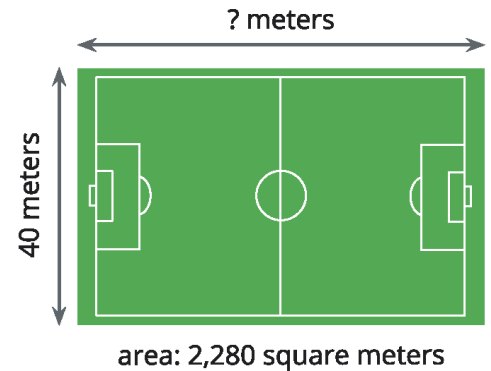




# Multiplying and Dividing Multi-digit Numbers



Student Workbook

$$2 \times 2 = 4$$

$$4 \times 2 = 8$$

$$8 : 4 = 2$$

$$4 - 2 = 2$$



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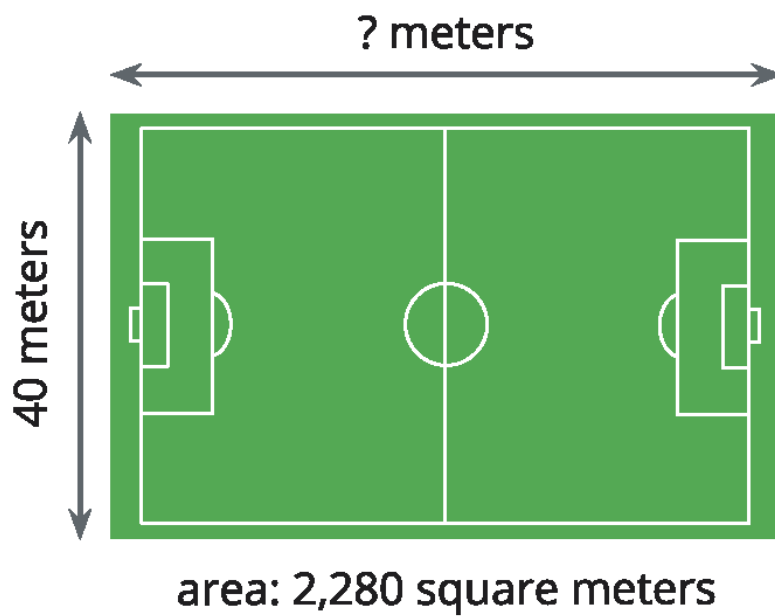
ISBN: 979-8-88970-916-9

# Multiplying and Dividing Multi-digit Numbers

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## Multiplying and Dividing Multi-digit Numbers Student Workbook

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# Lesson 1: Patterns that Grow

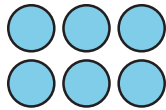
- Let's describe patterns and think about what might come next.

## Warm-up: Notice and Wonder: Sets of Circles

What do you notice? What do you wonder?

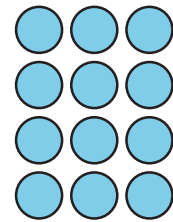


step 1



step 2

step 3



step 4

# 1.1: Bottle Cap Patterns

Han is arranging his bottle caps in a pattern. Here are the first two steps.



1. a. What might be the rule that Han has in mind? How do you think the pattern might continue?

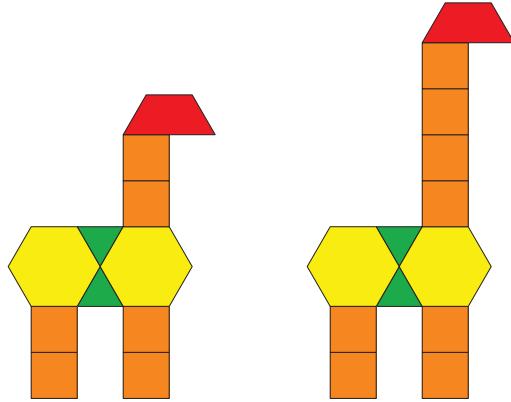
b. Describe or draw the next 2 steps.

2. Is there another possible rule?

3. For each rule that you found, write the numbers that represent the number of caps in step 1 through step 6.

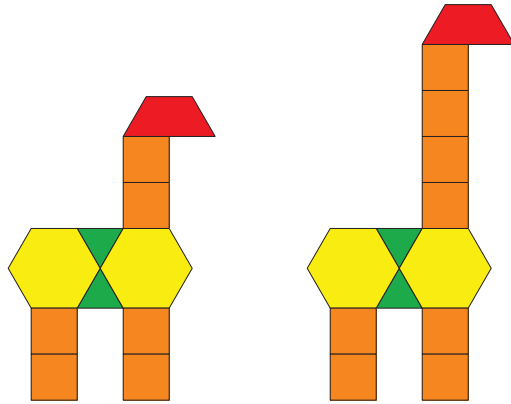
## 1.2: Taller and Taller

Jada used pattern blocks to make giraffes. Here are the first two steps. She continued to add 2 square blocks for each step that followed.



Partner A:

1. List the number of square blocks in each of the first five steps. Write two observations about the numbers.
2. Without drawing the giraffe, predict how many square blocks the tenth step will have. Explain or show your reasoning.
3. Will a step ever have 25 square blocks? Explain or show your reasoning.



Partner B:

1. List the total number of blocks in each of the first five steps. Write two observations about the numbers.
2. Predict how many total blocks the tenth step will have. Explain or show your reasoning.
3. Will a step ever have a total of 25 blocks? Explain or show your reasoning.

# Lesson 2: Patterns that Repeat

- Let's look at shapes that repeat by a rule and make some predictions about the patterns they create.

## Warm-up: How Many Do You See: Colorful Tiles

How many tiles do you see? How do you see them?



## 2.1: Patterns that Repeat

1. Here is a pattern made by arranging shapes.



- Look for as many features of patterns as you can and describe them to your partner.
- What rule might this pattern follow?
- Use the rule to extend the pattern so that it repeats one more time.

2. Create a new pattern that uses only a circle and one other shape, and that follows a new rule.



- Trade your pattern with your partner. Look for as many features of patterns as you can and describe them.
- What rule might your partner have followed to create their pattern?
- Use the rule to extend their pattern so that it repeats one more time.

## 2.2: Numbered Patterns

Here is the pattern of shapes you saw earlier.



1. Number the shapes 1 to 12.
2. Your teacher will assign you a shape. Write it in every blank space and answer the questions.

- a. What numbers were written for the \_\_\_\_\_s?
- b. If you extend the pattern, what numbers will be written for the next two \_\_\_\_\_s?
- c. What number will the tenth \_\_\_\_\_ have? Explain or show your reasoning.
- d. Will the 30th shape be a \_\_\_\_\_? Explain or show your reasoning.

## 2.3: Clare's Pattern

Clare created a pattern using 3 shapes—a triangle, a circle, and a square—that repeat in that order.

1. Draw the first 10 shapes in Clare's pattern.
2. Clare numbered her shapes. What numbers are the first 5 squares?
3. What rule is the numerical pattern following?
4. What is the 31st shape in Clare's pattern? Explain or show your reasoning.
5. Clare wants to use 40 shapes in her pattern and the last shape to be a square. Is this possible? Explain or show your reasoning.

# Lesson 3: From Visual Patterns to Numerical Patterns

- Let's look at numerical patterns we can write to describe patterns in rectangles.

## Warm-up: Number Talk: Patterns in Multiplication

Find the value of each expression mentally.

- $20 \times 3$

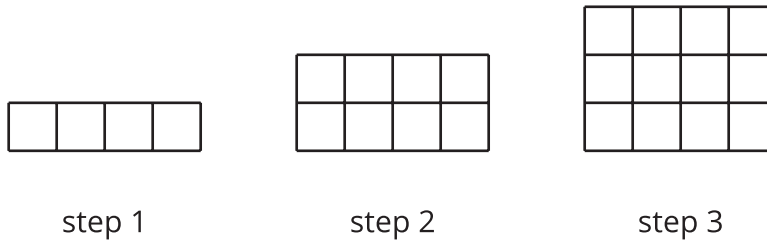
- $21 \times 3$

- $40 \times 3$

- $42 \times 3$

### 3.1: Growing Rectangles

Here is a pattern of rectangles that follows a rule.



- Priya says, "Each step increases by 1."
  - Noah says, "Each step increases by 4."
  - Lin says, "Each step increases by 2."
1. Can you think of possible reasons that all of them could be correct even though they describe the patterns differently?

2. Revise the statement made by each student so that what they mean is clearer and more precise.

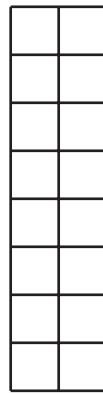


### 3.2: More Growing Rectangles

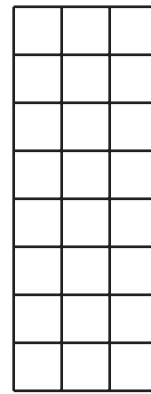
Here is another pattern of rectangles that also follows a rule.



step 1



step 2



step 3

1. The number list 1, 2, 3, \_\_\_\_, \_\_\_\_, \_\_\_\_ represents the number of vertical columns in the first six steps of the pattern. Complete the number list.
2. Find another feature of the rectangles that can be represented with a number list and would show a pattern. Write at least one list of numbers for the first six steps of that feature.

Feature: \_\_\_\_\_

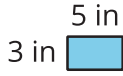
Number list: \_\_\_\_, \_\_\_\_, \_\_\_\_, \_\_\_\_, \_\_\_\_, \_\_\_\_

3. Without writing out all the numbers, predict the 30th number in your list. Explain your reasoning by completing this sentence frame:

I know that the 30th number is \_\_\_\_ because . . .

### 3.3: No Grid This Time!

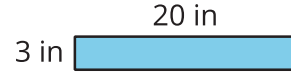
Here are steps 1 and 4 in a pattern of rectangles. One side length of the rectangle increases by 5 units each time.



step 1

step 2

step 3



step 4

1. Sketch the missing rectangles in steps 2 and 3. Label the sides with their lengths.
2. Write two numerical patterns that each represent the rectangles, from step 1 to step 6.

a. What are you representing? : \_\_\_\_\_

Numerical pattern: \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_

b. What are you representing? : \_\_\_\_\_

Numerical pattern: \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_, \_\_\_\_\_

3. For each of the following questions, if you answer yes, show how you know and state the step number. If you answer no, explain or show why not.

If the pattern continues:

a. Could 82 inches be a side length of a rectangle?

b. Could 300 square inches be the area of a rectangle in the pattern?

c. Could 100 inches be the perimeter of a rectangle in the pattern?

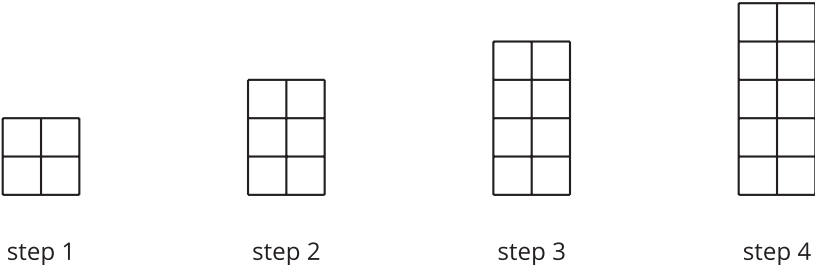
# Lesson 4: Numerical Patterns

- Let's explore numerical patterns.

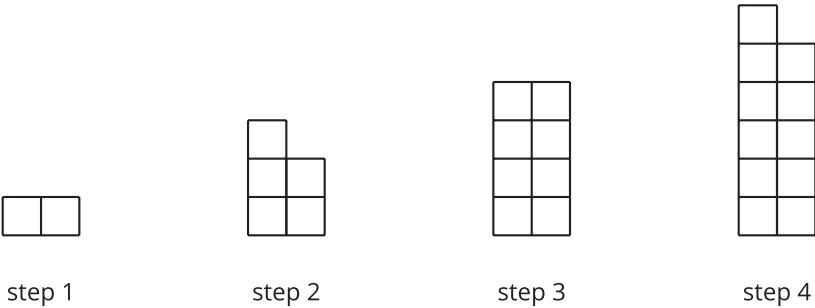
## Warm-up: Which One Doesn't Belong: Stacked Squares

Which one doesn't belong?

A



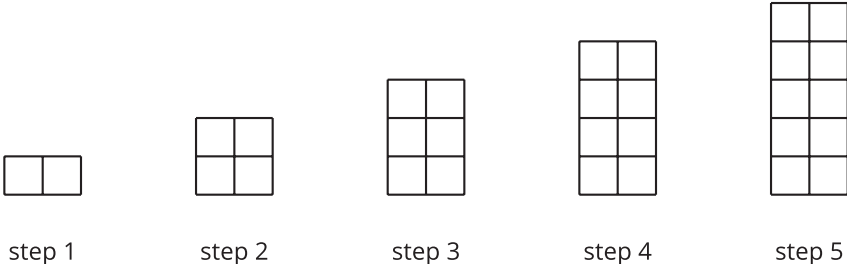
B



C

2, 4, 6, 8

D



## 4.1: Count by 10 and by 9

Andre's class is choral counting by 10 and then by 9. The left column shows the numbers they say when counting by 10.

1. Complete the right column with the first ten numbers the class will say when counting by 9.

What patterns do you notice about the features of the numerical patterns?  
Make at least two observations about each list of numbers.

counting by 10	counting by 9
10	
20	
30	
40	
50	
60	
70	
80	
90	
100	

2. For the numbers in the "counting by 10" column, why do you think:
  - a. the digits in the tens place change the way they do?
  
  
  
  
  
  
  
  
  
  
  - b. the digits in the ones place are the way they are?

3. For the numbers in the “counting by 9” column, why do you think the digits in the ones place change the way they do? Explain your reasoning.

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## 4.2: Count by 99

Andre's class did a choral count by 99. Here are the first six numbers they said.

1. Study the list of numbers. Make at least 3 observations about features of the pattern.

counting by 99
99
198
297
396
495
594

2. Extend the list with the next four multiples of 99. Be prepared to discuss how you know what numbers to write.
3. Why do you think the digits in the numbers change the way they do?



### 4.3: Count by 15

Elena counted by 15 and recorded the numbers she counted:

15      30      45      60      75      90

1. Write the next four numbers she'd record if she kept going.

2. What patterns do you see? Describe as many as you can.

3. Choose one pattern that you noticed and explain why you think it happens.

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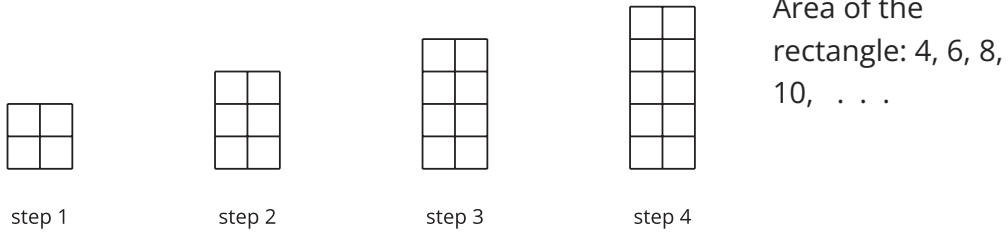
4. Could 250 be a number that Elena calls out if she continued to count by 15?  
Explain or show your reasoning.

## Section Summary

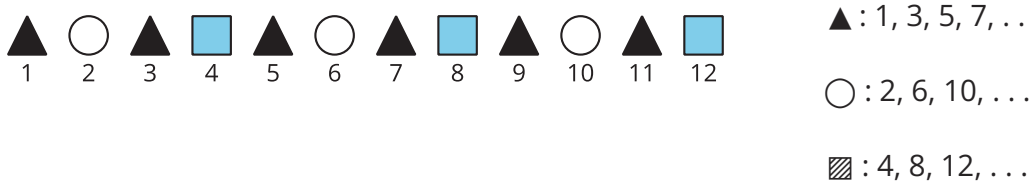
### Section Summary

In this section, we looked at different patterns of shapes and patterns of numbers. We saw shapes that grew or repeated by certain rules, and we used numbers to help us see how the shapes changed. Here are some examples of the patterns:

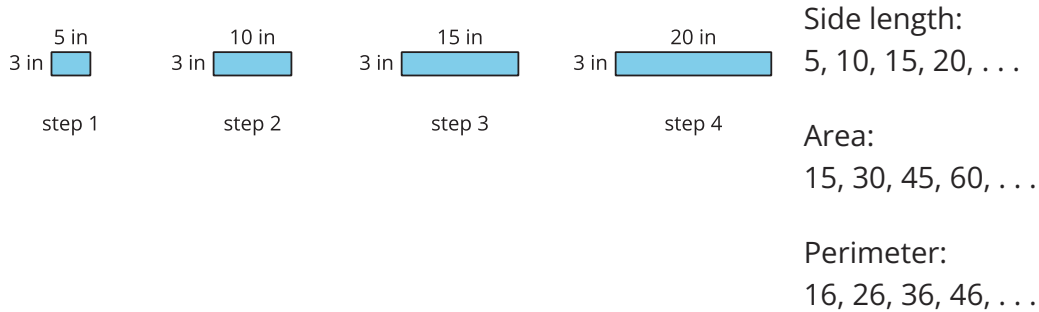
- Shapes that grow by a rule: add 1 row of equal-size squares



- Shapes that repeat by a rule: triangle, circle, triangle, square, repeat



- Rectangles that change by a rule: increase the length of the rectangle by 5 inches



- Numbers that change by a rule

- Add 9: 9, 18, 27, 36, 45
- Add 10: 10, 20, 30, 40, 50
- Add 99: 99, 198, 297, 396, 495
- Add 100: 100, 200, 300, 400, 500

We learned to extend the patterns by first finding their rule. Sometimes we can use addition and multiplication to represent a rule and then extend the pattern. Other times we can see how the digits in the numbers change to make predictions.

## Lesson 5: Products Beyond 100

- Let's find products beyond 100.

### Warm-up: Number Talk: A Number Times Some Multiple of 10

Find the value of each expression mentally.

- $8 \times 30$

- $5 \times 30$

- $10 \times 30$

- $15 \times 30$

## 5.1: Elena's Sticky Gift

Elena receives a sheet of fancy stickers as a gift.



How many stickers are there? Explain or show how you would find out without counting every sticker.

## 5.2: More and More Stickers

1. Elena has another sheet of stickers that has 9 rows and 21 stickers in each row. How many stickers does Elena have? Explain or show your reasoning.

2. Noah's sticker sheet has 3 rows with 48 stickers in each row. Andre's sticker sheet has 7 rows with 23 stickers in each row.

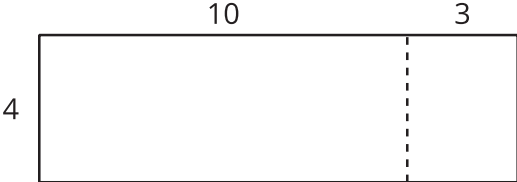
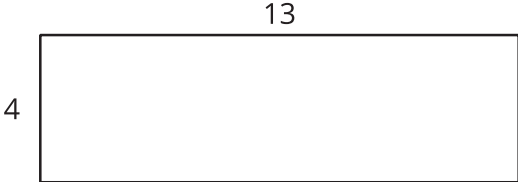
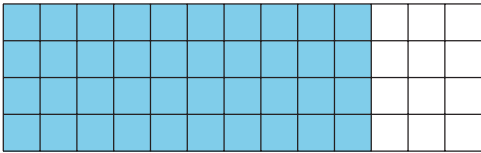
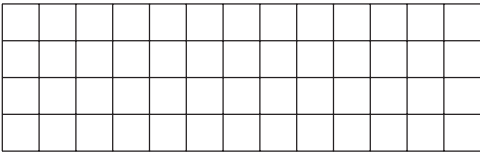
Who has more stickers? Explain or show your reasoning.

# Lesson 6: Multiply Two-digit Numbers and One-digit Numbers

- Let's multiply two-digit and one-digit numbers.

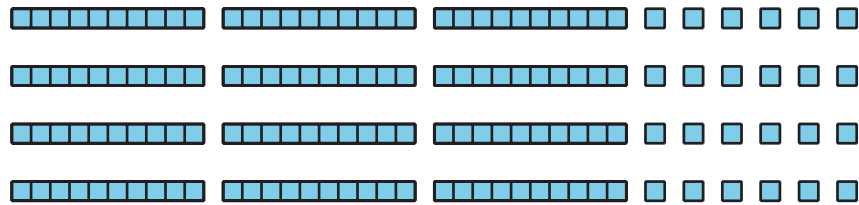
## Warm-up: Notice and Wonder: With and Without a Grid

What do you notice? What do you wonder?



## 6.1: Tyler's Diagrams

1. To find the value of  $4 \times 36$ , Tyler uses a base-ten diagram, as shown here.



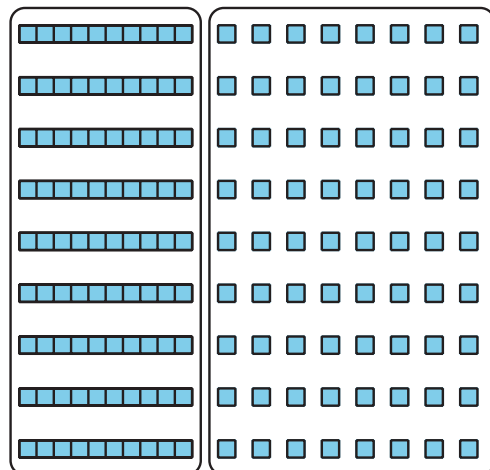
a. Where is the 36 in Tyler's diagram?

b. Where is the 4 in Tyler's diagram?

c. What is the value of  $4 \times 36$ ?

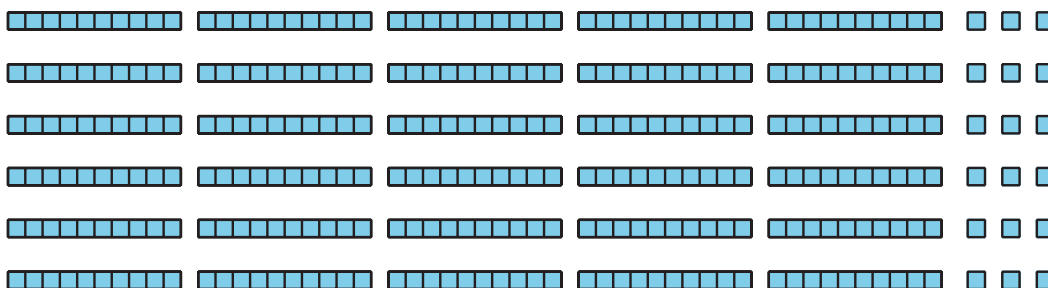
2. Here is a diagram Tyler made to find the value of  $9 \times 18$ .

Explain or show how his diagram helps him find the value of  $9 \times 18$ .



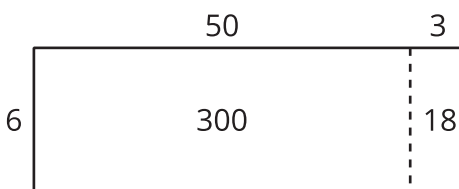
## 6.2: Two Kinds of Diagrams

1. Priya drew a base-ten diagram to multiply  $6 \times 53$ . She said it shows that the product can be found by adding 300 and 18.



- a. Where do you see 6 and 53 in her diagram?
- b. Where do you see 300 and 18 in Priya's diagram? What do they represent?

2. Han drew this diagram to multiply  $6 \times 53$ :



Where do you see 300 and 18 in his diagram? What do they represent?

3. Which diagram do you prefer for multiplying  $6 \times 53$ : Han's way or Priya's way? Explain your reasoning.

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4. Find the value of  $6 \times 53$ .

5. Draw a diagram to represent each multiplication expression. Then, find the value of each product.

a.  $6 \times 48$

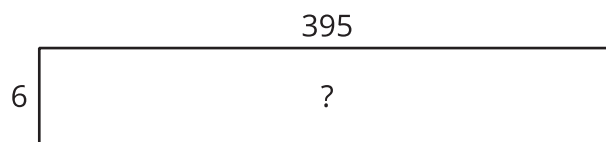
b.  $9 \times 67$

# Lesson 7: Multiply Three- and Four-digit Numbers by One-digit Numbers

- Let's multiply three- and four-digit numbers by one-digit numbers.

## Warm-up: Estimation Exploration: Mysterious Area

What is the area of the rectangle?

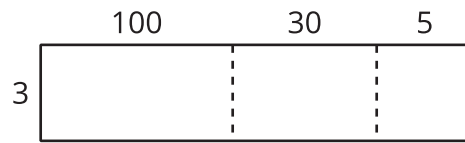


Record an estimate that is:

too low	about right	too high

## 7.1: Larger Numbers to Multiply

1. Clare drew this diagram.



a. What multiplication expression can be represented by the diagram?

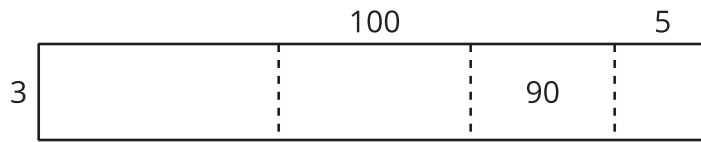
b. Find the value of the expression. Show your reasoning.

2. Consider the expression  $6 \times 252$ .

a. Draw a diagram to represent the expression.

b. Find the value of the expression. Show your reasoning.

3. Lin drew a diagram to represent  $3 \times 2,135$ .



- Complete Lin's diagram.
- Write an expression to represent the value of each part of the diagram.
- Find the value of  $3 \times 2,135$ . Show your reasoning.

## 7.2: Jada's Errors

1. Jada used a diagram to multiply  $3 \times 6,489$  and made a few errors.

	6,000	400	80	9
3	18	12	24	27

a. Explain the errors Jada made.

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b. Find the value of  $3 \times 6,489$ . Show your reasoning.

2. Find the value of  $5 \times 699$ . Show your reasoning.

3. Find the value of  $8 \times 4,973$ . Show your reasoning.

# Lesson 8: Multiply 2 Two-digit Numbers

- Let's multiply 2 two-digit numbers.

## Warm-up: Number Talk: Extra Groups

Find the value of each expression mentally.

- $20 \times 60$

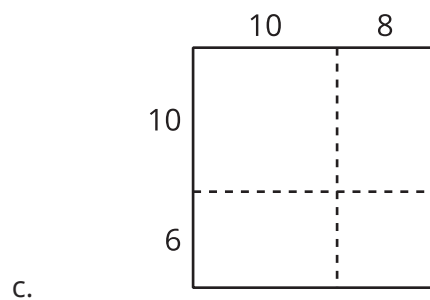
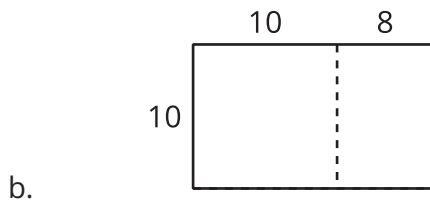
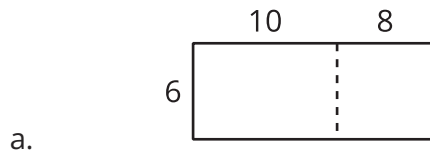
- $21 \times 60$

- $20 \times 62$

- $19 \times 60$

## 8.1: Two by Two

1. For each diagram, write a multiplication expression that the diagram can represent. Then, find the value of the expression. Use equations to show or explain your reasoning.



2. How are the diagrams alike? How are they different? Discuss with your partner.
3. Use a diagram to find each product.

a.  $13 \times 21$

b.  $25 \times 46$

## 8.2: Decompose by Place Value

These diagrams could be used to find the value of  $49 \times 57$ .

Diagram A

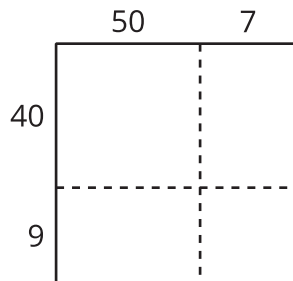
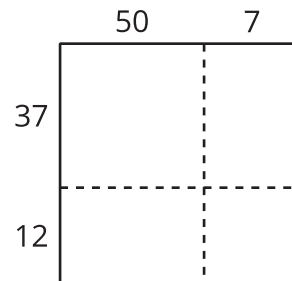


Diagram B



1. Which diagram is more helpful when finding the value of  $49 \times 57$ ? Why?

2. Use a diagram to find each product.

a.  $49 \times 57$

b.  $29 \times 55$

## Lesson 9: Recording Partial Products: One-digit and Three- or Four-digit Factors

- Let's analyze and try an algorithm that uses partial products.

### Warm-up: Which One Doesn't Belong: Expressions Galore

Which one doesn't belong?

A.  $7 \times 50$

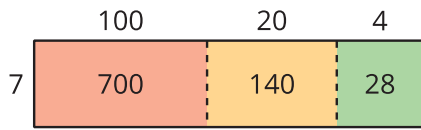
B.  $(3 \times 50) + (4 \times 50)$

C.  $(5 \times 10) \times 7$

D.  $50 + 50 + 50 + 50 + 50 + 50 + 50$

# 9.1: An Algorithm for Noah

1. Noah drew a diagram and wrote expressions to show his thinking as he multiplied two numbers.



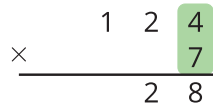
$$700 + 140 + 28 = 868$$

$$\begin{aligned} &7 \times 124 \\ &7 \times (100 + 20 + 4) \\ &(7 \times 100) + (7 \times 20) + (7 \times 4) \\ &700 + 140 + 28 \end{aligned}$$

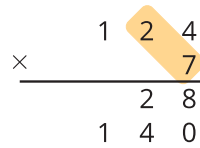
How does each expression represent Noah's diagram? Be prepared to share your thinking with a partner.

2. Later, Noah learned another way to record the multiplication, as shown here.

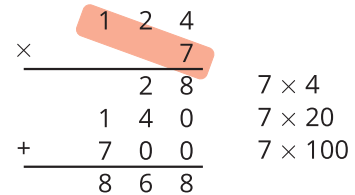
**Step 1**



**Step 2**

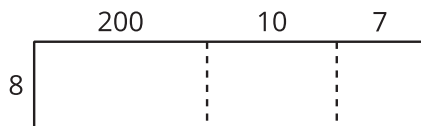


**Step 3**



Make sense of each step of the calculations and record your thoughts. Be prepared to explain Noah's steps to a partner.

3. Complete the diagram to find the value of  $217 \times 8$ . Use Noah's recording method to check your work.



$$8 \times 7$$

$$8 \times 10$$

$$+ \underline{\hspace{2cm}} \quad 8 \times 200$$



3. Find the value of each expression. For at least one expression, use the algorithm that Noah used. Show your reasoning.

a.  $4 \times 5,342$

b.  $7 \times 983$

# Lesson 10: Using Algorithms with Partial Products: 2 Two-digit Numbers

- Let's try to multiply two-digit numbers with an algorithm that uses partial products.

## Warm-up: Number Talk: Products

Find the value of each expression mentally.

- $30 \times 7$

- $15 \times 14$

- $50 \times 8$

- $25 \times 16$



## 10.2: Han's Multiplication Mishap

1. Decide with your partner who will find each product. Show your reasoning.

$$\begin{array}{r} \times \quad 19 \\ \hline \quad 32 \end{array}$$

$$\begin{array}{r} \times \quad 32 \\ \hline \quad 19 \end{array}$$

2. Here is Han's computation of  $51 \times 47$ .

$$\begin{array}{r} \times \quad 51 \\ \hline \quad 7 \\ \quad 35 \\ \quad 40 \\ + \quad 200 \\ \hline 282 \end{array} \quad \begin{array}{l} 7 \times 1 \\ 7 \times 5 \\ 40 \times 1 \\ 40 \times 5 \end{array}$$

a. What error or errors did Han make?

b. Show the correct computation for finding the value of  $51 \times 47$ .

$$\begin{array}{r} \times \quad 51 \\ \hline \quad 47 \end{array}$$

# Lesson 11: Partial Products and the Standard Algorithm

- Let's compare multiplication algorithms.

## Warm-up: Number Talk: The Value of the Digits

Find the value of each expression mentally.

- $5 \times 101$

- $5 \times 102$

- $5 \times 203$

- $5 \times 404$

## 11.1: Two Algorithms to Multiply

1. Here are two algorithms for finding the value of  $3 \times 713$ .

**Kiran**

$$\begin{array}{r} 713 \\ \times 3 \\ \hline 2,139 \end{array}$$

**Diego**

$$\begin{array}{r} 713 \\ \times 3 \\ \hline 9 \\ 30 \\ + 2,100 \\ \hline 2,139 \end{array}$$

Discuss with your partner:

- How are Kiran's algorithm and Diego's algorithm alike? How are they different?
- How do you think Kiran found the product 2,139?

2. Find the value of each product.

a.  $212 \times 4$

b.  $3 \times 4,132$

## 11.2: Algorithm Comparison

1. Analyze the two algorithms used to find the value of  $4 \times 223$ .

**Kiran**

$$\begin{array}{r} \phantom{\times} \phantom{2} \phantom{2} \phantom{3} \phantom{4} \\ \phantom{\times} \phantom{2} \phantom{2} \phantom{3} \phantom{4} \\ \phantom{\times} \phantom{2} \phantom{2} \phantom{3} \phantom{4} \\ \times \phantom{2} \phantom{2} \phantom{3} \phantom{4} \\ \hline 8 \phantom{9} \phantom{2} \end{array}$$

**Diego**

$$\begin{array}{r} \phantom{\times} \phantom{2} \phantom{2} \phantom{3} \phantom{4} \\ \phantom{\times} \phantom{2} \phantom{2} \phantom{3} \phantom{4} \\ \phantom{\times} \phantom{2} \phantom{2} \phantom{3} \phantom{4} \\ \times \phantom{2} \phantom{2} \phantom{3} \phantom{4} \\ \hline \phantom{2} \phantom{2} \phantom{3} \phantom{4} \\ \phantom{2} \phantom{2} \phantom{3} \phantom{4} \\ \phantom{2} \phantom{2} \phantom{3} \phantom{4} \\ + \phantom{2} \phantom{2} \phantom{3} \phantom{4} \\ \hline 8 \phantom{9} \phantom{2} \end{array}$$

a. How are Kiran and Diego's algorithms alike? How are they different?

b. Where is the 12 in Kiran's algorithm?

2. a. Try using Kiran's algorithm to find the value of  $512 \times 3$ .

b. Check your work using a different method.

# Lesson 12: Solve Problems Involving Multiplication

- Let's solve problems using what we learned about multiplication of whole numbers.

## Warm-up: What Do You Know About 1 Year?

What do you know about 1 year?



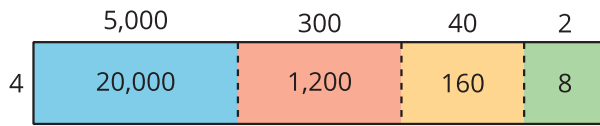


## Section Summary

### Section Summary

In this section, we learned to multiply factors whose products are greater than 100, using different representations and strategies to do so.

When working with multi-digit factors, it helps to decompose them by place value before multiplying. For example, to find the value of  $4 \times 5,342$ , we can decompose the 5,342 into its expanded form,  $5,000 + 300 + 40 + 2$ , and then use a diagram or an algorithm to help us multiply.



$$20,000 + 1,200 + 160 + 8 = 21,368$$

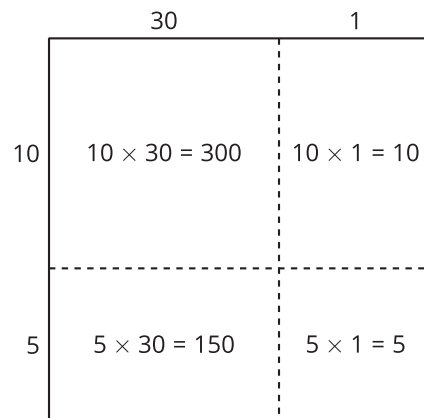
$$\begin{array}{r}
 \phantom{0}5,342 \\
 \times \phantom{000}4 \\
 \hline
 \phantom{000}8 \\
 \phantom{00}160 \\
 \phantom{0}1,200 \\
 + 20,000 \\
 \hline
 21,368
 \end{array}$$

In both the diagram and the algorithm, the 20,000, 1,200, 160, and 8 are called the partial products. They are the result of multiplying each decomposed part of 5,342 by 4.

We can do the same to multiply a two-digit number by another two-digit number.

For example, here are two ways to find the value of  $31 \times 15$ . The 31 is decomposed into  $30 + 1$  and 15 is decomposed into  $10 + 5$ .

$$\begin{array}{r}
 \phantom{0}31 \\
 \times \phantom{00}15 \\
 \hline
 \phantom{00}5 \\
 \phantom{0}150 \\
 \phantom{00}10 \\
 + 300 \\
 \hline
 465
 \end{array}$$



# Lesson 13: Situations Involving Equal-size Groups

- Let's interpret and solve division problems.

## Warm-up: Estimation Exploration: Lots of Paletas

How many paletas are in the case?



Record an estimate that is:

too low	about right	too high

## 13.1: Paletas for a Class Party

Diego's aunt is buying paletas, which are ice pops, for a class party. At the local market, paletas come in different flavors. She buys the same number of paletas of each flavor.



1. What mathematical questions can we ask about this situation?

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2. Here is an equation:

$$84 \div 7 = ?$$

In the situation about the class party, what questions could the equation represent?

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3. Find the answer to one of the questions you wrote. Show your reasoning.

## 13.2: More Snacks for a Class Party

1. Priya's mom made 85 gulab jamuns for the class to share. Priya gave 5 to each student in the class.

How many students are in Priya's class? Explain or show your reasoning.



2. Han's uncle sent in 110 chocolate-covered breadsticks for a snack. The students in Han's class are seated at 6 tables. Han plans to give the same number of breadsticks to each table.

How many breadsticks does each table get? Explain or show your reasoning.



# Lesson 14: Situations Involving Factors and Multiples

- Let's interpret and solve division problems beyond 100.

## Warm-up: Number Talk: Dividing by 7

Find the value of each expression mentally.

- $21 \div 7$

- $35 \div 7$

- $140 \div 7$

- $196 \div 7$



2. Kiran wants to know how many multiples of 7 are between 0 and 150.

a. He thinks he can use division to find out. Do you agree? Explain your reasoning.

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b. How many multiples will he find? Show your reasoning.

c. Is 150 a multiple of 7? Show how you know.

## 14.2: Jada's Mystery Number

Jada is writing multiples of a mystery number. After writing a bunch of numbers, she writes out 126.

- Mai says 6 is the mystery number.
- Priya says 8 is the mystery number.
- Andre says 9 could be the mystery number.

1. Which student do you agree with? Show your reasoning and include equations.

2. Jada gives one more clue: "If I keep writing multiples, I'll get to 153."

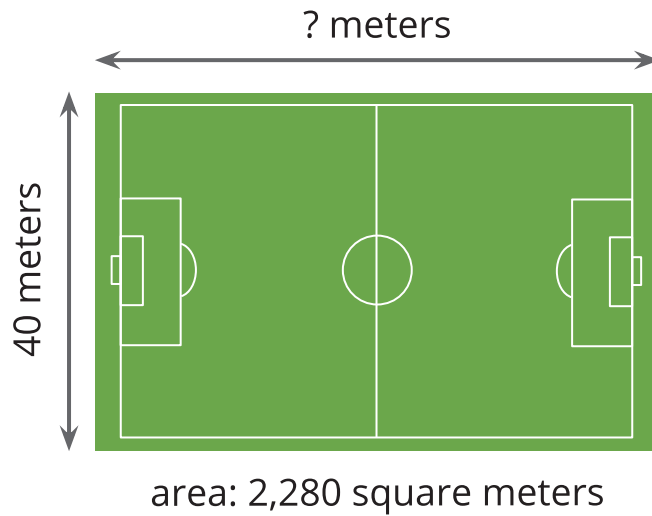
What is the mystery number? Explain or show your reasoning.

# Lesson 15: Situations Involving Area

- Let's divide to find the side length of a rectangle.

## Warm-up: Estimation Exploration: Area of a Soccer Field

Estimate: What is the length of the soccer field in meters?



Record an estimate that is:

too low	about right	too high



## 15.2: Tyler's Mural

Tyler is also creating a rectangular mural for the art club. He has 197 tiles for his mural. His mural is 6 tiles wide.

1. Will Tyler use all of his tiles in the mural? Explain your reasoning.

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2. How many tiles long is Tyler's mural? Show your reasoning using numbers, pictures, or words.

# Lesson 16: Base-ten Blocks to Divide

- Let's use base-ten blocks to divide.

## Warm-up: What Do You Know About Base-ten Blocks?

What do you know about base-ten blocks?



## 16.1: Blocks to Divide

Use the base-ten blocks to represent each expression. Then find the value of each expression.

1.  $488 \div 4$

2.  $104 \div 8$

## 16.2: Show Us Your Blocks

Find the value of each expression. Explain or show how you used base-ten blocks to find the value.

1.  $96 \div 4$

2.  $86 \div 2$

3.  $108 \div 9$

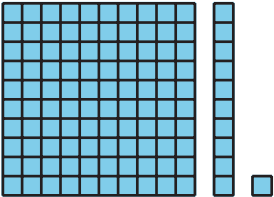
# Lesson 17: Base-ten Diagrams to Represent Division

- Let's divide using base-ten blocks or diagrams.

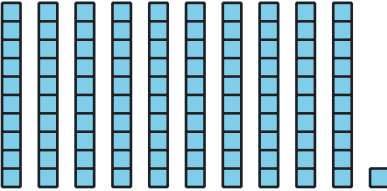
## Warm-up: Which One Doesn't Belong: Base-ten Diagrams

Which one doesn't belong?

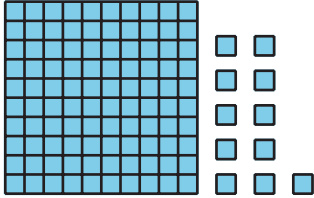
A



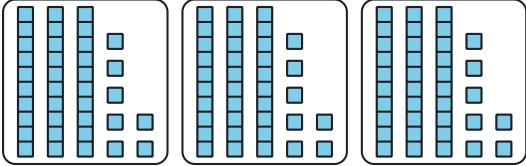
B



C



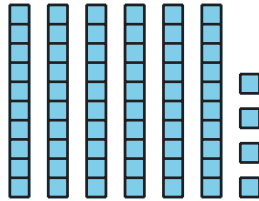
D



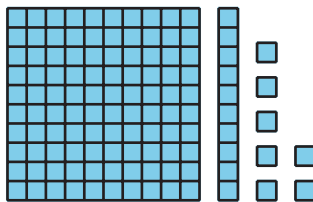
## 17.1: Divide with Diagrams or Blocks

1. Priya draws a base-ten diagram to find the value of  $64 \div 4$ . A rectangle represents 10. A small square represents 1.

Use the diagram (or actual blocks) to help Priya complete the division. Explain or show your reasoning.

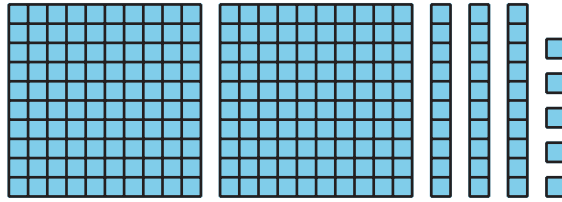


2. Use this base-ten diagram (or actual blocks) to find the value of  $117 \div 3$ .

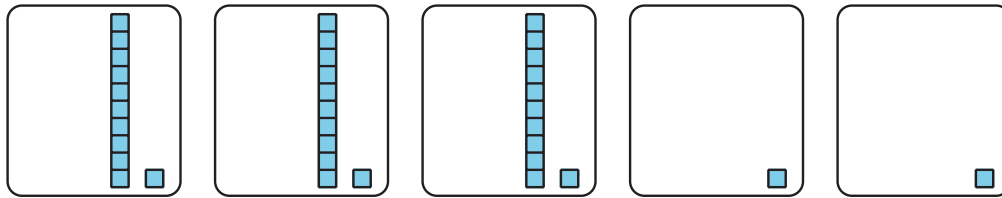


## 17.2: Help Noah Get Unstuck

1. This diagram represents 235.



To find  $235 \div 5$ , Noah draws the following diagram but then gets stuck.



He says, "There are not enough of the hundreds or the tens pieces to put into 5 groups."

Explain or show how Noah could find  $235 \div 5$  with his diagram.

2. Find the value of  $432 \div 6$ . Show your reasoning. Use base-ten diagrams or blocks if you find them helpful.

# Lesson 18: Divide with Partial Quotients

- Let's analyze and use an algorithm that uses partial quotients.

## Warm-up: Number Talk: Divide by 3

Find the value of each expression mentally.

- $90 \div 3$

- $96 \div 3$

- $960 \div 3$

- $954 \div 3$

## 18.1: Decompose Dividends

1. Find the value of  $465 \div 5$ . Show your reasoning. You may use base-blocks if you find them helpful.

2. Here's how Priya finds the value of  $465 \div 5$ .

$$400 \div 5 = 80$$

$$60 \div 5 = 12$$

$$5 \div 5 = 1$$

---

$$465 \div 5 = 93$$

- a. What has Priya done? Describe her steps.
- b. How is Priya's method similar to your method?
- c. Use Priya's method to find the value of  $428 \div 4$ .

## 18.2: Tyler's Method

Tyler uses a different method to find the value of  $465 \div 5$ . Let's compare Priya's and Tyler's work.

Priya's method

$$\begin{array}{r} 400 \div 5 = 80 \\ 60 \div 5 = 12 \\ 5 \div 5 = 1 \\ \hline 465 \div 5 = 93 \end{array}$$

Tyler's method

$$\begin{array}{r} \boxed{93} \\ 1 \\ 12 \\ 80 \\ 5 \overline{)465} \\ - 400 \quad 5 \times 80 \\ \hline 65 \\ - 60 \quad 5 \times 12 \\ \hline 5 \\ - 5 \quad 5 \times 1 \\ \hline 0 \end{array}$$

1. How are Priya and Tyler's methods alike? How are they different? List as many similarities and differences as you can find.

2. Why do you think Tyler uses subtraction in his method?

3. Show how Tyler might record the process of finding the value of  $428 \div 4$ .

# Lesson 19: Division With and Without Remainders

- Let's find quotients and remainders using an algorithm that uses partial quotients.

## Warm-up: Notice and Wonder: Equations with Hundreds

What do you notice? What do you wonder?

$$100 = 33 \times 3 + 1$$

$$200 = 66 \times 3 + 2$$

$$300 = 100 \times 3$$

$$400 = 133 \times 3 + 1$$

$$500 = 166 \times 3 + 1$$

$$600 = 200 \times 3$$

## 19.1: A Stack of Partial Quotients

Jada used partial quotients to find out how many groups of 7 are in 389.

Analyze Jada's steps in the algorithm.

$$\begin{array}{r} \boxed{55} \\ 8 \\ 7 \\ 40 \\ 7 \overline{)389} \\ - 280 \\ \hline 109 \\ - 49 \\ \hline 60 \\ - 56 \\ \hline 4 \end{array}$$

1.
  - a. Look at the three numbers above 389. What do they represent?
  
  
  
  
  
  
  
  
  
  
  - b. Look at the three subtractions below 389. What do they represent?
  
  
  
  
  
  
  
  
  
  
  - c. What is another way you can decompose 389 to divide by 7?

2. Is 389 a multiple of 7? Explain your reasoning.

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3. Use an algorithm that uses partial quotients to find out how many groups of 3 are in 702.

4. Is 702 a multiple of 3? Explain your reasoning.

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## 19.2: Andre and Elena's Work

Andre and Elena are dividing 2,316 by 5. Before they begin, Andre says, "I can already tell that there will be a remainder."

1. Without doing any calculations, decide if you agree with Andre. Explain your reasoning.

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2. Here is Andre and Elena's work. Each student made one or more errors. Identify the errors each student made. Then, show a correct computation.

Andre's Work

$$\begin{array}{r} \boxed{103} \\ 3 \\ 60 \\ 40 \\ 5 \overline{)2,316} \\ \underline{-2,000} \\ 316 \\ - 300 \\ \hline 16 \\ - 15 \\ \hline 1 \end{array}$$

Elena's Work

$$\begin{array}{r} \boxed{400} \\ 60 \\ 100 \\ 300 \\ 5 \overline{)2,316} \\ \underline{-1,500} \\ 816 \\ - 500 \\ \hline 316 \\ - 300 \\ \hline 16 \end{array}$$

### 19.3: Incomplete Calculations

Here are four calculations to find the value of  $3,294 \div 3$ , but each one is unfinished.

Complete at least two of the unfinished calculations. Be prepared to explain why you chose them.

A

$$\begin{array}{r}
 90 \\
 1,000 \\
 3 \overline{)3,294} \\
 \underline{-3,000} \quad 3 \times 1,000 \\
 294 \\
 \underline{- 270} \quad 3 \times 90
 \end{array}$$

B

$$\begin{array}{r}
 80 \\
 200 \\
 400 \\
 400 \\
 3 \overline{)3,294} \\
 \underline{-1,200} \quad 3 \times 400 \\
 2,094 \\
 \underline{-1,200} \quad 3 \times 400 \\
 894 \\
 \underline{- 600} \quad 3 \times 200 \\
 294 \\
 \underline{- 240} \quad 3 \times 80
 \end{array}$$

C

$$\begin{array}{l}
 600 \div 3 = \\
 600 \div 3 = \\
 600 \div 3 = \\
 600 \div 3 = \\
 600 \div 3 = \\
 270 \div 3 =
 \end{array}$$

D

$$\begin{array}{r}
 3,300 \div 3 = 1,100 \\
 \underline{- \quad 6 \div 3 = \quad 2}
 \end{array}$$

# Lesson 20: Interpret Remainders in Division Situations

- Let's solve problems involving division and interpret remainders.

## 20.1: Muffins and Seats

1. Two bakers at a bakery made 378 muffins. The muffins are put in boxes of 4.

- The first baker says they will need 94 boxes for all the muffins.
- The second baker says 95 boxes are needed.



Who do you agree with? Explain or show your reasoning.

2. An auditorium seats 258 people. The seats are arranged in rows of 9, but there is one short row with fewer than 9 seats.

How many rows of 9 seats are there? How many seats are in the shorter row?

## 20.2: Save for a Garden

1. A school needs \$1,270 to build a garden. After saving the same amount each month for 8 months, the school is still short by \$6.

How much did they save each month? Explain or show your reasoning.



2. Choose one of the following division expressions.

$$711 \div 3$$

$$3,128 \div 8$$

- a. Write a situation to represent the expression.

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- b. Find the value of the quotient. Show your reasoning.

- c. What does the value of the quotient represent in your situation?

## Section Summary

### Section Summary

In this section, we solved different problems that involve dividing whole numbers.

We recalled two ways of thinking about division. For example, suppose  $274 \div 8$  represents a situation where 274 markers are put into equal groups. The value of  $274 \div 8$  can tell us:

- how many markers are in each group if there were 8 groups, or
- how many groups can be made if there were 8 markers in each group.

We learned that the 274 in  $274 \div 8$  is called the **dividend**. We then explored different ways to find the value of a quotient (or the result of the division). For  $274 \div 8$ , we can:

- Divide by place value and think about putting 2 hundred, 7 tens, and 4 ones into 8 equal groups.
- Divide in parts and find partial quotients. For example, we can first find  $160 \div 8$  (which is 20), and then  $80 \div 8$  (which is 10), and then  $32 \div 8$  (which is 4).
- Think in terms of multiplication. For example, we can think of  $8 \times 20 = 160$ ,  $8 \times 10 = 80$ , and so on.

Here is one way to record division using partial quotients:

$$\begin{array}{r} \boxed{34} \\ 4 \\ 10 \\ 20 \\ 8 \overline{)274} \\ - 160 \quad 8 \times 20 \\ \hline 114 \\ - 80 \quad 8 \times 10 \\ \hline 34 \\ - 32 \quad 8 \times 4 \\ \hline 2 \end{array}$$

Sometimes a division results in a leftover that can't be put into equal groups or is not enough to make a new group. We call the leftover a **remainder**. Dividing 274 by 8 gives 34 and a remainder of 2.

# Lesson 21: Different Ways to Solve Problems

- Let's reason about and solve multi-step problems.

## Warm-up: Which One Doesn't Belong: Expressions with 5 or 90

Which one doesn't belong?

A.  $5 \times 90$

B.  $90 + 90 + 90 + 90 + 90$

C.  $(4 \times 90) + (1 \times 90)$

D.  $3 \times 3 \times 10 \times 5$

## 21.1: Going on a Field Trip

1. Forty-five students are going on a field trip to a museum. Tickets for the museum are \$18 each. Teachers have \$900 to cover tickets for the trip. Will this be enough to cover tickets for every student?

If yes, will there be any leftover money and how much?

If no, how much more money is needed?

2. Your teacher will show five strategies for answering the previous question. Analyze the strategies.

- a. Which strategy is closest to yours? With a partner, take turns explaining how your strategy is close to the poster you chose.

- b. Discuss a different strategy with your partner. Try using this strategy to find the value of  $14 \times 35$ .

## 21.2: A Trip to the Movies

Movie tickets are \$9 each. The theater sold the same number of tickets two days in a row.

The theater made \$3,132 from ticket sales on the first day.



1. Record and answer one question of your choice from the list the class generated. Discuss your strategy with your partner.

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2. Use the given information about movie tickets to complete the following statement:

\_\_\_\_\_ tickets were sold on the first and second days.

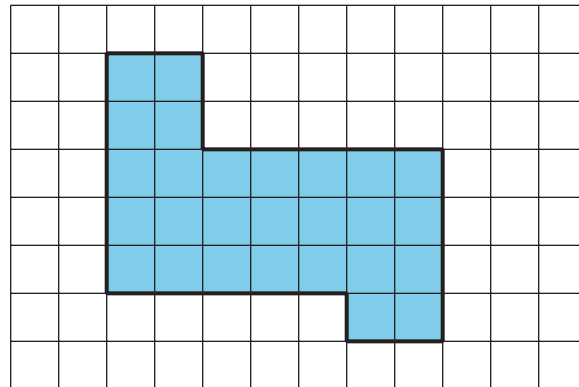
3. A medium drink is \$7 and small popcorn is \$5. If each ticket holder purchases popcorn and a drink, how much money will the theater collect from the sales of popcorn and drink?

# Lesson 22: Problems About Perimeter and Area

- Let's solve situations involving perimeter and area.

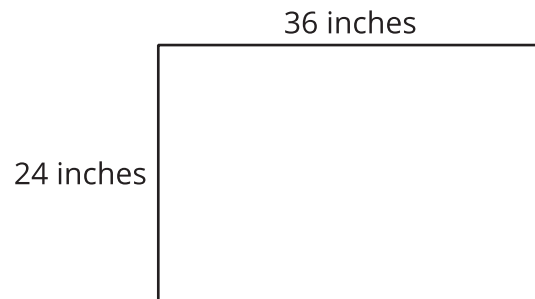
## Warm-up: How Many Do You See: Shaded Squares

How many shaded squares do you see? How do you see them?



## 22.1: Create a Class Banner

Jada's teacher bought a poster paper that measures 36 inches by 24 inches. Her plan is to cut it into pieces, rearrange them, and tape them back together to create a welcome banner that is 8 inches tall and 8 feet long.



1. Does she have enough paper to make the banner? Show your reasoning.

2. How many square inches is the poster paper?



# Lesson 23: Solve Problems with Many Operations

- Let's solve multi-step problems involving the four operations.

## Warm-up: True or False: Differences

Decide if each statement is true or false. Be prepared to explain your reasoning.

- $50,000 - 999 = 49,001$

- $4,799 = 5,000 - 311$

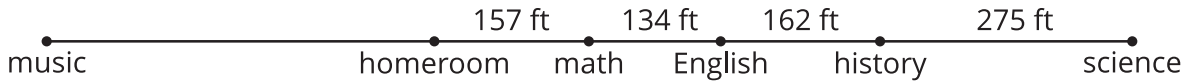
- $3,005 = 4,000 - 1,995$

- $2,000 - 1,234 = 1,876$

## 23.1: Back and Forth

Mai's cousin is in middle school. She travels from her homeroom to math, then English, history, and science. When she finishes her science class, she takes the same path back to her homeroom.

Mai's cousin makes the same trip 5 times each week. The distances between the classes are shown.



1. How far does Mai's cousin travel each round trip—from her homeroom to the four classes and back? Write one or more expressions or equations to show your reasoning.
2. Each week, Mai's cousin makes 3 round trips from her homeroom to her music class. The total distance traveled on those 3 round trips is 2,364 feet.  
  
How far away is the music room from her homeroom? Show your reasoning.
3. Mai thinks her cousin travels 2 miles each week just going between classes. Do you agree? Explain or show your reasoning.

## 23.2: Fitness Challenge

To motivate students to exercise, Han's school is holding a fitness challenge with prizes.



1. Han walked 32,550 steps in the first week. He walked the same number of steps every day. How many steps did Han walk each day? Show your reasoning.

2. The table shows the number steps Han took each week for the first three weeks. How much did the number of steps drop from the first week to the second week?

week 1	week 2	week 3	week 4
32,550	28,098	36,249	

3. If Han wants to meet the challenge, what is the fewest number of steps that he needs to take in week 4? Show your reasoning.

4. How do you know your answer to problem 3 is reasonable?

# Lesson 24: Assess the Reasonableness of Solutions

- Let's solve problems and assess the reasonableness of solutions.

## Warm-up: Notice and Wonder: Native American Languages

What do you notice and what do you wonder?



## 24.1: Do You Speak Navajo?

1. In the list of the ten most widely spoken Native American languages in the U.S., Navajo and Yupik are the most widely spoken.

a. How many more Navajo speakers are there than Yupik speakers? Show your reasoning.

language	number of speakers
Navajo	166,826
Yupik	19,750
Dakota	17,855
Apache	13,445
Keres	13,190
Cherokee	11,465
Ojibwa	9,735
Choctaw	9,635
Zuni	9,615
Pima	6,990

b. About how many times as many Navajo speakers as there are Yupik speakers? Show your reasoning.

2. Navajo, Apache, and Cherokee languages have been used during wartime to help the U.S. military keep its communications secure and incomprehensible to their enemies.

a. Based on the data here, how many people might have been able to understand the communications? Show your reasoning.

b. How do you know that your answer is reasonable?

3. Are there more Navajo speakers than the speakers of all the other nine languages combined? Explain or show how you know.

## 24.2: Languages in Philadelphia and Chicago

Philadelphia, the birthplace of the United States, is a diverse city and home to people of different backgrounds.

The table shows 2017 data on some types of languages spoken in Philadelphia and the numbers of people who speak them.

language	number of speakers in Philadelphia
English only	1,224,539
Spanish	127,352
Other Indo-European	6,750
Asian	364

1. Based on the data, are there more people in Philadelphia who only speak English or more people who speak a language other than English? Show how you know.

2. What is the difference between the number of people who speak only English and those who speak another language? Show how you know.

3. Chicago is a city with a similar population to Philadelphia.

This table shows data on some types of languages spoken by people in Chicago.

language	number of speakers in Chicago
English only	1,731,836
Spanish	422,568
Other Indo-European	25,777
Asian	1,005

a. How many more speakers of Spanish and other Indo-European languages are in Chicago than in Philadelphia? Show your reasoning.

b. How do you know your answer is reasonable?

## Section Summary

### Section Summary

In this section, we encountered problems that involve large numbers from different contexts and that could be solved with different strategies.

In the beginning, we saw at least five ways to find the product of 45 and 18: by multiplying and dividing in parts, using a series of equations, drawing diagrams, and more.

Later, we explored problems about measurements, with numbers up to four digits. We found that, often, the same problem could be solved using different operations.

For example, in the fitness challenge activity, Han took 32,550 steps in 7 days. We can find the number of steps he took each day by thinking in terms of multiplication (what number times 7 is 32,550?) or in terms of division (what is 32,550 divided by 7?).

We can also write different equations.

$$7 \times n = 32,550$$

$$32,550 \div 7 = n$$

To find out how many steps Han had to take to reach a goal of 120,000 steps if he had 96,897 steps, we can think in terms of addition (what number must be added to 96,897 to make 120,000?) or subtraction (what is the difference between 120,000 and 96,897?).

$$96,897 + n = 120,000$$

$$120,000 - 96,897 = n$$

# Lesson 25: Paper Flower Decorations

- Let's make patterns with paper flowers.

## Warm-up: How Many Do You See: Paper Flowers

How many do you see? How do you see them?



## 25.1: Paper Flower Construction

Follow these steps to make paper flowers:

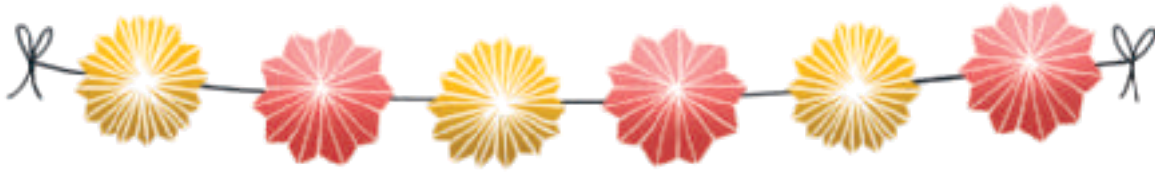
- Place 6 pieces of tissue paper on top of each other.
- Starting at one side, fold over about 1 inch, then fold in the opposite direction. Repeat with this accordion fold (like a paper fan) until you have a strip that is 1 inch wide and the length of the original paper.
- Tie a rubber band around the middle of the folded paper strip. Then, open up the folds.



- Carefully, one layer at a time, fold the layers up into the middle to make the petals.



## 25.2: Quinceañera Decorations



Priya and Jada are making paper flower garlands for their friend's quinceañera. Each garland uses 12 flowers.

1. Priya wants 2 big flowers, followed by 2 small flowers. Jada wants 1 big flower, followed by 2 small flowers. Use their patterns to draw the garlands.

2. Priya and Jada make 25 garlands of each type. How many large and small flowers will they need altogether?

3. Diego and Kiran also made flowers. They made a total of 155 flowers for garlands that require 16 flowers each. How many garlands can they make?

4. It takes 1 minute to cut the strips for a flower and 2 minutes to finish it. How long did it take Diego and Kiran to make the 155 flowers, if they each make about the same number of flowers?

## 25.3: Make Your Own Problems

1. Write a multi-step problem about making paper flowers.

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2. Exchange the problem with your partner and solve each other's problems.

## Section A: Practice Problems

### 1. Pre-unit

Here is a list of the first ten multiples of 5:

5, 10, 15, 20, 25, 30, 35, 40, 45, 50

- Circle the multiples of 10 in the list.
- What do you notice about where the multiples of 10 are on the list?
- Why do you think that is?

### 2. Pre-unit

Find the value of each expression.

a.  $14 \times 7$

b.  $13 \times 6$

c.  $23 \times 4$

d.  $85 \div 5$

**3. Pre-unit**

There are 418 students at Jada's school. There are 135 fewer students at Noah's school. How many students are there at Jada's and Noah's schools together? Explain or show your reasoning.

**4. Pre-unit**

a. What is the value of the digit 6 in each of the numbers?

i. 165

ii. 18,622

iii. 675,219

b. Complete this statement so that it is true:

The value of the 6 in 675,219 is \_\_\_\_\_ times that of the 6 in 165.

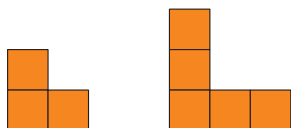
**5. Pre-unit**

Find the value of each sum and difference.

$$\begin{array}{r} 15,826 \\ + 44,371 \\ \hline \end{array}$$

$$\begin{array}{r} 33,517 \\ - 16,322 \\ \hline \end{array}$$

6. a. Mai follows a rule to build a pattern of square blocks. What might the next 2 shapes in Mai's pattern look like? Sketch the shapes or describe them.



- b. Will Mai's pattern ever use 20 squares? Explain your reasoning.

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(From Unit 6, Lesson 1.)

7. Han types the letters a, s, d, f and then repeats them in that order, over and over.

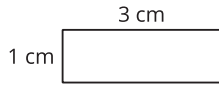
- a. What is the 5th letter Diego will type? What about the 10th? The 20th?

- b. If Diego numbers the letters he types, starting with 1 for the first a, what numbers will go with the first 6 f's he types?

- c. What do you notice about the numbers for the f's?

(From Unit 6, Lesson 2.)

8. Here is the first rectangle in a pattern. For each step in a pattern of rectangles, the short side stays the same and the long side grows by 2 centimeters.



- a. Draw the next 4 steps in the rectangle pattern. Include the length and width of each rectangle.
- b. Can the perimeter of the rectangle, in centimeters, be an even number?  
Explain your reasoning.

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- c. Can the area of the rectangle, in square centimeters, be an even number?  
Explain your reasoning.

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(From Unit 6, Lesson 3.)

9. a. Make a list of the first ten multiples of 8.
- b. What pattern do you see in the digits in the ones place? How are the digits changing?
- c. Why do you think it is changing that way?

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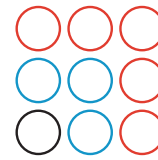
(From Unit 6, Lesson 4.)

**10. Exploration**

- a. Make a list of the multiples of 2, 3, 4, 5, 6, 7, 8, 9, 10. Stop when you get a multiple of 10. For example, for 2, the list is 2, 4, 6, 8, 10.
- b. What do you notice about your lists? Make some observations.

**11. Exploration**

Tyler draws this picture and writes the equation  
 $1 + 3 + 5 = 9$ .



- a. How do you think the equation relates to the picture?
  
  
  
  
  
  
  
  
  
  
- b. Tyler keeps drawing circles to make larger squares. How many new circles does he need to draw to make a 4-by-4 square, and then a 5-by-5 square?
  
  
  
  
  
  
  
  
  
  
- c. What pattern do you notice in the number of circles Tyler adds each time?
  
  
  
  
  
  
  
  
  
  
- d. Why do you think the number of circles is increasing that way?

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**12. Exploration**

A growing pattern of squares that makes rectangles is shown here.



step 1                  step 2                  step 3

a. Find the area and perimeter of the rectangles in steps 2 and 3.

step	number of squares	area of rectangle (square units)	perimeter of rectangle (units)
1	2	2	6
2			
3			

b. Consider patterns you see in the chart. Extend the values in each column as if the pattern continued in steps 4 and 5.

c. Draw the next two diagrams (for steps 4 and 5). Were your predictions for the area and perimeter of each rectangle correct?

d. How would you describe this pattern to a classmate?

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**13. Exploration**

Mai and Tyler are each making their own pattern.

Some of their pattern symbols are the same, some are different. The table shows the first six symbols in Mai's pattern and the first four in Tyler's pattern.

<b>Mai's pattern</b>	@	#	\$	@	#	\$		
<b>Tyler's pattern</b>	~	@	~	@				

a. What are the next two symbols in Tyler's pattern? Explain your reasoning.

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b. At what step do you think Mai and Tyler will next both draw the same symbol at the same time? Explain or show how you know.

## Section B: Practice Problems

1. Mai has a sheet of stickers with 23 rows and 8 stickers in each row.

a. Does Mai have more or less than 100 stickers? Explain your reasoning.

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b. Find how many stickers Mai has. Explain or show your reasoning.

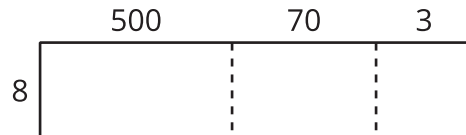
(From Unit 6, Lesson 5.)

2. Find the value of  $7 \times 64$ . Use a diagram if it is helpful.



(From Unit 6, Lesson 6.)

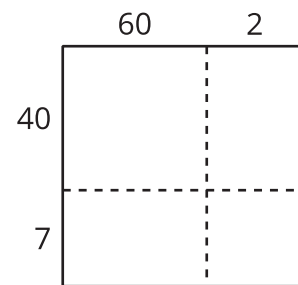
3. a. Use the diagram to find the value of  $8 \times 573$ .



b. Find the value of  $4 \times 3,516$ .

(From Unit 6, Lesson 7.)

4. a. Use the diagram to find the value of  $47 \times 62$ .



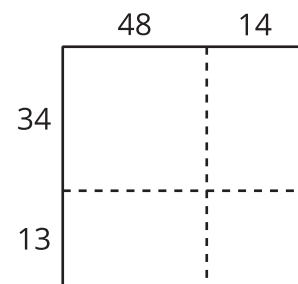
b. Would this diagram be helpful to find the value of  $47 \times 62$ ? Explain your reasoning.

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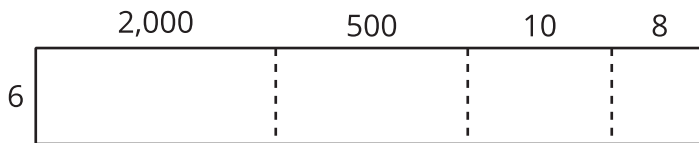
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(From Unit 6, Lesson 8.)

5. The diagram and calculations show two ways for finding the value of  $2,518 \times 6$ .



$$\begin{array}{r}
 2,518 \\
 \times \quad 6 \\
 \hline
 15,108
 \end{array}$$

a. How does each part of the vertical calculation relate to the diagram?

b. Find the value of  $3,172 \times 5$  using a method of your choice.

(From Unit 6, Lesson 9.)

6. Here is an incomplete calculation that uses partial products of  $65 \times 43$ .

a. Write multiplication expressions that the numbers 15, 180, 200, and 2,400 each represent. Then, find the value of  $65 \times 43$ .

$$\begin{array}{r}
 65 \\
 \times 43 \\
 \hline
 15 \\
 180 \\
 200 \\
 + 2,400 \\
 \hline
 \end{array}$$

b. Find the value of the product  $45 \times 38$ .

(From Unit 6, Lesson 10.)

7. Here is how Elena calculated the value of  $723 \times 3$ .

$$\begin{array}{r} 723 \\ \times 3 \\ \hline 2,169 \end{array}$$

a. Where does the 9 in Elena's calculation come from? What about the 6?

b. Where do the 2 and the 1 in calculation come from?

c. Use Elena's method to find the value of  $534 \times 2$ .

(From Unit 6, Lesson 11.)

8. There are 4,218 students in school district A. School district B has 3 times as many students as school district A. How many students are in school district B? Explain or show your reasoning.

(From Unit 6, Lesson 12.)

**9. Exploration**

Clare was double checking her answers for some products. Without doing the computation again, she knew that these answers were incorrect. How might Clare have known?

a.  $5 \times 5,783 = 27,914$

b.  $7 \times 8,419 = 54,253$

c.  $9 \times 9,999 = 99,999$

**10. Exploration**

Here is Mai's strategy to find the value of  $9 \times 8,235$ .

$$\begin{array}{r} 82,350 \\ - \quad 8,235 \\ \hline 74,115 \end{array}$$

a. Explain why Mai's method works.

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b. Use Mai's method to find the value of  $9 \times 6,789$ .

c. Find the value of  $9 \times 6,789$  using a strategy you learned. How is Mai's method like yours? How is it different than yours?

## Section C: Practice Problems

1.
  - a. If 5 pencils cost 95 cents, how much does each pencil cost? Explain or show your reasoning.
  
  
  
  
  
  
  
  
  
  
  - b. If 68 colored pencils are split evenly between 4 students, how many pencils does each student get? Explain or show your reasoning.

(From Unit 6, Lesson 13.)

2. Priya writes the multiples of a number and 63 is on her list. Priya's number is not 1.
  - a. What could Priya's number be? Explain your reasoning.

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- b. 112 is the last number on Priya's list. What is Priya's number? How many numbers are on Priya's list?

(From Unit 6, Lesson 14.)

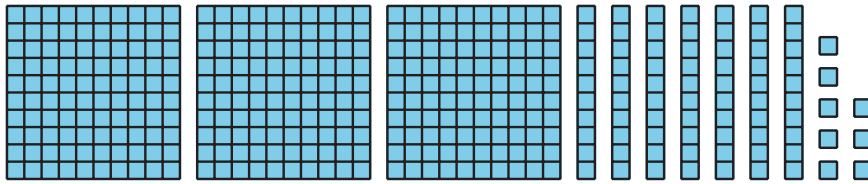
3. Clare has 194 square tiles. Can Clare put all of her tiles in 6 rows with the same number of tiles in each row? Explain or show your reasoning.

(From Unit 6, Lesson 15.)

4. A long, rectangular hallway is 8 feet wide and has an area of 368 square feet. How long is the hallway?
- Write a multiplication equation and a division equation that represent the situation.
  - Find the length of the hallway. Explain or show your reasoning.

(From Unit 6, Lesson 16.)

5. Here is 378 represented with base-ten blocks.



Use words, diagrams, or equations to show how to use the base-ten blocks to find the value of  $378 \div 6$ .

(From Unit 6, Lesson 17.)

6. Here are two incomplete calculations of  $864 \div 4$ . Complete each calculation to find the value of the quotient.

A

$$\begin{array}{r}
 800 \div 4 = 200 \\
 40 \div 4 = \\
 20 \div 4 = \\
 4 \div 4 = \\
 \hline
 864 \div 4 =
 \end{array}$$

B

$$\begin{array}{r}
 100 \\
 100 \\
 4 \overline{)864} \\
 \underline{- 400} \quad 4 \times 100 \\
 464 \\
 \underline{- 400} \quad 4 \times 100
 \end{array}$$

(From Unit 6, Lesson 18.)

7. a. Use partial quotients to find the value of  $637 \div 4$ .

b. If there are 637 toothpicks and 4 people, what could  $637 \div 4$  mean in this situation? What could each step you took in the algorithm mean?

c. What does the value of the quotient represent in the situation?

(From Unit 6, Lesson 19.)

8. There are 875 peaches at the orchard. Each box contains 9 peaches. How many boxes are needed for the peaches? Explain your reasoning.

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(From Unit 6, Lesson 20.)

### 9. Exploration

Consider the expression  $286 \div 5$ .

- a. Write a division story with a question that can be answered by finding the value of  $286 \div 5$ . Then, answer the question.

- b. Write a different story with a question that can be answered by finding the value of  $286 \div 5$  but with a different answer than your first story. Answer the question.

**10. Exploration**

Mai has a special way to see that 531 is a multiple of 9. She says, "Each hundred is 11 nines and 1 more and each ten is one nine and 1 more, so 531 is 58 nines and 9 more."

a. Make sense of and explain Mai's reasoning. Is 531 a multiple of 9?

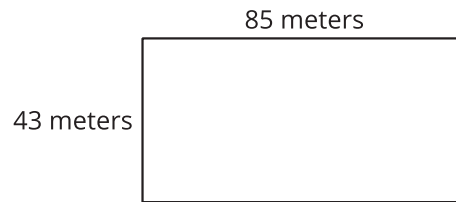
b. Use Mai's reasoning to decide if 648 is a multiple of 9.



2. The diagram shows the side lengths of a sports field.

a. What is the perimeter of the field?

Explain or show your reasoning.



b. What is the area of the field? Explain or show your reasoning.

(From Unit 6, Lesson 22.)

3. The table shows the number of students who have different numbers of pets at a school.

How many pets do all the students have together? Explain or show your reasoning.

number of pets	number of students
1	218
2	167
3	287
4	138

(From Unit 6, Lesson 23.)

4. A builder is covering the floor of a rectangular room that is 23 feet by 25 feet with tiles that are 1 foot by 1 foot. The tiles are sold in boxes of 12.

Diego says 59 boxes are needed to cover the floor, and that there will be a few leftover tiles.

a. Is Diego's answer reasonable? Explain or show your reasoning.

b. How many boxes of tiles would you get to cover this floor? Explain or show your reasoning.

(From Unit 6, Lesson 24.)

### 5. **Exploration**

Find a rectangular room at home or in the school.

a. Which unit would you choose for measuring the length and width: inches, feet, or yards? Explain your reasoning.



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