hline
65 & \\
-\ 60 & 5 \times 12 \\
\hline
5 & \\
-\ 5 & 5 \times 1 \\
\hline
0 &
\end{array}
$$

Section D: Let’s Put It To Work: Problem Solving with Large Numbers

Students solve a variety of problems that involve all four operations on multi-digit numbers. The problems can be approached in many ways, allowing students to choose their methods and representations strategically. Many of them also involve multiple steps.

Try it at home!

Near the end of the unit, ask your student to solve the following problems:

- 16×48
- $324 \div 6$

Questions that may be helpful as they work:

- Can you draw a diagram to help you solve the problem?
- Can you explain the steps of your algorithm?

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Unit Assessments

Check Your Readiness A, B, C and D

End-of-Unit Assessment

Assessment : Section A Checkpoint

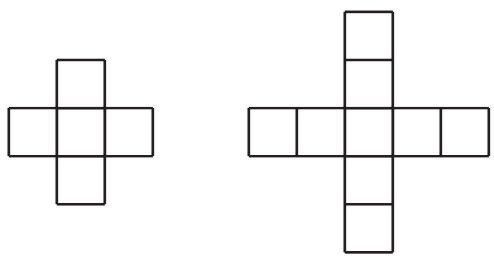
Problem 1

Goals Assessed

- Generate a number or shape pattern that follows a given rule.

Statement

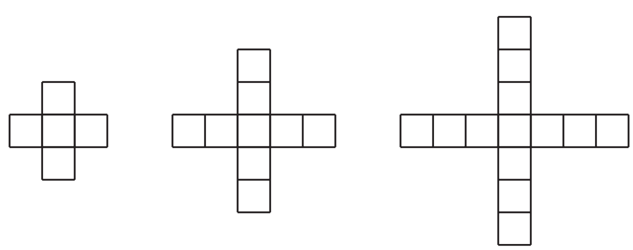
Here are the first two steps of a pattern of squares that look like + symbols. At each step, 4 squares are added.



step 1 step 2 step 3

1. Draw the next figure in the pattern.
2. How many squares will be needed in step 7? Explain or show your reasoning.
3. Will there be a figure that uses exactly 40 squares? Explain or show your reasoning.

Solution

1. 

   step 1 step 2 step 3

2. 29. Sample reasoning: There are 5 squares in step 1. Adding 4 squares 6 times takes us to step 7. $5 + (6 \times 4) = 5 + 24 = 29$.
3. No. Sample reasoning: The number of squares will always be an odd number because we start with an odd number and always add 4, an even number, each time.

Problem 2

Goals Assessed

- Generate a number or shape pattern that follows a given rule.
- Identify apparent features of a number pattern that were not explicit in the rule itself.

Statement

Here are the first three multiples of 8:

8, 16, 24, ...

Here are the first three multiples of 12:

12, 24, 36, ...

What are the next three numbers that are on both lists? Explain or show your reasoning.

Solution

48, 72, and 96. Sample reasoning:

- I extended the pattern until I found three more common numbers for the patterns.
- 24 is on both lists and going out 3 more places in the lists adds 24 more.

Assessment : Section B Checkpoint

Problem 1

Goals Assessed

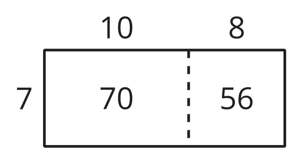
- Multiply a whole number of up to four digits by a one-digit whole number, and 2 two-digit numbers using strategies based on place value and the properties of operations.

Statement

Find the value of 18×7. Explain or show your reasoning.

Solution

$18 \times 7 = 126$. Sample reasoning:



$70 + 56 = 126$

Problem 2

Goals Assessed

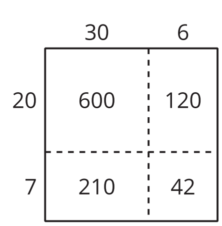
- Multiply a whole number of up to four digits by a one-digit whole number, and 2 two-digit numbers using strategies based on place value and the properties of operations.

Statement

Find the value of 27×36. Explain or show your reasoning.

Solution

972. Sample reasoning:



$600 + 120 + 210 + 42 = 972$

Problem 3

Goals Assessed

- Multiply a whole number of up to four digits by a one-digit whole number, and 2 two-digit numbers using strategies based on place value and the properties of operations.

Statement

Here is Noah’s work finding the value of 92×78.

Do you agree with Noah’s work? Explain or show your reasoning.

$$
\begin{array}{rcccc}
 & & & 9 & 2 \\
 & \times & & 7 & 8 \\
\hline
 & & & 1 & \\
 & & & 1 & 6 \\
 & & 7 & 2 & 0 \\
 & & & 1 & 4 \\
+ & & 6 & 3 & 0 \\
\hline
 & 1, & 3 & 8 & 0
\end{array}
$$

Solution

I disagree with Noah. Sample reasoning: He multiplied 2 and 90 by 7 instead of by 70. Those partial products should be 10 times as much, 140 and 6,300 instead of 14 and 630.

Assessment : Section C Checkpoint

Problem 1

Goals Assessed

- Use an algorithm that uses partial quotients to divide multi-digit numbers of up to four digits by one-digit divisors, resulting in numbers with or without a remainder.

Statement

A rectangular wall that is 8 feet tall is covered with 296 square feet of wallpaper.

How many feet long is the wall? Explain or show your reasoning. Use a diagram if it is helpful.

Solution

37 feet. Sample reasoning: $80 + 80 + 80 + 56 = 296$ and $10 + 10 + 10 + 7 = 37$

Problem 2

Goals Assessed

- Use an algorithm that uses partial quotients to divide multi-digit numbers of up to four digits by one-digit divisors, resulting in numbers with or without a remainder.

Statement

Find the value of $1{,}925 \div 7$ using an algorithm that uses partial quotients.

Solution

275. Sample reasoning:

$$
\begin{array}{r}
\boxed{275} \\
5 \\
70 \\
200 \\
7\overline{)1{,}925} \\
-1{,}400 \\
\hline
525 \\
-\ \ 490 \\
\hline
35 \\
-\ \ 35 \\
\hline
0
\end{array}
$$

Problem 3

Goals Assessed

- Use an algorithm that uses partial quotients to divide multi-digit numbers of up to four digits by one-digit divisors, resulting in numbers with or without a remainder.

Statement

Mai is putting together a photo album. She has 229 photos. Each page can hold 9 photos. How many pages of the photo album does Mai need for all the photos? Explain or show your reasoning.

Solution

26 pages. Sample reasoning: 229 divided by 9 gives 25 with a remainder of 4. This means that 25 pages will be full and there will be 1 page with only 4 photos.

$$
\begin{array}{r}
\boxed{25} \\
5 \\
20 \\
9\overline{)229} \\
-\ \ 180 \\
\hline
49 \\
-\ \ 45 \\
\hline
4
\end{array}
$$

Assessment : Section D Checkpoint

Problem 1

Goals Assessed

- Use the four operations to solve problems that involve multi-digit whole numbers and assess the reasonableness of answers.

Statement

A rectangular garden is 11 yards long and 9 feet wide. How many square feet is the area of the garden? Explain or show your reasoning.

Solution

297 square feet. Sample reasoning: I know that 11 yards is 33 feet so the area is 9×33 feet. I know $9 \times 3 = 27$ and $9 \times 30 = 270$ and $27 + 270 = 297$.

Problem 2

Goals Assessed

- Use the four operations to solve problems that involve multi-digit whole numbers and assess the reasonableness of answers.

Statement

Tyler scored 126 points in 9 basketball games.

1. If Tyler scored the same number of points each game, how many points did he score in each game? Explain or show your reasoning.
2. Diego scored 5 more points than Tyler did in each game. How many points did Diego score in the 9 games? Explain or show your reasoning.

Solution

1. 14 points. Sample reasoning: I know that $10 \times 9 = 90$ and $4 \times 9 = 36$ and $90 + 36 = 126$. So I added 10 and 4.
2. 171 points. Sample reasoning: $126 + (9 \times 5) = 126 + 45 = 171$.

Assessment : End-of-Unit Assessment

Problem 1

Students evaluate statements about the sequence of multiples of 3. Students who select C are probably just looking at the given numbers and not thinking about the next number in the sequence. Students who select E have probably also answered question B incorrectly or they do not recall what a multiple is. Students who do not select B or D have not understood that the rule for the sequence can be written using multiplication.

Statement

Here is the beginning of a list of multiples of 3:

3, 6, 9, 12, 15, 18, ...

Select **all** statements that are true about this list.

A. The terms in the list switch between even and odd.

B. The 10th term in the list is 30.

C. There are no multiples of 7 in the list.

D. The 100th term in the list is 300.

E. The next multiple of 5 in the list is 60.

Solution

["A", "B", "D"]

Aligned Standards

4.OA.C.5

Problem 2

Students estimate the value of the product of a four-digit number and a one-digit number. While they may calculate the product and find the closest of the choices they can also answer the question by rounding or estimating. Students who select A may be performing division rather than multiplication while answers B and D likely imply a misunderstanding of place value.

Statement

Which is the best estimate for the value of $7{,}395 \times 8$?

A. 1,000

B. 6,000

C. 60,000

D. 600,000

Solution

C

Aligned Standards

4.NBT.B.5

Problem 3

This item complements the previous one, addressing the operation of division. Once again, students can perform the division and then choose the closest answer but this is not necessary. Students who select A or C do not understand place value while students who select D have probably performed the wrong operation, multiplication instead of division.

Statement

Which is the best estimate for the value of $9{,}995 \div 5$?

A. 200

B. 2,000

C. 20,000

D. 50,000

Solution

B

Aligned Standards

4.NBT.B.6

Problem 4

Students find products of a one-digit and four-digit number and two two-digit numbers. Students may draw a diagram to show partial products or they may write equations or they may arrange their calculations vertically.

Statement

Find the value of each product. Show or explain your reasoning.

1. $8 \times 4{,}174$
2. 35×74

Solution

1. 33,392. Sample response:

$$
\begin{array}{r}
4{,}\,1\;7\;4 \\
\times \qquad\quad 8 \\
\hline
{\color{blue}1}\qquad\quad\; \\
3\;2 \\
5\;6\;0 \\
8\;0\;0 \\
+\;3\;2{,}\,0\;0\;0 \\
\hline
3\;3{,}\,3\;9\;2
\end{array}
$$

2. 2,590. Sample response:

$$
\begin{aligned}
30 \times 70 &= 2{,}100 \\
30 \times 4 &= 120 \\
5 \times 70 &= 350 \\
5 \times 4 &= 20 \\
35 \times 74 &= 2{,}590
\end{aligned}
$$

Aligned Standards

4.NBT.B.5

Problem 5

Students find quotients of three- and four-digit numbers by a one-digit divisor. They may use diagrams or vertical calculations, or they may add multiples and work their way up to the number. Small arithmetic errors in multiplication or subtraction may show that students need further practice with arithmetic outside of the context of division (although they may understand the meaning of division well).

Statement

Find the value of each quotient. Explain or show your reasoning.

1. $714 \div 6$
2. $3{,}626 \div 7$

Solution

1. 119. Sample response:

$$
\begin{array}{r}
600 \div 6 = 100 \\
60 \div 6 = \;\;10 \\
30 \div 6 = \;\;\;\;5 \\
24 \div 6 = \;\;\;\;4 \\
\hline
714 \div 5 = 119
\end{array}
$$

2. 518. Sample responses: First I took away 500 sevens from 3,626, then 10 sevens, and then 8 sevens. There was nothing left, so $3{,}626 \div 7 = 518$.

$$
\begin{array}{rrl}
 & \boxed{518} & \\
 & 8 & \\
 & 10 & \\
 & 500 & \\
7 & \overline{)3,626} & \\
- & 3,500 & 7 \times 500 \\
\hline
 & 126 & \\
- & 70 & 7 \times 10 \\
\hline
 & 56 & \\
- & 56 & 7 \times 8 \\
\hline
 & 0 &
\end{array}
$$

Aligned Standards

4.NBT.B.6

Problem 6

Students perform multiplication and division to solve problems about the same context. They can use any method to calculate. It is important for students to see multiplication and division together so that they understand how to identify the correct operation from what is happening in context.

Statement

1. Lin's school bought 275 packages of pens. There are 6 pens in each package. How many pens did Lin's school buy? Show your reasoning.
2. Another school bought a total of 1,734 pens. There are 6 pens in each package. How many packages of pens did this school buy? Show your reasoning.

Solution

1. 1,650. Sample response:

$$
\begin{array}{rrrrr}
 & & 2 & 7 & 5 \\
\times & & & & 6 \\
\hline
 & & & 3 & 0 \\
 & & 4 & 2 & 0 \\
+ & 1, & 2 & 0 & 0 \\
\hline
 & 1, & 6 & 5 & 0
\end{array}
$$

2. 289. Sample response:

$$\begin{array}{r l} \boxed{289} & \\ 9 & \\ 80 & \\ 200 & \\ 6\overline{)1{,}734} & \\ -1{,}200 & 6 \times 200 \\ \hline 534 & \\ -\ 480 & 6 \times 80 \\ \hline 54 & \\ -\ 54 & 6 \times 9 \\ \hline 0 & \end{array}$$

Aligned Standards

4.NBT.B.5, 4.NBT.B.6, 4.OA.A.3

Problem 7

Students use division to solve a problem where they need to interpret the remainder. In this situation, the quotient shows how many full bags of soil are needed for a project and the remainder means that part of another bag is needed. Students might say that 26 bags is sufficient and they can spread the soil around. This is also an acceptable answer, reflecting an understanding of the situation. The important point is for students to realize that only whole numbers make sense as answers so they either need to get an extra bag of soil or explain what they can do with one less bag.

Statement

Jada's family is getting soil for a garden. The garden will be 160 square feet. Each bag of soil covers 6 square feet. How many bags of soil does Jada's family need? Explain or show your reasoning.

Solution

27. Sample response: 20 bags covers 120 square feet. Then another 6 covers 36 more feet or 156 square feet total. That's not quite enough so they need one more bag. The 27 bags cover 162 square feet.

Aligned Standards

4.OA.A.3

Problem 8

Students solve a multi-step problem about tiling a rectangle with squares of different sizes. They will need to convert from feet to inches and then check whether or not the dimensions of the rectangle are multiples of the different tile side lengths in order to decide which tiles will work to cover the rectangle. The numbers for each problem are friendly but each problem requires a conversion from feet to inches and some calculations.

Statement

Clare is using square tiles to tile a bathroom. The bathroom is a rectangle that measures 8 feet by 5 feet. The tiles come in three sizes: 3 inches by 3 inches, 4 inches by 4 inches, and 8 inches by 8 inches.

1. How many tiles will Clare need if she uses only 3-inch tiles? Show your reasoning. Draw a diagram if it is helpful.
2. How many tiles will she need if she uses only 4-inch tiles? Show your reasoning. Draw a diagram if it is helpful.
3. Can Clare cover the space using only 8 inch by 8 inch tiles? Explain or show your reasoning.

Solution

1. 640. Sample response: A foot is 12 inches so a 1 foot by 1 foot square is 12 inches by 12 inches. I need 16 of the 3-inch tiles to cover one square foot. The whole area is 8 feet by 5 feet so is covered by 40 of these individual square feet. So she will need $40 \times 16 = 640$ of the 3-inch tiles.
2. 360. Sample response: A foot is 12 inches so a 1 foot by 1 foot square is 12 inches by 12 inches. I need 9 of the 4-inch tiles to cover one square foot. The whole area is 8 feet by 5 feet so is covered by 40 of these individual square feet. So she will need $40 \times 19 = 360$ of the 4-inch tiles.
3. No. Sample response: The rectangle is 8×12 by 5×12 inches, which is 96 inches by 60 inches. The side that’s 96 inches is a multiple of 8, so 12 of the 8 inch by 8 inch tiles will fill that side. But the side that’s 60 inches won’t work with these tiles because 7 of the tiles make 56 inches and then one more is 64 inches.

Aligned Standards

4.MD.A.1, 4.MD.A.3, 4.NBT.B.5

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Assessment Answer Keys

Check Your Readiness A, B, C and D

End-of-Unit Assessment

Assessment Answer Keys

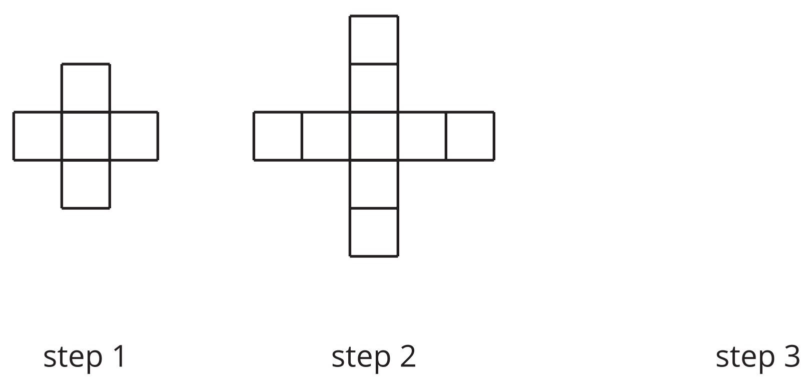
Assessment: Section A Checkpoint

Problem 1

Goals Assessed

- Generate a number or shape pattern that follows a given rule.

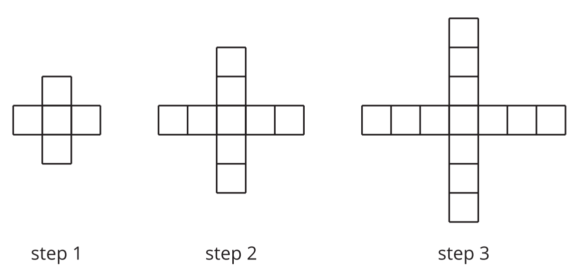
Here are the first two steps of a pattern of squares that look like + symbols. At each step, 4 squares are added.



a. Draw the next figure in the pattern.
b. How many squares will be needed in step 7? Explain or show your reasoning.
c. Will there be a figure that uses exactly 40 squares? Explain or show your reasoning.

Solution

a.



b. 29. Sample reasoning: There are 5 squares in step 1. Adding 4 squares 6 times takes us to step 7. $5 + (6 \times 4) = 5 + 24 = 29$.
c. No. Sample reasoning: The number of squares will always be an odd number because we

start with an odd number and always add 4, an even number, each time.

Problem 2

Goals Assessed

- Generate a number or shape pattern that follows a given rule.
- Identify apparent features of a number pattern that were not explicit in the rule itself.

Here are the first three multiples of 8:

8, 16, 24, ...

Here are the first three multiples of 12:

12, 24, 36, ...

What are the next three numbers that are on both lists? Explain or show your reasoning.

Solution

48, 72, and 96. Sample reasoning:

- I extended the pattern until I found three more common numbers for the patterns.
- 24 is on both lists and going out 3 more places in the lists adds 24 more.

Assessment: Section B Checkpoint

Problem 1

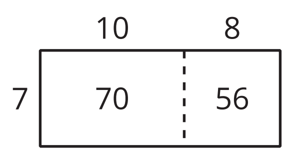
Goals Assessed

- Multiply a whole number of up to four digits by a one-digit whole number, and 2 two-digit numbers using strategies based on place value and the properties of operations.

Find the value of 18×7. Explain or show your reasoning.

Solution

$18 \times 7 = 126$. Sample reasoning:



$70 + 56 = 126$

Problem 2

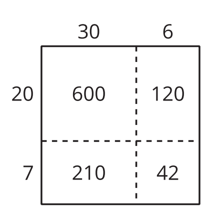
Goals Assessed

- Multiply a whole number of up to four digits by a one-digit whole number, and 2 two-digit numbers using strategies based on place value and the properties of operations.

Find the value of 27×36. Explain or show your reasoning.

Solution

972. Sample reasoning:



$600 + 120 + 210 + 42 = 972$

Problem 3

Goals Assessed

- Multiply a whole number of up to four digits by a one-digit whole number, and 2 two-digit numbers using strategies based on place value and the properties of operations.

Here is Noah's work finding the value of 92×78.

Do you agree with Noah's work? Explain or show your reasoning.

$$\begin{array}{r} 92 \\ \times\ \ 78 \\ \hline {\scriptstyle 1}\ \, \\ 16 \\ 720 \\ 14 \\ +\ \ 630 \\ \hline 1{,}380 \end{array}$$

Solution

I disagree with Noah. Sample reasoning: He multiplied 2 and 90 by 7 instead of by 70. Those partial products should be 10 times as much, 140 and 6,300 instead of 14 and 630.

Assessment: Section C Checkpoint

Problem 1

Goals Assessed

- Use an algorithm that uses partial quotients to divide multi-digit numbers of up to four digits by one-digit divisors, resulting in numbers with or without a remainder.

A rectangular wall that is 8 feet tall is covered with 296 square feet of wallpaper.

How many feet long is the wall? Explain or show your reasoning. Use a diagram if it is helpful.

Solution

37 feet. Sample reasoning: $80 + 80 + 80 + 56 = 296$ and $10 + 10 + 10 + 7 = 37$

Problem 2

Goals Assessed

- Use an algorithm that uses partial quotients to divide multi-digit numbers of up to four digits by one-digit divisors, resulting in numbers with or without a remainder.

Find the value of $1{,}925 \div 7$ using an algorithm that uses partial quotients.

Solution

275. Sample reasoning:

$$
\begin{array}{r}
\boxed{275} \\
5 \\
70 \\
200 \\
7\overline{)1{,}925} \\
-1{,}400 \\
\hline
525 \\
-\ \ 490 \\
\hline
35 \\
-\ \ \ 35 \\
\hline
0
\end{array}
$$

Problem 3

Goals Assessed

- Use an algorithm that uses partial quotients to divide multi-digit numbers of up to four digits by one-digit divisors, resulting in numbers with or without a remainder.

Mai is putting together a photo album. She has 229 photos. Each page can hold 9 photos. How many pages of the photo album does Mai need for all the photos? Explain or show your reasoning.

Solution

26 pages. Sample reasoning: 229 divided by 9 gives 25 with a remainder of 4. This means that 25 pages will be full and there will be 1 page with only 4 photos.

$$
\begin{array}{r}
\boxed{25} \\
5 \\
20 \\
9\overline{)229} \\
-\ \ 180 \\
\hline
49 \\
-\ \ \ 45 \\
\hline
4
\end{array}
$$

Assessment: Section D Checkpoint

Problem 1

Goals Assessed

- Use the four operations to solve problems that involve multi-digit whole numbers and assess the reasonableness of answers.

A rectangular garden is 11 yards long and 9 feet wide. How many square feet is the area of the garden? Explain or show your reasoning.

Solution

297 square feet. Sample reasoning: I know that 11 yards is 33 feet so the area is 9×33 feet. I know $9 \times 3 = 27$ and $9 \times 30 = 270$ and $27 + 270 = 297$.

Problem 2

Goals Assessed

- Use the four operations to solve problems that involve multi-digit whole numbers and assess the reasonableness of answers.

Tyler scored 126 points in 9 basketball games.

a. If Tyler scored the same number of points each game, how many points did he score in each game? Explain or show your reasoning.
b. Diego scored 5 more points than Tyler did in each game. How many points did Diego score in the 9 games? Explain or show your reasoning.

Solution

a. 14 points. Sample reasoning: I know that $10 \times 9 = 90$ and $4 \times 9 = 36$ and $90 + 36 = 126$. So I added 10 and 4.
b. 171 points. Sample reasoning: $126 + (9 \times 5) = 126 + 45 = 171$.

Assessment: End-of-Unit Assessment

Problem 1

Standards Alignments

Addressing 4.OA.C.5

Narrative

Students evaluate statements about the sequence of multiples of 3. Students who select C are probably just looking at the given numbers and not thinking about the next number in the sequence. Students who select E have probably also answered question B incorrectly or they do not recall what a multiple is. Students who do not select B or D have not understood that the rule for the sequence can be written using multiplication.

Here is the beginning of a list of multiples of 3:

3, 6, 9, 12, 15, 18, ...

Select **all** statements that are true about this list.

A. The terms in the list switch between even and odd.

B. The 10th term in the list is 30.

C. There are no multiples of 7 in the list.

D. The 100th term in the list is 300.

E. The next multiple of 5 in the list is 60.

Solution

["A", "B", "D"]

Problem 2

Standards Alignments

Addressing 4.NBT.B.5

Narrative

Students estimate the value of the product of a four-digit number and a one-digit number. While they may calculate the product and find the closest of the choices they can also answer the question by rounding or estimating. Students who select A may be performing division rather than multiplication while answers B and D likely imply a misunderstanding of place value.

Which is the best estimate for the value of $7{,}395 \times 8$?

A. 1,000

B. 6,000

C. 60,000

D. 600,000

Solution

C

Problem 3

Standards Alignments

Addressing 4.NBT.B.6

Narrative

This item complements the previous one, addressing the operation of division. Once again, students can perform the division and then choose the closest answer but this is not necessary. Students who select A or C do not understand place value while students who select D have probably performed the wrong operation, multiplication instead of division.

Which is the best estimate for the value of $9{,}995 \div 5$?

A. 200

B. 2,000

C. 20,000

D. 50,000

Solution

B

Problem 4

Standards Alignments

Addressing 4.NBT.B.5

Narrative

Students find products of a one-digit and four-digit number and two two-digit numbers. Students may draw a diagram to show partial products or they may write equations or they may arrange their calculations vertically.

Find the value of each product. Show or explain your reasoning.

a. $8 \times 4{,}174$

b. 35×74

Solution

a. 33,392. Sample response:

$$\begin{array}{rrrrrr} & & 4, & 1 & 7 & 4 \\ & \times & & & & 8 \\ \hline & & 1 & & & \\ & & & & 3 & 2 \\ & & & 5 & 6 & 0 \\ & & & 8 & 0 & 0 \\ + & 3 & 2, & 0 & 0 & 0 \\ \hline & 3 & 3, & 3 & 9 & 2 \end{array}$$

b. 2,590. Sample response:

$30 \times 70 = 2{,}100$
$30 \times 4 = 120$
$5 \times 70 = 350$
$5 \times 4 = 20$
$35 \times 74 = 2{,}590$

Problem 5

Standards Alignments

Addressing 4.NBT.B.6

Narrative

Students find quotients of three- and four-digit numbers by a one-digit divisor. They may use diagrams or vertical calculations, or they may add multiples and work their way up to the number. Small arithmetic errors in multiplication or subtraction may show that students need further practice with arithmetic outside of the context of division (although they may understand the meaning of division well).

Find the value of each quotient. Explain or show your reasoning.

a. $714 \div 6$

b. $3{,}626 \div 7$

Solution

a. 119. Sample response:

$$\begin{array}{r}
600 \div 6 = 100 \\
60 \div 6 = 10 \\
30 \div 6 = 5 \\
24 \div 6 = 4 \\
\hline
714 \div 5 = 119
\end{array}$$

b. 518. Sample responses: First I took away 500 sevens from 3,626, then 10 sevens, and then 8 sevens. There was nothing left, so $3{,}626 \div 7 = 518$.

$$\begin{array}{rrl}
 & \boxed{518} & \\
 & 8 & \\
 & 10 & \\
 & 500 & \\
7 & \big)\overline{3{,}626} & \\
 & -3{,}500 & 7 \times 500 \\
\cline{2-2}
 & 126 & \\
 & -70 & 7 \times 10 \\
\cline{2-2}
 & 56 & \\
 & -56 & 7 \times 8 \\
\cline{2-2}
 & 0 &
\end{array}$$

Problem 6

Standards Alignments

Addressing 4.NBT.B.5, 4.NBT.B.6, 4.OA.A.3

Narrative

Students perform multiplication and division to solve problems about the same context. They can use any method to calculate. It is important for students to see multiplication and division together so that they understand how to identify the correct operation from what is happening in context.

a. Lin's school bought 275 packages of pens. There are 6 pens in each package. How many pens did Lin's school buy? Show your reasoning.

b. Another school bought a total of 1,734 pens. There are 6 pens in each package. How many packages of pens did this school buy? Show your reasoning.

Solution

a. 1,650. Sample response:

$$\begin{array}{r} 275 \\ \times\quad 6 \\ \hline 30 \\ 420 \\ +\ 1{,}200 \\ \hline 1{,}650 \end{array}$$

b. 289. Sample response:

$$\begin{array}{rl} \boxed{289} & \\ 9 & \\ 80 & \\ 200 & \\ 6\,\overline{)1{,}734} & \\ -1{,}200 & 6 \times 200 \\ \hline 534 & \\ -\ 480 & 6 \times 80 \\ \hline 54 & \\ -\ 54 & 6 \times 9 \\ \hline 0 & \end{array}$$

Problem 7

Standards Alignments

Addressing 4.OA.A.3

Narrative

Students use division to solve a problem where they need to interpret the remainder. In this situation, the quotient shows how many full bags of soil are needed for a project and the remainder means that part of another bag is needed. Students might say that 26 bags is sufficient and they can spread the soil around. This is also an acceptable answer, reflecting an understanding of the situation. The important point is for students to realize that only whole numbers make sense as answers so they either need to get an extra bag of soil or explain what they can do with one less bag.

Jada's family is getting soil for a garden. The garden will be 160 square feet. Each bag of soil covers 6 square feet. How many bags of soil does Jada's family need? Explain or show your reasoning.

Solution

27. Sample response: 20 bags covers 120 square feet. Then another 6 covers 36 more feet or 156 square feet total. That's not quite enough so they need one more bag. The 27 bags cover 162 square feet.

Problem 8

Standards Alignments

Addressing 4.MD.A.1, 4.MD.A.3, 4.NBT.B.5

Narrative

Students solve a multi-step problem about tiling a rectangle with squares of different sizes. They will need to convert from feet to inches and then check whether or not the dimensions of the rectangle are multiples of the different tile side lengths in order to decide which tiles will work to cover the rectangle. The numbers for each problem are friendly but each problem requires a conversion from feet to inches and some calculations.

Clare is using square tiles to tile a bathroom. The bathroom is a rectangle that measures 8 feet by 5 feet. The tiles come in three sizes: 3 inches by 3 inches, 4 inches by 4 inches, and 8 inches by 8 inches.

a. How many tiles will Clare need if she uses only 3-inch tiles? Show your reasoning. Draw a diagram if it is helpful.

b. How many tiles will she need if she uses only 4-inch tiles? Show your reasoning. Draw a diagram if it is helpful.

c. Can Clare cover the space using only 8 inch by 8 inch tiles? Explain or show your reasoning.

Solution

a. 640. Sample response: A foot is 12 inches so a 1 foot by 1 foot square is 12 inches by 12 inches. I need 16 of the 3-inch tiles to cover one square foot. The whole area is 8 feet by 5 feet so is covered by 40 of these individual square feet. So she will need $40 \times 16 = 640$ of the 3-inch tiles.

b. 360. Sample response: A foot is 12 inches so a 1 foot by 1 foot square is 12 inches by 12 inches. I need 9 of the 4-inch tiles to cover one square foot. The whole area is 8 feet by 5 feet so is covered by 40 of these individual square feet. So she will need $40 \times 19 = 360$ of the 4-inch tiles.

c. No. Sample response: The rectangle is 8×12 by 5×12 inches, which is 96 inches by 60 inches. The side that's 96 inches is a multiple of 8, so 12 of the 8 inch by 8 inch tiles will fill that side. But the side that's 60 inches won't work with these tiles because 7 of the tiles make 56 inches and then one more is 64 inches.

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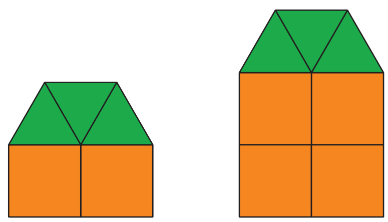
Lesson Cool Downs

Lesson 1: Patterns that Grow

Cool Down: Andre's House Pattern

Andre used pattern blocks to make houses in a pattern. For each new step, he adds a new “floor” made of squares. The triangles are used for the roof of the house.



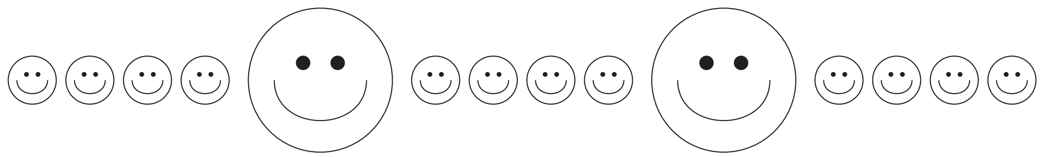
1. Draw the next step in Andre’s pattern.
2. If Andre continues the pattern:
    a. How many triangles will Andre use in the 15th house? Explain or show your reasoning.

    b. How many squares will Andre use in the 15th house? Explain or show your reasoning.

Lesson 2: Patterns that Repeat

Cool Down: Happy Faces

Diego created a pattern with smiley faces.



1. Extend Diego's pattern by drawing the next 5 shapes.

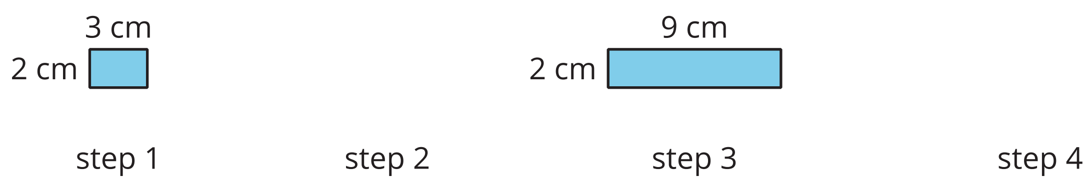
2. If Diego numbered the smiley faces, what numbers would he write for the first 5 large smiley faces?

3. Will the 42nd smiley face be a large one or a small one? Explain or show your reasoning.

Lesson 3: From Visual Patterns to Numerical Patterns

Cool Down: Another Set of Rectangles

Here are steps 1 and 3 in a pattern of rectangles where a side length grows by 3 centimeters each time.



1. Draw the missing rectangles in steps 2 and 4. Label the sides with their lengths.

2. Write a numerical pattern to represent the pattern. Explain how your numerical pattern represents the rectangles.

3. If the pattern continues, could 50 represent the side length or area of one of the rectangles? If so, which step? If not, why not? Explain or show your reasoning.

Lesson 4: Numerical Patterns

Cool Down: Count by 8

Kiran counted by 8 and recorded the numbers he counted:

8 16 24 32 40 48

Could 105 be a number that Kiran writes if he continued to count by 8? Explain or show your reasoning.

Lesson 5: Products Beyond 100

Cool Down: Rows of Seats

A theater has 8 rows of seats and 27 seats in each row. How many seats are in the theater? Show your reasoning.

Lesson 6: Multiply Two-digit Numbers and One-digit Numbers

Cool Down: Represent the Product

Find the value of 6×83. Use a diagram if it is helpful.

Lesson 7: Multiply Three- and Four-digit Numbers by One-digit Numbers

Cool Down: The Value of the Product

Find the value of 6×218. Show your reasoning.

Lesson 8: Multiply 2 Two-digit Numbers

Cool Down: What's the Product?

Find the value of 24×17. Explain or show your reasoning. Use a diagram if it helpful.

Lesson 9: Recording Partial Products: One-digit and Three- or Four-digit Factors

Cool Down: Partial Products

Find the value of $5 \times 1{,}023$. Show your reasoning.

Lesson 10: Using Algorithms with Partial Products: 2 Two-digit Numbers

Cool Down: Choose Your Own Strategy

Find the value of 15×43. Show your reasoning.

Lesson 11: Partial Products and the Standard Algorithm

Cool Down: Choose a Way to Multiply

Find the value of each product. Show your reasoning.

1. 4×798

2. $8 \times 2{,}864$

Lesson 12: Solve Problems Involving Multiplication

Cool Down: Leap Year

In a leap year, the month of February has 29 days. How many hours are in that month? Show your reasoning.

Lesson 13: Situations Involving Equal-size Groups

Cool Down: After the Class Party

After the class party, 6 students offer to wash 96 pieces of utensils (spoons and forks). Each student is washing the same number of utensils.

How many pieces of utensils does each student wash? Explain or show your reasoning.

Lesson 14: Situations Involving Factors and Multiples

Cool Down: Reaching 161 with Multiples

Mai is writing multiples of a number. When she reaches 161, she has written 7 numbers.

1. What number is Mai writing multiples of? Explain or show your reasoning.

2. What division equation can represent the question you just answered?

Lesson 15: Situations Involving Area

Cool Down: Sticky Notes on the Door

Jada's class is decorating their door with square sticky notes for their teacher. Each sticky note has a drawing or a message from a student.

The class used 234 square sticky notes to cover their classroom door completely, leaving no gaps or overlaps between the notes. It takes 9 square notes to cover the width of the door.

How many square notes does it take to cover the full height of the door? Show how you know.

Lesson 16: Base-ten Blocks to Divide

Cool Down: Division Reflection

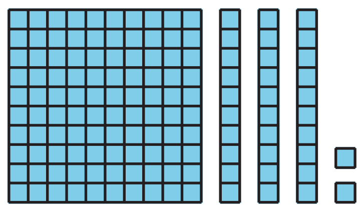
How was using the base-ten blocks helpful in your work today? How was it not helpful?

Lesson 17: Base-ten Diagrams to Represent Division

Cool Down: Find the Value of a Quotient

Find the value of $132 \div 4$. Show your reasoning.

The base-ten diagram represents 132. Use the diagram if you find it helpful.



Lesson 18: Divide with Partial Quotients

Cool Down: Subtract Groups

Priya and Tyler use different methods to find $430 \div 5$. Their work is incomplete. Complete Priya's and Tyler's work.

Priya's work

$$\begin{array}{r} 300 \div 5 = \\ 100 \div 5 = \\ 30 \div 5 = \\ \hline 430 \div 5 = \end{array}$$

Tyler's work

$$\begin{array}{rrl} & 6 & \\ & 20 & \\ & 60 & \\ 5 & \overline{)430} & \\ - & 300 & \quad 5 \times 60 \\ \hline \end{array}$$

What is the value of $430 \div 5$?

Lesson 19: Division With and Without Remainders

Cool Down: Find a Quotient

How many groups of 4 are in 1,865?

Use partial quotients to show your reasoning.

Lesson 20: Interpret Remainders in Division Situations

Cool Down: Miscounting?

Mai is reciting multiples of 6. The last number she calls out is 194. Clare says, “I think you may have made a mistake.”

Do you agree with Clare? Explain or show your reasoning.

Lesson 21: Different Ways to Solve Problems

Cool Down: Big Weekend at the Movies

A one-room movie theater has 278 seats. Its goal is to sell 2,600 tickets every weekend. The theater plays a movie 5 times each Saturday and 4 times each Sunday.

Last weekend, the movie theater was completely full for every movie played on Saturday and Sunday. Did the movie theater meet its goal?

Lesson 22: Problems About Perimeter and Area

Cool Down: Paper for a Banner

Han has a rectangular piece of paper that is 96 inches by 36 inches. He is using it to create a banner for Awards Day. Last year the banner measured 2,304 square inches.

1. Will the new banner fit in the same area that the old banner was? Show your reasoning.

2. What is the difference in square inches between the area of last year's banner and this year's banner?

Lesson 23: Solve Problems with Many Operations

Cool Down: Long-distance Driving

A truck driver needs to deliver goods to a city that is 2,654 km away.

1. If she drives 285 km each day, could she get there in 8 days? Show your reasoning.

2. In the first three days, the driver traveled 1,087 km. At the end of the fourth day, she has 972 km to go. How many km did she travel on the fourth day?

Lesson 24: Assess the Reasonableness of Solutions

Cool Down: The Children and the Elderly

Here are the data on the numbers of children and senior citizens in Philadelphia as of 2017.

age	number of people
under 5 years	107,736
5–14 years	184,323
15–17 years	53,530
65 years and over	203,007

1. As of 2017, what is the number of people under the age of 18 in Philadelphia?

2. How do you know your answer to problem 1 is reasonable?

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Instructional Masters

Instructional Masters for Multiplying and Dividing Multi-digit Numbers

address	title	students per copy	written on?	requires cutting?	card stock recommended?	color paper recommended?
Activity Grade4.6.21.1	Going on a Field Trip	1	no	yes	no	no
Center	Can You Draw It Stage 4 Recording Sheet	1	yes	no	no	no
Center	Five in a Row Multiplication and Division Stage 2 Gameboard	2	no	no	no	no
Center	Number Puzzles Mult Stage 1 Recording Sheet	1	yes	no	no	no
Center	Compare Stage 3 Multiplication Cards	2	no	yes	no	no
Center	Compare Stage 3-8 Directions	2	yes	no	no	no
Center	Compare Stage 3-8 Directions	2	yes	no	no	no
Center	Compare Stage 3-8 Directions	2	yes	no	no	no
Center	Five in a Row Multiplication and Division Stage 3 Gameboard	2	no	no	no	no
Center	Compare Stage 4 Division Cards	2	no	yes	no	no
Center	Rolling for Fractions Stage 2 Recording Sheet	1	yes	no	no	no
Center	Watch Your Remainder Stage 1 Recording Sheet	1	yes	no	no	no
Center	Watch Your Remainder Stage 1 Spinner	2	no	no	no	no
Center	Compare Stage 7 Cards	2	no	yes	no	no

Going on a Field Trip

A Clare

If tickets were $20 each, the cost would be 45×20 or 900.

Because $18 is $2 less than $20, we need to subtract 45×2 from 45×20, or subtract 90 from 900, which is 810.

Going on a Field Trip

B Kiran

$$
\begin{aligned}
10 \times 18 &= 180 \\
20 \times 18 &= 360 \\
40 \times 18 &= 720 \\
5 \times 18 &= 90 \\
45 \times 18 &= (40 \times 18) + (5 \times 18) \\
&= 720 + 90 \\
&= 810
\end{aligned}
$$

Going on a Field Trip

C Han

100 tickets cost 1,800.

50 tickets is half of 1,800, which is 900.

45 tickets is less than 50 tickets,
so they will have enough money.

Going on a Field Trip

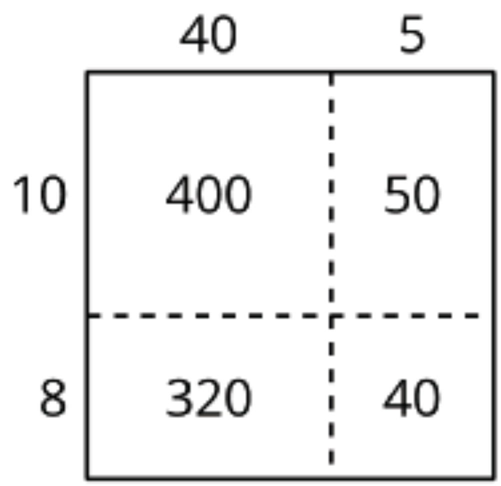
D Tyler

$$18 = 9 \times 2$$
$$2 \times 45 = 90$$

$$\begin{aligned} 18 \times 45 &= 9 \times 2 \times 45 \\ &= 9 \times 90 \\ &= 810 \end{aligned}$$

Going on a Field Trip

E Mai



$$400 + 320 + 50 + 40 = 810$$

Can You Draw It Stage 4 Recording Sheet

Directions:

- Partner A: Draw a rectangle and tell your partner either the area or the perimeter of your shape.
- Partner B:
    - Draw the rectangle you think your partner drew.
    - Earn 2 points if your rectangle matches your partners.
    - Earn 1 point if it doesn't match but has the correct area or perimeter.
- Take turns. The partner with the highest score at the end of 8 rounds wins.

round	drawing	points
1		
2		
3		
4		

Can You Draw It Stage 4 Recording Sheet

5		
6		
7		
8		

Five in a Row Multiplication and Division Stage 2 Gameboard

Directions:

- Partner A:
  - Put a paper clip on 2 numbers in the grey rows.
  - Multiply the numbers.
  - Cover the product of the 2 numbers with a counter.
- Partner B:
  - Move 1 of the paper clips, multiply the numbers, and cover the product with a counter.
- Take turns. The first partner to cover 5 squares in a row wins.

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

1	2	3	4	5

6	7	8	9

Puzzle 1

Fill in digits to make each equation true.
You may only use each digit (0–9) once.

$\boxed{1}\ \boxed{} \times \boxed{}\ \boxed{} = 230$
$\boxed{}\ \boxed{} \times \boxed{2}\ \boxed{5} = 425$
$\boxed{}\ \boxed{0} \times 31 = 1{,}550$
$\boxed{}\ \boxed{0} \times \boxed{}\ \boxed{0} = 2{,}400$
$\boxed{1}\ \boxed{} \times \boxed{2}\ \boxed{} = 522$

Puzzle 2

Fill in digits to make each equation true.
You may only use each digit (0–9) once.

$11 \times \square\,\boxed{2} = \boxed{3}\,\square\,\boxed{2}$
$\boxed{4}\,\square \times 20 = \boxed{9}\,\boxed{2}\,\square$
$\square\,\square \times 25 = 675$
$10 \times \square\,\square = 890$
$\square\,\boxed{1} \times \boxed{1}\,\square = 154$

Puzzle 3

Fill in digits to make each equation true.
You may only use each digit (0–9) once.

$\square\,\boxed{1} \times \boxed{1}\,\square = 1{,}349$
$\square\,\square \times 30 = 1{,}800$
$\square\,\boxed{5} \times \square\,\boxed{1} = 775$
$\boxed{4}\,\square \times \boxed{3}\,\square = 1{,}395$
$\boxed{3}\,\square \times 23 = \boxed{8}\,\boxed{7}\,\square$

Puzzle 4
Fill in digits to make each equation true.
You may only use each digit (0–9) once.

$\square\ \boxed{1} \times \boxed{1}\ \square = 610$
$\square\ \square \times 41 = 3{,}239$
$\square\ \boxed{7} \times \square\ \boxed{4} = 1{,}428$
$\boxed{5}\ \square \times \boxed{1}\ \square = 795$
$\boxed{1}\ \square \times 47 = \boxed{5}\ \boxed{6}\ \square$

Compare Stage 3 Multiplication Cards

Compare Stage 3

$$12 \times 9$$

Compare Stage 3

$$12 \times 7$$

Compare Stage 3

$$13 \times 7$$

Compare Stage 3

$$14 \times 6$$

Compare Stage 3

$$15 \times 6$$

Compare Stage 3

$$10 \times 20$$

Compare Stage 3

$$21 \times 4$$

Compare Stage 3

$$19 \times 5$$

Compare Stage 3 Multiplication Cards

Compare Stage 3

$$18 \times 5$$

Compare Stage 3

$$17 \times 4$$

Compare Stage 3

$$16 \times 6$$

Compare Stage 3

$$14 \times 7$$

Compare Stage 3

$$31 \times 3$$

Compare Stage 3

$$20 \times 4$$

Compare Stage 3

$$8 \times 9$$

Compare Stage 3

$$9 \times 7$$

Compare Stage 3 Multiplication Cards

Compare Stage 3 12×5	Compare Stage 3 13×4
Compare Stage 3 15×3	Compare Stage 3 9×5

Compare Stage 3-8 Directions

Directions:

- Split the deck between the players.
- Each player turns over a card.
- Compare the values. The player with the greater value keeps both cards.
- If the values are the same, each player turns over one more card. The player with the greater value keeps all four cards.
- Play until you run out of cards. The player with the most cards at the end of the game wins.

Record any sets of cards that are challenging to compare:

Compare Stage 3-8 Directions

Directions:

- Split the deck between the players.
- Each player turns over a card.
- Compare the values. The player with the greater value keeps both cards.
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Five in a Row Multiplication and Division Stage 3 Gameboard

Directions:

- Partner A:
  - Put a paper clip on 2 numbers in the grey rows. Multiply the numbers. Cover the product of the 2 numbers with a counter.
- Partner B:
  - Move 1 of the paper clips, multiply the numbers, and cover the product with a counter.
- Take turns. The first partner to cover 5 squares in a row wins.

252	294	450	351	360
378	252	312	336	405
315	405	288	273	351
390	273	378	324	450
360	450	294	360	252

12	13	14	15

21	24	27	30

Compare Stage 4 Division Cards

Compare Stage 4 $78 \div 6$	Compare Stage 4 $84 \div 7$
Compare Stage 4 $68 \div 4$	Compare Stage 4 $65 \div 5$
Compare Stage 4 $90 \div 6$	Compare Stage 4 $45 \div 15$
Compare Stage 4 $57 \div 19$	Compare Stage 4 $72 \div 18$

Compare Stage 4 Division Cards

Compare Stage 4

$$52 \div 13$$

Compare Stage 4

$$84 \div 12$$

Compare Stage 4

$$42 \div 7$$

Compare Stage 4

$$56 \div 8$$

Compare Stage 4

$$72 \div 9$$

Compare Stage 4

$$64 \div 8$$

Compare Stage 4

$$81 \div 9$$

Compare Stage 4

$$72 \div 3$$

Compare Stage 4 Division Cards

Compare Stage 4 $92 \div 4$	Compare Stage 4 $69 \div 3$
Compare Stage 4 $84 \div 4$	Compare Stage 4 $63 \div 3$

Rolling for Fractions Stage 2 Recording Sheet

- Each partner:
  - Roll 3 number cubes. Use the numbers to complete the expression. Write the product.
  - Check your partner's work to make sure you agree.
  - Determine the number of points each partner gets:
    - 2 points for creating an expression less than 1
    - 5 points for creating an expression greater than 1
    - 10 points for creating an expression that is equal to 1
- Repeat for the next round. The partner who has the most points once the recording sheet is full wins the game.

round	**equation**	**points**
1	$\square \times \frac{\square}{\square} =$	
2	$\square \times \frac{\square}{\square} =$	
3	$\square \times \frac{\square}{\square} =$	
4	$\square \times \frac{\square}{\square} =$	
5	$\square \times \frac{\square}{\square} =$	
6	$\square \times \frac{\square}{\square} =$	

Rolling for Fractions Stage 2 Recording Sheet

- Each partner:
    - Roll 3 number cubes. Use the numbers to complete the expression. Write the product.
    - Check your partner’s work to make sure you agree.
    - Determine the number of points each partner gets:
        - 2 points for creating an expression less than 1
        - 5 points for creating an expression greater than 1
        - 10 points for creating an expression that is equal to 1
- Repeat for the next round. The partner who has the most points once the recording sheet is full wins the game.

round	**equation**	**points**
1	$\square \times \frac{1}{\square} =$	
2	$\square \times \frac{1}{\square} =$	
3	$\square \times \frac{1}{\square} =$	
4	$\square \times \frac{1}{\square} =$	
5	$\square \times \frac{1}{\square} =$	
6	$\square \times \frac{1}{\square} =$	

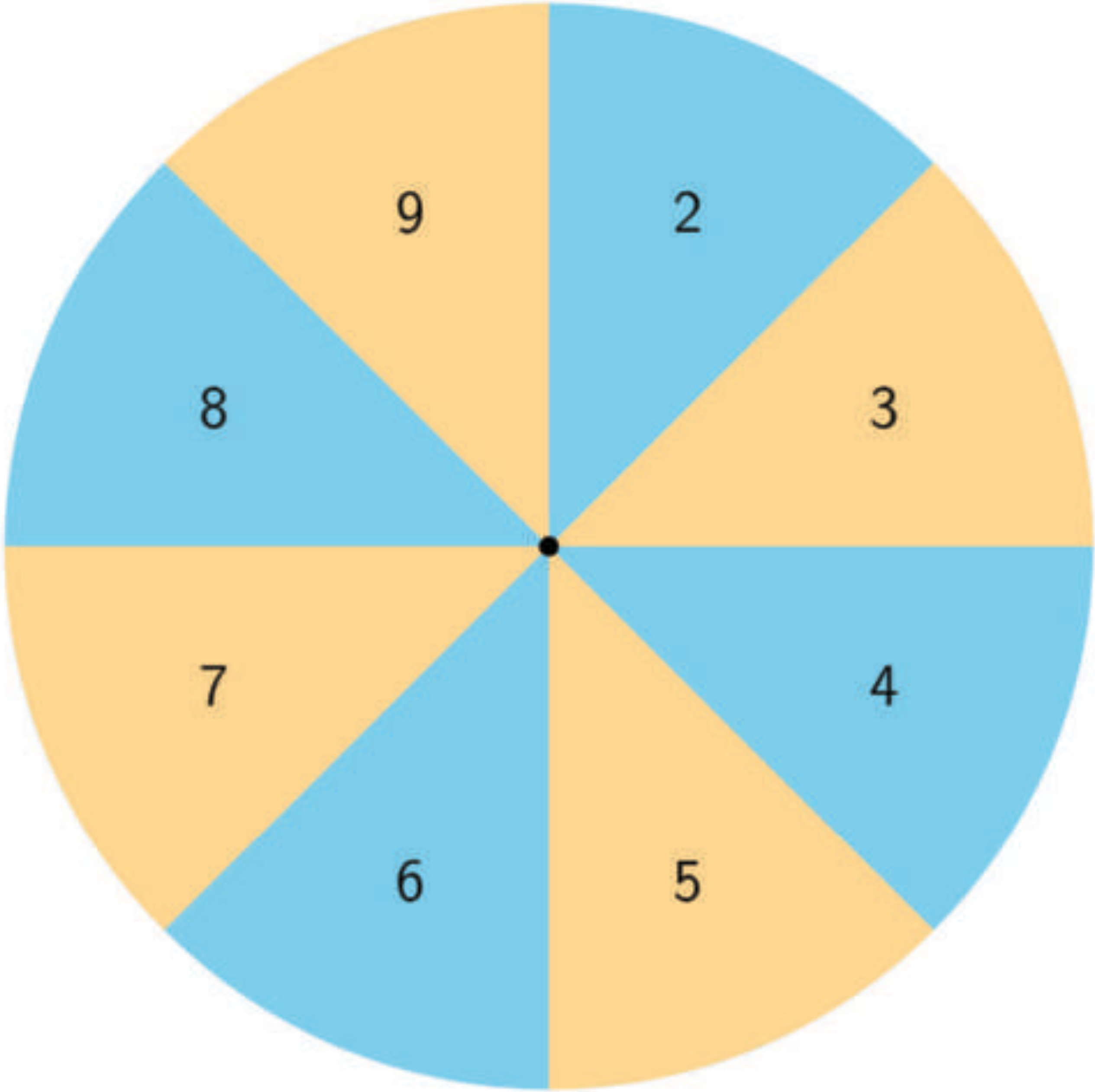
Watch Your Remainder Stage 1 Recording Sheet

Directions:

- Spin the spinner to get your one-digit divisor.
- Each partner:
  - Pick cards and use 3–4 of them to create a dividend.
  - Write a multiplication expression to represent the quotient. (For example, for $109 \div 9$ you would write $(9 \times 12) + 1$ and your score would be 1.)
  - Check your partner's work to make sure you agree.
  - Your score for the round is the remainder when you divide.
- Take new cards so that you have 4 cards to start the next round.
- The partner who has the fewest points once the recording sheet is full wins the game.

round	expression	points
1		
2		
3		
4		
5		
6		

Watch Your Remainder Stage 1 Spinner



Compare Stage 7 Cards

Compare Stage 7 $23{,}446 + 12{,}802$	Compare Stage 7 $43{,}921 + 102{,}392$
Compare Stage 7 $27{,}301 + 821{,}821$	Compare Stage 7 $91{,}234 + 89{,}001$
Compare Stage 7 $912{,}245 + 81{,}928$	Compare Stage 7 $82{,}391 + 28{,}319$
Compare Stage 7 $27{,}392 - 16{,}121$	Compare Stage 7 $86{,}954 - 42{,}321$
Compare Stage 7 $30{,}204 - 8{,}659$	Compare Stage 7 $100{,}000 - 72{,}734$

Compare Stage 7 Cards

Compare Stage 7 $182{,}492 - 18{,}652$	Compare Stage 7 $109{,}203 - 73{,}928$
Compare Stage 7 $8{,}354 \times 5$	Compare Stage 7 $5{,}294 \times 8$
Compare Stage 7 $9{,}263 \times 4$	Compare Stage 7 $4{,}826 \times 9$
Compare Stage 7 $7{,}934 \times 6$	Compare Stage 7 $6{,}839 \times 7$
Compare Stage 7 36×24	Compare Stage 7 28×42

Compare Stage 7 Cards

Compare Stage 7 54×25	Compare Stage 7 68×29
Compare Stage 7 74×56	Compare Stage 7 47×32
Compare Stage 7 $2{,}287 \div 3$	Compare Stage 7 $1{,}244 \div 4$
Compare Stage 7 $5{,}286 \div 6$	Compare Stage 7 $2{,}534 \div 5$
Compare Stage 7 $6{,}972 \div 3$	Compare Stage 7 $8{,}728 \div 4$

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ISBN: 979-8-88970-917-6